# MEASUREMENT OF REACTIVE POWER IN NON-SINUSOIDAL SUPPLY SYSTEMS

A DISSERTATION

submitted in partial fulfilment of the
requirement for the award of the degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
( Measurement & Instrumentation )



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# C E R T I F I C A T E

Certified that the M.E. dissertation entitled 'MEASUREMENT OF REACTIVE POWER IN NONSINUSOIDAL SUPPLY SYSTEMS' which is being submitted by Shri BIJENDRA KUMAR in partial fulfilment for the award of the degree of M.E. in ELECTRICAL ENGINEERING (Measurement and Instrumentation) of the UNIVERSITY OF ROORKEE, ROORKEE is arecord of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further certified that he has worked for a period of $4\frac{1}{2}$ months from January 6, 1984 to May 21, 1984 for preparing this dissertation.

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## A C K N O W L E D G E M E N T

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Bkumar

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# ABSTRACT

Various chapters which are enough to explain and analyse the heading of the dissertation.

CHAPTER (1) gives the idea about, why reactive power measurement is necessary for nonsinusoidal supply system.

CHAPTER (2) includes the literature review upto date, which is essentially needed in the whole work of dissertation.

CHAPTER (3) presents the various sources of nonsinusoidal supply and harmonic analysis and their suppression in nonsinusoidal supply.

CHAPTER (4) includes two different techniques define reactive power and measurement of reactive power for non-sinusoidal supply system. Also how power factor can be improved in each technique is described in this chapter. This chapter gives the complete reactive power analysis for nonsinusoidal supply system.

CHAPTER (5) has the illustration of both techniques by taking two numerical examples. Which technique is preferred mostly in relation with compensation of reactive power components and power factor improvements also discussed.

CHAPTER (6) gives the conclusion that one technique (i.e. Current Subdivision Technique) is useful than other technique in connection with complexity of the equations, instrumentation, accuracy of the different instrument in meter, and readiness of compensation of reactive power components by the Operator.

# C O N T E N T S

# 1. INTRODUCTION :

## IMPORTANCE AND UTILITY OF REACTIVE POWER MEASUREMENT FOR NON-SINUSOIDAL SUPPLY SYSTEM :

freq/ ac conversion?

The recent past development and increasing use of power semiconductor switching devices for frequency or magnitude conversion and control have been profound impact on modern electrical power technology. The main effect of this development is the frequent use of nonsinusoidal voltage sources supplying nonlinear loads at higher power level. Not only it is necessary to know accurately the active power being delivered to the load under nonsinusoidal conditions, means must also be provided to determine and control the reactive current or reactive power so that losses in the network can be minimised. One of the problems of switch controlled systems is the low power factor generally associated with them. Precision measurement of reactive power is important because its compensation increases the power factor. A low value of power factor indicates a number of drawbacks, most important of which are increased dimensions of the generating, transmitting, and switching equipments, and increased line losses.

Two different approaches viz. (1) Fourier Analysis Technique, (2) Current Subdivision Technique will be adopted later to define reactive power for nonsinusoidal supply system. In each technique how reactive power is measured by different reactive meters and how power factor is improved by different methods for nonsinusoidal supply system will also be described in detail.

## 2. LITERATURE REVIEW :

(1) SUGGESTED DEFINITION OF REACTIVE POWER AND POWER FACTOR IMPROVEMENT UNDER NONSINUSOIDAL CONDITIONS :

D. Sharon [1]; A. E. Emanual [2]; W. Shepherd & P. Zakikihani [3]; N. L. Kusters and W.J.M. Moore [4] worked under this heading.

(2) ELECTRIC WAVE DISTORTIONS : THEIR HIDDEN COST AND CONTAINMENT :

John R. Linders [5] presented this paper under this heading with detailed descriptions.

(3) HARMONIC ANALYSIS AND SUPRESSION FOR ELECTRICAL SYSTEMS SUPPLYING POWER CONVERTORS AND OTHER NONLINEAR LOADS :

David D. Shipp [6] worked in detail under this title.

(4) MEASUREMENT OF REACTIVE POWER FOR NONSINUSOIDAL SYSTEMS :

Ramon Aparicio Lopez, Juen Carlos Montano Asquerino and Guillermo Rodriguez-Izquierdo [7]; Leszek S. Czarneck [8], [9] worked under this heading and also suggested ahout reactive power meter for nonsinusoidal systems.

(5) A COURSE OF MODERN ANALYSIS :

E. Whitteker and G. Watson [10] described about fourier series analysis of nonsinusoidal wave in this book.

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## 3. PRACTICAL SITUATION OF NONSINUSOIDAL SUPPLY :

(1) NONLINEAR LOADS :

- (a) Rectifiers
- (b) Controlled rectifiers
- (c) Static frequency convertor
- (d) Saturable reactors
- (e) Arc furnaces
- (f) Electric arc welders

(2) PHASE UNBALANCE SOURCES :

- (a) Single-phase linear loads
- (b) Single-phase nonlinear loads
- (c) Open delta transformer
- (d) Untransposed flat configuration of conductor
- (e) Blown capacitor fuse

(3) EXCITING CURRENTS :

- (a) Normal condition
- (b) Overexcited condition
- (c) Ferroresonance

(4) RESONANCES :

- (a) Shunt
- (b) Series

(5) OTHER MISCELLANEOUS SOURCES

- (a) Switching surges
- (b) Lightning
- (c) Faults
- (d) Reversing, jogging and breaking loads
- (e) d.c. ~~and~~ in a.c. circuits

(1) NONLINEAR LOADS : The effect on the supply system of loads which include rectification will depend to a great extent on the d.c. circuit parameters. The effect of a smoothing conductance inductance in the d.c. side of thyristors causes sqare waves of current in the a.c. supply. This effect will also occure with uncontrolled rectifiers and in frequency convertors if they also use a smoothing ~~conduc-~~ inductor tance.

If the d.c. load includes only a smoothing capacitance rather than inductance, the a.c. input current will flow only during peak voltage part of each half cycle. This chopped current wave occures because the current can only flow during that part of the cycle when the a.c. supply voltage is greater than the d.c. voltage across capacitor. The r.m.s. value of the a.c. current will be a function of the average d.c. value required by the load. The duration of current flow for each half cycle will depend on the size of the capacitor. A large capacitor will smooth the d.c. very well. The a.c. current pulse will be short and hence of large magnitude.

When niether inductance nor capacitance is used to smooth the d.c., the a.c. will be less distorted. A full wave single phase rectifier, if loaded only with a resistance, will have negligible distortion in its a.c. supply current. When such loads are three-phase or controlled, as with a phase back thyristor, distortion in the current wave will result. The actual waveshape of a distorted current will depend on the relative phase angles of the harmonics to the fundamental as well as to the magnitude of each harmonic.

An additional type of distortion associated with the thyristors namely the'commutation notches'. These occure each half cycle on each phase on a typical six pulse system. They are caused by the fact that in order to control the output voltage it is necessary to force the conducting thyristors into a nonconducting condition at an unnatural point on the a.c. wave. In order to do this so as to commutate the current to the next thyristor it is necessary to momentarily place a short circuit on the a.c. supply for a few microseconds. In order to limit this short circuit current to an acceptable value, all thyristors rely on inductances in the a.c. load and system. The larger this inductance the less short circuit but the longer it must exist to cause proper commutation. These commutating notchds, which are the result of a necessary short circuitng of each phase of the system once or twice each cycle in a thyristor drive, can be major source of wave distortion. The frequencies involved in these commutating notches are in mid-audio frequency range and highor. They can cause radio interferenc (electromagnetic interference) as well as the wave distortion.

When the load to be controlled does not require d.c., saturable reactors may be used for the controlled device. These are iron-core inductances which have supplemental control windings. When a small amount of d.c. is applied to these extra windings the inductance is reduced significantly, thus permitting more a.c. current to flow. The dominant harmonic is the third, but when used in a balanced three-phase configuration, this largely cancel out.

The wave distortion caused by electric arc furnaces or welders is seldom analyzed on a frequency basis becquse of the random nature of the arc currents. These are single-phase phenomina on an instantaneous basis even though they may be three-phase devices and will average out over a brief period of time, as three-phase loads. This type of wave distortion is referred to as fliker because of its effect on illumination devices.

(2) PHASE UNBALANCE : Phase unbalance is treated as a form of wave distortion. The unbalanced loads or unbalanced phase impedences in the supply are the major causes. A single-phase-resistive load will cause exactly the same amount of phase unbalance, as a similar but highly reactive load, but with 90° difference in phase angle. This is true even though the apparent voltage of the loaded phase will not be the same in each case. This is because the unbalance in all cases is the I.Z drop caused by the single phase load, whereas the observed voltage reduction on the loaded phase is only the in-phase component of this product.

When a single-phase load is also nonlinear, such as a single-phase thyristor drive, there may be harmonic problems as well as phase unbalance. A single-phase thyrister load may have third order harmonics as well as others i.e. 5th, 7th, 11th, 15th etc.

Another unsuspected source of phase unbalance is a blown main fuse on a capacitor bank. Normally these do not blow for individual can failures and so are not usually monitored. However, should a main capacitor fuse open, an unbalance in-phase voltage result. If the capacitor ba nk was providing, say a four percent voltage correction in conjuction with its function of improving power factor. Then with one main fuse blown a two percent voltage unbalance will occure. This could result in an increase in an a.c. motor losses of about 8 %.

how?

(3) EXCITING CURRENTS AND FERRORESONANCE : Exciting currents do not normally cause any wave distortion of consequwnce. An exception would be when, due to a system upset, high voltage occures. Exciting currents increase rapidly with an increase in voltage. In fact, transformer standards specify that only 110 % name plate voltage, the transformer should not overheat without load. In other words at 110 % voltage the exciting losses may equal the normal full load losses of the transformer. At 130 % of rated voltage, exciting currents may approach actual full-load current. Such currents will contain over 50 % third and higher harmonics and the voltage will be seriously distorted.

Ferroresonance is another abnormality which occures only infrequently, but when it does, severe wave distortion and over voltage can occure. Ferroresonance can only occure when a conductor opens and one phase of a transformer becomes energized through some system capacitance. One or two hundred feet of cable can provide sufficient capacitance to bring on ferroresonance. Thus, a typical cause could be a fuse blowing on the cable entrance to an industrial service with a transformer of upto say, 5,000 KVA. The larger the transformer and the lower the voltage, the more capacitance required to cause ferroresonance. However, neither size nor voltage should be assumed to prevent the ferroresonance, because capacitive current as little as one percent of the transformer magnetizing current have been known to caus ferroresonance with an unloaded transformer. There is no exact amount of capacitance needed as with a true resonance. The nonlinear inductance of the transformer's magnetizing impedance can lead to ferroresonance over perhaps a 100/1 variation in capacitance. Ferroresonance once in existence, it will probably continue until some piece of equipment fails. Ferroresonance does not generally occure if the transformer is loaded to more than 30 % of its self-cooled rating. The larger the series capacitance, the more load that is needed on the transformer to prevent ferroresonance. Ferroresonance is not a single frequency phenomenon, and the likely frequencies are difficult to predict. Knowing the likely frequencies would provide no additional insight into their prevention. Prevention consists of system arrangements which preclude over-exciting a transformer through a series capacitance and maintaining load on the transformer, and when it does occure, immediately de-energizing the other phases of transformer.

(4) RESONANCES : True resonance are of a different nature. They are linear phenomena. There can be series or shunt resonance. When a shunt resonance exists, very little current is needed to build up a very large voltage. A series resonance, on the other hand, requires very little voltage for cause a high current to flow at the resonant frequency. If there were no nonlinear loads, resonance would never cause any problem.

The actual situation could be worsed since the energy in the commutating notches must ~~also~~ be considered. This energy will be absorbed by the system at the resonant frequency and will make the system 'ring' at this frequency following each notch, each half cycle, and on each phase.

Series resonance is generally a deliberately designed condition. It is used to prevent shunt resonance fro causing problems and to provide a controlled path for specific harmonics. When an inductance is placed in series with capacitor and the pair is ~~then~~ connected between phases, there will be series resonance at that frequency. When the reactance of the two devices are equal (but of opposite sign). At this frequency the impedence is very low, consisting of only equivalent resistance of capacitor and inductor. When resonated at a specific frequency the nonlinear load harmonic at that frequency will largely flow in this circuit and not out into the system. The resonant circuits are frequently called traps.

In addition to trapping a specific frequency, series resonant circuits act like inductances at all higher frequencies. Thus there is no possibility of shunt resonance occurring at higher frequency. However, at lower frequencies the trap acts like a capacitor of a size larger than actual and somewhat proportional to frequency. Thus, the resonant frequency is lowered by the trap inducance which is there to make up the series resonance.

Regardless of whether they are trapped or not, capacitors will always cause an increase in harmonic voltages for all harmonics below the shunt resonant frequency.

(5) OTHERS MISCELLANEOUS SOURCES : Most of the listed miscellaneous sources of wave distortion are not of a continuous nature. Their containment is well understood and they will not be discussed here except last item i.e. d.c. in a.c. circuits. When a minimum cost rectifier or thyristor design results in ~~possible~~ d.c. in a.c. circuit, problems can result. Since d.c. can not be transferred between windings of a transformer, it is logical that any d.c. is like an exciting current and not like a load current. When d.c. flows in a transformer, core saturation may occure and cause the exciting currents to increase greatly. Odd harmonics will appear and considerable voltage distortion will be apparent. These excess exciting currents may cause fuses to blow on small or instrument-type transformer. Very little d.c. in a.c. circuit can cause trouble because the effect is not related to the rated current of the circuit

but rather to only the one or two percent exciting current of the connected transformers.

## HARMONIC ANALYSIS AND SUPPRESSION IN NONSINUSOIDAL SUPPLY SYSTEM :

Nonsinusoidal supply system consists of several harmonics. Therefore, study of harmonics in it is necessary in correct measurement of various quantities. The prevailing sources of harmonics are ~~from~~ rectifiers, d.c. motor drives (convertors/invertors), uninterruptable power supplies (UPS), cycloconverters, ~~are~~ furnaces and/or any device with non-linear characteristics which derive their power from a linear/sinusoidal electrical system. Systems composed of these types of loads have the potential to develop harmonic related problems and are therefore prime candidates of harmonic analysis study. More recent problems involve the performance of computers, numerical controlled machines, telephone interference and other sophisticated electronic equipments, which are very sensitive to power line pollusion. These types of devices may respond incorrectly to normal inputs, give false signals or possibly not respond at all.

Harmonics are voltages and/or currents present on an electrical system at some multiple of the fundamental (normally 50 Hz) frequency. Typical values are the fifth (250 Hz), seventh (350 Hz), eleventh (550 Hz) and so on. To better understand harmonic related problems, it is necessary to understand how and where harmonics are generated which is explained earlier in sources of wave distortion.

Converters generate harmonic voltages and currents on both a.c. and d.c. sides. A converter of pulse number P generates harmonics principally of orders :

$h = Pq$ On the d.c. side and

$h = Pq \pm 1$ On the a.c. side

q being any integer. Most HV dc converters have pulse number 6 or 12. The amplitudes of harmonics decrease with increasing order, the a.c. harmonic current of order h is less than $I_1/h$, where $I_1$ is the amplitude of fundamental current.

Taking example of Fig. (1), a six-pulse, six-phase converter is shown in Fig. 1(a) and Fig. 1(b) indicates wavefo for the d.c. current and the corresponding a.c. line current. The square a.c. current waveform represents a distorted sinusoidal waveform rich in harmonic content which can be separated into component using fourier analysis technique. Fourier series for this waveform is

~~what is this Id?~~

what does this term indicate?

$$I_{ac} = \frac{2\sqrt{3}}{\pi} I_d \left(\cos\theta - \frac{1}{5}\cos 5\theta + \frac{1}{7}\cos 7\theta - \frac{1}{11}\cos 11\theta + \frac{1}{13}\cos 13\theta \ldots\right) \quad (1$$

The higher order terms are the harmonic components. A similar fourier analysis of the distorted sinusoidal waveforms of other harmonic generating equipment as mentioned previously will yield similar harmonic components.

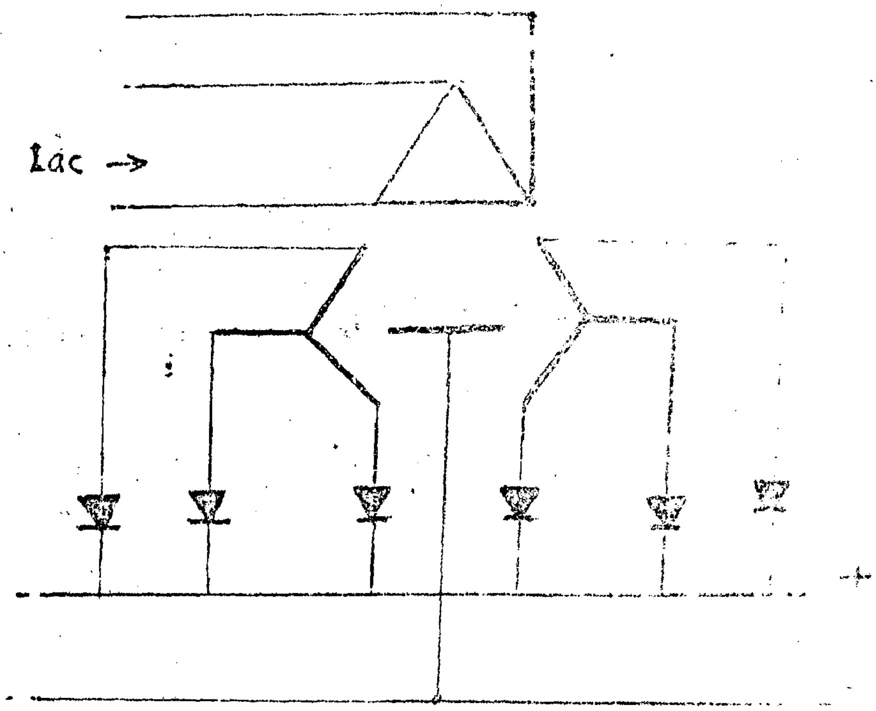



Fig 1 (a)

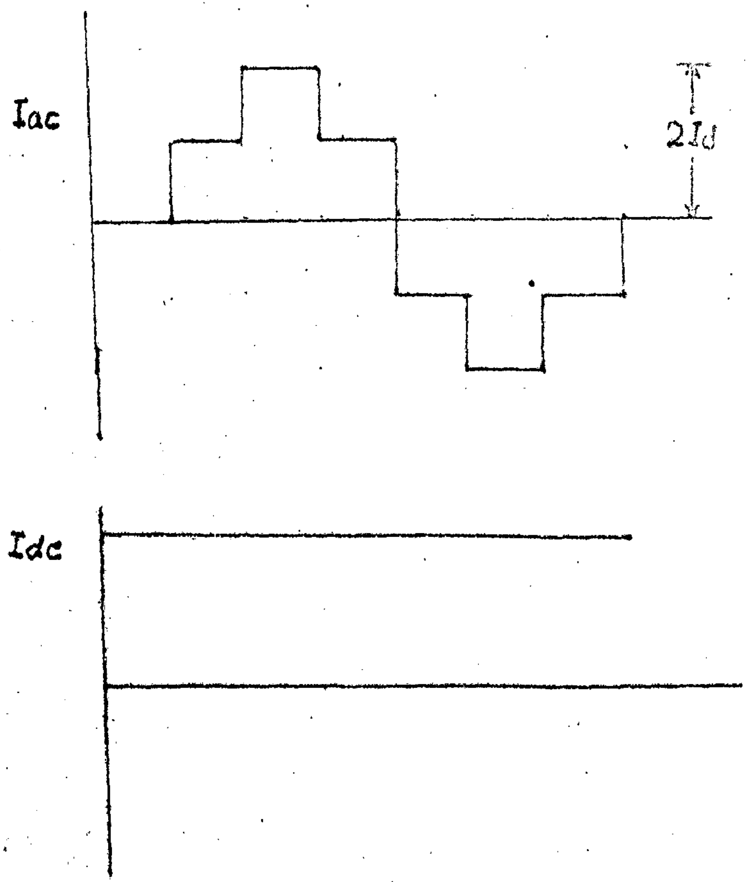



Fig 1 (b)

Rectifiers and other similar harmonic generating equipment are represented as current sources at each harmonic frequency (arc furnaces are represented as voltage sources). With reference to the previous fourrier expansion, the maximum theoretical harmonic current magnitude from each converter is equal to the fundamental frequency full load current magnitude divided by the order of harmonic.

These harmonic current magnitude are also functions of the number of converter pulses. The magnitude of the system harmonic voltage are a result of the harmonic current flowing back into the harmonic impedances of the a.c. system. The order of harmonic current is np $\pm$ 1, where n is any integer and p is the ~~number of~~ number of converter pulses. Thus, for a six pulse converter, the order of harmonics are fift , seventh, eleventh, thirteenth, seventeenth and ninteenth, etc. For a 12-pulse converter, the order of harmonics are 11th, 13th, 23rd, 25th, 35th, 37th etc. This procedure, using a higher number of phases for lower order harmonic cancellation, is referred to as phase multiplication. Although phase multiplication theoretically will cancel normal harmonics not of the order 'np $\pm$ 1', in practice both current magnitude and phase angle will deviate enough to allow only incomplete cancellation. Mostly 10 - 25 % of the maximum harmonic magnitude will remain. To be as realistic as possible, this factor sometimes referred to a ~~a~~ harmonic cancellation factor (HCF), should be included.

Additional reduction of the harmonic current magnitude is due to the series ~~conductive~~ inductive reactance between the harmonic source and the utility supply. The larger this inductive reactance is (commutating reactance) the more it impedes that particular harmonic generation. For example, the maximum 5th harmonic current magnitude available is $1/h = 1/5 = 0.2$ per cent or 20 per cent (commutating reactance = 0). However, due to significant commutating

cross reference

reactance, the actual magnitude may only be 17 %. This reduction may be referred to as commutating reactance factor (CRF).

The final factor reducing a pa rticular harmonic current is the per unit loading (LDF). If a converter is only 50 % loaded (fundamental component of current) then the harmonic current will only be 50 % of its maximum value on that system. For drives utilizing phase, retard (speed) control, this loading factor is also proportional to the a.c. fundamental current component, but is not necessarily proportional to the d.c. KW or h.p. output.

The total harmonic current value injected into the system by a particular device at each harmonic frequency is then,

$$I_h = (FLA/h)(LDF)(HCF)(CRF) \quad \ldots \quad \ldots \quad (2)$$

where, FLA is the fundamental full load amperes of the device and h is the order of harmonic.

ANALYSIS TECHNIQUES :

Basic system connections (one line diagram) and impedances established. A harmonic analysis study should analyse the system under study state conditions for normal power flow and harmonic current flow (sometimes referred to as harmonic load flow) for all harmonics being modelled for as many system switching conditions as required. A typical range of harmonic frequencies modeled may be from 5(250 Hz) to 37 (1850 Hz). The harmonic resonant point at a particular location will probably differ under each separate switching condition, so all normal modes of operation should be included.

Power factor correction capacitors are designed to continuously carry 135 % of their nameplate rated (fundamental) KVA or KVAC (capacitive), 110 % of their rated voltage and 180 % of their rated current, existing standard (135 % pending standard). This 'overload' capability provides margin for system over voltages and/or harmonic voltages which may occure. The total loading of a bank may be calculated as the sum of KVA loading of the fundamental and each harmonic. This may be expressed as

$$KVAC = \sum_h (V_h I_h) = \sum_h (V_h^2 / X_h) \quad \ldots \quad \ldots \quad (3)$$

Where,

h = fundamental or order of harmonic

V = fundamental or harmonic voltage

I = fundamental or harmonic current

X → ~~D~~ = fundamental or harmonic reactance

The capacitors must also have sufficient dielectric to withstand the anticipated peak voltages resulting from the fundamental and the harmonics. This peak voltage is pessimistically calculated as the arithmatic sum of all the component Peak voltages(not as r.m.s. value)

$$V_{peak} = \sum_h V_h$$

The peak is used because of the 'random' phase relationships that exist between the various harmonic components. The third loading factor is the total r.m.s. current which connections, bushings, and other components of the capacitor bank must handle. The current is calculated as

$$I_{rms} = \left( \sum_h I_h^2 \right)^{1/2} \qquad \ldots \qquad \ldots \qquad (4)$$

Additional problems can arise due to hafmonics in motors, lighting, ballast transformers and other similar equipment. These problems are essentially excessive heating due to circulating harmonic currents. To evaluate this effect, rms voltages rather than peak values are required. Therefore, rms voltage should also be calculated and printed throughout the system. Total rms voltage may be calculated using the following equations :

$$V_{rms} = \left( \sum_h V_h^2 \right)^{1/2} \qquad \ldots \qquad (5)$$

where h = fundamental or order of harmonic.

The detailed results of an harmonic analysis study should include the four main points mentioned above : total capacitor bank KVAC loading, peak voltage, rms current, and rms voltage. The voltage and current values should be provided at all critical systems, locations susceptible to harmonic problems, where the appropriate values may be compared to the devices rating in question.

SOLUTIONS TO HARMONIC PROBLEMS :

The primary solution to any harmonic related problem is accomplished by shifting the system resonant point to some other frequency not generated by the electrical equipment of the system. The simplest and least expensive method would be to alter or bypass system operating conditions and procedure which will lead to harmonic resonance. If this approach is impractical or undesirable then quite often additional apparatus is required.

The remedial measures involving additional equipment generally used to minimise harmonic effects include shunt LC filters located at the harmonic source and tuned to series resonance at the troublesome harmonics. This approach provides a low impedance path from the harmonic currents to flow with very little flowing back into the rest of the ac. system. However, a separate filter may be required for every major harmonic source. In other cases, where power factor connection capacitors in the a.c. system cause resonance at the generated harmonics, their location or size may be charged to eliminate the resonance, or series reactors may be added to detune them at the troublesome resonant frequency.

There are essentially three different major schemes which will accomplish adequate filtering. Economic considerations as well as the particular filtering requirements for each case will determine which scheme is the most desirable. Quite often utility requirements for harmonic content imposed upon their system will dictate which scheme is to be used. Fig. 2 indicates these schemes.

The least expensive and therefore most desirable, of these three schemes is that of Fig. 2(a). Generally, a carefully selected tuning reactor will be sufficient to relieve harmonic resonance the careful selection of the tuning reactor is stressed. If it is incorrectly collected, the harmonic frequency for which it is tuned will probably be lowered to acceptable levels, but another harmonic frequency may then become dominant with the resonant point, only being shifted to another harmonic frequency present on the system.

Fig. 2(b) is generally used when more stringent (Severe) harmonic content requirements are in effect. In Fig. 2(c) the two lowest and most troublesome harmonics are filtered out individually with one high-pass filter used to filter all higher order of harmonics above these two harmonics.

When harmonics appear to be the cause of system problems, it is desirable to determine the system harmonic resonance point. To determine this resonance point the short circuit capacity at each capacitor bank location is

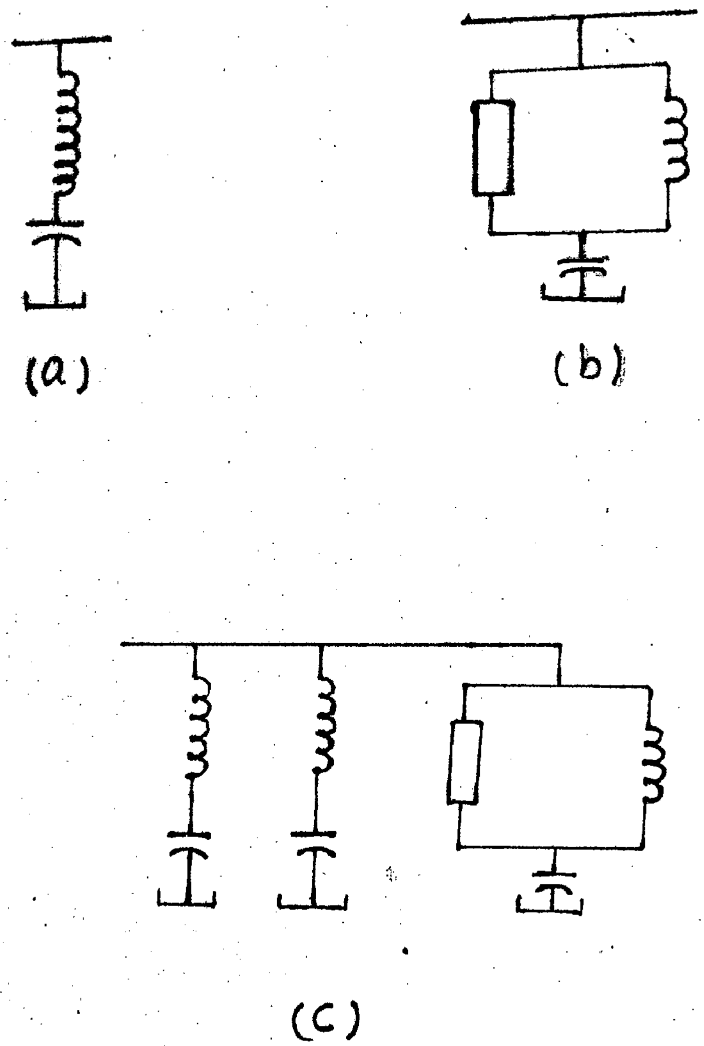



Fig - (2) Three Schemes for filtering the Harmonics required. The equation $h_r = \sqrt{MVA/MVAC}$ is a close approximation of this resonance point where $h_r$ is the resonance point per unit of the fundamental frequency, MVA is the short circuit capacity and MVAC is the Mvar rating of the unfiltered capacitor bank at that location. This location is very useful for an initial evaluation. If the resonance point is close to one of the harmonic frequencies present on the system, then possible harmonic related problems could occure.

## (4) REACTIVE POWER ANALYSIS :

Authors [1], [2], [3] presented the approach based on Fourier Analysis Technique and author [4] presented the approach based on 'Current Subdivision Technique', to define reactive power and improve the power factor for nonsinusoidal supply system. Authors [7], [8], [9] and author [4] described the measurement of reactive power under nonsinusoidal conditions based on 'Fourier Analysis Technique' and 'Current Subdivision Techniq ue' respectively. Therefore, reactive power analysis will be performed below by these two techniques.

### (4.1) FOURIER ANALYSIS TECHNIQUE :

#### (4.1.1) DEFINITION OF REACTIVE POWER AND POWER FACTOR IMPROVEMENT FOR NONSINUSOIDAL SUPPLY SYSTEM :

Let a nonsinusoidal voltage source is connected to nonlinear load. The instantaneous values v and i of the voltage and current, respectively, may be generally expressed as :

how to get these relations?

which are n groups ...



$$v = \sqrt{2}\sum_{1}^{n} V_n \operatorname{Sin}(n\omega t + \alpha_n) + \sqrt{2}\sum_{1}^{m} V_m \operatorname{Sin}(m\omega t + \alpha_m) \quad \ldots \quad (6)$$

The instantaneous current i contains the group of harmonics present in the voltage v and a further group p not present in the voltage, i.e.

$$i = \sqrt{2}\sum_{1}^{n} I_n \operatorname{Sin}(n\omega t + \alpha_n + \phi_n) + \sqrt{2}\sum_{1}^{p} I_p \operatorname{Sin}(p\omega t + \alpha_p + \phi_p) \quad \ldots \quad (7)$$

The average power P has the period T of the instantaneous vi product and contain only the n harmonic terms.

$$P = \frac{1}{T}\int_{0}^{T} vi\,dt = \sum_{1}^{n} V_n I_n \operatorname{Cos}\phi_n \quad \ldots \quad \ldots \quad (8)$$

?

If the voltage has a period $T_e$ and current has period $T_i$, the apparent power S at the supply terminals is given by

how are they defined

$$S^2 = \frac{1}{T_e}\int_{0}^{T_e} v^2\,dt \cdot \frac{1}{T_i}\int_{0}^{T_i} i^2\,dt \quad \ldots \quad \ldots \quad (9)$$

So
$$\frac{1}{T_e}\int_{0}^{T_e} v^2\,dt = \frac{1}{T_e}\int_{0}^{T_e}\Big[2\sum_{1}^{n} V_n^2 \operatorname{Sin}^2(n\omega t + \alpha_n)$$

$$+ 2\sum_{1}^{m} V_m^2 \operatorname{Sin}^2(m\omega t + \alpha_m) + 4\sum_{1}^{n} V_n \operatorname{Sin}(n\omega t + \alpha_n)\sum_{1}^{m} V_m \cdot$$

$$\operatorname{Sin}(m\omega t + \alpha_m)\Big]dt$$

$$= \frac{1}{T_o}\int_{0}^{T_e}\Big[\sum_{1}^{n} V_n^2\big(1 - \operatorname{Cos}^2 2(n\omega t + \alpha_n)\big) + \sum_{1}^{m} V_m^2\big(1 - \operatorname{Cos} 2(m\omega t + \alpha_m)\big)$$

$$+ 2\sum_{1}^{n}\sum_{1}^{m} V_n V_m\big(\operatorname{Cos}(n\omega t + m\omega t + \alpha_n + \alpha_m) - \operatorname{Cos}(n\omega t - m\omega t + \alpha_n - \alpha_m)\big)\Big]dt$$

$$\text{Therefore, } \frac{1}{T_e}\int_0^{T_e} v^2\,dt = \left(\sum_1^n V_n^2 + \sum_1^m V_m^2\right)$$

$$\text{Similarly } \frac{1}{T_i}\int_0^{T_i} i^2\,dt = \left(\sum_1^n I_n^2 + \sum_1^p I_p^2\right)$$

$$\text{So, } S^2 = \left(\sum_1^n V_n^2 + \sum_1^m V_m^2\right)\left(\sum_1^n I_n^2 + \sum_1^p I_p^2\right) \quad \ldots \quad (10)$$

unsafe statement

While P and S both have the dimensions of volts multiplied by amperes, they are totally different in character. The average power P has the physical nature of power, where S has no physical nature at all, but is a figure of merit representing the energy transfer capability of the load.

what do they physically represent ?

The total apparent power S may be resolved into three analytical component defined as the active apparent power $S_R$, the true ~~reactive~~ apparent power $S_x$ and the distortion power $S_D$, where

$$S_R^2 = \sum_1^n V_n^2 \sum_1^n I_n^2 \cos^2 \phi_n \neq P^2 \quad \ldots \quad (11)$$

$$S_X^2 = \sum_1^n V_n^2 \sum_1^n I_n^2 \sin^2 \phi_n \quad \ldots \quad (12)$$

$$S_D^2 = \sum_1^n V_n^2 \sum_1^p I_p^2 + \sum_1^m V_m^2\left(\sum_1^n I_n^2 + \sum_1^p I_p^2\right) \quad \ldots \quad (13)$$

The components $S_R$, $S_X$ and $S_D$ sum to S as follows :

$$S^2 = S_R^2 + S_X^2 + S_D^2 \quad \ldots \quad \ldots \quad (14)$$

If for example, the voltage contains harmonics of order 1, 2 and 3, and the resulting current contains harmonics of order 1, 2 and 4.

$$S_R^2 = (V_1^2 + V_2^2)(I_1^2 \text{Cos}^2\phi_1 + I_2^2 \text{Cos}^2\phi_2)$$

$$S_X^2 = (V_1^2 + V_2^2)(I_1^2 \text{Sin}^2\phi_1 + I_2^2 \text{Sin}^2\phi_2)$$

$$S_D^2 = (V_1^2 + V_2^2)I_4^2 + V_3^2(I_1^2 + I_2^2 + I_4^2)$$

can it be proved analytically

$$S^2 = (V_1^2 + V_2^2 + V_3^2)(I_1^2 + I_2^2 + I_4^2)$$

For linear loads the distortion power $S_D = 0$, but $S_R$, $S_X$ are unchanged. $S_R$ and $S_X$ may be thought of as being related to hypothetical current $i_R$ and $i_X$ flowing in the purely resistive and purely reactive impedance branches of the load equivalent circuit (from eqn. 7).

7

$$i_R = \sqrt{2}\sum_1^n I_n \text{Cos}\phi_n \ \text{Sin}(n\omega t + \alpha_n) \quad \ldots \quad (15)$$

$$i_X = \sqrt{2}\sum_1^n I_n \ \text{Sin}\ \phi_n \ \text{Cos}(n\omega t + \alpha_n) \quad \ldots \quad (16)$$

An instantaneous inductive current will have negative amplitude. Current $i_R$ and $i_X$ have the rms form. The formula $\sum_1^n V_n I_n \text{Sin}\ \phi_n$ has no scientific basis, but is an arbitrary mathematical statement so that

$$Q = \sum_1^n V_n \ I_n \ \text{Sin}\ \phi_n \neq S_X \quad \ldots \quad (17)$$

how?

It does not represent any real physical quantity, and circuit compensation of this formula would not result in maximum power factor operation.

Also, frequently quoted in the literature is the term distortion power D given by

$$D^2 = S^2 - P^2 - Q^2 \neq S_D^2 \quad \ldots \quad (18)$$

So, D like Q, is an arbitrary mathematical value without physical meaning. Certainly one may consider analytical components of S, but there is no scientific justification for chosing Q and D as defined in equation (17) and (18).

In any circuit, linear or nonlinear, regardlessof the waveform, the power factor is the factor by which the apparent power must be multiplied to obtain average power P. (Referred to [1]).

$$\text{Power factor} = \frac{\text{average power}}{\text{Apparent power}} = \frac{\frac{1}{T}\int_0^T vi\,dt}{V_{rms}\ I_{rms}}$$

$$= \frac{\sum_1^n V_n I_n \cos\phi_n}{\left[\sum_1^n V_n^2 + \sum_1^m V_m^2)(\sum_1^n I_n^2 + \sum_1^p I_p^2)\right]^{1/2}} \quad \ldots (19)$$

Power factor represents a figure of merit of the character of power consumption. Its low value indicates mainly poor utilisation of the source power capacity needed by the load.

If circuit compensation is sought in nonsinusoidal circuits, the maximum power factor is realised by the compensation of $S_X$, not $Q = \sum_1^n V_n I_n \sin\phi_n$. The serious drawback of the reactive power is that its compensation does not lead in general to maximum power factor operation. For example, Q is for nonlinear load, characterised by Eqn. (6) and (7) should reduce to zero by connecting a capacitance C in parallel with it. Then what is worse, the power factor may be deteriorated by the compensation.

To illustrate this, let the numerical data for the source voltage and the load current in connection with equation (6) and (7) will be taken later. Capacitance C is connected parallel with nonlinear load, is

$$C = -\frac{\frac{1}{\omega}\sum_{1}^{n} V_n I_n \sin \phi_n}{\sum_{1}^{n} V_n^2 n + \sum_{1}^{m} V_m^2 m} \qquad \ldots \quad (20)$$

Eqn. (20) yields a positive value for a load of inductive character. The individual phase angle of the resultant current harmonics are given by

$$\phi_n' = \tan^{-1} \frac{V_n n\omega C + I_n \sin \phi_n}{I_n \cos \phi_n},$$

$$\phi_m' = \tan^{-1} \frac{V_m m\omega C + 0}{0} = \tan^{-1} \infty = \pi/2 \quad \ldots \quad (21)$$

NUMERICAL EXAMPLE :

Given,

$V_1 = V_7 = V_9 = 100$ V   all other harmonics = 0

$I_1 = I_5 = 1$A   ---do---

$\phi_1 = -\pi/3$ rad $= -60^\circ$

$\omega = 314$ rad/sec.

Solution: .... From Eqn. (20), the capacitance C for total compensation of Q is

$$C = -\frac{1}{314} \times \frac{100 \times 1 \times \sin(-60^\circ)}{100^2 \times 1 + 100^2 \times 7 + 100^2 \times 9}$$

$$= 1.62 \times 10^{-6} \text{ farads}$$

On substituting in Eqn. (21), the phase angles of the resultant harmonics are

$$\phi_1' = -30.5^\circ, \qquad \phi_7' = \phi_9' = 90^\circ$$

Resultant effective values of the current harmonics are calculated from the expressions :

$$I_n' = (V_n^2\, n^2\omega^2 C^2 + 2V_n I_n\, n\omega C \sin\phi_n + I_n^2)^{1/2} \quad \ldots (22)$$

$$I_m' = (V_m\, m\omega C) \quad \ldots \quad \ldots (23)$$

Substituting in Eqn. (22) and (23), we obtain

$$I_1' = 0.956\ \text{A} \quad I_7' = 0.358\ \text{A}, \ I_9' = 0.459\ \text{A}$$

The calculated apparent powers before and after the compensation of Q are respectively

$$S = (100^2 + 100^2 + 100^2)^{1/2}(1^2 + 1^2)^{1/2} = 245\ \text{VA}$$

and

$$S = (100^2 + 100^2 + 100^2)^{1/2}(0.956^2 + 0.358^2 + 0.459^2 + 1^2)^{1/2}$$
$$= 260\ \text{VA}$$

As the average power is not affected by the addition of C, it is readily seen that the power factor has deteriorated, in this example, though the compensation of Q.

To eliminate this drawback, reactive power as given before will be assumed to be 'true reactive power' $S_X$, where

$$S_X^2 = \sum_1^n V_n^2 \sum_1^n I_n^2 \sin^2\phi_n \quad \ldots (24)$$

The word true may easily lead to the interpretation that a real physical entity is involved. The term $S_X$ can not be totally compensated by means of energy storage devices if the voltage is nonsinusoidal, but may be reduced to minimum value as $S_{Xmin}$. The justification claimed for this definition is that minimizing $S_X$ by means of (linear) storage devices leads to the maximum power factor. Power factor of unity can not be realised at all in a nonsinusoidal circuit.

However, an essential inadequacy of equation (24) is that it is in general, discontinuous, and therefore, can not be truly minimized. To appreciate this, a very large linear impedance is assumed to be added in parallel with the load. Although the current may be considered to be unaffected, the group of harmonics that is now present in both the voltage and current is (n + m). The new value of $S_X$ denoted $S'_x$ will be

$$(S'_x)^2 = (\sum_1^n V_n^2 + \sum_1^m V_m^2)(\sum_1^n I_n^2 \sin^2 \phi_n + \text{negligible terms}) \quad ..(25)$$

$S_x^2$ is therefore increased discontinuously by the term $\sum_1^m V_m^2 \sum_1^n I_n^2 \sin^2 \phi_n$. Although the minimisation of $S'_x$ by means of energy storage device leads, in fact, to a relative optimum power factor, the minimum value of $S'_x$ may be greater than the original $S_x$ before compensation.

To consider a <u>specific case</u>, the general expression for $S'_x$ after the addition of a capacitance C in parallel with the load is first written as

$$S_x' = \left(\sum_1^n V_n^2 + \sum_1^m V_m^2\right)^{1/2} X \left[\sum_1^n (V_n\, n\omega C + I_n \sin\phi_n)^2 + \sum_1^m V_m^2\, m^2\, \omega^2\, C^2\right]^{1/2} \quad \ldots \quad \ldots(26)$$

The minimization of eq. (26) with respect to C can be done as below :

$$\frac{dS_x'}{dC} = \frac{\left(\sum_1^n V_n^2 + \sum_1^m V_m^2\right)}{\left[\sum_1^n (V_n\, n\omega C + I_n \sin\phi_n)^2 + \sum_1^m V_m^2\, m^2\omega^2C^2\right]^{1/2}} X$$

$$\left[\sum_1^n 2V_n n\omega\, (V_n\, n\omega C + I_n \sin\phi_n) + \sum_1^m 2V_m^2\, m^2\omega^2 C\right]$$

for minimisation

$$\frac{dS_x'}{dC} = 0\,, \quad \sum_1^n 2\, V_n^2\, n^2\omega^2 C + \sum_1^n 2V_n I_n\, n\omega \,\mathrm{Sing}\, \phi_n + \sum_1^m 2V_m^2\, m^2\omega^2 C = 0$$

or

$$C = -\frac{\sum_1^n V_n n\, I_n \sin\phi_n\,)}{\left(\sum_1^n V_n^2\, n^2 + \sum_1^m V_m^2\, m^2\right)\omega} \quad \ldots\ldots \quad (27)$$

$$\therefore S_x' = \left(\sum_1^n V_n^2 + \sum_1^m V_m^2\right)^{1/2} \left[\left(\sum_1^n I_n^2 \sin^2\phi_n\right) + \sum_1^n V_n^2\, n^2\, \omega^2\, C^2 + \sum_1^m V_m^2\, m^2\, \omega^2\, C^2 + \sum_1^n 2V_n I_n n\omega C \sin\phi_n\right]^{1}$$

$$S'_{x\,min} = \left(\sum_1^n V_n^2 + \sum_1^m V_m^2\right)^{1/2} \left[\sum_1^n I_n^2 \sin^2\phi_n + \right.$$

$$\left[ \frac{\left(\sum_1^n V_n\, n\, I_n \sin\phi_n\right)^2}{\left(\sum_1^n V_n^2 n^2 + \sum_1^m V_m^2 m^2\right)} = \frac{\left(\sum_1^n 2V_n I_n\, n \sin\phi_n\right)^2}{\left(\sum_1^n V_n^2 n^2 + \sum_1^m V_m^2 m^2\right)} \right]^{1/2} \Bigg]$$

$$S'_{x\,min} = \left(\sum_1^n V_n^2 + \sum_1^m V_m^2\right)^{1/2} \left[\sum_1^n I_n^2 \sin^2\phi_n - \right.$$

$$\left. \frac{\left(\sum_1^n V_n I_n\, n \sin\phi_n\right)^2}{\sum_1^n V_n^2 n^2 + \sum_1^m V_m^2 m^2} \right]^{1/2} \quad \ldots\ldots\ldots \quad (28)$$

Substituting the above numerical data in Eqn. (24) and (28) we obtain

$V_1 = V_7 = V_9 = 100$ V.

$I_1 = I_5 = 1$A

$\phi_1 = -\pi/3$ rad $= (-60^\circ)$

$\omega = 314$ rad/sec.

$$S_x = [\,100^2\,(1^2 \times 0.867^2)]^{1/2} = 87 \text{ VA}$$

$$S'_{xmin} = (100^2 + 100^2 + 100^2)^{1/2} \times$$

$$\left[1^2 \times 0.867^2 - \frac{100^2 \times 1^2 \times 1^2 \times 0.867^2}{100^2 + 100^2 \times 7^2 + 100^2 \times 9^2}\right]^{1/2} = 149 \text{ VA}$$

$S'_{xmin}$ may be thus greater than the original $S_X$ before compensation. Therefore, the assertion that $S_X$ is minimised by power factor optimisation is not, in general, true.

NEW ANALYSIS OF THE APPARENT POWER :

To remedy the above solution, a new analysis of the apparent power [1] in terms of the components P, $S_Q$, $S_C$ is suggested as follows :

$$P = \sum_1^n V_n I_n \cos \phi_n \qquad \ldots (29)$$

$$S_Q = V_{rms} \left( \sum_1^n I_n^2 \sin^2 \phi_n \right)^{1/2} \qquad \ldots (30)$$

$$S_C = \left[ \sum_1^m V_m^2 \sum_1^n I_n^2 \cos^2 \phi_n + V^2_{rms} \sum_1^p I_p^2 + \right.$$

$$\left. \frac{1}{2} \sum_1^\beta \sum_1^\gamma (V_\beta I_\gamma \cos \phi_\gamma - V_\gamma I_\beta \cos \phi_\beta)^2 \right]^{1/2} \qquad \ldots (31)$$

where both β and γ vary from 1 to n.

In this analysis, P is the average power, $S_Q$ is designated the quadrature reactive power, and includes all like frequency and cross-frequency reactive effects associated with the quadrature components of the n group of harmonic currents and $S_C$ is designated the complementary reactive power and includes the cross-frequency reactive effects of :

(a) the inphase components of the harmonic currents in the n group.

(b) the harmonic currents in the p group . .

$P^2$, $S_Q^2$ and $S_C^2$ add up to the square of the apparent power, i.e.

$$[(\sum_1^n V_n^2 + \sum_1^m V_m^2)(\sum_1^n I_n^2 + \sum_1^p I_p^2)] = P^2 + S_Q^2 + S_C^2$$

One of the advantages of the above analysis over previous ones is that the component $S_Q$ is continuous, and its minimisation by parallel connection to the load of an optimum linear capacitance $C_{opt}$ of inductance $L_{opt}$ (depending on the load character) leads to a maximum power factor. The expression for $C_{opt}$ is from equation (27) is

$$C_{opt} = \frac{-\frac{1}{\omega}\sum_1^n V_n n I_n \sin \phi_n}{\sum_1^n V_n^2 n^2 + \sum_1^m V_m^2 m^2} \quad \text{----} \quad (32)$$

Similarly

$$L_{opt} = \frac{1}{\omega} \frac{\sum_1^n \frac{V_n^2}{n^2} + \sum_1^m \frac{V_m^2}{m^2}}{\sum_1^n V_n I_n \frac{1}{n} \sin \phi n} \quad \ldots \quad (33)$$

A further interesting property of the new analysis is that the addition of a lineqr capacitance or inductance in parallel with the load does not affect the component P and $S_C$. This confirms the above assertion that the capacitive or inductive minimisation of $S_Q$ results in the optimum power factor.

If for example a capacitance C is added in parallel to the load characterised by equation (6) and (7), the general expression for the apparent power in terms of C is

$$S = (\sum_1^n V_n^2 + \sum_1^m V_m^2)^{1/2} [\sum_1^n (I_n^2 + V_n^2 n^2\omega^2C^2 + 2V_nI_n n\omega C \sin\phi_n) + \sum_1^m V_n^2 m^2\omega^2C^2 + \sum_1^p I_p^2]^{1/2} \quad \ldots \quad \ldots \quad (34)$$

The differentiation of equation (34) with respect to C and equating to zero leads to the same $C_{opt}$ as that expressed by Eqn. (32). Similarly, when this procedure is used for an inductance connected in parallel with the load, equation (33) for $L_{opt}$ is obtained.

POWER factor properties in relation to capacitance compensation are shown in table No. 1.

Harmonic filters are widely used for power factor improvement in connection with d.c. transmission lines or links. This method can be extended to n voltage harmonics. The compensation circuit will have n branches in parallel, each branch with (n-1) filters in series with an adequate inertive element. Nonlinear capacitors and saturated reactors are the best suited element to compensate the harmonics generated by nonlinear loads.

POWER FACTOR PROPERTIES IN RELATION TO CAPACITANCE COJPENSATION

| | Sinusoidal voltage | | Nonsinusoidal voltage | |
|---|---|---|---|---|
| | Linear RX load | Nonlinear RX load | Linear RX load | Nonlinear RX load |
| Components of apparent power present in load | $S_R$ $S_X$ | $S_R$, $S_X$, $S_D$ | $S_R$, $S_X$ | $S_R$, $S_X$, $S_D$ |
| Power factor if $S_X$ is completely compensated. | $\frac{P}{S_R} = 1$ | $\frac{P}{\sqrt{(S_R^2 + S_D^2)}} < 1$ | $\frac{P}{S_R} < 1$ | $\frac{P}{\sqrt{(S_R^2+S_D^2)}} < 1$ |
| Can $S_X$ be completely compensated by capacitance (for inductance loads) ? | Yes | Yes | No | No |
| Highest power factor realisable by capacitance compensation. | $\frac{P}{S_R} = 1$ | $\frac{P}{\sqrt{(S_R^2+S_D^2)}} < 1$ | $\frac{P}{\sqrt{(S_R^2+S^2_{Xmin})}} < 1$ | $\frac{P}{\sqrt{(S_R^2+S_{Xmin}^2+S_D^2)}}$ |

This new quadrature reactive power definition $S_Q$ possesses clear advantage over previous definition, as in connection with power factor improvement. As the lossless devices are generally used for power factor compensation, the minimisation of apparent power leads directly to the optimum power factor.

An additional advantage of ~~the~~ this direct approach is that it uses the apparent power that is closely related to economic and physical factors. This is in contrast to the various reactive power definitions in which physical meaning are obscure or nonexistent. As is well known the apparent power represents the economic effort in terms of the generation, transmission and switching equipment as well as the losses necessary to supply the power to the given load. At the same time, it is equal to the maximum average active power that could be drawn from this effort.

### (4.1.2) REACTIVE POWER MEASUREMENT FOR NONSINUSOIDAL SUPPLY SYSTEM :

In some reactive power meters - such as electro-dynamometers with a large inductive impedance in the series with a movable coil, if one signal corresponding to the voltage or current is not a pure ~~sinwave~~ sine-wave, the accurate adjustment is a problem, and the error is usually large . Furthermore, if both signals are not sinusoidal the measurement will be totally incorrect. Precision measurement of rea reactive power is also important in order to standardize the reactive loads. Based on the above considerations, a new reactive power meter is described for nonsinusoidal supply systems. Following two different methods are described to measure reactive power for nonsinusoidal supply system.

(4.1.2.1) METHOD(I):

THEORY : This method [7] is based on the mathematical process of orthogonality that applies to functions that can be expressed by Fourier series.

Let us consider a system fed from a nonsinusoidal voltage source

$$v = \sum_{0}^{n} \sqrt{2}\, V_n \operatorname{Sin}(n\omega t - \alpha_n) \qquad \ldots (35)$$

with the current given by

$$i = \sum_{0}^{n} \sqrt{2}\, I_m \operatorname{Sin}(m\omega t - \beta_m) \qquad \ldots (36)$$

A frequency controlled Sin/Cos oscillator controlled by a staircase to get the harmonic frequencies of (35) and (36) makes it possible to get a double series of terms $A_1 \operatorname{Cos}\alpha_1$ and $A_1 \operatorname{Sin}\alpha_1$ where $A_1$ is a quantity proportional to the first harmonic amplitude in (35) and (36) and $\alpha_1$ is respective harmonic phase angle. Algebraic manipulation of these terms leads to the expression for reactive power.

Let Sin $r\omega t$ and Cos $r\omega t$ be the signals generated by the Sin/Cos oscillator, with $r = 1, 2, \ldots K$. If we multiply Sin $r\omega t$, Cos $r\omega t$ and v, i to obtain

$$K_1 = \sum_{0}^{n} \sqrt{2}\, V_n \operatorname{Sin}(n\omega t - \alpha_n) \operatorname{Sin} r\omega t \qquad \ldots (37)$$

$$K_2 = \sum_{0}^{n} \sqrt{2}\, V_n \operatorname{Sin}(n\omega t - \alpha_n) \operatorname{Cos} r\omega t \qquad \ldots (38)$$

$$K_3 = \sum_{o}^{n} \sqrt{2}\, I_m \operatorname{Sin}(m\omega t - \beta_m) \operatorname{Sin} r\omega t \qquad \ldots \quad (39)$$

$$K_4 = \sum_{o}^{n} \sqrt{2}\, I_m \operatorname{Sin}(m\omega t - \beta_m) \operatorname{Cost} r\omega t \qquad \ldots \quad (40)$$

If we integrated these functions between the limits O and $2\pi$ and impose the orthogonal condition, we obtain the following expression

$$\begin{bmatrix} v_n \\ i_{m=n} \end{bmatrix}_{n=r} \neq 0$$

In this case we get

$$L_1 = \frac{1}{2\pi} \int_o^{2\pi} K_1 \, d\omega t = \frac{1}{2\pi} \int_o^{2\pi} \sum_{o}^{n} \sqrt{2}\, V_n \operatorname{Sin}(n\omega t - \alpha_n) \operatorname{Sin} r\omega t \, d\omega t$$

$$= \frac{1}{2\pi} \int_o^{2\pi} \sum_{o}^{n} \frac{\sqrt{2}}{2} V_n [\operatorname{Sin}(n\omega t - \alpha_n) \operatorname{Sin} r\omega t] \, d\omega t$$

$$= \frac{1}{2\pi} \int_o^{2\pi} \sum_{o}^{n} \frac{\sqrt{2}}{2} V_n [\operatorname{Cos}((n-r)\omega t - \alpha_n) - \operatorname{Cos}((n+r)\omega t - \alpha_n)] d\omega t$$

Since $\operatorname{Sin} x \operatorname{Sin} y = \frac{1}{2}[\operatorname{Cos}(x-y) - \operatorname{Cos}(x+y)]$

$$= \frac{1}{2\pi} \int_o^{2\pi} \sum_{o}^{r} \frac{\sqrt{2}}{2} V_r [\operatorname{Cos}(-\alpha_r) - \operatorname{Cos}(2r\omega t - \alpha_r)] d\omega t$$

where $n = r = m$

$$= \frac{1}{2\pi} \int_o^{2\pi} \sum_{o}^{r} \frac{\sqrt{2}}{2} V_r [\operatorname{Cos} \alpha_r - \operatorname{Cos} 2r\omega t \operatorname{Cos} \alpha_r - \operatorname{Sin} 2r\omega t \operatorname{Sin} \alpha_r] d\omega t$$

$$= \frac{1}{2\pi} \sum_{o}^{r} \frac{\sqrt{2}}{2} V_r [(\operatorname{Cos} \alpha_r)\omega t - \frac{\operatorname{Sin} 2r\omega t}{2r} \operatorname{Cos} \alpha_r$$

$$+ \frac{\operatorname{Cos} 2r\omega t}{2r} \operatorname{Sin} \alpha_r]_o^{2\pi}$$

$$L_1 = \frac{\sqrt{2}}{2} V_r \operatorname{Cos} \alpha_r \qquad \ldots \qquad (41)$$

$$L_2 = \frac{1}{2\pi} \int_0^{2\pi} K_2 \, d\omega t =$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \sum_0^n \sqrt{2}\, V_n \operatorname{Sin}(n\omega t - \alpha_n) \operatorname{Cos} r\omega t \, d\omega t$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \sqrt{2}/2 \sum_0^n V_m[\operatorname{Sin}((n+r)\omega t - \alpha_n)$$

$$+ \operatorname{Sin}((n - r)\omega t - \alpha_n)] \, d\omega t$$

Since $\operatorname{Sin} x \operatorname{Cos} y = \frac{1}{2}[\operatorname{Sin}(x+y) + \operatorname{Sin}(x-y)]$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{\sqrt{2}}{2} \sum_0^r V_r[\operatorname{Sin}(2r\omega t - \alpha_r) - \operatorname{Sin} \alpha_r] \, d\omega t$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{\sqrt{2}}{2} \sum_0^r V_r[\operatorname{Sin} 2r\omega t \operatorname{Cos} \alpha_r - \operatorname{Cos} 2r\omega t \operatorname{Sin}\alpha_r$$

$$- \operatorname{Sin} \alpha_r] \, d\omega t$$

$$= \frac{1}{2\pi} \times \frac{\sqrt{2}}{2} \sum_0^r V_r\left[- \frac{\operatorname{Cos} 2r\omega t \operatorname{Cos} \alpha_r}{2r} - \frac{\operatorname{Sin} 2r\omega t \operatorname{Sin}\alpha_r}{2r}\right.$$

$$\left. - (\operatorname{Sin} \alpha_r)\omega t\right]_0^{2\pi}$$

$$L_2 = -\frac{\sqrt{2}}{2} V_r \operatorname{Sin} \alpha_r \qquad \ldots\ldots\ldots \qquad \ldots \qquad (42)$$

Similarly,

$$L_3 = \frac{\sqrt{2}}{2} I_r \operatorname{Cos} \beta_r \qquad \ldots\ldots\ldots \qquad \ldots \qquad (43)$$

$$L_4 = -\frac{\sqrt{2}}{2} I_r \operatorname{Sin} \beta_r \qquad \ldots\ldots \qquad \ldots \qquad (44)$$

It is easy to see that if we realize the operations

$$L_2 \times L_3 - L_1 \times L_4 \qquad \ldots \qquad (45)$$

or $\quad -\frac{1}{2} V_r I_r \operatorname{Sin} \alpha_r \operatorname{Cos} \beta_r + \frac{1}{2} V_r I_r \operatorname{Cos} \alpha_r \operatorname{Sin} \beta_r$

We get,

$$\frac{1}{2} V_r I_r \operatorname{Sin} (\beta_r - \alpha_r) = \frac{1}{2} V_r I_r \operatorname{Sin} \phi_r$$

which is the reactive power corresponding to the r harmonics Total reactive power is attained when summing algebraically partial reactive power of significant harmonics. The operation performed by

$$\frac{1}{2} \sum_{r=1}^{n} V_r I_r \operatorname{Sin} \phi_r \qquad \ldots \qquad (46)$$

is the reactive power, except for a scale factor.

BLOCK DIAGRAM :

Figure (3) is the block diagram of the reactive power meter. The system works with nonsinusoidal signals, expandable in Fourier series in which the fundamental frequency is 50 Hz.

Here signals v and i given by equations (35) and (36) are gated at the input of the multipliers 1, 2 and 3, 4 respectively. Frequency controlled Sin/Cos oscillator generates the singals Sin $r\omega t$ and Cos $r\omega t$ which are gated at the input of the multipliers 2, 3 and 1, 4 respectively. Output of these multipliers are the analog signals given by

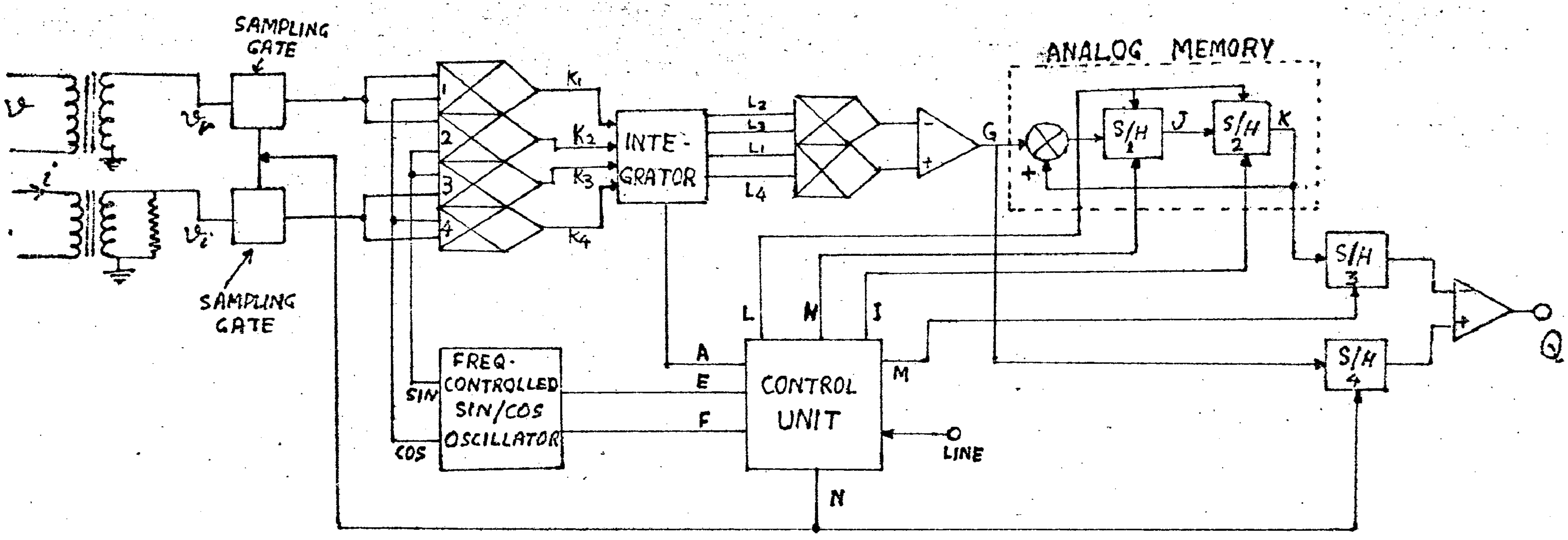



Fig (3) - Simplified Block Diagram of Reactive Power Meter

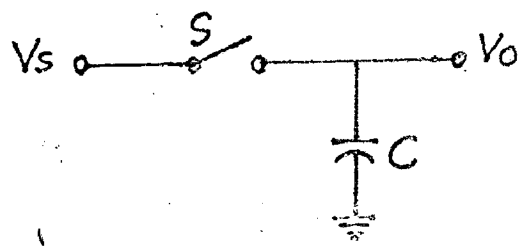



Fig (5) - Sample and Hold Circuit
(Schematic Diagram)

equations (37) to (40) denoted as $K_1$, $K_2$, $K_3$, $K_4$. The controlled unit is realized with IC-CMOS generates the different waveforms shown in Fig. (4) as presented by [7]. The clock frequency is obtained from the 50 Hz a.c. line. The staircase signal E controls the frequency of the Sin/Cos oscillator. Sequence F allows the introduction of initial conditions to Sin/Cos oscillator, and sequence A controls the integration period of analog signals $K_1$, $K_2$ $K_3$ and $K_4$. The analog signal $G = (L_3 \times L_2 - L_1 \times L_4)$ at the output of the differential circuit is the detected partial reactive power - in an imagined case, and must be accumulated and added to other contributions of significant components.

Two samples and hold (S/H) circuits and one summer closed loop perform the analog memory function that stores the total reactive power in consecutive steps. This can be observed in Fig. (3). A sample and hold circuit in its simplest form is a switch S in series with a capacitor as in Fig. (5). The voltage across capacitor tracks the input signal during the time $T_g$ when a logic control gate closes S, and holds the instantaneous value attained at the end of the interval $T_g$. When the control gate opens S.

The final values $Q_{r1}$, $Q_{r2}$, ...... of this signal corresponds to partial reactive power detected by the system. Sequence H controls the $S/H_1$ and the leading edges of sequence I controls the $S/H_2$ which samples the output signal J of $S/H_1$, the output signal K of $S/H_2$ is returned and

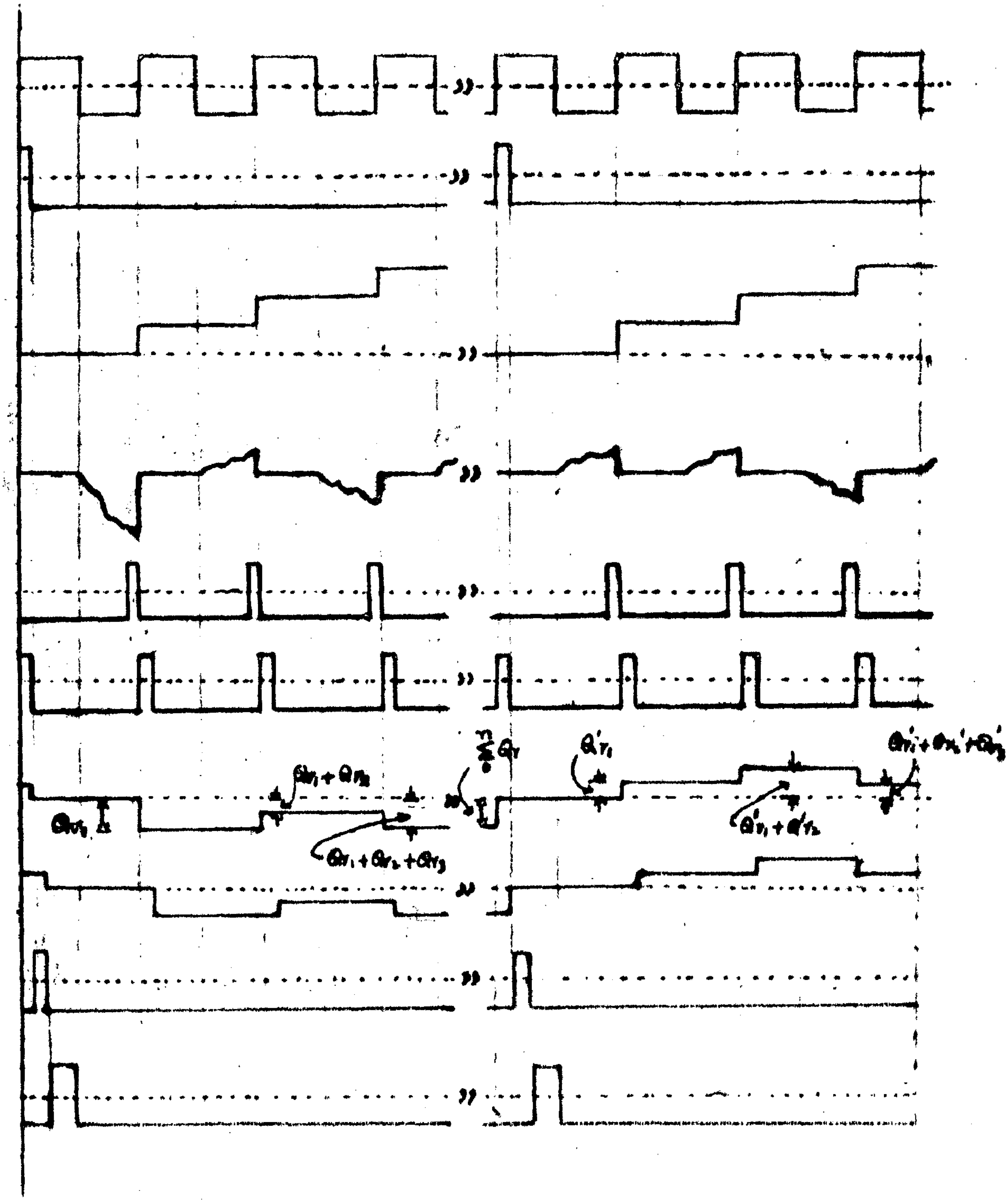


Fig (4) Waveforms corresponding to the Block Diagramm added to signal G for obtaining the accumulative value or reactive power. The reactive power repetition rate is determined by the pulses of sequence L. $S/H_3$ controlled by sequence M samples the output signal of the analog memory and holds it during the period of sequence L. Sequence N inhibits the input to the total system and controls $S/H_4$, which dötects the static error of the reactive power final value. This error is substracted from output of the $S/H_3$ by differential circuit and finally true reactive power is obtained at the output terminal of differential circuit.

(4.1.2.2) METHOD II :

Another method for measurement of reactive power $Q = \sum_{n=1}^{N} Q_m = \sum_{n=1}^{N} V_n I_n \sin \phi_n$ for nonsinusoidal supply system, consists of transforming the load voltage and the load current frequency spectra by multiplying both the voltage and the current of the load by pair of quadrature voltages. The frequency of these voltages is increased with reference to any chosen harmonic frequency $n\omega_1$ of the load voltage by a constant value $\omega_f$. The products obtained are next filtered by two mutually symmetrical band pass filters which pass the signals only in the vicinity of the frequency $\omega_f$. The mean alue of the filter output voltage product is proportional to the harmonics reactive power $Q_n$.

MEASUREMENT PRINCIPLE :

In order to select a singular harmonic component from the load voltage of frequency $\omega_1$ and instantaneous value

$$v = V_o + \sqrt{2} \sum_{n=1}^{N} V_n \; Cos(n\omega_1 t - \alpha_n) \qquad \ldots \quad (47)$$

with the aid of nonadjustable filter $F_1$, which passes the signals only in the vicinity of frequency $\omega_f$, the voltage $v$ is multiplied by the voltage $v_a$

$$v_a \triangleq \sqrt{2}\, V_a \; Sin\,(\, h\omega_1 + \omega_f)t \qquad \ldots \quad (48)$$

where $h$ is a number from the set $[1, 2 \ldots N]$. Thus we get

$$v_1 \triangleq m_1\, v\, v_a = \sqrt{2}\, m_1\, V_a\, V_o \; Sin\,(h\omega_1 + \omega_f)t$$

$$+ m_1\, v_a \sum_{n=1}^{N} V_n \; Sin\,[(h\omega_1 + \omega_f - n\omega_1)t + \alpha_n]$$

$$+ m_1 V_a \sum_{n=1}^{N} V_n \; Sin[(h\omega_1 + \omega_f + n\omega_1)t - \alpha_n] \quad \ldots \quad (49)$$

A component of frequency $\omega_f \neq 0$, selected from the voltage $v_1$, is related to only one $V_n$ value, indicated by the equality $n = h$, if for any number $n, h \in [0, 1, \ldots N]$

$$h\omega_1 + \omega_f - n\omega_1 \neq - \omega_f \qquad \ldots \quad (50)$$

It means that the frequencies

$$\omega_f = (n-h)\, \frac{\omega_1}{2} = r\, \frac{\omega_1}{2}\,, \; r \in [0, 1, \ldots N) \quad \ldots . \quad (51)$$

are inadmissible as filter frequencies. So its bandwidth B must be less than $\omega_1/2$. The center frequency $\omega_c$ of the filter may be taken as the mean value of any neighbouring inadmissible frequencies i.e. it may be equal to one of the values

$$\omega_c = r\frac{\omega_1}{2} + \frac{\omega_1}{4} \quad \ldots \quad \ldots \quad (52)$$

Then the deviation of differential frequency $h\omega_1 + \omega_f - n\omega_1$ from the center frequency $\omega_c$ of the filter must be limited by the following inequality.

$$(\omega_f - \omega_c) \leq \frac{B}{2} < \frac{\omega_1}{4} \quad \ldots \quad (53)$$

as illustrated in Fig. (6).

If in passband $|\omega - \omega_c| < B/2$ of the filter the magnitude of its frequency characteristics $K_1(J\omega)$ has a constant value $K_1$ i.e.

$$K_1(j\omega) \triangleq |K_1(j\omega)| e^{-j\theta_1(\omega)} \triangleq K_1 e^{-j\theta_1(\omega)} \quad \ldots \quad (54)$$

in which $\theta_1(\omega)$ denotes the phase shift characteristics of the filter, and if it attenuates the signals from beyond its band, pass efficiently enough, then the output voltage of the filter is

$$v_2 = K_1 m_1 V_a V_n \sin [\omega_f t + \alpha_n - \theta_1(\omega_f)] \quad \ldots \quad (55)$$

If the voltage v is applied to a load of

$$Z(jn\omega_1) \triangleq Z_n e^{j\phi_n} \quad \ldots \quad (56)$$

be impedance for harmonic frequencies, then the instantaneous current of the load is

$$i = I_o + \sqrt{2} \sum_{n=1}^{N} I_n \operatorname{Cos} (n\omega_1 t - \beta_n) \quad \ldots \quad (57)$$

with $(\alpha_n - \beta_n) = \phi_n$ or $\beta_n = \alpha_n - \phi_n$ ... (58)

Let us multiply this current by a voltage $v_b$ of the same frequency as the voltage $v_a$, but shifted with reference to it by an angle $\gamma_b$ i.e.

$$v_b \triangleq \sqrt{2} V_b \operatorname{Sin} [(h\omega_1 + \omega_f)t + \gamma_b] \quad \ldots \quad (59)$$

Their product $v_3$ has the form

$$v_c \triangleq m_2 \, i \, v_a = \sqrt{2} m_2 V_b I_o \operatorname{Sin}[(h\omega_1 + \omega_f)t + \gamma_b]$$

$$+ m_2 V_b \sum_{n=1}^{N} I_n \operatorname{Sin} [(h\omega_1 + \omega_f - n\omega_1)t + \beta_n + \gamma_b]$$

$$+ m_2 V_b \sum_{n=1}^{N} I_n \operatorname{Sin} [(h\omega_1 + \omega_f + n\omega_1)t - \beta_n + \gamma_b] \quad \ldots \quad (60)$$

where $m_2$ denotes a dimensional coefficient. If this voltage is filtered by the second filter $F_2$, similar to $F_1$, which for $|\omega - \omega_c| < B/2$ has the following frequency characteristics

$$K_2(j\omega) \triangleq K_2 e^{-j\theta_2(\omega)} \quad \ldots \quad (61)$$

Then its output voltage is

$$v_4 = K_2 m_2 V_b I_n \operatorname{Sin} [\omega_f t + \beta_n + \gamma_b - \theta_2(\omega_f)] \quad \ldots \quad (62)$$

The mean value of the product of output voltages $v_1$ and $v_2$ measured by a moving coil voltmeter.

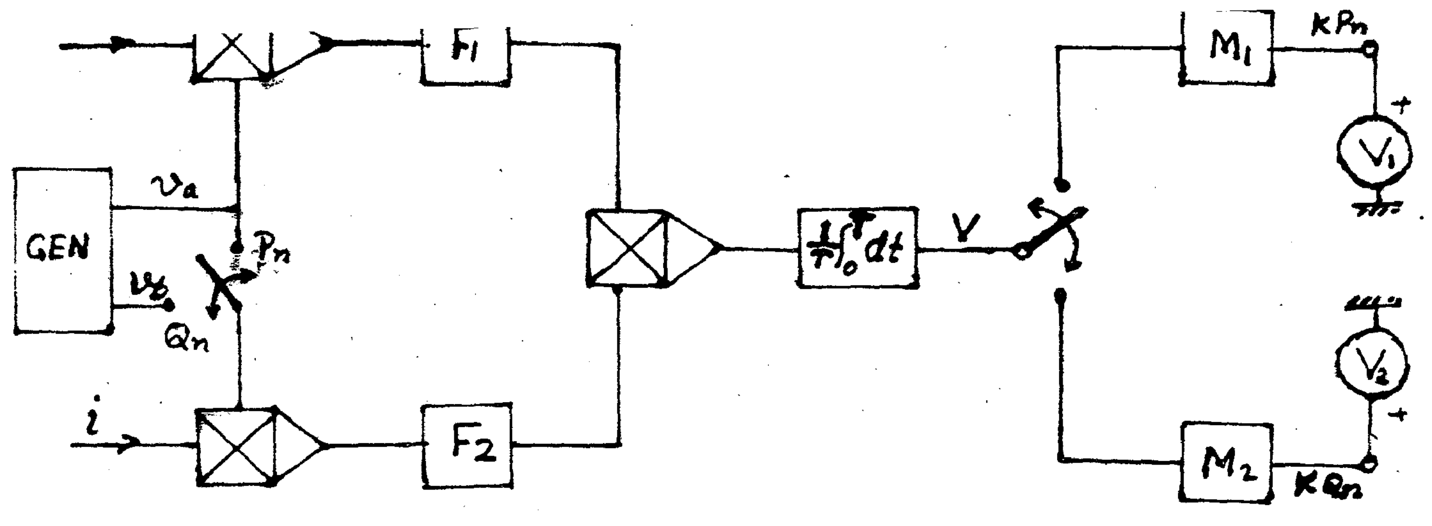



Fig (7) — The structure of harmonics Active/Reactive Power meter

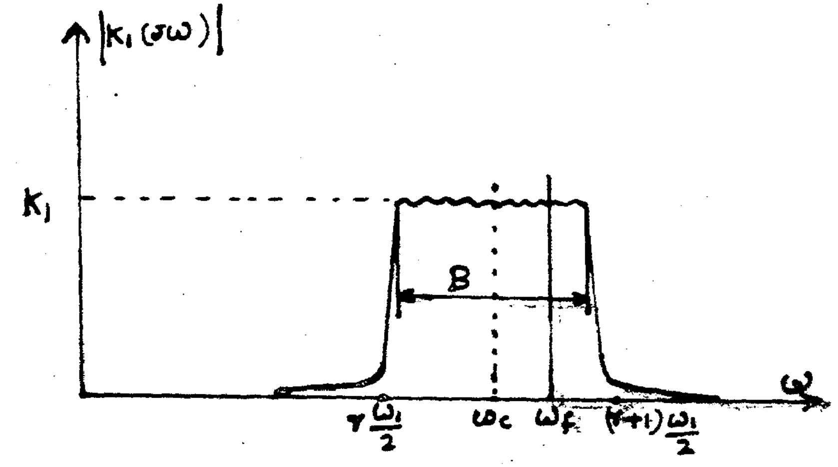



Fig (6) — A plot of the filter frequency characteristics magnitude

$$V \triangleq m_3 v_2 v_4 \triangleq \frac{1}{T}\int_0^T m_3 v_2 v_4 \, dt$$

$$\text{or } V = \frac{1}{2} K_1 K_2 m_1 m_2 m_3 V_a V_b V_n I_n \times$$

$$\text{Cos}\,[\alpha_n - \beta_n - \gamma_b - \theta_n(\omega_f) + \theta_2(\omega_f)] \quad \ldots \quad (63)$$

$$\text{For} \quad \gamma_b = \pi/2 \quad \ldots \quad (64)$$

$$\theta_1(\omega_f) = \theta_2(\omega_f) \quad \ldots \quad (65)$$

is proportional to reactive power $Q_n$ of the singular harmonic component of frequency $n\omega_1$

$$V = K V_n I_n \text{ Sin } \phi_n = K Q_n \quad \ldots \quad (66)$$

in which

$$K = \frac{1}{2} K_1 K_2 m_1 m_2 m_3 V_a V_b \quad \ldots \quad (67)$$

According to the above considerations, the $Q_n$ power meter has the structure shown in Fig. (7). Aside from the three multipliers and a moving coil voltmeter, the meter requires a source of two voltages, shifted in relation to each other $\pi/2$ phase angle, and a pair of narrow band pass filters of very similar phase shift ($\theta_1(\omega)$ and $\theta_2(\omega)$) characteristics. However, there are no special accuracy boundaries in order to fulfil these requirements. It is worth noting or worth considering, that if the load current is multiplied by the same voltage $v_a$, that the load voltage is multiplied by it, then the average network output voltage V is proportional to active power $P_n$ of a harmonic component, but not to the reactive power. If the meter is equipped with two memories $M_1$ and $M_2$ which store the output voltages V of the averaging network, then the meter simultaneously provides two voltages proportional to both the active and reactive powers.

## 4.2 CURRENT SUBDIVISION TECHNIQUE :

### 4.2.1 DEFINITION OF REACTIVE POWER AND POWER FACTOR IMPROVEMENT FOR NONSINUSOIDAL SUPPLY SYSTEM :

In this approach, the apparent power is divided into an active power component, a reactive power component, and a residual reactive power component. Essentially, taking the voltage as the reference, this resolves into dividing the current as

(1) ACTIVE CURRENT COMPONENT - which has the same waveform and phase as the voltage ( and thus the same waveform and phase as the current in a resistor with the same voltage across it).

(2)(a) INDUCTIVE REACTIVE CURRENT COMPONENT - which has the same waveform and phase as that of current in an inductor with the same voltage across it,

(b) CAPACITIVE REACTIVE CURRENT COMPONENT - which has the same waveform and phase as that of the current in capacitor with the same voltage across it.

(3) RESIDUAL REACTIVE CURRENT COMPONENT - either INDUCTIVE or CAPACITIVE being that which remains

of the total current after the active and the respedtive inductive or capacitive reactive current components have been extracted.

The actual magnitudes of the active, inductive reactive, capacitive reactive components in the total current are determined from the time averaged product of the current and the reference waveform. Each of these quantities can be either positive or negative. When the capacitive reactive component of the current is negative, it can be completely compensated by a shunt capacitor of suitable value. Similarly, when the inductive reactive component is negative, a shunt inductor can be used to accomplish the same result. The residual reactive component, having no reference, is always positive and compensation by means of positive components is not possible.

A similar breakdown can also be made of course, using the current as a reference, with essentially the same results. Howeve., since the voltage in a network is normally maintained more or less constant, with current as the variable, and since the current is the quantity involved in transmission network losses, the voltage will be retained as the reference here.

When sinusoidal conditions occure, the residual component of the current is zero and the inductive and capacitive reactive components are dqual in magnitude, but opposite in sign. Complete compensation for an inductive load can be achieved by the application of shunt capacitors and correspondingly, a capacitive loa capacitors and correspondingly a capacitive load can be compensated for with shunt inductor.

With a non-sinusoidal voltage and linear loads, the residual reactive component of the current is not necessarily zero and the inductive and capacitive reactive current components may be unequal, and both positive or opposite in sign. An inductive load will have a positive inductive reactive component and a negative capacitive reactive component, which is smaller in magnitude. Thus, only partial compensation can be achieved. A similar result is obtained with a capacitive load. With a combined inductive and capacitive load, the inductive and cpacitive reactive components may both the positive and then no compensation by passive linear components is possible.

With a sinusoidal voltage and a nonlinear load, the inductive and capacitive reactive currents components, if present, will be equal but opoosite in sign, and a residual reactive component will also be present. Complete compensation is therefore possible by passive means of the inductive or capacitive reactive component but not for the residual component.

With a nonsinusoidal voltage and nonlinear load, the inductive and capacitive reactive components, if present at all, may be both positive or opposite in sign, and not necessarily equal in magnitude. Partial compensation is possible but then only if a negative component is present.

MATHEMATICAL CONSIDERATIONS :

Let

$v$, $V$ be respectively the instantaneous and rms values of the voltage

$\dot{v}$, $\dot{V}$ be respectively the instantaneous and rms values of dv/dt

$\bar{v}$, $\bar{V}$ be respectively the instantaneous and rms values of the alternating component of $\int v\, dt$

$i$, $I$ be respectively the instantaneous and rms values of the current

APPARENT POWER :

The apparent power $S = VI$ ... ... (68)

ACTIVE POWER :

The active power $P = \frac{1}{T}\int_0^T vi\, dt$

or $P = \frac{1}{T}\int_0^T vip\, dt = VI_p$ ... (69)

Where ip and $I_p$ are respectively the instantaneous and rms ACTIVE CURRENTS (i.e., the active power components of the total current i) and

$$V^2 = \frac{1}{T}\int_0^T v^2\, dt \ ; \ I^2_p = \frac{1}{T}\int_0^T i_p^2\, dt$$

From Equation (69), rms active current

$$I_p = \frac{P}{V} = \frac{\left(\frac{1}{T}\int_0^T vi\, dt\right)}{V} \quad \text{...} \quad (70)$$

for rsistive load R, $i = i_p = v/R$

Multiplying by $V^2$

$$V^2 i_p = \frac{vV^2}{R} = vP, \quad \text{Since} \quad V^2/R = P$$

∴ the instantaneous active current i

$$i_p = \frac{Pv}{V^2} = v\left(\frac{1}{T}\int_0^T vi\,dt\right)/V^2 \qquad \ldots (71)$$

REACTIVE POWER :

The instantaneous REACTIVE CURRENT $i_q = i - i_p$

The rms REACTIVE CURRENT $I_q = \sqrt{(I^2 - I_p^2)}$

The REACTIVE POWER $Q = \sqrt{(S^2 - P^2)}$

The reactive current is further divided into either an inductive or capacitive component and a corresponding residual inductive or capacitive component. Referring to Eqn. (69) and (70), by definition

~~The rms inductive reactive current~~

The rms INDUCTIVE REACTIVE CURRENT

$$I_{ql} = \left(\frac{1}{T}\int_0^T \bar{v}i\,dt\right)/\bar{V} \quad \ldots\ldots\ldots \quad (72)$$

and its instantaneous value $i_{ql} = \bar{v}\left(\frac{1}{T}\int_0^T \bar{v}i\,dt\right)/\bar{V}^2 \quad \ldots(73)$

The INDUCTIVE REACTIVE POWER $Q_l = VI_{ql}$

$$= V\left(\frac{1}{T}\int_0^T \bar{v}i\,dt\right)/\bar{V} \qquad \ldots(74)$$

The rms CAPACITIVE REACTIVE CURRENT $I_{qc} = \left(\frac{1}{T}\int_0^T \dot{v}i\,dt\right)/\dot{V}$

In applying these concepts, it is convenient to define two composite frequencies

$$\omega_l = V/\bar{V}, \qquad \omega_c = \dot{V}/V$$

Thus, for a voltage -

$$v = \sqrt{2}\,[\,V_1 \sin(\omega t + \phi_1) + V_2 \sin(2\omega t + \phi_2) + \cdots\cdots$$

$$+ V_k \sin(k\omega t + \phi_k)]$$

$$V = \sqrt{(V_1^2 + V_2^2 + \cdots\cdots + V_k^2)}$$

$$\bar{V} = (1/\omega)\sqrt{[V_1^2 + (V_2/2)^2 + \cdots + (V_k/k)^2]}$$

$$\dot{V} = \omega\sqrt{[V_1^2 + (2V_2)^2 + \cdots + (kV_k)^2]}$$

$$\omega_1 = \omega V/\sqrt{[V_1^2 + (V_2/2)^2 + \cdots\cdots + (V_k/k)^2]}$$

$$\text{or} \quad \frac{\omega_1}{\omega} = V/\sqrt{[V_1^2 + (\frac{V_2}{2})^2 + \cdots\cdots + (\frac{V_k}{k})^2]}$$

$$\omega_c = \omega\sqrt{[V_1^2 + (2V_2)^2 + \cdots\cdots + (kV_k)^2]}/V$$

$$\text{or} \quad \frac{\omega_c}{\omega} = \sqrt{[V_1^2 + (2V_2)^2 + \cdots\cdots + (kV_k)^2]}/V$$

It is apparent that $\omega \leq \omega_1 \leq \omega_c$

The INDUCTIVE REACTIVE POWER $= VI_{ql} = \omega_1(\frac{1}{T}\int_0^T \bar{v} i \, dt)$

The CAPACITIVE REACTIVE POWER $= VI_{qc} = (\frac{1}{\omega_c})(\frac{1}{T}\int_0^T \dot{v} i \, dt)$

The main value of these composite frequencies however is that they can be used with either an inductance L or capacitance C to obtain the equivalent impedance under nonsinusoidal conditions. Thus for an inductance

$$i_{ql} = \frac{1}{L}\int v \, dt = \bar{v}/L$$

and $I_{ql} = \bar{V}/L = V/\omega_1 L$ (Since $\omega_1 = V/\bar{V}$ and

Z equivalent = $\omega$ L)

Similarly for a capacitance

$$I_{qc} = C\dot{V} = V\omega_c C \quad ( Z \text{ equivalent} = 1/\omega_c C)$$

The instantaneous RESIDUAL REACTIVE CURRENT either INDUCTIVE or CAPACITIVE, is obtained by subtracting the corresponding linear reactive currents $i_{ql}$ or $i_{qc}$ from the total reactive current $i_q$.

$$i_{qlr} = i_q - i_{ql} = i - i_p - i_{ql}$$

$$i_{qcr} = i_q - i_{qc} = i - i_p - i_{qc}$$

$$I_{qlr} = \sqrt{(I^2 - I_p{}^2 - I_{ql}{}^2)}$$

$$I_{qcr} = \sqrt{(I^2 - I_p{}^2 - I_{qc}{}^2)}$$

$$Q_{lr} = \sqrt{(S^2 - P^2 - Q_l{}^2)}$$

$$Q_{cr} = \sqrt{(S^2 - P^2 - Q_c{}^2)}$$



Application of this technique i.e. power factor improvement can be illustrated by considering the effect of adding capacitance C in parallel with an inductance L under nonsinusoidal voltage conditions. For such a circuit, the following relationships derived as shown below.

The inductive reactive component of the total current is given by

$$I_{ql} = V/\omega_1 L - V\omega_1 C$$

where $V/\omega_1 L$ is the inductive reactive component of the current through the inductance L and

$V\omega_1 C$ is the inductive reactive component of the current through capacitance C

$V\omega_1 C$ can be derived from equation (72)

$$I_{q1} = \frac{1}{T}\int_o^T \bar{v}i \, dt \,/\, \bar{V} \quad \text{and } i = C\frac{dv}{dt} = C\dot{v}$$

$$\therefore \quad I_{q1} \left(\frac{C}{T}\int_o^T \bar{v}\,\dot{v}\, dt\right)/\,\bar{V} = \left(\frac{C}{T}\int_o^T - v^2\, dt\right)/\,\bar{V}$$

$$= -CV^2 \,/\, \bar{V} = -\omega_1 CV$$

The remaining component of the current through C, which is the residual inductive reactive component $I_{q1r}$, is orthogonal to the inductive reactive component and can be obtained from the total current $V\omega_c C$ through C by the equation

$$I_{q1r} = \sqrt{[(V\omega_c C)^2 - (V\omega_1 C)^2]} = VC\sqrt{[(\omega_c)^2 - (\omega_1)^2]}$$

The equations for the capacitive reactive and residual capacitive reactive current can be derived similarly

$$I_{qc} = V\omega_c C - V/\omega_c L$$

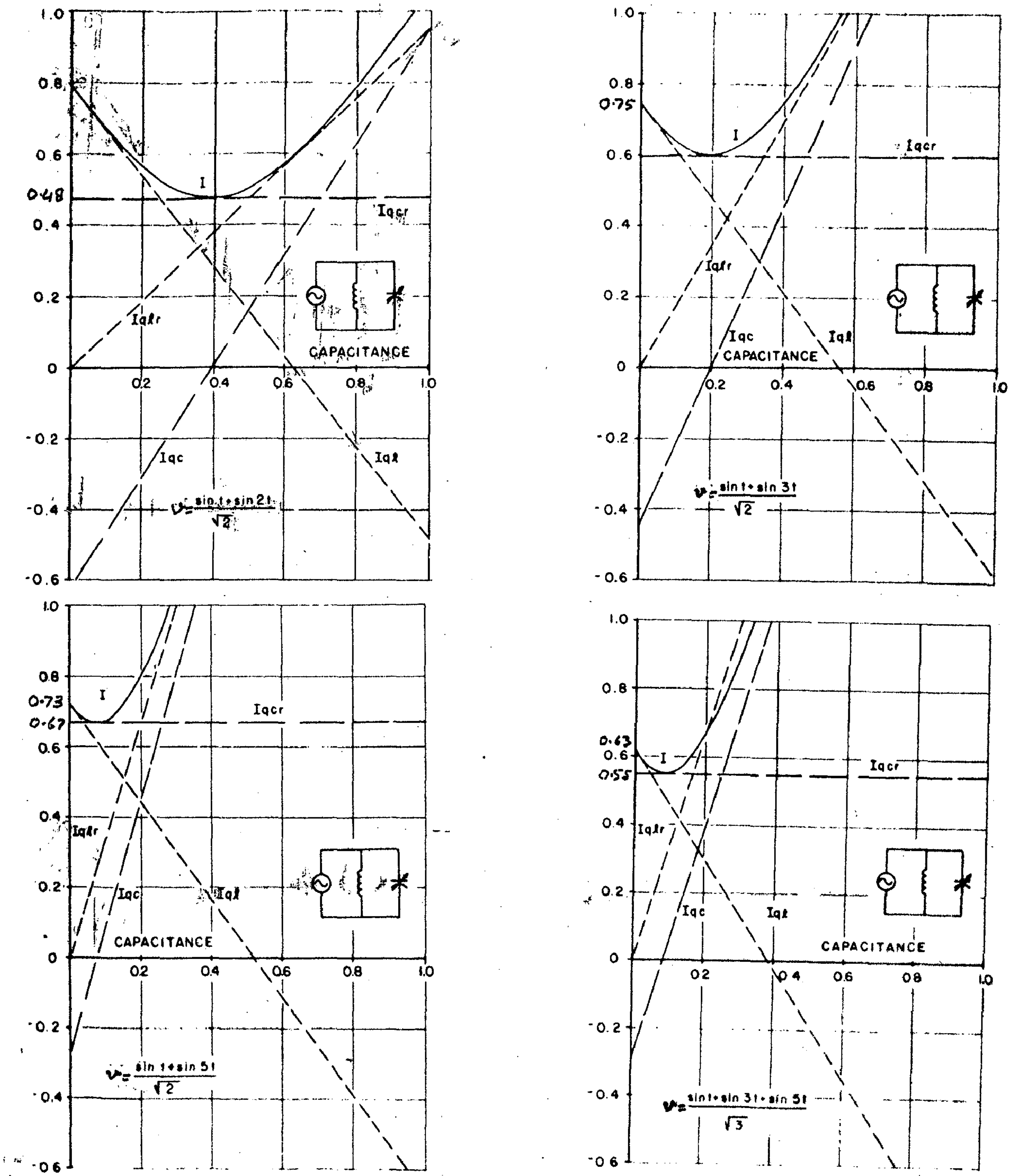
$$I_{qcr} = \sqrt{[(\frac{V}{\omega_1 L})^2 - (\frac{V}{\omega_c L})^2]} = (\frac{V}{L})\sqrt{[(\frac{1}{\omega_1})^2 - (\frac{1}{\omega_c})^2]}$$

$$I = I_q = \sqrt{(I_{q1}{}^2 + I_{q1r}{}^2)} = \sqrt{(I_{qc}{}^2 + I_{qcr}{}^2)}$$

Plots of these five currents ( i.e. $I_{ql}$, $I_{qlr}$, $I_{qc}$, $I_{qc}$ and $I(I_q)$) against variable C for the various combinations of fundamental and harmonic voltages are shown in Fig. (8). To plot the five currents following quantities should be calculated as below :

| Voltage | $V$ | $\bar{V}$ | $\dot{V}$ | $\omega_1$ | $\omega_c$ |
|---|---|---|---|---|---|
| $(\sin \omega t + \sin 2\omega t)/\sqrt{2}$ | 1 | $\frac{0.791}{\omega}$ | $1.581\omega$ | $1.265\omega$ | $1.58\omega$ |
| $(\sin \omega t + \sin 3\omega t)/\sqrt{2}$ | 1 | $0.745/\omega$ | $2.236\omega$ | $1.342\omega$ | $2.24\omega$ |
| $(\sin \omega t + \sin 5\omega t)/\sqrt{2}$ | 1 | $0.721/\omega$ | $3.606\omega$ | $1.387\omega$ | $3.61\omega$ |
| $(\sin \omega t + \sin \omega t + \sin 5\omega t)/\sqrt{3}$ | 1 | $0.619/\omega$ | $3.416\omega$ | $1.614\omega$ | $3.42\omega$ |

For comparing purposes, these plots have been normalized by setting $\omega = 1$ and $L = 1$ and varying C from 0 to 1. It is present, in each that at $C = 0$, a negative $I_{qc}$ is present, therefore compensation by adding C is possible. The optimum value is achieved when the total current I is a minimum and this occures when $I_{qc} = 0$. The actual reductions in total current I that are realized by adding the optimum value of C are listed below. Here resultant current I will be equal to residual capacitive reactive current $I_{qcr}$.




FIGURE(8) Effect of Capacitance in Parallel With An Inductance On The Current Components for Various Applied Voltages

PERCENT CURRENT REDUCTION WITH OPTIMUM COMPENSATION :

| Figure | Voltage | Percent reduction |
|---|---|---|
| 1 | $(\sin t + \sin 2t)/\sqrt{2}$ | $\frac{.8 - .48}{.8} \times 100 = 40\%$ |
| 2 | $(\sin t + \sin 3t)/\sqrt{2}$ | $\frac{.75 - .6}{.75} \times 100 = 20\%$ |
| 3 | $(\sin t + \sin 5t)/\sqrt{2}$ | $\frac{.73 - .67}{.73} \times 100 = 8\%$ |
| 4 | $(\sin t + \sin 3t + \sin 5t)/\sqrt{3}$ | $\frac{.63 - .55}{.63} \times 100 = 1$ |

so that by choosing the optimum value of capacitance C we can reduce total current I to minimum value as clearly shown in Fig.(8). Therefore, we can improve the power factor and thus minimizes the losses.

### 4.2.2 REACTIVE POWER MEASUREMENT FOR NONSINUSOIDAL SUPPLY SYSTEM :

A circuit measuring the various components of the current under distorted waveform conditions according to the proposed method of analysis is shown in Fig. (9). The current circuit accepts instantaneous values of the voltage v and current i and separates the current into its active component $i_p$ and reactive component $i_q$ (at switch position 1) or into the active ($i_p$), inductive ($i_{ql}$) and residual inductive ($i_{qlr}$) reactive components (at switch position 2) or into its active ($i_p$), capacitive ($i_{qc}$) and residual

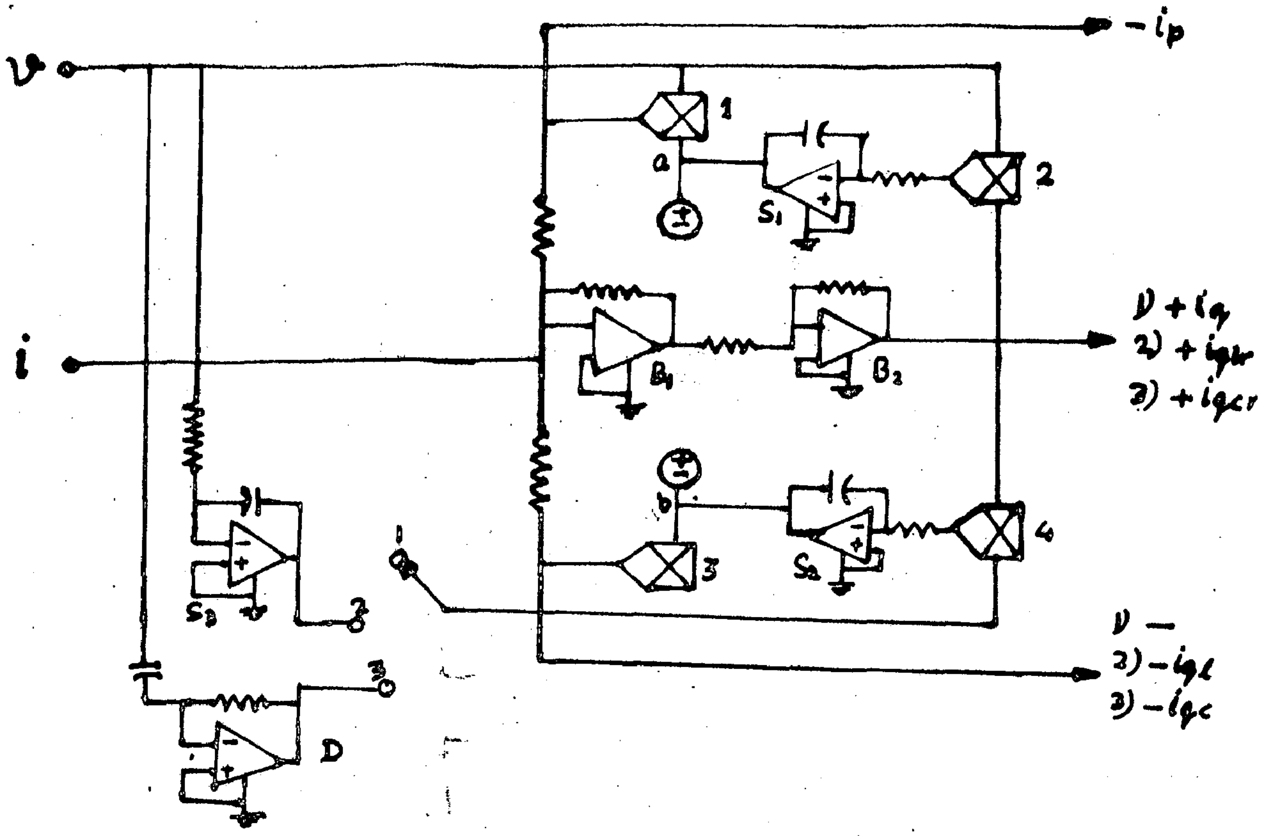



Fig - (9) Active / Reactive Current Analyzer

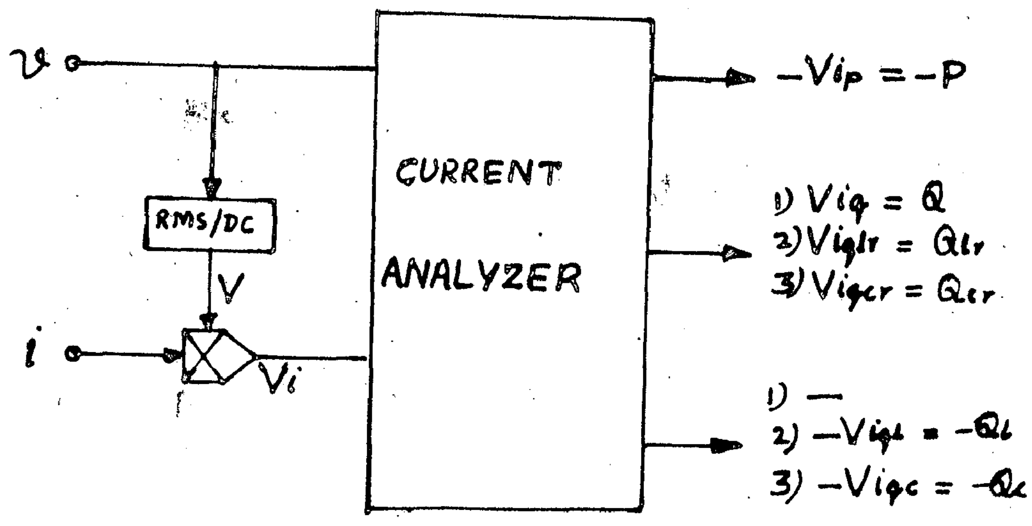



Fig - (10) Modification To Current Analyzer for Measurement of Reactive/Active Components of power.

capacitive ($i_{qcr}$) reactive component (at switch position 3). The sign of the active component $i_p$ is determined by dc polarity integrator connected to point a and that of the inductive or capacitive reactive component $i_{ql}$ and $i_{qc}$ respectively, by a dc polarity indicator at point p.

At switch position 1, multiplier 2 gives the output (vi and it is integrated in integration $S_1$. Output of the integrator $S_2$ is multiplied with $\bar{v}$ by multiplier 1 give result with polarity as ( $- i_p$) noting that multiplier 1 has gain of ($1/V^2$). At the same time + $i_q$ is obtained using operational amplifier $B_1$ as summer, at the output terminal of operational amplifier $B_2$.

At switch position 2, $- i_p$ is obtained in similar manner as above. After operation of integrator $S_3$, multiplier 4, integrator 3 and multiplier 3 of gain ($1/ \bar{V}^2$) inductive reactive current with negative polarity $- i_{ql}$ is obtained. At the same time after using of operational amplifier $B_1$ as summer, $i_{qlr} = (i - i_p - i_{ql})$ is obtained at the output terminal of operational amplifier $B_2$.

At switch position 3, using differentiation D and multiplier (3) of gain ($1/\dot{V}^2$) and integrator and multipliers as in same manner as at switch position 2 $- i_p$, $i_{qc}$, $i_{qcr} - (i - i_p - i_{qc})$ are obtained.

The circuit can be adapted to indicate the corresponding components of power by a simple modification at its input as shown in Fig. (10). Fig. (10) consists of arranging to multiply the input current i by the rms value V of the voltage v. All outputs of current analyser are then instantaneous currents components multiplied by V and thus indicate instantaneous power. Terminals are provided for displaying the corresponding instantaneous values on an oscilloscope.

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## 5. ILLUSTRATION OF BOTH TECHNIQUES BY NUMERICAL EXAMPLES :

EXAMPLES (I)

Suppose nonsinusoidal supply produces, triangular wave as shown in Fig. (11). One cycle of the wave can be expressed mathematically between $-\pi$ and $+\pi$. From Fig. (11) we have $f(\alpha)$ at an angle $\theta$ where $\tan\theta = \frac{V}{\pi}$

$$f(\alpha) = \frac{V}{\pi}\alpha \qquad 0 < \alpha < \pi$$

also $$f(\alpha) = +\frac{V}{\pi}\alpha \qquad -\pi < \alpha < 0$$

(Since $f(\alpha)$ will have a negative value in the interval $-\pi < \alpha < 0$)

(i) Constant term $a_o$ :

$$a_o = \frac{1}{2\pi}\int_0^{2\pi} f(\alpha)\,d\alpha$$

From Fig. (11) it is clear that in one cycle of the wave between $-\pi$ and $+\pi$ the positive area is equql to the negative area. Hence, $a_o = 0$

(ii) Coefficient of Cosine terms :

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\alpha)\,\text{Cos}\,n\alpha\,d\alpha = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\alpha)\text{Cos}\,n\alpha\,d\alpha$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{V}{\pi}\alpha\,\text{Cos}\,n\alpha\,d\alpha$$

$$= \frac{V}{\pi^2}\left[\left[\frac{\alpha\,\text{Sin}\,n\alpha}{n}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{\text{Sin}\,n\alpha}{n}\,d\alpha\right] = 0$$

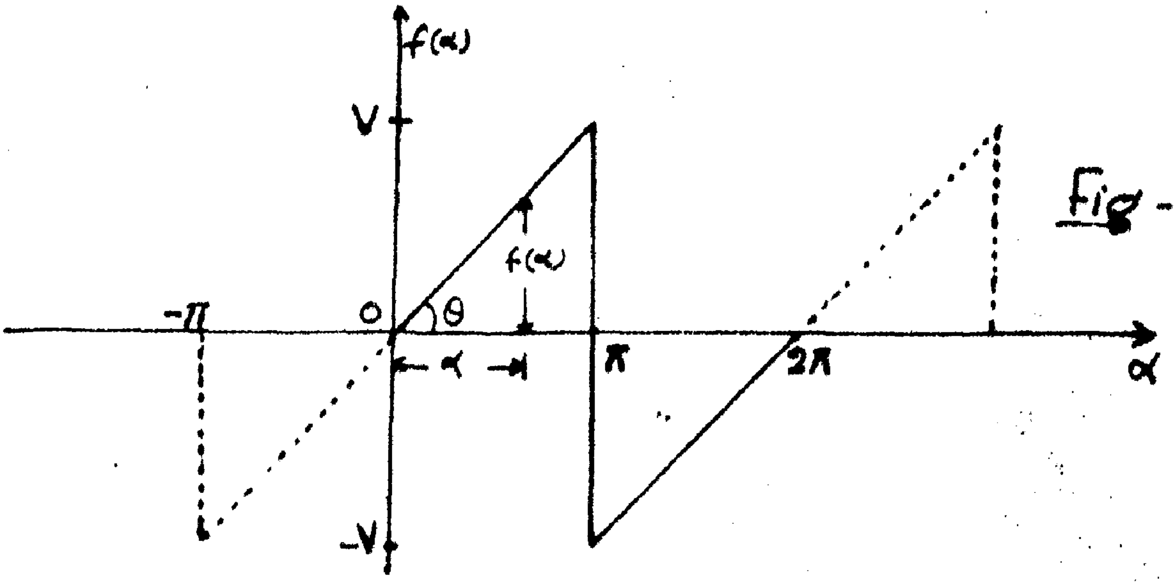


Fig - (11)

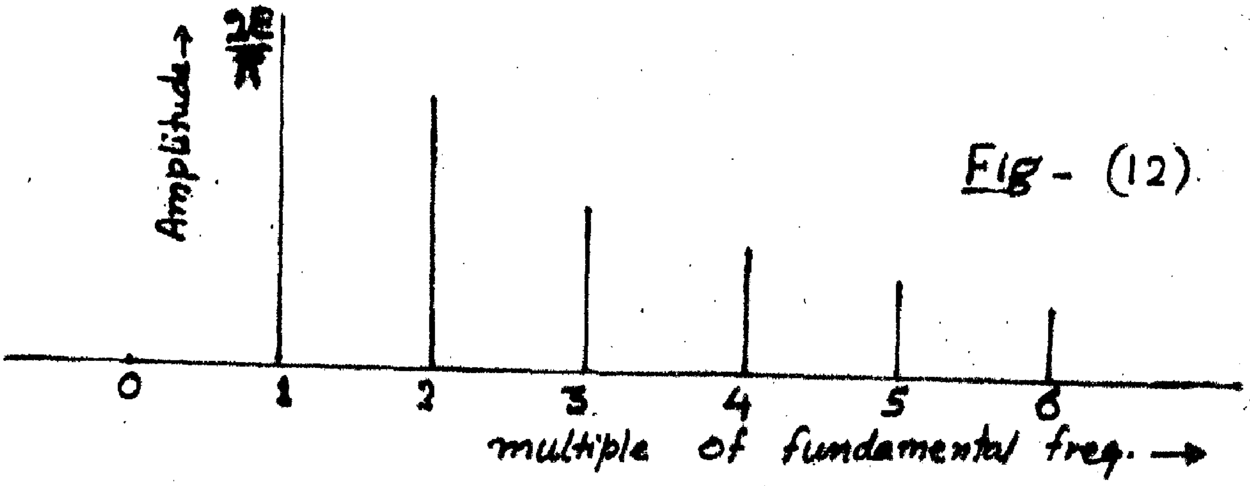


Fig - (12)

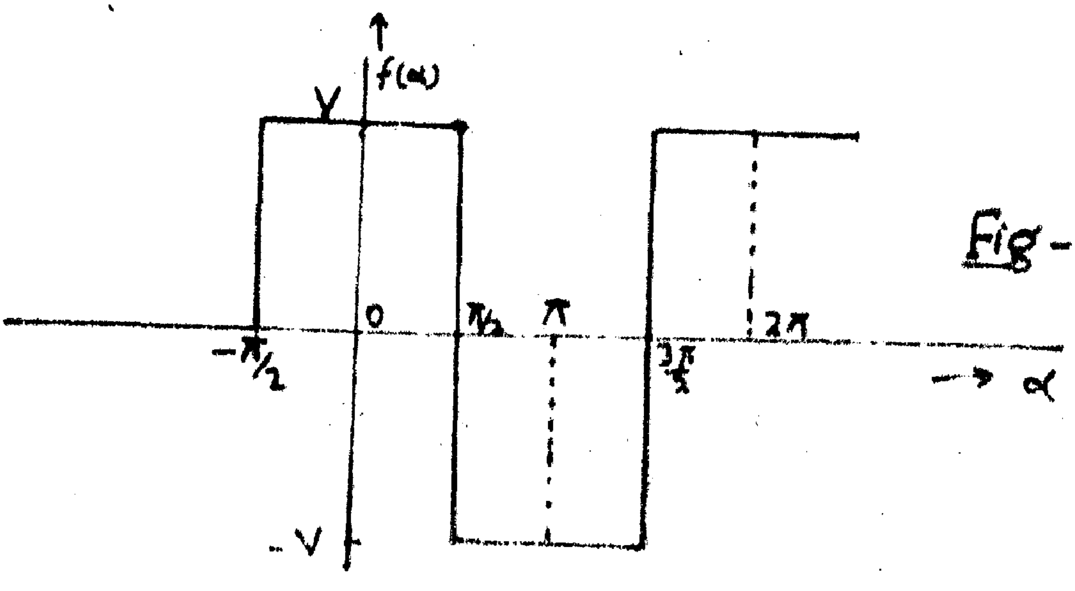


Fig - (13)

(iii) Coefficient of Sine terms :

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\alpha)\ \text{Sin}\ n\alpha\ d\alpha$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\alpha)\ \text{Sin}\ n\alpha\ d\alpha = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{V}{\pi}\ \alpha \cdot \text{Sin}\ n\alpha\ d\alpha$$

$$= \left[ -\frac{V}{n\pi^2}\ \alpha\ \text{Cos}\ n\alpha \right]_{-\pi}^{\pi} + \frac{V}{n\pi^2} \int_{-\pi}^{\pi} \text{Cos}\ n\alpha\ d\alpha$$

The integral vanishes since $n \neq 0$

Hence simplying

$$b_n = \frac{2V}{n\pi^2}\ (-1)^{n+1}$$

Thus the Fourier series

$$f(\alpha) = a_o + \sum_{n=1}^{n=\infty} a_n\ \text{Cos}\ n\alpha + \sum_{n=1}^{n=\infty} b_n\ \text{Sin}\ n\alpha$$

where,

$a_o$ = a constant

$b_n$, $a_n$ = amplitude of different harmonics

$\alpha$ = indpendent variable

$n$ = integer such as 1, 2, 3, etc.

Therefore,

$$f(\alpha) = \frac{2V}{\pi}\left[\text{Sin}\ \alpha - \frac{1}{2}\ \text{Sin}\ 2\alpha + \frac{1}{3}\ \text{Sin}\ 3\alpha - \frac{1}{4}\ \text{Sin}\ 4\alpha + ..\right]$$

$$v = \frac{2V}{\pi}\left[\text{Sin}\ \alpha - \frac{1}{2}\ \text{Sin}\ 2\alpha + \frac{1}{3}\ \text{Sin}\ 3\alpha - \frac{1}{4}\ \text{Sin}\ 4\alpha + ...\right]$$

Graph giving amplitudes of different harmonics is shown in Fig. (12).

Let $V = 100$ volts, $\alpha = \omega t$, $\omega = 400$ rad/sec.

$$\therefore \quad v = [63.66 \text{ Sin } 400\,t - 31.83 \text{ Sin } 800\,t + 21.22 \text{ Sin } 1200\,t$$
$$- 15.92 \text{ Sin } 1600\,t + 12.73 \text{ Sin } 2000\,t - 10.61$$
$$- 10.61 \text{ Sin } 2400\,t + \ldots\ldots ]$$

Suppose

$$i = [6.2 \text{ Sin } (400\,t + 26.6^{o}) - 2.5(800\,t + 56.3^{o}) +$$
$$1.2 \text{ Sin } (1200\,t + 68.2^{o}) - 0.56\text{Sin } (1600\,t + 72.4^{o})$$
$$+ 0.32 \text{ Sin } (2000\,t + 78.3^{o}) - 0.18 \text{ Sin } (2400\,t + 84.3^{o}) + \ldots\ldots ]$$

$$\because \quad V = \sqrt{(V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2 + V_6^2)}$$

$$\therefore \quad V = \sqrt{[(63.66)^2 + (31.83)^2 + (21.22)^2 + (15.92)^2 + (12.73)^2 + (10.61)^2]}$$

$$= 77.73 \text{ volts.}$$

$$\because \quad \overline{V} = \frac{1}{\omega} \sqrt{[V_1^2 + (\frac{V_2}{2})^2 + (\frac{V_3}{3})^2 + (\frac{V_4}{4})^2 + (\frac{V_5}{5})^2 + (\frac{V_6}{6})^2]}$$

$$\therefore \quad \overline{V} = \frac{1}{400} \sqrt{[(63.66)^2 + (31.83/2)^2 + (21.22/3)^2 + (15.92/4)^2 + (12.73/5)^2 + (10.61/6)^2]}$$

$$= 0.1655 \text{ volts.}$$

$$\because \quad \dot{V} = \omega \sqrt{[V_1^2 + (2V_2)^2 + (3V_3)^2 + (4V_4)^2 + (5V_5)^2 + (6\,V_6)^2]}$$

$$\therefore \quad \dot{V} = 400 \sqrt{[(63.66)^2 + (2 \times 31.83)^2 + (3 \times 21.22)^2 + (4 \times 15.92)^2 + (12.73 \times 5)^2 + (6 \times 10.61)^2]}$$

$$= 51131.58 \text{ volts.}$$

Two composite frequencies $\omega_1$, $\omega_c$ are determined as

$$\omega_1 = \frac{V}{\overline{V}} = \frac{77.73}{0.1655} = 469.67$$

$$\omega_c = \frac{\dot{V}}{V} = \frac{51131.58}{77.73} = 657.81$$

It is apparent that $\omega \leq \omega_1 \leq \omega_c$

INDUCTIVE REACTIVE POWER :

$$Q_1 = VI_{q1} = \omega_1 \left[ \frac{1}{T} \int_0^T \overline{v} i \, dt \right]$$

Suppose

$$v = \sqrt{2} \sum_{k=1}^{n} V_k \operatorname{Sin} (k\omega t + \phi_k)$$

and $i = \sqrt{2} \sum_{k=1}^{n} I_k \operatorname{Sin} (k\omega t + \phi_k + \beta_k)$

$$\overline{v} = \int v dt = \sqrt{2} \int \sum_{k=1}^{n} V_k \operatorname{Sin} (k\omega t + \phi_k) dt$$

$$= -\sqrt{2} \sum_{k=1}^{n} \frac{V_k}{k\omega} \operatorname{Cos} (k\omega t + \phi_k)$$

$$Q_1 = \omega_1 \left[ \frac{1}{T} \int_0^T \overline{v} i \, dt \right]$$

$$= \frac{2\omega_1}{\omega} \left[ \frac{1}{T} \int_0^T \sum_{k=1}^{n} \frac{V_k I_k}{k} \{ \operatorname{Cos}(k\omega t + \phi_k) \times \operatorname{Sin} (k\omega t + \phi_k + \beta_k) \} dt \right]$$

$$= \frac{\omega_1}{\omega} \left[ \frac{1}{T} \int_0^T \sum_{k=1}^{n} \frac{V_k\ I_k}{k} \left( \text{Sin}(2k\omega t + 2\phi_k + \beta_k) \right.\right.$$

$$\left.\left. + \text{Sin}\ (\beta_k)\right) dt \right]$$

$$Q_1 = -\frac{\omega_1}{\omega} \sum_{k=1}^{n} \left( \frac{V_k\ I_k}{k} \text{Sin}\ \beta_k \right)$$

CAPACITIVE REACTIVE POWER :

$$Q_c = VI_{qc} = (1/\omega_c)(1/T \int_0^T \dot{v} i\ dt)]$$

$$\dot{v} = dv/dt = d/dt\ [\sqrt{2} \sum_{k=1}^{n} V_k\ \text{Sin}\ (k\omega t + \phi_k)]$$

$$\dot{v} = \sqrt{2}\ \omega \sum_{k=1}^{n} kV_k\ \text{Cos}\ (k\omega t + \phi_k)$$

So, $Q_c = 1/\omega_c[1/T \int_0^T \dot{v} i\ dt]$

$$= \frac{2\omega}{\omega_c}[1/T \int_0^T \sum_{k=1}^{n} kV_k I_k(\text{Cos}(k\omega t + \phi_k)$$

$$\text{Sin}\ (k\omega t + \phi_k + \beta_k))dt]$$

$$= \frac{\omega}{\omega_c}[1/T \int_0^T \sum_{k=1}^{n} kV_k\ I_k(\text{Sin}(2k\omega t + 2\ \phi_k + \beta_k)$$

$$+ \text{Sin}\ (\beta_k))dt]$$

$$Q_c = \frac{\omega}{\omega_c} \sum_{k=1}^{n} (kV_k\ I_k\ \text{Sin}\ \beta_k)$$

$$\because Q_1 = -\frac{\omega_1}{\omega} [(V_1\ I_1\ \text{Sin}\ \beta_1) + (\frac{V_2 I_2\ \text{Sin}\ \beta_2}{2})$$

$$+ \cdots\cdots + (\frac{V_6\ I_6\ \text{Sin}\ \beta_6}{6})]$$

$$\therefore \quad Q_1 = -\frac{469.67}{400}[(176.73) + (33.10) + (7.88) + (2.12)$$

$$+ (0.80) + (0.32)]$$

$$= -259.43 \quad \text{vars}$$

$$\therefore \quad Q_c = \frac{\omega_c}{\omega_c}[(V_1 \, I_1 \sin \beta_1) + 2 \, V_2 \, I_2 \sin \beta_2) + \cdots\cdots$$

$$+ (6 \, V_6 \, I_6 \sin \beta_6)]$$

$$\therefore \quad Q_c = \frac{400}{657.81}[(176.73) + (132.4) + (70.92) + (33.92)$$

$$+(20.0) + (11.52)$$

$$= 250.77 \quad \text{vars}$$

APPARENT POWER

$$S = VI$$

where $V = \sqrt{(V_1^2 + V_2^2 + V_3^2 + \cdots\cdots + V_6^2)}$

$$= 77.73 \text{ volts.}$$

$$I = \sqrt{(I_1^2 + I_2^2 + I_3^2 + \cdots\cdots + I_6^2)}$$

$$= 6.82 \text{ Amp.}$$

$$S = 77.73 \times 6.82 = 530.49 \text{ watts.}$$

ACTIVE POWER

$$P = \frac{1}{T}\int_0^T vi \, dt$$

$$= \sum_{k=1}^{n} V_k \, I_k \cos \beta_k$$

$$= (V_1 I_1 \text{ Cos } \beta_1 + V_2 I_2 \text{ Cos } \beta_2 + \ldots\ldots + V_6 I_6 \text{ Cos } \beta_6)$$

$$= (352.92 + 44.16 + 9.46 + 2.68 + 0.83 + 0.19)$$

$$P = 410.24 \text{ Watts.}$$

RESIDUAL INDUCTIVE REACTIVE POWER :

$$Q_{1r} = \sqrt{(S^2 - P^2 - Q_1^2)}$$

$$Q_{1r} = 214.05 \quad \text{vars}$$

RESIDUAL CAPACITIVE REACTIVE POWER :

$$Q_{cr} = \sqrt{(S^2 - P^2 - Q_c^2)}$$

$$Q_{cr} = 224.69 \quad \text{vars}$$

Alternatively, reactive power canbe calculated by Fourier Analysis Technique as below :

$$S_x^2 = \sum_1^n V_n^2 \sum_1^n I_n^2 \sin^2 \beta_k$$

$$\text{or} \quad (V_{rms}) \left( \sum_1^n I_n^2 \sin^2 \beta_k \right)^{1/2}$$

$$= (V_1^2 + V_2^2 + V_3^2 + \ldots\ldots + V_6^2) \times$$

$$(I_1^2 \sin^2 \beta_1 + I_2^2 \sin^2 \beta_2 + \ldots + I_6^2 \sin^2 \beta_6)$$

$$= 6041.95 \times (13.69)$$

$$\therefore \; S_x = 287.60 \quad \text{vars}$$

EXAMPLE (II) :

Suppose nonsinusoidal supply is source of square wave as shown in Fig. (13). One cycle of the wave can be expressed mathematically between $-\pi/2$ to $3\pi/2$. From Fig. (13)

$$f(\alpha) = V \text{ for } -\pi/2 < \alpha < \pi/2$$

$$f(\alpha) = -V \text{ for } \pi/2 < \alpha < 3\pi/2$$

Fourier series for this wave form can be obtained as below :

(i) Constant term $a_o$ :

It can be easily seen that between limit 0 and $2\pi$ the positive area of the curve is equal to negative area of curve. Hence

$$a_o = \frac{1}{2\pi}\int_o^{2\pi} f(\alpha)d\alpha = 0$$

(ii) Coefficient of Cosine terms are :

$$a_n = \frac{1}{\pi}\int_o^{2\pi} f(\alpha) \text{ Cos } n\alpha \, d\alpha$$

$$= \frac{1}{\pi}[\int_{-\pi/2}^{\pi/2} (V) \text{ Cos } n\alpha \, d\alpha + \int_{\pi/2}^{3\pi/2} (-V) \text{ Cos } n\alpha \, d\alpha]$$

$$= \frac{1}{\pi}[[\frac{V \text{ Sin } n\alpha}{n}]_{-\pi/2}^{\pi/2} + [\frac{-V \text{ Sin } n\alpha}{n}]_{\pi/2}^{3\pi/2}]$$

$$= \frac{V}{n\pi}[\text{Sin } \frac{n\pi}{2} - \text{Sin}(-\frac{n\pi}{2}) - \text{Sin } \frac{3n\pi}{2} + \text{Sin } \frac{n\pi}{2}]$$

where,

$$n = 1, 2, 3, 4 \ldots\ldots$$

Choosing different values of n

$a_n = 0$ for $n = 2, 4, 6 \ldots\ldots$

$a_n = \frac{4V}{n\pi}$ for $n = 1, 5, 9, 13\ldots\ldots$

$an = -\frac{4V}{n\pi}$ for $n = 3, 7, 11, 15\ldots\ldots$

(iii) Coefficient of Sine terms are :

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\alpha)\ \text{Sin}\ n\alpha\ d\alpha$$

$$= \frac{1}{\pi}\left[\int_{-\pi/2}^{\pi/2} V\ \text{Sin}\ n\alpha\ d\alpha + \int_{\pi/2}^{3\pi/2} (-V)\text{Sin}\ n\alpha\ d\alpha\right]$$

$$= \frac{1}{\pi}\left[\left[\frac{-\text{Cos}\ n\alpha}{n}\right]_{-\pi/2}^{\pi/2} + \left[\frac{\text{Cos}\ n\alpha}{n}\right]_{\pi/2}^{3\pi/2}\right] = 0$$

Hence the expression for the square wave with the origin as indicated in Fig. (13) is

$$v = f(\alpha) = \frac{4V}{\pi}[\text{Cos}\ \alpha - \frac{1}{3}\ \text{Cos}3\alpha + \frac{1}{5}\ \text{Cos}\ 5\alpha - \frac{1}{7}\ \text{Cos}\ 7\alpha$$

$$+ \frac{1}{9}\ \text{Cos}\ 9\alpha - \frac{1}{11}\ \text{Cos}\ 11\alpha\ ]$$

Let V = 100 volts, $\alpha = \omega t$, $\omega$ = 400 rad/sec.

$$v = [127.32\ \text{Cos}\ 400\ t - 42.44\ \text{Cos}\ 1200\ t + 25.46\ \text{Cos}\ 2000\ t$$

$$- 18.19\ \text{Cos}\ 2800\ t + 14.15\ \text{Cos}\ 3600\ t + 11.57\ \text{Cos}4400$$

$$\text{Let}\ i = [\ 12.5\ \text{Cos}\ (400\ t + 20^\circ) - 4.2\ \text{Cos}\ (1200\ t + 30^\circ)$$

$$+ 2.4\ \text{Cos}\ (2000\ t + 40^\circ) - 1.1\ \text{Cos}(2800\ t + 50^\circ)$$

$$+ 0.6\ \text{Cos}\ (3600\ t + 60^\circ) + 0.2\ \text{Cos}\ (4400 + 70^\circ)]$$

Reactive Power can be calculated by current subdivision technique as below :

$$\because\quad V = \sqrt{(V_1^2 + V_3^2 + V_5^2 + V_7^2 + V_9^2 + V_{11}^2)}$$

$$\therefore\quad V = \sqrt{[(127.32)^2 + (42.44)^2 + (25.46)^2 + (18.19)^2 + (14.15)^2 + (11.57)^2]}$$

$$= 139.02\ \text{volts.}$$

$$\therefore \quad \bar{V} = \frac{1}{\omega} \sqrt{[V_1{}^2 + (\frac{V_3}{3})^2 + (\frac{V_5}{5})^2 + (\frac{V_7}{7})^2 + (\frac{V_9}{9})^2 + (\frac{V_{11}}{11})^2]}$$

$$\therefore \quad \bar{V} = \frac{1}{400} \sqrt{[(127.32)^2 + (\frac{42.44}{3})^2 + (\frac{25.46}{5})^2 + (\frac{18.19}{7})^2 + (\frac{14.15}{9})^2 + (\frac{11.57}{11})^2]}$$

$$= 0.321 \text{ volts}$$

$$\therefore \quad \dot{V} = \omega \sqrt{[V_1{}^2 + (3V_3)^2 + (5V_5)^2 + (7V_7)^2 + (9V_9)^2 + (11\,V_{11})^2]}$$

$$\therefore \quad \dot{V} = 400\sqrt{[(127.32)^2 + (3 \times 42.44)^2 + (5 \times 25.46)^2 + (7 \times 18.19)^2 + (9 \times 14.15)^2 + (11.57 \times 11)^2]}$$

$$= 124747.61 \text{ volts.}$$

Two composite frequencies are

$$\omega_1 = \frac{V}{\bar{V}} = \frac{139.02}{0.321}, \quad \omega_c = \frac{\dot{V}}{V} = \frac{124747.61}{139.02}$$

$$\omega_1 = 433.08 \qquad \omega_c = 897.34$$

It is apparent that $\omega \leq \omega_1 \leq \omega_c$

INDUCTIVE REACTIVE POWER :

$$Q_1 = VI_{q1} = \omega_1 \left( \frac{1}{T} \int_0^T \bar{v} i \, dt \right)$$

Let $v = \sqrt{2} \sum_{k=1}^{n} V_k \cos (k\omega t + \phi_k)$

$$i = \sqrt{2} \sum_{k=1}^{n} I_k \cos (k\omega t + \phi_k + \beta_k)$$

$$\bar{v} = \int v dt = \sqrt{2} \int \sum_{k=1}^{n} V_k \cos (k\omega t + \phi_k) dt$$

$$= \sqrt{2} \sum_{k=1}^{m} \frac{V_k}{k\omega} \sin (k\omega t + \phi_k)$$

$$\therefore \quad Q_1 = \frac{2\omega_1}{\omega}\left[\frac{1}{T}\int_0^T \sum_{k=1}^{n} \frac{V_k I_k}{k} (\mathrm{Sin}\,(k\omega t + \phi_k)\right.$$

$$\left.\mathrm{Cos}\,(k\omega t + \phi_k + \beta_k))dt\right]$$

$$= \frac{\omega_1}{\omega}\left[\frac{1}{T}\int_0^T \sum_{k=1}^{n} \frac{V_k I_k}{k} (\mathrm{Sin}\,(2k\omega t + 2\phi_k + \beta_k)\right.$$

$$\left. - \mathrm{Sin}\,(\beta_k)\ )dt\right]$$

$$= -\frac{\omega_1}{\omega} \sum_{k=1}^{n} \left(\left(\frac{V_k I_k}{k}\right) \mathrm{Sin}\,\beta_k\right)$$

CAPACITIVE REACTIVE POWER :

$$Q_c = VI_{qc} = \frac{1}{\omega_c}\left[\frac{1}{T}\int_0^T \dot{v} i\, dt\right]$$

$$\dot{v} = \frac{dv}{dt} = \frac{d}{dt}\left[\sqrt{2} \sum_{k=1}^{n} V_k\, \mathrm{Cos}\,(k\omega t + \phi_k)\right]$$

$$= -\sqrt{2}\ \omega \sum_{k=1}^{m} kV_k\, \mathrm{Sin}\,(k\omega t + \phi_k)$$

$$= -\frac{2\omega}{\omega_c}\left[\frac{1}{T}\int_0^T \sum_{k=1}^{n} kV_k I_k (\mathrm{Sin}\,(k\omega t + \phi_k) \mathrm{Cos}(k\omega t + \phi_k + \beta_k))dt\right]$$

$$= -\frac{\omega}{\omega_c}\left[\frac{1}{T}\int_0^T \sum_{k=1}^{n} kV_k I_k (\mathrm{Sin}\,(2k\omega t + 2\phi_k + \beta_k) - \mathrm{Sin}\,\beta_k)dt\right]$$

$$= \frac{\omega}{\omega_c} \sum_{k=1}^{n} (k\, V_k\, I_k\, \mathrm{Sin}\,\beta_k)$$

$$\therefore \quad Q_1 = -\frac{\omega_1}{\omega}\left[(V_1\, I_1\, \mathrm{Sin}\,\beta_1) + \left(\frac{V_3\, I_3\, \mathrm{Sin}\,\beta_3}{3}\right)\right.$$

$$\left. + \cdots\cdots + \left(\frac{V_{11}\, I_{11}\, \mathrm{Sin}\,\beta_{11}}{11}\right)\right]$$

$$Q_I = -\frac{433.08}{400}\left[(544.33) + 29.71 + 2.86 + 2.19 + 0.94 + 0.2\right]$$

$$= -633.63 \text{ vars}$$

$$\therefore \quad Q_c = \frac{\omega}{\omega_c}\left[(V_1\ I_1\ \sin\beta_1) + (3\ V_3\ I_3\ \sin\beta_3)\right.$$

$$\left. + (5\ V_3\ I_3\ \sin\beta_5) + \ldots\ldots + (11\ V_{11}\ I_{11}\ \sin\beta_{11})\right]$$

$$= \frac{400}{897.34}\left[546.33 + 267.39 + 196.5 + 107.31 + 76.14 + 24.2\right]$$

$$= 541.99 \text{ vars}$$

APPARENT POWER : $S = VI$

Where, $V = \sqrt{(V_1^2 + V_3^2 + \ldots\ldots + V_{11}^2)} = 139.02$ volts

$I = \sqrt{(I_1^2 + I_3^2 + \ldots\ldots + I_{11}^2)} = 13.46$ Amps.

$\therefore$ $S = 139.02 \times 13.46 = 1871.21$ Watts.

ACTIVE POWER :

$$P = \frac{1}{T}\int_0^T vi\ dt = \sum_{K=1}^{n} V_k\ I_k\ \cos\beta_k$$

$$= (V_1\ I_1\ \cos\beta_1 + V_3\ I_3\ \cos\beta_3 + \ldots\ldots + V_{11}\ I_{11}\cos\beta_{11})$$

$$= (1495.52 + 154.37 + 46.81 + 12.86 + 4.25 + 0.79)$$

$$= 1714.6 \text{ Watts.}$$

RESIDUAL INDUCTIVE REACTIVE POWER :

$$Q_{lr} = \sqrt{(S^2 - P^2 - Q_l^2)}$$
$$= 400.1 \text{ vars}$$

RESIDUAL CAPACITIVE POWER :

$$Q_{cr} = \sqrt{(S^2 - P^2 - Q_c^2)}$$
$$= 517.5 \text{ vars}$$

Alternatively Reactive Power can be calculated by ~~Fusion~~ Fourier Analysis Technique as below :

$$S_X^2 = \sum_1^n V_n^2 \sum_1^n I_n^2 \sin^2 \beta_k$$

or $$S_X = (V_{rms}) \left( \sum_1^n I_n^2 \sin^2 \beta_k \right)^{1/2}$$

$$= (V_{rms}) [ I_1^2 \sin^2 \beta_1 + I_3^2 \sin^2 \beta_3 + \ldots$$

$$+ I_{11}^2 \sin^2 \beta_{11} ]^{1/2}$$

$$= (139.02)[18.28 + 4.41 + 2.38 + .71 + 0.27 + 0.03 ]^{1/2}$$

$$= 709.9969 = 710 \text{ vars.}$$

Reactive power is calculated from both techniques (current subdivision technique) and Fourier Analysis Technique). In current subdivision technique maximum realizable compensation of reactive power is possible than Fourier analysis technique, to improve the power factor. In current subdivision technique complete compensation of inductive or capacitive reactive components ($Q_l$ or $Q_c$) is possible by means of capacitor or inductor of optimum value connected across supply terminals of nonsinusoidal load. Here residual components ($Q_{lr}$ or $Q_{cr}$) are not compensated at all. Compensation in inductive or capacitive components minimize the supply current and thus maximise the power factor. In Fourier analysis technique reactive power ($S_X$) can be minimized by connecting the capacitor of optimum value across supply terminal to a value $S_{Xmin}$, but will not be fully compensated. Therefore, we can say that reactive power definition by current subdivision technique is more adequate than definition by Fourier analysis technique in relation to power factor correction.

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## 6. CONCLUSION

In readtive power analysis two techniques (i.e. Fourier Analysis Technique and Current Subdivision Technique) are adopted. To measure and control reactive power requires that it should be sui ably defined. Above two techniques define the reactive power in different manner. In Fourier analysis technique there are some formidable equations which are not only ~~oper~~ difficult to instrument but which are also of little use to the utility operator. An alternative technique i.e. Current Subdivision Technique which is readily susceptable to being instrumented and which will provide the operator with direct indicator of whether reactive power can be reduced by what means and by how much. In view of the power factor correction, reactive power definition by current subdivision technique is more advantageous because compensation of reactive power by using either shunt reactors or capacitors is maximum obtainable in this technique than other technique.

In each technique, structure of reactive power meters for nonsinusoidal supply system and measurement of reactive power by these meters are explained clearly. From comparison of these two techniques, Current Subdivision Technique has relatively simple instrumentation rather than other technique. In this technique various components can be segregated and measured and the results can be readily applied by power engineers to realize the maximum obtainable compensation for reactive power. In such a meter, terminals for displaying the corresponding values of various component on an oscilloscope. Reactive power meter based on Fourier analysis technique is effected by accuracy of quadrature oscillators and multipliers. Also meter accuracy depends upon the filter properties.

Concluding above discussion current subdivision technique is better approach than Fourier analysis technique in all respect.

## REFERENCES

[1] D. SHARON - 'Reactive Power definitions and power factor improvement in nonlinear systems', Proc. Inst. Elec. Emg., Vol. 120, No. 6, pp 704-706, June 1973.

[2] A. E. EMANUAL - 'Energy factors in power systems with nonlinear loads' Arch. Elbtrotech., Vol. 59, pp 183-189, 1977.

[3] W. SHEPHERD AND ZAKIKHANI - 'Suggested definition of reactive power for nonsinusoidal systems', Proc. Inst. Elec. Eng., Vol. 119, No. 9, pp. 1361-1362, Sept. 1972.

[4] N. L. KUSTERS AND W. J. M. MOORE - 'On the definition of reactive power under nonsinusoidal conditions', IEEE Trans. Power App. Syst., Vol. PAS-99, No. 5, pp. 1845-1854, Sept./Oct. 1980.

[5] JOHN, R. LINDERS - 'Electrical Wave distortion : Their hidden cost and containment,' IEEE Trans. Ind. Appl. Vol. IA-15, No. 5, pp 458-471. Sept./Oct. 1979.

[6] DAVID D. SHIPP - 'Harmonic Analysis and Suppression for electrical systems supplying static power converters and other nonlinear loads,' IEEE Trans. Ind. Appl. Vol. IA-15, No. 5, pp. 453-458, Sept./Oct. 1979.

[7] R. A. LOPEZ, J.C.M. ASQUERINO AND G. RODRIGEZ-IZGUIERDO - 'Reactive Power meter for nonsinusoidal system' IEEE Trans. Instrum. and Meas., Vol. IM-26, No.3, pp 258-260, Sept. 1977.

[8] L. S. CZARNECKI - 'Measurement principle of a reactive power meter for nonsinusoidal system', IEEE Trans. Instrum. Meas. Vol. IM-30, No. 3, pp. 209-212, Sept. 1981.

[9] L. S. CZARNECKI - 'Measurement of the individual harmonics reactive power in nonsinusoidal systems', IEEE Trans. on Instrum. and Meas., Vol. IM-32, No. 2, pp. 383-384, June 1983.

[10] E. WHITTEKER AND G. WATSON - 'A Course of Modern Analysis.' New York, Macmillan, 1947.

[11] E.W. KIMBARK - 'Direct Current Transmission'. Vol. 1, Chapter 8, pp. 295-301. New York, Wiley 1971.