BER PERFORMANCE OF 2D-SPREADING CODES IN MC-DS-CDMA WITH NAKAGAMI-m FADING

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

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By

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report, titled **"BER Performance of 2D Spreading Codes in MC-DS-CDMA with Nakagami-m Fading"** being submitted in partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems**, in the Department of Electronics and Computer Engineering, Indian Institute of Technology, Roorkee, is an authentic record of my own work carried out from July 2010 to June 2011, under guidance and supervision of **Mr. S. Chakravorty**, Assistant Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology, Roorkee.

The results embodied in this dissertation have not submitted for the award of any other Degree or Diploma.

Date:30June 2011

Place: Roorkee

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CERTIFICATE

This is to certify that the statement made by the candidate is correct to the best of my knowledge and belief.

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) combined with code division multiple access (CDMA) makes the multicarrier (MC-) CDMA systems one of the promising candidates for the next generation mobile communication systems. MC-CDMA systems have advantage of both OFDM and CDMA i.e. high spectral efficiency and robustness to fading. MC-DS-CDMA systems also offer both temporal and frequency diversities, support narrowband interference suppression and require a lower chip rate than that of single carrier systems utilizing the same bandwidth.

Although MC-DS-CDMA systems are promising candidates for high data rate services, the multiple access interference (MAI) problems inherent to the single-carrier (SC-) DS-CDMA system also exists and is the limitation to its achievable capacity. To reduce the level of MAI, spreading codes with good correlation properties are required.

In this dissertation, one of the three basic MC CDMA schemes, namely MC-DS-CDMA, is combined with 2-dimensional spreading code. Two families of 2-dimensional codes have been considered, a family of Orthogonal Variable Spreading Factor (OVSF) codes and other one Complete Complementary Codes (CCC). These codes can be recursively generated using Kronecker Product. A multi-rate system is considered having users with two data rates. Simulations have been carried out for BER analysis of these codes for MC-DS-CDMA in Nakagami-m fading channel. Finally, BER performance of 2D-Orthogonal Variable Spreading Factor codes and 2D-Complete Complementary Codes are compared.

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Chapter 1

INTRODUCTION

The number of mobile and Internet subscribers is growing rapidly on a global scale, and with this growth comes new and improved wireless services and increased Quality of Service demands. Beyond third generation mobile cellular communications aim at achieving high data rates to facilitate high quality voice communications and reliable wireless multimedia services (IP-based services). To accommodate the growth of both subscribers and multimedia service standards, there is a strong need for the continued evolution of mobile cellular communications technology. This growth trend is expected to continue in the coming years with people relying more and more on the internet and the convenience of wireless communications services.

The amount of radio frequency spectrum assigned to mobile cellular communications is limited and consequently bandwidth is a precious communications commodity. Parallel to developing new technologies to support innovative wireless services, it is also imperative to make efficient use of channel resources, namely the allocated bandwidth. The focus of mobile cellular technologies evolution therefore remains to fit as many subscribers onto the network as possible without exceeding the available bandwidth and maintaining the desired QoS requirements.

1.1 Background and Motivation

Spread spectrum code division multiple access (CDMA) approaches have been proposed for a variety of digital cellular mobile and wireless personal communications systems. Conventionally CDMA systems [1] use a spreading signal to reduce the effects of co-channel interference. Each user is assigned a spreading signal that is approximately orthogonal to the other spreading signals. The desired signal is recovered by correlating the incoming signal with the spreading signal assigned to the user. The interference due to other users is reduced in the correlation process and appears as noise to the receiver. The ability of CDMA to reduce the effects of co-channel interference allows for more users to transmit on the same frequency band at the same time. Some of the advantages of spread spectrum are low probability of intercept, narrowband interference rejection, anti-jamming, random access.

The CDMA air interface is used in both 2G and 3G networks. The basis of 3G cellular systems has been around DS-CDMA; this basis is evident from the global appearance of W-CDMA in 3G technologies. As the number of mobile subscribers continues to surge along with the evolution of wireless services, the next generation of mobile communications, the fourth generation (4G), will also need to evolve in order to support even higher data rates and network capacity. W-CDMA is seen as the dominant multiple access scheme for the air interface in 3G systems standardization and will remain a candidate for 4G standardization. Since the recent popularity of orthogonal frequency division multiplexing (OFDM) and multicarrier modulation (MCM) techniques, multicarrier code division multiple access (MC-CDMA) is emerging as a possible candidate for the air interface multiple access scheme of 4G technologies.

To support high speed and multi-rate data services in next generation mobile communication systems, two approaches are under investigation: variable length spreading and multi-code techniques. Variable length spreading CDMA employs multiple spreading factors for multi-rate transmissions, while multi-code CDMA allocates multiple codes to high rate services. Multicarrier schemes exhibit a better narrowband interference suppression effect, along with robustness to fading, as compared to single carrier systems [2]. For example, OFDM, a multicarrier scheme is used in IEEE 802.11a wireless local area networks. However, higher data rates are expected in mobile communication systems beyond 3G. One possible technology is MC-DS-CDMA [3]. In general, a given frequency spectrum is divided into M frequency bands (or carriers), and each carrier is used to convey a time-spreading code sequence. This kind of MC/DS-CDMA schemes has several advantages over single-carrier DS-CDMA systems including:

- Narrowband interference suppression: It is a basic characteristic of CDMA system that a narrowband interferer appears as noise with low power spectral density after de-spreading at the receiver side. Thus effectively suppressing interference.

- Multipath fading robustness: In MC-DS-CDMA, data is spread is time domain and then transmitted over multiple carriers, thus offering both temporal and frequency diversity.
- Lower chip rate: Lower chip rate is due to the fact that the length of the time-spreading code sequences in MC/DS-CDMA systems is reduced by a factor of M. It is because M carriers are used to convey the code sequence(s) simultaneously, as opposed to single-carrier systems that only employ one carrier and one long code sequence per user.
- Multicarrier system requires lower-speed parallelized signal processing in contrast to faster-speed serialized signal processing required in single-carrier systems.

Thus, it offers both temporal and frequency diversities, supports narrowband interference suppression, and require a lower chip rate than that of single carrier systems utilizing the same bandwidth [3][4].

Motivated by the advantages over its single carrier counterparts, multi-carrier systems is current topic of research, where OFDM and CDMA are combined with two-dimensional spreading. Compared to the SC- systems, the MC- systems are able to utilize radio resource more flexibly – a crucial feature for the next- generation systems to provide a wide variety of services and QoS.

1.2 2 D Orthogonal Variable Spreading Factor (2D-OVSF) codes and 2 D Complete Complementary Codes (2D CCC)

In order to support variable rates of data multimedia in CDMA system, a set of orthogonal codes with different lengths must be used, because the rate of information varies and the available bandwidth is fixed. It is possible to support higher data rates in direct sequence CDMA (DS-CDMA) systems by assigning multiple fixed-length orthogonal codes to a user. In an alternative CDMA scheme which is known as OVSF-CDMA, a single Orthogonal Variable Spreading Factor (OVSF) code is assigned to each user. In this case, a higher data rate can be accessed by using a lower spreading factor. The data rates provided are always a power of two with respect to the lowest-rate codes. OVSF codes assignment has significant impact on the code utilization of the system.

2D OVSF codes were proposed by W.C. Kwong et al. [5] .These are constructed by generalizing one-dimensional (1D) OVSF codes and they possess ideal correlation properties. 2D OVSF codes can be generated recursively by using a tree structure [5]. Orthogonality of these spreading codes is employed to improve the bandwidth efficiency and interference-rejection capability of the MC-DS-CDMA systems. The 2D OVSF codes preserve the orthogonality between code matrices of different spreading factors in an OVSF code tree, supporting multimedia services with a variety of data rates. By adopting 2D-OVSF spreading, the multiuser channel is transformed into orthogonal single-user channels irrespective of user's data rates.

Complete Complementary Codes are derived from complementary sets of sequences. Such sequences were first studied by Golay [6]. Further, the properties of complementary sequences were studied by a number of authors, notably Turyn [7], Welti [8] and Taki et al [9]. Unlike 2D OVSF-Codes, individual rows (or sequences) of Complete Complementary Codes are not orthogonal, but all of the rows (or sequences) are required to maintain the orthogonality between any two code matrices. In other words, in 2D- Complete Complementary codes sum of correlation is ideal instead of correlation itself.

Spreading codes with good correlation properties are basic requirement of CDMA system. If auto-correlation side-lobes are zero, then there will be no self interference, while zero cross-correlation guarantees no multiple access interference in multi-user scenario. In a single-carrier DS-CDMA system, it is not possible to construct a set of 1D spreading code sequences with zero cyclic autocorrelation side-lobes and zero cross-correlation, however, as shown in [7] and [8], it is possible to construct such a class of 2D spreading codes. MC-DS-CDMA in conjunction with 2D-spreading codes provides a number of advantages over SC-DS-CDMA as mentioned in previous section. BER performance of MC-DS-CDMA with 2D spreading codes has been studied by authors [10] [11] in AWGN, Rician and Rayleigh fading channel. In [10], authors have presented analytical and simulation results for BER performance of 2D-OVSF codes in AWGN, Rician and Rayleigh channel. In [11], Bell and Tseng have evaluated performance of mutually orthogonal complementary codes in MC-DS-CDMA in AWGN and

Rayleigh channel. As we know, Nakagami-m fading is generalized multipath fading channel which can be approximated to a Rician or Rayleigh channel by setting its parameter to appropriate values. The Nakagami-m distribution [12] is a versatile statistical distribution, which is capable of modeling a variety of fading environments, such as land mobile, as well as indoor mobile multipath propagation channels and ionospheric radio links.

1.3 Statement of Problem

In this dissertation, two families of 2D spreading codes i.e. 2D OVSF and 2D CCC codes are examined for their BER performance in MC-DS-CDMA system with Nakagami-m fading. The objectives of this dissertation are as follows:

- Review of various MC-CDMA schemes.
- Study of 2-Dimensional Orthogonal Variable Spreading Factor (2D-OVSF) codes and 2-Dimensional Complete Complementary Codes (2D-CCC).
- Examining BER performance of these 2D-OVSF and 2D-CCC codes in conjunction with MC-DS-CDMA in Nakagami-m fading for various values of fading factor.
- Comparison of BER performances of 2D-OVSF codes and 2D-CCC codes.

1.4 Organization of the Report

Including this introductory chapter the dissertation report is organized in seven chapters. Chapter 2 presents concept of MC-CDMA systems along with discussion of various families of MC-CDMA. Chapter 3 introduces 2D Orthogonal Variable Spreading Factor (OVSF) Codes and describes their statistical properties and methods of construction. Chapter 4 presents Complete Complementary Codes (CCC) and their statistical properties and method of synthesis. Chapter 5 deals with the system model used for the simulation purpose and also provide simulation results for the BER performance of the 2D-OVDF codes and CCC codes. Finally Chapter 6 concludes the report, and suggestions for future work are also given in this chapter

Chapter 2

MC-CDMA SYSTEMS

Code-division multiple access (CDMA) is a multiplexing technique where a number of users simultaneously and asynchronously access a channel by modulating and spreading their information-bearing signals with pre-assigned signature sequences. CDMA technique has been considered to be a candidate to support multimedia services in mobile radio communications, because it has its own capabilities to cope with asynchronous nature of multimedia data traffic, to provide higher capacity over conventional access techniques and to combat the hostile channel frequency selectivity.

Multicarrier modulation scheme, often called orthogonal frequency-division multiplexing (OFDM), has drawn a lot of attention in the field of radio communications. This is mainly because of the need to transmit high data rate in a mobile environment which makes a highly hostile radio channel. Multicarrier CDMA, which is a combination of OFDM and CDMA have been subject to extensive research.

2.1 Overview of Multi-Carrier CDMA

Multi-carrier Code Division Multiple Access (MC-CDMA) represents a fusion of two radio access techniques, namely Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA), thus also known as OFDM-CDMA. Combination of OFDM and CDMA were initially proposed by several groups of author in the early 1990's.

OFDM, the technology at the heart of digital broadcast television and radio, and also now accepted as the next generation standard for wireless LAN systems, solves the difficult inter-symbol interference problem encountered with high data rates across multipath channels. By dividing the bandwidth into many small orthogonal frequencies (efficiently achievable using the fast Fourier transform), the data can be transmitted across multiple narrowband channels, which suffer only from flat fading.

CDMA, the technology chosen for the third generation of mobile phone networks exploits the diversity in the radio channel to improve performance and allows better spectral efficiency and easier base station placement compared to second generation systems. Multicarrier CDMA systems combine many aspects of these two technologies to provide a communication system that has the advantages of both.

Three types of new multiple access techniques based on OFDM-CDMA were proposed namely, multicarrier CDMA (MC-CDMA), multicarrier DS-CDMA (MC-DS-CDMA) and Multitone CDMA (MT-CDMA). MC-CDMA was developed by N.Yee, J-P. Linnartz and G. Fettweis [13], K. Fazel and L. Papke [14], and A. Chouly, A. Brajal and S. Jourdan [15]; Multicarrier DS-CDMA by V. DaSilva and E. S. Sousa [16]; and MT-CDMA by L. Vandendorpe [17].

In the original MC-CDMA system proposed in [13], each data bit is transmitted in parallel on multiple independently fading subcarriers. The CDMA coding is applied across the carriers, with each carrier modulated by a single code chip. The simulation results for this system assumed that each subcarrier fades independently, an assumption that is not strictly speaking true for OFDM-based systems. In practice, a degree of diversity equal to the number of carrier cannot be assumed, as the diversity will be dependent on the physical channel.

In [16], authors had proposed a combination of OFDM and CDMA called Multicarrier Direct Sequence (MC-DS-CDMA). This system differs significantly in that the modulated code chips are spread in time and therefore each subcarrier is used as a narrowband channel for transmitting DS-CDMA signals. To achieve frequency diversity, the same DS-CDMA spread data bit is usually transmitted over each subcarrier in parallel and the received signals are combined to give a more robust data estimate.

Multitone CDMA [17] is another combination of OFDM and CDMA techniques. In this case the baseband OFDM (also termed discrete Multitone) modulation is performed first, and then this modulated signal is spread with a CDMA code. Both of these techniques use elements very closely related to DS-CDMA systems, such as RAKE receivers, which perform detection by correlating the received signal with a replica of the spreading code.

2.2 Variants of OFDM-CDMA

A brief overview of multi-carrier techniques along with their transmitter/receiver structure and power spectrum is given below.

2.2.1 MC-CDMA

MC-CDMA conveys the transmitted signals using a number of subcarriers. In MC-CDMA the transmitter spreads each parallel sub-stream of data generated with the aid of serial-to-parallel (S-P) conversion across N_p number of subcarriers using a given N_p-chip spreading code, $\{c[0], c[1], \ldots, c[N_p - 1]\}$. As seen in Fig. 2.1, the transmitted MC-CDMA signal using BPSK modulation can be expressed as

$$s_{MC}(t) = \sqrt{\frac{2P}{UN_p}} \sum_{i=-M}^{M} \sum_{u=0}^{U-1} \sum_{j=0}^{N_p-1} b_i[u]c[j]p_{T_s}(t-iT_s)\cos[2\pi(f_c + f_{jU+u})t] \qquad (2.1)$$

where P and f_c represent the transmitted power and carrier frequency respectively and U represents the number of bits that are S-P converted. Each transmitted symbol contains U data bits, $2M + 1$ represents the number of U-bit symbols conveyed by a transmitted data burst, and $b_i[u] \in \{+1, -1\}$ represents the u^{th} bit of the i^{th} transmitted symbol. As seen in Eq.2.1, the total number of subcarriers is $Q = UN_p$. Let $f_0 < f_1 < \ldots < f_{UN_p-1}$, then the N_p number of subcarriers $\{f_{jU+u}\}: j=0\ldots N_p-1$ having the maximum possible frequency spacing between any two subcarriers are used for conveying the N_p chips of the sub-stream $b_i[u]$ for $u = 0, 1 \ldots, U-1$. T_s represents both symbol duration and chip duration, since in MC-CDMA the symbol duration and chip duration assume the same value.

The number of users supported by MC-CDMA depends on both the processing gain and the cross-correlation characteristics of the different spreading codes. However, since in MC-CDMA each subcarrier signal is assumed to be experiencing flat fading, no multipath-induced inter-symbol interference (ISI) is imposed on the subcarrier signals, which would impair their autocorrelation. Hence, the number of users supported by MC-CDMA remains independent of the auto-correlation characteristics of the spreading codes. If, however, the MC-CDMA system's transmission bit rate is high [18], the transmitted signal would experience frequency selective fading. In this case S-P conversion (as shown in fig.2.1) invoking a high number of input data bits and a high number of subcarriers can be employed to prevent the subcarriers from experiencing frequency- selective fading.

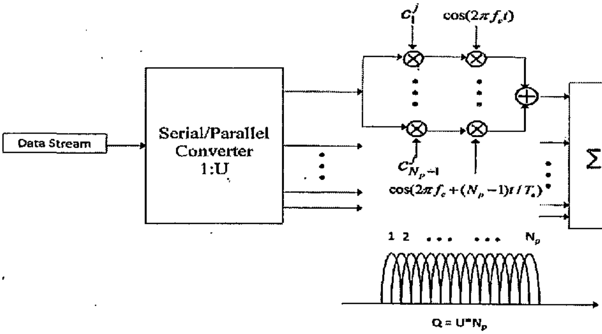

Fig.2. 1 Transmitter and Power Spectrum of MC-CDMA system

The receiver structure for MC-CDMA is shown in fig.2.2. As seen in the diagram, the received signal is combined in frequency domain collecting all the received energy scattered in frequency domain. With MC-CDMA, the data symbol is transmitted over N narrowband subcarriers with each subcarrier being encoded with 0 or 180 phase offset. The resulting signal has a pn-coded structure in the frequency domain. If the number of and spacing between subcarrier is appropriately chosen, it is unlikely that all the subcarrier will be located in a deep fade and consequently frequency diversity is achieved. As a MC-CDMA signal is composed of N narrowband subcarrier signals, each of which has symbol duration much larger than the delay spread, a MC-CDMA signal will not suffer from inter-chip interference and ISI. In addition, multiple access may be possible by having different users transmit at the same subcarrier but with a different pn-code that is orthogonal to the codes of other users.

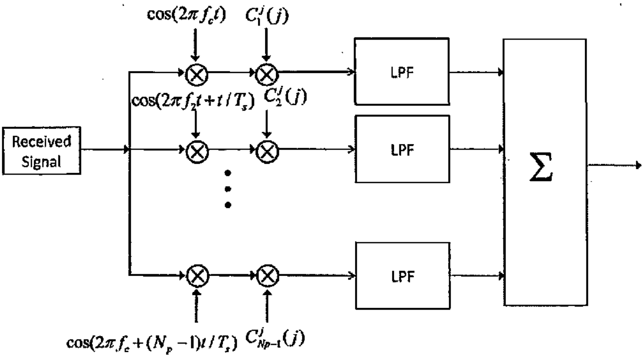

Fig.2. 2 Receiver of MC-CDMA system

2.2.2 MT-CDMA

MT-CDMA scheme is similar to MC-CDMA in the sense that the incoming bit stream is divided into a number of different bit streams. The MT-CDMA transmitter spreads the S/P converted data streams in frequency domain using a given spreading code. The spreading of each stream is done with a much longer spreading sequence relative to the MC case. This results in substantial subcarrier spectral overlap, in contrast to the MC case where subcarrier spectra are essentially non-overlapping. As with the MC schemes, typically the MT approach also uses a constant bandwidth for each of the subcarriers. The key distinction between these techniques is the amount of spectral overlap among subcarriers—MC has little overlap, with subcarrier orthogonality over chip time T_c, whereas MT has much overlap, with subcarrier orthogonality over symbol time T_s. As a result, resistance to narrowband interference is reduced compared to basic MC and to the other MC-SS systems since such an interferer will affect most subcarriers as opposed to only a small subset. Also, as each subcarrier is a wideband DS-SS signal therefore fading cannot be flat and they don't fade independently.

Since, the MT-CDMA scheme uses longer spreading codes in proportion to the number of subcarrier, as compared with a normal (single carrier) DS-CDMA scheme, therefore, the system can accommodate more users than the DS-CDMA scheme. Frequency selective fading destroys the orthogonality among users and therefore multiuser detection (MUD) is unavoidable. As a result, given the involved processing complexity, this transmission technique is more feasible for the uplink. The transmitter and power spectrum of MT-CDMA scheme is shown in fig. 2.3.

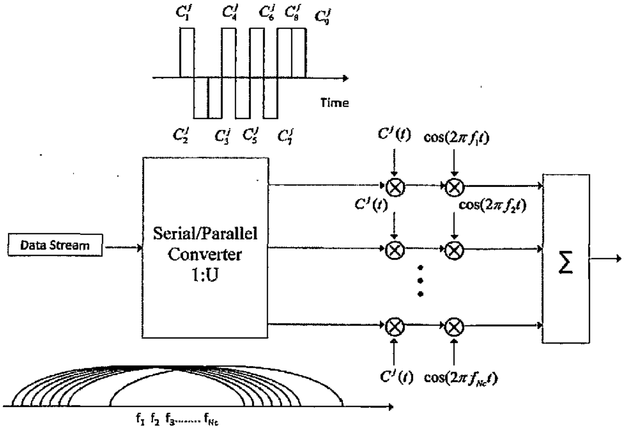

Fig.2. 3 Transmitter and Power Spectrum of MT-CDMA system

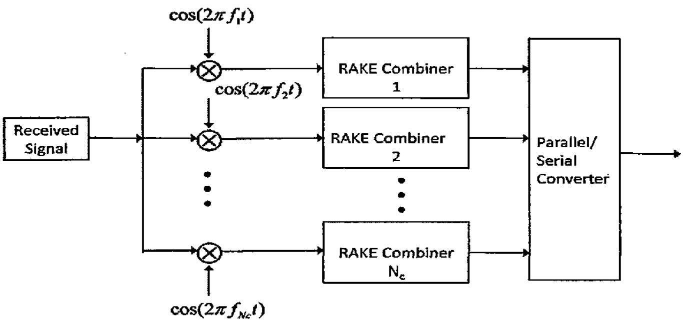

Fig.2. 4 Receiver structure of MT-CDMA

The receiver structure of MT-CDMA scheme is shown in fig. 2.4. It consists of N_c RAKE receivers for each subcarrier. Synchronization is difficult at the receiver side, and both code and multicarrier frame sync is required. Although the receiver can use all subcarriers to establish code synchronization, the lack of orthogonality and of independent fading of subcarriers makes the task more difficult.

2.2.3 MC-DS-CDMA

The Multicarrier DS-CDMA transmitter spreads the SP-converted data streams using a given spreading code in the time domain so that the resulting spectrum of each subcarrier can satisfy the orthogonality condition with the minimum frequency separation [10]. This scheme was originally proposed for uplink communication channel, because the introduction of OFDM signaling into DS-CDMA scheme is effective for the establishment of a quasi-synchronous channel. Fig. 2.3 show the Multicarrier DS-CDMA transmitter of the j^{th} user and the power spectrum of the transmitted signal, here *GMD* denotes the processing gain, N_c the number of sub-carriers, and $C^j(t) = [C^j_1 \; C^j_2 \ldots C^j_{GMD}]$ the spreading code of the j^{th} user.

Receiver structure of MC-DS-CDMA is shown in fig.2.6. Received signal r (t) is first demodulated in each subcarrier. After correlating with spreading sequence, decision is taken for the received symbol. Finally parallel to serial conversion is performed to recover original data stream.

In [2], a multicarrier based DS CDMA scheme with a larger subcarrier separation is proposed in order to yield both frequency diversity improvement and narrow band interference suppression. In addition, a Multicarrier based DS-CDMA scheme, which transmits the same data using several subcarriers, is proposed in [16]. Each subcarrier must undergo nonselective fading in order to benefit from the advantage of simple equalization of MC systems. As a result, only short spreading sequences can be used. Frequency selective channels won't destroy the orthogonality among users since each subcarrier is constrained to undergo flat fading. Therefore, MC-DS-CDMA is well suited for the uplink, as opposed to MC-CDMA which is considered more suitable for the downlink. Both frame and chip synchronization is necessary, however, due to the longer chip duration code synchronization is easier compared to DS-SS.

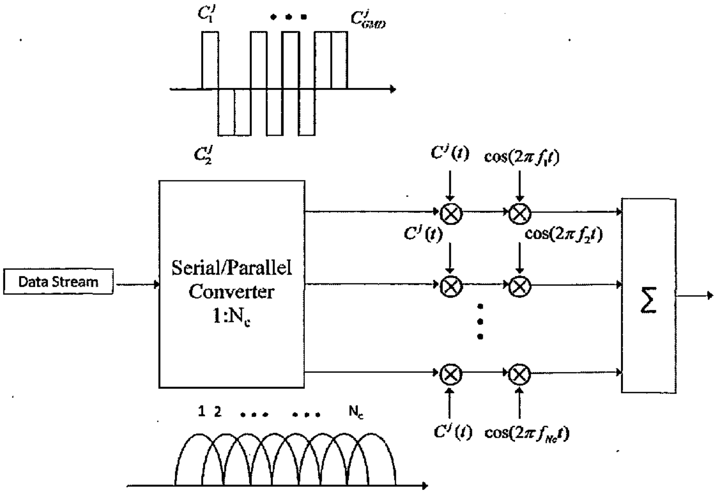

Fig.2. 5 Transmitter and Power Spectrum of MC-DS-CDMA system

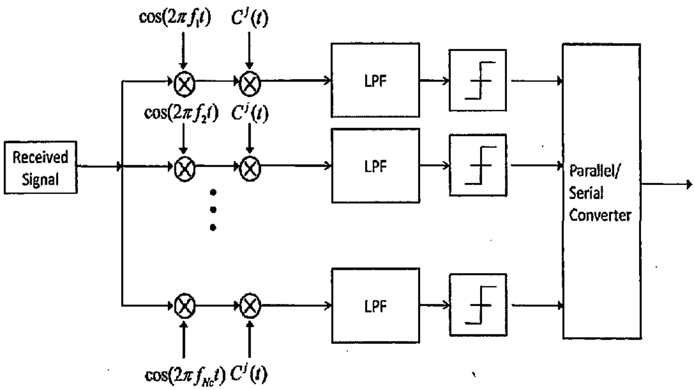

Fig.2.6 Receiver structure of MC-DS-CDMA

MC-DS-CDMA is capable of supporting ubiquitous communications in environments as diverse as indoor, open rural, suburban, and urban areas [3]. This is achieved by avoiding or at least mitigating the problems imposed by the different dispersion fading channels associated with the above-mentioned diverse communication environments.MC-DS-CDMA guarantees that the combined subcarrier signals experience independent fading. It is capable of mitigating the requirements of high-chip-rate based signal processing, as encountered in SC DS-CDMA. This is achieved by introducing computationally efficient fast Fourier transform (FFT)-based parallel processing, carrying out modulation/demodulation for all subcarriers in a single FFT step. MC-DS-CDMA is also capable of mitigating the worst-case peak-to-average power fluctuation experienced, since with the advent of using DS spreading of the subcarriers we have a significantly decreased number of subcarriers from MC-CDMA.

In MC-DS-CDMA the orthogonality of the spreading codes remains un-impaired by fading-induced dispersion, since each subcarrier signal experiences flat fading. Therefore, for downlink transmission, users using different spreading codes can be detected with near single-user performance without employing MUD. The desired signal can be detected using conventional low complexity single-user detectors.

Performance

In [19], authors have derived exact BER performance of asynchronous MC-DS-CDMA under Nakagami-m fading. They have used bpsk modulation and random spreading sequences. Fig. 2.7 and Fig. 2.8 shows the BER performance under Rayleigh and Nakagami-m (m=2) channel. In their work, authors have compared Standard Gaussian Approximation of MAI with exact expressions for MAI using Characteristic functions. In fig 2.7 and fig 2.8 performance of MC-DS-CDMA for number of carriers U=1, 4, 16 and spreading length L=7, 31 is plotted. As seen in the figures, by increasing spreading length L from 7 to 31, an improvement in performance is observed. In Rayleigh channel, achievable BER at 15 dB is in range of 5×10^{-2} for number of subcarrier U=1, 4, 16, while in Nakagami-m (m = 2) BER is in range of 6×10^{-3} at 15 dB. However both plots saturate as SNR is increased, i.e. no further advantage is achieved by increasing SNR.

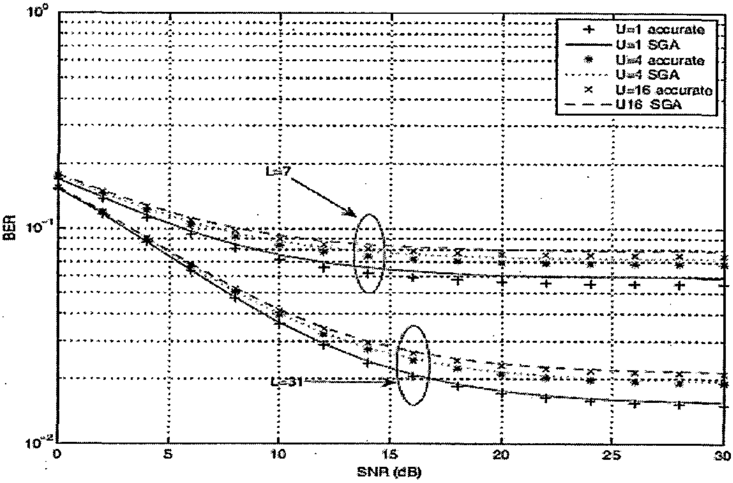

Fig. 2.7 BER performance of MC-DS-CDMA in Rayleigh Fading

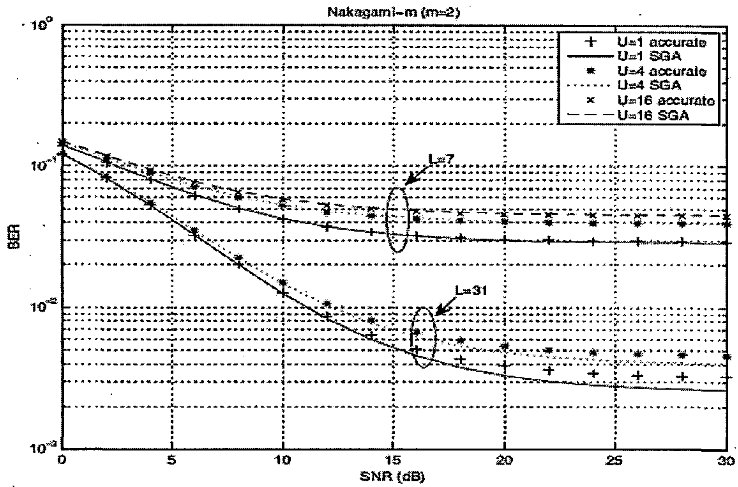

Fig. 2.8 BER performance of MC-DS-CDMA in Nakagami-m Fading (m = 2)

2.3 System Features Comparison

System features of three Multicarrier schemes is summarized and compared with DS-CDMA in Table 2.1 [18]. A rectangular pulse shape is used for both DS-CDMA and Multicarrier schemes. The required bandwidths of MC-CDMA and Multicarrier DS-CDMA schemes are almost half as wide as that of DS-CDMA scheme, and the MT-CDMA scheme has almost the same bandwidth as the DS-CDMA scheme. However, when the Nyquist filter with a small roll-off factor is used in the DS-CDMA scheme, the required bandwidths of MC-CDMA and Multicarrier DS-CDMA schemes become comparable with that of DS-CDMA scheme.

Table 2.1 System Features Comparison of DS-CDMA and three Multicarrier CDMA schemes

Parameters	DS-CDMA	MC-CDMA	MC-DS-CDMA	MT-CDMA
Symbol duration at subcarrier	T_S	$N_C T_S / G_{MC}$	$N_C T_S$	$N_C T_S$
Number of subcarriers	1	N_C	N_C	N_C
Processing gain	G_{DS}	$G_{MC}=G_{DS}$	$G_{MD}=G_{DS}$	$G_{MT}=N_C G_{DS}$
Chip duration	T_S / G_{DS}		$N_C T_S / G_{MD}$	$N_C T_S / G_{MT}$
Subcarrier separation		$1/T_S$	$G_{MD}/N_C T_S$	$1/N_C T_S$
Required Bandwidth (main lobe)	G_{DS}/T_S (Nyquist filter roll-off = 0)	$(N_C+1)G_{MC}/N_C T_S$	$(N_C+1)G_{MD}/N_C T_S$	$(N_C-1+2G_{MT})/N_C T_S$

Chapter 3

2-D ORTHOGONAL VARIABLE SPREADING FACTORS (OVSF) CODES

As mentioned earlier, 2D OVSF codes were first proposed by W.C. Kwong et al. [5]. Orthogonal Variable Spreading Factors (OVSF) Codes are the most common family of 2D spreading codes. They are preferred because they have all the properties i.e. orthogonality and correlation properties, that are required for a good spreading codes. In addition to it, since they have variable spreading factors, they can be used in multi-rate applications for matching data rates.

2D OVSF codes can be generated recursively by using a tree structure. The new codes offer both variable length spreading and multi-code capabilities. Since they possess zero cyclic auto-correlation and cross-correlation properties, they can be used as both the channelization and scrambling codes [5]. In other words, with the 2D OVSF codes, the two-layered spreading scheme is not necessarily needed and the variable length spreading and multi-code techniques used for multi-rate transmission can be applied to multicarrier systems.

3.1 Definition

A set of 2D spreading code sequences, C, with the even (odd) autocorrelation side-lobes and even (odd) cross-correlation functions of at most λ_a^e (λ_a^o) and λ_c^e (λ_c^o), respectively, is a collection of (+1, −1) $M \times N$ matrices, such that [5]:

- (Even/Odd Autocorrelation) For any matrix $A_{M\times N}^{(i)} \in C$ and an integer $\tau \in (0,N)$, the absolute values of the 2D binary discrete even and odd autocorrelation side-lobes of $A_{M\times N}^{(i)}$ are no greater than integers λ_a^e and λ_a^o
- (Even/Odd Cross-correlation) For any two distinct matrices $A_{M\times N}^{(i)}$ and $A_{M\times N}^{(j)}$ and an integer $\tau \in [0,N)$, the absolute values of the 2D binary discrete even and odd cross-correlation function of $A_{M\times N}^{(i)}$ and $A_{M\times N}^{(j)}$ are no greater than integer λ_c^e and λ_c^o

The code set C is said to be *orthogonal* if and only if $\lambda_a^e = \lambda_a^o = \lambda_c^e = \lambda_c^o = 0$. The codes used in this thesis are orthogonal by definition.

3.2 Correlation Properties

Orthogonality in 2D domain

Orthogonality is one of the most important properties for a sequence to be used as spreading codes. We are all familiar with orthogonality in one dimension which can be stated as: Two vectors are called orthogonal if their inner product is zero for all shifts, but in coding practice it is required that the inner product of each sequence must be zero for all shift except zero lag. In other words, the cross-correlation functions should be zero for all delay and auto-correlation function should have single peak at zero lag over one period of code.

To extend this definition of orthogonality in two dimensional domain, define auto-correlation and cross-correlation functions [5] as follows:

Let ***C*** be a set of 2D spreading codes that consist of a number of M×N matrices designated as $A_{M\times N}^{(i)}$. Then auto-correlation and cross-correlation is defines as:

Auto-correlation

There are two types of auto-correlation defined for 2D spreading codes: odd and even auto-correlation. The definitions of "even" and "odd" are based on the transmission pattern of two consecutive data bits. While the even represents the case of a "+1" followed by another "+1" or a "-1" followed by another "-1", and the odd represents for the case of a "+1" followed by an "-1" or vice versa.

- *Even Auto-correlation*

  For any matrix $A_{M\times N}^{(i)} \in \boldsymbol{C}$ and an integer t Є (0, N), the absolute value of the 2D binary discrete even auto-correlation is defined as in eqn. (3.1) and should be less than a number λ_a^e

$$| \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} a_{k,l}^{(i)} * a_{k,l\oplus t}^{(i)} | \leq \lambda_a^e \tag{3.1}$$

where $a_{k,l}^{(i)}$ Є {+1, -1} is an element of $A_{M \times N}^{(i)}$ at the kth row and lth column, and ⊕ denotes modulo-N addition.

- *Odd Auto-correlation*

The odd auto-correlation for the same set is defined as and should be less than λ_a^o .

$$| \sum_{m=0}^{M-1} \sum_{n=0}^{N-t-1} a_{m,n}^{(i)} * a_{m,n\oplus t}^{(i)} + \sum_{m=0}^{M-1} \sum_{n=N-t}^{N-1} a_{m,n}^{(i)} * (-a_{m,n\oplus t}^{(i)}) | \leq \lambda_a^o \tag{3.2}$$

Cross –correlation

As in case of auto-correlation, two types of cross-correlation can be defined for 2D spreading codes: odd and even cross-correlation. Let there are two distinct matrices $A_{M \times N}^{(i)}$ and $A_{M \times N}^{(j)}$ from the same set ***C*** and t Є [0.N) then cross-correlations are defined as below:

- *Even Cross-correlation*

  It is defined as

$$| \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} a_{k,l}^{(i)} * a_{k,l\oplus t}^{(j)} | \leq \lambda_c^e \tag{3.3}$$

  this modulus should be less than a number λ_c^e .

- *Odd Cross-correlation*

  Odd cross-correlation is defined in the following equation:

$$|\sum_{m=0}^{M-1}\sum_{n=0}^{N-t-1} a_{m,n}^{(i)} * a_{m,n\oplus t}^{(j)} + \sum_{m=0}^{M-1}\sum_{n=N-t}^{N-1} a_{m,n}^{(i)} * (-a_{m,n\oplus t}^{(j)})| \leq \lambda_c^o \tag{3.4}$$

Now coming back to the issue of orthogonality in 2D as described above, the two code matrices are said to be orthogonal if and only if all the above correlation coefficients are identically zero [10] that is:

$$\lambda_a^e = \lambda_a^o = \lambda_c^e = \lambda_c^o = 0 \tag{3.5}$$

3.3 Construction of 2D-OVSF codes

There are two families of 2D-OVSF codes namely, Square family and Rectangular family. In square family number of rows are equal to the number of columns while in rectangular family the number of columns is larger than number of rows. The number of columns represents the number of chips or spreading factor and number of rows represents the number of sub-carrier. Thus, in square family number of sub-carrier is equal to the spreading factor but in rectangular family spreading factor is greater than number of sub-carrier. For the generation of these codes, kronecker product is used. In general, small 2D-OVSF codes of dimension 2×2 are chosen and then higher codes are generated using repeated Kronecker Product of previously generated codes.

3.3.1 Construction of Square Code Matrix $A_{2^k \times 2^k}^{(i)}$

Initially we start with k=1, i.e. cardinality N= 2^k = 2 and generate first 2D OVSF codes which are two orthogonal matrices as shown below:

$$A_{2\times 2}^{(1)} = \begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix} \qquad A_{2\times 2}^{(2)} = \begin{pmatrix} +1 & -1 \\ +1 & +1 \end{pmatrix} \tag{3.6}$$

Now the next code matrices for k=2 i.e. N=4, are generated by taking Kronecker Product of the above two matrices that is:

$$A^{(1)}_{4\times4} = A^{(1)}_{2\times2} \otimes A^{(1)}_{2\times2} = \begin{pmatrix} +1\begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix} & +1\begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix} \\ +1\begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix} & -1\begin{pmatrix} +1 & +1 \\ +1 & -1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{pmatrix}$$

$$A^{(2)}_{4\times4} = A^{(2)}_{2\times2} \otimes A^{(1)}_{2\times2} = \begin{pmatrix} +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \\ +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \end{pmatrix}$$

$$A^{(3)}_{4\times4} = A^{(1)}_{2\times2} \otimes A^{(2)}_{2\times2} = \begin{pmatrix} +1 & -1 & +1 & -1 \\ +1 & +1 & +1 & +1 \\ +1 & -1 & -1 & +1 \\ +1 & +1 & -1 & -1 \end{pmatrix}$$

$$A^{(4)}_{4\times4} = A^{(2)}_{2\times2} \otimes A^{(2)}_{2\times2} = \begin{pmatrix} +1 & -1 & -1 & +1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & +1 & +1 \end{pmatrix} \tag{3.7}$$

In general, with k>1, the 2D OVSF code of size N×N for $N=2^k$ can be generated by a recursive formula [10]:

$$A^{(2i-1)}_{2^k\times 2^k} = A^{(1)}_{2\times 2} \otimes A^{(i)}_{2^{k-1}\times 2^{k-1}} = \begin{pmatrix} A^{(i)}_{2^{k-1}\times 2^{k-1}} & A^{(i)}_{2^{k-1}\times 2^{k-1}} \\ A^{(i)}_{2^{k-1}\times 2^{k-1}} & -A^{(i)}_{2^{k-1}\times 2^{k-1}} \end{pmatrix}$$

$$A^{(2i)}_{2^k\times 2^k} = A^{(2)}_{2\times 2} \otimes A^{(i)}_{2^{k-1}\times 2^{k-1}} = \begin{pmatrix} A^{(i)}_{2^{k-1}\times 2^{k-1}} & -A^{(i)}_{2^{k-1}\times 2^{k-1}} \\ A^{(i)}_{2^{k-1}\times 2^{k-1}} & A^{(i)}_{2^{k-1}\times 2^{k-1}} \end{pmatrix} \tag{3.8}$$

The code tree for the generation of above mentioned square code matrices is shown in fig. 3.1. The code tree has root at $A^{(1)}_{2\times 2}$ and $A^{(2)}_{2\times 2}$ is shown up to k=3 depth.

These square 2D OVSF code matrices satisfy the condition of orthogonality that is they have auto-correlation side-lobes and cross-correlation equal to zero.

3.3.2 Construction of Rectangular code matrix $A^{(i)}_{2^k\times 2^{k+\alpha}}$ with $\alpha \geq 1$

Since k and α are positive integers we suppose that $M=2^k$ and $N=2^{k+\alpha}$. Then the generation of second type i.e. rectangular 2D OVSF code matrices of cardinality $M=2^k$ using $A^{(i)}_{M\times N}$ starting with k=1 is done.

$$A^{(1)}_{2\times 4} = [\, A^{(1)}_{2\times 2} \;\; A^{(2)}_{2\times 2} \,] = \begin{pmatrix} +1 & +1 & +1 & -1 \\ +1 & -1 & +1 & +1 \end{pmatrix}$$

$$A^{(2)}_{2\times 4} = [\, A^{(1)}_{2\times 2} \;\; -A^{(2)}_{2\times 2} \,] = \begin{pmatrix} +1 & +1 & -1 & +1 \\ +1 & -1 & -1 & -1 \end{pmatrix} \tag{3.8}$$

This procedure is continued with M =2 and N = $2^{1+\alpha}$ for a given $\alpha \geq 1$ according to below given equations:

$$A^{(1)}_{2\times 2^{1+\alpha}} = [\, A^{(1)}_{2\times 2^{\alpha}} \;\; A^{(2)}_{2\times 2^{\alpha}} \,]$$

$$A^{(2)}_{2\times 2^{1+\alpha}} = [\, A^{(1)}_{2\times 2^{\alpha}} \;\; -A^{(2)}_{2\times 2^{\alpha}} \,] \tag{3.9}$$

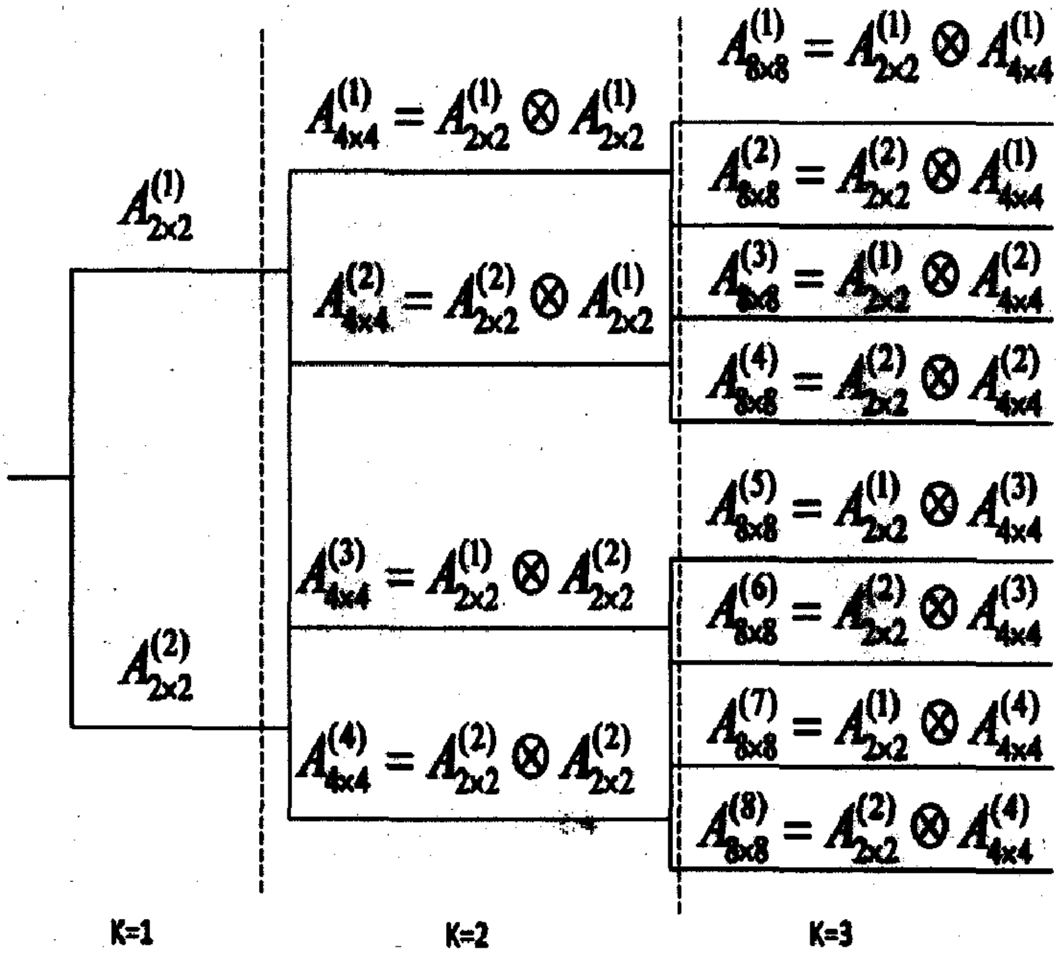

Fig. 3.1 Code Tree for Square 2D OVSF Codes

In general, with k > 1, the recursive equations for generation of rectangular 2DOVSF are given by:

$$A^{(2i-1)}_{2^k \times 2^{k+\alpha}} = A^{(1)}_{2\times 2} \otimes A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} = \begin{pmatrix} A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} & A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} \\ A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} & -A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} \end{pmatrix}$$

$$A^{(2i)}_{2^k \times 2^{k+\alpha}} = A^{(2)}_{2\times 2} \otimes A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} = \begin{pmatrix} A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} & -A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} \\ A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} & A^{(i)}_{2^{k-1}\times 2^{k-1+\alpha}} \end{pmatrix} \quad (3.10)$$

The code tree for this type of codes is shown below for α=1. The tree is shown up to k=3 depth and has roots $A^{(1)}_{2\times 4}$ and $A^{(2)}_{2\times 4}$. These rectangular 2D OVSF code matrices satisfy the condition of orthogonality that is they have auto-correlation side-lobes and cross-correlation equal to zero.

3.4 Generation and Properties of $A^{(i)}_{16\times 64}$ and $A^{(i)}_{16\times 256}$

In this section, we present the codes that are used in this dissertation for the purpose of simulations. We have used two sets of rectangular 2D OVSF codes; first one is 16-by-64 code matrix and second is 16-by-256 code matrix. Both these matrices are for low data rate.

16-by-64 code matrix is generated by using algorithm described in section 3.3, for $\alpha = 2$ and $k = 4$ i.e. firstly we have to use eqn.(4.5) two times to set $\alpha = 2$ to generate $A^{(1)}_{2\times 8}$ and $A^{(2)}_{2\times 8}$ and then traverse code tree by applying eqn.(4.6) four times to reach $k = 4$ level in the code tree structure. Correlation properties of these codes are verified in MATLAB as shown in fig. 3.3 and fig. 3.4. In fig {}, autocorrelation of $A^{(1)}_{16\times 64}$ and cross-correlation of $A^{(1)}_{16\times 64}$ and $A^{(2)}_{16\times 64}$ for various values of lag is shown. As seen in the graph, these codes possess ideal correlation properties.
Following the same procedure to generate 16-by-256 code for $\alpha = 4$ and $k = 4$, using eqn. (4.5) four times to generate $A^{(1)}_{2\times 32}$ and $A^{(2)}_{2\times 32}$ and then eqn. (4.6) four times to reach fourth level in code

tree in fig. 3.5. In fig. 3.4, autocorrelation of $A^{(1)}_{16\times256}$ and cross-correlation of $A^{(1)}_{16\times256}$ and $A^{(2)}_{16\times256}$ are shown for various values of lag.

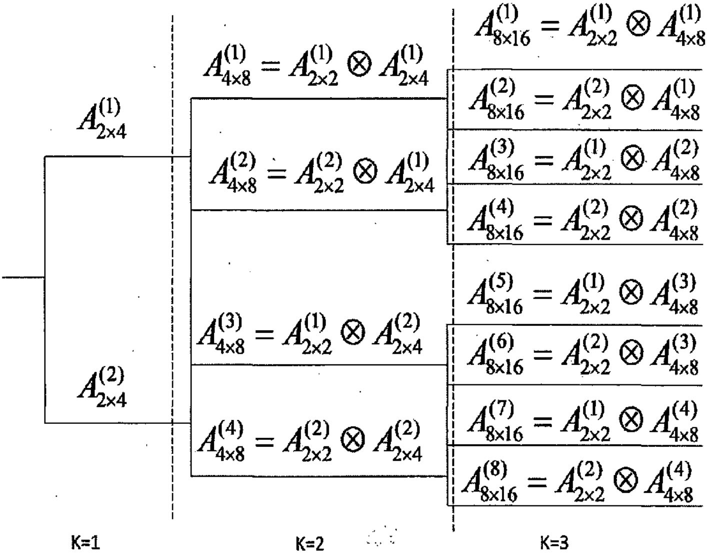

Fig. 3. 2 Code Tree for Rectangular 2D OVSF Codes

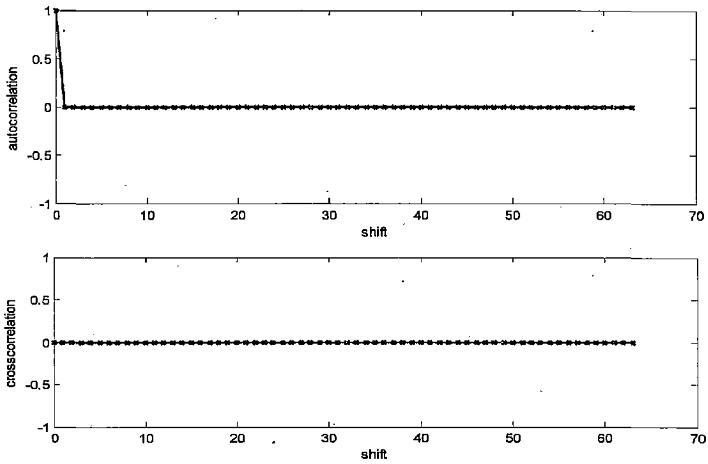

Fig. 3. 3 Auto-correlation and cross-correlation of $A^{(1)}_{16\times64}$ and $A^{(2)}_{16\times64}$

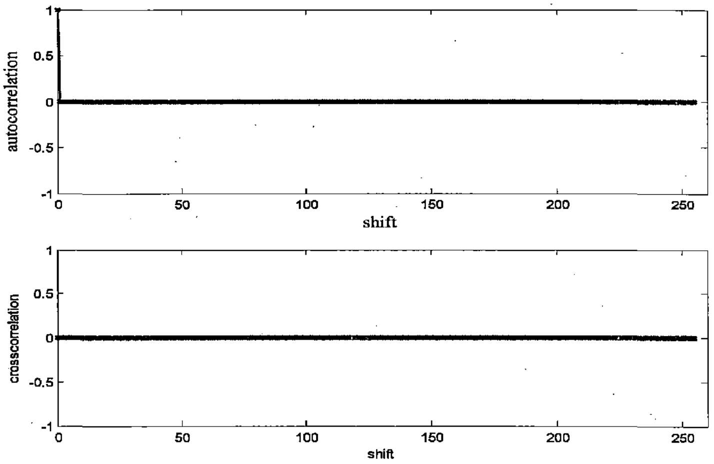

Fig. 3. 4 Auto-correlation and cross-correlation of $A^{(1)}_{16\times256}$ and $A^{(2)}_{16\times256}$

3.5 Code Assignment and Performance under Fading

Code Assignment

2D OVSF codes can support multi-rate users in MC-DS-CDMA. A higher data rate user can be assigned with a code of spreading factor smaller than the spreading of low rate user. Thus, high rate users are always assigned code from lower level of code tree.

As we know, codes in the same level of tree are orthogonal to each other, so codes from any one level can be assigned to a user. Also two codes from different level of code tree are orthogonal if one of them is not a descendent of the other. A code X is called mother code of any other code Y (child code) if X lies in the same path from code Y to root of the tree structure. Thus if a code is already assigned to a user then any of its child code cannot be assigned.

As shown in fig.3.5, if code $A^{(1)}_{4\times16}$ (marked grey in the tree) is currently assigned to a user, then the code $A^{(1)}_{2\times8}$, $A^{(1)}_{8\times32}$ and $A^{(2)}_{8\times32}$ cannot be allotted to a new user, all other codes except root node can be assigned to a new user.

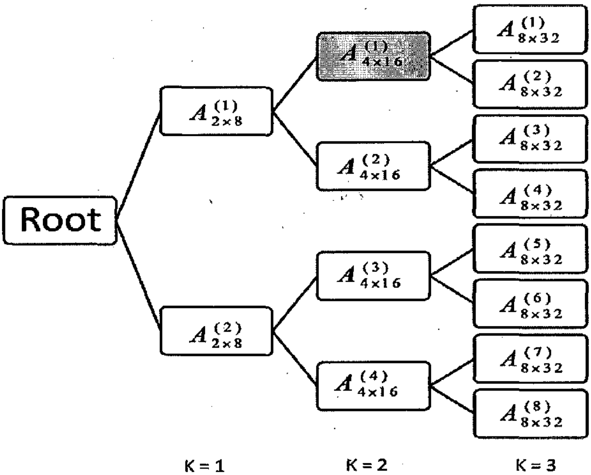

Fig. 3. 5 Code Tree for $A^{(i)}_{2^k\times2^{k+\alpha}}$ for $\alpha = 2$ and code assignment

Performance under Fading

In [10] authors have evaluated performance of these 2D-OVSF codes in AWGN, Rician and Rayleigh fading channel condition. In AWGN and Rician (weak fading) channel, these codes provide advantage over traditional MC-DS-CDMA employing 1D codes. Level of MAI is significantly reduced because 2D-OVSF codes retains their orthogonality in these channels, thus data rate can be effectively increased and system can support more simultaneous users for a given BER. BER performance, as evaluated by above authors is shown in fig{}. Here LR means low rate user and code assigned to it is 16-by-64 matrix generated in above section from k = 4 level of code tree, HR means high rate user having twice of data rate of LR and code assigned to it is 8-by-32 from the same tree but at level k = 3. In this system, author has considered 16 user including both HR and LR. Performance is calculated for both the user. The legend, for example, "HR=10, LR=6, HR" means that there are 10 higher-rate users and 6 lower-rate users, and the higher-rate users' performance is analyzed. As shown in the figure, the performances of the LR and HR users are almost identical, independent of the code length. It is because the 2-D OVSF codes are orthogonal and the code orthogonality cannot be destroyed by a non-fading channel. The argument of longer code length adding more diversity is not applicable in this case because of no MAI created by the 2-D OVSF codes. The MRC and EGC reception methods give the same performance in the AWGN non-fading channel because the weights of the diversity branches in both methods are all unity (i.e., $g_m = 1$). The 2-D OVSF codes are effective in the Rician channel because the fading effect is weak and the code orthogonality is mostly preserved. In frequency selective fading channel, orthogonality of codes is destroyed and hence this scheme does not gain any advantage over conventional MC-DS-CDMA employing 1D code.

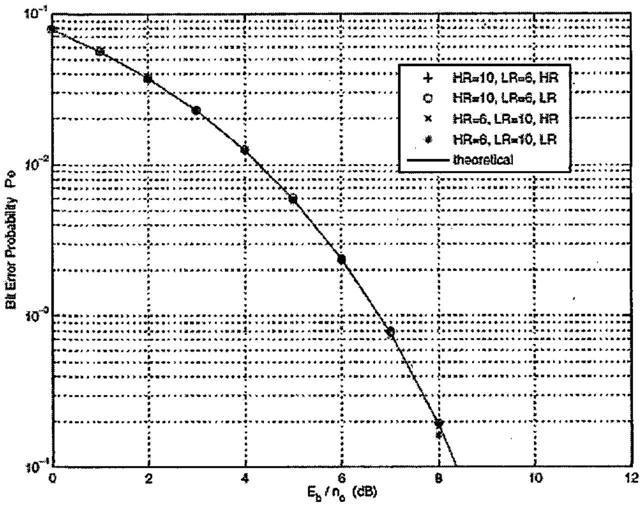

Fig 3. 6 Performance of 2D-OVSF codes in AWGN channel [15]

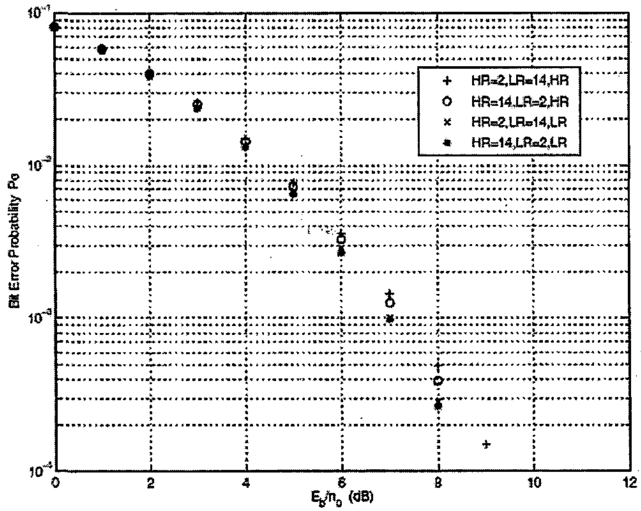

Fig. 3. 7 Performance of 2D-OVSF codes in Rician channel for

rice factor R = 10dB with MRC reception [10]

Chapter 4

2-D COMPLETE COMPLEMENTARY CODES (CCC)

Complementary codes were first studied by Golay [6], Turyn [7] and Taki et al. [9]. The pairs of code-words from these codes have auto-correlation functions equal to zero for all even shifts except zero shift. The concept of complementary codes was further developed by Tseng and Lui [20], Sivaswamy [21] and Frank [22]. Suehiro and Hatori [23] extended the concept to families of Complete Complementary Codes whose auto-correlation is equal to zero for all shifts except zero shift and the cross-correlation is zero for all shifts. Suehiro and Hatori proposed and (M, N, L) complete complementary code that consists of M sets of N sequences of length L. They also proposed the construction of the (N, N, N^2) complete complementary codes and then extends the concept to give construction of (M, N, MN) and (N, N, MN) complete complementary codes.

4.1 Preliminaries

Before defining Complete Complementary Codes, it is required that we understand the basic terminology of Complementary sequences.

- Complementary Pairs

Let $\{A_1, A_2\}$ be a sequence of +1's and -1's. Let $\psi(A_1,l)$ denotes the auto-correlation of A_1 at lag l, then pair of sequences $\{A_1, A_2\}$ is called a complementary pair, if

$$\psi(A_1,l)+\psi(A_2,l)=0 \qquad \forall l\neq 0 \tag{4.1}$$

Complementary pairs were first introduced by Golay [5], where the pairs are over the binary alphabet $\{1,-1\}$, as known as Golay pairs.

- Complementary Sets

A set of m sequences $\{A_1, A_2, \ldots\ldots, A_m\}$, each having length n, is called a complementary set if

$$\sum_{i=1}^{m}\psi(A_i,l)=0 \qquad 1\le l\le n-1 \qquad (4.2)$$

A complementary set can be represented using a complementary set matrix

$$A=\begin{bmatrix} r_1\\ r_2\\ \vdots\\ r_m\end{bmatrix}=\begin{bmatrix} c_0\\ c_1\\ \vdots\\ c_{n-1}\end{bmatrix}^T=\begin{bmatrix} a_{1,0} & a_{1,1} & \cdots & a_{1,n-1}\\ a_{2,0} & a_{2,1} & \cdots & a_{2,n-1}\\ \vdots & \vdots & \ddots & \vdots\\ a_{m,0} & a_{m,1} & \cdots & a_{m,n-1}\end{bmatrix} \qquad (4.3)$$

where the row sequences, $r_i = (A_{i,0}, A_{i,1} \ldots\ldots A_{i,n-1})$, $1 \le i \le m$, are complementary sequences, that is,

$$\sum_{i=1}^{m}\psi(r_i,l)=0 \qquad 1\le l\le n-1 \qquad (4.4)$$

$c_j = (A_{1,j}, A_{2,j}, A_{3,j}, \ldots, A_{m,j})$, $0 \le j \le n-1$, are column sequences.

- Mutually Orthogonal Complementary Sets

Complementary sets $\{A_1, A_2, \ldots, A_m\}$ and $\{B_1, B_2, \ldots, B_m\}$ are mates if

$$\sum_{i=1}^{m}\psi(A_i,B_i,l)=0 \qquad 1-n\le l\le n-1 \qquad (4.5)$$

The mutually orthogonal (MO) complementary set is a collection of complementary sets in which any two are mates. Let M (k, m, n) denote the Mutually Orthogonal Complementary set consisting of k complementary sets each having m complementary sequences of length n. For binary sequences, k cannot exceed m, that is, the maximum number of Mutually Orthogonal Complementary set is equal to the number of complementary sequences in a set. Hence, M (m, m, n) is called a Mutually Orthogonal Complementary of order m [23].

4.2 Definition

Let an (M, N, L) family $A = \{A_1, A_2, \ldots\ldots A_M\}$ having M matrices of N rows and L columns. We denote A_j in matrix form as:

$$A_j = \begin{bmatrix} a_1^j \\ a_2^j \\ \vdots \\ a_N^j \end{bmatrix}$$

Then, the (M, N, L) family is called an (M, N, L) complete complementary codes [24] if they satisfy:

(i)
$$\psi(A_j, A_j, l) = \sum_{i=1}^{N} \psi(a_i^j, a_i^j, l)$$
$$= \begin{cases} D, & l = 0 \\ 0, & l = \pm 1, \pm 2, \cdots \pm (L-1) \end{cases} \qquad \forall 1 \le j \le M \qquad (4.6)$$

where D is a positive constant independent of *i*.

(ii) For every $1 \le j, k \le M$

$$\psi(A_j, A_k, l) = \sum_{i=1}^{N} \psi(a_i^k, a_i^k, l) = 0 \qquad l = 0, \pm 1, \pm 2, \cdots \pm (L-1) \qquad (4.7)$$

4.3 Synthesis of Complete Complementary Codes

In this section, we describe some method to synthesize complementary sets and complete complementary codes. Firstly, let us define some operations on sequence that will facilitate in understanding synthesis of complementary sets, as shown in Table 4.1. Let A be a sequence of +1's and -1's, h is a variable having value either +1 or -1.

4.3.1 Synthesis of complementary sequences

There are a number of methods to synthesize complementary set of sequences from smaller family. Some of them are briefly described below. Their proofs can be found in [20].

(i) From a complementary set of sequences $(A_i, 1 \le i \le p)$ other complementary sets can be obtained by

(ii) reversing any number of the sequences in the set, i.e. performing operation $\tilde{A}_i$ on any number of sequence

(iii) performing $-A_i$ on any number of the sequences in the set, and

(iv) negating alternate elements in all sequences in the set.

Table 4.1 Operations over sequences for Synthesis of CCC codes

Symbol	Operation
$\tilde{A}$	Reverse of sequence A
$-A$	Negation of A
$-\tilde{A}$	Reverse of negation of A
A^h	$= \begin{cases} A & if\, h=+1 \\ -A & if\, h=-1 \end{cases}$
AB	Concatenation of sequence A and B
$A \circ B$	Interleaving of sequence A and B
A^*	Sequence with element of odd subscripts of A and B
A^{**}	Sequence with element of even subscripts of A and B

2. Let $(A_i, 1 \le i \le p)$ be a complementary set.

Then $(A_i^*, A_i^{**}, 1 \le i \le p)$ is also a complementary set that is choosing elements with odd subscripts and even subscripts. The new set generated has twice the number of sequences than the original set and length of sequences is halved.

3. Let $(A_i, 1 \le i \le p)$ be a complementary set and $(B_i, 1 \le i \le p)$ one of its mates. Then $(A_i \circ B_i, 1 \le i \le p)$ is also a complementary set.

4. Let $(A_i, 1 \le i \le p)$ be a complementary set. Let H be a $q \times p$ orthogonal matrix, whose elements are either +1 or-1. Then

$$(A_1^{h_{11}} A_2^{h_{12}} \cdots A_p^{h_{1p}}, \cdots\cdots A_1^{h_{q1}} A_2^{h_{q1}} \cdots A_p^{h_{qp}})$$

$$\equiv\left(\prod_{j=1}^{p} A_j^{h_{ij}}, 1 \le i \le q\right) \tag{4.8}$$

is a complementary set of q sequences. For example, let (A, B) be a complementary set of sequences and orthogonal matrix H is given by:

$$H\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 1 \\ -1 & -1 \end{bmatrix}$$

Then $\{AB, A(-B), (-AB), (-A)(-B)\}$ is a set of complementary sequences.

5. Let (A, B) and (X, Y) be two complementary sets of sequences. Then the set ($A^X B^Y, A^{\tilde{Y}} B^{-X}$) is also a complementary set

4.3.2 Synthesis of complete complementary codes

Jin and Koga [24] have shown that if (M_1, N_1, L_1) and (M_2, N_2, L_2) complete complementary codes are given; we can easily construct a larger family by using Kronecker Product. Also an (N, N, N) complete complementary code can be obtained using only N[th] DFT matrix FN and the cyclic shifts of the rows. Both methods are briefly explained below.

1. *Using DFT Matrix:*

Let us consider the DFT (discrete Fourier transform) matrix. For an arbitrary integer $N \ge 2$ the N^{th} DFT matrix is defined by

$$F_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \cdots & \omega^{(N-1)^2} \end{bmatrix} \tag{4.9}$$

where $\omega = e^{i\frac{2\pi}{N}}$. For each k = 0, 1, ... $N - 1$, the matrix $F_N^{(k)}$ is obtained by repeating the cyclic shift with respect to the rows for k times, i.e.

$$F_N^{(1)} = \begin{bmatrix} 1 & \omega & \omega^2 & \vdots & \omega^{N-1} \\ 1 & \omega^2 & \omega^4 & \vdots & \omega^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{2(N-1)} & \cdots & \omega^{(N-1)^2} \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \tag{4.10}$$

Then, $S=\{F_N^{(k)}: k=0,1,\cdots,N-1\}$ is complete complementary code. For example consider the case of N = 3, by definition,

$$F_N^{(0)} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4=\omega \end{bmatrix}$$

$$F_N^{(1)} = \begin{bmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{bmatrix}$$

$$F_N^{(0)} = \begin{bmatrix} 1 & \omega^2 & \omega \\ 1 & 1 & 1 \\ 1 & \omega & \omega^2 \end{bmatrix} \tag{4.11}$$

where $\omega = e^{i\frac{2\pi}{3}}$ then $S=\{F_N^{(0)},F_N^{(1)},F_N^{(2)}\}$ gives a (3, 3, 3)-complete complementary code. We can verify their correlation properties by using $\omega^3=1; 1+\omega+\omega^2=0; \omega^2=\omega$.

2. Using Kronecker Product

Let us consider two families of complementary codes given by (M_p, N_p, L_p) complete complementary code P = {P_1, P_2, . . . , P_{Mp} } and (M_q, N_q, L_q) complete complementary code Q = {Q_1,Q_2, . . . ,Q_{Mq} },

Then S is an (M_pM_q, N_pN_q, L_pL_q)-complete complementary code given by

$$S=\{S_{ij}: 1\le i\le M_p; 1\le j\le M_q\}$$

Where

$$S_{ij}=P_i\otimes Q_j \tag{4.12}$$

$\otimes$ denote Kronecker Product of two matrices.

4.4 Generation of (16,16,256) Complete Complementary Code

In this dissertation, we have used Kronecker Product method to generate higher families of complementary codes. The two families P and Q are identical (2, 2, 4) complementary codes i.e. $P = Q = \{P_1, P_2\}$, where P_1 and P_2 are given by

$$P_1 = \begin{bmatrix} +1 & +1 & +1 & -1 \\ +1 & -1 & +1 & +1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} +1 & +1 & -1 & +1 \\ +1 & -1 & -1 & -1 \end{bmatrix} \tag{4.13}$$

Now applying eqn. (4.12), we get S (4, 4, 16) complementary code as

$S = \{S_1, S_2, S_3, S_4\}$

Where

$$S_1 = P_1 \otimes P_1 = \begin{bmatrix} +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 \\ +1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 \\ +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 \\ +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & +1 \end{bmatrix}$$

$$S_2 = P_1 \otimes P_2 = \begin{bmatrix} +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 \\ +1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 \\ +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 \end{bmatrix}$$

$$S_3 = P_2 \otimes P_1 = \begin{bmatrix} +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 \\ +1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 \end{bmatrix}$$

$$S_4 = P_2 \otimes P_2 = \begin{bmatrix} +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 \\ +1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 \\ +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & +1 & -1 \\ +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & +1 \end{bmatrix}$$

In the next step, P = {P_1, P_2} and S = {S_1,S_2,S_3,S_4} are chosen to generate next higher family R= {R_i: i = 1, 2,…,8} of (8, 8, 64) complementary codes. Similar process is followed to generate finally (16, 16, 256) complementary code that are used as signature codes in the simulation.

The correlation properties of complementary code S = {S_1, S_2, S_3, S_4} are verified through MATLAB simulations. Fig.4.1 shows the auto-correlation of matrix S_1 for each row and then sum of auto-correlation. Fig.4.2 shows cross-correlation of each row of code matrix S_1 and S_2 and finally sum of cross-correlation. As seen clearly in the figure, sum of auto-correlation has single peak at zero lag and sum of cross-correlation is identically zero for all lag.

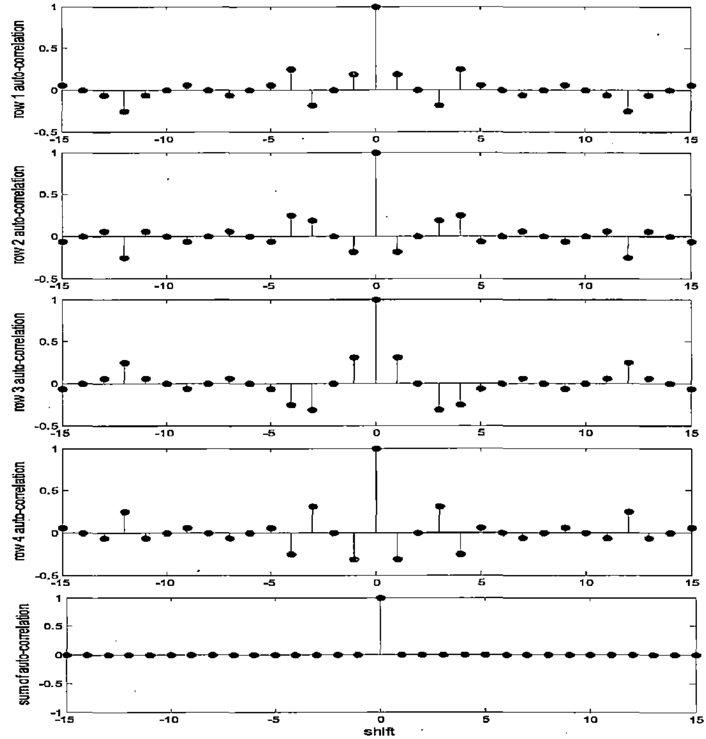

Fig.4.1 Auto-correlation of rows and Sum of auto correlation for family *S* (4, 4, 16)

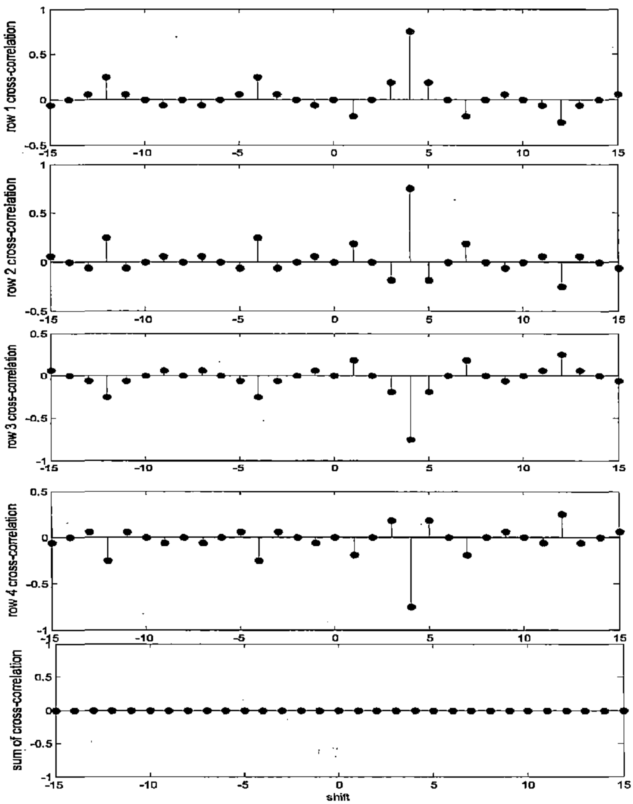

Fig 4.2 Cross-correlation of rows and Sum of cross correlation for family *S* (4, 4, 16)

4.5 Performance in Fading

In [11], authors have investigated Complete Complementary Codes for performance in AWGN and fading channels. Referring to fig.4.3, performance of MC-DS-CDMA with Complete Complementary codes outperforms the single channel system with m-sequences of length 62.

The bit-error probability remains the same as the number of interfering users increases, assuming perfect carrier synchronization, provided there must be orthogonal codes available to allot additional users. In AWGN and Rician channel, the data rate can be increased for each user. Actually, for the ideal AWGN channel, the data rate per user can be as much as M times higher than that of SC-system without significant increase in system complexity. The MAI is reduced, so system can support more users for a given average error probability constraint. It is important to point out that although severe constraints on the existence of a set of M spreading sequences instead of a single spreading sequence for each user, the number of users this system can support (capacity) is comparable to their system assigning a single sequence per user.

MC-DS-CDMA with Complete Complementary codes system is vulnerable to frequency-selective fading, and it performs the same as the SC-system in Rayleigh frequency-selective fading. This is because the Rayleigh fading channel is completely phase non-coherent, so it destroys the properties of complementary code sets.

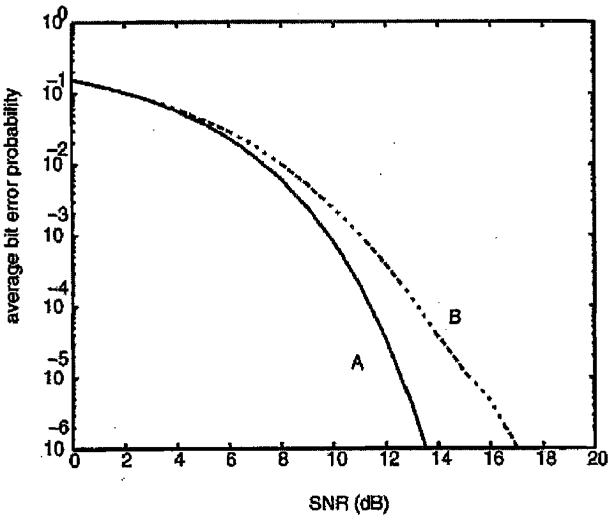

Fig. 4.3 Performance comparison of MC-DS-CDMA with Complementary codes and SC-DS-CDMA with m-sequences in AWGN channel [3]

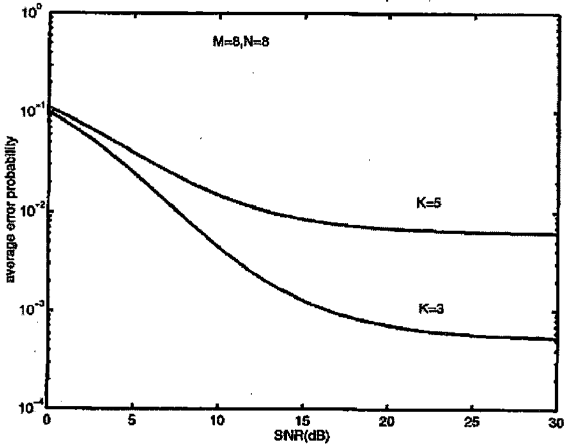

Fig. 4.4 Performance of MC-DS-CDMA with Complementary codes in Rayleigh channel for k=3, 5 users [3]

Chapter 5

SYSTEM MODEL AND RESULTS

In this chapter, we present the system model and simulation set-up of MC-DS-CDMA employing 2 dimensional spreading codes. Implementation of transmitter and receiver structure is done by using IFFT and FFT in MATLAB.

5.1 System Model

A MC-DS-CDMA system is considered with non overlapping carriers to avoid unnecessary self-interference [10]. Transmitter model for k[th] user is shown in fig.5.1.

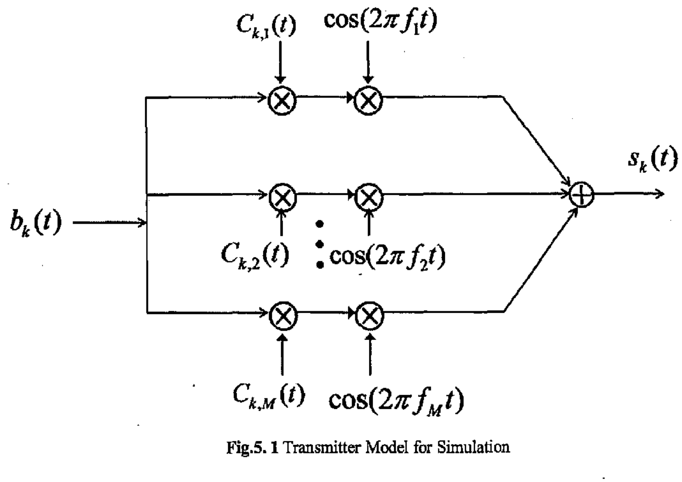

Fig.5. 1 Transmitter Model for Simulation

Here $b_k(t)$ is the user data with bit interval T_b,

C_k is the 2-dimensional M-by-N spreading code matrix assign to the k[th] user,

$C_{k,i}$ represents the i^{th} row of code matrix C_k.

M is the number of subcarrier in the system,

N is the spreading factor of the user,

ω_i is the i^{th} subcarrier frequency i =1.....M.

Firstly, user data $b_k(t)$ is multiplied (spreading) by $C_{k,i}$ row of code matrix of length N and then modulated by the i^{th} carrier {by multiplying by $\cos(2\pi f_i t)$ }. The transmitted signal of k^{th} user can be written as:

$$s_k(t) = \sum_{i=1}^{M} b_k(t) C_{k,i}(t) cos(2\pi f_i t) \tag{5.1}$$

There are K simultaneous users depending upon maximum number of signature sequence available in the code family. These users can transmit on different data rates. Higher data rates can be provided by using smaller spreading factor. Since, same data is transmitted in all the subcarriers, this diversity can be exploited at the receiver by employing diversity reception method i.e. EGC and MRC. The receiver of the k^{th} user is shown in fig.5.2.

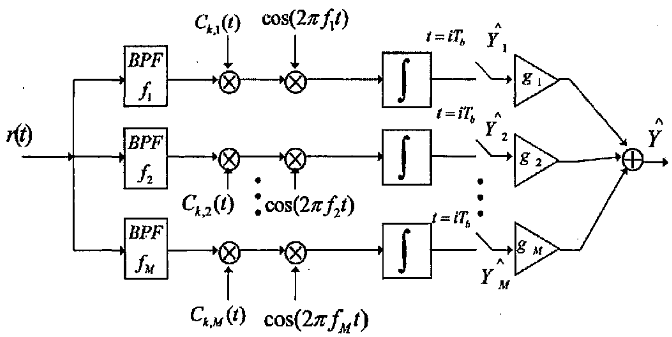

Fig.5. 2 Receiver Model for Simulation

Receiver employs band-pass filter/ RAKE receiver combination. It uses M band pass filters centered at each subcarrier frequency. Firstly, received signal is separated in M subcarrier by filtering through band pass filter, and then dispreading is done by multiplying $C_{k,i}$. The resultant signal is then demodulated and then signal is weighted according to the method of diversity combining. Hard decision in taken on the variable $\hat{Y}$ which is linear combination of filtered and de-spread signal $\hat{Y}_i$. Expression for $\hat{Y}$ is given by:

$$\hat{Y} = \sum_{i=1}^{M} g_i \hat{Y}_i \tag{5.2}$$

where g_i is the weight of i^th branch of RAKE receiver. In equal gain combining (EGC), all g_i's are set equal to unity. This method is simple but provides suboptimal performance. If maximum ratio combining (MRC) reception method is used, g_i's are set to corresponding fading amplitude i.e. $g_i = \alpha_i$ to provide maximum output signal-to-noise ratio.

5.2 Nakagami-m Fading

The Nakagami-m distribution is suitable for describing statistics of mobile radio transmission in complex media such as the urban environment [12]. In practice, it has proved very useful due to an easy manipulation and a wide range of applicability of various approximations. Since the Nakagami-m random process is defined as an envelope of the sum of 2m independent Gauss random processes, the Nakagami-m distribution is described by the pdf [12]:

$$p_z(z,\Omega) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m z^{2m-1}\exp\left(-\frac{m}{\Omega}z^2\right) \qquad z > 0, m \geq \frac{1}{2} \tag{5.3}$$

where z is the received signal level, Γ(.) is the gamma function, m is the parameter of fading depth (fading figure), defined as:

$$m = \frac{E^2[z]}{Var[z^2]} \tag{5.4}$$

while Ω □ is average signal power:

$$\Omega = E[z^2] \tag{5.5}$$

Parameter m controls the severity or depth of the amplitude fading. In particular, when the parameter of fading depth is m=1, the Nakagami-m distribution is reduced to the familiar Rayleigh distribution, while the case $m = 0.5$ corresponds to the unilateral Gauss distribution. Value of m less than one correspond to fading more severe than Rayleigh fading and value greater than one correspond to fading less severe than Rayleigh fading. The case $m \to \infty$ describes the channel without fading. For $m > 1$, Nakagami-m distribution approximates Rician distribution. PDF of Nakagami-m distribution for various values of m is shown in fig 5.3.

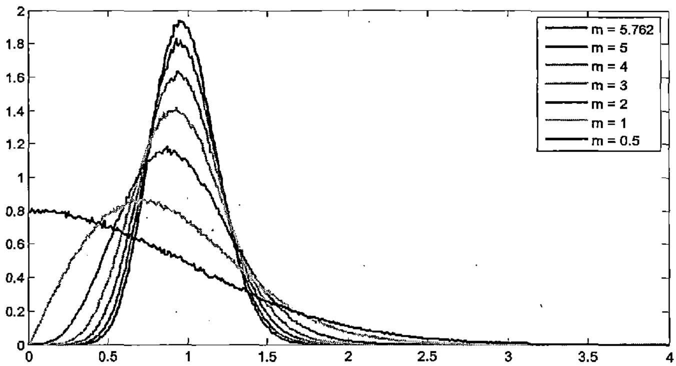

Fig.5. 3 Probability Density Function of Nakagami-m

We choose the Nakagami-m channel model for reasons of generality, and because the other models can be described by the Nakagami-m distribution by an appropriate choice of relevant parameters.

5.3 Results and Discussions

In this section, we present simulation results and discussion for BER performance of 2D-OVSF and 2D- CCC codes. Two families of 2D-OVSF codes of different spreading factors and one family of 2D-CCC codes are considered in the simulations. For both the codes, multi-user environment and users with two different rates has been considered. Channel is modeled as Nakagami-m flat fading channel with m = 1,2,3,4 and 5. For m = 5.762 Nakagami-m channel approximates Rician channel with Rice factor R = 10 dB. This case is also considered in the simulation. The simulations were carried out in MATLAB. Parameters shown in Table 5.1 are common to all simulations.

Table 5. 1 Common Parameters for Simulation of System

Parameter	Value
Number of Channels in System	16
Number of Low Rate User	10
Number of High Rate User	6
Performance Measure	Low Rate User BER

5.3.1 2D-OVSF codes

Codes used in simulations are generated in MATLAB as explained in section 3.4. Firstly, we will consider spreading code matrices with $\alpha = 2$ where α is defined in section 3.3. Simulations are carried out by using parameters shown in Table 5.2.

Table 5.2 Simulation Parameters for 2D-OVSF Codes of family $\alpha = 2$

Parameter	Value
Low Rate User Spreading Factor	64
High Rate User Spreading Factor	32
Low Rate User Code Matrices	$A^{(i)}_{16\times64}$
High Rate User Code Matrices	$A^{(i)}_{8\times32}$

BER performance of Low Rate User is plotted against E_b/N_o for m=1, 2, 3, 4 and 5 in fig.5.4. As seen in the fig. 5.4, for m=1 BER achieved at 30 dB E_b/N_o is 10^{-3}. This is due to orthogonality of 2D-OVSF codes is lost in Rayleigh fading and thus there is no advantage of using 2-dimensional spreading codes as compared to system using 1-dimensional spreading codes [2]. However, as the value of m is increased BER performance get improved because higher the value of m gives less severe fading.

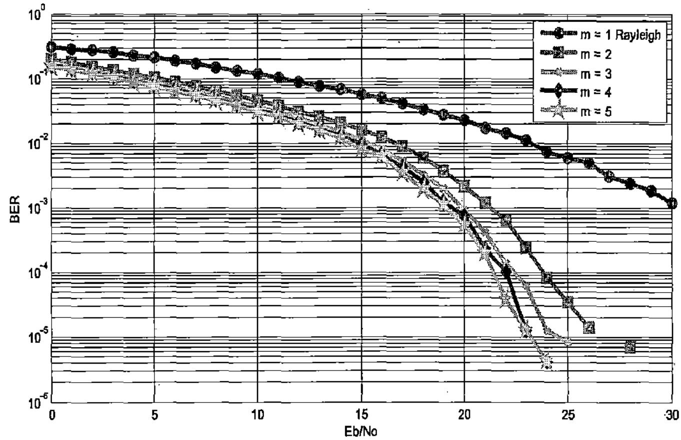

Fig.5. 4 BER Performance of LR user in Nakagami-m channel using 2D OVSF codes N = 64

Fig.5.5 shows the performance comparison of BER in Rayleigh channel (m = 1) and Rician channel (m = 5.762) for Rice factor of R= 10 dB. Since, in Rician channel, orthogonality of 2D-OVSF code is preserved achieved BER is 10^{-3} at 15 dB as compared to 30 dB in Rayleigh channel. Thus, these codes are well suited for Rician fading.

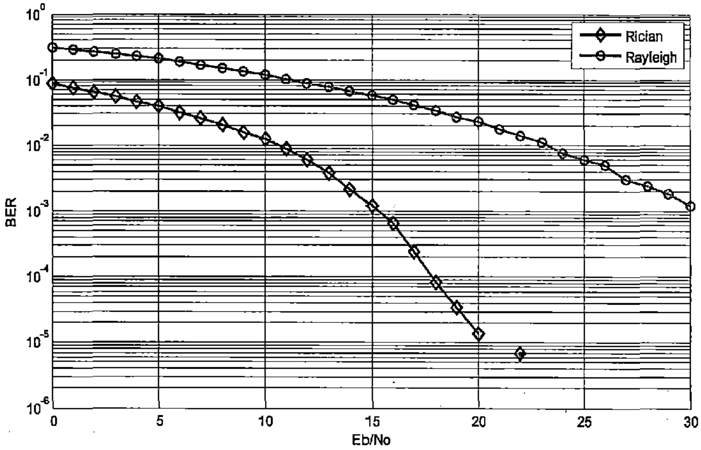

Fig.5. 5 Comparison of BER performance of 2D-OVSF codes in Rician and Rayleigh fading for N=64

For 2D-OVSF code family with $\alpha = 4$, simulation parameters are provided in Table 5.3, rest of the parameters are same as in Table 5.1. BER performance in Nakagami-m fading is plotted in fig.5.6.

Table 5. 3 Simulation Parameters for 2D-OVSF Codes of family $\alpha = 4$

Parameter	Value
Low Rate User Spreading Factor	256
High Rate User Spreading Factor	128
Low Rate User Code Matrices	$A^{(i)}_{16\times 256}$
High Rate User Code Matrices	$A^{(i)}_{8\times 128}$

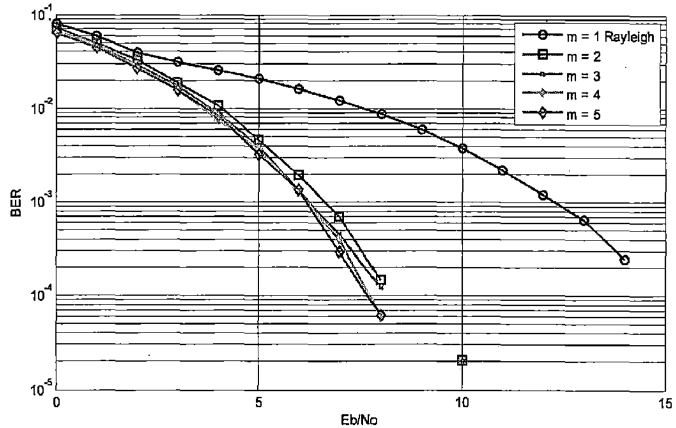

Fig.5. 6 BER Performance of 2D-OVSF codes in Nakagami-m channel for N=256

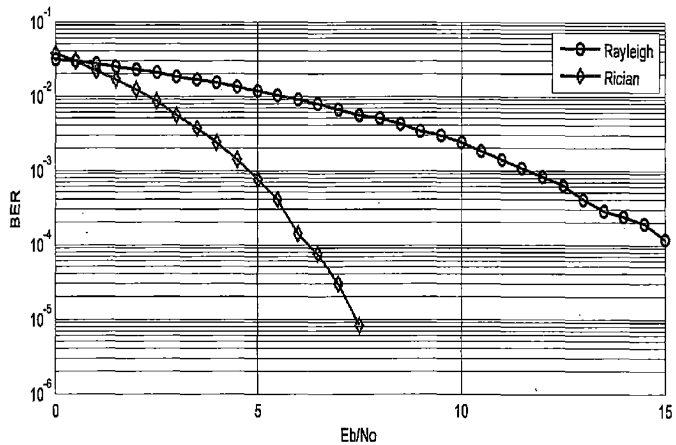

Fig.5. 7 Comparison of BER performance of 2D-OVSF codes in Rician and Rayleigh fading for N=256

It is observed from fig.5.4 and fig.5.6 that by increasing the spreading factor four times (from L= 64 to 256), there is much improvement in the BER performance. For m = 2, achieved BER is 10^{-3} at 7 dB for N = 256 as compared to 22 dB for N = 64. Similar performance is obtained for other values of 'm' showing improvement of about 15 dB. Fig. 5.7 shows comparison of BER performance in Rician and Rayleigh channel.

5.3.2 2D-CCC codes

2D-Complete Complementary Codes used in simulation are constructed as described in section 4.5. A similar system model is considered as in case of 2D-OVSF codes. Parameters of simulation are same as shown in Table 5.1 and Table 5.3. Fig.5.8 shows BER performance of Low Rate user in Nakagami-m channel for different values of m. BER of 10^{-3} is achieved in range of 7 dB to 8 dB for m = 2, 3, 4 and 5. In 2D-Complete Complementary codes also, performance is improved as we increase the value of m. Fig.5.9 shows BER performance of 2D-CCC in Rician and Rayleigh fading. BER of 10^{-3} is achieved at 7 dB and 12.5 dB for Rician and Rayleigh fading.

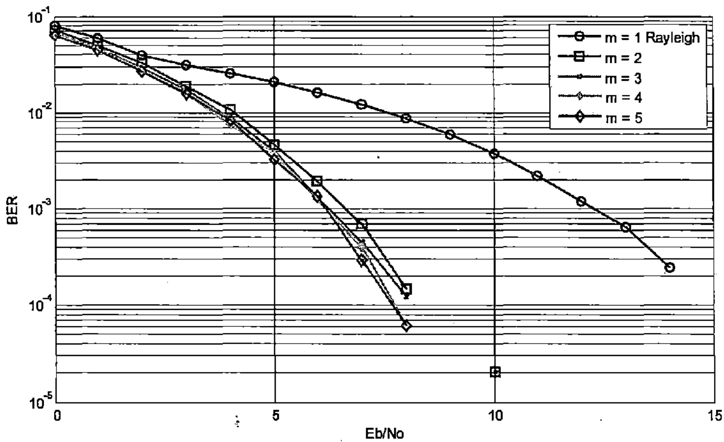

Fig.5. 8 BER performance of 2D-CCC codes in Nakagami-m fading N = 256

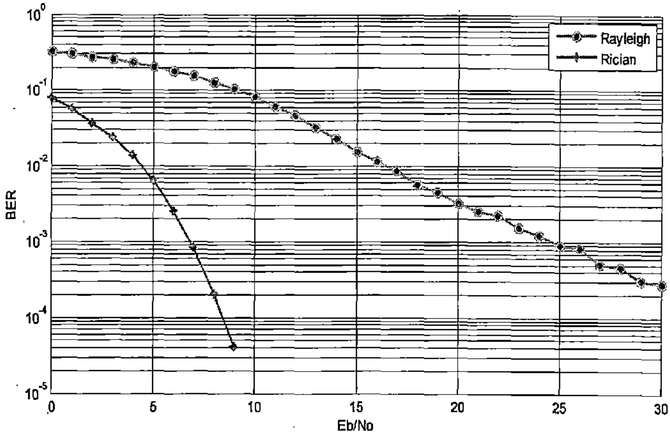

Fig.5. 9 Performance comparison of 2D-CCC codes in Rayleigh and Rician fading

5.3.3 Performance comparison of 2D-OVSF and 2D-CCC codes

As seen in the fig.5.6 and fig.5.8, both the codes perform well in Nakagami-m fading for values of m greater than 2. However, for m = 1 both the codes lost their orthogonality in Rayleigh fading and hence performance is reduced. Both these codes are suited for Rician fading channel. Also for values of m greater than 2, 2D-OVSF codes slightly perform better, about 1 dB at 10^{-3} BER, than 2D-CCC codes. The reason behind this is that 2D-OVSF codes have ideal correlation properties, whereas in 2D-CCC codes sum of correlation is ideal but not actual correlation.

Chapter 6

CONCLUSIONS AND FUTURE WORKS

Spreading sequences is an integral part of a CDMA system. Work presented in this dissertation is focused on the study and performance of 2-dimensional spreading codes in MC-DS-CDMA. A multi-rate multi-user MC-DS-CDMA system is considered with Nakagami-m fading. 2-D Orthogonal Variable Spreading Factor codes and 2-D Complete Complementary Codes are generated in MATLAB and their correlation properties are verified.

Performance of 2D OVSF codes and 2D CCC codes in Nakagami-m fading is analyzed for different fading factors (m = 1, 2, 3, 4, 5 and 5.762). As seen in the results, they perform well in Rician fading and Nakagami fading for values of m greater than two but in Rayleigh channel, they do not give any advantage compared to conventional DS-CDMA system. This is due to the reason because orthogonality of these codes is not maintained in Rayleigh Fading.

By varying spreading factor of 2D-OVSF codes from 64 to 256 in Rayleigh channel, an improvement of 15 dB is observed for target BER of 10^{-3} . On comparing OVSF codes and CCC codes of same length 256, it is observed that OVSF codes have an improvement of 1 dB at target BER 10^{-3} in Nakagami fading except for m = 1 case (Rayleigh fading).

Future Work

In this section, we suggest some areas which can continue the work performed in this dissertation.

As both of the codes considered in this dissertation do not remain orthogonal in Rayleigh fading, hence interference due to multiple access is not zero. To reduce the level of interference between users, power control can be applied in the system.

Since, these codes are suited in Rician fading and Nakagami fading (for m greater than 2), codes which can retain their orthogonality in Rayleigh channel can be investigated or designed.

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