OPTIMIZATION OF LOSSES IN THREE-PHASE VARIABLE FREQUENCY INVERTER-FED INDUCTION MOTORS

DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the degree of

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ELECTRICAL ENGINEERING

(Power Apparatus and Electric Drives)

by

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October, 1978

To

Smt. AMBABAI

My Mother

CERTIFICATE

Certified that the dissertation entitled, 'OPTIMIZATION OF LOSSES IN THREE - PHASE VARIABLE FREQUENCY INVERTER-FED INDUCTION MOTORS', which is being submitted by Shri Chandidas Bhojraj Deshpande in partial fulfilment of the requirements for the degree of MASTER OF ENGINEERING in ELECTRICAL ENGINEERING (Power Apparatus and Electric Drives) of the University of Roorkee, Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further certified that he has worked for a period of about Seven months from March to September 1978 for preparing this dissertation at this University.

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C. B. DESHPANDE

ABSTRACT

The losses of a 3-phase squirrel-cage induction motor excited from a bridge inverter are investigated here. Formulation of the losses for variable operating frequency taking into account the presence of time harmonics in the voltage waveform is carried out based on the well-known relations available for fixed frequency operation.

Next, flow charts have been developed for determination of the losses with constant Volts / Hz and constant-flux modes of operation of the motor on sinusoidal as well as non-sinusoidal supply voltage waveforms. Computer programs based on the flow charts have been used to calculate the losses for a 5 h.p. motor with constant full load torque over a frequency range from 0.2 to 1.0 per unit normal (rated) frequency. The effect of variation of equivalent-circuit parameters on the losses of the motor is also studied from the computed results.

Finally, a flow chart for optimization of the losses of the inverter-fed induction motor has been developed using the incremental search technique. The quantities which are considered as variables are the number of conductors per stator slot, the area of cross-section of the stator conductor and the core length. A program prepared on the basis of this flow chart, has been used to obtain suitable values of the design variables for the 5 h.p. motor taken as an example so that the total losses of the motor are minimized.

Performance of the motor over operating frequency range of 0.2 to 1.0 per unit normal value is compared with the optimized equivalent. The method of optimization used is quite general and can be easily extended to include few more constraints or a few more design variables.

C O N T E N T S

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LIST OF SYMBOLS

A_b	Area of rotor bar in cm^2.
A_s	Area of cross-section of stator conductor in cm^2.
B	Flux density in wb/m^2.
B_C (B_{CR})	Stator (Rotor) core flux density in wb/m^2.
B_g	Average flux density over effective gap area in wb/m^2.
$B_m(1)$	Maximum flux density in wb/m^2 for fundamental frequency.
$B_m(K)$	Maximum flux density in wb/m^2 for K^{th} time harmonic.
B_T (B_{TR})	Stator (Rotor) teeth flux density in wb/m^2.
C_E	Cross-section of end ring in cm^2.
C_L (C_O)	Full-load (No-load) pulsation loss constant, numeric.
C_{S1} (C_{S2})	Stator (rotor) iron loss factor in watts per cubic inch for rotor (stator) slot frequency.
C_{db}	Skin-effect ratio for rotor bars at the stator slot frequency (which is 2 $s_{1p}f$ at synchronous speed), numeric.
D (D_O)	Stator air-gap (outside) diameter in cm.
D_R	Mean diameter of end rings in cm.
E_{LC} (E_{LCO})	End loss factor (Coefficient).
I_1 (I_2)	Stator (Rotor) fundamental frequency current in amp.

I_K (I_{2K})	Stator (Rotor) K^{th} harmonic current in amp.
I_{har}	R.M.S. harmonic current in stator ... amp.
I_M	Stator No-load current of fundamental frequency in amp.
K_1 (K_2)	Stator (Rotor) Carter's factor numeric.
$K_{2q} \pm 1$	Pitch times distribution factor of the stator winding for the $(2q \pm 1)$ field, numeric.
K	Order of time harmonic.
K_{d_1} (K_{p_1})	Distribution (pitch) factor for the stator for fundamental field numeric.
K_i	Saturation factor ... numeric.
K_{pf}	Pole face constant ... numeric.
K_r (K_x)	Skin-effect factor for rotor resistance (reactance) at time harmonic frequency ... numeric.
K_s	Skin-effect ratio for rotor bars at the phase belt frequency (which is 2 q f at synchronous speed), numeric.
L	Core length -in cm.
L_B	Length of rotor bar in cm.
L_i	Net iron length in cm.
L_{mt}	Mean length of stator turn .... in cm.
N	Rotor speed in r.p.m.
N_S	Synchronous speed of the fundamental field in r.p.m.
P	No. of pole pairs.

R_1 (R_2)	Stator (Rotor) per phase resistance in ohms.
$R_{2b}(R_{2r})$	Rotor bar (end-ring) rest. ref. to stator in ohms per phase.
R_{2K}	Rotor resistance corrected for skin effect at K^{th} time harmonic in ohms per phase.
R_{CLC} (R_{SLC})	Rotor core (surface) loss coefficient.
R_M (R_o)	Shunt (series) equivalent magnetizing resistance to represent core loss in ohms per phase.
R_{SNC}	Rotor no-load surface loss coefficient.
$S_1(S_2)$	No. of stator (Rotor) slots.
$S_{1P}(S_{2P})$	No. of stator (Rotor) slots / pole.
$S_{CLC}(S_{SLC})$	Stator core (surface) loss coefficient.
S_{KCO}	Skew-leakage loss constant.
$S_{iT}(S_{iTR})$	Specific iron loss in stator (rotor) teeth in watts/Kg
$S_{iC}(S_{iCR})$	Specific iron loss in stator (rotor) core in watts / Kg
T_1	No. of turns / phase in stator.
T	Torque in Nm.
V_1	R.M.S. phase voltage for fundamental frequency in volts.
V_5 (V_K)	R.M.S. phase voltage for 5^{th} (K^{th}) time harmonic in volts.
W_1 (W_2)	Stator (Rotor) copper loss on non-sinusoidal supply voltage in watts.

W_{2K}	Rotor copper loss for K^{th} time harmonic in watts.
W_3 (W_1)	Stator iron loss on non-sinusoidad (sinusoidal) supply voltage in watts.
W_4 (W_{ZZ})	Zig-zag flux loss in rotor for non-sinusoidal (sinusoidal) voltage waveform in watts.
W_5 (W_{SE})	End leakage flux loss for non-sinusoidal (sinusoidal) voltage waveform in watts.
W_6 (W_{SL_1})	Stator surface loss on non-sinusoidal (sinusoidal) voltage waveform in watts.
W_7 (W_{SL_2})	Rotor surface loss due to non-sinusoidal (sinusoidal) voltage waveform in watts.
W_8 (W_K)	Skew leakage flux loss for non-sinusoidal (sinusoidal) voltage waveform in watts.
W_9 (W_B)	Phase-belt leakage flux loss on non-sinusoidal (sinusoidal) supply voltage in watts.
W_{10} (W_{iR})	Rotor iron-loss on non-sinusoidal (sinusoidal) supply voltage waveform in watts.
W_{11}	Friction and windage loss in watts.
W_{SO}	Rotor surface loss at no-load in watts.
W_T	Stator Tooth Width in cm.
W_T	Total losses on non-sinusoidal supply in watts.
W_{TC} (W_{TT})	Weight of stator core (teeth) in Kg .
W_{TCR} (W_{TTR})	Weight of rotor core (teeth) in Kg .
W_{iK} (W_{iRK})	Stator (Rotor) iron loss for K^{th} time harmonic in watts.
X_1 (X_2)	Stator (Rotor) leakage reactance at fundamental frequency in ohms per phase.

X_{11} (X_{22})	Total stator (rotor) reactance at fundamental frequency in ohms per phase.
X_{2K}	Rotor leakage reactance at fundamental frequency corrected for skin-effect due to K^{th} time harmonic in ohms per phase.
X_{S1} (X_{S2})	Stator (Rotor) slot leakage reactance at fundamental frequency in ohms per phase.
X_M (X_{MO})	Shunt (series) equivalent magnetizing reactance to represent magnetizing current in ohms per phase.
Z	No. of conductors per stator slot.
a_s	Area of section of stator conductor in cm^2.
b_{bar} (b_s)	Width of the rotor bar (slot) in cm.
d_c (d_{SS})	Depth of stator core (slot) in cm.
f_1 (f)	Normal (Operating) frequencyin Hz.
f_{2K}	K^{th} harmonic rotor frequency in Hz.
g (g_e)	Actual (Effective) air-gap length in cm.
h (h_e)	Actual (Effective) height of the rotor bar in cm.
q	Number of phases.
s_1 (s)	Fundamental slip at normal (operating) frequency in p.u.
s_K	K^{th} time harmonic slip in p.u.
α	Angle of skew in radians.
β	Flux pulsation, numeric
λ_1(λ_2)	Stator (Rotor) slot pitch in cm.
σ	Ratio of skew to one stator slot pitch, numeric

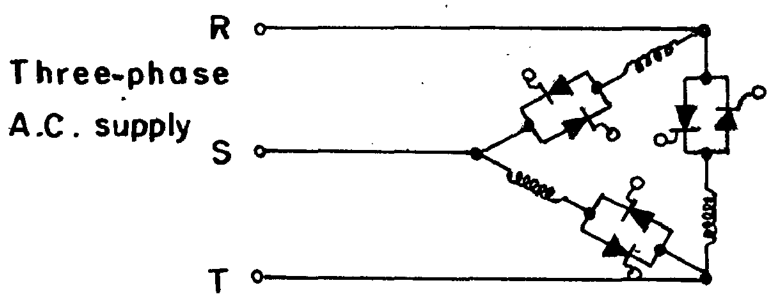
INTRODUCTION

The squirrel-cage induction motor has long been accepted as a dependable work horse for constant-speed applications. This has been its traditional role under conditions of fixed frequency and fixed voltage operation. This basic characteristic, viz. to run at a virtually constant speed close to synchronous, of the simple, cheap and reliable machine has long been a challenge to designers who have sought to devise variable speed schemes.

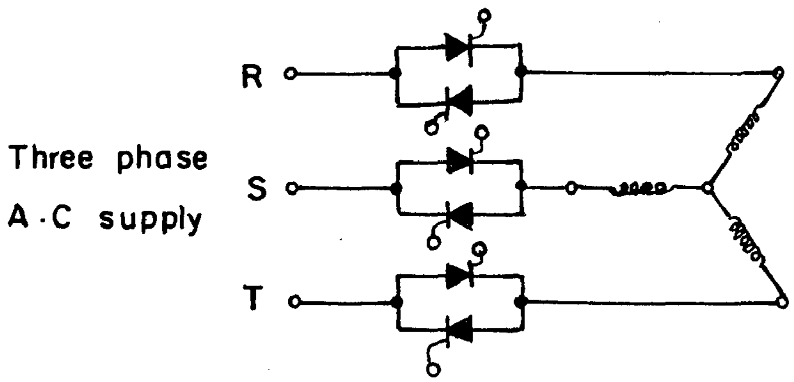
For cases where speed variation is necessary, the d.c. drive has been widely adopted and has proven successful.[25] In recent years the d.c. supply has been obtained from the a.c. network by means of static converters which permit the controlled rectification of the alternating voltage so that a variable direct voltage is provided for the armature. The application of direct voltage makes the stepless variation of speed of the d.c. motor possible and precise speed control is achieved by adopting closed-loop feed back methods.

The d.c. drive does have disadvantages,[25,36] however, such as the need for maintenance (replacement of worn brushes) and a relatively large flywheel effect. The limit values for output and speed of the d.c. machine are much lower than for the induction machine. Thus, in many cases, the tasks set can be carried out better with the induction motor. One requirement for this is a suitable incoming supply to the induction motor to give it the characteristics of a d.c. drive. The advent of silicon power thyristors around 1960 led to the development of static converters and thus made the stepless variation of speed of induction motor possible with practically no switching losses[25,9,14,29]. Progress made in the field of solid-state components, combined with advances in static converter and control techniques has put a new emphasis on the use of induction motor as a high performance variable speed drive and has opened new vistas for its applications[6,10,17,36].

Polyphase induction motors used in static variable-frequency drive systems are versatile torque transducers having operating characteristics and features which meet the requirements of modern variable speed drive systems[6]. Some of these characteristics are the capability for operation at very low and high speeds, at high torque overloads, in a constant horse power versus speed mode, and in the negative torque range for dynamic braking. Attractive features of induction motors include easy availability of wide ranges in horse power, voltage and speed ratings. In addition they have size, weight, torque-to-inertia ratios and cost advantages when compared to d.c. motor drives. Consideration will generally be given to a.c. drives of this kind[6,9,36] where for certain drive configurations and conditions of operation they offer the opportunity for important cost savings over their d.c. counterparts. For example :



(a)



(b)

Fig. 1 - Stator voltage control of Induction motor for

(a) Delta-connection

(b) Star-connection

i) Multiple number of motors (three or more) operating from a common power supply.

ii) Environmental conditions requiring the use of enclosed or explosion proof motors.

iii) High speed motors.

iv) Very low speed motors for direct connection to avoid drive train wind up and back lash.

v) Motors subjected to severe vibration or shock, excessive heat or moisture.

vi) Severe weight or space limitations on motor.

vii) Hollow shaft motors for overhung mounting or for feeding water, air, steam etc. through the motor shaft to the driven equipment.

viii) In-accessibility of motor for maintenance and inspection for very long periods.

ix) Outputs higher than the maximum ratings of d.c. motors.

A. HISTORICAL DEVELOPMENT OF THYRISTORIZED INDUCTION MOTOR DRIVES.

As a result of availability of SCRs interest in variable speed induction motor drives has shown a tremendous increase and in the recent years many new techniques suitable for the adjustment of the speed of the a.c. motors have been developed.

One such method is the variable voltage control by thyristors[33] used for obtaining subsynchronous speeds from the induction motor. In this scheme, normally three pairs

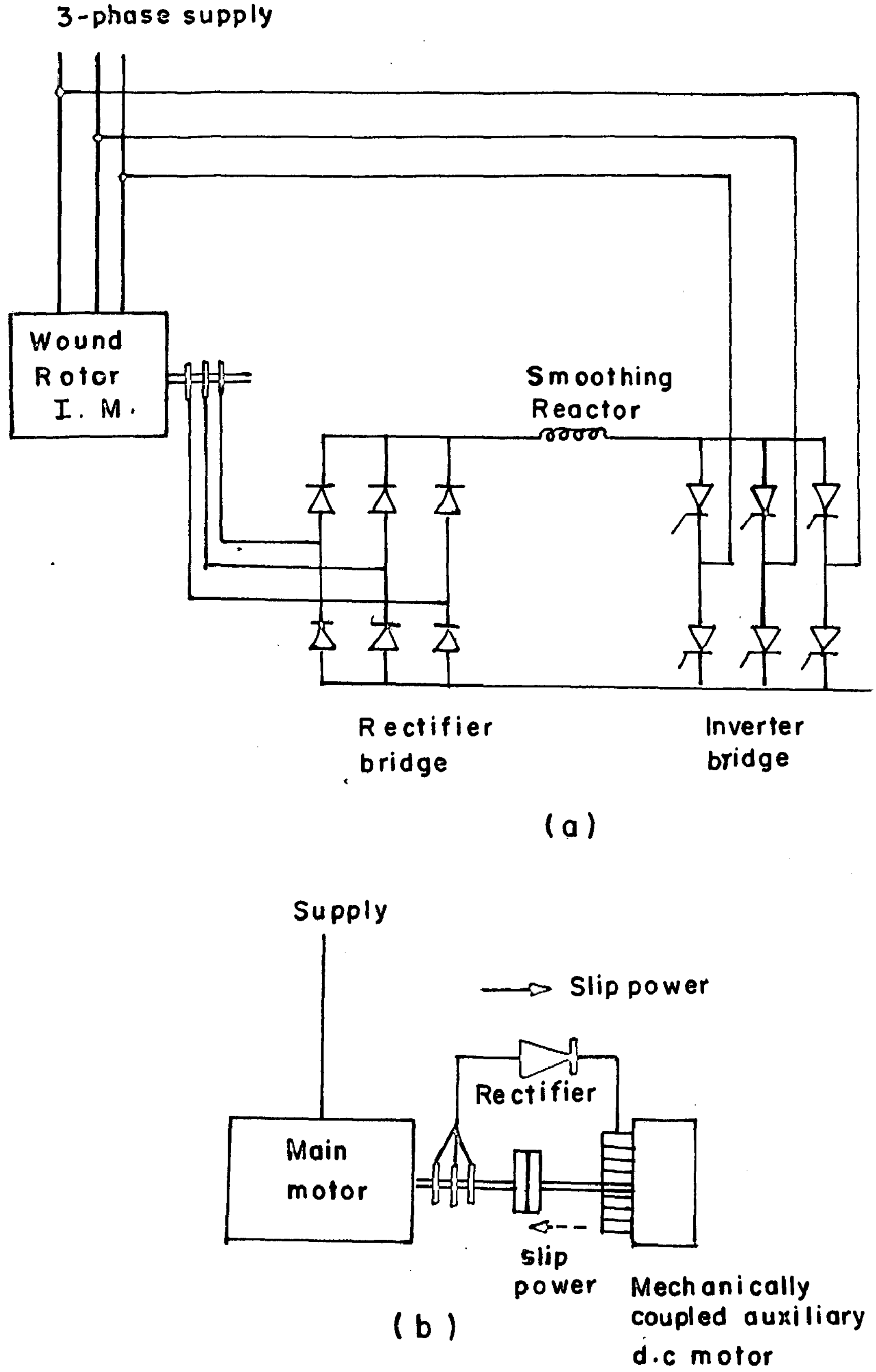


Fig . 2 - Basic circuit diagrams of

(a) Static converter cascade

(b) Modified Kramer drive

of anti-parallel SCRs are installed one in each line as shown in Fig.1 and the firing angles are symmetrically controlled to smoothly regulate the stator voltage of the motor. Thus, the effective voltage delivered to the motor can be varied from zero to full supply voltage. Stator voltage control eliminates the complex circuitry of the variable-frequency schemes and so is cheaper to install. However, the operating efficiency is poor, and derating is necessary at low speeds to avoid overheating due to excessive current and reduced ventilation. Other limitations[32] of the scheme are unsuitability for constant torque operation and problem of thyristor turn off at low speeds. This scheme is used for pump and blower type speed control where starting torque requirement is low and load torque usually increases with speed.[33] Other applications include fractional horse power drives and a.c. powered cranes and hoists[32].

Another method of speed control of induction motors applicable only in the case of slip ring type is the static slip-power recovery drive. There are basically two schemes, namely (i) static converter cascade[40] called as Scherbius static scheme and (ii) modified Kramer-drive[8] as shown in Fig.2. In the case of static Scherbius-scheme, a three phase bridge-rectifier connected in the rotor circuit of induction motor feeds rectified slip power through the smoothing inductor to the thyristor inverter. The inverter returns the rectified slip power to the a.c. supply. In the case of static Kramer-drive, a mechanically coupled auxiliary d.c. motor is supplied with rectified slip-frequency energy from the main motor slip rings. In this case, thus, the slip power is recovered not as an electrical power returned to the supply system but as mechanical torque developed by the d.c. motor. Static Scherbius-scheme is finding more acceptance in recent years, particularly for a limited sub-synchronous range of speed variation.[40]

These two schemes are having a number of limitations. The static Scherbius-scheme has a low efficiency and poor power-factor, which can only be improved at greater complexicity and cost[32]. Also there are difficulties in obtaining super synchronous speed in this method. In static Kramer-drive, one additional machine is required which is not desirable in some applications and also it increases the cost of the drive.

The most versatile method giving efficient wide-range speed control is variable-frequency control of induction motor. The motor torque is produced by interaction of rotor and stator flux. This flux is created by current flow in both the stator and rotor. In order to maintain a constant torque, current flow should be constant. Since motor impedance decreases with the reduction of applied frequency, the applied voltage should vary in a direct relationship with the applied frequency if constant torque

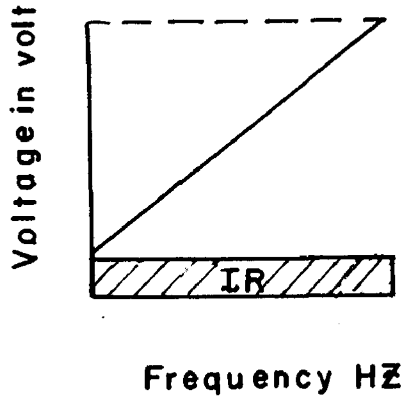


Fig. 3 - A.C motor voltage versus frequency

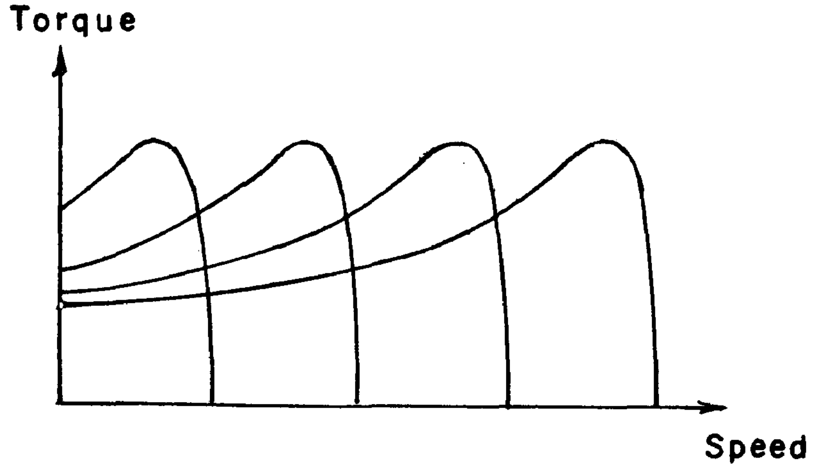


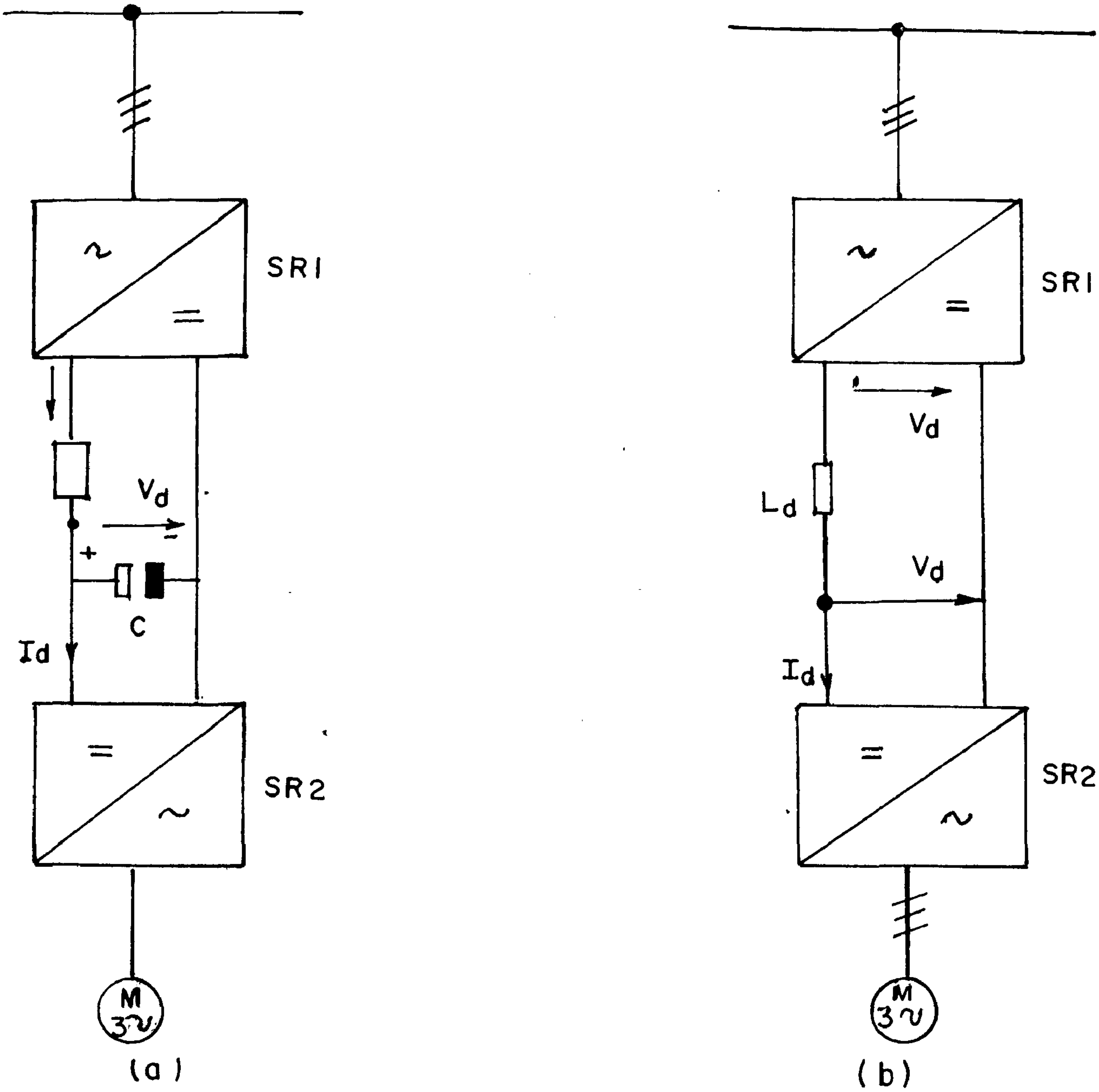
Fig. 4 - Induction motor torque speed characteristics at different supply frequencies and constant air-gap flux

capability at the output shaft of the motor is to be maintained. Thus voltage / frequency = constant.

Fig.3. is a plot for this characteristic which shows that the applied voltage is reduced as the frequency is decreased. It should be noted that the voltage-frequency ratio is not constant down to zero but requires a boost at low frequencies to compensate for the stator resistance drop IR, which becomes significant with respect to the total applied voltage[46]. Fig.4. shows the torque characteristics of the motor at several different stator frequencies, break down torque being maintained constant due to constant air-gap flux. These motor characteristics are suitable for driving a constant-torque load at variable speed.

Ideal power sources for the frequency-control method are static frequency converters[9], which can provide a three-phase a.c. system of variable frequency and voltage from an a.c. system of constant voltage and frequency or from a d.c. system. Various converter types and control methods, each differing in scope and principle, have been developed to permit economically optimal solutions to be found for a wide range of applications[46,25,17].

Static frequency converters can be subdivided into two basic sub-groups: d.c. link converters and cycloconverters. In d.c. link converters, the power taken from the a.c. system is first of all converted into d.c. power by a system-



Fig, 5 - Basic circuit diagrams of
(a) The constant voltage d-c link converter
(b) The constant current " "

side converter and then back into a.c. power of the required voltage, frequency and phase number by a machine side converter[9] as shown in Fig.5. These two converters are decoupled by a link containing a capacitor or reactor, depending on whether the link is to be of the direct voltage type or direct current type. The system-side converter is generally a standard rectifier and the machine-side converter is a thyristorized inverter. The output frequency of the d.c. link converter is determined by the rate at which the inverter thyristors are triggered into conduction. This is controlled by the reference oscillator and logic circuits, which generate and distribute firing pulses in the correct sequence to the various thyristors. Thus the output frequency may be controlled from zero to several hundred hertz. At the end of its conduction period, each thyristor must be turned off by an auxiliary commutating circuit. In general, output voltage is controlled by adjusting the amplitude of the inverter square wave output or by controlling its duty cycle or pulse width[36]. The inverter systems can be designated as

(i) Variable input transformer.

(ii) Variable output transformer.

(iii) Variable voltage input (phase controlled rectifier or chopper).

(iv) Pulse-width (Phase shift).

(v) Pulse-width modulated.

Out of these the last three only are truely static schemes. In the case of phase shift control, which requires two similar inverters operated at the same frequency from the same d.c. supply; the inverter outputs are combined in a transformer, and voltage control is obtained by phase-shifting the output of one inverter relative to the other. In the case of pulse-width modulated control, the alternating output voltage of a static inverter is rapidly switched on and off several times during each half cycle. The magnitude of the fundamental output voltage is controlled by variation of the total on-time during a half-cycle. In the three schemes mentioned above commutation and triggering logic and circuitry are quite complex.

The constant-voltage d.c. link converter is particularly suitable for group drives with induction machines and also in special cases for wide range of applications[25]. In the case of single-motor drives, more advantages are offered by converters with a direct-current link[9]. One major drawback of d.c. link converters is that the output voltage waveform is usually nonsinusoidal.

In the case of cycloconverters, the network frequency is directly converted to a lower frequency[32] without intermediate rectification, the output voltage being formed from the sections of the input voltage. The cycloconverters are not generally favoured for variable-speed induction motor drives due to the following drawbacks.

(i) The maximum output frequency must be less than about one third or one-half of the input frequency for reasonable power output and efficiency.

(ii) The cycloconverter requires a large number of thyristors and its control circuitry is more complex than that employed in many d.c. link converters; and

(iii) The cycloconverter has a low input power-factor, particularly at reduced voltages.

Typical variable-frequency induction motor drive applications are :

(i) Drives in textile industry, for example pumps, fans and agitator drives in the bleaching and dyeing areas.

(ii) Multimotor drives such as conveyor system.

(iii) Drives requiring high starting torque such as crushers and mixers.

(iv) For driving hoists, lifts, machine tools etc.

(v) Drives in chemical plant or food stuff industry.

(vi) Drives for sub-merged pumps where reliability, maintenance, weight or dimensions are prime factors.

(v) For driving electric traction vehicles.

The brief review of thyristorized induction motor drives here suggests that for small and medium size induction motors, the only economical way of controlling the speed is to use a d.c. link converter scheme with the motor.

B. STATE OF ART

The static variable-frequency induction motor drive has introduced numerous problems. The components of the electronic converter are much more sensitive than the machines to over-currents caused by the load or by system faults, and to over-voltages and other irregularities of the three-phase supply; system design and protection are affected by this increased vulnerability.

The cooperation with electronic frequency converter has far-reaching consequences[6,12,14,20,23,24,26] for the concerned a.c. motor, viz. the induction motor. Induction motors excited with static frequency converters almost invariably are subjected to non-sinusoidal voltage wave-form and the presence of time harmonics in the applied voltage results in currents at the harmonic frequencies. These currents result in additional and some times rather large losses depending upon the harmonic content of the supply. The additional losses increase the heating of a given machine and may lead to a reduction of its available continuous output. If these losses are high, relative to the fundamental losses, it is necessary to know their nature, locations and relationship to the motor-and converter-design features.

In the year 1966, Jain[24] discussed the contribution of harmonics to copper losses and torque and concluded that the torque of induction motor is affected by negligible amount because of voltage wave shape.

Klingshirn and Jordan[26] have also given the method of calculation of harmonic currents by using equivalent circuit approach. They have given a simple method to account for the effect of magnetic saturation. In this paper, the losses are separated into various components and it is shown that the harmonic losses are nearly independent of motor load. It is also shown that the losses of an induction machine with non-sinusoidal waveform impressed across its terminals can be markedly different from its sinusoidal losses depending upon the harmonic content of the impressed waveform.

In the year 1968 Chalmers and Sarkar[12] demonstrated the importance of losses due to skew-leakage fluxes in the case of induction motors having non-sinusoidal waveforms. It is pointed out that when the frequencies of either fundamental or of the largest harmonics are increased, it may become desirable to use unskewed construction.

Mclean and others[31] have investigated the design of induction motor required to achieve a good performance when fed with non-sinusoidal voltage waveform. It is indicated that increasing the number of phases and utilizing full-pitched coils will improve the efficiency, and a 9-phase system gives almost identical performance figures to a similar sinusoidally fed 3-phase machine.

Jacovides[23] has emphasised that the Fourier analysis provides a good method and sufficiently accurate calculation of induction motor performance when the latter is subjected to non-sinusoidal source. However it is pointed out that the method should not be used for predicting current waveforms.

Largiader[29] and Klautschek[25] have reported that the additional losses are about 20 percent of the fundamental wave losses for the induction motor which is fed from a static inverter. 'Off-the-shelf' motor, hence, cannot be arbitrarily applied to an inverter and the design of motor has to be modified. Certain design recommendations are also given by Largiader, such as :

(i) application of new types of insulating materials with higher thermal stability

(ii) use of low-resistance rotor, and

(iii) enlarging cooling surfaces and/or increasing the ventilation.

It is also reported[14] that the freedom from skin effects is to be preferred in the case of squirrel cage induction motor when the supply contains switching effects, because here the skin effect is not only useless but even detrimental.

The optimization of losses of inverter-fed induction motor has been done by Tsivitse and Klingshirn[42] for the supply conditions, viz. frequency and voltage, when a standard motor is used.

When a standard motor is operated on a non-sinusoidal supply, to avoid overheating because of additional losses it is necessary to reduce the continuous power and torque output of motor[32].

Other important effect on the operation of an induction motor excited from a static inverter is the production of pulsating torques, giving rise to torque fluctuations particularly at low speeds. Because of the mass of the rotor these torque fluctuations are smoothed out considerably. One means of reducing these torque fluctuations is to increase the leakage reactance which permits the flow of harmonic currents to be contained within a given limit[29,21].

The appraisal of literature shows that the operation of a polyphase induction motor fed from a static frequency-converter is modified considerably in comparison with its operation from a sinusoidal voltage source. Presence of time harmonics in the applied voltage waveform gives rise to additional losses and pulsating torques in the motor. The additional losses are about 20 percent of the fundamental losses for the motor. Hence there is a necessity of taking measures against these detrimental influences on the machine performance. It is essential to recognize the presence of time harmonics in the supply waveform at design stage and to scale the parameters of the induction motor, suitably, for variable frequency operation.

C. OUTLINE OF PRESENT WORK

In the present work, optimization of losses of induction motor, fed from a static frequency converter, has been carried out by using direct search method.

In Chapter I of this dissertation there is a discussion of various formulae for calculation of the losses of induction motor on non-sinusoidal voltage sources.

Chapter II deals with the performance of induction motor for variable frequency operation. A flow chart for calculation of equivalent-circuit parameters is developed. Another flow chart for calculation of losses from the equivalent-circuit parameters and suitable design data has been developed. The variation of losses with frequency for sinusoidal as well as non-sinusoidal voltage sources is studied by running programs on IBM - 1620 digital computer. The effect of various equivalent-circuit parameters on the total losses and other performance factors such as efficiency, fundamental slip, total input current etc. for variable frequency operation of the motor is discussed.

The optimization of losses of induction motor fed from an electronic frequency converter is considered in Chapter III. A flow chart for optimization using direct search technique of incremental search type has been developed and programs were run on IBM - 1620 digital computer to obtain optimized design.

The performance of the motor for the optimized parameters is compared with the normal design.

Finally , conclusions have been drawn from the results obtained. Limitations of the present method as well as the scope for further work is also indicated.

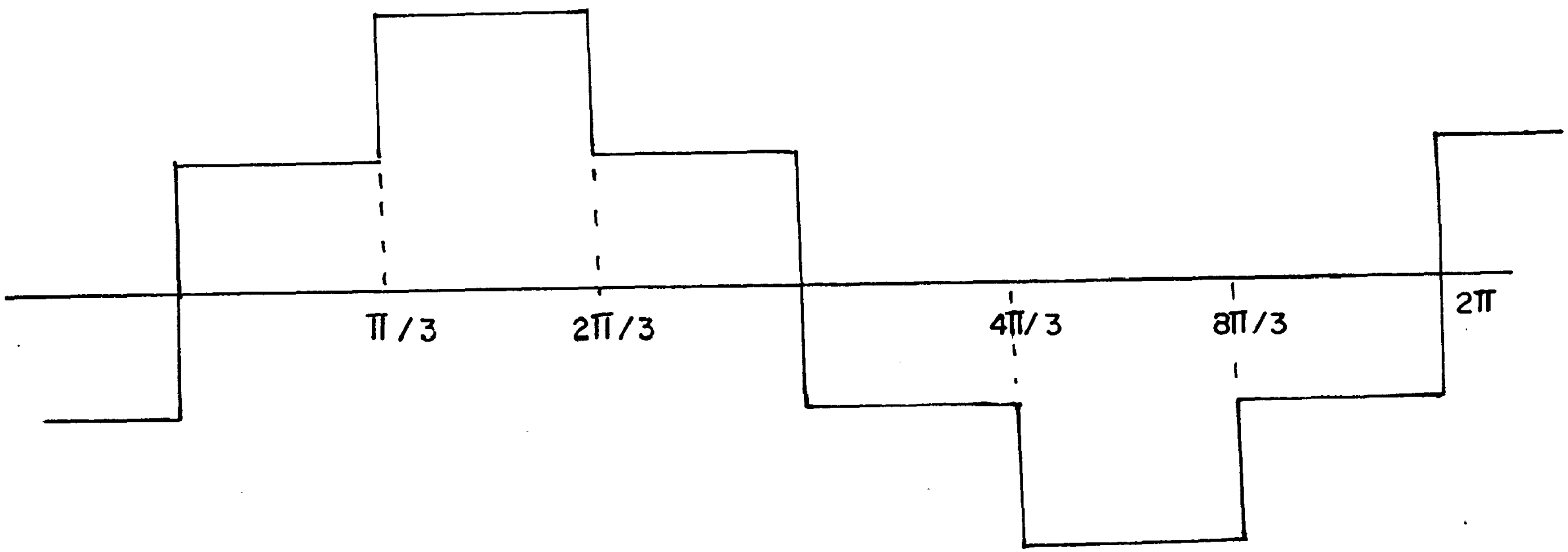


Fig. 1.1 - Output voltage waveform of a three-phage bridge inverter

CHAPTER I

POLYPHASE INDUCTION MOTOR LOSSES ON NON-SINUSOIDAL VOLTAGE SOURCES

INTRODUCTION

Static converters which are being increasingly used to obtain flexible performance characteristics from robust but inherently constant speed cage induction motors, have the output voltage and current waveforms rich in harmonics. These harmonics have a detrimental effect on the motor performance.

In this chapter a method of predicting currents and losses in the presence of time harmonics is presented. The work of Klingshirn and Jordon[26] has become handy in this case. The various well established loss formulae[2,3,4,12,39] have been modified to account for time harmonic effects.

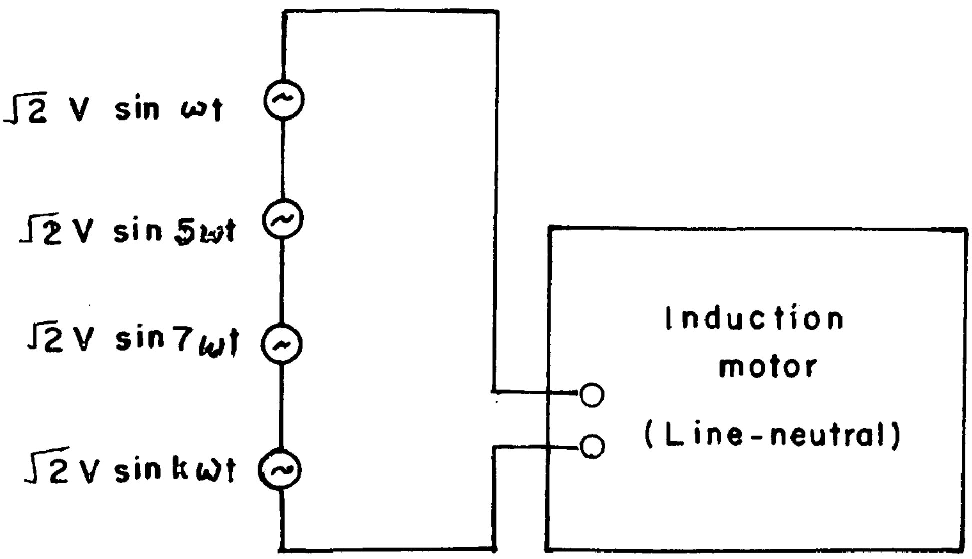
The following assumptions have been made to get simpler relations;

(i) Saturation of iron parts of the machine is neglected.

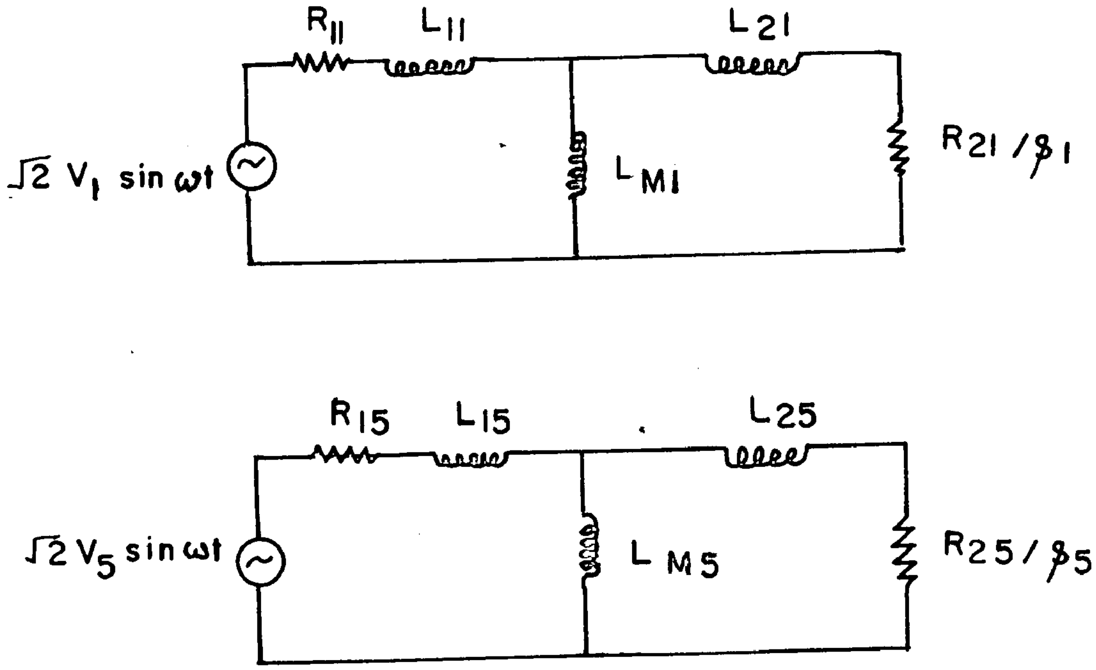
(ii) The non-sinusoidal output voltage from three-phase bridge inverter is balanced.

1.1. METHOD OF ANALYSIS

Most of the voltage waveforms produced by static frequency converters are made up of a series of discrete steps. Fig.1.1. displays a six step waveform which is quite common as the output waveform of three-phase inverters.



(a)



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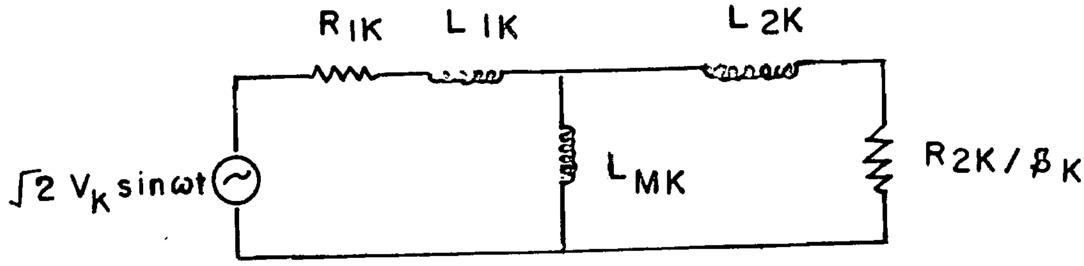


(b)

Fig. 1.2(a)-Non sinusoidal excitation of Induction motor per phase

(b)-Equivalent circuit for induction motor with non-sinusoidal excitation.

Assuming that the voltage waveform is known and its Fourier analysis has been obtained, the general expression for the impressed voltage v(t) is :

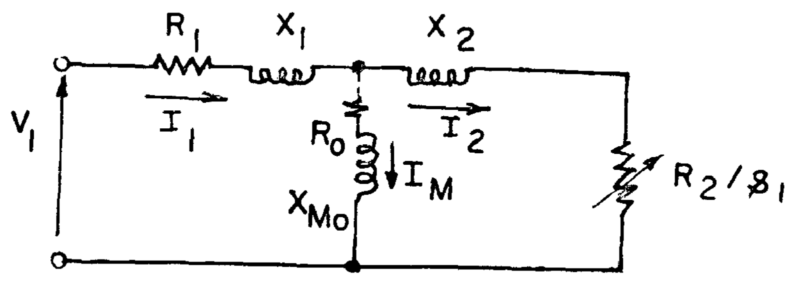
$$v(t) = \sqrt{2} \Big[V_1 \operatorname{Sin} wt + V_5 \operatorname{Sin} 5\, wt + V_7 \operatorname{Sin} 7\, wt + \ldots\ldots\ldots\ldots + V_K \operatorname{Sin} K\, wt \Big] \ldots \qquad (1.1)$$

equation (1.1) is the voltage waveform most frequently encountered with three-phase induction motors. It does not contain any even harmonics or harmonics which are divisible by three. This waveform is assumed here.

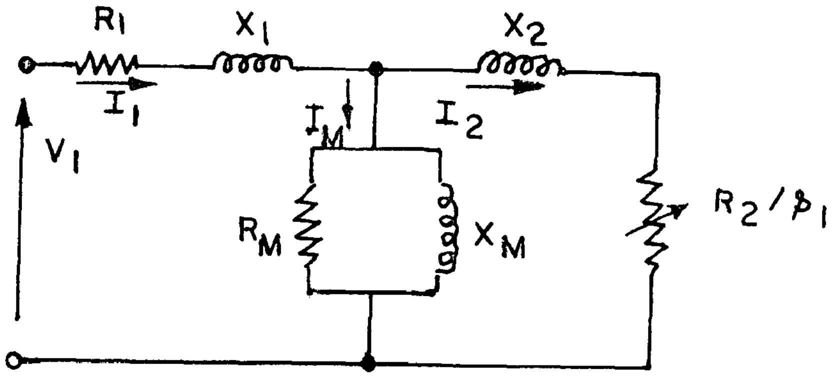
As the magnetic saturation is neglected, the motor may be regarded as a linear device. The analysis of performance of the motor can proceed as if there were a series of independent generators all connected in series supplying the motor as shown in Fig.1.2(a). Each generator would represent one of the voltage terms in equation (1.1). The conventional induction motor equivalent-circuit is very useful in calculating the performance of a motor under steady state operating conditions. Since each harmonic current will be independent of all of the others, a series of independent equivalent circuits, one for each harmonic, can be used to calculate the complete steady state performance of an induction motor with non-sinusoidal voltages applied. This equivalent-circuit arrangement is shown in Fig.1.2(b).

1.2. DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS

The conventional equivalent-circuit of induction



(a)



(b)

Fig. 1.3 - Induction motor fundamental frequency equivalent circuits

motor for fundamental frequency of f_1 Hz is shown in Fig.1.3. It is necessary to know the values of equivalent circuit parameters for calculation of currents at fundamental frequency as well as at the harmonic frequencies. Hence the calculation of equivalent circuit parameters from design variables is discussed in the following paragraphs. Various well established formulae as developed by Alger[4] are used for this purpose.

Stator Resistance Per Phase

For the calculation of stator per phase resistance we have to determine the mean length of turn, which can be expressed as[38]:

$$L_{mt} = \left(2\,L + 2.3 \times \frac{\pi D}{2\,P} + 24 \right) \text{ in cms.} \qquad \ldots \qquad (1.2)$$

The per phase stator resistance is then given by:

$$R1 = 2.095 \times 10^{-6} \times \frac{T_1 \times L_{mt}}{a_s} \text{ in ohms.} \qquad \ldots \qquad (1.3)$$

Rotor Resistance Per Phase

Assuming that the bar and end rings are made of copper, and a skew of one slot pitch, we have to first determine the length of the bar according to the formula[28]

$$L_B = \left[L \,/\, \cos\left(\frac{\pi \times 2\,P}{S_1}\right) + 2.4 \right] \text{ in cms.} \qquad \ldots \qquad (1.4)$$

and then determine the rotor resistance[4] referred to the stator per phase as :

$$R_2 = R_{2b} + R_{2r} \text{ in ohms.} \quad \ldots \quad (1.5)$$

where

$$R_{2b} = \frac{4.19 \times 10^{-6} \times q\,(K_{p_1} K_{d_1})^2\, T_1^2 \times 2\, L_B}{S_2 \times A_b} \quad \ldots \quad (1.6)$$

and

$$R_{2r} = \frac{4.19 \times 10^{-6} \times q\,(K_{p_1} K_{d_1})^2\, T_1^2\, D_R}{\pi\, C_E\, P^2} \quad \ldots \quad (1.7)$$

Magnetizing Reactance

The formula for calculation of magnetizing reactance[4] at fundamental frequency of f_1 Hz is :

$$X_M = \frac{2.51\, q\, f_1\, T_1^2\, K_{p_1}^{\,2}\, K_{d_1}^{\,2}\, DL}{K_1 \times g_e \times P^2 \times 10^8} \text{ in ohms} \quad \ldots \quad (1.8)$$

The ratio $K_1 = \dfrac{\text{Total ampere-turns}}{\text{Air gap ampere-turns}}$

is called the saturation factor of the magnetic circuit. Procedure for calculation of the saturation factor is outlined in Appendix (6).

Primary Leakage Reactance

Conventionally, the primary leakage reactance takes into account (i) primary slot leakage ; (ii) Coil end leakage (iii) zig-zag and phase-belt leakage ; and (iv) Skew leakage. So the total primary leakage reactance is

$$X_1 = X_{S1} + X_{Z/2} + \frac{X_C}{2} + X_{\kappa} / 2 \quad \ldots \quad (1.9)$$

where

X_{S1} = Primary or stator slot leakage reactance.

X_Z = Zig zag leakage reactance.

X_C = Coil end leakage reactance.

$X\alpha$ = Skew reactance.

The stator slot leakage reactance X_{S1} at fundamental frequency of f_1 Hz can be expressed as[4] :

$$X_{S1} = \frac{3.16 \, f_1 \, q \, L \, T_1^2 \times 10^{-7} \times Ps_1}{S_1} \text{ in ohms} \quad \ldots (1.10)$$

Here P_{S1} is the permeance coefficient of stator slot. For a stator slot with known dimensions, this can be easily calculated[4]. The zig-zag reactance at fundamental frequency of f_1 Hz as given by the following equation is suggested as a good value to use[4].

$$X_Z = \frac{\pi^2 X_M}{12} \left[\frac{(6/K_1 - 1)}{5 \times (S_1/2\,P)^2} F_{SC} + \frac{(6/K_2 - 1)}{5 \times (S_2/\,2P)^2} \right] \text{ ohms} \quad (1.11)$$

where K_1 and K_2 are Carter's factors for stator and rotor respectively. It is usually sufficient to take F_{SC} as 1.

The belt leakage reactance is taken as zero for squirrel-cage motors[4].

The coil end reactance at the fundamental frequency of f_1 Hz is conveniently obtained by the following expression currents on the degree of saturation of the iron must be taken into account. It is shown[26] that the presence of time harmonics in the supply voltage, causes a reduction in the values of reactances due to increased degree of saturation of the iron parts. Thus the values of leakage reactances at fundamental frequency are reduced by 15 percent and the magnetizing reactance at the fundamental frequency by 4 persent as indicated by Klingshirn and Jordon[26]. Then these values are used for calculation of the performance of the motor on non-sinusoidal supply.

1.3. CALCULATION OF FULL LOAD SLIP

Assuming that full load torque in Newton-meters is given, then the well known torque expression can be rewritten to calculate the fundamental slip as shown below.

The torque, in newton meters[43], developed by a polyphase induction motor at fundamental frequency neglecting, friction and windage, is :

$$T = \frac{\frac{Pq}{2\pi}\left[\frac{V_1}{f_1}\right]^2 \times s_1 f_1 \, (R_0{}^2 + X_{MO}^2) / R_2}{\left[R_1 + R_0 + \left\{\frac{R_1 R_0 + (X_{MO}^2 - X_{11}\, X_{22})}{R_2}\right\} s_1\right]^2 + \left[X_{11} + \frac{s_1}{R_2}\left\{R_1 X_{22} + (X_1 + X_2) R_0\right\}\right]^2} \qquad \ldots\ldots\ldots\ldots(1.17)$$

where $X_{11} = X_{MO} + X_1$ ... ... (1.18)

$X_{22} = X_{MO} + X_2$ ... ... (1.19)

It is known[26] that the contribution of harmonic torque to steady state torque is negligible. Hence the

where

X_{S1} = Primary or stator slot leakage reactance.

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where K_1 and K_2 are Carter's factors for stator and rotor respectively. It is usually sufficient to take F_{SC} as1.

The belt leakage reactance is taken as zero for squirrel-cage motors[4].

The coil end reactance at the fundamental frequency of f_1 Hz is conveniently obtained by the following expression

$$X_C = \frac{7\ f_1\ q\ T_1^2 D}{p^2\ 10^8}\ (K_{p_1} - 0.3)\ \text{ohms} \quad \ldots \quad (1.12)$$

For a given angle α of skew expressed in radians, the skew leakage reactance at the fundamental frequency is expressed as[4].

$$X_\alpha = \frac{\alpha^2}{12}\ X_M \quad \text{ohms} \quad \ldots \quad \ldots \quad (1.13)$$

Rotor Leakage Reactance

The rotor leakage reactance referred to the stator per phase at the fundamental frequency is given as follows[4].

$$X_2 = X_{S2} + X_{2e} \quad \text{ohms} \quad \ldots \quad \ldots \quad (1.14)$$

where $X_{2e} = \frac{1}{2}\ (X_Z + X_C + X_\alpha)$ ohms ... (1.15)

In equation (1.14), the only quantity that remains to be found out is X_{S2}, the rotor slot leakage reactance. This rotor slot leakage reactance at fundamental frequency is determined by a similar expression as is used for X_{S1}, viz.

$$X_{S2} = 3.16\ f_1\ q\ L\ T_1^2 \times 10^{-7} \times \frac{K_{p1}^2\ K_{d1}^2\ P_{s2}}{K_{p_2}^2\ K_{d_2}^2\ S_2} \quad \text{ohms} \quad \ldots \quad (1.16)$$

for a squirrel - cage winding $K_{p_2} = K_{d_2} = 1$ and P_{S2} is determined in a similar fashion as P_{S1}.

In order to arrive at appropriate values for the reactances in the equivalent-circuit with motor operating on non-sinusoidal voltage, the effects of harmonic voltages and currents on the degree of saturation of the iron must be taken into account. It is shown[26] that the presence of time harmonics in the supply voltage, causes a reduction in the values of reactances due to increased degree of saturation of the iron parts. Thus the values of leakage reactances at fundamental frequency are reduced by 15 percent and the magnetizing reactance at the fundamental frequency by 4 persent as indicated by Klingshirn and Jordon[26]. Then these values are used for calculation of the performance of the motor on non-sinusoidal supply.

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where $X_{11} = X_{MO} + X_1$ ... ... (1.18)

$X_{22} = X_{MO} + X_2$ ... ... (1.19)

It is known[26] that the contribution of harmonic torque to steady state torque is negligible. Hence the equation (1.17) is rewritten in modified form to calculate full load slip, when full load torque is known.

Let D_1, D_2, D_3, D_4 be some constants defined as

$$\left.\begin{aligned} D_1 &= \frac{T \times 2\pi \times f_1 \times R_2}{(R_0'^2 + X_{M0}^2) \times V_1^2 \times P \times q} \\ D_2 &= R_1 + R_0 \\ D_3 &= \frac{R_1 R_0 + (X_{M0}^2 - X_{11} X_{22})}{R_2} \\ D_4 &= \frac{R_1 X_{22} + (X_1 + X_2) R_0}{R_2} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (1.20)$$

Then the equation No.(1.17) can be written as

$$D_1 (D_3^2 + D_4^2) s_1^2 + \left[2 D_1 (D_2 D_3 + X_{11} D_4) - 1.0\right] s_1 + (D_2^2 + X_{11}^2) = 0 \quad (1.21)$$

If
$$\left.\begin{aligned} Y_1 &= D_1 (D_3^2 + D_4^2) \\ Y_2 &= D_2^2 + X_{11}^2 \\ Y_3 &= 2 D_1 (D_2 D_3 + X_{11} D_4) - 1.0 \end{aligned}\right\} \quad \ldots \quad (1.22)$$

Then equation (1.21) further gets simplified to

$$Y_1 s_1^2 + Y_3 s_1 + Y_2 = 0 \quad \ldots \quad (1.23)$$

This quadratic equation can be solved by usual method and the value of s_1 computed.

1.4. HARMONIC EQUIVALENT CIRCUITS

The fundamental equivalent circuit of Fig.1.3. may be adapted for the K^{th} harmonic voltage and current as shown

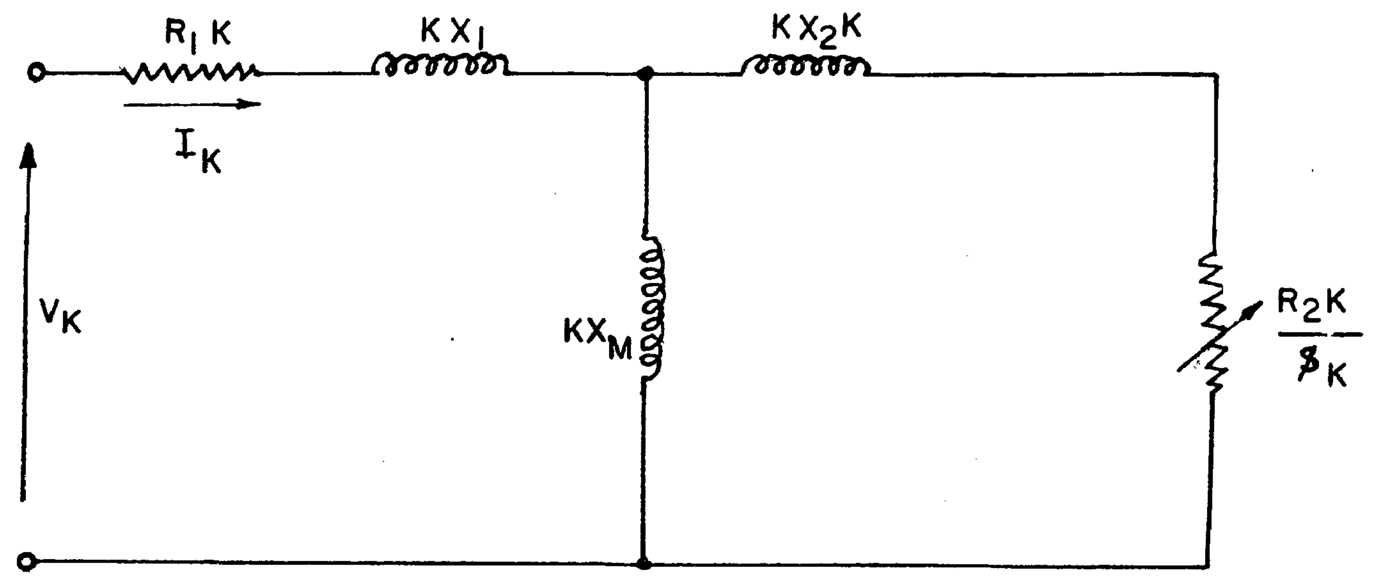

Fig. 1.4 - K^{th} harmonic equivalent circuit

in Fig. 1.4. In this case, the magnetizing resistance branch representing iron losses is neglected, since this branch contributes very little to the stator harmonic current. The effect of increase in iron losses is, however, considered separately as discussed in iron loss calculations, to be followed. The other differences between this circuit (Fig.1.4.) compared to the circuit at fundamental frequency are those needed to take account of the harmonic frequencies. Thus for a time harmonic of order K we have to modify the fundamental equivalent circuit in the following manner.

(a) All reactances are evaluated at the harmonic frequency Kf_1.

(b) The slip is the harmonic slip.

(c) Skin effect is taken into account in calculating the secondary resistance and leakage reactance.

Thus,

If the synchronous speed of fundamental field = N_S

Synchronous speed of the K^{th} harmonic = $K\,N_S$

If rotor speed = N

$$\text{Fundamental slip } s_1 = \frac{N_s - N}{N_s} \qquad \ldots \qquad (1.24)$$

K^{th} harmonic slip for a forward rotating harmonic field is

$$s_K = \frac{K\,N_S - N}{K\,N_s} \qquad \ldots \qquad \ldots \qquad (1.25)$$

and for a backward rotating field

$$s_K = \frac{K\ N_s + N}{K\ N_s} \qquad \ldots \qquad \ldots \qquad (1.26)$$

In general, therefore $s_K = \dfrac{K \pm (1-s_1)}{K}$ ... (1.27)

For the K^{th} harmonic, rotor frequency,

$$f_{2K} = s_K\ Kf_1$$

$$= \left[K \pm (1 - s_1)\right] f_1 \qquad \ldots \qquad (1.28)$$

The modification of the values of rotor resistance and reactance for accounting the skin effect is done in the following manner.

For a time harmonic of K^{th} order in the supply voltage waveform, the values of rotor resistance and reactance are expressed as[4,27]:

$$R_{2K} = K_r\ R_{2b} + R_{2r} \quad \text{ohms} \qquad \ldots \qquad (1.29)$$

$$X_{2K} = K_x\ X_{2S} + X_{2e} \quad \text{ohms} \qquad \ldots \qquad (1.30)$$

where $K_r = \dfrac{he\ (\text{Sinh}\ 2\ he + \text{Sin}\ 2\ he)}{(\text{Cosh}\ 2\ he - \cos\ 2\ he)}$ ... (1.31)

$$K_x = \frac{3}{2\ he} \times \frac{\text{Sinh}\ 2\ he - \text{Sin}\ 2\ he}{\text{Cosh}\ 2\ he - \text{Cos}\ 2\ he} \qquad \ldots \qquad (1.32)$$

and he being given by :

$$he = 0.138\ h \sqrt{f_{2K} \times \frac{b_{bar}}{b_s}} \qquad \ldots \qquad \ldots \qquad (1.33)$$

Here h is the height of the bar and b_{bar} and b_s are the bar and slot widths respectively, all these quantities being expressed in cms.

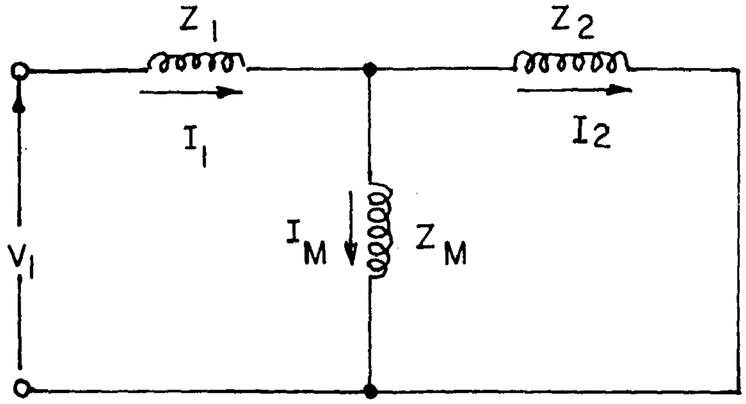

Fig.1·5 - Generalized equivalent circuit

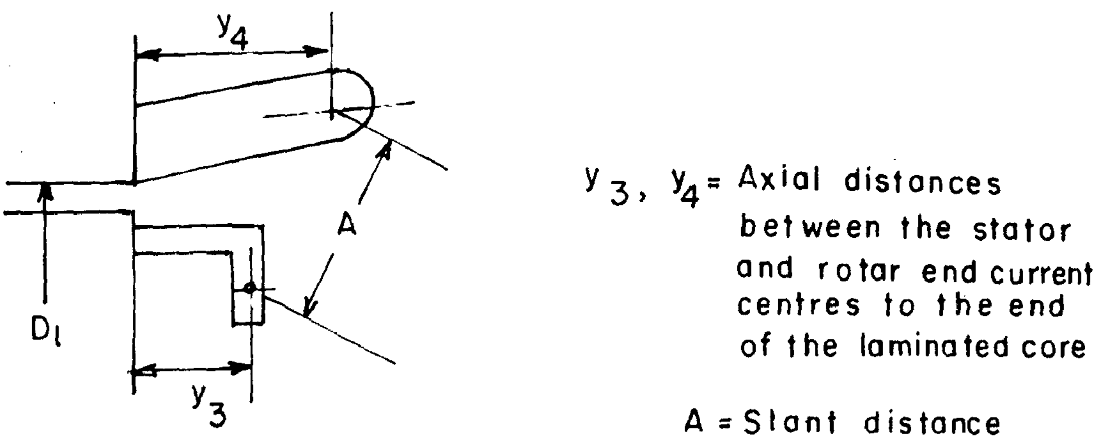

Fig.1.6 - End structure of an induction machine geometry of the end winding.

1.5. CALCULATION OF CURRENTS

Fundamental Currents

Following method of calculation (currents) due to Veinott[43] is utilized here. Fig. 1.5. shows a more general form of equivalent circuit for Fig. 1.3. By setting up simultaneous equations and solving them, it is found that the currents, in terms of impedances, are :

$$I_1 = \frac{V(Z_m + Z_2)}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \quad \ldots \quad \ldots \quad (1.34)$$

$$I_2 = \frac{V Z_m}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \quad \ldots \quad \ldots \quad (1.35)$$

$$I_M = \frac{V Z_2}{Z_1 Z_m + Z_1 Z_2 + Z_2 Z_m} \quad \ldots \quad \ldots \quad (1.36)$$

It is now a simple matter to substitute individual resistances and reactances into the above equations. This done, the following equations are obtained :

$$I_1 = \frac{A_5 + A_6/s_1 + j\,A_7}{U + j\,W} \quad \ldots \quad \ldots \quad (1.37)$$

$$I_2 = \frac{A_5 + j\,A_8}{U + j\,W} \quad \ldots \quad \ldots \quad (1.38)$$

$$I_M = \frac{A_6/s_1 + j\,A_{10}}{U + j\,W} \quad \ldots \quad \ldots \quad (1.39)$$

Where $U = A_1 + A_2/s_1$ ... ... (1.40)

$W = A_3 + A_4/s_1$ ... ... (1.41)

The values of A_1 , A_2 , ... etc. are given in Table 2.1.

TABLE NO. 2.1

A CONSTANTS FOR POLYPHASE MOTORS

A Constant	Expression for Constant
A_1	$-\left(X_1 + X_2 \dfrac{X_{MO}}{X_{MO} + X_2}\right) + \dfrac{R_1 R_0}{(X_{MO} + X_2)}$
A_2	$\dfrac{(R_1 + R_0)R_2}{X_{MO} + X_2}$
A_3	$\dfrac{R_0(X_1 + X_2)}{X_{MO} + X_2} + R_1$
A_4	$\dfrac{(X_{MO} + X_1)R_2}{(X_{MO} + X_2)}$
A_5	$\dfrac{V_1 R_0}{(X_{MO} + X_2)}$
A_6	$\dfrac{V_1 R_2}{(X_{MO} + X_2)}$
A_7	V_1
A_8	$\dfrac{V_1 X_{MO}}{X_{MO} + X_2}$
A_{10}	$\dfrac{V_1 X_2}{(X_{MO} + X_2)}$

The values of X_{MO} and R_0 are calculated from X_M and R_M in the following way.

$$R_0 = \frac{X_M^2}{R_M} \cdot \frac{1}{\left[1 + (X_M / R_M)^2\right]} \quad \ldots \quad \ldots \quad (1.42)$$

$$X_{M0} = X_M \quad \frac{1}{1 + (X_M / R_M)^2} \quad \ldots \quad \ldots \quad (1.43)$$

Harmonic Currents

The calculation of harmonic currents from harmonic equivalent circuits is done exactly in the same way as for fundamental currents. The only difference being that in this case R_0 is zero and the values of X_{M0}, X_1, X_2, R_1, R_2 etc. are replaced by their suitable values for the particular harmonic. Then A-constants are determined and from them the harmonic currents.

Hence total harmonic r.m.s. current is given by

$$I_{har} = \sqrt{I_5^{\,2} + I_7^{\,2} + I_{11}^{\,2} + I_{13}^{\,2} + \ldots\ldots + I_K^{\,2}}$$

$$= \sqrt{\left[\sum_{5}^{\infty} I_K^2\right]} \quad \ldots \quad \ldots \quad (1.44)$$

1.6. MOTOR LOSSES WITH NON-SINUSOIDAL SUPPLIES

The polyphase induction motor operated with non-sinusoidal supply voltage has the usual motor losses, and some additional losses due to the harmonics. These are discussed in the following paragraphs.

1.6.1. COPPER LOSSES

Stator Copper Loss

The stator I^2R loss is given by the usual equation with an additional term to account for the loss due to harmonic currents :

$$W_1 = q\left[(I_1^2 + I_{har}^2)R_1\right] \quad \ldots \quad \ldots \quad (1.45)$$

The skin effect is neglected in the case of small motors.

Rotor Copper Loss

Since the rotor resistance is a function of the harmonic frequency, the rotor copper loss is calculated independently for each harmonic. In general, for the K^{th} harmonic.

$$W_{2K} = q(I_{2K})^2 R_{2K} \quad \ldots \quad \ldots \quad (1.46)$$

where I_{2K} is the K^{th} harmonic rotor current, and R_{2K} is the corresponding rotor resistance, corrected for skin effect. The total harmonic copper loss is then obtained as a summation of the harmonic contributions. Thus total rotor copper loss is :

$$W_2 = q\, I_2^2 R_2 + \sum_5^n q\, (I_{2K})^2 R_{2K} \quad \ldots \quad (1.47)$$

1.6.2. IRON LOSSES

Stator Iron Loss

The stator iron loss is a function of the flux density in the stator core and teeth. A formula given by Vickers[47] for the iron loss per kilogram for lohys is :

$$W_1/Kg = 0.014322\ B^{1.8}\ f_1^{1.6} \text{ watts} \quad \ldots \quad (1.48)$$

where B is in wb/m^2, and f_1 is in Hz.

If average flux density in stator teeth is B_T and core flux density is B_C, then the specific iron loss for stator teeth and core will respectively be given as :

$$S_{1T} = 0.014322\ (B_T \times 1.57)^{1.8}\ f_1^{1.6} \quad \ldots \quad (1.49)$$

and $$S_{1c} = 0.014322\ (B_C)^{1.8}\ f_1^{1.6} \quad \ldots \quad (1.50)$$

Since 1 cm^3 of iron weighs 7.9 gm[47] the weight of stator teeth and core is respectively given by :

$$W_{TT} = 7.9 \times 10^{-3} \times S_1 \times d_{SS} \times W_T \times L_1 \quad \ldots \text{Kg} \quad (1.51)$$

and $$W_{TC} = 7.9 \times 10^{-3} \times \pi (D_o - d_c) \times d_c \times L_1 \quad \ldots \text{Kg} \quad (1.52)$$

and so iron loss in stator for fundamental frequency is expressed as :

$$W_1 = S_{1T} \times W_{TT} + S_{1C} \times W_{TC} \quad \ldots \quad (1.53)$$

The same procedure for calculation of harmonic iron loss is to be repeated excepting, here the values of respective harmonic flux densities in place of B_T and B_C and harmonic frequency Kf_1 in place of f_1 is to be substituted in equations (1.49) and (1.50). For the six step waveform assumed, the voltage of K^{th} harmonic is $\frac{1}{K}$ times that of fundamental and so the K^{th} harmonic flux density is $1/K^2$ times the fundamental flux density, i.e.

$$B_m(K) = \frac{1}{K^2} B_m(1) \qquad \ldots \qquad \ldots \qquad (1.54)$$

Thus if W_{iK} represents the stator iron loss for K^{th} harmonic, then the total iron loss in the stator upto n^{th} order harmonic would be :

$$W_3 = W_i + \sum_{5}^{n} W_{iK} \qquad \ldots \qquad \ldots \qquad (1.55)$$

Rotor Iron Loss

For fundamental frequency operation the iron loss in the rotor is usually negligible. If desired, this can be calculated with the help of equation (1.48) as follows :

$$S_{iTR} = 0.014322\,(B_{TR} \times 1.57)^{1.8}.\,(s_1 f_1)^{1.6} \qquad (1.56)$$

and $S_{iCR} = 0.014322\,(B_{CR})^{1.8}.\,(s_1 f_1)^{1.6}$ (1.57)

and so the iron loss in rotor for fundamental frequency operation is

$$W_{iR} = S_{iTR} \times W_{TTR} + S_{iCR} \times W_{TCR} \qquad \ldots \qquad (1.58)$$

The iron loss in rotor due to harmonic frequencies, however, is considerably increased and should be taken into account. In this case, the harmonic slip (which is nearly unity) in place of s_1, harmonic frequency in place of f_1 and harmonic flux densities in place of respective flux densities are to be substituted in equations (1.56) and (1.57). The rotor iron loss upto n^{th} order harmonic in supply would then be :

$$W_{10} = W_{1R} + \sum_{5}^{n} W_{1RK} \qquad \ldots \qquad \ldots \qquad (1.59)$$

where W_{1RK} = Rotor iron loss for K^{th} order harmonic

1.6.3. FRICTION AND WINDAGE LOSS

This loss is not influenced by the harmonics in voltage waveform. When the slip or frequency changes, however, then this loss is changed. An approximate expression, developed on the lines of Yermekova[48] is given below.

$$W_{11} = 0.016 \times HP \times 746 \times (1 - s_1) \qquad \ldots \qquad (1.60)$$

1.6.4. STRAY-LOAD LOSSES

The stray-load losses are defined as the excess of the total losses actually occuring in a motor at a given load current over the sum of the calculated I^2R losses for that current, the no-load core loss, and friction and windage loss. They are caused by the magnetomotive forces (m m f) of the motor-load currents, which divert some of the no-load magnetic flux into leakage paths, thereby creating flux pulsations and eddy current losses in the laminations, the conductors, and adjacent metal parts. Following components of the stray-load loss are considered here.

(i) The losses in the motor end structure, due to end leakage flux.

(ii) High-frequency rotor surface losses.

(iii) High-frequency stator surface losses.

(iv) The high-frequency tooth pulsation and rotor I^2R losses, due to zig-zag leakage flux.

(v) The six times frequency rotor I^2R losses, due to circulating currents induced by the stator belt leakage flux.

(vi) The extra iron losses in motors with skewed slots, due to skew leakage flux.

End Leakage Flux Losses

These losses are due to eddy currents set up in the end structure of the machine by leakage fluxes which enter the laminations in an axial direction, and also penetrate the end fingers, flanges and other adjacent metal parts. They are larger when the coil over hang is long, the distance between the centers of the peripheral currents in the stator and rotor end windings is large, and when other metal parts are close to the end turns. The part of the end-leakage flux that causes most of the end loss is due to the peripheral components of the stator and rotor end turn currents, since this flux flows in radial planes. The equation given by Alger et al[3] for calculation of stator end loss is :

$$W_{SE} = 0.3\, q\, I_1^{\,2}\, f_1\, E_{Lc} \qquad \ldots \qquad \ldots \qquad (1.61)$$

where

$$E_{Lc} = \frac{0.15748\ q\ (K_{d_1}\ K_{p_1})^2 \times D \times T_1^2 \log(1 + \frac{A^2}{4y_3\ y_4})}{10^7\ P^2} \qquad (1.62)$$

where A = Slant distance in cms between the assumed center of the stator and rotor peripheral currents.

y_3, y_4 = Axial distances between the stator and rotor end-current centers to the end of the laminated core in cms.

Fig.(1.6) shows the end-winding geometry on which this formula is based.

To adapt this equation to the motor with time harmonics in the stator current, it is necessary to apply it to the fundamental, and to each harmonic separately. The total loss is then the sum of these individual components :

$$W_{SE} = 0.3\ E_{Lc}\ q \sum_{K=1}^{n} I_K^2\ K.f \qquad \ldots \qquad (1.63)$$

An end loss like that described above for the stator also occurs in the rotor end laminations. For the motor operated at its normal slip, the frequency of the fundamental component of rotor leakage flux is very low, and the loss is negligible. However each harmonic will produce a loss. This is taken to be equal to the stator end loss for that harmonic[26].

Thus, the final equation for end loss on non-sinusoidal supply becomes :

$$W_5 = 0.3\ E_{Lc}\ q\ I_1^2 f_1 + 2 \times 0.3 \times E_{Lc}\ q \sum_{K=5}^{n} I_K^{2}\ K f_1 \qquad (1.64)$$

Rotor Zig-Zag Flux Loss

This loss is due to pulsating flux in the rotor teeth due to slot permeance and slot m m f harmonics. This flux induces currents in the rotor bars, with an I^2R loss resulting. The analysis by Alger et al leads to an approximate equation :

$$W_{ZZ} = q\ C_{db}\ R_{2b}\ \ C_0\ I_M^2 + C_L\ I_1^2 \qquad \ldots \qquad (1.65)$$

The authors have given curves to find machine constants C_0 and C_L in their paper.

The general expression for the loss factor C_L is[11] :

$$C_L = \left(\frac{4}{\pi^2}\right)^2 \left\{ \left[\frac{S_{2P}^2}{(2S_{1P}+1)^2} \operatorname{Sin}^2 \left(\frac{2S_{1P}+1}{2S_{2P}}\right)\pi \right]^2 + \left[\frac{S_{2P}^2}{(2S_{1P}-1)^2} \operatorname{Sin}^2 \frac{(2S_{1P}-1)}{2S_{2P}}\pi \right]^2 \right\} \qquad \ldots \qquad (1.66)$$

and the expression for C_0 is given as[4]

$$C_0 = \frac{K^2(S_1 - P)^2}{\left[\frac{(S_1-P)^2\pi^2}{2\,P\,S_{2P}^2}\operatorname{cosec}^2\frac{(S_1-P)}{2\,P\,S_{2P}} - 2\,P + K(S_1 - P)\right]^2}$$

$$+ \frac{K^2(S_1 + P)^2}{\left[\frac{(S_1+P)^2\pi^2}{2\,P\,S_{2P}^2}\operatorname{cosec}^2\frac{(S_1+P)}{2\,P\,S_{2P}} - 2\,P + K(S_1+P)\right]^2}$$

.... (1.67)

where S_{1P} = No. of stator slots per pole

S_{2P} = No. of rotor slots per pole

and $K = \frac{\text{Permeance ripple}}{\text{Fundamental ripple}} = \frac{\beta}{2-\beta}$ ... (1.68)

Also β, the flux pulsation, is the ordinate of Fig.(1.7).

The parameter C_{db} is a constant to account for the deep-bar effect at the slot harmonic frequency, which is $2\,S_{1P}\,f_1$ Hz.

With time harmonics present in the stator current, I_M is taken to be the fundamental no-load current, and I_1 is the entire stator current, including harmonics. Thus on non-sinusoidal supply, the zig-zag loss becomes[26]:

$$W_4 = q\,C_{db}\,R_{2b}\left[C_0\,I_M^2 + C_L\,(I_1^2 + I_{har}^2)\right] \quad \ldots \quad (1.69)$$

Surface Losses

The dips in the stator flux distribution around the airgap due to the slot openings, and also the steps in the m.m.f. distribution due to the concentration of current

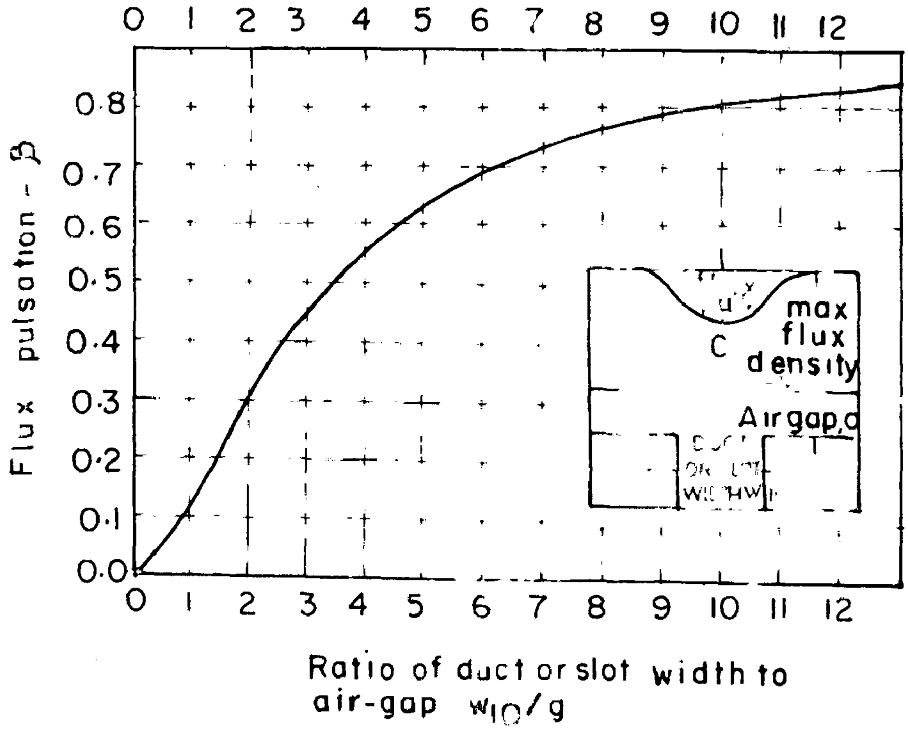

Fig. 1.7 - Flux pulsation due to slot or duct openings

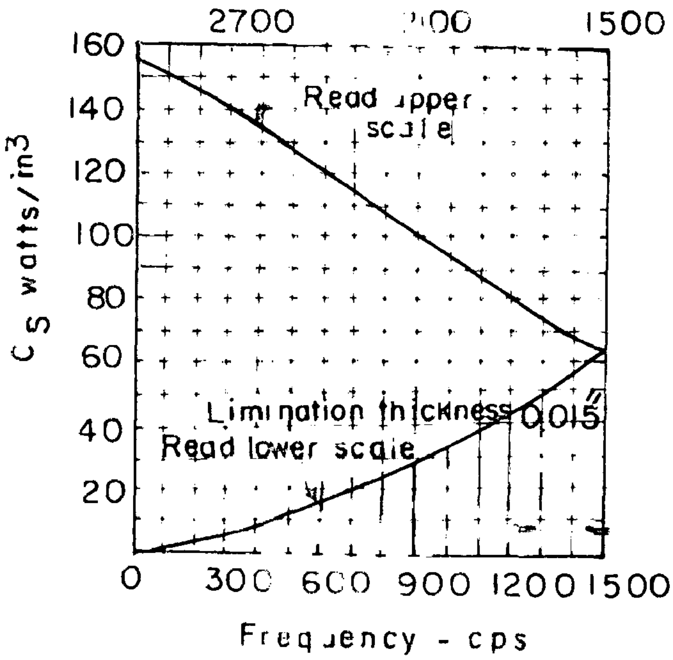

Fig 1.8 - High frequency steel loss factor, 2.5 % silicon steel.

in the slots, cause slot frequency pulsations in the radial flux density around the air-gap. These pulsations cause eddy current losses in the laminations that are usually called surface losses. The rotor surface loss due to load currents is therefore[3] :

$$W_{SL2} = 0.0508 \; D \; L \left(\frac{I_1}{S_{1P} \; I_M}\right)^2 B_g^{\,2} \, C_{S2} \, \lambda_1 \qquad \ldots \qquad (1.70)$$

By analogy with the above equation, the stator surface loss under load is

$$W_{SL_1} = 0.0508 \; D \; L \left(\frac{I_1}{S_{2P} \; I_M}\right)^2 B_g^{\,2} \; C_{s1} \, \lambda_2 \qquad \ldots \qquad (1.71)$$

Where B_g = Average flux density over the effective gap area in wb / m^2

C_{S_2} = Rotor iron loss factor, from Fig.(1.8) for stator slot frequency ($2 \; S_{1P} f_1$)

λ_1 = Stator slot pitch in cms.

C_{S_1} = Stator iron loss factor, from Fig.(1.8) for rotor slot frequency ($2 \; S_{2P} \; f_1$)

λ_2 = Rotor slot pitch in cms.

Approximate expressions for determining C_{S_1} and C_{S_2} analytically are obtained as

$$C_{S_1} = 0.00123 \; (2S_{2P}f_1)^{1.48} \qquad \ldots \qquad (1.72)$$

and

$$C_{S_2} = 0.00123 \; (2S_{1P}f_1)^{1.48} \qquad \ldots \qquad (1.73)$$

The expressions for rotor and stator surface losses are adapted for non-sinusoidal operation as under :

$$W_7 = 0.0508 \; D \; L \; \frac{(I_1^{\,2} + I_{har}^2)}{(S_{1P} \; I_M)^2} \; B_g^{\,2} \; C_{S_2} \; \lambda_1 \qquad \ldots \qquad (1.74)$$

and
$$W_6 = 0.0508 \; D \; L \; \frac{(I_1^{\,2} + I_{har}^2)}{(S_{2P} \; I_M)^2} \; B_g^{\,2} \; C_{S_1} \; \lambda_2 \qquad \ldots \qquad (1.75)$$

Since the time average value of flux density is the same as without harmonics[26], the only change in expressions (1.74) and (1.75) in comparison with (1.70) and (1.71) respectively is that total r.m.s. current including harmonics is substituted in place of I_1.

Phase-Belt Leakage Flux Loss

The low-order harmonic fields due to the phase belts of the stator winding also induce currents in the rotor. For these fields, the impedance of a squirrel-cage rotor is small in comparison with the magnetizing reactance when the motor is operating at full speed. Thus the stray loss due to belt leakage is given by[3] :

$$W_B = q \; I_1^2 \; K_s \; R_{2b} \left[\frac{K_{2q-1}^2 + K_{2q+1}^2}{K_{p_1}^2 \; K_{d_1}^2} \right] \qquad \ldots \qquad (1.76)$$

where K_s = ratio of $\dfrac{\text{a.c. resistance}}{\text{d.c. resistance}}$ for the rotor bars at the phase-belt frequency which is $2q \; f_1$ at synchronous speed.

$K_{2q \pm 1}$ = Pitch times the distribution factor of the stator winding for the $(2q \pm 1)$ field.

The phase-belt loss with time harmonics present in the supply is taken as

$$W_9 = q\ K_S\ R_{2b} \left[\frac{K^2_{2q-1} + K^2_{2q+1}}{K^2_{p_1}\ K^2_{d_1}} \right] \left[I_1^2 + I^2_{har} \right] \quad \ldots \quad (1.77)$$

Skew Leakage Flux Loss

Ideally, the fundamental m.m.f.s. produced by the stator and rotor load currents of an induction motor are equal and opposite, and produce no additional flux in the air-gap. However, in a machine with skewed rotor slots, the small phase displacement between these fundamental m m f s at the ends of the machine produces an increase in the radial gap flux density, which increases with distance from the machine centre[11]. This is known as the skew leakage flux and it produces a corresponding iron loss in the stator core and teeth at fundamental frequency. Thus skew leakage loss is[3] :

$$W_K = \frac{\delta^2\ \pi^2}{12\ S^2_{1P}} (I_2 / I_M)^2 \text{ (Stator iron loss + Rotor surface loss at no load)} \quad \ldots \quad (1.78)$$

where δ = ratio of skew to one stator slot pitch (normally = 1)

The rotor surface loss at no load is given by[3]

$$W_{SO} = 0.0508\ D\ L\ B_g^2\ K_{Pf}\ C_{S2}\ \lambda_1 \quad \ldots \quad (1.79)$$

The skew leakage loss on non-sinusoidal supply voltage, by modifying the equation (1.78), is taken as :

$$W_8 = \frac{\sigma^2 \pi^2}{12\ S_{1P}^2} \ \frac{(I_2^2 + I_{2har}^2)}{I_m{}^2} (W_3 + W_{SO}) \qquad \ldots \qquad (1.80)$$

Where

I_{2har} is the total harmonic rotor current.

1.7. <u>RESUME</u>

A method for calculation of induction motor losses on non-sinusoidal supplies has been formulated in this chapter. The simple method of steady state analysis used here, consists of performing a Fourier analysis of the supply waveform and then obtaining the motor response as a summation of the responses for the fundamental component and the various harmonics. The method is very much suited for a study of the increased machine losses on non-sinusoidal voltage waveforms.

CHAPTER II

VARIABLE FREQUENCY OPERATION OF A THREE PHASE CAGE INDUCTION MOTOR

INTRODUCTION

In this chapter calculation of losses of three phase cage induction motor for variable frequency operation is considered. The losses are determined for constant full load torque condition. The variation of losses with frequency at constant volts / Hz mode of operation for sinusoidal as well as non-sinusoidal voltage waveforms is studied. Also variable frequency operation at constant-flux mode with non-sinusoidal supply voltage waveform is discussed. Three flow-charts, one for determination of equivalent circuit parameters; second one for determination of losses on sinusoidal as well as non-sinusoidal supply voltage waveform with constant volts / Hz mode of operation and a third one for determination of losses on non-sinusoidal voltage with constant-flux mode of operation have been developed. Computer programs based on these flow-charts were run on I B M - 1620 digital computer and results obtained are compared and discussed. In addition to this the effect of variation of equivalent-circuit parameters on the total losses and other performance factors such as fundamental slip, efficiency and total input current for constant volts/Hz mode of operation with non-sinusoidal supply voltage is studied.

2.1. CONSTANT VOLTS / Hz MODE OF OPERATION

This mode of operation is commonly used in simple open-loop systems. In order to obtain a constant torque at various frequencies of operation a constant air-gap flux is necessary. This can be achieved by keeping the ratio E_1/f_1 constant, where E_1 is the r.m.s. phase e.m.f. and f_1 is the normal operating frequency. If stator leakage impedance is small, then fundamental applied voltage V_1 is approximately equal to E_1 and consequently the air gap flux is nearly constant when the ratio V_1/f_1 has a fixed value. This is the constant volts/Hz mode of working.

Operation of the machine is considered as constant torque drive upto the normal frequency. Thus operating frequency is given as :

$$f = W f_1 \qquad \ldots \qquad \ldots \qquad (2.1)$$

where $$W \leq 1.0 \qquad \ldots \qquad \ldots \qquad (2.2)$$

and the operating voltage is :

$$V = W \cdot V_1 \qquad \ldots \qquad \ldots \qquad (2.3)$$

2.2. LOSS COEFFICIENTS

In chapter I, various formulae for calculation of losses of induction motor in the presence of time harmonics are already discussed. When variable frequency operation is being considered some of these formulae are to be rewritten in different form , for convenience, by defining certain loss-coefficients as indicated below. The loss coefficient is the term used here for that part of the loss formula which is unaffected by variation of the operating frequency.

Stator Core Loss Coefficient

Combining equations (1.49), (1.50) and (1.53) in chapter I, we get the iron loss in stator for fundamental frequency of f_1 Hz as

$$W_1 = 0.014322 \left[(B_T \cdot 1.57)^{1.8} \cdot W_{TT} + (B_C)^{1.8} W_{TC} \right] \cdot f_1^{1.6} \quad \ldots (2.4)$$

For variable frequency operation, f_1 will be replaced by f in the above formula. As ratio V_1/f_1 is having a fixed value and the flux densities in various iron parts are approximately constant, we can rewrite the above equation for variable frequency f as

$$W_1 = S_{CLC} \cdot f^{1.6} \quad \ldots \quad \ldots \quad (2.5)$$

where

$$S_{CLC} = 0.014322 \left[(B_T \cdot 1.57)^{1.8} W_{TT} + (B_C)^{1.8} W_{TC} \right] \quad \ldots \quad (2.6)$$

and is called as stator core loss coefficient.

The total iron loss in presence of non-sinusoidal supply voltage given by equation (1.55) can be rewritten in terms stator core loss coefficient as

$$W_3 = S_{CLC} \cdot f^{1.6} + S_{CLC} \sum_{K=5}^{n} \left(\frac{1}{K^2}\right)^{1.8} \cdot (Kf)^{1.6}$$

$$\text{or} \quad W_3 = S_{CLC} \cdot f^{1.6} + S_{CLC} \sum_{K=5}^{n} (f^{1.6}/K^2) \quad \ldots \quad (2.7)$$

Rotor Core Loss Coefficient

Defining rotor core-loss coefficient as :

$$R_{CLC} = 0.014322 \left[(B_{TR} \times 1.57)^{1.8} W_{TTR} + (B_{CR})^{1.8} W_{TCR} \right] \ldots (2.8)$$

the expression (1.58) for rotor iron loss for fundamental frequency of f_1 Hz simplifies to :

$$W_{1R} = R_{CLC} \times (sf)^{1.8} \quad \ldots \quad \ldots \quad (2.9)$$

for operating frequency of f Hz.

where s is the fundamental slip corresponding to the operating frequency. Now, the total iron loss on non-sinusoidal supply in the rotor as expressed in equation (1.59) will be

$$W_{10} = R_{CLC} \cdot (sf)^{1.8} + R_{CLC} \sum_{K=5}^{n} (s_K\, Kf)^{1.6} \cdot \left(\frac{1}{K^2}\right)^{1.8}$$

$$\text{or } W_{10} = R_{CLC} \cdot (sf)^{1.8} + R_{CLC} \sum_{K=5}^{n} (s_K\, f)^{1.6} / K^2 \quad \ldots \quad (2.10)$$

where s_K is the slip corresponding to K^{th} time harmonic.

Surface Loss Coefficients

Stator surface loss coefficient is defined as

$$S_{SLC} = \frac{0.0508 \cdot D.L \cdot B_g^2 \ \lambda_2}{(S_{2P})^2} \quad \ldots \quad (2.11)$$

Similarly the rotor surface loss coefficient is expresses as:

$$R_{SLC} = \frac{0.0508\ D\ L\ B_g^2\ \lambda_1}{(S_{1P})^2} \quad \ldots \quad (2.12)$$

The expressions(1.70), (1.71), (1.74), (1.75) for the stator and rotor surface losses for sinusoidal as well as non-sinusoidal operations, thus are reduced to

$$W_{SL_1} = S_{SLC}\ (I_1/\ I_M)^2\ C_{S_1} \quad \ldots \quad (2.13)$$

$$W_{SL_2} = R_{SLC}\ (I_1/\ I_M)^2\ C_{S_2} \quad \ldots \quad (2.14)$$

$$W_6 = S_{SLC} \frac{(I_1^2 + I_{har}^2)}{I_M^2}\ C_{S_1} \quad \ldots \quad (2.15)$$

and $W_7 = R_{SLC} \dfrac{(I_1^2 + I_{har}^2)}{I_M^2}\ C_{S_2}$ ... (2.16)

The no load rotor surface loss given by equation (1.79) would be rewritten

as $W_{So} = R_{SNC} \cdot C_{S_2}$ ... (2.17)

where $R_{SNC} = 0.0508\ D\ L\ B_g^2\ K_{Pf}\ \lambda_1$ ... (2.18)

and is called as Rotor no-load surface loss coefficient.

<u>Skew Leakage Loss Coefficient</u>

Using the defining equation for skew leakage loss coefficient as

$$S_{KCO} = \frac{\sigma^2\ \pi^2}{12\ S_{1P}^2} \quad \ldots \quad \ldots \quad (2.19)$$

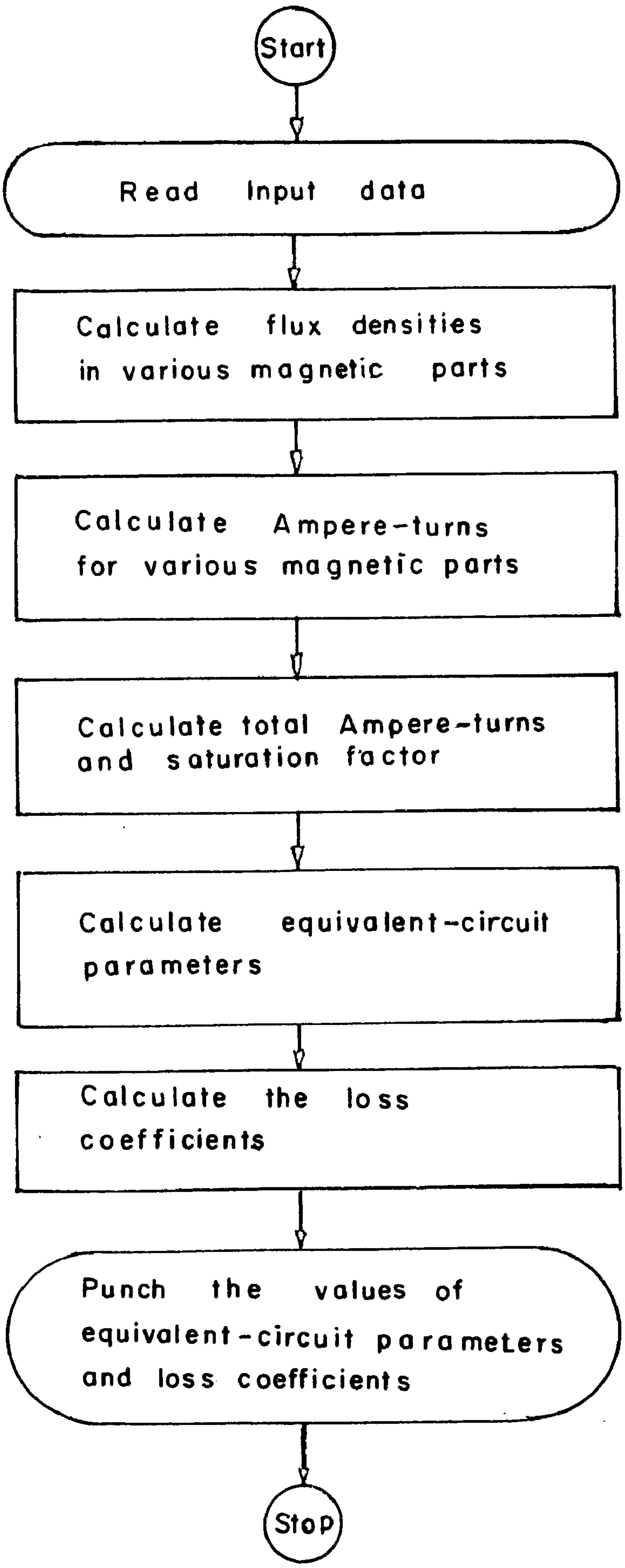

Fig .2.1 A flow chart for calculation of equivalent-circuit parameters and loss coefficients.

the skew leakage flux losses for sinusoidal and non-sinusoidal operation expressed in equations (1.78) and (1.80) are obtained as

$$W_K = S_{KCO} (I_2 / I_M)^2 (W_1 + W_{SO}) \quad \ldots \quad (2.20)$$

and $$W_9 = S_{KCO} \frac{(I_2^2 + I_{2har}^2)}{I_M^2} (W_3 + W_{SO}) \quad \ldots \quad (2.21)$$

End Leakage Loss Coefficient

The end leakage loss coefficient is expressed as

$$E_{LCO} = 0.3 (E_{LC})(3) \quad \ldots \quad \ldots \quad (2.22)$$

and so the end leakage losses on sinusoidal as well as non-sinusoidal supply given by equations (1.61) and (1.64) get transformed to :

$$W_{SE} = (E_{LCO}) I_1^2 f \quad \ldots \quad \ldots \quad (2.23)$$

and $$W_5 = E_{LCO} I_1^2 f + 2 E_{LCO} \sum_{K=5}^{n} I_K^2 K f \quad \ldots \quad (2.24)$$

for operating frequency of f Hz.
Where I_K is the K^{th} harmonic stator current.

2.3. FLOW CHART FOR DETERMINATION OF THE EQUIVALENT-CIRCUIT PARAMETERS AND THE LOSS COEFFICIENTS

To study the effect of variation of fundamental frequency on total losses and components of the total losses, it is essential to determine the equivalent circuit parameters and loss coefficients. A simple flow chart to achieve this

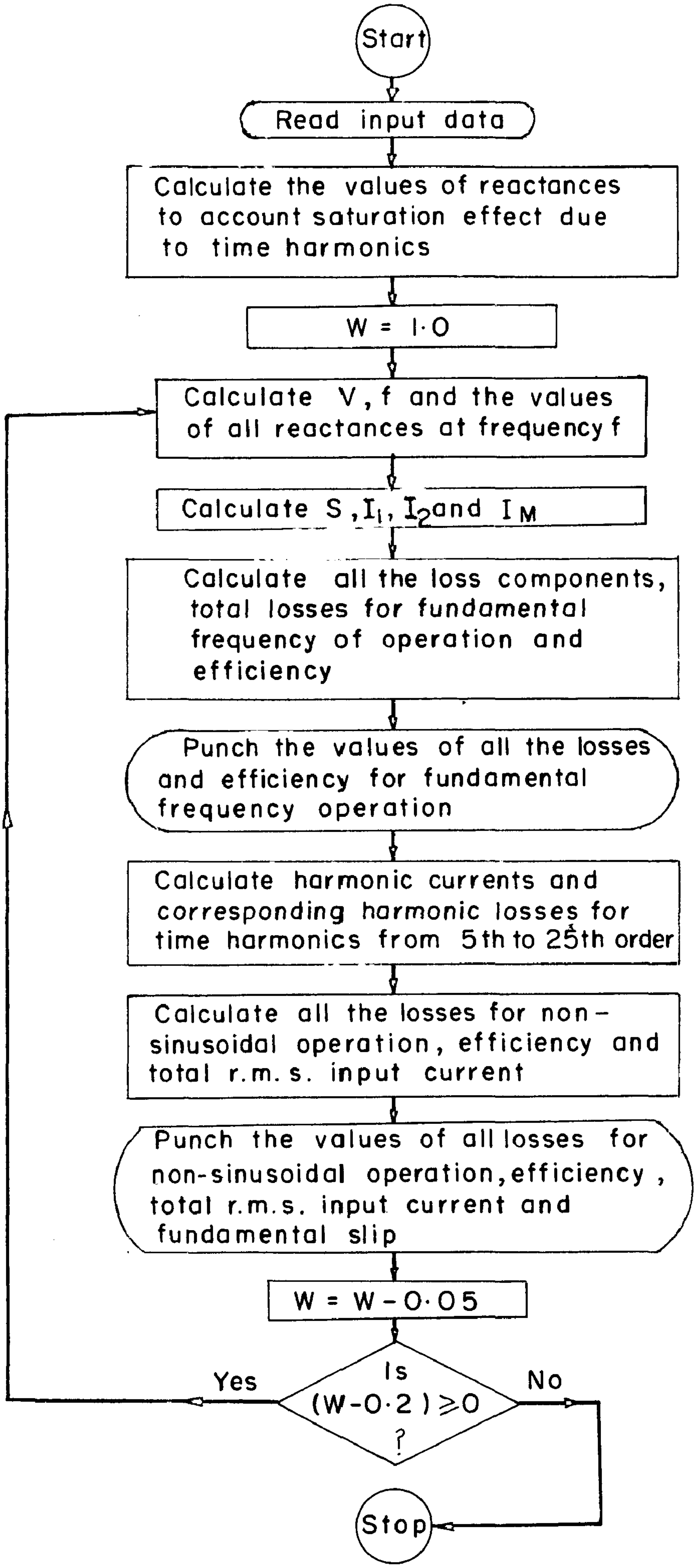

Fig. 2·2 A flow chart for determination of losses at different frequencies for constant volts/Hz mode of operation objective is shown in Fig.2.1.

The input data for a computer program based on this flow chart includes name-plate data of the machine, stator and rotor dimensions, winding details etc. The detailed input data has been tabulated in table No.I and is given in Appendix No.7. The values of various items are taken from design sheet of a squirrel cage motor of normal design[38].

The output data for this program consists of the values of various loss coefficients and the equivalent-circuit parameters. A separate program for these calculations was found necessary to save computational time.

2.4. DETERMINATION OF LOSSES FOR CONSTANT VOLTS/Hz MODE AT DIFFERENT FREQUENCIES OF OPERATION

A flow chart as shown in Fig.2.2. is developed to calculate the losses at different operating frequencies when the volts/Hz ratio is kept constant. Both sinusoidal and non sinusoidal voltage waveforms have been considered. The motor is assumed to develop full-load torque for all operating frequencies. The flow chart is self explanatory. First the values of all the reactances in the equivalent - circuit are recalculated for accounting saturation due to time harmonics as discussed in section 1.2, of chapter I. Then the determination of losses proceeds as shown in Fig. 2.2.

The input data for a computer program based on this flow chart comprises of the output data from the program

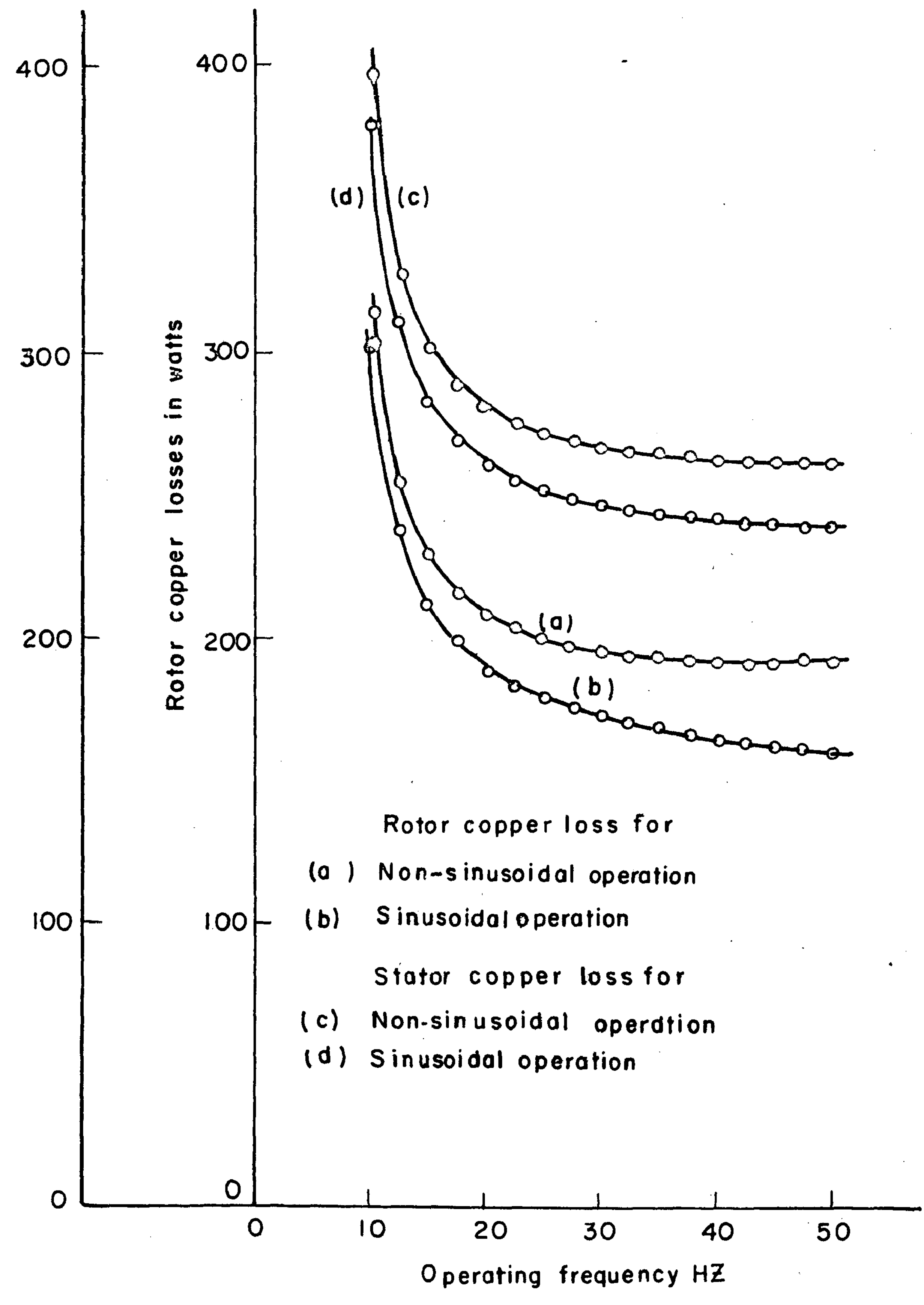

Fig. 2.3 - Variation of copper losses with frequency for constant V/f operation.

based on Fig. 2.1, plus name plate data and a part of design data like number of stator and rotor slots, depth of the rotor conductor, ratio of conductor width to slot width in the rotor, pulsation and belt-leakage loss constants etc. of the motor[38] as listed in table No.II given in Appendix No.8.

The output data for this program consists of the values of all losses, efficiency and total r.m.s. current at different operating frequencies, for sinusoidal as well as non-sinusoidal voltage waveforms with constant Volts/Hz mode of operation. A comparison of operations with sinusoidal and non-sinusoidal supply voltages is done in the following paragraphs. One point to be noted here is that the operating frequency range for the motor was found to be from 10 Hz to 50 Hz with the normal values of the parameters; and below 10 Hz, the motor was incapable of producing full-load torque with constant Volts / Hz mode of working.

2.4.1. PERFORMANCE ON VARIABLE FREQUENCY OPERATION

Here the effect of variation of operating frequency on the total losses, fundamental slip, efficiency and total r.m.s. current is considered. Computed losses of the motor for sinusoidal and non-sinusoidal supply voltages are represented graphically and thus a comparison of the performance on these voltage waveforms is effected.

Copper Losses

In Fig. 2.3. is illustrated variation of copper losses of the motor with operating frequency. It is obvious that for a given frequency the copper losses with non-sinusoidal voltage waveform are more than the corresponding losses with sinusoidal supply voltage. Percentage increase in rotor copper losses with non-sinusoidal voltage source over corresponding losses on sinusoid supply is about 19 percent on normal frequency which is progressively reduced as the operating frequency is reduced. At 10 Hz, the lowest operating frequency, an increase of only about 5 percent is obtained in rotor copper losses. This is because of two reasons. At higher values of operating frequency the deep bar effect in rotor due to harmonics causes more increase in the rotor resistance and more decrease in the leakage reactance of the rotor than at the lower operating frequency. Rotor harmonic current and also the stator harmonic current is governed by the values of the leakage reactances at particular harmonic frequency. Thus the harmonic currents are more at higher values of the operating frequency than the corresponding ones at lower operating frequency. Hence there is considerable increase in rotor copper losses on non-sinusoidal supply at higher values of the operating frequency as seen in the Fig. 2.3. In the case of stator copper losses the stator resistance is not affected by skin effect due to harmonics at higher operating frequency. The increase in stator harmonic currents at higher operating frequency, however, causes an increase in the stator copper losses on non-sinusoidal supply in comparison with the losses on sinusoidal voltage source. The figures obtained for the increase of stator copper losses on non-sinusoidal supply over the losses on sinusoidal supply vary from about 9 percent to 4 percent with the variation of operating frequency from 50 Hz to 10 Hz.

Another interesting point to be noted here is that for constant Volts/Hz mode of working, even with sinusoidal supply voltage, the rotor and stator losses increase progressively as the operating frequency is reduced. The increase in the losses due to variation of the operating frequency from 50 Hz to about 25 Hz is not significant. As the operating frequency is reduced below 20 Hz, however, the increase in the losses is very high and at 10 Hz the stator and rotor copper losses on sinusoidal supply increase respectively by about 60 to 90 percent over their values at the normal frequency. This is due to the following reasons. The decrease in the operating frequency causes progressive reduction in the reactances whereas the stator resistance is unaffected by this variation. So at low operating frequencies there is considerable reduction in the air-gap flux due to increased influence of the stator resistance. Again the fundamental slip is considerably increased at low frequencies as shown in Fig. 2.7. These two factors cause very much increase in the fundamental rotor current at low operating frequency as the motor is assumed to operate with a constant load torque. The increase in fundamental rotor current is responsible for

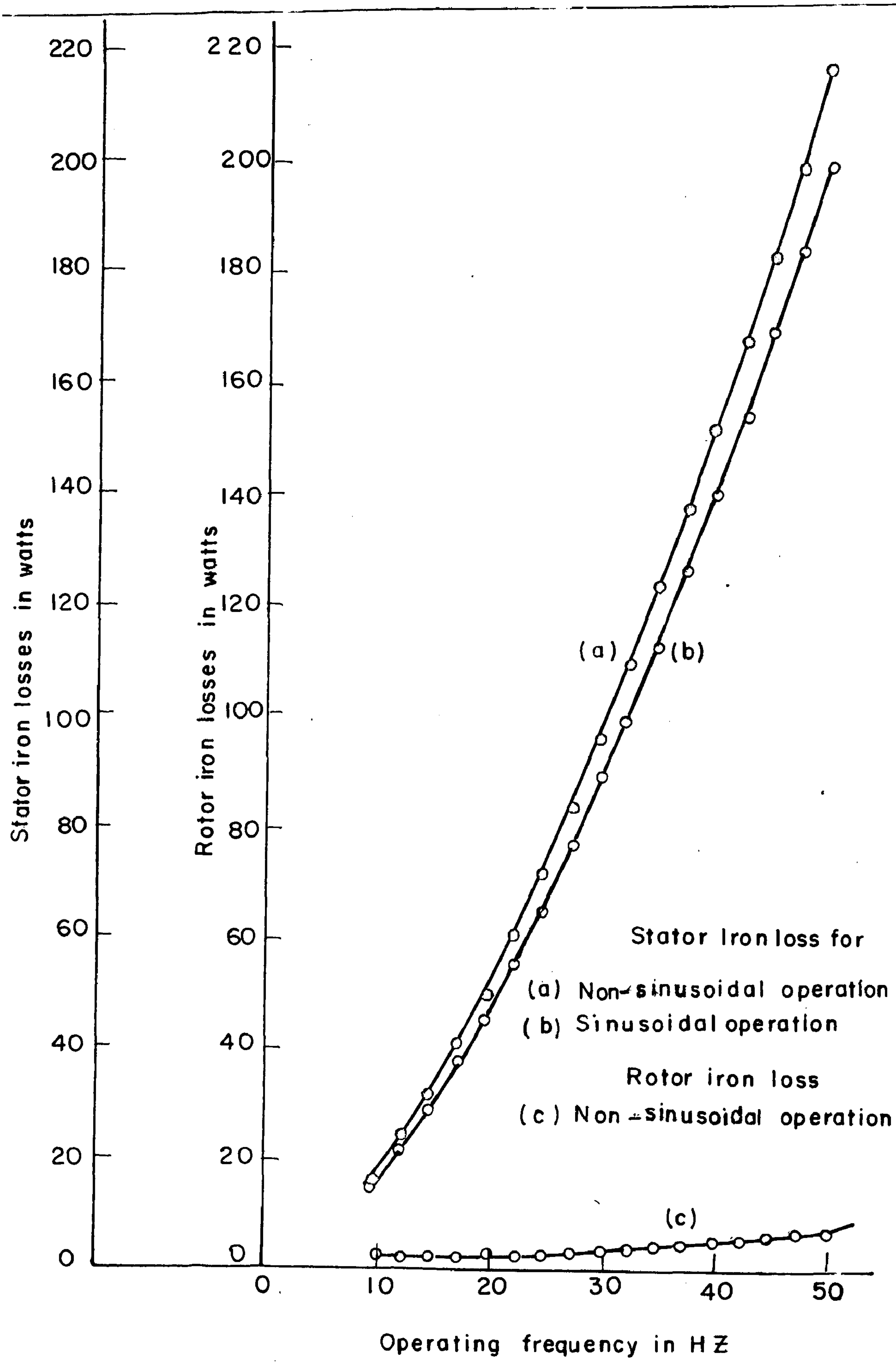

Fig. 2.4 - Variation of Iron losses with frequency for constant V/f operation.

substantial increase in the stator fundamental current. The net result is, thus, very large increase in the copper losses at the lowest operating frequency over the normal frequency of operation as indicated above.

Iron Losses

The influence of operating - frequency variation on the iron losses for sinusoidal as well as non-sinusoidal supply voltages with constant Volts/Hz mode of operation is shown in Fig. 2.4. Increase in the stator iron losses is about 8.5 percent with non-sinusoidal voltage waveform over the sinusoidal ones for all the operating frequencies. This is easily explained by equation (2.7). Again the iron losses progressively decrease with decrease of the operating frequency as they are directly dependent on the frequency as illustrated by equations (2.5) and (2.7).

The rotor core losses are insignificant for fundamental frequency operation. In comparison they are considerably increased with non-sinusoidal supply voltage waveform. These losses progressively decrease with the operating frequency but at the lowest value of the operating frequency, they again increase because of substantial increase in the fundamental slip.

Stray-Load and Friction and Windage Losses

The variation of stray-load and friction and windage losses with the operating frequency is shown in Fig. 2.5.

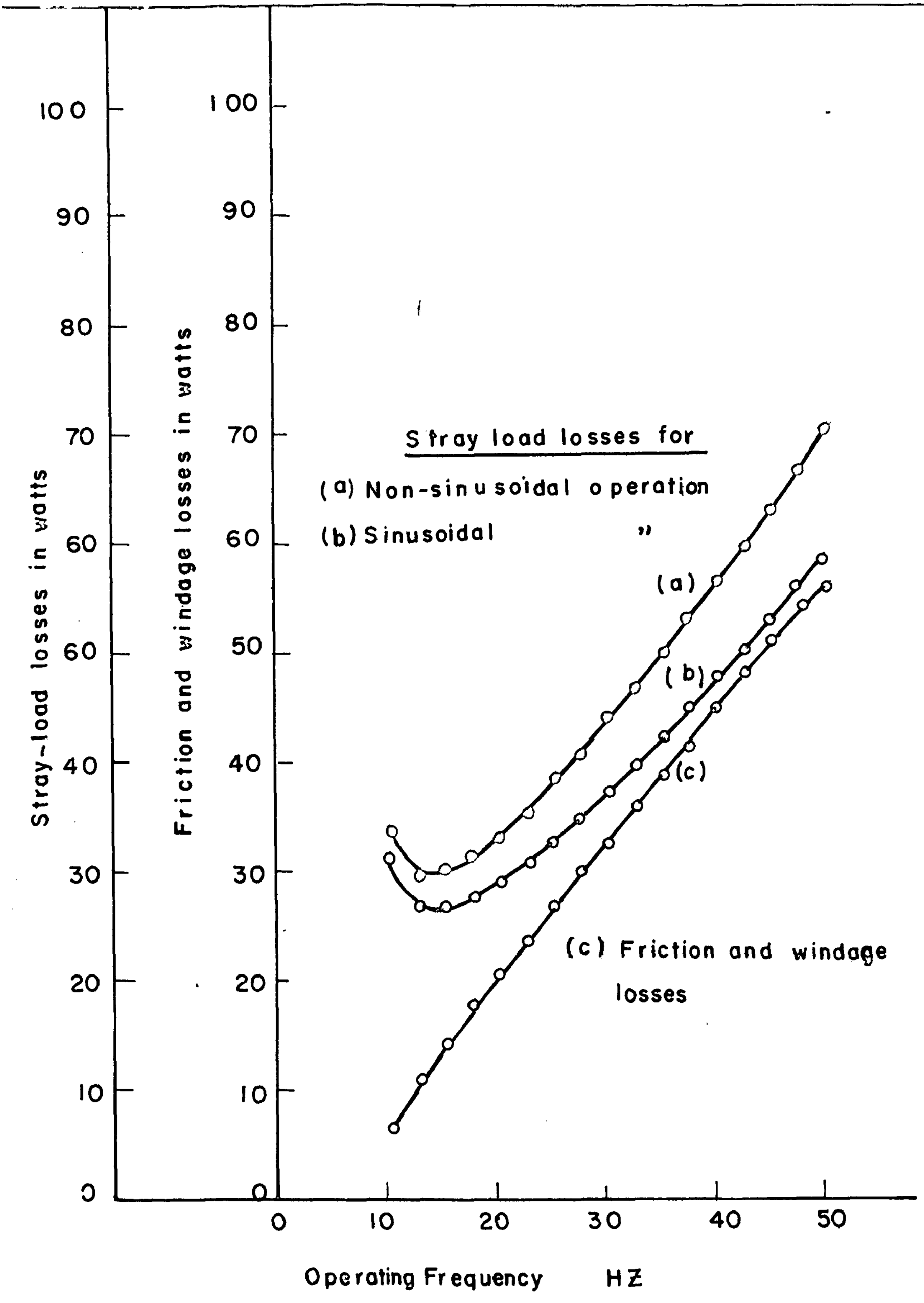

Fig.2.5 - Variation of stray-load losses and friction and windage losses with frequency for constant V/f operation.

The friction and windage losses are dependent upon the operating frequency and the fundamental slip as given by equation (1.60) in chapter I, however, they are not influenced by harmonics in the supply voltage. When the operating frequency is decreased, the fundamental slip increases as shown in Fig. 2.7. Hence these losses will clearly decrease with the reduction in operating frequency as illustrated in Fig. 2.5(c). This decrease will be more in the low frequency range because of the large values of slip.

The influence of operating frequency on the stray-load losses for sinusoidal as well as non-sinusoidal supply voltages is depicted by curves (a) and (b) of Fig. 2.5. The stray load-losses on non-sinusoidal voltage waveform are greater than for the sinusoidal ones for a given operating frequency, as is the case with all other losses. The increase in these losses on non-sinusoidal supply is about 19 percent over the ones with sinusoidal voltage source at the normal frequency and it decreases to about 7.5 percent at the lowest operating frequency of 10 Hz. The stray load losses progressively decrease with the reduction of operating frequency upto certain value (15 Hz) and then they start increasing again. All the stray-load losses are functions of the currents in the motor. In addition some them are dependent on the iron loss factors and skin effect factors due to space harmonics. Hence when the operating frequency is reduced considerably, the fundamental currents shoot up and these losses start increasing.

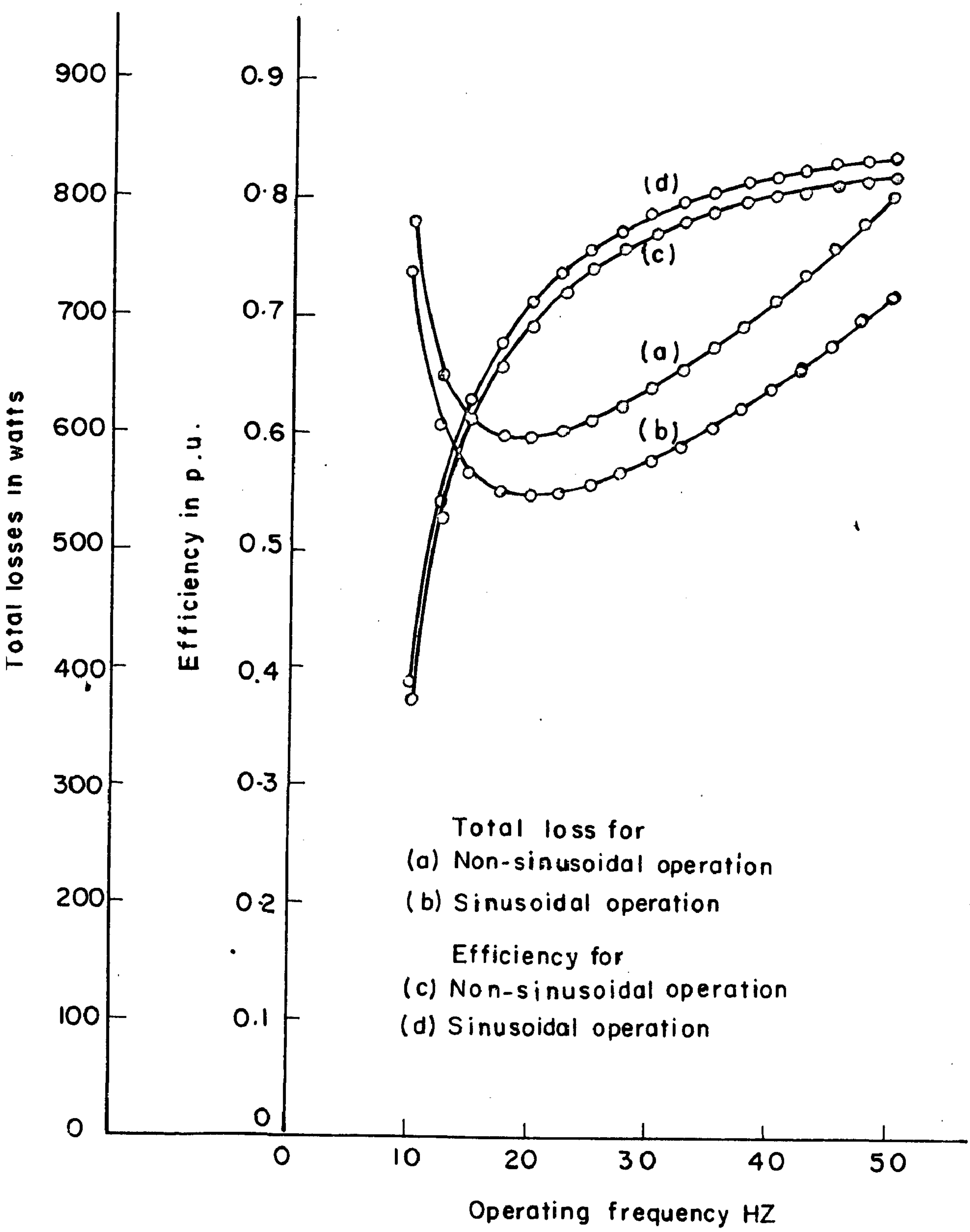

Fig. 2·6 - Variation of total losses and efficiency with frequency for constant V/f operation.

Total Losses and Efficiency

The effect of variation of the total losses and efficiency with the operating frequency is shown in Fig.2.6. The curves (a) and (b) in this figure illustrate the variation of the total losses with operating frequency. It is seen from the curves (a) and (b) of Fig. 2.6. that the total losses for sinusoidal as well as non-sinusoidal supply voltages decrease consistently with the reduction of operating frequency upto a certain value of the operating frequency (20 Hz). When the frequency is decreased further, they start increasing again. This can be easily explained as follows. When the operating frequency starts decreasing from the normal value, the reduction in iron losses, stray-load losses, friction and windage losses dominates over the increase in the copper losses of the machine up to certain value of the operating frequency and so the net result is the decrease of the total losses with the operating frequency. When the operating frequency decreases further the increase in the copper losses dominates over the decrease in the other components of the losses. Thus the total losses again start increasing. As expected, the percentage increase in the total losses with non-sinusoidal operation over the ones with sinusoidal supply is found to be maximum (about 12 percent) at the normal frequency and it reduces continuously with the reduction of the operating frequency.

The efficiency-variation with the operating frequency is shown by the curves (c) and (d) of Fig. 2.6. As it is

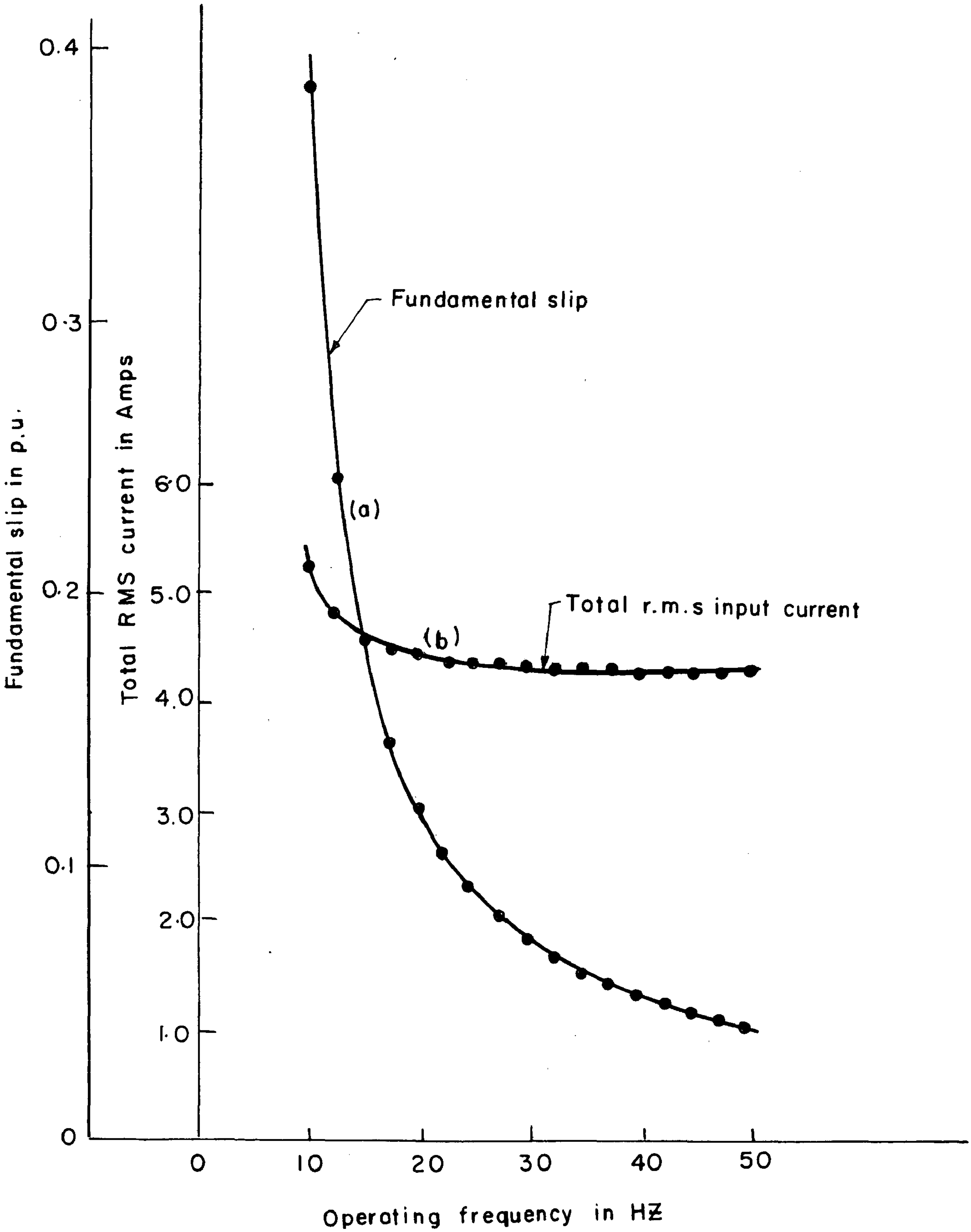

Fig. 2.7 - Variation of total r m s current and fundament. slip with frequency for constant V/f operation.

clear from the study of various loss-curves, the efficiency decreases both for sinusoidal and non-sinusoidal working with the reduction of operating frequency. The efficiency for non-sinusoidal operation is obviously less than the sinusoidal operation and the maximum reduction in efficiency with non-sinusoidal operation is found to be at the normal frequency (of 50 Hz).

Effect of Frequency-variation on the Fundamental Slip and Total R.M.S. Input Current

Curves (a) and (b) of Fig. 2.7. represent respectively the effect of frequency-variation on the fundamental slip and the total r.m.s. input current. As seen from curve (a), the fundamental slip increases with decrease in operating frequency. Initially (upto 30 Hz) the increase in the slip is gradual but when the operating frequency is reduced considerably, (below 20 Hz) the slip starts shooting up. This is due to the reason that at very low value of operating frequency the air-gap flux is reduced to such an extent that to produce a constant load torque, the fundamental slip has to increase substantially.

The curve (b) of Fig. 2.7. shows that the total r.m.s. current increases with the reduction in the operating frequency. The increase in the total current is negligibly small initially (i.e. upto 30 Hz) and then moderate upto certain value (i.e. 20 Hz) of the operating frequency. When the operating frequency is very low, (i.e. below 15 Hz) the increase in the current is considerable. The reason for this is already explained while discussing the effect of frequency-variation on the copper losses.

From the discussion given in the preceeding paragraphs it becomes amply clear that, with the constant Volts/Hz operation the performance of the motor deteriorates at low operating frequencies. Also the total losses on non-sinusoidal operation are the largest at the normal frequency. Increase in the losses causes a reduction in the efficiency by about 2 percent.

2.5. INFLUENCE OF VARIATION OF EQUIVALENT-CIRCUIT PARAMETERS ON PERFORMANCE

Other aspect of the study was to consider the effect of variation of the equivalent-circuit parameters on the total losses, efficiency and fundamental slip of the motor for constant Volts/Hz mode of operation with non-sinusoidal supply voltage waveform. For this purpose one parameter was varied at a time while all others were kept constant at their normal values. The computations were carried out by making suitable modifications in the program for calculation of the losses on non-sinusoidal supply voltage with constant Volts/Hz mode of operation. The modifications in the program include the following:

(1) Feeding three values of the parameter to be varied, viz. values above and below the normal value and the normal value.

(2) Insertion of a 'DO LOOP' to obtain the performance at all the values of the parameter.

(3) Increase in step size of the variable operating frequency, to save computational time, and

(4) Punching out only the desirable output i.e. the total losses, efficiency and the fundamental slip.

Thus, modified programs were run on I B M - 1620 digital computer and the results are listed in tables 2.1 to 2.5.

Stator Resistance

Effect of variation of the stator resistance on the total losses is clearly shown by Table No. 2.1. The results listed are for three values of the stator resistance R_1, namely the normal value and values giving 30 percent variation on both sides of the normal value. By studying the table it can be concluded that the total losses decrease when the stator resistance is decreased and vice-versa for a given operating frequency. Obviously the effect of reduction of the resistance on the efficiency would be to increase the latter for a particular frequency, as shown in the table. For a given operating frequency the fundamental slip increases with increase of the stator resistance.

Table No. 2.1

Effect of Variation of Stator per Phase Resistance on Performance

S.No.	Stator Resistance per phase - ohms		Fundamental Frequency - Hz	Total losses - Watts	Efficiency in p.u.	Fundamental p.u. slip
1.		3.33	50.0	719.62	0.836	0.0403
2.		3.33	40.0	624.80	0.823	0.0512
3.	0.7 R_1	3.33	30.0	543.68	0.797	0.0701
4.		3.33	20.0	484.14	0.738	0.112
5.		3.33	10.0	497.94	0.526	0.279
1.		4.75	50.0	807.46	0.820	0.0414
2.	Normal	4.75	40.0	715.32	0.802	0.0531
3.	Value	4.75	30.0	639.99	0.769	0.0738
4.	R_1	4.75	20.0	597.15	0.693	0.1217
5.		4.75	10.00	772.05	0.378	0.3874
1.		6.17	50.0	898.92	0.803	0.0427
2.	1.3 R_1	6.17	40.0	810.96	0.781	0.0552
3.		6.17	30.0	744.90	0.740	0.0781
4.		6.17	20.0	732.00	0.644	0.135

Magnetizing Reactance

The influence of variation of the magnetizing reactance, on the total losses, efficiency and slip is shown in Table No. 2.2. It can be seen that the effect of increase of the magnetizing reactance is beneficial on the performance of the motor as the total losses decrease, the efficiency improves and the fundamental slip decreases for a given operating frequency. It can, however, be pointed out that the increase in the parameter value by 30 percent from normal value does not produce considerable improvement in the motor performance, whereas a reduction of 30 percent from the normal value of the parameter gives rise to significant deterioration of the performance of the motor. This can be attributed to considerable increase in the magnetizing current and consequently to the increase in copper losses.

Table No. 2.2

Effect of Variation of Magnetizing Reactance (at normal frequency of 50 Hz) on Performance.

Sl. No.	Magnetizing Reactance at normal frequency-ohms		Fundamental Frequency Hz	Total losses Watts	Efficiency in p.u.	Fundamental slip - p.u.
1.		323.22	50.0	800.78	0.821	0.0408
2.		323.22	40.0	704.49	0.805	0.0522
3.	$1.3X_M$	323.22	30.0	625.61	0.773	0.0727
4.		323.22	20.0	579.76	0.700	0.1197
5.		323.22	10.0	744.60	0.391	0.377
1.		248.63	50.0	807.46	0.820	0.0414
2.		248.63	40.0	715.32	0.802	0.0531
3.	Normal	248.63	30.0	639.99	0.769	0.0738
4.	Value	248.63	20.0	597.15	0.693	0.1217
5.	X_M	248.63	10.0	772.05	0.378	0.3874
1.		174.04	50.0	849.67	0.812	0.0426
2.	$0.7X_M$	174.04	40.0	759.76	0.792	0.0546
3.		174.04	30.0	686.06	0.756	0.0760
4.		174.04	20.0	644.35	0.676	0.1256
5.		174.04	10.0	837.77	0.351	0.4102

Stator Leakage Reactance

The stator leakage reactance (at normal frequency) is varied in three steps viz. normal value and 20 percent increase or decrease in the normal value. The variation of this parameter does not produce significant variation in the total losses, efficiency, and fundamental slip as shown by Table No.2.3. Increase in the value of the parameter produces decrease in the total losses for a given operating frequency upto certain minimum value (30 Hz) of the latter. This is because of the decrease in the harmonic currents and consequently harmonic copper losses. When the operating frequency reaches a very low value (below 20 Hz), however, the increase in the value of the parameter has a detrimental effect on the total losses, efficiency, etc. This is due to the fact that in the low frequency range of operation, for a given operating frequency, the increase in the value of the parameter causes increase in the fundamental slip which is more important contrarary to its effect on the latter, in the higher frequency range. This gives rise to increase in the fundamental copper losses and consequently total losses for low operating frequencies, when the value of the parameter is increased.

Table No. 2.3

Effect of Variation of Stator per Phase Reactance (at normal frequency of 50 Hz) on Performance.

Sl. No.	Stator per phase reactance at normal frequency - ohms.		Fundamental Frequency Hz.	Total losses Watts	Efficiency - p.u.	Fundamental slip - p.u.
1.		7.283	50.0	816.27	0.818	0.0407
2.	$0.8X_1$	7.283	40.0	722.21	0.801	0.0521
3.		7.283	30.0	644.70	0.768	0.0725
4.		7.283	20.0	598.72	0.693	0.1194
5.		7.283	10.0	754.80	0.389	0.374
1.		9.103	50.0	807.46	0.820	0.0414
2.	Normal Value	9.103	40.0	715.32	0.802	0.0531
3.	X_1	9.103	30.0	639.99	0.769	0.0738
4.		9.103	20.0	597.15	0.693	0.1217
5.		9.103	10.0	772.05	0.378	0.3874
1.		10.924	50.0	803.13	0.821	0.0422
2.	$1.2X_1$	10.924	40.0	712.34	0.803	0.0541
3.		10.924	30.0	638.64	0.769	0.0752
4.		10.924	20.0	598.44	0.692	0.1242
5.		10.924	10.0	794.82	0.365	0.4032

Rotor Leakage Reactance

The effect of variation of the rotor per phase reactance (at normal frequency) by 20 percent on both sides of the normal value on the total losses, efficiency etc. is presented in the Table No.2.4. This parameter variation is having similar effect as the variation of stator leakage reactance on the motor performance and so the same comments are also applicable in this case.

Table No.2.4

Effect of Variation of rotor per phase Reactance (at normal frequency of 50 Hz) on Performance

Sl. No.	Secondary per phase rectance at normal frequency- ohms		Fundamental Frequency - Hz	Total losses - Watts	Efficiency - p.u.	Fundamental slip p.u.
1.		7.457	50.0	816.74	0.818	0.0413
2.	$0.8X_2$	7.457	40.0	723.10	0.801	0.0528
3.		7.457	30.0	646.00	0.768	0.0734
4.		7.457	20.0	600.49	0.692	0.1209
5.		7.457	10.0	758.32	0.386	0.3789
1.		9.321	50.0	807.46	0.820	0.0414
2.		9.321	40.0	715.32	0.802	0.0531
3.	Normal	9.321	30.0	639.99	0.769	0.0738
4.	Value	9.321	20.0	597.15	0.693	0.1217
5.	X_2	9.321	10.0	772.05	0.378	0.3874
1.		11.186	50.0	801.87	0.821	0.0416
2.	$1.2X_2$	11.186	40.0	710.89	0.803	0.0533
3.		11.186	30.0	636.97	0.770	0.0742
4.		11.186	20.0	596.42	0.693	0.1226
5.		11.186	15.0	612.62	0.605	0.1836

Rotor Per Phase Resistance

Table No.2.5. gives the effect of variation of the rotor resistance by 30 percent on both sides of the normal value. The variation of rotor resistance produces similar effect on the total losses, efficiency and fundamental slip of the motor as is produced by the variation of the stator resistance. Hence similar comments are applicable in this case also.

Table No.2.5

Effect of Variation of Rotor Per Phase Resistance on Performance

Sl. No.	Rotor per phase Resistance - ohms		Fundamental Frequency -Hz	Total Losses - Watts	Efficiency - p.u.	Fundamental - p.u.
1.		3.007	50.0	741.93	0.834	0.0290
2.	0.7 R_2	3.007	40.0	650.88	0.819	0.0371
3.		3.007	30.0	575.69	0.791	0.0517
4.		3.007	20.0	529.90	0.726	0.0852
5.		3.007	10.0	672.69	0.454	0.2712
1.		4.296	50.0	807.46	0.820	0.0414
2.	Normal	4.296	40.0	715.32	0.802	0.0531
3.	Value	4.296	30.0	639.99	0.769	0.0738
4.	R_2	4.296	20.0	597.15	0.693	0.1217
5.		4.296	10.0	772.05	0.378	0.3874
1.		5.585	50.0	872.85	0.806	0.0539
2.	1.3 R_2	5.585	40.0	779.59	0.786	0.0690
3.		5.585	30.0	704.09	0.747	0.0959
4.		5.585	20.0	664.12	0.660	0.1582
5.		5.585	10.0	870.88	0.304	0.5036

2.6. CONSTANT-FLUX OPERATION

A high torque throughout the speed range of the motor can be achieved by maintaining a constant air-gap flux. In order to keep the air-gap flux constant, the ratio E_1 / f_1 should have a fixed value, where E_1 is the per phase e.m.f. at the normal frequency of f_1 Hz. Thus when the operating frequency is given by equation (2.1), the e.m.f. per phase corresponding to the operating frequency of f Hz is expressed as

$$E = (E_1 / f_1) f \qquad \ldots \qquad \ldots \qquad (2.25)$$

A flow chart for determination of the losses on non-sinusoidal voltage waveform with constant-flux mode is developed as shown in Fig. 2.8. This flow chart is very similar to the one shown in Fig. 2.2, except suitable modifications required for constant-flux operation. These modifications are as follows.

In the case of constant flux-operation, the value of E_1/f_1 ratio is to be found out first, for which the knowledge of full load torque, the fundamental slip corresponding to the normal frequency (of f_1 Hz) and the equivalent-circuit parameters is necessary. This fundamental slip, obtained from the output data of the program based on Fig.2.2, along-with other quantities is fed as input data. Once E_1/f_1 ratio is known, the value of e.m.f. E is calculated from equation (2.25) for a given operating frequency. Then the stator, rotor and

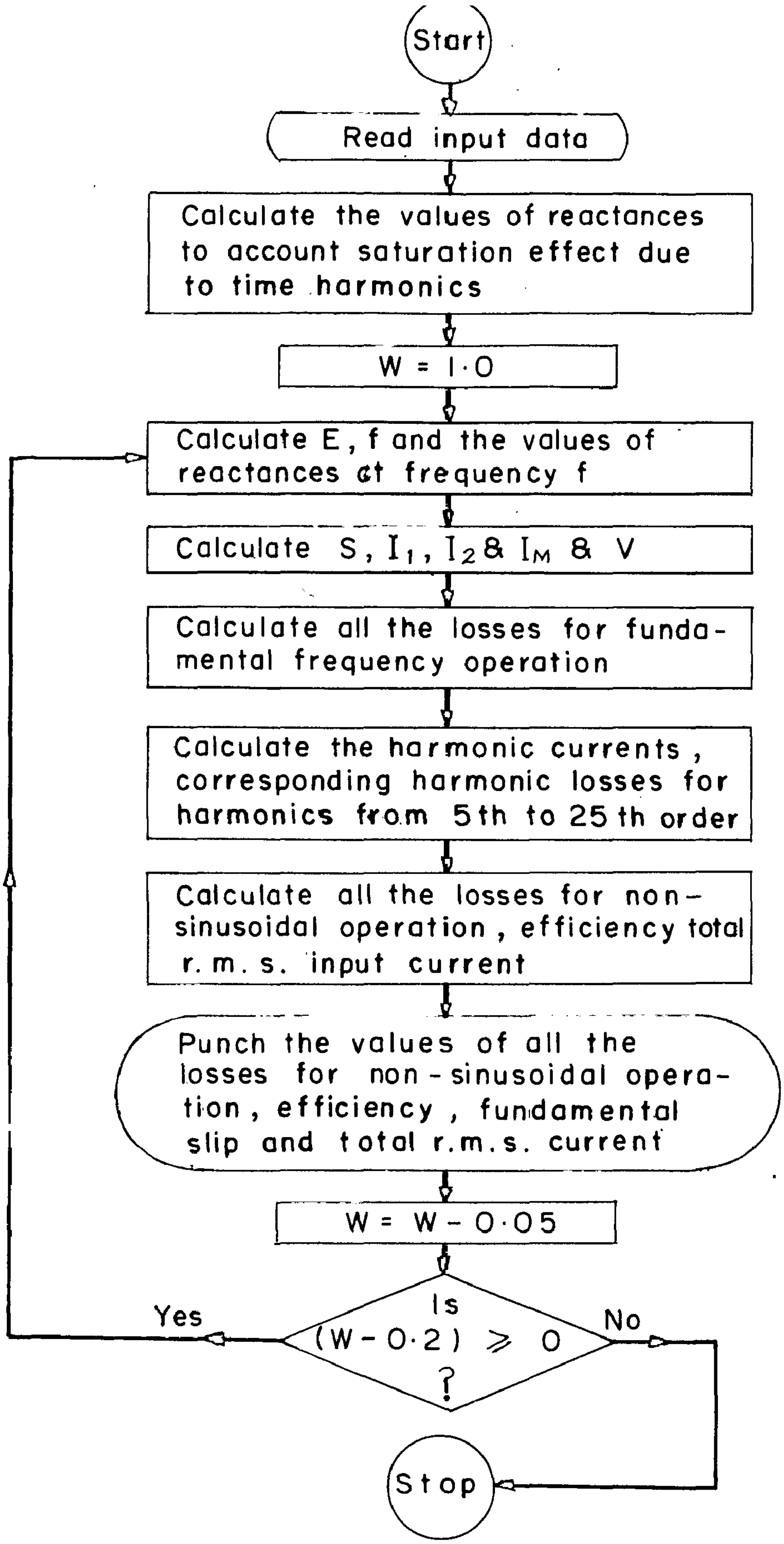

Fig. 2·8 A flow chart for determination of losses at different operating frequencies for constant-flux mode of operation no-load currents (in the stator) are calculated from the equivalent-circuit of the motor by the usual method. When the value of stator current becomes known, the stator per phase applied voltage V can be calculated. On the other hand for constant Volts/Hz mode of operation, the operating voltage V at the operating frequency of f Hz is first calculated and then from this the various currents are determined.

Rest of the procedure, namely calculation of losses, efficiency etc. is the same for the two programs.

The input data of the program for constant-flux operation is the same as that for constant Volts/Hz mode of operation. As already mentioned, this table is given in Appendix No.8.

The output data for this program comprises of the total losses, efficiency, fundamental slip, operating voltage corresponding to the operating frequency and total r.m.s. input current. The results obtained are represented in graphical form and compared with the operation of the motor on non-sinusoidal voltage waveform with constant Volts/Hz mode of working as given in the following paragraphs. With the normal values of the equivalent-circuit parameters, it was seen that here the operating frequency range can be from 5 Hz (or even slightly lesser) to 50 Hz, contrary to the lower limiting value of operating frequency of 10 Hz only in the case of constant Volts/Hz mode of working.

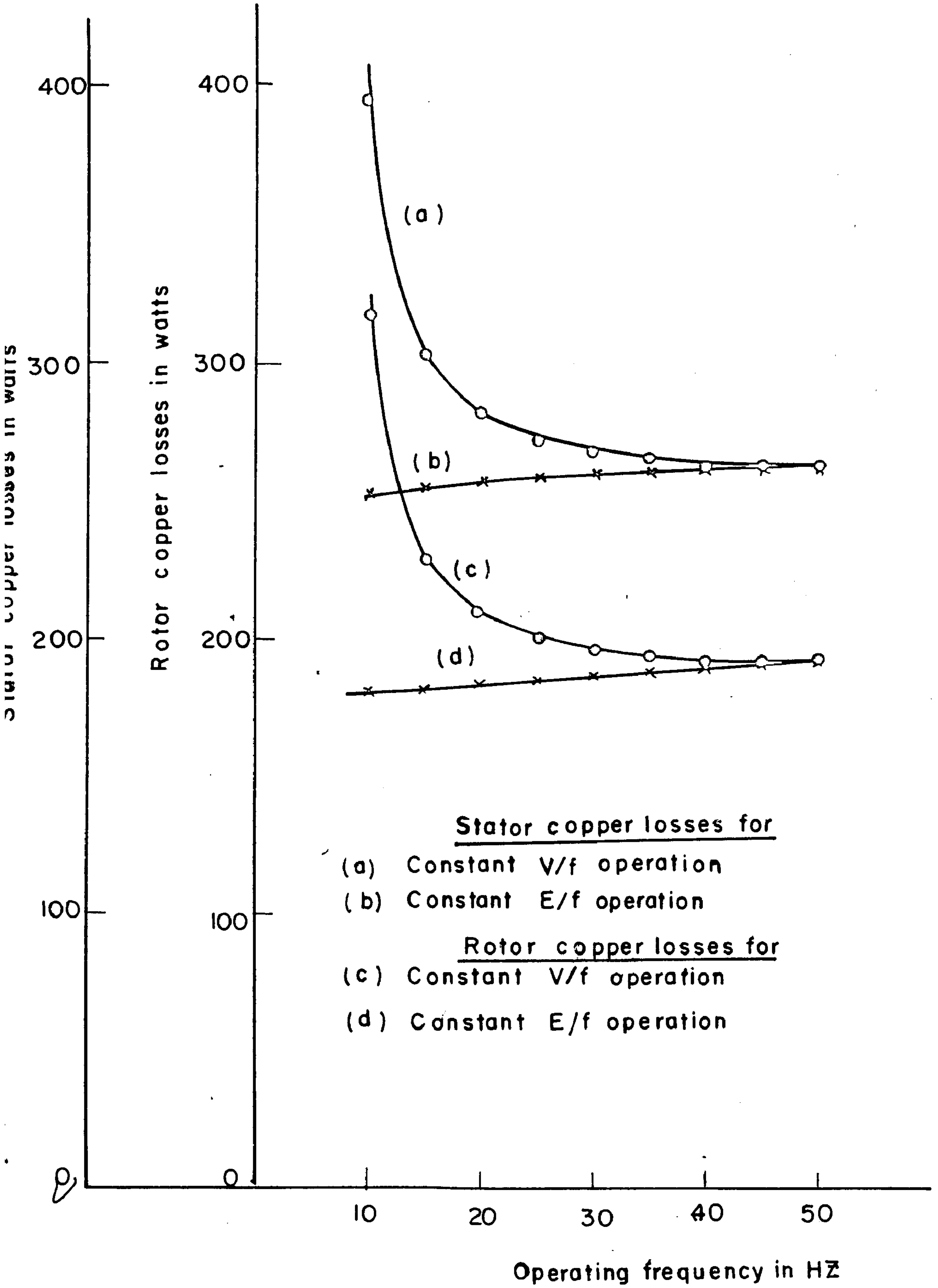

Fig. 2.9 - Variation of copper losses with frequency for constant V/f and contant E/f operations on non-sinusoidal supply voltage wave forms.

2.6.1. COMPARISON OF PERFORMANCE ON CONSTANT-FLUX AND CONSTANT VOLTS/Hz OPERATION WITH NON-SINUSOIDAL SUPPLY VOLTAGES

Copper Losses

It has already been discussed that the copper losses in the case of constant Volts/Hz mode of working with non-sinusoidal supply voltage increase when the operating frequency is reduced. In comparison with this, the stator and rotor copper losses of the motor with constant-flux operation on non-sinusoidal voltage source decrease with the reduction of operating frequency as shown in Fig.2.9. The decrease in copper losses with reduction of operating frequency for constant-flux operation can be explained as follows. In the case of constant-flux operation the rotor fundamental current remains absolutely constant for all operating frequencies as the motor is assumed to produce a constant load torque. The stator fundamental current, however, decreases slightly with the reduction of frequency because of the reduction in iron loss current component. Again the contribution of rotor and stator harmonic currents to the respective total r.m.s. currents in the rotor and stator is maximum at the normal frequency. As the operating frequency is reduced, the harmonic currents in the stator and rotor are reduced as explained in section 2.4.1. of this chapter. The net result is that the copper losses decrease with the reduction of operating frequency in the case of constant-flux operation.

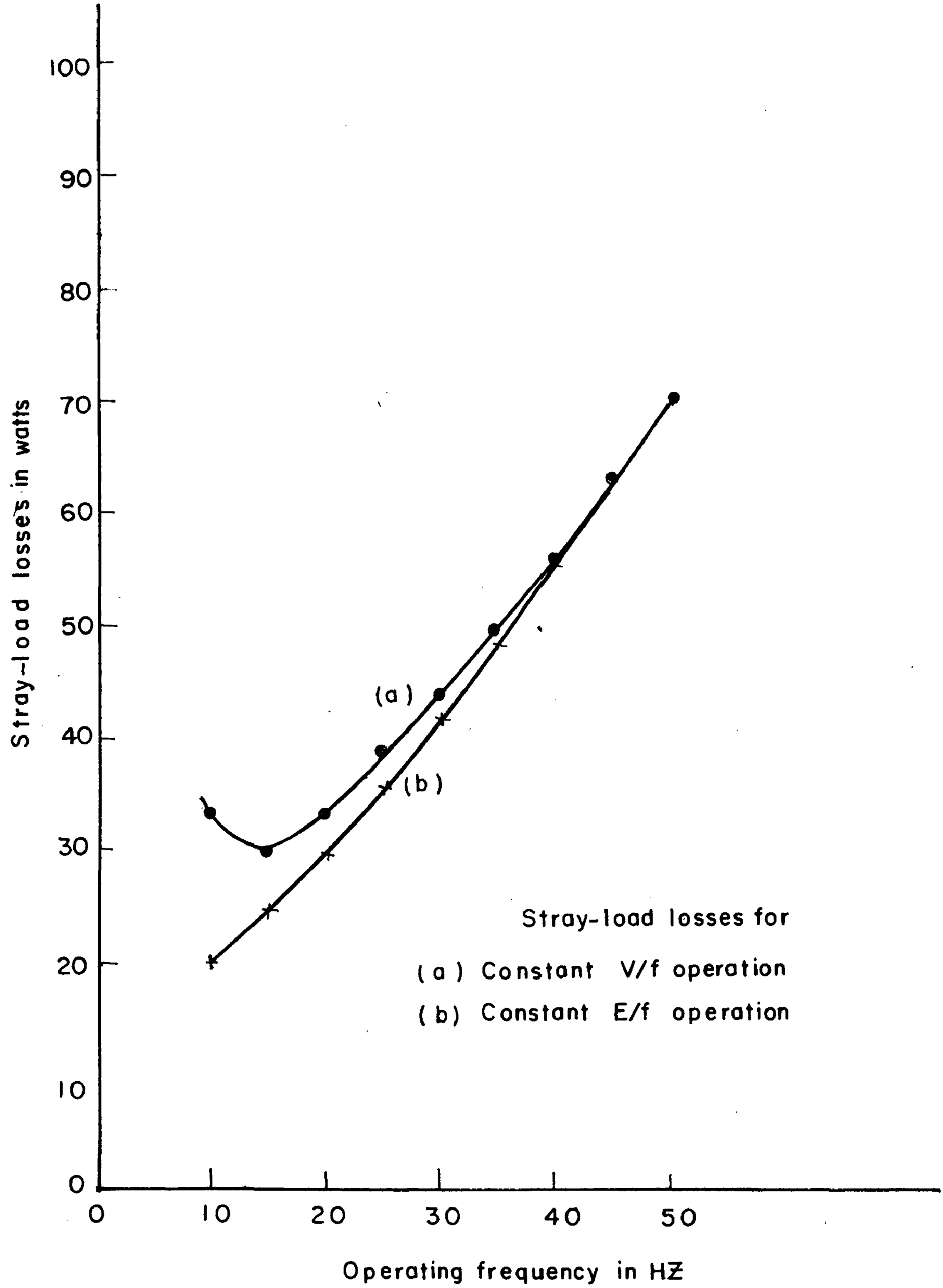

Fig.2.10-Variation of stray-load losses with operating frequency for constant V/f and constant E/f operations on non-sinusoïdal supply voltage waveform.

Stray Load Losses

Fig. 2.10. shows the variation of stray-load losses with frequency for constant Volts/Hz and constant-flux operations. As already seen in section 2.4.1. of this chapter, the stray-load losses for constant volts/Hz mode of operation, decrease with the reduction of the operating frequency upto certain value of the latter and when the frequency is reduced further, the stray-load losses start increasing. In the case of constant-flux operation, these losses progressively decrease with the reduction of the operating frequency. As seen earlier, the stray load losses depend upon the currents, flux and frequency of the machine in addition to its dimensions. In the case of constant-flux operation the flux is constant and the total r.m.s. currents in stator and rotor decrease with the reduction in the operating frequency as explained in the earlier paragraph. Thus the stray-load losses decrease with the reduction of the operating frequency in the case of constant-flux operation.

Iron and Friction and Windage Losses

Comparison between the iron losses, and friction and windage losses both for constant Volts/Hz and constant-flux operations on non-sinusoidal supplies can be very easily done by referring to the Table No.2.6. As shown in the table, the stator iron-losses for both constant Volts/Hz and constant-flux operations are same for a given operating frequency.

Actually, there is some reduction in the flux with reduction the operating frequency in case of constant Volts/Hz operation and so these losses should be slightly less than the ones with constant-flux operation. The equality in stator iron losses for a given frequency with both the modes of working has occured because while formulating the expression for calculation of these losses with constant Volts/Hz operation, the flux densities in various magnetic parts of the motor are considered to be approximately constant as discussed in section 2.2 of this chapter.

For a given frequency, the rotor iron losses with constant-flux operation are slightly less than the ones with constant Volts/Hz mode of working. Again, the Table 2.6 shows that for a given frequency, the friction and windage losses with the constant-flux operation are more than these losses on constant Volts/Hz mode of working. This has occured because for a given frequency, the fundamental slip is less in constant-flux operation in comparison with its value on constant Volts/Hz operation. So the rotor iron losses, which depend on the slip and the friction and windage losses, which depend on the speed of the motor are respectively less and more with constant-flux operation than their values with constant Volts/Hz mode of operation.

Total Losses

The total losses in the case of constant-flux operation progressively decrease with the reduction of

Table No. 2.6

Variation Iron Losses and Friction Losses with Frequency for Constant V/f and Constant E/f Operations on Non-Sinusoidal Supply Voltages.

Sl. No.	Frequency Hz	Stator Iron Losses in Watts		Rotor Iron Losses in Watts.		Friction & Windage Losses in Watts.	
		For constt. V/f opern.	For constt. E/f opern.	For constt. V/f opern.	For constt.E/f opern.	for constt.V/f opern.	for constt. E/f opern.
1.	50	218.26	218.26	7.51	7.51	57.21	57.21
2.	45	184.40	184.40	6.42	6.41	51.21	51.24
3.	40	152.73	152.73	5.41	5.39	45.21	45.27
4.	35	123.35	123.35	4.47	4.44	39.20	39.30
5.	30	96.39	96.39	3.62	3.56	33.17	33.34
6.	25	72.00	72.00	2.86	2.78	27.10	27.37
7.	20	50.38	50.38	2.21	2.08	20.97	21.40
8.	15	31.80	31.80	1.73	1.48	14.65	15.43
9.	10	16.62	16.62	1.77	0.99	7.31	9.46

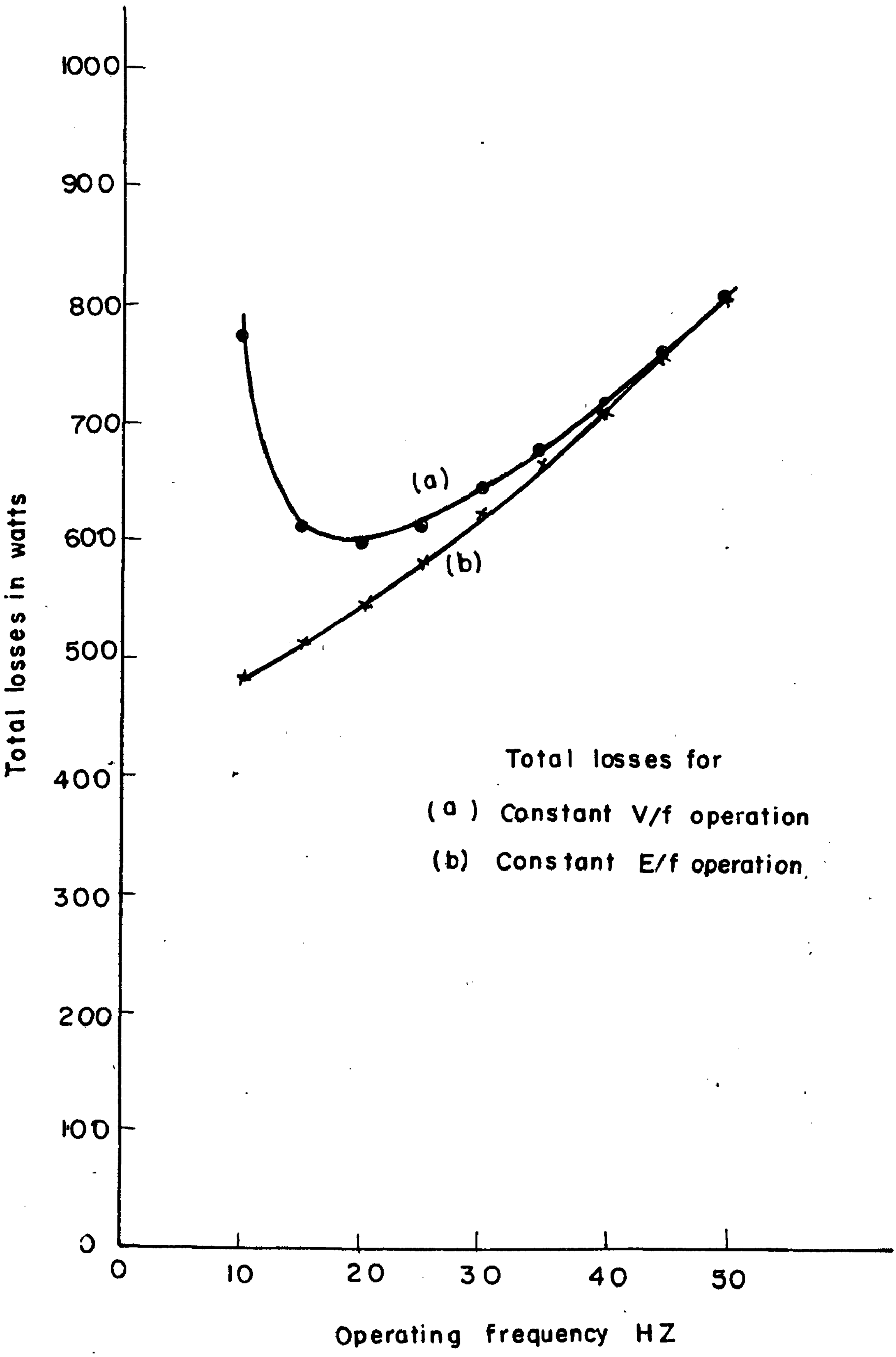

Fig. 2.11 - Variation of total losses with operating frequency for constant V/f and constant E/f operations on non-sinusoidal supply voltages.

operating frequency as is shown in Fig.2.11. It is clear from the above discussion that all the components of the total losses in constant-flux operation decrease with the reduction of the operating frequency. Naturally, therefore, the total losses also decrease with the reduction of operating frequency in the case of constant-flux operation on non-sinusoidal supply voltage. When the variation of the total losses with frequency on constant-flux operation is compared with that of the constant Volts/Hz operation, the advantages of constant-flux operation become evident. The performance of the motor on low operating frequencies for the constant Volts/Hz mode of working deteriorates considerably, whereas, that for the constant-flux mode shows a significant improvement.

2.7. RESUME

In this chapter the study of the various losses for sinusoidal as well as non-sinusoidal supply voltages with constant Volts/Hz mode of working as well as constant-flux mode of operation is carried out in considerable detail. This study indicates that the total losses on non-sinusoidal supply are more than those on the sinusoidal one. In case of constant Volts/Hz mode of working, the increase in the total losses with non-sinusoidal supply voltage is as much as 12 percent over their value with sinusoidal voltage source, which occurs at the normal frequency. The low frequency performance of the motor with the constant Volts/Hz mode of working is very poor, so far as the efficiency, total losses, and fundamental slip are concerned. There is a considerable improvement in the performance of the motor for whole operating frequency range with constant-flux mode of working. As the total losses increase considerably with non-sinusoidal supply voltages, there is a necessity of optimizing them for improving the performance of the motor and limiting the temperature rise under this condition. The stator as well as rotor resistance values have a predominant effect on the total losses. In addition, the losses are governed by the values of the flux densities in various magnetic parts. Hence it is necessary to select suitable design variables which influence significantly the total losses and then optimization of the total losses can be carried out. This aspect is considered in the next chapter.

CHAPTER III

OPTIMIZATION OF LOSSES OF INVERTER-FED THREE PHASE SQUIRREL-CAGE INDUCTION MOTOR

INTRODUCTION

In this chapter, optimization and various factors regarding use of the optimization techniques for electrical machine design problems are briefly discussed. A procedure is developed for the use of Direct Search Technique of optimization to obtain optimized (minimized) value of total losses of an inverter-fed three-phase cage induction motor. Number of conductors per stator slot, (or number of turns per stator phase), area of cross-section of stator conductor and core (stack) length are chosen as variables and their optimized values corresponding to the lowest value of total losses of the motor are obtained using the optimization procedure. The flux density in stator teeth has been used as the main constraint in the optimization, which is reflected in determination of the limiting values of the variables. Another constraint imposed is the stator slot space factor. The performance factors, namely various losses, full load torque, fundamental slip corresponding to the torque, efficiency, and all the equivalent circuit parameters corresponding to the optimum design of the motor are determined.

3.1. OPTIMIZATION AND ITS BACKGROUND

Optimization is an act of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, the important problem faced is the allocation of scarce resources - men, machines, and raw materials - in such a way that the end product (or products) meets certain specifications, while at the same time some objective function such as profit is maximized. Thus, the optimization can be defined as a collective process of finding the set of conditions required to achieve the best results from a given situation[18].

3.1.1. APPLICATION OF OPTIMIZATION

Optimization in its broadest sense can cover a wide range of examples and applications. Some typical applications[37] are given below :

(1) Design of air-craft and aerospace structures for minimum weight.

(2) Finding the optimal trajectories for space vehicles.

(3) Design of civil engineering structures like frames, foundations, bridges, towers, chimneys and dams for minimum cost.

(4) Optimum design of electrical machinery, e.g. the electrical machines can be designed for minimum losses, minimum weight or minimum cost as per the requirements.

(5) Planning of maintenance and replacement of equipment to reduce operating costs.

(6) Controlling the waiting and idle times and queueing in production line to reduce the costs.

(7) Design of optimum pipe line networks for process industries.

(8) Shortest route taken by a salesman visiting different cities during one tour.

(9) Analysis of statistical data and building empirical models from experimental results to obtain the most accurate representation of a physical phenomenon.

(10) Optimum design of control systems.

(11) Inventory control.

3.1.2. DESIGN VARIABLES

In many design problems there are several possible alternative design concepts. Within these design concepts, there are variables which specify the dimensions, proportions and other details of the item. Thus, the numerical quantities[18] for which values are to be chosen in producing a design are called design variables.

3.1.3. DESIGN CONSTRAINTS

A design which meets all the requirements placed on it, is called a feasible design.

The design restrictions[37] that must be satisfied in order to produce an acceptable design are collectively called as constraints.

3.1.4. THE OBJECTIVE FUNCTION

Of all feasible designs, some are 'better' than others. There must be some quality that the better designs have more of than the less desirable ones do. If this quality can be expressed as a computable function of the design variables, we can consider optimizing to obtain a best design. The function[18] with respect to which the design is optimized is called the objective function.

3.1.5. MATHEMATICAL PROGRAMMING PROBLEM

After making the appropriate engineering judgements and defining all the necessary functions and limitations, the general optimization problem can be formulated[22] as follows.

It is desired to determine values for n variables $x_1, x_2, \ldots\ldots, x_n$ which satisfy the m inequalities or equations

$$g\ (x_1, x_2, \ldots\ldots, x_n) \left\{ \leqslant\ ,\ =\ ,\ \geqslant \right\} b_i\ , \qquad \ldots \quad (3.1)$$
$$i = 1, 2, \ldots\ldots, m$$

and, in addition, maximize or minimize the function

$$Y = f(x_1, \ldots\ldots\ldots, x_n) \qquad \ldots \qquad \ldots \qquad (3.2)$$

The restrictions (3.1) are called the constraints, and (3.2) is called the objective function.

3.1.6. AVAILABLE OPTIMIZATION TECHNIQUES

In the case of electrical machine design problem, generally, the constraint equations as well as the objective function are non-linear. In this case the optimization problem can be classified under two main categories, viz., unconstrained optimization and constrained optimization. Some of the various techniques[5,18,37] for solution of the problem are :

A - Unconstrained Optimization

(i) Gradient Methods : steepest descent.

(ii) Random Search Method.

(iii) Univariate Methods.

(iv) The Pattern Move.

(v) Powell's Method : conjugate directions.

(vi) Rosenbrock's Method of rotating axes.

(vii) Quadratic interpolation.

(viii) Second Order Method : Newton's procedure.

B - Constrained Optimization

(i) Lagrange multipliers.

(ii) Method of feasible directions.

(iii) Penalty function methods.

(iv) By transformation of variables.

(v) Constraint Approximation methods.

(vi) Direct search method.

3.1.7. CONSIDERATIONS IN SELECTING AN OPTIMIZATION METHOD

When conditions require the inclusion of optimization as a step in the design process, a decision regarding the choice of a method is an essential feature. There are a number of factors[18] which enter into this decision. Some of these are :

(i) The man hours and other costs necessary to develop the program.

(ii) The running time costs to solve the desired optimization problems.

(iii) The expected reliability of the program in finding the desired solution.

(iv) The flexibility of the program ; whether it can be used in different ways on the same general problem.

(v) The generality of the program or parts of it. Whether it can be used to solve other problems.

(vi) The ease with which the program can be used and its output interpreted.

3.2. SELECTION OF A SUITABLE TECHNIQUE FOR OPTIMIZATION OF LOSSES OF STATIC INVERTER-FED, 3-PHASE INDUCTION MOTOR

In order to optimize a system, it is necessary to obtain a suitable mathematical model representing the system.

The model may be one of the two types. First, a mathematical model prepared to represent the process system, in which analytical relationships, together with appropriate restrictions, define the response of the process. Alternativ-ely a model can be developed where the response of output is considered for varying inputs. A model of this type is called a 'Black Box Model', since only the range of input is to be selected. Sometimes, especially for more complex mathematical models, a system adequately represented by a mathematical model is treated in the second manner , so that the outputs are determined numerically for a given set of input values.

The mathematical model for the optimization of losses of inverter-fed, three-phase cage induction motor is very complex due to the inter-relationship of a large number of design parameters and their conflicting effects on the losses of the machine. Therefore, a mathematical model of the latter type is selected. The 'Direct Search Method' is used which is most suitable for this type of mathematical model. In the 'incremental search' technique based on the above method, an ordered search is conducted from a given starting point. As pointed out by Foultan and others[19], this method is the only one likely to succeed where so many non-linearities are involved. Although, this method is not as fast as other indirect methods, yet the method involves lesser man hours (and hence cost to develop the program), ensures generality and provides ease for application. Another consideration in selection of this technique is that two of the three selected design variables are discrete variables.

3.2.1. SELECTION OF VARIABLES

In the case of small induction motors, the use of standard laminations is preferred for manufacturing purposes. Here it is assumed that the standard laminations are to be used for the design of the induction motor. Once the set of laminations is decided, the quantities like air-gap diameter, number of stator and rotor slots, depth of stator and rotor core, the dimensions of the slots, length of air-gap etc. are fixed and the only variables available are three, viz. number of conductors per stator slot, area of cross-section of a conductor in stator slot and core length. So these variables are considered in the present study.

It is clear from the equations (1.3), (1.5), (1.8), (1.9) and (1.14) discussed in chapter I, that the equivalent circuit parameters are very much sensitive to the change in number of stator turns per phase. Another factor which affects the above parameters considerably, is the core length. For a given load torque, as seen earlier, the fundamental slip and various motor currents are governed by the equivalent-circuit parameters. Hence the copper losses and stray-load losses are indirectly controlled by the values of these parameters. Copper losses, in addition are controlled by the values of the resistances in the equivalent-circuit. The iron losses, on the other hand, are governed by the flux densities in various magnetic parts. The flux densities in turn are dependent on the values of the core length and the number of stator turns per phase. Thus the variables selected, viz., the stack-length and the number of stator conductors per slot influence the objective function i.e. total losses, in a very much significant manner. The third variable namely, the area of cross-section is not an independent one. The dimensions of the stator slot are fixed, so when the number of conductors in this slot is varied, the area of cross-section will have to be adjusted accordingly, so that the space factor remains more or less the same.

3.2.2. OBJECTIVE FUNCTION

The term 'design optimization' only has a meaning in the context of a given specification. In the present case, the optimum machine is defined as that inverter-fed induction motor which has minimum total losses for a given specification. So the objective function is the total losses of the motor, which can be expressed as :

$$W_T = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10} + W_{11} \qquad \ldots \qquad \ldots \qquad (3.3)$$

The procedure for calculating all the loss components from W_1 through W_{11} , as already outlined in chapter I, consists of three main steps as follows:

(i) Determine the equivalent-circuit parameters from design variables and calculate the fundamental slip for constant full load torque condition.

(ii) Determine the currents for fundamental frequency as well as for the time harmonics.

(iii) Calculate the loss components and compute total losses, the objective function, as per equation (3.3).

3.2.3. CONSTRAINTS

The main constraint considered in this work is the limiting value of stator teeth flux density. The value of the flux density is limited so that the teeth are not over saturated and the maximum value has been taken to be 1.5 Wb/m^2. The flux density in teeth will automatically fix the minimum values of core length and number of conductors per stator slot. The constraints on the variation of the number of stator conductors per slot obtained as above are used for different core lengths.

Stator slot space factor is recognized as the second constraint. This factor is assumed to remain more or less constant to about 0.4. When the number of conductors per stator slot are allowed to vary between the limits as mentioned above, the conductor cross-sectional area is adjusted suitably to keep the space factor very near to the assumed value.

3.3. DESIGN SPECIFICATIONS

Here, the following quantities are considered as specified for the motor.

(a) Operating voltage and number of phases.

(b) Normal frequency.

(c) Number of poles.

(d) Full load power output at the normal frequency.

(e) Full load torque.

The term normal frequency used above and explained earlier means the maximum value of the operating frequency for constant torque operation, which is generally 50 Hz.

The specifications of the motor[38], considered as an example for optimization, are given in appendix No.9. For this particular motor the limits of variation of the design variables based on the constraints discussed in previous section are found to be as follows :

(i) Number of conductors per stator slots

— varied from 74 to 54 in step of 1.

(ii) Area of cross-section of stator conductor

— varied in three steps, viz.

for conductors between 74 to 68 — the cross-section is taken as 0.01038 cm^2

for conductors between 67 to 61 — the cross-section is taken as 0.0117 cm^2

and for conductors between 60 to 54 — the cross-section is taken as 0.01314 cm^2

(iii) Minimum value of core length

— 8 cm .

3.4. OPTIMIZATION PROCEDURE

As seen in chapter II maximum value of the total losses occurs at the normal frequency for constant Volts/Hz mode or constant-flux mode of operation on non-sinusoidal supply waveforms. Consequently the total losses are first optimized (i.e. minimized) at the normal frequency by using incremental search technique and then with the optimized parameters obtained, the performance of the motor for complete operating frequency range is checked.

Essentially, the procedure of optimization used here consists of the following steps :

(1) Determination of the equivalent-circuit parameters, for the first value of the number of stator conductors per slot and the stator conductor cross-section when core length is specified along with other design data of the machine.

(2) Calculation of fundamental slip and the fundamental currents.

(3) Evaluation of harmonic currents for harmonics from 5^{th} to 25^{th} order and then the determination of harmonic losses.

(4) Determination of component and total losses on non-sinusoidal supply voltage.

(5) Repeating the procedure with all the values of number of conductors, and at the same time picking

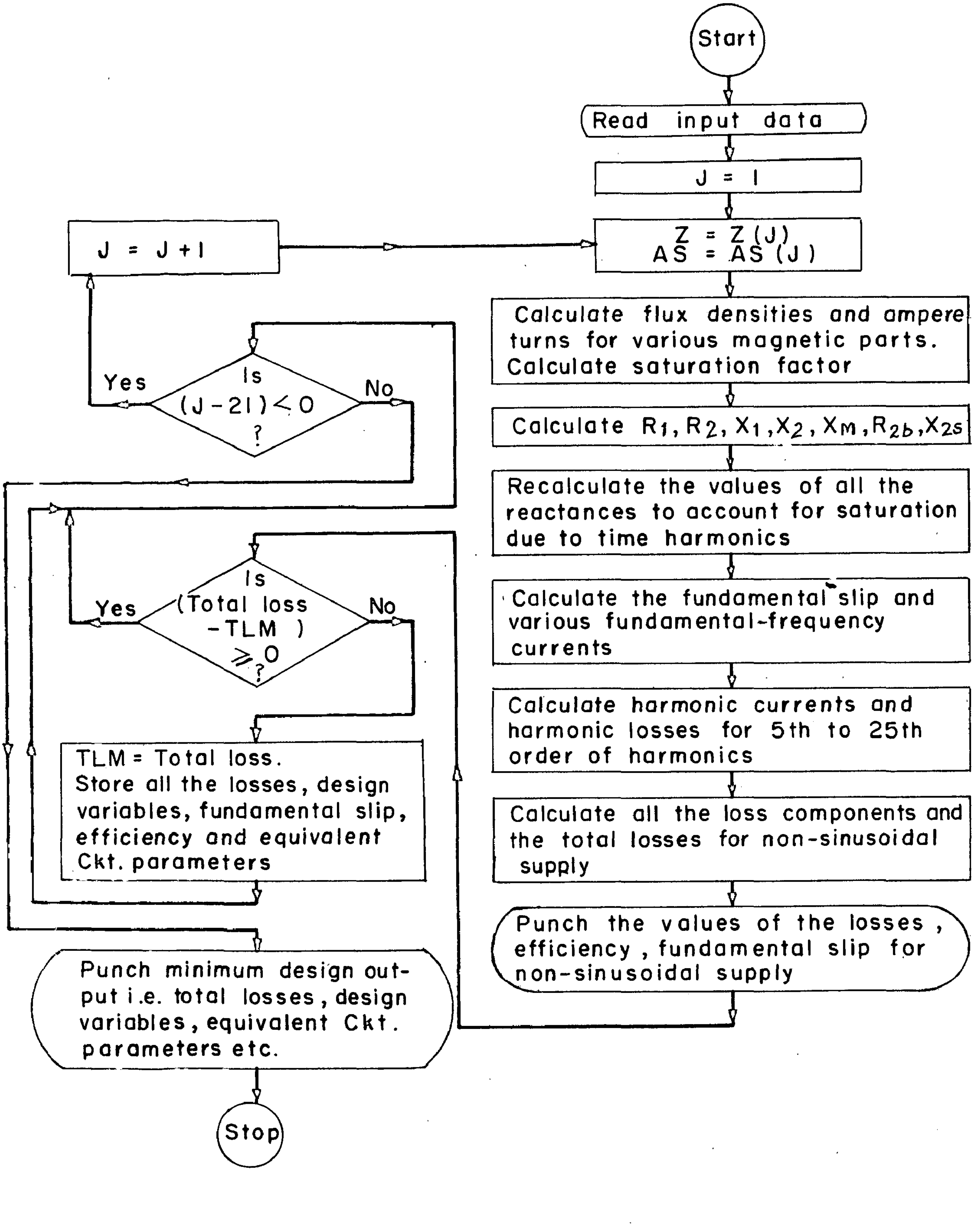

Fig. 3·1 A flow chart for optimization of total losses by incremental search technique the minimum value of the total losses.

For a given core-length, thus, a set of values of the total losses and the minimum value of the total losses is obtained. For different values of the core-lengths, by repeating the above procedure different sets of values of the total losses along with a set of minimum values, one for each core length is obtained. Finally out of the minima the smallest value is apparent and so is easily picked out.

A flow chart for the optimization procedure described above is shown in Fig. 3.1. A computer program based on this flow chart is developed. A number of runs of this program on IBM - 1620 digital computer gave the desired optimum solution of the problem. The input data for the computer program is listed in Table No.III given in Appendix No.9. It consists of the specifications of the motor, stator and rotor dimensions, winding details, and values of the design variables. The output of the program comprises of set of values of the total losses as pointed out earlier. This output data is represented by graphs and then the results are discussed in the paragraphs to follow.

3.5. EFFECT OF DESIGN VARIABLES ON THE TOTAL LOSSES

In the following paragraphs is considered the effect of design variables on the objective function, viz. the total losses of the induction motor excited from non-sinusoidal voltage source. The three selected design variables, as

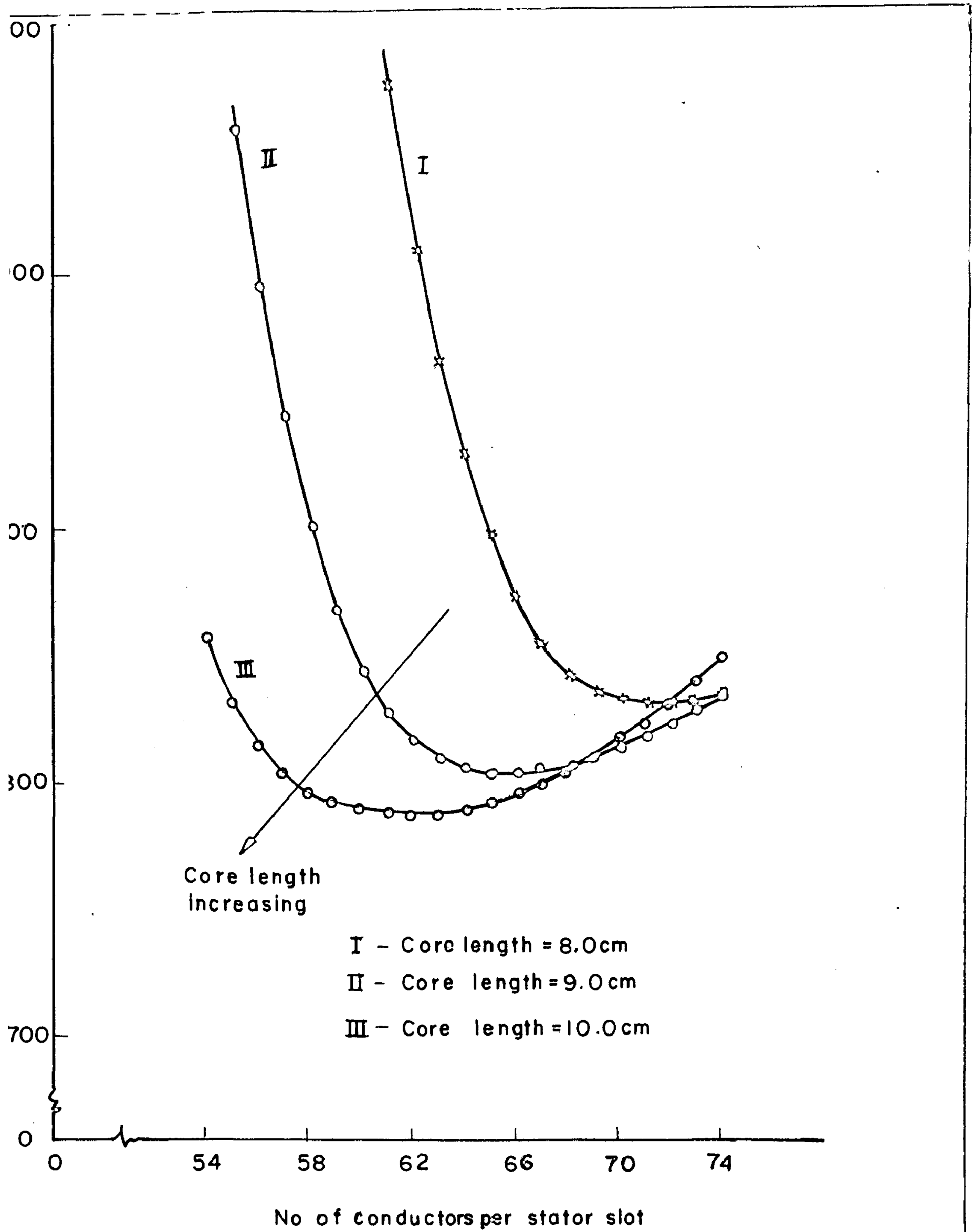

Fig. 3.2 - Variation of total losses with no. of conductors per stator slot for various core lengths, cross section of stator conductor constant (at 0.0117 cm^2)

already discussed, are the number of conductors per stator slot, core length and the area of cross-section of a stator conductor, which are used for this purpose. These variables are respectively denoted as z, C_L and A_s and the symbols are used in the discussion, wherever necessary, for convenience.

Fig. 3.2. shows how the total losses vary with z, for three different values of core lengths, when A_s is kept constant. It can be seen from the curves of Fig. 3.2. that for a given core length, as z is increased continuously from the lower specified limit, the total losses start decreasing first, then reach a minimum value and finally start increasing again. This can be explained as follows. For the lower specified value of z, the iron losses are unusually high due to increased flux densities in various magnetic parts of the motor. Thus in this case the iron losses dominate over the other components of total losses resulting in the increase of the latter. On the other hand, for upper specified limit of z, the copper losses are very high in comparison with all other loss components and cause increase of the total losses. Hence the minimum value of the total losses is obtained for a value of z between the two limiting values, when the copper and iron losses of the machine are having moderate values. The stray-load and friction and windage losses definitely play a role in deciding the minimum value of the total losses, however, predominant role is played here by the iron and

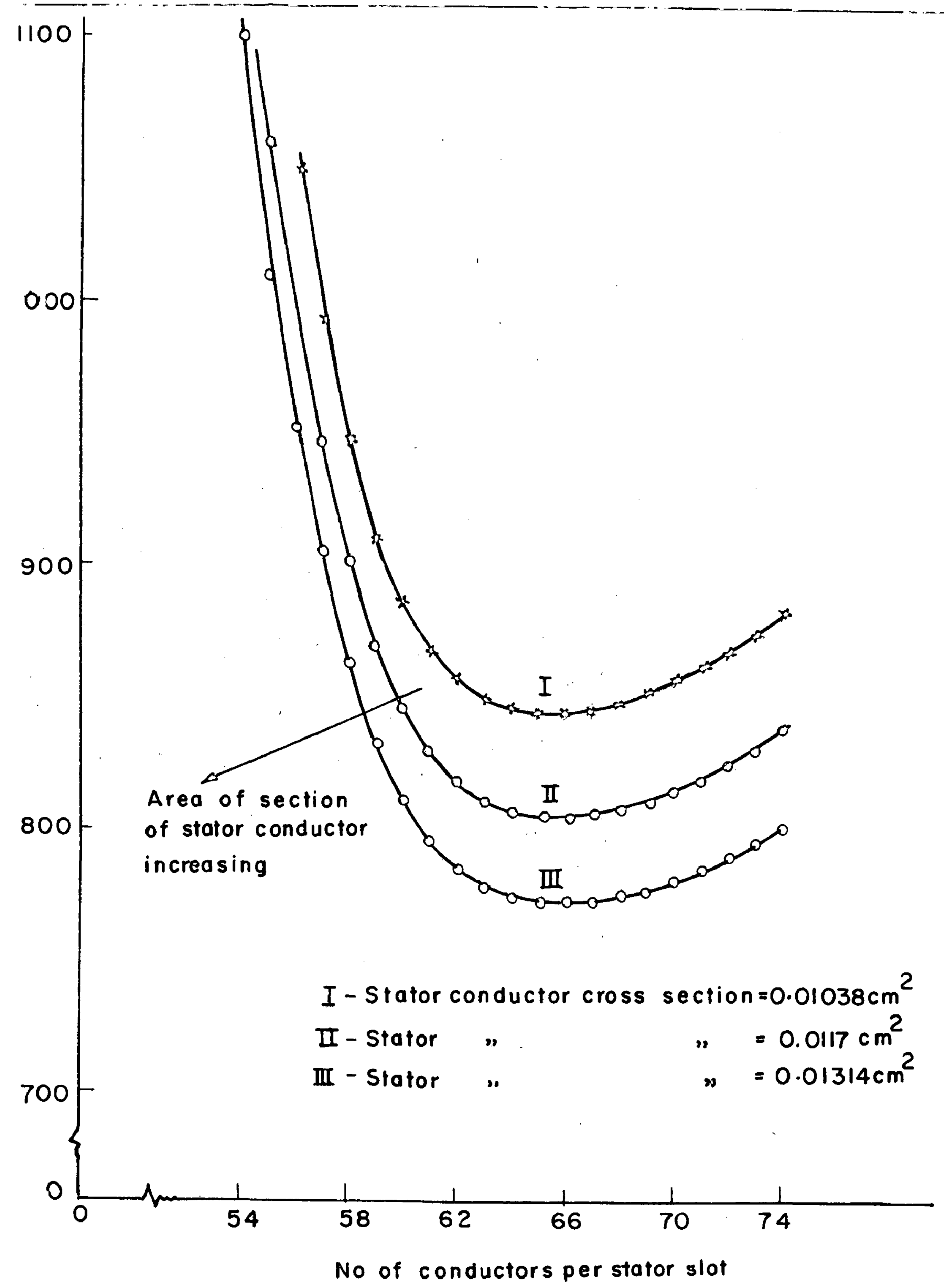

Fig. 3.3 -Variation of total losses with no. of conductors/stator slot for various cross sections of stator conductor, core length constant (at 9.0 cm).

copper losses. Further examination of the curves of Fig.3.2. reveals that the minimum value of the total losses is different for different core lengths and occurs at different values of z. Thus, it is seen that when the core length is increased, the minimum value of the total losses is reduced and is obtained for smaller value of z.

In Fig. 3.3. is illustrated the influence of the cross-sectional area of stator conductor A_s, on the total losses. The curves of total losses against z, obtained for three values of A_s with constant core length have a similar nature to those of Fig. 3.2. The point to be noted here is that the minimum value of the total losses gets reduced when A_s is increased and vice-versa for a given core length. Obvious reason for this is that the stator copper losses are affected by the value of A_s and these losses being a major component of the total losses influence the minimum value of the latter.

The manner in which the core length affects the total losses of the motor is clearly demonstrated by Fig.3.4. Examination of the curves represented in this figure shows that for a particular value of z with A_s constant, when the core length is increased consistently from lower limit of the latter, the total losses start decreasing first, reach a minimum and then start increasing again. The reason for this is the dominance of the iron losses over all other los-

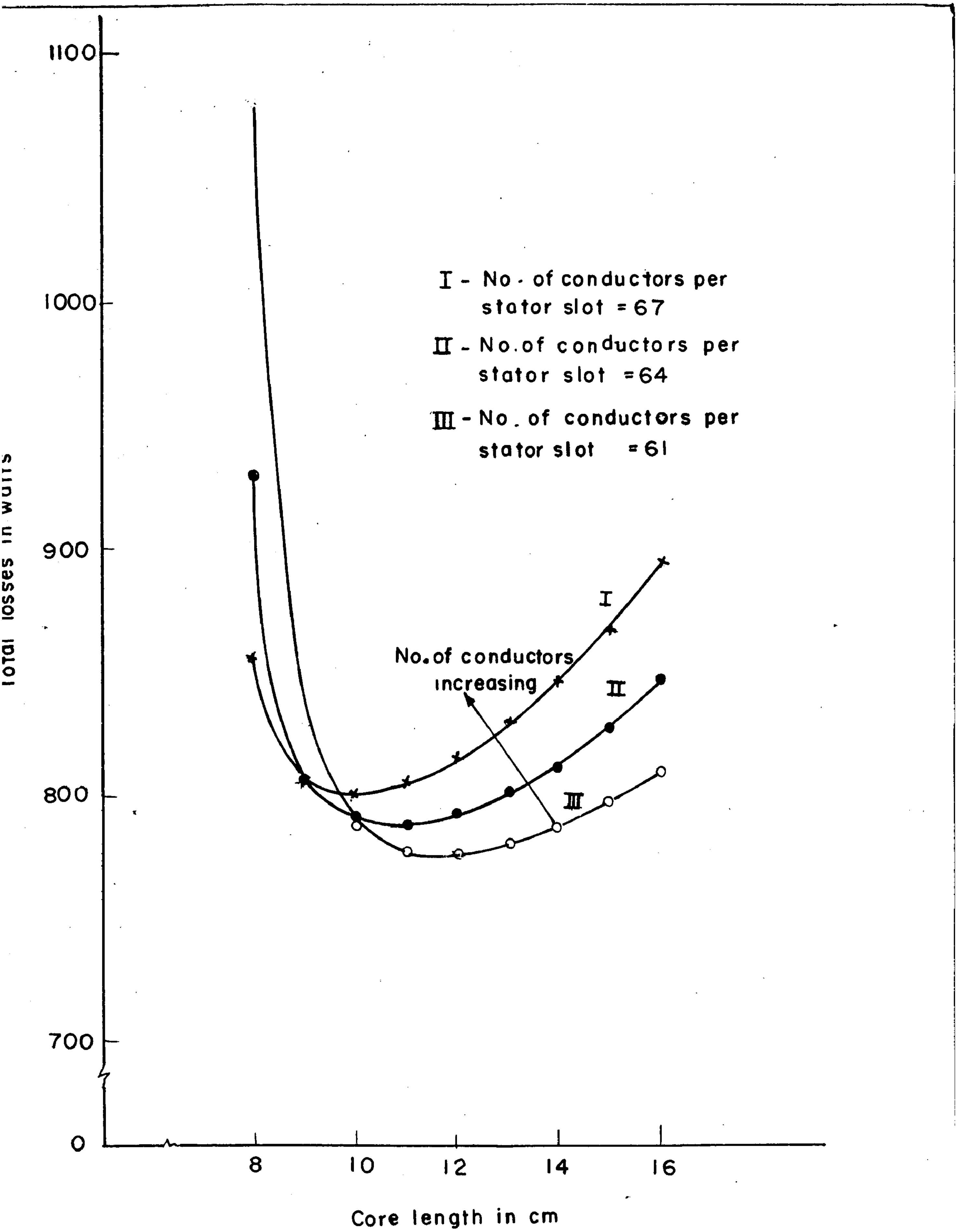

Fig 3.4 - Variation of total losses with core length for various no. of conductors per stator slot, cross section of stator conductor constant (at $0.0117 cm^2$)

components for lower values of the core length, whereas for the higher values of the core length, the copper losses dominate over all the other losses and thus resulting in the increased value of the total losses in either case. The minimum of the total losses is obtained, when these two major component losses have moderate values as explained earlier. Thus the curves represented here are having the same general nature as those of Fig. 3.2. Another observation, made earlier, which can be made again from the study of curves of Fig. 3.4. is that as z is decreased, the minimum value of the total losses gets reduced and occurs at increased value of the core length.

To summarize, the study indicates that to get the smallest value out of the minima of the total losses, the core-length should be increased, the number of conductors per stator slot should be decreased and the area of cross-section of stator conductor should be increased.

3.6. THE OPTIMIZED DESIGN

Results of the previous section indicate the way to arrive at the optimal solution of the problem. To achieve this objective, lower and upper limits of z were decided as discussed earlier and the values of z between these two limits were divided into three sets each one having a different value of A_s such that the space factor for stator slot varies between 0.38 to 0.35 approx. With these values of z and A_s , computations for the total losses were carried out for various core lengths. Placing no constraint on core length, the computations were carried out till the smallest value of the minima of total losses is obtained. Results of these computations are listed in Table No. 3.1. Examination of the table shows that the smallest of the minima occurs for a certain core length (15 cm here) and if the core length is varied on either side of this value, then the minimum value of the total losses starts increasing.

Table No. 3.1

Variation of Total Losses with Number of Conductors per Stator Slot for Various Core Lengths, the Area of Cross-section of Stator Conductor Being Adjusted Suitably with the Number of Conductors.

. o.	No. of stator conductors per slot	Area of section of stator conductor in cm^2	Total Losses in Watts for core length = 10.0 cm	for core length = 11.0 cm	for core length = 12.0 cm	for core length = 13.0 cm	for core length = 14.0 cm	for core length = 15.0 cm	for core length = 16.0 cm
1	74	0.01038	896.2	916.8	942.7	974.4	1016.5	1062.4	1112.6
2	73	"	885.9	904.6	928.1	957.1	991.3	1037.8	1083.7
3	72	"	876.6	893.1	914.4	940.8	972.1	1012.9	1056.8
4	71	"	868.0	883.3	901.5	925.5	954.1	995.3	1031.8
5	70	"	860.2	873.0	889.4	911.2	937.3	971.5	1008.5
6	69	"	853.1	863.6	878.5	897.7	921.4	952.8	986.6
7	68	"	846.7	854.9	867.9	885.1	906.6	931.8	964.5
8	67	0.0117	801.6	806.4	816.1	829.7	847.1	867.9	895.2
9	66	"	797.3	799.9	807.9	819.8	835.3	854.1	879.0
0	65	"	793.9	794.2	800.9	810.6	824.3	841.2	863.8
1	64	"	791.2	789.3	794.0	802.4	814.1	829.2	847.1
2	63	"	789.4	785.1	787.8	794.7	804.7	818.1	834.1
3	62	"	788.8	781.8	782.5	787.7	796.2	807.8	822.1
4	61	"	789.1	779.2	777.9	781.4	788.3	798.3	810.9
5	60	0.01314	759.6	746.9	743.1	744.4	748.8	756.2	766.1
6	59	"	762.4	746.5	740.6	740.2	743.3	749.1	757.5
7	58	"	767.5	746.9	738.9	736.7	738.4	742.8	749.7
8	57	"	775.0	748.6	738.2	734.1	734.3	737.3	742.8
9	56	"	785.8	751.6	738.3	732.5	731.0	732.6	736.7
0	55	"	800.8	755.8	739.6	731.7	728.5	728.8	731.5
1	54	"	824.2	761.4	741.8	731.8	726.8	725.8	727.1

Note :- Minimum values of the total losses are underlined.

A comparison of the optimized design and the normal design of the induction motor is carried out in Table No.3.2. as given below.

Table No.3.2.

A Comparison Between the Normal Design and Optimized Design of the Induction Motor.

S.No.	Item	Normal Machine	Optimized Machine.
1.	Core length in cm.	9.0	15.0
2.	No. of conductors per stator slot.	64.0	54.0
3.	Area of cross-section of stator conductor in cm^2.	0.0117	0.01314
4.	Stator slot space factor approx.	0.37	Approx. 0.36
5.	Stator per phase resistance in ohms.	4.75	4.19
6.	Rotor resistance referred to stator, in ohms per phase.	4.30	4.18
7.	Stator leakage reactance at the normal frequency(-50 Hz)	9.10	10.62
8.	Rotor leakage reactance referred to stator in ohms per phase at the normal frequency (50 Hz).	9.32	10.88
9.	Magnetizing reactance at the normal frequency in ohms per phase.	248.6	342.7
10.	Total loss on non-sinusoidal operation in watts for normal operating frequency in watts.	810.0	725.8
11.	Percentage reduction in the total losses for normal operating frequency.	-	10.5
12.	Efficiency for full load torque at the normal frequency in p.u.	0.82	0.835
13.	Fundamental slip at the normal frequency for constt. full load torque operation in p.u.	0.0414	0.0398
14.	Total r.m.s. input current in Amps.	4.30	4.06

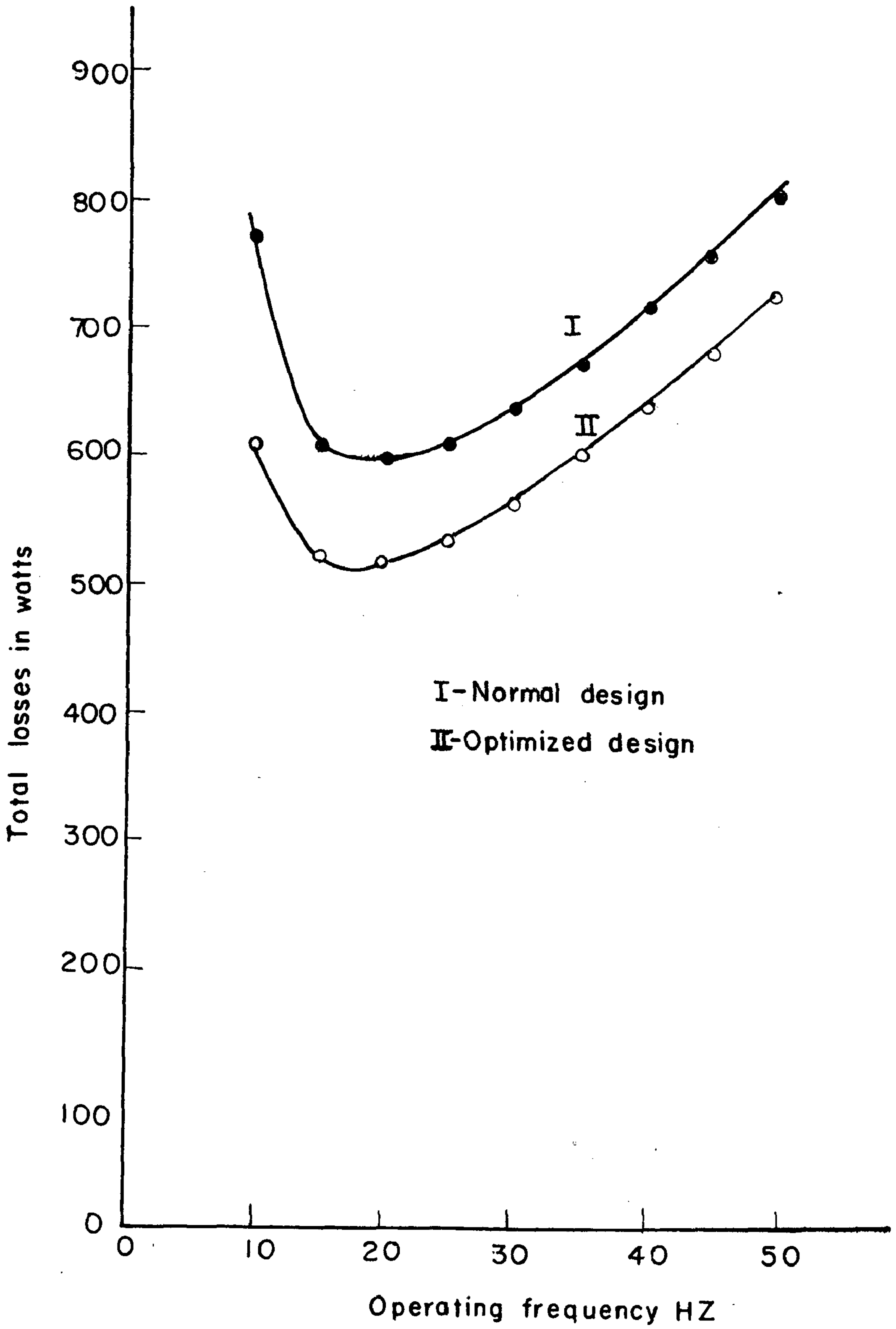

Fig.3.5 - Variation of totat losses with frequency for constant V/f operation in the case of (I) normal design (II) optimized design of the motor.

As seen from Table No.3.3., the total losses are reduced by 10.5 percent, efficiency increased by 1.5 percent and the total r.m.s. input current reduced (by 5.6 percent) in the case of the optimized design in comparison with the normal design and thus there is overall improvement in the performance of the motor with optimized design.

Another question that should be answered is whether the optimized design is capable of giving better performance over the normal design of motor for whole range of operating frequencies. Figs. 3.5 and 3.6 answer this question very well. In Fig. 3.5 is shown the variation of the total losses with operating frequency for constant Volts/Hz mode of working of normal and optimized designs of the motor. It is seen that for all the frequencies in the operating range, there is a marked reduction in the total losses with the optimized design. In fact the percentage reduction in the total losses for low operating frequencies is more and so the low frequency performance of the motor is improved with the optimized design.

Fig. 3.6. shows a comparison of the total losses on constant-flux operation with normal as well as optimized designs of the motor for different operating frequencies. The optimized design shows a superior performance with this method of operation also.

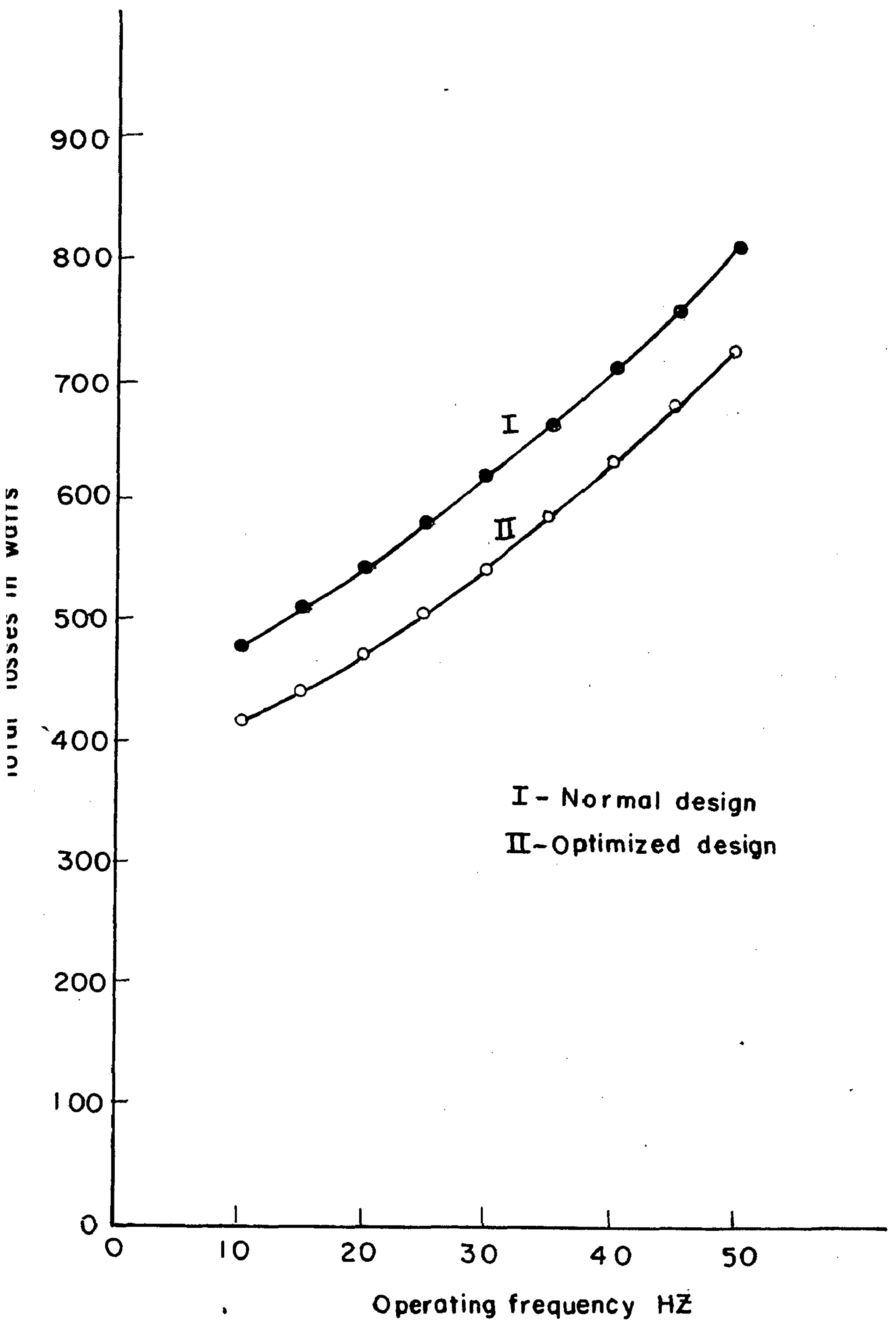

Fig . 3.6 - Variation of total losses with frequency for constant flux operation in the case of (I) normal design (II) optimum design of the motor.

3.7. RESUME

Optimization of the total losses of induction motor excited from non-sinusoidal voltage source has been accomplished by direct search method of the incremental search type. Three design variables, viz. no. of conductors per stator slot, core length and the cross-section of the stator conductor are used for this purpose. Limiting values of the stator teeth flux density and space-factor of the stator slot are considered as the two main constraints of the problem from which, the allowable variation in the number of conductors per stator slot and corresponding variation in cross-section of the conductor is determined. Accordingly by fixing the limits of variation on the number of conductors per stator slot and the cross-section of the stator conductor, a set of minima of total losses is obtained by varying the core length. The smallest of the minima, gives the optimum solution of the problem. For this optimized design the performance of the motor is compared with the normal design for whole range of operating frequencies and it has been shown that the performance of the optimized design is superior to the normal design. The reduction in the total losses obtained by optimized design over the normal design is 10.5 percent at the normal frequency and it is even more at the smaller values of the operating frequency in the case of the example considered.

CONCLUSIONS

Developments in the field of power electronics have led to the use of static converters becoming increasingly important for varying the speed of three-phase squirrel-cage induction motors. The versatile, wide range variable frequency drive has brought to the fore many new problems. One such problem of significance is the increased losses of the motor due to non-sinusoidal nature of output voltage of the static converter.

In the present work, flow charts have been developed to study systematically the losses of induction motor for constant Volts/Hz and constant-flux modes of operation on sinusoidal as well as non-sinusoidal supply voltage waveforms. Computer programs based on the above flow charts have been used to calculate the losses of a 5 h.p. induction motor used as an illustration. The losses of the motor have been determined for the two modes of operation over operating frequency range of 10 to 50 Hz with the assumption that the motor drives a load of constant full-load torque. The results of computations have indicated that :

(i) The additional losses due to time harmonics in supply voltage form a considerable percentage (12 percent for the case considered) of the total losses at the normal operating frequency. These losses decrease as the operating frequency is reduced from the normal value.

(c) The decrease in the magnetizing reactance produced an increase of 6 percent in the total losses, whereas the increase in the value of the parameter caused only one percent reduction in the total losses.

(d) The stator and rotor leakage reactances produced only 2 to 1 percent variation in the total losses.

The detailed study of the losses has pointed out that the additional losses due to non-sinusoidal supply voltage waveform deserve a proper consideration and they should be minimized so that the motor operates satisfactorily when connected to a static inverter for variable frequency operation.

The optimization of the losses of the induction motor has been carried out by using incremental search technique based on direct search method. A flow chart has been developed for the process of optimization. The computer program based on the above flow-chart has been used to find the suitable values of the three selected variables, viz. number of conductors per stator slot, area of cross-section of the stator conductor and core length ; so that minimum total losses are obtained for the motor. For the 5 h.p. motor used an example, a reduction of 10.5 percent in the total losses is achieved by using this method. Also the optimized design of the motor when compared with its normal design shows an overall improvement in the motor performance over the entire operating frequency range.

CONCLUSIONS

Developments in the field of power electronics have led to the use of static converters becoming increasingly important for varying the speed of three-phase squirrel-cage induction motors. The versatile, wide range variable frequency drive has brought to the fore many new problems. One such problem of significance is the increased losses of the motor due to non-sinusoidal nature of output voltage of the static converter.

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(i) The additional losses due to time harmonics in supply voltage form a considerable percentage (12 percent for the case considered) of the total losses at the normal operating frequency. These losses decrease as the operating frequency is reduced from the normal value.

(ii) For all operating frequencies the performance of the motor is much better with constant-flux mode of operation in comparison with the constant Volts/Hz mode of operation.

(iii) Performance of the motor deteriorates considerably at very low operating frequencies (so far as losses, fundamental slip, efficiency etc. are concerned) with constant Volts/Hz mode of working.

The effect of variation of equivalent-circuit parameters on the total losses, fundamental slip, efficiency of the motor has also been studied. It has been brought out that stator and rotor resistances influence the total losses considerably. The decrease of magnetizing reactance has been found to produce increase in the total losses, whereas the increase in the value of the parameter has produced very small improvement in the performance of the motor. The other two equivalent-circuit parameters, viz. stator and rotor reactances have been found to affect the total losses by a very small percentage. Thus for the motor taken as an example, when the variation of the equivalent-circuit parameters was confined to 30 percent variation on both sides of the normal value, the effect on the total losses has been found to be as follows:

(a) The variation of stator resistance produced 11 percent variation in the total losses.

(b) The variation of rotor resistance caused 8 percent variation in the total losses.

(c) The decrease in the magnetizing reactance produced an increase of 6 percent in the total losses, whereas the increase in the value of the parameter caused only one percent reduction in the total losses.

(d) The stator and rotor leakage reactances produced only 2 to 1 percent variation in the total losses.

The detailed study of the losses has pointed out that the additional losses due to non-sinusoidal supply voltage waveform deserve a proper consideration and they should be minimized so that the motor operates satisfactorily when connected to a static inverter for variable frequency operation.

The optimization of the losses of the induction motor has been carried out by using incremental search technique based on direct search method. A flow chart has been developed for the process of optimization. The computer program based on the above flow-chart has been used to find the suitable values of the three selected variables, viz. number of conductors per stator slot, area of cross-section of the stator conductor and core length ; so that minimum total losses are obtained for the motor. For the 5 h.p. motor used an example, a reduction of 10.5 percent in the total losses is achieved by using this method. Also the optimized design of the motor when compared with its normal design shows an overall improvement in the motor performance over the entire operating frequency range.

Scope for Future Work

Further work of this problem can be taken up on the following lines.

(1) An optimized design of induction motor obtained by using the procedure developed here can be used to fabricate the motor. The motor can, then, be tested to verify the results of the method.

(2) For using standard frames in the fabrication of the motor, limits on the values of core length can be imposed and the optimal solution within this constraint can be achieved by a few runs of the program of optimization.

(3) Variation of core length in smaller steps can be included in the above program of optimization by insertion of one more 'do loop' and search for optimal solution can be carried out with the help of a present generation computer in preference to IBM- 1620 digital computer.

(4) Calculation of the cost of active materials can be included in the program of optimization and so the comparison between different solutions (a set of minima of total losses for various core lengths) can be effected. Thus the proper design as per requirements can be selected.

(5) Instead of using only the three variables mentioned above, a few more variables could be used and then by using faster optimization techniques a complete optimized design of the motor can be obtained.

REFERENCES

1. Ahmad, S.,'Optimization of Induction Motor Design Using Digital Computer', M.E. Dissertation, University of Roorkee, Roorkee, October 1972.
2. Alger, P.L.,'Induced High-frequency Currents in Squirrel-case Windings', A.I.E.E. Trans.,part III, Vol.76, October 1957, p.724.
3. Alger, P.L., Angst, G., and Davies, E.,'Stray-load losses in polyphase induction machines', A.I.E.E. Trans., PartIIIA, June 1959, pp. 349-357.
4. Alger,P.L.,'The Nature of Induction Machines', Gordon and Breach, N.Y.-1965.
5. Aoki, M.,'Introduction to Optimization Techniques', Macmillan Company, N.Y. - 1971.
6. Barnes, E.C.,'Performance and Characteristics of Induction Motors for Solid State Variable-frequency Drives', IEEE Trans. on Ind. and Gen. Applic., Vol. IGA-7, No.2,March/April 1971, pp.212-217.
7. Berg, G.J., and Sarkar, A.K.,'Speed Change of Induction Motors with Variable Frequency Supply', IEEE Trans. on Power App. and Systems, Vol. PAS-90, March 1971, pp.500-508.
8. Bland, R.J., Hancock, N.N., and Whitehead R.W.,'Considerations Concerning a Modified Kramer System', Proc. IEE, Vol.110, No.12, December 1963, pp.2228-2232.
9. Böhm, K., and Wesselak, F.,'Variable Speed a.c. Drives with Static Frequency Converter Feed', Siemens Review, No.3,1972, pp.126-129.

10. Brenneisen, J., Futterlieb, E., Muller, E., and Schulz, M., 'A New Converter Drive System for a Diesel-electric Locomotive with Asynchronous Traction Motors', IEEE Trans. on Ind. Applic., Vol. IA-9, No.4, July/August 1973, pp. 483-491.

11. Chalmers, B.J., 'Electromagnetic Problems of a.c. Machines', Chapman and Hall Ltd., 1965.

12. Chalmers, B.J., and Sarkar, B.R., 'Induction Motor Losses Due to Non-sinusoidal Supply Waveforms', Proc. IEE, Vol.115, No.12, December 1968, pp. 1777-1782.

13. Chalmers, B.J., and Narain, C.K., 'High Frequency No Load Losses of Cage Induction Motor', IEEE Trans. on Power App. and Systems, Vol.PAS - 89, 1970, pp. 1043-1049.

14. De Jong, H.C.J., 'A.C. Motor Design with Conventional and Converter Supplies', Claredon Press, Oxford, 1976.

15. Elger, H., and Weiss, M., 'A Sub-synchronous Static Converter Cascade for Variable-speed Boiler Feed Pump Drives', Siemens Review, No.10, October 1968, pp.405-407.

16. Floter, W., and Ripperger, H., 'Field Oriented Closed-loop Control of an Induction Machine with the New Transvector Control System', Siemens Review, No.6, June 1972, pp.248-252.

17. Fornasieri, J., and Girardin, G., 'Frequency Controlled Drives with Squirrel Cage Motors', Brown Boveri Review, No.9, September 1971, pp.407-415.

18. Fox, R.L., 'Optimization Methods of Engineering Design', Addison-Wesley, Reading, Mass., 1971.

19. Fulton, N.N., Slater, R.D., and Wood, W.S., 'Design Optimization of Small Three-phase Induction Motor', Proc. IEE, Vol.123, No.2, February 1976, pp.141-144.

20. Garg, O.P., and Saxena, R.B., 'Losses in Inverter Fed Wide Speed Range Three Phase Induction Motor', Electrical India, 30th April, 1975, p. 5.

21. Garg, O.P., 'Performance and Torque Pulsations Minimization of Inverter Fed Three Phase Induction Motor', Ph.D. Thesis, Birla Institute of Technology and Science, Pilani, February 1976.

22. Hadley, G., 'Non-linear and Dynamic Programming', Addison-Wesley Publishing Company, Inc., London - 1964.

23. Jacovides, L.J., 'Analysis of Induction Motor Drives with Non-sinusoidal Supply Voltage Using Fourier Analysis', IEEE Trans. on Ind. Applic., Vol. I A-9, No.6, November/December 1973, pp.741-746.

24. Jain, G.C., 'The Effect of Voltage Wave Shape on the Performance of a Three-phase Induction Motor', IEEE Trans. on Power App. and Systems, Vol. PAS-83, June 1966, pp. 561-566.

25. Klautschek, H., 'Induction Motor Drives with Constant-Current d.c. - Link Frequency Converters', Siemens Review, No.10, October 1976, pp. 427-432.

26. Klingshirn, E.A., and Jordan, H.E., 'Polyphase Induction Motor Performance and Losses on Non-sinusoidal Voltage Sources', IEEE Trans. on Power App. and Systems, Vol.PAS-87, No.3, March 1968, pp. 624-631.

27. Kostenko, M., and Pitrovsky, L., 'Electrical Machines' . Part II, Foreign Languages Publishing House, Moscow.

28. Kuhlmann, J.H., 'Design of Electrical Apparatus', Third Edition, Chapman and Hall Ltd., 1950.

29. Largiader, H., 'Design Aspects of Induction Motors for Traction Applications with Supply Through Static Frequency Changers', Brown Boveri Review, No.4, 1970, pp. 152-167.

30. Lippo, T.A., and Turnbull, F.G., 'Analysis and Comparison of Two Types of Square Wave Inverter Drives', IEEE Trans. on Ind. Applic., Vol.IA-11, No.2, March/April 1975, pp. 137-147.

31. Mclean, G.W., Nix, G.F., and Alwash, S.R., 'Performance and Design of Induction Motors with Square Wave Excitation', Proc. IEE, Vol.116, No.8, August 1969, pp.1405-1411.

32. Murphy, J.D., 'Thyristor Control of a.c. Motors', Pergamon Press Ltd., Oxford, 1973.

33. Paice, D.A., 'Induction Motor Speed Control By Stator Voltage Control', IEEE Trans. on Power App. and Systems, Vol.PAS-87, No.2, February 1968, pp.585-590.

34. Paice, D.A., 'Speed Control of Large Induction Motors By Thyristor Converters', IEEE Trans. on Ind. and Gen.Applic., Vol.IGA-5, No.5, September/October 1969, pp.545-551.

35. Plunkett, A.B., and Lippo, T.A., 'New Method of Induction Motor Torque Regulation', IEEE Trans. on Ind. Applic., Vol.IA-12, No.1, January/February 1976, pp.47-55.

36. Pollack, J.J.,'Some Guide Lines for the Application of Adjustable Speed a.c. Drives', IEEE Trans. on Ind.Applic., Vol. IA-9, No.6, November/December 1973, pp.704-710.

37. Rao, S.S.,'Optimization-theory and Applications', Wiley Eastern Ltd., Bombay, 1978.

38. Say, M.G.,'Performance and Design of a.c. Machines', English Language Book Society, London, 1958.

39. Schwartz, K.K.,'Survey of Basic Stray Load Losses in Squirrel Cage Induction Motors', Proc. IEE, Vol.III, No.9, September 1964, pp.1565-1574.

40. Shepherd, W., and Stanway, J.,'Slip Power Recovery in An Induction Motor By the Use of a Thyristor Inverter', IEEE Trans. on Ind. and Gen. Applic., Vol.IGA-5, No.1, January/February 1969, pp.74-82.

41. Still, A., and Siskind C.S.,'Elements of Electrical Machine Design' Third Edition, McGraw-Hill Book Company, Inc.,N.Y. 1954.

42. Tsivitse, P.J., and Klingshirn, E.A.,'Optimum Voltage and Frequency for Polyphase Induction Motors Operating With Variable Frequency Power Supplies', IEEE Trans. on Ind. and Gen. Applic., Vol. IGA-7, No.4, July/August 1971, pp. 480-487.

43. Veinott, C.G.,'Theory and Design of Small Induction Motors', McGraw-Hill Book Company, Inc., N.Y.-1959.

44. Veinott, C.G.,'Synthesis of Induction Motor Designs on The Digital Computers',AIEE Trans.,Part III,Vol.79,1960, p.12.

45. Veinott, C.G., 'Induction Machinery Design and the Digital Computer', AIEE Trans. on Power App. and Systems, Vol.76, February 1957, pp. 1509-1516.

46. Veres, R.P., 'New Inverter Supplies for High Horse Power Drives', IEEE Trans. on Ind. and Gen. Applic. Vol.IGA-6, No.2, March/April 1970, pp.121-127.

47. Vickers, H., 'The Induction Motor', Second Edition, Sir Issac. Pitman and Sons Ltd., London - 1953.

48. Yermekova, R.A., 'On The Effectiveness of Induction Motor Performance in the Presence of Voltage and Frequency Deviations', Electric Technology (U.S.S.R.), Vol.I, 1972, pp.136-142.

APPENDIX(1)

```
*************************************************************************
        PROGRAM FOR DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS
                    OF 3-PH. SQ. CAGE INDUCTION MOTOR
*************************************************************************
C  C  CALCULATION OF SATURATION FACTOR C.B. DESHPANDE
      READ 2..V1,F1,WF,P,CL,DA,SS,RS
      READ 21,GA,SSO,RSO,DU,DSS,DSR,DC,DCR
      READ 21,WSR,WT
   21 FORMAT(8F10.4)
      READ 1,AS,PW,Z,AB,CE,DR,P1,P2
       READ 1,C1,CP
1     FORMAT(8F10.3)
      TP=Z*SS/6.0
      PHI=V1/(4.44*WF*F1*TP)
      BG=PHI*P*0.63661E4/(CL*DA)
      BG60=1.36*BG
      Y1=3.141592*DA/SS
      Y2=3.141592*(DA-2.0*GA)/RS
      CF1=Y1*(5.0*GA+SSO)/(Y1*(5.0*GA+SSO)-SSO*SSO)
      CF2=Y2*(5.0*GA+RSO)/(Y2*(5.0*GA+RSO)-RSO*RSO)
      CFD=CL/(CL-DU/(1.0+2.5*GA))
      GAE=GA*CF1*CF2*CFD
      ATG=8000.0*BG60*GAE
      BT=2.0E4*PHI*P/(0.9*(CL-DU)*WT*SS)
      BT6=1.36*BT
      BC=PHI/(1.8E-4*DC*(CL-DU))
      CLS=3.141592*(DA+2.0*DSS+DC)/(6.0*P)
      WTR=3.141592*(DA-2.0*GA-4.0*DSR/3.0)/RS-WSR
      BTR=2.0E4*PHI*P/(0.9*(CL-DU)*WTR*RS)
      BTR6=1.36*BTR
      BCR=PHI/(1.8E-4*(CL-DU)*DCR)
      CLR=3.141592*(DA-2.0*GA-2.0*DSR-DCR)/(6.0*P)
      K=0
   22 K=K+1
      GO TO (23,24,27,28),K
   23 B=BT6
      GO TO 31
   24 B=BC
      GO TO 31
   27 B=BTR6
      GO TO 31
   28 B=BCR
   31 IF(B-0.8)61,61,62
   61 AT=100.0*(B-0.1)+60.0
      GO TO(91,92,93,94),K
   62 IF(B-1.05)63,63,64
   63 AT=280.0*(B-0.8)+130.0
      GO TO (91,92,93,94),K
   64 IF(B-1.2)65,65,66
   65 AT=666.67*(B-1.05)+200.0
      GO TO (91,92,93,94),K
   66 IF(B-1.3)67,67,68
   67 AT=1000.0*(B-1.2)+300.0
      GO TO (91,92,93,94),K
   68 IF(B-1.4)69,69,71
   69 AT=2000.0*(B-1.3)+400.0
```

```
      GO TO (91,92,93,94),K
 71   IF(B-1.46)72,72,73
 72   AT=3333.34*(B-1.4)+600.0
      GO TO (91,92,93,94),K
 73   IF(B-1.5)74,74,75
 74   AT=5000.0*(B-1.46)+800.0
      GO TO (91,92,93,94),K
 75   IF(B-1.55)76,76,77
 76   AT=12000.0*(B-1.5)+1000.0
      GO TO (91,92,93,94),K
 77   IF(B-1.67)78,78,79
 78   AT=20000.0*(B-1.55)+1600.0
      GO TO (91,92,93,94),K
 79   AT=38461.5*(B-1.67)+4000.0
      GO TO (91,92,93,94),K
 91   ATST=AT
      GO TO 22
 92   ATSC=AT
      GO TO 22
 93   ATRT=AT
      GO TO 22
 94   ATRC=AT
      TAST=ATST*DSS*0.01
      TASC=ATSC*CLS*0.01
      TART=ATRT*DSR*0.01
      TARC=ATRC*CLR*0.01
      TAT=ATG+TAST+TASC+TART+TARC
      SF=TAT/ATG
      PUNCH 101,SF
101   FORMAT(5X,E15.8)
      R1=2.095E-6*TP*(2.0*CL+3.6128308*DA/P+24.01/AS
      ATG=TP*TP*WF*WF
      BL=CL/COSF(6.2831852*P/SS)+2.4
      R3=25.14E-6*ATG*BL/(RS*AB)
      ATST=4.0011561E-6*ATG*DR/(CE*P*P)
      R2=R3+ATST
      Y1=7.53E-8*F1*ATG*DA*CL
      Y2=SF*GAE*P*P
      XM=Y1/Y2
      SA=21.0E-8*F1*TP*TP*DA*(PW-0.3)/(P*P)
      Y1=(6.0/CF1-1.0)*P*P/(1.25*SS*SS)
      Y2=(6.0/CF2-1.0)*P*P/(1.25*RS*RS)
      RA=0.82246*(Y1+Y2)*XM
      AN=3.2898667*XM*P*P/(SS*SS)
      ATST=0.5*(SA+RA+AN)
      X1=ATST+9.48E-7*F1*CL*TP*TP*P1/SS
      X3=9.48E-7*F1*CL*ATG*P2/RS
      X2=X3+ATST
      ELCO=0.42519E-7*ATG*DA*C1/(P*P)
      Y1=7.11E-3*SS*WT*DSS*(CL-DU)
      Y2=22.336723E-3*(DA+DC+2.0*DSS)*DC*(CL-DU)
      SA=1.5707963*BT
```

```
      RA=BC
      SCLC=0.014322*(Y1*SA**1.8+Y2*RA**1.8)
       B=3.1415926*(DA-2.0*GA-DSR)/RS-WSR
       Y1=7.11E-3*RS*DSR*B*(CL-DU)
       Y2=22.336723E-3*(DA-2.0*GA-2.0*DSR-DCR)*DCR*(CL-DU)
      SA=1.5707963*BTR
      RA=BCR
      RCLC=0.014322*(Y1*SA**1.8+Y2*RA**1.8)
      ATG=BG*CF1*CF2*CFD
      ATST=0.63837*(ATG*P)**2.0*CL*DA
      SSLC=ATST*(DA-2.0*GA)/(RS**3.0)
      RSLC=ATST*DA/(SS**3.0)
      RSNC=0.15959*ATG**2.0*CL*DA*(DA/SS)*CP
      SKCO=3.2898667*(P/SS)**2.0
      PUNCH 11,X1,R1,X2,R2,XM
       PUNCH 11 ,X3,R3
      PUNCH 11,ELCO,SCLC,RCLC,SSLC,RSLC
      PUNCH 11,RSNC,SKCO
11     FORMAT(5X,5E15.8)
      STOP
      END
```

APPENDIX(2)

```
*************************************************************************
                 PROGRAM FOR DETERMINATION OF LOSSES
                 OF INVERTER FED 3-PH INDUCTION MOTOR
                      WHEN V/F RATIO IS CONSTANT
*************************************************************************
C  C  CALCULATION OF LOSSES OF INDUCTION MOTOR ON NON SINUSOIDAL SUPPLY
       READ1,V1,F1,P,SS,RS
      READ1,H,R,CO,CZ,CB,TF,HP
       READ1,SCLC,ELCO,SSLC,RSLC,RSNC,SKCO,RCLC
      READ 1,X1,R1,X2,R2,X3,R3,XM
1      FORMAT(8F10.3)
      PUNCH 2,X1,R1,X2,R2,XM
2     FORMAT(5X,5E15.7)
      X1=0.85*X1
      X2=0.85*X2
      X3=0.85*X3
      XM=XM/1.04
       W=1.0
12     V=W*V1
       F=W*F1
        X1W=W*X1
       X2W=W*X2
       X3W=W*X3
       XMW=W*XM
       W3=SCLC*F**1.6
      Y1=3.0*V*V/W3
      B=(1.0+(XMW/Y1)**2)
       RM=XMW*XMW/(Y1*B)
       XMW=XMW/B
       X11=XMW+X1W
       X22=XMW+X2W
       SA=R1+RM
       B=X1W+X2W
       RA=(R1*RM+XMW*XMW-X11*X22)/R2
       AN=(R1*X22+RM*B)/R2
       ATG=RM*RM+XMW*XMW
      D=2.094395*TF*R2*F/(P*V*V)
      Y2=D*(SA*SA+X11*X11)/ATG
      Y1=D*(RA*RA+AN*AN)/ATG
      E=2.0*D*(SA*RA+AN*X11)/ATG-1.0
       S=(-E-SQRTF(E*E-4.0*Y1*Y2))   /(2.0*Y1)
      Y1=0.47746*P*V*V*S*(ATG/(R2*F))
       E=(SA+S*RA)*(SA+S*RA)
      Y2=(AN*S+X11)**2
       T=Y1/(E+Y2)
        E=X22*S
       D=-(X1W+X2W*XMW/X22)
       Y1=SA*R2/E+R1*RM/X22+D
          Y2=R1+RM*B/X22+X11*R2/E
       D=SQRTF(Y1*Y1+Y2*Y2)
       D=V/D
       B=R2/E
       Y1=RM/X22
```

```
      SA=D*SQRTF((B+Y1)**2+1.0)
      RA=D*SQRTF(ATG)/X22
      AN=D*SQRTF(B*B+(X2W/X22)**2)
      ATG=SA*SA
      W1=3.0*R1*ATG
      W2=3.0*R2*RA*RA
      D=0.138*H*SQRTF(R*SS*F/P)
      RM=2.0*D
      X11=0.5*EXPF(RM)
      X22=0.5*EXPF(-RM)
      B=X11-X22
      E=X11+X22
      S1=D*(B+SINF(RM))/(E-COSF(RM))
      W4=3.0*S1*R3*(CO*AN*AN+CZ*ATG)
      W5=ELCO*F*ATG
      D=0.138*H*SQRTF(R*6.0*F)
      RM=2.0*D
      X11=0.5*EXPF(RM)
      X22=0.5*EXPF(-RM)
      B=X11-X22
      E=X11+X22
      S2=D*(B+SINF(RM))/(E-COSF(RM))
      W9= 3.*S2 * ATG * R3 * CB
      W10= RCLC *(S*F)**1.6
      W11=HP*746.0*0.016*(1.0-S)*W
      E=(SA/AN)**2
      CS1=0.00123*(RS*F/P)**1.48
      CS2=0.00123*(SS*F/P)**1.48
      W6=SSLC*CS1*E
      W7=RSLC*CS2*E
      WO=RSNC*CS2
      W8=SKCO*(RA/AN)**2   *(W3+WO)
      PUNCH25,W1,W2,W3,W4,W5
      PUNCH25,W11,E,TL,T,S
      PUNCH25,W6,W7,W8,W9,W10
      PUNCH25,W11,E,TL,T,S
      SCHC=0.0
      I=5
      HARQ=0.0
      NCNT=0
      HASQ=0.0
50    DO10II=I,25,6
      HO=II
      IF(II-25)250,250,300
250   IF(NCNT)70,70,60
70    HM=(HO+1.0-S)
      GOTO100
60    HM=(HO-1.0+S)
100   VH=V/HO
      SH=HM/HO
      B=0.138*H*SQRTF(R*HM*F)
      AT=2.0*B
      Y1=0.5*EXPF(AT)
```

```
      Y2=0.5*EXPF(-AT)
      D=Y1-Y2
      E=Y1+Y2
      Y1=SINF(AT)
      Y2=COSF(AT)
       RM=E-Y2
      X11=B*(D+Y1)/RM
      X22=(1.5/B)*(D-Y1)/RM
      R2H=R2+(X11-1.0)*R3
      X2H=HO*(X2W+(X22-1.0)*X3W)
      X1H=HO*X1W
      XMH=HO*W*XM
      XSH=X2H+XMH
      X22=XSH*SH
      B=-(X1H+X2H*XMH/XSH)+R1*R2H/X22
      E=R1+(XMH+X1H)*R2H/X22
      D=SQRTF(B*B+E*E)
       B=VH/D
      SAH=B*SQRTF((R2H/X22)**2+1.0)
      RAH=B*XMH/XSH
      HASQ=HASQ+SAH*SAH
      HARQ=HARQ+RAH*RAH
      W2=W2+3.0*R2H*RAH*RAH
      W5=W5+2.0*ELCO*(F*HO)*SAH*SAH
      W10=W10+RCLC*(HM*F)**1.6/(HO**3.6)
      SCHC=SCHC+1.0/(HO**2.0)
10    CONTINUE
300   I=I+2
      NCNT=NCNT+1
      IF(I-7)50,50,200
200   W3=W3*(1.0+SCHC)
      W1=W1+3.0*R1*HASQ
      W4=W4+3.0*S1*R3*CZ*HASQ
      B=HASQ/AN**2
      W6=W6+SSLC*CS1*B
      W7=W7+RSLC*CS2*B
      B=(RA*RA+HARQ)/AN**2
      W8=SKCO*B*(W3+WO)
      W9=W9+3.0*S2*R3*HASQ *CB
      TL=W1+W2+W3+W4+W5+W6+W7+W8+W9+W10+W11
      B=T*6.2831852*F*(1.0-S)/P-W11
      E=B/(B+TL)
      TOC=SQRTF(HASQ+ATG)
      PUNCH25,W1,W2,W3,W4,W5
      PUNCH25,W6,W7,W8,W9,W10
      PUNCH 25,TL,S,F,TOC,E
25    FORMAT (5X,5E15.7)
      W=W-0.1
      IF(W-0.2)15,12,12
15    STOP
      END
```

APPENDIX(3)

```
*****************************************************************************
                    PROGRAM FOR DETERMINATION OF LOSSES
                    OF INVERTER FED 3-PH INDUCTION MOTOR
                        WHEN E/F RATIO IS CONSTANT
*****************************************************************************
C   CALCULATION OF LOSSES OF IND.MOTOR-CONST.FLUX OPERATION-C.B.D.
    READ1,V1,F1,P,SS,RS
    READ1,H,R,CO,CZ,CB,TF,HP
    READ1,SCLC,ELCO,SSLC,RSLC,RSNC,SKCO,RCLC
    READ 1,X1,R1,X2,R2,X3,R3,XM
    FORMAT (8F10.3)
    X1=0.85*X1
    X2=0.85*X2
    X3=0.85*X3
    XM=XM/1.04
    SFL=0.03978133
    F2=SFL*F1
    X2S=X2*F2/F1
    A=R2*R2+X2S*X2S
    B=F2*R2
    EBYF=SQRTF(TF*2.094395*A/(P*B))
    W=1.0
12  F=W*F1
    E1=EBYF*F
    S=F2/F
    X1W=W*X1
    X2W=W*X2
    X3W=W*X3
    XMW=W*XM
    W3=SCLC*F**1.6
    Y1=3.C*E1*E1/W3
    B=(1.0+(XMW/Y1)**2)
    RM=XMW*XMW/(Y1*B)
    XMW=XMW/B
    D=SQRTF(XMW*XMW+RM*RM)
    AN=E1/D
     D=SQRTF(X2W*X2W+(R2/S)**2)
    RA=E1/D
    A1=RM*R2/S-XMW*X2W
    A2=XMW*R2/S+RM*X2W
    A3=RM+R2/S
    A4=XMW+X2W
    A1=A1/A4
    A2=A2/A4
    A3=A3/A4
    B=1.0+A3*A3
    RE=(A1*A3+A2)/B
    XE=(A2*A3-A1)/B
    D=SQRTF(RE*RE+XE*XE)
    SA=E1/D
    PUNCH 4,SA,RA,AN
  4 FORMAT(5X,3E15.6)
```

```
      RE=RE+R1
      XE=XE+X1W
      B=SQRTF(RE*RE+XE*XE)
      V=SA*B
      ATG=SA*SA
      W1=3.0*R1*ATG
      W2=3.0*R2*RA*RA
      D=0.138*H*SQRTF(R*SS*F/P)
      RM=2.0*D
      X11=0.5*EXPF(RM)
      X22=0.5*EXPF(-RM)
      B=X11-X22
      E=X11+X22
      S1=D*(B+SINF(RM))/(E-COSF(RM))
      W4=3.0*S1*R3*(CO*AN*AN+CZ*ATG)
      W5=ELCO*F*ATG
      D=0.138*H*SQRTF(R*6.0*F)
      RM=2.0*D
      X11=0.5*EXPF(RM)
      X22=0.5*EXPF(-RM)
      B=X11-X22
      E=X11+X22
      S2=D*(B+SINF(RM))/(E-COSF(RM))
      W9=3.0*S2*ATG*R3*CB
      W10=RCLC*(S*F)**1.6
      W11=HP*746.0*0.016*(1.0-S)*W
      E=(SA/AN)**2
      CS1=0.00123*(RS*F/P)**1.48
      CS2=0.00123*(SS*F/P)**1.48
      W6=SSLC*CS1*E
      WO=RSNC*CS2
      W7=RSLC*CS2*E
      SCHC=0.0
      I=5
      HARQ=0.0
      NCNT=0
      HASQ=0.0
50    DO10II=I,25,6
      HO=II
      IF(II-25)250,250,300
  250 IF(NCNT)70,70,60
   70 HM=(HO+1.0-S)
      GO TO 100
   60 HM=(HO-1.0+S)
  100 VH=V/HO
      SH=HM/HO
      B=0.138*H*SQRTF(R*HM*F)
      AT=2.0*B
      Y1=0.5*EXPF(AT)
      Y2=0.5*EXPF(-AT)
      D=Y1-Y2
      E=Y1+Y2
      Y1=SINF(AT)
```

```
      Y2=COSF(AT)
      RM=(E-Y2)
      X11=B*(D+Y1)/RM
      X22=(1.5/B)*(D-Y1)/RM
      R2H=R2+(X11-1.0)*R3
      X2H=HO*(X2W+(X22-1.0)*X3W)
      X1H=HO*X1W
      XMH=HO*W*XM
      XSH=X2H+XMH
      X22=XSH*SH
      B=-(X1H+X2H*XMH/XSH)+R1*R2H/X22
      E=R1+(XMH+X1H)*R2H/X22
      D=SQRTF(B*B+E*E)
      B=VH/D
      SAH=B*SQRTF((R2H/X22)**2+1.0)
      RAH=B*(XMH/XSH)
      HASQ=HASQ+SAH*SAH
      HARQ=HARQ+RAH*RAH
      W2=W2+3.0*RAH*RAH*R2H
      W5=W5+2.0*ELCO*(F*HO)*SAH*SAH
      W10=W10+RCLC*(HM*F)**1.6/(HO**3.6)
      SCHC=SCHC+1.0/(HO**2)
   10 CONTINUE
  300 I=I+2
      NCNT=NCNT+1
      IF(I-7)50,50,200
  200 W3=W3*(1.0+SCHC)
      W1=W1+3.0*R1*HASQ
      W4=W4+3.0*S1*R3*CZ*HASQ
      B=HASQ/AN**2
      W6=W6+SSLC*CS1*B
      W7=W7+RSLC*CS2*B
      B=(RA*RA+HARQ)/AN**2
      W8=SKCO*B*(W3+WO)
      W9=W9+3.0*S2*R3*CB*HASQ
      WS=W4+W5+W6+W7+W8+W9
      TL=W1+W2+W3+W4+W5+W6+W7+W8+W9+W10+W11
      B=TF*6.2831852*F*(1.0-S)/P-W11
      E=B/(B+TL)
      TOC=SQRTF(HASQ+ATG)
      PUNCH 25,W1,W2,W3,W4,W5
      PUNCH 25,W6,W7,W8,W9,W10
      PUNCH 25,W11,S,TOC,TL,E
      PUNCH 25,WS,F,V
   25 FORMAT(5X,5E15.8)
      W=W-0.1
      IF(W-0.2)15,12,12
   15 STOP
      END
```

APPENDIX(4)

```
*******************************************************************************
                        PROGRAM FOR OPTIMIZATION OF LOSSES
                       OF INVERTER FED 3-PH INDUCTION MOTOR
*******************************************************************************
C   C  OPTIMIZATION OF LOSSES OF INDUCTION MOTOR -C.B.DESHPANDE
       DIMENSION Z(21),AS(21)
       DO101M=1,21
101    READ1,Z(M),AS(M)
1      FORMAT(2F10.4)
       READ2,DA,CL,SS,SSO,RS,RSO,DC,DCR
       READ2,GA,DSS,DSR,WT,WSR,H,R
       READ2,CE,DR,BL,AB,P1,P2
       READ2,WF,PW,CP,C1,CB,CO,CZ
       READ2,V1,F1,P,HP,TF
2      FORMAT(8F10.3)
       Y1=3.1415926*DA/SS
       B=5.0*GA+SSO
       Y2=Y1*B
       CF1=Y2/(Y2-SSO*SSO)
       D=DA-2.0*GA
       Y1=3.1415926*D/RS
       B=5.0*GA+RSO
       Y2=Y1*B
       CF2=Y2/(Y2-RSO*RSO)
       GAE=GA*CF1*CF2
       WTR=3.1415926*(D-4.0*DSR/3.0)/RS-WSR
       SA=DA+2.0*DSS+DC
       CLS=0.5236*SA/P
       E=0.014322*F1**1.6
       B=WT
       STIC=0.00711*SS*DSS*B*E
       SCIC=0.0223367*SA*DC*E
       SA=D-2.0*DSR-DCR
       CLR=0.5236*SA/P
       B=3.1415926*(D-DSR)/RS-WSR
       RTIC=0.00711*RS*DSR*B*E
       RCIC=0.0223367*SA*DCR*E
       B=(CF1*CF2)**2
       SA=0.00123*(RS*F1/P)**1.48
       RA=0.00123*(SS*F1/P)**1.48
       E=0.63837*B*P*P*DA
       SSLC=SA*E*D/RS**3
       RSLC=RA*E*DA/SS**3
       RSNC=0.15959*B*DA*(DA/SS)*CP*RA
       SKCO=3.2898667*(P/SS)**2
       ELC=0.42519*F1*WF*WF*DA*C1/(P*P)
       B=0.138*H*SQRTF(R*SS*F1/P)
       SA=2.0*B
       RA=0.5*EXPF(SA)
       AN=0.5*EXPF(-SA)
       D=RA-AN
       E=RA+AN
       CDB=B*(D+SINF(SA))/(E-COSF(SA))
       B=0.138*H*SQRTF(R*6.0*F1)
```

```
      SA=2.0*B
      RA=0.5*EXPF(SA)
      AN=0.5*EXPF(-SA)
      D=RA-AN
      E=RA+AN
      BLCO=B*(D+SINF(SA))/(E-COSF(SA))
      B=P*P
      Y3=(6.0/CF1-1.0)*B/(1.25*SS*SS)
      Y4=(6.0/CF2-1.0)*B/(1.25*RS*RS)
      WFLC=HP*746.0*0.016
      TLM=10000.0
14    DO11J=1,21
      TP=Z(J)*SS/6.0
      SA=V1/(4.44*WF*F1*TP)
      BG=SA*P*0.6366154/(CL*DA)
      D=1.36*BG
      ATG=8000.0*D*GAE
      BT=2.0E4*SA*P/(0.9*CL*WT*SS)
      RA=1.36*BT
      BC=SA/(1.8E-4*DC*CL)
      BTR=2.0E4*SA*P/(0.9*CL*WTR*RS)
      AN=1.36*BTR
      BCR=SA/(1.8E-4*CL*DCR)
      K=0
22    K=K+1
      GOTO(23,24,27,28),K
23      B=RA
      GOTO31
24    B=BC
      GOTO31
27    B=AN
      GOTO31
28    B=BCR
31     IF(B-0.8)61,61,62
61    AT=100.0*(B-0.1)+60.0
      GOTO(91,92,93,94),K
62    IF(B-1.05)63,63,64
63    AT=280.0*(B-0.8)+180.0
      GOTO(91,92,93,94),K
64    IF(B-1.2)65,65,66
65    AT=666.67*(B-1.05)+200.0
      GOTO(91,92,93,94),K
66    IF(B-1.3)67,67,68
67    AT=1000.0*(B-1.2)+300.0
      GOTO(91,92,93,94),K
68    IF(B-1.4)69,69,71
69    AT=2000.0*(B-1.3)+400.0
      GOTO(91,92,93,94),K
71    IF(B-1.46)72,72,73
72    AT=3333.34*(B-1.4)+600.0
      GOTO(91,92,93,94),K
73    IF(B-1.5)74,74,75
74    AT=5000.0*(B-1.46)+800.0
```

```
       GOTO(91,92,93,94),K
75     IF(B-1.55)76,76,77
76     AT=12000.0*(B-1.5)+1000.0
       GOTO(91,92,93,94),K
77     IF(B-1.67)78,78,79
78     AT=20000.0*(B-1.55)+1600.0
       GOTO(91,92,93,94),K
79     IF(B-1.8)80,80,81
80     AT=38461.5*(B-1.67)+4000.0
       GOTO(91,92,93,94),K
81     AT=60000.0*(B-1.8)+9000.0
       GOTO(91,92,93,94),K
91     Y1=AT
       Y1=Y1*DSS*0.01
       GOTO22
92     Y2=AT
       Y2=Y2*CLS*0.01
       GOTO22
93     X11=AT
       X11=X11*DSR*0.01
       GOTO22
94     X22=AT
       X22=X22*CLR*0.01
       D=Y1+Y2+X11+X22+ATG
       SF=D/ATG
       SA=2.0*CL+3.6128308*DA/P+24.0
       R1=2.095E-6*TP*SA/AS(J)
       B=TP*TP*WF*WF
       Y1= COSF(6.2831852*P/SS)
       BL=CL/Y1+2.4
       R3=25.14E-6*B*BL/(RS*AB)
       AT=P*P
       SA=4.0011561E-6*B*DR/(CE*AT)
       R2=R3+SA
       SA=7.53E-8*F1*B*DA*CL
       RA=SF*GAE*AT
       XM=SA/RA
       ATG=TP*TP
       SA=21.0E-8*F1*ATG*DA*(PW-0.3)/AT
       RA=0.82246*XM*(Y3+Y4)
       AN=3.2898667*XM*AT/(SS*SS)
       SA=0.5*(SA+RA+AN)
       RA=9.48E-7*F1*CL
       X1=SA+RA*ATG*P1/SS
       X3=RA*B*P2/RS
       X2=SA+X3
       B=1.5707963*BT
       W3=(STIC*B**1.8+SCIC*BC**1.8)*CL
       X1W=0.85*X1
       X2W=0.85*X2
       X3W=0.85*X3
       XMW=XM/1.04
        Y1=3.0*V1*V1/W3
       B=(1.0+(XMW/Y1)**2)
        RM=XMW*XMW/(Y1*B)
```

```
       XMW=XMW/B
       X11=XMW+X1W
       X22=XMW+X2W
       SA=R1+RM
      RA=(R1*RM+XMW*XMW-X11*X22)/R2
       B=X1W+X2W
       AN=(R1*X22+RM*B)/R2
       ATG=RM*RM+XMW*XMW
      D=2.094395*TF*R2*F1/(P*V1*V1)
      Y2=D*(SA*SA+X11*X11)/ATG
      Y1=D*(RA*RA+AN*AN)/ATG
      E=2.0*D*(SA*RA+AN*X11)/ATG-1.0
       S=(-E-SQRTF(E*E-4.0*Y1*Y2))   /(2.0*Y1)
      Y1=0.47746*P*V1*V1*S*(ATG/(R2*F1))
       E=(SA+S*RA)*(SA+S*RA)
       Y2=(AN*S+X11)**2
       T=Y1/(E+Y2)
        E=X22*S
       D=-(X1W+X2W*XMW/X22)
       Y1=SA*R2/E+R1*RM/X22+D
         Y2=R1+RM*B/X22+X11*R2/E
       D=SQRTF(Y1*Y1+Y2*Y2)
       B=R2/E
       Y1=RM/X22
      D=V1/D
       SA=D*SQRTF((B+Y1)**2+1.0)
       RA=D*SQRTF(ATG)/X22
      AN=D*SQRTF(B*B+(X2W/X22)**2)
       W2=3.0*R2*RA*RA
      ELCO=ELC*TP*TP*1.0E-7
      W5=ELCO*SA*SA
      RCLC=(RTIC*(BTR*1.5707963)**1.8+RCIC*BCR**1.8)*CL
      W10=RCLC*S**1.6
      HASQ=0.0
      I=5
      HARQ=0.0
      SCHC=0.0
      NCNT=0
50    DO10II=I,25,6
      HO=II
      IF(II-25)250,250,300
250   IF(NCNT)70,70,60
70    HM=(HO+1.0-S)
      GOTO100
60    HM=(HO-1.0+S)
100   VH=V1/HO
      SH=HM/HO
      B=0.138*H*SQRTF(R*HM*F1)
      AT=2.0*B
       Y1=0.5*EXPF(AT)
      Y2=0.5*EXPF(-AT)
      D=Y1-Y2
      E=Y1+Y2
```

```
      Y1= SINF(AT)
      Y2= COSF(AT)
      RM=E-Y2
      X11=B*(D+Y1)/RM
      X22=(1.5/B)*(D-Y1)/RM
      R2H=R2+(X11-1.0)*R3
      X2H=HO*(X2W+(X22-1.0)*X3W)
      X1H=HO*X1W
      XMH=HO*XM/1.04
      XSH=X2H+XMH
      X22=XSH*SH
      B=-(X1H+X2H*XMH/XSH)+R1*R2H/X22
      E=R1+(XMH+X1H)*R2H/X22
      D=SQRTF(B*B+E*E)
      B=VH/D
      SAH=B*SQRTF(1.0+(R2H/X22)**2)
      RAH=B*XMH/XSH
      HASQ=HASQ+SAH*SAH
      HARQ=HARQ+RAH*RAH
      SCHC=SCHC+1.0/(HO**2.0)
      W2=W2+3.0*R2H*RAH*RAH
      W5=W5+2.0*ELCO*HO*SAH*SAH
      W10=W10+RCLC*HM**1.6/(HO**3.6)
10    CONTINUE
300   I=I+2
      NCNT=NCNT+1
      IF(I-7)50,50,200
200   W3=W3*(1.0+SCHC)
      ATG=SA*SA
      Y2=HASQ+ATG
      B=RA*RA
      D=B+HARQ
      E=Y2/(AN*AN)
      W1=3.0*R1*Y2
      W4=3.0*CDB*R3*(CO*AN*AN+CZ*Y2)
      AT=BG*BG*CL
      A=AT*E
      W6=SSLC*A
      W7=RSLC*A
      WO=RSNC*AT
      W8=SKCO*(W3+WO)*D/(AN*AN)
      W9=3.0*BLCO*Y2*R3*CB
      W11=WFLC*(1.0-S)
      TL=W1+W2+W3+W4+W5+W6+W7+W8+W9+W10+W11
      B=T*6.2831852*F1*(1.0-S)/P-W11
      E=B/(B+TL)
      PUNCH25,W1,W2,W3,W4,W5
      PUNCH25,W6,W7,W8,W9,W10
      PUNCH25,W11,E,TL,T,S
25    FORMAT(5X,5E15.8)
      IF(TL-TLM)20,11,11
20    TLM=TL
      W1M=W1
      W2M=W2
      W3M=W3
```

```
      W4M=W4
      W5M=W5
      W6M=W6
      W7M=W7
      W8M=W8
      W9M=W9
      W10M=W10
      W11M=W11
      TM=T
      SM=S
      CLM=CL
      ZM=Z(J)
      ASM=AS(J)
      R1M=R1
      R2M=R2
      X1M=X1
      X2M=X2
      XMM=XM
11    CONTINUE
15    PUNCH110,TLM,CLM,ZM,ASM
110   FORMAT(5X,4HTLM=E15.7,4HCLM=F7.3,3HZM=F5.1,4HASM=F10.5)
      PUNCH120,W1M,W2M,W3M,W4M,W5M
      PUNCH120,W6M,W7M,W8M,W9M,W10M
      PUNCH120,W11M,TM,SM
      PUNCH120,R1M,R2M,X1M,X2M,XMM
120   FORMAT(5X,5E15.7)
      STOP
      END
```

APPENDIX(5)

****

LIST OF SYMBOLS

USED IN COMPUTER PROGRAMS

****

A1=INTER MEDIATE VARIABLE
A2=INTER MEDIATE VARIABLE

A3=INTER MEDIATE VARIABLE
A4=INTER MEDIATE VARIABLE
AB=AREA OF ROTOR BAR
AN=NO LOAD CURRENT

AN =INTERMEDIATE VARIABLE
AS=CROSS-SECTION OF STATOR CONDUCTOR
AT=AMPERE TURNS
ATG=AMPERE TURNS FOR AIR-GAP

ATG=INTERMEDIATE VARIABLE
B =INTERMEDIATE VARIABLE
BC=STATOR-CORE FLUX DENSITY
BCR=ROTOR-CORE FLUX DENSITY
BG=AVERAGE AIR-GAP FLUX DENSITY
BL=ROTOR BAR LENGTH
BLCO=DEEP BAR COEFF. FOR BELT-LEAKAGE LOSS
BT=AVERAGE STATOR-TEETH FLUX DENSITY
BTR=AVERAGE ROTOR-TEETH FLUX DENSITY AT 1/3 POSITION
C1=END WINDING GEOMETRY CONSTANT
CB=BELT-LEAKAGE LOSS CONSTANT
CDB=DEEP BAR COEFF. FOR ROTOR ZIG-ZAG LOSS
CE=CROSS-SECTION OF END-RINGS
CF1=CARTERS FACTOR FOR STATOR
CF2=CARTERS FACTOR FOR ROTOR
CFD=CARTERS FACTOR FOR DUCTS
CL=CORE LENGTH
CLR=MAGNETIC PATH LENGTH THROUGH ROTOR CORE
CLS=MAGNETIC PATH LENGTHTHROUGH STATOR CORE
CO=NO-LOAD PULSATION LOSS CONSTANT
CP=POLE FACE CONSTANT
CS1=STATOR SURFACE LOSS FACTOR
CS2=ROTOR SURFACE LOSS FACTOR
CZ=FULL-LOAD PULSATION LOSS CONSTANT
D =INTERMEDIATE VARIABLE
DA=AIR-GAP DIAMETER
DC=DEPTH OF STATOR CORE
DCR=DEPTH OF ROTOR CORE
DR=MEAN DIAMETER OF END-RINGS
DSR=DEPTH OF ROTOR SLOT
DSS=DEPTH OF STATOR SLOT
DU=NO. OF DUCTS

E1=PER PHASE E.M.F.
E=EFFICIENCY
E =INTERMEDIATE VARIABLE
EBYF=E/F RATIO
ELCO=END LOSS COEFFICIENT
F1=NORMAL FREQUENCY
F2=ROTOR FREQUENCY
F=OPERATING FREQUENCY
GA=AIR-GAP LENGTH
GAE=EFFECITIVE AIR-GAP LENGTH
H=DEPTH OF ROTOR CONDUCTOR
HARQ=ROTOR HARMONIC CURRENT SQUARE
HASQ=STATOR HARMONIC CURRENT SQUARE
HM =INTERMEDIATE VARIABLE
HO=ORDER OF TIME HARMONIC
HP=HORSE POWER OUT PUT AT NORMAL FREQUENCY
I =INTERMEDIATE VARIABLE
NCNT=INTERMEDIATE VARIABLE
P1=PERMEANCE COEFFICIENT OF STATOR
P2=PERMEANCE COEFFICENT OF ROTOR
P=NO. OF POLE PAIRS
PHI=FLUX PER POLE
PW=PITCH FACTOR OF PRIMARY
R1=STATOR RESISTANCE PER PHASE
R2=ROTOR RESISTANCE REFERRED TO STATOR PER PHASE
R3=ROTOR BAR RESISTANCE REFERRED TO STATOR
R2H=HARMONIC ROTOR REST. REFERRED TO STATOR
R1M=STATOR RESISTANCE PER PHASE FOR MIN. TOTAL LOSSES
R2M=ROTOR PER PHASE RESISTANCE FOR MIN. TOTAL LOSSES
R=RATIO OF COND. WIDTH TO SLOT WIDTH OF ROTOR
RA=ROTOR CURRENT
RA =INTERMEDIATE VARIABLE
RAH=ROTOR HARMONIC CURRENT
RCIC=INTERMEDIATE VARIABLE
RCLC=ROTOR CORE LOSS COEFFICIENT
RE=INTER MEDIATE VARIABLE
RM=MAGNETISING RESISTANCE
RM =INTERMEDIATE VARIABLE
RS=NO. OF ROTOR SLOTS
RSLC=ROTOR SURFACE LOSS COEFFICIENT
RSNC=ROTOR NO LOAD SURFACE LOSS COEFFICIENT
RSO=ROTOR SLOT OPENING
RTIC=INTERMEDIATE VARIABLE
S1=DEEP-BAR COEFF. FOR ROTOR ZIG-ZAG LOSS
S2=DEEP-BAR COEFF. FOR BELT LEAKGE LOSS
S=FUNDAMENTAL SLIP
SA=STATOR CURRENT
SA =INTERMEDIATE VARIABLE
SAH=STATOR HARMONIC CURRENT
SCHC=STATOR HARMONIC CORE LOSS CONSTANT
SCIC=INTERMEDIATE VARIABLE
SCLC=STATOR CORE-LOSS COEFFICIENT
SF=SATURATION FACTOR
SFL=FULL LOAD SLIP
SH=HARMONIC SLIP

SKCO=SKEW LEAKAGE LOSS COEFFICIENT
SM=FUNDAMENTAL SLIP FOR MIN. TOTAL LOSSES
SS=NO. OF STATOR SLOTS
SSLC=STATOR SURFACE LOSS COEFFICIENT
SSO=STATOR SLOT OPENING
STIC=INTERMEDIATE VARIABLE
T=FULL LOAD TORQUE
TARC=AMPERE-TURNS FOR ROTOR CORE
TART=AMPERE-TURNS FOR ROTOR TEETH
TASC=AMPERE-TURNS FOR STATOR CORE
TAST=AMPERE-TURNS FOR STATOR TEETH
TAT=TOTAL AMPERE-TURNS
TF=FULL-LOAD TORQUE
TL=TOTAL LOSSES
TLM=MINIMUM TOTAL LOSSES
TM =INTERMEDIATE VARIABLE
TOC=TOTAL R.M.S. IN PUT CURRENT
TP=NO. OF STATOR TURNS/PHASE
V1=RATED VOLTAGE OF THE MOTOR
V=OPERATING VOLTAGE
VH=HARMONIC VOLTAGE
W1=STATOR COPPER LOSS
W2=ROTOR COPPER LOSS
W3=STATOR IRON LOSS
W4=ROTOR ZIG-ZAG LOSS
W5=END LOSS
W6=STATOR SURFACE LOSS
W7=ROTOR SURFACE LOSS
W8=SKEW LEAKGE LOSS
W9=BELT LEAKGE LOSS
W10=ROTOR IRON LOSS
W11=FRICTION AND WINDAGE LOSS
W1M=STATOR CU. LOSSES FOR MIN. TOTAL LOSSES
W2M=ROTOR CU. LOSSES FOR MIN. TOTAL LOSSES
W3M=STATOR IRON LOSSES FOR MIN. TOTAL LOSSES
W4M=ROTOR ZIG-ZAG LOSSES FOR MIN. TOTAL LOSSES
W5M=END LOSSES FOR MIN. TOTAL LOSSES
W6M=STATOR SURFACE LOSSES FOR MIN. TOTAL LOSSES
W7M=ROTOR SURFACE LOSSES FOR MIN. TOTAL LOSSES
W8M=SKEW LEAKGE LOSSES FOR MIN. TOTAL LOSSES
W9M=BELT LEAKGE LOSSES FOR MIN. TOTAL LOSSES
W10M=ROTOR IRON LOSSES FOR MIN. TOTAL LOSSES
W11M=FRICTION AND WINDAGE LOSSES FOR MIN. TOTAL LOSSES
WF=WINDING FACTOR
WO=NO LOAD ROTOR SURFACE LOSS
WS=STRAY-LOAD LOSSES
WSR=WIDTH OF ROTOR SLOT
WT=WIDTH OF STATOR TEETH
WTR=WIDTH OF ROTOR TEETH AT 1/3 POSITION
X1=STATOR REACTANCE PER PHASE AT NORMAL FREQUENCY
X2=ROTOR REACTANCE REFERRED TO STATOR PER PHASE AT NORMAL FREQUENCY
X3=ROTOR SLOT REACTANCE REF. TO STATOR/PH AT NOR. FREQ.
X11=INTERMEDIATE VARIABLE

X22=INTERMEDIATE VARIABLE
X1H=HARMONIC STATOR REACTANCE
X2H=HARMONIC ROTOR REACT. REFERRED TO STATOR
X2S=ROTOR LEAKAGE REACTANCE AT SLIP FREQUENCY
X1M=STATOR LEAKGE REACTANCE FOR MIN. TOTAL LOSSES
X2M=ROTOR LEAKGE REACTANCE FOR MIN. TOTAL LOSSES
X1W=INTERMEDIATE VARIABLE
X2W=INTERMEDIATE VARIABLE
X3W=INTERMEDIATE VARIABLE
XE=INTER MEDIATE VARIABLE
XM=MAGNETISING REACTANCE PER PHASE AT NORMAL FREQUENCY
XMH=HARMONIC MAGNETISING REACTANCE
XMM=MAGNETISING REACTANCE FOR MIN. TOTAL LOSSES
XMW=INTERMEDIATE VARIABLE
XSH=INTERMEDIATE VARIABLE
Y1 =INTERMEDIATE VARIABLE
Y2 =INTERMEDIATE VARIABLE
Y3=INTER MEDIATE VARIABLE
Y4=INTER MEDIATE VARIABLE
Z=NO. OF CONDUCTORS PER STATOR SLOT

APPENDIX (6)

CALCULATION OF THE SATURATION FACTOR

It is necessary to determine the air-gap ampere-turns and total ampere-turns for calculation of the saturation factor. The various steps in the process are outlined below:

(i) Compute Carter's factors for stator and rotor, respectively denoted by K_1 and K_2, from the expressions below :

$$K_1 = \frac{y_{s_1}\,(5g + w_{10})}{y_{s_1}\,(5g + w_{10}) - w_{10}^2} \qquad \ldots \qquad (I)$$

$$\text{and } K_2 = \frac{y_{s_2}\,(5g + w_{20})}{y_{s_2}\,(5g + w_{20}) - w_{20}^2} \qquad \ldots \qquad (II)$$

Where y_{s_1} and y_{s_2} are stator and rotor slot pitches respectively, g is the air-gap length and w_{10} and w_{20} are the stator and rotor openings respectively.

(ii) Compute effective air-gap length from the expression:

$$g_e = g\,K_1\,K_2 \;\ldots\ldots \text{ in cm}\ldots. \qquad \ldots \qquad (III)$$

(iii) Compute average air-gap flux density and from that its value at 30° from the pole centre. The air-gap ampere-turns are calculated from

$$AT_g = 8000 \quad B_{30}\; g_e \qquad \ldots \qquad \ldots \qquad (IV)$$

(iv) Evaluate flux densities in different iron parts of the magnetic circuit. Then the stator and rotor teeth flux densities are calculated at 30° from the pole

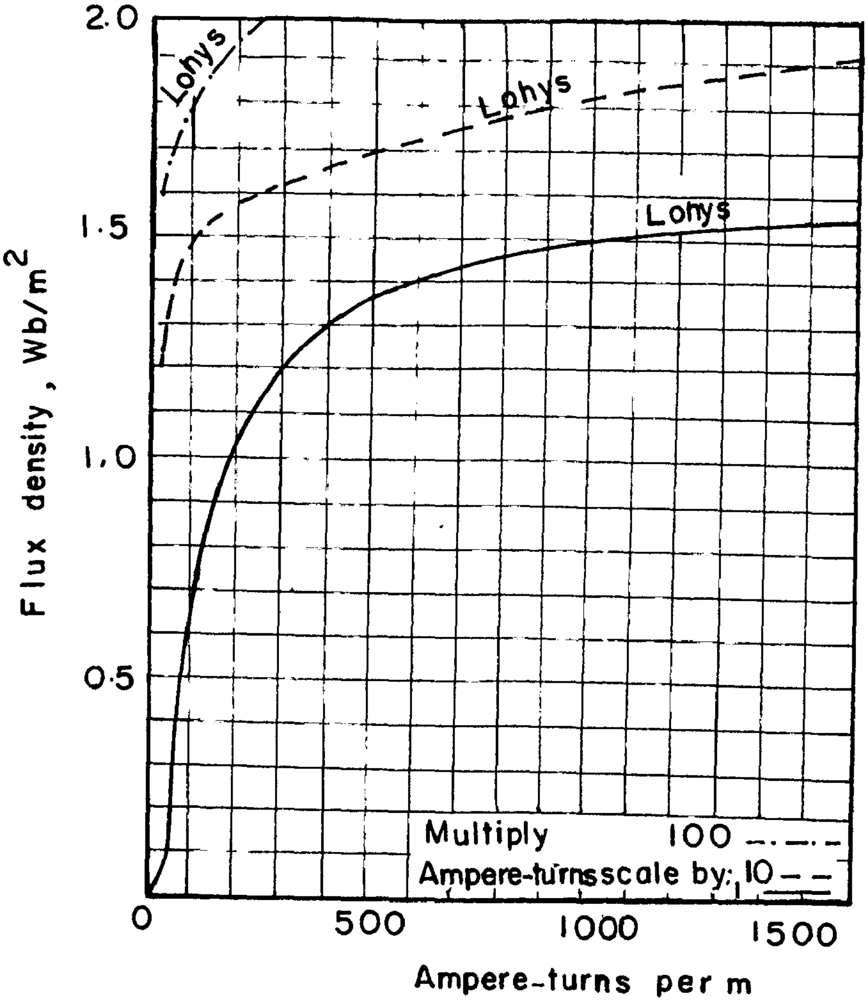

Fig.I- Magnetization cure for Lohys

centre. The ampere-turns per meter for iron parts are taken from magnetization curve of Fig. I and multiplied by length of the path to obtain total ampere turns.

(v) Saturation factor is determined from

$$K_1 = \frac{\text{Total } A_T}{\text{Air-gap } A_T} \quad \ldots \quad \ldots \quad (V)$$

as given in Chapter I.

The magnetization curve shown in Fig.I is for Lohys[38] and for feeding the data of this curve to a digital computer, the curve is approximated by following straight lines[1].

(a) $x = 100\,(y - 0.1) + 60$ for flux densities between 0.1 to 0.8 wb/m^2

(b) $x = 280\,(y - 0.8) + 130$ for flux densities between 0.8 to 1.05 wb/m^2

(c) $x = 666.67(y-1.05) + 200$ for flux densities between 1.05 to 1.2 wb/m^2

(d) $x = 1000(y - 1.2) + 300$ for flux densities between 1.2 to 1.3 wb/m^2

(e) $x = 2000(y - 1.3) + 400$ for flux densities between 1.3 to 1.4 wb/m^2

(f) $x = 3333.34(y-1.4) + 600$ for flux densities between 1.4 to 1.46 wb/m^2

(g) $x = 5000(y-1.46) + 800$ for flux densities between 1.46 to 1.5 wb/m^2

(h) $x = 12000\ (y-1.5) + 1000$ for flux densities between 1.5 to 1.55 wb/m^2.

(i) $x = 20000\ (y-1.55) + 1600$ for flux densities between 1.55 to 1.67 wb/m^2

(j) $x = 38461.5(y-1.67) + 4000$ for flux densities between 1.67 to 1.8 wb/m^2

(k) $x = 60000\ (y-1.8) + 9000$ for flux densities above 1.8 wb/m^2

APPENDIX (7)

The data given below pertains to a 5-h.p., 400 - Volts, 3-phase, 50-Hz, 4-pole, delta connected squirrel-cage induction motor taken as an example[38].

Table No.I

Input Data for Computer Program for Calculation of Equivalent-Circuit Parameters and Loss-Coefficients.

Sl. No.	Input Quantity	Symbol	Value
1.	Rated voltage	V 1	400.0
2.	Rated frequency	F 1	50.0
3.	Winding factor	W F	0.96
4.	No. of pole-pairs	P	2.0
5.	Core-length	C L	9.0
6.	Air-gap diameter	D A	15.0
7.	No. of stator slots	S S	36.0
8.	No. of rotor slots	R S	30.0
9.	Air-gap length	G A	0.045
10.	Stator slot opening	S S O	0.3
11.	No. of ducts	D U	0.0
12.	Rotor slot opening	R S O	0.1
13.	Depth of stator slot	D S S	2.4
14.	Depth of rotor slot	D S R	1.05
15.	Depth of stator core	D C	2.1
16.	Depth of rotor core	D C R	2.96
17.	Width of slot in rotor	W S R	0.65

Contd.....

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Sl. No.	Input Quantity	Symbol	Value
18.	Width of stator teeth	W T	0.60
19.	Area of stator conductor	A S	0.0117
20.	Pitch factor for stator winding	P W	1.0
21.	No. of conductors/stator slot	z	64.0
22.	Area of rotor bar	A B	0.46
23.	Area of cross-section of end rings.	C E	1.2
24.	Mean diameter of end rings	D R	11.7
25.	Permeance coefficient for stator slots	P 1	1.7
26.	Permeance coefficient for rotor slots	P 2	1.65
27.	End Winding-geometry constant	C 1	0.2672
28.	Pole face constant	C P	0.3

Appendix (8)

The data given below pertains the same motor, name plate details of which are mentioned in Appendix No.7.

Table No. II

Input Data for Determination of Losses at Different Frequencies for Constant Volts/Hz Mode of Working.

Sl. No.	Input Quantity	Symbol	Value
1.	Rated voltage.	V 1	400.0
2.	Normal frequency	F 1	50.0
3.	No. of pole-pairs	P	2.0
4.	No. of stator slots	S S	36.0
5.	No. of rotor slots	R S	30.0
6.	Depth of rotor conductor	H	0.875
7.	Ratio of conductor width to slot width in rotor	R	0.885
8.	No-load pulsation loss constant	C 0	0.03028
9.	Full-load pulsation loss constant	C z	0.00139
10.	Belt-leakage loss constant	C B	0.0855
11.	Full-load torque	T F	24.8
12.	Rated horse power output at normal frequency	H P	5.0
13.	Stator core loss coefficient	S C L C	0.385
14.	End loss coefficient	E L C 0	0.0058
15.	Stator surface loss coefficient	S S L C	0.0557
16.	Rotor surface loss coefficient	R S L C	0.0324
17.	Rotor no-load surface loss coefficient	R S N C	0.789

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Sl. No.	Input Quantity	Symbol	Value
18.	Skew leakage loss coefficient	S K C O	0.0102
19.	Rotor core loss coefficient	R C L C	0.144
20.	Stator per phase leakage reactance at normal frequency	X 1	9.10
21.	Stator per phase resistance	R 1	4.75
22.	Rotor per phase leakage reactance at normal frequency referred to stator	X 2	9.32
23.	Rotor per phase resistance referred to stator	R 2	4.30
24.	Rotor slot reactance at normal frequency referred to stator	X 3	3.19
25.	Rotor bar resistance referred to stator per phase	R 3	2.97
26.	Magnetizing reactance at normal frequency.	X M	248.6

APPENDIX (9)

The specifications of the motor taken as an example for optimization are as follows :

(1)	Operating voltage	400.0	Volts
(2)	No.of phases	3.0	
(3)	Normal frequency	50.0	Hz
(4)	Number of poles	4.0	
(5)	Full load torque output at normal frequency.	5.0	h p
(6)	Full load torque	24.8	Nm.

The detailed input data of this motor used in optimization program is given below in a tabular form.

Table No. III

Input Data for Optimization Program

S.No.	Input Quantity	Symbol	Value
1.	Air-gap diameter	D A	15.0
2.	Core length	C L	Varied between 8.0 to 16.0 in step of 1.0 for different sets.
3.	No. of stator slots	S S	36.0
4.	Stator slot opening	S S O	0.3
5.	No. of rotor slots	R S	30.0
6.	Rotor slot opening	R S O	0.1
7.	Depth of stator core	D C	2.1
8.	Depth of rotor core	D C R	2.96
9.	Air-gap length	G A	0.045

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S.No.	Input Quantity	Symbol	Value
10.	Depth of stator slot	D S S	2.4
11.	Depth of rotor slot	D S R	1.05
12.	Width of stator teeth	W T	0.6
13.	Width of rotor slot	W S R	0.65
14.	Depth of rotor conductor	H	0.875
15.	Ratio of conductor width to slot width in rotor	R	0.885
16.	End ring cross-section	C E	1.2
17.	Mean diameter of end rings	D R	11.7
18.	Area of rotor copper-bar	A B	0.46
19.	Permeance coeff. of stator slot.	P 1	1.7
20.	Permeance Coeff. of rotor slot	P 2	1.65
21.	Stator winding factor	W F	0.96
22.	Stator pitch factor	P W	1.0
23.	Pole face constant for rotor	C P	0.3
24.	Belt-leakage loss constant	C B	0.0855
25.	No-load pulsation loss constant	C O	0.03028
26.	Full-load pulsation loss constant	C z	0.00139
27.	End winding geometry constant	C 1	0.2672
28.	Normal operating voltage	V 1	400.0
29.	Normal operating frequency	F 1	50.0
30.	Number of pole pairs	P	2.0
31.	Rated power output in h.p. at the normal frequency	H P	5.0

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S.No.	Input Quantity	Symbol	Value
32.	Full-load torque output	T F	24.8
33.	No. of stator conductors per slot	z	Varied between 74.0 $\pm_0$ 54.0 in step of 1.0 for each core length.
34.	Area of cross-section of stator conductor	A S	Varied in three steps as 74.0 to 68.0 conductors — 0.01038 67.0 to 61.0 conductors - 0.0117 60 to 54 conductors - 0.01314