SEQUENCING AND SCHEDULING OF WATER RESOURCES SYSTEMS

A DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING

By

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SEPTEMBER, 1982.

CERTIFICATE

Certified that the dissertation entitled "SEQUENCING AND SCHEDULING OF WATER RESOURCES SYSTEMS" which is being submitted by Mr. D. SANKARJI in partial fulfilment of the requirements for the award of the Degree of Master of Engineering in ELECTRICAL ENGINEERING (SYSTEMS ENGINEERING & OPERATION RESEARCH) of the UNIVERSITY OF ROORKEE, ROORKEE (U.P.) is the record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

It is further certified that he has worked for a period of eight months from January 1982 to August 1982 for preparing this dissertation at this University.

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ACKNOWLEDGEMENT

I would like to express my sincere and profound gratitude to my supervisor, Mr. S. Kumar, for the inspiration and valuable guidance throughout the course of this work. I am also thankful to the Head of the Department of Electrical Engineering for extending all facilities of the department for carrying out the work.

Thanks are also extended to Engineer-in-Chief's Branch, Army Head Quarters, New Delhi, who sponsored me to do Master of Engineering Course in this University.

I am also thankful to all my friends who provided inspiration, when most needed.

D. JANKARJI

S Y N O P S I S

This work opens with the definition of the field of water resources systems, emphasising its physical and natural components. The introduction explains the complexity of most problems in this field. It therefore needs a broad approach to their analysis and solution which is offered by systems engineering. It is briefly described in the context of design and operation of water resources systems. A review on technical papers on water resources planning is presented in Chapter II.

Among various problems pertaining to development of water resources engineering, is the sequencing and scheduling problem, which is the topic of this thesis work. Chapter III deals with the deterministic case of the problem when the demand is assumed to increase only after every time interval, the interval being one year. It is assumed here that the optimal sizing and locations of the projects have already been decided by the decision makers. The scheduling problem is concerned with optimal selection of available projects, and to decide when to construct each of selected projects with minimum overall expenditure. Sequencing is a special case of scheduling problem where only sequencing of construction is decided. 0-1 integer programming formulation is used in this respec and implicit enumeration algorithm is used to solve a scheduling problem of 4 projects. Later the problem with stochastic water demand is solved using same implicit enumeration technique in Chapter IV.

One of the main components of the water supply system is the reservoir. The release policies of the reservoir - the amount of water to be released in each month or season - has to be decided which results in optimum operational cost. A system of reservoir operated along with an aquifer is taken for analysis and to evaluate the release policies in Chapter V. Howard's policy iteration technique is used to solve the problem and to arrive at the optimal release policies.

The computer program for implicit enumeration technique used for solving 0-1 integer programming problem is given in Appendix - A. A subroutine developed by the author, simplifies the work necessary to prepare the data for the main program, is given in Appendix B.

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1. INTRODUCTION

1 Details of Water Resources System

Man's quest for a better use of available water is as old mankind itself. Water is a renewable resource that follows in ture, a path called the hydrological cycle. Man has always tried tap the hydrological cycle at one or more points in order to ilise the water for a variety of purposes. His attempts to take vantage of certain aspects of hydrological cycle gave rise to ter resources projects of a wide range of sophistication. In eir simplest form, such projects are nothing more than primitive cilities for the storage of rainwater in cisterns or digging of lls for exploitation of ground water. In their most sophisticat rm, water resources projects involve a complex of multipurpose ructures that regulate stream flows, recharge ground water uifers, generate hydroelectric power, protection from floods etc e multi-purpose projects are defined as "engineering works which rve more than one principal purpose, and where the value of nefits accruing from each such purpose is commensurate with the rtion of total cost allotted to it -. In between these two tremes - primitive water storage facilities and intricate ltiple-purpose projects - there are many possible ways in ich water resources may be developed or utilised. Each of these ys represents a solution to a programming problem.

It is easy therefore to realise that the increasing complex water resources systems gives rise to a host of problems onnected with the development, control, allocation, treatment, itilization, and reuse of water. The analysis and solution of hese problems from the field of 'water resources engineering'.

The main problem in water resources engineering and perhaps the problem in its most general formulation, arises from the fact that water is often available at times, in location and of a quality different from those which define the demand for it. In addition to this, the amount of water available may be at variance with those required for certain economic activities. Considering this situation, which can be characterised by the maldistribution of water in time and in space coupled with its often undesired quality, these major questions arise :

1. What system has to be built in order to minimise this discrepancy existing between the natural supply of water (in time, space and quality) and the demand for it ?

2. To what extent should the water resources be developed and how extensive should be the region serviced by the system ?

3. Once it is built, how should the system be operated so as to achieve a given set of objectives in the best possible way ?

The order in which these questions are developed here correspond to the chronological sequence in which the answers wer attempted for planning, development and operation of water resources system.

1.2 Physical Components of Water Resources System :

Comprehensive plans for the construction and operation of dams, canals, and so on have been completed and are being implemented in a number of countries. These plans do not necessarily cover all aspects of water utilization; some aspects are and will remain of no interest to some countries. Plans in some countries may be formulated mainly for hydropower generation and agriculture requirements whereas the importance may be for flood control or drinking water in other countires. And it may be navigations projects to improve the transportation network in some other places. In the arid regions the need for water for agricultural purposes has always exceeded the rainfall supply. Ground water supplies in those regions might be rapidly depleted. The 'shortage' of present water resources will force the planning of complex schemes to transfer excess water to the areas of water shortage.

A system in general is an arbitrary isolated combination of elements (abstract and arbitrary subdivisions) of the real world. Usually the elements correspond to the physical components of the real world, as illustrated in Fig. 1.2.1 for a river basin - components such as rivers, dams, sources of water and users of water. In many countries the development of water resources began with a major attention to river basin development.

There are some countries and regions in some countries where the natural sources of stream flow is very limited compared to the

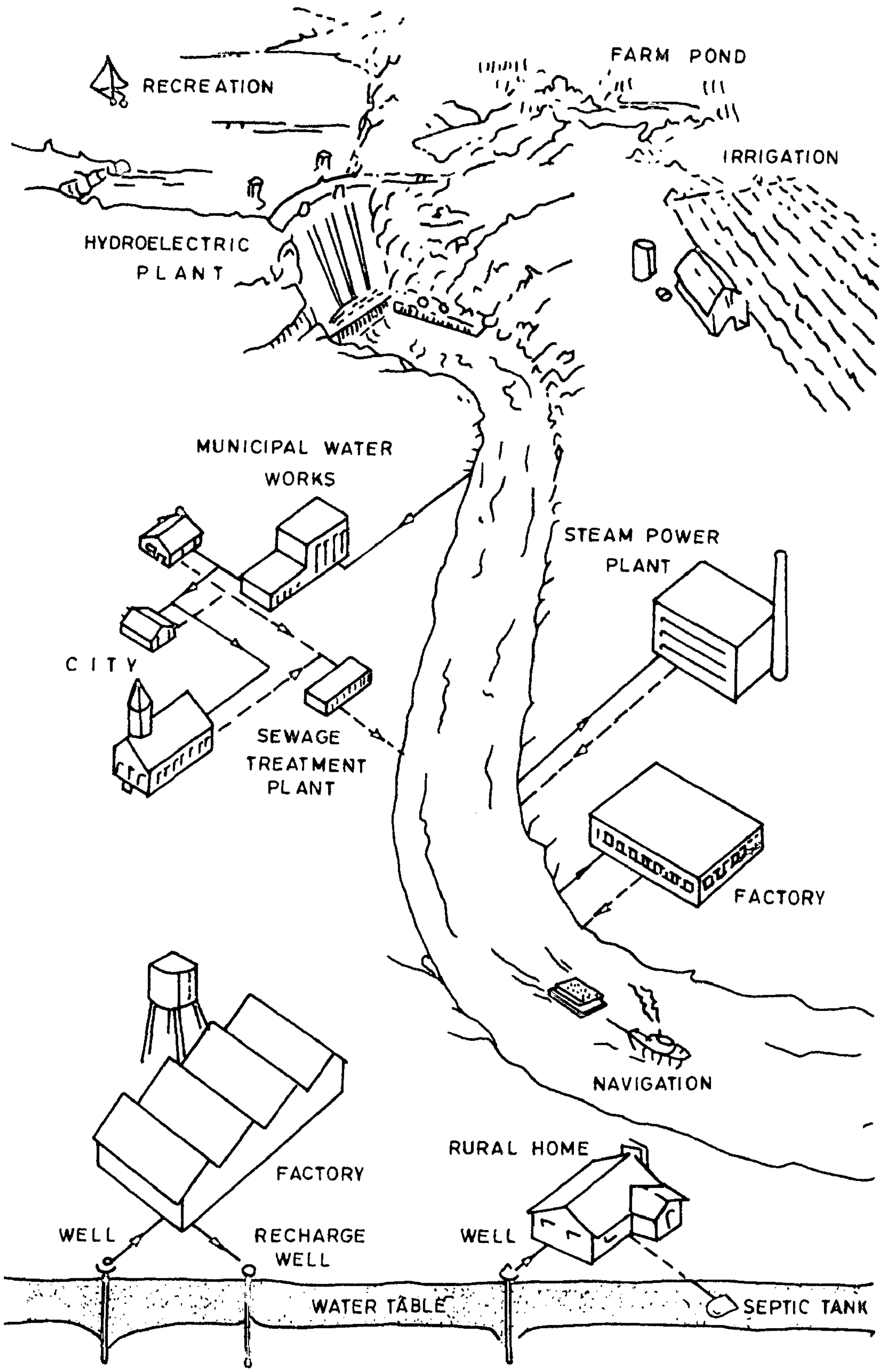

FIGURE.1.2.1 PHYSICAL COMPONENTS OF A RIVER BASIN

demand. The planning of resources is then diverted on exploitation ground water through shallow wells, deep wells. The shortage of resources makes the planners to think of conservation of water and leads to the planning of waste water reclamation. Several classes of waste water can be reclaimed and reused particularly where the impurities are degradable organic matter. The used water are trapped wherever possible, purified and recirculated to the consumers. The littoral states find themselves in abundant resource of sea water. Desalination plants are planned for recoupment of water supply. The regions having no further potential natural source of water, plan to import water from other regions. Canals are dug or giant pipes are laid to carry water. All the above alternative schemes all put together or in various combination may be the goal of water resources development authority. A typical water resource development scheme is shown in Fig. 1.2.2.

The figure schematically shows the water resource system proposed for two demand points (areas) separated geographically by 40 miles. This distance is divided into 10 zones. Within this distance there are 18 sources of water. The wells x_i (i = 1, 2, 3, 4, 5) supply water to Area 1 and the wells x_i (i = 6, 7, 8, 9 and 10) to Area 2. There is an electrodyalisis plant located midway between the two areas. Fresh surface water is available near Area 1. A desalination plant is located near Area 2. Both areas have got Advanced Waste Recycle System. A complimentary cross over system is proposed which can carry surface water to Area 2 and the desalinated water to Area 1 from 2. This system can be studied for a flexible and encompassing analysis of alternative source of water.

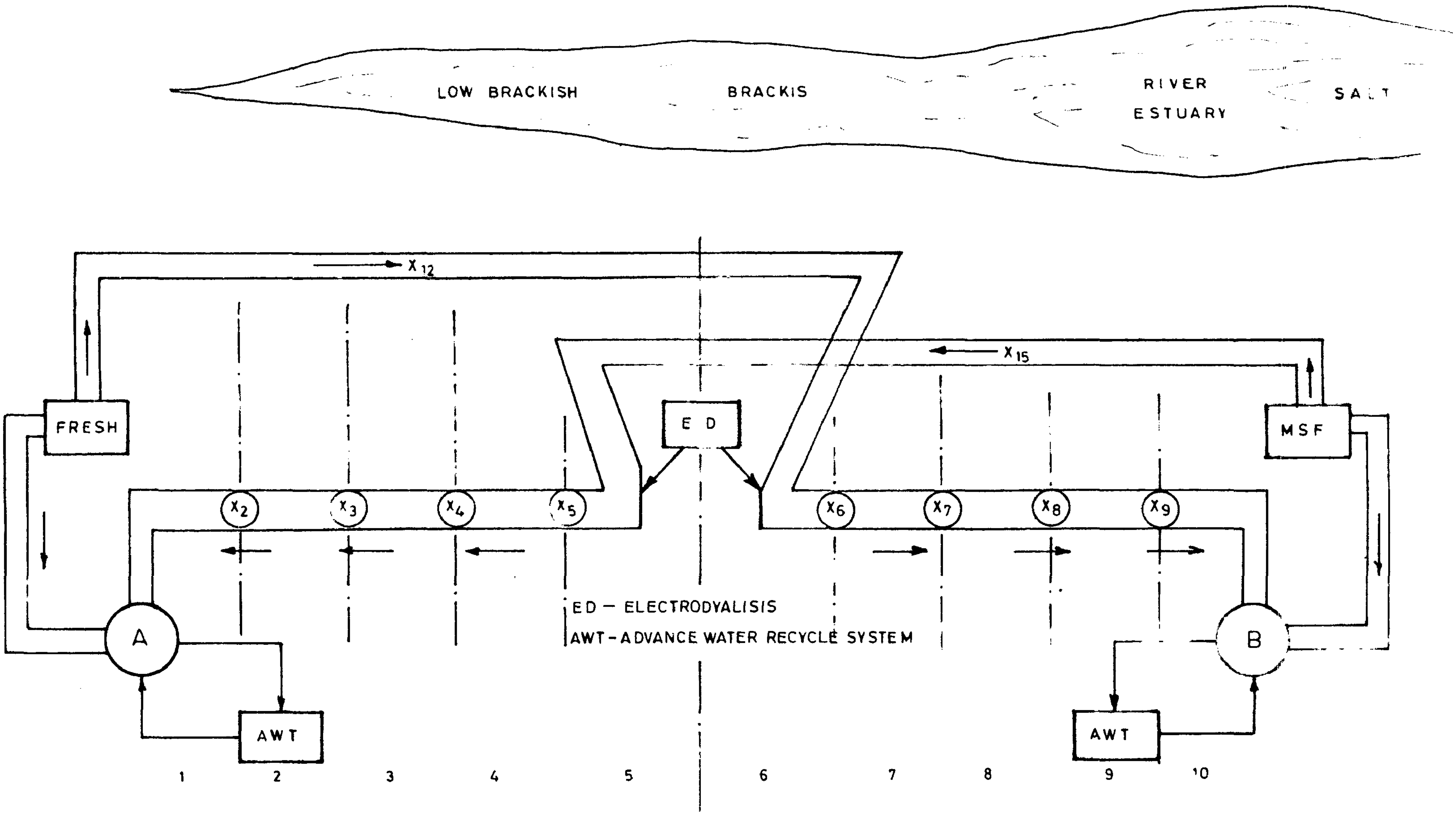

FIGURE 1.2.2 A TYPICAL WATER RESOURCES DEVELOPMENT PROJECTS

1.3 Nature of Water Resources System :

Water resources system in many respects defy rational description. To those living in arid environment water resources system mean food and fibre, to those in humid areas the system brings to mind wells and water works, protection against floods, hydroelectric power. To the outdoor-recreation-minded, the systems are virtual emporiums for fishing, boating, swimming, camping and other activities. To the engineer the systems may be dams, diversions, tunnels, pumping, electric power plant, weirs, spillways etc. To the lawyer, a water resources system is a device for the implementation of water rights. There is hardly one water resources project in which legal overtones are absent. To the economist, the field is predominantly economic efficiency, income redistribution, stimulation of economic growth, pricing policies, etc. The scale of investment required water sources development system, the long economic life of the system, supplying the need of different combination of users, integration of rivers and regional areas - all these characteristics tend to keep the water resources system under the Government control. Thus, political factors also play a vital role in selection and operation of a water resource system.

Water resources system indeed include all these points of view. They are physical, they are sociological; they are biological, economic, political, legal, agricultural as well. Thus water resources engineering covers the overlap and extends also into the realms of natural sciences (Mathematical, hydrology etc.) and social sciences (economics, sociology, public administration). The relative ease with which one of these aspects might be quantifiable, as compared to another, does not in any way reflect a correspondingly greater importance. The water resources systems engineer has to recognise and integrate these many quantitative and nonquantitative dimensions of the system to the greatest extent possible.

1.4 <u>The Systems Approach to Water Resources Problems</u> :

The design of water resources system is so complex a problem that a broad approach is required to its analysis and solution. Among the factors playing a major role are water demands, water supply, water quality, available technology, management framework and data gathering facilities. All of them have to be attuned to each other and to a set of objectives which are either spelled out explicitely or included implicitely in the formulation of the problem.

For a plan to be really comprehensive, it should aim at the optimum development of all resources of a river basin including land, water and other natural resources. To optimise the benefits from the invested capital, projects cannot be evaluated by considering each element in the project individually. Projects can be envisioned, planned for and designed correctly only when they are considered as components of overall development within a region. In the context of the development of water resources, unified development is achieved by multiunit, multipurpose systems; that is each unit of the system, such as a dam, serve many purposes - irrigation, energy, flood control and reation - and also can be combined with other units in the tem to serve the total demands on the system.

This broad perspective of water resources problems requires oval approach. Perhaps, the most important advance made in ent years in water resources was the adoption of systems ineering to the analysis of problems in the field. Systems ineering, is in a sense, the attacking a complex problem on a ad front. Variables describing components or states of a tem can be defined and relationships between them represented ough equations in a mathematical model. These relationships, ther linear or nonlinear can be properly evaluated by a variety techniques, some of which have been made possible by the advance computer science. Systems analysis is undertaken in order to o ~~realised~~ rational decisions in so far as possible as to optimal design, ation or operation of a physical system. As might be expected, tem analysis is primarily useful in dealing with planning blems that are sufficiently complex that no one man can be sidered an 'expert' on every aspect of the situation.

To provide a better perspective on the systems approach a water resources system, we first see as to how the systems lysis takes place in general. Several ~~plans~~ phases can be distinguishe h feedback possible from any phase to an earlier phase.

1. The first phase of systems analysis consists of erstanding and translating into quantitative terms the objective performance requirements sought for the system in relation to environment in which it operates.

2. The next phase is to formulate quantitatively (e.g., by a flow diagram) the structure and boundaries of the system.

3. Then a mathematical model has to be prepared for the system that includes all the possible interrelations between the variables that can be quantified. All the quantifiable constraints must be included in the model, in addition to the functions yielding the input-output relations between the variables.

4. The coefficients in the model must be estimated and the desired input relations specified.

5. The model should be validated in light of the objectives established by step 1.

Fig. 1.4.1 shows conceptually the cyclic nature of these phases.

Finally, taking into account all the phases of the analysis in as much detail as possible (but always being limited by the cost involved and by lack of information) the analyst is prepared to use the model for

(1) <u>Economic experimentation</u> - The system time scale can be compressed by computer simulation so that existing and proposed operations can be examined in relatively reasonable times.

(2) <u>Extrapolation</u> - Extreme ranges of operating conditions can be examined without incurring damages that might arise in real physical system.

(3) <u>Study of commutability and evaluation of alternate policies</u> - Elements of the system can be rearranged, new factors

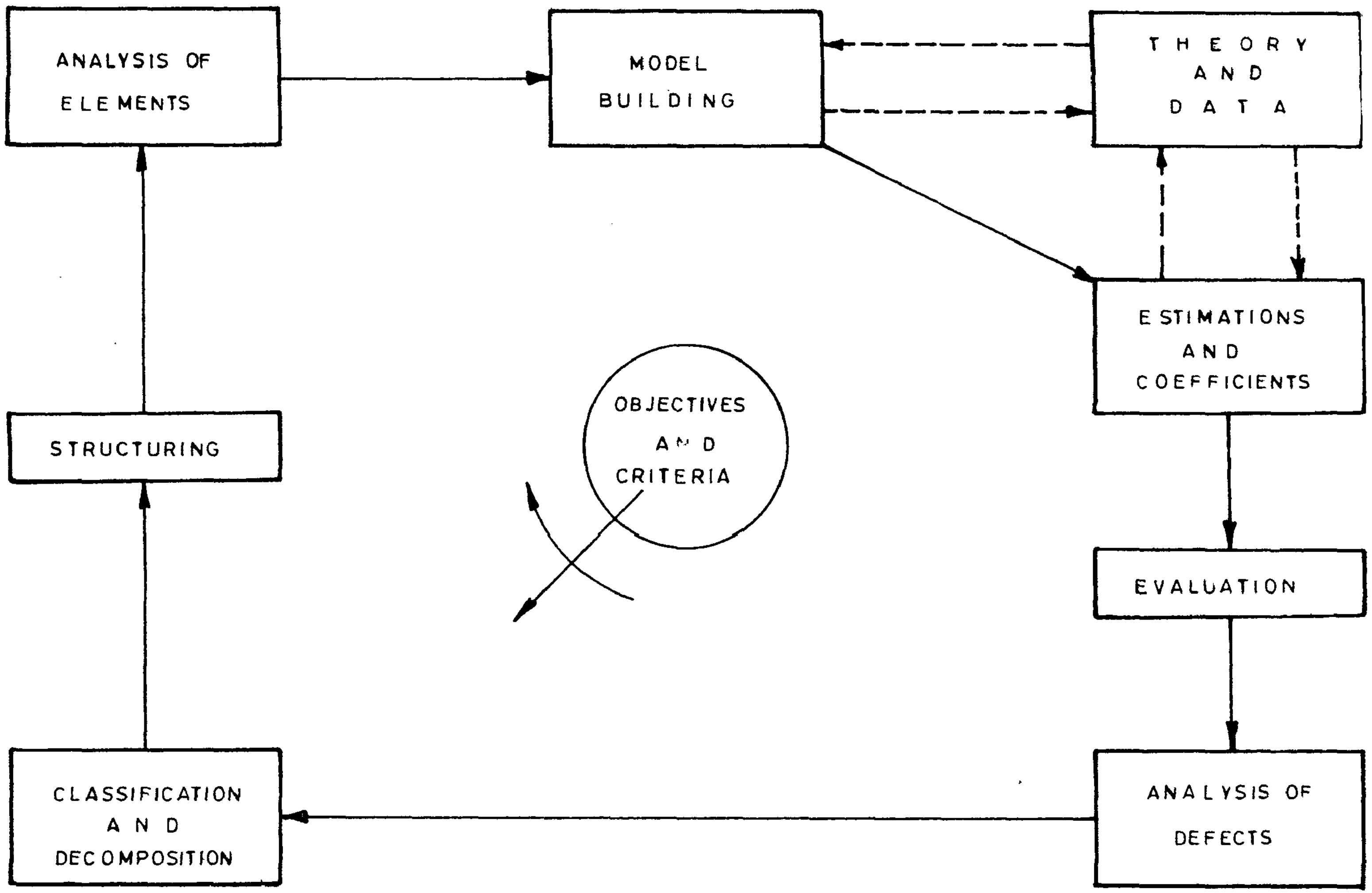
ANALYSIS OF ELEMENTS
MODEL BUILDING
THEORY AND DATA
ESTIMATIONS AND COEFFICIENTS
OBJECTIVES AND CRITERIA
STRUCTURING
EVALUATION
CLASSIFICATION AND DECOMPOSITION
ANALYSIS OF DEFECTS

FIGURE 1.4.1

introduced, and the design of future system evaluated.

(4) Effect of stochastic variables - Random effects can be introduced with known statistics for the random variables.

(5) Sensitivity - The effects on outputs of changes in individual and joint variables and parameters can be examined.

All of the above use of model pertain to design and operation without actually undertaking the physical construction of the system.

Now considering the procedural steps involved in system analysis and using the model development, the system analyst tries to find answers for the following questions in addition.

1. What alternative system can be considered ?
2. What effects or consequences are imposed on the users by the various prospective system ?
3. How reliable are the estimates of system costs and revenues ? And how does the level of reliability affect the choice of alternatives ?
4. Are the consequences of the system alternatives measured, evaluated and presented to the decision makers ? Because the practice of analysing essentially a single alternative and submitting it to the lawmakers for a 'yes' or 'no' decision is not in the best interest of the profession or general public.

We shall see how to relate the concepts of systems analysis to the problems of water resources management. Planning for the development of water resources of a river basin, for example, requires examination of the following interrelated activities :

1. The identification of objectives for the water resources system.

2. Selection of the various structures involved in the system viz. dams, reservoirs, canals, pumping stations - their optimum size and location.

3. Sequencing the construction of the projects thus decided so as to achieve optimum economy over the planning period.

4. Optimum operating policy of the system. Or the optimum scheduling of reservoir discharge, storage etc.

In this work, the activities 1, 3 and 4 will be considered. These activities are further elaborated as follows :

In general the main objective of a water resources development project is to maximise the benefits or the regional welfare. The term 'region' is employed to denote a geographical area ranging in size from a small farming field to an extensive river basin. The objective can be (a) making meeting required water demands in future of different category of users like industrial, irrigation, hydroelectric power plants, (b) to meet required water quality which is measured in terms of dissolved oxygen and intangible benefits like recreation, aesthetics, flood control.

Now, assuming that the outcome of each of 3 and 4 activities can be expressed as benefit of reward R, we can say R is a function of two sets of variables which we would like to optimize. The constraints of this optimization problems will be the objectives outlined earlier.

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CHAPTER - II

REVIEW OF TECHNICAL PAPERS ON WATER RESOURCES EXPANSION

We have seen in previous chapter, the complex nature of water resources systems, its many phases of uses and the necessity to introduce the concept of system analysis for planning and operation of the system. It is to be remembered that neither water resources nor their many uses were created with mathematical analysis in mind. There is probably no standardised method of analysis whose structure conforms to the real world of water resources well enough that it can be taken as a general mathematical model for its optimisation.

Among the earliest comprehensive formulations of problems in water resources development, utilisation and conservation was the report of the United States' President Water Resources Policy Commission in 1950. This report gave impetus to further investigations and generated sufficient interest to start research activities in a number of universities. A number of workers at the University of California in 1955 began studying the problem of optimisation of water resources systems. The earliest mention of concern in this area is perhaps a problem suggested by W.A. Hall in the book on dynamic programming by R. E. Bellman. One of the first product of the research was a fundamental paper showing the analysis of sequential multistage decision processes in water resources engineering, through the application of dynamic programming.

With the passage of time, the concept of comprehensive planning for water and other natural resources have gained acceptance rapidly. Because, large water supply systems involves crores of rupees in capitalinvestment and careful planning is essential. The growing sophistication of computing machinery has not only made possible the rapid and accurate performance of tedious and repetitive calculations, but has also been a catalytic factor in the evolution of several mathematical theories and methods. We shall review the latest development of various methodologies.

The problems can be divided broadly into two classifications (i) planning the sequencing and scheduling of water supply projects; i.e. when or whether to build each dam or canal, (ii) the operating schedule of the reservoir or dams. Nevertheless both these types can also combined to find optimal solution.

Water needs are generally estimated on the basis of expected growth in demand over a future period. In many instances, there are a number of possible independent projects which, in the aggregate are expected to meet the requirements upto some future date. The decision on sequence of construction is based on the present minimum cost which is influenced by the slope of the water demand curve, the discount rate applicable, and the relative costs capacities and operation of the water supply facilities envisaged.

It is always beneficial to postpone a project till it is needed to meet the demand. For, the interest on the amount invested is a loss till the project is put into operation. Thus if C_i is the capital cost of the project i, the interest rate is r and the project will be constructed after a time interval t, then the present value of the capital cost is $C_i(1 + r)^{-t}$. The time value of the capital as expressed in the discount factor makes the problem of project selection dynamic, that is, the selectionof expansion projects depends upon previous selections and upon time. Dynamic project scheduling can be classified into four distinct categories which are depicted in Table 2.1.

TABLE 2.1.

		Time	
		Continuous	discrete
project size	Continuous	I	II
	Discrete	III	IV

A scheduling problem will be referred to as a Single - Location Project Scheduling problem (SLPSP) if the geographical value of projects is not taken implicitly into account. The length of the scheduling or planning horizon also characterises a project scheduling problem. Planning horizons are considered to be either finite or infinite. In a type I SLPSP (see table2.1) for linearly growing demand functions, Manne[4] showed that for the infinite planning horizon, the optimal scheduling policy was to construct equal-sized projects at regular intervals. A concave project-capital cost-capacity relationship of the form $C(J) = KJ$ was assumed, along with a demand function of $D(t) = at$. Kuhich[4] continued Manne's approach and extended it to planning for the finite horizon. He obtained results for both linear demand functions and geometric demand functions of the form

$$D(t) = Do.e^{at}$$

If one assumes that the time periods are discrete, the objective function of the minimum cost type II SLPSP can be written as

$$\min \sum_{t=0}^{T} \sum_{t=1}^{N} C_i t(S_i) X_i t$$

where,

$C_i t\ (S_i)$ = Capital cost of building project i in year t at a scale (S_i)

$X_i t$ = decision to build project i in year t.

The problem is difficult to solve, because of the multiplicative factors of $C_i t(S_i)$ and $X_i t$. Some techniques of nonlinear integer programming have been developed[6]. As an alternative to assuming the availability of a continuous scale of project sizes, it may make more sense to limit consideration to one or more sized projects for each possible site. This is possible because (i) project cost information is usually found with a particular project scale and configuration in mind; (ii) funding and political decisions with respect to a project are usually made, given a fixed project configuration. This discrete project size configuration is particularly applicable to water resources systems. Type III and Type IV SLPSP fall under this category. In this type III and IV SLPSPs it is assumed that a finite set of N feasible projects have been prescreened, technically and economically. Economic feasibility is determined usually by investigating an individual project's ratio of discounted benefits to discounted costs as explained by Moss[4]. In type III problems, the water demand function D(t) over the planning horizon is a continuous, nondecreasing function. In the type IV SLPSP D(t) = Dt is a constant, annually projected requirement over the planning horizon, nondecreasing over the index set of planning years (0, 1, ..., T). The time increment for capacity is taken to be one year and it is assumed that the projected demands are also made on an annual requirement basis.

Butcher, Haimes & Hall[20] developed a Dynamic Programming algorithm to solve the type IV SLPSP. They made use of Bellman's[21] 'principle of optimality' to formulate a recursive relationship between the adjacent stages. Working on the same model Morin[22] developed a modification of Butcher et al. algorithm to solve more general scheduling problem.

The type IV SLPSP problem can be formally stated as follows :

Given

1. A demand function, D(t) = Dt, of discrete values defined over an index set $t \in \{0, 1, \ldots T\}$, with $D_{t+1} \geq D_t$
2. A set of projects $N = \{1, 2, \ldots, N\}$; the ith project has an annual capacity q_i and a capital construction cost C_i; each is constant over the planning horizon.

3. A discount factor on all construction cost, $(1 + r)^{-t}$, $t \in \{0, 1, \ldots, T\}$, where r is a constant over the planning horizon.

Find a schedule of N out of M projects :

$S_N = ((1), t_{(1)}); ((2), t_{(2)}); \ldots ; ((N), t_{(N)})$
where (i) denotes the ith ordered project built in the schedule S_N; j denotes the jth project; $t_{(i)}$ denotes the construction time of the ith ordered project. Thus the present value cost (PVC) of the schedule,

$$PVC(S_N) = \sum_{i=1}^{N} C_{(i)} (1 + r)^{-t_{(i)}}$$

is minimized and the available capacity always equals or exceeds the demand D_t. $S = \{B, A\}$ implies $P_{(1)} = P_B$ and $P_{(2)} = P_A$.

Williams and Chiu[9] proposed to write this problem in a 0-1 integer programming formulation.

Thomas - L - Morin[10] used integer programming formulation for sequencing and scheduling of the projects. He has considered multiple demand functions by including a demand constant for each type of requirement at each year t. Multidimensional projects were then included easily into the IP formulation. Constraint equation was written in general as

$$\sum_i Q_{im} X_i(t) \geqslant D_m(t) \; ; \forall t \in (0, T)$$

$$m = 1, 2 \ldots, M$$

He has further shown the advantage of the I.P. formulation in the case in which certain type of projects interdependencies, financial and other constraints can be incorporated. For example, of project '1' cannot be built before project 'n' we simply append the following constraint to the Integer Program;

$$\sum_{k=0}^{t} (x_{nk} - x_{1k}) \geq 0 \quad t = 0, 1, .. T$$

If there is a limitation (B_t) of the capital expenditure in period t, another constraint can be appended to incorporate this requirement

$$\sum_{i=1}^{N} C_i \cdot x_i t \leq B_t$$

Both Morin and Williams however have pointed out the excessive computational requirements of I.P. problem. Even a small problem involving 4 projects, 3 demands and a 50 year planning horizon would have in excess of 200 decision variables and 150 constraints.

A common problem in the optimization of regional water resources system is to identify an appropriate trade-off between comprehensiveness and simplicity of description. Advanced system-analysis techniques allow an exact description of the system but their application is usually restricted to fairly small size problems. Simple techniques, such as linear programming allow the planner to deal with large scale problems, but only after rather drastic simplifications in their formulations. Clausen[10] offered quadratic programming which is seen as a compromise; the mathematics and numerics is rather simple, but non-linearities can be considered.

Here the objective function (which is the cost function) is of the type;

$$C(x_j) = A_j x_j^{\alpha_j}, \quad A_j > 0, \; 0 < \alpha_j < 1 \quad \ldots \quad (2.1)$$

where $C(x_j)$ is cost associated with treatment of water quantity x_j and A_j, α_j are constants. However, in a planning situation reliable information on the actual cost structure is difficult to obtain. A reasonable compromise is to work with quadratic cost functions.

$$C(x_j) = (a_j - 1/2\, b_j\, x_j)x_j \quad \ldots \quad (2.2)$$

The optimization procedure is based upon minimization of the quadratic objective function. If power cost functions are provided as input to the model, these will be approximated by quadratic functions of the type given in equation (2.2) in prescribed intervals around even current values of the variables. These fitting procedure is performed by linear regression on the cost function derivatives yielding the parameters a_j and b_j in equation (2.2) as simple analytical functions of the original parameters A_j and α_j in equation (2.1). The entire procedure is repeated until the optimal solution is interior in the prescribed fitting intervals usually in a few iterations.

Two questionable features of many existing studies of capacity expansion planning problems in water resources are (1) the modelling of the expansion as deterministic process and (2) the lack of control over the incidence of system failure. System failure is related to the occurrence of shortfalls in water supply. Robert J. Hinks[12] used a sequential decision model as a general framework for the formulation of stochastic demand capacity expansion problems subject to reliability constraints designed to regulate the incidence of system failure. Various different types of constraints were investigated and incorpora-ted into a dynamic programming solution procedure by means of

Lagrangian theory. The constraints considered were (1) the expected value of the total number of system failures during the entire planning interval should not exceed α ; (2) the probability of failure during any year t in the planning interval should not exceed β_t, $0 \leq \beta_t \leq 1$; (3) the probability of N or more failures during the entire planning interval should not exceed γ , $0 \leq \gamma \leq 1$.

Knudsen & Rosbjerg[13] developed a new Dynamic program algorithm for sequencing and scheduling problem. The model takes into account the capital investment and also the operational costs. Stochastic demand was considered. The mathematical formulations were formed with chance constraint model and penalty model.

Reservoir operating policies form another important water resources scheduling problem. The operating policy is described as follows : An operator of a water supply system has to decide from time to time how immediate demands on the system are to be met. Without detailed foreknowledge of inflows to and demands on the system in future he cannot guarantee to find the best way of using the system. All he can hope to do is to devise a policy which will be the best, given the uncertainty. In meeting these demands he may have several objectives in mind, such as cost minimization, maximization of benefits, etc.

Harley & Chidley[14] described a method of using deterministic dynamic programming to determine the long term operating policy for a reservoir system. The long term policy is defined to be the policy which is to be used as a matter of routine without foreknowledge of future events. The same authors used policy iteration technique and value iteration techniques to a two-reservoir operation problem.

Now going back to the operator of the system, he may operate the system with intuition which comes out of experience. This intuition is a calculated assessment of past experience combined with what he thinks the future might bring. But in a complex system it may not be all obvious how to operate the system to achieve the required goals. The problem become complex with more than only two reservoirs operating in conjunction meeting various demands. A decomposition method for the long term selecting of reservoirs in series was presented[15]. The author discussed a method for weekly operating policies of a power system of n reservoirs in series. The method takes into account the stochasticity of river flows. It consists of rewriting or decomposing the stochastic nonlinear optimization problem of n state variables as n-1 problems of two state variables which are solved by Dynamic programming. The release policy obtained with this method for reservoir i is a function of the water content of the reservoir and of the total amount of potential energy stored in the down stream reservoirs.

Optimization of a water resources system must appropriately mesh the modelling of the system with the optimization technique used. If the system model is linear, many effective optimization technique exist. But if the model is nonlinear in the objective function and/or constraints, very few optimization methods exist. Himmelblau[16] described how a water resources system, including water quantity and quality can be modelled to form a non-linear programming problem. The problem of water quality was solved by two techniques (a) Generalized Reduced Gradient Method, and (b) a Conjugate Gradient Projection Method.

A number of techniques are available in tackling water resources problems. It is for the system analyst to use any of the techniques available which can be most suitable for a specific problem, from the view point of simplicity and efficiency.

CHAPTER - III

SEQUENCING AND SCHEDULING OF WATER RESOURCES SYSTEMS - DETERMINISTIC CASE

3.1 The nature of the problem

Water needs are generally estimated on the basis of expected growth in demand over a future period. In many instances, there are a number of possible independent projects which, in the aggregate are expected to meet the requirements upto some future date. Each project because of economical consideration, and the supply-demand criterion will most probably be constructed when needed. If the demand is a linear function of time and all projects have about the same maximum capacity, the effect of interest rate on capital cost is to dictate the construction of the unit with the lowest unit cost first. However, this simple criterion will not be sufficient if the demands are not linear and output of projects are different.

Let us illustrate this point with a simple example of two projects. The demand say, increases at the rate of 1/3 units per year to 10 units over a period of 30 years. Let the annual discount rate be 5%.

If project No. 1 has a capacity of 4 units and a cost C_1 of Rs. 500,000, its unit cost is Rs. 125000/- for project 2 of 6 units capacity the cost is assumed as Rs. 600,000/- thus with a unit cost of Rs. 100,000/-. We assume project No. 1 is constructed first. With its capacity of 4 units, it can meet the demand till a period of 12 years after which the second project should be ready to meet the demand. The total present cost of building these two projects will be $500000 + 600,000\,(1.05)^{-12}$ = Rs. 834,080.

If project 2 is constructed first, the project 1 is needed to be commissioned only after 18 years. The present value of cost in this case will be

$$600{,}000 + 500{,}000\ (1.05)^{-18} = \text{Rs. } 807{,}750$$

Thus, it is economical to built the project 2 first i.e. the project with less per unit cost, or the largest project.

In the problem above if the project 2 has a capacity of 6 units and a cost Rs. 700,000 the its unit cost is Rs. 116,667. In that case, the present cost when project 1 is constructed first is

$$500{,}000 + 700{,}000\ (1.05)^{-12} = \text{Rs. } 890760$$

and $$700{,}000 + 500{,}000\ (1.05)^{-18} = \text{Rs. } 907750$$

when project 2 is constructed first.

Here it is economical to build the project No. 1 first i.e. the project with higher unit cost or the smallest project. Thus unit cost is not a satisfactory criterion. The effect of slope of water demand curve also plays significant role in sequencing of projects. To illustrate, we modify the second example problem by introducing a demand rate of 1 unit per year. Now if the first project with 4 units output is constructed first, the project 2 will need to be in operation after 4 years.

Thus the total present value of the cost is

$$500000 + 70{,}000\ (1.05)^{-4} = \text{Rs. } 1075390$$

whereas the total cost when project 2 is constructed first is

$$700{,}000 + 500{,}000\ (1.05)^{-6} = \text{Rs. } 1073100$$

Thus the optimum decision is to construct the larger project first changing the optimal decision when the demand increase was 13 unit per year.

We conclude from these examples the decision of sequencing the projects based on minimum present value of the total cost is equally influenced by the slope of water demand curve, the interest rate, the relative costs and capacities of the water supply project. The problem will be much more complex when the projects are more than two and there are multiples of demands. We, therefore, need an optimization technique to solve these problems.

3.2 Assumptions :

1. The discrete project scheduling and sequencing will be formulated.

2. We assume a finite set of N feasible project which has been prescreened to be politically, technically and economically admissible.

3. The capital cost of construction C_i, and the estimated annual yield of a project, Q_i are assumed to be known and of fixed values over the planning horizon.

4. The demand function D(t) is assumed. There may be number of demands on each project viz., irrigation, power generation, industrial. D(t) is nondecreasing over the index set of planning years (0, 1, ..., T). The time increment for capacity is taken to be 1 year and it is assumed that project demands are also made on an annual requirement basis. Probabilistic demand is considered separately.

5. It is assumed cost C_i is discounted to a single period of time; that is, the time at which the project is considered operational

and able to supply its full capacity. Project "Warm-up" is not considered in the formulation. It must be answered that ample 'lead time' is available to design and construct any one of the projects.

6. It is assumed initially that operational unit cost over a project life time are minor with respect to capital outlays. It is estimated capital costs of construction may be 500 times greater than annual operation and maintenance costs. Thus operational unit costs are dropped from consideration.

7. Projects under consideration were independent of each other. Possible interdependencies are (i) financial, (ii) political, and (iii) physical or hydrological.

8. Differences in the quality and usefulness of water yielded by each project is not considered; that is, one unit of supply from any project is equivalent to that of any other in terms of its ability to satisfy the demand.

It may be very difficult to formulate a model for project scheduling where all the above assumptions are released.

3.3 Mathematical Model :

1. Continuous Time Case

The difference between scheduling and sequencing problem is because of one assumption. In scheduling problem we have to select only a subset of N considered projects and sequence their construction. We can say the output from all projects will be equal to or more than the demand at the end of planning horizon. i.e.

$$\sum_{i=1}^{N} Q_i \geq D(t) \qquad \text{for } t = 0, 1, \ldots, T$$

A special case of this scheduling problem occurs when we make the assumption that total capacity of N projects equals the demand at the end of planning period, i.e.

$$\sum_{i=1}^{N} Q_i = D(t)$$

which means only sequencing of project construction is involved. All the projects have got to be completed at the end of planning horizon to meet the demand.

Let $\emptyset$ be the set of development projects (1, 2, ..., N). Each project $i \in \emptyset$ is described by an $(M + 1)$ - tuple $(C_i, Q_{i1}, Q_{i2}, \ldots, Q_{iM})$, in which, C_i is its capital cost of construction and Q_{ij} is the capacity (output) of the ith project to satisfy the jth demand. An n-project sequence, S_n consists of an ordered subset $(K_1, K_2, \ldots, K_n)$ of the n ($\leq N$) projects from $\emptyset$.

The M-dimensional project scheduling problem is to find a sequence S_n and a set of associated completion time so as to

$$\min \sum_{i \in S_n} C_i e^{-rt_i} \qquad \text{subject to}$$

$$\sum_{i \in \phi} Q_{ij} x_i(t) \geq D_j(t) \qquad \forall t \in (0, T)$$

$$\sum_{i \in \phi} a_{i2}\, x_i(t) \geq D_2(t) \qquad \forall t \in (0, T)$$

$$\vdots \qquad \vdots$$

$$\sum_{i \in \phi} a_{iM}\, x_i(t) \geq D_M(t) \qquad \forall t \in (0, T)$$

where

r = continuous interest rate,

t_i = construction completion time of the ith development project,

$x_i(t)$ = a unit step function which is equal to zero if $t < t_i$ and is equal to 1 if $t \geq t_i$

$D_j(t)$ = projected increase in demand for the jth resource from time '0' to time 't' (Here it is assumed that initial capacity is equal to the initial demand)

T = length of planning horizon

Thus the problem is to find that subset of development projects together with their associated completion time.

2. Discrete Model :

In the continuous time model, the demand is assumed to increase continuously over the planning period. The time period if discretised to one year each, will result in discrete model. The demand is increased only at the end of each year time interval and during each interval the demand for water is constant. The D(t) is given as step function for discrete time formulation. The model is to find,

$$x = (x_{ik}), \quad i = 1, 2, \ldots N;$$
$$k = 0, 1, \ldots T$$

so as to

$$\text{Min} \sum_{i=1}^{N} C_{ik}\, x_{ik} \qquad \ldots\ldots \qquad (3.3.1)$$

where $C_{ik} = Ci\, e^{-rk}$ ... ... (3.3.2)

C_{ik} being the present value of the cost of project i to be built after time k.

subject to the conditions

$$\sum_{k=0}^{T} x_{ik} \le 1 \quad , \quad i = 1, 2, .. N \qquad \ldots \qquad (3.3.3)$$

The constraint (3.3.3) ensures that each project is considered only once for construction. And,

$$\sum_{i=1}^{N} \sum_{k=0}^{t} Q_{i1}\, x_{ik} \geqslant D_1(t), \quad t = 0, 1, \ldots, T$$

$$\sum_{i=1}^{N} \sum_{k=0}^{t} Q_{i2}\, x_{ik} \geqslant D_2(t), \quad t = 0, 1, \ldots\ T$$

$$\vdots \qquad\qquad \vdots \qquad\qquad (3.3.4)$$

$$\sum_{i=1}^{N} \sum_{k=0}^{t} Q_{iM}\, x_{ik} \geqslant D_M(t), \quad t = 0, 1, \ldots, T$$

$$x_{ik} = 0 \text{ or } 1 \quad \forall i, k \quad \ldots\ldots \quad (3.3.5)$$

The equations (3.3.4) are the constraints for demands, that the combined output of all projects (Q_{ij}) corresponding to the type of demand $D_j(t)$ should be more than the demand. These constraints are constructed for the project of multidimension in nature. Which means the demand is not for a single purpose, but for multipurpose, the demand being $D_j(t)$, $j = 1, 2, \ldots, M$.

The zero-one variable x_{ik} of equation 3.3.5 takes the value 1 if project 'i' is completed in period 'k' and takes the value 0, otherwise.

This 0-1 integer programming formulation is used to solve a practical multidimensional scheduling problem which will be described in section 3.5.

A few more constraints can be added to the model if required. For example, if project 'l' cannot be built before project 'n', we introduce the constraint.

$$\sum_{k=0}^{t} (x_{nk} - x_{lk}) \geq 0 \qquad t = 0, 1, \ldots, T$$

If the capital expenditure is to be limited to β_t in a period t, the constraint will be

$$\sum_{i=1}^{N} c_i \, x_i t \leq \beta_t$$

3.4 <u>Solution Technique</u> :

The implicity enumeration approach is used to solve the problem. The implicit enumeration algorithm is only applicable to the special case where each variable takes on the value 0 or 1.

One procedure to solve 0-1 IP model is to examine every possible combination of the variables set equal to 0 and 1. This is total or explicit enumeration. The combination that satisfies all the constraints and minimizes the objective function is declared the optimal solution. However, this requires examining 2^n combinations of the variables which is very difficult when the number of variable are more. Hence, it would be useful to have a procedure that would systematically examine only a small subset of all possible combinations of the variables before reaching an optimal solution. This is what the implicit enumeration algorithm does.

Basically, the implicit enumeration algorithm begins with setting all the variables to zero and systematically sets certain variables to take the value of 1 until a feasible solution is obtained. The algorithm systematically looks at various combinations of the variables set equal to zero and one that can possibly improve on the best feasible solution to date, till the optimum solution is reached. Many combinations that cannot possibly lead to a better feasible solution are not examined and thus said to be implicitly enumerated.

Algorithm :

The 0-1 integer program has the form

$$\text{minimize } Z = \sum_{j=1}^{N} C(J) * X(J)$$

$$\text{subject to } Y(I) = -B(I) + \sum_{j=1}^{N} A(I, J) * X(J) \geq 0$$

$$I = 1, 2, \ldots, M$$

$$X(J) = 0 \text{ or } 1$$

$$J = 1, 2, \ldots\ldots, N$$

where $C(J) \geq 0$; $J = 1, 2, \ldots, N$

The notation used is :

FREE = the set of subscripts of the variables that have not been specified to be 0 or 1.

NFREE = the set of subscripts of the variables that have been specified to be 0 or 1. If an element of NFREE is negative, the corresponding variable has been specified to be 0, otherwise, it has been specified to be 1. The first element (left most) in NFREE corresponds to the first variable specified to be 0 or 1. Likewise the second element in NFREE corresponds to the second variable specified to be 0 or 1 etc.

ZMIN = the value of the objective function corresponding to the best feasible solution to date

VC = the set of violated constraints.

T = the variables in FREE that have

(a) An objective function coefficient less than BOUND,

where, $BOUND = ZMIN - \sum_{I \in NØFREE} C(I)*X(I)$

(b) A positive coefficient in some constraint in VC

$\sum_{I \in FREE}$ = sum over all subscripts in FREE

STEP 1

Set FREE = (1, 2, ... , N)

NFREE = ∅, the empty set

ZMIN = 10^{10}

STEP 2

Calculate $Z = \sum_{I \in NFREE} C(I)*X(I)$

Note that some of the X(I)'s in the above sum may be specified to have the value 0.

STEP 3

Evaluate each constraint G(I), (I=1, 2, .., M) using the NFREE variables each set equal to 0. If each of the constraint are feasible (satisfied), then the values of variables used to evaluate the constraints constitute a feasible solution.

Let VC denote the set of violated constraints.

STEP 4

If VC is empty, go to step 12; otherwise, go to step 5.

STEP 5

Set BOUND = ZMIN − Z

STEP 6

Select the FREE variables that have a chance to make all of the constraints feasible. That is let T be the set of variables in FREE that have

1. A positive coefficient in some constraint in VC
2. An objective function coefficient < BOUND

A violated constraint can only be made 'more' infeasible by setting to 1 a variable with negative coefficient in the constraint, so only variables with a positive coefficient in a given constraint have a chance to make the constraint feasible (≥ 0). Likewise, a variable X(K) in FREE such that

$$\sum_{I \in UFREE} C(I)*X(I) + C(K) \geq ZMIN$$

should not be considered for inclusion in UFREE since the feasible solution corresponding to ZMIN is already atleast as good.

STEP 7

If T is empty, go to step 21; otherwise go to step 8.

STEP 8

For each constraint in VC

Set to 1 the FREE variables in T that have positive coefficients in the given constraint.

Set the UFREE variables equal to their specified values.

STEP 9

If any of the constraints are still violated, go to step 11; otherwise go to step 10.

STEP 10

Remove from FREE and add to UFREE the variable in T that would minimize the total distance from feasibility over all constraints. This process is covered in detail in steps 10A - 10C.

STEP 10-A

For each variable, say X(K), in T, evaluate each constraint

Q(I), (I = 1, 2, ..., M), using the NFREE variables with their specified values, X(K) = 1, and the remaining FREE variables each set equal to 0.

STEP 10-B

Sum the negative results from step 10-A and let ASUM be the absolute value of sum. The absolute value of each negative result is the amount the corresponding constraint must be increased to be feasible. Hence, ASUM represents in some sense the total distance from feasibility using X(K) = 1.

STEP 10-C

Remove from FREE and add to NFREE the variable in T that has the smallest total distance from feasibility (the smallest ASUM). Go to step 2.

STEP 11

If NFREE is empty, go to step 21; otherwise no feasible completion of the partial solution represented by NFREE has a smaller value than the current ZMIN, so go to step 16.

STEP 12

The variables in NFREE with their specified values, along with the variables in FREE set equal to 0, form a complete solution. Go to step 13.

STEP 13

If Z < ZMIN, go to step 14; otherwise, go to step 15.

STEP 14

Set ZMIN = Z. Save the complete solution and go to step 15.

STEP 15

Back track. If DFREE is empty, the feasible solution X(I) = 0 (I = 1, 2, ..., N) is optimal, so go to step 20; otherwise go to step 16.

STEP 16

If the last element in DFREE is negative, go to step 18; otherwise go to step 17. The rightmost element in DFREE is considered to be the last element in DFREE.

STEP 17

Make the last (rightmost) element in DFREE negative and go to step 2. The variable corresponding to the last element has been specified to be 1 (corresponding subscript in DFREE has been positive). We now specify the variable to be zero (change the sign of the last element in DFREE to minus).

STEP 18

If all elements in DFREE are negative, an optimal solution has been reached, so go to step 20; otherwise go to step 19.

STEP 19

Make the rightmost positive element in DFREE negative and remove the remaining elements to the right from DFREE. Add the dropped elements to FREE. Go to step 2.

STEP 20

The complete solution corresponding to ZMIN is optimal. If

ZMIN = 10^{10}, no feasible solution. Print results. Stop.

STEP 11

No feasible solution to the problem. Stop.

The flow chart for the algorithm is given in Fig. 3.4.1.

Computer Program for the algorithm :

The program used from reference (5) is given as Appendix (A). The following notations are used in the program.

M	-	Total number of constraints
N	-	No. of variables
MLLT	-	Number of < OR = constraints
MGLT	-	Number of > OR = constraints
MET	-	Number of = constraints
MTYPE	-	0 for minimization problem
		1 for maximization problem
CODE(I)	=	0 if < OR = constraint
		1 if > OR = constraint
		2 if = Constraint
B(I)	=	Constant in the constraint
A(I, J)	=	Coefficients in the constraints
C(J)	=	Cost coefficients of objective function.

The program will solve both maximization and minimization problems with positive or negative cost coefficients. The problem is presented to the program in its original form and the program takes care of the necessary bookwork to get it in the standard

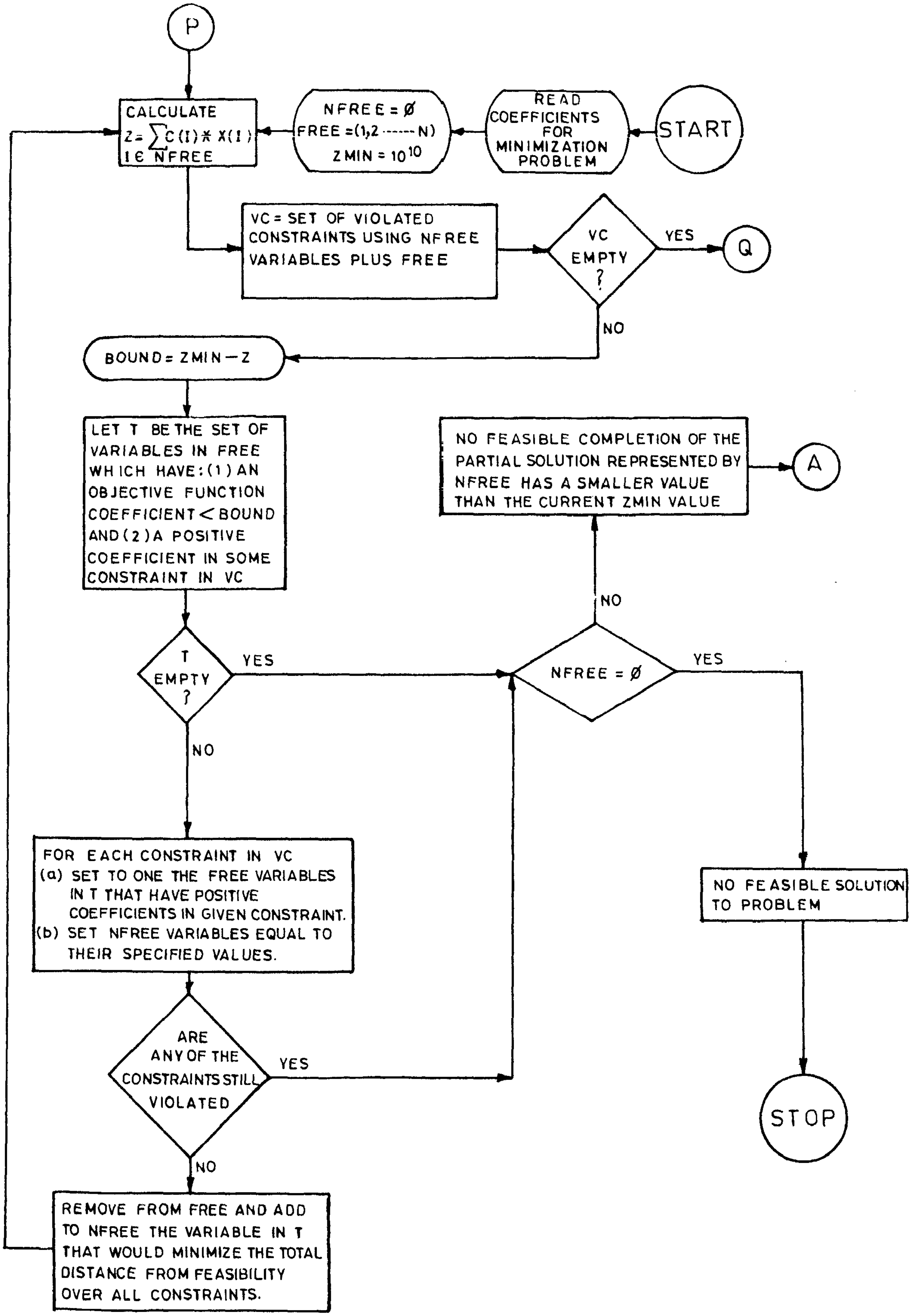

FIGURE. 3.4.1 FLOWCHART FOR ALGORITHM

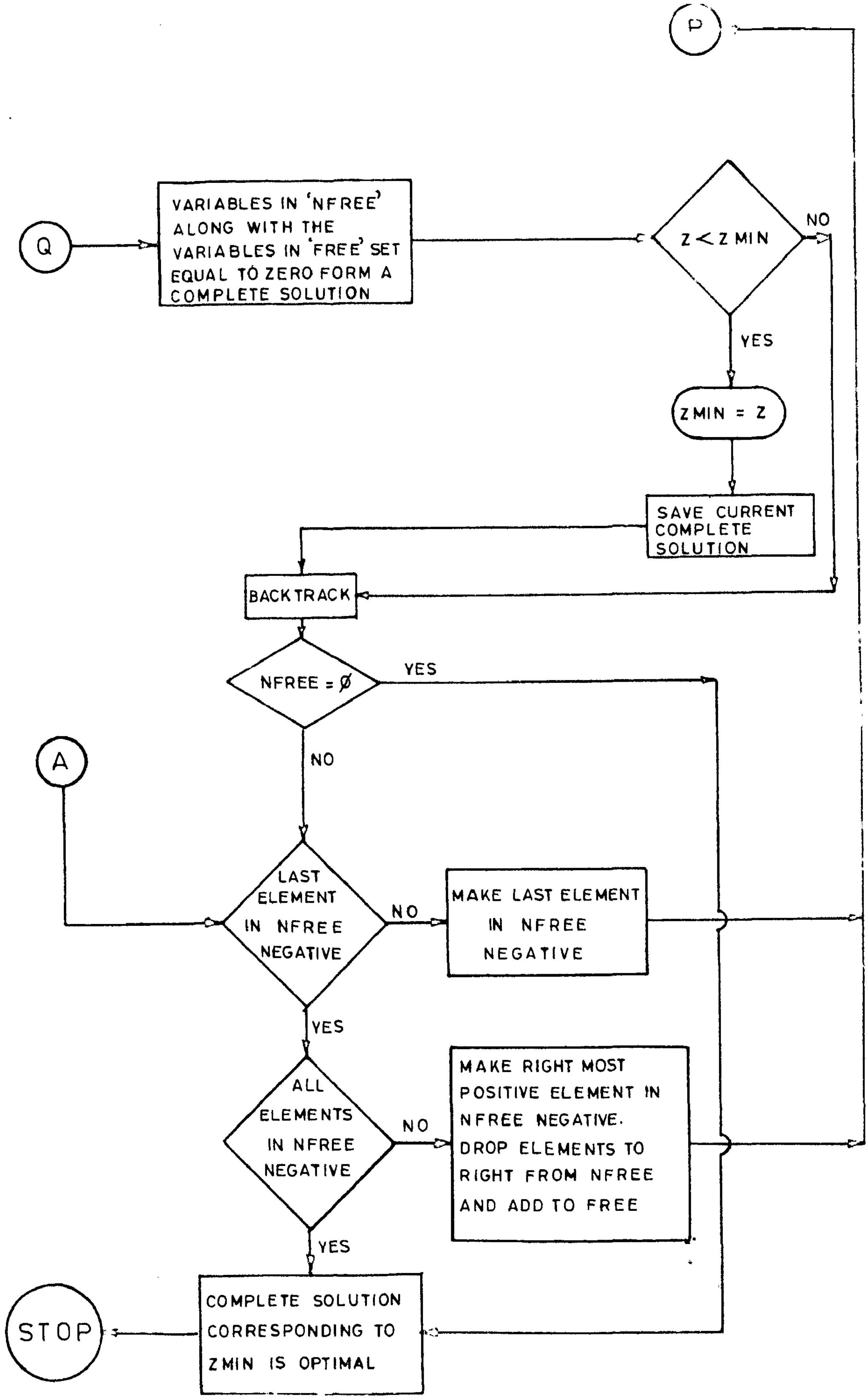

FIGURE 3.4.1 FLOWCHART FOR ALGORITHM

form for the implicit enumeration algorithm. The only requirement is that the objective function and/or the constraints must be multiplied by proper constants to make all the input data in the integer form.

<u>Subroutine</u>

The integer programming has N(T + 1) decision variables and (M + N (T + 1)) constraints. Thus even a small problem inviting 4 projects, 3 demands and 10 years planning horizon would have 44 decision variables and 33 constraints. The constraint coefficients (A(I, J) will be 44 x 33 = 1452.

This enormous data to be punched, and checked will involve a long time. Lot of calculation involved in finding out the cost coefficient of the objective function, using equation (3.3.2) and the demand in each year. Further almost the whole process has to be repeated if the problem need to be solved for different planning horizon.

To ease this preliminary work, a subroutine is developed by the author and is given in appendix (B). It is enough to feed the data direct from the problems. The subroutine calculates and prepares the data in proper form that is acceptable to the main program.

3.5 Numerical Example No. 1

A four project sequencing and scheduling problem as given in Table 3.5.1 is taken up as a sample problem. The demand projection for the problem is assumed to be of the form

$$D_j(t) = a_j t^2 + b_j t \qquad t = 0, 1.., 10 \quad \ldots \quad (3.5.1)$$

TABLE 3.5.1

Project No. i	Capital cost C_i (10^6 $)	OUTPUTS (10^3 acre-foot) Q_{i1}	Q_{i2}	Q_{i3}	Demand Projection Parameters j	a_j	b_j
1	10.500	2.0	12.0	220.0	1	- 0.222	2.456
2	57.600	36.0	208.0	2360.0	2	0.015	0
3	25.500	16.0	45.0	925.0	3	2.071	0
4	33.200	13.0	82.0	2670.0			

The unit of $D_j(t)$ is 10^3 acre-foot. $D_j(t)$ is calculated for all the three types of demands (j = 1, 2, 3) and for 10 years planning period. Both $D_j(t)$ as well as C_{ik} using equation (3.3.2) are calculated in the subroutine. A_{ij} correspond to A(I, J) in the program, D(t) to B(I) and C_{ik} to C(J).

Give all these information, the problem is to select projects out of the four to supply the demands and fix up the date of their construction in sequence to achieve overall minimum present value of the total cost.

Numerical Example No. 2

A two project problem with 3 types of demands is taken for solution. The capital cost, outputs and demand coefficients are given in Table 4.4.2.

Project No. i	Capital cost $ 10^6	OUTPUT 10^3 acre-feet			Demand projection parameters		
		q_{i1}	q_{i2}	q_{i3}	j	a_j	b_j
1	37.900	88.0	33.0	2300.0	1	0	52.592
2	214.000	2004.0	495.0	20700.0	2	- 0.165	15.927
					3	- 7.603	836.364

3.6 <u>Results</u> :

The results obtained from the computer run is given in Table 3.6.1 for example 1. The program prints the results of the optimal value of the function ZMIN and the values of the variables yielding ZMIN. ZMIN is the present value of the optimal cost of the projects, from the values of the variables yielding the value 1, the sequencing of the projects as well as the completion dates of construction can be derived directly.

The problem that was taken up results in ZMIN equal to $ 46 x 10^6. The variables yielding 1 are x_{24} and x_{42}. This means, project No. 3 must be ready in the first year and project No. 4 be ready on the eighth year, from the beginning of the planning period. The interpretation is as follows :

TABLE 3.6.1.

The variable x_{ik}, $i = 1, 2, \ldots 4$ and $k = 0, 1, \ldots, 10$ are converted to single subscript variables totalling 44.(4 x 11) and numbered in that order as x_1, x_2 etc. $x_{1,0}$ corresponds to x_1, $x_{1,10}$ corresponds to x_{11}, $x_{2,0}$ to x_{12} etc. Thus the results x_{24} and x_{42} correspond to $x_{3,1}$ and $x_{4,8}$ from which it is known that project No. 3 in the first year and project No. 4 in eighth year must be ready.

This problem needed 3 minutes 40.89 seconds CPU time DEC-20 computer system.

Results obtained for example 2 from computer run is given in Table 3.6.2 It can be seen that the optimum cost is 0 203 x 10^6. Only one project i.e. No. 2 is to be completed in the first year. The CPU time taken is 13.51 seconds.

[illegible]

VARIABLE	1	[illegible]
VARIABLE	2	[illegible]
VARIABLE	3	[illegible]
VARIABLE	4	[illegible]
VARIABLE	5	[illegible]
VARIABLE	6	[illegible]
VARIABLE	7	[illegible]
VARIABLE	8	[illegible]
VARIABLE	9	[illegible]
VARIABLE	[illegible]	[illegible]
VARIABLE	11	[illegible]
VARIABLE	12	[illegible]
VARIABLE	13	[illegible]
VARIABLE	14	[illegible]
VARIABLE	15	[illegible]
VARIABLE	16	[illegible]
VARIABLE	17	[illegible]
VARIABLE	[illegible]	[illegible]
VARIABLE	19	[illegible]
VARIABLE	2[illegible]	[illegible]
VARIABLE	21	[illegible]
VARIABLE	22	[illegible]

[illegible] ...CTIO[illegible] IS 203563

TABLE . 3.6.2.

CHAPTER - IV

SEQUENCING AND SCHEDULING OF WATER RESOURCES SYSTEMS - STOCHASTIC CASE

4.1 Nature of the Problem

So far we assumed the future demand of water is deterministic. But in reality it must be regarded as a stochastic variable, because the future demand cannot be predicted with certainty. The deterministic function D(t) is replaced by a series of stochastic variables $\tilde{D}(t)$, which of course will influence the optimal division to be taken. We can define

$$P\left\{\tilde{D}(t) \leq C\right\} = F_t(C)$$

i.e. $F_t(C)$, $t = 0, 1, \ldots, T$ is the probability distribution function of the demand in time period t. On this basis, we could try to set up the model for future requirements.

One possibility is to assume $\tilde{D}(t)$, $t = 0, \ldots, T$ independent, normally distributed random variable with mean μ_t and standard deviation σ_t ; μ_t and σ_t in general being increasing function of t. We can formulate the optimal scheduling under stochastic demand as a 'Chance Constraint Model'.

4.2 The Chance Constraint Model

When the consequences of shortage is difficult to evaluate in economic terms, the chance constraint linear programming represents an alternate way of taking stochastic future into account. As the name suggests this technique can be used to solve problems involving chance constraints, that is constraints having finite probability of being violated. The constraints are permitted to be violated by a specified probability. The decision makers have to express their preferences against shortage in the shape of confidence level $1 - \alpha$, defined by requiring that the probability of shortage in a given time period must not exceed α.

The constraint equation in the linear programming problem is

$$\sum_{j=1}^{n} a_{ij} x_j \geqslant b_i, \quad i = 1, 2, \ldots m. \qquad \ldots \quad (4.2.1)$$

Let p_i be the probability of that this constraint could be violated. Let $\bar{b}_i$ and $var(b_i)$ denotes the mean and variance of the normally distributed random variable, b_i; that is,

$$P\left[\sum_{j=1}^{n} a_{ij} x_j \leq b_i\right] \leqslant p_i \qquad \ldots \quad (4.2.2)$$

This can be written as ,

$$P\left[\sum_{j=1}^{n} \frac{a_{ij} X_j - \bar{b}_i}{\sqrt{Var(b_i)}} \leqslant \frac{b_i - \bar{b}_i}{\sqrt{Var(b_i)}}\right] \leqslant p_i$$

or,

$$1 - P\left[\frac{\sum_{j=1}^{n} a_{ij} X_j - b_i}{\sqrt{Var(b_i)}} \leqslant \frac{b_i - b_i}{\sqrt{Var(b_i)}}\right] \geqslant 1 - p_i$$

or

$$P\left[\frac{b_i - \bar{b}_i}{\sqrt{\operatorname{Var}(b_i)}} \leq \frac{\sum_{j=1}^{n} a_{ij} X_j - \bar{b}_i}{\sqrt{\operatorname{Var}(b_i)}}\right] \geqslant 1 - p_i \qquad \ldots \quad (4.2.3)$$

If E_i represents value of the standard normal ~~variate~~ variate at which $(1 - p_i) = \phi(E_i)$, can be stated the equation (4.2.3) according to definition of probability

$$\phi\left[\frac{\sum_{j=1}^{n} a_{ij} - \bar{b}_i}{\sqrt{\operatorname{Var}(b_i)}}\right] \geqslant \phi(E_i) \qquad \ldots \quad \ldots \quad (4.2.4)$$

which ϕ is the cumulative probability of the distribution

This is possible only if

$$\frac{\sum_{j=1}^{n} a_{ij} X_j - \bar{b}_i}{\sqrt{\operatorname{Var}(b_i)}} \geqslant E_i \qquad \ldots \quad (4.2.5)$$

or,

$$\sum_{j=1}^{n} a_{ij} X_j - \bar{b}_i - E_i\sqrt{\operatorname{Var}(b_i)} \geqslant 0 \qquad \ldots \quad (4.2.6)$$

Thus the stochastic linear programming problem can be stated in its equivalent deterministic problem using equation (4.2.6).

We can introduce this theory to our Integer Programming formulation of the previous section as follows.

$$\text{Minimize} \quad \sum_{i=1}^{N} C_{ik}\, x_{ik} \quad \text{subject to} \quad \ldots \qquad (4.2.7)$$

$$\sum_{k=0}^{T} x_{ik} \leq 1 \qquad \ldots \qquad (4.2.8)$$

$$\sum_{i=1}^{N} \sum_{k=0}^{t} Q_{i1}\, x_{ik} - \bar{D}_1(t) - \sigma_{D_1(t)} . E_i \geq 0$$

$$\sum_{i=1}^{N} \sum_{k=0}^{t} Q_{i2}\, x_{ik} - \bar{D}_2(t) - \sigma_{D_2(t)} . E_i \geq 0$$

$$\vdots \qquad\qquad \vdots \qquad\qquad (4.2.9)$$

$$\sum_{i=1}^{N} \sum_{k=0}^{t} Q_{iM}\, x_{ik} - \bar{D}_M(t) - \sigma_{D_M(t)} . E_i \geq 0$$

$$x_{ik} = 0 \text{ or } 1 \; ; \; \forall\, i, k \qquad \ldots \qquad (4.2.10)$$

and $C_{ik} = C_i e^{-rk}$

4.3 <u>Solution Technique</u>

Since the stochastic case sequencing and scheduling problem is formulated in 0-1 integer programming model, the Implicit Enumeration algorithm is used to solve this type of problems. The listing of the ~~program is given in~~ Appendix (C). The subroutin used in the deterministic case problem, mentioned in Chapter III is not used here. The data is directly entered in the main program itself.

4.4 Numerical Example

The 4 projects example is again taken and given below in Table 4.4.1.

Table 4.4.1

Project No.	Capital cost (C_i)($ x 10^6)	Output in (acre feet x 10^3) Q_{i1}	Q_{i2}	Q_{i3}
1	10.500	2.0	12.0	220.0
2	57.600	56.0	208.0	2360.0
3	25.500	16.0	45.0	925.0
4	33.200	13.0	82.0	2300.0

The mean demand during every year and for every category of demand are assumed. Three demand categories are assumed. The demands are given in Table 4.4.2.

Table 4.4.2

Mean Demand Projections over 10 years

$D_1(0) = 0$	$D_2(0) = 0$	$D_3(0) = 0$
$D_1(1) = 1.5$	$D_2(1) = 0.07$	$D_3(1) = 1.2$
$D_1(2) = 3$	$D_2(2) = 0.27$	$D_3(2) = 5$
$D_1(3) = 3.5$	$D_2(3) = 0.6$	$D_3(3) = 11$
$D_1(4) = 5.5$	$D_2(4) = 1$	$D_3(4) = 19.3$
$D_1(5) = 7$	$D_2(5) = 1.6$	$D_3(5) = 30$
$D_1(6) = 8.3$	$D_2(6) = 2.5$	$D_3(6) = 44.5$
$D_1(7) = 9.3$	$D_2(7) = 3.4$	$D_3(7) = 60$

$D_1(8)$	=	11	$D_2(8)$	=	4.3	$D_3(8)$	=	80
$D_1(9)$	=	12	$D_2(9)$	=	5.5	$D_3(9)$	=	100
$D_1(10)$	=	13.2	$D_2(10)$	=	6.7	$D_3(10)$	=	120

The planning horizon, T = 10 years, the interest rate is 5%. The units of D(t) and a_{ij} are 10^3 acre-foot and 10^3 acre-foot respectively. The standard deviation is taken as one tenth of the mean val ue of demand.

It is further assumed that the constraints are to be satisfied with a probability of at least 0.9.

$$\phi(E_i) = 0.9$$

$\therefore$ $E_i = 1.3$ from the table of standard normal variate.

With all these data given and assumed, the problem to select and sequencing of the projects out of the four given in order to achieve the minimum present value of the cost.

4.5 <u>Results</u>

The results obtained from the computer is given in Table 4.5.1 It can be seen the optimum cost is \$ 24.25×10^6. The projects selected are project No. 3 to be completed at the beginning of second year. Only one project (Project No. 3) is selected, which is sufficient to meet all the demands till 10 years time.

This problem required 1 minute 48.84 seconds CPU time of DEC-20 computer system.

TIMAL SOLUTION

VARIABLE	1	HAS	VALUE	OF	0
VARIABLE	2	HAS	VALUE	OF	0
VARIABLE	3	HAS	VALUE	OF	0
VARIABLE	4	HAS	VALUE	OF	0
VARIABLE	5	HAS	VALUE	OF	0
VARIABLE	6	HAS	VALUE	OF	0
VARIABLE	7	HAS	VALUE	OF	0
VARIABLE	8	HAS	VALUE	OF	0
VARIABLE	9	HAS	VALUE	OF	0
VARIABLE	10	HAS	VALUE	OF	0
VARIABLE	11	HAS	VALUE	OF	0
VARIABLE	12	HAS	VALUE	OF	0
VARIABLE	13	HAS	VALUE	OF	0
VARIABLE	14	HAS	VALUE	OF	0
VARIABLE	15	HAS	VALUE	OF	0
VARIABLE	16	HAS	VALUE	OF	0
VARIABLE	17	HAS	VALUE	OF	0
VARIABLE	18	HAS	VALUE	OF	0
VARIABLE	19	HAS	VALUE	OF	0
VARIABLE	20	HAS	VALUE	OF	0
VARIABLE	21	HAS	VALUE	OF	0
VARIABLE	22	HAS	VALUE	OF	0
VARIABLE	23	HAS	VALUE	OF	0
VARIABLE	24	HAS	VALUE	OF	1
VARIABLE	25	HAS	VALUE	OF	0
VARIABLE	26	HAS	VALUE	OF	0
VARIABLE	27	HAS	VALUE	OF	0
VARIABLE	28	HAS	VALUE	OF	0
VARIABLE	29	HAS	VALUE	OF	0
VARIABLE	30	HAS	VALUE	OF	0
VARIABLE	31	HAS	VALUE	OF	0
VARIABLE	32	HAS	VALUE	OF	0
VARIABLE	33	HAS	VALUE	OF	0
VARIABLE	34	HAS	VALUE	OF	0
VARIABLE	35	HAS	VALUE	OF	0
VARIABLE	36	HAS	VALUE	OF	0
VARIABLE	37	HAS	VALUE	OF	0
VARIABLE	38	HAS	VALUE	OF	0
VARIABLE	39	HAS	VALUE	OF	0
VARIABLE	40	HAS	VALUE	OF	0
VARIABLE	41	HAS	VALUE	OF	0
VARIABLE	42	HAS	VALUE	OF	0
VARIABLE	43	HAS	VALUE	OF	0
VARIABLE	44	HAS	VALUE	OF	0

THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS 2425

TABLE 4.5.1

CHAPTER - V

OPTIMAL SCHEDULING OF RESERVOIRS

5.1 Nature of the problem

A storage reservoir is one of the most important elements in a water resource system. The design of reservoirs must take into account the stochastic aspect of the inflows. The optimal development of water resources is conditional also on the establishment of appropriate operating policies. The operating policy is a time schedule of releases from reservoirs, of pumpages from acquifers and/or reservoirs and of acquifer recharge operations. The establishment of such schedules which indicate quantities of water to be effected at different points in time, is an important problem in water resources engineering. The problem is, of course, the selection of the operating procedures that will best achieve the stated objectives of the development scheme.

The operating rules were practised on the basis of personal judgement alone. No alternate procedure were tested. The rules were (1) store all inflow unless needed to meet a target output, (2) when available, release water from storage to fulfil immediate needs, (3) study all damaging floods on record in the flood control analysis.

The objective of operation of the reservoir may be meeting the required demand and in doing so achieve cost minimization or maximization of benefits. The operator of the system looks at the state of the system and has to decide on what releases and transfers have to be made at that time. The state of the system means the level of water in the reservoir and the season in which the release has to be made. The operating policy can be seen to to the set of decisions of each possible state of the system.

In a complex system it may not be at all obvious how to operate the system to achieve the required goals. The system becomes complex when two or more reservoirs are linked and operated. Linked reservoirs should perform better than those worked singly. The objective of operating linked reservoirs is to increase the complementarity among them.

Furthermore it is recognised that operating procedures are sequential decision problems and have to be treated as such. These problems take account of the fact that a decision is likely to have consequences that extend over a considerable period of time. The consequence of a decision regarding the release of water from a reservoir is not only the yield of that release but also the yields from a sequence of releases following the first one. The optimisation technique of dynamic programming is well suited to solving sequential decision problems. But considering the uncertainty of inflows into the reservoir, a variation of this technique called stochastic dynamic programming is very useful method in solving such problems. The details of this can be seen[16].

5.2 Solution Technique

The natural inflow to the reservoirs are probabilistic in nature. The process of change of states of reservoirs (levels) can be termed as a semi Markovian process. The probability of transition from state i at stage n, to state j at stage n-1 is denoted by p_{ij}. The set of probabilities for N states in the system is conveniently represented by the transition matrix P,

$$P = \begin{bmatrix} p_{11} \cdots\cdots & p_{1j} \cdots\cdots & p_{1N} \\ \vdots & & \\ p_{i1} \cdots\cdots & p_{ij} \cdots\cdots & p_{iN} \\ \vdots & & \\ p_{N1} \cdots\cdots & p_{Nj} \cdots\cdots & p_{NN} \end{bmatrix} \qquad \ldots (5.2.1)$$

The elements in row i are the probabilities of transition to stage j. Thus,

$$0 \leq p_{ij} \leq 1 \qquad \text{and} \qquad \sum_j p_{ij} = 1$$

The probability of being at stage j at stage n-1 denoted by $\pi_{n-1}(j)$ is determined from multiplying the probability and of being in state i at stage n by the transition probability p_{ij} and then summing over all states at stage n. Thus,

$$\pi_{n-1}(j) = \sum_{i=1}^{N} p_{ij}\,\pi_n(i) \qquad j = 1, \ldots, N$$
$$n = 1, \ldots, N$$

With every change of state, a reward or cost is created. Corresponding to P there is a cost matrix

$$C = \begin{bmatrix} c_{11} \cdots & c_{1j} \cdots\cdots & c_{1N} \\ \vdots & \vdots & \\ c_{i1} \cdots & c_{ij} \cdots\cdots & c_{iN} \\ \vdots & & \\ c_{N1} \cdots & c_{Nj} \cdots\cdots & c_{NN} \end{bmatrix} \quad \ldots \quad (5.22)$$

The total expected cost from a ,n stage process expressed recursively,

$$c_n(i) = \sum_{j=1}^{N} p_{ij}\left[c_{ij} + c_{n-1}(j)\right]; \quad n = 2, \ldots N \quad \ldots \quad (5.2.3)$$

If there are a number of decisions available at each state, we can introduce the decision making into the multistage Markovian Model by permitting a choice among several transitions and cost matrices. A decision variable $d_n = k$, $k = 1, \ldots K$ designates the choice of the kth transition matrix and kth cost matrix. The probability of transition is denoted by $p_{ij}(d_n)$ and the cost by $c_{ij}(d_n)$. The expected cost from n stages starting in state i, is then

$$c_n(i, d_n, \ldots d_1) = \sum_{j=1}^{N} p_{ij}(d_n)\left[c_{ij}(d_n) + c_{n-1}(j, d_{n-1}, \ldots d_1)\right] \quad \ldots \quad (5.2.$$

Applying recursive optimization to minimize the expected cost (or maximize the reward) and denoting the minimized expected cost from n stages by $f_n(i)$ we have

$$f_n(i) = \min \sum_{j=1}^{N} p_{ij}(d_n)\left[c_{ij}(d_n) + f_{n-1}(j)\right] \qquad \ldots \quad (5.2.5)$$

$$d_n = 1, 2, \ldots k$$

The term $\sum_{j=1}^{N} p_{ij}(d_n)\, c_{ij}(d_n)$ is just the expected cost from stage n, and is denoted by $q_i(d_n)$

Howard[7] called the solution of this equation as ' Value Iteration '. The equation are solved again and again till the $(n+1)^{th}$ stage value is same as that of nth stage. But there are some reservation about this criterion. It was stated that major changes can occur after this point. The policy iteration that will be described in succeeding paras is found to be a better technique to arrive at the optimum policy.

Instead of value iteration, where the convergence is not certain, it is better to have a direct search of all possibilities and compare the expected costs and arrive at the solution which gives the minimum value. In that case, for N states and K alternatives for each state, we have to reach K^N possibilities which is very laborious even for 4 states and 3 alternatives problem. The answer to this question is policy iteration. In this out of the various alternatives available one set of alternatives are chosen as preliminary policy. For this set of alternative the optimum cost is calculated using the equation

$$C_i(n+1) = q_i + \sum_{j=1}^{N} p_{ij}\, C_j(n) \qquad i = 1, 2, \ldots, N \qquad \ldots (5.2.6)$$

Then in each state, the expected cost for all alternatives are calculated. If the cost is minimised further for any alternative k in the state i, then for the alternative in the preliminary policy, the new alternatives are taken as the new policy for the process. Again this new policy is used to evaluate the expected gain and policy improvement. This is continued till it is found there is no improvement in the decision already taken and the final policy is the optimum long term policy. The iteration cycle process consisting of policy evaluation and policy improvement is known as Policy Iteration[7].

5.3 <u>Mathematical Model</u> :

The state space is specified by levels in the reservoir (i) and the seasons (S). The transition matrix P

Seasons

$$P = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} & S_1 & & \\ & & S_2 & \\ & & & S_3 \\ S_4 & & & \end{array} \right] \end{array} \qquad \ldots (5.3.1)$$

Seasons

Since the transition can take place from one season to the succeeding seasons the entries are zeroes other than the submatrices. However, the submatrices $\bar{S}$ will have non-zero entries p_{ij}

The stochastic dynamic programming equation considering the discounting factor for present day cost for the expected cost is

$$C_i(n+1) = q_i + \sum_{j=1}^{n} p_{ij}(d_n)\, C_j(n) \qquad \ldots \qquad (5.3.2)$$

where d_n is the subscript denoting the decision variable and β is discounting factor.

By matrix notation equation (5.3.2) can be written as

$$\begin{bmatrix} \bar{C}_1(n+1) \\ \bar{C}_2(n+1) \\ \bar{C}_3(n+1) \\ \bar{C}_4(n+1) \end{bmatrix} = \begin{bmatrix} \bar{q}_1 \\ \bar{q}_2 \\ \bar{q}_3 \\ \bar{q}_4 \end{bmatrix} + \beta \begin{bmatrix} & \bar{U}_1 & & \\ & & \bar{U}_2 & \\ & & & \bar{U}_3 \\ \bar{U}_4 & & & \end{bmatrix} \begin{bmatrix} \bar{C}_1(n) \\ \bar{C}_2(n) \\ \bar{C}_3(n) \\ \bar{C}_4(n) \end{bmatrix} \qquad \ldots \qquad (5.3.3)$$

where $\bar{C}_1$, $\bar{C}_2$, $\bar{C}_3$, $\bar{C}_4$ are vectors of cost for the states in each season.

Expanding equation (5.3.3) and putting $\bar{T} = \beta\bar{U}$ gives,

$$\begin{aligned} \bar{C}_1(n+1) &= \bar{q}_1 + \bar{T}_1\bar{C}_2(n) \\ &\vdots \\ \bar{C}_4(n+1) &= \bar{q}_4 + \bar{T}_4\bar{C}_1(n) \end{aligned} \qquad \ldots \qquad (5.3.4)$$

As n becomes large, $C_r(n+1) = C_r(n)$ and equation (5.3.4) can be written,

$$\begin{aligned} \bar{C}_1 &= \bar{q}_1 + \bar{T}_1\bar{C}_2 \\ &\vdots \\ \bar{C}_4 &= \bar{q}_4 + \bar{T}_4\bar{C}_1 \end{aligned} \qquad \ldots \qquad (5.3.5)$$

Or, we can write,

$$\bar{C}_1 = \bar{q}_1 + \bar{T}_1(\bar{q}_2 + \bar{T}_2\,\bar{q}_3 + \bar{T}_3(\bar{q}_4 + \bar{T}_4\bar{C}_1)$$

$$= \bar{q}_1 + \bar{T}_1\bar{q}_2 + \bar{T}_1\bar{T}_2\bar{q}_3 + \bar{T}_1\bar{T}_2\bar{T}_3\bar{q}_4 + \bar{T}_1\bar{T}_2\bar{T}_3\bar{T}_4\bar{C}_1$$

Putting

$$\bar{R}_1 = \bar{T}_1\bar{T}_2..\bar{T}_4 = \bar{S}_1\bar{S}_2\bar{S}_3\bar{S}_4 \text{ and} \quad \ldots \quad (5.3.6)$$

$$\bar{Q}_1 = \bar{q}_1 + \bar{T}_1(\bar{q}_2 + \bar{T}_2(\bar{q}_3 + \bar{T}_3\bar{q}_4) \quad \ldots \quad (5.3.7)$$

we obtain

$$\bar{C}_1 = \bar{Q}_1 + \bar{R}_1\bar{C}_1$$

Or,

$$\bar{C}_1 = (\bar{I} - \bar{R}_1)^{-1}\,\bar{Q}_1 \quad \ldots \quad (5.3.8)$$

The equation (5.3.8) can be solved for $\bar{C}_1$

The equation (5.3.8) gives the constant present day total cost for the system for the decisions (operating rules) assumed initially. From $\bar{C}_1$ we can find the cost of $\bar{C}_2$, $\bar{C}_3$ etc. using equation (5.3.5)

For normal iterative stochastic dynamic programming, when there are k policy options are available in each state, the optimal decision is one which gives $f_n(i)$,

$$f_n(i) = \text{Min} \sum_{j=1}^{N} p_{ij}(d_n)\left[c_{ij}(d_n) + f_{n-1}(j)\right] \quad \ldots \quad (5.3.9)$$

$$d_n = 1,\ 2,\ \ldots\ k$$

where $\sum_{j=1}^{n} D_{ij}(d_n)\ C_{ij}(d_n) = q_i(d_n)$

Substituting the values found for season', C_1, for $f_{n-1}(j)$ in equation (5.3.9) gives the minimum cost $\bar{C}_4$. If these values differ from that for $\bar{C}_4$ found from equation (5.3.5), the new cost and the corresponding decision is used to find $\bar{C}_3$ and $\bar{C}_2$. Once the better policy is arrived at the procedure is carried over from the beginning. When the values for $\bar{C}_r$ found from equations (5.3.5) and (5.3.9) are same the procedure is stopped.

5.4 <u>Example problem</u> :

We shall take a simple system of a single reservoir and acquifer. The data assumed are as below :

Reservoir volume V_a = 30,000 mg (million gallons)

Reservoir (contents) States SA = 0, 10000, 20,000, 30,000 m.g.

Release decisions RA = 5000, 8000, 12000 mg.

Maximum Release from acquifer RB = 10,000 mg.

<u>Costs</u> :

Release from reservoir = Rs. 10,000/m.g.

Release from acquifor = Rs. 2000/m.g.

Penalty cost as deficit to demand = Rs. 2500/m.g.

Spillage cost = Rs. 0/m.g.

Present worth factor or Discount factor = ~~0.985~~ 0.95 per month

Inflows to the Reservoir and the Corresponding Probabilities etc. in two seasons

Table 5.1

Season 1		Season 2	
Inflow (m.g.)	Probability	Inflow (m.g.)	Probability
5000	0.3	1000	0.2
6000	0.5	4000	0.6
10000	0.2	8000	0.2

Demands are

Table 5.2

Season 1 (mg.)	Season 2 (mg.)
12,000	16,000

The release policy decisions for the two seasons are arbitrarily taken, for both acquifer and the reservoir, to meet the given demand. They are :

Table 5.3

Season 1			Season 2		
Decision number	Release from reservoir (m.g.)	Release from acquifer (m.g.)	Decision Number	Release Reservoir (m.g.)	Release acquifer from acquifer (m.g.)
1	5000	7000	4	6000	10000
2	8000	4000	5	8000	8000
3	12000	0	6	12000	4000

From the data, the calculation for each season, state (reservoir volume) done for total expected cost of operation, the end state (reservoir volume at the end of the season) and the associated probability. If the end state is not the same as any of the assumed state (0, 10,000, 20,000, 30,000), the probability is apportioned to the nearest states. The calculations are tabulated below :

<u>Season 1</u> : Demand = 12,000 m.g. (see table 5.1 and 5.2). Required release from reservoir and acquifer for the decision number be seen from Table 5.3. All costs in thousands of rupees.

TABLE 5.4

States	Release Decision Number	Inflow	Available water in Reservoir	Actual Release (Reservoir)	Deficit	Deficit cost	Reservoir Release cost	Acquifer release cost	Total cost (TC)	Prob. (i)	End State 4 – 5	End state prob. apportion $\rho_{i,j}$	Expect cost over transition (q_i) Σ(TC)	Transition probabilities $p_{i,j}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	5000	5000	5000	0	0	5000	14000	19000	0.3	0	$P_0 = 0.3$		$p_{o,o} = 0.71$
		8000	8000	5000	0	0	5000	14000	19000	0.5	3000	$P_{o,o} = 0.35$ $P_{o,1} = 0.15$	19000	$p_{o,1} = 0.29$ $p_{o,2} = 0$
		120000	12000	5000	0	0	5000	14000	19000	0.2	7000	$P_{o,o} = 0.06$ $P_{o,1} = 0.14$		$p_{,3} = 0$
	2	5000	5000	5000	5000	7500	5000	8000	20500	0.3	0	$P_{o,o} = 0.3$		$p_{o,o} = 0.92$
		8000	8000	8000	0	0	8000	8000	16000	0.5	0	$P_{o,o} = 0.5$	17350	$p_{o,1} = 0.08$
		12000	12000	8000	0	0	8000	8000	16000	0.2	4000	$P_{o,o} = 0.12$ $P_{o,1} = 0.08$		$p_{o,2} = 0.0$ $p_{o,3} = 0$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		5000	5000	5000	7000	17500	5000	0	22500	0.3	0	$p_{o,1} = 0.3$		$p_{o,o} = 1$
	3	8000	8000	8000	4000	10000	8000	0	22000	0.5	0	$p_{o,o} = 0.5$	20150	$p_{o,1} = 0$
		12000	12000	12000	0	0	12000	0	12000	0.2	0	$p_{o,o} = 0.2$		$p_{o,2} = 0$ $p_{o,3} = 0$
		5000	15000	5000	0	0	5000	14000	19000	0.3	10000	$p_{1,1} = 0.3$		$p_{1,o} = 0$
	1	8000	18000	5000	0	0	5000	14000	19000	0.5	13000	$p_{1,1} = 0.35$ $p_{1,2} = 0.15$	19000	$p_{1,1} = 0$ $p_{1,2} = 0.25$
		10000	20000	5000	0	0	5000	14000	19000	0.2	15000	$p_{1,1} = 0.1$ $p_{1,2} = 0.1$		$p_{1,3} = 0$
10000		5000	15000	8000	0	0	8000	8000	16000	0.3	7000	$p_{1,o} = 0.09$ $p_{1,1} = 0.21$		$p_{1,o} = 0.09$
	2	8000	18000	8000	0	0	8000	8000	16000	0.5	10000	$p_{1,1} = 0.5$	14000	$p_{1,1} = 0.87$ $p_{1,2} = 0.04$ $p_{1,3} = 0$
		10000	20000	8000	0	0	8000	8000	16000	0.2	12000	$p_{1,1} = 0.16$ $p_{1,2} = 0.04$		
		5000	15000	12000	0	0	12000	0	12000	0.3	3000	$p_{1,o} = 0.21$ $p_{1,1} = 0.09$		$p_{2,o} = 0$ $p_{2,1} = 0$
	3	8000	1000	12000	0	0	12000	0	12000	0.5	6000	$p_{1,o} = 0.2$ $p_{1,1} = 0.3$	12000	$p_{2,2} = 0.75$ $p_{2,3} = 0.25$
		10000	20000	12000	0	0	12000	0	12000	0.2	8,000	$p_{1,o} = 0.04$ $p_{1,1} = 0.16$		

1	2	3	4	11	12	13	14	15
20000	1	5000	25000	0.3	20000	$p_{1,2} = 0.3$	19000	$p_{2,0} = 0$
		8000	28000	0.5	23000	$p_{2,3} = 0.25$ $p_{2,2} = 0.15$		$p_{2,1} = 0$ $p_{2,2} = 0.75$
		10000	30000	0.2	25000	$p_{2,2} = 0.1$ $p_{2,3} = 0.1$		$p_{2,3} = 0.25$
	2	5000	25000	0.3	17000	$p_{2,1} = 0.09$ $p_{2,2} = 0.21$	16000	$p_{2,0} = 0$
		8000	28000	0.5	20000	$p_{2,2} = 0.5$		$p_{2,1} = 0.09$ $p_{2,2} = 0.87$
		10000	30000	0.2	22000	$p_{2,2} = 0.16$ $p_{2,3} = 0.04$		$p_{2,3} = 0.04$
	3	5000	25000	0.3	12000	$p_{2,1} = 0.21$ $p_{2,2} = 0.09$	12000	$p_{2,0} = 0$ $p_{2,1} = 0.45$
		8000	28000	0.5	16000	$p_{2,1} = 0.2$ $p_{2,2} = 0.3$		$p_{2,2} = 0.55$ $p_{2,3} = 0$
		10000	30000	0.2	18000	$p_{2,1} = 0.04$ $p_{3,2} = 0.16$		
30000	1	5000	35000	0.3	30000	$p_{3,3} = 0.2$	19000	$p_{3,0} = 0$ $p_{3,1} = 0$
		8000	38000	0.5	30000	$p_{2,3} = 0.5$		$p_{3,2} = 0$
		10000	40000	0.2	30000	$p_{3,3} = 0.5$		$p_{3,3} = 1$
	2	5000	35000	0.3	27000	$p_{3,2} = 0.09$ $p_{3,3} = 0.21$		$p_{3,0} = 0$ $p_{3,1} = 0$
		8000	38000	0.5	30000	$p_{3,3} = 0.5$		$p_{3,2} = 0.09$
		10000	40000	0.2	30000	$p_{3,3} = 0.2$		$p_{3,3} = 0.91$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		5000	25000	5000	0	0	5000	14000	19000	0.3	20000	$p_{1,2} = 0.3$		$p_{2,o} = 0$
	1	8000	28000	5000	0	0	5000	14000	19000	0.5	23000	$p_{2,3} = 0.25$ $p_{2,2} = 0.15$	19000	$p_{2,1} = 0$ $p_{2,2} = 0.75$
		10000	30000	5000	0	0	5000	14000	19000	0.2	25000	$p_{2,2} = 0.1$ $p_{2,3} = 0.1$		$p_{2,3} = 0.25$
		5000	25000	8000	0	0	8000	8000	16000	0.3	17000	$p_{2,1} = 0.09$ $p_{2,2} = 0.21$		$p_{2,o} = 0$
20000	2	8000	28000	8000	0	0	8000	8000	16000	0.5	20000	$p_{2,2} = 0.5$	16000	$p_{2,1} = 0.09$ $p_{2,2} = 0.87$
		10000	30000	8000	0	0	8000	8000	16000	0.2	22000	$p_{2,2} = 0.16$ $p_{2,3} = 0.04$		$p_{2,3} = 0.04$
		5000	25000	12000	0	0	12000	0	12000	0.3	12000	$p_{2,1} = 0.21$ $p_{2,2} = 0.09$		$p_{2,o} = 0$ $p_{2,1} = 0.45$
	3	8000	28000	12000	0	0	12000	0	12000	0.5	16000	$p_{2,1} = 0.2$ $p_{2,2} = 0.3$	12000	$p_{2,2} = 0.55$ $p_{2,3} = 0$
		10000	30000	12000	0	0	12000	0	12000	0.2	18000	$p_{2,1} = 0.04$ $p_{3,2} = 0.16$		
		5000	35000	5000	0	0	5000	14000	19000	0.3	30000	$p_{3,3} = 0.2$		$p_{3,o} = 0$
	1	8000	38000	5000	0	0	5000	14000	19000	0.5	30000	$p_{2,3} = 0.5$	19000	$p_{3,1} = 0$ $p_{3,2} = 0$
		10000	40000	5000	0	0	5000	14000	19000	0.2	30000	$p_{3,3} = 0.5$		$p_{3,3} = 1$
30000	2	5000	35000	8000	0	0	8000	8000	16000	0.3	27000	$p_{3,2} = 0.09$ $p_{3,3} = 0.21$		$p_{3,o} = 0$ $p_{3,1} = 0$
		8000	38000	8000	0	0	8000	8000	16000	0.5	30000	$p_{3,3} = 0.5$		$p_{3,2} = 0.09$
		10000	40000	8000	0	0	2000	8000	16000	0.2	30000	$p_{3,3} = 0.2$		$p_{3,3} = 0.91$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		5000	35000	12000	0	0	12000	0	12000	0.3	23000	$p_{3,2} = 0.21$ $p_{3,3} = 0.09$		$p_{3,0} = 0$ $p_{3,1} = 0$
	3	8000	38000	12000	0	0	12000	0	12000	0.5	26000	$p_{3,2} = 0.2$ $p_{3,3} = 0.3$	12000	$p_{3,2} = 0$ $p_{3,3} = 0.45$
		10000	40000	12000	0	0	12000	0	12000	0.2	28,000	$p_{3,2} = 0.4$ $p_{3,3} = 0.16$		$p_{3,3} = 0.55$

Season No. 2 : Demand = 16000 m.g. Required release from reservoir and acquifer for the decision number be see from Table 5.3.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		1000	1000	1000	5000	12500	1000	20000	33500	0.2	0	$p_{o,o} = 0.2$		$p_{o,o} = 0.96$
	4	4000	4000	4000	2000	8000	4000	20000	29000	0.6	0	$p_{o,o} = 0.6$	29300	$p_{o,1} = 0.04$ $p_{o,2} = 0$
0		8000	8000	6000	0	0	6000	20000	26000	0.2	2000	$p_{o,o} = 0.16$ $p_{o,1} = 0.04$		$p_{o,3} = 0$
		1000	1000	1000	7000	17500	1000	16000	34500	0.2	0	$p_{o,o} = 0.2$		$p_{o,o} = 1$
	5	4000	4000	4000	4000	10000	4000	16000	30000	0.6	0	$p_{o,o} = 0.6$	29700	$p_{o,1} = 0$ $p_{o,2} = 0$
		8000	8000	8000	0	0	8000	16000	24000	0.2	0	$p_{o,o} = 0.2$		$p_{o,3} = 0$
		1000	1000	1000	11000	27500	1000	8000	36500	0.2	0	$p_{o,o} = 0.2$		$p_{o,o} = 1$
	6	4000	4000	4000	8000	20000	4000	8000	32000	0.6	0	$p_{o,o} = 0.6$	31700	$p_{o,1} = 0$
		8000	8000	8000	4000	10000	6000	8000	26000	0.2	0	$p_{o,o} = 0.2$		$p_{o,2} = 0$ $p_{o,3} = 0$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10000		1000	11000	6000	0	0	6000	20000	26000	0.2	5000	$p_{1,o} = 0.1$ $p_{1,1} = 0.1$		$p_{1,o} = 0.22$ $p_{1,1} = 0.74$ $p_{1,2} = 0.04$ $p_{1,3} = 0$
	4	4000	14000	6000	0	0	6000	20000	26000	0.6	8000	$p_{1,o} = 0.12$ $p_{1,1} = 0.48$	26000	
		8000	18000	6000	0	0	6000	20000	26000	0.2	12000	$p_{1,1} = 0.16$ $p_{1,2} = 0.04$		
		1000	11000	8000	0	0	8000	16000	24000	0.2	3000	$p_{2,o} = 0.14$ $p_{1,2} = 0.06$		$p_{1,o} = 0.38$ $p_{1,1} = 0.62$ $p_{1,2} = 0$ $p_{1,3} = 0$
	5	4000	14000	8000	0	0	8000	16000	24000	0.6	6000	$p_{1,o} = 0.24$ $p_{1,1} = 0.36$	24000	
		8000	16000	8000	0	0	8000	16000	24000	0.2	10000	$p_{1,1} = 0.2$		
		1000	11000	11000	1000	2500	11000	8000	21600	0.2	0	$p_{1,o} = 0.2$		$p_{1,o} = 0.76$ $p_{1,1} = 0.24$ $p_{1,2} = 0$ $p_{1,3} = 0$
	6	4000	14000	12000	0	0	12000	8000	20000	0.4	2000	$p_{2,o} = 0.48$ $p_{1,1} = 0.12$	20300	
		8000	18000	12000	0	0	12000	6000	20000	0.2	6000	$p_{1,o} = 0.08$ $p_{1,1} = 0.12$		
		1000	21000	6000	0	0	6000	20000	26000	0.2	15000	$p_{2,1} = 0.1$ $p_{2,2} = 0.1$		$p_{2,o} = 0$ $p_{2,1} = 0.22$ $p_{2,2} = 0.74$ $p_{2,3} = 0.04$
	4	4000	24000	6000	0	0	6000	20000	26000	0.6	18000	$p_{2,1} = 0.12$ $p_{o,2} = 0.48$	26000	
		8000	28000	6000	0	0	6000	20000	26000	0.2	22000	$p_{2,2} = 0.16$ $p_{2,3} = 0.04$		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		1000	21000	8000	0	0	8000	10000	24000	0.2	13000	$P_{2,1} = 0.14$ $P_{2,2} = 0.06$		$P_{2,0} = 0$ $P_{2,1} = 0.38$
	5	4000	24000	8000	0	0	8000	16000	24000	0.6	16000	$P_{2,1} = 0.24$ $P_{2,2} = 0.36$	24000	$P_{2,2} = 0.62$ $P_{2,3} = 0$
20000		6000	28000	8000	0	0	8000	16000	24000	0.2	20000	$P_{2,2} = 0.2$		
		1000	21000	12000	0	0	12000	8000	20000	0.2	9000	$P_{2,0} = 0.02$ $P_{2,1} = 0.18$		$P_{2,0} = 0.02$
	6	4000	24000	12000	0	0	12000	8000	20000	0.6	12000	$P_{2,1} = 0.48$ $P_{2,2} = 0.12$	20000	$P_{2,1} = 0.74$ $P_{2,2} = 0.24$ $P_{2,3} = 0$
		8000	28000	12000	0	0	12000	8000	20000	0.2	16000	$P_{2,1} = 0.08$ $P_{2,2} = 0.12$		
		1000	31000	6000	0	0	6000	20000	26000	0.2	25000	$P_{3,2} = 0.1$ $P_{3,3} = 0.1$		$P_{3,0} = 0$ $P_{3,1} = 0$
	4	4000	34000	6000	0	0	6000	20000	26000	0.6	28000	$P_{3,2} = 0.12$ $P_{3,3} = 0.48$	26000	$P_{3,2} = 0.22$ $P_{2,3} = 0.78$
		8000	38000	6000	0	0	6000	20000	26000	0.2	30000	$P_{3,3} = 0.2$		
30000		1000	31000	8000	0	0	8000	16000	24000	0.2	23000	$P_{3,2} = 0.14$ $P_{3,3} = 0.06$		$P_{3,0} = 0$ $P_{3,1} = 0$
	5	4000	34000	8000	0	0	8000	16000	24000	0.6	26000	$P_{3,2} = 0.24$ $P_{3,3} = 0.36$	24900	$P_{3,2} = 0.38$ $P_{3,4} = 0.62$
		8000	38000	8000	0	0	8000	16000	24000	0.2	30000	$P_{3,3} = 0.2$		

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		1000	31000	12000	0	0	12000	8000	20000	0.2	19000	$p_{3,1} = 0.02$ $p_{3,2} = 0.18$		$p_{3,0} = 0$ $p_{3,1} = 0.02$
	6	4000	34000	12000	0	0	12000	8000	20000	0.6	22000	$p_{3,2} = 0.48$ $p_{3,3} = 0.12$	20000	$p_{3,2} = 0.74$ $p_{3,3} = 0.74$
		8000	38000	12000	0	0	12000	8000	20000	0.2	26000	$p_{3,2} = 0.08$ $p_{3,3} = 0.12$		

The required release from reservoir can be seen from Table 5.3 for the corresponding release decision number. Actual release from the reservoir (Column 6) will be the same unless the available water in the reservoir is less than the required release. If it is less, deficit occurs and the deficit cost is entered under column 7. The four discrete states (reservoir contents) assumed are 0, 10000, 20000, and 30000 (or 0, 1, 2, 3). If the end state is not equal to any of these states, then the probability of the end state is apportioned to the nearby discrete states. (Column 13). Transition probabilities of different states are added from column 13 and given under column 15.

If column (4) - (5) comes to more than 30000 the capacity of the reservoir, the excess water is spilled over. The cost of spillage is taken as 0. The maximum value of end state can be 30000.

From Table 5.4 the probabilities p_{ij} and the expected costs are summarized for all states and decisions and given in Table 5.5.

Table 5.5

Season 1

Starting State	Decision Number	Probabilities of end State 0	10000	20000	30000	Expected cost qi
0	1	0.71	0.29	0	0	19000
	2	0.92	0.08	0	0	17350
	3	1	0	0	0	20150
10000	1	0	0.75	0.25	0	19000
	2	0.09	0.04	0.04	0	16000
	3	0.45	0.45	0	0	12000
20000	1	0	0	0.75	0.25	19000
	2	0	0.09	0.87	0.04	16000
	3	0	0.45	0.55	0	12000
30000	1	0	0	0	1	19000
	2	0	0	0.09	0.91	16000
	3	0	0	0.45	0.55	12000

Season 2

Starting State	Decision Number	0	10000	20000	30000	Expected cost qi
0	4	0.96	0.04	0	0	29300
	5	1	0	0	0	29700
	6	1	0	0	0	31700
10000	4	0.22	0.74	0.04	0	26000
	5	0.38	0.62	0	0	24000
	6	0.76	0.24	0	0	30000
20000	4	0	0.22	0.74	0.04	26000
	5	0	0.38	0.62	0	24000
	6	0.02	0.74	0.24	0	20000
30000	4	0	0	0.22	0.78	26000
	5	0	0	0.38	0.62	24000
	6	0	0.02	0.74	0.24	20000

Examining Table 5.5, we can arrive at the preliminary policy for each state that gives the minimum expected cost. The policy is tabulated in Table 5.6.

Table 5.6

Res. Level	Decision Season 1	Decision Season 2
0	2	4
10000	3	6
20000	3	6
30000	3	6

These decisions are now used to set up the $\overline{P}$ and $\overline{q}$ matrices :

$\overline{P}$

Starting state		Season 1: 0	Season 1: 10000	Season 1: 20000	Season 1: 30000	Season 2: 0	Season 2: 10000	Season 2: 20000	Season 2: 30000
Season 1	0					0.92	0.08	0	0
	10000					0.45	0.55	0	0
	20000					0	0.45	0.55	0
	30000					0	0	0.45	0.55
Season 2	0	0.96	0.04	0.00	0				
	10000	0.76	0.24	0	0				
	20000	0.02	0.74	0.24	0				
	30000	0.00	0.02	0.74	0.24				

$\bar{q}$ =

	Starting state	
season 1	0	17350
	10000	12000
	20000	12000
	30000	12000
season 2	0	29300
	10000	20300
	20000	20000
	30000	20000

$$\bar{R}_1 = 0.95^2 \begin{bmatrix} 0.92 & 0.08 & 0 & 0 \\ 0.45 & 0.55 & 0 & 0 \\ 0 & 0.45 & 0.55 & 0 \\ 0 & 0 & 0.45 & 0.55 \end{bmatrix} \begin{bmatrix} 0.96 & 0.04 & 0 & 0 \\ 0.76 & 0.24 & 0 & 0 \\ 0.02 & 0.74 & 0.24 & 0 \\ 0 & 0.02 & 0.74 & 0.24 \end{bmatrix}$$

$$= 0.9 \begin{bmatrix} 0.941 & 0.056 & 0 & 0 \\ 0.95 & 0.15 & 0 & 0 \\ 0.35 & 0.52 & 0.13 & 0 \\ 0.009 & 0.341 & 0.52 & 0.13 \end{bmatrix}$$

[illegible]

$$= \begin{bmatrix} 0.8469 & 0.0504 & 0 & 0 \\ 0.855 & 0.135 & 0 & 0 \\ 0.315 & 0.468 & 0.117 & 0 \\ 0.0081 & 0.3069 & 0.468 & 0.117 \end{bmatrix}$$

$$(\bar{\bar{I}} - \bar{\bar{R}}) = \begin{vmatrix} 0.1531 & -0.0504 & 0 & 0 \\ -0.855 & 0.865 & 0 & 0 \\ -0.315 & -0.468 & 0.883 & 0 \\ -0.0081 & -0.3069 & -0.468 & 0.883 \end{vmatrix}$$

$$\bar{Q}_1 = \begin{vmatrix} 17350 \\ 12000 \\ 12000 \\ 12000 \end{vmatrix} + 0.95 \begin{vmatrix} 0.92 & 0.08 & 0 & 0 & 29300 \\ 0.45 & 0.55 & 0 & 0 & 20800 \\ 0 & 0.45 & 0.55 & 0 & 20000 \\ 0 & 0 & 0.45 & 0.55 & 20000 \end{vmatrix}$$

$$= \begin{vmatrix} 44501 \\ 35132 \\ 31125 \\ 31000 \end{vmatrix}$$

$$\underbrace{\begin{bmatrix} 0.1531 & -0.0504 & 0 & 0 \\ -0.855 & 0.865 & 0 & 0 \\ -0.315 & -0.468 & 0.883 & 0 \\ -0.0081 & -0.3069 & -0.468 & 0.883 \end{bmatrix}}_{\bar{I}-\bar{R}_1} \underbrace{\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}}_{\bar{C}_1} = \underbrace{\begin{bmatrix} 44501 \\ 35132 \\ 31128 \\ 31000 \end{bmatrix}}_{\bar{Q}_1}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{bmatrix}$$

Now $\bar{C}_2 = \bar{q}_2 + \bar{\Pi}_2 \bar{C}_1$

$$\begin{vmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{vmatrix} = \begin{vmatrix} 29300 \\ 20300 \\ 20000 \\ 20000 \end{vmatrix} + 0.95 \begin{vmatrix} 0.96 & 0.04 & 0 & 0 \\ 0.76 & 0.24 & 0 & 0 \\ 0.02 & 0.74 & 0.24 & 0 \\ 0 & 0.03 & 0.74 & 0.24 \end{vmatrix} \begin{vmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{vmatrix}$$

$$\begin{vmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{vmatrix} = \begin{vmatrix} 457582 \\ 454852 \\ 470651 \\ 448104 \end{vmatrix}$$

The results above for $\bar{C}_1$ and $\bar{C}_2$ give the present day expected costs of operating the system for the two reasons for the policy assumed in the beginning. Now we proceed further to see if these costs are optimum, if not how to improve them. For this we use the dynamic programming recursive equation

$$C_i(n+1)^f = q_i^f + \sum_j p_{ij}^f \quad C_j(n) \qquad i = 5, 6, 7, 8$$

where f is decision number

There are two stages season 1 and season 2. The optimization is carried out the 2nd season, using the expected cost of season 1 found above and p_{ij}'s from Table55 for season 2 for each state (5, 6, 7 and 8) of the season 2. For state 5, decision 4,

$$c_5(n+1)^4 = 29300 + 0.95\,(0.96 \quad 0.04 \quad 0 \quad 0)\begin{bmatrix}449503\\482511\\451343\\446149\end{bmatrix}$$

$$= 457582$$

$$c_5(n+1)^5 = 29700 + 0.95\,(1 \quad 0 \quad 0 \quad 0)\begin{bmatrix}449503\\482511\\451343\\446149\end{bmatrix}$$

$$= 456727$$

$$c_5(n+1)^6 = 31700 + 0.75\,(1 \quad 0 \quad 0 \quad 0)\begin{bmatrix}449503\\482511\\451343\\446149\end{bmatrix}$$

$$= 458727$$

The optimal value for state 5 is 456727 which is the improvement of the cost found for V_5 from V_2, which is 457580. Decision 5 is optimal at present.

Similar calculations are done for state 6, 7 and 8 in season 2.

$$c_6(n+1)^4 = 26000 + 0.95\,(0.22 \quad 0.74 \quad 0.04 \quad 0)\begin{bmatrix}449503\\482511\\451343\\446149\end{bmatrix}$$

$$= 477442$$

$$C_6(n+1)^5 = 24000 + 0.95\,(0.38 \quad 0.62 \quad 0 \quad 0)\begin{vmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{vmatrix}$$

$$= 470469$$

$$C_6(n+1)^6 = 20300 + 0.95\,(0.76 \quad 0.24 \quad 0 \quad 0)\begin{vmatrix} 449508 \\ 482511 \\ 451343 \\ 446149 \end{vmatrix}$$

$$= 45485$$

The optional cost for state 6 is 454852 from $\bar{V}_2$. That is decision No. 6, remains without change.

<u>For State 7</u>

$$C_7(n+1)^4 = 26000 + 8.95\,(0 \quad 0.22 \quad 0.74 \quad 0.04)\begin{vmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{vmatrix}$$

$$= 461092$$

$$C_1(n+1)^5 = 24000 + 0.95\,(0 \quad 0.38 \quad 0.62 \quad 0)\begin{vmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{vmatrix}$$

$$= 464027$$

$$C_7(n+1)^6 = 20,000 + 0.95\,(0.02 \quad 0.74 \quad 0.24 \quad 0) \begin{bmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{bmatrix}$$

$$= 470652$$

The minimum cost is 461092 of policy 4.

<u>For State 8</u>:

$$C_8(n+1)^4 = 26000 + 0.95\,(0 \quad 0 \quad 0.22 \quad 0.78) \begin{bmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{bmatrix}$$

$$= 450926$$

$$C_8(n+1)^5 = 24000 + 0.95\,(0 \quad 0 \quad 0.38 \quad 0.62) \begin{bmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{bmatrix}$$

$$= 449716$$

$$C_8(n+1)^6 = 20000 + 0.95\,(0 \quad 0.02 \quad 0.74 \quad 0.24) \begin{bmatrix} 449503 \\ 482511 \\ 451343 \\ 446149 \end{bmatrix}$$

$$= 448184$$

The optional cost is 448184 and decision is 6. The result of this procedure for season 2 is as follows :

State	New Decision	New Cost
c_5	5	456727
c_6	6	454852
c_7	4	461092
c_8	6	448184

Using the new policy for season 2, the optimization is carried out for the season 1.

$$c_1(n+1)^7 = 19000 + 0.95\,(0.71 \quad 0.29 \quad 0 \quad 0) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 462374$$

$$c_1(n+1)^2 = 17350 + 0.95\,(0.92 \quad 0.08 \quad 0 \quad 0) \begin{vmatrix} 456727 \\ 454852 \\ 401092 \\ 448184 \end{vmatrix}$$

$$= 451098$$

$$c_1(n+1)^3 = 20150 + 0.95\,(1 \quad 0 \quad 0 \quad 0) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 454040$$

The optimal cost is 449503 and the decision is 2.

<u>For State 2</u> :

$$C_2(n+1)^1 = 19000 + 0.95\,(0 \quad 0.75 \quad 0.25 \quad 0) \begin{vmatrix} 456127 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 452591$$

~~$C_2(n+1)^2 = 16000 + 0.75\,(0.09 \quad 0.87 \quad 0.04 \quad 0)$~~

$$\begin{matrix} 456727 \\ 454852 \\ 461092 \\ 441152 \end{matrix}$$

$= $ ~~452591~~

$$C_2(n+1)^2 = 16000 + 0.75\,(0.09 \quad 0.87 \quad 0.04 \quad 0) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 448214$$

$$C_2(n+1)^3 = 12000 + 0.95\,(0.45 \quad 0.55 \quad 0 \quad 0) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 444911$$

The minimum cost is 444911 for decision No. 3

For state 3 :

$$c_3(n+1)^1 = 19000 + 0.95\,(0 \quad 0 \quad 0.75 \quad 0.25) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 453971$$

$$c_3(n+1)^2 = 16000 + 0.95\,(0 \quad 0.09 \quad 0.87 \quad 0.04) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 453012$$

$$c_3(n+1)^3 = 12000 + 0.95\,(0 \quad 0.45 \quad 0.55 \quad 0) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 447369$$

The optimal cost comparing $\tilde{C}_1$ is 447369 for decision No. 3

For state 4 :

$$c_4(n + 1)^1 = 19000 + 0.95\,(0 \quad 0 \quad 0 \quad 1) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 444775$$

$$c_4(n+1)^2 = 16000 + 0.95\,(0 \quad 0 \quad 0.09 \quad 0.91) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 442878$$

$$c_4(n+1)^3 = 12000 + 0.95\,(0 \quad 0 \quad 0.45 \quad 0.55) \begin{vmatrix} 456727 \\ 454852 \\ 461092 \\ 448184 \end{vmatrix}$$

$$= 443293$$

The optimal cost comparing $\bar{C}_1$ is 442878 with corresponding decision No. 2.

So, the new policy to be used is

State (Reservoir) level	Decision	
	Season 1	Season 2
0	2	5
10000	3	6
20000	3	4
30000	2	6

Using this new policy the $\bar{P}$ and $\bar{q}$ matrices are again set up from the data in Table 5.5 and similar calculations are carried out. The final policy is reached when the optimum cost for $\bar{C}_r$ is the same as found using the equation

$$C_i(n+1)^f = q_i^f + \sum_j p_{ij}^f \, G_j(n)$$

In this problem the optimum policy was arrived after 5 iterations. The optimum policy is

State (Reservoir level)	Decision	
	Season 1	Season 2
0	1	4
10000	3	6
20000	3	6
30000	3	6

CHAPTER - VI

C O N C L U S I O N

The work presented in this dissertation is oriented towards optimal sequencing and scheduling of water resources systems. The model presented was based on zero-one integer programming code for both deterministic and stochastic types of demands. The integer programming model was used with the purpose of ease of programming and is quite flexible for any modifications required. It can be used to model a wide range of technological relationship.

In almost all cases of programming codes for the sequencing and scheduling problems, the computational time requirement becomes very large as the number of projects increases. In the problems examined in this thesis, the projects were assumed to be independent of each other. If these are project interdependence, that can be used to reduce the problem size. For example if project No. 3 must precede No. 5, the problem can be solved with project No. 5 omitted until such time the project No. 3 is sequenced. By this sort of arrangement the initial problem size can somewhat be reduced.

The success of a model depends on to a large extent on the proper data input. In real situations the demand pattern of water and inflow of water through streams into the reservoirs are stochastic. It is better to have a near accurate analysis of inflow-demand patterns. There are a few papers published on forecasting these characteristics.

The sequencing problem example considered only the capital costs of the project. In systems where the operation involves pumped storage systems, pumping from acquifer etc. the operations costs form a considerable part of the capital expenditure annually. Therefore, it becomes necessary to construct the model including the operational costs which can be solved either by mixed integer programming code or by Dynamic Programming.

In the initial chapter in particular and in subsequent chapters the numerous problems involved in water resources system expansion have been mentioned. The sequencing problem is only one of them. It is for the system analyst to use any solution technique suiting the problem in hand from the view point of better efficiency, and economy.

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APPENDIX- A

```
C
C
C
C
      C ,C ,P,C
      I (CG(2 ),X( ),Y(2 ),C (2 ),
      I C(2 ), (1 ),(1 ,2 ), ( ,2 ),  (2 ),  (2 )
      I  (2 ), (2 ),Z ,Z, ,S ,Z G,( )
      C  ( , , , G , TYP , , T , )
      TYP
   45 FOR AT(5X,'T   E   COEFFICIENTS   ',//15X,'
     1OFF  > =C  ',/15X,'C  > > =C  ',/15X,'
     2D   >   =C  ',//)
   55 FOR AT(  C   A(I,1) A(I,2) A(I,3) A(I, ) (I, )
     1A(I,6) A(I,7) A(I, )',/)
      D  55 I=1,
      TYPE 51,I,C (I), (I)
   51 FOR AT (I3,1 ,I5)
      TYPE 52,( (I,J),J=1, )
   52 FORMAT('+',15X,   /(15X,   ))
   45 CONTINUE
      IF( TYP . . ) GO   35
      TYP   36
   36 FOR AT(// 5X,'   C  EFFICIENTS   T  E O  ECTI  AL  F   C   ,
     1TO    I   ',/)
      GO TO 37
   35 TYPE 38
   38 FOR AT(//5X,'T   COEFFICIENTS   O  JECTIVE  UNCTION  F
     1TO    X   ZE  AR :',/)
   37 TYPE 39,(C(J),J=1, )
   39 FOR AT(15X,  /15X,  )
C     STEP    TO   E     TS   AXI
C
      IF( TYP . . ) GO T
      DO   J=1,
   60 C(J)=-C(J)
   99 CS =
      DO 100 J=1,
      IF (C(J). . ) GO T  105
      CS =CS +C(J)
      C(J)=-C(J)
      FLAG(J)=1
      DO 110 I=1,
      B(I)= (I)- (I,J)
      A(I,J)=- (I,J)
  110 CONTINUE
      GO TO 1
  105 FLAG(J)=
  100 CONTINUE
      K1= +1
      J=
      IF(  .  . ) GO T  3
      1= +
      C  ( )=1
        ( I)=
      DO 2  J=1,
      A( , )=
  205 CONTI
      DO 2  I=1,
      IF (C  (I). . ) G  TO 2
      B( 1)= ( )- (I)
      C  (I)=1
      DO 21  J=1,
  210 A( ,J)=A( ,J)-A(I,J)
  200 CONTI
  300 IF ( . . )  1=
      DO 31  I=1,
      IF (C  (I). .2) G  TO 319
      IF (C  (I). .1) G  TO 32
      Q(I,K1)= (I)
      DO 315 J=1,
  315 Q(I,J)=- (I,J)
      GO TO 3
  320 Q(I, )=- (I)
      DO 325 J=1,
  325 Q(I,J)= (I,J)
  31  CONTINUE
  321 FOR AT(//  ,'C   E   CONSTRAINTS',/)
      DO 322 I=1,
      TYPE 323,I, (I, )
  323 FOR AT(I3, 3)
      TYPE 52,( (I,J),J=1, )
  322 CONTINUE
  235 FOR AT(// ' STEP  =   PARTIAL SOLUTIONS  (  )', , '   ',/)
      DO   I=1,
      F  (I)=I
        (I)=
```

```
120   C
      Z   =
      ZEI  G=
          =
201   C    T
          P=   P+1
      S  =
      D  22   I=1,
      IF (     (I).   .  )     T   24
      IF(      (I).LE.  ) G  TO 22
      I = F    (I)
      S  =S  +C(I )
220   C    T
230   Z=S
350   C   T
      D  36  I=1,
       Y(I)=
360   C   T
      Y(K1)=1
      DO 37  I=1,
      IF( F    (I).LE.  ) G  TO 370
      I =  F   (I)
      Y(I )=1
370   CO  T
      DO 375  =1,
      S   =
      DO 38   =1,
      S  =S   +C(I,J)*Y(J)
380   CO  T
      IF(       .  ) G  T  385
      VC( )=1
       G  T   375
385   VC(I)=
375   CO  T
      D      I=1,
      IF(VC(I).  .1) G  T   5
400   C  TI
      GO TO
500       I=Z I -Z
C      TYPE 2 6,      ,      (1),I=1, )
206   FO   A   (I ,  X,   ,  (   )X,   16/))
C      TYPE 2 7,
207   F      (  +  ,7 X,  )
      DO      I=1,
      T(J)=
      IF(     (J).  .  ) G
      IF(C( J).G    .   ) G
      DO 61   I=1,
      IF(VC(I).  .  )
      IF (C(I,J).  .  ) G       3
610   C  TI
      G
630   T(J)=
6     C    T
      D  7    =1,
      IF (T(J).  .1) G
7     C  TI
      G  TO
8     C  T
          1   J=1,
810   Y(J)=
      DO  3    =1,
      IF ( F    (J).LE. ) G  T  3
      I = F    ( )
      Y(I )=1
830   CONTI
      Y(K1)=1
      D  84  I=1,  1
      IF(  C(I).  .  )
      S  =
      D   85  J=1,
      IF(C(J).  .1.A   .  (I,J).G   .    .   .K1) Y(J)=1
      S  =S  +C(I,J)*Y(J)
      IF(C(J).  .1.  .C(I,J).G  .   .   .  .K1) Y(J)=
850   C   TI
      IF(
840   CO  T
          =1
      D   1   =1,
      IF(T( ).  .1) G
      K    =1
         92    J=1,
      IF(   .         ) Y( J)=
920   CO  T
      Y(       )=1
      D  93   J=1,
      IF( F   (JJ).L  .  ) G   T    93
      I =     (JJ)
```

```
              ( (  )=
      930   C
            Y(  )=
            A  (     )=
              (     I=1, I
            S  =
            D  95    =  ,
      950   S   =S   +(  ,  )*Y( I)
            IF(S   .  . )
            A   (  (    )=     (  (    I)=S
      940   C
            A   (     )=
            IF(  -      )       , 9  , 942
      941    I =
            G    (  9  )
      942    I =
      943   IF(  I  .   .       (       ))K  =
      910   C
            FRE (    )=
            D   96    =  ,
            IF (  R   (I).    )     T     97
      960   CONTI
      970   FREE(I)=
            GO TO 2  1
     1100   DO 111   I=1,
            IF( F   (  ).    )  G   TO 1
     1110   CONT
            GO T
     1200   CONTI
            D   121   I=1,
     1210   CCS(I)=
            D  122   I=1,
            IF(    (I).   .  )  G   TO 122
            I =     (  )
            CCS(I  )=
     1220   CO T
            IF(Z   T.    )  G        1
            GO TO 15
     1400   Z  =Z
            Z L G=1
            D   141   I=1,
            LAS (I)=CCS(I)
     1410   CO T
     1500   CO  T
C             YPE  2  , STEP, (   S (I),I=1, )
C            TYPE  2 7,Z
            D  151   I=1,K
            IF( F   (I).  .  )  G  TO 16
     1510   CONT
            G  TO 2 1
     1600   CO T
            K =  =1
            D   161   I=1,KK
            II=I+1
            IF( F   (II).EQ. )  GO TO 162
            GO TO 1610
     1620   KO  T =1
            IF(  (I).LT.  )  G   TO 1
            GO TO 17
     1610   CO TI
            KO  T =
            IF(  R  ( ).LT. )  GO TO 16
            GO TO 17
     1700   FR  (KOU TR)=- FRE (KOU TR)
            GO TO 2 1
     1800   CO TI
            KOU  R=KOU TR+1
            D  181   I=1,
             =K     -I
            IF(  .GE. )  GO TO 2
            IF(    (  ).GE.  )  G  TO 19
     1810   C   T
     1900       (  )=-     (  )
             I= +
            D   191   I=  ,K
            IF( F   (I).   .  )  G  TO 2 1
            I =I   ( F   (I))
              F   (I)=
            F   (I )=
            IF(I . .K)  GO T  2 1
     1910   C  T
     2 1    C   T
            IF(Z   G. . )  G     2
            D   2 2  I=1,K
            IF(    (I).  . )  G     2 2
            L   ( )=1-   (I)
     2020   C
                Z 7       =I,
```

```
      [illegible]
2025  F[illegible](I[illegible],//[illegible],/)
      [illegible] =[illegible]
      TY[illegible],I,[illegible](I)
2040  F[illegible](5X,'[illegible]T[illegible]',I3,'[illegible]',I2)
2030  C[illegible]
         I =[illegible]+CSI[illegible]
      IF([illegible].[illegible].[illegible]) [illegible] 2035
      [illegible] I =-[illegible]I
2035  TY[illegible],[illegible]I
2050  FO[illegible](//[illegible]X,'[illegible] CRITICAL V[illegible]',
     [illegible])
2070     C[illegible]
         G[illegible] 2[illegible])
2100  TYP[illegible] 211[illegible]
2110  [illegible](// 5X,'[illegible]RESSURE SOL[illegible]')
2[illegible]  ST[illegible]
      E[illegible]
```

APPENDIX- B

```
C       EVALUATION OF COFFTS IN IP PROBLEM
          SUBROUTINE COFFTS(M,K,NLET,NGET,NET,NTYPE,NP,NTIME,NDEM,
     1    A,CODE,B,IC)
          INTEGER CODE(200),A(200,400),B(200)
          DIMENSION X(3),Y(3),IP(200),S(10),C(400),IC(400)
          OPEN(UNIT=1,DEVICE='DSK',FILE='DSJ3.DAT')
        READ(1,20)M,K,NLET,NGET,NET,NTYPE,NP,NTIME,NDEM
          TYPE*,M,K,NLET,NGET,NET,NTYPE,NP,NTIME,NDEM
20      FORMAT(9I5)
C       LESS THAN OR EQUAL TO CONSTRAINR COFFTS
        DO 24 I=1,NP
        CODE(I)=0
        B(I)=1
        DO 23 N=1,NP
          DO 22 J=((N-1)*(NTIME+1)+1),(N*(NTIME+1))
        IF (I.EQ.N) GO TO 21
        A(I,J)=0
        GO TO 22
21      A(I,J)=1
22      CONTINUE
23      CONTINUE
24      CONTINUE
C       GREARER THAN OR EQUAL TO CONSTRAINTS COFFTS
        DO 33 N=1,NDEM
        READ (1,*)X(N),Y(N)
          TYPE *,X(N),Y(N)
          ID=0
          DO 30 I=((NP+N)+(N-1)*NTIME),((NP+N)+N*NTIME)
          ID=ID+1
        CODE(I)=1
        IP(I)=(ID-1)*(X(N)*(ID-1)+Y(N))
        B(I)=IP(I)
        DO 28 L=1,NP
          DO 27 J=((L-1)*(NTIME+1)+1),(L*(NTIME+1))
          IF(J.GT.((L-1)*(NTIME+1)+ID)) GO TO 10
        IF (ID-1) 25,25,26
25      READ (1,*)S(L)
          TYPE*,S(L)
26      A(I,J)=S(L)
        GO TO 27
10      A(I,J)=0
27      CONTINUE
28      CONTINUE
30      CONTINUE
33      CONTINUE
C       OBJECTIVE FUNCTION COFFTS
          J=0
        DO 35 N=1,NP
        READ (1,5)C(N)
5         FORMAT(F8.1)
          TYPE*, C(N)
          DO 34 IT=1,(NTIME+1)
        J=1+J
        C(J)=C(N)*(EXP(-0.05*(IT-1)))
        IC(J)=C(J)
          TYPE *,IC(J)
34      CONTINUE
35      CONTINUE
          CLOSE(UNIT=1)
          RETURN
        END
```