GENERATION PLANNING UNDER UNCERTAINTIES

A DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the Degree

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(System Engineering and Operation Research)

By

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CERTIFICATE

Certified that the dissertation entitled "Generation planning under uncertainties" which is being submitted by Mr. S.P. SINGH in partial fulfilment of the requirements for the award of the degree of Master of Engineering in Electrical Engineering (System Engg. and operation Research) of the University of Roorkee, Roorkee (U.P.) is the record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

It is further certified that he has worked for a period of 10 months from January 1980 to October 1980 for preparing this dissertation at this University.

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ABSTRACT

A number of mathematical programming methods have been applied for the generation expansion planning problems by various authors (as described the review work). The cost of operation of energy produced depends upon the following factors. A part of the cost is directly proportional to the power injected at the generating Buses. A part is proportional to square of the power at these buses and there is fixed cost. Considering the above the generation planning problem shall be formulated as quadratic programming problem. The constraints are that the sum of the generated powers at these Buses is greater than or equal to the total demand at the load buses. At each Bus the power that can be injected is limited by maximum value.

The Beales algorthim is applied to the generation planning problem for which the results are available by other method. In this method the quadratic cost function is represented by an upper Triangular matrix. This results in saving of Computer space, compare to other type of programming. Therefore the memory is comparatively

not more as the constraint matrix is represented in the same way except that Zeroth row of A (Constraint) matrix is not used. In this method these are no artificial constants used for optimization purpose. A feasible basic solution has to be chosen in this problem for choosing initial values of injected power are taken equal to the original injected power (for the previous stage) plus additional demand distributed equally among generating buses. It is found that for the five Bus system to which this algorithm is applied the optimal solution is obtained in one iteration only.

The uncertainity of generation is taken into account in the following way. The loss of load probability at each generating bus is calculated using the recurssive convolution integral equation. The injected power at each bus is consider as equivalent load. The probability of this load exceeding the installed capacity gives the LOLP.

CONTENTS

INTRODUCTION

1.1 Power System Planning:

Power system planning may be defined as the optimal expansion schedule of facilities in an electric utility system to meet future requirements. The planner has to decide to what capacity the facilities to be expanded, where to be located and when the expansion to be completed. The study generally includes the expansion of existing facilities and in stallation of new facilities such as installation of new power plants, Transmission lines, Sub-Transmission lines, Substation and feeders etc.

The estimation of future expansion is very complex. In planning studies one has to consider the electrical and economic relationship in the system and adjacent area, character of growth demand, the non-linearity of relationship between characteristics, rating and operating conditions of the plants and network and probabilistic nature of data.

The power system expansion and installation involves huge investments. Therefore the alternative expansion policies available for meeting system requirements should be selected very carefully. Additionally, the expansion of facilities should be within the budgetary limitations. The selected expansion policy should provide cheap and reliable supply of power to consumers. The constraints that may be imposed on planning are following:

i) Security of system: Security of system means that no overloading of circuits is permitted under steady state operating conditions.

ii) Capacity Constraints: These are due to the physical limits on the maximum capacity available for expansion at the power plant sites or the number and type of circuits that can be added in the right of ways in the system.

iii) Power output constraints: The maximum power output of any plant can not be exceeded the installed capacity.

iv) Choice of variables: The variables in the planning problem represent the expansion of generation facilities, Transmission facilities, distribution facilities and the power flow through the circuits. These variables may be discrete or continuous. For example, the capacity expansion of power plants may be selected from descrete unit sizes or it can be continuously varied. The type of cost function which represents the generation expansion capital costs may require the use of mixed variables for its formulation. The representation of Transmission network expansion in the problem formulation requires generally descrete variables. The variables associated with the output of gnerators are represented by continuous variables. These variables are non-negative since in the formulation, it is assumed that no retirement of existing facilities occurs during the planning horizon. The power system planning may be devided into the following:

1. Generation planning
2. Transmission planning
3. Distribution planning

1.1.1 Generation Planning

The general object of power system planning problem is to ensure the provision of reliable supply of power to consumer at the lowest possible cost. As the part of this generating plant expansion must be determined which will ensure that given standard of reliability can be met, and which promises the lowest cost of solution available. The planning engineer has to consider a number of uncertainities while designing the system. The uncertain factor have a great influence on the planning decisions. These uncertainities arise from different sources, among them are the random nature of forecast, money values and the availability of capital. The aim of planning engineer is to design a system that provides a reliable power of supply at the lowest possible cost to consumer.

Under uncertain conditions, it is customary to provide adequate reserve capacity i.e. generation level in excess of the expected demand requirements to meet the fluctuation in demands. In a practical Generation System Planning the number of variables is very large. Further more the capacity can be added in descrete increments only. The planner is confronted with multitude of technical and economical constraints (such as system reliability requirements, Geographical and budgetary limitation). The equation relating capital and operating costs to the system confeguration are usually non-linear. Therefore the planning problem is very complex even without considering the difficulty posed by uncertainities about future events. The planner needs to have access to an appropriate computational technique. This would give him a unified approach to many different system expansion problems and would enable him to arrive at optimal long-term plan directly without usual trial and error method. In other words long-term system planning problem should be formulated as a problem of mathematical optimization which could be programmed and executed on a digital computer.

The selection of a suitable computational technique for an outlined problem poses severe difficulties. The presence of uncertainities, descrete nature of problem and large number of constraints seem to rule out all known optimization methods such as gradient procedures, linear programming and branch and bound algorithms. But the dynamic programming is the exception. But for the large number variables would make the dynamic programming approach computationally infeasible since more than few variables, both the computer storage requirements and computation time become excessive.

The objective of the generation planning in power system, is to what generating is to built, where the additional capacity is to be located and when it is to be completed in power system for a given future demand forecast considering the uncertainities in the demand and unit outage.

1.1.2 Transmission Planning

Historically, transmission planning has followed gneration system planning because the construction time of Transmission facilities is much shorter than for generation facilities and transmission planning depends upon knowledge of the location and capacity of both generation and demand centres. The main aim of ~~generation~~ Transmission planning is to develope a Transmission network in an electric utility system capable of meeting future generation and load conditions and no branch should be overloaded while Transmitting the power from generation centre to demand centre under emergency or normal modes. The system emergency mode is due to outage of generator unit and or Transmission Lines. The Transmission expansion planning therefore seek a minimum cost plan where in new lines or increase of existing capacities are decided so that no overloaded lines should exist under steady state conditions for future demands.

As the system is in continuous operation, the energy loss in network is continuous phenomenon and therefore system having more losses would prove to be more costly in long run. There are some model consider the effect of system losses. These model design the system efficiently and consider not only the investment costs and the fuel and the costs of the power plants, but also the investment costs incurred due to energy losses in the network. Therefore large losses due to continuous operation in system may be avoided by above model.

The violation of voltage and reactive power generation specification may occur under heavily loaded or emergency condition of operation. The model considering the active flows in formulation, may violate other system specifications such as magnitude of Bus voltage and reactive power generation. To overcome these problems there are some models, determine the Bus voltage, Tap setting of Transformer and Static Capacitor allocation in the power system for maintaining the system voltage and reactive power output of Generator within the specified limits under normal or emergency mode of power system.

1.1.3 Distribution Planning

The objective of Distribution planning[14] is to develope a distribution system which will provide economic, reliable and safe electric energy to end users. This objective is usually approached by generating alternatives for expansion of distribution and evaluating these alternative for economy, reliability and safety.

The planner may state his problem as attempt to minimize the cost of Sub-Transmission, Substation, feeders etc and the cost of losses. Planner is usually constrained by permissible value of voltage, flicker, short circuit duty and continuity of service. To meet these objectives the distribution planner may have to consider additions to the Sub-Transmission networks, location and size of network, service area of substation, location of breakers and switches, size of feeders and laterals, location of capacitors and voltage regulators and the loading of Transformer and feeders. There are certain factors over which distribution planner has no control. These include of course, when the customers, demand energy, frequency and duration of outage, the cost of equipment, labour and money and regulation imposed by State or Central Government.

LITERATURE REVIEW

Baldwin[1] and other proposed the simulation technique for additional generator installation date. The authors have suggested that the utility management not only must provide adequate service to its consumer but also must avoid overinvestment in spare equipment. To get these objectives a level of service of reliability is specified i.e. to state numerically the acceptable risk of shortage from inadequate installed generation capacity. Then overinvestment has been avoided by installing no more than enough equipment to maintain this level.

The technique presented by authors is the new ways to evaluate reserve adequacy and subsequently to arrive at installation date for new capacity. The techniques are based on the use of simulated daily reserve margin available from operational games studies.

The author has concluded that the average percent margin alone should not be used a measure of service of reliability without studying despersion. The dispersion of margin, as caused by capacity and Load fluctuations, is an important factor in evaluating real risk. The second conclusion was that the required installation date for a new unit is really a random quantity with certain statistical properties. If the new unit is scheduled based on average expected risk, actual circumstances as they develope may or may not be required the unit before the date schedule.

Fitzpatric[2] has developed a series of programs for a small scale computer such as the IBM 650, to optimize the selection of future addition to generate capacity. In proposed technique the annual load curve has been adapted to propagate the series of new generator requirement curves. These curves giving new generation needs by type for future years, are then available for use in other programs mentioned which develope real expansion pattern and all cost associated with them.

One of the difficulty in attempting to develope an economical generator expansion pattern has been the necessity of studying enormous number of plans. The author has developed a series of programs for small scale digital computer, such as IBM 650 to optimize the selection of future generating capacity additions, thereby minimising the number of plans to be studied.

Henault[3] have formulated a mathematical optimization problem for determination of least costly expansion of power system in the presence of uncertainity about the future loads. The expansion policy has been determined, such that the investment decision is based on an up-to-date estimate of the system requirements.

The main feature of the proposed technique is, presence of uncertainity has been formulated as mathematical optimization problem, that can be solved by stochastic dynamic programming. The planning decision have been made at regular interval. The objective function of the problem is,

Min.

$$J_t(x_t, d_{t+1}, t) = E\{\sum_{K=t}^{K=T-1} [L'(x_K, x_{K+1}, K) + L''(x_{K+1}, d_{K+1}, K) + L'''(x_{K+1}, d_{K+1}, K)] + I(x_T, T)\} \quad \ldots (2.1)$$

Subject to

$$x_{t+1} = x_t + u_t^{(i,j)} \quad \ldots (2.2)$$

$$\bar{x}_{t+1} \geq d_{t+1} \quad \ldots (2.3)$$

where, L' = investment cost, depends upon the investment decision u_t

L'' = operating cost and is function of equipment state x_t and average demand in sub period t

L''' = penalty cost, associated with constraint violation. A function of the form L''' is introduced to penalize the planner for temporary lack of reliability

x_t = decision variable for configuration i

x_{t+1} = decision variable for configuration j

$u_t^{(i,j)}$ = the value of decision variable x_t at the time t, when the decision is to change configuration i to j configuration

$\overline{x}_{t+1}$ = Max. power that can be delivered by configuration x_{t+1} with largest capacity of Transmission Service

d_{t+1} = demand in sub period t+1

$I(x_T, T)$ = investment component of terminal cost

T = terminal period

The symbol E in eqn.(2.1) denotes the expectation taken over the random variables, d_{t+2} , d_{t+3}, d_{T-1} .

In order to obtain the meaningful decision policy the author have made some important assumptions at the out set of program.

1. Divide the planning period into a number of stages at which the decision are to be made.

2. Organise the equipment cost and technical data in a systematic and consistant manner to form a basis for a computerized data bank.

3. Assign discounting factors and equipment salvage value.

4. Define stochastic growth model, and assign probability distribution for the random variables (Demand, Generation etc)

5. Define the Reliability Criteria rigrously.

By these systematic effort spent in prepration of input data, the computer print out provides a great deal of important information. For example the expected total costs for any state and planning stage are obtained together with the optimal investment policy. The additional cost incurred when decision maker is unable to follow the optimal policy is easily calculated. The author believe that the problem formulation given can be generalized and extended. The assumption which are followed by authors are,

1. Different equipment addition leadtime can be considered stochastic for each individual equipment addition.

2. More than one load can be considered, stochastic and uncertainities in other parameter i.e. generation cost etc. can be taken into account by defining additional variables.

3. Because the demand do not have to known in advance at each planning stage, can be past records only.

The limitations of the approach are the maximum number of random variables (i.e. demands) and alternative system configuration the program can efficiently handle.

Guy[(4)] has described a method for determining the most economical generator commitment policy and loading schedule for a days operation of an electric utility system while maintaining a desired level of reliability. Generating units are scheduled to supply the system load for a day. The author has used a constrained search technique to determine which unit should shut down or start up in future hours to minimize the system fuel costs, including the start upcosts. The start up and shutdown times of generating units have been determined to maintain a desired system reliability. The security measure has used been is the probability that the available capacity at any time is greater than system load at that time.

The application of proposed technique results in generation schedule which meets system reliability requirement and yields minimum fuel costs.

Booth[5] has described a procedure for determining the optimal expansion plan for the expansion of generation facilities of a power system over a long period of time. He has combined, A method of production costing based on probabilistic simulation methods, with an advanced dynamic programming formulation of the problem in order to treat uncertainity in a systematic manner.

The planning problem has been formulated in a manner suitable for solution using conventional dynamic programming as follows.

The objective function is

Min. the cost fn.

$$J \Delta \sum_{t=1}^{t=T-1} \frac{1}{(1+2)} + [aC_t(1 + b)^t + O_t(1 + C)^t] \quad \ldots (2.4)$$

subject to

$$x_{t+1} = u_t \quad \ldots \quad t = 1, \ldots, T-1 \qquad \ldots (2.5)$$

$$d_{t+1} = f(d_t, W_t, t) \qquad \ldots (2.6)$$

where i = discount rate

C_t = total capital in service in period t

O_t = operating cost in period t

a = annual capital charge rate

b = rate of escalation of capital

C = rate of escalation of operating charges

and W_t = random variable

x_t = state of system in period t

X_t = set of allowable states in period t,

u_t = decision made in period t for the configuration in period t+1

U_t = set of allowable decisions in period t

d_t = demand in period t

T = planning period

and $f(.)$ = general nonlinear function

The cost function contain the random variables. Then cost of J is randomly distributed. Therefore for stochastic dynamic programming, the cost fn. is

$$J_{PF} = \frac{E}{W_1 \ldots W_{T-1}} [J] \qquad \ldots (2.7)$$

where $W_1 \overset{E}{\ldots} W_{T-1}$ denotes the expected value of J given the probability distributions $W_1 \ldots W_{T-1}$. The author has applied a simplified approach "the open loop feed back approach", which consists of reducing the problem to a series of deterministic open loop optimisations, which can be solved by either forward or backward dynamic programming methods. These deterministic optimizations are used to determine a decision schedule u_t^* ..., u_{T-1}^* , however, only the decision u_t^* is actually employed.

Mathematically, the procedure is to define

$$C_s' = E [C_s]$$

$$O_s' = E [O_s]$$

and

$$J_{OIF} = \sum_{S = t}^{S=T-1} \left(\frac{1}{1+i}\right)^S [aC_s' (1 + b)^s + O_s' (1 + C)^s] \quad \dots (2.8)$$

where the expectation of C_s and O_s is conditional on the information available at time t.

The advantage of the procedure used is the ability to combine the constraints of system reliability with the determination of cost of production in a fast and efficient algorithm, together with the reduced state description in a dynamic programming formulation. The use of probabilistic simulation method combined with a dynamic programming procedure used in an "Open loop feed back" mode allow a rational approach to the problem of uncertainity in future. The approach is capable of extension to the case where both generation and Transmission system may be treated is one expansion problem.

The limitation of technique is that because of nature of problem no one single optimum solution can found.

EVANS[6] has proposed an algorithm for optimal generation planning. Author has used quadratic programming to minimize the discounted value of operating and capital costs, subject to constraints on unit sizes, demand requirement and send out requirements. The advantage of the technique are that it is easy to use and its efficiency is not significantly affected by the problem size. The principal application of the proposed technique is for financial planning beyond the horizon, where detailed engineering design and planning has established the scheduling of generating units.

SULLIVAN[7] presented a comprehensive generation reserve planning technique using probabilitic load and generator model coupled with standard techniques. The technique used is simple and is ~~combine~~ Convenient for analyzing the reserve requirement of both isolated and interconnected systems in which load forecast uncertainity, interconnection transfer capability and unit maintenance may be easily included.

The author has included several simple test cases in discussion to clarify the concepts as well as computational procedures. In each instance test cases appear to illustrate the ease and effect on system reliability of, including forecast uncertainity, unit maintenance and interconnection transfer capability in generation reserve calculation.

OATMAN and HAMANT[8] have developed a approach to optimized generation planning based on the development of large number of expansion pattern comprising all possible combinations of selected units. Essential to the practical utility of this approach are certain algorithms that enable large number of unit, Long range expansion pattern to be generated automatically, utilizing minimum of production cost, studies to hold computer core requirements and computation time within reasonable bounds.

The objective is to determine the generation expansion plan for which the following is minimum as compared with other generation expansion pattern to have a optimum expansion pattern,

$$TAC = I.C. + P.C. + O/M \qquad \ldots (2.9)$$

where TAC = total annual cost

I.C. = product of initial investment and some fixed charge, or capatilization, rate

P.C. = total system fuel cost

O/M = operating and maintenance cost

In the dynamic programming approach the pattern of units having minimum total annual cost is not always optimum. Because the interdependance between fuel cost penalties for units already installed and those to be installed in future is neglected. The fuel cost penalty depends upon the number of hours the unit will be required to run. In addition, eschtion factors which vary for different type of fuel may alter the relative economics of the operating unit.

The author has presented program for the dynamic approach. The program described in this approach provides easily and efficiently, a wealth of information concerning the interaction of future and existing capacity, that the number of possible expansion patterns may reach cosmic proportions only emphasizes the futility in expecting a single generation plan to be valid under all condition of planning period.

The author believed that more rational approach is to measure the sensitivity of resultant pattern to the variations in those parameter which most effect the future generation mixes. These parameter include load forecasts, system load factors fuel prices and plant investment cost. In dynamic expansion program any of these factors may be altered by simple substitution of one or two data cards.

Rogers[9] have developed a model for determining the optimal generation installation program for an electric power system. A new type of dynamic programming is combined with widely used production cost model to optimize, in separate but related procedures, the sequence of unit types and timing of each installation. One of the feature of the proposed model is the separate optimizations, by which an efficient procedure is designed to choose among (various possible installation programs with different types of plan). The optimal installation times reflect the way the engineering and economic characteristic of each plant type enhance those of the existing system to reduce the cost of energy.

Swey and Dalezinn[10] have formulated the mathematical model for long range planning of Generation and Transmission. The mathematical model (Linear Mixed Integer Programming) deals the problem of selecting an expansion plan over a planning horizon for an electric utility system. The objective function that, is chosen is minimization of present value of capital investment cost associated with the construction of power plant and transmission line plus the operating cost of the system. The restriction associated with problem are the requirement to satisfy the forecast demands of the system for electrical energy plus the physical restriction that result from having limited capital resource and plant site limitation.

In general such a mathematical model can be represented as follows,

Minimize

$$aX + bZ \qquad \ldots (2.10)$$

Subject to

$$AX + BZ \leq C \qquad \ldots (2.11)$$

$$X = 0 \text{ or } 1 \qquad \ldots (2.12)$$

$$Z \geq 0$$

where A and B are matrices and a,b,C,X,Z are vectors of appropriate dimensions.

The model presented by author has provided construction expansion of the schedule for power plant, and transmission line i.e. what capacity should build, where the additional capacity is located and when it is to be completed. Additionally an appropriate schedule for plant and transmission line is provided.

The limitation of this model is that it is linear and can not give more accurate results.

FORMULATION OF THE PROBLEM

The formulation of generation planning problem consists of the formulation of objective function and constraints. The problem formulated may linear or nonlinear.

2.1 Formulation of generation planning problem

The planning horizon is divided into stages and it may be assumed that the demand forecast is available for each stage. The planning engineer has to decide the generation level at every stage so that the demand requirements are satisfied. The decision to be taken at each stage of planning period is the choice of plant size or unit types to installed to what level of generation in previous stage. Using Quadratic Programming the cost function which is function of injected power is minimized. For minimum value of cost function the injected power at each bus is determined. The injected power at each bus determine, what level of capacities to be added among the capacities available for expansion into the capacities already working. The loss of load probability is calculated at each bus. If the loss of load probability is greater than certain preselected value than add more unit to satisfy the reliability requirements. The cost function which to be minimized consists of the following terms.

(i) Capital costs : The problem of generation planning is the problem of adding the number (depending on size) and the type of units or power plants. This requires the capital investments for installation of the units or power plants.

Let X represents a combination of unit type (or plant size) for installation at any stage. Let the number of unit types or the power plants in the system be NG. The i^{th} component of X_i of x represents the i^{th} type of unit or the plant size chosen for installation at any stage. Associated with X_i , let the capacity of the plant be GX_i . The capital cost associated with X_i is given by,

$$C_{fi}(X_i) = \left(\frac{1+I}{1+r}\right)^t \left[\frac{(1+r)^{T_i+1} - 1}{(1+r)^{T_i+1}}\right] (CC_i)\; a_{oi}\, GX_i \qquad \ldots (3.1)$$

where t = time (in years) at which investment is made

I = inflation rate

r = interest rate

T_i = life span (in years) of the unit (plant)

CC_i = carrying cost

a_{oi} = cost (mu) per MW capacity of the unit (plant) of type i

Total cost associated with the vector X is therefore given by

$$C_f(X) = \sum_{i=1}^{NG} C_{fi}(X_i) \qquad \ldots (3.2)$$

(ii) Operating cost: The operating cost of system is dependent on the output of the individual plant or units. As an approximation the operating cost may be assumed to be function of the expected value of plant outputs. The operating cost can therefore be obtained from the solution of the following.

Min.

$$F'(\underline{X}) = \sum_{i=1}^{NG} f_i(P_i) \qquad (3.3)$$

subject to

$$\sum_{i=1}^{NG} P_i = \bar{D}_m \qquad (3.4)$$

$$0 \leq P_i \leq P_{gi}^{m} \qquad i = 1 \,..\, NG$$

where, $\bar{D}_m$ = mean value of demand at any stage

P_{gi}^{m} = max. power limit to be generated at the any stage.

The present cost function considering the interest and inflation rate is,

$$F(X) = \left(\frac{1+I}{1+r}\right)^t F'(X) \qquad (3.5)$$

Another model for determining the operating cost is based on plant or unit is classified as base load, peak load, or midrange type. Accordingly the capacity factor (CF_i) for the i^{th} type unit is defined and the annual operating cost is calculated according to formula.

$$F_i(X_i) = 8.76 \left(\frac{1+I}{1+r}\right)^t \left[\frac{(1+r)^{t_i+1} - 1}{(1+r)^{t_i+1}} \right] (CF_i)\beta_i \; GX_i \qquad \dots (3.6)$$

where, CF_i = capacity factor for the unit of type i

β_i = cost (mu) per GWH output

GX_i = expected value of capacity of unit of type i

Hence total operating cost is given by,

$$F(X) = \sum_{i=1}^{NG} F_i(X_i) \qquad \dots (3.7)$$

The above model presented are not in quadratic form. Therefore a model approximating the costs, equivalent quadratic cost function has been followed in the present case.

2.2 Generation quadratic programming problem

The Beales method[12] is applied to the generation planning problem. The objective function is quadratic cost function of injected power. The constraints are linear function of injected power. In this method quadratic function is represented by an upper triangular matrix. The constraint matrix is represented in the same way except that Zeroth row of the matrix is not used. Thus reducing the computer memory.

<u>Statement of the problem</u>

Minimise the cost function

$$f(P_i) = \sum_{i=1}^{NGB} a_i + b_i P_i + C_i P_i^2 \qquad \ldots (3.8)$$

subject to

$$\sum_{i=1}^{NGB} P_i \geq \sum_{j=1}^{NL} P_{Lj} \qquad \ldots (3.9)$$

$$P_{gi}^{m} - P_i \geq 0 \qquad i = 1, \ldots, NGB \qquad \ldots (3.10)$$

where, P_i = injected power at i^{th} bus

a_i = fixed part of the cost at bus i

b_i = linear part of the cost at bus i

C_i = quadratic coefficient of cost at bus i

NGB = number of generating buses

NL = number of load buses

P_{gi}^{m} = maximum power limit at generating bus i

P_{Lj} = load at the load bus j

SOLUTION TECHNIQUE

4.1 Review of techniques

The techniques that may be used in planning problems are as follows:

4.1.1 Dynamic programming: In the planning problems the decision have to be made sequentially at different points in time, at different points in space and at different levels. Such problems in which decisions to be made sequentially are sequential decision problem. These decisions are to be made at number of stages, are referred as multistage decision problem. The dynamic programming is the mathematical technique well suited for optimization for mustistage decision problem.

The multistage decision problem having N variables is represented as a sequence of N single variable problem which are solved successively. In most of cases, these N subproblems are easier to solve than the original problem. The decomposition of N subproblems is done in such a manner that the optimal solution of the original N variable can be obtained from the optimal solutions of the N one-dimensional problems. Any of the particular optimization technique can not be used for optimization of N-single variable problems. It may range from a simple enumeration process to a differential calculus or a nonlinear programming techniques. Further, the problem has to be relatively simple so that the set of resultant equation can be solved either analytically or numerically. The non-linear programming techniques can be used to solve slightly more complicated multistage decision problem. But their application requires the variables to be continuous and prior knowledge about the region of global minimum or maximum. In all these cases the introduction of stochastic variability makes the problem extremely complex renders the problem unsolvable except by using some sort of approximation like chance constrained optimization. Dynamic programming can also deal with descrete variables, nonconcave noncontinuous, and non-differentiable functions.

4.1.2 <u>Integer Programming</u>: In many situation the problem variables are permitted to take any fractional value. In many cases it is very difficult to round off solution without violating any of the constraints. Frequently the rounding off certain variables requires substantial change in the value of some other variables in order to satisfy all constraints. The round off solution may give a value of objective function that is very far from original optimum value. All these difficulties are avoided by solving optimization problem as integer programming problem.

In optimization problem all the variables are constrained to take only integer value the optimization problem is all integer programming problem. When some variables only are restricted to take only integer value, the optimization problem is mixed integer programming problem. Among the several techniques available for solving the all integer and mixed integer linear programming problems, the cutting plane method, the branch and bound algorithm(16) can be used.

4.1.3 Stochastic Programming: Stochastic programming deal with situation where some or all of the parameters are described by stochastic variables rather than deterministic quantities.

Depending upon the nature of equations involved in problems, a stochastic optimization problem is a stochastic linear or dynamic or non-linear programming problem. The basic idea used in solving any stochastic programming problem is to convert equivalent deterministic problem. The resulting deterministic problem is solved by familiar techniques like linear, geometric, dynamic and non-linear programming.

4.1.4 Constrained Non-Linear Programming: The generation planning problem are usually non-linear constrained optimization problem, because the equation relating capital and operating costs to system configuration are non-linear. The constraint that are imposed on planning problem are linear in nature usually. Such optimization problem which can be stated in standard form as, Find x such that $f(x) \rightarrow$ minimum

and $g_j(x) < 0$ $j = 1,2, \ldots m$

There are many techniques available for the solution of a constrained non-linear programming problem. All these methods are classified into two broad categories, namely, direct methods and indirect methods. In the direct methods, the constraints are handled in an explicit manner where as most of indirect methods, the constrained problem is solved as a sequence of unconstrained minimization problem.

4.1.4.1 Direct methods:

Constraint approximation methods: In these methods, the non-linear objective function and constraints are linearized about some point and the approximating linear programming is solved by using linear programming techniques. The resulting optimum solution is then used to construct a new linear approximation which will again be solved by using linear programming techniques. The procedure is continued until specified convergence creteria are satisfied. There are two methods namely, the cutting plane method and the approximate programming method which work on this principle

Methods of feasible directions: The methods of feasible direction are those which produce an improving succession of usable feasible directions. A feasible direction is one along which at least a small step can be taken without leaving the feasible domain. A usuble feasible direction is feasible direction along which the objective function value can be reduced at least by small amount. Each iteration consists of two important steps in the method of feasible directions. The first step consists of finding a usable feasible direction at a specified point and second step consist of determining a proper step length along the usable feasible direction found in the first step.

4.1.4.2 Indirect Methods

Transformation of variables: Some of the constrained optimization problem have their constraints expressed as simple and explicit function of decision variables. In such cases it may possible to make a change of variable such that the constraints are automatically satisfied. In some other cases it may be possible to know, in adxance which constraint will be active at the optimum solution. In these cases, it may use particular constraint equation $g_j(x) = 0$, to eliminate some of variables from the problem.

Penalty Function methods: There are two type of penalty function methods - the interior penalty function method and the exterior penalty function method. In both type of method the constrained problem is transferred into a sequence of unconstrained minimization problem such that constrained minimum can be obtained by solving the sequence of unconstrained minimization problems.

In the interior penalty function method the sequence of unconstrained minima lie in the feasible region and thus it converges to the constrained minimum from interior of the feasible region. In the exterior methods the sequence of unconstrained minima lie in the infeasible region and converges to a desired solution from exterior of the feasible region.

4.1.5 Quadratic programming: In the planning problem where the objective function can be approximated in quadratic form and the constraint implemented to the problem are linear, then the quadratic programming can be used to minimise the cost function. It is a special case of non-linear programming. The quadratic programming can be solved by Lagrange multiplier technique. For such problems Beale's method has also been used.

4.2 BEALE'S METHOD

The Beale method(11) belongs to the geometrically most illustrative methods for quadratic optimization. The convex quadratic objective function has the form

$$Q\,(x_1, \ldots., x_n) = Q(x)$$

Subject to Linear Constraints

$$AX = a_{.o} \qquad \ldots (4.1)$$

$$X > o$$

The Beale method is started with any feasible basic solution of system (4.1). The system of equations in (4.1) are solved with respect to chosen basic variables. If these are assumed to be first m variables, i.e. $x_1, \ldots, x_m$, this leads to

$$x_g = d^1_{go} + \sum_{h=1}^{n-m} d^1_{gh} Z_h \quad (g = 1, \ldots, m) \quad .. (4.2)$$

where, $Z_h = x_{m+h}$

The upper index refers to first approximation. Because of the particular choice, the basic variables assume the value $d^1_{go} \geqslant o$ at the initial point of approximation. The variables on the right side of (4.2) the independent or vanishing variables. Those on the left side are the dependent or basic variables.

Using equation (4.2) basic variables can be eliminated from Q. For the sake of simplicity the following notation is recommended,

$$Q(x_1, \ldots, x_n) = Q^1 (Z_1 \ldots, Z_{n-m})$$

$$= C^1_{oo} + 2 \sum_{i=1}^{n-m} C^1_{io} Z_i + \sum_{h=1}^{n-m} \sum_{i=1}^{n-m} C^1_{hi} Z_i Z_h$$

$$= C^1_{oo} + \sum_{i=1}^{n-m} C^1_{oi} Z_i + \sum_{h=1}^{n-m} (C^1_{ho} + \sum_{i=1}^{n-m} C^1_{hi} Z_i) Z_h$$

$$= (C^1_{oo} + C^1_{oi} Z_1 + \dots + C^1_{o,n-m} Z_{n-m}).1$$
$$+ (C^1_{10} + C^1_{11} Z_1 + \dots + C_{1,n-m} Z_{n-m}) Z_1$$
$$\vdots$$
$$+ (C^1_{ho} + C^1_{h1} Z_1 + \dots + C^1_{h,n-m} Z_{n-m}) Z_h$$
$$\vdots$$
$$+ (C^1_{n-m,o} + C^1_{n-m,1} Z_1 + \dots + C^1_{n-m,n-m} Z_{n-m}) Z_{n-m} \qquad \dots (4.3)$$

In equn. (4.3) the symmetry $C^1_{ih} = C^1_{hi}$ holds, and further

$$\frac{1}{2} \frac{\partial Q^1}{\partial Z_h} = C^1_{ho} \qquad (h = 1, \dots n-m) \qquad \dots (4.4)$$

Clearly, the value of Q at the first approximation is equal to C^1_{oo}. If for chosen Trial Point

$$\frac{\partial Q^1}{\partial Z_h} \geq 0$$

From Kuhn-Tucker Conditions[11] this point already represents the optimal solution, since every increase of independent variable could increase the value of Q^1. However for certain Z_h

$$\frac{\partial Q^1}{\partial Z_h} < 0 \qquad \text{i.e. } C^1_{ho} < 0 \qquad \dots (4.5)$$

holds at trial point. It is possible to improve the Q-value by making Z_h (i.e. independent variable) positive.

Suppose this happen for h = 1 i.e. for Z_1 , then, if Z_1 increases other basic variable of course also change. As in case of linear optimization, the question arises how much the variable Z_1, should increase. In the quadratic case there may be following possibilities.

Case 1. Let Z_1 increases until one of the basic variable disappears

Case 2. $\frac{\partial Q^1}{\partial Z_1}$ become zero before one of the basic variables does in which case Z_1 is of course increased only until $\frac{\partial Q^1}{\partial Z_1} = o$, otherwise the value of objective function would increase.

In case 1 the constraint system has to solve again for new basic variables. These are then substituted in the objective function, and the second approximation is obtained.

In case 2 introduce a new variable u_1 by

$$u_1 = \frac{1}{2} \frac{\partial Q^1}{\partial Z_1} \qquad \ldots (4.6)$$

u_1 is not sign restricted and is called the first free variable. As second approximation choose that point at which the first free variable disappears, together with all independent variables except that one which has entered the basis i.e. except Z_1 . Again, the constraint system and the objective function are newly rearranged whereby the free variable u_1 is included among the independent variables

$$u_1 = c^1_{10} + \sum_{h=1}^{n-m} c^1_{1h} Z_h = \frac{1}{2} \frac{\partial Q^1}{\partial Z_1} \qquad \ldots (4.7)$$

For the new dependent variable Z_1

$$Z_1 = -\frac{c^1_{10}}{c^1_{11}} + \frac{1}{c^1_{11}} u_1 - \sum_{h=2}^{n-m} \frac{c_{1h}}{c^1_{11}} Z_h$$

$$= d^2_{10} + d^2_{11} u_1 + \sum_{h=2}^{n-m} d^2_{1h} Z_h$$

If this equation is used to eliminate Z_1 from system of equations (4.2) it follows

$$x_g = d^2_{go} + d^2_{g1} u_1 + \sum_{h=2}^{n-m} d^2_{gh} Z_h \quad (g = 1,2,\ldots m, m+1) \qquad \ldots (4.8)$$

In the same way Z_1 is also eliminated from Q_1 .

With the second approximation proceed once again with the same rule as described before. The Kuhn-Tucker condition for the free variable has the form

$$\frac{\partial Q^2}{\partial u_1} = o$$

If the derivative with respect to a free variable is not zero then Q can be lowered by varying u_1 positively or negatively. More precisely, if the derivative with respect to u_1 is larger than zero, u_1 has to become negative or vice versa.

Beale's Algorithm : Beale's method determines the minimum of a definite quadratic form subject to linear constraints. The determination of a feasible basic solution of constraint system has nothing to do with quadratic optimization, and is therefore assumed here to have been accomplished before hand. As a result, the linear constraint will have the form

$$a_{io} + \sum_{k=1}^{n} a_{ik} x_k \geqslant o \qquad \text{with } a_{io} \geqslant o \qquad \ldots (4.9)$$

These are stored row wise in the first through the m^{th} row of simplex tableau a . The o^{th} row of constraint matrix is not used since symmetric quadratic objective function C requires an (n+1) x (n+1) array as storage area (not just one row).

However, for reasons of storage economy, only the upper triangle of C is stored in the array c, in the form of densely packed row segments. The element c_{ik} therefore corresponds to c [(2 x n-i+1) x i/2 + K] . This necessitates two prepration of two tableaus shown in Fig. (4.1).

Storage allocation: Internally the program expands tableau

A by (n+1) additional rows. Therefore this space has to reserved when declaring A (but not for the input routine The dimension of storage area of A has to be (m+n+2) x (n+1).

Computational approach: Corresponding to each exchange of variables (dependent - independent) in the constraint tableau A

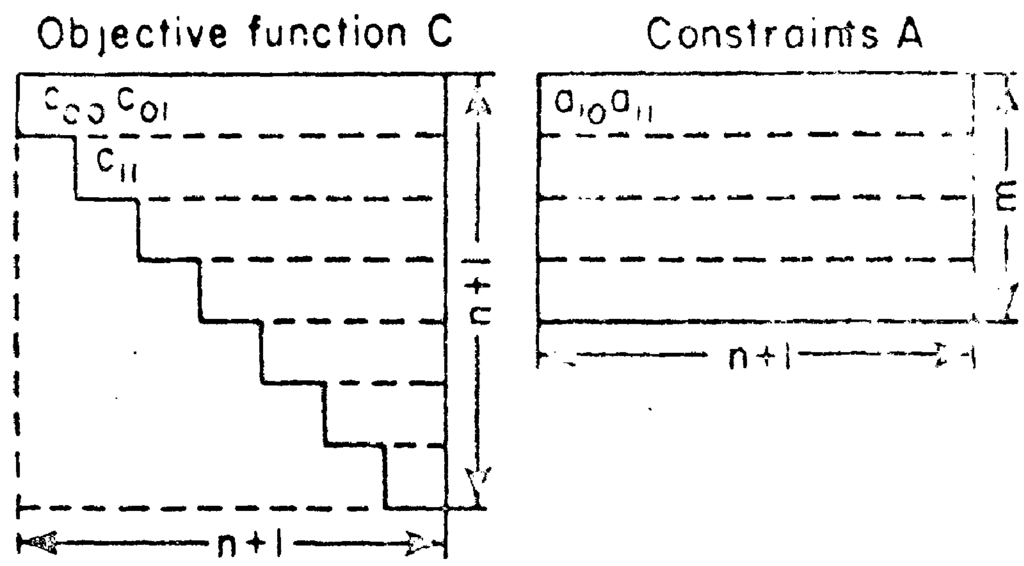


Fig. 4.1. Schematic representation of the storage of the objective function and the constraints in Beale's method.

therefore there is a corresponding exchange in a quadratic objective function C . This latter exchange is handled by means of subroutine mp9.

4.3 Reliability Calculations

To determine whether a generation expansion plan satisfies a desired level of reliability generally two reliability indicies are used, loss of load probability, (LOLP), and expected value of demand not served, ϵ(DNS). In the present case the loss of load probability has been calculated.

4.3.1 Loss of Load Probability : Effective load[13] of the system gives the relationship between the system load and generating units. The actual units may replaced by ficticous perfectly reliable units and fictitious random load, whose probability density functions are the outage capacity density function of units.

The effective load, be defined by

$$Le = L + \sum_i L_{oi} \qquad \ldots \qquad (4.10)$$

where, L is the fictitious random load

L_{oi} is random outage load of the i^{th} unit.

When $L_{oi} = C_i$, the net demand injected into the system for i^{th} unit is zero, just as it would be if actual C_i were forced offline. Also the installed capacity of the system be given by

$$IC = \sum_i C_i \qquad \ldots \qquad (4.11)$$

In the special case where actual units are 100 percent reliable, $L_{oi} = 0$, $i = 1, \ldots G$ and $Le = L$. Unfortunately this case never occurs, so one forced to have F(Le) from F(L) and f_{oi} (L_{oi}). Since Le is the sum of independent random variables, L and L_{oi}, $i = 1 \ldots G$ whose distributions are known, therefore F(Le) is obtained using the recursive convolution equation

$$F^{i}(Le) = \int_{L_{oi}} F^{i-1} (Le - L_{oi})\, f_o(L_{oi})\, dL_{oi} \qquad \ldots (4.12)$$

where, $F^{i}(Le)$ be the effective load probability distribution with the outage of first i units. Convolved in obiously,

$$F^{i}(Le) = \begin{matrix} F(L) & \text{for } i = o \\ F(Le) & \text{for } i = G \end{matrix}$$

Since f_{oi} is the descrete density function equn. (4.12) becomes,

$$F^{i}(Le) = \sum_{L_{oi}} (Le - L_{oi})\, f_{oi}\, (L_{oi}) \qquad \ldots (4.13)$$

L_{oi} denote the descrete value L_o can assume.

Because the outage capacity of unit may defined as two stage stochastic process.

$$f_{oi}(L_{oi} = o) = pi$$

$$f_{oi}(L_{oi} = C_i) = qi$$

The equn. (4.13) simplifies further,

$$F^{i}(Le) = F^{i-1}(Le)Pi + F^{i-1}(Le - C_i)qi \quad i = 1 \ldots G$$

The above equation gives the probability of existing the load Le .

4.3.2 <u>Reliability Analysis for Isolated System</u>: The loss of load probability is the popular method for generation system reliability analysis. It must be remember that the goal of system planner is to select several expansion plans from perhaps a dozen feasible plans that satisfy the desired reliability creteria established in electric utility. Thus the basic problem is to evaluate week by week the variation in reliability, as new units are added to supply the growing load, to determine plans that have acceptable reliability characteristics. For each week in the study period, care must be taken to simulate the anticipated maintenance schedules because these schedules drastically influence system reliabilit Carrying out the steps outlined enables the system planner to identify quickly the expansion plans that are acceptable and should be evaluated and compared on an economic basis.

In systems in which peak demand is very pronounced and peaking units are used extensively during the peak periods, it is usually necessary to define two load shape for each week or month, one load shape, would define peak hours, in which peakers are needed, and other would define the off peak period, in which only mid range and base load units are required. The delineation of peak and off peak periods usually results in a more accurate representation of system. The reliability analysis procedure is same for every week.

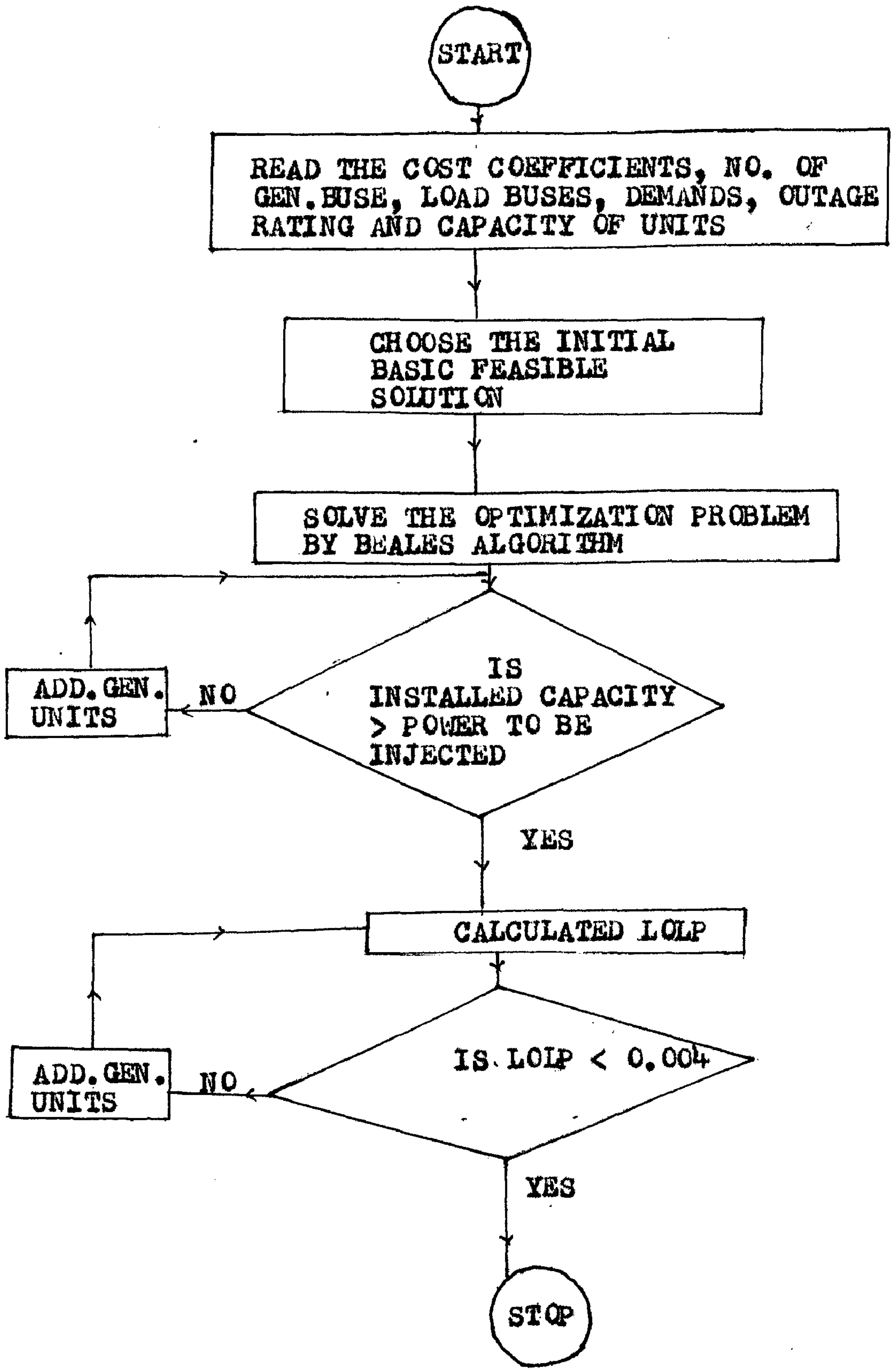
4.4 Developement of algorithm for Generation Planning

Using Beale's technique an algorithm has been developed. The steps of the algorithm are as follows.

1. Formulate the quadratic cost function with coefficients a_i, b_i, c_i as in 4.2 .
2. Choose initial feasible solution as follows. The injected power at each bus for the present stage is taken equal to the injected power for the previous stage plus increase in demand distributed equally at all the buses.
3. Find new optimized injected powers by Beale's quadratic programming method.
4. Add the generating units at the generating buses so that the installed capacity at these buses is sufficient to inject this power.
5. Calculate the LOLP by eqn. (4.14). If the LOLP < 0.004 capacity at these buses are sufficient to take into account uncertainities, otherwise add generating units so that LOLP < 0.004.

The LOLP has been calculated manually. The steps used in algorithm are also accompanied with Flow Chart.

4.4.1 Flow Chart



The computer programme for the Beale's Method is given in index.

4.5 Application

The algorithm is applied to the five bus system. A fine bus system shown in Fig. 4.2 is considered for expansion. The bus, 1,2,3 are generating buses and 4,5 are load buses. The existing and future demands are given in Table 4.1 . The fixed investment and expansion costs of the plant are given in Table 4.2 . The operating costs are assumed to be of the form given by

$$f_i(P_{oi}) = b_i P_i + c_i P_i^2 \qquad i = 1,2,3, \ \ldots$$

The operating cost coefficients are given in Table 4.2. The Table 4.3 gives the existing plant output, installed and maximum capacities.

TABLE 4.1

BASE MVA = 200

Load Bus No.	Demand	
	Existing (p.u.)	Future (p.u.)
4	0.5	0.8
5	0.7	1.0

TABLE 4.2

Gen.Bus No.	Fixed Cost in money unit (mu)	Cost of capacity expansion mu/MW	Operating cost coefficien	
			b_i mu(p.u. output)	c_i mu/(p.u.output)
1	100	1	2.4506	0.0041
2	125	1	2.49218	0.00207
3	50	1	2.117	0.00379

TABLE 4.3

Gen. Bus. No.	Output (p.u.)	Existing installed capacity (p.u.)	Maximum site capacity(p.u.)
1	0.15	0.3	5.0
2	0.6	0.6	1.0
3	0.45	0.6	10

The shape of load duration curve is assumed to be as shown in Fig. 4.3. The load probability distribution curve is shown in Fig. 4.4, in which $F_k(L)$ is probability of load existing for period k, and Θ is the max. demand.

The number of units available, their probability of outage, cost of unit, their capacities are as follows.

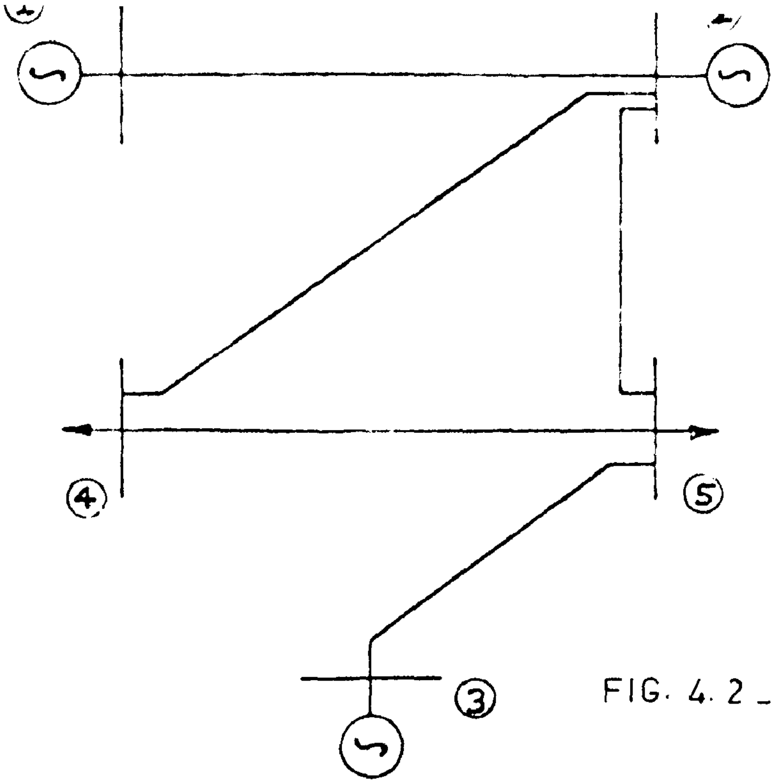


FIG. 4.2 _ 5 BUS SYSTEM

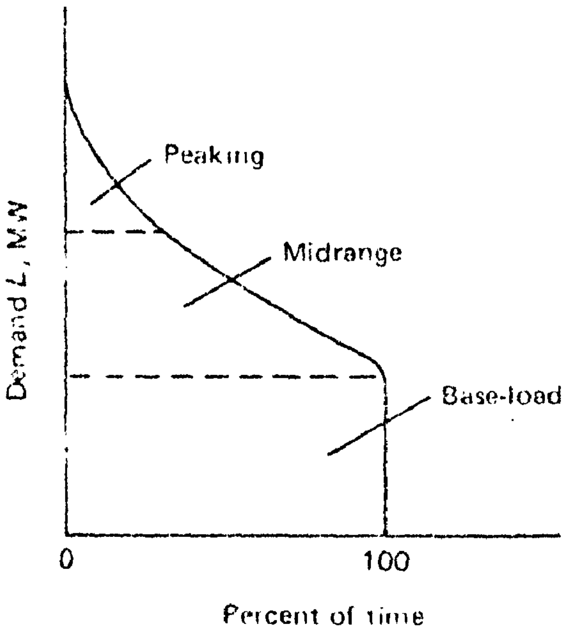


FIG. 4.3 _ TYPICAL LOAD DURATION CURVE

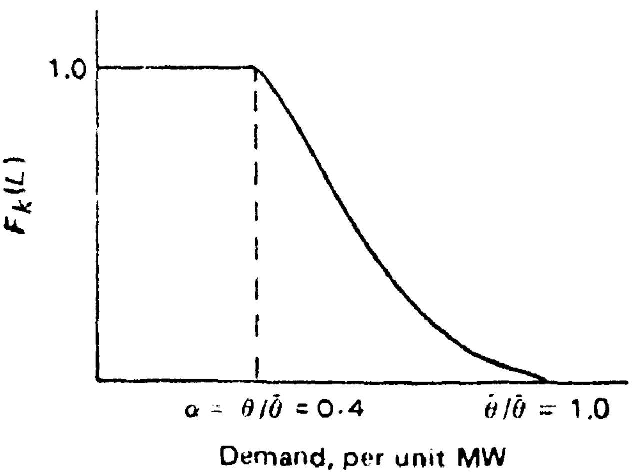


FIG. 4.4 _ LOAD DISTRIBUTION CURVE

TABLE 4.4

Gen.Bus No.	Number of units
1	3
2	4
3	4

TABLE 4.5

Gen. Bus No.	Outage rate	Cost (mu)	Carrying cost	Capacity in MW
1	0.02	60	0.2	25
	0.02	60	0.2	25
	0.02	60	0.2	25
2	0.02	110	0.2	50
	0.02	110	0.2	50
	0.02	110	0.2	50
	0.02	60	0.2	25
3	0.02	60	0.2	25
	0.02	60	0.2	25
	0.02	110	0.2	50
	0.02	60	0.2	25

The unit available for expansion are as follows

TABLE 4.6

Gen.Bus No.	No. of units
1	7
2	3
3	6

The outage rate, cost of unit, carrying cost and capacity of these units are given in Table 4.7.

TABLE 4.7

Gen. Bus No.	Outage rate	Cost (mu)	Carrying cost	Capacity in MW
	0.02	110	0.2	50
	0.02	60	0.2	25
	0.02	60	0.2	25
1	0.02	110	0.2	50
	0.02	60	0.2	25
	0.02	60	0.2	25
	0.02	60	0.2	25
	0.02	110	0.2	50
2	0.02	60	0.2	25
	0.02	110	0.2	50
	0.02	60	0.2	25
	0.02	60	0.2	25
	0.02	60	0.2	25
3	0.02	110	0.2	50
	0.02	110	0.2	50
	0.02	110	0.2	50

The results obtained where as follows. The injected power at each buses is given in Table 4.8.

Table 4.8

Gen. Bus No.	Injected Power in MW
1	90.71617
2	130.40883
3	138.87500

The above values are must same as the results obtained in reference (15).

Initial costs = 7411.3787 mu

The reserves depending upon loss of load probability are as follows.

If the loss of load probability is greater than 0.004 the additional generation capacity is added.
The cost of final injected power is (i.e. operating cost) is

$$\sum_{i=1}^{NGB} a_i + b_i P_i + c_i P_i^2 = a_1 + b_1 P_1 + c_1 P_1^2$$

$$= a_2 + b_2 P_2 + c_2 P_2^2$$

$$= a_3 + b_3 P_3 + c_3 P_3^2$$

$$= 1258.3486 \text{ mu.}$$

The loss of load probability at various buses are as follows:

TABLE 4.9

Gen. Bus No.	Installed Capacity MW	LOLP
1	125	0.68915×10^{-3}
2	225	0.24693×10^{-2}
3	175	0.103930×10^{-2}

Therefore the units to be added at generating buses are as follows:

TABLE 4.10

Gen.Bus No.	No. of units	Capacity in MW
1	1	50
2	1	50
3	2	25

Cost of unit additions = 2800 mu

Total Final Costs = 1058.348 mu

Cost of Expansion = Final Cost - Initial Cost

= 3146.6093mu

CONCLUSIONS AND SUGGESTIONS

The quadratic programming problem formulated can be applied to solve generation planning problems. The Beale's algorithm is applied. The memory required is less as cost function coefficients are represented by the upper triangular matrix, as no artificial constants are used for optimization purpose, and the A (Constraint matrix), C (Cost coefficient matrix) are transferred according to method similar to simplex linear programming algorithm. The method works faster. For the five bus system the results are obtained in 3.02 seconds (including compilation time) and one iteration. Loss of load probability incorporated takes into account the uncertainties involved due to forced outages of various generating units. The reserves of generating capacities needed are calculated on LOLP basis, which is the more popular method of reliability analysis of generating system.

Suggestions:

The formulation presented for generation planning can be extended to the transmission and distribution planning problems. The cost function of transmission planning problem may capital cost of transmission line and cost of transmission losses. The constraint imposed may be security constraints i.e. no overloading of circuits be permitted. In distribution planning problem may having objective function of the cost of substations, Sub-Transmission lines and cost of losses. The constraints may be permissible value voltage, no overloading of circuits and constraint on the number and type of circuit that can be added. As the dimensionality problems are not inherent in this technique it should be possible to analyse large systems.

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APPENDIX

```
C     MAIN PROGRAM GENERATION SYSTEM PLANNIM QP METHOD
      COMMON/THREE/NU,NA,NC,NNC,QO,,PO,QO1,PO1,CCO,CC,AO,AO1,CCF,CF,RR
      DIMENSION AC(3),BC(3),D(3),DM(5,2),POI(3),PO1I(3),PO2I(3),IPRT2(15
     1),CA(120),AA(120)
      DIMENSION COP(3),XTOL(3),DIF(5)
      READ(1,2000) NB,NGB,NL,NY,NTJ
      PRINT 2000,NB,NGB,NL,NY,NTJ
      READ(1,2000)(AC(I),BC(I),D(I),I=1,NGB)
      PRINT 2000,(AC(I),BC(I),D(I),I=1,NGB)
      READ(1,2000)RINT,RINF
      PRINT 2000,RINT,RINF
      READ(1,2000)(PGIM(I),I=1,NGB)
      PRINT 2000,(PGIM(I)=I,1,NGB)
      READ(1,2000)((DM(I,J),I=1,NY),J=1,NL)
      PRINT2000,((DM(I,J),I=1,NY),J=1,NL)
      READ(1,2000)(POI(I),I=1,NGB)
      PRINT 2000,(POI(I),I=1,NGB)
      READ(1,2000)(PO2I(I),I=1,NGB)
      PRINT 2000,(PO2I(I),I=1,NGB)
      NGB2=2*NGB
      M=NGB2+2
      M1=M+1
      N=NGB2
      N1=NGB+1
      IZSR=NGB2+1
      IT=1
      ISSR=1
      IT1=IT+1
      ITRN=1
      NBB=M+N+1
      N2=N1+NGB
      N12=N1+1
      DO212I=1,NGB
  212 PO1I(I)=POI(I)
      RINF=1.0+RINF
      RINT=1.0+RINT
      R=(RINF/RINT)*IT
      CCF=0.0
      R1=((RINF**NTJ-1)/RINT**NTJ)
      DO 118 J=1,NGB
      NUU=NU(J)
      DO 118 I=1,NUU
      ANC=NC(J,I)
  118 CCF=CCF+R*R*CC(J,I)*AO(J,I)*ANC
   77 R=(RINF/RINT)*IT
      DO 6 I=1,NGB
      AC(I)=R*AC(I)
      BC(I)=R*BC(I)
    6 D(I)=R*D(I)
      DO 8 I=1,NGB
    8 COP(I)=AC(I)+BC(I)*PO1I(I)+D(I)*PO1I(I)**2
      GCOIN=0.0
```

```
      DO 18 I=1,NGB
   18 GCOIN=GCOIN+COP(I)
      GCOIN=CCF+GCOIN
      PRINT 119,GCOIN
      TYPE119,GCOIN
  119 FORMAT(13HINITIAL COST=,E20.5)
      ITRN=0
  410 CA(1)=0.0
      I1I=IZSR**2
      DO 416 I=1,I1I
  416 CA(I)=0.0
      I2I=(NBB+1)*IZSR
      DO417I=1,I2I
  417 AA(I)=0.0
      DO 11 I=1,NGB
      DP(I+NGB)=0.
   11 CA(1)=CA(1)+AC(I)+BC(I)*PO1I(I)+D(I)*PO1(I)**2
      CA(1)=CA(1)+CCF
      N1=NGB+1
      DO 9 KH=2,N1
      KH1=KH-1
    9 CA(KH)=BC(KH1)*0.5+D(KH1)*PO1I(KH1)
      DO 403 I=N12,N2
  403 CA(I)=-CA(I-NGB)
      DO 10 I=1,N
      KH=(2*NGB2-I+1)*I/2+I+1
      IF(I.GT.NGB)CA(KH)=D(I-NGB)
      IF(I.LE.NGB)CA(KH)=D(I)
   10 CONTINUE
      DO 404 I=1,NGB
      KH=(2*NGB2-I+1)*I/2+(NGB+I)+1
  404 CA(KH)=-D(I)
      PRINT 5000,(CA(I),I=1,I1I)
      IT1=IT+1
      DO114I=1,N1
  114 AA(I)=0.0
      K=(NGB+1)*IZSR+1
      AA(K)=0.0
      DO13I=1,NGB
      KH=K+I
      KH2=KH+NGB
      AA(KH2)=-1.0
   13 AA(KH)=1.0
      DO 14 I=1,NGB
      KH=I*IZSR+I+1
      AA(KH)=-1.0
      KH2=KH+NGB
      AA(KH2)=-AA(KH)
      KH=I*IZSR+1
      AA(KH)=PGIM(I)-PO2I(I)
```

```
   14 CONTINUE
      DO405I=1,NGB
      KH=IZSR*(NGB+1+I)+1
      KH1=KH+I
      KH2=KH1+NGB
      AA(KH)=PO2I(I)-PO1I(I)
      AA(KH1)=1.0
      AA(KH2)=-1.0
  405 CONTINUE
      NBB1=NBB+1
      DO 16 I=1,NBB1
      K1=(I-1)*IZSR+1
      K2=I*IZSR
      TYPE 5000,(AA(K),K=K1,K2)
   16 PRINT 5000,(AA(K),K=K1,K2)
      TYPE 3002
      PRINT 3002
 3002 FORMAT(@CONSTRAINT MATRIX@)
      CALL BEALE (IZSR,ISSR,N,M,CA,AA,IFALL,IPRT2)
      IF(ITRN..GT.0.AND.IFALL.NE.0) GO TO 411
      DO 15 I=1,NBB1
      K1=(I-1)*IZSR+1
      K2=I*IZSR
      TYPE 5000,(AA(K),K=K1,K2)
   15 PRINT 5000,(AA(K),K=K1,K2)
      IF(IFALL-1)307,306,307
  307 DO 305 I=1,NBB
      IF(IPRT2(I)-NGB2)309,309,305
  309 II=IPRT2(I)
      I1=(I-0)*IZSR+1
      DP(II)=AA(I1)
  305 CONTINUE
      DO 407 I=1,NGB
      DIF(I)=DP(I)-DP(I+NGB)
      DIF(I)=ABS(DIF(I))
  407 PO2I(I)=PO2I(I)+DP(I)-DP(I+NGB)
      TYPE 3001
      PRINT 3001
 3001 FORMAT(@INJECTED POWERS@)
      DO 315 I=1,NB
      IF(I.GT.NL) GO TO 314
      P(I)=-DM(IT1,I)
      GO TO 315
  314 I1=I-NL
      P(I)=PO2I(I1)
      PRINT 2000,P(I)
      TYPE 2000,P(I)
  315 CONTINUE
      XX=DIF(1)
      DO 408 I=2,NGB
```

```
      IF(XX.LT.DIF(I))XX=DIF(I)
  408 CONTINUE
      ITRN=ITRN+1
      PRINT 409,ITRN
  409 FORMAT(@ITERNO=@,I4)
      IF(ITRN.EQ.21)GO TO 411
      IF(XX.GT..001)GO TO 410
  411 DO 318 I=1,NGB
  318 COP(I)=AC(I)+BC(I)*PO2I(I)+D(I)*PO2I(I)**2
      GFCON=0.0
      DO 418 I=1,NGB
  418 GCOFN=GCOFN+COP(I)
      GCOFN=CCF+GCOFN
      PRINT 218,GCOFN
      TYPE 218,GCOFN
  218 FORMAT(//11HFINAL COST=,E20.5)
      COSEXP=GCOFN-GCOIN
      PRINT 84,COSEXP
      TYPE 84,COSEXP
   84 FORMAT(21H**COST OF EXPANSION=,E20.6)
  306 PRINT 4000
      TYPE 4000
 4000 FORMAT(9H*QP FAILS)
  601 STOP
      END
```

```
      SUBROUTINE BEALE(IZSR,ISSR,N,M,C,A,IFALL,IPRT2)
      DIMENSION C(120),A(120),IPRT1(15),IPRT2(15),LIST(15),ABLIST(15),
     1L1(15),L2(15)
      INTEGER ABLIST,V,S,Z,T,R
      REAL MAX
      LOGICAL B1
      DO 1K=1,N
      L1(K)=K
    1 IPRT1(K)=K
      DO 2I=1,M
      L2(I)=I
    2 IPRT2(I)=N+I
      L10=N
      L20=M
      DO 3 K=1,N
    3 ABLIST(K)=0
      LISTO=0
      M1=M
 1000 IF(LISTO.EQ.0)GO TO 1001
       CALL MP8(C,0,0,1,LIST,LISTO,KP,MAX)
      IF(MAX.NE.0.)GO TO 2000
 1001 CALL MP5(C,L1,L10,Q1,KP,0,0,1)
       IF(Q1.LT.0.)GO TO 2000
      IFALL=0
      RETURN
 2000 KH=KP*(N+1)-((KP-1)*KP)/2+1
       MAX=0.
      IF(C(KH).GT.0.)MAX=C(KP+1)/C(KH)
        V=-1
      IF(MAX.GT.0.)V=1
      CALL MP2(A,L2,L20,IP,IZSR,ISSR,KP,Q1,N,V)
      IF(IP.NE.0.OR.MAX.NE.0.)GO TO 3000
      IFALL=1
       RETURN
 3000 IF(IP.EQ.0..OR.Q1.GT.ABS(MAX))GO TO 4000
      I=IPRT1(KP)
      IPRT1(KP)=IPRT2(KP)
      IPRT2(IP)=I
      CALL MP3(A,1,M,0,N,IP,KP,IZSR,ISSR,1,1)
      B1=.TRUE.
      CALL MP9(A,C,IP,KP,N,M1,IZSR,ISSR,B1)
      GO TO 6000
 4000 ABLIST(KP)=ABLIST(KP)+1
       T=0
      R=0
4001  IF(LISTO.EQ.0)GO TO 5000
      DO 4002 I=1,LISTO
 4002 IF(LIST(I).EQ.KP)GO TO 4003
      GO TO 5000
 4003 Z=0
```

```
      KKP=KP+1
      DO 4004 S=1,KKP
      IS=S-1
      KH=T*IZSR+IS*ISSR+1
      KH1=KP+Z+1
      A(KH)=C(KH1)
 4004 Z=Z+N-IS
      NKP=N-KP
      IF(NKP.LT.1)GO TO 4005
      DO 4006 S=1,NKP
      KH=T*IZSR+(KP+S)*ISSR+1
      KH1=KP*(N+1)-(KP*(KP-1))/2+S+1
 4006 A(KH)=C(KH1)
 4005 IP=T
      CALL MP3(A,R,M,O,N,IP,KP,IZSR,ISSR,1,1)
      B1=.FALSE.
      CALL MP9(A,C,IP,KP,N,M1,IZSR,ISSR,B1)
      GO TO 1000
 5000 LISTO=LISTO+1
      LIST(LISTO)=KP
      IPRT2(M+1)=IPRT1(KP)
      IPRT1(KP)=N+M1+KP
      L2(M+1)=M+1
      L20=M+1
      DO 5001 S=1,L10
 5001 IF(L1(S).EQ.KP)T=S
      DO 5003 S=T,L10
 5003 L1(S)=L1(S+1)
      L10=L10-1
      M=M+1
      T=M
      R=1
      GO TO 4001
 6000 IF(IPRT2(IP).LE.N+M1)GO TO 1000
      IPRT2(IP)=IPRT2(M)
      L20=L20-1
      L10=L10+1
      L1(L10)=KP
      LISTO=LISTO-1
      DO 6001 K=1,LISTO
 6001 IF(LIST(K).EQ.KP)GO TO 6003
      GO TO 6004
 6003 DO 6005 I=K,LISTO
 6005 LIST(I)=LIST(I+1)
 6004 NN=N+1
      KH1=IP*IZSR+1
      KH2=M*IZSR+1
      DO 6002 K=1,NN
      KHO=(K-1)*ISSR
      KH3=KH1+KHO
```

```
      KH4=KH2+KHO
6002  A(KH3)=A(KH4)
      M=M-1
      GO TO 1000
      END
      SUBROUTINE MP2(A,L2,L20,IP,IZSR,ISSR,KP,Q1,N,IV)
      DIMENSION A(120),L2(15)
      V=IV
      IP=0
      IF(L20.LT.1)RETURN
      DO 1I=1,L20
      KH=L2(I)*IZSR+1
      KH1=KH+KP*ISSR
    1 IF(V*A(KH1).GT.0.)GO TO 2
      RETURN
    2 Q1=V*A(KH)/A(KH1)
      IP=L2(I)
      IZ=I+1
      IF(IZ.GT.L20)RETURN
      DO 3I=IZ,L20
      KH=L2(I)*IZSR+1
      KH1=KH+KP*ISSR
      IF(V*A(KH1).LE.0.)GO TO 3
      Q=V*A(KH)/A(KH1)
      IF(Q.GE.Q1)GO TO 4
      IP=L2(I)
      Q1=Q
      GO TO3
    4 IF(Q.NE.Q1)GO TO 3
      IO=L2(I)
      DO 5 K=1,N
      KHO=IP*IZSR+K*ISSR+1
      KH2=IP*IZSR+KP*ISSR+1
      KH=IO*IZSR+K*ISSR+1
      QP=V*A(KHO)/A(KH2)
      QO=V*A(KH)/A(KH1)
      IF(QP.LT.QO)GO TO 3
    5 IF(QO.LT.QP)GO TO 6
    6 IP=IO
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MP3(A,IO,I1,KO,K1,IP,KP,IZSR,ISSR,IP1,IP2)
      DIMENSION A(120)
      KH=IP*IZSR+KP*ISSR+1
      PIV=1./A(KH)
      IIO=IO+1
      II1=I1+1
      KKO=KO+1
      KK1=K1+1
      DO 1 II=IIO,II1
      I=II-1
      IF(I.EQ.IP)GO TO 1
      KHO=I*IZSR+KP*ISSR+1
      IF(IP2.EQ.1)A(KHO)=A(KHO)*PIV
      DO 2 KK=KKO,KK1
      K=KK-1
      IF(K.EQ.KP)GO TO 2
      KH1=I*IZSR+K*ISSR+1
      KH2=IP*IZSR+K*ISSR+1
      A(KH1)=A(KH1)-A(KH2)*A(KHO)
    2 CONTINUE
    1 CONTINUE
      IF(IP1.NE.1)GO TO 4
      DO 5 KK=KKO,KK1
      K=KK-1
      KH2=IP*IZSR+K*ISSR+1
    5 IF(K.NE.KP)A(KH2)=-A(KH2)*PIV
    4 IF(IP2.EQ.1)A(KH)=PIV
      RETURN
      END


      SUBROUTINE MP5(A,L1,L10,Q1,KP,IZNR,IZSR,ISSR)
      DIMENSION A(120),L1(15)
      KP=L1(1)
      KH=IZNR*IZSR+1
      KHO=KH+L1(1)*ISSR
      Q1=A(KHO)
      IF(L10.LT.2)RETURN
      DO 1K=2,L10
      KHO=KH+L1(K)*ISSR
      IF(A(KHO).GE.Q1)GO TO 1
      Q1=A(KHO)
      KP=L1(K)
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MP8(A,IZNR,IZSR,ISSR,LIST,LISTO,KP,AMAX )
      DIMENSION A(120),LIST(15)
      KP=LIST(1)
      KH=IZNR*IZSR+1
      KHO=LIST(1)*ISSR+KH
      AMAX=A(KHO)
      IF(LISTO.LT.2)RETURN
      DO 1 K=2,LISTO
      KHO=KH+LIST(K)*ISSR
      IF(ABS(AMAX).GE.ABS(A(KHO)))GO TO 1
      KP=LIST(K)
      AMAX=A(KHO)
    1 CONTINUE
      RETURN
      END


      SUBROUTINE MP9(A,C,IP,KP,N,M1,IZSR,ISSR,B1)
      DIMENSION A(120),C(120)
      LOGICAL B1
      IN=N+1
      DO 9000 IRR=1,IN
      IR=IRR-1
      IF(IR.EQ.KP)GO TO 9000
      IZ=0
      IF(IR.GT.KP)IZ1=KP-1
      IF(IR.LT.KP)IZ1=IR
      IZZ1=IZ1+1
      DO 9002 ISS=1,IZZ1
      IS=ISS-1
      KH=IR+IZ+1
      KH1=IP*IZSR+IR*ISSR+1
      KHO=KP+IZ+1
      C(KH)=C(KH)+A(KH1)*C(KHO)
 9002 IZ=IZ+N-IS
      IT=IZ+KP
 9000 CONTINUE
 9100 IKP=KP+1
      IF(IKP.GT.N)GO TO 9300
      DO 9102 IS=IKP,N
      DO 9103 IR=IS,N
      KH=IZ+N-KP+IR+1
      KH1=IP*IZSR+IR*ISSR+1
      KHO=IT+IS+1-KP
 9103 C(KH)=C(KH)+A(KH1)*C(KHO)
 9102 IZ=IZ+N-IS
 9200 DO 9201 IR=IKP,N
      KH=IT+IR+1-KP
      KH1=IP*IZSR+IR*ISSR+1
 9201 C(KH)=C(KH)+A(KH1)*C(T+1)
 9300 IZ1=0
      DO 9301 IRR=1,Kp
```

```
      IR=IRR-1
      IZ=0
      KH=KP+IZ1+1
      KH1=IP*IZSR+IR*ISSR+1
      STORE=C(KH)+A(KH1)*C(T+1)
      DO 9302 ISS=1,IRR
      IS=ISS-1
      KH=IP*IZSR+IS*ISSR+1
      KH1=IRR+IZ
      C(KH1)=C(KH1)+A(KH)*STORE
 9302 IZ=IZ+N-IS
 9301 IZ1=IZ1+N-IR
 9400 IF(IKP.GT.N)GO TO 9500
      DO 9401 IR=IKP,N
      IZ=0
      IRR=IR+1
      DO 9401 ISS=1,IRR
       IS=ISS-1
      IF(IS.NE.KP)GOTO 9402
      IZ=IZ+1
      GO TO 9401
 9402 KH=IRR+IZ
      KH1=IP*IZSR+IS*ISSR+1
      KHO=IT+IR+1-KP
      C(KH)=C(KH)+A(KH1)*C(KHO)
 9401 IZ=IZ+N-IS
 9500 IF(.NOT.B1)GO TO 9600
      IZ=0
      KHO=IP*IZSR+KP*ISSR+1
      DO 9501 ISS=1,KP
      IS=ISS-1
       KH=IKP+IZ
      KH1=IP*IZSR+IS*ISSR+1
      C(KH)=C(KH)+A(KH1)*C(T+1)
 9501 IZ=IZ+N-IS
      DO 9503 IR=KP,N
      KH=IR+IT+1-KP
 9503 C(KH)=C(KH)*A(KHO)
      IZ=0
      DO 9504 ISS=1,IKP
      IS=ISS-1
      KH=IKP+IZ
      C(KH)=C(KH)*A(KHO)
 9504 IZ=IZ+N-IS
 9600 IZ=0
      DO 9601ISS=1,KP
      IS=ISS-1
      KH=KP+IZ+1
      C(KH)=0.
9601  IZ=IZ+N-IS
      C(T+1)=1./C(T+1)
      IF(IKP.GT.N)GO TO 9602
      DO 9603 IR=IKP,N
      KH=IR+IT+1-KP
 9603 C(KH)=0.
 9602 RETURN
      END
```