ANALYSIS AND APPLICATION OF THERMISTOR BRIDGE CIRCUITS IN FLOW MEASUREMENT

A DISSERTATION

Submitted in partial fulfilment
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in
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(Measurement and Instrumentation)

By

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CERTIFICATE

Certified that the dissertation entitled 'ANALYSIS AND APPLICATION OF THERMISTOR BRIDGE CIRCUIT IN FLOW MEASUREMENT' which is being submitted by Sri Ashok Kumar Singh in partial fulfilment for the award of the Degree of Master of Engineering in Electrical Engineering(Measurement and Instrumentation) of University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of about 14 months with effect from August, 1980 to September, 1981 for preparing this dissertation at this University.

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Roorkee
October 13, 1981.

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CONTENTS

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ABSTRACT

The dissertation deals with the analysis and applications of thermistor bridge circuits. All important and useful thermistor bridge circuits namely Wheatstone (WB), Astable multivibrator (AMB), and Astable-Monostable multivibrators (AMB-MMB) have been considered for analysis. It has been mentioned that AMB-MMB circuit is more sensitive and linear as compared to WB and AMB circuits for similar input/output conditions. Some useful mathematical relations have been derived for 4x5 combinations of transducers circuit by carrying out generalized analysis. Expressions are given for the thermistor resistance for linearizing and balancing conditions for five different transducer probe configurations Piecewise linearized model approach has been applied to a WB circuit by dividing R-T characteristic of an actual NTC thermistor in three segments. The flow rate has been related to thermistor parameter change for showing its application in flow metering. The response time and sensitivity considerations have been investigated using a numerical analysis method. It has been shown that there is decrease in the rise time with increase in velocity. Some other interesting applications of thermistor bridge circuits are also given. Lastly, the conclusions drawn from the present work and the problems to be solved in future are given.

NOMENCLATURE

AFD	allowed fractional deviation
AMB	astable multivibrator bridge
B	thermistor constant in ^{o}K, it depends upon the material of the thermistor
CFT	constant frequency triggering
FD	fractional deviation
G	gain
MMB	monostable multi-vibrator bridge
NTC	negative temperature coefficient
PTC	positive temperature coefficient
R_o	cold resistance or thermistor resistance at $T=T_o$
R_m	resistance of output meter
$R(T)$ or R_T	thermistor resistance at temperature $T^{o}K$
S_A	sensitivity of AMB circuit
S_W	sensitivity of WB circuit
T_o	ambient temperature
T_i	datum point
VFT	variable frequency triggering
WB	wheatstone bridge circuit
x	normalized temperature ($\triangleq$ T/T_o)
x_i	normalized datum point
y	ratio of hot to cold-resistance of a thermistor

$\vert_{\dots}$	denotes .. is substituted in y
	temperature coefficient of resistance
	normalized thermistor constant ($\triangleq$ b/T_o)
	equivalent to
$)\vert$.=.	replaced by
	implies 'changed to '

CHAPTER 1

INTRODUCTION

Amongst different types of temperature sensing devices (e.g. thermocouples and metallic resistance thermometers), thermistor (THERMally sensitive resISTOR) is a relatively sensitive, small, rugged, inexpensive and simple device. It is passive [1]. Although, the thermistor principle is more than 150 years old, the present concept came into existence in early thirties and become popular by the name of regulating resistor. The first commercial thermistor was made of UO_2 in Germany in 1932 by OSRAM and was marketed under the trade name URDOX. Thereafter,UO_2 was replaced by $MgTiO_3$ (Spinel), reduced with H_2. The degree of reduction and admixture of HgO were controlling the resistivity and temperature dependence of these types of units. In 1936, PHILIPS followed with another type of semiconducting resistor made from silicon or ferrosilicon sintered together with inorganic binders and sold under the name STARTO tube. Both products URDOX and STARTO were sealed into bulbs filled with H_2,N_2, or Ar to prevent oxidation and thereby making them less suitable for temperature measurement because of high distance between sensor and the environment [1,2] . In 1940, BELL TELEPHONE LABORATORIES developed compensating resistance element to compensate resistance changes in transmission lines due to temperature variations. PHILIPS reentered the field in the year 1942 with a number of oxide systems in which iron oxides of different valence state were the conductive component. From 1950's to date, the art of controlling resistivity and its temperature dependence in transition metal Oxide systems has been advanced together with improved long range stability, interchangeability and miniaturization. So, the development of variety of bead types in glazed glass encapsulated form, had taken place.

1.1 THERMISTOR - TYPES, PACKAGES AND CHARACTERISTICS

The thermistor is a two terminal device having temperature-dependent resistance as its fundamental property.[1-32] One type with falling resistance characteristics is said to have negative temperature coefficient (NTC) of resistance. Fig. 1.1 shows resistance-temperature characteristics of a NTC type thermistor material. Note from the curve that a decrease in resistance (i.e. resistance ratio) of approximately 90 :1 results from a change in temperature from $0^{o}C$ to $100^{o}C$. Note also the nonlinearity of the curve over much of its length. This inherent non-linearity in its exponential relationship has always been considered the main disadvantage of thermistor.

On the other hand, a thermistor with rising characteristics is said to have positive temperature coefficient (PTC) of resistance . Earlier, the high PTC resistors were termed 'Thermistors'. Lateron, for all resistors with high positive or negative temperature coefficients, the term 'Thermistors'

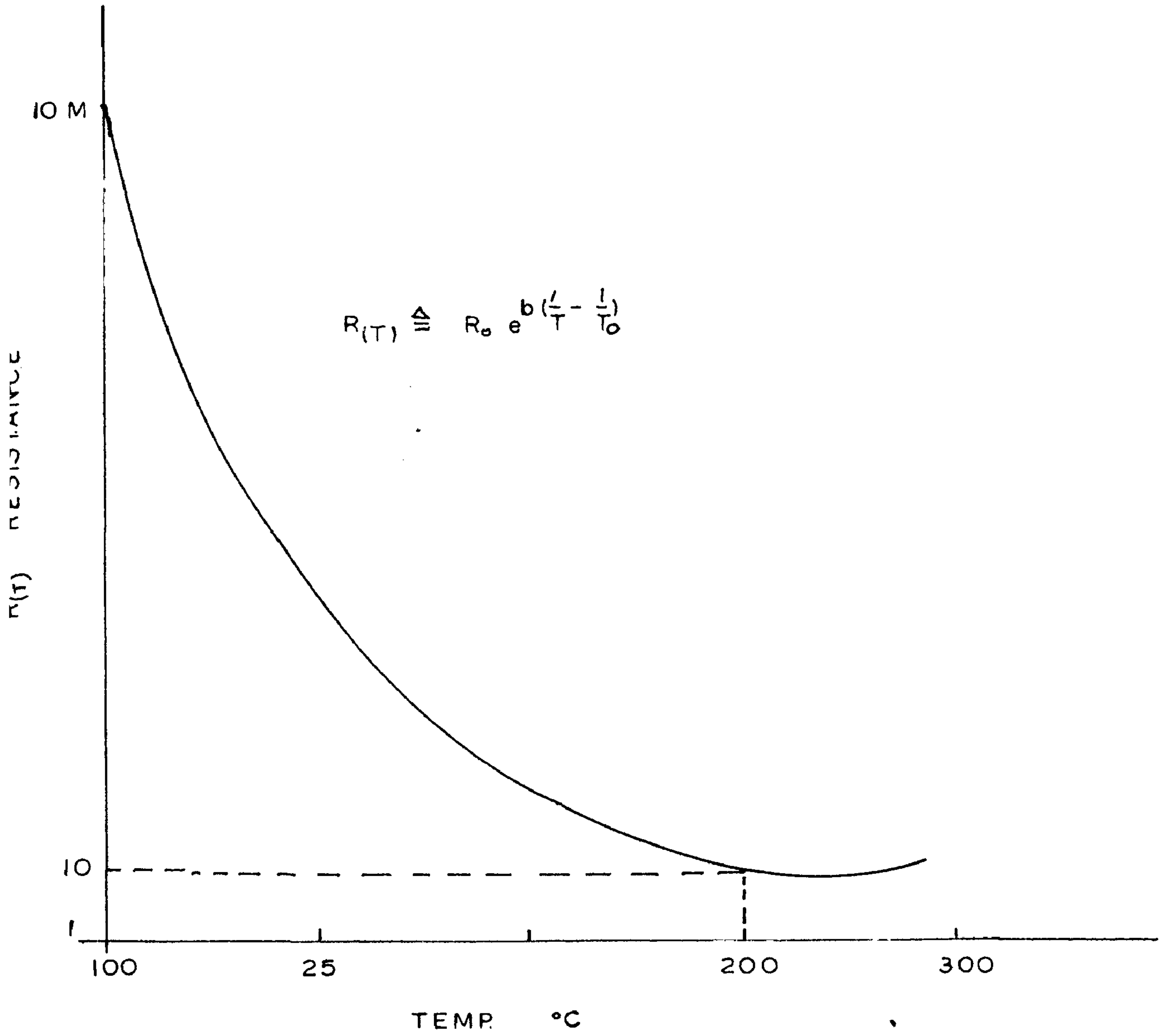


FIG. 1.1 RESISTANCE TEMPERATURE (R-T) CHARACTERISTIC OF NTC THERMISTOR

was used with a prefix or suffix of PTC or NTC. The static voltage versus current characteristics of PTC and NTC units are given in Fig. 1.2. Note that the V-I characteristics of PTC and NTC are rotated 90^{o} against each other, the voltage peak becomes a current peak and the negative branch of the voltage characteristics a slope of decreasing current. A PTC thermistor, because of its switching characteristic, is mainly used as a switch for control and protection purposes. In this dissertation we have considered only NTC thermistors as they find their applications in measurements and control. NTC thermistors, therefore, will now onwards be mentioned simply as 'Thermister' [1,2]

On the basis of type of heating, the thermistors are also termed as directly or indirectly - heated type. In the first type (also known as self heated), heating is produced either by the ambient temperature or by the passage of current through thermistor or both. In an indirectly heated type, heating is produced mainly by an electric heater element such as a wire filament built into the thermistor. Here, some heating may also result from power dissipated into the thermistor, but the effects of ambient temperature are minimized, if not eliminated, by enclosing both the thermistor and heater element in a bulb which may be evacuated [1].

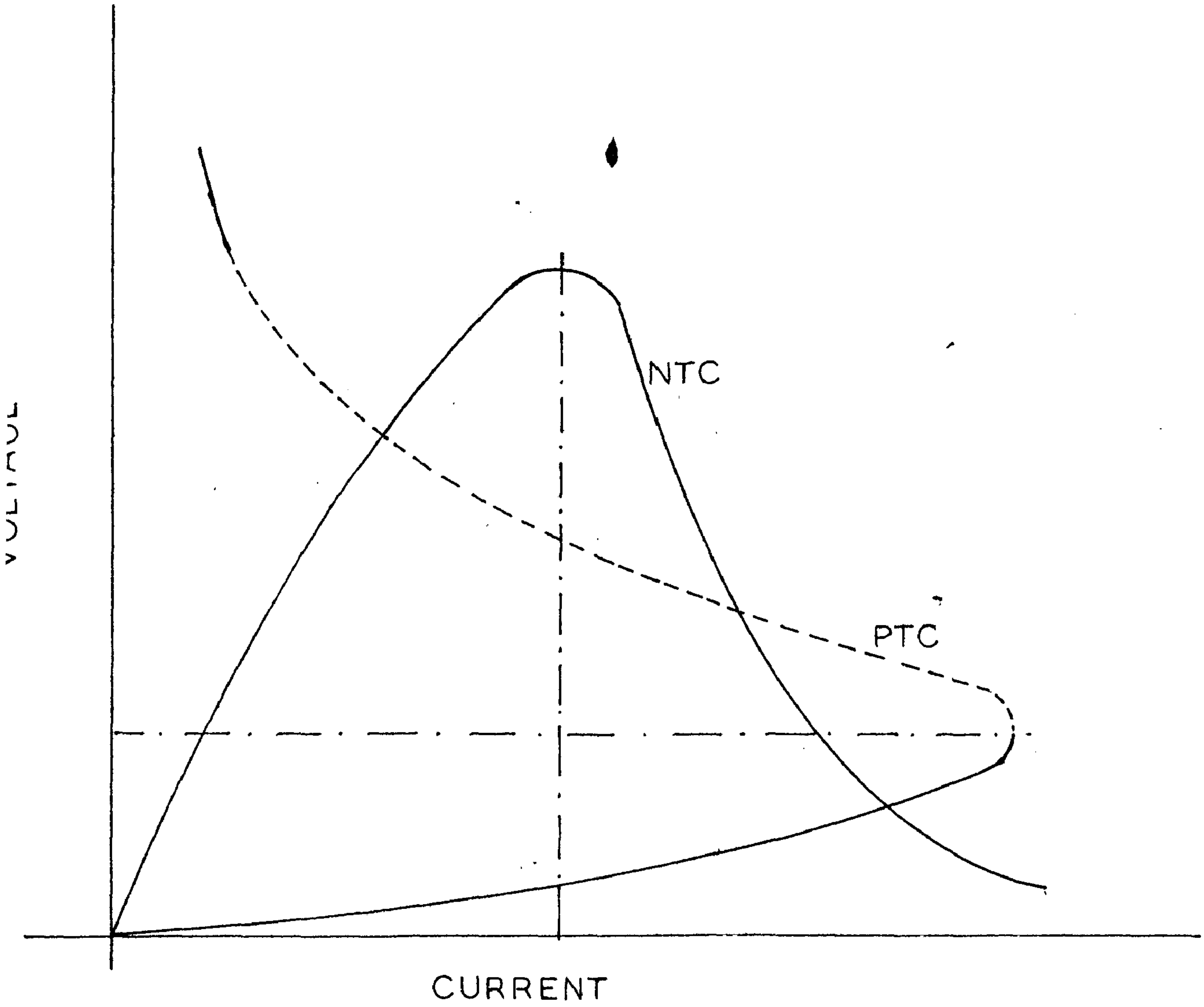


FIG. 1.2 STATIC VOLTAGE VS CURRENT CHARACTERSTICS

Thermistors are available in various types of packages depending upon its rating and end uses. Some of the common configurations are disc, rod, bead, washer, epoxy moulded and thin films [2]. Disc type thermistors find application in temperature compensation in transistor amplifier circuitry . On the other hand,rod types are used for surge supression, glass enveloped bead type for amplitude control in electronic circuitry and glass coated bead types and glass probe bead types for temperature measurement and control. The areas where measurement of temperature is exploited are aeronautics, agriculture, airconditioning, biology photography , machine operation etc. A detailed description of thermistor applications have also been considered and are given in Chapter 6 of this dissertation.

The thermistor has certain interesting characteristics such as cold resistance, hot resistance, temperature coefficient of resistance and resistance ratio. The static V-I characteristics and resistance versus temperature characteristics have already been discussed earlier.

The resistance of a thermistor in which electrical power dissipation is zero is called the zero power or cold resistance. The resistance value is specified for a given room temperature (e.g. 25°C) and is the nominal resistance rating of the thermistor. Thus, the 100 Ohm thermistor means that the thermistor has an R(T) value of 100 Ohm.

On the other hand hot resistance is the resistance of a heated thermistor. The temperature coefficient of resistance 'α' is defined as [2] -

$$\alpha = \frac{1}{R(T)} \cdot \frac{d\,R(T)}{dT} \qquad (1.1)$$

where R(T) denotes thermistor resistance.

1.2 FLOW MEASUREMENT - AN IDEA

The factors that determine the thermal conductivity of the medium surrounding the thermistor, and therefore, its dissipation constant, also determine the thermistor resistance for a given selfheated power. Thus, the dissipation constant plays the basic role behind a number of thermistor applications in various diverse areas such as level and pressure-gauges, flowmeter and chromatography [2]. Mac Donald [1] first studied the various factors involved in the operation of self heated thermistors, while Pearson et al and Rasmussen [1] pioneered the work in the field of flow-rate meters for gases and liquids. While measuring the flow with the help of thermistors, the principle involves heating of thermistor through current. The thermistor, thus heated, is raised to a temperature higher than that of flowing medium. The change in velocity of flowing medium produces a heat transfer from thermistor surface, consequently change occurs in its temperature and resistance. This change is electrically measured easily through a bridge or other circuits. A brief discussion on the topic and relation between power dissipated in the sensor with flow is given in Chapter 5.

1.3 PROBLEM FORMULATION AND DISCUSSION ON VARIOUS CHAPTERS

In this dissertation, the proposed problem has been studied for its various relevent aspects. Briefly, they are -

a) the generalized analysis of thermistor - WB and AMB circuits.

b) design of a thermistor - WB circuit utilizing the analysis carried out in (a),

c) linearized piecewise approximation of thermistor characteristic and derivation of a relation for gain in a simple WB circuit and finally comparing it with the existing general expression,

d) development of a computer programme for (c) using FORTRAN-IV

e) study of behaviour of thermistor used as a sensor in small flow measuring problems.

f) flow measurement using thermistor in bridge configuration,

g) different applications of thermistor in measurement.

Chapter 2 is devoted to the general introduction of existing techniques of analysis of thermistor - WB,-AMB and -MMB circuits. It also includes a discussion on

merits and demerits of the different thermistor bridge circuits. Chapter 3 discusses a) the generalized analysis and b) the design of a ~WB thermistor circuit using the concepts in a). On the other hand c) and d) described as above, have been taken up in Chapter 4. The Computor program thus developed is successfully run over DEC 2050 Computor and its listing is given in the Appendix. A study of behaviour of thermistor used as a sensor in small flow measuring problems is taken up in Chapter 5 . It also envisages the problem of flow measurement using thermistor in bridge configuration. Finally, Chapter
include some other applications
6 and 7 summarizes the various conclusions, derived from the present work, alongwith diversified applications of a thermistor used as a sensor or transducer element. Some interesting problems for future work are also suggested.

CHAPTER II

AN OVERVIEW OF THERMISTOR BRIDGE CIRCUITS

A thermistor bridge circuit may be an ac or dc bridge circuit with thermistor in its one or more arms. In this chapter, some of the commonly used bridge circuits for temperature measurement are described. Such circuits are having following two distinct advantages and, therefore, are preferable in use in comparison with the simple series and/or parallel network configurations [21]. Here -

a) the measurement of change in parameters such as voltage and/or current is accurate and easy, and

(b) cancellation of dc term in the output is possible. It is synonym to ob-tain a balance or null reading for some input quantity of interest.

Section 2.1 considers a simple thermistor Wheatstone bridge (WB) circuit while sections 2.2 and 2.3 describe astable multivibrator (AMB) and monostable multivibrator (MMB) - bridge circuits. Their relative advantages and disadvantages are also discussed.

2.1 THERMISTOR WB CIRCUIT

A simple Wheatstone bridge circuit is shown in Fig.2.1 with the temperature sensor (thermistor) in one of its arms. At some temperature T_o, to balance the bridge or to obtain a null in the output meter, the ratio R_4/R_3 is controlled such that [3,20,22] -

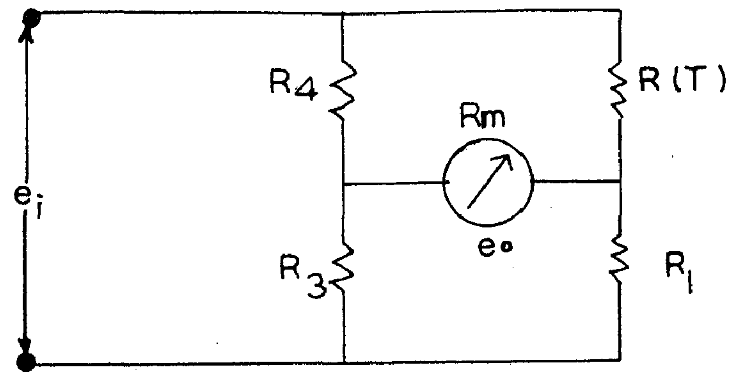
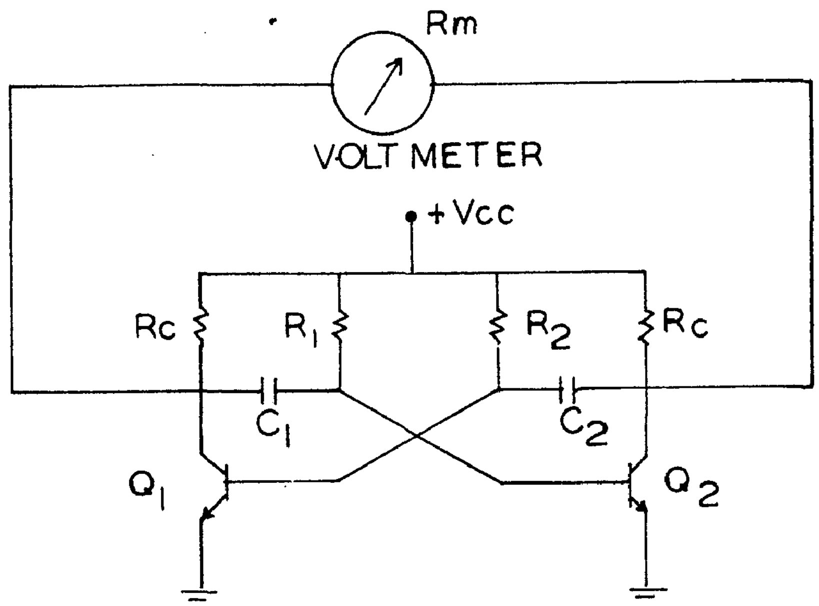
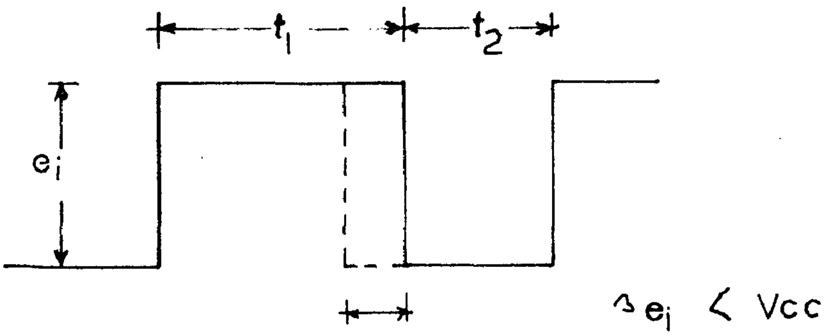


FIG. 2.1 A THERMISTOR WB CIRCUIT



$R_1 = R(T)$, R_2 IS USED FOR BALANCE

(a)



(b)

2.2 (a) & (b) THERMISTOR AMB CIRCUIT

$$R_4R_1 = R_3 \; R(T)\Big|_{T=T_o} = R_3R_o \qquad (2.1)$$

Resistance R_1 is used, on the other hand, for controlling the linearity of the output in the desired temperature range.

Consider a simple case where all resistances in the bridge circuit are equal to R. The bridge is, then, balanced and output voltage is zero. In case R(T) = R changes to R(T) = R + ΔR , then

$$\frac{e_o}{e_i} = R \left[\frac{1}{2R} - \frac{1}{2R+\Delta R} \right]$$

$$= \frac{\Delta R/2}{\Delta R + 2R}$$

$$= \frac{\Delta R / 4R}{1 + \frac{\Delta R}{2R}} \qquad (2.2)$$

(if $R_m >> R$, where R_m is the resistance of the output meter) Equation (2.2) states that the deviation from ideal linearity is $\Delta R/2R$ and the bridge sensitivity is given as [5,21]

$$S_w = \frac{d}{dR} \left[\frac{e_o}{e_i} \right]$$

$$= \frac{1}{4R} \qquad (2.3)$$

Thus, on account of its simplicity and convenient physical form a bridge circuit is normally used as a temperature transducer.

2.2 THERMISTOR AMB CIRCUIT

The astable or free-running multivibrator, when used as a bridge has the following useful features [4,5,6]:-

i. The bridge output is in a form ideally suited to recording directly on magnetic tape or to transmission via telegraph or radio-link to a remote monitoring station.

ii. Even if the signals become degraded in transmission it can easily be restored.

iii. The circuit alternately samples two variables and so both may be monitored together with their sum and difference.

iv. Certain temperature sensors which are voltage sensitive and not operating correctly in a conventional bridge can satisfactorily be used in this circuit.

v. The circuit sensitivity is higher than that of the Wheatstone bridge (WB).

vi. If C_1, C_2 are proportional to two quantities and R_1, R_2 to two other quantities, then their product C_1R_1 and C_2R_2 can be compared (i.e., a product bridge) using this type of bridge circuit.

Apart from several advantages listed above an AMB circuit offers some disadvantages. Following two precautions should normally be considered.

i. Temperature difference between circuit components must be avoided. This requirement applies to conventional bridges also, but the use of semi-conductor transistors in this circuit makes temperature difference especially undesirable.

ii. Matched transistors must be used. These are available as difference amplifier pairs, selected for matched current amplification factor and minimum ΔV_{BE} over a wide temperature range.

The astable multivibrator as shown in Fig. 2.2(a) generates a rectangular waveshape at the collectors of the two transistors by switching regularly between two astable states. The thermistor R(T) with capacitor C_1 determines the period t_1 (Fig. 2.2(b) as

$$t_1 = C_1 R(T) \ln 2 \qquad (2.4a)$$

Similarly, the period t_2 is given by

$$t_2 = C_2 R_2 \ln 2 \qquad (2.4b)$$

These relationships are perfect, provided stray capacitances are small and R(T) and R_2 are much larger than collector resistance R_c and also if leakage currents including that in external loads are negligible [5,6,23]. It was shown by Maher [4] that the astable multivibrator, Fig. 2.2(a), can be used as the bridge circuit with a capacitance transducer C or with a resistance transducer R. A meter connected to the collectors of transistors Q_1 and Q_2 is a standard dc voltmeter measuring the difference of average voltage of the collectors. If the resistance R_m is very high, the approximate average voltage between collectors is the ratio of e_o/e_i as given below :

$$\frac{e_o}{e_i} = \frac{t_1-t_2}{t_1+t_2} \qquad (2.5)$$

where the periods t_1 and t_2 of rectangular pulses are given by (2.4a) and (2.4b).

Consider a simple case where $C_1 = C_2 = C$ and $R(T) = R_2 = R$ at some pre-determined temperature. Under this condition $t_1 = t_2 = t$ (say) and $e_o = 0$. Because of the changes in temperature, the thermistor resistance R(T) changes and hence t varies. Assume that this variation is Δt . Using (2.5) , the approximate average voltage between collectors is

$$\frac{e_o}{e_i} = \frac{\Delta t}{2t + \Delta t} \qquad (2.6)$$

Substituting for Δt from (2.4a), equation (2.6) is re-written as

$$\frac{e_o}{e_i} = \frac{C \Delta R \; \ln 2}{2 \, CR \ln 2 \; + C \Delta R \; \ln 2}$$

$$= \frac{\Delta R/2R}{1 + \Delta R/2R} \qquad (2.7)$$

By comparing equation (2.7) with an equal arm wheatstone bridge relation (2.2), it is justified that a free running or astable multi may be used as a bridge circuit. Equation (2.7) states that the nonlinearity of the output or the deviation from ideal linearity is $\Delta R/2R$, which is same as thermistor WB circuit. The AMB circuit sensitivity is given as ,

$$S_A = \frac{d}{dR}\left[\frac{e_o}{e_i}\right]$$

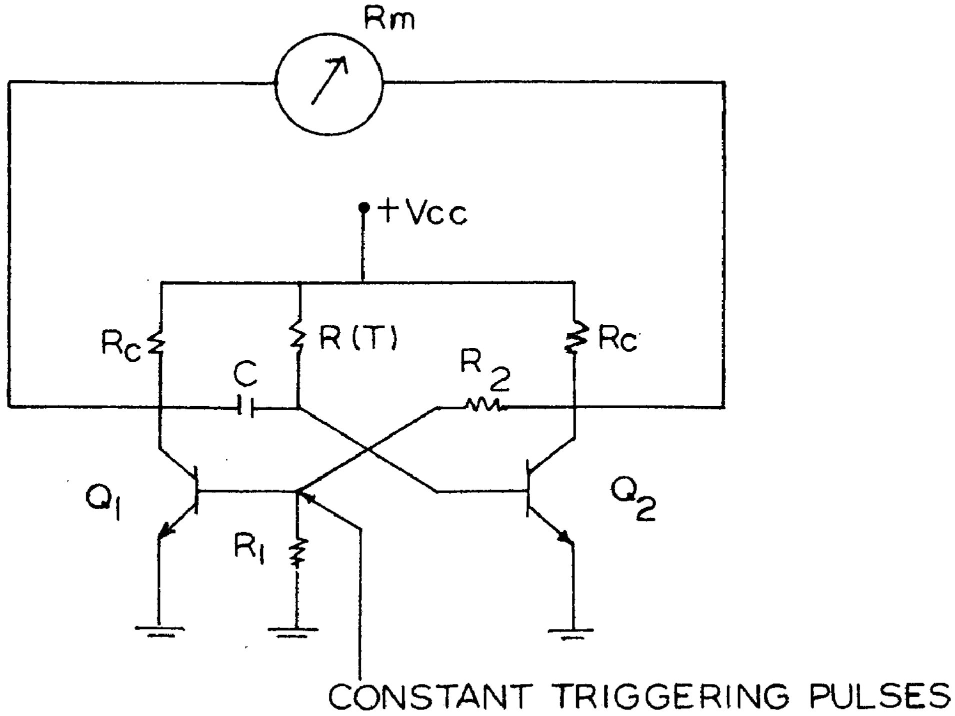
$$= \frac{1}{2\,R} \qquad (2.8)$$

From (2.3) and (2.8) , $S_A / S_W = 2$. Thus, the sensitivity of the astable multivibrator bridge is twice higher compared with the Wheatstone bridge.

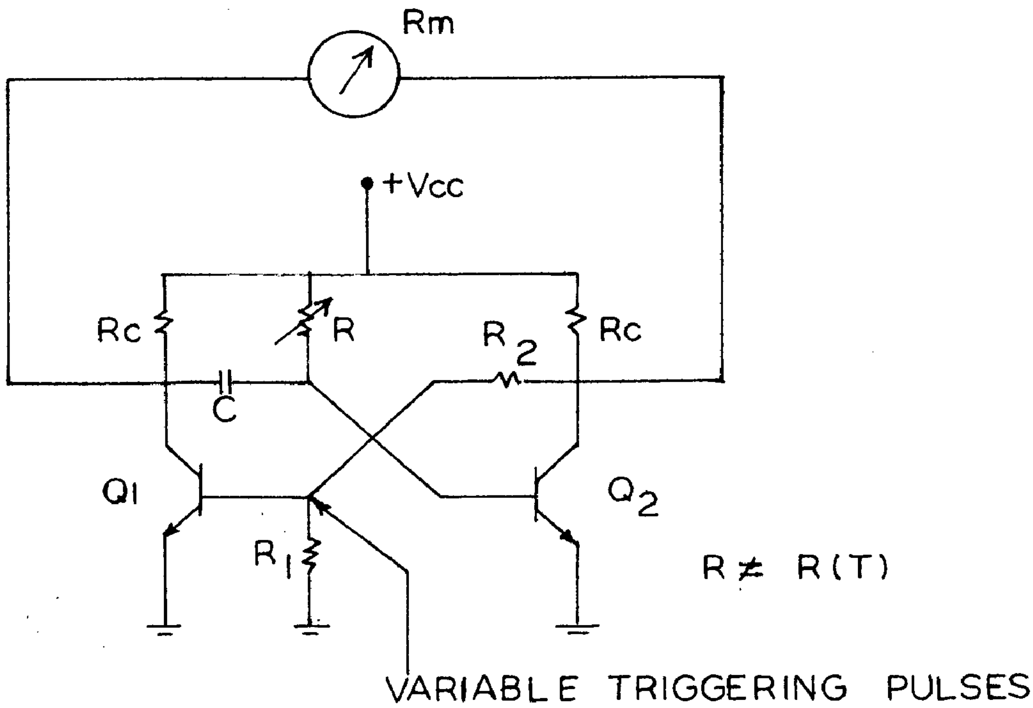
2.3 THERMISTOR MMB CIRCUIT

Monostable multivibrators are also used for making thermistor bridge circuits. Two advantages of the monostable multivibrator bridge circuit over the Wheatstone and astable bridges are1)linearity for great changes of transducer impedance & 2) higher sensitivity. The disadvantages of the monostable bridge compared with the astable one are greater complexity, and the possibility of dividing the frequency of triggering pulses [5,8,10,11]

Two configurations of a thermistor MMB circuit (or probe) for measuring the temperature are given in Fig. 2.3. The first part of the circuit, not shown in figure, is invariably an astable multivibrator which is used to supply either constant or variable triggering pulses to monostable multivibrator probes. To generate constant triggering pulses, the timing resistances in an astable multi are considered simple resistive elements whose values are independent of temperature. In such a case, the thermistor is assumed to be connected in the monostable multi, that follows astable multi, as shown in Fig. 2.3(a). On the other hand, Fig. 2.2(a) is used for generating variable frequency triggering pulses. The MMB that follows this astable multi is given in Fig. 2.3(b). Note that a variable frequency triggered (VFT) monostable multivibrator probe does not contain any thermistor in its configuration. It has been mentioned in literature that a constant



(a)



(b)

FIG. 2.3 (a) & (b) THERMISTOR MMB CIRCUIT freq triggering (CFT) yields a PWM waveform in the output of MMB circuit, while a VFT gives rise to a PPM waveform.

The MMB with CFT has a linear resistance voltage characteristic in the wide range. On the other hand a VFT-MMB gives a higher sensitivity [5,8,17-19] . It is shown that the thermistor MMB is approximately 2 times more sensitive than AMB and 4 times more sensitive than WB [5,12].

2.4 CONCLUSIONS

In this chapter some of the commonly used thermistor bridge circuits have been described in brief and their relative merits and demerits have been considered. It is observed that a monostable multivibrator bridge is more sensitive and linear for changes of transducer impedance or temperature across thermistor. However, from balancing point of view the basic Wheatstone bridge offers advantage over multivibrator bridge circuits.

CHAPTER III

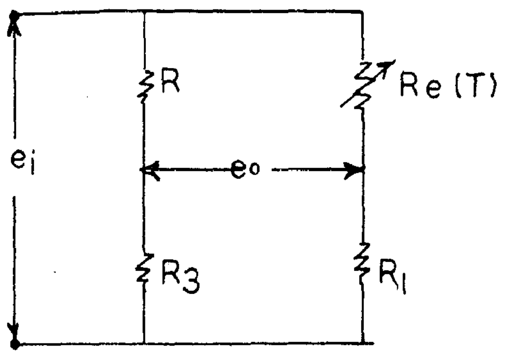
GENERALIZED VIEW OF THERMISTOR BRIDGE CIRCUITS

Because of their high sensitivity and convenient physical form, a number of thermistor bridge circuits have been developed in the past . Some of them are shown in Fig. 3.1 . These circuits are compared on the basis of their relative gains, linearity, balance, and simplicity. In this chapter, firstly general expressions are obtained for above parameters and then they are analyzed and adjusted in such a way that a comparison between various bridge circuits becomes trivial.

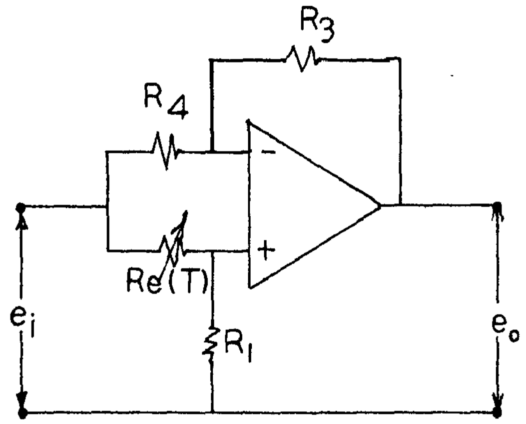
To add to generality, five different configurations of thermistor probe are also considered . In the end, a thermistor bridge circuit has been designed for temperature measurement using generalized parameters.

3.1 GENERALISED ANALYSIS OF THERMISTOR BRIDGE CIRCUIT

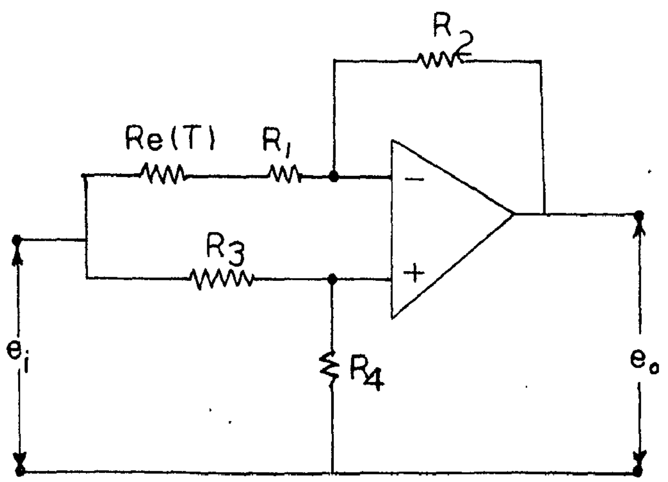
For obtaining a generalized analysis, consider the thermistor bridge circuits of Fig. 3.1 . The expressions for gain G for circuits CT-1 through CT-4 is mentioned in Table 3.1. Thermistor resistance Re(T) is the equivalent value, defined separately for five different probes configurations shown in Fig. 3.2, and is listed in Table 3.2.



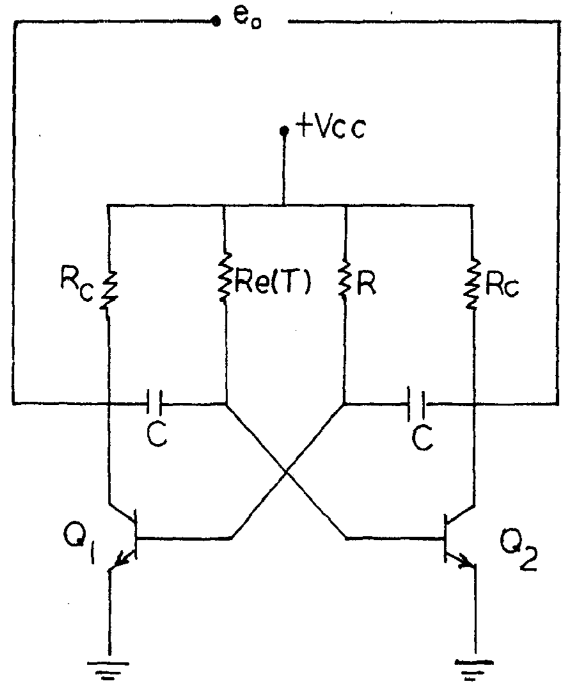
(i) CT-1 THERMISTOR WB CIRCUIT



(ii) ACTIVE WB CIRCUIT - I (CT-2)



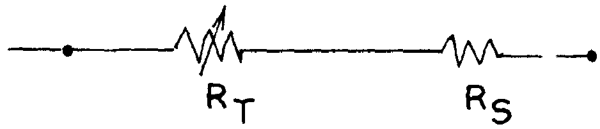
(iii) ACTIVE WB CIRCUIT- II (CT-3)



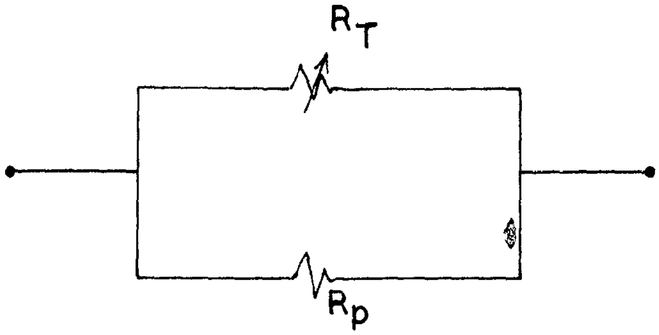
(iv) THERMISTOR AMB CIRCUIT (CT-4)

FIG. 3.1 DIFFERENT BRIDGE CIRCUITS CONSIDERED FOR GENERALISED ANALYSIS

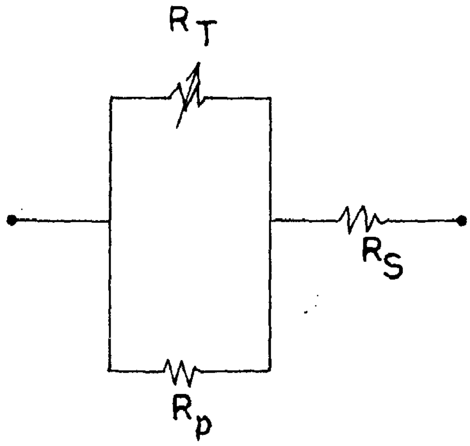
(i) P-1 SIMPLE THERMISTOR



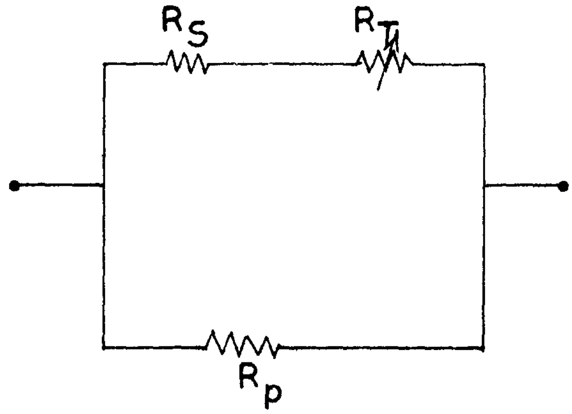
(ii) P-2 THERMISTOR WITH A SERIES RESISTANCE



(iii) P-3 THERMISTOR WITH A SHUNT RESISTOR



(iv) P-4 THERMISTOR WITH PARALLEL & SERIES RESISTANCE



(v) P-5 THERMISTOR WITH A SERIES & PARALLEL RESISTANCE

FIG. 3.2 DIFFERENT THERMISTOR PROBE CONFIGURATION

Table 3.1

Expression for gain, G and Values of transformation coefficients a_o, a_1, b_o, b_1

(See Appendix -2)

Circuit	$G \triangleq e_o / e_i \; (= V_T/V_p)$	ao	a_1	bo	b_1
CT -1	$\dfrac{R_3 Re(T) - R_1 R_4}{(R_3+R_4)\,(R_1+Re(T))}$	$-R_1R_4$	R_3	$R_1(R_3+R_4)$	R_3+R_4
CT-2	$\dfrac{R_1R_4 - R_3 Re(T)}{R_4\,(R_1+Re\,(T))}$	R_1R_4	$-R_3$	$R_1 R_4$	R_4
CT-3	$\dfrac{R_4(Re(T)+R_1) - R_2R_3}{(R_3+R_4)\,(R_1+Re(T))}$	$R_1R_4 - R_2R_3$	R_4	$R_1(R_3+R_4)$	(R_3+R_4)
CT -4	$\dfrac{R_1 - Re(T)}{R_1+Re(T)}$	R_1	-1	R_1	$+1$

Table 3.2

Equivalent thermistor resistance Re(T) and Values of transformation Coefficients A_1, A_2, A_3 and A_4

Probe	Re (T)	A_1	A_2	A_3	A_4
P-1	R_T	1	o	o	1
P-2	$R_s + R_T$	1	R_s	o	1
P- 3	$R_p R_T/(R_p + R_T)$	R_p	o	1	R_p
P -4	$\frac{R_T(R_s+R_p) + R_s R_p}{R_p + R_T}$	$R_s + R_p$	$R_s R_p$	1	R_p
P -5	$\frac{R_p(R_s+R_T)}{R_T + R_s + R_p}$	R_p	$R_s R_p$	1	$R_s + R_p$

To have a general analysis of above 4x5 thermistor bridge circuits, the gain expression are modified in the following form

$$G = \frac{a_o + a_1 \; Re\,(T)}{b_o + b_1 \; Re\,(T)} \qquad (3.1)$$

The constants a_o, a_1, b_o; b_1 are defined in Table 3.1. Similarly, a general expression for Re(T) is considered and is given as -

$$Re(T) = \frac{A_1 R_T + A_2}{A_3 R_T + A_4} \qquad (3.2)$$

with the coefficients A_1, A_2, A_3 and A_4 shown in Table 3.2. Equations (3.1) and (3.2) are advantageous as they are simple and may be handled easily to obtain a general analysis of 4x5 different circuits considered here. Differentiating (3.1) with respect to 'T', we have -

$$\frac{dG}{dT} = \frac{[b_o + b_1 Re(T)]\, a_1 - [a_o + a_1 Re(T)] b_1}{[\, b_o + b_1 \; Re(T)\,]^2} \cdot \left[\frac{\partial Re(T)}{\partial T}\right]$$

$$= \frac{NR_T}{T^2 D^2} \qquad (3.3)$$

where $N = b(a_1 b_o - b_1 a_o) \quad (A_2A_3 - A_1A_4)$

and $D = R_T(b_1A_1 + b_oA_3) + b_1A_2 + b_oA_4$

The second and third order derivatives are obtained from (3.1) as -

$$\frac{d^2G}{dT^2} = \frac{N}{(TD)^4}\left[- bR_TD^2 - 2TR_TD^2 + 2bR_T^2D(b_1A_1+b_oA_3)\right] \tag{3.4}$$

and,

$$\frac{d^3G}{dT^3} = \frac{N}{T^8D^6}\left\{ T^4D^3 \left[\frac{b^2R_TD}{T^2} + \frac{b^2\ R_T^2}{T^2}(b_1A_1 + b_oA_3) - 2R_TD \right.\right.$$

$$+ \frac{2b}{} \frac{R_TD}{T} + \frac{2b\ R_T^2}{T}(b_1A_1 + b_oA_3) - \left.\frac{4b^2R_T^2}{T^2}(b_1A_1+b_oA_3)\right]$$

$$-[-bR_TD - 2T\ R_TD + 2bR_T^2\ (b_1A_1+b_oA_3)]$$

$$\left.[4T^3D^3 + 3T^4D^2R_T(-b/T^2)(b_1A_1+ b_oA_3)\ ]\right\} \tag{3.5}$$

The derivations of (3.4) through (3.6) are given in Appendix 1. For the above analysis, the thermistor is assumed to be modeled by

$$R_T = R_o \exp\ [b(1/T - 1/T_o)] \tag{3.6}$$

where R_T is thermistor resistance at temperature T ; T_o is the selected reference themperature ^oK ; b is a constant of the thermistor material, a typical value being 3600 K . By defining

$$G\ \Big|_{T = T_o} \longrightarrow G(T_o) \tag{3.7a}$$

$$\frac{dG}{dT}\Big|_{T= T_o} \longrightarrow G'(T_o) \tag{3.7b}$$,

$$\left.\frac{d^2G}{dT^2}\right|_{T = T_o} \longrightarrow G''(T_o) \qquad (3.7c)$$

$$\left.\frac{d^3G}{dT^3}\right|_{T = T_o} \longrightarrow G'''(T_o) \qquad (3.7d)$$

We may write (3.7) from (3.1) through (3.4) as follows

$$G(T_o) = \frac{R_o(a_oA_3 + a_1A_1) + (a_oA_4 + a_1A_2)}{R_o(b_oA_3 + b_1A_1) + (b_oA_4 + b_1A_2)} \qquad (3.8a)$$

$$G'(T_o) = \frac{\acute{R}_o N}{T_o^2 D_o^2} \qquad (3.8b)$$

$$G''(T_o) = \frac{N}{T_o^4 D_o^3} \left[-bR_oD_o - 2T_oR_oD_o + 2bR_o^2 (b_1A_1 + b_oA_3)\right] \qquad (3.8c)$$

where, $D_o = R_o(b_1A_1 + b_oA_3) + b_1A_2 + b_oA_4$

For linearity of gain, $G''(T_o) \equiv 0$ (see section 3.2) Under this condition -

$$G'''(T_o) = -\frac{b^2 N R_o}{2T_o^6 D_o^2}$$

$$= -\frac{b^2}{2T_o^4} G'(T_o) \qquad (3.8d)$$

Similarly, it can be shown that the fourth-order derivative evaluated at $T = T_o$ under the linearity constraint is given by

$$G''''(T_o) = \frac{2b^2}{T_o^5} G'(T_o) \qquad (3.8e)$$

Equation (3.8) is quite simple. Note that it has been presented in a way which exploits the basic mathematical similarities of 4 x 5 one thermistor temperature transducer circuits.

3.2 LINEARIZING AND BALANCING CONDITIONS

[A] Linearization -

From (3.1), it is obvious that gain G is a nonlinear function of T, as Re(T) and hence R_T does not change linearily with respect to temperature. In order to represent a perfect straight line characteristic from the above equation, it should be of the following form :

$$G = G(T_o) + m(T - T_o) \qquad (3.9)$$

where m represents the desired slope of the straight line and is called linear gain coefficient. Obviously, it means that -

(a) $\left.\frac{dG}{dT}\right|_{T = T_o} \longrightarrow G'(T_o)$ must be equal to constant ''m''

and

(b) $\left.\frac{d^nG}{dT^n}\right|_{T=T_o}$ should be equal to zero for all $n \geqslant 2$.

Condition (b) simply states that second - third - and all higher-order derivatives should be zero in the operating temperature range ; in case a linear output change with temperature change is desired. Since there are not enough independent parameters to enable this, only $G''(T_o)$ is made equal to zero for 4x5 one thermistor bridge circuits considered in this chapter. Hence, the linearity condition simplifies into

$$G''(T_o) \equiv 0 \tag{3.10}$$

Substituting (3.10) in (3.8c), we have

$$(b + 2T_o)D_o = 2\ b\ R_o\ (b_1A_1 + b_oA_3) \cdots$$

or,
$$\frac{2b}{b + 2T_o} = \frac{R_o(b_1A_1 + b_oA_3) + b_1A_2 + b_oA_4}{R_o(b_1A_1 + b_o A_3)}$$

which reduces to

$$\frac{b_1A_2 + b_oA_4}{R_o(b_1A_1 + b_oA_3)} = \frac{b - 2T_o}{b + 2\ T_o}$$

If $M_o = (b - 2T_o) / (b+2T_o)$; we have the ratio (b_o/b_1) is obtained as

$$\frac{b_o}{b_1} = \frac{M_o\ R_o\ A_1 - A_2}{A_4 - M_o\ R_o\ A_3} \tag{3.11}$$

Note that M_o is dependent on the thermistor type and the selected reference temperature T_o. From Table 3.1, $b_o/b_1 = R_1$. Thus, to obtain a linearization in the gain, R_1 may be changed such that (3.11) is satisfied. Specific expressions for R_1 is obtained for probes P-1 to P-5 by substituting the values of transformation coefficients $A_1 - A_4$ in (3.11) and are listed in Table 3.3.

Table 3.3

Expressions for R_1 to linearize gain G in circuits CT-1 to CT-4 with different probe configurations

Circuit	CT-1 to CT-4
P-1	$R_1 = M_o R_o$
P-2	$R_1 = M_o R_o - R_S$
P-3	$R_1 = \dfrac{M_o R_o R_p}{R_p - M_o R_o}$
P-4	$R_1 = \dfrac{M_o R_o R_p}{R_p - M_o R_o} - R_S$
P-5	$R_1 = \dfrac{M_o R_o R_p - R_S R_p}{R_S + R_p - M_o R_o}$

The linear gain coefficient $G'(T_o)$, defined in (3.9), now, be computed from (3.8b) by properly substituting the linearity condition and the values of transformation coefficients a_o, a_1, b_o and b_1 in it. Expressions of $G'(T_o)$ obtained as above for circuits CT-1 through CT-4 are listed below :

CT-1 From equation (3.8b), substituting for N and D_o,

$$G'(T_o) = \frac{R_o b}{T_o^2} (A_2A_3 - A_1A_4) \frac{(a_1b_o - b_1a_o)}{[b_o(R_oA_3+R_4) + b_1(R_oA_1+A_2)]^2}$$

Table 3.1 gives $a_o = -R_1R_4$, $a_1 = R_3$, $\frac{b_o}{R_1} = b_1 = R_3+R_4$,

Therefore -

$$G'(T_o) = \frac{R_o b}{T_o^2} (A_2A_3 - A_1A_4) \frac{R_1}{[R_1(R_oA_3+A_4) + R_oA_1 + A_2]^2}$$

Substituting for R_1 from (3.11), we have

$$G'(T_o) = \frac{b}{R_oT_o^2} \frac{(M_oR_oA_1-A_2)(A_4-M_oR_oA_3)}{(A_2A_3-A_1A_4)(1+M_o)^2} \tag{3.12a}$$

CT-2

$$G'(T_o) = \frac{-(1+R\ /R_4)}{R_oT_o^2(1+M_o)^2} \frac{(M_oR_oA_1-A_2)(A_4-M_oR_oA_3)}{(A_2A_3-A_1A_4)} \tag{3.12b}$$

CT-3

$$G'(T_o) = \frac{b\ R_2R_3}{R_oT_o^2(1+M_o)^2(R_3+R_4)} \frac{(A_4-M_oR_oA_3)^2}{(A_2A_3-A_1A_4)} \tag{3.12C}$$

CT-4

$$G'(T_o) = \frac{2b}{R_oT_o^2(1+M_o)^2} \frac{(A_4-M_oR_oA_3)(M_oR_oA_1-A_2)}{(A_2A_3-A_1A_4)} \tag{3.12d}$$

Equations (3.12) can be manipulated, in turn, by substituting the various values of coefficients A_1-A_4 from table 3.2 to find 20 different expressions for $G'(T_o)$. These linear gain coefficients are helpful in obtaining a linearized design of thermistor bridge circuits.

[B] Balancing conditions:

Using the staight line or linear representation (3.9) of gain , it is obvious that G is balanced (i.e. zero output is obtained) at $T=T_o$ if

$$G(T_o) \equiv 0 \qquad (3.13a)$$

It explains that an instrument with center zero scale indicator will show zero reading at T_o. Substituting (3.13a) in (3.8a), we have

$$R_o\ (a_oA_3 + a_1A_1) + a_oA_4 + a_1A_2 = 0$$

Which may be rewritten as

$$(a_o/a_1) = -\frac{R_oA_1 + A_2}{R_oA_3 + A_4}$$

$$= -\ Re\ (T_o) \qquad (3.13b)$$

Where $Re\ (T_o) = Re\ (T)\ |_{T=T_o}$

Note that (3.13b) is valid only for CT-1 to CT-3. These are the conventional bridge circuits. They have sufficient parameters (e.g. R_3 and R_4) which may be controlled independently to balance the circuit, in case one of their arms, R_1, is being varied to obtain linearity in response using (3.13b), the specific experessions for balancing circuits CT-1 to CT-3 are given below-

CT-1 and CT-2

$$R_4/R_3 = Re\ (T_o)/R_1 \tag{3.14a}$$

Where, $(a_o/a_1) = - R_1\ (R_4/R_3)$

CT-3

$$\frac{R_1R_4 - R_2R_3}{R_4} = -Re\ (T_o)$$

or $$(R_4/R_3) = \frac{R_2}{R_1}\ (1 + Re(T_o)/R_1)^{-1} \tag{3.14b}$$

With a thermistor AMB circuit there are only two independent parameters namely R_1 and $Re(T)$ or in general, time constants R_1C_1 and Re $(T)C_2$. Using (3.11) -

$$R_1 = \frac{M_oR_oA_1 - A_2}{A_4 - M_oR_oA_3} \tag{3.15a}$$

i.e. R_1 is varied according to above equation to obtain a linear response. On the other hand, if balance is desired, (3.13) simplifies into

$$R_1 = Re\ (T_o)$$
$$= \frac{R_oA_1 + A_2}{A_4 + R_oA_3} \tag{3.15b}$$

Obvioulsy, (3.15a) and (3.15b) can not go simultaneously and hence balancing of linearized AMB circuit CT-4 is impossible at $T = T_o$.

Theorem 3.1 Linearized AMB circuit is balanced at temperature T_b, such that $T_b > T_o$.

Proof: To prove the above theorem, consider CT-4 with probe P-1, where $R_1 = M_o R_o$. Substituting thevalue for R_1 in (3.1), we have

$$G = \frac{M_o R_o - Re(T)}{M_o R_o + Re(T)} \quad ; \; a_o = b_o = R_1 \text{ and } a_1 = -b_1 = 1$$

$$= \frac{M_o R_o - R_T}{M_o R_o + R_T} \quad , \; A_1 = A_4 = 1 \text{ and } A_3 = A_2 = 0$$

For G to be zero, at temperature T_b ; $M_o R_o = R_T \big|_{T=T_b}$ Hence

$$M_o = e^{b\left(\frac{1}{T_b} - \frac{1}{T_o}\right)}$$

or , $T_o/T_b = 1 + T_o/b \; \ell n \; M_o$

Which can be manipulated into

$$T_b/T_o = \left(1 + \frac{T_o}{b} \ell n \; M_o\right)^{-1} \qquad (3.16)$$

Note that the RHS of (3.16) is always greater than unity. Hence, the theorem is proved.

This deviation of balance point in positive direction for a linearized AMB circuit is shown graphically in Fig. 3.4

3.3 CALCULATION OF FRACTIONAL DEVIATION

Using Taylor series expansion for (3.1) around $T = T_o$, we have

$$G(T_o + h) = G(T_o) + h\,G'(T_o) + \frac{h^2}{2!}G''(T_o) + \frac{h^3}{3!}G'''(T_o) + \frac{h^4}{4!}G''''(T_o) + \ldots\ldots \qquad (3.17)$$

Where h is an increment of temperature from a particular value T_o. From section 3.2, we know that

$G(T_o) \equiv 0$, for balancing the WB circuit

and $G''(T_o) \equiv 0$, for linearizing the response

Substituting these conditions in (3.17) -

$$G(T_o+h) = h\,G'(T_o) + \frac{h^3}{3!}G'''(T_o) + \frac{h^4}{4!}G''''(T_o) + \ldots\ldots \qquad (3.18a)$$

The terms h^3, h^4 etc. indicates a departure of G from linearity and may be regarded as error terms ε_3, ε_4 etc. ε_4 is negligible compared with ε_3 for small values of h, since

$$\varepsilon_4/\varepsilon_3 = \frac{h^4}{4!}G''''(T_o) \Big/ \frac{h^3}{3!}G'''(T_o)$$

$$= \frac{h}{4}G''''(T_o) \,/\, G'''(T_o)$$

From (3.8d) and (3.8e) -

$$\varepsilon_4 / \varepsilon_3 = - \frac{h}{4} \frac{2b^2}{T_o^5} G'(T_o) / \left(\frac{b^2}{2T_o^4} G'(T_o) \right)$$

$$= - h/T_o \qquad (3.19)$$

Equation (3.19) shows that ε_4 is less than 3.5 % if T_o = 298 ^oK and h be selected as 10^o. Thus, the fourth- and all higher - order derivatives in (3.18a) may be neglected in comparison with third-order derivative, as the error introduced by them is negligible. Rewriting (3.18) as

$$G(T_o + h) \cong hG'(T_o) + \frac{h^3}{3!} G'''(T_o) \qquad (3.18b)$$

The fractional deviation (FD) is defined by [3] -

$$|FD| = \left| \frac{G(T_o+h) - hG'(T_o)}{hG'(T_o)} \right| \qquad (3.20)$$

For (3.18b), using (3.20) -

$$|FD| = \frac{h^3}{3!} \frac{G'''(T_o)}{hG'(T_o)}$$

$$= \frac{h^2 b^2}{12 T_o^4} \qquad (3.21)$$

Equation (3.21) relates fractional deviation to thermistor parameter b , ambient temperature T_o and increment h. Let $(h_{max}/T_o) = (T_{max} - T_o)/ T_o$ be the maximum normalised temperature change to produce allowed deviation |AFD|, then (3.21) gives -

$$\left(\frac{h_{max}}{T_o}\right) = \frac{T_{max} - T_o}{T_o} = \frac{\sqrt{12\,|FD|}}{(b/T_o)}$$

$$\left(\frac{T_{max}}{T_o} - 1\right) = \frac{2T_o}{b}\sqrt{3\,|FD|} \qquad (3.22)$$

Another interesting term, called gain - bandwidth product GBW is introduced and is defined as [3]-

$$GBW = |G'(T_o)| \left(\frac{T_{max}}{T_o} - 1\right) \qquad (3.23)$$

Equation (3.23) can be modified for CT-1 to CT-4 by substituting the results for $G'(T_o)$ discussed in (3.12a) through (3.12d) respectively.

3.4 DESIGN OF A THERMISTOR BRIDGE CIRCUIT

Consider the design problem given in [3]. According to this, a one-thermistor temperature trandsucer is required that operates over the temperature range $-25\,^{o}C < T < 35^{o}C$ with miximum gain-slope deviation $|FD|$ of 10% and maximum output voltage magnitude about 1.0V.

As is obvious, the extreme temperature limits are $248^{o}K$ ($= T_{min}$) and $308^{o}K$ ($= T_{max}$). The selected reference temperature, T_o , is

$$T_o = \frac{T_{max} + T_{min}}{2} = 278^{o}K \text{ or } 5^{o}C.$$

Using (3.22) -

$$\left(\frac{T_{max}}{T_o} - 1\right) = \frac{2T_o}{b} \sqrt{3\ |FD|}$$

We have

$$b = \frac{(308/278 - 1)}{2 \times 278 \sqrt{3 \times 0.1}}$$

$$\cong 2780$$

For simplicity, let us consider the design of parameters for CT-2 with probe P-1. Equation (3.12b), is, therefore, modified by using the transformation coefficients $A_1 - A_4$ from Table 3.2 and it is given as -

$$G'(T_o) = \frac{-b(1+R_3/R_4)}{R_o T_o^2 (1+M_o)^2} \left(\frac{M_o R_o}{-1}\right)$$

$$= \frac{M_o}{(1+M_o)^2} \left(\frac{b}{T_o^2}\right)\left(1 + \frac{R_3}{R_4}\right) \tag{3.25}$$

Rewriting the linearity and balancing conditions in CT-2 with P-1 , we have

Balancing condition : $R_4/R_3 = R_o/R_1$ (3.26a)

Linearity condition : $R_1 = M_o\ R_o$ (3.26b) where $M_o = \frac{b-2T_o}{b+2T_o} = \frac{2780 - 2 \times 278}{2780 + 2 \times 278}$

$= \frac{10 - 2}{10 + 2}$

$= 8/12.$

Consider a 5 K thermistor having b = 2780 as obtained in (3.24) . Therefore ,

$R(25^{o}C) = 5\ K$

To calculate $R(5^{o}C)$ and hence R_o, use (3.6) such that

$R_o = R(5^{o}C) = 11.2\ K$, since $T_o = 5^{o}C$, $T = 25^{o}C$.

Note that bridge output is zero at $T = 278\ ^{o}K$ (or $5^{o}C$)

Thus, using (3.26 b).

$R_1 = \frac{8}{12} \times 11.2\ K \cong 7.5\ K$

and (3.26a) -

$R_4 = \frac{1}{M_o}\ R_3$

$= \frac{12 \times 10\ K}{8}$, if R_3 is assumed to be of 10 K

$= 15\ K.$

Taking (3.25) , (3.26a) and (3.26b) , we have

$$1 + \frac{R_3}{R_4} = 1 + M_o$$

and ; $$G'(T_o) = \frac{M_o}{1 + M_o} \cdot \frac{b}{T_o^2}$$

$$= \frac{8/12}{(1 + 8/12)} \quad \frac{2780}{(278)^2}$$

$$= 4/278.$$

Since, a maximum of 1.0 V output is required, the corresponding input voltage is given by (3.9)

$$\frac{e_o}{e_i} = G\Big|_{T=max}$$

$$= G'(T_o)(T-T_o)\Big|_{T = T_{max}}$$

$$= G'(T_o)\ (T_{max} - T_o)$$

Therefore,

$$\frac{e_o}{e_i} = \frac{4}{278} \times T_o\ \left(\frac{T_{max}}{T_o} - 1\right)$$

$$= 4\ (\ 308/278 - 1)$$

$$= 0.432$$

and $e_i = 1/\ 0.432 \cong 2.3\ V$

The designed values for the active thermistor bridge circuit CT - 2 with probe P-1 are summarized in Table 3.4 below

Table 3.4

Design results for CT-2

e_i	2.3V
R_1	7.5 K
R_3	10 K (assumed)
b	2780
$R(25^oC)$	5 K (assumed.
R_4	15K

3.5 CONCLUSION

In this chapter, a generalized view of analysing 4x5 one thermistor temperature transducer circuits is described. Note that the sensitivity considerations for various bridge circuits are considered in Chapter 2. The gain vs T/T_o curves, shown in Fig.(3.3) and (3.4) for WB and AMB - circuits with various values of b, reveal that an AMB circuit is more sensitive than a WB circuit. These curves are plotted using the data

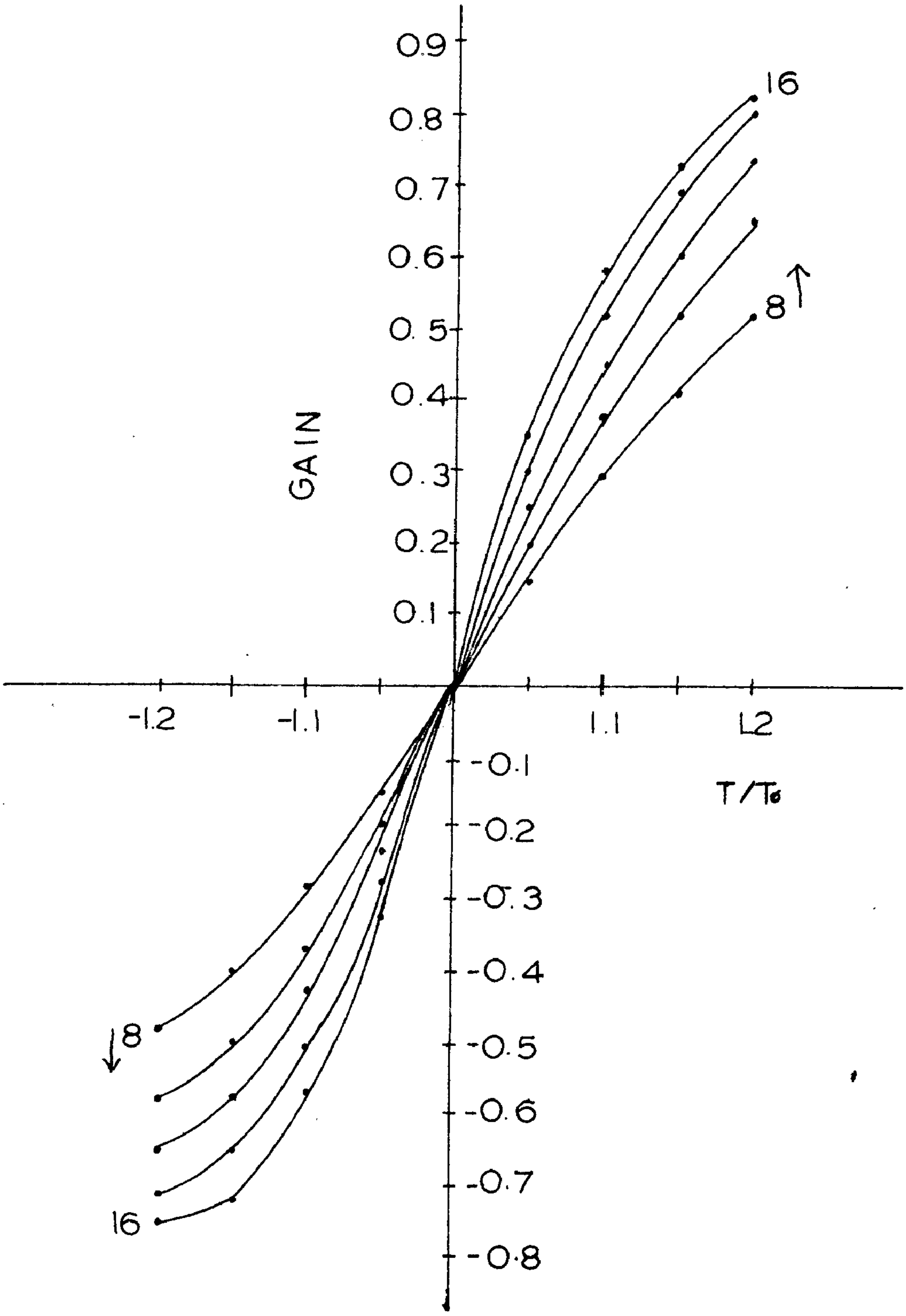


FIG. 3.3 PLOT FOR GAIN VS T/To
FOR P-1 & CT-2 (WB)

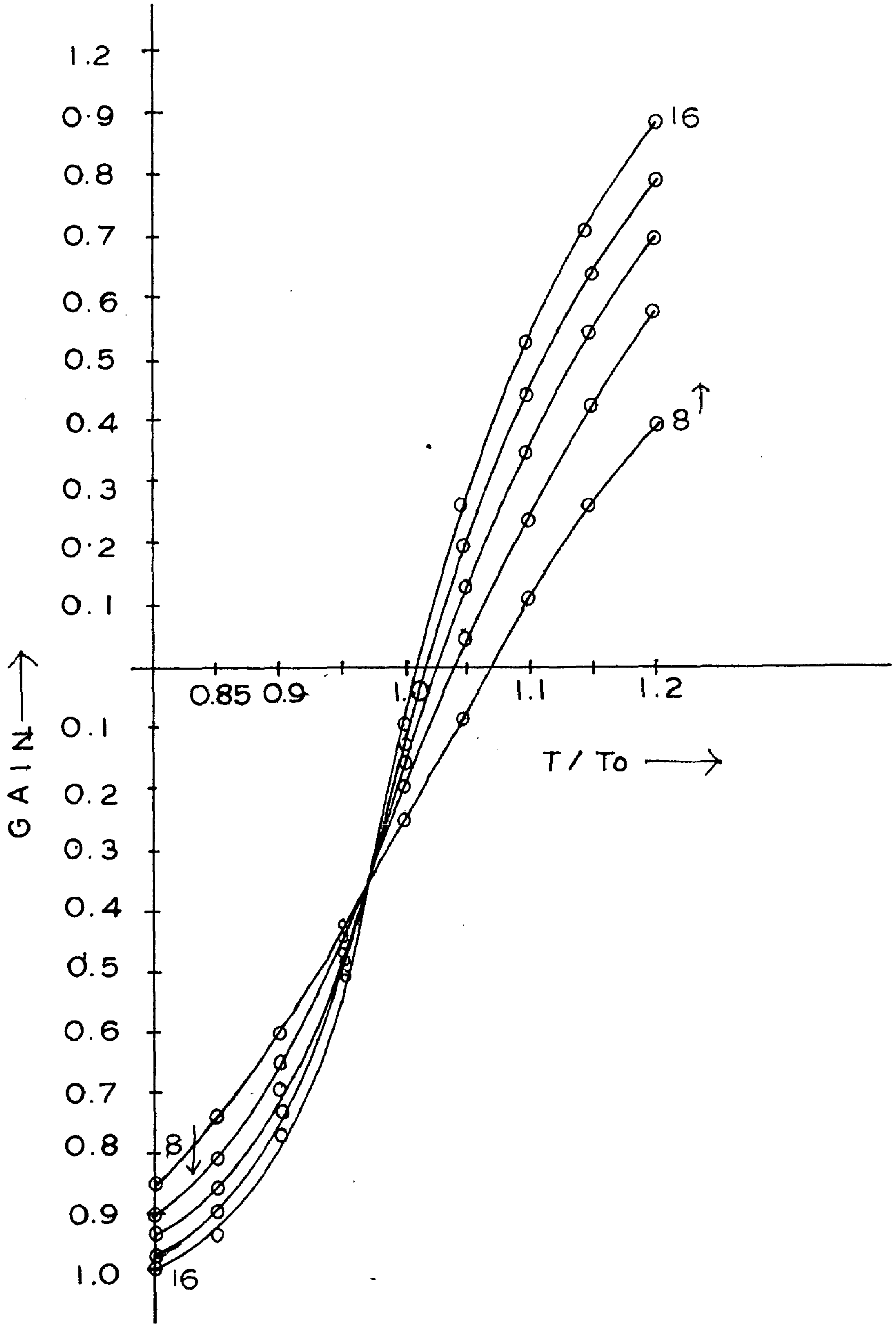


FIG. 3.4 PLOT OF GAIN VS T/To FOR P1 & CT-4 (AMB)

given in Tables 3.5 and 3.6 respectively which are obtained from the general expressions derived in this chapter. Lastly, a design problem is also considered and the results are summarized in Table 3.4.

Table. 3.5

Values of Gain for CT-2 (Using Appendices 3 and 4)

$$G = M_o(1-R_T/R_o)/(M_o+R_T/R_o)$$

T/T_o \ b/T_o	8	10	12	14	16
0.80	-0.48	-0.58	-0.65	-0.71	-0.75
0.85	-0.40	-0.50	-0.58	-0.65	-0.71
0.90	-0.28	-0.37	-0.44	-0.51	-0.57
0.95	-0.15	-0.19	-0.23	-0.27	-0.32
1.00	0	0	0	0	0
1.05	0.15	0.20	0.25	0.30	0.35
1.10	0.29	0.37	0.45	0.52	0.58
1.15	0.41	0.52	0.61	0.69	0.73
1.20	0.52	0.65	0.74	0.81	0.82

Table 3.6

Values of Gain for CT-4 and P-1 (Using Appendices - 3 and 4)

T/T_o \ b/T_o	8	10	12	14	16
0.80	-0.85	-0.90	-0.93	-0.96	-0.97
0.85	-0.74	-0.80	-0.85	-0.89	-0.92
0.90	-0.60	-0.64	-0.68	-0.72	-0.76
0.95	-0.43	-0.42	-0.44	-0.46	-0.48
1.00	-0.25	-0.20	-0.17	-0.14	-0.12
1.05	-0.06	-0.05	0.13	0.20	0.27
1.10	0.11	0.24	0.35	0.46	0.53
1.15	0.26	0.43	0.54	0.65	0.73
1.20	0.40	0.58	0.69	0.79	0.84

CHAPTER IV

ANALYSIS OF THERMISTOR WB CIRCUIT USING A LINEAR MODEL

In this chapter a piecewise linear model is first proposed for the nonlinear temperature characteristic of the thermistor (given in Fig. 1.1). This model is, in turn, used to calculate a gain in a simple Wheatstone Bridge circuit. A plot of gain Vs temperature is also made for a thermistor and it is shown to be balanced at a number of different temperatures, as against the balancing at single temperature of conventional thermistor WB circuit.

4.1 PIECEWISE LINEAR MODEL OF THERMISTOR

A system or process is said to be linear if it obeys the principle of superposition [24]. A differential equation which is linear does not contain any terms which are products of or powers of the variable and its derivatives. From Fig. 1.1 it is obvious that a thermistor has a nonlinear resistance temperature characteristics as it varies exponentially . If such an element is taken in one of the arms of a Wheatstone Bridge to measure the temperature the response of bridge becomes nonlinear too. In literature, various methods exist for obtaining a linearization into the response thus obtained. Beakley [21] and Olsen and

Brumley [9] have concentrated on linearization of the thermistor characteristic itself while Broughton [3] and other contemporary authors [11,14,16,18] have suggested a Taylor series representation of the response and evaluating the linearity condition by substituting the second order derivative in expansion as zero. Here, a piecewise linear model for thermistor characteristic is suggested. Linearization essentially consists of replacing the actual operating characteristic function by tangents at various temperatures called datum points.

The temperature characteristics of a thermistor is given by

$$R_T = R_o \, e^{b\left(\frac{1}{T} - \frac{1}{T_o}\right)} \qquad (4.1a)$$

Equations (4.1a) can be normalized as

$$y = e^{\beta\left(\frac{1}{x} - 1\right)} \qquad (4.1b)$$

where $y \equiv R_T/R_o$

$\beta \equiv b/T_o$

and $x \equiv T/T_o$

Let us assume that a linear operation of thermistor be required in the temperature range of -35^oC to 85^oC. For ease of operation and having piecewise linear model, divide the total range into three subranges each of $40^{\circ}C$ span. Accordingly -

Region I	$-35^{\circ}C$	to	$5^{\circ}C$	
Region II	$5^{\circ}C$	to	$45^{\circ}C$	(4.2)
Region III	$45^{\circ}C$	to	$85^{\circ}C$	

To develop a linear model for various ranges, consider the datum point as T_i. The different values of $T_i (i = 1,2,3)$ for above regions are listed in Table 4.1. Note that

$$T_2 = \frac{T_1 + T_3}{2}$$

where T_2 is equal to the ambient temperature T_o. From (4.1b)

$$\frac{dy}{dx} = e^{\beta(\frac{1}{x} - 1)} (-\beta/x^2)$$

Therefore,

$$\left.\frac{dy}{dx}\right|_{x = x_i} = e^{\beta(\frac{1}{x_i} - 1)} \left(-\beta/x_i^2 \right) \qquad (4.3)$$

where $x_i = T_i / T_o$. Substituting for x_i in (4.3), we have -

$$\left.\frac{dy}{dx}\right|_{x=x_i} = \begin{cases} -(\beta/0.7496)\ e^{\beta(0.155)} & ,\ \text{Region I} \\ -\ \beta & ,\ \text{Region II} \\ -(\beta/1.2864)\ e^{-\beta(0.1183)} & ,\ \text{Region III} \end{cases} \qquad (4.4a)$$

Use (4.1b) to obtain the values of y at various datum points as

$$\left. y \right|_{x=x_i} = e^{\beta(\frac{1}{x_i} - 1)}$$

$$= \begin{cases} e^{0.155\beta} & ,\ \text{Region I} \\ 1 & ,\ \text{Region II} \\ e^{-0.1183\beta} & ,\ \text{Region III} \end{cases} \qquad (4.4b)$$

Applying the tangent approximation technique [24],the linear model for thermistor characteristic is

$$y - \left. y \right|_{x=x_i} = \left.\frac{dy}{dx}\right|_{x=x_i} (x - x_i) \qquad (4.5)$$

Equation (4.5) may be simplified for the different regions as -

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Region I

$$y = e^{0.155\beta\left[-\frac{\beta}{0.7496}(x - 0.8658) + 1\right]} \qquad (4.6a)$$

Region II

$$y = -\beta(x - 1) + 1 \qquad (4.6b)$$

Region III

$$y = e^{-0.1183\beta\left[-\frac{\beta}{1.2864}(x - 1.1342) + 1\right]} \qquad (4.6c)$$

Table 4.1

Datum Points for Different regions

Region	I	II	III
Temperature range	$238^{\circ}K$ to $278^{\circ}K$ $-35^{\circ}C$ to $5^{\circ}C$	$278^{\circ}K$ to $318^{\circ}K$ $5^{\circ}C$ to $45^{\circ}C$	$318^{\circ}K$ to $358^{\circ}K$ $45^{\circ}C$ to $85^{\circ}C$
Normalised temperature range	0.7986 to 0.9329	0.9329 to 1.0671	1.0671 to 1.2013
	$T_1 = -15^{\circ}C$	$T_2 = 25^{\circ}C = T_o$	$T_3 = 65^{\circ}C$
Datum Point T_i	258° K	298° K	338° K
Normalised datum point $x_i \triangleq T_i/T_o$	0.8658	1.000	1.1342

Table 4.2 Values of γ using (4.1 b)

	$T^{o}K$	238	248	258	268	278	288	298	308	318	328	338	348	358
	X	.7986	.8322	.8658	.8993	.9329	.9664	1.000	1.0336	1.0671	1.1007	1.1342	1.1678	1.2013
β	8	7.52	5.02	3.46	2.45	1.78	1.32	1.00	0.77	0.61	0.48	0.39	0.32	0.26
	10	12.45	7.51	4.71	3.06	2.05	1.42	1.00	0.72	0.53	0.40	0.31	0.24	0.19
	12	20.62	11.24	6.42	3.83	2.37	1.52	1.00	0.68	0.47	0.33	0.24	0.18	0.13
	14	34.15	16.82	8.76	4.80	2.74	1.63	1.00	0.63	0.42	0.28	0.19	0.13	0.10
	16	56.55	25.17	11.94	6.00	3.16	1.75	1.00	0.59	0.37	0.23	0.15	0.10	0.07

		Region	I			Switching Point I		Region II			
	X	.7986	.8322	.8658	.8993	(x',y')	.9329	.9664	1.0000	1.0336
	8	5.93	4.70	3.46	2.30	(.9137,1.69)	1.54	1.27	1.00	0.73
	10	8.93	6.82	4.71	2.74	(.9106,1.89)	1.67	1.34	1.00	0.66
β	12	13.33	9.88	6.42	3.21	(.9078,2.11)	1.81	1.40	1.00	0.60
	14	19.75	14.25	8.76	3.64	(.9051,2.33)	1.94	1.47	1.00	0.53
	16	28.06	20.51	11.94	3.96	(.9026,2.56)	1.07	1.54	1.00	0.46

		Switching Point II	Region III				
	X	(x",y")	1.0671	1.1007	1.1342	1.1678	1.2013
	8	(1.0525,.58)	0.55	.47	.39	.31	.23
	10	(1.0491,.51)	0.47	.39	.31	.23	.15
β	12	(1.0468,.44)	0.39	.32	.24	.17	.09
	14	(1.0445,.38)	0.33	.26	.19	.12	.05
	16	(1.0433,.31)	0.28	21	.15	.09	.03

Thus, we observe that the thermistor described by (4.1b) is approximated by linear equations (4.6a) through (4.6c) in different temperature ranges. Tables (4.2) and (4.3) list the values of y obtained from (4.1b) and (4.6) respectively. A picewise linearization of thermistor characteristic curve is also plotted for a typical value of β and is shown in Fig. 4.1.

4.2 CALCULATION OF GAIN IN THERMISTOR WB CIRCUIT

Consider the bridge circuit shown in Fig. 2.1. The gain G is

$$G \triangleq \frac{e_o}{e_i} = \frac{R_3 R_T - R_1 R_4}{(R_1+R_T)(R_3+R_4)} \tag{4.7}$$

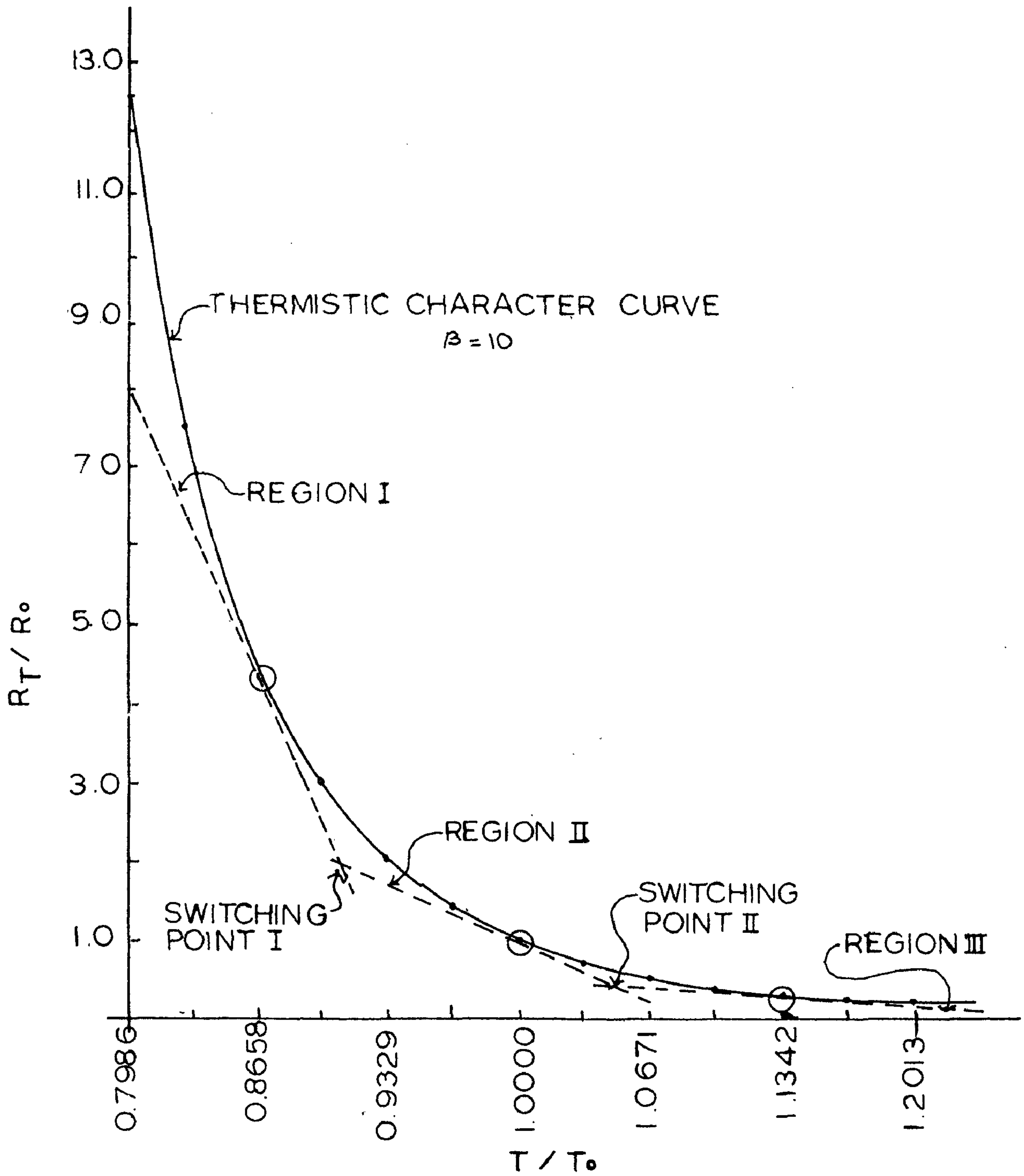
To balance gain at datum point $T_i (i = 1,2,3)$, we have

$$R_1 R_4 = R_3 R_{T_i} \quad , \text{ where } R_{T_i} = R_T \Big|_{T=T_i} \tag{4.8a}$$

or

$$(R_4/R_3) = (R_{T_i}/R_1) \tag{4.8b}$$

The ratio R_{T_i}/R_o is calculated in (4.4b) as $Y\Big|_{x=x_i}$, for $i = 1,2,3$. Substituting (4.8b) in (4.7) we get



FI.G 4.1 PIECE WISE LINEARIZATION OF THERMISTOR CHARACTERSTIC CURVE

$$G = \frac{y - y\big|_{x = x_i}}{\left(y + \dfrac{R_1}{R_{T_i}}\, y\,\Big|_{x=x_i} \right)\left(1 + \dfrac{R_{T_i}}{R_1} \right)} \qquad (4.9)$$

To compare the results obtained in (4.9) with (4.12) below, assume [3]

$$\frac{R_1}{R_{T_i}} = \frac{\beta - 2x_i}{\beta + 2x_i} \quad , \text{ where } x_i = T_i/T_o \qquad (4.10)$$

From (4.9) and (4.10)

$$G = \left[\frac{\beta - 2x_i}{2\beta}\right] \frac{y - y\big|_{x=x_i}}{\left[y + \left[\dfrac{\beta - 2x_i}{\beta + 2x_i}\right] y\,\Big|_{x = x_i} \right]} , \qquad (4.11)$$

Note that equation (4.11) has been derived assuming a piecewise linear model for thermistor . The schematic of such a linearized thermistor WB circuit is slightly different than Fig. 2.1 and is shown in Fig. 4.3

The another approach of linearization for the desired temperature range, gain G as defined in (4.7) is approximated by a Taylor series expansion about $x=1$

ASSUME A THERMISTOR HAVING $\beta = 10$

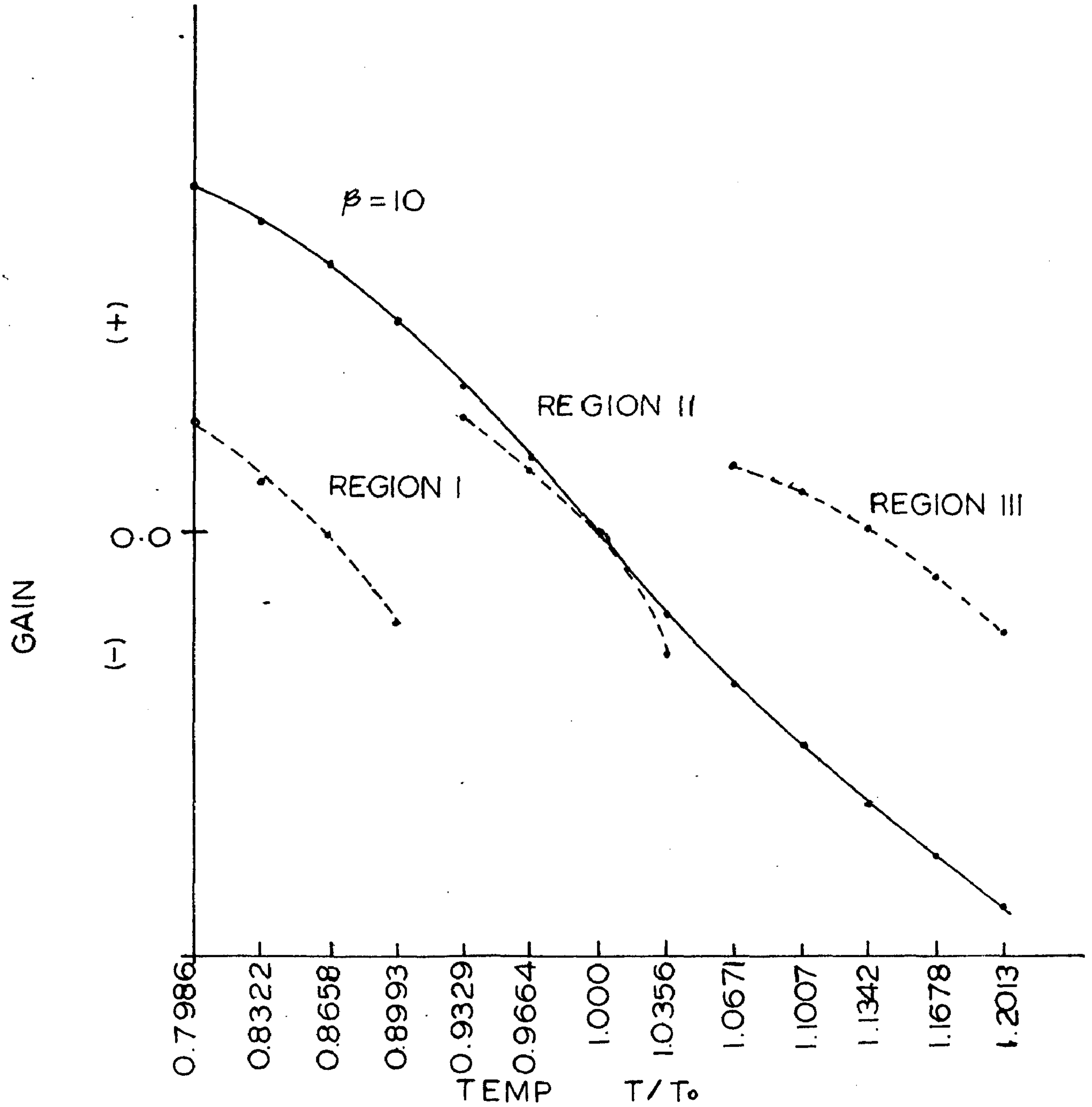


FIG. 4.2 TEMP VS GAIN VARIATION OF LINEAR THERMISTOR

(corresponding to $T = T_o$). and an appropriate condition is derived by equating to zero the coefficients of second and higher order derivatives, if possible [3]. Using balancing and linearity constraints, equation (4.7) is modified as -

$$G = \left[\frac{\beta-2}{2\beta}\right] \cdot \frac{y-1}{\left[y + \left(\frac{\beta-2}{\beta+2}\right)\right]} \qquad (4.12)$$

4.3 COMPARISON

For comparing (4.12) with (4.11) , consider a typical value of normalized thermistor constant β as 10. The values obtained for G frompiecewise linearization and Taylor series expansion concepts are listed in Table 4.4.

A plot of gains for various temperatures is also made and is shown in Fig. 4.2. It is clear from the figure that gain is balanced at a number of different temperatures known as datum points, as against the balancing at single temperature of conventional thermistor WB circuit discussed in Chapter 3.

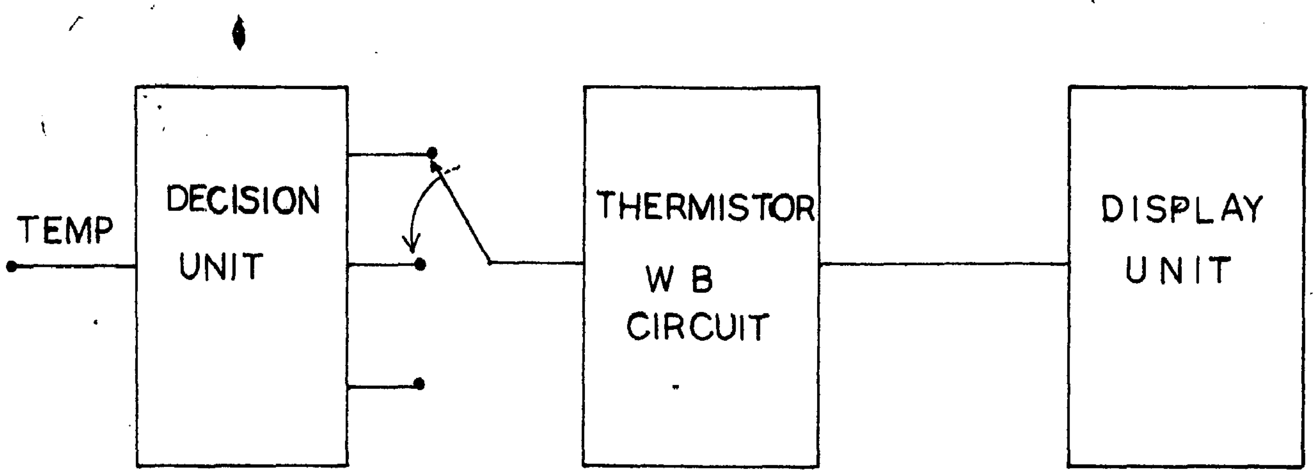


FIG. 4.3 SCHEMATIC DIAGRAM OF THE LINEARISED THERMISTOR WB CIRCUIT

Table 4.4

Values of gain, G for $\beta = 10$

	Region I				Region II			
x	0.7986	0.8322	0.8658	0.8993	0.9329	0.9664	1.0000	1.0336
Piece-wise linearized G	0.11	0.06	0	-0.09	0.12	0.07	0	-0.11
Taylor series Exp.G	0.35	0.32	0.28	0.22	0.15	0.08	0	-0.08

	Region III				
x	1.0671	1.1007	1.1342	1.1678	1.2013
Piece-wise linearized G	0.07	0.04	0	-0.04	-0.10
Taylor series Exp.G	-0.16	-0.22	-0.28	-0.33	-0.38

CHAPTER V

FLOW MEASUREMENT AND THERMISTOR BRIDGE CIRCUITS

Out of various applications of a thermistor bridge circuit, fluids' velocity measurement is a typical one. It is primarily used in the areas of physiological and pharmacological research, and blood flow measurement where 'a little' flow occurs. In such circuits, a self and separately heated mode of operation of thermistor is utilized. As discussed in section 1.2, the operation of a thermistor is based on the dependence of the dissipation coefficient on physical parameters of the fluid, and in particular on its velocity. The variation of the heat exchanged with the fluid per unit time modifies the temperature of the device and then its resistance. Consequently, the fluid velocity may be determined by means of a simple electrical measurement. Section 5.1 describes mathematically the behaviour of a thermistor placed in flow as a sensing element. First, some of the definitions are considered which make the text self-sufficient. Second, a mathematical analysis for thermistor behaviour is presented. It is based on [1,13,15,16]. Section 5.2 investigates a technique for the response time and sensitivity considerations of thermistor flowmeters. An example is solved to illustrate the method. A simple flowmeter employing thermistor bridge circuit is described in Section 5.3.

5.1 BEHAVIOUR OF THERMISTOR IN FLOW

DEFINITION 5.1 : The dissipation - coefficient or - constant ''k'' is defined as the power input necessary to raise the temperature of the sensor $1^{O}K$ above ambient temperature. It depends on the thermodynamic properties of the fluid and in particular on its velocity with respect to the thermistor [15].

DEFINITION 5.2 : The thermal time constant 'τ' represents the time required for a thermistor to change 63.2% of the total temperature difference between its initial and final temperature when subjected to a temperature change under zero power condition [1].

DEFINITION 5.3 : As discussed in section 1.1, the temperature coefficient of resistance 'α' is defined as

$$\alpha = \frac{1}{R(T)} \frac{dR(T)}{dT} \qquad (5.1)$$

where R(T) denotes thermistor resistance.

To describe the thermal behaviour of a thermistor placed in a flow, consider the following heat-transfer equation for a self-heated sensor

$$C \frac{dT}{dt} = P - k\ (T - T_a) \qquad (5.2)$$

with, C , thermal capacity of the thermistor

T temperature of the thermistor body

T_a temperature of the surrounding fluid

P dissipated power in the sensor and is given by $i^2R(T)$ where 'i' is the constant bias current feeding the thermistor.

In (5.2) , if the feeding power P is removed, we have

$$C \frac{dT}{dt} = -K\ (T - T_a) \qquad (5.3a)$$

Changing temperature difference $(T-T_a)$ by T' we have

$$C \frac{dT'}{dt} + k\,T' = 0$$

$$\frac{dT'}{dt} + \frac{k}{C}\,T' = 0 \qquad (5.3b)$$

which, after integration, has the solution

$$T - T_a = (T_o - T_a)\ e^{-kt/C} \qquad (5.3c)$$

Equation (5.3) represents the temperature signal of the self-heated thermistor in a fluid at rest. Using definition 5.2, the thermal time constant -

$$\tau \equiv C/k \qquad (5.4a)$$

and $$\frac{T - T_a}{T_o - T_a} = \frac{1}{e} = 0.368$$

Since 'k' is a function of fluid velocity v, it usually denoted as k(v). At rest, v = 0 , hence (5.4) is rewritten as

$$\tau_o = C/k(0) \tag{5.4b}$$

For simplicity of the derivation, consider the piece-wise linearized model of thermistor (see Chapter 4). The thermistor resistance R(T) , in general, is approximated as

$$R(T) = R_o[1 + \alpha(T - T_o)] \tag{5.5}$$

where 'α' is resistance temperature coefficient of thermistor, R_o and T_o are the resistance and temperature values at fluid velocity equal to zero. Note that (5.5) is valid for small deviations from the initial temperature. Substituting for 'P' in (5.2) , we have

$$C \frac{dT}{dt} = i^2R_o[1+ \alpha(T-T_o)] - k (T - T_a)$$

$$= P_o(1-\alpha T_o) + P_o\alpha T - kT + kT_a \tag{5.6a}$$

where $P_o(= i^2R_o)$ is the dissipated power in the sensor at zero fluid velocity. Rewriting (5.6) as

$$\frac{C}{k(o)} \frac{dT}{dt} + \frac{k(v) - Po\alpha}{k(o)} T = \frac{Po(1-\alpha To)}{k(o)} + \frac{k(v)}{k(o)} T_a \quad (5.6b)$$

The normalized time constant of the thermistor signal [16] is defined by

$$\bar{\tau}'(v) = \frac{k(o)}{k(v) - Po\,\alpha} \quad (5.7\text{ a})$$

Other normalized parameters are

$$\bar{m} = \frac{P_o(1 - \alpha T_o)}{k(o)} \quad (5.7\text{ b})$$

and $\bar{\tau}(v) = k(o)/k(v)$ (5.7 c)

Substituting the results of (5.7 a) through (5.7c) in (5.6 b), we have -

$$\frac{dT}{d(t/\tau_o)} + \frac{T}{\bar{\tau}'(v)} = \bar{m} + \frac{T_a}{\bar{\tau}(v)} \quad (5.6\text{ c})$$

Equation (5.6 c) can be manipulated into

$$\frac{dT}{d\bar{t}} + \frac{T - T_a}{\bar{\tau}'(v)} = \bar{m} + (T_a \left(\frac{1}{\bar{\tau}(v)} - \frac{1}{\bar{\tau}'(v)}\right) \quad (5.6\text{ d})$$

With $\bar{t} = t/\tau_o$. Replacing temperature difference $(T - T_a)$ by T', (5.6 d) is rewritten as

$$\frac{dT'}{d\bar{t}} + \frac{T'}{\bar{\tau}'(v)} = \bar{m} + T_a \left(\frac{1}{\bar{\tau}(v)} - \frac{1}{\bar{\tau}'(v)}\right) \quad (5.6\text{e})$$

Note that equation (5.6 e) is the **normalized** version for the expression (5.2). It is similar to (5.3b). However , (5.6e) is quite general as it considers the effect of feeding power P and hence the behaviourof thermistor. The response of (5.6e) for a velocity step from 0 to v is given from [15] as -

$$T' = C_2 + C_1 \exp\left(-t/\bar{\tau}'(v)\right) \tag{5.8}$$

with

$$C_2 = \bar{\tau}'(v)\left[\bar{m} + T_a\left(\frac{1}{\bar{\tau}(v)} - \frac{1}{\bar{\tau}'(v)}\right)\right]$$

and $C_1 = T_o - T_a - C_2$

In this approximation the time constant $\bar{\tau}'(v)$ of the temperature signal is dependent on the fluid velocity, temperature coefficient and resistance value of thermistor at the initial temperature (see (5.7a)). Therefore,even being a linear equation, the time response to a velocity step depends both on the amplitude and the direction of the velocity variation [15]. Equation (5.6e) isvalid for small deviations from the initial temperature. Two nonlinear phenomenon occur when there are larger increments in the thermistor temperature. They are [15] -

(a) dependence of the dissipation coefficient 'k' on v and $(T-T_a)$;

(b) exponential temperature dependence of the thermistor resistance.

For (a), the influence of the dependence of k on $(T-T_a)$ and v (i.e., $k(v,T-T_a)$) is examined in [15,16]. The equation which describes the thermistor behaviour in case (a) remains formally equal to (5.6e) except that $\bar{\tau}'$ and $\bar{\tau}$ also depend on $(T-T_a)$ and not only on v . This implies that

$$\tau'(v,\ T-T_a)\ ,\ \text{and}\ \ \bar{\tau}(v,T-T_a)$$

are introduced in (5.6e) in place of $\bar{\tau}'(v)$ and $\bar{\tau}(v)$ respectively.

For (b), it is known that a thermistor resistance varies exponentially with the temperature according to the relation

$$R(T) = R_s\ e^{b(\frac{1}{T} - \frac{1}{T_s})}$$

with, b characteristic constant of the thermistor

R_s thermistor resistance at the reference temperature T_s

This introduces a further nonlinear term in (5.2), which then assumes the form

$$C\frac{dT}{dt} = P_o \exp\left(B/T - B/T_s\right) - k(v,T-T_a)(T-T_a) \quad)(5.9)$$

Equation (5.9) , thus, is the general mathematical description for the behaviour of thermistor placed in a flow as a sensing element.

5.2 RESPONSE TIME AND SENSITIVITY CONSIDERATION

For simplicity of analysis, consider the linear case (5.8). It describes the temperature signal of the self-heated thermistor in a fluid at velocity 'v'. To determine the response time, it is sufficient to find out the rise time defined as the time in which the temperature signal (in response to velocity step of amplitude 'v') rises from 10% to 90% of its steady state velue (5.10). Using (5.8), we have

$$T'\Big|_{t\to\infty} = C_2 \tag{5.10}$$

Since $C_1 \exp(-t/\bar{\tau}'(v)) \to 0$. Let 10% and 90% of steady state value be obtained at t_1 and t_2 respectively. Thus, mathematically

$$T'\Big|_{t=t_1} = 0.1\ C_2 \tag{5.11a}$$

and

$$T'\Big|_{t=t_2} = 0.9\ C_2 \tag{5.11b}$$

with $t_r = t_2 - t_1$, where 't_r' is the rise time. From (5.11a), (5.11b) and (5.8), we have

$$0.1\ C_2 = C_2 + C_1 \exp(-t_1/\bar{\tau}'(v)) \tag{5.12a}$$

$$0.9\ C_2 = C_2 + C_1 \exp(-t_2/\bar{\tau}'(v)) \tag{5.12b}$$

Equations (5.12a) and (5.12b) simplify as -

$$-0.9\ C_2 = C_1 \exp(-t_1/\bar{\tau}'(v)) \qquad (5.12c)$$

$$-0.1\ C_2 = C_1 \exp(-t_2/\bar{\tau}'(v)) \qquad (5.12d)$$

Considering the ratio of (5.12c) and (5.12d), we get

$$9 = \exp[(t_2-t_1)/\bar{\tau}'(v)]$$

therefore,

$$\ln 9 = (t_2-t_1)/\bar{\tau}'(v)$$

and, hence

$$\begin{aligned} t_r &= \bar{\tau}'(v) \ln 9 \\ &= \frac{2.2\ k(0)}{k(v) - \alpha P_o} \quad ; \text{ using } (5.7a) \end{aligned} \qquad (5.13)$$

Here, $P_o = i^2 R_o$ and $k(v)$ is given by [15] -

$$k(v) = a_1 + a_2\ v^{1/2} \qquad (5.14)$$

where the parameters a_1 and a_2 assume the values 2.4 mW/K and 0.22 mW $S^{1/2}$ / K $cm^{1/2}$. Dividing (5.13) by a factor 2.2, the normalized rise time is given by

$$\bar{t}_r = \frac{k(0)}{k(v) - P_o \alpha} \qquad (5.15)$$

It is obvious that $k(v)$ is an always increasing function of velocity (see (5.14)). Thus, (5.15) decreases with the increase in velocity. In the following an example is solved to illustrate the statement.

Example 5.1 Consider a spherical boron thermistor with the characteristics [15,16] given in Table 5.1 below :

Table 5.1

Shape	Spherical
Diameter	1 mm
$R(25^{o}C)$	2K
$b(25^{o}C)$	$3300^{o}K$
$\alpha(25^{o}C)$	$-0.038\ ^{o}K^{-1}$

Without any loss of generality, it is assumed that $R_o = 2K$ and $T_o = 298^{o}K$ for a thermistor placed in a static fluid. Using (5.14), we have

$$k(0) = a_1$$
$$= 2.4 \text{ m W/K}$$

and $k(v) = 4.6$ mW/K

For a velocity step of $v = 100$ cm/sec. Substituting the values of $k(0)$, $k(v)$ and α in (5.15), the various values of normalized rise time for different bias current 'i' feeding the thermistor are summarized in Table 5.2.

Table 5.2

i	0 mA	10 mA	20 mA	30 mA	4o mA
$\bar{t}_r$	0.52	0.20	0.07	0.03	0.019

A plot of normalized rise time of thermistor temperature response to an air velocity step of 100 cm/sec. (starting from zero velocity) as a function of the bias current is made and is given in Fig. 5.1. Note that the thermistor shows a relatively high response time of the order of some seconds. Obviously, it is a great limitation in the use of thermistor as flowmeters. References [15,16] describe another important aspect namely the sensitivity consideration in thermistor flowmeters. Two types of sensitivities viz. thermistor-voltage and resistance sensitivities have been defined for 'Δv' velocity step and curves show their basic nature. It is concluded that there is a saturation effect both in sensitivity and in response velocity,' it is always convenient, however, to bias the thermistor at high currents.

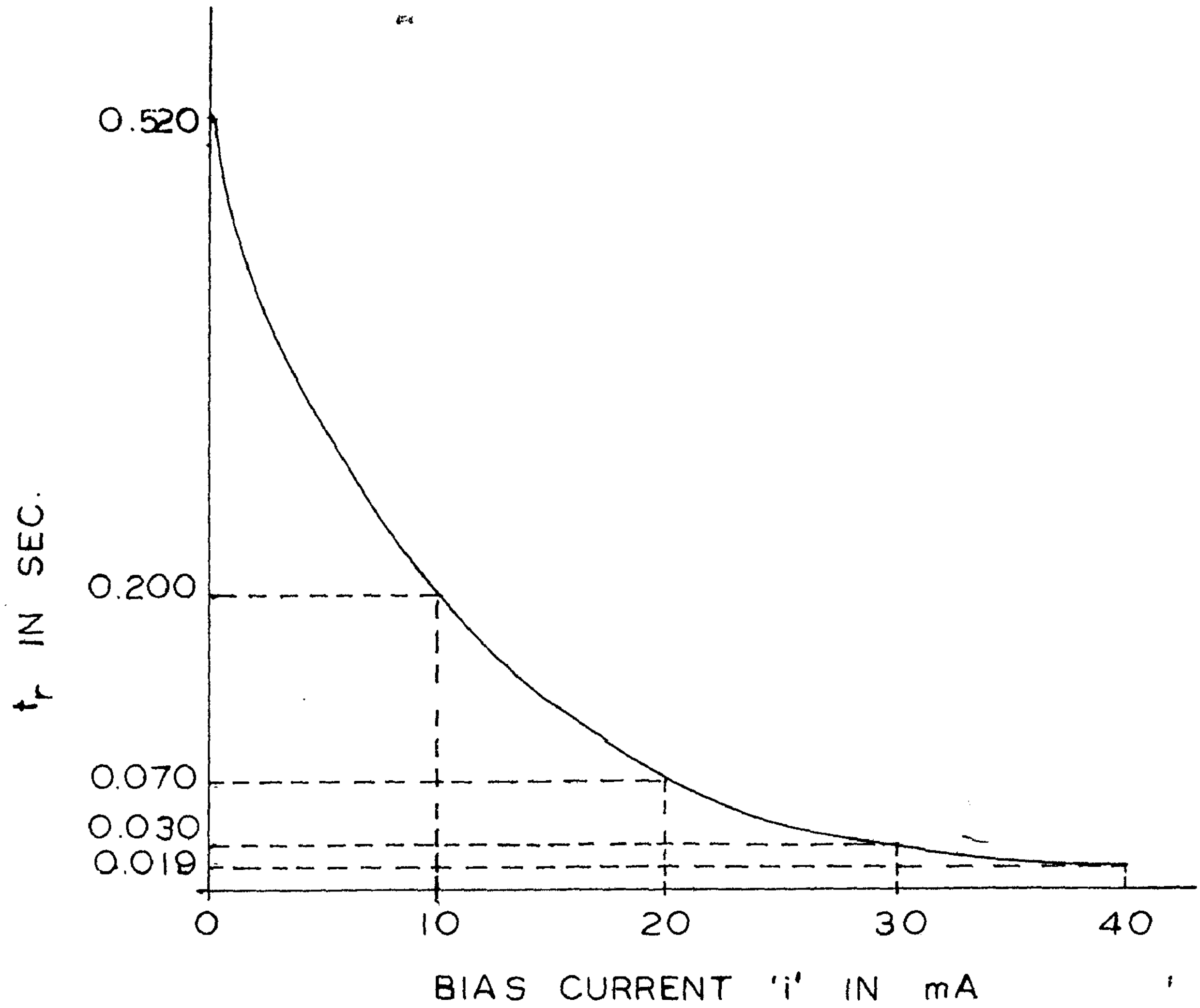
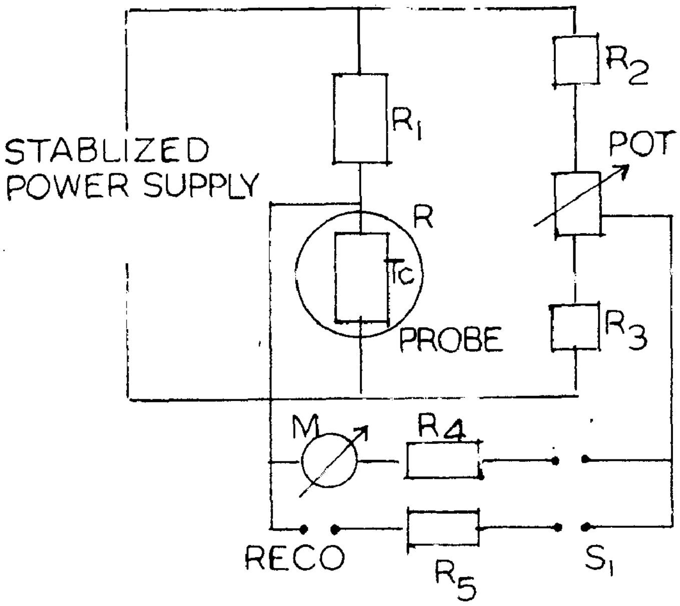


FIG. 5.1 NORMALISED RISE TIME AS A FUNCTION OF BIAS CURRENT

5.3 FLOWMETER WITH THERMISTOR BRIDGE CIRCUIT

The bridge circuit usedfor measurement of flow of gases within range 0.2 - 2.0 litres/hour and liquid within range of 0.0004 - 0.4 litres/ hour is given in Fig. 5.2(a). The bridge circuit is formed by R, R_1, R_2 and R_3 where R is a bead thermistor encapsulated in a glass tube as shown in Fig. (5.2b) and the unit is termed as probe. This probe is a glass measuring tube with inner diameter of 0.4 cm and a length of 6 cm into which the bead thermistor is inserted from top. The bridge is fed from the stabilized power supply. The switch S_1 is used to connect into the diagonal of the bridge either to the meter M, built into the instrument or the recorder 'RECO'. Before starting the bridge potentiometer 'POT' is adjusted to balance the bridge at zero velocity of the surrounding medium. Since the resistance of heated thermistor used as a pickup depends not only on velocity but also on the temperature of the surrounding medium, the elimination of temperature variation is of utmost importance. Here temperature stabilizer has been used for the purpose. The flowing medium because of the length of coiled tube is heated to the temperature of bath before it enters to the tube containing thermistors. When measuring hthe liquid flow, it is desirable to reduce the temperature of the bath and, thus , of flowing liquids below 20^{o}C



$R_1 = 40K\Omega = R_2$

$R_4 = 20K\Omega$

$R_5 = 20K\Omega$

$M = 40\mu A \quad 5K\Omega$

FIG 5.2(a) THERMISTOR FLOWMETER BRIDGE CIRCUIT

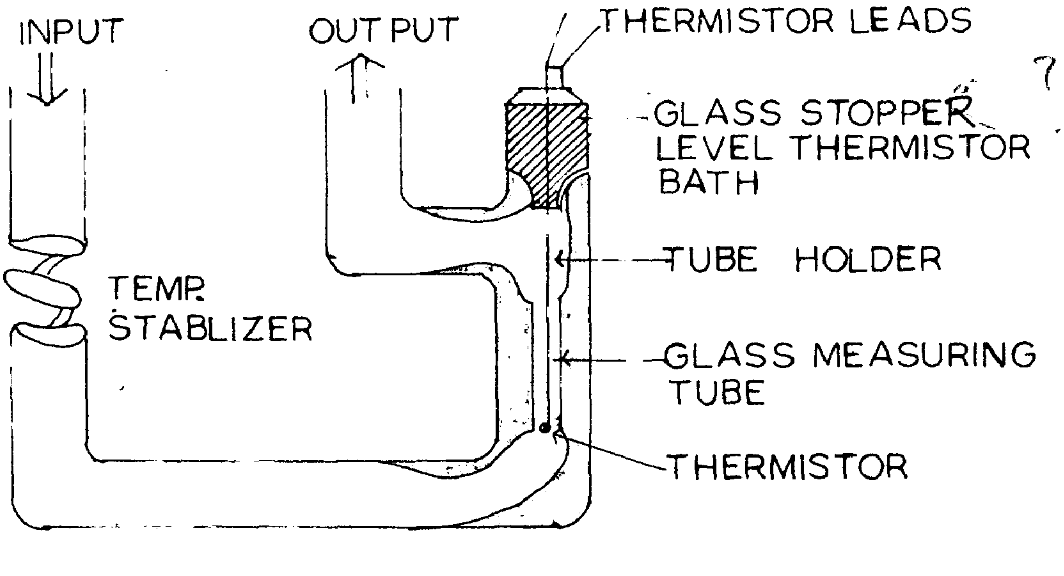


FIG. 5.2 THE PROBE

in order to prevent the air bubbles from springing upon the thermistor bead.

5.4 CONCLUSION

This Chapter investigates in detail the behaviour of thermistor placed in a 'small' flow. For simplicity of analysis, linear approximation of thermistor characteristic has been considered. However, for practical considerations the non-linear effects must be taken into account and an optimization of the thermistor performance in terms of sensitivity and response speed need be considered too. In the last, a flowmeter employing a thermistor bridge circuit is described.

CHAPTER VI

SOME OTHER APPLICATIONS OF THERMISTOR BRIDGE CIRCUIT

Chapter 2, 3 and 4 describe in detail the analysis and design of thermistor bridge circuits which are primarily used for temperature measurement. A typical example, where a self heated thermistor is applied for 'small' flows is considered in Chapter 5. Besides measuring the above two parameters viz temperature and flow, thermistor bridge circuits also find their applications in vast area of instrumentation [1,23,25,30,31,32]. In this Chapter, a brief review is made of some of the areas and the basic principle involved therein is discussed. Sections 6.1 through 6.3 describe the application of thermistor bridge circuit for the measurement of non-electrical quantities. The remaining chapter, however, lists the principles of measurement of electrical parameters like voltage and current.

6.1 PRESSURE MEASUREMENT

The V-I characteristic of a thermistor is utilized for measuring small pressures upto 2.0 Torrs. Here, a pressure change is equivalent to the movement of V-I characteristics along the constant resistance line without any significant change of shape (refer Fig. 6.1).

A simple bridge circuit based on the above principle is described in [29,31] to measure the

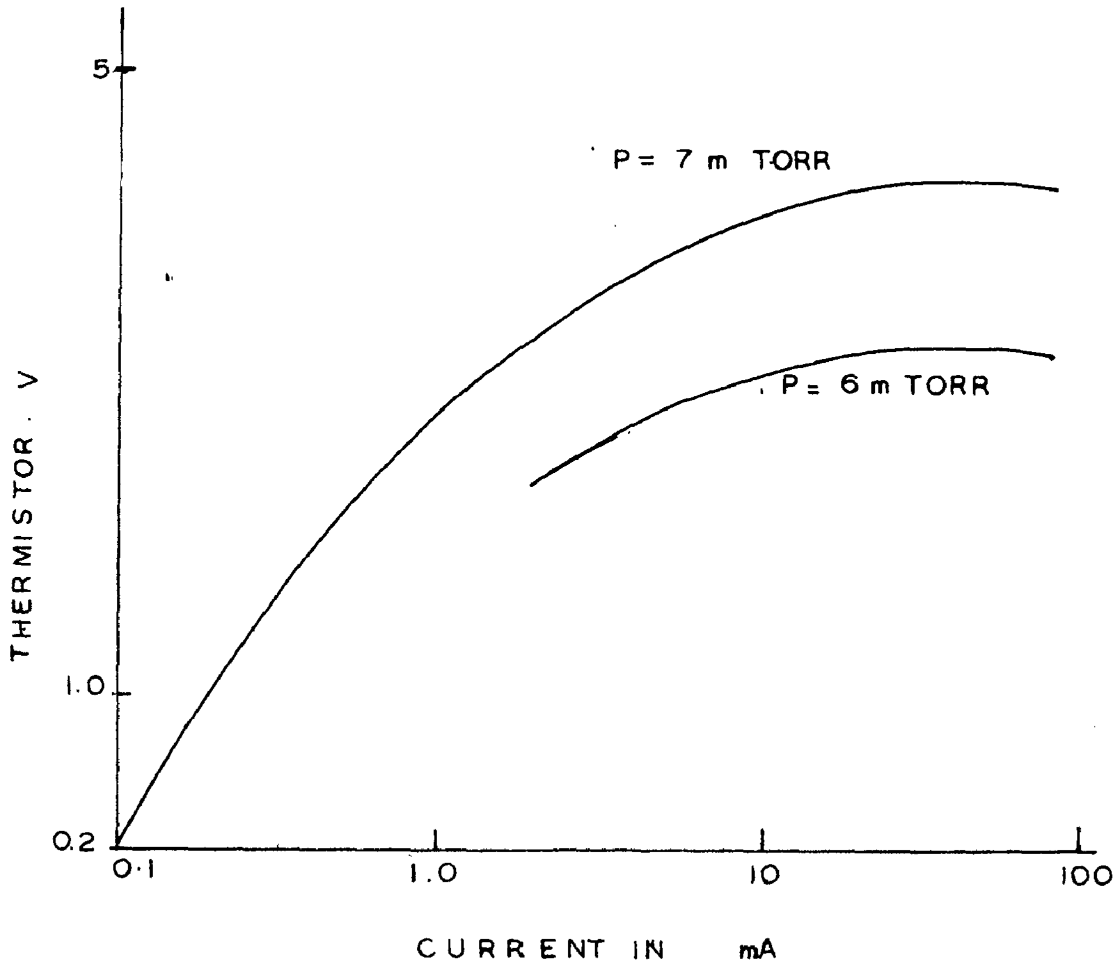


FIG. 6.1 V I CHARACTERSTICS OF THERMISTOR

pressure. Note that a bead type thermistor is normally used in such applications. A calibration curve between the bridge output and the measured pressure is also discussed and is shown in Fig. 6.2.

6.2 THERMOMETRIC ANALYSIS

Thermistor bridge circuit may be used as a method for determining the concentration of a substance in solution by virtue of temperature changes which occurs due to addition of reactants to it. If the volumetric rates of flow of the two solutions are kept equal, the temperature change due to the reaction is a measure of the concentration of the weaker solution under test. The molarity of solution under test is given by [30] .

$$N = (K/H) [2 T_p - T_w - T_z]$$

where, T_w , T_z and T_p are temperatures of reactant, test and product solutions respectively, K is the identical thermal capacity and H the heat of reaction. A typical thermistor bridge circuit used for the purpose is described in [30] . The bridge balancing operation may be made automatic by replacing the Galvanometer with a servo-amplifier which, in turn, operates a servomotor coupled to the shaft of the potentiometer. This arrangement helps in continuous display of solutions -

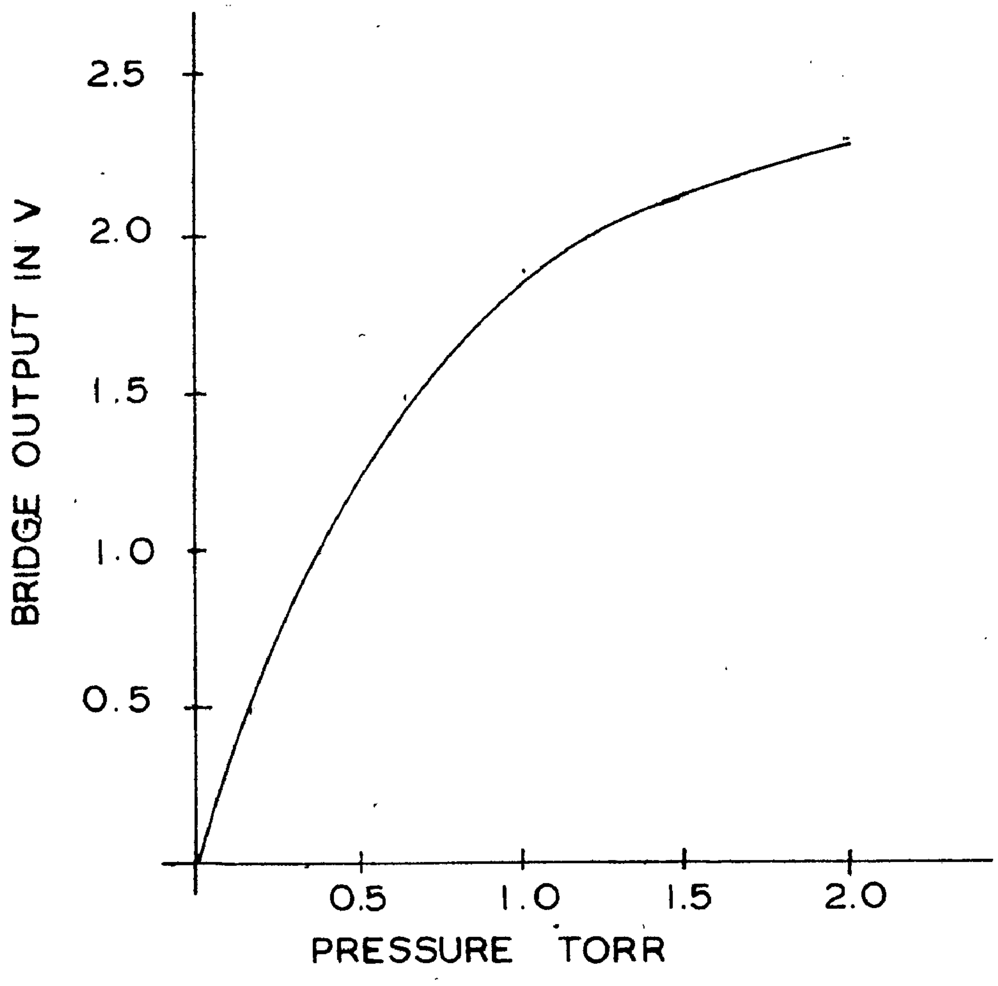


FIG. 6.2 GRAPH BETWEEN PRESSURE AND BRIDGE OUTPUT

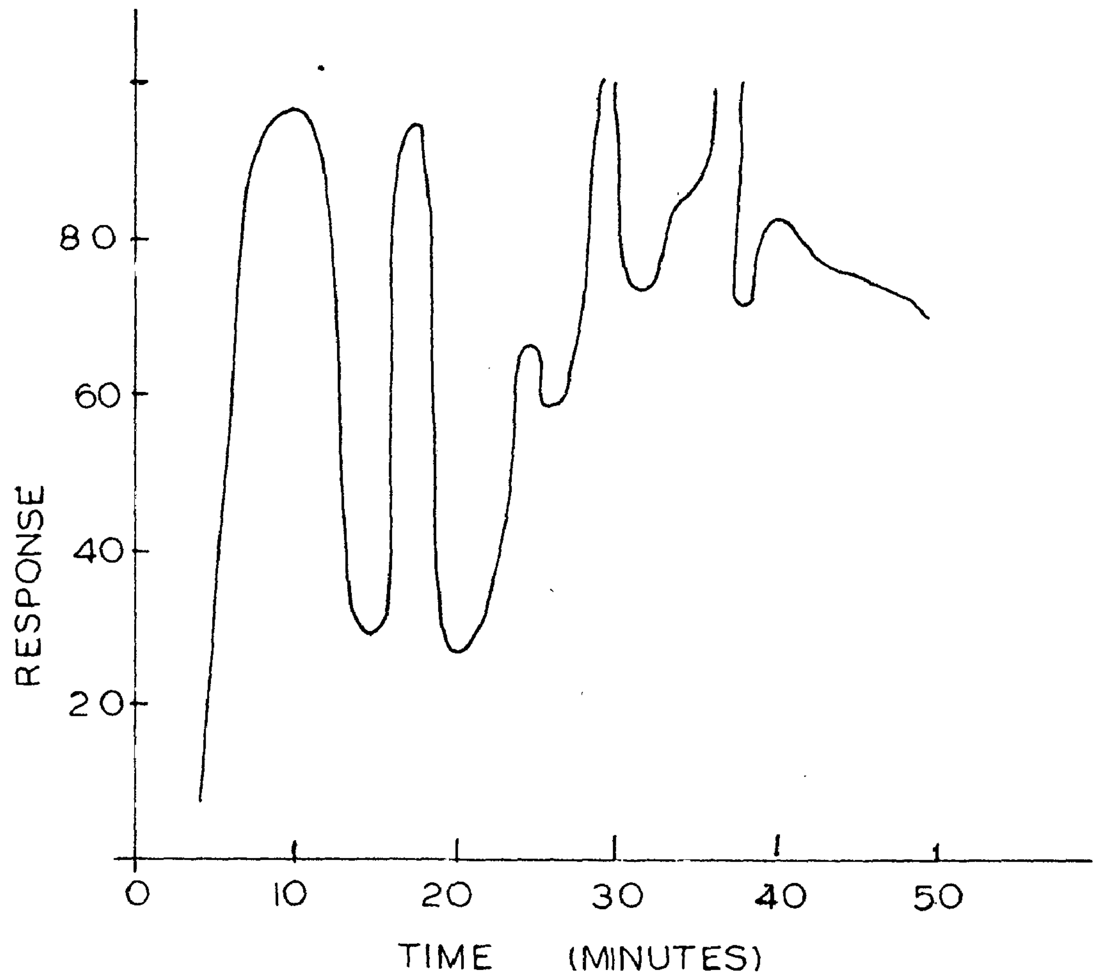


FIG. 6.3 GAS CHROMATOGRAM OF 70 PPM ISOPROPYL ALCOHAL IN BENZENE

concentration suitable for on line process control applications.

6.3 GAS ANALYSIS AND GAS CHROMATOGRAPHY

An interesting application of a thermistor bridge circuit is for trace analysis by Gas -Chromatography [28]. Here two thermistors, used to sense the data, work as a part of a Wheatstone bridge. With the carrier gas flowing and bridge at balance, the sample is injected by means of a syringe into the gas stream. The vapourized sample is passed into the column and the component of mixture is carried through the column at different rates. They arrive separately at the exit of the column and the detecting cell. The appearance of the sample vapour in detecting cell produce a change in thermistor temperature and a corresponding resistance change proportional to the concentration of solute vapour in the carrier gas. The unbalance in the Wheatstone bridge resulting from the resistance change is recorded as chromatogram. Fig. 6.3 shows a chromatograph of 70 ppm isopropyl alcohol in benzene [1,28].

6.4 RF VOLTAGE AND CURRENT MEASUREMENT

Besides non-electrical applications discussed in sections 6.1 through 6.3, a thermistor bridge is conveniently applied for measuring electrical parameters such as voltage, current and power at high frequencies. These measurements are important in the area of audio and video communications.

The measurement of voltage at HF, UHF and VHF ranges is derived from the knowledge of power and impedance. The RF power bridge measures the power dissipated in the thermistor. The thermistor resistance alongwith the dissipapated power , then, determines the voltage. A detailed description of the method is given in [27,32]. A self balancing bridge for RF voltage standard consists of a bridge circuit and a thermistor mount. The rf voltage at the output port of thermistor mount is determined from the readings of the digital voltmeter connected across the bridge using dc substitution technique. Experimental results obtained in [32] demonstrates the fact that a thermistor bridge may be used as the primary standard in UHF range with an accuracy comparable with the power meter method.

The measurement of RF current using a thermistor bridge circuit utilizes the heating effect of current. The temperature, thus, rising by an amount is proportional to the mean square value of input current. Because of large temperature coefficient of resistance, the thermistor resistance decreases considerably and, hence, the

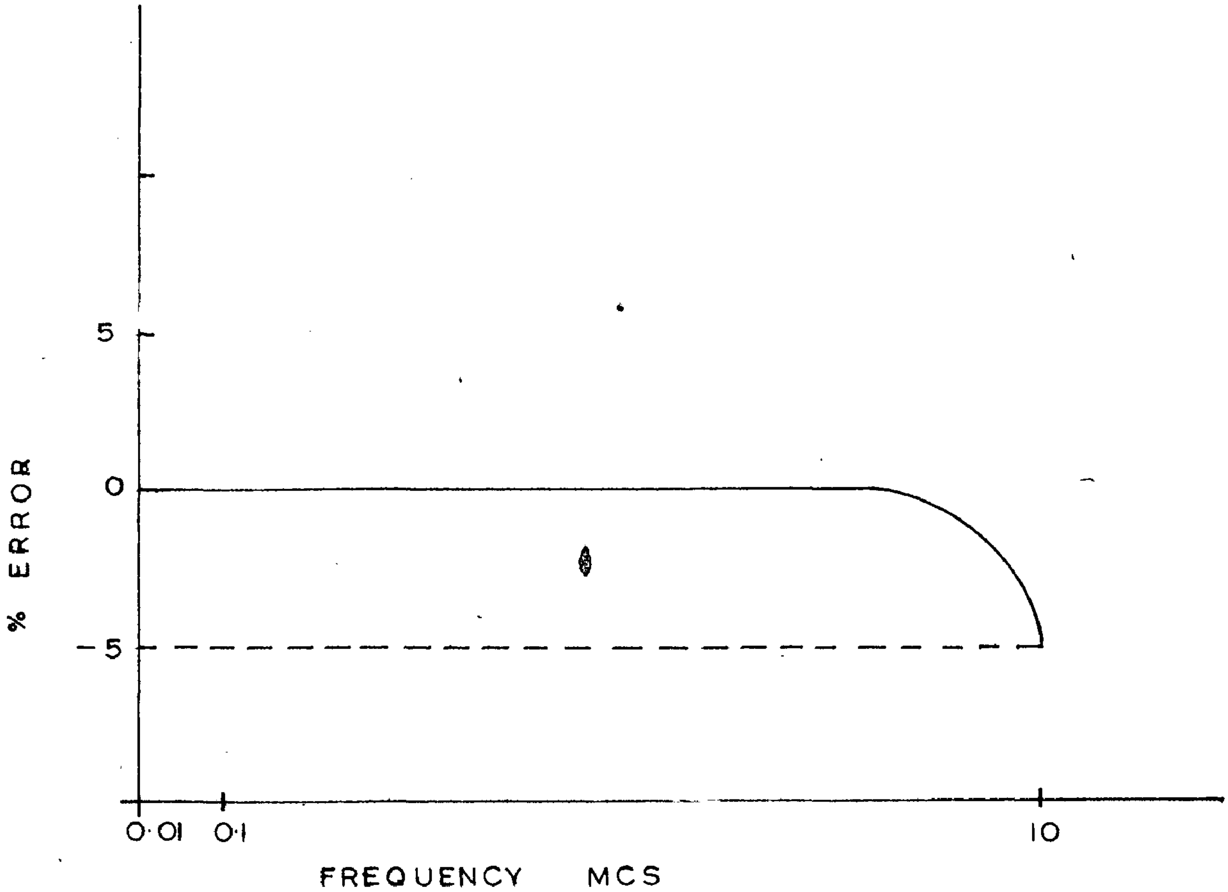


FIG. 6.4 FREQUENCY VS % ERROR

bridge becomes unbalanced causing a proportionate dc voltage to appear at the input of dc amplifier. Note that a dc amplifier is primarily used to raise the accuracy of the system. The output of the amplifier is fed to a dc ammeter which measures the proportionate response of bridge and hence the rms value of input ac current [23,26] .

The method described above indicates the rms value of current upto 10 mA [26] . In an essentially linear scale, the frequency characteristics of the instrument is shown in Fig. 6.4, from which it is obvious that the relative error is - 4 % at 10 MHz. Above that the accuracy falls on account of the inductive-effect in the input circuit [26] .

6.5 AMPLITUDE MODULATION

The simple thermistor Wheatstone bridge finds its application in the area of communication also. It is used as a modulator to obtain an amplitude-modulated signal. Here, a separately heated thermistor is utilized. The heater terminal is excited by low frequency modulating signal causing a proportionate change in thermistor resistance. The voltage corresponding to carrier frequency is applied across the bridge and the modulated - output is picked-up from the opposite terminals in the bridge. Note that the inertia of heater and heat transfer to thermistor limit the modulating frequency [25].

6.6 CONCLUSION

From the foregoing discussions it is evident that a thermistor bridge circuit has its numerous applications in the discipline of instrumentation. Efforts are in progress to have linear output response from bridge circuits and to reduce response time of thermistor to increase the sensitivity. Thermistor bridge is suitably utilized for RF power measurement/standardisation of RF voltage and a lot of work is in progress in this area too.

CHAPTER VII

SUMMARY AND CONCLUSION

The proposed problem has been studied in this dissertation for its various relevent aspects. Briefly, the features of our investigation are as follows:-

1) Generalized analysis of thermistor bridge circuits considered in Chapter 3 has the advantage of studying 4 x 5 bridge configurations simultaneously.

2) proposed design of a thermistor-WB circuit using the concepts of generalized analysis (Chapter-3).

3) development of a linearized piecewise approximation of thermistor characteristic curve and its application for determining the expression of gain for a thermistor-WB circuit (Chapter-4). Intitutively, an instrument working on above principle and having provision of manual or automatic range selection will be more linear in comparison with the existing system.

4) study of behaviour of thermistor used as a sensor in flow measuring problems (Chapter 5).

5) review of some of the interesting areas where a thermistor bridge circuit is utilized (Chapter -6).

In short, an attempt has been made to tackle the problem from various relevant aspects and to give some reasonable answer in each case. Still, there remains a large number of problems to be solved.

Particularly, they are -

1) investigating in detail the features of 4 x 5 bridge configurations discussed here and discussing their application- oriented suitability.

2) performing error analysis in Chapter 4 and verifying the notion of linearity of piecewise approximation.

3) detailed study of applications of thermistor bridge circuits.

REFERENCES

1. Herbert B. Sachse, Semiconducting temperature sensors and their applications, John Wiley and Sons, New York 1975.

2. R F Turner, ABC's of thermistors, Foulsham-Shams, 1970.

3. M B Broughton, Analysis and design of almost linear one thermistor temperature transducers, IEEE Inst. and Measurement, March 1974.

4. F. Maher, The multivibrator bridge for temperature measurement, J. Sci. Instrum, Vol.44, pp 531, July 1967.

5. D. Stankovic and M Simic, Monostable multivibrator as the bridge circuit with linear characteristic, IEEE Trans. on Instrumentation and Measurement, Feb. 1972, p.66

6. K. Holm, Thermistor thermometer based on an astable multivibrator, Electron. Engg., Dec 1968, p. 700.

7. D. Stankovic, Note on thermistor thermometer nonlinearity, J. Phys E. Sci. Inst. Vol. 6, Dec. 1973, p. 1237.

8. D K Stankovic, Linearized thermistor MV bridges for temperature measurement, IEEE Instrumentation and Measurement, June 1974, p 179.

9. D K Stankovic, conversion of fluid thermal parameters into frequency and time by means of thermistors, IEEE Instrumentation and Measurement March 1975, p 66.

10. M Ikeuchi et al., A Linear temperature to frequency converters, ibid, Sept. 1975, p 233.

11. S. Natarajan, Widely linear temperature to frequency converters, ibid, p 235.

12. D. Patranabis and PC Sen, A linear temperature to frequency converter, Institution of Engineers, E T, Nov. 1972, p 74.

13. J.P. Dujardin, A Linearizing amplifier for a thermal flow meter equipped with thermistors, Medical and Biological Engineering, May 1973, p 356.

14. J. Stockert and E R Nave, Operational amplifier circuit for linearizing temperature readings from thermistors, IEEE Biomedical Engineering, March 1974, p 164.

15. A Taroni and G. Zanarini Dynamic behaviors of thermistor Flowmeters, IEEE Industrial Electronics and Control Engg., Aug. 1975, p 391.

16. A Taroni and G. Zanarini, Sensitivity and response time of thermistor flow meters, ibid, Nov. 1975, p. 566.

17. D K Stankovic and J. Elazar Thermistor MV as the T/F converter and as a bridge for temperature measurement, IEEE Instrumentation and Measurement, March 1977, p 41.

18. S. Natarajan and B B Bhattacharya, Temperature to time converters, ibid, p 77.

19. D K Stankovic and J. Elazar, Thermistor MV bridge with the variable balancing point position, IEEE Trans. on Inst and Measurement, Dec. 1977, p 358.

20. M. Boel and B. Erickson, Corellatin study of a thermistor thermometer, Rev. Sci. Inst. Sept. 1965, p 904

21. W R Beakley, The design of thermistor thermometer with linear calibration J. Sci Instrum. June 1951, p 176.

22. F.E. Terman, and J.M. Pettit, Electronic Measurements, Mc Graw Hill, 1965.

23. K.K. Aggarwal and Suresh Rai, Waveshaping and digital circuit, Khanna Publishers, New Delhi 1978.

24. J.Schwarzenback and F F Gill, System Modelling and Control, Mc Graw Hill, 1976.

25. B P Lathi, An Introduction to Communication System, John Wiley and Sons. 1965.

26. H B Wood, An r.m.s. milliammeter of Novel design for the measurement of current from Zero to Video frequencies, Journal Scientific Instrument, Vol. 31. 1954 p 125.

27. G U Sorger et al, An R F Voltage standard for Receiver Calibration IRE Transaction on Instrumentation, Vol. I-10(1), 1961, p 9.

28. C E Bennet et al, Thermistor bridge circuit for trace analysis by gas chromatography, Analytical Chemistry Vol. 30(5), 1958 p 898.

29. H H H Green, Single Crystal Silicon Carbide thermistors for low pressure measurement, Journal Scientific Instrument. Vol. 42, 1965, p. 342.

30. PT Priestley, A Multiple thermistor indicator for thermometric Analysis Journal of Scientific Instrument Vol. 42, 1965.

31. C J Berry, Low pressure measurement with thermistors Journal of Scientific Instrument Vol.44, 1967,p 83.

32. F. Uchiyama K. Yamamura, and I. Yokoshina ,Precision R F Voltage standard using thermistor bridge circuit covering HF-UHF range, IEEE Transactions on Instrumentation and Measurements , Vol. IM-27, 1978, p 385.

Appendix- 1

Derivation of Differential Equations

Consider the generalized gain expression

$$G = \frac{a_o + a_1\,Re(T)}{b_o + b_1\,Re(T)} \qquad (A-1.1)$$

where,
$$Re(T) = \frac{A_1 R_T + A_2}{A_3 R_T + A_4} \qquad (A-1.2)$$

FIRST-ORDER DERIVATIVE -

$$\frac{dG}{dT} = \frac{[b_o+b_1 Re(T)]\; a_1 \dfrac{dRe(T)}{dT} - [a_o+a_1 Re(T)]\; b_1 \dfrac{dRe(T)}{dT}}{[\,b_o + b_1\,Re(T)]^2}$$

$$= \frac{a_1[b_o+b_1 Re(T)] - b_1[a_o+a_1 Re(T)]}{[b_o+b_1 Re(T)]^2} \left[\frac{d\,Re(T)}{dT}\right]$$

$$= \frac{a_1 b_o - b_1 a_o}{[b_o+b_1 Re(T)]^2} \cdot \left[\frac{dRe(T)}{dT}\right] \qquad (A-1.3)$$

Using (A-1.2), we have

$$\frac{dRe(T)}{dT} = \frac{(A_3R_T+A_4)A_1 \dfrac{dR_T}{dT} - (A_1R_T+A_2)A_3 \dfrac{dR_T}{dT}}{(A_3R_T+A_4)^2}$$

$$= \frac{A_1(A_4+A_3R_T) - A_3(A_2+A_1R_T)}{(A_3R_T+A_4)^2} \quad \frac{dR_T}{dT}$$

With

$$\frac{dR_T}{dT} = - \frac{b}{T^2} R_o \, e^{-b\left(\frac{1}{T} - \frac{1}{T_o}\right)}$$

$$= - \frac{bR_T}{T^2} \; ; \text{ since } R_T = R_o \, e^{-b\left(\frac{1}{T} - \frac{1}{T_o}\right)}$$

Therefore,

$$\frac{dRe(T)}{dT} = - \frac{bR_T}{T^2} \frac{A_1A_4 - A_2A_3}{(A_3R_T + A_4)^2} \qquad \text{(A-1.4)}$$

Substituting (A-1.4) and (A-1.2) in (A-1.3);the first order derivative is

$$\frac{dG}{dT} = \frac{(a_1b_o - b_1a_o)(A_1A_4 - A_2A_3)(-b R_T)}{T^2(b_o + b_1 \, Re(T))^2 \, (A_3R_T + A_4)^2}$$

$$= - \frac{bR_T}{T^2} \frac{(a_1b_o - b_1a_o)(A_1A_4 - A_2A_3)(A_3R_T + A_4)^2}{[b_o(A_3R_T+A_4) + b_1(A_1R_T+A_2)]^2 \, (A_3R_T+A_4)^2}$$

$$= - \frac{bR_T}{T^2} \frac{(a_1b_o - b_1a_o)(A_1A_4 - A_2A_3)}{[R_T(b_1A_1 + b_oA_3) + (b_1A_2 + b_oA_4)]^2} \qquad \text{(A-1.5)}$$

SECOND AND HIGHER-ORDER DERIVATIVES

For finding out expressions for $\frac{d^nG}{dT^n}$; $n \geqslant 2$, consider (A-1.5) as

$$\frac{dG}{dT} = \frac{bR_T}{T^2} \quad \frac{(a_1 b_o - b_1 a_o)(A_2 A_3 - A_1 A_4)}{D^2} \qquad \text{(A-1.6)}$$

where , $D = R_T (b_1 A_1 + b_o A_3) + (b_1 A_2 + b_o A_4)$.

$$\frac{d^2G}{dT^2} = \frac{b(a_1 b_o - b_1 a_o)(A_2 A_3 - A_1 A_4)}{T^4 D^4} \left[\frac{dR_T}{dT}(T^2 D^2) - R_T D^2 \frac{d}{dT}(T^2) - R_T T^2 \frac{d}{dT}(D^2) \right] \qquad \text{(A-1.7)}$$

Note that -

$$\frac{dR_T}{dT} = - \frac{b\ R_T}{T^2}$$

$$\frac{d}{dT}(T^2) = 2\ T$$

and,

$$\frac{d}{dT}(D^2) = 2D \frac{dD}{dT}$$

$$= 2D(b_1 A_1 + b_o A_3) \frac{dR_T}{dT}$$

$$= \frac{- 2b\ R_T D(b_1 A_1 + b_o A_3)}{T^2}$$

Substituting these values in (A-1.7) , we have

$$\frac{dG^2}{dT^2} = \frac{b(a_1b_o - b_1a_o)\ (A_2A_3 - A_1A_4)}{T^4D^4}[-\ b\ R_T\ D^2 - 2\ T\ R_T\ D^2$$
$$+2bR_T^2\ D(b_1A_1 + b_oA_3)\ ]$$

$$= \frac{R_Tb\ (a_1b_o - b_1a_o)(A_2A_3 - A_1A_4)}{T^4D^3}[-bD-2TD$$
$$+\ 2bR_T(b_1A_1 + b_oA_3)\ ]$$

(A-1.8a)

Comparing it with equation (A-1.6), it follows that

$$\frac{dG^2}{dT^2} = \frac{[-bD\ -2TD\ +2bR_T\ (b_1A_1 + b_oA_3)\ ]}{DT^2}\ \frac{dG}{dT}$$

(A-1.8b)

For linearity, the desired condition is

$$\frac{d^2G}{dT^2}\Big|_{T=T_o} = 0$$

Therefore,

$$[-bD - 2TD + 2bR_T\ (b_1A_1 + b_oA_3)]\ \Big|_{T=T_o} = 0$$

$$R_T\ \Big|_{T = T_o} = R_o$$

and,

$$D \Big|_{T=T_o} = R_o \; (b_1A_1+b_oA_3) + (b_1A_2+ b_oA_4)$$

$$= D_o \; \text{(say)}$$

which simplifies into

$$- bD_o - 2T_oD_o + 2bR_o(b_1A_1+b_oA_3) = 0$$

i.e.,

$$\frac{2b}{b + 2\,T_o} = \frac{D_o}{R_o(b_1A_1+ b_oA_3)}$$

$$= \frac{R_o(b_1A_1+ b_oA_3) + b_1A_2 + b_oA_4}{R_o(b_1A_1 + b_o\; A_3)}$$

Subtracting denominator from the numerator , we have,

$$\frac{b - 2T_o}{b + 2T_o} = \frac{b_1A_2 + b_oA_4}{R_o(b_1A_1 + b_oA_3)}$$

Let $(b- 2T_o)/ (b+2T_o) = M_o$; thus

$$M_oR_o(b_1A_1+ b_oA_3) = b_1 A_2+ b_oA_4$$

which simplifies to

$$b_o/ b_1 = \frac{A_2 - R_oM_oA_1}{R_oM_oA_3 - A_4} \qquad \text{(A-1.9)}$$

Equation (A-1.9) defines the condition of linearity in terms of thermistor parameter b and hence M_o.

For finding out the third-order derivative, we start from (A-1.8a) -

$$\frac{d^3G}{dT^3} = \frac{2\,b(a_1b_o - b_1a_o)\,(A_2A_3 - A_1A_4)}{T^8\,D^6}$$

$$\Bigg[T^4D^3\,\Big[-bD\frac{dR_T}{dT} - bT\frac{d}{dT}(D) - 2D - 2T\frac{d}{dT}(D) + 2b(b_1A_1 + b_oA_3)\frac{d(R_T^2)}{dT}\Big] - \Big[bR_TD - 2TR_TD + 2bR_T^2(b_1A_1 + b_oA_3 \Big[D^3\frac{d}{dT}(T^4) + T^4\frac{d(D^3)}{dT}\Big] \Bigg]$$

with

$$\frac{dR_T}{dT} = -\frac{b}{T^2}R_T$$

$$\frac{d(D)}{dT} = (b_1A_1 + b_oA_3)\,dR_T/dT$$

$$\frac{d(R_T^2)}{dT} = 2\,R_T\frac{dR_T}{dT}$$

$$\frac{d(T^4)}{dT} = 4\,T^3$$

$$\frac{d(D^3)}{dT} = 3D^2\frac{dD}{dT}$$

Therefore,

$$\frac{d^3G}{dT^3} = \frac{N}{T^8\ D^6}\left[T^4D^3\ [\frac{b^2R_TD}{T^2} + \frac{b^2R_T^2}{T^2}(b_1A_1 + b_oA_3) - 2\ R_TD \right.$$

$$+\frac{2bR_TD}{T} + \frac{2bR_T^2}{T}(b_1A_1 + b_oA_3) - \frac{4b^2R_T^2}{T^2}(b_1A_1 + b_oA_3)]$$

$$-[b\ R_TD - 2TR_TD + 2bR_T^2(b_1A_1 + b_oA_3)]$$

$$\left. [4T^3D^3 + 3\ T^4D^2R_T(-b/T^2)(b_1A_1 + b_oA_3)] \right]$$

(A-1.10)

where $N = b(a_1b_o - b_1a_o)\ (A_2A_3 - A_1A_4)$

Note that equation (A-1.10), in turn, can be used for finding out higher-order derivatives.

Appendix - 2

EXPRESSIONS FOR GAIN IN CT-1 THROUGH CT-4

[A] For bridge circuits CT-1 through CT-3, the gain G, is defined as

$$G \equiv e_o/e_i \qquad (A\text{-}2.1)$$

Equation (A-2.1) is, in turn, used for finding out the expressions for gain in thermistor WB circuits. Thermistor WB circuit (CT-1) :-

$$e_o = \frac{e_i}{R_3 + R_4} R_3 - \frac{e_i}{R_1 + R_e(T)} R_1$$

$$= \frac{e_i}{(R_3+R_4)(R_1+R_e(T))} [R_3(R_1+R_e(T)) - R_1(R_3+R_4)]$$

$$= \frac{e_i}{(R_3+R_4)(R_1+R_e(T))} [R_3R_e(T) - R_1R_4]$$

Therefore, using (A-2.1)

$$G = e_o/e_i$$

$$= \frac{R_3R_e(T) - R_1R_4}{(R_3+R_4)(R_1+R_e(T))} \qquad (A\text{-}2.2)$$

Active WB circuit-I (CT-2) :

Consider the equivalent circuit of CT-2 as shown in Fig. A-2.1. Note that

$$v_i = - v_1 + v_2 \qquad (A\text{-}2.3)$$

with

$$v_2 = \frac{e_i}{R_1 + R_e(T)} R_1 ; \text{ and}$$

$$\frac{e_i - v_1}{R_4} = \frac{v_1 - e_o}{R_3}$$ Using KCL equation. It modifies into

$$v_1 = \frac{R_3 R_4}{R_3 + R_4} \left[\frac{e_i}{R_4} + \frac{e_o}{R_3} \right]$$

Therefore,

$$e_o = A v_i$$

Hence

$$\frac{e_o}{A} = v_i \approx 0 \ ; \text{ since } A \to \infty , \text{ for an ideal OPAMP.}$$

Thus, $v_1 = v_2$ to satisfy $v_i = 0$. It signifies that-

$$\frac{e_i R_1}{R_1 + R_e(T)} = \frac{R_3 R_4}{R_3 + R_4} \left(\frac{e_i}{R_4} + \frac{e_o}{R_3} \right)$$

$$= \frac{e_i R_3}{R_3 + R_4} + \frac{e_o R_4}{R_3 + R_4}$$

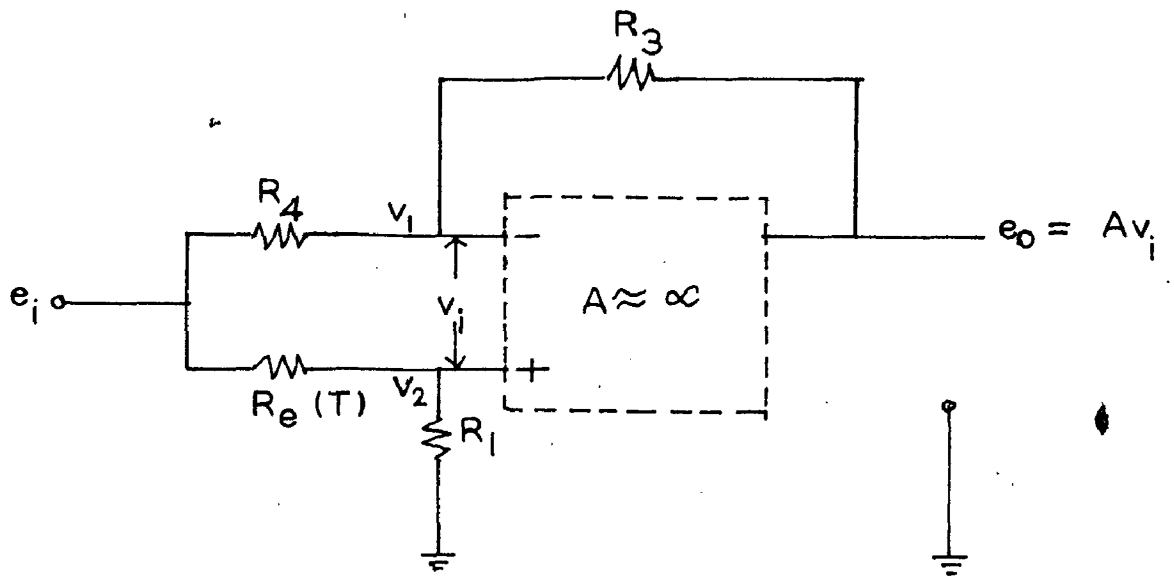


FIG. A-2.1 EQUIVALENT CIRCUIT OF CT-2

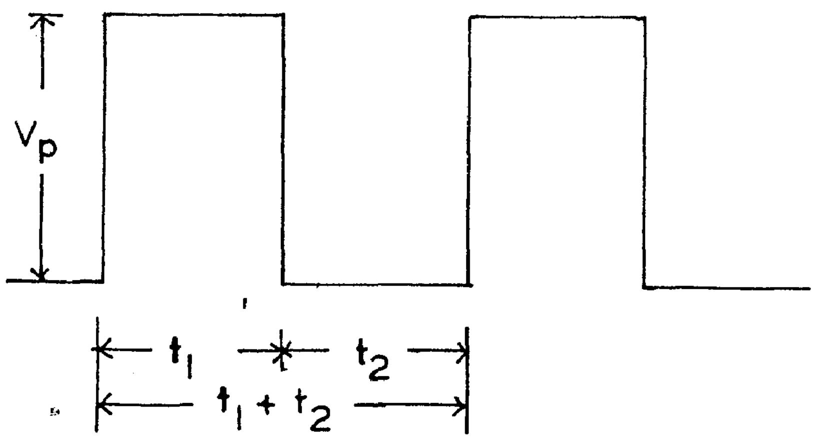


FIG. A-2.2 AMB (CT-4) OUTPUT

which simplifies as

$$G = e_o/e_i$$

$$= \frac{R_1R_4 - R_3\, R_e\ (T)}{R_4\ \ (R_1 + R_e\ (T))} \qquad (A-2.4)$$

Active WB circuit-II (CT-3) :

Using the analysis of CT-2 it is obvious that $v_1 = v_2$. Here

$$\frac{e_i - v_1}{R_1 + R_e(T)} = \frac{v_1 - e_o}{R_2}$$; using KCL equation, and (A-2.5a)

$$v_2 = \frac{e_i}{R_3 + R_4} R_4 \qquad (A-2.5b)$$

Equation (A-2.5a) gives the value for v_1 as

$$v_1 = \left(\frac{e_i}{R_1+R_e(T)} + \frac{e_o}{R_2}\right) \frac{R_2(R_1+R_e(T))}{R_1+R_2+R_e(T)}$$

$$= \frac{e_i\ \ R_2}{R_1+R_2+R_e(T)} + \frac{e\ (R_1+R_e(T))}{R_1+R_2+R_e(T)} \qquad (A-2.5c)$$

Using (A-2.5b) and (A-2.5c), we have

$$\frac{e_i\ R_4}{R_3 + R_4} - \frac{e_i\ R_2}{R_1+R_2+R_e(T)} = \frac{e_o(R_1 + R_e(T))}{R_1+R_2+R_e(T)}$$

Therefore

$$\frac{e_o(R_1+R_e(T))}{R_1+R_2+R_e(T)} = e_i\left[\frac{R_1R_4 + R_2R_4 + R_4R_e(T) - R_2R_3 - R_2R_4}{(R_3+R_4)(R_1+R_2+R_e(T))}\right]$$

Hence, gain 'G' is (see, A - 2.1) -

$$G = e_o/e_i$$

$$= \frac{R_4(R_1+R_e(T)) - R_2R_3}{(R_3+R_4)(R_1+R_e(T))} \qquad \text{(A-2.6)}$$

[B] We know that, because of variation in temperature T, thermistor resistance and hence thetime period of AMB circuit changes. Initially, at $T = T_o$, the multivibrator is properly balanced, and

$V_p\, t_1 = V_p\, t_2$, where, V_p is the collector pulse amplitude. (A-2.7)

For $T \gtrless T_o$, the AMB is unbalanced and the output swing V_T indicated in the voltmeter connected across the collectors of two transistors is given by (refer Equation(2.5)

$$V_T = \frac{V_p\, t_2 - V_p\, t_1}{t_1 + t_2} \qquad \text{(A-2.8)}$$

with $t_1 = C\, R_e(T)\, \ln 2$, and

and $t_2 = C\, R_1\, \ln 2$

Equation (A-2.8) is simplified by substituting t_1 and t_2 as-

$$\frac{V_T}{V_p} = \frac{R_1 - R_e(T)}{R_1 + R_e(T)} \qquad \text{(A-2.9)}$$

Let G of AMB circuit be defined as

$$G = V_T / V_p$$

Then, using (A-2.9) , we have

$$G = \frac{R_1 - R_e\ (T)}{R_1 + R_e\ (T)} \qquad \text{(A-2.10)}$$

Equations (A-2.2), (A-2.4), (A-2.6) and (A-2.10) are summarized in Table 3.1.

Appendix - 3

Consider the expression for gain, G, for CT-2 from Table 3.1 as

$$G = \frac{R_1 R_4 - R_3 \, Re(T)}{R_4 \, (R_1 + Re(T))}$$

which simplifies into

$$G = \frac{R_1 (R_4 / R_3) - R_T}{(R_4/R_3) \; (R_1 + R_T)} \qquad \text{(A-3.1)}$$

since $Re(T) = R_T$ for probe P-1. To modify G, substitute the linearity and balancing conditions in (A-3.1). These are-

$R_1 = M_o R_o$; Linearity condition (Table 3.2)

$R_4/R_3 = Re(T_o)/R_1$

$= R_o/R_1$; Balancing condition (Equation (3.14))

Now, equation (A-3.1) is modified as -

$$G = \frac{R_1(R_o/R_1) - R_T}{(R_o/R_1) \; (R_1+R_T)}$$

$$= \frac{R_o - R_T}{R_o \; (1 + R_T/R_1)}$$

$$= \frac{1 - (R_T/R_o)}{1 + (R_T/M_o R_o)}$$

$$= \frac{M_o(1-R_T/R_o)}{M_o + R_T/R_o} \qquad \text{(A-3.2)}$$

Equation (A-3.2) has been used for finding out various values of gain G with different values of M_o. The values of G are given in Table 3.5 and plotted as Fig. 3.3.

The expression for gain of AMB circuit CT-4 can similarly be modified by substituting $R_1 = M_o R_o$. Thus;

$$G = \frac{R_1 - Re(T)}{R_1 + Re(T)} \quad ; \quad \text{see Table 3.1}$$

$$= \frac{M_o R_o - R_T}{M_o R_o + R_T} \quad ; \quad \text{since } Re(T) = R_T \text{ for probe P-1}$$

$$= \frac{M_o - (R_T/R_o)}{M_o + (R_T/R_o)} \qquad \text{(A-3.3)}$$

Equation (A-3.3) has been used to find out the response of AMB circuit CT-4 with P-1 as shown in Fig. 3.4

Appendix - 4

Values of R_T/R_o using $R_T/R_o = e^{b/T_o\,(T_o/T\;-1)}$
for various values of 'b' ($T_o = 298^oK$)

$M_o = \frac{b-2T_o}{b+2T_o}$	0.6	0.67	0.71	0.75	0.78
b/T_o / T/T_o	8	10	12	14	16
0.80	7.39	12.18	20.09	33.12	54.60
0.85	4.10	6.05	8.67	12.43	17.81
0.90	2.43	3.04	3.74	4.66	5.81
0.95	1.52	1.65	1.82	2.01	2.23
1.00	1.00	1.00	1.00	1.00	1.00
1.05	0.68	0.61	0.55	0.50	0.45
1.10	0.48	0.41	0.34	0.28	0.24
1.15	0.35	0.27	0.21	0.16	0.12
1.20	0.26	0.18	0.13	0.09	0.07

Appendix - 5

```
C     COMPUTATIØN ØF  NØRMALIZED THERMISTOR RESISTANCE AND
      GAIN ØF  WHEATSTØNE BRIDGE
      DIMENSIØN  X(3),Y(3),GN(3),GD(3)
C     XIND REPRESENTS NØRMALIZED TEMP. INTERVAL IN DEGREES
      READ 100, XIND
      PRINT 200, XIND
      XINT=XIND/298
      X(1)=0.8658
      X(2)=1.
      X(3)=1.1342
      BETA=8.
      DØ8 II=1,6
      PRINT 300,BETA
      Y(1)=EXP(0.155*BETA)
      Y(2)=1.
      Y(3)=EXP(-1.*0.1183*BETA)
      DØ 1 I=1,3
      X2=2.*X(I)
      GN(I)=(BETA-X2)/2./BETA
      GD(I)=Y(I)*(BETA-X2)/(BETA+X2)
      CØNTINUE
      PRINT 400
      PRINT 500
      AX=0.7986
      AXF=0.9137
      AXFF=AXF+XINT
      AY=Y(1)*(1.-BETA*(AX-X(1))/0.7496)
      GAIN=GN(1)*(AY-Y(1))/(AY+GD(1))
      PRINT 600,AX,AY,GAIN
      AX=AX+XINT
      IF(AX.GT.AXFF)GØ TØ 3
      GØ TØ 2
      PRINT 700
      PRINT 500
      AX=0.9329
      AXF=1.0525
      AXFF=AXF+XINT
```

```
4      AY=1.+(1.-AX)*BETA
       GAIN=GN(2)*(AY-Y(2))/(AY+GD(2))
       PRINT 600,AX,AY,GAIN
       AX=AX+XINT
       IF(AX.GT.AXFF)GΦTΦ 5
       GΦ TΦ 4
5      PRINT 800
       PRINT 500
       AX=1.0671
       AXF=1.2013
       AXFF=AXF+XINT
6      AY=Y(3)*(1.-BETA*(AX-X(3))/1.2864)
       GAIN=GN(3)*(AY-Y(3))/(AY+GD(3))
       PRINT 600, AX, AY, GAIN
       AX=AX+XINT
       IF(AX.GT.AXFF)GΦ TΦ 7
       GΦ  TΦ  6
7      BETA=BETA+2.
8      CΦNTINUE
100    FΦRMAT(F3.1)
200    FΦRMAT(/' INTERVAL ΦF X VALUES IN DEGREES=',F8.1)
300    FΦRMAT(/' 2b BETA=',F8.1)
400    FORMAT('1b. RESULTS FΦR REGION 1'////)
500    FΦRMAT('9b,AX 14b AY 12b GAIN'///)
600    FΦRMAT(1X,3E 16.7)
700    FΦRMAT ('1b RESULTS FΦR REGIΦN 2'////)
800    FΦRMAT ('1b RESULTS FΦR REGIΦN 3'////)
       STΦP
       END
```

APPENDIX - 6

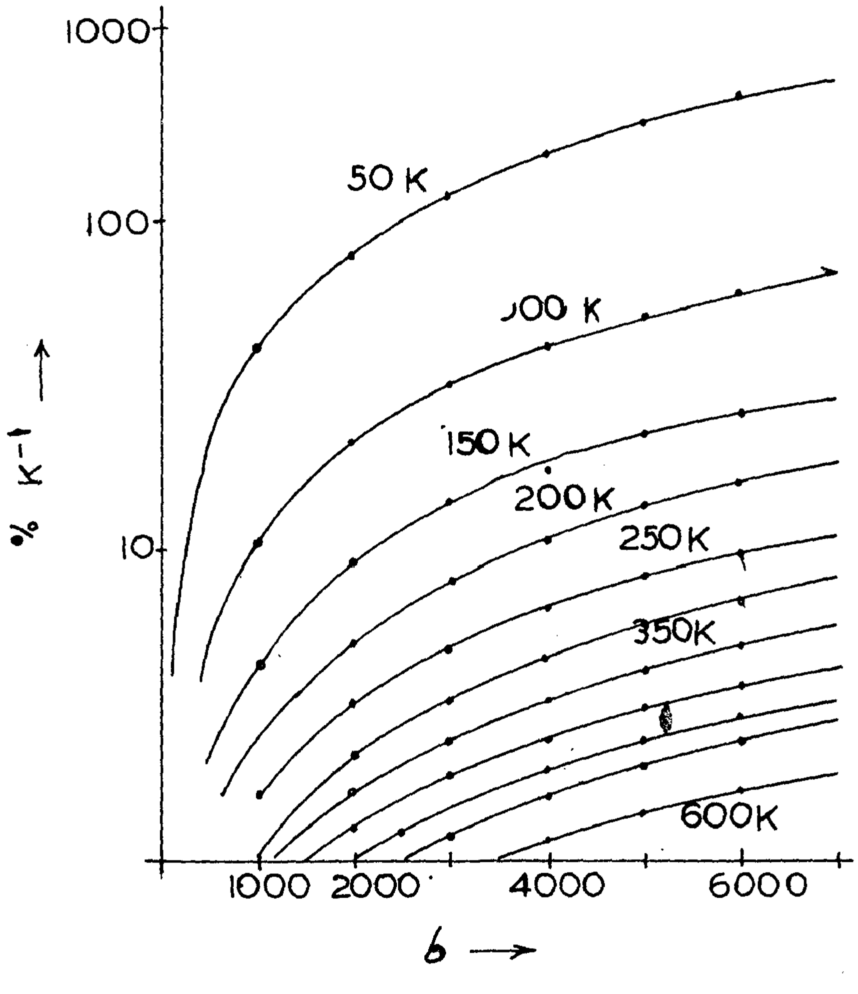


FIG. A-1 TEMPERATURE COEFFICIENT IN $\% K^{-1}$ FOR TEMPERATURES BETWEEN 50 AND 600 K AS FUNCTION OF b

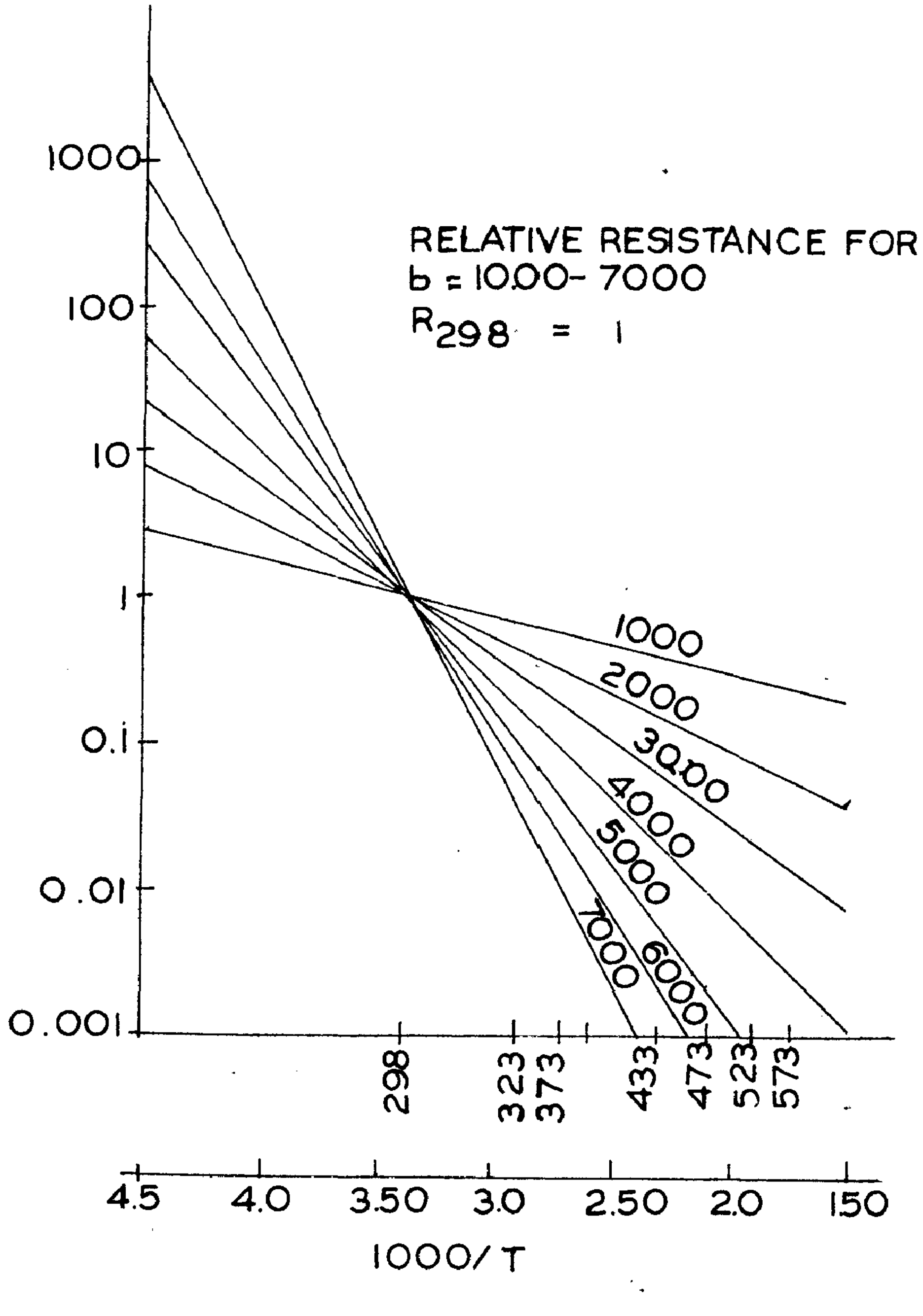


FIG. A-2 RELATIVE RESISTANCE VALUE R(T)/R298 FOR MATERIAL CONSTANTS FROM 1000 - 7000 K

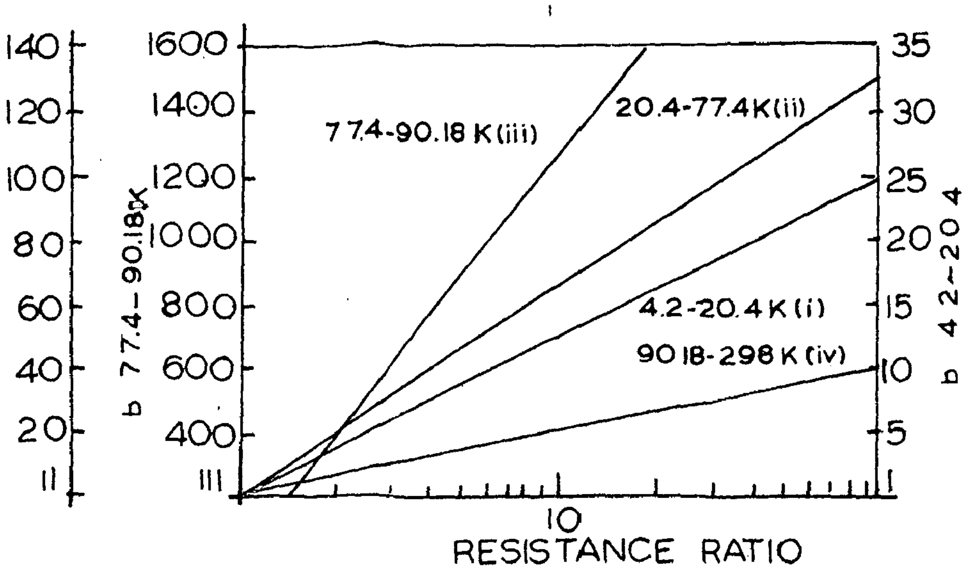


FIG. A-3 MATERIAL CONSTANT b AS FUNCTION OF THE RATIO FOR TEMP. INTERVALS i 4.2-20.4K, ii 20.4-77.4K, iii 77.4-90.18K, iv 90.18-273.15 K

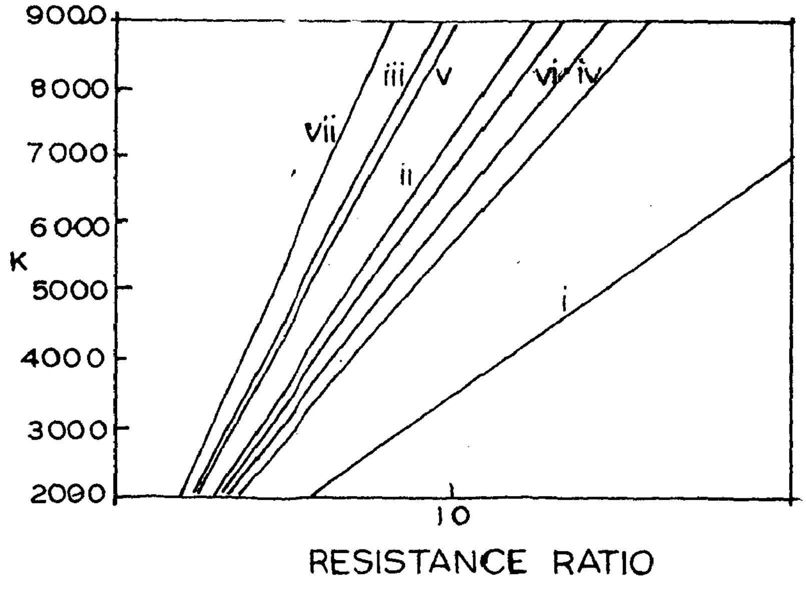


FIG. A-4 MATERIAL CONSTANT b AS A FUNCTION OF THE TEMPERATURE INTERVALS i 298-333K, ii 373-423K iii 423-473K iv 473-573K v 573-673K vi 673-873K vii 873-1073K