EQUIVALENT CIRCUIT OF INDUCTION MOTOR WITH UNSYMMETRICAL UNBALANCE ON THE SECONDARY SIDE

A DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the degree

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in

ELECTRICAL ENGINEERING

(Electrical Machine Design)

By

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C_E_R_T_I_F_I_C_A_T_E

Certified that the dissertation entitled, "EQUIVALENT CIRCUIT OF INDUCTION MOTOR WITH UNSYMMETRICAL UNBALANCE ON THE SECONDARY SIDE", which is being submitted by Shri Shyam Manohar Goel in partial fulfilment of the requirements for the degree of MASTER OF ENGINEERING in ELECTRICAL ENGINEERING (Electrical Machine Design) of the University of Roorkee, Roorkee is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further certified that he has worked for a period of about ..16....months from..April.78 to...July.79. for preparing this dissertation ~~at this University~~.

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A C K N O W L E D G E M E N T S

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A B S T R A C T

In the present work, a steady-state equivalent circuit has been developed for the general case of rotor unbalance in an induction motor using three different methods based on symmetrical components concept, Generalised Rotating-Field Theory and Generalised Electrical Machines Theory. The equivalent circuits so derived have been found compatible and represent the same circuit.

It has been shown that the equivalent circuit developed in the present study is of general nature and covers all special cases studied by earlier authors. It is found valid in all cases of symmetrical or unsymmetrical rotor unbalance. The effect of variation of resistance or reactance or both in the rotor circuit over the torque/slip characteristic can be conveniently studied.

Using the circuit, fast and economical computerised calculations can be carried out for a large number of asymmetrical cases for computing the performance characteristics of the machine. The equivalent circuit can also be used for predicting the transient behaviour of the machine.

C_O_N_T_E_N_T_S

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SYMBOLS

V, V_t	Supply voltage and Thevenin's open-circuit voltage per phase respectively.
V_{2o}, V_{2+}, V_{2-}	Zero-, positive-and negative-sequence components respectively, of the voltage across the sequence-components of the extra impedances added in the rotor circuit.
E_1, E_2	Induced e.m.f. per phase in stator and rotor windings respectively
I_{1+}, I_{1-}	Stator positive-and negative-sequence currents respectively.
I_{21-}, I_{22-}	Negative-sequence rotor currents
I_{11-}, I_{12-}	Negative-sequence stator currents
I_1, I_2	Stator and Rotor currents for machine with one winding each in stator and rotor.
I_1 , I_2	Stator and rotor currents matrices
R_1, R_2	Stator and Rotor resistances per phase respectively.
X_1, X_2	Stator and Rotor reactances per phase respectively at line frequency
X_m	Magnetizing reactance
R_s, X_s	Resistance and reactance of the supply lines.
Z_a, Z_b, Z_c	Extra impedances Z_a, Z_b, and Z_c added in the rotor circuit respectively.
Z_{m+}, Z_{m-}	Positive-and negative-sequence machine impedances.
Z_o, Z_+, Z_-	Zero-, positive-and negative-sequence components of external rotor impedances respectively
Z_t	Series impedance obtained by Thevenin's Theorem

Z	Machine Impedance
X_{1f}, X_{1b}	Self reactance of the stator winding due to the forward and backward fields.
X_{2c}	External impedance added in the rotor circuit
X_f, X_b	Self reactance of the reference winding due to the forward and the backward fields.
f, f_r	Supply and shaft frequencies
f_1, f_2	Stator and rotor frequencies
N, N_r	Synchronous and actual speed
s	Fractional slip
, r	Synchronous and actual speed, electrical rad/sec.
a	$e^{j2\pi/3}$ operator.
R_1, R_2	Rotor-and stator-resistance matrix
X_1, X_2	Rotor and stator leakage-reactance matrices
M_1	Stator mutual-impedance-coefficient matrix due to forward field.
M_2	Rotor mutual-impedance-coefficient matrix due to forward field.
M_{12}	Stator-to-rotor mutual impedance-coefficient matrix due to forward field.

Subscripts

f, b	forward and backward fields.
+, -, o	positive-, negative and zero-sequence components

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INTRODUCTION

INTRODUCTION

Generally the performance characteristics of induction motors have been obtained based upon the assumption of symmetrical phase windings and balanced supply voltages.

Due to technological or operational impeffections, electrical machines may not be perfectly symmetrical. Technological asymmetries are sometimes created intentionally. However, due to practical considerations it is generally difficult to design an absolutely symmetrical machine. Unbalanced voltages may occur owing to line disturbances or unsymmetrical load distribution. Unbalance on rotor side may exist due to improper short-circuiting of starting resistances during acceleration or a lead from slip-ring may be open. Unbalance on the rotor side may also be created for speed control[8]. Simultaneous unbalance on both sides of the air-gap due to a blown fuse, a contact failure or inclusion of unbalanced external impedance in addition to rotor unbalance, results in the phenomenon of multiple reflections. This results in unequal flow of currents leading to reduced torque and increase in copper losses. Further, excessive voltages in the windings may damage the insulation and saturate the magnetic circuit to a high degree. Thus one finds that assymentry in inductionmotor may not be completely avoided; it may be either intentional or due to faulty operating conditions.

The unbalanced operation of induction machine has been a subject matter of considerable interest over the years. The operation with an unbalanced secondary circuit was first investigated by Georges[1]. The Georges phenomenon is the operation of a 3-phase wound-rotor induction motor at half-synchronous speed and is obtained by unbalancing the rotor circuit. This phenomenon has been used to advantage in the starting of synchronous motors. The half-speed property was subsequently explained on a non-mathematical basis by Lamme[2] using the theory of two oppositely rotating fields arising from the single-phase secondary winding. The effect of unequal external rotor resistances was studied by Jones[10]. The equivalent circuits for certain particular cases of unbalance were derived by Garbino etal[20]. Methods to remove certain disadvantages in the reduced-speed operation, such as low power factor and decreased breakdown torque, were presented by Barton etal[15]. Leung etal[25] derived the equivalent circuit due to unsymmetrical rotor-winding connections. Brown etal[13], using symmetrical component theory, established a general method of analysis for the operation of induction motor having asymmetrical primary connections. Brown etal[23] proposed a generalized rotating-field theory where the aggregate behaviour of the machine was obtained through the summation of effects produced by each phase. Jha etal[26] developed a generalised rotating-field theory of induction machines having asymmetrical windings on both the stator and rotor. The phase-by-phase approach of this generalised theory made to develop an unified approach for a m/n winding induction machine with no constraints either on the number of turns or on the relative displacement of the winding axes. Recently Vas[30] derived a new equivalent circuit for the general case of rotor unbalance. The positive-and negative-sequence circuits are coupled through a four-port network consisting of current or voltage controlled generators.

The fact that in a number of cases, induction machines work under asymmetrical operational conditions, made necessary to study the operation of unsymmetrical machines. The studies made by various authors so far as described in the preceding paragraph cover a smaller range of unbalanced operating conditions with symmetrical rotor unbalance. The circuits so developed consider only the simple and special cases viz., operation with one rotor phase open-circuited[2] and equal resistances included in two phases[15]. Furthermore, all these studies consider only the variation of external resistances added in the rotor circuit and the reactance has been neglected. Therefore, it is necessary to develop a general method for performance evaluation of induction machine with all possible cases of rotor unbalance. Therefore, it has been the purpose of the present work to develop a general method for the computation of operational characteristics(transient and steady-state). The equivalent circuit of a machine furnishes a very convenient and easy method for predetermination of its performance. In the present work a general equivalent circuit has been derived. Using this single-circuit, fast and economical computerised calculations can be carried out for a large number of asymmetrical cases, as only one network is to be solved by using routine network-solving procedures available at most computer systems. The equivalent circuit developed also presents the physical concepts clearly. The method of symmetrical components has been used for the development of equivalent circuit for wound-rotor induction motor under rotor unbalance created by connecting unequal impedance in the rotor circuit. The equivalent circuits derived so far in special cases of rotor unbalance have been shown as particular cases obtained from the general equivalent circuit so obtained.

A chapter-wise summary of the work presented in the thesis is outlined here. In chapter I, theory of symmetrical components has been used to develop the complete general equivalent circuit by superposing the positive-and negative-sequence equivalent circuits. The generalized rotating-field theory of an m-n winding induction machine, has been extended in chapter II to develop equivalent circuit of a wound rotor induction motor under general case of rotor unbalance. The concepts underlying the generalised machine theory have been utilised in chapter III to develop the general equivalent circuit of the machine. All these equivalent circuits have been shown compatible representing the same general circuit. In chapter IV, conclusions and scope for further work is highlighted.

CHAPTER I

STEADY-STATE EQUIVALENT CIRCUIT WITH GENERAL ROTOR UNBALANCE

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STEADY-STATE EQUIVALENT CIRCUIT WITH GENERAL ROTOR UNBALANCE

INTRODUCTION

With the aid of the methed of symmetrical components and the knowledge of a balanced induction motor, it is possible to analyse the motor with general rotor unbalance on a mathematical basis. In this chapter, a steady-state equivalent circuit is derived for a three-phase induction motor with symmetrical stator and rotor asymmetry created by adding unequal impedances in the secondary circuit. Positive-and negative-sequence circuits are developed and combining the two, the complete steady-state equivalent circuit of the motor is obtained. The circuit has been shown to be valid for any case of symmetrical or asymmetrical rotor unbalance and helps in writing the general equations determining the performance of the motor.

The following assumptions have been made;

(i) The stator and rotor windings are symmetrical and unbalance created by adding unequal external impedances in the rotor circuit.

(ii) The three-phase supply voltage is balanced and sinusoidal.

(iii) Saturation and iron losses are neglected.

1.1 POSITIVE-SEQUENCE EQUIVALENT CIRCUIT

When a balanced three-phase voltage of positive-sequence V_{1+} and frequency f is impressed on the symmetrical stator winding of a three-phase induction motor, the positive-sequence currents flowing through the stator winding set up a sinusoidally distributed magnetic field in the air-gap rotating forward at speed ω radian per sec. relative to the stator. This field induces an e.m.f. in the stator as well as in the rotor. The former emf tends to balance the applied voltage while the latter circulates unbalanced currents in the rotor windings. The field rotates forward relative to the rotor circuit with a speed $s\omega$ and induces in its winding a positive-sequence e.m.f. sE_2 and frequency sf and causes positive-sequence currents I_{2+} to flow in the secondary impedances. The stator and rotor positive-sequence currents satisfy the m.m.f. balance equation so that the difference of their m.m.fs. just produces the positive-sequence flux.

For the rotor circuit,

$$I_{2+} = \frac{sE_2 - V_{2+}}{R_2 + jsX_2}$$

$$= \frac{E_2 - \frac{V_{2+}}{s}}{\frac{R_2}{s} + jX_2} \qquad \ldots(1.1)$$

The positive-sequence equivalent circuit assuming unity turn ratio is shown in Fig. 1.1.

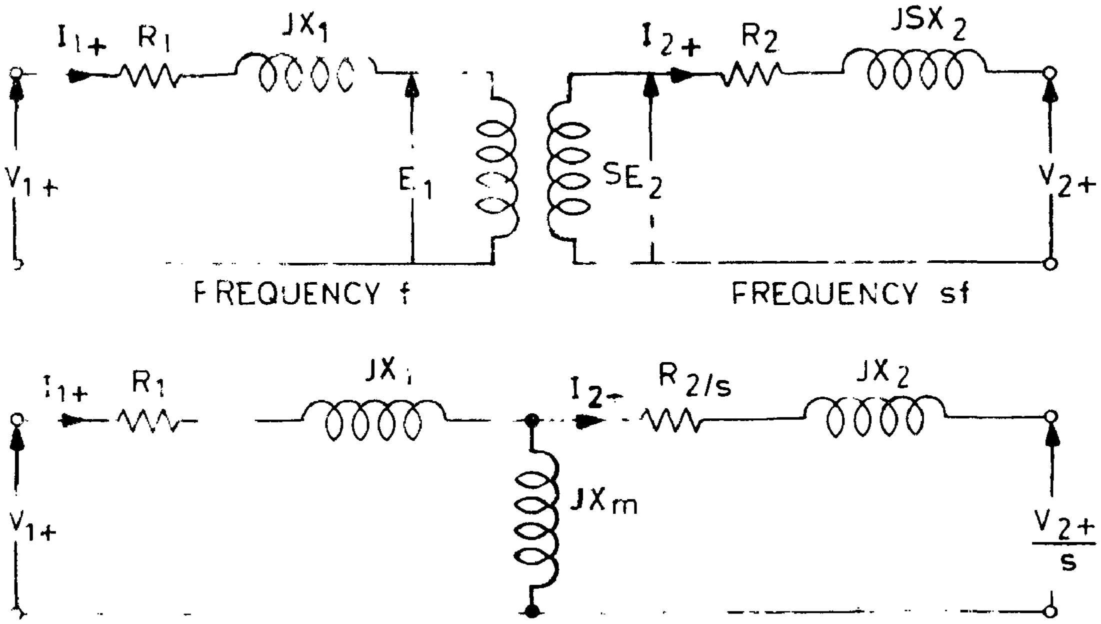


FIG.1.1 POSTIVE-SEQUENCE EQUIVALENT CIRCUIT

1.2 NEGATIVE-SEQUENCE EQUIVALENT CIRCUIT

The rotor voltages and currents are of slip frequency sf. By resolving the currents into symmetrical components, the positive-sequence m.m.f. will be rotating at a speed $s\omega$ relative to the rotor, or a speed $s\omega + \omega_r = s\omega + (1-s)\omega = \omega$ relative to the stator. The stator field reacts with the rotor positive-sequence current to develop a torque T_+, which accelerates the rotor and thus decreases the relative speed $(\omega - \omega_r)$ of the stator field with respect to the rotor winding. A similar principle applies to the negative-sequence field, arising from the negative-sequence currents in the rotor and rotating at a speed $-s\omega$ relative to the rotor or a speed $s\omega - \omega_r = s\omega - (1-s)\omega = (2s-1)\omega$ relative to the stator. This negative-sequence rotor field will induce stator currents of frequency $(2s-1)f$ and produces a torque T_-. It follows that T_- will aid T_+ to speed up the motor if $(2s-1) > 0$ and vice versa.

The negative-sequence field having a speed $(2s-1)\omega$ relative to the stator, induces in the stator winding a negative-sequence e.m.f. of $(2s-1)f$ frequency which in turn circulates negative-sequence current in the balanced stator impedance. The path of this current is through the resistance and reactance of the stator winding and the power lines supplying the induction motor. It is likely that the impedance offered to the flow of these currents by the supply lines will be small but it may not be negligible, so that the impedance

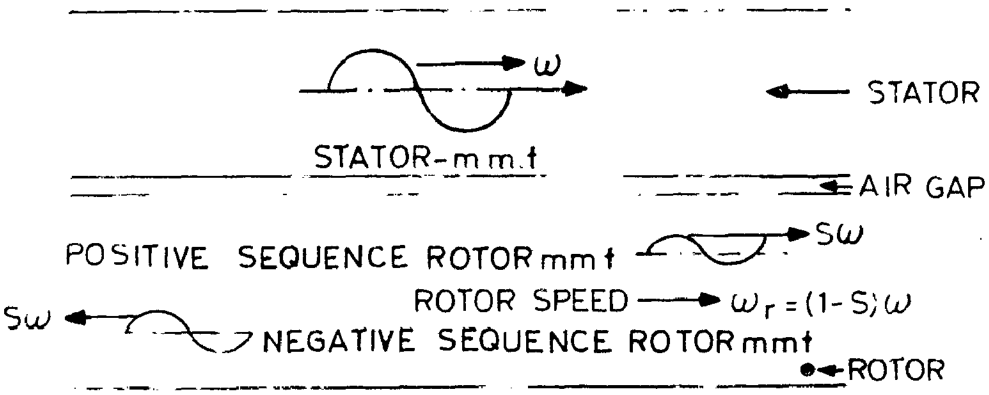


FIG.1.2 RELATIVE DIRECTION OF ROTOR AND STATOR mmf COMPONENTS.

of the power lines will be considered while deriving the negative-sequence equivalent circuit.

For the rotor circuit,

$$I_{2-} = \frac{V_{2-} - sE_2}{R_2 + jsX_2}$$

$$= \frac{\frac{V_{2-}}{s} - E_2}{\frac{R_2}{s} + jX_2} \qquad \ldots(1.2)$$

Also for the stator circuit,

$$I_{1-} = \frac{(2s-1)E_1}{(R_1+R_s) + j(2s-1)(X_1 + X_s)}$$

$$= \frac{E_1}{\frac{(R_1+R_s)}{(2s-1)} + j(X_1 + X_s)} \qquad \ldots(1.3)$$

where R_s and X_s are the resistance and reactance of the power lines. The negative-sequence equivalent circuit, assuming a turn ratio of unity, is shown in Fig. 1.3.

1.3 POSITIVE- AND NEGATIVE-SEQUENCE MACHINE IMPEDANCES

The positive- and negative-sequence impedances (Z_{m+}, Z_{m-}) of the machine are easily obtained from the positive-and negative-sequence equivalent circuits as given in Fig. 1.1 and Fig. 1.3 respectively and are given as under

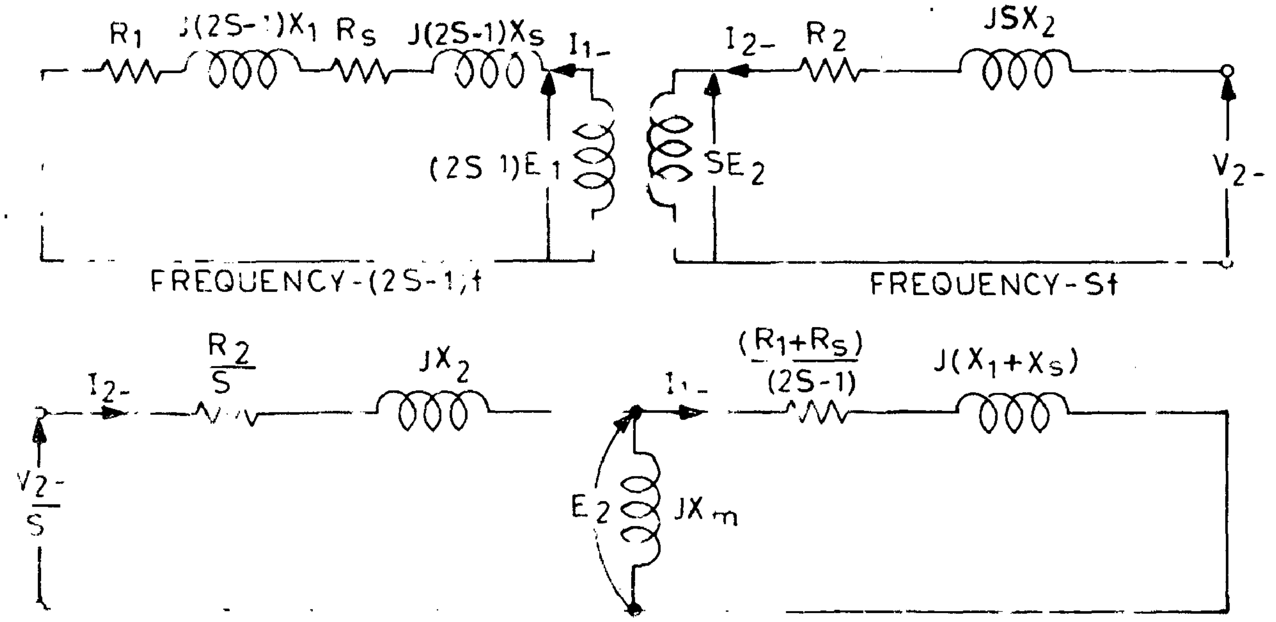


FIG.1.3 NEGATIVE-SEQUENCE EQUIVALENT CIRCUIT

$$\frac{Z_{m+}}{s} = \frac{jX_m\ (R_1+jX_1)}{R_1+j(X_1+X_m)} + \frac{R_2}{s} + jX_2 \qquad \ldots(1.4)$$

and

$$\frac{Z_{m-}}{s} = \frac{jX_m\left[\dfrac{(R_1+R_s)}{(2s-1)} + j(X_1+X_s)\right]}{\dfrac{(R_1+R_s)}{(2s-1)} + j(X_1+X_s+X_m)} + \frac{R_2}{s} + jX_2 \qquad \ldots(1.5)$$

The circuits pertaining to the positive- and negative-sequence machine impedances are shown in Fig. 1.4.

1.4 SIMPLIFICATION OF THE POSITIVE-SEQUENCE EQUIVALENT CIRCUIT BY THE THEVENIN'S NET-WORK THEOREM

The Thevenin's theorem, when applied across the points a, b in the positive-sequence equivalent circuit, permits the replacement of the circuit to the left of a, b by a circuit having a voltage source V_t in series with an impedance Z_t and thereby the circuit of Fig. 1.5(a) simplifies to that given in Fig. 1.5(b). Here V_t is the open-circuit voltage appearing across the points a,b and the impedance Z_t is that reviewed from the same points when the voltage source in the network is short-circuited.

Referring to Positive-sequence equivalent circuit simplified by Thevenin's Theorem, Fig. 1.5(a),

$$V_t = \frac{jX_m}{R_1+j(X_1+X_m)} \cdot V_{1+} \qquad \ldots(1.6)$$

$$Z_t = R_t + jX_t$$

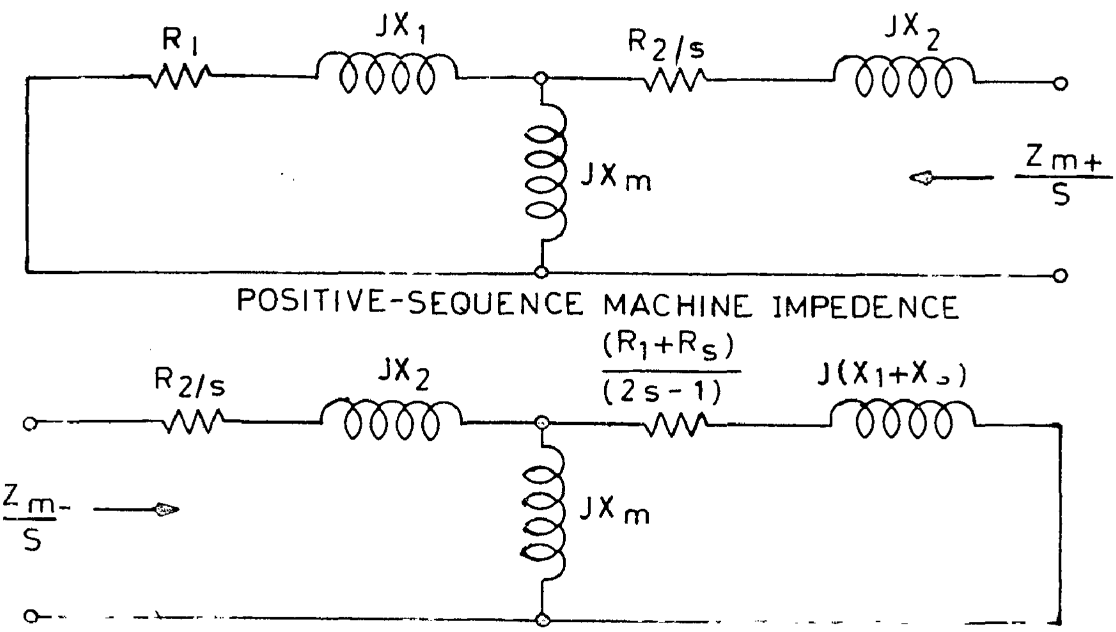


FIG.1.4 NEGATIVE-SEQUENCE MACHINE IMPEDENCE

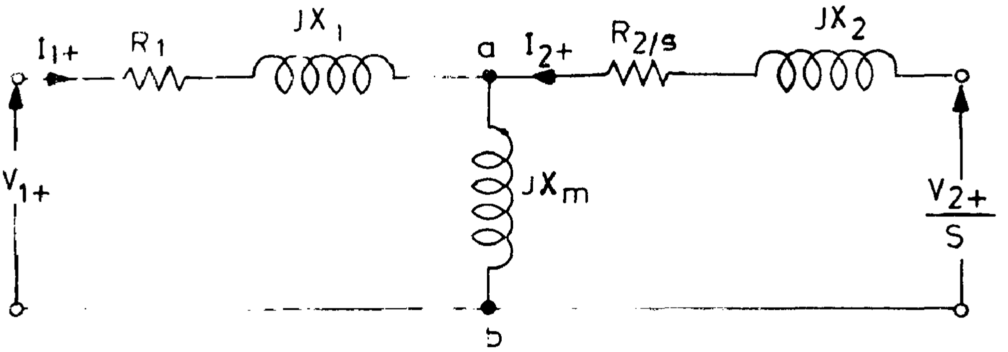


FIG.1.5(a) POSITIVE-SEQUENCE EQUIVALENT CIRCUIT

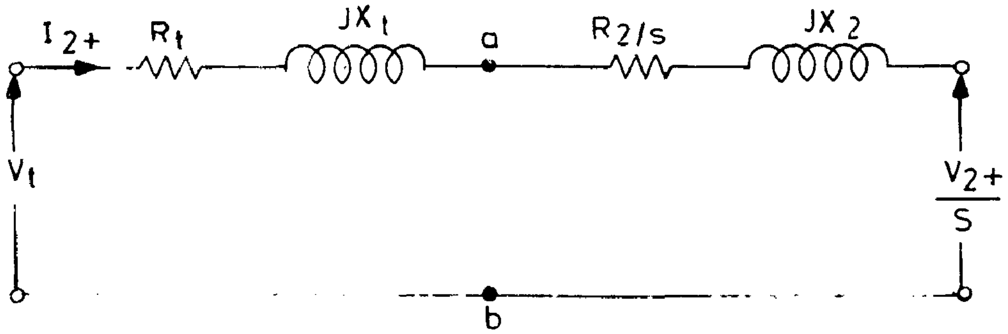


FIG.1.5(b) POSITIVE-SEQUENCE EQUIVALENT CIRCUIT
SIMPLIFIED BY THEVENIN'S THEOREM

$$= \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} \qquad \ldots(1.7)$$

and $$V_t = \frac{V_{2+}}{s} + \left[(R_t + \frac{R_2}{s}) + j(X_t + X_2)\right] \cdot I_{2+}$$

$$= \frac{V_{2+}}{s} + \frac{Z_{m+}}{s} I_{2+}$$

$$V_{2+} = sV_t - Z_{m+} \cdot I_{2+} \qquad \ldots(1.8)$$

1.5 EXTERNAL ROTOR SEQUENCE-IMPEDANCES

The rotor asymmetry of general nature is created by inserting unequal external impedances Z_a, Z_b and Z_c in the rotor phases a, b and c respectively, the rotor is assumed star connected. Resolving the external rotor impedances into sequence components, the zero-, positive- and negative-sequence components of external rotor impedances are given by-

$$Z_o = \frac{1}{3} (Z_a + Z_b + Z_c) \qquad \ldots(1.9)$$

$$Z_+ = \frac{1}{3} (Z_a + aZ_b + a^2 Z_c) \qquad \ldots(1.10)$$

$$Z_- = \frac{1}{3} (Z_a + a^2 Z_b + aZ_c) \qquad \ldots(1.11)$$

1.6 ROTOR-SEQUENCE VOLTAGES:

The zero, positive- and negative-sequence components of the voltage across the rotor sequence impedances Z_o, Z_1 and Z_2 may be given by-

$$V_{2o} = Z_o I_{2o} + Z_- I_{2+} + Z_+ I_{2-} \qquad \ldots(1.12)$$

$$V_{2+} = Z_+ I_{2o} + Z_o I_{2+} + Z_- I_{2-} \qquad \ldots(1.13)$$

$$V_{2-} = Z_- I_{2o} + Z_+ I_{2+} + Z_o I_{2-} \qquad \ldots(1.14)$$

V_{2o}, V_{2+} and V_{2-} in the above equations are the zero-, positive- and negative-sequence components of the rotor phase voltage.

In this case, the star point is assumed to be isolated and as such the zero-sequence currents will not be present. From the positive- and negative-sequence equivalent circuits, it is evident that if the rotor positive-sequence current flowing out of the impedance is assigned a positive sign, the rotor negative-sequence current flowing into the impedance will be assigned a negative sign. Since the star point is isolated, the equation (1.12) vanishes as the zero-sequence voltage V_{2o} is zero. Substituting $I_{2o} = 0$ and affixing a positive sign before I_{2+} and a negative sign before I_{2-} in equations (1.13) and (1.14), the following equations are obtained.

$$V_{2+} = Z_o I_{2+} - Z_- I_{2-} \qquad \ldots(1.15)$$

$$V_{2-} = Z_+ I_{2+} - Z_o I_{2-} \qquad \ldots(1.16)$$

Rearranging equation (1.16),

$$\frac{I_{2-}}{I_{2+}} = \frac{Z_+}{Z_o + Z_{m-}} \qquad \ldots(1.17)$$

where

$$Z_{m-} = \frac{V_{2-}}{I_{2-}} , \quad \text{the negative-sequence impedance of the machine} \qquad ...(1.18)$$

Substituting I_{2-}, from equation (1.17) in (1.15),

$$\frac{V_{2+}}{s} = I_{2+} \cdot \frac{Z_{2+}}{s} \qquad ...(1.19)$$

where,

$$\frac{Z_{2+}}{s} = \frac{Z_o}{s} - \frac{\frac{Z_+}{s} \cdot \frac{Z_-}{s}}{\frac{Z_o}{s} + \frac{Z_{m-}}{s}} \qquad ...(1.20)$$

1.7 ROTOR-SEQUENCE CURRENTS

Expressions for the rotor positive- and negative-sequence currents I_{2+} and I_{2-} are easily obtained from equations (1.8), (1.15), (1.16) and (1.17).

From equation (1.8),

$$I_{2+} = \frac{sV_t - V_{2+}}{Z_{m+}}$$

Substituting for V_{2+} from equation (1.15),

$$I_{2+} = \frac{sV_t - (Z_o I_{2+} - Z_- I_{2-})}{Z_{m+}}$$

Substituting for I_{2-} from equation (1.17) and simplifying,

$$I_{2+} = \frac{s(Z_o + Z_{m-})}{(Z_o + Z_{m+})(Z_o + Z_{m-}) - Z_+ Z_-} \cdot V_t \qquad ...(1.21)$$

Substituting for I_{2+} from equation (1.21) in equation (1.17),

$$I_{2-} = \frac{sZ_+}{(Z_o+Z_{m+})(Z_o+Z_{m-}) - Z_+Z_-} \cdot V_t \qquad \ldots(1.22)$$

Rearranging equation (1.21),

$$I_{2+} = \frac{V_t}{\frac{Z_{m+}}{s} + \frac{Z_o}{s} - \frac{\frac{Z_+}{s} \cdot \frac{Z_-}{s}}{\frac{Z_o}{s} + \frac{Z_{m-}}{s}}}$$

$$= \frac{V_t}{\left(\frac{Z_{m+}}{s} + \frac{Z_{2+}}{s}\right)} \qquad \ldots(1.23)$$

where,

$$\frac{Z_{2+}}{s} = \frac{R_{2+}}{s} + jX_{2+}$$

$$= \frac{Z_o}{s} - \frac{\frac{Z_+}{s} \cdot \frac{Z_-}{s}}{\frac{Z_o}{s} + \frac{Z_{m-}}{s}} \qquad \ldots(1.24)$$

Here $\frac{Z_{2+}}{s}$ can be interpreted as the load impedance such that the current I_{2+} passing through it causes a voltage drop equal to $\frac{V_{2+}}{s}$. The equivalent circuit pertaining to equation (1.23) is shown in Fig. 1.6.

1.8 ROTOR-SEQUENCE VOLTAGES

Expressions for the rotor positive- and negative-sequence voltages V_{2+} and V_{2-} are easily obtained from

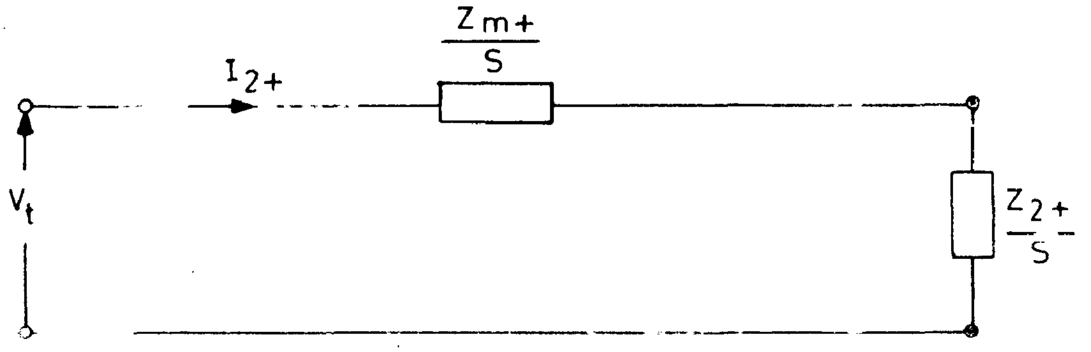


FIG.1.6(a) EQUIVALENT CIRCUIT PERTAINING TO EQUATION(1.23)

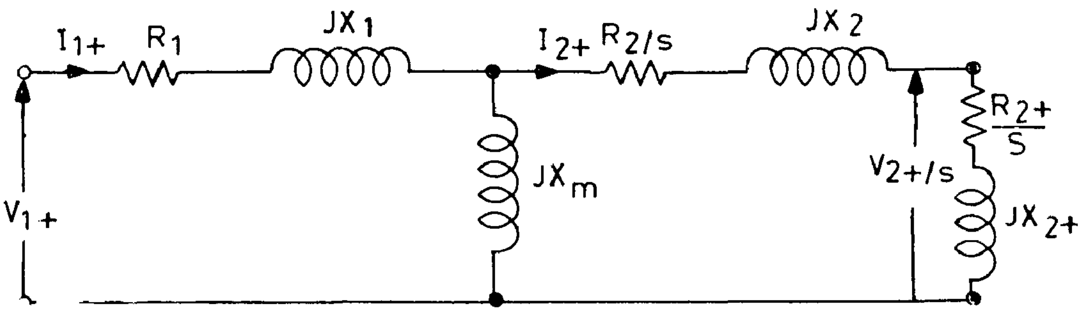


FIG.1.6(b) COMPLETE EQUIVALENT CIRCUIT.

equations (1.15), (1.16), (1.21) and (1.22).

Substituting for I_{2+} and I_{2-} from equation (1.21) and (1.22) in equation (1.15),

$$V_{2+} = \frac{s\,[Z_0(Z_0+Z_{m-}) - Z_+ . Z_-]}{(Z_0+Z_{m+})\ (Z_0+Z_{m-}) - Z_+ . Z_-} . V_t \qquad \ldots(1.25)$$

Substituting for I_{2+} and I_{2-} from equations (1.21) and (1.22) in equation (1.16),

$$V_{2-} = \frac{sZ_+ \ . \ Z_{m-}}{(Z_0 + Z_{m+})(Z_0+Z_{m-}) - Z_+ . Z_-} . V_t \qquad \ldots(1.26)$$

1.9 DEVELOPMENT OF STEADY-STATE EQUIVALENT CIRCUIT WITH GENERAL ROTOR UNBALANCE

The equations (1.4), (1.5), (1.12), (1.13), (1.14) and (1.24) and the equivalent circuit of Fig. 1.6(a) show the nature of the circuit of induction motor and helps to develop the complete equivalent circuit for the general case of rotor unbalance.

From the Fig. 1.6(a) pertaining to the equivalent circuit of induction motor, the machine impedance Z is given by,

$$Z = \frac{Z_{m+}}{s} + \frac{Z_{2+}}{s} \qquad \ldots(1.27)$$

Substituting from equation (1.24),

$$Z = \frac{Z_{m+}}{s} + \frac{Z_o}{s} - \frac{\frac{Z_+}{s} \cdot \frac{Z_-}{s}}{\frac{Z_o}{s} + \frac{Z_{m-}}{s}} \qquad \ldots(1.28)$$

Substituting the values of Z_o, Z_+ and Z_- from equations (1.9), (1.10) and (1.11) in equation (1.28) and simplifying

$$Z = \frac{Z_{m+}}{s} + \frac{Z_c}{s} + \frac{\frac{Z_{m-}}{s} \cdot \frac{(Z_a+Z_b-2Z_c)}{3s} + \frac{(Z_a \cdot Z_b - Z_c^2)}{3s^2}}{\frac{(Z_a+Z_b+Z_c)}{3s} + \frac{Z_{m-}}{s}} \qquad \ldots(1.29)$$

Equation (1.29) can be represented as follows-

$$Z = \frac{Z_{m+}}{s} + \frac{Z_c}{s} + \frac{Z_p}{s} + 3\frac{Z_q}{s} \qquad \ldots(1.30)$$

where

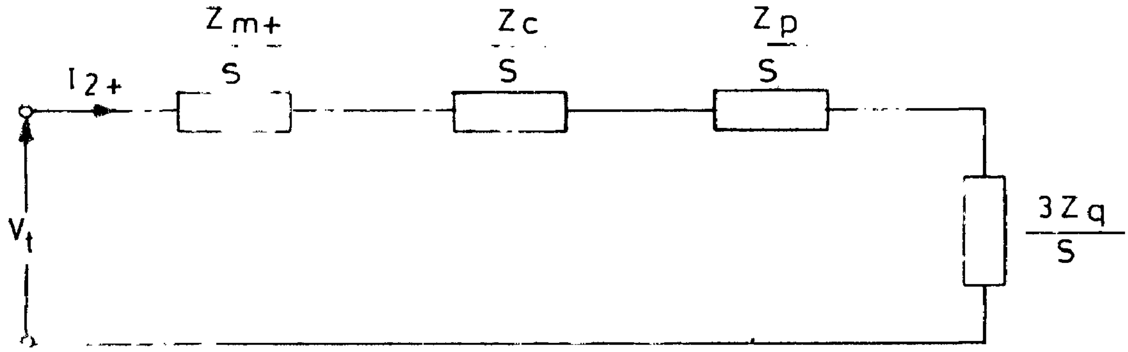
$$\frac{Z_p}{s} = \frac{\left[\frac{Z_{m-}}{s} + \frac{(Z_b+Z_c)}{2s}\right]\left[\frac{2Z_a - (Z_b+Z_c)}{6s}\right]}{\left[\frac{Z_{m-}}{s} + \frac{Z_b+Z_c}{2s}\right] + \left[\frac{2Z_a - (Z_b+Z_c)}{6s}\right]} \qquad \ldots(1.31)$$

and
$$3\frac{Z_q}{s} = \frac{\left[\frac{3Z_{m-}}{s} + \frac{(2Z_a+Z_b+3Z_c)}{2s}\right]\left[\frac{(Z_b-Z_c)}{2s}\right]}{\left[\frac{3Z_{m-}}{s} + \frac{(2Z_a+Z_b+3Z_c)}{2s}\right] + \left[\frac{(Z_b-Z_c)}{2s}\right]} \qquad \ldots(1.32)$$

The equivalent circuit of the induction motor as represented by equation (1.30) is shown in Fig. 1.7.

The circuits pertaining to impedance terms $\frac{Z_p}{s}$, and $\frac{Z_q}{s}$ are shown in Fig. 1.8.

Inserting the circuits pertaining to impedance terms $\frac{Z_p}{s}$ and $\frac{Z_q}{s}$ from Fig. 1.8 in the equivalent circuit of Fig. 1.7,



EQUIVALENT CIRCUIT OF INDUCTION MOTOR AS REPRESENTED BY EQUATION (1.30)

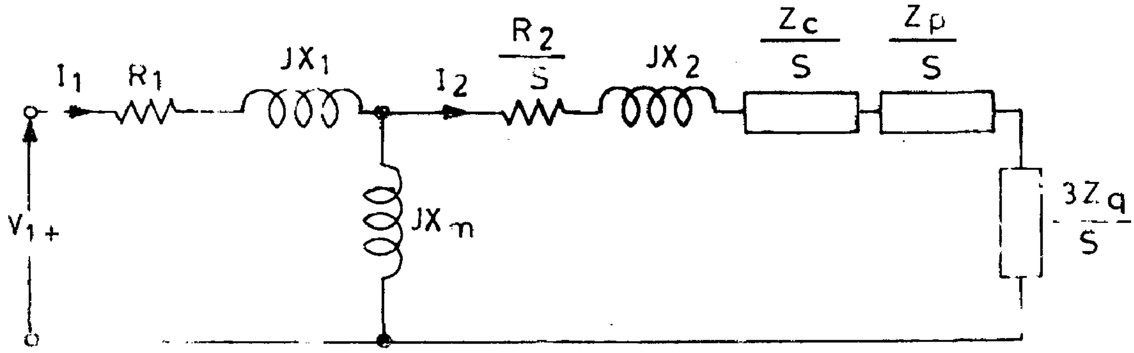
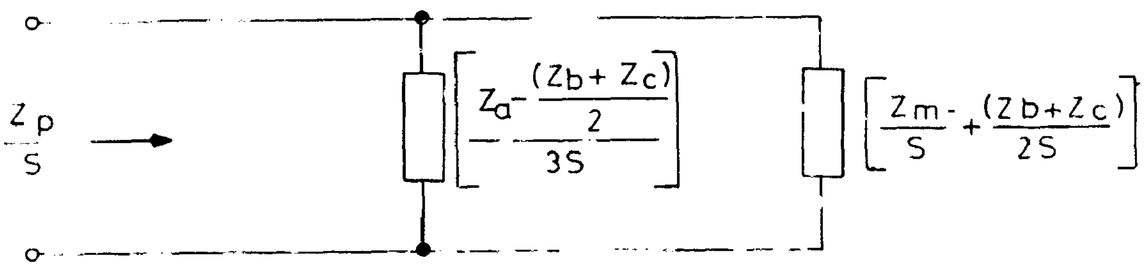
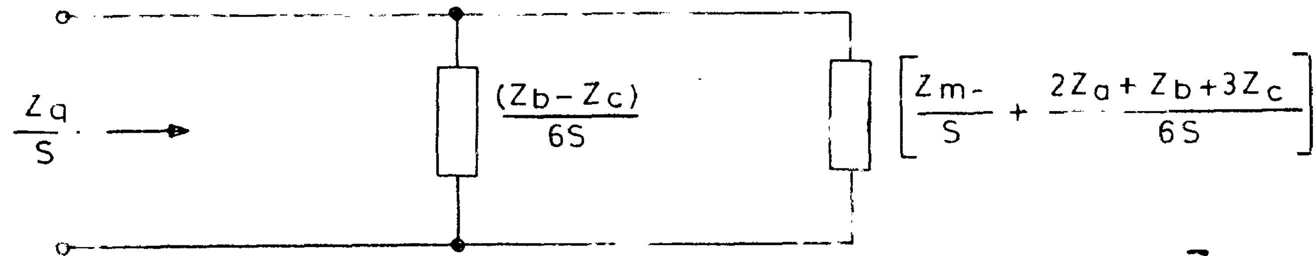


FIG.1.7 EQUIVALENT CIRCUIT OF INDUCTION MOTOR



CIRCUIT PERTAINING TO IMPEDENCE TERM $\frac{Z_p}{S}$



CIRCUIT PERTAINING TO IMPEDENCE TERM $\frac{Z_q}{S}$

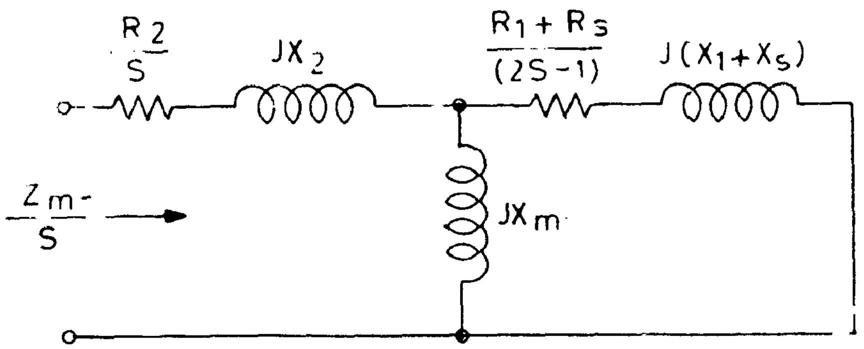


FIG.1.8 CIRCUIT PERTAINING TO IMPEDENCE TERM $\frac{Z_{m-}}{S}$

the complete equivalent circuit of the induction motor with general case of rotor unbalance is obtained and shown in Fig. 1.9.

1.10 COMPATIBILITY OF THE EQUIVALENT CIRCUIT TO THOSE REPORTED EARLIER BY SEVERAL AUTHORS:

The equivalent circuit obtained in the previous section is very general in nature and is applicable to the calculation of the behaviour of induction machines under several types of unbalanced operation. The equivalent circuits for specific cases of rotor unbalance, reported earlier by several authors, can be easily deduced from the general equivalent circuit. A few of these cases are given in the following sub-sections.

1.11 SYMMETRICAL STATOR WITH ONE ROTOR-PHASE OPEN-CIRCUITED

In this case the external rotor impedance in phases a,b and c are ∞, zero and zero respectively. The equivalent circuit is obtained by substituting $Z_a = \infty$, $Z_b = Z_c = 0$ in the general equivalent circuit of Fig. 1.9 and is shown in Fig. 1.10. This is a well known case discussed by Lamme[3].

1.12 SYMMETRICAL STATOR WITH THE SAME EXTERNAL IMPEDANCES IN THE TWO ROTOR PHASES

In this case, the external rotor impedance in phases a,b and c are Z_a, Z_b and Z_b respectively. The equivalent

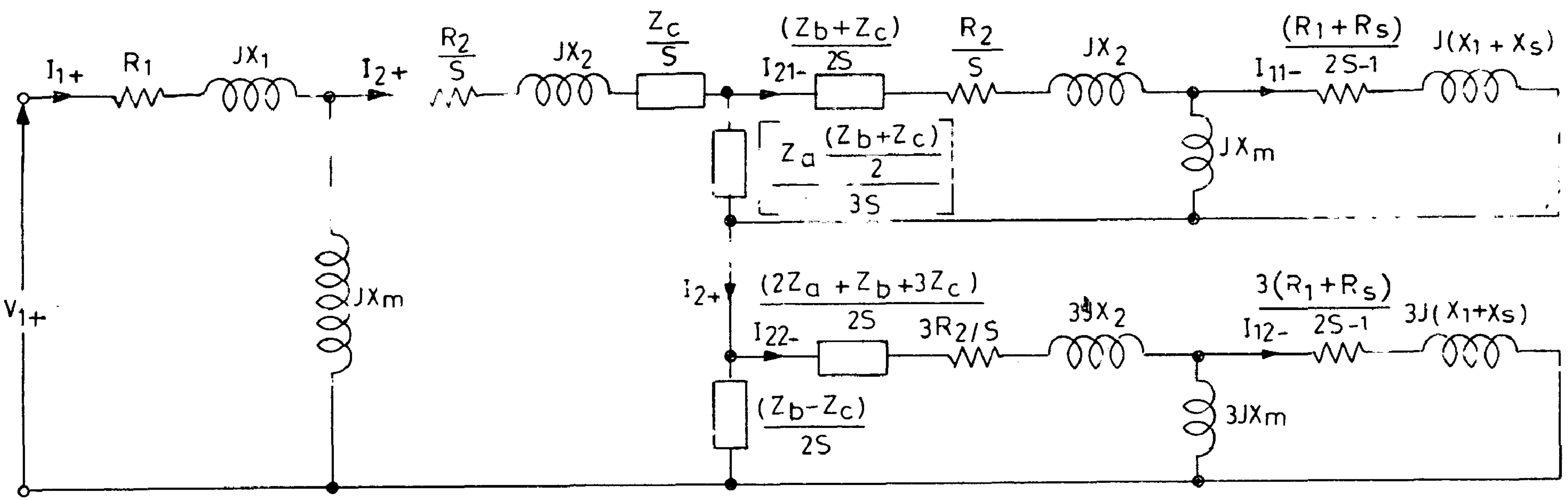
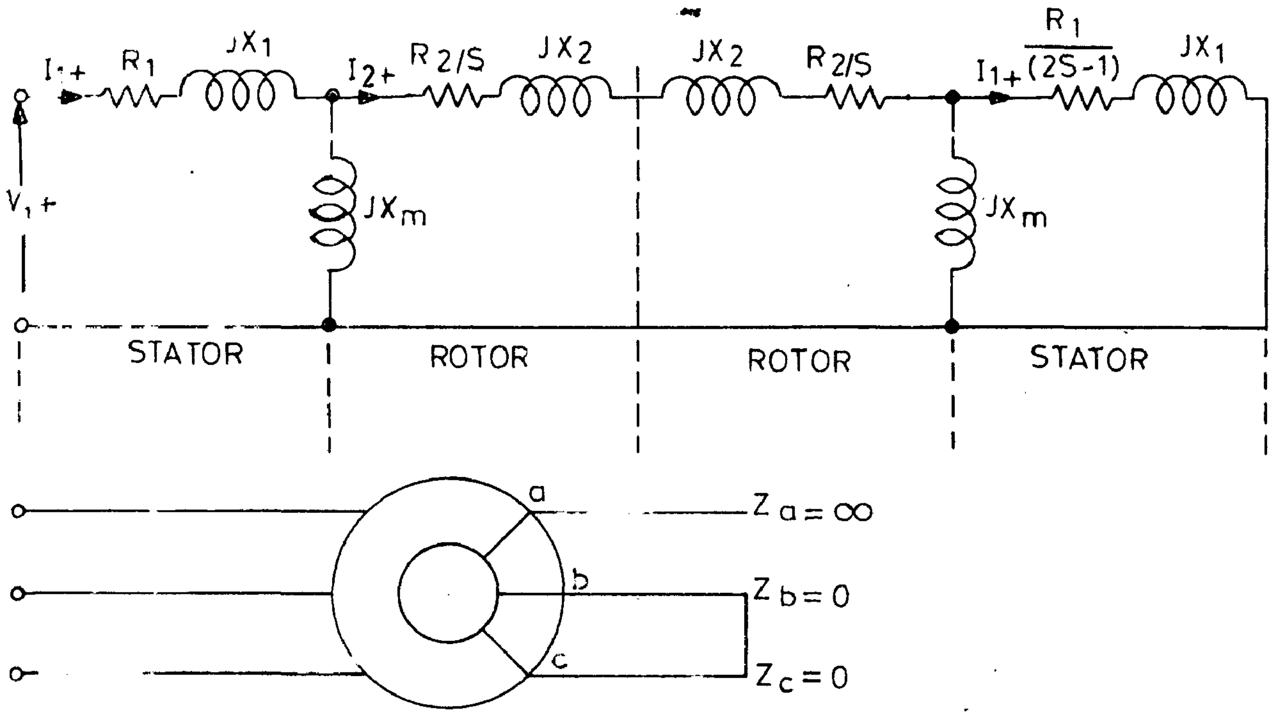


FIG.1.9 EQUIVALENT CIRCUIT OF INDUCTION MOTOR UNDER GENERAL ROTOR UNBALANCE



THE TOTAL IMPEDENCE OF THE CIRCUIT IS GIVEN BY

$$Z = R + JX_1 + \cfrac{JX_m}{1 + \cfrac{JX_m}{2(R_{2/S} + JX_2) + \cfrac{JX_m}{1 + \cfrac{JX_m}{\frac{R_1}{(2S-1)} + JX_1}}}}$$

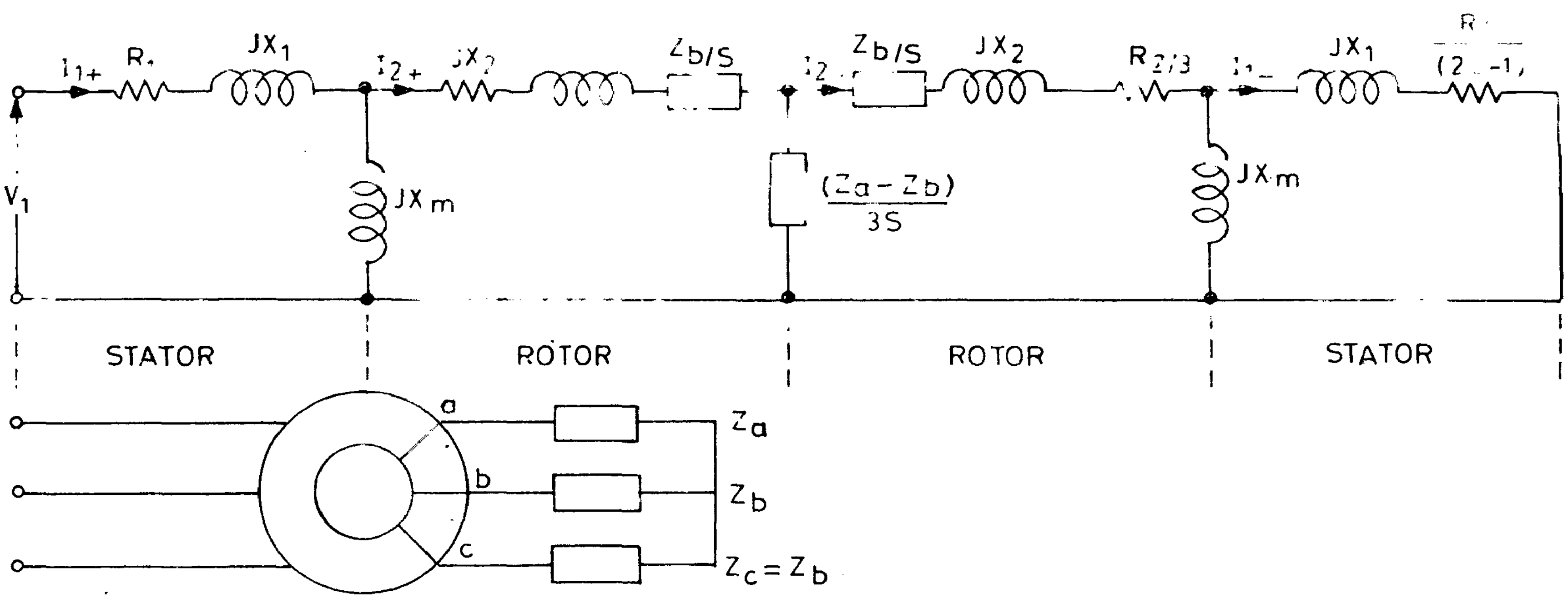
FIG.1.10 EQUIVALENT CIRCUIT OF MOTOR WITH ONE ROTOR PHASE OPEN CIRCUITED circuit is obtained by substituting $Z_b = Z_c$ in the general equivalent circuit of Fig. 1.9 and is shown in Fig. 1.11. This is a well known case discussed by Barton etal[6].

1.13 SYMMETRICAL STATOR AND SHORT-CIRCUITED SYMMETRICAL ROTOR FED FROM A BALANCED SUPPLY

This is the very common case for the performance of the balanced three-phase induction motor. The equivalent circuited is obtained by substituting in Fig. 1.9, $Z_a = Z_b = Z_c = 0$ and is given in Fig. 1.12.

1.14 NATURE OF SYMMETRIES

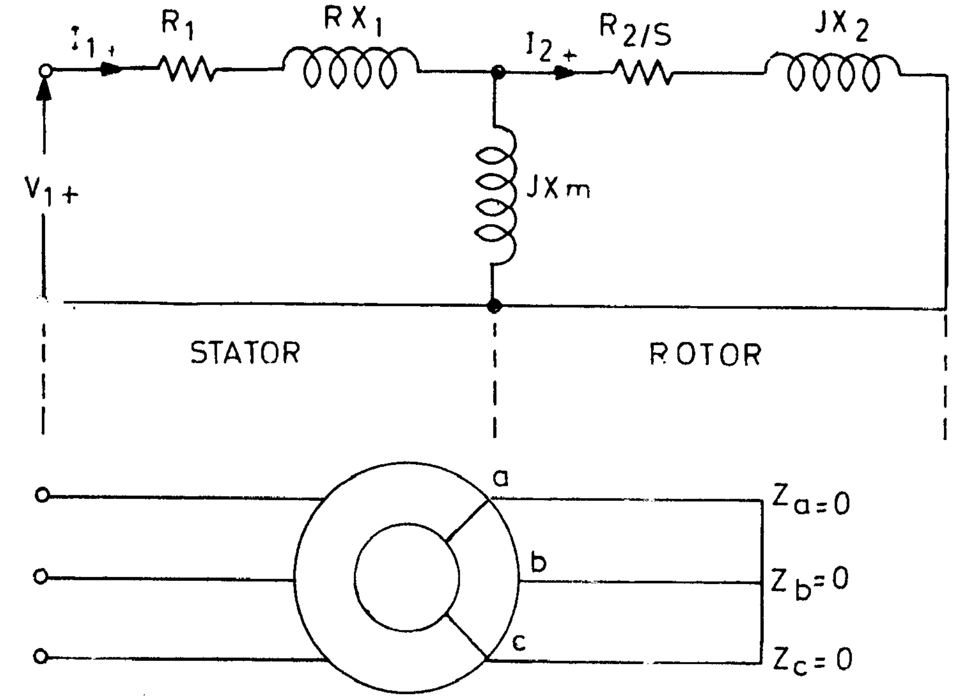
On examining the equivalent circuits in several specific cases of rotor unbalance, it is observed that all these cases could be divided in two categories of rotor unbalance-symmetrical and asymmetrical unbalance. In case of symmetrical unbalance, the external impedance added in the two rotor phases is symmetrical with respect to the third one while in the case of unsymmetrical unbalance all the three external impedances are unequal and no symmetry exists with respect to any phase. The circuit in the former case is simple and symmetrical about the air-gap. In the latter case, there is no symmetry about the air-gap and the equivalent circuit is complex.



THE TOTAL IMPEDENCE OF THE CIRCUIT IS GIVEN BY

$$Z = R_1 + JX_1 + \cfrac{JX_m}{1 + \cfrac{JX_m}{(R_{2}/S + JX_2) + \cfrac{Z_b}{S} + \cfrac{(Z_a - Z_b)/3S}{1 + \cfrac{(Z_a - Z_b)/3S}{\cfrac{R_2}{s} + JX_2 + \cfrac{Z_b}{3} + \cfrac{jx_m}{1 + \cfrac{jx_m}{\cfrac{R_1}{(2S-1)} + jx_1 - jx_1}}}}}}$$

FIG. 1.11 EQUIVALENT CIRCUIT OF THE MOTOR WITH THE SAME EXTERNAL IMPEDANCES IN THE TWO ROTOR PHASES.



THE TOTAL IMPEDENCE OF THE CIRCUIT IS GIVEN BY:

$$Z = R_1 + JX_1 + \cfrac{JX_m}{1 + \cfrac{JX_m}{R2/S + JX_2}}$$

FIG.1.12 EQUIVALENT CIRCUIT OF THE MOTOR UNDER BALANCED OPERATION

1.15 ALTERNATIVE FORM OF EQUIVALENT CIRCUIT

From equation (1.19),

$$\frac{V_2}{s} = \left[\frac{Z_0}{s} - \frac{\frac{Z_+}{s} \cdot \frac{Z_-}{s}}{\frac{Z_0}{s} + \frac{Z_{m-}}{s}}\right] I_{2+} \qquad \ldots(1.33)$$

This represents the circuit shown in Fig. 1.12. Inserting this circuit in the complete equivalent circuit of Fig. 1.6(b), alternative form of the circuit is obtained as shown in Fig. 1.13.

1.16 TORQUE EXPRESSION

The voltage equations for the circuit shown in Fig. 1.9 may be written as follows for the six meshes:

$$V_{1+} = (Z_1+jX_m)I_{1+} - jX_mI_{2+} \qquad \text{Mesh I} \quad \ldots(1.34)$$

$$0 = -jX_mI_{1+} + \left[\frac{Z_2}{s} + \frac{(Z_a+Z_b+Z_c)}{3s} + jX_m\right] I_{2+}$$

$$- \frac{2Z_a-Z_b-Z_c}{6s} I_{21-} - \frac{(Z_b-Z_c)}{2s} I_{22-} \qquad \text{Mesh II} \quad (1.35)$$

$$0 = - \frac{(2Z_a-Z_b-Z_c)}{6s}I_{2+} + \left[\frac{(Z_a+Z_b+Z_c)}{3s} + \frac{Z_2}{s} +jX_m\right] I_{21-}$$

$$-jX_m I_{11-} \qquad \text{Mesh III} \quad \ldots(1.36)$$

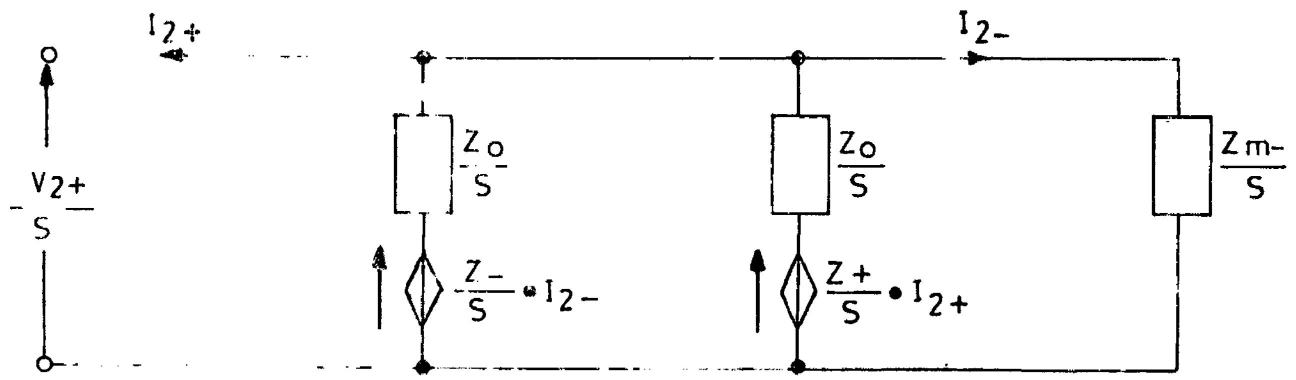


FIG. 1.13 EQUIVALENT CIRCUIT REPRESENTED BY EQUATION (1.33)

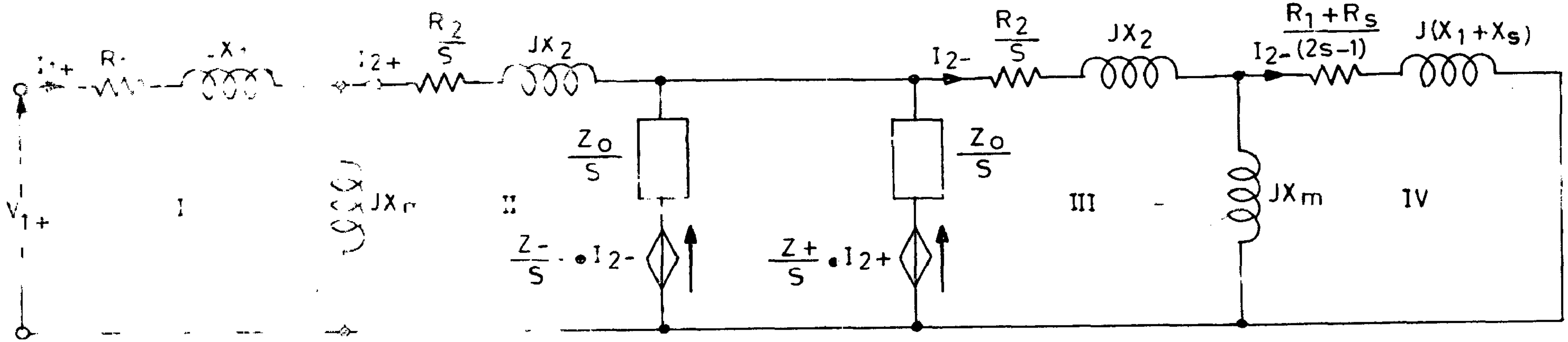


FIG.1.14 ALTERNATIVE FORM OF COMPLETE EQUIVALENT CIRCUIT

$$0 = -\frac{(Z_b - Z_c)}{2s} I_{2+} + \left[\frac{3Z_2}{s} + \frac{(Z_a + Z_b + Z_c)}{s} + 3jX_m\right] I_{22-}$$

$$-3jX_m I_{12-} \qquad \text{Mesh IV} \qquad ...(1.37)$$

$$0 = -jX_m I_{21-} + \left(\frac{Z_1}{2s-1} + jX_m\right) I_{11-} \qquad \text{Mesh V} \qquad ...(1.38)$$

$$0 = -3jX_m I_{22-} + 3\left(\frac{Z_1}{2s-1} + jX_m\right) I_{12-} \qquad \text{Mesh VI} \qquad ...(1.39)$$

Writing the voltage equations for the six meshes in the matrix form,

$$[V] = [Z] \cdot [I] \qquad ...(1.40)$$

where

$$[V] = \begin{bmatrix} V_{1+} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad [I] = \begin{bmatrix} I_{1+} \\ I_{2+} \\ I_{21-} \\ I_{22-} \\ I_{11-} \\ I_{12-} \end{bmatrix}$$

and

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots\cdots & Z_{16} \\ Z_{21} & Z_{22} & Z_{23} & \cdots\cdots & Z_{26} \\ Z_{31} & Z_{32} & Z_{33} & \cdots\cdots & Z_{36} \\ Z_{41} & Z_{42} & Z_{43} & \cdots\cdots & Z_{46} \\ Z_{51} & Z_{52} & Z_{53} & \cdots\cdots & Z_{56} \\ Z_{61} & Z_{62} & Z_{63} & \cdots\cdots & Z_{66} \end{bmatrix}$$

In the impedance matrix, elements are defined as follows-

$$Z_{11} = Z_1 + jX_m$$

$$Z_{12} = Z_{21} = Z_{53} = Z_{35} = -jX_m$$

$$Z_{56} = Z_{65} = -3jX_m$$

$$Z_{66} = 3Z_{55} = 3 \quad \frac{Z_1}{(2s-1)} + jX_m$$

$$Z_{44} = 3Z_{33} = 3Z_{22} = 3 \left[\frac{Z_2}{s} + \frac{(Z_a+Z_b+Z_c)}{3s} + jX_m \right]$$

$$Z_{23} = Z_{32} = -\frac{(2Z_a-Z_b-Z_c)}{6s}$$

$$Z_{14} = Z_{42} = -\frac{(Z_b - Z_c)}{2s}$$

and remaining elements of the matrix are all zero.

The voltage equations given above are obtained by making reference to the rotor frame as asymmetry is created on the rotor side. This is necessary to obtain frequency-equivalence for the sequence components:

Substituting $p = js\omega$, the operational impedance matrix $Z(p)$ of the machine is obtained as given below-

$$Z(p) = \begin{bmatrix} Z_{11}(p) & Z_{12}(p) & Z_{13}(p) & \cdots\cdots & Z_{16}(p) \\ Z_{21}(p) & Z_{22}(p) & Z_{23}(p) & \cdots\cdots & Z_{26}(p) \\ Z_{31}(p) & Z_{32}(p) & Z_{33}(p) & \cdots\cdots & Z_{36}(p) \\ Z_{41}(p) & Z_{42}(p) & Z_{43}(p) & \cdots\cdots & Z_{46}(p) \\ Z_{51}(p) & Z_{52}(p) & Z_{53}(p) & \cdots\cdots & Z_{56}(p) \\ Z_{61}(p) & Z_{62}(p) & Z_{63}(p) & \cdots\cdots & Z_{66}(p) \end{bmatrix} \qquad \ldots(1.41)$$

Various elements of the operational impedance matrix $Z(p)$ are defined as below:

$$Z_{11}(p) = R_1 + (L_1 + L_m)(p + j\omega_r)$$

$$Z_{12}(p) = -L_m(p + j\omega_r)$$

$$Z_{46}(p) = 3Z_{35}(p) = 3Z_{21}(p) = -3L_m p$$

$$Z_{44}(p) = 3Z_{33}(p) = 3Z_{22}(p) = 3\left[R_2 + \frac{R_a + R_b + R_c)}{3} + \left(L_2 + L_m + \frac{(L_a + L_b + L_c)}{3}\right.\right.$$

$$Z_{23}(p) = Z_{32}(p) = -\frac{(2R_a - R_b - R_c)}{6} - \frac{(2L_a - L_b - L_c)}{6}p$$

$$Z_{24}(p) = Z_{42}(p) = -\frac{(R_b - R_c)}{2} - \frac{(L_b - L_c)}{2}p$$

$$Z_{64}(p) = 3Z_{53}(p) = -3L_m(p - j\omega_r)$$

$$Z_{66}(p) = 3\ Z_{55}(p) = 3R_1 + 3(L_1 + L_m)(p - j\omega_r)$$

and the remaining elements are all zero.

The torque expression is given by-

$$T = Re\left[I_t^{*}\,\omega\, G I\right] \qquad ...(1.42)$$

where G, the co-efficients of ω_r in Z(p) matrix, is given by-

$$G = \begin{bmatrix} j(X_1+X_m) & -jX_m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & jX_m & 0 & -j(X_1+X_m) & 0 \\ 0 & 0 & 0 & 3jX_m & 0 & -3j(X_1+X_m) \end{bmatrix} \qquad ...(1.43)$$

The torque expression for the machine on simplification is given by,

$$T = Re\,(V_{1+}\cdot \overset{*}{I}_{1+}) - R_1 I_{1+}\cdot \overset{*}{I}_{1+} + \frac{R_1}{(2s-1)}\, I_{11-}\cdot \overset{*}{I}_{11-} + \frac{3R_1}{(2s-1)}\cdot I_{12-}\cdot \overset{*}{I}_{12-} \qquad ...(1.44)$$

1.17 VARIATION OF TORQUE

The torque expression given by the equation (1.44),

$$T = Re(V_{1+}\cdot \overset{*}{I}_{1+}) - R_1 I_{1+}\cdot \overset{*}{I}_{1+} + \frac{R_1}{(2s-1)}\, I_{11-}\cdot \overset{*}{I}_{11-} + \frac{3R_1}{(2s-1)}\cdot I_{12-}\cdot \overset{*}{I}_{12-}$$

may be represented by,

$$T = T_p + T_n \qquad \ldots(1.45)$$

where T_p = Positive-sequence torque

$$= \quad Re(V_{1+} \cdot \overset{\circ}{I}_{1+}) - R_1 I_{1+} \cdot \overset{\circ}{I}_{1+}$$

and T_n = Negative-sequence torque

$$\frac{R_1}{(2s-1)} I_{11-} \cdot \overset{\circ}{I}_{11-} + \frac{3R_1}{(2s-1)} \cdot I_{12-} \cdot \overset{\circ}{I}_{12-}$$

While obtaining the net torque T from equation (1.45), the sign of negative-sequence torque T_n has to be taken into account, which will depend upon the value of slip. Whether T_n helps or opposes T_p can be easily ascertained by considering the motion of negative-sequence m.m.f. with respect to the stator conductors, Fig. 1.2. For slips less than 0.5, this m.m.f. moves in the same direction as the rotor causing a torque in the stator tending to accelerate the stator in the same direction as that of the rotor. The reaction torque, therefore, opposes the positive-sequence torque.

$$T = T_p - T_n \quad \text{for slips less than } 0.5 \qquad \ldots(1.46)$$

This can be verified from equation (1.45) which yields negative values for T_n for $(2s-1) < 0$ i.e. for $s < 0.5$.

As the slip increases, the rotor slows down and the negative-sequence m.m.f. gains speed with respect to the motor. At half synchronous speed the rotor and the m.m.f speeds are equal and opposite so that the relative speed of the negative-sequence m.m.f with respect to the stator is zero and no negative-sequence torque is produced. This is in agreement that for negative-sequence torque to be zero, $(2s-1) = 0$ i.e., slip is equal to 0.5. When the slip is greater than 0.5, i.e., below half synchronous speed, the m.m.f due to negative-sequence current acts in opposition to that of the rotor. The reaction torque thus aids that produced by the positive-sequence current and

$$T = T_p + T_n \quad - \text{ for slips greater than } 0.5 \qquad \ldots(1.47)$$

This can be verified from equation (1.45) which yields positive values for T_n for $(2s-1) > 0$ i.e. for $s > 0.5$. The direction of the positive- and negative-sequence torques at different speeds and corresponding slips are summarised in Table 1.

TABLE 1

Speed r	Slip s	Torque direction	
		Positive-sequence	Negative-sequence
$\omega_r < \frac{1}{2}\omega$	$s > \frac{1}{2}$	Forward	Forward
$\omega_r = \frac{1}{2}\omega$	$s = \frac{1}{2}$	Forward	Zero
$\frac{1}{2}\omega < \omega_r < \omega$	$\frac{1}{2} > s > 0$	Forward	Backward
$\omega_r = \omega$	$s = 0$	Zero	Zero
$\omega_r > \omega$	$s < 0$	Backward	Backward

The general equivalent circuit and the torque expression derived above enable the complete performance of the machine with asymmetric rotor predicted. The first mesh in the equivalent circuit of Fig. 1.13 shows the main stator current I_{1+} at supply frequency f, the second and the third meshes show the referred rotor currents I_{2+} and I_{2-} at slip frequency sf and the fourth, the negative-sequence stator current I_{1-} at a frequency $(2s-1)f$. The torque expression shows that between stand still $(s=1)$ and half speed $(s=\frac{1}{2})$, all the torque terms are positive. Between half speed and synchronous $(s=0)$, some of the terms in the torque expression are positive and some become negative. A sudden change in the torque/slip characteristic at half speed may therefore, be anticipated. The equivalence circuits presents the physical consideration clearly. The supply voltage in the stator produces a rotating field at synchronous speed which induces a slip-frequency current in the rotor. Since the rotor is asymmetric, two equal and opposite rotating fields at slip-frequency sf are produced. The one synchronises with the main rotating field from the stator and the other induces in the stator balance current at a frequency $(2s-1)f$. These currents circulate through the supply lines. The machine behaves likes two direct-coupled normal induction motors. The stator of first is connected through the supply. The rotors are connected in series with two phases interchanged. The stator of the second is short-circuited.

The shape of the torque/slip characteristic with asymmetric rotor is shown in Fig. 1.14.

1.18 LOSSES IN THE MACHINE

The losses in the circuit shown in Fig. 1.9 represent per phase the total input to the machine, so that if the actual copper losses are subtracted then the difference must represent the output. Division by (1-s) then gives the torque in synchronous watts.

Considering the equivalent circuit of Fig. 1.9, Power input per phase or per phase circuit losses

$$= I_{1+}^2 \cdot R_1 + I_{2+}^2 \frac{(R_2+R_c)}{2} + I_{21-}^2 \left(\frac{R_2}{s} + \frac{R_b+R_c}{2s}\right)$$

$$+ (I_{2+} - I_{21-})^2 \cdot \frac{(2R_a-R_b-R_c)}{6s} + I_{11-}^2 \cdot \frac{R_1}{(2s-1)}$$

$$+ 3I_{22-}^2 \cdot \left(\frac{R_2}{s} + \frac{2R_a+R_b+3R_c}{6s}\right) + 3I_{12-}^2 \cdot \frac{R_1}{(2s-1)}$$

$$+ (I_{2+} - I_{22-})^2 \frac{(R_b-R_c)}{2s} \qquad \ldots(1.48)$$

Actual losses per phase

$$= I_{1+}^2 R_1 + I_{2+}^2 (R_2+R_c) + I_{21-}^2 \left(R_2 + \frac{R_b+R_c}{2}\right)$$

$$+ (I_{2+}-I_{21-})^2 \cdot \frac{(2R_a-R_b-R_c)}{6} + I_{11-}^2 \cdot R_1$$

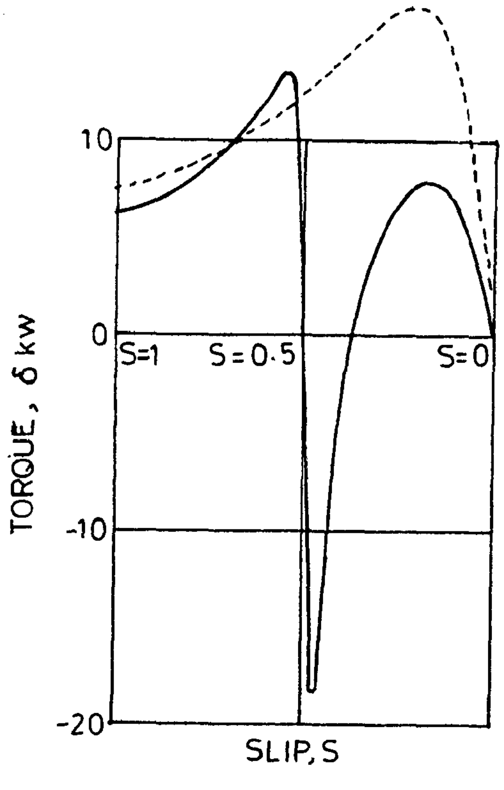


FIG. 3.2 TORQUE SLIP CURVE FOR SINGLE-PHASE ROTOR OPERATION WITH NORMAL CHARACTERSTIC-BROKEN LINE.

$$+ 3I_{22-}^2 \cdot \left(\frac{R_2}{s} + \frac{2R_a+R_b+3R_c}{6}\right) + 3I_{12-}^2 \cdot R_1$$

$$+ (I_{2+}-I_{22-})^2 \cdot \frac{(R_b-R_c)}{2} \qquad \ldots(1.49)$$

$$\text{output} = I_{2+}^2 \frac{(1-s)}{s}(R_2+R_c) + I_{21-}^2 \cdot \frac{(1-s)}{s}\left(R_2+\frac{R_b+R_c}{2}\right)$$

$$+ (I_{2+}-I_{21-})^2 \frac{(1-s)}{s} \cdot \frac{(2R_a-R_b-R_c)}{6} + I_{11-}^2 \cdot \frac{2R_1(1-s)}{(2s-1)}$$

$$+ 3I_{22-}^2 \cdot \frac{(1-s)}{s}\left(R_2+\frac{2R_a+R_b+R_c}{6}\right) + 6I_{12-}^2 \cdot \frac{R_1}{(2s-1)}$$

$$+ (I_{2+} - I_{22-})^2 \cdot \frac{(1-s)}{s}\frac{(R_b - R_c)}{2} \qquad \ldots(1.50)$$

$$\text{Torque} = \frac{(R_2+R_c)}{s} I_{2+}^2 + \left[\frac{R_2}{s} + \frac{(R_b + R_c)}{2s}\right] I_{21-}^2$$

$$+ 3\left[\frac{R_2}{s} + \frac{2R_a+R_b+3R_c}{6s}\right] I_{22-}^2 + \frac{2R_1}{(2s-1)} \cdot I_{11-}^2 + \frac{6R_1}{(2s-1)} I_{12-}^2$$

$$+ \frac{(R_b-R_c)}{2s}(I_{2+}-I_{22-})^2 + \frac{(2R_a-R_b-R_c)}{6s}(I_{2+}-I_{21-})^2 \qquad \ldots(1.51)$$

This torque expression is found to be the same as given by the equation (1.44).

CHAPTER II

EQUIVALENT CIRCUIT OF INDUCTION MOTOR-GENERALISED ROTATING FIELD THEORY

CHAPTER II

EQUIVALENT CIRCUIT OF INDUCTION MOTOR-GENERALISED ROTATING FIELD THEORY

INTRODUCTION

Generalised rotating-field concepts have been used in this chapter to extend the analysis to the case of induction motor with general rotor unbalance. The interaction of a single-phase stator winding with a single-phase winding on the rotor has been studied and the aggregate behaviour of the machine obtained through the summation of the effects of each phase. The current flowing in each winding produces forward and backward rotating fields, which in turn reacts with all the windings on the machine. The performance of the machine is determined from the voltage equations of the stator and rotor windings. These equations can be easily interpreted to obtain the complete equivalent circuit of the machine with asymmetric rotor. A new equivalent circuit has also been developed and is shown compatible with the earlier circuit developed in chapter I.

In the following analysis, supply voltage has been assumed balanced and sinusoidal, stator and rotor windings symmetrical and rotor unbalance created by adding unequal impedances in the circuit. The saturation and iron losses are neglected.

2.1 ANALYSIS OF AN INDUCTION MACHINE HAVING A SINGLE-PHASE WINDING ON THE STATOR AND A SINGLE-PHASE SHORT-CIRCUITED WINDING ON THE ROTOR:

Before proceeding with the generalised case of a three-phase machine with general rotor unbalance, it will be proper first to study the interactions between a single-phase stator winding and a single-phase rotor winding rotating with an angular speed of ω_r electrical radians per sec.

2.2 FIELDS PRODUCED BY THE INTERACTION OF STATOR AND ROTOR CURRENTS

When the single-phase winding on the stator is excited by a single-phase sinusoidal voltage at frequency $f = \frac{\omega}{2\pi}$ cycles/sec., the current flowing in the winding produces an m.m.f., which, acting on the magnetic circuit of the machine produces an alternating magnetic flux. The current flowing in the short-circuited rotor-winding reacts with the stator current to produce a resultant air-gap field which may be resolved into a series of component rotating fields, all with the same number of poles but with different speeds. The forward and backward-rotating fields produced by the stator current of fundamental frequency f rotate in space at $\pm$ electrical rad/sec., and hence induce, in the rotor, currents of frequency $f-f_r$ and $f+f_r$, where $f_r = \frac{\omega_r}{2\pi}$ cycles/sec. Each of these currents flowing in the single-phase rotor winding produces forward and backward fields, the two

The stator-voltage equation for fundamental frequency current is,

$$V = I_1(R_1 + jX_1 + jX_{1f} + jX_{1b})$$

$$-jI_{2(f-f_r)}\sqrt{X_{1f}\cdot X_{2f}} - jI_{2(f+f_r)}\sqrt{X_{1b}\cdot X_{2b}} \qquad \ldots(2.1)$$

The bracketed subscripts against the currents refer to their frequencies for identification purposes.

The rotor-voltage equation for current of frequency $(f-f_r)$ when referred to the stator is,

$$0 = I_{2(f-f_r)}(R_2\cdot \frac{f}{f-f_r} + jX_2 + jX_{2f} + jX_{2b}$$

$$-jI_{1(f)}\sqrt{X_{1f}\cdot X_{2f}} - jI_{1(f-2f_r)}\sqrt{X_{2b}\cdot X_{1f}} \qquad \ldots(2.2)$$

The stator-voltage equation for currents of frequency $f_1 = (f + kf_r)$, where k is even but not equal to zero is,

$$I_{1(f_1)}\ (R_1 \frac{f}{f_1} + jX_1 + jX_{1f} + jX_{1b}) - jI_{2(f-f_r)}\sqrt{X_{1f}\cdot X_{2b}}$$

$$-jI_{2(f_1-f_r)}\ \sqrt{X_{1b}\cdot X_{2f}} = 0 \qquad \ldots(2.3)$$

and the rotor-voltage equation for currents of frequency $f_2 = f-f_r$ is,

$$I_{2(f_r)}(R_2 \frac{f}{f_2} + jX_2 + jX_{2f} + jX_{2b}) - jI_{1(f_1-f_r)}\sqrt{X_{1b}\cdot X_{2f}}$$

$$-jI_{1(f_2+f_r)}\sqrt{X_{2b}\cdot X_{1f}} = 0 \qquad \ldots(2.4)$$

forward fields (an induced field has been considered forward when it is in the direction of the inducing field and backward if it is in the opposite direction) have speeds in space of $\pm \omega$ electrical rad./sec., and are stationary individually with respect to the inducing stator fields, but the two backward fields have velocities in space of $-\omega \pm 2\omega_r$, inducing currents of frequency $f \pm 2f_r$ in the stator winding. These in turn produce forward and backward fields, the forward being stationary with respect to the inducing fields and the backward producing another series of fields.

This production of inducing fields due to high frequency currents goes on ad infinitum. The current in the stator has the frequencies f, $f \pm 2f_r$, $f \pm 4f_r, \ldots$, i.e. of the order of $f \pm kf_r$, where k is even (including zero), while the current in the rotor has the frequencies $f \pm f_r$, $f \pm 3f_r, \ldots$, i.e. of the order $f + kf_r$ where k i odd. The speeds of rotating fields and reaction frequencies for currents are shown in Fig. 1.1

2.3 VOLTAGE EQUATIONS

It may be noted that the rotor winding is on short-circuit for currents of all frequencies, while the stator winding is on short-circuit for all frequencies except the fundamental. The mutually coupled coils have self reactances as expressed and unity coefficient of coupling.

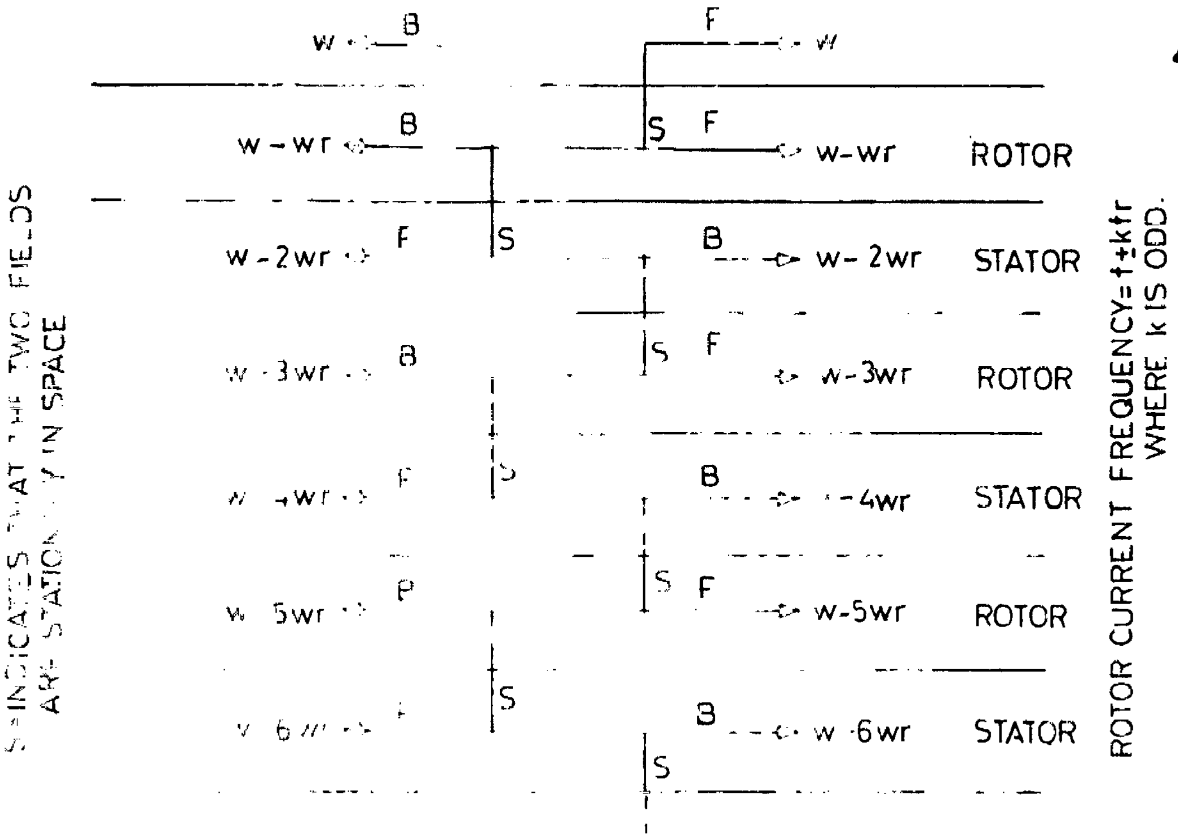


FIG.2.1(a). STATOR FORWARD FIELD

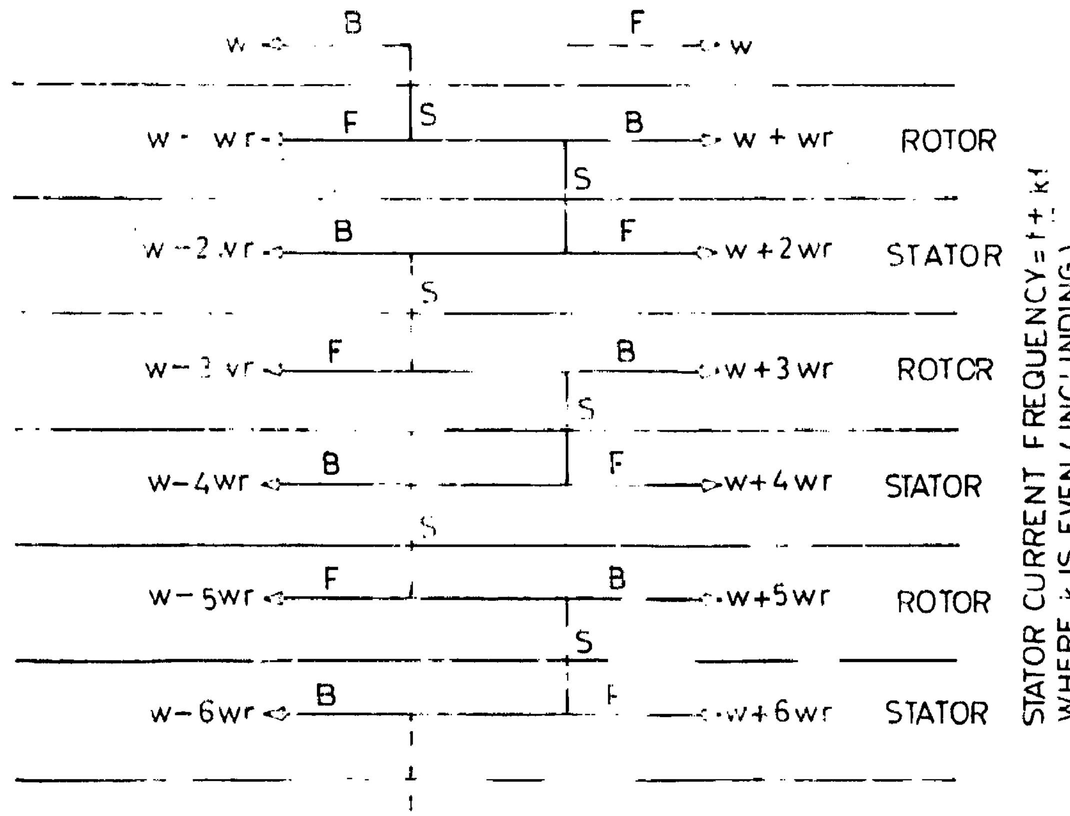


FIG.2.1(b). STATOR BACKWARD FIELD

2.4 EQUIVALENT CIRCUIT

The equations (2.1 to (2.4) show the nature of the equivalent circuit and is shown in Fig. 2.3. The equivalent circuit contains an unending chain of forward and backward field loops showing an infinite number of stages of interaction between the stator and the rotor.

2.5 FREQUENCY OF REACTIONS

The following points may be noted from the above analysis:

(a) The presence of asymmetry in the stator and the rotor (in the form of single-phase winding on each) leads to the induction of currents of specific frequencies f_1 in the stator and f_2 in the rotor given by $f_1 = f \pm kf_r$, where k is even, and $f_2 = f \pm kf_r$, where k is odd.

(b) Voltages of frequency f_1 are induced in the stator winding by rotor currents of frequency $f_1 - f_r$ and $f_1 + f_r$.

(c) Voltages of frequency f_2 are induced in the rotor winding by stator currents of frequency $f_2 - f_r$ and $f_2 + f_r$.

Keeping in view, the above interaction between a stationary and a rotating winding, the analysis can be

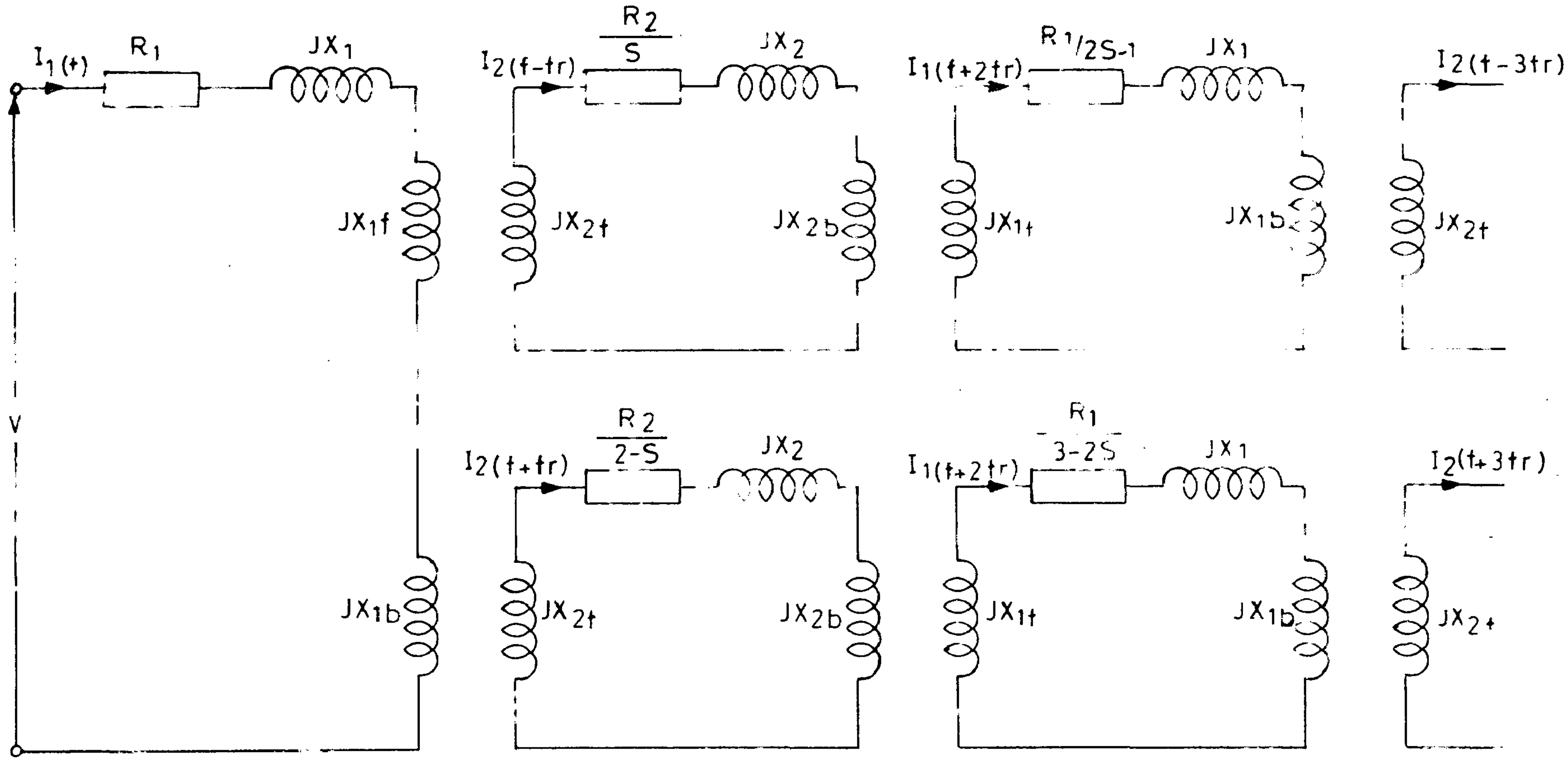


FIG.2.2 **Equivalent Circuit for Equation 1 to 4**

extended to induction motor having three windings on the stator and on the rotor. The current flowing in each winding may be considered to produce forward and backward rotating fields, which in turn would react with all the windings on the machine. The aggregate behaviour of the machine can then be expressed through the voltage equations of the stator and rotor windings for all frequencies present.

2.6 ANALYSIS OF INDUCTION MOTOR WITH GENERAL ROTOR UNBALANCE

Assuming the stator symmetrically wound and the rotor asymmetrical, the balanced currents flowing through the stator windings will produce no backward rotating field and the first loop of the equivalent circuit would consist only of the mutual reactance due to the forward field. Owing to the rotor asymmetry, the stator forward field would induce unbalanced currents of frequency $f-f_r$ in the short-circuited rotor windings, which in turn produce both forward and backward rotating fields. The forward field produced by the rotor currents is synchronous with the inducing field, while the backward field induces balanced currents of negative-sequence and frequency $(f-2f_r)$ in the symmetrical stator winding. Since the stator currents are balanced, these will produce no backward field, and hence further induction of currents in the rotor will cease. The equivalent circuit of Fig. 2.3 will therefore, close at the third loop of $(f-2f_r)$ stator frequency and is shown in Fig. 2.4.

2.7 VOLTAGE EQUATIONS

Consider the case when the stator winding is symmetrical and rotor asymmetry is introduced through unequal external star connected impedance Z_a, Z_b and Z_c in series with the rotor windings. The stator and rotor mutual-impedance-coefficient matrices due to forward field, which are functions of winding turns and their respective space displacements, are given as follows:

$$[M_1] = [M_2] = [M_{12}] = \begin{bmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{bmatrix} \quad \ldots(2.5)$$

For the symmetrical-stator machine excited by a balanced supply, the stator carries only the frequencies f and $(f - 2f_r)$, while the rotor carries only the frequency $(f - f_r)$.

The three voltage equations in the matrix form are:

$$[V_1]_f = ([Z_1] + j[M_1] X_f)[I_1]_f - j[M_{12}] X_f [I_2]_{(f-f_r)} \quad \ldots(2.6)$$

$$0 = [V_1]_{f-2f_r} = ([Z_1]_{f-2f_r} + j[M_1]^{\circ} X_b)[I_1]_{(f_1-2f_r)} - [M_{12}]^{\circ} X_b [I_2]_{(f-f_r)} \quad \ldots(2.7)$$

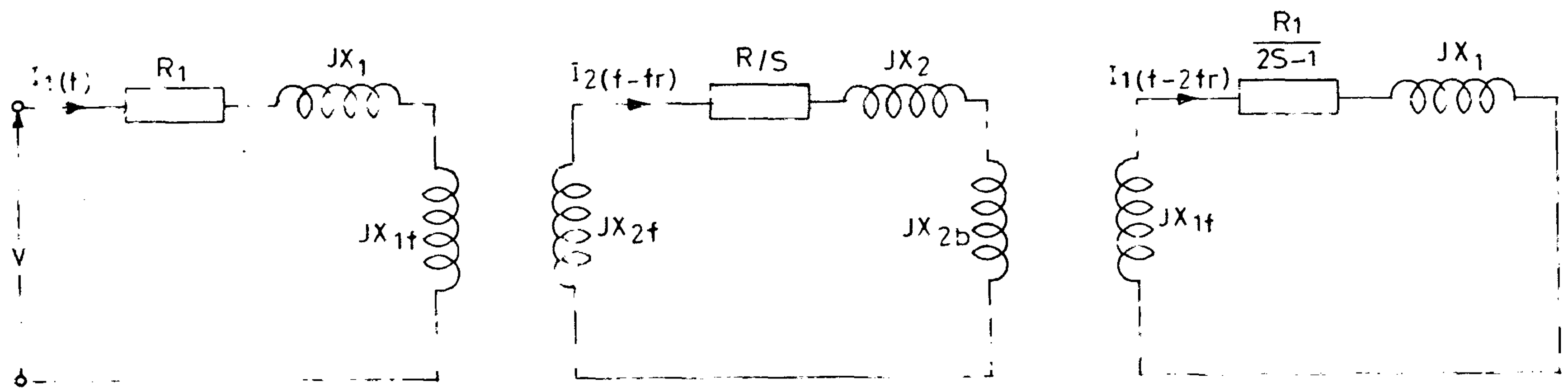


FIG 2.3 EQUIVALENT CIRCUIT OF MACHINE WITH ASYMMETRICAL ROTOR

$$0 = [V_2]_{(f-f_r)} = \left([Z_2]\frac{f}{f-f_r} + j\,[M_2]\,X_f + j\,[M_2]^{*}\,X_b \right) [I_2]_{(f-f_r)}$$

$$-j\,[M_{12}]_t\,X_f\,[I_1]_f - j\,[M_{12}]^{*}_t\,X_b\,[I_1]_{(f-2f_r)}$$

...(2.8)

Where $[Z_1]$ is the stator impedance diagonal matrix, having same elements (R_1+jX_1); $[M_1]\,X_f$ and $[M_1]^{*}\,X_b$ are the stator mutual-reactance matrices due to the forward and the backward field; $[M_{12}]\,X_f$ and $[M_{12}]^{*}\,X_b$ are the mutual-reactance matrices between the stator and rotor windings due to the forward and the backward fields; Z_{2e} is the total rotor impedance diagonal matrix (including the external impedances added in the rotor phases) having elements Z_2+Z_a, Z_2+Z_b and Z_2+Z_c and $Z_2 = (R_2+jX_2)$; $[M_2]\,X_f$ and $[M_2]^{*}\,X_b$ are the rotor mutual-reactance matrices due to the forward and the backward fields, and $[M_{12}]_t\,X_f$ and $[M_{12}]^{*}_t\,X_b$ are the mutual-reactance matrices between the rotor and the stator windings due to the forward and backward fields.

2.8 EQUIVALENT CIRCUIT

The equations (2.6), (2.7) and (2.8) show the nature of the equivalent circuit. Premultiplying these equations by the 3-phase symmetrical-components transformation matrix S_3

$$S_3 = \frac{1}{3} \begin{matrix} \\ 0 \\ + \\ - \end{matrix} \begin{matrix} a & b & c \\ \end{matrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

and discarding zero-sequence voltages and currents as these are absent. On simplification, following equations are obtained:

$$V_1 = (R_1 + jX_1)I_{1+} + jX_m(I_{1+} - I_{2+}) \quad \ldots(2.9)$$

$$0 = \frac{V_{1-}}{(2s-1)} = (\frac{R_1}{2s-1} + jX_1)I_{1-} + jX_m(I_{1-} - I_{2-}) \quad \ldots(2.10)$$

$$0 = \frac{V_{2+}}{s} = (\frac{R_2}{s} + jX_2)I_{2+} + jX_m(I_{2+} - I_{1+}) + \frac{Z_0}{s}I_{2+} + \frac{Z_-}{s} I_{2-} \quad \ldots(2.11)$$

$$0 = \frac{V_{2-}}{s} = (\frac{R_2}{s} + jX_2)I_{2-} + jX_m(I_{2-} - I_{1-}) + \frac{Z_+}{s} \cdot I_{2+} + \frac{Z_0}{s} I_{2-} \quad \ldots(2.12)$$

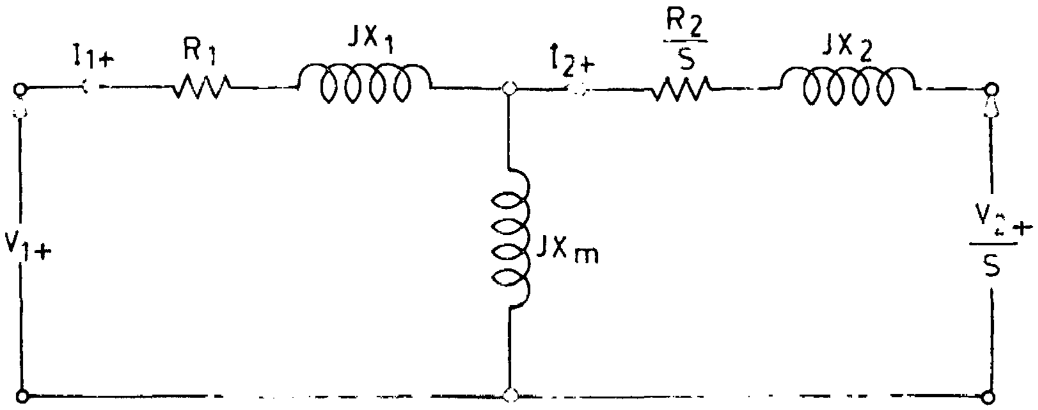
Where Z_0, Z_+ and Z_- are the zero-, positive- and negative-sequence impedance components of the external impedances Z_a, Z_b and Z_c added in the rotor circuit, and are given by,

$$Z_0 = \frac{1}{3}(Z_a + Z_b + Z_c) \quad \ldots(2.13)$$

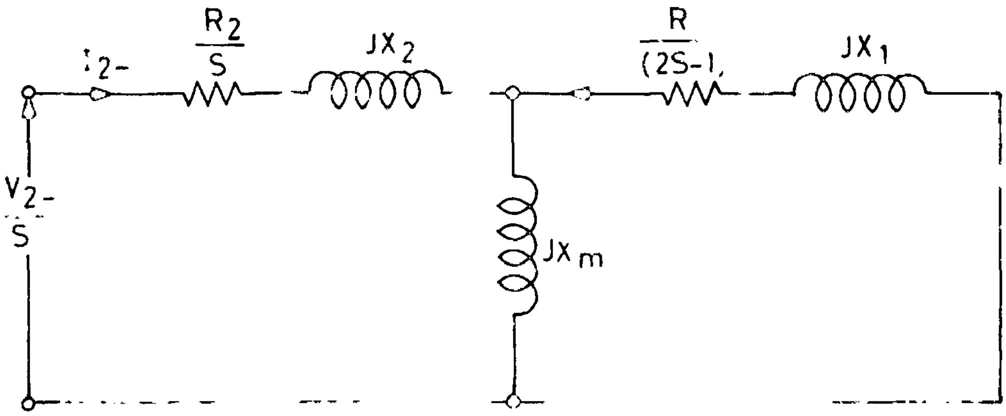
$$Z_+ = \frac{1}{3}(Z_a + aZ_b + a^2 Z_c) \quad \ldots(2.14)$$

$$Z_- = \frac{1}{3}(Z_a + a^2 Z_b + aZ_c) \quad \ldots(2.15)$$

Equations (2.9) to (2.12) represent the equivalent circuit shown in Fig.2.5.



POSITIVE-SEQUENCE EQUIVALENT CIRCUIT



NEGATIVE SEQUENECE EQUIVALENT CIRCUIT

FIG. 2.4

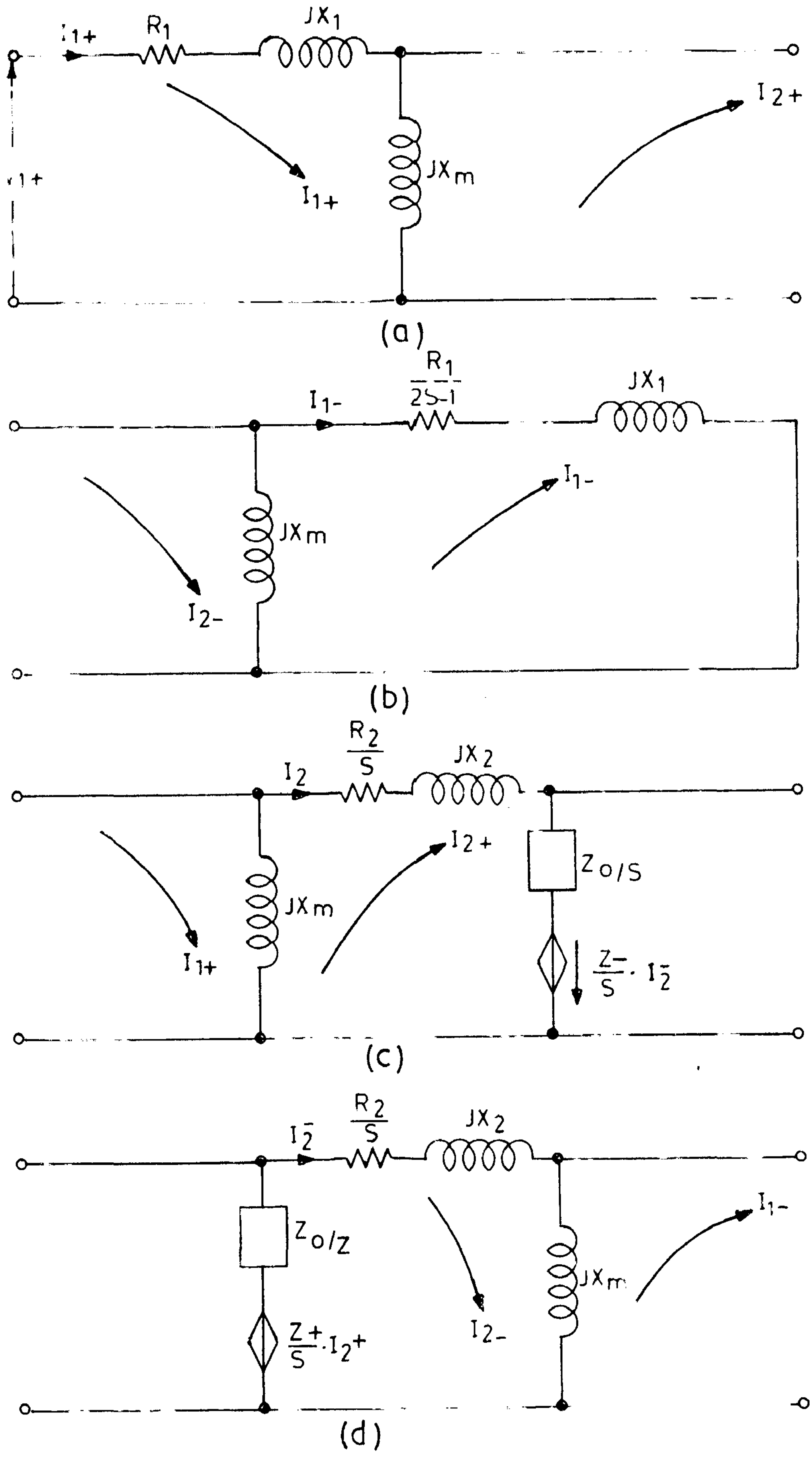


FIG. 7.5 A Sequence-circuits

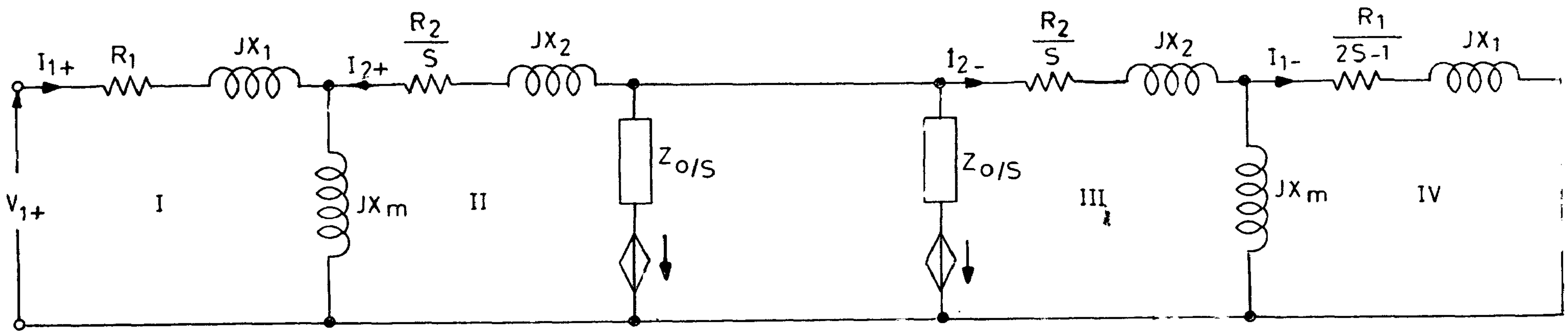


FIG.2.5B EQUIVALENT CIRCUIT OF INDUCTION MOTOR WITH GENERAL ROTOR UNBALANCE.

2.9 COUPLING NETWORK

The rotor-sequence voltage equations given by equations (1.12), (1.13 and (1.14) may be expressed in the matrix form as follows,

$$\begin{bmatrix} V_{20} \\ V_{2+} \\ V_{2-} \end{bmatrix} = \begin{bmatrix} Z_0 & Z_- & Z_+ \\ Z_+ & Z_0 & Z_- \\ Z_- & Z_+ & Z_0 \end{bmatrix} \cdot \begin{bmatrix} I_{20} \\ I_{2+} \\ I_{2-} \end{bmatrix} \qquad \ldots(2.16)$$

where V_{20}, V_{2+}, V_{2-} and I_{20}, I_{2+}, I_{2-} are the symmetrical components of the rotor voltages and currents and Z_0, Z_+, Z_- are the symmetrical components of the external impedances Z_a, Z_b and Z_c added in the rotor circuit. As the rotor is star-connected, zero-sequence currents are absent. Therefore, neglecting zero-sequence component, voltage equation may be written,

$$\begin{bmatrix} \frac{V_{2+}}{s} \\ \frac{V_{2-}}{s} \end{bmatrix} = \begin{bmatrix} \frac{Z_0}{s} & \frac{Z_-}{s} \\ \frac{Z_+}{s} & \frac{Z_0}{s} \end{bmatrix} \cdot \begin{bmatrix} I_{2+} \\ I_{2-} \end{bmatrix} \qquad \ldots(2.17)$$

The voltage expression given by equation (2.17) represents a four-port network consisting of two controlled voltage generators and represents the circuit shown in Fig. 2.6.

The complete equivalent circuit of the machine Fig.2.5 may be splitted in three component circuits as shown in

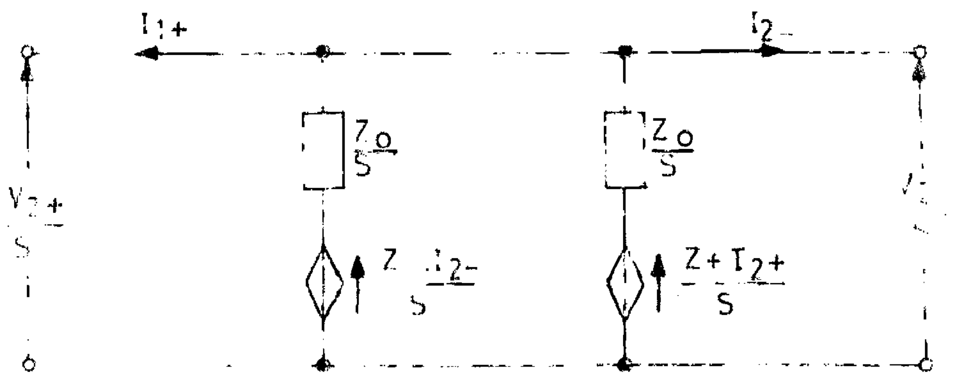
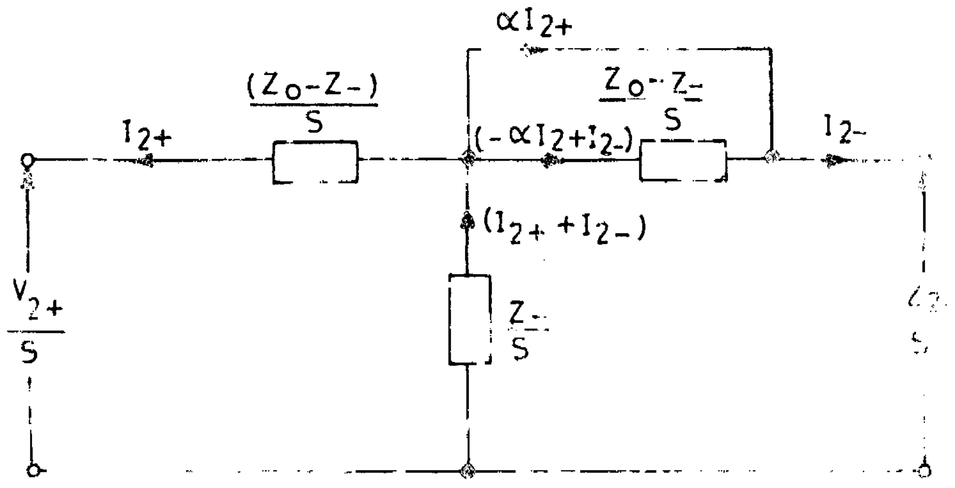


FIG.2.6 T-TYPE COUPLING NETWORK CONSISTING OF TWO CONTROLLED VOLTAGE GENERATORS



$$\alpha = \frac{(Z_- - Z_+)}{(Z_o - Z_-)}$$

FIG.2.8 T-TYPE COUPLING NETWOK CONSISTING OF ONE CONTROLLED GENERATOR ONLY

Fig. 2.7. It is observed that the circuits Fig. 2.7(a) and (b) correspond to positive-and negative-sequence circuits already derived in chapter I, Fig. 1.13 and the third circuit is seen to be the coupling network connecting the positive- and negative-sequence circuits.

2.10 ALTERNATIVE FORM OF EQUIVALENT CIRCUIT

The coupling network can be of T or π type, and from each of these types, two different networks can be realized. One of them, the π-type coupling network shown in Fig. 2.6, consists of the symmetrical component impedances and two voltage generators controlled by the positive-and negative-sequence rotor currents. The second one, the T-type coupling network as shown in Fig. 2.8, consists of one voltage generator only controlled by the positive-sequence rotor current.

By combining the equivalent circuits, given in Fig. 1.13 and Fig. 2.8, the complete equivalent circuit of the asymmetrical machine is obtained and is shown in Fig. 2.9.

2.11 TORQUE EXPRESSION

Voltage equations for the circuit shown in Fig. 2.5 may be written as follows for the four meshes,

$$V_{1+} = (Z_1 + jX_m)I_{1+} + jX_m\ I_{2+} \qquad \text{Mesh I} \qquad \ldots(2.18)$$

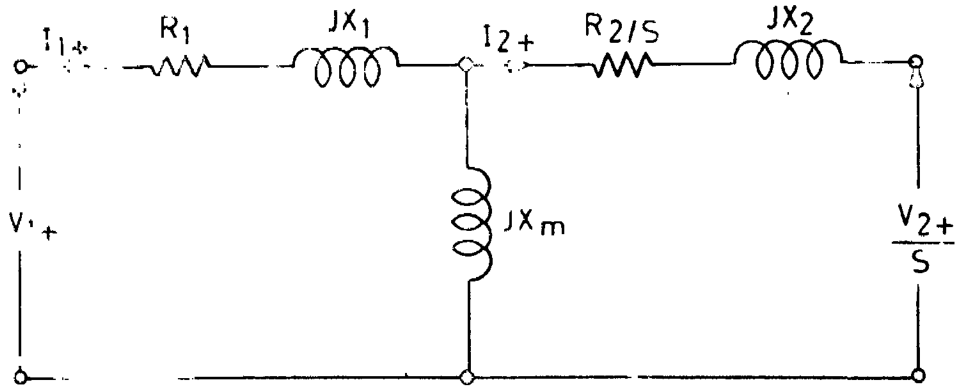


FIG. 2.7 (a) POSITIVE-SEQUENCE CIRCUIT

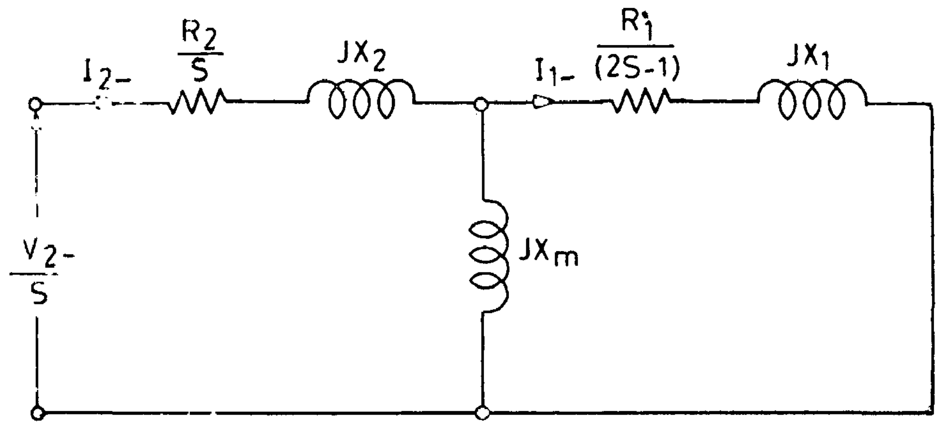


FIG. 2.7 (b) NEGATIVE SEQUENCE CIRCUIT

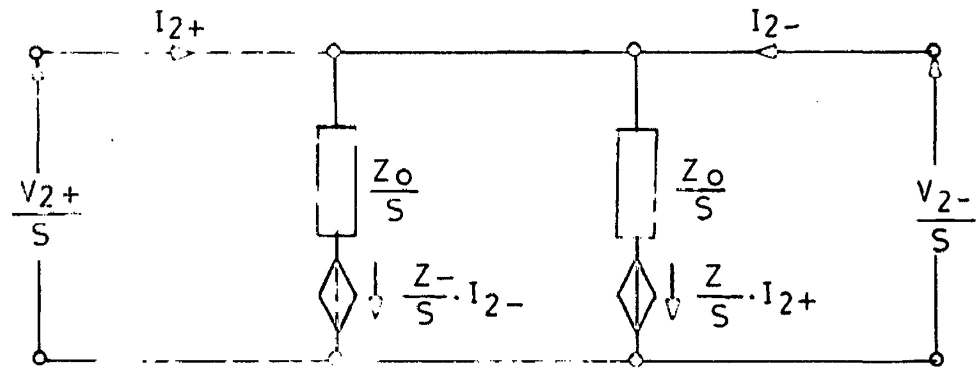
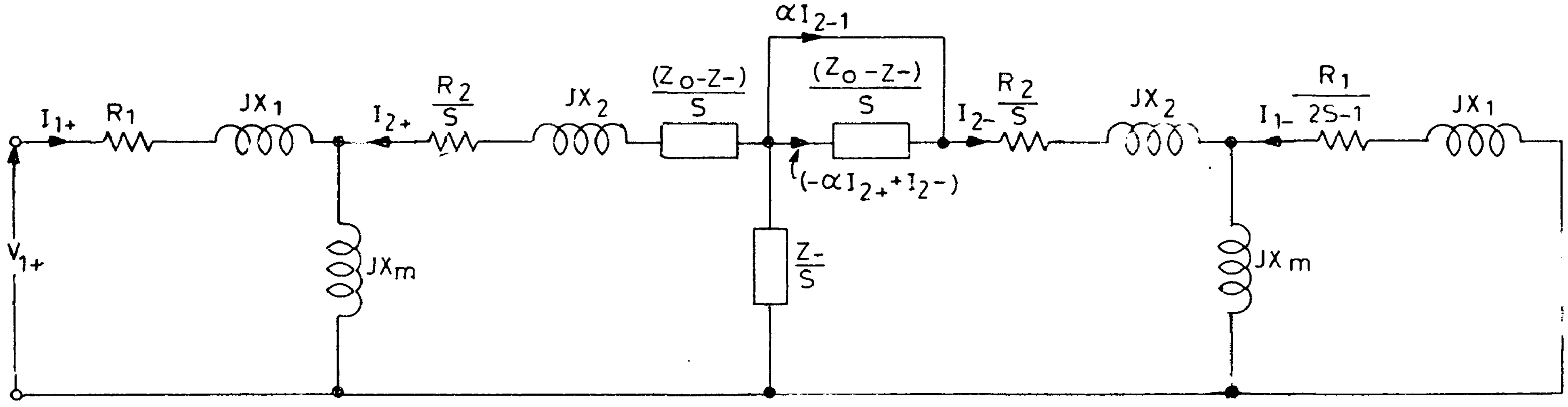


FIG. 2.7(c) COUPLING CIRCUIT.



$$\alpha = \frac{(Z_- - Z_+)}{(Z_o - Z_-)}$$

FIG.2.9 COMPLETE STEADY STATE EQUIVALENT CIRCUIT OF A SYMMETRICAL INDUCTION MOTOR.

$$0 = jX_m\, I_{1+} + \left[\frac{(Z_2+Z_0)}{s} + jX_m\right] \cdot I_{2+} + \frac{Z_-}{s}\, I_{2-} \qquad \text{Mesh II} \quad \ldots(2.19)$$

$$0 = \frac{Z_+}{s}\, I_{2+} + \left[\frac{(Z_2+Z_0)}{s} + jX_m\right] \cdot I_{2-} + jX_m\, I_{1-} \qquad \text{Mesh III} \quad \ldots(2.20)$$

$$0 = jX_m I_{2-} + \left[\frac{Z_1}{(2s-1)} + jX_m\right] \cdot I_{1-} \qquad \text{Mesh IV} \quad \ldots(2.21)$$

Writing the voltage equations for the four meshes in the matrix form,

$$[V] = [Z] \cdot [I] \qquad \ldots(2.22)$$

where $[V] = \begin{bmatrix} V_{1+} \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\quad I = \begin{bmatrix} I_{1+} \\ I_{2+} \\ I_{2-} \\ I_{1-} \end{bmatrix}$

and $[Z] = \begin{bmatrix} Z_1+jX_m & jX_m & 0 & 0 \\ jX_m & \frac{(Z_2+Z_0)}{s}+jX_m & \frac{Z_-}{s} & 0 \\ 0 & \frac{Z_+}{s} & \frac{(Z_2+Z_0)}{s}+jX_m & jX_m \\ 0 & 0 & jX_m & \frac{Z_1}{(2s-1)}+jX_m \end{bmatrix}$

The voltage equations given above are obtained by making reference to the rotor frame as asymmetry is created on the rotor side. This is necessary to obtain frequency-equivalence for the sequence components.

Substituting $p = js\omega$, the operational impedance matrix $Z(p)$ of the machine is obtained as given below-

$$Z(p) = \begin{bmatrix} R_1+(L_1+L_m)\cdot(p+j\omega r) & L_m(p+j\omega r) & 0 & 0 \\ L_m p & (R_2+R_o)+j(L_2+L_o+L_m)p & R_-+jL_-p & 0 \\ 0 & R_++jL_+p & (R_2+R_o)+j(L_2+L_o+L_m)p & L_m p \\ 0 & 0 & jL_m(p-j\omega r) & R_1+j(L_1+L_m)\cdot(p-j\omega r) \end{bmatrix} \quad ..(2.23$$

The torque expression is given by,

$$T = Re\left[I_t^* \omega GI\right]$$

where G is the coefficients of ω_r in $Z(p)$

$$G = \begin{bmatrix} j(X_1+X_m) & jX_m & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -jX_m & -j(X_1+X_m) \end{bmatrix} \quad ....(2.24)$$

The torque expression for the machine on simplification is given by,

$$T = Re\,(V_{1t}\cdot\dot{I}_{1+}) - R_1 I_{1+}\cdot\dot{I}_{1+} + \frac{R_1}{(2s-1)} I_{1-}\cdot\dot{I}_{1-} \quad ...(2.25)$$

CHAPTER III

STEADY-STATE EQUIVALENT CIRCUIT-GENERALISED MACHINE THEORY

CHAPTER III

STEADY-STATE EQUIVELENT CIRCUIT-GENERALIZED MACHINE THEORY

INTRODUCTION

The analysis of the performance of the induction motor under unbalanced conditions of operation forms an interesting application of the methods of generalized machine theory. In the following, steady-state equivalent circuit is derived for a polyphase induction motor with symmetrical stator winding and unbalance created by adding unequal impedances in the rotor circuit. The analysis employs the transformations from three phase to symmetrical components and then to rotor reference frame. The complete equivalent circuit so obtained is valid for any case of symmetrical or asymmetrical rotor unbalance and helps in writing the general equations determining the performance of the machine.

In the analysis, it is assumed that the three-phase supply voltage system is balanced and sinusoidal, the stator and rotor windings symmetrical. The air-gap of the machine is constant, saturation and iron losses are neglected. The external impedances of the rotor differ and are Z_a, Z_b and Z_c respectively.

3.1 MACHINE IMPEDANCE

The windings of the induction motor may be represented diagrammatically as shown in Fig. 3.1 and in the first instance it is assumed that both ends of each of the stator and rotor phase windings are accessible. These are six voltage equations and each of the six voltages depends upon all of the six currents. The impedance matrix therefore consists of 36 non-zero terms. To simplify the presentation of these equations, compound matrices will be used, the suffixes 1 and 2 being used for the stator and rotor windings respectively. The individual phase windings will be distinguished by capital letters ABC for the stator and by small letters a b c for the rotor. The clockwise direction of rotation will be taken as positive.

In compound matrix form, the voltage equation reads,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \ldots(3.1)$$

3.11 STATOR/STATOR IMPEDANCE Z_{11}

The stator resistance and inductance matrices are given by,

$$[R_1] = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_1 & 0 \\ 0 & 0 & R_1 \end{bmatrix} \qquad \ldots(3.2)$$

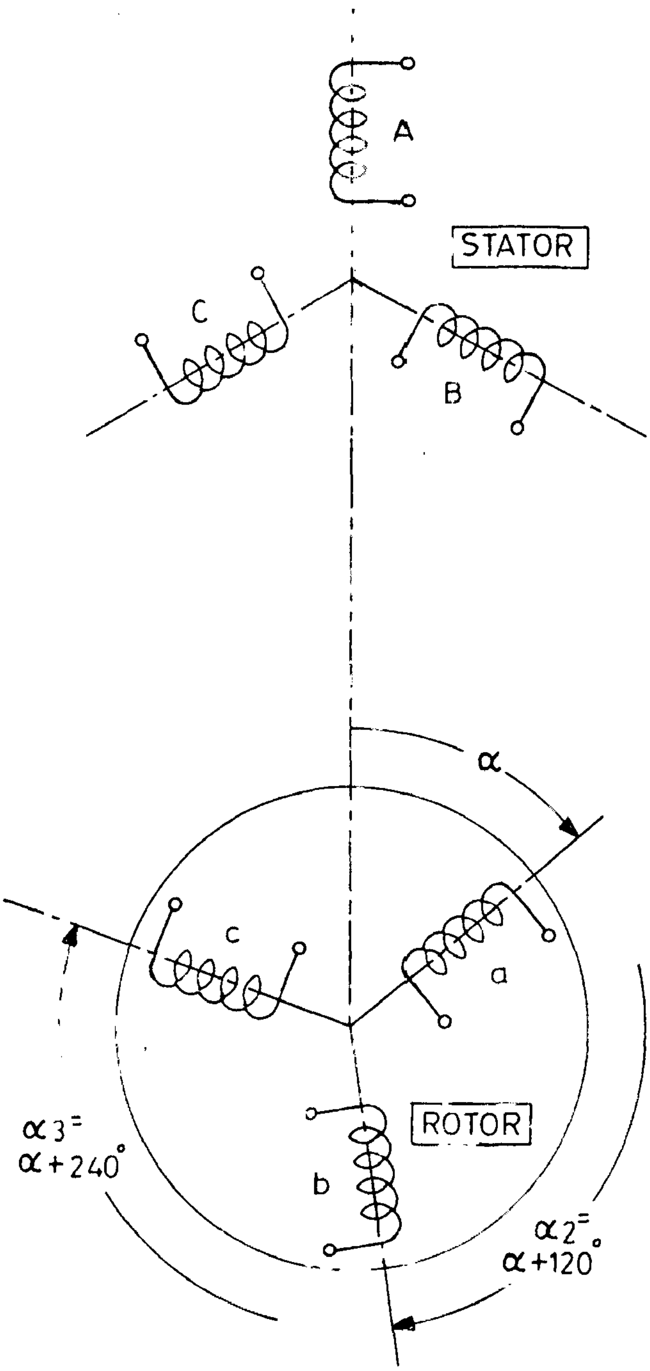


FIG. 3.1 DIAGRAMMATIC REPRESENTATION OF THE THREE PHASE SLIP-RING INDUCTION MOTOR.

$$[L_1] = \begin{bmatrix} L_{1\ell} & 0 & 0 \\ 0 & L_{1\ell} & 0 \\ 0 & 0 & L_{1\ell} \end{bmatrix} + L_{1s} \begin{bmatrix} 1 & \cos 120^\circ & \cos 240^\circ \\ \cos 120^\circ & 1 & \cos 120^\circ \\ \cos 240^\circ & \cos 120^\circ & 1 \end{bmatrix} \quad \ldots(3.3)$$

where R_1, L_1 and L_{1s} are the resistance, coefficients of leakage-and self-inductances of a phase of the stator winding respectively and L_1, the inductance of a phase.

The stator self-impedance matrix is given by,

$$Z_{11} = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} A & B & C \\ \left[\begin{matrix} R_1+(L_{1\ell}+L_{1s})p \\ -\frac{1}{2}L_{1s}p \\ -\frac{1}{2}L_{1s}p \end{matrix}\right. & \begin{matrix} -\frac{1}{2}L_{1s}p \\ R_1+(L_{1\ell}+L_{1s})p \\ -\frac{1}{2}L_{1s}p \end{matrix} & \left.\begin{matrix} -\frac{1}{2}L_{1s}p \\ -\frac{1}{2}L_{1s}p \\ R_1+(L_{1\ell}+L_{1s})p \end{matrix}\right] \end{matrix} \quad \ldots(3.4)$$

3.12 ROTOR/ROTOR IMPEDANCE Z_{22}

The rotor resistance and inductance matrices are given by,

$$[R_2] = \begin{bmatrix} R_2+R_a & 0 & 0 \\ 0 & R_2+R_b & 0 \\ 0 & 0 & R_2+R_c \end{bmatrix} \quad \ldots(3.5)$$

$$\text{and } [L_2] = \begin{bmatrix} L_{2\ell} & 0 & 0 \\ 0 & L_{2\ell} & 0 \\ 0 & 0 & L_{2\ell} \end{bmatrix} + L_{2sb} \begin{bmatrix} 1 & & \cos 240^\circ \\ \cos 120^\circ & 1 & \cos 120^\circ \\ \cos 240^\circ & \cos 120^\circ & 1 \end{bmatrix} \quad \ldots(3.6)$$

where R_2, L_2 and L_{2s} are the resistance, coefficients of linkage inductance-and self-inductance of a phase of the rotor winding respectively and L_2, the inductance of a phase; Z_a, Z_b and Z_c are the external impedances added in the a, b and c phases of the rotor winding given by,

$$\begin{aligned} Z_a &= R_a + L_a p \\ Z_b &= R_b + L_b p \\ \text{and} \quad Z_c &= R_c + L_c p \end{aligned} \qquad \ldots(2.7)$$

The rotor self-impedance matrix is given by,

$$Z_{22} = \begin{matrix} \\ a \\ b \\ c \end{matrix} \begin{matrix} a & b & c \\ \begin{bmatrix} (R_2+R_a)+(L_a+L_{2\ell}+L_{2s})p & -\frac{1}{2}L_{2s}p & -\frac{1}{2}L_{2s}p \\ -\frac{1}{2}L_{2s}p & (R_2+R_b)+(L_b+L_{2\ell}+L_{2s})p & -\frac{1}{2}L_{2s}p \\ -\frac{1}{2}L_{2s}p & -\frac{1}{2}L_{2s}p & (R_2+R_c)+(L_c+L_{2\ell}+L_{2s})p \end{bmatrix} \end{matrix} \qquad \ldots(3.8)$$

3.13 STATOR/ROTOR IMPEDANCE Z_{12}

The stator-to-rotor mutual-impedance-matrix is given by,

$$Z_{12} = M \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} a & b & c \\ \begin{bmatrix} p\cos\alpha & p\cos(\alpha+120^\circ) & p\cos(\alpha+240^\circ) \\ p\cos(\alpha+240^\circ) & p\cos\alpha & p\cos(\alpha+120^\circ) \\ p\cos(\alpha+120^\circ) & p\cos(\alpha+240^\circ) & p\cos\alpha \end{bmatrix} \end{matrix} \qquad \ldots(3.9)$$

3.14 ROTOR/STATOR IMPEDANCE Z_{21}

The impedance matrix Z_{21} is the impedance Z_{12} with its elements transposed, that is with rows and columns interchanged.

$$Z_{21} = Z_{12t} \qquad \ldots(3.10)$$

3.2 SYMMETRICAL COMPONENTS TRANSFORMATION

Before transformation by the Symmetrical Components Transformation C_1, it is convenient to derive the general form of the transformed impedance matrix. The complete transformation in compound form is,

$$C = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} C_1 & 0 \\ 0 & C_1 \end{bmatrix} \end{array} \qquad \ldots(3.11)$$

so that the transformed impedance matrix becomes,

$$Z' = C_t^* \, Z \, C = \begin{bmatrix} C_{1t}^* \, Z_{11} C_1 & C_{1t}^* \, Z_{12} C_1 \\ C_{1t}^* \, Z_{21} C_1 & C_{1t}^* \, Z_{22} C_1 \end{bmatrix} \qquad \ldots(3.12)$$

The Symmertical Components Transformation C_1 is given by,

$$C_1 = \frac{1}{\sqrt{3}} \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} 0 & + & - \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \end{array} \qquad \ldots(3.13)$$

3.21 STATOR/STATOR IMPEDANCE Z_{11}'

The transformed stator self-impedance matrix is given by,

$$Z_{11}' = C_{1t}^{*}\ Z_{11}C_1 = \frac{1}{\sqrt{3}} \begin{matrix} 0 \\ + \\ - \end{matrix}\overset{\begin{matrix} A & B & C \end{matrix}}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}} \circ$$

$$\cdot \begin{matrix} A \\ B \\ C \end{matrix}\overset{\begin{matrix} A & B & C \end{matrix}}{\begin{bmatrix} R_1+(L_{1\ell}+L_{1s})p & -\frac{1}{2}L_{1s}p & -\frac{1}{2}L_{1s}p \\ -\frac{1}{2}L_{1s}p & R_1+(L_{1\ell}+L_{1s})p & -\frac{1}{2}L_{1s}p \\ -\frac{1}{2}L_{1s}p & -\frac{1}{2}L_{1s}p & R+(L_{1\ell}+L_{1s})p \end{bmatrix}} \cdot \frac{1}{\sqrt{3}} \begin{matrix} A \\ B \\ C \end{matrix}\overset{\begin{matrix} 0 & + & - \end{matrix}}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}}$$

on simplification,

$$Z_{11}' = \begin{matrix} 0 \\ + \\ - \end{matrix}\overset{\begin{matrix} 0 & + & - \end{matrix}}{\begin{bmatrix} R_1+L_{1\ell}p & 0 & 0 \\ 0 & R_1+(L_{1\ell}+3/2L_{1s})p & 0 \\ 0 & 0 & R_1+(L_{1\ell}+3/2L_{1s})p \end{bmatrix}} \quad \ldots(3.14)$$

3.22 ROTOR/ROTOR IMPEDANCE Z_{22}'

The transformed rotor self-impedance matrix is given by,

$$Z_{22}' = C_{1t}^{*}\ Z_{22}C_1$$

$$= \frac{1}{\sqrt{3}} \begin{matrix} 0 \\ + \\ - \end{matrix}\overset{\begin{matrix} \alpha & b & c \end{matrix}}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}} \cdot \begin{matrix} a \\ b \\ c \end{matrix}\overset{\begin{matrix} a & b & c \end{matrix}}{\begin{bmatrix} (R_2+R_a)+(L_a+L_{2\ell}+L_{2s})p & -\frac{1}{2}L_{2s}p & -\frac{1}{2}L_{2s}p \\ -\frac{1}{2}L_{2s}p & (R_2+R_b)+(L_b+L_{2\ell}+L_{2s})p & -\frac{1}{2}L_{2s}p \\ -\frac{1}{2}L_{2s}p & -\frac{1}{2}L_{2s}p & (R_2+R_c)+(L_c+L_{2\ell}+L_{2s})p \end{bmatrix}},$$

$$\cdot \frac{1}{\sqrt{3}} \begin{matrix} & 0 & + & - \\ a & 1 & 1 & 1 \\ b & 1 & a^2 & a \\ c & 1 & a & a^2 \end{matrix}$$

$$Z'_{22} = \begin{matrix} & 0 & + & - \\ 0 & (R_2+R_0)+(L_{2\ell}+L_0)p & R_-+L_-p & R_++L_+p \\ + & R_++L_+p & (R_2+R_0)+(L_{2\ell}+L_0\frac{3}{2}L_{2s})p & R_-+L_-p \\ - & R_-+L_-p & R_++L_+p & (R_2+R_0)+(L_{2\ell}+L_0+\frac{3}{2}L_{2s})p \end{matrix}$$

...(3.15)

3.23 STATOR/ROTOR IMPEDANCE Z'_{12}

Substituting,

$$\cos\alpha = \tfrac{1}{2}\left[e^{j\alpha} + e^{-j\alpha}\right],$$

$$\cos(\alpha + 120^\circ) = \tfrac{1}{2}\left[e^{j(\alpha + \frac{2\pi}{3})} + e^{-j(\alpha + \frac{2\pi}{3})}\right] = \tfrac{1}{2}\,(ae^{j\alpha} + a^2e^{-j\alpha})$$

and $$\cos(\alpha + 240^\circ) = \tfrac{1}{2}\left[e^{j(\alpha + \frac{4\pi}{3})} + e^{-j(\alpha + \frac{4\pi}{3})}\right] = \tfrac{1}{2}\,(a^2e^{j\alpha} + ae^{-j\alpha})$$

in the stator-rotor mutual-impedance-matrix given by equation (3.9), the result is,

$$Z'_{12} = \frac{M}{2} \begin{matrix} & a & b & c \\ A & p(e^{j\alpha}+e^{-j\alpha}) & p(ae^{j\alpha}+a^2e^{-j\alpha}) & p(a^2e^{j\alpha}+ae^{-j\alpha}) \\ B & p(a^2e^{j\alpha}+ae^{-j\alpha}) & p(e^{j\alpha}+e^{-j\alpha}) & p(ae^{j\alpha}+a^2e^{-j\alpha}) \\ C & p(ae^{j\alpha}+a^2e^{-j\alpha}) & p(a^2e^{j\alpha}+ae^{-j\alpha}) & p(e^{j\alpha}+e^{-j\alpha}) \end{matrix}$$

..(3.1

The transformed stator-to-rotor mutual-impedance-matrix Z'_{12} is given by,

$$Z'_{12} = C_{1t}^{*} Z_{12} C_1$$

$$Z'_{12} = \frac{1}{\sqrt{3}} \begin{array}{c} \\ 0 \\ + \\ - \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \end{array} \cdot$$

$$\cdot \frac{M}{2} \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} p(e^{j\alpha} + e^{-j\alpha}) & p(ae^{j\alpha} + a^2 e^{-j\alpha}) & p(a^2 e^{j\alpha} + ae^{-j\alpha}) \\ p(a^2 e^{j\alpha} + ae^{-j\alpha}) & p(e^{j\alpha} + e^{-j\alpha}) & p(ae^{j\alpha} + a^2 e^{-j\alpha}) \\ p(ae^{j\alpha} + a^2 e^{-j\alpha}) & p(a^2 e^{j} + ae^{-j}) & p(e^{j\alpha} + e^{-j\alpha}) \end{bmatrix} \end{array} \cdot$$

$$\cdot \frac{1}{\sqrt{3}} \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} 0 & + & - \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ a & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \end{array}$$

$$Z'_{12} = \frac{3M}{2} \begin{array}{c} \\ 0 \\ + \\ - \end{array} \begin{array}{c} \begin{array}{ccc} 0 & + & - \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & pe^{j\alpha} & 0 \\ 0 & 0 & pe^{-j\alpha} \end{bmatrix} \end{array} \qquad \ldots(3.17)$$

3.24 ROTOR/STATOR IMPEDANCE Z'_{21}

Using the relation,

$$Z'_{21} = Z^{\circ}_{12t}$$

the transformed rotor-stator mutual-impedance-matrix is given by,

$$Z'_{21} = \frac{3M}{2}\begin{array}{c} \\ 0 \\ + \\ - \end{array}\begin{array}{c} \begin{array}{ccc} 0 & + & - \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & pe^{-j\alpha} & 0 \\ 0 & 0 & pe^{j\alpha} \end{bmatrix} \end{array} \quad \ldots(3.18)$$

3.25 COMPLETE TRANSFORMED IMPEDANCE MATRIX Z'

It is now possible to combine the four component transformed impedance matrices together to form the complete transformed matrix. The result is,

$$Z' = \begin{array}{c|cccccc} & 0 & + & - & 0 & + & - \\ \hline 0 & R_1+L_{1\ell}p & 0 & 0 & 0 & 0 & 0 \\ + & 0 & R_1+(L_{1\ell}+\frac{3}{2}L_{1s})p & 0 & 0 & \frac{3M}{2}pe^{j\alpha} & 0 \\ - & 0 & 0 & R_1+(L_{1\ell}+\frac{3}{2}L_{1s})p & 0 & 0 & \frac{3M}{2}pe^{-j\alpha} \\ 0 & 0 & 0 & 0 & (R_2+R_0)+(L_{2\ell}+L_0)p & R_-+L_-p & R_++L_+p \\ + & 0 & \frac{3M}{2}pe^{-j\alpha} & 0 & R_++L_+p & (R_2+R_0)+(L_{2\ell}+L_0+\frac{3}{2}L_{2s})p & R_-+L_-p \\ - & 0 & 0 & \frac{3M}{2}pe^{j\alpha} & R_-+L_-p & R_++L_+p & (R_2+R_0)+(L_{2\ell}+L_0+\frac{3}{2}L_{2s})p \end{array}$$

. . . (3.19)

3.3 REFERENCE FRAME TRANSFORMATION C_2

Before the voltage equations for the asymmetrical machine may be written, it is important to observe that if symmetrical components are used decomposition of them can be carried out in the usual way only in a reference frame fixed to the asymmetrical side. The reason for this is that the frequencies of the positive-and negative-sequence components will be equal only in this reference frame.

For ensuring frequency-equivalence for the positive- and negative-sequence components, these components will be obtained in the rotor reference frame as unbalance is created on the rotor side.

In compound matrix form, the voltage equation is given by,

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} \cdot \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} \qquad \ldots(3.20)$$

The stator will be transformed by the Reference Frame Transformation C_2, the rotor will be left unchanged. The complete transformation in compound form is

$$C = \begin{array}{c} \\ 1 \\ 2 \end{array}\overset{\begin{array}{cc} 1 & \quad 2 \end{array}}{\begin{bmatrix} C_2 & 0 \\ 0 & 1 \end{bmatrix}} \qquad \ldots(3.21)$$

so that the transformed impedance matrix becomes

$$Z'' = C_t^* Z' C = \begin{bmatrix} C_{2t}^* Z_{11}' C_2 & C_{2t}^* Z_{12}' \\ Z_{21}' C_2 & Z_{22}' \end{bmatrix} \qquad ...(3.22)$$

The Reference Frame Transformation C_2 is given by,

$$C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{j\alpha} & 0 \\ 0 & 0 & e^{-j\alpha} \end{bmatrix} \qquad ...(3.23)$$

and

$$C_{2t}^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j\alpha} & 0 \\ 0 & 0 & e^{j\alpha} \end{bmatrix} \qquad ...(3.24)$$

3.31 STATOR/STATOR IMPEDANCE Z_{11}''

Considering each element of the transformed impedance matrix in turn, the stator/stator element is given by,

$$Z_{11}'' = C_{2t}^* Z_{11}' C_2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j\alpha} & 0 \\ 0 & 0 & e^{j\alpha} \end{bmatrix} \cdot \begin{bmatrix} R_1 + L_{1\ell} p & 0 & 0 \\ 0 & R_1 + (L_{1\ell} + \frac{3}{2} L_{1s}) p & 0 \\ 0 & 0 & R_1 + (L_{1\ell} + \frac{3}{2} L_{1s}) p \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{j\alpha} & 0 \\ 0 & 0 & e^{-j\alpha} \end{bmatrix}$$

$$Z_{11}'' = \begin{bmatrix} R_1+L_{1\ell}p & 0 & 0 \\ 0 & R_1+e^{-j\alpha}(L_{1\ell}+\frac{3}{2}L_{1s})pe^{j\alpha} & 0 \\ 0 & 0 & R_1+e^{j\alpha}(L_{1\ell}+\frac{3}{2}L_{1s})pe^{-j\alpha} \end{bmatrix} \qquad ...(3.25)$$

The differential operator p in Z_{11}'' operates in addition upon the current terms, since Z_{11}'' is only part of the original expression Z 2.

$$\therefore \quad pe^{j\alpha}\cdot i = j\, e^{j\alpha}\cdot p\alpha\cdot i + e^{j\alpha}\cdot pi$$

$$= e^{j\alpha}\, j\,\omega_r i + e^{j\alpha}\cdot pi$$

$$\text{since} \quad p\alpha = \omega_r$$

$$\therefore \quad pe^{j\alpha} = e^{j\alpha}(p+j\omega_r)$$

similarly $pe^{-j} = e^{-j\alpha}(p-j\omega_r)$.

Substituting these, the complete matrix is given by

$$Z_{11}'' = \begin{bmatrix} R_1+L_{1\ell}p & 0 & \\ 0 & R_1+(L_{1\ell}+3/2L_{1s})(p+j\omega_r) & 0 \\ 0 & 0 & R_1+(L_{1\ell}+3/2L_{1s})(p-j\omega_r) \end{bmatrix} \qquad ...(3.26)$$

3.32 STATOR/ROTOR IMPEDANCE Z_{12}''

The stator-to-rotor element Z_{12}'' of the impedance matrix is given by,

$$Z_{12}'' = C_{2t}^{*}\ Z_{12}' = \frac{3M}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j\alpha} & 0 \\ 0 & 0 & e^{j\alpha} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & pe^{j\alpha} & 0 \\ 0 & 0 & pe^{-j\alpha} \end{bmatrix}$$

$$= \frac{3M}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & e^{-j\alpha}\ pe^{j\alpha} & 0 \\ 0 & 0 & e^{j\alpha}\ pe^{-j\alpha} \end{bmatrix} = \frac{3M}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & (p+j\omega_r) & 0 \\ 0 & 0 & (p-j\omega_r) \end{bmatrix}$$

...(3.27)

3.33 ROTOR/STATOR IMPEDANCE Z_{21}''

The rotor-to-stator element of the impedance matrix is given by,

$$Z_{21}'' = Z_{21}' C_2$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{3M}{2}pe^{-j\alpha} & 0 \\ 0 & 0 & \frac{3M}{2}pe^{j\alpha} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{j\alpha} & 0 \\ 0 & 0 & e^{-j\alpha} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{3M}{2}p & 0 \\ 0 & 0 & \frac{3M}{2}p \end{bmatrix}$$

...(3.28)

3.34 ROTOR/ROTOR IMPEDANCE Z''_{22}

As the rotor/rotor element is not transformed, it remains unchanged.

3.35 COMPLETE TRANSFORMED IMPEDANCE MATRIX Z''

It is now possible to combine the four component transformed impedance matrices together to form the complete transformed matrix. The result is,

$$
Z'' = \begin{array}{c} \\ s_o \\ s_+ \\ s_- \\ r_o \\ r_+ \\ r_- \end{array}
\begin{array}{cccccc}
s_o & s_+ & s_- & r_o & r_+ & r_- \\
R_1+L_{1\ell}p & 0 & 0 & 0 & 0 & 0 \\
0 & R_1+(L_{1\ell}+\frac{3}{2}L_{1s})(p+j\omega_r) & 0 & 0 & \frac{3M}{2}(p+j\omega_r) & 0 \\
0 & 0 & R_1+(L_{1\ell}+\frac{3}{2}L_{1s})(p-j\omega_r) & 0 & 0 & \frac{3M}{2}(p-j\omega_r) \\
0 & 0 & 0 & (R_2+R_o)+(L_{2\ell}+L_o)p & R_-+L_-p & R_++L_+p \\
0 & \frac{3M}{2} & 0 & R_++L_+p & (R_2+R_o)+(L_{2\ell}+L_o+\frac{3}{2}L_{2s})p & R_-+L_-p \\
0 & 0 & \frac{3M}{2}p & R_-+L_-p & R_++L_+p & (R_2+R_o)+(L_{2\ell}+L_o+\frac{3}{2}L_{2s})p
\end{array}
\qquad \ldots(3.29)
$$

When the fourth row and column are moved up into second position, a compound matrix of the form results,

$$Z'' = \begin{bmatrix} Z_0 & \\ & Z \end{bmatrix}$$

which possesses the important property of having terms on the leading diagonal only. It follows that the two systems represented by two voltage equations

$$V_0 = Z_0 I_0$$

have no connection or reaction between them, and may therefore be treated quite independently. Further, zero-sequence current in the rotor will be absent as there is no external (slip-ring) connection to the star-point. The stator zero-sequence current will also be absent as there is no three-phase four-wire supply It follows that for normal conditions of operation, zero-sequence currents cannot flow. The zero-sequence voltage equations may therefore be neglected.

The impedance matrix given by equation (3.29) transforms to

$$
Z'' = \begin{array}{c} \\ s_+ \\ \\ s_- \\ \\ r_+ \\ \\ r_- \end{array}
\begin{array}{c}
\begin{array}{cccc} \qquad s_+ \qquad & \qquad s_- \qquad & \qquad r_+ \qquad & \qquad r_- \qquad \end{array} \\
\begin{bmatrix}
R_1+(L_{1\ell}+\frac{3}{2}L_{1s})(p+j\omega_r) & 0 & \frac{3M}{2}(p-j\omega_r) & 0 \\
0 & R_1+(L_{1\ell}+\frac{3}{2}L_{1s})(p-j\omega_r) & 0 & \frac{3M}{2}(p-j\omega_r) \\
\frac{3M}{2}p & 0 & (R_2+R_0)+(L_{2\ell}+L_0+\frac{3}{2}L_{2s})p & R_-+L_-p \\
0 & \frac{3M}{2}p & R_++L_+p & (R_2+R_0)+(L_{2\ell}+L_0+\frac{3}{2}L_{2s})p
\end{bmatrix}
\end{array}
\qquad \ldots(3.30)
$$

Rearranging equation (3.30),

$$
Z'' = \begin{array}{c} \\ s_+ \\ \\ r_+ \\ \\ r_- \\ \\ s_- \end{array}
\begin{array}{c}
\begin{array}{cccc} \qquad s_+ \qquad & \qquad r_+ \qquad & \qquad r_- \qquad & \qquad s_- \qquad \end{array} \\
\begin{bmatrix}
R_1+(L_{1\ell}+\frac{3}{2}L_{1s})(p+j\omega_r) & \frac{3M}{2}(p+j\omega_r) & 0 & 0 \\
\frac{3M}{2}p & (R_2+R_0)+(L_{2\ell}+L_0+\frac{3}{2}L_{2s})p & R_-+L_-p & 0 \\
0 & R_++L_+p & (R_2+R_0)+(L_{2\ell}+L_0+\frac{3}{2}L_{2s})p & \frac{3M}{2}p \\
0 & 0 & \frac{3M}{2}(p-j\omega_r) & R_1+(L_{1\ell}+\frac{3}{2}L_{1s})(p-j\omega_r)
\end{bmatrix}
\end{array}
\qquad \ldots(3.31)
$$

Since the frequency reference has been made to rotor side,

$$p = js\omega$$

$$\therefore (p - j\omega_r) = j(2s-1)\omega$$

$$\text{and } (p + j\omega_r) = j\omega$$

Substituting these in equation (3.31),

$$Z'' = \begin{array}{c} \\ s_+ \\ r_+ \\ r_- \\ s_- \end{array} \begin{array}{cccc} s_+ & r_+ & r_- & r_+ \\ \left[\begin{array}{c} R_1 + j\omega(L_{1\ell} + \frac{3}{2}L_{1s}) \\ \frac{3}{2} js\omega M \\ 0 \\ 0 \end{array} \right. & \begin{array}{c} \frac{3}{2} j\omega M \\ (R_2 + R_o) + j\omega s (L_{2\ell} + L_o + \frac{3}{2}L_{2s}) \\ R_+ + js\omega L_+ \\ 0 \end{array} & \begin{array}{c} 0 \\ R_- + j\omega s L_- \\ (R_2 + R_o) + js\omega (L_{2\ell} + L_o + \frac{3}{2}L_{2s}) \\ \frac{3}{2} j(2s-1)\omega M \end{array} & \left. \begin{array}{c} 0 \\ 0 \\ \frac{3}{2} js\omega M \\ R_1 + j(2s-1)\omega (L_{1\ell} + \frac{3}{2}L_{1s}) \end{array} \right] \end{array}$$

...(3.32)

Substituting further,

$$\omega(L_{1\ell} + \tfrac{3}{2}L_{1s}) = X_1 + X_m$$

$$\omega(L_{2\ell} + \tfrac{3}{2}L_{2s}) = X_2 + X_m$$

and $\frac{3}{2}\omega M = X_m$,

the impedance matrix given by equation (3.32) becomes

$$Z'' = \begin{array}{c|cccc} & s_+ & r_+ & r_- & s_- \\ \hline s_+ & R_1+j(X_1+X_m) & jX_m & 0 & 0 \\ r_+ & jsX_m & (R_2+R_o)+js(X_2+X_o+X_m) & R_-+jsX_- & 0 \\ r_- & 0 & R_++jsX_+ & (R_2+R_o)+js(X_2+X_o+X_m) & jsX_m \\ s_- & 0 & 0 & j(2s-1)X_m & R_1+j(2s-1)(X_1+X_m) \end{array}$$

or

$$Z'' = \begin{array}{c|cccc} & s_+ & r_+ & r_- & s_- \\ \hline s_+ & Z_1+jX_m & jX_m & 0 & 0 \\ r_+ & jsX_m & (Z_2+Z_o)+jsX_m & Z_- & 0 \\ r_- & 0 & Z_+ & (Z_2+Z_o)+jsX_m & jsX_m \\ s_- & 0 & 0 & j(2s-1)X_m & Z_1+j(2s-1)X_m \end{array} \qquad \ldots.(3.33)$$

.4 COMPLETE EQUIVALENT CIRCUIT

From the impedance matrix given by equation (3.33), the voltage equations may be written,

$$V_{1+} = (Z_1+jX_m)I_{1+} + jX_m\,I_{2+} \qquad \ldots(3.34)$$

$$0 = jX_m I_{1+} + \left[\frac{Z_2 + Z_0}{s} + jX_m\right] I_{2+} + \frac{Z_-}{s} I_{2-} \qquad \ldots(3.35)$$

$$0 = \frac{Z_+}{s} I_{2+} + \left[\frac{Z_2 + Z_0}{s} + jX_m\right] I_{2-} + jX_m I_{1-} \qquad \ldots(3.36)$$

$$0 = jX_m I_{2-} + \left[\frac{Z_1}{(2s-1)} + jX_m\right] I_{1-} \qquad \ldots(3.37)$$

These voltage equations are the same as given by equations (2.18),(2.19), (2.20) and (2.21) and may be interpreted to represent the complete equivalent circuit already obtained in chapter I and II shown in Fig. 2.9.

CHAPTER IV

CONCLUSIONS AND SCOPE FOR FURTHER WORK

CHAPTER IV

CONCLUSIONS AND SCOPE FOR FURTHER WORK

C_O_N_C_L_U_S_I_O_N_S

The method of symmetrical components has been found very effective in deriving a steady-state equivalent circuit for the general case of asymmetrical rotor unbalance. Using the circuit computerised calculations can be made to obtain operating characteristics of the motor under various rotor unbalance conditions.

The equivalent circuits have been developed using three different methods based on Symmetrical Components Concept, Generalised Rotating Field Theory and Generalised Machine Theory. The equivalent circuits so obtained have been found compatible.

It has been observed that the equivalent circuits derived in the previous works pertaining to cases of symmetrical unbalance form the particular cases and are easily deduced from the general equivalent circuit developed in the present work.

All cases of rotor unbalance are seen to belong to two types-symmetrical and asymmetrical. In case of symmetrical unbalance, external impedance is symmetrical about one phase and the equivalent circuit so derived is quite simple and possesses symmetry about the air-gap. In case of asymmetrical unbalance, the external impedance is not symmetrical about any phase. If possesses no symmetry about the air-gap and the equivalent circuit is complex.

The equivalent circuit developed in the present work consists of controlled generators. However, in cases of symmetrical unbalance these controlled generators are absent which greatly simplifies the circuit.

While developing the general equivalent circuit it has been observed that the decomposition into symmetrical components can be carried out in the usual way only when the reference frame is fixed to the asymmetrical side. This is required to ensure frequency-equivalence of the positive- and negative-sequence components.

Using the general equivalent circuit so developed, fast and economical computerised calculations can be carried out in all asymmetrical cases as only one network is to be solved by using routine network-solving procedures available at most computer systems.

The derived equivalent circuit can be meaningfully employed for predicting the transient behaviour of the machine at constant speed.

The general equivalent circuit can be conveniently used to compute the reactance values to be added in the rotor circuit to obtain pre-determined torque-speed characteristic.

SCOPE FOR FURTHER WORK

The present work unfolds further scope as outlined below:

FREQUENCY DEPENDANCE OF WINDING RESISTANCE

The winding resistance does not remain constant but varies with frequency. The relationship has been found to be linear and can be easily obtained from blocked rotor tests over a frequency range. This may be accounted to compute the performance characteristics accurately.

REACTANCE DEPENDANCE ON MAGNETIC SATURATION

The reactance of the machine does not remain constant but varies with magnetic saturation. It has a higher value at low saturation and decreases considerably with increased saturation in the machine. It may be accounted to predict the characteristics accurately.

TRANSIENT OPERATION AT CONSTANT SPEED

The derived equivalent circuit can be meaningfully employed for the study of constant-speed transients. For carrying out this study, operational impedance of the coupling, positive-and negative-sequence circuits must be used.

TRANSIENT OPERATION AT VARYING SPEED

The differential equations governing the transient behaviour of the asymmetrical machine may be written in state-variable form by using Park-vector technique. The derived equations can be directly solved by a digital computer.

EQUIVALENT CIRCUIT OF ASYMMETRICAL SQUIRREL CAGE INDUCTION MOTOR

The method outlined in the present work for the development of equivalent circuit may be extended in case of asymmetrical squirrel cage induction motor. Such asymmetries may be present due to die-casting difficulties, impurities in the casting material or due to broken rotor bars giving unequal rotor impedances.

R_E_F_E_R_E_N_C_E_S

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