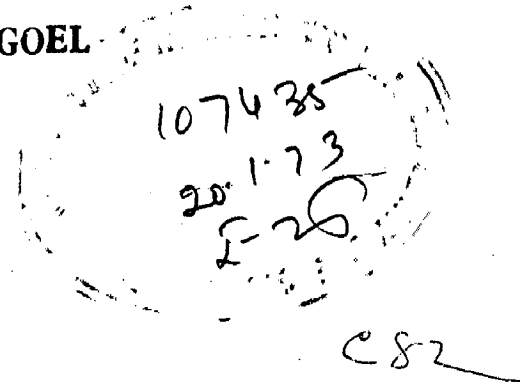


**DESIGN AND PERFORMANCE EVALUATION  
OF  
SOME INDUSTRIAL INSTRUMENTATION  
USING THERMORESISTIVE SENSING  
ELEMENTS**

*A Dissertation*  
*submitted in partial fulfilment*  
*of the requirements for the degree*  
*of*  
MASTER OF ENGINEERING  
*in*  
MEASUREMENT AND INSTRUMENTATION

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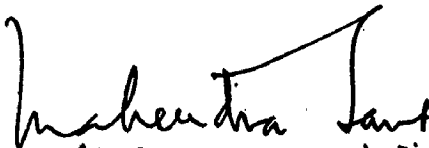
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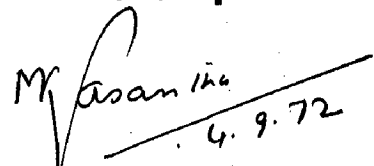


## C E R T I F I C A T E

Certified that the dissertation entitled " DESIGN AND PERFORMANCE EVALUATION FO SOME INDUSTRIAL INSTRUMENTATION USING THERMORESISTIVE SENSING ELEMENTS" which is being submitted by Shri N.K. Goel in partial fulfilment for the award of the Degree of Master of Engineering in Electrical Measurements and Instrumentation of Electrical Engineering of the University of Roorkee, Roorkee is a record of the student's own work carried out by him under our super vision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of seven months from January 1972 to July 1972 for preparing this dissertation at this University.

  
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Finally, he would like to thank every body whosoever has directly or otherwise influenced his work.

A B S T R A C T

This dissertation deals with those theoretical and practical aspects of thermoresistive semiconductor type elements which influence markedly the design and performance of instrumentation systems using these elements as sensors. The Thermoresistive elements considered in Part I of this work are the thermistors. After discussing exhaustively, the D.c., a.c. and transient characteristic of these elements, the various techniques of linearising, their inherently non-linear characteristics, have been considered in detail. The various industrial applications, classified on the basis of their different characteristics, have next been discussed with special emphasis on the design considerations. The practical aspect of this part convey the design and fabrication of a gas analyser, prototype model, and a digital temperature and liquid level indicator. The latter by virtue of its simplicity, reliability and high speed of response has potential applications in industrial control system.

Part II of the dissertation deals with the use of bipolar junction transistors as thermal sensors. After discussing the temperature sensitive characteristic of these elements, design and fabrication of an electronic thermometer using a transistor sensor is described which is reliable, sensitive, linear, and



simple to manufacture and has uses in the industrial and medical fields.

## P R E F A C E

Thermo resistive semiconductor type elements -whose resistance is extremely sensitive to temperature variations are today finding increasing application in the field of instrumentation. The elements discussed in this dissertation are thermistors and transistors.

The first two chapters of the work deal with the electrical representation of thermistors as electric circuit elements and provide the necessary background to the understanding of the principles on which their various applications are based. In particular, in Chapter II, after introducing their static D.C. characteristic, the small signal equivalent circuits have been established followed by an exhaustive treatment of the transient behaviour of these elements under both low level and high level signal conditions. The third chapter is devoted to the linearisation of the thermistor characteristic which is inherently non-linear with special emphasis on the design aspects. Selection factors and the testing of the thermistors are covered in the fourth Chapter.

Whereas the applications of thermistors in instrumentation are almost legion ranging in such diverse fields process, industries, medicine and space instrumentation systems, where they have established themselves, the use of transistors as sensors on the other hand is still in the developmental stage. Even the properties

of thermistors have still not been fully explored and newer applications of these versatile elements are still being found. In particular the introduction of posistors have opened up new vistas in their applications in modern instrumentation system. The ever members of the transistor family have wide potentialities of being used in instrumentation systems as transducers and more will definitely be heard of them in this, their new role in the coming years. The extreme sensitivity of bipolar junction transistor parameters to heat have been successfully exploited for precise and rapid measurement of temperature.

Although measurement or indication of temperature is an obvious use, it has to be recognised that the resistance of thermistor depends on both the ambient temperature and the internally dissipated power. These properties are exploited in applications as temperature measurement and control, gas analysis and velocity measurement, temperature and liquid level indication, and pressure measurement, etc. which are discussed in detail in Chapters five and six. In addition the complete design and fabrication details of a gas analyser and anemometer and a digital temperature and liquid level indicator, have been discussed fully. A transistor electronic thermometer has been fabricated using transistor as a sensing element which is reliable, sensitive and

simple to manufacture, gives a linear scale, and has wide uses in industrial and medical fields.

Chapter VII deals with those upcoming applications of directly heated thermistors which are dependent on their small signal, nonlinear and transient characteristics.

In the last chapter after discussing the nature of the dependence of the various parameters of junction transistor on temperature complete details have been given of the design of a electronic transistor thermometer.

# C O N T E N T S

Chapter	Page
CERTIFICATE	i
ACKNOWLEDGEMENT	ii
ABSTRACT	iii
PREFACE	v
 <u>P A R T I</u>  	
THERMISTOR AS A THERMOSENSITIVE ELEMENT	
I	INTRODUCTION
	1.1 Terminology
	1.2 Forms
	1.3 Electrical Representation
	1.4 Applications
II	D.C., A.C. AND TRANSIENT CHARACTERISTIC
	2.1 D.C. Characteristic
	2.1.1 Introduction
	2.1.2 Resistivity
	2.2 Specific definitions
	2.2.1 Temperature coefficient of resistance
	2.2.2 Dissipation constant, K
	2.2.3 Thermal time constant
	2.2.4 Power sensitivity
	2.2.5 Temperature sensitivity
	2.2.6 Half temperature
	2.2.7 Voltage turnover
	2.3 A.C. and Transient Characteristics
	Introduction
	2.4 Low level analysis
	2.4.1 General small signal analysis
	2.4.2 Small signal equivalent circuits
	2.4.3 Low level transient analysis
	2.5 High level analysis for Newtonian Cooling
	2.5.1 A.C. Signals
	2.5.2 Medium and Low frequency analysis
	2.5.3 High Frequency approximation
	2.5.4 Transient Analysis
III	LINEARISATION OF THERMISTOR CHARACTERISTIC
	3.1 Introduction
	3.2 Shaping of the resistance-Temperature characteristic

## Contents Contd..

3.3	Method of equalising thermistors	41
3.4	Thermistor linearization	43
3.4.1	Principle	43
3.4.2	Theory	44
3.4.3	Design Considerations	45
3.4.4	Linearity	46
3.5	Linearisation of temperature-output characteristic	48
3.5.1	Direct reading bridges	48
3.5.2	Linearisation	48
3.6	Linearisation of potentiometer output	50
3.6.1	Two thermistor network	51
3.6.2	Three thermistor network	52
3.7	Linearisation of the output of bridge Network	52
3.8	Linearising of self-heated thermistor	57
3.8.1	Introduction	57
3.8.2	Principle of linearisation	57
3.9	Conclusions	58

## IV SELECTION CONSIDERATIONS AND TESTING OF THERMISTORS

4.1	Selection factors	60
4.1.1	Introduction	60
4.1.2	Type selection factors	60
4.1.3	Power rating	63
4.2	Method of mounting the thermistors	65
4.3	Temperature stability	66
4.3.1	N.T.C. Thermistors	66
4.4	Testing of thermistors	67
4.4.1	Introduction	67
4.4.2	Determination of B & K	68

## V APPLICATION BASED ON DISSIPATION CONSTANT

5.1	Introduction	70
5.2	Effect of ambient on K	70
5.3	Application based on the rate of flow	71
5.4	Application to Instrument design	74
5.4.1	In Bathy Thermography	74
5.4.2	In Low pressure measurement	76
5.4.3	In gas analysis and chromatography	78
5.4.4	Other applications based on rate of flow	80
5.5	Design details of simple gas analyser and anemometer	80
5.5.1	Introduction	80
5.5.2	Basic principle	80
5.5.3	Constructional details and design considerations	81
5.5.4	Thermistor selection	83

<b>VI</b>	<b>APPLICATION BASED ON RESISTANCE TEMPERATURE CHARACTERISTIC</b>	
6.1	Introduction	85
6.2	Temperature measurement	86
6.3	Incorporation of the thermistor in a Wheatstone bridge	87
6.4	Constant sensitivity bridge	90
6.5	Self balancing thermometer	90
6.6	Differential thermometers	92
6.7	Types of displays in thermistor thermometry	93
6.8	Temperature sensing and control	95
6.8.1	Digital temperature indicator	95
6.8.2	Liquid level indicator	97
6.8.3	Design considerations	98
6.9	Temperature controller	99
6.9.1	Column oven	99
6.9.2	Electronic Controller for the oven	100
6.10	Temperature compensation	100
6.11	Thermistor thermometer based on an astable multivibrator	102
6.11.1	Introduction	102
6.11.2	Principle of operation	102
6.11.3	Linearity	103
6.11.4	Maximum error due to nonlinearity	104
6.11.5	Stability	105
<b>VII</b>	<b>MISCELLANEOUS APPLICATIONS OF DIRECTLY HEATED THERMISTORS</b>	
7.1	Introduction	107
7.2	The thermistor as a low frequency circuit element	107
7.2.1	Phase-advancing (differentiating) circuit	107
7.3	Limiter (regulator, compressor)	111
7.4	Expander	112
7.5	Time-delay relay	113
7.6	Sequential switching circuit	113
7.7	Selector switching circuit	114
7.8	R-C Oscillator stabilisation	115
7.9	Automatic gain control	116
7.10	Compensation of transistor d-c bias	116
7.11	Low frequency multivibrator	117
7.12	Motor protection against temperature variation	118
7.13	Motor protection against over load	119

P A R T II

VIII	TRANSISTOR AS THERMORESISTIVE ELEMENT	
8.1	Introduction	120
8.2	Variation of parameter with temperature	120
8.3	Transistor electronic thermometer	121
8.3.1	Principle	121
8.3.2	Design considerations	123
8.3.3	Calibration	124
8.3.5	Discussion	125
8.3.6	Null and deflection method	126
	C O N C L U S I O N	128
	B I B L I O G R A P H Y	129-131



# PART I

## THERMISTOR AS THERMOSENSITIVE ELEMENT

# CHAPTER I

## INTRODUCTION

# INTRODUCTION

## 1.1 TERMINOLOGY

Thermistor is an acronym for thermally-sensitive resistors. The electrical resistance of the thermistor is a function of its temperature, which depends on both the ambient temperature and the atmosphere and on the internal power dissipation. Thermistors are made from semiconducting material and temperature coefficients between about -5% and + 60% per K are normally encountered. The sign of the temperature coefficient distinguishes the basic types of thermistor.

Negative temperature coefficient (N.T.C) thermistors are the more common. The more common in the N.T.C. thermistors category are the oxide thermistors. These can be made with room temperature resistances ranging from a few ohms to megohms and the room temperature coefficient of resistance usually lies between -4% and -5% per K. For normal N.T.C. thermistors the dependence of resistance on temperature approximately follows an exponential law. This is also true for single-crystal silicon carbide thermistors, which have application over the temperature range -100 to +300 °C.

Positive temperature coefficient (P.T.C) thermistors are made from two kinds of materials : compounds having the barium titanate structure ( one trade name for these is 'Posistors') and diamond-lattice semiconductors such as silicon (a trade name for these is 'Silistors'). The temperature range in which the barium titanate type of thermistors has a positive temperature coefficient can be very large ( $\approx 60\%$  per K). Diamond-lattice type thermistors have a much smaller temperature coefficient of resistance ( $\approx 0.8\%$  per K for silicon), but this is more uniform and applicable over a wider temperature range.

The majority of thermistors are two-terminal devices although three and four-terminal devices are made. The two and three-terminal thermistors are referred to as directly heated types, since the thermistor material itself absorbs power directly from the circuit in which it is connected. Four terminal thermistors are termed indirectly heated types. These have a separate heater in thermal, but not electrical, contact with the thermistor material.

## 1.2 FORMS

Thermistors are made in a variety of shapes and sizes, although they are normally small. Various geometries have been developed to suit particular applications.

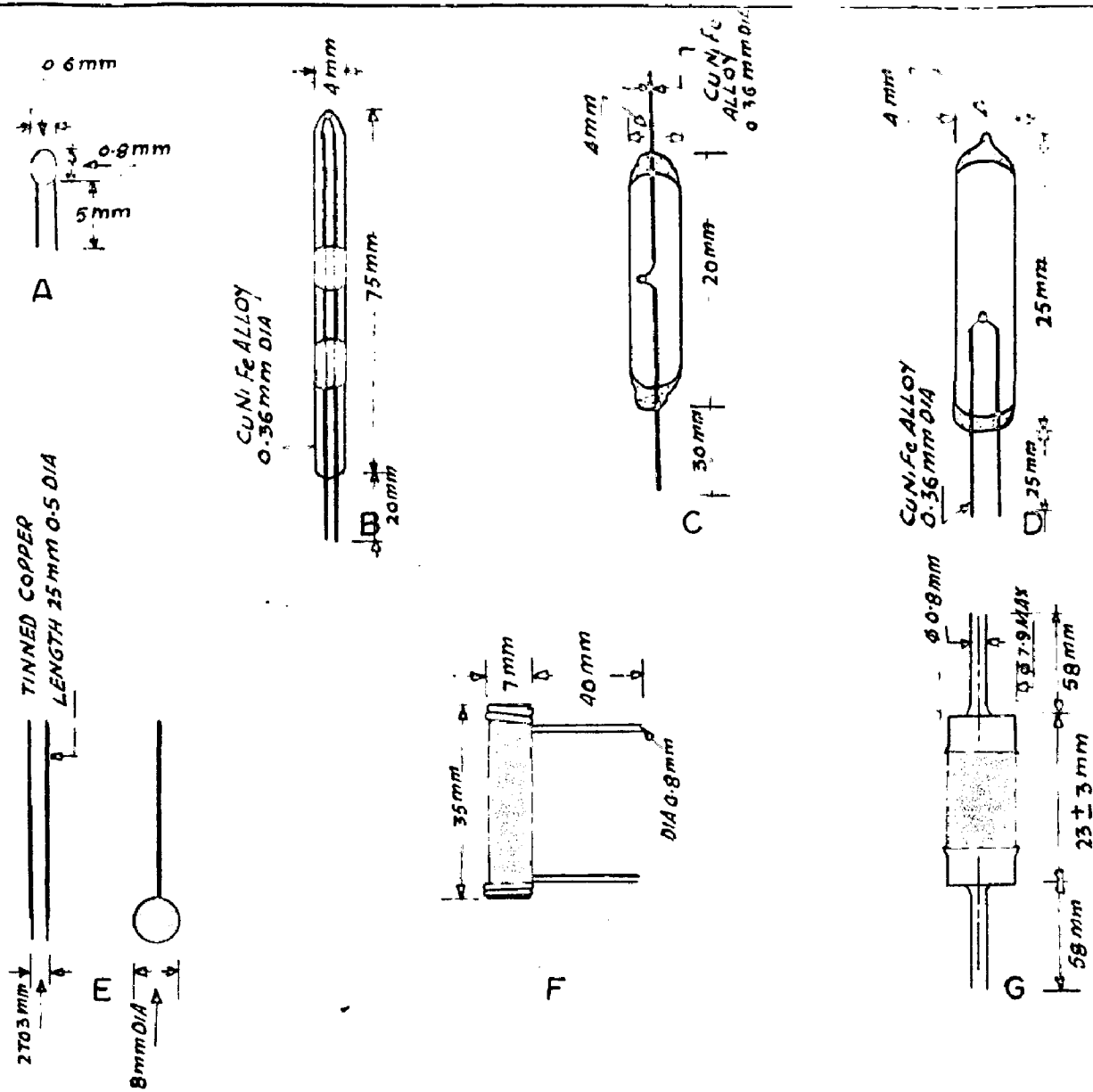


FIG. 1.1 DIFFERENT FORMS OF THERMISTOR PRODUCED BY SEM

- A GLASS COATED BEAD TYPE
- B GLASS PROBE BEAD TYPE
- C/D GLASS ENVELOPE BEAD TYPE
- E DISC TYPE
- F/G ROD TYPE

These are referred to as beads (encapsulated in different ways), discs, washers, rods and thin films. Examples are shown in Fig. 1.1.

### 1.3 ELECTRICAL REPRESENTATION

To avoid complication, consideration should be given to the specification of the properties of standard two-terminal thermistors. This is done in terms of the following quantities :

$R_a$  = The 'zero power' d.c. resistance of the thermistor at the ambient temperature,  $T_a$  (K) : This is the resistance measured when the applied power is so small that there is negligible self-heating.

$R, R(T)$  = The d.c. resistance under active conditions,  $T$  being absolute temperature of the thermistor,

$I$  = The current through the thermistor,

$V$  = The voltage across the thermistor,

$P=VI$  = The power dissipated in the thermistor.

In the steady state condition the equation relating the electrical to thermal power is

$$P = VI = K ( T - T_a ) \quad (1.1)$$

Here  $K$  is known as either the thermal conductance or dissipation constant, and is commonly quoted in  $mw/k$ ,  $K$  is determined by both the construction of the thermistor and its operational environment. Provided that

$T - T_a \ll T_a$ ,  $K$  is independent of  $T$  to a first order. Transient conditions are governed by the following equation ;

$$C \frac{dT}{dt} + K (T - T_a) = P \quad (1.2)$$

where  $C$  is the thermal capacity of the thermistor.

This equation may be written in the form

$$\frac{dT}{dt} + \frac{T - T_a}{\tau} = \frac{P}{C} \quad (1.3)$$

Here  $\tau$  is the thermal time constant of the thermistor which specifies its thermal inertia.

#### 1.4 APPLICATIONS

These are related to the various characteristic relationships between  $I, V, P, K$  and  $T_a$ . The basic thermistor material property is a temperature dependent resistance. However, the material temperature can be influenced in different ways. One is by the ambient temperature,  $T_a$ . The basic thermistor material property is a temperature dependent resistance. However, the material temperature can be influenced in different ways. One is by the ambient temperature,  $T_a$ . Another is by self heating due to internal dissipation of power, this can either be the result of connection into an electrical circuit

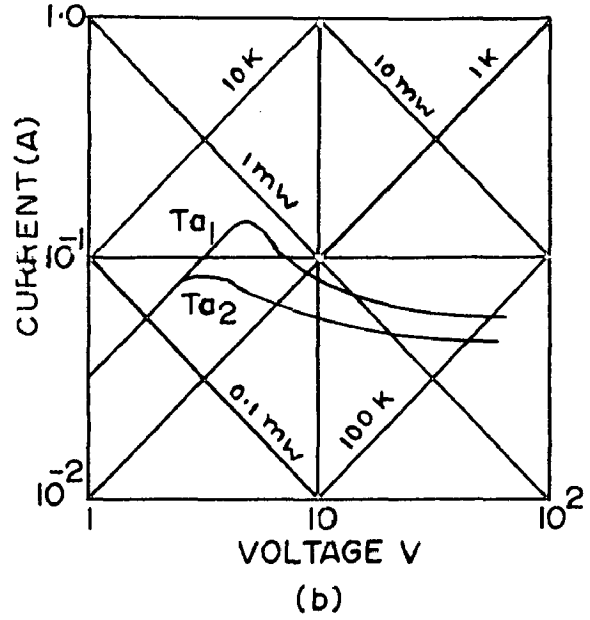
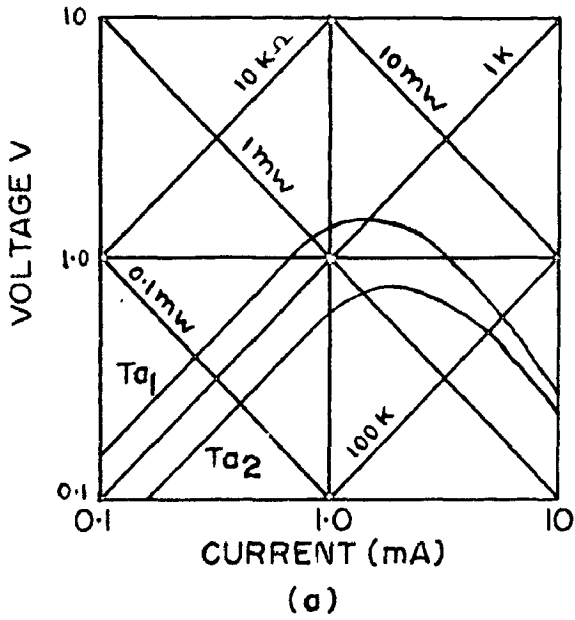


FIG.1.2 VOLTAGE/CURRENT CHARACTERISTICS OF THERMISTOR AS A FUNCTION OF AMBIENT TEMPERATURE,  $T_a$  THE DISSIPATION CONSTANT AND CIRCUIT ARE INVARIANT AND  $T_{a2} > T_{a1}$   
 (a) N.T.C. (b) P.T.C.

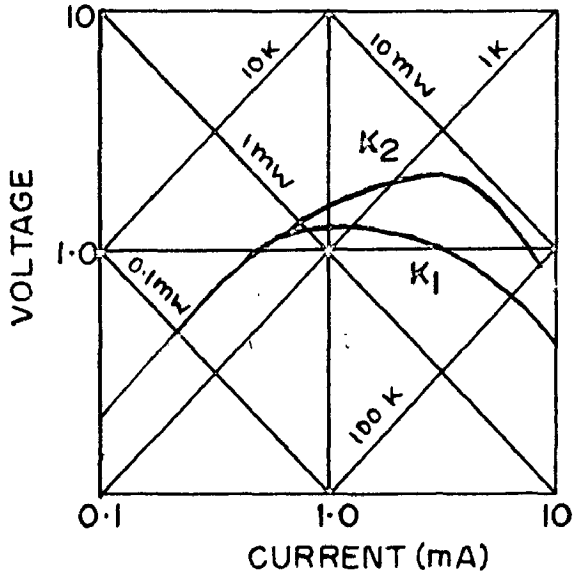


FIG.1.3 VOLTAGE/CURRENT CHARACTERISTICS OF AN N.T.C. THERMISTOR, WITH DISSIPATION CONSTANT AS PARAMETER,  $T_a$  AND THE CIRCUIT ARE INVARIANT AND  $K_2 > K_1$

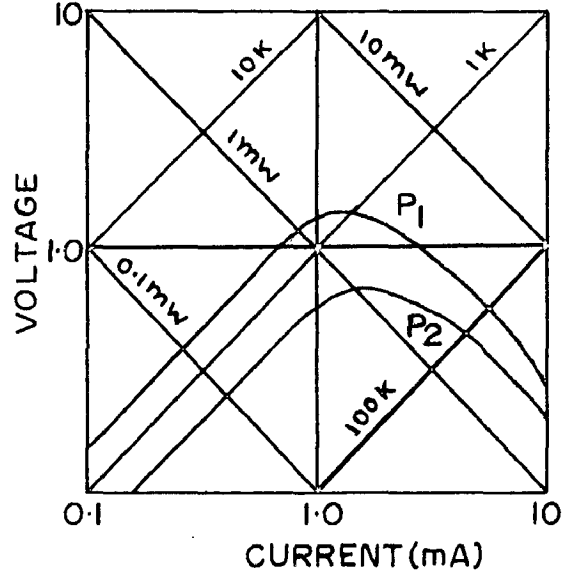


FIG.1.4 VOLTAGE/CURRENT CHARACTERISTICS OF AN N.T.C. THERMISTOR, ABSORBED RADIANT POWER AS PARAMETER.  $K_1 T_a$  AND THE CIRCUIT ARE INVARIANT AND  $P_2 > P_1$



or the absorption of electromagnetic radiation, usually in the microwave and infrared regions of the spectrum. Further, when there is significant self heating, the material temperature is not a constant for a given power dissipation, since the dissipation constant  $K$  is influenced by ambient conditions. Other applications stem from the thermal inertia, specified by thermal time constant  $t_c$ . These include both transient and small-signal conditions.

Typical V-I characteristics for N.T.C. and P.T.C. thermistors are shown in Fig. 1.2a and b respectively. Here the parameter is  $T_a$ ; the environment and hence  $K$  are invariant and there is no incident radiation. At low power levels, so that there is negligible self heating, Ohm's law is obeyed. As self heating occurs so that N.T.C. thermistor resistance falls, whereas that of the P.T.C. thermistor rises. For most practical devices the power rating is such that the characteristic 'turnover' phenomenon occurs - i.e. there is a voltage maximum for N.T.C. and a current maximum for P.T.C. devices. In both cases, beyond turnover, the incremental resistance is negative. It may be noted here that for some P.T.C. thermistors a second turnover occurs due to contact non linearity. Applications based

on these characteristics are as follows : measurement and control of temperature, compensation of change of resistance with temperature in other circuit components; oscillator amplitude and/or frequency regulation, amplifier gain or level stabilisation and equalisation, voltage regulation (N.T.C. types), current regulation (P.T.C. types), speech volume limiting, expansion and compression; switching.

If the ambient temperature is kept constant then a further family of characteristics can be generated with the dissipation constant as the parameter, as shown in Fig. 1.3. For a given thermistor,  $K$  can be varied by connecting it to the heat sinks having different thermal capacities, by changing the density of the gaseous environment, or the rate of flow of a surrounding fluid or gas. The following applications stem from these characteristics, vacuum manometers, flow meters, fluid velocity measurement, thermal conductivity analysis, gas detectors, gas chromatography, liquid level measurement, control and alarm.

A third set of static characteristics is shown in Fig. 1.4. Here the ambient temperature and dissipation constant are invariant and the parameter is absorbed radiant power.

# CHAPTER 2

**D. C. , A. C. & TRANSIENT  
CHARACTERISTIC**

## D.C., A.C. AND TRANSIENT CHARACTERISTICS

### 2.1 D.C. CHARACTERISTICS

#### 2.1.1 Introduction

In the subsequent discussion of d.c. characteristics, Newtonian cooling will be assumed, so that the steady-state relation between electrical power,  $P$ , dissipated in the thermistor material as heat loss per second is

$$P = VI = K (T - T_a) \quad (2.1)$$

with  $K$  independent of  $T$  for  $T - T_a \ll T_a$ . It is implicit that all parts of the thermistor are assumed to be at the same temperature,  $T$ . For large values of  $T - T_a$  it is necessary to use the more complicated expression.

#### 2.1.2 Resistivity

Following the established theory for single-crystal semiconductors, a similar semiquantitative approach can be used to explain the macroscopic dependence of resistivity on temperature of oxide semiconductors. Considering, for example, n-type material. If the density of valence electron sites in such a semiconductor is  $N$ , then at any absolute temperature  $T$  a fraction  $n/N$  of these will be free. The temperature

dependence of  $n$  is governed by an activation energy  $E_g$ , corresponding to an effective energy gap of about 0.5V in practice, and in thermodynamic equilibrium

$$n^2 = (N-n) F \exp (-E_g/kT) \quad (2.2)$$

where  $k$  is Boltzmann's constant and  $F$  is a coefficient which is slightly dependent on temperature. For  $n \ll N$ , as is the case in practice

$$n \simeq \sqrt{(FN) \exp (-E_g/2kT)} \quad (2.3)$$

In an n-type semiconductor the conductivity  $\sigma$  is given by  $n e \mu$ , where  $\mu$  is the electron mobility,  $\mu$  depends on the particular scattering mechanisms involved, but can be represented over limited ranges of temperature by a power-dependence on temperature, as follows :

$$\mu \propto T^d \quad (2.4)$$

where  $d$  is a small negative number. Equations 2.3 and 2.4 can be combined to obtain the equation for resistivity

$$\rho = A T^{-c} \exp (B'/T) \quad (2.5)$$

Here  $c$  is a small positive or negative number or zero,  $B'$  usually lies between 1500 and 6000 K and  $A$  can have a wide range of values. For analytical purposes Equation 2.5 can also be written as,

$$\rho = \rho_c \exp (B/T) \quad (2.6)$$

From Equation 2.1,  $T = \left( \frac{P}{K} + T_a \right)$ , it follows that Equation 2.6 can be re-written as,

$$P = P_c \exp \left[ \frac{B}{\frac{P}{K} + T_a} \right] \quad (2.7)$$

$\rho$  and  $\rho_c$  are related to corresponding terminal resistance values  $R$  and  $R_c$  by the same form factor so that we may write, correspondingly,

$$R = R_c \exp (B/T) = R_c \exp \left[ B / \left( P/K + T_a \right) \right] \quad (2.8)$$

For 'zero power' condition ( $P \approx 0$ ),  $R = R_a$ , corresponding to ambient temperature  $T_0$  and it follows that

$$R_a = R_c \exp (B/T_a) \quad (2.9)$$

$$\text{and } R = R_a \exp \left[ B \left( \frac{1}{T} - \frac{1}{T_a} \right) \right] \quad (2.10)$$

## 2.2 SPECIFIC DEFINITIONS

### 2.2.1 Temperature Coefficients of Resistance

The temperature coefficient of resistance,  $\alpha$ , may be defined as,

$$\alpha = \frac{1}{R} \frac{dR}{dT} = \frac{d}{dT} \left( \log_e \frac{R}{R_a} \right) \quad (2.11)$$

Hence it follows from Equation 2.10 that

$$\alpha = - \frac{B}{T^2} \quad (2.12)$$

### 2.2.2. Dissipation Constant K

This was introduced in Equation 2.1 and may be formally defined as the power required for a unit temperature rise. It is alternatively known as the thermal conductance. K depends on the environment of the thermistor and on its mounting as well as on its ratio of surface to volume and the thermal conductivity of the lead wires and envelope. The conditions under which it is measured should therefore be specified. Since cooling is not strictly Newtonian, nor is the thermistor at a uniform temperature, K is not strictly constant: it increases slightly with increasing T. An empirical relation has been given by Smith<sup>1</sup> to relate power loss to temperature rise,

$\Delta T = T - T_a$  ; when  $\Delta T$  is not small, this is

$$P = K (\Delta T + k'' (\Delta T)^6) \quad (2.13)$$

where  $k''$  is of the order of  $3 \times 10^{-12}$ . Values of K range from about  $10 \mu W K^{-1}$  for a bead, mounted in vacuum,  $50 \mu W K^{-1}$  for a vacuum-backed flake thermistor to  $50 m W K^{-1}$  for large rods or discs mounted on a heat sink.

### 2.2.3 Thermal Time Constant

It is the quotient of the thermal capacity  $C$  and dissipation constant  $K$ . The thermal capacity is the product of mass and specific heat and is therefore governed by the volume and density. A large mass and small dissipation constant means a large  $\tau$  and vice-versa. The cooling curve of the thermistor may be obtained by switching over at  $t = 0$  to a constant current supply of small magnitude, so that  $I^2 R$  was at all times negligible and the voltage across the thermistor was determined as a function of time; thence  $R$ ,  $T$  and  $\tau$  are obtained.

### 2.2.4 Power Sensitivity<sup>2</sup>

From Equation 2.1 and 2.10 we can write

$$-\log_e \left( \frac{R}{R_0} \right) = \frac{B(T - T_a)}{T T_a} = \frac{BP}{T_a(P + K T_a)} \quad (2.14)$$

The power sensitivity, which specifies the change of resistance with power dissipation, is defined as,

$$p = - \frac{1}{R} \frac{dR}{dP} = - \frac{d}{dP} \left[ \log_e \left( \frac{R}{R_0} \right) \right] \quad (2.15)$$

so that on substitution for  $\log_e (R/R_0)$  from Equation 2.14 we obtain,

$$p = \frac{B}{K T_a^2} \frac{1}{(1 + P/K T_a)^2} = \frac{p_0}{\left(1 + \frac{P}{K T_a}\right)^2} \quad (2.16)$$



Here  $p_0 = \frac{B}{K T_a^2}$  is the power coefficient of resistance when  $P/kT_a = 1$ . For high sensitivity  $B/k$  should clearly be as large as possible. It is evident from Equation 2.16 that the sensitivity falls to 25% of its maximum value when  $P = K T_a$ ; i.e. at  $T = 2T_a$ .

### 2.2.5 Temperature Sensitivity<sup>2</sup>

The temperature coefficient of resistance is defined as follows :

$$\alpha = \frac{1}{R} \frac{dR}{dT} = \frac{d}{dT} \left[ \log_e \left( \frac{R}{R_a} \right) \right] \quad (2.17)$$

Hence differentiating equation 2.14 with respect to  $T$ , we find that

$$\alpha = - \frac{B}{\left( \frac{P}{K} + T_a \right)^2} \quad (2.18)$$

which reduces to  $-B / T_a^2$  when  $P/k \ll T_a$ .

### 2.2.6 Half Temperature<sup>2</sup>

It is useful to know the temperature range,  $\Delta T$ , which will halve or double the resistance to its value at an arbitrary reference temperature  $T_a$ . From Equation 2.14

$$\frac{B}{T + \Delta T} - \frac{B}{T_a} = \pm \log_e 2 \quad (2.19)$$

so that

$$\Delta T = \frac{\log_e 2}{-\alpha_a \pm \frac{\log_e e^2}{T_a}} \quad (2.20)$$

Here  $\alpha_a$  is the value of  $\alpha$  at  $T_a$ ; The plus sign applies to doubling the resistance and the minus sign to halving it.

### 2.2.7 Voltage Turnover<sup>2</sup>

The 'peak' or 'turnover' voltage can be obtained from Equation 2.14 by replacing  $R$  by  $V^2/P$  and then determining  $dV/dP = 0$ .

This leads to

$$P_m^2 + P_m K (2T_a - B) + K^2 T_a^2 = 0 \quad (2.21)$$

where  $P_m$  is the power dissipation at turnover. Equation 2.21 may be solved to give,

$$\frac{P_m}{K} = \frac{B}{2} - T_a \pm \frac{B}{2} \left(1 - \frac{4T_a}{B}\right)^{1/2} = T_m - T_a \quad (2.22)$$

where  $T_m = \frac{B}{2} \left[1 - \left(1 - \frac{4T_a}{B}\right)^{1/2}\right]$

By using a binomial expansion for  $(1 - 4T_a/B)^{1/2}$  we find

$$P_m \approx \frac{1}{P_0} \left(1 + \frac{2T_a}{B}\right) \approx \frac{1}{P_0} \quad (2.23)$$

$$\text{and } T_m \approx T_a \left(1 + \frac{T_a}{B}\right) \quad (2.24)$$

Since  $B$  lies between 1500 and 600 K,  $T_m$  lies some 15-60 K above  $T_a$  when  $T_a$  is room temperature.

## A.C. AND TRANSIENT CHARACTERISTICS

### 2.3 INTRODUCTION

The thermal inertia, specified by the thermal time constant,  $\tau$ , gives rise to interesting and useful relationship between terminal voltage and current. It is convenient to distinguish between low and high level conditions. In the former, the applied excitation, (for example a sine wave or step function of current and voltage), is small in relation to the biasing parameters, but in the latter is not. For low-level conditions small signal electrical circuits can be derived.

### 2.4 LOW -LEVEL ANALYSIS

#### 2.4.1. General Small-Signal Analysis

Small-signal equivalent circuits for biased thermistors were derived by Ekelöf and Kihlberg in 1954 and independently by Burgess<sup>3</sup> in 1955. Burgess's treatment is followed here.

Let  $V$  and  $I$  be the instantaneous voltage and current applied to the thermistor, whose instantaneous temperature is  $T$ . The initial assumption made is that

$T$  is single valued and uniform throughout the device, so that internal temperature gradients, such as will arise to some degree in practice, are ignored.

If the rate of loss of heat is determined solely by the instantaneous excess Temperature,  $\theta = T - T_a$ , above the ambient temperature  $T_a$ , then the balance of power equation is

$$C \frac{d\theta}{dt} + f(\theta) = P = IV \quad (2.25)$$

which reduces to

$$C \frac{d\theta}{dt} + K\theta = IV \quad (2.26)$$

for Newtonian cooling, which will be assumed in this section. In these equations  $C$  is the heat capacity of the thermistor and will be assumed independent of  $T$ , and  $K$  is the dissipation constant or thermal conductance.

$I$ ,  $V$  and  $\theta$  will be assumed to have steady components, with relatively small a.c. components at angular frequency  $\omega$ , as follows :

$$I = I_0 + I_1 \exp(j\omega t) \quad (2.27a)$$

$$V = V_0 + V_1 \exp(j\omega t) \quad (2.27b)$$

$$T - T_a = \theta = \theta_0 + \theta_1 \exp(j\omega t) \quad (2.27c)$$

The mean current and voltage,  $I_0$  and  $V_0$ , respectively, define a point on the steady - state characteristic for

the ambient temperature  $T_a$  : this point is also on the isothermal curve for temperature  $T_a + \theta_0$ . The quantities  $I_1$ ,  $V_1$  and  $\theta_1$  are in general complex, carrying both amplitude and phase information. The corresponding expression for instantaneous power is, approximately,

$$P = P_0 + P_1 \exp(j\omega t) = IV = I_0 V_0 + (I_0 V_1 + I_1 V_0) \exp(j\omega t) \quad (2.28)$$

if  $|I_1|$ ,  $|V_1| \ll I_0$ ,  $V_0$ , respectively. Substitution from Equation 2.27c and 2.28 into Equation 2.26 gives

$$K \theta_0 = I_0 V_0 \quad (2.29a)$$

$$\theta_1 (K + j\omega C) = I_0 V_1 + I_1 V_0 = P_1 = \theta_1 y \quad (2.29b)$$

here  $y = P_1 / \theta_1$  is the 'thermal admittance'. This may be expressed as,

$$y = \frac{C}{\tau} + j\omega C \quad (2.30)$$

where  $\tau = C/K$  is the thermal time constant of the thermistor; i.e. the time taken for the excess temperature to fall to  $1/e$  of its initial value if the d.c. and a.c. sources are removed. Now the alternating component of current  $I_1$  is determined by both  $V_1$  and  $\theta_1$  through the isothermal a.c. conductance  $g$  and the current temperature coefficient,  $h$  :

$$I_1 = \left( \frac{\partial I}{\partial V} \right)_T V_1 + \left( \frac{\partial I}{\partial T} \right)_V \theta_1 = g V_1 + h \theta_1 \quad (2.31)$$

The sign of  $h$  distinguishes N.T.C. and P.T.C. thermistors:

$h > 0$  signifies N.T.C and  $h < 0$ , P.T.C. The a.c. admittance is

$$Y(\omega) = \frac{I_1}{V_1} = \frac{gy + hI_0}{y - hV_0} \quad (2.32)$$

At zero frequency this becomes the pure conductance

$$Y_0 = G_0 = \frac{gK + hI_0}{K - hV_0} \quad (2.33)$$

$G_0$  is simply the slope of the steady-state current-voltage characteristic. Another way of looking at this is to consider small and infinitely slow increments  $\Delta I$  and  $\Delta V$  along the steady state characteristic, accompanied by a temperature change  $\Delta \theta$ , so that

$$K\Delta\theta = I_0\Delta V + V_0\Delta I \quad (2.34a)$$

$$\Delta I = g\Delta V + h\Delta\theta \quad (2.34b)$$

These equations may be combined to give  $G_0 = \Delta I / \Delta V$ , in the form of Equation 2.32.

At infinite frequency the a.c. admittance is

$$Y_\infty = G_\infty = g \quad (2.35)$$

This is again purely real because instantaneous temperature of the device cannot follow the rapid variation of electrical power.

The general form of admittance,  $Y(\omega) = G(\omega) + jB(\omega)$ , may be expressed in terms of  $G(\omega)$  and  $B(\omega)$  as follows :

$$\begin{aligned}
 G(\omega) &= \frac{g\omega^2 C^2 + (gC/\tau + hI_0)(C/\tau - hV_0)}{\omega^2 C^2 + (C/\tau - hV_0)^2} \\
 &= g + \frac{h(I_0 + gV_0)(C/\tau - hV_0)^2}{\omega^2 C^2 + (C/\tau - hV_0)^2} \quad (2.36a)
 \end{aligned}$$

$$B(\omega) = \frac{\omega Ch(gV_0 + I_0)}{\omega^2 C^2 + (C/\tau - hV_0)^2} \quad (2.36b)$$

The corresponding formulae for the impedance representation,

$$Z(\omega) = 1/Y(\omega) = R(\omega) + jX(\omega), \text{ are,}$$

$$R(\omega) = \frac{g\omega^2 C^2 + (C/\tau - hV_0)(gC/\tau + hI_0)}{g^2\omega^2 C^2 + (gC/\tau + hI_0)^2} \quad (2.37a)$$

$$X(\omega) = \frac{\omega C h(gV_0 + I_0)}{g^2\omega^2 C^2 + (gC/\tau + hI_0)^2} \quad (2.37b)$$

Using the identity  $g \equiv I_0/V_0$ , which is valid for linear isothermals, which arise from N.T.C. Thermistors, it follows from Equation 2.33 that

$$hI_0 = \frac{gK(G_0 - g)}{(G_0 + g)} \quad (2.38a)$$

$$hV_0 = \frac{K(G_0 - g)}{(G_0 + g)} \quad (2.38b)$$

Making use of identities  $G_0 = 1/R_0$ ,  $g = 1/R_\infty$  and  $T = C/K$ , together with the above expressions for  $hI_0$  and  $hV_0$ , we obtain alternative forms for Equations 2.35 and 2.37 as follows :

$$G(\omega) = \frac{\frac{\omega^2 T^2}{R_\infty} + \frac{4 R_0}{(R_0 + R_\infty)^2}}{\omega^2 T^2 + \frac{4 R_0^2}{(R_0 + R_\infty)^2}} \quad (2.36c)$$

$$B(\omega) = \frac{\frac{2\omega T (R_\infty - R_0)}{R_\infty (R_\infty + R_0)}}{\omega^2 T^2 + \frac{4 R_0^2}{(R_0 + R_\infty)^2}} \quad (2.36d)$$

$$R(\omega) = \frac{R_0 + \frac{\omega^2 T^2 (R_\infty + R_0)^2}{4 R_\infty}}{1 + \frac{\omega^2 T^2 (R_\infty + R_0)^2}{4 R_\infty^2}} \quad (2.37c)$$

$$X(\omega) = \frac{\frac{\omega T (R_\infty^2 - R_0^2)}{2 R_\infty}}{1 + \frac{\omega^2 T^2 (R_\infty + R_0)^2}{4 R_\infty^2}} \quad (2.37d)$$

Both the impedance and admittance loci, when plotted in



the complex plane, are semicircles with diameter on the real axis, if  $g, c, h$  and  $\tau$  are independent of frequency. This is clear if, for example, the expression for  $Z(\omega)$  is rewritten in the form

$$Z(\omega) = R_{\infty} - \frac{(R_{\infty} - R_0)}{1 + j\omega\tau \left[ \frac{R_{\infty} + R_0}{2R_{\infty}} \right]} \quad (2.39)$$

If  $G_0$  is negative, there is a critical frequency defined by,

$$\omega_0^2 = \left( \frac{hV_0}{c} - \frac{1}{\tau} \right) \left( \frac{hI_0}{gC} + \frac{1}{\tau} \right) = \frac{4gG_0}{\tau^2 (G_0 + g)^2} \quad (2.40)$$

The terminal conductance will be zero and  $I_1$  and  $V_1$  will be in phase quadrature. This situation is depicted by the admittance locus on the left of Fig. 2.1 for an N.T.C thermistor. From the geometry it is apparent that the susceptance at  $\omega_0$  is given by

$$B(\omega_0) = (-G_0 g)^{1/2}$$

The condition that  $G_0$  is negative is  $h > c / \tau V_0$  for an N.T.C. thermistor and  $h < -gC / \tau I_0$  for a P.T.C. thermistor. The right hand locus in Fig. 2.1

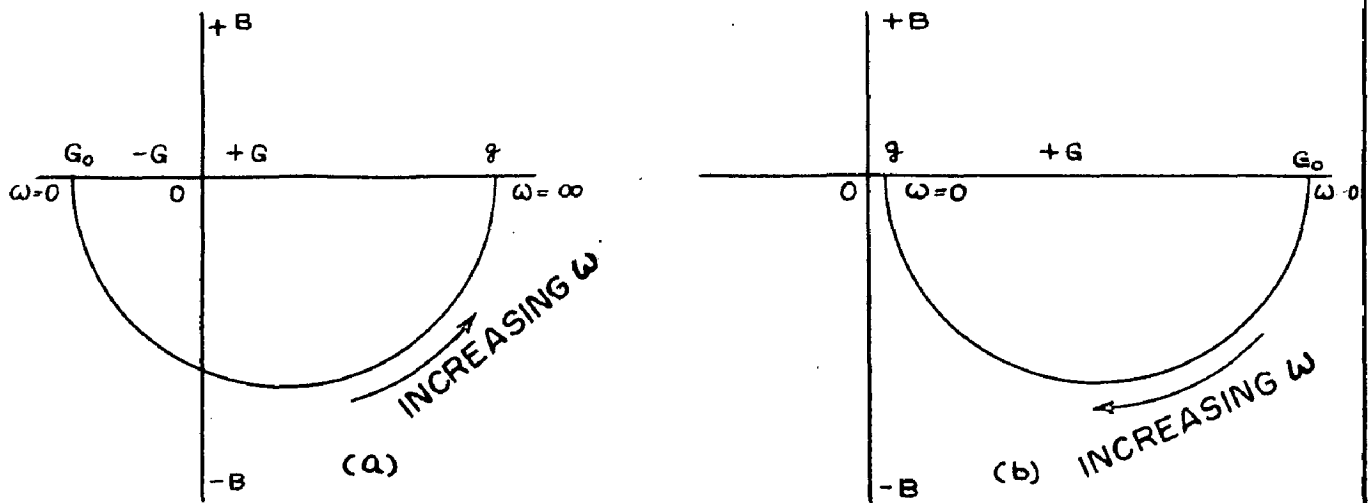


FIG.2.1 SMALL SIGNAL ADMITTANCE LOCI FOR AN N.T.C. THERMISTOR BIASED (a) BEYOND TURN OVER (b) BELOW TURN OVER.

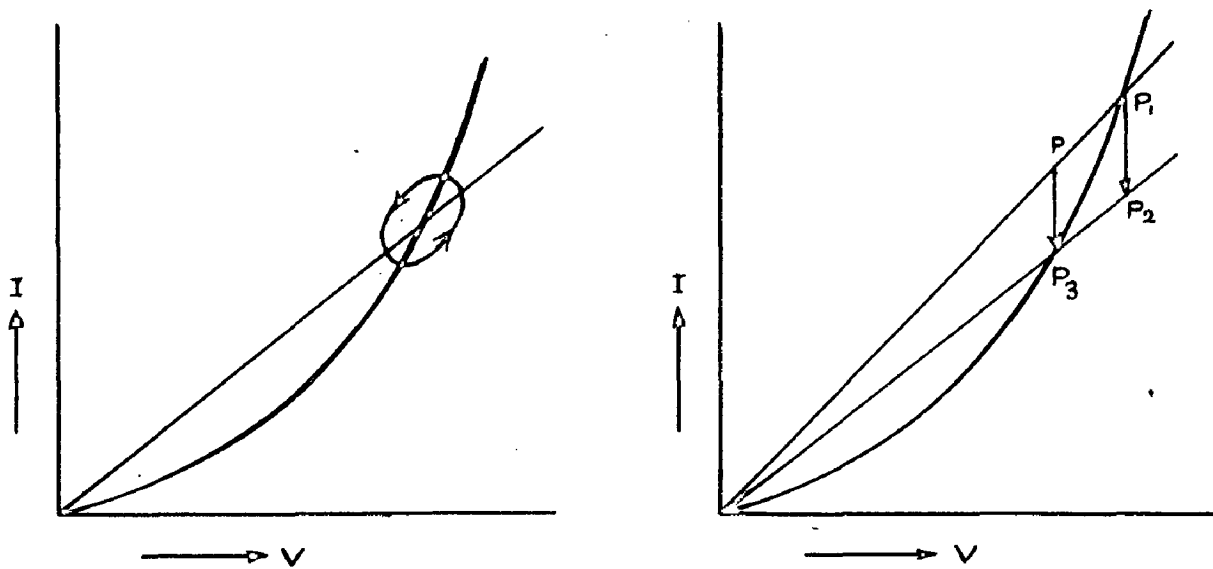


FIG.2.2 LOCUS OF THE OPERATING POINT FOR (a) SINUSOIDAL AND (b) SQUARE WAVE EXCITATION OF AN N.T.C. THERMISTOR.

is for an N.T.C thermistor biased below turnover, so that  $G_0$  is positive.

It is instructive to consider the load line approach for small-signal a.c. operation. At very low frequencies, such that  $\omega \ll 1/\tau$ , the temperature is cophasal with IV and the locus of the operating point is a small arc of the static characteristic (straight line for the small-signal case being considered, corresponding to the slope conductance,  $G_0$ ). At high frequencies, such that  $\omega \gg 1/\tau$ , the power variation is so rapid that the temperature cannot change and isothermal conditions prevail. The locus is then a section of isothermal characteristic. In the intermediate frequency range the instantaneous temperature lags the instantaneous power dissipation with a phase angle corresponding to that of thermal admittance  $Y$ , so giving rise to an approximately elliptical locus as shown in Fig. 2.2a. The major axis of the ellipse may have either a positive or negative slope according to the amplitude and frequency of the excitation. It will be noted that the direction in which the ellipse is traced out is anticlockwise, for bias beyond turnover. That this is so may be seen by reference to Fig. 2.2.b. If a small instantaneous departure from the established operating point is produced by a

ge change then initially the thermistor temperature  
 before and the operating point follows the ohmic  
 from the origin from  $P$  to  $P_1$ . As the thermistor  
 up so its resistance falls and the operating  
 moves to  $P_2$ . If a small voltage excitation is  
 ed then the operating point moves viz  $P_3$  back to  
 he lead of voltage change over current change is  
 istic of the inductive behaviour already esta-  
 ed. For P.T.C. thermistors the converse is true  
 he locus is traced in a clockwise direction if  $I$   
 are plotted as in the same way. At the critical  
 ency such that the admittance is a pure susceptance  
 urrent lags the voltage by  $\pi/2$  radians and  
 es of the ellipse are parallel to the  $I$  and  $V$   
 respectively.

### Small Signal Equivalent Circuits

Equivalent circuits corresponding to the analytical  
 ssions of equations 2.36 and 2.37 are shown in  
 2.3. In the upper part of the figure two forms  
 own for N.T.C. thermistors ( $h > 0$ ,  $G_0 > g$ ) and  
 , two corresponding equivalent circuits for P.T.C.  
 istors ( $h < 0$ ,  $G_0 < g$ ). It is convenient to  
 fy the elements in terms of  $R_0$ ,  $R_{\infty}$  and  $\tau$ , since  
 may readily be determined experimentally.  $R_0$  and

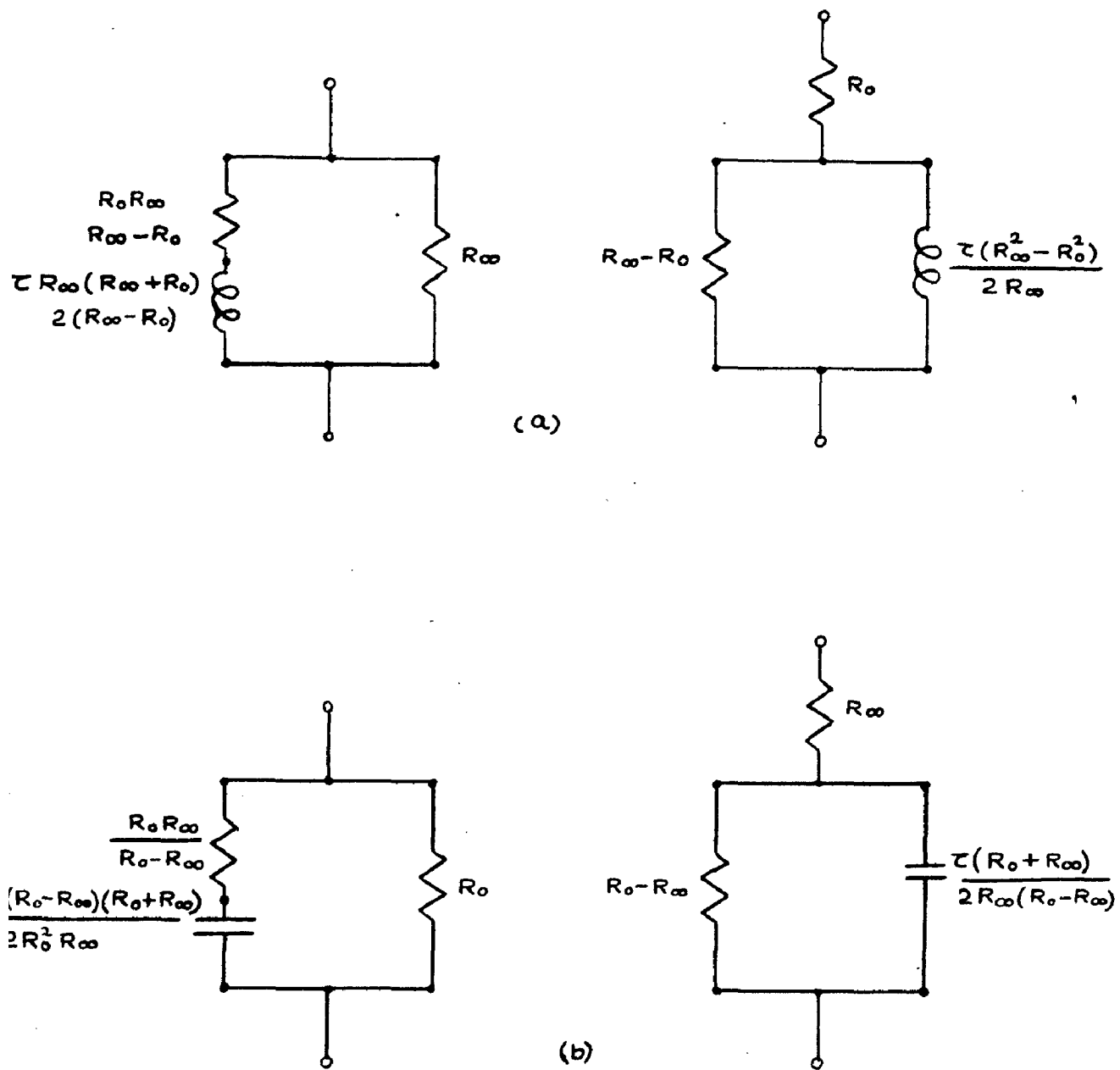


FIG.2.3 SMALL SIGNAL EQUIVALENT CIRCUITS OF THERMISTOR  
 (a) N. T. C. (b) P. T. C.

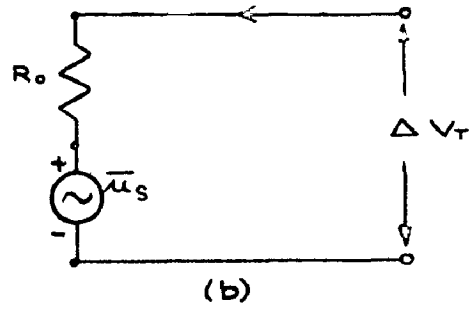
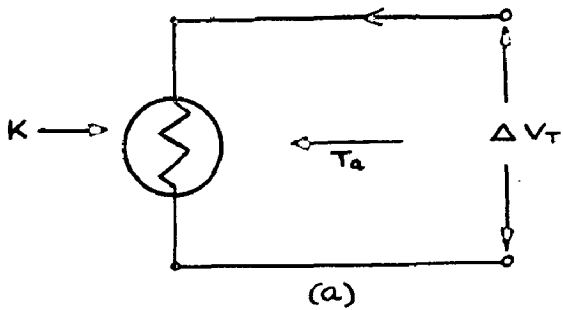
$R_{\infty}$  are very simply obtained from d.c. measurements.  $\tau$  can be calculated from the measured effective time constant introduced in the next section.

In some applications in which the operating point is defined by an external source of emf and a series resistor, changes in it can be brought about by changes in ambient temperature  $T_a$  and / or dissipation constant  $K$ . When such changes are relatively small an appropriate small-signal equivalent circuit can be readily devised if the changes in  $K$  or  $T_a$  are very slow in relation to  $1/\tau$ .

The form of equivalent circuit is similar to those for the vacuum triode. It is shown in Fig. 2.4. The resistance,  $R_o$ , is simply the slope of the static characteristic at the operating point, defined as

$$R_o = \left. \frac{\delta V}{\delta I} \right|_{K, T \text{ const.}}$$

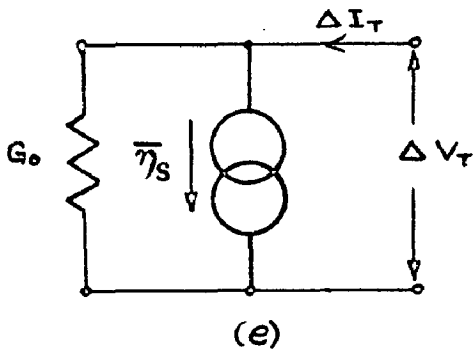
The 'amplification' factor is comprised of two parts, in general, since excitations of both dissipation factor and ambient temperature are possible. For N.T.C thermistors (current controlled devices), these are defined respectively as,



$$(c) \Delta V_T = R_o \Delta I_T + \mu_s$$

$$(d) \mu_s = \mu_K \Delta K + \mu_{T_a} \Delta T_a$$

(S = STIMULUS)



$$(f) \Delta I_T = G_o \Delta V_T + \bar{\eta}_s$$

$$(g) \bar{\eta}_s = \eta_K \Delta K + \eta_{T_a} \Delta T_a$$

FIG. 2·4 SMALL SIGNAL REPRESENTATION OF THERMISTORS (a-d, N.T.C., e-g, P.T.C.): (a) CIRCUIT (b) EQUIVALENT CIRCUIT (c) SMALL-SIGNAL EQUATION (d) DEFINITION OF  $\mu_s$  (e) EQUIVALENT CIRCUIT (f) SMALL-SIGNAL EQUATION (g) DEFINITION OF  $\bar{\eta}_s$

$$\mu_K = \left. \frac{\delta V}{\delta K} \right|_{I, T_a \text{ const.}}, \quad \mu_{T_a} = \left. \frac{\delta V}{\delta T_a} \right|_{I, K \text{ const}}$$

It is obvious from the characteristics of Fig. 1.3 and 1.2a respectively that  $\mu_K$  is positive and  $\mu_{T_a}$  is negative.  $\mu_K$  can be derived by differentiating the two expressions for  $V$ ,  $V = IR$  and  $V = K(T - T_a)/I$  with  $R$  defined as in Equation 2.10,

$$\frac{\delta V}{\delta K} = - \frac{BV}{I^2} \frac{dT}{dK} = \frac{T - T_a}{I} + \frac{K}{I} \frac{dT}{dK}$$

Elimination of  $dT/dK$  gives,

$$\mu_K = \frac{V}{1 - K/\alpha P} \frac{1}{K} \quad (2.41a)$$

where  $\alpha$  as defined in Equation 2.11 is negative. Similarly it can be shown that

$$\mu_{T_a} = - \frac{V}{1 - K/\alpha P} \frac{1}{T - T_a} \quad (2.41b)$$

The overall relation between the small-signal quantities is shown in Fig. 2.4.

A similar equivalent circuit can be devised for P.T.C Thermistors. In this case, however, since we are concerned with voltage controlled devices (current is



a single valued function of voltage) it is more appropriate to define quantities  $\eta_K$  and  $\eta_{T_a}$  as follows :

$$\eta_K = \left. \frac{\delta I}{\delta K} \right|_{V, T \text{ const}}; \quad \eta_{T_a} = \left. \frac{\delta I}{\delta T_a} \right|_{V, K \text{ const.}}$$

Here  $\eta_K$  is positive and  $\eta_{T_a}$  negative. It is also more appropriate to use the slope conductance,  $G_0$  of the static characteristic at the operating point, defined as  $(\delta I / \delta V) |_{K, T_a \text{ const.}}$  in the equivalent circuit. This, and the terminal small-signal current-voltage relationship, expressed in terms of small-signal parameters, is shown in Fig. 2.4b.

### 2.4.3 Low Level Transient Analysis

We consider a simple series circuit comprising a thermistor, a resistor of resistance  $S$  and battery of e.m.f.  $V$  with an additional series e.m.f.  $\Delta V \ll V$ , which can be switched in series with  $V$ . In the initial steady state condition determined by  $V$ , we can adapt Equation 2.2.6 to obtain,

$$K (T_{\infty} - T_a) = V^2 \frac{R_{\infty}}{(R_{\infty} + S)^2} \quad (2.42)$$

where  $T_{\infty}$  is the thermistor temperature and  $R_{\infty}$  is the

corresponding d.c. (isothermal) resistance. On applying the step of e.m.f.,  $\Delta V$ , the equation defining the transient conditions is

$$C \frac{dT}{dt} + K (T - T_{\infty}) = (V + \Delta V)^2 \frac{R'_{\infty}}{(R_{\infty} + S)^2} - V^2 \frac{R_{\infty}}{(R_{\infty} + S)^2} \quad \dots(2.43)$$

For small temperature differences,  $T - T_{\infty}$ , we may suppose that  $R'_{\infty}$ , the instantaneous value of  $R_{\infty}$ , has the form,

$$R'_{\infty} = R_{\infty} \left[ 1 + \alpha (T - T_{\infty}) \right] \quad \dots(2.44)$$

Here  $\alpha = -B / T_{\infty}^2$  is the temperature coefficient of resistance at the working point. Substitution for  $R'_{\infty}$  in Equation 2.43 and neglect of second order terms in  $T - T_{\infty}$  leads to

$$C \frac{dT}{dt} + (T - T_{\infty}) \left[ K - (V + \Delta V)^2 \frac{R_{\infty}}{(R_{\infty} + S)^2} - \frac{B}{T_{\infty}^2} \frac{(R_{\infty} - S)}{(R_{\infty} + S)} \right] = (V^2 + 2V\Delta V) \frac{R_{\infty}}{(R_{\infty} + S)^2} \quad \dots(2.45)$$

By inspection it can be seen that the effective thermal time constant for the circuit is

$$\tau_e = \frac{1 - (V + \Delta V)^2 \frac{R_\infty}{(R_\infty + S)^2} \frac{B}{K T_\infty^2} \frac{(R_\infty - S)}{(R_\infty + S)}}{\dots} \quad \dots(2.46)$$

We have assumed that for small temperature differences  $R'_\infty - R_\infty$  is linearly related to  $T - T_\infty$ , so that the time constant governing the rate of change of  $R'_\infty$  is likewise  $\tau_e$ . Furthermore, since the changes in  $R'_\infty$  are small, the rate of change of current is also governed by  $\tau_e$ , which may therefore be formally identified with the effective time constant for the circuit,  $\tau_c$ .

$\tau_c$  specifies the rate of change of resistance with time and is the parameter of importance for circuit applications. This will now be derived for N.T.C. Thermistors. We have two basic equations relating to I and V;

$$P = IV = I^2 R_\infty = V^2 / R_\infty = K (T_\infty - T_a) \quad (2.47)$$

$$R_\infty = V/I = R_c \exp ( B/T_\infty ) \quad (2.48)$$

From the total differential of equation 2.47 we obtain

$$\frac{dV}{dI} = \frac{K}{I_\infty} \frac{dT}{dI} - \frac{V_\infty}{I_\infty}$$

and from the corresponding differential of equation 2.48,

$$\frac{dV}{dI} = \frac{V dI}{I^2} = \frac{-B}{T_{\infty}^2} \frac{V_{\infty}}{I_{\infty}} dT$$

Combining these leads to

$$\frac{dV}{dI} \frac{K T_{\infty}^2}{V_{\infty} B I_{\infty}} \left[ \frac{V_{\infty}}{I_{\infty}} - \frac{I}{I_{\infty}} \frac{dV}{dI} \right] = \frac{V_{\infty}}{I_{\infty}}$$

which may be put into the form

$$\frac{K T_{\infty}^2}{B I_{\infty}^2 R_{\infty}} = \frac{R_{\infty} + R_0}{R_{\infty} - R_0}$$

and because

$$I_{\infty} = \frac{V_1}{R_{\infty} + S} \approx \frac{V_1 + \Delta V}{R_{\infty} + S}$$

This becomes

$$\frac{K T_{\infty}^2 (R_{\infty} + S)^2}{B (V_1 + \Delta V)^2 R_{\infty}} = \frac{R_{\infty} + R_0}{R_{\infty} - R_0} \quad (2.49)$$

Substituting in equation 2.45 yields

$$T_c = T \frac{(R_{\infty} + R_0)}{2 R_{\infty}} \frac{(R_{\infty} + S)}{(R_0 + S)} \quad (2.50)$$

For  $S = \infty$ , Equation 2.50 reduces to ,

$$\tau_c = \tau_e = \tau \frac{(R_\infty + R_0)}{2 R_\infty} \quad (2.51)$$

which is the property of the thermistor itself.

If  $R_0$  and  $R_\infty$  are known and  $\tau_e$  is measured for a range of values of  $S$ ,  $\tau$  can be calculated.

Equation 2.50 may be derived directly after using either of the equivalent circuits in the upper part of Fig.2.3 to represent the thermistor in the circuit which has been discussed.

## 2.5 HIGH LEVEL ANALYSIS FOR NEWTONIAN COOLING

Provided that the instantaneous resistance of the thermistor can be specified to a good approximation in a relatively simple closed form, then it is possible to obtain approximate predictions of circuit behaviour under large signal conditions.

### 2.5.1 A.C. Signals

We will consider a simple series circuit at ambient temperature  $T_a$ , comprising a sinusoidal source of e.m.f.  $e(t) = E \sin \omega t$ , a resistance  $S$  and an N.T.C. thermistor, whose instantaneous resistance is specified as,

$$R(T) = R_c \exp (B/T) \quad (2.52a)$$

$$\text{or } R(\theta) = R_c \exp \left[ \frac{B}{(T_a + \theta)} \right] \quad (2.52b)$$

The analytical technique depends on the frequency range. For 'medium' and 'low' frequencies such that  $\omega\tau$  is of the order of or less than unity a computer solution is required but at high frequencies,  $\omega\tau \gg 1$ , an approximate analysis is possible.

### 2.5.2 Medium and Low Frequency Analysis

The instantaneous current in the circuit is

$$i(t) = \frac{E \sin \omega t}{S + R(T)}$$

and the instantaneous power dissipated in the thermistor is

$$P = R(T) i^2(t) = \frac{R(T) E^2 \sin^2 \omega t}{(S + R(T))^2} \quad (2.53)$$

Substitution of Equation 2.53 into Equation 2.26 gives

$$C \frac{d\theta}{dt} + K\theta = \frac{R\theta}{(S+R(\theta))^2} E^2 \sin^2 \omega t \quad (2.54)$$

Integrating with respect to  $t$  brings the equation to a form suitable for analogue computation. Introduction

of the identities

$$g(\theta) = \frac{R(\theta)}{(s + R(\theta))^2}$$

$$h(t) = E^2 \int_0^t \sin^2 \omega t dt$$

leads to

$$C \theta - \theta(0) = K \int_0^t \theta dt = \int_0^t g(\theta) dh(t) \quad (2.55)$$

Here  $g(\theta)$  and  $h(t)$  are known functions and furthermore  $\theta(0) = 0$  if the e.m.f. is connected at  $t = 0$  and the thermistor temperature at this instant is  $T_a$ . Equation 2.55 can readily be set up on a differential analyser and a solution for  $\theta(t)$  found. Corresponding time-variations of  $R, i$  and  $v$  can then be calculated.

### 2.5.3 High-Frequency Approximation

If  $\omega\tau > 1$  then the instantaneous resistance will vary very little over a period. For the same series circuit considered above it is then permissible to write the following expressions for the instantaneous voltage and current at the thermistor terminals

$$v(t) = (\bar{V} + \Delta V) \sin \omega t \quad (2.56a)$$

$$i(t) = (\bar{I} + \Delta I) \sin \omega t \quad (2.56b)$$

Here  $\bar{V}$  and  $\bar{I}$  represent 'average' peak values of voltage and current, which approximately specify  $v(t)$  and  $i(t)$ , respectively.  $\Delta V$  and  $\Delta I$  represent in-phase and time dependent amplitude around  $\bar{V}$  and  $\bar{I}$ , respectively. It follows that

$$\bar{V} + \bar{I} s = \bar{E} \quad (2.57a)$$

$$\Delta V + \Delta I s = 0 \quad (2.57b)$$

and the instantaneous resistance of the thermistor is given by

$$R(t) = \frac{V(t)}{I(t)} \approx \frac{\bar{V}}{\bar{I}} \left[ 1 + \frac{\Delta V}{\bar{V}} - \frac{\Delta I}{\bar{I}} \right]$$

$$\text{or } R(t) = \bar{R} + \Delta R \quad (2.58)$$

To proceed further an expression for  $\Delta R$  must be obtained from the power-balance equation, 2.26,

This may be written,

$$P - K\theta = C \frac{d\theta}{dt} = C \frac{d\theta}{dR} \frac{dR}{dt} \quad (2.59)$$

The relative variations in  $\theta$  are much smaller than in  $P = VI$ , so we can put,

$$P - K\theta \approx P - K\bar{\theta} = P - \bar{P} \quad (2.59a)$$

From equation 2.59 an expression for  $dR$  can be obtained

$$dR = \frac{(P - \bar{P})}{C} \frac{dR}{d\theta} dt \quad (2.60)$$

Since  $dR/d\theta$  can be identified with  $(\overline{dR/d\theta}) = \bar{R} \bar{K}/\bar{P}$ ,



in the present degree of approximation, this becomes

$$dR = \frac{(P-\bar{P})}{\bar{P}} \frac{\bar{F} \bar{R}}{\bar{T}} \quad (2.61)$$

where  $T$  has also been replaced by its mean value  $\bar{T}$ .

Integration gives

$$R = \bar{R} \left[ 1 - \frac{\bar{F}}{\bar{T}} \int_0^t \left( \frac{P}{\bar{P}} - 1 \right) dt \right] \quad (2.62)$$

$$\text{Hence } \frac{\Delta R}{\bar{R}} = - \frac{\bar{F}}{\bar{T}} \int_0^t \left( \frac{P}{\bar{P}} - 1 \right) dt \quad (2.63)$$

The applied power is given by

$$P = VI \simeq \bar{V} \bar{I} \sin^2 \omega t$$

$$\text{or } P = \bar{P} (1 - \cos 2\omega t) \quad (2.64)$$

where  $\bar{P} = \bar{V} \bar{I} / 2$ . Substitution in Equation 2.63 and subsequent integration gives

$$\frac{\Delta R}{\bar{R}} \approx \frac{\bar{F}}{2 \omega \bar{T}} \sin 2 \omega t \quad (2.65)$$

Now from Equations 2.57 and 2.58 we can obtain

$$\frac{\Delta V}{\bar{V}} = \left(1 - \frac{\bar{V}}{E}\right) \frac{\Delta R}{\bar{R}} = \frac{S}{(S+\bar{R})} \frac{\Delta R}{\bar{R}} \quad (2.66a)$$

$$\frac{\Delta I}{\bar{I}} = - \frac{\bar{V}}{E} \frac{\Delta R}{\bar{R}} = - \frac{\bar{R}}{(S+\bar{R})} \frac{\Delta R}{\bar{R}} \quad (2.66b)$$

Introducing  $\Delta V$  and  $\Delta I$  from Equation 2.66 into Equation 2.56 and paying regard to Equation 2.65, leads to the following final expressions for  $V$  and  $I$  :

$$V(t) = \bar{V} \left( \sin \omega t + \frac{D}{(1+\alpha)} (\cos \omega t - \cos 3 \omega t) \right) \quad (2.67a)$$

$$I(t) = \bar{I} \left( \sin \omega t - \frac{D}{(1+\alpha)} (\cos \omega t - \cos 3 \omega t) \right) \quad (2.67b)$$

Where  $\alpha = \bar{R} / S$

and  $D = \bar{F} / 4 \omega \tau$ .

The current lags the voltage at the fundamental frequency as expected for the N.T.C. thermistor, which we have already seen has inductive properties for small signals. The magnitude of the phase angle is

$$\theta = \tan^{-1} \frac{D}{1 - \frac{D^2 \alpha}{(1+\alpha)^2}} \quad (2.68)$$

or  $\theta \simeq D$  if  $D \ll 1$

If  $S = \infty$ , i.e.,  $\alpha = 0$ , only the voltage is distorted.

$$V = \bar{V} \left( \sin \omega t + D (\cos \omega t - \cos 3 \omega t) \right) \quad (2.69)$$

correspondingly if  $S = 0$ , i.e.,  $\alpha = \infty$  only the current is distorted :

$$I = \bar{I} \left( \sin \omega t - D (\cos \omega t - \cos 3 \omega t) \right) \quad (2.70)$$

#### 2.5.4 Transient Analysis

It is convenient to use a semi-graphical approach. The same series circuit as analysed above will be considered here for the case in which, at  $t = 0$  the circuit e.m.f. is changed from  $E_1$  to  $E_2$  and the series resistance from  $S_1$  to  $S_2$ . These conditions are shown by the corresponding load lines I and II drawn on the static characteristic for an N.T.C. thermistor at ambient temperature  $T_a$ , in Fig. 2.5. For  $t = 0$  the circuit is in the stationary state, represented by point 1 ( $V_1, I_1$ ), which is the intersection between the thermistor V-I characteristic and load line I, determined by the equation

$$V + S_1 I = E_1 \quad (2.71)$$

The thermistor resistance  $R_1 = V_1/I_1$  is on the isothermal (linear) characteristic from the origin, which passes through point 1. At  $t = 0$  the change in  $E$  and  $S$  to the new values  $E_2$  and  $S_2$  force the dynamic point  $V, I$  to move from load line I to load line II. Because of the thermal inertia of the thermistor, abrupt changes of thermistor resistance are not possible, so at  $t = 0$ , the operating point is constrained to move along the isothermal characteristic passing through point I to the corresponding

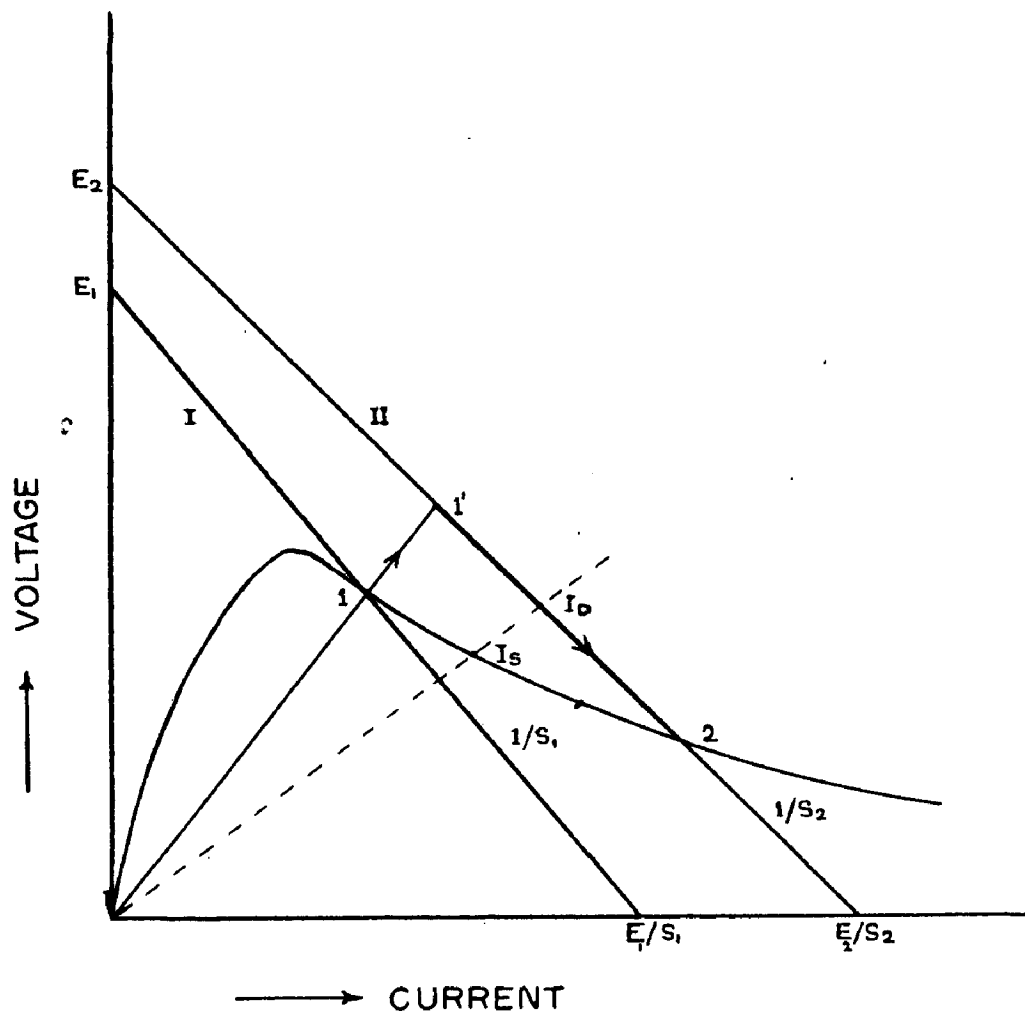


FIG. 2.5 STATIC AND DYNAMIC CHARACTERISTICS OF A N.T.C. THERMISTOR FOR TRANSIENT ANALYSIS.

point 1' on the load line II. Hence the thermistor voltage and current change instantaneously from  $V_1, I_1$  to  $V_1', I_1'$ .

At the same time the power supplied increases from  $V_1 I_1$  to  $V_1' I_1'$ , while the rate of heat loss is unchanged. Hence for  $t > 0$  the thermistor temperature increases and its static resistance decreases. Therefore the dynamic operating point moves down towards on load line II from 1' until it reaches point 2., its intersection with the static characteristic, which represents the final stationary state.

The dynamic characteristic is therefore represented by 1-1'-2. Since instantaneous changes in  $E$  and  $S$  have been assumed it is only necessary to determine the movement of the dynamic point from 1' to 2. This is governed by the time-dependence of the thermistor excess temperature,  $\theta$ , or the corresponding inverse function, which can be written, by integration of Equation 2.26 ;

$$t = \int_{\theta_1}^{\theta} \frac{\tau(\theta) d(K\theta)}{(P(\theta) - K\theta)} \quad (2.72)$$

Here  $\theta_1$  is the excess temperature corresponding to the initial operating point. The integral can readily be evaluated graphically if  $\tau(\theta)$  is known. Values of  $P(\theta) - K\theta$  required for denominator are obtained by subtracting the product of the thermistor current and voltage at the intersection of resistance lines from the origin and the static characteristic (e.g.  $I_s$ ) from the corresponding intersection of this line and the dynamic characteristic (correspondingly  $ID$ ).

Alternatively it can be assumed that the curve of  $P(\theta) - K\theta$  versus  $K\theta$  is represented by a series of straight lines, so that the piecewise constant derivative  $c = d(P(\theta) - K\theta) / d(K\theta)$  can be introduced. Then.

$$\frac{d(K\theta)}{(P(\theta) - K\theta)} = \frac{1}{c} \frac{d(P(\theta) - K\theta)}{(P(\theta) - K\theta)} \quad (2.73)$$

Integration of Equation 2.72 between two points a and b on a straight part of the  $(P(\theta) - K(\theta))$  versus  $K\theta$  curve yields,

$$t_b - t_a = \frac{\tau}{C_{ab}} \log_e \left[ \frac{P(\theta)_b - K\theta_b}{P(\theta)_a - K\theta_a} \right] \quad (2.74)$$

If, in particular, the internal power dissipation is

small,  $c = -1$  and Equation 2.74 becomes

$$t_b - t_a = -\tau \log_e \frac{\theta_b}{\theta_a}$$

$$\text{or } \theta_b = \theta_a \exp \left[ - (t_b - t_a) / \tau \right] \quad (2.75)$$

# CHAPTER 3

## LINEARISATION OF THERMISTOR CHARACTERISTIC



## LINEARISATION OF THERMISTOR CHARACTERISTIC

3.1 INTRODUCTION

The negative temperature coefficient of the thermistor has great sensitivity to temperature and good stability against all other environmental changes, but it is inherently non-linear. The non-linear characteristic of the thermistor is a definite drawback which tends to limit the use of the sensor for a variety of the applications for which otherwise its characteristics are admirable suitable. In particular this is so for temperature measurement and compensation. Since in these applications the thermistor is almost invariably operated in the zero internal dissipation mode. The problem of linearising under these conditions will be discussed in detail. In the following pages emphasis will be laid on determining analytically the extent to which linearisation is possible within a specified range of temperature, with the various methods outlined in the next section. The author deals with the various techniques of linearising the two types of characteristics, viz., (i) resistance temperature characteristic and (ii) temperature-output voltage due to the change in the thermistor resistance owing to the change in temperature.

### 3.2 SHAPING OF THE RESISTANCE-TEMPERATURE CHARACTERISTIC

In general the resistance-temperature characteristics of commercial thermistors do not have the form required for a specific application. Under certain conditions it is, however, possible to modify the characteristic so that it approximates a desired form. This is done using a series and shunt-connected resistors. If a simple design procedure is to be used the requirements are that the thermistor temperature is equal to the ambient temperature  $T_a$  and the passive resistors are also at  $T_a$ . It is also implicit that the thermistor obeys Ohm's Law which is valid for N.T.C. thermistors at low power levels.

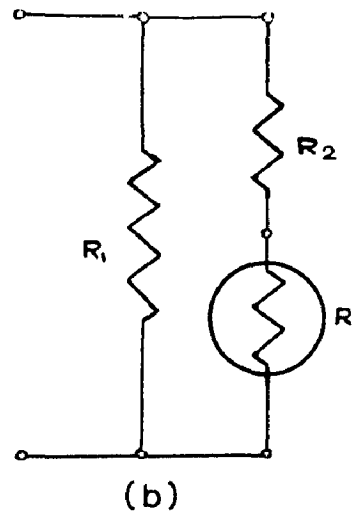
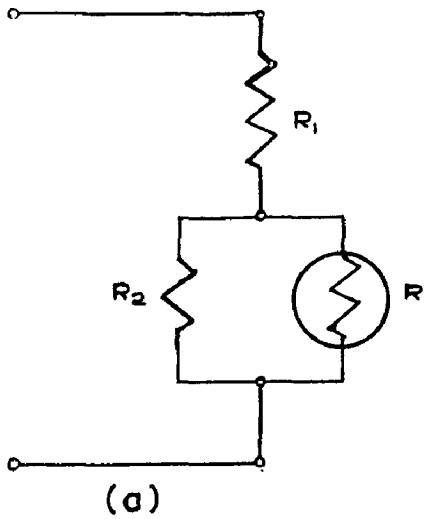
Shaping networks with two fixed resistors are shown in Fig. 3.1. For (a) the terminal resistance  $R_T$  is

$$R_T = \frac{R_1 R_2 + (R_1 + R_2) R}{R_2 + R} \quad (3.1)$$

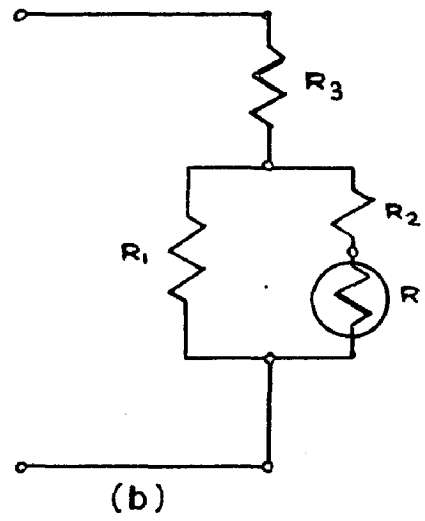
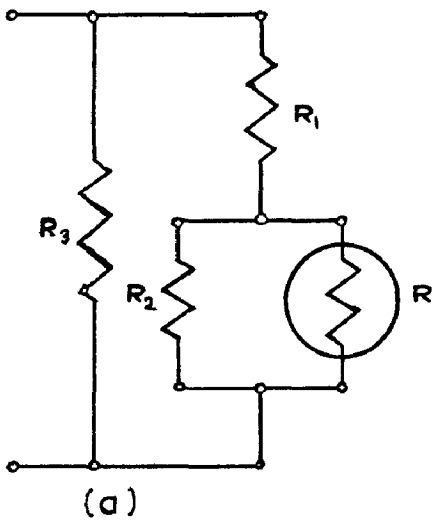
and for (b)

$$R_T = \frac{R_1 R_2 + R_1 R}{R_1 + R_2 + R} \quad (3.2)$$

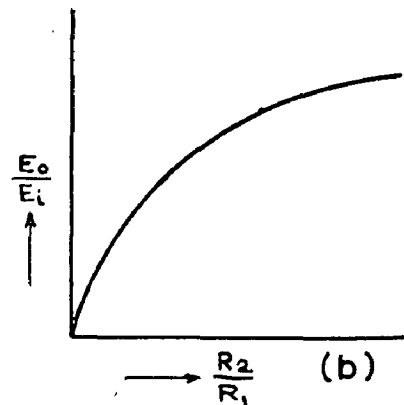
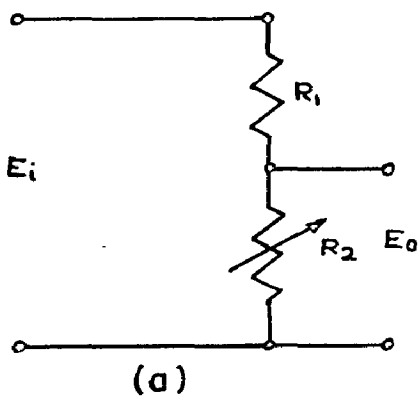
The variation of  $R_T$  with temperature will always be less than the corresponding variation of  $R$  itself. Furthermore an increase in  $R$  will always cause an increase in  $R_T$ . These facts may be expressed mathematically as



IG.3.1 SHAPING NETWORKS WITH TWO FIXED RESISTORS.



G.3.2 MORE REFINED SHAPING NETWORKS WITH THREE FIXED RESISTORS.



IG. 3.4 (a) POTENTIAL DIVIDER ARRANGEMENT

(b) TRANSFER FUNCTION OF A POTENTIAL DIVIDER IS A NON LINEAR FUNCTION OF  $R_2$ .

follows -

$$R_T = \frac{a + bR}{C + R} \quad (3.3)$$

with positive coefficients  $a, b,$  and  $C$ . Geometrically, this corresponds to a rectangular hyperbola. More complicated networks involving more than two passive resistors can be used. Examples are shown in Fig. 3.2. These define a terminal resistance having the general form of equation 3.1, but permit a better approximation to any desired curve.

### 3.3 METHOD OF EQUALISING THERMISTORS

Godin<sup>19</sup> in 1961 described a bridge for direct measurement of temperature differences. This instrument only measures temperature differences accurately if two identical thermistors are used. Two similar thermistors can be transformed into the equivalent of two nearly identical thermistors, by using series and parallel resistances.

At a given temperature  $T_1$ , two similar thermistors can always be made identical. In practice this identity can be made to hold over a considerable range. Considering two thermistors C and D whose resistances

$R_C$  and  $R_D$  are given by the formulae

$$R_C = A_C \exp(B_C / T) \text{ and } R_D = A_D \exp(B_D / T) \quad (3.4)$$

The symbols should be chosen so that at  $T_1$ ,

$$-\frac{dR_D}{dT} > -\frac{dR_C}{dT} \quad (3.5)$$

If  $R_D$  is placed in parallel with fixed resistance  $X$ , the combined resistance  $R_E$  will be given by

$$R_E = \frac{K R_D}{K + R_D} \quad (3.6)$$

Differentiating with respect to  $T$  gives

$$\frac{dR_E}{dT} = -\frac{B_D R_D K^2}{T^2 (X + R_D)^2} \quad (3.7)$$

$$\text{and } \frac{dR_C}{dT} = -\frac{B_C R_C}{T^2} \quad (3.8)$$

At temperature  $T_1$  make  $dR_E / dT = dR_C / dT$

Therefore

$$B_C R_C (X + R_D)^2 = B_D R_D X^2$$

$$X = \frac{B_C R_C R_D + R_D (B_C B_D R_C R_D)^{1/2}}{B_D R_D - B_C R_C} \quad (3.9)$$

To make the thermistor identical at  $T_1$  a resistance  $S$  must be placed in series with either  $R_C$  or  $R_E$ . Depending on the circumstances

$$S = \pm (R_C - R_E) \quad \text{both at } T_1 \quad (3.10)$$

### 3.4 THERMISTOR LINEARIZATION<sup>41</sup>

#### 3.4.1 Principle

The R-T characteristic of an N.T.C. thermistor obeys an exponential law. From this it follows that the conductance against temperature characteristic of a thermistor is also non-linear. It has been found that a linear resistance against temperature characteristic can be achieved over a limited temperature range if the thermistor is shunted with a suitable fixed resistance (Fig. 3.3a). Similarly, it has been found that a linear conductance against temperature characteristic can be achieved over a limited temperature range if a suitable fixed resistor is connected in series with the thermistor (Fig. 3.3b).

A thermistor plus shunt-resistance network has a point of inflection where the rate of change of resistance with temperature is at a maximum. It is about this point of inflection that a more linear change in resistance is obtained.

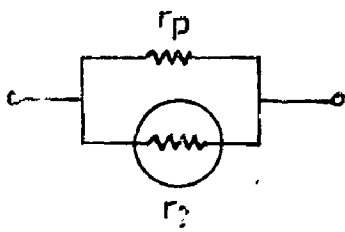


FIG. 3-3(a) A THERMISTOR PLUS SHUNT-RESISTOR NETWORK SUCH AS THIS CAN BE USED TO OBTAIN A LINEAR RESISTANCE-VERSUS-TEMPERATURE CHARACTERISTIC.

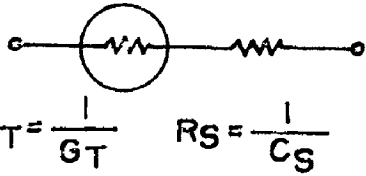


FIG. 3-3(b) A LINEAR CONDUCTANCE VERSUS TEMPERATURE CHARACTERISTIC CAN BE OBTAINED WITH A RESISTOR CONNECTED IN SERIES WITH THE THERMISTOR.

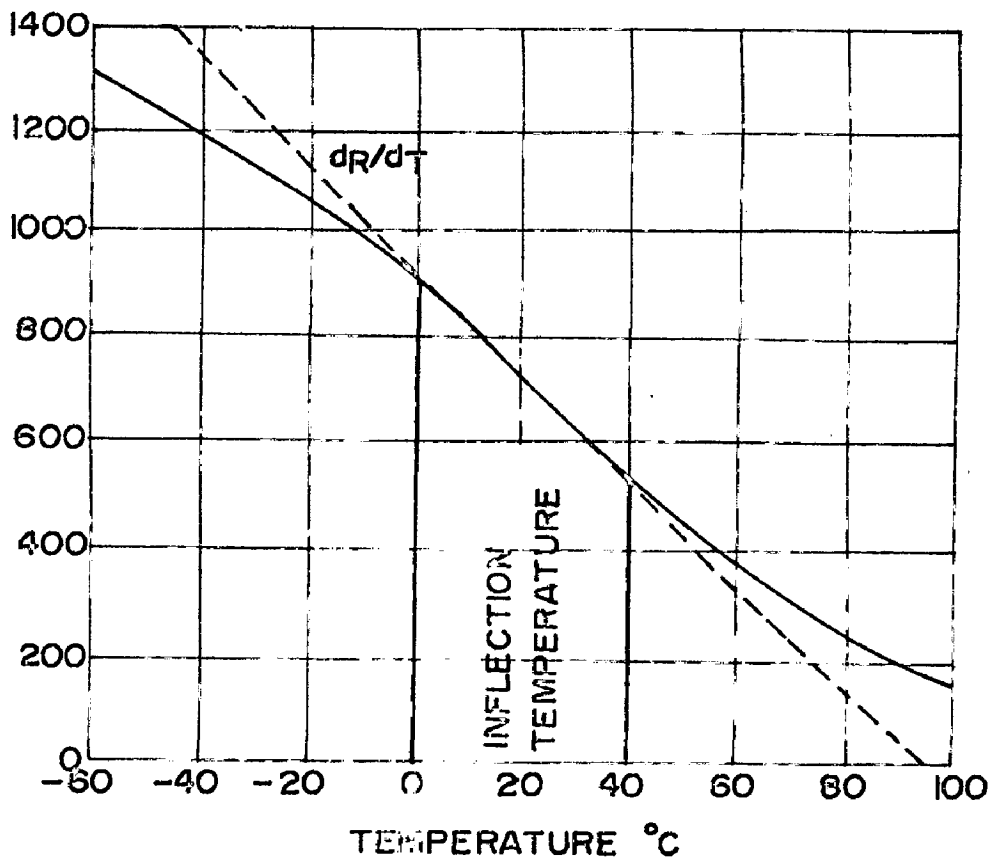


FIG. 3-3(c) THE PARALLEL NETWORK SHOWN IN FIG. 3-3(a) HAS A RESISTANCE-VERSUS-TEMPERATURE CHARACTERISTICS AS SHOWN HERE. IN THIS EXAMPLE  $r_p = 1,346 \Omega$ ,  $B = 3,000^\circ K$  AND RESISTANCE OF  $2,000 \Omega$  AT  $20^\circ C$ .

### 3.4.2 Theory

Considering the network of Fig. 3.3(a) . The total resistance of the network is

$$R = \frac{R_T r_p}{R_T + r_p} \quad (3.11)$$

The condition that point of inflection lies at

temperature  $T_1$

$$\left. \frac{d^2 R}{dT} \right|_{T = T_1} = 0$$

Therefore by the applying the condition in Equation 3.11, we get,

$$r_p = - R_{T_1} \left[ \frac{B - 2 T_1}{B + 2 T_1} \right] \quad (3.12)$$

and

$$\frac{dR}{dT} = - \frac{R_{T_1} (B - 2 T_1)^2}{4 B T_1^2} \quad (3.13)$$

where

$r_p$  = parallel resistance

$R_{T_1}$  = resistance of the thermistor at temperature  $T_1$  .

$T_1$  = inflection temperature in  $^{\circ}$  K.



$T_i$  A thermistor and a series resistor network has an inflection point where the maximum rate of change of conductance with temperature occurs. It is about this point of inflection that a more linear change in conductance is obtained. Formulae relating to this point of inflection have been derived from which component values can be determined.

$$\frac{dG}{dT} = G_{T_i} \frac{(B + 2T_i)}{4B T_i^2} \quad (3.14)$$

$$G_s = G_{T_i} \left[ \frac{B + 2T_i}{B - 2T_i} \right] \quad (3.15)$$

where  $G_s$  = series conductance

$G_{T_i}$  = Thermistor conductance at temperature  $T_i$

$T$  = Inflection temperature  $^{\circ}K$

### 3.4.3 Design Considerations

From the basic thermistor equation, the temperature coefficient of the thermistor has been shown to be  $-(B/T^2)$ , therefore, at temperature  $T_i$  the temperature coefficient is  $-(B/T_i^2)$ . If now we look at equation 3.12 it can be seen that unless  $T_i$  is high or  $B$  is very low the temperature coefficient of the parallel network at  $T_i$ , relative to the thermistor, can be considered as being approximately

equal to  $-(B/4(T_1)^2)$ , therefore the resistance change required will be approximately  $\pm 1/4$  of the resistance change of the thermistor. If  $dR/dT$  of the network is known, then an approximate value of thermistor resistance at the inflection temperature or mid range temperature, can be determined for an estimated value of  $(B/T^2)$ , from which the nearest standard thermistor can be obtained to give the calculated thermistor resistance value at  $T_1$ . In this way a thermistor resistance can be determined fairly quickly, for say, a compensation problem, where a linear resistance change is necessary. The shunt resistance can be determined from equation 3.11.

#### 3.4.4 Linearity

The linearity obtained worsens as the temperature deviation from  $T_1$  is increased, thus the degree of linearity over a given temperature range will depend on the extent of the range. Over a  $\pm 20^\circ\text{C}$  range, linearity suitable for most practical purposes can be achieved. Better linearity over a wider temperature range is possible by considering, instead of the slope  $\frac{dR}{dT}$  from Equation 3.12 as drawn in Fig. 3.3(C), a slope swung about the inflection axis, which will cross the network resistance against temperature characteristic at three points, including the inflection point. In this way deviation about the slope is somewhat reduced.

Expanding the resistance  $R$  of equation 3.11 into an infinite series about the inflection temperature  $T_1$  as follows :

$$R(T) = R(T_1) + hR'(T_1) + \frac{h^2}{2!} R''(T_1) + \dots + \frac{h^n}{n!} R^n(T_1) \quad \dots(3.16a)$$

On substituting the value of  $r_p$  and substituting the value of higher derivative in Equation 3.16a,

$$R(T) = R(T_1) + hR'(T_1) - \frac{h^3 B^2}{2T_1^4} R''(T_1) + \frac{h^4 B^2}{K_E T_1^5} R'''(T_1) \quad (3.16b)$$

where  $h = T - T_1$  and  $K_E < 10$  the terms in  $h^3$ ,  $h^4$  etc. indicate departure from linearity and may be regarded as error terms  $\epsilon_3$ ,  $\epsilon_4$  etc.  $\epsilon_4$  is negligible compared  $\epsilon_3$  for small values of  $h$ , since,

$$\frac{\epsilon_4}{\epsilon_3} = \frac{h^4 B^2}{K_E T_1^5} \bigg/ \frac{h^3 B^2}{2 T_1^4} = \frac{2h}{K_E T_1} \quad (3.17)$$

This shows that for all values of  $h < 20$  and  $T_1 > 200$  K,  $\epsilon_4 \leq \frac{1}{10} \epsilon_3$ , since  $K_E \geq 2$ . Therefore only  $\epsilon_3$

need be considered for calculating the error in linearity.

### 3.5 LINEARISATION OF TEMPERATURE-OUTPUT CHARACTERISTIC

#### 3.5.1 Direct Reading Bridges

Early bridges have been described by Deeter<sup>35</sup>, Herington and Handley<sup>36</sup>, and Greenhill and Whitehead<sup>37</sup>. No specific attempt was made to obtain a linear thermometer. However in recent years attempts have been made by some investigators<sup>16, 39</sup> to produce a linear thermometer in which the thermistor is operated in the zero internal dissipation mode.

#### 3.5.2 Linearisation

Linearisation of inherently non-linear dependence of current and voltage of an N.T.C. thermistor in series with a resistance  $r$  and e.m.f.  $E$  can be considered analytically. Assuming that the thermistor resistance  $R$  was given by Equation 2.7, i.e.  $R = R_C \exp(B/T)$ . The current  $I$  is given by

$$I = \frac{E}{R + r} \quad (3.18)$$

Expanding the current into an infinite series about the working temperature  $T_0$  as follows :

$$I(T) = I(T_0) + hI'(T_0) + \frac{h^2 I''(T_0)}{2!} + \dots + \frac{h^n I^{(n)}(T_0)}{n!} \quad (3.19)$$

where  $h = (T - T_0)$  and  $I', I'', \dots, I^n$  are the first, second and  $n$ th derivatives of  $I$  with respect to  $T$ .

The approach to linearisation was based on making the second derivative on the right hand side of Equation 3.19 zero. The condition, obtained by differentiating Equation 3.18 twice with respect to  $T$  is

$$r = \frac{B - 2T_0}{B + 2T_0} R_0 \quad (3.20)$$

where  $R_0 = R_c \exp(B/T_0)$ . On substituting this value for  $r$  into higher derivatives in Equation 3.19 this becomes,

$$I(T) = I_0 + hI'(T_0) - \frac{h^3 B^2}{3! \cdot 2T_0^4} I''(T_0) + \frac{h^4 2B^2}{4! T_0^5} I'''(T_0) \quad (3.21)$$

The terms in  $h^3$ ,  $h^4$  etc indicate departure from linearity and may be regarded as error terms,  $\epsilon_3$ ,  $\epsilon_4$  etc.  $\epsilon_4$  is negligible compared with  $\epsilon_3$  for small values of  $h$ ,

since

$$\frac{\epsilon_4}{\epsilon_3} = \frac{2 h^4 B^2}{4! T_0^5} I''(T_0) \bigg/ \frac{h^3 B^2}{3! \cdot 2 T_0^4} I''(T_0) = -\frac{h}{T_0} \quad (3.22)$$

This shows, for example, that at all values of  $T_0 > 200$  K,  $\epsilon_4 < 0.1 \epsilon_3$ , if  $h < 20$  K. Hence only  $\epsilon_3$  need be considered. On this basis Beakley showed

that if it is required to construct a thermometer to measure temperature from 290 K to 310 K with a thermistor for which  $B = 3000$  K, the departure from linearity has a maximum value of  $0.03^{\circ}\text{C}$ . If an error of  $0.1^{\circ}\text{C}$  is permissible then a range from 285 K to 315 K may be covered.

### 3.6 LINEARISATION OF POTENTIOMETER OUTPUT

Another technique of linearisation was suggested by Harruff<sup>40</sup> considering the potentiometer divider circuit of Fig. 3.4, where  $R_2$  might be a thermistor. The output voltage is a nonlinear function of  $R_2$ :

$$E_0 = E_1 / \left( 1 + \frac{R_1}{R_2} \right) \quad (3.23)$$

If the values of  $R_2$  necessary to give uniform (linear) increments of output voltage change are plotted against a temperature scale, then the characteristic of a typical thermistor can be added in Fig. 3.5 to show what linearisation must accomplish. It would appear to be easy just to raise the high temperature end of the thermistor curve with a series resistor and reduce its overall slope with a shunt to match the required characteristic.

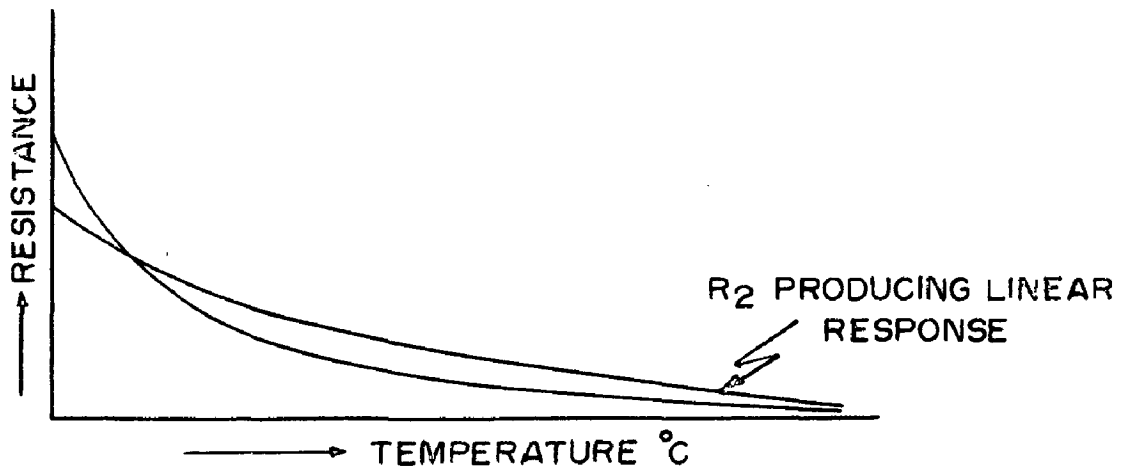
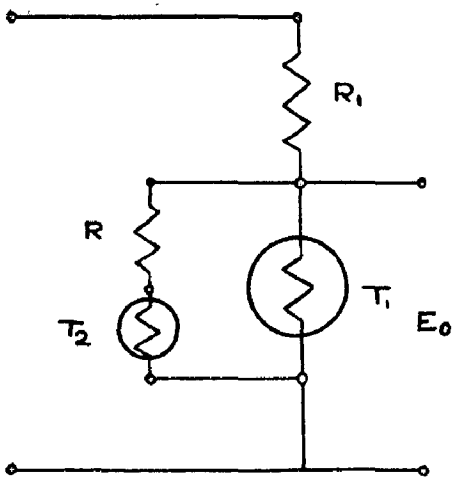


FIG. 3.5 THE CURVE OF WHICH GIVES LINEAR POTENTIAL DIVIDER OUTPUT AS A PRETENDED FUNCTION OF TEMPERATURE LOOKS SIMILAR TO THE CURVE OF A TYPICAL THERMISTOR.



G. 3.6 DUAL ELEMENT NETWORK

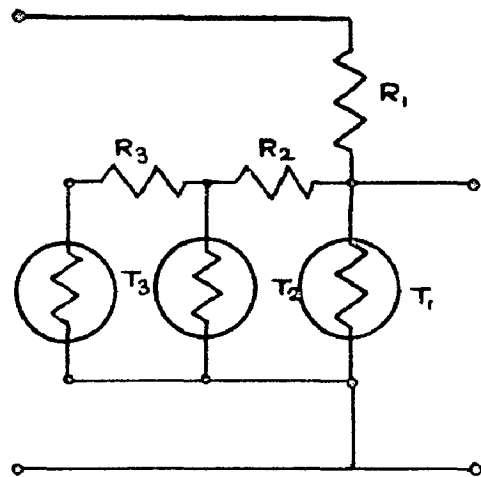


FIG. 3.7 TRIPLE ELEMENT NETWORK.

This approach turns out to afford an exact match at a maximum of three points, while the discrepancies between the curves are usually large with any economically reasonable network of passive resistors. The reason is not hard to find. The curve of  $R_2$  is a rational function, while the characteristic of a negative temperature coefficient thermistor is an exponential function. The two curves just are not of the same shape

### 3.6.1 Two Thermistor Network

The second temperature sensitive element added in Fig. 3.6 provides a much closer match to the required  $R_2$  curve than a network of constant resistor can. Values can be assigned to the circuit elements by solving the set of equations.

$$E_0 + n \Delta E_0 = E_1 n \frac{RT_1 (RT_2 + R)}{RT_1 (RT_2 + R) + R_1 (RT_1 + RT_2 + R)} \quad (3.24)$$

where  $n = 0, 1, 2, 3, \dots, E_0/\Delta E_0$

$$RT = RT_0 \exp \left[ B \left( \frac{1}{T} - \frac{1}{T_0} \right) + C \left( \frac{1}{T} - \frac{1}{T_0} \right)^2 \right] \quad (3.25)$$

$$\text{Absolute temperature, } T = T_0 + n \Delta T$$

B and C are constants for a given thermistor, but not necessarily the same for both the thermistors. A more



illuminating solution technique is a kind of mechanised trial and error. A computer programme can be written to calculate  $R$  and  $R_1$ , given characteristics of the two thermistors.

### 3.6.2 Three-Thermistor Network

Still closer linearisation is possible with a third sensitive element, as shown in Fig. 3.7. The greater the number of independently variable elements in the network, the more closely it will approximate the desired curve. Therefore, as sensitive elements are added to the circuit, linearity may be improved and/or temperature span widened.

### 3.7 LINEARISATION OF THE OUTPUT OF BRIDGE NETWORK

The resistance-temperature characteristic of a thermistor can be linearised by connecting a non-temperature sensitive, resistance in parallel. Farhi and Groves<sup>38</sup> recommended the shunt to have a resistance equal to that of the thermistor at the mid point of the temperature range over which linearization is required. Nordon and Bainbridge<sup>18</sup> suggested a different value of shunt resistor using Beakley approach which could give more accurate linear characteristic of thermistor. They used two such linearised identical thermistors in the two arms of Wheatstone bridge. The bridge is balanced with both thermistors at the same temperature

and it will remain balanced at any other temperature within the range of linearisation provided the temperature of both thermistors remain equal. The arrangement is shown in Fig. 3. 8. Temperature can be measured in terms of the resistance  $S$  which will be a linear function of temperature.

Instead of calibrating the resistance in terms of temperature Scott<sup>34</sup> linearised and calibrated the output of the bridge in terms of temperature.

The approach used is outlined below (i)

- (i) Assume the resistance - temperature characteristic of the sensing device is of the form,

$$R = R_0 \left( 1 + \frac{\theta}{A + B\theta} \right) \quad (3.26)$$

where  $R$  is the resistance of the device at temperature  $\theta$  and  $A$  and  $B$  are constants. It will be shown that the resistance changes of this form produce a truly linear output from a properly designed bridge. Fit this equation to three experimental points from the characteristic of the sensing device.

- (ii) Estimate the errors directly as the difference between the assumed equation and the experimental curve of the sensing device.

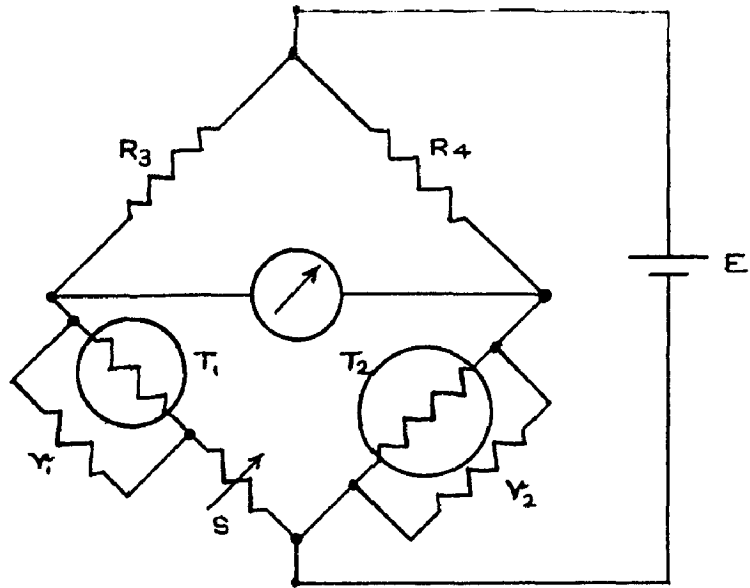


FIG. 3·8 A BRIDGE CIRCUIT

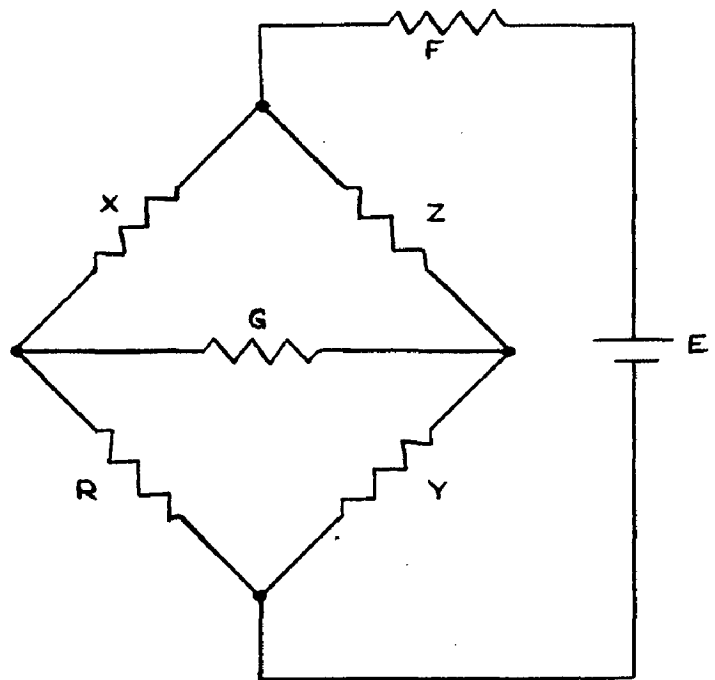


FIG. 3·9 A BRIDGE CIRCUIT

(iii) Equate the non-linear output terms to zero and from this equation obtain the desired values of the circuit parameters.

It is possible to linearise outputs by the use of simple shunt or series arrangements but this is not normally done in practice. Rather the elements of the bridge network are used to provide the shunt or series resistance.

The general case based on the circuit of Fig.3.9 is considered. Assuming that the internal resistance of battery is zero, the current  $I$  through arm  $G$  is given by ,

$$I = \frac{E (YX - RZ)}{F(R+Y)(Z+X) + YZ(R+X) + XR(Y+Z) + G [ F(R+Y+Z+Z) + (R+X)(Y+Z) ]}$$

(3.27)

Now allowing  $R$  to change to  $R(1 + \alpha)$  where  $\alpha$  is dimensionless. Furthermore it is required that  $\alpha$  is of the form  $\theta / (A + B\theta)$  where  $\theta$  is a variable causing changes in  $R$ , and  $A$  and  $B$  are constants. Now  $\alpha$  will appear in both numerator and denominator. Both numerator and denominator are multiplied by  $A + B\theta$  to leave the numerator as a linear function of  $\theta$ . In the denominator non- $\alpha$  terms will be multiplied by  $A + B\theta$  whilst  $\alpha$  terms will be replaced by  $\theta$ . If

now the  $\Theta$  terms of the denominator are equated to zero the following expression results :

$$\left[ F \{ (R + Y)B + R \} (Z+X) + YZ \{ (R+X)B + R \} + XR (Y+Z) (B+1) \right] + GF \left[ \{ B(R+Y+Z+X) + R \} + \{ (R+X)B + R \} (Y+Z) \right] = 0 \quad (3.28)$$

which is the condition for I to be linear function of  $\Theta$ . This has been done by equating the non-linearity producing terms to zero.

Now putting  $G = \eta R$ ,  $F = \beta R$  and  $Z = X$ ,  $Y = R$

$X = \xi R$  and solving equation 3.28 for B.

whence,

$$B = \frac{2\beta\xi + \eta\beta + (1+\xi)\eta + (2+\xi)\xi}{4\beta\xi + 2\xi(1+\xi) + \eta\beta(2+2\xi) + (1+\xi)^2\eta} \quad (3.29)$$

For  $\beta = 0$  (i.e. battery arm resistance zero) this degenerates to

$$B = \frac{(1+\xi)\eta + (2+\xi)\xi}{(1+\xi)^2\eta + 2\xi(1+\xi)} \quad (3.30)$$

For  $\eta = \infty$  (i.e. infinite impedance output indicator)

$$B = \frac{\beta + 1 + \xi}{\beta (2 + 2\xi) + (1 + \xi)^2}$$

For  $\beta = 0$  and  $\eta = \infty$

$$B = \frac{1}{1 + \xi} \quad (3.31)$$

Series shunt constant corrections - Considering arm R in Fig. 3.9, comprising an element Q shunted by a resistance P, and allowing any resistance change to occur in Q such that Q changes to  $Q(1 + \alpha_0)$ . Hence

$$R(1 + \alpha) = \frac{PQ}{P+Q} (1+\alpha) = \frac{P(1 + \alpha_0)Q}{Q(1 + \alpha_0) + P} \quad (3.32)$$

from which,

$$\alpha = \frac{P \alpha_0}{Q(1 + \alpha_0) + P} \quad (3.33)$$

Now allowing  $\alpha_0$  to be of the form  $\theta / (C + D\theta)$  hence,

$$\alpha = \frac{\theta}{\left[ \frac{P+Q}{P} C + \frac{Q + (P+Q)D}{P} \theta \right]} \quad (3.34)$$

or compared with  $\alpha = \frac{\theta}{A + B\theta}$  used earlier.

$$A = \frac{P+Q}{P} C, \quad B = \frac{Q + (P+Q)D}{P}$$

$$\text{or } D = \frac{PB - Q}{P+Q} \quad (3.35)$$

For a similar series arrangement, with the element Q and the series resistance S it can be found that

$$B = \frac{Q + S}{Q} D \quad \text{or} \quad D = \frac{Q}{Q + S} B \quad (3.36)$$

### 3.8 LINEARISING OF SELF HEATED THERMISTOR

#### 3.8.1 Introduction

In some cases inadequate response may be obtained at low bridge voltages. It is always tempting to increase the supply voltage to the thermistor in an attempt to improve its sensitivity, however, the equations developed in the preceding sections became invalid as the self heating increases and a different approach to linearisation must be taken.

It can be shown that in the self heated mode, a point of inflection in the thermistor current versus temperature curve still exists and the possibility arises of linearising the thermistor about this point.

#### 3.8.2 Principle of Linearisation

Considering a simple circuit having a thermistor and a resistor connected across a supply  $V_s$ . The resistance of the thermistor is  $A e^{B/T}$  at absolute temperature T, where A and B are constants, and P be

the fixed series resistor.  $V$  is the voltage drop across thermistor and  $I$  is the current flowing through the circuit. From the knowledge of thermal dissipation constant of the thermistor, it is possible to derive  $V-I$  curves for the device at any given ambient temperature,  $T$ . A temperature span is chosen over which approximate linearity is desired and the mid and extreme temperature of this span ( $T_0$ ,  $T_1$ ,  $T_2$  respectively, where  $T_2 - T_0 = T_0 - T_1$ ) are used to calculate three  $V-I$  curves. The load line of the circuit is then placed over the curve either graphically or via the computer and moved around. Unit equal lengths are stepped out along itself in going from  $T_2$  to  $T_0$  and from  $T_0$  to  $T_1$ . The resulting values of  $P$  and  $V_s$  imply that the departure from linearity is zero at  $T_0$ ,  $T_1$  and  $T_2$  and hopefully the error between these extremes is small.

### 3.9 CONCLUSIONS

#### (a) Linearising the R-T characteristics -

(1) Farhi and Groves suggested the shunt to have a resistance equal to that of the thermistor at the mid point of the temperature range over which linearisation is required. In all probability this thumb rule was arrived at after experimental verification.



(ii) However theoretically it can be shown that a shunt resistance equal to  $R_{T_1} \left[ \frac{B-2T_1}{B+2T_1} \right]$  (which is a condition for point of inflection to occur at the mid of the temperature range over which linearity is desired) will effect linearisation of the R-T characteristic over a range on either side of the point of inflection.

(b) Linearising I- output Voltage Characteristic -

(i) Linearisation of the output voltage versus temperature characteristic can be effected by potentiometric arrangement. The more the number of parallel branches the more will be the linearity achieved.

(ii) Linearity can also be achieved by incorporating the thermistor in one arm of the bridge.

# CHAPTER 4

## SELECTION CONSIDERATIONS AND TESTING OF THERMISTORS

## SELECTION FACTORS AND TESTING OF THERMISTORS

4.1 SELECTION FACTORS<sup>4</sup>4.1.1 INTRODUCTION

It has already been discussed that the thermistors are thermally sensitive resistors that exhibit a resistance change of about 4 percent per degree centigrade (NTC thermistors). They have a stable, ceramic-like structure consisting mostly of metallic oxides, and are commercially available in a variety of shapes such as disks, washers, rods and beads. Different types of thermistors have its own merits and demerits. Size, response time, maximum rated temperature and precision of manufacture are some of the factors involved in selecting the proper type of thermistor.

4.1.2. Type Selection Factors

The response time of a thermistor is closely related to its size and also to the mounting arrangements and operating conditions. The smallest bead type unit has a diameter of 0.007 inch without glass coating. The same bead with glass coating is only 0.015 inch in diameter. This unit has a fast thermal time

constant of about 1 sec. when supported by its own leads in still air. The time constant is less in moving air, or in oil, or when the unit is embedded in another object whose temperature is to be sensed. Some of the large washer type thermistors (for example, 3/4 inch in diameter) have thermal time constant of 2 or 3 minutes. This figure can be reduced by bolting the washer thermistor to a thermal sink.

Maximum rated temperature depends on whether a unit is glass coated and whether it has attached leads. Disks and rods with leads are usually rated at 125°C continuous ambient temperature. The same unit without leads are rated at 150°C. Glass coated beads will withstand 300°C continuously. The types with soldered leads are limited by the melting point of the solder. Types without soldered leads will not be destroyed by exposure to high temperatures for short periods, but may show a slight permanent resistance shift.

The basic thermistor is sintered in air at a temperature between 1200 and 1400 °C during manufacture. Disks, washers and rods are metalized at a temperature of 650°C and beads are glass coated at 850°C so that the maximum ratings are conservative.

The precision with which the various types of thermistors can be manufactured is another factor in the choice of proper type. Discs washers and rods are usually furnished with a 10 % resistance tolerance at a specified temperature, while beads are furnished with a 15 or 20 percent resistance tolerance at a specified temperature. Discs and washers can be furnished to a greater precision but with a slight increase in the cost, since they can be ground in manufacture, reducing their diameter, and thus adjusting their resistance. Thermistors discs have been furnished in quantity with a resistance accuracy of 0.25 percent. Bead types are difficult to produce to higher precision because of their small size, and consequently are usually only furnished at higher precision by simple selection and at consequent increase in price.

The tolerance discussed above are the variations encountered from unit to unit. A given unit will repeat its performance indefinitely over a 100°C range without detectable hysteresis or variation. The tolerances may seem high compared to the tolerances normally associated with wire resistance thermometers, however, the ten times higher temperature coefficient of the thermistor permits ten times the resistance

variation for the same temperature accuracy.

In terms of temperature accuracy, discs, rods, and washers are usually furnished with a given resistance within about  $2^{\circ}\text{C}$  of a specified temperature. Discs have been furnished with a precision of  $0.05^{\circ}\text{C}$ .

Another type of precision has to do with the difference from unit to unit in following the nominal resistance - temperature curve specified by manufacturer. This departure from the curve (or tolerance on temperature coefficient) is about plus or minus  $0.5^{\circ}\text{C}$  over a  $100^{\circ}\text{C}$  range for discs and rods, and about plus or minus  $2.5^{\circ}\text{C}$  over a  $100^{\circ}\text{C}$  range for beads. Again this tolerance refers to variations from unit to unit, a given unit reproduces without detectable change.

#### 4.1.3. Power Rating

The factor most commonly overlooked in the applications of thermistors is the power level at which the unit is operated. Normally the thermistors must be operated at much lower power levels than other resistors would be when an ordinary resistor is rated at one watt, this rating implies that one watt will not damage the resistor even though it may heat it as

much as  $100^{\circ}\text{C}$  above ambient temperature. A thermistor of the same size may also not be damaged by one watt, but the fact that it has been heated to  $100^{\circ}\text{C}$  above ambient temperature is not conducive to accurate measurement of the ambient temperature.

As a result, the maximum power rating of a thermistor is usually not as important as a knowledge of its actual temperature rise when dissipating a given amount of power. There is a proportionality between the dissipating power and the temperature rise, that is usually expressed as a dissipation constant of so many milliwatts of electrical power per resultant degree centigrade rise in temperature. The exact specification of the dissipation constant is difficult since it depends on the environment and method of mounting the thermistor. The value given is usually that which applies when the thermistor is suspended by its own leads in still air. The same thermistor in oil, or in moving air, or clamped to some other object may have a dissipation constant ten or more times the value given for still air.

In actual design there is no objection to allowing the thermistor to run slightly hotter than its surroundings, since this method achieves maximum output or sensitivity. However, temperature rise should be restricted to at most a few times. The accuracy of

temperature to which the measurement is to be made. Otherwise, undesirable effects such as long warm-up time or variation in readings can result.

#### 4.2 METHOD OF MOUNTING THE THERMISTORS<sup>5</sup>

Two 6 cm lengths of 40 SWG enamelled copper wire should be taken and the ends be cleaned and tinned with 40/60 alloy multicore solder for about 1/2 mm. All traces of flux should be removed and the wire be inserted in a no.1 hypodermic needle with the soldered tips projecting 1 cm from the needle tip.

The thermistor should be attached to a wire frame with shellac to facilitate soldering the assembly is performed on a flat glass plate with components held in position with small pillets of plasticine. The platinum-iridium / copper joints were made with a minimum of solder. The surplus thermistor leads are then trimmed to the joint and the plasticine removed. Great care is necessary in performing this operation. The copper leads are then drawn into the hypodermic needle until the thermistor bead lay centrally on the needle tip.



A small quantity of ohmaline air-drying varnish should be applied to the tips and be allowed to dry for two days. The terminal ends of the copper lead wires are sealed to the hyperdermic head by dropping a small quantity of varnish into the head between the leads while the hypodermic is suspended in the vertical position, tip downwards. The probe should be left in this position for forty eight hours to allow the varnish to harden, and then should be tested for continuity and insulation resistance.

A more mechanically robust seal might result if a thermosetting resin, such as araldite, are used.

#### 4.3 TEMPERATURE STABILITY

It is of paramount importance in a measuring instrument that the scale reading does not change significantly after calibration. Thermistors used for thermometry in particular should therefore be aged<sup>6</sup>. The following is a summary of the conclusions which may be drawn concerning long and short term accuracy.

##### 4.3.1. N.T.C. Thermistors

(a) Measurement of small temperature differences over short periods of time can be very accurate. In a

calorimeter 0.0002 K has been claimed<sup>6</sup> in a temperature difference of 0.01°C . A short term stability of 5 parts in 10<sup>6</sup>, corresponding to an accuracy of temperature of 0.0001 K has also been reported.

(b) For large temperature differences, over a short period of time Beck has quoted an accuracy of the order of 0.01°C in a temperature difference of 6-12°C.

(c) Long term stability depends upon previous history of the thermistor and whether it has been 'aged'. It may be affected by thermal shock or large temperature changes.

(d) A short term stability of  $\pm 0.2$  K has also been achieved for thermistors with especially low resistivity, made to operate at liquid-nitrogen boiling point (-253 K) . However, if long term stability better than  $\pm 10^{\circ}$  C is required periodic recalibration is necessary.

#### 4.4 TESTING OF THERMISTORS

##### 4.4.1 Introduction

In the earlier pages, the selection factors of the thermistor, which a designer of Electronic Systems (in which the thermistor is an inherent part) has often

to take into account, are discussed. A brief summary of the methods used for testing the thermistors in so far as their performance characteristics are concerned.

#### 4.4.2 Determination of B and K<sup>7</sup>

The representation of R in Equation 2.14 readily leads to a determination of B and K from measured experimental values of R. Two approaches will be outlined. In the first R is measured for three values of applied power, which are also measured. Denoting these by the suffices 1, 2, 3 we obtain,

$$\log_e R_1 - \log_e R_2 = \frac{B(P_2 - P_1)/K}{(P_1/K + T_a)(P_2/K + T_a)} \quad (4.1)$$

$$\log_e R_2 - \log_e R_3 = \frac{B(P_3 - P_2)/K}{(P_2/K + T_a)(P_3/K + T_a)} \quad (4.2)$$

Dividing and solving for K we find

$$K = \frac{1}{T_a} \frac{P_1(P_2 - P_3)\log_e R_1 + P_2(P_3 - P_1)\log_e R_2 + P_3(P_1 - P_2)\log_e R_3}{(P_2 - P_3)\log_e R_1 + (P_3 - P_1)\log_e R_2 + (P_1 - P_2)\log_e R_3} \quad (4.3)$$

K being known, B can be calculated from Equation 2.14.

The second approach is graphical, From Equations 2.9 and 2.10 we can write,

$$\log_e R_a = \log_e R_c + B / T_a$$

$$\log_e R = \log_e R_c + B / \left( \frac{P}{K} + T_a \right)$$

so that

$$\left[ \log_e \frac{R_a}{R} \right]^{-1} = \frac{T_a}{B} \left[ 1 + \frac{T_a K}{P} \right] \quad (4.4)$$

Hence a plot of  $\left[ \log_e (R/R_a) \right]^{-1}$  against  $P^{-1}$  has a slope of  $T_a K$  and the intercept on the ordinate axis is  $T_a/B$ .

# CHAPTER 5

**APPLICATIONS BASED ON  
DISSIPATION CONSTANT**

## APPLICATIONS BASED ON VARIATION OF DISSIPATION CONSTANT

### 5.1 INTRODUCTION

The dissipation constant or thermal conductance,  $K$ , is not a property of the thermistor alone, although it is affected by its geometry. It is markedly influenced by the ambient atmosphere. As is well known, heat can be lost from a body by conduction, convection and radiation. As far as thermistors are concerned the 'atmosphere' for applications involving variation of dissipation constant, is either liquid or gaseous. These applications are diverse and cover many areas of scientific interest and engineering practice. They may be summarised as in the following subdivisions, anemometers, flow-meters and fluid-velocity meters, manometers (vacuum gauges), thermal conductivity analysis and gas-chromatography.

### 5.2 EFFECT OF AMBIENT ON $K$

At the atomic or molecular level, heat exchange between a solid body and surrounding gaseous or liquid ambient atmosphere is the result of collisions of the free particles of the surrounding medium with the solid. The rate at which heat is lost from such a body at a higher temperature than its surrounding medium clearly

depends on whether this is stationary or moving. In the latter case the collision rate is higher and the rate of heat loss is correspondingly increased. For gaseous atmospheres the thermal conductivity of gas increases linearly with pressure (from nearly vacuum conditions) upto an air-equivalent pressure of about 200  $\mu$ , in the so-called molecular-flow range, but becomes independent of pressure at pressures somewhat in excess of this when the probability of molecular collisions increases significantly.

5.3 APPLICATIONS BASED ON THE RATE OF FLOW <sup>8,9</sup>

These include anemometers and flow and fluid-velocity meters. Rasmussen's<sup>8</sup> approximate analysis will be followed here to indicate the interaction between K and the medium. His interest was in producing an improved bathy thermograph and an accurate oceanographic velocity meter, so both ambient temperature  $T_a$  and fluid velocity are considered as variables. The heat transfer equation 2.26 is

$$C \frac{dT}{dt} + K (T - T_a) = P \tag{2.26}$$

The approximations made are ;

- (a) the initial bead temperature is  $T_0$ , the corresponding thermistor resistance is  $R_0$  and,

$$R = R_0 \left[ 1 + \alpha_0 (T - T_0) \right] \quad (5.1)$$

$\alpha_0$  being the temperature coefficient of resistance at  $T_0$ , (Valid if  $T - T_0 \ll T_0$ );

(b) there is a constant heating current  $I$ .

On substitution in Equation 2.26 for  $P = I^2 R$ , with  $R$  defined as in Equation 5.1, the following equation is obtained,

$$\frac{dT}{dt} + \frac{T}{\tau'} = m + \frac{T_a}{\tau} \quad (5.2)$$

where,

$$\frac{1}{\tau'} = \frac{(K - P_0 \alpha_0)}{C}$$

$$m = \frac{P(1 - \alpha_0 T_0)}{C}$$

$$\frac{1}{\tau} = \frac{K}{C}$$

(with  $K$  a function of velocity,  $v$ , and  $\tau'$ ,  $\tau$  and  $T_a$  functions of time.

The usefulness of equation 5.2 may be demonstrated by determining the behaviour in a simple case in which the ambient temperature gradient  $g = \frac{dT_a}{ds}$  is constant. It is assumed that a steady state exists at  $t = 0$ , so that  $P_0 = K(T_0 - T_a)$  is the initial condition. The solution of Equation 5.2 is,



$$T = T_0 + \frac{\tau_1^2}{\tau} g v ( e^{-t/\tau'} - 1 ) + \frac{\tau'}{\tau} g v t \quad (5.3)$$

It is important to note that the reduction in effective time constant represented by  $\tau'/\tau$  is the result of the dissipation of power in the thermistor, and is specified by

$$\frac{\tau'}{\tau} = \frac{1}{[1 + B P_0 / T_0^2 K(v)]} \quad (5.4)$$

The reduced time constant, which means a more rapid response, is accompanied by a reduced sensitivity to fluid temperature variations and an increased sensitivity to speed variations.

A knowledge of the dependence of  $K$  on the relative speed,  $v$ , of the fluid is necessary for a more general solution of Equation 5.2.

Under steady-state conditions we have from Equation 2.1

$$T = T_a + \frac{P}{K(v)} \quad (5.5)$$

$T$  can also be expressed as follows, from the basic thermistor resistance - temperature Equations (2.10 and 2.11)

$$T = \frac{\alpha_a T_a^2}{\alpha_a T_a - \log_e \left( \frac{R}{R_a} \right)} \quad (5.6)$$

These two expressions can be equated and adjusted to give,

$$K(v) = \frac{P}{T_a} \left[ \frac{\alpha_a T_a}{\log_e \left( \frac{R}{R_a} \right)} - 1 \right] \quad (5.7)$$

Hence  $K(v)$  can be determined from easily-measured variables. A suitable experimental arrangement for determining  $K(v)$  from this equation is outlined by Rasmussen<sup>8</sup>. The thermistor is supported at the tip of the horizontal arm from a vertical shaft, which can be rotated at known velocities within a large Dewar jar containing the fluid in question.

#### 5.4 APPLICATION TO INSTRUMENT DESIGN

##### 5.4.1 In Bathythermography

The theory presented above has been widely implemented. Rasmussen's particular interest was in the design of a bathythermograph, illustrated by the schematic diagram of Fig. 5.1. In this instrument a thermistor, heated by a constant current, is carried down into the depths of the sea by a weight suspended from a light cable. The integrator gives an output voltage proportional to the length of cable and the thermistor voltage drop is amplified to give the

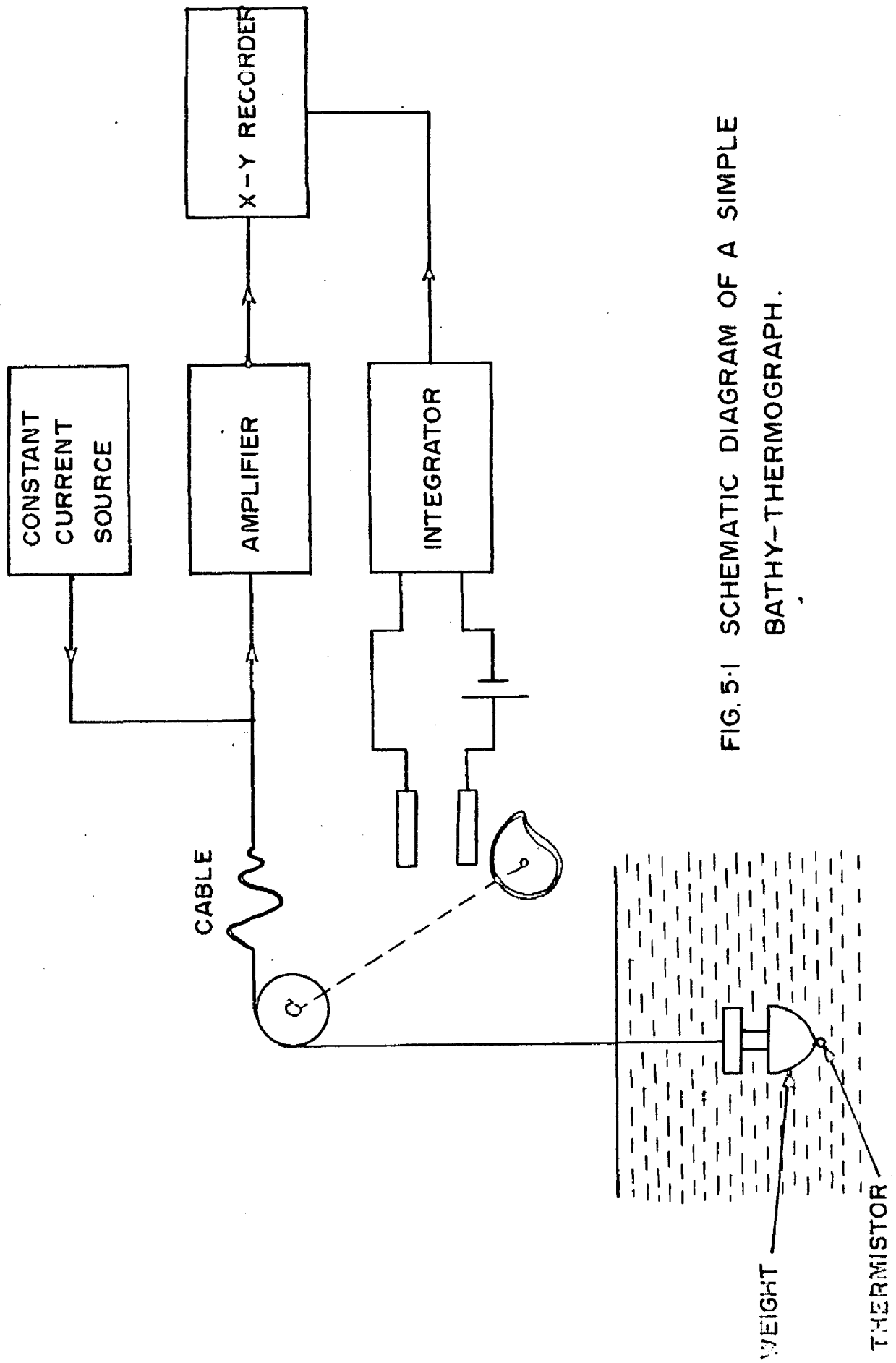


FIG. 5-1 SCHEMATIC DIAGRAM OF A SIMPLE BATHY-THERMOGRAPH.

the necessary electrical signals to operate an X-Y recorder. If a diving rate of 5 ft/s, a maximum temperature gradient of 0.25 °C/ft and a variational temperature accuracy of 0.05°C are specified then the design considerations are as follows. From Equation 5.3

$$\frac{dT}{dt} = 0.15 (1 - e^{-t/\tau'}) \quad (5.8)$$

On the assumption that the rate of change attains 99% of the maximum in an interval corresponding to a temperature change of 0.05 deg C an approximate value of time constant  $\tau'$  is obtained of 0.014 S. Since the time constant in air is nearly 25 times that in water a thermistor having a time constant in air of approximately 0.355 S is required, i.e. a bead-type thermistor is indicated. A standard range of bead type thermistor is indicated. A standard range of bead type thermistors has a thermal time constant of about 0.55 S in air; i.e. about 0.02 S in water. This is greater than the effective time constant  $\tau'$ , which is 0.014 S; hence the required reduction can be determined from Equation 5.3 as follows ,

$$\frac{0.014}{0.020} = \frac{3}{7} = \frac{BP}{K(T_0 + P/K)^2} \quad (5.9)$$

In solving this equation the maximum expected value of

$T_0$  should be considered. Taking  $K = 1.45 \text{ mW/K}$  (in water) and  $B = 3600 \text{ K}$ , with an ambient temperature of  $290 \text{ K}$ , it follows that the heating power required is  $\approx 22.5 \text{ mW}$ . This corresponds to a thermistor temperature of about  $305.5 \text{ K}$ , i.e.  $\approx 15.5 \text{ K}$  above ambient.

#### 5.4.2 In Low Pressure Measurement

At normal pressures both the viscosity and thermal conductivity of gases are independent of the pressure. At low pressure ( $\propto$  number of molecules/cm<sup>3</sup>), such that mean free paths of molecules are of the order of or greater than the dimensions of the enclosure, the thermal conductivity is proportional to the pressure. Considering a hot surface at temperature  $T_s$  surrounded by a monatomic gas at low pressure in an enclosure at ambient temperature,  $T_a$ . Molecules striking the hot surface with incident temperature  $T_i$  are re-emitted or reflected with a mean energy corresponding to a temperature  $T_r < T_r \leq T_s$ . On this basis an accommodation coefficient,  $\alpha$ , can be defined,

$$\alpha = \frac{T_r - T_i}{T_s - T_i} \quad (5.10)$$

It is established that in such collisions the mean energy transferred to or from a surface at  $T$ , per molecule, is given by  $E = 2kT$ . Hence if  $y$  represents

the number of incident molecules per  $\text{cm}^2$ . The net energy transfer per second is

$$\begin{aligned}
 E_0 &= \nu 2k (T_r - T_i) \\
 &= \frac{1}{4} n v_i 2k (T_r - T_i) \\
 &= \frac{1}{2} \frac{P v_i}{T_i} (T_r - T_i) \\
 &= \frac{\alpha}{2} \frac{P v_i}{T_i} (T_s - T_i) \quad (5.11)
 \end{aligned}$$

This follows since a molecule of mass  $m$  approaching the surface with incident velocity  $v_i$ , rebounding with the same speed will experience a momentum change of  $2m v_i$  and if  $\nu$  molecules of density  $n$  per  $\text{cm}^3$  strike an area of  $1 \text{ cm}^2$  per second the total impulse exerted on the area per unit time is  $2 m \nu v_i$ , pressure  $P$ , is defined as the rate at which momentum is imparted per  $\text{cm}^2$ , so that  $P = 2 m \nu v_i$ , and it can also be shown that  $P = n K T_i$ .

The application of heated thermistors to vacuum measurement is obvious. Various instruments have been described<sup>10,11,12,13,14</sup> for the measurement in the pressure range  $1-10^{-7}$  mm Hg, although some do not cover the whole range. That due to Bradley<sup>10</sup> does. It consists a bead type thermistor sealed into the end of

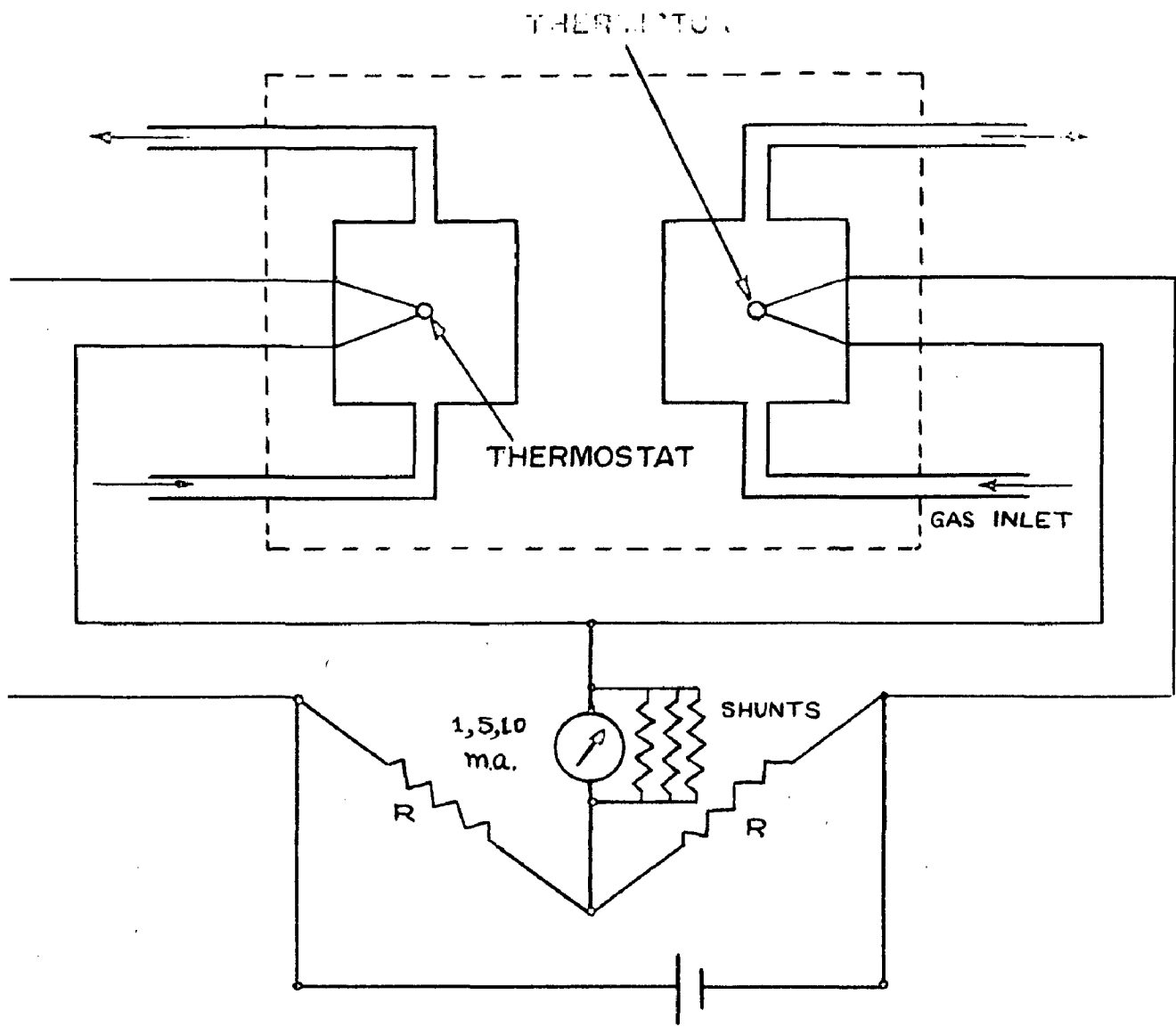


FIG. 5·2 CIRCUIT DIAGRAM OF ANEMOMETER.

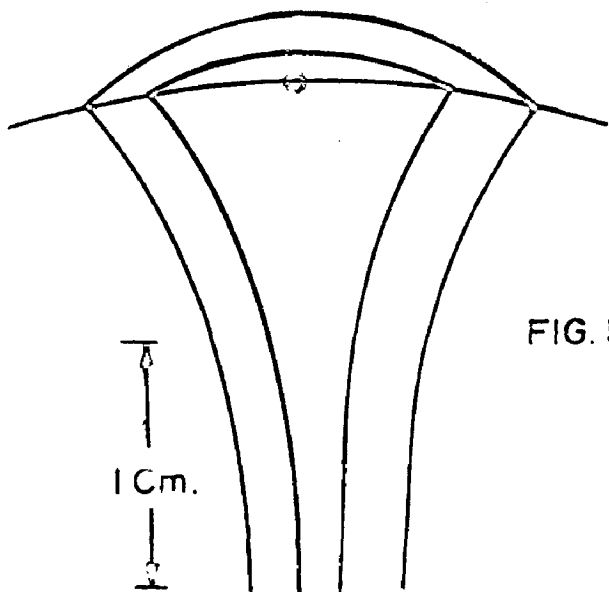


FIG. 5·3 BRADLEY'S MCLERD GAUGE HEAD.

the closed capillary as shown in Fig. 5.3. After calibration the thermistor may be used directly in the range  $1-10^{-3}$  mm Hg. Pressure in the lower range are measured by compressing the gas in the small chamber enclosing the thermistor until the pressure rises to  $10^{-3}-1$  mm Hg. The pressure in the chamber is then determined and the original very low pressure can be calculated from the compression ratio. Bradley's gauge used a Wheatstone bridge with a balancing thermistor in series with a resistance in an adjacent arm to the sensing thermistor. Varicak and Saftic<sup>11</sup> covered the range  $1-10^{-6}$  mm Hg directly by depositing miniature thermistors on thin metal foils.

An associated application which depends on the measurement of change of low pressure with time is in the determination of molecular weight by effusimetry<sup>14</sup>. This involves the release of the gas, whose molecular weight is to be determined, by effusion through a small orifice into a chamber which is initially evacuated.

#### 5.4.3 In Gas Analysis and Chromatography

Chromatography emerged as a chemical analytical technique. The technique has been extended to gases and is known as 'gas chromatography'. There are two variants.



In the first- 'gas liquid partition chromatography', G.L.P., or 'gas liquid chromatography' GLC, the absorbing agent is a liquid, distributed on a solid support, in the second, gas solid chromatography, GSC, the absorbent is a solid, but GSC has proved to be of less value than GLC. Gas-liquid chromatography can be used to determine the make-up of a gaseous mixture. In the latter case an inert gas is used to transport the unknown sample into the column and through it. The emergence from the column of a particular constituent of a gaseous mixture is sensed, for a fixed rate of carrier gas flow, by its intrinsic thermal conductivity. Hence if a means of measuring instantaneous thermal conductivity is available, both the qualitative and quantitative indication of the constituents is possible.

The sensing head for GLC is a thermal-conductivity cell. This is shown schematically in Fig. 5.2. It consists of a metallic block, of large heat capacity, through which both the input and output gaseous streams from the column flow. Holes bored transversely to the direction of 'gas flow' with appropriate seals, contain matched thermistors which are connected electrically in the usual wheatstone bridge circuit. In view of the sensitivity of the thermistor resistance to temperature and gas flow rate both of these have to be controlled.

#### 5.4.4 Other Applications Based on Rate of Flow

Several papers have been written on the application of thermistors to anemometry. These include instrumentation for measuring wind speed and forced flow under laboratory conditions. As discussed earlier in detail the dissipation constant is a function of velocity of the gas in ambient medium. It is this characteristic which is exploited in anemometry.

### 5.5 DESIGN DETAILS OF A SIMPLE GAS ANALYSER AND ANEMOMETER

#### 5.5.1 Introduction

For demonstrating the application of thermistors in gas analysis a simple thermistor device has been designed and fabricated which can be used both for gas analysis and with slight modification for anemometry purposes.

#### 5.5.2. Basic Principle

The device essentially exploits that property whereby the dissipation constant and therefore the resistance of a thermistor depends on the thermal conductivity of the medium and the velocity of the ambient in which it is placed. The basic arrangement shown in schematic form in Fig. 5.2 essentially consists of two

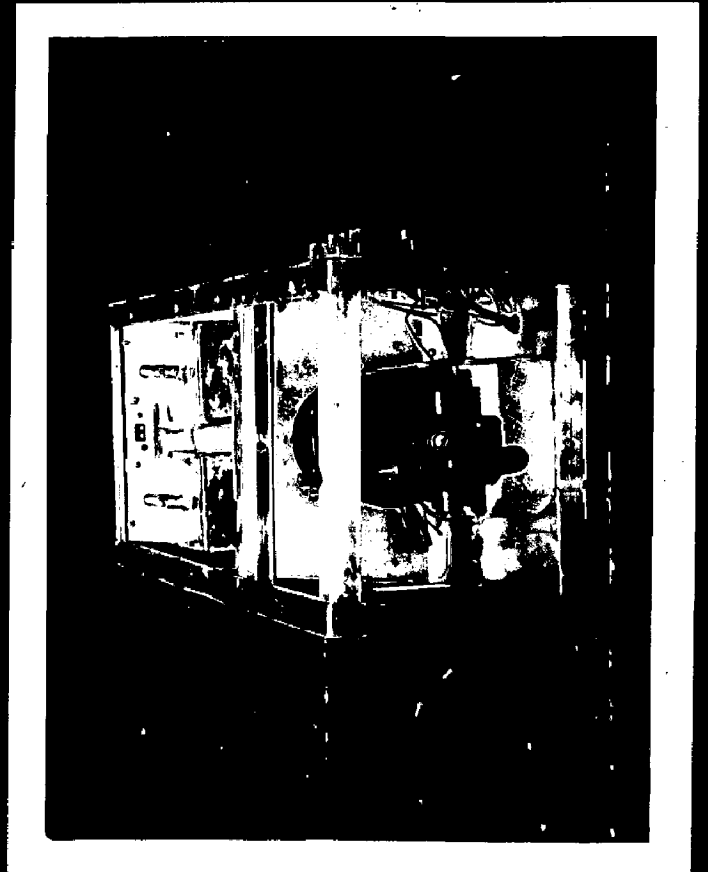
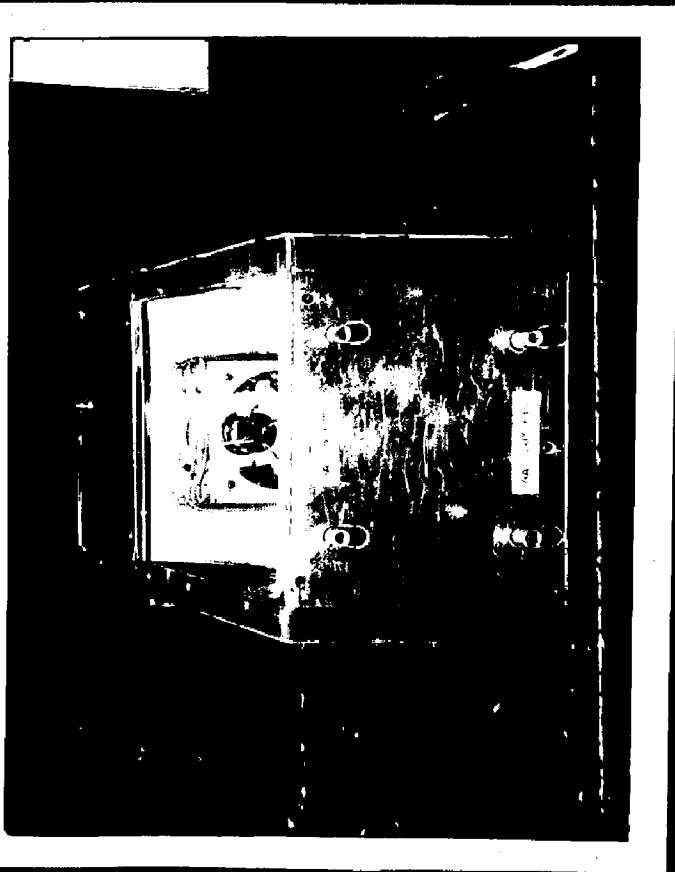
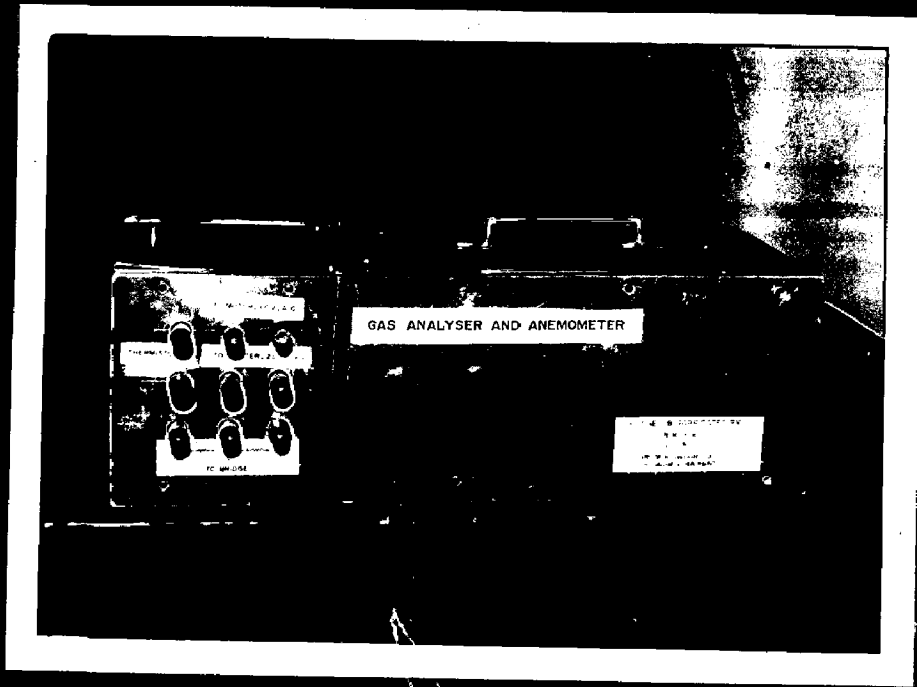
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chambers in which are housed two matched thermistors. One of the chambers and hence the thermistor is maintained under reference conditions which are kept constant. These conditions are the constituents of the ambient medium and its velocity and temperature. This thermistor will henceforth be referred to as the reference thermistor. The other chamber houses the sensing thermistor which is surrounded either by the gas under analysis or by the medium whose velocity is to be measured. These two thermistors form the adjacent arms of a Wheatstone bridge, the output of which is a measure of the measurand. Initially the two chambers are kept under reference conditions and the bridge output is balanced so that the bridge output is zero. Next the gas to be analysed is introduced into the sensing chamber. Depending upon the thermal conductivity of this gas the dissipation constant and therefore the resistance of the sensing thermistor will undergo a change causing an unbalanced output voltage to appear from across the bridge which can be calibrated in terms of either the percentage concentration of a particular gas or gases or the velocity of flow, depending on whether the device is being used as a gas analyser or as an anemometer respectively.

### 5.5.3. Constructional Details and Design Considerations

Thermostat is a rectangular chamber of 1' x 8" x 6" size made of aluminium. It is insulated by means fibre

sheet to make the heat loss by conduction and radiation as small as possible. The thickness of the aluminium sheet used in the construction of the chamber is deliberately kept large ( $1/8$ " ) so that it has a large thermal capacity consequently reducing the temperature variations inside the chamber in response to rapid changes in the environment temperature. The chamber is heated by means of a heater coil placed on one of its sides. Air is made to circulate by means of revolving blades attached to a small motor shaft so that there is no temperature gradient in the chamber and the temperature inside it is maintained nearly constant. The blades are shaped in such a way - details of which are clearly visible in the plate 1 - that air is made to circulate radially. Two brass chambers (3" cube) are placed on the opposite end of the heater coil. These containers must be exactly similar in size and shape and must have the same thermal capacity, which will provide uniform and equal heating of the two thermistors under similar gas flow conditions. The choice of the chamber material was decided from considerations of thermal capacity and constructional facilities. In order to render temperature inside the chamber insensitive to ambient temperature variations a high thermal capacity material was required.



From this consideration and the fact that brass lended itself easily and economically to brazing it was decided to use brass.

The shape of the chamber ideally must be such that the thermistor is equally spaced from all the sides of the chamber. From this consideration the ideal shape is spherical, but on account of fabrication diffuculties the next best shape, viz., cubical was selected.

A few exploded views of the device are shown in plate 1, 2 and 3.

#### 5.5.4 Thermistor Selection

The specification of the thermistors suitable for this application should be such that they have very small time constant, (preferably of the order of 1 sec. or even less) be impervious to gas and moisture and good stability CB should be reasonably constant with time) . From these considerations the type of the thermistor selected was epoxy coated bead type. From considerations of accuracy it is absolutely essential that the two thermistors be as exactly matched as possible so that at all temperatures the temperature coefficients are nearly the same and the value of B, the material constant. Use of matched thermistor will thus ensure

that under identical ambient conditions in the two chambers the value of the thermistor resistance will be the same.

# CHAPTER 6

**APPLICATION BASED  
ON RESISTANCE  
TEMPERATURE  
CHARACTERISTIC**



## APPLICATIONS BASED ON RESISTANCE TEMPERATURE CHARACTERISTICS

### 6.1 INTRODUCTION

One of the extensive fields of application of thermistors is in temperature measurement and sensing, and also temperature control and alarm systems. They can be used as well to compensate for changes in resistance of the circuit components having a temperature coefficient of resistance of opposite sign, N.T.C. thermistors, in conjunction with passive linearising resistors, are mainly used for this purpose to compensate for the positive increase of resistance with temperature of metallic conductors. The features of thermistors which send them to these applications are :

- (a) a large temperature coefficient of resistance
- (b) small size
- (c) the ability to withstand electrical and mechanical stresses
- (d) the ability to operate over a wide range of temperature
- (e) the large range of resistance values which can be realised.

For temperature measurement the non-linear resistance-temperature characteristic of N.T.C thermistors

is a disadvantage, but techniques are available for changing this to a linear one which have been discussed in detail in Chapter III.

## 6.2 TEMPERATURE MEASUREMENT

Generally a thermistor is chosen for temperature measurement where remote indication and alarm are desired, digital indication is required, small size is essential, ruggedness is necessary or where a small temperature difference is involved. A platinum resistance thermometer would be chosen where long term stability is of the highest importance or where the temperature is too high for a thermistor ( $>350$  K). Thermocouples are only useful for measurement of large temperature differences as they have a lower sensitivity, or for measurement of temperature in the very high ranges for which thermistors are unsuitable.

For a simple thermistor system it is sufficient to monitor the current through a series combination of a passive resistor and a thermistor, fed from a constant voltage supply. However, the majority of applications involve the use of a Wheatstone bridge to eliminate the standing current, with thermistor or preferably the thermistor and the compensating network occupying one arm, the other arms ideally are temperature insensitive resistors.

Direct reading thermistor bridges give an indication of temperature either in terms of the balanced current in the detector or a calibrated resistance arm, which is adjusted to give a balance at a fixed temperature.

The other type of bridge, which is useful for many applications, such as thermoelectric chemical analyses and dew-point hygrometry, is designed to measure temperature differences.

Common to both types of bridge is the problem of decrease of rate of change of (N.T.C) Thermistor resistance with increasing temperature. This leads to a temperature dependent sensitivity. A second problem is the spread of constants for thermistors, even of a given type.

### 6.3 INCORPORATION OF THE THERMISTOR IN A WHEATSTONE BRIDGE

It will be seen that when a thermistor is incorporated in a Wheatstone bridge in which the other components are passive, there is an unbalanced voltage across the detector when the resistance of the thermistor changes with temperature and so is the change in the

detector current  $I_g$ . The circuit to be considered is shown in Fig. 6.1. The three equations defining this network are :

$$\left. \begin{aligned} E_b &= R_1(I_1 + I_g) + RI_1 \\ E_b &= (R_2 + R_3)I_2 + R_2I_g \\ 0 &= RI_1 - I_2R_3 + R_gI_g \end{aligned} \right\} \quad (6.1)$$

Eliminating  $I_1$  and  $I_2$  gives

$$I_g = \frac{E_b(R_1R_3 - R_2R)}{R(R_1R_2 + R_1R_3 + R_2R_3 + R_2R_g + R_3R_g) + R_1(R_2R_3 + R_2R_g + R_3R_g)} \quad (6.2)$$

which may be rewritten as,

$$I_g = \frac{E_b(l - mR)}{pR + q} \quad (6.3)$$

where  $l, m, p, q$  are constants. An alternative form is

$$I_g + \text{Constant} = \frac{E_b(l/p + mq/p^2)}{R + q/p} \quad (6.4)$$

which may be recast as,

$$I = \frac{E_b s}{R + r} \quad (6.5)$$

where  $s$  is a constant which may be termed a sensitivity-reduction factor ;

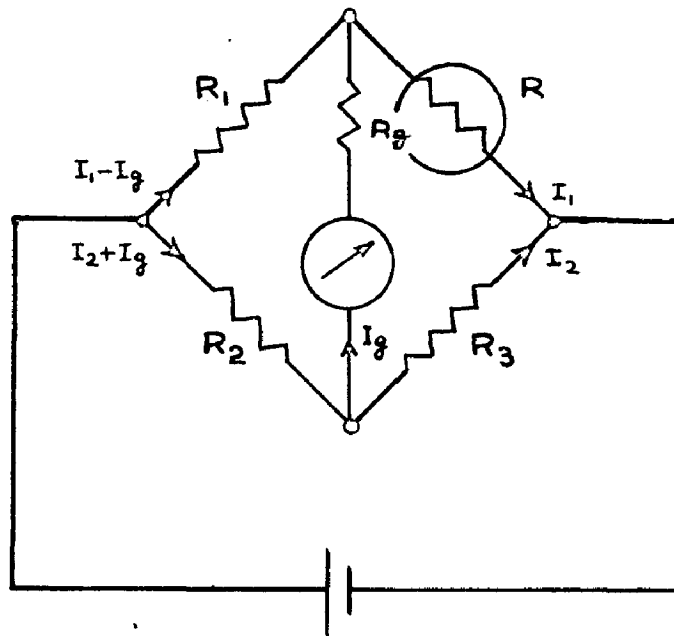


FIG. 6-1 WHEATSTONE BRIDGE CIRCUIT FOR TEMPERATURE MEASUREMENT .

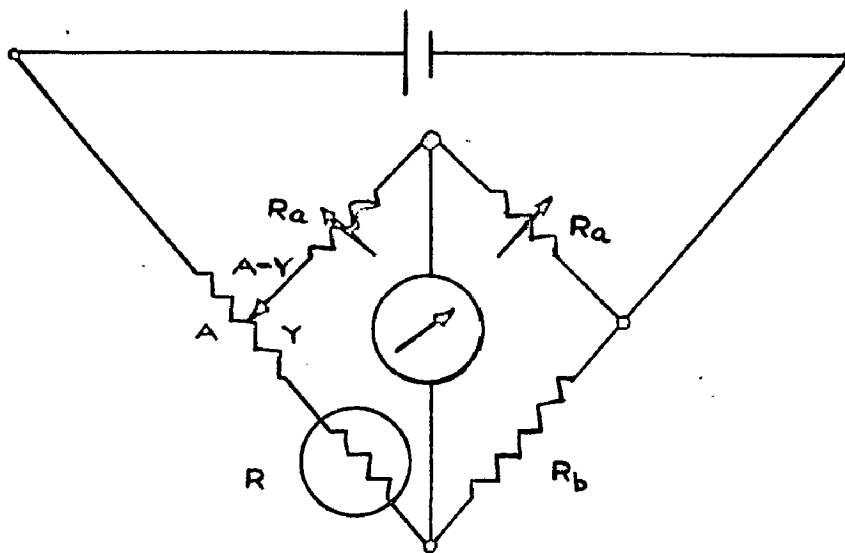


FIG. 6-2 THERMISTOR BRIDGE GIVING NEAR CONSTANT SENSITIVITY OVER A LIMITED RANGE OF TEMPERATURE .

$$S = \left( \frac{l}{p} + \frac{mq}{p^2} \right) \quad (6.6)$$

Equation 6.5 can be made linear in  $T$ . It is analogous to equation 3.18 and is linear in  $T$  for the same conditions that equation 3.18 is linear in  $T$ , i.e.

$$r = \frac{q}{p} = \frac{r_1 \left[ r_g(r_2 + r_3) + r_2 r_3 \right]}{(r_1 + r_g)(r_2 + r_3) + r_2 r_3} = \frac{r_1 x}{r + x} \quad (6.7)$$

The relationship between galvanometer current and temperature can be shown to be

$$\frac{dI_g}{dT} = \frac{SE_0(B + 2T_0)^2}{4T_0^2 B^2 R_0} \left( 1 - \frac{h_0^2 B^2}{12 T_0^4} \right) \quad (6.8)$$

where  $h_0$  represents the limits of range of thermometer.

In the design of direct-reading thermometer  $r$  is calculated from Equation 3.20 and  $S$  from equation 6.6. If a current limiting resistor is used in series with the battery supplying the bridge the expression for galvanometer current again is of the form of equation 3.18 and a linear response is approximated.

It is implicit in the above discussion that self-heating of the thermistor is insignificant. The effect of small self-heating is considered by Beakley.

#### 6.4 CONSTANT SENSITIVITY BRIDGE

Since the rate of change of (N.T.C) thermistor resistance with temperature decreases with temperature, the sensitivity of the simple bridge considered so far decreases as the working temperature increases. Pltts and Priestley<sup>15</sup> have described a bridge in which near constant sensitivity can be achieved over a limited range of temperature. Their circuit is shown in Fig. 6.2. The novel feature is a tapped resistance A, part of which is in series with the thermistor and the remainder in series with the supply to the bridge. With this arrangement it has been shown, to a first approximation, that  $dI_g/dT$  is independent of R and hence of T if

$$\frac{2 R_a R_b}{(R_a + R_b)} \approx R_b - A$$

#### 6.5 SELF BALANCING THERMOMETER

Priestley has described a self-balancing bridge in which the d.c. out of balance signal from the bridge is fed to a servo-amplifier, which drives a servo-motor, which is connected to a balancing ten-turn

potentiometer in one arm. This has a straight-through spindle and one end of this is connected to digidial. Priestley incorporated the thermistor in shunt with a resistance  $S$  of value

$$S = R_0 \frac{(B - 2T_0)}{(B + 2T_0)} \quad (6.9)$$

which gives a second differential coefficient of zero in the Taylor expansion of the resistance of the combination as a function of  $T$  about  $T_0$ .  $R_0$  is the thermistor resistance at  $T_0$  and  $B$  the material constant, (cf. Beakley's<sup>16</sup> corresponding criteria for a resistance in series with the thermistor).

An earlier thermometer for measuring small temperature changes on a short term basis was described by Greenhill and Whitehead<sup>17</sup>, who achieved an accuracy of  $0.0002^\circ\text{C}$  in  $0.01^\circ\text{C}$ . The sensing thermistor was incorporated in a d.c. Wheatstone bridge with a reflecting galvanometer detector. The incident light to this was chopped using a slotted rotating disc to produce a.c. at about 5 KHz. The reflected light impinged on a photoelectric cell after passing the edge of a wide slit, was amplified and produced an output which was fed back to a second thermistor in an adjacent arm, thereby heating it. The resistance of this second thermistor depends on the amplitude of the a.c. output



from the amplifier, which in turn depends upon the amount of off-balance current in the galvanometer. The output a.c. of the amplifier is recorded and after calibration of the sensing thermistor can be used as a measure of temperature change.

#### 6.6 DIFFERENTIAL THERMOMETERS

For some applications the measurement of temperature difference, rather than the direct measurement of temperature, is required. A Wheatstone bridge is again to be preferred, but now incorporating two thermistors in adjacent arms as sensing elements. Ideally these should be perfectly matched, not only at the operating temperature, but over the range of temperature difference to be measured, this condition does not arise naturally so equilibration is necessary.

In their bridge Nordon and Bainbridge<sup>18</sup> first linearised each thermistor using shunt resistors and then equalised the combined resistance at a given temperature, by connecting a resistance in series with one of the combinations. The different temperature sensitivities were compensated by passing different constant currents through each arm, of such values that the same potential variations per unit of temperature change occurred across each. A high impedance detector was

used in the bridge to permit this. A linear dependence on temperature, with a maximum error of 0.5 % was obtained over a range of 50 K and with a maximum error of 0.05% over a range of 18 K about ambient.

Godin<sup>19</sup> used a somewhat different approach to equalisation. Only one of the thermistors was shunted by a resistance and the value of this was determined by equating the first temperature-coefficient of resistance of the combination with that of the other thermistor. Equalisation of resistance at the operating temperature was achieved by connecting a small resistance either in series with the shunt combination or the other thermistor.

Godin<sup>20, 21</sup> described the design of two differential bridges. The first used two uncompensated thermistors within the bridge and was based on analysis for zero temperature-coefficient of detector galvanometer current and maximum sensitivity to differential temperature changes. This involved non-linear scaling and a variation of sensitivity with temperature, unless the operation was close to the design temperature. In the second design equalisation was considered.

## 6.7 TYPES OF DISPLAYS IN THERMISTOR THERMOMETRY

Electronic thermometers employing thermistors

as the sensing element can display the measurand in a variety of forms - the more important of which are analog.digital or graphical.

In electronic analog thermometers the end device is either a voltmeter, ammeter or a galvanometer. Some of the outstanding advantages of this form of display are :

1. Simplicity and low cost
2. Capable of displaying continuously the measure of the measurand.
3. They have withstood the test of time.

On the other hand the fast expanding digital form of display is superior to its analog counterpart in following respects :

1. Greater over all accuracy.
2. Display in digital form eliminates reading errors and saves operator's time in taking measurements.
3. Lends itself admirably to digital telemetry systems which are definitely superior to its analog counterpart.
4. Capable of communicating with digital computers which are increasingly being used for both data reduction and automatic control systems.

## 6.8 TEMPERATURE SENSING AND CONTROL<sup>22</sup>

There are many applications in which the temperature dependence of thermistor resistance is used as a sensor for purposes of initiating control, rather than as a means of measurement. In control applications the error signal is generally amplified and either operates a relay (ON-OFF control) or controls the conduction period of a thyatron, silicon-controlled-rectifier or the operation of a transistor.

While a thermistor and its required associated equipment are more expensive than a standard thermostat, the thermistor has certain advantages in temperature control applications. The most important advantage is the ease of adjustment of the control point.

### 6.8.1 Digital Temperature Indicator<sup>23</sup>

In digital temperature indicator a temperature sensitivity thermistor is used to control the current through a voltage divider and, consequently, the voltage drop across each half of the voltage divider. The circuit uses two lamps to indicate the temperature range of the ambient air. The digital temperature indicator can indicate relative temperature (cold-warm-hot)

from 50 to 60°C . When the relative temperature is cold, only lamp  $L_1$  is illuminated, when the relative temperature is warm, both lamps  $L_1$  and  $L_2$  are illuminated. When the relative temperature is hot only lamp  $L_2$  is illuminated. See Fig. 6.3.

To adjust the digital temperature indicators, maintain the thermistor at the desired temperature and adjust warm adjust resistor  $R_5$  so that lamp  $L_2$  just lights. Now, with the thermistor still maintained at the desired temperature, adjust hot adjust resistor  $R_1$  so that lamp  $L_1$  will just turn off.

When the digital temperature indicator is properly adjusted, the resistance in parallel with neon lamp  $L_2$  will be lower than the resistance in parallel with neon lamp  $L_1$  if the ambient air temperature is cold. This is due to the temperature coefficient of the thermistor  $R_6$ . The resistance of  $R_6$  is directly proportional to the temperature. Since the same current flows through  $R_1$ ,  $R_2$ ,  $R_5$  and  $R_6$ , a larger voltage is dropped across  $L_1$  than across  $L_2$ . The higher voltage is just sufficient to illuminate  $L_1$ . When the ambient temperature becomes warm, the resistance of the thermistor  $R_6$  increases just enough so that the resistance in parallel with

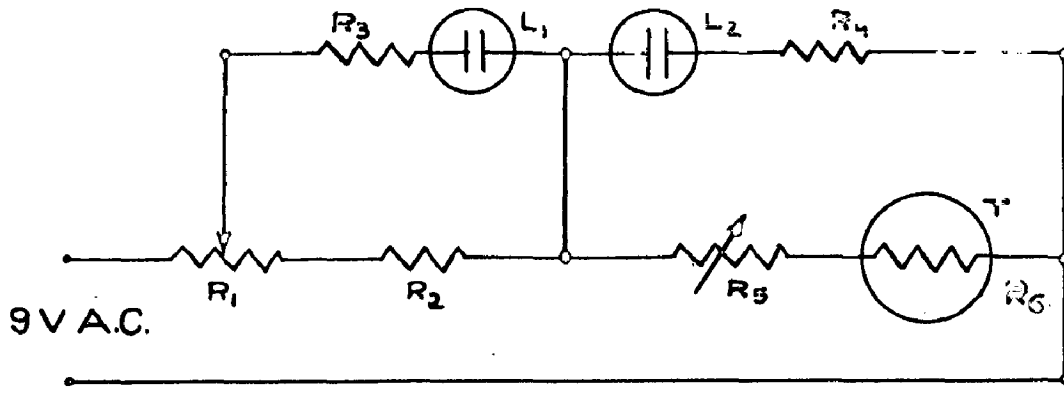


FIG. 6.3  $R_1$  - HOT ADJUST,  $R_5$  - WARM ADJUST  
 $L_1$  AND  $L_2$  - LAMPS

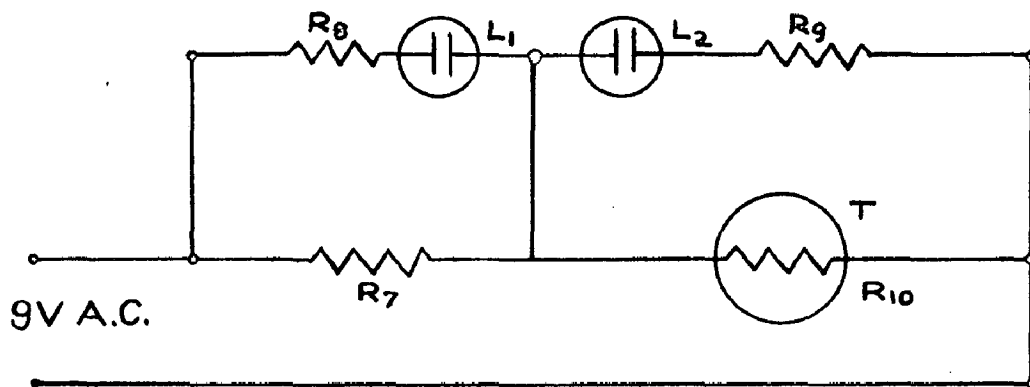


FIG. 6.4 LIQUID LEVEL INDICATOR  
 TO SCHMITT TRIGGER

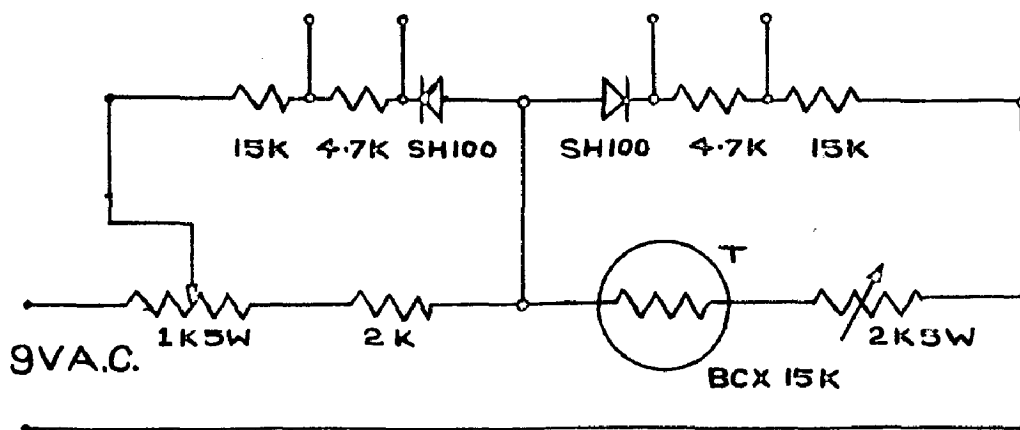


FIG. 6.5 PRACTICAL CIRCUIT OF DIGITAL TEMPERATURE INDICATOR.

each lamp is the same. Therefore the same voltage is dropped across each lamp and both of them will be illuminated. When the temperature becomes hot, the resistance across  $L_2$  becomes greater than the resistance across  $L_1$ , and only lamp  $L_2$  is illuminated.

### 6.8.2 Liquid-Level Indicator

The liquid-level indicator uses a temperature sensitive thermistor  $R_{10}$  immersed in the liquid as the liquid level sensor. With thermistor  $R_{10}$  immersed in the liquid, indicator lamp  $L_1$  is illuminated. See Fig. 6.4. If the liquid level drops below the thermistor the resistance of the thermistor is increased, causing indicator lamp  $L_1$  to extinguish and indicator lamp  $L_2$  to be illuminated. When the liquid level drops below the thermistor level, there is a 10 to 30 second delay before low indicator lamp  $L_2$  illuminates. When the liquid level rises above the thermistor level, there is a 2 to 10 second time delay before normal indicator lamp  $L_1$  illuminates. Both lamps are extinguished from 1 to 5 seconds during the transition period.

When thermistor  $R_{10}$  is immersed in liquid, its resistance is relatively low and more voltage is dropped

across resistor  $R_7$ , then across the thermistor. This high voltage across  $R_7$  is sufficient to illuminate neon lamp  $L_1$ ; When the liquid level drops below the thermistor, its resistance increases and more voltage is dropped across it than across  $R_7$ . Thus neon lamp  $L_2$  is illuminated and  $L_1$  is extinguished. Due to fairly slow temperature change of the thermistor, there is a slight delay encountered in the operation of the indicator lights.

The author has fabricated the temperature and liquid level indicator using the same principle. The practical circuit of Fig. 6.3 is shown in Fig. 6.5. The thermistor used is BCX 15 K. The diodes in the branch ab and bc are incorporated such that the current flowing in one branch does not flow in the other. The circuit is fed with 9 V a.c. The voltage across bd and be is fed to the base of Schmitt trigger. The Schmitt Trigger is designed for 6 V d.c. supplied by constant voltage source. The complete circuit is given in Fig. 6.6. The voltages drop across bd and be are sufficient to trigger the Schmitt circuit at a set value of temperature.

### 6.8.3 Design Considerations

The thermistor circuit is designed such that the current in branch ab does not flow in branch bc and vice-



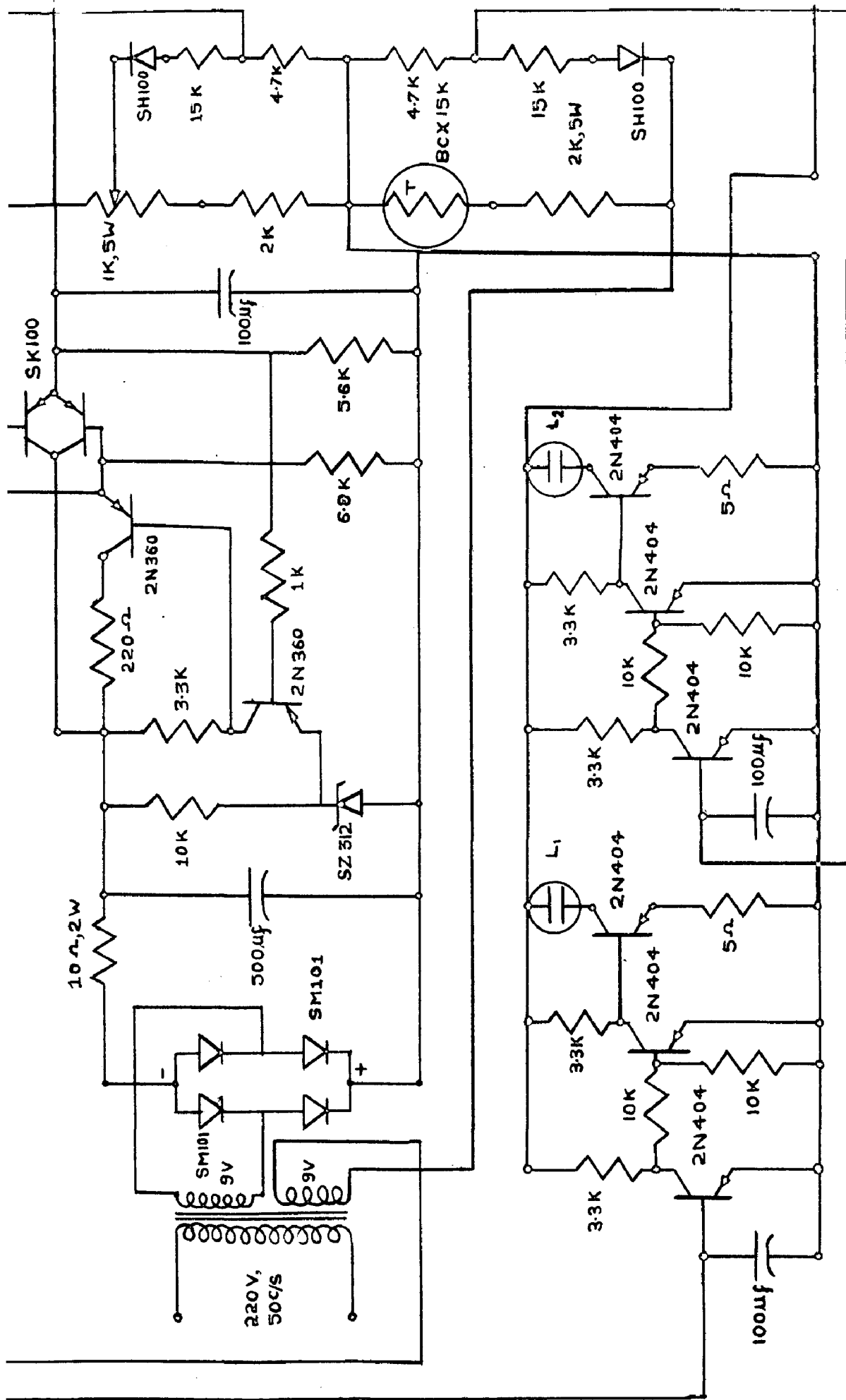


FIG. 6.6 COMPLETE CIRCUIT DIAGRAM OF DIGITAL TEMPERATURE AND LIQUID LEVEL INDICATOR.

versa, and the potential of point e and d should always be negative with respect to b for the current flowing for each half cycle through the two branches alternatively. In order to achieve this two diodes in the direction shown in Fig. 6.5 are connected. Moreover the resistance of the branches ab and bc are kept sufficiently large as compared to their parallel branch. The resistances db and be are such that the potential drop across them is sufficient to operate the Schmitt trigger.

Constant voltage supply source is designed for 6 V, 250 mA current. The voltage is made constant by using a zener. The complete circuit is shown in Fig.6.6 In order to raise the power rating of the supply two power transistors SK100 are used in parallel.

Schmitt trigger is designed to operate at .2 V.

## 6.9 TEMPERATURE CONTROLLER<sup>29</sup>

It consists of two units :(a) column oven, and (b) electronic controller for the oven.

### 6.9.1 Column Oven

This is a chamber where a constant temperature is maintained. Inside the oven are incorporated the

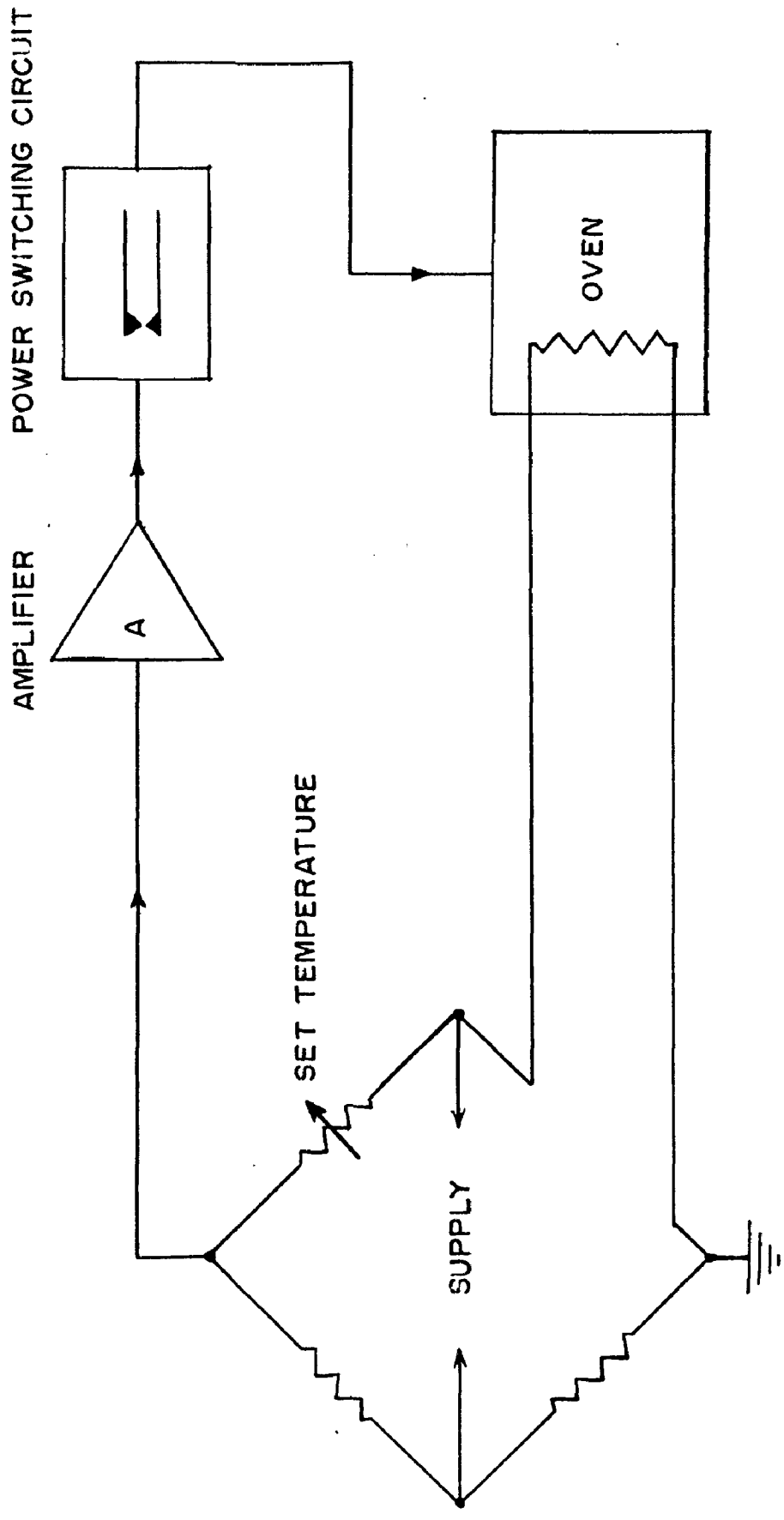


FIG. 6.7 SCHEMATIC DIAGRAM OF TEMPERATURE CONTROLLER.

following :

(a) A blower Unit - fitted to the left hand side of the oven. The blower is operated by means of a fractional horse power motor mounted horizontally outside the oven.

(b) Heater assembly - consists of one 1000 watts heater wired round the blower and connected to the supply operated by a relay.

#### 6.9.2. Electronic Controller for the Oven

A sensor unit, a thermistor, is mounted on the wall of the column oven on the right hand side. This sensor unit is connected in a Wheatstone Bridge circuit operated from a 6 V A.C. supply. The change in the resistance of the sensor corresponding to the change in the temperature of the oven results in an out of balance in the bridge voltage. This voltage is amplified by means of an amplifier and is fed to the relay, which in turn operates the heater. The Schematic circuit diagram is shown in Fig. 6.7.

#### 6.10 TEMPERATURE COMPENSATION<sup>22</sup>

A thermistor can be used to compensate an electrical device for undesired variations in performance that are caused by temperature changes in some other components. The most common occurrence of this kind is the change in resistance of a copper coil with temperature.

For example, millivoltmeters are subject to errors in calibration with changes in temperature, and consequently resistance, of the moving coil. A suitable thermistor network (usually consisting thermistor and associated shunt resistor) can be connected in series with the coil so that the overall resistance is constant despite temperature changes.

In a resolver (angular-position data transmitter) manufactured by the Control Engineering Corporation, bead type thermistor is directly embedded directly in the coils of the unit and not only compensate for changes in resistance of the copper windings, but also simultaneously correct for variations in the magnetic properties of the core material. Variations in the 'Q' of the IF transformers of certain Air Force radio receivers is prevented by the inclusion of a suitable thermistor in the circuit.

In battery chargers, the proper point to switch from a high charging rate to a low rate depends on battery temperature. The Berg Gibson Manufacturing Co. uses a thermistor in series with a voltage sensitive relay to perform this function in their charger. The thermistor circuit energizes the relay at a proper voltage in accordance with temperature.

## 6.11. THERMISTOR THERMOMETER BASED ON AN ASTABLE MULTIVIBRATOR<sup>24, 25, 26</sup>

### 6.11.1 Introduction

An electronic thermometer is described which is reliable, sensitive, linear and simple to manufacture, and has uses in industry, medicine, etc.

### 6.11.2 Principle of Operation

Fig. 6.8 shows an astable multivibrator with a moving coil meter connected between the output collectors. When  $R_{B1} = R_{B2}$  the mean current through the meter is zero, (Fig. 6.9) as  $t_1$  equal  $t_2$ , and the pointer is at zero. By lowering the value of one of the base resistors  $R_B$ ,  $t_1$  or  $t_2$  will be shorter. The mean current will therefore diverge from zero and the meter will show a reading, Fig. 6.10. It is easily seen that

$$i_{\text{mean}} = \left[ \frac{(t_1 - t_2)}{(t_1 + t_2)} \right] i_{\text{max}}$$

$$\text{or } i_{\text{mean}} / i_{\text{max}} = (t_1 - t_2) / (t_1 + t_2)$$

If  $R_{B2}$  is replaced by a thermistor (N.T.C) then, if  $T_1$  is the lowest temperature to be measured and  $T_2$  is the highest

$$R_{B1} = R_{\text{NTC}}(T_1)$$

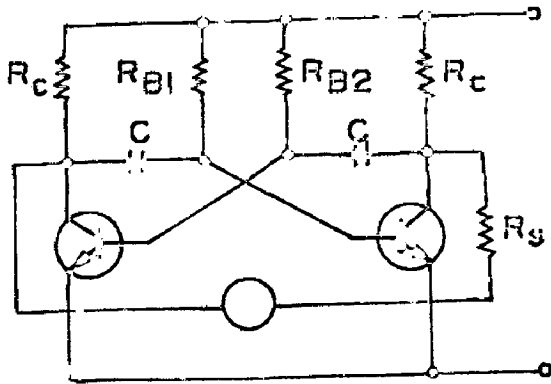


FIG. 6-8

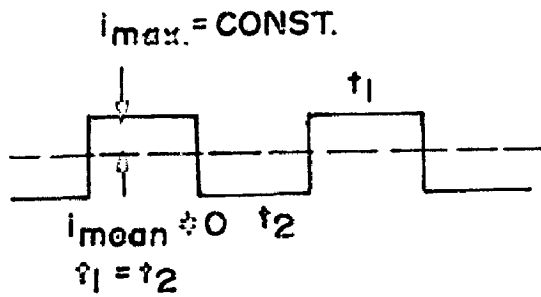


FIG. 6-9

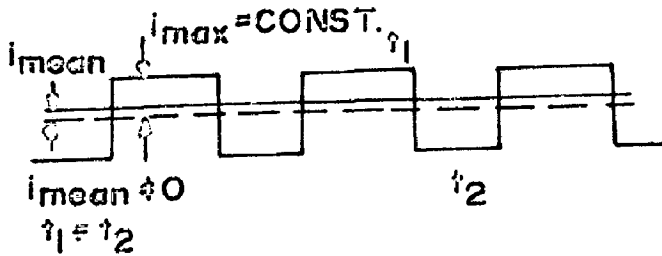


FIG. 6-10

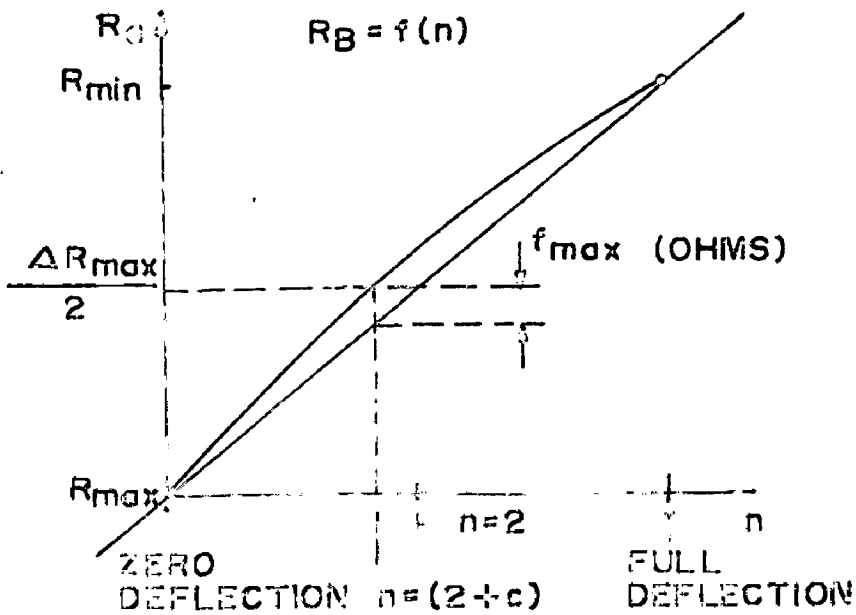


FIG. 6-11

Since

$$\begin{aligned} (t_1 - t_2) / (t_1 + t_2) &= (R_{B1} - R_{NTC}) / (R_{B1} + R_{NTC}) \\ &= \Delta R / (2R_{B1} - \Delta R) \end{aligned}$$

then  $i_{\text{mean}} = \left[ (R_{B1} - R_{NTC}) / (R_{B1} + R_{NTC}) \right] i_{\text{max}}$

and  $R_{NTC} = R_{B1} \frac{1 - (i_{\text{mean}} / i_{\text{max}})}{1 + (i_{\text{mean}} / i_{\text{max}})}$

$i_{\text{mean}}$  is defined to be equal to the meter current at full deflection, and one can therefore replace  $i_{\text{mean}}$  by  $i_{\text{mean}}/n$ . Then the value of  $R_{NTC}$  which gives a pointer movement equal to  $1/n$  of full deflection is

$$R_{NTC} = R_{B1} \frac{n - (i_{\text{mean}} / i_{\text{max}})}{n + (i_{\text{mean}} / i_{\text{max}})} \quad (6.10)$$

where,  $i_{\text{mean}} / i_{\text{max}}$  is a constant.

### 6.11.3 Linearity

Let  $C = i_{\text{mean}} / i_{\text{max}}$

then by using Equation (6.10)

$$R_{B1} - \Delta R = R_{B1} \left[ (n - C) / (n + C) \right]$$



$$\text{and } \Delta R = 2 R_{B1} C / (n+C) \quad (\infty \cong n \cong 1) \quad (6.11)$$

It is seen that a condition of linearity is that  $C \ll n$ ,

or  $C \ll 1$  as  $n_{\min} = 1$

From Equation 6.11

$$C = \Delta R \cdot n / (2R_{B1} - \Delta R)$$

To keep  $C$  small one must ensure that  $R_{B1} \gg \Delta R$

#### 6.11.4 Maximum Error Due to Non-Linearity

Equation 6.11 gives,

$$\Delta R = 2 R_{B1} / (n/C + 1)$$

It is required to find an expression which gives the maximum error for  $R_{NTC}$  when  $n$  is chosen arbitrarily.

Now

$$\left[ \frac{2 R_{B1}}{n/C + 1} - f \right] n = \frac{2 R_{B1}}{1/C + 1}$$

where  $|f|$  is the difference between the theoretical linear value of  $R_{NTC}$  and the value obtained, and therefore,

$$f = \frac{2 R_{B1} (n-1)}{(n/C + n) (n/C + 1)} \quad \text{ohms}$$

The differential coefficient for this expression is

$$\frac{df}{dn} = \frac{2R \left[ \left( \frac{n}{C} + n \right) \left( \frac{n}{C} + 1 \right) - (n-1) \left( \frac{1}{C} + 1 \right) \left( \frac{2n}{C} + 1 \right) \right]}{\left[ \left( \frac{n}{C} + n \right) \left( \frac{n}{C} + 1 \right) \right]^2}$$

and by setting  $df/dn = 0$

$$n = 2 + (1+C)/(1+1/C) = 2 + C \quad (6.12)$$

and  $2 \gg C$

The maximum error due to non-linearity thus lies very near the middle of the scale, (exactly, when  $\Delta R = \Delta R_{\max}/2$ )

The maximum error (See Fig. 6.11) will be ,

$$f_{\max} = \frac{2R_{Bl}}{2/C + 1 + 1} - \frac{\Delta R_{\max}}{2 + C}$$

$$\text{or } f_{\max} = \Delta R_{\max} \left( \frac{1}{2} - \frac{1}{2+C} \right) \text{ ohms}$$

#### 6.11.5 Stability

It is seen that a change in battery voltage has no influence on the meter zero deflection, as  $t_1 = t_2$

But (see Fig. 6.8)

$$i_{\text{mean}} = \frac{t_1 - t_2}{t_1 + t_2} \cdot \frac{1}{R_c + R_s} E$$

and therefore variations in supply voltage must be taken

into consideration, i.e. a stabilized voltage should be used. Care must be taken not to pass too much current through the base N.T.C. resistor, but to work on the linear scale in the voltage-current characteristics of these resistors. Because of the pulsed current, higher maximum values than shown in the characteristics can be used.

# CHAPTER 7

## MISCELLANEOUS APPLICATIONS OF DIRECTLY HEATED THERMISTORS

## MISCELLANEOUS APPLICATIONS OF THERMISTORS

7.1 INTRODUCTION

In this Chapter various applications of both NTC and PTC thermistors will be considered which are based on either their small signal or their nonlinear or transient properties all of which have been considered in detail in earlier chapters. The small signal properties of the N.T.C. type thermistors can be exploited for using it as a variable radio frequency resistor and those of both N.T.C. and P.T. C types for phase shifting, differentiating and integrating applications. Non linear applications include oscillator amplitude and low frequency regulation, amplifier gain or level stabilization, voltage and current limiting or regulation, volume limiting and signal expansion and compression. Transient applications are in overload protection, surge suppression, time delay in relay circuits and switching.

Some of these applications which have potentialities for further development and popularization will be outlined in this Chapter.

7.2 THE THERMISTOR AS A LOW FREQUENCY CIRCUIT ELEMENT<sup>27</sup>7.2.1 Phase-Advancing (Differentiating) Circuit

The use of an N.T.C. thermistor will be considered for the simple circuit shown in Fig. 7.1 a and in equivalent

form in 7.1b . The direct current bias circuit is not shown. The voltage transmission is given by

$$T_d = \frac{V_2}{V_1} = \frac{R_1 + R_o + j\omega\tau(R_1 + R_\infty)(R_\infty + R_o) / 2R_\infty}{R + R_1 + R_o + j\omega\tau(R + R_1 + R_\infty)(R_\infty + R_o) / 2R_\infty} \quad (7.1)$$

The locus of the tip of the corresponding vector is a semicircle as shown in Fig. 7.2. Two conditions are portrayed. On the left the d.c. bias is such that  $R_1 + R_o$  is positive ; the phase shift is therefore less than  $90^\circ$  at all frequencies. On the right the thermistor is biased sufficiently for beyond turnover that  $R_1 + R_o$  is negative ; phase advance up to  $180^\circ$  can be achieved in this case.

The circuit time constant is (see section 2.4.3)

$$\tau_c = \tau \frac{(R_\infty + R_o)(R + R_1 + R_\infty)}{2R_\infty(R + R_1 + R_o)} \quad (7.2)$$

A choice of additional series and shunt resistances enables the circuit time constant to be adjusted to any desired value. For the circuit configuration shown in Fig. 7.3. it is ,

$$\tau_c = \tau \frac{(R_\infty + R_o)(R_s + R_\infty)}{2R_\infty(R_s + R_o)} \quad (7.3)$$

where  $R_s$  is the resistance presented to the thermistor :

$$R_s = R_2 + R_3 (R + R_1) / (R_3 + R_1 + R) \quad (7.4)$$

If the circuit is to be stable  $R_s$  must be greater than  $R_0$  and for the time constant to be large  $R_s \approx R_0$ .

Hence the larger is the demanded time constant the greater is the possibility of instability.

To illustrate a possible design procedure it will be assumed that the thermistor impedance is specified and values of  $R$ ,  $R_1$ ,  $R_2$  and  $R_3$  have to be chosen to define a required ratio of high to low frequency transmission  $T_{dr} = T_{d\infty} / T_{d0}$ , and a circuit time constant  $\tau_c = x\tau_e$  where  $\tau_e$  is the 'effective thermal time constant' of the thermistor, defined in Equation 2.51 .

Hence,

$$x\tau_e = \left[ \frac{R_s + R_\infty}{R_s + R_0} \right] \tau_e \quad (7.5)$$

and

$$R_s = \frac{R_\infty - x R_0}{x - 1} \quad (7.6)$$

Since  $R_0$  and  $R_\infty$  have been prescribed,  $R_s$  is fixed. The transmission ratio is given by ,

$$T_{dr} = \frac{(R_y + R_\infty) (R_s + R_0)}{(R_y + R_0) (R_s + R_\infty)} = \left[ \frac{R_y + R_\infty}{R_y + R_0} \right] \frac{1}{x} \quad (7.7)$$

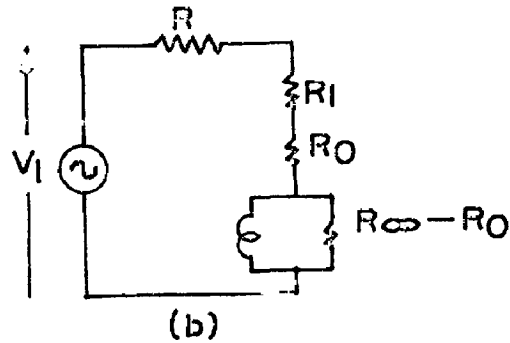
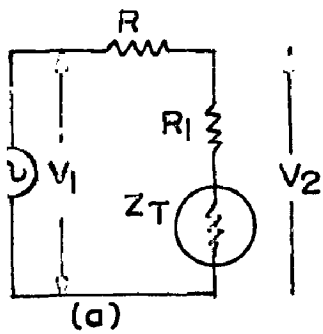


FIG. 7.1 (a) PHASE ADVANCING CIRCUIT INCORPORATING N.T.C. THERMISTOR (b) EQUIVALENT CIRCUIT

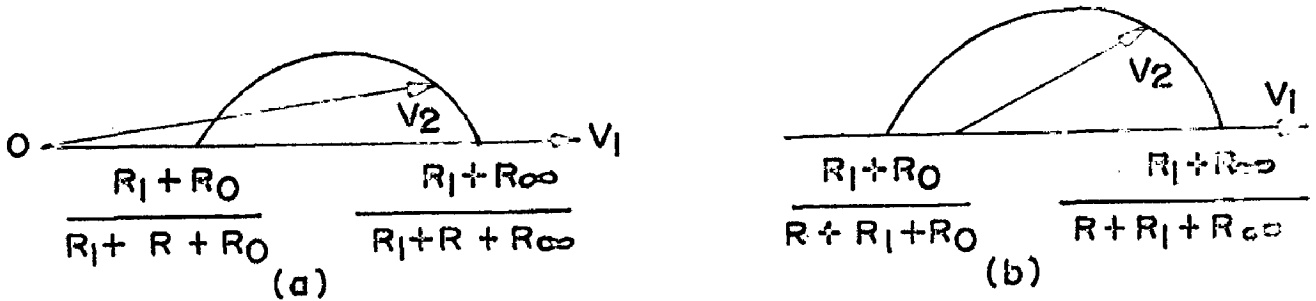


FIG. 7.2 PHASE ADVANCE TRANSMISSION (a)  $R_1 + R_0 > 0$   
(b)  $R_1 + R_0 < 0$

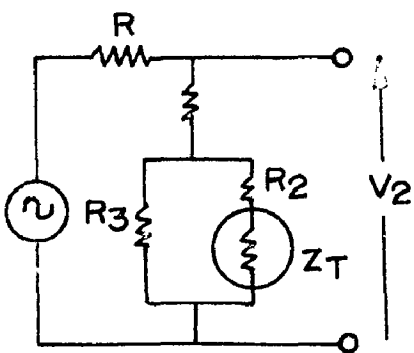


FIG. 7.3  
GENERAL PHASE-ADVANCING NETWORK, INCORPORATING N.T.C. THERMISTOR

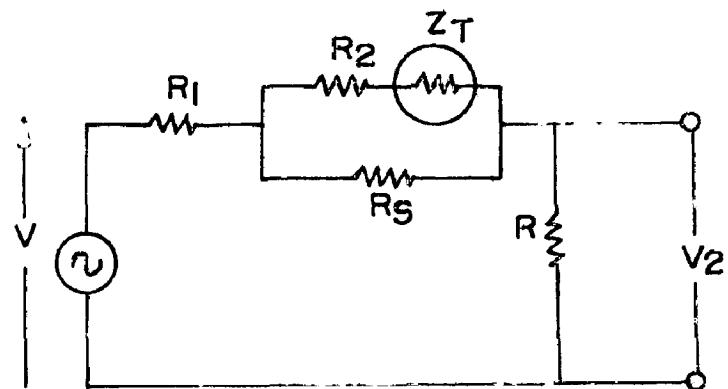


FIG. 7.4  
GENERAL PHASE-RETARDING (INTEGRATING) NETWORK, INCORPORATING N.T.C. THERMISTOR.



where

$$R_y = R_2 + R_1 R_3 / (R_1 + R_3) \quad (7.8)$$

is therefore also fixed. R should be made as small as possible, so that the attenuation will be small. The minimum value is that of the voltage source. Either  $R_1$ ,  $R_2$  or  $R_3$  may be given an arbitrary value, say zero and the others calculated using Equations 7.6 and 7.8.

### 7.2.2 Phase Retarding (integrating) circuit

Again an N.T.C Thermistor will be used by way of example. This general form of circuit is shown in Fig. 7.4. The transmission is

$$T_1 = \frac{V_2}{V_1} = 1 - \frac{V_3}{V_1} = 1 - T_d \quad (7.9)$$

It follows that the circuit time constant is the same as that of the differentiating network considered above. In this case the resistance  $R_s$  presented to the thermistor is given by

$$R_s = R_2 + \frac{R_3(R_1 + R)}{R + R_1 + R_3} = \frac{R_\infty - x R_0}{x-1} \quad (7.10)$$

The ratio  $T_{ir}$  of the high to low frequency transmission is

$$\begin{aligned}
 T_{1r} &= \frac{1-T_{di}}{1-T_{do}} = \frac{1 - \frac{(R_y + R_\infty)(R_3 + R_1)}{(R_s + R_\infty)(R_3 + R_1 + R)}}{1 - \frac{(R_y + R_0)(R_3 + R_1)}{(R_s + R_0)(R_3 + R_1 + R)}} \\
 &= \frac{1}{x} \frac{(R_2 + R_3 + R_\infty)}{(R_2 + R_3 + R_0)} \quad (7.11)
 \end{aligned}$$

From this,

$$R_2 + R_3 = \frac{R_\infty - x T_{1r} R_0}{x T_{1r} - 1} \quad (7.12)$$

Equations 7.6 and 7.12 may be used to design the integrating network.

### 7. 3 LIMITER (REGULATOR, COMPRESSOR)<sup>28</sup>

The nonlinear resistance of the thermistor may be employed in simple circuits for signal limiting, peak compression, and voltage regulation.

Fig. 7.5 shows a circuit of this type which is seen to resemble the variator limiter circuit. Resistor R and the thermistor form a voltage divider with the upper output terminal of the circuit connected to the tap between them. The thermistor and the resistor are chosen so that, at normal desired output voltage, the

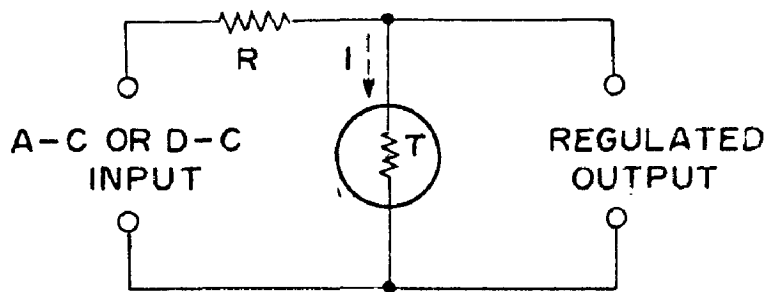


FIG.7.5 LIMITER (REGULATOR COMPRESSOR)

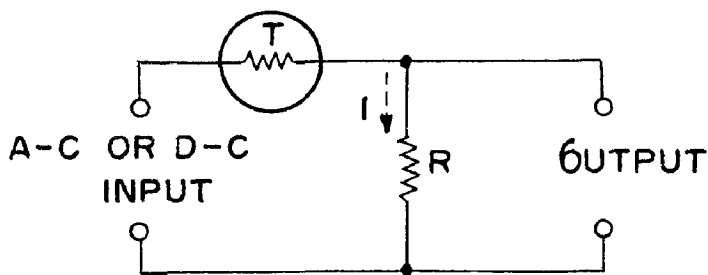


FIG.7.6 EXPANDER

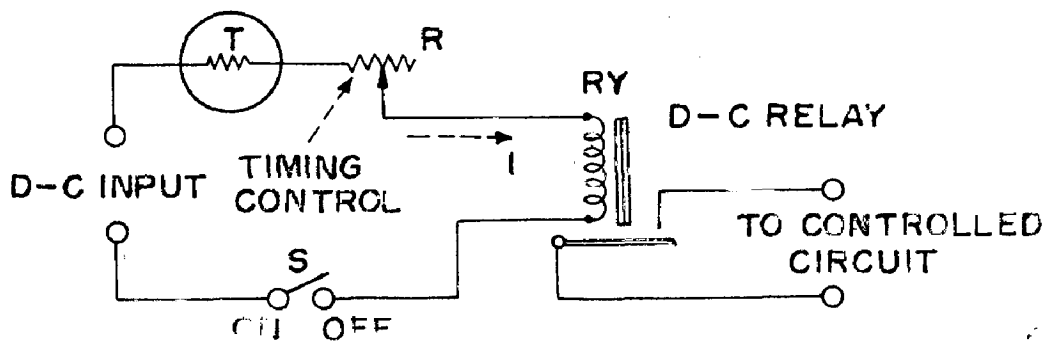


FIG.7.7 TIME - DELAY RELAY.

thermistor resistance is high. If the input voltage then increases, current  $I$  increases and lowers the resistance of the thermistor. This, in effect, causes the tap to move down the voltage divider, reducing the output voltage to its original level. In this way the output voltage is stabilized.

Several of these single stages may be cascaded for increased voltage regulation or signal compression.

#### 7.4 EXPANDER<sup>28</sup>

In Fig. 7.6 the thermistor is connected ahead of resistor  $R$ . The resistance of the latter is chosen low with respect to the thermistor resistance, so that the thermistor resistance will be the most effective in determining the current  $I$ . The output voltage is the drop  $IR$  across the resistor.

A small change in input voltage produces a large change in thermistor current, and this, in turn, produces a large change in the output voltage drop across resistor  $R$ . This circuit must not be misconstrued as an amplifier; it magnifies the ratio of change of input voltage but not the absolute voltage. The input voltage itself actually is reduced by the voltage-divider action of the resistor and thermistor in series.

Several of these single stages may be cascaded for increased expander action.

### 7.5 TIME-DELAY RELAY<sup>28</sup>

The time-delay effect evidenced by a thermistor after switching on its current may be utilized to delay the pickup of a relay. Fig. 7.7 shows the simple circuit. The thermistor is connected in series with the relay coil. A time interval follows closure of switch S before the current I reaches a level high enough to actuate the relay.

A large number of time-delay characteristics are available in commercial thermistors. The delay interval with a given thermistor may be controlled over a reasonable range by adjustment of rheostat R. When switch S is opened, the relay drops out immediately.

### 7.6 SEQUENTIAL SWITCHING CIRCUIT<sup>28</sup>

After switch S is closed in the circuit shown in Fig. 7.8, the flow of current into the various loads  $RL_2$  to  $RL_5$  starts at various times, depending upon the time-delay characteristics of the thermistors  $T_1$  to  $T_4$ . By suitable choice of the thermistors, this action may be made sequential. Thus  $RL_1$  is energized immediately, since there is no thermistor in this leg

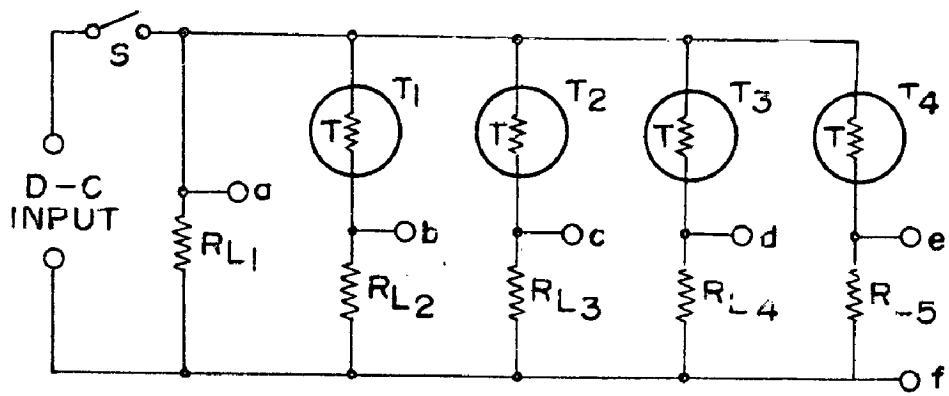


FIG.7.8 SEQUENTIAL SWITCHING CIRCUIT

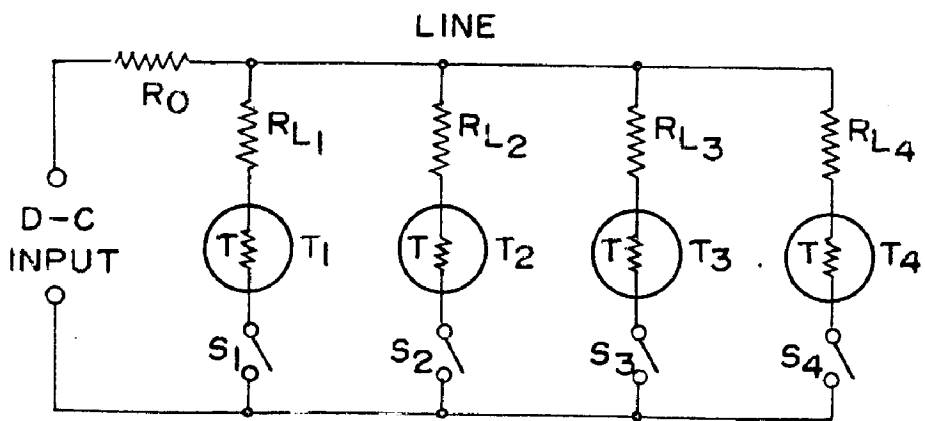


FIG.7.9 SELECTOR SWITCHING CIRCUIT

of the circuit, and if the time delay of each thermistor is longer than that of the preceding one, the other legs will operate in sequence ( $R_{L2}$ , then  $R_{L3}$ , next  $R_{L4}$ , and finally  $R_{L5}$ ).

If this circuit is employed for driving or triggering other circuits,  $RL_1$  to  $RL_5$  may be actual resistors, and output voltage taken from terminals a to e and the common terminal f.

#### 7.7 SELECTOR SWITCHING CIRCUIT<sup>28</sup>

In Fig. 7.9, when any one switch  $S_1$  to  $S_4$  is closed, operating current will flow through the corresponding load  $RL_1$  to  $RL_4$ , but only negligible current (zero current) will flow through any other load if its switch also is closed. The ON leg prevents any other legs in the circuit from being switched on, thus only one leg can be ON at a time.

The thermistors are chosen so that their resistances with respect to the accompanying load resistance will permit maximum current flow. When one thermistor is conducting heavily, the voltage drop across the common series resistance  $R_0$  reduces the voltage at all other thermistors to a level too low for any of the others to conduct heavily at the same time. Only when the switch in the conducting leg is opened will the starting condition be restored and another leg be

operable.

Although mechanical switches are shown here for simplicity in illustration, they might be electronic switches.

28

### 7.8 R-C OSCILLATOR STABILIZATION

The Wien-bridge oscillator circuit is widely used in low-distortion, R-C tuned audio and supersonic signal generators at frequencies upto one megacycle.

Fig. 7.10 shows a portion of the oscillator circuit. In this arrangement, positive feedback for oscillation is transmitted through  $C_3$  and the R-C tuning circuit  $R_1 - C_1 - R_2 - C_2$ . Negative feedback for stability is provided through  $C_3$ ,  $R_2$  and the thermistor. The cathode of tube  $V_1$  is tapped to the junction of  $R_2$  and the thermistor. The non-linear resistance of the thermistor automatically regulates the amount of negative feedback and stabilizes the cathode voltage, since very large changes in the thermistor current result in only small changes, in the cathode voltage- the voltage drop across the thermistor.



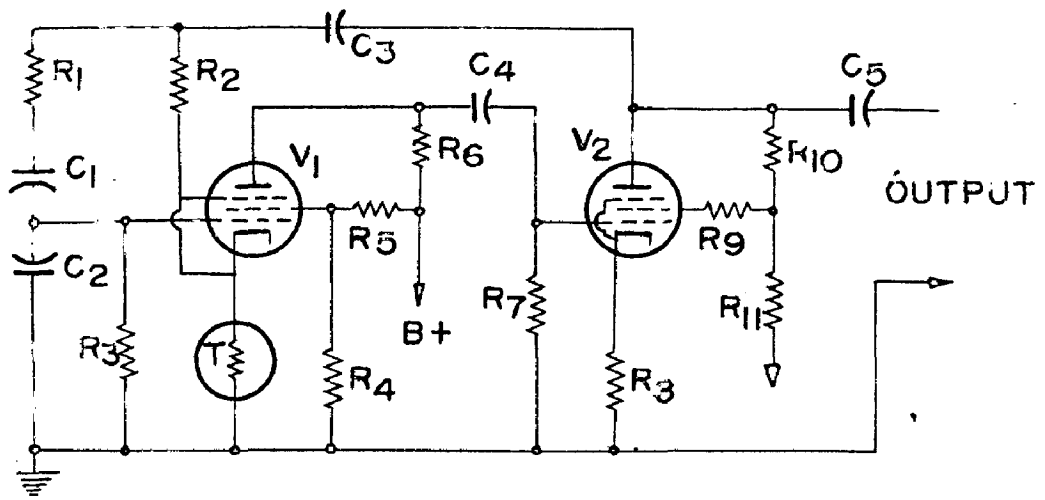


FIG.7-10 R-C OSCILLATOR STABILIZATION.

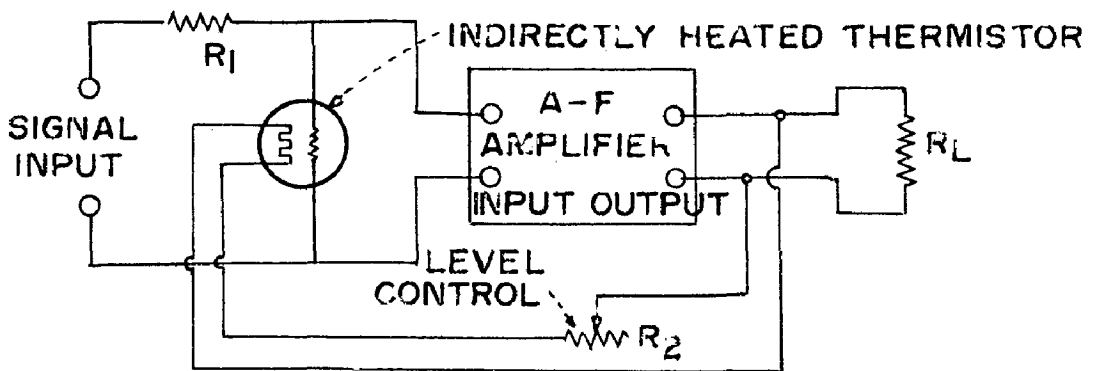


FIG.7-11 AUTOMATIC GAIN CONTROL

## 7.9 AUTOMATIC GAIN CONTROL<sup>28</sup>

Fig. 7.11 shows one arrangement for automatic gain control of an audio amplifier. Here an input-signal voltage divider is formed by  $R_1$  and a thermistor in series. This is an indirectly heated thermistor, and its heater element is connected to the low-impedance output of the amplifier.

When the output signal rises above a predetermined level in response to an input-signal increase, the heater element is energized. This heats the thermistor and lowers its resistance, causing the voltage divider to lower the signal presented to the amplifier input terminals. The signal level at which this action occurs is governed by the setting of rheostat  $R_2$ . When the input and output signal fall, the opposite action takes place. In this way, the gain is stabilized at a desired level.

The output characteristics of the amplifier must be such that connection of the heater element of the thermistor does not introduce distortion or serious output power loss.

## 7.10 COMPENSATION OF TRANSISTOR D-C BIAS<sup>30</sup>

To prevent thermal runaway and the possible destruction of the transistors, and to maintain the

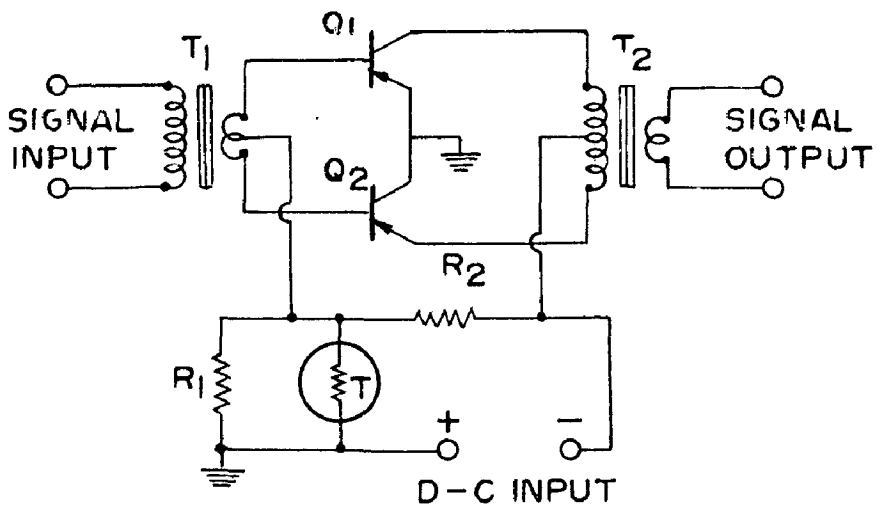


FIG.7.12 COMPENSATION OF TRANSISTOR BIAS.

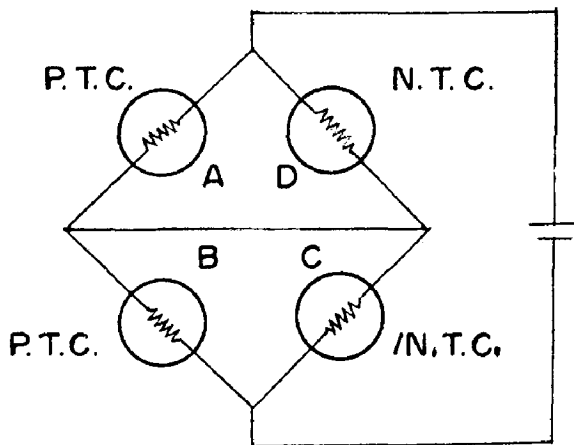


FIG.7.13 LOW FREQUENCY MULTIVIBRATOR INCORPORATING P.T.C. AND N.T.C. THERMISTORS.

proper operating point of a transistor circuit, the d.c. base bias must be stabilized. The transistor currents increase with temperature, so automatic temperature compensation is imperative.

Fig. 7.12 shows the use of a thermistor to stabilize a transistorized power amplifier against collector - current variations due to temperature. Here the thermistor is shunted across the lower leg  $R_1$  of the base-bias voltage divider  $R_1 R_2$ . The thermistor is mounted close to the transistors so as to experience the same temperature environment. As the temperature increases, the resistance of the thermistor decreases. This lowers the total resistance of the lower leg of the voltage divider and reduces the d-c base voltage, lowering the collector current to its initial, safe low value.

#### 7.11 LOW FREQUENCY MULTI VIBRATOR

The instability of series-connected P.T.C thermistors has been utilised in the circuit shown in Fig. 7.13 to generate relaxation oscillations with a period ranging from 30 to 40 S. The cold resistances of the N.T.C. thermistors are nominally equal to and high, whereas those of the P.T.C. thermistors are nominally equal and low. When the voltage is

first applied the current will flow almost entirely through arms A and B. Suppose A starts to heat first, as already described this will be regenerative. Eventually the voltage across A will rise almost to the applied voltage. This will cause the N.T.C. thermistor,  $D_1$  to be heated and its resistance to fall, again a regenerative process, which progressively diminishes the current in A and increases that in P.T.C. thermistor B. The resistance of D eventually becomes very small and the majority of the applied voltage then appears across B. With a proper choice of cold values of resistance A will now have returned to its original low resistance value so that as B's resistance approaches its maximum value the current will shift from the path through B and D to that through A and C and the cycle then repeats.

#### 7.12 MOTOR PROTECTION AGAINST TEMPERATURE VARIATION

Small ventilator-cooled electric motors often operate under conditions in which dust accumulates in the ventilator over a long period and often leads to overheating and failure. A safeguard can be provided by connecting a P.T.C. thermistor in series with the motor, mounted in thermal contact with its winding.

The load line of the motor normally intersects the thermistor static characteristic in its linear region. If overheating occurs, however, the peak current of the thermistor characteristic is depressed and design is such that this peak falls below the load line so that switching occurs to an intersection at low current and high voltage, before the motor is permanently damaged.

#### 7.13. MOTOR PROTECTION AGAINST OVERLOAD

If an electric motor is overloaded or braked while running at its rated voltage then the winding may again be damaged due to Joule heating. Decreasing the speed increases the armature current and this is equivalent to decreasing the effective resistance. This can be prevented from causing permanent damage by connecting a P.T.C. thermistor in series with motor, so that the normal operating position is with an intersection of motor load line with a linear part of the characteristic, and overload, represented by a rotation of the load line clockwise about the applied voltage as a pivot, trips to a high voltage, low current operating point in the nonlinear region.

# PART II

## CHAPTER 8

### TRANSISTOR AS THERMORESISTIVE ELEMENT

## TRANSISTOR AS A THERMO-RESISTIVE ELEMENT

### 8.1 INTRODUCTION

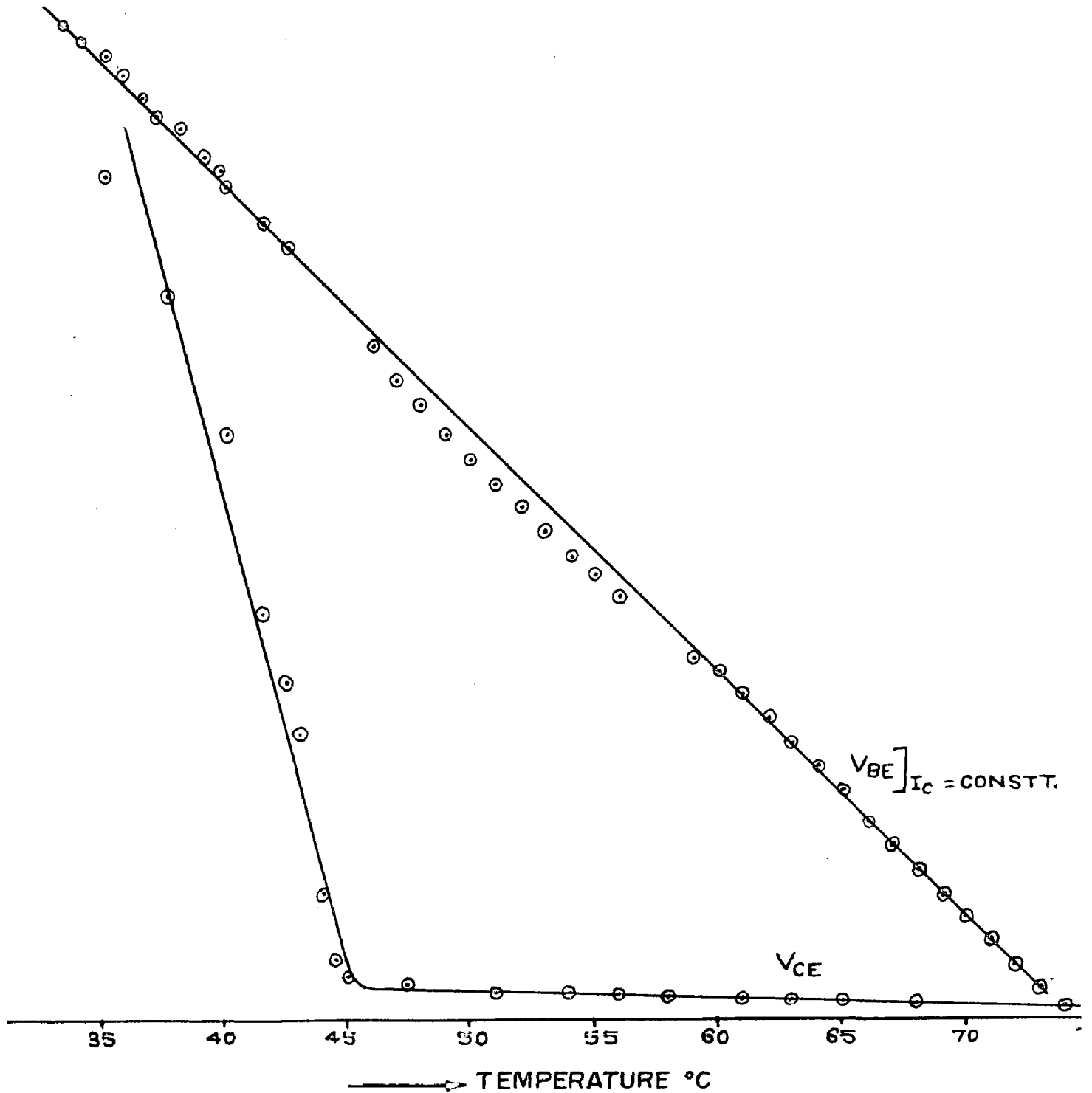
Characteristics of the transistor are greatly affected by thermal variations. To some extent each of the parameters of any transistor equivalent circuit exhibits temperature sensitivity. Transistor resistance decreases with the increase in temperature and vice-versa in the direction of current flow. This sensitivity of transistor resistance to heat changes the base current, collector current and the current gain  $h_{FE}$ .

It has long been regarded as a drawback that transistors are temperature sensitive in their normal use, however, this adverse temperature instability can be used to advantage in the transistor electronic thermometer.

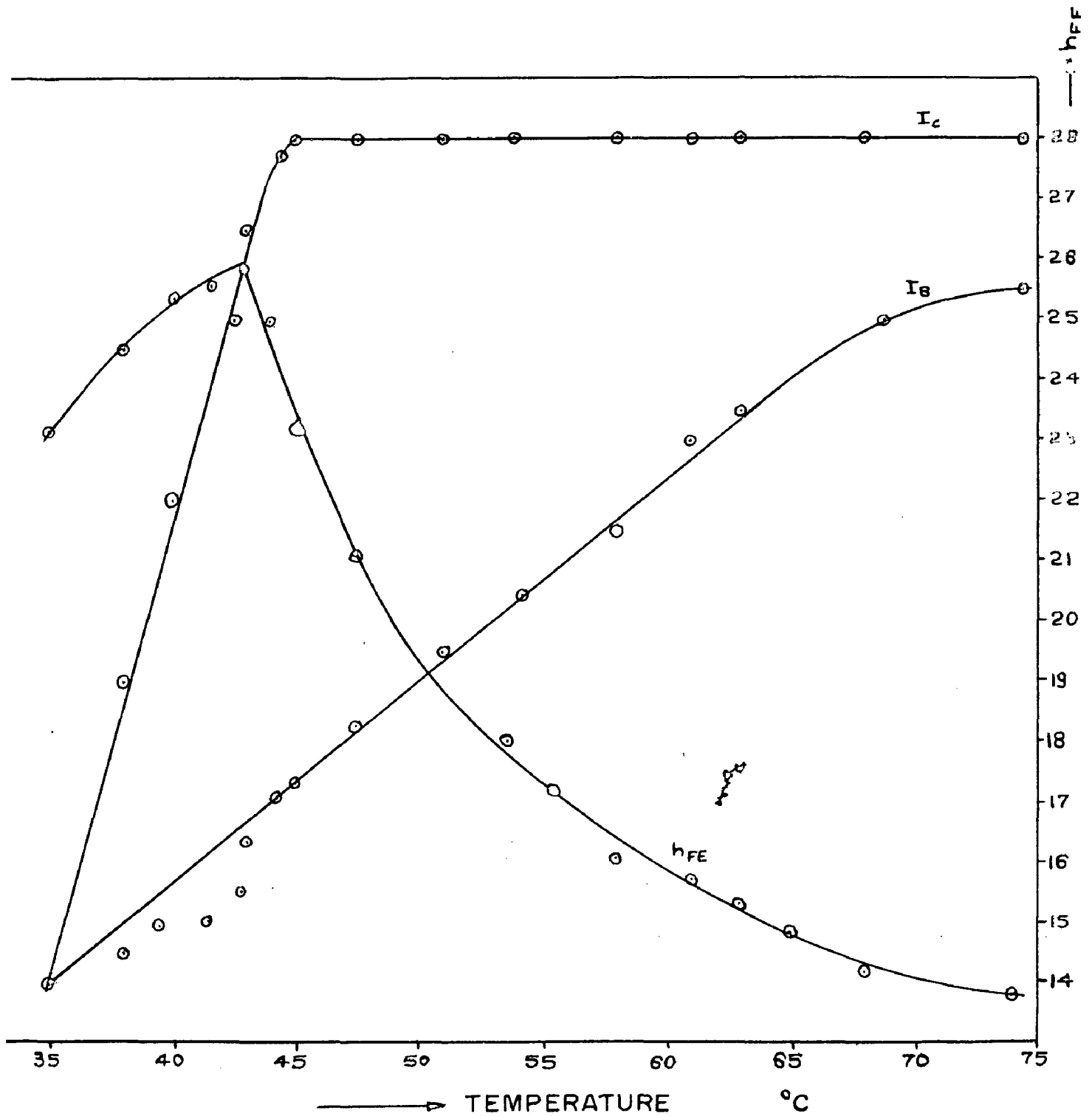
### 8.2 VARIATION OF PARAMETER WITH TEMPERATURE

To some extent each of the parameters of any transistor equivalent circuit exhibits temperature sensitivity. When the internal or junction temperature varies over a considerable range, definite steps must be taken to compensate a circuit for changes in parameter values.





B-1 TEMPERATURE VERSUS (I) BASE TO EMITTER VOLTAGE AND (II) COLLECTOR TO EMITTER VOLTAGE OF TRANSISTOR 2N 404



3.2 TEMPERATURE VERSUS (I) BASE CURRENT, (II) COLLECTOR CURRENT AND (III) CURRENT GAIN  $h_{FE}$  OF TRANSISTOR 2N404.

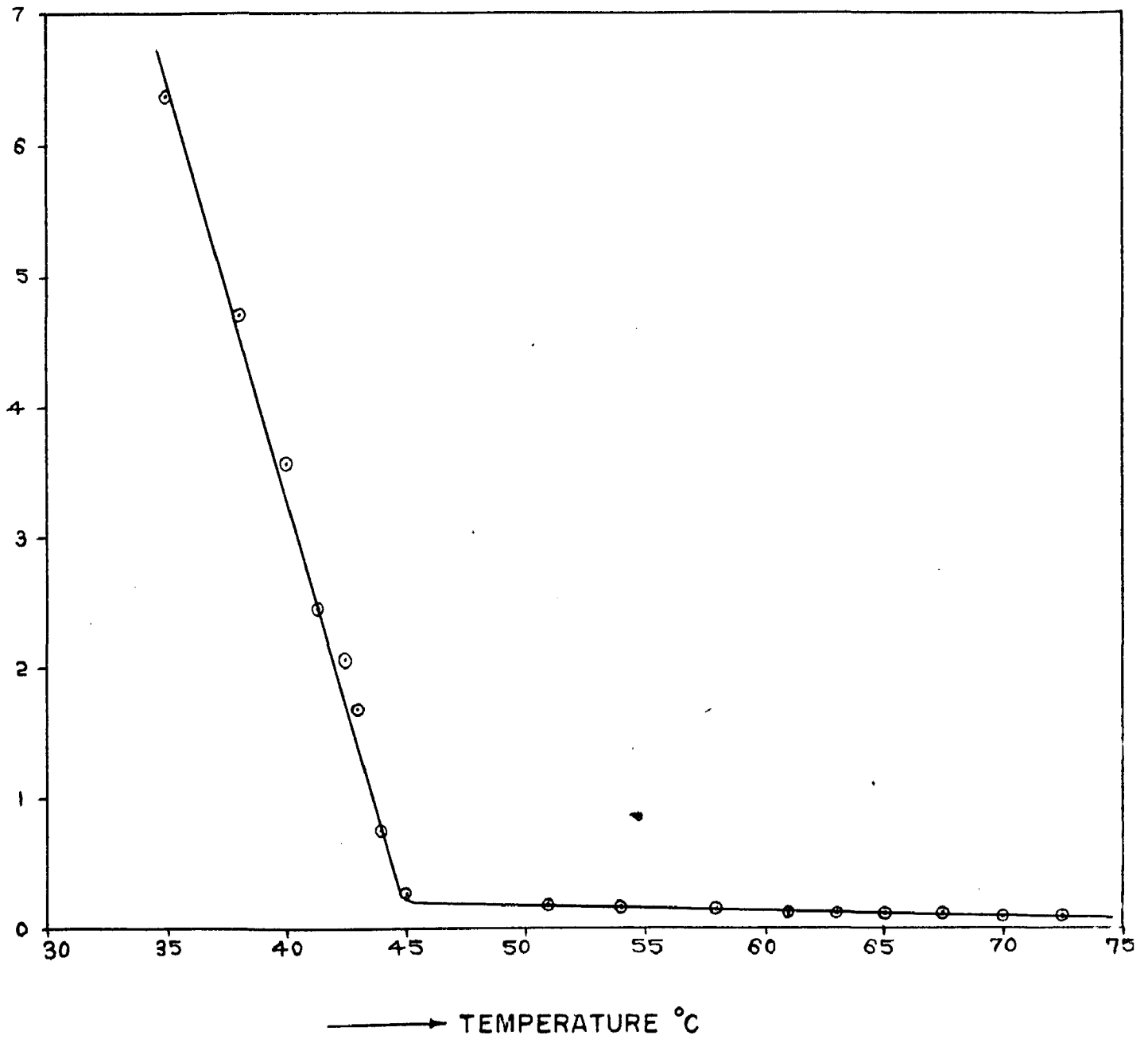


FIG. 8.3 TEMPERATURE VERSUS EMITTER TO COLLECTOR RESISTANCE OF THE TRANSISTOR 2N404 .

A number of curves showing the variation of base to emitter voltage, collector to emitter voltage, base current, collector current, current gain and emitter to collector resistance with temperature are shown in Fig. 8.1, 8.2 and 8.3.

Fig. 8.1 shows that emitter to base voltage decreases linearly with the increase of temperature when the collector current is kept constant. This is true for germanium as well as for silicon transistors in general. The change for germanium transistor is about  $-1.8$  mv per  $^{\circ}\text{C}$ , while for silicon is  $-1.7$  mv per  $^{\circ}\text{C}$ . The change is  $1.01667$  mv per degree centigrade for a germanium 2N404 transistor,  $V_{CE}$  decrease linearly with temperature rise upto about  $45^{\circ}\text{C}$  and then the degree of decrease becomes very low as the transistor goes to near saturation.

Fig. 8.2 shows the variation of  $I_C$  and  $I_B$  with temperature. Both increase with temperature  $I_C$  becomes constant when the transistor saturates. Similarly  $R_{CE}$  decreases with temperature rise until  $I_C$  becomes constant, when the change in  $R_{CE}$  is negligible as shown in Fig. 8.3.

### 8.3 TRANSISTOR ELECTRONIC THERMOMETER<sup>31,32</sup>

#### 8.3.1 Principle

Variation of transistor  $V_{BE}$  characteristic with temperature is linear over a wide range of temperature.

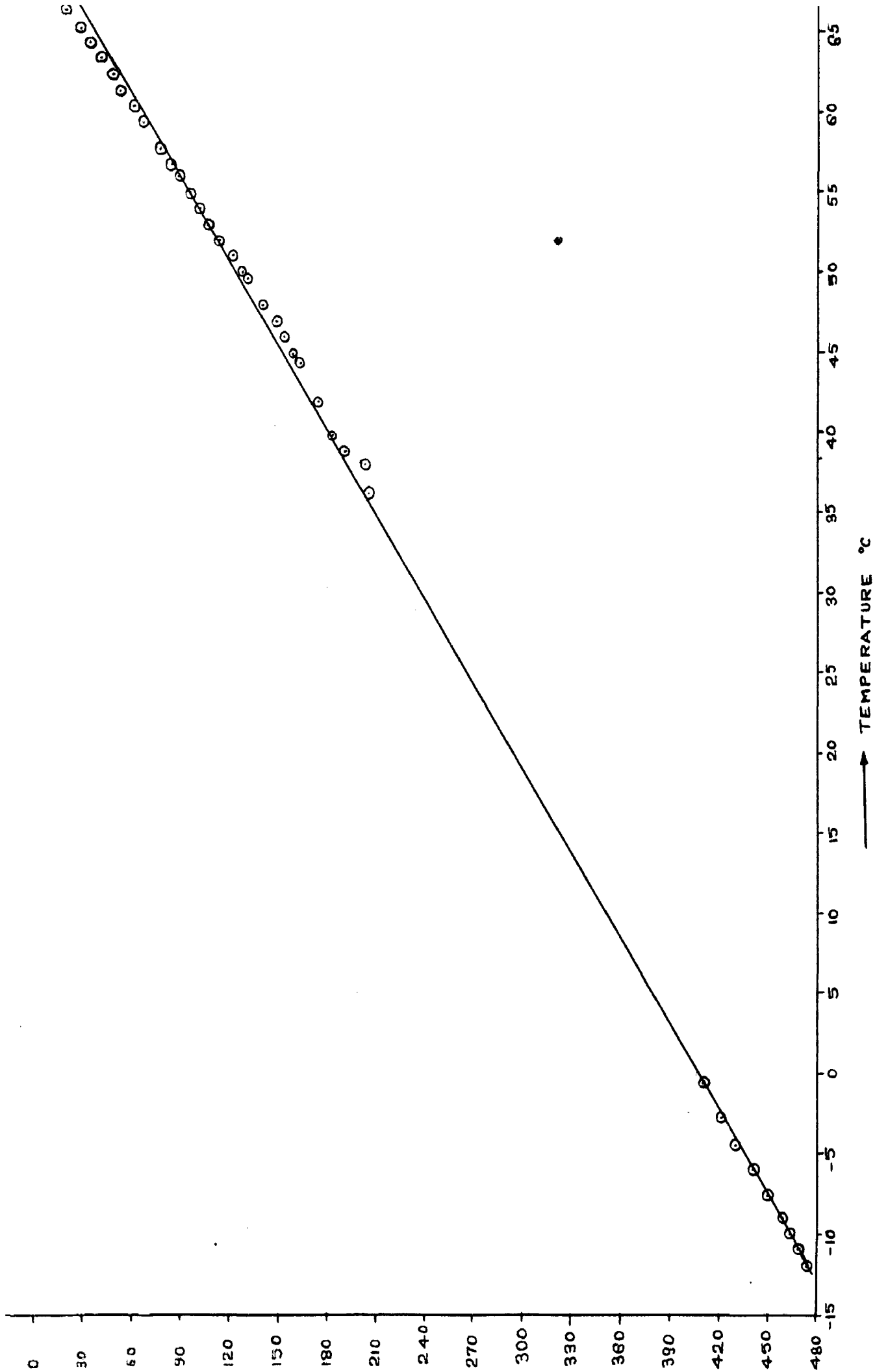


FIG. 8-5 TEMPERATURE VS. RESISTANCE R CHARACTERISTIC OF TRANSISTOR 2N404.

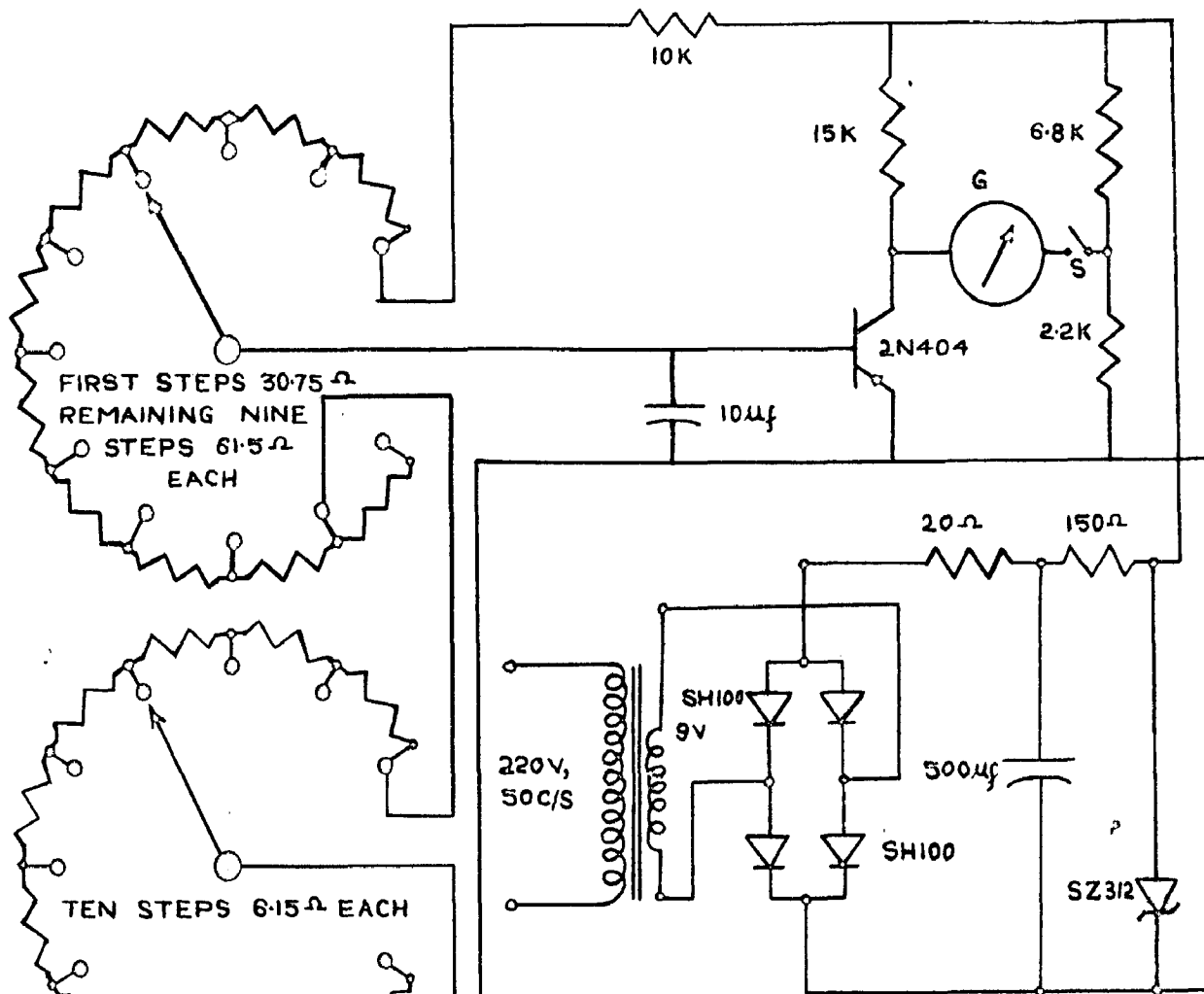


FIG. 8.4 COMPLETE CIRCUIT DIAGRAM OF TRANSISTOR ELECTRONIC THERMOMETER.

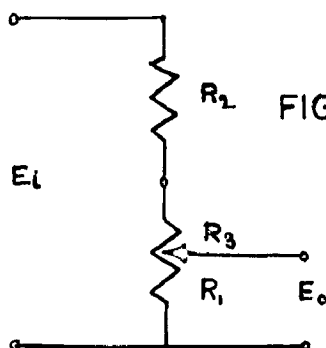
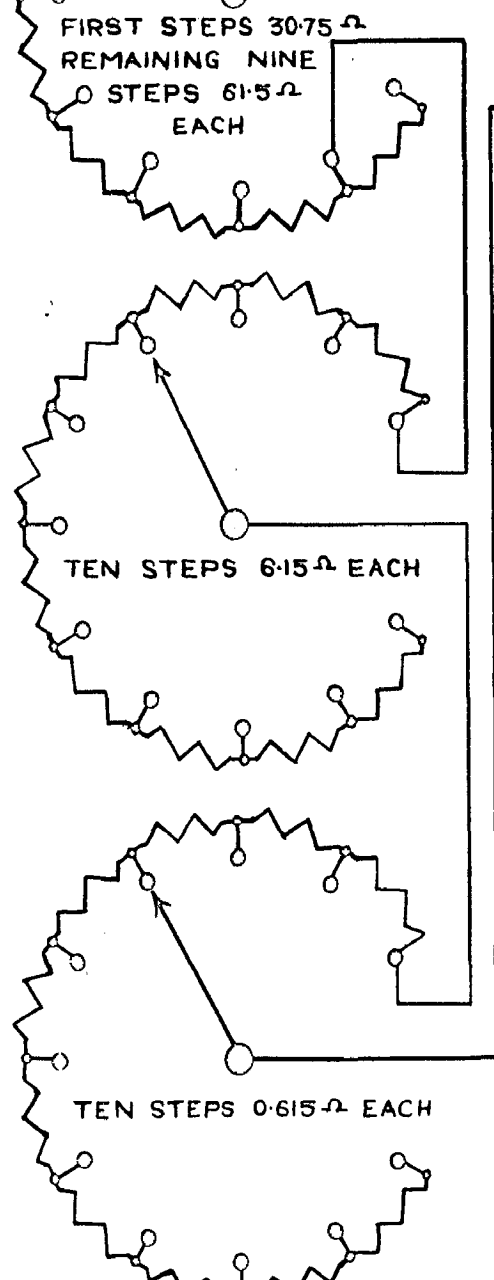


FIG. 8.6 SIMPLE POTENTIAL DIVIDER CIRCUIT.

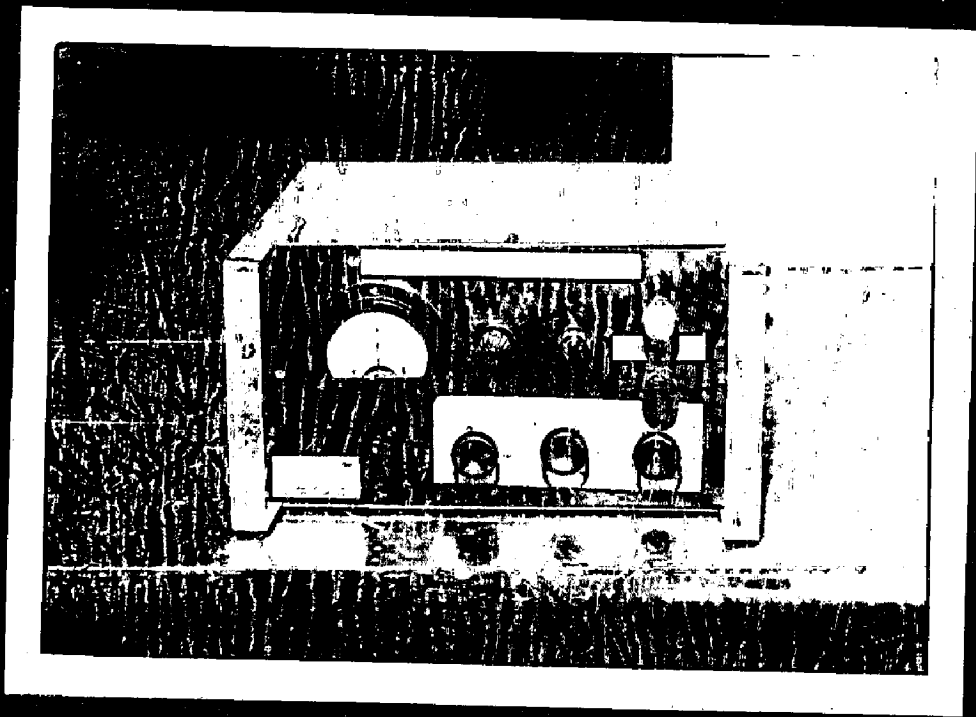
This variation can be made to control the deflection of a meter which in turn can be calibrated directly in degrees.

To maintain constant collector current, the base bias must be increased as the temperature decreases and conversely. The relationship between base bias and temperature is linear over a wide range of temperatures and if the base for a constant collector current is calibrated in degrees, it becomes a temperature scale. For a typical germanium transistor (2N404), variations in bias is about 1.01667 mv per degree centigrade. The transistor thermometer can accurately measure the small base voltage increments corresponding to temperature changes for calibration into degrees of temperature.

A highly sensitive and accurate transistor thermometer having high speed response is shown in Fig. 8.4. This circuit can be used for house hold and outdoor applications. The temperature sensing element is a 2N404 transistor.

Referring to Fig. 8.4, switch  $S_1$  applies power to the transistor. This is a 6 V d.c. supply stabilized by means of a zener. Circuit constants will change for other values of voltage.

A bias voltage in steps representing degree C is provided for the d.c. input to the transistor, bias voltage increments to maintain a given collector voltage





are about 1.01667 mv per deg. C. The potential divider includes  $R_1$ ,  $R_2$  and  $R_3$ .

Potentiometer  $R_1$  is calibrated for scale divided into ten steps each containing 61.5 ohms and representing  $10^\circ\text{C}$ . First step of  $R_1$  has 30.75 ohms and represents  $5^\circ\text{C}$ . Similarly  $R_2$  has 10 steps each 6.15 ohms and calibrated for  $1^\circ\text{C}$  and  $R_3$  having 10 steps each 0.615 ohm and representing  $0.1^\circ\text{C}$ .

With a change of 6.15 ohms in the bias resistance, the bias voltage varies by 1.01667 mv.

### 8.3.2 Design Considerations

As has been discussed above  $1^\circ\text{C}$  is denoted by a 6.15 ohms variation in the bias resistance. Therefore temperature can be calibrated in terms of resistances. Temperature in decade form i.e. steps having equivalent temperature of  $0.1^\circ\text{C}$ ,  $1^\circ\text{C}$  and  $10^\circ\text{C}$ , have been obtained with the help of  $R_1$ ,  $R_2$  and  $R_3$ .  $R_4$  is selected so that a collector voltage of about 1 V is obtained when the transistor temperature is  $30^\circ\text{C}$ .  $R_6$  and  $R_5$  are so chosen that the potential drop across  $R_6$  is about 1 V. The 0-50 microampere galvanometer reads zero when point A and B are at the same potential. If the temperature of the transistor changes, the base biasing decades and  $R_3$  are adjusted for zero reading of the meter. The reading of the base bias then gives the temperature of the transistor

probe and the indication of the bias is calibrated to read temperature.

### 8.3.3 Calibration

Tests made with the thermostat producing temperature below 0°C and above ambient temperature indicated linearity of base voltage versus temperature. It had also been noted that bias resistance versus temperature curve is also linear if the resistances are connected in potential divider form so as to keep the total resistance across supply constant. The curve is shown in Fig. 8.5.

The theory that the linear variation of resistance corresponds to linear variation of output voltage, in a potential divider arrangement, can be explained by referring to Fig. 8.6 .

$$\text{Current } I = \frac{V_{in}}{R_1 + R_2 + R_3} = \text{constant since } R_1 + R_2 + R_3 \text{ is constant}$$

$$\text{and } V_{out} = I R_1 \propto R_1$$

Therefore if  $R_1$  varies linearly,  $V_{out}$ , too, varies linearly.

### 8.3.4 Performance Specifications

- a. Accuracy of the instrument is 1%
- b. Least count is 0.1°C
- c. The range of the instrument is -15°C to 70°C
- d. Repeatability of the instrument is 70%.

e. Base to emitter resistance of the transistor is inversely proportional to temperature from  $-15^{\circ}\text{C}$  to  $80^{\circ}\text{C}$ .

f. Sensitivity of the instrument is 6.15 ohms per degree centigrade variation of the base to emitter bias resistance of the transistor.

### 8.3.5 Discussion

Transistor electronic thermometer can be used for indoor and outdoor applications. Other thermometers which are also portable are the mercury thermometer and the thermistor electronic thermometer. The disadvantages in case of mercury thermometer over transistor electronic thermometers are the following viz., (i) it can not be used for quick variations of temperature, (ii) it takes more time to reach the mercury column to the required temperature, and (iii) the accuracy is less. Thermistor electronic thermometer is more sensitive to temperature than the transistor electronic thermometer, but the change of resistance of thermistor with temperature is non linear.

Other devices of temperature measurement are platinum resistance thermometer and the thermocouple. Although both the devices can measure temperature over a large span (upto  $1000^{\circ}\text{C}$  in case of platinum resistance thermometer and upto  $2000^{\circ}\text{C}$  in case of thermocouple) with greater accuracy, these are not portable and are very costly and the maintenance is difficult.

### 8.3.6 Null and Deflection Methods<sup>33</sup>

A useful classification with regard to the mode of operation of instruments separates devices by their operation of a null or a deflection principle. In a deflection type device the measured quantity produces some physical effect that engenders a similar but opposing effect in some part of the instrument. The opposite effect is closely related to some variable (usually a mechanical displacement or deflection) that can be directly observed by some human sense. The opposite effect increases until a balance is achieved, at which point the 'deflection' is measured and the value of the measured quantity inferred from this.

In contrast to the deflection type device, a null-type device attempts to maintain deflection at zero by suitable application of an effect opposing that generated by the measured quantity. Necessary to such an operation are a detector of unbalance and a means (manual or automatic) of restoring balance. Since deflection is kept at zero (ideally), determination of numerical values requires accurate knowledge of the magnitude of the opposing effect.

Upon comparing the null and deflection methods of measurement, we note that, in the deflection instrument,

accuracy depends on the calibration of the scale whereas in the null instrument it depends on the accuracy of the standard scale to which the measurand is compared. The accuracy attainable by the null method is of a higher level than that of the deflection method. Another advantage of null methods is the fact that, since the measured quantity is balanced out, the detector of unbalance can be made very sensitive because it need cover only a small range around zero. Also the detector need not be calibrated since it must detect only the presence and direction of unbalance and not the amount. On the other hand, a deflection instrument must be larger, more rugged, and thus less sensitive if it is to measure large magnitudes.

The disadvantages of null methods appear mainly in dynamic measurements. By the use of automatic balancing devices (such as the instrument servomechanism) the speed of null methods may be improved considerably, and the instruments of this type are of great importance.

Keeping in view the above mentioned advantages and disadvantages of the two types of measurements, null method was considered better than deflection method for transistor electronic thermometer.

# CONCLUSION

CONCLUSION

- (a) Thermistors as temperature sensors for precise measurement of temperature are best fitted for measurement of very small temperature differences of the order of  $0.01^{\circ}\text{C}$  over a limited temperature range (typical order being  $20^{\circ}\text{C}$ ) This is on account of the high temperature coefficient and the fact that substantial linearity of the R-T characteristic can be achieved only over a range of about  $40^{\circ}\text{C}$ .
- (b) They are most popular as sensors of such quantities as temperature, low pressure, thermal conductivity, liquid and gas flow, for control of these quantities and such other applications where linearity of the sensor characteristic is not a stringent requirement.
- (c) It has been established in the present work that the transistors (preferably germanium transistors) have a great scope of being used as temperature sensors in "Electronic thermometers" which have the advantages of greater accuracy, speed of response ruggedness, and linearity over the conventional thermometers. In particular electronic thermometer using a transistor as sensor has advantages of accurate measurement of low temperatures.

B I B L I O G R A P H Y

1. SMITH, O.J.M. "Thermistors : Part I ; Static characteristics and Part II ; Dynamic characteristics", Rev.Scient.Instrum. Vol-21, p 344, 351, 1950.
2. SCARR, R.W.A. and SETTERINGTON, R.A. "Thermistors, their Theory, Manufacture and Application", Proc. Instn.Elect. Engrs., Vol. 107B, p 395, 408, 1960.
3. BURGESS, R.E. "The a.c. admittance of Temperature-dependent circuit elements", Proc.Phys.soc., Vol 68, p 766, 1955.
4. BENNETT, FRANK K, "Which type of thermistor?" Prod. Engg. Vol 25, No. 3 , p 182 March 1954.
5. BELFIELD, W. and JOHNSON, R.W., " A Thermistor probe" J. Scient. Instrum. Vol. 34 , p 209, May 1957.
6. BECK, A., " The stability of thermistors" J. Scient.Instrum. Vol 33, p 16, 1956.
7. BECKER, J.A., GREEN, G.B. and PEARSON, C.L., "Properties and uses of thermistors - thermally-sensitive resistors", Trans. An.Inst. elect.Engrs., Vol 65, p 711, 1946.
8. RASMUSSEN, R.A. " Application of thermistors to measurements in moving fluids", Rev. Scient. Instrum., Vol 33, p.38, 1962.
9. VEPREK " A thermistor flow meter " J. Scient.Instrum.Vol.40, p-66, 1963.
10. BRADLEY, R.S., " A Thermistor McLeod Gauge for a pressure range 1 -  $10^{-7}$  mm of mercury", J.Scient.Instrum., Vol 31, p129, 1954.
11. VARICK, M. and SAFTIC, B. " Principle of a Semiconductor manometer in the pressure range of 1 to  $10^{-6}$  mm Hg" Rev. Scient. Instrum., Vol. 30, p 391, 1959.
12. FLANICK, A.P. and AINSWORTH, J.E. " Thermistor pressure gauge design", Rev. Scient. Instrum., Vol 32, p. 356, 1961.
13. BERRY, C.J. " Low pressure measurement with thermistors", J. Scient.Instrum., Vol 44, p-83, 1967.
14. ZEMANY, P.D. "Molecular weight determinations by effusiometry" Rev. Scient. Instrum., Vol 23, p 176, 1952.



15. PITTS, E. and PRIESTLEY, P.T. " Constant sensitivity bridge for thermistor thermometers," J.Scient Instrum., Vol 39, p 75, 1962.
16. BEAKLEY W.R. " The design of thermistor thermometers with linear calibration," J.Scient.Instrum., Vol. 28, p 176, 1951.
17. GREENHILL, E.B. and WHITEHEAD, J.R. " An apparatus for measuring small temperature changes in liquids" J.Scient.Instrum. Vol 26, p 92 , 1949.
18. NORDON, P and BAINBRIDGE, N.W. " The use of unmatched thermistors for the measurement of temperature difference under varying ambient conditions", J.Scient.Instrum.Vol 39, p 399, 1962.
19. GODIN, M.C. " A Method of equalising thermistors" J. Scient. Instrum. Vol. 39, p 241, 1962.
20. GODIN, M.C. " Simple bridge for the direct measurement of temperature differences", J.Scient.Instrum., Vol. 38, p.330, 1961.
21. GODIN, M.C. " A modified thermistor bridge", J.Scient Instrum., Vol 40, p. 500, 1963.
22. BENNETT, J.K. " How to use thermistors" Prod.Engg., Vol-25, p.200, April 1954.
23. LYTEL, ALLEN " Handbook of Transistor circuits", W.Foulsham & Co. Ltd, England.
24. MAHER , "The multivibrator bridge for temperature measurement" J. Scient.Instrum. Vol 44, p 531, July 1967.
25. LÖVBARG , " A linear temperature to frequency convertor" J. Scient.Instrum. Vol 42, p.611, 1965.
26. KARL HOLM, " Thermistor thermometer based on an astable multivibrator" Electronic Engg. Vol 40, p. 700 Dec, 1968.
27. CANDY, C.J.N., " The specification of the properties of the thermistor as a circuit element in very low frequency systems" Proc. Instn. elect. Engrs., Vol 103 B, p 398, 1955.
28. TURNER " Semiconductor devices" HOLT, RINEHART and WINSTON, Inc. New york, 1961.
29. TAYLOR, A.H. " Thermistor thermostats" Electronics, Vol 23 p 154 , 1950.
30. WHEELER, A.J. " Thermistors Compensate Transistor amplifiers" Electronics, Vol 30, p 169, 1957.
31. BARTON, L.E. " Measuring Temperature with diodes and transistors Electronics, Vol. 35, p. 38, August 1962.

32. BLATT, S. Green " Transistor becomes sensor in temperature regulator", Electronics, Vol 37, 28, p 65, 1964.
33. DOEBELIN ERNEST O, " Measurement systems: application and design" McGraw Hill Book Company, 1966.
34. SCOTT, " Linearising of output of bridge network" J. Scient.Instrum., Vol 41, p 458, July, 1964.
35. DEETER, E.L. "Null temperature bridge, " Electronics, Vol 21, p 180 , 1948.
36. HERINGTON, C.F.G. and HANDLEY, R. " The use of thermistors for recording small temperature differences", J. Scient. Instrum. Vol 25 , p 434 , 1948.
37. Green Hill, E.B. and WHITEHEAD, J.R. " An apparatus for measuring small temperature changes in liquids" J. Scient. Instrum., Vol -26, p. 92, 1949.
38. FARHI, S. and GROVES, S. Trans. Amer.Inst.Elect.Engrs. No. 55, p 246, 1961.
39. BOWMAN, M.J. " On the linearity of a thermistor thermometer" Radio Electron Engg., Vol 39, p 209, 1970.
40. HARRUFF, R.W. " Linear temperature scale from thermistor sensors" Instrum. Tech., Vol 14 , No.6 , Jan-July 1966.
41. BRYCE, C.H., HOLE , V.H.R. " Measurement and Control by Thermistor" Industrial Electronics, Vol 5, p 294, July 1967.