COMPREHENSIVE PROTECTION OF POWER TRANSMISSION LINE, USING ON-LINE DIGITAL COMPUTER

A DISSERTATION

Submitted in partial fulfilment of
the requirements for the award of the degree
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in
ELECTRICAL ENGINEERING
(Power System Engineering)

By

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CERTIFICATE

Certified that the dissertation entitled, "Comprehensive Protection of Power Transmission line, Using On-line Digital Computer", which is being submitted by Sri C.B. Sanghavi in partial fulfilment for the award of the Degree of Master of Engineering in Electrical Engineering (Power System Engineering) of University of Roorkee, Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is to further certify that he had worked for 6 months & 20 days. months from 1st Jan. 80 to 21st July 80. for preparing this dissertation at this University.

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ABSTRACT

Utilising an on-line digital computer to perform protection, switching and data collection functions of modern high voltage transmission system is attracting attention.

The idea of computers for primary protection is already brought out. The different methods of computer aided protection schemes like overcurrent, under voltage, frequency and impedance relays are developed. The problem at present, with this specific application is that the computer sampling rate must be very high in order to provide reliable tripping signal within one cycle after fault initiation. The faulted line must be tripped very fast, thus this situation calls for a very high speed and reliable computer.

Due to many advantages resulting from the use of computer for primary protection like adaptable relay characteristics, immunity from spurious transients and high frequency components, there has been many attempts made recently to adopt on-line digital computers for system primary protection schemes.

The digital protection system detects a power system disturbance by comparing the latest set of sampled data with the corresponding set obtain 1-cycle earlier; if sample differs more than a set amount, the fault processing programmes are executed.

The fault processing programmes have an algorithm that determines the type of fault, a routine that selects the voltage and current combinations for fault calculation.

The present dissertation is a critical review of comprehensive protection of high voltage power transmission line using on-line digital computer, and to indicate the feasibility of developing a complete protection scheme.

In present state of art, computers are applied for both ON and OFF line work on power system. The primary protection by computers appears to be long way off, however back-up protection and automatic switching control by the digital computers appear to be promising.

The relaying trends with digital computer where hardware cost for a given level of capacity has been dropping, and where software application and knowledge has been advancing.

The present dissertation work is attempting to give only new ideas for application of computers for transmission line protection along with how to implement it.

CONTENTS

Page No.

Certificate

Acknowledgements

Abstract (i)

CHAPTER : 1 - INTRODUCTION 1-11

1:1 : Introduction 1

1:2 : Digital methods of relaying 3

1:3 : Application of computers to decision making 5

1:3:1 Centralised protection scheme using computer 6

1:4: Computer Control 8

1:4:1 Primary protection with on-line computer 9

1:4:2 Off-line protection 10

1:4:3 Back-up protection 10

1:5: General remark 10

CHAPTER : 2 - SAMPLING FOR COMPUTER PROTECTION OF TRANSMISSION : 12-26

2:1 : General 12

2:2 : Methods of analysis 13

2:3 : Physical Quantity Transform 13

2:4 : Sample function 15

2:4:1 Sampled function frequency spectrum 17

2:4:2 Even and odd square wave 17

2:4:3 Sample derivative function 19

2:5 : Effects of harmonics 24

2:6 : Sampling for computer 25

contd...

Page No.

CHAPTER : 3 - **SIGNAL PROCESSING USING DIGITAL COMPUTER** 27-38
3:1 : Analog processing 27
3:2 : Digital processing 29
3:2:1 Response Selection 30
3:2:1 Time locked filter pair 32
3:2:3 Refinements 34
3:3 : Line fault detection 36
3:4 : Frequency Domain Analysis 38

CHAPTER : 4 - **IMPEDANCE MEASUREMENT USING DIGITAL COMPUTER.** 39-51
4:1 : On-line computers 39
4:2 : Impedance calculation based on peak values 40
4:3 : Computation of impedance using integrals of voltage & current 41
4:4 : Fourier analysis method 42
4:5 : Preductive calculations for peak values 46
4:5:1 Selection of voltave and current pairs 48
4:5:2 Tripping criteria 50

CHAPTER : 5 - **HIGH SPEED PROTECTION OF POWER SYSTEM USING A DIGITAL COMPUTER:** 52-64
5:1 : Relaying philosophy 52
5:2 : System Hardware 55
5:3 : System Software 58
5:4 : Relaying program 60

CHAPTER : 6 - **AN ALGORITHM FOR DIGITAL IMPEDANCE CALCULATIONS** 65-79
6:1 : General 65
6:2 : Mathematical background 67
6:3 : Calculating impedances 71
6:4 : Digital Impedance calculation algorithm 72
6:5 : Appendix I 78

contd...

Page No.

CHAPTER : 7 - CONCLUSIONS 80-85

7:1 : Comparison of different methods 80
7:2 : Selecting the relay reach limit 83
7:3 : Advantages of digital protection 83
7:4 : The current research work 84

BIBLIOGRAPHY.................................... 86-88

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CHAPTER - 1

INTRODUCTION

1.1. Introduction: The possibilities of utilising an on-line digital computer; to perform protection, switching and data collection functions of modern high voltage transmission system and substations is attracting attention. Of these functions protective relaying is likely the most exacting interms computer hardware of facilities and speed. Rockefeller[1] proposes a complete group of programme for protection both internal (transformers, busbars) and external (transmission lines) to substation. The increasing size, capacity and complexity of power systems, the necessity of highly reliable, sensitive and selective high speed protection scheme is widely felt. However, it is not possible to satisfy these requirements with conventional relaying. Recently a good amount of work has been done on development of high speed relays with the help of electron circuits.

Even with these sophistications, it is not possible to achieve the desired accuracy and selectively. Mann and Morrison[10,11] have concentrated upon the protection of transmission line by digital computer, as part of a larger project embracing a range of substation functions.

The earlier work concentrated only on the development of an individual protection scheme or relay rather than a complete protection system for transmission line. One more draw-back of the present relaying scheme is a large amount of build in redundancy, which unnecessarily increases the protection system cost.

It is felt that the modern on line stored program digital computers can offer a better solution to overcome these problems compared to the present day systems, these computers offer attractive compactness and flexibility and with the development of modern hardware technology, its cost will also go down. Such on-line digital computers are already being used for various operations in modern power systems, but it remains to use them for system protection.

With a view to establish more firmly the feasibility of digital computer requirements for the protection, not only for a single line but also for a group of transmission lines in a substation, the program described has been coded for a somewhat more recent computers which incorporates some of the features found to be desirable. The object of research projects is to develop a practical protection system by the digital techniques, using readily available digital hardwares. The basic relaying algorithms were developed and tested with digitally simulated data. Later, a computer and data acquisition equipment were obtained for real-time protection of the digital system on transmission line model in laboratory. The test results indicate the feasibility of developing a complete two terminal protection system.

1.2. Digital methods for relaying:

There are three different schemes for the application of digital computers in the power system protection.

1 - To instal a special purpose computer for each station.

2 - To use static relays for fault detection and a computer for decision making.

3 - To use centrally located computor for fault detection and decision making.

In the first scheme, various signals from the desired location are fed to the high speed A/D convertor, output of Analog/Digital is fed to digital computor. The computer calculates from these sampled inputs, the peak current, peak voltage; frequency; phase angle; power on each line etc. Once these values are known any type of relay, over-current, over-voltage, directional, impedance under-frequency and so on can be simulated with the help of appropriate software.

From the available information computer decides the type and location of fault and trips the particular line. Most of the relays can be realised with conventional criteria. However distance relay needs different approach to utilise the digital techniques to full extent. Digital methods are given to determine the impedance seen by the relays.

The line currents and voltages are sampled, simultaneously 8 to 16 times per cycle (480 to 960 samples per second) by the data acquisition equipment. The sampled data is digitised and stored directly into the memory of a computer for processing by the protection program. The computer is the computational unit of the system and performs the fault calculations and relaying logic.

The digital protection system detects a power system disturbance by comparing the latest set of sampled data with the corresponding obtained one cycle earlier. If corresponding set obtained differs by more than a set amount, the fault processing program are executed. The fault processing programs include an algorithm that determines the fault type, a routine that selects combinations of voltages and currents to be used in fault calculation and the basic fault calculation algorithm.

The digital protection system uses a directional - comparison blocking scheme for its tripping logic. Usually, power line carrier provides the communication link between terminals. The trip logic utilises R, L values in derived from the fault calculation algorithm, depending upon the relative position of the R, L values in the relay characteristic, appropriate action is taken. For phase-to-ground fault, a zero-sequence direction routine is added to determine the direction of the ground fault. The information provided by this routine is used to assure more reliable protection when ground fault occurs with high fault resistance.

1.3. Application of computers to decision making:

The present relaying systems have built in redundancy because it is felt that the duplication always improves reliability. We have carrier relays, supervised by distance relays which we in turn backed up by over current relays.

The problem posed in this section is - Is there any logical way to say this scheme of protection is optimal one for the system? The logical choice is made fromapplication of decision theory to the outputs of various relays. For a particular type of fault and location, all the relays can be neighbourhood of system give any one of the three following information:

a - Yes : there is a fault, trip the line

b - No : there is no fault do not trip the line

c - No information: the relay can not decide, whether there is a fault or not.

Special types of correction codes are to be developed for the successful transmission of these signals.

The above information is given by many relays since we have introduced redundancy in the system. Based on this conflicting signals, a decision has to be taken regarding what to do with the system. This requires decision making console; which gets random information from various relays, process them; measure the uncertainty and sends proper signals for tripping the line. In this scheme the digitalcomputer is used as final decision making unit. As soon as the conventional relay detect a fault, the computer is completely used for protection only. For the remaining period when system is healthy, the computer can be used for various purposes.

1.3.1 - Centralised protection scheme using computer:

This system is based on the assumption that highly reliable communication channels are available. In this scheme only one centrally located computer is used to which continuous signals from all lines are fed. The signals from the receiving ends of the lines are also brought. For this carrier frequency line communication, microwave channels or laser beam can be used. The operation of this scheme is similar to the first scheme and any type of relaying function can be realised, since the information regarding both ends of the line is available with the computer; there is no necessity to realise a distance relay characteristic inthis scheme. In addition to just sending tripping signals to the faulted lines; the computer also estimates the behaviour of the rest of the system when these faulted lines are removed.

If the system is likely to fall out of step, the computer sends correcting signals to inhibit the development swings. The same computer can also be used for recording the pre-fault and post-fault information, whether there is a mal operation of the system.

Evidently to perform all these functions on-line at such high speeds required for protection a reasonably larger high power speed computer is required. In order to ensure the safety of the system a bare minimum of conventional back-up relays are provided and normally the computer provides the primary relaying with the advent of reliable and economical techniques of communication this scheme may prove to be more ideal one.

1.4. Computer control:

To power system operation :

The application of computers to the power system operationhas been discussed and because of reliability of computer operation and its ability to initiate a particular operation in certain sequence which can be pre-programmed the computer control has become mandatory to perform complex operation.

The generator start up and shut down is one of such operation where the on-line computers are being employed. The same on-line computer can be used for monitoring and check up of various system constraints like temperature, system pressure, bearing oil properties and initiate an alarm or shut down depending upon the state of emergency. Similarly computers are also used for automatic generation control to maintain system frequency and certain tie-line loadings at pre-specified values. The same control can also be used to schedule generation at different stations to minimise the overall cost. In all these operation the system variable change slowly and the computer is required to scan the input data at reasonably low sampling rate so that between two samples there is adequate time for the computer to do the required calculations and to give out the required control signals. Further even if due to any reason the computer is out of service the manual control can be resorted to immediately with no serious consequences. This is the case with many process control computers.

1.4.1 - Primary protection with on-line computer:

The idea of using computers for primary protection is already brought out. The different methods of computer aided protection schemes like overcurrent, under-voltage, frequency and impedance relays are developed. The problem at present, with this specific application is that the computer sampling rate must be very high in order to provide a reliable tripping signal with-in one cycle after fault initiation. Further the possibility of going to manual control in case the computer fails is not there, as the faulted line must be tripped very fast. Thus this situation calls for a very high speed and reliable computer. However, due to many advantages resulting from the use of computer for primary protection like adoptable relay characteristics, immunity from spurious transients and high frequency components, there has been many attempts made recently to adopt on-line digital computers for system primary protection scheme; it shows the recent interest in this direction and how the computer can be used for primary protection.

1.b.2 - Off-line protection:

In addition to the on-line applications, there are many off-line applications of the computers to the power system problems. Some of these are load flow, short circuit, stability and reliability. Since these are not directly related to power system protection detailed discussion on these topics will not be taken up here. The contingency evaluation under various types of outages the system assessment is also one of the important off-line applications.

1.b.3 - Back-up protection:

Use of computers for back-up protection deserves serious consideration, since this system can operate slowly the computer has enough time to make necessary computations to permit isolation of the faulted system. This is an important aspect of the computer added back up protection. Planned back-up trip is necessary, computer can also help in rerouting (restoring ?) the supply in emergency by automatic switching at substations.

1.5. General remark:

Since on-line digital computers are already being used for various operations inpower systems, it is felt that there is no reason to bar its applications for the system protection. In present state of art, computers are

both for ON and OFF line work on power systems. The
protection by computers appears to be long way off;
back-up protection and automatic switching control
igital computers appear to be promising. There are
modules available, for specific aspect of protection.
ent research work in this area is attempting to give
ideas for application of computers for transmission
tection along with how to implement it. These works
ortant guide for protection schemes of future. The
relaying techniques, collect the samples of line
and voltages at high sampling rate by data acquisition
it. The sampled data is digitized and processing is
computer by the protection program. The computer is
utational unit performs fault calculations and
logic. The fault calculation algorithm determines
trical location of fault by calculating circuit
ers R (resistance) and L (inductance). The digital
ion system uses a directional comparison blocking
for tripping logic. The digital protection system
a power system disturbance by comparing the latest
sampled data with the corresponding set obtain 1-cycle
if sample differs more than a set amount, the fault
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ne that selects the voltage and current combinations
lt calculation and the basic fault calculation algorithm.

CHAPTER -2

SAMPLING FOR COMPUTER PROTECTION OF TRANSMISSION LINES

2.1. General:

A digital system to completely overlay a power system is of considerable interest since it could provide a facility to monitor, control and protect the power system on an integrated and co-ordinated basis. The use of digital computers in power systems for data logging and analysis, alarm processing, contigency evaluation and economic dispatch is well established. Some work has been done on power system digital control. However interest in the area of digital protection of transmission lines has occurred to the extent of a trial installation[10,11,15,18].

The main component of transmission line protection is the impedance relay. Since impedance is the complex value of the physical quantity, current or voltage, by sampling with two orthogonal functions of the variable time. It is the purpose of this chapter to examine some of the orthogonal functions suitable for digital relay sampling[2], and to compare the methods chosen to the presently used by the analysis in the frequency domain[4]. The orthogonal functions used are a sample and its derivative. Other orthogonal functions and second derivatives of a variable with its integral are orthogonal.

2.2. Method of analysis:

Sampling of a function can be defined as taking a weighted mean of some physical quantity over a narrow range of variable. This is also defination of convolution. Convolution in time domain is analogous to multiplication in the frequency domain. Therefore, to determine the effects of sampling of a physical quantity over a limited range of variable.

The procedure is as follows:

1. Transform the function of the physical quantity into the frequency domain.
2. Transform the function of sample variable into the frequency domain.
3. Multiply the transforms together to obtain the frequency spectrum of the sampled function and examine this spectrum.

2.3. Physical quantity transform:

During the normal operation voltage $v(t)$, and the current $i(t)$ can be described by sinusoidal functions as follows:

$$v(t) = V_m \sin 2\pi ft \qquad (1)$$

$$i(t) = I_m \sin (2\pi ft - \phi) \qquad (2)$$

where t is time

V_m is peak voltage

I_m is peak current

f is frequency, and

ϕ is phase lag.

However, on occurance of fault, a d.c. component and harmonics of current are likely. For short duration after fault the current $i_F(t)$ can be approximated as follows:

$$i_F(t) = I_{d.c.} + I_m \sin(2\pi ft - \phi) + I_3 \sin(6\pi ft - \phi) \qquad ($$

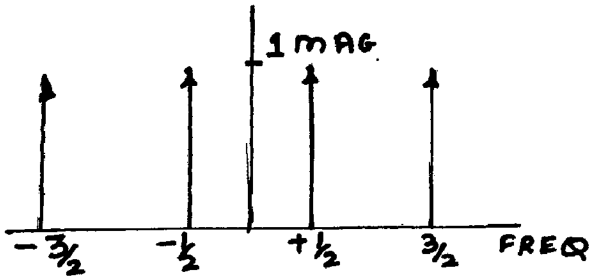
where

$I_{d.c.}$ is d.c. component

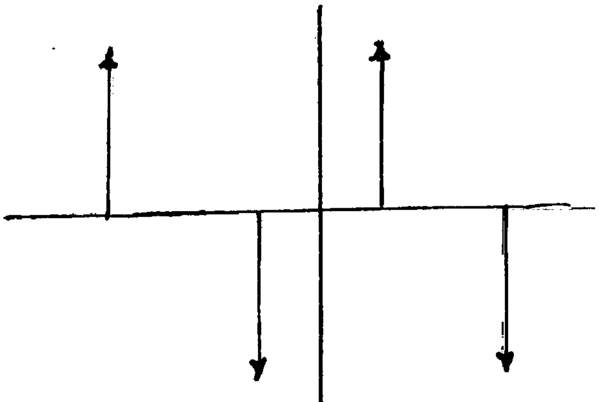
I_3 is third harmonic and above third harmonic are ignored.

The transform of this equation (3) the general form into real and imaginary axis transforms are shown in Figure 2.1(a) and 2.1(b).

The function is scaled so that the length of period is 1. Therefore 1 on the frequency scale is equal to 60 Hz. Since the magnitude of each frequency depends upon the exact system analysed, the components are shown to equal one for illustrative purpose only.



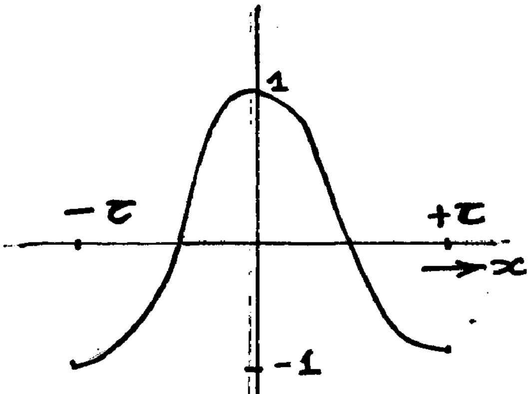
(a) Real axis



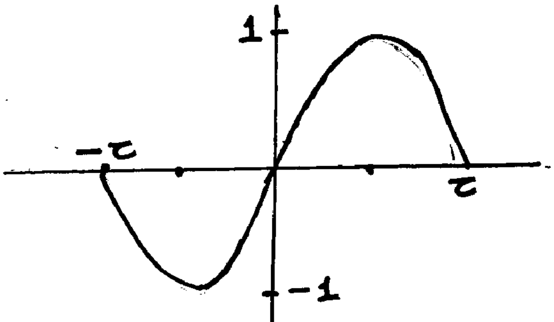
(b) imaginary axis

FIGURE 2:1

FAULT CURRENT TRANSFORM.



(a) $\cos \pi x$



(b) $\sin \pi x$

FIGURE 2·2

Time Domain orthogonal functions.

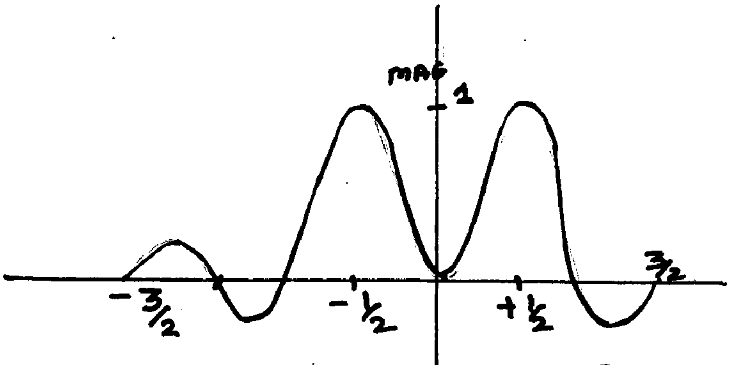
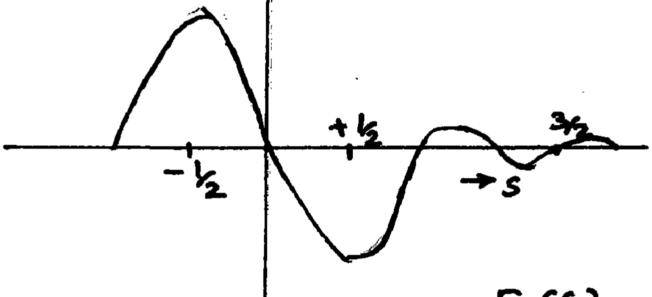


Fig 2·3 (a)· $F_c(S)$



2·3(b) $F_s(S)$.

FIGURE 2:3 FOURIER TRANSFORM OF SINE COSINE FUNCTIONS.

2.4. Sample functions:

A short listed set of orthogonal functions may be chosen to include the following:

1. Sine and cosine functions
2. Odd and even square waves
3. A value and its derivative (integral);

analysis and testing is limited to these cases.

2.4.1 - Cosine sine functions:

For incorporation into a sample system for computer relaying the length of function in the time domain must be limited. Since the 60 Hz impedance is to be used the period from $-\tau$ to $+\tau$ is $\frac{1}{60}$ seconds. The same function to be transformed is given in Figure 2.2(a) and 2.2(b).

The fourier transform $F_c(s)$ of cos πx is

$$F_c(s) = \int_{-\tau}^{+\tau} \cos \pi x \cdot e^{-j2\,sx} \cdot dx$$

Since cos x is an even function

$$F_c(s) = \int_{-\tau}^{+\tau} \cos \pi x \cdot \cos 2\pi xs \cdot dx$$

From trigonometric identities above equation becomes

$$F_c(s) = \tfrac{1}{2} \cos 2\pi x (s + \tfrac{1}{2})\, dx + \tfrac{1}{2} \cos 2\pi x (s-\tfrac{1}{2})\, dx$$

Solving $F_c(s) = \dfrac{\sin 2\pi\tau(s+\frac{1}{2})}{2\pi\tau(s+\frac{1}{2})} + \dfrac{\sin 2\pi\tau(s-\frac{1}{2})}{2\pi\tau(s-\frac{1}{2})}$

from scaling we have

$$\tau = 1$$

So that

$$F_c(s) = \tfrac{1}{2} \operatorname{sinc} 2(s+\tfrac{1}{2}) + \tfrac{1}{2} \operatorname{sinc} 2(s-\tfrac{1}{2}) \qquad (5)$$

where

$$\operatorname{sinc} s = \frac{\sin \pi s}{\pi s}$$

The Fourier transfer $F_s(s)$ of sin x is

$$F_s(s) = \int_{-\tau}^{+\tau} \sin \pi x \cdot e^{-j2\pi xs} \cdot dx$$

Since sin x is an odd function

$$F_s(s) = \int_{-\tau}^{+\tau} j \sin \pi x \cdot \sin 2\pi xs \cdot dx$$

Solving,

$$F_s(s) = j \frac{\sin 2\pi\tau(s-\frac{1}{2})}{2\pi\tau(s-\frac{1}{2})} - j\tau \frac{\sin 2\pi\tau(s+\frac{1}{2})}{2\pi\tau(s+\frac{1}{2})}$$

which is written as

$$F_s(s) = \tfrac{j}{2} \operatorname{sinc} 2(s-\tfrac{1}{2}) - \tfrac{j}{2} \operatorname{sinc} 2(s+\tfrac{1}{2}) \qquad (6)$$

Equations (5) and (6) are the Fourier transforms of the x domain functions shown in Figures 2.2(a) and 2.2(b).

2.4.1 - Sampled function frequency spectrum:

With the change of variable from ξ to f multiplication of the components shown in Figures 2.1(a) and Fig.2.3(a give the real axis frequency spectrum obtained by using the cosine function as a sample function. Figure 2.1(b) and 2.3(b) give the resultant imaginary axis spectrum. It shows that cosine sine functions have a high rejection to dc and to high order harmonics. Limiting the length, however, flattens and shifts the frequency spectrum around the fundamental. For the cosine and sign waveforms the displacement is to frequency slightly above or below 60 Hz respectively.

2.4.2 - Even and odd square wave:

The even and odd waves shown in figure 2.4(a) and 2.4(b) are orthogonal at 60 Hz. The importance of this function in computer relaying is that real and imaginary components of the physical quantity sampled are obtained through addition rather than multiplication. The Fourier transform, $F_e(s)$, of even square wave is

$$F_e(s) = \int_{-\tau}^{-\tau/2} -e^{-j2\ sx} \cdot dx + \int_{-\tau/2}^{+\tau/2} e^{-j2\pi sx} \cdot dx - \int_{\tau/2}^{\tau} e^{-j2\pi sx} \cdot dx$$

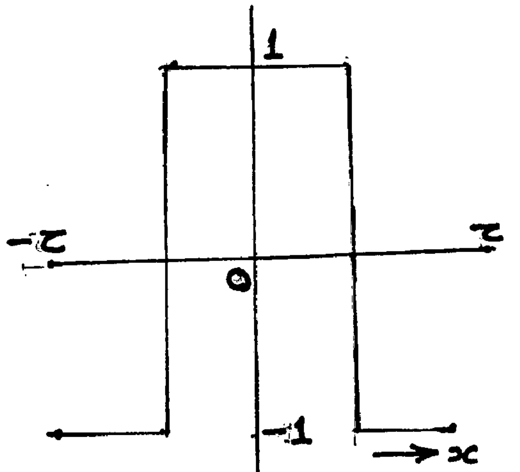


FIGURE 2:4 (a) even

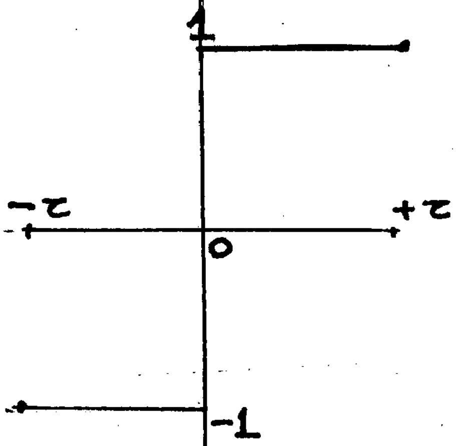
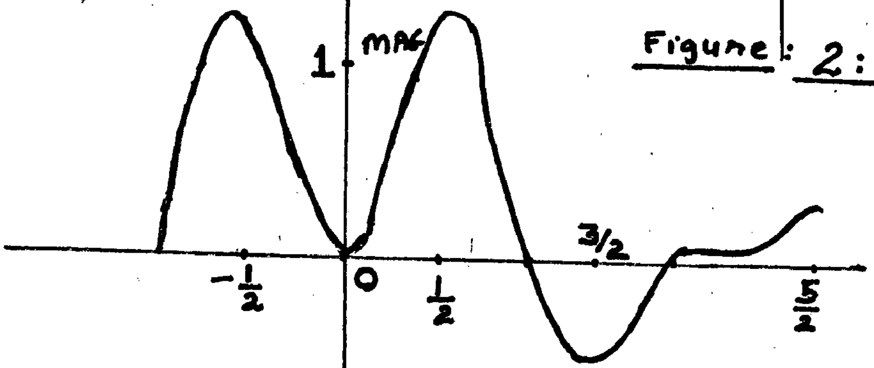
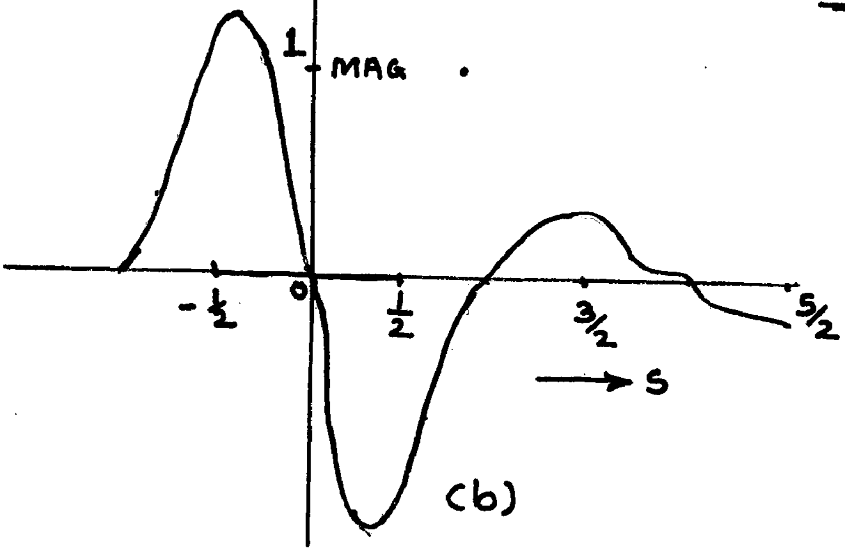


Figure: 2:4 (b) odd orthogonal square waves



(a)

FIGURE 2·5 (a) $F_e(S)$



(b)

2·5 (b) $F_o(S)$

orthogonal square wave transform

Square wave sample and current transforms:

With a change of variable from s to f convolution of square wave sample and current wave is obtained by multiplication of Fig.2.5 and Fig.2.1. The real and imaginary components are obtained fromthese two Figures by taking parts (a) and (b) respectively. It can be said that the square waves also have a high rejection to dc and tend to limit high order harmonics. Although attenuation of harmonic is limited, the relative magnitude between its real and imaginary components is approximately equal. At 60 Hz the per unit magnitude is greater than 1 as a result of the sampling transform. Thus absolute values of current and voltage are affected which may require rescaling of the analog inputs.

2.4.3 - Sample-derivative function:

The sample-derivative function is based upon the principle that a physical quantity i(t) is expressed as,

$$i(t) = I_m \sin(2\pi f t - \phi)$$

has an orthogonal derivative, i'(t),

$$i'(t) = 2 f \cdot I_m \cos(2\pi f t - \phi)$$

and i'(t) can be evaluated in the discrete case from a differential equation.

evaluating

$$F_e(s) = 2\tau \frac{\sin \pi s \tau}{\pi s \tau} - \tau \frac{\sin 2\pi s \tau}{\pi s \tau}$$

Further simplication gives

$$F_e(s) = 2\ \mathrm{s}\underline{\mathrm{in\ cs}} - \mathrm{si}\underline{\mathrm{nc}\quad 2s} \qquad (7)$$

The Fourier transform, $F_o(s)$ of odd square wave is

$$F_o(s) = \int_{-\tau}^{0} -e^{-j2\pi s x}\, dx + \int_{0}^{\tau} e^{-j2\pi s x} \cdot dx$$

Equating equation becomes

$$F_o(s) = 2j \frac{\sin^2 \pi s \tau}{\pi s \tau}$$

Simplifying for $\tau = 1$

$$F_o(s) = -2j \cdot \mathrm{sin}\underline{\mathrm{c}\ \ S}\ \sin \pi s \qquad (8)$$

Equations (7) and (8) are Fourier transform functions of x domain functions shown in figure 2.4(a) and 2.4(b).

These transform are shown in figure 2.5(a) and 2.5(b).

Square wave sample and current transforms:

With a change of variable from s to f convolution of square wave sample and current wave is obtained by multiplication of Fig.2.5 and Fig.2.1. The real and imaginary components are obtained fromthese two Figures by taking parts (a) and (b) respectively. It can be said that the square waves also have a high rejection to dc and tend to limit high order harmonics. Although attenuation of harmonic is limited, the relative magnitude between its real and imaginary components is approximately equal. At 60 Hz the per unit magnitude is greater than 1 as a result of the sampling transform. Thus absolute values of current and voltage are affected which may require rescaling of the analog inputs.

2.4.3 - Sample-derivative function:

The sample-derivative function is based upon the principle that a physical quantity i(t) is expressed as,

$$i(t) = I_m \sin (2\pi ft - \phi)$$

has an orthogonal derivative, i'(t),

$$i'(t) = 2 f \cdot I_m \cos (2\pi ft - \phi)$$

and i'(t) can be evaluated in the discrete case from a differential equation.

This is an important function in computer relaying for number of reasons. Firstly, it is the present method applied inpractical applications. Secondly, only present and immediately preceding samples are required. It should be noted here that modifications to this method may require three or more samples. None the less the number of samples required is minimal. Lastly the determination of instantaneous impedance utilises a straight forward technique. Running sums and such techniques are required to obtain instantaneous impedance in all other sample functions considered.

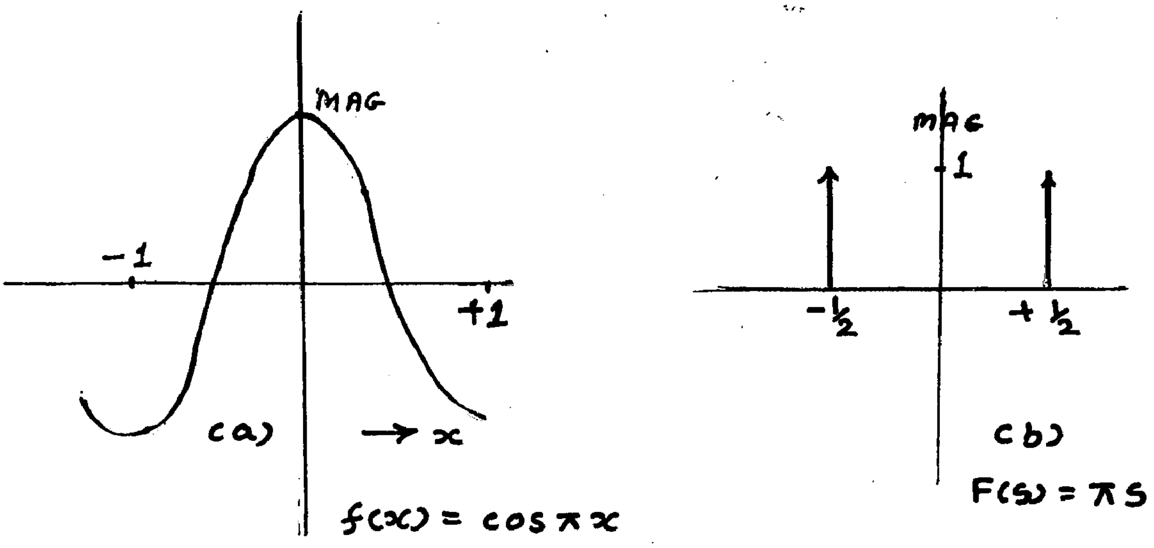
Now the equation $F_c(s) = \int_{-\tau}^{+\tau} \cos\pi x \cdot e^{-j2\pi sx}\, dx$ has limit from $-\tau$ to $+\tau$, the limits of integration are extended to cover the interval from $-\infty$ to ∞. The result is the well known Fourier transform, F(s), of the function f(x) = cos x

$$F(s) = \int_{-\infty}^{+\infty} \cos\pi x \cdot e^{-j2\pi sx} \cdot dx = \pi(s)$$

The transform is shown in Figure 2.6(a) and (b). As x increases s decreases, for s=o, f(x)=1

$$F(s) = \int_{-\infty}^{+\infty} e^{-j2\pi sx} \cdot dx = \delta(s)$$

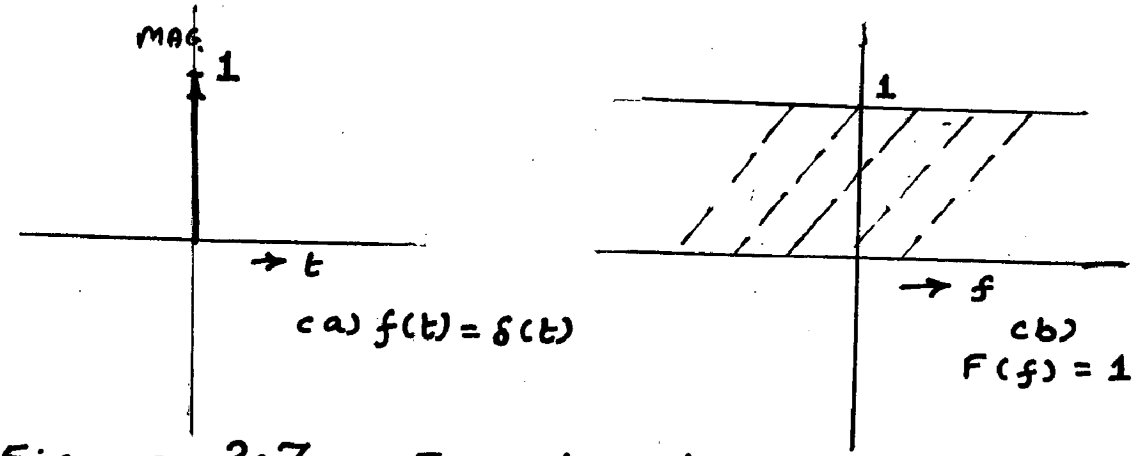
$$f(t) = \int_{-\infty}^{\infty} e^{-j2\pi ft} \cdot df = \delta(t)$$



(a) f(x) = cos πx

(b) F(s) = πs

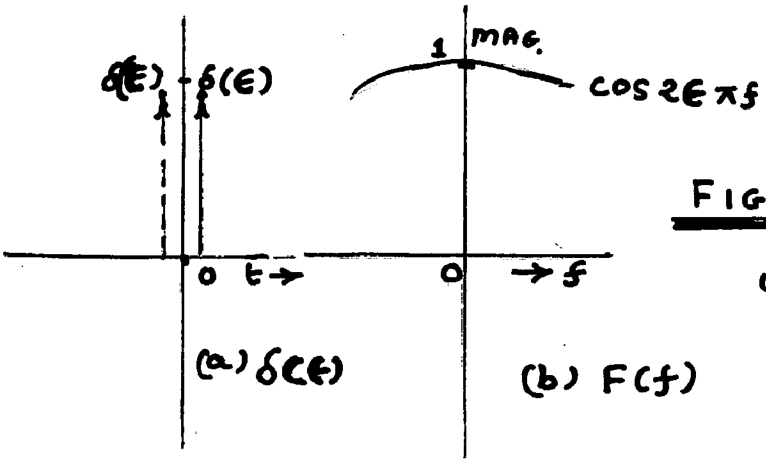
FIGURE 2.6 Cosine transform pair



(a) f(t) = δ(t)

(b) F(f) = 1

FIGURE 2.7 Impulse transform pair.



(a) δ(ε)

(b) F(f)

FIGURE 2.8 delayed impulse transform pair

This equation is shown in figure 2.7 and here $F(f) = 1$

(t) implies a unit pulse when $t = o$.

For an impulse at $t = \epsilon$ where ϵ is small value, the transform pair is shown in Figure 2.8.

If (t) is interpreted as a sample function, for a sample interval 'a' that is discrete, the time domain function can be expressed as $f(t) = \delta(na)$

Under the condition that

$$na = t - kT \quad \text{where} \quad k = o,\ 1, 2 \ldots\ldots\ldots$$

such that $o \leqslant na \leqslant T$ where T is the period.

The Fourier transform of $\mathcal{F}\delta(na)$ of the discrete sample function, $\delta(na)$, is

$$\delta(na) = \cos(na \pi f)$$

The sample value of the current, $i(t)$, as expressed can be now written as,

$$i(t) = (na)$$

and can be evaluated in the frequency domain by multiplying the transforms of the functions such that

$$na = t - kT - \emptyset - \frac{\pi}{2}$$

Similarly, the transform of the finite difference equation is evaluated. Taking the transform,

$F(s)$, of $f(s) = \sin \pi x$

$$F(s) = \int_{-\infty}^{+\infty} \sin \pi x \;\; e^{-j2\pi xs} \cdot dx \;=\; j\, I_1(s).$$

This transform pair is shown in Figure 2.9. It then follows for a change of variable

$$f(t) = \sin \pi t$$

$$F(t) = j\, I_1\,(f)$$

The finite difference of $f(t)$ taken over an interval b is

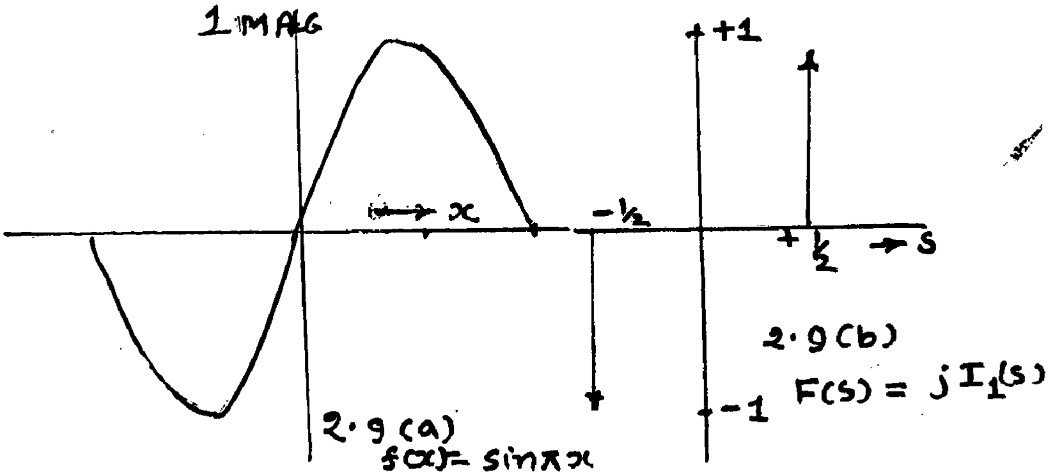
$$\Delta f(t) = f(t+\tfrac{b}{2}) - f(t-\tfrac{b}{2})$$

But this is function shown in Figure 2.9(b), so that the finite difference is the equivalent of convolution.

That is $\quad \Delta f(t) = I_1(t)^{\circ}\; f(t)$

Convolution is multiplication in frequency domain. Thus the finite difference of a function of time expressed, can be evaluated in frequency domain by a shift of axis such that

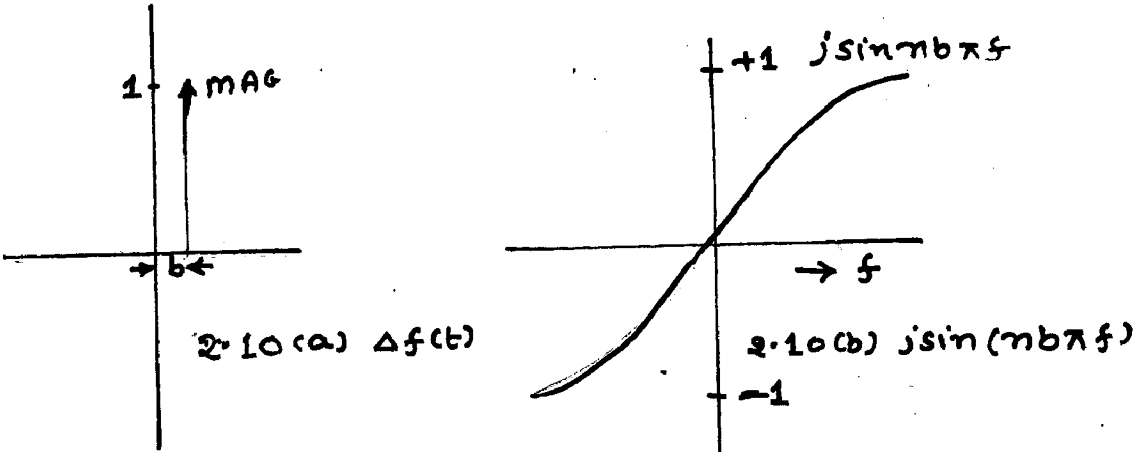
$$f(t) = f(t - \phi - \frac{\pi}{2}) \;-\; f(t - \phi - \frac{\pi}{2} - b)$$



2·9 (a) $f(x) = \sin\pi x$

2·9 (b) $F(s) = jI_1(s)$

FIGURE 2·9 sine transform pair.



2·10 (a) $\Delta f(t)$

2·10 (b) $j\sin(nb\pi f)$

FIGURE 2·10 FINITE DIFFERENCE TRANSFORM.

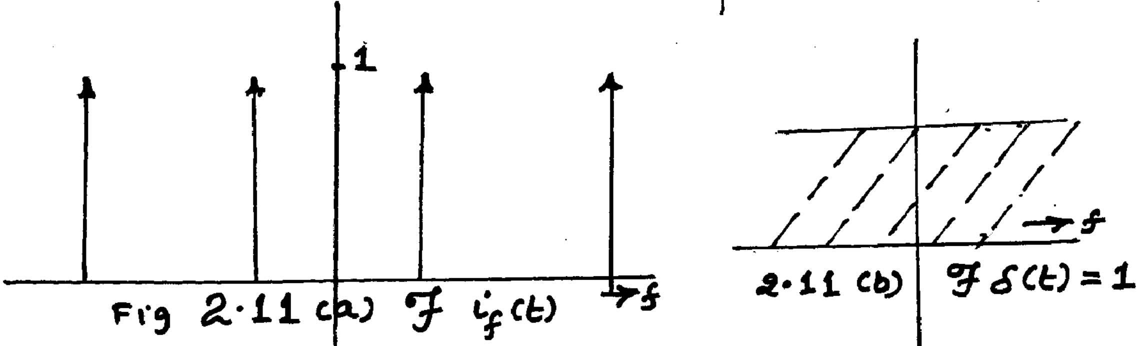
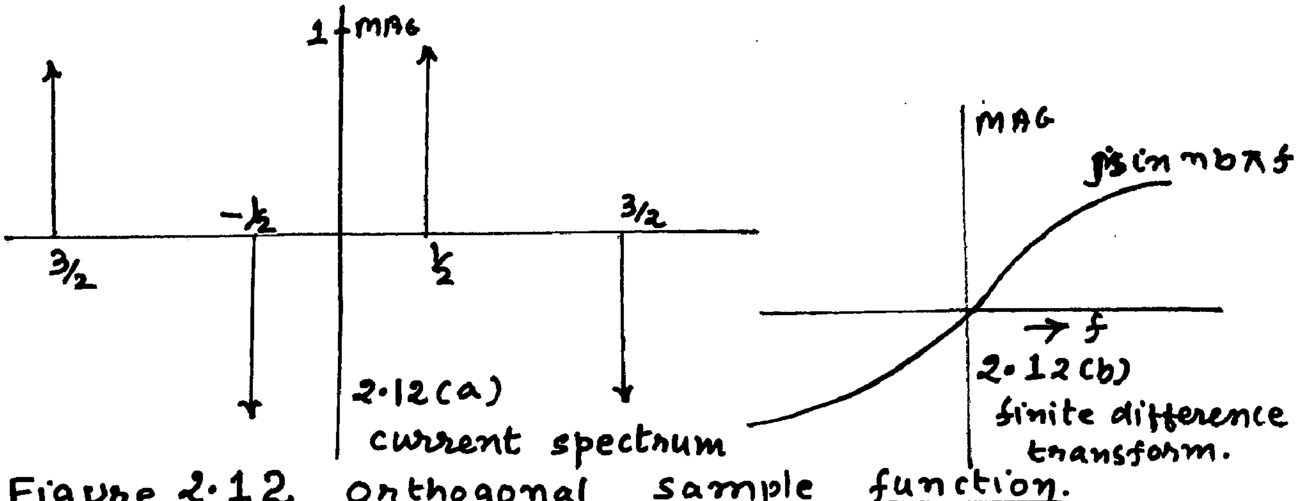


Fig 2·11 (a) $\mathcal{F}\, i_f(t)$

2·11 (b) $\mathcal{F}\,\delta(t) = 1$

FIGURE 2·11 Synchronous SAMPLE TRANSFORM.



2·12 (a) current spectrum

2·12 (b) finite difference transform.

Figure 2·12 orthogonal sample function.

where $\frac{b}{2}$ represents a further shift to account for the method used in digital relay system. The Fourier transform F(f) of $\Delta f(t)$ of above equation is

$$F(f) = j \sin (nb\pi f)$$

where $nb = t-kT = \phi - \frac{\pi}{2} = \delta \cdot b$

The finite difference transform pair is shown in Figure 2.10.

Figure 2.10(b) indicates that faster sampling increases the period of $j \sin (nb\pi f)$. Thus the product of $j \sin (nb\pi f)$ and Fourier transform of the 60 Hz current on the imaginary axis gives a decrease in the value. This is as expected since decreasing the time between samples causes small absolute change of variable.

The sampling techniques used in the relaying can be examined from the frequency domain view point.

For a sample taken exactly in phase

$$\cos (na\pi f) = 1$$

so that the value on the real axis is

$$\delta (f) \; I_n \cos (na\pi f) = I_n$$

The imaginary axis value is

$$2 X_1(f) \; j \; I_n \sin (nb\pi f) \approx 0.$$

The approximate sign arises from the delay of $b/_2$. For the following samples, the value of the sample function $\delta(na)$ decreases in the frequency domain and the value of the finite difference transform $j \sin(nb\pi f)$ increases until finally the Fourier transform of (na) vanishes and

$$\mathcal{F}\Delta f(t) = j\,Im$$

This is a different way of looking at sampling: This gives a clear picture of the harmonics and helps to clarify the filtering requirements and therefore the delay.

2.5. Effects of harmonics:

In the event of a fault harmonics occur, the two limiting cases from above can be studied. Case 1 is taken as the time when instantaneous value of current is maximum. Case 2 is taken when the instantaneous value of current is zero.

In general the harmonics which occur depend upon the low pass filter cut off frequency. It is assumed that the filter cut-off frequency is approximately equal to 180 to 200 Hz to permit observation of the effects of the third harmonic.

Case 1 occurs at the maximum instantaneous current so that the transform of the sample function $\delta(t)$ is a constant and normalised this constant is 1. The transform of the finite difference can be taken as zero. The transform of the fault current from Fig.2.1(a) and the sample function from Fig.2.7(b) are then synchronised and are shown in Fig.2.11.

This function does not reject either the d.c. or third harmonic for this case 1, as one might expect. However, there are values of na for which the third harmonic is attenuated by the sample function.

Case 2 occurs at near zero instantaneous current so that the finite difference has maximum effect. The transforms of current and finite difference from Figure 2.1(b) and 2.10(b) respectively, shown in Figure 2.12.

For the case shown, which represents small values of nb, the fundamental is attenuated to a greater extent than the third harmonic. A value of nb can be chosen to increase the fundamental and decrease the third harmonic. This means an increase in sample interval or the storage of previously sampled values. These are strictly illustrated cases, the trend when harmonics are present is discussed. It can be also shown that in cases when $0 < na < \frac{\pi}{2}$ and harmonics are present an error in phase determination occurs.

2.6. Sampling for computer:

This chapter illustrates a method of analysis for determining the sample functions for computer relaying. From analysis it is shown that functions with high rejection to the d.c and high harmonics follow the system impedance. Functions without high rejection introduce errors in impedance determination which under certain conditions may speed correct tripping. The different types of techniques useful for online implementation of impedance calculation for transmission line protection using digital computer can be developed. Usually the Fourier analysis method is used to compute the line voltage and current fundamental frequency amplitudes from a set of samples over one full period of system frequency; and samples are weighted by a set of predetermined coefficients which are pre-fed to computer.

CHAPTER - 3

SIGNAL PROCESSING USING DIGITAL COMPUTERS

3.1. Analog processing:

The transmission line and power system protection function dictates the character of voltage and current signal processing requirement in the replacement of conventional instrumentation, and relays by digital computers. The details of a signal processing technique derived from frequency domain consideration are explained in this chapter. The signal processing technique is designed on the basis of a frequency domain analysis of transmission line protection requirements, and such factors are on analog filtering, data window size and sampling frequency are chosen by the frequency characteristics. Performance of signal processing can be demonstrated by its application to transmission line fault detection.

Transmission line fault detection can be achieved by noting a change in the measured system impedance. The location of the fault relative to measuring point may be determined from the steady state value of post fault impedance. Therefore the sufficient information should be contained in a small bandwidth around the power system frequency. All steady state information is at power frequency and higher frequencies are only required transiently to permit a sufficiently fast change between pre-fault and post fault conditions. Protection may therefore be considered in part, as notch filtering process in the frequency domain.

The value of highest frequency required is of considerable importance. It is necessary for

f_s —— $2f_{max}$(1) where f_s = sampling frequency

$_{max}$ = maximum frequency present in sampled signal.

The current and voltage signals must be band-limited by an analog low pass filters, prior to sampling. Failure to meet the restriction of equation (1) may lead to errors in data samples.

If a low cost RC filter (Figure 3.1) is used, the cut-off or half power frequency f_c will be given by,

$$f_c = \frac{1}{2\ RC} \qquad (2)$$

The rise time t_r (10 - 90 %) of this filter to a step input is given by,

$$t_r = RC\ (-\ln 0.1 + \ln 0.9) = 2.2\ RC \qquad (3)$$

combining equations (2) and (3)

$$f_c = \frac{0.35}{t_r} \qquad (4)$$

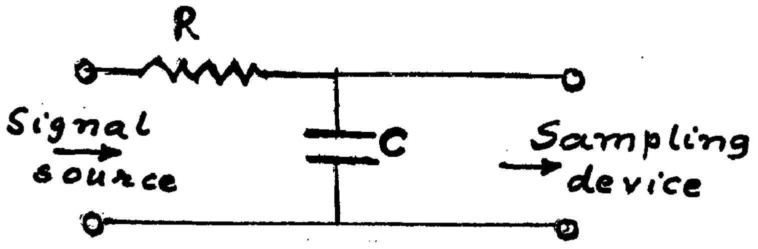
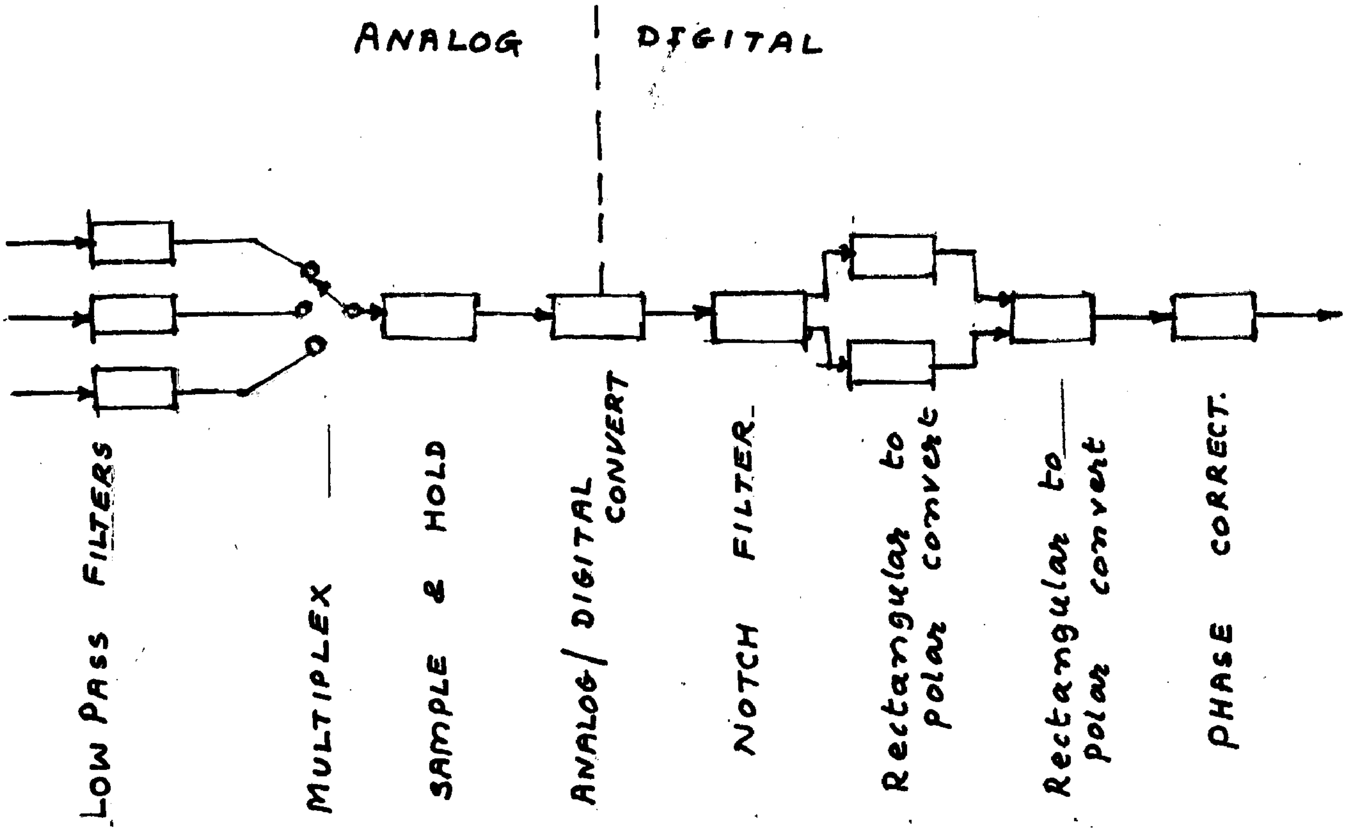


FIGURE 3:1: RC LOW PASS FILTER.



URE 3:2: OVER ALL SIGNAL PROCESSING SCHEME.

The minimum rise time required can be determined on speed requirements of protection. The shorter detection times have less advantages due to relative speed of circuit breaker and trip circuit operation. At least 4 samples per cycle eliminates errors also results in an inter sample delay consistent with the selected rise time. At this sample rate, data is available to perform two or three impedance calculations within the maximum fault detection time. The fault detection time of 1 cycle (15 to 17 ms) and allowing time for computation and trip decision checking and rising time, the cut-off frequency to an 85 Hz is more suitable. The low sample rate and simple filtering reflect directly in the cost of implementation.

3.2. Digital processing:

Processing of the sampled signals in the computing system is also analysed on a frequency domain basis. The design of a digital algorithm on frequency basis is known as digital filtering. This technique is well utilised in communication engineering. In power engineering field and in transmission line protection we can use this technique.

The basic description of digital filtering is explained here. Digital filtering is essentially the process of multiplying signal samples by samples of impulse response of the filter. The filter response will be larger than one sample interval so the signal or filter does not violate the constraints of equation (1). The filter response will be sampled more than once and the process will involve several point-by-point multiplications and a summation of the results for each output sample generated. A succeeding output sample will be generated by all the input data samples by one position past the constant samples. This process is discrete form of convolution.

$$\text{if} \quad Y_{(jw)} = H_{(jw)} \cdot X_{(jw)}$$

where $A(jw) \triangleq$ Fourier transform of $a(t)$

$$\text{then} \quad y(t) = h(t) \circ x(t) \qquad (5)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau) \cdot d\tau$$

3.2. Response selection:

The band of frequencies around 60 Hz willbe passed by a filter with 60 Hz sine wave impulse response. The response must be finite in length. The truncation points will affect frequency response, so they may be chosen on this basis.

For a truncated sinusoid impulse

$$H(w) = \int_{-B}^{+B} \cos(\Omega t).\ \exp(-jwt)\ dt$$

where $B = TP/2$ (6)

$$= \frac{\sin\ (1-\omega/\Omega)\ p}{(\Omega - \omega)} + \frac{\sin\ (1+\omega/\Omega)\ p}{(\Omega + \omega)}$$

where Ω = impulse response sinusoid frequency

p = number of cycles in impulse response

To determine the effects of dc offset in fault currents, $H(0) = 0$

$$\text{which is} = \frac{2 \sin(\pi p)}{\Omega} = 0 \qquad (7)$$

The frequency response 60 Hz and 85 Hz should be as uniform as possible to permit acceptable rise times.

$$\frac{H(2\pi 85)}{H(2\pi 60)} = \frac{H(1.42\,\Omega)}{H(\Omega)} = \frac{1}{p}\ \frac{\sin(-0.42\pi p)}{-0.42} + \frac{\sin(2.42\pi p)}{2.42} \qquad (8)$$

Maximum gain at 85 Hz together with uniformity requirements are obtained for p=1. In short, it can be decided to combine data samples covering 1 cycle to calculate the impedance.

The choice of data window duration appears to be an important factor. The 60 Hz digital filter for 240 Hz sampling frequency is defined by,

$$Y_k = \sum_{n=1}^{4} X_{n+k-4} \quad \sin\frac{\pi n}{2} \tag{9}$$

where x_i = i'th input sample.

y_i = i'th output sample.

In presence of a decaying d.c. offset component inthe signal, a data window of one sample period plus one half cycle of the fundamental frequency enables the phasor computationto be performed with good accuracy.

3.2.2 - Time locked filter pair:

The computation required for digital filtering is significantly reduced if the equation (9) is replaced by equation(10

$$y_k = \sum_{n=1}^{4} x_{n+k-4} \quad \sin\frac{\pi(n+k)}{2} \tag{10}$$

The filter coefficients are periodic, they may be moved in step with incoming signal samples so that the each sample is associated with a single coefficient only.

In addition the use of equation (10) results in a frequency translation so that a 60 Hz input results in d.c output. The disadvantage of using equation (10) instead of (9) is that the filter outputs a re on input signal phase and its amplitude. The phase difference can be eliminated additionally calculating

$$z_k = \sum_{n=1}^{4} x_{n+k-4} \frac{\cos\pi(n+K)}{2} \qquad (11)$$

$$c_k \angle \phi_k = z_k - jy_k \qquad (12)$$

That is, a phasor quantity is formed by input signal by combining the outputs of two orthogonal filters. This procedure is followed by anlogy after noting similarity between equation(10) and the fundamental sine coefficient of a Fourier series analysis. A more general and rigorous consideration based on orthogonality requirements are discussed earlier.

The simplicity of equation (11) and (12) for filtering and phasor formation is that the entire algorithm reduces to calculating one of the equations at each sampling rate.

$$\begin{aligned} c_k \angle \phi_k &= (-x_{k-2} + x_k) - (x_{k-3} - x_{k-1}) \\ c_{k+1} \angle \phi_{k+1} &= (-x_{k-2} + x_k) - j(-x_{k-1} + x_{k+1}) \\ c_{k+2} \angle \phi_{k+2} &= (x_k - x_{k+2}) - j(-x_{k-1} + x_{k+1}) \qquad (13) \\ c_{k+2} \angle \phi_{k+3} &= (x_k - x_{k+2}) - j(x_{k+1} - x_{k+3}) \end{aligned}$$

Note that no products are required and further more that only one of two components is recalculated at each sampling instant.

3.2.3 - Refinements:

<u>Phase correction</u> :- It is possible to eliminate the requirement of simultaneous sampling of sample quantities. Instead of that sample signals sequencially and apply phase correction by computation. This correction, θ is added by modifying the filter coefficients in equations(10) and (11).

$$y_k = \sum_{n=1}^{4} x_{n+k-4} \sin \frac{\pi(n+k)+\theta}{2} \qquad (14)$$

$$z_k = \sum_{n=1}^{4} x_{n+k-4} \frac{\cos\pi(n+k)+\theta}{2} \qquad (15)$$

OR modify equation (12)

$$c_k \underline{/\phi_k + \theta} = z_k - jy_k \qquad (16)$$

The choice of technique depends upon the use of the phasor quantity.

The sequential sampling rather than simultaneous sampling can reduce costs considerably since a single sample/hold amplifier can be switched between a large number of signals rather than requiring one amplifier per signal. Sequential sampling capacity also increases the opportunities for multiple use of signals since phase can be assigned relative to any reference after the act of sampling.

Ripple removal :- In practice, the time locked filter pair impulse frequency will not exactly match the power system frequency. This is due to equipment tolerances. This will result with a low amplitude ripple superimposed on equation (10) and (11). The ripple will be at a frequency equal to sum of the system and notch filter frequencies and is close to 120 Hz. This may be removed by a stage of digital low pass filter. Two point averaging response can be choosen. This gives sample equation corresponding to (13).

$$
\begin{aligned}
2\,C_k \angle \phi_k &= (x_{k-4} - 2x_{k-2} + x_k) - j\,(2\,x_{k-3} - 2\,x_{k-1}) \\
2\,C_{k+1} \angle \phi_{k+1} &= (-2\,x_{k-2} + 2\,x_k) - j\,(x_{k-3} - 2\,x_{k-1} + x_{k+1}) \\
2\,C_{k+2} \angle \phi_{k+2} &= (-\,x_{k-2} + 2\,x_k - x_{k+2}) - j\,(-2x_{k-1} + 2x_{k+1}) \\
2\,C_{k+3} \angle \phi_{k+3} &= (-2x_{k+2} + 2x_k) - j\,(-x_{k-1} + 2x_{k+1} - x_{k+3}) \qquad (17
\end{aligned}
$$

This signal processing technique could be adopted to every sensed current and voltage in transmission line protection and in the switching station. The phasor results may be directly used for any metering, alarming and controlling function and for equipment and transmission line protection via differential comparison of impedance calculations. The overall signal processing scheme is shown in Figure 3.2.

3.3. Line fault detection:

Classical system protection engineering techniques show that the three phase transmission line is characterised by six impedances. Three impedance s are associated with a single phase to ground and theother three impedances are between phase pairs.

$$Z_{pn} = \frac{V_{pn}}{I_p + KI_o} \qquad (18)$$

where $K = \frac{Z_0}{Z_1} - 1$

p = phase designation

V_{pn} = phase-neutral voltage

I_p = phase current

I_o = zero sequence current

$\frac{Z_0}{Z_1}$ = ratio of zero sequence to positive sequence impedance for line.

and

$$Z_{p_1p_2} = \frac{V_{p1n} - V_{p2n}}{I_{p1} - I_{p2}} \qquad (19)$$

Line fault detection and location is accomplished at each sampling instant by comparing each of these six impedances with predetermined zone of impedance value. The equations (13) and (17) must be synchronized with the power system voltage or current. The location of an impedance within the zone indicates the occurrence and location of a corresponding fault. Impedance values outside the zone indicate either no fault or faults beyond the intended reach of the measuring point.

It is anticipated that considerable electric system operational can be accomplished with digital computer protection. Operating impedance zone shaping can be defined without the usual physical restrictions of conventional protective relays. Off-line digital impedance relay data can be used with refined estimates of line and fault parameters for on-line use.

The availability of impedance results one cycle after the fault is adequate for the majority of transmission line applications; there is a substantial number of installations where critical fault clearing time are low and higher operating speed is necessary. One disadvantage of using 4 samples per cycle is, to prevent false detection, at least 3 successive calculations are necessary to produce impedance values within the protected zone before a trip signal is sent to circuit breaker. If sampling rate is increased, it shows that equation(1 and (17) becomes more complicated, since the sine and cosine factors (10) and (11) can be no longer evaluated at integral multiples of $\pi/2$.

3.4 Frequency domain analysis:

Frequency domain analysis of the signal processing requirements of transmission line protection by digital computer has led a scheme which eliminates many problems. The division between analog and digital processing functions and the choice of the data time window have been rationalized. A simple analog low pass filter, non-simultaneous sampling capacity and sampling frequency close to the theoretical minimum can be proved effective in providing signal processing suitable for high speed transmission line protection. The processed phasor quantities derived from each signal are also useful for other switching station functions besides line protection. The frequency domain analysis approach is adptable for a single centralised computer at each station or multiple processors, each associated with a single piece of power apparatus.

CHAPTER - 4

IMPEDANCE MEASUREMENT USING DIGITAL COMPUTER

4.1. On-line computers:

On-line computers for system protection are already in the use as backup relays in many transmission systems. With the advent of high speed relays using solid state devices, attention is directed towards the application of digital computers for protection. These computers not only sense an abnormal operation in the system within the protection zone but also take a proper decision to rectify the trouble. This logical decision is essential particularly when the system is large and normally operates at its critical load. With the growing reliability of computer hardware and high speed of operation, it is possible that the computers can to used on-line, for primary relaying and the conventional relaying systems will take place of backup protection.

If computer is fed with the sampled values of voltage and current signals derived from the transmission line, it will be possible to compute peak current, peak voltage, phase angle and impedance in each cycle. Here the computer can function as an over current relay,under voltage relay, reverse power relay or a distance relay. In fact any type of conventional relay characteristics can be realised from the computer.

It is necessary for the computer tostore the sample values at least over a period of 1 cycle of the system frequency. With these values it computes all the relevant information required for system checkup. All these computations have to be completed before the next sample is fed. With the arrival of the new sample the first sampled value in the previous ensemble is delted and the same computation is again made over the present set of samples. This procedure is continuously repeated by the on-line computer[10,19].

4.2. Impedance calculation based on peak values:

The method of obtaining the peak current and peak voltage is quite simple. The calculation of impedance based on the peak values and the phase angle based on zero crossing will be erroneous. This method requires digital computer for implementation. Among the sample values of voltage and current over a period of one cycle, the positive and negative peak amplitudes are found and their average magnitude is taken to represent V_{max} or I_{max}.

$$V_{max} = \frac{|V_{peak+ve}| + |V_{peak-ve}|}{2}$$

$$I_{max} = \frac{|I_{peak+ve}| + |I_{peak-ve}|}{2}$$

$$Z = \frac{V_{max}}{I_{max}} \qquad (1)$$

The time difference (T_{+ve}) and (T_{-ve}) are computed for each cycle. The average of these gives a measure of the phase angle

$$\text{Phase angle} = \left[\frac{(T_{+ve}) + (T_{-ve})}{2}\right]\omega \qquad (2)$$

The ω is angular frequency. These computations are repeated for every one cycle of faulted wave forms. The computed values of Z impedance and ϕ phase angle in each cycle are given in Table 4.1:

	impedance	phase angle
1 cycle	0.592	108°
	0.578	99°
2 cycle	0.578	90°
	0.563	90°
3 cycle	0.582	81°

4.3. Computation of impedance using the integrals of voltage and current:

If the voltage and current signals are assumed sinusoidal

$$v = V_m \sin wt$$

$$i = I_m \sin (wt - \phi) \qquad (3)$$

then integrals over a period of one cycle of rectified wave forms are given by,

$$\int_{\frac{\alpha}{\omega}}^{\frac{2\pi+\alpha}{\omega}} |v|\ dt = 4\ V_m$$

$$\int_{\frac{\alpha}{\omega}}^{\frac{2\pi+\alpha}{\omega}} |i|\ dt = 4\ I_m \qquad (4). \quad \text{for any}\ \alpha$$

The effect of higher frequencies present in the transient wave forms, will be almost nullified due to integration. The impedance measured by this method will be more accurate than the one measured from individual samples. This method has one drawback that it is not easy to obtain the phase angle in this case. The percentage variation of impedance over the mean is much less than the variation in impedance obtained by first method.

4.4. Fourier analysis method:

The impedance and phase angle, from the ensemble of samples over period of one cycle is done by Fourier analysis. The ensemble of samples over a period of one cycle is assumed to repeat periodically and Fourier analysis is performed on the ensemble values of these samples.

The method of obtaining the phasor representation from data samples uses Fourier transform equations and in this case it is with advantage to have sample rate a multiple of fundamental frequency. Thus nx60 where (n = 3,4.....) are acceptable sampling rate[20].

Data window is the time span covered by sample set needed to execute the computation procedure. The smaller the data window, the faster is the response of computer procedure. In most Fourier Transform calculation a data window of one period of fundamental frequency is considered to be desirable. Half cycle data window is also acceptable. In the presence of a decaying d.c. component in the signal, a data window of one sample period plus one half cycle of fundamental frequency enables the phasor computations to be performed with acceptable accuracy.

Table 4.2

Sample rate	Min. data window response in ms
240	12.5 ms
360	11.1
480	10.4
720	9.7
960	9.4

The rate of 720 Hz is attractive as the corresponding Fourier coefficients are 0, ± 1, $\pm \frac{1}{2} \pm \frac{\sqrt{3}}{2}$. All the coefficients are easily obtained in integral arithmetic, even the last coefficient has an adequate binary representation of five terms. The digital harmonic filter equations corresponding to this sampling rate are executed quite easily. Fourier transform or digital harmonic filter equations for signal with constant d.c offsets are as follows:

Fourier transform of N data samples taken over one period of fundamental frequency produces the phasor estimate for the fundamental frequency.

$$y_c = \sqrt{2}\, Y_c = \frac{2}{N} \sum_{k=1}^{N} y_k \frac{\cos \pi k}{N}$$

$$y_s = \sqrt{2}\, Y_s = \frac{2}{N} \sum_{k=1}^{N} y_k \frac{\sin 2\pi k}{N} \qquad (5)$$

for N = 12

$$6\, y_c = -(y_6 - y_{12}) + \frac{1}{2}(y_2 - y_8 - y_4 - y_{10}) + \frac{\sqrt{3}}{2}(y_1 - y_7 - y_5 - y_1$$

$$6\, y_s = -(y_3 - y_9) + \frac{1}{2}(y_1 - y_7 + y_5 - y_{11}) + \frac{\sqrt{3}}{2}\cdot$$

$$\cdot (y_2 - y_8 + y_4 - y_{10}) \qquad (6)$$

With six data samples, a somewhat poor estimate will be obtained.

$$6\,\bar{y}_c = -2\,y_6 + (y_2 - y_4) + \sqrt{3}\,(y_1 - y_5)$$

$$6\,\bar{y}_s = -2\,y_3 + (y_1 + y_5) + \sqrt{3}\,(y_2 + y_4) \qquad (7)$$

The amplitude and phase angle of fundamental component is obtained as follows. The relay input and waveforms are sampled and stored over a period of the fundamental frequency. Assuming the input waveform to be repetative over this period, fourier analysis techniques are applied to the sampled values of the input to obtain fundamental amplitude.

$$F_1(t) = \sqrt{(a_1^2 + b_1^2)}\ \sin\left(wt - \tan^{-1}\frac{a_1}{b_1}\right) \qquad (8)$$

where w is the angular frequency. In above equation,

$$a_1 = \frac{X}{2\pi}\left(f_o + 2f_1\cos x + 2f_2\cos x + \ldots\ldots\ldots 2f_{n-1}\cos(n-1)x + f_n\cos nx\right)$$

and

$$b_1 = \frac{X}{2\pi}\left(2f_1\sin x + 2f_2\sin 2x \ldots\ldots\ldots\ldots 2f_{n-1}\sin(n-1)x\right) \qquad (9)$$

where x is sampling interval in radians.

$n = \frac{2\pi}{x}$ and $f_0, f_1 \ldots\ldots\ldots f_n$ are the amplitude of input wave form at the sampling instants.

Thus the fundamental amplitude and phase angle of voltage and current waveforms applied to the relay are obtained. After this the magnitude of impedance, reactance and resistance of the line upto fault can be evaluated. Using these values of impedance, reactance and resistance, the location and zone of the fault can be determined. The digital circuits can perform logical operations on the relaying quantities and any tripping characteristics for the relay can be realized.

4.5. Predictive calculation of peak values:

In this method the peak values of the voltage and current wave forms are calculated for each sample.

Let $v = V_m \sin wt$

then $V_m = \left\{ v^2 + \left(\frac{v^1}{w}\right) \right\}^{\frac{1}{2}}$

and similarly

$$I_m = \left\{ i^2 + \left(\frac{i^1}{w}\right)^2 \right\}^{\frac{1}{2}} \qquad (10)$$

where v and i are the voltage and current samples and v^1 and i^1 are their time derivatives. These phase angle ϕ is given by

$$\phi = \left[\tan^{-1} \frac{wi}{i'} - \tan^{-1} \frac{wv}{v'} \right] \qquad (11)$$

The impedance calculation of the transmission line makes use of the trigonometric identities

$$v = V_{pk} \sin (wt + \phi + \delta)$$
$$i = I_{pk} \sin (wt + \delta).$$

where v and i can be measured leaving four unknowns; V_{pk} I_{pk} ϕ and $(wt + \delta)$. In order to obtain the four unknowns; four equations are needed to solve; samples may be taken at known time intervals. The angle equivalent of this time interval will be referred as Δ. Equations for samples taken at time t_{n-2}, t_{n-1} and t_n are,

$$v_{n-2} = V_{pk} \sin (wt_n + \phi + \delta - 2\Delta) \qquad i_{n-2} = I_{pk} \sin (wt_n + \delta - 2\Delta)$$

$$v_{n-1} = V_{pk} \sin (wt_n + \phi + \delta - \Delta) \qquad i_{n-1} = I_{pk} \sin (wt_n + \delta - \Delta)$$

$$v_n = V_{pk} \sin (wt_n + \phi + \delta) \qquad i_n = I_{pk} \sin (wt_n + \delta). \qquad (12$$

This sets of equations are expanded with the use of trigonometric substitutions to find the apparent resistance to the fault (r_f) and apparent reactance to fault (x_f) :

$$r_f = \frac{2\, v_{n-1} \cdot i_{n-1} - v_n \cdot i_{n-2} - v_{n-2} \cdot i_n}{2\, (i_{n-1}^2 - i_{n-2}\, i_n)} \qquad (13)$$

$$x_f = \frac{v_{n-1} \cdot i_n - v_n \cdot i_{n-1}}{i_{n-1}^2 - i_{n-2} \cdot i_n} \sin(\Delta) \qquad (14)$$

The equation of r_f is independent of the sampling rate and therefore it is not affected by system frequency. X_f is dependent on the sin(Δ) constant and will be sensitive to a change in system frequency. If greater accuracy is required, the compensation can be done by storing various values of sin(Δ).

4.5.1 - Selection of voltage current pairs:

The voltage current pair used to calculate r_f and x_f changes depending upon the type of fault being checked for and the presence or absence of zero voltage condition. When no zero voltage condition is detected, phase to ground voltages and zero sequence compensated phase currents are used to detect single phase to ground faults while phase to phase voltages and currents are used to check for all other fault combinations.

During zero voltage condition, the voltages and currents are selected to determine whether the cause is a potential circuit failure, a fault outside, or bonified fault inside the protected line section.

The selected voltage-current pairs give a number proportional to the source impedance, when three consecutive calculations result in an indication that a fault exists in the protected line section, the line is tripped.

A three phase zero voltage condition requires special attention since there is no reference to indicate the location of a fault. Voltage obtained prior to the fault can be used for this purpose, but the phase relationship becomes incorrect, due to the phase shift of the voltages near the fault. The trip-or- no trip decision is made on the current magnitude. This means the magnitude of each phase current is calculated and checked against a given maximum. If all three magnitudes exceed this maximum, a trip signal is initiated.

The peak current squared I_{pk}^2 is calculated before a single phase to ground fault calculation is performed

$$I_{pk}^2 = \frac{i_{n-1}^2 - i_{n-2} \cdot i_n}{\sin^2(\Delta)} \qquad (15)$$

This I_{pk}^2 number is then compared to the peak current squared of one of the other phases. If I_{pk}^2 for phase under considera-

-tion is greater than selected the fault is assumed to be of phase to phase rather than single phase to ground type. In this case the single phase to ground impedance calculations are not performed.

4.5.2 - Tripping criteria:

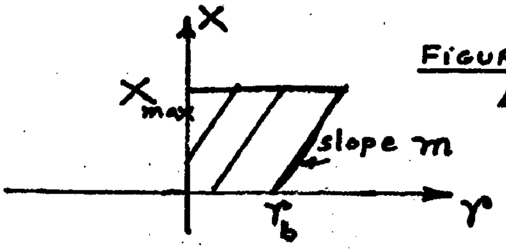


FIGURE 4:5:1

The r-x characteristic for the digital relay's will not be circle, unlike the conventional electromechanical relays where it is usually circle. There are three reasons for this r-x characteristic (Figure 4.5:1)

First, it is simpler to implement the linear characteristic. Second, a correctly designed linear characteristic canprovide a good arc coverage and it can be tailored to prevent tripping under maximum load conditions. Finally, it can beused for out of step relaying with additional logic. The characteristic to be made use of initially will be similar to Figure (4.5:1). This can be made more complex if necessary.

In operation, values of r and x which are calculated using the equations described would be checked to see if the sample point lies within the shaded area of Fig.4:5:1

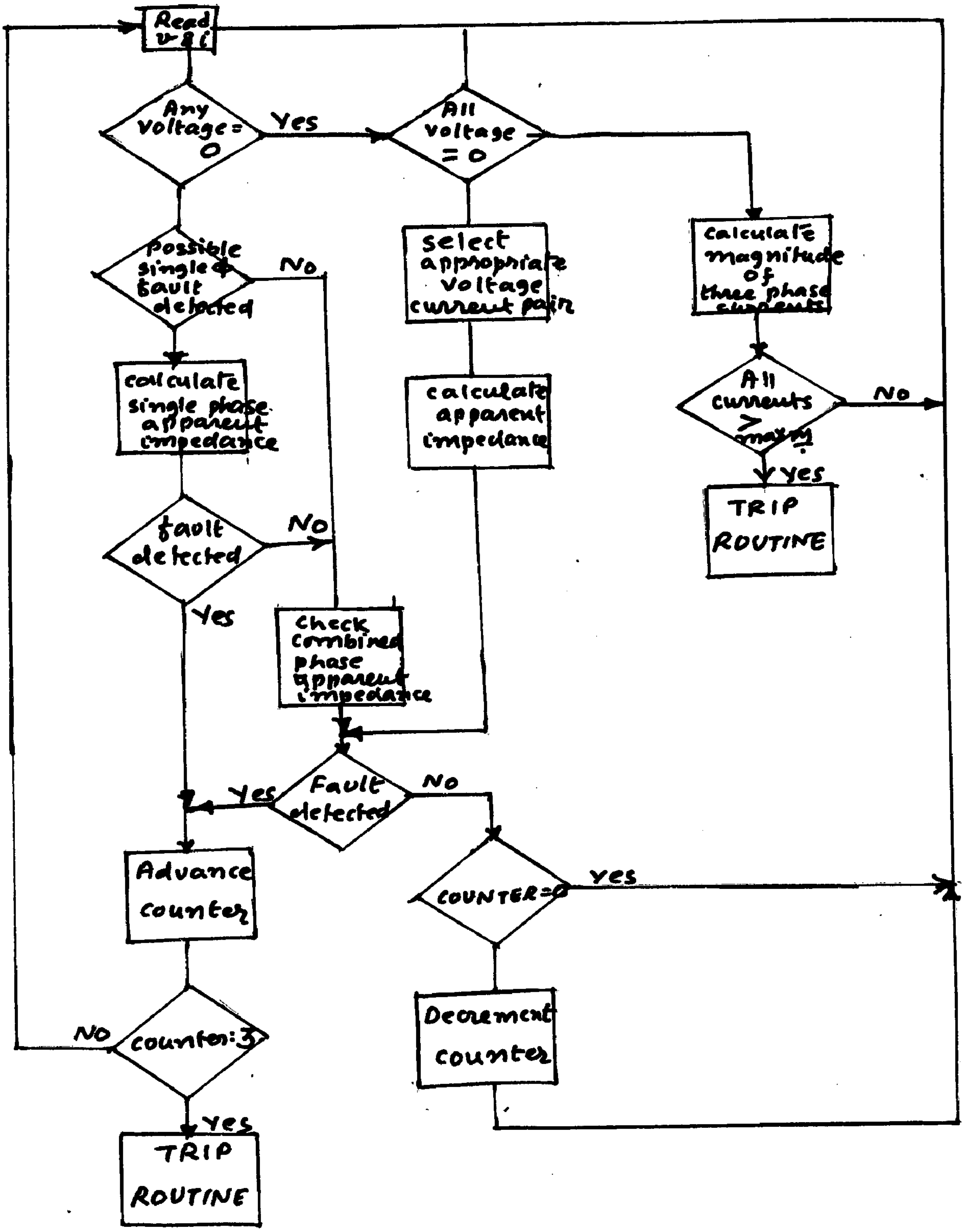


FIGURE 4:5:2 FLOW CHART FOR DIGITAL RELAY SIMULATION.

A program can be written to simulate the operation of computer as a digital relay. This simulation is limited to determine the output which would result for a given input. The flow chart of simulation program is given in Figure 4.5:2.

CHAPTER - 5

HIGH SPEED PROTECTION OF POWER SYSTEM USING A DIGITAL COMPUTER

5.1. Relaying philosophy:

Increasing interest is being shown in the use of the digital computers for protection. Given adequate speed of sampling and conversion to digital form for processing, problems remain of calculating the fault conditions within a defined zone of transmission line, particularly when harmonics and noise are present.

Fault must be cleared fast and selectively. To avoid generator instability which could readily lead to a wide spread blackout, a three phase short circuit near a generating stationmust be isolated fromthe healthy parts of system in about 0.15 second. In this interval the relays must energise the trip coil of circuit breaker, wait about 0.07 sec. to see if appropriate breakers open, and trip the other breakers if one of the proper fails to interrupt the fault current. Of course whether a stability problem exists in a given application, the objective is to clear fast, minimising damage and disturbance to loads.

The possibility of utilising an on-line digital computer to perform protection, switching and data collection of modern high voltage transmission system and substations is attracting increasing attention. This chapter describes the digital computer programme for protection of three phase transmission line. This programme detects the presence of disturbance, classifies the fault into one of the six fault types and using a method of impedance calculation determines modulus and phase of the faulted line.

Accuracy of calculation and reliability of detection are determined from fault data for a model transmission line. The method is shown to be suitable for extension to use in multiline situations. The relaying trends with digital computer where the hardware cost for a given level of capacity has been dropping, and where software application and knowledge has been advancing. Most of the analog function can be duplicated digitally, and the digital filtering of signal is commonplace. The tremendous software job, as well as other seen and unseen problems, time-shared; real time; stored-program digital computer wide perform protection function as well as control and data acquisition. To sense a fault or to locate it all electrical relays use one or more of the following:

1. Level detection : Fault cause observations in currents and voltages. At fault inception a sudden rise or drop of voltage can occur. Current and voltage peaks can

change in magnitude, with respect to prefault condition. Abnormally high current; low frequency, voltage level detection is usually done.

2. Magnitude comparison: A magnitude of voltage compared to line current provides a basis for distance measurement; i.e. how far away fault is located or percentage differential relay compares two currents.

3. Angle comparison: A current falling within 180° band in relation to a reference voltage may be used to indicate direction of fault-power flow. The movement of centre zero wattmeter needle left or right, for example indicates the direction of real power flow.

For transmission stations the computer must convert to digital form, the instantaneous value of sets of currents and sets of voltages. Each is sampled every 0.5 ms with such a fast rate the station control cabling must be well shielded to minimise both magnetic and capacitive coupling. Voltage fault detector looks for one set of voltage for each KV level as determined by potential selection logic. Use of voltage offers faster detection, since voltages can change magnitude instantaneously while current cannot. It ignores currents to minimise prefault duty since there are many more currents than voltages. One method proposed by Slemon, Robertson and Ramamoorty[8] is based on calculation of the fundamental components of voltage and current wave forms by a Fourier technique requiring evenly spaced samples taken over a complete cycle of the system frequency. From the fundamental components, the impedance magnitude and angle seen from the relaying point can be calculated. Another method described by Rockfeller[1] for practical installation is based on the work of Mann and Morrison[10,11] uses samples to predict the peak values of fault voltage and current as well as the phase angle between the peaks. These techniques envolve a form of digital filtering, an application which has been highly developed in tele-communication, but be useful for high speed power system line protection. A measurement and calculation time equivalent to about half a cycle (10 ms for 50 to 60 Hz) and measurement error not more than 5 % of line reactance to fault point is sought.

5.2. System hardware:

The computor system looks only at the currents and voltages of one three phase transmission line and thus capable of controlling the one related circuit breaker. The components of computer contains the cabinet process control computer, tele type writer, paper tape reader, punch, an analog signal conditioning package, digitizer, data buffer interface, A/D control unit and power supply.

Figure 5.1 gives an overlook at the system Raw a.c. information passes through signal conditioning to A/D sub-system where it is sampled and converted under Data buffer (SPM : stratch pad memory) control circuits, then transferred to computer sub-station for processing. The KSR type writer and programmer's console provide facilities for logging of desired data by the computer; as well as a convenient means of execution control, software generation loading and modification, also program check out. The software initiates contact outputs from the computer, they are used for reclosing as well as operation of various trouble alarms. The static auxilary trip relay 94 provides a thyristor - controlled high-speed output for circuit breaker tripping. The power supply unit can provide uninterupted power to the entire system from 3ϕ a.c. supply or from 110V d.c. station battery via invertor.

Signal conditioning : The peripheral sub-systems include the analog-input connections and signal conditioning (SPA/ACP) features. The low pass filters is RC-type; with phase delay of about 30 degrees and the cutoff frequency is 200 Hz. The need for these filters is related to the A/D sampling rate of 720 Hz. For incoming transients at 180 Hz or 360 Hz, the sampling frequency corresponds to twice the 720 Hz frequency of sample wave form. The result

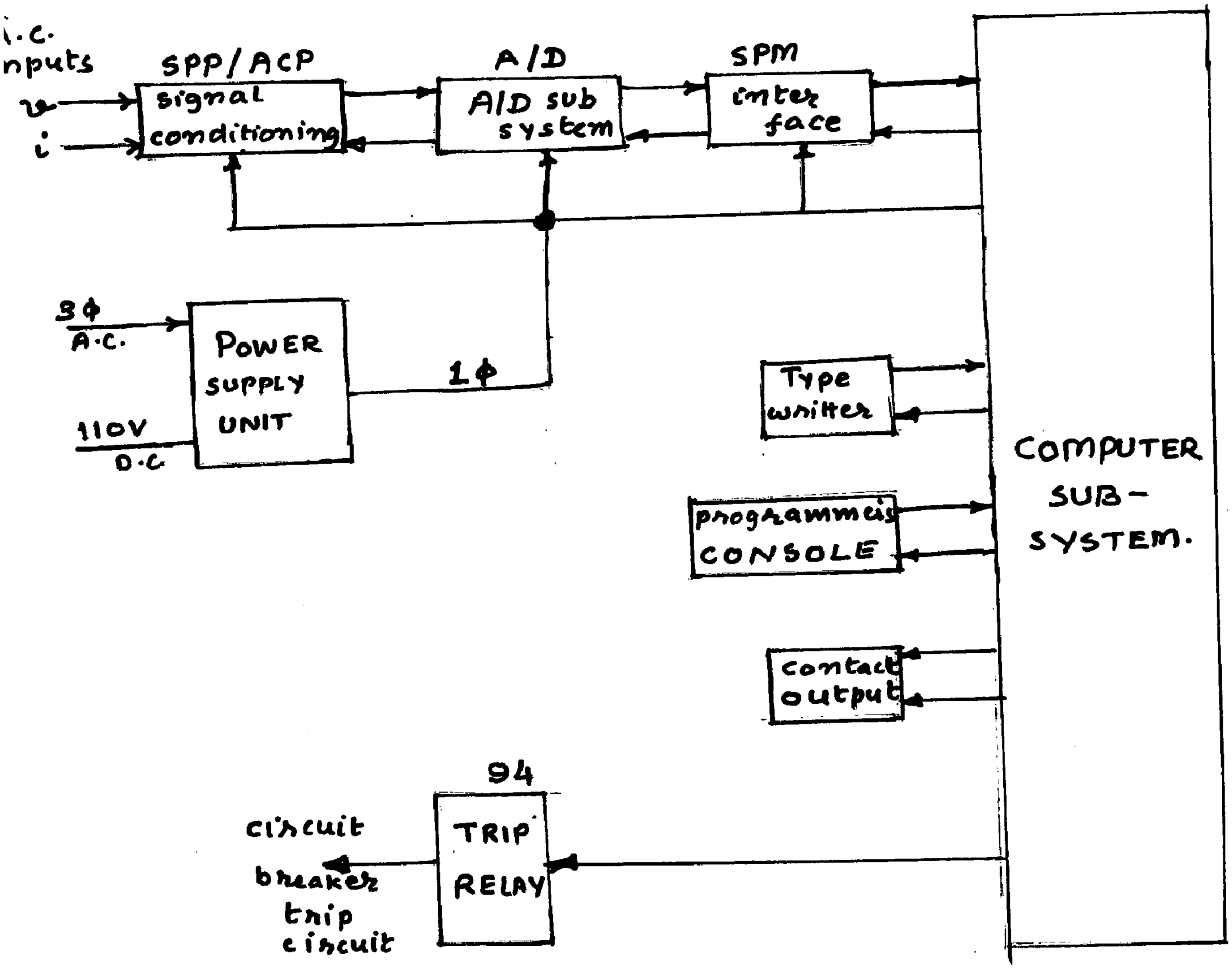


FIGURE 5:1 MAJOR COMPONENTS AND SUB-SYSTEM.

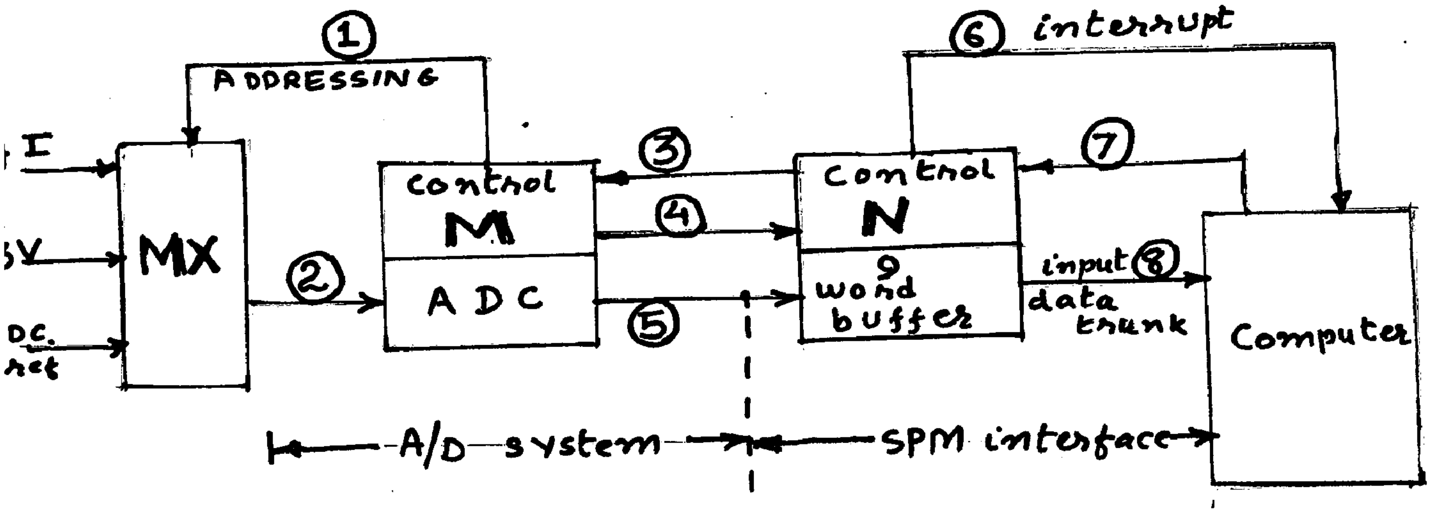


FIGURE : 5:2 ADC SYSTEM and COMPUTER INTERFACE.

is that all frequencies below 360 Hz are properly reconstructed and interrupted from the sampled data, but all those above this frequency will be aliased, i.e. they will appear in the sampled data as false. It is important that filters must be used to remove those misleading high frequencies.

Analog-to-Digital conversion :

Operation of A/D sub-system is shown in Figure 5.2. A crystal controlled oscillator in Control N section of the SPM interface sets the sampling rate 720 Hz (one sample-set every 1.356 ms). Each pulse (3) from oscillator causes control M of A/D converter (ADC) to address (1) the first channel of multiplier MX. The residual current is sampled and hold amplifier converts (2) to digital form; and stored(5) in the first location of 9-word buffer. Control M signals control N (4) to send an "A/D complete" request interrupt (6) to computer central processing unit. Control N places the contents of SPM on direct data input trunk (8). CPU then performs the input, storing the digital quantities, it also signal control N(7) to place next SPM on the data trunk. The A/D system uses 37 μs to sample and convert quantity and store results in SPM. The 333 μs is the totaltime. The entire process is repeated 1.326 MS (720 Hz) later. Eight Mx channels are used for a.c. quantities (3 phase currents, 3 phase-to-ground voltages and the residual current which is read twice), while precisely regulated 8-volt reference is connected to the last channel. The software can use this last input for periodic testing of ADC calibration.

Out-puts : A software talk initiates 4 ms contact output to reclose the breaker. Other contacts outputs operate annunciations to alarm following tripping or for A/D system trouble or computer hardware failure.

Power supply : The reliable performance of the computer depends upon an uninterrupted a.c. power supply. The 110V station battery d.c. supply is kept in constant operation to assure power availability. An inverter supplies power for entire system.

5.3. System software:

Software system include standard executive program, on-line tasks for logging testing and control, off-line tasks for diagnosis, more detailed testing and data generation, and an interrupt handling program which embodies the high speed relaying function.

There are process input - output routines for handling contact inputs and outputs and various interrupts. The system executive or monitor program directs all software functions. The standard software includes elaborate programmer's

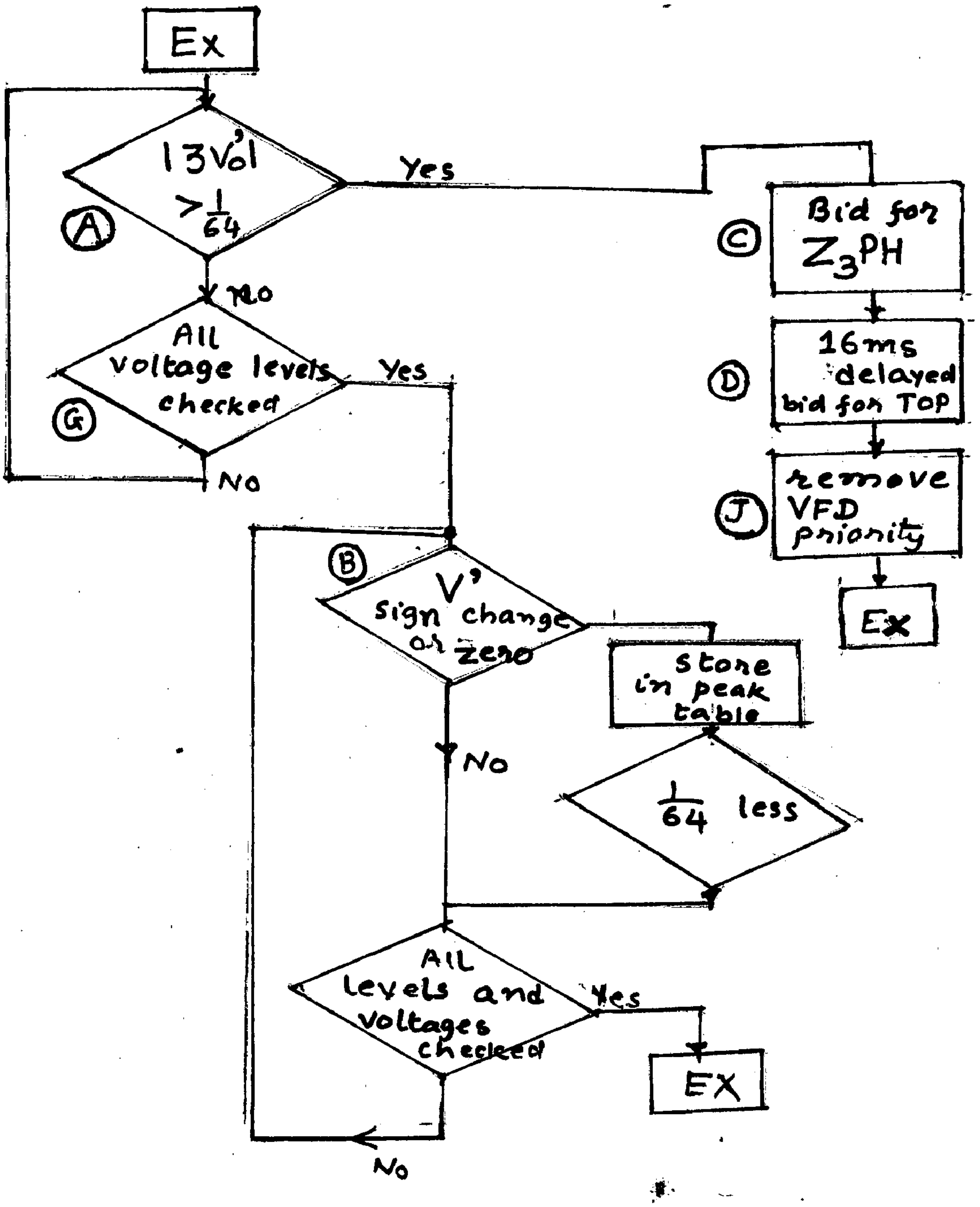


FIGURE 5·3 Voltage Fault Detection.

console section, used to investigate, modify, input and output the contents of memory. The system library contains commonly used utility subroutines may be called by any program such as real arithmetic multiplication.

VFD: Voltage fault detection: VFD processes each voltage sample as shown in Figure 5.3. Block A looks for a significant jump in $3V_o$ since the last sample. Block A makes one such check per voltage level unless fault is indicated sooner.

Block B looks for a sign change or zero value in the first difference of phase to ground voltage (one check for each of three voltages on each voltage level, unless a fault is detected sooner). The block E stores measured value as peak. If this value is at least $\frac{1}{85}$ p.u. less than the previous peak, a fault is detected. Either A or F enters bid C for selected fault routines as well as 16 ms delayed bid for TOP (turn-off fault program) VFD then removes its own priority. VFD will not turn on until TOP turns off the fault routine.

This will occur in 16 ms unless atleast one of the differential or line routines finds some indication of fault.

5.4. Relaying program:

The flow chart shows the normal course of software events. A series of 9 direct channel inputs reads the sample set from SPM and stores it in currently used raw data table, except d.c value which is stored separately. Periodically an out-off step detector checks the phase currents for indications of a developing swing, otherwise logic proceeds directly to a section which uptodates and organises the raw data tables.

Under normal condition, the next step is ground fault detector logic, which examines the residual current samples. If a disturbance is not detected here, phase fault detector PFD is run. This program, on each pass, makes predictions for the values of each phase current 6 sample sets later (a little less than one half cycle) and compares the present values with those predicted 6 samples ago. The sample of those currents predictions must be fabricated by combination of the present values and their rate of change. Figure 5.5 shows the way in which disturbances are detected. Part (a) portrays the normal steady state condition. Part (b) sudden increase in current, part (c) sudden decrease and part (d) phase shift with no change in magnitude. The predictions fall out-of-line with the waveform are once again detected by PFD routines. If out of step has already operated PFD is desensitized so that out of swing will be ignored. The ground fault detector

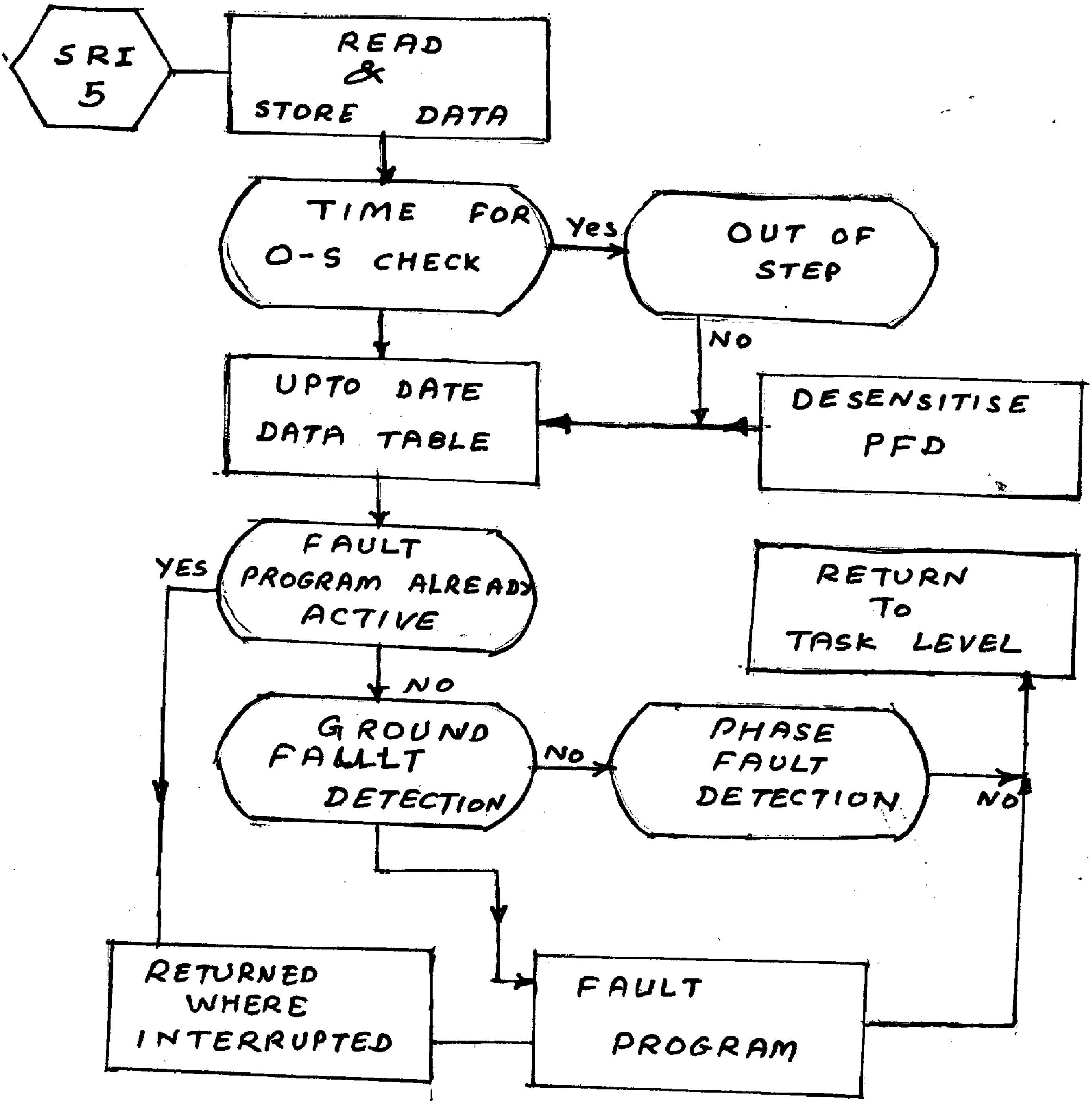


FIGURE 5:4 THE FLOW CHART.

can still operate, allowing full ground protection during swings, in addition severe phase fault still can be immediately detected.

If both fault detectors have been executed with negative results, the computer returns control to program which was interrupted, or, if none; to the task scheduler routine. The next SRI # 5 will signal more new data for processing.

The purpose of fault program (Figure 5.5) is calculation of apparent line impedances for the directional distance checking. A fault type analysis (FTA) routine attempts to find characteristics of fault which can aid processing. It looks for severe instantaneous over current, (b) low line-to-ground or line-to-line voltage indicating faulted phase(s) for distance check, (c) high phase or residual currents also indicating faulted phases and types of fault, and (d) a voltage phase reversal due to capacitive faults.

If none of these severe conditions are found, ground distance and phase distance checks are made on all phases, using the zone 3 reach characteristic shown in Figure 5.7. For either results (b) or (d) a memory voltage must be used for directional sensing. If any phase shows a

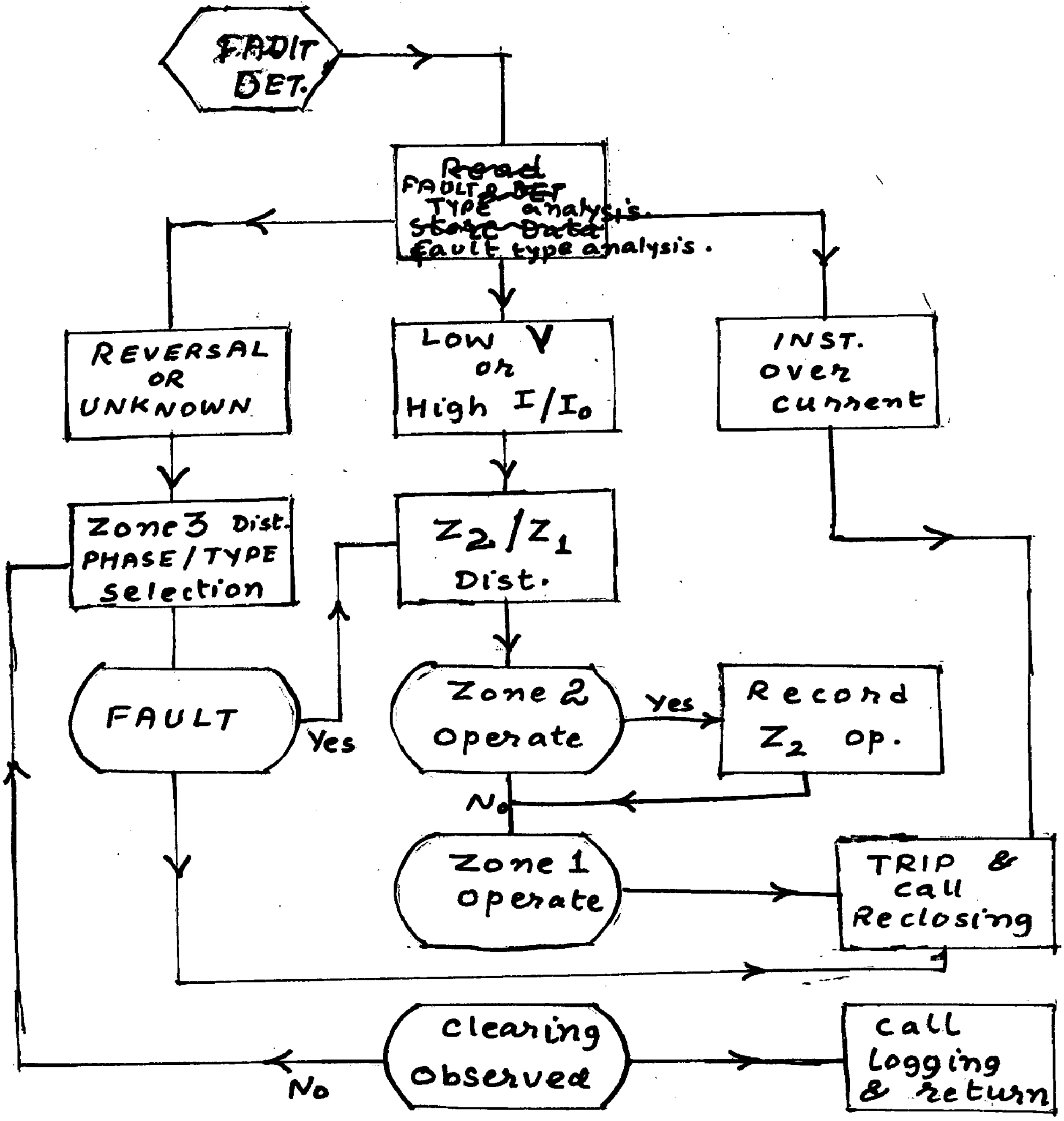


FIGURE 5:5 FAULT PROGRAM.

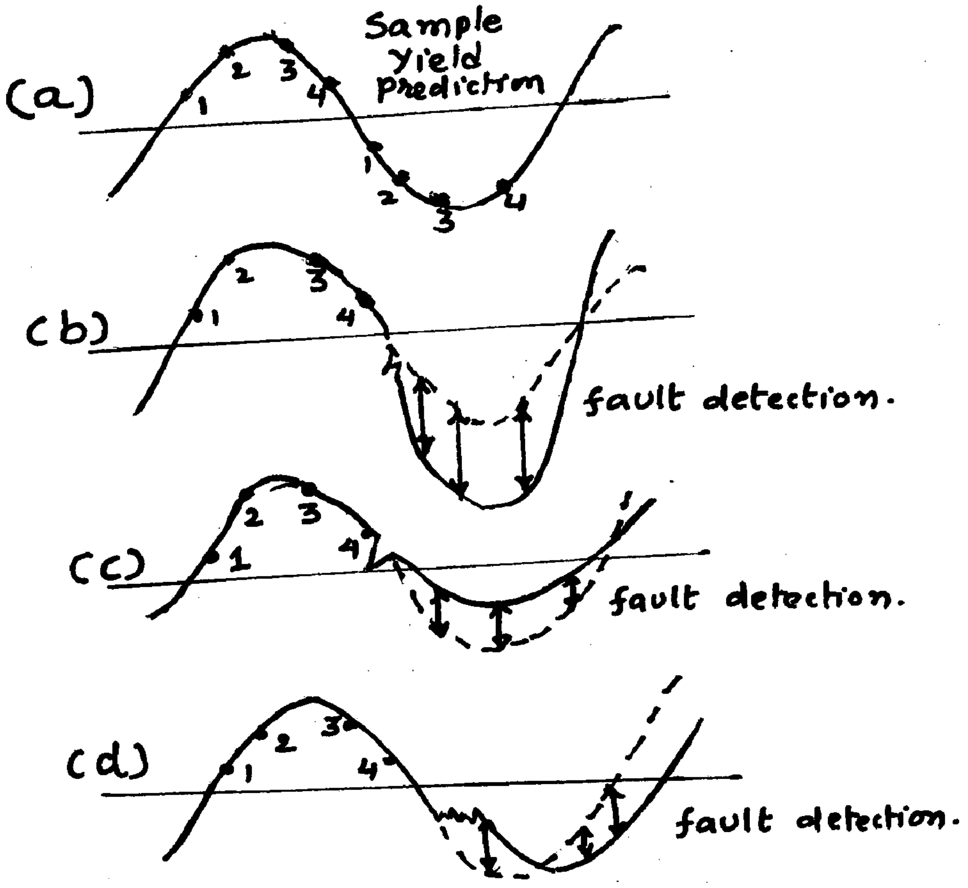


FIGURE : 5:6

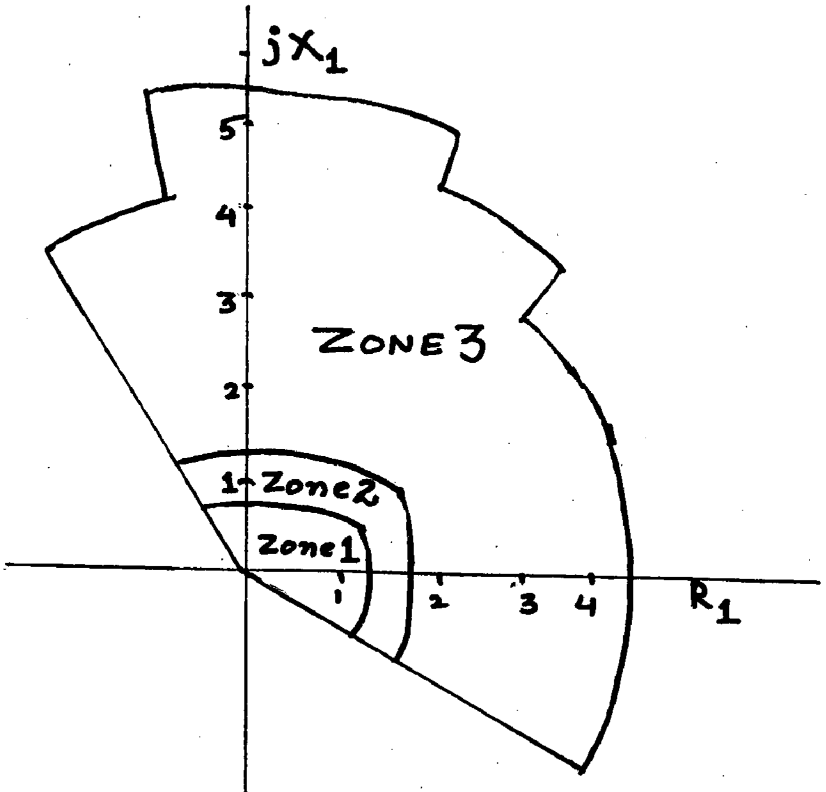


FIGURE 5:7 R-X reach characteristic

fault located within zone 3, a check is made strictly on most severely faulted phase (s) with either ground or phase distance logic, Using zone 1 or zone 2 reach characteristic of Fig.5.7.

If the type of fault analysis results in (b) or (c) the logic proceeds directly to zone 2/1 check on apparently faulted phase using ground or phase distance approximate. For a distance check, the fault program calculates the apparent inductance and compares it to the reach characteristic for the zone of interest. A positive sign on the results indicates that this impedance is inside the zone. After a trip output in Figure 5.6, the reclosing task is bid for. A number of tasks for performing relay-related functions, logging and diagnostic are included in software system, of these special interest tasks are 6 and 7 in Table 5.1.

Table 5.1

System tasks

Priority	Name	On/off line	Function
F	Sync.	On	Performs system timing
C	Reclosing	On	Initiated after trip output. Performs software time delay then recloses breaker. Resets trip.
A	A/D Fail. Logging	On	Periodically logs and alarms if calibration is out of limits.

8	Programmer's console	On	Performs memory multiplications in command.
7	Paper-tape fault logging	On	Punches reserved faulted data tables following internal faults.
6	Teletype fault logging	On	Prints sequence-of-events information following fault-program-run. Analyses reserved tables for internal faults. Resets logging functions of fault program.
5	A/D and input Diagnostic	Off	Performs elaborate statistical analysis of A/D converter calibration or input waveform.
3	ZLM 1 Gen.	Off	Generates relay reach characteristic tables.
2	Data-Gen and Diagnostic	Off	Generates mathematical look up used in impedance calculation. Generates tables of simulated fault data for relay program testing.
0	Look	On	Initiates timing functions, performs check of A/D calibration, Performs hardware checks.

Included in above table are a number of tasks for performing relay related functions, logging and diagnostics. This is described in descending order of priority, of special interest are logging tasks 6 and 7. The fault program initiates task 6 after every fault program execution. If fault was internal (zone 2 or zone 1), task 7 is also run. It logs raw tables of analog data, reserved above the fault point and ckt clearing, on paper tape via punch in programmer's ionsole. This indicates the technical feasibility of high speed relaying by computer for protecting a single transmission line in a laboratory.

CHAPTER -6

ALGORITHM FOR DIGITAL IMPEDANCE CALCULATION

6.1. General:

An algorithm suitable for calculating impedance for digitised voltages and currents sampled at relay location is developed here. Each input is assumed to be composed of a decaying d.c component and components of fundamental and harmonic frequencies. Parameters of a digital filter are used to compute the real and imaginary components of voltage and current phasors. Impedances as seen from a relay location are calculated. Transmission lines form major part of a power system. Different types of relays are used to protect these lines. The most commonly used relays are from the family of distance relays. These relays basically evaluate the impedance looking into the transmission line from the voltages and currents at the relay location. The impedance is assumed to be proportional to the distance from the relay to the fault and this determines if the fault is in the relay's prospective zone.

The conventional impedance relays are either electromagnetic or solid state type. Some relays which use digital processor for computing impedance and making decision have been developed. The algorithms used to calculate apparent impedance used in those relays can be categorised into four groups. The first group is developed assuming that the waveform presented to relay is pure sinusoid. The second group of algorithms use~~d~~ Fourier analysis and third group use digital filters to extract the fundamental frequency information from the inputs. The lastgroup of algorithms numerically solve a differential equation which describes the behaviour of the transmission line. Mann and Morrison[11] proposed an algorithm which uses the fact that the amplitude of a sinusoid can be determined from its value and it's rate of change at any instant, and this rate of change can be calculated by using differential equations. Gilcrest, Rockfeller and Udren[13,15] used the first and second derivatives to calculate the peak values of the sinusoids. The third type of algorithms use digital filters to extract information concerning the fundamental frequency, Hope and Umamaheswar[2] used finite impulse response filters. The last group of algorithms have been proposed and used by Morrison[10], Ranjbar and Cory[14], these algorithms num erically solve the differential equation representing the transmission line by a series of R-L model.

6.2. Mathematical background:

In digital relays voltage and current outputs of tranducers are processed and converted to millvolts level. The function of the processor is two fold, one is to suppress surges travelling from power system to the relay and other is to block high frequencies from reaching the relays. Process and signals of the millivolt level are converted to numerical values by analog to digital converters.

The output of digital filter are the real and imaginary components of fundamental frequency phasor. These components are used to calculate the impedance as seen from the relay location.

The output of CCVT or voltage transformer during fault can be numerically expressed as:

$$v(t_1) = K_1 e^{-t/\tau} + \sum_{n=1}^{N} K_{2n} \sin (n w_0 t_1 + \theta_n) \qquad (1)$$

The time constant τ depends upon X/R ratio of the system but is also affected by arc resistance which varies from fault to fault. In practice, even harmonics are not present in the fault voltages and currents. Also higher order harmonics are blocked from reaching the relay by signal conditioning equipment which usually includes analog filters. The cut-off frequency of these filters is determined by over all design considerations for the relay.

It is possible to expand $e^{-\frac{t_1}{\tau}}$

$$e^{-\frac{t_1}{\tau}} = 1 - \frac{t_1}{\tau} + \frac{1}{2!}\left(\frac{t_1}{\tau}\right)^2 - \frac{1}{3!}\left(\frac{t_1}{\tau}\right)^3 \ldots\ldots\ldots \qquad (2)$$

Using first three terms of this series

$$v(t_1) = K_1 - \frac{k_1}{\tau}\, t_1 + \frac{k_1}{2\;{}^2}\, t_1^2 + K_{21} \sin (v_o t_1 + \theta_1)$$
$$+ K_{23} \sin (3\, w_o t_1 + \theta_3) \ldots\ldots \qquad (3)$$

$$v(t_1) = K_1 - \frac{k_1}{\tau} t_1 + \frac{k_1}{2\tau^2}\, t_1^2 + (K_{21} \cos \theta_1) \sin w_o t_1$$
$$+ (K_{21} \sin \theta_1) \cos w_o\, t_1 + (K_{23} \cos \theta_3) \sin 3w_o t_1$$
$$+ (K_{23} \sin \theta_3) \cos 3w_o t_1 \ldots\ldots\ldots\ldots\ldots \qquad (4)$$

Let $x_1 = k_1$ $\quad x_2 = k_{21} \cos \theta_1$ $\quad x_3 = k_{21} \sin \theta_1$

$x_4 = k_{23} \cos \theta_3$ $\quad x_5 = k_{23} \sin \theta_3$

$x_6 = -\frac{k_1}{\tau}$ $\quad x_7 = \frac{k_1}{(\tau_1)^2}$

$$a_{11}=1 \qquad a_{12} = \sin(w_o t_1) \qquad a_{13} = \cos(w_o t_1)$$

$$a_{14} = \sin(3w_o t_1) \qquad a_{15} \cos(3w_o t_1)$$

$$a_{16} = t_1 \qquad a_{17} = t_1^2 \qquad (5)$$

The equation (6) can be obtained by making these substitution in equation (4).

$$v(t_1) = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 + a_{17}x_7 \quad \dots\dots\dots\dots\dots\dots \quad (6)$$

the next sample of voltage at time t_2 where $t_2 = t_1 + \Delta t$ can be expressed by similar equation followed,

$$v(t_2) = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 + a_{27}x_7 \quad \dots\dots\dots\dots \quad (7)$$

Proceeding in this manner, m equations can be generated from m samples of voltage received fromthe system at the specified rate, which can be written in matrix form

$$\underset{m\times 7}{[A]}\ \underset{7\times 1}{[X]} = \underset{m\times 1}{[V]} \quad \dots\dots\dots\dots\dots (8)$$

The elements of A matrix depend on the time reference and the sampling rate used, and can be predetermined in an off line mode.

The equation (8) is a set of m equations in seven unknown.

Premultiplying both sides of equation (8) with left pseudo-inverse of $[A]$, the value of the unknowns can be obtained as follows:

$$\underset{7\times 1}{[X]} = \left[\underset{7\times m}{[A]^{\dagger}}\ \underset{m\times 7}{[V]}\right]^{-1} \cdot \underset{7\times m}{[A^{T}]} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (9)$$

where

$[A]^{\dagger}$ is pseudo-inverse of $[A]$ and is defined as

$$\underset{7\times m}{[A]^{\dagger}} = \left[\underset{7\times m}{[A^{T}]}\ \underset{m\times 7}{[A]}\right]^{-1} \cdot [A]^{T}$$

The concept of pseudo-inverse provides least error squares solution of unknowns; seven in this case, from m independent equation $(m>7)$. If the voltages and currents are expressed similar to equation (4), the second and third rows of $[A]^{\dagger}$ can be used to determine the real and reactive components of voltages and currents.

6.3. Calculating impedances : as seen from a relay location -

Define the real and reactive components of a voltage and current fundamental frequency determined by the approach stated as followed:

$$x_{2v} = V_{pk} \cos \theta_v \qquad x_{3v} = V_{pk} \sin \theta_v$$

$$x_{2i} = I_{pk} \cos \theta_i \qquad x_{3i} = I_{pk} \sin \theta_i$$

then

$$R = \mathrm{Re}\left[\frac{V}{I}\right] = \frac{x_{2v}\, x_{2i} + x_{3v}\, x_{3i}}{x_{2i}\, x_{2i} + x_{3i}\, x_{3i}} \qquad (10)$$

$$X = \mathrm{Im}\left[\frac{V}{I}\right] = \frac{x_{3v}\, x_{2i} - x_{2v}\, x_{3i}}{x_{2i}\, x_{2i} + x_{3i}\, x_{3i}} \qquad (11)$$

Each sample obtained from A/D converter yields an equation. Since there are seven unknowns atleast seven samples are required to compute the values of unknown. Coefficient of [A] matrix are first calculated assuming seven samples. The matrix is then inverted.

If 720 Hz is sampling rate, and number of samples used is increased to eight which increase [A] matrix to 8x7. The elements of [A] matrix are again calculated. The matrix $[A]$ is not square matrix, so its pseudo inverse is obtained. The largest and smallest numerical values among the elements of the second and third rows of $[A]^{\dagger}$ are tabulated. From practical study, it was concluded that using nine samples of each signal obtained at 720 Hz would be better. The elements of $[A]^{\dagger}$ already calculated and tabulated for second and rows are used to compute the real and imaginary components of voltage and current phasors; using equation (10) and (11). The data can be generated from a digital model of a transmission line.

This describes algorithm for developing a digital filter which explicitly takes account of the decaying d.c. components in the system voltages and currents. The concept of pseudo-inverse has been used in developing the algorithm is described above. It is shown that the proposed approach can effectively calculate the impedance from fault data obtained from a power system.

6.b. Digital impedance calculation algorithm:

A digital impedance calculation algorithm which does not require prior filtering of transmission line voltage and current signals to remove either transient d.c components or the transient high frequency components before calculating the impedance of theline. Computational requirements of the algorithm are presented and compared to those of a corresponding algorithm which requires prior filtering out of the transient high frequency components before the impedance of the line is calculated.

The post-fault line conditions on the line are described by

$$V = R_L + L_L \frac{di}{dt} \quad \ldots\ldots\ldots\ldots \quad (12)$$

To determine R_L and L_L from the voltage and current wave forms containing power frequency and d.c. offset components. If samples of the line voltage and current wave-forms are taken at uniform time increment h = t and if we replace the derivative in equation (12) by its central finite difference for eachof the two successive sets of samples, one gets equations (13) and (14) by applying conditions of equation (12) to these sample sets.

$$\left[\begin{array}{c|c} i_k & \frac{i_{k+1} - i_{k-1}}{2h} \\ \hline i_{k+1} & \frac{i_{k+2} - i_k}{2h} \end{array}\right] \begin{bmatrix} R_L \\ L_L \end{bmatrix} = \begin{bmatrix} V_K \\ V_{k+1} \end{bmatrix} \quad \ldots\ldots\ldots \quad (13)$$

where i_k, V_K are K^{th} samples of current and voltage waveforms, or $\quad A\,\underline{P} = \underline{V} \quad$ .... (14)

where $\underline{P} = \begin{bmatrix} R_L \\ L_L \end{bmatrix}, \qquad V = \begin{bmatrix} V_k \\ V_{k+1} \end{bmatrix}$

and A is the coefficient matrix of R_L and L_L in the equation(13). Equation (14) can be solved for R_L and L_L to give the solutions given by equation (15) and (16),

$$R_L = \frac{V_k\,(i_{k+1} - i_k) - V_{k+1}\,(i_{k+1} - i_{k-1})}{i_k\,(i_{k+2} - i_k) - i_{k+1}\,(i_{k+1} - i_{k-1})} \quad \ldots\ldots \quad (15)$$

$$L_L = 2h\,\frac{i_k\,V_{k+1} - i_{k+1}\,V_k}{i_k\,(i_{k+2} - i_k) - i_{k+1}\,(i_{k+1} - i_{k-1})} \quad \ldots\ldots \quad (16)$$

Line reactance is given by X_L in then given by equation (17) where n is number of samples per power frequency cycle of voltage and current waveforms.

$$X_L = \omega L_L = \frac{2\pi}{T}\cdot L_L = \frac{2\pi}{nh}\cdot L_L \quad \ldots\ldots\ldots\ldots\ldots \quad (17)$$

or

$$X_L = \frac{4\pi}{n}\;\frac{i_k\,V_{k+1} - i_{k+1}\,V_k}{i_k\,(i_{k+2} - i_k) - i_{k+1}\,(i_{k+1} - i_{k-1})} \quad \ldots\ldots\ldots (18)$$

A sampling rate is taken $n = 16\pi$, then

$\frac{4\pi}{n} = \frac{1}{4}$ and this multiplication by $\frac{1}{4}$ in equation (17) can be accomplished by a two bit right shift of the data X_L which is faster than a multiplication.

The main reason for introducing this new algorithm, equations 13 or 14 or 15 and 16 is primarily because a single equivalent section transmission line model is used instead of a lumped parameter transmission line model.

$$V = R_L\,(i-i_c) + L_L \cdot \frac{d(i-i_c)}{dt}, \qquad i_c = c\,\frac{dv}{dt} \tag{19}$$

or

$$V = R_L\, i + L_L \frac{di}{dt} - R_L C \frac{dv}{dt} - L_L C \frac{d^2v}{dt^2} \tag{20}$$

By selecting four successive sets of voltages and current samples and replacing the derivatives in equation (20) by corresponding finite differences one gets equation (21) for the four sample sets.

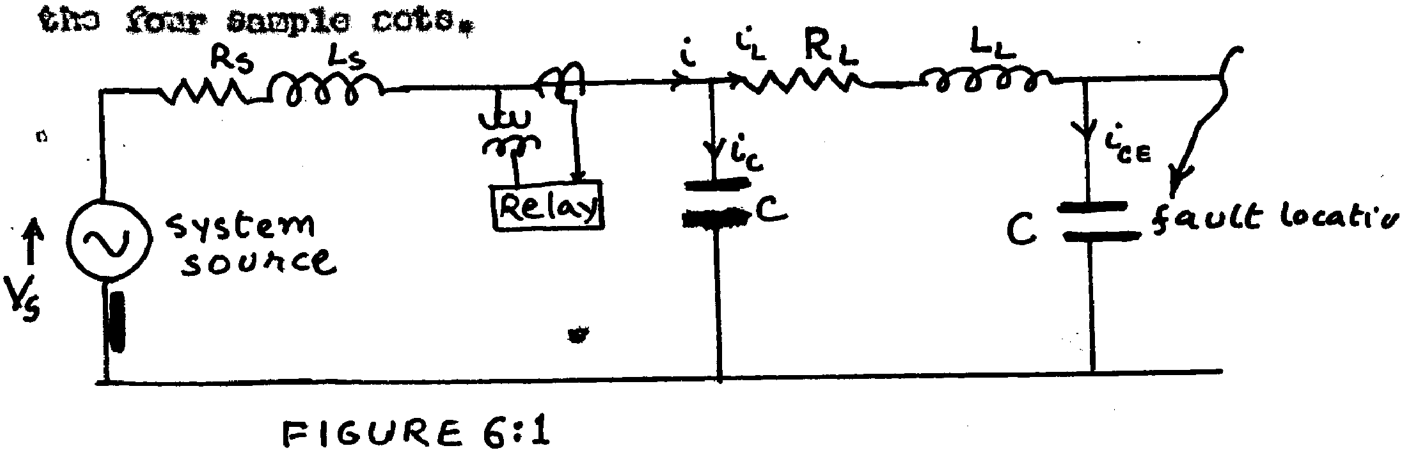


FIGURE 6:1

$$\left[\begin{array}{c|c|c|c} i_k & \dfrac{i_{k+1} - i_{k-1}}{2h} & \dfrac{-V_{k+1} - V_{k-1}}{2h} & \dfrac{-V_{k+1} - 2V_k + V_{k+1}}{h^2} \\ \hline i_{k+1} & \dfrac{i_{k+2} - i_k}{2h} & \dfrac{-V_{k+2} - V_k}{2h} & \dfrac{-V_{k+2} - 2V_{k+1} + V_k}{h^2} \\ \hline i_{k+2} & \dfrac{i_{k+3} - i_{k+1}}{2h} & \dfrac{-V_{k+3} - V_{k+1}}{2h} & \dfrac{-V_{k+3} - 2V_{k+2} + V_{k+1}}{h^2} \\ \hline i_{k+3} & \dfrac{i_{k+4} - i_{k+2}}{2h} & \dfrac{-V_{k+4} - V_{k+2}}{2h} & \dfrac{-V_{k+4} - 2V_{k+3} + V_{k+2}}{h^2} \end{array}\right] \left[\begin{array}{c} R_L \\ \hline L_L \\ \hline CR_L \\ \hline CL_L \end{array}\right] = \left[\begin{array}{c} V_k \\ \hline V_{k+1} \\ \hline V_{k+2} \\ \hline V_{k+3} \end{array}\right] \qquad \text{............(21)}$$

The equation (21) can be expressed in partitioned form as equation (22)

$$\left[\begin{array}{c|c} \underline{A} & \underline{B} \\ \hline \underline{C} & \underline{D} \end{array}\right] \left[\begin{array}{c} \underline{P} \\ \hline \underline{CP} \end{array}\right] = \left[\begin{array}{c} \underline{V_1} \\ \hline \underline{V_2} \end{array}\right] \qquad \text{.................(22)}$$

where

$$\underline{P} = \left[\begin{array}{c} R_L \\ \hline L_L \end{array}\right], \quad \underline{V_1} = \left[\begin{array}{c} V_K \\ \hline V_{K+1} \end{array}\right], \quad \underline{V_2} = \left[\begin{array}{c} V_{K+2} \\ \hline V_{K+3} \end{array}\right]$$

and A B C and D are 2x2 submatrices of the coefficient matrix in equation (21).

Since vectors $\underline{P} = \begin{bmatrix} R_L \\ L_L \end{bmatrix}$ and $C\underline{P} = \begin{bmatrix} CR_L \\ CL_L \end{bmatrix}$

are linearly related in equation (22). This equation (22) can be reduced to equation (23) or (24) by matrix reduction, and the unknown capacitance is eliminated inprocess as well.

$$\left[\underline{A} - \underline{B}\, D^{-1} C\right] \cdot \underline{P} = \left[\underline{V_1} - B\, D^{-1}\, \underline{V_2}\right] \quad \ldots\ldots\ldots\ldots\ldots (23)$$

or

$$\left[A^{1}\, \underline{P}\right] = \left[V^{1}\right] \quad \ldots\ldots\ldots\ldots\ldots \quad (24)$$

where

$$A^{1} = [A - BD^{-1}C], \quad V^{1} = \underline{V_1} - \underline{B}\, D^{-1}\, \underline{V_2} \quad \text{and} \quad \underline{P} = \begin{bmatrix} R_L \\ L_L \end{bmatrix}$$

A digital impedance calculation algorithm (equation (24)) based on a single equivalent π section of model has been developed. The model enables the calculation of impedance from samples of voltage and current waveforms containing high frequency transient components associated with this π model; without requiring that those high frequency transient components first be filtered out of the waveforms.

Appendix I

$\left[[A][A]\right]^{-1}\ [A]^{T}$ is the left pseudo-inverse of A.

Consider m linear equations of n unknown as follows:

$$\underset{m\times n}{[A]}\underset{n\times 1}{[X]} = \underset{m\times 1}{[Y]} \qquad m>n \qquad (I)$$

premultiplying both sides by $[A]^{T}$

$$\underset{n\times m}{[A]^{T}}\ \underset{m\times n}{[A]}\ \underset{n\times 1}{[X]} = \underset{n\times m}{[A]^{T}}\ \underset{m\times 1}{[Y]} \qquad (II)$$

$\left[[A]^{T}\ [A]\right]^{-1}$ is an nxn matrix.

premultiplying both sides of equation II by

$\left[[A]^{T}\ [A]\right]^{-1}$, the vector of unknown $[X]$ is obtained as follows:

$$[X] = [A]^{T}\cdot[A]^{-1}\cdot[A]^{T}\cdot[Y] \qquad (III)$$

comparing (I) and (III) we can write

$\left[[A]^{T}\ [A]\right]^{-1}\ [A]^{T}$ as left pseudo-inverse of $[A]$

The expansion form of equation (II) is

$$\begin{bmatrix} \Sigma a_{i1}\ a_{i1} & \Sigma a_{i1}\ a_{i2} & \cdots\cdots & \Sigma a_{i1}\ a_{in} \\ \Sigma a_{i2}\ a_{i1} & \Sigma a_{i2}\ a_{i2} & \cdots\cdots & \Sigma a_{i2}\ a_{in} \\ \vdots & & & \\ \Sigma a_{in}\ a_{i1} & \Sigma a_{in}\ a_{i2} & \cdots\cdots & \Sigma a_{in}\ a_{in} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \Sigma a_{i1}\ y_i \\ \Sigma a_{i2}\ y_i \\ \\ \Sigma a_{in} y_i \end{bmatrix}$$

All summations in this equation is for i from 1 to m. This equation is also known as least errors square equation. Since equation III is the solution of equation II, the least error squares equation, it is obvious that the pseudo-inverse approach provides the least error squares solution.

CHAPTER - 7

CONCLUSION

7.1. Comparison of different methods:

The few methods for calculation of impedance and phase angle are described.

a - impedance calculation based on peak value.

b - computation of impedance using integrals of voltage and current.

c - Fourier analysis method.

d - predictive method.

A relative comparison of their accuracy is made. All these methods require a digital computer for implementation.

(i) The calculation of impedance based on first method (a), the peak values and phase angle based on zero crossing will be with more errors. This is because of the presence of transients in the line voltage and current immediately after the fault.

(ii) The V_{max} and I_{max} based, on first method, are obtained from the sampled values of $V_{positive}$ and $V_{negative}$ or

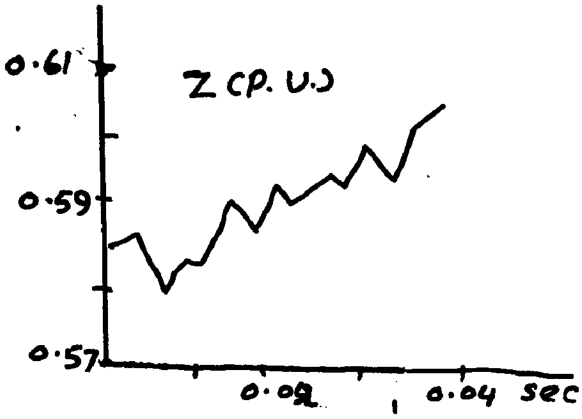


FIGURE 7·1 IMPEDANCE VARIATION

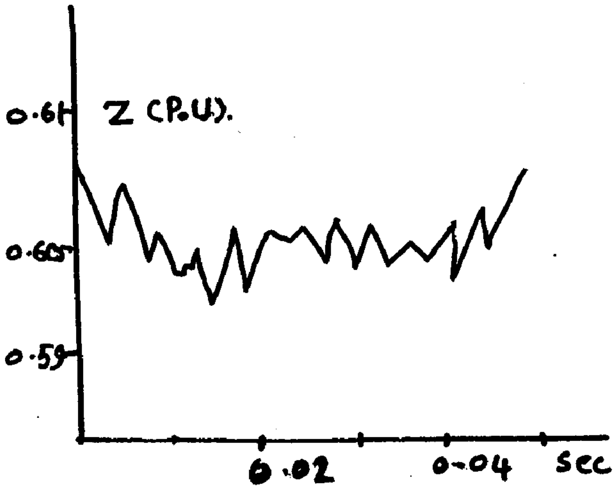


FIGURE 7:2 (a)

Impedance variation

Fourier Analysis Method.

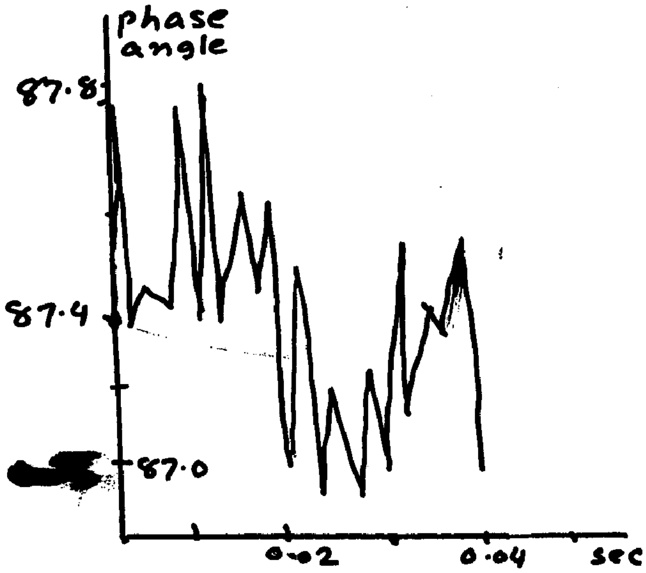


FIGURE 7:3 (b)

Phase angle variation

Fourier Analysis Method.

$I_{positive}$ and $I_{negative}$ amplitudes and the average magnitude is taken to represent the V_{max} and I_{max} respectively. This is not accurate.

(iii) Similarly the time difference of $V_{peak + ve}$ and $I_{peak + ve}$ is (T_{+ve}) and time between $V_{peak-ve}$ and $I_{peak-ve}$ (T_{-ve}) are computed. The average of these two gives the measure of phase angle. This is also erroneous because of transients in the line voltage and current immediately after the fault.

(iv) Due to integration in second method (b), the effect of higher frequencies present in the transient wave forms will be nullified. This give more accurate impedance measurement, than the impedance measurement by individual samples. In this integral method it is not easy to obtain phase angle.

(v) The percentage variation over the mean integrals of voltage and current methodis much less than impedance calculation based on peak values. This is shown in figure 7.1 and it is compared with table 4.1.

(vi) Fourier Analysis method explained, is used to compute the impedance and phase angle from both voltage and current. It canbe stated that the Z and ∅ obtained by this

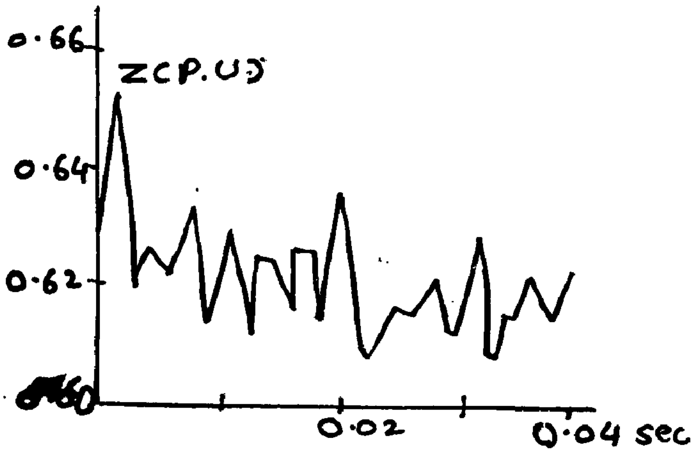


FIGURE : 7:4 Impedance variation
preductive calculation

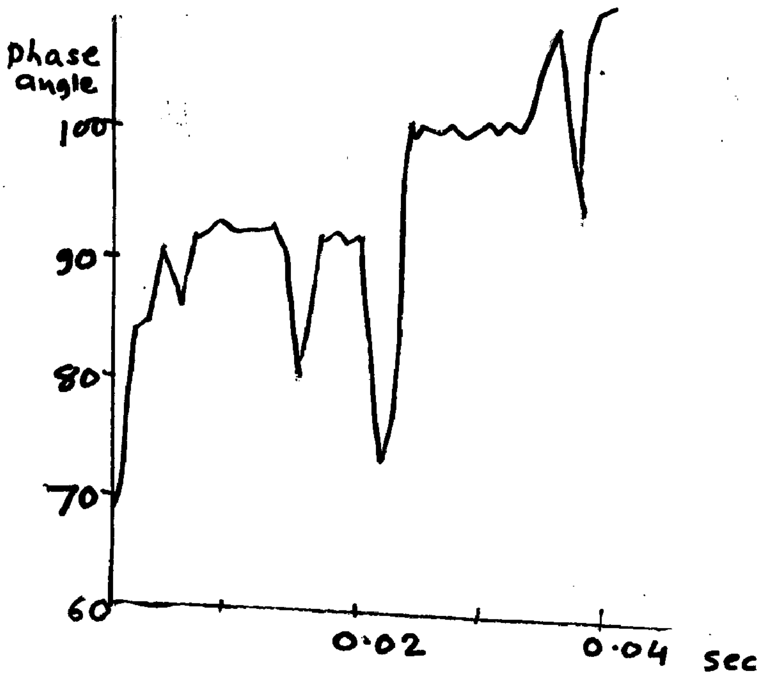


FIGURE 7:5 Phase angle variation
(preductive calculation)

method for faulted circuit is fairly constant compair to all other methods. The variation of Z and ϕ as function of time, is shown in figure 7.2 and 7.3. This method gives fairly constant value of impedance and phase angle.

(vii) To compare the predictive method with that given by Fourier analysis method, the Z-impedance in p.u. value and phase angle ϕ are averaged over a period of one cycle. The variation of Z and ϕ is plotted in Figure 7.4 and 7.5. It can be seen that the predictive method is relatively inaccurate compared to Fourier Analysis technique.

(viii) To evaluate the correct impedance of the line by different methods, it is necessary to have atleast a full cycle of samples of line voltage and current immediately after a fault. Hence this type of relay will have an inherent time delay of one cycle. The percentage over-reach of this relay will be negligible and therefore, the whole section can be protected in the first zone time.

(ix) Computing the impedance of faulted line by digital computer with different methods have been compared, of these the one using Fourier analysis method over an ensemble of samples covering a period of one cycle of system frequency can give good results.

II Selecting the relay reach limit:

The total job with digital computer consists of:

(i) fault detection (FD)

(ii) protective zone selection (PZS), which line or apparatus appear faulted.

(iii) fault-type analysis (FTA) which a.c. signal relevant to a particular protective zone should be processed in detail.

(iv) fault location (FL) - positive identification of which protective zones are faulted.

Fault resistance modifies, apparent impedance presented to relaying system for simplicity this resistance is represented on R-X complex plane. A computer analysis of the specific application, for selecting the relay reach limit can be provided at 0, 20, 40, 60, 80 and 100 % of distance.

III - Advantages of digital protection:

Based on preceding, we can make conclusion regarding digital transmission line protection.

(a) a digital protection can be made as fast and accurate compare to conventional relaying scheme.

(b) digital relaying techniques can provide security against tripping on loss of potential circuits while allowing for correct relay operation during a valid zero fault in protected zone.

(c) phenomena such as magnetizing inrush, third harmonic currents and sensor response are not big problems.

(d) operation within the possible range of power system frequency encounterable will not create a serious reach accuracy problem.

(e) a digital computer with its available hardware is capable of protecting one transmission line with 4 millisecond nominal clearing time.

Multi-line protection might be practically reached by more use of fault detection routines.

IV - The current research work:

(i) The object of the digital protection is to develop techniques and algorithms for relaying system that equalled or exceeded the performance of existing solid-state relaying system.

(ii) One can onset the project and commercially available digital equipment can be used. This approach can make it possible to develop the protection system without extensive hardware development. This will also help to evaluate the use of commercial minicomputers in power system protection.

(iii) The digital protection project can meet its objectives. The project can successfully demonstrate the capability and performance of digital techniques for transmission line protection. The system can be installed in the field, to operate with speed and security comparable to that of present advanced relaying systems.

(iv) The basic algorithm for R, L calculation has use for distance determination, by using stored prefault voltages, and a zero sequence direction unit for direction during phase and ground faults. The results of inductance calculations give accurate fault location which can be of value in assessing transmission line damage.

(v) Futures activity in digital relaying will be concerned with extending these techniques in other applications for protective relaying and development of cost effective digital systems by applying newly developed large scale integrated semiconductor devices.

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