ANALYSIS AND PERFORMANCE OF SINGLE PHASE INDUCTION MOTOR SUBJECTED TO THYRISTOR CONTROL

A DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
POWER APPARATUS AND ELECTRIC DRIVES

By

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Aug. 76-78

Our knowledge is the amassed though
and experience of innumerable minds.

EMERSON

CERTIFICATE

Certified that the dissertation entitled "ANALYSIS AND PERFORMANCE OF SINGLE PHASE INDUCTION MOTOR SUBJECTED TO THYRISTOR CONTROL" which is being submitted by Mr. Vijai Prakash Agrawal in partial fulfilment for the award of the degree of Master of Engineering in Power Apparatus and Electric Drives of Electrical Engineering Department, University of Roorkee, Roorkee is a record of students own work carried out by him under my supervision and guidance. The matter embodied in this Dissertation has not been submitted for award of any other degree or diploma.

This is further to certify that he has worked for a period of six months from April 1978 to September 1978 for preparing this Dissertation at this University of Roorkee.

24/10/78

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ACKNOWLEDGEMENT

The author considers it his privilege to express his profound indebtedness to Dr. D. R. KOHLI, Professor of Electrical Engineering, his supervisor for this dissertation for all the expert guidance, help and inspiration that he has received at his hand so abundently throughout the period of this work. He will for long cherists the many fruitful discussions he had with him and the meticulous thoroughness with which he has gone through the manuscript of his work. The author for long remember his almost religious advocacy of the finer point of technical writing and for his critical observations which have contributed significantly in raising the quality of the present work.

He also takes this opportunity to express his thankfulness to other staff members of the department, particularly members of P.A.E.D. group for the productive discussions he had with them.

The author also places on record his gratefulness to Dr. P. Mukhopadhyay, Professor and Head for the many facilities he has received from him and his department.

Last but no means the least the cooperation and help of the departmental technical staff in particular Mr. Ganguli & Mr. Rawat is gratefully acknowledged.

The author is also thankful to his friends who helped him directly or indirectly.

OCTOBER 1978 VIJAI P AGRAWAL

ABSTRACT

Various industrial and domestic applications require a small power variable speed drive. The requirement is beautifully met with phase controlled single phase induction motor. Thyristor is used as power control equipment for this purpose, efficiency, power factor and compactness being its major advantages. The only draw back is application of non-sinusoidal voltage at motor terminals, it alters behaviour of single phase induction motor altogether. Therefore there is need of predicting motor performance under non-sinusoidal condition.

Solid state controls are recent one therefore not much work is available especially for single phase induction motor. Conventional approaches fail under such circumstances or become very complex. Previously using state-space approach the performance has been calculated, but the results obtained are indirect one and computer program is also complex. In this dissertation a simpler computer program has been developed using circuit approach (Chapter 3). Performance is computed and compared with performance obtained by a practical set up. (Chapter 4). Performance studies are made and improvement in the design is suggested (Chapter 5).

CONTENTS

Chapter				Page

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CHAPTER - 1

INTRODUCTION

With the near universal application of a.c. system of distribution of electrical energy for light and power, the field application of a.c. motors has widened considerably during recent years. In fact most of the new products have entirely been possible due to advances made in the design of fractional horse power motors. The major draw back of single phase induction motor is complete absence of starting torque. Once started it is capable of running without auxiliary devices. The starting method is used to distinguish one type of single phase induction motor from the other. Some of these are

1. Shaded pole
2. Resistor or reactor split phase motor
3. Capacitor start induction run motor
4. Capacitor run motor
5. Repulsion start induction run.

For any substantial power rating capacitor start induction run and capacitor run motors are the only ones frequently used. The studies presented in this dissertation are restricted to the former type only, method is suggested for the other type also.

1.1. CAPACITOR-START INDUCTION RUN MOTOR

Capacitor start induction run motor use a high capacity electrolytic type capacitor for starting service only. A centrifugal switch or relay is employed in the starting winding circuit. Starting torques of 200 to 350 percent of full load torque are feasible with acceptable starting currents. Ratings from 100 W to 10 KW are used industrially.

<u>Capacitor run motor</u> : Permanent-split capacitor use oil paper foil capacitor, permanently connected in the auxiliary winding circuit for both starting and running duty. Comparatively low values of capacitance are necessary for good running performance. Such low values however result in low starting torque - values of 35 to 50 percent of full load torque are common. Greatest volume of production is for fans, unit heaters and similar applications.

Two-value capacitor motor use both type of capacitor described, with the electrolytic type cut out as soon as the motor reaches about 70% of synchronous speed. This provides good starting torque with quite pulsation free running operation.

1.2. <u>NECESSITY OF SPEED CONTROL</u>

There are numerous applications of small power variable speed drive in modern industry and domestic appliances. For example in small lathes, the electric drive controlling rotational speed of the job must, have its speed set or adjusted in accordance with the kind of metal to be worked, the quality and kind of cutting tool to be used, the size of work piece and other factors. Similarly the speed at which the induced draft fan in a boiler house should operate will have to be adjusted in accordance with

1.3. SINGLE PHASE INDUCTION MOTOR SPEED CONTROL

A.C. motor runs at a speed which is determined principally by frequency of the a.c. supply. Speed control is feasible by changing supply frequency but is not economically justifiable. On fixed frequency supply the torque developed by the induction motor at a given slip varies approximately as the square of applied voltage, and steady state operation occurs when the motor torque balances the load torque. Consequently the rotor slip is determined by load torque and applied voltage. The line voltage control can be accomplished by introducing variable resistance or tapped reactance coil in series with stator main winding. In first case power loss occurs in control element and second type of control results in over all poor power factor of the drive. Size of the control element also increases with the increase in power rating of the machine. Thyristor used as a control elements eliminate all these draw backs. Now triacs and S.C.R.'s are fastly replacing the conventional controls.

1.4. NEED FOR ANALYSIS WHEN SOLID STATE CONTROLLER IS USED

Solid state power device introduce switching transients in the machine as well as in the power line. Generally non-sinusoidal voltage is applied across the machine. This deteriorates machine performance and affects the power system as a whole. It necessitates redesigning of motor for particular type of controller or vice-versa. Designers ultimate object is to optimise the over all drive, therefore, performance study for quality of fuel being fired, its moisture and ash content, the boster firing conditions and steam output required of the boiler unit. In can be achieved by changing angle of the blades but mechanical considerations are in favour of controlling its speed itself[2].

Speed of domestic fan is required to be adjusted depending upon so many human and environmental factors. The drive for a record player should have various fixed speeds depending upon record's speed. In so many other applications, the drive must have speed control to attain high productivity, proper operation and high quality of product.

Previously a.c. commutator motors or d.c. motors were employed in such applications. They suffer from their inherent draw backs of higher maintenance cost, spark over during commutation, involved construction and higher maintenance cost. The advent of silicon power thyristors around 1960 added new impetus to the development of variable speed scheme by making variable frequency & variable voltage power supply a practical possibility by providing a versalite and economic solid state power control element. Now squirrel cage induction motor basically a simple and reliable machine can be used instead, along with power control equipment. Numerous low power motor speed control using simple voltage control are already in use and currently efficient control for motors in thousands of horse power are under consideration[3].

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1.5. REVIEW OF PREVIOUS WORK

There is not much literature available as these controls are very recent ones. Switching conditions are also very complicated from mathematical point of view. Conventional approaches are no more valid as physical conditions are entirely different. Considerable work has been done on the analysis of 3-phase induction motor subjected phase angle control[4-5]. However much work has not so far been done for single phase induction motor. The problem in this case is more difficult because machine is an unbalanced 2-phase motor. Consideration of capacitor in the auxiliary winding further complicates the situation by increasing order of the differential equations. P.C. Krause[6-7] has used analog computer (differential analyzer) to study the electrical and mechanical transients as well as steady state operation of phase angle controlled thyristor speed controller. The ~~development of analog~~ development of analog computer simulation which enables one to study dynamic and steady state performance of an induction motor has been reported[7]. Besides analog simulation switching functions can easily be described by a Fourier series method[8]. Steady state solution of individual frequency components can be determined, total solution can be obtained through superposition. At low speeds this method is well applicable but at near full load speed applied voltage waveform for the entire cycle is difficult to determine because of presence of counter e.m.f. when SCR's are blocking.

Novotny and Fath[9] have done pioneering work in this direction. They have developed the mathematical model of the machine subjected to phase angle control. Analysis has been done only for steady state conditions using state-space approach, and numerical methods for the solution of resulting equations. The results obtained analytically are compared with experimental results. A difference of about ± 20% between experimental and analytical results have been observed, which the author have attributed to usual difference between experimental and practical results. Using the same approach Hammersworthy and Klang[10] have also given the steady state performance characteristics. The only difference between the two is that the latter authors have simplified the equations of Novotny & Fath. The computed results have been compared with those obtained by Novotny & Fath. The two agree, which is naturally expected because the equations are same. The results reported in both the papers are in terms of conduction period of the thyristor, since conduction period is not a direct independent variable (in contrast firing angle is a direct independent variable) no useful interpretation of results relating to practical condition is possible.

1.6. OUR APPROACH

In the approach presented a mathematical model is developed for a motor with single winding in stator. Depending upon power factor angle control is obtained if $\alpha > \phi$. In the controlable region two different modes of thyristor operation are evident namely conducting mode and non conducting mode. Circuit equations are altered accordingly for these two modes of operation. A method is given for solving these equations for steady state condition. A program is developed to obtain complete characteristic at different speeds and firing angles. Constraints imposed by the thyristor are in corporated in the program itself. Initially unknown circuit conditions are established through iterative process and performance is calculated there after.

Performance characteristics obtained for a typical motor are critically discussed and effect of variation rotor resistance is studied. This is followed by some suggestion for improvement of motor design.

CHAPTER - 2

METHODS OF SPEED CONTROL USING S.C.R's

Speed of an induction motor can be changed by changing supply frequency, supply voltage or both. But voltage control is more economical for single phase fractional horse power motors.

2.1. STATIC FREQUENCY CHANGER

In the static a.c system adjustable speed is achieved by applying adjustable frequency control to an a.c motor by means of a static converter. This converter system changes fixed line frequency supply to variable voltage variable frequency supply. The break down torque is maintained constant by keeping V/F ratio constant so that air gap flux is constant. It is suitable for driving a constant-torque load at variable speed. If stator voltage remains constant as frequency is varied the air gap flux and breakdown torque decreases with increasing frequency, suitable for applications where a torque is required at stand still and low speeds and smaller torque is sufficient for high speed running, constant horse power operation.

Static frequency converter has the best load speed regulation despite the fact that it operates open loop[8]. This system is economical for very large power drives or for very small power drives. The reason being that in case of large power drive cost of control equipment is a reasonable proportion of overall cost of the drive. However for medium power drives the cost of control equipment form a major part of total cost of drive, which therefore becomes uneconomical. For small power drive it is again economically justifiable as cheap low power frequency changer can be built using power transistors. This system has very large speed control range.

2.2. VOLTAGE CONTROL

A good speed control range can be obtained by varying supply voltage of high rotor resistance squirrel cage induction motor. For constant torque load rotor copper losses & current are given by

$$\text{rotor copper loss} \propto S$$

$$\text{current} \propto S / R^{\frac{1}{2}}$$

but for fan type of load

$$\text{rotor copper loss} \propto (1 - S)^2 S$$

$$\text{current} \propto \frac{(1 - S)(S)^{\frac{1}{2}}}{R^{\frac{1}{2}}}$$

where S is slip and R is rotor resistance.
Therefore when used for fan type of loads losses and current both always remain within tolerable limits.

The simplicity of the thyristor voltage control and inherent high reliability of the device provide opportunity for an extremely reliable equipment design. Closed loop schemes which continuously adjust the motor voltage are readily applied to thyristor voltage control circuits[3].

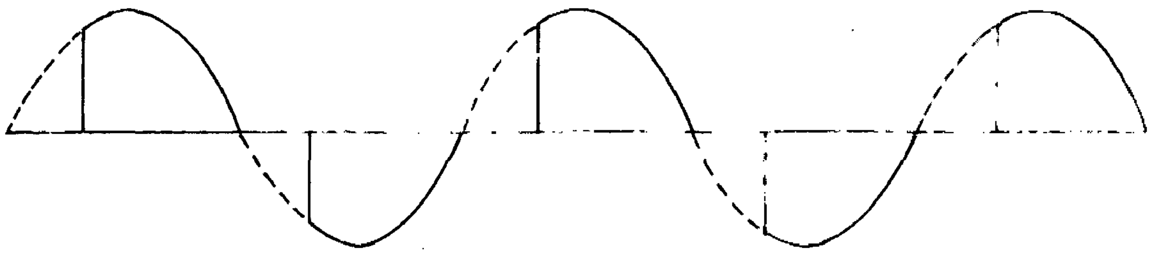

FIG 2.1 VOLTAGE WAVEFORM OF PHASE ANGLE CONTROL
$\alpha = 60°$

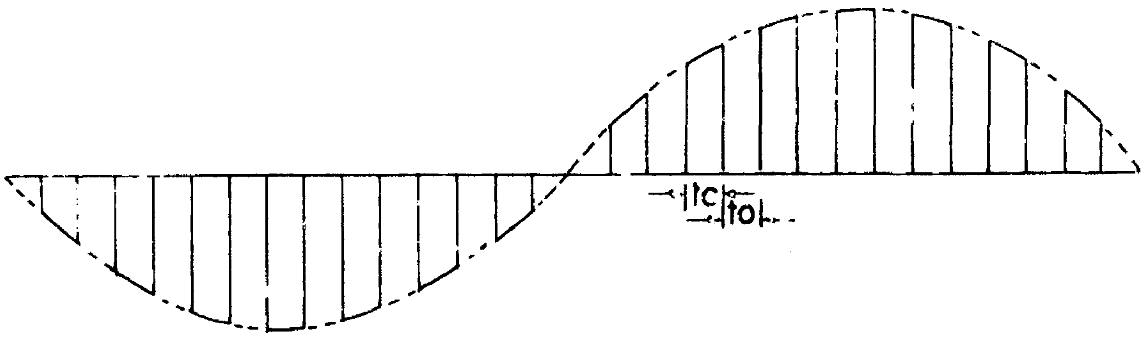

FIG.2.2 VOLTAGE WAVEFORM OF CHOPPER CONTROL 50%

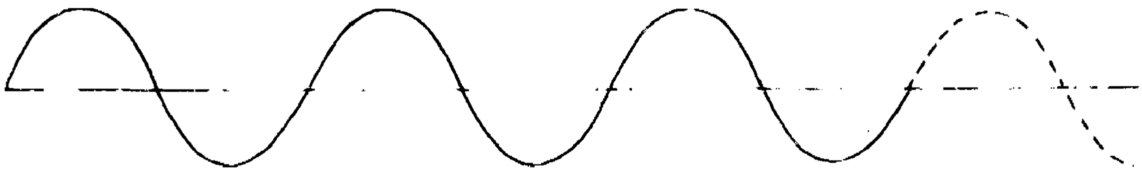

FIG.2.3 VOLTAGE WAVEFORM OF INTEGRAL CYCLE CONTROL 75%

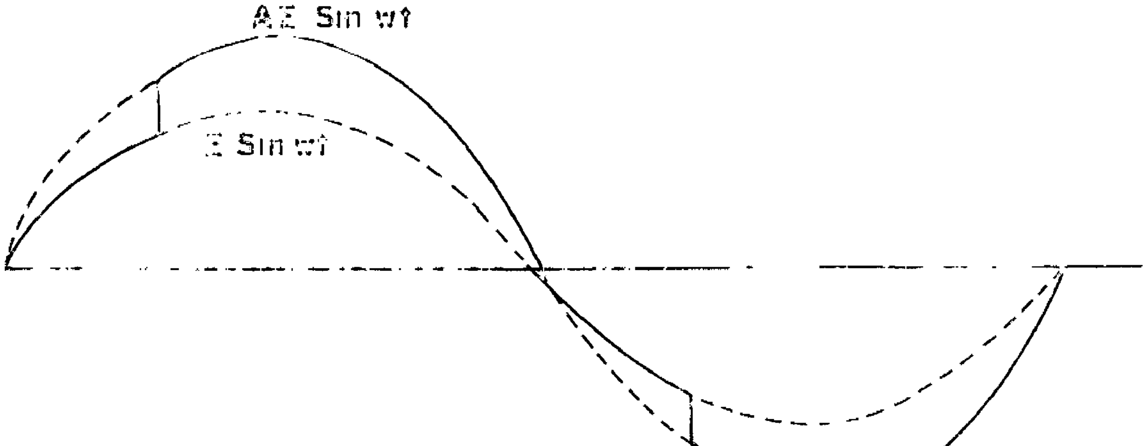

FIG.2.4 VOLTAGE WAVEFORM OF SYNCRONOUS TAP CHANGING CONTROL.

2.2.1. Phase Angle Control : In this case terminal voltage of the motor is varied by controlling the firing points of a pair of back to back connected thyristors. By phase delaying the firing points, only part of the voltage in each cycle is made available thereby reducing the r.m.s. value of voltage applied to the motor. The voltage wave shape obtained in such type of control is shown in FIG. 2.1.

Its control circuit is simple and sufficiently large range can be obtained for high rotor resistance induction motor. But it has high harmonic contents in output voltage wave for large delay angles. Also it introduces significant radio frequency interferences[12].

2.2.2. A.C. Chopper Control (Pulsed supply) : Power is switched on and off several times with in one cycle. The mean load voltage is controlled by changing the on period during an operating cycle i.e. $t_c / t_c + t_o$, voltage wave shape is shown in FIG. 2.2 .

It has faster speed of response and harmonic content is lower. But it has disadvantages of higher cost due to more elaborate control and power circuitry, and greater radio frequency interference which restricts its use in domestic equipments[12].

2.2.3. Integral Cycle Control : The supply is given to the motor for complete a few cycles and cut off for a subsequent few cycles. This type of control introduces severe torque and speed pulsation, vibration and noise. No radio frequency interference is caused because voltage build up follows sine wave and is not abrupt. Simple burst firing techniques are used. The voltage wave applied to the motor is shown FIG. 2.3 . Stepless speed control is not in possible when using integral cycle techniques.

2.2.4. Synchronous Tap Changing : In systems which have an output transformer and require only small adjustments of the voltage synchronous tap changing is often preferable to ordinary phase control, output, wave form is shown in FIG. 2.4. It gives a good performance but the cost is very high, justifiable only in case of large machines.

Out of above four methods, for small single phase induction motors phase angle control technique appears to be more suitable because of

(i) Simple control circuitry

(ii) Low cost

(iii) Resonable good performance characteristics.

2.3. DERATING OF MOTOR

Single phase induction motor supplied through a phase angle controller is subjected to non-sinusoidal voltage wave form. The time harmonics present in the applied voltage results in currents at harmonic frequencies, which cause additional losses augmented by skin effect. More over even when there is no current in stator circuit some circulating current flows in the rotor resulting in additional losses. Depending upon dissipation conditions temperature of the motor may rise above prescribed limits and may endanger motor life if proper safeguard are not provided. To keep temperature rise of the motor within prescribed limits the motor has to be derated.

The temperature rise of the machine depends upon total losses (I^2R loss, hyteresis & eddy current loss, friction and windage loss) and rate of dissipation of heat. With phase angle control I^2R losses can be easily computed once the r.m.s. value of currents is known (in stator and rotor both). The friction loss at the new speed can also be easily computed if friction loss vs speed curve have been obtained. However it is not possible to exactly determine iron losses. For their determination the variation of flux in various parts of the circuit needs to be known. In the analysis of single phase induction motor because of nature of problem one is constrained to use circuit approach where in iron losses are difficult to account for. Therefore for such machine it is difficult to determine the extent of derating necessary without making approximations of doubtful validity.

Designing the motor with a large leakage inductance or by inserting external series reactor in stator helps in attenuating harmonic currents[8]. However the use of additional reactor reduces the overall p.F.

2.4. SYMMETRICAL AND NON-SYMMETRICAL WAVE

Because of practical limitations of firing circuits (unmatched components) or harmonics already present in the a.c. supply a non-symmetrical voltage wave may appear at the output

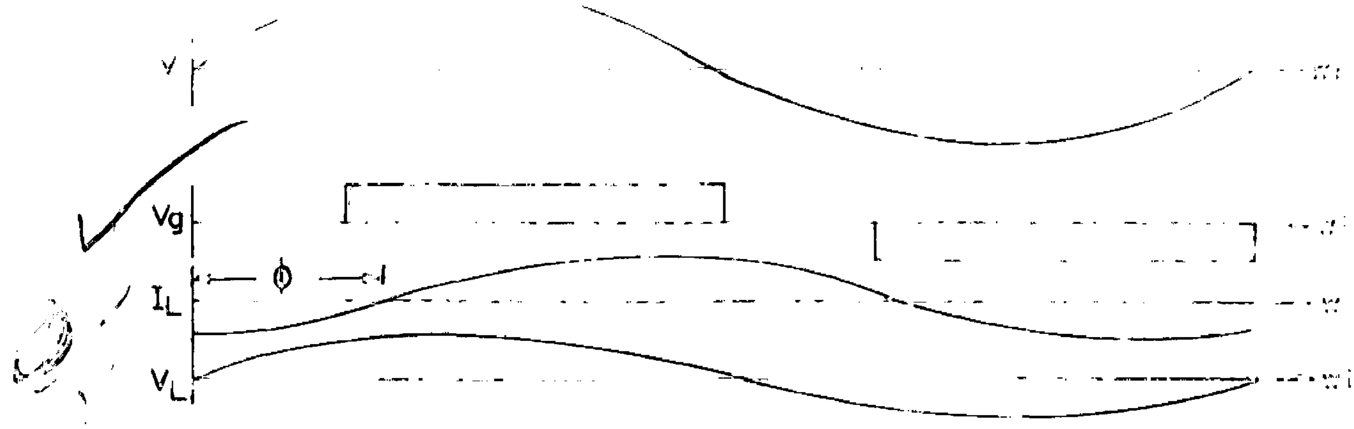

FIG.2.5 WAVEFORM FOR THYRISTOR CONTROL OF SERIES R-L LOAD $\alpha < \phi$

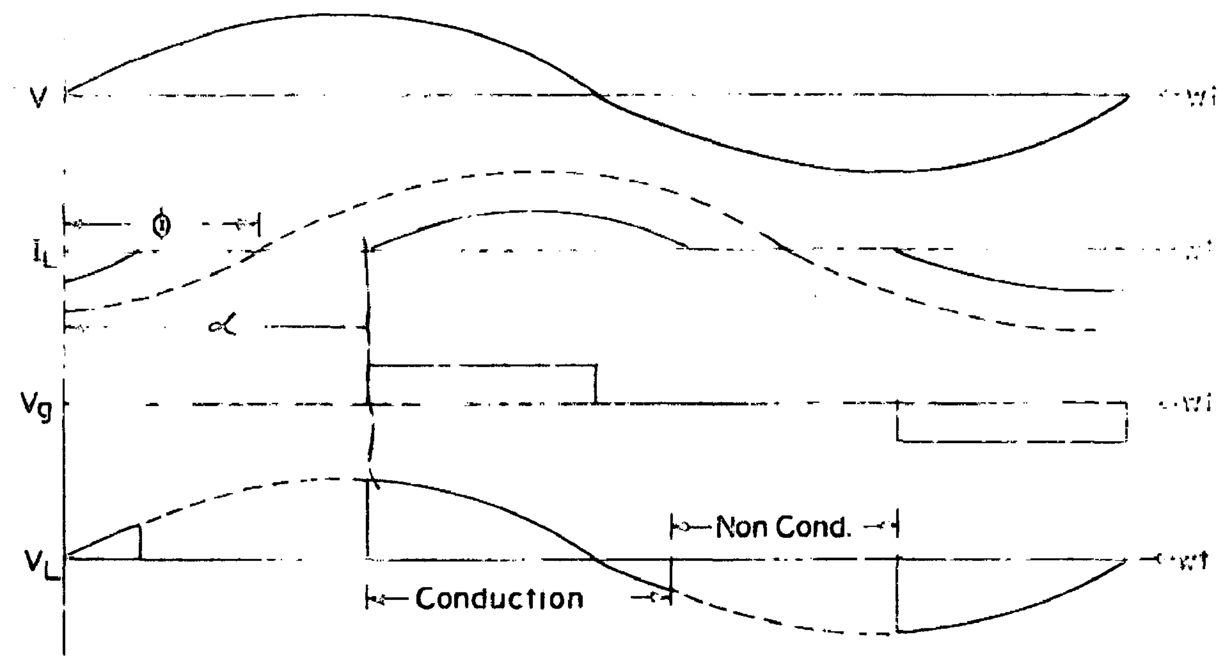

FIG.2.6 DISCONTINUOUS CONDUCTION.

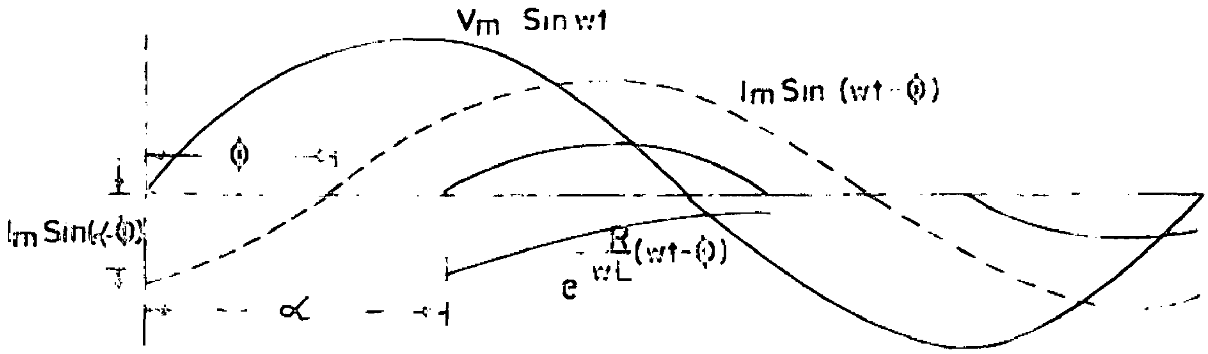

FIG.2.7 WAVEFORMS FOR THYRISTOR CONTROL OF SERIES R-L LOAD $\alpha > \phi$

terminals[21]. Application of such voltage wave will deteriorate machine performance because even harmonics will also be present in addition to odd harmonics in voltage wave and presence of d.c. component may cause saturation in the magnetic circuit. Therefore application of only symmetrical wave is advisable. Incidently it simplifies the analysis also. Solution for one half cycle may be extended to the other half cycle or performance for only one half cycle need be calculated.

2.5. THE TWO MODES OF OPERATION WITH PHASE ANGLE CONTROL

Depending upon the values of triggering angle and slip two distinct modes of operation are obtained. Power factor of the motor depends upon slip and range of firing angle control in turn depends upon power factor angle ϕ .

2.5.1. Continuous Current Conduction : When the thyristors are triggered by long pulses at a triggering angle α less than or equal to sinusoidal phase angle ϕ of load impedance, sinusoidal operation results[13]. A typical steady state cycle is shown in FIG. 2.5. The use of single short pulse when $\alpha < \phi$ could cause only one thyristor to conduct because the continuation of conduction after the end of the voltage half cycle ensures that gating of the reverse thyristor would not have any effect. The thyristor pair would then act as a rectifier.

2.5.2. Discontinuous Current Conduction : When the triggering angle α is greater than the load phase angle ϕ the current occurs in discontinuous, non-sinusoidal pulses, as shown in FIG. 2.8. A number of interesting facts hide in this case. The on set of the non-sinusoidal load current pulse always coincides with triggering angle. Conduction current is found to cease prior to the end of the sinusoidal current cycle. The range of possible load current wave form then varies from the dotted sinusoidal of FIG. 2.7 when $\alpha \leq \phi$ to pulse of currents with their leading edge at firing angle α of the applied voltage cycle and their conduction approaches zero as α approaches 180°.

It is discontinuous mode of operation which is important from control point of view.

CHAPTER - 3

APPROACH TO ANALYSIS

Application semiconductor switching elements to induction motor speed control has created a need for a fast and accurate means of predicting motor performance under non-sinusoidal excitation inherent in these systems. In many case this excitation takes the form of alternate periods of normal sine wave excitation and period of open circuit excitation.

The difficulty in the analysis is chiefly due to the current constraints introduced by thyristor in each half cycle for a given speed and triggering angle in steady state is a complex function of motor parameters and the value of rotor currents at the instant of triggering; this conduction period is initially unknown. The stator e.m.f. induced during the blocking period of thyristor is also undefined initially. Different analysis available treat the situation in different manner. Simplicity of the solution accuracy of simulation of practical conditions are the major criteria to judge them.

The speed control of a single phase induction motor with pair of back to back connected thyristors in main winding is extremely simple. As natural commutation is used no capacitors are required except for starting purpose. For analysis only main winding is being considered.

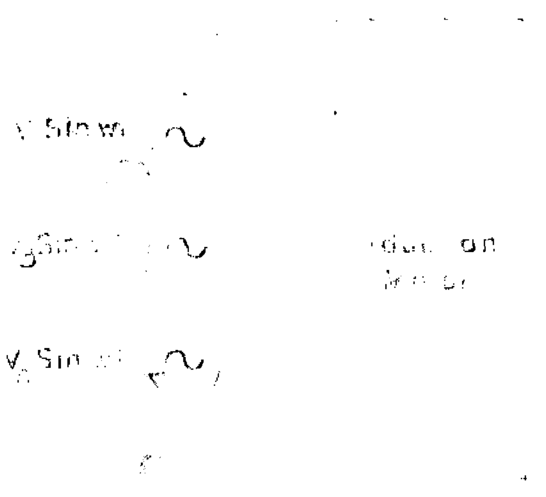

FIG 3.1 [illegible] VOLTAGE [illegible] SINUSOIDAL COMPONENTS

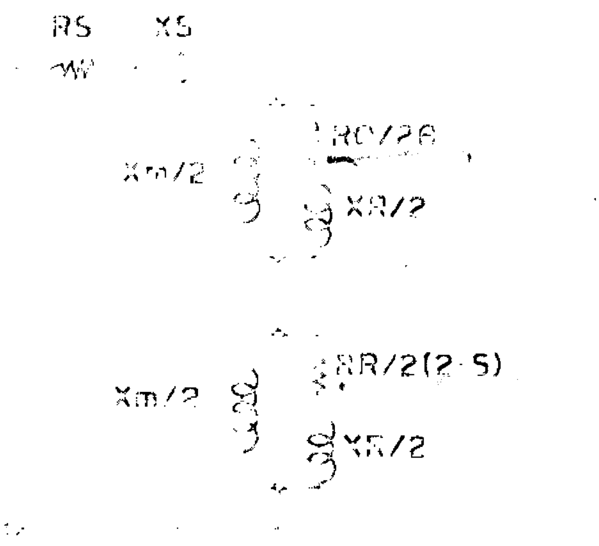

FIG 3.2 EQUIVALENT CIRCUIT OF [illegible] PHASE INDUCTION MOTOR

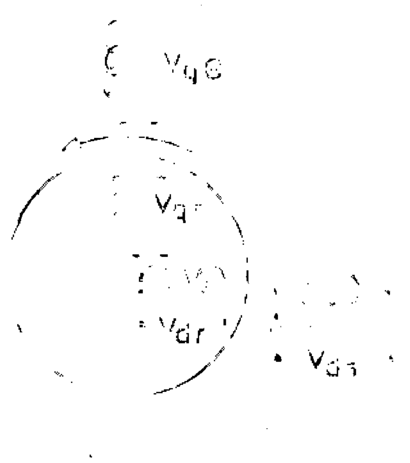

FIG 3.3 INDUCTION MOTOR REPRESENTATION IN QUASI HOLONOMIC REFERENCE FRAME

3.1. METHOD OF FOURIER SERIES ANALYSIS

Impressed voltage wave form is analysed by Fourier Series, the general expression for impressed voltage $v(t)$ is

$$v(t) = \sqrt{2}\,(V_1 \sin \omega t + V_3 \sin 3\omega t + V_5 \sin 5\omega t + V_K \sin K\omega t) \quad \ldots \quad (3.1$$

Analysis of the performance of motor can be proceeded as if there were a series of independent generators[11] all connected in series supplying the motor as shown in FIG. 3.1.

The equivalent circuit of single phase induction motor for sinusoidal operation (fundamental) may be reduced to simple series R-L load. In the form of series circuit

$$R_{eq} = R_s + \frac{R_r}{2}\left(\frac{X_m}{2}\right)^2 \left[\frac{1/s}{\left(\frac{R_r}{2s}\right)^2 + X^2} + \frac{1/2-s}{\left(\frac{R_r}{2(2-s)}\right)^2 + X^2} \right] \quad \ldots \quad 3.2$$

$$X_{eq} = X_s + \left[\frac{\left(\frac{R_r}{2s}\right)^2 + \frac{X_r X}{2}}{\left(\frac{R_r}{2s}\right)^2 + X^2} + \frac{\left(\frac{R_r}{2(2-s)}\right)^2 + \frac{X_r X}{2}}{\left(\frac{R_r}{2(2-s)}\right)^2 + X^2} \right] \frac{X_m}{2} \quad \ldots \quad 3.3$$

$$\phi = \tan^{-1} \frac{X_{eq}}{R_{eq}}$$

Where $X = \frac{X_r}{2} + \frac{X_m}{2}$

R_s = Resistance of stator main winding in ohm

R_r = Rotor resistance reflected to stator in ohm

X_s = Leakage reactance of stator in ohm

X_r = Leakage reactance of rotor reflected to stator in ohms

X_m = Mutual reactance in ohms

α = Firing angle in degrees

x = extinction angle in degrees

ϕ = Power factor angle.

Now treating induction motor as passive R-L load the extinction angle x for a certain firing angle α may be determined from the following equation[4] which can be solved only by successive iteration

$$\sin(x-\phi) - \sin(\alpha-\phi)\, e^{-\cot\phi.(x-\alpha)} = 0 \quad \ldots \quad 3.4$$

Once the extinction angle is known wave form of the impressed voltage is known which can be resolved into fundamental and other harmonic components.

$$av_0 = 0 \quad \ldots \quad 3.8$$

$$av_1 = \frac{\sqrt{2}\,EV}{2}\left[\cos 2\alpha - \cos 2x\right]$$

$$bv_1 = \frac{\sqrt{2}\,V}{2}\left[2(x-\alpha) - \sin 2x + \sin 2\alpha\right] \quad \ldots \quad 3.9$$

$$cv_1 = \sqrt{av_1^2 + bv_1^2}$$

$$\psi v_1 = \tan^{-1}\frac{av_1}{bv_1}$$

nth harmonic component is given by

$$avn = \frac{\sqrt{2}\,V}{2}\left[\frac{2}{n+1}\left\{\cos(n+1)\alpha - \cos(n+1)x\right\} - \frac{2}{n-1}\left\{\cos(n-1)\alpha - \cos(n-1)x\right\}\right] \quad \ldots \quad 3.10$$

$$bvn = \frac{\sqrt{2}\,V}{2}\left[\frac{2}{n+1}\left\{\operatorname{Sin}(n+1)\,\infty - \operatorname{Sin}(n+1)\,x\right.\right.$$

$$\left.\left.-\frac{2}{n+1}\operatorname{Sin}(n-1)\,\infty - \operatorname{Sin}(n-1)\,x\right\}\right]$$

$$cvn = \sqrt{avn^2 + bvn^2}$$

$$\Psi n = \tan^{-1}\frac{avn}{bvn} \qquad 3.13$$

Where

avo = Amplitude of d.c. voltage component

av_1 = Peak amplitude of sine wave component for fundamental

bv_1 = Peak amplitude of cos wave component for fundamental

cv_1 = Peak amplitude of fundamental wave

Ψv_1 = Phase angle for fundamental wave

and

avn = Peak amplitude of sine wave component nth harmonic

bvn = Peak amplitude of cos wave component nth harmonic

cvn = Peak amplitude of nth harmonic wave

Ψn = Phase angle for nth harmonic wave.

The effect of each harmonic voltage is investigated separately. The analysis is carried out as if there were a row of motors arranged on the same shaft and therefore rotating at the same speed N but fed from circuits with different voltages V_1, V_3, V_5, V_7 ... V_n of different frequencies F, 3F, 5F, 7F ...nF respectively where n is the order of harmonic.

The speed of rotation of nth harmonic m.m f is n NS. Where NS is synchronous speed for fundamental. Therefore the slip of the rotor corresponding to nth order harmonic m.m.F. is given by

$$Sn = 1 - \frac{(1 - S_1)}{n} \qquad \ldots \qquad 3.14$$

For each of the motor conceived for a particular harmonic, an equivalent circuit is set up which differs from the circuit for the fundamental only in its parameters. The current and torque from each of the equivalent circuit are summed up to get total response of the machine.

$$I = \sqrt{I_1^2 + I_3^2 + I_5^2 + \ldots} = \sqrt{\sum I_n^2} \qquad \ldots \qquad 3.15$$

$$T = T_1 + T_3 + T_5 + \ldots = \sum T_n \qquad \ldots \qquad 3.16$$

This approach is very involved one requiring laborious computations. If high accuracy is required a large no of harmonic voltage components have to be considered. Further the back e.m.f. generated in the stator winding during non-conduction period can not be directly obtained.

3.2. MATHEMATICAL MODEL OF SINGLE PHASE INDUCTION MOTOR

In the analysis of drives fed from thyristor controlled supplies classical models like steady state equivalent circuit are generally not useful because the applied voltage may be discontinuous and non-sinusoidal / non-uniform. Circuital models are perhaps the most useful in such cases because these can be easily established and are easily subjected to mathematical treatment. These can be used either for dynamic, transient or steady state analysis.

For the analysis purpose in the present case model is taken in Quasi-holonomic reference frame FIG. 3.3. Speed fluctuations are not being considered. Only transformer voltage appear in stator winding, both speed and transformer voltages appear in rotor winding.

The voltage equations in quasi-holonomic reference frames are of the form

$$[V] = [R][i] + [L]\,p\,[i] + [G]\,\dot{\theta}\,[i] \qquad \ldots \qquad 3.17$$

$\dot{\theta}$ = speed of rotation of rotor = Wr

L = inductance matrix

G = torque matrix

The complete voltage and current equations can be written in the matrix form as.

$$\begin{bmatrix} V_{ds} \\ V_{dr} \\ V_{qr} \\ V_{qs} \end{bmatrix} = \begin{bmatrix} R_{ds}+L_{ds}p & M_d p & 0 & 0 \\ M_d p & R_{dr}+L_{dr}p & L_{qr}W_r & M_q W_r \\ -M_d W_r & -L_{dr}W_r & R_{qr}+L_{qr} & M_q p \\ 0 & 0 & M_q p & R_{qs}+L_{qs}p \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \\ i_{qs} \end{bmatrix}$$

In the above matrix

Rds, Rdr, Rqr, Rqs = resistance of respective winding

Lds, Ldr, Lqr, Lqs = self inductance of the respective windings.

SINGLE PHASE MOTOR WITH SINGLE WINDING & TWO SHORT CIRCUITED ROTOR WINDING

im=CM, ird=CD, irq=CQ

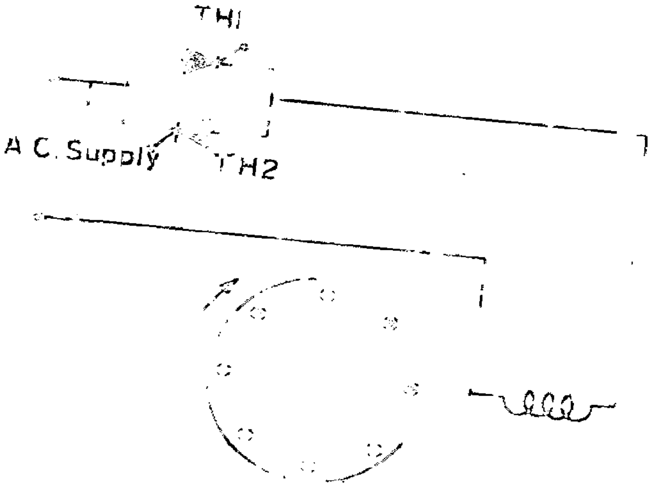

FIG.3.4 ACTUAL POWER CIRCUIT

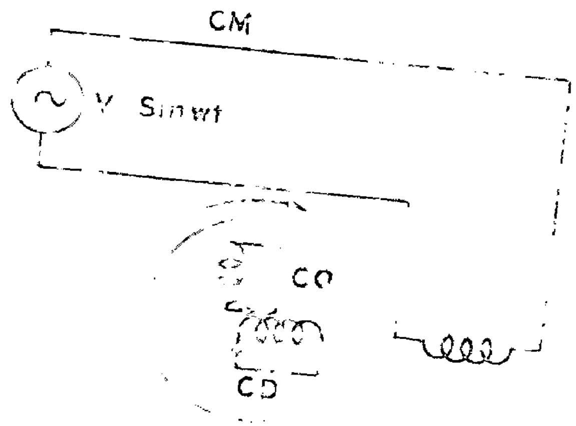

FIG 3.5 CONDUCTION MODE REPRESENTATION.

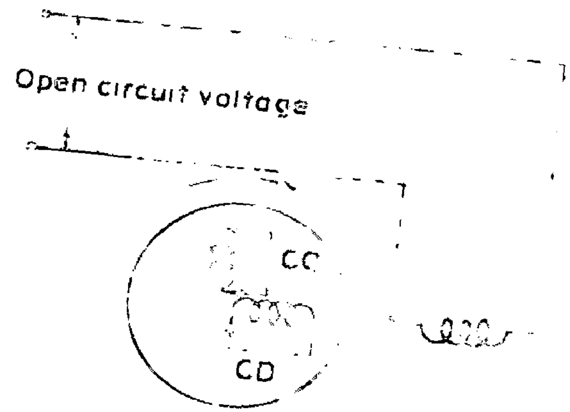

FIG 3.6 NON CONDUCTION MODE REPRESENTATION.

Md = Mutual inductance between the windings of d - axis

Mq = Mutual inductance between the windings of q - axis

For balanced rotor machines these equations can be further simplified.

$$R_{dr} = R_{qr} = R_r, \quad L_{dr} = L_{qr} = L_r$$

The simplified voltage and current equations will be given as

$$\begin{bmatrix} V_{ds} \\ V_{dr} \\ V_{qr} \\ V_{qs} \end{bmatrix} = \begin{bmatrix} R_{ds}+L_{ds}\,p & M_d\,p & 0 & 0 \\ M_d\,p & R_r+L_r\,p & L_r\,W_r & M_q\,W_r \\ -M_d\,W_r & -L_r\,W_r & R_r+L_{qr}\,p & M_q\,p \\ 0 & 0 & M_q\,p & R_{qs}+L_{qs}\,p \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \\ i_{qs} \end{bmatrix} \qquad \text{- 3.22} \;\; \text{..3.25}$$

3.2.1. Normal Operation : A single phase capacitor start induction motor with its stator voltage controlled by a pair of back to back connected thyristors in series with main winding is considered (FIG. 3.4).

When running with only main winding in the circuit i.e. qs open and rotor windings short circuited, the above equations reduce to

$$\begin{bmatrix} V_m \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_m+L_m p & M\,p & 0 \\ M\,p & R_r+L_r p & L_r\,W_r \\ -M\,W_r & -L_r\,W_r & R_r+L_r p \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{dr} \\ i_{qr} \end{bmatrix} \qquad \text{... 3.26} \;\; \text{... 3.28}$$

Where $R_m = R_{ds}$ = resistance of main winding

$L_m = L_{ds}$ = reactance of main winding

$M = M_d$ = mutual inductance between stator & rotor.

When subjected thyristor control the operation has to be considered separately in two parts for conducting and non-conducting mode. The following assumptions may be validly made

1. The turn-on and turn-off times of thyristors are negligible.
2. The hold on current of the thyristors are negligibly small.
3. The forward drop in the thyristors are negligible.

3.2.2. <u>Conducting Mode</u> : When ever any one thyristor conducts the main winding voltage is same as supply voltage ignoring the forward drop in the conducting thyristor. The speed fluctuations due to switching of thyristor are smoothed by the inertia of the rotor and of the connected load FIG. 3.5. During conducting mode equ 3.23 - 3.28 apply with $V_m = V \sin(\omega t + \alpha)$

$$\begin{bmatrix} V \sin(\omega t + \alpha) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_m + L_m p & M p & - \\ M p & R_r + L_r p & -L_r \omega_r \\ + M \omega_r & + L_r \omega_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_m \\ i_{dr} \\ i_{qr} \end{bmatrix} \quad \ldots 3.29 \; \ldots 3.31$$

(With reverse direction of rotation)

$$[v] = [R][i] + [L]\, p\, [i] + [G]\, \omega_r\, [i]$$

$$= \left\{ [R] + [G]\, \omega_r \right\} [i] + [L]\, p\, [i]$$

$$p[i] = -[L]^{-1} \left\{ [R] + [G]\, \omega_r \right\} [i] + [L]^{-1} [v]$$

$$V_m = V \sin(\omega t + \alpha) \Big|_{\omega t = 0}^{\omega t = \beta}$$

The above equations will be valid throughout non-conduction period.

3.2.3. <u>Solution Using Circuit Approach</u> : The solution for conducting and non-conducting periods are obtained by solving simultaneous differential equations (3.29 - 3.31) & (3.35 - 3.38). Initially relaxed electric and magnetic circuit is considered. In subsequent cycles rotor currents are established which are treated as initial conditions for the next cycle[16].

The solution obtained for the conducting period is of the form.

$$i_m = C_{11}\, e^{s_1 t} + C_{12} e^{s_2 t} + C_{13} e^{s_3 t} + C_{14} \sin(\omega t + \phi_1) \qquad \ldots \quad 3.40$$

$$i_{dr} = C_{21} e^{s_1 t} + C_{22} e^{s_2 t} + C_{23} e^{s_3 t} + C_{24} \sin(\omega t + \phi_2)$$

$$i_{qr} = C_{31} e^{s_1 t} + C_{32} e^{s_2 t} + C_{33} e^{s_3 t} + C_{34} \sin(\omega t + \phi_3) \qquad \ldots \quad 3.41$$

For non-conduction period

$$i_{dr} = D_{21}\, e^{s_4 t} + D_{22}\, e^{s_5 t} \qquad \ldots \quad 3.42$$

$$i_{qr} = D_{31}\, e^{s_4 t} + D_{32}\, e^{s_5 t} \qquad \ldots \quad 3.43$$

The coefficients C_{11}, C_{12} .... D_{31}, D_{32} are initial condition dependent terms.

At firing angle α one thyristor is switched on, sinusoidal voltage is applied across the main winding, circuit is

When subjected thyristor control the operation has to be considered separately in two parts for conducting and non-conducting mode. The following assumptions may be validly made

1. The turn-on and turn-off times of thyristors are negligible.
2. The held on current of the thyristors are negligibly small.
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$$\begin{bmatrix} V\sin(Wt+\alpha) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_m+L_mp & Mp & - \\ Mp & R_r+L_rp & -L_rW_r \\ +MW_r & +L_rW_r & R_r+L_rp \end{bmatrix} \begin{bmatrix} i_m \\ i_{dr} \\ i_{qr} \end{bmatrix} \quad \begin{matrix} \ldots 3.29 \\ \\ \ldots 3.31 \end{matrix}$$

(With reverse direction of rotation)

$$[v] = [R][i] + [L]\,p[i] + [G]\,W_r\,[i]$$

$$= \{[R] + [G]\,W_r\}[i] + [L]\,p\,[i]$$

$$p[i] = -[L]^{-1}\{[R] + [G]\,W_r\}[i] + [L]^{-1}[v]$$

$$V_m = V\sin(Wt + \alpha)\Big|_{Wt=0}^{Wt=\beta}$$

$$\begin{bmatrix} \frac{di_m}{dt} \\ \frac{di_{dr}}{dt} \\ \frac{di_{qr}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{L_r R_m}{D_1} & -\frac{M R_r}{D_1} & \frac{M L_r W_r}{D_1} \\ -\frac{M R_m}{D_1} & \frac{L_m R_r}{D_1} & -\frac{L_m L_r W_r}{D_1} \\ -\frac{M W_r}{L_r} & -W_r & -\frac{R_r}{L_r} \end{bmatrix} \begin{bmatrix} i_m \\ i_{dr} \\ i_{qr} \end{bmatrix} + \begin{bmatrix} -\frac{L_r}{D_1} \\ \frac{M}{D_1} \\ 0 \end{bmatrix} V \sin(Wt + \alpha) \qquad \ldots 3.32 \; \ldots 3.34$$

The above equations are valid as long as thyristor is in conducting mode.

<u>Blocking Mode</u> : When current in the main winding goes below the held on current of the conducting thyristor, it switches to the blocking mode FIG. 3.6. No current is allowed to flow in the main winding until the next thyristor is triggered in the subsequent half cycle. The system equations pertaining to OFF period is obtained by imposing current constraint $i_m = 0$.

The resulting modelm is

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_r + L_r p & -L_r W_r \\ L_r W_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} \qquad \ldots 3.35 \; \ldots 3.36$$

induced voltage

$$e_m = M\, p\, i_m \qquad \ldots 3.37$$

System equation can be written as

$$\begin{bmatrix} \frac{di_{dr}}{dt} \\ \frac{di_{qr}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_r}{L_r} & W_r \\ -W_r & -\frac{R_r}{L_r} \end{bmatrix} \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} \qquad \ldots 3.38 \; \ldots 3.39$$

The above equations will be valid throughout non-conduction period.

3.2.3. <u>Solution Using Circuit Approach</u> : The solution for conducting and non-conducting periods are obtained by solving simultaneous differential equations (3.29 - 3.31) & (3.35 - 3.38). Initially relaxed electric and magnetic circuit is considered. In subsequent cycles rotor currents are established which are treated as initial conditions for the next cycle[16].

The solution obtained for the conducting period is of the form.

$$i_m = C_{11} e^{a_1 t} + C_{12} e^{a_2 t} + C_{13} e^{a_3 t} + C_{14} \sin(\omega t + \phi_1) \quad \ldots \quad 3.40$$

$$i_{dr} = C_{21} e^{a_1 t} + C_{22} e^{a_2 t} + C_{23} e^{a_3 t} + C_{24} \sin(\omega t + \phi_2)$$

$$i_{qr} = C_{31} e^{a_1 t} + C_{32} e^{a_2 t} + C_{33} e^{a_3 t} + C_{34} \sin(\omega t + \phi_3) \quad \ldots \quad 3.41$$

For non-conduction period

$$i_{dr} = D_{21} e^{a_4 t} + D_{22} e^{a_5 t} \quad \ldots \quad 3.42$$

$$i_{qr} = D_{31} e^{a_4 t} + D_{32} e^{a_5 t} \quad \ldots \quad 3.43$$

The coefficients C_{11}, C_{12} .... D_{31}, D_{32} are initial condition dependent terms.

At firing angle as one thyristor is switched on, sinusoidal voltage is applied across the main winding, circuit is relaxed initially (zero initial conditions). Equations (3.42 .. 3.44) are valid till iqs goes to zero. toN is obtained by solving the equation

$$C_{11}e^{s_1 t} + C_{12}e^{s_2 t} + C_{13}e^{s_3 t} + C_{14}\,\mathrm{Sin}\,(Wt + \phi_1) = 0 \qquad \ldots \quad 3.44$$

If toN period of half cycle continuous conduction results no control.

toN period of half cycle discontinuous conduction control is obtained.

Rotor currents are obtained at t = toN , these current decay as governed by equations (3.42, 3.43).

After end of the period of half cycle another thyristor fires, the rotor current at the end of half cycle are treated as initial condition for next conduction period. The process continues till steady state is achieved. Instantaneous torque may be obtained from.

$$T = M\, i_{m} \cdot i_{qr} \qquad \ldots \qquad \ldots \qquad 3.45$$

This approach is again very much involved. Good results are expected if one is interested in evaluating dynamic performance of the machine.

3.2.4. <u>State Space Technique</u> : State-space modelling of systems poses notational convenience, facilitates application of modern control theory and simplifies digital simulation[9]. The initial condition state mector pertaining to steady state operating conditions of the induction motor is directly solved for and the need for simulation of thyristors are also avoided. State space modelling of the form $[\dot{X}] = [A].[X] + [B].[u]$ is employed and by using a steady-state criterion the initial condition state vector which would give rise directly to steady state is determined. The order of system is three for a normal single phase induction motor and five for a capacitor run motor. If the model of the form $[\dot{X}] = [A].[X]$ is used, the corresponding system size will go up to five and seven respectively[10].

<u>State Equations</u> :

Linear State equations for single phase induction motor during conduction period is given by

$$[\dot{X}] = [A].[X] + [b].[u] \qquad \ldots \qquad 3.46$$

During non conduction period the linear state equations are obtained as.

$$[\dot{X}] = [B].[X] \qquad \ldots \qquad 3.47$$

Where

$$[\dot{X}] = \begin{bmatrix} \frac{di_m}{dt} \\ \frac{di_{dr}}{dt} \\ \frac{di_{qr}}{dt} \end{bmatrix} \qquad [A] = \begin{bmatrix} \frac{L_r R_m}{D_1} & -\frac{M R_r}{D_1} & \frac{M L_r W_r}{D_1} \\ -\frac{M R_m}{D_1} & \frac{L_m R_r}{D_1} & -\frac{L_m L_r W_r}{D_1} \\ -\frac{M W_r}{L} & -W_r & -\frac{R_r}{L} \end{bmatrix} \qquad [b] = \begin{bmatrix} -\frac{L_r}{D_1} \\ \frac{M}{D_1} \\ 0 \end{bmatrix}$$

$$[X] = \begin{bmatrix} i_m \\ i_{dr} \\ i_{qr} \end{bmatrix} \qquad [B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{R_r}{L_r} & W_r \\ 0 & -W_r & \frac{R_r}{L_r} \end{bmatrix}$$

<u>Initial Condition State Vector</u> : The analysis seeks a steady state initial condition state vector X1 such that if the integration of motor differential equation is started from this vector, the steady state condition would be directly obtained. When such a vector is traceable the need for integrating from arbitrary initial conditions until steady state is reached can be avoided and it is enough to integrate through half a cycle. Also no simulation of thyristor characteristic is necessary.

Assuming time $t = 0$ at the beginning of non-conducting mode, the solution to eqn (3.37) is given by FIG. 3.7 .

$$\left[X(t)\right] = \exp\left\{[B]\ t\right\}\left[X1\right] \qquad \ldots \qquad 3.48$$

The vector X2 at the end of non-conducting mode $t = T_{OFF}$ is then obtained from equation (3.48).

$$\left[X2\right] = \left[X(t_{OFF})\right] = \exp\left\{[B]\ T_{OFF}\right\}\ \left[X1\right] \qquad \ldots \qquad 3.49$$

The solution pertaining to conducting mode is given by

$$\left[X(t)\right] = \exp\left\{[A]\ (t-T_{OFF})\right\}\left[X2\right] + \int_{T_{OFF}}^{t} \exp\left\{[A]\ (t-n)\right\}[b]\,u(n)\ dn \qquad \ldots 3.50$$

where n is a dummy variable.

The vector X3 at the end of conducting mode is obtained from eqn (3.50) as

$$\left[X3\right] = \left[X(T)\right] = \exp\left\{[A]\ T_{on}\right\}\left[X2\right] + \exp\left\{[A]\ T\right\}\int_{T_{OFF}}^{T} \exp\left\{-[A]\ n\right\}[b]\ u(n)\ dn \qquad \ldots \qquad 3.51$$

where T is time corresponding to half a period.

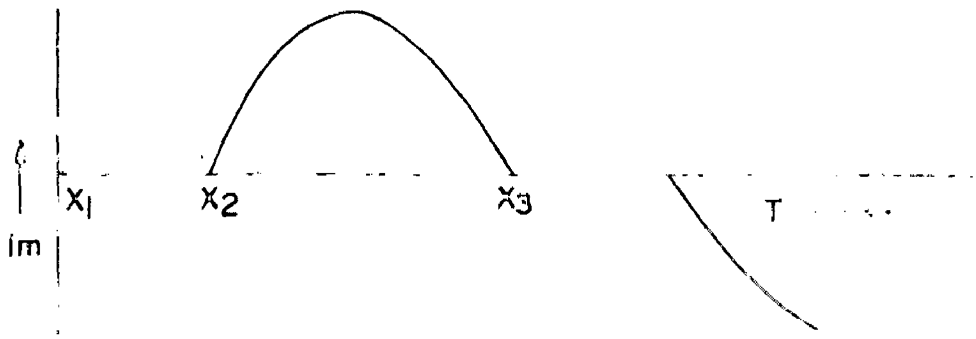


FIG.3.7 MAIN WINDING CURRENT Vs TIME ILLUSTRATING SWITCHING POINTS.

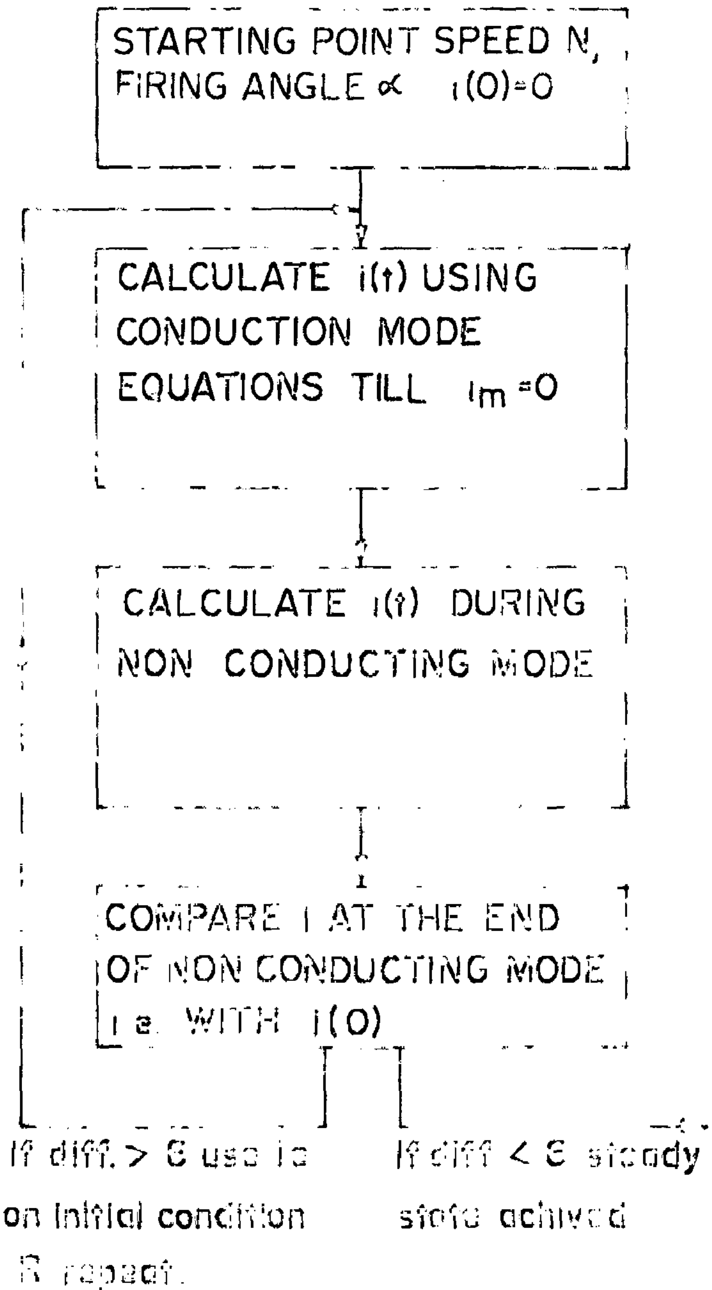


FIG.3.8 BLOCK REPRESENTATION OF ITRATIVE PROCESS INVOLVED FOR ESTABLISHING INITIAL CONDITIONS.

For steady-state

$$[X3] = -[X1] \qquad \ldots \quad 3.52$$

Combining equation (3.49),(3.51) and (3.52) the desired initial condition vector for direct steady-state becomes.

$$[X1] = -[D]^{-1} \exp\{[A]T\} \int_{T_{OFF}}^{T} \exp\{-[A]m\}.[b]u(m)\,dm \qquad \ldots 3.53$$

where

$$[D] = [I] + \exp\{[A]\,T_{on}\}\exp\{[B]\,T_{OFF}\} \qquad \ldots \quad 3.54$$

The solution for X1 given by equation (3.53) is however in complete since

$$u(m) = V_m \sin(W_m + \delta) \qquad \ldots \quad 3.55$$

where δ remains undefined.

The value of δ may be evaluated by assuming δ and T_{on} as independent variable. Firing angle α is determined from

$$\exp(-R\beta/X) = \sin(\alpha + \beta - \theta)/\sin(\alpha - \theta)$$

$$= W_o T_{on}$$

a good starting value may be taken $\alpha = \pi - \beta$.

Now is obtained from.

$$\delta = \alpha - W_o T_{OFF}$$

Although the solution for X1 as given by eqn (8) is now complete. It is an approximate solution and needs correction which is done by iterations[10].

Once initial vector is know, state equation may be solved to obtain, torque, back e.m.f. and losses etc. This approach gives sufficiently good results and makes efficient use of computer time.

3.3. APPROACH USED IN THIS DISSERTATION

In the analysis presented in this dissertation, the circuit model as represented by the equations (3.32 - 3.34) & (3.38 - 3.39) are used. Since analysis relates to steady state operation, no speed fluctuations. Initially unknown circuit conditions are established by iterative process show in FIG.3.8. It is further explained as follows :-

1. Initially motor is considered to be rotating at certain speed and no currents are flowing in the stator and rotor circuits.
2. At certain firing angle α thyristor is switched on, line voltage appears across the stator winding. Currents start setting up in the stator and rotor circuit. Runge Kutta method is applied to obtain solution of this problem. Values at an increment are calculated from some initial values, again those calculated values are treated as initial values for next increment. Second order Runge Kutta method gives sufficient accuracy for this purpose. Certain fixed increment length is taken which can be reduced as extinction point is approached. This point is detected by checking value of main winding current. Conduction equations (3.32 - 3.34) are valid till main winding current goes zero.

3. Since current through thyristor can not go negative, equations for non-conduction (3.23 - 3.25) should be taken up as the extinction point is crossed. These equations are similarly solved till the other thyristor is triggered for negative half cycle. The rotor currents decay during non-conduction period.

4. At the end of period for half cycle the signs of rotor currents are reversed. They are compared with initial values, if difference is large they are taken as initial values for next iteration, otherwise steady state is considered to has been achieved.

Voltage current and other performance characteristic can be easily determined therefrom. The process is further made clear by the flow chart given in FIG. 4.1.

3.3.1. <u>Induced e.m.f. in the Main Winding During Non-conduction</u> :

An induced e.m.f. appears across the main winding terminals when both the thyristors are in the blocking mode. This back e.m.f. is due to rotor currents. An expression for this induced e.m.f. e_m is obtained.

$$e_m = M\, p\, i_{d}$$

It is preferable to get this equation in terms of actual currents rather than their derivative.

$$e_m = M\left(W_r i_{qr} - \frac{R_r}{L_r} i_{dr}\right)$$

3.3.2. <u>Developed Torque</u> : The torque developed by a single phase induction motor is not steady even for sinusoidal operation. With thyristor switching torque remains zero during non-conduction mode assuming auxiliary winding is open. The instantaneous developed torque is

$$\text{Torque} = M\, i_m\, i_{qr}$$

developed power

$$P = M\, \omega_r i_m\, i_{qr}$$

3.3.3. Ohmic Loss of the Motor : Since all the currents during conduction and non-conduction are known Total I^2 R losses can be easily determined.

$$\text{Total } I^2 R \text{ loss} = \frac{1}{T} \int_0^T \left\{ Rm(i_m)^2 + Rr(i_{dr})^2 + Rr(i_{qr})^2 \right\} dt$$

Separate loss determination may be of interest if effect of parameter variation is to be studied.

T is period for half voltage-wave cycle.

R.M.S. and Average Value of Current : Once main winding current wave shape is known r.m.s. and average value of current can be easily determined

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^{Ton} i_m^2\, dt}$$

$$I_{av} = \frac{1}{T} \int_0^{Ton} i_m\, dt$$

Efficiency : Iron and rotational loss are considered to be constant. From total losses efficiency can be determined as follows :-

$$n\,\% = \frac{\text{Power developed}}{\text{Power developed + Ohmic losses + Iron loss and rotational loss}} \times 100$$

3.4. MERITS OF PRESENTED APPROACH OVER RAMAMOORTHY'S APPROACH

Though both of the approaches start from similar circuit equations but method of solving them is entirely different. The approach used here is new and has several advantages compared to Ramamoorthy[10] and Novotny & Fath[9].

(1) Performance characteristics are obtained which are in terms of speed and firing angle variation. For phase controlled drive these are practically observable variables. In Ramamoorthy's approach the performance is obtained in terms of speed and conduction period. Conduction period is not a physically observable parameter. Therefore this approach is practically more realistic.

(2) If motor is running at certain speed and supply goes off momentarily, then reswitching transients may become important. Such a study is possible in this case just by observing the behaviour of currents immediately after reclosing of the switch.

(3) Its computer program is very simple and it does not require large memory. Performance studies can be done even on a small compute such as TDC 312.

(4) Ramamoorthy's approach involves iteration at two steps, one for firing angle determination and other for correcting initial state vector. Therefore it may not have superiority over the present approach as far as computer time is concerned.

(5) In dynamic analysis of induction motor the overall situation becomes very complicated. This simple program can be easily modified to be of use in transient (dynamic) studies.

The accuracy can be easily improved by reducing the step length and steady state check criterion. Fourth order Runge Kutta method can be used for further improving the results.

Analysis for capacitor run induction motor can be done on similar lines using the matrices given by Ramamoorthy and Ilango[10].

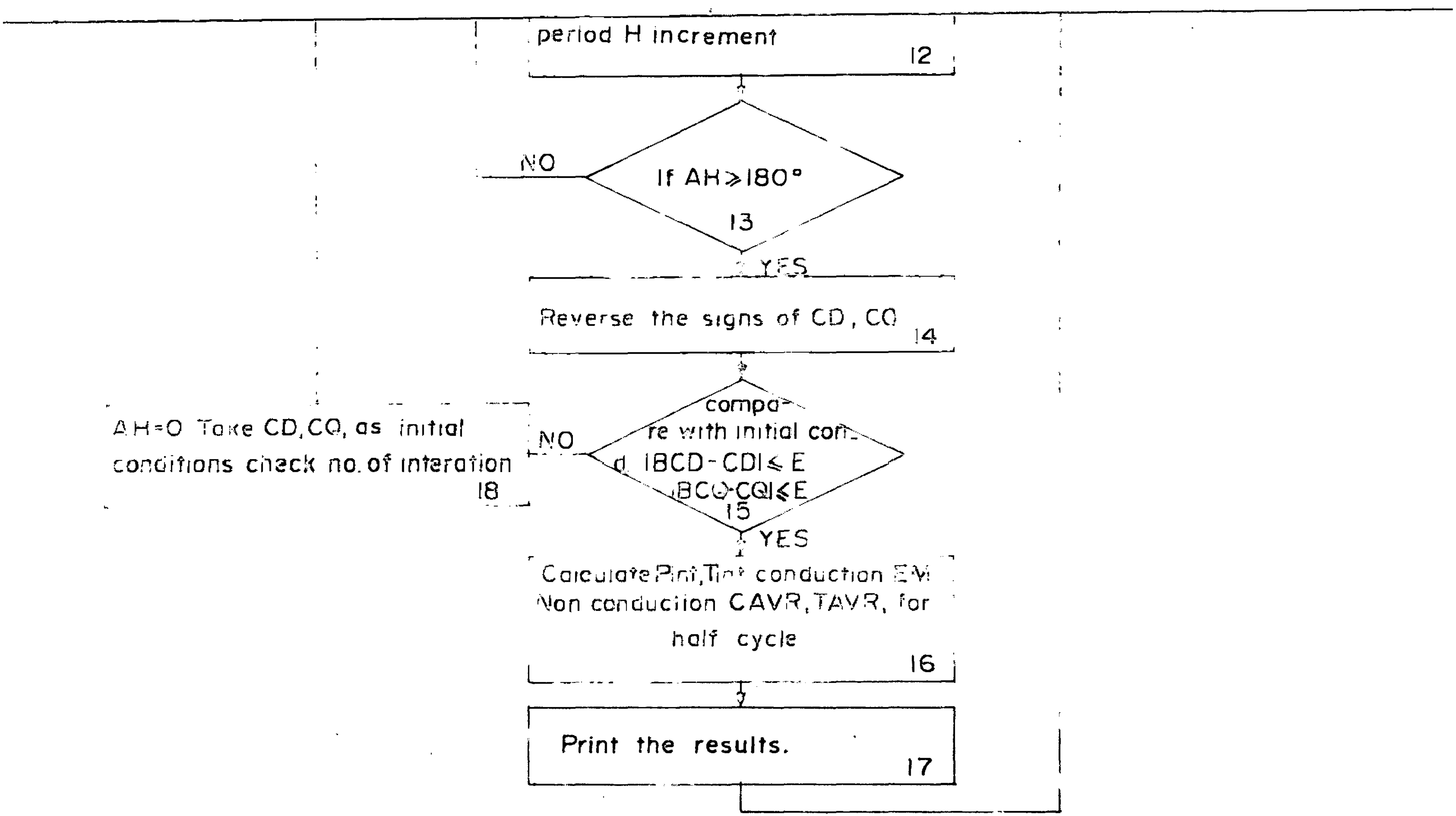


FIG.4.1 FLOW CHART FOR THE ANALYSIS OF SINGLE PHASE SINGLE WINDING INDUCTION MOTOR WITH PHASE ANGLE CONTROL.

CHAPTER - 4

FLOW CHART FOR THE PROBLEM

Using circuit equations for the conduction period (3.32 - 3.34) and equations for non-conduction period (3.35-3.39), a computer program has been developed using the approach suggested in section 3.3.

4.1. DISCONTINUOUS CURRENT CONDUCTION

Flow chart for the discontinuous current conduction is given in FIG. 4.1. Control is obtained only in this mode of operation. Several steps in the flow chart need explanation.

Step 1 : Machine parameter are given. The mutual and self inductances can be easily obtained from

$$M = X\phi / Wr$$

$$L_s = L_r = (X_1 + X\phi) / Wr$$

in case auxiliary winding is present

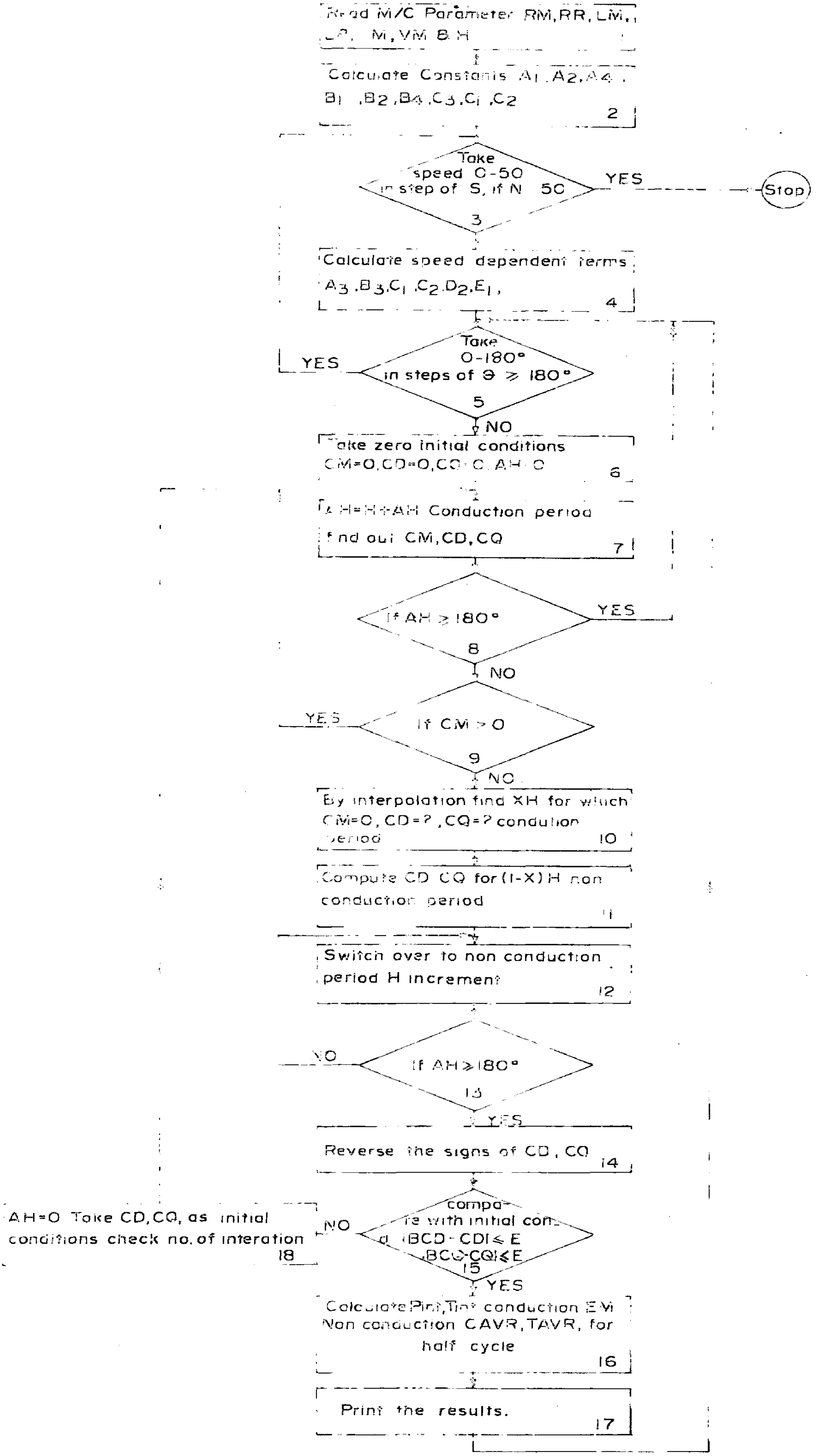
$$L_a = (X\phi\, a_n^2 + X_{1a}) / Wr$$

$$a = \frac{N_a}{N}$$

$$VM = \sqrt{2}\ \text{(r.m.s. voltage of the supply)}$$

$$H = \text{Step length in sec.}$$

Step 2 : The coefficients of [A], [B] & [D] matrix which do not have speed dependent terms are given as



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Step 1 : Machine parameter are given. The mutual and self inductances can be easily obtained from

$$M = x\phi/Wr$$

$$Ls = Lr = (x_1 + x\phi)/Wr$$

in case auxiliary winding is present

$$La = (x\phi\, a_b^2 + x_{1a}) / Wr$$

$$a = \frac{N_a}{N}$$

$$Vm = \sqrt{2}\ (\text{r.m.s. voltage of the supply})$$

$$H = \text{Step length in sec.}$$

Step 2 : The coefficients of [A] , [b] & [D] matrix which do not have speed dependent terms are given as

$A_1 = Lr . Rs/D$

$A_2 = - M . Rr/D$

$A_4 = - Lr\ W1/D$

$B_1 = - M . Rs/D$

$B_2 = Ls.Rs/D$

$B_4 = M . W1/D$

$C_3 = - Rr/Lr$

$D_1 = - Rr/Lr$

$D_3 = - Rr/Lr$

$D = M.M - Ls.Lr$

Step 3 : Some speed is chosen.

$Wr = S$ = Speed given in electrical radians / Sec.

Wo = Supply frequency in electrical radians / Sec.

= 314.159 for 50 Hz supply.

Step 4 : The speed dependent terms of matrix A & B are calculated.

$A_3 = M.Lr.Wr / D$

$B_3 = - Ls.\ Lr.\ Wr / D$

$C_1 = - M.Wr / Lr$

$C_2 = - Wr$

$D_2 = Wr$

$B_1 = Wr$

Step 6 : Initially circuit is considered to be relaxed. Zero initial conditions are taken.

Step 7 : First order linear differential equations for conducting mode are taken. Using second order Runge Kutta method increments are calculated for H step length.

$u_1 = (A1.i_m + A2.\ i_{rd} + A3.i_{rq} + A4.\text{Sin}\ \infty).H$

$v_1 = (B1.i_m + B2.i_{rd} + B3.i_{rq} + B4.\text{Sin}\ \infty).H$

$w_1 = (C1.i_m + C2.i_{rd} + C3.i_{rq}).H$

$i_{m1} = i_m + u_1$

$i_{rd1} = i_{rd} + v_1$

$i_{rq1} = i_{rq} + w_1$

$\infty(\text{new}) = \infty\ (\text{old}) + H.Ws$

$u_2 = (A1.i_{m1} + A2.i_{rd1} + A3.i_{rq1} + A4.\text{Sin}\ \infty).H$

$v_2 = (B1.i_{m1} + B2.i_{rd1} + B3.i_{rq1} + A4.\text{Sin}\ \infty).H$

$w_2 = (C1.i_{m1} + C2.i_{rd1} + C3.i_{rq1}).H$

$i_m(\text{new}) = i_m(\text{Old}) + \frac{1}{2}(u_1 + u_2)$

$i_{rd}(\text{new}) = i_{rd}(\text{Old}) + \frac{1}{2}(w_1 + v_1)$

$i_{rq}(\text{new}) = i_{rq}(\text{Old}) + \frac{1}{2}(v_1 + w_2)$

AH = AH + H (it keeps check of the time)

Step 9 : If even after the end of period for half cycle main winding current does not become zero, it means there is condition of continuous current conduction and control is lost. $\infty < \phi$

Step 10 : If between the consecutive points of computation, the current changes its sign, in order to get extinction point linear interpolation is used to obtain value of time at which current passes through zero.

$$x = -\ i_m/u$$

x H will be the increment when current pass through zero or point of extinction. Conducting equations are valid only upto this point.F

Step 11 : For the rest of the interval of duration H(1-X) non-conducting mode equations have to be taken up.because just after extinction point conducting mode equations are no longer valid.

Step 12 : For non-conducting mode period (starting from time when $i_m \rightarrow 0$ and ending at point $Wt = \pi$) the following first order linear differential equations are solved using. Second order Runge Kutta method.

$$X_1 = (D_1.i_{rd} + D_2.i_{rq}).H$$

$$Y_1 = (E_1.i_{rd} + E_2.i_{rq}).H$$

$$i_{rd1} = i_{rd} + X_1$$

$$i_{rq1} = i_{rq} + Y_1$$

$$X_2 = (D_1.i_{rd1} + D_2.i_{rq1}).H$$

$$Y_2 = (E_1.i_{rd1} + E_2.i_{rq1}).H$$

$$i_{rd}(new) = i_{rd}(Old) + \tfrac{1}{2}(X_1 + X_2)$$

$$i_{rq}(new) = i_{rq}(Old) + \tfrac{1}{2}(Y_1 + Y_2)$$

Step 14 : At the end of the period for half cycle signs of the rotor currents are reversed. Instead of solving for negative half cycle the same positive cycle is solved using them as initial conditions. Thus length of the program is reduced.

Step 15 : Final values at the end of the period for half cycle are compared with the starting values. If they exceed certain limit next iteration is performed. Error check affects the accuracy of the result. If check criterion is satisfied it means steady state has been achieved.

Step 16 : Integration is done to obtain average torque, average current, r.m.s. current and total copper loss. Back emf is also determined.

Step 18 : A check is provided on the no. of iderations performed. If required steady state is not obtained within prescribed no. of iteration results are printed out. It is in order to check computer from calculating infinitely.

Intermediate results can be easily obtained by introducing PRIN T statement in step 7 and step 12.

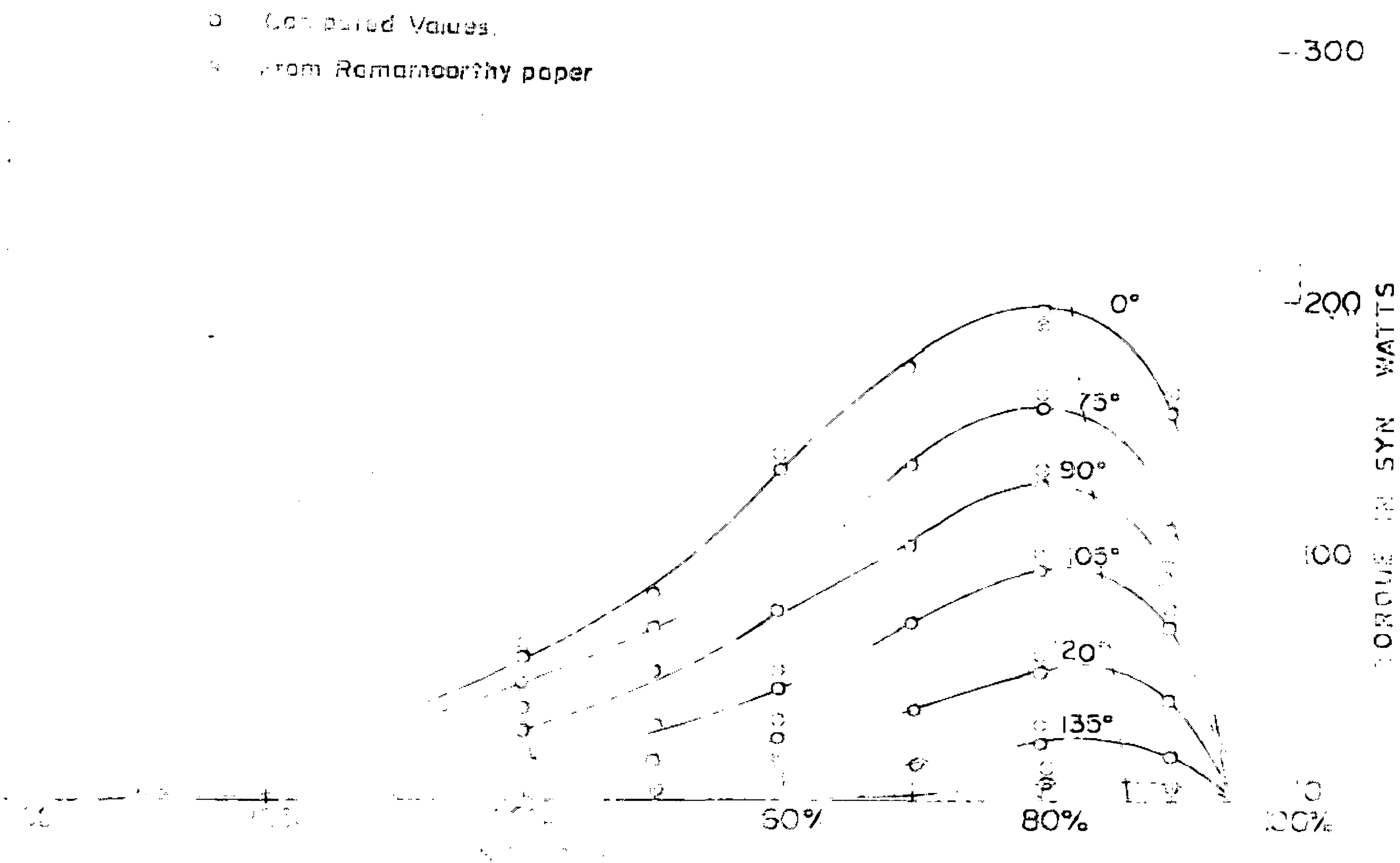
4.2. CONTINUOUS CURRENT CONDUCTION

Though this region is not important from control point of view. But to obtain complete performance characteristics the same approach can be applied with a minor change. At step 8 when period of half cycle is complete it goes to step 14 directly and the iterations continue till steady state is reached. Computer program in Fortran IV is given in Appendix II.

4.3. VERIFICATION OF RESULTS

Validity of any approach can be proved best if its results are in conformity with results obtained from a practical set up or with other analytical results which in term have been verified.

4.3.1. Verification of Ramamoorthy's Results : Ramamoorthy[10] has given certain performance characteristics for the same data as used by Novotny and Fath[9] in their earlier work.



[illegible] SPEED [illegible] NOVOTANY & FATH

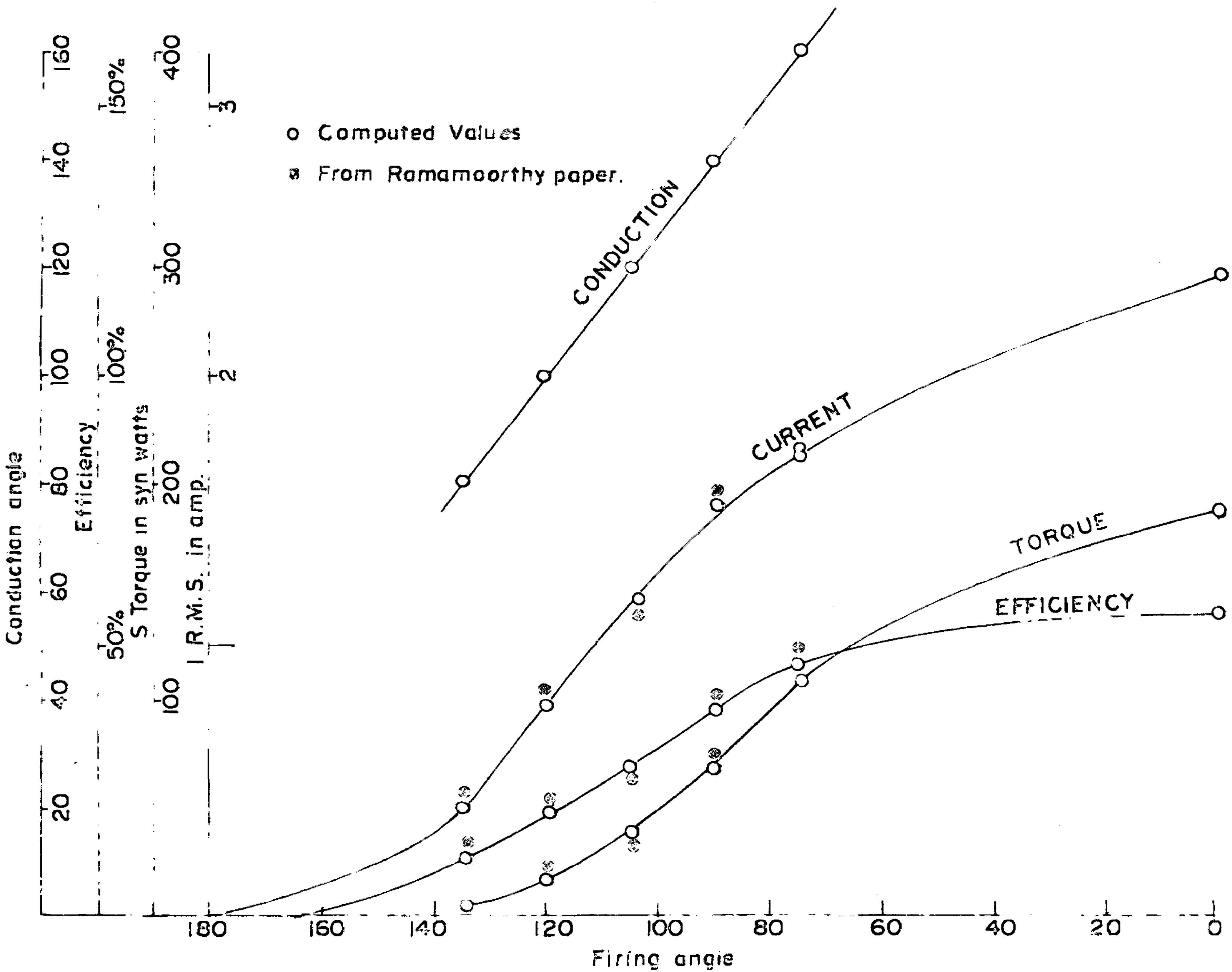


FIG.4.3 PERFORMANCE CURVES FOR SINGLE PHASE INDUCTION MOTOR.

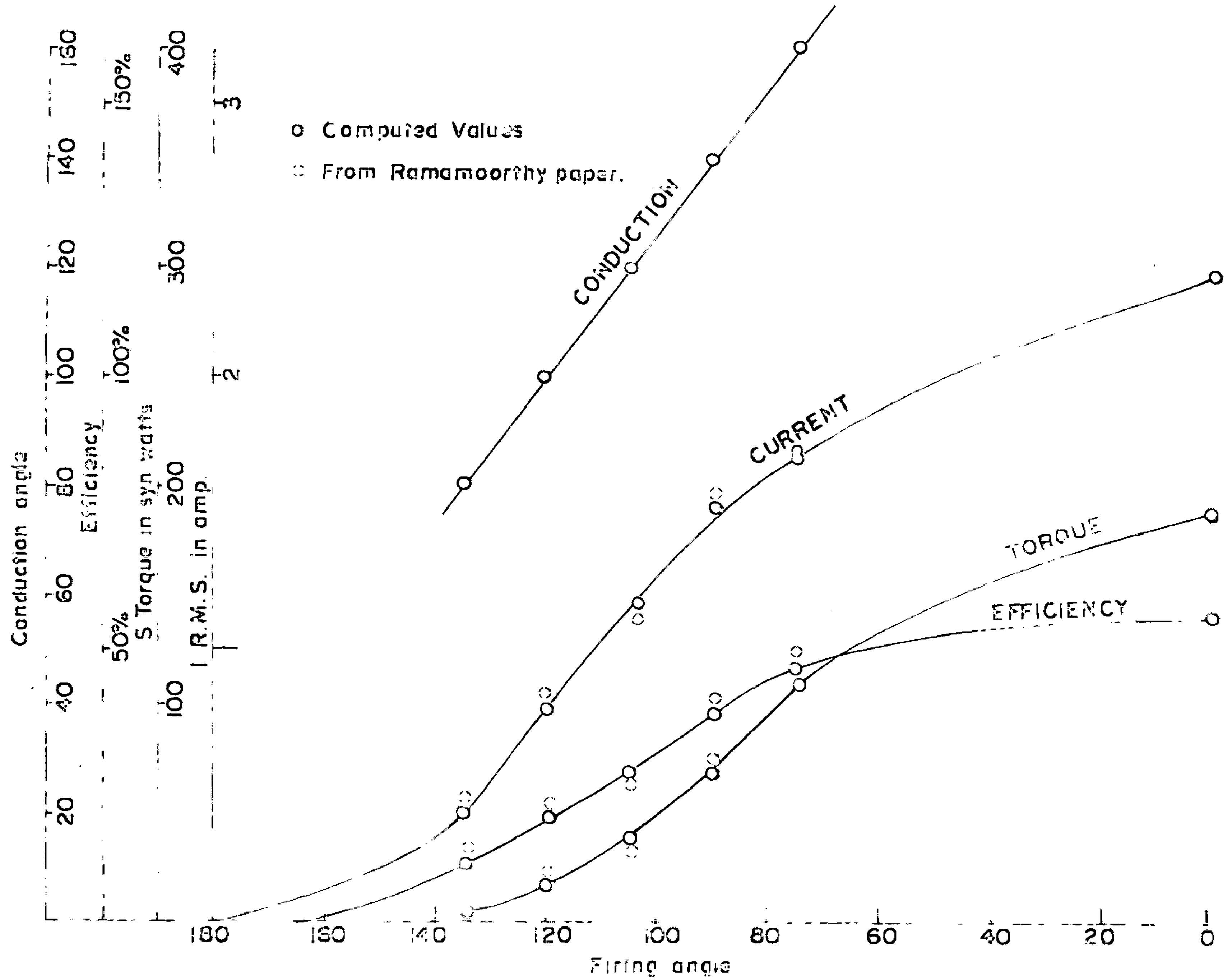


FIG.4.3 PERFORMANCE CURVES FOR SINGLE PHASE INDUCTION MOTOR.

These results relate to a machine with following specifications and parameters.

Name plate Data

1/3 h p   1100 r/min   60 Hertz

208 to 230 Volts   22 amperes   5 uF (running and/or starting capacitor)

Type S C   continuous duty 50°C rise

Machine constants

RM = 15.32 ohms   Xl = 12.93 ohm   as = 1.2

Rr = 24.73 ohms X_M = 164.91 ohms XC.530.5 ohm

Ra = 56.21 ohms X_{la} = 17.32 ohms   V = 230 V

The mutual and self inductances can be determined from

$$M = X_M / Ws$$

$$LM = Lr = (X_1 + X_M) / Ws$$

$$La = (X M a_s^2 + X_{la}) / Ws$$

Ls = Lr = 0.5 H   LA = 0.676 H   M = 0.4375 H

The torque-speed curve FIG.4.2 and curves for variation of efficiency, torque conduction angle and RMS current with firing angle FIG 4.3 are plotted. These are very close to as given by Ramamoorthy (10).

4.4. DESCRIPTION OF PRACTICAL SET UP

A pair of back to back connected thyristors (2 N 3670) is used in the main winding of single phase induction motor. A snubber circuit is used to protect thyristor from voltage surges

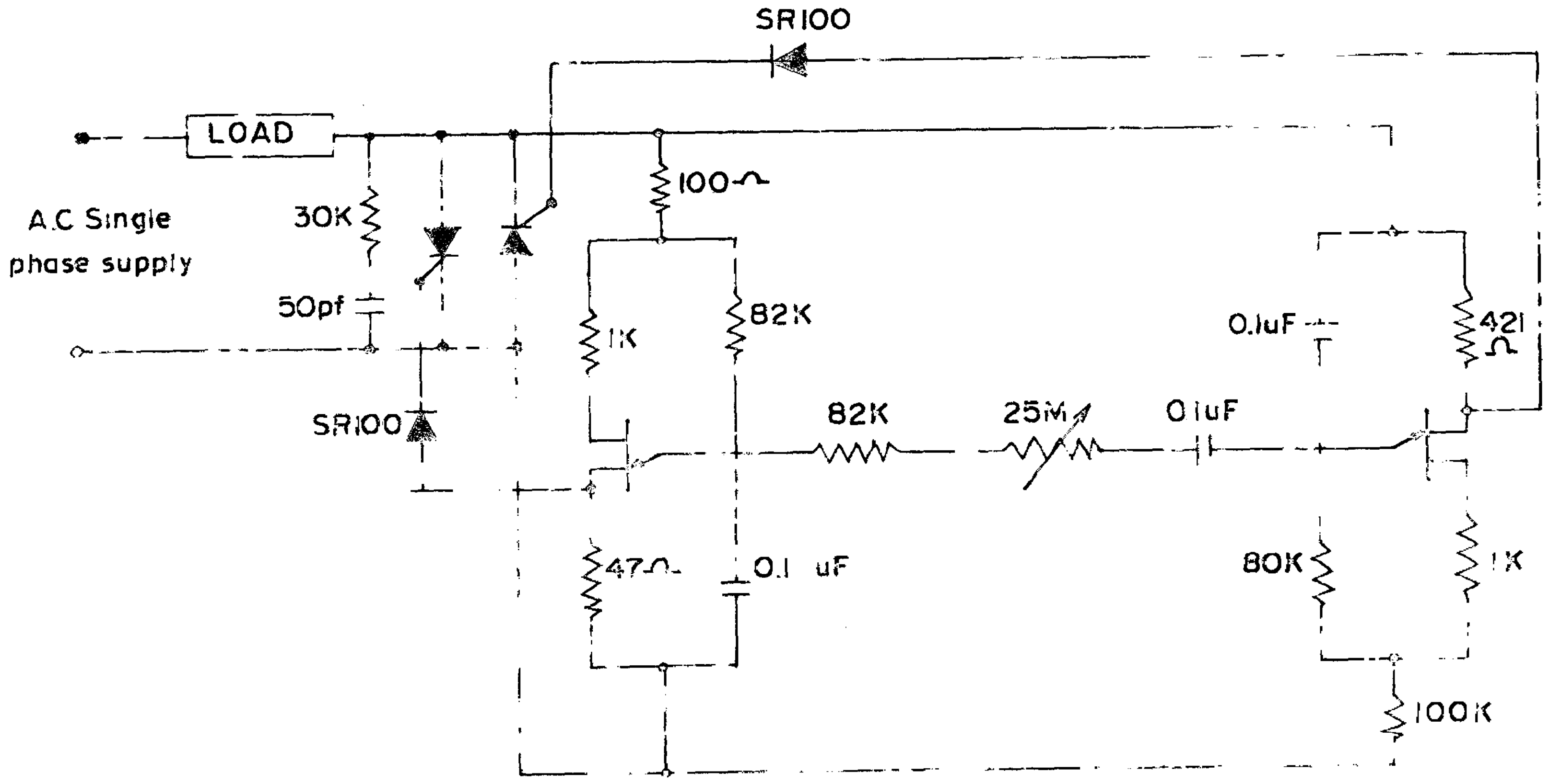


FIG.4.4 FIRING CURRENT FOR BACK TO BACK CONNECTED THYRISTOR EMPLOYED IN PHASE ANGLE CONTROL.

in the line. Two UJT relaxation oscillators operating in anti-parallel mode are used to obtain triggering pulses for the thyristor pair. They operate in positive and negative part of the cycle alternatively and are synchronized with line. Phase angle control from 5° - 175° is obtained through this circuit. Detailed circuit is shown in FIG. 4.4.

To obtain complete torque speed characteristic of induction motor it is coupled with a d.c. shunt motor which is connected to supply. In the stable region it is made to operate as a generator and in unstable region its behaviour changes from motor to generator at equilibrium point. Thus the combined characteristics of the drive enables us to obtain unstable region also. Since d.c. motor was not of sufficient capacity to maintain equilibrium condition in the unstable region, complete characteristic for large firing angle could only be obtained.

Name plate date of experimental induction motor H P = 0.5 (Watts 380)

R P M 1440	Phase 1	Cycles 50
Volts 220/230	Amps 38	
Rating Cont	Class A insulation.	

From short circuit and open circuit test the machine constants are obtained as described in[17].

R_M = 4.2 ohm X 1 = 85 ohm

R_r = 6.1 ohms X M = 100 ohms.

The self and mutual inductances are

L_M = L_r = 0.3453 H M = 0.3182 H

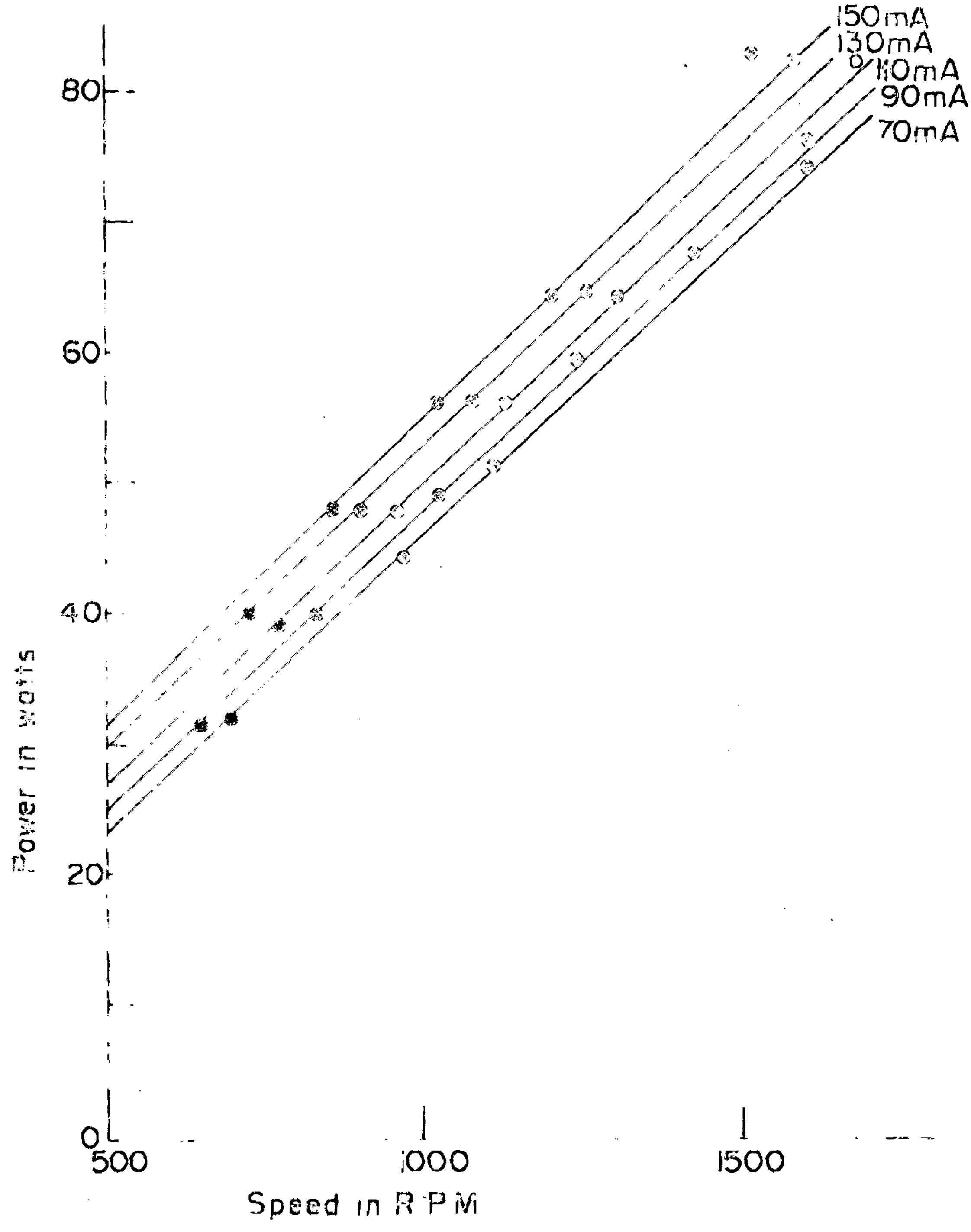


FIG 4.5 SPEED Vs CONSTANT LOSSES (Iron+Friction)

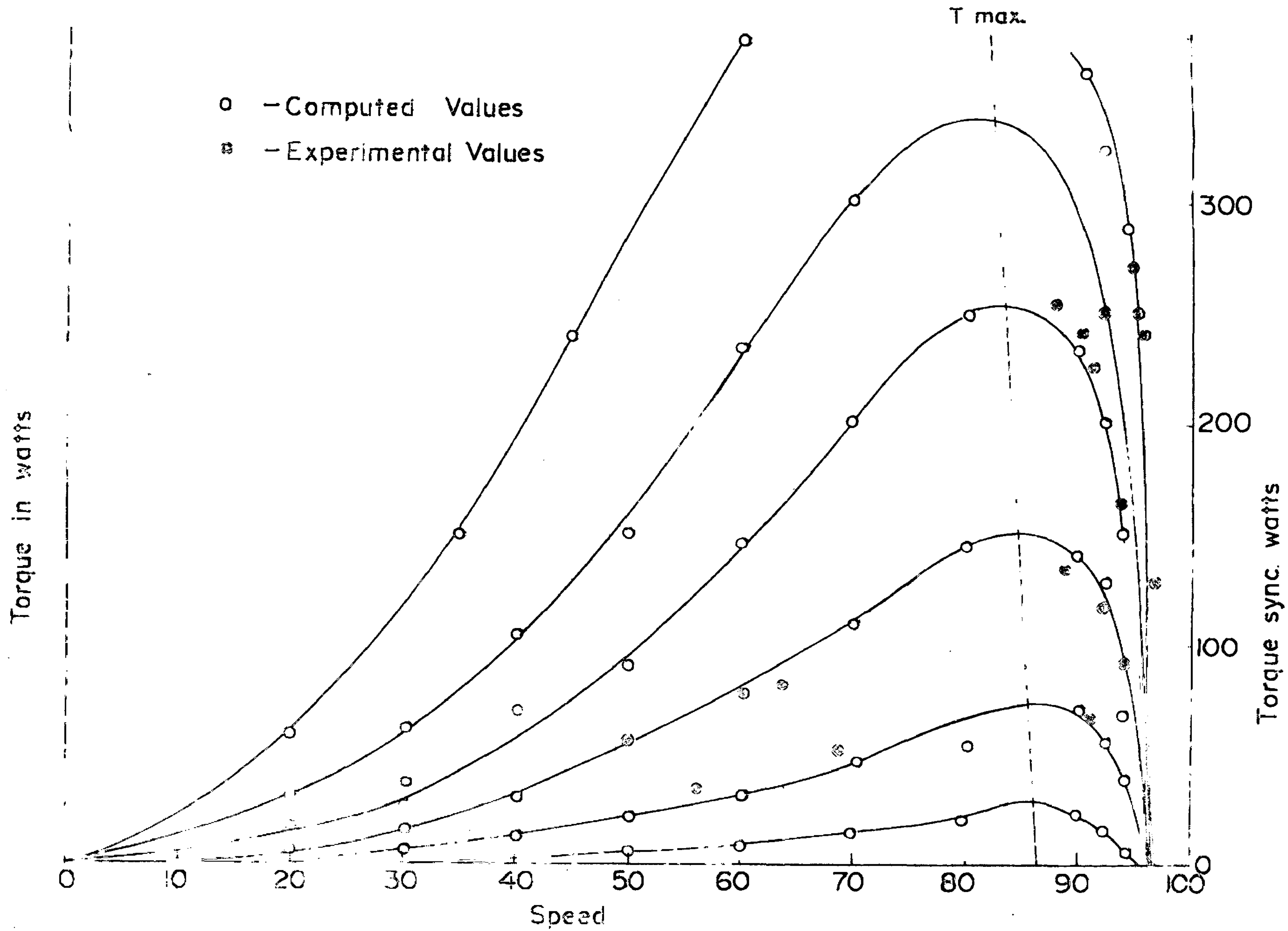


FIG. 4.6 TORQUE SPEED CURVE FOR EXPERIMENTAL MOTOR.

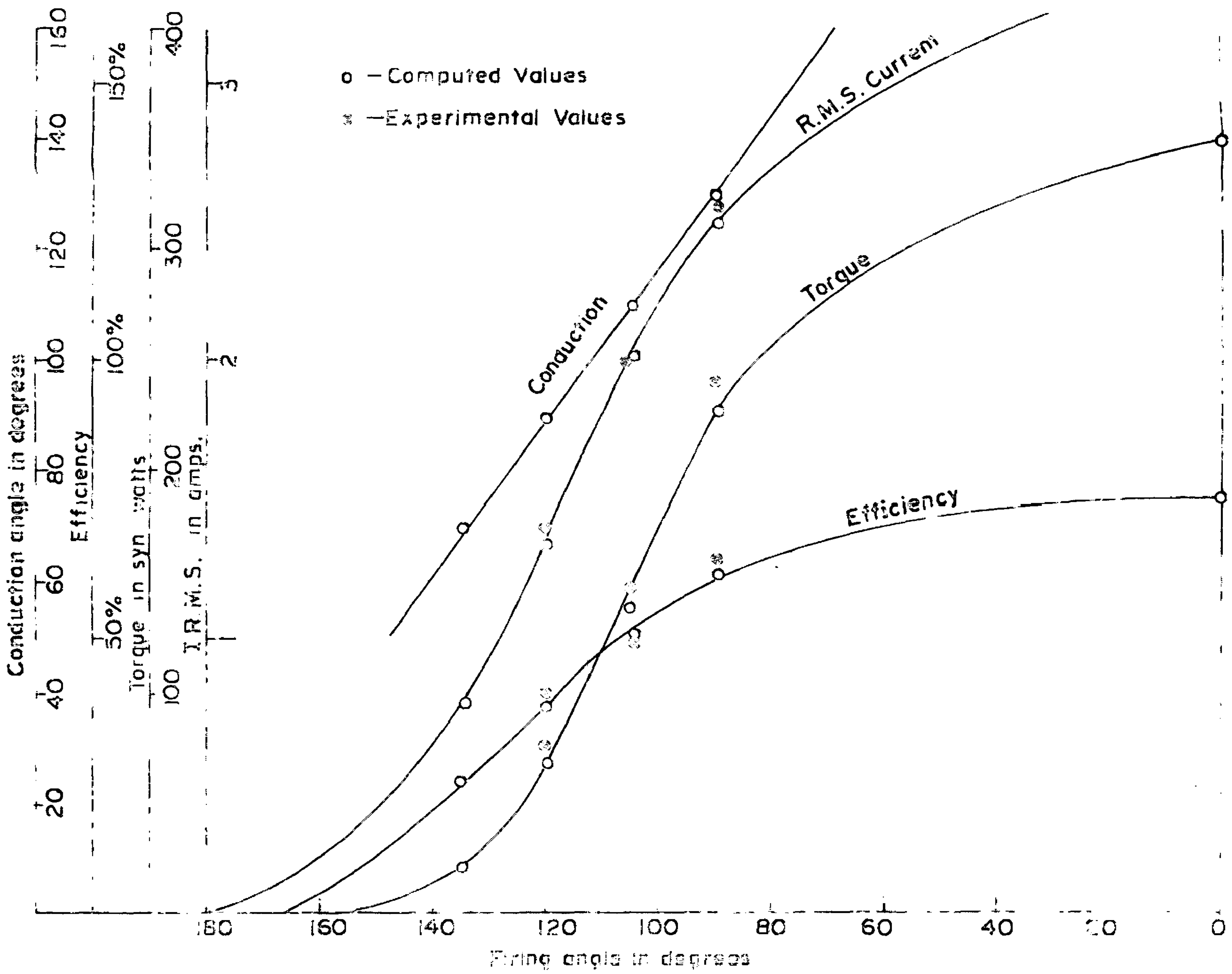


FIG.4.7 PERFORMACE CURVE FOR EXPERIMENTAL MOTOR

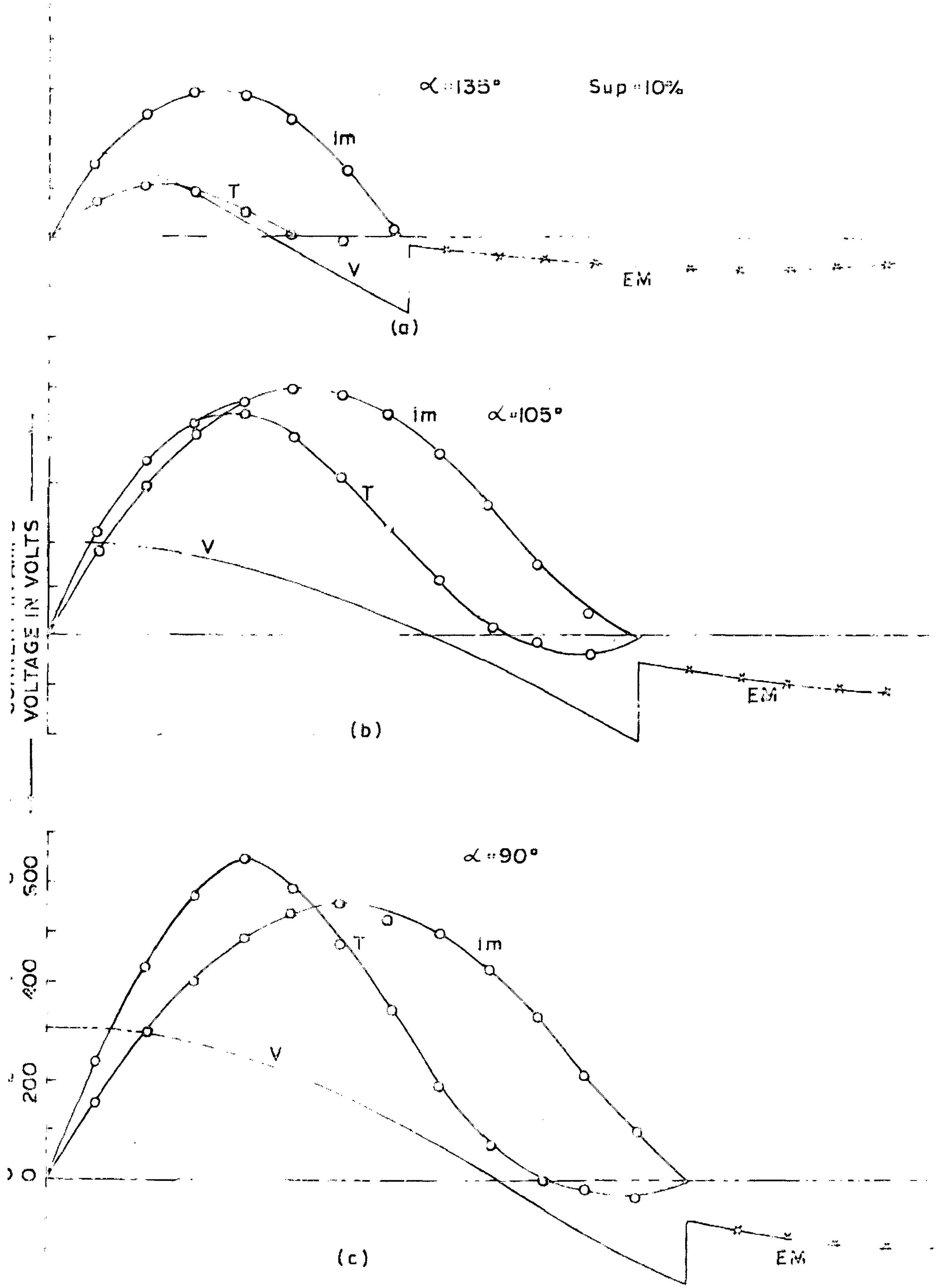


FIG.4.8 WAVE FORMS OF CURRENT, VOLTAGE, TORQUE IN A SINGLE PHASE INDUCTION MOTOR

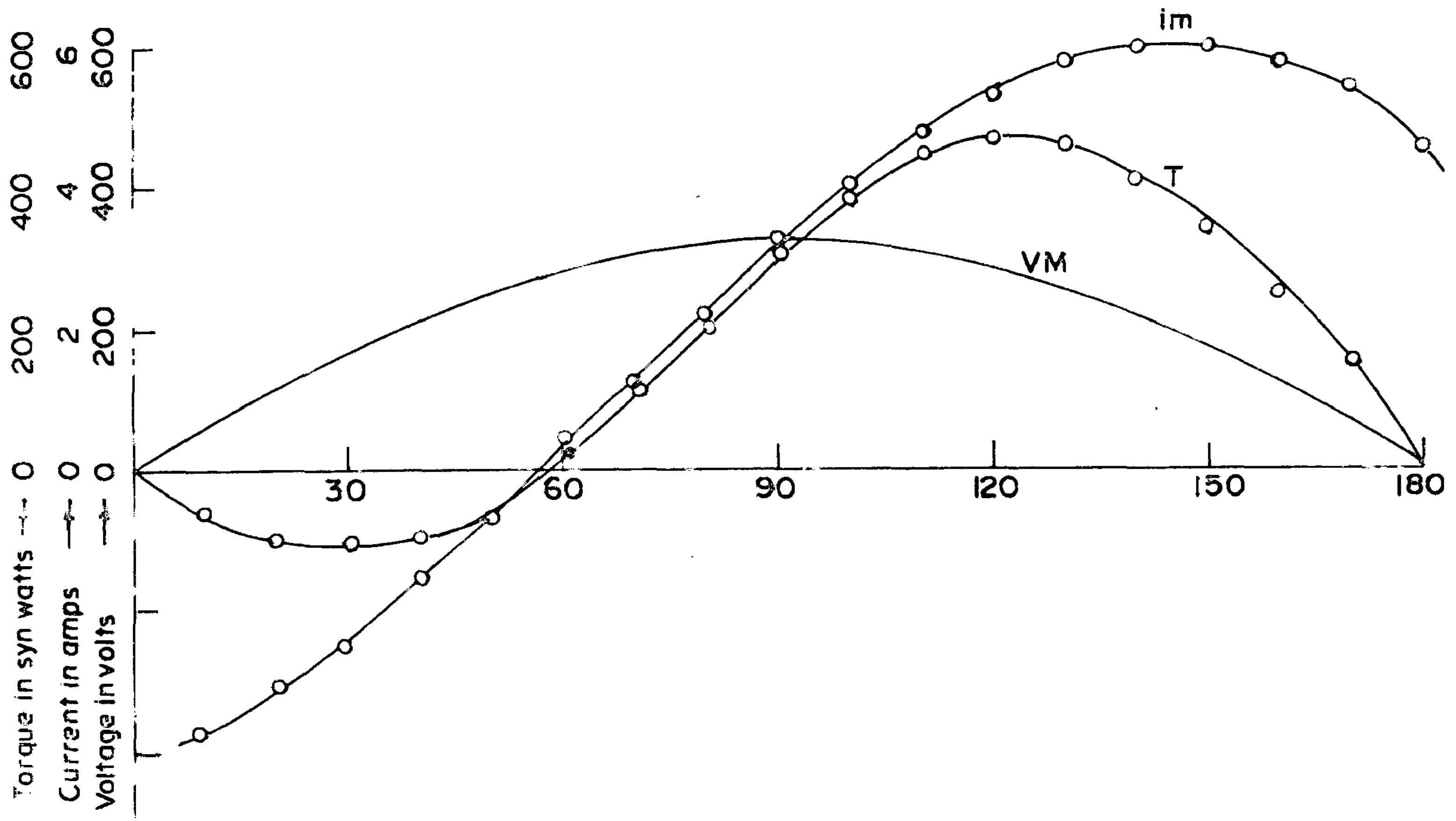
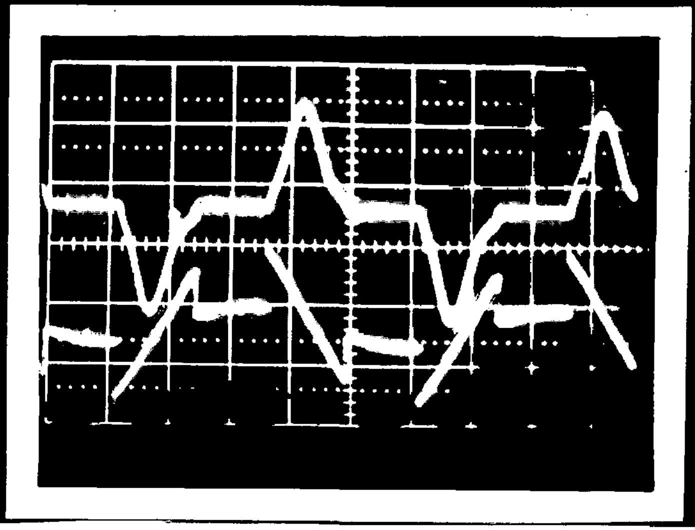
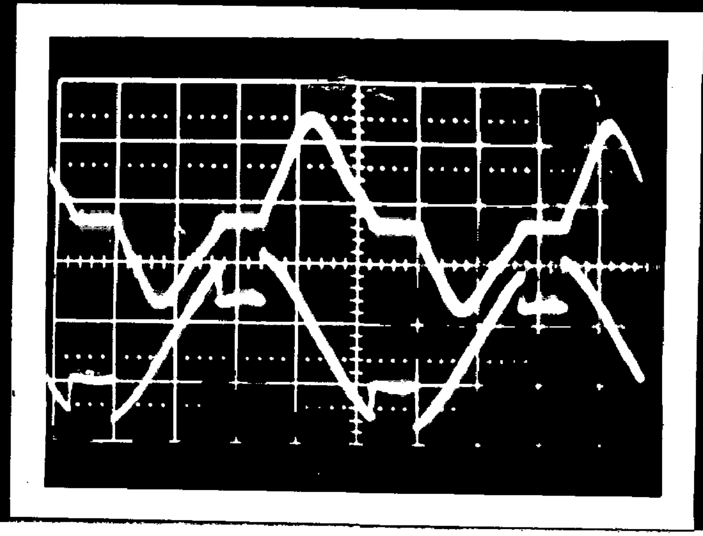


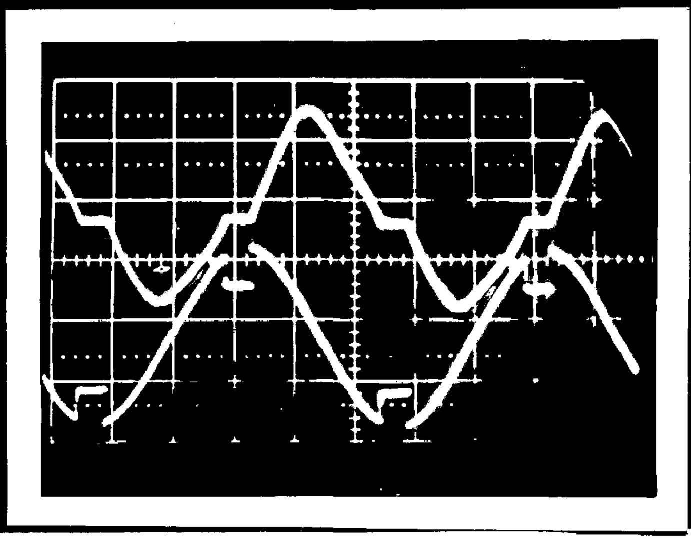
FIG.4.9 CURRENT, VOLTAGE & TORQUE WAVE FORM FOR CONTINUOUS CONDUCTION.



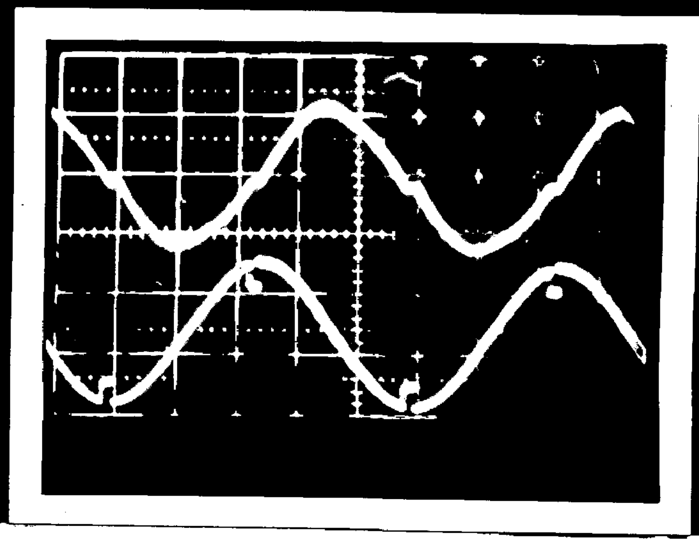
PH(a) Current & Voltage waveforms at $\alpha = 135^\circ$ S = 10% Discontinuous cond.



PH(b) Current & Voltage waveforms at $\alpha = 105^\circ$ S = 10% Discontinuous cond.



PH(c) Current & Voltage waveforms at $\alpha = 90^\circ$ S = 10% Discontinuous cond.



PH(d) Current & Voltage waveforms at $\alpha = 75^\circ$ S = 10% continuous conduction.

Name plate data of coupled d.c. machine

R.P.M. = 1500 H P = ¼

Volts = 220 Current = 1.35 Amp

Rating continuous

4.4.1. <u>Verification from practical set up</u> : , A.C. motor is left unexcited and power is fed from d.c. side. Speed vs constan losses. Curves are determined for different field currents. Now when a.c. motor is excited and d.c. machine acts as gene its shaft power can be determined as

4.5. PERFORMANCE CHARACTERISTIC

Torque vs speed and characteristics for torque vs angle, conduction period vs firing angle, efficiency vs firi angle, rms current vs firing angle at a particular slip are calculated. Calculated characteristics are shown in FIG.4.6 FIG.4.7. Points obtained experimentally are marked on these characteristics. It is seen that computed results are close enough to experimentally obtained results. Deviation of 10% i observed, 20% deviation was also reported by Novotny and Fa

4.6. WAVE SHAPES

Voltage, current and torque wave shapes are plott against time for different firing angles FIG.4.8(a), FIG.4.8 FIG.4.8(c) & FIG.4.9 (one half cycle only). There nature is similar to that shown by Ramamoorthy[10]. The nature can agai compared with the experimentally recorded current and voltage forms shown in photographs PH(a), PH(b), PH(c) and PH(d).

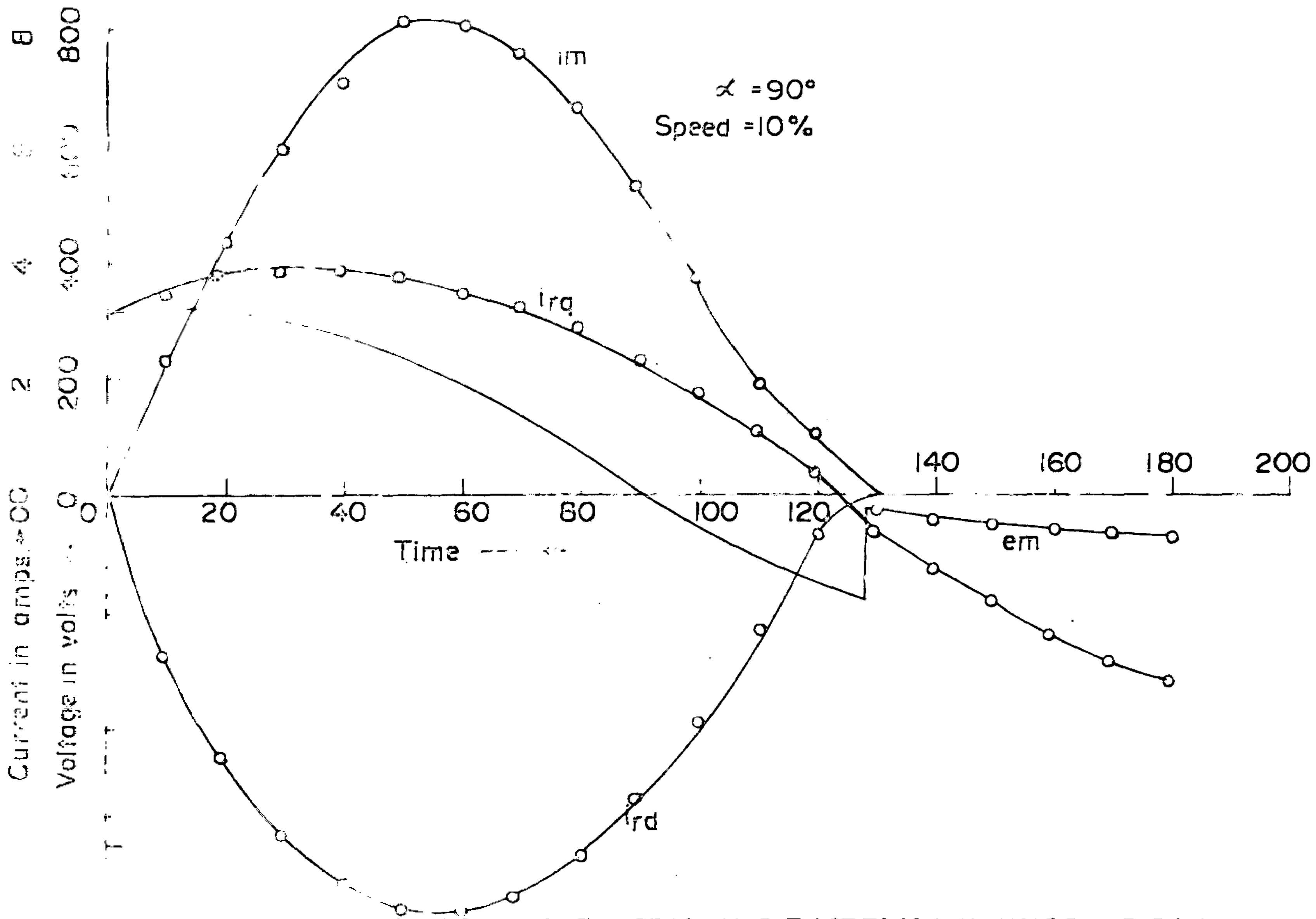


FIG 4.10 CURRENT & VOLTAGE WAVE FORM IN DIFFERENT WINDINGS OF SINGLE PHASE INDUCTION MOTOR.

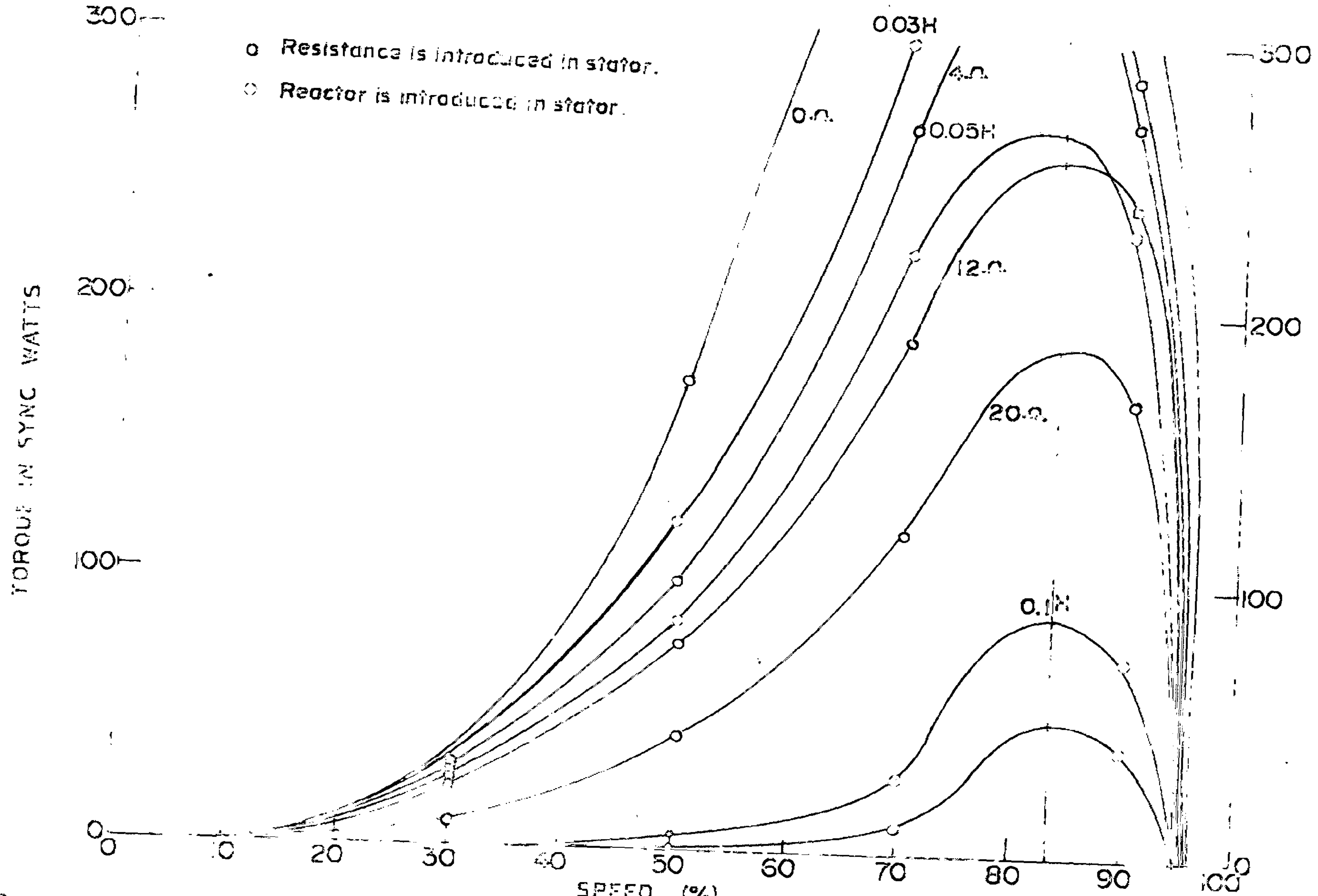


FIG 4.? TORQUE SPEED CURVE WHEN RESISTOR OR REACTOR IS INTRODUCED IN STATOR

In FIG(d) perfectly continuous wave could not be obtained because of ON-OFF period of thyristors in the circuit. FIG.4.10 shows nature of different stator and rotor currents (non observable) which gives clear picture of instantaneous torque during conducting period and back emf during non-conducting period.

4.7. SPEED CONTROL BY INTRODUCING RESISTOR OR REACTOR

Torque speed characteristic of single phase induction motor for different resistors or reactors in stator main winding are shown in FIG. 11. These elements are normally used in conventional method of speed control.

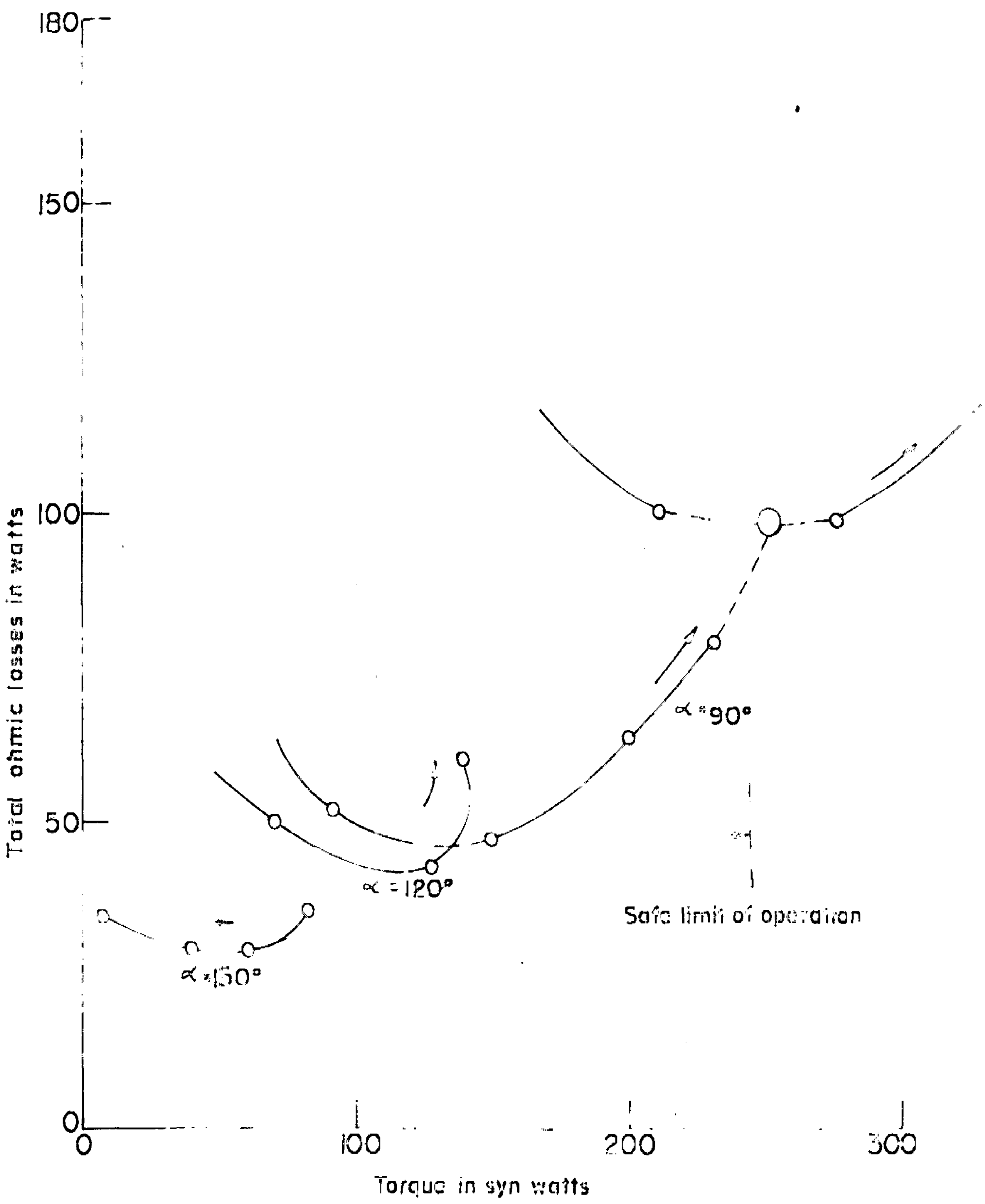


FIG.5.1 OHMIC LOSSES Vs TORQUE

CHAPTER - 5

PERFORMANCE STUDIES

The following aspects of motor performance under phase control operation are considered of interest.

5.1. TORQUE-SPEED CHARACTERISTICS

When a resistor or a reactor is used for speed control, maximum torque occurs at certain fixed speed, while in case of phase angle control peak torque occurs at successively higher speeds as firing angle is increased. This is not desirable from speed control application point of view as it reduces range of speed control.

High rotor resistance gives better speed control range but for lower rotor resistance not much speed variation could be obtained at full load. Therefore a high rotor resistance is recommended from control range point of view.

5.2. OHMIC LOSSES

Ohmic losses normally decrease as firing angle is increased as shown in FIG.5.1. Loss curve have a minima at certain speed, on either side of which the losses increase considerably. It is observed that for a particular torque the losses increase the losses for the same torque under continuous condition condition. Furthermore speed also falls down for the same torque production.

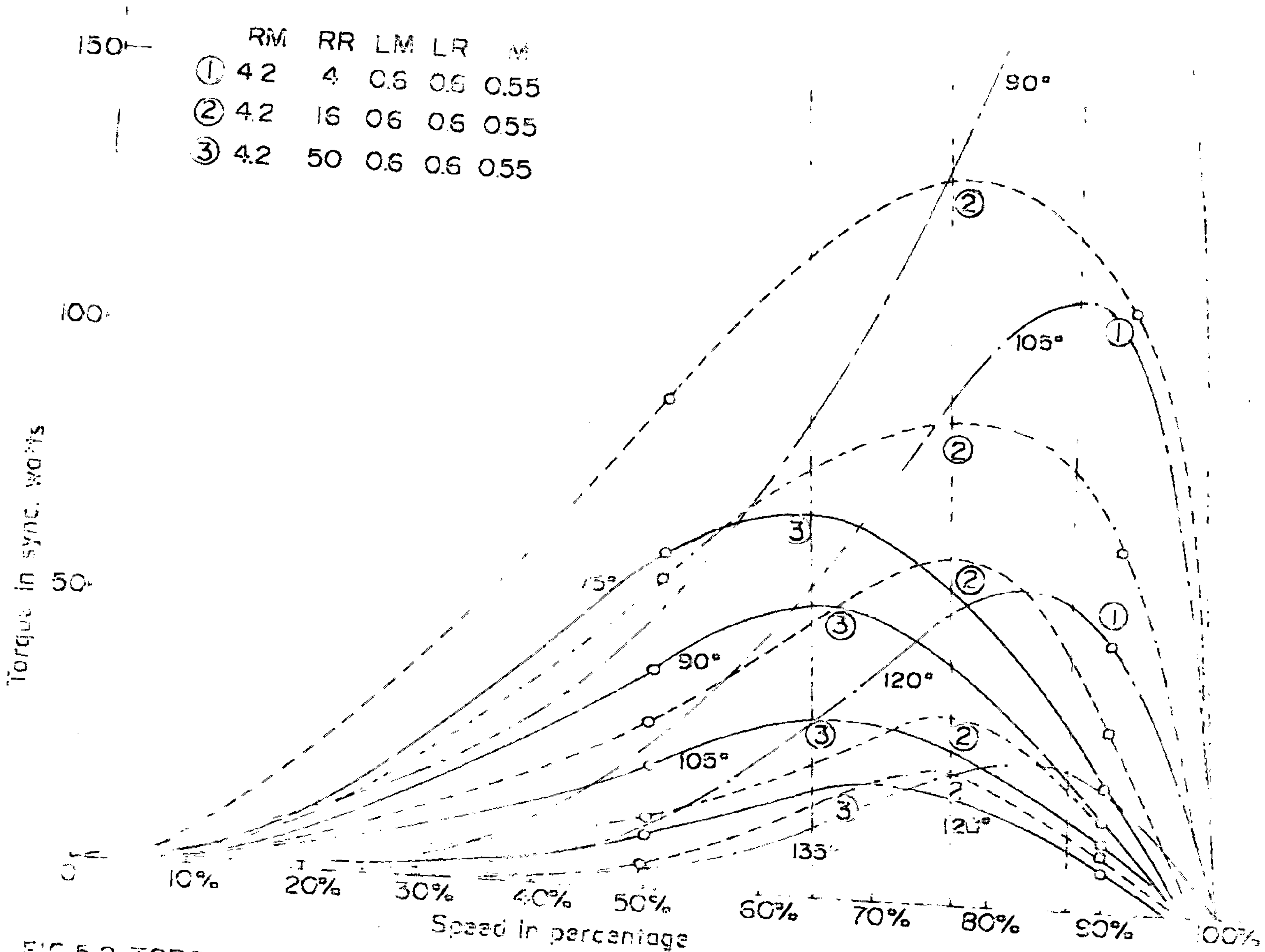


FIG 5.2 TORQUE SPEED CHARACTERISTIC FOR DIFFERENT ROTOR RESISTANC

5.3. DERATING OF MACHINE

It is evident from FIG.5.1 that for same torque production at increased firing angle the ohmic losses increase. Moreover iron loss are also expected to increase for higher firing angles because of higher harmonic contents. The cooling conditions are also deteriorated as reduced speed results in poorer dissipation The net result is that to operate motor safely without violating temperature conditions maximum power delivered by the machine has to be reduced. A definite guideline could not be given because of the difficulties mentioned in section 2.3.

5.4. CONTROL RANGE

Larger control range is obtained if the motor has a rotor with higher resistance. But break down torque also decreases as rotor resistance is increased therefore range obtained is not as large as it is in case of 3-phase induction motor.Still large rotor resistance is desirable from control point of view. Range of Firing Angle Control : Control circuit gains control at lesser firing angle in case of machine having high rotor resistance FIG. 5.2 which results in smoother speed control.

5.5. EFFICIENCY

Efficiency of motor decreases as firing angle is increased FIG.4.7. Efficiency is higher in case of lower rotor resistance. Therefore it is disadvantageous to use machine having high rotor resistance from efficiency considerations.

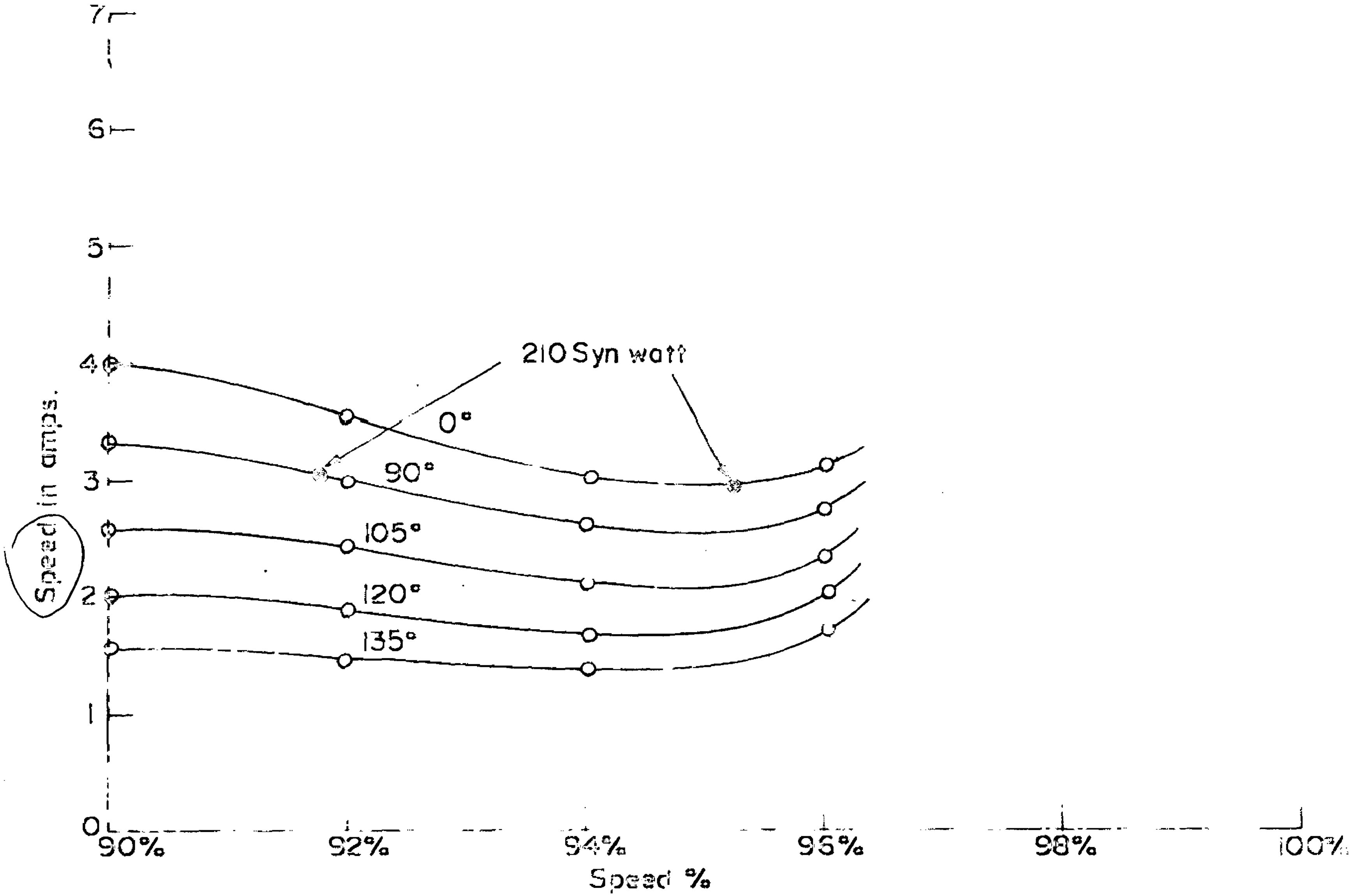


FIG. 5.3 CHARACTERISTIC FOR CURRENT Vs SPEED

5.6. R.M.S. CURRENT

Current vs. speed curve for different firing angle also show a minima at certain speed FIG. 5.3, in fact at that speed machine has highest power factor. For the production of same torque more current flows at higher firing angle and at reduced speed.

5.7. INSTANTANEOUS TORQUE

The variation of instantaneous torque with time has almost the same nature as in an induction motor with continuous conduction FIG. 4.8 & FIG. 4.9. The torque is clearly seen to consist of an average unidirection component superposed by an alternating component. The frequency of alternating component depends upon the conduction period. It increases with decrease in conduction period. Reduction in average torque is obtained by reduction in peak torque and not by developing a region of negative instantaneous torque. Therefore phase angle control will not necessarily lead to excessive vibration and associated motor noise.

5.8. EFFECT OF CHANGE OF ROTOR RESISTANCE

As rotor resistance is increased break down torque appears at reduced speeds and its magnitude is also decreased. This behaviour is contrary to 3-phase induction motor where break down torque remains constant with variation in rotor resistance and only its point of occurance changes. Increase in rotor resistance obviously reduces the efficiency, but a larger range of firing angle and larger range of speed control are obtained

FIG.5.2. The net result is that control remains smooth.

5.9. IMPROVEMENTS IN THE DESIGN

To obtain large speed variation rotor resistance may be kept high, since single phase induction motors are used for very small power application. The reduction in efficiency can be tolerated over the gain of speed control range. Large air gap or reactor in main winding may be used to reduce radio frequency interference.

Application : Phase angle control is successfully employed in fan characteristic loads. Since at lower speed the torque requirement is low therefore current and ohmic losses always remain with in tolerable limits.

For drives needing precise constant speed closed loop system is feasible using phase angle control with 1-phase induction motor. This would be considerably superior to constant speed drives using other types of motor.

CHAPTER - 6

SCOPE FOR FURTHER WORK

The work presented in the dissertation may be extended to several other situations. The areas of further work are suggested as follows:-

(1) In order to obtain greater accuracy in analysis representing phenomenon such as saturation and skin effect in rotor bars by allowing motor parameters to vary with speed results in good correlation for normal operation.

(2) Dynamic Studies: The present work can be used as sub-routine in the major program of dynamic study. At one speed torque is calculated acceleration is found out. Speed can be extrapolated assuming developed torque to be constant. Then at new speed torque developed and acceleration are recalculated which are kept constant during next increment. This dynamic study of the motor can be determined by step by step solution.

(3) The same approach can be extended to capacitor run induction motor by considering control element in the main winding and auxiliary winding having a capacitor in series.

(4) By combining the curves for capacitors run motor and single winding motor one can obtain T-S characteristic. For capacitor start normal run induction motor. So long as centrifugal switch is closed T-S curves for capacitor run motor are valid. After the speed at which centrifugal switch is released curves of single motor winding are valid. Similarly characteristics for capacitor

start and capacitor run (two capacitors are used) induction motor can be obtained by different capacitor run characteristics. Or decision to operate on different programs can taken by computer at the critical speed when change in the motor circuit is done through a switch.

(5) Thyristor rating requirements can also be found by observing the current and voltage peaks and the rate of rise $\frac{dv}{dt}$, $\frac{di}{dt}$.

CHAPTER - 7

CONCLUSION

The method of analysis presented is useful for determining the steady state performance of 1-phase induction motor subjected to thyristor control. The method of solution of equations is simpler than those presented by earlier authors. The method can be easily extended for studies under transient conditions and for capacitor start and capacitor run motors. It is believed that this method of solution requires lesser computer time and sufficiently good accuracy ± 10%.

Performance characteristics for various values of firing angle have been theoretically computed and compared with experimental results. The two show close agreement. The important conclusion that can be drawn from performance characteristics are as below :-

(1) Large range of speed variation is obtained in case of 1-phase induction motor having large rotor resistance, but the range is somewhat reduced because of reduction in break down torque. Comparatively larger range is obtained in case of phase controlled 3-phase induction motor.

(2) Higher efficiency is obtained compared to resistor controlled induction motor.

(3) Torque of the induction motor varies smoothly with firing angle. Therefore no serious threat of machine noise is expected.

Simple closed loop scheme for precise speed control of the drive is feasible therefore it has its wide application in low power constant speed drives.

REFERENCES

1. Taylor, E.O., 'Performance and Design of A.C. Commutator Motor' (Book) A.H. WHEELER & CO. INDIA.

2. Chilkin, M., 'Electric Drives' (Book) MIR Publishers Moscow, 76.

3. Paice, P.A., 'Induction Motor Speed Control by Stator Voltage Control' IEEE Trans. on PAS Vol.87 pp.585-590 Feb. 1968.

4. Ramamoorthy, M., Ilango, B., 'Steady State Analysis of Thyristor Controlled Three Phase Induction Motor' IEEE Trans. PAS 1974 p.1200.

5. Ramamoorthy, M., 'Steady State Analysis of Phase Controlled Induction Motors with Isolated Neutrals' IEEE Trans. IECI 23 No.2 p.179 May 1976.

6. Krause, P.C., 'A Constant Frequency Induction Motor Speed Control' Proc. National Electronics Conference Vol.20 pp. 361-365 1964.

7. Krause, P.C., 'Simulation of Unsymmetrical Two-Phase Induction Machine', IEEE Trans. PAS Vol.84 pp.1025-1037, November 1965.

8. Murphy, J.M.D., 'Thyristor Control of A.C. Motors' (Book) Pergamon Press.

9. Novotny, D.W. and Fath A.F., 'The Analysis of Induction Machine Controlled by Series Connected Semiconductor Switch', 1968 IEEE Trans. PAS 87 p.597,605.

10. Ramamoorthy, M. and D. Ilango, 'Application of State Space Technique to Steady State Analysis of Thyristor Controlled Single Phase Induction Motor' INT.J. CONTROL, 1973 Vol.18 No.2 pp.353-363.

11. Klingshirn, E.A. and Jordan H.E., 'Polyphase Induction Motor Performance and Losses on Non-sinusoidal Voltage Sources' IEEE Trans. P.A.S. Vol.87 pp.624-631, March 1968.

12. Mazda, F.F., 'Thyristor Control' (Book) Newnes - Butterworths.

13. Shepherd, W., 'Thyristor Control of A.C. Circuits' (Book) Bradford University Press. p. 77.

14. Gauper, A.H., Harden J.D., 'Power Supply Aspects of Semi-M.C. Guerzio Conductor Equipment' IEEE Spectrum Vol.8 pp.32-43, Oct.1971.

15. Kohli, D.R., 'Circuit Approach to Modelling & Analysis of Driven Motor', A Short Term Course on 'Analysis and Performance of Thyristor Controlled Drives', EED, University of Roorkee, June 1972.

16. Kohli, D.R., 'Transients in Rotor-fed Shunt Commutator Motor Schrage Type' Ph.D. Thesis, June 1969, E.E.D., University of Roorkee, INDIA.

17. Kohli, D.R. and Jain S.K., 'A Course in Electrical Machinery Experiments' (Book) 1974, Asia Publishing House, INDIA.

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APPENDIX - I

```
C.C.    ANALYSIS OF SINGLE PHASE INDUCTION MOTOR DISCONTINUOUS
        CONDUCTION
        READ 40, R1, RR, AL1, ALR, AM, VM, H
   40   FORMAT (7 F 10.8)                          FORTRAN IV
        D = A M * A M - AL1 * ALR
        A1 = ALR * R1 / D
        A2 = - A M * RR / D
        A4 = - A L R * VM / D
        D1 = - A M * R R / D
        D2 = AL1 * RR / D
        D4 = A M * V M / D
        C 3 = - RR / A L R
        D1 = C 3
        D 2 = D1
        DO 100N = 1, 10
        ZN = N-1
        S P D = 5 * Z N
        PRINT 501 , S P D
  501   FORMAT (4 H SPD = F 10.5)
        S = SPD * 2 * 3.14159
        A3 = A M * A L R * S / D
        B3 = - A L M * A L R * S/D
        C1 = - A M * S / ALR
        C 2 = - S
        D2 = S
```

Contd....

Contd.....

```
          E1 = D 2
          DO 200L =  2 , 10
          ZL = L-1
          ALP = 15 . * ZL
          PRINT 502, ALP
    502   FORMAT (4 H A L P = F 10.5)
          B L P = ALP * 3.14159/180
          I = 1
          C M = 0
          C D = 0
          C Q = 0
          M = 0
     10   M = M+1
          IF (M-20) 9,301,200
      9   BCD = CD
          BCQ = CQ
          ALP = BLP
          SI   = SIN(ALP)
          AH = 0
     11   AH = H + AH
          IF (AH - 0.01) 16, 200, 200
     16   U 1 = (A 1 * C M + A 2 * C D + A3 * CQ+A4*SI) *H
          V 1 = (B 1 * C M + B 2 * C D + B3 * CQ+B4*SI) *H
          W 1 = (C 1 * C M + C 2 * C D + C3 * C Q) * H
          CM1 = C M + U1
          CD1 = C D + V1
```

Contd........

Contd.......

```
        C Q 1  =  C Q + W 1
        A L P = A L P + H * 314.159
        S I = S I N (ALP)
        U 2 = (A1 * C M1+A2* CD1 + A3 * CQ1 + A4 * SI)*H
        V 2 = (B1 * CM1 + B 2 * CD1 + B3*CQ1 + B4 * SI)*H
        W 2 = (C1 * CM1 + C2 * CD1 + C3 * CQ1)*H
        U = (U1 + U2) * .5
        V = (V1 + V2) * .5
        W = (W1 + W2) * .5
        CM2 = CM + U
        IF (CM2) 18, 12, 17
17      CM = CM2
        CD = CD + V
        CQ = CQ + W
        PRINT 50, CM, CD, CQ
50      FORMAT (3 E 16.8)
        G O TO (11,14),I
18      X = - CM/U
        CD = CD + V*X
        CQ = CQ + W*X
        X1 = (D1*CD+E2*CQ)*H*(1.-X)
        Y1 = (E1*CD+E2*CQ)*H*(1.-X)
        CD = CD+X1
        CQ = CQ + Y1
        GO TO (12,14), I
```

Contd........

```
12      X1 = (D1*CD+D2*CQ)*H
        Y1 = (E1*CD+E2*CQ)*H
        CD1 = CD+X1
        CQ1 = CQ + Y1
        X2 = (D1*CD1+D2*CQ1)*H
        Y2 = (E1*CD1+E2*CQ1)*H
        CD=  CD+.5*(X1+X2)
        CQ = CQ +.5*(Y1+Y2)
        AH = H+AH
        PUNCH 60,CD,CQ
60      FORMAT(2E16.8)
        GO TO (22,16),I
22      IF(AH-.01)12,13,13
13      CD = -CD
        CQ =-CQ
        CS = C.
        IF(ABSF(BCD-CD)-.I)23,22,10
23      IF(ABS(BCQ-CQ)-.1)301,301,10
301     I=2
        CAVR=0.
        CRMS=0.
        TAVR=0.
        GO TO 9
```

Contd......

```
   14     TEN = AM * CM * CQ * S
          SS  = CM*CM*TM + RR*(CD*CD + CQ*CQ)
          PRINT 503, CM, TEN
          CAVR = CAVR + CM * H / 0.01
          TAVR = TAVR + TEN * H / 0.01
          CMIS = CMIS + CM*CM*H / 0.01
          AOSS = AOSS + SS*H / 0.01
          IF (CM) 11, 24, 24
   24     CMIS = SQRT (CMIS)
          PRINT 504 CAVR, TAVR, CMIS
  504     FORMAT (5 H CAVR =, E 16.8, 6H TAVR =, E 16.8,
                          6H CMIS =, E 16.8)
          GO TO 12
   15     EM = (S * CQ + CS * CD) * AM
          PRINT 505 EM
  505     FORMAT (4H EM =, E 16.8)
          SS = RR * (CD*CD + CQ*CQ)
          AOSS = AOSS + SS * H / 0.01
          IF (AM - 0.01) 12, 25, 25
   25     PRINT 506, AOSS
  506     FORMAT (6H AOSS =, 16.8)
  300     CONTINUE
  100     CONTINUE
          STOP
          END
```

APPENDIX II

```
C C         ANALYSIS OF SINGLE PHASE INDUCTION MOTOR
            CONTINUOUS CURRENT                    FORTRAN IV
            READ 40, RM, RR, ALM, ALR, AM, VM, H
   40       FORMAT (7F 10.8)
            D = AM * AM - ALM * ALR
            A1 = ALR * RM / D
            A 2 = - AM * RR / D
            A4 = - ALR * VM / D
            B1 = - AM * RM / D
            B2 = ALM * RR / D
            B 4 = AM * VM / D
            C 3 = - RR / ALR
            DO 100 N = 2, 10
            ZN = N - 1
            SPD = 5. * ZN
            PRINT 501, SPD
  501       FORM AT (4H SPD =, F 10.6)
            S = SPD * 2 . * 3.14159
            A3 = ALM * ALR *S / D
            B3 = - ALM * ALR * S/D
            C1 = - AM * S / ALR
            C2 = - S
            BLP = 0
            I = 1
            CM = 0
```

Contd.....

Contd.....

```
         CD = 0
         CQ = 0
         M  = 0
10       M  =  M + 1
         IF (M - 11) 9, 301, 100
 9       BCD = CD
         BCQ = CQ
         ALP = BLP
         SI  = SIN (ALP)
11       IF (AH - 0.01) 16,13,13
16       U1  = (A1 * CM + A2 * CD + A3 * CQ + A4 * SI)* H
         V1  = (B1 * CM + B2* CD + B3 * CQ + B4 * SI)* H
         W1  = (C1 * CM + C2* CD + C3 * CQ)* H
         CM1 = CM + U1
         CD1 = CD + V1
         CQ1 = CQ + W1
         ALP = ALP + H * 314.159
         SI  =  SIN (ALP)
         U2  = (A1 * CM1 + A2 * CD1 + A3 * CQ1 + A4 * SI)* H
         V2  = (B1 * CM1 + B2 * CD1 + B3 * CQ1 + B4 * SI)* H
         W2  = (C1 * CM1 + C2 * CD1 + C3 * CQ1)* H
         U  = (U1 + U2) * .5
         V  = (V1 + V2) * .5
         W  = (W1 + W2) * .5
         CM = CM + U
```

Contd......

Contd.......

```
      CD = CD + V
      CQ = CQ + U
      AH = H + AH
      GO TO (11, 14), I
13    CI = - CI ; CD = -CD; CQ = -CQ
      IF (ABS (BCD - CD) - 0.01) 23, 23, 10
23    IF (ABS (BCQ - CQ) - 0.01) 301, 301, 10
301   I = 2
      TAVR = 0
      CRMS = 0
      LOSS = 0
      AH = 0
      GO TO 9
14    TIN = AMI * CI * CQ * S
      SS = CI * CI * RS + RR * (CD*CD + CQ*CQ)
      PRINT 503 , CI, TIN
503   FORMAT (2E 16.8)
      TAVR = TAVR + TIN * H / 0.01
      CRMS = CRMS + CI *CI * H / 0.01
      LOSS = LOSS + SS*H / 0.01
      IF (AH - 0.01) 13, 504, 504
504   PRINT 505, TAVR
505   FORMAT (6H TAVR =, E 16.8)
      CRMS = SQRT (CRMS)
      PRINT 104, CRMS, LOSS
104   FORMAT (6 H  CRMS =, E 16.8, 6 H LOSS =, E 16.8)
100   CONTINUE
      STOP
      END
```