ON SOME ASPECTS OF SYSTEM RELIABILITY OPTIMIZED DESIGN

A DISSERTATION

Submitted in partial fulfilment
of the requirements for the award of the Degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(System Engg. & Operations Research)

By
GURBACHAN LAL MADAAN

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE—247672
1978

CERTIFICATE

Certified that the dissertation entitled 'ON SOME ASPECTS OF SYSTEM RELIABILITY OPTIMIZED DESIGN' which is being submitted by Mr. Gurbachan Lal Madaan in partial fulfilment of the requirements for the degree of Master of Engineering in Electrical Engineering (System Engineering and Operation Research) of the University of Roorkee, is a record of the student's own work carried out by him under our supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of over *five* ~~and a half~~ months, from January 1978 to May 1978 for preparing this dissertation for the Master of Engineering Degree, at this university.

R.B. Misra 1/6/78 Lecturer Electrical Engg. Deptt. University of Roorkee, Roorkee	K.B. Misra, Professor Electrical Engg. Deptt. University of Roorkee, Roorkee

ABSTRACT

The present work deals with the optimal design of a system from the reliability point of view. Due to increased complexity and sophistication in the modern systems, the system reliability always tends to decrease. Therefore, some means must be provided for enhancing the system reliability. System reliability can be improved by employing structural redundancy at the subsystem level, and/or by practising planned maintenance and repair schedules. This , however, is constrained by resources which are limited and pose a problem to the design and maintenance engineer. Consequently, the problem of optimal allocation of redundancies and of the optimal number of repair-crews arises. The constraints on the system are the overall cost, weight, availability of the system and, power consumption, etc. An attempt has been made to find the optimal allocation of redundancies and the optimal number of repair crews, for maximizing the system reliability or the system availability.

In chapter I, up-to-date, literature about the aspects of reliability optimization has been surveyed and is reviewed in brief.

The availability models of the maintained systems with different repair facilities are discussed in chapter II. A problem of finding the optimal number of repair-crews is

solved by assuming the necessary data. The problem of finding the optimal maintenance interval is also discussed in brief.

The mathematical modelling of the optimal design of a system having active redundancy is presented in section 2.5. The problem of redundancy allocation to the series and bridge systems has been discussed. The computer programs for these problems are developed in FORTRAN-II and a number of problems have been solved on IBM 1620 and TDC-312 computers.

Chapter III deals with the optimization techniques for maximization of the system reliability subject to the given constraints. The techniques discussed in this chapter are Variational method, Penalty function method, Lagrange's Multiplier Method, and Lawler and Bell's optimization method. The constrained reliability problem is converted into unconstrained problem by the use of penalty function and is solved by the steepest ascent method. Lawler and Bell's method is made use of in solving the constrained redundancy problem by transforming the problem into binary variables. At the end, various methods discussed have been compared so as to help the system designer to apply the appropriate techniques. Future avenues of research are also discussed, in short.

ACKNOWLEDGEMENT

The author wishes to express his profound gratitude and indebtness to Dr. K.B. Misra, Professor and Sri R.B. Misra, Lecturer, both in the Department of Electrical Engineering, University of Roorkee, for their inspiring guidance, encouragement, whole-hearted co-operation and pains-taking efforts at all stages of this work. Their critical suggestions have helped a lot in steering through all the impediments.

Thanks are also due to Dr. J. Sharma, O.I./C, Computer Lab. and the staff of the Computer Centre, S.E.R.C., Roorkee, for providing the necessary facilities in carrying out the work.

At last, but not the least, the author is grateful to Dr. P. Mukhopadhyay, Professor and Head, and Prof. L.M. Ray, Group Leader, System Engg. and Operations Research, Electrical Engineering Department for providing the various facilities in the department.

Gurbachan Lal Madaan

C O N T E N T S

CHAPTER I

INTRODUCTION AND REVIEW OF THE LITERATURE

The reliable performance of a system for a mission under various conditions is of utmost importance in many industrial, military and everyday life situations. Although, the qualitative concepts of reliability are not new, its quantitative aspects have been developed over the past two decades. Such development has resulted from the increasing needs for highly reliable systems and safer, cheaper components.

Reliability is a serious concern to the systems e engineer. In the first place, he must be concerned with the consequences of system failure. Frequent failures or extended periods of down-time may result in a complete lack of system capability. Secondly, there are high maintenance costs. It is reported that in U.S.A., it costs the Army services about two dollars per year to maintain every dollar spent on electronic equipment. A third aspect of reliability problem is safety. This problem is extremely important in the design of aircrafts and systems for manned space flight.

The increased complexity, sophistication and automation in modern systems has made the reliability problem more acute, because as the systems become increasingly complex, the reliability also tends to decrease. Therefore, some means must be employed to increase system reliability. This can be done by the following methods :

1. By reducing the complexity of the system
2. By increasing the reliability of components through a product improvement program
3. By using structural redundancy
4. By practising planned maintenance and repair schedules

Reducing the complexity of the system may yield poor steady state and transient response of the system and reduced accuracy. The product improvement program demands the use of improved package and shielding techniques, derating etc. Although these techniques result in reduced failure rate of the component, but require more time for design and special state of art of production. This makes the cost of part improvement program higher as compared to a redundant component. By employing structural redundancy at the subsystem level, keeping specific system topology, one can provide theoretically unity system reliability. Structural redundancy may involve the use of two or more identical components, so that when one fails, others are available in such a way that the system is able to perform the specified task in the presence of some faults in the components. For example, the aeroplanes usually employ two to four engines in redundancy.

Redundancy may be classified under three broad catagories : Active redundancy, standby redundancy and voting redundancy. In active redundancy, all the redundant paths (units) are continuously energised while the system operates. If the redundant unit does not perform any function and comes into operation only when the primary unit fails, this type of redundancy is called standby redundancy. In the third type of redundancy, three or more units operate in conjunction with a switch which selects the unit with agreeing outputs if they constitute a majority. This type of redundancy is commonly used in computer applications.

When the cost of repair in money as well as in time is less in comparison with the cost of equipment, it is economical to consider system repair. It may be possible that at a time, more than one component fail simultaneously. This requires more than one crew in order to increase the operating time of the equipment. For optimal design, a mathematical model is developed using Markov chains and the optimal repair crew are found out.

Several authors have considered the optimal redundancy allocation problem using various formulations and computational techniques, Moscowitz and Mc Lean |8| considered the problem of maximizing reliability with one constraint i.e. cost using a variational method. Gorden |9| also considered the problem of single constraint. Kettelle |10| provided a computational approach for maximizing reliability subject to a cost constraint. However, Proschan and Bray |11| extended Kettelle's method to include more than one constraint viz. cost, weight, etc. Bellman and Dreyfus |12| sketched a dynamic programming method for maximizing system reliability, given specific constraint values. Fan et al. |13| used the discrete maximum principle for maximizing reliability. Federowiez and Mazumdar |14| formulated the redundancy allocation problem in the form of Geometric programming problem to obtain approximate solutions. Tillman and Littschwager |15| developed a method for maximizing reliability or minimizing cost subject to several constraints by using an Integer programming formulation. Ghare and Taylor |16| maximized the reliability of parallel redundant systems by a Branch and Bound procedure. Lawler and Bell |17| described a simple, easily programmed method for solving discrete optimization problems with monotone objective function and completely arbitrary constraints. Misra |18| applied the method of |17| to optimize system reliability or cost subject to multiple c constrints. Misra |19| later on used least square formulation for maximizing system reliability. Banerjee and Rajamani |20| used the parametric approach to solve reliability problem. Misra and Sharma |21| applied Geometric programming technique to the reliability problem. Sharma and Venkateswaran |22| presented a simpler method with no assurance of obtaining the true optimum. Luis |23| presented a procedure of solving non-linear programming problems which first finds, a pseudo-solution to the problem and then uses direct search in the neighbourhood of pseudo solution to find the optimum point. Nakagawa and Nakashima |24| determined the optimal redundancy allocation by using a more reliable candidate at the stage that has the greatest value of the weighted sensitivity function. Tillman et al |25| applied Hooke and Jeeves pattern search technique in combination with the heuristic approach by Aggarwal |26| to solve the mixed integer nonlinear programming problem in which the system reliability is to be maximized as a function of component reliability level and the number of components used at each stage.

The introduction of maintenance is one of the major options for increasing system effectiveness. Morse |4| considered a 1 - unit system with repair and preventive maintenance (pm) and derived the optimum pm policy maximizing the steady-state availability of the system. Graver |27| and Srinivasan |28| obtained the Laplace-stielt es transform of the time distribution to the first system failure for a 2 -unit standby redundant system. T. Nakagava and S. Osaki |29| considered the two unit standby system with repair and pm. Balagurusamy and Misra |30| used the concepts of minimal cut sets and minimal tie sets to assess the availability and other parameters such as failure frequency, mean down time, etc. of a repairable m-order system comprising units with unequal failure and repair rates.

CHAPTER II

PROBLEM FORMULATION

2.1 NOMENCLATURE : Unless otherwise stated, the following symbols are used in this dissertation report -

k = Number of stages or subsystems in a system

m_j = Number of standby components

n_j = Number of redundant components in the j th stage

r_j = Reliability of the j th type component, $0 < r_j < 1$

q_j = Unreliabioity of the j th type component $0 < q_j < 1$

R_j = Reliability of the j th stage, $0 < R_j < 1$

R_s = System Reliability, $0 < R_s < 1$

Q_s = System unreliability, $0 < Q_s < 1$

a_{ij} = Resources requirement associated with each component of j th stage.

b_i = Total amount of resources available for the i th type of constraint.

s = Number of constraints on the system

t = Mission time

β_j = Failure rate of the j th type component

α_j = Repair rate of the j th type component

r = Number of repairmen available

R_o = Minimum reliability of each stage

$p_i(t)$ = Probability of being in i th stage

A^{ss} = System inherent availability (steady state), $0 \leq A^{ss} \leq 1$

2.2 SYSTEM MODELS

Any system can be classified in the following categories -

1. N-stage series system (1 - out of N:F), showin in Fig.1.

The functional operation depends upon the proper operation of all system components. Such systems are also referred to as chain models or weakest link models, since the system fails as soon as the weakest component fails.

$$R_s(t) = \prod_{i=1}^{N} R_i(t) \tag{2.1}$$

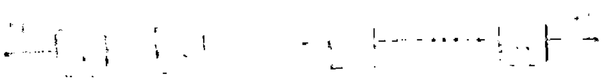
2. M-stage parallel system (1 out of M:G), shown in Fig.2.

These are M paths connecting the input to the output and all components must fail for the system to fail. Such systems are also known as rope models, since the system fails when all the components fail and its behaviour is thus akin to that of a rope, which breaks when all the fibers break.

$$R_s(t) = 1 - \prod_{i=1}^{M} \left[1-R_i(t) \right] \tag{2.2}$$

3. Mixed series parallel system shown in Fig. 3. N components are connected in series and M such series connections are connected in parallel to form the system. The reliability of this type of system can be found by decomposing into series and parallel subsystems.

4. Mixed parallel-series system, shown in Fig. 4, N stages are connected in series, and components are connected in parallel at each stage. The reliability of this type of



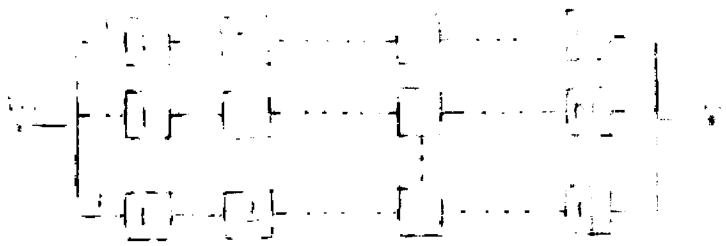
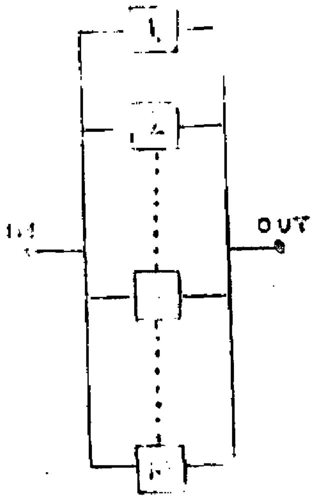


FIG.3 :



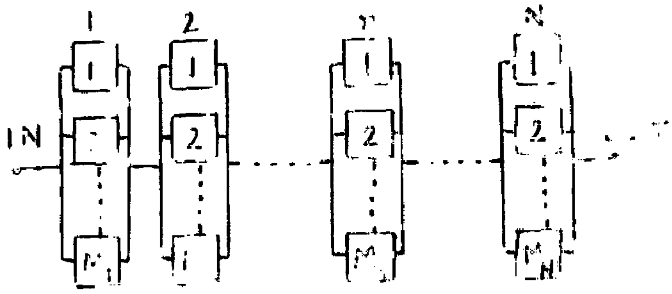
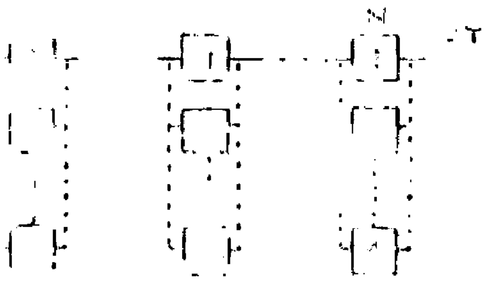


FIG.4 :



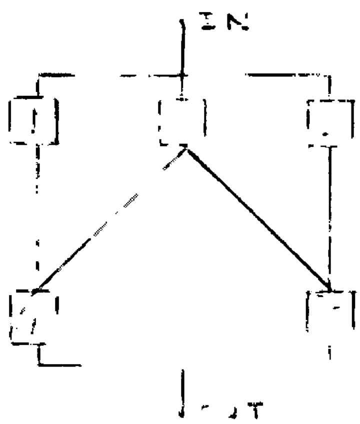
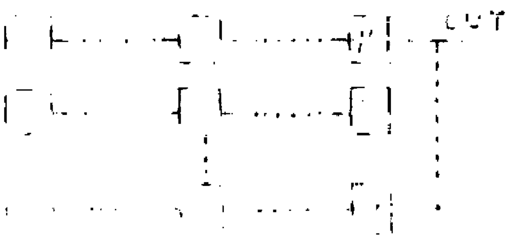
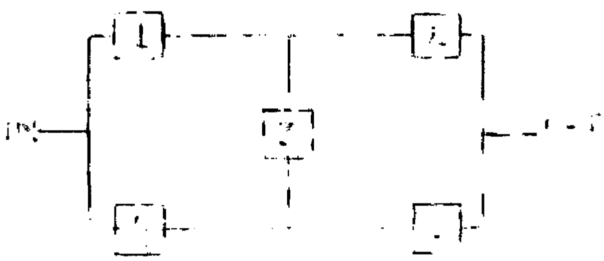


Fig.7:





system can be found out by decomposing into series and parallel subsystems.

5. Element standby system, shown in Fig.5. It has the same form as a mixed parallel series system. However, the parallel components are not all active at the same time.

6. Standby system, shown in Fig.6. It has the same form as a mixed series-parallel system. However, the parallel M series subsystems are not all active at the same time.

7. Non-series-parallel system, shown in Fig. 7. The reliability can be evaluated by using conditional probabilities or other approaches.

8. Complex bridge network system, shown in Fig.8. It is one of the complex reliability systems in the form of the bridge network:-

The reliability of such a system can be found by different methods viz. star delta transformation, factoring theorem method and by the method of inspection. By applying the method of inspection, the reliability is found below :-

Forward Paths : 1-2, 4-5, 1-3-5, 2-3-4.

Paths with one loop : 1-2-3-4, 1-2-4-5, 2-3-4-5. 1-2-3-5, 1-3-4-5.

Paths with two loops : 1-2-3-4-5 (two paths)

$$R_s = r_1r_2+r_4r_5+r_1r_3r_5+r_2r_3r_4-r_1r_2r_3r_4-r_1r_2r_4r_5-r_2r_3r_4r_5-r_1r_2r_3r_5-r_1r_3r_4r_5+2r_1r_2r_3r_4r_5 \quad (2.3)$$

$$R_s = r_1 r_2 + r_4 r_5 + r_1 r_3 r_5 + r_2 r_3 r_4 - r_1 r_2 (r_3 r_4 + r_4 r_5) - r_2 r_3 (r_4 r_5 + r_1 r_5) - r_3 r_4 (r_1 r_5) + 2 r_3 (r_1 r_2)(r_4 r_5) \tag{2.4}$$

2.3 SYSTEM MAINTAINABILITY PROBLEM :

In this topic, the problem of developing mathematical models for the reliability of systems that can be maintained while in use, is considered. A Markovian approach is employed for describing the stochastic behaviour under a variety of failure ans repair conditions.

Since failed equipments are restored to operation in a finite time, the figure-of-merit of the system's reliability is called availability. It is employed to determine the probability that the system is in an acceptable state at any time t, given that it was fully operating at t = 0.

Availability is classified into three catagories -

1. Point availability :- It is defined to be the probability that the system is in an up state (i.e. either operating or operable) at a specified time.
2. Internal availability :- It is the expected fractional amount of an interval of specified length that the system is in an up state.
3. Inherent or Steady State Availability :- It is defined to be the expected fractional amount of time in a continuum of operating time that the system is in an up-state.

Inherent availability is commonly referred to as the uptime ratio or limiting availability.

Systems in which allowable repair could be advantageous include simple parallel and standby systems. Series systems with repair offer no increase in reliability since as soon as a component fails, the system has failed, however, if the objective is to keep the system operating as much as possible during a specified period of time, then repair would be a valuable aid in meeting this objective.

2.3.1 Availability Model of Single Equipment Systems :-

This model can be designated by two states : State 0, the system is operating and state 1, the system is failed and under repair. Since the conditional probability of failure in t, t+dt is βdt, and the conditional probability of completing a repair in t, t+dt is αdt, the following transition matrix can be made

(states at t+dt)

$$P = \begin{array}{c} \text{States} \\ \text{at} \\ t \end{array} \begin{array}{c} 0 \\ \\ 1 \end{array} \begin{bmatrix} 1-\beta & \beta \\ & \\ \alpha & 1-\beta \end{bmatrix}$$

The probability of being in state 0 at time t+dt is

$$P_0(t+dt) = P_0(t)(1-\beta dt)+P_1(t)\alpha dt \tag{2.5}$$

The probability of being in state 1 at time t+dt is

$$P_1(t+dt) = P_0(t)\beta dt+P_1(t)(1-\alpha dt) \tag{2.6}$$

From equations (2.5) and (2.6)

$$P_0'(t) = -\beta P_0(t) + \alpha P_1(t) \tag{2.7}$$

$$P_1'(t) = \beta P_0(t) - \alpha P_1(t) \tag{2.8}$$

For the steady state behaviour

$$P_i = \lim_{t \to \infty} P_i(t)$$

This means that the steady-state behaviour can be found by setting the derivatives $P_i'(t)$ equal to zero.

From (2.7) and (2.8)

$$0 = -\beta P_0 + \alpha P_1 \tag{2.9}$$

$$0 = \beta P_0 - \alpha P_1 \tag{2.10}$$

Also $$1 = P_0 + P_1 \quad \left[\because \sum_{i=0}^{n} P_i = 1 \right] \tag{2.11}$$

Solving (2.9), (2.10) and (2.11), the solutions are

$$P_0 = \frac{\alpha}{\beta+\alpha} \tag{2.12}$$

$$P_1 = \frac{\beta}{\beta+\alpha} \tag{2.13}$$

2.3.2 Availability model of (k,n) Systems with parallel repairs :-

Having described the single equipment model, the generalized availability of model of system with (k,n) components, in which at least k components should operate for

for the operation of system, is given:

The following assumptions are used in developing the model:

1. Component failures are statistically independent.
2. The components have only two states, either operating or non operating.
3. Failure and repair times have exponential distribution. Failure detection and switching devices are perfect.
4. Multiple repair facility exists.
5. Repairs begin immediately on first-come first-served basis.
6. The probability of more than one failure in the interval t, t+dt is of order O(dt).

If at any time t, the system is in state i, then the probability that during (t,t+dt) the transition $i \to i+1$, for $0 \leq i \leq n-k$, occurs equals $\beta_i dt + O(dt)$ and the probability of $i \to i-1$, for $= \leq i \leq n-k+1$, equals $\alpha_i dt + O(dt)$. Since the total system failure occurs when the system is in state n-k+1, the only transitions of interest are $i \to i+1$ for $i = 0,1,\ldots,$ n-k-1, and $i \to i-1$ for $i = 1,2,\ldots\ldots,$ n-m.

$$\therefore P_r\left[i \to i+1 \text{ in } (t,t+dt)\right] = \binom{n-i}{1}\beta dt(1-\beta dt)^{n-i-1} + O(dt) \quad (2.14)$$

$$= (n-i)\beta dt + O(dt) \quad (2.15)$$

Since $(1-\beta dt)^{n-i-1} = 1-(n-i-1)\beta dt + O(dt)$,

$$\therefore \beta_i = \begin{cases} (n-i)\beta & \text{if } 0 \leq i \leq n-k \\ 0 & \text{otherwise} \end{cases} \tag{2.16}$$

Similarly,

$$\alpha_i = \begin{cases} i\alpha & \text{if } 1 \leq i \leq r \\ r\alpha & \text{if } r+1 \leq i \leq n-k+1 \\ 0 & \text{otherwise} \end{cases} \tag{2.17}$$

Then, the transition probability,

$$p_i(t+dt) = p_i(t)(1-\beta_i dt)(1-\alpha_i dt)+p_{i-1}(t)\beta_{i-1}dt(1-\alpha_{i-1}dt)$$
$$+ p_{i+1}(t)\alpha_{i+1}dt(1-\beta_{i+1}dt) + 0(dt) \tag{2.18}$$
$$= p_i(t)(1- (\beta_i+\alpha_i)dt) + p_{i-1}(t)\beta_{i-1}dt +$$
$$+ p_{i+1}(t)\beta_{i+1}dt \tag{2.19}$$

or

$$\frac{p_i(t+dt)-p_i(t)}{dt} = -(\beta_i+\alpha_i)p_i(t) + \beta_{i-1}p_{i-1}(t) +$$
$$+ \alpha_{i+1}p_{i+1}(t) \tag{2.20}$$

letting $dt \to 0$

$$p_i'(t) = -(\beta_i+\alpha_i)p_i(t) + \beta_{i-1}p_{i-1}(t) + \alpha_{i+1}p_{i+1}(t) \tag{2.21}$$
$$\text{for } 0 \leq i \leq n-k+1$$

Using equation (2.17), we obtain

$$p_0'(t) = - n\beta p_0(t) + \alpha p_1(t) \tag{2.22}$$

$$p_i'(t) = -(\,(n-i)\beta + i\alpha)\,p_i(t) + (n-i+1)\beta\, p_{i-1}(t) + (i+1)\alpha\, p_{i+1}(t) \qquad (2.23)$$

for $1 \leq i \leq r$

$$p_i'(t) = -(\,(n-i)\beta + r\alpha)\,p_i(t) + (n-i+1)\beta p_{i-1}(t) + r\alpha\, p_{i+1}(t) \qquad (2.24)$$

for $r \leq i \leq n-k$

$$p_{n-k+1}' = -r\alpha\, p_{n-k+1}(t) + k\beta\, p_{n-k}(t) \qquad (2.25)$$

The steady state solution is found by defining

$$p_i = \lim_{t \to +\infty} p_i(t) \quad \text{for } i = 0,1,\ldots,n-k+1 \qquad (2.26)$$

Taking the limit of both sides of (2.21) as $t \to +\infty$, we obtain,

$$0 = -(\beta_i+\alpha_i)p_i + \beta_{i-1}p_{i-1} + \alpha_{i+1}p_{i+1} \qquad (2.27)$$

for $i = 0,1,\ldots,n-k+1$.

Solving equations (2.22) to (2.25),

$$0 = -n\beta p_0 + \alpha p_1 \qquad (2.28)$$

$$0 = -\left[\,(n-1)\,\beta + \alpha\,\right] p_1 + n\beta p_0 + 2\alpha p_2 \qquad (2.29)$$

$$0 = -\left[\,(n-2)\beta + 2\alpha\,\right] p_2 + (n-1)\,\beta p_1 + 3\alpha p_3 \qquad (2.30)$$

From (2.28), (2.29) and (2.30)

$$p_1 = \frac{n\beta}{\alpha}\, p_0$$

$$p_2 = \frac{1}{2\alpha}\left[\left[(n-1)\beta + \alpha \right] \frac{n\beta}{\alpha} p_0 - n\beta \; p_0\right]$$

$$= \frac{1}{2\alpha} \left[\frac{(n-1)\beta}{\alpha} \right] n\beta \; p_0$$

$$= \frac{n(n-1)}{2} \left(\frac{\beta}{\alpha} \right)^2 p_0$$

$$p_3 = \frac{1}{3\alpha} \left[\left\{ (n-2)\beta + 2\alpha \right\} \frac{n(n-1)}{2} \left(\frac{\beta}{\alpha} \right)^2 - (n-1)\beta \frac{n\beta}{\alpha} \right] p_0 .$$

$$= \frac{1}{3\alpha} \left| \frac{n(n-2)\beta \; (n-1)}{2} \left(\frac{\beta}{\alpha} \right)^2 \right| p_0$$

$$= \frac{n(n-1)(n-2)}{!3!} \left(\frac{\beta}{\alpha} \right)^3 p_0$$

$$= \frac{n!}{3! \; (n-3)!}$$

In general,

$$p_i = \frac{n!}{i! \; (n-i)!} \left(\frac{\beta}{\alpha} \right)^i \; p_0$$

$$= \binom{n}{i} \left(\frac{\beta}{\alpha} \right)^i p_0 \tag{2.31}$$

For $r \leq i \leq (n-k)$

$$p_{r-1} = \binom{n}{r-1} \left(\frac{\beta}{\alpha} \right)^{r-1} p_0 \tag{2.32}$$

$$p_r = \binom{n}{r} \left(\frac{\beta}{\alpha} \right)^r p_0 \tag{2.33}$$

$$0 = p_r' = -\left\{ (n-r)\beta + r\alpha \right\} p_r + r\alpha p_{r+1} + (n-r+1)\beta \; p_{r-1} \tag{2.34}$$

$$0 = p'_{r+1} = -\left[(n-r-1)\beta + r\alpha\right] p_{r+1} + r\alpha p_{r+2} + (n-r)\beta p_r \tag{2.35}$$

From equation (2.34)

$$p_{r+1} = \frac{1}{r\alpha}\left[\left\{(n-r)\beta + r\alpha\right\}\binom{n}{r}\left(\frac{\beta}{\alpha}\right)^r - (n-r+1)\beta\binom{n}{r-1}\left(\frac{\beta}{\alpha}\right)^{r-1}\right]p_0$$

$$= \frac{n(n-1)\ldots\ldots\ldots(n-r)(n-r+1)}{r!\ .\ \alpha^r\ (r\alpha)^1}\ \beta^{r+1}\ p_0$$

Similarly solving equation (2.35)

$$p_{r+2} = \frac{n(n-1)\ldots\ldots\ldots(n-r+1)\ \beta^{r+2}}{r!\ \ \alpha^r\ (r\alpha)^2}\ p_0$$

In general,

$$p_i = \frac{n(n-1)\ldots\ldots\ldots(n-i+1)}{r!\ \alpha^r\ (r\alpha)^{i-r}}\ \beta^i\ p_0 \tag{2.36}$$

Since $\sum\limits_{j=1}^{n-k+1} p_i = 1$, we have

$$p_0 = \left[1 + \sum_{j=1}^{r}\binom{n}{j}\left(\frac{\beta}{\alpha}\right)^j + \sum_{j=r+1}^{n-k+1}\frac{n(n-1)\ldots(n-j+1)}{r!\ \alpha^2\ (r\alpha)^{j-r}}\ \beta^i\right]^{-1} \tag{2.37}$$

For parallel redundant system

$k = 1$

and for series redundant system,

$k = n$

Equations for these systems can be had from equations (2.31),(2.36) and (2.37)

2.4 OPTIMAL MAINTENANCE POLICIES

Failure of a system during actual operation is sometimes costly and/or dangerous. Therefore, it is important to maintain the operating system preventively before failure (e.g. inspection, overhaul or repair if needed).

In this topic an attempt is made to find or characterize optimum maintenance policies: that is to seek the number of a specific class of maintenance policies that minizes total cost, maximizes availability, or in general attains the best value of the prescribed objective function.

Barlow and Proschan |1| considered the maintenance policies by governing the scheduling of replacement of equipment so as to forestall failure during operation. They aimed at achieving maximum operational readiness by inspection policies through the model where failure is known only through checking. Further they considered more complicated formulations in which decisions concerning replacement, repair and inspection are made at each successive step. The Markovian model is considered, and the decision depends only on the information concerning the present state of the system and not on its past history.

In all the replacement models to be considered, a cost c_1 is suffered for each failed item which is replaced ; this includes all costs resulting from failure and its replacement. It is assumed that failures are instantly detected and replaced. A cost $c_2 < c_1$ is suffered for each nonfailed item which is exchanged. Let $N_1(t)$ denote the number of failures during $[0,t]$ and $N_2(t)$ denote the number of exchanges of nonfailed items during $[0,t]$, we may express theexpected cost during $[0,t]$ as

$$C(t) = c_1 E|N_1(t)| + c_2 E|N_2(t)|$$

If we interpret c_1 as the mean time to replace a failed component and c_2 as the mean time to replace a non-failed component. Then $C(t)$ becomes the expected down time in $[0,t]$. The replacement policy minimizing $C(t)$ will then maximize limiting availability.

2.4.1 OPTIMAL MAINTENANCE INTERVAL

Optimum maintenance interval can minimise total cost of maintenance and repair |3|. The reliability can be set at the level acceptable to a particular form of operation by adjusting the maintenance interval. If no hazard is involved, and failure can be tolerated, then the corrective maintenance interval is that which produces the minimum operational cost.

If maintenance labour costs are of paramount importance, then the possibility of achieving 100 percent reliability by means of standby redundancy may become an attractive - financial proposition. This is equally true where the cost of having plant out of commission is very high, and a small amount of down-time would cost more than the time cost of

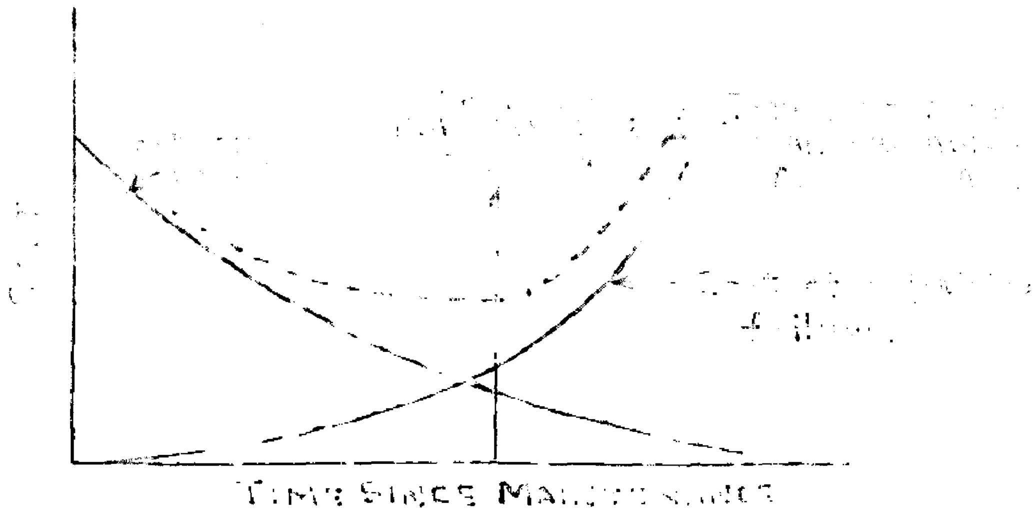
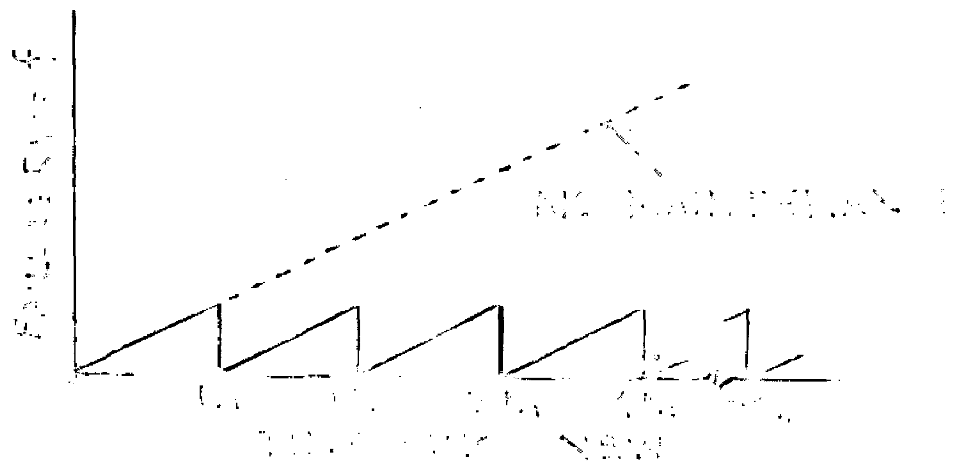


FIG. 9 : OPTIMUM MAINTENANCE [illegible]
CAN MINIMIZE TOTAL COST [illegible]
AND REPAIR



[illegible] EFFECT OF [illegible]
[illegible] LINEAR FAILURE [illegible]

either keeping passive redundant standby equipment idle for much of the time or running active redundant equipment at part load for much of the time.

<u>Theoretical relationships between R and M</u> :

Failure rate $f_1 = F(t)$

where t = time since new

Failure rate $f_2 = F(t-nt_m)$

where t_m = interval during maintenance

n = No. of maintenance operations carried out.

This assumes that maintenance is carried out which restors the equipment to the as new condition.

Reliability = R = 1- unreliability, and for equipment being maintained at intervals :-

$$R = 1 - \frac{1}{N} \int_0^T f_2 \, dt$$

$$= 1 - \frac{1}{N} \int_0^T F(t-nt_m) \, dt$$

where T = total time of separation of each unit.

and N = No. of units.

Consider now a linear relationship between failure rate and time since last overhaul :

$$F(t) = kt$$

$$F(t-nt_m) = k(t-nt_m) \quad \text{Fig. 7}$$

$$\text{Reliabioity} = 1 - \frac{1}{N} \int_0^T k(t-n\ t_m)dt$$

where $T = nt_m$

$$= 1 - \frac{k}{2N} n\ t_m^2$$

which shows that as maintenance interval t_m tends to 0, reliability tends to 1 andas t_m tends to ∞, reliability tends to 0. Similarly, expressions can be derived for other relationships between failure rate and time.

2.4.2 OPTIMAL REPAIR CREW PROBLEM

In many situations, it is not always economically feasible or desirable to repair equipments as they fail. Rather, one may decide to wait until m out of n equipments have reached a failed state. The optimum policy frequently depends upon the cost of the maintenance policy determatives and their effects on system reliability. For example, in the case of a two equipment redundant system, the following pollicies can be considered :

1. Have two repair crews and repair each equipment as it fails.
2. Have one repair crew and repair each equipment as it fails.
3. Have one repair crew and repair the equipments when the system fails.

Associated with each policy is a different availability and maintenance cost. These will be the cost of the repair crews and the cost of replacement for failed equipments. The repair crew cost of policy 1 will be about twice as high as the other alternatives. The replacement cost of policy 3 will be less than the other alternatives since repairs will be made less frequently. The availability of each of the alternate policies |28| is

Policy	System Availability
1	$\frac{\alpha^2 + 2\beta\alpha}{\alpha^2 + 2\beta\alpha + \beta^2}$
2	$\frac{\alpha^2 + 2\beta\alpha}{\alpha^2 + 2\beta\alpha + 2\beta^2}$
3	$\frac{3\alpha^2 + 2\beta\alpha}{3\alpha^2 + 3\beta\alpha + \beta^2}$

Suppose β = .005 per hour and α = 1.0 per hour. Then under each policy respectively, cumulative down-time in a 10,000 hour period will be .248 hour, 0.496 hour and 16.667 hour respectively. If the penalty cost per unit down-time is Rs. 10,000, the cost of a single repair crew is Rs. 5 per unit time and the repair action is to replace a failed equipment with a new one costing Rs.2,000 discarding the failed one.

The cost of single repair crew for 10,000 hour period will be Rs.50,000.

For policy 1, we would expect to be replacing an equipment every 100 hours, since each equipment has an MTBF of 200 hours. Therefore, over a 10,000 hour period, we would expect to make 100 replacements, at a cost of Rs.2,000,00 . For policy 2, the results are approximately the same. For the third policy, replacements would be made on an average of every 75 hours. Therefore, in a 10,000 hour period, we shall replace both equipments at a cost of Rs.3,000,00. The expected costs for the various policies are given in the Table 1.

Table 1

Policy	Expected penalty cost(Rs)	Repair crew cost in Rs.	Expected Replacement cost in Rs.	Expected Total cost in Rs.
1	2,480	100,000	200,000	302,480
2	4,960	50,000	200,000	254,960
3	166,667	50,000	300,000	516,667

From the table, we find that policy 2 is the least expensive. Therefore, for this problem, it is advisable to have one repair crew and repair each equipment as it fails.

2.4.2.1 Example

Consider another problem of finding the optimal repair crew policy. Consider three equipment parallel redundant system which can be in states 0,1,2 or 3 as defined in section 2.3.1. Let us assume that there is no loss when the system is in state 0, a loss of Rs.500 per unit time when the system is in state 1, a loss of Rs.2,000 per unit time when the system is in state 2 and a loss of Rs.5,000 per unit time when the system is in state 3. Let us assume the case of independent servicing. Then, the three policies can be considered :

Policy 1 - Assign one repair crew
Policy 2 - Assign two repair crews
Policy 3 - Assign three repair crews.

Let the cost of repair crew be Rs.5 per hour and the failure and repair rates of each equipment be .005 and and 1.0 per hour respectively. Then from section 2.3.1.2, the amount of time, the system spends in each state for the three policies is given in Table 2.2, and the numerical values are calculated in Table 2.3 for a period of 10,000 hours duration. The expected penalty costs and repair costs for three policies are given in Table 2.4. From Table 2.4, we find that the best policy is to employ one repair crew. Therefore, the optimal number of repair crew for this problem is only one.

Table 2.2

Policy	Proportion of time in each state			
	State 0	State 1	State 2	State 3
One repair crew	$\frac{\alpha^3}{\alpha^3+3\beta\alpha^2+6\beta^2\alpha+6\beta^3}$	$\frac{3\beta\alpha^2}{\alpha^3+3\beta\alpha^2+6\beta^2\alpha+6\beta^3}$	$\frac{6\beta^2\alpha}{\alpha^3+3\beta\alpha^2+6\beta^2\alpha+6\beta^3}$	$\frac{6\beta^3}{\alpha^3+3\beta\alpha^2+6\beta^2\alpha+6\beta^3}$
Two repair crews	$\frac{2\alpha^3}{2\alpha^3+6\beta\alpha^2+6\beta^2\alpha+3\beta^3}$	$\frac{6\beta\alpha^2}{2\alpha^3+6\beta\alpha^2+6\beta^2\alpha+2\beta^3}$	$\frac{6\beta^2\alpha}{2\alpha^3+6\beta\alpha^2+6\beta^2\alpha+3\beta^3}$	$\frac{3\beta^3}{4\alpha^3+12\beta\alpha^2+12\beta^2\alpha+6\beta^3}$
Three repair crews	$\frac{\alpha^3}{\alpha^3+3\beta\alpha^2+3\beta^2\alpha+\beta^3}$	$\frac{3\beta\alpha^2}{\alpha^3+3\beta\alpha^2+3\beta^2\alpha+\beta^3}$	$\frac{3\beta^2\alpha}{\alpha^3+3\beta\alpha^2+3\beta^2\alpha+\beta^3}$	$\frac{\beta^3}{\alpha^3+3\beta\alpha^2+3\beta^2\alpha+\beta^3}$

Table 2.3

'olicy	Expected time in 10,000 hours			
	State 0	State 1	State 2	State 3
ıe repair 'ew	9850.76	147.7614	1.4776	0.0059
ro repair 'ews	9851.48	147.7722	0.73881	0.00098
ıree repair 'ews	9851.49	147.7723	0.73886	0.00098

Table 2.4

Policy	Expected Penalty cost in Rs.	Expected Maintenance cost in Rs	Expected Total cost in Rs.
One repair Crew	76,865	50,000	126,865
Two repair Crews	75,448	100,000	175,448
Three repair Crews	75,370	150,000	225,370

2.5 OPTIMAL REDUNDANCY ALLOCATION PROBLEM

Malfunctioning of any constituent part of a system causes loss of money and time due to system interruption. Therefore the system must have high reliability. Since the individual components can not be made 100 percent reliable, the use of redundancies is often restorted to. The use of redundancies, however, is constrained by the resources available to the design engineer.

Several authors (Barlow, Bellman and Dreyfus, Federowicz and Mazumdar, Ghare and Taylor, Jensen, Messinger and Shooman, Misra, Sharma, Mizukami Koichi, Moskowitz and Mc Lean, Myers and Enrick, Tillman, Tillman and Littis-chwager) have considered the above problem using various formulations and computational techniques. The methods available are gradient methods, variational methods, dynamic programming, integer programming branch and bound methods geometric programming, etc. The redundancy allocation is an integer programming problem when the allocations are allowed to take only integer values. The system reliability function is a non-linear function. The constraints can be normally linear or nonlinear.

The variational method and the discrete maximum principle, although being versatile, offer only an approximate solution. Geometric programming also provides an approximate solution after many simplifying assmptions. In most of the approximate methods, the decision variables are treated as being continuous and the final integer solution is obtained by rounding off the real solution to the nearest integers.

2.5.1 SERIES SYSTEM REDUNDANCY ALLOCATION

The various assumptions to be made for the analysis are :-

1. The failure of any subsystem or module results in the system failure.
2. The failures of the subsystems are statistically independent.
3. The failure distribution of the component is exponential with failure rate β_j .

2.5.1.1 Statement of the Problem :

Assuming that there are k subsystems or stages (in series) in a system. Then the system reliability is

$$R_s(n) = \prod_{j=1}^{k} R_j(n_j) \tag{2.38}$$

where,

$R_j(n_j)$ = Reliability of j th stage

n_j = Number of redundant components in j th stage.

Since the use of redundancy is limited by the availability of resources, the optimal design problem can be stated as, Maximize system reliability

$$R_s(n) = \prod_{j=1}^{k} R_j(n_j) \tag{2.39}$$

Subject to the constraints

$$\sum_{j=1}^{k} G_{ij}(n_j) \leq b_i \tag{2.40}$$

$n_j \geq 1$ and integers ,

$i = 1,2,\ldots\ldots,s$

where $G_{ij}(n_j)$ is the i th type resources requirement for j th stage and b_i is the total amount of resources available for the i th type of constraint.

It is nonlinear integer programming problem. For solving this problem , the expression for reliability of the j th stage is required. The reliability at the j th stage can be increased by putting the components in parallel. Let stage j have a set of n_j components connected in parallel, each having the probability of failure as q_j. Then the expression for reliability is

$$R_s(n) = \prod_{j=1}^{k} R_j(n_j) \tag{2.41}$$

where, $R_j(n_j) = 1 - q_j^{\,n_j}$

$$\therefore \quad R_s(n) = \prod_{j=1}^{k} (1-q_j^{\,n_j}) \tag{2.42}$$

A parallel redundant system is shown in Fig. 11.

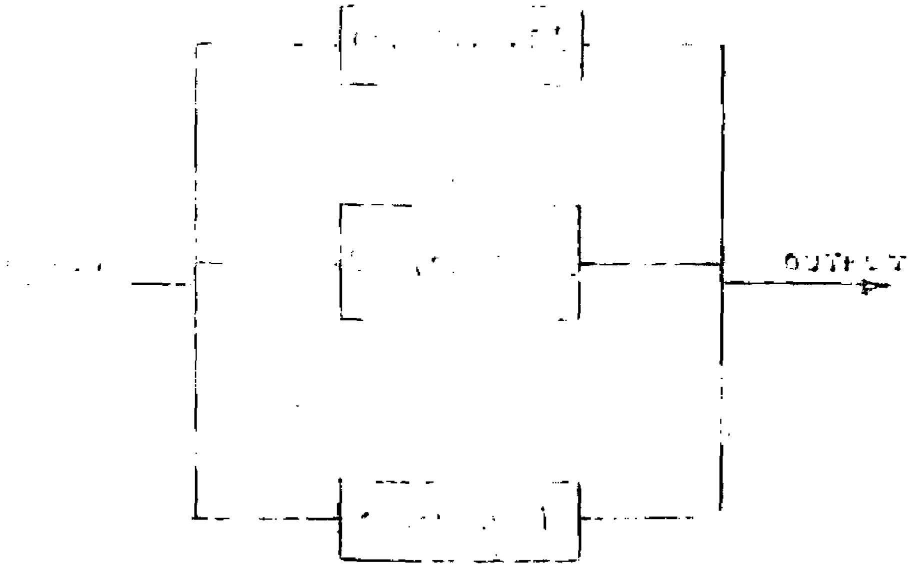


FIG. 11: A PARALLEL REDUNDANT SUBSYSTEM

[illegible]

Fig. 12 shows how the reliability of the system increases with the number of redundant components having exponential failure distribution.

2.5.1.2 <u>General solution of redundancy allocation problem</u> :

Assuming linear constraints on n in equation (2.40), if the constraint is on cost, equation (2.40) becomes

$$\sum_{j=1}^{k} C_{ij}\, n_j \leq C_i \qquad (2.41)$$

$$i = 1,2,\ldots\ldots,s$$

where $C_{ij} > 0$ and each C_i shows the allowable limit of cost, weight or volume, etc up to s constraints. The problem can therefore be stated as : the selection of n such that R(n) is maximum subject to the constraints given in equation (2.41).

An approximate solution of problem (2.39) can be rapidly and easily obtained by generating an incomplete family of undominated allocations.

To describe the concept of undominated redundancy allocation, we say n^o is undominated if $R(n) > R(n^o)$ implies $C_i(n) > C_i(n^o)$ for same i, whereas $R(n) = R(n^o)$ implies either $C_i(n) > C_i(n^o)$ for same i or $C_i(n)\ C_i(n^o)$ for all i,

where $C_i(n) = \sum_{j=1}^{k} C_{ij} n_j$

Taking logarithm on both sides of equation (2.39),

$$\text{Log } R_s(n) = \sum_{j=1}^{k} \text{Log } R_j(n_j) \qquad (2.42)$$

Since Log X is a monotone - increasing function of x, the problem of maximizing $R_s(n)$ is equivalent to maximizing log R(n).

The procedure for generating an incomplete family of undominated allocation can be summarised as follows :

Starting with redundancy allocation of (1,1,...,1), one adds a new component to that stage which yields greatest improvement in system reliability for the cost incurred in placing it. This continues till any one constraint is violated. If $\log R_j(n)$ is concave, each redundancy allocation generated by above procedure is undominated |1|. To prove that $\log (R_n)$ is a concave function of n, one can show that

$$\delta^2 \log R_j(n) = \delta^2 \log (1 - q_j^{\,n})$$

$$= \log \frac{(1-q_j^{\,n+2})(1-q_j^{\,n})}{(1-q_j^{\,n+1})^2} \qquad (2.43)$$

where $\delta \log R_j(n) = \log R_j(n+1) - \log R_j(n)$

The denominator is large than numerator as

$$(1-q_j^{\,n+1})^2 - (1-q_j^{\,n+2})(1-q_j^{\,n}) = q_j^{\,n}(q_j-1)^2$$

$$> 0$$

Therefore $\delta^2 \log R_j(n) < 0$, so also $\log R_s(n)$, as the sum of concave functions is again a concave functionl

$$\text{Hence} \quad \log R_s(n) = \sum_{j=1}^{k} \log R_j(n_j)$$

is concave

2.5.1.3 Example

Single Cost Factor

Assuming that there is only one constraint in (2.42), i.e. cost of the item, the procedure for generating allocations will be to calculate desirability factor F_j for each stage given by

$$F_j = \frac{\delta \log R_j(n_j)}{c_{j1}} = \frac{1}{c_{j1}} \left[\log R_j(n_j+1) - \log R_j(n_j) \right] \tag{2.44}$$

Retaining the index j_0 for which F_{j0} is maximum amongst the stages, a component is added to that stage to find new allocation. If maximum occurs for more than one index, the lowest has been chosen for allocation.

Taking numerical example |10|, in which data runs as,

Stage j	1	2	3	4
Reliability	0.8	0.7	0.75	0.85
cost	1.2	2.3	3.4	4.5

Table 2.5 gives the complete information about the undominated allocations. It may be noted that allocations are given for the system and actual redundancy allocation can be found by substracting (1, 1, 1, 1) from the system allocations.

Table 2.5 - Single Cost Allocation

stem lo- tion	System Reliability	System Cost	Desirability Factors			
			F_1	F_2	F_3	F_4
1 1 1	0.3570	11.4	0.15194	0.11407	0.06563	0.03106
1 1 1	0.4284	12.6	0.02732	0.11407	0.06563	0.03106
2 1 1	0.5569	14.9	0.02732	0.02910	0.06563	0.03106
2 2 1	0.6961	18.3	0.02732	0.02910	0.01435	0.03106
2 2 2	0.8005	22.8	0.02732	0.02910	0.01435	0.00431
3 2 2	0.8560	25.1	0.02732	0.00836	0.01435	0.00431
3 2 2	0.8845	26.3	0.00536	0.00836	0.01435	0.00431
3 3 2	0.9287	29.7	0.00536	0.00836	0.00348	0.00431
4 3 2	0.9468	32.0	0.00536	0.00248	0.00348	0.00431
4 3 2	0.9529	33.2	0.00107	0.00248	0.00348	0.00431
4 3 3	0.9715	37.7	0.00107	0.00248	0.00348	0.00064
4 4 3	0.9831	41.1	0.00107	0.00248	0.00086	0.00064
4 3	0.9887	43.4	0.00107	0.00074	0.00086	0.00064
4 3	0.9900	44.6	0.00021	0.00074	0.00086	0.00064
5 3	0.9929	48.0	0.00021	0.00074	0.00022	0.00064
5 3	0.9946	50.3	0.00021	0.00022	0.00022	0.00064
5 4	0.9974	54.8	0.00021	0.00022	0.00022	0.00010
5 4	0.9979	57.1	0.00021	0.00007	0.00022	0.00010
6 4	0.9987	60.5	0.00021	0.00007	0.00005	0.00010
6 4	0.9989	61.7	0.00004	0.00007	0.00005	0.00010
6 5	0.9994	66.2	-	-	-	-

2.5.1.4 Minimum number of Components at each stage for desired reliability

Consider the problem of redundancy allocation to each stage for getting a reliability of at least R_0 at minimum system cost. It is assumed that the redundant equipments will be in parallel arrangement. It is not difficult to see that the reliability of each stage must be greater than R_0. Therefore, the minimum number of equipments required in each stage can be found by solving

$$R_0 = 1 - (1 - r_j)^{k_j} \tag{2.45}$$

or

$$(1 - r_j)^{k_j} = 1 - R_0 \tag{2.46}$$

Taking logarithm on both sides of equation (2.46)

$$k_j \log(1-r_j) = \log (1-R_0)$$

or

$$k_j = \frac{\log(1-R_0)}{\log(1-r_j)} \tag{2.47}$$

If a linear cost relationship is assumed, where the cost of the j th stage is $c_{j,k} = C_j k_j$, then the minimum relative increase in cost per stage to reach reliability R_0 is

$$\frac{c_{j,k}}{C_j} = \frac{\log (1-R_0)}{\log(1-r_j)} \tag{2.48}$$

Thus the relative increase in cost is proportional to the ratio of logarithms of the desired unreliability and the unreliability with no redundancy.

Moskowitz and Mc Lean |8| have given a variational solution to the allocation problem for linear cost functions. The solutions are

$$k_j = \frac{\log(1-R_0^{a_j})}{\log(1-r_j)} \qquad (2.49)$$

where,

$$a_j = \frac{C_j/\log(1-r_j)}{\sum_{i=1}^{k} \left(\frac{C_i}{\log(1-r_i)} \right)} \qquad (2.50)$$

2.5.1.5 Example

Suppose the required level of system reliability R_0 is 0.99 and the stage reliabilities without redundancy are 0.80, 0.70, 0.80 and 0.70 respectively. Assuming that the cost of a single equipment in each stage is Rs.2000, Rs.3000 and Rs.1000 respectively. The data is given in Table 2.5 and the minimum number of redundant equipments per stage to meet system reliability is calculated in column no.5.

Table 2.6 - Basic Data For Allocation Problem

Stage	Cost per Equipment in Rs.	Equipment Reliability	Equipment Unreliability	Min.no.of redundant equp.per stage ro meet system reliability
1	2000	0.80	0.20	3
2	3000	0.70	0.30	4
3	1500	0.80	0.20	3
4	1000	0.70	0.30	4

2.5.2 Bridge System Redundancy Allocation

Making the same assumptions as in section 2.3.2.1, the optimal redundancy is found by the heuristic algorithm. A computer program of the algorithm is written FORTRAN -II and a number of problems are solved on IBM 1620 computer. The reliability of the system with initial allocation(1,1,1,1,1) is calculated in section 2.2. Equation (2.4) can be implemented for calculating the initial reliability of the system. A component is to be added to the stage where its addition produces the greatest ratio of increase in reliability to the increase in cost. The desirability factors F_i's are calculated for each stage, as

$$F_i = \frac{\delta R_s}{\delta C_s} \tag{2.51}$$

Then a component is added to the stage that gives the maximum of the F_i's. Again, the reliability is computed for the new configuration and the procedure is repeated till the desired constraint on the cost is violated.

From equation (2.4)

$$R_s = r_1r_2+r_4r_5+r_1r_3r_5+r_2r_3r_4-r_1r_2(r_3r_4+r_4r_5)-r_2r_3(r_4r_5+r_1r_5)$$
$$-r_3r_4(r_1r_5)+2r_3(r_1r_2)(r_4r_5)$$

and the system cost

$$C_s = C_1 + C_2 + C_3 + C_4 + C_5$$

Try redundancy at stage 1, and calculate new system reliability expression, R'_s.

Then increase in reliability per unit system reliability

$$= \frac{R_s' - R_s}{R_s} \tag{2.52}$$

Increase in system cost per unit system cost

$$= \frac{C_1}{C_s}$$

$$\therefore \quad F_1 = \frac{R_s' - R_s}{R_s} \cdot \frac{C_s}{C_1}$$

Calculate F_2, F_3, F_4 and F_5. Find maximum of these F_i's. Let it be F_j. Then put one redundant component in stage j and compute the reliability. Again find the best allocation and repeat the procedure till the constraint is violated. Then the preceeding allocation gives the optimal number of components in each stage.

CHAPTER III

OPTIMIZATION TECHNIQUES

The reliability optimization problem is a nonlinear integer programming problem. The methods for solving this problem are classified into two groups, one which includes methods that require simple formulation and yield approximate results and the other which includes methods that ate complicated but yield an exact integer solution to the problem |21|. The procedure to be used for the solution of the reliability problem depends on the accuracy of the results and the cost of obtaining them because the system designer has to solve several alternatives and alteration in the design parameter from other technical considerations. The redundancy allocation problem does not require an exact solution as the objective function of the system reliability is a well behaved nonlinear function and the linear constraints need not be considered ' too tight ' to relax at the design stage.

3.1 VARIATIONAL METHOD

In section 2.5.1, from equation (2.38),

$$R_s = \prod_{j=1}^{k} R_j \tag{3.1}$$

and the basic system cost

$$C_o = \sum_{j=1}^{k} c_j \tag{3.2}$$

The problem is to find redundancy allocation which gives minimum cost for the specified system reliability of R_s. Denoting the number of elements in stage j by m_j the reliability of stage j can be written as

$$R_j = 1 - q_j^{m_j} \tag{3.3}$$

where $q_j = 1-r_j$, r_j is the reliability of each element in j th stage and R_j is the reliability of m_j such elements in parallel.

Introducing another variable a_j defined by

$$R_j = R_s^{a_j} \tag{3.4}$$

It can be shown that a real positive number a_j between 0 and 1, can always be found to satisfy (3.4). Then from (3.3) and (3.4) each m_j can be written as

$$m_j = \frac{\log(1-R_j)}{\log q_j} = \frac{\log(1-R_s^{a_j})}{\log q_j} \tag{3.5}$$

and the system cost and reliability can be given by

$$C_s = \sum_{j=1}^{k} m_j c_j = \sum_{j=1}^{k} \frac{c_j \log(1-R_s^{a_j})}{\log q_j} \tag{3.6}$$

$$R_s = \prod_{j=1}^{k} R_j = \prod_{j=1}^{k} R_s^{a_j} = R_s^{\left[\sum_{j=1}^{k} a_j\right]} \tag{3.7}$$

For (3.7) to be valid,

$$a = \sum_{j=1}^{k} a_j = 1 \tag{3.8}$$

It is possible to optimise cost with reliability. This occurs for distribution of a_j's which gives stationary value for the ratio C_s/R_s. The distribution of a_j's is to be found which satisfies

$$\delta\left(\frac{C_s}{R_s}\right) = 0 \quad \text{or} \quad \frac{\delta C_s}{C_s} - \frac{\delta R_s}{R_s} = 0 \tag{3.9}$$

subject to the constraint that $\delta a = \sum_{j=1}^{k} \delta a_j = 0$ (3.10)

If λ is a real constant then simultaneous solution of (3.6), (3.7) and (3.8) and

$$\frac{\delta C_s}{C_s} - \frac{\delta R_s}{R_s} - \lambda\,\delta a = 0 \tag{3.11}$$

will provide the distribution of a_j's for stationary value of C_s/R_s. Now

$$\delta R_s = R_s(a+\delta a) - R_s(a)$$

$$= R_s\left[\sum_{j=1}^{k}(a_j+\delta a_j)\right] - R_s\left[\sum_{j=1}^{k} a_j\right] = R_s(R_s{}^{[\Sigma\delta a_j]} - 1)$$

since $\delta a = \Sigma\,\delta a_j = 0$

therefore $\delta R_s / R_s = 0$ (3.12)

Similarly the variation of C_s with a is given by,

$$\delta C_s = C_s(a+\delta a) - C_s(a)$$

$$\delta C_s = \sum_{j=1}^{k} \frac{c_j}{\log q_j} \log \left[1 - R_s^{(a_j + \delta a_j)} \right] - \sum_{j=1}^{k} \frac{c_j}{\log q_j} \cdot \log(1 - R_s^{a_j})$$

$$= \sum_{j=1}^{k} c_j' \log \left[\frac{1 - R_s^{(a_j + \delta a_j)}}{1 - R_s^{a_j}} \right] \tag{3.13}$$

where $c_j' = c_j / \log q_j$

If it is assumed that R_s isquite high, i.e. very close to unity and Q_s is very small

$$\delta C_s = \sum_j c_j' \log \left[\frac{1-(1-Q_s)^{a_j + \delta a_j}}{1-(1-q_j)^{a_j}} \right]$$

$$= \sum_j c_j' \log \left[1 + \frac{\delta a_j}{a_j} \right]$$

then to the first approximation

$$\frac{\delta C_s}{C_s} \approx \frac{\sum_j c_j \frac{\delta a_j}{a_j}}{C_s} \tag{3.14}$$

Substituting (3.12) and (3.14) in (3.11) yields

$$\sum_i \frac{c_j'}{C_s a_j} \delta a_j - \lambda \sum_j \delta a_j = 0 \tag{3,15}$$

For (3.15) to hold good $\quad a_j = \dfrac{c'_j}{\lambda C_s}$ (3.16)

Solving for λ , from $\sum_j a_j = 1$

$$\therefore \lambda = \sum_j \frac{c_j'}{C_s} \qquad (3.17)$$

Substituting (3.17) in (3.16),

$$a_j = \frac{c_j'}{c_0'} = \frac{c_j/\log q_j}{\sum_i c_i/\log q_i} \qquad (3.18)$$

Therefore minimum cost can be obtained for the distribution a_j given by (3.18) and substitution of (3.18) in (3.5) yields the values of m_j $(j = 1,\ldots,k)$: the elements in each stage with the total cost as

$$C_s = \sum_{j=1}^{k} m_j c_j = \sum_{j=1}^{k} c_j \frac{\log(1-R_s^{a_j})}{\log q_j}$$

3.1.1 Procedure for calculating optimum allocations

The general procedure for determining the optimum allocation can be outlined as follows :

1. Using the cost and reliability data about each element type a_j's using equation (3.18) are calculated and the calculated values can be checked by finding their sum which should be equal to unity, i.e.

$$\sum_{j=1}^{k} a_j = 1$$

2. For the given system reliability R_s and unreliabilities of each element type one can calculate the values of m_j's the probable number of elements in each stage, using equation (3.5).

3. Usually the values so calculated for m_j's will not be integers and as the m_j's can only have integer values, so the values of m_j's obtained in step 2 are rounded off to the lower integer values.

4. Now as the reliability of the system will fall short of the given system reliability due to truncation of the values of m_j's the further improvement in system reliability can be obtained by adding successively the element types that yield minimum increase in cost for a certain increase in reliability.

5. Therefore the desirability factors F_j's for each stage are calculated as defined by

$$F_j = \frac{\delta R_s / R_s}{c_j / C_s}$$

where, F_j = the desirability factor for adding a unit or element to the j th group ;

R_s, C_s = system reliability and cost before adding the unit to j th group ;

C_j = cost of adding a unit to j th stage.

However it can be shown that $\delta R_s / R_s = \delta R_j / R_j$ (3.20)

Here R_j is the reliability of j th group before the addition of new unit to that stage and δR_j is the increase in reliability of that stage after new unit has been added. Therefore (3.19) can be written as

$$F_j = \frac{\delta R_j / R_j}{c_j / C_s} \tag{3.21}$$

To show (3.20) holds good one can write that

$$R_s = \prod_{j=1}^{k} R_j$$

and the reliability of the system R_s', after a unit to j th stage has been added will be

$$R_s' = \frac{1}{R_j} \left(\sum_{j=1}^{k} R_j \right)(R_j + \delta R_j) = \frac{R_s(1 + \delta R_j)}{R_j}$$

also $\delta R_s = R_s' - R_s$, therefore,

$$\delta R_s = R_s \frac{\delta R_j}{R_j}$$

or $$\frac{\delta R_j}{R_j} = \frac{\delta R_s}{R_s}$$

6. Once all F_j's have been calculated in step 5, a new element is added to the stage j for which the F_j calculated is maximum.

7. New reliability and cost of the system is calculated. If the reliability of the system is now more than or equal to the given reliability, the allocation obtained so far is the optimum value, otherwise the steps 5 and 7 are

repeated till the system reliability is at least equal to R_g or greater than this.

3.2 PENALTY FUNCTION METHOD

In this method, the constrained problem is converted into an unconstrained problem by the use of a penalty function, which is added to the constrained problem.

The reliability problem is

$$-\log R_s(n) = -\sum_{j=1}^{k} \log R_j(n_j) \tag{3.22}$$

subject to the constraints

$$\sum_{j=1}^{k} G_{ij}(n_j) \leq b_j \qquad i = 1,2,\ldots,s \tag{3.23}$$

$$n_j > 0 \text{ and integer}$$

The equivalent unconstrained problem can be written as Minimize

$$F(n, r) = -\log R_s(n) + r_p \sum_{i=1}^{s} \left[b_i - \sum_{j=1}^{k} G_{ij}(n_j) \right]^{-1} \tag{3.24}$$

where r_p is a parameter called as penalty factor. A sequence of positive values of r_p which are strictly decreasing to zero, are used for minimizing (3.24). It results in a sequence of minimum points which converges to the constrained minimum of the $-\log n\, R_s(n)$. If the optimal solution is integral, then problem is solved. Otherwise, a non-integral variable, say n_j, is chosen which has highest fractional part , dn_j. A new constraint is incorporated in in the original problem which can be written as

$$n_j \geq | n_j | + 1 \tag{3.25}$$

where $|n_j|$ is the integral portion of the n_j. The new problem is again solved in the similar way as original unconstrained problem. If new problem converges, n_j is set as $|n_j| + 1$; otherwise, as $|n_j|$. The same procedure is repeated for other variables.

3.2.1 Algorithm of the Method

1. Select an initial value of $r_p > 0$ and an interior point n^o. Set $l = 0$
2. If n^l nearly minimizes $F(n,r_p)$, go to step 6, otherwise calculate direction vectors d_j

$$d_j = \frac{\partial F(n,r_p)}{\partial n_j} \qquad j = 1,2,\ldots,k$$

3. Choose stepsize t_1 that minimizes $F(n^l + t_1 d^l , r_p)$
4. Calculate new trial point

$$n_j^{l+1} = n_j^l + t_1 d_j^l \qquad j = 1,2,\ldots,k$$

5. Set $l = l+1$ and go to step 2.
6. Check convergence. If solution is optimal go to step 7; else replace r_p by str_p, where $0 < st \leq 1$ and go to step 2 with $l = 0$.

7. Choose that variable which has greatest dn_j and add the following constraint in the problem

$$n_j \geq |n_j| + 1$$

8. Repeat step 2 - 5. If problem converges set $n_j = |n_j| + 1$; otherwise, $n_j = |n_j|$ and remove j th stage from calculation.

9. If all variables are tried, stop ; else, go to step 7.

The initial value of r_p should be such that

$$r_p^o = F_c \frac{-\log n \, R_s(n)}{\sum_{i=1}^{s} \left[b_i - \sum_{j=1}^{k} G_{ij}(n_j) \right]^{-1}} \tag{3.26}$$

where F_c is $0.01 < F_c < 1$.

3.3 LAGRANGE'S MULTIPLIER METHOD

Consider a simple n unit series structure. Let the reliability of individual units be $x_1, x_2, \ldots, x_n$ and that of the structure be x_s.

Then unreliability of components will be $y_1, y_2, \ldots, y_n$ where,

$$y_1 = 1 - x_1$$

$$y_2 = 1 - x_2$$

$$\vdots$$

$$y_n = 1 - x_n$$

Define parameter ϕ_i as

$$\phi_i = \frac{y_i}{x_i} = \frac{1-x_i}{x_i} \tag{3.27}$$

Then

$$x_i = \frac{1}{\phi_i + 1} \tag{3.28}$$

For the given series system

$$x_s = x_1 . x_2 , \ldots \ldots , x_n \tag{3.29}$$

or $$(1+\phi_s) = (1+\phi_1)(1+\phi_2) \ldots \ldots (1+\phi_n) \tag{3.30}$$

For n = 2

$$\phi_s = \phi_1 + \phi_2 + \phi_1\phi_2$$

Once ϕ_s is known from (3.30), the system reliability is obtained from

$$x_s = \frac{1}{1 + \phi_s} \tag{3.31}$$

In most reliability studies, the components have a relatively high value of 'x' i.e. $\phi << 1$.

∴ Equation (3.30) can be approximated by

$$\phi_s = \sum_{i=1}^{n} \phi_i \tag{3.32}$$

Similarly for n unit paralle structure

$$\phi_p = \prod_{i=1}^{n} \phi_i \tag{3.33}$$

Now consider the redundancy problem in which, at the

j th stage n_j is the total number of components in parallel.

∴ System parametric equation becomes

$$\phi_s = \Sigma \; \phi_j^{n_j} \tag{3.34}$$

The problem is to minimise ϕ_s (i.e. to maximize system reliability R) subject to the linear constraint

$$\Sigma \; c_j n_j \leq C \; .$$

Introduce the Lagrangian Multiplier to give the unconstrained promulation as

$$L = \Sigma \; \phi_j^{n_j} + \lambda [\Sigma \; c_j n_j - C] \tag{3.35}$$

The conditions of optimality are

$$\frac{\partial L}{\partial n_j} = 0 \tag{3.36}$$

and

$$\frac{\partial L}{\partial \lambda} = 0 \tag{3.37}$$

Differentiating (3.35) w.r.t. y_j and λ

$$\frac{\partial L}{\partial n_j} = \text{Ln} \; \phi_j \, . \, \phi_j^{n_j} + c_j = 0 \tag{3.38}$$

$$\frac{\partial L}{\partial \lambda} = \Sigma \; c_j n_j - C = 0 \tag{3.39}$$

From (3.38)

$$n_j = a_j \; \text{Ln} \; \lambda + b_j \tag{3.40}$$

where

$$a_j = \frac{1}{Ln\phi_j} \tag{3.41}$$

$$b_j = \frac{Lnk_j}{Ln\phi_j} \tag{3.42}$$

$$k_j = \frac{-c_j}{Ln\phi_j} \tag{3.43}$$

From (3.39) and (3.40)

$$\lambda = e^s \tag{3.44}$$

where

$$s = \frac{C - \Sigma c_j b_j}{\Sigma c_j a_j}$$

Once λ_j is obtained, n_j can be calculated from equation (3.40). n_j so obtained by treating as a continuous variable, is approximated to the nearest integer to get a near-optimum solution. The algorithm of the method is given in reference |20|

3.3.1 EXAMPLE

Consider the problem of maximizing the reliability of the following 4 -stage system

Stage j	Cost c_j	Reliability R_j
1	4.0	0.90
2	3.0	0.85
3	2.0	0.80
4	1.0	0.70

Cost constraint is 30 units.

Following the procedure of section 3.3

$$\phi_1 = \frac{0.1}{0.9} = 0.111 \qquad \text{Ln}\,\phi_1 = -2.19$$

$$\phi_2 = 0.176 \qquad \text{Ln}\,\phi_2 = -1.737$$

$$\phi_3 = 0.250 \qquad \text{Ln}\,\phi_3 = -1.386$$

$$\phi_4 = 0.428 \qquad \text{Ln}\,\phi_4 = -0.849$$

$$k_1 = \frac{-C_1}{\text{Ln}\phi_1}$$

$$= \frac{-4}{-2.19} = 1.826 \,, \qquad \text{Ln}\,k_1 = 0.602$$

$$k_2 = 1.727 \qquad \text{Ln}\,k_2 = 0.546$$

$$k_3 = 1.443 \qquad \text{Ln}\,k_3 = 0.367$$

$$k_4 = 1.178 \qquad \text{Ln}\,k_4 = 0.164$$

$$a_1 = \frac{1}{\text{Ln}\phi_1} \qquad b_1 = \frac{\text{Ln}k_1}{\text{L}\phi_1}$$

$$= -0.457 \qquad = -0.275$$

$$a_2 = -0.576 \qquad b_2 = -0.314$$

$$a_3 = -0.721 \qquad b_3 = -0.265$$

$$a_4 = -1.178 \qquad b_4 = -0.193$$

s is calculated as

$$s = \frac{30-(-1.1-0.942-0.53-0.193)}{-1(1.828+1.728+1.442+1.178)}$$

$$s = \frac{32.765}{-6.176} = -5.305$$

$$\therefore \lambda = e^{-5.305}$$

$$= 0.00496$$

$y_1 = a_1 \ln \lambda + b_1$

$= 2.425 + (-0.275)$

$= 2.15 — 2$

$y_2 = 3.056 - 0.314$

$= 2.742 — 3$

$y_3 = 3.826 - 0.265$

$= 3.56 — 4$

$y_4 = 6.25 - 0.195$

$= 6.057 — 6$

Total system cost = 2(4) + 3(3) + 4(2) + 6(1)

= 8 + 9 + 8 + 6

= 31

$\therefore$ The cost constraint is violated.

The Lagrangian Multiplier λ is decremented by 0.001 and the procedure is repeated.

New $\lambda = 0.00396$

The optimal allocation is found to be (2,3,4,5) and the optimal system reliability is 0.98269.

3.4 LAWLER AND BELL'S DISCRETE OPTIMIZATION METHOD

Lawler and Bell |17| described a simple, easily programmed method for solving discrete optimization problems with monotone objective functions and arbitrary (possibly non-convex) constraint. The problem can be stated as

$$\text{Minimize } Z = g_0(x)$$

subject to

$$\begin{aligned} g_{11}(x) - g_{12}(x) &\geq 0 \\ g_{21}(x) - g_{22}(x) &\geq 0 \\ &\vdots \\ g_{n1}(x) - g_{m2}(x) &\geq 0 \end{aligned} \qquad (3.46)$$

In general, the type of problems that can be solved by this method should be put in the form

Minimize $g_0(x)$ subject to the constraint of the form

$$g_{i1}(x) - g_{i2}(x) \geq 0, \qquad i = 1,2,\ldots,r$$

where

$$x = (x_1, x_2, \ldots, x_n)$$

and $x_j = 0 \text{ or } 1$, $(j = 1,2,\ldots,n)$

with the restrictions that each of the functions $g_0, g_{11}, \ldots$ $\ldots g_{12}, \ldots, g_{m2}$ is monotone non-decreasing in each of the variables $(x_1, x_2, \ldots, x_n)$.

Here it is possible to transform non-negative integers into binary variables also and if necessary, an arbitrary objective function of the form

$$\text{minimize } g_0(x)$$

can be replaced by a monotone non-increasing objective function by the formulation as

$$\text{minimize } \bar{Z}$$

subject to

$$\bar{Z} - g_0(x) \geq 0 \qquad (3.47)$$

Vector x is 'binary' in the sense that each x_j is either 0 or 1. $x \leq y$ if and only if $x_j \leq y_j$ for $j = 1,\ldots,n$. This is the vector partial ordering. There is also the lexicographic or numerical ordering of these vectors obtained by identifying with each x, the integer value

$$N(x) = x_1(2^{n-1}) + x_2(2^{n-2}) + \ldots + x_n(2^0) \qquad (3.48)$$

Numerical ordering is a refinement of the vector partial ordering i.e. $x \leq y$ implies $N(x) \leq N(y)$; however, $N(x) \leq N(y)$ does not imply $x \leq y$.

The method is basically is a search method, which starts with $x = (0,0,\ldots,0)$ and examine the 2^n solution vectors in the numerical ordering described above. Further the labor of examination is considerably cut down by following certain rules. As the examination proceeds one can retain the least costly up-to-date solution. If $\hat{x}$ is the solution having 'cost' $g_0(\hat{x})$ and x is the vector being examined then the following steps indicate the conditions under which certain vectors may be skipped.

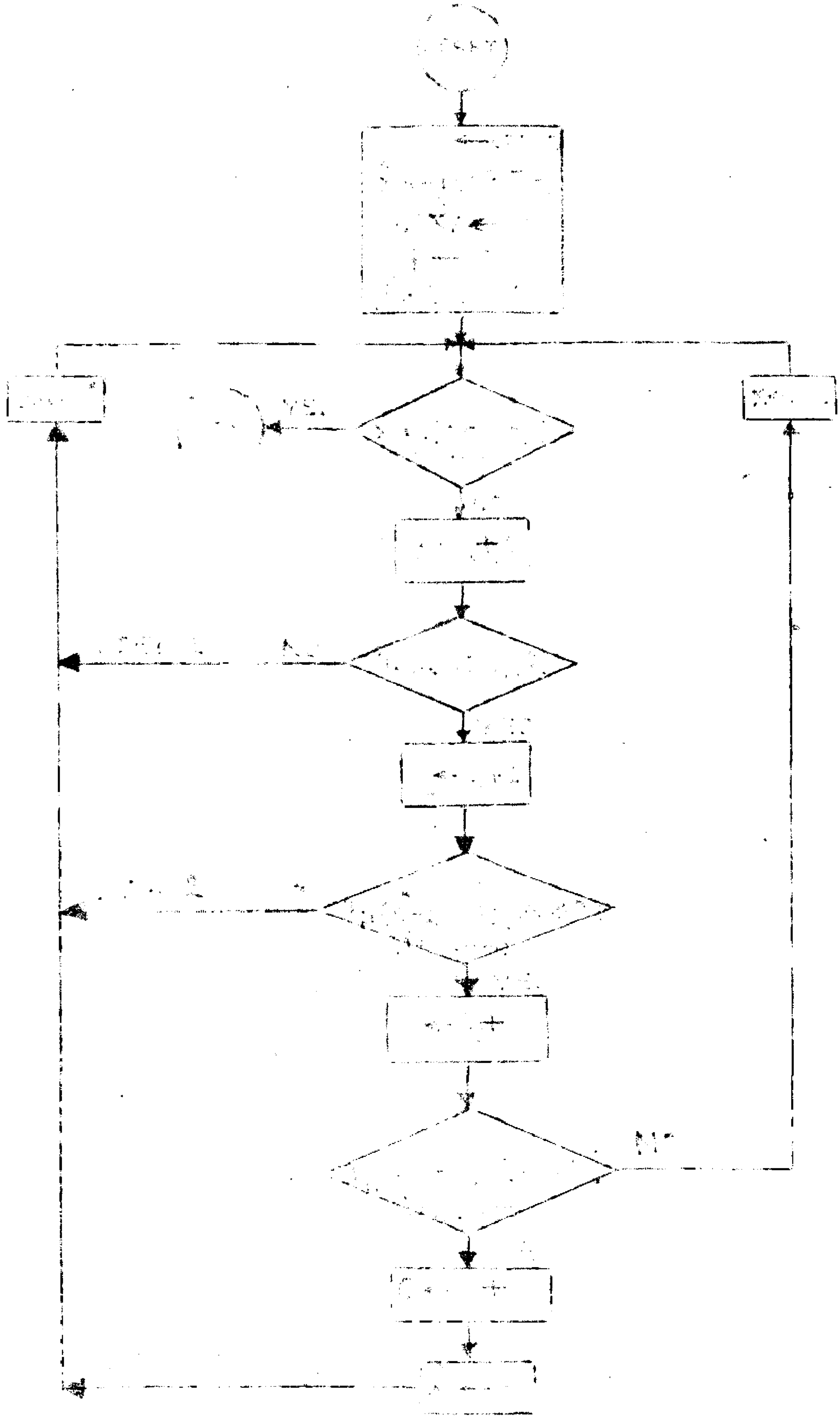
1. Test if $g_0(x) \nleq \hat{g}_0(\hat{x})$. If YES, skp to x^* and repeat the operation, otherwise proceed to step 2.

2. Examine whether $g_{i1}(x^*-1) - g_{i2}(x)$ 0 for $i = 1,\ldots,r$. If YES, prđceed to step 3 ; otherwise skip to x^* and go to step 1.

3. Further, if $g_{i1}(x) - g_{i2}(x) \geq 0$ $(i=1,\ldots,r)$, replace $\hat{x}$ by x and skip to x^*; otherwise change x to $x+1$, In either case further execution is transferred to step 1. Lawler and Bell [17] call the above steps of the algorithm skipping rules 1,3,2 , respectively. Following the above rules, all the vectors are examined and scanning continues until a vector having maximum numerical order, viz. $(1,1,\ldots,1)$, is found. In case one has skipped to a vector having numerical order higher than $(1,\ldots,1)$, designate this state by 'overflow' and terminate the procedure. The least 'costly' vector recorded provides the optimum solution.

One should not be overwhelmed by the number of trials. In practice the number of vectors to be examined may be quite small. For example, in an 11 -variable problem with a total of 2^{11} solution vectors, only 42 vectors were examined.

x^* is the first vector following x in the numerical order that has the property x x^*. For any x, x^* is calculated on a computer by treating x as a binary number and then substracting 1 from it. Logically OR x and $x-1$ to obtain $x^* -1$. Finally add 1 to obtain x^*.



x^* can also be found out by the following method |31|.

Let the right most position of one in x be u and the position of rightmost zero to the left of u be v. Then x^* vector can be calculated from x by

1 - putting $x^*_v = 1$

2 - putting $x^*_i = 0$ for $v+1 \leq i \leq u$

3 - putting $x^*_i = x_i$ for $1 \leq i \leq v-1$

where us is the total length of vector x, rememberig that x is written as $(x_1, x_2, \ldots, x_n)$.

It is found that the second procedure is efficient with respect to the computer time.

3.4.1 ADVANTAGES OF THE ALGORITHM

This algorithm has been found of great interest in solving variety of problems arising in the reliability area and those that defy solution by any standard procedure. This is due to the following reasons :

1. The algorithm provides and exact integer solution.
2. The procedure is very simple and involves only the functional evaluations. No partial derivatives or positivity tests are required.
3. There exists a possibility of reducing the number of searches and the search does not so greatly increase with the number of variables.
4. The memory requirements are extremely smaller thanas

with other methods.

5. A very large problem can also be solved in various passes as the search can be broken up in the range of binary variables.

3.4.2 OPTIMAL ALLOCATION PROBLEMS

Considering a series-parallel system with statistically independent componets and assuming that -

1. The system has n stages in series, i.e., the system fails if any stage fails.
2. Each stage has several identical components in parallel to provide the redundancy. The reliability of each component is known.
3. For the j th stage, if $1+m_j$ components (each with reliability p_j) are used, the stage reliability is

$$R_j(m_j) = 1-(1-p_j)^{1+m_j} \tag{3.49}$$

The overall reliability of the system is

$$R_s = \prod_{j=1}^{n} R_j(m_j) = \prod_{j=1}^{n} 1-(1-p_j)^{1+m_j} \tag{3.50}$$

or

$$Z = \sum_{j=1}^{n} \phi_j(m_j) \tag{3.51}$$

where

$$\phi_j(m_j) = \mathrm{Ln}\, R_j(m_j) \quad \text{and} \quad Z \equiv \mathrm{Ln}\, Rs$$

This form is more convenient to use since each term of the sum depends on a single variable. Moreover, since $\phi_j(m_j)$ is a monotone increasing concave function of m_j, maximizing R_s is equivalent to maximizing $\ln R_s = X$. Two situations will be considered here.

1. minimizing the cost of a system, given that the system reliability is not less than a preassigned value; the cost function and constraints may be any arbitrary functions.
2. maximizing the system reliability subject to given constants ; the constraints need not be linear.

In the following sectons, we will show low different redundancy optimization problems can be formulated as an integer programming problem with zero-one type variables, so that it is easier and more economical to solve them using the Lawler-Bell algorithm than by any other method.

3.4.2.1 EXAMPLE

Consider a system consisting of two stages. The reliability, cost and weight parameters of the components are given below. It is required to find the optimal number of parallel components to be employed in each stage to increase the system reliability. The total cost and weight of the system should not exceed 40 and 30 units respectively.

Stage	1	2
Component Reliability	0.91	0.96
Cost	9	6
Weight	5	8

The problem can be written as

Maximize $\text{Ln } R_s(n)$

$$\text{Ln}R_s(s) = \text{Ln}(1-.09^{m_1+1})+\text{Ln}(1-.04^{m_2+1})$$

or Minimize

$$g_0(m) = -\text{Ln}(1-.09^{m_1+1}) - \text{Ln}(1-.04^{m_2+1}) \qquad (3.52)$$

subject to the constraints

$$g_1(m) = 25-9m_1 -6m_2 \geq 0 \qquad (3.53)$$

$$g_2(m) = 17-5m_1-8m_2 \geq 0 \qquad (3.54)$$

Since the objective function is non-increasing, the following situations will be made by once more replacing usual binary variables x'_{ij} with $(1-x_{ij})$. Subscript i and j refer to the constraint and stage reppectively.

Before m_1 and m_2, the non-negative integer variables, can be transformed to the variables of zero- one type, it is necessary to estimate their maximum values. From (3.53) and (3.54), we find that $m_1 \leq 3$ and $m_2 \leq 3$. Therefore, we make the following substitution,

$$\begin{aligned} m_1 &= x'_{11} + 2x'_{12} \\ &= (1-x_{11}) + 2(1-x_{12}) \\ &= 3-x_{11} - 2x_{12} \end{aligned} \qquad (3.55)$$

$$\begin{aligned} m_2 &= x'_{21} + 2x'_{22} \\ &= (1-x_{21}) + 2(1-x_{22}) \\ &= 3-x_{21} - 2x_{22} \end{aligned} \qquad (3.56)$$

where x_{ij} is either 0 or 1

Substituting these values of m_1 and m_2,

$$g_0(x) = -\ Ln(1-0.09^{4-x_{11}-2x_{12}}) - Ln(1-.04^{4-x_{21}-2x_{22}})$$

$$g_{11}(x) = 9x_{11}+18x_{12}+6x_{21}+12x_{22}-20$$

$$-g_{12}(x) = g_{21}(x) = 0$$

$$g_{22}(x) = 5x_{11}+10x_{12}+8x_{21}+16x_{22}-22$$

Following the rules of Lawler-Bell algorithm, the solution sequence is given in Table 3.1.

Table 3.1

Test Vector				Comments
x_{22}	x_{12}	x_{11}	x_{21}	
0	0	0	0	Skip to x* through step 2
0	0	0	1	Skip to x* through step 2
0	0	1	0	Skip to x* through step 2
0	1	0	0	Change x to x+1 through step 3.
0	1	0	1	Skip to x* through step
0	1	1	0	Change x → x+1 through step 3
0	1	1	1	g_0=0.09437, skip to x* through step 3

Table contd..

Table 3.1 contd.

Test Vectors				Comments
x_{22}	x_{12}	x_{11}	x_{21}	
1	0	0	0	Change $x \to x+1$ through step 3.
1	0	0	1	Change $x \to x+1$ through step 3.
1	0	1	0	Change $x \to x+1$ through step 3.
1	0	1	1	$g_0 = 0.04155$, skip to x^*
1	1	0	0	$g_0 = 0.00973$, skip to x^*

The optimal solution obtained is

$$x_{22} = x_{12} = 1 \text{ and } x_{21} = x_{11} = 0$$

From equation (3.55) and (3.56)

$$m_1 = 3-2 = 1$$
$$m_2 = 3-2 = 1$$

Therefore, the optimum number of parallel components to be added to the existing-ones, are one in each stage, with maximum reliability as 0.9903.

The basic consideration in the design of a complex system is that its reliability should be high. The reliability of modern system, being sophicated, needs special consideration. The reliability of the constituent components is insufficient to meet the system reliability goal. Therefore, some means must be provided for increasing the system reliability. One can obtain high reliability for the system by providing as many redundancies as possible, but to ensure that this does not become too costly, heavy or bulky system. Therefore, the question of optimization of system reliability with respect to cost, weight or volume arises. The other problem is to minimize the system down-time by resorting to planned maintenance of the equipments. Therefore, the question of optimum maintenance policies arises. Both of these aspects are covered in this dissertation work, through the mathematical models.

The reliability problem has form of nonlinear integer programming problem having integer variables. A few methods of optimization of such problems are presented in chapter III. The methods discussed are the variational method, the penalty function method, Lagrange's Multiplier Method and the Lawler and Bell's optimization method. Other methods of optimization are also available. Variational method is easier to obtain for single constraint problems, and it provides near optimum or optimum solution convenie-ntly, fast and without much complexity. When the reliability problem has a number of constraints and approximate solution is required, the use of penalty function approach can be made as explained in section 3.2. This method provides continuous solution and has fast convergence. For obtaining the solution by Lagrangian Multplier method, one has to try several values of the Lagrangian Multipliers before arriving at a correct value. The exact integer solution of the reliability provlem is obtained from Lawler and Bell's algorithm. The nonlinear integer problem is converted into zero-one nonlinear programming problem. This method is easily programmable and the memory requirements are extremely smaller as with the other methods. There exists a possibility of reducing the number of searches and the search does not so greatly increase with the number of variables. Therefore, a very large problem can also be solved in various passes as the search can be broken up in the range of binary variables. The method to be used for the solution of reliability problem depends on the accuracy of the results and the cost of obtaining them.

In future, due to the advent of 'space age' the system will demand more sophisticated equipments for communications, command and control, missile and satellite launching and navigation. Many new techniques will be needed to cope with these new problems. A continuing growth in the problems of system maintenance is anticipated. Therefore, the problems of systems planning may be expected to become more important in the future and to require the development of advanced techniques for decision-making. It is hoped that the present work may help in the design of tomorrow's systems.

R E F E R E N C E S

1. Barlow, R.E. and Proschan, F.
Mathematical Theory of Reliability.
John Wiley and Sons, Inc., New York, 1965.

2. Bazovsky, I.
Reliabiàity Theory and Practice',
Prentice Hall, Englewood Cliffs, 1961.

3. Brook, R.H.W.
Reliability Concepts in Engineering Manufacture!
John Wiley and Sons, Inc., New York, 1972

4. Morse, P.M.
Queues, Inventory and Maintenance.
John Wiley and Sons, Inc., New York 1958.

5. Rau, J.G.
Optimization and Probability in System Engineering.
Von Nostrand Reinhold Company, New York, 1970.

6. Sandler, G.H.
System Reliability Engineering.
Prentice Hall, Englewood Cliffs, N.J.,1963.

7. Taha, H.A.
Operations Research, an Introduction.
Macmillan Publishing Co.,Inc.,New York, 1976.

8. Moskowitz Mc Lean
Some Reliability Aspects of System Design.
IRE Trans. on Reliability and Quality Control,
PGRQC -8, Sept. 1956,pp.7-35.

9. Gordon, K.
Optimum Component Redundancy for Maximum System Redundancy.
Operation Research, vol.5, April 1957,pp.229-243.

10. Kettelle, J.D. (Jr.)
Least Cost Allocation of Reliability Investment.
Ops.Res., vol.10,pp.229-265,Mar-Apr. 1967.

11. Proschan, F. , Bray, T.H.
Optimum Redundancy under Multiple Constraints.
Ops.Res., vol.13, Sept-Oct. 1965

12. Bellman, R.E. and Dreyfus, S.E.
Dynamic Programming and the Reliability of Multi-component Devices.
Ops.Res., vol. 6, Mar-Apr. 1959, pp.250-206.

13. Fan, L.T., Wang, C.S. , Tillman, F.A. and Hwang, C.K.
Optimization of Systems Reliability
IEEE Trans. Rel. vol. R-16, Sept. 1967, pp.81-86.

14. Federowicz and Mazumdar
Use of Geometric Programming to Maximize Reliability Achieved by Redundancy.
Ops.Res.,vol.16, Sept. 1968, pp. 948-994.

15. Tillman, F.A. and Littschwager, J.M.
Integer Programming Formulation of Constrained Reliability Problems.
Management Science, vol.13, July 1967, pp.887-899.

16. Ghare, P. and Taylor, R.
Optimal Redundancy for Reliability in Series Systems.
Ops. Res.,vol.17, Sept. 1969, pp. 838-847.

17. Lawler, E.L. and Bell, M.D.
A Method of Solving Discrete Optimization Problems.
Ops. Res., vol. 14, Nov-Dec.1966, pp. 1098-1112.

18. Misra, K.B.
A Method of Solving Redundancy Optimization Problems.
IEEE Trans. Rel. vol. R-20, August 1971, pp. 117-120.

19. Misra, K.B.
Least Square Approach for System Reliability Optimization.
International Journal on Control, vol.17, No.1, 1973, pp. 199-207.

20. Banerjee, S.K. and Rajamani, K.
Optimization of System Reliability using a Parametric Approach.
IEEE Trans. Rel. vol.R-22, April 1973, pp. 35-39.

21. Misra, K.B. and Sharma, J.
A New Geometric Programming Formation for a Reliability Problem.
International Journal on Control, vol.18, Sept.1973, pp. 497-503.

22. Sharma, J and Venkateswaran, K.V.
A Direct Method for Maximizing the System Reliability.
IEEE Trans. Rel. vol R-20, Nov. 1971, pp.256-259.

23. Luis, R.
Optimization of System Reliability by a new Nonlinear Integer Programming Procedure.
IEEE Trans. Rel. vol. R-24, April 1975, pp.14-16.

24. Nakagawa, Y and Nakashima, K.
A Heuristic Method for determining Optimal Reliability Allocation.
IEEE Trans. Rel. Aug. 1977, vol. R-26, pp.156-161.

25. Tillman, F.A. ., Hwang, C and Kuo, W.
Determining Component Reliability and Redundancy for Optimum System Reliability.
IEEE Trans. Rel. vol.R-26, pp.162-165.

26. Aggarwal, K.K. , Gupta, J.S. and Misra, K.B.
A New Heuristic Criterion for Solving a Redundancy Optimization Problem.
IEEE Trans. Rel. vol.R-24, April 1975 , pp. 86-87.

27. Graver, D.P.
Time to Failure and Availability of Paralleled System with Repair.
IEEE Trans. Rel.,vol. R-12, June 1963, pp.30-38.

28. Srinivasan, V.S.
The Effect of Standby Redundancy in System's Failure with Repair Maintenance.
Opr.Res.,vol.14, Dec.1966, pp.1024-1036.

29. Nakagawa T and Osaki, S.
Optimum Preventive Maintenance Policies for 2 Unit Redundant System.
IEEE Trans. Rel. vol.R-23, No.2, June 1974,pp.86-91.

30. Balagurusamy, E , and Misra, K.B.
Availability and Failure Frequency of Repairable m-order Systems.
International Journal on Systems Sciences, No.11,vol.7, 1976, pp.1209-1215.

31. Sharma, J.D.
Optimization of Reliability of a Series System.
Ph.D. Thesis 1974, University of Roorkee, Roorkee.

32. Misra, R.B.
A note on Optimal Redundancy Allocation for a Bridge Network.
Accepted for Publication in IEEE Trans. Rel.

33. Nakagawa, T.
Optimum Preventive Maintenance Policies for Repairable Systems.
IEEE Trans. Rel., vol. R-26, No.3, Aug.1977,pp.168-172.

34. Madaan, G.L.
Optimal Inspection and Maintenance Schedules for Multicomponent Systems.
A Seminar Report 1977, Dec., University of Roorkee, Roorkee.

35. Misra, K.B.
Redundancy Allocation in Electronic Relay Circuits.
Ph.D. Thesis, 1970 University of Roorkee, Roorkee.

APPENDIX -A

```
C  RELIABILITY OPTIMIZATION  N SERIES SYSTEM BY REDUNDANCY,G L MADAAN
   DIMENSION R(5),F(5),C(5),M(5),RS(5),RS1(5),Q(5)
   READ 10,CSG
   PUNCH10,CSG
   DO9ITER=1,3
   READ10,(R(I),I=1,5)
   PUNCH10,(R(I),I=1,5)
10 FORMAT (5F10.5)
   READ 10,(C(I),I=1,5)
   PUNCH10,(C(I),I=1,5)
   DO 1I=1,5
   M(I)=1
1  Q(I)=1.-R(I)
   K=0
   DO 7J=1,5
7  RS1(J)=0.
   CS=0.
   DO 5J=1,5
   AMJ=M(J)
 5 CS=CS+C(J)*AMJ
8  DO 2I=1,5
3  RS(I)=RS1(I)
   RSI=RS(I)
   RS1(I)=R(1)*R(2)*R(3)*R(4)*R(5)
   RSI1=RS1(I)
   M(I)=M(I)+1
   K=K+1
   IF(K-2)12,13,13
12 RI=R(I)
13 R(I)=1.-Q(I)**M(I)
   GO TO(3,4),K
 4 K=0
   M(I)=M(I)-2
   R(I)=RI
   FI=(RSI1-RSI)/RSI
2  F(I)=FI*CS/C(I)
   PUNCH 20,(F(I),I=1,5)
20 FORMAT (4E16.8)
   FM=F(1)
   IM=1
   DO 6J=2,5
   IF (F(J)-FM)6,6,11
```

CONTINUED ON THE NEXT PAGE

```
 11 FM=F(J)
    IM=J
 6  CONTINUE
    M(IM)=M(IM)+1
    R(IM)=1.-(Q(IM)**M(IM))
    CS=CS+C(IM)
    RST=RS1(IM)
    PUNCH 30,(M(J),J=1,5)
 30 FORMAT(13H ALLOCATION= ,5I5)
    PUNCH40,CS,RST
 40 FORMAT(26H COST AND RELIABILITY ARE ,2(2X,E15.8))
    PUNCH50
 50 FORMAT(26H COST AND RELIABILITY ARE ,2(2X,E15.8))
    PUNCH500,(R(J),J=1,5)
500 FORMAT (4(2X,E16.8))
    IF(CS-CSG)8,9,9
     CONTINUE
     STOP
    END
```

APPENDIX B

```
C  RELIABILITY OPTIMIZATION  N SERIES SYSTEM BY REDUNDANCY,G L MADAAN
   PROGRAM ACCEPTEDZ       36930  42280      59339  59999

60.00000
  .65000     .65000     .65000     .65000     .65000
 3.50000    3.20000    4.00000    5.00000    6.50000
0.22199993E+01  0.24281242E+01  0.1 424994E+01  0.15539995E+01
0.11953845E+01
ALLOCATION=      1     2     1     1     1
COST AND RELIABILITY ARE    0.25400000E+02   0.15663922
COST AND RELIABILITY ARE
  0.65000000        0.87750000        0.65000000        0.65000000
  0.65000000
0.25400000E+01  0.72025515      0.22225000E+01  0.17780004E+01
0.13676923E+01
ALLOCATION=      2     2     1     1     1
COST AND RELIABILITY ARE    0.28900000E+02   0.21146295
COST AND RELIABILITY ARE
  0.87750000        0.87750000        0.65000000        0.65000000
  0.65000000
0.74925945      0.81950253      0.25287505E+01  0.20230004E+01
0.15561538E+01
LLOCATION=      2     2     2     1     1
OST AND RELIABILITY ARE    0.32 00000E+02   0.28547500
OST AND RELIABILITY ARE
  0.87750000        0.87750000        0.87750000        0.65000000
  0.65000000
0.85296285      0.93292812      0C74634250      0.23030000E+01
0.17715384E+01
LLOCATION=      2     2     2     2     1
OST AND RELIABILITY ARE    0.37900000E+02   0.38539125
OST AND RELIABILITY ARE
  0.87750000        0.87750000        0.87750000        0.87750000
  0.65000000
).98259237      0.10747104E+01  0.85976857      0.68781466
).20407692E+01
LOCATION=      2     2     2     2     2
)ST AND RELIABILITY ARE    0.44400000E+02   0.52027819
)ST AND RELIABILITY ARE
  0.87750000        0.87750000        0.87750000        0.87750000
```

CONTINUED ON THE NEXT PAGE

```
  0.87750000
 0.11511108E+01  0.12590275E+01  0.10072222E+01  0.80577762
 0.61982906
ALLOCATION=     2    3    2    2    2
COST AND RELIABILITY ARE    0.47600000E+02   0.56748861
COST AND RELIABILITY ARE
  0.877 0000        0.95712500        0.87750000        0.87750000
  0.87750000
 0.12340738E+01  0.43311809      0.10798146E+01  0.86385172
 0.66450146
ALLOCATION=     3    3    2    2    2
COST AND RELIABILITY ARE    0.51100000E+02   0.61898294
COST AND REL+ABILITY ARE
  0.95712500        0.95712500        0.87750000        0.87750000
  0.87750000
 0.42511134      0.46496553      0.11592131E+01  0.92737052
 0.71336180
ALLOC+TION=     3    3    3    2    2
COST AND RELIABILITY ARE    0.55100000E+02   0.67514992
COST AND RELIABILITY ARE
  0.95712500        0.95712500        0.95712500        0.87750000
  0.87750000
 0.45838802      0.50136190      0.40108930      0.99996288
 0.76920221
ALLOCATION=     3    3    3    3    2
COST AND RELIABILITY ARE    0.60100000E+02   0.73641352
COST AND RELIABILITY ARE
  0.95712500        0.95712500        0.95712500        0.95712500
  0.87750000
    +RROR LC-2 IN  STATEMENT 0000 + 05 L. L.
```

APPENDIX -B

```
95.
 .85        .85        .85        .85        .85
 3.         4.         5.         6.         7.

C  RELIABILITY OPTIMIZATION  N SERIES SYSTEM BY REDUNDANCY,G L MADAAN
   PROGRAM ACCEPTEDZ     36930  42280     59339  59999

95.00000
  .85000    .85000    .85000    .85000    .85000
 3.00000   4.00000   5.00000   6.00000   7.00000
0.12499998E+01  0.93749987      0.74999990      0.62499991
0.53571421
LLOCATION=     2    1    1    1    1
OST AND RELIABILITY ARE    0.28000000E+02   0.51026110
OST AND RELIABILITY ARE
  0.97750000       0.85000000        0.85000000        0. 5000000
  0.85000000
0.18260870      0.10500000E+01  0.84000000      0.70000000
0.59999995
LLOCATION=     2    2    1    1    1
OST AND -ELIABILITY ARE    0.32000000E+02   0.58680027
OST AND RELIABILITY ARE
  0.97750000       0.97750000        0.85000000        0.85000000
  0.85000000
0.20869556      0.15652167      0.95999986      0.79999993
0.68571422
LLOCATION=     2    2    2    1    1
OST AND RELIABILITY ARE    0.37000000E+02   0.67482030
OST AND RELIABILITY A-E
  0.97750000       0.97750000        0.97750000        0.85000000
  0.85000000
0.24130441      0.18097831      0.14478265      0.92500000
0.79285708
LLOCATION=     2    2    2    2    1
OST AND RELIABILITY ARE    0.43000000E+02   0.77604335
OST AND RELIABILITY ARE
  0.97750000       0.97750000        0.97750000        0.97750000
  0.85000000
0.28043464      0.21032598      0.16826078      0.14021732
0.92142850
LOCATION=     2    2    2    2    2
OST AND RELIABILITY ARE    0.50000000E+02   0.89244985
```

CONTINUED ON THE NEXT PAGE

```
COST AND RELIABILITY ARE
  0.97750000        0.97750000        0.97750000        0.97750000
  0.97750000
0.32608685      0.24456513      0.19565222      0.16304351
0.13975158
ALLOCATION=      3    2    2    2    2
COST AND RELIABILITY ARE    0.53000000E+02   0.90991082
COST AND RELIABILITY ARE
  0.99662500        0.97750000        0.97750000        0.97750000
  0.97750000
0.50853920E-01  0.25923922      0.20739138      0.17282615
0.14813661
ALLOCATION=      3    3    2    2    2
COST AND RELIABILITY ARE    0.57000000E+02   0.92771343
COST AND RELIABILITY ARE
  0.99662500        0.99662500        0.97750000        0.97750000
  0.97750000
0.546916 0E-01  0.41018730E-01  0.22304352      0.185 6950
0.15931671
LLOCATION=      3    3    3    2    2
OST AND REL+ABILITY ARE    0.62000000E+ 2   0. 4586435
OST AND RELIABILITY ARE
  0.996 2500        0.99662500        0.99662500        0.97750000
  0.97750000
0.59489206E-01  0.44616905E-01  0.35693654E-01  0.20217388
0.17329190
LLOCATION=      3    3    3    3    2
OST AND RELIABILITY ARE    0.68000000E+02   0.96437039
OST AND RELIABILITY AR+
  0.99662500        0.99662500        0.99662500        0.99662500
  0.97750000
0.65245993E-01  0.48934495E-01  0.3 147736E-01  0.32623113E-01
0.19006214
LOCATION=      3    3    3    3    3
OST AND RELIABILITY ARE    0.75000000E+02   0.98323851
OST AND RELIABILITY ARE
  0.99662500        0.99662500        0.99662500        0.99662500
  0.99662500
0.71962446E-01  0.53971835E-01  0.43177620E-01  0.35981350E-01
0.30841157E-01
LOCATION=      4    3    3    3    3
OST AND RELIABILITY ARE    0.78000000E+02   0.98606876
OST AND RELIABILITY ARE
  0.9 949380        0.99662500        0.99662500        0.99662500
  0.99662500
```

CONTINUED ON THE NEXT PAGE

```
 0.11193722E-01  0.56131165E-01  0.44904932E-01  0.37420776E-01
 0.32074951E-01
ALLOCATION=     4    4    3    3    3
COST AND RELIABILITY ARE   0.82000000E+02   0.98890718
COST AND RELIABILITY A-E
   0.99949380        0.99949380        0. 9662500        0.99662500
   0.99662500
 0.11767427E-01  0.88255702E-02  0.47207742E-01  0. 9339646E-01
 0.33719697E-01
ALLOCATION=     4    4    4    3    3
COST AND RELIABILITY ARE   0.87000000E+02   0.99175377
COST AND RELIABILITY ARE
   0.99949380        0.99949380        0.99949380        0.99662500
   0.99662500
 0.12485084E-01  0.93638132E-02  0.74910506E-02  0.41738348E-01
 0.35775 52E-01
ALLOCATION=     4    4    4    4    3
COST AND RELIABILITY ARE   0.93000000E+02   0.99460854
COST AND RELIABILITY ARE
   0.99949380        0.99949380        0.99949380        0.99949380
   0.99662500
 0.13346155E-01  0.10009616E-01  0.80076930E-02  0.66730775E-02
 0.38243185E-01
ALLOCATION=     4    4    4    4    4
COST AND RELIABILITY ARE   0.10000000E+03   0C99747154
COST AND RELIABILITY ARE
   0.99949380        0.99949380        0.99949380        0.99949380
   0.99 49380
     ERROR LC-2 IN  STATEMENT 0000 + 05 L. L.
```

APPENDIX -C

```
C   C  RELIABILITY OPTIMIZATION IN BRIDGE SYSTEM BY REDUNDANCY,G L MADAAN
       DIMENSION R(5),F(5),C(5),M(5),RS(5),RS1(5),Q(5)
       READ 10,CSG
       PUNCH10,CSG
       DO9ITER=1,3
       READ10,(R(I),I=1,5)
       PUNCH10,(R(I),I=1,5)
    10 FORMAT (5F10.5)
       READ 10,(C(I),I=1,5)
       PUNCH10,(C(I),I=1,5)
       DO 1I=1,5
       M(I)=1
     1 Q(I)=1.-R(I)
       K=0
       DO 7J=1,5
     7 RS1(J)=0.
       CS=0.
       DO 5J=1,5
       AMJ=M(J)
     5 CS=CS+C(J)*AMJ
     8 DO 2I=1,5
     3 RS(I)=RS1(I)
       RSI=RS(I)
       X=R(1)*R(2)
       Y=R(3)*R(4)
       Z=R(4)*R(5)
       V=R(2)*R(3)
       T=R(1)*R(5)
       W=R(3)*R(5)
       U=R(2)*Y+R(1)*W-X*(Y+Z)-V*(Z+T)-Y*T
       RS1(I)=X+Z+U+2.*R(3)*X*Z
       RSI1=RS1(I)
       M(I)=M(I)+1
       K=K+1
       IF(K-2)12,13,13
    12 RI=R(I)
    13 R(I)=1.-Q(I)**M(I)
       GO TO(3,4),K
     4 K=0
       M(I)=M(I)-2
       R(I)=RI
       FI=(RSI1-RSI)/RSI
     2 F(I)=FI*CS/C(I)
       PUNCH 20,(F(I),I=1,5)
    20 FORMAT (4E16.8)
       FM=F(1)
       IM=1
```

CONTINUED ON THE NEXT PAGE

```
      DO 6J=2,5
      IF (F(J)-FM)6,6,11
   11 FM=F(J)
      IM=J
    6 CONTINUE
      M(IM)=M(IM)+1
      R(IM)=1.-(Q(IM)**M(IM))
      CS=CS+C(IM)
      RST=RS1(IM)
      PUNCH 30,(M(J),J=1,5)
   30 FORMAT(13H ALLOCATION= ,5I5)
      PUNCH40,CS,RST
   40 FORMAT(26H COST AND RELIABILITY ARE ,2(2X,E15.8))
      PUNCH50
   50 FORMAT(26H COST AND RELIABILITY ARE ,2(2X,E15.8))
      PUNCH500,(R(J),J=1,5)
  500 FORMAT (4(2X,E16.8))
      IF(CS-CSG)8,9,9
9        CONTINUE
         STOP
      END
```

APPENDIX -D

```
C  C  RELIABILITY OPTIMIZATION IN BRIDGE SYSTEM BY REDUNDANCY,G L MADAAN

120.
0.6        0.6        0.6        0.6        0.6
 4.0       6.0        5.0        7.0        3.0
120.00000
   .60000     .60000     .60000     .60000     .60000
  4.00000    6.00000    5.00000    7.00000    3.00000
80.78602620E-02 53.85735080E-02 20.96069868E-02 46.16344354E-02
10.77147016E-01
ALLOCATION=      1     1     1     1     2
COST AND RELIABILITY ARE     2.80000000E+01    7.44768000E-01
+OST AND RELIABILITY ARE
  60.00000000E-02   60.00000000E-02   60.00000000E-02   60.00000000E-02
  84.00000000E-02
  .198528    .054528   -.041472    .294528    .313728
 7.52513534E-02 30.02835782E-02 18.7099767 E-02 56.67440061E-02
42.73266305E-02
ALLOCATION=      2     1     1     1     2
COST AND RELIABILITY ARE     3.20000000E+01    8.27251200E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   60.00000000E-02
  84.00000000E-02
  .177331   -.140236   -.162892    .029107    .223411
31.90641971E-02 39.68992248E-02 22.6731652 E-02 34.01993355E-02
42.54189295E-02
ALLOCATION=      2     1     1     1     3
COST AND RELIABILITY ARE     3.50000000E+01    8.60244480E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   60.00000000E-02
  93.60000000E-02
  .274375   -.043192   -.096952    .052807    .320455
33.10924819E-02 29.82207104E-02 25.28751640E-02 38.95331708E-02
17.89824213E-02
ALLOCATION=      2     1     1     2     3
COST AND RELIABILITY ARE     4.20000000E+01    9.27263232E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   84.00000000E-02
  93.60000000E-02
  .183478   -.081788   -.060899    .033564    .367030
15.74564302E-02 19.79197829E-02 12.06219779E-02 17.34620755E-02
23.22050996E-02
ALLOCATION=      2     1     1     2     4
COST AND RELIABILITY ARE     4.50000000E+01    9.42642892E-01
+OST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   84.00000000E-02
  97.44000000E-02
```

CONTINUED ON THE NEXT PAGE

```
 .044287  -.254065  -.275569   .217087   .387506
15.87707866E-02 14.50324735E-02 12.47640621E-02 18.87731707E-02
 7.89281339E-03
ALLOCATION=     2    1    1    3    4
COST AND RELIABILITY ARE    5.20000000E+01    9.70323333E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   93.60000000E-02
  97.44000000E-02
 .005734  -.2 2582  -.264227   .211619   .408365
76.03519604E-03 95.94353152E-03 59.81667221E-03 84.76602430E-03
11.61310788E-02
ALLOCATION=     2    1    1    3    5
COST AND RELIABILITY ARE    5.50000000E+01    9.76824373E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   93.60000000E-02
  98.97600000E-02
 -.050820  -.342725  -.351326   .285920   .417448
75.54274639E-03 69.38008071E-03 60.00910934E-03 90.18312981E-03
48.80538976E-03
ALLOCATION=     2    1    1    4    5
COST AND RELIABILITY ARE    6.20000000E+01    9.88036219E-01
+OST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   97.44000000E-02
  98.97600000E-02
 -.066592  -.350624  -.347282   .284089   .426148
35.89709199E-03 45.36374650E-03 28.52817271E-03 40.20294953E-03
55.56126246E-03
ALLOCATION=     2    1    1    4    6
COST AND RELIABILITY ARE    6.50000000E+01    9.90692503E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   97.44000000E-02
  99.59040000E-02
 -.089354  -.378878  -.382319   .313953   .429924
 5.33009979E-03 32.51706197E-03 28C1 482480E-03 42.24470371E-03
23.23741168E-03
ALLOCATION=     2    1    1    5    6
+OST AND RELIABILITY ARE    7.20000000E+01    9.95199589E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   98.97600000E-02
  99.59040000E-02
 -.095719  -.382117  -.380780   .313277   .433461
16.60719001E-03 20.99916925E-03 13.25079496E-03 18.63288434E-03
25.83889906E-03
ALLOCATION=     2    1    1    5    7
COST AND RELIABILITY ARE    7.50000000E+01    9.96271042E-01
+OST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   98.97600000E-02
  99.83616000E-02
 -.104846  -.393450  -.394826   .325245   .434994
16.23671932E-03 14.95657636E-03 12.97482569E-03 19.42683355E-03
10.75462928E-03
ALLOCATION=     2    1    1    6    7
COST AND RELIABILITY ARE    8.20000000E+01    9.98077452E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.59040000E-02
  99.83616000E-02
```

```
 -.107401  -.394758  -.394223   .324985   .436418
 5.53210643E-04 95.52995786E-04 60.36216933E-04 84.80625049E-04
11.77628138E-03
ALLOCATION=     2    1    1    6    8
COST AND RELIABILITY ARE    8.50000000E+01   9.98507463E-01
COST AND RELIABILITY A-E
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.59040000E-02
  99.93446400E-02
 -.111056  -C399296  -.399847   .329775   .437035
73.48108655E-04 67.71051327E-04 58.75854477E-04 87.94063173E-04
48.80745561E-04
ALLOCATION=     2    1    1    7    8
COST AND RELIABILITY ARE    9.20000000E+01   9.99230599E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.83616000E-02
  99.93446400E-02
 -.112079  -C399822  -.399608   .329673   .437606
33.87529990E-04 42.84812991E-04 27.08885124E-04 38.04556731E-04
52.85889305E-04
ALLOCATION=     2    1    1    7
+OST AND RELIABILITY ARE    9.50000000E+01   9.99402832E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.83616000E-02
  99.97378560E-02
 -.113542  -.401638  -.401858   .331589   .437853
*2.82799215E-04 30.25404511E-04 26.2576 050E-04 39.29184313E-04
21.82925841E-04
ALLOCATION=     2    1    1    8    9
COST AND RELIABILITY ARE    1.02000000E+02   9.99692178E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.93446400E-02
  99.97378560E-02
 -.113951  -.401848  -.401763   .331549   .438082
15.01905167E-04 18.99798391E-04 12.01322202E-04 16.86992848E-04
23.44339890E-04
ALLOCATION=     2    1    1    8   10
COST AND RELIABILITY ARE    1.05000000E+02   9.99761108E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.93446400E-02
  99.98951424E-02
 -.114536  -.402575  -.402663   .332315   .438181
14.50947421E-04 13.37255004E-04 11.60674794E-04 17.36710286E-04
96.52498712E-05
ALLOCATION3     2    1    1    9   10
COST AND RELIABILITY ARE    1.12000000E+02   9.99876861E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.97378560E-02
  99.98951424E-02
 -.114700  -.402659  -.402625   .332299   .438272
65.95916686E-05 83.43469400E-05 52.76378670E-05 74.09106048E-05
10.29699399E-04
ALLOCATION=     2    1    1    9   11
COST AND RELIABILITY ARE    1.15000000E+02   9.99904439E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.97378560E-02
  99.99580569E-02
 -.114934  -.402950  -.402985   .332606   .438312
63.55838903E-05 58.57938510E-05 50.84525437E-05 76.07739443E-05
42.29005890E-05
ALLOCATION=     2    1    1   10   11
COST AND RELIABILITY ARE    1.22000000E+02   9.99950743E-01
COST AND RELIABILITY ARE
  84.00000000E-02   60.00000000E-02   60.00000000E-02   99.98951424E-02
  99.99580569E-02
 -.115000  -.402983  -.403070
```

APPENDIX -D

```
C  C  RELIABILITY OPTIMIZATION IN BRIDGE SYSTEM BY REDUNDANCY,G L MADAAN
      PROGRAM ACCEPTEDZ      36930  43320      59259  59999

 75.00000
  .80000    .80000    .80000    .80000    .80000
  4.00000   5.00000   2.00000   7.00000   3.00000
 0.19466291      0.15573033      0.94382020E-01  0.11123595
 0.25955055
ALLOCATION=     1    1    1    1    2
COST AND RELIABILITY ARE    0.24000000E+02   0.94515200
COST AND RELIABILITY ARE
   0.80000000        0.80000000        0.80000000        0.80000000
   0.96000000
 0.19891657      0.55124592E-01  0.72806065E-01  0.13223958
 0.57204763E-01
ALLOCATION=     2    1    1    1    2
COST AND RELIABILITY ARE    0.28000000E+02   0.97648640
COST AND RELIABILITY ARE
   0.96000000        0.80000000        0H80000000        0.80000000
   0.96000000
 0.44924495E-01  0.69999540E-01  0.86911 00E-01  0.49999671E-01
 0.59899326E-01
ALLOCATION=     2    1    2    1    2
COST AND RELIABILITY ARE    0.30000000E+02   0.98254840
COST AND RELIABILITY ARE
   0.96000000        0.80000000        0.96000000        0.80000000
   0.96000000
 0.48338075E-01  0.45923232E-01  0.18512064E-01  0.32802308E-01
 0C64450766E-01
ALLOCATION=     2    1    2    1    3
COST AND RELIABILITY ARE    0.33000000E+02   0.98888100
COST AND RELIABILITY ARE
   0.96000000        0.80000000        0.96000000        0.80000000
   0.99200000
 0.52939002E-01  0.16599429E-01  0.20446443E-01  0.36264798E-01
 0.14087033E-01
ALLOCATION=     3    1    2    1    3
COST AND RELIABILITY ARE    0.37000000E+02   0.99522650
COST AND RELIABILITY ARE
```

CONTINUED ON THE NEXT PAGE

```
   0.99200000        0.80000000        0.96000000        0.80000000
   0.99200000
 0.11794551E-01  0.18865333E-01  0.23438532E-01  0.13475238E-01
 0.15727307E-01
ALLOCATION=      3    1    3    1    3
COST AND RELIABILITY ARE    0.39000000E+02    0. 9648740
COST AND RELIABILITY AR+
   0.99200000        0.80000000        0.99200000        0.80000000
   0.99200000
 0.12485832E-01  0.11973718E-01  0.49332784E-02  0.85526558E-02
 0.16649080E-01
ALLOCATION=      3    1    3    1    4
COST AND RELIABILITY ARE    0.42000000E+02    0.99776360
COST AND RELIABILITY ARE
   0.99200000        0.80000000        0.9 200000        0.80000000
   0.99840000
 0.13443815E-01  0.42860252E-02  0.53354315E-02  0.92186160E-02
 0.35822110E-02
ALLOCATION=      4    1    3    1    4
COST AND RELIABILITY ARE    0.46000000E+02    0.99904110
COST AND RELIABILITY ARE
   0.99840000        0.80000000        0.99200000        0.80000000
   0.99840000
 0.29422212E-02  0.47075540E-02  0.58752335E-02  0.33625385E-02
 0.39214266E-02
ALLOCATION=      4    1    4    1    4
COST AND RELIABILITY ARE    0.48000000E+02    0.99929630
COST AND RELIABILITY ARE
   0.99840000        0.80000000        0.99840000        0.80000000
   0.99840000
 0.30729622E-02  0.29492752E-02  0.12272636E-02  0.21059390E-02
 0.40972830E-02
ALLOCATION=      4    1    4    1    5
COST AND RELIABILITY ARE    0.51000000E+02    0. 9955220
COST AND RELIABILITY ARE
   0.99840000        0.80000000        0.99840000        0.80000000
   0.99968000
 0.32654620E-02  0.10439274E-02  0.13061848E-02  0.22377162E-02
 0.86738840E-03
ALLOCATION=      5    1    4    1    5
COST AND RELIABILITY ARE    0.55000000E+02    0.99980820
COST AND RELIABILITY ARE
   0.99968000        0.80000000        0.99840000        0.80000000
   0.99968000
 0.70138450E-03  0.11255158E-02  0.14027690E-02  0.80393987E-03
 0.93701303E-03
```

CONTINUED ON THE NEXT PAGE

```
ALLOCATION=      5    1    5    1    5
COST AND RELIABILITY ARE    0.57000000E+02   0.99985920
COST AND RELIABILITY ARE
   0.99968000       0.80000000       0.99968000       0.80000000
   0.99968000
 0.72970272E-03  0.70005856E-03  0.29074092E-03  0.50004182E-03
 0C97293696E-03
ALLOCATION=      5    1    5    1    6
COST AND RELIABILITY ARE    0.60000000E+02   0.99991040
COST AND RELIABILITY ARE
   0.99968000       0.80000000       0.99968000       0.80000000
   0. 9993600
 0.76806880E-03  0.24482192E-03  0.30302715E-03  0.52719008E-03
 0.20601844E-03
ALLOCATION=      6    1    5    1    6
COST AND RELIABILITY ARE    0.64000000E+02   0. 9996160
COST AND REL+ABILITY ARE
   0.99993600       0.80000000       0.9 968000       0.80000000
   0.99993600
 0.16480632E-03  0.25984996E-03  0.32961264E-03  0.18835008E-03
 0.22187517E-03
ALLOCATION=      6    1    6    1    6
COST AND RELIABILITY ARE    0.66000000E+02   0.99997190
COST AND RELIABILITY ARE
   0.99993600       0.80000000       0.9 993600       0.80000000
   0C99993600
 0.16830471E-03  0.16104451E-03  0.6 301945E-04  0.116 1756E-03
 0.22660635E-03
ALLOCATION=      6    1    6    1    7
COST AND RELIABILITY ARE    0.69000000E+02   0.99998220
COST AND RELIABILITY ARE
   0.99993600       0.80000000       0.99993600       0.80000000
   0.99998720
 0.17422808E-03  0.57961030E-04  0.69001225E-04  0.12124500E-03
 0.43700776E-04
ALLOCATION=      7    1    6    1    7
COST AND RELIABILITY ARE    0.73000000E+02   0.99999230
COST AND RELIABILITY ARE
   0.99998720       0.80000000       0.99993600       0.80000000
   0.99998720
 0.38325292E-04  0.61320470E-04  0.80300615E-04  0.43800335E-04
 0.48667040E-04
ALLOCATION=      7    1    7    1    7
COST AND RELIABILITY ARE    0.75000000E+02   0.99999450
+OST AND RELIABILITY ARE
   0.99998720       0.80000000       0.99998720       0.80000000
   0.99998720
```

APPENDIX -E

```
C   C  G)L)MADAAN,UNDOMINATED ALLOCATIONS (MULTI-CONSTRAINTS
       DIMENSION R(10),Q(10),C(10),W(10),M1(10),M2(10),F(10)
C      N=NO OF STAGES,CG=GIVEN COST,WG=GIVEN WEIGHT
       READ1,N,CG,WG
1      FORMAT(I3,2F10.5)
       READ 2,(R(I),C(I),I=1,N)
2      FORMAT(7F10.5)
       READ 2,(W(I),I=1,N)
       PUNCH2,(R(I),C(I),I=1,N)
       PUNCH2,(W(ID,I=1,N)
       DO3I=1,N
       Q(I)=1.-R(I)
       M1(I)=1
3      M2(I)=2
       K=1
       A1=0.25
       A2=0.75
12     CS=0.
       PUNCH2,A1,A2
       WS=0.
       RS=1.
       DO4I=1,N
       AM1=M1(I)
       CS=CS+C(I)*AM1
       WS=WS+W(I)*AM1
       GOTO(13,14),K
13     RS=RS*R(I)
       GOTO4
14     QI=Q(I)
       M11=M1(I)
       RP=(1.-QI**M11)
       RS=RS*RP
4      CONTINUE
       PUNCH5,RS,CS,WS
5      FORMAT(3F10.5)
       PUNCH6,(M1(I),I=1,N)
6      FORMAT(8I5)
       IF(CS-CG)10,10,11
10     IF(WS-WG)15,15,11
15     DO7I=1,N
       QI=Q(I)
       M11=M1(I)
       M22=M2(I)
```

CONTINUED ON THE NEXT PAGE

```
      CI=C(I)
      WI=W(I)
      D=A1*CI+A2*WI
 7    F(I)=(LOGF(1.-QI**M22)-LOGF(1.-QI**M11))/D
      PUNCH2,(F(I),I=1,N)
      X=F(1)
      N1=N-1
      J1=1
      DO9J=1,N1
      IF(X-F(J+1))8,8,9
      X=F(J+1)
      J1=J+1
 9    CONTINUE
      PUNCH1,J1,F(J1)
      M1(J1)=M1(J1)+1
      M2(J1)=M2(J1)+1
      K=2
      GOTO 12
 11   A1=A1+0.25
      A2=1.-A1
      DO21I=1,N
      M1(I)=1
 21   M2(I)=2
      K=1
      IF(A1-1.)12,12,20
 20   STOP
      END
```