RELIABILITY EVALUATION OF COMPLEX NETWORKS

A DISSERTATION
submitted in partial fulfilment
of
the requirements for the award of the Degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(System Engineering & Operations Research)

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CERTIFICATE

This is to certify that the dissertation entitled "Reliability Evaluation of Complex Networks" which is being submitted by Shri SUSHIL KUMAR AGARWAL, in partial fulfilment of the requirement for the award of Degree of Master of Engineering in Electrical Engineering (System Engineering and Operations Research) of University of Roorkee, Roorkee is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in it has not yet been submitted so far, for the award of any Degree or Diploma.

This is to certify that he has worked for a period of about 2 years from January 1976 to December 1977 for the preparation of this dissertation for Master of Engineering of this University.

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A C K N O W L E D G E M E N T

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C O N T E N T S

A B S T R A C T

In the present dissertation, reliability and the various terms associated with it are discussed, then various existing methods of reliability evaluation are discussed and compared.

Also the computer programmes for four of the existing methods are modified and run on T.D.C. 312 and IBM 1620 computers. The results of each computer run alongwith the programme listings are appended in the appendices.

CHAPTER - 1

INTRODUCTION

In the past reliability has been recognised only as a qualitative aspect. Engineering systems earlier were so simple that an acute need for quantitative study of reliability was not felt. Reliability as a pressing need was recognised only after Second World War, when several studies revealed some startling results, which served as an impetus for further investigations. Now, reliability has become a recognized engineering discipline, with its own methods, procedures, and techniques. A high degree of reliability is an absolute necessity when dealing with modern complex system such as space mission and Air Craft Systems. In such systems, the failure of a part or component results not only in the loss of the failed item but most often results in the loss of some larger assembly or system, of which it is a part. The reliability of such costly and sophisticated systems has to be ensured before these are actually commissioned.

The most accepted definition for Reliability is : "It is the probability that a device will operate satisfactorily for a given period of time in its intended application". The word "device" in this definition may mean component, block, subsystem, equipment, or complete system in a particular application. The definition includes the term "probability", which indicates the use of a quantitative measure for reliability. In addition to the probabilistic aspec the definition involves three other considerations : "Satisfactory operation, length of time, and intended application".

There has to be a definition of what constitutes a satisfactory operation. Certainly, an equipment does not necessarily have to be totally inoperative for it to be unsatisfactory. "What constitutes satisfactory performance ? " has to be defined for a meaningful measure of the reliability.

The length of time of operation is more definitive. A mission is defined as covering some specific length of time.

The last consideration, intended application, must also be a part of the reliability definition. Any equipment is designed to operate in a given manner under particular sets of conditions. These include environmental conditions (temperature, pressure, humidity, acceleration, vibration, shock, acoustic noise etc.) and operation conditions (voltage, current torque, and corrosive atmosphere etc.) which will be encountere in manufacturing, transportation, storage and use.

A well designed, well engineered thoroughly tested, and properly maintained equipment should never fail in operatio However, experience shows that even the best design, manufacturing, and maintenance efforts do not completely eliminate the occurrence of failures. Reliability distinguishes three characteristic types of failures which may be inherent in the equipment and occur without any fault on the part of the operat

First, there are 'early failures', which occur early in the life of an equipment and in most cases result from poor manufacturing and quality control techniques. Such failures can be eliminated by "debugging" process and are not considered in this study.

Secondly, there are "wear out failures", which are caused by the wearout of parts. These failures are a symptom of component aging. To avoid those failures, the parts of an equipment are designed for a longer life than the intended life of the equipment.

Thirdly, there are so-called "Chance failures" which are caused by sudden stress accumulations beyond the design strength of the component. These failures occur at random intervals, irregularly and unexpectedly. These are most common types of failures during the useful operational life of an equipment. Therefore, maximum attention in the reliability literature has been given to these "Chance" or "Catastrophic" failures.

Use of this probabilistic definition of reliability, helps in determining, at most, the reliability of very simple components. But reliability engineering has its domain much beyond this. A prior knowledge of the reliability of any complex equipment or system is necessary even before it is put to actual operation. Probability, though itself a statistical concept, has an exact mathematics governing it. Hence, to evaluate the reliability of a system, whatsoever complex, al we need is to know the reliabilities of basic components buildi the system and the operational relationship of these components in building up the system. This operational relationship is depicted through the use of reliability block diagrams, also called as logic diagrams.

System reliability evaluation depends on the type of reliability block diagram. While in the case of series parallel networks, it is a simple matter to evaluate the system reliability; such an evaluation may be quite involved in the case of non-series parallel networks. Most of the existing methods for the reliability evaluation of those systems, which result in non-series parallel reliability block diagrams are discussed in next chapter. The methods are illustrated with examples and a critical comparison of these methods is also giv in that chapter. It is observed from a survey of these methods that there does not exist a simple reliability evaluation technique which can be easily computorized and results into a simple reliability expression.

CHAPTER - 2

DEFINITIONS RELATED WITH RELIABILITY

Some of the definitions which are frequently used in Reliability Studies are discussed below :

2.1 RELIABILITY BLOCK DIAGRAM :

A block diagram which depicts the operational relationship of various elements in a physical system, as regards the success of the overall system, is called Reliability Block Diagram. While the system diagram depicts the physical relationship of the system elements, the reliability block diagram shows the functional relationship and indicates which elements must operate successfully for the system to accomplish its intended function. The function which is performed may be the simple action of a switch which opens or closes a circuit or may be a very complex activity such as the guidanc of a space craft.

2.2 SERIES PARALLEL RELIABILITY BLOCK DIAGRAM :

Two blocks in a block diagram are shown in series if the failure of either of them results in system failure. In a series block diagram of many blocks (e.g. Fig. 2.1), it is imperative that all the blocks must operate successfully for system success.

Similarly, two blocks are shown in parallel in the block diagram, if the success of either of these results in

FIG. 2-A

FIG. 3-A

FIG. 4-A NON-SERIES PARALLEL … DIAGRAM

System Success. In a parallel block diagram of many blocks (e.g. Fig. 2.2), successful operation of any one or more blocks ensures system success.

A block diagram, in which both the above connections are used, is termed as series parallel Block Diagram. A closely related structure is an r out of K structures. Such a block diagram represents a system of K components in which any r must be good for system to operate successfully. A simple example of such a type of system is a piece of stranded wire with K strands in which at least r are necessary to pass the required current. Such a block diagram can not be recognized without a description inscribed on it (as in FIG. 2.3). Parallel reliability block diagram can be described as a special case of this type with r equal to unity.

2.3 NON-SERIES PARALLEL BLOCK DIAGRAM :

A block diagram which can not be completely described through series or parallel operational relationships, is called a non-series parallel block diagram. Most of the physical syste result into a non-series parallel block diagram. A typical non-series parallel block diagram is shown in Fig. 2.4.

2.4 BRANCH :

A component in a physical system is shown as a segment in the reliability block diagram, which is called Branch. If a branch in the block diagram can be traversed in one direction only, the corresponding branch is called directed branch and its direction is indicated by an arrow on the block diagram. If, however, the branch can be traversed in either direction, then it is called an undirected branch. No arrow is placed on such a branch. In Fig. 2.4, branches a, b, c, f, g and h are directed branched while branches d and e are undirected branches

2.5 NODE :

Interconnection of two or more branches is called a node. A node, which has all branches incident from it, is called Source or input node and a node which has all branches incident upon it, is called sink or output node. In the diagram of Fig. 2.4, node n_1 is input node while node n_2 is output node.

2.6 PATH :

A path is a sequence of branches of the reliability block diagram from input node to output node such that the succeeding node of any branch is the same as preceding node of the next branch. A minimal path is one which satisfies the above property but no subset of this set of branches satisfies the same. Term "tie set" is also used interchangeably with path. In the diagram of Fig. 2.4, some of the tie sets are abc, adgh, fgh and adgec.

2.7 Cut Set :

A Cut set is a set of branches which when cut will not allow any path from input node to output node. A minimal cut s is one which satisfies this property but no subset of this has this property. In the diagram of Fig. 2.4, some of the cut set are : bdf, ceg, adg and beh.

2.8 FLOW GRAPH :

Flow graph is a weighted graph corresponding to the reliability block diagram, in which weight of each branch is the reliability of the corresponding block in reliability block diagram. In a flow graph, every branch must be directed. Therefore, a particular reliability block diagram may give rise to more than one flow graphs. Four such graphs for the reliability block diagram of Fig. 2.4 are shown in Fig. 2.5. A subgraph is any subset of the branches of the flow graph. A Loop in a flow graph is a sequence of branches (disregarding direction) such that the succeeding node of every branch is the same as the proceding node of the next branch and further the succeeding node of the last branch in the sequence is the same as the preceding node of the first branch. For example, $p_b\ p_d\ p_g\ p_e$ is a loop in the flow graphs of Fig. 2.5

2.9 CONNECTION MATRIX :

A connection matrix (c) is a mathematical representatio of the reliability block diagram. If there are n nodes in a reliability diagram, connection matrix is of the order n x n such that :

$$C_{ij} = \begin{cases} X; \text{ if branch } X \text{ is connected from node } i \text{ to node } j. \\ 0; \text{ Otherwise} \end{cases}$$

The connection matrix (C) corresponding to the block diagram of Fig. 2.4 is given as :

$$(C) = \begin{bmatrix} 0 & 0 & a & f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d & b & 0 \\ 0 & 0 & 0 & 0 & 0 & g \\ 0 & 0 & 0 & 0 & 0 & e \\ 0 & h & 0 & 0 & 0 & 0 \end{bmatrix} \quad \ldots (2.1)$$

Some of the properties of this matrix are :

2.9.1 All diagonal entries of (C) are zero, as there are no self loops in the reliability block diagram.

2.9.2 If there are no parallel branches in the reliability block diagram, each entry of (C) will have at most only one element.

2.9.3 All elements of the column of (C) corresponding to the source node are zero.

2.9.4 All elements of the row of (C) corresponding to the sink node are zero.

MODES OF FAILURES :

Modes of failure of various components in a physical
should be carefully considered before arriving at its
ility block diagram. It is necessary because the effec
ch of these failure modes on the system success may be
different.

A simple system of a series connection of two diodes
g. 2.6), has a relinbility diagram which is mode depend
des have a tendency to fail by open circuit, then the
ility block diagram is shown in Fig. 2.7. But if these
have a tendency to fail by short circuit, then the
ility block diagram is shown in Fig. 2.8. System
ility calculation will be quite different in both the

If a physical component has a tendency to fail in
han one mode, then it is conventional to draw a seperat
diagram considering each mode of failure. System failu
ility is then calculated for each mode seporately by
ing the corresponding block diagram. Assuming that the
ent modes of failures are independent, system unreliabi
algebraic sum of all these failure probabilities. Ove
reliability is then immediately known.

Most predominant modes of failure in may system
ents (for example electronic components) are open circu
ort circuit failures. These two modes being dual of ea

other, reliability block diagram for short circuit mode will be the dual of reliability block diagram for open circuit mode and vice versa. The dual reliability block diagram can be drawn from a given block diagram as follows :

2.10.1 Place a dot in each loop of the block diagram, and number these dots. These correspond to nodes in the dual block diagram.

2.10.2 Place a dot above the block diagram which corresponds to the IN node on the dual diagram; place a dot below the block diagram corresponding to the OUT node of the dual diagram.

2.10.3 Draw dotted lines connecting these dots and intersectin one and only one branch at a time. In the dual diagram, draw the branches between nodes corresponding to these dotted lines.

The procedure is illustrated for the diagram of fig. 2. in Fig. 2.9 and the dual reliability block diagram is shown in Fig. 2.10.

2.11 FAULT TREE :

In extremely Complex Systems, it sometimes becomes impossible to develop a reliability block diagram and, therefor an exact mathematical model. The difficulty arises not from any deficiency in the mathematics used but from the complexity of the system. The electrical subsystem of a manned space craf for instance, involved more than 2,000 separate success paths for just one phase of the mission. In such complex system, the concept of a fault tree is used and then the system is analysed using simulation techniques.

A fault tree is a tool for reliability analysis of complex system and is particularly useful for evaluation of system failure caused by multiple component failures by providing a convenient and efficient format for the problem description. A fault tree is a logical representation of the inter-relationship of various events occurring within a complex system such as a missile or a nuclear reactor. It is constructe using events interconnected by logic 'gates'. Each gate indicat the relationship between a set of input events and an output event. The input events are considered to be causes of the output event. Output events from most gates serve as input events to other ates. An output event, which is not the output of any gate, is a basic input event. Only a few types of logic gates are used and the logic of each is simple and completely defined.

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CHAPTER - 3

EXISTING TECHNIQUES OF RELIABILITY EVALUATION

3.1 FUNDAMENTAL PRINCIPLE OF RELIABILITY EVALUATION :

System reliability calculations are based on,

3.1.1 As precise as possible a measurement of the reliability of the components used in the system environment

3.1.2 The calculation of the reliability of complex combinations of these components.

Some of the existing techniques for making this calculation are discussed in this chapter. The techniques may be deterministic (either exact or approximate) or simulation methods.

3.2 ASSUMPTIONS FOR RELIABILITY EVALUATION :

The following assumptions are made for the evaluation of system reliability.

3.2.1 Each element may be represented as a two terminal device.

3.2.2 All elements are always operating. It implies that no standby redundancy is being used.

3.2.3 The states of all elements are statistically independent implying that the failure of one element does not affect the probability of failure of other elements.

3.2.4 The state of each element and of the entire system is either good (operating) or bad(failed).

3.2.5 Nodes of the system are assumed perfectly reliable.

Even by making use of these assumptions, the evaluation of reliability in a general system is quite involved and depends on the nature of reliability block diagram. In the case of a series block diagram of Fig. 2.1 system reliability expression can be written as,

$$R = \prod_{i=1}^{n} p_{x_i} \qquad \ldots \quad (3.1)$$

where, p_{x_i} is the reliability of component x_i. Similarly, for the parallel block diagram of Fig. 2.2 and an r out of K structure of Fig. 2.3, system reliability expression can be given in (3.2a) and (3.2b) respectively.

$$R = 1 - \prod_{i=1}^{n} (1 - p_{x_i}) \qquad \ldots \quad (3.2a)$$

$$R = \sum_{j=r}^{K} \binom{K}{j} p_{x_i}^{j} (1-p_{x_i})^{k-j} \quad .. \quad (3.2b)$$

No such simple formulas are possible in the case of general non-series parallel block diagram. It is, however, desirable to simplify a given block diagram as far as possible by making use of above relations. Therefore, it is assumed in the following discussion that there are no simple series or parallel connections in the block diagram and there is only one branch between any pair of nodes. Reliability evaluation of such diagrams is discussed in the following sections. For the discussion ~~are~~ purposes we will take the reliability block diagram as shown in Fig. 3.1.

3.3 METHODS OF RELIABILITY EVALUATION :

The existing methods of reliability evaluation are discussed below :

3.3.1 PATH ENUMERATION :

In almost all the reliability evaluation algorithms, a knowledge of either all the paths or the cut sets is necessary Many techniques exist for enumeration of all minimal paths in a general block diagram. These methods vary slightly with each other and are based on the connection matrix of the block diagram and its powers. If (C) is the connection matrix of a block diagram, then element in i, j position of $(C)^n$ gives all paths from i to j of size n

Example 3.3.1.1

For the block diagram of Fig. 3.1

$$(C) = \begin{bmatrix} 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b & 0 & c \\ 0 & d & e & 0 \end{bmatrix} \quad \ldots \quad (3.3a)$$

methods for the reliability evaluation in case of general systems having non-series parallel reliability block diagrams.

3.3.2.1.1 EXHAUSTIVE SEARCH METHOD :

This method is also known as event space method and is one of the most primitive and straight forward technique of reliability evaluation. If there are n number of branches in a block diagram, then there are 2^n possible states of the system. The method consists of listing all these states and sorting out those states in which system is a success. The system reliability expression is then written as sum of the probabilities of these successful states. For the diagram of Fig. 3.1, all the 32 possible states alongwith system output in each case, are shown in Table 3.1.

The reliability expression is then directly written in equation 3.4. This gives 16 success states of the system, thus reliability of the system is

$$
\begin{aligned}
R = \; & q_a q_b p_c p_d q_e + q_a q_b p_c p_d p_e + q_a p_b p_c q_d p_e \\
& + q_a p_b p_c p_d q_e + q_a p_b p_c p_d p_e + p_a q_b q_c p_d p_e \\
& + p_a q_b p_c p_d q_e + p_a q_b p_c p_d p_e + p_a p_b q_c q_d p_e + p_a p_b q_c q_d \\
& + p_a p_b q_c p_d q_e + p_a p_b q_c p_d p_e + p_a p_b p_c q_d q_e + p_a p_b p_c q_d p_e \\
& + p_a p_b p_c p_d q_e + p_a p_b p_c p_d p_e \qquad \ldots \qquad (3.4)
\end{aligned}
$$

$$(C)^2 = \begin{bmatrix} 0 & ab + cd & ce & ae \\ 0 & 0 & 0 & 0 \\ 0 & ed & 0 & 0 \\ 0 & eb & 0 & 0 \end{bmatrix} \quad \ldots \quad (3.3b)$$

$$(C)^3 = \begin{bmatrix} 0 & ceb + aed & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \ldots \quad (3.3c)$$

Different paths in the block diagram, therefore, are ab, cd, ceb and aed.

Other methods of finding paths are based on graph theory, but have no distinct advantages over the above methods.

3.3.2 RELIABILITY EVALUATION :

Methods for evaluating the Reliability can broadly be classified into two categories :

(i) Deterministic methods

(ii) Simulation methods

3.3.2.1 DETERMINISTIC METHODS :

This section presents some of the deterministic methods for the reliability evaluation in case of general systems having non-series parallel reliability block diagrams.

3.3.2.1.1 EXHAUSTIVE SEARCH METHOD :

This method is also known as event space method and is one of the most primitive and straight forward technique of reliability evaluation. If there are n number of branches in a block diagram, then there are 2^n possible states of the system. The method consists of listing all these states and sorting out those states in which system is a success. The system reliability expression is then written as sum of the probabilities of these successful states. For the diagram of Fig. 3.1, all the 32 possible states alongwith system output in each case, are shown in Table 3.1.

The reliability expression is then directly written in equation 3.4. This gives 16 success states of the system, thus reliability of the system is

$$
\begin{aligned}
R = {} & q_a q_b p_c p_d q_e + q_a q_b p_c p_d p_e + q_a p_b p_c q_d p_e \\
& + q_a p_b p_c p_d q_e + q_a p_b p_c p_d p_e + p_a q_b q_c p_d p_e \\
& + p_a q_b p_c p_d q_e + p_a q_b p_c p_d p_e + p_a p_b q_c q_d p_e + p_a p_b q_c q_d \\
& + p_a p_b q_c p_d q_e + p_a p_b q_c p_d p_e + p_a p_b p_c q_d q_e + p_a p_b p_c q_d p_e \\
& + p_a p_b p_c p_d q_e + p_a p_b p_c p_d p_e \qquad \ldots \quad (3.4)
\end{aligned}
$$

3.3.2.1.2 CANONICAL EXPANSION METHOD :

In this method, determination of all the paths in the block diagram is essential. After knowing all the m paths, system success function can be written as :

$$S = T_1 \cup T_2 \cup \ldots \cup T_m \quad \ldots \quad \ldots \quad (3.5)$$

where, T_i indicates the successful operation of all elements in path i. Function S is then expanded into canonical form. In the canonical expansion, each term is mutually exclusive from all other terms. Therefore, by simply changing the Bootean variables A, $\bar{A}$ to the probability variables pA, qA respectively and changing the connective variable from logical union to algebraic summation, reliability expression immediately follows.

For the block diagram of Fig; 3.1 paths have been deter determined as ab, cd, aed and ceb. Therefore,

$$S = AB \cup CD \cup AED \cup CEB \quad \ldots \quad \ldots \quad (3.6)$$

Expanding this into Canonical form,

$$S = AB(C\cup\bar{C})(D\cup\bar{D})(E\cup\bar{E}) \cup CD(A\cup\bar{A})$$
$$(B\cup\bar{B})(E\cup\bar{E}) \cup AED(B\cup\bar{B})(C\cup\bar{C}) \cup$$
$$CEB(A\cup\bar{A})(D\cup\bar{D})$$

$$= \overline{AB}CD\overline{E} \cup \overline{AB}CDE \cup \overline{A}BC\overline{D}E \cup \overline{A}BCD\overline{E}$$
$$\cup \overline{A}BCDE \cup A\overline{BC}DE \cup A\overline{B}CD\overline{E} \cup A\overline{B}CDE \cup$$
$$AB\overline{CDE} \cup AB\overline{CD}E \cup AB\overline{C}D\overline{E} \cup AB\overline{C}DE \cup$$
$$ABC\overline{DE} \cup ABC\overline{D}E \cup ABCD\overline{E} \cup ABCDE \qquad \ldots \quad (3.7)$$

Reliability expression therefore is given as :

$$R = q_a q_b p_c p_d q_e + q_a q_b p_c p_d p_e + q_a p_b p_c q_d p_e + q_a p_b p_c p_d q_e$$
$$+ q_a p_b p_c p_d p_e + p_a q_b q_c p_d p_e + p_a q_b p_c p_d q_e$$
$$+ p_a q_b p_c p_d p_e + p_a p_b q_c q_d q_e + p_a p_b q_c q_d p_e$$
$$p_a p_b q_c p_d q_e + p_a p_b q_c p_d p_e + p_a p_b p_c q_d q_e$$
$$+ p_a p_b p_c q_d p_e + p_a p_b p_c p_d q_e + p_a p_b p_c p_d p_e \quad \ldots (3.8)$$

It may be observed that this expression is exactly same as given in (3.4). A computerised algorithm of this method was given by D.B. Brown (6).

3.3.2.1.3 PROBABILITY CALCULUS METHOD :

This method is based on a simple probability law,

$$Pr\{X \cup Y\} = Pr\{X\} + Pr\{y\} - Pr\{X \cap Y\} \qquad \ldots \quad (3.9)$$

Since Reliability is defined as the probability of success of the system. Therefore,

$$R \triangleq Pr\{S\} = Pr\{T_1 \cup T_2 \cup \ldots T_m\} \qquad \ldots \quad (3.10)$$

Now Reliability expression can be derived by the repeated application of (3.9). For the block diagram of

Fig. 3.1 system success function is given by (3.6) therefore,

$$R = Pr\left\{AB \cup CD \cup AED \cup CEB\right\} \quad \ldots \quad (3.11)$$

Now, $Pr\left\{AB\right\} = p_a p_b$, $Pr\left\{AED\right\} = p_a p_d p_e$

$Pr\left\{CD\right\} = p_c p_d$, $Pr\left\{CEB\right\} = p_b p_c p_e$

By the application of (3.9)

$$Pr\left\{AB \cup CD\right\} = p_a p_b + p_c p_d - p_a p_b p_c p_d$$

$$Pr\left\{AB \cup CD \cup AED\right\} = p_a p_b + p_c p_d - p_a p_b p_c p_d + p_a p_d p_e - p_a p_b p_d p_e - p_a p_c p_d p_e + p_a p_b p_c p_d p_e$$

$$Pr\left\{AB \cup CD \cup AED \cup CEB\right\} = p_a p_b + p_c p_d + p_a p_d p_e + p_b p_c p_e - p_a p_b p_c p_d - p_a p_b p_d p_e - p_a p_c p_d p_e - p_a p_b p_c p_e - p_b p_c p_d p_e + 2 p_a p_b p_c p_d p_e \quad ..(3.12$$

In a slight variation of this method, reliability expression of a system can also be written as :

$$R = 1 - \prod_{i=1}^{m} \left[1 - Pr\left\{T_i\right\}\right] \quad \ldots \quad (3.13)$$

For the evaluation of (3.13, Poincare gave a gene ral relation. In this method, the reliability expression can be written as :

$$R = S_1 - S_2 + \ldots\ldots + (-1)^{m-1} S_m \quad \ldots \quad (3.14)$$

where, S_1 = Sum of probabilities of all paths

S_2 = Sum of probabilities of all inter sections of two paths.

— — — — — — — — —

S_m = Sum of probabilities of all inter sections of m paths.

For the block diagram of Fig. 3.1, paths have been determined as ab, cd, ade and ceb.

Therefore, $S_1 = p_a p_b + p_c p_d + p_a p_d p_e + p_b p_c p_e$

$$S_2 = p_a p_b p_c p_d + p_a p_b p_d p_e + p_a p_b p_c p_e + p_a p_c p_d p_e + p_b p_c p_d p_e + p_a p_b p_c p_d p_e$$

$$S_3 = 4\, p_a p_b p_c p_d p_e$$

$$S_4 = p_a p_b p_c p_d p_e$$

Substituting these values of S_1 through S_4 into (3.14), we have :

$$R = p_a p_b + p_c p_d + p_a p_d p_e + p_b p_c p_e - p_a p_b p_c p_d - p_a p_b p_d p_e - p_a p_b p_c p_e - p_a p_c p_d p_e - p_b p_c p_d p_e + 2\, p_a p_b p_c p_d p_e \quad \ldots \quad \ldots \quad (3.15)$$

The result is the same as expressed by relation (3.12) earlier.

3.3.2.1.4 FLOW GRAPH METHOD :

In this method, firstly all the flow graphs correspondi to a given reliability block diagram are drawn. Reliability expression can then be written as :

$$R = F_0 - F_1 + F_2 \quad \ldots \quad \ldots \quad (3.16)$$

where F_0 = Sum of probabilities of all forward paths.

F_1 = Sum of probabilities of all subgraphs with one loop.

F_2 = Sum of probabilities of all subgraphs with two loops and so on.

For the diagram of Fig. 3.1, there are two flow graphs possible, which are shown in Fig. 3.2. For this problem all forward paths are shown in the Fig. 3.3, and all subgraphs with one loop are shown in the Fig. 3.4. There are only two subgraphs with two loops shown in Fig. 3.2. itself. From these figures,

$$F_o = p_a p_b + p_c p_d + p_a p_d p_e + p_b p_c p_e$$

$$F_1 = p_a p_b p_c p_e + p_a p_c p_d p_e + p_a p_b p_d p_e + p_b p_c p_d p_e + p_a p_b p_c p_d$$

$$F_2 = p_a p_b p_c p_d p_e + p_a p_b p_c p_d p_e$$

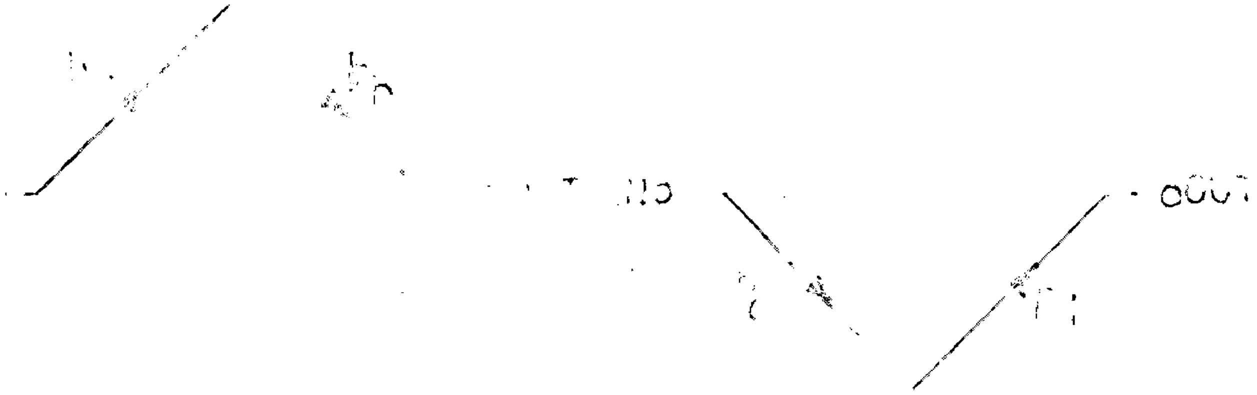
Substituting these values of F_0 through F_2 into (3.16), we have

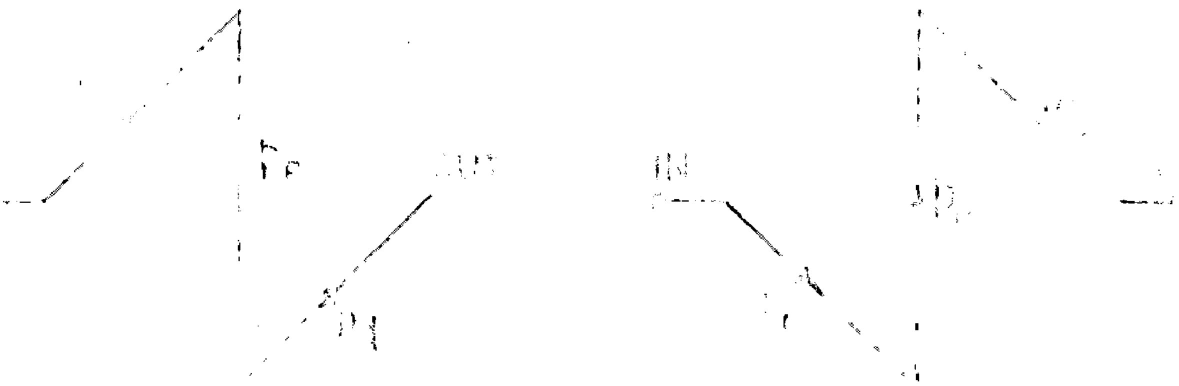
$$R = p_a p_b + p_c p_d + p_a p_d p_e + p_b p_c p_e - p_a p_b p_c p_e - p_a p_b p_d p_e - p_b p_c p_d p_e - p_a p_b p_c p_d - p_a p_c p_d p_e + 2\, p_a p_b p_c p_d p_e \quad \ldots \quad (3.17$$

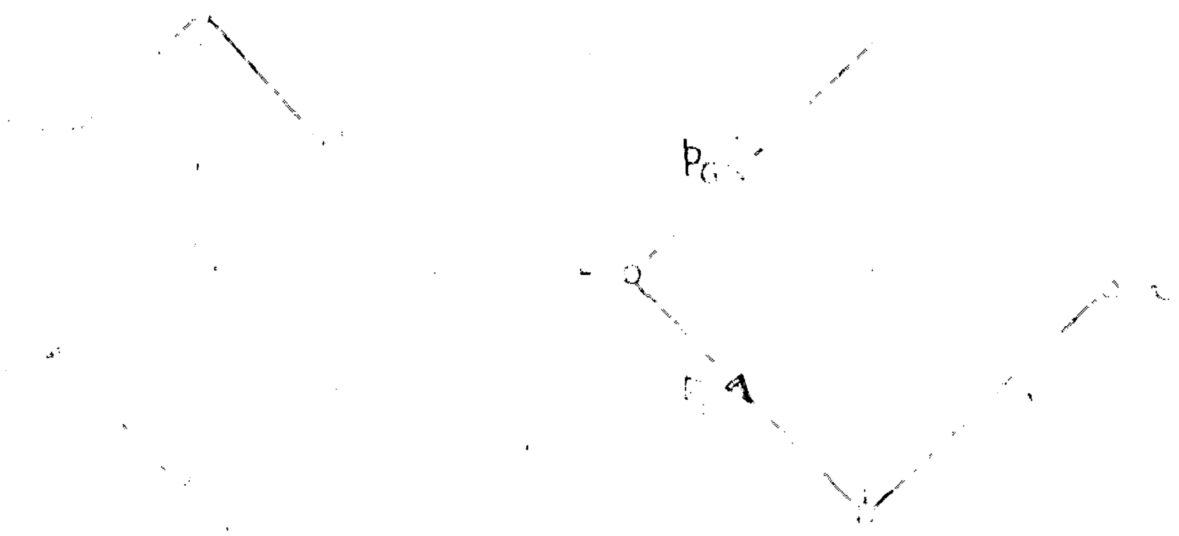
Reliability expression derived in (3.17) is the same as that derived earlier in (3.15).

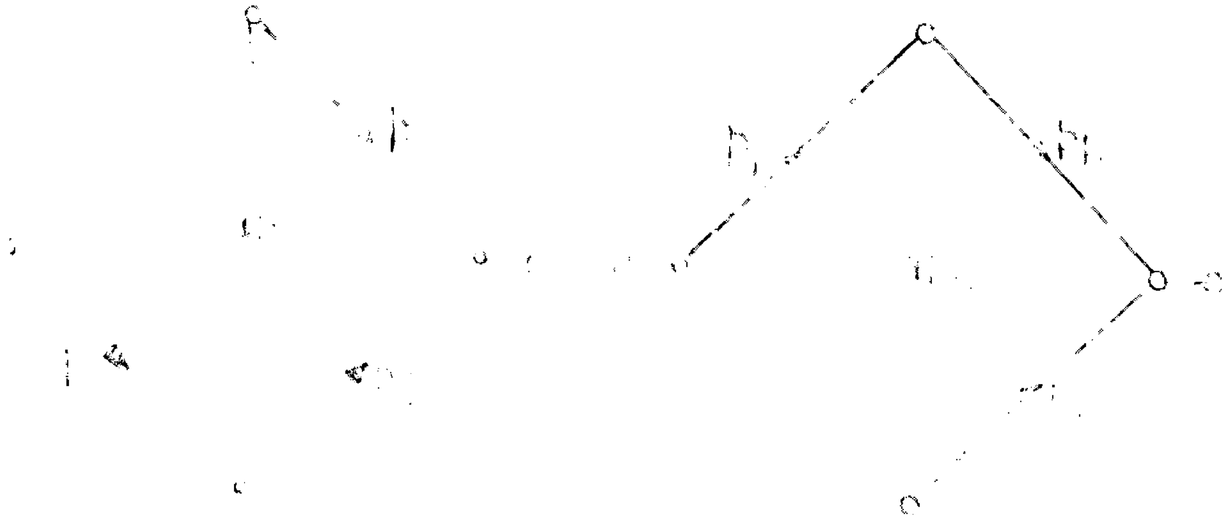
3.3.2.1.5 <u>BAYE'S THEOREM METHOD</u> :

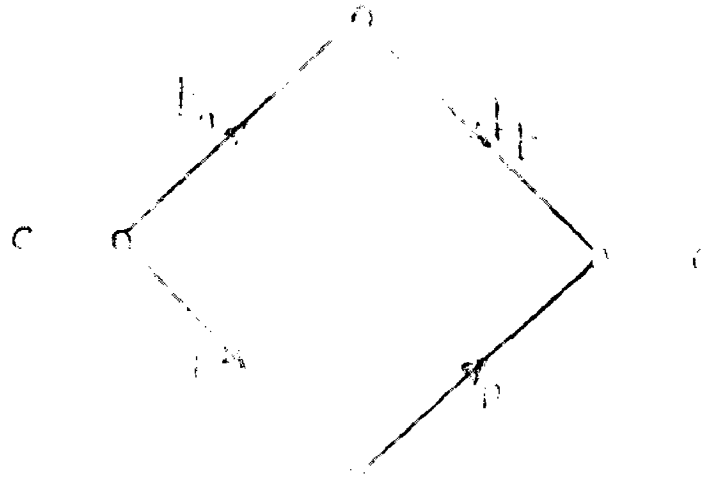
Baye's theorem states : If X is an event which depends upon one of two mutually exclusive events Y_i and Y_j of which











one must necessarily occur, then the probability of occurrence of A is given by

$$Pr\left\{A\right\} = Pr\left\{A/Y_i\right\} Pr\left\{Y_i\right\} + Pr\left\{A/Y_j\right\} Pr\left\{Y_j\right\} \quad \ldots \quad (3.18)$$

When used in the reliability application, Y_i and Y_j are respectively taken as the success and failure of a critical element, Z. Key result then is :

$$R \triangleq Pr\left\{S\right\} = Pr\left\{S/Z\right\} p_z + Pr\left\{S/\bar{Z}\right\}(1-p_z) \quad \ldots \quad (3.19)$$

Where $Pr\left\{S/Z\right\}$ is the probability of success of the system when component Z is good; while $Pr\left\{S/\bar{Z}\right\}$ is the probability of success of the system when component Z is bad.

For the block diagram of Fig. 3.1, key element is e, therefore,

$$R = Pr\left\{S/E\right\} p_e + Pr\left\{S/\bar{E}\right\} (1 - p_e) \quad \ldots \quad (3.20)$$

$Pr\left\{S/E\right\}$ and $Pr\left\{S/\bar{E}\right\}$ can be evaluated easily using the block diagrams shown in Fig. 3.5. These are series parallel diagrams. Therefore,

$$Pr\left\{S/E\right\} = (p_a + p_c - p_a p_c)(p_b + p_d - p_b p_d) \quad \ldots \quad (3.21)$$

$$Pr\left\{S/\bar{E}\right\} = p_a p_b + p_c p_d - p_a p_b p_c p_d \quad \ldots \quad (3.22)$$

Substituting these results (3.21) and (3.22) into (3.20), reliability expression of the system is obtained. This expression also is exactly same as derived earlier by the flow graph method.

3.3.2.1.6 PROBABILITY MAP METHOD :

This method presented for the reliability evaluation of a general system by using a probability map, is similar to Karanaugh map used in Boolean algebra. In this method, system success function S is plotted on probability map and then groups are formed taking care that no term is covered in more than one group. System success function is then written in an alternate equivalent form S(dis) such that all its terms are mutually disjoint. System reliability expression then immediately follows. For the block diagram of Fig. 3.1, System success function is given in (3.6). This is shown plotted in Fig. 3.6 along with the formation of groups. From a knowledge of these groups, S(dis) can be written as :

$$S(dis) = CD \cup AB\bar{C} \cup ABC\bar{D} \cup A\overline{BC}DE \cup \bar{A}BC\bar{D}E \quad \ldots \quad (3.23)$$

As all the terms are mutually exclusive in (3.23), reliability expression is given as :

$$R = Pr\{S(dis)\} = p_c p_d + p_a p_b p_c + p_a p_b p_c p_d + p_a p_b p_c p_d p_e + p_a p_b p_c p_d p_e \quad \ldots \quad (3.24)$$

Equation (3.24) gives yet another expression for the reliability of the system having a reliability block diagram of Fig. 3.1.

3.3.2.1.7 CUT-SET APPROACH :

It is also possible to use a knowledge of cut-sets of the block diagram for evaluating the unreliability (and hence reliability) of a system. If there are j number of cut-sets in all; the unreliability expression of the system can be written as :

$$Q = Pr\left\{\overline{K}_1 \cup \overline{K}_2 \ldots \cup \overline{K}_i \ldots \cup \overline{K}_j\right\} \quad \ldots \quad (3.25)$$

where $\overline{K}_i$ indicates the failure of all branches in cut-set i. System reliability is, then $R = 1 - Q$. As relation (3.25) for the evaluation of system unreliability, is similar to (3.10), for the evaluation of system reliability, all the methods of reliability evaluation can be extended for use with cut-sets also.

This approach requires enumeration of all cut-sets in th block diagram. This can be conveniently done by a number of methods. Nelson (9) has also described a method for the determination of all cuts from the knowledge of all paths, the computer programme for which will be discussed in next chapter.

3.3.2.1.8 METHODS OF BOUNDS :

The method of bounds is a limiting value procedure for determining the reliability of a system. To obtain an approximate value for reliability, we obtain an upper bound R_U and a lower bound R_L for system reliability.

$$R = Pr\left\{T_1 \cup T_2 \ldots \cup T_m\right\} \leqslant \sum_{i=1}^{m} Pr\, T_i$$

Therefore,

$$R_U = \sum_{i=1}^{m} Pr\left\{T_i\right\} \quad \ldots \quad (3.26)$$

Relation (3.26) sets an upper limit for the reliability of a system and is a good approximation in the low reliability region. Similarly, using cut-sets, a lower limit can be defined.

$$R = 1 - Pr\left\{\overline{K}_1 \cup \overline{K}_2 \cup \ldots \overline{K}_j\right\} \geqslant 1 - \sum_{i=1}^{j} Pr\left\{\overline{K}_i\right\}$$

Therefore, $R_L = 1 - \sum_{i=1}^{j} Pr\left\{\overline{K}_i\right\}$ ... (3.27)

Equation (3.27) is a lower limit of the system reliabili and is quite a good approximation in the high reliability region

The method was extended by Nelson (9) to obtain a closer approximation to system reliability. Instead of defining a unique upper and lower bound, they define a set of successively lower upper bounds and successively higher lower bounds to reach as close to system reliability as desired.

3.3.2.2 SIMULATION METHOD :

Simulation, or Monte Carlo methods, are used for system reliability prediction when an exact mathematical model can not be developed economically or when it becomes too complex to permit timely evaluation, of reliability. In such methods an analogous stochastic process is set up which behaves as much like the actual system as possible. The model process is then observed, and the results are tabulated and treated as if they were experimental data representing the actual proble . The key f ature in Monte Carlo methods is the generation of random numbers within the computer.

CHAPTER - 4

COMPARISON OF VARIOUS METHODS

4.1 METHOD COMPARISON :

In the previous sections, some general techniques for the reliability evaluation of systems have been presented. Deterministic as well as simulation methods are discussed. Simulation methods are very useful for the analysis of Complex System.

The deterministic techniques of Section 3.3.2.1 are exact except Method of bounds (Section 3.3.2.1.8). Method of bounds is a limiting value procedure and in certain cases is an excellent time saving substitute for the more lengthy mathematically exact procedures. Cut-set approach (Section 3.3.2.1.7 is another basic approach for the reliability evaluation of a system. It allows us to evaluate system reliability by making use of all the cut sets of the system rather than tie sets (or paths). The labour required for the enumeration of all cut sets is comparable to the labour for enumeration of all paths. Therefore, the only attraction for this approach of reliability evaluation is in those cases where the number of cut sets may be less than the number of tie sets. It has been shown that this is actually the case when the average number of branches incident on the node in a reliability block diagram is more than four. Therefore, this approach of evaluation has an advantage when the number of interconnecting branches is large. In the following paragraph a critical comparison is given for the exact deterministic methods based on tie set approach (Section 3.3.2.1.1 through 3.3.2.1.6).

Exhaustive search method for the reliability evaluation of a system (Section 3.3.2.1.1) has the advantage of being extremely simple. It does not also require the knowledge of all paths in the diagram of the system. The method, however, requires the analysis of all possible states of the system. Reliability expression is quite involved and needs many computations for its numerical evaluation. For a simple 5 element system, with a reliability block diagram in Fig. 3.1, there are 32 possible states out of which 16 are success states. Reliability expression (3.4) requires 64 multiplications and 15 summations for numerical évaluation of reliability. If T_s is the time for one summation on the computer and, T_m is the time for one multiplication, then the total computational time in this case will be :

$$T_c = 64\, T_m + 15\, T_s \qquad \ldots \qquad (4.1)$$

In a typical digital computer, T_m is about ten times more than T_s. (In IBM 1620, T_m = 12,512 μ Sec., and T_s = 1,200 μ Sec.). Using $T_m = 10\, T_s$, relation (4.1) becomes :

$$T_c = 655\, T_s \qquad \ldots \qquad (4.2)$$

The number of possible states and hence the complexity of expression increases very rapidly with an increase in the number of elements.

For a system with 20 elements, total number of states will be above 1 million, thus rendering the method impracticabl for large systems.

Direct canonical expansion method (Section 3.3.2.1.2) does not require the enumeration of all possible states of the system, but starts with a knowledge of all paths of the system. The final derived expression, however, is as complicated as given by exhaustive search method.

Probability Calculus method (Section 3.3.2.1.3) is based on simple probability theorem (3.9) and by the repeated application of this theorem, an expression for the system reliability is derived. For the system having block diagram of Fig. 3.1, system reliability expression using this method is given in (3.12). The expression requires 26 multiplications and 9 summations. Computational time is then given as :

$$T_c = 26\ T_m + 9\ T_s = 269\ T_s \qquad \ldots \qquad (4.3)$$

The method has the disadvantage of the large number of repetitions of application of (3.9) for large systems.

Total computational time in this case has been reduced to about one third the previous requirement and this ratio improves further for large systems.

Flow graph method of section 3.3.2.1.4 requires the enumeration of all directed flow graphs of the reliability block diagram. Therefore, all forward paths and all subgraphs with various numbers of loops have to be enumerated. For the system with block diagram of Fig. 3.1, there are two flow graphs. In all 11 subgraphs (4 forward path, 5 subgraphs with the one loop, 2 subgraphs with two loops) have to be considered. The method is convenient only if the number of interconnecting branches is small. The reliability expression derived by this method, however, is the same as derived by probability calculus method.

Baye's theorem method (Section 3.3.2.1.5) also involves the repeated applications of Baye's theorem. The number of such repetitions increase with the number of interconnecting branches. Also, the final reliability expression is as involved as given by flow graph method.

Probability map method (Section 3.3.2.1.6) results in a reliability expression which is extremely simple. For the diagram of fig. 3.1, system reliability expression is given in (3.24). It requires 14 multiplications and 4 summations total computational time therefore is :

$$T_c = 14 \; T_m + 4 \; T_s = 144 \; T_s \qquad \ldots \; (4.4)$$

Computational time, therefore, has been further cut down by about 50%. This is a significant advantage as a particular reliability expression might be used many a times while designing reliable systems. A simplified expression has an added advantage if reliability of the system is to be expressed not at one time but as a function of time. Probability map method, therefore is very convenient tool for the reliability evaluation of a system. But the method being graphical can not be easily computerized. Moreover the method is convenient only if the number of variables is not more than six. No convenient format of Karnaugh graph exists for more than six variables. In a general system, the number of constituent elements will invariably exceed six. Therefore, this method although very convenient and efficient for small systems, can not be used for large systems.

4.2 ERROR COMPARISON :

4.2.1 DEPENDENCE OF ERROR :

It is desirable to compare the various methods from the point of view of errors also. Since the component reliability data, being statistical, can be true only within certain limits, system reliability value will also be true within certain error. The error in the final result will obviously depend upon,

4.2.1.1 The error in the component values and

4.2.1.2 System reliability expression

4.2.2 ERROR ANALYSIS :

Therefore, the error in final results is different for various methods. The error analysis is based on the following theorems.

4.2.2.1 The absolute error in the summation of certain terms is equal to the summation of the absolute errors in these terms.

4.2.2.2 The relative error in the multiplication of certain terms is equal to the summation of the relative errors in these terms. Absolute error, if desired, can be found by multiplying the relative error with the product.

In the following analysis, it is assumed that all component reliability values have an absolute error of Δ e. All unreliability values (q's) are also, therefore, having an absolute error of Δ e. For simplification, all reliability (unreliability) values are assumed equal to p (or q). It is further assumed, that the value of the error Δ e is quite small.

In the exhaustive search method (3.3.2.1.1) or canonical expansion method (section (3.3.2.2.1), reliability expression in the case of block diagram of Fig. 3.1, is given in (3.4). Using the above theorem, error in system reliability, Δ R is, therefore, given as :

$$\Delta R = \Delta e \Big[(5/p)p^5 + 5(4/p + 1/q)p^4 q + 8(3/p + 2/q)p^3 q^2 + 2(2/p + 3/q)p^2 q^3 \Big]$$

$$\text{or } \Delta R = \Delta e \, (10\, p^4 + 36\, p^3 q + 30\, p^2 q^2 + 4p\, q^3) \quad \ldots (4.5)$$

In the high reliability region ($p = 1$), this error is given by :

$$\Delta R \approx 10\, \Delta e \quad \ldots (4.6)$$

In probability Calculus method (Section 3.3.2.1.3), flow graph method (Section 3.3.2.1.4), and Baye's theorem method (Section 3.3.2.1.5), the reliability expression for this system is given in (3.12). Error in system reliability, by the application of these methods, is given by :

$$\Delta R = \Delta e \Big[2(2/p)p^2 + 2(3/p)p^3 + 5(4/p)p^4 + 2(5/p)p^5 \Big]$$

$$\text{or, } \Delta R = \Delta e(10\, p^4 + 20\, p^3 + 6\, p^2 + 4p) \quad \ldots \quad \ldots (4.7)$$

In the high reliability region, this approximates to :

$$\Delta R \approx 40\, \Delta e \quad \ldots (4.8)$$

In the probability map method (Section 3.3.2.1.6), system reliability expression is given in (3.24). Error in reliability by this method is :

$$\Delta R = \Delta e \Big[(2/p)p^2 + (2/p + 1/q)p^2 q + (3/p + 1/q)p^3 q + 2(3/p + 2/q)p^3 q^2 \Big]$$

or, $\Delta R = (p^3 + p^2 + 2p + 2pq + 3p^2q + 6\ p^2q^2 + 4\ p^3q)\Delta e$

... (4.9)

In the high reliability region, this error approximates to,

$\Delta R \approx 4\ \Delta e$ ... (4.10)

Various comparison criteria are summarised in the form of table 4.1.

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CHAPTER - 5

IMPROVED ANALYTIC TECHNIQUES OF RELIABILITY EVALUATION

5.1 A FAST METHOD FOR PATH AND CUT ENUMERATION :

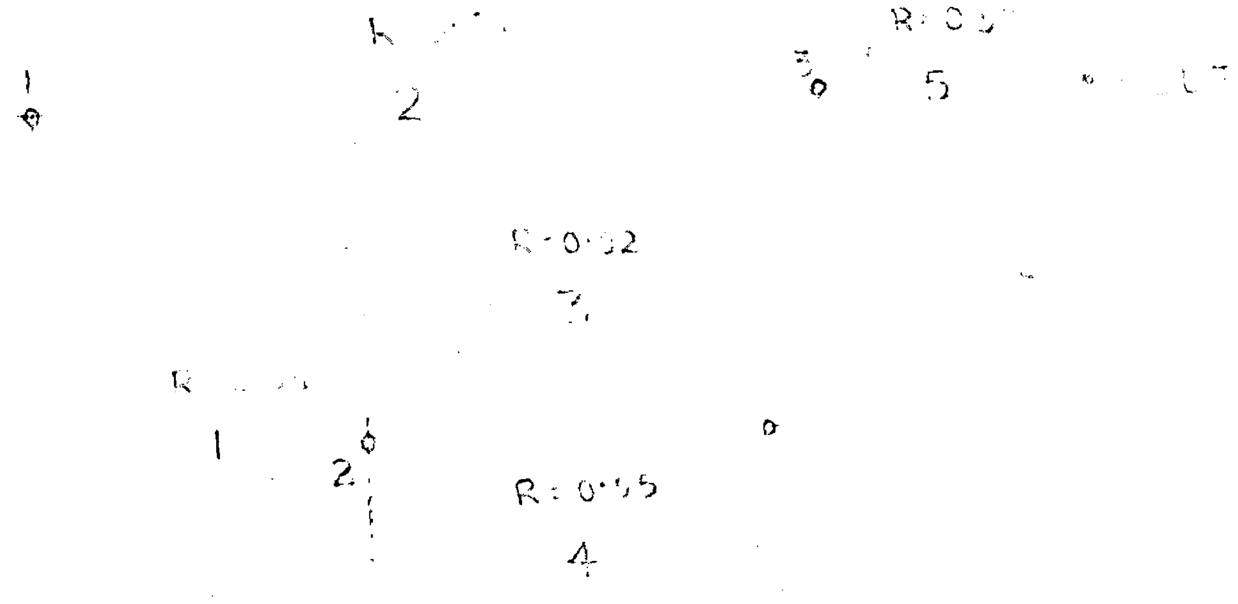
As observed in the last chapter, the first step in almost all the reliability evaluation algorithms is the determination of all paths in the reliability block diagram. A method for finding out all minimal paths and then based on this all minimal cuts is discussed below. Then also computer programme is developed for it which will be discussed in the next chapter. Now this programme can further be divided in two parts depending upon the generation of paths and cuts.

5.1.1 GENERATION OF PATHS :

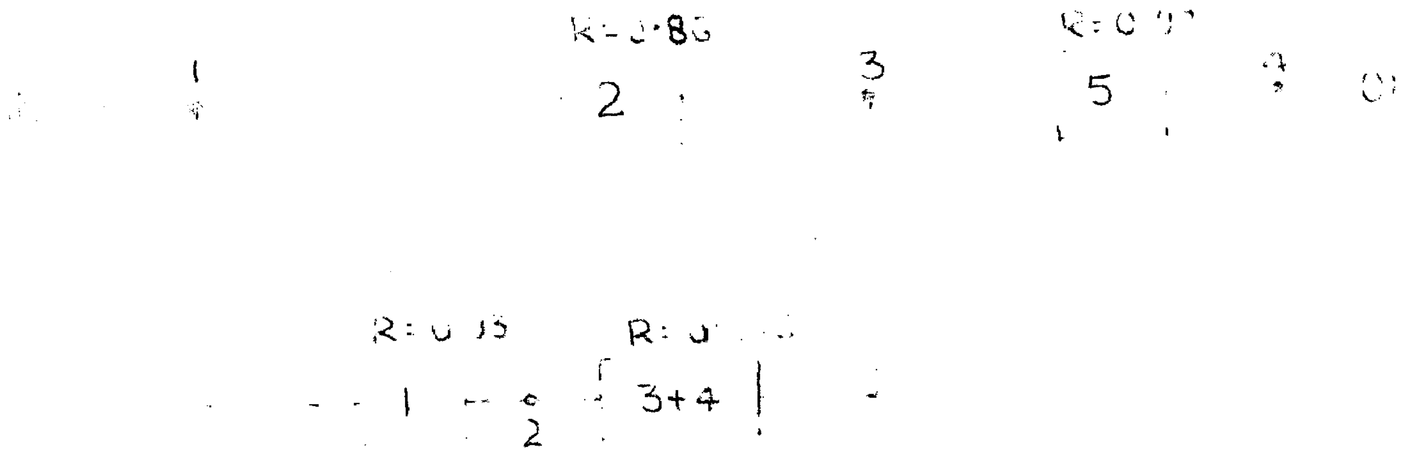
Here a listing of the elements in the system, their predecessors, and the probability of successful operation of each element are the inputs. Then backwards check ups are done from output node towards input node in order to find out minimal success paths. The procedure will be more clear from the following example.

5.1.1.1 Example :

Consider the Block Diagram of Fig. 5.1. Here there is only one branch touching node 4, which is one end of element 5. So element 5 will be common in all paths. Then there are three branches touching node 3 (which is the other end of node 5).



5.1 - CI[illegible] FOR COMPUTE[illegible] R[illegible]



IG. 5.2 - RE[illegible]ED BLOCK DIAGRAM OF FIG 5.1

Thus forming three paths containing elements 2, 3, 4 respectively; Also each containing 5. Now we consider node at other ends of new elements 2, 3, 4, which are node 1, 2, 2 respectively. Again we do the same analysis as discussed previously. In this way we proceed until input node is reached Thus getting all the minimal success paths.

5.1.2 GENERATION OF CUT SETS :

Here a simple procedure using Boolean logic is used for obtaining a matrix identifying the minimal cuts of the system from the matrix containing the paths. The procedure will be mo: clear from the following example.

5.1.2.1 Example :

Again considering the Block Diagram of Fig. 5.1. The matrix containing the minimal paths is :

$$P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \ldots (5$$

Where the tie sets are $P_1 = (1, 3, 5)$ (indicated by 1's in columns 1, 3 and 5 of the first row), $P_2 = (2, 5)$, $P_3 = (1, 4, 5)$. Now consider the column vectors (1, 0, 1), (0, 1, 0) etc., of the path matrix P. For a single element to be a cut, it must be in each path, i.e. its column vector in P must be the unit vector (1, 1, 1). Note that element 5 is th only single element cut. In general, if P_c denotes the C^{th} column vector of an I-path matrix, and if the elements

$$P_{Ci} = 1, \text{ for all } i = 1, 2 \ldots I$$

then the corresponding element C is a single element cut. If $P_{Ci} = 0$ for some i in each column vector, then there are no single element cut, and one must proceed to look for two element cuts.

For two element cuts consider for $C \neq d$

$$P_{Ci} + P_{di}$$

Where the + indicates the logic sum or union. If,

$$P_{Ci} + P_{di} = 1, \text{ for all } i = 1, 2, \ldots I$$

then elements C and d form a two element cut.
For example, $P_{1i} + P_{2i} = 1$, for all $i = 1, 2 \ldots I$
and hence elements (1, 2) form a cut.

This procedure continues until all possible cuts of order 1, 2 ..... n (n being the number of elements in the system) have been exhausted or until only unit vectors are obtained in the vector unions as described. At each stage all the non-minimal cuts are eliminated by using the following approach. After a possible cut of order M has been identified, it is checked against all cuts of order (M - 1), (M-2) ... 1, by using Boolean logic for intersection, i.e. the "AND" operation, for the multi lication of two vectors. If the cut of order N contains a cut of smaller order, the vector product would be equal to the order of the smaller cut. In this case the cut of order N would be eliminated because it is non-minimal.

In this way the above steps describe how the programme identifies minimal cuts.

5.2 AN ALGORITHM FOR THE RELIABILITY EVALUATION OF REDUNDANT NETWORKS :

An algorithm is presented for evaluation of reliability of any redundant network. It uses the properties of diagraphs and is especially suitable for the computer analysis of large complex networks. A method for deriving the reliability expression for any type of network is also described. Also a computer programme in Forgo is developed for the reliability evaluation of Series-parallel networks.

5.2.1 METHOD :

Before proceeding to evaluate the overall reliability of redundant networks it is usually advantageous to combine all parallel elements between nodes i and j using Boolean algebra rules and replace them by an equivalent link having reliability C_{ij} connecting i and j. If there are n parallel elements,

$$C_{ij} = Pr \left\{ \bigcup_{k=1}^{n} E_k \right\} = 1 - \prod_{k=1}^{n} (1 - p_{ij}) \quad \ldots \quad (5.2)$$

Where E_k is the event that the k^{th} element is good.

A weighted connection matrix (C) is obtained with the property that for any non-zero entry in (C) there exists one and only one branch between any two nodes. Initially all elements of (C) are initialized to zero, and therefore only non-zero entries are transferred to (C).

Once the parallel branches have been grouped together and a wieghted connection matrix (C) developed, the equivalent network will have fewer branches, i.e. the number of non-zero entries of (C).

e.g. the matrix (C) for the net work of Fig. 5.1 will be,

$$(C) = \begin{bmatrix} 0 & C_{12} & C_{13} & 0 \\ 0 & 0 & C_{23} & 0 \\ 0 & 0 & 0 & C_{34} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \ldots \quad (5.3)$$

where $C_{12} = p_1$ etc.

The reduced network corresponding to (5.3) will be as shown in Fig. 5.2 with the values of the corresponding probabilities indicated.

In Series - parallel networks, the elements can either be in series or in parallel. Fig. 5.2 obtained after reduction (combining the parallele elements only) does not contain any two or more edges across a pair of nodes but may have nodes to which only two edges are connected (one IN and other OUT). This type of nodes are called Series node and will now be eliminated from the reduced diagraph which is given by

$$Pr\left\{\bigcap_{k=1}^{m} E_k\right\} = P_{Series} = \prod_{k=1}^{m} P_k \quad \ldots (5.4)$$

For m elements in series.

So for the given Fig. 5.2 while eliminating node 2 the product is transferred to the entry of C_{13} and added to the existing value using parallel combination rules, i.e.

$$C_{13\ new} = C_{13\ old} + C_{12}\, C_{23} - C_{13\ old}\, C_{12}\, C_{23} \quad \ldots (5.5)$$

In general, if node k has element C_{ik} IN and element C_{kj} OUT, then an entry $C_{ij} = C_{ik} \cdot C_{kj}$ is transferred to the location (i, j) and is added to existing value using,

$$C_{ij\ new} = C_{ij\ old} + C_{ik}\, C_{kj} - C_{ij\ old}\, C_{ik}\, C_{kj} \quad \ldots (5.6)$$

However, the entries C_{ik} and C_{kj}, once they have been used and the node k has been eliminated, are made zero.

The information about the node, needed for the elimination process just described, can be had through the use of what is called a degree matrix, $\Delta \equiv (d_{ij})$. There are two degree matrices defined for a diagraph (D), one is out degree matrix (Od(D)), which has only diagonal entries Od_{ii}, indicating the number of branches "going out" or directed away from the node i The other matrix is in-degree matrix (Id(D)) for graph (D).

This matrix also has diagonal entries Id_{ii} indicating the number of branches "coming in" or directed towards the node i. It is easier to understand that Od_{ii} is the total number of non zero entries of the row corresponding to node i, in (C). (Od_{ii} would have been $= \sum_{j=1}^{n} C_{ij}$ where C_{ij} were either 1 or 0 if we had not attached weights to the individual elements).

Similarly, Id_{ii} for node i will be the total number of non-zero entries corresponding to i^{th} column of matrix (C) e.g. just before elimination, (Od) and (Id) for network of Fig. 5.1 will be

$$(Od) = \begin{bmatrix} 2 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{bmatrix} (Id) = \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & 2 & \\ & & & 1 \end{bmatrix} \quad .. (5.$$

It is interesting to note that $Id_{11} = 0$ if node 1 is a source node and $Od_{44} = 0$ if node 4 is a sink node.

The elements of matrices (Od) and (Id) will keep on changing as the elimination proceeds. Finally when all intermediate nodes have been eliminated there will be only one entry in (Od), for our example, $Od_{11}=1$, the rest of the entries will be zero. The same applies to (Id), which will also have only one entry $Id_{44} = 1$.

Since matrices (Od) and (Id) have only diagonal entries, it is economical to find a simpler way of storing them in memory. We can make use of the column corresponding to a source node of (C) for storing the diagonal elements of (Od) and the row corresponding to a sink node may be utilized for storing the diagonal elements of (Id) because both these column and row have zero entries throughout, always. Incidental the element of (C) corresponding to a sink or source entry will always be zero, therefore overlapping of (Od) and (Id) elements at the corner is no problem because $Od_{sink} = Id_{source} = 0$ e.g. Fig; 5.1 will have (C) just before the elimination process.

$$C \equiv \left[\begin{array}{c|ccc} 2 & 0.93 & 0.86 & 0 \\ 1 & 0 & 0.996 & 0 \\ 1 & 0 & 0 & 0.98 \\ \hline 0 & 1 & 2 & 2 \end{array}\right] \quad \ldots (5.8$$

As is evident from (5.8) we have been able to save a lot of space by combining the features of three matrices (C), (Od) and (Id). It is also easier to find total number of non-zero entries in any row and enter it in first column of that row and vice versa.

It was pointed out earlier that the elimination Starts with the node i that has $Od_{ii} = Id_{ii} = 1$. After eliminating and updating the entries of (C), again we look for the node that has in degree = out degree = 1. This goes on till all such nodes have been exhausted and finally the only entry in (C) left will be that of $C_{source, sink}$, which will be the total transmittance (reliability) of the network. The changes in (C) as nodes 2 and 3 are eliminated are presented in (5.9) for the example under discussion.

$$\left[\begin{array}{c|ccc} 1 & 0 & 0.9897 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0.98 \\ \hline & 0 & 1 & 1 \end{array}\right]$$

After node 2 is eliminated

$$\left[\begin{array}{c|ccc} 1 & 0 & 0 & 0.9699 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline & 0 & 0 & 1 \end{array}\right] \quad \ldots (5.9)$$

After node 3 is eliminated.

5.2.2 ALGORITHM IN BRIEF :

The steps involved in the algorithm can be summarized as follows :

5.2.2.1 Draw a diagraph for the network assigning proper direction to the elements, and numbers to the nodes and elements.

5.2.2.2 From the data, a weighted connection matrix is developed after combining the parallel elements across any two nodes.

5.2.2.3 Define Od_{ii} and Id_{ii} for each node.

5.2.2.4 Eliminate the node k that has $Od_k = Id_k = 1$.

5.2.2.5 Transfer the product $C_{ik}\, C_{kj}$ to (i, j) entry and modify the old C_{ij} entry using,

$$C_{ij}\text{ new} = C_{ij}\text{ old} + C_{ik}\, C_{kj} - C_{ij}\text{ old}\, C_{ik}\, C_{kj}$$

Also make the entries $C_{ik} = C_{kj} = 0$

5.2.2.6 Check whether all the intermediate nodes have been eliminated, if not, go to 5.2.2.3 othersise, print out the element $C_{source,\ sink}$ and Stop.

This algorithm has a unique advantage of being fast and direct and requires minimum extra information or manipulation. All informations are contained in (C).

5.3 ALGORITHM FOR RELIABILITY EVALUATION OF A REDUCIBLE NETWORK USING SPARSE MATRIX TECHNIQUE :

Here another algorithm for computing the reliability of a reducible network is presented, The computer storage is considerably reduced by numbering the nodes, in a special order and by not storing the complete date, so it can handle large syste

5.3.1 METHOD :

The numbering of the nodes of the graph is done manually in linear order, that is, all the nodes which are directed towards node k are numbered less than k, and all the nodes which are directed away from the node k are numbered greater than k. If the reliability matrix (weighted connection matrix) of this graph is formed, the lower diagonal and diagonal entries of this matrix will be zero. To save computer memory the upper diagonal entries of this matrix are stored in linear form . If i and j are the starting and finishing node of an edge respectively, the entry in row i and column j of the reliability matrix is stored in location m of the array R where,

$$m\ (i,\ j) = (i-1)\ n + j - (i(i + 1))/2 \ \ldots\ldots \quad (5.10)$$

Therefore the memory requirement to store the reliability matrix is $n(n-1)/2$.

If there are J parallel elements between a node pair, there are J xxxxxk entries of that node pair. The total number of rows in the table will be equal to the number of elements in the system. All entries of array R are initially set to zero. The manually prepared table is entered into the computer. The location of the element corresponding to each row of the table in the array R is calculated by (5.10).

The reliability of the element is stored in the location k with the help of the following relation.

$$R_k - R_k + r - R_k \cdot r \quad \ldots\ldots \qquad (5.11)$$

This takes into account for any parallel elements in the original network. The reliability matrix is now formed.

For furthdr reduction of the network, the indegree and outdegree of a node (defined as number of edges into and out of the node respectively) are calculated. The number of non-zero entries in a column of the reliability matrix indicates the indegree of the corresponding node. The outdegree of a node is equal to the number of non-zero entries in the corresponding row of the reliability matrix. The indegree and outdegree of each node are stored in arrays Id_i and Od_j respectively. The nodes to be eliminated first will be those whose indegree and outdegree is one. Let k be such a node, assume it is connected to node i and j, the entries of the array R are modified by the following relation :

$$R_{i1} = \begin{cases} R_{i2} \cdot R_{i3}, & \text{if there is no element between node i and j (i.e. } i_2 \text{ and } i_3 \text{ were in series)} \quad \ldots (5.12). \\ R_{i1} + (R_{i2} \cdot R_{i3}) - R_{i1} \cdot (R_{i2} \cdot R_{i3}), & \text{otherwise (i.e., } i_2 \text{ and } i_3 \text{ were in parallel)} \quad \ldots (5.13) \end{cases}$$

where $i_1 \equiv m(i, j) \quad \ldots \quad (5.14)$

$i_2 \equiv m(i, k) \quad \ldots \quad (5.15)$

$i_3 \equiv m(k, j) \quad \ldots \quad (5.16)$

With the successive reduction of such nodes, the original network will reduce to a network having a single element and the reliability of this element will be the reliability of the network. If the network is not reducible, the algorithm prints an error message. Step by step we can describe the algorithm as follows.

5.3.2 ALGORITHM IN BRIEF.

~~5.3.2.1~~ Set array $R_k (k = 1, 2 \ldots. k)$ equal to zero.
Set array Od_i and Id_i $(i = 1, \ldots n)$ equal to zero.

5.3.2.2 Set $k = 1$

5.3.2.2.1 Read data table, that is, i (starting node) j (finishing node), r(reliability of the element connected between node i and node j). If $i < j$, stop with message "Nodes are wrongly numbered".

5.3.2.2.2 Calculate m by (5.10) and modify R_m with the help of (5.11)

5.3.2.2.3 If $k < L$, set $k \leftarrow k + 1$ and go to step 5.3.2.2.1

5.3.2.3 Set $i = 1$

5.3.2.3.1 Set $j = i + 1$

5.3.2.3.2 Calculate m by (5.10). If $R_m > 0$, set $Od_i \leftarrow Od_i + 1$ and $Id_j \leftarrow Id_j + 1$

5.3.2.3.3 If $i < (n-1)$ Set $i \leftarrow (i + 1)$ and go to step 5.3.2.3.

5.3.2.4 Set $i = 1$

5.3.2.4.1 If $Od_i \neq 1$, go to step 5.3.2.4.3

5.3.2.4.2 If $Id_i = 1$, set $k = i$ and go to step 5.

5.3.4.3 If $i \geq (-1)$, stop. Otherwise set $i \leftarrow (i + 1)$ and go to step 5.3.[illegible].4.1

5 5.3.2.5 Set $i = 1$

5(a) 5.3.2.5.1 Calculate i_2 with the help of (5.15) set $j1 = i$ If $R_{i1} > 0$, Go to step 5.3.2.6

5(b) 5.3.2.5.2 If $i \geq k$, stop. Otherwise set $i \leftarrow (i + 1)$ and go to step 5.3.2.5.1

6 5.3.2.6 Set $i = k + 1$

7 5.3.2.7 Set $j_2 = 1$ and Calculate i_3 by (5.16). If $R_{i3} > 0$, go to step 5.3.2.8.

7(a) 5.3.2.7.1 If $i > n$, stop, otherwise set $i \leftarrow (i + 1)$ and go to step 5.3.2.7.

8 5.3.2.8 Calculate $m_1 = n (j_1 - 1) + j_2 - (j_1 + 1)/2$. If $R_{m1} > 0$, modify R_{m1} with the help of (5.13) set $Od_{j1} \leftarrow Od_{j1} - 1$ and $Id_{j2} \leftarrow Id_{j2} - 1$ and go to next step. Otherwise modify R_{i1} by (5.12).

9 5.3.2.9 Set R_{i2} and R_{i3} to zero and go to step 5.3.2.4.1.

The advantage of this method is this that here the computation time is considerably reduced by avoiding

(a) The scanning of data table for combining parallel connected elements.

and (b) The calculation of indegree and outdegree of each node after eliminating a series node.

5.4 AN EFFICIENT ALGORITHM FOR RELIABILITY EVALUATION USING ANALYTICAL APPROACH :

Using the ideas of set theory a new technique is introduced which allows the elimination of unwanted subsets of T_i (Minimal path) itself, without the need of break up each term into constituent states.

5.4.1 METHOD

System success function S is written as :

$$S = T_1 \cup T_2 \; \ldots \; \cup T_m \qquad \ldots \quad (5.17)$$

The aim is to rewrite S as S(dis) with all its terms mutually exclusive. The procedure for doing this becomes fast if the paths are enumerated in a way that the path having fewest branches is listed first and so son.

To select T_2(dis) from T_2, we expand T_2 about B_1 (corresponding to a branch b_1) which is contained in T_1 but not contained in T_2 as :

$$T_2 = T_2 B_1 \cup T_2 \overline{B}_1 \qquad \ldots \quad (5.18)$$

Now if $(T_2 B_1) \subset T_1$, $(T_2 B_1)$ is dropped from further consideration, because it is already included. Otherwise it is further expanded about B_2 and so on. If $(T_2 \overline{B}_1) \cap T_1 = \Phi$, $(T_2 \overline{B}_1)$ is disjoint with T_1. If however, this is not true, this subset also is further expanded about B_2 and so on. Ultimately, we shall find all subsets of T_2 which are disjoint with T_1. Union of all these subsets is T_2(dis.)

Similarly, we find T_i(dis) for all i such that T_i(dis) $T_j = \Phi$ for all $j < i$. This step is fastest if we first expand T_i about a set corresponding to that branch which has occurred most often in earlier paths. Then, S(dis) is given as :

$$S(dis) = T_1 \cup T_2(dis) \quad \ldots \quad \cup T_m(dis) \ldots \qquad (5.19)$$

Reliability expression then immediately follows :

5.4.2 <u>ALGORITHM IN BRIEF</u> :

The above steps can be organised in the form of an algorithm as :

5.4.2.1 Write a Boolean expression for S

5.4.2.2 Define an n_n dimensional vector $\vec{E_i}$ ($i = 1, 2, \ldots m$) Corresponding to T_i such that k^{th} element of this vector is 1, If that particular branch b_k is contained in T_i, and 0 otherwise.

5.4.2.3 Define another vector $\vec{V_j}$ which adds up all previous $\vec{E_i}$

$$\vec{V_j} = \sum_{i \leq j} \vec{E_i};\ i, j = 1, 2, \ldots m$$

5.4.2.4 Let i = 2

5.4.2.5 If there are any non-zero entries in $\vec{V_1}$ Corresponding to zero entries, in $\vec{E_1}$, record their positions in order of their ascending magnitude in $\vec{V_1}$. Let these be

k_1, k_2 ... k_r. This ordering helps in getting the minimum expression fast, as already explained.

5.4.2.6 Decompose $\vec{E}_i$ in two components $\vec{E}_i(k_1)$ and $\vec{E}_i(k_1)$. This corresponds to expanding T_i about k_1. $\vec{E}_i(k_1)$ and $\vec{E}_i(\bar{k}_1)$ are formed by replacing 0 in position k_1 in E_i by 1 and -1 respectively. If $\vec{E}_i$ (k_1) contains 1's in <u>all</u> those positions where there have been 1's in <u>any</u> $\vec{E}_j$ $(j < i)$, then $\vec{E}_i(k_1)$ is DROPPED from further analysis because it is already included in a previous path. If $\vec{E}_i(\bar{K}_1)$ contains -1 in <u>any</u> position where there is 1 in $\vec{E}_j$ for <u>all</u> $j < i$, then $\vec{E}_i(\bar{k}_1)$ is RETAINED as a disjoint subset. If $\vec{E}_i(k_1)$ is not dropped and/or if $\vec{E}_i(\bar{k}_1)$ is not retained, then there are further decomposed about k_2 and so on; carrying out the dropping and retaining tests at each step. Union of the retained components of $\vec{E}_i$ is T_i(dis).

5.4.2.7 Repeat steps 5.4.2.5 & 5.4.2.6 for $i = 3, 4 \ldots m$

5.4.2.8 Rewrite 3 as :

$$S(\text{dis}) = T_1 \cup T_2(\text{dis}) \ldots \cup T_m(\text{dis})$$

5.4.2.9 Change the Boolean variables x and $\bar{x}$ to reliability. Variables p_x and q_x ; change (U) in Boolean expression to (+) in algebraic expression to get the reliability expression.

In this way we have discussed four methods for assisting for the calculation of Reliability of a system. The computer codes for all these methods have been developed. Computer Code for one of the method is run on TDC-312 and for all the methods the Computer Codes are run on IBM 1620 for the system, shown in block diagram 5.1. The results along with computer codes are appended in appendices from I to V.

—

CHAPTER - 6

CONCLUSIONS

The most important aspect of system reliability discipline have been extensively studied. This aspect is :

System reliability Evaluation

Existing reliability evaluation techniques applicable to general systems have been critically studied. Same example has been solved by all the methods for bringing conformity of results and a meaningful comparison of the techniques. Apart from the complexity of techniques, final derived reliability expression differs in complexity in different cases. Computational time required for the numerical evaluation of this reliability expression and absolute error in the final result is also compared for each method.

Based on these methods one method for evaluation of mineral success path and minimal cuts is discussed in details. Also a computer code in Forgo for the said method is developed. The complete computer code alongwith the results obtained for the system shown in Fig. 5.1 is appended in Appendix I.

Based on the properties of diagraphs a technique for the reliability Evaluation of redundant network is discussed. Also a computer code in Forgo is developed for the said technique, which is tested on IBM-1620 for the system shown in Fig. 5.1. The computer code along with result is appended in Appendix II.

Another method for reliability evaluation using sparse matrix technique is discussed. This method has advantage over previous method that here only non zero entries of the connection matrix is stored, thus saving computer memory locations. For this method the computer Codes for TDC-312 and IBM 1620 are developed and these are run for the system of Fig. 5.1. The computer code for TDC-312 alongwith the result is appended in appendix III and computer code for IBM-1620 alon with the result is appended in appendix IV.

For a general system a successful attempt is made for the analytic specification of the reliability expression using the concepts of set theory. In this technique, each term of system success function is treated as a set and is broken into a few subsets. Some of these subsets may be dropped as these are completely contained in other sets of the success function. Remaining subsets may be retained as these are disjoint with other sets. A union of retained subsets is the disjoint portion of that term of success function. Thus, it is possibl to write system success function in a way that all its terms are mutually disjoint without a breakup into all success states The method is organised in the form of an algorithm which is very simple and easily computerizable. The computer code for this method is developed for IBM-1620, in Forgo and it has been successfully run for the system shown in Fig. 5.1. Appendix V shows the computer code for this method along with the results obtained from computer run.

These methods as discussed above will be of great help for the evaluation of the reliability of complicated systems.

TABLE - 3.1

LISTING OF ALL STATES FOR THE BLOCK DIAGRAM OF FIG. 3.1

S. No.	A	B	C	D	E	S
1	0	0	0	0	0	0
2	0	0	0	0	1	0
3	0	0	0	1	0	0
4	0	0	0	1	1	0
5	0	0	1	0	0	0
6	0	0	1	0	1	0
7	0	0	1	1	0	1
8	0	0	1	1	1	1
9	0	1	0	0	0	0
10	0	1	0	0	1	0
11	0	1	0	1	0	0
12	0	1	0	1	1	0
13	0	1	1	0	0	0
14	0	1	1	0	1	1
15	0	1	1	1	0	1
16	0	1	1	1	1	1
17	1	0	0	0	0	0
18	1	0	0	0	1	0
19	1	0	0	1	0	0
20	1	0	0	1	1	1
21	1	0	1	0	0	0
22	1	0	1	0	1	0
23	1	0	1	1	0	1
24	1	0	1	1	1	1
25	1	1	0	0	0	1
26	1	1	0	0	1	1
27	1	1	0	1	0	1
28	1	1	0	1	1	1
29	1	1	1	0	0	1
30	1	1	1	0	1	1
31	1	1	1	1	0	1
32	1	1	1	1	1	1

TABLE - 4.1.

(Computational Time and Absolute Error Comparison)

Method	Block Diagram of Fig. 3.1 T_c	E	Remarks
Exhaustive Search Method	655 T_s	10 Δe	Applicable only for very small systems.
Canonical Expansion Method			
Probability Calculus Method	269 T_s	40 Δe	Methods useful only if number of undirected branches is small.
Flow Graph Method			
Baye's theorem Method			
Probability Map Method	144 T_s	4 Δe	Applicable if number of element is less than six.

APPENDIX - I

```
C  C   ANALYSIS OF MINIMAL SUCCESS PATHS AND CUT SETS  SUSHIL B.H.E.L.
       DIMENSION IP(10,10),IC(10,10), IUN(10),IB(10), BOUND(10)
        DIMENSION PROB(10),IO(20),ITABL(30,10),IPRED(10,10),IACTI(10)
       DIMENSION INCOE(10,10)
  1    READ1500,N
C      ZERO ARRAYS
        DO401I=1,10
       IUN(I)=0
       IB(I)=0
       BOUND(I)=0.0
       PROB(I)=0.0
        DO401J=1,10
       IC(J,I)=0
 401   IP(J,I)=0
       READ 1510,(PROB(I),I=1,N),EPSIL
       PUNCH1000,N,(I,PROB(I),I=1,N)
       DO100I=1,10
       DO100J=1,10
 100   IP(I,J)=0
       JB=N+1
       DO104I=1,JB
       READ102,IACTI(I)
       DO104II=1,N
       READ102,IPRED(I,II)
       LL=IACTI(I)
 104   ITABL(LL,II)=IPRED(I,II)
       J=1
       NP=1
       IP(1,1)=25
 106    J=J+1
       KIC=0
       ICONT=0
       DO128I=1,NP
       IF(IP(I,J-1))107,107,108
 107   KIC=KIC+1
       GO TO 128
 108   K=IP(I,J-1)
       M=1
       IP(I,J)=ITABL(K,M)
 109   M=M+1
       IF(ITABL(K,M))111,128,111
 111   ICONT=ICONT+1
       KBC=NP+ICONT
       JL=J-1
       DO112KK=1,JL
 112   IP(KBC,KK)=IP(I,KK)
       IP(KBC,J)=ITABL(K,M)
       GO TO 109
 128   CONTINUE
       IF(KIC-NP)113,114,114
 113   NP=NP+ICONT
       GO TO 106
 114   PUNCH1010,NP
       DO450I=1,NP
       DO400J=1,10
 400   IUN(J)=0
```

```
      DO430J=1,10
      K=IP(I,J)
      IF(K)430,430,410
410   IF(K-25)420,430,430
420   IUN(K)=1
430   CONTINUE
       DO440J=1,10
440   IP(I,J)=IUN(J)
450   CONTINUE
      DO4I=1,NP
      K=0
      DO3J=1,N
      IF(IP(I,J))3,3,2
2     K=K+1
      IO(K)=J
3     CONTINUE
      PUNCH1020,I,(IO(J),J=1,K)
4     CONTINUE
C     *** DETERMINE SYSTEM CUTS
C       *** CHECK FOR SINGLE ELEMENT CUTS
      K=1
      DO30J=1,N
      DO20I=1,NP
      IF(IP(I,J))30,30,20
20    CONTINUE
      IC(K,J)=1
      K=K+1
30    CONTINUE
C       *** CHECK FOR DOUBLE ELEMENT CUTS
      N1=N-1
      IF(N1)571,571,31
31    DO90I=1,N1
      I1=I+1
      DO90J=I1,N
      IDUM=0
      DO40L=1,NP
      IF(IP(L,I))601,602,601
602   IF(IP(L,J))601,603,601
601   ITICK=1
      GO TO 704
603   ITICK=0
704   IDUM=IDUM+ITICK
40    CONTINUE
      IF(IDUM-NP)90,50,90
50    IC(K,I)=1
      IC(K,J)=1
      K1=K-1
      IF(K1)71,71,51
51    DO70L=1,K1
      IDUM=0
      JDUM=0
      DO60M=1,N
      IF(IC(K,M))604,605,604
604   IF(IC(L,M))606,605,606
605   ITICK=0
      GO TO 607
606   ITICK=1
```

```
 607   IDUM=IDUM+ITICK
       JDUM=JDUM+IC(L,M)
 60    CONTINUE
       IF(IDUM-JDUM)70,80,70
 70    CONTINUE
 71    K=K+1
       GO TO 90
 80    IC(K,I)=0
       IC(K,J)=0
 90    CONTINUE
C        ***CHECK FOR TRIPLE ELEMENT CUTS THAT ARE MINIMAL
       N2=N-2
       IF(N2)571,571,91
 91    DO180I=1,N2
       I1=I+1
       DO180J=I1,N1
       I2=J+1
       DO180L=I2,N
       IDUM=0
       DO120M=1,NP
       IF(IP(M,I))608,609,608
 609   IF(IP(M,J))608,610,608
 610   IF(IP(M,L))608,611,608
 608   ITICK=1
       GO TO 612
 611   ITICK=0
 612   IDUM=IDUM+ITICK
 120   CONTINUE
       IF(IDUM-NP)180,130,180
 130   DO135II=1,N
 135   IC(K,II)=0
       IC(K,I)=1
       IC(K,J)=1
       IC(K,L)=1
       K1=K-1
       IF(K1)171,171,131
 131   DO170M=1,K1
       IDUM=0
       JDUM=0
       DO140IJ=1,N
       IF(IC(M,IJ))613,614,613
 613   IF(IC(K,IJ))615,614,615
 614   ITICK=0
       GO TO 616
 615   ITICK=1
 616   IDUM=IDUM+ITICK
       JDUM=JDUM+IC(M,IJ)
 140   CONTINUE
       IF(IDUM-JDUM)170,150,170
 150   IC(K,I)=0
       IC(K,J)=0
       IC(K,L)=0
       GO TO 180
 170   CONTINUE
 171   K=K+1
 180   CONTINUE
C        ***ALL MINIMAL CUTS OF THIRID ORDER OR LESS HAVE BEEN DETERMINI
 571   NC=K-1
       PUNCH 1030,NC
       DO 195 I=1,NC
```

```
      K=0
      DO190 J=1,N
      IF(IC(I,J))190,190,185
185   K=K+1
      IO(K)=J
190   CONTINUE
      PUNCH 1020 ,I,(IO(J),J=1,K)
195   CONTINUE
1500 FORMAT(10I5)
1510 FORMAT(8E10.4)
102   FORMAT(6I5)
1000 FORMAT(/7X,17H CIRCUIT CONTAINS,I3,9H ELEMENTS/11X,8H ELEMENT,
    115X,12H PROBABILITY/11X,7H NUMBER,16X,11H OF SUCCESS/(I15,F26.4)
1010 FORMAT(17X,26H TIE SETS OR SUCCESS PATHS,I3/12X,5H PATH,15X,
    116H ELEMENT NUMBERS/)
1020 FORMAT(I15,8X,5I5)
1030 FORMAT(17X,9H CUT SETS,I3/12X,8H CUT SET,15X,
    116H ELEMENT NUMBERS/)
         STOP
      END
    5
.9300E 00 .8600E 00 .9200E 00 .9500E 00 .9800E 00 .1000E-03
    1
   -1
    0
    0
    0
    0
    2
   -1
    0
    0
    0
    0
    3
    1
    0
    0
    0
    0
    4
    1
    0
    0
    0
    0
    5
    2
    3
    4
    0
    0
   25
    5
    0
    0
```

```
   0
   0
C  C  ANALYSIS OF MINIMAL SUCCESS PATHS AND CUT SETS  SUSHIL B.H.E.L.
      PROGRAM ACCEPTEDZ      36930  54620     58509  59999


        CIRCUIT CONTAINS  5 ELEMENTS
          ELEMENT                  PROBABILITY
          NUMBER                   OF SUCCESS
            1                       .9300
            2                       .8600
            3                       .9200
            4                       .9500
            5                       .9800
              TIE SETS OR SUCCESS PATHS  3
           PATH                  ELEMENT NUMBERS
            1             2    5
            2             1    3    5
            3             1    4    5
              CUT SETS  3
           CUT SET                 ELEMENT NUMBERS
            1             5
            2             1    2
            3             2    3    4
0     STOP END AT S. 1030 + 01 L.  Z
```

```
C  C   MAIN PROGRAM  SUSHIL B.H.E.L.
       DIMENSION IK(10),JK(10),P(30),C(10,10)
        READ10,NN,NE
 10    FORMAT(2I3)
       READ20,(IK(I),JK(I),P(I),I=1,NE)
 20     FORMAT(5(2I2,F10.6))
        DO1I=1,NN
        DO1J=1,NN
 1      C(I,J)=0.0
        JJ=2
        N1=NN-1
        DO2I1=1,N1
        DO3J1=JJ,NN
       PO=0.
       DO 13 I=1,NE
       IF(IK(I)-I1)13,14,13
 14    IF(JK(I)-J1)13,15,13
 15    PI2=P(I)
       IK(I)=0
       JK(I)=0
       QO=1.-PO
       PO=PO+QO*PI2
 13     CONTINUE
 3      C(I1,J1)=C(I1,J1)+PO
        JJ=JJ+1
 2      CONTINUE
        PUNCH 30,((C(I,J),J=1,NN),I=1,NN)
        N2=NN-2
        DO8NM=1,N2
       K=1
       DO 21 I=1,N1
       M=0
       DO 22 J=2,NN
       IF(C(I,J))23,22,23
  23   M=M+1
  22   CONTINUE
       C(I,1)=M
   21  CONTINUE
       DO 41 J=2,NN
       L=0
       DO51 I=1,N1
       IF(C(I,J))61,51,61
 61    L=L+1
 51    CONTINUE
       C(NN,J)=L
 41    CONTINUE
       DO 71 I=2,N1
       IF(C(I,1)-1.)71,81,71
 81    IF(C(NN,I)-1.)71,91,71
 91    K=I
 71    CONTINUE
        IF(K-1)9,9,11
 9      PUNCH60
 60    FORMAT(24H NO SERIES PARALLEL CASE)
        GO TO 12
 11     PUNCH 50
 50    FORMAT(26H MATRIX BEFORE ELIMINATION)
       PUNCH30,((C(I,J),J=1,NN),I=1,NN)
        PUNCH40,K
 40     FORMAT(25H NODE TO BE ELIMINATED IS,I4)
```

```
      DO4I=1,N1
      IF(C(I,K))5,4,5
    4 CONTINUE
  5   IX=I
      DO6J=2,NN
      IF(C(K,J))7,6,7
    6 CONTINUE
    7 JX=J
      CX=C(IX,K)*C(K,JX)
      C(IX,JX)=CX+C(IX,JX)*(1.-CX)
      C(IX,K)=0.0
      C(K,JX)=0.0
 8    CONTINUE
      PUNCH31,C(1,NN)
      GO TO 32
 31   FORMAT(27H RELIABILITY OF THE SYSTEM=,F10.6)
 12   PUNCH 30,((C(I,J),J=1,NN),I=1,NN)
 30   FORMAT(6F10.6)
 32   STOP
      END
  4  5
 1 2   .93     1 3   .86     2 3   .92     2 3   .95     3 4   .
C  C  MAIN PROGRAM  SUSHIL B.H.E.L.
      PROGRAM ACCEPTEDZ      36930  43230     59129  59999

 0.000000   .930000   .860000  0.000000  0.000000  0.000000
  .996000  0.000000  0.000000  0.000000  0.000000   .980000
 0.000000  0.000000  0.000000  0.000000
 MATRIX BEFORE ELIMINATION
 2.000000   .930000   .860000  0.000000  1.000000  0.000000
  .996000  0.000000  1.000000  0.000000  0.000000   .980000
 0.000000  1.000000  2.000000  1.000000
 NODE TO BE ELIMINATED IS   2
 MATRIX BEFORE ELIMINATION
 1.000000  0.000000   .989679  0.000000  0.000000  0.000000
 0.000000  0.000000  1.000000  0.000000  0.000000   .980000
 0.000000  0.000000  1.000000  1.000000
 NODE TO BE ELIMINATED IS   3
 RELIABILITY OF THE SYSTEM=   .969886
0      STOP END AT S. 0032 + 00 L.  Z
```

A P P E N D I X - I$_{II}$

```
0:   C    RELIABILITY EVALUATION OF SERIES PARALLEL SYSTEM.
          DIMENSION R(16), IC(4), IR(4)
          READ NN,NL
          N1=NN-1
          N2=NN-2
          INR=(NN*N1)/2
          DO10I=1,INR
          R(I)=0.0
10:       CONTINUE
          DO20I=1,NN
          IR(I)=0
          IC(I)=0
20:       CONTINUE
          DO30I=1,NL
          READ II, IJ, RR
          IF(II - IJ)25, 21, 21
21:       WRITE 41
          GO TO 180
25:       IND = (II-1)*NN + IJ - (II + 1))/2
          R(IND) - R (IND) + RR - R(IND) * RR
30:       CONTINUE
          DO500I =I1, N1
          I1 = I + 1
          DO50J = I1, NN
          IND = (I-1)* NN + J - (I + 1))/2
          IF (R(IND))40, 50, 40
40:       IR (I) = IR (I) + 1
          IC(J)  = IC (J) + 1
50:       CONTINUE
          IL = 0
60:       DO90I = 1, NL
          IF (IR)I) - 1) 90, 70, 90
70:       IF (IC(I) - 1) 90, 80, 90
80:       K = 1
          WRITE 61, K
          GO TO 100
90:       CONTINUE
          WRITE  51
          GO TO 180
100:      DOI05I = 1, N1
          IND = (I - 1) * NN + K - (I*(I+1))/2
          I1 = 1
          I.(R(IND))110, 105, 110
105:      CONTINUE
110:      I$ = IND
          K1 = K + 1
          DO120I = K1,
          JJ = I
          IND = (K-1)   .. + 1 - (K*(K + 1))/2
          IF(R(IN..)),130, 12., 13 .
```

```
120      CONTINUE
130:     J1 = IND
         ZZ = R(I1) * R (J1)
         IND = (I1-1) * NN + JJ - (II * (II + 1)/2
         IF(R(IND)150, 140, 150
140:     R(IND) = ZZ
         GO TO 160
150:     R(IND) = R(IND) + ZZ - R(IND) * ZZ
         IR(II) = IR(II) - 1
         IC(JJ) = IC(JJ) - 1
160:     R(I1) = 0.0
         R(j1) = 0.0
         IR(K) = 0
         IC(K) = 0
         IL = IL + 1
         IF(IL - N2)60, 2 170, 170
170:     WRITE 31, R(N1)
31:      FORMAT(/, 'SYSTEM RELIABILITY = ', E)
41:      FORMAT(/, 'NODES ARE WRONGLY NUMBERED')
51:      FORMAT(/, 'SYSTEM HAS NONSERIES PARALLEL STRUCTURE')
61:      FORMAT(/, 'NODE TO BE ELIMINATED', I)
180:     STOP
         END

4 5 1 2 .93 1 3 .86 2 3 .92 2 3 .95 3 4 .98

C: C     RELIABILITY EVALUATION OF SERIES PARALLEL SYSTEM
≠
NODE TO BE ELIMINATED+2
NODE TO BE ELIMINATED+3

SYSTEM RELIABILITY = 9.698856E-1
# #
```

A P P E N D I X - IV

```
C   C   RELIBILITY EVALUATION OF SERIES PARALLEL SYSTEM  SUSHIL B.H.E.L.
        DIMENSION R(300),IC(30),IR(30)
        READ 1000,NN,NL
C       NN=NUMBER OF NODES,NL=NUMBER OF ELEMENTS
        N1=NN-1
        N2=NN-2
        INR=(NN*N1)/2
        DO10I=1,INR
 10      R(I)=0.0
        DO20I=1,NN
        IR(I)=0
 20     IC(I)=0
        DO30I=1,NL
        READ2000,II,IJ,RR
C       II=STARTING NODE,IJ=FINISHING NODE,RR=RELIABILITY OF ELEMENT CO
C       BETWEEN II AND IJ NODES
        IF(II-IJ)25,21,21
 21     PUNCH4000
        GO TO 180
 25     IND=(II-1)*NN+IJ-(II*(II+1))/2
        R(IND)=R(IND)+RR-R(IND)*RR
 30     CONTINUE
C       CALCULATE INDEGREE AND OUTDEGREE OF EACH NODE
        DO50I=1,N1
        I1=I+1
        DO50J=I1,NN
        IND=(I-1)*NN+J-(I*(I+1))/2
        IF(R(IND))40,50,40
 40     IR(I)=IR(I)+1
        IC(J)=IC(J)+1
 50     CONTINUE
C       FIND THE NODE K TO BE ELIMINATED
        IL=0
 60     DO90I=1,NN
        IF(IR(I)-1)90,70,90
 70     IF(IC(I)-1)90,80,90
 80     K=I
        PUNCH6000,K
        GO TO 100
 90     CONTINUE
        PUNCH5000
        GO TO 180
C       ELIMINATE THE KTH NODE AND MODIFY THE ARRAY    R
 100    DO 105 I=1,N1
        IND=(I-1)*NN+K-(I*(I+1))/2
        II=I
        IF(R(IND))110,105,110
 105    CONTINUE
 110    I1=IND
        K1=K+1
        DO120I=K1,NN
         JJ=I
        IND=(K-1)*NN+I-(K*(K+1))/2
        IF(R(IND))130,120,130
```

```
120   CONTINUE
130   J1=IND
      ZZ=R(I1)*R(J1)
      IND=(II-1)*NN+JJ-(II*(II+1))/2
      IF(R(IND))150,140,150
140   R(IND)=ZZ
      GO TO 160
150   R(IND)=R(IND)+ZZ-R(IND)*ZZ
      IR(II)=IR(II)-1
      IC(JJ)=IC(JJ)-1
160   R(I1)=0.0
      R(J1)=0.0
      IR(K)=0
      IC(K)=0
       IL=IL+1
      IF(IL-N2)60,170,170
170   PUNCH3000,R(N1)
1000   FORMAT(2I5)
2000  FORMAT(2I5,F10.6)
3000  FORMAT(10X,20H SYSTEM RELIABILITY=,F10.6)
4000  FORMAT(27H NODES ARE WRONGLY NUMBERED)
5000  FORMAT(40H SYSTEM HAS NONSERIES PARALLEL STRUCTURE)
6000  FORMAT(10X,25H**NODE TO BE ELIMINATED**,I5)
180   STOP
      END
    4     5
    1     2     .93
    1     3     .86
    2     3     .92
    2     3     .95
    3     4     .98
C  C  RELIBILITY EVALUATION OF SERIES PARALLEL SYSTEM  SUSHIL B.H.E.L
      PROGRAM ACCEPTEDZ      36930  45590      59239  59999

          **NODE TO BE ELIMINATED**     2
          **NODE TO BE ELIMINATED**     3
           SYSTEM RELIABILITY=    .969886
0     STOP END AT S. 0180 + 00 L.  Z
```

APPENDIX - V

```
C  C    RELIABILITY  EVALUATION  SUSHIL B.H.E.L.
        DIMENSION LE(10,10),LT(10,10),NK(10),KP(10),KL(10),MA(10)
        DIMENSION MB(10,10),JMB(10),LOB(10,10),RR(10)
C       LE(I,J)=MATRIX OF PATH
C       NE=NUMBER OF ELEMENTS,NP=NUMBER OF PATHS
        READ10,NE,NP
 10     FORMAT(2I5)
        READ11,((LE(I,J),J=1,NE),I=1,NP)
 11     FORMAT(30I2)
        DO12I=1,NP
        DO12J=1,NE
        LT(I,J)=0
        DO12K=1,I
 12     LT(I,J)=LT(I,J)+LE(K,J)
        PUNCH13
 13     FORMAT(19H RETAIN THIS VECTOR)
        DO 91J=1,NE
        IG=1
 91     LOB(IG,J)=LE(1,J)
        PUNCH14,(LOB(IG,J),J=1,NE)
 14     FORMAT(10I5)
        DO234I=2,NP
        LAT=0
        MAT=1
        MPR=0
        DO17J=1,NE
        IF(LE(I,J))17,15,17
 15     IF(LT(I,J))16,17,16
 16     MPR=MPR+1
        NK(MPR)=LT(I,J)
        KP(MPR)=J
 17     CONTINUE
        ND=1
        NDK=1
        DO22JA=1,NE
        MA(JA)=LE(I,JA)
 22     MB(MAT,JA)=LE(I,JA)
 23     KT=KP(ND)
        MA(KT)=1
        MB(MAT,KT)=-1
        ND=ND+1
 73     IF(ND-MPR)51,51,59
 51     ID=I-1
        NKN=0
        DO 55L=1,ID
        DO53K=1,NE
        IF(LE(L,K)-1)53,52,53
 52     IF(MA(K)-1)55,53,55
 53     CONTINUE
 59      NKN=1
        GO TO 56
 55     CONTINUE
 56     IF(NKN-1)24,26,24
 24     MAT=MAT+1
        DO25MR=1,NE
 25     MB(MAT,MR)=MA(MR)
        GO TO 23
 26     PUNCH27
```

```
 27    FORMAT(17H DROP THIS VECTOR)
       PUNCH14,(MA(KK),MK=1,NE)
 28    LAT=LAT+1
       DO29LR=1,NE
 29    JMB(LR)=MB(LAT,LR)
       NDK=NDK+1
 03    IF(NDK-MPR)61,61,66
 61    IR=I-1
       MPM=0
       DO65L=1,IR
       DO64K=1,NE
       IF(JMB(K)+1)64,62,64
 62    IF(LE(L,K)-1)64,63,64
 63    GO TO 65
 64    CONTINUE
       GO TO 67
 65    CONTINUE
 66    MPM=1
 67    IF(MPM-1)30,32,30
 30    MAT=MAT+1
       DO31LK=1,NE
       MA(LK)=JMB(LK)
 31    MB(MAT,LK)=JMB(LK)
       PUNCH 101
 101   FORMAT(9H CONTINUE)
       PUNCH14,(MA(LK),LK=1,NE)
       ND=NDK
       GO TO 23
 32    PUNCH13
       IG=IG+1
       DO92LP=1,NE
 92    LOB(IG,LP)=JMB(LP)
       PUNCH14,(LOB(IG,LP),LP=1,NE)
       IF(LAT-MAT)28,234,234
 234   CONTINUE
C      RR=RELIABILITY OF THE ELEMENT,R=SYSTEM RELIABILITY
       READ 93,(RR(KK),KK=1,NE)
 93    FORMAT(6F10.6)
       R=0.0
        DO94KLT=1,IG
       SUM=1.0
       DO 96 KT=1,NE
        IF(LOB(KLT,KT))95,96,97
 95    SUM=SUM*(1.0-RR(KT))
        GO TO 96
 97    SUM=SUM*RR(KT)
 96    CONTINUE
 94    R=R+SUM
       PUNCH 90,R
 98    FORMAT(20H SYSTEM RELIABILITY=,F10.6)
       STOP
       END
   5    3
 0 1 0 0 1 1 0 1 0 1 1 0 0 1 1
   .93      .86      .92      .95      .98
C  C  RELIABILITY  EVALUATION  SUSHIL B.H.E.L.
```

PROGRAM ACCEPTEDZ 36930 47770 58889 59999

```
RETAIN THIS VECTOR
   0    1    0    0    1
DROP THIS VECTOR
   1    1    1    0    1
RETAIN THIS VECTOR
   1   -1    1    0    1
DROP THIS VECTOR
   1    1    0    1    1
CONTINUE
   1   -1    0    1    1
DROP THIS VECTOR
   1   -1    1    1    1
RETAIN THIS VECTOR
   1   -1   -1    1    1
SYSTEM RELIABILITY=   .969886
0     STOP END AT S. 0098 + 01 L.  Z
```

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