OPTIMAL MAINTENANCE STRATEGIES FOR UNRELIABLE SYSTEMS

A DISSERTATION
submitted in partial fulfilment of
the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
SYSTEM ENGINEERING & OPERATIONS RESEARCH

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C E R T I F I C A T E

Certified that the dissertation entitled 'OPTIMAL MAINTENANCE STRATEGIES FOR UNRELIABLE SYSTEMS', which is being submitted by Sri N.V. Patharkar in partial fulfilment of the requirements for the award of degree of Master of Engineering (System Engineering and Operations Research) of University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of five months and twenty days from 5th February, 1977 to 25th July, 1977 for preparing this dissertation for the Master of Engineering degree, at this University.

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ABSTRACT

The high cost and extraordinary demands made of modern systems have provoked the development and implementation of maintenance policies for stochastically failing equipment. The practical need for maintenance policies has stimulated theoretical interest and in many cases has led to the development of policies that possess theoretical novalty and practical importance.

The aim of this dessertation is to find optimal maintenance policies for equipment subjected to breakdown over load and deterioration.

Chapter one deals with Introduction and Review of Literature.

Chapter two introduces optimal maintenance policy for equipment subject to breakdown or deterioration. Finite time horizon and Infinite time horizon, both have been considered and problems are solved.

Chapter three considers the case of optimal maintenance strategies when changes of states are semi-markovian. Only the catastropic failure of the equipment is considered and the solution of the problem is obtained.

In chapter four the maintenance stragegy for a cumulative damage model is discussed.

CONTENTS

1. INTRODUCTION AND REVIEW OF LITERATURE

1.1 INTRODUCTION

Man has been performing maintenance operations since the begining of time. In the past, human activities were largely accomplished through the use of an individual's own skills of physical capabilities. The body's built in process of self healing and regeneration considerably minimised any direct maintenance requirements. In recent times, the development of complex systems has tremendously expanded man's capabilities but it has also increased the maintenance requirements. In 1975, United States electric utilities spent over $ 3 billion for maintenance. This represented more than 100 percent increase in five years. Almost $ 2 billion was for generating plant maintenance. In addition to these direct out-of-pocket costs, hidden costs are generally incurred when generating units are unavailable.

The trend in the modern technology is to intensify the use of the plant and machinery. Production rate have increased and at the same time, due to extended operation, the oppertunities for maintenance have been reduced. In continuous or semicontinuous industrial process or flow-line production the failure of a unit is costly and some times dangerous. If some preventive measures are adopted, the failure can be avoided.

The equipment fails either due to the continuous deterioration or catastropic failure. In either case we have to choose some course of action. Thus the primary function of maintenance is to control the condition of equipment. Some of the problems associated with this include determination of

a) Inspection frequencies
b) Overhaul intervals i.e. part of a preventive maintenance policy.
c) Whether or not to do repairs i.e. a breakdown maintenance policy or not.
d) Replacement rules for components.
e) Replacement rules for capital equipment.
f) Maintenance crew sizes.
g) Spares provisioning rules.

Problems within these areas can be classed as being deterministic or probabilistic.

Deterministic problems are those in which the timing and the out-come of the maintenance action are assumped to be known with certainty. For example, we may have equipment which is not subject to failure but whose operating cost increases with use. To reduce this operating cost a replacement can be performed. See Fig.1.

Probabilistic problems are those where the timing and outcome of the maintenance action depend on chance. In the simplest situation the equipment may be described as being Good or Failed. The probability law describing changes from Good to Failed may be described by the distribution of time between completion of the maintenance action and failure. Determination of maintenance decisions involves a problem of decision under one main source of uncertainty namely : it is impossible to predict with certainty when a failure will occur or more generally when the transition from one state of the equipment to another will occur. A further source of uncertainty is that it may be impossible to determine the state of the equipment, either Good, Failed or some where between, unless a definite maintenance action is taken, such as inspection.

The first step for choosing the maintenance action, is to determine the objective of the study. Once the objective is determined, whether as maximise profit per unit time, minimize total maintenance cost, minimise downtime per unit

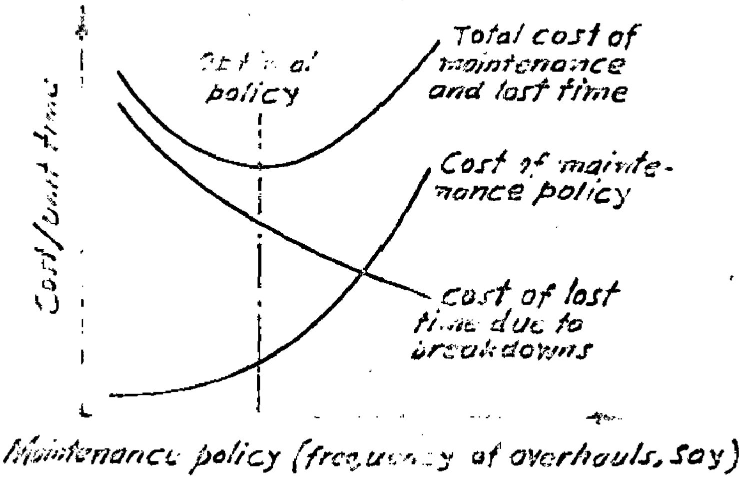

Total cost of maintenance and lost time
policy
Cost of mainte-nance policy
cost of lost time due to breakdowns
Cost/unit time
Maintenance policy (frequency of overhauls, say)

FIG. 2

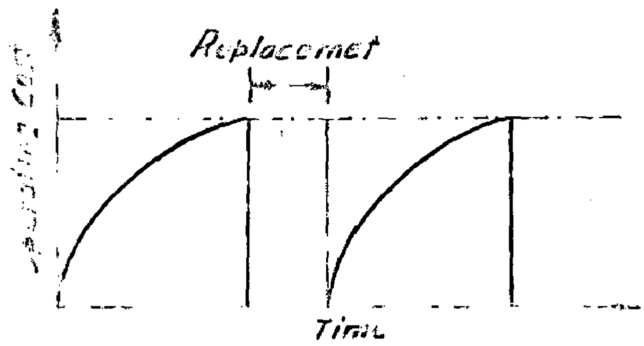

Replacemet
Time

FIG. 1.

time, etc., an evaluative mathematical model can be constructed which enables management to determine the best way to operate the system to achieve the required objective Fig. 2 illustrates the type of approach taken through using a mathematical model to determine the optimal frequency of overhauling a piece of plant by balancing the input (maintenance cost) of the maintenance policy against its output (reduction in downtime).

1.2 REVIEW OF LITERATURE

A commonly considered replacement policy is the policy based on age (age replacement). Such a policy is in force if a unit is always replaced at the time of failure or T hours after its installation, whichever occours first; T is a constant unless otherwise specified. If T is a random variable, we have the random age replacement policy. Campbell discussed the problem of group replacement to decide whether street lamps should be replaced regularly or as they failed.Under this policy the unit is replaced at time KT (K = 1, 2,....), and at failure. Clearly the cost per lamp of replacing all lamps at once is less than the cost of replacing each lamp as if fails. The cost of the additional lamps required for preventive maintenance must be balanced against the cost of the additional failures that occur if replacement is postponed.

The number of the situations in which either of the above policies can be applied are limited since both suffer serious drawbacks. The age-replacement policy cannot take advantage of periods when replacement is particularly cheap, and it is difficult to schedule several replacements at the same time. After an in service failure, the block- replacement policy may result in the replacement of a nearly new component. To over-come this last drawback, Crookes[c] introduced policy which, if replacement opportunity was imminent, meant that a component failing in service was not replaced and the system remained idle. The effect of the policy is to reduce the number of replacements, at the expense of idle time. Woodman[16] considered a method by which the above drawback can be avoided. In fact the policy that was developed include the age and block replacement policies as special cases. The improvement is brought about by the simple expedient of replacing a component after a failure, but not always replacing it when a replacement opportunity occurs. Such policies, where replacement is not obligatory at every opportunity, are called optional policies.

Bartholomew[3] developed a two-stage replacement strategies model. In this policy, the failures in one group are replaced by new items and those in the other group are replaced by items already operating in the first group. But this two stage replacement strategy is of limited advantage. A generalised theory of multi-stage replacement strategies is discussed by Naik and Nair[12]. They showed it to be a more economical in many practical cases and with the aid of this policy, it is possible to arrive at the optimum number of stages.

Thompson[14] developed the model for the gradual deterioration of a system of machines with age, which can be partly offset via preventive maintenance. He considered the salvage value of the machine at the time t, S(t) satisfying the differential equation

$$dS(t)/dt = -d(t) + f(t)u(t)$$

where d(t) is the obsolescence function (in dollars) substracted from S at time t.

f(t) is Maintenance effectiveness function (in dollars) at the time t added to S per doller spent on maintenance.

u(t) is the maintenance function (in dollars) satisfying the constraint $0 \leq u(t) \leq U$

The performance index was to optimise the discounted profit during the life of the machine plus the discounted salvage value at time T, where T is the sale date of the machine.

This model has two drawbacks-

1) The simple linear equation of the model may not completely represent the observed economic behaviour.

(2) Extension of Thomson's model for a single machine to a system of machines involve the assumption that all machines are brought at the same time. If they are brought at different times, the above equation may be written for separate machines to determine individual preventive maintenance policies. Otherwise a policy of the form $u(t) = 1$, for all machines give rise to the problem of maintenance allocation. Avoiding the above drawbacks, Sarma and Alam[1] applied the optimal control theory to consider the effect of deterioration and intermittent breakdowns on the maintenance policies. But the maintenance strategy shown by Sarma and Alam, is not also practicable. According to them, the maintenance should be continuous over some period, and afterwords there should be no maintenance action.

A replacement problem for a cumulative damage model was discussed by Toshio Nakagawa[15]. Hanon Luss[9] deals with the maintenance policies when deterioration can be observed by inspections. He developed a markovian model in which the holding times in various states are exponentially distributed. Several papers deal with models where deterioration can be observed. However most of them concentrate on replacement policies that assume that the system state is always known. Kao[11] studied optimal replacement rules when changes of state are semi-markovian. To consider all the possible maintenance actions i.e. overhaul, repair, replace, no action for a model is a must now-a-days.

2. OPTIMAL OVERHAUL/ REPAIR/ REPLACE MAINTENANCE POLICY FOR EQUIPMENT.

An attempt has been made to determine a decision rule be taken to minimise the total cost of maintenance and lost production over Finite time and Infinite time horizon.

2.1 INTRODUCTION

An overhaul is a restrorative maintenance action which is taken before an equipment has reached a defined failed state, while a repair is made after the failed state has occurred. It should be noted that the failed state does not necessarily mean that equipment has 'broken down' in the usual sense that it ceases completely to function but it may be in a failed state because items, say, are being produced outside specific tolerance limits. Overhaul and

Repair both improve the condition of the equipment but there is a gradual deterioration over time and then replacement of the complete equipment is made. Fig. 3 illustrates the usual consequences of overhauls and repairs. Jardine[10] discussed such maintenance policies without the discount factor. Optimum policy depends on the discount rate. In determining the optimum operating policy for an operational system, it is often desirable to discount rewards received in the future. Unfortunately the magnitude of this discounting is usually not known preciously, and the question arises; How is the optimum policy affected by variations in discount rate?. The optimum policy and its variations with discount rate will depend, of course, on the mathematical model used to describe the system

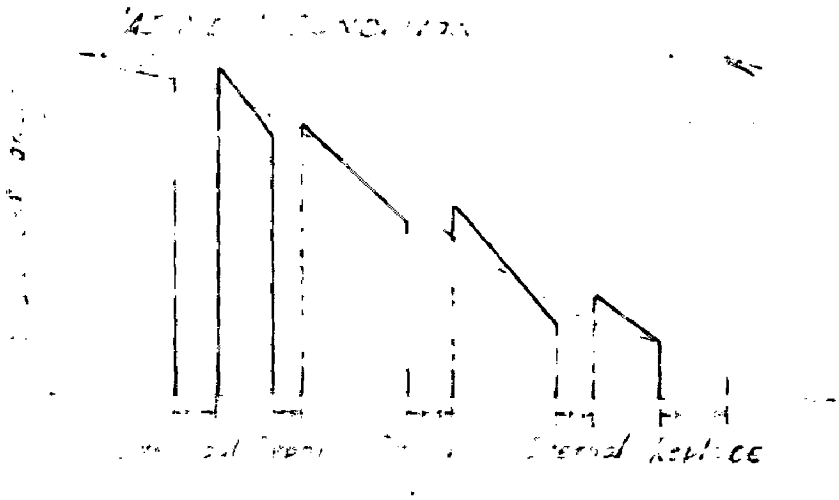
2.2 MODEL I - EQUIPMENT SUBJECT TO BREAKDOWN : DISCOUNTED COST WITH FINITE TIME HORIZON.

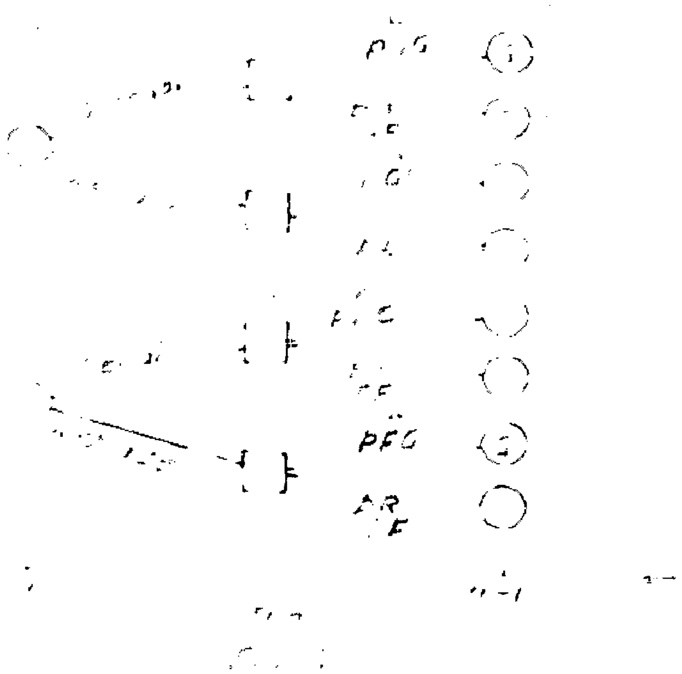
2.2.1 Assumptions

1. Overhaul or repair action does not return equipment to as new condition.
2. The decisions can only be made at discrete points in time.

2.2.2 Formation of Problems

1. I is the state of the equipment (Good or Failed) at the start of the period.





2. J is the state of the equipment (Good or Failed) at the end of a period.
3. "a" is the action which is taken at the start of a period (in this case overhaul, repair or replace).
4. $p^{a}IJ$ is the probability that the equipment will go from state I to J in one period if action "a" is taken.
5. $C^{a}IJ$ is the cost per period of going from state I to state J if action "a" is taken. In this case this will be the cost of overhaul C_o, repair C_r, or replacement C_R, and a cost C_L associated with lost production if equipment fails during the period.
6. N is the number of possible states.
7. α is the discount factor.
8. fn(I) is the minimal expected total discounted cost, with n periods to go and starting in state I.
9. The objective is to determine a combined overhaul/repair/replace policy to minimise the total present value cost associated with these actions, and any consequential production losses, over the next n periods of time.

The cost of the first decision, at the start of the n^{th} period is $C^{a}IJ$ if action "a" is taken and we result in state J. But we would only result in the state J with probability $p^{a}IJ$. There are a number of results that could occur if action "a" is taken, therefore the expected cost resulting from action "a" is

$$\sum_{J=1}^{N} C^{a}IJ\ p^{a}IJ$$

where N is the number of possible states at the end of a period.

At the end of period we are in state J, with (n-1) periods to operate. The minimal total expected discount cost over this remaining time is $f_{n-1}(J)$. Again, the equipment is brought in state J with probability $p^{a}IJ$ and therefore expected discounted cost is

$$\alpha \sum_{J=1}^{N} p^{a}IJ\ f_{n-1}(J)$$

Thus, starting in state I, with n periods to go, taking action a and resulting in state J, the expected total discount cost over n periods is,

Expected cost of first decision + Expected present value cost

$$= \sum_{J=1}^{N} C^{a}IJ\ p^{a}IJ + \alpha \sum_{J=1}^{N} p^{a}IJ\ f_{n-1}(J) \qquad (2.1)$$

Since, we wish to minimise the expected total present value cost, we wish to take the best action "a" when in state I with n periods to go. The best action is that one which minimises equation (2.1). The resulting minimal total expected present value cost $f_n(I)$ and best action "a" can be obtained from the following recurrence relation

$$f_n(I) = \min_a \left[\sum_{J=1}^{N} C^a_{IJ}\, p^a_{IJ} + \alpha \sum_{J=1}^{N} p^a_{IJ}\, f_{n-1}(J) \right] \quad n \geq 1 \tag{2.2}$$

Equation (2.2) can be solved recursively with the starting condition $f_0(I) = 0$, then

$$f_1(I) = \min_a \left[\sum_{J=1}^{N} C^a_{IJ}\, p^a_{IJ} \right] \tag{2.3}$$

2.2.3 Numerical Example

Let a piece of equipment whose performance at any time can be characterised by only two states, i.e. N = Good (G) or Failed (F). There are three possible actions which can be taken, i.e. a = overhaul (o), repair (r), replace (R). If the equipment is in condition G, it can either be overhauled or replaced. If the equipment is in condition F, then it can either be repaired or replaced. See Fig. 4. The appropriate transition probabilities and the cost per period are given in the table 2.1. We want to determine the optimal maintenance policy such that the expected total present value cost over four future periods of time is minimised. Assume the discount factor of 0.8.

Table - 2.1

Condition at the start of period	Decision	Condition at end of period: Good	Condition at end of period: Failed	Cost per period condition at end of period: Good	Cost per period condition at end of period: Failed
Good	Overhaul	$p^{o}_{GG} = 0.75$	$p^{o}_{GF} = 0.25$	$C^{o}_{GG} = 200$	$C^{o}_{GF} = 1200$
	Replace	$p^{R}_{GG} = 0.95$	$p^{R}_{GF} = 0.05$	$C^{R}_{GG} = 500$	$C^{R}_{GF} = 1500$
Failed	Repair	$p^{r}_{FG} = 0.60$	$p^{r}_{FF} = 0.40$	$C^{r}_{FG} = 100$	$C^{r}_{FF} = 1100$
	Replace	$p^{R}_{FG} = 0.95$	$p^{R}_{FF} = 0.05$	$C^{R}_{FG} = 500$	$C^{R}_{FF} = 1500$

Equation (2.3) is

$$f_1(I) = \min_a \left| \sum_{J=1}^{N} C^a_{IJ} \, P^a_{IJ} \right|$$

when I = G, with one period to go, there are two possible maintenance actions :

$$f_1(G) = \min \left| \begin{array}{ll} \sum_{J=1}^{N} C^O_{GJ} \, P^O_{GJ} & \text{Overhaul} \\ \sum_{J=1}^{N} C^R_{GJ} \, P^R_{GJ} & \text{Replace} \end{array} \right|$$

considering the decision to overhaul, then :

$$\sum_{J=1}^{N} C^O_{GJ} \, P^O_{GJ} = 450$$

considering the decision to replace, then :

$$\sum_{J=1}^{N} C^R_{GJ} \, P^R_{GJ} = 550$$

$$\therefore \quad f_1(G) = \min \left| \begin{array}{c} 450 \\ 550 \end{array} \right| = 450$$

and so the best decision for minimizing total expected present value cost is to overhaul.

When I = F, with one period to go :

$$f_1(F) = \min \begin{vmatrix} \sum_{J=1}^{N} C^r_{FJ}\, P^r_{FJ} & \text{repair} \\ \sum_{J=1}^{N} C^R_{FJ}\, P^R_{FJ} & \text{replace} \end{vmatrix}$$

$$= \min \begin{vmatrix} 500 \\ 550 \end{vmatrix} = 500, \ \text{repair}$$

With two periods of time to go, equation (2.2) becomes :

$$f_2(I) = \min_{a} \left[\sum_{J=1}^{N} C^a_{IJ}\, P^a_{IJ} + \alpha \sum_{J=1}^{N} P^a_{IJ} f_1(J) \right]$$

when $I = O$, with two periods to go :

$$f_2(O) = \min \begin{vmatrix} C^o_{OO}P^o_{OO} + C^o_{OF}P^o_{OF} + \alpha P^o_{OO}f_1(O) + \alpha P^o_{OF}f_1(F) \\ C^R_{OO}P^R_{OO} + C^R_{OF}P^R_{OF} + \alpha P^R_{OO}f_1(O) + \alpha P^R_{OF}f_1(F) \end{vmatrix}$$

$$= \min \begin{vmatrix} 820 \\ 952 \end{vmatrix} \ \text{Overhaul}$$

when $I = F$, with two periods to go

$$f_2(F) = \min \begin{vmatrix} 876 \\ 952 \end{vmatrix} \ \text{Repair}$$

Table 2.2 is ~~W~~ constructed for values of n upto 4.

Table - 2.2

DISCOUNT FACTOR $\alpha = 0.8$

Periods to go : n	4		3		2		1	
State of Equipment at start of period I	G	F	G	F	G	F	G	F
Action to take at start of period	Overhaul	Repair	Over-haul	Repair	Over-haul	Repair	Over-haul	Repair
Expected present value cost	1354.9	1412	1117	1173.9	820	876	450	500

It is found that even if we change the discount factor from 1 to .1, the same optimal decisions as obtained above, should be taken.

2.3 MODEL II - EQUIPMENT SUBJECT TO BREAK DOWN : DISCOUNTED COST WITH INFINITE TIME HORIZON.

2.3.1 Assumptions - As in the section 2.2.1

2.3.2 Formulation of Problem

1. The parameters I, J, a, p^a_{IJ} , C^a_{IJ}, N and α are as defined in section 2.2.2.
2. g is the long run average maintenance cost.
3. V(I) is the cost which depends on the state of the equipment at the start of the operation.
4. We wish to determine the optimal maintenance policy over a long period of time. So our objective is to minimise expected total discounted cost per unit time.

$f_n(I)$, the minimal total expected discounted cost over the n periods of time then, as $n \to \infty$

$$f_n(I) \to ng + V(I)$$

i.e. $f_n(I)$ is composed of two parts - a steady state part ng and a transient part V(I) which depends upon the starting state.

Therefore for sufficiently large n,

$$f_n(I) = \min_a \left[\sum_{J=1}^{N} C^a{}_{IJ}\, p^a{}_{IJ} + \alpha \sum_{J=1}^{N} p^a{}_{IJ} f_{n-1}(J) \right]$$

From eqn. (2.2)

$$= ng + V(I)$$

Now, $f_{n-1}(J) = (n-1)g + V(J)$ and therefore

$$ng + V(I) = \min_a \left[\sum_{J=1}^{N} C^a{}_{IJ} p^a{}_{IJ} + \alpha \sum_{J=1}^{N} p^a{}_{IJ} \cdot (n-1)g + \alpha \sum_{J=1}^{N} p^a{}_{IJ}\, V(J) \right]$$

i.e. $g(n + \alpha - \alpha n) + V(I)$

$$= \min_a \left[\sum_{J=1}^{N} C^a{}_{IJ}\, p^a{}_{IJ} + \alpha \sum_{J=1}^{N} p^a{}_{IJ}\, V(J) \right] \quad (2.4)$$

Since $\sum_{J=1}^{N} p^a{}_{IJ} = 1$

Expression (2.4) is a system of N equations in N+1 unknowns.

2.3.3 SOLUTION TECHNIQUE

The solution of (2.4) can be obtained by using the algorithm developed by Howard[7]. According to this algorithm

i) Choose some arbitrary policy

ii) If there are N possible states, let $V(N) = 0$

iii) Solve the N equations of expression (2.4) to give the average long-term gain g and the relative values V(I) of various starting states.

iv) For each condition, I, and using V(I) values obtained in step (iii), find the alternative, a, which minimises

$$\sum_{J=1}^{N} C^{a}_{IJ}\, p^{a}_{IJ} + \alpha \sum_{J=1}^{N} p^{a}_{IJ}\, V(J) \qquad (2.5)$$

v) Using the policy obtained in step (iv) repeat the step (iii) until the optimal policy is determined. This is specified when g is minimized and will be the case when the policies on two successive iterations are identical.

2.3.4 Example

Consider the same numerical example given in the article 2.2.3.

Using Howard's Algorithm,

i) Let us choose the following policy.
If the equipment is in state G at the start of the period, then replace.
If the equipment is in state F at the start of period, then replace.

ii) There are two possible conditions G and F, so let V(F) = 0.

iii) For $n = 1$, expression (2.4) becomes

$$g(1 + 0.8 - 0.8) + V(G) = 550 + 0.8 \times 0.95 \times V(G) + 0.8 \times 0.05\, V(F)$$

$$g(1 + 0.8 - 0.8) + V(F) = 550 + 0.8 \times 0.95\, V(G) + 0.8 \times 0.05\, V(F)$$

which gives B $V(G) = 0$, $g = 550$

iv) For each condition find the best alternative using equation (2.5)

If equipment is in condition G, (2.5) becomes :

$$\min \begin{vmatrix} 450 + 0.8\,(0.75 \times 0 + 0.25 \times 0) \\ 550 + 0.8\,(0.95 \times 0 + 0.05 \times 0) \end{vmatrix}$$

$$= \min \begin{vmatrix} 450 \\ 550 \end{vmatrix} \leftarrow \text{Overhaul}$$

If in condition F, (2.5) becomes

$$\min \begin{vmatrix} 500 + 0.8\,(0.6 \times 0 + 0.4 \times 0) \\ 550 + 0.8\,(0.95 \times 0 + 0.05 \times 0) \end{vmatrix}$$

$$= \min \begin{vmatrix} 500 \\ 550 \end{vmatrix} \leftarrow \text{Repair}$$

Therefore, at the end of first iteration the new policy is :

If in condition G at start of period then overhaul.

If in condition F at start of period then repair.

v) Using above policy solve expression (2.4) using $V(F) = 0$

$$g(2 + 0.8 - 1.6) + V(G) = 450 + 0.8 \times 0.75\, V(G) + 0.8 \times 0.05\, V(F)$$

$$g(2 + 0.8 - 1.6) + V(F) = 500 + 0.8 \times 0.6\, V(G) + 0.8 \times 0.4\, V(F)$$

which gives $V(G) = -56.8$ and $g = 427.28$

vi) For each condition find best alternatives using the values of $V(G)$ and g obtained in previous step, and using (2.5).

If in condition G, (2.5) becomes

$$\min \begin{vmatrix} 450 + 0.8\ (0.75(-56.8) + 0.25 \times 0) \\ 550 + 0.8\ (0.95(-56.8) + 0.05 \times 0) \end{vmatrix}$$

$$= \min \begin{vmatrix} 415.9 \\ 506.9 \end{vmatrix} \leftarrow \text{Overhaul}$$

If in condition F, (2.5) becomes

$$\min \left| \begin{array}{l} 500 + 0.8(0.6(-56.8) + 0.25 \times 0) \\ 550 + 0.8(0.95(-56.8) + 0.05 \times 0) \end{array} \right|$$

$$= \min \left| \begin{array}{c} 472.8 \\ 506.9 \end{array} \right| \leftarrow \text{repair}$$

Therefore, at the end of second itemation the new policy is :

If in condition G at start of period then overhaul

If in condition F at start of period then repair.

Thus the optimal decisions have been determined since the same policy has been obtained on two successive iterations.

The resulting average cost per period is 427.28. In the above problem, by changing the values of discount factor, we get the same policy.

2.4 MODEL III - EQUIPMENT SUBJECT TO DETERIORATION : FINITE TIME HORIZON

2.4.1 Assumptions

1. The deterioration of the equipment can be inspected and measured.
2. The state transition probabilities can be obtained from the failure data and operating experience.
3. See the assumptions in 2.2.1

2.4.2 Formulation of Problem

1. $I = 0, 1, 2, \ldots N$ represents the state of the equipment at the start of period. State " 0 " means the equipment is in good state " N " denotes the failed states.
2. J is the state of teh equipment at the end of period. $J = 0, 1, \ldots\ldots, N$.
3. The parameters a, P^a_{IJ}, C^a_{IJ} as defined in section 2.2.2
4. $f_n(I)$ is the minimal expected total cost with n periods to go and starting in state I.
5. The objective is to determine a combined overhaul/ repair/ replace policy to minimise the total cost associated with these actions over the next n periods of time.

   As before discussed in the Model, the recurrence relation can be written as

$$f_n(I) = \min_a \left[\sum_{J=0}^{N} C^a_{IJ} \; P^a_{IJ} + \sum_{J=0}^{N} P^a_{IJ} f_{n-1}(J) \right] \quad n \geq 1.$$

2.4.3 Numerical Example

Let a piece of equipment whose performance at any time can be characterized by 4 states i.e. $N = 0, 1, 2, 3$. " 0 " means the equipment is in good state, " 3" means it is in failed state. 1 and 2 denotes the deterioration states.

The appropriate transition probabilities are given in Table 2.3. The cost per period is given in Table 2.4. The optimal decisions obtained are as shown in Table 2.5 for the values of n upto 3.

Table 2.3

Condition at start of period	Decision	Condition at the end of period 0	1	2	3
0	Overhaul	0.70	0.15	0.1	0.05
	Replace	0.95	0.03	0.02	0.01
1	Overhault	0.65	0.2	0.1	0.05
	Replace	0.95	0.03	0.02	0.01
2	Overhaul	0.60	0.25	0.1	0.05
	Replace	0.95	0.03	0.02	0.01
3	Repair	0.65	0.2	0.1	0.05
	Replace	0.95	0.03	0.02	0.01

Table 2.4

Cost per period C^{a}_{IJ}

Condition at start of period	Decision	Condition at end of period 0	1	2	3
0	Overhaul	$C^{O}_{00} = 200$	$C^{O}_{01} = 500$	$C^{O}_{02} = 700$	$C^{O}_{03} = 1200$
	Replace	$C^{R}_{00} = 500$	$C^{R}_{01} = 700$	$C^{R}_{02} = 1000$	$C^{R}_{03} = 1500$
1	Overhaul	$C^{O}_{10} = 300$	$C^{O}_{11} = 600$	$C^{O}_{12} = 800$	$C^{O}_{13} = 1300$
	Replace	$C^{R}_{10} = 500$	$C^{R}_{11} = 800$	$C^{R}_{12} = 1100$	$C^{R}_{13} = 1500$
2	Overhaul	$C^{O}_{20} = 400$	$C^{O}_{21} = 700$	$C^{O}_{22} = 900$	$C^{O}_{23} = 1400$
	Replace	$C^{R}_{20} = 500$	$C^{R}_{21} = 800$	$C^{R}_{22} = 1100$	$C^{R}_{23} = 1500$
3	Repair	$C^{r}_{30} = 450$	$C^{r}_{31} = 750$	$C^{r}_{32} = 950$	$C^{r}_{33} = 1450$
	Replace	$C^{R}_{30} = 500$	$C^{R}_{31} = 800$	$C^{R}_{32} = 1100$	$C^{R}_{33} = 1500$

Table 2.5

Periods to go : n	3				2				1			
State of equipment at start of period I	0	1	2	3	0	1	2	3	0	1	2	3
Action to take at start of period	Overhaul	Overhaul	Replace	Replace	Overhaul	Overhaul	Replace	Replace	Overhaul	Overhaul	Replace	Replace
Expected future cost : $f_n(I)$	1123.03	1244.35	1268.97	1268.97	735.9	856.4	894.93	894.93	345	460	536	536

2.5 MODEL IV - EQUIPMENT SUBJECT TO DETERIORATION : INFINITE TIME HORIZON

With the assumptions made in the model 3, and following the formulation procedure given in the model no.2, we get the equation

$$g + V(I) = \min_{a} \left[\sum_{J=1}^{N} C^{a}_{IJ} P^{a}_{IJ} + \sum_{J=1}^{N} P^{a}_{IJ} \, V(J) \right]$$

The problem given in the section 2.4.3 is solved by the Howard's Algorithm outlined in the model no. II.

The initial policy chosen was -

1. If equipment in state 0 at start of period then overhaul
2. If in state 1 at start of period then overhaul.
3. If in state 2 at start of period then replace.
4. If in state 3 at start of period then replace.

The same policy has been obtained on two successive iterations showing these to be the optimal decisions.

The resulting average cost per period is 386.95.

3. OPTIMAL MAINTENANCE STRATEGIES WHEN CHANGES OF STATE ARE SEMI - MARKOVIAN

The maintenanse strategies for the equipment subject to catastropic failure has been discussed. We consider a semi-Markovian model in which the holding times in the various states are assumed exponentially distributed. The optimization criterion is the expected average cost per unit time.

3.1 INTRODUCTION

In the past decade a substantial body of literature on the maintenance of stochastically failing system has been developed. There exists a specific class of problems that are generally be referred to as Markovian Replacement models. Under the Markovian formulation, the deterioration process of a system as indicated by the change of underlying states, is represented by the transition probability matrix of a Markov-chain.

In using a Markov chain to model a multistate deterioration process, we assume that the performance of the system does not age i.e. the probability of making a transition to a less desirable state does not increase in time. However, the aging phenomenon is observed in practice. One way to overcome this difficulty is through the use of semi-markov process. Semimarkovian process can be applied to model systems that exihibit two different types of degradation. (1) Gradual deterioration (2) catastropic failure. This class of problems has been discussed by Barlow and Proschan[2], and Kao[11]. Kao took into consideration only replacement policy for gradually deteriorating unit.

3.2 DISCRETE-TIME SEMI-MARKOV PROCESS

The Markov model has the property that a transition is made at every time instant. There exist a general class of processes where the time between transitions may be several of the unit-time intervals, and where this transition time can depend on the transition that is made. This process retains enough of the Markovian properties to deserve the name of a "Semi-Markov" process. The stay in any state is described by an integer valued random variable that depends on the state presently occupied and on the state to which the next transition will be made. This process forms a convenient formulation for physical systems.

3.2.1 Holding times and Waiting times

Whenever a process enters a state i, it determines the next state j to which it will move according to state i's transition probabilities $p_{i1}, p_{i2}, \ldots, p_{iN}$. However, after j has been selected but before making this transition from state i to state j, the process "holds" for a time τ_{ij} in state i.

Thus, holding times τ_{ij} are positive, integer-valued random variables, each governed by a probability mass function $h_{ij}(\cdot)$ called the holding time mass function for a transition from state i to state j. Thus,

$$P\left\{\tau_{ij} = m\right\} = h_{ij}(m), \qquad \begin{array}{l} m = 1, 2, \ldots \\ i = 1, 2, \ldots, N \\ j = 1, 2, \ldots, N, \end{array}$$

we assume that the mean $\bar{\tau}_{ij}$ of all holding time distributions are finite and that all holding times are at least one time unit in length

$$h_{ij}(0) = 0$$

we shall find it useful to develop additional notation for the holding time behaviour. We use $\leq h_{ij}(\cdot)$ for the cumulative probability distribution of τ_{ij},

$$\leq h_{ij}(n) = \sum_{m=0}^{n} h_{ij}(m) = P\left\{\tau_{ij} \leq n\right\}.$$

and $> h_{ij}(\cdot)$ for the complementry cumulative probability distribution of τ_{ij}.

$$> h_{ij}(n) = \sum_{m=n+1}^{\infty} h_{ij}(m) = 1 - \leq h_{ij}(n)$$

$$= P\left\{\tau_{ij} > n\right\}$$

Suppose that now the process enters state i and chooses a successor state j, but we as observers do not know the successor chosen. The probability mass function assign to the τ_i spent in i, we shall then call $w_i(.)$.

$$w_i(m) = \sum_{j=1}^{N} p_{ij}\, h_{ij}(m) = P\left\{\tau_i = m\right\}$$

That is, the probability that the system will spend m time units in state i if we do not know its successor state is the probability that it will spend m time units in state i if its successor state j is multiplied by the probability its successor state is j and summed over all possible successor states.we call τ_i, the waiting time in state i, and $w_i(.)$ the waiting time probability mass function. The mean waiting time $\overline{\tau}_i$ is related to the mean holding time $\overline{\tau}_{ij}$ by

$$\overline{\tau}_i = \sum_{j=1}^{N} p_{ij}\, \overline{\tau}_{ij}$$

3.2.2 <u>State Occupancy Costs</u>

When the system is holding in state i, an occupancy cost of A_i units is incurred per unit time. In particular, A_0 may be considered as the maintenance cost per unit time of a new system and A_L the cost of system failure.

3.2.3 Cost Rates

Let r_i^a denote the expected cost per occupancy of state i while decision a is in force. This is related to the fixed cost, variable cost, and mean waiting time as

$$r_i^a = \text{Fixed cost} + \text{Variable cost} \times \text{Mean waiting time.}$$

The cost rate q_i^a is defined as the expected cost per unit time in state i while decision a is being executed and can be found from

$$q_i^a = \frac{r_i^a}{\bar{\tau}_i^a}$$

3.3 DEVELOPMENT OF MODEL

1. Let the performance of the equipment is characterised by a finite number of states G and F. State G means the equipment is in Good state and F means the equipment is in failed state.
2. The change of state is assumed to follow a discrete time semi-markov process.
3. Possible maintenance action a is overhaul or Replace or Repair.
4. p_{ij}^a denotes the probability of transition from i^{th} state to j^{th} state, when action a is taken.

   $i = G, F.$ $j = G, F$
5. Let τ_{ij}, a integer random variable be the holding time

in the state i before making transition to state j. The corresponding holding time mass function is $h_{ij}(.)$.

6. $\overline{\tau}_i$ denotes the mean waiting time in state i.
7. When the maintenance action is chosen as the replacement, we replace the system with a fixed cost of R_i units. This fixed cost of R_i may be the purchase cost of the new equipment minus the salvage value of the old equipment. Similarly let O_i units and E_i units represent the fixed cost for overhaul and repair respectively. Let a variable cost D_i units per unit time is needed for each maintenance action. This variable cost D_i may consist of the cost of lost production.
8. For our model, r_i^a the expected cost per occupancy of state i is,

$$r_G^o = O_G + D_G \cdot \overline{\tau}_G^o \qquad \text{For overhaul}$$

$$r_G^R = R_G + D_G \, \overline{\tau}_G^R \qquad \text{For replace}$$

$$r_F^r = E_F + D_F \, \overline{\tau}_F^r \qquad \text{For repair}$$

9. It is assumped that the maintenance in the form of overhaul or repair does not return equipment to the as new-state.
10. The objective is to minimise the expected average maintenance cost per unit time.

If V_i is the cost which depends on the state of the equipment at the start of operation, g is the average maintenance cost per unit time, the relative value equation for the above defined process can be written (see Howard pp 869)

$$V_i + g\,\bar{T}_i^{a} = r_i^{a} + \sum_{j=1}^{N} p_{ij}^{a}\;V_j \qquad (3.1)$$

To find the optimal maintenance strategy we use the following solution technique.

3.4 SOLUTION TECHNIQUE

Suppose, we use an arbitrary stationary policy and find the gain and relative values of this policy by solving equation (3.1)

$$V_i + g\,\bar{T}_i = r_i + \sum_{j=1}^{N} p_{ij}V_j \quad , \quad i = 0,F$$

With the relative value of state N, equal to zero.

We then solve equation (3.1) for g.

$$g = \frac{r_i^{a}}{\bar{T}_i^{a}} + \frac{1}{\bar{T}_i^{a}}\left[\sum_{j=1}^{N} p_{ij}V_j - V_i\right] \qquad i = 0,\ F$$

or in terms of the cost rate

$$g = q_i^{a} + \frac{1}{\bar{T}_i^{a}}\left[\sum_{j=1}^{N} p_{ij}\,V_j - V_i\right] \qquad (3.2)$$

The heuristic development of the policy iteration procedure then follows. If the result for the right hand side of equation (3.2) is lower than the one for our arbitrary policy, we would have some tentative indication that the alternative " a " in state i would lead to lower maintenance cost for the process than does the alternative used in state i under the arbitrary policy. Solve equation 3.1 for new policy and repeat the entire process. We continue to do this until the test quantity indicates that there is no reason to change the decision in any state.

The iteration cycle appears in Fig. 5. The upper box, policy evaluation, finds the average value of maintenance cost and relative values for a particular policy. The lower box, policy improvement, finds a new policy that is improvement over the original policy by using the values of the original policy. Then this new policy is evaluated, and so on.

POLICY EVALUATION

For the present policy solve

$$v_i + g\,\overline{T}_i = q_i\,\overline{T}_i + \sum_{j=1}^{N} p_{ij}v_j \; ; \; i = 1,2,\ldots,N$$

with $v_N = 0$, for the value of g, and the relative values $v_1, v_2, \ldots, v_{N-1}$.

POLICY IMPROVEMENT

For each state i find the alternative a that minimizes

$$\Gamma_i^{a} = q_i^{a} + \frac{1}{\overline{T}_i^{a}}\left[\sum_{j=1}^{N} p_{ij}^{a}\, v_j - v_i\right]$$

using the relative values v_i of the previous policy. Make this alternative the new decision in state i. Repeat for all states to find the new policy

Fig.5 - The Iteration Cycle.

3.5 Numerical Example

Consider a equipment whose performance at any time can be characterised by only two states, either Good (G) or Failed (F). Let the holding time distributions in each state are exponentially distributed as follows -

$$h_{GG}(m) = 1/5\ e^{-m/5} \qquad m = 1, 2, ..$$

$$h_{GF}(m) = 1/6\ e^{-m/6} \qquad m = 1, 2, ..$$

$$h_{FG}(m) = 1/4\ e^{-m/4} \qquad m = 1, 2, ..$$

$$h_{FF}(m) = 1/3\ e^{-m/3} \qquad m = 1, 2, ..$$

$$\therefore\ \bar{\tau}_{GG} = 5, \quad \bar{\tau}_{GF} = 6, \quad \bar{\tau}_{FG} = 4, \quad \bar{\tau}_{FF} = 3$$

The other data required is as shown in table 3.1 and 3.2

Table 3.1 (HYPOTHETICAL DATA)

Item	State G	State F
Fixed Replacement Cost	5	6
Fixed Maintenance Cost (Overhaul or Repair)	2	3
Variable maintenance Cost D_i	2	5
Mean time for Replacement	1	3

Table 3.2

state	Alternative	Transition Probability		Mean waiting time	Expected occupancy cost	Expected cost rate
i	a	p_{IG}^{a}	p_{IF}^{a}	$\bar{\tau}_i^{a}$	r_i^{a}	q_i^{a}
G	Overhaul	0.8	0.2	5.2	12.4	2.38
	Replace	1	0	1	7	7
F	Repair	0.85	0.15	3.85	22.25	5.8
	Replace	1	0	3	21	7

EVALUATION :

Let us choose the initial policy as -

If in state G, then replace

If in state F, then replace.

For this policy, we write

$$V_G + g\,\bar{\tau}_G^{R} = r_G^{R} + p_{GG}^{R}\,V_G + p_{GF}^{R}\cdot V_F$$

$$V_F + g\,\bar{\tau}_F^{R} = r_F^{R} + p_{FG}^{R}\,V_G + p_{FF}^{R}\cdot V_F$$

setting $V_F = 0$,

$$\therefore\ V_G + g = 7 + V_G$$

$$3g = 21 + V_G$$

Solving, we get $g = 7$, $V_G = 0$

Thus, the average cost per unit time under the policy [replace, replace] is 7, and the relative cost of starting in state Good rather than in state Failed is 0.

First policy Improvement :-

Present policy [replace, replace]

Evaluation : $g = 7$, $V_G = 0$, $V_F = 0$

State	Alternative	Test Quantity
i	a	$\Gamma_i^a = q_i^a + \frac{1}{\bar{T}_i^a}\left[\sum_j p_{ij}^a V_j - V_i \right]$
G	Overhaul	2.38 ←
	Replace	7
F	Repair	5.8 ←
	Replace	7

Next policy is [Overhaul, Repair]

with $V_F = 0$, the policy evaluation equantions are

$$V_G + 5.2\, g = 12.4 + 0.8\, V_G$$

$$3.85\, g = 22.25 + 0.85\, V_G$$

Solving, $g = 2.89$, $V_G = -13.14$, $V_F = 0$

Second Policy Improvement :

State	Alternative	Test Quantity
G	Overhaul	2.885 ←
	Replace	7
F	Repair	2.82 ←
	Replace	3.27

∴ Next policy is [Overhaul, Repair]

Since the same policy has been obtained on two successive iterations, the optimal policy is [Overhaul, Repair] i.e. when the equipment is in Good state, maintenance action is of overhauling, and for the equipment in Failed state, repair it.

The expected average cost of maintenance per unit time = 2.89.

4. MAINTENANCE STRATEGY FOR A CUMULATIVE DAMAGE MODEL

In this chapter systems which fail due to degradation caused by the cumulative damage are considered. The optimal replacement level of damage which minimises the total expected cost per unit time for an infinite time span is obtained.

4.1 INTRODUCTION

During the power shortage crisis in the capital three months ago, the name of the Badarpur Power Generating station was in light. There were three generating units, one failed in October 1976, but it was not commissioned for necessary repair actions. The result was that the two remaining units were overstressed to meet the demand and the whole plant was out of action in the month of March. It is of great importance to avoid failure of the item when its failure during operation is costly and dangerous. It is wise to have some preventive maintenance policy.

TOSHIO NAKAGAWA[15] has considered a replacement problem for the car tyres. We extend his theorems to our model.

4.2 FORMULATION OF PROBLEM

Let, there are three equipments in parallel which share the equal load suppose by chance one of the equipments has failed. We assume that the remaining two equipments meet with the demand at the cost of it's overloading. As the equipment is subject to Over-stresses, there is a continuous deterioration of the equipment.

Assumptions

i) The total amount of damage suffered for the equipment is known at any time.

ii) The equipment can fail only when the total amount of damage exceeds a prespecified failure level K.

We are using the level of damage as a replacement indicator because we know the total amount of damage at any time. We make a replacement of the item when the total amount of damage exceeds a level K_1 ($0 \leq K_1 \leq K$). That is, we investigate the total amount of damage to the equipment after each blow. If the damage exceeds K_1 and is less than K, we exchange the equipment before it has failed, if the damage exceeds K it has failed and we replace it, and otherwise, we leave it alone.

We introduce the following costs.

i) Cost C_1 occurs when a failed equipment is replaced. This includes all its cost resulting from the failure and its replacement.

ii) C_2 cost is incurred when a nonfailed item is replaced before failure.

$C_2 < C_1$.

iii) $C(t)$ be the total expected cost during $(0,t)$.

We want to find out the value of K_1^* which minimises the expected cost per unit time for an infinite time span,

Let us define the following cumulative process. Let, the random variable X_i ($i = 1, 2, \ldots$) are associated with a sequence of inter-arrival time between successive blows. Let the random variable W_i ($i = 1, 2, \ldots$) denote the amount of damage produced by i^{th} blow.

We assume that W_i are non-negative, independent, and identically distributed, and that W_i is independent of X_j ($j \neq i$)

Assume that $P_r \left\{ X_i \leq t \right\} = F(t)$

$$P_r \left\{ W_i \leq x \right\} = G(x) \quad (i = 1, 2, \ldots$$

and the Renewal function $M(x) = \sum_{j=1}^{\infty} G^j(x)$

consider the cycle from the begining of the equipment operation to replacement.

The probability that the equipment is replaced after failure i.e. the total amount of damage exceeds the failure level K, given that the total amount of damage just before failure was less than K_1, is given by

$$A_1(K_1) = P_r \left\{ W_1 > K \right\} + \sum_{j=2}^{\infty} P_r \left\{ W_1 + W_2 + \ldots + W_{j-1} \leq K_1 \right.$$

$$\left. \text{and } W_1 + W_2 + \ldots + W_j > K \right\}$$

The first term on R.H.S. is the probability that the amount of damage exceeds a level K at the first blow and the second term is that the total amount of damage is u ($0 \leq u \leq K_1$) at the $(j-1)^{th}$ blow and then exceeds a level K at the next blow.

$$\therefore A_1 (K_1) = 1 - G(K) + \sum_{j=2}^{\infty} \int_0^{K_1} \left[1 - G(K - u) \right] dG^{j-1}(u)$$

(4.1)

From definition of the renewal function M(x), we have

$$A_1(K_1) = 1 - G(K) + \int_0^{K_1} \left[1 - G(K - u) \right] dM(u) \qquad (4.2)$$

In a similar fashion, the probability that the equipment is replaced before failure, i.e. the total amount of damage is between K_1 and K, and is

$$A_2(K_1) = P_r\left\{K_1 < W_1 \leq K\right\} + \sum_{j=2}^{\infty} P_r\left\{W_1 + W_2 + \dots + W_{j-1} \leq K_1\right.$$

$$\left.\text{and } K_1 < W_1 + W_2 + \dots + W_j \leq K\right\}$$

$$= G(K) - G(K_1) + \int_0^{K_1}\left[G(K-u) - G(K_1-u)\right]dM(u)$$

$$(4.3)$$

Further the meantime of one cycle i.e. the meantime that the total amount of damage exceeds a level K_1 for the first time is,

$$\sum_{j=1}^{\infty} \int_0^{\infty} t\, P_r\left\{W_1 + W_2 + \dots + W_{j-1} \leq K_1 \text{ and } W_1 + W_2 \dots\right.$$

$$\left.\dots + W_j > K \mid X_1 + X_2 + \dots + X_j \leq t\right\} dP_r\left\{X_1 +\right.$$

$$\left.X_2 + \dots + X_j \leq t\right\}$$

$$= \sum_{j=1}^{\infty}\left[G^{(j-1)}(K_1) - G^{(j)}(K_1)\right]\ (j/\lambda)$$

$$= \left[1 + M(K_1)\right]/\lambda \qquad (4.4)$$

where $1/\lambda = E[X_i]$ and $G^{0}(K_1) = 1$ for $K_1 \geq 0$.

It is easily shown that the total expected cost per cycle is equal to the total expected cost per unit of time for an infinite time span (Ross).

$$C(K_1) = \lim_{t\to\infty} \frac{\hat{C}(t)}{t} = \frac{C_1A_1(K_1) + C_2A_2(K_1)}{\text{mean time of one cycle}} \tag{4.5}$$

Substituting (4.2), (4.3) and (4.4) into (4.5) and using the relation

$$M(K_1) = G(K_1) + \int_0^{K_1} G(K_1 - u)\, dM(u) \tag{4.6}$$

we have,

$$C(K_1) = \frac{C_2 + (C_1 - C_2)\left\{1 - G(K) + \int_0^{K_1}[1 - G(K-u)]\, dM(u)\right\}}{[1 + M(K_1)]/\lambda} \tag{4.7}$$

which is a function of K_1. In particular cases, we have

$$C(0) = \lambda C_1[1 - G(K)] + \lambda C_2 G(K) \tag{4.8}$$

$$C(K) = \frac{\lambda C_1}{[1 + M(K)]} \tag{4.9}$$

Note C(0) represents the expected cost when the equipment is always replaced at the first blow, irrespective of the amount of damage and C(K), the expected cost that the equipment is replaced only after failure.

In order to obtain K_1^* ($0 \le K_1^* \le K$) which minimises the expected cost rate $C(K_1)$ in (4.7), we assume that G(x) has a density. Then we have the following theorem given by

Nakagawa which gives the optimum replacement policy.

THEOREM:

i) If $M(K) > C_2/(C_1 - C_2)$ then there exists a unique K_1^* ($0 < K_1^* < K$) which satisfies the following eqn.

$$\int_{K-K_1}^{K} [1 + M(K - u)] \, dG(u) = C_2/(C_1 - C_2) \qquad (4.10)$$

ii) If $M(K) \leq C_2/(C_1 - C_2)$ then $K_1^* = K$ i.e. a policy of replacement only after failure is optimum.

If there exist a unique K_1^* ($0 < K_1^* < K$) for case (i), the expected cost rate is

$$C(K_1^*) = \lambda(C_1 - C_2) [1 - G(K - K_1^*)] \qquad (4.11)$$

and for case (ii), it is given in (4.9).

We can further obtain the following.

$M(K) \geq \theta K - 1$, where $1/\theta = E[W_i]$ (see Barlow and Proschan)

Thus, if $K > C_2/[\theta(C_1 - C_2)]$, then there exists a unique K_1^*($0 < K_1^* < K$) satisfying (4.10).

4.3 Example

Consider a replacement of a equipment where the damage to it is a function of working hourse under overstress condition.

If under overstress condition working hours exceeds $K = 1500$ hrs, the equipment is regarded as failed. The working hours per time unit is assumed to obey the exponential distribution with mean $1/\theta$ i.e. $G(x) = 1 - e^{(-\theta x)}$. Let C_2 represent the usual replacement cost of the equipment. A cost C_1 includes all the costs resulting from the failure of equipment in service. C_1 will be greater than C_2 since there is a risk of accidents. From theorem if $K > 1/[\theta(C_1 + C_2 - 1)]$ then there exists K_1^* satisfying

$$\theta K_1^* \exp[-\theta (K-K_1^*)] = \frac{1}{(C_1/C_2-1)}$$

we replase the equation when total working hours exceeds K_1^* hours before failure. Table 4.1 shows the values of K_1^* when C_1/C_2 and $1/\theta$ are specified.

Table 4.1

$K = 1500$ Hrs.

$1/\theta$	$C_1/C_2 = 2$ Optimal Replacement level K_1^*	$C_1/C_2 = 4$ Optimal Replacement level K_1^*	$C_1/C_2 = 5$ Optimal Replacement level K_1^*
10	1450	1440	1436
20	1415	1396	1388
30	1388	1353	1343
40	1362	1318	1306
50	1335	1282	1265
60	1310	1251	1235

finite time horizon and Infinite time horizon respectively. Chapter third represents the optimal maintenance strategies for the equipment when changes of states are Semi-Markovian. Only catastropic failure is considered. Method of obtaining optimal policy is presented. The objective criterion was to minimise the expected cost per unit time. In chapter four, we discussed a problem of a cumulative damage model for an equipment, which is replaced at a certain level of damage before failure or at failure whichever occurs first.

In first and third models of chapter two, we have considered finite time horizon. In practice, the period of time over which we wish to optimize our maintenance decisions may be very long and so we may be interested in determining what the best decisions are and what is the resulting cost when the period is infinite. Second and fourth models deal with this problem.

The cumulative damage model discussed in a chapter four is very practicable. The problem of replacement decision to be taken, when the total expected cost due to failures exceeds a critical cost level or when the total down time exceeds a critical down time level, can be formulated with the help of this model.

The author feels the need of investigation in the following areas -

Two stage and multi stage maintenance strategies with the semi-markovian process.

The maintenance strategies for the system consisting of equipments with depedent failures.

REFERENCES

1. Alam, M., Sharma, V.V.S., 'Optimal Preventive Maintenance Policy for an Equipment Subject to Deterioration and Random Failure', IEEE Trans. Syst., Man, Cybern., Vol. SMC-4, pp 72-75, March 1974.

2. Barlow, R.E., and Prochan, F., 'Mathematical Theory of Reliability', John Wiley and Sons. 1965.

3. Bartholomew, D.J.,'Two-stage Replacement Strategies', Operational Research Quarterly, Vol. 14(1963), pp 71.

4. Bellman, R., and Dreyfus, S., 'Applied Dynamic Programming' (Oxford University Press, 1962).

5. Cox, D.R., Renewal Theory (Methuen/Wiley 1962).

6. Crooks, P.C.I., 'Replacement Strategies', Operational Research Quarterly, Vol. 14, No. 2.

7. Howard, R.A., 'Dynamic Probabilistic Systems' Vol. I : Markov Models, John Wiley and Sons., 1971.

8. Howard, R.A., 'Dynamic Probabilistic Systems', Vol. II, Semi-Markov and Decision Processes, John Wiley and Sons,1971.

9. Hanon Luss, 'Maintenance Policies when Deterioration can be Observed by Inspections', Operation Research, Vol. 24, No.2, March-April 1976.

10. Jardine, A.K.S., 'Maintenance, Replacement and Reliability', Pitman Publishing, 1973.

11. Kao, E.P., 'Optimal Replacement Rules when Changes of States are Semi-Markovian', Operation Research Vol. 21, 1973, pp. 1231 - 1249.

12. Naik, M.D. and Nair, K.P.K., 'Multistage Replacement Strategies' Operation Research Vol. 13, 1965.

13. Ross, 'Applied Probability Models with Optimisation Applications'.

14. Thomson, G.L., 'Optimal Maintenance Policy and Sale Date of a Machine', Management Science, Vol. 14, pp 543-550, May 1968.

15. Toshio Nakagawa, 'On a Replacement Problem of a Cumulative Damage Model', Operational Research Quarterly, Vol. 27.4, p 895.

16. Woodman, R.C., 'Replacement Policies for Components that Deteriorate', Operational Research Quarterly, Vol. 18, No.2 (1967)

5. CONCLUSION AND REMARKS

This dissertation work is an attempt in presenting mathematical models for decision making problem arising in the maintenance of an equipment. Illustrative numerical examples have been solved.

In second chapter, four models have been presented for maintenance of the equipment and optimal policies are obtained. First model considers the maintenance policy for the equipment subject to breakdown with finite time horizon. The objective was to minimise the expected total discounted cost. Second model deals with the maintenance policy for an equipment subject to breakdown considering discounting factor and infinite time horizon, minimising the expected discounted cost per unit time. Third and Fourth model deals with the maintenance policy for an equipment subject to deterioration considering