OPTIMAL DESIGN AND DEVELOPMENT
OF
PROPORTIONAL-INTEGRAL DERIVATIVE CONTROLLER

A DISSERTATION
submitted in partial fulfilment of
the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(MEASUREMENT & INSTRUMENTATION)

By

PARMOD KUMAR

DERARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE (INDIA)
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C E R T I F I C A T E

Certified that the dissertation entitled "OPTIMAL DESIGN & DEVELOPMENT OF PROPORTIONAL-INTEGRAL-DERIVATIVE CONTROLLER" which is being submitted by Sri PARMOD KUMAR in partial fulfilment for the award of the degree of Master of Engineering in Measurement & Instrumentation, of the University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is to further certify that he has worked for a period of about 6½ months from July 1975 to January 20, 1976 for preparing the dissertation for Master of Engineering at the University of Roorkee.

ROORKEE.

Dated: Jan. 31, 1976.

(S.C. SAXENA)
Lecturer
Department of Electrical Engg.
University of Roorkee,
Roorkee.

A C K N O W L E D G E M E N T S

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_S_Y_N_O_P_S_I_S_

Optimization principles are of undisputed importance in modern design and system operation. Here in this dissertation, Regulatory technique has been used to solve the problem and in designing a practical p-i-d controller.

To start with different controllers and various modes of their operation have been considered. Comparitive study of different controllers have also been discussed.

In the second chapter optimization principles are used for the linear regulatory system . Merits and shortcommings of applications of this theory are also included. The theory has been applied to design optimal p-i-d controller for the furnace temperature control.

Optimal settings could also be found-out by using emperical formulae. All such emperical formulae which have practical applicability have been discussed in third chapter.

An electronic p-i-d controller have been designed by using optimal theory. Test results are given. Keeping the experimental tolerances in view practical and theoritical results were approximately same.

C O N T E N T S

I N T R O D U C T I O N

CHAPTER - 1

1.1 CONTROLLER :

Unique specifications imposed on the performance of the control systems are chiefly dictated by the precise nature of the particular job to be handled by the system. Firstly, it is reasonable requirement that the transient component should not be unbounded as it approches infinity for any bounded input. This is the typical condition of absolute stability for a linear system. As such, condition of absolute stability is not complete and sufficient in deciding the performance of the system. A control system may be extremely oscillatory though absolutely stable. In cases of this type it will be desirable that the oscillations should settle down, within the accepted limits.

Concerning the overall performance of the system main function of the controller employed in system is to attain this very specofic requirement. Thus automatic controller is an instrument utilized to maintain the controlled variable at the desired value. The automatic controller must be designed to prevent the load disturbances affecting the value of the controlled variable. Most importantly, therefore, the control of load disturbances had always been of prime importance in the automatic process control systems.

1.2 Modes of Controller :

The automatic controller, including its measuring means, determines the value of the controlled variable, compares the actual value to the desired value,

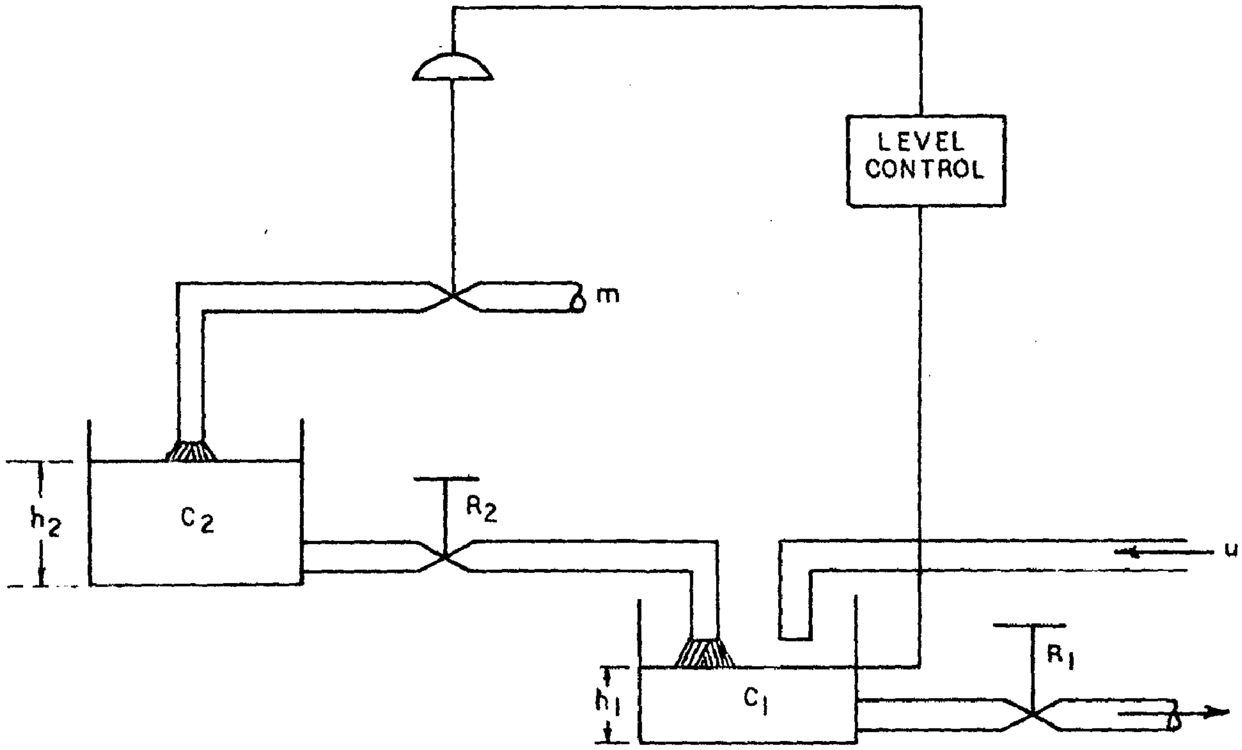


FIG. 1 LEVEL CONTROL PROBLEM

determines the deviation and produces the counter action necessary to maintain the smallest possible deviation. The method by which the automatic controller produces the counteraction is called the mode of control or control action.

In analyzing a specific control problem a choice based on economic factors must be made among the various control actions. Generally speaking, the more difficult the control problem, the more complicated the controlling means becomes. This does not at all mean that a complicated automatic controller is necessary to produce good automatic control. On the contrary, the simplest control devices are often capable of providing a high quality of control.

Each of the control is applicable to process having certain characteristics. Here, the effectiveness of the various modes of control by comparing the responses to a load change on a process shown in Fig.1 has considered.

The process equation is

$$C = \frac{K_1}{(T_1 S+1)(T_2 S+1)} m + \frac{K_1}{T_1 S+1} u$$

where C is the controlled variable = head in the lower vessel

R_1 = resistance of lower outlet valve.

T_1 = time constant of lower vessel = $R_1 A_1$.

T_2 = time constant of upper vessel = $R_2 A_2$.

m = manipulated variable = inflow to upper vessel.

u = load variable = inflow to lower vessel.

If the vessel time constants are equal, T = 20 secs. As shown in Fig below a change of inflow to lower vessel may result in appreciable deviation for several minutes. The following comments apply to each type of control. The numbers correspond to the numbered covers of Fig.2.

1. Proportional - derivative Control : It provides the smallest maximum error because the derivative part of the response allows the proportional sensitivity to be increased to a high value. The stabilization time is smallest because of the derivative action. Offset is allowed but is only half that experienced without derivative action.

2. Proportional - integral - derivative : It has the next smallest maximum deviation and offset is eliminated because of the integral action. However, the addition

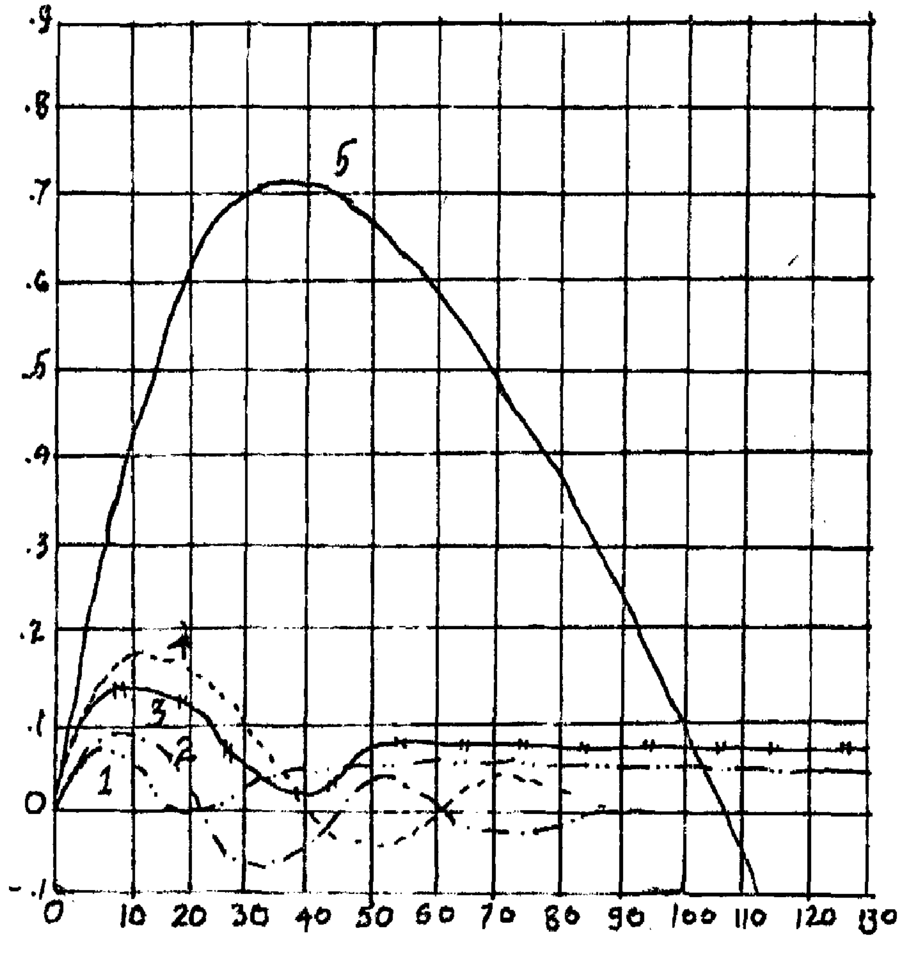


FIG. 2 COMPARISION OF DIFFERENT MODES

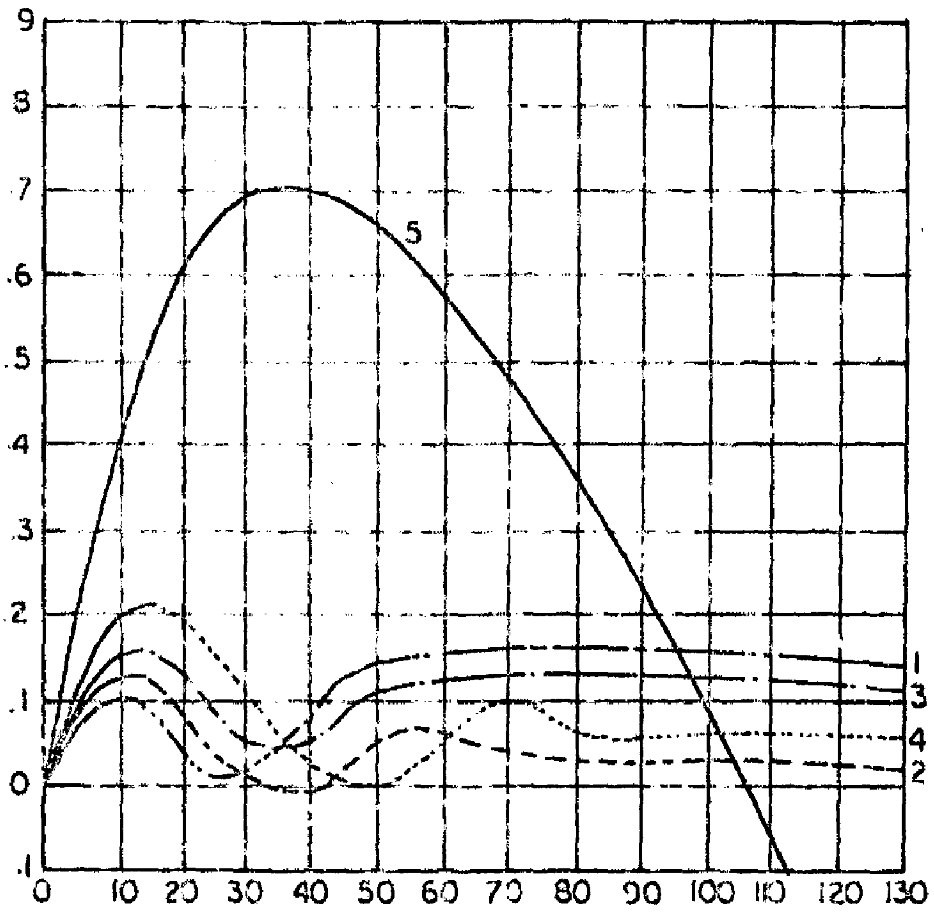


FIG.2 COMPARISION OF DIFFERENT MODES

of integral action markedly increases the stabilization time.

3. Proportional control : It has large maximum deviation than controllers with derivative action because of the absence of this stabilizing influence. Offset is also large.

4. Proportional - integral control : It has no offset of the integral action. The unstabilizing influence of integral response is reflected in the large maximum deviation and persisting deviation.

5. Integral control : It is best suited for the control of processes having little or no energy strorage and the results of the comparison are not representative of all integral control. However, on this process, the results indicate a large maximum deviation and a long stabilization time.

With the result of this comparison in mind, it is logical to ask why proportional - integral derivative control action is not universal employed. The answer is generally based on economic reasons, because each additional control action usually requires an additional piece of equipment that must be purchased, installed and maintained. In addition, each control action may require adjustment of a parameter such as proportional sensitivity integral time or derivative time. This often requires considerable installation and maintence time in order to obtain the proper adjustment of parameters.

OPTIMAL DESIGN OF "p-i-d" CONTROLLER

CHAPTER - II

2.1 Optimum Controller :

The word optimum is associated with system design in technical literature. The optimum control system is defined as the control system that minimizes a given error index for a given dynamic process and subject to given design constraints. It should be noted that a optimum system is defined in terms of mathematical model of the design problem. If the mathematical model is altered, then the minimization leads to a different control system. A typical change in the mathematical model arises from the adjustment of weight factors in the error index, a procedure which may be required in order to satisfy all the design specifications simultaneously.

Parameters such as gains and time constants are adjusted, when the configuration of the controller is assumed fixed and a limited number of parameters are adjusted to minimize the error index. Parameter optimization is the selection of these parameters to minimize the error index.

When the system optimization results in linear controller, the controller is termed the linear optimum controller. On the other hand when the optimum controller is in fact non-linear but one constrains

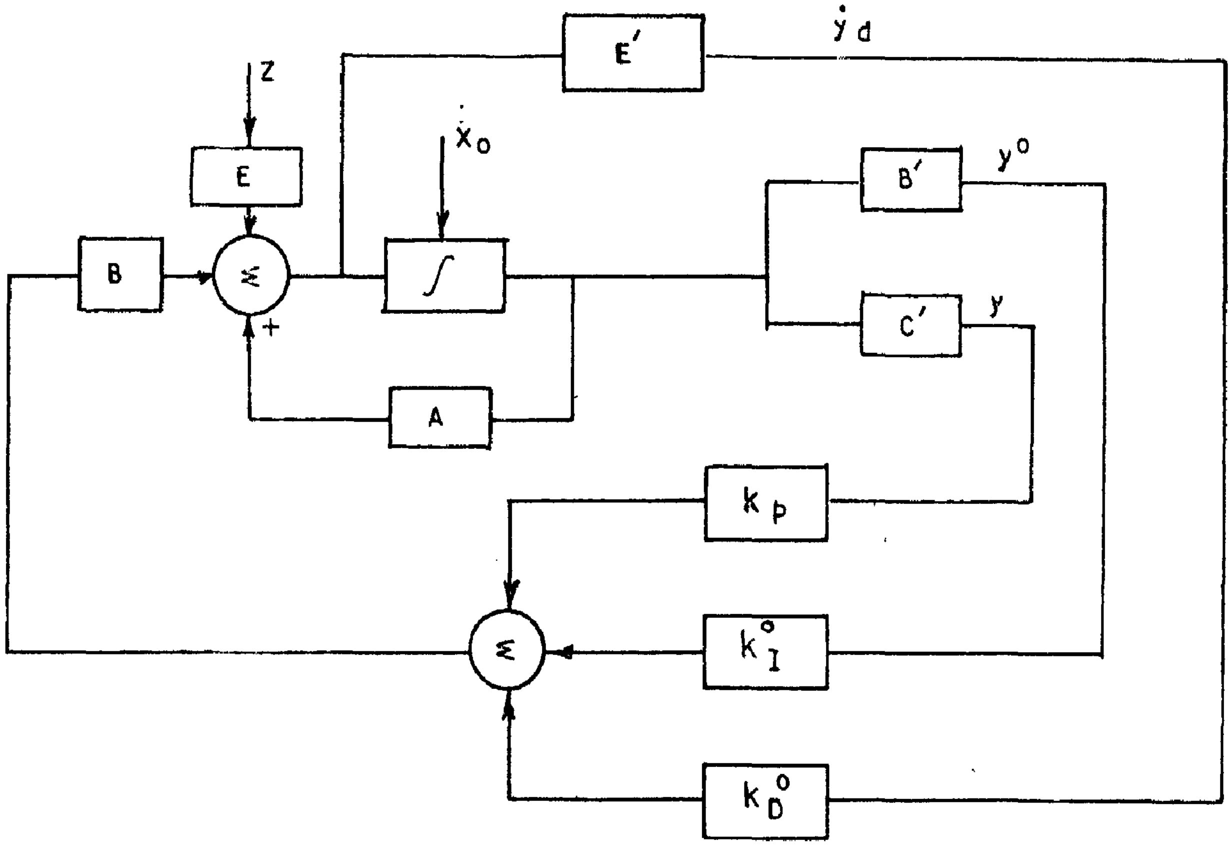


FIG. 3 REGULATOR PROBLEM

oneself in investigating only linear controller , the resulting linear controller is termed the "Optimum Linear Controller".

There are two types of problems in optimal control :

1. Regulator Problem.
2. Tracking or Servomechanism Problem.

1. Regulator Problem : Suppose that initially the plant output or any of its derivative is nonzero. Provide a plant input to bring the output and its derivatives to zero. In other words, the problem is to apply a control to take the plant from a non-zero state to the zero state. This problem may typically occur where the plant is subjected to unwanted disturbances that perturb its output. (e.g. a radar control system with antenna subject to wind gusts).

2. Tracking or Servomechanism Problem : Suppose that the plant output or derivative is required to track some prescribed function. Provide a plant input that will cause this tracking (e.g. when a radar antenna is to track aircraft such a control is required).

2.2 Design as a Linear Regulator : Consider a linear time invariant system

$$\dot{x}(t) = Ax(t) + Bu(t) + FZ(t) \qquad ....(1)$$

where $x(t_0) = x_0$

and define three measurable system outputs

$$y(t) = C'x(t)$$

$$y^0(t) = D'x(t) \qquad ....(2)$$

$$y^d(t) = E'x(t)$$

Where x(t) is an 'n' state vector, u(t) is an 'm' control vector, Z(t) is a 'k' disturbance vector, y(t) is an 'r' output, y^0 is a 'p' vector of the outputs specified to have zero steady state and y^d is a q vector of outputs to be used in derivative feedback. A, B, F, C', D' and E' are constant - coefficient matrices of appropriate dimensions. It is assumed that $0<q\leq r\leq n$, $p\leq n$ and $k\leq n$.

The control "u^0" is assumed to the output of a 3 term controller with input y, y^o and y^d so that

$$\overset{o}{u}(t) = -k_p y(t) - k_i \int_{t_0}^{t} y^o(t)dt - k_d \dot{y}_d(t) + u_o \qquad ....(3)$$

where k_p, k_i and k_d are the proportional, integral and derivative feedback gain matrices of the dimensions $m \times r$, $m \times p$ and $m \times q$ respectively, and u_o is the m - dimensional initial control vector. Such a regulatory is shown in Fig. 3.

from eqn. (1) and eqn. (2) it follows that

$$\dot{y}_d(t) = E'\dot{x}(t) = E'(Ax(t) + Bu(t) + FZ(t)) \quad .....(4)$$

So that the p-i-d control law, equation (3), becomes

$$u^o(t) = -k_pC'x(t) - k_i\int_{t_o}^{t} D'x(t)dt - k_dE' \quad Ax(t)$$
$$+ Bu(t) + FZ(t) + u_o$$

If $Z(t)$ is assumed to be slowly varying quantity or a constant i.e $Z(t) = Z =$ constant. then

$$u^o(t) = -(k_pC' + k_d E'A)\,x(t) - k_iD'\int_{t_o}^{t} x(t)dt + u_o$$
$$- k_dE'B\dot{u}(t)$$

$$\dot{u}^o(t) \quad (I_m + k_dE'B) = -\Big[(k_pC' + k_dE'A)\,x(t)$$
$$+ k_iD'\int_{t_o}^{t} x(t)dt + u_o$$

$$u^{o}(t) = -\left(I_m + k_d E'B \right)^{-1} \left(k_p C' + k_d E'A \right) x(t)$$

$$+ k_i D' \quad x(t)dt \quad + u_o \left(I_m + k_d E'B \right)^{-1}$$

$$u^{o}(t) = -\left(I_m + k_d E'B \right)^{-1} \left(K_p C' + k_d E'A \right) x(t)$$

$$+ k_i D' \quad x(t)\,dt \quad + u_o^{o} \qquad \ldots\ldots\ldots(5)$$

where $u_o^{o} = \left(I_m + k_d E'B \right)^{-1} u_o$ ....(6)

The control law equation (5) in a more compact form

$$u^{o}(t) = -k_p^{*}\, x(t) - k_i^{*}\, D' \quad x(t)dt + u_o^{o} \qquad \ldots\ldots(7)$$

Where new m x n and m x p dimentional, proportional and integral gain matrices k_p^{*} and k_i^{*} are given by

$$k_p^{*} = \left(I_m + k_d E'B \right)^{-1} \left(k_p C' + k_d E'A \right)$$

$$k_i^{*} = \left(I_m + k_d E'B \right)^{-1} k_i$$

Equation (7) shows that for a given system, equations (1) and (2) any p-i-d output constrained controller equation (3) can be reduced to a "p-i" controller with complete - state proportional feedback and integral feedback limited to the same output variable $y^o(t)$ as used in p-i-d control law, eqn. (3). It also shows that optimum complete state feedback regulator performance can be achieved with an incomplete - feedback realisation, by introducing a derivative control action.

Now, the augmented with "p" integrals of outputs y^o, The augmented (n + p) th - order system for Z(t) - constant, is then defined by

$$\hat{\dot{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}\,\hat{u}(t), \quad \hat{x}(t_o) = x_o \qquad \ldots\ldots\ldots(9)$$

where

$$\hat{x} = \begin{bmatrix} x - x_{ss} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ v - v_{ss} \end{bmatrix} \qquad \hat{u} = u - u_{ss}$$

$$v = v_o + \int_{t_o}^{t} y^o(t)dt = y_o + D'\int_{t_o}^{t} x(t)dt$$

$$\hat{A} = \begin{bmatrix} A & 0 \\ D' & 0 \end{bmatrix} \qquad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

and the subscript ss designates steady - state values.

By associating the system model, equation (9) with the quadratic performance index

$$J = \frac{1}{2}\int_{t_0}^{\infty} x'(t)\, Q\, x(t) + u'(t)\, Ru(t) \; dt \quad \text{.....(10)}$$

where

$$Q = Q' = \begin{bmatrix} Q_n & 0 \\ 0 & Q_p \end{bmatrix} \geq 0$$

$Q_n = \hat{Q}_n \geqslant 0$ is an n x n constant matrix.

$Q_p = \hat{Q}_p > 0$ is a p x p constant matrix .

and $\hat{R} = \hat{R} > 0$ is an m x m constant matrix .

The solution for the feedback - gain - matrices k_p^* and k_1^* in equation (7) can be obtained, provided that the conditions of complete controllability and observability of the triple $\left[\hat{A}, \hat{B}, \hat{Q}^{\frac{1}{2}}\right]$ are satisfied:

Then,

$$k_p = R^{-1} B' P_{11}$$

$$\text{.......(11)}$$

$$k_1^* = R^{-1} B' P_{12}$$

Where P_{11} and P_{12} are n x n and n x p dimensional submatrices of an (n + p) x (n + p) constant matrix.

$$\hat{p} = \hat{p}\begin{bmatrix} p_{11} & p_{12} \\ p_{12}' & p_{22} \end{bmatrix} \qquad \ldots\ldots\ldots\ldots(12)$$

The unique positive definite solution of an (n+p) th algebraic matrix Riccati equation

$$\hat{P}\hat{A} + \hat{A}'\hat{P} - \hat{P}\hat{B}R^{-1}\hat{B}'\hat{P} + Q = 0 \qquad \ldots.(13)$$

The asymptotically stable optimal closed - loop system obtained by substitution of equation (7) in equation (1), also represents the closed - loop system when the p-i-d control law equation (3) is applied to the system in equation (1).

$$\dot{x}(t) = Ax(t) + B\left[-k_p^{*}\, x(t) - k_i^{*}\, D'\int_{t_0}^{t} x(t)\, dt + u_0^{o}\right] + FZ(t)$$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix} = \begin{bmatrix} A & B \\ -k_p^o A - k_i^o D & k_p^o B \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} F \\ -k_p^o F \end{bmatrix} z(t)$$

and

$$\begin{bmatrix} x(t_o) \\ u(t_o) \end{bmatrix} = \begin{bmatrix} x_o \\ u_o^o \end{bmatrix} \qquad \ldots\ldots(14)$$

where $u_o^o = -\hat{R}^{-1} B' P_{12} v_o$

The p-i-d controller realisation equation (3) requires the feedback - gain matrices k_p, k_i and k_d to be determined by the solution of the matrix equation (8). The number of unknowns is $mr + mp + mq = m(p+r+q)$ while the number of equations is $mn + mp = m(n+p)$. In general such a system have solution if

$$r + q \geqslant n \qquad \ldots\ldots\ldots\ldots(15)$$

i.e. when the sum of numbers of proportional and derivative action is greater or equal to the system order. For $r + q = n$, the unique realisation of a p-i-d controller is

$$\begin{bmatrix} k_p & k_d \end{bmatrix} = k_p^* \begin{bmatrix} \dfrac{C'}{E'A - E'Bk_p} \end{bmatrix}$$

$$k_i = \begin{bmatrix} I_m + k_d E' B \end{bmatrix} k_i^* \qquad \text{......(16)}$$

On the other hand, for $r + q \quad n$, the realisation of an optimum p-i-d regulator may exist, but the solution for the gain matrices k_p, k_i and k_d is not unique. It can be expressed as

$$\left.\begin{aligned} \begin{bmatrix} k_p & \vdots & k_d \end{bmatrix} &= \begin{bmatrix} k_p^* & \vdots & 0 \end{bmatrix} \begin{bmatrix} C' & \vdots & G \\ \hline E'A - E'Bk_p^* & \vdots & H \end{bmatrix}^{-1} \\ \text{and} \quad k_i &= (I_m + k_d E'B) k_i^* \end{aligned}\right\} \quad \text{---} (\ $$

Where arbitrarily chosen matrices G (of the dimension $r \times (r + q - n))$ and H (of the dimension $q \times (r + q - n)$) ensure the regularity of the inverted part in the first

of equation (17), the dimension of which is
$(r + q) \times (n + q)$

Short Comings of the above Method : Although industrial regulator design is nearly always based on linear models of the processes to be controlled, such models are always inaccurate. Inaccuracies may arise from approximations made in the theoretical description of the process, from linearising a non-linear model to obtain one more amenable to analysis from errors in parameters identification and from many other sources.

Advantages : If we are given a system by equations (1) and (2) in which the disturbances is unmeasurable, stability and output regulation may be achived by the state and output feedback.

<u>Numerical Problem</u> :

Design a p-i-d controller using linear regulatory theory for a second - order system. The system parameters are

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad C_a' = C_f = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$C_b' = E_b' = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

with the weighting matrices

$$R = 1 \qquad Q_n = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \qquad q_b = 1 \qquad \ldots\ldots\ldots(18)$$

Solution : To find the control law, we must first find k_p^* and k_i^* i.e. P, which can be found from the Riccati equation

$$\hat{P}\hat{A} + \hat{A}'\hat{P} - \hat{P}\hat{B}R^{-1}\hat{B}'\hat{P} + Q = 0$$

Here

$$\hat{A} = \begin{bmatrix} A & 0 \\ D & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{A}' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad R = 1 \qquad R^{-1} = 1$$

$$Q = Q' = \begin{bmatrix} Q_n & 0 \\ & \\ 0 & Q_p \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Riccati equation becomes

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} - \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

Now, simplifying the above matrices, we get

$$\begin{bmatrix} -p_{12}+p_{13} & p_{11} & 0 \\ -p_{22}+p_{23} & p_{21} & 0 \\ -p_{32}+p_{33} & p_{31} & 0 \end{bmatrix} + \begin{bmatrix} -p_{21}+p_{31}, & -p_{22}p_{33}, & -p_{23}+p_{33} \\ p_{11} & p_{12} & p_{13} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} p_{12}p_{21} & p_{12}p_{22} & p_{12}p_{23} \\ p_{22}p_{21} & p_{22}^2 & p_{22}p_{23} \\ p_{32}p_{21} & p_{32}p_{22} & p_{32}p_{23} \end{bmatrix} = - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \ldots($$

Adding the L.H.S. matrices and then equating corresponding elements with R.H.S. matrix and simplifying, we get

$p_{11} = 5 \qquad p_{21} = 2 \qquad p_{31} = 2$

$p_{12} = 2 \qquad p_{22} = 2 \qquad p_{32} = 1$

$p_{13} = 2 \qquad p_{23} = 1 \qquad p_{33} = 3$

$\therefore$ Optimum gain

$$k_p^* = R^{-1} B' P_{11} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \end{bmatrix}$$

$$k_i^* = R^{-1} B' P_{12}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 1$$

$\therefore$ Optimum control law (equation 7)

$$u(t) = - k_p^* x(t) - k_i^* D' \quad x(t)\, dt + u_o^o$$

becomes

$$u(t) = - 2\, x_1(t) - 2\, x_2(t) - \int_{t_o}^{t} x_1(t) dt + u_o \quad \ldots\ldots(20)$$

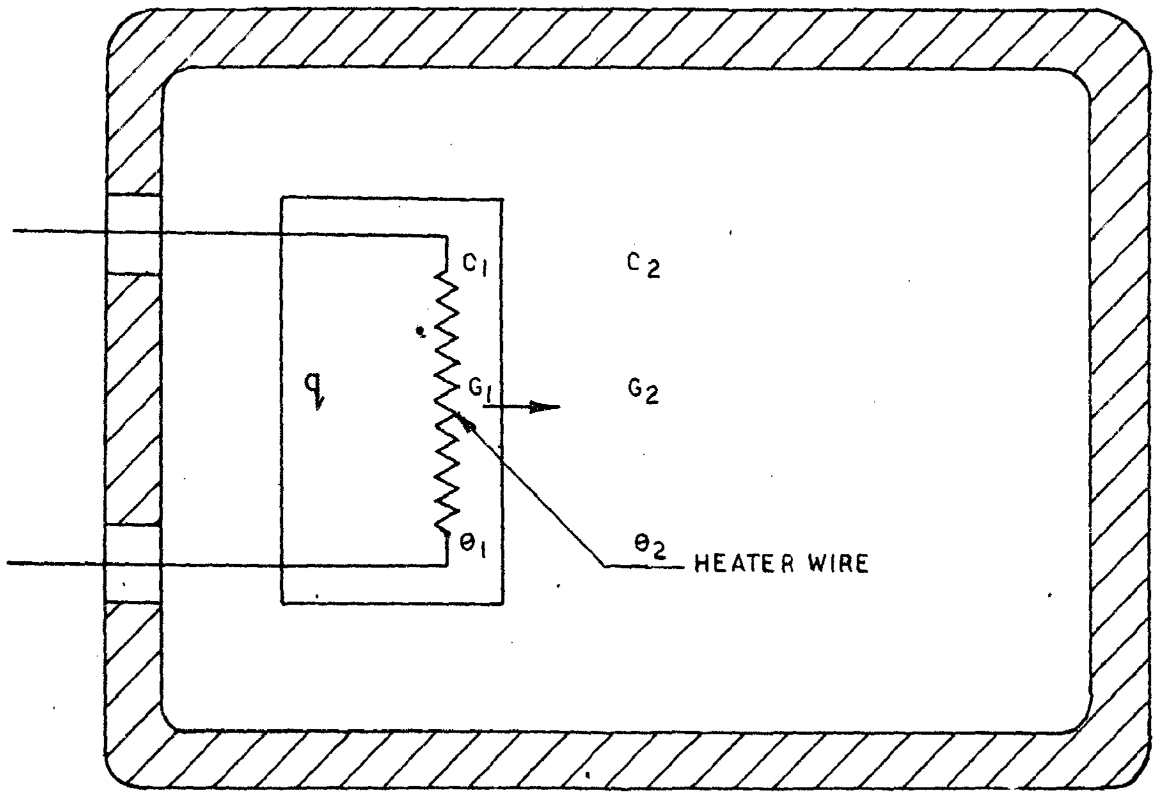


FIG. 4 ELECTRICAL HEATED FURNACE

2.3 ~~Application to Industrial Furnace~~ :

Temperature is one of the most important process variable. So here an electrical heated furnace is considered to control its temperature i.e. we are interested in designing a controller for the Muffal furnace. The specification of furnace are given below.

Heater wire of Kanthal A-1 (N_1 - Chromium alloy) 14 SGW; input 440 V, 3 o, 50 Hz. capacity 10 Kw.

Thermal capacity of Heater wire C_1 = 65 B.Th.U/°F
Thermal conductivity of Heater wire G_1 = 0.32 B.Th.U/°min./°F
Thermal capacity of heating chamber C_2 = 80 B.Th.U/°F
Thermal conductivity of chamber G_2 = 0.25 B.Th.U./min/°F

Now, before analyzing the problem let us consider the following assumptions :

1. The heater has a negligible inductance.
2. The mass of the metal in heater resistance is small.
3. The lead wire from the voltage source to the heater may be assumed to conduct negligible heat away from the coil.

An electrical heated furnace is shown in Fig.4. Let the thermal capacity and thermal conductance of the heater be C_1 and G_1 respectively. Then heat is transferred from heater to the surrounding fluid according to the relation.

$$q_1 = G_1 (\theta_1 - \theta_2) \qquad \text{..........(21)}$$

where q_1 is the heat transmitted from chamber 1 to 2.

θ_1 - is the difference between heater element temperature and the ambient temperature.

θ_2 - is the difference between the temperature of the heating chamber and the ambient temperature.

Now, combining and equating the rate of input heat with the rate of the output heat, we have

$$q = q_1 + C_1 \frac{d\theta_1}{dt} \qquad \text{.........(22)}$$

where $C_1 \dfrac{d\theta_1}{dt}$ is the storage of heat

from (21) and (22) we get

$$q = C_1 \frac{d\theta_1}{dt} + G_1 (\theta_1 - \theta_2) \qquad \text{.......(23)}$$

where q is the total heat rate of the heating element embedded in the heater. Also, similarly for heating chamber

$$q_1 = G_1 (\theta_1 - \theta_2) = G_2 \theta_2 + C_2 \frac{d\theta_2}{dt} \qquad \ldots\ldots(24)$$

rearranging equation (23) and (24) we obtain

$$q = C_1 \frac{d\theta_1}{dt} + G_1 \theta_1 - G_1 \theta_2$$

$$\text{or} \quad \frac{d\theta_1}{dt} = \frac{1}{C_1} q + \frac{G_1}{C_1} \theta_1 - \frac{G_1}{C_1} \theta_2 \quad \ldots\ldots(25)$$

$$\text{and} \quad 0 = - G_1\theta_1 + C_2 \frac{d\theta_2}{dt} + (G_1 + G_2) \theta_2$$

$$\text{or} \quad \frac{d\theta_2}{dt} = \frac{G_1}{C_2} \theta_1 + \frac{(G_1 + G_2)}{C_2} \theta_2 \quad \ldots\ldots(26)$$

let $\theta_1 = x_1$ $\theta_2 = x_2$ and $q = u$

then

$$\dot{x}_1 = \frac{G_1}{C_1} x_1 - \frac{G_1}{C_1} x_2 + \frac{1}{C_1} u$$

$$\ldots\ldots(27)$$

$$\dot{x}_2 = \frac{G_1}{C_2} x_1 - \frac{G_1 + G_2}{C_2} x_2 + 0.u$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{G_1}{C_1} & -\frac{G_1}{C_1} \\ \frac{G_1}{C_2} & \frac{G_1 + G_2}{C_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} \\ 0 \end{bmatrix} u \quad \ldots (\ldots$$

$$\therefore \quad A = \begin{bmatrix} G_1/C_1 & G_1/C_1 \\ G_2/C_2 & \frac{G_1 + G_2}{C_2} \end{bmatrix}$$

$$B = \begin{bmatrix} 1/C_1 \\ 0 \end{bmatrix}$$

Substituting the value of G_1, G_2, C_1 and C_2 we get

$$A = \begin{bmatrix} .005 & -.005 \\ .004 & .007 \end{bmatrix} = 10^{-3} \begin{bmatrix} 5 & -5 \\ 4 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} .015 \\ 0 \end{bmatrix} = 10^{-3} \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

with weighting matrices

$$\hat{R} = 1 \quad Q_n = 10^{-3} \begin{bmatrix} 1 & 0 \\ 00 & 0 \end{bmatrix} Q_p = 1 \cdot 10^{-3}$$

Now, from the Riccati equation

$$\hat{P}\hat{A} + \hat{A}'\hat{P} - \hat{P}\hat{B}R^{-1}\hat{B}'\hat{P} + Q = 0 \qquad \text{.......(29)}$$

$$\text{where} \quad \hat{A} = \begin{bmatrix} A & 0 \\ D & 0 \end{bmatrix} = 10^{-3} \begin{bmatrix} 5 & 5 & 0 \\ 4 & 7 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad \hat{A}^{1} = 10^{-3} \begin{bmatrix} 5 & 4 & 1 \\ -5 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} = 10^{-3} \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix}$$

$$R = 1 \quad \therefore \quad R^{-1} = 1$$

$$Q = \hat{Q} = \begin{bmatrix} Q_n & 0 \\ 0 & Q_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, the Riccati equation becomes

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} 5 & -5 & 0 \\ 4 & 7 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 4 & 1 \\ -5 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

$$-10^{-1}\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}\begin{bmatrix} 1.5 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1.5 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

on simplifying, we get

$$\begin{bmatrix} 5p_{11} + 4p_{12} + p_{13} & -5p_{11} + 7p_{12} & 0 \\ 5p_{21} + 4p_{22} + p_{23} & - p_{21} + 7p_{22} & 0 \\ 5p_{31} + 4p_{32} + p_{23} & -5p_{31} + 7p_{32} & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 5p_{11} + 4p_{21} & 5p_{12} + 4p_{22} + p_{32} & 5p_{13} + 4p_{23} + p_{33} \\ -5p_{11} + 7p_{31} & -5p_{12} + 7p_{22} & -5p_{13} + 7p_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$-2.25 \times 10^{-1} \begin{bmatrix} p_{11}^2 & p_{11}p_{12} & p_{11}p_{13} \\ p_{11}p_{21} & p_{12}p_{21} & p_{13}p_{31} \\ p_{11}p_{31} & p_{12}p_{31} & p_{31}p_{13} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \quad \ldots($$

Summing the matrices on L.H.S. and then equating the elements on both side, we get

$$p_{31} \cdot p_{13} = 1 \quad \text{(Comparing 33 element)}$$

$$\text{or} \quad p_{13} = \frac{1}{p_{31}} \qquad \ldots\ldots\ldots(31)$$

$$-5p_{13} + 7p_{23} - .225\, p_{13}\, p_{31} = 0 \quad \text{(comparing 23)}$$

$$\text{or} \quad -5p_{13} + 7p_{23} - .225 = 0 \qquad \ldots\ldots\ldots(32)$$

$$-5p_{13} + 4p_{23} + p_{33} - .225\, p_{11}p_{13} = 0 \quad \text{(comparing 13)}$$

$$\ldots\ldots\ldots(33)$$

$$-5p_{31} + 7p_{32} - .225\, p_{12}p_{31} = 0$$

$$\text{or} \quad -5 + 7p_{32}\, p_{13} - .225\, p_{12} = 0 \qquad \ldots\ldots\ldots(34)$$

$$-5p_{21} + 7p_{22} - 5p_{12} + 7p_{22} + .225\, p_{12}p_{21} = 0 \qquad \text{......(35)}$$

(comparing 22)

$$-5p_{11} + 7p_{12} + 5p_{12} + 4p_{22} - .225\, p_{11}p_{12} = 0 \qquad \text{......(36)}$$

(comparing 12)

$$5p_{31} + 4p_{32} + p_{23} - .225\, p_{11}p_{31} = 0 \text{ (comparing 31)}$$

$$\text{or} \quad 5 + 4p_{32}p_{13} + p_{23}p_{13} -.225\, p_{11} = 0 \qquad \text{.......(37)}$$

$$5p_{21} + 4p_{22} + p_{23} -5p_{11} + 7p_{31} - .225\, p_{11}p_{21} = 0$$

(comparing 21)

or

$$5p_{21}p_{13} + 4p_{22}p_{13} + p_{23}p_{13} - 5p_{11}p_{13} + 7 -.225\, p_{11}p_{21}p_{13} \qquad \text{.......(38)}$$

$$5p_{11} + 4p_{12} + p_{13} + 5p_{11} + 4p_{21} + p_{31} - .225\, p_{11}^2 + 1 = 0$$

$$\text{or} \quad 5p_{11}p_{13} + 4p_{12}\, p_{13} + p_{13}^2 + 5p_{11}p_{13} + 4p_{21}p_{13} + 1$$

$$- .225\, p_{13}p_{11}^2 + p_{13} = 0 \qquad \text{.........(39)}$$

Now, from the eqn. (31), (32) we get

$$p_{31} = \frac{1}{p_{13}}$$

and $p_{23} = 0.7\, p_{13} + .045$ ....(40)

Hence, substituting these values in the above equations, we get

$-5p_{13} + 4\,(.7p_{13} + .045) + p_{33} - 0.225\, p_{11}p_{13} = 0$

or $-5p_{11} + 2.8\, p_{13} + 0.18 + p_{33} - 0.225\, p_{11}p_{13} = 0$ ...(41)

$-5 + 7p_{32}\, p_{13} - .225\, p_{12} = 0$

or $p_{12} = 32\, p_{32}\, p_{13} + 22$ ....(42)

$-5p_{21} + 7p_{22} - 5p_{12} + 7p_{22} + .225\, p_{12}p_{21} = 0$ ......(43)

$-5p_{11} + 7p_{12} + 5p_{12} + 4p_{22} - .225\, p_{11}p_{12} = 0$ ......(44)

$5 + 4p_{32}\, p_{13} + p_{13}\,(.7p_{13} + .045) - .225\, p_{11} = 0$

or $5 + 4p_{32}p_{13} + .7p_{13}^2 + .045\, p_{13} - .225\, p_{11} = 0$ ....(45)

$5p_{21}p_{13} + 4p_{22}p_{13} + p_{23}p_{13} - 5p_{11}p_{13} + 7 - .225\, p_{11}p_{21}p_{13} = 0$

.....(46)

$$5p_{11}p_{13} + 4p_{12}p_{13} + p_{13}^2 + 5p_{11}p_{13} + 4p_{21}p_{13} + 4p_{21}p_{13}$$

$$- .225p_{13}p_{11} + p_{13} = 0 \qquad \ldots\ldots\ldots(47)$$

Now substituting the value $p_{12} = 32\, p_{32}p_{13} + 22$ in equations from (41) to (47) and simplifying, we get

$$-160p_{32} - 110 + 7p_{32}p_{21} = 0 \qquad \ldots\ldots\ldots(48)$$

$$14p_{22} - 160p_{32} - 110 + 7p_{32}p_{21} = 0 \qquad \ldots\ldots\ldots(49)$$

$$-5p_{11} + 12(32p_{32} + 22) + 4p_{22} - .225p_{11}(32p_{32} + 22) = 0$$
$$\ldots\ldots\ldots(50)$$

$$5 + 4p_{32}p_{13} + .7p_{13}^2 + .045p_{13} - .225p_{11} = 0 \qquad \ldots\ldots\ldots(51)$$

$$5p_{21}p_{13} + 4p_{22}p_{13} + p_{23}p_{13} - 5p_{11}p_{13} + 7 - .225\, p_{11}p_{21}p_{13} =$$
$$\ldots\ldots\ldots(52)$$

$$5p_{11}p_{13} + 4p_{13}\ (32p_{32}p_{13} + 22) + p_{13}^2 + 5p_{11}p_{13} + 4p_{21}p_{13}$$

$$- .225\, p_{13}p_{11}^2 + p_{13} = 0$$

$$10\, p_{11}p_{13} + 128\, p_{13}^{2}p_{32} + 88p_{13} + p_{13}^{2} + 4p_{21}p_{13}$$

$$-.225\, p_{13}p_{11}^{2} + p_{13} = 0 \;\ldots\ldots \qquad \ldots\ldots\ldots\ldots (53)$$

Again, we have from eqn. (48)

$$7p_{32}p_{21} = 160p_{32} + .110 \qquad \ldots\ldots\ldots\ldots (54)$$

$$p_{21} = 28.6 + \frac{15.9}{p_{32}} \qquad \ldots\ldots\ldots\ldots (55)$$

$$p_{21} \quad 28 + \frac{16}{p_{32}} \qquad \ldots\ldots\ldots\ldots (56)$$

Now, substituting the valve of p_{21} in equation (49) we get

$$p_{22} = 0 \qquad \ldots\ldots\ldots\ldots (57)$$

Substituting the value of p_{21} from equation (56) and p_{22} from (57) we get from equations (50) to (53)

$$384\, p_{32} + 264 - 7p_{11}p_{32} = 0$$

$$\therefore \quad p_{11} = \frac{38}{p_{32}} + 52 \qquad \ldots\ldots\ldots\ldots (58)$$

Also, $5 + 4p_{32}p_{13} + 0.7p_{13}^2 + .045p_{13} - .225p_{11} = 0$ ....(59)

$$5p_{13}\left(21 + \frac{16}{p_{32}}\right) + p_{23}p_{13} - 5p_{11}p_{13} + 7 - .225p_{11}p_{13}\left(21 + \frac{16}{p_{32}}\right) = 0 \quad ....(60)$$

$$10\,p_{11}p_{13} + 138p_{13}^2p_{33} + 90p_{13} + p_{13}^2 + 40p_{13}\left(21 + \frac{16}{p_{32}}\right) - .225\,p_{13}\,p_{11}^2 = 0 \quad ....(61)$$

Again, substituting the value of p_{11}, from equation (58) in equations from (59) to (61) and simplifying, we get

$$p_{13} = \frac{28\,p_{32} + 16}{118 + 166\,p_{32}} \quad ....(62)$$

and $112\,p_{32}^3 - 644\,p_{32}^2 - 1128\,p_{32} - 944 = 0$ ......(63)

By heat and trial method, we get

$$p_{32} \simeq 7.3$$

$$\therefore \quad p_{13} = \frac{28\ p_{32} + 16}{118 + 166\ p_{32}} = 1.55$$

$$p_{32} = 7.3$$

$$p_{13} = 1.55$$

$$p_{22} = 0$$

$$p_{11} = 57.1$$

$$p_{12} = 266$$

$$p_{21} = 23.2$$

$$p_{31} = 0.6$$

$$p_{23} = 1.13$$

$$\therefore \quad k_p^* = R^{-1}\ B'\ p_{11}$$

$$k_1^* = R^{-1}\ B'\ P_{12} \qquad \text{.........(64)}$$

$$k_p^* = 10^{-3} \begin{bmatrix} 15 & 0 \end{bmatrix} \begin{bmatrix} 57.1 & 266 \\ 23.2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 0 \end{bmatrix} \begin{bmatrix} .0571 & .266 \\ .0232 & 0 \end{bmatrix}$$

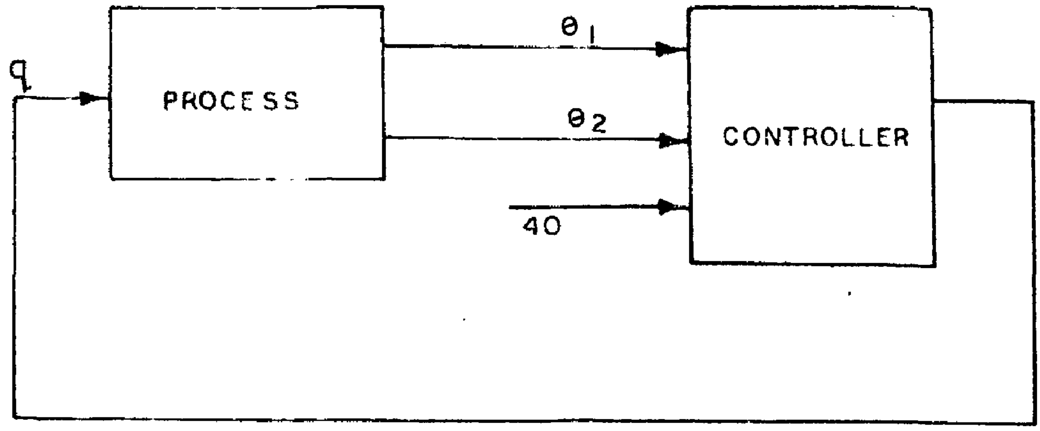


FIG . 5 (a)

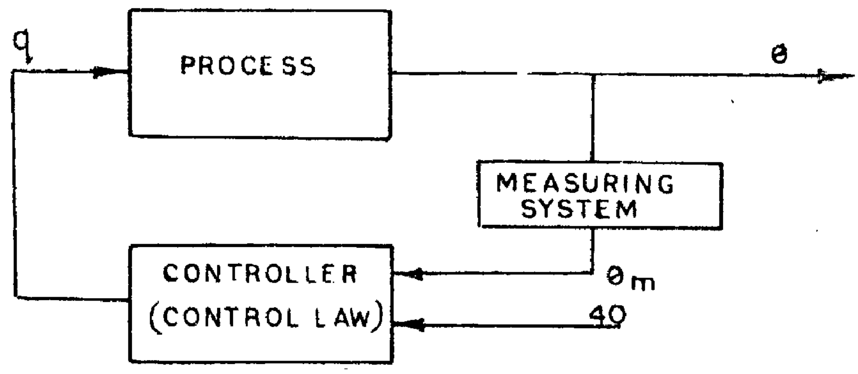


FIG . 5 (b)

BLOCK DIAGRAMS

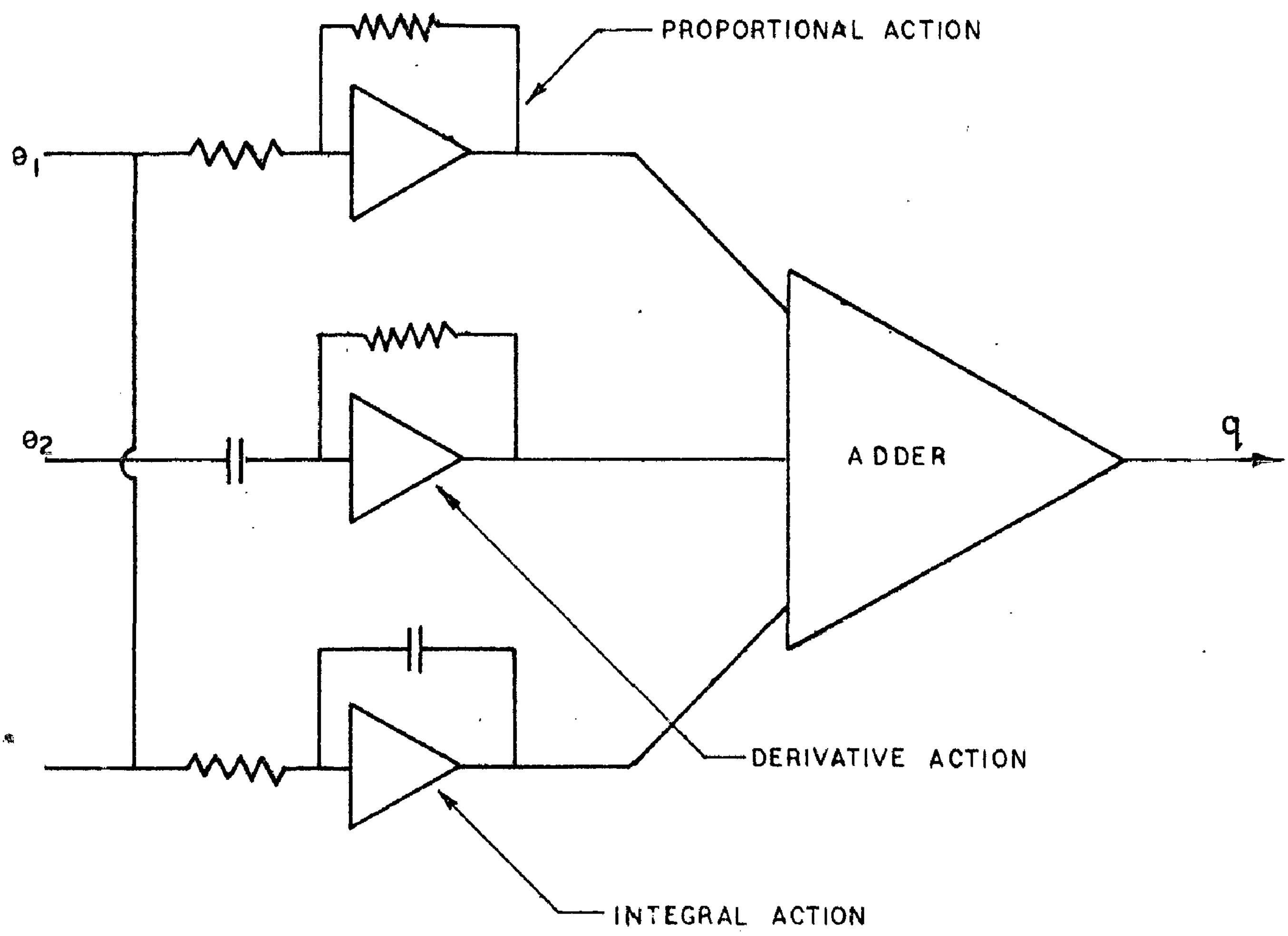


FIG. 6 SIMULATION OF CONTROL LAW

$$k_p^* = \begin{bmatrix} .86 & 4 \end{bmatrix} \qquad \text{.......(65)}$$

$$K_1^* = 10^{-3} \begin{bmatrix} 15 & 9 \end{bmatrix} \begin{bmatrix} 266 \\ 1.13 \end{bmatrix} \cong 4 \qquad \text{........(66)}$$

$\therefore$ Optimum control lew becomes

$$u(t) = -\ 0.86\ x_1(t) - 4\ x_2(t) - 4\ \ \ x_1(t)dt + u_o \ \text{....(67)}$$

Therefore, the optimum control lew becomes

$$q = -\ 0.86\theta_1(t) - 4\ \theta_2(t) - 4\ \ \ \theta_1(t)\ dt + u_o$$

$$= -\ \ .86\ \theta_1(t) + 4\ \theta_2(t) - 4\ \ \ \theta_1(t)\ dt \qquad \text{...(68)}$$

Hence, the above control system can be shown as in Fig.5.

OPTIMAL SETTING OF CONTROLLERS

CHAPTER - III

3.1 <u>Using Emperical Formulas</u> :

There are numerous methods of choosing good setting for controllers, however, when three responses are present, the situation is much more complex and consequently, only a few of the many possible approaches :

1. Ziegler - Nichols Method : Ziegler and Nichols have given a rule of thumb for adjusting the proportional sensitivity "S" reset rate 'I' and derivative rate "D" in terms of the reaction curve of the process .

$$S = \frac{1.2}{R_r \; L_r} \qquad I = \frac{0.5}{L_r} \qquad D = 0.5 L_r$$

Where L_r is the effective lag in minutes and R_r is the reaction rate in cm. per minute pon kg/cm.

2. Cohen and Coon Method : In 1953 Cohen and Coon published "Optimum" controller settings, which are corrections to the Ziolglor - Nichols formulas. They took into account the self - regulation of the process. This was done by introducing an index of self - regulation

$$u = \frac{R_r \; L_r}{n} ,$$

which can be determined from the process reaction curve. If the process has no self regulation, m is very large, and u = o. The cohn - Coon formulas, which are based on an analytical investigation, are the following

$$\text{Proportional response sensitivity} = \frac{1}{R_r L_r}\left(1 + \frac{1}{3}u\right)$$

proportional + reset response

$$\text{Sensitivity} = \frac{0.9}{L_r R_r} (1 + \frac{1}{11\,u}$$

$$\text{Reset rate} = \frac{0.3}{L_r}\left(\frac{1 + \frac{11}{5}u}{1 + \frac{1}{11}u} \right)$$

proportional + Derivative response

$$\text{Sensitivity} = \frac{1.2}{L_r R_r}\left(1 + \frac{1}{8}u \right)$$

$$\text{Derivative time} = 0.27\, L_r \frac{1 - \frac{1}{3}u}{1 + \frac{1}{8}u}$$

Proportional + Derivative responses for a non-interacting controller

$$\text{Sensitivity} = \frac{1.35}{L_r R_r} \left(1 + \frac{1}{5} u \right)$$

$$\text{Reset rate} = \frac{0.4}{L_r} \left(\frac{1 + \frac{3}{5} u}{1 + \frac{1}{5} u} \right)$$

$$\text{Derivative time} = 0.37 L_r \left(\frac{1}{1 + \frac{1}{5} u} \right)$$

When the process has no self - regulation, the expression in parentheses all reduces to unity, and the formulas give about the same settings as the Ziegle Nichols results.

Another method of Ziegler and Nichols consists of cutting out the reset and derivative actions and turning up the proportional sensitivity until continous cycling occurs. The ultimate values of proportional sensitivity and periodic so determined may be used as a basis for choosing good controller settings.

$$S = 0.6\, S_u \qquad I = \frac{2.0}{P_u}\,; \quad D = \frac{P_u}{8}$$

The ultimate values may also be obtained from the Bode plot of the open loop.

These methods are valuable guides to selecting good controller adjustments, because they are easy to apply if either the reaction curve has been run or the ultimate values have been found in one manner or the other.

3.2 <u>Procedure of Setting</u> :

Proportional reset rate controllers do not always have three separate adjustments. Two of these three adjustments may be synchronized so that the adjustment of only two is required. In some controllers a fixed rate time is incorporated so that it requires no adjustment. If the two adjustments are proportional band and reset rate, the procedure is as follows :

1. Set the reset rate to zero or to as low a value as possible.
2. Determine the proportional band.
3. Check the proportional band setting and while doing the note the period of cycling in minutes.

4. Set the reset rate to one divided by the period of cycling in minutes.
5. Again check the stability of control on a recovery and trim each adjustment as need.

If the separate adjustments are included, the procedure may be as follows :

1. Set the reset rate and rate time to zero or to as low value as possible.
2. Determine the optimum proportional Band and while doing so note the period of cycling in minutes.
3. Set the rate time to one eight of the cycling in minutes.
4. Reduce the proportional band by one fifth.
5. Check the setting and note the new period of cycling in minutes.
6. Set the reset rate to one divide by the new period of cycling in minutes as found in step 5.
7. Check these setting and trim if necessary.

DESIGN AND DEVELOPMENT OF ELECTRONIC p-i-d CONTROLLER

CHAPTER - IV

4.1 Introduction :

In early 1950s severly new types of electronic controllers have been developed which are fully competive with pneumatic instruments for process applications. The major adventages of these instruments are negligble transmission lag, the absence of dead zones due to friction or hystersis in moving parts, capability of operation at low temperature and compatability with other electrical devices such as electronic measurement transducers, data processing equipment and computers. The major disadvantage in process work is need for intrinsically safe electricel equipment in hazardous areas. However, now this problem has overcome. It is, however, usually necessary to convert the final electrical output of the instrument in to an air pressure signal to operate a conventional pnumatic diaphragm valve in most applications since there is, as yet, no really satisfactory electrically actuated equivalent to the diaphragm valve.

4.2 Design Considerations :

The importance of designing the control system to suit the characteristics of the process can not be overlooked. The selection of the controller depends on the operating requirements of the process and the tolerances permitted in the performance, e.g. the maximum off set, the minimum deviation and the maximum recovery time which can be allowed. If offset can not be tolerated, then the intergal action must be included in the controller. Since this is the only way of eliminating offset or reducing it to a negligible amount in the presence of significant changes in load.

Derivative action is generally called for when the process has a large number of storage elements and thus considerable transfer lag. If a small offset is not critical to the operation, then it may be possible to omit the integral action and the use of derivative will depend on the other factors and also on whether a small enough offset can be obtained from proportional control alone without increased gain usually available with the added derivative action.

4.3 Design :

There is no basic design of Electronic controller, although certain basic electrônic circuit can be identified in commercial instruments. The input and output signals are usually direct currents of a few milli-amperes in magnitude although no standard range has yet been agreed. The principal component of those instruments is high gain operational amplifier, with a gain usually

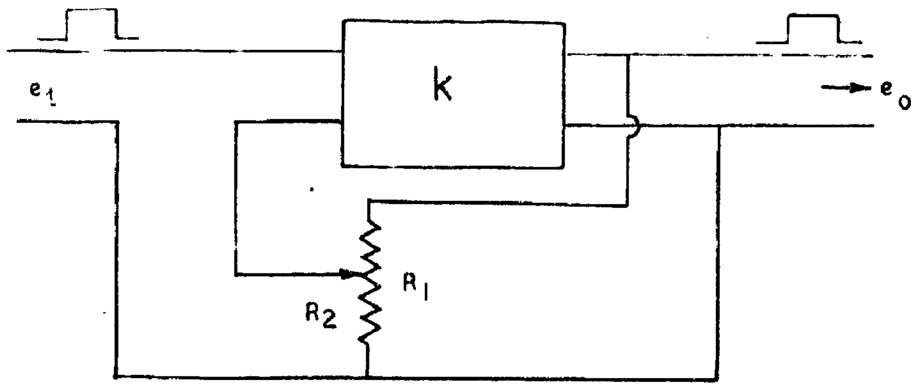
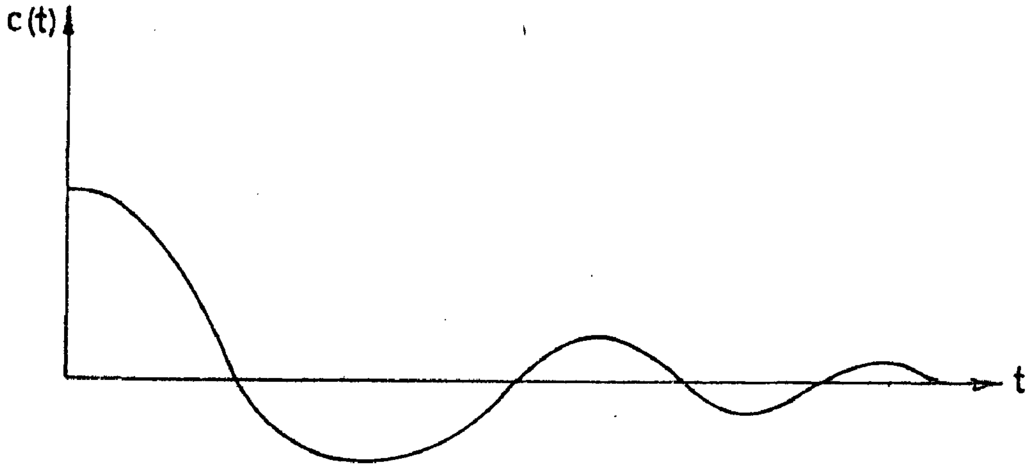
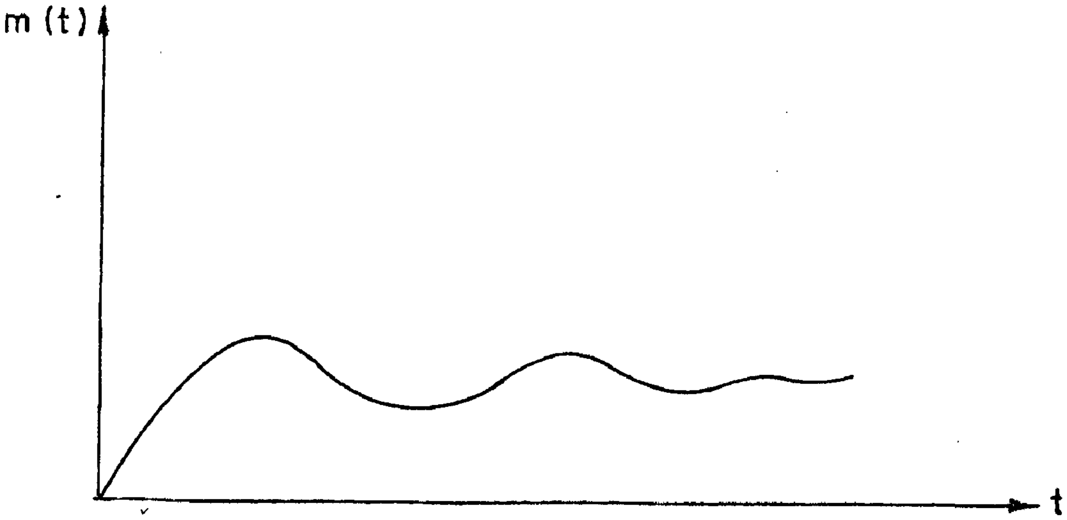


FIG. 7 PROPORTIONAL CONTROLLER



(a) ERRO SIGNAL



(b)

FIG. 8 OUTPUT SIGNAL FROM THE CONTROLLER exceeding 1,000:1. The gain of the proportional circuit does not depend on the gain of the amplifier but upon the ratio of the input and feedback resistors or capicitors and thus can be very accurately calibrated. The error signal is generated by passing the input current from a suitable measuring instrument through a resistor of about 500 ohm, so generating an input voltage which is subtracted from a desired value voltage set by a voltage dividing resistor.

1. Proportional Controllers : An electronic proportional controller is an amplifier which receives a small voltage signal and produces a voltage output at a higher power level. A schematic diagram of such a controller is shown in Fig.1.

for this controller

$$e_o = K\left(e_i - e_o \frac{R_2}{R_1} \right), \quad K \quad \frac{R_2}{R_1} \quad 1$$

Thus the transfer function of this controller is

$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{R_1}{R_2} = K_p$$

K_p is the gain of the proportional controller. The gain K_p can be adjusted by changing the ratio of resistance (R_1/R_2) in the feedback circuit.

2. Proportional-plus-Integral Electronic Controller : In the proportional control of a plant whose transfer function does not possesses an integral 1/s, there is a steady state error or offset in the response to a step input. Such an offset can be eliminated if the integral control is included in the controller.

In the intergal control of a plant, the control signal, the output signal from the controller, at any instant is the area under the actuating error signal curve up to that instant. The control signal m(t) can have non-zero value when the actuating error signal e(t) is zero, as shown in Fig. 2.

Fig. shows the principal of obtaining proportional plus integral control action in electronic controller. Essentially, we insert an appropriate circuit in the feed back path to generate the desired control action. The transfer function of the controller may be obtained as follow :

$$\frac{E_f(s)}{E_o(s)} = \frac{R_1 C_1 s}{R_1 C_1 s + 1} = \frac{R_1}{R_1 + \frac{1}{C_1 s}}$$

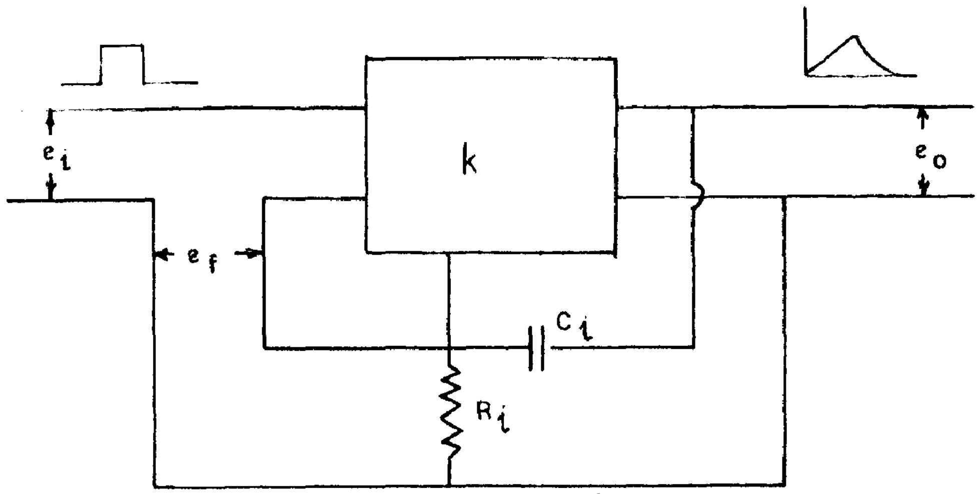


FIG. 9 PROPORTIONAL PLUS INTEGRAL CONTROL ACTION

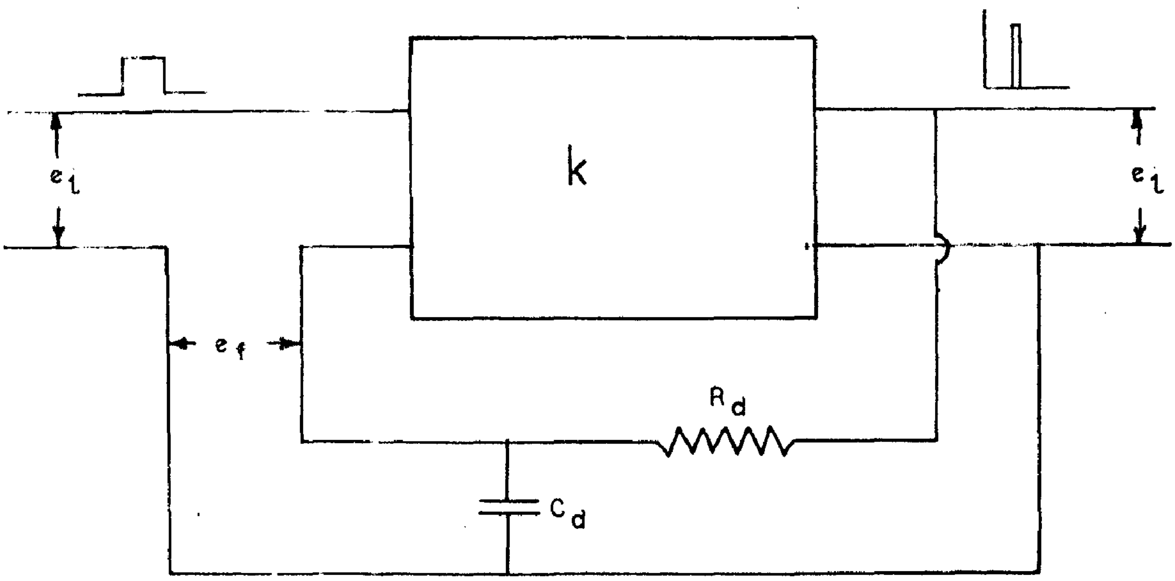


FIG. 10 PROPORTIONAL PLUS DERIVATIVE CONTROL ACTION

$$\left[E_i(s) - E_f(s) \right] K = E_o(s)$$

Hence, for $\left| \dfrac{KR_i C_i S}{R_i C_i S + 1} \right| \gg 1$

$$\frac{E_o(s)}{E_i(s)} = \frac{K(R_i C_i S + 1)}{K R_i C_i S + R_i C_i S + 1}$$

$$= \frac{R_i C_i S + 1}{R_i C_i S} = 1 + \frac{1}{T_i S}$$

Where $T_i = R_i C_i$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = 1 + \frac{1}{T_i S}$$

3. Proportional + Derivative Controller : Derivative control action, when added to a proportional controller,

provides a means of obtaining a controller with high sensitivity. An advantage of using derivative control action is that it responds to the rate of change of actuating error and can produce a significant correction before the magnitude of the actuating error becomes too large. Derivative control thus anticipates the actuating error, initiates an early corrective action and tends to increase stability.

Although derivative control does not effect the steady state errer directly it adds damping to the system and thus permits the use of a large value of the gain K, which will result in an improvement in the steady state accuracy.

Because derivative control operates on the rate of change of the actuating errer and not the actuating errer itself, therefore, this mode is never used alone. It is always used with proportional or proportional plus integral action.

Fig. shows the principal of obtaining proportional plus derivative action in electronic controller. Essentially we insert an appropriate circuit in the feedback path to generate the desired control action. The transfer function of the controller may be obtained as follow :

$$\frac{E_f(s)}{E_o(s)} = \frac{1}{R_d\ C_d\ S + 1}$$

$$\left[E_i(s) - E_f(s)\right] K = E_o(s)$$

Hence, for $\left|\dfrac{K}{R_d\ C_d\ S + 1}\right| \gg 1$

$$\frac{E_o(s)}{E_i(s)} = \frac{K\ (R_d\ C_d\ S + 1)}{R_d\ C_d\ S + 1 + K} = R_d\ C_d\ S + 1$$

$$\frac{E_o(s)}{E_i(s)} = T_d\ S + 1$$

Where $T_d = R_d\ C_d$

4. <u>Proportional-plus-Integral-plus-Derivative Controller</u> (p-i-d Controller) :

p-i-d controller is essentially a compromise between the advantages and disadvantages of p-i controller and advantages of p-d controller. There is no off set owing to the integral action, and the

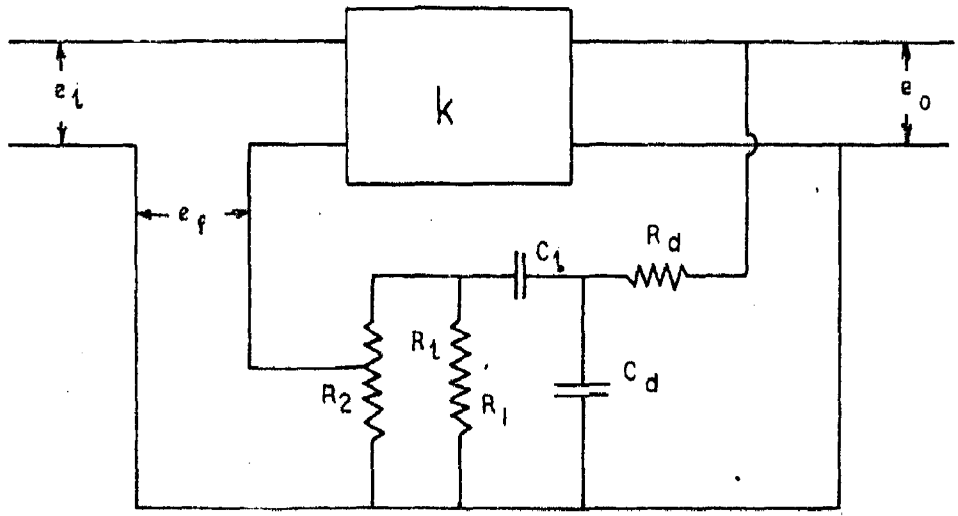


FIG. 11 (a) PROPORTIONAL PLUS INTEGRAL PLUS DERIVATIVE CONTROLLER

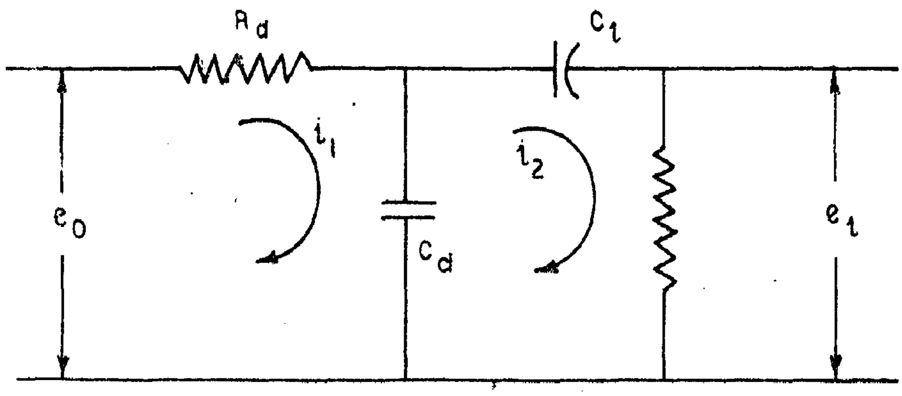


FIG. 11(b) EQUIVALENT CKT. DIAGRAM OF ABOVE CKT. FIG. 11(b) FOR ANALYSIS stabilizing effect of the derivative allows the gain to be increased, so reducing the maximum deviation and increasing the speed of response compared to 'p' and p-i controller. The destablizing effect of the integral action, however, will not permit such a large increase in gain as with p-d controller. Thus the maximum derivation and recovery time are not quite as good as with p-d controller.

Integral controller used alone has been omitted from these comparisons as this mode of control is best suited for processes with little or no capacitance. Feed - back circuit used in the controller is shown in Fig. Fig shows equivalent ckt. of Fig.

Applying Kirchoff's law to the above ckt. we get the equations for the above feedback circuit are

$$\frac{1}{C_d s}\left[I_1(s) - I_2(s)\right] + R_d\, I_1(s) = E_o(s) \qquad \ldots.(69)$$

$$\frac{1}{C_d\, s}\left[I_2(s) - I_1(s)\right] + \frac{1}{C_i\, s}\, I_2(s) + R_i I_2(s) = 0 \qquad \ldots..(70)$$

Simplifying eqn. (69) we get

$$I_1(s)\left[\frac{1}{C_d\ S} + R_d\right] = E_o(s) + \frac{I_2(s)}{C_d\ S} \qquad \ldots\ldots(71)$$

Substituting the value of $I_1(s)$ from Eqns. (71) in (70) we get

$$\frac{1}{C_d\ S}\ I_2(s) - \frac{\left(E_o(s) + \dfrac{I_2(s)}{C_d\ S}\right)}{\dfrac{1 + R_d\ C_d\ S}{C_d\ S}} +$$

$$\frac{1}{C_i\ S}\ I_2(s) + R_i\ I_2(s) = 0$$

$$\text{or}\quad I_2(s)\left[\frac{1}{C^d\ S} - \frac{1}{1 + R_d\ C_d\ S} + \frac{1}{C_i\ S} + R_i\right]$$

$$= \frac{E_o(s)}{1 + R_d\ C_d\ S}$$

$$I_2(s) \quad \frac{R_1\ C_1\ C_d\ S^2 + R_d\ C_d\ R_1\ C_1\ C_d\ S + C_d S + C_d^{\ 2}\ S^2}{C_d\ C_1\ S^2\ (\ 1 + R_d\ C_d\ S\ ).}$$

$$\therefore \quad \frac{I_2(s)}{E_o(s)} = \frac{C_1\ S}{R_1\ C_1\ R_d\ C_d\ S^2 + (R_1\ C_1 + R_d\ C_d + R_d\ C_1)\ S + 1}$$

or

$$\frac{E_1(s)}{E_o\ (s)} = \frac{R_1\ C_1\ S}{R_1\ C_1\ R_d\ C_d\ S^2 + (R_1\ C_1 + R_d\ C_1 + R_d\ C_d)\ S + 1}$$

.....(72)

from Fig. we also have

$$K(\ e_1 - e_f\ ) = e_o \quad \text{and} \quad e_f = e_1\ \frac{R_2}{R_1}$$

$\therefore$ we obtain,

$$\left[E_o(s) - \frac{R_2}{R_1} \left(\frac{R_1\ C_1\ S\ E_o(s)}{R_1\ C_1\ R_d\ C_d\ S^2 + (R_1\ C_1 + R_d\ C_d)\ S + 1} \right) \right] K$$

$$= E_o(s)$$

$\therefore$ Transfer function $\dfrac{E_o(s)}{E_i(s)}$ is

$$\frac{E_o(s)}{E_i(s)} = \frac{KR_1\left[R_1 C_1 R_d C_d S^2 + (R_1 C_1 + R_d C_1 + R_d C_d)S + 1\right]}{KR_2 R_1 C_1 S + R_1\left[R_1 C_1 R_d C_d S^2 + R_1 C_1 + R_d C_d \quad S + 1\right]}$$

.......(73)

If the loop gain is very much greater than unity, then this last equation may be simplified to give

$$\frac{E_o(s)}{E_i(s)} = \frac{R_1\left[R_1 C_1 R_d C_d S^2 + (R_1 C_1 + R_d C_1 + R_d C_d) S + 1\right]}{R_2 R_1 C_1 S}$$

$$= K_p \left[T_d S + \left(1 + \frac{R_d}{R_1} + \frac{T_d}{T_1}\right) + \frac{1}{T_1 S} \right]$$

where $K_p = \dfrac{R_1}{R_2}$ $\quad T_d = R_d C_d$ $\quad$ and $T_1 = R_1 C_1$

Definding $\alpha = 1 + \dfrac{R_d}{R_1} + \dfrac{T_d}{T_1}$

then
$$\frac{E_o(s)}{E_i(s)} = K_p \left(1 + \frac{T_d}{} S + \frac{1}{T_i S}\right) \quad \ldots.(74)$$

The value of depends upon the calibration of the potentiometer, cut off frequency etc. The above equation can be writen

$$\frac{E_o(s)}{E_i(s)} = K_p \left(1 + T_d S + \frac{1}{T_i S} \right) \quad \ldots..(75)$$

rewriting the control law equation (68),

$$\frac{q}{\theta} = 0.86 \left(1 + 4.6S + \frac{4.6}{S} \right) \quad \ldots...(68)$$

Now comparing the above two equations (75) & (68), we get

$$K_p = 0.86$$

$$T_d = 4.6$$

$$T_i = \frac{1}{4.6} = 0.22$$

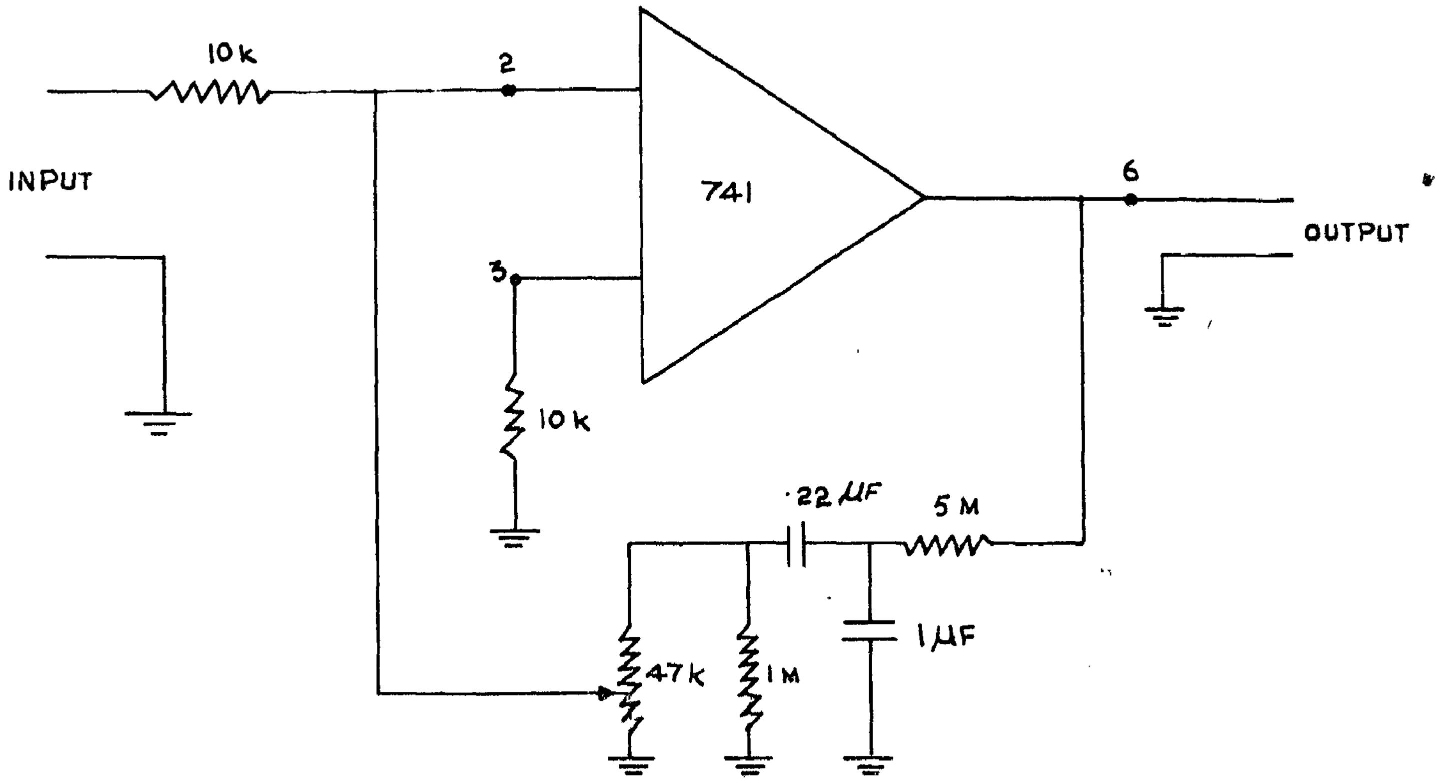
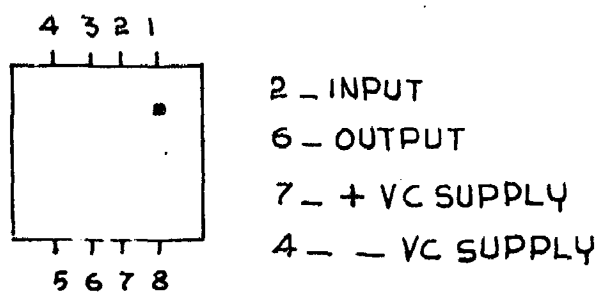


FIG.12. DESIGNED p-i-d CONTROLLER



OPERATIONAL AMPLIFIER _ BLOCK DIAG.

Now, if we take R_d = 5 M ohm then C_d 1 u F

R_1 = 1 M ohm then C_1 = .22 u F.

4.4 <u>Procedure of Setting and Testing</u> :

1. Set the derivative time to zero and the integral time to imfinity (as large as possible) by a loop gain of 5 and test the stability by moving the set point. Select a value of gain that gives a desired transient response.

2. Turn in derivative time and increase loop gain about 25%. Try several related values of derivative time and loop gain until a transient response with good stability and minimum period of oscillation is obtained.

3. Turn off the derivative action and decrease loop gain by 35% from last step. Set the integral time to give a good return on a load change.

4. Turn in derivative time to 30% more than the value set in 2. Set integral time to double that value found in step 3. Set the proportional sensitivity above 20% higher in step 3. Try several values near these settings to see if a better response can be obtained. use the highest derivative time possible because this will improve loop gain and provide smaller period of oscillation.

5. Take a print of the input output signal and compare it with theoretical one.

CONCLUSIONS

The prototype model of p-i-d controller is developed and tested in the Laboratory by giving the input from the signal generator and output of the controller was given to CRO. The output response of the controller was is displayed and noted down on CRO screen. The curve on CRO is found to be approximately same within the limit as theoretical one

However, the better response can be achieved by using computer. But in India, Factories can not bear this high expenditure.

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