# GENERATION ALLOCATION IN LARGE POWER SYSTEMS

A DISSERTATION

*submitted in partial fulfilment of the requirements for the award of the degree*

*of*

MASTER OF ENGINEERING

*in*

ELECTRICAL ENGINEERING
(System Engineering and Operations Research)

By

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## CERTIFICATE

Certified that the disseration entitled 'Generation Allocation in Large Power Systems' which is being submitted by Shri Mahesh Chandra Verma, in partial fulfilment for the award of degree of MASTER OF ENGINEERING (ELECTRICAL) IN SYSTEM ENGINEERING AND OPERATION RESEARCH, Of University of Roorkee, is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of eleven months from Jan. 84 to July 84 and Sept. 84 to Dec. 84, for preparing this dissertation of Master of Engineering Degree at this University.

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# NOTATIONS

The list of principal symbols used in the thesis is given below:

N = total no. of buses,

NL = total no.of load buses,

NG = total no.of generator buses,

NB = total no.of branches or lines,

T = total no.of transformers,

$V_i$ = voltage magnitude at bus i,

$\delta_i$ = voltage phase angle at bus i,

$P_{Gi}$ = active power generation at bus i,

$P_{Gi}$ = reactive power generation at bus i,

$P_{Li}$ = active power load at bus i,

$Q_{Li}$ = reactive power load at bus i,

$P_i$ = net active power injection at bus i,

$Q_i$ = net reactive power injection at bus i,

$r_L$ = resistance of the line connecting buses i and j,

$x_L$ = reactance of the line connecting buses i and j,

Y = bus admittance matrix

$G_{ij}+j\ B_{ij}$ = i j$^{th}$ element of the bus admittance matrix,

$A_{1i}, A_{2i}, A_{3i}$ = cost function constants associated with $i^{th}$ generator,

$V_i^{min}$ = minimum allowable voltage magnitude at bus i,

$V_i^{max}$ = maximum allowable voltage magnitude at bus i,

$P_{Gi}^{min}$ = minimum allowable active power generation of $i^{th}$ generator,

$P_{Gi}^{max}$ = maximum allowable active power generation of $i^{th}$ generator,

$Q_{Gi}^{min}$ = minimum allowable reactive power generation of $i^{th}$ generater,

$Q_{Gi}^{max}$ = maximum allowable reactive power generation of $i^{th}$ generater,

$t_i$ = tap ratio of transformer i,

$t_i^{min}$ = minimum allowable tap ratio of transformer i,

$t_i^{max}$ = maximum allowable tap ratio of transformer i,

U = vector of independent or control variables,

X = vector of dependent or state variables,

f = objective function or the optimization criterion.

# ABSTRACT

Generation allocation problem is very important because of huge system operating costs with the increase in size and complexity, the simple criterion of 'equal incremental cost' operation of generators does not give optimal operating conditions with desired accuracy and hence more sophisticated methods are now used to find the optimal flow of active and reactive power in the systems.

In the present study, the active and reactive powers are allocated amongst the system sources such that the defined criterion is optimally satisfied. The criteria or the objectives for optimization could be total system operating cost, system losses to describe a desired behaviour of the system. The objective are optimized such that the system power flow equations and limit constrained imposed upon the variables by the system operating condition and design considerations are satisfied. Because of large number of variables and constrained involved, and both the objective function and constrained being non-linear, the problem is quite a challenging one from computational consideration.

To reduce the complexity and size of the problem we have made use of variable decomposition and problem decomposition approach. The complete problem of optimal generation happens to be a constrained non-linear programming problem. A new approach is suggested in this work to transform the constrained problem into unconstrained one.

The complete generation allocation problem has been decomposed into two sub-problem of active and reactive power optimization. The decomposition utilizes the physical properties of the electric power networks and helps in solving the two important problems independently. The active power optimization is done by minimizing system operating costs, with bus voltage magnitudes held constant at optimal system bus voltage level evaluated by minimizing the real power losses and reactive generation at each bus with bus voltage phase angles as constraint, subject to above defined equality and inequality constraint. The minimization is carried out by Hen-Powell method which is simple and fast convergent.

The general purpose computer programs have been developed for the problems undertaken in the present study and are tested and run on DEC 2050. The storage requirement is appreciably reduced by made use of sparse technique. Two systems i.e. 6-bus and 26-buses have been taken for carrying out the various studies relating to the problem discussed above and the results are reported in IV Chapters.

# C O N T E N T S



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# INTRODUCTION

There are basically two requirements for the satisfactory operation of an electrical power system. One of them is that the electric power generation should be sufficient to meet the prevailing demand on the system continuously. The energy supplied to the consumer should be within the given limits of voltage and frequency and should have continuity of service and dependable operation. At the same time the overall cost of the generation should be minimum.

In this thesis a method is presented for finding the operating policy of the power system such that the cost of generation is minimum and simultaneously system losses are minimum.and the operating variables are within limit.

In the first chapter, literature on this problem is reviewed.

In the second chapter, generation allocation problem is discussed as a nonlinear programming problem. The basic assumption for the solution of these problem is that the system is operating under steady state conditions.

In the third chapter, problem formulation and the optimization technique used is presented. The Hen-Powell, and reduced gradient method are used to solve nonlinear constrained problem.

And in the last chapter we have considered a 6 - bus and 26 - bus sample systems and are solved the test problem by above mention method.

# CHAPTER-I

## LITERATURE REVIEW

Over the years, numerous methods have been proposed for optimizing the power flow problem.

The first major step in the development of a method of co-ordinating incremental fuel costs and incremental transmission losses was presented in 1949 by E.E. George, H.W. Page and J.B.Ward [56] in their use of the network analyzer to prepare predicted plant loading schedules for a large power system. At the same time the electrical engineering staff of the American Gas and Electric Service Corporation, also with the aid of the network analyzer, developed a method of modifying the incremental fuel costs of the various plants on an incremental slide Rule in order to account for transmission losses. Next, the American Gas and Electric Service corporation, in cooperation with the General Electric Company, successfully employed transmission loss formulas and punched-card machines for the preparation of penalty factor charts to be used in the economic scheduling of generation [20]. The incremental production cost of a given plant multiplied by the penalty factor for that plant gives the incremental cost of power delivered to the system load from that plant.

In 1952 a paper entitled 'Evaluation of Methods of Coordinating Incremental Fuel costs and Incremental Transmission Losses' [21] presented.

1. A mathematical analysis of various methods of coordinating incremental fuel costs and incremental transmission losses.
2. An evaluation of the error introduced in optimum system operation by assumptions involved in determining a loss formula.
3. An evaluation of the saving to be obtained by coordinating incremental fuel costs and incremental transmission losses.

Progress in the analysis of the economic operation of a combined thermal and Hydroelectric power system was reported by the Hydro-electric Power Commission of ontario and the General Electric Company in the paper, 'Short-Range Economic Operation of a Combined thermal-Hydroelectric power system [37].

An iterative method of calculating generation schedules suitable for the use of a high speed automatic digital computer has been described in the paper entitled ''Automatic digital computer Applied to Generation scheduling', by A.F.Glimn, R. Habermann, Jr.,L.K. Kirch-mayor and R.W. Thomas [4]. For a given load the computer calculates and tabulates incremental cost of received power, to total transmissi losses, total fuel input, penalty factors and received load alongwit the allocation and summation of generation.

Till now many new approach have been taken account for the optimization of the power flow problem.

In year 1970, M. Ramamoorthy and J.Gopala Rao [23] proposed a method for optimum load flow using the penalty function approach. They have taken some function for minimization as proposed by Demnul Tinny in 1968. Equating constraints are eliminated from the minimizi function and the load demand is satisfied by suitably defining a set of variables. The constraints considered are inequality constraints on maximum and minimum powers and voltages at buses. The main features of the method are elimination of equality constraints fould computationally good from the point of convergence of solution and ease in programming. Shen and Laeyhton [51] have formulated system operating conditions as a dual linear programming problem. The sub-optimal model allows fast solution to be obtained dependably by applying the revised simple linear programming method. Problem include same constraints as in [25]. The features of this method are fast speed of convergence and low computer storage.

J.T. Day [16] has formulated the optimal load flow problem with linear programming using linear unit cost/output. He has considered start up costs, running and maintenance cost with the constraints available units, number of units running at any given time satisfying spining requirements, hydro constraints reflecting energy and capacity limitation and interior capacity consideration. Both real and reactive power schedules are obtained by a method which minimizes the fuel cos while satisfying the network power equation. This method was proposed by R.B. Gungar, N.E. Tsang and Webb [26]. Minimum fuel cost is obtain by requiring the total differential of the cost to be zero with the saving bus power treated as a dependent variable. System losses are expressed in terms of net bus powers. The main features of this metho are that solution obtained in very few iteration and also requires les memory.

Sasson, Trenino and Aboytes [2] have proposed a method which is a simple alternation of Newton's Method. The main features are faster convergence than with normal newton's method and a control of convergence gurantees a non divergent solution.

Bree, Dommel, Peschon and Pawell have described a method which minimizes the operating cost of an interconnected power system constrained by prescribed area interchange control [17]. Roy Billinton and Sachdeva [30] have proposed a suboptimum method which comprises of three steps (i) Load flow solution (ii) Optimal voltage evaluation and (iii) Real power optimization with optimally determined system voltage: The large problem has been decomposed into two small problems. The real and reactive power optimization problem has been solved by the proposed technique applying gradient method and the penalty factor approach. A.M. Sasson, Fo Viloria and F.A. Boytes [2] have formulated the optimal power flow problem and solved by using Hessian Matrix. Rei and Husdrof [10] have formulated as a quadratic programming problem and solved using Wolf's algorithm. This method is capable of handlin both equality and inequality constraints, on P,Q and V and can solve the load flow as well as the economic load flow problem. The use of penalty factor is not necessary. Sterling and Nicholson [11] have developed linear programming with a quadratic function for generation and transmission losses. Beal's algorithm used for real power dispatch and gradient technique is used to allocate reactive power subject to nodal voltages and reactive power limits. The main feature is that it can be used for on-line scheduling using analogue/hybrid computer.

R. Padmore [29] has presented a practical and efficient method for the economic dispatch of generation with explicit-recognition of line security limits. The optimization problem is solved by gradient projection method.

A method proposed by O. Alsae and B.Stott [24] which incorporated exact outage contingency into the Dommel-Tinney method. The controllab system quantities in the base cable problem (i.e. generated nw, contro voltage magnitude, transformer taps, are optimized according to some objective function so that no limit on the other quantity (i.e. generator MVAR and current loading, transformer circuit loading, load bus voltage magnitude, angular displacement) occur in either the base or contingency case system operating condition.

H.H. Happ [15] has used Jaceobian Matrix for the solution of optimal generation allocation problem. The major advantages is that its inherent simplicity and rapid convergence behaviour which are characteristic particularly important for ON line implementation.

Waight G. James, Farrokh Albyyeh and Anian Bose [54] have proposed a method that combines dynamic and linear programming technique such that the real operational constraints of reserve margins and camp rates are optimally met by the resulting generation schedules. Dillon, Tharam S. [8] have developed a method for sensitivity analysis that allows the recomputation of a new schedule for not the large variation in system conditions for less frequent resolution of the total optimal power flow problem as well as quick calculation of new schedule if system is in an alert state Kathleen M. Sidenblad [52] have proposed a new probablistic production cost methodology by using the concept of an expected incremental generation cost curve. Thus method is useful for production costing in traditional system and those involving storage. In this method he has used linear programmin approach.

R.R. Shoults and D.T. Shun [27] have developed a method by decomposing the optimal power flow formulation into a P-problem (P-$\delta$, real power model) and a Q-problem (Q-V, reactive power model) by using the decoupling principle. This method simplifies the formulatic improves computation time and permits a certain flexibility in the type of calculation desired. The nonlinear Gradient Method is used in a decomposed manner. Burchett, Robert C. [5] have proposed an advanced power flow methodology for optimally dispatching all active and reactive power in power system. They have given technique to improve the solution algorithm, the handling of penalty function and the power solution optimization methodology. They have provided a new algorithm for the determination of an optimal step length and for scaling of control variables gradient. Isoda [14] have given a me hod for the conventional load dispatching determination among thermal units known as a noval load dispatching method which takes into accou the response capabilities of thermal units and short term future load demand. Bur Rurg T.[6] has developed a method in order to reschedule the real power generation to satisfy a set of load change, a linear programming is used and solved by simplex method. The method is iterative with the property that it does not require the use of penalty factors or determination of step size. Which can cause conver gence difficulties. This method has potential for on line applicatio at practical time intervals.

L. Roy and N.D. Rao [19] have developed a new method by using cartesian coordinate for the formulation of economic dispatch problem and reclassification of state and control variables associated with generator buses. This formulation results in exact equality constraint model in which the coefficient matrix is real, sparse, diagona dominent, smaller is size and need be computed and factorised once on in each gradient step. R.A. Smith and R.D. Shults [28] have proposed a generation planning tool that evaluates operation performance of proposed generation expansion plans. E. Houser and G. Irisarri [9] have developed a method that takes advantages of the mild non-linearties of the problem in attempting to change the weight of the constraint quantities in such away so as to both alleviates the current overloads and avoid the criterian of new ones due to the change of the controls. Jolissunt, Arvantidis and Linenberge [85] have developed a decomposition method in which real and reactive power equation are decomposed. Two subproblem are formed one corresponding to real power equation and the other to the reactive power equation. The two problems are alternatively solved untill the desired accuracy is achieved.

# CHAPTER II

## GENERATION ALLOCATION PROBLEM

There are basically two objectives of power system. The primary objective is to provide adequate uninterrupted supply of power of certain quality to meet all the demand of the customers. This implies that the generation must be adjusted, in real time to match prevailing demand. And the other objective, to be achieved as long as it is consistent with continuity of service and dependable operation, is to generate the required total output at minimum overall cost. The achievements of these two objective involve many complicate studies that are interrelated.

### 2.1. SYSTEM DESCRIPTION

Power stations are connected to different load centres and/ or to other generating stations by means of high voltages transmission network to form what is known as power system. The operation of different power stations is automatically related to each other so that the objective of continuity of service and high economy are achieved for a far-flung networ as a whole. This interconnection provides an opportunity to co-ordinate the operation of many generating stations of a network so that prevailing customer demand is fulfilled, power interchanges with neighbouring networks or systems are established and maintained by using tie lines, and the output of available alternative sources are maintained at such levels as will provide optimum overall economy.

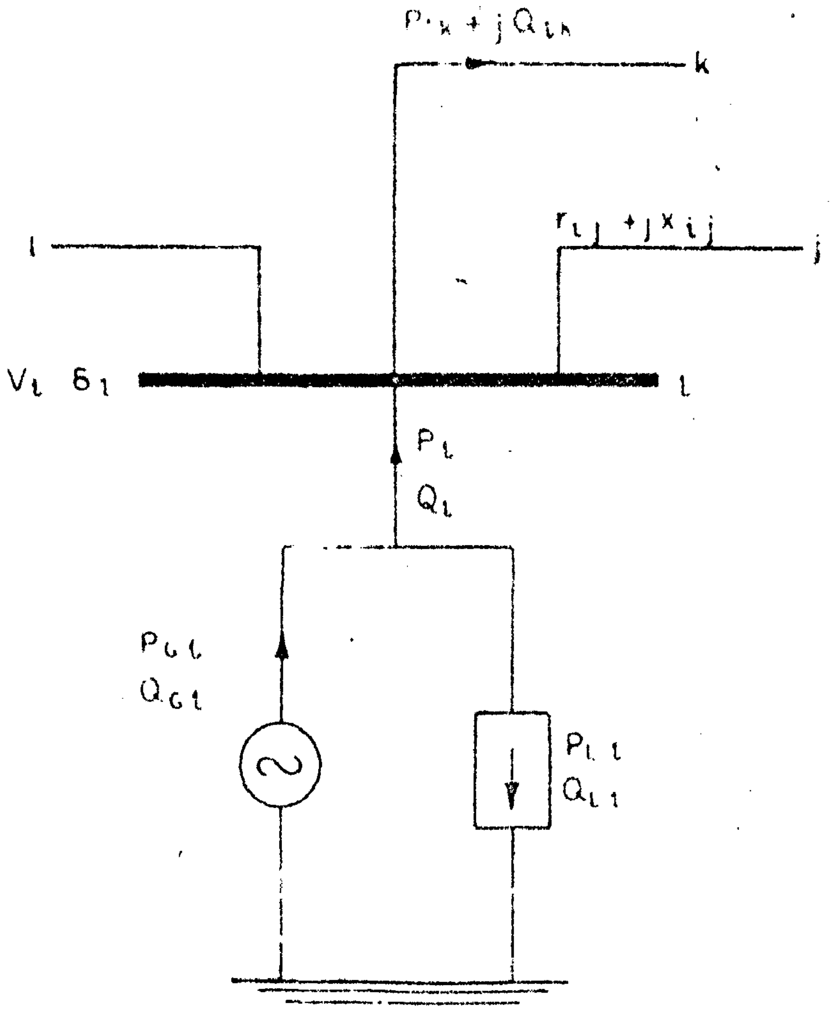

FIGURE 2.1 NODE DESCRIPTION

The mathematical description of the system is given by linear circuit theory. The power flow pattern in the system mainly depends on the load and generation distribution and the network configuration. Each system node or bus can be described completely as in Fig.2.1. where the lines j,k and $\ell$ are the electrical interconnections of node i with nodes j,k and $\ell$ .

In most power-system studies, one of the basic problem is to find the voltage magnitude and phase angles at system nodes to meet a given schedule of generation and loads. In the past this problem has been solved with digital computers using iterative method. The iterative method converge slowly and are subject to ill-conditioned situations. Now-a-days there are elimination method for power flow solution.

A basic requirement of the system operation is that the active and reactive power balance must be satisfied at each node of the system. This can be met by proper choice of node voltage magnitudes and phase angles such that the steady state power flow equations are satisfied.

$$P_{Gi}-P_{Li}=P_i(V,\delta)=V_i \sum_{j=1}^{N} V_j[G_{ij} \cos(\delta_i-\delta_j)+B_{ij} \sin(\delta_i-\delta_j)] \quad \ldots (1)$$

$$Q_{Gi}-Q_{Li}=Q_i(V,\delta)=V_i \sum_{j=1}^{N} V_j[G_{ij} \sin(\delta_i-\delta_j)-B_{ij} \cos(\delta_i-\delta_j)]$$

$$i=1,\ldots N \quad \ldots (2)$$

$P_i$ and $Q_i$ are non-linear function of V and $\delta$ .

The conservation law of energy must be satisfied, requiring the satisfaction of the following equations

$$\sum_{i=1}^{NG} P_{Gi} - \sum_{i=1}^{N} P_{Li} - P_{Loss} = 0 \qquad \ldots (3)$$

$$\sum_{i=1}^{NG} Q_{Gi} - \sum_{i=1}^{N} Q_{Li} - Q_{Loss} = 0 \qquad \ldots (4)$$

Equation (1) to (4) defines what is known as power generation allocation problem.

Each node of the system is characterised by four variables, P,Q,V and $\delta$ of which two are specified and the other two must be found. Depending upon which variables are specified, the node can be devided into three types:

(a) Slack node- with V, $\delta$ specified and P,Q unknown. Also for convenience the phase angle of slack node is taken as reference with $\delta = 0$.

(b) Load node or P,Q node with P,Q specified and V, $\delta$ unknown.

(c) Generator node or P,V node- with P,V specified and Q,$\delta$ unknown.

For the solution of generation allocation problem the choice of the values of specified variables of system nodes is made somewhat arbitrarily from the experience of the system operators. The values for the real power generation were generally determined from economic loading of the units. The simple method of loading determination is by the technique of equal incremental cost which is based on the assumption that system transmission losses are negligible.

## 2.2. OPTIMAL GENERATION ALLOCATION PROBLEM

The óptimal generation allocation problem is extremely important both from economic and system operating considerations. The optimization criterian chosen for the determination of optimal generation scheduling is decided by the state in which system is operating and should be optimized such that the power flow equation (1) and (2) are satisfied. The must also satisfy the following inequality constraints arising from system operating limits and design consideration.

1. The voltage magnitude at each node must lie between specified upper and lower limits,

$$V_i^{min} \leq V_i \leq V_i^{max} \quad , i = 1, \ldots N \qquad \ldots (5)$$

2. The power output at any generator should not exceed its rating nor should it below that necessary for stable boiler operation,

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad i = 1, \cdots NG \qquad \ldots (6)$$

3. The reactive power output of any reactive power source must lie between its specified upper and lower limits,

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \quad i = 1, \cdots NG \qquad \ldots (7)$$

4. Transformer tap position are variable but no tap position shall be outside the allowable range,

$$t_i^{min} \leq t_i \leq t_i^{max} \quad i = 1, \cdots T \qquad \ldots (8)$$

5. The power flowing in each line is such that it does not violate the maximum permissible phase angle difference between the connecting nodes,

$$\theta_{ij} \geq |\delta_i - \delta_j| \quad , \quad i = 1, N-1 \quad j = i+1, N \qquad \ldots (9)$$

The mathematical objective function and its constraints in concise term in terms of X, U (where X is state variable and U is control variable) can be written as

$$\text{Min} \quad f(X,U) \qquad \ldots (10)$$

subject to

$$g(X,U) = 0$$

$$h(X,U) \geq 0$$

$$X^{min} \leq X \leq X^{max}$$

$$U^{min} \leq U \leq U^{max} .$$

2.3. TYPES OF GENERATION ALLOCATION PROBLEM

There are three type of generation allocation problem according to the operating state of the system. These three operating states are discussed along with the usual optimization criterian adopted during these system operating states.

2.3a. NORMAL OPERATING STATE

The power system is said to be in normal operating state, when all the equality and inequality constraints of the optimization problem defined in equation (10) are satisfied. Under normal operating state of the system all the demands are met such that all the components are loaded within acceptable limits. The objective under this mode of operation is usually cost of operation for a given period of time. The length of this period will depend on the type of sources available in the power system. In this study static optimization of the generation allocation problem is done based on the assumption that active and reactive load at each node is known with its constant mean value during the scheduling period. The general problem of optimal generation scheduling is formulated in Chapter III.

2.5b. EMERGENCY OPERATING STATE

A power system is said to be an emergency mode of operation when one or more of the security-related inequality constraints are being violated. In this state customer demands are not fully met or apparatus is overloaded. The security level is practically non-existent. If emergency control action are not taken in time or are ineffective, the system then starts to disintegrate. In this state equality and inequality constraints have been isolated and major portion of the system load would be lost. This mode of operation occurs when a severe disturbance such as a large load changes, a loss of generator, a loss of circuit between heavy importing/exporting section experienced in the system.

There are three type of emergency state. The first type, which is known as steady state emergency, occurs when after a disturbance, the power system remains in steady state and continues in operation but with the operating constraints not fully met. The second type of emergency is known as transient oscillations and is associated with varying $\delta$ but almost constant f. The third type of emergency occurs when, because of disturbence, the power system becomes unstable, during this time both the operating and the load constraints are not being met. This type of emergency condition is known as dynamic instability.

The objective function of the scheduling problem during this mode of operation is to minimize the 'inconvenience' experienced by the customers, rather than to minimize the cost of generation. The convenience can be modelled by a function, which may consist of terms depending on the load actually curtailed, deviation from scheduled voltage, deviation from scheduled frequency, equipment overload resulting in reduced life etc.

The inconvenience function f mathematically can be expressed as

$$f = \sum_{i=1}^{N} \phi_i [P_L^o - P_{Li}, V_i^o - V_i, P_{Gi}^o - P_{Gi}, \ldots] \qquad \ldots (11)$$

where $\phi_i$ - is the penalty function at node i

$P_{Li}^o$ , $V_i^o$ , $P_{Gi}^o$ - are the demanded load, nominal voltage and scheduled generation at node i.

$P_{Li}$, $V_i$, $P_{Gi}$ - are the actual values of these quantities during emergency mode at node i.

In literature the following objectives have been proposed for optimization during emergency mode of operation [51-53].

$$f_1 = \sum_{i=1}^{N} \frac{(P_{Li}^o - P_{Li})^2}{\epsilon_i \quad P_{Li}^o} \qquad \ldots (12)$$

$$f_2 = \sum_{i=1}^{N} \frac{P_{Li}^o - P_{Li}}{\epsilon_i \quad P_{Li}^o} \qquad \ldots (13)$$

$$f_3 = -\sum_{i=1}^{N} \epsilon_i \, P_{Li} \qquad \ldots (14)$$

where $\epsilon_i$ are chosen according to the priority for a given load at a particular node.

## 2.3c. RESTORATIVE OPERATING STATE

In the restorative state, the some of the equality constraints are violated and all the inequality constrainst are satisfied. The system attain a new normal operating state with the remaining generation and transmission resources. For this state the system could transit either to the alert or normal state depending on circumstances The objective of the restorative mode of operation is to bring the system from its deteriorate state to a desired post-fault steady state as quickly as possible, so that customer 'Inconveneince', is minimized. A general form of the inconvenience function is

$$f = \int_{0}^{T} \sum_{i=1}^{N} \phi_i \, [\breve{P}_{L_i}(t) - P_{L_i}(t),\ \breve{V}_i(t) - V_i(t),\ \breve{P}_{G_i}(t) - P_{G_i}(t), \ldots ] \quad \ldots (15$$

where

$\breve{P}_{Li}(t), P_{Li}(t)$ - are load demanded and actually supplied at node i at time t.

$\breve{V}_i(t)$, $V_i(t)$ - are nominal and actual voltage at node i at time t.

$\breve{P}_{Gi}(t), P_{Gi}(t)$ - are desired and actual generation at node i, at time t

$\phi_i$ - is penalty function at node i

T - is time marking the end of restoration.

## 2.4. SYSTEM SECURITY

In practice, we try to maintain a system in the normal operating state. But it is not sufficient because under certain conditions, the occurance of some disturbances may cause the system to go into an emergency such as the overloading of lines and violation of voltage limits. So in this framework, the security of the system will next be defined.

A set of most probable disturbances (contingencies) is first specified. This set of disturbances may consist of the following:

(a) a single line out,

(b) a loss of a generator,

(c) sudden loss of a load,

(d) sudden change of flow in an inter-tie,

(e) a three phase-fault in the system.

Suppose that a power system in the normal operating state is subjected to the set of disturbances in the specified set, the system remain in the operating state, then system is said to be secure, otherwise it is insecure.

# CHAPTER 3

## PROBLEM FORMUNATION AND SOLUTION TECHNIQUE

The general mathematical problem as described in Chapter 2, is given below-

Minimize the generation cost function

$$f(PG) \tag{1}$$

subject to

$$g_p(V,\delta,t,\phi) = 0 \tag{2}$$

$$g_q(V,\delta,t,\phi) = 0 \tag{3}$$

$$h(P,Q,V,\delta,t,\phi) \leq 0 \tag{4}$$

As in a power system the ratio of reactance to resistance of the lines is generally of high value. So the real power at a bus does not change appreciably for a small change in the magnitude of bus voltages. Similarly the reactive power does not change appreciably for small change in the phase angle of the bus voltages. These characteristics of power system enable the problem to be broken into two sub-problems, resulting in saving in memory requirements and increasing the speed of solution.

### 3.1. FORMULATION OF SUB-PROBLEM I

The problem is to minimize the operating cost function, which is the function of real power and bus voltage angle keeping bus voltage magnitude constant, while satisfying both the power flow equality constraints and the upper and lower bound inequality constraints on the variables. The objective is to adjust the system variables i.e. active generation and phase angles of bus voltages such that the over all cost of generation is minimum. The objective function can be written as-

$$f_1 = \underset{P_{Gi}}{\text{Min}} \sum_{i=1}^{NG} (a_i + b_i\, P_{Gi} + C_i\, P_{Gi}^2) \qquad (5)$$

where

$P_{Gi}$ - is the active power generation of ith generator.

$a_i, b_i$ and $c_i$ - are the cost coefficient of ith generator.

NG - is the total number of generators.

PROBLEM CONSTRAINTS

There are two type of optimum power flow constraints, the equality constraints and inequality constraints.

EQUALITY CONSTRAINTS

There are two equality constraints, that the sum of all injection at a bus must be set equal to zero as given below:

$$P_{Gi} - P_{Li} - P_i\,(V,\delta) = 0 \qquad (6)$$

$$Q_{Gi} - Q_{Li} - Q_i\,(V,\delta) = 0 \qquad (7)$$

IN-EQUALITY CONSTRAINTS

The following variables are to be held within a specified range.

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \qquad (8)$$

Equality constraints on the reactive power flow given by equation (7) are not tight constraints, therefore they can be augmented in the objective function with a penalty, so that the reactive power flow deviations are restricted.

Now the Sub-problem I for the minimization of cost of generation can be rewritten as

Minimize

$$f_1 = \sum_{i=1}^{NG} (a_i + b_i P_{Gi} + C_i P_{Gi}^2) + K \sum_{i=1}^{N} (Q_{Gi} - Q_{Li} - Q_i)^2 \qquad (9)$$

Subject to

$$P_{Gi} - P_{Li} - P_i(V,o) = 0, \quad i=1,N \qquad (10)$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad , \quad i = 1,NG \qquad (11)$$

where K is the penalty factor.

## 3.2. FORMULATION OF SUB-PROBLEM II

The system voltage magnitudes and bus reactive powers should be such that the system losses and total reactive generation are minimized. Therefore the sub-problem II can be formulated as

Minimize

$$f_2 = \sum_{L=1}^{NL} - G_L [(V_i - V_j)^2 + V_i V_j (o_i - o_j)^2] + K_2 \sum_{i=1}^{NG} (Q_{Gi})^2 \qquad (12)$$

Subject to

$$P_{Gi} - P_{Li} - P_i(V,o) = 0 \ , \quad i = 1,N \qquad (13)$$

$$Q_{Gi} - Q_{Li} - Q_i(V,o) = 0 \ , \quad i = 1,\ldots, N \qquad (14)$$

$$V_i^{min} \leq V_i \leq V_i^{max} \quad , \quad i = 1,\ldots NG \qquad (15)$$

Variation in real power flow in the line due to change in $Q_{Gi}$ and V is very less. Therefore they are augmented in the objective function with a penalty term $K_1$ . Therefore subproblem II can be restated as

Min.

$$f_2 = \sum_{L=1}^{NL} -G_L[(V_i - V_j)^2 + V_i V_j (o_i - o_j)^2] + K_1 \sum_{i=1}^{N} (P_{Gi} - P_{Li} - P_i$$

$$(V,o))^2 + K_2 \sum_{i=1}^{NG} (Q_{Gi})^2 \qquad (16a)$$

Sub. to

$$Q_{Gi} - Q_{Li} - Q_i(V,\delta) = 0 \quad i = 1,N \tag{16b}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \quad , \quad i = 1, NG \tag{16c}$$

3.3. SOLUTION TECHNIQUE

For the large power system the system variable become large. Therefore to reduce the size of the problem we decompose system variables in to two groups i.e. state variable X and control variable U. For the sub-problem I the state variables are taken $\delta_i$, i = 1,N-1, $P_{GN}$ and control variables $P_{Gi}$, i = 1, NG-1 and for the sub-problem II $v_i$, i = 1, NL, $QG_i$, i=1, NG and $V_i$, i=1,NG are the state and control variables respectively.

Since in both sub-problem I and sub-problem II, the dependent variable (say X) enters into function F, but it is related to control variables (say U) through the equality constraints. eqn.(11) and (14) (say g),. This may create problem to solve the large power system. To remove this difficulty we have used Reduced Gradient Method.

3.4. CALCULATION OF REDUCED GRADIENT

At the initial point $U^o$, $X^o$, we may expand F and g for a small variations du and dx by Taylor series neglecting higher degree terms

$$\partial F = \frac{dF}{dU^o} \partial u + \frac{dF}{dX^o} \partial X \tag{17}$$

and

$$\frac{dg}{dX^o} \partial X + \frac{dg}{dU^o} \partial U = 0 \tag{18}$$

where the derivatives may be interpretted as-

$\frac{dF}{dU}$ - is an m-row of component $\frac{\partial F}{\partial U_i}$

$\frac{dF}{dX}$ - is an n-row of component $\frac{\partial F}{\partial X_i}$

$\frac{dg}{dX}$ - is an nxn Jacobian matrix of elements $\frac{\partial g_i}{\partial X_j}$

$\frac{dg}{du}$ - is an nxm Jacobian matrix of elements $\frac{\partial g_i}{\partial U_i}$

m - is the total number of control variables U

n - is the total number of state variables X

From (19)

$$\partial X = -[\frac{dg}{dX^o}]^{-1}[\frac{dg}{dU^o}]\partial u \qquad (20)$$

putting the value of oX from (20) to (18), we have-

$$\partial F = [\frac{dF}{dU^o} - [\frac{dF}{dX^o}][\frac{dg}{dX^o}]^{-1}[\frac{dg}{dU^o}]]\partial u \qquad (21)$$

The reduced gradient ∂F/∂u at the point $U^o$, $X^o$, is defined as a row vector

$$\frac{\partial F}{\partial U^o} = \frac{dF}{dU^o} - [\frac{dF}{dX^o}][\frac{dg}{dX^o}]^{-1}[\frac{dg}{dU^o}] \qquad (22)$$

This gradient is called 'reduced' because the equality constraints (17) g continue to be satisfied when a small change ∂u is made. So the equality constraints in both the sub-problem vanishes.

In equation (22) the R.H.S. may be written as

$$\frac{\partial F}{\partial U^o} = \frac{dF}{dU^o} - \lambda^T[\frac{dg}{dU^o}] \qquad (23)$$

where $\lambda^T = [\frac{dF}{dX^o}] [\frac{dg}{dX^o}]^{-1}$

or $\lambda^T [\frac{dg}{dX^o}] = \frac{dF}{dX^o}$ (23)

$\lambda^T$ is a row vector of $1 \times m$.

Equation (23) is a linear simultaneous equation. So for a large power system, it is necessary to solve hundreds of simultaneous equations of this form, for the value of $\lambda$.

General form of equation (23) can be written as

$\underline{A}\, X = \underline{b}$ (24)

There are many method to solve this equation for X. But they are having some limitations. As the direct inversion method of solving the equations requires $n^2$ storage locations and $n^3$ arithmatic operation. Gaussian Elimination method of solving such equation requires $n^3/3$ equations but more or less same intermediate storage location is required as compared to direct inversion methods.

The effective and most widely used method of manipulating coefficient matrices to solve simulation linear equations is that associated with triangulation of matrices or triangular decomposition

The LH method of factorization consists of expressing the coefficient matrix A as a product of two factor matrices, such that

$A = LH$

where L - is a lower triangular matrix.

H - is a higher (or upper) triangular matrix which has unit element on its diagonal.

if $A \underline{X} = \underline{b}$ are the set of simultaneous equations

Then $L\,H\,\underline{X} = \underline{b}$

Letting $H\,\underline{X} = \underline{Y}$

$L\,\underline{Y} = \underline{b}$

Since L is a lower triangular matrix, Y can be found from L and $\underline{b}$ by forward substitution and hence H is an upper triangular matrix the unknown vector $\underline{X}$ can be found from H and Y by backward substitution. Where

$$L = L_1^{-1}\,L_2^{-1}\,.\,.\,.\,L_n^{-1}$$

<u>LDH factorisation</u>

This method expresses original coefficient matrix A as a product of three factor matrices such that

$$A = L'DH$$

where

L' - is a lower triangular matrix which has unit elements on its diagonal.

H - is a higher (or upper) triangular matrix which has unit elements on its diagonal.

D - is a diagonal matrix which has zero off-diagonal elements.

If $A\,\underline{X} = \underline{b}$ is a set of equations

$$A = L'\,DH$$

then $L'DH\,\underline{X} = \underline{b}$

Letting $H\,\underline{X} = \underline{Y}$

and $D\,\underline{Y} = \underline{Y}'$

then $L'Y' = \underline{b}$

## 3.5. BIFACTORISATION TECHNIQUE

The technique of product form of inverse and triangular factorisation have been considered into a technique called bifactorisation. This method is particularly suitable for sparse coefficient matrices that have dominant and Nonzero diagonal elements that are either symmetrical or if not symmetrical, have a symmetrical sparsity structure.

The method is based on finding 2n factor matrices for an nth order problem such that the product of these factor matrices satisfying the requirement.

$$L^{(n)}\ L^{(n-1)}\ldots L^{(2)}\ L^{(1)}\ A\ R^{(1)}\ R^{(2)}\ldots R^{(n-1)}\ R^{(n)} = U \qquad (25)$$

where A - is original coefficient matrix.

L - is left hand factor matrix.

R - is right hand factor matrix.

U - unit matrix of order n.

Premultiply equation (25) by inverse of $L^{(n)}$, $L^{(n-1)}$,... $L^{(2)}$ $L^{(1)}$ consecutively gives

$$AR^{(1)}\ R^{(2)}\ldots R^{(n-1)}R^{n} = (L^{(1)})^{-1}(L^{(2)})^{-1}\ldots(L^{(n-1)})^{-1}\ (L^{n})^{-1} \qquad (26)$$

Post multiplying by $L^{(n)}$, $L^{(n-1)}$,..., $L^{(2)}$ and $L^{(1)}$ consecutively gives

$$AR^{(1)}\ R^{(2)}\ldots R^{(n-1)}\ R^{n}\ L^{(n)}\ L^{(n-1)}\ldots\ L^{(2)}\ L^{(1)} = U \qquad (27)$$

Finally premultiplying eqn.(27) by $A^{-1}$ gives

$$R^{(1)}\ R^{(2)}\ldots R^{(n-1)}\ R^{n}.L^{n}\ L^{(n-1)}\ldots\ L^{(2)}\ L^{(1)} = A^{-1} \qquad (28)$$

Although the original coefficient matrix of equation is symmetrical the superimposed factor of triangular decomposition are asymmetrical, whereas those of bifactorisation are symmetrical.

To determine the factor matrices L and R the following intermediate matrices are introduced.

$$A = A^{(o)}$$
$$A^{(1)} = L^{(1)}\ A^{(o)}\ R^{(1)}$$
$$A^{(2)} = L^{(2)}\ A^{(1)}\ R^{(2)}$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$A^{(K)} = L^{(K)}\ A^{(K-1)}\ R^{(K)}$$
$$A^{(n)} = L^{(n)}\ A^{(n-1)}\ R^{(n)}$$

The L and R factor matrices at $K^{th}$ step are given by

$$L_{iK}^{(K)} = -\frac{a_{iK}^{(K-1)}}{a_{KK}^{(K-1)}} \qquad (i=K+1,\ldots,n) \tag{29}$$

$$R_{Kj}^{(K)} = -\frac{a_{Kj}^{(K-1)}}{a_{KK}^{(K-1)}} \qquad (j=K+1,\ldots,n) \tag{30}$$

$$a_{ij}^{K} = a_i \tag{31}$$

In case of bifactorisation only left hand side matrices need be stored. But in case of triangular decomposition both upper and lower triangular matrices need be stored, which nearly doubles the required storage space and indexing information. Thus inherent problem can be overcome by further decomposing LH form triangulation into LDH form. The storage is now reduced to that of bifactorisation but to achieve this additional set of operation is required which itself decreases its efficiency. Whereas in triangular decomposition the product of two factor matrices gives the original coefficient matrix, in bifactorisation the product of factor gives the inverse.

So by using bifactorisation technique we are calculating the value of λ. and the derivatives of function w.r.t. u.

## 3.6. UPDATING OF CONTROL VARIABLE

Updating of the control variables is done with the help of Han-Powell method.

Each iteration of Han-Powell begins with an estimate, $U_K$ say, of the required vector of variables, an n x n positive definite matrix, $B_K$ say (which can be regarded as an approximation to the second derivative matrix of the Lagrangian function of the minimization problem, and a set ($\mu_i$; i=1,2,...m) of non-negative parameters that are used after modification in line search. On most iterations the search direction, $d_K$ , say, is the vector $\underline{d}$ that minimizes the quadratic function

$$Q(\underline{d}) = F(\underline{X}_K) + \underline{d}^T \underline{\nabla} F(\underline{X}_K) + \frac{1}{2} \underline{d}^T B_K \underline{d} \qquad (36)$$

subject to the linear constraints

$$\left.\begin{array}{ll} C_i(\underline{X}_K) + \underline{d}^T \underline{\nabla} C_i(\underline{X}_K) = 0, & i=1,2,\ldots,m' \\ C_i(\underline{X}_K) + \underline{d}^T \underline{\nabla} C_i(\underline{X}_K) \geq 0, & i=m'+1,\ldots,m \end{array}\right] \qquad (37)$$

However, $\underline{d}_K$ is modified if the constraints (37) are inconsistent. Having calculated $\underline{d}_K$ , a positive multiplier $\alpha_K$ is chosen, and $U_{K+1}$ is given by

$$U_{K+1} = \underline{U}_K + \alpha_K \underline{d}_K \qquad (38)$$

Finally $B_{K+1}$ is calculated from $B_K$ and from a change in gradient of an estimate of the Lagrangian function by B.F.G.S. formula.

We give particular attention to the choice of $\alpha_K$. It depends on the parameters ($\mu_i$, $i = 1,2,\ldots,m$), but they may differ from the values that were given at the start of iteration, because on some iteration they are revised at the end of the quadratic programming calculation that determine $\underline{d}_K$ . The step length $\alpha_K$ has to satisfy the condition

$$\omega_K(\underline{U}_K + \alpha_K \underline{D}_K) < \omega_K(\underline{U}_K), \tag{39}$$

where $\omega_K$ is the function

$$\omega_K(\underline{U}) = F(\underline{U}) + \sum_{i=1}^{m'} \mu_i |C_i(\underline{u})| + \sum_{i=m'+1}^{m} \mu_i \max\,[0, -C_i(\underline{U})] \tag{40}$$

In Hen-Powell method, the quadratic expression (36) and (37) is solved by simplex method. SIMPLEX METHOD [38] mathematically is explained below by taking general equation.

### 3.7. SIMPLEX METHOD FOR QUADRATIC PROGRAMMING

A quadratic programming model is defined as follows

$$\text{minimize} \quad X_o = \underline{C}\,\underline{X} + \underline{X}^T \underline{D}\,\underline{X} \tag{41}$$

subject to

$$\underline{A}\,\underline{X} \leq P_o \,, \; X \geq 0 \tag{42}$$

where

$$\underline{X} = (x_1, x_2, \ldots, x_n)^T$$

$$\underline{C} = (C_1, C_2, \ldots, C_n)$$

$$\underline{P}_o = (b_1, b_2, \ldots, b_n)^T$$

$$\underline{A} = \begin{matrix} a_{11} & \cdots & a_n \\ \vdots & & \vdots \\ a_{m1} & & a_{mn} \end{matrix}$$

$$\underline{D} = \begin{matrix} d_{11} & \cdots & d_{1n} \\ \vdots & & \vdots \\ d_{n1} & & d_{nn} \end{matrix}$$

The function $\underline{X}^T \underline{D}\, \underline{X}$ defines a quadratic form where $\underline{D}$ is symmetric. The matrix $\underline{D}$ is assumed positive definite if for minimization. This mear $X_o$ is strictly convex im $\underline{X}$ for minimization. The constraints are assumed linear in this case which guarantees as convex solution space.

The solution to this problem is secured by direct application of the Kuhn-Tucker necessary conditions.

Kuhn-Tucker Condition

The Kuhn-Tucker necessary conditions is used for identifying stationary points of a nonlinear constrained problem subject to inequality constraints. The development is based on the Lagrangian method. These conditions are also sufficient under certain limitation. The Lagrangian function is thus given by

$$L\,(\underline{X},\underline{S},\underline{\lambda}) = f(\underline{X}) - \underline{\lambda}\,[g(\underline{X})+\underline{S}^2] \qquad (43)$$

Given the constraints

$$g(\underline{X}) \leq 0 \qquad (44)$$

The Kuhn-Tucker conditions necessary for $\underline{X}$ and $\underline{\lambda}$ to be a stationary point of the above minimization problem is given below.

$$\underline{\lambda} \geq 0, \qquad (45)$$

$$\underline{\nabla}\, f(\underline{X}) - \underline{\lambda}\,\underline{\nabla}\, g(\underline{X}) = 0 \qquad (46)$$

$$\lambda_i\, g_i\,(\underline{X}) = 0,\ i=1,2,\ldots,\ m \qquad (47)$$

$$g(\underline{X}) \leq 0 \qquad (48)$$

Since $X_o$ is strictly convex and the solution space is convex set, these conditions are also sufficient for a global optimum.

The problem may be rewritten as

minimize
$$X_o = \underline{C}\,\underline{X} + \underline{X}^T\,\underline{D}\,\underline{X} \tag{49}$$
subject to
$$\underline{G}(\underline{X}) = \begin{pmatrix}\underline{A}\\ -\underline{I}\end{pmatrix}\underline{X} - \begin{pmatrix}\underline{P}_o\\ \underline{0}\end{pmatrix} \leq 0 \tag{50}$$

Let

$\underline{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$, and $\underline{U} = (\mu_1, \mu_2, \ldots, \mu_n)^T$

be the Lagrange multipliers corresponding to the sets of constraints $\underline{A}\,\underline{X} - \underline{P}_o \leq 0$ , and $-\underline{X} \leq 0$, respectively. Application of the Kuhn-Tucker condition immediately yields-

$$\underline{\lambda} \geq 0, \quad \underline{U} \geq 0 \tag{51}$$

$$\underline{V}\,X_o - (\underline{\lambda}^T, \underline{U}^T)\underline{V}\,\underline{G}(\underline{X}) = \underline{0} \tag{52}$$

$$\lambda_i(b_i - \sum_{j=1}^{n} a_{ij}\,X_j) = 0, \quad i=1,2,\ldots,m \tag{53}$$

$$\mu_i\,X_j = 0, \qquad j=1,2,\ldots,n \tag{54}$$

$$\underline{A}\,\underline{X} \leq \underline{P}_o, \qquad -\underline{X} \leq 0 \tag{55}$$

now

$$\underline{V}\,X_o = \underline{C} + 2\underline{X}^T\underline{D} \tag{56}$$
$$V\,\underline{G}(\underline{X}) = \begin{pmatrix}\underline{A}\\ -\underline{I}\end{pmatrix} \tag{57}$$

Let $\underline{S} = \underline{P}_o - \underline{A}\,\underline{X} \geq 0$ be the slack variables of the constraints. The above conditions reduce to

$$-2\underline{X}^T\underline{D} + \underline{\lambda}^T\underline{A} - \underline{U}^T = \underline{C} \tag{58}$$

$$\underline{A}\,\underline{X} + \underline{S} = \underline{P}_o \tag{59}$$

$$\mu_j x_j = 0 = \lambda_i\,S_i, \quad \text{for all } i \text{ and } j$$
$$\underline{\lambda}\,,\ \underline{U}\,,\ \underline{X}\,,\ \underline{S} \geq 0 \tag{60}$$

Observing that $\underline{D}^T = \underline{D}$ , the transpose of first set of equation is

$$-2\ \underline{D}\underline{X} + \underline{A}^T\ \underline{\lambda} - \underline{U} = \underline{C}^T \tag{61}$$

Hence the above necessary conditions may be combined as

$$\left(\begin{array}{c|c|c|c} -2\underline{D} & \underline{A}^T & -\underline{I} & \underline{O} \\ \hline \underline{A} & \underline{O} & \underline{O} & \underline{I} \end{array}\right) \begin{array}{c} \underline{X} \\ \underline{\lambda} \\ \underline{U} \\ \underline{S} \end{array} = \begin{array}{c} \underline{C}^T \\ \underline{P}_o \end{array} \tag{62}$$

$$\mu_j\ X_j = 0 = \lambda_i S_i\ , \text{ for all i and j}$$

$$\underline{\lambda}\ ,\ \underline{U}\ ,\ \underline{X},\ \underline{S} \geq \underline{O} \tag{63}$$

Except for the conditions $\mu_j\ X_j = 0 = \lambda_i\ S_i$, the remaining equation are linear functions in $\underline{X}, \underline{\lambda}, \underline{U}$ and $\underline{S}$. The problem is thus equivalent to solving a set of linear equation, while satisfying the additional conditions $\mu_j X_j = 0 = \lambda_i\ S_i$. The solution may be obtained by using Phase I of the two phase method.

The only restriction here is that the condition $\lambda_i S_i = 0 = \lambda_j X_j$ should always be maintained. This means if $X_i$ is in the basic solution at positive level, $S_i$ cannot become basic at positive level. Similarly $\mu_j$ and $X_j$ cannot be positive simultaneously. Phase-I will end in usual manner with the sum of artificial variables equal to zero only if the problem has a solution space.

Hessian Matrix $B_{K+1}$ is obtained by updating $B_K$ with the help of Broyden-Fletcher-Goldfarb-Shamno (B.F.G.S.) formula, is given by

$$B_{K+1} = B_K - \frac{1}{S_K^T B_K S_K} B_K S_K S_K^T B_K + \frac{1}{Y_K^T S_K} Y_K Y_K^T \tag{64}$$

where - $S_K$ vector denote the change in during the $K^{th}$ iteration (i.e. $S_K = U_{K+1} - U_K$)

$Y_K$ denote the change in gradient ($Y_K = G_{K+1} - G_K$)

Now the steps of the method can be written as

**Step-I** Read system data, tole, FCO (initial cost of generation assume very high value)

**Step-II** Set INF = -1

**Step-III** Find the value of $Q_{Gi}$, $P_{GN}$, $V_i$, and $\delta_i$ by performing the load flow.

**Step-IV** Calculate the derivative of function $F_1$ and derivatives of its equality constraints w.r. to state variables (i.e. $\delta_i$, i=1, N -1, $P_{GN}$).

**Step-V** Calculate the value of $\lambda$ by using Bifactorisation technique.

**Step-VI** Calculate the value and derivatives of function $F_1$ and its inequality constraints w.r. to control variables (i.e. $P_{Gi}$,i=1, $N_{G-1}$) respectively.

**Step-VII** Minimize the cost of generation w.r. to $P_{Gi}$ using Hen Powell method.

**Step-VIII** IF INF = 0 go to III

**Step-IX** Calculate the value of function FC. IF $|FCO-FC| \leq$ tole, go to Step-XVI-

**Step-X** Set FCO=FC and INF = -1.

**Step-XI** Calculate the derivatives of function $F_2$ and derivatives of its equality constraints w.r. to state variables ($V_i$, i=1,NL, $Q_{Gi}$.. ...,i=1, NG).

**Step-XII** Calculate the value of $\lambda$ by using Bifactorisation technique.

**Step-XIII** Calculate value and derivatives of function $F_2$ and value and derivatives of its inequality constraints w.r. to control variables (i.e. $V_i$, i=1,NG).

**Step-XIV** Minimize the system losses by using Han power method w.r. to $V_i$, i=1, NG.

**Step-XV** IF INF = 0 go to Step (X) otherwise go to Step-II.

**Step-XVI** Print ($P_{Gi}$, $Q_{Gi}$, i=1,NG) and ($V_i$, $\delta_i$, i=1,N), FC, system losses and stop.

In this chapter we have taken two examples of 6-bus and 26-bus systems and solved by using discussed technique. And results are also tabulated.

## TEST PROBLEM-1

Considering a 6-bus system shown in Fig.4.1. System data, load conditions, limit constraints and cost coefficient are given in Table (1) to Table (4) and results are tabulated in Table (5).

### TABLE -1(a)

Line parameters and charging data on 100 MVA Base

| Bus No. | | Line Parameters | | |
|---|---|---|---|---|
| i | j | r (p.u.) | x (p.u.) | $y_c$ (p.u.) |
| 1 | 5 | 0.0250 | 0.1682 | 0.2593 |
| 2 | 3 | 0.1021 | 0.4980 | 0.4984 |
| 2 | 6 | 0.2130 | 0.8957 | 0.2406 |
| 3 | 4 | 0.0328 | 0.1325 | 0.0325 |
| 4 | 5 | 0.0238 | 0.2108 | 0.3017 |
| 4 | 6 | 0.1191 | 0.2704 | 0.0328 |
| 5 | 6 | 0.1494 | 0.3692 | 0.0412 |

where $2y_c$ is the shunt admittance of the line.

### TABLE-1(b)

| Bus No. | | Transformer data | | |
|---|---|---|---|---|
| i | j | r(p.u.) | x(p.u.) | Ratio |
| 1 | 5 | 0.0 | 0.1682 | 1.03 |
| 2 | 6 | 0.0 | 0.8957 | 0.96 |
| 5 | 6 | 0.0 | 0.3692 | 0.98 |

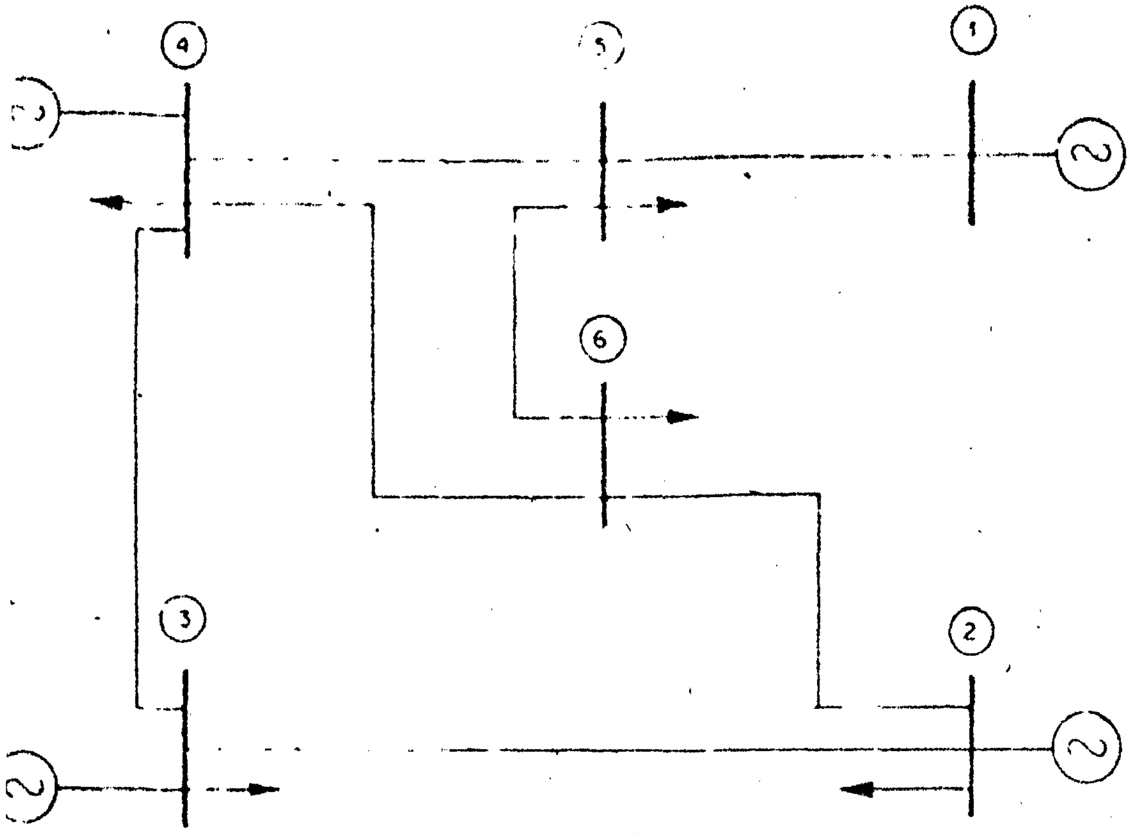


FIGURE 4.1 6-BUS SYSTEM

## TABLE-2

### System load data on 100 MVA base

| Bus No. i | LOAD $P_{Li}$ (p.u.) | $Q_{Li}$ (p.u.) |
|---|---|---|
| 1 | 1.6998 | 0.0904 |
| 2 | 0.9167 | 0.1500 |
| 3 | 2.4718 | 0.5249 |
| 4 | 0.0000 | 0.0000 |
| 5 | 0.3765 | 0.3024 |
| 6 | 0.7719 | 0.3884 |

## TABLE-3

### Limit constraints imposed upon various variables on 100 MVA base

| Bus No. i | $P_{Gi}^{(max)}$ (p.u.) | $P_{Gi}^{(min)}$ (p.u.) | $Q_{Gi}^{(max)}$ (p.u.) | $Q_{Gi}^{(min)}$ (p.u.) | $V_i^{(max)}$ (p.u.) | $V_i^{(min)}$ (p.u.) |
|---|---|---|---|---|---|---|
| 1 | 0.63 | 0.30 | 0.98 | 0.0 | 1.05 | 0.90 |
| 2 | 0.70 | 0.50 | 0.81 | -0.81 | 1.05 | 0.90 |
| 3 | 4.0 | 3.80 | 2.26 | -1.1 | 1.05 | 0.90 |
| 4 | 1.70 | 1.50 | 1.39 | -1.38 | 1.05 | 0.90 |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.90 |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.90 |

TABLE -4

System cost data

| Bus No. i | $A_{1i}$(Rs.) | $A_{2i}$(Rs./MW) | $A_{3i}$(Rs./MW$^2$) |
|---|---|---|---|
| 1 | 0.00 | 0.2100 | 0.000000 |
| 2 | 144.43 | 0.4564 | 0.001495 |
| 3 | 0.00 | 0.2100 | 0.000000 |
| 4 | 44.00 | 1.2718 | 0.005500 |

TABLE-5

Optimal system voltage level for minimum cost of generation and minimum system losses

| Bus No. i | Voltage Magnitude $V_i$(p.u.) | Bus phase angle $\delta_i$(rad.) | Reactive power generation $Q_{Gi}$(p.u.) | Real power generation $P_{Gi}$(p.u.) |
|---|---|---|---|---|
| 1 | 1.0281 | 0.1736 | 0.00 | 0.00 |
| 2 | 1.0082 | 0.0460 | 0.00 | 0.00 |
| 3 | 1.0500 | 0.0864 | -0.2534 | 0.700 |
| 4 | 1.0482 | 0.5241 | 0.3063 | 3.800 |
| 5 | 0.9813 | 0.4407 | -0.0069 | 1.500 |
| 6 | 1.0176 | 0.0000 | 1.3242 | 0.3633 |

Total active power generation = 5.3633 p.u.

Total cost of generation = Rs. 191.86

Total system loss = 0.2512 p.u.

TEST PROBLEM-2

Now considering another problem of 26-bus system as shown in Fig.4.2. The system data, load conditions, limit constraints and cost coefficient are given in Table (6) to Table (9) respectively and results are given in Table (10).

TABLE-6(a)

Line parameters and charging data on 100 MVA base

| Bus No. | | Line parameters | | |
|---|---|---|---|---|
| i | j | r (p.u.) | x (p.u.) | $y_c$ (p.u.) |
| 13 | 26 | 0.00 | 0.0131 | 0.00 |
| 26 | 16 | 0.00 | 0.0392 | 0.00 |
| 16 | 20 | 0.00 | 0.432 | 0.00 |
| 20 | 26 | 0.00 | 0.314 | 0.00 |
| 2 | 10 | 0.00 | 0.015 | 0.00 |
| 9 | 10 | 0.1494 | 0.3392 | 0.824 |
| 9 | 12 | 0.0658 | 0.1494 | 0.0364 |
| 12 | 26 | 0.0533 | 0.121 | 0.0294 |
| 9 | 14 | 0.0618 | 0.2397 | 0.0638 |
| 11 | 14 | 0.0676 | 0.262 | 0.0698 |
| 24 | 26 | 0.061 | 0.521 | 0.059 |
| 6 | 26 | 0.0513 | 0.1986 | 0.053 |
| 6 | 24 | 0.0129 | 0.0532 | 0.0148 |
| 7 | 24 | 0.0906 | 0.3742 | 0.0874 |
| 6 | 7 | 0.0921 | 0.3569 | 0.095 |
| 11 | 21 | 0.0513 | 0.2118 | 0.0496 |
| 8 | 11 | 0.0865 | 0.3355 | 0.0894 |
| 17 | 21 | 0.0281 | 0.1869 | 0.0474 |
| 8 | 22 | 0.0735 | 0.2847 | 0.0758 |
| 17 | 22 | 0.0459 | 0.3055 | 0.0774 |
| 1 | 4 | 0.0619 | 0.2401 | 0.0638 |
| 4 | 22 | 0.061 | 0.2365 | 0.063 |
| 23 | 22 | 0.0 | 0.0305 | 0.0 |
| 15 | 1 | 0.0 | 0.0147 | 0.0 |
| 2 | 13 | 0.0086 | 0.0707 | 0.6034 |
| 1 | 7 | 0.0199 | 0.0785 | 0.0808 |
| 15 | 23 | 0.0107 | 0.0617 | 0.8942 |
| 2 | 25 | 0.0074 | 0.0608 | 0.5186 |
| 1 | 3 | 0.0 | 0.392 | 0.0 |
| 19 | 3 | 0.0 | 0.145 | 0.00 |
| 5 | 22 | 0.0 | 0.175 | 0.0 |
| 5 | 18 | 0.0 | 0.154 | 0.0 |

## TABLE-6(b)

| Bus No. | | Transformer data | | |
|---|---|---|---|---|
| i | j | r (p.u.) | x (p.u.) | Ratio |
| 18 | 26 | 0.0 | 0.0181 | 1.08 |
| 26 | 16 | 0.0 | 0.0892 | 0.96 |
| 2 | 10 | 0.0 | 0.0150 | 1.08 |
| 28 | 22 | 0.0 | 0.0805 | 0.97 |
| 15 | 1 | 0.0 | 0.0147 | 0.89 |
| 1 | 8 | 0.0 | 0.0892 | 0.98 |
| 19 | 8 | 0.0 | 0.145 | 0.98 |
| 5 | 22 | 0.0 | 0.175 | 0.99 |
| 5 | 18 | 0.0 | 0.154 | 1.08 |

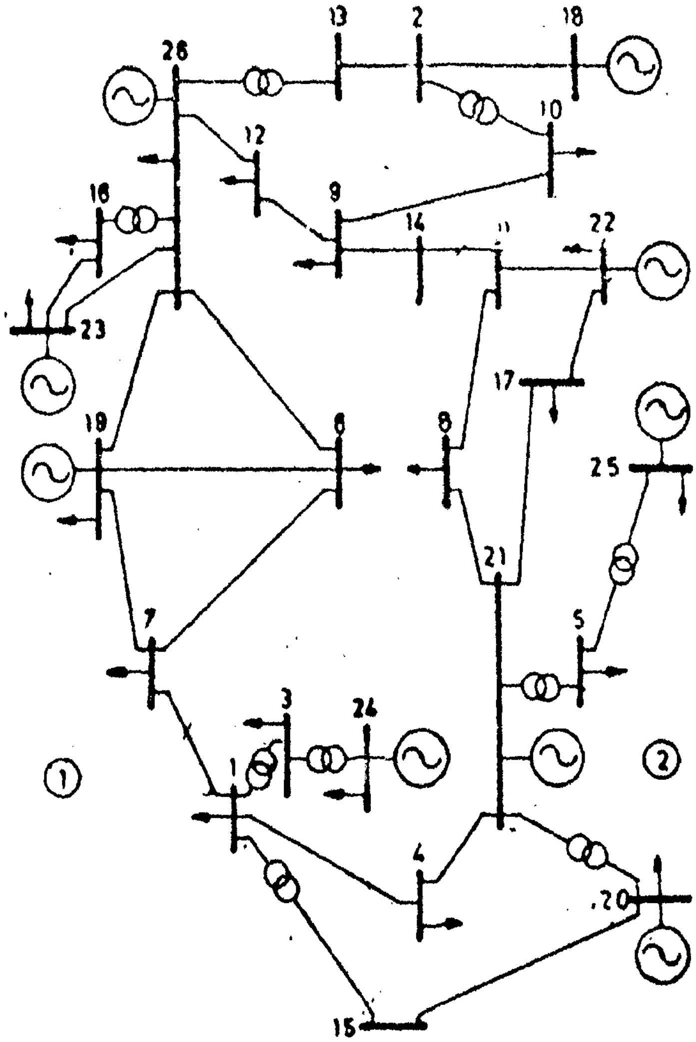


FIGURE 4.2 26-BUS SYSTEM

TABLE-7

System load data on 100 MVA base

| Bus No. i | Load $P_{Li}$ (p.u.) | $Q_{Li}$ (p.u.) |
|---|---|---|
| 1 | 0.82 | 0.21 |
| 2 | 0.0 | 0.0 |
| 3 | 0.57 | 0.17 |
| 4 | 0.48 | 0.21 |
| 5 | 0.43 | 0.11 |
| 6 | 0.4 | 0.1 |
| 7 | 1.11 | 0.27 |
| 8 | 0.23 | 0.06 |
| 9 | 0.67 | 0.21 |
| 10 | 1.02 | 0.27 |
| 11 | 0.43 | 0.14 |
| 12 | 0.43 | 0.12 |
| 13 | 0.0 | 0.0 |
| 14 | 0.0 | 0.0 |
| 15 | 0.0 | 0.0 |
| 16 | 1.31 | 0.3 |
| 17 | 0.03 | 0.01 |
| 18 | 0.0 | 0.0 |
| 19 | 0.05 | 0.0 |
| 20 | 0.04 | 0.0 |
| 21 | 0.56 | 0.0 |
| 22 | 0.0 | 0.0 |
| 23 | 0.0 | 0.0 |
| 24 | 0.0 | 0.0 |
| 25 | 0.0 | 0.0 |
| 26 | 0.0 | 0.0 |

## TABLE-8

**Limit constraints imposed upon various variable on 100 MVA base**

| Bus No. i | $P_{Gi}^{(max)}$ (p.u.) | $P_{Gi}^{(min)}$ (p.u.) | $Q_{Gi}^{(max)}$ (p.u.) | $Q_{Gi}^{(min)}$ (p.u.) | $V_i^{(max)}$ (p.u.) | $V_i^{(min)}$ (p.u.) |
|---|---|---|---|---|---|---|
| 1 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 3 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 4 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 7 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 8 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 9 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 10 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 11 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 12 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 13 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 14 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 15 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 16 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 17 | 0.0 | 0.0 | 0.0 | 0.0 | 1.05 | 0.95 |
| 18 | 1.0 | 0.5 | 2.0 | -2.0 | 1.05 | 0.95 |
| 19 | 0.5 | 0.0 | 2.0 | -2.0 | 1.05 | 0.95 |
| 20 | 0.5 | 0.0 | 2.0 | -2.0 | 1.05 | 0.95 |
| 21 | 0.5 | 0.0 | 2.0 | -2.0 | 1.05 | 0.95 |
| 22 | 1.5 | 1.0 | 2.0 | -2.0 | 1.05 | 0.95 |
| 23 | 3.0 | 2.7 | 2.0 | -2.0 | 1.05 | 0.95 |
| 24 | 2.0 | 1.3 | 2.0 | -2.0 | 1.05 | 0.95 |
| 25 | 3.0 | 2.7 | 2.0 | -2.0 | 1.05 | 0.95 |
| 26 | 3.0 | 1.0 | 2.0 | -2.0 | 1.05 | 0.95 |

TABLE-9

System cost data

| Bus No. $i$ | $A_{1i}$ Rs. | $A_{2i}$ Rs./MW | $A_{3i}$ Rs./$MW^2$ |
|---|---|---|---|
| 18 | 44.0 | 127.18 | 0.55 |
| 19 | 0.0 | 21.0 | 0.04 |
| 20 | 144.43 | 45.546 | 0.1495 |
| 21 | 0.0 | 21.0 | 0.04 |
| 22 | 44.0 | 127.18 | 0.55 |
| 23 | 0.0 | 21.0 | 0.05 |
| 24 | 144.43 | 45.46 | 0.14959 |
| 25 | 10.0 | 21.0 | 10.0 |
| 26 | 0.0 | 21.0 | 0.08 |

## TABLE-10

**Optimal system voltage level for minimum cost of generation and minimum system losses**

| Bus No. | Voltage Magnetude $V_i$ (p.u.) | Bus phase angle $\delta_i$ (rad.) | Reactive power generation $Q_{Gi}$ (p.u.) | Real power generation $P_{Gi}$ (p.u.) |
|---|---|---|---|---|
| 1 | 1.0254 | 0.0663 | 0.0000 | 0.0000 |
| 2 | 1.0634 | 0.0842 | 0.0000 | 0.0000 |
| 3 | 1.0407 | 0.0504 | 0.0000 | 0.0000 |
| 4 | 0.9800 | 0.0838 | 0.0000 | 0.0000 |
| 5 | 0.9913 | 0.2235 | 0.0000 | 0.0000 |
| 6 | 1.014 | 0.0455 | 0.0000 | 0.0000 |
| 7 | 0.9988 | 0.0079 | 0.0000 | 0.0000 |
| 8 | 0.9641 | 0.0388 | 0.0000 | 0.0000 |
| 9 | 0.9783 | -0.1108 | 0.0000 | 0.0000 |
| 10 | 1.0348 | 0.0627 | 0.0000 | 0.0000 |
| 11 | 0.9513 | -0.0904 | 0.0000 | 0.0000 |
| 12 | 0.9731 | -0.0759 | 0.0000 | 0.0000 |
| 13 | 1.0430 | 0.0143 | 0.0000 | 0.0000 |
| 14 | 0.9736 | -0.1034 | 0.0000 | 0.0000 |
| 15 | 0.9213 | 0.0944 | 0.0000 | 0.0000 |
| 16 | 1.0344 | -0.0457 | 0.0000 | 0.0000 |
| 17 | 0.9783 | 0.0383 | 0.0000 | 0.0000 |
| 18 | 0.9763 | 0.3055 | 0.1087 | 0.5000 |
| 19 | 1.0233 | 0.0671 | 0.0270 | 0.1755 |
| 20 | 1.0146 | -0.0266 | -0.0196 | 0.0000 |
| 21 | 0.9650 | -0.0646 | 0.0252 | 0.1665 |
| 22 | 1.0092 | 0.2112 | 0.1664 | 1.0000 |
| 23 | 0.9763 | 0.2298 | 0.1429 | 2.7000 |
| 24 | 1.0250 | 0.0831 | -0.0079 | 1.3000 |
| 25 | 1.0606 | 0.2314 | -0.4676 | 2.7000 |
| 26 | 1.0068 | 00.0000 | -0.0454 | 0.3087 |

Total active power generation = 8.8457 p.u.

Total cost of generation = Rs. 837.90

Total system loss = 0.293 p.u.

## CHAPTER-5

## CONCLUSIONS

In the present study new problem formulation is proposed for Generation allocation problem. The objective of the suggested formulation is to minimize the mismatches between the specified and calculated power at the system bus so as to provide complete insight to the whole system which greatly helps in taking the corrective decisions. The technique should find its application in power system planning. The solution technique used for this problem is simple.

Optimal Generation Allocation problem is very important because of huge system operating costs. At the same time it involves large number of problem variables and limit constraints. Variable decomposition and problem decomposition is utilised in order to reduce its size. Variable decomposition approach devides the problem variables i to control and state variables and uses sensitivity relation to incorporate state variables with control variables, determined by optimization technique. This reduces the total storage requirement because of reduction in the size of the matrices. Problem decompositio approach decomposes the Generation allocation problem into two sub-problem of active power and reactive power optimization which can be solved independently. For their formulation use is made of P-$\delta$ and Q-V coupling thus reducing the size of sub-problem. Active power optimization is achieved by minimizing the active power generation ~~is generation~~ with constant bus voltage magnitude and thus giving optimal system bus voltage phase angles. These bus voltage phase angle are held constant for reactive power optimization in which the total active power loss and reactive generation at each bus is minimized for obtaining the bus voltage magnitude. The optimization process can be terminated at any stage and the results will be better than the initial operating conditions. The storage requirements for the sub-problem is reduced as the problem variables and number of constraints are reduced.

Generation allocation problem is constrained nonlinear programming problem having nonlinear objective function subject to nonlinear equality and inequalities constraints. In this study use is made for the new formulations for transforming the generation allocation problems into unconstrained form using reduced gradient and bifactorization technique. We have solved these unconstrained problem by using Hen-Powell method which is simple and faster in convergence. And because of sparse technique The storage requirement is also reduced.

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## APPENDIX 'A' LINKNET- A STRUCTURE FOR COMPUTER REPRESENTATION OF NETWORKS

In the development of any algorithm which deals with a network the programmer must decide on how the network information should be stored in computer memory. The decision is particularly important for the analysis of the large sparse networks which occur in power system studies. Often, it will largely determine the computer memory requirements and it may also significantly affect the processing of the algorithm.

Most of the structures have been developed for particular types of network algorithm and have a limited range of application. However, for a wide variety of network problems there are a number of common features which are desirable in a storage structure. These can be summarized as:

(i) Use of a small amount of computer memory

(ii) Processing of the network information should be facilitated; i.e. the branches and nodes connected to any given node should be easily scanned

(iii) The structure should easily reflect network changes; e.g. the addition or removal of branches

(iv) The structure should be basically simple and easy to program

The LINKNET structure ~~(see Reference 5 and Acknowledgements)~~ has been designed as a general purpose structure for representing networks in a computer. It incorporates each of the desirable features which are listed in a balanced manner. The structure derives its name from linked-lists which it applies to obtain some of these features. The usefulness of the LINKNET structure has been demonstrated by its application in the programming of a wide variety of network algorithms.

THE LINKNET STRUCTURE

We assume that the nodes and branches in the network are numbered, either manually or by the computer. Typically in a power system network the nodes are numbered manually but the branch numbering is left to the computer. The properties of the network are divided into node properties, branch properties and topological properties. The node and branch properties are stored in a fairly standard fashion. For each node or branch property a one-dimensional array is allocated and each position in the array is identified with th node or branch having the corresponding number..

The topological properties of the network are represented by specifying the connections between the nodes and the branches. Firstly, we assume that the ends of each branch are numbered as follows; ends of branch 1 are numbered 1 and 2, ends of branch 2 are numbered 3 and 4 etc. Thus, the branch end numbers may be derived from a branch number as:

$$END = f(BRANCH) = 2.BRANCH - 1$$

and,

$$END = g(BRANCH) = 2.BRANCH$$

Conversely a branch number may be derived from either of its end numbers using,

$$BRANCH = h(END) = (END + 1)/2$$

In this relationship the integer round off is used to obtain the two to one mapping between branch ends and branches. The topology of the network can now be defined by constructing a linked-list of the branch ends which are connected to each node. For each node we define a pointer:

LIST(NODE) = The first branch end on the list from NODE. For each branch end we define a pointer, NEXT(END) = The next branch end on the list after END. The last branch end on the list for each node is indicated when NEXT(END) = O. The LIST(NODE) and NEXT(END) pointers are sufficien to define the network topology uniquely. They allow the branches connec ted to any node to be directly obtained using the procedure:

| | | |
|---|---|---|
| Initialize, | END | = LIST(NODE) |
| then set, | BRANCH | = h(END) |
| and | END | = NEXT(END) until NEXT(END) = O. |

In network computations it is often necessary to obtain the nodes which are connected to a given node. This operation is facilitated by defining an additional pointer for each branch end:

FAR(END) = the node at the far or opposite end of the branch. The nodes connected to any given node can now be obtained using the procedur

| | | |
|---|---|---|
| Initialize, | END | = LIST(NODEA) |
| then set, | NODEB | = FAR(END) |
| and | END | = NEXT(END) until NEXT(END) = O. |

The successive values of NODEB will be the nodes which are connected to NODEA. It may also be required to obtain the nodes at the ends of a given branch. These can be obtained as follows:

ENDA = f(BRANCH)

ENDB = g(BRANCH)

NODEA = FAR(ENDB)

NODEB = FAR(ENDA)

=The pointers LIST(NODE), NEXT(END) and FAR(END) form the framework of the LINKNET structure. The LINKNET structure is the ease with which the structure can be modified to reflect the addition or removal of network branches. This is in fact one of the main features which distinguishes the LINKNET structure from more conventional structures such as double entry line tables.