ON SOME ASPECTS OF COMPONENT TOLERANCE IN NETWORK-DESIGN

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C E R T I F I C A T E

Certified that the Dissertation entitled, "ON SOME ASPECTS OF COMPONENT TOLERANCE IN NETWORK- DESIGN' which is being submitted by Shri Gurbax Singh in partial fulfilment for the award of the degree of Master of Engineering in System Engineering and Operations Research of Electrical Engineering of University of Roorkee, Roorkee is a record of the student's own work carried out by him under our supervision and guidance. The matter embodied in this Dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of eight months from January 1975 to July 1975 and April 1976 for preparing this dissertation at this University.

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A C K N O W L E D G E M E N T

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A B S T R A C T

In the mass production of circuits where 100 per cent yield is not required, but an estimate of yield based on individual component behaviour assists in production decisions the tolerance analysis becomes the final phase of circuit design.

Practically it is very difficult to have the precise valued elements. More the precision, higher is the cost, so to reduce the overall circuit cost, the slackness in element values is allowed. But higher the slackness, more is the probability of circuit response to violate the specified limits i.e., the circuit fails. So to avoid circuit failure, tolerance analysis is required. As cost and the circuit response are the functions of tolerance, we optimise the cost of the circuit subject to circuit respond constraints.

In this study two itrative algorithms are presented for the tolerance assignment in a given frequency domain. The first algorithm is for continuous tolerance case and the second is for the discrete tolerance case. The presented algorithms can be used for both types of circuits whose element values are either correlated or uncorrelated and has the feature of allowing the designer to specify a circuit yield of less than 100 per cent and can handle any type of probability distribution. Two problems have been considered. The first problem is of a band pass filter circuit to illustrate the

case of uncorrelated parameters for both continuous and discrete element tolerance assignment with normal distribution. And the second problem is of a three transistor low pass amplifier used to illustrate the case of integrated circuits, i.e., with correlated parameters.

C O N T E N T S

INTRODUCTION

The final step in most circuit design is a tolerance analysis, i.e., a study of the effect of parameter variation on circuit behaviour. There are two distinctive applications where such information is vital: (1) in the space and Military electronics field, where it is surely worth the effort to eliminate any possibility of failure, and (2) in the mass production of circuits, where 100 per cent yield is not required, but an estimate of yield based on individual component behaviour assists in production decisions. Also when the system response needed is to be within some tolerable limits, it becomes possible for the individual elements to have its values to lie within some tolerable limits. For mass production it is very uneconomical to produce elements with precise values, the cost of a component decreases with the increase of tolerance limits. So for the economical mass production of components, tolerance analysis is very important.

Totally unpredictable failures can be prepared for only by redundancy in the circuit design. Therefore, our concern in this thesis is with failures that are predictable in some sense. Such failures tend to arise in one of the following ways.

(i) Catastropic failure, which is abrupt and results from large changes in certain components due to environmental stress such as radiation, shock, or temperature extreme; if the change in component values is known, these stresses can be simulated by analyzing the network with new element values;

(2) Drift failures, which result from small gradual changes in component values; the component changes typically can be predicted to lie within certain tolerance limits; such tolerance information is then used in design to avoid failure;

(3) Random failure, occurring in manufacturing process due to a circuit containing an excess of marginal components; associated with each component is a nominal value and a probability distribution giving the likelihood the value lies within a range near the nominal; this statistical information can also be used in the design phase.

When electrical circuits are designed for mass production, it is essential that a high percentage of the manufactured circuits operate within prescribed specifications, i.e., a high yield of manufactured circuits must be obtained. In order to meet the yield objective and also produce circuits at as low a cost as possible, it is necessary to optimize both the circuit response tolerances and the circuit cost.

Generally there are two types of tolerance assignment problems. The first type is where the circuit elements can only be assigned a finite number of discrete tolerance values. Such a situation is encountered in the design of circuits using lumped components. The second type of problem is where the circuit element tolerance may be assigned any value, i.e., it is a continuous variable. This situation arises in the design of integrated circuits.

Various algorithms have been developed for the assignment of element tolerances. Monte carlo tolerance analysis has proven to be a useful tool in evaluating the effects of component tolerances and environmental variations on electrical circuit performance. The method involves 'constructing' samples of the circuit inside the computer useing element values, that obey the manufacturing statistics, analysing these samples and forming empirical distributions of performance. One common outcome of the process is the prediction of yield. Monte carlo analysis hence forth referred to as TAP, is an open loop structure. At the conclusion of the tap run one observes yield and is faced with one of two:situations:

(1) the yield is too low. With this result the designer knows he must change his tolerances. Unfortunately TAP gives him little information as to which tolerances to change and by how much.

(2) Yield is adequate. Here the designer may be satisfied

by the design but he obtains little help in determining weather a cheaper set of tolerances might not give equally satisfactory yield. An algorithm for the discrete case has been proposed by Karafin[4]. This technique forms tables of pairs of discrete tolerance values for which circuit performance remains acceptable. An efficient tree search procedure is then employed to find the set of tolerance values which satisfied the performance criterion and yields the least cost. A worst case and a Monte Carlo analysis are used to verify this choice of tolerances. If either of these tests fails, the next least costly set of tolerances is tried. This method assumes the element values are uncorrelated and that a 100 percent yield is desired. Seth and Roe[5] describes an algorithm for continuous tolerance case which minimizes circuit cost. The algorithm makes use of higher order moment equations to predict response variations from element tolerances and a final verification of the resulting design useing a Monte-Carlo analysis. A similar approach has also been presented by Pinel and Roberts[6]. In particular, the tolerance assignment problem is recast into a nonlinear programming problem which uses the Fletcher-Powell optimization algorithm to minimize circuit cost. The algorithm yields a worst case design as a by-product. Again, this algorithm assumes a desired yield of 100 percent. A.R. Therbjornsen and S.W. Director[7] in their paper assigned two new iterative algorithms for tolerance assignment in the frequency domain. The first algorithm is for the continuous tolerance case and has the feature of allowing the designer to specify a circuit yield of less than 100 percent. For circuits whose element values are uncorrelated, the algorithm can handle any type of probability distribution. Normal distributions are assumed for circuits having correlated elements. A Monte Carlo analysis[8], which take into account element value correlation, is incorporated into the main iteration loop. The frequent use of Monte Carlo analysis is made less economically objectionable by useing a variable sample size, the sample size remains small until the algorithm nears convergence, which time the sample size is increased to obtain the needed accuracy. Use of a Monte Carlo analysis is justified by the fact that it is the most dependable of the tolerance analysis methods. The second tolerance assignment algorithm is for the discrete tolerance case in which the circuit elements have uncorrelated values.

Here in this present study, two more iterative algorithms are presented for the tolerance assignment in a given frequency domain. The first algorithm is for continuous tolerance case and the second is for the discrete tolerance case, for both type of circuits whose element values are either correlated or uncorrelated and has the feature of allowing the designer to specify a circuit yield of less than 100 per cent and can handle any type of probability distribution.

Introduction to

(i) Tolerance Analysis from the Sensitivity approach

Let a network variable f(i.e. V or I) be considered a function of parameters p_i which have nominal values p_i^o:

$$f = f(p_1, p_2, \ldots, p_n)$$
$$= f(p) \qquad (1)$$

For small changes Δp_i, the change in f is

$$\Delta f = \sum_{i=1}^{n} \Delta p_i \left.\frac{\partial f}{\partial p_i}\right|_{p_i = p_i^o} \qquad (2)$$

Such derivative information is typically used in one of three ways:

1. If the Δp_i are known precisely, Δf may be estimated directly from equation (2)

2. If the Δp_i are assumed to lie within the tolerance limits ϵ_i, i.e.,

$$-\epsilon_i \leq \Delta p_i \leq \epsilon_i$$

Then the largest possible variation in f may be estimated as

$$\Delta f_{max} = \sum_{i=1}^{n} (\pm \epsilon_i) \frac{\partial f}{\partial p_i}$$

Where the sign is chosen the same as that of the derivative; if Δf_{max} represents acceptable behaviour, the circuit is said to be a worst-case design.

3. If Δp_i are described by probability distributions, then

a distribution for Δf may be estimated by the method of moments.

(2) Independence and correlation

i. Two random variables X and Y and said to independent iff

$$Pr\{[a \leq X \leq b \text{ and } c \leq Y \leq d]\} = Pr\{[a \leq X \leq b]\} Pr\{[c \leq Y \leq d]\}$$

ii. The correlation coefficient associated with two random variables X and Y is defined (for unclassified data)

$$\rho_{XY} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{n \, \sigma_{\bar{X}} \, \sigma_{\bar{Y}}}$$

where $\bar{X}$, $\bar{Y}$, $\sigma_{\bar{X}}$ and $\sigma_{\bar{Y}}$ are the means and standard deviations associated with X and Y respectively.

iii. Independent variables are uncorrelated, but uncorrelated variables need not be independent.

iv. If the variance of h(x) is σ^2, the variance of h(ax) is $a^2\sigma^2$. Where h(x) is the probability density distribution function of random variable x. This property shows the effect of scaling.

v. If random variables X_1, $X_2, \ldots, X_n$ with variances σ_1^2, σ_2^2, $\ldots, \sigma_n^2$ are described by the correlation matrix

$$\underline{R} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & & & \\ \rho_{m1} & \rho_{n2} & \cdots & 1 \end{bmatrix}$$

Then the variance σ_X^2 of the random variable

$$X = \sum_{i=1}^{n} X_i$$

is given by

$$\sigma_X^2 = \underline{\sigma}^T \underline{R}\ \underline{\sigma}$$

Where $\underline{\sigma} = \left[\sigma_1 , \sigma_2 \ldots \sigma_n\right]^T$.

If $\underset{\sim}{R} = \underset{\sim}{I}$ so that the variables are uncorrelated, this equation becomes

$$\sigma_X^2 = \sum_{i=1}^{n} \sigma_i^2$$

(3) Method of Moments

Returning to the equation

$$\Delta f = \sum_{i=1}^{n} \Delta p_i \frac{\partial f}{\partial p_i} \qquad (4)$$

Suppose we are given the variances σ_i^2 of the distributions associated with Δp_i and are interested in the variance σ_f^2 associated with the variable Δf. If the partial derivatives are real numbers, they may be considered as scaling factors for the distributions of Δp_i. According to the statement (3) we may define the scaling factor $a_i = \partial f / \partial p_i$, so that $\Delta p_i' = a_i p_i$ and the standard deviation σ_i' of $\Delta p_i'$ is

$$\sigma_i' = a_i \sigma_i$$

But now from equation (4), Δf is the sum of random variables $\Delta p_i'$. Therefore

$$\sigma_f^2 = \left[\frac{\partial f}{\partial p_1}\sigma_1 \ \ldots \ \frac{\partial f}{\partial p_n}\sigma_n\right] \underset{\sim}{R} \begin{bmatrix} \frac{\partial f}{\partial p_1}\sigma_1 \\ \vdots \\ \frac{\partial f}{\partial p_n}\sigma_n \end{bmatrix}$$

$$= \left[S_1\sigma_1 \ , \ S_2\sigma_2, \ \ldots \ S_n\sigma_n\right] \underset{\sim}{R} \begin{bmatrix} S_1\sigma_1 \\ S_2\sigma_2 \\ \vdots \\ S_n\sigma_n \end{bmatrix}$$

$$= \underline{\sigma} \begin{bmatrix} S_1^2, & \rho_{12}S_1S_2, & \ldots & \rho_{1n}S_1S_n \\ \rho_{21}S_2S_1, & S_2^2, & \ldots & \rho_{2n}S_2S_n \\ \vdots & & & \\ \rho_{n1}S_nS_1, & \rho_{n2}S_nS_2, & \ldots & S_n^2 \end{bmatrix} \underline{\sigma}^T$$

Where $S_i = \dfrac{\partial f}{\partial p_i}$, the system sensitivity w.r.t p_i.

After multiplying the matrices we get

$$\sigma_f^2 = \sum_{i=1}^{n} S_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} \rho_{ij}\, S_iS_j\sigma_i\,\sigma_j$$

If all the variables are uncorrelated

$$\text{i.e. } \rho_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

we get

$$\sigma_f^2 = \sum_{i=1}^{n} S_i^2 \sigma_i^2$$

Thus, the variance of the response may be estimated from the variance of element variations if the sensitivities are given.

STATEMENT OF PROBLEM

(A) Statement of the problem:

Assume we are given a circuit and a set of n nominal element values, denoted by p_k^o, $k = 1,2,\ldots,n$. The magnitude of the response of this network at a set of m discrete frequency points within some frequency range of interest is denoted by A_i^o; $i = 1,2, \ldots,m$. The set of responses $\left\{A_i^o \mid i = 1,2,\ldots,m\right\}$ is called 'nominal circuit response'. We are interested in determining the set of element tolerances $\left\{t_k \mid k=1,2,\ldots,n\right\}$ (in percent). Such that 1) A yield specification is met, i.e. a given percentage of circuits have response $\left\{A_i \mid i=1,2,\ldots,m\right\}$ which deviates no move than some specified amount from the nominal response $\left\{A_i^o \mid i = 1,2,\ldots,m\right\}$ for all possible values of the circuit elements

$$\left\{p_k = (1 \pm \epsilon t_k/100)p_k^o \,\middle|\, 0 \leq \epsilon \leq 1,\ k = 1,2, \ldots,n\right\}; \text{ and}$$

2) A minimum cost of the circuit is achieved.

Before coming to the actual problem we introduce some additional notation. Let $\hat{A}_i^U$ and $\hat{A}_i^L$ denotes the specified upper and lower limits on the magnitude of the circuit response, respectively, at the ith frequency points. Observe that

$$\hat{A}_i^L \leq A_i^o \leq \hat{A}_i^U$$

(B) The Performance function:

In this method we minimize the circuit cost w.r.t., the element standard deviation, such that the magnitude of the circuit response remains within the specified limit,i.e., we fix some constrains which keeps into account the circuit response limits as well as the yield. The cost of the circuit element is roughly inversely proportional to its tolerance, total circuit cost, the cost function, is represented by

$$f(t) = \sum_{k=1}^{n} \frac{\alpha_k}{t_k} \qquad \ldots \qquad (1)$$

Where α_k is the cost factor of the kth element

It is important to recognize the relation between tolerance and standard deviation. If σ_{pk} denotes the standard deviation associated with the kth element, then the element tolerance and the standard deviation are related by

$$t_k = \beta_k \sigma_{pk}$$

Where β_k is a constant which depends upon the probability distribution associated with the element value. For a normal distributtion, essentially all possible values lie with in $3\sigma_{pk}$ points. The most extreme values the kth element can assume are

$$p_k^o \pm 3\sigma_{pk} = p_k^o(1 + \frac{t_k}{100})$$

So that for this case

$$t_k = 300\, \sigma_{pk}/p_k^o$$

Now we turn to the yield requirement. Let σ_{A_i} denote the standard deviation of the magnitude of the circuit response at the ith frequency and $\hat{\sigma}_{A_i}$ denote the desired (i.e., maximum allowable) value of σ_{A_i}. The desired $\hat{\sigma}_{A_i}$ is calculated from the specified upper and lower limits of response at the frequency and taking yield into consideration. For a normal distribution a curve is plotted for yield against the ratio of half the total response tolerance to the desired standard deviation of the response. From the curve for a given yield, corresponding ratio(γ) is traced out, which intern gives the desired $\hat{\sigma}_{A_i}$

$$\sigma_{A_i} = \frac{T_i}{\gamma} \qquad \ldots \qquad (3)$$

Where T_i is half the total response tolerance and is

$$T_i = \frac{1}{2}\,(\hat{A}_i^U - \hat{A}_i^L) \qquad \ldots \qquad (4)$$

The response standard deviations σ_{A_i} may be approximated from knowledge of the element value standard deviations with a truncated form of the propogation of the variance equation(4)

$$\sigma_{A_i}^2 = \sigma_p^T \left[X^i \right] \sigma_p \qquad \ldots \qquad (5)$$

where $\sigma_p = (\sigma_{p_1}, \sigma_{p_2}, \ldots, \sigma_{p_n})^T$ and $\left[X^i\right]$ is a symmetric matrix. The element of $\left[X^i\right]$ are

$$x^i_{jk} = \rho_{jk}\, S^i_j\, S^i_k \qquad \ldots \qquad (6)$$

Where ρ_{jk} is the correlation coefficient between the jth and kth elements, and S^i_j and S^i_k are the sensitivities w.r.t. the jth and kth element at ith frequency

$$\text{Where} \quad S^i_k = \left.\frac{\partial A_i}{\partial p_k}\right|_{p^o_k} \qquad \ldots \qquad (7)$$

The partial derivatives are easily and economically evaluated using the modified adjoint circuit technique |2|

For the required yield specification to be met, in all cases

$$\sigma^2_{A_i} \leq \hat{\sigma}^2_{A_i} \qquad \ldots \qquad (8)$$

σ_{A_i} and $f_1(t)$, useing eq.(2) are function of σ_{pk}.

The total performance function is

$$\text{Min} \quad f_1(t) = f(\sigma_p) = \sum_{k=1}^{n} \frac{\alpha_k}{t_k}$$

Subject to

$$\sigma^2_{A_i} \leq \hat{\sigma}^2_{A_i} \;\Big|\; i = 1,2, \ldots, m.$$

Minimization of the Performance Function

A good first estimate of the best element standard deviations results from minimization of the cost function subject to the given constraints. The solution of the problems involving large systems is hampered by size; the problem is simply too big. In such cases, a common approach is to decompose the original problem into subproblems, and solve these. For the interaction between the subproblems, it proposes multilevel approach. For the constrained optimization, Lagrangian function is formed, as for the problem

$$\text{minimize} \quad f(\sigma_p)$$

$$\text{Subject to} \qquad \sigma_{A_i}^2(\sigma_p) \leq \hat{\sigma}_{A_i}^2 \qquad i = 1,2, \ldots m$$

Given this problem, we define a Lagrangian function

$$L(\sigma_p, u) = f(\sigma_p) + u(\sigma_A^2(\sigma_p) - \hat{\sigma}_A^2)$$

where $\sigma_A^2(\sigma_p = (\sigma_{A_1}^2(\sigma_p), \sigma_{A_2}^2, \ldots, \quad (\sigma_{A_m}^2(\sigma_p)),$

$$u = (u_1, u_2, \ldots, u_m)$$

and a second Lagrangian problem

$$\text{minimize} \quad L(\sigma_p, u)$$

with $u \geqslant 0$, related to the primal by some theorms.

For any set of values of u we minimize the Lagrangian function and we get the optimal values of σ_p as σ_p^*. Using these values we form a dual function

$$h(u) = L(\sigma_p^*, u) = \min_{\sigma_p} L(\sigma_p, u)$$

We maximize the dual function w.r.t. u, and the process repeats until a saddle point is achieved where the Lagrangian and the dual are equal i.e.

$$\min_{\sigma_p} \max_{u \geq 0} L(\sigma_p, u) = \max_{u \geq 0} \min_{\sigma_p} L(\sigma_p, u)$$

For h(u) differentiable, a steepest ascent algorithm, modified to handle the constraints $u \geq 0$, may be used to maximize h(u). This leads to a solution procedure

1. Choose initial values $u_0 \geq 0$, Step 1, i=0,1,2,..proceeds as foll(
2. Solve the Lagrangian problem with $u = u_i$, obtaining a solution $\sigma_p(u_i)$. In the seperable case, this may be accomplished by solving the subproblems.
3. Form the dual function $h(u_i) = L(\sigma_p(u_i), u_i)$ and its gradient $\nabla h(u_i) = \sigma_A^2(\sigma_p(u_i)) - \hat{\sigma}_A^2$
4. Define a direction of search S_i by

$$S_i^k = \begin{cases} \left.\dfrac{\partial h}{\partial u^k}\right|_{u_i} & \text{if } u_i^k > 0 \\ \max\left\{0, \left.\dfrac{\partial h}{\partial u^k}\right|_{u_i}\right\} & \text{IF } u_i^k = 0 \end{cases} \qquad k = 1, 2, \ldots, m.$$

Choose a new vector u_{i+1} by

$$u_{i+1} = u_i + \alpha_i S_i$$

The step size α_i must be selected so that

$$h(u_{i+1}) > h(u_i)$$

If h is differentiable there exists $\alpha_i > 0$ satisfying the above unless u_i maximizes h. A common procedure is to choose α_i to maximize

$$g(\alpha) = h(u_i + \alpha S_i)$$

subject to the constraints $\alpha_i \geq 0$ and $u_{i+1} > 0$

5. Return to step 2, stopping when $\alpha_i \simeq 0$.

If $g(\alpha)$ is to be maximized, it must be evaluated a number of times, requiring a solution of the lagrangion problem each time. Suggested procedure for performing this one-dimensional search are found in Fletcher and Powell[9] and Lasden and Waren[10]. Although theoretical convergence of this algorithm is eve-dently an open question at this time, the procedure is widely used and will generally converge to a global solution for convex problems.

This gradient procedure may be viewed as a coordination algorithm for a second level coordinator, whose task is to solve the dual problem given values of h(u) and $\nabla h(u)$. The first level units solve the subproblems and provide the values of h and ∇ h.

The Discrete Tolerance Design Algorithm

A second algorithm has been developed to assign tolerance values for the case, when the tolerances are discrete variables. The continuous tolerance design algorithm is used first to find t_c^A an approximate solution for the element tolerances. The continuous tolerance case is the idle case as the tolerance values may be in fractions to strictly satisfy the given set of constraints. So the optimum continuous set given the least possible cost for the given set of cost function and constraints so the optimum descrete tolerance set will have the cost $\geqslant$ the cost obtained by continuous tolerance set.

Also it is assumed that this locally optimum set of tolerances(t_c^A) is close to the desired locally optimum discrete tolerance set. So we round off or truncate the optimum continuous tolerance set in such a way so that it satisfies the constraints. For this set t^A, cost is found out. Now as the calculation of cost for a given set of tolerances is easier than the checking up of the constraints, We form a table of all the possible sets of tolerances which we get with combination of tolerances having values in neibourhood of the values the set of tolerances t^A and having cost between the costs, the sets t^A, tc^A has . From this table, then we, first take the set which has the least cost and check it for the yield specifications. if it satisfies the yield specifications, then neglect all other sets, because they are more costly, and we will try to form if possible some other sets with lesser cost. If this set violates the constraints, then we check the next least costly set, this process goes on until we get a least costly set which satisfies the yield requirements. This set of tolerances gives the optimal discrete tolerance set.

Design Examples: Continuous Case

Two examples are presented to illustrate the continuous tolerance design algorithm. The first circuit is the band pass fitter of Fig.1. The circuit has the nominal gain versus frequency response shown in Fig.2. We desire a set of element tolerances such that 100 percent of the manufactured circuits have responses that are within ± 1.0dB of the nominal response curve. Theother data for this example are given in the table 1.

The second circuit is the three-transister integrated circuit amplifier of Fig 3. The nominal gain versus frequency response curve and the allowed tolerance is shown in Fig.4a. We desire a yield of 100 per cent and assume that all elements have normal distribution. All the resistors are assumed to be interdependent, with a correlation coefficient of 0.9. The transistor β'S are also assumed to be interdependent with a correlation coefficient of 0.8. The correlation between resistors and transistors β'S is assumed to be negligible. Other datas for this example are given in Table 7.

Discrete Tolerance Design Example

To illustrate the discrete tolerance design algorithm, the beand-pass filter of Fig 1 is used. The solution of example 1 is used as the approximate tolerance set t^A. Other data for this third example is given in Table 1.

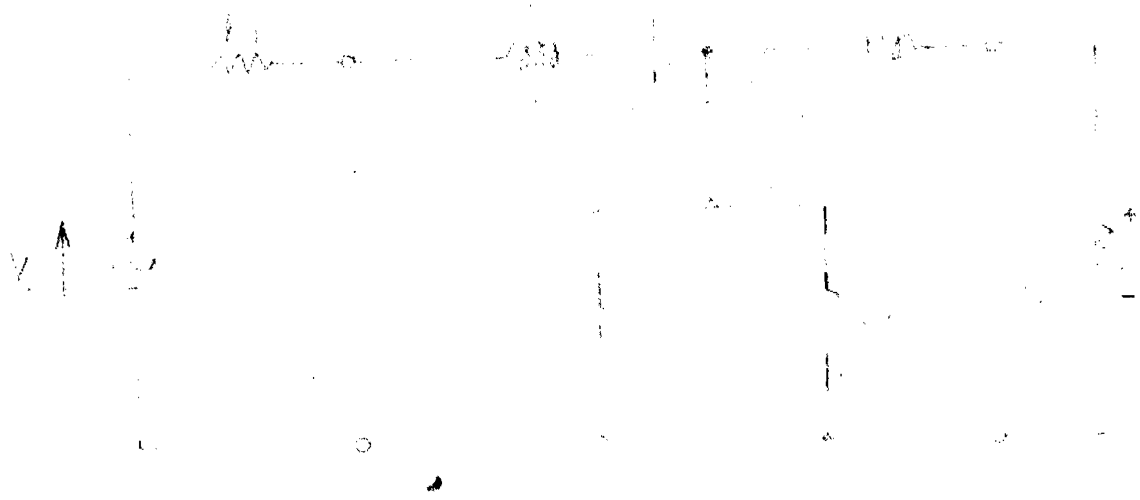

Fig.1. Band Pass Filter Circuit

Frequency

Fig.2. Response to Frequency Curve

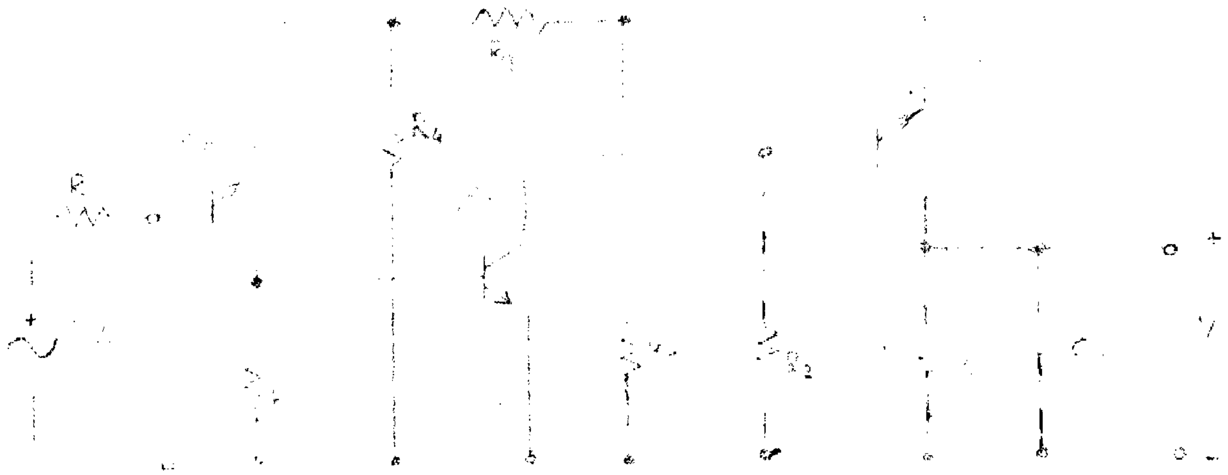

Fig.3 Three Transistor Low Pass Amplifier Used to Illustrate the Case of Integrated Circuits.

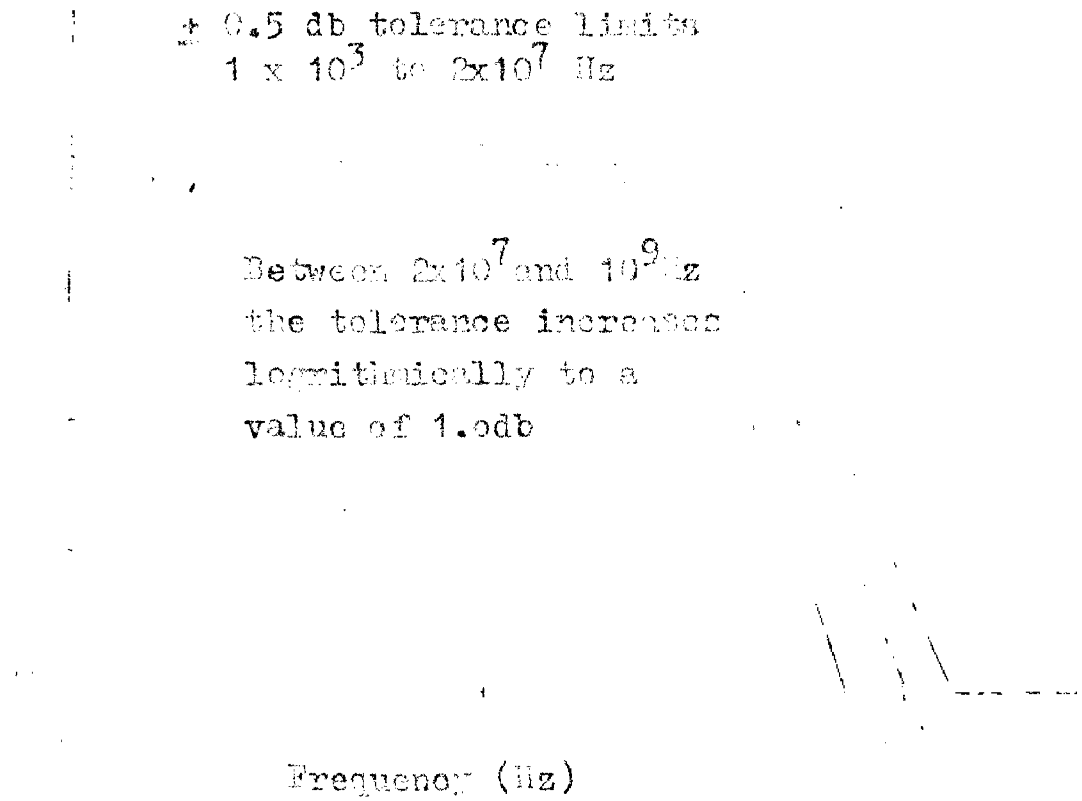

Fig.4. Response to Frequency Curve.

EXAMPLE 1

Table 1

k	Element	Nominal value p_k	Upper Tolerance Limit T_k^U	Lower Tolerance Limit T_k^L	Cost Factor α_k
1	L_1	5.0mH	50 percent	2 percent	5
2	L_2	5.0mH	50 "	2 "	5
3	L_3	1.25mH	50 "	2 "	5
4	C_4	0.25µF	50 "	2 "	2
5	C_5	0.25µF	50 "	2 "	2
6	C_6	[illegible].0 µF	50 "	2 "	2

The gain of the circuit of Fig.1 is

$$A = \frac{V_2}{V_1} = \frac{R_2}{\left\{\left[R_1 + R_2\right]\right\} + (X_5 - X_8)\left\{\left[R_1(X_4 - X_7) + R_2(X_3 - X_6)\right]\right\} + j\left[(X_3 + X_4 - X_6 - X_7) + (X_8 - X_5)\left\{\left[R_1 R_2 - (X_3 - X_6)(X_4 - X_7)\right]\right\}\right]}$$

∴ The Absolute gain of the circuit(Nominal response value)

$$\left|\frac{V_2}{V_1}\right| = \frac{R_2}{\sqrt{\left[(R_1 + R_2) + (X_5 - X_8)\left\{\left[R_1(X_4 - X_7) + R_2(X_3 - X_6)\right]\right\}\right]^2 + \left[(X_3 + X_4 - X_6 - X_7) + (X_8 - X_5)\left\{\left[R_1 R_2 (X_3 - X_6)(X_4 - X_7)\right]\right\}\right]^2}}$$

Where

$X_3 = wL_3$ $\quad X_6 = 1/wC_6$

$X_4 = wL_4$ $\quad X_7 = 1/wC_7$

$X_5 = 1/wL_5$ $\qquad$ $X_8 = w\,C_8$

For the response to frequency curve, we considered ten different frequency points and the corresponding nominal response values are computed taking the nominal element values.

The maximum allowable upper and lower response values are computed from the maximum tolerance limit of ± 1.0DB as under

Let Nominal response value A_i in db = $20 \log_{10} A_i = x$ say -i

∴ Upper response value A_i' in db = $20 \log_{10} A_i' = x+1$ -ii

and Lower " " A_i'' " = $20 \log_{10} A_i'' = x-1$ -iii

From i, ii and iii, we get

$A_i = 10^{x/20}$, $A_i' = 10^{(x+1)/20}$ i and i $A_i'' = 10^{(x-1)/20}$

$\frac{A_i'}{A_i} = 10^{1/20}$ or $A_i' = A_i \times 10^{1/20} = 1.122\,A_i$

$\frac{A_i''}{A_i} = 10^{-1/20}$ or $A_i'' = A_i \times 10^{-1/20} = 0.8912\,A_i$

A_i, A_i' and A_i'' are given in the table 2.

Considering the normal distribution, the standard deviation

$$\hat{\sigma}_i = \frac{A_i' - A_i''}{6}$$

Hence the maximum allowable response variance given by $\hat{\sigma}_i^2$ is also tabulated in table 2.

Table 2

Response No. i	Frequency H_Z	Nominal Response Value A_i	Upper Response Value A_i'	Lower Response Value A_i''	Max. Allowable Response Variance $\hat{\sigma}_i^2$
1	1003	0.002278	0.002556	0.002030	7.689×10^{-9}
2	2071	0.03505	0.03933	0.03124	1.8167×10^{-6}
3	3134	0.3286	0.3687	0.2929	1.5989×10^{-4}
4	3598	0.4838	0.5428	0.4312	3.467×10^{-4}
5	4530	0.5000	0.5610	0.4456	3.7×10^{-4}
6	5446	0.4938	0.5541	0.4401	3.61×10^{-4}
7	6253	0.3814	0.4279	0.3399	2.151×10^{-4}
8	6856	0.2388	0.2679	0.2128	8.444×10^{-5}
9	9465	0.04091	0.0459	0.03646	2.474×10^{-6}
10	19080	0.002757	0.003093	0.002457	1.122×10^{-8}

Now, we know that the total cost is

$$C = \sum_{k=1}^{6} \frac{\alpha_k}{t_k} \qquad \ldots \qquad (i)$$

We have to optimise this function under the following constraints.

The actual responsevariance for frequency i $\leq$ The max. allowable response variable for frequency i.

i.e. $\sum_{k=1}^{6} s_{ik}^2 \, \sigma_{pk}^{\ 2} \leq \hat{\sigma}_i^2$ $\quad (i = 1,2,\ldots,10)$ .. (ii)

Now we have $t_k = \sigma_{pk} \cdot \beta_k$; and for Normal Distribution,

$\beta_k = 300/p_{ok}$ ... (iii)

From ii and iii we get

$$\sum_{k=1}^{6} s_{ik}^2 \; \frac{p_{ok}^2 \; t_k^2}{(300)^2} \leq \hat{\sigma}_i^2$$

Multiplying both sides by $(3000)^2$, we get

$$\sum_{k=1}^{6} (10 s_{ik} p_{ok})^2 \; t_k^2 \leq (3000 \; \hat{\sigma}_i)^2$$

or $\sum_{k=1}^{6} S_{ik}^2 \; t_k^2 \leq \sigma_{A_i}^2$ $\quad (i = 1,2,\ldots,10)$ ... (iv)

Where S_{ik} is the sensitivity constant $= 10 \; p_{ok} \cdot s_{ik}$

σ_{A_i} is the response variance constant $= 3000 \; \hat{\sigma}_i$

S_{ik} and σ_{A_i} are computed for different values of i and k and are given in the table 3.

For the fulfilment of constraints of equations(ii) the equations(iv) must satisfy. So we will optimise the cost function under the constraints (iv).

From i and iv the Lagrangian function so formed is

$$L(t_k, u) = \sum_{k=1}^{6} \alpha_k / t_k + \sum_{i=1}^{10} u_i \left(\sum_{k=1}^{6} S_{ik}^2 \; t_k^2 - \sigma_{A_i}^2 \right)$$

OR $$L(t_k,u) = \sum_{k=1}^{6} (\alpha_k/t_k + t_k^2 \sum_{i=1}^{10} u_i S_{ik}^2) - \sum_{i=1}^{10} u_i \sigma_{A_i}^2$$

For optimal values, after differenting, and simplification we get

$$t_k = \left(\frac{\alpha_k}{2 \sum_{i=1}^{10} u_i S_{ik}^2} \right)^{1/3}$$

Putting the values of α_k, and also $S_{i1} = S_{i2}$ and $S_{i4} = S_{i5}$ we get

$$t_1 = t_2 = \left[\frac{2.5}{\sum_{i=1}^{10} u_i S_{i1}^2} \right]^{1/3} ; \quad t_4 = t_5 = \left[\frac{1}{\sum_{i=1}^{10} u_i S_{i4}^2} \right]^{1/3}$$

$$t_3 = \left[\frac{2.5}{\sum_{i=1}^{10} u_i S_{i3}^2} \right]^{1/3} ; \quad t_6 = \left[\frac{1}{\sum_{i=1}^{10} u_i S_{i6}^2} \right]^{1/3}$$

$$\partial h/\partial u_i = \sum_{k=1}^{6} S_{ik}^2 t_k^2 - \sigma_{A_i}^2$$

Table 3

i	$\sigma_{A_i}^2$	s_{i1}^2	s_{i2}^2	s_{i3}^2	s_{i4}^2	s_{i5}^2	s_{i6}^2
1	0.0692	1.42×10^{-6}	1.42×10^{-6}	6.09×10^{-4}	5.75×10^{-4}	5.75×10^{-4}	1.5×10^{-6}
2	16.35	8.9×10^{-3}	8.9×10^{-3}	0.27	0.198	0.198	0.012
3	14.39	6.56	6.56	84.3	28.0	28.0	19.8
4	3120	11.68	11.68	136.0	28.5	28.5	55.7
5	3330	12.6	12.6	49.5	12.35	12.35	50.8
6	3249	23.3	23.3	54.4	10.9	10.9	116.0
7	1936	34.4	34.4	31.6	9.26	9.26	117.5
8	760	15.1	15.1	7.02	2.8	2.8	37.8
9	22.27	0.28	0.28	0.02	0.0144	0.0144	0.393
10	0.101	8.4×10^{-4}	8.4×10^{-4}	2.72×10^{-6}	2.65×10^{-6}	2.65×10^{-6}	9.08×10^{-4}

Case I: Continuous tolerance assignment

Subcase I: No lower tolerance limit of 2 per cent.

The tolerance limits are computed by taking the different values of constants in different iterations following the algorithm discussed earlier. The different iterations are tabulated in table 4.

Table 4

Iterations	I	II	III	IV	V	VI	VII	VIII
u_1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
u_4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_7	0.0	0.0005	0.00075	0.00085	0.00093	0.00097	0.00098	0.000985
u_8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_{10}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_1	7.25	4.72	4.26	4.12	4.02	3.97	3.96	3.955
t_2	7.25	4.72	4.26	4.12	4.02	3.97	3.96	3.955
t_3	3.1	2.92	2.85	2.82	2.8	2.792	2.79	2.788
t_4	3.29	3.13	3.06	3.03	3.01	3.0	3.0	3.0
t_5	3.29	3.13	3.06	3.03	3.01	3.0	3.0	3.0
t_6	3.7	2.33	2.1	2.03	1.98	1.995	1.95	1.948
C	4.75	5.97	6.365	6.507	6.616	6.666	6.676	6.68

Contd...

Table 4 (Contd.)

Iterations	I	II	III	IV	V	VI	VII	VIII
$\partial h/\partial u_1$	- 0.0507	-0.0527	-0.0534	- 0.0537	- 0.054	- 0.054	- 0.0541	- 0.0541
$\partial h/\partial u_2$	- 8.54	-9.683	-10.08	- 10.2	-10.31	-10.34	-10.355	-10.362
$\partial h/\partial u_3$	658.3	229	92	53.6	18.5	6.0	2.0	0.0
$\partial h/\partial u_4$	782	-577	-811	-882	-943	-962	-970	-973
$\partial h/\partial u_5$	574	-1826	-2014	-2055	-2113	-2129	-2133	-2136
$\partial h/\partial u_6$	1536	- 897	-1242	-1346.5	-1417	-1448	-1457	-1462
$\partial h/\partial u_7$	3790	693	261	136.5	52	12.5	3.6	- 0.4
$\partial h/\partial u_8$	1474.6	235	64.3	15.5	- 18.2	-32.2	- 37.5	-39
$\partial h/\partial u_9$	13.1	- 7.17	-9.86	- 10.73	- 11.27	-11.5	- 11.56	-11.61
$\partial h/\partial u_{10}$	-0.00022	-0.0585	- 0.0664	-0.069	-0.0702	-0.0708	- 0.0801	-0.0802

Subcase II: There is a lower tolerance limit of 2 per cent. First four iterations remain same as in subcase I and other iterations are tabulated in Table 5.

Table 5

Iterations	IV	V	VI	VII	VIII	IX
u_1	0.0	0.0	0.0	0.0	0.0	0.0
u_2	0.0	0.0	0.0	0.0	0.0	0.0
u_3	0.001	0.001	0.001	0.001	0.00099	0.000985
u_4	0.0	0.0	0.0	0.0	0.0	0.0
u_5	0.0	0.0	0.0	0.0	0.0	0.0
u_6	0.0	0.0	0.0	0.0	0.0	0.0
u_7	0.00085	0.00093	0.00098	0.001	0.00102	0.001025
u_8	0.0	0.0	0.0	0.0	0.0	0.0
u_9	0.0	0.0	0.0	0.0	0.0	0.0
u_{10}	0.0	0.0	0.0	0.0	0.0	0.0
t_1	4.12	4.02	3.96	3.94	3.915	3.91
t_2	4.12	4.02	3.96	3.94	3.915	3.91
t_3	2.82	2.80	2.79	2.785	2.785	2.79
t_4	3.03	3.01	3.0	2.995	2.995	3.0
t_5	3.03	3.01	3.0	2.995	2.995	3.0
t_6	2.03	2.0	2.0	2.0	2.0	2.0

Contd...

Table 5(Contd.)

Iterations	IV	V	VI	VII	VIII	IX
C	6.507	6.603	6.647	6.662	6.683	6.685
$\partial h/\partial u_1$	- 0.0537	- 0.054	-0.0541	-0.0541	-0.0541	-0.0541
$\partial h/\partial u_2$	-10.2	-10.3	-10.362	-10.384	-10.37	-10.36
$\partial h/\partial u_3$	53.6	20.2	4.7	- 0.8	- 1.1	0.7
$\partial h/\partial u_4$	-882	- 938.2	-961.2	-968.2	-969.2	-968.2
$\partial h/\partial u_5$	-2055	- 2108.8	-2125.3	-2130.8	-2133	-2134.3
$\partial h/\partial u_6$	-1346.5	- 1408	-1437.1	-1446.2	-1451.1	-1453
$\partial h/\partial u_7$	136.5	61.2	24.5	13.3	1.9	-0.2
$\partial h/\partial u_8$	15.5	- 15	-30.9	- 36.2	- 40.9	- 41.3
$\partial h/\partial u_9$	-10.73	- 11.23	-11.52	- 11.6	- 11.69	- 11.72
$\partial h/\partial u_{10}$	- 0.069	- 0.07	- 0.071	- 0.071	- 0.071	- 0.0716

Case II : Discrete Tolerance Assignment

In this algorithm, discrete values of tolerances are assigned to the different elements. The discrete optimal set of tolerances lies in the close neighbourhood of the continuous optimal set of tolerances. So first, we find this discrete set of tolerances, which satisfies the constraints and its cost becomes the upper limit of cost and the optimal cost obtained in the continuous tolerance case becomes the lower limit of cost for discrete tolerance calculations. Lower limit on the element tolerance value is of 2 per cent. So, we form all the possible set of tolerances, which satisfy the cost constraints.

In this case the lower limit on the cost is 6.685 units and the upper limit on the cost is 7.33 corresponding to the set $\{4,4,2,3,3,2\}$

As there are two set of cost coefficients, so, we form the subsets by permutation of three element tolerances. The subsets so formed are tabulated in table 6-A with their costs corresponding to different cost coefficients. The subsets so formed, having one combination are grouped together as these have the same cost, and are represented by an alphabet as shown in table 6A.

The actual sets of discrete tolerances satisfying the cost constraints are formed by combinationsof two subsets.

The cost of the set is the sum of the individual costs of two subsets. The cost of the first subset in the combination corresponds to $\alpha_k = 5$, and the cost of second subset in the combination corresponds to $\alpha_k = 2$. The sets so obtained are tabulated in table 6-B, with their costs.

From this table, first we take the set having the minimum cost and check it for system constraints. If it satisfies the constraints, then this set is the optimal set of tolerances, if not we neglect this set and try the next set, now having the minimum cost. Proceeding in this way we get the optimal set of discrete tolerances.

For this problem the optimal set is $\left\{\left[4,4,3,2,2,2\right]\right\}$ having optimal cost = 7.17.

Table 6 - A

Groups of subsets of tolerances having one combination	Cost of subsets $\alpha_k = 5$	$\alpha_k = 2$
a = (2, 2, 2)	7.5	3.0
b = (2,2,3),(2,3,2),(3,2,2)	6.67	2.67
c = (2,2,4),(2,4,2),(4,2,2)	6.25	2.5
d = (2,2,5),(2,5,2),(5,2,2)	6.0	2.4
e = (2,3,3),(3,2,3),(3,3,2)	5.83	2.33
f = (2,3,4),(2,4,3),(3,2,4) (3,4,2),(4,2,3),(4,3,2)	5.42	2.16
g = (2,3,5),(2,5,3),(3,2,5) (3,5,2),(5,2,3),(5,3,2)	5.15	2.06
h = (2,4,4),(4,2,4),(4,4,2)	5.00	2.00
i = (2,4,5),(2,5,4),(4,2,5) (4,5,2),(5,2,4),(5,4,2)	4.75	1.9
j = (2,5,5),(5,2,5),(5,5,2)	4.5	1.8
k = (3,3,3)	5.0	2.0
l = (3,3,4),(3,4,3),(4,3,3)	4.6	1.84
m = (3,3,5),(3,5,3),(5,3,3)	4.35	1.74
n = (3,4,4),(4,3,4),(4,4,3)	4.15	1.66
o = (3,4,5),(3,5,4),(4,3,5), (4,5,3),(5,3,4),(5,4,3)	3.90	1.56
p = (3,5,5),(5,3,5),(5,5,3)	3.65	1.46
q = (4,4,4)	3.75	1.5
r = (4,4,5),(4,5,4),(5,4,4)	3.5	1.4
s = (4,5,5),(5,4,5),(5,5,4)	3.25	1.3
t = (5,5,5)	3.0	1.2

Table 6-B

Combination of subsets	Cost	Combination of subsets	Cost	Combination of subsets	Cost
d,s	7.3	g,n	6.81	k,e	7.33
d,t	7.2	g,O	6.70	k,f	7.16
e,q	7.33	h,e	7.33	k,g	7.06
e,r	7.23	h,f	7.16	k,h	7.0
e,s	7.13	h,g	7.06	k,i	6.9
e,t	7.03	h,h	7.0	k,j	6.8
f,l	7.26	h,i	6.9	k,k	7.0
f,m	7.16	h,j	6.8	k,l	6.84
f,n	7.08	h,k	7.0	k,m	6.74
f,O	6.98	h,l	6.84	l,b	7.27
f,p	6.88	h,m	6.74	l,c	7.1
f,q	6.92	i,c	7.25	l,d	7.0
f,r	6.82	i,d	7.15	l,e	6.93
f,s	6.72	i,e	7.08	l,f	6.76
g,f	7.31	i,f	6.91	m,b	7.02
g,g	7.21	i,g	6.81	m,c	6.85
g,h	7.15	i,h	6.75	m,d	6.75
g,i	7.05	i,k	6.75	m,e	6.68
g,j	6.95	j,b	7.17	n,a	7.15
g,k	7.15	j,c	7.0	n,b	6.82
g,l	6.99	j,d	6.9	o,a	6.9
g,m	6.89	j,e	6.83	q.a	6.75

EXAMPLE 2

Table 7

K	Element	Nominal Value p_k	Upper Toler. Limit T_k^U	Lower Toler. limit T_k^L percent	Cost factor α_k	Element depend on	Correlation coefficient ρ	Slope of linear Relationship
1	R_1	10.6K	50percent	2	1.0	R_2	0.9	1.0
2	R_2	6 K	50 "	2	1.0	-	-	-
3	R_3	1.9K	50 "	2	1.0	R_2	0.9	1.0
4	R_4	290Ω	50 "	2	1.0	R_2	0.9	1.0
5	R_5	290Ω	50 "	2	1.0	R_2	0.9	1.0
6	R_6	2.5 K	50 "	2	1.0	R_2	0.9	1.0
7	C_7	1.0pf	50 "	2	1.0	-	-	-
8	β_8	120	60 "	10	1.0	β_9	0.8	1.0
9	β_9	120	60 "	10	1.0	-	-	-
10	β_{10}	120	60 "	10	1.0	β_9	0.8	1.0

For the response to prequency curve, we considered eighteen different frequency points and the corresponding nominal response values are computed taking the nominal element values.

The maximum allowable upper and lower response values are computed from the max. tolerance limit of ± 0.5 db for a frequency range of 1×10^3 to 2×10^7 H_Z and between 2×10^7 to 10^9 H_2 the tolerance increases logrithmically from ± 0.5db to ± 1.0db.

From the upper and lower response limits the maximum allowable response variance is computed. Table 7 shows the element values their upper and lower tolerance limits cost factor and correlation coefficient and Table 8 shows the nominal response values and maximum allowable response variances at different frequency points.

Response no. i	Frequency Hz	Nominal Response Value A_i	Upper Response Value A'_i	Lower Response Value A''_i	Max. Allowable Response variance $\hat{\sigma}_i^2$
1	$1x10^3$	72.45	76.74	68.39	1.937
2	$2x10^6$	72.57	76.91	68.55	1.941
3	$4x10^6$	72.92	77.27	68.87	1.960
4	$6x10^6$	73.47	77.80	69.34	1.988
5	$8x10^6$	74.21	78.61	70.06	2.033
6	$1.0x10^7$	75.08	79.52	70.88	2.072
7	$1.2x10^7$	76.01	80.54	71.78	2.133
8	$1.4x10^7$	76.88	81.47	72.61	2.178
9	$1.6x10^7$	77.53	82.13	73.20	2.244
10	$1.8x10^7$	77.72	82.32	73.37	2.233
11	$2.0x10^7$	77.21	81.75	72.86	2.192
12	$7.1568x10^7$	8.172	8.731	7.655	$3.222x10^{-2}$
13	$2.263x10^8$	0.3521	0.3791	0.3271	$7.755x10^{-5}$
14	$3.811x10^8$	0.08093	0.08784	0.07452	$4.944x10^{-6}$
15	$5.358x10^8$	0.03183	0.03486	0.02909	$9.267x10^{-7}$
16	$6.905x10^8$	0.01838	0.02029	0.01666	$3.667x10^{-7}$
17	$8.435x10^8$	0.01361	0.01515	0.01223	$2.378x10^{-7}$
18	$1.0x10^9$	0.01300	0.01266	0.01007	$1.867x10^{-7}$

For this problem the cost function is

$$C = \sum_{k=1}^{10} \alpha_k/t_k \qquad \ldots \qquad (1)$$

And the constraints are

$$\sum_{k=1}^{10} s_{ik}^2 \sigma_{pk}^2 + \sum_{k=1}^{10} \sum_{j=k+1}^{10} 2\rho_{jk} s_{ij} \cdot s_{ik} \sigma_{pj} \sigma_{pk} \leq \hat{\sigma}_i^2$$

$$(i = 1,2,\ldots,18) \qquad \ldots \qquad (2)$$

As in the previous example substituting $t_k \cdot p_{ok}/300$ for σ_{pk} in (2) we get

$$\sum_{k=1}^{10} s_{ik}^2 \frac{p_{ok}^2 \; t_k^2}{300} + \sum_{k=1}^{10} \sum_{j=k+1}^{10} 2\rho_{jk} s_{ij} \frac{p_{oj} \cdot t_j}{300} \cdot s_{ik} \frac{p_{ok} t_k}{300} \leq \hat{\sigma}_i^2$$

Multiplying both sides by $(300)^2$, we get

$$\sum_{k=1}^{10} (s_{ik} p_{ok})^2 \; t_k^2 + \sum_{k=1}^{10} \sum_{j=k+1}^{10} (2\rho_{jk} s_{ij} p_{oj} \; s_{ik} \; p_{ok}) t_k t_j \leq (300 \hat{\sigma}_i)^2$$

$$\text{OR} \sum_{k=1}^{10} S_k^{i\,2} t_k^2 + \sum_{k=1}^{10} \sum_{j=k+1}^{10} S_{jk}^i \; t_j \; t_k \leq \sigma_{A_i}^2 \qquad \ldots \qquad (3)$$

$$(i = 1,2,\ldots,18)$$

Where

S_k^i is the sensitivity constant at ith frequency $= s_{ik} p_{ok}$

S_{jk}^i is the sensitivity constant at ith frequency$= 2\rho_{jk} s_{ij} s_{ik} p_{oj} p_{ok}$

and σ_{A_i} is the response variance " " $= 300\,\hat{\sigma}_i$

S_k^{i2}, S_{jk}^i and $\sigma_{A_i}^2$ are computed for different values of i and k and are given in Table 9.

From (2) and (3), we get the Lagrangian function as

$$L(t_k,u)=\sum_{k=1}^{10}\frac{\alpha_k}{t_k}+\sum_{i=1}^{18}u_i\left\{\left[\left(\sum_{k=1}^{10}S_k^{i\,2}\,t_k^2+\sum_{k=1}^{10}\sum_{j=k+1}^{10}S_{jk}^{i}\,t_j t_k\right)-\sigma_{A_i}^2\right]\right\}$$

For optimal values, by differentiating and after some simplifications we get:

$$t_k=\left[\frac{\alpha_k}{\sum_{i=1}^{18}u_i\left\{\left[2\,S_k^{i\,2}+\sum_{\substack{j=1\\ j\neq k}}^{10}S_{jk}^{i}\,\frac{t_j}{t_k}\right]\right\}}\right]^{1/3}$$

and

$$\frac{\partial h}{\partial u_i}=\sum_{k=1}^{10}S_k^{i\,2}\,t_k^2+\sum_{k=1}^{10}\sum_{j=k+1}^{10}S_{jk}^{i}\,t_j\,t_k-\sigma_{A_i}^2$$

Initially in the first iteration, for calculating t_k we assume some ratios of t_j/t_k, then these values of t_k are reused for calculating the more accurate values of t_k. After one iteration for calculating the values of t_k, use the values of tolerances of preceeding iteration for the ratio of t_j/t_k. When we reaches the optimal value to have the more accurate answers, reuse, values thus obtained for the ratio of t_j/t_k for the same iteration. Different iterations are tabulated in table 10.

Table 9 The values given in the right hand top corners are the powers of ten by which the corresponding values are to be multiplied.

i	s_{i1}^2	s_{i2}^2	s_{i3}^2	s_{i4}^2	s_{i5}^2	s_{i6}^2	s_{i7}^2	s_{i8}^2	s_{i9}^2	s_{i10}^2
1	$1.05^{\times 10^{1}}$	1.13^{1}	3.122^{2}	4.15^{2}	4.1^{2}	5.25^{2}	1.3^{-6}	4.4^{-1}	8.72^{-1}	2.92^{-1}
2	1.05^{1}	1.14^{1}	3.13^{2}	4.16^{2}	4.12^{2}	5.26^{2}	5.19^{0}	4.43^{-1}	8.75^{-1}	2.93^{-1}
3	1.06^{1}	1.15^{1}	3.16^{2}	4.2^{2}	4.16^{2}	5.3^{2}	2.09^{1}	4.47^{-1}	8.84^{-1}	2.96^{-1}
4	1.08^{1}	1.16^{1}	3.21^{2}	4.27^{2}	4.22^{2}	5.35^{2}	4.75^{1}	4.54^{-1}	8.97^{-1}	3.00^{-1}
5	1.1^{1}	1.19^{1}	3.28^{2}	4.35^{2}	4.3^{2}	5.42^{2}	8.56^{1}	4.63^{-1}	9.15^{-1}	3.06^{-1}
6	1.13^{1}	1.21^{1}	3.35^{2}	4.46^{2}	4.41^{2}	5.50^{2}	1.36^{2}	4.73^{-1}	9.37^{-1}	3.13^{-1}
7	1.16^{1}	1.25^{1}	3.44^{2}	4.57^{2}	4.52^{2}	5.58^{2}	1.98^{2}	4.85^{-1}	9.6^{-1}	3.21^{-1}
8	1.18^{1}	1.27^{1}	3.52^{2}	4.67^{2}	4.62^{2}	5.64^{2}	2.73^{2}	4.97^{-1}	9.82^{-1}	3.29^{-1}
9	1.2^{1}	1.3^{1}	3.58^{2}	4.75^{2}	4.7^{2}	5.65^{2}	3.57^{2}	5.05^{-1}	9.99^{-1}	3.34^{-1}
10	1.21^{1}	1.3^{1}	3.59^{2}	4.77^{2}	4.72^{2}	5.59^{2}	4.47^{2}	5.08^{-1}	10.04^{-1}	3.36^{-1}
11	1.19^{1}	1.28^{1}	3.55^{2}	4.71^{2}	4.66^{2}	5.43^{2}	5.36^{2}	5.0^{-1}	9.9^{-1}	3.31^{-1}
12	1.34^{-1}	1.44^{-1}	3.97^{0}	5.28^{0}	5.22^{0}	2.95^{0}	3.73^{1}	5.62^{-3}	1.11^{-2}	3.71^{-3}
13	2.48^{-4}	2.67^{-4}	7.37^{-3}	9.8^{-3}	9.7^{-3}	9.09^{-4}	1.15^{-1}	1.05^{-5}	2.06^{-5}	6.89^{-6}
14	1.31^{-5}	1.41^{-5}	3.9^{-4}	5.18^{-4}	5.12^{-4}	1.78^{-5}	6.37^{-3}	5.52^{-7}	1.09^{-6}	3.64^{-7}
15	2.02^{-6}	2.18^{-6}	6.03^{-5}	8.01^{-5}	7.92^{-5}	1.41^{-6}	1.0^{-3}	8.52^{-8}	1.68^{-7}	5.63^{-8}
16	6.76^{-7}	7.28^{-7}	2.01^{-5}	2.67^{-5}	2.64^{-5}	2.85^{-7}	3.35^{-4}	2.85^{-8}	5.61^{-8}	1.88^{-8}
17	3.71^{-7}	3.99^{-7}	1.1^{-5}	1.46^{-5}	1.45^{-5}	1.05^{-7}	1.84^{-4}	1.56^{-8}	3.08^{-8}	1.03^{-8}
18	3.38^{-7}	3.64^{-7}	1.0^{-5}	1.34^{-5}	1.32^{-5}	6.82^{-8}	1.68^{-4}	1.42^{-8}	2.81^{-8}	9.4^{-9}

Contd. 42.

Table 9 (contd).

i	S^i_{21}		S^i_{23}	S^i_{24}	S^i_{25}	S^i_{26}	S^i_{98}	S^i_{910}	$\sigma^2_{A_i}$
1	6.21	0	3.38	3.9	3.88	4.39	9.92	8.05	174300
2	6.22	0	3.39	3.91	3.89	4.4	9.95	8.10	174700
3	6.24	0	3.43	3.95	3.93	4.43	1.008	8.15	176400
4	6.38	0	3.48	4.01	3.99	4.49	1.02	8.20	178900
5	6.51	0	3.55	4.09	4.07	4.57	1.04	8.50	183000
6	6.66	0	3.63	4.19	4.16	4.65	1.065	8.65	186500
7	6.83	0	3.72	4.29	4.27	4.74	1.09	8.90	192000
8	6.99	0	3.81	4.39	4.37	4.82	1.12	9.10	196000
9	7.11	0	3.87	4.47	4.44	4.87	1.134	9.23	202000
10	7.14	0	3.89	4.49	4.46	4.86	1.14	9.29	201000
11	7.05	0	3.84	4.43	4.4	4.75	1.125	9.16	197280
12	7.9	-2	4.3	4.96	4.93	3.71	1.26	1.025	2900
13	1.47	-4	7.99	9.21	9.16	2.81	2.36	1.91	6.98
14	7.74	-6	4.22	4.87	4.84	9.02	1.24	1.01	0.445
15	1.2	-6	6.53	7.53	7.49	1.0	1.915	1.56	0.0834
16	3.99	-7	2.18	2.51	2.5	2.59	6.4	5.18	0.033
17	2.19	-7	1.19	1.38	1.37	1.16	3.5	2.85	0.0214
18	2.0	-7	1.09	1.26	1.25	8.97	3.2	2.60	0.0168

Table 10

Iteration	I	II	III	IV	V	VI	VII
u_1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_{10}	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_{11}	1.0×10^{-5}	5.0×10^{-6}	2.5×10^{-6}	2.0×10^{-6}	1.1×10^{-6}	1.05×10^{-6}	1.04×10^{-6}
u_{12}	0.0	0.0	0.0	2.5×10^{-6}	60×10^{-6}	66.5×10^{-6}	66.5×10^{-6}
u_{13}	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_{14}	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_{15}	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_{16}	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_{17}	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u_{18}	0.0	0.0	0.0	0.0	0.0	0.0	0.0
t_1	14.8	16.65	23.5	25.1	27.2	27.05	27.1
t_2	7.6	9.58	11.1	15.3	16.6	16.58	16.6
t_3	4.98	6.28	7.9	8.27	8.97	8.95	8.96
t_4	4.56	5.75	7.24	7.53	8.17	8.15	8.16
t_5	4.58	5.78	7.28	7.57	8.21	8.17	8.20
t_6	4.36	5.5	6.92	7.35	8.38	8.41	8.44
t_7	4.53	5.72	7.2	6.3	5.62	5.47	5.475

Contd..44.

Table 10(Contd.)

Iteration	I	II	III	IV	V	VI	VII
t_8	36.1	45.5	57.3	60.0	60.0	60.0	60.0
t_9	32.1	40.5	51.0	45.5	50.3	50.2	50.2
t_{10}	39.9	50.3	60.0	60.0	60.0	60.0	60.0
C	1.406	1.115	0.8867	0.829	0.8252	0.8093	0.80723
$\partial h/\partial u_1$	-120090	-88350	-37800	-28400	-9160	-11450	-11180
$\partial h/\partial u_2$	-120300	-88550	-37700	-28350	-9050	-11310	-11020
$\partial h/\partial u_3$	-121800	-89900	-38900	-30900	-8750	-10990	-10750
$\partial h/\partial u_4$	-123500	-91100	-39400	-31500	-8670	-10530	-10330
$\partial h/\partial u_5$	-126400	-93300	-40400	-30500	-8580	-10420	-10130
$\partial h/\partial u_6$	-128300	-94200	-40000	-30400	-8220	- 8530	- 8340
$\partial h/\partial u_7$	-132000	-96950	-41000	-30850	-7990	- 8170	- 8000
$\partial h/\partial u_8$	-134200	-98000	-40200	-29900	-6210	- 6530	- 6240
$\partial h/\partial u_9$	-138800	-101700	-43000	-31750	-6980	-7560	- 7335
$\partial h/\partial u_{10}$	-136965	- 99404	-39600	29640	-2872	- 3615	- 3450
$\partial h/\partial u_{11}$	-132140	- 90580	-33180	-24800	+ 713	- 165	+ 2.0
$\partial h/\partial u_{12}$	-1591.6	- 825	+ 395	+ 867	+62.4	- 1.89	+ 1.2
$\partial h/\partial u_{13}$	- 4.23	- 2.62	+ 0.05	-0.012	-0.02	-0.025	-0.02
$\partial h/\partial u_{14}$	- 0.291	- 0.201	- 0.0555	-0.072	-0.091	-0.103	-0.0908
$\partial h/\partial u_{15}$	-0.0587	-0.04425	- 0.02115	- 0.0265	-0.0309	-0.0331	-0.0301
$\partial h/\partial u_{16}$	-0.02343	0.01783	- 0.00885	- 0.0115	-0.0143	-0.0173	-0.0154
$\partial h/\partial u_{17}$	-0.01473	-0.01082	- 0.00457	- 0.00716	-0.0109	-0.0135	-0.0116
$\partial h/\partial u_{18}$	-0.01245	-0.00991	-0.00583	-0.00673	-0.00735	-0.00805	-0.0079

CONCLUSION

The present study deals with the tolerance assignment problem in the electronic circuits having discrete or continuous components tolerances. In other algorithms where the tolerance to different elements are assignmented such that the circuit response remains with in the specified limits, there is a large possibility of getting tight tolerances, which inturn gives higher circuit cost. In this algorithm we directly optimise the cost w.r.t. the element tolerance subject to the circuit response constraints, therefore there is no possibility of tight tolerances.

In the presented method, the problem is divided in number of subproblems each having a single variable. This reduces the computational time considerably. Beside this due to the decomponent of problem, the computer memory requirement is very less. By solving subproblems simultaneously on a computer or computers operating in parallel, the computation time can further be reduced.

REFERENCES

1. K.Geher,'Theory of Network Tolerances', McGraw-Hill Book Company, N.Y., 1971.

2. Donald A. Calahan,'Computer-Aided Network Design', McGRaw-Hill book company, N.Y. 1967.

3. Leon S. Lasdon,'Duality and decomposition in mathematical programming',IEEE Trans. on System Science and Cybernetics, July 1968, Vol. SSC-4, No.2, pp.86-99.

4. B.J. Karafin,'Optimum assignment of component tolerances for electrical networks,' Bell syst. Tech. J., Apr.1971, Vol.50, No.4, pp.1225-1242.

5. A.K. Seth and P.H. Roe,'Selection of Component tolerances for optimum circuits reproducibility,' in Proc. 1971 IEEE Int. Symp. Electrical Network Theory(London, England, Sept. 1971), pp.105-106.

6. J.F. Pinel and K.A. Roberts,'Tolerance assignement in linear networks using nonlinear programming,' in Proc. 1972 IEEE Int. Symp. Circuit Theory(Universal City, Calif. Apr. 1972),pp.179-183).

7. A.R. Thorbjornsen and S.W. Director,'Computer aided tolerance assignment for linear circuit with correlated elements', IEEE Trans. on Circuit Theory, Sept, 1973, Vol.CT-20, No.5, pp.518-524.

8. D.G. Mark and L.H. Stemper,'Variability analysis', Electro-Technol.,N.Y. July 1965, pp.37-48.

9. R. Fletcher and M.J.D. Powell,'A rapidly convergent descent method for minimization', Computer J., Vol.6, July 1963,p.163.

10. L.S. Lardon and A.D. Waren,'Mathematical programming for optimal design', Electro-Technol, Nov. 1967,pp.53-71.

11. M.G. Rezai-Falehr and G.C. Temes,'Statistical Large tolerance analysis of nonlinear circuits in the time domain', IEEE Trans. on Circuits and Systems, Jan.1975, Vol.CAS-22, No.I,pp.15-21.

12. J.F. Pinel,'Tolerance assignement on worst case basis treated as NLP.', T-CT 72 Sep.,pp475-479.

13. R.N. Gadenz,'A Method for the Computation of Large Tolerance Effects', IEEE Trans. on Circuit theory, Nov.1973,pp.704.

14. J.W. Bandler and P.C. Liu,'Automated network design with optimal tolerances', IEEE Trans. on C.S., Vol. CAS-21, No.2, March 1974, pp.219-222.