ENERGY DETECTION AND CYCLIC PREFIX BASED CORRELATION DETECTION FOR COOPERATIVE SPECTRUM SENSING

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report entitled, "**ENERGY DETECTION AND CYCLIC PREFIX BASED CORRELATION DETECTION FOR COOPERATIVE SPECTRUM SENSING**" towards the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems**, submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from July 2010 to June 2011, under the guidance of **Mr.S.Chakravorty, Assistant Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee.**

I have not submitted the matter embodied in this dissertation for the award of any other Degree.

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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ABSTRACT

Cognitive radio is a technology that provides the unlicensed users access to the bands of spectrum that is not temporarily utilized by the licensed users. These unused bands are known as spectrum holes. The process of sensing the environment to identify these spectral holes is known as spectrum sensing. Cooperation among unlicensed users for spectral sensing can be used to combat the hidden terminal problem and fading.

In this dissertation work, cooperative spectrum sensing based on energy detection and cyclic prefix based correlation detection is studied for an OFDM primary system. For both these detection methods, data fusion is used at the fusion center for cooperative spectrum sensing. The receiver operating characteristics is studied for fading and shadowing channels. Data fusion is applied by maximizing modified deflection coefficient for energy detection and by Neyman-Pearson likelihood ratio test for cyclic prefix based correlation detection. The average number of statistics required for these fixed sample size tests are compared with the sequential probability ratio test.

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LIST OF ABBREVATIONS

CSMA	Carrier Sense Multiple Access
TDMA	Time Division Multiple Access
OFDM	Orthogonal Frequency Division Multiplexing
QAM	Quadrature Amplitude Modulation
SPRT	Sequential Probability Ratio Test
LLR	Log Likelihood Ratio

Chapter 1
INTRODUCTION

Cognitive radio is a novel technology which improves the spectrum utilization by allowing unlicensed networks to borrow unused radio spectrum from licensed networks or to share the spectrum with the licensed networks. Currently the radio spectrum is allocated statically and it has been found that such static allocation may result in underutilization of the spectrum [1]. The radio frequency spectrum is a scarce resource and cognitive radio technology can be used to utilize the available spectrum opportunistically. Primary users are the licensed users and secondary users are unlicensed users. Primary users have higher priority or legacy rights on the usage of a specific part of the spectrum. The secondary users exploit this spectrum in such a way that they do not cause interference to primary users. [2]. Cognitive radio is an extension of software radio paradigm. Mitola and Maguire stated that "Radio etiquette is the set of RF bands, air interfaces, protocols, and spatial and temporal patterns that moderate the use of radio spectrum. Cognitive radio extends the software radio with radio-domain model-based reasoning about such etiquettes" [3]. Haykin [4], stated that "cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (i.e., its outside world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming radio frequency (RF) stimuli by making corresponding changes in certain operating parameters (e.g., transmit power, carrier frequency, and modulation strategy) in real time, with two primary objectives in mind: 1) Highly reliable communications whenever and wherever needed; and 2) Efficient utilization of the radio spectrum."

1.1. Spectrum Sensing

One of most critical components of cognitive radio technology is spectrum sensing. Spectrum sensing refers to indentifying the bands of spectrum which are not utilized by primary users. These unused bands are known as spectrum holes. By sensing and adapting to the environment, a cognitive radio is able to utilize the spectrum holes and serve its users without causing harmful interference to primary users. Secondary users shall vacate the spectrum as soon as they find that primary user is active in a particular spectral band. Various methods such as energy detection, matched filtering and feature detection can be used for spectrum sensing [2]. The spectrum sensing for the identification of the primary user has certain challenges. For spectrum sensing purposes, secondary user terminals need high sampling rate, high resolution analog to digital converters with large dynamic range and high speed signal processors. The primary users may become active in a frequency band at any point of time. Hence the sensing methods must identify the presence of primary user in a band within a specified duration. The sensing frequency determines how often the environment needs to be sensed. It depends on the changing characteristics of primary user. For example, the presence of a TV channel in a particular spectral band does not change frequently. Hence the sensing frequency can be relaxed. The sensing frequency also depends on the interference tolerance of the primary user. For example, a cognitive radio network exploiting public safety bands needs to have higher sensing frequency [2].

If prior knowledge of primary user signal is available at secondary user, the optimal detector for spectrum sensing in stationary Gaussian noise is a matched filter [1]. This is a coherent detector which correlates a known signal with the received signal. However this detector cannot be applied for spectrum sensing if the primary signal type is unknown. Alternatively, an energy detector may be employed for primary user detection. This detector measures the energy of the received primary signal and compares it with a threshold. It does not require any prior knowledge of the primary user signal. However it has some drawbacks. If noise variance is not accurately known, the energy detector has poor performance in low SNR conditions. Another challenging issue is the inability to differentiate the interference from other secondary users sharing the same channel and primary user [1]. A cyclic prefix based correlation detector can be used to detect an OFDM primary signal based on correlation in a OFDM symbol due to the presence of cyclic prefix [5].

1.2. Cooperative Spectrum Sensing

The wireless channel between primary user and the secondary users may suffer from multipath fading and shadowing. This can make the spectrum sensing results unreliable. Shadowing and fading may result in the hidden primary user problem similar to the hidden terminal problem in carrier sense multiple access (CSMA) [2] This is shown in figure (1.1). Due to this, the cognitive radio can cause interference to primary users. Cooperation of the secondary users can improve the spectrum sensing performance in such environments. Cooperation results in sensing diversity gain, provided by the multiple cognitive radios. Even though one cognitive radio may fail to detect the signal of the primary user, there are chances for other cognitive radios to detect it. Cooperative sensing decreases the probabilities of miss detection and false alarm considerably. Cooperative spectrum sensing can be implemented in two ways-centralized and distributed [2]. In centralized sensing, a central unit known as fusion center collects the sensing information from the cognitive radios and indentifies the spectral holes. In distributed sensing the cognitive radios share information among each other and each cognitive radio makes its own decision of identifying the spectral holes. The cooperative sensing can be applied in two approaches - decision fusion and data fusion. In decision fusion based approach, a binary decision is made at each of secondary users about the presence of primary user. The fusion center collects these binary decisions and makes a final decision regarding the presence of primary user signal. In data fusion approach, secondary users shall send the statistics obtained based on spectrum sensing to the fusion center. For example in an energy detector, the energy values are the statistics obtained at secondary users. The fusion center shall combine these statistics received from secondary users to make a decision regarding the presence of primary user.

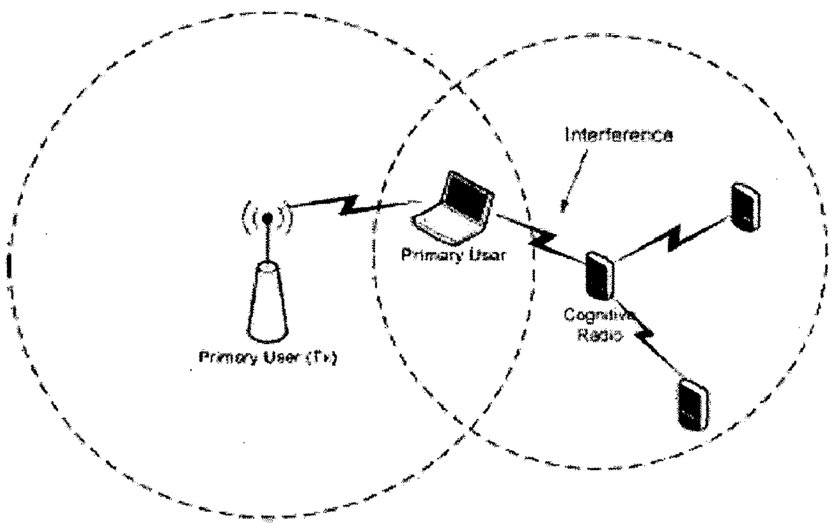

Figure 1.1: Illustration of Hidden Primary User Problem [2]

1.3. Problem Statement

The main objectives of this dissertation are

1) To study and evaluate cooperative spectrum sensing based on energy detection for fading and shadowing channels with modified deflection coefficient method for

data fusion and to compare the average statistics required with sequential probability ratio test.

2) To study and evaluate cooperative spectrum sensing based on cyclic prefix based correlation detection for fading and shadowing channels with Neyman-Pearson likelihood ratio test for data fusion and to compare the average statistics required with sequential probability ratio test.

1.4. Organisation of the Report

This dissertation report consists of 5 Chapters including this introductory chapter.

Chapter 2 provides an overview of cooperative spectrum sensing approaches. The reliability issues and tradeoffs in cooperative spectrum sensing are discussed. It also discusses the cooperative spectrum sensing for multiband systems.

In Chapter 3, cooperative spectrum sensing based on energy detection is discussed. This Chapter discusses about local sensing in a secondary user, modified deflection coefficient based data fusion and sequential detection methods applied for data fusion. The simulation results of the local and cooperative sensing in slow frequency selective Rayleigh fading channels and shadowing channels are presented. The simulation results of average of number of statistics of the modified deflection coefficient based data fusion are compared with the sequential detection methods.

In Chapter 4, cooperative spectrum sensing based on cyclic prefix based correlation detection is discussed. This Chapter includes the discussion about local sensing in a secondary user and comparison of the data fusion based on Neyman-Pearson likelihood ratio test with the sequential detection method. The simulation results of local sensing and cooperative sensing in slow frequency selective Rayleigh fading channels and shadowing channels are shown. A simulation result when the cyclic prefix length and synchronization information are known is also shown. The cyclic prefix based correlation detection and energy detection in the presence of noise uncertainty is studied.

The dissertation concludes with the Chapter 5.

Chapter 2

OVERVIEW OF COOPERATIVE SPECTRUM SENSING

Cooperative spectrum sensing improves the reliability of spectrum sensing by providing diversity gain. This chapter provides an overview of cooperative spectrum sensing approaches. Some techniques to improve the reliability of cooperative sensing and tradeoffs in cooperative sensing are discussed. An optimal wideband sensing framework for jointly detecting multiple bands both at the fusion center is also discussed.

2.1. Hypothesis Testing

For cooperative spectrum sensing, a binary hypothesis test needs to be performed to infer the presence of primary user. Let the hypothesis H1 indicate the presence of the primary user and H0 indicate the presence of the spectrum hole. Let $y = [y_1, y_2, \ldots y_N]$ be the vector of N observations under the hypothesis H0 or H1. Let $P(y \mid H0)$ indicate the probability density function under the hypothesis H0 and $P(y \mid H1)$ indicate the probability density function under the hypothesis H1. The following hypothesis tests can be applied at the fusion center.

2.1.1. Bayes Test

The Bayes test minimizes the average cost and is given by [6]

$$L(y) = \frac{P(y \mid H1)}{P(y \mid H0)} \underset{H0}{\overset{H1}{\gtrless}} \frac{P(H0)(C_{10} - C_{00})}{P(H1)(C_{01} - C_{11})} \qquad (2.1)$$

The cost value C_{ij} represents the cost of deciding H_i is true when H_j is present. The Bayesian criterion requires the priori probabilities of H0 and H1. These priori probabilities may not be available for spectrum sensing problems.

2.1.2. Neyman-Pearson Test

For Neyman-Pearson test, the objective is to maximize the probability of detection for a target probability of false alarm α. The Neyman-Pearson test can be performed with the following likelihood ratio test as[7]

$$L(y) = \frac{P(y \mid H1)}{P(y \mid H0)} \underset{H0}{\overset{H1}{\gtrless}} \gamma \qquad (2.2)$$

where γ is the threshold that is set such that the probability of false alarm is a fixed value α.

2.1.3. Generalized Likelihood Ratio Test

A composite hypothesis test is applied when some parameters of the probability density functions are unknown. A generalized likelihood ratio test is a composite hypothesis test that does not require a prior knowledge of unknown parameters. These unknown parameters are estimated by maximum likelihood estimation as follows[8]

$$\begin{aligned} \hat{\theta}_0 &= \max_{\theta_0} \mathrm{P}(y \mid H0, \theta_0) \\ \hat{\theta}_1 &= \max_{\theta_1} \mathrm{P}(y \mid H1, \theta_1) \end{aligned} \qquad (2.3)$$

where θ_0 and θ_1 are the set of unknown parameters under H0 and H1 respectively. The generalized likelihood ratio test can be written as

$$L(y) = \frac{P(y \mid \hat{\theta}_1, H1)}{P(y \mid \hat{\theta}_1, H0)} \underset{H0}{\overset{H1}{\gtrless}} \gamma \qquad (2.4)$$

where γ is the threshold. The generalised likelihood ratio test may be applied for spectrum sensing if there are unknown parameters like noise variance. The generalised likelihood ratio test has been applied for spectrum sensing in [9] and [10].

2.1.4. Sequential Probability Ratio Test

The likelihood ratio tests specified in (2.1) to (2.4) are fixed sample size detectors. In fixed sample size detectors, a fixed number of observations or samples are used. Sequential detection requires random number of samples depending on the observation sequence. A sequential test known as sequential probability ratio test (SPRT) requires lesser average number of samples for detection compared to fixed sample size tests. After every observation, SPRT may accept H0, may reject H0 or may continue with the test procedure with the next observations. SPRT can be applied for spectrum sensing to reduce the average delay of detection of spectrum holes. SPRT has disadvantages compared to fixed sample size tests. For SPRT, though the average number of samples required is lesser, occasionally the number of samples required before the final decision is made may be large. SPRT requires probability density functions of the observations both under H0 and H1. [11],[12]

2.2. Cooperative Sensing Approaches

In this section the two approaches of combining the sensing information at the fusion center - decision fusion and data fusion - are discussed.

2.2.1 Local Sensing Framework

Let us consider a cognitive radio network with K secondary users. Let us assume that the channel between the primary user and the fusion center is a Rayleigh flat fading channel. Secondary user i (i = 1,2,..K) formulates the following binary hypothesis test problem [7]

$$H0: x_i(t) = v_i(t) \qquad (2.5)$$
$$H1: x_i(t) = h_i s(t) + v_i(t)$$

where t is the time index

$x_i(t)$ is the received signal at i[th] secondary user

h_i is the channel gain between the primary transmitter and the i[th] secondary user

$v_i(t) \sim CN(0, \sigma_i^2)$ is complex circularly symmetric Gaussian noise with mean 0 and variance σ_i^2

H0 refers to absence of primary user signal

H1 refers to presence of primary user signal.

The key metrics of spectrum sensing are [1]
(i)Probabilities of correct detection given by P[Decision =H1|H1] and P[Decision =H0|H0],
(ii)Probability of false alarm given by P[Decision =H1|H0] and
(iii)Probability of miss detection given by P[Decision =H0|H1]

Specifically, $P(H0 \mid H0)$ is the probability that the secondary users successfully identify the unoccupied spectral segment and is an important measure of opportunistic spectrum utilization. Likewise $P(H0 \mid H1)$ is the probability that the secondary users cause harmful interference to the primary users. The objective of cooperative spectrum sensing is to maximize the $P(H0 \mid H0)$ while maintaining $P(H0 \mid H1)$ as low as possible.[7]

Let $x_i = [x_i(1), x_i(1), ...x_i(N)]^T$ where N is the number of samples used for sensing. Let $T_i(\mathbf{x_i})$ represent the test statistic obtained by spectrum sensing for the i^{th} secondary user. As an example, let us consider energy detector for spectrum sensing at the i^{th} secondary user. The decision rule at the secondary user is given by[7]

$$T_i(x_i) = \sum_{t=1}^{N} | x_i(t) |^2 \underset{H0}{\overset{H1}{\gtrless}} \gamma_i \qquad (2.6)$$

where $T_i(x_i)$ is the energy at the i^{th} secondary user and γ_i is the local threshold at the i^{th} secondary user.

2.2.2. Decision Fusion

In decision fusion approach, each cognitive radio makes a binary decision locally on whether the primary user is present or not. The fusion center combines these decisions made by secondary users to obtain a final decision. The optimal decision fusion rule is based on likelihood ratio test according to Neyman-Pearson criterion [7]. Let u_i denote the individual decision of each cognitive radio. Let u_i be '0' if H0 is inferred and let u_i be '1' if H1 is inferred. Let $\boldsymbol{u} = [u_1, u_2, ...u_K]^T$ denote the vector of decisions from individual secondary users. Let $P(\boldsymbol{u} | H0)$ and $P(\boldsymbol{u} | H1)$ represent the probability distributions for H0 and H1 respectively. Then the likelihood ratio test is given by

$$L(\boldsymbol{u}) = \frac{P(\boldsymbol{u} | H1)}{P(\boldsymbol{u} | H0)} \underset{H0}{\overset{H1}{\gtrless}} \gamma^* \qquad (2.7)$$

where γ^* denotes the optimal threshold at the fusion center.

Sub optimal decision fusion rules can also be applied. Counting rule is a sub optimal rule that can be used. It counts the number of secondary users that infer the presence of primary user and compares it to a threshold. The fusion center infers that the primary user signal is transmitted, if n secondary users out of K secondary users infer H1 ("n out of K" rule) [1]. This rule is an AND rule, if the fusion center infers that the primary user signal is transmitted when all secondary users infer H1 (n=K). This is an OR rule if the fusion center infers that the primary user signal is transmitted, if any one of the secondary users infer H1 (n=1). The OR rule is a conservative rule as it infers H1 even if one of the secondary users infer H1. The OR rule has the highest probability of detection compared to other rules and it is shown in figure(2.1).

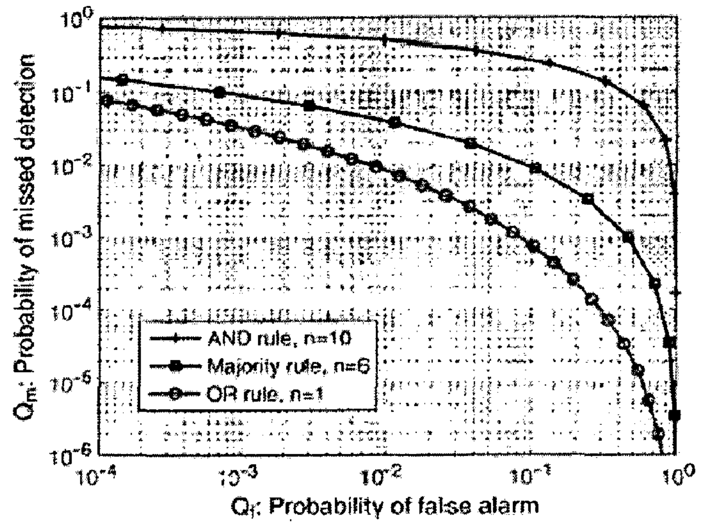

Figure 2.1: Cooperative spectrum sensing performance with decision fusion rules (n=1,6,10) over Rayleigh fading channels with average SNR =10dB for ten secondary users.[1]

In [13], the problem of minimizing the total error rate of cooperative spectrum sensing under different conditions of "n out of K" rule is considered. Let P_f be probability of false alarm and the P_d be the probability of detection at the secondary users. Let us assume that secondary users employ energy detection. At each secondary user, let the threshold be λ and let the average SNRs be $\bar{\gamma}$. For "n out of K rule", the probability of false alarm of the cooperative sensing system is given by,[13]

$$Q_f = \sum_{l=n}^{K} {}^{K}C_l P_f^l (1-P_f)^{K-l} \qquad (2.8)$$

The probability of missed detection of the cooperative sensing system is given by,

$$Q_m = 1 - \sum_{l=n}^{K} {}^{K}C_l P_d^l (1-P_d)^{K-l} \qquad (2.9)$$

Three optimal rules for evaluating the optimal value of n, optimal energy detection threshold and optimal number of secondary users are presented in [13].

1. Optimal Voting Rule: For a fixed value of K, the optimal value of n given by n_{opt} can be found by minimizing the total error rate $Q_f + Q_m$ with respect to n. The optimal value of n is given by

$$n_{opt} = \min\left(K, \left\lceil \frac{K}{1+\alpha} \right\rceil\right) \qquad (2.10)$$

where $\alpha = \dfrac{\ln(P_f / 1 - P_m)}{\ln(P_m / 1 - P_f)}$.

- If P_f and P_m are of the same order i.e $\alpha \approx 1$, the optimal value of n is K/2.
- The OR rule is optimal if $\alpha \geq K-1$. This means that $P_f \leq P_m^{K-1}$. Hence for large K, $P_f \ll P_m$ .This can be obtained when the threshold λ is large.
- The AND rule is optimal if $\alpha \to 0$. This implies $P_m \ll P_f$ .This can be obtained when the threshold λ is small.

2. Optimal Energy Detection Threshold: For a fixed value of K, n and SNR $\bar{\gamma}$, the optimal value of threshold λ^* is obtained by minimizing the total error rate $Q_f + Q_m$ with respect to λ.[13]

The above results are can be explained with figure 2.2. From figure 2.2, it can be seen that the total error rate is dependent on the threshold. For a fixed very small threshold the optimal rule is an AND rule(n=K). For a fixed large threshold the optimal rule is OR rule(n=1).

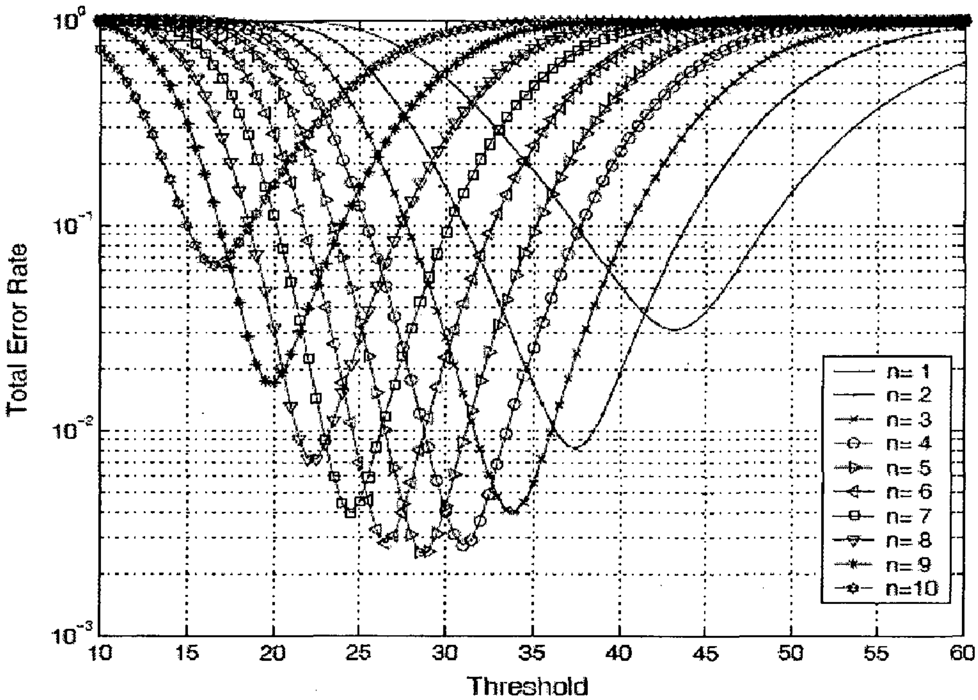

Figure 2.2 : Total error rate of cooperative sensing in 10dB AWGN channel.(For voting rules n=1,2....10,K=10)[13]

3. Optimal Number of Secondary Users: If the number of secondary users in a cooperative spectrum sensing system is large, the transmission of decisions from secondary users may result in overhead in bandwidth and sensing time. If SNR $\bar{\gamma}$ and threshold λ are known, the least number of secondary users required for cooperative sensing to achieve a target error bound $Q_f + Q_m \leq \varepsilon$ can be found [13].

From figure (2.3), it can be found that the total error rate depends on the number of cooperating secondary users given by, K. If the target error rate is restricted as $Q_f + Q_m \leq 0.01$, it can be found from figure (2.3) that the smallest numbers of secondary users to get the error rate target are 12, 17, and 32 for SNR values of 20, 15, and 10 dB respectively.[13]

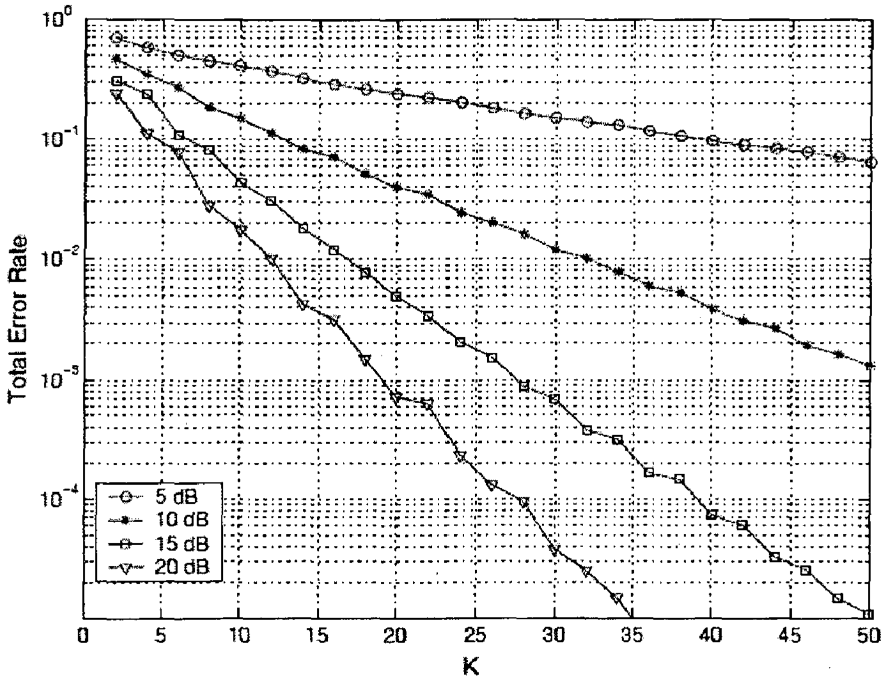

Figure 2.3: Total error rate of cooperative spectrum sensing versus number of secondary users in Rayleigh fading with SNR = 5, 10, 15, 20 dB; optimal voting rule applied; detection threshold set fixed at $\lambda = 20$ [13]

A linear quadratic decision fusion method is proposed in [14] to consider the correlation between secondary users in cooperative sensing. Let $\boldsymbol{u} = [u_1, u_2, ...u_K]^T$ denote the vector of decisions from K secondary users. This method provides a suboptimal solution to the decision fusion problem by using the partial statistical knowledge given by the second-order statistics of the local decisions under H1 and the fourth-order statistics under H0. This method uses a deflection criterion of the form [14]

$$D_T = \frac{[E(T(X) \mid H1) - E(T(X) \mid H0)]^2}{Var(T(X) \mid H0)} \tag{2.11}$$

Here $T(X)$ represents a linear quadratic test statistic of the form [14]

$$T(X) = h^T X + X^T MX \tag{2.12}$$

where h is a vector of length K and M is a KxK square matrix.

Here the components of the vector X is given by

$$X_i = \log\left(\frac{q_1(u_i)}{q_0(u_i)}\right) - E_0\left[\log\left(\frac{q_1(u_i)}{q_0(u_i)}\right)\right] \tag{2.13}$$

where $q_j(\,)$ denotes the probability distribution of u_i under the hypothesis H_j and E_0 represents the expectation operator under H_0. The objective is to find the linear quadratic function of the form (2.12) to maximize the deflection criterion in (2.11). Based on the deflection criterion, the linear quadratic detector compares a linear quadratic function of the local decisions with a predetermined threshold and achieves better probability of detection of the primary user with a higher value of deflection. The proposed scheme outperforms the counting rule in correlated shadowing.

2.2.3. Data Fusion

The data fusion approach involves the cognitive radios to send the observed test statistics to the fusion center [7]. The fusion center then makes the final decision based on the statistics collected from secondary users. The secondary users send the summary statistics $y = [T_1(x_1), T_2(x_2), ...T_K(x_K)]^T$ to the fusion center in which an optimal likelihood ratio test can be performed as [7]

$$L(y) = \frac{P(y \mid H1)}{P(y \mid H0)} \underset{H0}{\overset{H1}{\gtrless}} \gamma^* \tag{2.14}$$

where γ^* denotes the optimal threshold for a desired probability of false alarm. From the central limit theorem, it can be found that, for energy detection, y is asymptotically normally distributed for large N with [7]

$$y \sim N(\mu_0, \Sigma_0) \text{ under H0}$$

$$y \sim N(\mu_1, \Sigma_1) \text{ under H1} \qquad (2.15)$$

where μ_0 and Σ_0 are mean vector and covariance matrix of y under H0 and

μ_1 and Σ_1 are mean vector and covariance matrix of y under H1.

A sub optimal rule based on the linear combination of the test statistics of the secondary users can be used. Specifically, the test statistic can be chosen to be of the form [7]

$$L(y) = w^T y \underset{H0}{\overset{H1}{\gtrless}} \gamma \qquad (2.16)$$

where w is the weight vector representing the contribution of the individual nodes to the global decision and γ is the threshold. For example, if a secondary user has high SNR value, the statistics of that secondary user should be assigned a larger weighting coefficient. Since the linear combination of several Gaussian random variables is still Gaussian, the probability of detecting a spectrum hole is [7]

$$P(H0 \mid H0) = 1 - Q\left(\frac{\gamma - \mu_0^T w}{\sqrt{w^T \Sigma_0 w}}\right) \qquad (2.17)$$

The probability of miss detection is

$$P(H0 \mid H1) = 1 - Q\left(\frac{\gamma - \mu_1^T w}{\sqrt{w^T \Sigma_1 w}}\right) \qquad (2.18)$$

The weights w and the threshold γ can be found by maximizing the probability of detecting the spectral hole subject to a constraint on the interference probability as follows[7]

$$\max_{w,\gamma} P(H0 \mid H0) \qquad (2.19)$$

$$\text{such that } P(H0 \mid H1) \le \varepsilon$$

This problem can be solved using optimization methods. Other sub optimal linear fusion rules like maximal ratio combiner, equal gain combiner and maximum modified deflection coefficient based combiner can also be applied.

A modified deflection coefficient based combiner is proposed in [15]. This involves in finding the weight vector w that maximizes the modified deflection coefficient given by,

$$\max_{w} d_m^2(w) = \frac{[E(L(y) \mid H1) - E(L(y) \mid H0)]^2}{\mathrm{var}(L(y) \mid H1)} \qquad (2.20)$$

such that $\|w\|^2 = 1$

For a maximal ratio combiner, the weights are equal to the signal to noise ratios at the secondary users [16]. In [16], a softened two bit hard combination scheme for energy detection is considered. In one bit decision fusion schemes, the energy values are divided into 2 regions with a single threshold. However the probability of detection of the primary user and probability of detecting a spectrum hole can be increased if the observed energy values are divided into 4 regions. This is a 2 bit hard scheme with 3 thresholds λ_1, λ_2 and λ_3. This is shown in figure (2.4).

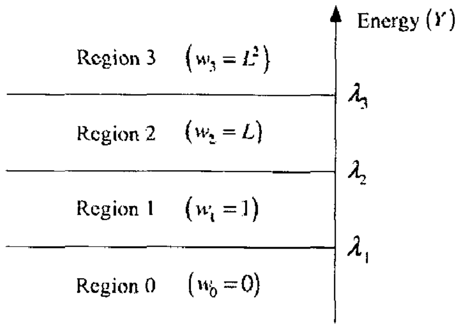

Figure 2.4: Principle of two-bit hard combination scheme[16]

The primary signal is inferred to be present if one of the observed energy is in region 3, if L energy values are in region 2 or L^2 energy values are in region 1, where L is a design parameter. The weights of the regions are given by $w_0 = 0$, $w_1 = 1$, $w_2 = L$ and $w_3 = L^2$. The weighted summation is given by [16]

$$N_c = \sum_{i=0}^{3} w_i N_i \qquad (2.21)$$

where N_i denotes the number of secondary users with energy values in the region i. The primary signal is inferred to be present if $N_c \geq L^2$. With an overhead of one more bit, this 2 bit hard combination scheme has better performance compared to OR rule.[16]

2.3. Reliability of Sensing and Reporting Channels

2.3.1 Reliability of Sensing Channel

The sensing channel refers to the channel between the primary users and fusion center. The effects of fading and shadowing on spectrum sensing are studied in [17]. Cooperative spectrum sensing is used to combat these channel effects in the sensing channel. The effect of the correlation in shadowing is considered in [17], [18]. The closely located secondary users may experience similar shadowing effects. The effect of correlation due to shadowing can reduce the diversity gain due to cooperation. It is shown that the probability of missed opportunity of utilizing the spectrum holes (probability of false alarm P(H1|H0)) is lower bounded due to the correlated shadowing effects. This lower bound on probability of missed opportunity due to correlated shadowing for distances D=100m and D=200m between secondary users is shown in figure (2.5). It is shown that having a small number of secondary users over a large distance may be more effective than a large number of closely located users in correlated shadowing scenarios.

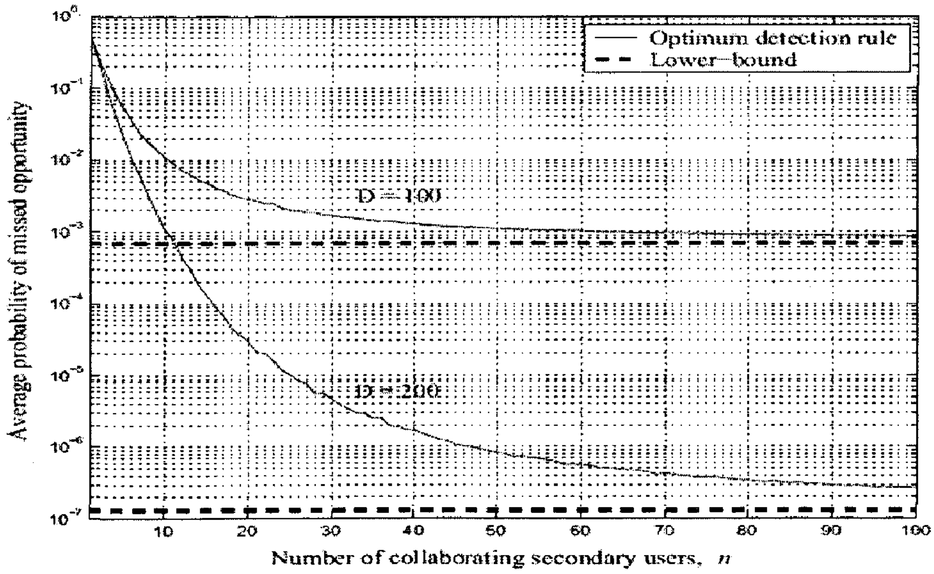

Figure 2.5: Average probability of missed opportunity versus the number of collaborating secondary users[18]

2.3.2 Reliability of Reporting Channel

The reporting channels between secondary users and fusion center can exhibit fading resulting in erroneous transmission of sensing information to fusion center [1]. This can be illustrated by considering OR rule based decision fusion at fusion center. Let the number of secondary users be K. For the secondary user i, let P_f^i be the probability of false alarm and P_m^i be the probability of miss detection. Let P_e^i denote the probability of error over the reporting channel between secondary users and fusion center. In the presence of reporting channel errors, the probability of false alarm of the cooperative sensing system for OR rule is,[1]

$$Q_f = 1 - \prod_{i=1}^{K} [(1-P_f^i)(1-P_e^i) + P_f^i P_e^i] \qquad (2.22)$$

The probability of miss detection of the cooperative sensing system for OR rule [1] is,

$$Q_m = \prod_{i=1}^{K} [P_m^i(1-P_e^i) + (1-P_m^i)P_e^i] \qquad (2.23)$$

Let us assume that every secondary user has an identical probability of false alarm and experiences identical and independent fading in reporting channel. Then $P_e^i = P_e, \forall i = 1, 2, ...K$. The probability of false alarm of the cooperative sensing system is lower bounded by $\bar{Q}_f$ as follows [1]

$$Q_f \geq \bar{Q}_f = 1 - (1-P_e)^K \qquad (2.24)$$

(2.20) was derived from (2.18) by considering that Q_f increases with P_f and $Q_f \geq \min Q_f = \lim_{P_f \to 0} Q_f$.

For small values of P_e, (2.20) becomes [1]

$$Q_f \geq KP_e \qquad (2.25)$$

This shows that the probability of false alarm has a lower bound due to reporting channel errors which does not depend on the sensing channel between primary user and secondary users.

2.4. Robustness of Cooperative Spectrum Sensing

The clustering approach, the censoring approach and relay assisted approach are some of the robust techniques that can be employed for cooperative spectrum sensing. This section discusses these techniques.

2.4.1 Clustering Approach

In clustering approach, secondary users are configured into many clusters [1], [19]. The channel between any two secondary users in a cluster is assumed to be perfect as these secondary users are close to each other. The secondary user which has the highest SNR in the reporting channel is chosen as the cluster head. Secondary users report the sensing information to the cluster head. Cluster head makes a preliminary decision regarding the presence of primary user using decision fusion or data fusion methods. These preliminary decisions are forwarded to the fusion center which makes a final decision. This is a form of selection diversity as the best link(the link between cluster heads and fusion center), which has the highest SNR is chosen for transmission of decisions. This approach is shown in figure(2.6) The errors due to reporting channel between secondary user and the fusion center can be reduced by this approach. The bandwidth of the control channel can be reduced as the number of decision bits transmitted to the fusion center is reduced.

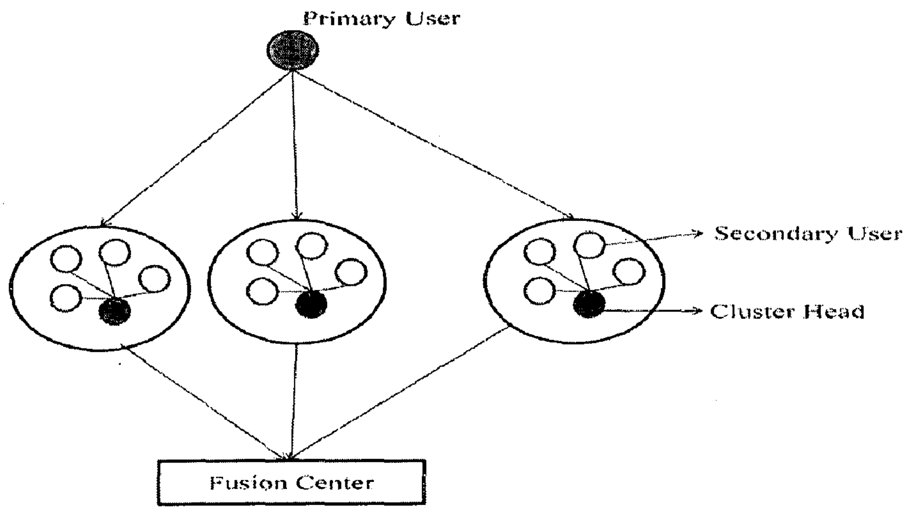

Figure 2.6: Clustering Approach [1],[19]

2.4.2 Censoring Approach

As the number of secondary users increase in a cognitive radio network, the number of decision bits sent to the fusion center increase. This increases the bandwidth of the control channel used for the transmission of these bits. At a secondary user, the decisions are made by comparing a local test statistic to a threshold γ. This test statistic is the energy value if the energy detector is employed in secondary users. The decisions in the vicinity of this threshold are not reliable. A censoring approach for transmission of decision bits is proposed in [20].

The secondary users whose statistics fall in the ambiguous region $[\gamma_1,\gamma_2]$ around the threshold γ are neglected. The decision bits of secondary users whose statistics are out of this ambiguous region are reported to the fusion center. The decision of a secondary user is 1 if the statistic is higher than γ_2 and the decision of a secondary user is 0 if the statistic is lower than γ_1. This censored approach reduces the average number of bits sent to the fusion center as the unreliable information is censored.

An energy efficient cooperative spectrum sensing is proposed in [21]. The cognitive radio network shall use a combination of sleeping and censoring to reduce the energy consumption. In this scheme, when in sleep mode, each secondary user switches off its sensing transceiver and incurs no observation costs or transmission costs. It also censors the transmission of statistics to the fusion center, if the statistics fall within the ambiguous region $[\gamma_1,\gamma_2]$. Let C_{si} and C_{ti} be the energy consumed by the i[th] secondary user in sensing and transmission, respectively. The average energy consumed for the cooperative sensing network is given by,[22]

$$C_T = (1-\mu)\sum_{i=1}^{N}(C_{si} + C_{ti}(1-\rho)) \tag{2.26}$$

where ρ denotes the censoring rate and μ is the sleeping rate of the secondary users.

The censoring rate ρ is the probability of the test statistic to fall in the ambiguous region given by $P[\gamma_1 < E_i < \gamma_2]$ where E_i is the test statistic of the i[th] secondary user. Hence the parameter ρ can be written in terms of γ_1 and γ_2. The objective is determine the optimum sleeping rate μ and the censoring thresholds γ_1 and γ_2 to minimize the average energy consumed C_T, subject to constraints on probability of false alarm $P_f \le \alpha$ and probability of detection $P_d \ge \beta$. Here α and β are pre-specified design parameters. The energy optimization problem is given by,[22]

$$\begin{aligned} &\min_{\mu,\gamma_1,\gamma_2} \ \mathrm{C_T} \\ &\textit{such that } P_f \le \alpha, P_d \ge \beta \end{aligned} \tag{2.27}$$

2.4.3 Relay Assisted Approach

A relay assisted cooperative spectrum sensing scheme based on amplify and forward protocol is proposed in [22]. A cognitive radio network with two secondary users U1 and U2 is considered as in figure (2.7). These secondary users are assumed to be operating in a fixed time division multiple access (TDMA) mode to send data to the fusion center. Secondary users shall vacate the band as soon as possible if primary user starts using that band. If a secondary user, say U1, is far away from a primary user, the primary user signal power received by U1 will be weaker compared to the secondary user U2 which is nearby the primary user. The secondary user, which receives weaker primary user power levels, may take longer time to detect the primary signal. Cooperation among the secondary users shall reduce this detection time of the secondary user with weaker primary signal power levels and improve the agility of the network. Secondary users U1 and U2 shall transmit data in successive slots using amplify and forward relaying as shown in figure (2.8).

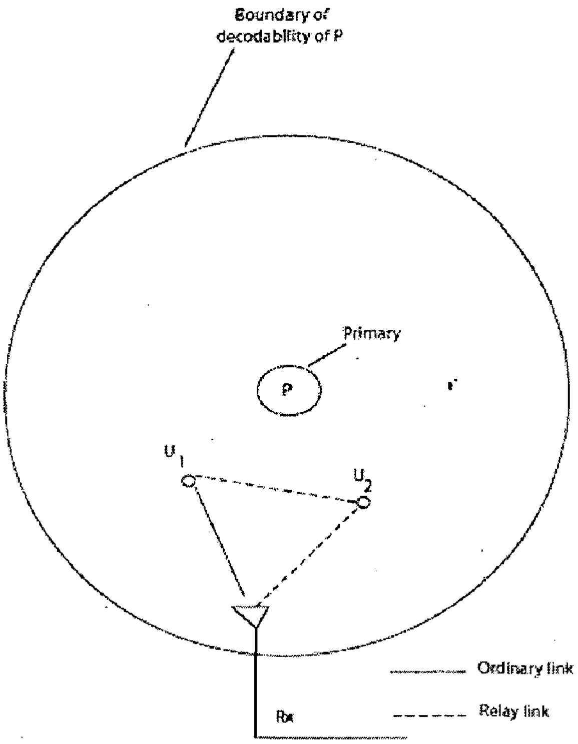

Figure 2.7: Relay assisted cooperative sensing[22]

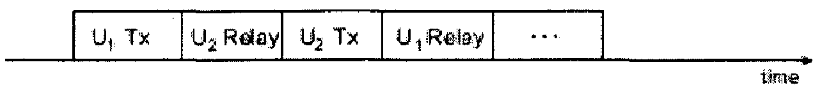

Figure 2.8: Relaying protocol used.[22]

In the first time slot T_1, U1 transmits and U2 listens. At time slot T_1, the signal received by U2 from U1 is given by, [22]

$$y_2 = \theta h_{p2} + a h_{12} + w_2 \tag{2.28}$$

where h_{pi} denotes the instantaneous channel gain between the primary user and U_i,

h_{12} denotes the instantaneous channel gain between U1 and U2

w_2 denotes the additive complex Gaussian noise,

a denotes the data sent from U1

θ denotes the primary user indicator; $\theta = 1$ implies the primary user presence and $\theta = 0$ implies the primary user absence.

In the next time slot T_2, U2 relays the message and U1 shall listen to its own message relayed by U2. The signal received by U1 from U2 is [22]

$$\begin{aligned} y_1 &= \sqrt{\beta_1} y_2 h_{12} + \theta h_{p1} + w_1 \\ &= \sqrt{\beta_1} h_{12} (\theta h_{p2} + a h_{12} + w_2) + \theta h_{p1} + w_1 \end{aligned} \tag{2.29}$$

where β_1 represents the scaling factor used by U2 for the relayed message. Secondary user U_1 can cancel the message part and is left with the signal [22]

$$Y = \theta H + W \tag{2.30}$$

where $H = h_{p1} + \sqrt{\beta_1} h_{12} h_{p2}$ and $W = w_1 + \sqrt{\beta_1} h_{12} w_2$.

Hence given the observation Y, the detection problem at U1 is to identify if $\theta = 0$ or $\theta = 1$. This relay assisted cooperative spectrum sensing scheme shall improve the probability of detection of the primary user. Compared to non cooperative systems, this cooperative sensing scheme provides agility improvements as it reduces the detection time given by the number of slots for detection.

2.5. Tradeoffs in Cooperative Spectrum Sensing

2.5.1 Sensing-Throughput Tradeoff

The sensing duration is an important parameter that can determine the sensing accuracy. A longer sensing duration can increase the probability of detection of the primary user but result in longer waiting time for secondary users [23]. Let the sensing duration be τ and T be the fixed frame duration. The cognitive radio network periodically senses for every frame duration T. The time available for the cognitive radio network for data transmission is $T-\tau$. Let C_0 be the throughput of the cognitive radio network when it operates in the absence of primary users and C_1 be the throughput in the presence of primary users. For example, let us consider one point to point transmission in the secondary network and SNR of this secondary link is $SNR_s = P_s / N_0$, where P_s is the received signal power of the secondary user and N_0 is the noise power. Let P_p be the interference power of primary user measured at the secondary user receiver. Let us assume that the primary and secondary user signals are Gaussian, white and independent of each other. Then the throughputs C_0 and C_1 are given by [23],

$$\begin{aligned} C_0 &= \log_2(1+SNR_s) \\ C_1 &= \log_2\left(1+\frac{P_s}{P_p+N_0}\right) = \log_2\left(1+\frac{SNR_s}{1+SNR_p}\right) \end{aligned} \tag{2.31}$$

where $SNR_p = P_p / N_0$. Let $P(H_0)$ be the probability that primary user is present and $P(H_1)$ be the probability that primary user is absent. Let γ be the detection threshold of the secondary user.

The average throughput of the secondary network is given by [23]

$$R(\tau) = \frac{T-\tau}{T} C_0 (1-P_f(\gamma,\tau)) P(H_0) + \frac{T-\tau}{T} C_1 (1-P_d(\gamma,\tau)) P(H_1) \tag{2.32}$$

where the probability of false alarm P_f and the probability of detection P_d depend on threshold γ and sensing duration τ.

We can find that as the sensing time τ increases, the throughput reduces. However, as the sensing time τ increases, the probability of false alarm reduces for a given probability of detection. This improves the throughput as the probability of detecting a spectral hole increases. The objective of the sensing-throughput tradeoff problem is to find the optimal τ such that the throughput is maximized with a constraint on the probability of detection of the primary user. It can be formulated as[23]

$$\begin{aligned} &\max_{\tau} \quad R(\tau) \\ &\text{such that } P_d(\gamma,\tau) \geq \bar{P}_d \end{aligned} \tag{2.33}$$

where $\bar{P}_d$ is the minimum target probability of detection of the primary user.

The sensing-throughput tradeoff is considered for "k out of N" rule based cooperative spectrum sensing in [24]. For "k out of N" rule based cooperative spectrum sensing the average throughput of the cognitive radio network is given by

$$R(\tau,k,\gamma)=\frac{T-\tau}{T}C_0(1-Q_f(\tau,k,\gamma))P(H_0)+\frac{T-\tau}{T}C_1(1-Q_d(\tau,k,\gamma))P(H_1) \quad (2.34)$$

where Q_f refers to the probability of false alarm of the cooperative spectrum sensing system and Q_d refers to the probability of detection of the cooperative spectrum sensing system. These probabilities are functions of sensing duration τ, threshold γ and the value of k chosen for "k out of N" rule. The sensing throughput tradeoff optimization for "k out of N" rule is formulated as

$$\begin{aligned}&\max_{\tau,k,\gamma} \quad R(\tau,k,\gamma)\\&\text{such that } \; Q_d(\tau,k,\gamma)\geq \bar{Q}_d\end{aligned} \quad (2.35)$$

where $\bar{Q}_d$ is the minimum probability of detection that the fusion center needs to achieve to protect the primary user.

2.5.2 Cooperation-Processing Tradeoff

Cooperative sensing improves the probability of detection of the primary user by providing diversity gain. The sensing time of individual secondary users can be reduced as the number of cooperating secondary users increase. However, as the number of cooperating secondary users increase, the transmission time of the local sensing statistics or decisions to fusion center results in overhead. There exists a tradeoff between local processing time and time for sending the sensing results to the fusion center.[25]

This tradeoff can be formulated as the following optimization problem[25]

$$\begin{aligned}&\min_{\tau,n} \quad T_S=\tau+nT_C\\&\text{such that } \; Q_d(\tau,n)|_{SNR=SNR_{\min}}=1-\beta\\&\qquad Q_f(\tau,n)=\varepsilon\\&\qquad n\geq 1,\ \text{n integer}\end{aligned} \quad (2.36)$$

where T_S is the total time consumed for cooperative sensing

τ is the sensing duration,

T_c is the time consumed for polling the secondary user by the fusion center

Q_d is the probability of detection of cooperative sensing system

Q_f is the probability of false alarm of cooperative sensing system

n is the number of secondary users

β and ε determine the constraints on probability of detection and probability of false alarm respectively

SNR_{min} is the minimum SNR at which the cognitive radio network has to detect the primary user.

2.6. Multiband Joint Detection

Wideband sensing refers to identifying spectral holes over multiple frequency bands over a wide range of frequency. Multiband joint detection refers to jointly detecting multiple spectral holes of wideband spectrum. A spatial-spectral joint detection method is proposed in [7],[26]. This is a cooperative wideband sensing scheme which jointly detects multiple bands by exploiting the spatial diversity provided by multiple secondary users. This is a scheme based on the linear combination of statistics received from secondary users for each of the K subbands. Let us consider M secondary users sensing K subbands. Let $T_{k,i}$ be the statistics for the k[th] subband obtained by the i[th] secondary user. Each secondary user sends the statistics to the fusion center. For the subband k, the sensing results of M secondary users can be represented by the vector $\boldsymbol{Y}_k = [T_{k,1}, T_{k,2}, \ldots T_{k,M}]^T$. At the fusion center, for each subband, the sensing results are linearly combined through a weight vector $\boldsymbol{w}_k = [w_{k,1}, w_{k,2}, \ldots w_{k,M}]^T$ as[7]

$$z_k = \boldsymbol{w}_k^T \boldsymbol{Y}_k = \sum_{i=1}^{M} w_{k,i} Y_{k,i} \qquad (2.37)$$

The test statistic z_k is compared to a threshold γ_k for each subband as[7]

$$z_k \underset{H_{0,k}}{\overset{H_{1,k}}{\gtrless}} \gamma_k \qquad k = 1, 2, \ldots. K \tag{2.38}$$

An optimal spatial-spectral joint optimization problem can be formulated as the problem of maximizing the aggregate throughput $R(\gamma, W)$ of the cognitive radio network subject to the constraints on aggregate interference $I(\gamma, W)$, maximum interference in each subband and minimum opportunistic utilization in each subband. This optimization problem is given by[7]

$$\begin{aligned}
\max_{\gamma, W} \quad & R(\gamma, W) = \sum_{k=1}^{K} r_k P(H_{0,k} \mid H_{0,k}, \gamma_k, w_k) \\
\text{such that} \quad & I(\gamma, W) = \sum_{k=1}^{K} c_k P(H_{0,k} \mid H_{1,k}, \gamma_k, w_k) \le \varepsilon \\
& P(H_{0,k} \mid H_{1,k}, \gamma_k, w_k) \le \alpha_k, \quad k = 1, 2, \ldots. K \\
& P(H_{0,k} \mid H_{0,k}, \gamma_k, w_k) \ge \beta_k, \quad k = 1, 2, \ldots. K
\end{aligned} \tag{2.39}$$

where $\gamma = [\gamma_1, \gamma_2, \ldots.. \gamma_K]^T$ is the threshold vector

$W = [w_1, w_2, \ldots. w_K]$ is the matrix of weights for K subbands and M secondary users

α_k is the maximum allowable interference in the subband k and

β_k is the minimum opportunistic utilization in the subband k.

Chapter 3

COOPERATIVE SPECTRUM SENSING BASED ON ENERGY DETECTION

Energy detector based approach is a common way of spectrum sensing because of its low computational and implementation complexities [2]. The energy detector is optimal if the secondary users have limited knowledge about primary user signal [29]. The energy of the band pass filtered signal is compared to a threshold to determine whether the primary signal is present. The challenges with energy detector based sensing are selection of the threshold for detecting primary users, inability to differentiate interference from primary users and noise, and poor performance under low signal-to-noise ratio (SNR) values. The noise power uncertainty that exists due the difference between actual and the estimated noise powers also degrade the probability of detection of the energy detector [27]. This chapter describes the cooperative spectrum sensing of single band of Orthogonal Frequency Division Multiplexing (OFDM) primary signals using energy detection. A sub optimal method based on modified deflection coefficient [15] is applied for data fusion and the performance is studied for various channels. The average number of statistics required for the modified deflection coefficient based method is compared to sequential probability ratio test (SPRT) [11], [12] based data fusion and detection.

3.1. Local Sensing

The primary system is assumed to be an OFDM system consisting of narrowband subcarriers each modulated by quadrature amplitude modulation (QAM). An OFDM signal is constructed by feeding symbols to an Inverse Fast Fourier Transform (IFFT) through serial to parallel conversion [28]. If C(0),C(1),...,C(T_d-1) are T_d complex QAM symbols, then the output of the IFFT is

$$c(t) = \frac{1}{\sqrt{T_d}} \sum_{f=0}^{T_d-1} C(f) e^{\frac{j2\pi ft}{T_d}} \qquad (3.1)$$

where t is a discrete time index , f is a discrete frequency index and T_d represents number of useful symbols. T_c symbols are added in front of the block as the cyclic prefix to form an

OFDM block $[C(T_d - T_c),\ldots\ldots C(T_d - 1), C(0),\ldots\ldots, C(T_d - 1)]$ which is of size $T_s = T_d + T_c$. A transmitted frame may contain several such blocks. Let us denote the symbols of the transmitted OFDM frame by s(t). Let H_0 be the null hypothesis indicating that the primary signal is absent and H_1 be the alternate hypothesis that the primary signal is present. Then the hypothesis testing problem is written as

$$\begin{aligned} H0:\ x_i(t) &= w_i(t) \\ H1:\ x_i(t) &= \sum_{l=0}^{P-1} h_i(l)s(t-l) + w_i(t) \end{aligned} \tag{3.2}$$

where $h_i(l)$ with $l=0,1,\ldots.P-1$ represents the channel taps. The fading channel is assumed to be slow with $h_i(l)$ constant during the interval of observation. By the Central Limit Theorem, c(t) is approximately Gaussian, since it is a linear combination of T_d signals [10]. Also E[C(f)] =0 and E[s(t)]=0. If the channel gains are random and uncorrelated then for different hypothesis the probability distribution [28] shall be

$$\begin{aligned} H0:\ x_i(t) &\sim CN(0, \sigma_{w,i}^2) \\ H1:\ x_i(t) &\sim CN(0, \sigma_{w,i}^2 + v_i\sigma_s^2) \end{aligned} \tag{3.3}$$

where σ_s^2 is primary signal power, $\sigma_{w,i}^2$ is the noise power and $v_i = \sum_{l=0}^{P-1} |h_i(l)|^2$ is the total gain at each secondary user. The energy of the received signal $x_i(t)$ of M samples is computed as [28]

$$Y_i = \frac{1}{M}\sum_{t=0}^{M-1} |x_i(t)|^2 \tag{3.4}$$

Using central limit theorem for correlated variables [28], it can be shown that

$$Y_i \sim \begin{cases} N\left(\sigma_{w,i}^2, \dfrac{\sigma_{w,i}^4}{M}\right) & H0 \\ N\left((\sigma_{w,i}^2 + v_i\sigma_s^2), \dfrac{(\sigma_{w,i}^2 + v_i\sigma_s^2)^2}{M}\right) & H1 \end{cases} \tag{3.5}$$

Based on Neyman-Pearson criterion, the decision rule of each secondary user is given by [28]

$$Y_i \underset{H0}{\overset{H1}{\gtrless}} \gamma_c \tag{3.6}$$

where γ_i is the decision threshold for each secondary user. The probability of false alarm and probability of detection at each secondary user is [15]

$$P_{f,i} = P(Y_i > \gamma_i \mid H0) = Q\left[\frac{\gamma_i - E(Y_i \mid H0)}{\sqrt{Var(Y_i \mid H0)}}\right] \tag{3.7}$$

$$P_{d,i} = P(Y_i > \gamma_i \mid H1) = Q\left[\frac{\gamma_i - E(Y_i \mid H1)}{\sqrt{Var(Y_i \mid H1)}}\right] \tag{3.8}$$

3.2. Global Data Fusion

3.2.1 Modified Deflection Coefficient Based Data Fusion

To allow multiple secondary users to collaborate, the test statistics are transmitted to the fusion center via a dedicated error free control channel. The global statistics can be computed as linear combination of Y_i [15] as follows

$$Y_c = \sum_{i=1}^{M} w_i Y_i = \boldsymbol{w}^T \boldsymbol{Y} \tag{3.9}$$

where $\boldsymbol{w} = [w_1, w_2, ...w_K]$, $w_i>0$ is the weight vector to control the global spectrum detector. The combining weight for the signal from a particular user represents its contribution to the global decision [15]. For example, if a cognitive radio generates a high-SNR signal that may lead to correct detection on its own, it should be assigned a larger weighting coefficient. For those secondary users experiencing deep fading or shadowing, their weights are decreased in order to reduce their negative contribution to the decision fusion. Since Y_i are normal random variables, their linear combination is also normal. Y_c has mean [15]

$$\overline{Y}_c = E(Y_c) = \begin{cases} \boldsymbol{\sigma}^T \boldsymbol{w} & H0 \\ (\boldsymbol{\sigma} + \sigma_s^2 \boldsymbol{g})^T \boldsymbol{w} & H1 \end{cases} \tag{3.10}$$

where $g = [v_1, v_2, ...v_K]^T$ represents the channel gains for each of the secondary users and $\sigma = [\sigma^2_{w,1}, \sigma^2_{w,2}, ...\sigma^2_{w,K}]^T$ represents the vector of noise variances of the secondary users.

Y_c has the variance [15]

$$Var(Y_c) = E(Y_c - \bar{Y}_c)^2 = \mathbf{w}^{\mathrm{T}} E[(\mathbf{Y}\text{-}\bar{\mathbf{Y}})(\mathbf{Y}\text{-}\bar{\mathbf{Y}})^T]\, \mathbf{w} \tag{3.11}$$

For the different hypotheses the variances are

$$\begin{aligned} Var(Y_c \mid H0) &= \mathbf{w}^{\mathrm{T}} E[(\mathbf{Y}\text{-}\bar{\mathbf{Y}}_{H0})(\mathbf{Y}\text{-}\bar{\mathbf{Y}}_{H0})^T \mid H0]\, \mathbf{w} \\ &= \sum_{i=1}^{K} (\frac{{\sigma_{w,i}}^4}{M}) w_i^2 \\ &= \mathbf{w}^{\mathrm{T}} \Sigma_{H0} \mathbf{w} \end{aligned} \tag{3.12}$$

with $\Sigma_{H0} = diag(\frac{{\sigma_{w,1}}^4}{M}, \frac{{\sigma_{w,2}}^4}{M}, ...\frac{{\sigma_{w,K}}^4}{M})$

$$\begin{aligned} Var(Y_c \mid H1) &= \mathbf{w}^{\mathrm{T}} E[(\mathbf{Y}\text{-}\bar{\mathbf{Y}}_{H1})(\mathbf{Y}\text{-}\bar{\mathbf{Y}}_{H1})^T \mid H1] \mathbf{w} \\ &= \sum_{i=1}^{K} (\frac{({\sigma_{w,i}}^2 + v_i {\sigma_s}^2)^2}{M}) w_i^2 \\ &= \mathbf{w}^{\mathrm{T}} \Sigma_{H1} \mathbf{w} \end{aligned} \tag{3.13}$$

with $\Sigma_{H1} = diag(\frac{({\sigma_{w,1}}^2 + v_1 {\sigma_s}^2)^2}{M}, \frac{({\sigma_{w,2}}^2 + v_2 {\sigma_s}^2)^2}{M}, ...\frac{({\sigma_{w,K}}^2 + v_K {\sigma_s}^2)^2}{M})$

Since Σ_{H1} is positive semi-definite and diagonal, its square root can be written as

$$\mathbf{\Sigma}_{H1}^{1/2} = diag(\sqrt{\frac{({\sigma_{w,1}}^2 + v_1 {\sigma_s}^2)^2}{M}}, \sqrt{\frac{({\sigma_{w,2}}^2 + v_2 {\sigma_s}^2)^2}{M}}, ...\sqrt{\frac{({\sigma_{w,K}}^2 + v_K {\sigma_s}^2)^2}{M}}) \tag{3.14}$$

The weight vector can be obtained by maximizing the modified deflection coefficient [15]

$$d_m^2 = \frac{[E(Y_c \mid H1) - E(Y_c \mid H0)]^2}{\mathrm{var}(Y_c \mid H1)} = \frac{(\sigma_s^2 \mathbf{g}^{\mathrm{T}} \mathbf{w})^2}{\mathbf{w}^{\mathrm{T}} \Sigma_{H1} \mathbf{w}} \tag{3.15}$$

The modified deflection coefficient is maximized under the unit-norm constraint on the weight vector [15],

$$\max_{w} d_m^2(w) \tag{3.16}$$

under the constraint $||w||^2 = 1$

This problem is solved as follows. Applying the linear transformation $w' = \Sigma_{H1}^{1/2} w$ we get,[15]

$$\begin{aligned} d_m^2(w) &= \frac{\sigma_s^4 w'^T \Sigma_{H1}^{-T/2} g g^T \Sigma_{H1}^{-1/2} w'}{w'^T w'} \\ &\leq \sigma_s^4 \lambda_{\max} \\ &= \sigma_s^4 \, || \Sigma_{H1}^{-T/2} g ||^2 \end{aligned} \tag{3.17}$$

where the inequality follows from Rayleigh Ritz inequality and the equality is achieved if

$$w' = \Sigma_{H1}^{-T/2} g \tag{3.18}$$

which is the eigen vector of the positive semi-definite matrix $\Sigma_{H1}^{-T/2} g g^T \Sigma_{H1}^{-T/2}$ corresponding to the maximum eigen value. The optimum solution for **w** [15] is

$$w = \frac{\Sigma_{H1}^{-1/2} w'}{|| \Sigma_{H1}^{-1/2} w' ||} \tag{3.19}$$

which maximises d_m^2. To enforce $E(Y_c|H1) > E(Y_c|H0)$ the weight vector is considered as

$$w^o = sign(g^T w) w \tag{3.20}$$

Considering the linear rule at the fusion center with a threshold γ_c, we have the global decision rule

$$Y_c \underset{H0}{\overset{H1}{\gtrless}} \gamma_c \tag{3.21}$$

The probability of false alarm and probability of detection with the global decision rule is given by

$$P_f = Q\left(\frac{\gamma_c - \sigma^T w}{\sqrt{w^T \Sigma_{H0} w}}\right) \quad (3.22)$$

$$P_d = Q\left(\frac{\gamma_c - (\boldsymbol{\sigma} + \sigma_s^2 \mathbf{g})^T \mathbf{w}}{\sqrt{\mathbf{w}^T \Sigma_{H1} \mathbf{w}}}\right) \quad (3.23)$$

3.2.2 Sequential Detection

In sequential detection the fusion center receives the log likelihood ratio (LLR) from each secondary user sequentially. A sequential detection scheme known as sequential probability ratio test (SPRT) is utilized. After receiving each statistics Y_i from the i^{th} secondary user, the log likelihood ratio (LLR) L_i is calculated and a sequential hypothesis test is performed. If sufficient evidence is available to conclusively decide either of the hypotheses, the test procedure is terminated. Otherwise the hypothesis test is continued. The test procedure after receiving 'k' statistics [11],[12] is

$$\sum_{i=1}^{k} L_i \le \log B \quad \text{Decide H0}$$

$$\sum_{i=1}^{k} L_i \ge \log A \quad \text{Decide H1} \quad (3.24)$$

Otherwise, take the next user statistics

where,

$$A = \frac{1-\beta}{P_f}$$

$$B = \frac{\beta}{1-P_f}$$

$$L_i = \ln\frac{P(Y_i \mid H1)}{P(Y_i \mid H0)} = \ln\left(\frac{\text{var}(Y_i \mid H0)}{\text{var}(Y_i \mid H1)}\right) + \left(-\frac{(V_i - E(V_i \mid H1))^2}{(2 * \text{var}(V_i \mid H1))} + \frac{(Y_i - E(Y_i \mid H0))^2}{(2 * \text{var}(Y_i \mid H0))}\right)$$

Here $\beta = 1 - P_d$. The mean and variances of Y_i under the two hypothesis is found from (3.5). The number of log likelihood ratios used to form the decision, $k=K_s$ is a random variable. Performance of a sequential detector can be expressed in terms of the average sample number. The average sample number for the sequential probability ratio test is defined as the number of samples (statistics) required on average for arriving at a decision under either hypotheses. Under the two hypotheses assuming that the signal and noise powers are same in each secondary user, the average numbers of samples for the SPRT [11], [12] are,

$$E[K_s \mid H0] = \frac{P_f \log A + (1-P_f)\log B}{E[L_i \mid H0]}$$
$$E[K_s \mid H1] = \frac{(1-\beta)\log A + \beta \log B}{E[L_i \mid H1]} \tag{3.25}$$

The average sample number for the sequential probability ratio test is given by

$$K_m = \max\{E[K_s \mid H0], E[K_s \mid H1]\} \tag{3.26}$$

By simulations, we obtain the average sample number required for sequential probability ratio test and for modified deflection coefficient based data fusion and compare them.

3.3. Results and Discussion

The performance results for a spectrum sensing are described in terms of the receiver operating characteristics plot for various channels. The parameters used for the simulation for local sensing are given in the table 3.1. These parameters are same as that used in [5].

Table 3.1: Simulation parameters for the local sensing with energy detection

Number of OFDM blocks	100
OFDM symbol type	16 QAM
Size of IFFT (Td)	32
Cyclic prefix length (T_c)	8
Number of OFDM samples	$100(T_d+T_c)=4000$

3.3.1. Local sensing

The receiver operating characteristics of the energy detector for a single cognitive radio for AWGN, slow frequency selective Rayleigh fading, shadowing, correlated shadowing and slow frequency selective Rayleigh fading with shadowing channels is simulated. The slow frequency selective Rayleigh fading channel of channel order 6 is simulated by complex Gaussian coefficients with exponential power delay profile. For simulating shadowing effects, the SNR of the user is selected randomly from a Gaussian distribution with mean SNR=-17 dB and standard deviation of 5 dB for each realization.

It can be seen from figure (3.1) that the slow fading channel degrades the performance compared to AWGN. For shadowing channel, the energy detector has lower probability of detection for higher probabilities of false alarm compared to AWGN channel. For AWGN+Shadowing+Fading channels the energy detector has lower probability of detection compared to AWGN+Shadowing due to the Rayleigh fading effects.

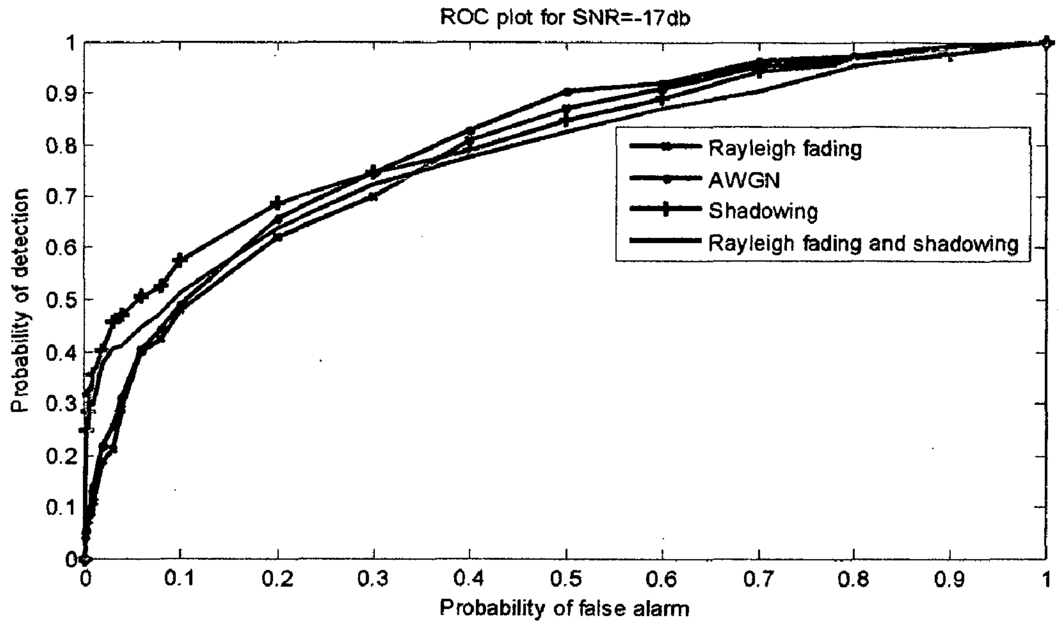

Fig 3.1 Energy Detection-Receiver Operating Characteristics Plot for Local Sensing

3.3.2. Cooperative sensing

From figure (3.2) we find that cooperative sensing through the modified deflection coefficient method can improve the sensing performance for various channels. 5 secondary users are used for the simulations.

In figure (3.3) we compare the average number of secondary user statistics of the sequential detection based data fusion to that of the modified deflection coefficient based data fusion (a fixed sample size based method) for AWGN channels. For the sequential detection the limit for false alarm rate is set to 0.05 and the limit for the probability of miss detection is set to be 0.05. The sequential detection is truncated at a maximum number of user statistics of 100. It can found from figure (3.3) that the average number of secondary user statistics required is lesser for sequential detection.

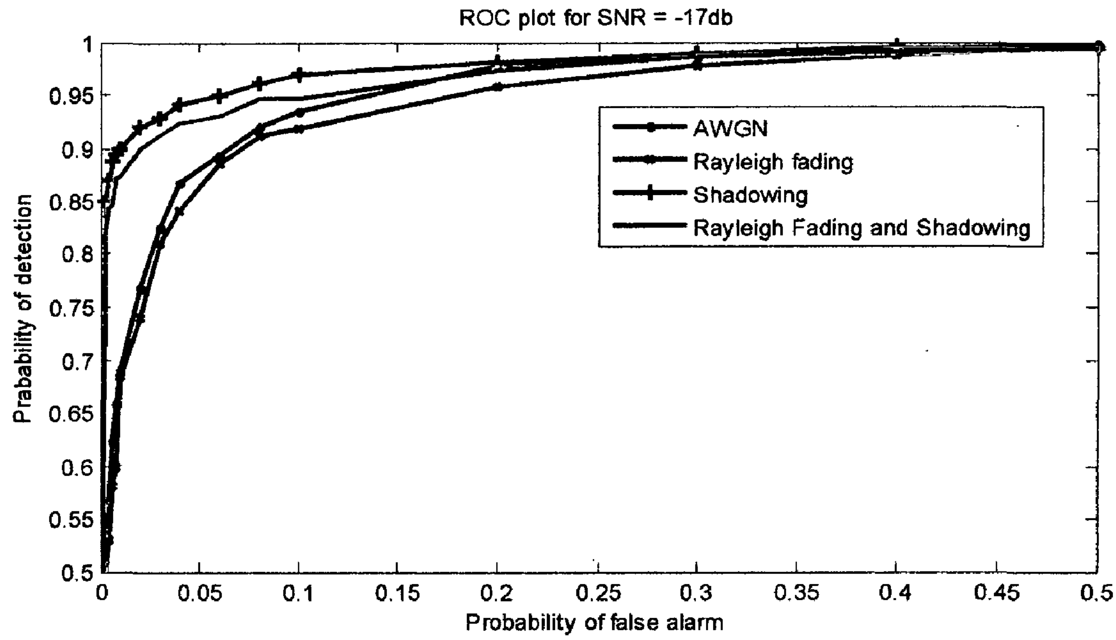

Figure 3.2 Receiver Operating Characteristics Plot for Cooperative Sensing based on Modified Deflection Coefficient (5 secondary users)

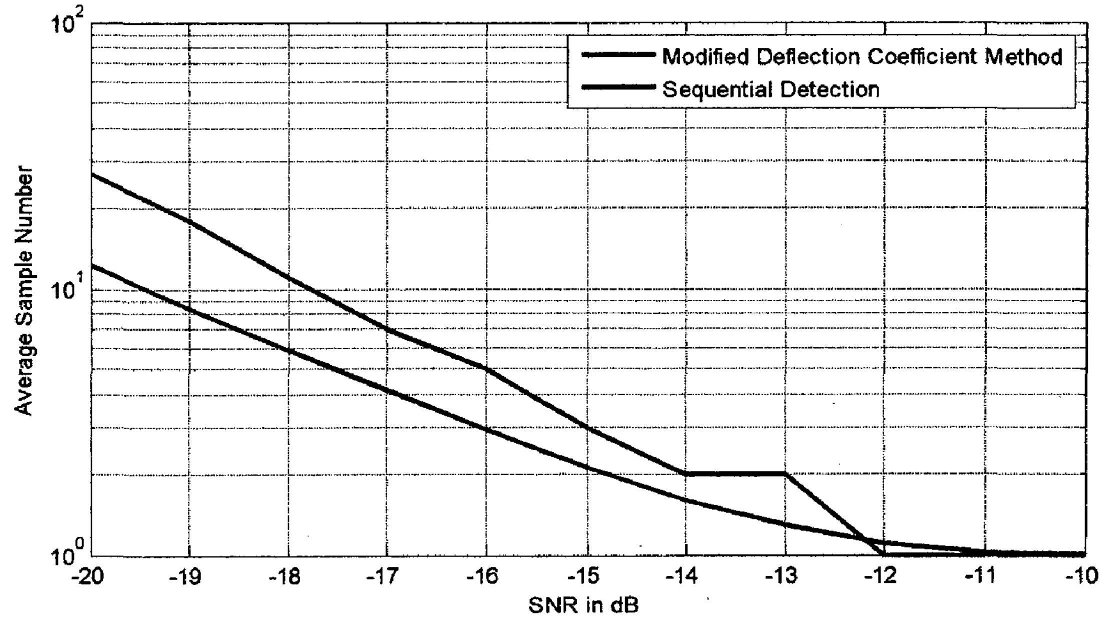

Fig 3.3 Comparison of Sequential Detection with the Fixed Sample Size Modified Deflection Coefficient Method

In figure (3.4) the performance of cooperative sensing in the presence of correlated shadowing is plotted for 2 users. This study of correlated shadowing is also reported in [17],[18]. The shadowing correlation would degrade performance of collaborative sensing when collaborating users are close. This is because the closely located users are likely to experience similar shadowing effects. The correlation function due to shadowing is $\rho(d) = \exp(-ad)$ where 'a' is a constant depending upon the environment [17]. The value of 'a' used for simulation is 0.1204 considering urban environment. The correlated shadowing is simulated by considering SNR of the users given by a correlated normal distribution with mean -17 dB, standard deviation σ =5dB and covariance matrix of $\sigma^2\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$

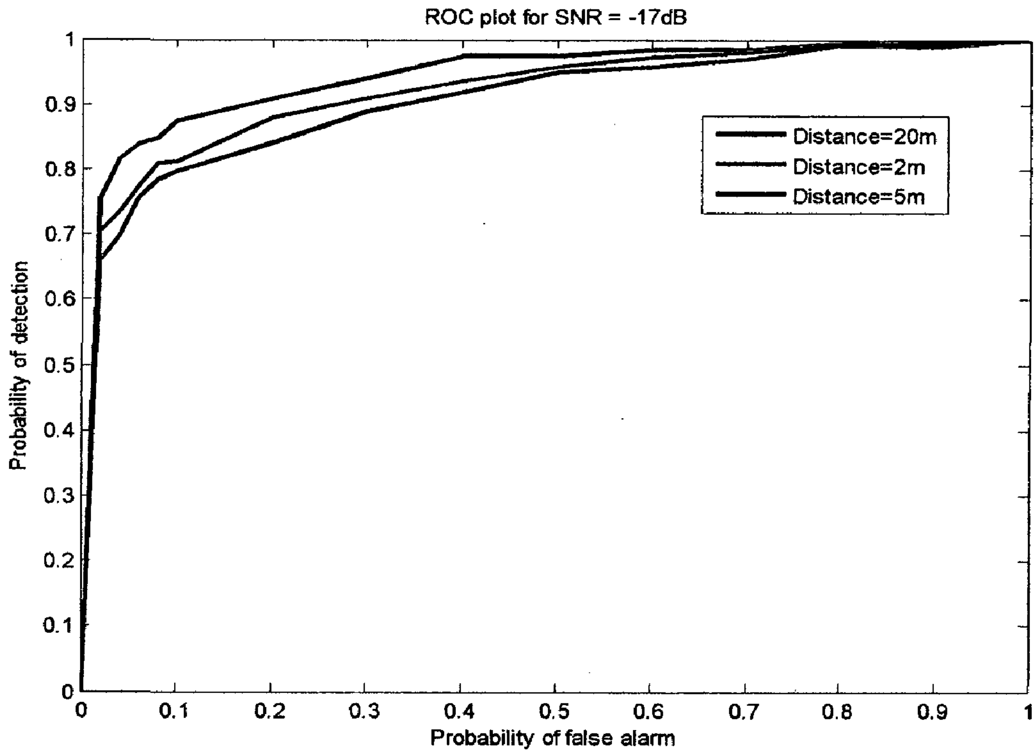

Fig 3.4 Energy Detection-Receiver Operating Characteristics Plot for the Correlated Shadowing Channels (for 2 secondary users for various distances).

Chapter 4

COOPERATIVE SPECTRUM SENSING BASED ON CYCLIC PREFIX BASED CORRELATION DETECTION

Orthogonal Frequency Division Multiplexing (OFDM) is used in various applications such as digital television, audio broadcasting, wireless networking and broadband internet access. Hence the problem of sensing the OFDM primary signals is important. The presence of cyclic prefix in the OFDM primary signal can be exploited for spectrum sensing [5]. For OFDM symbols the autocorrelation coefficient is non zero at a lag of $\pm T_d$ samples, where T_d is the number of samples of useful symbol duration in the OFDM block. A study of this cyclic prefix based autocorrelation detector proposed in [5] is done for fading and shadowing channels. For global fusion of the test statistics a fixed sample size based likelihood ratio test is applied. This is compared with sequential detection based data fusion.

4.1. Local Sensing

The primary user is assumed to transmit an OFDM signal same as that defined in section 3.1. Let H0 be the null hypothesis indicating that the primary signal is absent and H1 be the alternate hypothesis that the primary signal is present. First an AWGN channel is considered. The hypothesis testing problem is written as [5]

$$\begin{aligned} H0: x(t) &= w(t) \\ H1: x(t) &= s(t) + w(t) \end{aligned} \quad (4.1)$$

where x(t) is the received signal in a secondary user and w(t) is the complex circular additive white Gaussian noise. By Central limit theorem if the IFFT size is large s(t) will complex Gaussian distributed.

$$\begin{aligned} &\text{H0: x(t)} \sim \text{CN}(0, \sigma_s^2) \\ &\text{H1: } x(t) \sim \text{CN}(0, \sigma_s^2 + \sigma_w^2) \end{aligned} \quad (4.2)$$

Since $x(t) = x_r(t) + jx_i(t)$ is a circularly symmetric Gaussian random variable, real and imaginary parts of x(t) are distributed as, [5]

$$\begin{aligned} x_r(t) &\sim N(0, \sigma_x^2/2) \\ x_i(t) &\sim N(0, \sigma_x^2/2) \end{aligned} \tag{4.3}$$

For a cyclic prefix decoder, the value of the autocorrelation coefficient $\rho(\tau) = E(x(t)x(t+\tau)]/E[x(t)x^*(t)]$ for lags of $\tau = \pm T_d$ under the two hypotheses is [5]

$$\begin{aligned} &\text{H0: } \rho(\pm T_d) = 0 \\ &\text{H1: } \rho(\pm T_d) = \rho_1 \end{aligned} \tag{4.4}$$

where $\rho_1 = \frac{Tc}{T_d + T_c}\frac{\sigma_s^2}{\sigma_s^2 + \sigma_w^2} = \frac{Tc}{T_d + T_c}\frac{SNR}{1+SNR}$.

Here $\text{SNR} = \frac{\sigma_s^2}{\sigma_w^2}$. (4.5)

For a real valued coefficient $\rho(\tau) = \rho(-\tau)$. Hence only $\rho(T_d)$ is considered. The observation is considered over several OFDM samples i.e., $x(0), x(1), \ldots. x(M + T_d - 1)$ where M>>T_d. Two real random vectors $\mathbf{z}_1$ and $\mathbf{z}_2$ are formed given by

$$\mathbf{z}_1 = [x_r(0)\ x_i(0)\ x_r(1)\ x_i(1)\ \ldots\ldots x_r(M-1)\ x_i(M-1)] \tag{4.6}$$

$$\mathbf{z}_2 = [x_r(T_d)\ x_i(T_d)\ x_r(T_d+1)\ x_i(T_d+1)\ \ldots\ldots x_r(M+T_d-1)\ x_i(M+T_d-1)]$$

Here $x_r(t)$ and $x_i(t)$ are the real and imaginary parts of x(t). Due to the circular symmetry assumption, the zero mean random variables $x_r(t)$ and $x_i(t)$ and are independent and identically distributed. The random variables $z_1(t)$ and $z_2(t)$ which are t^{th} component of the vectors $\mathbf{z}_1$ and $\mathbf{z}_2$ respectively, are jointly Gaussian under both the hypotheses with probability densities given by [5]

$$f(z_1(t), z_2(t) \mid H0) = \frac{1}{2\pi\sigma_0^2}\exp\left\{-\frac{1}{2}\left[\frac{z_1^2(t) + z_2^2(t)}{\sigma_0^2}\right]\right\} \tag{4.7}$$

$$f(z_1(t), z_2(t) \mid H1) = \frac{1}{2\pi\sigma_1^2\sqrt{1-\rho_1^2}}\exp\left\{-\frac{1}{2(1-\rho_1^2)}\left[\frac{z_1^2(t) - 2\rho_1 z_1(t)z_2(t) + z_2^2(t)}{\sigma_1^2}\right]\right\}$$

where $\sigma_1^2 = (\sigma_s^2 + \sigma_w^2)/2$ and $\sigma_0^2 = \sigma_w^2/2$. The likelihood ratio test for the hypothesis test is given as [5]

$$\Lambda = \prod_{t=0}^{2M-1} \frac{f(z_1(t), z_2(t) \mid H1)}{f(z_1(t), z_2(t) \mid H0)} \tag{4.8}$$

$$= \frac{\sigma_0^{2M}}{\sigma_1^{2M}(1-\rho_1^2)^M} \exp\left\{ -\frac{1}{2}\left(\frac{1}{(1-\rho_1^2)\sigma_1^2} - \frac{1}{\sigma_0^2} \right) \sum_{t=0}^{2M-1} (z_1^{\,1}(t) + z_2^{\,2}(t)) + \frac{\rho_1 \sum_{t=0}^{2M-1} z_1(t) z_2(t)}{(1-\rho_1^2)\sigma_1^2} \right\},$$

Since $z_1(t)$ and $z_2(t)$ are identically distributed random variables, $E[z_1^2(t)]=E[z_2^2(t)]$. The likelihood estimate of $E[z_1^2(t)]$ based on **z1** and **z2** is given by [5]

$$\hat{\sigma}_z^2 = \frac{1}{4M} \sum_{t=0}^{2M-1} (z_1^{\,2}(t) + z_2^{\,2}(t)) \tag{4.9}$$

But $\hat{\sigma}_z^2$ can be written in terms of the observations $[x_r(1)\ x_i(1)\ x_r(2)\ x_i(2)\ ......x_r(M)\ x_i(M)]$ as [5]

$$\hat{\sigma}_z^2 = \frac{1}{2(M+T_d)} \sum_{t=0}^{M+T_d-1} (x_r^{\,2}(t) + x_i^{\,2}(t)) = \frac{1}{2(M+T_d)} \sum_{t=0}^{M+T_d-1} |x(t)|^2 \tag{4.10}$$

The maximum likelihood estimate $\hat{\rho}_{ML}$ of the autocorrelation coefficient ρ from vectors $\mathbf{z_1}$ and $\mathbf{z_2}$ is [5]

$$\hat{\rho}_{ML} = \frac{\frac{1}{2M} \sum_{t=0}^{2M-1} x_r(t) x_r(t+T_d) + x_i(t) x_i(t+T_d)}{\hat{\sigma}_z^2} = \frac{\frac{1}{M} \sum_{t=0}^{M-1} R\{x(t) x^*(t+T_d)\}}{\hat{\sigma}_z^2} \tag{4.11}$$

where R{} denotes real part of complex number. Using approximation $\hat{\sigma}_z^2 \approx \sigma_1^2 \approx \sigma_0^2$ for low SNR case we get, [5]

$$\Lambda = \frac{\exp\left\{ -2M\left[\frac{\rho_1^2}{1-\rho_1^2} \right] + \frac{2M\rho_1\hat{\rho}_{ML}}{1-\rho_1^2} \right\}}{(1-\rho_1^2)^M} \tag{4.12}$$

The log likelihood ratio (LLR) is given as [5]

$$L = \ln(\Lambda) = -M\ln(1-\rho_1^2) + 2M\frac{\rho_1(\hat{\rho}_{ML} - \rho_1)}{1-\rho_1^2} \tag{4.13}$$

Therefore, when using the LLRT, the alternative hypothesis for the present hypothesis test is decided if $L > \eta_1$ where η_1 is the threshold of the detector. Equivalently, H1 is decided if

$$\hat{\rho}_{ML} > \frac{(1-\rho_1^2)}{2M\rho_1}(\eta_1 + M\ln(1-\rho_1^2)) + \rho_1 = \eta_l \tag{4.14}$$

Thus $\hat{\rho}_{ML}$ is the required test statistic for the hypothesis test. If $\hat{\rho}_{ML}$ is the symbol correlation coefficient for 2M symbols, from a real valued Gaussian distribution with correlation ρ , then $\frac{\sqrt{2M}(\hat{\rho}_{ML}-\rho)}{1-\rho^2}$ is asymptotically distributed according to N(0,1). [5] This can be applied to the two hypotheses as follows [5]

$$\begin{aligned} H0&: \lim_{M\to\infty} \sqrt{2M}\hat{\rho}_{ML} \xrightarrow{d} N(0,1) \\ H1&: \lim_{M\to\infty} \sqrt{2M}\hat{\rho}_{ML} \xrightarrow{d} N(\sqrt{2M}\rho_1,(1-\rho_1^2)^2) \end{aligned} \tag{4.15}$$

where $\rho_1 = \frac{Tc}{T_d+T_c}\frac{\sigma_s^2}{\sigma_s^2+\sigma_w^2}$ and '$\xrightarrow{d}$' denotes the convergence in distribution. Using these distributions, we can approximate the distribution of the test statistic for sufficiently large M as[5]

$$\begin{aligned} H0&: \hat{\rho}_{ML} \sim N(0,\frac{1}{2M}) \\ H1&: \hat{\rho}_{ML} \sim N(\rho_1,\frac{(1-\rho_1^2)^2}{2M}) \end{aligned} \tag{4.16}$$

This can be considered as Neyman-Pearson detection problem to satisfy a constant false alarm constraint. The probability of false alarm is given by [5]

$$P_f = P(\hat{\rho}_{ML} > \eta_l \mid H0) = Q\left(\sqrt{2M}\eta_l\right) \tag{4.17}$$

For a false alarm rate of P_f, the threshold of the detector can be evaluated as

$$\eta_l = (1/\sqrt{2M})Q^{-1}\left(P_f\right) \tag{4.18}$$

The probability of detection [5] is given by

$$P_d = P(\hat{\rho}_{ML} > \eta_l \mid H1) = Q\left(\frac{\eta_l - \rho_1}{\sqrt{\frac{(1-\rho_1^2)^2}{2M}}}\right) \tag{4.19}$$

4.1.1 Knowledge of Cyclic Prefix

If the length of the cyclic prefix (Tc) and the synchronization information (position of the cyclic prefix in an OFDM block) is known, the performance of the detector can be improved. The maximum likelihood of the autocorrelation coefficient can be rewritten as[5]

$$\hat{\rho}_{ML} = \frac{\frac{1}{M}\sum_{t=0}^{M-1} R\{x(t)x^*(t+T_d)\}}{\hat{\sigma}_z^2} = \frac{\frac{1}{M}\sum_{n=0}^{N_s-1}\sum_{t=0}^{T_s-1} R\{x(nT_s+t)x^*(nT_s+t+T_d)\}}{\hat{\sigma}_z^2} \tag{4.20}$$

where $N_s = M/T_s$ is the number of OFDM blocks over which autocorrelation coefficient is estimated and $T_s = T_c + T_d$ is the number of symbols in an OFDM block. In absence of information related to the cyclic prefix, the numerator is obtained by summing the product over a sliding window of length T_s for an OFDM block and again adding these sums for N_s blocks. If length of the cyclic prefix and the position of the cyclic prefix in an OFDM block is known then the sliding window can be reduced to T_c symbols for each block as follows [5]

$$\hat{\rho} = \frac{\frac{1}{M}\sum_{n=0}^{N_s-1}\sum_{t\in CP} R\{x(nT_s+t)x^*(nT_s+t+T_d)\}}{M_1\hat{\sigma}_z^2} \tag{4.21}$$

where CP denotes the cyclic prefix samples and $M_1 = T_c N_s$. The distributions under different hypotheses are given by [5]

$$\begin{aligned} H0 &: \hat{\rho}_c \sim N_r(0, \frac{1}{2M_1}) \\ H1 &: \hat{\rho}_c \sim N_r(\rho_c, \frac{(1-\rho_c^2)^2}{2M_1}) \end{aligned} \tag{4.22}$$

where $\hat{\rho}_c = SNR/(1+SNR)$.The Neyman-Pearson detection can be applied for the given probability of false alarm. The probability of detection is found to be higher if the cyclic prefix length and synchronization is known.

4.1.2 Local Sensing for Multipath Channels

Under H0 and H1 the received signal x(t) can be written as[5]

$$\begin{aligned} H0: x(t) &= w(t) \\ H1: x(t) &= \sum_{l=0}^{P-1} h(l)s(t-l) + w(t) \end{aligned} \quad (4.23)$$

where h(l) with l=0,1,....P-1 represents the channel taps. Channel taps are assumed to be independent of each other, of the transmitted data s(t) and of the noise w(t). The autocorrelation coefficient for two hypotheses is [5]

$$\begin{aligned} H0&: \rho = 0 \\ H1&: \rho = \rho_2 \end{aligned} \quad (4.24)$$

$$\begin{aligned} \text{where } \rho &= E(x(t)x(t+T_d)]/E[x(t)x^*(t)] \\ \rho_2 &= E(x(t)x(t+T_d)\,|\,H1]/E[x(t)x^*(t)\,|\,H1] \\ &= \frac{Tc}{T_d+T_c}\frac{\delta\sigma_s^2}{\delta\sigma_s^2+\sigma_w^2} \\ &\text{with } \delta = \sum_{l=0}^{P-1} E[|h(l)|^2] \end{aligned} \quad (4.25)$$

If s(t), h(t) and w(t) are circularly symmetric and Gaussian, x(t) is circularly symmetric and Gaussian. Similar to the AWGN case, $\hat{\rho}_{ML}$ can be calculated by (4.11). The asymptotic distributions for $\hat{\rho}_{ML}$ under the two hypotheses are [5]

$$\begin{aligned} H0&: \hat{\rho}_{ML} \sim N_r(0, \frac{1}{2M}) \\ H1&: \hat{\rho}_{ML} \sim N_r(\rho_2, \frac{(1-\rho_2^2)^2}{2M}) \end{aligned} \quad (4.26)$$

Using these distributions the Neyman-Pearson detector can be designed as in (4.17) and (4.18) with ρ_1 replaced by ρ_2. The SNR for multipath channels is given by $\frac{\delta\sigma_s^2}{\delta\sigma_s^2+\sigma_w^2}$.[5]

4.2. Global Data Fusion

4.2.1 Neyman-Pearson Likelihood Ratio Test

If K_f is the number of samples required for the fixed sample size test, the likelihood ratio test based on Neyman-Pearson criterion can be written as [5]

$$\begin{aligned}&\sum_{n=1}^{K_f} L_n < \eta_3, \quad \text{Decide H0}\\ &\sum_{n=1}^{K_f} L_n \geq \eta_3, \quad \text{Decide H1}\end{aligned} \tag{4.27}$$

where η_3 is the threshold of the detector. Equivalently the test statistic $T_f = 2M\sum_{n=1}^{K_f}\frac{\rho_n\hat{\rho}_n}{1-\rho_n^2}$ can be used. This statistic T_f contains only the variable terms of L_n. The fixed sample size test can be [5] written as

$$\begin{aligned}&T_f < \eta_f, \quad \text{Decide H0}\\ &T_f \geq \eta_f, \quad \text{Decide H1}\end{aligned} \tag{4.28}$$

where $\eta_f = \eta_3 + M\sum_{n=1}^{K_f}\log(1-\rho_n^2) - 2M\sum_{n=1}^{K_f}\frac{\rho_n^2}{(1-\rho_n^2)}$

Since $\hat{\rho}_n$ is Gaussian distributed, T_f which is a sum of Gaussian distributed random variable will have the distributions,[5]

$$\begin{aligned}&H0: T_f \sim N(0,\sigma_{f0}^2)\\ &H1: T_f \sim N(m_f,\sigma_{f1}^2)\end{aligned} \tag{4.29}$$

where

$$m_f = 2M\sum_{n=1}^{K_f}\frac{\rho_n^2}{(1-\rho_n^2)} \tag{4.30}$$

$$\sigma_{f0}^2 = 2M\sum_{n=1}^{K_f} \frac{\rho_n^2}{(1-\rho_n^2)^2} \tag{4.31}$$

$$\sigma_{f1}^2 = 2M\sum_{n=1}^{K_f} \rho_n^2 \tag{4.32}$$

4.2.2 Sequential Detection

For sequential detection the fusion center receives the log likelihood ratios from each secondary user sequentially. After receiving each log likelihood ratio from a secondary user, a sequential probability ratio test (SPRT) is performed. The channel is assumed to an AWGN channel. If sufficient evidence is available to conclusively decide either of the hypotheses, the test procedure is terminated. Otherwise the hypothesis test is continued. The number of samples required for sequential probability ratio test can be compared with the Neyman-Pearson fixed sample size test. The test procedure after receiving 'k' statistics is [5],[11]

$$\sum_{n=1}^{k} L_n \le \log B \text{ Decide H0}$$

$$\sum_{n=1}^{k} L_n \ge \log A \text{ Decide H1} \tag{4.33}$$

Otherwise, take the next user statistics

where,

$$A = \frac{1-\beta}{P_f}$$

$$B = \frac{\beta}{1-P_f} \tag{4.34}$$

and $L_n = -M\log(1-\rho_n^2) + 2M\frac{\rho_n(\hat{\rho}_n - \rho_n)}{1-\rho_n^2}$

Here $\beta = 1 - P_d$ and $\hat{\rho}_n$ is the autocorrelation coefficient of the nth user. The number of log likelihood ratios used to form the decision, $k=K_s$ is a random variable. Performance of a sequential detector can be expressed in terms of the average sample number. The average sample number for the sequential probability ratio test is defined as the number of samples (statistics) required on average for arriving at a decision under either hypotheses. Under the two hypotheses, the average numbers of samples for sequential probability ratio test when $\rho_n = \rho_1 \forall n$ are, [5],[11]

$$E[K_s \mid H0] = \frac{P_f \log A + (1-P_f)\log B}{E[L_n \mid H0]}$$
$$E[K_s \mid H1] = \frac{(1-\beta)\log A + \beta \log B}{E[L_n \mid H1]} \tag{4.35}$$

The mean values of the LLRs under the different hypothesis is given by [9]

$$E[L_n \mid H0] = -M\log(1-\rho_n^2) - 2M\frac{\rho_n^2}{1-\rho_n^2}$$
$$E[L_n \mid H1] = -M\log(1-\rho_n^2) \tag{4.36}$$

The Average sample number for the SPRT is given by[9]

$$K_m = \max\{E[K_s \mid H0], E[K_s \mid H1]\} \tag{4.37}$$

4.3. Results and Discussion

The parameters of OFDM primary signal used for simulation are same as that used in chapter 3 for energy detection. They are repeated here in table (4.1) for convenience.

Table (4.1) Parameters for local sensing with cyclic prefix based correlation detection

Number of OFDM blocks	100
OFDM symbol type	16 QAM
Size of IFFT (Td)	32
Cyclic prefix length (T_c)	8
Number of OFDM samples	$100(T_d+T_c)=4000$

4.3.1. Local sensing

The cyclic prefix correlation detector is simulated for AWGN, slow frequency selective Rayleigh fading, shadowing, correlated shadowing and slow frequency selective Rayleigh fading with shadowing channels. The slow frequency selective Rayleigh channel of channel order 6 is simulated by complex Gaussian coefficients with exponential power delay profile with the normalisation $\delta = \sum_{l=0}^{P-1} E[|h(l)|^2] = 1$. For simulating shadowing effects, the SNR of the user is selected randomly from a Gaussian distribution with mean SNR=-10 dB and standard deviation of 5 dB for each realization.

It can be seen from figure (4.1) that the slow frequency selective Rayleigh fading channel degrades the performance compared to AWGN. It can be seen that for higher probability of false alarm, the detection probability in shadowing channel is lower than that of AWGN channel due to the shodowing effects. For AWGN+Shadowing+Fading channels the cyclic prefix based correlation detection has lower probability of detection compared to AWGN+Shadowing due to the Rayleigh fading effects. These results are similar to that in [5] for shadowing channels and. AWGN channels.

Figure (4.2) shows the effects of exploiting information regarding the cyclic prefix length and its position on the detector performance. The results confirm to that in [5].In presence of this additional information, the performance of the scheme improves considerably. This is the best performance we can get from the cyclic prefix based autocorrelation detection and it serves as an upper bound on the performance of the practical detectors where only partial or no information about the cyclic prefix is available.

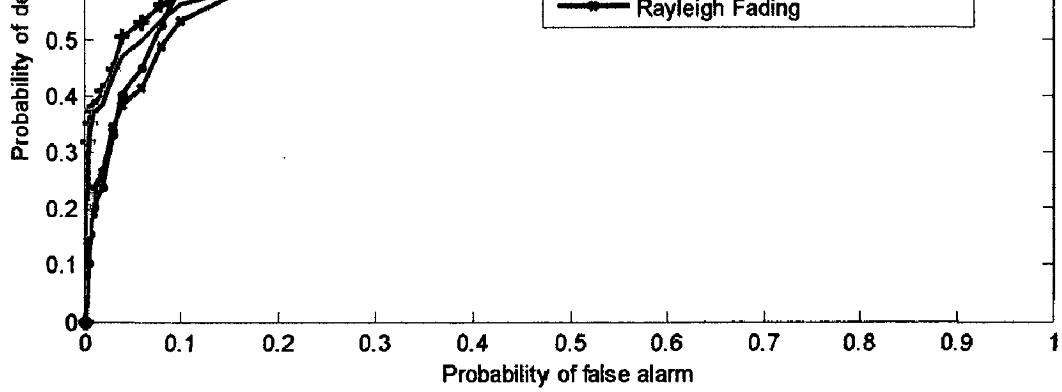

Figure 4.1 Cyclic Prefix Correlation Detection- Receiver Operating Characteristics Plot for Local Sensing

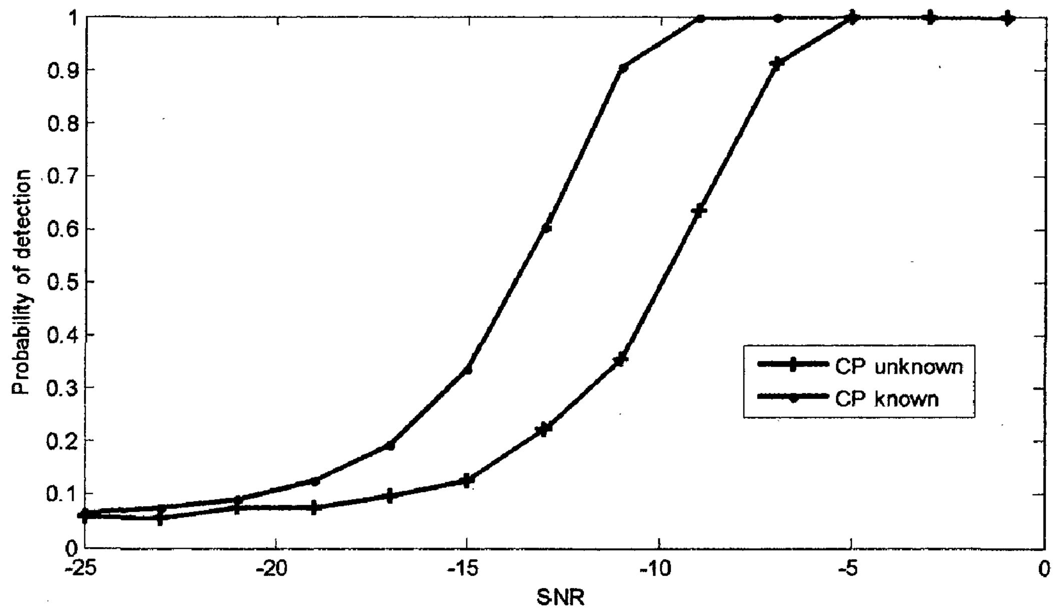

Figure 4.2 Effect of Knowledge of Cyclic Prefix-Plot for the probability of detection (as function of SNR for a probability of false alarm P_f =0.05.)

4.3.2. Cooperative sensing

From figure (4.3) and figure (4.4) it can be found that the cooperative sensing through the cyclic prefix based autocorrelation detector can improve the sensing performance for various channels. 5 secondary users are used for the simulations.

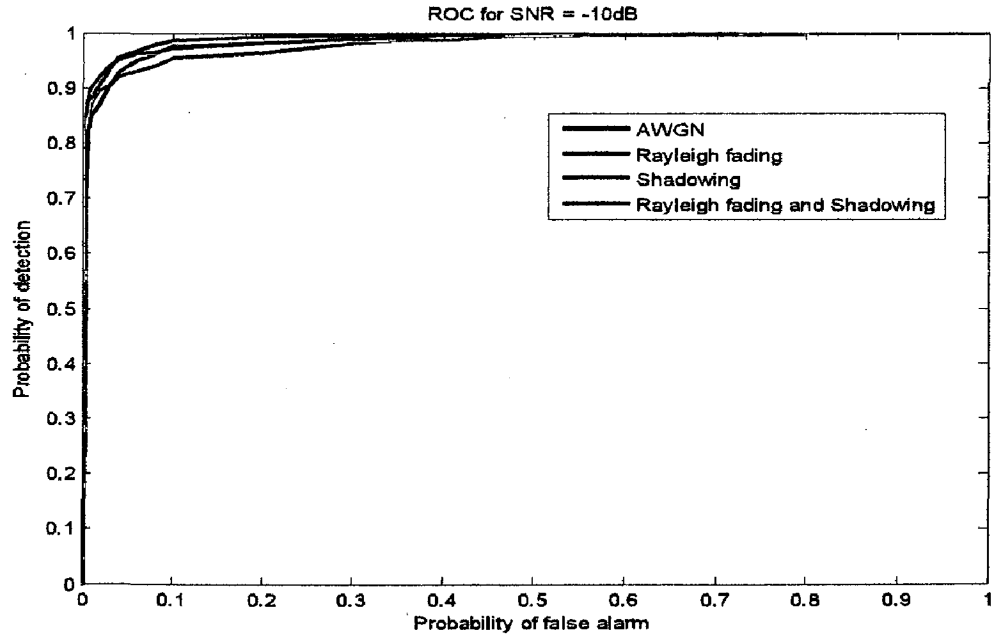

Figure 4.3 Cyclic Prefix Correlation Detection-Receiver Operating Characteristics Plot for cooperative sensing

In figure (4.5) the average number of secondary user statistics of the sequential detection based data fusion is compared to that of the fixed sample size detection. For sequential detection, the limit for false alarm rate is set to 0.05 and the limit for the probability of miss detection is set to be 0.05. The sequential detection is truncated at a maximum number of user statistics of 1000. It is assumed that each user transmits a single log likelihood ratio per detection period to the fusion center. It can found from figure (4.5) that the average number of secondary user statistics is lesser for sequential detection compared to fixed sample size detectors.

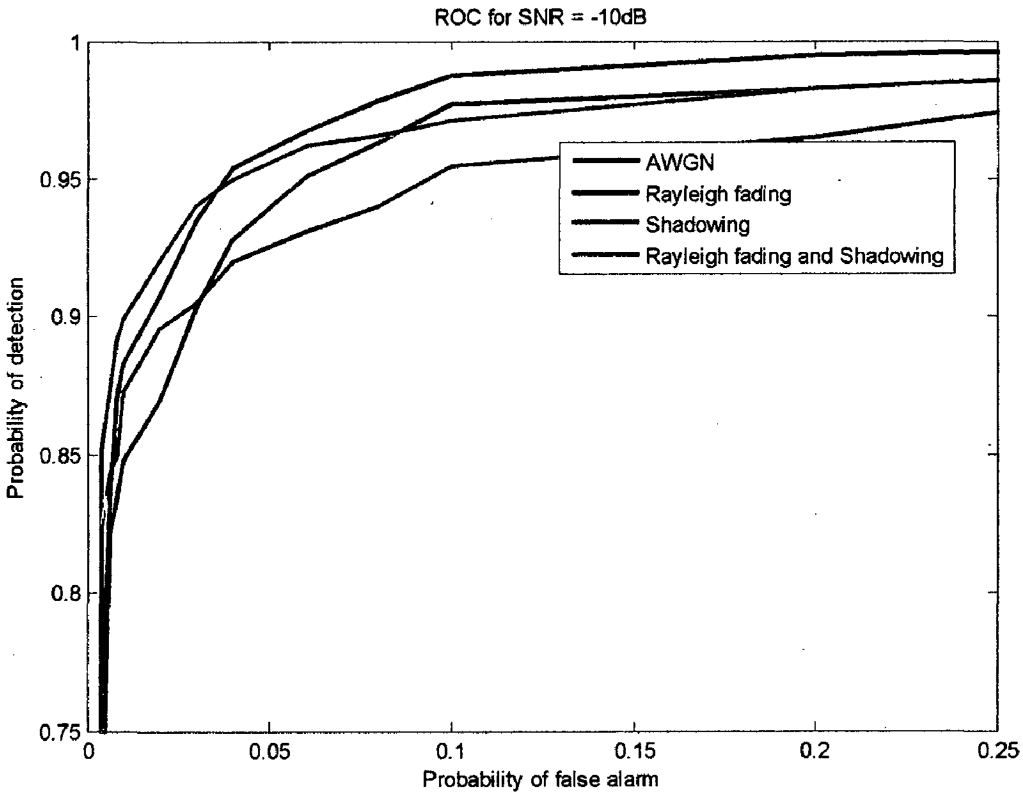

Figure 4.4 Receiver Operating Characteristics Plot for cooperative sensing (Zoomed for low probability of false alarm values)

In figure (4.6) the performance of cooperative sensing in the presence of correlated shadowing is studied for 2 users. The shadowing correlation would degrade performance of collaborative sensing when collaborating users are close. This is because the closely located users are likely to experience similar shadowing effects. The correlation function due to shadowing is $\rho(d) = \exp(-ad)$ where 'a' is a constant depending upon the environment

[17],[18]. The value of used for simulation is 0.1204 considering urban environment. The correlated shadowing is simulated by considering SNR of the users given by a correlated normal distribution with mean -10 dB and standard deviation σ=5dB and covariance matrix of $\sigma^2\begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}$

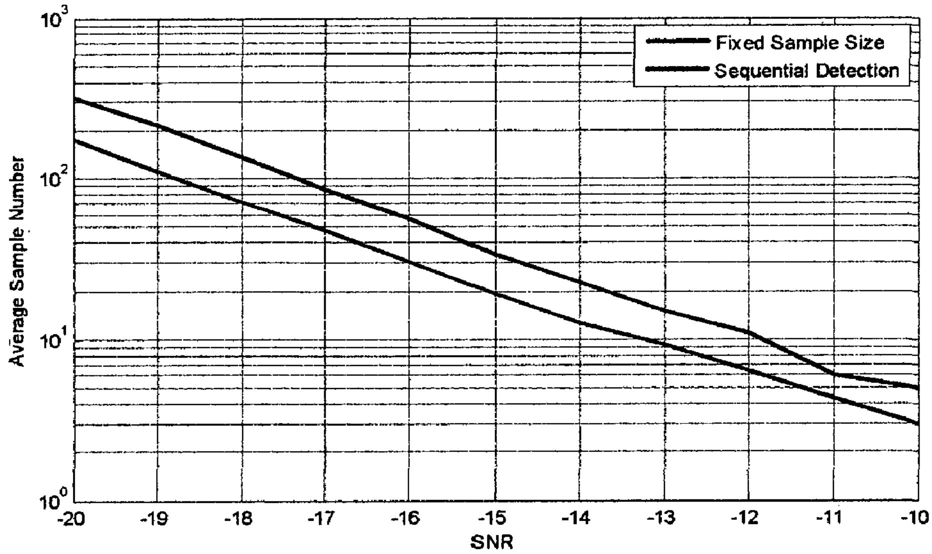

Figure 4.5 Comparison of the sequential detection with fixed sample size detection

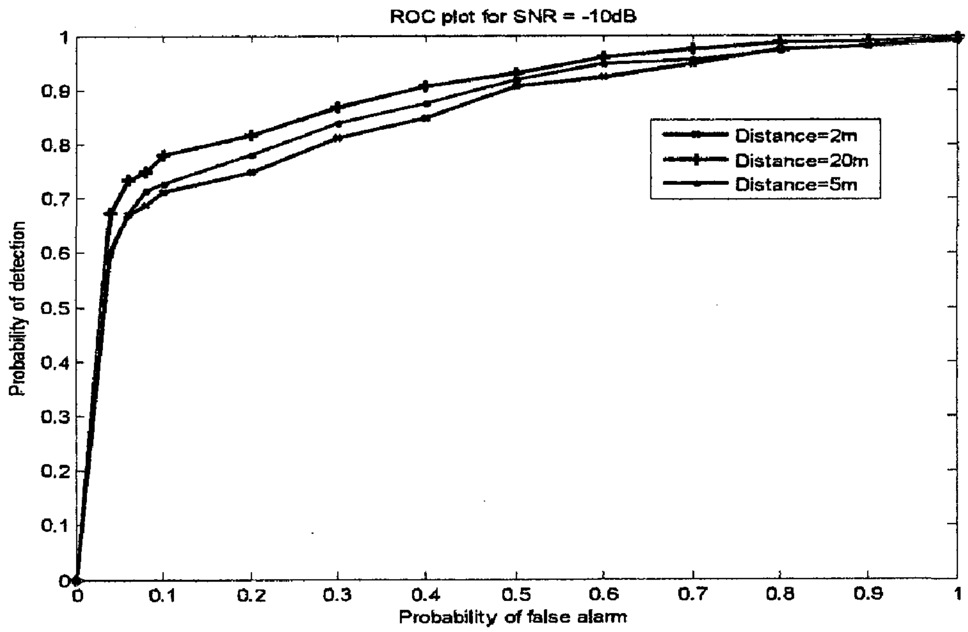

Figure 4.6 Receiver Operating Characteristics Plot for the Correlated Shadowing Channels (for 2 secondary users for various distances).

4.3.3. Comparison of Energy Detector and Cyclic Prefix based Correlation Detector

In practical systems the exact value of the noise variance is unknown, due to temperature variations, calibration errors, etc. The performance of the energy detector degrades in the presence of noise uncertainty [27]. The actual noise power could be in the range $[\sigma_w^2 / \rho, \sigma_w^2 \rho]$, where ρ denotes the uncertainty level and σ_w^2 is the nominal noise power. The noise uncertainty expressed in dB considered for simulation is 10 $\log \rho$=0.5dB. The actual noise variance deviates .5 dB from the true noise variance. Figure (4.7) provides a comparison of the energy detection and the cyclic prefix based correlation detection as a function of SNR. It can be seen that the energy detector has higher probability of detection than the cyclic prefix based correlation detector when there is no noise uncertainty and the performance of the energy detector degrades for a noise uncertainty of 0.5 dB. The cyclic prefix based correlation detector however is unaffected by the noise uncertainty. This comparison of energy detection and the cyclic prefix based correlation detection in the presence of noise uncertainty was also done in [9], [10].

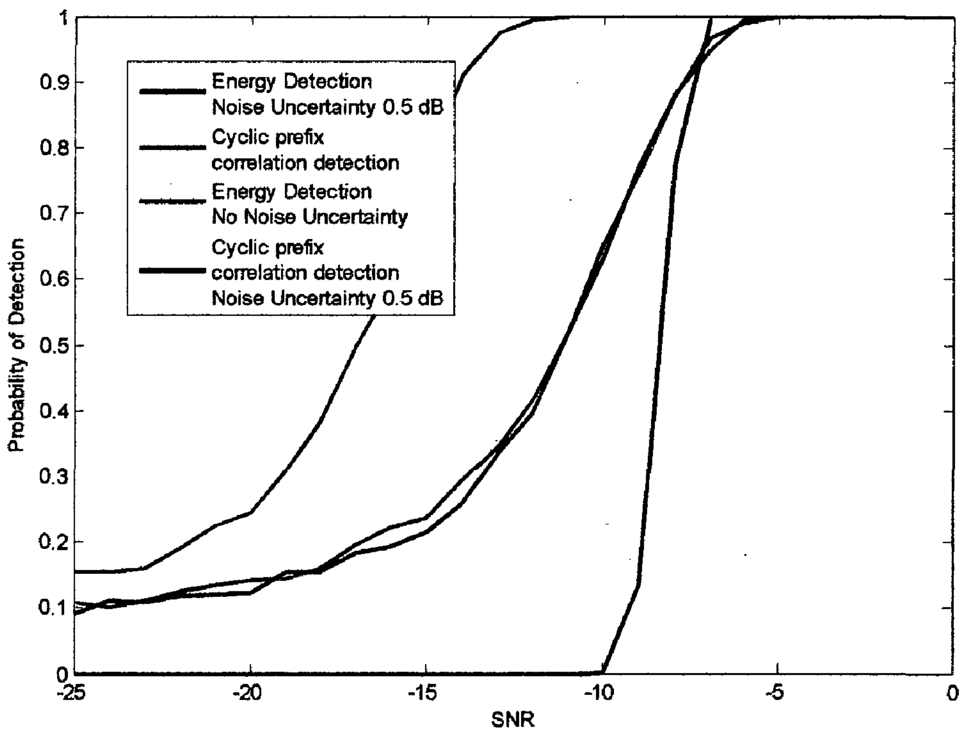

Figure 4.7 Comparison of the cyclic prefix based correlation detection and the energy detection for a single cognitive radio in the presence of noise uncertainty.

Chapter 5
CONCLUSIONS

Cognitive radio is a novel solution that reduces the spectrum scarcity by allowing secondary users to utilize the unused bands of spectrum without interfering with primary users. Spectrum sensing is a critical component of cognitive radio technology to identify the spectrum holes. Secondary users should reduce the interference to primary users and simultaneously improve the spectrum utilization. Cooperative spectrum sensing can be used to provide diversity to the detection of primary users and reduce the effects of fading and shadowing.

In this dissertation, cooperative spectrum sensing based on energy detection and cyclic prefix based correlation detection was studied for fading and shadowing channels. The data fusion was employed at the fusion center to combine the statistics from the secondary users. It was verified by simulation that data fusion based on sequential method required lesser number of average statistics. The energy detector and the cyclic prefix based correlation detection were compared for AWGN channels. The energy detector is affected by the noise power uncertainty and the cyclic prefix based correlation detection is unaffected by the noise power uncertainty. Hence the cyclic prefix based correlation detection can be employed for identifying the OFDM based primary users.

Future Work

Cooperative spectrum sensing was studied for single band systems. For wideband systems, as the number of bands is large, the control channel bandwidth for transmitting the statistics may increase. Future work may be done for wideband systems. Future work may also be done by considering the throughput, sensing duration, the bandwidth for control channel for these detectors. Research may also be done considering the errors in the reporting channel between the cognitive radios and the fusion center.

REFERENCES

[1] K Ben Letaief., Wei Zhang, "Cooperative communications for cognitive radio networks," *Proceedings of the IEEE*, vol.97, no.5, pp.878-893, May 2009.

[2] Tevfik Yucek and Huseyin Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Communication Surveys and Tutorials*, Vol.11, No.1, First Quarter 2009.

[3] J. Mitola and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Personal Communication*, vol. 6, pp. 13–18, Aug. 1999.

[4] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal of Selected Areas Communication*, vol. 23, pp. 201–220, Feb. 2005.

[5] Sachin Chaudhari, Visa Koivunen, and H. Vincent Poor, "Autocorrelation-based decentralized sequential detection of OFDM Signals in cognitive Radios," *IEEE Transactions on Signal Processing*, vol. 57, no. 7, July 2009, pp. 2690–2700.

[6] Sunmin Lim, Hoiyoon Jung, Myung Sun Song, "Cooperative spectrum sensing for IEEE 802.22 WRAN system," *Proceedings of 18th Internatonal Conference on Computer Communications and Networks*, 2009. pp.1-5, 3-6 August 2009

[7] Zhi Quan, Shuguang Cui, H Poor, A Sayed, "Collaborative wideband sensing for cognitive radios," *IEEE Signal Processing Magazine*, vol.25, no.6, pp.60-73, November 2008.

[8] Yonghong Zeng, Ying-Chang Liang, Anh Tuan Hoang, and Rui Zhang, "A Review on Spectrum Sensing for Cognitive Radio: Challenges and Solutions," *EURASIP Journal on Advances in Signal Processing*, vol. 2010, Article ID 381465, pp. 1–15, 2010.

[9] S Bokharaiee, H.H Nguyen, E. Shwedyk, "Blind spectrum sensing for OFDM-based cognitive radio systems," *IEEE Transactions on Vehicular Technology*, vol.60, no.3, pp.858-871, March 2011

[10] Erik Axell, Erik G. Larsson, "Optimal and sub-optimal spectrum sensing of OFDM signals in known and unknown noise variance," *IEEE Journal on Selected Areas in Communication*, Vol 29, No 2, pp.290-304, February 2011

[11] Wald, Abraham, "Sequential analysis," New York: John Wiley and Sons, 1947.

[12] Mourad Barkat, "Signal detection and estimation," Boston, MA: Artech House, 2005.

[13] Wei Zhang, R Mallik, K Letaief, "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol.8, no.12, pp.5761-5766, December 2009

[14] J. Unnikrishnan, V.V Veeravalli, "Cooperative sensing for primary detection in cognitive radio," *IEEE Journal of Selected Topics in Signal Processing*, vol.2, no.1, pp.18-27, Feb. 2008

[15] Zhi Quan, Shuguang Cui, A.H Sayed., "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol.2, no.1, pp.28-40, Feb. 2008.

[16] Jun Ma, Guodong Zhao, Ye Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol.7, no.11, pp.4502-4507, November 2008

[17] A Ghasemi, E.S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks*, 2005., pp.131-136, 8-11 Nov. 2005.

[18] A Ghasemi, E.S. Sousa, "Asymptotic performance of collaborative spectrum sensing under correlated log-normal shadowing," *IEEE Communications Letters,* vol.11, no.1, pp.34-36, Jan. 2007

[19] Chunhua Sun, Wei Zhang, K Ben, "Cluster-based cooperative spectrum sensing in cognitive radio systems," *IEEE International Conference on Communications, 2007*, pp.2511-2515, 24-28 June 2007

[20] Chunhua Sun, Wei Zhang, K.B Letaief, "Cooperative spectrum sensing for cognitive radios under bandwidth constraints," *IEEE Wireless Communications and Networking Conference, 2007,* pp.1-5, 11-15 March 2007.

[21] S.Maleki, A.Pandharipande, G.Leus, "Energy-efficient distributed spectrum sensing for cognitive sensor networks," *IEEE Sensors Journal,* vol.11, no.3, pp.565-573, March 2011

[22] G. Ganesan, Ye Li, "Cooperative spectrum sensing in cognitive radio, part I: two user networks," *IEEE Transactions on Wireless Communications*, vol.6, no.6, pp.2204-2213, June 2007

[23] Ying-Chang Liang, Yonghong Zeng, E.C.Y Peh, Anh Tuan Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Transactions on Wireless Communications,* vol.7, no.4, pp.1326-1337, April 2008

[24] E.C.Y Peh, Ying-Chang Liang, Yong Liang Guan, Yonghong Zeng, "Optimization of cooperative sensing in cognitive radio networks: A sensing-throughput tradeoff view," *IEEE Transactions on Vehicular Technology,* vol.58, no.9, pp.5294-5299, Nov. 2009

[25] A. Ghasemi, E. S. Sousa, "Spectrum sensing in cognitive radio networks: The cooperation-processing tradeoff," *Wireless Communication Mobile Computing*, vol. 7, no. 9, pp. 1049–1060, Nov. 2007

[26] Zhi Quan; Shuguang Cui; Sayed, A.H.; Poor, H.V.; , "Optimal multiband joint detection for spectrum sensing in cognitive radio networks," , *IEEE Transactions on Signal Processing,* vol.57, no.3, pp.1128-1140, March 2009

[27] Rahul Tandra and Anant Sahai "SNR walls for signal detection," *IEEE Journal of Selected Topics in Signal Processing*, Vol. 2, No.1, February 2008

[28] A Jayaprakasam, V Sharma, C.R Murthy, P Narayanan, "Cooperative spectrum sensing algorithms for OFDM systems with frequency selective channels," *International Conference on Signal Processing and Communications* (SPCOM), pp.1-5, 18-21 July 2010.

[29] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits on cognitive radio," in *Proceedings of Allerton Conference on Communication Control and Computing*, Oct. 2004, pp. 131–136.