# MAINTENANCE SCHEDULING OF HYDROTHERMAL POWER STATIONS

*A DISSERTATION*

*Submitted in partial fulfilment*
*of the requirements for the award of the Degree*
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*By*


PARMOD KUMAR TANEJA





DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
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# C E R T I F I C A T E

Certified that the dissertation entitled 'MAINTENANCE SCHEDULING OF HYDRO THERMAL POWER STATIONS', which is being submitted by Shri Parmod Kumar Taneja in partial fulfilment of the requirement for the award of the degree of Master of Engineering in Electrical Engineering (System Engineering and Operations Research) of the University of Roorkee, Roorkee, is a record of students' own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further certified that he has worked for 6½ months from April 15th, 1978 to oct 31st 1978 for preparing this dissertation at this University.

ROORKEE

DATED NOVEMBER 13 ,1978

(Dr. J. Sharma )
Reader in Electrical Engineering
Deptt. of Electrical Engineering
University of Roorkee, Roorkee

# A B S T R A C T

The increasing size and complexity of power systems have introduced the need for a more systematic approach to the determination of maintenance schedules for power generating facilities. The application of mathematical programming technique in the solution of maintenance scheduling problems is of a great interest and importance to utilities. The present work is primarily concerned with the development of mathematical models and maintenance scheduling algorithms for a decision making situations arising in the functioning of a power plant or group of power plants under centralized administration.

First of all, the problem of preventive maintenance scheduling is discussed. In it number of objective criteria for optimal maintenance scheduling of generators is discussed. The criteria are based on reliability indices, deviation from a desired schedule, constraint violations and minimum cost functions. A mathematical model for finding maintenance scheduling ~~the~~ policies of generating units is developed in the presence of uncertainities. This problem is an integer programming having only o - 1 variables. A method based on lexicographic enumeration technique is developed for its solution. The operation of the method is exemplified by application to a realistic system.

The special structure of the mathematical model is also exploited for developing a new, simple and efficient direct search optimisation technique. The method developed is capable of taking in to account all the complex constraints and hence results in a practically implementable solution, if a feasible solution exists. The applicacy of the algorithm is demonstrated for the risk levelization and reserve levelization problems.

Next a technique for scheduling generation maintenance based on the frequency and duration method for reliability evaluation of power systems is presented. The method permits the weekly risk created by removing units from service to be minimised. It allows all practical constraints imposed on the system to be included, and, if necessary, continuously updated. It uses an approximate technique for the evaluation of frequency and duration of the outages. The basic concept of the approximate method is to split availability, rate of departure and hence the frequency of occurence, immediately adjacent to the exact state. A typical generating system is analysed to illustrate the generation maintenance scheduling technique.

Finally the problem of corrective maintenance scheduling is presented for optimal allocation of spare units. In this the steady state availability of a repairable system with stand-by units is maximised under constraints of total cost and weight. In the end, avenues of future research are discussed.

## ACKNOWLEDGEMENTS

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Parmod Taneja

PARMOD KUMAR TANEJA

## LIST OF ILLUSTRATIONS

## LIST OF SYMBOLS

The following is the list of main symbols, which is common for the text. The other symbols used at specific locations are defined separately in the text.

| | | |
|---|---|---|
| $x_{ij}$ | = | maintenance on unit i starting in jth week |
| $b_i$ | = | resources available for the ith type of constraints. |
| $C_{ij}$ | = | Cost coefficients for the binary variables $x_{ij}$ |
| $S^T$ | = | A non redundant solution set |
| $L_{ej}$ | = | Equivalent load for interval j |
| $L_{pj}$ | = | Predicted peak load for interval j |
| $R_j$ | = | Reserve available for jth period |
| $A_i$ | = | Availability of state i |
| $\lambda_{+i}$ | = | Rate of transition out of a given capacity state i to one where more capacity is available. |
| $\lambda_{-i}$ | = | Rate of transition in downward direction of state i. |
| $\lambda$ | = | Constant failure rate |
| $\mu$ | = | Constant repair rate |
| fo(x) | = | Any objective function to be optimised |
| $P_o$ | = | Steady state probability of state zero |
| $e_i$ | = | Constant replacement rate of i unit |
| $L_i$ | = | Number of spare i units. |

# CONTENTS



Contd.

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# CHAPTER - I

## INTRODUCTION

### BACKGROUND OF THE PROBLEM

As power system generating facilities become larger, diversified and more complex, the need for rigorous control and monitoring of the maintenance of generating facilities increases. The reliability of operation, the production costs and the capital expenditure are all affected by the methods used to schedule maintenance. In prevailing situations, technological, environmental and competitive factors interact in a complicated fashion and it becomes difficult to arrive at a decision regarding the maintenance scheduling problems. The maintenance scheduling requires the preparation of a time table, a plan or a program or a scheme which gives a maintenance procedure for the system concerned. In tackling simple problems, human judgement can be used in the preparation of maintenance schedules. But in the solution of large intricate maintenance problems, human judgement alone is no longer applicable. This requires the development of mathematical programming models and techniques to perform the maintenance scheduling problem in an optimum way, so as to meet the overall objective of providing reliable electric services to the customers at minimum cost.

A generating system itself consists of a group of major subsystems which are functionally inter-related with each other in concepts, operations and objectives. The problems of importance faced by the utility are identified and posed to the system engineer. Thus a realistic appraisal of the specifies of the problems is obtained by system analyst. Now it is the task of system analyst to prepare a suitable mathematical model which adequately describes the functional relationship between variables and which must give good predictions of the behaviour of the system in future time periods.

The effectiveness of maintenance scheduling problem depends upon the following -

(1) The selection of independent or decision variables.

(2) Selection of an objective function, that is, the quantity to be maximised or minimised as a function of the independent or decision variables.

(3) Specification of the limits, or constraints, on the values of independent variables so as to give a feasible realizable solution.

(4) The input data requirement are also specified.

Then the selection of the most appropriate technique of solution depends on the nature of objective function, constraints and types of decision variables choosen in order to simplify the task of decision making.

OBJECTIVES AND SCOPE

The objective of the maintenance scheduling is to develope mathematical models and solution techniques to find the time periods and sequences of maintenance of generating units, so that cost of maintenance is reduced within the set of different constraints connected with manpower availability, sequencing and security of the system. It also includes the probabilistic approach for adjusting the maintenance schedule in the presence of uncertainities associated with forced outage of generating units and error in demand forecast due to seasonal and socio economic changes.

The reliability of the generating system, production cost and capital expenditure on a power system are all affected by the maintenance outage of generating facilities. Also the lower cost per installed Megawatts for the new larger units will not necessarily result in the lower life cycle cost of the system. This is due to facts that generating unit forced outage rates have increased tremendously as the unit size increases. Thus to keep the same probability of meeting daily peaks, the increase in the total installed reserve requirement can more than offset the lower capital cost per megawatt associated with larger units. Thus better planning of maintenance scheduling of generating units results in two major areas of saving. Firstly, better maintenance scheduling would allow the most efficient unit to be available at right time which helps in controlling the forced outage rates and causing reduction in fuel usages. Also efficient maintenance planning can postpone the construction cost involved in generation expansion program. Thus the main purpose of maintenance scheduling of generating units is to satisfy all the constraints on the system while keeping it as reliable as possible, that is, still satisfying the load demand with a reasonable assurance of continuity.

Early attempts in the generator maintenance scheduling have been made to develope heuristic algorithms according to different objectives. An earlier work in this direction has been done by Christiaanse and Palmer using an objective of maximisation of minimum net reserve. Garver used the same to levelization of risk by replacing the capacity of unit with effective capabilities. Later Zurn and Quintana proposed the method known as the 'Group Sequential Scheduling'. Here successive approximate dynamic programming is used to produce the schedules. Gerard T. Egan and Morztyn used branch and bound method to solve the scheduling problems with various objectives. Patton and Ali have attempted to minimise the risk of failing to meet the load demand. According to them the units are assigned according to the priority list, that is, the most difficult unit is adjusted first for maintenance scheduling and so on.

These methods possess the advantages of being able to take account of the constraint that appear in the problem.

However these approaches suffer from the twin failings of not guaranteeing to find a feasible solution when one exists.

Dopazo and Merril have used zero - one integer programming with a variety of objectives. In it Bala's additive algorithm is used to prepare maintenance schedules. A limitation of this is that each unit must be maintained exactly once during the time period of interest. Furthermore, it may be difficult to introduce some of the complex constraints that exist in the maintenance scheduling problems.

The present work is primarily concerned with the development of mathematical models and solution techniques to solve the maintenance scheduling problems. First of all the problem of preventive maintenance scheduling is discussed. The task of scheduling preventive maintenance involves specifying dates at which man power is to be allocated to an overhaul of a major functional element or group of elements. The problem becomes an involved one, when a large number of generating units are to be scheduled for maintenance in the multiperiod scheduling horizon. Also there is an element of uncertainty associated with the reserve available for maintenance purposes due to errors in demand forecast. Then the total available generation is also a random variable due to random failure of generating units.

Therefore first section is concerned with the development of a mathematical model for preventive maintenance which includes all the possible set of constraints. Also the uncertainities associated with the errors in demand forcast and generator outages are considered. The correspondingly chance constraints are transformed in to a linear deterministic equivalent by using the chance constraint programming technique. A lexicographic enumeration technique which is based on Lawler and Bell algorithm for discrete optimisation problems is used for the solution of preventive maintenance scheduling problems. In this method certain infeasible solution vectors which helps in increasing the efficiency of scheduling algorithm are skipped by using some rules.

Next the problem of maintenance scheduling is presented considering the reserve and risk levelization criteria. This criteria includes the problem of levelization of reserve and risk, which reduces the expenditure on maintenance during the operating life of the units. This includes the forced outage rates of generating units and the variation of risk as a function of system peak loads. In this the different objectives are consider to level the reserve throughout the year. It helps in enhancing the reliability of generating units. The uncertainities in predicting peak load forecast are consider to determine the net reserve margins. A new direct search optimisation technique to solve this scheduling problem is developed by exploiting its special structure.

In the next section the problem of maintenance scheduling is solved by considering the frequency and duration of the outages of the generating units as objective function. Its main feature is that during a particular week if a limit is imposed on risk level, it includes that an additional constraint. A method is presented for the calculation of generating system and load model and to combine them to produce a capacity reserve model at particular interval with the risk indices for loss of load at that period.

In the end the problem of corrective maintenance scheduling is presented. To have built in maintenance in the system at the design stage is referred to as the problem of corrective maintenance scheduling. In this the maintainability analysis is included in the design phase. Such an analysis has to be extended up to subsystem levels. The objective is aimed at maximizing steady state availability or minimization of cost subject to the attainment of a specific level of availability. Availability is adjusted by the number of spare allowed. Other measure of system goodness are considered, viz failure rate, weight, price, mean replacement time, and mean replacement cost of a unit. The analysis result in the optimal allocation of spare units having non-zero replacement time. An algorithm is developed for the solution of this problem. It exploits the special structure of the problem.

# CHAPTER - II

## PREVENTIVE MAINTENANCE SCHEDULING IN PRESENCE OF UNCERTAINITIES

### INTRODUCTION

The task of scheduling preventive maintenance( that is routine or planned) involves specifying dates at which manpower is to be allocated for overhaul of a major functional element or group of elements. The scheduling interval between two successive maintenance events is decided taking into consideration type and the state of the unit to be maintained. In real situations, manual scheduling is both difficult and tedious because of the large number of generating units and the associated variation in the required demand and generation, which are random in nature due to the forced outages of generating units.

Heuristic algorithms have been developed for obtaining a maintenance schedule according to different objectives. In the early stages, adhoc-computer algorithms were developed in an attempt to do scheduling automatically. The need for automation results to revise maintenance schedules time to time, arising from unforeseen circumstances, such as forced outages of units, unexpected delays in installation of new unit unavailability of manpower or change in the load forecast. But the main drawbacks that come in the quick revision of schedules in these methods are as follows -

(1) They may fail to find a schedule satisfying problem constraints, even when one does exist.

(2) While they implicitly incorporate a criterion of goodness, they do not necessarily find the best schedule in terms of these criteria.

(3) The criterion of goodness is limited to either qualising net reserve or equalising an approximation to 'Loss of Load Probability'.

In this Chapter, a mathematical model is developed for the maintenance scheduling problem by considering the random naturē of demand and generation outages. A method based on lexicographic enumeration technique is presented for the solution of this problem.

## 2.1 COST FUNCTIONS FOR OPTIMAL SCHEDULING

Depending upon the choice of system analyst and subject to the appropriate data the importance of optimal preventive maintenance scheduling of the generating units consists of several classes of objective criterion. Some of the important classes of objective criterion are given below -

(i) Maximise the system reliability under conditions of load uncertainities and forced outages of units.

(ii) Minimise operating cost.

(iii) Minimum deviation from the required maintenance schedule.

(iv) Penalty criterion for constraint violation

(v) Minimum cost function criterion.

RELIABILITY CRITERION

The most important class of objective criterion for maintenance scheduling is based upon the optimisation of the system reliability. If this requirement is met, then a given security can be achieved with a smaller installed capacity. This leads to the reduction in capital investment. Various types of reliability indices are chosen for maintenance scheduling of generating units depending on the nature of the components whether stochastic, semi-stochastic or deterministic.

The simplest and oldest form of expressing system reliability is in terms of the expected peak net reserve, i.e. the difference between the expected peak demand and the net available installed capacity to supply the demand. This figure of merit therefore ignores both the daily demand variations and the generating unit forced outage rates. If the peak demand is given deterministically, this index is then purely deterministic.

Reliability index chosen as expected duration of unmet demand or the expected probability of loss of load is also commonly used. Here a simplyfying assumption is often made to compute the loss of load probability by neglecting the daily load variations and thus only considering the peak load distribution along the considered time period. Thus it provides the probability of loss of load given that the demand is at its peak.

Another reliability index is chosen as the expected lack of modified net reserve. It is identical to the expected lack of net reserve except for the fact that generating units are replaced by their effective load carrying capabilities and predicted load by their equivalent load.

Minimisation or maximisation of the reliability indices throughout the planning interval may or may not provide levelled system reliability. Therefore the more effective optimisation procedure is the minimisation (or maximisation) of the maximum (minimum) reliability index. It prevent the large variation of reliability indices during the planning interval.

OPERATING COST CRITERION

In assessing the operating cost two factors are of importance : the energy production cost and the maintenance cost. The later is of importance if planned outages are allowed to vary within given limits. The calculation of energy production cost is difficult and time consuming. This is because of the probablistic elements involved in the total generation due to the random failure of generating units.

DEVIATION FROM THE MAINTENANCE SCHEDULE

The deviation from the maintenance schedule criterion considers various objective aspects, namely maintenance urgency, ideal maintenance sequence of various units, ideal preventive maintenance schedules and changes in previously established schedules. For preventive maintenance and desired outage sequences of various units one wishes to minimize the deviation from ideal dates.

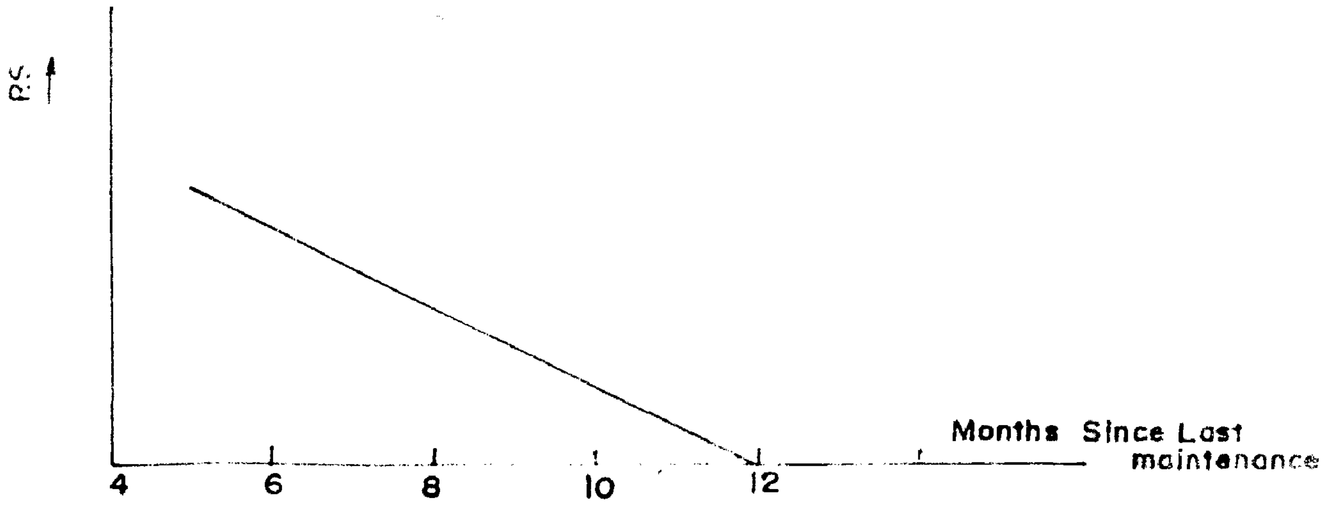



FIG. 2·1 (a) MAINTENANCE INVESTMENT LOST BY MAINTAINING UNIT TWO EARLY

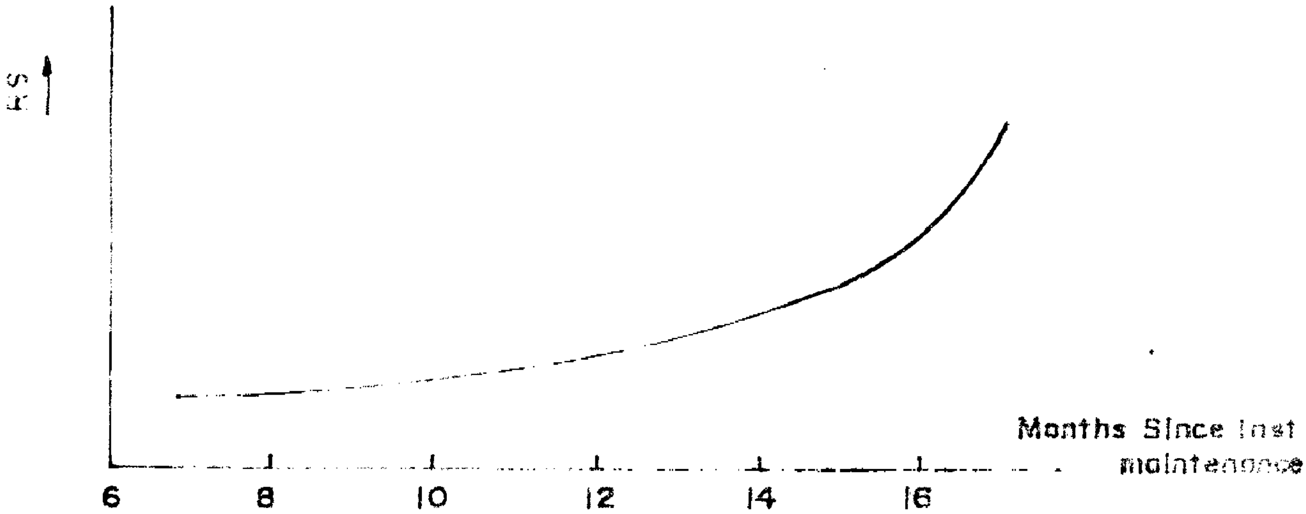



FIG. 2·1 (b) EXPECTED OUT OF POCKET COST IN TERMS OF TIME SINCE PREVIOUS MAINTENANCE

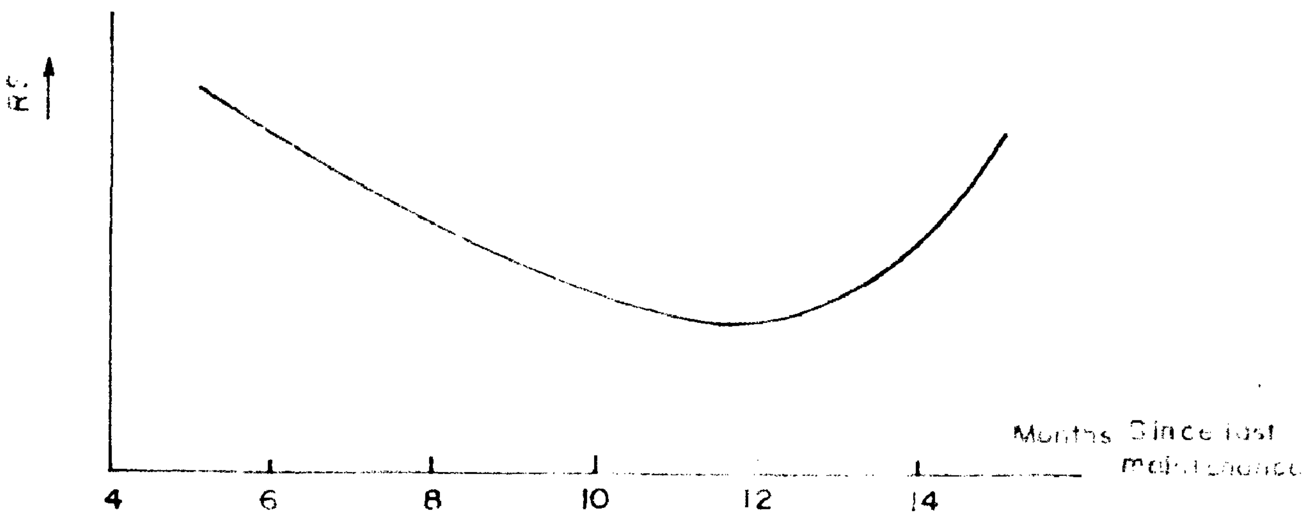



FIG. 2·1 (c) TOTAL MAINTENANCE COST IN TERMS OF TIME SINCE PREVIOUS MAINTENANCE

Also one attempts to minimise deviation from the previously established maintenance schedule which becomes partially infeasible due to forced outages leading to early maintenance.

PENALTY CRITERION FOR CONSTRAINT VIOLATION

Optimal preventive maintenance scheduling of $g$ generators involves a constrained optimisation problem. It is sometimes impossible to obtain a feasible solution without violating constraints. Among the infeasible solutions one attempts to search for the least infeasible one accomodating with penalty criterion imposed upon constraint violations. Consider here for example the minimum lateness penalty schedule criterion. Table 2.1 shows a possible penalty cost function for three unit system given .

TABLE 2.1

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 |

For each generating unit there is a penalty of zero imposed for beginning maintenance during the Ist allowed week and penalty of one and two imposed respectively if maintenance begin during the 2nd or 3rd week. The schedule that minimises the cost function is the minimum lateness penalty schedule.

## MINIMUM COST FUNCTION CRITERION

In the designing of preventive maintenance schedule the particular component of interest is 'how much cost occur for maintenance'. This is of importance if planned outages are allowed to vary within limits. Generally, the maintenance planning is done on an annual basis. By getting maintenance, one purchases a twelve-month smooth operation of the unit. Fig. 2.1(a) shows that there is a cost associated with maintaining a unit too early. The figure shows that by performing maintenance ~~for~~ too early on units one is throwing away rest of 12 months operation duration purchased at the time of last maintenance. Fig. 2.1(b) represents that expected maintenance costs will rise sharply if maintenance is delayed too long. This is due to the negligence on the part of maintenance crew who have to do the maintenance. Fig. 2.1(c) is the sum of the costs of (a) and (b). The optimal time to begin maintenance on this unit is clearly shown in Fig. 2.1(c).

## 2.2 MATHEMATICAL MODELLING

The problem of preventive maintenance scheduling involves the determination of timings and sequences of the outages of the generating units over a specified period, such that a minimum level of specified security is achieved and costs involved are minimised. Here a mathematical model for preventive maintenance is as given :

Minimise cost of maintenance

$$\sum_{i \in N} \sum_{j \in T} C_{ij} X_{ij} \quad \ldots (2.1)$$

Where N is the total number of generating units and T is the total number of planning weeks.

Subject to -

$$\sum_{i \in N} d_i x_{ij} \leq b_i \quad \ldots (2.2)$$

Where $x_{ij} = 0$ or $1$ ... (2.3)

here $C_{ij}$ are the cost coefficients, $d_i$ are the limits on the constraint coefficients. Variable $x_{ij}$ is equal to 1 if maintenance on unit i starts on $j^{th}$ week otherwise it is zero if maintenance does not start on it. In accordance with the general accepted terminology its solution vector is S and is given by = $(x_{11}, x_{12} \ldots, x_{21}, x_{22} \ldots, x_{31}, \ldots x_{n1}, \ldots x_{n}$

DATA REQUIREMENT

In order to produce a maintenance scheduling procedure that results in practically implementable schedules, it is essential that the numerous and complex constraints which limit the choice of scheduling times are incorporated in to the solution method. Therefore the following data must be known while considering the maintenance aspects of the units -

(1) Total number of units and capacity of each.

(2) The number and duration of desired maintenance outage for each unit in the system.

(3) Earliest available beginning maintenance period and latest available beginning period.

(4) Gross reserve available for maintenance purposes in a particular week.

(5) The inhibited period if any, for each unit during which maintenance can't be performed. This is to satisfy the seasonal limitations constraints and it also includes any firm prescheduled outage date for that unit.

As an example consider a simple three unit system maintained during a time horizon of four weeks. The unknowns associated with this are given in Table 2.2. The variables $x_{ij}$'s assigned to each unit are also given. $x_{ij}$ gives maintenance of unit i starts in $j^{th}$ week.

TABLE 2.2

UNKNOWN ASSOCIATED WITH THE PROBLEM

| Unit | Capacity | Allowed period | Outage duration weeks | Associated variables |
|---|---|---|---|---|
| 1 | 80 | 1 to 4 | 2 | $x_{11}$ $x_{12}$ $x_{13}$ |
| 2 | 110 | 1 to 3 | 1 | $x_{21}$ $x_{22}$ $x_{23}$ |
| 3 | 50 | 2 to 4 | 2 | $x_{32}$ $x_{33}$ |

## CONSTRAINTS ON A MAINTENANCE SCHEDULE

For producing an optimal maintenance schedule, it is necessary to incorporate numerous and even complex constraints which limit the choice of scheduling algorithm. The first of these constraints is the requirement that the generators must be overhauled regularly. This is necessary to keep their efficiency at a reasonable level, to keep the incidence of forced outage low, and to prolong the life of generators. This periodicity is incorporated by the constraint that each unit is maintained once in a year by specifying the maximum and minimum time the generator may run without maintenance. This constraint can be written as

$$\sum_{i \in N} \sum_{j \in T} x_{ij} = 1 \qquad \ldots (2.4)$$

A resource constraint is a limit on the resources available for maintenance at any given time. Only a limited number of generators can be maintained at a time due to availability of limited number of resources.

For example if there is restriction on the percentage of reserve MW available for maintenance, then we have to see tha while scheduling maintenance, whether the reserve is available for maintenance or not. This restriction can be termed as a resource constraint and written mathematically as

$$\sum_{i \in N} d_i \, x_{ij} \leq R \qquad \ldots (2.5)$$

Where R is the reserve available for maintenance purposes and $d_i$ is capacity of $i^{th}$ generating unit.

A sequence constraint is expressed as unit m must be taken down exactly k weeks after unit l comes back on line.

i.e. $x_{lj} - x_{m,j+k} \leq 0$

In addition to the constraint that only one unit can be maintained at a time, many other constraint can be forced on the maintenance scheduling of a unit.

CALCULATIONS OF RESERVE CONSIDERING RANDOM DEMAND AND GENERATION OUTAGES

The uncertainities in the demand forecast is due to the factors such as the nature of load, seasonal changes, socio-economic growth rate, model chosen to describe the growth rate etc. The future demand may therefore be described by an appropriate probability function f(D), which defines the probability distribution of demand over the entire range. If the generating system consists of number of individual demands, each one is governed by a suitable distribution function, the total demand probability distribution is obtained by the convolution of the individual density functions. It is generally assumed that demand can be defined by a normal distribution. It is specified in terms of an expected value $\bar{D}$ and a variance $\sigma_D$ to represent error in forecasting. It is given as -

$$\bar{D} = \int_{-\infty}^{\infty} D\, f\,(D)\; d\, D$$

$$\sigma_D^{\,2} = \int_{-\infty}^{\infty} (D-\bar{D})^2\; f(D)\; dD$$

There are many different types of generators in service and they are randomly forced off-line because of technical reasons. Random outages or availability of a generation is described by a discrete probability distribution function. Since a power plant consists of number of same or different types of units, the availability of the plant is determined by the convolution of the distribution function of its individual units. The total installed capacity can be described by a distribution function f(G) which is obtained by convolving the distribution functions describing the outputs of the power plant in the system. Let the power plant output is described by an equivalent normal distribution with $\bar{G}$ mean value and its deviator $\sigma_G$. The mean value $\bar{G}$ and the deviations $\sigma_G$ are calculated with the help of outage rates of generating units. These are given by the expression as follows -

$$\bar{G} = n(1-q)\; A$$
$$\sigma_G^{\,2} = nq\;(1-q)\; A^2$$

Where n = number of identical units

q = outage rate of each generating unit

A = power rating of each unit.

Sometimes these two distributions are unacceptable, e.g. when loads have a large variance or generating units are dissimilar. Therefore the data are represented by a series of discrete values expressed in terms of their individual probabilities of occurence. These are given by the expression -

Mean value $m = \sum_{i=1}^{m} x_i \, p_i$

Variance $= \sum_{i=1}^{m} (x_i - m)^2 \, p_i$

Where $x_i$ is the $i^{th}$ discrete value having a probability of occurrence of $p_i$ and m is the total number of discrete values.

Reserve is given by the difference between installed capacity and demand. Since generation and demand are random variables, therefore, the reserve is also a random variable having normal distribution with mean value $\bar{R}$ and deviation $\sigma_R$. These are calculated from the mean value and variance of demand and generation respectively, and are given as -

$$R = G - D = \frac{1}{\sigma_R\sqrt{2\pi}} \exp \left[ 1/2 \, \frac{(R-\bar{R})^2}{\sigma_R^2} \right]$$

$$\bar{R} = \bar{G} - \bar{D}$$

and $\sigma_R = \sqrt{\sigma_G^2 + \sigma_D^2}$

i.e.

$$P\left[\sum_{i=1}^{N} a_{ij}\ x_{ij} \leq R_j\right] = P\left[\sum_{j=1}^{N} \frac{a_{ij}\ x_{ij} - \bar{R}_j}{\sqrt{Var\ (R_j)}} \leq \frac{R_j - \bar{R}_j}{\sqrt{Var\ R_j}}\right] \geq b_i$$

... (2.7)

Where $\frac{R_j - \bar{R}_j}{\sqrt{Var\ R_j}}$ is a standard normal variate with a mean of zero and variance of one. Thus the probability of realizing $\sum_{j=1}^{N} d_i\ x_{ij}$ smaller than or equal to $R_j$ can be written as

$$P\left[\sum_{j=1}^{N} d_i\ x_{ij} \leq R_j\right] = \phi\left(\frac{(R_j - \bar{R}_j)}{\sqrt{Var\ (R_j)}}\right)$$

Where $\phi$ represent cumulative distribution function of the standard normal distribution evaluated at x. If $e_i$ denotes the value of the standard normal variate at which

$$\phi\ (e_i) = p_i$$

Then the constraint in equation (2.7) can be written as

$$\phi\left[\sum_{j=1}^{N} \frac{d_i\ x_{ij} - \bar{R}_j}{\sqrt{Var\ (R_j)}}\right] \geq \phi\ (e_i)$$

These inequalities will be satisfied only if

$$\frac{\sum_{j=1}^{N} d_i\ x_{ij} - \bar{R}_j}{\sqrt{Var\ R_j}} \leq e_i$$

CONVERSION OF STOCHASTIC CONSTRAINT IN TO ITS DETERMINISTIC EQUIVALENT

The constraint 2.5 which is the resources constraint can be divided in to a set of deterministic and probabilistic constraints as follows -

$$P\left[\sum_{i \in N} d_i \; x_{ij} \leq R_j\right] \geq b_i \qquad \ldots (2.6)$$

Equation (2.6) is interpreted as constraining the unconditional probability to be no smaller than $b_i$. In other words there is a probability of $b_i$ that $R_j$ is at least as large as $\sum_{i=1}^{N} d_i \; x_{ij}$, where $b_i$ lies between 0 and 1. The above equation is called Chance Constraints because they impose restrinction on probability.

DEVELOPMENT OF DETERMINISTIC EQUIVALENT

Here the equation 2.6 has been changed in to its deterministic equivalent with the help of stochastic programming model. For simplicity we assume that random variable $R_j$ is normally distributed with known mean and standard deviation. Let $\bar{R}_j$ and Var $(R_j)$ denote the mean and variance of normally distributed random variable $R_j$.

$$\text{And} \quad \sum_{j=1}^{N} d_i \; x_{ij} = c_i \text{ (say)}$$

Then constraints can be expressed as $P\left[c_i \leq R_j\right] \geq b_i$

or $$\sum_{j=1}^{N} d_i \, x_{ij} - \bar{R}_j \leq e_i \sqrt{Var \, R_j}$$

In general we can write that

$$\sum_{j=1}^{N} d_i \, x_{ij} \leq R_j + e_i \sqrt{Var \, R_j} \qquad \ldots (2.8)$$

This means that the probabilistic constraint is equivalent to deterministic linear constraint and is given as in equation (2.8).

## 2.3 OPTIMISATION TECHNIQUE

At any time each generating facilities may be either in operation or under maintenance. The total number of possible states of generating system is $2^n$. Where n is given by the number of units multiplied by total planning period. A simple exhaustive search method will require scanning of $2^n$ states, which will consume lot of time. Hence a better approach is needed in order to find optimal schedule which not only reduces the number of states to be enumerated but also increase the efficiency of the solution.

Here a lexicographic enumeration technique is used for the solution of maintenance scheduling problem. The problem using this technique can be put as -

$$\text{Maximise } f_o(x) \qquad \ldots (2.9)$$

Sub. to - $f_{j1}(x) - f_{j2}(x) \leq 0 \; (j = 1, 2, \ldots N)$

and size of vector $x = (x_{11}, x_{12}, \ldots x_{nt})$

Where

$$x_{ij} = \begin{bmatrix} 1 & \text{if maintenance on unit i starts } j^{th} \text{ week} \\ 0 & \text{otherwise} \end{bmatrix}$$

Also each of the function $\left[f_{j1}(x) \ldots . f_{jn}(x)\right]$ to be monotone nonincreasing in each of the variable. A function is monotone nonincreasing if

$$x' \geq x^2 \qquad h(x') \leq h(x^2)$$

In this technique some rules are used to skip a large number of non promising solution vectors. They are

Rule 1

If $f_o(x) \leq f_o(x)$ skip to $x^*$

Where $x^*$ be considered The first solution vector succeeding an arbitrary vector x in the numerical ordering such that $x \leq x^*$.

This means that $x \leq x + 1 \leq \ldots\ldots \leq x - 1$

The steps to get $x^*$ are as follows -

(1) Subtract one from x to obtain $x^* - 1$.

(2) Logically 'or' x and x-1 to obtain $x^* - 1$.

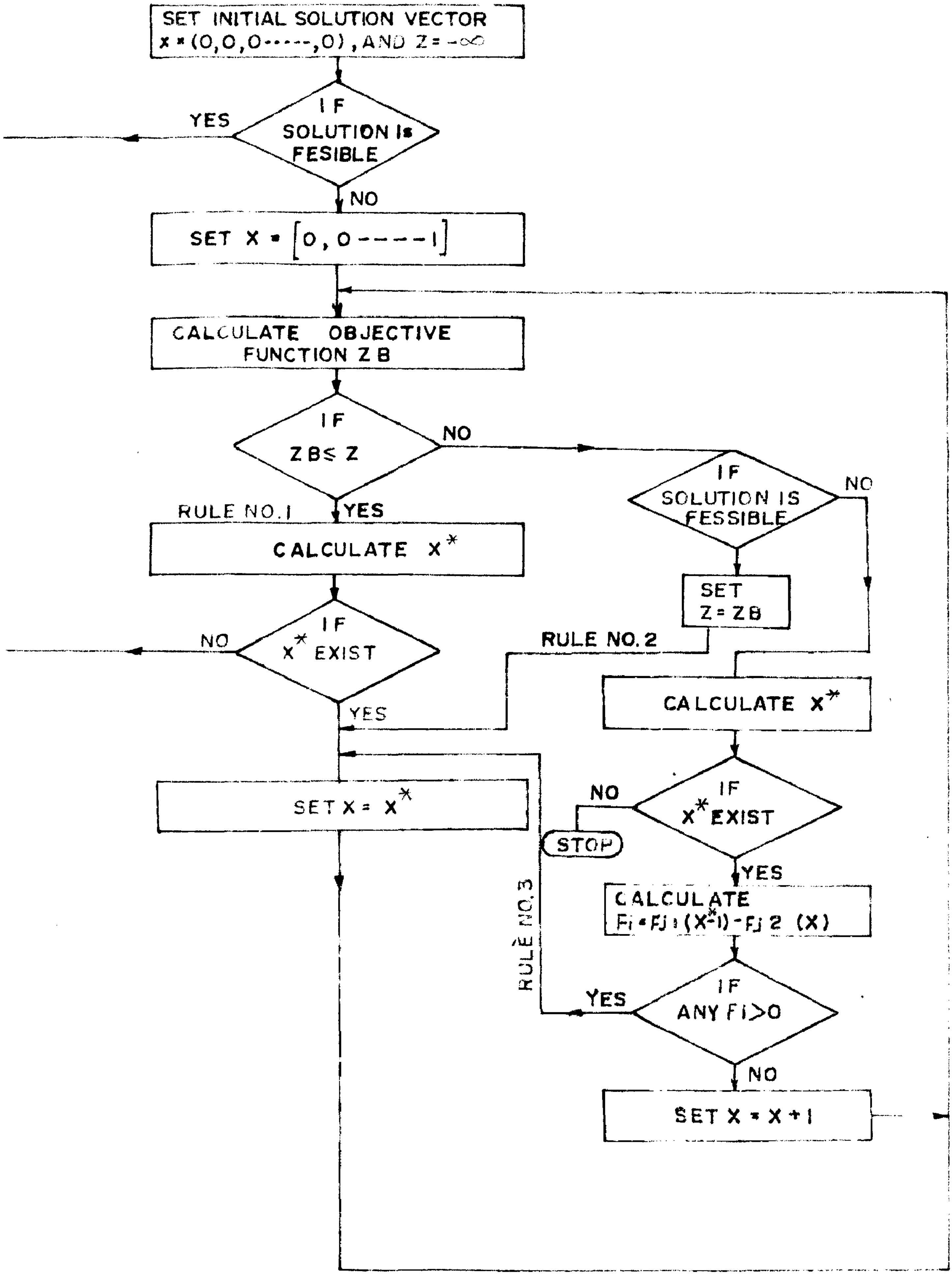
(3) Add one to $x^* - 1$ to obtain $x^*$.

For example let x = (00101) in order to get $x^*$ we perform according to above steps as

(i) $x - 1 = 00100$

(ii) $x^* - 1 = 00101$

(iii) $x^* = 00110$




G.2 PREVENTIVE MAINTENANCE SCHEDULING USING LEXICOGRAPHIC ENUMERATION TECHNIQUE

Justification :- Because $f_o$ is monotone nonincreasing none of the vectors x+1, x+2.... can be more costly than $\hat{x}$.

Rule 2 :- If x is a feasible solution skip to $x^*$

Justification:-

Same as above.

Rule 3 :- If $f_{j1}(x^*-1) - f_{j2}(x) \geq 0$ skip to $x^*$

Justification :- With respect to vectors in the interval $[x, x^*-1]$, $x^*-1$ minimise $f_{j1}$ and x maximise $f_{j2}(x)$ due to monotonically non-increasing property. Hence if this is the case that $f_{j1}(x^*-1) - f_{j2}(x) > 0$, there will be no vector x' in the interval such that

$$f_{j1}(x') - f_{j2}(x') \leq 0$$

The above rules form the basis for the development of lexicographic enumeration Technique. This algorithm describe as given and flow chart of it is given in Fig.2.2.

2.3.1. Lexicographic Enumeration Algorithm

Step 1 : If $x = [0,0.....0]$ is feasible to (2.9) it is also optimal otherwise let $x = [0,0.....1]$ and $\underline{f}$ (possibly $-\infty$) be the best lower bound on $f_o(x)$. Go to step 2.

Step 2 : If $f_o(x) \leq \underline{f}$ go to step 5 otherwise go to step 3.

Step 3 : If x is feasible to (2.9) let $\underline{f} = f(x)$ and go to step 5 otherwise go to step 4.

Step 4 : If $x^*$ exist and for some j

$f_{j1}(x^*-1) - f_{j2}(x) > 0$ go to step 5.

Otherwise if $x = [1, \ldots\ldots 1]$ go to step 6

If $x \neq [1, 1, \ldots\ldots 1]$ let x be the vector satisfying

$p(x) = p(x) + 1$, but $x = x$ and go to step 2.

Step 5 : If $x^*$ does not exist go to step 6. Otherwise let $x = x^*$ and go to step 2.

Step 6 : Terminate if $\bar{f} = (-\infty)$ has no feasible solution. Otherwise the solution that yield $\underline{Z}$ is optimal.

## 2.4 EXAMPLES

A simple three unit example as given in Table 2.2 is solved to show the actual operation of scheduling technique. The objective function choosen is that of minimum Lateness penelty schedule. Now as there is element of uncertainity associated with the reserve available for maintenance purposes due to error in demand forecast and random failure of generating units. The reserve available for maintenance purposes is given in Table 2.4.

TABLE 2.4

| Week No. | Mean value of Reserve | Variance |
|---|---|---|
| Ist | 150 | 20 |
| IInd | 170 | 25 |
| IIIrd | 180 | 16 |
| IVth | 120 | 16 |

Now as given in (2.5) along with the equation (2.7) the resources constraint can be written as

$$P\left[\sum_{i=1}^{N} d_i \; x_{ij} \leq R_j\right] \geq .99$$

Where .99 is the probability that $\sum_{i=1}^{N} d_i \; x_{ij}$ lesser than $R_j$.

Therefore these constraint along with objective function is written as

Minimise

$$Z = x_{12} + 2\,x_{13} + x_{22} + 2\,x_{23} + x_{32} + x_{33}$$

Subject to : (1) Each unit must be maintained once.

i.e. $x_{11} + x_{12} + x_{13} = 1$

$$x_{21} + x_{22} + x_{23} = 1$$

$$x_{32} + x_{33} = 1$$

Sequencing Constraint : Maintenance on 3rd unit begins immediately after maintenance on Ist is completed.

i.e.

$$x_{21} - x_{32} = 0$$

$$x_{22} - x_{33} = 0$$

$$x_{23} = 0$$

Resources Constraint : Now the value of standard normal variable at which probability is .99 is 2.33. Therefore these constraints are written as

TABLE - 2.5

SOLUTION OF EXAMPLE 2.4 USING LEXICOGRAPHIC ENUMERATION TECHNIQUE

| S No. | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{32}$ | $x_{33}$ | $x^o$ | $x^o$-1 | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 00000010 | 00000001 | Constraint not satisfied, Put x = x |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 00000100 | 00000011 | 〃 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 00000100 | 00000011 | Constraint violated skip to x = $x^o$ |
| 4 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 00001000 | 00000111 | 〃 |
| 5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 00010000 | 00001111 | Put x = $x^o$+1 |
| 6 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 00001010 | 00001001 | Put x = $x^o$+1 |
| 7 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 00001100 | 00001011 | Constraint violated skip to x = $x^o$ |
| 8 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 00010000 | 00001111 | 〃 |
| 9 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 00100000 | 00011111 | Put x = $x^o$+1 |
| 10 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 00010010 | 00010001 | Constraint violated skip to x = $x^o$ |
| 11 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 00010100 | 00010011 | Put x = $x^o$+1 |
| 12 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 00010100 | 00010011 | Put x = $x^o$+1 |
| 13 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 00011000 | 00010111 | Constraint violated skip to x = $x^o$ |
| 14 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 00100000 | 00011111 | 〃 |
| 15 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 010000000 | 01111111 | Put x = $x^o$+1 |
| 16 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 00100010 | 00100001 | Constraint violated skip to x = $x^o$ |
| 17 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 00100100 | 00100011 | Put x = $x^o$+1 |
| 18 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 00100100 | 00100011 | Constraint violated skip to x = $x^o$ |
| 19 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 00101000 | 00100111 | 〃 |
| 20 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 00110000 | 00101111 | Put x = $x^o$+1 |
| 21 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 00101010 | 00101001 | Constraint violated skip to x = $x^o$ |
| 22 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 000101100 | 01010011 | Put x = $x^o$+1 |
| 23 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 00101100 | 00101011 | Constraint violated skip to x = $x^o$ |
| 24 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 00110000 | 00101111 | 〃 |
| 25 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 01000000 | 00111111 | Put x = $x^o$+1 |
| 26 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 00110010 | 00110001 | Constraint violated skip to x = $x^o$ |
| 27 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 00110100 | 00110011 | Feasible Z = -2, Skip to x = $x^o$ |
| 28. | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 00111000 | 00110111 | Constraint violated skip to x = $x^o$ |
| 29. | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 01000000 | 00111111 | 〃 |
| 30 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 10000000 | 01111111 | Put x = $x^o$+1 |
| 31 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 01000010 | 01000000 | Put x = $x^o$+1 |
| 32 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 01000100 | 01000011 | Put x = $x^o$+1 |
| 33 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 01000100 | 01000011 | Constraint violated skip to x = $x^o$ |
| 34 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 01001000 | 01000111 | 〃 |
| 35 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 01010000 | 01001111 | x = x 〃 |
| 36 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 01001010 | 01001001 | 〃 |
| 37 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 01100000 | 01011111 | Put x = $x^o$+1 |
| 38. | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 01010010 | 01010001 | Constraint violated skip to x = $x^o$ |
| 39 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 01010[illegible]00 | 01010011 | Feasible [illegible], skip to x = $x^o$ |
| 40 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 01011000 | 01010111 | Constraint violated skip to x = $x^o$ |
| 41 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 01100000 | 01011111 | 〃 |
| 42 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 10000000 | 01[illegible]11111 | 〃 |
| 43 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | Put x = $x^o$+1 |
| 44 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 10000010 | 10000001 | Put x = $x^o$+1 |
| 45 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 10000100 | 10000011 | Put x = $x^o$+1 |
| 46 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 10001000 | 10000111 | Constraint violated skip to x = $x^o$ |
| 47 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 10010000 | 10001111 | Put x = $x^o$+1 |
| 48 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 10001010 | 10001001 | Feasible Z = -2, skip to x = $x^o$ |
| 49 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 10001100 | 10001011 | Feasible Z = -1, skip to x = $x^o$ |
| 50. | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 10010000 | 10001111 | Constraint violated skip to x = $x^o$ |
| 51 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 10100000 | 10011111 | Put x = $x^o$+1 |
| 52 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 10010010 | 10010001 | Constraint violated skip to x = $x^o$ |
| 53 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 10010100 | 10010011 | 〃 |
| 54 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 10011000 | 10010111 | 〃 |
| 55 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 10100000 | 10011111 | 〃 |
| 56 | 1 | 0 | [illegible] 1 | 0 | 0 | 0 | 0 | 0 | TERMINATE | | 〃 |
| 57 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | | | |

$$80\ x_{11} + 110\ x_{21} \leq 150 + 2.33\sqrt{20} \qquad = 162.8$$

$$80\ x_{12} + 110\ x_{22} + 50\ x_{32} \leq 170 + 2.33\sqrt{25} \quad = 181.65$$

$$80\ x_{13} + 110\ x_{23} + 50\ x_{33} \leq 180 + \sqrt{16}\ .233 = 189.32$$

$$80\ x_{13} + 50\ x_{33} \leq 120 + 2.33\sqrt{16} \qquad = 129.32$$

Now for use of lexicographic enumeration technique these must be monotically non increasing. Therefore the above stated problem can be written as

Maximise :

$$Z = -x_{12} - 2x_{13} - x_{22} - 2x_{23} - x_{32} - x_{33}$$

Subject to :

$$-(-x_{11} - x_{12} - x_{13} + 1) \qquad = 0 \qquad (1)$$

$$-(-x_{21} - x_{22} - x_{23} + 1) \qquad = 0 \qquad (2)$$

$$-(-x_{32} - x_{33} + 1) \qquad = 0 \qquad (3)$$

$$-(-80x_{11} - 110x_{21} + 162.8) \leq \qquad = 0 \qquad (4)$$

$$-(-80x_{12} - 110x_{22} - 50x_{32} + 181.65) \leq 0 \qquad (5)$$

$$-(-80x_{13} - 110x_{23} - 50x_{33} + 189.32) \leq 0 \qquad (6)$$

$$-(-80x_{13} - 50x_{33} + 129.32) \qquad \leq 0 \qquad (7)$$

and

$$-x_{32} - (-x_{21}) \leq 0 \qquad ( \qquad (8)$$

$$-x_{33} - (-x_{22}) \leq 0 \qquad (9)$$

$$- (-x_{23}) \leq 0 \qquad (10)$$

The solution of above problem using lexicographic enumberation technique is given in Table No.2.5. The

TABLE - 2.6

SOLUTION OF EXAMPLE 2.4b USING LEXICOGRAPHIC ENUMERATION TECHNIQUE

| Sl. No. | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{32}$ | $x_{33}$ | $x^*$ | $x^*-1$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 0 0 0 0 0 0 0 | 0 1 1 1 1 1 1 1 | |
| 35 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | Put $x = \hat{x}$ |
| 36. | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 0 0 0 0 0 1 0 | 1 0 0 0 0 0 0 1 | Put $x = \hat{x}$ |
| 37 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 0 0 0 0 1 0 0 | 1 0 0 0 0 0 1 1 | Put $x = \hat{x}$ |
| 38. | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 0 0 0 0 1 0 0 | 1 0 0 0 0 0 1 1 | Constraint violated skip to $x = x^*$ |
| 39. | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 0 0 1 1 0 0 0 | 1 0 0 0 0 1 1 1 | ″ |
| 40 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 0 0 1 0 0 0 0 | 1 0 0 0 1 1 1 1 | Put $x = \hat{x}$ |
| 41 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 0 0 0 1 0 1 0 | 1 0 0 0 1 0 0 1 | To obtain $z = -2$, skip to $x = x^*$ |
| 42 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 0 0 0 1 1 0 0 | 1 0 0 0 1 0 1 1 | Constraint violated skip to $x = x^*$ |
| 43 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 0 0 1 0 0 0 0 | 1 0 0 0 1 1 1 1 | ″ |
| 44 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 0 1 0 0 0 0 0 | 1 0 0 1 1 1 1 1 | ″ |
| 45 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 1 0 0 0 0 0 0 | 1 0 1 1 1 1 1 1 | ″ |
| 46 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | Terminate | | ″ |

optimal solution found is

$$x_{12} = x_{21} = x_{32} = 1$$

and $$x_{11} = x_{22} = x_{23} = x_{33} = x_{13} = 0$$

i.e. the maintenance on Ist unit begins in 2nd week, second on first and third on 2nd and corresponding the maximum value of objective function is $Z = -1$.

The more constraints can be added in the above stated example, one of the important constraint is that maintenance can't start simultaneously on all units. We can write this constraint as

$$-(-x_{11} - x_{21} + 1) \leq 0 \qquad (11)$$

$$-(-x_{12} - x_{22} - x_{32} + 1) \leq 0 \qquad (12)$$

$$-(-x_{13} - x_{23} - x_{33} + 1) \leq 0 \qquad (13)$$

The optimal solution of this using the lexicographic technique is given in Table No.2.6. In this there are only two feasible scheduled which are also optimal and is given by -

(i) $x_{11} = 1$, $x_{22} = 1$, $x_{33} = 1$

That is maintenance on Ist, 2nd and 3rd unit is started in Ist, 2nd and 3rd week respectively.

(ii) $x_{13} = 1$, $x_{21} = 1$ and $x_{32} = 1$

i.e. maintenance on Ist unit begin in third week, 2nd on Ist and on third it begin in 2nd week.

## CHAPTER - III

## GENERATOR MAINTENANCE SCHEDULING USING LEVELIZED RISK CRITERIA

### INTRODUCTION

In generator maintenance scheduling problems, the most important objective is to maximise system reliability throughout the year. There are several types of objective criterion used to fulfil the above requirement. One of them is based on the assumption that reliability of the system is maximised if the reserve is levelised throughout the year. Second is that the levelization of risk also increases the reliability of the system. Levelization of risk over the entire scheduling period is done by computing the effective capability of generating units.

Also in generator maintenance scheduling problems, it is highly convenient for the maintenance scheduler to select as far as possible, the most efficient algorithm to suit the particular needs of a given generator maintenance scheduling problems. Here a new algorithm is developed which exploit the specialized structure of the problem. First the algorithm for maintenance scheduling is developed it is then utilized in the problems of reserve and risk levelization.

### 3.1 OPTIMIZATION TECHNIQUE THEORY

From the mathematical modelling of preventive maintenance scheduling problem developed in equations 2.4 to 2.6 has a specialized form. It is signified by the constraint that each unit must be maintained once during the allowed period which form a part of the constrained set. That is

$$\sum_{i \in N} \sum_{j \in T} x_{ij} = 1 \qquad (3.1)$$

Stated in wards equation (3.1) says that summation of all the variables corresponding to each stage is unity. Thus in any enumerated solution vector only one variable for each stage will be unity and all the remaining will be zero. Even if the requirement of maintaining a unit is twice such a requirement is modelled by replacing the single unit by two equivalent units of the same capacity and maintenance is seperated by a fixed time horizon. Therefore, by using this property large number of infeasible solutions are never generated and the direct search optimisation technique should be in a position to generate the remaining set of non-redundant solution vectors.

Now any solution vector Q is composed of N independent subvectors, where N is the number of stages in the problem. Each subvector further consist of $j_i$ variables where $j = (1,2,..T)$ is the number of variables or components in $i^{th}$ subset. The subscripts of x gives the number of subvector $(i=1,2,... N)$ and $j = 1,2,.... T$ gives the number of variables in any subset.

Now if there are j components in $i^{th}$ subset, then these correspond to j locations which are to be occupied by $a_i$ objects. Then the number of ways in which $a_i$ objects are occupied at $j_i$ locations without repetition is termed as permutation and is given by

$$^{a_i}_{a_i}P_{ji} = \frac{\lfloor j_i}{\lfloor j_i - a_i}$$

Therefore for N subvectors the number of possible solution vector S is given by as follows -

$$S = {}^{a_1}P_{j1} \times {}^{a_2}P_{j2} \cdots\cdots {}^{a_N}P_{jN}$$

$$= \frac{\lfloor j_1}{\lfloor j_1 - a_1} \times \frac{\lfloor j_2}{\lfloor j_2 - a_2} \cdots\cdots\cdots \frac{\lfloor j_N}{\lfloor j_N - a_N}$$

Now here each variable in $i^{th}$ subset is unity i.e. $a_1 = a_2 \cdots = a_N = 1$

Therefore total no. of solution vector S is given by

$$S = j_1 \times j_2 \cdots\cdots j_n = \prod_{i=1}^{N} j_i \quad \text{Where } j = (1,2,\ldots T)$$

3.1.1. Development of Direct Search Optimisation Procedure

Now it is desirable that for the generation of solution vectors some systematic technique should be adopted so as to give the total number of solution vectors in a systematic way. For this an efficient technique is developed and is given below -

Let us consider that there are N subvectors and each subvectors has j variable e.g. $X_{ij}$ denotes the $i^{th}$ subvector having j variables, Now the initial solution vector S has its right most element as unity in each subvectors and all other elements are zero. This serves as the reference point for generating further solutions. Now keeping right hand side elements up to $x_{i-1,j}$ as unity, shift the unity entry of $i^{th}$ subvector one position towards left hand side such that now $x_{i,j-1}$ has a unity entry in $i^{th}$ subvector at $(j-1)^{th}$ place and all other are zero in it. This gives the 2nd possible solution vector. Thus keeping $x_{i-1,j}$ unity element at their original places, shift systematically the unity element of $i^{th}$ subvector towards left hand side, till it occupies $x_{i1}$ position in $i^{th}$ subset.

After this the unit element of $x_{i-1,j}$ shifts towards left hand side as that it now occupies $x_{i-1,j-1}$ position and unit element in the $i^{th}$ subvector goes to its original place i.e. at extreme right hand side. Again shift systematically the unit element of $i^{th}$ subvector toward left hand side. Repeat the whole process until unit element of i-1 and i subvector occupies the $x_{i-1,1}$ and $x_{i1}$ position respectively. Now when this condition is reached the unit element of $x_{i-2,j}$ is shifted one position toward left and $x_{i-1,1}$ and $i_1$ to their original positions. In this way the whole procedure is repeated until unit element in every subvectors occupies the extreme left hand position. Thus in this way we get all the possible solution vectors having unit element in each subvectors. By using the above proposed technique a large number of infeasible solution vectors are removed in the ~~enunciation~~ enumeration process.

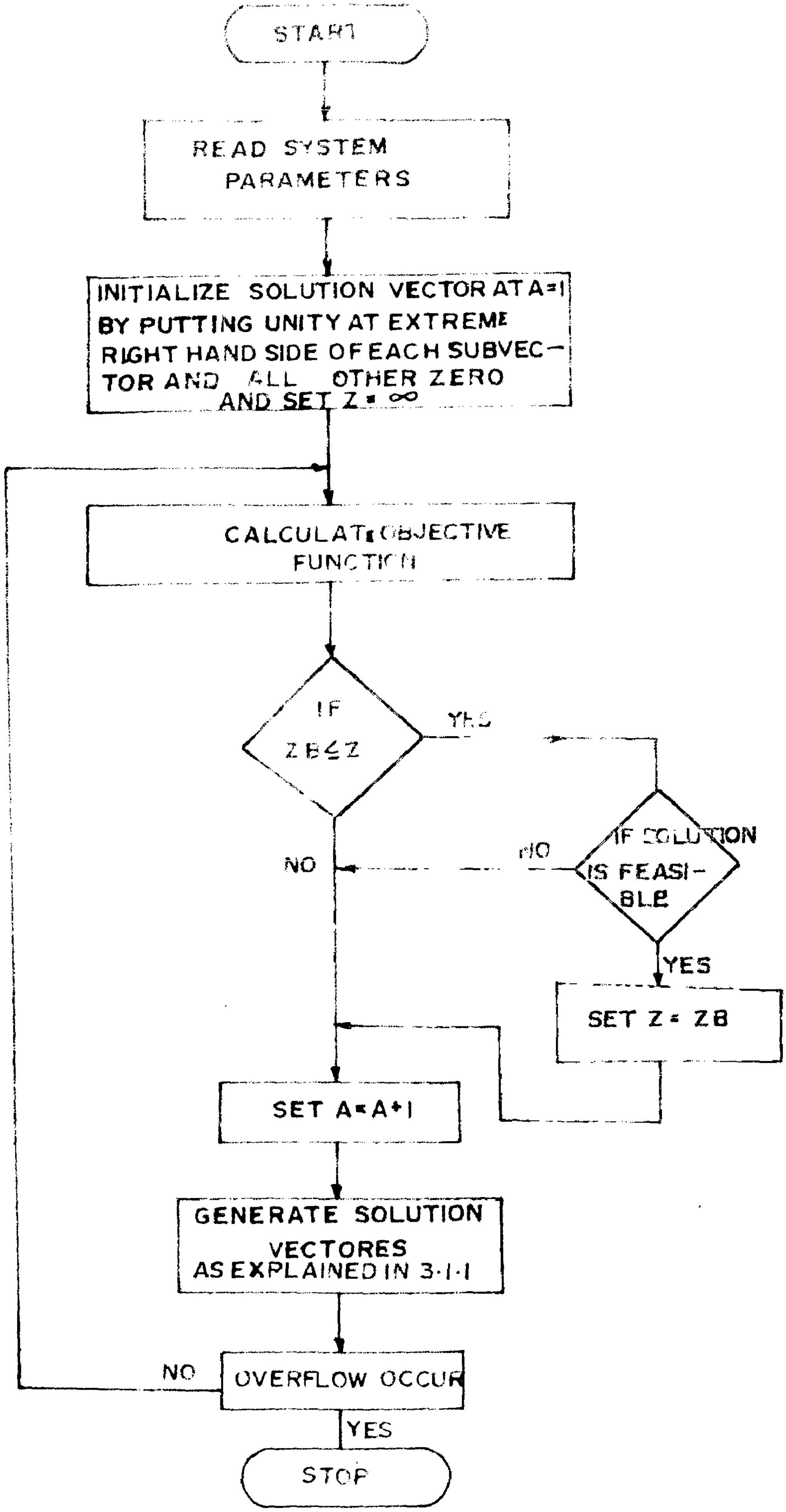



FIG.3.1 DIRECT SEARCH OPTIMISATION TECHNIQUE FOR MAINTENANCE SCHEDULING

3.1.2. Algorithm

The complete algorithm for maintenance scheduling is explained as follows -

(1) Read system Parameters - Cost coefficients, constraints and limits imposed on them.

(2) Initialize solution vector such that the extreme right hand side element of each subvector is unit and all others are zero.

In this case A = 1 and $Z = \infty$, where A is the number of solution vector and Z is the objective function to be optimised.

(3) Calculate objective function ZB and check if it is lesser than Z. If Yes go to Step 4. Otherwise move to Step 6.

(4) Search for a feasible solution. If it does not exist go to Step 6 otherwise to Step 5.

(5) Set Z = ZB minimum and move to Step 6.

(6) Generate solution vectors by the procedure explained already. If overflow occur stop, otherwise go to Step 3.

The complete procedure for the direct search optimisation technique is given in Fig. 3.1. Using this procedure the example ( First ) of Chapter II is solved and is given in Table 3.1. Here total number of possible solution vectors are given by

$$\prod_{j=1}^{N} = 3 \times 3 \times 2 = 18$$

TABLE - 3.1

SOLUTION OF EXAMPLE 2.4 USING DIRECT SEARCH OPTIMISATION TECHNIQUE

| S.No. | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{32}$ | $x_{33}$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | Constraints not satisfied |
| 2 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | Constraints are violated |
| 3 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | Infeasible Z = [illegible] |
| 4 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | Constraints not satisfied |
| 5 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | Constraints not satisfied |
| 6 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | Feasible Z = 2 |
| 7 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | Infeasible constraints violated |
| 8 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | Infeasible constraints violated |
| 9 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | Constraints are violated |
| 10 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | Constraints are violated |
| 11 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | Constraints are violated |
| 12 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | Solution is infeasible |
| 13 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | Solution is infeasible |
| 14 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | Solution is infeasible |
| 15 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | Feasible Z = 2 |
| 16 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | Infeasible |
| 17 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | Constraints violated |
| 18 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | Constraints are violated |

Therefore the optimal solution found is same as in example 2.1, also the total number of solution vectors skipped here are more than that in previous method. Thus this technique increases the efficiency of direct search optimisation.

3.1.3. Advantages

The optimisation technique developed above has the following advantages -

(1) Here every move is in forward direction and no back-tracking is required.

(2) A large number of solution vectors which do not satisfy the problem format are never generated.

(3) It not only gives optimal solution but also the set of all feasible schedules available.

(4) Since the constraint that each unit should be maintained once is satisfied the total number of solution vectors are reduced greatly.

Using the above technique the problem of levelization of reserve and risk is solved. The complete procedure for it is described in the succeeding section.

3.2. RESERVE LEVELIZING

In levelling the reserve the basic assumption is that it will maximise system reliability. Reserve levelizing prevent large variations in reserve permitted. This variation occurs when the systems minimum reserve determined by one interval is different from the other. Thus the objective here is to maximise the single variable, which in this case is the minimum net reserve for all time periods. Intuitively the larger the minimum net reserve, the greater the system reliability. The problem of reserve levelizing can be written as

$$\text{Maximise Min } (R_j = j = 1,2,\ldots\ T)$$

$$\text{or} \quad \text{Minimise} \sum_{j=1}^{T} (R_j)^2 \qquad \ldots (3.1)$$

$$\text{Where} \quad R_j = S - L_{pj} - \sum_{i=1}^{N} c_i\, x_{ij}$$

Where $R_j$ = Reserve available for maintenance at $j^{th}$ time period.

$S$ = gross system capacity

$C_i$ = Reserve loss due to maintenance activity i

$L_{pj}$ = Predicted peak load for interval j.

Also there are some periods during the year when the load can be predicted more accurately than in others. It would be desirable to perform more maintenance in periods of accurate prediction so as to avoid the possibility of large load variations at other times reducing the effective reserve. Thus in 3.1, it would be best to use a value of predicted load which has a constant probability of exceeding the actual load, i.e. $L_{pj}$ is defined such that the probability $P\left[L_j \leq L_{pj}\right] \geq .99$ for each interval, Where $L_j$ is the actual load for interval j. This is again converted in to its deterministic equivalent by using chance constrained programming as already described in the previous chapter. Thus the problem of reserve levelizing reduces to minimization of

Minimise $$\sum_{j=1}^{T} (S - L_{pj} - \sum_{R=1}^{N} C_i\, x_{ij})^2$$

Sub. to $\sum_{j \in T} a_i\, x_{ij} \leq b_i$ $(i = 1,2,\ldots N)$. It leads to the greatest possible amount of maintenance schedule, as any extra maintenance will increase the sum of the squares of the reserve.

3.2.1. Sample Application

The system consists of three units each having capacity of 50, 20 and 10 MW respectively. Maximum predicted load for each interval is 10, 20 and 55 MW with variance 5, 5, and 10 respectively. The constraint on the maintenance schedule is

(1) There must be exactly one maintenance done on each unit in the given method.

(2) The load must always be met, i.e. -ve reserve is not allowed.

Now converting the probabilistic constraint in to its deterministic equivalent the values of $L_1$, $L_2$ and $L_3$ comes respectively by using equation 2.8 as

$$L_1 = 10 + 2.33\sqrt{5} = 15.2100$$

$$L_2 = 20 + 2.33\sqrt{5} = 25.2100$$

$$L_3 = 55 + 2.33\sqrt{10} = 62.36$$

So in the light of above constraint and cost functions the problem is formulated as

TABLE - 3.2

PROBLEM : RESERVE LEVELIZATION

| | $x_{11}$ | $x_{12}$ | $x_{12}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{31}$ | $x_{32}$ | $x_{33}$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | Infeasible constraint 6th is violated |
| 2 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | " |
| 3 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | " |
| 4 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | " |
| 5 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | " |
| 6 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | " |
| 7 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | " |
| 8 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | " |
| 9 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | " |
| 10 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | " |
| 11 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | " |
| 12 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | " |
| 13 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | Constraint 5 & 6 violated |
| 14 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | " |
| 15 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | " |
| 16 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | Feasible Z=2083.45 |
| 17 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | Constraint are violated |
| 18 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | Feasible Z = 1544.4578 |
| 19 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | Infeasible constraint 6th is violated. |
| 20 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | " |
| 21 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | " |
| 22 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | Feasible Z = 1487.45 |
| 23. | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | Feasible Z = 114445 |
| 24 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | Feasible Z = 1544.45 |
| 25 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | Constraint set is violated |
| 26 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | " |
| 27 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | " |

$$\text{Minimise } (64.79 - 50\, x_{11} - 20\, x_{21} - 10\, x_{31})^2 + (54.79 - 50\, x_{12} - 20 x_{22} - 10\, x_{32})^2 + (17.64 - 50\, x_{13} - 20\, x_{23} - 12\, x_{33})^2$$

Sub. to -

$$x_{11} + x_{12} + x_{13} = 1 \qquad (1)$$

$$x_{21} + x_{22} + x_{23} = 1 \qquad (2)$$

$$x_{31} + x_{32} + x_{33} = 1 \qquad (3)$$

$$50\, x_{11} + 20\, x_{21} + 10\, x_{31} \leq 64.79 \qquad (4)$$

$$50\, x_{12} + 20\, x_{22} + 10\, x_{32} \leq 54.79 \qquad (5)$$

$$50\, x_{13} + 20 x_{23} + 10 x_{33} \leq 17.64 \qquad (6)$$

The problem is solved using the direct search optimisation technique and its solution is given in ~~Chart~~ Table No. 3.2 The optimal value for maintenance scheduling is as $x_{11} = 1$, $x_{22} = 1$, $x_{32} = 1$.

$$x_{12} = x_{13} = x_{21} = x_{23} = x_{31} = x_{33} = 0$$

3.3 RISK LEVELIZING

The risk levelizing approach seeks to levelize risk. The risk levelizing approach is better than the reserve levelizing approach due to its more accurate treatment towards the objectives. In fact the reserve levelizing is same as the risk lev-elizing provided capacity forced outage probabilities are linearly related to capacity forced outage magnitudes. The loss of load probability method is used to compute the risks, which is used to calculate the effective load carrying capability of generating units, i.e. the load ~~carr~~ that the unit may carry within the designated reliability. The effective capacity of a unit is a function of the generator forced outage rates as well as the reliability characteristic of the system.

The procedure adopted in calculating the effective capability of generating units is given below -

(i) Capacity Outage Probability Table

The first requirement in the process of calculating each unit effective capability is to built a capacity outage probability table. The capacity outage probability table gives the probability of having a certain number of megawatts or more on forced outage. In this each unit is assumed to be connected in parallel and merged in to one another to permit the development of a capacity outage table. Thus there are total $2^n$ possible states in capacity outage table where n is the total number of units.

Example -

Consider a power system having the following generating units and the forced outage rates as given in Table 3.3.

TABLE 3.3

| Capacity | Force Outage Probability | Success Probability |
|---|---|---|
| 100 | 0.1 | 0.9 |
| 70 | 0.05 | 0.95 |
| 50 | 0.09 | 0.91 |

From these availability and outage rates, Table 3.4 is obtained as given

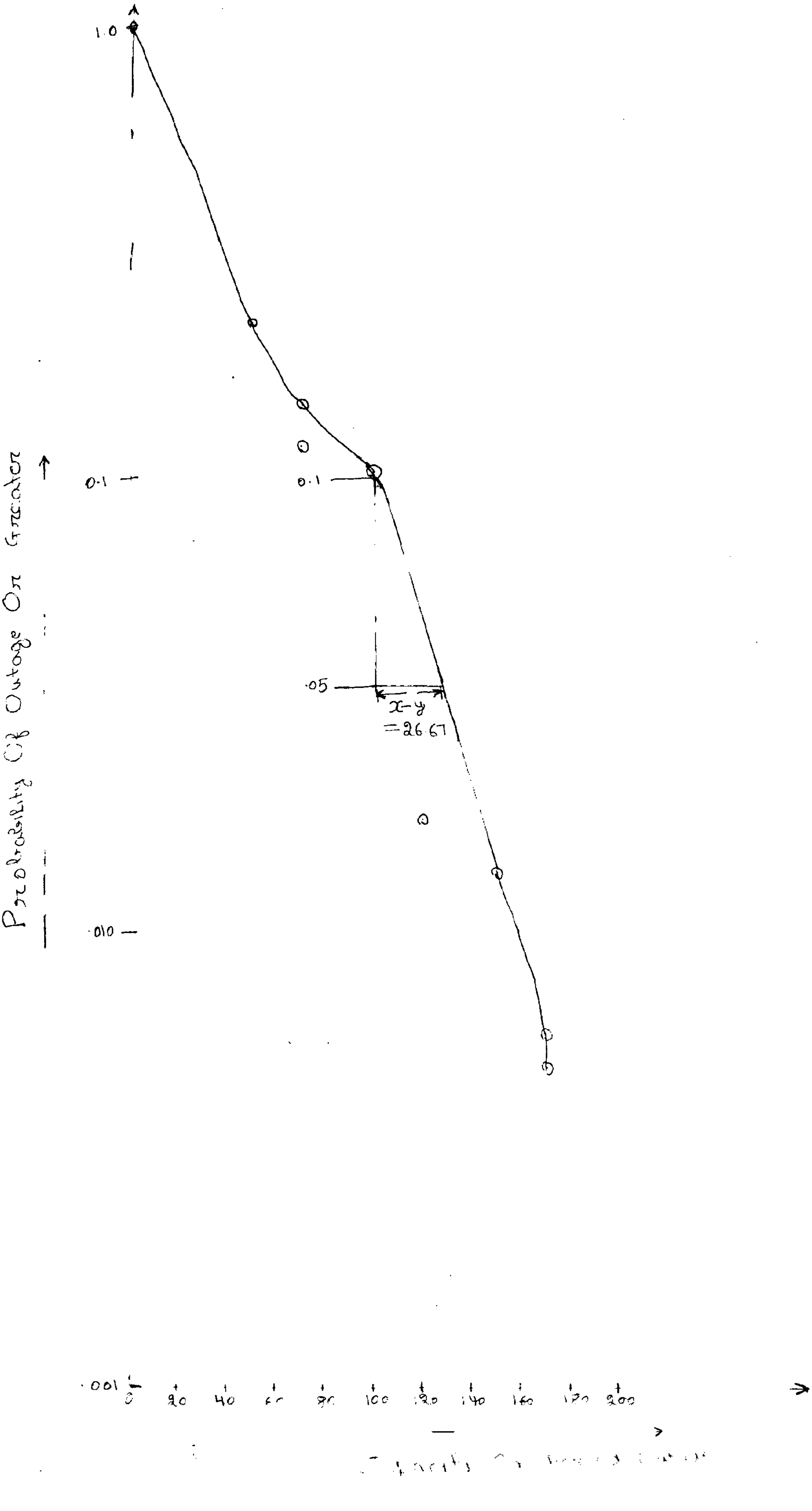
TABLE 3.4

| State | Outage | Probability of Outage | Cumulative Probability or outage |
|---|---|---|---|
| 1 | 0 | 0.77805 | 1.00 |
| 2 | 50 | 0.07695 | 0.22195 |
| 3 | 70 | 0.04095 | 0.14500 |
| 4 | 100 | 0.08645 | 0.10405 |
| 5 | 120 | 0.00405 | 0.01760 |
| 6 | 150 | 0.00855 | 0.01355 |
| 7 | 170 | 0.00455 | 0.00500 |
| 8 | 220 | 0.00045 | 0.00045 |

It gives the probability of outage of certain MW or more on forced outage. Thus for example probability of outage of 100 MW or more is .10405. Similarly for other states also the probability of outage can be found.

(ii) Estimating System Characteristic

Next step requires the calculations of system characteristic m. It is defined as the megawatt variation in capacity outage that will increase the probability by a multiple of 2.718. The general expression to calculate m is given in Appendix I. To calculate system characteristic graphically a straight line is drawn on the semi-log plot



Probability Of Outage Or Greater
1.0
0.1
·010
·001
0.1
·05
x-y
=26.67
0
20
40
60
80
100
120
140
160
180
200

of capacity outage versus the cumulative probability of outage. The straight line is generally drawn between the 0.1 probability level and .1/260 level. These are selected on assuming that there are at the most 260 week days peak loads. However any level of risk could be substituted if desired and the two points adjusted accordingly. Thus from the semilog plot as shown in Fig. 3.2, the value of m given according to Appendix I is

$$m = 28 / \ln \frac{.1/.35}{} = 26.67 \text{ MW}$$

(iii) Calculating Effective Capability

Effective capability of a unit may be estimated once the characteristic m is determined. The analytical treatment for estimating the effective load carrying capability of generating units is given in Appendix II. According to it the effective capability is given as

$$C^{*} = C - m \ln \left[ (1-r) + r\, e^{c/m} \right]$$

Where

$C^{+}$ = effective capability of unit

C = actual unit rating

r = forced outage rate of generating unit

m = megawatt of reserve decrease that will increase the risk 2.718 times.

Thus effective load carrying capabilities of the three units found by using above equation are as given in Table 3.5.

TABLE 3.5

| No. | Rating MW | Forced Outage Rate | Effective Capability |
|---|---|---|---|
| 1 | 100 | 0.1 | 56.24 |
| 2 | 70 | 0.05 | 56.80 |
| 3 | 50 | 0.09 | 39.25 |

From above table it is clear that removal of any unit for maintenance (for example 100 MW unit) has an effect on risk equivalent to increasing the load equal to effective load carrying capability (56.24 MW) of that unit.

(iv) Adjusting Maintenance Schedules

Once the effective load carrying capabilities of generating units are found with the help of data available the objective of maximising system reliability while scheduling maintenance is done by levelling the risk. Here these capacities are substituted for actual capacities and the method proceeds in exactly the same manner as in levelizing reserve technique. Thus here the objective function is expressed as

$$\text{Levelize } R_j \quad \left[ j = 1, 2, \ldots.. T \right]$$

Where

$$R_j = S' - L_{ej} - \sum_{i=1}^{N} C_i' \, x_{ij}$$

Where $L_{ej}$ = equivalent load for the interval j which has the same risk as the actual load. This is introduced because of the fact that two peak loads in different intervals might have different variation in the load level and thus have different risks.

The equivalent load is calculated by calculating the risk associated with each of daily peak loads. Then it is averaged. This average risk is used to compute equivalent risk as -

$$L_{ej} = \text{Largest load} + m\ (\text{Ln Average Risk})$$

Also $C'_i$ = effective load carrying capability of unit i.

$S'$ = effective capacity of the system

$C'_i$ and $S'$ are calculated taking into account the forced outage rates.

The levelling objective can be expressed as

$$\text{Maximize min} \left[ R'_j, \quad j = 1,2,\ldots\ldots T \right]$$

or

$$\text{Minimise} \sum_{i=1}^{T} (R'_j)^2$$

$$\text{Sub. to} \sum_{j=1}^{T} a_i\, x_{ij} \leq b_i \quad \left[ i = 1,2,\ldots\ldots N \right]$$

Now here the constraints are same as in the reserves levelizing problem. It also include an additional constraint that during a particular interval maintenance can started over any one unit only. Also the equivalent loads during each interval is given as 70, 100 and 80 MW respectively.

TABLE - 3.6

PROBLEM : LEVELIZATION OF RISK

| | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{31}$ | $x_{32}$ | $x_{33}$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | Infeasible constraint 9th is violate |
| 2 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | " |
| 3 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | " |
| 4 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | " |
| 5 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | Constraint 8 is violated |
| 6 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | " |
| 7 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | Constraints are violated |
| 8 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | Infeasible Z = 1076.3426 |
| 9 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | Infeasible constrainsts are violated |
| 10 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | " |
| 11 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | " |
| 12 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | " |
| 13 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | " |
| 14 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | " |
| 15 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | " |
| 16 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | " |
| 17 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | " |
| 18 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | " |
| 19 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | " |
| 20 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | FeasibleZ=1088.5842 |
| 21 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | Infeasible constraints 4 is violate |
| 22 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | Infeasible constraint 5 is violated |
| 23 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | " |
| 24 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | " |
| 25 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | " |
| 26 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | " |
| 27 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | " |

So in the light of above constraints the problem is formulated as -

Minimise

$$(82.29 - 56.24\, x_{11} - 56.80\, x_{21} - 39.25\, x_{31})^2$$
$$+(52.29-56.24\, x_{12}-56.80\, x_{22}-39.25\, x_{32})^2+(72.29-56.24\, x_{13}$$
$$-56.80\, x_{23} - 39.25\, x_{33})^2$$

Sub. to -

$$x_{11} + x_{12} + x_{13} = 1 \quad (1)$$
$$x_{21} + x_{22} + x_{23} = 1 \quad (2)$$
$$x_{31} + x_{32} + x_{33} = 1 \quad (3)$$
$$56.24\, x_{11}+56.80\, x_{21}+39.25\, x_{31} \leq 82.29 \quad (4)$$
$$56.24\, x_{12}+56.80\, x_{22}+39.25\, x_{32} \leq 52.29 \quad (5)$$
$$56.24\, x_{13}+56.80\, x_{23}+39.25\, x_{33} \leq 72.29 \quad (6)$$
$$x_{11} + x_{21} + x_{31} = 1 \quad (7)$$
$$x_{12} + x_{22} + x_{32} = 1 \quad (8)$$
$$x_{13} + x_{23} + x_{33} = 1 \quad (9)$$

Here only two feasible solution are found and optimal solution is as given in Table 3.6. According to that the possible levelization of risk is done when

$x_{13} = x_{21} = x_{32} = 1$ and all other are zero.

# CHAPTER - IV

## GENERATOR MAINTENANCE SCHEDULING USING FREQUENCY AND DURATION CRITERIA

### INTRODUCTION

There are at present several methods for calculating the reliability of generating units. The standard frequency and duration technique would be particularly helpful when a reliability assessment has to be carried out for power systems, as in the case of scheduling generator maintenance that minimize risk for the system. In such cases the risk resulting from every plausible maintenance schemes must be evaluated before making a choice.

The standard frequency and duration method can be tedious due to the large number of calculations involved. Here a new and efficient algorithm that minimises the risk is used for generator maintenance scheduling problems. For this first, an approximate frequency and duration technique is presented. This rounds many states of generating system model, thus avoiding full scale mathematical optimisation which is very tedious and time consuming for large number of generating units. After this the load model is used and combined with generating system model to compute margin states, availabilities and risk indices. Then the proposed generator maintenance scheduling technique is outlined and assesed.

## 4.1 GENERATING SYSTEM MODEL

In modelling the generating system, the units are assumed to be connected in parallel. Each unit is defined by a given maximum capability, mean up-time and mean down-time, enabling to calculate the availabilities of any state, or the combinations of states which are of interest. Each unit, in turn, may be merged into a generation system model to permit the development of a capacity model. This model is characterized by the existence of various amounts of capacity available and the stationary probabilities. The availabilities, failure rates and repair rates are related with mean up-time and down-time by the relations given below -

$m = 1/\lambda$ mean up time (days)

$r = 1/\mu$ mean down time or repair time (days)

Availability $= A = m/(m+r)$

Where $\lambda$ and $\mu$ are failure and repair rate per unit time.

Consider a set of three generating units in parallel. Therefore total possible states are $2^3$. The description of these eight states is given in Table 4.1.

Where $\lambda + i$ = rate of transition out of a given capacity state i to one where more capacity is available.

$\lambda - i$ = rate of transition in downward direction of state i.

TABLE 4.1

POSSIBLE STATES OF THREE REPAIRABLE GENERATORS IN PARALLEL

| State No. | $G_1$ | $G_2$ | $G_3$ | Rate of Departure $\lambda - i$ | $\lambda + 1$ |
|---|---|---|---|---|---|
| 1 | Up | Up | Up | $\lambda_1 + \lambda_2 + \lambda_3$ | 0 |
| 2 | Up | Up | Down | $\lambda_1 + \lambda_2$ | $u_3$ |
| 3 | Up | Down | Up | $\lambda_1 + \lambda_3$ | $u_2$ |
| 4 | Up | Down | Down | $\lambda_1$ | $u_1 + u_2$ |
| 5 | down | Up | Up | $\lambda_2 + \lambda_3$ | u |
| 6 | Down | Up | Down | $\lambda_2$ | $u_1 + u_3$ |
| 7 | Down | Down | Up | $\lambda_3$ | $u_1 + m_2$ |
| 8 | Down | Down | Down | 0 | $u_1 + u_2 + u_3$ |

Example

The complete data for three generating units in parallel is given in Table 4.2.

TABLE 4.2

DESCRIPTION OF THE PROBLEM

| Unit | No.of crews | Capacity | Mean time days Up | Down | Maintenance data Duration weeks | Earliest starting week | Latest poss ble startin week |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 30 | 99 | 1.0 | 4 | 15 | 26 |
| 2 | 1 | 50 | 98.5 | 1.5 | 3 | 13 | 26 |
| 3 | 2 | 35 | 99.0 | 7.0 | 4 | 7 | 13 |

Therefore from this we find that the Generators $G_1$, $G_2$ and $G_3$ have the following characteristics -

$$\lambda_1 = 1/99 \text{ per day} = 0.010101 \text{ per day}$$

$$\lambda_2 = 1/98.5 \text{ per day} = 0.010152 \text{ per day}$$

$$\lambda_3 = 1/99 \text{ per day} = 0.010101 \text{ per day}$$

$$\mu_1 = 1.0 \text{ per day}, \mu_2 = .66666 \text{ per day}, \mu_3 = 1.0 \text{ per day}$$

| Availability of On State | Availability of Off State |
|---|---|
| $p_1 = 0.99$ | $q_1 = 0.01$ |
| $p_2 = 0.985$ | $q_2 = 0.015$ |
| $p_3 = 0.99$ | $q_3 = 0.01$ |

If in the construction of exact capacity state availability table, identical capacity states exists then the only way that a system can transit at any instant of time from one capacity state to another with the same available capacity is that one generator is repaired and another becoming faulty at the same instant. Thus from the availability, mean up-time and mean down-time Table 4.3 which gives capacity state model is obtained.

TABLE - 4.3

CAPACITY STATE MODEL

| State No. | Capacity | Availability | Cumulative Availability | $\lambda_{down}$ | $\lambda_{up}$ |
|---|---|---|---|---|---|
| 1 | 115 | 0.9654 | 1.00 | 0.03042 | 0 |
| 2 | 85 | 0.00975 | 0.0346 | 0.020242 | 1 |
| 3 | 80 | 0.00975 | 0.02485 | 0.0202 | .66666 |
| 4 | 65 | 0.0147 | 0.0151 | 0.0101 | 1.66666 |
| 5 | 50 | 0.000098 | 0.0004 | 0.020242 | 1 |
| 6 | 35 | 0.000148 | 0.000302 | 0.010142 | 2 |
| 7 | 30 | 0.000148 | 0.000154 | 0.0101 | 1.66666 |
| 8 | 0 | 0.0000015 | 0.000006 | 0 | 2.66666 |

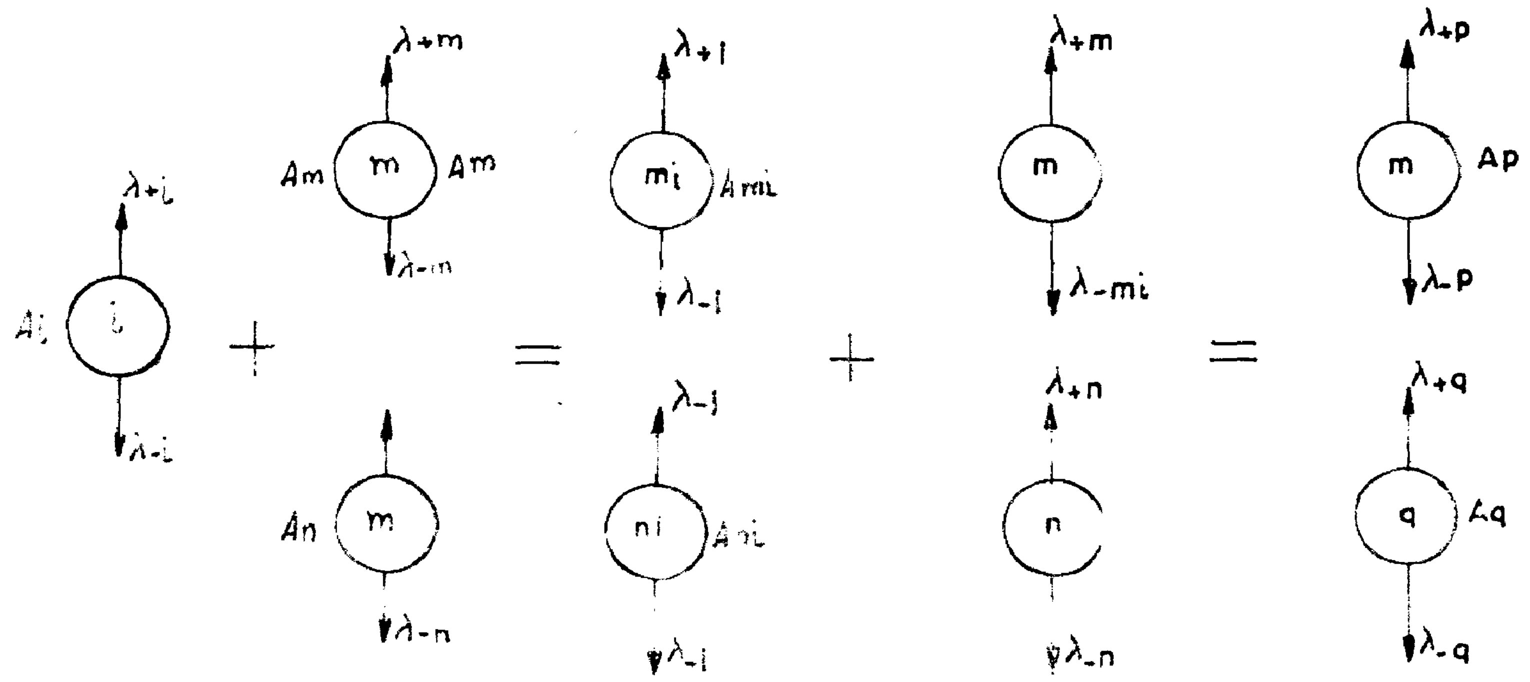



FIG.4.1 ROUTING TECHNIQUE

Rounding Technique

For $n$ number of units, the number of existing capacity states is $2^n$. If this is combined with load model to get capacity reserve model, the calculations become complicated and require too much time.

The basic concepts of rounding technique is to choose in advance a step length (in megawatts) that will predetermine the number of capacity states, and hence the size of the table and the value of each state. However, in this case, a capacity state created by a unit or combination of units may not correspond to any of the predetermined round states. Here the availability and rate of departure must be split between the two rounded states immediately adjacent to the exact state.

As for example consider the state i in capacity table model (capacity $C_i$, Availability $A_i$ rate of departure $\lambda + i$ and $\lambda - i$). Assume that $C_i$ is not equal to any of the rounded state. Let the two rounded states adjacent to $C_i$ be $C_m$ and $C_n$ such that $C_m > C_i > C_n$. The rounding process is illustrated in Fig. 4.1.

The availability $A_i$ is split proportionally to the numerical difference between $C_i$ and $C_m$ and between $C_i$ and $C_n$ as given -

$$A_{mi} = A_i \times \frac{C_i - C_n}{C_m - C_n}$$

and $$A_{ni} = A_i \times \frac{C_m - C_i}{C_m - C_n}$$

Although state i is split but the combination of units by which it was created is not altered, that is, the rates of departure remains same. Now the identical capacity states are combined and the capacity, availability and rate of departure are given as -

(1) Capacity

$$C_p = C_m, \qquad C_q = C_n$$

(2) Availabilities

$$A_p = A_m + A_{mi}, \qquad A_q = A_n + A_{ni}$$

(3) Rate of Departure

$$\lambda + p = \frac{A_m \lambda + m + A_{mi} \lambda + i}{A_p} \qquad \lambda - p = \frac{A_m \lambda - m + A_{mi} \lambda - i}{A_p}$$

$$\lambda + q = \frac{A_n \lambda + n + A_{ni} \lambda + i}{A_q} \qquad \lambda - q = \frac{A_n \lambda - n + A_{ni} \lambda - i}{A_q}$$

The size of cumulative capacity state model is reduced greatly with the help of above described equations. Therefore by applying the rounding tëchnique to the cumulative capacity state model of Table 4.3, the solution given in Table 4.4 is found and is given as -

TABLE - 4.4

CUMULATIVE CAPACITY STATE MODEL

| State No. | Capacity MW | Availability | Cumulative Availability | $\lambda_{up}$ | $\lambda_{down}$ |
|---|---|---|---|---|---|
| 1 | 115 | 0.972225 | 1.00 | 0 | .03022 |
| 2 | 65 | 0.027375 | 0.27775 | 1.2729 | .07230 |
| 3 | 50 | 0.0002898 | 0.0004 | 1.56038 | .01359 |
| 4 | 0 | 0.0001048 | 0.0001102 | 1.82219 | .010042 |

4.2 LOAD MODELLING

For measuring the reliability of the system, generation capacity model alone is not sufficient as it only measures the reliability of generating system. It would seem that a more adequate measure would be one that incorporates a criterion of the expected load pattern. The reliability model used for maintenance purposes must incorporate the calculations of availability of cumulative capacity reserve margin states. The system model gives whether the generating capacity or load is in excess or not during a particular period. It also gives the probability of existence of reserve margin and its frequency.

For this a load model is used. This model represents the daily load cycle as a sequence of peak loads $L_i$, each of a mean duration of e day interspread with period averaging (1-e) day of a fixed, light load. The load model is based on following assumptions -

(1) Daily loads in a period will be represented by a set of N load levels.

(2) The load model is assumed statistically independent.

(3) Load state transition occurs independently of generation state transition.

(4) The mean duration of peak loads is the fraction of a day.

(5) The sequence of daily peak load is a random sequence of N load levels.

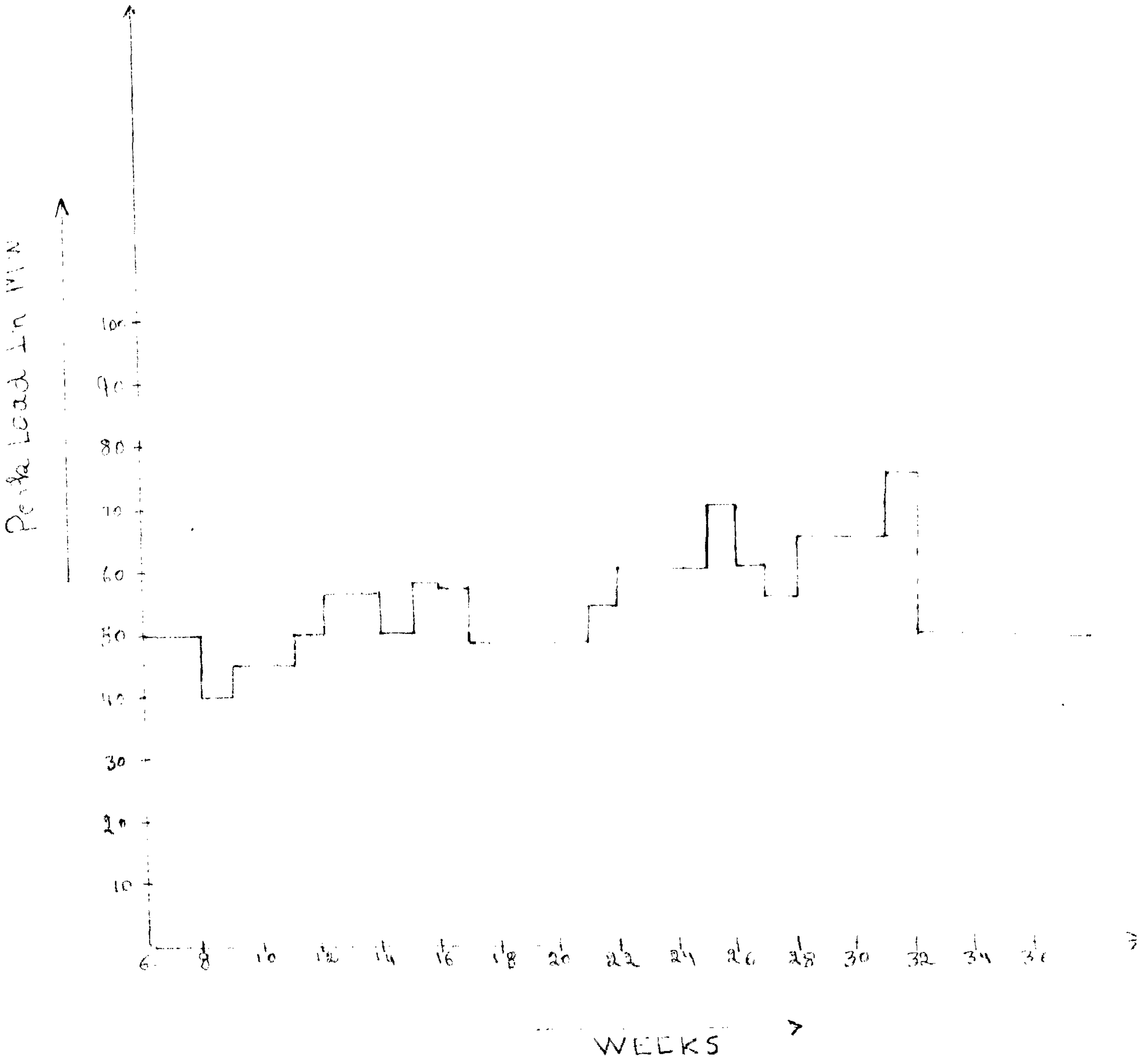



Fig 4.2 Data for Load model

These assumptions allow the construction of a load model. Now calculating the availability of the particular peak load level $L_i$ in the interval D days long is given by

$$A_i = e\ n_i/D$$

Where e is the fraction of a day on average having this peak load level and $n_i$ is the expected number of occurrences of $L_i$ in days. The mean duration of load level $L_i$ is e day.

To illustrate the procedure from the graph of load characteristics during a period of 15 to 18th week, peak loads of 58, 57 and 48 MW are expected to occur $7^{th}$, $7^{th}$ and 14th day respectively. The duration e of each peak load is 1/3 day. Now here the interval length is 28 days. Using the assumptions given above the results are presented and are given in table No.4.5.

TABLE - 4.5

LOAD MODEL

Intermal Length D = 365 days    Exposure Factor e = 1/3 day

| State No. | Load $L_i$ (MW) | Occurence $n_i$(days) | Availability $A_i = n_i\ e/D$ | Departure rates $\lambda_{up}$ | $\lambda$ down=1/e |
|---|---|---|---|---|---|
| 1 | 58 | 7 | 0.0063926 | 0 | 3 |
| 2 | 57 | 7 | 0.0063926 | 0 | 3 |
| 4 | 48 | 14 | 0.012785 | 0 | 3 |

It is desired to solve the reliability of combined system consisting of generation and load model. Both of these are assumed statistically independent and combined into a single system. Reserve or margin is the difference between the available capacity and load. That is we can say that a margin state $m_k$ is the combination of load state $L_i$ and capacity state $C_j$ i.e.

$$m_k = C_j - L_i$$

In order to calculate data on a cumulative margin basis, it also requires the een calculation of rate of departure from $m_k$ to larger and smaller margin states. Now the rate of transfer from a given margin state to one where larger margin is available, is equal to the rate of transfer upward in capacity plus the rate of transfer downward in load.

$$\lambda_{+k} = \lambda_{+c} + \lambda_{-L}$$

$$\lambda_{-k} = \lambda_{-c} + \lambda_{+L}$$

Also the availability of the margin states is given by

$$A_k = A_j A_i$$

Combined, they yield the occurrence frequency of these exact states and is given by

$$f_k = A_k (\lambda_{+k} + \lambda_{-k})$$

Thus on combining the load model and generating system model we get the margin availability table. From this the probability of failure to supply load, that is, system have a negative reserve is given by the sum of the availabilities of these states in the margin-availability table that has a negative reserve margin and this serves the purpose of risk involves while optimising maintenance scheduling of generating units.

## 4.3 MAINTENANCE SCHEDULING TECHNIQUE

This technique is based on the calculation of all alternative schedules, discarding those that violate the constraints, and to select the minimum risk schedule from the remaining set of feasible schedules. Its basic concept is to arrange units or group of units serviced by the same crew in increasing order of flexibility in maintenance scheduling problem. Thus in this way, the units that are more difficult to schedule are considered first. This is due to the fact that they need longer maintenance and/or because they have a larger capacity, causing in increasing the system risk when they are on outage.

### Data Requirement

Following data is necessary to tackle the required maintenance scheduling problem of generating units -

(1) Characteristic of each generating unit - Capacity, mean up time and mean down time.

(2) The number and duration of desired maintenance outage.

(3) The crew assigned to each unit.

(4) The inhibited period, if any, for each unit during which maintenance can't be performed.

(5) The peak load with expected duration of it.

All these are given in Table 4.2 and load data is given in Fig. 4.2. These are sufficient for optimising the maintenance scheduling algorithm with the desired risk level.

4.3.1. ALGORITHM

(1) Choose the required criterion for ordering of generating units in decreasing difficulty of scheduling. The scheme used here is to order the crews in decreasing (total capacity allocated to them x total duration of maintenance) and then order the units within each of them in decreasing (capacity x duration of maintenance).

(2) Calculate the generating system model using the rounding technique.

(3) Consider each unit in turn from the priority list established in (1) and perform steps 4 to 7.

(4) Establish which dates are possible for maintenance of that unit. For each of the possible starting dates, perform step 5. However if all possible dates are eliminated in the process, the algorithm is to be moved backward one unit, rescheduling it to the date corresponding to the next risk index on the list established in step 6 and then process is continued from step 6 onward.

(5) Determine the weeks during which the unit will be out of service if maintenance starts on the date being considered in this step. For each of these weeks do the following

(i) Modify the generation model calculated in step 2, to take in to account the withdrawal of this unit and any other prescheduled unit from service. Use the efficient algorithm described in Appendix III.

(ii) Calculate the load model for the week in question from the load data.

(iii) From the cumulative generation and exact load models of (i) and (ii) calculate first -ve cumulative margin value and calculate its corresponding availability or frequency.

Repeat (i), (ii) and (iii) for all weeks during which the unit is assumed out of service and evaluate the risk index R by summing all the risk indices found in (iii). In this either frequency or availability may be chosen as risk index.

(6) Repeat step 5 for each possible starting dates and average all risk indices R in increasing order. The first one of the list is now chosen.

(7) Select the starting date corresponding to the risk index chosen in 6 as the firm starting date for the unit being considered and consider the maintenance to be firmly scheduled for that unit.

(8) Repeat step 3 to 7 until all units that are to be calculated for maintenance are considered.

The above algorithm used to solve example stated in table 4.2 is as shown.

Here according to step (1) crews are arranged in decreasing order of (total capacity x duration) and units are arranged in decreasing order of (capacity x duration). Preceeding in this way we get the Table 4.6 as follows -

TABLE 4.6

| Crew | Capacity of Unit | Duration (Weeks) | Earliest possible starting date (Weeks) | Latest Starting date (Weeks) |
|---|---|---|---|---|
| 1 | (i) 30 MW | 4 | 15 | 26 |
| | (ii) 50 MW | 2 | 13 | 26 |
| 2 | (iii) 35 MW | 4 | 7 | 13 |

Now according to priority list choose 30 MW unit Ist, as system risk is effected maximum by it, then 50 MW and 35 MW respectively. Step 2 is already performed and the calculated generating system model is given in Table 4.4. Now unit having capacity 30 MW is chosen and its maintenance can be started between 15th to 26th week and maintenance interval is four weeks. Then for taking 30 MW unit from the generating system we have to modify the generating system model. Thus according to Appendix 3 the availability of 65 and 50 MW is to be modified as

$$C_j \geq C_i - C_k$$

Where $C_j = 50$ MW, $C_i = 65$ MW $C_k = 30$ MW

$\therefore$ The new value of

$$A'_i = \frac{A_i - A'_j\,(1-A_k)}{A_k} \quad \text{and} \quad A'_j = \frac{A_j - A'_i\,(1-A_k)}{A_k} \qquad \ldots(i)$$

$$\therefore \quad A'_i = A_i - \left[A_j - A'_i(1-A_k\right]\left[\frac{1-A_k}{A_k}\right] \qquad \ldots(ii)$$

Once $A'_i$ is calculated the value of $A'_j$ is easily found.

Now here the value of $A_k = 0.000154$

$A_i = 0.027775$ and $A_j = 0.0004$

Substituting these values in (ii) and (i) we get

$A'_i = 0.00039578$ and $A'_j = 0.028317$

Thus the modified generating ~~table~~ system model after withdrawl of 30 MW unit is as given in Table 4.7.

TABLE - 4.7

| State No. | Capacity | Cumulative Availability |
|---|---|---|
| 1 | 115 | 1.00 |
| 2 | 65 | 0.0003956 |
| 3 | 50 | 0.028317 |
| 4 | 0 | 0.0001102 |

Now the load model for 15th to 19th week is already calculated and is given in Table No.4.5.

Then cumulative generation and exact load models are combined together to give a single system. The data in the table include all combinations of load, capacity, margin in megawatts and availability. Now the risk involved or total probability of failure is the sum of the availabilities of those states which have- -ve reserve margin. Thus here the calculated risk gives the probability of failure if the maintenance is performed between 15th to 18th week. The complete procedure to calculate risk is given in Table 4.8.

Now from the Table 4.8 the calculated risk R is given by the sum of the availabilities having -ve reserve margin. Therefore R = Risk = 0.00018101 + 0.00018101 + 0.00000070446 + 0.00000070446 = 0.00036483. Thus risk if maintenance of 30 MW performed between 15th to 18th week is 0.00036483.

Similarly if maintenance starts on 16th week we get that the risk involved is same as above i.e. 0.00036483. But if it starts on 17th week risk R = 0.0000028178 and if it starts on 18th week R = 0.00018541. Similarly calculating these values up to 26th week and on arranging the risk in increasing order we get the Table No. 4.9.

TABLE - 4.9

| Sl.No. | Maintenance Starting week | Risk | S.No. | Maintenance Starting week | Risk |
|---|---|---|---|---|---|
| 1 | 17th | .0000028178 | 7 | 21st | .00072688 |
| 2 | 18th | .00018545 | 8 | 23rd | " |
| 3 | 15th | .00036483 | 9 | 26th | " |
| 4 | 16th | .00036483 | 10 | 22nd | .00072933 |
| 5 | 19th | .000368 | 11 | 24th | .00072941 |
| 6 | 20th | .000557 | 12 | 25th | .00072941 |

TABLE - 4.8

MARGIN STATES

| GENERATION DATA | | | LOAD DATA | | | |
|---|---|---|---|---|---|---|
| | | | $i$ = | 1 | 2 | 3 |
| | | | $L_i$ = | 58 | 57 | 48 |
| | | | $A_i$ = | 0.0063926 | .0063926 | 0.012785 |
| $j$ | $C_j$ | $A_j$ | | | | |
| 1 | 115 | 1.0 | M = | 57 | 58 | 67 |
| | | | A = | .0063926 | 0.0063926 | 0.0063926 |
| 2 | 65 | .0003957 | M = | 7 | 8 | 17 |
| | | | A = | .0000025245 | .0000025295 | .0000050577 |
| 3 | ~~.028317~~ 5° | .028317 | M = | -8 | -7 | 2 |
| | | | A = | .00018101 | .00018101 | .00036203 |
| 4 | 0 | .0001102 | M = | -58 | -57 | -48 |
| | | | A = | .00000070446 | .00000070446 | .0000014089 |

Thus to keep the risk minimum maintenance on 30th MW unit must starts on 17th week. Now choose 50 MW unit next which is to be maintained by the same crew between 13th to 26th week. Here 17th to 20th weeks are excluded as these weeks are considered to be firm maintenance dates for 30 MW Unit.

Now again modify the generation model by withdrawing 50 MW unit with the already prescheduled unit and repeat the same procedure. On repeating the procedure we get that the minimum risk occurs if maintenance starts either on 13th or 14th week and is equal to .00018242. Now only 35 MW unit is left and its maintenance is to be performed between 7th to 13th week with the total duration of maintenance equal to four weeks. Now again the generating system model is modified by taking account of the withdrawl of this unit and the risk index is arranged in the increasing order. We find that the maintenance on it can be started in any week between 7th to 9th week and corresponding risk is .0000028178.

Therefore the complete solution is that start maintenance on 30 MW unit on 18th week, on 50MW unit either on 13th or on 14th week and on 35 MW unit at any time between seventh to ninth week.

## 4.3.2. FEATURES OF MAINTENANCE SCHEDULING ALGORITHM

This technique has the following main advantages -

(1) In it by using fast rounding technique the size of the generation table model is reduced considerably. If the generation table size is reduced by n states, then ~~number~~ reserve margin states is reduced by n x number of load levels.

(2) The proposed maintenance schedule is flexible enough so that it may be able to take any set of additional constraints.

(3) Sometimes it is desired to limit the weekly risk to a certain level. This can be done by simple selection process and adding additional constraint to the weekly risk imposed.

(4) If any forced outage of unit occurs then the maintenance on it has to be performed without waiting for the date reserved for it. Thus here the maintenance on this is adjusted on firm prescheduled date and algorithm is repeated for the other units.

(5) Here the units may be chosen according to any priority list. The risk increases proportionately and the yearly risk is levelized by similar amount. Even if the units are randomly choosen for adjusting maintenance schedules, they do not seem to increase risk drastically.

# CHAPTER - V

## CORRECTIVE MAINTENANCE SCHEDULING BY OPTIMAL ALLOCATION OF SPARE UNITS

### INTRODUCTION

Problems of corrective maintenance scheduling require that a system should be designed to have built-in maintenance as far as possible. Such an approach reduces the expenditure on maintenance during the operating life of the system and also increases the reliability. In designing systems with regard to reliability and repairability, typical consideration involves tradeoffs between mean time to repair, mean time to replace, and system mean time to failure. Criterion of goodness depends upon the cost and availability. The tradeoff techniques are not only helpful in reducing the downtime of a repairable systems but also enhances the availability by allocating the spare units. Subject to different set of constraints, a system analyst is posed with the problem of preparing the best schedule out of host of available alternatives in allocating the spare units so that the availability is maximised. Thus the steady-state availability of a repairable system with cold standby and nonzero replacement time is maximised under constraints of total cost and weight. Similarly the cost can be minimised under constraints of steady-state availability and total weight.

4.1 MODELLING

For the maintainability analysis of generating units, various models are used in the reliability evaluation of a system. Some of the important reliability models are : standby systems, systems subjected to two types of failure, standby system with repair facilities. Here for allocating the spare units, standby system with repair facilities model is considered. The following assumptions are used in developing this model :

(1) System is as good as new after any type of maintenance (or replacement) is performed.

(2) The failure rate $\lambda_i$, repair rate $\mu_i$ and replacement rate $\theta_i$ are constant.

(3) There is one repair facility for each i unit at the repair section.

(4) Spare units do not fail and also the failure of units in the system are independent.

For deriving the expression for steady state availability consider a single system consisting of n subsystems and also let i subsystem have one i unit and $L_i$ spare units with a constant failure rate $\lambda_i$ and repair rate $\mu_i$ as shown in Fig. 5.1(a). When a fault occur in a system due to failure of any subsystem component, the component is disconnected and the repair is performed. The failed component is replaced by a spare unit and the other component when repaired, kept as a standby component. Now here in i subsystem having $L_i$ spare units, there are $2\ L_{i+1}$ different states. The i subsystem in

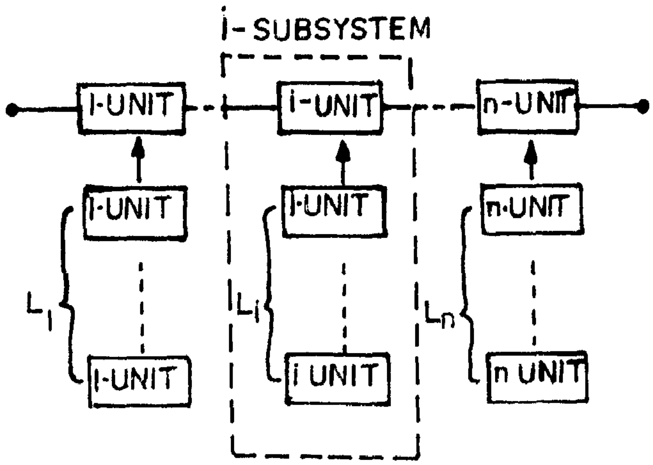



FIG. 5·1(a) THE SYSTEM ALLOCATED $L_i$ SPARE i-UNITS
($i=1, 2, \ldots, n$)

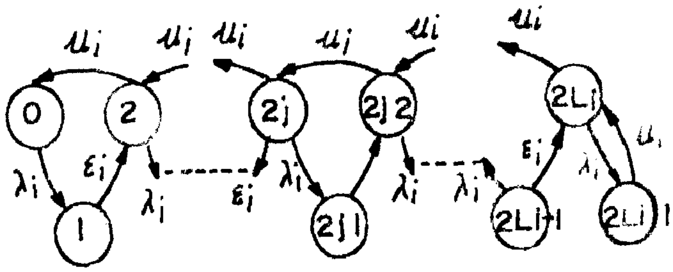


FIG.5·1(b) THE TRANSITION DIAGRAM OF THE i -SUBSYSTEM an even stable is up and odd state is down. The complete detail is as shown in Table 5.1.

TABLE - 5.1

STATES OF i SUBSYSTEM

| i Unit | | | Number of Spare Units | |
|---|---|---|---|---|
| States | Up | Down | Up | Down |
| 0 | 1 | 0 | $L_i$ | 0 |
| 1 | 0 | 1 | $L_i$ | 0 |
| 2 | 1 | 0 | $L_{i-1}$ | 1 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 2j | 1 | 0 | $L_{i-j}$ | j |
| 2j+1 | 0 | 1 | $L_{i-j}$ | j |
| 2j+2 | 1 | 0 | $L_{i-j-1}$ | j+1 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $2L_i$ | 1 | 0 | 0 | $L_i$ |
| $2L_{i+1}$ | 0 | 1 | 0 | $L_i$ |

The transition diagram of it is shown in Fig. 5.1(b)

In the steady state number entering each state is equal to the number leaving it. Thus the balance set of equations as derived from Fig. 5.1(b) is given below

$$0 = -\lambda_i P_o + \mu_i P_2 \qquad (1)$$

$$0 = \lambda i P_{2j} - \theta_i P_{2j+1} \; (j = 0,1,2,\ldots L_{i-1}) \qquad (2)$$

$$0 = \theta_i P_{2j+1} - (\lambda_i + \mu_i) P_{2j} + \mu P_{2j+1} \qquad (3)$$

$$0 = \theta_i P_{2L_{i-1}} - (\lambda_i + \mu_i) P_{2Li} + \mu_i P_{2Li+1} \quad (4)$$

$$0 = \lambda_i P_{2Li} - \mu_i P_{2Li+1} \quad (5)$$

$$\sum_{j=1}^{2L_i+1} P_j = 1 \quad (6)$$

Where $P_j$ is the steady state probability for stable j and $\theta_i$ is the constant replacement rate of i unit. Now from equation (1) and (3) by the method of inductions the steady state probability for state $P_{2j}$ is obtained and given as

$$P_{2j} = (\lambda_i/\mu_i)^j P_o \; P_{rj} = (0 \ldots\ldots L_i) \quad (7)$$

Also

$$P_{2j+1} = \frac{\lambda_i}{\theta_i} P_{2j} = \lambda_i/\theta_i \; (\lambda_i/\mu_i)^j P_o \quad (8)$$

$$P_{2L_i+1} = \left(\frac{\lambda_i}{\theta_i} + \lambda_i/\mu_i\right) P_{2L_i} \quad (9)$$

Now as $\sum_{j=1}^{L_i+1} P_j = 1$, therefore steady state probability at state zero is derived from equations 7 to 9 and is given by

$$P_o = 1 \Big/ \left[1 + (\lambda_i/\theta_i + \lambda_i/\mu_i) \sum_{j=0}^{L_i} (\psi_i)\right] \quad (10)$$

## 5.2 PROBLEM FORMULATION

The problem of optimal allocation of spare units for maximum system availability must include the system mission time and the following properties of each unit : failure rate, weight, price, mean repair time and cost, mean repair time and cost, mean replacement time and cost. The problem is formulated as non -linear integer programming problem as -

SAMPLE EXAMPLE

The solution technique of the problem is explained with the help of the following examples.

The system consisting of three different stages connected in series. The availability of the system is to be maximised subject to the cost and weight constraints using the optimal allocation of spare units with repair facilities. The data associated with the problem is shown in Table 5.2.

TABLE - 5.2

DATA ASSOCIATED WITH THE EXAMPLE

| | 1 Unit | 2 Unit | 3 Unit | Constraints |
|---|---|---|---|---|
| $\lambda_i$ | 0.05 | 0.02 | 0.03 | $T = 100$ |
| $C_i$ | 20 | 60 | 40 | $W_s = 33$ |
| $q_i$ | 2 | 6 | 4 | $C_s = 724$ |
| $C'_i$ | 40 | 50 | 60 | $n = 3$ |
| $W_i$ | 5 | 4 | 3 | |
| $Y_i$ | 1 | 4 | 3 | |
| $t_i$ | 10 | 20 | 10 | |

Now using equations 11 to 15b and the data as given in Table 5.2 the problem can be written as

Maximise - Availability A(L) which is given by the expression

$$Z = \left[\frac{1 - .5^{L_1+1}}{.5 + .55(1 - .5^{L_1+1})}\right]\left[\frac{1 - .4^{L_2+1}}{.6 + .48(1 - .4^{L_2+1})}\right]\left[\frac{1 - .3^{L_3+1}}{.7 + .39(1 - .3^{L_3+1})}\right] \quad \ldots (18)$$

Maximise $A(L)$ (11)

Subject to : $C(L) \leq C_s$ and $W(L) \leq W_s$ (12)

Where $A(L)$ is the steady state system availability and is derived from the equations 7 to 9 for a system composed of n independent subsystems, which are functionally in series and having $L_i$ spare units in $i^{th}$ subsystem is given by the expression

$$A(L) = \prod_{i=1}^{n} A_i \qquad (13)$$

$$\text{Where} \quad A_i = \sum_{j=0}^{L_i} P_{2j} \qquad (14)$$

and values of $P_{2j}$ is calculated by using expressions 7 to 10.

$C(L)$ is the cost constraints $C_s$ is the maximum cost of the system group. $C(L)$ is given by the expression as

$$C(L) = T \sum_{i=1}^{n} \lambda i \; (q_i + c_i) + \sum_{i=1}^{n} (L_i) \; C'_i \qquad (15a)$$

Where T is the system mission time, $C_i$ and $C'_i$ are the mean repair cost of i unit and price of an i unit respectively and $q_i$ is the mean replacement cost.

$W_s$ = Upper bound of system weight

and $W(L)$ weight constrained is given by $\sum_{i=1}^{n} L_i \, w_i$ (15b)

Where $w_i$ is the weight of i unit.

Another important version of the problem is also given by

Minimise $C(L)$ (16)

Subject to $A(L) \geq A_s$ and $W(L) \leq W_s$ (17)

Sub. to

(i) Cost constraints -

$$200 - 40\, L_1 - 50\, L_2 - 60\, L_3 \geq 0 \qquad (19)$$

(ii) Weight Constraints

$$21 - 5\, L_1 - 4\, L_2 - 3\, L_3 \geq 0 \qquad (20)$$

Where $L_1$, $L_2$, $L_3$ are the number of standby components used in Ist, 2nd and 3rd stage respectively. From the design consideration upper and lower bound on all standby components are known. Here the upper and lower bounds are given as that $L_1$ lies between 0 and 2 and $L_2$ and $L_3$ between one and three respectively.

## 5.2.1. ALGORITHM AND SOLUTION

For the solution of above stated problem, direct search optimisation technique, as developed earlier is used. Here the non-ve integer variables involved in the problem can be transferred in to problem involving binary variables.

Integer variables converted in to 0 - 1 variables must be able to satisfy the property that each subset occupy with a unity element. Here the $L_i$ s are converted into problem format by the property as given by the expression as given

$$L_i = \sum_{j=\underline{L_i}}^{\overline{L_i}} j\, L_{ij} \quad \left[ i = 1,2,\ldots\, L_i \right] \qquad (21)$$

Where $\overline{L_i}$ = Upper bound on variable $L_i$

$\underline{L_i}$ = Lower bound on variable $L_i$

## TABLE - 5.3

### SOLUTION FOR ALLOCATING SPARE UNITS

| | $L_{10}$ | $L_{11}$ | $L_{12}$ | $L_{21}$ | $L_{22}$ | $L_{23}$ | $L_{31}$ | $L_{32}$ | $L_{33}$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | Infeasible constant 25 is not satisfied. |
| 2 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | " |
| 3 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | " |
| 4 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | " |
| 5 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | " |
| 6 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | " |
| 7 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | " |
| 8 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | " |
| 9 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | " |
| 10 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | " |
| 11 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | " |
| 12 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | " |
| 13 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | " |
| 14 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | " |
| 15 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | Feasible Z=.63246 |
| 16 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | Infeasible constraints violated. |
| 17. | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | " |
| 18 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | Feasible Z=.58039 |
| 19. | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | Infeasible constraints violated |
| 20. | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | " |
| 21. | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | " |
| 22. | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | " |
| 23. | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | Feasible Z=.51874 |
| 24 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | Feasible Z=.49645 |
| 25 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | Infeasible constraints violated. |
| 26. | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | Feasible Z=.47598 |
| 27. | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | Feasible Z=.45553 |

$$L_{ij} = \begin{bmatrix} 1 & \text{if } i^{th} \text{ stage has } j \text{ spare units} \\ 0 & \text{otherwise} \end{bmatrix}$$

$$\text{and} \quad \sum_{j=0}^{L_i} L_{ij} = 1 \qquad (22)$$

Therefore the above stated problem can be written by putting the values of $L_1$, $L_2$ and $L_3$ from equations 21 and 22 is given as

$$\text{Maximise} \quad A(L) \qquad (23)$$

$$\text{Subject to :} \quad 200 - 40\,(0\,L_{10} + 1\,L_{11} + 2\,L_{12}) - 50(L_{21} + 2L_{22} + 3L_{23})$$

$$- 60\,(L_{31} + 2\,L_{32} + 3\,L_{33}) \geq 0 \qquad (24)$$

$$21 - 5\,(0L_{10} + L_{11} + 2L_{12}) - 4(L_{31} + 2L_{32} + 3L_{33})$$

$$- 3\,(L_{31} + 2\,L_{32} + 3\,L_{33}) \geq 0$$

$$L_{10} + L_{11} + L_{12} = 1 \qquad (26)$$

$$L_{21} + L_{22} + L_{23} = 1 \qquad (27)$$

$$L_{31} + L_{32} + L_{33} = 1 \qquad (28)$$

Now using the direct search optimisation technique the result optimal solution found from table 5.3 is

$$L_{11} = L_{22} + L_{31} = 1$$

$$L_{10} + L_{21} = L_{23} = L_{32} = L_{33} = 0$$

The result show that for the maximum availability the number of standby components with stage Ist, 2nd and 3rd are one, two and one respectively. The system availability achieved with this arrangement is 0.63246.

# CHAPTER-VI

## CONCLUSIONS

The application of mathematical programming technique in the solution of complex maintenance scheduling problems is of a great interest and importance to the utilities. The present work is an attempt in presenting mathematical models and algorithms for preventive and corrective maintenance scheduling of generating units.

First of all, the problem of preventive maintenance scheduling is discussed. In it several classes of objective criteria for optimal preventive maintenance scheduling of power generating units have been presented. Then a mathematical model is developed for preventive maintenance scheduling of generating units in the presence of uncertainities due to error in the demand forecast and generator outages. A method for the solution of the problem of scheduling maintenance of generating units in a power system has been developed. It employs the lexicographic enumeration technique for the solution. Certain rules incorporated in it help in skipping large number of infeasible solution vectors and hence enhance the effectiveness of lexicographic enumeration technique.

The properties of the mathematical model are also exploited by evolving a new and efficient direct search optimisation technique. This method is based on simple logic and by employing this procedure the region of search for finding the optimal solution is drastically reduced. It is capable of incorporating all of the different and complex constraints. Hence the solution obtained is always practically implementable. Illustrative examples of risk levelization and reserve levelization are given using the above proposed technique.

Next, another maintenance scheduling algorithm ~~algorithm scheduling~~ is presented based on frequency and duration criterion. This uses a fast rounding technique. It helps in reducing the large number of states in generating system model. Therefore while performing calculations, the size of the margin states is reduced greatly which is helpful in increasing the efficiency of the scheduling algorithm.

Finally a mathematical description of the corrective maintenance scheduling problems is presented. In this a mathematical model for allocating of spare units with repair facilities is discussed. This analysis is of value in building the inventory for various components in power systems to satisfy the system reliability. The problem model is critically analysed. The direct search optimization techniques is presented for finding the optimal allocation of spare units.

Further work would include, optimising a wide range of decisions in context with the problem. To mention a few, these are, pool coordination of resources within the utility and with the neighbouring utilities. Instead of levelizing reserves, it may be economically beneficial for a particular utility to vary these reserves. It could buy less expensive energy from neighbouring utilities during some portion of the year, while selling surplus energy during other periods. Along the same vein, it is intended to analyse the impact of forced outage and daily unavailability rates on net reserve and production cost.

In short author has tried to present different mathematical models and technique of analysis for optimal preventive and corrective maintenance scheduling problems. It is hoped that the work reported would be useful to the management for developing optimal maintenance policies as regard to generating systems.

## APPENDIX - I

### ESTIMATION OF SYSTEM CHARACTERISTIC

System characteristic is defined as the megawatt of reserve decrease that will increase the risk 2.718 times, that is megawatt variation in capacity outage accompanies by a multiple of 2.718 increase in probability. Here assumption is made that the risk expressed in terms of installed reserve is given as -

$$P_x = A\ e^{-x/m} \qquad (i)$$

Where

$P_x$ = cumulative probability of having x MW or more on outage

x = installed reserve

m = system characteristic

A = proportionately constant

This approximation is based on the past historical data availables.

Let $P_y$ be the cumulative probability of having y MW or more on outage such that $y > x$. This is expressed as

$$P_y = A\ e^{-y/m} \qquad (ii)$$

Now here probability of risk in x MW system is more than y MW system.

Divide the larger risk by the smaller risk we get

$$\frac{P_x}{P_y} = \frac{A_e^{-x/m}}{A_e^{-y/m}} = e\ (y-x)\ m$$

Taking the natural logarithms on both sides we get

$$\frac{y - x}{m} = L_n(P_x/P_y)$$

$$m = (y-x)/\{L_n(P_x/P_y) \qquad (iii)$$

Therefore for estimating the system characteristic risk is plotted as a function of capacity outage versus probability on a semilog paper and a straight line is drawn between the designated probability level. Then value of capacity outage is seen corresponding to the value of risk levels and system characteristic is calculated as given in equation (iii).

## APPENDIX - 2

### EXPRESSION FOR CALCULATING EFFECTIVE CAPABILITY

Effective capability of a unit is defined as the load which the unit is able to carry with a designated reliability. $P_x$ is the probability of outage of x MW or more on forced outage. Suppose any new unit is added in to the system say c MW is added, then the cumulative probability of outage of x MW depends upon the two possible conditions for the new unit, that is if the unit is in service or on forced outage. Thus if r is the forced outage rate of c MW unit, the new cumulative probability of outage of x MW is given by the sum of the two components given below -

(i) When Unit in Service - Here the probability of outage is given by (1-r) multiplied by the probability of outage of x MWs.

(ii) Unit is on Forced Outage - Now when unit is on forced ~~alen~~ outage with a probability of r then total probability of outage of x MW is given by r multiplied by $P_{x-c}$. Where $P_{x-c}$ is the probability of outage of x-c megawatt or greater.

Therefore from the total probability of outage of x megawatt with the addition of new unit is given by -

$$P_x = (1-r)\, P_x + r\, P_{x-c} \qquad (i)$$

However if there is no change of load by adding c megawatt unit then the value of risk will be given at a reserve of x+c MW and is given as

$$P_{x+c} = (1-r)\, P_{x+c} + r\, P_x \qquad (ii)$$

Now according to equation (i) of Appendix - I

$$P_{x+c} = A\ e^{-(x+c)/m} = P_x\ e^{-c/m}$$

Substituting this in above equation we get

$$P_{x+c} = \left[(1-r)\ e^{-c/m} + r\right] \quad P_x \qquad \text{(iii)}$$

The reserve say (y) is increased proportionally with the addition of new unit, so that probability of outage remains same.

Therefore $\quad P_{x+y} = P_x \qquad$ (iv)

The expression for the new probability of outage in terms of old function is found by substituting x+y instead of x in (i)

$$\therefore \quad P_{x+y} = (1-r)\ P_x + r\ P_{x+y-c} \qquad \text{(v)}$$

now $\quad P_{x+y} = P_x\ e^{-y/m} \qquad$ (vi)

and $\quad P_{x+y-c} = P_x\ e^{-(y-c)/m} \qquad$ (vii)

Substituting this in to equation ( v) we get

$$P_{x+y} = \left[(1-r)\ e^{-y/m} + r\ e^{-(y-c)/m}\right] P_x \qquad \text{(viii)}$$

Substituting this equation in (iv) we get

$$\left[(1-r) + re^{\,c/m} \quad e^{-y/m} = 1\right]$$

Taking the log on both sides we get

$$\text{Ln} \quad (1-r) + re^{c/m} \quad + (-y/m) = 0$$

$$\therefore \quad y = m\ \text{Ln} \quad (1-r) + r\ e^{c/m}$$

Now the load carrying capability of the unit having capacity c and forced outage rate r is given as

$$c^{*} = c-y = c-m\ \text{Ln} \quad (1-r) + re^{-c/m}$$

Where $c^{*}$ is the effective load carrying capability of new unit.

# APPENDIX - III

## EFFICIENT TECHNIQUE FOR REMOVAL OF UNITS

When any unit is removed from the generating system model for maintenance purposes, the whole generating system model is modified. Billinton and Singh suggested that generation model can be modified by applying the suitable technique. According to them if any unit having capacity $C_k$ and average failure and repair rates are $\lambda_k$ and $u_k$ respectively with availability $A_k$ can be removed from a system and the generation model can be modified accordingly. Therefore if $C_i$ and $C_j$ are the capacity states including that of $C_k$ such so that $C_j$ greater or equal to $C_i - C_k$ then the new modified states are given with the following cumulative availabilities.

$$A'_i = \frac{A_i - A_j ( 1 - A_k)}{A_k} \qquad (i)$$

and $$A'_j = \frac{A_j - A'_i ( 1 - A_k)}{A_k} \qquad (ii)$$

Where $A'_i$ and $A'_j$ are new cumulative availability index for units i and j taken account of withdrawal of unit k. On solving the above two equation the value of $A'_i$ and $A'_j$ can be easily calculated.

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