MODAL CONTROL OF LONG TRANSMISSION LINES

A DISSERTATION

Submitted in partial fulfilment
of the requirements for the award of the Degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(Power System Engineering)

By
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CERTIFICATE

Certified that the dissertation entitled ''MODAL CONTROL OF LONG TRANSMISSION LINES'', which is being submitted by Shri CHILUKURI TIRUPATI RAYUDU, in partial fulfilment of the requirements for the award of the degree of MASTER OF ENGINEERING in ELECTRICAL ENGINEERING (POWER SYSTEM ENGINEERING) at the University of Roorkee, Roorkee, is a record of candidate's own work carried out by him under the supervision and guidance of the undersigned. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further certified that he has worked for a period of ...7........... months for preparing this dissertation at this University.

ROORKEE

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ROORKEE. (CH. T. RAYUDU)

ABSTRACT

The modal control technique has been applied to the system of equations to calculate the responses at the intermediate points of a long transmission line when the line is divided into number of equal sections. Those responses have been compared with the responses obtained from the RC transmission line equation of diffusion type. It has been assumed that a transmission line is a leakage free non-inductive cable.

The modal control theory, long transmission line, representation of a transmission line in nominal T and π and a loss free line have been defined in introduction chapter.

The review of the literature has been given in chapter two, explaining all the necessary existing tools used in the present problem. They are formation of state equation, feedback compensation technique, network synthesis (which is enough to design an RC transmission line from the driving point transfer impedance function), an analogy to an RC transmission line from heat conduction through a slab and why at all modelling of a system is required.

The mathematical methods used in the present problem are given in the third chapter. It comprises with the solutions of RC line equation $\frac{\partial X}{\partial t} = \alpha^2 \frac{\partial^2 X}{\partial x^2}$, a state equation $\dot{X} = AX + Bu$ and state equation with feedback compensation $\dot{X} = MX + Br$ where $M=(A - BK^T)$.

The transmission line, designed from the driving point transfer impedance function (which has been calculated in the third chapter) has been represented in chapter four.

The results obtained for different number of orders such as 4, 5 and 6 for the state equation as well as for the RC line equation ~~equation~~ are shown graphically and discussed in chapter five. These graphs are compared to each other and it has been concluded that the state space technique with feedback compensation is the most advantageous one.

C O N T E N T S

CHAPTER - 1

INTRODUCTION

The modal control theory is applied to long transmission lines for getting the responses at the intermediate points over the length of the line. In a general sense the theory of modal control may be regarded as being part of linear systems theory as seen from the state space point of view[23]. This state space approach on modern control theory is a new approach to the analysis and design of complex control systems. It has its own advantages over the conventional control theory, applicable to multi-input and multi output systems which may be linear or non-linear, time invariant or time varying, essentially in a time-domain approach. A linear dynamic system equation is written in state space form as $\dot{X} = AX + Bu(t)$ where X is a state vector, u is control input and A and B are matrices with appropriate dimensions. In order to gain insight into the dynamic behavior of a system, it is helpful to make a coordinate transformation in to a new state vector Z through T as $X = TZ$ so that the coefficient matrix of the state vector will be in a canonical form exhibiting directly its eigen values on the main diogonal. T should be a non singular constant square matrix and the matrix T is called the modal matrix when it is selected so that $T^{-1}AT$ is diogonal[24]. The canonical form used here is called Jordan canonical form. The modal control theory is applicable both for lumped parameter systems and distributed parameter systems.

The eigen values of the coefficient matrix 'A' of the state equation $\dot{X} = AX + Bu$ are the roots of the characteristic equation $|\lambda I - A| = 0$. A linear state variable feedback method can be applied to the control system to change the response characteristics.

Thus the modal control approach is employed to improve the response of the transmission line by shifting the eigen values of the uncontrolled system in to the appropriate locations. The ratio of the system output to input in laplace transformation of this state equation is called transfer function. Then the parameters of the transmission line are designed from the transfer function by applying the network synthesys approach.

The term transmission line refers to a system of conductors used to transmit electric power from a source to a load[1]. These lines are basically divided into two types, short and long lines. by comparing with the wave length (λ) of the waves to be propagated over them. An electrically long line has a length of the order of the wave length on a reasonable fraction of it (at least $\frac{\lambda}{10}$). Thus a line of four cm. is electrically long for a wave of 3000 M Hz current (10 cm wave) and is too short for a 50 c/s current wave (6×10^6 m wave). A line to be long at 50 Hz must have its actual length of about 100 Kmr or more.

In order to take account[2] of the fact that the current and voltage vary along a line, it is assumed that each elementry line section, however short, has a resistance, an inductance, a shur capacitance and a leakage conductance between the wires, then this line is regarded as a network with distributed parameters as shown in Fig.(1.1).

For voltages up to say 20 KV, that the line capacitance can be ignored, but as the voltage and length of the line increases, capacitance becomes of gradually increasing importance. Voltages up to 100 KV satisfactory solutions can be obtained from T and π representation of the transmission lines. The capacitance is distributed over the entire length of the line but for a simple treatment its effect can be approximately taken into account by assuming that it is lumped in the forms of condensers shunted accross the line at one or more points.

The two common methods are nominal T and π. In the nominal T method[1] the whole of the line capacity is assumed to be concentrated at the middle point of the line, and half the line resistance and reactance to be lumped on either side as shown in Fig.(1.2.1). In the π method the capacitance is split into two halves, which are situated at either end of the line as in Fig.(1.2.2).

Referring to nominal T

$$E_{b1} = E_{b2} + I_r \left(\frac{Z}{2}\right) \text{ where } Z = R + JX$$

$$I_c = Y E_{b1} = Y(E_{b2} + I_r Z/2)$$

$$I_s = I_r + I_c = I_r \left(1 + \frac{YZ}{2}\right) + YE_{b2}$$

$$E_a = E_{b1} + I_s Z/2 = E_{b2} \left(1 + \frac{YZ}{2}\right) + I_r\left(Z + \frac{YZ^2}{4}\right)$$

in nominal π $$I_{c2} = \frac{Y}{2} E_{b1}$$

$$I = I_r + I_{c2} = I_r + \frac{Y}{2} E_{b1}$$

$$E_a = E_{b1} + IZ = E_{b1}\left(1 + \frac{YZ}{2}\right) + I_r Z$$

$$I_{c1} = \frac{Y}{2} E_a = \frac{Y}{2} \left[E_{b1}\left(1 + \frac{YZ}{2}\right) + I_r Z\right]$$

$$I_s = I + I_{c1} = E_{b1} Y \left[1 + \frac{YZ}{4}\right] + I_r \left[1 + \frac{YZ}{2}\right]$$

If these nominal T and nominal π methods are adopted to long lines there will be considerable error. The line impedance and admittance are therefore, considered uniformly distributed over the line length. As shown in Fig.(1.3) a small length 'dx' is considered on the transmission line then by applying Rigorous solution method the voltage and current equations are written as

$$E = E_{b1} \cosh r x + I_R Z_c \sinh r x$$

$$I = I_R \cosh r x + \frac{E_{b1}}{Z_c} \sinh r x$$

By substituting l for x we get the equations for voltage and current at sending end as

$$E_s = E_{b1} \cosh rl + I_R Z_c \sinh r l$$
$$I_s = I_R \cosh rl + \frac{E_{b1}}{Z_c} \sinh r l$$

where r is $\sqrt{Z\,Y}$ (propagation constant)

Z_c is $\sqrt{Z/Y}$ (characteristic impedance).

An idealization[2] of a practical line is nothing but a dissipationless line or loss free line. By making the series resistance and shunt conductance are equal to zero, we could obtain a dissipationless line in which there is no energy loss. At radio frequencies, short lines have the property such that R and G are negligible in comparison with X_L and X_C. Therefore the two-wire and co-axial transmission lines used in radio Engineering are often treated as loss free lines with sufficient accuracy. The propagation constant and characteristic impedance for a loss less line are as follows.

$$r = w\sqrt{L\,C}$$

$$Z_c = \sqrt{L/C}$$

However the loss may be negligible in transmission lines but it can not be eliminated completely.

The response calculations of long transmission lines at intermediate points is done by two different methods and then compared. One of those two methods is the representation of transmission line by one dimensional transmission line equation of diffusion type and the other one is the approximate representation of state equation.

The basic parameters of a transmission line are resistance (R), capacitance (C), inductance (L) and conductance (G). The transmission line considered here for modelling is leakage free noninductive cable where inductance (L) and conductance (G) are assumed to be zero. The transmission line of this type is represented mathematically by one-dimensional RC transmission line equation in the form

$$\frac{\partial X}{\partial t}(x,t) = \alpha^2 \frac{\partial^2 X}{\partial x^2}(x,t) \qquad (1.1)$$

where $\alpha^2 = \frac{1}{R\,C}$

By taking into account the initial, final as well as boundary conditions the responses of the equation (1.1) has been calculated for a step input u(t) at different equal distances over the length of transmission line. These responses are compared with the responses obtained from an approximate representation of transmission line state equation of the form

$$\dot{X} = A X + B u(t) \qquad (1.2)$$
$$Y = C^T X$$

for the same step input u(t) at the same points over the transmission line.

The modelling of transmission line has done by applying the modal control technique through state variable feedback. The effect of state variable feedback on systems of the form given in (1.2) is to replace u(t) by $(r - K^T X)$ where K's are feedback gain coefficients. The closed loop equations of the system then be written as

$$\dot{X} = (A - B K^T) X + B r$$
$$\dot{X} = M X + B r \qquad (1.3)$$
$$Y = C^T X$$
$$\text{Say } M = A - B K^T$$

By the application of this feedback technique the poles are shifted to the desired locations. The responses calculated from the closed loop system for the same step input r(t) at the same states. Those responses are compared both with open loop state equation and RC transmission line equation (1.2) and (1.1) respectively.

The modal control approach is employed to improve the response of the transmission line by shifting the eigen values of the uncontrolled system in to the desired locations.

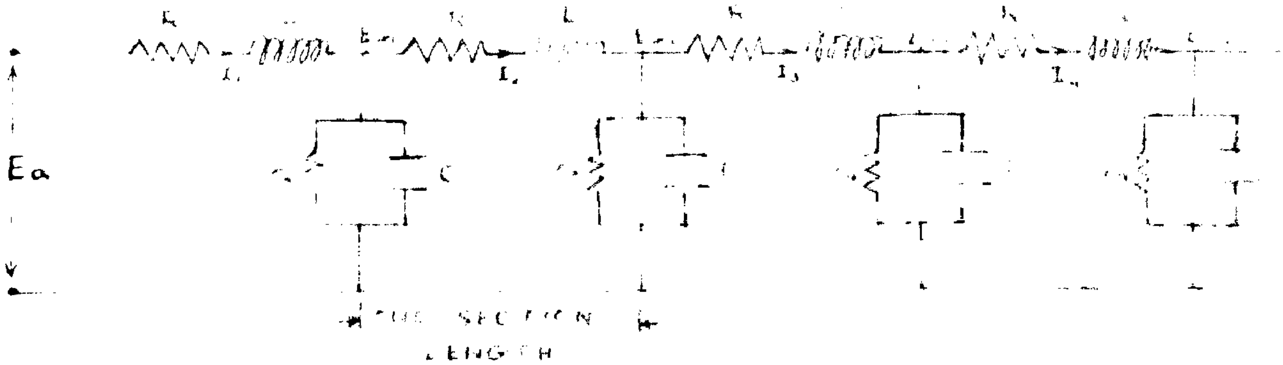
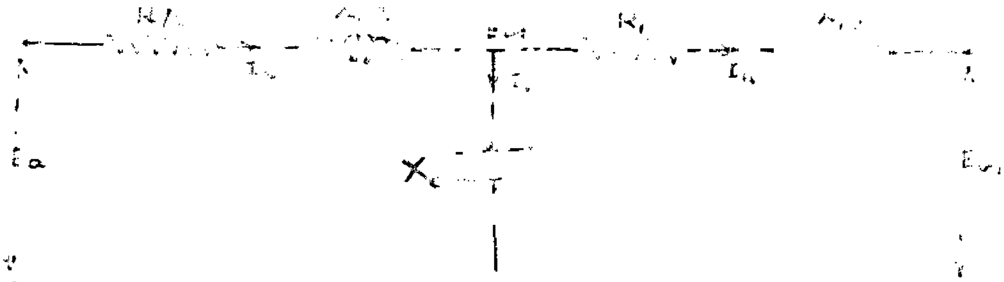


FIG.



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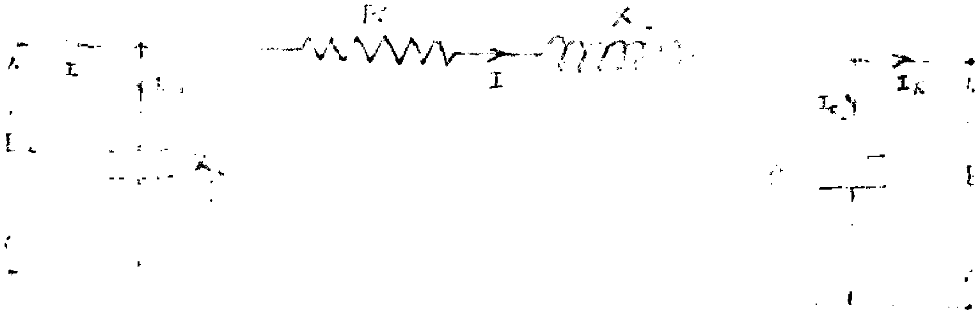

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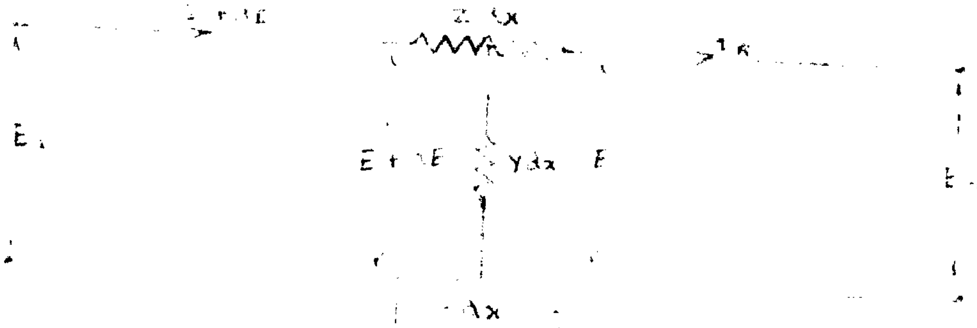


FIG.

CHAPTER - 2

REVIEW OF THE EXISTING LITERATURE

2.1 Necessity of Modelling the System[3]

Modelling of any physical system is a necessary preliminary to make it perfect before bringing the system in to practical use. The modelling of any complex system facilitâtes to prepare all corresponding data in the laboratory itself.

In electrical systems simulation began with the creation of calculating boards or network analysers later those have gradually been perfected and by now are semi-automatic and automatic. These modelling principles which partly relate to physical modelling, but their main task is to serve as mathematical models. These models will provide the solutions to the equations formed by describing the steady or transient states of electrical systems. These dynamic or physical models are found necessary in their development, not only as the means for solving equations but also as an experimental base which replaces such natural investigations in actual systems. Those which can check theoretical propositions by the criterion of practice or in conditions closely similar to actual can check, adjust and work out any forms of apparatus.

In the process of experiments on models, the investigations carried out on similar systems do not in principle ensure a rigorous one to one relation between results even when total similarity is realized. This is conditioned by the fact that absolute identity of concrete phenomena represented in different space-time domains is a mathematical abstraction and in actual problems it is absent. The differential equation which describes the law governing the flow of a multiplicity of associated phenomena is a mathematical model of some averaged phenomenon, even within the limits of one and same modelling structure. Its concrete realization differ one to another owing to random variations in the physical reproduction of the coefficients of the equation. In many cases the process being investigated depends on the pre-history of the process at the moment in time taken as the initial instant.

Another aspect of the impossibility of producing an exact model is linked with the fact that actual accuracy is determined by the depth of cognition of the original. The errors in the determination of the parameters are depend on the initial simplifying assumptions, observational errors and so on. The errors in production of the model can have a real influence on the results of modelling. Further more, the existance of definite difference between the model and the original is an indispensable condition for the realizability of those functions which are imposed on the model. The problems of evaluating the accuracy of the physical modelling structure are connected with the fact that the reproduction of the process being modelled. It is accompanied by errors in the determination and reproduction of the similarity criteria. The random variations of the parameters of the model are depending on the features and structure of the model.

In connection with the development of experimental investigations which have regarded to the inaccuracy of the initial data, new ways of using dynamic models are intended. Here it is possible to regard the model itself as the object of investigation. Thus any investigation on a model leads to the appearance of new, more precise, theoritical propositions. The complicated problems of modelling have required automation to be applied to the experimentation carried out on the model. Such automisation enables testing to be carried out more quickly and speeds up the issue of results, thereby imparting qualitatively new attributes to the physical model. Increasingly extensive use is being made of combined models which contain elements of physical modelling along with elements of mathematical modelling.

The use of digital computers enables new properties and possibilities to be imparted to the old methods. A great complex problems can be solved by using computers and also the systems having number of variables can be handled easily. The role of the methods of modelling is growing with the development of modern methods of investigation and with the mathematisation of these mathods.

Therefore mathematical modelling is a necessary preliminary to the analysis of any physical system. The results of the analysis depend entirely on how accurately the system is described mathematically.

.2 Modal Control Theory Applied To Lumped And Distributed Parameter Systems.

Rosenbrock, H.H. appears to have been the first to propose modal control of a process.

To introduce the basic principle of modal control an ideal case may be considered[24] where all elements of state vector X(s) are directly measurable (C = I) and the control vector B u(t) is produced by a non-singular B from an n.vector u(t). A control linear dynamic system on the state equation is described by

$$\dot{X} = A\,X + B\,u(t) \qquad (2.2.1)$$

and the output equation is written as

$$Y = C^T X \qquad (2.2.2)$$

where X is a state vector, u(t) is input vector, Y is an output vector and A, B and C are matrices with appropriate dimensions. Since the state variables are not unique, the intention is to transform the state vector X to a new state vector Z, or into model modain by means of a constant, square, non singular matrix T, so the state equation (2.2.1) above becomes as

$$\begin{aligned} X &= T\,Z \\ \dot{X} &= T\,\dot{Z} \\ \dot{Z} &= T^{-1} A\,T\,Z + T^{-1} B\,u \\ \dot{Z} &= \wedge Z + B' u. \end{aligned} \qquad (2.2.3)$$

$$\text{where } \wedge = T^{-1} A\,T$$

$$B' = T^{-1} B$$

The eigen values are same for the original system and for the transformed equations. So the eigen values are invariant in a linear transformation. The matrix T is called a model matrix when it is selected, so that $T^{-1} A\,T$ is diagonal. For an nth order system

$$T^{-1} AT = \wedge = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix} \qquad (2.2.4)$$

A great reduction in the elements of A is obtained when A is reduced to the Jordan canonical form Λ by the above coordinate transformation. The matrix Λ possesses the same invariant characteristics as A but is much easier to manupulate. Hence transforming A into simplest possible form makes it possible to investigate intrinsic properties of the dynamic system with the minimum of manupulations.

When all eigen values of the control object are real and distinct only these eigen values appear as diagonal elements of 'Λ' as in (2.2.4). D AZZO, J.J. and C.H. Houpis[22] have discussed three methods for obtaining the modal matrix T for the case of distinct eigen values. If duplications exist in the eigen values then it is necessary to use the Schwartz form. The Schwartz form is the name for this form of the Jordan canonical matrix. If the system has conjugate complex eigen values a modified canonic form of transformation X = T Q W is used in 2.2.3 where Z = Q W.

$$\text{Let} \quad \lambda_1 = \sigma + J\,W$$
$$\lambda_2 = \sigma - J\,W$$

be a pair of conjugate complex eigen values where σ and w are both real and $w > 0$. The first eigen vector V' that corresponds to λ_1 is also complex. Let α and β be real vectors determined by $V' = \alpha + J\beta$. Now the first two columns of T matrix can be replaced by the first two conjugate complex eigen vectors α and β respectively.

$$T = [\,\alpha\,,\,\beta\,,\,\ldots\ldots\,]$$

with the modification in the transformation the resulting Λ or

2.2.4 matrix will take the form

$$\Lambda = \begin{bmatrix} \sigma & w & 0 & \cdots \\ -w & \sigma & 0 & \cdots \\ 0 & \cdot\cdot & \lambda_3 & \cdots \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdots \end{bmatrix}$$

The modified steps of the matrix reduction just shown for the first two eigen values, apply to a complex pair placed at the ith and i + 1 st location without any basic change in computation. The complex eigen values are rarely occur in practice. As in the present problem of interest that all eigen values of the control object are real and distinct then the 'Λ' matrix is diagonal That is the desined or prescribed closed loop modal domain matrix of the control object with modal control, where the diagonal elements are prescribed closed loop eigen values of the controlled system.

The modal control of a class of distributed parameter systems governed by partial differential equations of the diffusion type have been presented by Porter , B and Bradshaw, A. For both ideal control laws and practical control laws have been derived for the assignment of one real time domain eigen value[25] and also arbitrary values to several of the time-domain eigen values of distinct type[26]. In the case of single eigen value suggested to use sensors to limit the departures introduced from the ideal modal control to the practical modal control of distributed parameters. McGlothin[27] used a generalized eigen function integration method to develop a modal control model of a class of distributed parameterm systems. These systems described by scalar parabolic partial differential equation in one space dimension with mixed boundary conditions, the control being exercised at the boundary of the system. This general method is useful in studying decoupling and pole allocation in the class of distributed systems considered. In Crossely's modal control theory[28] the loop gains of a single input system may be readily calculated using a simple formula for the case when the system matrix has a number of sets of confluent eigen values. But the Jordan canonical form must not contain two or more Jordan blocks associated with the same eigen value location in the λ plane. This restriction is relaxed in the case of multi input systems by allocating a different control input to each such repeated block.

The closed loop poles of a control system having a single controlling input can be put in desired locations providing all state variables are measurable and all modes are both observable and controllable. Under these ideal conditions modal control will be satisfactory if there are necessary number of manupulations to control the dominant modes.

.3 State Equation And State Variable Feed Back

.3.1. Formulation of state equation : In general, most physical systems are non-linear in nature and since it is true that an exact mathematical description of any system valid under all conditions is too difficult, if not impossible to formulate. One of the most difficult problems confronting presently is the problem of obtaining an adequate mathematical description of the physical system.

However, for most physical systems the approximate behavior is generally known and on the strength of this knowledge one can make various simplifying assumptions with the result that an exact mathematical model becomes quite unnecessary.

There are three well known methods of describing physical systems mathematically[9]

1) The transfer function model.
2) The state variable model and
3) The component connection model.

1) The transfer function is defined as the ratio of the Laplace transform of the output to input. The transfer function may or may not describe the system completely. In other words only those modes of the system which are commanded from the input and observed from the output terminals, appear in the transfer function.

2) The state variable description for a system is of the type

$$\dot{X} = f(x,u,t) \qquad y = g(x,u,t) \tag{2.3.1}$$

And now consider the class of linear time invariant systems those can be described by the pair of vector equations

$$\dot{X} = A\,X + B\,u(t) \qquad Y = C^T X \tag{2.3.2}$$

where

u is input

Y is output

X is state vector.

This is more elegant and easily amenable for numerical solution with the help of a digital computer. A wide range of choices of state variables and the coefficient matrices are possible giving the same input-output relationship. Another advantage of this metho one state variable model may be converted into another by a non-singular coordinate transformation.

3) Lastly the component connection model is more natural as most engineering systems are constructed by the interconnection of components or sub-systems. The component connection model has never come into popular use because of the lack of a form which is general enough for most practical problems.

The most innovative aspect of modern system theory is undoubtedly the prevalence of state space models for dynamical systems. This has provided a frame work which is at the same time extremely general, offers many advantages and yields concrete and specific practical results much more directly than other methods were able to provide.

A dynamic system[7] consisting of a finite number of lumped elements may be described by ordinary differential equations in which time is the independent variable. By use of vector matrix notation, an nth order differential equation may be expressed by a first-order vector-matrix differential equation. If n elements of the vector are a set of state variables, then the vector-matrix differential equation is called a state equation. This can be explained by taking an example of nth order system in which the forcing function does not involve derivative terms.

Consider the following nth order system

$$y^n + a_1 y^{n-1} + a_2 y^{n-2} + \dots + a_{n-1}\dot{y} + a_n y = u \qquad (2.3.3)$$

Noting that the knowledge of $y(0)$, $\dot{y}(0) \dots y^{n-1}(0)$, together with the input $u(t)$ for $t \geqslant 0$, determines completely the future behavior of the system, we may take $y(t)$, $\dot{y}(t), \dots, y^{n-1}(t)$ as a set of n state variables.

Let us define

$$\begin{aligned} X_1 &= y \\ X_2 &= \dot{y} \\ &\vdots \\ X_n &= y^{n-1} \end{aligned}$$

Then equation (2.3.3) can be written as

$$\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= X_3 \\ &\vdots \\ \dot{X}_{n-1} &= X_n \\ \dot{X}_n &= -a_n X_1 - \dots - a_1 X_{n-1} + u \end{aligned}$$

or $\quad \dot{X} = A\,X + B\,u \qquad (2.3.4)$

Where $X = \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 & 0 \dots & 0 \\ 0 & 0 & 1 \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -a_n & -a_{n-1} & & -a_1 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$

The output equation becomes

$$Y = [1 \quad 0 \ldots 0] \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{bmatrix}$$

$$Y = C^T X$$

$$\text{where} \quad C = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Therefore the first order differential equation (2.3.4) is the state equation. Similarly a state equation can also be developed to a transmission line in which only resistance in series and capacitance in shunt are connected. The transmission line is provided with a current source at the sending end and the receiving end is kept open. The states of the equation (2.3.4) are calculated by developing a computer program which will be discussed clearly in the next chapter.

3.2. Pole shifting by feedback compensation : One of the most popular techniques for altering the response characteristics of a control system is the application of linear state variable feedback. There is considerable interest at present in the design of feedback controllers for multivariable systems to meet various performance objectives. Several techniques have been developed to evaluate the feedback gain matrix in time domain as well as in frequency domain to meet the desired system responses. If the system has a single input, then the feedback gain matrix is unique as in the present work, where as for multi input case, many solutions for the feedback gain matrix exist.

A method has been developed[8], in which the feedback coefficient matrix has been calculated from the transfer function of a closed loop system in order to shift the poles in to the pre-assigned locations and also suggested to replace the Zeros if any in the closed loop transfer function by the poles at the some locations. This method is to determine for a pair of dominant complex poles which will result in a step response satisfactory for the desired use of the system. A linear state variable feed back method is suggested by Brockett, R.W.[10] to obtain any desired pole configuration consistant with the dimension of the system when the system equations are controllable and observable. An expression for the transfer function of a system described by a set of first order differential equations is proposed in which it not only relates the poles and zeros to the eigen values of matrices but also makes it possible to compute the transfer function without matrix inversion. A feedback compensator is provided in cascade to obtain arbitrary pole placement [11,12] using state variable feedback for controllable and observable system of linear time invariant multi-input,multi-output plant. In the technique suggested by Solheim[13] it is possible to designate the closed loop poles at preassigned arbitrary locations and at the same time minimise a quadratic performance index with a preassigned weighting matrix for the control input vector. Wonham[14] has shown that the controllable system can be forced to satisfy any set of eigen values by employing appropriate linear feed back. Hence the control problem becomes that of designing a feedback matrix such that the closed loop system satisties or approaches these eigen value requirements.

Retallack and Mac Farlane[15] gave a procedure for shifting a sub set of eigen values of the original system. It was pointed out by the same authors that their approach can not be used for satisfying a criterion other than a simple allocation of closed-loop poles since the gain matrix has rank one. Vittal Rao[16] described a procedure for evaluating the state feedback matrix of a linear system for which only r of the n system eigen values need to be suitably shifted along with the minimisation of an appropriate quadratic performance index. The remaining n-r eigen values of the original system are not disturbed, and are passed on to the resultant feedback system. A characterization is given for the class of all closed loop eigen vector sets which can be obtained with a given set of distinct closed loop eigen values[17] and for non-distinct eigen values[18] used state feedback. Graupe[19] attempted to design an appropriate weighting matrices for the performance index which provides an algorithm for deriving a diagonal state weighting matrix according to the eigenvalue requirements. The solution given employs matrix differential calculas and a static gradient minimization sub-routine to minimize an eigenvalue error criterion. However the choice of feed back matrix is not unique, unless the system is of single input.

For the present work the single input feedback, satisfying eigenvalue requirements is considered. The method applied for finding out feedback gain matrix is as follows.

For example consider a single input third order controllable system as follows[19]

$$\dot{X} = AX + Bu \qquad \text{say } u \text{ is scalar}$$

$$\text{where } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \; B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

with the linear feedback control policy

$$u = -K^T X = -[K_1X_1 + K_2X_2 + K_3X_3]$$

We require the eigenvalues of the system in closed loop be $\lambda_1 \; \lambda_2 \; \lambda_3$ consequently K_i are derived such that

$$\det (A - B K^T - \lambda_i I) = 0$$

$$\det (M - \lambda_i I) = 0$$

$$i = 1, 2, 3$$

$$\therefore M = (A - B K^T)$$

$$\text{Hence } \det \begin{bmatrix} (a_{11} + K_1b_1 - \lambda_i) & (a_{12} + K_2b_1) & (a_{13} + K_3b_1) \\ (a_{21} + K_1b_2) & (a_{22} + K_2b_2 - \lambda_i) & (a_{23} + K_3b_2) \\ (a_{31} + K_1b_3) & (a_{32} + K_2b_3) & (a_{33} + K_3b_3 - \lambda_i) \end{bmatrix} \qquad (2.3.5)$$

Thus the characteristic equation of (2.3.5) represents a set of three linear equations (for i = 1,2,3) in K_i, with three unknowns.

Consequently in the single input case, a unique solution for K exists that brings the closed loop system to satisfy any set of eigenvalues λ_i. A similar set of n equations exists for any nth order state vector, giving the n components of K for the single input case.

The plant we consider for compensation by state variable feedback is in a state variable formulation as shown in Fig.(2.3.1)

$$\dot{X} = A X + B u \tag{2.3.6}$$

The plant output is assumed given by

$$Y = C^T X$$

In state variable feedback we assume that the entire state vector X is available for feedback. The feedback system is shown in Fig.(2.3.2).

The quantity for feedback is $K^T X$ and substracted from the system input r, to get an input to the plant which is given by

$$u = r - K^T X \tag{2.3.7}$$

Substituting (2.3.7) in (2.3.6)

$$\begin{aligned} \dot{X} &= A X + B(r - K^T X) \\ &= (A - B K^T)X + B r \\ &= M X + B r \qquad (2.3.8) \end{aligned}$$

Say $M = A - B K^T$

The equation (2.3.8) is the equation for the Fig.(2.3.2) i.e. after applying the state variable feedback.

4 Net Work Synthesis

A large percentage of networks in control systems are to consist of only resistors and capacitors because of the low frequency of interest. The vast majority of synthesis problems in the design of feedback control systems demand realization of the prescribed transfer functions by networks consisting entirely of resistors and capacitors. Inductors are avoided because of the excessive weight and size required by the low frequencies of interest. In this problem basically it is assumed that a leakage free non-inductive cable, where the conductance (G) and inductance(L) are equal to Zero. So here the network synthesis is mainly concerne with RC transmission line.

In linear networks if any two of the three quantities[30] the network, the excitation and the response are given the third may be found. When the excitation and the response are given and it is required to determine a network the problem is defined as synthesis when the response is required to find out when the other two are given is called network analysis. In analysis the solution is unique where as in synthesis however solutions are not unique and there may exist no solution at all. If there is any solution to a given problem, there are an indefinite number of other solutio from which a choice may be made.

For one terminal pair networks, only one voltage and one current are identified and so only one network function can be defined i.e. the driving-point impedance function or simply the impedance of a network Z(s). The reciprocal of the impedance function is the driving-point admittance function or the admittance Y(s). For two terminal-pair network two currents and two voltages are identified. A large number of network functions are possible in this case, four different quantities taken two at a time gives six possible network functions[29]. Those six transfer functions are the driving point impedance (1) at input terminals (2) at output terminals (3) transfer admitance function (4) transfer impedance function (5) voltage ratio transfer function,Δ These network functions are important in synthesis because they may be used to describe models which approximate actual systems. The various voltage and current transforms in a given network are related to each other by network functions which are quotients of rational polynomials in the complex frequency variable S. A general transform function of this form may be written for impedance

$$Z(s) = \frac{p(s)}{q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \ldots\ldots\ldots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \ldots\ldots\ldots + b_{m-1} s + b_m}$$

where a and b coefficients are real constants, n is the degree of p(s) if $a_0 \neq 0$ and m is the degree of the denominator polynomial q(s) if $b_0 \neq 0$. The network function Z(s) can be written after factorising both numerator and denominator polynomial as

$$Z(s) = \frac{p(s)}{q(s)} = \frac{a_o}{b_o} \frac{(s - Z_1)(s - Z_2) \ldots\ldots\ldots (s - Z_n)}{(s - p_1)(s - p_2) \ldots\ldots\ldots (s - p_n)}$$

where the roots of p(s) = 0, Z_1, Z_2 ...... Z_n are the zeros of the transfer function and the roots of q(s) = 0, p_1 , p_2 ,....p_m are the poles of the transfer function.

Δ and (6) current ratio transfer function.

If a rational function[29] has simple poles restricted to the finite negative real axis where it has positive real residues and if its value for $s = \infty$ is non negative and finite then it satisfies the necessary conditions for being the driving point impedance of an RC network. Further more the poles and zeros alternate along the axis. The lowest critical frequency is a pole and the heighest critical frequency is a zero. This above discussion is concerned exclusively with the characteristics of driving point impedance functions. Similarly for driving point admittance functions can also be realized as RC net work functions but only change is the roles of poles and zeros are simply interchanged. The poles and zeros still alternate along the axis, but for an admittance function the lowest critical frequency is a zero, the heighest a pole and all residues are negative.

Most of the techniques for the synthesis of transfer functions are based at least in part on the synthesis of driving point impedance or admittance functions. There are four fundamental methods[20] by which networks for RC impedance functions can be synthesized. Each method depends on the technique of writing the function in such a form that a suitable network configuration and element values can be determined by inspection.

First and Second of Foster : The first Foster form is also called as partial-fraction expansion of Z(s) and second Foster form is partial-fraction expansion of Y(s)/S . The poles are removed one at a time until the function no longer possesses singularities i.e. until the function is simply a constant. But each step in the reduction must meet two requirements : the removed quantity must be recognizable as a simple impedance and the remainder must be an RC impedance function.

<u>First and Second Cauer Forms</u> : These are called continued fraction expansion about infinity and about zero respectively. In this cauer form also the net-work is realised by removing the poles of the transfer function, but it is some what different from the Foster like form of removal of poles.

The first cauer form is applied for the realisation of the RC transmission line in the present problem from the transfer impedance Z(s). This is also said to be the consistent removal of components at $s = \infty$.

<u>The First Cauer Form of RC Network Realization</u> : The RC driving point impedance is a quotient of unfactored polynomials in which all powers of s are present expressed as Z(s) for an nth order syst

$$Z(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_n + b_0} \qquad (2.4.1)$$

This impedance Z(s) has finite non zero values at both $s = 0$ and $s = \infty$ which is its most general form and all coefficients must be positive. For getting the ladder development of an RC network an initial series resistance having a finite non zero positive real value at $s = \infty$ is removed from the Z(s) function. The remaining impedance say $Z_1(s)$ is now zero at $s = \infty$ so the reciprocal $Y_1(s) = \frac{1}{Z_1(s)}$ has a pole at $s = \infty$. Removal of this pole produces a shunt capacitance. The remainder function say $Y_2(s)$ has a positive real non zero value at $s = \infty$ because this value must be larger than the non negative zero frequency value. Hence the reciprocal

$Z_2(s) = \frac{1}{Y_2(s)}$ is again like Z(s). This $Z_2(s)$ has a finite non zero positive real value that can be subtracted to begin a second cycle in the ladder development exactly like the one just completed. If the process is to be repeated until the remainder is reduced to a constant yields the continued fraction representation of cauer first form.

$$Z(s) = R_1 + \cfrac{1}{C_2 s + \cfrac{1}{R_3 + \cfrac{1}{C_4 s + \cdots + \cfrac{1}{R_{n-1} + \cfrac{1}{C_n s + \cfrac{1}{R_{n+1}}}}}}} \qquad (2.4.2)$$

Although n+2 coefficients appear in expression (2.4.1) for Z(s), only n + 1 of them are independent, since the constant multiplier is the ratio of a_n/b_n. Hence the development procedure will yield n + 1 elements equal the number of finite non zero critical frequencies plus one. The ~~resulting network for the expression (2.4.2) is shown in Fig.(2.4.1)~~. It is a ladder of series resistors and shunt capacitors, the first element is a resistor when $Z(\infty)$ is non zero and is a capacitor when $Z(\infty)$ is zero. The last element is a resistor when Z(0) is finite and is a capacitor when Z(s) has a pole at the origin. Thus the elements, of the network from the transfer impedances are determined.

2.5 RC Transmission Line Equation of Diffusion Type

A transmission line equation is formed by considering the two basic parameters resistance and capacitance. The remaining two basic parameters inductance and conductance are assumed to be zero because it is considered a leakage free nonductive cable. Therefore the RC transmission line equation is written mathematically as

$$\frac{\partial X(x,t)}{\partial t} = \alpha^2 \frac{\partial X(x,t)}{\partial x^2} \qquad (2.5.1)$$

where $\alpha^2 = \frac{1}{R\,C}$.

This equation can be derived by taking a simple example of one dimensional heat conduction. This illustrates many features of the control of distributed parameter systems.

A metal slab is considered[5] bounded by two infinite parallel planes as shown in Fig.(2.5.1). Assumed that on one side of the metal slab is perfectly insulated and the temperature distribution through out the slab is controlled by applying heat uniformely over the other side of the metal slab. The behavior of the temperature θ at distance x and time t is given by one dimensional diffusion equation when all conditions assumed are uniform with respect to other two coordinates y and z.

$$\frac{\partial \theta}{\partial t} = \alpha^2 \frac{\partial^2 \theta}{\partial x^2}$$

where α^2 is the diffusivity of the material of the slab. The systems dynamic behaviour can be computed by taking a finite number of variables to represent the continuous terperature distribution.

By dividing the slab in to number of slices of finite thickness an electrical RC network can be derived. For example the slab is divided into two slices then the electrical model is represented as in Fig.(2.5.2).

As shown in Fig.(2.5.2) "u" represents the temperature applied to the heated face of the slab, and θ_1 and θ_2 represent the temperatures at the centres of the two slices.

When it is considered as an electrical transmission line the u becomes step input and θ_1 and θ_2 become responses X_1 and X_2 at the intermediate points when the line is divided in to number of sections.

The initial condition $X_o(x,o) = o$

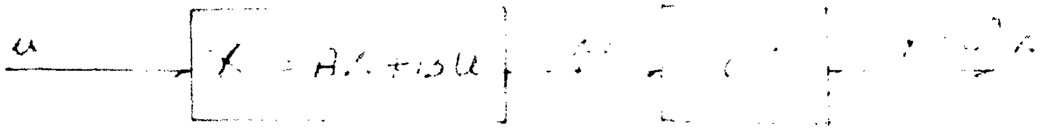
$$o \leq x \leq 1 \qquad o \leq t \leq \infty \tag{2.5.2}$$

and boundary conditions are

$$\frac{\partial X}{\partial x}(x,t) \Big/_{x=0} = -u(t)$$

$$\frac{\partial X}{\partial x}(x,t) \Big/_{x=1} = 0$$

The solution of the equation (2.5.1) will be given in the next Chapter (3.1) by using the boundary conditions given here.



3.1

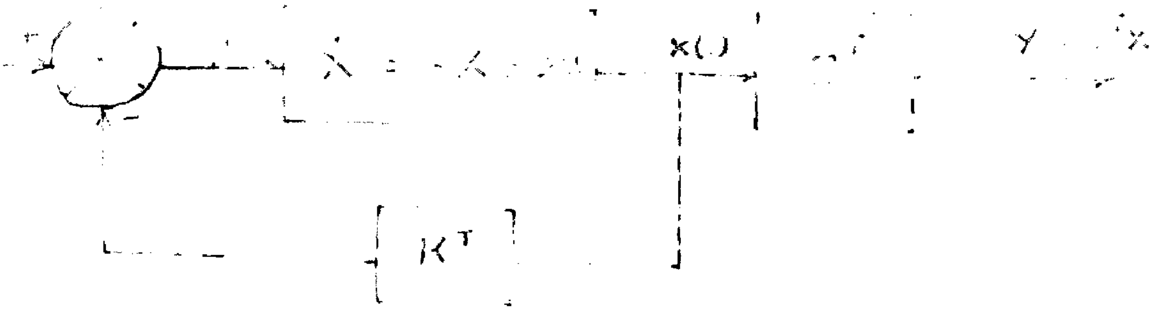
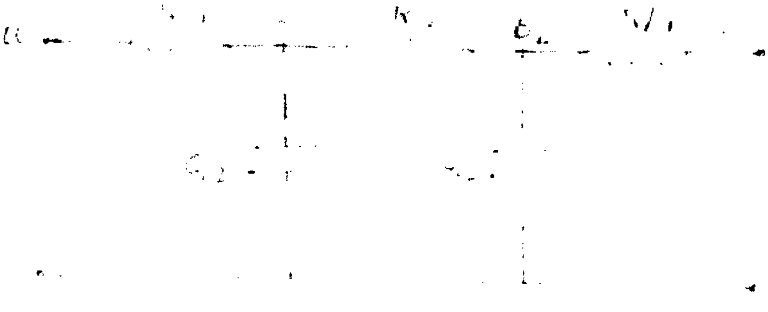


FIG.



CHAPTER - 3

MATHEMATICAL METHODS USED IN PRESENT PROBLEM

This chapter presents the mathematical techniques used for the present problem.

.1 One Dimensional RC Transmission Line Equation

Consider the transmission line equation[4] as given in (1.1)

$$\frac{\partial X(x,t)}{\partial t} = \alpha^2 \frac{\partial^2 X(x,t)}{\partial x^2} \tag{3.1.1}$$

and also this is the equation for leakage free non-inductive cable where G and L are equal to zero.

Then $$\alpha^2 = \frac{1}{R\ C} \tag{3.1.2}$$

X(x,t) represented as X for simplicity

$$\frac{\partial X}{\partial t} = \frac{1}{R\ C} \frac{\partial^2 X}{\partial x^2} \tag{3.1.3}$$

Let us calculate the problem for unity value of α^2 for simplicity. The per unit values can be extended to the actual transmission line.

$$\frac{\partial X}{\partial t} = \frac{\partial^2 X}{\partial x^2} \quad \text{Assumed } \alpha^2 = 1 \tag{3.1.4}$$

Taking Laplace transformation on both sides of the equation with respect to t.

$$S\ X(x,s) - X_0(x,0) = \frac{\partial^2 X}{\partial x^2}(x,s) \tag{3.1.5}$$

and $\frac{\partial X(x,s)}{\partial x} \Big/_{x = 0} = - u(s) = -1/s$

$$\frac{\partial X}{\partial x}(x,s) \Big/_{x=1} = 0 \qquad (3.1.6)$$

$X_0(x,0) = 0$ from 2.5.2

Also $SX(x,s) = \frac{\partial^2 \theta(x,s)}{\partial x^2}$ (3.1.7)

Solution of (3.1.7) is thus given by

$$X = A \sinh \sqrt{S}\, x + B \cosh \sqrt{S}\, x \qquad (3.1.8)$$

at $x = 0$ $\frac{\partial X}{\partial x} = -1/s$ from (3.1.6)

$$\frac{\partial X}{\partial x} = A \cosh \sqrt{S}\, x . \sqrt{S} + B \sinh \sqrt{S}\, x . \sqrt{S} = - u(s)$$

$$= \sqrt{S}\,(A \cosh \sqrt{S}\, x + B \sinh \sqrt{S}\, x) = - u(s)$$

$x = 0$ $\sqrt{S}\, A = - u(s)$

$$A = - u(s) \frac{1}{\sqrt{S}} = - \frac{u(s)}{\sqrt{S}}$$

at $x = 1$

$$\frac{\partial X}{\partial x} = 0$$

$$= \sqrt{S}\,(A \cosh \sqrt{S} + B \sinh \sqrt{S}\,) = 0$$

$$B = - A \frac{\cosh \sqrt{S}}{\sinh \sqrt{S}} = \frac{u(s)}{\sqrt{S}} \frac{\cosh \sqrt{S}}{\sinh \sqrt{S}}$$

The solution of (3.1.7) is given as

$$X = - \frac{u(s)}{\sqrt{S}} \sinh \sqrt{S}\, x + \frac{u(s)}{\sqrt{S}} \frac{\cosh \sqrt{S}}{\sinh \sqrt{S}} \cosh \sqrt{s}\, x$$

or $X = \frac{\cosh \sqrt{S} \cosh \sqrt{S}\, x - \sinh \sqrt{S}\, x \sin \sqrt{S}}{\sqrt{S} \sinh \sqrt{S}}\, u(s)$

or $X = \frac{\cosh \sqrt{S}\,(1 - x)}{\sqrt{S} \sinh \sqrt{S}}\, u(s)$ (3.1.9)

The poles of (3.1.9) are given when denominator equated to zero

$$\sqrt{S}\ \sinh \sqrt{S} = 0$$

This gives

$$S = -K^2 \pi^2 \quad ; \quad K = 0, 1, 2, \ldots\ldots \qquad (3.1.10)$$

These are the actual eigen values of the transmission line. The solution of (3.1.9) in t form may be written by writing (3.1.9) in the form[6]

$$X(x,t) = \int_0^t \left(2 \sum_{K=1}^{\infty} \cos K\pi x \, e^{-K^2\pi^2(t-\xi)}\right) u(\xi)\, \partial\xi$$

$$X(x,t) = +2 \sum_{K=1}^{\infty} \cos K\pi x \frac{[1-e^{-K^2\pi^2 t}]}{K^2 \pi^2} u(t) \qquad (3.1.11)$$

Assumed the expansion of X(x,t) in coordinates x and time t, in convergent series in the sense of Weinberger[4]. This gives the complete solution of the system (3.1.5 and 3.1.6). For finding out the responses 'X' at the intermediate points of 'x' on the transmission line for a unit step input u(t) computer program is written which is given in Appendix (3.1.1.). For convenience the total length (x) of the transmission line is taken as unity and it is divided into equal lengths. The value of x depends upon the x numbersx of sections made. In the present problem it is divided intc 5, 6 and 7 sections and the intermediate points or states are always less than one to the number of sections. The results are shown graphically. The responses of interest are at the intermediat points and are named as X_1 , X_2 and so on.

2 Transmission Line Equations in State Space Form

Fourth Order

One dimensional differential equations may be written for the transmission line to represent in state equation form.

$$\frac{\partial X}{\partial t} = \frac{\partial X^2}{\partial x^2} \quad \text{from} \quad (3.1.4) \tag{3.2.1}$$

The differential equations can be written in Taylor Series form at a point on the transmission line when it is divided in to equal length 'h' as

$$X\ (x+h,t) = X(x,t) + \frac{\partial X}{\partial x} h + \frac{1}{2} \frac{\partial^2 X}{\partial x^2} h^2 \tag{3.2.2}$$

$$X\ (x-h,t) = X(x,t) - \frac{\partial X}{\partial x} h + \frac{1}{2} \frac{\partial^2 X}{\partial x^2} h^2 \tag{3.2.3}$$

By adding (3.2.2) and (3.2.3) then

$$\frac{\partial^2 X}{\partial x^2} = \frac{1}{h^2} [X(x + h,\ t) + X\ (x - h,\ t) - 2\ X\ (x,\ t)] \tag{3.2.4}$$

say $x = n\ h$ where h is section length and n is number of sections divided. The general equation is written as

$$\frac{\partial X}{\partial t} = \frac{1}{h^2} [X(nh + h,t) + X(nh - h,t) - 2X(nh,\ t)] \text{ since } \frac{\partial^2 X}{\partial x^2} = \frac{\partial X}{\partial t} \tag{3.2.5}$$

substitute for n = 1, 2, 3,... and soon in (3.2.5)

n is always greater than one , to the intermediate points. For 4 intermediate points n is 5 and so on. The value of h is equal to the length of the line divided by number of sections made. Just as RC transmission line equation here also the transmission line is divided as 5,6 and 7 and then compared.

$$\frac{\partial X(nh,t)}{\partial t} = \frac{1}{h^2}\left[X(nh + h,t) + X(nh - h,t) - 2X(nh, t)\right] \qquad (3.2.5)$$

Substitute for n = 1, 2, 3 and 4

$$\left.\frac{\partial X(h,t)}{\partial t}\right/_{n=1} = \frac{1}{h^2}\left[X(2h,t) + X(0,t) - 2X(h,t)\right]$$

$$\left.\frac{\partial X(2h,t)}{\partial t}\right/_{n=2} = \frac{1}{h^2}\left[X(3h,t) + X(h,t) - 2X(2h,t)\right] \qquad (3.2.6)$$

$$\left.\frac{\partial X(3h,t)}{\partial t}\right/_{n=3} = \frac{1}{h^2}\left[X(4h,t) + X(2h,t) - 2X(3h,t)\right]$$

$$\left.\frac{\partial X(4h,t)}{\partial t}\right/_{n=4} = \frac{1}{h^2}\left[X(5h,t) + X(3h,t) - 2X(4h,t)\right]$$

Say $X(h,t) = X_1$ $\quad X(2h,t) = X_2$ $\quad X(3h,t) = X_3$

$X(4h,t) = X_4$

$$\frac{\partial X}{\partial t} = \dot{X}$$

$$\left.\frac{\partial X}{\partial x}\right/_{n=0} = -u(t)$$

$$\left.\frac{\partial X}{\partial x}\right/_{n=1} = 0 \qquad \text{from (2.2.3)}$$

Where $\frac{\partial X}{\partial x} = \frac{X(x+h,t) - X(x,t)}{h}$

$$\left.\frac{\partial X}{\partial x}\right/_{X=0} = \frac{X(h,t) - X(0,t)}{h} = -u(t)$$

$$X(0,t) = h\,u(t) + X(h,t) \qquad (3.2.7)$$

$$\left.\frac{\partial X}{\partial x}\right/_{X=1} = \frac{X(1 + h,t) - X(1,t)}{h} = 0$$

$$X(5h,t) = X(4h,t) \qquad (3.2.8)$$

By substituting (3.2.7) and (3.2.8) in (3.2.6) then these equations can be written as $\dot{X} = AX + Bu(t)$

or

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} -25 & 25 & 0 & 0 \\ 25 & -50 & 25 & 0 \\ 0 & 25 & -50 & 25 \\ 0 & 0 & 25 & -25 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

Since $h = \frac{1}{n}$ where $n = 5$

$$\text{where } A = \begin{bmatrix} -25 & 25 & 0 & 0 \\ 25 & -50 & 25 & 0 \\ 0 & 25 & -50 & 25 \\ 0 & 0 & 25 & -25 \end{bmatrix} ; \; B = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (3.2.9)$$

The eigen values of this A matrix can be calculated[7]

$$\text{as } |A - \lambda I| = 0 \; ; \quad \begin{bmatrix} -(25+\lambda) & 25 & 0 & 0 \\ 25 & -(50+\lambda) & 25 & 0 \\ 0 & 25 & -(50+\lambda) & 25 \\ 0 & 0 & 25 & -(25+\lambda) \end{bmatrix} = 0 \qquad (3.2.10)$$

The characteristic equation of $|A - \lambda I| = 0$ is

$$\lambda^4 + 150\lambda^3 + 6250\lambda^2 + 62500\lambda = 0$$

A computer programme is written for finding out eigen values of this A matrix. It can be extended to the any order of A matrix and is given in Appendix (3.2.1). The eigen values of 4th order A matrix are given below.

$\lambda_1 = 0.00 \quad \lambda_2 = -14.64 \quad \lambda_3 = -50.00 \quad \lambda_4 = -85.35$

The responses at intermediate points X_1, X_2, X_3 and X_4 are calculate through a computer program (Runge Kutta method) which is given in Appendix (3.2.2) and compared with those responses from RC transmission line equation at the same points of distances by drawing graphs shown in graph numbers from 1 to 4.

As already explained the state feedback method, the state equation after feeding the states through feedback coefficient is as follows

$\dot{X} = M X + B r$ from (1.3)

where $M = (A - B K^T)$

M, A, B and K are matrices with appropriate dimension. The characteristic equation of M matrix may be written as

$$|M - \lambda I| = 0$$

$$|A - B K^T - \lambda I| = 0 \qquad (3.2.11)$$

The eigen values of this M matrix and the eigen values of the transmission line RC network equation are same. The feedback coefficients (K's) are calculated by substituting the eigen values from (3.1.10) where K = 0, 1, 2, 3 in (3.2.11) instead of λ. The matrices A and B are known.

$$|A - B K^T - \lambda I| = |M - \lambda I| = 0$$

$$B\,K^T = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} [K_1\ K_2\ K_3\ K_4] = \begin{bmatrix} 5K_1 & 5K_2 & 5K_3 & 5K_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$|M - \lambda I| = \begin{bmatrix} -(25 + 5K_1 + \lambda) & 25 - 5K_2 & -5K_3 & -5K_4 \\ 25 & -(50 + \lambda) & 25 & 0 \\ 0 & 25 & -(50+\lambda) & 25 \\ 0 & 0 & 25 & -(25+\lambda) \end{bmatrix}$$

The characteristic equation of $|M - \lambda I|$ is written as

$$\lambda^4 + (5K_1 + 150)\lambda^3 + (625\,K_1 + 125\,K_2 + 6250)\lambda^2 + (18750K_1$$
$$+ 9375K_2 + 3125K_3 + 62500)\lambda + (K_1 + K_2 + K_3 + K_4)\,78125 = 0$$

Now poles are shifted to the required locations. A computer program for finding out K's from the characteristic equation of M is written and the program is given in Appendix (3.2.3)

The feedback coefficients are as follows.

$K_1 = -2.364 \quad K_2 = 0.007 \quad K_3 = 5.238 \quad K_4 = -2.881$

Therefore the M matrix may be written as

$$M = \begin{bmatrix} -13.180 & 24.965 & -26.190 & 14.400 \\ 25.0 & -50.0 & 25.0 & 0.0 \\ 0.0 & 25.0 & -50.0 & 25.0 \\ 0.0 & 0.0 & 25.0 & -25.0 \end{bmatrix}$$

Again the responses X_1 X_2 X_3 and X_4 for this feedback matrix are calculated as in the case of A matrix and compared with the previous responses.

$$\dot{X} = M\,X + B\,r(t)$$
$$Y = C^T\,X$$

For finding out the network of the above equation (which is to be discussed in 4th Chapter) the transfer impedance is calculated for a current step input.

Applying Laplace transformation as

$$SX(s) - X(0) = MX(s) + BR(s)$$
$$(SI - M)\,X(s) = BR(s)$$

Since $X(0) = 0$

$$Y(s) = C^T\,X(s)$$
$$X(s) = (SI - M)^{-1}\,BR(s)$$
$$Y(s) = C^T(SI - M)^{-1}\,BR(s)$$
$$Z(s) = \frac{Y(s)}{R(s)} = C^T\,(SI - M)^{-1}\,B$$
$$Z(s) = C^T(SI - M)^{-1}\,B \qquad (3.2.12)$$

By knowing the values of C^T, M and B the transfer impedance may be calculated as

$$Z(s) = \frac{5\,S^3 + 750\,S^2 + 31250\,S + 312500}{S^4 + 138.56S^2 + 4774\,S^2 + 34613\,S} \qquad (3.2.13)$$

n = 6, 5th order.

The same procedure is applied by taking n = 6 for comparison purpose.

$$\frac{dX(nh,t)}{dt} = \frac{1}{h^2}\,[X(nh + h,t) + X(nh-h,t) - 2X(nh,t)] \text{ from } (3.2.5)$$

n varies from 1 to 5 since the number of sections are six. The state equation of the form $\dot{X} = AX + Bu(t)$ is written as in (3.2.6) by applying the same boundary conditions.

here $X(\dot{5}\,h,t) = X5$

$X(6\,h,t) = X(5\,h,t)$

$\dot{X} = AX + Bu(t)$ $\qquad h = \frac{1}{n} \quad n = 6$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} -36 & 36 & 0 & 0 & 0 \\ 36 & -72 & 36 & 0 & 0 \\ 0 & 36 & -72 & 36 & 0 \\ 0 & 0 & 36 & -72 & 36 \\ 0 & 0 & 0 & 36 & -36 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$\text{where } A = \begin{bmatrix} -36 & 36 & 0 & 0 & 0 \\ 36 & -72 & 36 & 0 & 0 \\ 0 & 36 & -72 & 36 & 0 \\ 0 & 0 & 36 & -72 & 36 \\ 0 & 0 & 0 & 36 & -36 \end{bmatrix} ; \; B = \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The characteristic equation is obtained as

$$|A-\lambda I| = \lambda^5 + 288\lambda^4 + 27216\lambda^3 + 933148\lambda^2 + 8398080\lambda = 0$$

The eigen values or roots of this equation are calculated from the computer program given in Appendix (3.2.1).

$\lambda_1 = 0.0 \quad \lambda_2 = -13.75 \quad \lambda_3 = -49.75 \quad \lambda_4 = -94.249 \quad \lambda_5 = -130.249$

As in the case of 4th order the responses are calculated at

X_1, X_2, X_3, X_4 and X_5 for comparison with the responses of transmission line equation, graphs are drawn and shown from graph numbers 5 to 9.

Now after applying feedback technique the state equation becomes

$$\dot{X} = MX + Br \quad \text{from (1.3)}$$

$$M = [A - B\,K^T]$$

The characteristic equation of M is $|M - \lambda I| = 0$

$$|A - B\,K^T - \lambda I| = 0 \qquad \text{from (3.2.11)} \qquad (3.2.14)$$

$$K^T = [K_1\ K_2\ K_3\ K_4\ K_5]$$

$$B\,K^T = \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [K_1\ K_2\ K_3\ K_4\ K_5] = \begin{bmatrix} 6\,K_1 & 6\,K_2 & 6\,K_3 & 6\,K_4 & 6\,K_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

These K's are calculated by substituting the transmission line eigen values from (3.1.10) where K = 0,1,2,3 and 4 in (3.2.14) instead of λ.

$$|M - \lambda I| = \begin{bmatrix} -(36+6K_1+\lambda) & 36-6K_2 & -6\,K_3 & -6\,K_4 & -6\,K_5 \\ 36 & -(72+\lambda) & 36 & 0 & 0 \\ 0 & 36 & -(72+\lambda) & 36 & 0 \\ 0 & 0 & 36 & -(72+\lambda) & 36 \\ 0 & 0 & 0 & 36 & -(36+\lambda) \end{bmatrix} = 0$$

The characteristic equation of $|M - \lambda I|$ is written as

$$\lambda^5 + (6K_1 + 288)\,\lambda^4 + (1512K_1 + 216K_2 + 27216)\,\lambda^3 +$$
$$(116640\,K_1 + 38880\,K_2 + 7776\,K_3 + 933120)\,\lambda^2 +$$
$$(2799360\,K_1 + 1679616\,K_2 + 839808\,K_3 + 279936\,K_4 + 8398080)\,\lambda$$
$$+ (K_1 + K_2 + K_3 + K_4 + K_5)\,10077696 = 0$$

For finding out K's a computer programme is used which is given in Appendix (3.2.3) and the poles are shifted to the desired locations.

The feedback coefficients for 5th order are as follows

$K_1 = 1.3499$ $\quad K_3 = 22.7725$ $\quad K_5 = 6.5389$

$K_2 - 12.3258$ $\quad K_4 = -18.3356$

Therefore the M matrix may be written as

$M = (A - B\,K^T)$ in which A, B and K matrices are known.

$$M = \begin{bmatrix} -44.10 & 109.92 & -136.62 & 109.98 & -39.24 \\ 36.0 & -72.0 & 36.0 & 0.0 & 0.0 \\ 0.0 & 36.0 & -72.0 & 36.0 & 0.0 \\ 0.0 & 0.0 & 36.0 & -72.0 & 36.0 \\ 0.0 & 0.0 & 0.0 & 36.0 & -36.0 \end{bmatrix}$$

Now the state equation after feeding back, is

$$\dot{X} = M\,X + B\,r$$

The eigen values of this M matrix and the first five eigen values of the transmission line equation are same. The responses of this state equation X_1, X_2, X_3, X_4 and X_5 are calculated by the computer programme given in Appendix (3.2.2).

The graphs are drawn for responses verses time as given in graph numbers from 5 to 9. The transfer impedance is calculated for this 5th order state equation as in (3.2.12).

$$Z(s) = \frac{6\ s^4 + 1728\ s^3 + 163331\ s^2 + 5598928\ s + 50384642}{s^5 + 296\ s^4 + 26593\ s^3 + 788386\ s^2 + 5465924\ s} \qquad (3.2.15)$$

The transmission line representation will be shown in the next Chapter.

n=7 Sixth Order

The same method is extended to the next order.

Here $n = 7 \qquad h = \frac{1}{n}$.

The general equation is written from (3.2.5) as

$$\frac{\partial X(nh,t)}{\partial t} = \frac{1}{h^2}\left[X(nh + h,t) + X(nh - h,t) - 2X(nh,t)\right]$$

Here n varies from 1 to 6 since the number of sections are seven. The state equation of the form $\dot{X} = AX + Bu(t)$ is written as in (3.2.6) by applying the same boundary conditions.

$$X(5\,h,t) = X_5 \qquad\qquad X(6\,h,t) = X_6$$

and also $\quad X(7\,h,t) = X(6\,h,t)$

$$\therefore \quad \dot{X} = AX + Bu(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} -49 & 49 & 0 & 0 & 0 & 0 \\ 49 & -98 & 49 & 0 & 0 & 0 \\ 0 & 49 & -98 & 49 & 0 & 0 \\ 0 & 0 & 49 & -98 & 49 & 0 \\ 0 & 0 & 0 & 49 & -98 & 49 \\ 0 & 0 & 0 & 0 & 49 & -49 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$\text{where } A = \begin{bmatrix} -49 & 49 & 0 & 0 & 0 & 0 \\ 49 & -98 & 49 & 0 & 0 & 0 \\ 0 & 49 & -98 & 49 & 0 & 0 \\ 0 & 0 & 49 & -98 & 49 & 0 \\ 0 & 0 & 0 & 49 & -98 & 49 \\ 0 & 0 & 0 & 0 & 49 & -49 \end{bmatrix} ; \; B = \begin{bmatrix} 7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The eigen values of this A matrix are obtained by writing characteristic equation as $|A - \lambda I| = 0$

$$|A - \lambda I| = \lambda^6 + 490\,\lambda^5 + 86445\,\lambda^4 + 6589256\,\lambda^3 + 201801728\,\lambda^2 + 1695188200\,\lambda = 0$$

Again the computer program given in appendix (3.2.1) is applied for finding out the eigen values of A matrix.

The eigen values are as follows

$\lambda_1 = 0.0 \quad \lambda_2 = -13.13 \quad \lambda_3 = -49.0 \quad \lambda_4 = -98.0$

$\lambda_5 = -147.0 \quad \lambda_6 = -182.87$

As in the previous two cases the responses X_1, X_2, X_3, X_4, X_5 and X_6 are calculated by using the computer programme given in appendix (3.2.2) and compared with the responses obtained from the transmission line equation at the same points. The graphs are drawn for all the states shown in graph numbers starting from 10 to 15.

The feedback matrix M is written in state equation form as

$$\dot{X} = MX + Br$$

$$\text{where } M = (A - B\,K^T)$$

The characteristic equation of M is $|M - \lambda I| = 0$

$$|A - B\,K^T - \lambda I| = 0 \quad \text{from (3.2.11)} \qquad (3.2.16)$$

$$K^T = [K_1\ K_2\ K_3\ K_4\ K_5\ K_6]$$

$$B\,K^T = \begin{bmatrix} 7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [K_1\ K_2\ K_3\ K_4\ K_5\ K_6] = \begin{bmatrix} 7K_1 & 7K_2 & 7K_3 & 7K_4 & 7K_5 & 7K_6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

These feedback coefficients are calculated by substituting the first six eigen values of transmission line from (3.1.10) where $K = 0, 1, 2, 3, 4$ and 5 in (3.2.16) instead of λ.

$$|M - \lambda I| = \begin{bmatrix} -(49+7K_1+\lambda) & (49-7K_2) & -7K_3 & -7K_4 & -7K_5 & -7K_6 \\ 49 & -(98+\lambda) & 49 & 0 & 0 & 0 \\ 0 & 49 & -(93+\lambda) & 49 & 0 & 0 \\ 0 & 0 & 49 & -(98+\lambda) & 49 & 0 \\ 0 & 0 & 0 & 49 & -(98+\lambda) & 49 \\ 0 & 0 & 0 & 0 & 49 & -(49+\lambda) \end{bmatrix} = 0$$

The characteristic equation of $|M - \lambda I|$ is written as

$$\lambda^6 + (7K_1 + 490)\lambda^5 + (3087K_1 + 343K_2 + 86436)\lambda^4 +$$
$$(470596K_1 + 117649K_2 + 16807K_3 + 6588344)\lambda^3 +$$
$$(288240005K_1 + 12353145K_2 + 4117715K_3 + 823543\,K_4 +$$
$$201768030)\lambda^2 + (605304100K_1 + 403536070K_2 + 242121640K_3 +$$
$$121060820K_4 + 40353607K_5 + 1694851540)\lambda + (K_1 + K_2 + K_3 + K_4 + K_5 + K_6)$$
$$1977326600 = 0$$

For finding out K's a computer programme is used which is given in Appendix (3.2.3) and the poles are shifted to the desired locations. The feedback coefficients (K's) for 6th order are as follows.

$K_1 = 7.5499 \quad K_2 = -29.4037 \quad K_3 = 39.7829 \quad K_4 = -2402.4920$

$K_6 = -4756.4220, \quad K_5 = 7140.985$

Therefore the M matrix may be written as $M = (A - B\,K^T)$ in which A, B and K matrices are known.

$$M = \begin{bmatrix} -101.850 & 254.828 & -278.480 & 16817.430 & -49986.895 & 33294.954 \\ 49.0 & -98.0 & 49.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 49.0 & -98.0 & 49.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 49.0 & -98.0 & 49.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 49.0 & -98.0 & 49.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 49.0 & -49.0 \end{bmatrix}$$

The state equation after feeding back is

$$\dot{X} = M\,X + B\,r$$

The eigen values of this M matrix and the first six eigen values of the transmission line equation are same.

The transfer impedance is calculated for this 6th order state equation as in (3.2.12)

$$s) = \frac{7S^5 + 3430S^4 + 605052S^3 + 46118578S^2 + 1412391500S + 11864370000}{S^6 + 543S^5 + 99650S^4 + 7350048S^3 + 199992314S^2 + 1348662200S} \quad (3.2.1$$

The representation of the transmission lines for all these transfer impedances will be discussed in the 4th Chapter.

CHAPTER - 4

REPRESENTATION OF TRANSMISSION LINE

In general the transmission lines are means to transmit power from the generating terminals to consumer centres. The basic parameters of the transmission lines are Resistance (R), inductance (L), conductance (G) and capacitance (C). In fact all these parameters are distributed over the length of the transmission line. The resistance and inductance are in series with the line and conductance and capacitance are in shunt with the lines. ~~By showing~~ All these parameters in a single line representation of a transmission line was shown in Fig.(1.1). For the sake of calculations these parameters are to be made lumped as shown in Figs.(1.2.2) either with T or π representation respectively. Here the conductance term is neglected. Distributed parameters are introduced as shown in Fig.(1.3) because of inaccuracy in the calculations of long transmission lines with the lumped parameter T and π representation.

The design of the transmission line from the transfer impedance function as given in the equation (3.2.12) is mainly concerned in this Chapter. After applying the feedback compensation the transfer impedance function Z(s) is written as

$$Z(s) = C^T(SI - M)^{-1} B.$$

Basically the transmission line is assumed that it is a leakage free noninductive cable with $G = 0$ and $L = 0$. So only two parameters left are resistance in series with the line and capacitan across the line. So the above transfer impedance function has to satisfy the conditions, to be as an RC network function in ladder form. If it has simple poles restricted to the finite negative real axis where it has positive real residues, then this function is realized as an RC network function. If all these conditions are satisfied then the first cauer method of continued fraction expansion about infinity will be applied as to get the network. This method was clearly explained in Chapter 2 under the heading of network synthesis. As given by the equation of the form in 2.4.2 the coefficient associated with the frequency term S is the value of capacitor and the constant term is resistor. A component at the beginning and ending of the designed transmission line can be seen by testing the transfer impedance function Z(s) at S = 0 and $S = \infty$. [The cauer form representation of an RC transmission line for an nth order transfer function was shown in Fig.(2.4.1)] When the transmission line is divided into number of sections with equal distances then all resistances are equal and all capacitances are equal.

$$R_1 = R_3 = \cdots\cdots = R_{n+1}$$
$$C_2 = C_4 = \cdots\cdots = C_n.$$

In a general way a unit length of transmission line is considered for simplicity in calculations and later on this can be extended to any length of the line. Similarly the resistance and capacitance are also calculated for per unit values.

The aim of the problem is to find out the responses at the intermediate points of the transmission line. So the number of intermediate points are always less than one to the number of sections divided over the length of the line. n represents number of sections. The transfer impedance function Z(s) when the line is divided into five number of equal sections is given in (3.2.13) as

$$Z(s) = \frac{5\,s^2 + 750\,s^2 + 31250\,s + 312500}{s^4 + 138.56\,s^2 + 4774\,s^2 + 36613\,s}$$

This transfer function can also be written as in (2.4.2) after applying the first cauer form to get the component values in RC network

$$Z(s) = \frac{1}{5}s + \cfrac{1}{\cfrac{1}{2.288} + \cfrac{1}{\cfrac{1}{9.169}s + \cfrac{1}{\cfrac{1}{5.746} + \cfrac{1}{\cfrac{1}{29.833}s + \cfrac{1}{4.2 + \cfrac{1}{\cfrac{1}{72.98}s}}}}}}\ .$$

where $C_2 = 0.2\ f$

$C_4 = 0.109\ f$

$C_6 = 0.0335\ f$

$C_8 = 0.0137\ f$

$R_3 = 0.437$ ohm

$R_5 = 0.174\ \Omega$

$R_7 = 4.2\ \Omega$

The transmission line represented with the above values is shown in Fig.(4.1).

When the transmission line is divided into six number of equal sections the transfer function from (3.2.15) is as

$$Z(s) = \frac{6\,s^4 + 1728\,s^3 + 163331\,s^2 + 5598928\,s + 50384642}{s^5 + 296\,s^4 + 26593\,s^3 + 788386\,s^2 + 5465924\,s}$$

Or

$$Z(s) = 0.166\,S + \cfrac{1}{0.75 + \cfrac{1}{0.0036\,S + \cfrac{1}{1.36 + \cfrac{1}{0.044\,S + \cfrac{1}{2.428 + \cfrac{1}{0.01\,S + \cfrac{1}{4.098 + \cfrac{1}{0.007\,S.}}}}}}}}$$

$C_2 = 0.166$ f

$C_4 = 0.0036$ f $R_3 = 0.75$ ohm

$C_6 = 0.044$ f $R_5 = 1.36$ ohm

$C_8 = 0.01$ f $R_7 = 2.428$ ohm

$C_{10} = 0.007$ f $R_9 = 4.098$ ohm

The transmission line represented for 5th order feedback compensated transfer function is shown in Fig. (4.2).

The transfer impedance Z(s) when the line is divided in to seven number of sections is given in (3.2.17) as

$$(s) = \frac{7S^5 + 3430S^4 + 605052S^3 + 461185S^2 + 1412391500S + 11864370000}{S^6 + 542.8S^5 + 99650S^4 + 7350048S^3 + 199992314S^2 + 1348662200S}$$

Or

Z(s) can also be written in the form (2.4.2). From that the component values are as follows.

$C_2 = 0.142$ f

$C_h = 0.031$ f $R_3 = 0.132$ ohm

$C_6 = 0.042$ f $R_5 = 0.634$ ohm

$C_8 = 0.0006$ f $R_7 = 6.221$ ohm

$C_{10} = 0.013$ f $R_9 = 3.126$ ohm

$C_{12} = 0.0005$ f $R_{11} = 5.448$ ohm

The transmission line with these values is represented in Fig.(4.3). In all three cases the first and last elements are capacitors. This method may be extended for any number of sections divided over the length of transmission line.

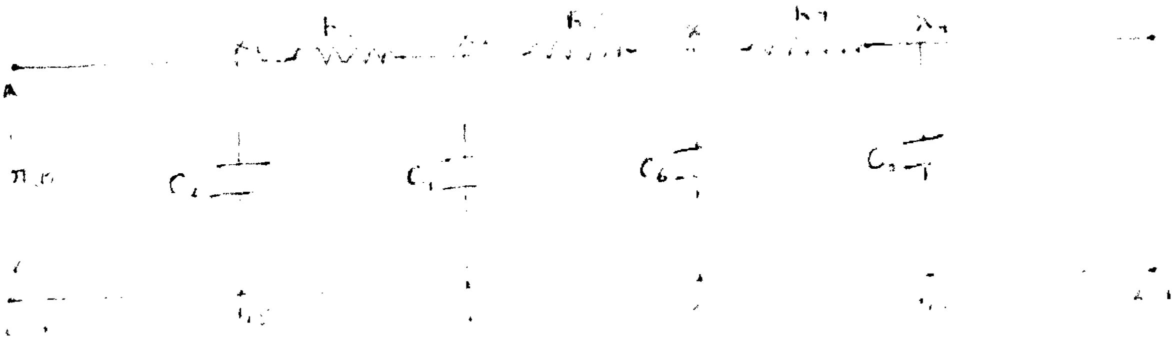
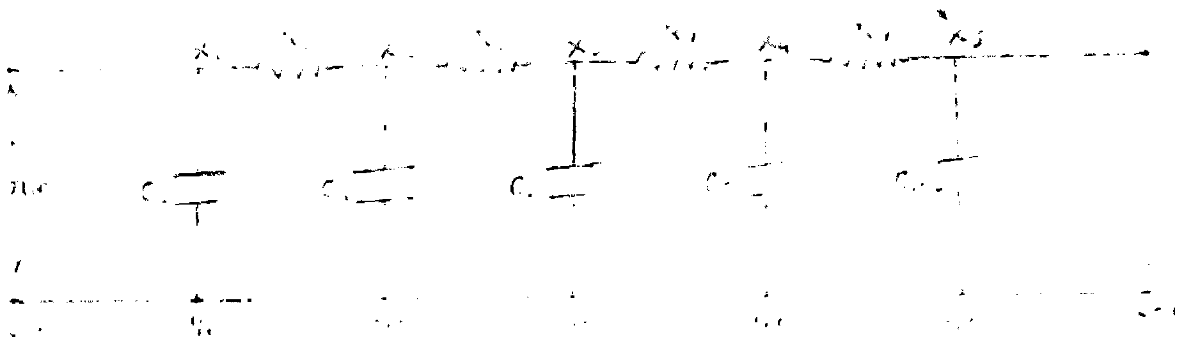


FIG [illegible] 1



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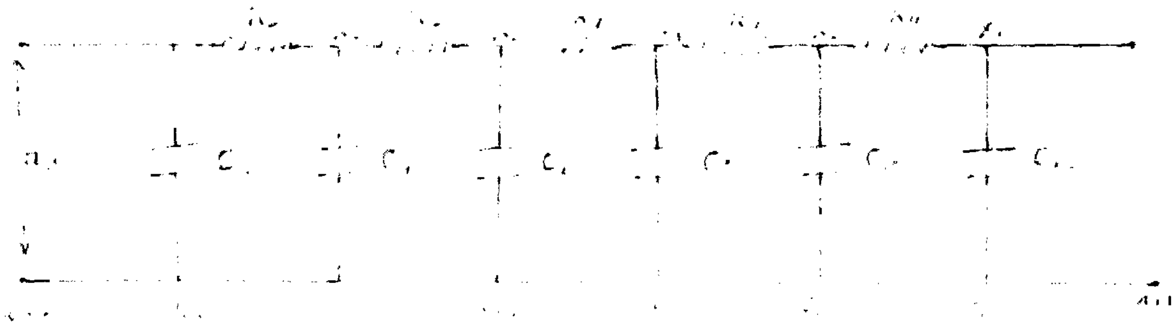


FIG [illegible] 3.

CHAPTER - 5

RESULTS AND DISCUSSION

The results are shown graphically for the RC transmission line equation of diffusion type. The responses shown, are at the intermediate points of interest over the length of the transmission line. Also seen how these responses vary when the line is divided into more different number of equal sections. For example when the line is divided into five equal sections in length, there will be four intermediate points by excluding the input and output terminals. These responses are called for convenience as X_1, X_2, X_3 and X_4. When the line is divided into six number of equal lengths there are five intermediate points called as X_1, X_2, X_3, X_4 and X_5 and so on. When the response at the first state 'X_1' is compared in both the cases, the response X_1 in the second set builds up quickly than in the first set of X_1 in the equal time internal probably because of less distance. The velocity of wave propagation is almost equal to the velocity of light. Even with this velocity of propagation the response over the transmission line builds up at the second junction 'X_2' only after first junction 'X_1' assumed and so on.

By taking the response of the RC transmission line equation as reference, another approximate model is developed by applying state space technique. The order of the state equation depends on the number of sections made. It is always less than one to the number of sections. Because we are not interested in the output state at the output terminal. The responses on the states of this state equation are calculated and compared with those of RC transmission line at the same intermediate points. From the graphs it is seen that the responses from the approximate state equation are comparitively poor with the RC network responses. The eigen values of the state equation are more negative compared with the reference approach eigen values.

Now the poles of the state equation are shifted exactly to the poles of RC transmission line equation by applying the state variable feedback compensation technique. The eigen values of state equation and that of RC transmissionline are same. In order to shift these eigen values to appropriate locations a feed back controllor gain matrix $K^T = [K_1\ K_2\ .....]$ is designed. Again the responses of state equation after poles shifted to the desired locations are calculated and conpared with the two responses already calculated. In each graph all the three responses are shown at the same intermediate point when the line is divided into equal sections in length. As indicated in the graphs, 'D' represents the response of the RC transmission line equation of diffusion type, 'A' represents the state equation response without feddback and 'M' with feedback. It is seen from the graphs the response after feedback compensation improves slightly even over the diffusion type transmission line response. It is not necessary to go for another new method to get the responses at the intermediate points just for this slight improvement. But the state space technique is a recent trend and it has more advantageous over the conventional type of approach. Thus the state space approach has at least three major advantages compared to stright input-output analysis. They are (1) cenceptional clarity, (2) greater information about the system itself and (3) computational convenience.

FIG 2

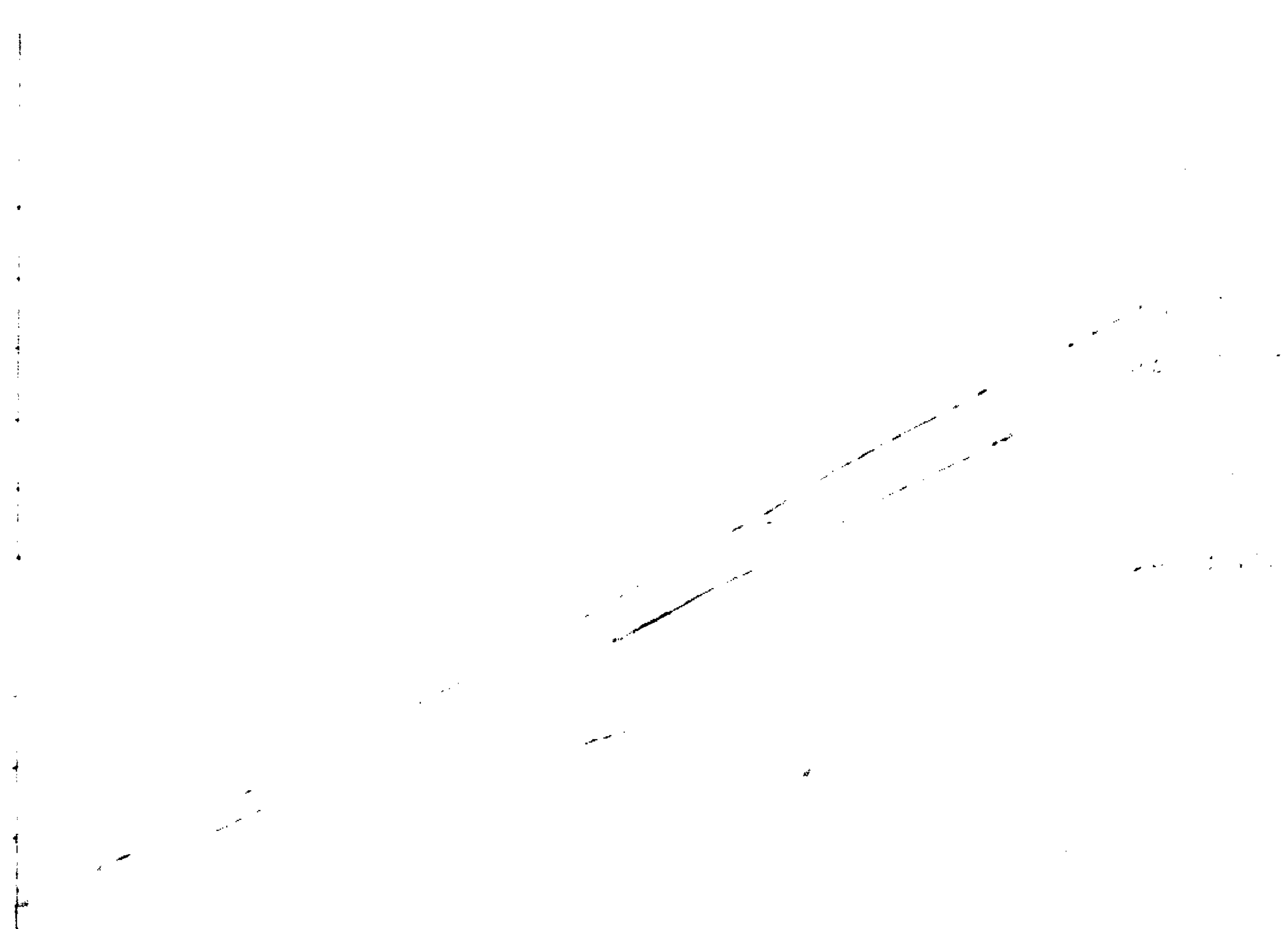


FIG. 4

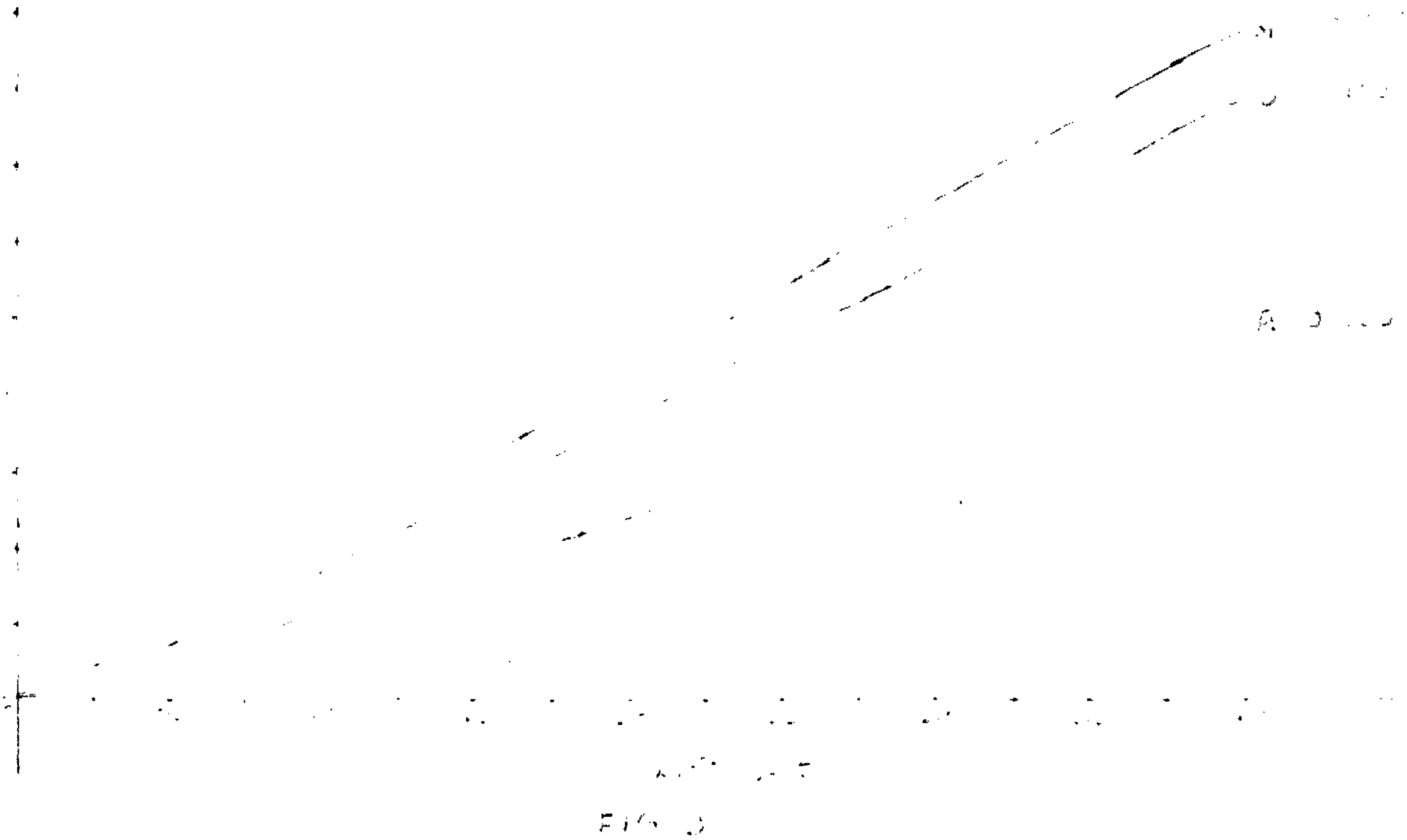
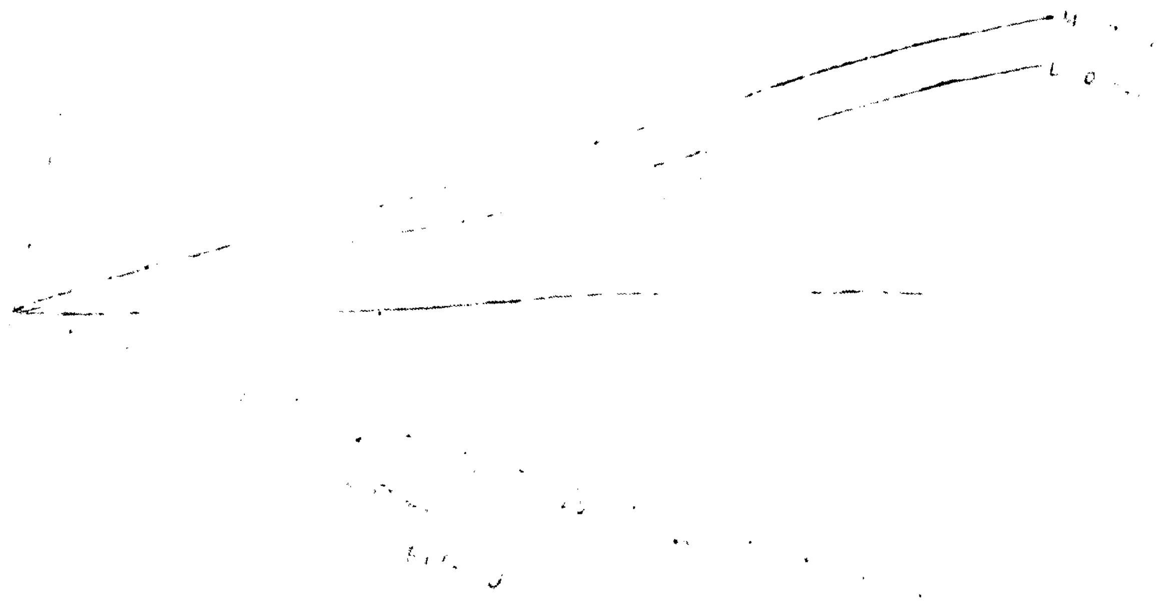


FIG. 5

FIG.



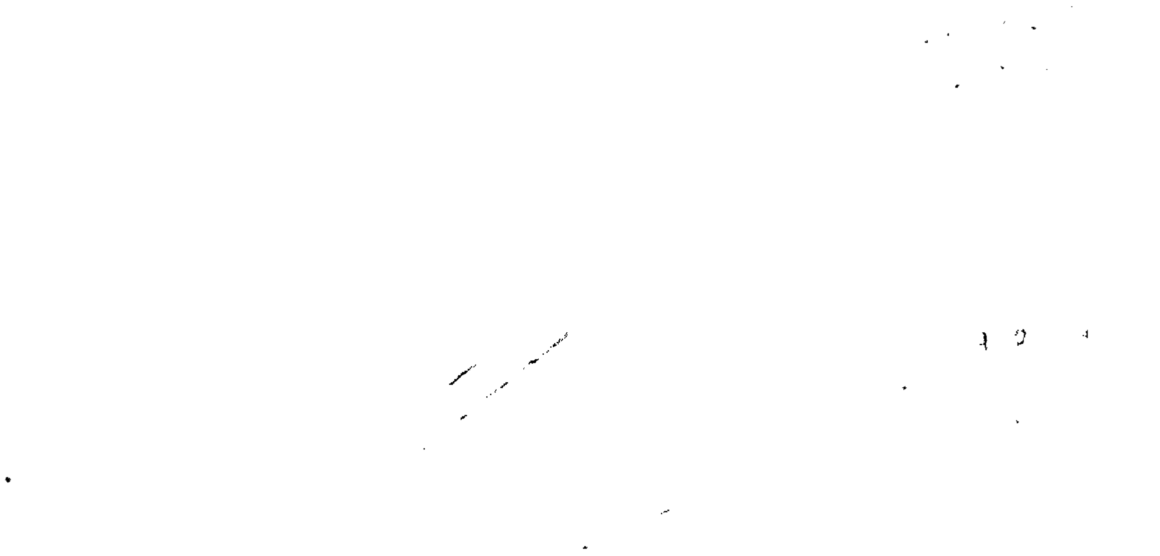


FIG 11

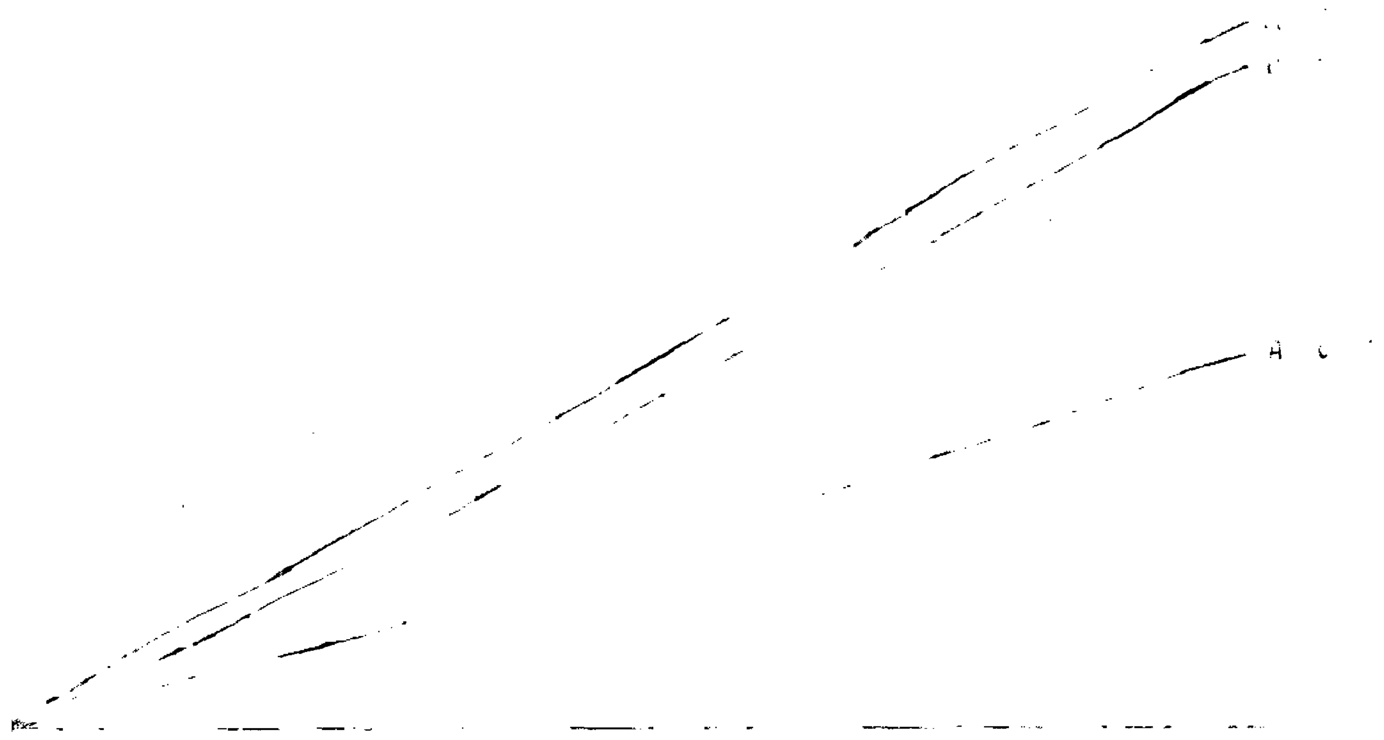


FIG 13

FIG 14

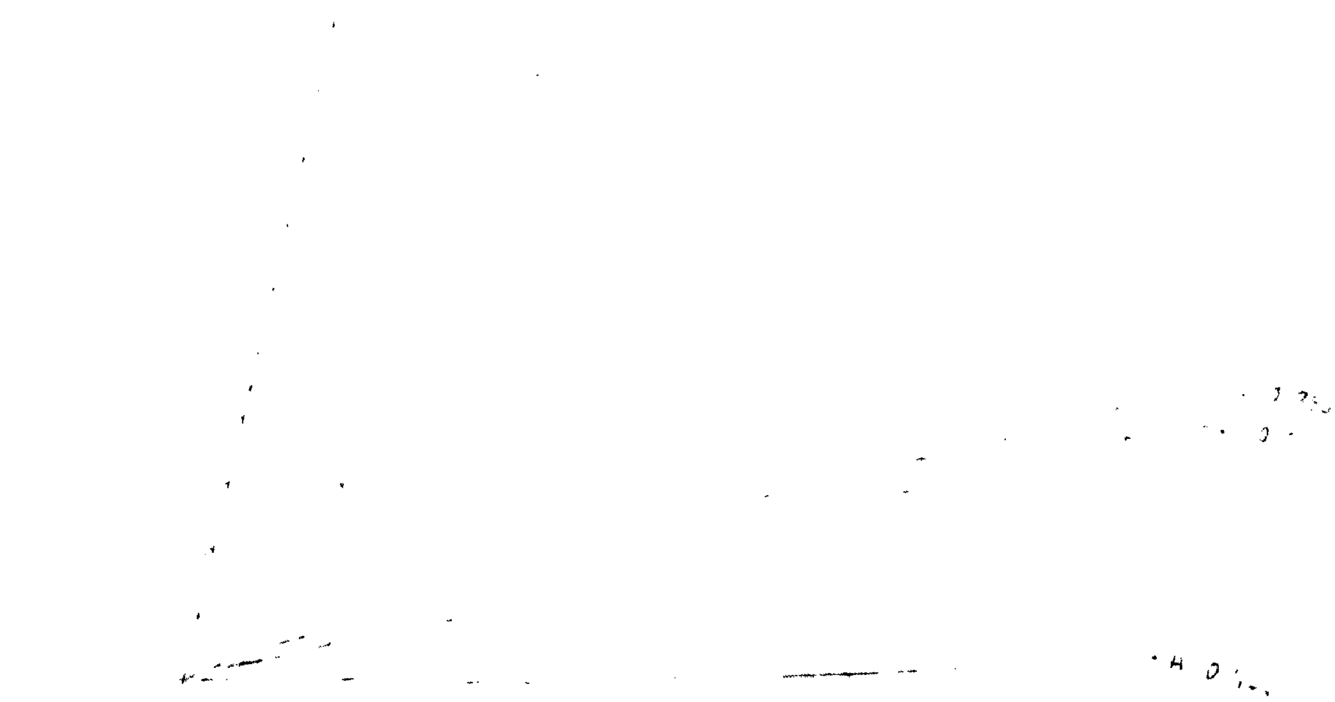


FIG 15

FIG 14

CONCLUSION

The results obtained from the state space technique with feedback and without feedback are compared with the results of a reference RC transmission line equation of diffusion type.

It is clear from the graphs that the results from state equation without feedback are poor where as with feedback compensation they are better comparitively with those obtained from RC transmission line equation.

As the number of sections of the transmission line increases, the responses with RC line equation and state equation with feedback are coming closer because number of eigen values considered are more.

Other advantages of this modal control theory are (1) conceptual clarity (2) greater information about the system and (3) computational convenience.

This recent technique has produced good responses out of all. So in conclusion it is suggested that it may be suitably applied to various systems for getting better responses.

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A P P E N D I X

3.1.1

```
C  C  CALCULATION OF Y AT VARIOUS POINTS OF X BY RAYUDU
      PI=3.1416
      N=6
      PUNCH 2
   2  FORMAT(5X,1HX,15X,1HT,15X,1HY)
  20  FORMAT(5X,E10.4,5X,E10.4,5X,E15.5)
      X=0.6664
   5  T=0.1
   6  Y=0.0
      DO 10 K=1,N
      AK=K
      PKX=AK*PI
      P2K2=PKX*PKX
      PK2X=-(P2K2)*T
      PKX=PKX*X
  10  Y=Y+2.0*(COSF(PKX)*(1.0-EXPF(PK2X)))/P2K2
      Y=T+Y
      PUNCH 20,X,T,Y
      T=T+0.1
      IF(T-5.0)6,6,15
  15  X=X+0.1666
      IF(X-0.833)5,5,30
  30  STOP
      END
```

3.2.1

```
C  C  EIGEN VALUE AND EIGEN VECTOR CALCULATION BY RAYUDU
      DIMENSION A(10,10),B(10,10)
      READ 1,N
    1 FORMAT(I7)
      READ2,((A(I,J),J=1,N),I=1,N)
    2 FORMAT(12F6.2)
      CALL EIGEN(A,B,N,10)
      PUNCH 4
    4 FORMAT(20X,13HEIGEN  VALUES)
      PUNCH3,(A(I,I),I=1,N)
      PUNCH 5
    5 FORMAT (20X,14HEIGEN  VECTORS)
      PUNCH 3,((B(I,J),J=1,N),I=1,N)
    3 FORMAT(2X,4E13.6)
      STOP
      END
```

3.2.3

```
C  C  DETERMINING K BY RAYUDU
      DIMENSION A(10,11),B(10)
      READ 1,N
      READ 2,C1,C2,C3,C4,C5,C6,C7,C8,C9,C11,C12,C13,C14,C15,C16,C17,
     1C18,C19,C21,C22,C23
      PI=9.87
      DO 5 I=1,N
      M=I-1
      XM=M
      XMS=XM**2
      XS=XMS*(-PI)
      A(I,1)=C1*XS**(N-1)+C2*XS**(N-2)+C3*XS**(N-3)+C4*XS**(N-4)+
     1C5*XS**(N-5)+C6
      A(I,2)=C7*XS**(N-2)+C8*XS**(N-3)+C9*XS**(N-4)+C11*XS**(N-5)+C6
      A(I,3)=C12*XS**(N-3)+C13*XS**(N-4)+C14*XS**(N-5)+C6
      A(I,4)=C15*XS**(N-4)+C16*XS**(N-5)+C6
      A(I,5)=C17*XS**(N-5)+C6
      A(I,6)=C6
    5 B(I)=-(XS**N+C18*XS**(N-1)+C19*XS**(N-2)+C21*XS**(N-3)+
     1C22*XS**(N-4)+C23*XS**(N-5))
      N1=N+1
      DO 6 I=1,N
    6 A(I,N1)=B(I)
      CALL SOLEQN (A,N,10)
      PUNCH 3
      PUNCH 4,(A(I,N1),I=1,N)
    1 FORMAT (I2)
    2 FORMAT (5F15.2)
    3 FORMAT (2X,14HDESIRED ANSWER)
    4 FORMAT (2X,6E13.6)
      STOP
      END
```

3.2.2

```
C C  SIMULTANEOUS LINEAR DIFFERENTIAL EQUATIONS (X DOT=A*X+F*W)-
C    SOLUTION BY RUNGE-KUTTA 4TH ORDER METHOD*SUBRATO*17692*RAYUDU
C    ALSO DETERMINATION OF OUTPUT YO=C*X+G*W
     DIMENSION X(6),D(6),Z(24),V(12),U(12),Y(2),B(4),A(36),F(12),W(2)
     DIMENSION YO(4),C(24),G(8)
     B(1)=.5
     B(2)=.5
     B(3)=1.
     B(4)=0.
 100 READ 1001,N,LL,M
     NL=N*LL
     NN=N*N
     READ 1002 ,S,TO,TF
     READ 1001,MM
     MN=MM*N
     ML=MM*LL
     READ 1002,(C(I),I=1,MN)
     READ 1002,(G(I),I=1,ML)
     READ1002,(F(I),I=1,NL)
     READ 1002,(W(I),I=1,LL)
     READ 1002,(X(I),I=1,N)
     READ 1002,(A(I),I=1,NN)
     PRINT 2001,N,LL,M,S,TO,TF
     PRINT 2002,TO
     PRINT 2003,(X(I),I=1,N)
     PRINT 2004
     PRINT 2003,(A(I),I=1,NN)
     PRINT 2005,(W(I),I=1,LL)
     PRINT 2006
     PRINT 2003,(F(I),I=1,NL)
     T=TO
     EP=2.*ABS (TF-TO)
     E3=EP
     DT=S
     Y(1)=T
     DO  101  I=1,N
 101 D(I)=0.
     DO 102 I=1,N
     DO 102 J=1,N
     IJ=(I-1)*N+J
 102 D(I)=D(I)+A(IJ)*X(J)
     DO 110 I=1,N
     DO 110 J=1,LL
     IJ=(I-1)*LL+J
 110 D(I)=D(I)+F(IJ)*W(J)
     DO 103 I=1,N
     V(I)=D(I)
 103 U(I)=X(I)
     L=0
 104 KI=-N
     DO 108 K=1,4
     KI=KI+N
     DO 105 I=1,N
     IK=KI+I
     Z(IK)=DT*V(I)
 105 X(I)=U(I)+B(K)*Z(IK)
     T=B(K)*DT+Y(1)
```

```
      DO 106 I=1,N
  106 D(I)=0.
      DO 107 I=1,N
      DO 107 J=1,N
      IJ=(I-1)*N+J
  107 D(I)=D(I)+A(IJ)*X(J)
      DO 120 I=1,N
      DO 120 J=1,LL
      IJ=(I-1)*LL+J
  120 D(I)=D(I)+F(IJ)*W(J)
      DO 108 I=1,N
  108 V(I)=D(I)
      DO 109 I=1,N
      I2=I+N
      I3=I2+N
      I4=I3+N
  109 U(I2)=U(I)+(Z(I)+2.*(Z(I2)+Z(I3))+Z(I4))/6.
      Y(2)=Y(1)+DT
      DO 111 I=1,N
      I2=I+N
  111 X(I)=U(I2)
      T=Y(2)
      DO 112 I=1,N
  112 D(I)=0.
      DO 113 I=1,N
      DO 113 J=1,N
      IJ=(I-1)*N+J
  113 D(I)=D(I)+A(IJ)*X(J)
      DO 130 I=1,N
      DO 130 J=1,LL
      IJ=(I-1)*LL+J
  130 D(I)=D(I)+F(IJ)*W(J)
      DO 114 I=1,N
      I2=I+N
  114 V(I2)=D(I)
      E=ABS (TF-Y(1))
      IF (E-EP) 115,115,126
  115 EP=E
      KT=1
  116 E=ABS(TF-Y(KT))
      IF(E-E3) 117,123,123
  117 E3=E
      KN=(KT-1)*N
      DO 118 I=1,N
      KM=KN+I
  118 X(I)=U(KM)
      DO 119 I=1,N
  119 D(I)=0.
      DO 121 I=1,N
      DO 121 J=1 ,N
      IJ=(I-1)*N+J
  121 D(I)=D(I)+A(IJ)*X(J)
      DO 140 I=1,N
      DO 140 J=1,LL
      IJ=(I-1)*LL+J
  140 D(I)=D(I)+F(IJ)*W(J)
      L=L+1
      IF (L-M) 123,122,123
```

```
 122 PRINT 2007,Y(KT)
     PRINT 2003,(X(I),I=1,N)
     L=0
     DO 150 I=1,MM
     YO(I)=0.
     DO 150 J=1,N
     IJ=(I-1)*N+J
 150 YO(I)=YO(I)+C(IJ)*X(J)
     DO 160 I=1,MM
     DO 160 J=1,LL
     IJ=(I-1)*LL+J
 160 YO(I)=YO(I)+G(IJ)*W(J)
     PRINT 2003,(YO(I),I=1,MM)
 123 KT=KT+1
     IF (KT-2) 116,116,124
 124 Y(1)=Y(2)
     DO 125 I=1,N
     I2=I+N
     V(I)=V(I2)
 125 U(I)=U(I2)
     GO TO 104
 126 PRINT 2008
     GO TO 100
1001 FORMAT (3I2)
1002 FORMAT (8F10.5)
2001 FORMAT (2X,2HN=,I2,3HLL=,I2,2HM=,I2,2HS=,E13.6,3HTO=,E13.6,3HTF=,
    1F13.6)
2002 FORMAT (2X,13HSTATES AT TO=,E13.6)
2003 FORMAT (2X,6E13.6)
2004 FORMAT (2X,13HSYSTEM MATRIX)
2005 FORMAT (2X,22HDISTURBANCE QUANTITIES,2E13.6)
2006 FORMAT (2X,18HDISTURBANCE MATRIX)
2007 FORMAT (2X,15HSTATES AT TIME=,E13.6)
2008 FORMAT (2X,12HPROGRAM ENDS)
     END
```