SWITCHING SURGE OVER VOLTAGE STUDY FOR EHV POWER SYSTEMS

A DISSERTATION

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(Power Systems Engineering)

By

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CERTIFICATE

Certified that the dissertation entitled 'SWITCHING SURGE OVER VOLTAGE STUDY FOR EHV POWER SYSTEMS', which is being submitted by Sri Vinay Pant in partial fulfilment for the award of the degree of Master of Engineering in Electrical Engineering (Power System Engineering), of University of Roorkee, Roorkee is a record of student's own work carried out by him under my guidance and supervision. The matter embodied in this dissertation has not been submitted for the award of any other degree.

This is to further certify that he has worked for a period of about 10 months from January 15, 1982 to October 30, 1982 in preparing this dissertation for the Master of Engineering Degree at this University.

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ABSTRACT

In this thesis an attempt has been made to calculate switching surge overvoltage using Laplace transform technique.

The merits and demerits of various methods for the calculation of switching surge overvoltage, namely, field tests, analog and digital techniques, have been discussed in Chapter-I.

A comparative study of different digital techniques has been made in Chapter-II. Subsequently an indepth study of the Laplace transform technique and its application towards

(i) unloaded line energization,

(ii) energization of resistance loaded line,

(iii) energization of inductance loaded line, are given.

Case study of a system for various line conditions has been done and simulated on DEC-2050 Computer System in FORTRAN-IV (a listing of program is given in Appendix-II). The results obtained have been plotted on CALCOM plotter, and are given in Chapter-V.

The concluding chapter, namely Chapter-VI, discusses the future scope of work in the related field.

NOMENCLATURE

x	line length
R_0	zero sequence resistance of the line per unit length
L_0	zero sequence inductance of the line per unit length
C_0	zero sequence capacitance of the line per unit length
R_1	positive sequence resistance of the line per unit length
L_1	positive sequence inductance of the line per unit length
C_1	positive sequence capacitance of the line per unit length
E_1	voltage of phase-1 with respect to ground
E_2	voltage of phase-2 with respect to ground
E_3	voltage of phase-3 with respect to ground
I_1	current in the conductor of phase-1
I_2	current in the conductor of phase-2
I_3	current in the conductor of phase-3
$[\Omega]$	surge impedance matrix of the system
$[L_g]$	generator inductance matrix
$[R_g]$	generator resistance matrix
$[L_L]$	load inductance matrix
$[R_L]$	load resistance matrix

CONTENTS

CHAPTER-I

INTRODUCTION

The fundamental requirement for reliable and uninterrupted power system operation is the elimination of disturbances to as great extent as possible. Such disturbances may be caused by overvoltages which exceed the insulation level and hence lead to flashovers. It is not an economic proposition to raise the insulation level of high voltage power systems to such an extent as to withstand all possible overvoltages: instead the latter must be restricted to a certain level. Infact UHV voltage levels are economically feasible only if some type of transient voltage control is used. Therefore, the prerequisite to a better system design is an indepth knowledge of various types of overvoltages that can occur in a power system, and their effect on the system insulation level.

1.1 OVERVOLTAGES IN POWER SYSTEMS

The various types of overvoltages that may arise on a transmission network as classified for the purpose of insulation co-ordination are given below. The definitions given to these classifications relate essentially to the wave shape of the overvoltage rather than to their origin.

(a) Lightning overvoltages: They have fast wavefronts and are usually generated by lightning strokes.

(b) Switching overvoltages: They have slower wavefronts and can be generated during the switching of lines, transformers, reactors and the occurrence of faults.

(c) Temporary overvoltages: They have frequency near to or an harmonic of the power frequency. Undamped overvoltages of power frequency may be produced on load rejection and during switching of lines or cables with relatively high charging currents. Overvoltages which may be only slightly damped and which may persist from a few cycles to a few seconds with a frequency equal to the supply frequency or one of its harmonics, may be encountered when transformers are energised from networks with certain configurations and parameters.

Overvoltages originating from more than one of the above causes may occur in rapid succession but only in exceptional cases simultaneously. Various causes may lead to an earthfault or a switching operation. A lightning stroke may, but need not, cause an earth fault. In all cases, however, an earthfault results in a switching operation to clear the fault. Generally a switching operation in a power system changes the state of the system from those conditions existing prior to the switching to those existing after the operation. The transients thus generated usually exhibit complex waveforms for which the fundamental frequency usually lies in the range 100 Hz to 1000 Hz but in some cases a very steep voltage rise or collapse can occur. In UHV and EHV systems there are a number of switching operations [1] which require special consideration as they may lead to magnitudes of the switching transients which influence the choice of the system insulation level. Moreover, with the increasing voltage of transmission systems switching surge overvoltages determine the insulation design rather than lightning overvoltages, as considerable technological progress has been made in controlling the magnitude of lightning overvoltages. Thus the determination of the magnitudes and waveshapes of switching surge overvoltages is imperative for an economical design of power system.

1.2 METHODS FOR DETERMINING SWITCHING SURGE OVERVOLTAGES

The methods for determining switching surge overvoltages, that can occur in a power system, can be broadly classified as

1.2.1 Field Tests

1.2.2 Analog or Model Methods

1.2.3 Digital or Analytical Methods.

1.2.1 Field Tests: Some field test have been reported in the literature [2,3,4]. These are reliable ways of determining the switching surge overvoltages on a line, as they take into account all the practical factors that can affect the surges. Tests are carried out on existing or experimental lines, and the surge magnitude and waveshape is recorded. Direct study of these processes in an actual network is possible only on very rarest occassions, as a system is either not available for such involved measurements or is still in the designing stage. The extensive field investigations to cover all possible system configurations are prohibitively expensive and time consuming. Moreover, the results obtained by field tests on a particular system can not be generalized for all the systems.

1.2.2 Analog or Model Methods: The technique is essentially that of designing an electrical model or analog of a dynamic system in such a manner that measurement on the model gives useful and proportional information about the actual system. The computing tools available for such type of studies are

1.2.2.1 Transient Network Analyzer (TNA)

1.2.2.2 Electronic Differential Analyzer (EDA).

1.2.2.1 Transient Network Analyzer (TNA) [5]: The TNA has been and is still the 'work horse' of the switching surge overvoltage studies. It comes close to being a direct electrical model of the system represented and is therefore easy to understand. It is much faster than other tools, usually operating in real time, though time scaling can be used.

The TNA extends to transient conditions the idea of the steady state analyzer or a-c calculating board. On TNA the equivalent network is built up with inductors, resistors, capacitors, coupling transformers, sources of sinusoidal e.m.f. and synchronous switches. Conventional resistors and capacitors are satisfactory in TNA models, but specially designed inductors are used to simulate frequency dependent characteristics as closely as possible to that of the real network elements. The transmission lines are represented by a tandem connection of three-phase π-units, and the ground return path is built up by series and parallel connected inductors and resistors. The number of π-units required to represent a line has to be chosen carefully as an insufficient number of π-units can lead to unwanted distortion on overvoltage waveshape and affect the maximum overvoltage peak [6]. The output of TNA is observed on oscilloscopes.

1.2.2.2 Electronic Differential Analyzer (EDA): This analyzer is well suited for solving electrical transient problems in lumpy circuits, and it is especially attractive for investigations of the affects of varying one, or more of circuit elements over a range of values.

The EDA, also known as Analog Computer (ANACOM), comprises a variety of units. They are integrators, inverters, summers, coefficient potentiometers and signal generators. In addition to these devices it has a display unit.

The physical system is represented by its differential and algebraic equations, and EDA basically solves the representative differential equations. Although, best suited for lumpy circuits, its use for circuits with distributed constants is also possible. Thomas and Hedin [7] have used EDA to solve switching surge problems involving single phase transmission lines by travelling wave method. The simulation was achieved by constructing a multichannel pure transport delay time unit which is not a standard component of EDA. This is capable of storing surge waveshapes of arbitrary form, operating on them, and delivering them back to EDA after a preselected time interval. For three phase circuits the amount of equipment required is considerable. This approach is therefore limited to relatively simple circuit arrangements.

1.2.3 Digital or Analytical Methods: The application of the digital computer to power system transient studies has been and remains a burgeoning field of endeavor. The appeal of digital computer is its ability to process a vast amount of data in a systematic way, and do so in an extremely short time. The computer is very adept at storing, retreiving, operating on, and restoring volumes of data. System transient studies can be stated in these terms, for they are concerned with describing events in space and time at many different locations, which may be set down as a large number of differential equations. Many techniques varying in mathematical approach and sophistication have been developed for solving the transient problem on digital computers.

The solutions for a large number of cases, as required for rational system design, can be computationally expensive. Hence in order to strike a balance some accuracy has to be sacrificed[8]. A co-ordinated use of TNA and digital computers can be economical for such studies [9]. To reduces computing time, a hybrid computation system has been developed in which the switching surge is simulated on TNA and the digital computer is used for data processing and control of TNA [10].

With the continuous development of system modelling techniques on TNA and digital computers, the results obtained from them show good agreement in general to the field tests. Some discrepancy occurs because of factors which can affect the accuracy of switching surge calculations. Basically three possible sources of error must be considered:

- Incomplete knowledge of the parameters of the real system
- Simplifications of the equivalents of the network elements
- Limitation of model simulation on TNA's and the limitations of mathematical simulation in digital programmes.

CHAPTER-II

REVIEW OF DIGITAL TECHNIQUES FOR CALCULATION OF SWITCHING SURGE OVERVOLTAGES

For switching surge overvoltage determination many computer programs are being implemented with the intention of minimizing the computer running time while improving the theoretical and technical quality of the solution. The digital simulation of a physical process is achieved by (1) formulating a mathematical model of the process (2) computing an approximate solution to the equation. Naturally the accuracy of the results obtained depends both on the fidelity of the model and the errors generated by the computation procedure. The various techniques developed for solving the transmission line transient problems are as follows:

2.1 Schynder-Bergeron Method.

2.2 Lattice Diagram Method.

2.3 Fourier Transform Method.

2.4 X-Transform Method.

2.5 Z-Transform Method.

2.6 System Approach Method.

2.7 Laplace Transform Method.

2.1 SCHYNDER-BERGERON METHOD

This method was first visualized as a graphical method for the calculation of transients in penstocks. This graphical method was modified to render it applicable to digital computers by Frey et.al.[11]. They studied few very simple cases and the calculations and computation time reported was quite large. In this method a relation is established between the voltage and the current at each end of the lines depending upon the voltage and the current at the opposite end, including transit time. The distributed parameter circuit elements are sectioned using a basic time interval. Initial conditions define the voltages existing at all busbars and hence at intermediate points. Surge propagation is initiated by connecting all sources to the circuit to be energized. The voltage and current is computed at each discrete point for every basic time interval. The overhead line parameters are in the form of modal surge impedances and attenuation factors are included approximately by introducing series resistance into the modal domain.

2.2 LATTICE DIAGRAM METHOD

This method is a digital computer adaptation of a graphical method of Bewley's lattice diagram [12]. The application of this method to single phase representations has been described by Barthold and Carter [13]. This method is capable of accomodating any specified input waveshape, real or complex line terminations, any system configuration. Basically this method is an application of superposition combined with an ingenious system of book keeping. The calculations are made in terms of the voltage wave increments which travel on the line comprising of the equivalent circuit and the behaviour of these travelling waves at junction and terminations is determined by reflection and refraction coefficients. They have assumed that the incoming unit wave proceeds through the discontinuity undiminished, but generates a new wave equal to the reflection coefficient at the instant it reaches the discontinuity.

This new wave emanates from the discontinuity in both directions, and the sum of the original wave and the newly generated wave represents the total response of the discontinuity to the impinging wave. The response of a complex network, in a similar manner, can be represented as the superposition of the undiminished transmission of the original input, plus similar transmission of secondary wave components generated as the original wave arrives at each bus in the system. The secondary waves produce a third generation of waves; and the process continues ad infinitum. Although this method is basically applicable to distributed parameter elements such as lines and cables it has been extended to include lumped parameters of generators, transformers and capacitor banks [14]. They have been represented as transmission line stubs , while certain non-linear elements are expressed by piecewise linear techniques.

This method has been extended to three phase circuits [9]. Whereas for single phase calculations the reflection and refraction coefficients are calculated from the individual line surge impedances for three phase calculations these surge impedances are replaced by surge impedance matrices and in this way the mutual effects between phases are included in calculation. The surge impedance matrix used to represent the transmission line is calculated at the predominant frequency of the transient or if this is not known, at a frequency based on the travel time of the line being switched.

The computer memory storage and running time required by this method are quite high.

2.3 THE FOURIER TRANSFORM METHOD

When a switching operation takes place, the elements of power system are subjected to voltages and currents having a wide range of frequency. The values of some electrical parameters do not remain constant but exhibit frequency dependency. While for some parameters (generator- and transformer-inductance, positive sequence line inductance) this variation is small or even negligible; other parameters (generator transformer resistance, line resistance, zero sequence line resistance and inductance) show a substantial variation with frequency, which is owing to skin effects and earth penetration [6]. Carson has shown that the mutual coupling, distortion and attenuation of travelling waves on the transmission line are frequency dependent. Hence the frequency dependence of parameters should be taken into account in the calculation of switching surges. This suggests the use of Fourier transforms method.

Fundamentally this method requires the calculation of the response of the system over a range of frequencies and the use of the inverse Fourier transform to transform the response from the frequency domain into time domain.

Fourier transform method has certain disadvantages associated with it. The analytical evaluation of inverse transform is very difficult to obtain, however, it can be evaluated numerically by integrating it within a finite range. This truncation of infinite range can give rise to Gibbs oscillations, which are quite pronounced and slow to die, and integrand to peak if the step length is large [16]. The results thus obtained will be peaky in nature and will not represent the true nature of system response.

The remedy to these problems as suggested by Day et.al.[17] is to use modified Fourier transform. The unwanted oscillations are removed by incorporating 'Sigma' factor in the transform.Battison et.al.[18] and Day [19] have demonstrated the use of this method for single phase and three-phase systems. Wedephol [20] has used a method which combines the modified Fourier transform and the steady-state theory of natural modes for the solution of line transient problem and discussed the problem of non-simultaneous closure of circuit breaker poles.

The disadvantage of Fourier transform method is the anticipated excessive computer running time resulting both from the calculation of frequency dependent transmission line solution of the problem at each frequency and also the multiple integration required to numerically evaluate inverse fourier transform. It also requires considerable data from the system which frequently are not available[21].

2.4 X-TRANSFORM METHOD

This method has been used by Raghavan and Sastry [22] for the switching surge overvoltage calculations. In this method reflection and refraction coefficients at all points of discontinuity and surge travel times of different lines are calculated. They are then represented by a block diagram. The transfer function of the system is determined with the help of system signal flow graph. The X-transform of the output surge is deduced using the transfer function. The surge voltage is found out by carrying the inverse X-transform. The main drawback of this method is the cumbersome and difficult evaluation of inverse X-transform for complex functions.

2.5 Z-TRANSFORM METHOD

In this method, used by Humpage [23] the transmission-line forward-impulse response and surge impedance function, initially formed in frequency domain, are mapped into Z-plane by bilinear transformation. They are then transformed into time domain, thereafter the formulation is wholly in the time domain and the sequences in solution, to which steps of transformation through the Z-plane lead, are of recursive form. It was found that this transformation is one which introduces a form of distortion error [24]. High accuracy in response function definition is achieved over an initial range of frequency beyond which the error progressively increases. This can be avoided by choosing a step length which minimises the error over the frequency range relevant to the electromagnetic transient made of system operation. But this leads to very high computer time. The other solution as suggested by Humpage [24] is to synthesize the transmission line forward impulse response and surge impedance function directly into the Z-plane. This method also leads to Z-plane function of lower order than those of previous work [23] and to longer step settings Both measures have considerably reduced the total computing time.

This method is still in a developing stage and has been applied to simple case only.

2.6 SYSTEM APPROACH METHOD

As the power system consists of a large number of elements, the differential equations describing the system are quite large. This renders their solution quite difficult. This difficulty is overcome by a nodal terminal approach put forward by Semlyen[25].

The simplification is due to the fact that while the total number of state variables in the power system are very large, the system is organized hierarchically, in an orderly way, by components with a small number of terminals interconnected algebraically by sparse matrix.

The mathematical description for any component of a power system is given by convolution using impulse matrix. A modelling by impulse response matrices is advantageous for complex components since it provides a simple input-output relationship involving few variables. They have developed norton type models for system components. The drawback of this method is the lengthy digital computions of impulse matrices requiring large computer running time.

2.7 LAPLACE TRANSFORM METHOD

This method for solution of travelling waves by laplace transform has been described by Uram et.al.[26,27]Application of Laplace transforms to equations for phase voltages produces six independent second order ordinary differential equations for voltages in terms of distance. They are separated by transforming the voltages into independent modes, which travel on line without interaction. On the assumption that the propagation coefficients are linear in the laplace operator; a simplified form of wave transmission results. For each mode, a wave launched onto one end of the line appears attenuated and delayed but undistorted. The phase voltage waves, however, are distorted since the modes have different velocities and attenuation factors. The mode with an earth return path travels at about three-quarters speed of the order modes, which is nearly that of light. Once the modal waves are known, the phase voltages are found by adding the forward and backward modal waves and using the inverse modal transformation.

Of the methods discussed so far the methods most commonly used are Schnyder-Bergeron, Fourier Transform, Lattice diagram, and Laplace transform as they are in a more developed stage as compared to other method. In assessing the differences between the above method of digital calculation, the most obvious basic difference is on the question of frequency dependence. The Schnyder-Bergeron and Laplace transform use fixed frequency parameters while the Fourier method accepts the continuous variation of parameters with frequency. The lattice diagram technique lies in between the two extremes as the earth responses are modified using Carsons formula. The Laplace transform method requires the minimum computer running time as compared to other methods. The results obtained are comparable with the results obtained from TNA and other methods [9].

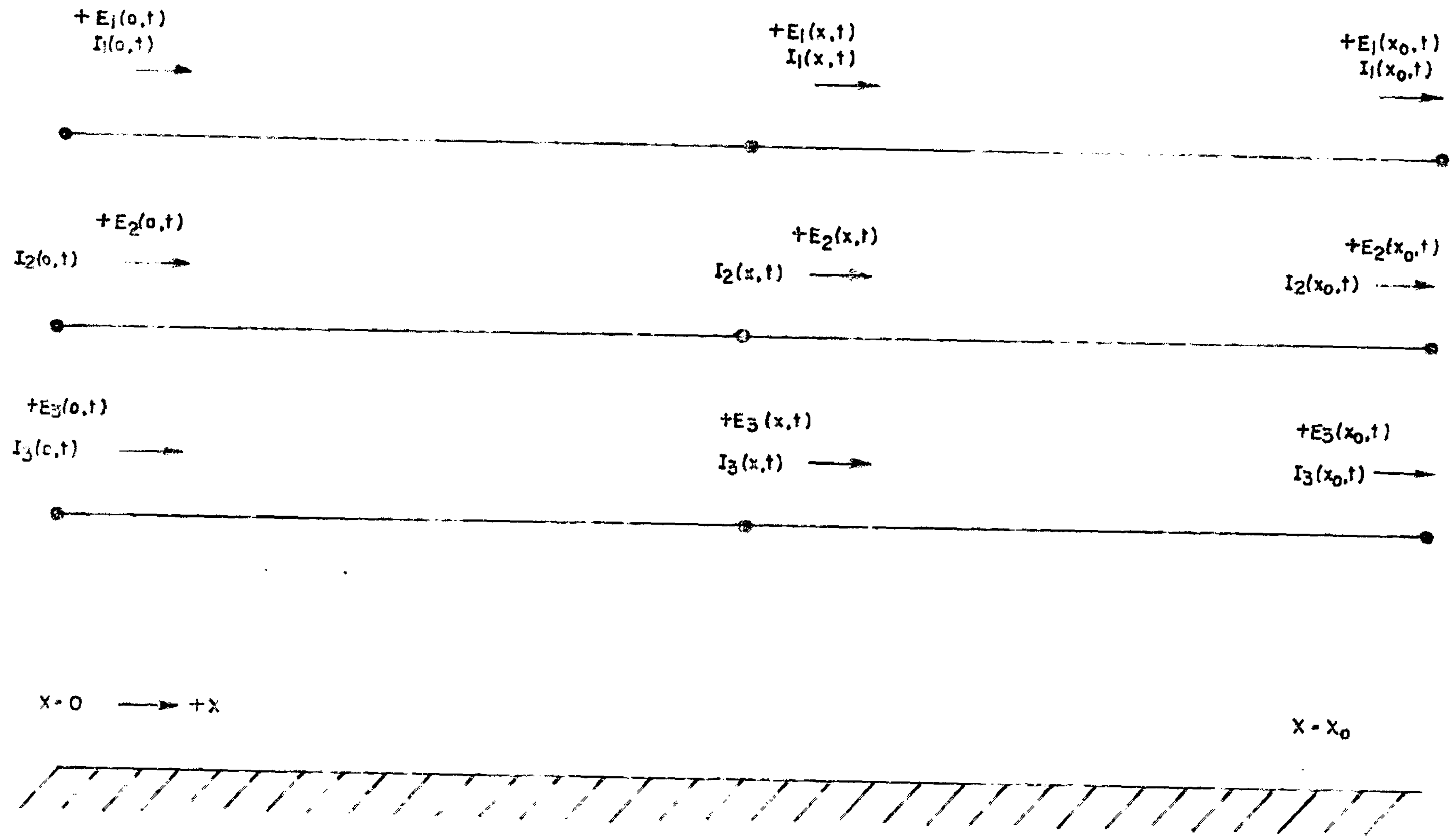

FIG.3.1 THREE PHASE TRANSMISSION LINE WITH EARTH RETURN.

CHAPTER-III

LAPLACE TRANSFORM TECHNIQUE FOR CALCULATION OF SWITCHING SURGE OVERVOLTAGES

3.1 3-PHASE LINE AND EQUIVALENT CIRCUIT

Consider a 3-phase transmission line consisting of three individual conductors parallel to earth as shown in Fig.[3.1].The left end of the line is considered as the sending end, where the generators are connected, while the recieving end, at the right hand, is at a distance of x_o miles. Normally the load will be considered here. The voltage with respect to ground at any point along the line, and the currents in conductors are to be determined. Each of these are functions of two variables: position x along the line, measured from some reference point, and the time t, measured from some reference time. The terminations at the recieving end will provide the boundary conditions necessary for solving the system equations.

The equivalent circuit, of a differential element of line used is shown in Fig.[3.2].In the circuit the overhead conductors are described by their positive sequence parameters, while the effects of the ground return are accounted for with their zero sequence parameters. The distributed parameter elements are used.

3.2 TRANSMISSION-LINE VOLTAGE AND CURRENT EQUATIONS

For the equivalent circuit shown in Fig.[3.2] two sets of describing equations can be derived. The first is found by applying Kirchoff's voltage law to the loop formed by each conductor

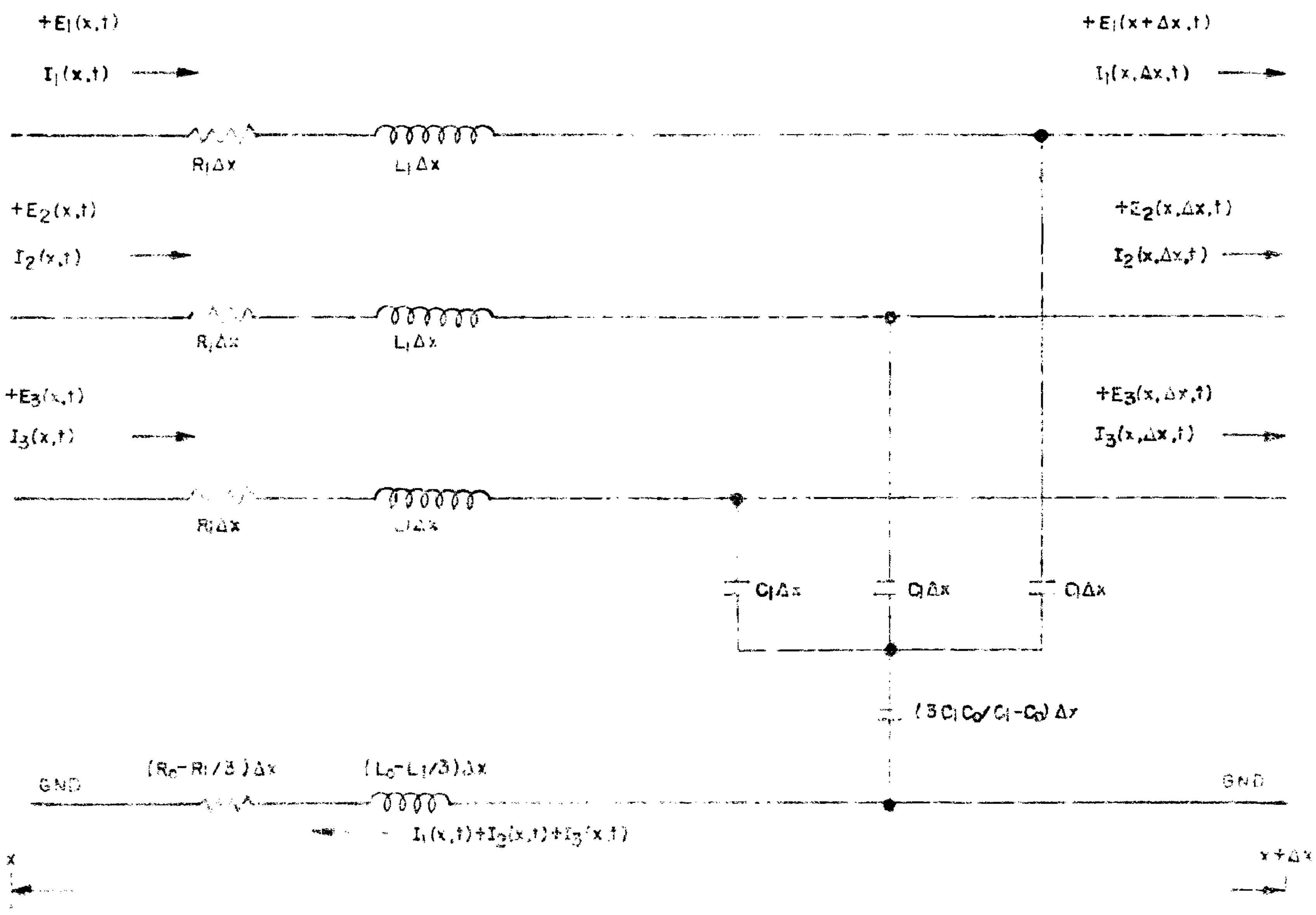

FIG 3.2 EQUIVALENT CIRCUIT FOR THREE PHASE TRANSMISSION LINE.

and ground, while a second set is written using Kirchoff's current law at the junction of each conductor with the capacitive branch to ground.

After rearranging the equations they may be arranged as follows:

$$
\left.
\begin{aligned}
-\frac{\partial E_1(x,t)}{\partial X} &= \tfrac{1}{3}\{[(R_o+L_o\tfrac{\partial}{\partial t})+2(R_1+L_1\tfrac{\partial}{\partial t})]I_1(x,t)+[(R_o+L_o\tfrac{\partial}{\partial t}) \\
&\quad -(R_1+L_1\tfrac{\partial}{\partial t})]I_2(x,t)+[(R_o+L_o\tfrac{\partial}{\partial t})-(R_1+L_1\tfrac{\partial}{\partial t})]I_3(x,t)\} \\
-\frac{\partial E_2(x,t)}{\partial X} &= \tfrac{1}{3}\{[(R_o+L_o\tfrac{\partial}{\partial t})-(R_1+L_1\tfrac{\partial}{\partial t})]I_1(x,t)+[(R_o+L_o\tfrac{\partial}{\partial t}) \\
&\quad +2(R_1+L_1\tfrac{\partial}{\partial t})]I_2(x,t)+[(R_o+L_o\tfrac{\partial}{\partial t})-(R_1+L_1\tfrac{\partial}{\partial t})]I_3(x,t)\} \\
-\frac{\partial E_3(x,t)}{\partial X} &= \tfrac{1}{3}\{[(R_o+L_o\tfrac{\partial}{\partial t})-(R_1+L_1\tfrac{\partial}{\partial t})]I_1(x,t)+[(R_o+L_o\tfrac{\partial}{\partial t}) \\
&\quad -(R_1+L_1\tfrac{\partial}{\partial t})]I_2(x,t)+[(R_o+L_o\tfrac{\partial}{\partial t})+2(R_1+L_1\tfrac{\partial}{\partial t})]I_3(x,t)\}
\end{aligned}
\right\} \quad (3.1)
$$

$$
\left.
\begin{aligned}
-\frac{\partial E_1(x,t)}{\partial t} &= \tfrac{1}{3}[(\tfrac{1}{c_o}+\tfrac{2}{c_1})\frac{\partial I_1(x,t)}{\partial X}+(\tfrac{1}{c_o}-\tfrac{1}{c_1})\frac{\partial I_2(x,t)}{\partial X} \\
&\quad +(\tfrac{1}{c_o}-\tfrac{1}{c_1})\frac{\partial I_3(x,t)}{\partial X}] \\
-\frac{\partial E_2(x,t)}{\partial t} &= \tfrac{1}{3}[(\tfrac{1}{c_o}-\tfrac{1}{c_1})\frac{\partial I_1(x,t)}{\partial X}+(\tfrac{1}{c_o}+\tfrac{2}{c_1})\frac{\partial I_2(x,t)}{\partial X} \\
&\quad +(\tfrac{1}{c_o}-\tfrac{1}{c_1})\frac{\partial I_3(x,t)}{\partial X}] \\
-\frac{\partial E_3(x,t)}{\partial t} &= \tfrac{1}{3}[(\tfrac{1}{c_o}-\tfrac{1}{c_1})\frac{\partial I_1(x,t)}{\partial X}+(\tfrac{1}{c_o}-\tfrac{1}{c_1})\frac{\partial I_2(x,t)}{\partial X} \\
&\quad +(\tfrac{1}{c_o}+\tfrac{2}{c_1})\frac{\partial I_3(x,t)}{\partial X}]
\end{aligned}
\right\} \quad (3.2)
$$

The partial differentials appearing in the equations (3.1) and (3.2) are converted into ordinary differentials by taking the Laplace transform with reference to time. The Laplace transform relationship resulting are

$$\left.\begin{aligned} \mathcal{L}\left(\frac{\partial E_1(x,t)}{\partial X}\right) &= \frac{dE_1(x,s)}{dx} \\ \mathcal{L}\left(\frac{\partial I_1(x,t)}{\partial t}\right) &= sI_1(x,s) - I(x,o^+) \end{aligned}\right\} \qquad (3.3)$$

Thus by taking the Laplace transform of the equations (3.1) and (3.2) and using the relationships of equation (3.3) with the assumption of zero initial condition, the following set of equations, which have been expressed in compact matrix form, results.

$$-\begin{bmatrix} \frac{dE_1(x,s)}{dx} \\ \frac{dE_2(x,s)}{dx} \\ \frac{dE_3(x,s)}{dx} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} (Z_o+2Z_1) & (Z_o-Z_1) & (Z_o-Z_1) \\ (Z_o-Z_1) & (Z_o+2Z_1) & (Z_o-Z_1) \\ (Z_o-Z_1) & (Z_o-Z_1) & (Z_o+2Z_1) \end{bmatrix}\begin{bmatrix} I_1(x,s) \\ I_2(x,s) \\ I_3(x,s) \end{bmatrix} \qquad (3.4)$$

$$-\begin{bmatrix} E_1(x,s) \\ E_2(x,s) \\ E_3(x,s) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} (\frac{1}{Y_o}+\frac{2}{Y_1}) & (\frac{1}{Y_o}-\frac{1}{Y_1}) & (\frac{1}{Y_o}-\frac{1}{Y_1}) \\ (\frac{1}{Y_o}-\frac{1}{Y_1}) & (\frac{1}{Y_o}+\frac{2}{Y_1}) & (\frac{1}{Y_o}-\frac{1}{Y_1}) \\ (\frac{1}{Y_o}-\frac{1}{Y_1}) & (\frac{1}{Y_o}-\frac{1}{Y_1}) & (\frac{1}{Y_o}+\frac{2}{Y_1}) \end{bmatrix}\begin{bmatrix} \frac{dI_1(x,s)}{dx} \\ \frac{dI_2(x,s)}{dx} \\ \frac{dI_3(x,s)}{dx} \end{bmatrix} \qquad (3.5)$$

The positive and zero-sequence impedances and admittances have been defined in equations(3.4) and (3.5) as follows:

$$\left.\begin{aligned} Z_o &= R_o + sL_o \\ Z_1 &= R_1 + sL_1 \\ Y_o &= sC_o \\ Y_1 &= sC_1 \end{aligned}\right\} \qquad (3.6)$$

The 3-phase transmission line relations can be expressed more explicitly by writing the equations (3.4) and (3.5) in an even more compact form as

$$-\frac{d}{dx}[E] = \frac{1}{3}[Z_A][I] \qquad (3.7.1)$$

$$-[E] = \frac{1}{3}[Z_B]\frac{d}{dx}[I] \qquad (3.7.2)$$

The voltages and current matrices of equations (3.7), which are 3-element column vectors, are expressed in the Laplace domain, and thus are function of the variable s as well as distance x from a reference point on the line.

3.3 GENERAL SOLUTION OF 3-PHASE TRANSMISSION LINE EQUATIONS

To obtain voltages and currents on the transmission line the pair of simultaneous matrix differential equations (3.7.1) and (3.7.2) need to be solved. This can be accomplished by eliminating either the voltage or current matrix, finding a solution for the remaining quantity, and the substituting this solution back in either of the two equations to obtain the complete set of voltages and currents.

To eliminate current matrix equation (3.7.1) is differentiated with respect to x and equation (3.7.2) is substituted in it. The resultant equation is given by

$$\frac{d^2}{dx^2}[E] - [Z_A][Z_B]^{-1}[E] = [0] \qquad (3.8)$$

Let $\quad [\alpha] = [Z_A][Z_B]^{-1}$ hence

$$\frac{d^2}{dX^2}[E] - [\alpha][E] = [0] \qquad (3.9)$$

where

$$[\alpha] = \frac{1}{3}\begin{bmatrix} (Z_oY_o+2Z_1Y_1) & (Z_oY_o-Z_1Y_1) & (Z_oY_o-Z_1Y_1) \\ (Z_oY_o-Z_1Y_1) & (Z_oY_o+2Z_1Y_1) & (Z_oY_o-Z_1Y_1) \\ (Z_oY_o-Z_1Y_1) & (Z_oY_o-Z_1Y_1) & (Z\ Y+2Z_1Y_1) \end{bmatrix}$$

The equation (3.9) represents three component equations involving the line voltages. It is difficult to solve this equation as it involves various combinations of the voltages of each line. To illustrate this, the equation (3.9) may be expanded into its components

$$\left.\begin{aligned} \frac{d^2E_1(x,s)}{dx^2} - \alpha_{11}\,E_1(x,s) - \alpha_{12}\,E_2(x,s) - \alpha_{13}\,E_3(x,s) &= 0 \\ \frac{d^2E_2(x,s)}{dx^2} - \alpha_{21}\,E_1(x,s) - \alpha_{22}\,E_2(x,s) - \alpha_{23}\,E_3(x,s) &= 0 \\ \frac{d^2E_3(x,s)}{dx^2} - \alpha_{31}\,E_1(x,s) - \alpha_{32}\,E_2(x,s) - \alpha_{33}\,E_3(x,s) &= 0 \end{aligned}\right\} \qquad (3.10)$$

The difficulty in solving this equation exists in the mathematical coupling between the voltages. To simply the solution the off-diagonal coefficient of this equation will have to be made identically zero. The equation, then resulting, will be soluable as it would contain one voltage and its ordinary second-order derivative only.

The actual transmission line voltages [E] are transformed linearly into a new set of variables [F] by a transformation

$$\left.\begin{array}{l}[E] = [T][F] \\ [F] = [T]^{-1}[E]\end{array}\right\} \quad (3.11)$$

Here the elements of the square (3 x 3) transformation matrix [T] are numerical constants.

Let the transformation of co-ordinates in equation (3.11) be substituted into the matrix differential equation (3.9), describing the transmission line voltages. After appropriate manipulation this yields

$$\frac{d^2}{dx^2}[F] - [T]^{-1}[\alpha][T][F] = 0 \quad (3.12)$$

To obtain the desired results the coefficient matrix product in equation (3.12) is diagonal. Indicating the co-efficients as as

$$[\xi] = [T]^{-1}[\alpha][T] \quad (3.13)$$

Replacing the coefficient matrix product in equation (3.12) by $[\xi]$ we get

$$\frac{d^2}{dx^2}[F] - [\xi][F] = 0 \quad (3.14)$$

Since $[\xi]$ is diagonal equation (3.14) can be expanded in to its component form as

$$\left.\begin{array}{l}\dfrac{d^2F_1(x,s)}{dx^2} - \xi_{11}\ F_1(x,s) = 0 \\ \dfrac{d^2F_2(x,s)}{dx^2} - \xi_{22}\ F_2(x,s) = 0 \\ \dfrac{d^2F_3(x,s)}{dx^2} - \xi_{33}\ F_3(x,s) = 0\end{array}\right\} \quad (3.15)$$

The coefficients ξ_{ii} remain to be determined. Briefly, the procedure consist of first finding the eigen-values, or characteristic roots, of the matrix $[\alpha]$; then, from these, the eigenvectors corresponding to each eigenvalues. The proper transformation matrix [T] is then composed of three column vectors, which are proportional to the eigenvectors of $[\alpha]$. The result of these matrix operations is the following transformation matrix [T] and its inverse

$$\left.\begin{aligned} [T] &= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix} \\ [T]^{-1} &= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ -1 & 2 & -1 \end{bmatrix} \end{aligned}\right\} \qquad (3.16)$$

The transformation in equation (3.16) is not unique because two of the eigenvalues are identical. Therefore, other transformations exist which will satisfy the system requirements.

The expansion of equation (3.13), using equations (3.9) and (3.16) results in

$$[\xi] = [T]^{-1}[\alpha][T] = \begin{bmatrix} Z_oY_o & 0 & 0 \\ 0 & Z_1Y_1 & 0 \\ 0 & 0 & Z_1Y_1 \end{bmatrix} \qquad (3.17)$$

Thus the $[\xi]$ matrix is diagonal and insertion of this matrix into the system relations given by equation (3.15) leads to a set of three differential equations to be solved

$$\left.\begin{aligned} \frac{d^2F_1(x,s)}{dx^2} - (Z_oY_o)F_1(x,s) &= 0 \\ \frac{d^2F_2(x,s)}{dx^2} - (Z_1Y_1)F_2(x,s) &= 0 \\ \frac{d^2F_3(x,s)}{dx^2} - (Z_1Y_1)F_3(x,s) &= 0 \end{aligned}\right\} \quad (3.18)$$

Since the coefficients (Z_oY_o) are not functions of displacement x, equation (3.18) may be solved for the transformed co-ordinates.

$$\left.\begin{aligned} F_1(x,s) &= K_{11}\,\varepsilon^{-\sqrt{Z_oY_o}\,x} + K_{12}\,\varepsilon^{+\sqrt{Z_oY_o}\,x} \\ F_2(x,s) &= K_{21}\,\varepsilon^{-\sqrt{Z_1Y_1}\,x} + K_{22}\,\varepsilon^{+\sqrt{Z_1Y_1}\,x} \\ F_3(x,s) &= K_{31}\,\varepsilon^{-\sqrt{Z_1Y_1}\,x} + K_{32}\,\varepsilon^{+\sqrt{Z_1Y_1}\,x} \end{aligned}\right\} \quad (3.19)$$

The constants of integration K_{ij} must be determined before attempting an inverse transformation into time domain. These constants are dependent on the boundary conditions existing at each end of line. Rewriting equation (3.19) in matrix form as

$$[F] = [K_1\varepsilon_x^-] + [K_2\varepsilon_x^+] \quad (3.20)$$

where

$$[K_1\,\varepsilon_x^-] = \begin{bmatrix} K_{11}\,\varepsilon^{-\sqrt{Z_cY_o}\,x} \\ K_{21}\,\varepsilon^{-\sqrt{Z_1Y_1}\,x} \\ K_{31}\,\varepsilon^{-\sqrt{Z_1Y_1}\,x} \end{bmatrix} \quad (3.21)$$

$$[K_2 \varepsilon_x^+] = \begin{bmatrix} K_{12}\, \varepsilon^{+\sqrt{Z_oY_o}\; x} \\ K_{22}\, \varepsilon^{+\sqrt{Z_1Y_1}\; x} \\ K_{32}\, \varepsilon^{+\sqrt{Z_1Y_1}\; x} \end{bmatrix} \qquad (3.21)$$

Finally, the solution for the actual line voltage may be expressed as:

$$[E] = [T][F] = [T][K_1 \varepsilon_x^-] + [T][K_2 \varepsilon_x^+] \qquad (3.22)$$

To obtain the line current the equation (3.7.1) is solved for the line current

$$[I] = -3[Z_A]^{-1} \frac{d}{dX} [E] \qquad (3.23)$$

Differentiating equation (3.22) and then substituting it in equation (3.23) provides the general solution for line current as

$$[I] = [T][\Omega]^{-1}[K_1 \varepsilon_x^-]-[T][\Omega]^{-1}[K_2 \varepsilon_x^+] \qquad (3.24)$$

The matrix $[\Omega]$ is diagonal and is composed of transmission line parameters.

$$[\Omega] = \begin{bmatrix} \Omega_o & 0 & 0 \\ 0 & \Omega_1 & 0 \\ 0 & 0 & \Omega_1 \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{Z_o}{Y_o}} & 0 & 0 \\ 0 & \sqrt{\frac{Z_1}{Y_1}} & 0 \\ 0 & 0 & \sqrt{\frac{Z_1}{Y_1}} \end{bmatrix} \qquad (3.25)$$

3.3.1 <u>Characteristic Impedance and Attenuation Constant</u>: The matrix $[\Omega]$ is the characteristic impedance of the 3-phase line. This is a quantity which is independent of the line voltages or currents and is a function of line parameters only. Although the exact form of the characteristic impedance is complex, the reduction of the exact expression can be made using the assumptions normally made in power transmission line work.

From the equation (3.25)

$$\Omega_o = \sqrt{\frac{R_o + sL_o}{G_o + sC_o}} \qquad (3.25.1)$$

For zero conductance it becomes

$$\Omega_o = \sqrt{\frac{L_o}{C_o}}\left(1+\frac{R_o}{sL_o}\right)^{1/2} = \sqrt{\frac{L_o}{C_o}}\left(1+\frac{1}{2}\frac{R_o}{sL_o}-\frac{1}{8}\frac{R_o^2}{s^2L_o^2}+\frac{1}{16}\frac{R_o^3}{s^3L_o^3}-\ldots\right) \qquad (3.25.2)$$

substituting $s = j\omega$ the first three terms of the infinite series become

$$\Omega_o = \sqrt{\frac{L_o}{C_o}}\left[1+\frac{1}{8}\left(\frac{R_o}{L_o\omega}\right)^2-\frac{1}{2}\,j\left(\frac{R_o}{L_o\omega}\right)\right] \qquad (3.26)$$

Choosing 50 Hz as the lowest frequency of interest, it is observed that for the system data used the terms other than unity have negligible effect on the magnitude and phase of Ω_o. Therefore all terms other than unity in the series are neglected. Hence

$$\Omega_o = \sqrt{\frac{L_o}{C_o}}$$

similarly

$$\Omega_1 = \sqrt{\frac{L_1}{C_1}} \qquad (3.27)$$

$$\gamma_o = \sqrt{(R_o+sL_o)(G_o+sC_o)} \qquad (3.28)$$

$$\gamma_o = s\sqrt{L_oC_o}\,\left(1+\frac{R_o}{sL_o}\right)^{1/2} = \sqrt{L_oC_o}\,\left(s+\frac{1}{2}\frac{R_o}{L_o}-\frac{1}{8}\frac{R_o^2}{sL_o^2}+\frac{1}{16}\frac{R^3}{s^2L_o^3}-\ldots\right) \qquad \ldots (3.28.1)$$

substituting $s = j\omega$ the first three terms of the infinite series become

$$\gamma_o = \sqrt{L_oC_o}\,\left\{\frac{1}{2}\frac{R_o}{L_o} + j\omega\left[1+\frac{1}{8}\left(\frac{R_o}{L_o\omega}\right)^2\right]\right\} \qquad (3.28.2)$$

using 50 Hz as a lower limit on frequency, evaluation demonstrates that the first two terms of the series should be retained, which yields

$$\gamma = s\sqrt{L_oC_o} + \frac{1}{2}R_o\sqrt{C_o/L_o} \qquad (3.28.3)$$

Rewriting the exponents of equation (3.19) gives

$$\left.\begin{aligned} \varepsilon^{-\gamma_o x} &= \epsilon^{-\frac{R_o}{2}\sqrt{\frac{C_o}{L_o}}\,x}\ \varepsilon^{-\sqrt{L_oC_o}\,sx} \\ \varepsilon^{-\gamma_1 x} &= \epsilon^{-\frac{R_1}{2}\sqrt{\frac{C_1}{L_1}}\,x}\ \varepsilon^{-\sqrt{L_1C_1}\,sx} \end{aligned}\right\} \qquad (3.29)$$

These exponentials have two parts; one involves constants and line displacement, while the other involves the Laplace variable s.The first of these terms decreases with distance along the line and thus represents attenuation of voltage and current. The second represents a delay factor since it is a term of the form $\varepsilon^{-\sqrt{LC}\,xs}$. This, from classical Laplace transform theory, produces a finite time delay of $\sqrt{LC}\,x$ seconds.

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3.4 PARTICULAR SOLUTION FOR 3-PHASE TRANSMISSION LINE EQUATIONS

To find particular solutions for the transmission-line voltages and currents, the matrix constants of integration occurring in equation 3.22 and 3.24 have to determined. Since this needs the evaluation of the boundary conditions at each end of the line it will be necessary to substitute the condition x = o at the sending end and $x = x_o$ at the recieving end. The resulting forms are:

$$\left.\begin{aligned} [E_o] &= [T][K_1] + [T][K_2] \\ [I_o] &= [T][\Omega]^{-1}[K_1] - [T][\Omega]^{-1}[K_2] \end{aligned}\right\} \quad (3.30)$$

$$\left.\begin{aligned} [E_{xo}] &= [T][K_1\, \varepsilon^{-}_{xo}] + [T][K_2\, \varepsilon^{+}_{xo}] \\ [I_{xo}] &= [T][\Omega]^{-1}[K_1\, \varepsilon^{-}_{xo}] - [T][\Omega]^{-1}[K_2\, \varepsilon^{+}_{xo}] \end{aligned}\right\} \quad (3.31)$$

Let

$$[A] = \begin{bmatrix} A_{11} \\ A_{12} \\ A_{13} \end{bmatrix} = [K_1\, \varepsilon^{-}_{xo}] = \begin{bmatrix} K_{11}\, \varepsilon^{-\gamma_o x_o} \\ K_{12}\, \varepsilon^{-\gamma_1 x_o} \\ K_{13}\, \varepsilon^{-\gamma_1 x_o} \end{bmatrix} \quad (3.32)$$

and $$[B] = \begin{bmatrix} B_{21} \\ B_{22} \\ B_{32} \end{bmatrix} = [K_2\, \varepsilon^{+}_{xo}] = \begin{bmatrix} K_{21}\, \varepsilon^{+\gamma_o x_o} \\ K_{21}\, \varepsilon^{+\gamma_1 x_o} \\ K_{23}\, \varepsilon^{+\gamma_1 x_o} \end{bmatrix} \quad (3.33)$$

Since equation (3.33) has a positive exponential, it is desirable to rewrite the expression in such a way that a negative exponential occurs. This may be accomplished as follows:

$$[K_2] = \begin{bmatrix} K_{21} \\ K_{22} \\ K_{23} \end{bmatrix} = [B\,\varepsilon^{-}_{xo}] = \begin{bmatrix} B_{21}\,\varepsilon^{-\gamma_o x_o} \\ B_{22}\,\varepsilon^{-\gamma_1 x_o} \\ B_{32}\,\varepsilon^{-\gamma_1 x_o} \end{bmatrix} \quad (3.34)$$

Suppose that the first element of the matrices in the equations (3.32) and (3.34) are expanded and written as follows:

$$\left.\begin{aligned} A_{11}(s) &= K_{11}(s)\,\varepsilon^{-\frac{R_o}{2}\sqrt{\frac{C_o}{L_o}}\,x_o}\,\varepsilon^{-\sqrt{L_oC_o}\,x_o\,s} \\ K_{21}(s) &= B_{21}(s)\,\varepsilon^{-\frac{R_o}{2}\sqrt{\frac{C_o}{L_o}}\,x_o}\,\varepsilon^{-\sqrt{L_oC_o}\,x_o\,s} \end{aligned}\right\} \quad (3.35)$$

The leading exponential in these equations is a real number and represents the attenuation along the line. However the second exponential is dependent on the variable s. The basic theory of Laplace transforms defines such a situation as a delay function which must be zero for a finite time.

The inverse laplace transform of the general function of the type $G(s) = \varepsilon^{-T_o s}\,G_1(s)$ will be

$$g(t) = g_1(t)\;U\;(t-T_o) \quad (3.36)$$

where $U(t-T_o)$ is delayed unit step.

On the similar lines the constant inequation (3.35) can be written in time domain as:

$$\left.\begin{aligned} A_{11}(t) &= \varepsilon^{-\frac{R_o}{2}\sqrt{\frac{C_o}{L_o}}\,x_o}\;K_{11}(t-\sqrt{L_oC_o}\,x_o)U(t-\sqrt{L_oC_o}\,x_o) \\ K_{21}(t) &= \varepsilon^{-\frac{R_o}{2}\sqrt{\frac{C_o}{L_o}}\,x_o}\;B_{21}(t-\sqrt{L_oC_o}\,x_o)U(t-\sqrt{L_oC_o}\,x_o) \end{aligned}\right\} \quad (3.37)$$

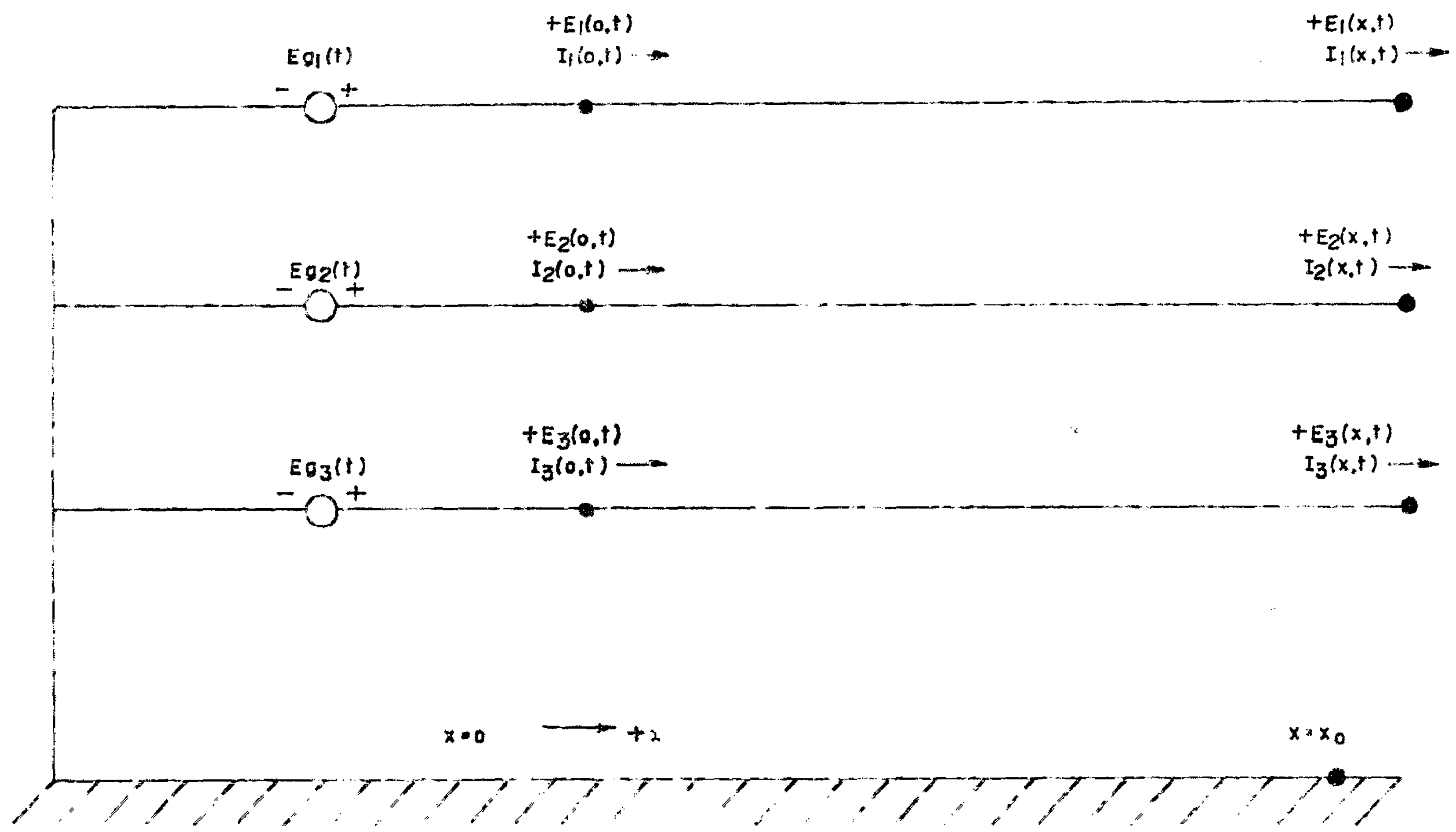


FIG.3.3 THREE PHASE LINE WITH RECEIVING END OPEN AND GENERATION AT SENDING END.

The constants $A_{11}(t)$ and $K_{21}(t)$ must be zero for $\sqrt{L_oC_o}\ x_o$ seconds after which they will have the form of $K_{11}(t)$ and $B_{21}(t)$ respectively.

In an identical manner the other elements of equations (3.32) and (3.34) can be written in time domain.

The transmission-line voltages and current at each end of the line now may be written, using the definitions in equation (3.32) and (3.34), as

$$\left.\begin{aligned} [E_o] &= [T][K_1] + [T][K_2] \\ [I_o] &= [T][\Omega]^{-1}[K_1] - [T][\Omega]^{-1}[K_2] \end{aligned}\right\} \quad (3.38)$$

$$\left.\begin{aligned} [E_{xo}] &= [T][A] + [T][B] \\ [I_{xo}] &= [T][\Omega]^{-1}[A] - [T][\Omega]^{-1}[B] \end{aligned}\right\} \quad (3.39)$$

3.5 INCREMENTAL SOLUTION OF TRANSMISSION LINE EQUATIONS

3.5.1 Unloaded Line: To illustrate the method of solving transmission line equations (3.38) and (3.39) a simple system will be considered. Suppose that the voltages at the sending end and the currents at the recieving end of line are known. This would be the case, for instance, if a 3-phase generator were connected at the sending end and the line were open at the recieving end Fig[3.3]. Then the quantities to be determined are the currents at the sending end and the voltages at the recieving end of the line.

Suppose that the first part of the equation (3.38) is solved for the matrix $[K_1]$ and the second part of equation (3.39) is solved for the matrix $[B]$ as follows:

$$\left.\begin{aligned} [K_1] &= [T]^{-1}[E_o] - [K_2] \\ [B] &= [A] - [\Omega][T]^{-1}[I_{xo}] \end{aligned}\right\} \quad (3.40)$$

Performing the inverse Laplace transformation to the time domain yields

$$\left.\begin{aligned}[K_1(t)] &= [T]^{-1}[E(0,t)] - [K_2(t)] \\ [B(t)] &= [A(t)] - [\Omega][T]^{-1}[I(x_o,t)]\end{aligned}\right\} \quad (3.41)$$

The equation (3.41) is solved on an incremental basis with time starting at t = o and increasing in increments Δt. For first few increments $[K_2(t)]$ and $[A(t)]$ must be zero since they are delay functions hence the above equation can be evaluated for $[K_1(t)]$ and $[B(t)]$ since the term on the right hand are either known or are zero. These values of $[K_1(t)]$ and $[B(t)]$ are stored for evaluation of delay function when the time delay is over. Once the delay functions are no longer zero, the proper value of $[A(t)]$ and $[K_2(t)]$ are determined from the past value of $[K_1(t)]$ and $[B(t)]$. These are substituted in equation (3.41) to provide the present value of $[K_1(t)]$ and $[B(t)]$ which are stored for further determination of delay functions.

Solution for the unknown sending end currents and recieving end voltages may now be obtained from the inverse Laplace transform of second part of equation (3.28) and the first part of the equation (3.29).

$$\left.\begin{aligned}[I(0,t)] &= [T][\Omega]^{-1}[K_1(t)] - [T][\Omega]^{-1}[K_2(t)] \\ [E(x_o,t)] &= [T][A(t)] + [T][B(t)]\end{aligned}\right\} \quad (3.42)$$

3.5.2 <u>Resistive Load Termination</u>: Consider a transmission line terminated at the receiving end with a resistive load which may be balanced or unbalanced and assume that the sending end is connected to an infinite bus Fig. (3.4). The boundary conditions

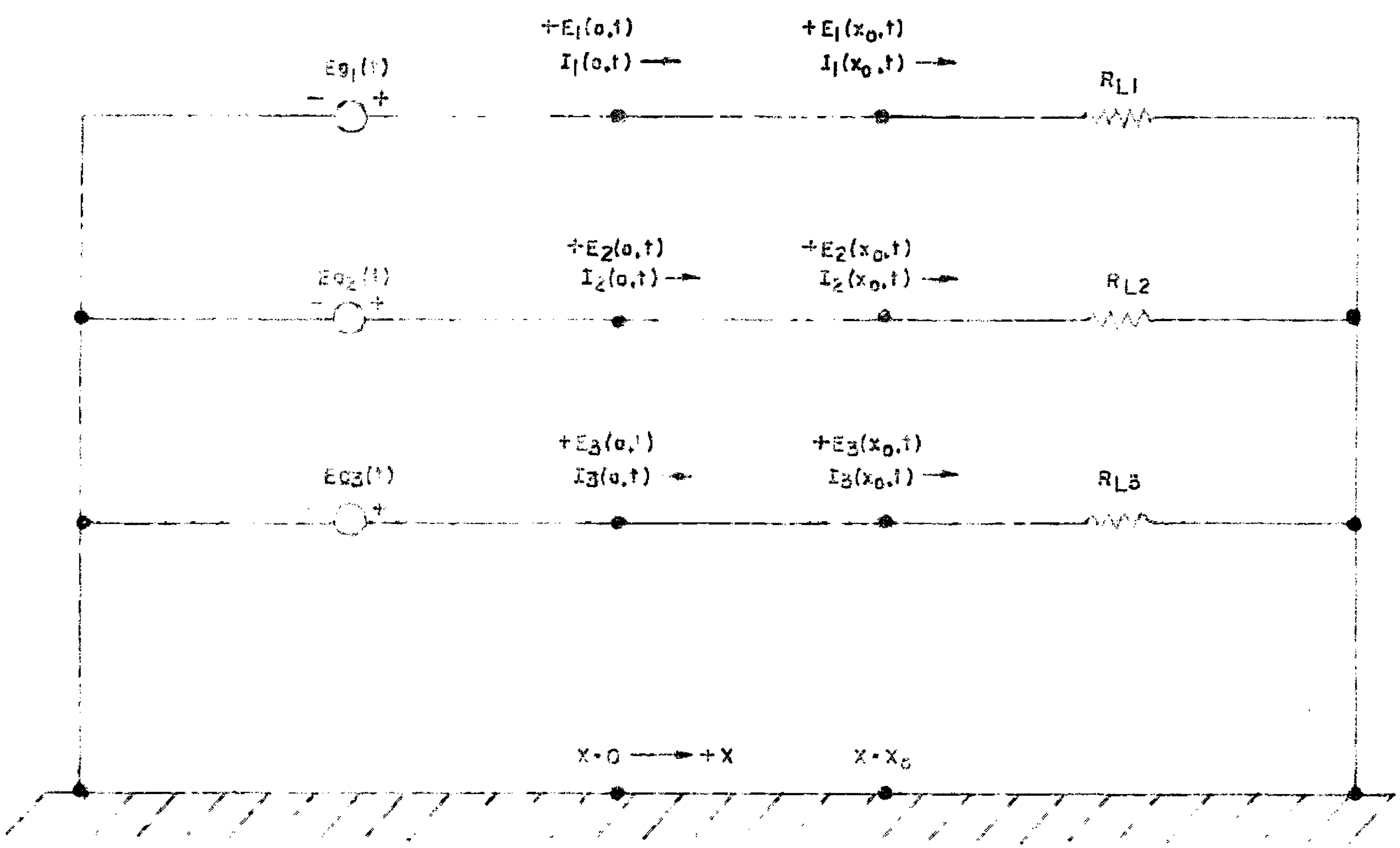


FIG.3.4 TRANSMISSION LINE WITH RESISTIVE LOAD AND INFINITE BUS.

for such a system thus consists of known voltages at the sending end while the unknown receiving-end voltages and currents are related by the load resistances. Using simplified notations the relation at the receiving-end can be written as:

$$\left.\begin{aligned} [E_{xo}] &= [R_L][I_{xo}] \\ \text{where} \quad [R_L] &= \begin{bmatrix} R_{L1} & 0 & 0 \\ 0 & R_{L2} & 0 \\ 0 & 0 & R_{L3} \end{bmatrix} \end{aligned}\right\} \tag{3.43}$$

substituting the expressions for receiving-end voltage and currents from equation (3.39) after transformation into time domain and using simplified notations we get

$$[T][A] + [T][B] = [R_L][T][\Omega]^{-1}[A] - [R_L][T][\Omega]^{-1}[B] \tag{3.44}$$

solving this equation for [B] matrix we obtain

$$[B] = \left[[T]^{-1}[R_L][T][\Omega]^{-1} + [\mathcal{I}]\right]^{-1} \times \left[[T]^{-1}[R_L][T][\Omega]^{-1} - [\mathcal{I}]\right][A] \tag{3.45}$$

Here the matrix $[\mathcal{I}]$ is the diagonal unit matrix.

Matrix $[K_1]$ can be determined from the first part of equation (3.40) while the delay matrices can be calculated from the stored value of $[K_1]$ and [B]. The receiving-end voltage and current can be found out by substituting the values of the calculated matrices into the following equations:

$$\left.\begin{aligned} [E_{xo}] &= [T]\left[[A]+[B]\right] \\ [I_{xo}] &= [T][\Omega]^{-1}\left[[A]-[B]\right] \end{aligned}\right\} \tag{3.46}$$

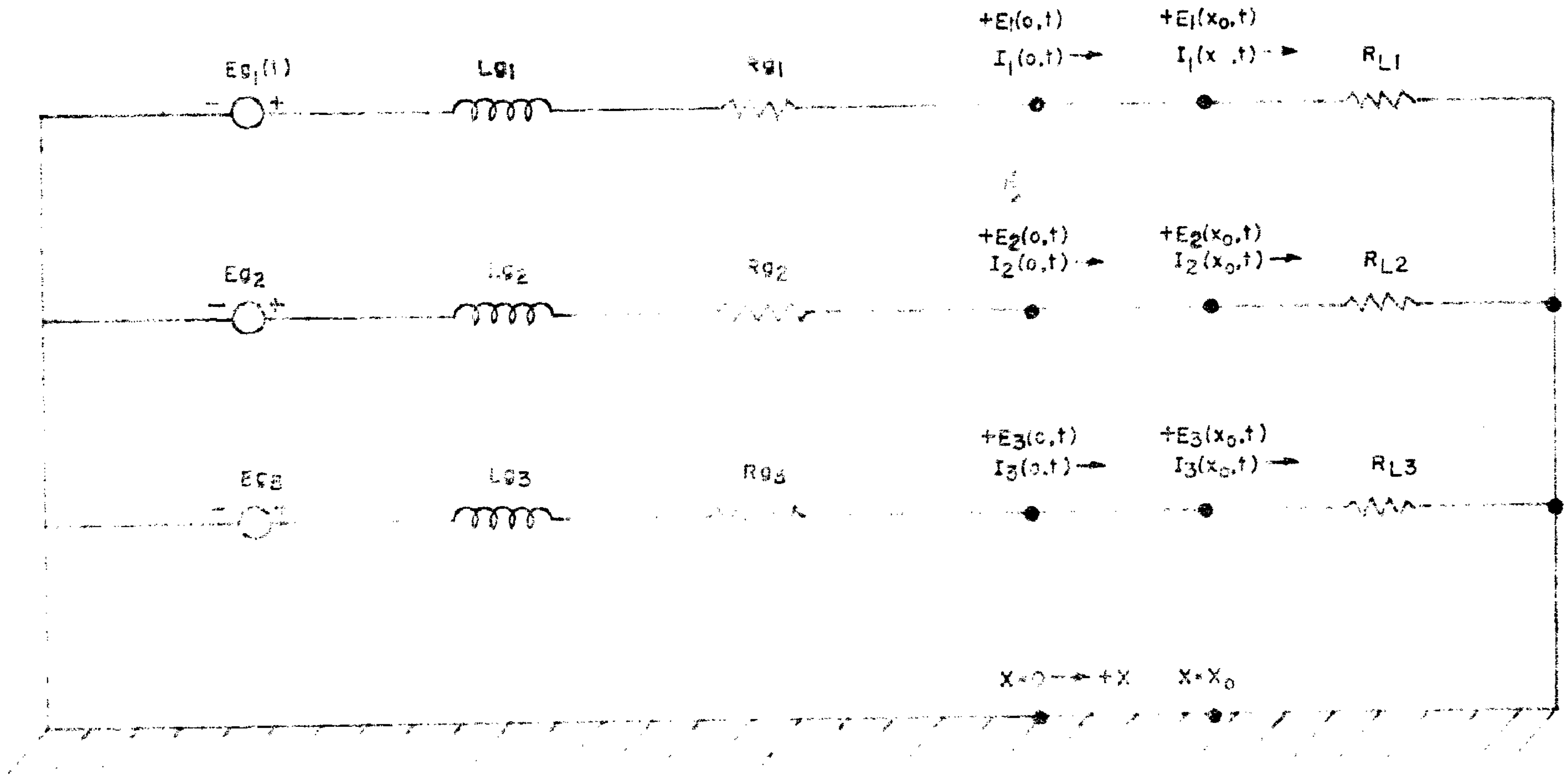


FIG. 3.5 TRANSMISSION LINE WITH RESISTIVE LOAD AND GENERATOR WITH SERIES IMPEDANCE.

3.5.3 Inclusion of Generator Impedance: In the cases discussed so far, it was assumed that the line is energized from an infinite bus source. A more realistic form of representation is necessary, taking into account the source representation. In dealing with switching surge overvoltages, the source side plays a very important role in the shape of the wave-form and the magnitude of the overvoltage. Hence the generator impedance is considered at the sending end as shown in Fig.3.5. The load resistances may be balanced or unbalanced, while the generator impedances may be symmetrical or not.

The unknown quantities are the transmission-line voltages and currents at both ends. The known quantities are the generator voltages and the constraints which have been imposed on the system by the series impedances at the sending end and the load resistances at the receiving-end.

The general transmission line equations are the equations (3.38) and (3.39) transformed into time domain. At the receiving of the system, the boundary equations are identical to equation (3.43), hence the matrix [B] can be calculated from equation (3.45). The matrix $[K_1]$ may be evaluated by writing the boundary equations at the sending end of the line. Referring to Fig.3.5, these may be expressed in matrix form as follows:

$$\left.\begin{aligned} [E_g] &= [L_g]\frac{d}{dt}[I_o] + [R_g][I_o] + [E_o] \\ \text{where} \quad [L_g] &= \begin{bmatrix} L_{g1} & 0 & 0 \\ 0 & L_{g2} & 0 \\ 0 & 0 & L_{g3} \end{bmatrix} \end{aligned}\right\} \qquad (3.46)$$

$$[R_g] = \begin{bmatrix} R_{g1} & 0 & 0 \\ 0 & R_{g2} & 0 \\ 0 & 0 & R_{g3} \end{bmatrix} \tag{3.46}$$

To determine $[K_1]$, the time domain transformed equations of sending end matrices $[E_o]$ and $[I_o]$, obtained from equations (3.38) are substituted into the differential equation (3.46).

$$[E_g] = [L_g]\,\frac{d}{dt}\,[T][\Omega]^{-1}\Big[[K_1]-[K_2]\Big]+[R_g][T][\Omega]^{-1}\Big[[K_1]-[K_2]\Big] + [T]\Big[[K_1]+[K_2]\Big] \tag{3.47}$$

Rearranging and solving equation (3.47) for $[K_1]$ we get

$$\frac{d}{dt}[K_1] = \frac{d}{dt}[K_2]+[V]\Big[[T]^{-1}[E_g]-[W][K_1]+[X][K_2]\Big]$$

where

$$[V] = \Big[[T]^{-1}[L_g][T][\Omega]^{-1}\Big]^{-1}$$

$$[W] = \Big[[T]^{-1}[R_g][T][\Omega]^{-1}+[\mathcal{I}]\Big] \tag{3.48}$$

$$[X] = \Big[[T]^{-1}[R_g][T][\Omega]^{-1}-[\mathcal{I}]\Big]$$

The matrix $[K_1]$ can be solved by using a numerical method for the solution of such differential equation. Details of solution procedure is given in Appendix-I. The delay matrices are evaluated as before and once all the matrices are known the unknown quantities can be determined from the generalized transmission line equation (3.46).

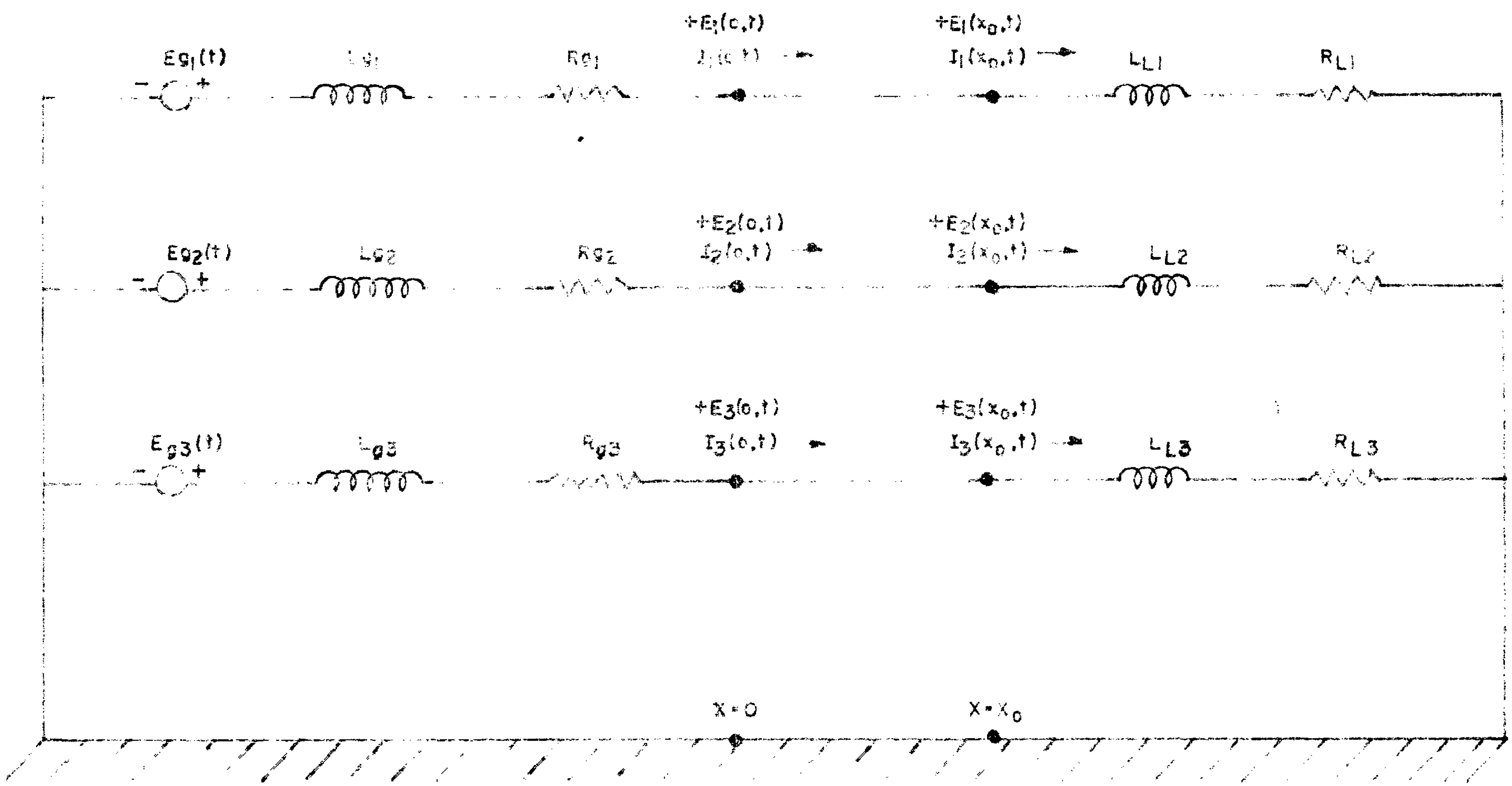


FIG.3.6 TRANSMISSION LINE WITH INDUCTIVE LOAD AND GENERATOR WITH SERIES IMPEDANCE.

3.5.4 Inductive Load Terminations: To include the lagging power factor loads at receiving end consider the Fig.3.6, where the load and the source impedance are assumed unbalanced for generality. The line voltages and currents at both ends are unknown while the generator voltages are known as function of time. At sending end of the line the situation is identical to the resistive termination case thus, solving the differential equation (3.48) results in matrix $[K_1]$.

At the receiving end the relationship between the voltages and currents is governed by the load resistance and the inductance. This may be written in matrix form as

$$[E_{xo}] = [L_L] \frac{d}{dt} [I_{xo}] + [R_L][I_{xo}] \tag{3.49}$$

where

$$[L_L] = \begin{bmatrix} L_{L1} & 0 & 0 \\ 0 & L_{L2} & 0 \\ 0 & 0 & L_{L3} \end{bmatrix}$$

$$[R_L] = \begin{bmatrix} R_{L1} & 0 & 0 \\ 0 & R_{L2} & 0 \\ 0 & 0 & R_{L3} \end{bmatrix}$$

Substitution of the transformed equation (3.38) and (3.39) in to equation(3.49)for the values of receiving end quantities leads to:

$$[T]\Big[[A]+[B]\Big]=[L_L]\frac{d}{dt}[T][\Omega]^{-1}\Big[[A]-[B]\Big]+[R_L][T][\Omega]^{-1}\Big[[A]-[B]\Big] \quad \ldots(3.50)$$

Rearranging the equation and solving for [B] gives

$$\left.\begin{aligned} \frac{d}{dt}[B] &= \frac{d}{dt}[A] + [\alpha]\Big[[\beta][A] - [\gamma][B]\Big] \\ \text{where} \\ [\alpha] &= \Big[[T]^{-1}[L_L][T][\Omega]^{-1}\Big]^{-1} \\ [\beta] &= \Big[[T]^{-1}[R_L][T][\Omega]^{-1} - [g]\Big] \\ [\gamma] &= \Big[[T]^{-1}[R_L][T][\Omega]^{-1} + [g]\Big] \end{aligned}\right\} \qquad (3.51)$$

This differential equation must be solved in order to evaluate the [B] matrix: this is in addition to the differential equation at the sending end for $[K_1]$ equation 3.48. The method of solving these equations has been discussed in Appendix-I.

CHAPTER-IV

SYSTEM CONSIDERED AND CASES STUDIED

4.1 SYSTEM CONSIDERED

A Hydro-electric power station with 4 units of 165 MW each in stage 1 has been provided at Dehar under the Beas Project. This power station is connected by a 400 KV single circuit transmission line to Panipat at a distance of 260 km. A study of switching surge overvoltage of this system is done hereunder for 2 units of 165 MW in operation.

4.2 SYSTEM DATA

4.2.1 Line Parameters: The line comprises of twin conductor bundles per phase and two galvanised steel overhead wires. The line has a delta configuration. The zero - and positive - sequence parameters of the line are as given below:

Positive sequence Parameters	
L_1 mH/Km	1.0143
C_1 nF/Km	11.3040
R_1 mΩ/Km	29.2560
Time Delay ms	0.8800

Zero sequence Parameters	
L_0 mH/Km	3.1200
C_0 nF/Km	7.7618
R_0 mΩ/Km	230.9300
Time Delay ms	1.2700

4.2.2 Generator Parameters: In these studies the generator has been represented by its subtransient reactances. Its value is 0.1147 p.u. on 100 MVA base. For two generators the source is represented by the parallel combination of generator and transformer impedances.

4.2.3 Generator Transformer Parameters: The transformer impedance for 3 x 60 MVA, $\frac{11/400}{\sqrt{3}}$ KV has an average value of 15 % on 180 MVA base.

4.3 CASES STUDIED

4.3.1 Unloaded Line Energization: In this case the line is assumed to be open at the receiving end. This is simulated by inserting a resistance of 10^6 pu at receiving end in all the three phases. The three poles of the sending end circuit breaker are assumed to close simultaneously at t = 0.

4.3.2 Unloaded Line Energization With Non Simultaneous Closure of Circuit Breaker Poles: The three poles of a circuit breaker do not close simultaneously, due to mechanical tolerances and prestrikes, but close within a 'pole-span' in a random manner. The pole span which is the characteristic of the circuit breaker is the time between the first pole and last pole to close. There can exist a very large number of pole closing sequences for different pole closing spans and study of all the possibilities is very time consuming. In this case, just to demonstrate the affect of non-simultaneous closure of circuit breaker pole, a pole closing span of 90° has been selected, with pole A closing at 45°, pole B at 90°, pole C at 135° from the t = 0 reference point.

4.3.3 Unloaded Line Energization With Pre-insertion Circuit Breaker Resistances: To reduce the overvoltages, line is energized via closing resistors. These resistances are shorted out of the circuit after a pre-determined time. In this study the value chosen for pre-insertion resistance is 400 Ω and the insertion time is 10 milliseconds.

4.3.4 Line Energization With Balanced Resistive Load at Receiving End: A balanced load of 1 p.u. in each phase is assumed at the receiving end. In this case the receiving and sending end breakers are assumed to close simultaneously.

4.3.5 Load Rejection at the Receiving End: In the study of the overvoltage due to load rejection at the receiving end the following procedure is adopted. The system is first allowed to reach the steady state. After this condition is reached the phase currents are monitered and when the current of any of the three phases, passes through zero that phase is opened, and this process continues till all the phases are open.

4.3.6 Unbalanced Conditions: An extreme unbalanced condition, wherein the line is completely open at the receiving end while one phase of the sending end is also open has been studied. In practice this might correspond is a situation in which one pole of the breaker at the sending end did not close.

The last experimental case of the resistive load is an extreme unbalance at the receiving end of the line. In this case one phase is assumed open while the others have 1.0 per-unit loads.

4.3.7 Line Energization With Balanced Inductive Load at the Receiving End: In this case a balanced inductive load of 1 p.u. consisting of 0.6 p.u. resistance and 0.8 p.u. reactance has been considered at the receiving end.

4.3.8 Line Energization With Unloaded Transformer at the Receiving End: In the study of overvoltage due to energization of the line with unloaded transformer at the receiving end, only the transformer magnetizing inductance has been considered neglecting all the losses. The value of transformer magnetizing reactance is 125 p.u. on 100 MVA base.

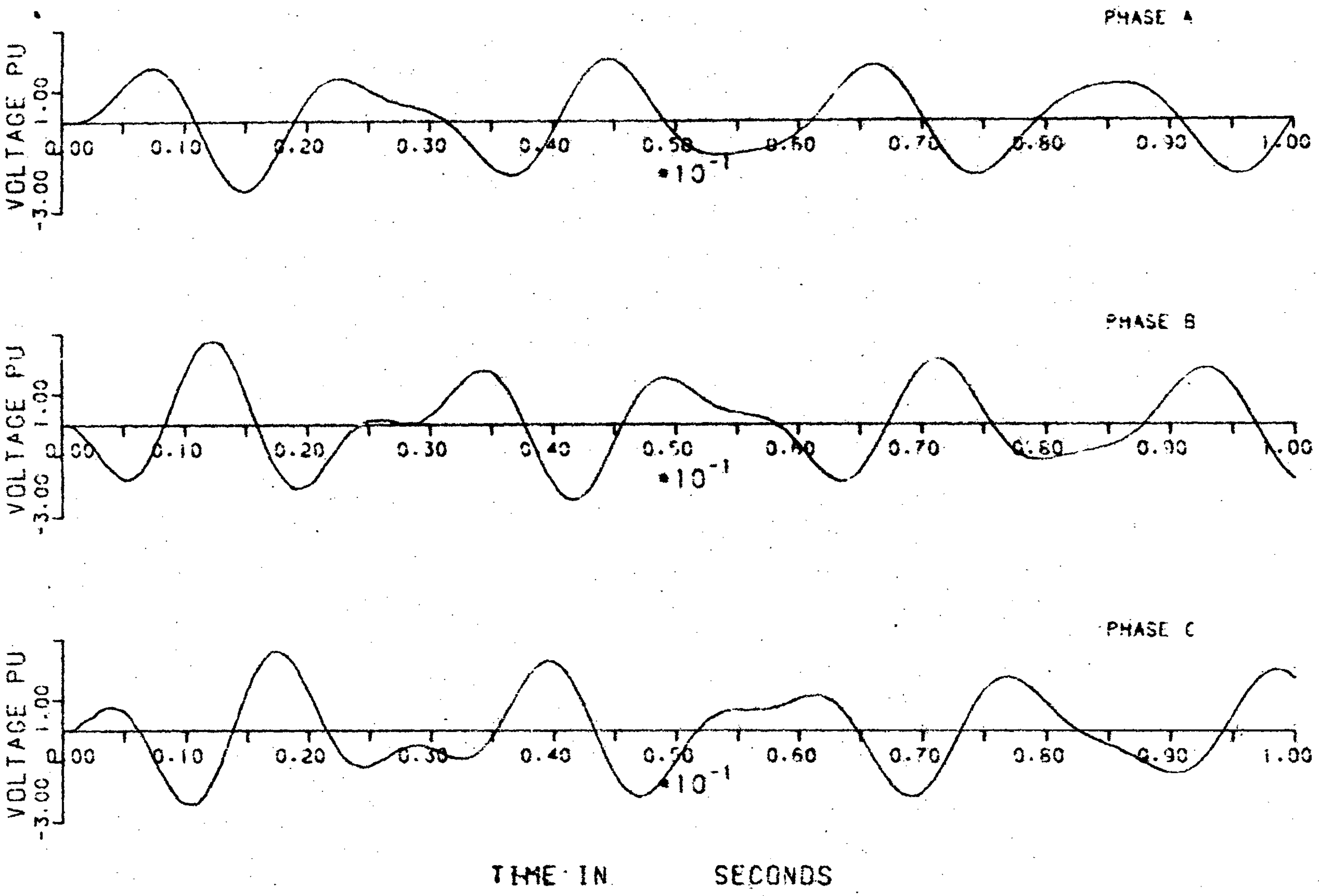
CHAPTER-V

RESULTS AND DISCUSSION

5.1 RESULTS

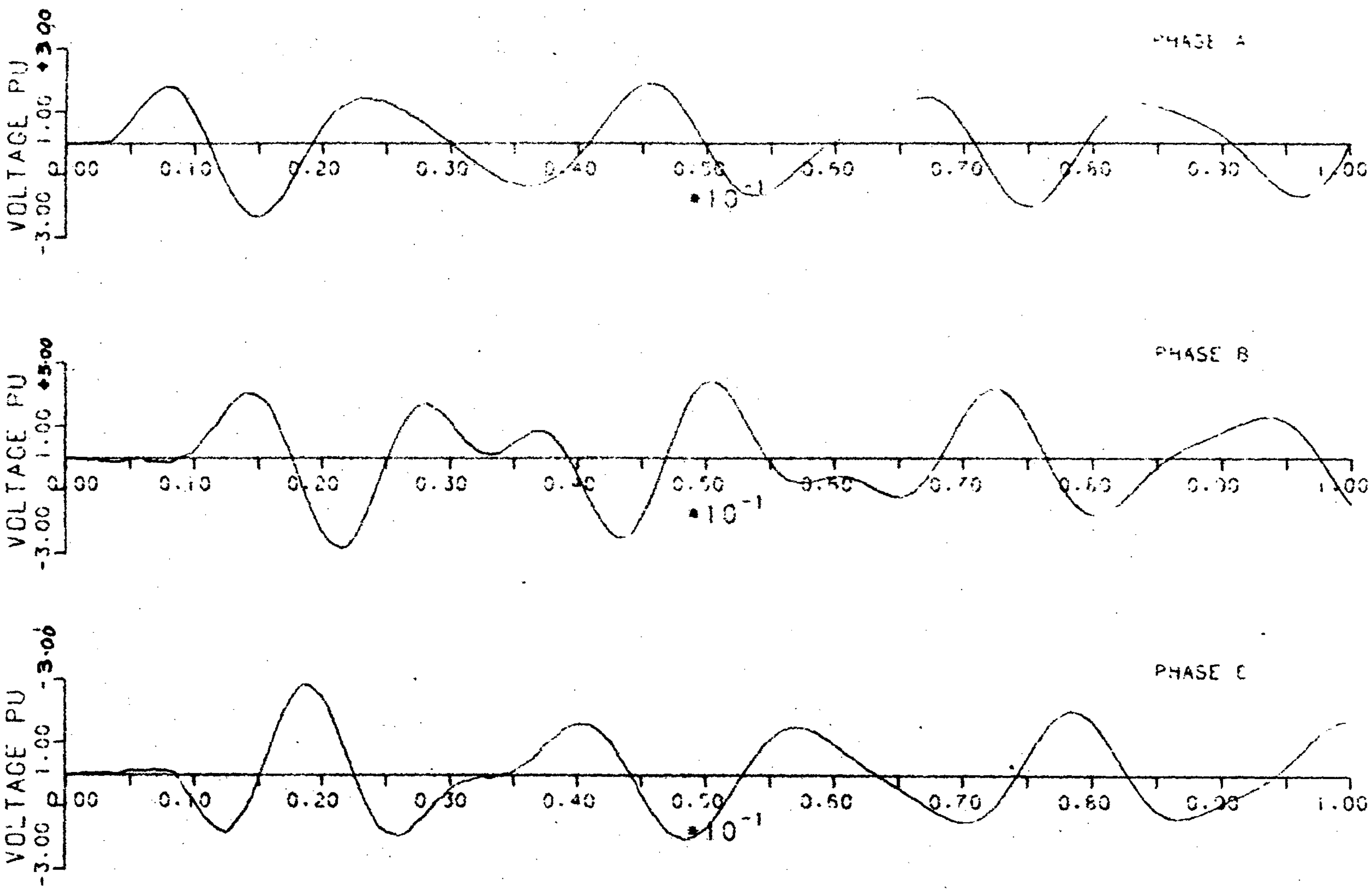
The maximum switching surge overvoltage occurring for different cases mentioned in Chapter-IV are given below:

Case	Maximum Overvoltage PU	Time of Occurrence (Milli seconds)	Ref. Fig. No.
Unloaded line energization with simultaneous closure of circuit breaker poles	2.76	12.35	5.1
Unloaded line energization with non-simultaneous closure of circuit breaker poles	2.88	18.80	5.2
Resistance energization of unloaded line	2.25	21.25	5.3
Line energization with balanced resistive load at receiving end	1.45	13.30	5.4
Load rejection	2.35	36.55	5.5
Unbalanced case with one breaker pole open at sending end for unloaded line	2.82	12.15	5.6
Unbalanced case with one load phase open at receiving end	2.57	17.10	5.7
Line energization with balanced inductive load	1.64	9.55	5.8
Line energization with unloaded transformer at receiving end	2.80	12.35	5.9



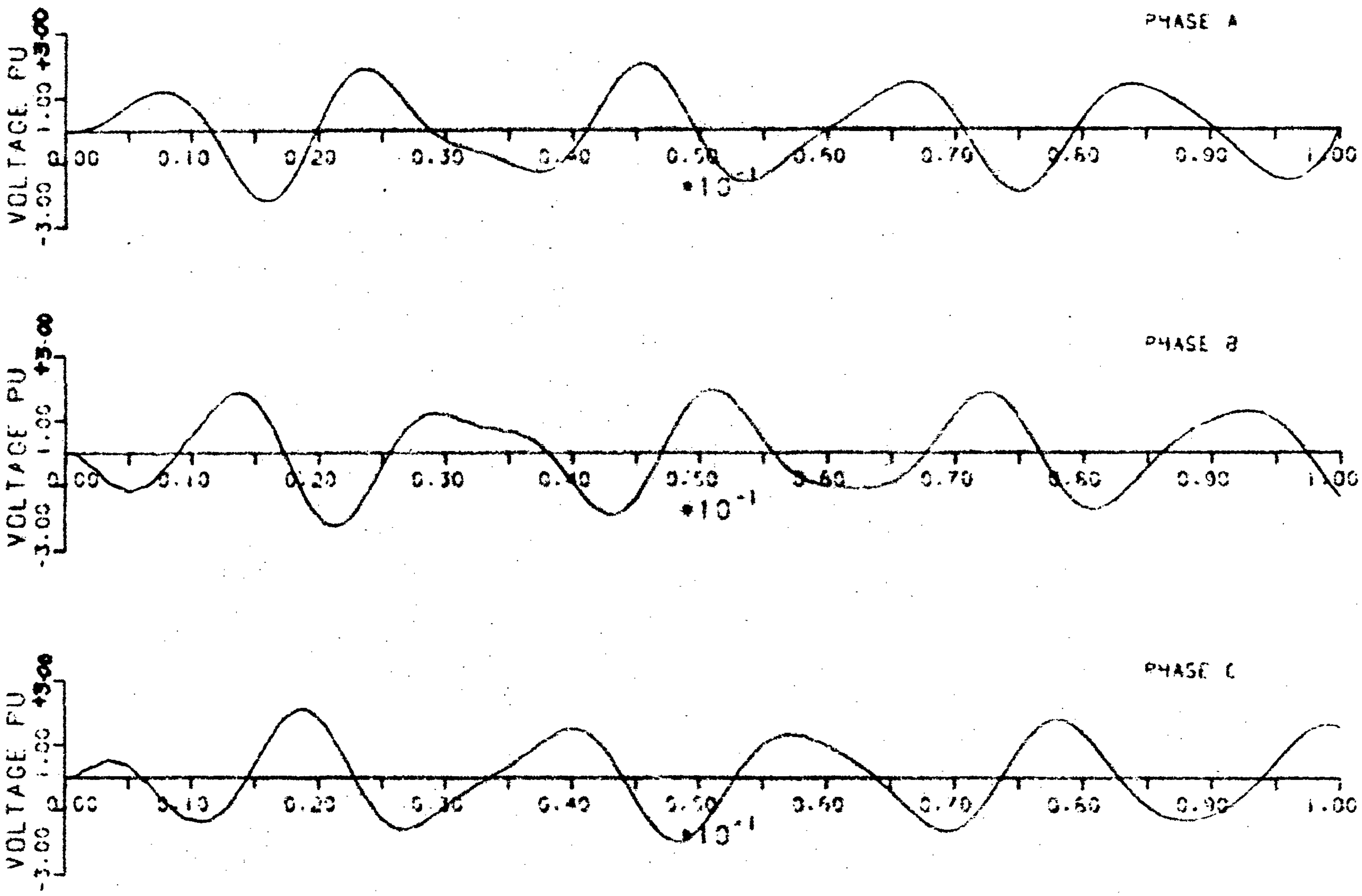
PHASE A
VOLTAGE PU
1.00
-3.00
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
*10⁻¹
PHASE B
VOLTAGE PU
1.00
-3.00
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
*10⁻¹
PHASE C
VOLTAGE PU
1.00
-3.00
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
*10⁻¹
TIME IN SECONDS

FIG 5.1



PHASE A
PHASE B
PHASE C
VOLTAGE PU
+3.00
1.00
-3.00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
*10^{-1}
TIME IN SECONDS

FIG 5.2



TIME IN SECONDS

FIG 5-3

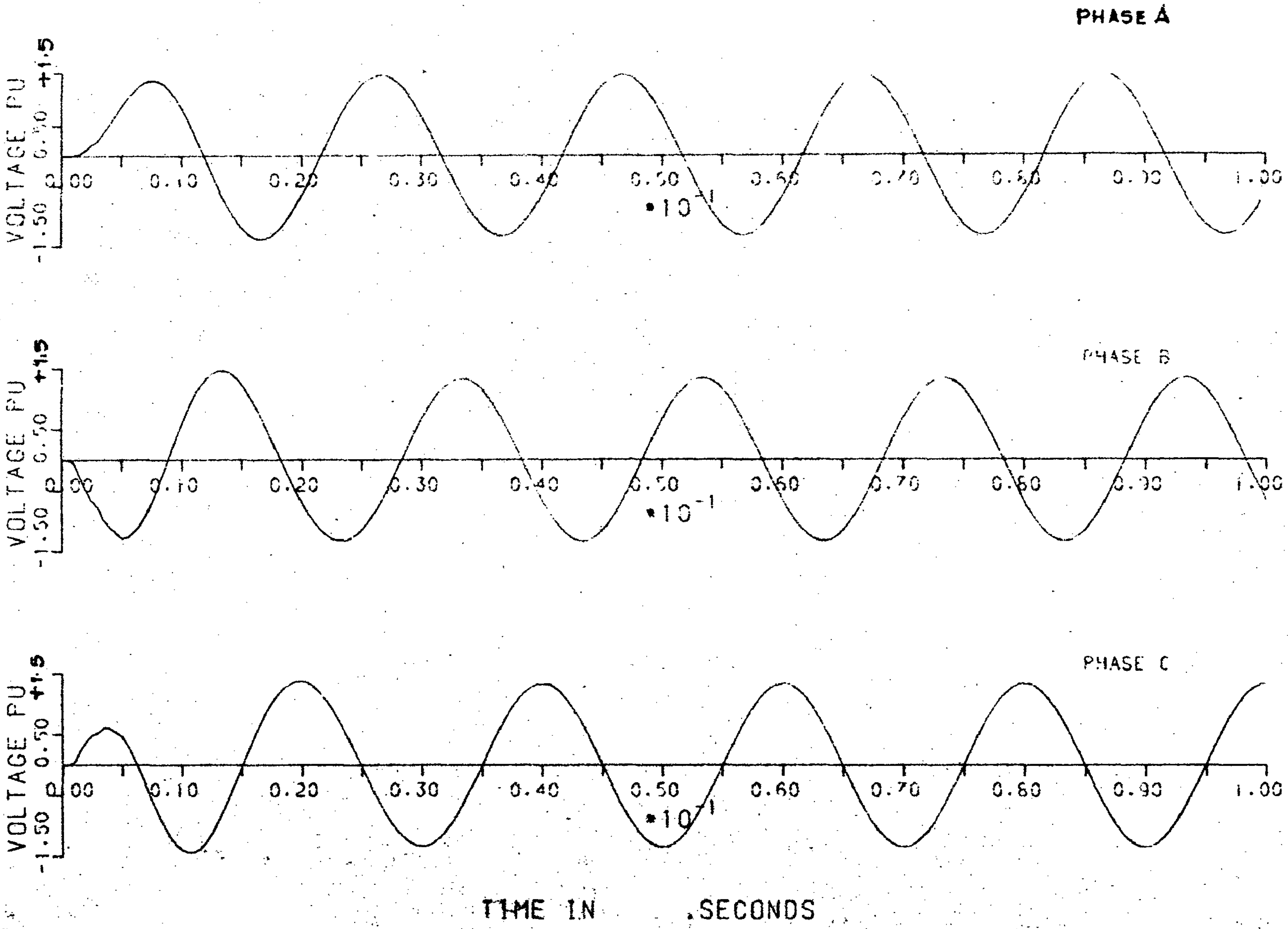

PHASE A
VOLTAGE PU
+1.5
0.50
0
-1.50
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
*10^-1
PHASE B
VOLTAGE PU
+1.5
0.50
0
-1.50
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
*10^-1
PHASE C
VOLTAGE PU
+1.5
0.50
0
-1.50
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
*10^-1
TIME IN .SECONDS

FIG 5.4

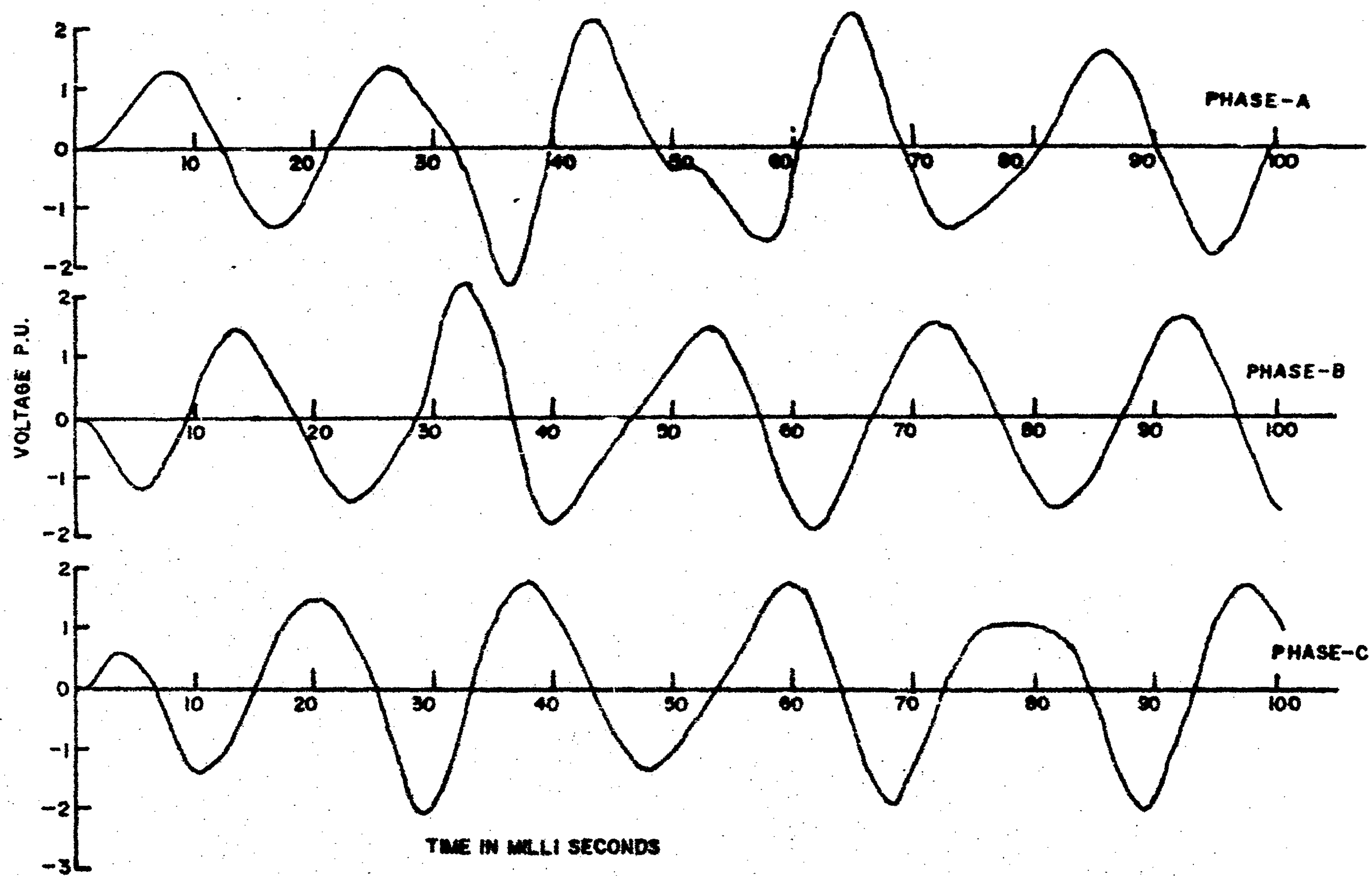
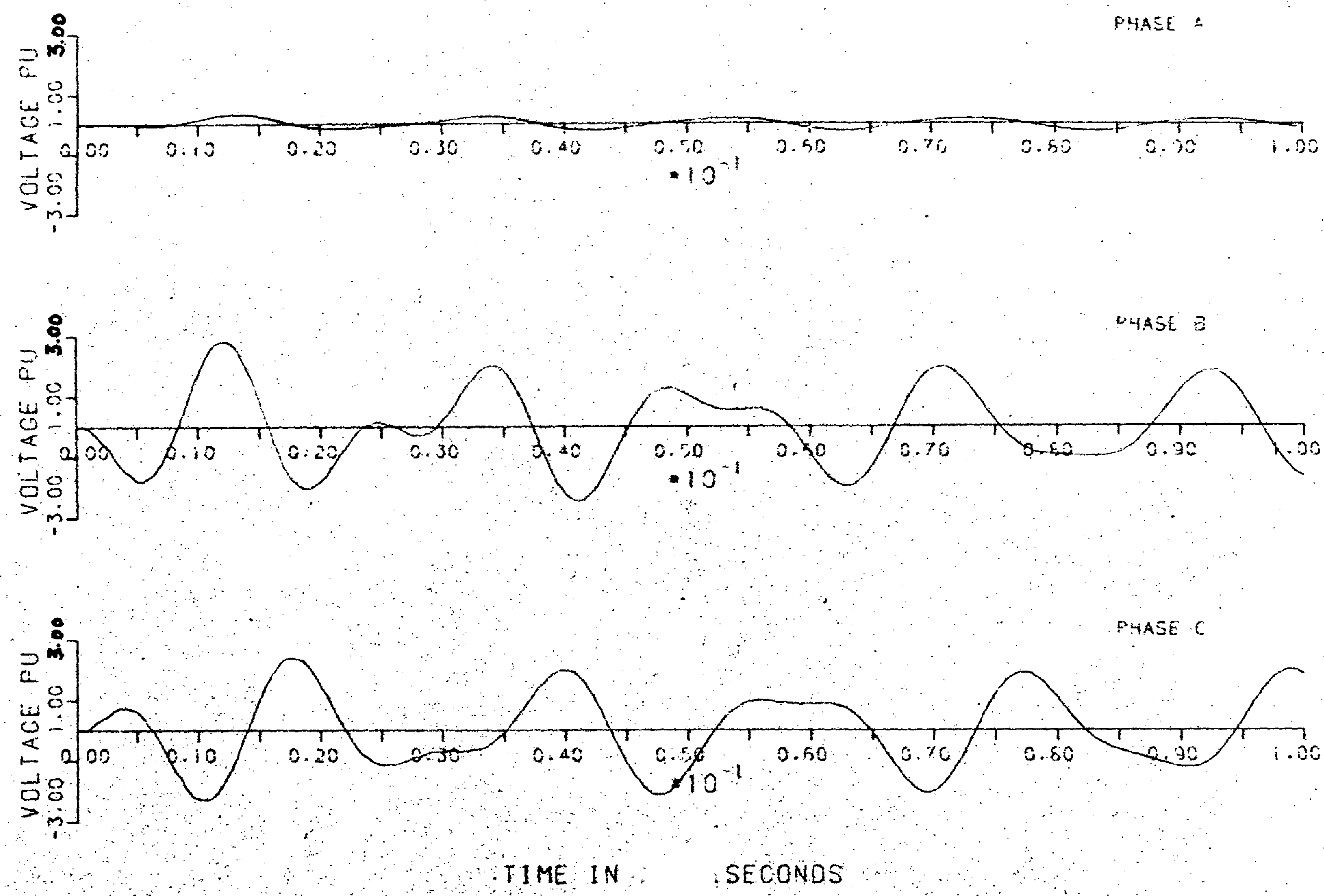


FIG.5.5



PHASE A
PHASE B
PHASE C
VOLTAGE PU
3.00
1.00
-3.00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
*10^-1
TIME IN SECONDS

FIG.5.6

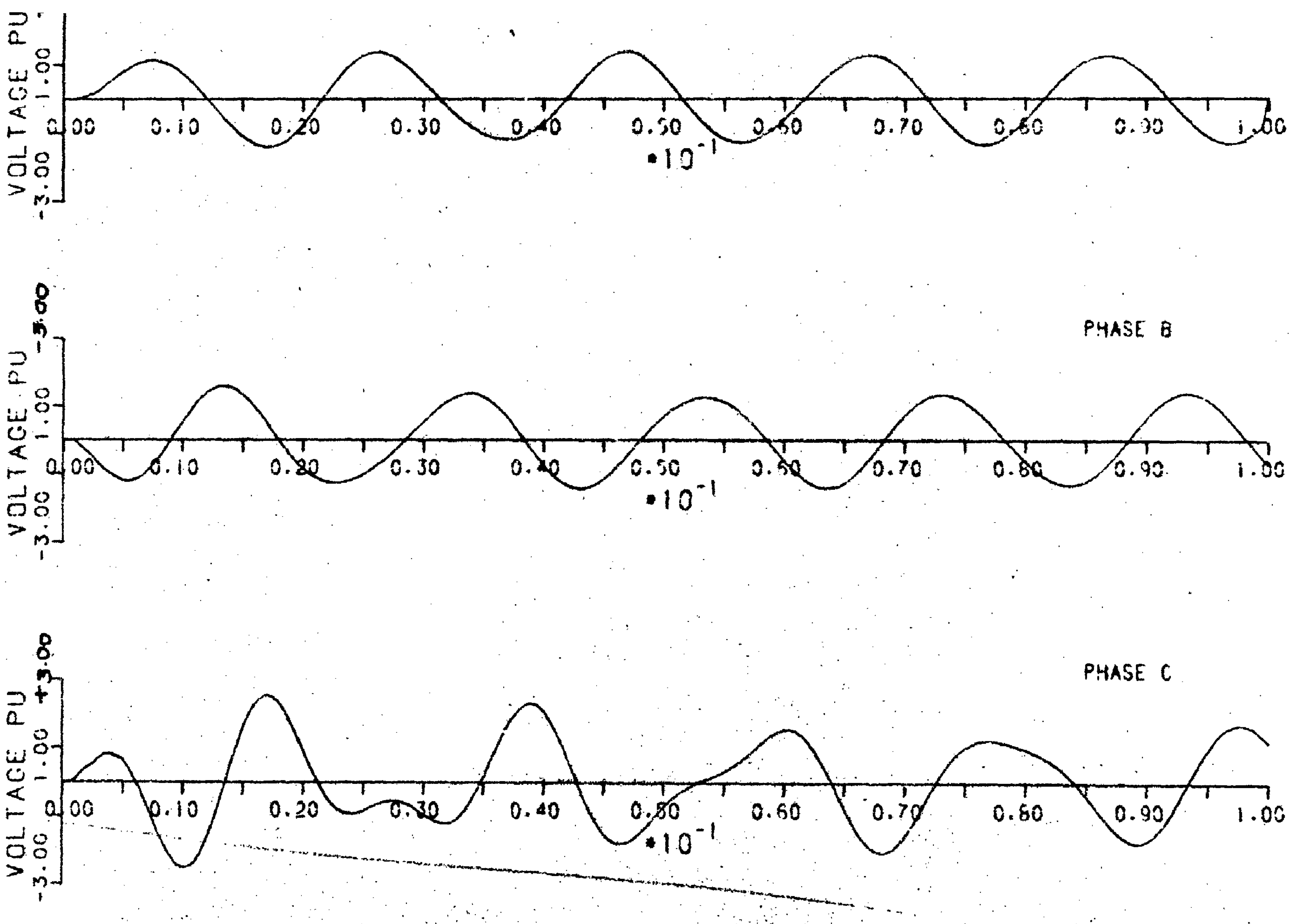

VOLTAGE PU
1.00
-3.00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
*10^-1
PHASE B
PHASE C
+3.00
TIME IN SECONDS

FIG 5.7

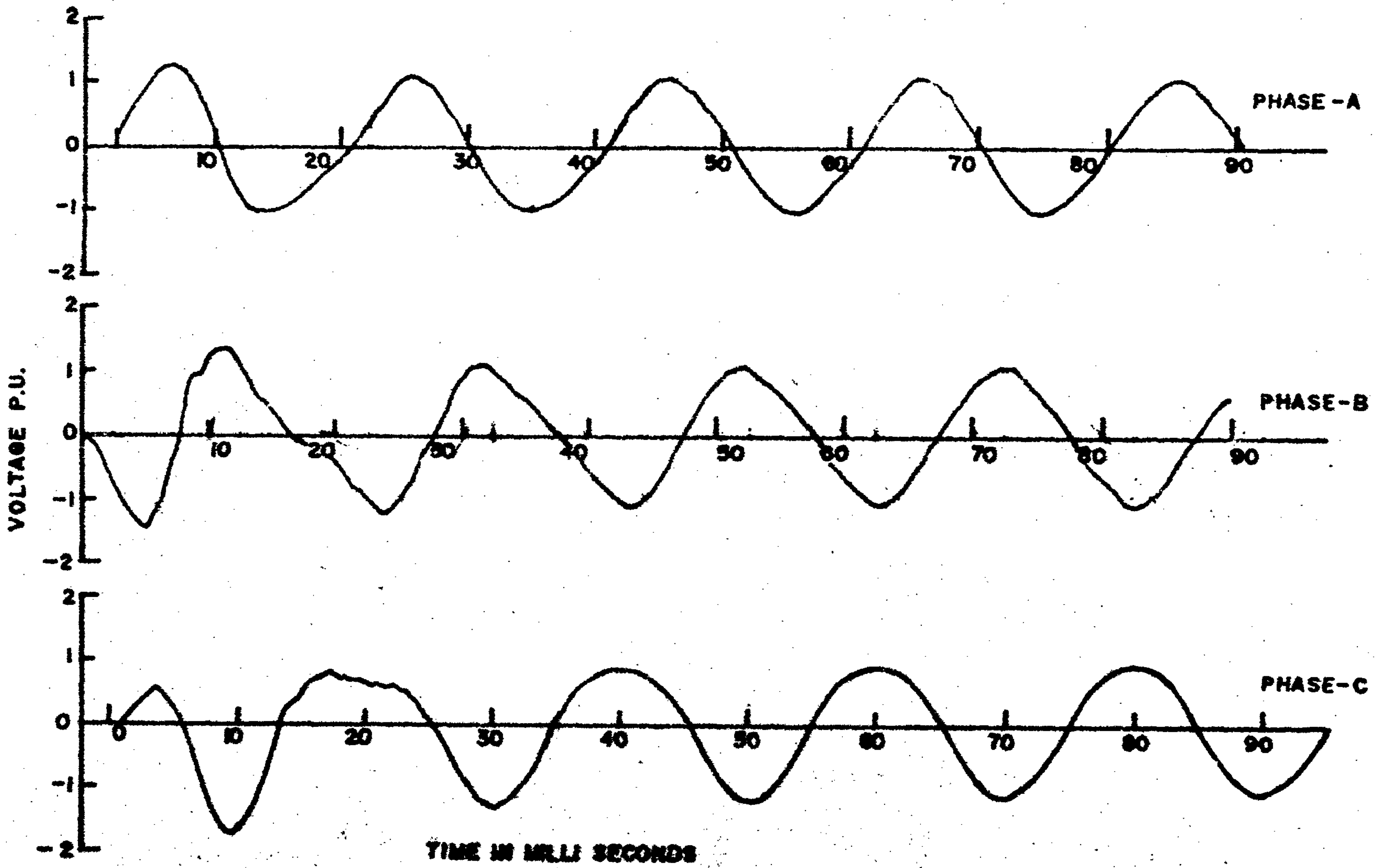


FIG.5.8

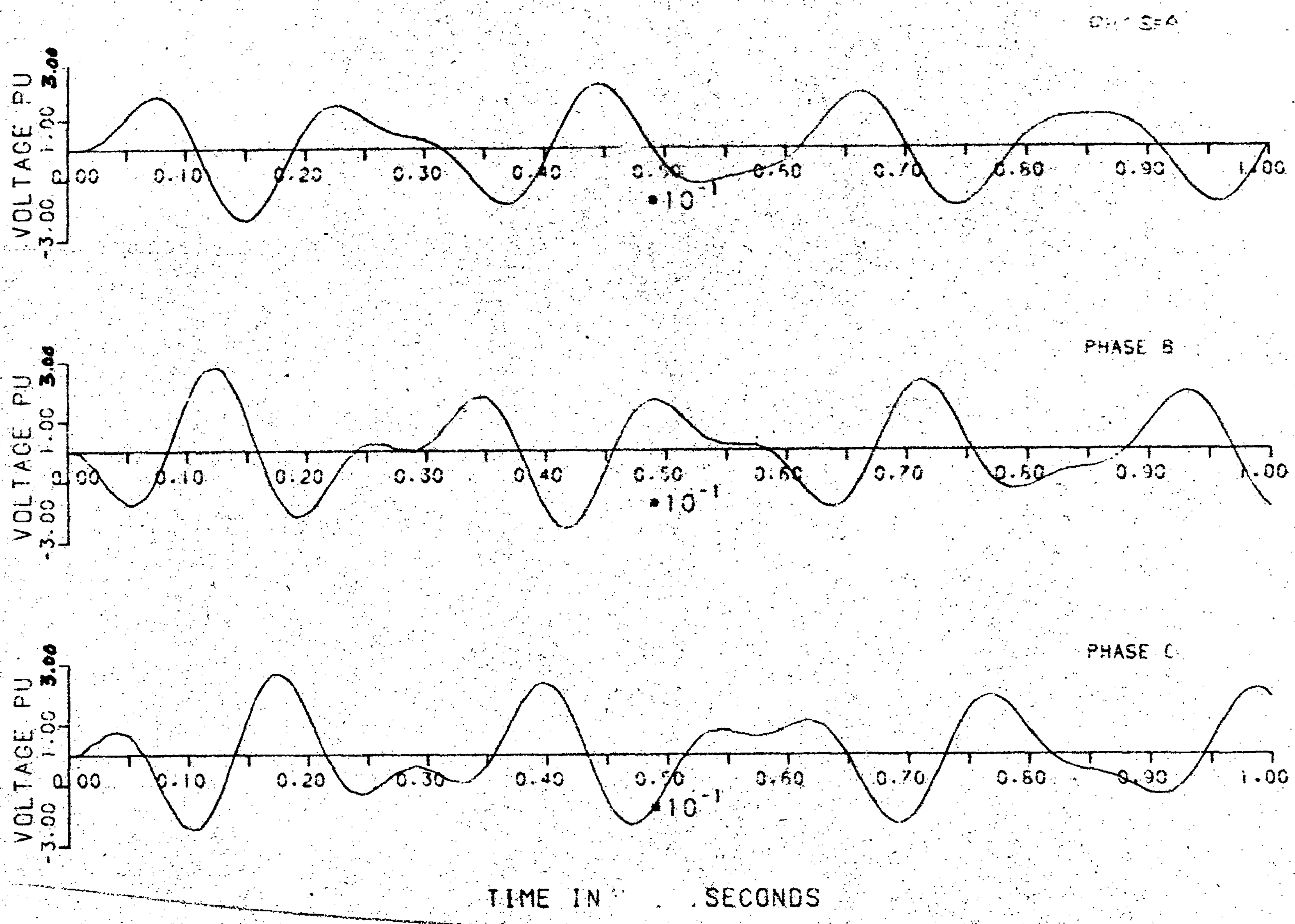

PHASE B
PHASE C
VOLTAGE PU
3.00
1.00
-3.00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
*10⁻¹
TIME IN SECONDS

FIG.5.9

5.2 DISCUSSION

For unloaded line energization, assuming simultaneous closure of circuit breaker poles, severe transients can be observed. These are quite erratic and take a long time to settle down to recognizable sinusoidal pattern. The extreme transient response is because of the open circuit at the receiving end, which leads to severe reflections of the travelling wave.

When energizing an unloaded line and considering non-simultaneous closure of breaker poles, a higher voltage peak is obtained as compared to the case of simultaneous closure of breaker poles. This may be due to the fact, that as phase A circuit breaker is closed first, followed other phases, the lines of open phases are charged from phase-A line before their circuit breakers close. Now, if phase-B (or phase-C) circuit breaker is closed impressing a power source voltage of polarity opposite to that of charging voltage, a higher abnormal voltage results.

When the line is energized via pre-insertion resistances the overvoltage peaks are reduced. The responses are not as irregular as in the previous cases. The resistance reduces the initial voltage step injected into the line which in turn, results in lower overvoltage peaks.

When a loaded line is energized, the overvoltage peaks are further reduced. This is due to the fact, that the reflections from the line end are reduced because of finite termination impedance. The response of first phase is quite smooth and damps out quickly, while the second and third phases have rough edges before the transients disappears. The reason for this is that the first phase is excited at zero voltage, which then continued sinusoidally. On the other hand, the second and third phases are excited with sudden step of voltages, because their phase angles are -120° and -240°. The voltages settle down to about 1.3 pu, which is higher than at the sending end, because of the charging current drawn by line capacitance.

For the unbalanced case of one breaker pole open at the sending end of the unloaded line, the response of other two phases is quite irregular. A small voltage is obtained at the receiving end of the open phase, even though the source on this phase is not connected. This effect results from the capacitive coupling between the lines producing some voltage at the receiving end of the open phase.

For the extreme unbalance at the receiving end, the severest transients are observed for the open phase, indicating severe reflections from the open end. For the other phases the transients settle sooner than the open phase.

When the line with inductive load termination is energized, irregular waveshapes are observed initially. The voltages finally settle down to 1 pu indicating that lagging load currents tend to cancel the leading line charging current, resulting in reduced terminal voltages.

CHAPTER-VI

CONCLUSIONS

As seen from the comparative study given in the last chapter we infer that the most severe overvoltages are obtained for the unloaded line energization with non-simultaneous closure of circuit breaker poles. The overvoltages resulting from the simultaneous closing of circuit-breaker poles for energizing line are relatively lower. Hence reduction in the maximum overvoltage is possible by decreasing the pole closing span. The advantage of pre-insertion resistance for energizing the line is clearly reflected in the results obtained. The studies show that steady state voltage exceeds 1.pu for unloaded lines. To control this excess voltage rise adequate shunt compensation should be provided. Preventive measures must be taken to ensure that transformers connected to long lines are not energized when they are not loaded, as excessive overvoltages are obtained in such cases.

It is observed for loaded lines that overvoltage peak is minimum when the power factor is unity, and the steady state voltage is excess of rated voltage of 1 pu. For inductive loads, vice-versa situation is observed; i.e. the overvoltage peak is higher and steady state voltage is nearly 1 pu. This indicates that while switching on any load appropriate compensation should be provided to make the power factor as close to unity as possible, thus reducing the peak overvoltage.

FUTURE SCOPE OF WORK

As the circuit-breaker pole closing span has a significant influence on the magnitude of switching surge overvoltage, the worst possible pole-span and pole-closing-sequence should be determined by studying the results for different combination of random pole-closing spans and pole-closing-sequences. An optimum value of pre-insertion resistance and insertion time needs to be determined to help in reducing the peak overvoltage.

The method may be extended to integrated power systems.

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APPENDIX - I

For resistive load termination the following equations were derived in Chapter III.

$$\frac{d}{dt}[K_1(t)] = \frac{d}{dt}[K_2(t)]+[V]\{[T]^{-1}[E_g]-[W][K_1(t)]+[X][K_2(t)]\} \quad (A.1)$$

$$[B(t)] = [Z][A(t)] \quad (A.2)$$

$$[A(t)] = a_t[K_1(t-T)]U(t-T) \quad (A.3)$$

$$[K_2(t)] = a_t[B(t-T)]U(t-T) \quad (A.4)$$

where

$$[Z] = \Big[[T]^{-1}[R_L][T][\Omega]^{-1}+[\mathcal{I}]\Big]^{-1} \times \Big[[T]^{-1}[R_L][T][\Omega]^{-1} - [\mathcal{I}]\Big][A]$$

a_t = attenuation factor

$U(t-T)$ = delayed unit step function

The differential equation (A.1) contains two unknown variable $[K_1]$ and $[K_2]$. To solve this equation $[K_2(t)]$ is eliminated from equation (A.1). Substituting (t) by (t-T) in equations (A.2) and (A.3) gives

$$[B(t-T)] = [Z][A(t-T)] \quad (A.5)$$

$$[A(t-T)] = a_t[K_1(t-2T)]U(t-2T) \quad (A.6)$$

Eliminating $[B(t-T)]$ from equations (A.4) and (A.5) we get

$$[K_2(t)] = a_t[Z][A(t-T)]U(t-T) \quad (A.7)$$

Elimination of $[A(t-T)]$ from equations (A.6) and (A.7) gives

$$[K_2(t)] = a_t^2[Z][K_1(t-2T)]U(t-2T) \quad (A.8)$$

Differentiating equation (A.8) w.r.t. t and substituting the value of $[K_2(t)]$ and $\frac{d}{dt}[K_2(t)]$ in equation (A.1) we get

$$\frac{d}{dt}[K_1(t)] = a_t^2[Z]\,\frac{d}{dt}[K_1(t-2T)]U(t-2T)+[V]\{[T]^{-1}[E_g]-[W][K_1(t)] + a_t^2[X][Z][K_1(t-2T)]U(t-2T)\} \quad (A.9)$$

This differential equation is solved by Runge-Kutta-Gill method [27] with an time interval of 50 μ seconds.

For inductive load terminations following equation was derived in Chapter III for determining matrix [B(t)]

$$\frac{d}{dt}[B(t)] = \frac{d}{dt}[A(t)] + [\alpha][\beta][A(t)] - [\alpha][\Upsilon][B(t)] \quad (A.10)$$

This differential equation is to be solved along with the differential equation (A.1). Differentiating equation (A.3) and (A.4) and substituting the values of [A(t)], $[K_2(t)]$ and their differentials into equations (A.1) and (A.10) we get

$$\frac{d}{dt}[K_1(t)] = a_t\,\frac{d}{dt}[B(t-T)]U(t-T)+[V]\{[T]^{-1}[E_g]-[W][K_1(t)] + a_t[X][B(t-T)]U(t-T)\} \quad (A.11)$$

$$\frac{d}{dt}B(t) = a_t\,\frac{d}{dt}[K_1(t-T)]U(t-T)+a_t[\alpha][\beta][K_1(t-T)]U(t-T) - [\alpha][\Upsilon][B(t)] \quad (A.12)$$

Replacing (t) by (t-T) in equation (A.12) and eliminating [B(t-T)] from equation (A.11) we get, on rearranging

$$\frac{d}{dt}[K_1(t)] = a_t^2\,\frac{d}{dt}[K_1(t-2T)]U(t-2T)+a_t^2[\alpha][\beta][K_1(t-2T)]U(t-2T) + a_t\Big[[V][X]-[\alpha][\Upsilon]\Big][B(t-T)]U(t-T)+[V]\{[T]^{-1}[E_g] - [W][K_1(t)] \quad (A.13)$$

Equation (A.12) and (A.13) are solved on Digital Computer by Runge-Kutta-Gill Method [27] with a time interval of 50μ seconds.

APPENDIX - II

The computer program for the calculation of Switching Surge Overvoltage was written in FORTRAN - IV. The various subroutines used are

RKGILL	This solves the differential equations by Runge-Kutta-Gill method.
DERIVR	This subroutine calculates the derivative functions for resistance load cases.
DERIVT	This subroutine calculates the derivative function for inductive load case.
EX	This subroutine calculates the voltages and currents at the receiving end of the line.
PROBY	This subroutine incorporates the changes in the system parameters and recalculates the constant matrices after the specified time.
CALMAT	This subroutine calculates the constant matrices.
EG	This function subroutine generates the sinusoidal generator voltages.

Apart from these, standard subroutines for matrix inversion (INV) and matrix multiplication (MUL) are used. The computer time required for the execution of this program varies from 14.0 to 17.0 seconds depending on the case studied.

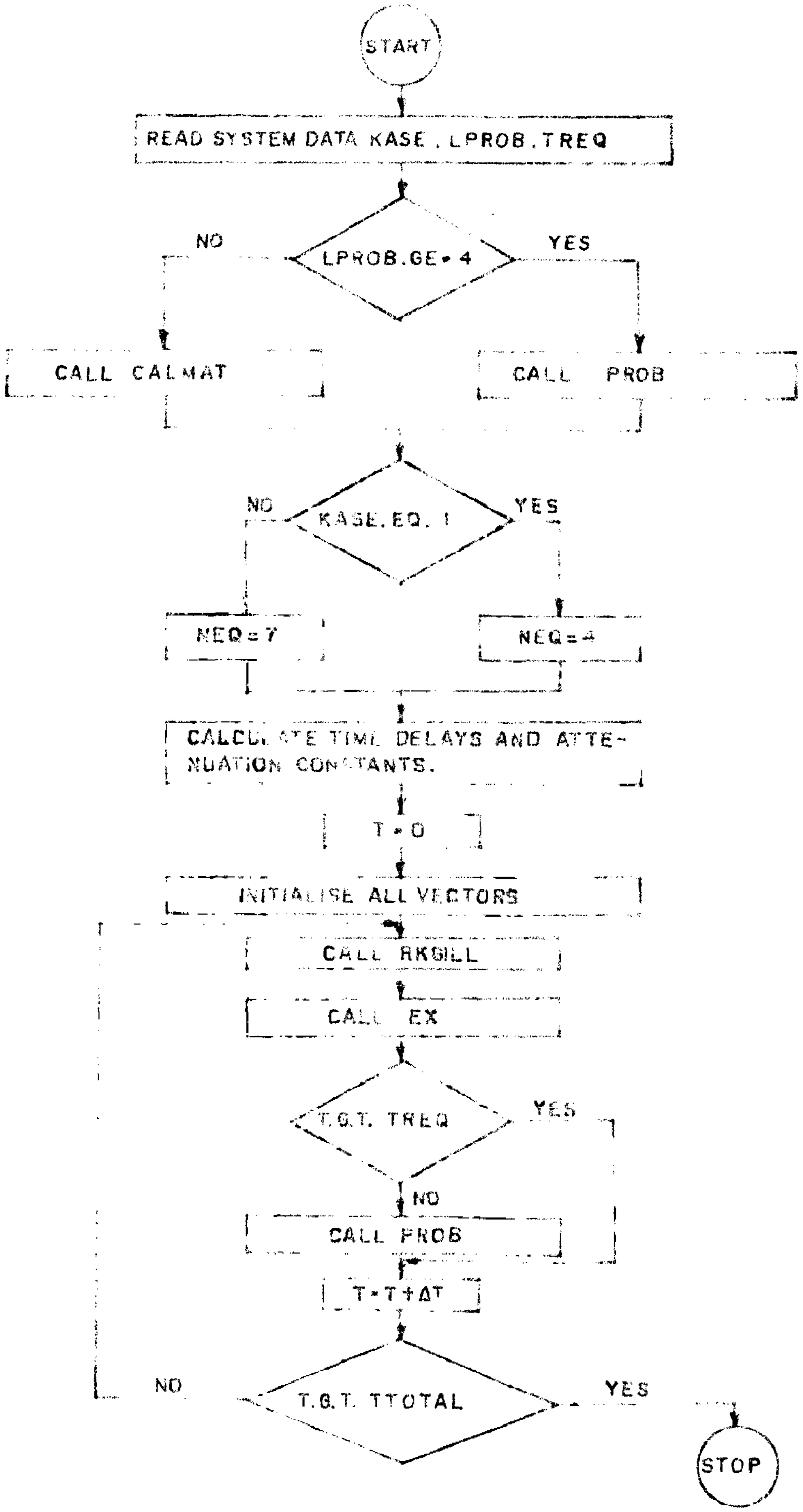

FIG. I.I FLOW CHART FOR CALCULATION OF SWITCHING SURGE OVER VOLTAGE BY LAPLACE TRANSFORM[redacted] METHOD.

```
C PROGRAM TO CALCULATE SWITCHING SURGE OVERVOLTAGE BY LAPLACE
C TRANSFORM METHOD
C KASE=1 FOR RESISTIVE LOAD CASES
C KASE=2 FOR INDUCTIVE LOAD CASES
C LPROB IS THE SELECTOR FOR VARIOUS CASES TO BE STUDIED
C NEQ= NEMBER OF EQUATIONS TO BE SOLVED
C TREQ= TIME SPECIFIED FOR CHANGE IN CIRCUIT PARAMETERS
C ATP,ATO ARE ATTENUATION CONSTANTS
C TT,TD ARE THE TIME DELAYS ASSOCIATED WITH SEQUENCE PARAMETERS
C H=TIME INCREMENT FOR SOLVING DIFF.EQUATIONS
       DIMENSION Y(7),YDOT(7),E(7),YD(3)
       COMMON/STORE/II,MD,ND,XS(2001,6),XDS(2001,6)
       COMMON/CONST/CNA(4),CNB(4),CNC(4)
       COMMON/STAB/A(3),B(3),TD(3),TT(3),AB(3),EXO(3),MO,NO,EPU
       COMMON/INPUT/TMAT(3,3),DINVT(3,3),RL(3,3),ALL(3,3),ALG(3,3)
       3,RO,R1,PLO,PL1,CO,C1,XO,EM,FREQ,KASE,LPROB,TREQ,CI(3)
       4,RG(3,3),RCB(3,3),DU(3,3),TPS1,TPS2,TPS3,RCBO(3,3)
       COMMON/STT/VW(3,3),VTI(3,3),VXZ(3,3),Z(3,3),VXM(3,3),
       6ALBT(3,3),ALGM(3,3),OHM(3,3),DINVO(3,3),TEMP1(3,3)
       COMMON/ATTEN/ATO,ATP,ATOS,ATPS
       OPEN(UNIT=1,DEVICE='DSK',FILE='Z.DAT')
      READ(1,*) RO,R1,PLO,PL1,CO,C1,XO,EM,FREQ
       READ(1,*) TREQ,KASE,LPROB,ROPEN,PS1,PS2,PS3
       READ(1,*)((TMAT(I,J),J=1,3),I=1,3)
       READ(1,*)((DINVT(I,J),J=1,3),I=1,3)
       READ(1,*)((RL(I,J),J=1,3),I=1,3)
       READ(1,*)((ALL(I,J),J=1,3),I=1,3)
      READ(1,*)((ALG(I,J),J=1,3),I=1,3)
      READ(1,*)((RG(I,J),J=1,3),I=1,3)
      READ(1,*)((RCB(I,J),J=1,3),I=1,3)
      READ(1,*)((RCBO(I,J),J=1,3),I=1,3)
       READ(1,*)((DU(I,J),J=1,3),I=1,3)
       IO=1
       IF(LPROB.GE.4) GO TO 9
       CALL CALMAT
       GO TO 4
)      CALL PROB(Y,IO)
       CNA(1)=.5;CNA(2)=.29289322;CNA(3)=1.70710678
       CNA(4)=.16666666;CNB(1)=2.;CNB(2)=1.;CNB(3)=1.
```

```
      CNB(4)=2.;CNC(1)=.5;CNC(2)=.29289322;CNC(3)=1.70710678
      CNC(4)=.5
4     NEQ=7
      IF(KASE.EQ.1) NEQ=4
      H=FREQ
      TD(1)=SQRT(PL1*C1)*XO
      TD(2)=SQRT(PLO*CO)*XO
      TT(1)=2.*TD(1)
      TT(2)=2.*TD(2)
     II=1
      MD=TT(1)/H+0.5
      ND=TT(2)/H+0.5
      MO=TD(1)/H+0.5
      NO=TD(2)/H+0.5
      EPU=EM*SQRT(2./3.)
      ATO=EXP(-0.5*RO*SQRT(CO/PLO)*XO)
      ATP=EXP(-0.5*R1*SQRT(C1/PL1)*XO)
      ATOS=ATO*ATO
      ATPS=ATP*ATP
      PRINT 222,ATO,ATP,ATOS,ATPS
222   FORMAT(3X,4F20.10)
      PRINT 10,MD,ND,MO,NO,KASE,LPROB,TREQ,EPU
      TYPE 10,MD,ND,MO,NO,KASE,LPROB,TREQ,EPU
10    FORMAT(2X,6I5,2X,2F20.10)
      DM=360.*50.
      TPS1=PS1/DM;TPS2=PS2/DM;TPS3=PS3/DM
      PRINT 1000,TPS1,TPS2,TPS3,PS1,PS2,PS3
1000  FORMAT(3X,6F15.10)
     DO 1 I=1,NEQ
     YDOT(I)=0.
      Y(I)=0.
      E(I)=0.
1     CONTINUE
     YDOT(1)=1.0
      T=0.
      DO 7 I=1,3
      S=0.
      DO 6 J=1,3
      JJ=J
```

```
6       S=S+VTI(I,J)*EG(JJ,T)
        YD(I)=S
        XDS(II,I)=YD(I)
7       CONTINUE
        DO 2 I=1,2001
       CALL RKGILL(Y,YDOT,E,H,NEQ)
        CALL EX(Y)
        IF(Y(1).GT.(TREQ+2.5E-06).AND.Y(1).GT.(TPS3+50.E-06)) GO TO 2
        CALL PROB(Y,IO)
2       CONTINUE
       STOP
       END
C SOLUTION OF DIFF. EQN. BY RUNGE -KUTTA GILL METHOD
       SUBROUTINE RKGILL (Y,YDOT,E,H,NEQ)
       DIMENSION YDOT(7),Y(7),E(7)
        COMMON/STORE/II,MD,ND,XS(2001,6),XDS(2001,6)
        COMMON/CONST/CNA(4),CNB(4),CNC(4)
        COMMON/STAB/A(3),B(3),TD(3),TT(3),AB(3),EXO(3),MO,NO,EPU
        COMMON/INPUT/TMAT(3,3),DINVT(3,3),RL(3,3),ALL(3,3),ALG(3,3)
        3,RO,R1,PLO,PL1,CO,C1,XO,EM,FREQ,KASE,LPROB,TREQ,CI(3)
        4,RG(3,3),RCB(3,3),DU(3,3),TPS1,TPS2,TPS3,RCCBO(3,3)
        COMMON/STT/VW(3,3),VTI(3,3),VXZ(3,3),Z(3,3),VXM(3,3),
        6ALBT(3,3),ALGM(3,3),OHM(3,3),DINVO(3,3),TEMP1(3,3)
       II=II+1
       DO 1J=1,4
        IF(KASE.NE.1) GO TO 8
        CALL DERIVR(Y,YDOT,H,NEQ,K,KK)
        GO TO 9
8        CALL DERIVT(Y,YDOT,H,NEQ,K,KK)
9       DO 1 I=1,NEQ
       X=CNA(J)*(YDOT(I)-CNB(J)*E(I))
       Y(I)=Y(I)+H*X
       E(I)=E(I)+3.0*X-CNC(J)*YDOT(I)
1       CONTINUE
        IF (KASE.NE.1) GO TO 10
         CALL DERIVR(Y,YDOT,H,NEQ,K,KK)
        GO TO 11
10        CALL DERIVT(Y,YDOT,H,NEQ,K,KK)
11       DO 2 I=2,NEQ
```

```
      J=I-1
      XDS(II,J)=YDOT(I)
 2    XS(II,J)=Y(I)
      RETURN
      END
C CALCULATION OF THE DERIVATIVE OF FUNCTION FOR INDUCTIVE LOADS
      SUBROUTINE DERIVT(Y,YDOT,H,NEQ)
      DIMENSION Y(7),YDOT(7),AK(3),AKD(3),AKDS(3),BS(3),BDS(3),AKS(3)
       COMMON/STORE/II,MD,ND,XS(2001,6),XDS(2001,6)
       COMMON/CONST/CNA(4),CNB(4),CNC(4)
       COMMON/STAB/A(3),B(3),TD(3),TT(3),AB(3),EXO(3),MO,NO,EPU
       COMMON/INPUT/TMAT(3,3),DINVT(3,3),RL(3,3),ALL(3,3),ALG(3,3)
       3,RO,R1,PLO,PL1,CO,C1,XO,EM,FREQ,KASE,LPROB,TREQ,CI(3)
       4,RG(3,3),RCB(3,3),DU(3,3),TPS1,TPS2,TPS3,RCBO(3,3)
       COMMON/STT/VW(3,3),VTI(3,3),VXZ(3,3),Z(3,3),VXM(3,3),
       6ALBT(3,3),ALGM(3,3),OHM(3,3),DINVO(3,3),TEMP1(3,3)
       COMMON/ATTEN/ATO,ATP,ATOS,ATPS
      T=Y(1)
      DO 2 I=2,4
      M=I-1
      S=0.0
      DO 1 J=1,3
       JJ=J
1     S=S-VW(M,J)*Y(J+1)+VTI(M,J)*EG(JJ,T)
2     YDOT(I)=S
      IF(T.LT.TD(1)) GO TO 15
      K=II-MO
      DO 3 I=2,3
      L=I+3
3     BS(I)=ATP*XS(K,L)
      IF(T-TD(2)) 4,5,5
4     BS(1)=0.0
      GO TO 6
5     K=II-NO
      BS(1)=ATO*XS(K,4)
5     DO 8 I=2,4
      M=I-1
      S=YDOT(I)
      DO 7 J=1,3
```

```
 7      S=S+VXM(M,J)*BS(J)
 8      YDOT(I)=S
        IF(T.LT.TT(1)) GO TO 15
        K=II-MD
        DO 9 I=2,3
        AK(I)=ATPS*XS(K,I)
 9      AKD(I)=ATPS*XDS(K,I)
        IF(T-TT(2)) 10,11,11
10      AK(1)=0.
        AKD(1)=0.
        GO TO 12
11      K=II-ND
        AK(1)=ATOS*XS(K,1)
        AKD(1)=ATOS*XDS(K,1)
12      DO 14 I=2,4
        M=I-1
        S=YDOT(I)
        DO 13 J=1,3
13      S=S+ALBT(M,J)*AK(J)
14        YDOT(I)=S+AKD(M)
15      DO 17 I=5,7
        M=I-4
        S=0.
        DO 16 J=1,3
16      S=S-ALGM(M,J)*Y(J+4)
17      YDOT(I)=S
        IF(T.LT.TD(1)) RETURN
        K=II-MO
        DO 18 I=2,3
        AKS(I)=ATP*XS(K,I)
18       AKDS(I)=ATP*XDS(K,I)
        IF(T-TD(2)) 19,20,20
19      AKS(1)=0.
        AKDS(1)=0.
        GO TO 21
20      K=II-NU
         AKS(1)=ATO*XS(K,1)
        AKDS(1)=ATO*XDS(K,1)
21      DO 23 I=5,7
```

```
      M=I-4
      S=YDOT(I)
      DO 22 J=1,3
22     S=S+ALBT(M,J)*AKS(J)
23    YDOT(I)=S+AKDS(M)
      RETURN
      END
C CALCULATION OF DERIVATIVE FUNCTION FOR RESISTIVE LOADS
      SUBROUTINE DERIVR(Y,YDOT,H,NEQ,K,KK)
       DIMENSION Y(4),YDOT(4),AD(3)
       COMMON/STORE/II,MD,ND,XS(2001,6),XDS(2001,6)
       COMMON/CONST/CNA(4),CNB(4),CNC(4)
       COMMON/STAB/A(3),B(3),TD(3),TT(3),AB(3),EXO(3),MO,NO,EPU
       COMMON/INPUT/TMAT(3,3),DINVT(3,3),RL(3,3),ALL(3,3),ALG(3,3)
       3,RO,R1,PLO,PL1,CO,C1,XO,EM,FREQ,KASE,LPROB,TREQ,CI(3)
       4,RG(3,3),RCB(3,3),DU(3,3),TPS1,TPS2,TPS3,RCBU(3,3)
       COMMON/STT/VW(3,3),VTI(3,3),VXZ(3,3),Z(3,3),VXM(3,3),
       6ALBT(3,3),ALGM(3,3),OHM(3,3),DINVO(3,3),TEMP1(3,3)
       COMMON/ATTEN/ATO,ATP,ATOS,ATPS
      T=Y(1)
      DO 2 I=2,NEQ
      S=0.0
      M=I-1
      DO 1 J=1,3
       JJ=J
 1    S=S-VW(M,J)*Y(J+1)+VTI(M,J)*EG(JJ,T)
 2    YDOT(I)=S
      IF(T.LT.TT(1)) RETURN
CC     PRINT 9
9      FORMAT(2X,'ENTERY')
      K=II-MD
      DO 3 I=2,3
       A(I)=ATPS*XS(K,I)
 3    AD(I)=ATPS*XDS(K,I)
      IF(T-TT(2)) 4,5,5
      A(1)=0.0
 4    AD(1)=0.0
      GO TO 6
 5    KK=II-ND
```

```
      A(1)=ATOS*XS(KK,1)
      AD(1)=ATOS*XDS(KK,1)
C      PRINT 10
10     FORMAT(2X,'ENTRY 2')
 6    DO 8 I=2,NEQ
      M=I-1
      S=YDOT(I)
      DO 7 J=1,3
 7     S=S+VXZ(M,J)*A(J)+Z(M,J)*AD(J)
 8    YDOT(I)=S
      RETURN
      END
C GENERATION OF FORCING FUNCTION
      FUNCTION EG(J,T)
       PI=3.1415926
       OMGA=314.15926
       EM=326.59862
      IF(J-2) 1,2,3
 1    EG=EM*SIN(OMGA*T)
      RETURN
2       EG=EM*SIN(OMGA*T-2.*PI/3.)
       RETURN
3     EG=EM*SIN(OMGA*T+2.*PI/3.)
      RETURN
      END
C CALCULATION OF RECIEVING END VOLTAGE AND CURRENT
       SUBROUTINE EX(Y)
       DIMENSION Y(7),AMB(3)
       COMMON/STORE/II,MD,ND,XS(2001,6),XDS(2001,6)
       COMMON/CONST/CNA(4),CNB(4),CNC(4)
       COMMON/STAB/A(3),B(3),TD(3),TT(3),AB(3),EXO(3),MO,NO,EPU
       COMMON/INPUT/TMAT(3,3),DINVT(3,3),RL(3,3),ALL(3,3),ALG(3,3)
       3,RO,R1,PLO,PL1,CO,C1,XO,EM,FREQ,KASE,LPROB,TREQ,CI(3)
       4,RG(3,3),RCB(3,3),DU(3,3),TPS1,TPS2,TPS3,RCBO(3,3)
       COMMON/STT/VW(3,3),VTI(3,3),VXZ(3,3),Z(3,3),VXM(3,3),
       6ALBT(3,3),ALGM(3,3),OHM(3,3),DINVO(3,3),TEMP1(3,3)
       COMMON/ATTEN/ATO,ATP,ATOS,ATPS
       T=Y(1)
       IF(T.LT.TD(1)) GO TO 12
```

```
      L=II-MO
      DO 1 I=2,3
      A(I)=ATP*XS(L,I)
1     CONTINUE
      IF(KASE.NE.2) GO TO 23
      B(2)=Y(6)
      B(3)=Y(7)
23    IF(T-TD(2)) 2,3,3
2     A(1)=0.0
      IF(KASE.NE.2)GO TO 24
      B(1)=0.0
      GO TO 6
3     LL=II-NO
      A(1)=ATO*XS(LL,1)
      IF(KASE.NE.2) GO TO 24
      B(1)=Y(5)
      GO TO 6
24    DO 26 I=1,3
      S=0.0
      DO 25 J=1,3
25    S=S+Z(I,J)*A(J)
      B(I)=S
26    CONTINUE
6     DO 9 I=1,3
      AMB(I)=A(I)-B(I)
9     AB(I)=A(I)+B(I)
      DO 7 I=1,3
      S=0.0;R=0.0
      DO 8 J=1,3
      R=R+TEMP1(I,J)*AMB(J)
8     S=S+TMAT(I,J)*AB(J)
      EXO(I)=S/EPU
      CI(I)=R
7     CONTINUE
C     IF(T-10.E-03) 11,11,12
12    LPRI=LPRI+1
      IF(LPRI-10)13,14,13
14    LPRI=0
  11  PRINT 10,T,EXO(1),EXO(2),EXO(3),CI(1),CI(2),CI(3)
```

```
10      FORMAT(2X,F7.4,5X,6F15.10)
876     FORMAT(1X,F7.4,3F14.10)
13      IF(ABS(ZMAX1).GE.ABS(EXO(1))) GOTO21
        ZMAX1=EXO(1)
        TOC1=T
 21     IF(ABS(ZMAX2).GE.ABS(EXO(2))) GOTO15
        ZMAX2=EXO(2)
        TOC2=T
 15     IF(ABS(ZMAX3).GE.ABS(EXO(3))) GOTO16
        ZMAX3=EXO(3)
        TOC3=T
16      IF(II-2001)20,22,22
22      PRINT17,ZMAX1,TOC1
17      FORMAT(/,4X,'MAX OV PH1=',F12.6,2X,'AT TIME =',E12.6,/)
        PRINT 18,ZMAX2,TOC2
18      FORMAT(/4X,'MAX OV PH2=',F12.6,2X,'AT TIME=',E12.6,/)
        PRINT 19,ZMAX3,TOC3
        TYPE 876, TOC3,ZMAX1,ZMAX2,ZMAX3
        WRITE(21,876) TOC3,ZMAX1,ZMAX2,ZMAX3
19      FORMAT(/,4X,'MAX OV PH3=',F12.6,2X,'AT TIME=',E12.6,/)
20      RETURN
        END
C SELECTION OF DIFFERENT CASES
        SUBROUTINE PROB(Y,I0)
        COMMON/STORE/I1,MD,ND,XS(2001,6),XDS(2001,6)
        COMMON/CONST/CNA(4),CNB(4),CNC(4)
        COMMON/STAB/A(3),B(3),TD(3),TT(3),AB(3),EXO(3),MO,NO,EPU
        COMMON/INPUT/TMAT(3,3),DINVT(3,3),RL(3,3),ALL(3,3),ALG(3,3)
        3,RO,R1,PLO,PL1,CO,C1,XO,EM,FREQ,KASE,LPROB,TREQ,CI(3)
        4,RG(3,3),RCB(3,3),DU(3,3),TPS1,TPS2,TPS3,RCBO(3,3)
        COMMON/STT/VW(3,3),VTI(3,3),VXZ(3,3),Z(3,3),VXM(3,3),
        6ALBT(3,3),ALGM(3,3),OHM(3,3),DINVO(3,3),TEMP1(3,3)
        COMMON/ATTEN/ATO,ATP,ATOS,ATPS
        DIMENSION Y(7)
        GO TO (1,2,3,4,5,6,7) LPROB
1       IF((Y(1)+2.5E-07).LT.TREQ) RETURN
        IF(ICB.NE.0) RETURN
        DO 11 I=1,3
        RCB(I,I)=0.0
```

```
      RG(I,I)=0.0
 11   CONTINUE
      CALL CALMAT
      ICB=ICB+1
      RETURN
 2    IF((Y(1)+2.5E-07).LT.TREQ) RETURN
      IF(IR.NE.0) RETURN
      DO 12 I=1,3
      RL(I,I)=1190.25E+06
 12   CONTINUE
      CALL CALMAT
      IR=IR+1
      RETURN
 3    IF((Y(1)+2.5E-06).LT.TREQ) RETURN
      IF(IC1.NE.0) GO TO 41
      IF(ABS(CI(1)).GT.0.01) GO TO 41
      RL(1,1)=1190.25E+06
      CALL CALMAT
      IC1=IC1+1
 41   IF(IC2.NE.0) GO TO 42
      IF(ABS(CI(2)).GT.0.01) GO TO 42
      RL(2,2)=1190.25E+06
      CALL CALMAT
      IC2=IC2+1
 42   IF(IC3.NE.0) GO TO 43
      IF(ABS(CI(3)).GT.0.01) GO TO 43
      RL(3,3)=ROPEN
      CALL CALMAT
      IC3=IC3+1
 43   RETURN
 4    IF(I4.NE.0) GO  TO 30
      DO 14 I=1,3
      RL(I,I)=RL(I,I)+ROPEN
 14   CONTINUE
      CALL CALMAT
      I4=I4+1
 30   IF((Y(1)+25.E-07).LT.TREQ) RETURN
      IF(I5.NE.0) GO TO 31
      DO 15 I=1,3
```

```
      RL(I,I)=RL(I,I)-ROPEN
15    CONTINUE
      CALL CALMAT
      I5=I5+1
31    RETURN
5     IF(I0.NE.1) GO TO 19
      DO 20 I=1,3
      RG(I,I)=RCBO(I,I)
20    CONTINUE
      CALL CALMAT
      I0=I0+1
19    IF(Y(1).GT.TPS1) GO TO 17
      GO TO 18
17    IF(I1.EQ.0) GO TO 29
      GO TO 18
29    RG(1,1)=0.0
      CALL CALMAT
      I1=I1+1
18    IF(Y(1).GT.TPS2) GO TO 21
      GO TO 22
21    IF(I2.EQ.0) GO TO 23
      GO TO 22
23    RG(2,2)=0.0
      CALL CALMAT
      I2=I2+1
22    IF(Y(1).GT.TPS3) GO TO 24
      GO TO 25
24    IF(I3.EQ.0) GO TO 26
      GO TO 1
26    RG(3,3)=0.0
      CALL CALMAT
      I3=I3+1
25    RETURN
6     IF(I6.NE.0) GO TO 32
      RCB(1,1)=RCBO(1,1)
      RL(1,1)=ROPEN
      CALL CALMAT
      I6=I6+1
32    RETURN
```

```
 7         IF(I7.NE.0) GO TO 33
           RL(3,3)=ROPEN
           I7=I7+1
           CALL CALMAT
 33        RETURN
           END
C CALCULATION OF CONSTANT MATRICES
           SUBROUTINE CALMAT
           DIMENSION TEMP2(3,3),TEMP3(3,3),TEMP4(3,3)
           1,TEMP5(3,3),TEMP6(3,3),TEMP7(3,3),TEMP8(3,3),TEMP9(3,3)
           2,TEMP10(3,3),V(3,3),W(3,3),X(3,3),BT(3,3),GM(3,3),TIL(3,3)
           3,ALP(3,3),ALPI(3,3),VX(3,3)
           COMMON/INPUT/TMAT(3,3),DINVT(3,3),RL(3,3),ALL(3,3),ALG(3,3)
           3,R0,R1,PL0,PL1,C0,C1,X0,EM,FREQ,KASE,LPROB,TREQ,CI(3)
           4,RG(3,3),RCB(3,3),DU(3,3),TPS1,TPS2,TPS3,RCB0(3,3)
           COMMON/STT/VW(3,3),VTI(3,3),VXZ(3,3),Z(3,3),VXM(3,3),
           6ALBT(3,3),ALGM(3,3),OHM(3,3),DINVO(3,3),TEMP1(3,3)
           COMMON/ATTEN/ATO,ATP,ATOS,ATPS
           DO 1 I=1,3
           DO 1 J=1,3
           OHM(I,J)=0.
 1         CONTINUE
          OHM(1,1)=SQRT(PL0/C0)
          T1=SQRT(PL1/C1)
          DO 2 J=2,3
          OHM(J,J)=T1
   2     CONTINUE
           CALL INV(OHM,DINVO)
          CALL MUL(TMAT,DINVO,3,TEMP1)
           CALL MUL(DINVT,RL,3,TEMP2)
           CALL MUL(TEMP2,TEMP1,3,TEMP3)
           DO 3 I=1,3
           DO 3 J=1,3
           TEMP4(I,J)=TEMP3(I,J)+DU(I,J)
           TEMP5(I,J)=TEMP3(I,J)-DU(I,J)
 3         CONTINUE
           CALL INV(TEMP4,TEMP6)
           CALL MUL(TEMP6,TEMP5,3,Z)
          CALL MUL (DINVT,ALG,3,TEMP7)
```

```
      CALL MUL (TEMP7,TEMP1,3,TEMP8)
      CALL INV (TEMP8,V)
       DO 7 I=1,3
       RG(I,I)=RG(I,I)+RCB(I,I)
7      CONTINUE
      CALL MUL (DINVT,RG,3,TEMP9)
      CALL MUL (TEMP9,TEMP1,3,TEMP10)
      DO 4 IK=1,3
      DO 4 JK=1,3
      W(IK,JK)=TEMP10(IK,JK)+DU(IK,JK)
      X(IK,JK)=TEMP10(IK,JK)-DU(IK,JK)
       BT(IK,JK)=TEMP5(IK,JK)
       GM(IK,JK)=TEMP4(IK,JK)
  4   CONTINUE
       CALL MUL (V,W,3,VW)
       CALL MUL (V,X,3,VX)
       CALL MUL(V,DINVT,3,VTI)
       CALL MUL(VX,Z,3,VXZ)
       CALL MUL(DINVT,ALL,3,TIL)
       CALL MUL(TIL,TEMP1,3,ALPI)
       CALL INV(ALPI,ALP)
       CALL MUL(ALP,BT,3,ALBT)
       CALL MUL(ALP,GM,3,ALGM)
       DO 6 I=1,3
       DO 6 J=1,3
       VXM(I,J)=VX(I,J)-ALGM(I,J)
6      CONTINUE
       RETURN
       END
      SUBROUTINE MUL(D,E,M,X)
C TO CALCULATE MATRIX MULTIPLICATION
      DIMENSION D(3,3),E(3,M),X(3,3)
       DO 10I=1,3
       DO 10J=1,M
      X(I,J)=0.
      DO 10 K=1,3
      X(I,J)=X(I,J)+D(I,K)*E(K,J)
  10  CONTINUE
      RETURN
```

```
      END
      SUBROUTINE INV(X,A)
C THIS CALCULATES MATRIX INVERSE
      DIMENSION X(3,3),A(3,3),B(3,6)
      DO 1000 J=1,3
      DO 1000 K=1,3
      B(J,K)=X(J,K)
 1000 CONTINUE
      N1=6
      N2=4
       DO 10M=1,3
      DO 20 N=4,6
      B(M,N)=0
20     MN=M+3
 10   B(M,MN)=1.
      DO 100 J=1,3
      IF(B(J,J).EQ.0.) GO TO 102
       DO 100 M=1,3
      IF(M.EQ.J) GO TO 100
      C=B(M,J)
       C=C/B(J,J)
      DO 105 N=1,N1
105    B(M,N)=B(M,N)-B(J,N)*C
 100   CONTINUE
      DO 106 J=1,3
      R=1./B(J,J)
      DO 106 N=N2,N1
 106  B(J,N)=B(J,N)*R
      DO 1001 J=1,3
      DO 1001 K=1,3
 1001 A(J,K)=B(J,K+3)
 102     RETURN
      END
```