# OPTIMAL NETWORK EXPANSION

A DISSERTATION
submitted in partial fulfilment
of the requirements for the award of the Degree
of
MASTER OF ENGINEERING
in
POWER SYSTEM ENGINEERING

*By*
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# C E R T I F I C A T E

Certified that the dissertation entitled "OPTIMAL NETWORK EXPANSION" which is being submitted by Shri RAM SHABAD in partial fulfilment for the award of the degree of Master of Engineering in Power System Engineering of Electrical Engineering of University of Roorkee, Roorkee is a record of the student's own work carried out by him under my supervision and guidance. The matter embodied in this Dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of six months from February 1976 to July, 1976 for preparing this Dissertation at this University.

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Aug., 1976. (RAM SHABAD)

## CONTENTS

A B S T R A C T

Starting with geographical positions of thd substations, which are to be interconnected, it has been shown that a set of equations can be obtained which are solvable by linear-programming techniques to obtain a minimum cost network design. Any security of supply conditions considered necessary can be incorporated into the design equations. One of the earliest proposals to formulate the criteria of power system design, in such a way that the problem could be solved by an automatic optimisation process, was put forward by Knight[2] in 1961. He used new method of linear programming to minimise an economic objective function of the electrical network, subject to a set of linear inequalities representing security requirements. The next approach to the planning of electrical-power networks is mixed integer linear programming based on interpretation of fixed cost transportation-type models which include both network security and cost of network losses.

The rewards for optimising the design of electrical power system increase with the size of the system but unfortunately severity of the problem increases very rapidly with the size of the system. Despite the

improvements in the size and speed of digital computers used for this purpose, it is always useful to reduce the storage requirement. To achieve this requirement piecewise optimization method is designed for use in the optimization of large linear problems.

Examples are considered for the illustration of the discussed methods. In order to solve the integer programming problem Lexicographical, Enumeration methods are used. In the II chapter a computational procedure for system planning is presented. This procedure combines and optimizes load flow, reliability analysis and economic evaluation.

In the III chapter Optimization techniques have been discussed with their advantages and disadvantages. Flow charts and the computer programmes have been developed for Enumeration, Lexico raphical and bounded ~~value~~ variable problems. Examples based on the reference[2] are given and solved, using the above computer programmes. A flow chart with computer programme is developed to write the security constraints by computer itself to avoid the error and complexity occurring in data preparation.

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# INTRODUCTION

Many papers and books have been written over the past sixty years on the analysis and performance of electrical power-system networks. D.C. and A.C. network analysis have been used to supplement hand computation in steady-state networks analysis since about 1925. Since 1952, digital computers have been increasingly employed to obtain numerical solutions to steady state and transient network equations. All this work has been concentrated on analysing the performance of networks and very little has been done on the mathematical design of networks. To illustrate the point, a network may be proposed which has a circuit between two substations A and B. The methods used for analysing purpose enable one to predict what the voltage and power-flow conditions on this circuit are likely to be and what its cross-section should be. They do not give any assistance in saying whether, infact, a circuit should be provided between A and B or not. The aim could be to obtain a minimum-cost outline design of a network to supply a number of load points from a number of supply points.

The method could be based on thd use of the technique of linear programming, developed by economists and mathematicians during and since the second world war. The problem involves the consideration of a number of variables whose relationship with each other is defined by a set of linear equations (or constraints), the number of variables being unequal to the number of constraints, and subject to the general condition that the variables should be non-negative. There may be a large number of solutions satisfying the constraints, and the problem is to find which of these solutions has some preferred characteristic, say minimum cost. Normal algebric method can not be used to solve such a problem, and linear programming enables optimum solution first to be obtained in a systematic manner and secondly to be identified when it is obtained. In order to solve the linear network equations three types of linear programming approach can be used. (1) Noninteger programming[2], (2) Mixed integer programming[19], (3) Pure integer programming[2].

The method using mixed-integer programming approach to the planning of electrical-power networks is based on interpretation of fixed-cost transportation-type models[14], and include both network securing and costs network losses. Both single period and multiperiod planning problems[19] can be considered.

The planning of electrical-power network is a complex process in which the application of computing techniques has grown steadily for high voltage transmission system into the more recent applications in distribution system analysis.

Alternative attempts to devise automatic computer methods have often resorted to a heuristic approach[3], and more recently, the immergence of advanced facilities for computer-aided design has stimulated the development of effective interactive computing methods for systems design[4,5]. The methods have ignored the time dependence of practical planning proposals, and the question of optimal sequencing of development has tended to be treated separately[6] although the combination of dynamic programming with heuristic network synthesis has been reported[7] and linear programming has been used for time phased planning of generation system.

A mixed integer programming model for optimal-power network planning that permits the dynamic requirements of the problem to be represented as a natural extension of network synthesis is described. Although according to requirements, present day computers are growing in size and speed, one is still faced with limited computational facilities. In the linear programming field, decomposition algorithm[8] has been developed which can deal with piecewise solution. But this cannot deal efficiently with the addition of constraints in the interconnecting systems. Kron[9] has developed diakoptical optimization algorithm for transportation problem.

In computational procedure for system planning described in Chapter II, Sec. (8) load flow, reliability analysis and economic evaluation are combined together. Actually planning the expansion of a high-voltage transmission system involves deciding which new lines will enable the system to satisfy forthcoming loads with the required degree of reliability. Since these decisions involve considerable investment and operating costs, the planner will wish to keep all costs as low as possible. The difficulties of the problem come from the tremendous number of possible alternatives, the need to make the best use of information about future loads, and the complexity of the reliability constraints. The outages of some specific combinations of lines must not at any time overload any other line in the system. The recent literature on power system reliability has shown the importance of sound planning in satisfying future demand. In view of the extremely high investment costs of electric power systems, it is desirable to have procedures for adding the right kind of equipment at the right time in the right location to achieve the desired level of reliability and quality of service at lowest cost over a long range.

Linear programming[12-14] deals with the minimization of a linear function in which the variables are non-negative and constrained by a system of linear equations. In an all integer method the problem is stated with given integer coefficients and all calculation result in integer coefficients at each iteration. For general purpose Lexicographical approach appears to be good tool to solve the integer programming problem. In special problem Enumeration method can be used conveniently but the convergence of this approach is not assured. In case the solution does not exist, the computer will not be able to give any such indication. In actual practice it is useful for the problem where the variables are zero or one and is very very useful where only one or two lines are to be added to a existing power system satisfying the previous security and reliability constraints. The advantage of the Lexicographical algorithm is that if the solution is not existing, the computer will directly indicate it and its convergence is assured in finite number of iterations provided the solution is existing. Next advantage of this method is that memory requirement for the long problem is not going to be increased excessively as compared to Enumeration method. The memory requirement for Enumeration method increases tremendously with the size of problem, the speed being faster. In case where the variables have upper bounded level, bounded variable interior programmes based on Lexicographical approach may be used. The flow chart and computer programmes have been developed. For all the methods discussed above and it may not be possible to say which method will be superior because for a particular problem, a particular approach will be advantageous. The selection of the procedure depends on the experience and requirement of the problem keeping the computational facility available in mind. In present work the programmes for Lexicographical algorithm, Enumeration method and bounded variable problems are programmed on computer which can be used according to requirements. Examples are solved using these methods and results are presented.

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All the three cases arise in order to increase the availability of supply to consumers. In each case, one is presented with a geographical disposition of substations which require connecting together at minimum cost satisfying the desirable conditions for security of supply etc. As the number of substations increase, the ways in which these connections can be made in a technically satisfactory manner will become large. The art of system design lies in choosing the scheme which is both technically and economically the best or at least makes a reasonable compromise between the two requirements.

(2) Criterion for Formulation of Network Design Equations :

The criteria used in the formulation of design equations for optimal solution by linear programming methods are given as :

(a) A criterion which gives a minimum-cost design.

(b) A criterion which gives a desi-gn with minimum circuit length.

(3) Formulation by the Minimum Cost Criterion using Pure Integer Programming :

(3.1) Outline of Method :

Firstly it is necessary to consider what supply

# CHAPTER - II

## FORMULATION OF OPTIMAL NETWORK EXPANSION PROBLEM USING LINEAR PROGRAMMING METHODS

### (1) Important Features of Power-Supply Network :

### (1.1) Requirements of a Power-supply Network :

A power supply network may be designed so that it will transmit given amounts of electrical power and satisfying the conditions given below :

(a) The cost of the network to be constructed and operated should be as small as possible.

(b) The continuity of supply afforded by the network should not be less than the minimum acceptable and this minimum acceptable limit for continuity depends on size and type of the load.

(c) So far as it concerns to generating station, the connections provided should give adequate capacity out of the station under those circuit outages and load conditions assessed as technically and economically justified during the design study.

(d) The necessary operational and control facilities required to obtain satisfactory performance from the network should be consistent with the facilities normally available for a network supplying loads of the size and type concerned.

(e) In case w here the extension of the power system is required, extension of the network should be possible.

(f) There should be no risk of harm to plant or personnel under normal or fault condition.

In the present work, the aim is concentrated to fulfil the requirements (a) to (c), and (e). Further it has been shown that how these requirements are achieved using linear programming.

## (1.2) Difficulties in Network Design :

At present due to complexity of the power system, engineers even in advanced countries have to face the problem of interconnecting a number of substations by a network which will satisfy with the conditions given in Section (1.1). Such cases may arise in :

(a) The provision of distribution methods (medium and high voltage) to supply new housing states.

(b) The reinforcement of existing distribution netoorks by superimposed subtransmission networks.

(c) The reinforcement of existing subtransmission networks by superimposed transmission networks.

All the three cases arise in order to increase the availability of supply to consumers. In each case, one is presented with a geographical disposition of substations which require connecting together at minimum cost satisfying the desirable conditions for security of supply etc. As the number of substations increase, the ways in which these connections can be made in a technically satisfactory manner will become large. The art of system design lies in choosing the scheme which is both technically and economically the best or at least makes a reasonable compromise between the two requirements.

(2) Criterion for Formulation of Network Design Equations :

The criteria used in the formulation of design equations for optimal solution by linear programming methods are given as :

(a) A criterion which gives a minimum-cost design.

(b) A criterion which gives a design with minimum circuit length.

(3) Formulation by the Minimum Cost Criterion using Pure Integer Programming :

(3.1) Outline of Method :

Firstly it is necessary to consider what supply conditions the proposed network has to satisfy and then the attempt can be made to formulate the design conditions mathematically. These will depend primarily on the network voltage. In the case of medium-voltage networks it is not always economical to provide a full duplicate supply, but particular attention must be paid to the voltage regulation. On the higher voltage netowrks a failure of the network will affect many consumers and for this reason duplication of supply is more important, Here our attention is mainly concentrated towards the designí of high voltage distribution methods, sub-transmission networks and the lower voltage transmission, networks. At these voltages, the maintenance of supply is the most important consideration. Other factors to be considered may be avoidance of excessive expenditure on switchgear and of too many circuits along any one route. It is to be kept in mind that as the number of circuits along any particular route will increase, the cost investment on switchgear will also increase with the number of circuits along that particular poute.

While writing the constraints, or network equations, as a starting point it is assumed that unless there is no any restriction (natural or man made), a path $P_{ij}$ for one or more circuits exists between every pair of substations $S_i$ and $S_j$. The aim is to design a value 0, 1, 2... to each path $P_{ij}$ indicating that the optimum design requires 0, 1, 2 .... circuits between substation $S_i$ and $S_j$.

In order to do this, inequalities specifying security conditions (Sec. 3.2) and any other design conditions considered necessary, such as limitation of the number of circuits into substations (Sec. 3.3) or along any given rules (Sec 3.4) are written down in terms of the possible paths. These linear inequalities are then used as constraint inequalities subject to which a cost function known as objective function of the network, again in terms of possible paths, is minimized. Generally the values obtained for the $P_{ij}$ will be non-integer, and it is necessary to employ some methods which will produce an integer valued solution.

(3.2) <u>Commission due to Security</u> :

In the beginning while writing the security of supply inequalities it is necessary to bear in mind that one has no knowledge of the final network connections. It is, therefore, necessary to specify minimum connections to every possible group of substations to ensure that all groups in the final design will have adequate circuit capacity connected to them.

Many supply authorities have standardized transformer sizes and overhead line and cable ratings. It is possible to say that two circuits will supply upto, say, 3 substations, 4 substations will require 3 circuits, and so on. Alternatively, if substation loads differ appreciably, and particularly if the design is to incorporate connections to generating stations, it is necessary to estimate the circuit capacity required into every possible group of substations. Thus security of supply conditions can be specified as :

(a) Each load substation must have at least $h_1$ circuits connected into it.

(b) Each possible group of two substations must have at least $h_2$ circuits connected into it.

(c) Each possible group of 3 load substations must have at least $h_3$ circuits connected into it.

and so on, for every possible group of load substations of all sizes upto and including all the load substations.

This set of conditions will produce the following inequalities.

$$\sum_{i=1 \neq 1}^{r} p_{1i} \geq h_1 \quad \text{for load substation } S_1 \qquad \ldots \qquad (1.1)$$

$$\sum_{i=1 \neq l_1, l_2}^{r} P_{il1} + \sum_{i=1 \neq l_1, l_2}^{r} P_{il2} \geqslant h_2$$

for load substations $S_{l1}$, $S_{l2}$ ... (1.2)

where, $S_1 \ldots S_m$ are supply substations

$S_n \ldots S_r$ are load substations

h is number of circuits required into a group of S substations

S is number of substations in a group

If the proposed network is also to interconnect generating stations, it is necessary to add a set of equations to specify that each generating station, every group of generating stations and evdry group of generating stations and load substations has sufficient circuit capacity connected to it to ensure that génoration is not restricted by lack of circuit capacity.

When the network is supplied only from one supply substation, it is not necessary to specify any circuit connection at this substation because the final security of supply equation (for all the load substations) will show what total circuit capacity is required into all the load substations and therefore out of the supply substation.

If more than one supply substation is to be connected to the network, it may be necessary to specify at least a certain number of circuits out of each of the supply substations.

(3.3) Constraint Due to Switchgear

Intension of the supply engineers is also towards the reduction of capital investment in switchgear, which may account for 40-50% of the cost of substation. Various methods used to reduce the capital investment in switchgear take the form of controlling more than one piece of equipment from one circuit-breaker. Now it is more important from the aspect of network design to consider a limit for number of circuits which can be controlled at one substation. This limit may be 2, 3 or 4 circuits.

Thus if it is desired to limit the maximum number of circuits controlled at any load substation to say, k, a set of inequalities can be written down as follows :

$$\sum_{i=1 \neq l}^{r} P_{il} \leq k \quad \text{for load substation } S_l \quad \ldots \quad \ldots \quad (4)$$

This type of inequality could also be used to ensure that excessive fault levels would not occur on the proposed network when a fault infeed over circuits is known to be very approximately constant. Supply substations must be focal points of a network and hence any limitation of the number of circuits connected into these is not often justified except for fault level control.

## (3.4) Constraint Due to Limitation of Intersubstation Circuits

Sometimes it would be desirable to ensure that all circuits shall provide as much opportunity as possible for connection into future substations and to satisfy this requirement many supply engineers consider that the number of circuits along any route should be restricted. This criterion will lead to a set of inequalities of the form

$$P_{ij} \leqslant w \qquad \ldots \qquad \ldots \qquad (5)$$

where w is the number of circuits along any one route which it is not wished to exceed.

If substations $S_i$, $S_j$ and $S_k$ are practically in line, inequalities of the form

$$P_{ij} + P_{ik} \leq w \qquad \ldots \qquad (6)$$

$$P_{ik} + P_{jk} \leq w \qquad \ldots \qquad (7)$$

will again ensure that a given number of circuits along paths between these substations is not exceeded.

## (3.5) Network Cost Function

The network cost function which has to be minimised will be

$$f = \sum_{i=1 \neq j}^{r} \sum_{j=1 \neq i} C_{ij} P_{ij} \quad \cdots \qquad (8)$$

This equation implies that the cost of providing circuits between two points is proportional to the number of circuits. This is true for single-circuit overhead lines and also applicable to underground cables. But it will only be approximately true for multi circuit overhead lines.

The cost, $C_{ij}$, of a circuit between substations $S_i$ and $S_j$ should include the cost of the controlling switchgear plus a ~~proposition~~ proportion of the establishment and civil engineering costs for substations $S_i$ and $S_j$. Therefore, it appears to be necessary to assume the number of circuits to be connected into a substation in the final design in order to account the switchgear, establishment and civil engineering costs. If this assumption is not correct, it is unlikely to affect the final design as the error in the cost will be small in relation to the total circuit cost.

## (4) Specification of Circuit Capacity into Substation Groups for Integer Programming

### (4.1) Distribution and Subtransmission Networks

Difficulty arises in the decision of circuit capacity that should be provided to the larger groups of substations. This is due to possible poor load sharing between a number of circuits. It may also be necessary to assume an outage of more than one circuit.

Only experience of load flows on networks and a knowledge of field statistics can enable a good selection of the required circuit capacity. It has been suggested that actual substation loads should be considered. Particularly in the case if they vary widely, the total load of the group can be turned into equivalent numbers of circuits for writing into the constraints, as follows :

$$h = N1 + M1$$

where N1 is the first integer greater than or equal to $S_L/S$ and M1 is a small integer say, equal to one (for from supply)

$S_L$ is total load in a group of substations

S = Maximum rating of circuits on proposed network

## (4.2) Transmission Networks

In the case of transmission networks, the circuit capacity into a group of substations can be calculated as the sum of two components, the planned transfer and the interconnection capacity. The planned transfer is given by the difference between the group load and the group generation.

The individual circuit capacities being known and the fault risk to be guarded having been decided, the number of circuits required into a group can be taken as

$$h = N_2 + M_2 \qquad \ldots \qquad (9)$$

where $N_2$ is the first integer greater than or equal to (planned transfer $\pm$ interconnection capacity)
and $M_2$ is small integer (for firm supply)

It is quite apparent that the major difficulty in applying this method is due to the large number of constraint inequalities needed to specify a design. The writing of the constraints if extremely tedious with the increase of number of substations to be interconnected. Some ways in which these difficulties might be minimised are suggested below :

## (5) Problems Associated with the Design Equations

### (5.1) Problems of the Solution

The limiting feature in the size of the problem which can be solved is the computer storage. The linear programming solution requires the storage and manipulation of a matrix slightly greater than the number of equations in one axis and the number of variables in the other axis.

If $a_1$ is the number of equations and upper bounded inequalities and b is the number of variables, a computer solution can be obtained by the reduced simplex method if

$$(a_1 + \text{small integer})(b_1 + \text{small integer}) \quad \text{computer storage} \qquad \ldots \qquad (10)$$

when the constraints are in the form of lower bounded inequalities

$$(a_2 + \text{small integer})(b_1 + \text{small integer}) \quad \text{computer storage}$$

where $a_2$ is the number of lower bounded inequalities and $b_1$ number of variables

### (5.2) Network - Design Matrix Size

With m supply and n load substations to be connected to the network, there will be $m + n_{c_2}$ possible paths between the substations, or the number of paths will be

$$1/2\ (m + n)\ (m + n - 1) \quad \ldots \quad \ldots \quad (11)$$

If all the design conditions proposed are to be included, there will be n upper-bounded inequations specifying maximum numbers of circuits at each load substation, and $m + n_{c_2}$ upper bounded inequalities specifying maximum numbers of circuits along any path. There will be $n_{c_1}$ lower-bounded inequalities specifying security conditions to each load substation, $n_{c_2}$ inequalities specifying security conditions as discussed to all possible groups of two load substations, and so on. Hence the total lower-bounded inequalities required to specify security conditions is

$$n_{c_1} + n_{c_2} + n_{c_3} + \ldots \quad \ldots + n_{c_{n-1}} + n_{c_n} = 2^n - 1 \quad \ldots (12)$$

The total number of constraint inequalities will therefore be

$$(n + m + n_{c_2})_{\text{upper bounded}} + (2^{n-1} - 1)_{\text{lower bounded}}$$

in terms of $m + n_{c_2}$ variables.

e.g. A network design to supply 8 load substations from 2 supply stations would require the setting down of 308 inequalities in te terms of 45 possible paths.

Practical experience in the application of network design has indicated that the majority of load substations in the final design, in fact, that the circuits per path will rarely exceed two. Thus it is considered that inequalities for limiting circuits into substations and along path are, in practice unnecessary.

As the greatest number of constraint inequalities result from the specification of minimum numbers of circuits into substation groups. But many of these constraints are over satisfied in practice, and could therefore be omitted from the design equation. This will be the case if paths between some substations are considered to be impossible or undesirable for circuit construction. In general, if there are $n_1$ load substations with no direct connections to $n_2$ other load substation, there is no need to consider security conditions to any group composed of one or more substation from the $n_1$ group and one or more substations from the $n_2$ group.

(5.3) <u>Use of Special Computer Programme</u>

Once the number of circuits required to be connected into groups of substations of various sizes or load has been decided, the writing of the constraints inequalities is a purely mechanical process. It has already been shown that the solution of a problem of any size requires the use of a computer and therefore writing of the constraints also by the computer will be preferable. The saving in time and reduction of possible errors in coding the large matrices for computer solution would also be significant. In order to solve this problem a separate programme can be developed very easily to prepare data for linear programme or data preparation programme may be itself a piece of the linear programme. The second choice will be preferable with time point of view.

A more promising approach is thought to be the use of Kron's 'tearing technique' discussed latter on in order to reduce the memory requirement and consequently for the optimization of the larger systems. When the solutions obtained in the examples were substituted in the initial design inequalities, it is found that 80% of these are over satisfied and therefore these may be omitted. An algorithm is given below for the data preparation of the Lexicographical, bounded value problem and Enumeration methods. Programme is also developed using this algorithm for the security constraints up to the security of the groups of two substation. Developed approach is capable of writing the constraints of higher groups in the same fashion as shown below.

# FLOW-CHART FOR DATA PREPARATION

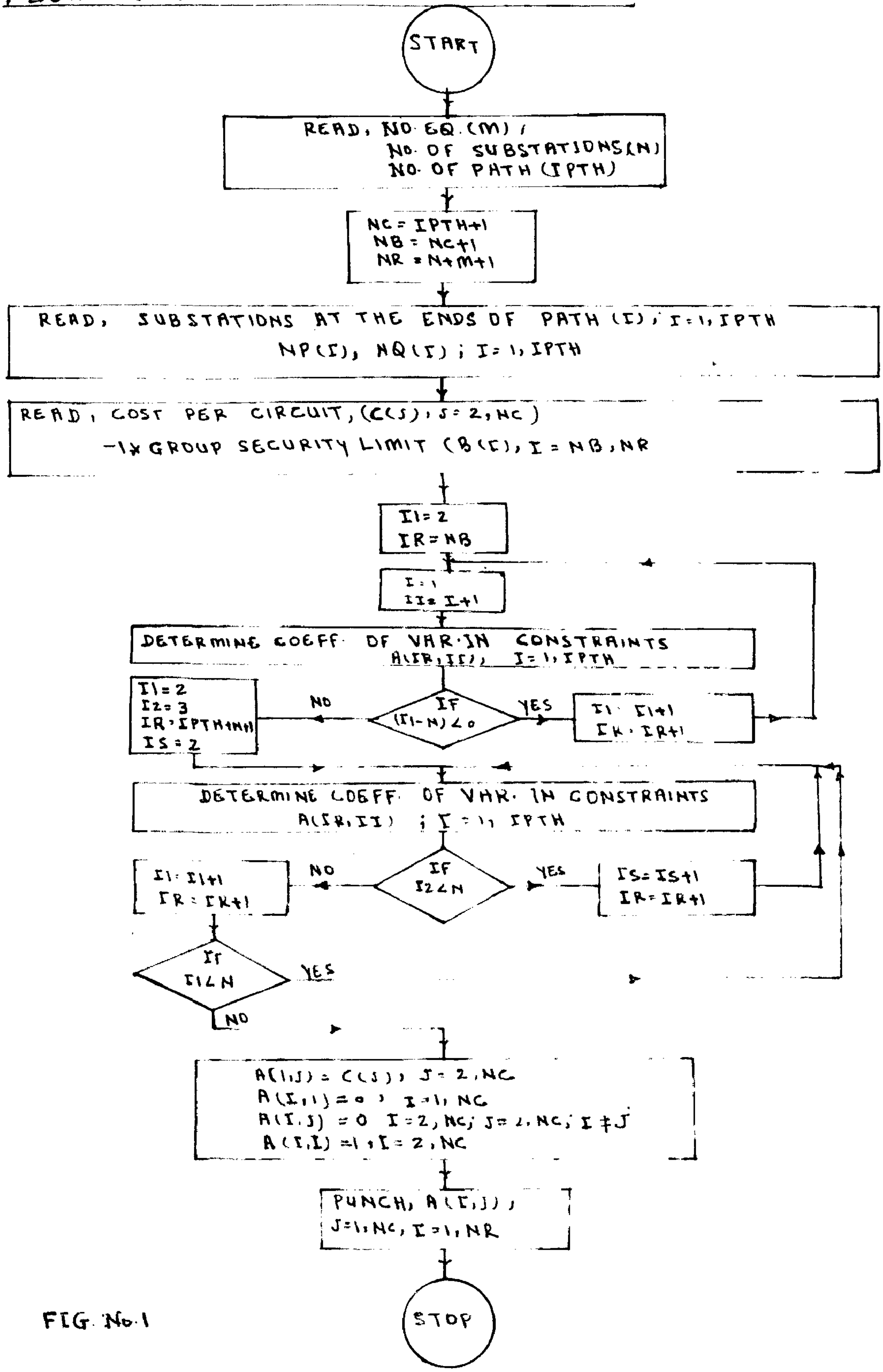



FIG. No.1

## 5.4 Algorithm for Data Preparation of Lexicographical Method

1. Develop a tableau by listing the paths, number of substations

| NP(I) | NQ(I) |
|---|---|
| 1 | 2 |
| . | . |
| . | . |
| 1 | N |
| 2 | 3 |
| . | . |
| . | . |
| 2 | N |
| 3 | 4 |
| . | . |
| . | . |
| 3 | N |
| . | . |
| . | . |
| (N-1) | N |

where NP(I) and NQ(I) are the two substations at the end of path (I).

2. Security of each load station lead to

$$\sum_{i=1 \neq l_1}^{r} P_{il} \geq h_1 \quad \text{for load substation } S_1$$

3. $\sum_{i=1 \neq l_1, l_2}^{r} P_{il1} + \sum_{i=1 \neq l_1, l_2}^{r} \geqslant h_2$ for load substation $S_{l1}, S_{12}$

and so on.

Flow chart is given for above algorithm indicating the whole technique IN FIG. NO. (1)

(6) Design by a Minimum Circuit Length Criterion

While designing an high-voltage rural network, practically it has been observed that it is advantageous ~~design~~ to design a network to interconnect a number of points so that each point has at least one connection and total circuit length is as small as possible.

Suppose we have to interconnect r points. Hence (r - 1) circuits are required and to specify at least one circuit into each point the following inequalities can be written down.

$$\sum_{i=1 \neq j}^{r} P_{ij} \geqslant 1 \quad \ldots \quad \ldots \quad (13)$$

$$\sum_{i=1 \neq j}^{r} \sum_{j=1 \neq i}^{r} P_{ij} = r - 1 \quad \ldots \quad (14)$$

$$P_{ij} \leqslant 1 \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

The length of interconnecting circuits to be minimised will be

$$f = \sum_{i=1 \neq j}^{r} \sum_{j=1 \neq i}^{r} l_{ij} \; P_{ij} \qquad \ldots \qquad (16)$$

where

$P_{ij}$ is number of circuits along path ij

$l_{ij}$ is length of path ij

(7) <u>Formulation by the minimum cost criterion using mixed integer Programming</u>

(7.1) <u>Fixed COST Transportation Model</u> :

Fixed cost transportation problem can be modelled by a 2-stage cost function, consisting of a fixed charge f and an incremental cost c that represents the marginal cost per unit. For x units, the fixed cost Z is given by

$$Z = f + cx \qquad \text{if } x > 0$$

$$\text{or} \qquad Z = 0 \qquad \text{if } x = 0 \qquad \ldots \qquad (17)$$

This cost function is expressed in mixed integer programming form by introducing an integer dummy variable P1 which can take only the values zero or one, so that if $x > 0$, $P1 = 1$, and if $x = 0$, $P1 = 0$.

For oxample,

$$Z = fp_1 + cx$$

subject to ... ... (18)

$$0 \leq Mp_1 - x$$

whore M is an upper bound on x and is made equal to the maximum carrying capacity.

(7.2) <u>Application of Fixed Cost Transportation model to Network Synthesis.</u>

Let us consider a planning problem in which a set of substations are to be connected togethor at minimum cost by making the best selection of available paths. The feasible directions of power flow in a path will be referred as 'routos' Thus a path will contain two routes.

(7.3) <u>Cost Function</u>

If the opdrating costs and energy losses are neglected, the cost of establishing a connection ona path is simply equal to initial fixed capital cost of the cispcuit or circuites installed. Thus the cost of the whole network consisting of m1 paths is

$$f = \sum_{J=1}^{m1} K_j p_1 \qquad \ldots \qquad \ldots \qquad (19)$$

whero m1 is total number of paths, the pj are intogor variables rostricted to the values zoro or one, if only one circuit per

Path is permitted, or zero, one or two, if two circuits per path are permitted; and so on. This cost function is to be minimised satisfying the security constraints (or in other words subject to the power-transfer capability of the network being adequate).

(7.4) <u>Constraints</u> :

Firstly power flows into each substation i must be satisfied. For a load substation

$$C_n(i, k)\,(P_j' - P_j) = D_i \qquad \ldots \qquad (20)$$

For a sup ly substation

$$C_n\,(i, k)\,(P_j - P_j') = A_i \qquad \ldots \qquad (21)$$

$$J \leftarrow UN(i)$$

where $D_i$, $A_i$, $P_j$, $P_j' \geqslant 0$ and

$$C_n(i, k) = +1, \; i < k$$

$$= -1, \; i > k$$

k being the number of node connected by branch j to node i, and $P_j$ and $P_j'$ being the power flows from node i to node k. Secondly the power flowing along each path must not be greater than the maximum capacity of the circuit or circuits.

$$0 \leq M_j p1_j - P_j - P_j' \qquad \ldots \qquad (22)$$

## 7.5 Inclusion of Costs of Losses

The annual cost of $I^2R$ losses in a given circuit is linearly related to the peak power loss by the loss-load factor. For future annual peak-power flows, therefore, this annual cost may be capitalised and represented as a quadratic function of peak-power flow (Fig. 1).

### Linearised $I^2R$ losses

The resulting nonlinear cost curve can be represented in the linear cost function by piecewise linearisation, where the power-flow variables for circuit j, $P_j$ and $P_j'$ are replaced by subsidiary variables $P_{aj}$, $P_{bj}$, $P_{cj}$, $P_{aj}'$, $P_{bj}'$, $Pc_j'$ etc. to represent power flows at each of the cost rates $C_{aj}$, $C_{bj}$, $C_{cj}$ etc., the number of cost rates depending on the accuracy required. For circuit j, the loss costs appear in the cost function as

$$C_{aj}\ P_{aj} + C_{aj}\ P_{aj}' + C_{bj}\ P_{bj} + C_{bj}\ P_{bj}' + C_{cj}\ P_{cj} + C_{cj}\ P_{cj}'$$

and the constraints of equation 22 are replaced by

$$0 \leq M_a p1_j - P_{aj} - P'_{aj}$$

$$0 \leq M_b p1_j - P_{bj} - P'_{bj}$$

$$0 \leq M_c p1_j - P_{cj} - P'_{cj}$$

where, $M_a$, $M_b$, $M_c$ represent power-flow limits for each of the loss-cost rates.

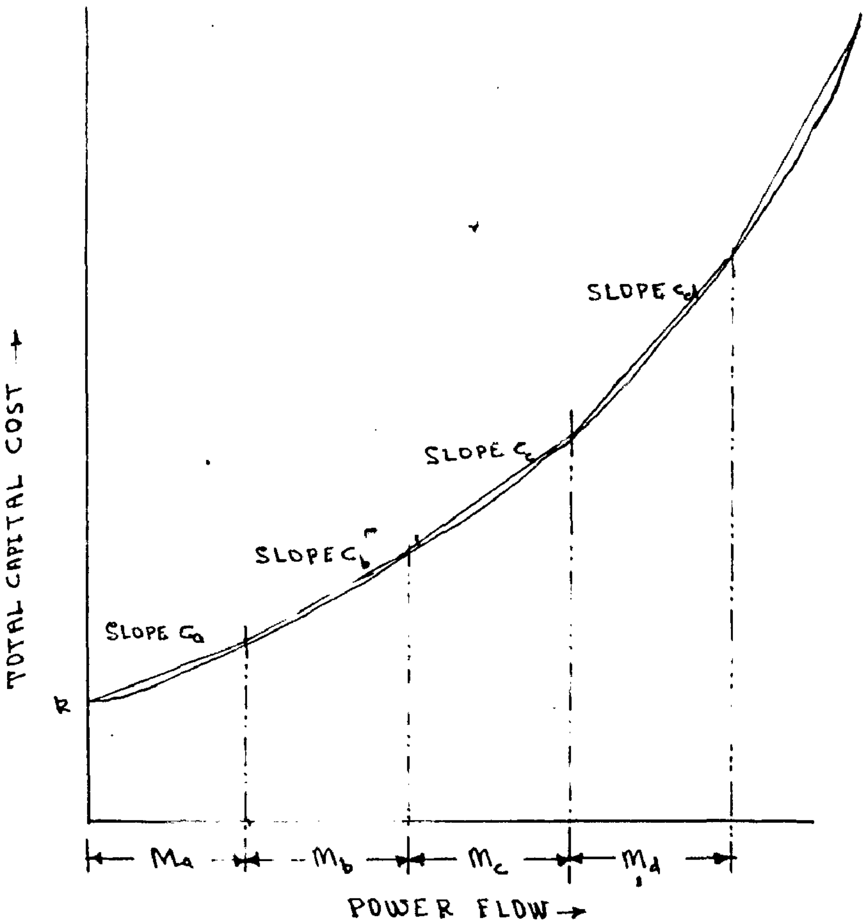



Fig. (2)

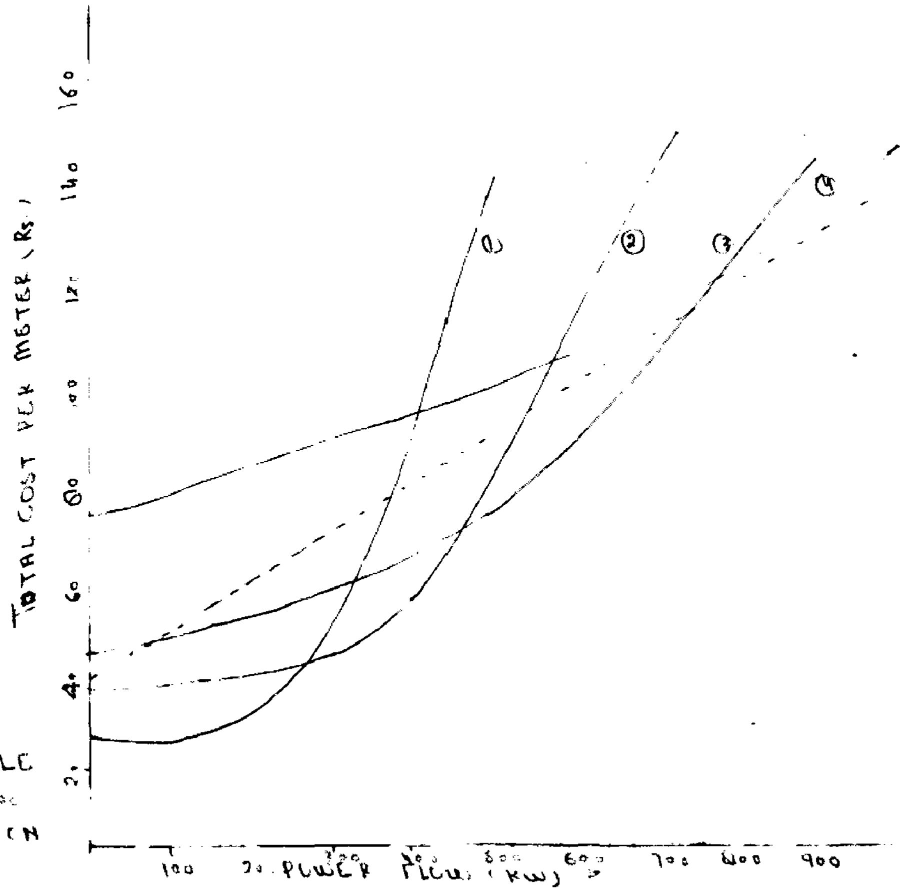



Fig 3 :
GENERLISED COSTS FOR POWER FLOW IN LOW VOLTAGE DISTRIBUTION CABLE OF mm² CROSS SECTION

## (7.6) Interpretation of Model for Low Voltage Network Design

The general form of fig. (1) suggest the idea of a special cost function for the low-voltage distribution systems, where security constraints are not required, and where the cost of losses can be quite significant. Though differnt types of cable or line are used, a general power-flow cost function can be derived that is made up of a single fixed cost plus a linear unit charge, without reference to thd type of conductor to be used, as shown in Fig. 2.

## 7.7 Time Phased Planning Models

Time phase planning model permits the dynamic requirements of the problem that is a natural extension of networks.

For multiple time intervals, each of the zero/one integer variables $p1_j$ is replaced by a specified ordered set of zero/one integer variables $D_{jt}$. Each member of the set relates to a specific time interval.

Since power flow will change with time, the number of power-flow variables is multiplied by T (For T intervals) the fixed-cost portion of the cost function is of the form

$$f = \sum_{j=1}^{m1} \sum_{t=1}^{T} K_{jt} D_{jt} \quad \ldots \qquad \ldots (24)$$

where $L_{jt}$ is present-valued cost of installing circuit J in the interval t. Power-flow constraints (security constraints) are as before, but increased in number by the factor T, because a full set of constraints must be included for each interval.

## (8) Formulation Combining Security and Reliability Constraints together

### (8.1) Requirement

In the linear integer, and mixed integer formulation discussed previously in ~~Chapters-I~~ Section (1-7) minimum cost design of interconnected system, considering the security constraints has been obtained. Operational cost also has been reduced. But it is not sure at all that a system satisfying the security constraints would be reliable also upto the desired degree of reliability. Availability and reliability are two different criteria. A system serving upto a required degree of availability is not bound to be reliable upto desired degree of reliability. But so for concerned to transmission system, it must be reliable upto desired degree of reliability, otherwise, even a minimum cost design may not enable us to escape from the various types of trouble.

### 8.2 A.C. Power Flow Equations

A.C. Power flow equation between two adjacent nodes i and j is given by

$$P_i - JQ_i = E_i^* I_i = |E_i| \angle - d_i \left[ y_{ii} \; |E_i| \; \angle \; \theta_{ii} + d_i + |y_{ij}| \; |E_j| \angle \theta_{ij} + d_j \right] \quad --- \quad (25)$$

where, $P_i$ = Active component of power sent from node i to node j

$Q_i$ = Reactive Component of power --do--

$E_i$, $E_j$ = VOltages at node i and j respectively.

$d_i$, $d_j$ = Voltages phase angle at node i and j respectively.

$$y_{ii} = y_{jj} = \frac{1}{Z_{ij} \angle \theta_{ij}}$$

$$y_{ij} = - \frac{1}{Z \angle \theta_{ij}}$$

$Z_{ij}$ = Impedence magnitude of line (i, j)

$\theta_{ij}$ = Phase angle of impedence $Z_{ij}$

$\theta_{ii}$ = $-\theta_{ij}$

From Eq. (25)

$$P_i = E_i^2 \, y_{ii} \, \text{Cos} \, \theta_{ii} + E_i E_j y_{ij} \, \text{Cos}(\theta_{ij} + d_j - d_i)$$

$$Q_i = E_i^2 \, y_{ii} \, \text{Sin} \, \theta_{ii} + E_i E_j \, y_{ij} \, \text{Sin} \, (\theta_{ij} + d_j - d_i)$$

or $$P_i = \frac{E_i^2}{Z_{ij}} \text{Cos} \, \theta_{ii} - \frac{E_i E_j}{Z_{ij}} \text{Cos} \, (\theta_{ij} + d_{ij}) \quad \ldots \quad (26)$$

$$Q_i = \frac{E_i^2}{Z_{ij}} \text{Sin} \, \theta_{ii} - \frac{E_i E_j}{Z_{ij}} \text{Sin} \, (\theta_{ij} + d_{ij}) \quad .. \quad (27)$$

where, $d_{ij} = d_i - d_j$

(8.3) A.C. Power Flow Model :

Injection of active power at node i

$$I_i = \sum_j -\frac{E_i E_j}{Z_{ij}} \cos(\theta_{ia} + d_{ia}) + \sum_j \frac{E_i^2 \cos\theta_{ii}}{Z_{ij}} \quad \ldots (28)$$

$$i = 1, \ldots\ldots n$$

where j is the index of the node directly connected with node i

Neglecting the resistance of branch (ij)

$$I_i = \sum_j \frac{E_i E_j}{Z_{ij}} \sin d_{ij} \quad \ldots \quad \ldots (29)$$

$$i = 1, 2, \ldots\ldots n$$

By Taylor's series expansion simplified A.C. power-flow equation can be obtained neglecting the higher order term of $d_{ij}$

Thus,

$$I_i = \sum_{j \in U(i)} \frac{E_i E_j}{Z_{ij}} (d_i - d_j) \quad \ldots (30)$$

where U(i) is set of all the nodes adjacent to node i

Eq. (30) may be written as

$$I_i = \sum_{j \in U(i)} C_{ij} (d_i - d_j) \quad \ldots \quad \ldots (31)$$

where $C_{ij}$ is capacity of branch ij (to be defined)

Also for conveniency it may be written that for the line 'a' from node i to j, line flow $P_{ija} = C_{ija} (d_i - d_j)$

Branch flow

$$C_{ij} = \sum_{a=1}^{S} \frac{E_{ia} \; E_{ja}}{Z_{ija}} \qquad \ldots \qquad \ldots \quad (32)$$

where $S$ = number of circuits in line a

$E_{ia}, E_{ja}$ = Voltages at node i and j for line a

$Z_{ij}$ = Impedance for line a between node i and j.

Let us define $\phi_k = \theta_i - \theta_j$ , $K = 1, 2 \ldots . r$ ... (33)

where k refers to branch k (which has been oriented from node i to node j).

The maximum loading of an EHV line can be formulated either as a maximum line flow or as a maxi mum ~~line~~ phase angle difference between its end nodes. Thoro We may suppose that for each line 'a' of each branch k, the permissible flow $\overline{P}_{ka} = C_{ka} \; \phi_{ka}$

$$k = 1, \ldots . \; r \text{ and } 'a' \in S(k)$$

where S(k) is set of all parallel lines in branch k

$C_{ka}$ = capacity of line 'a' of branch k

$\phi_{ka}$ = Voltage phase angle difference of line 'a' for branch k.

Hence inequality for line flow is

$$P_{ka} \quad \bar{P}_{ka} \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

Similarly a positive angular limit $\bar{\phi}_{ka}$ is given by

$$\phi_{ka} < \bar{\phi}_{ka} \qquad k = 1, \ldots \hat{z} \text{ and } a \in S(k) \ldots (35)$$

During the formulation we have to keep in mind that (1) If several lines of the same length are compared, the cost per unit capacity tends to decrease as capacity increases, (2) Several parallel lines are more reliable than a single line of equal admittance.

To choque a configuration, a series of outage tests must be conducted. For each of these tests a certain combination of lines should be temporarily removed from the model and the angular differences $\phi_k$ are computed using (26), (32) and (33). Let $\phi_k^{(m)}$ be the angle difference in branch k when test m is conducted. After M test has been performed, the maximum voltage phase-angle difference that can ever appear in branch k during the outage tests may be defined as

$$\phi_k^{\circ} = \text{Max} \quad |\phi_k^m| \quad k = 1, \ldots r \text{ and } m \in (1, \ldots m) \ldots (34)$$

If now we have

$$\phi_k^{\circ} \leqslant \bar{\phi}_k \qquad k = 1, \ldots r \qquad \ldots \qquad \ldots (35)$$

then the system is declared secure. If not, it means that overloads appear during some outage tests, and for each branch k, the maximum overload is ( $\phi_k^*$ - $\bar{\phi}_k$), whenever this number is positive. Since these overloads are unacceptable, it is necessary to remedy them by appropriate transmission capacity additions.

## (9) Diakoptic approach to the optimization for linear model of electrical networks

### (9.1) Requirement

Despite the improvements in the size and speed of digital computers used to solve the linear programmes, it is very necessary to find out the methods which require reduced storage capacity or fast convergence to solution or both. In order to get the first requirement, a piecewise method has been given below to solve the large linear programmes.

### (9.2) Power Flow Equations

Power flow through a branch may be expressed in terms of the voltage magnitude and phase angle in the non-linear form as given below :

$$P_{ij} = \frac{E_i E_j \sin (\theta_i - \theta_j)}{X_{ij}} \qquad \ldots \qquad (36)$$

$P_{ij}$ is the power flow through branch ij

$X_{ij}$ is the reactance of the branch ij (resistance neglected)

$E_i$ and $E_j$ are the voltage magnitudes at node i and j respectively

$\theta_i$ and $\theta_j$ are the voltage angles at node i and j respectively

Linearization of the power-flow equation is done, for the application of the linear programming. A linearization scheme is obtained by the application of following assumptions

(a) $V_i, V_j = 1$

(b) $\sin(\theta_i - \theta_j) = \theta_i - \theta_j$

Hence the power flow equation (36) is converted into the form

$$P_{ij} = \frac{\theta_i - \theta_j}{X_{ij}}$$

$$= \frac{\theta_i - \theta_j}{y_{ij}} \qquad \ldots \qquad \ldots \qquad (37)$$

where $y_{ij} = \frac{1}{X_{ij}}$

## (9.3) Problem Formulation for the Piecewise Solution

Let us consider the network shown in Fig. 3.

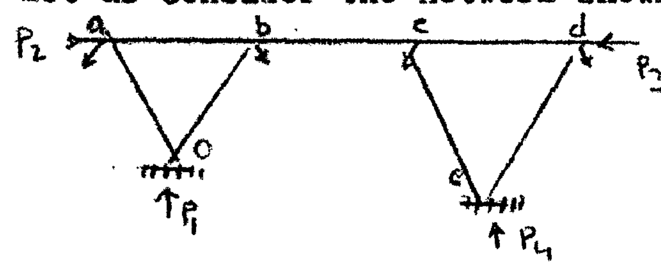
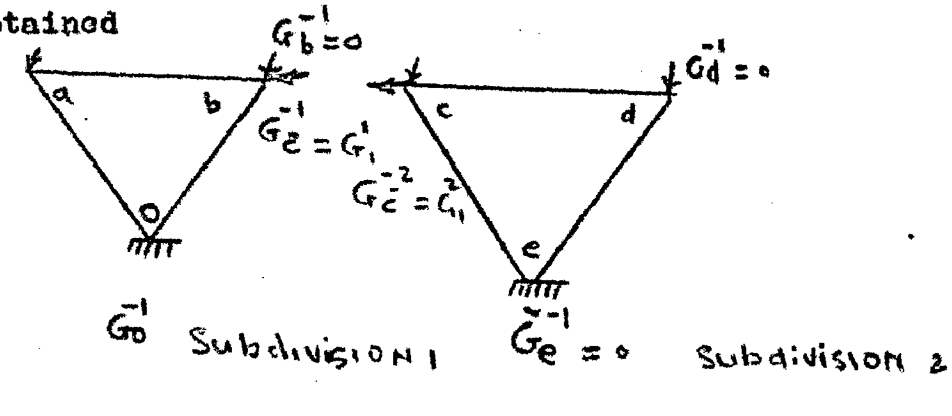



Fig.(3)

Bysplåtting, say, node O into two, the network shown in Fig. 4 is obtained



The shaded nodes in fig. 4 are reference nodes for each subdivision. The nodal equation for subdivision may be written as

$$
\begin{aligned}
y_{oo}\,\theta^1{}_o - y_{oa}\theta^1{}_o - y_{ob}\theta^1{}_b \qquad\qquad\qquad\qquad & + P'_{go} - P'_{lo} + G_o^{-1} = 0 \\
- y_{ao}\theta^1{}_o - y_{aa}\theta^1{}_a - y_{ab}\theta^1{}_b \qquad\qquad\qquad & + P^1{}_{ga} - P^1{}_{la} + G_a^{-1} = 0 \\
- y_{bo}\theta^1{}_o - y_{ba}\theta^1{}_a - y_{bb}\theta^1{}_b - y_{bc}\theta^1{}_c & + P^1{}_{gb} - P^1{}_{lb} + G_b^{-1} = 0 \\
- y_{cb}\theta^1{}_b + y_{cc}\theta^1{}_c & + P^1{}_{gc} - P^1{}_{l_c} + G_c^{-1} = 0
\end{aligned}
$$

... (38)

where $P^1{}_{gi}$ is total generated power at node i

$P^1{}_{li}$ is load at node i

$G^{-1}{}_i$ is power injection due to tearing at node i

and subscript 1 indicates subdivision 1.

Excluding the equation a t the reference node from the above set of equations, we get following matrix equation :

$$- y^1\theta^1{}_o + Y^1\theta^1 + P^1{}_g - P^1{}_l + G^{-1} = 0 \quad \ldots \qquad (39)$$

The total generated power at the ith node $P^1{}_{gi}$ may be related to the output of the individual generators $P^1$ as

$$P^1{}_{gi} = \sum_{j=1}^{m} C_{ij} \; P^1{}_j$$

where m is the number of individual generators and $C_{ij}$ is equal to 1 if generator j is at node and zero otherwise. In matrix form the above Eq. can be written as

$$P^1_g = C^1_p P^1 \quad \dots \quad \dots \quad (40)$$

where $C^1_p$ is topological matrix

The node power injection $G^{-1}$ may be related to the power transfor $G^1$ through the split nodes

$$G^{-1} = C^1_g G^1 \quad \dots \quad (41)$$

where $C^1_g$ is a topological matrix.

Substituting from Eq. (40) and (41) into Eq. (39) we get

$$- y^1\theta^1_o + Y^1\theta^1_1 + C^1_p P^1 + C^1_g G^1 - P^1_l = 0 \quad \dots \quad (42)$$

The equation at the reference which was eliminated may now be replaced by the relationship which sums the total power transferred into a subdivision to zero. Thus we get

$$S^1_p P^1 + S^1_g G^1 - S^1_l P^1_l = 0 \quad \dots \quad \dots \quad (43)$$

where S are summation row vectors with all the elements equal to one.

The power output of each generator is constrained between an upper limit and lower limit giving

$$P^1_m \leq P^1 \leq P^1_M \quad \dots \quad \dots \quad (44)$$

Let $P^1_e$ is the amount of generation above the minimum unit, Eq. (42), (43) and (44) may be written as :

$$-y^1\theta^1_o + Y^1\theta^1 + C^1P\, P^1_e + C^1_G G^1 - p^1_1 + C_p\, p^1_M = 0 \qquad \ldots (45)$$

$$S^1_p\, P^1_e + S^1_G G^1 - S^1_1 P^1_1 + S^1_p\, P^1_m = 0 \qquad \ldots (46)$$

$$0 \quad P^1_o \quad P^1_M - P^1_m \qquad \ldots \qquad \ldots (47)$$

The relation for the thermal limits on the lines is

$$-\frac{t_{ij}}{y_{ij}} \leq \theta_i - \theta_j \leq \frac{t_{ij}}{y_{ij}}$$

In general the thermal limits for subdivision may be expressed as

$$-T^1 \leq C^1_t\theta^1 \leq T^1 \qquad \ldots \qquad \ldots (48)$$

where $C^1_t$ is branch nodal incidence matrix

The relations for subdivision 2 may be written in similar manner by changing the superscript 1 to 2 in Eq. (45) to (48).

From the above relations (45) to (48) it is clear that the constraints for every subdivision are written in terms of the variables of every subdivision separately.

The constraints of the split node, however, link these constraints together.

The constraints for the above example may be written as :

$$G^1_1 - G^2_1 = 0$$

$$\bar{\theta}_c - \theta_c = 0$$

or in general form

$$C_g G = 0 \quad \ldots \quad \ldots \quad ~~(53)~~ \; (49)$$

and $$C_\theta \theta = 0 \quad \ldots \quad \ldots \quad ~~(54)~~ \; (50)$$

Constraints (45) to (50) may be expressed in matrix form to give the following programming tableau.

<table>
<tr><td>CONST. 45, 46, 47, 48</td><td></td></tr>
<tr><td>.</td><td>CONST. FOR 2nd SUB DIV.</td></tr>
<tr><td colspan="2">CONSTRAINT 49, 50</td></tr>
</table>

Tableau 1

The objective function of the linear programming to be considered is the generation cost which may be expressed in linear form as

$$\text{Cost} = C^t_g P$$

or $$\text{Cost} = C^t_g P_m + C^t_g P_e$$

Since the first term is constant the function to be minimised is

$$\text{Cost} = C^t_g P_e \ldots \quad \ldots \quad (51)$$

The general linear programming problem have inequality constraints. In these cases, we convert inequalities to equalities using the slack variables. For this purpose the coefficient of slack variables in objective function is taken equal to zero.

So far in the case of Optimal network expansion problems', we have t~~o~~ find out minimum cost design satisfying the security constraints. Thus our aim is to find the number of circuits between each pair of substations to be interconnected. The number of the circuits along all the possible paths will be variables appearing in the constraints. As we have already discussed that linear programming give a solution in which variables are not restricted to be integer. Thus integer linear programming which gives the solution consisting of all integer values will be recommended for this purpose.

## 2. Integer Programming

In an all integer method the problem is stated with given integer coefficients; all calculation result in integer coefficients at each iteration.

### 2.1 Optimality Theory for Integer Programming

We have to solve integer programming problem satisfying the conditions (1), where $a_{ij}$, $b_i$ and $C_j$ are given integers. The method used to find the optimal solution of Eq.(1) consists

# CHAPTER - III

## SOLUTION TECHNIQUES USING DIGITAL COMPUTER:

### 1. Linear Programming

A large number of problems in mathematical programming are solved as linear programmes. The general form of linear programming problem is :

To find $x_j \geqslant 0$ for $J = 1, 2, \ldots n$ that minimize Z

when

$$\sum_{j=1}^{n} C_j X_j = Z$$

$$\sum_{j=1}^{n} a_{ij} X_j = b_i \qquad \ldots \quad (1)$$

where $a_{ij}$ $b_i$ and $C_j$ are given constants. The linear form Z is called objective function.

The solution to the problem usually results in fractional $X_j$ values. The integer programming problem arises when the $X_j$ variables are restricted to integer values.

The general linear progammme is always defined in terms of minimization, when we have to maximize $Z = \sum C_j X_j$, we simply convert it to a minimization problem by minimizing $-Z$.

of making a series of changes of variables to achieve the transformation

$$X_j = d_j + \sum_{k=1}^{n} d_{jk} y_k \quad j=1,2\ldots n \ \ldots \quad (2)$$

The integer constants, $d_j$, $d_{jk}$ are developed in an iterative procedure during the solution of the problem. The initial transformation is established by writing Eq. (1) in parametric form as

$$Z = \sum_{j=1}^{n} C_j y_j$$

$$x_j = y_j \geqslant 0 \quad J = 1,2,\ldots\ldots \quad (3)$$

$$x_{n=i} = -b_i + \sum_{j=1}^{n} a_{ij} y_j \quad 0, \quad i = 1,2\ldots m$$

The variables $x_{n+i}$ for $i = 1,2, \ldots m$, are surplus variables. Eliminating $x_j$ from ~~(1)~~, putting (2), we get the equivalent programme:

To find integer $y_j \geqslant 0$ for $J = 1, 2, \ldots n$ that minimize Z, when

$$Z = \bar{Z}_o + \sum_{J=1}^{n} \bar{C}_j y_j$$

$$X_j = d_j + \sum_{k=1}^{n} d_{jk} y_k \quad 0 \quad J = 1,2\ldots n \quad \ldots (4)$$

$$X_{n+i} = -\bar{b}_i + \sum_{j=1}^{n} \bar{a}_{ij} y_j \quad 0, \quad i = 1,2\ldots m$$

The constants $\bar{Z}_o$, $\bar{C}_j$, $\bar{b}_i$ and $\bar{a}_{ij}$ are developed as result of transformation with

$$\bar{Z}_o = \sum_{j=1}^{n} C_j d_j$$

$$\bar{C}_j = \sum_{k=1}^{n} C_k d_{kj}, \quad j = 1,2 \ldots 1n \qquad \ldots \qquad (5)$$

$$\bar{b}_i = b_i - \sum_{j=1}^{n} a_{ij} d_j, \quad i = 1,2 \ldots m$$

$$\bar{a}_{ij} = \sum_{k=1}^{n} a_{ik} d_{kj}, \quad i = 1,2 \ldots m; \; j = 1,2, \ldots n$$

Theorem 1

If the constants $\bar{C}_j \geqslant 0$, $d_j \geqslant 0$, and $b_i \leq 0$ for all i and j, then the minimal solution to Eq. (4) is given by

$$Z = \bar{Z}_o \quad y_j = 0 \quad \text{for } J = 1,2 \ldots n.$$

## 3. Lexicographical Method

### 3.1 Improving A Nonoptimal Solution

Let us consider that all $C_j \geqslant 0$ we can write (4) as

$$x = B + \sum_{j=1}^{n} A_j y_j \qquad \ldots \qquad \ldots (6)$$

where $x_j$ is a column vector with components $Z, x_1, \ldots x_{n+m}$

B is a column vector with components $\bar{Z}_o$, $d_1$, $d_2 \ldots d_n$ $-\bar{b}_1 \ldots -\bar{b}_m$

and $A_j$ is a column vector with components $\bar{C}_j$, $d_{1j}$, $d_{2j}, \ldots d_{nj}$, $\bar{a}_{1j}$, $\bar{a}_{2j}$, .... $\bar{a}_{mj}$.

Initially (3) is also in the form of (6) with B components

$$0, 0, \ldots 0, -b_1, -b_2, \ldots -b_m$$

and $A_j$ components $C_j$, 0, 0, .... 0, 1, 0, ... 0, $a_{1j} \ldots a_{mj}$.
The 1 appears as the $(J+1)^{th}$ component of $d_j$

Lexicographic Ordering:

In order to ensure a finite algorithm we use lexicographic ordering in considering $A_j$ and B vectors.

A vector R is defined as lexigoraphically greater than zero or lexicopositive, if R has at least one nonzero component, the first of which is positive. A vector R is less than vector S in the lexicographic sense, if the vector S minus R is ~~lexicographic~~ lexicopositive

If (3) is written in the form of (6), we see that $A_j > 0$ because all $C_j > 0$. Suppose at some iteration we have achieved a form like (6) where $A_j > 0$ and one or more components of B are negative. At this stage optimality conditions of Theorem 1 are not fulfilled. Select a row from (6) with negative B components. Let the inequality be

$$\sum_{j=1}^{n} a_j y_j \geq b_o \quad \ldots \qquad (7)$$

where $-b_o$ is the negative component of B. FOR THE inequality to have a solution, at least one of the $a_j$ is positive. Taking D a positive number, any value $a_{j/D}$ may be written as

$$\frac{a_j}{D} = \left(\frac{a_j}{D}\right) - \frac{r_j}{D}, \quad 0 \leq r_j < D \quad \ldots \quad (8)$$

where (a) denotes the smallest integer greater than or equal to a. Thus after dividing by D in (7) and using (8) we have

$$\sum_{j=1}^{n} P_j y_j \geqslant \frac{b_o}{D} + \frac{1}{D} \sum_{j=1}^{n} r_j y_j$$

~~where $P_j = (a_j/D)$ and $\frac{1}{D} \sum_{j=1}^{n} r_j y_j \geqslant 0$;~~

Hence we have $\sum_{j=1}^{n} P_j y_j \geqslant \frac{b_o}{D}$ ... ... (9)

Since the left hand side of (9) can have only an integer value, then

$$\sum_{j=1}^{n} P_j y_j \geqslant q \quad \ldots \quad (10)$$

where $q = (b_o/D)$

It is desirable to make a change of variables for $y_s$, where $a_s > 0$; then if D is chosen so that $D \geqslant A_s$ FROM (10), we get,

$$y_s = q - \sum_{j \neq s} P_j y_j + y_s^1 \quad \ldots . \quad (11)$$

where integer $y_s^1 \geqslant 0$ represents the surplus variable in (10). If $y_s$ from (11) is substituted into (6), we have

$$x = B^1 + \sum_{j=1}^{n} A^1_j \, \vartheta_j \qquad \ldots (12)$$

The coefficients of 12 are

$$A^1_j = A_j - P_j A_s \qquad J \neq s$$

$$A^1_s = A_s \qquad \ldots (13)$$

$$B^1 = B + qA_s$$

We require that the

$A^1_j$ remain lexicopositive,

Thus

$$A_j - P_j A_s > 0, ~~J \neq s~~ \ldots \qquad \ldots (14)$$

Condition given by (14) enables us to determine the index S and a value for D, which produces $P_j$ values. If $J^+$ is defined as the set of indices J where $a_j > 0$ from (7), the index S is chosen by the rule

$$A_s = \text{Lexicographically min } A_j \quad J \in J^+$$

or $\quad A_s = 1 - \min A_j, \quad J \in J^+$

while developing the computer programme index S is taken as LMIN for easiness.

$A_s$ is the lexicographically smallest of the $A_j$ for $J \in J^+$. We define $l_{min}$ to be lexicographic minimum. Define integer value $U_j$ as the largest integer that maintain $A_J - U_J A_s$ lexicopositive for $J \in J^+$ also take $U_s = 1$. Condition (14) is fulfilled if $P_s \leq U_j$.

Now we are able to determine D. If $D = a_k/U_k$ for some index $k \in J^+$, then $P_k = U_k$. If $D \geqslant a_k/U_k$, then $P_k$ can only be reduced and condition (14) holds.

It is an important factor to be considered that in order to reduce the number of iterations, q is made as large as possible to bring a greatest change in B. Thus to produce a large q value, D is made as small as possible, Hence we find $D_1$ by the rule

$$D_1 = \max_{J \in J^+} \frac{a_j}{U_j}$$

It is notable that $D_1$ may be fractional and that D $a_s$ so that $P_s$ is unity in (10).

When (12) is developed, we denote B' and $A'_j$ values to be the current B and $A_j$ values. Hence (12) is of the same form as (6), and repeat the process begun with inequality (7). Eventually a form is developed where B has no negative components. At this point the form (6) is like (4) and the condition of theorem 1 are fulfilled. The optimal solution to (1) is then produced by the first ( n + 1) components of B.

## FLOW CHART FOR LEXICOGRAPHICAL METHOD

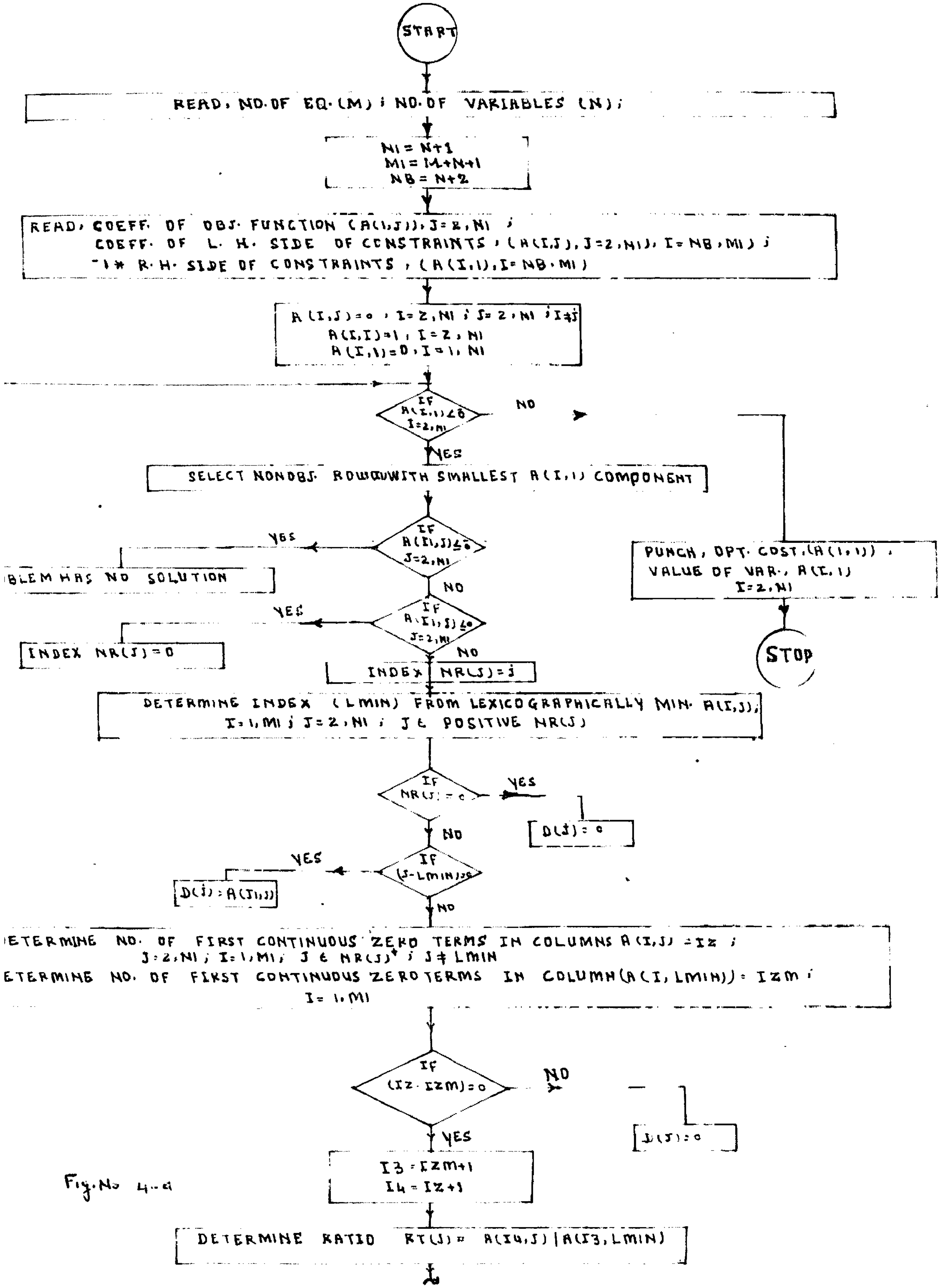



Fig. No 4.a

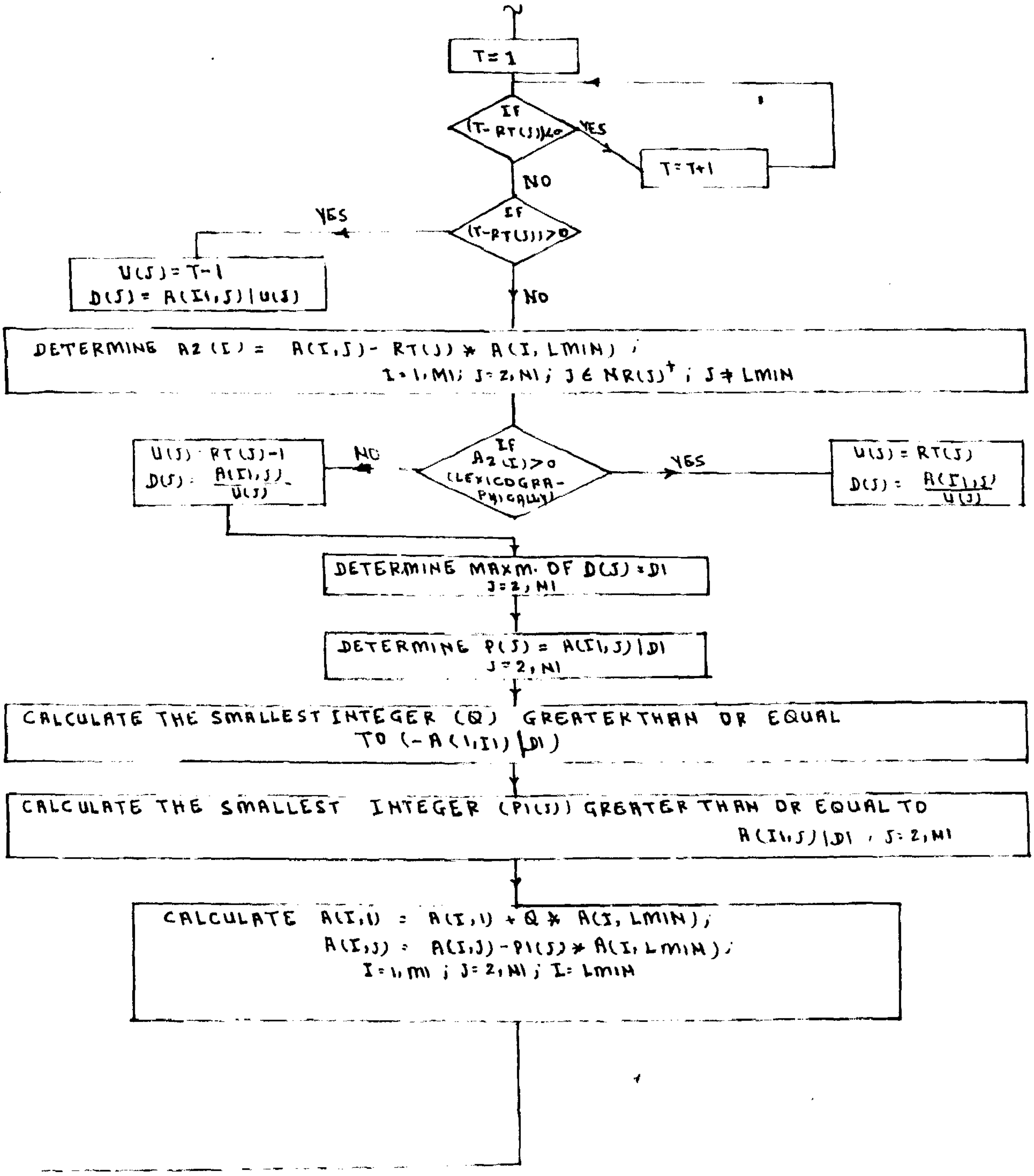

T = 1
IF (T-RT(J))<0
YES
T = T+1
NO
IF (T-RT(J))>0
YES
U(J) = T-1
D(J) = A(I1,J)|U(J)
NO
DETERMINING A2(I) = A(I,J) - RT(J) * A(I, LMIN) ;
I = 1, M1; J = 2, N1; J ∈ NR(J)+ ; J ≠ LMIN
IF A2(I) > 0 (LEXICOGRAPHICALLY)
NO
U(J) = RT(J)-1
D(J) = A(I1,J) / U(J)
YES
U(J) = RT(J)
D(J) = A(I1,J) / U(J)
DETERMINE MAXM. OF D(J) = D1
J = 2, N1
DETERMINING P(J) = A(I1,J)|D1
J = 2, N1
CALCULATE THE SMALLEST INTEGER (Q) GREATER THAN OR EQUAL
TO (-A(1,I1)|D1)
CALCULATE THE SMALLEST INTEGER (P1(J)) GREATER THAN OR EQUAL TO
A(I1,J)|D1 , J = 2, N1
CALCULATE A(I,1) = A(I,1) + Q * A(I, LMIN) ;
A(I,J) = A(I,J) - P1(J) * A(I, LMIN) ;
I = 1, M1 ; J = 2, N1 ; I = LMIN

Fig. No. 4(b)

3.2 Algorithm 1

?. Develop a tableau by listing the column B, $A_1$, $A_2$, ... n.
Initially $\bar{Z}_0 = 0$, $d_j = a$, $\bar{b}_i \triangleq b_i$; $\bar{C}_j = C_j \geqslant 0$, $d_{ii} = 1$, $d_{ij} = o$ for $i \neq j$; $\bar{a}_{ij} = a_{ij}$ Go to 2

2. If $d_j \geqslant 0$ for $J = 1, 2, \ldots n$

and $\bar{b}_i \leq 0$ for $i = 1, 2, \ldots, m$

the minimal solution is $Z = \bar{Z}_0$ $x_j = d_j$ doe $J = 1, 2, \ldots n$ Stop
Otherwise select the nonobjective row with the smallest B component. Suppose the row i$ - $b_o$, $a_1$, $a_2$, ... $a_n$. Define $J^+$ as the set of indices J where $a_j \geqslant 0$. If all $a_j \leq o$, the problem has no solution; stop otherwise, go to 3.

Flow chart for algorithm (1), taking into account the modification given in section (5) is drawn on front side. Fig No. 4a and 4-b

3. Determine index S from $A_s = 1 - \min_{s \in J} + A_j$. Find the largest integer $U_j$ that maintains $A_j - U_j A_s > o$ for $J \in J^+$ $J \neq S$. If $A_s$ and $A_j$ begin with an unequal number of zeros, take $U_j = \infty (D_j = o)$. Otherwise, suppose the first non-zero terms are $e_j$ and $e_s$. If $e_s$ does not divide $e_j$, take $U_j = (e_J/e_s)$, where (a) denotes the greatest integer less than or equal to a.

If $e_s$ divide $e_j$, then $U_j = e_J/e_s$ if $A_j - (e_J/e_s)A_s > o$ and $U_j = e_j/e_s - 1$ otherwise. Also $U_s = 1$. Take $D_1 = \max_{J \in J^+} a_j/U_j$ Go to 4.

Calculate $q = (b_o/D_1)$, $P_j = (a_i/D_1)$ and new column values $B^1 = B + qA_s$ and $A^1_j = A_j - P_jA_s$ For $J \neq S$. Designate $B^1$ and $A^1_j$ to be the current B and $A_j$. Return to 2.

4. Bounded Variable Problems

So far we are concerned to optimal network expansion, and hence come into need of restricting the number of circuits along a particular path say one or two. Thus we are interested in solving the integer programming problems when the variables have upper bound restraints. We have to find the solution of (1) with the addition constraints $x_J \leq Am_j$ For $J = 1,2,..n$, where $Am_j$ are given integer values.

The solution of the bounded variable problem can be obtained by Algorithm 1 with the additional inequalities $-x_j \geq -Am_j$ But the problem wi ll grow considerably large. Hence a different approach is given below.

Suppose at some ~~inter~~ iteration we have achieved a form like (6), where $A_j \geq o$ and one or more components of B have value $d_j < Am_j$

Select one such row from (6) and have

$$x_j = d_j + \sum_{K=1}^{n} d_{jk}y_k \quad \ldots \quad \ldots \quad (15)$$

$$\leq Am_j$$

The $y_k$ must satisfy

$$- \sum_{k=1}^{n} d_{jk} y_k \geq d_j - Am_j \qquad \ldots \qquad (16)$$

If we define $a_k$ and $b_o$ by

$$a_k = - d_{jk}, \quad k = 1,2,\ldots, n$$

$$b_o = d_j - Am_j$$

then (32) is exactly of the form given by (7). We develop transformation (11) and the new form of (6) given by (12). The lexicographic property of the $A_j$ is maintained. The bounded variable problem is solved by the use of (16).

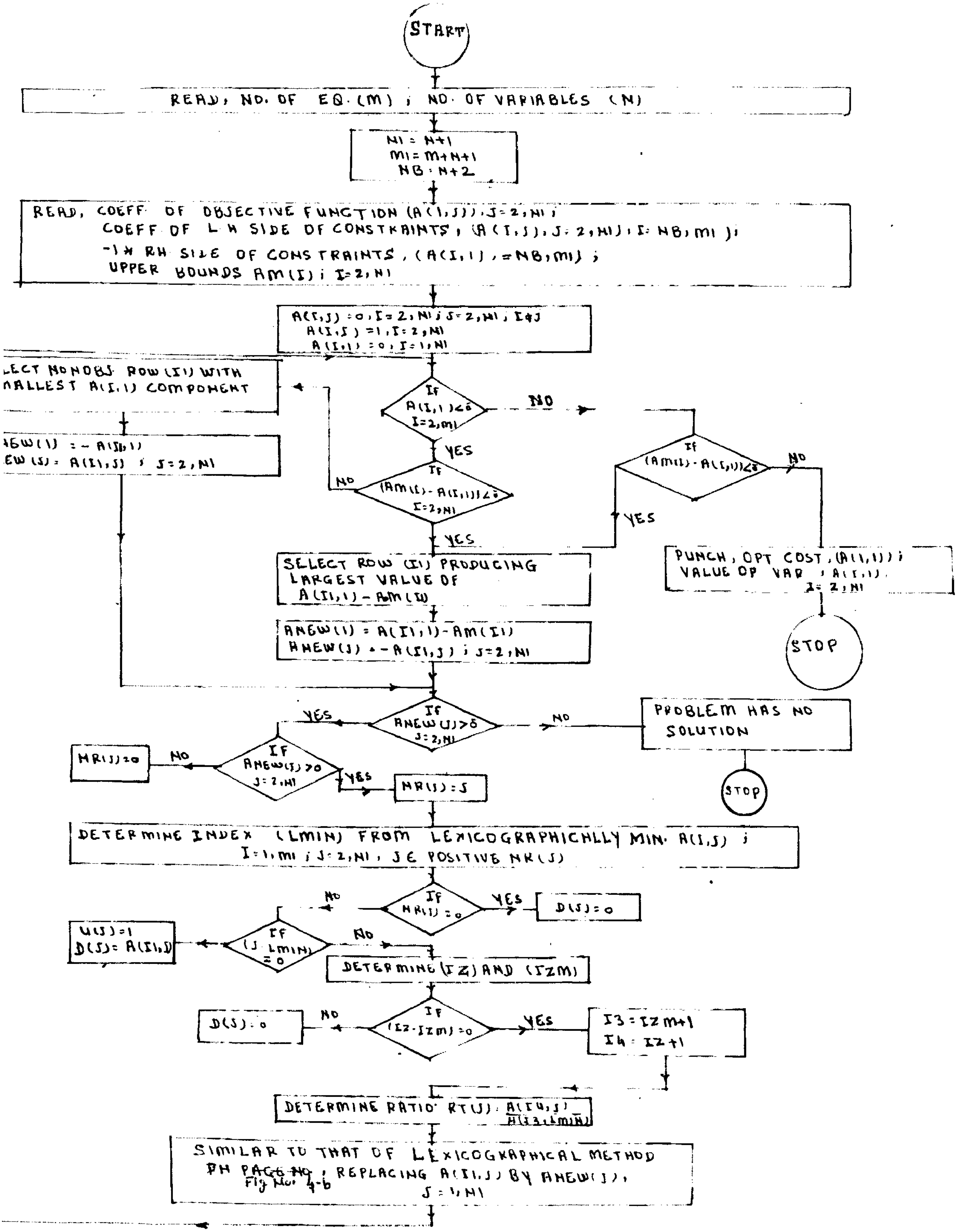
Algorithm 2

1. Same as step 1 of Algorithm 1 with the additional listing of $Am_1$, $Am_2$, ... $Am_n$.

2. (a) If $o \leq d_j \leq Am_j$ for $J = 1,2,\ldots,n$ and $\bar{b}_i \leq o$ for $i = 1,2,\ldots,m$, the minimal solution is $Z = \bar{Z}_o$, $x_j = d_j$ for $J = 1,2,\ldots,n$; stop. Otherwise go to 2(b)

   (b) If $d_j \leq Am_j$ for $J = 1,2,..n$, go to 2(c). Otherwise, select the row with $d_j$ component of B that produces the largest value of $d_j - Am_j > o$. Take $b_o$ $d_j - Am_j$ $a_k = - d_{jk}$ for $K = 1,2,\ldots n$, As the row picked. Go to 2(d).

# FLOW-CHART FOR BOUNDED VARIABLE PROBLEM



(c) Select the nonobjective row with the smallest B component. Suppose the row is $-b_o, a_1, a_2 \ldots a_n$ Go to 2(d).

(d) Define $J^+$ as the set of indices j where $a_j$ o. If all $a_j$ o, the problem has no solution; stop. Otherwise, go to 3.

3. Same as 3 of Algorithm 1

4. SAme as 4 of Algorithm 1

Flow chart for algorithm (2), taking into account the modification given in section (5) is drawn on front page - Fig No. 5

## 5. Modification in Algorithms for Development of computer programme

In present work, the vector B and $A_j$ for J = 1,2,...n are written in the form of a matrix. A(I,j). for I=1,2,.., M1 and J = 1,2,...N1

where

$$N1 = n + 1$$

$$M1 = N + M + 1$$

n is number of variables

and m is number of constraints.

## 6. Enumeration METHOD

Sometimes while solving the optimization problem, an enumeration problem has advantages over other methods. It is quite useful if the value of the variables is small say, zero or one. For the higher value of the variables existing in the solution, the memory requirement will be large. Of course, Since the variables take on only discrete values, they can be listed easily.

We have to find integer values of $x_j$ o for $J = 1,2,\ldots n$ that minimise Z when

$$\sum_{j=1}^{n} C_j x_j = Z$$

$$\sum_{j=1}^{n} a_{ij} x_j \geqslant b_i, \quad i = 1,2,\ldots, m \quad \ldots \quad (1)$$

To solve (1) directly by enumeration method, we begin by finding all the values of Z from

$$c_1x_1 + c_2x_2 + c_3x_3 + \ldots\ldots C_nx_n \quad \ldots \qquad (2)$$

that are produced by non negative integer values of $x_j$ where the $C_j$ are positive numbers. We find all feasible values of Z as a monotonic increasing sequence. For a feasible value of Z, say $Z_o$, we also find $x_j$ values that produce Z.

In the process of developing monotonic sequence of the Z, we obtain the solution to (1) when the smallest Z value in the sequence has corresponding $x_j$ values that satisfy the constraints. Thus the enumeration of (2) is performed in order of increasing values and stopped when the constraints are satisfied.

OW-CHART FOR ENUMERATION METHOD

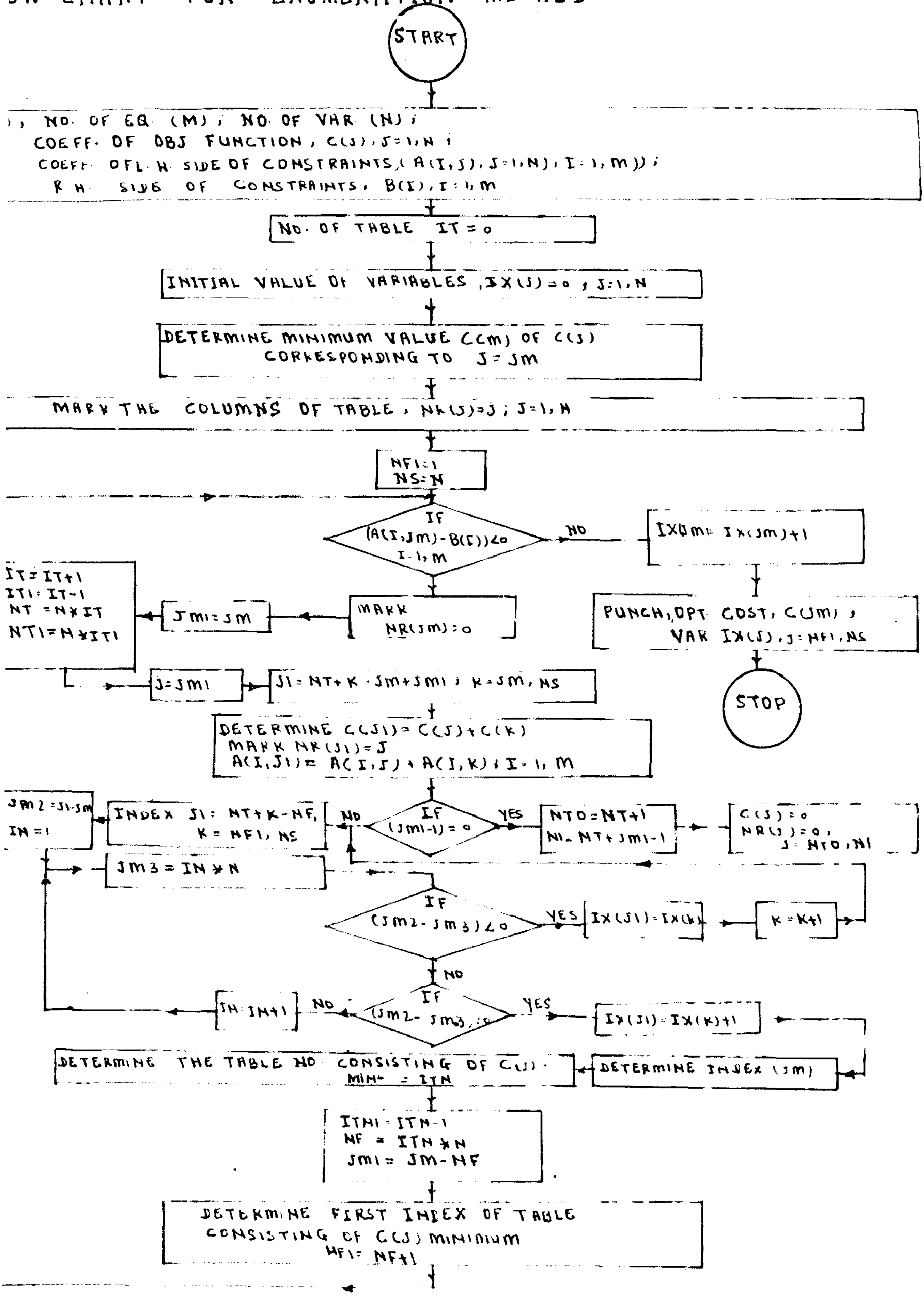



Fig. No. (6)

Theorem 2

If $Z_o$ is a feasible value for $Z = \sum_{j=1}^{n} C_j X_j$ that is produced by integer values $X^o_j$ for $J = 1,2,..,n$, then other feasible values of Z are produced by

$$Z_o = Z_o + C_j \quad \text{for } J = 1,2,\ldots, n$$

Flow chart for the Enumeration method is given on ~~front page~~ Fig No. 6

## 7. Branch and Bound Method

The branch and bound procedure for the solution to integer programming problems is useful for the problems having few variables. For the problem consisting of many variables, however, it requires extremely large computer storage capacity. This procedure is applicable to mixed integer programming problem also.

### 7.1 Branch and Bound solution Strategies Investigated

In the network synthesis even for mixed integer model, approximately 35% of the variables to be determined are restricted to integer values only, mainly zero or one. The speed with which these variables can be found effectively ~~mainly~~ determines the computer time. The progressively improving sequence of feasible solutions, produced during the search, has advantages in power-network design where it is often difficult to represent all constraints accurately, and a range of good suboptimal alternatives may therefore be of greater value than the true optimum.

We have to solve the mixed integer programming problem

$x_j \geqslant o$ for $J = 1,2 \ldots n$ that minimize $Z$ when

$$\sum_{j=1}^{n} C_j X_j = Z$$

$$\sum a_{ij} X_j = b_i \qquad i = 1,2,\ldots m. \qquad \ldots (16)$$

$$X_j \leqslant m_j \qquad j = 1,2\ldots n.$$

To handle the mixed integer case where only some of the $X_j$ variables are restricted to be integers. The remaining variables can be integers or fractions and are never picked to be the integer restricted variables only. Let us consider (16) linear programme by relaxing the integer constraints on the $X_j$. The resulting problem can be solved by the simplex method, which converts the equation (16) to an equivalent problem i.e. to find nonnegative integers $x_j$ $m_j$ that minimize $Z$ when

$$- Z + \sum_{j=m+1}^{n} \bar{C}_j X_j = - \bar{Z}_o$$

$$x_i + \sum_{j=m+1}^{n} \bar{a}_{ij} X_j = \bar{b}_i \quad i = 1,2,\ldots m \quad \ldots \quad (17)$$

where the first m variables and the last (n - m) variables have been arbitrarily selected as the basic and non basic variables respectively. The values of the objective function and basic variables are given by

$$Z'_o = \bar{Z}_o + \sum_{J \notin U} \bar{C}_j \; m_j$$

$$b'_i = \bar{b}_i - \sum_{J \notin U} a_{ij} \; m_j$$

where V is the index set of nonbasic variables.

If the continuous solution has all $b'_i$ (to be integer) as integer programme (16) is solved. If any of the $b'_i$ are fractional, we start the free enumeration. We consider the zero node of a tree as corresponding to the fractional solution with objective value $Z'_o$. Integer $X_i$ must satisfy $x_i \leq (b_i)$ or $X_i \geq (b'_i) + 1$. We branch to level one of the tree by adding either the upper or the lower bound to the constraints of (17). Then we find a new continuous minimum for Z. Then we follow the other branch. The minimal integer solution occurs for one of the branches. Let us assume that after some minimization we are at level r of the tree. We have a form like (17) with the additional constraints $o \leq P_j \leq X_j \leq \bar{m}_j$ ; $\bar{P}_j$ and $m_j$ are lower and upper bounds, respectively. Initially, $\bar{P}_j = o$, $\bar{m}_j = m_j$.

The values of the objective function and basic variables are given by

$$Z'_o = \bar{Z}_o + \sum_{j \in U} \bar{C}_j \; \bar{m}_j + \sum_{j \in L} \bar{C}_j \; \bar{P}_j \quad \ldots (18)$$

$$b'_i = \bar{b}_i - \sum_{j \in U} \bar{a}_{ij} \; \bar{m}_j - \sum_{j \in L} \bar{a}_{ij} \; \bar{P}_j$$

where $Z'_o$ is objective value.

$b'_i$ is ith basic variable ($\bar{P}_i \leq b'_i \leq m'_j$)

and L is the index set of nonbasic variables at their lower bounds.

—

# CHAPTER - IV

## PROBLEM FORMATIONS AND RESULTS

### Example 1

A design is required to connect four 132/33 K.V. Substations $S_2$ - $S_5$ located at suitable points within a specific area to one 275/132 K.V. substation $S_1$ established at a particular place say, 'A', where a firm source of power is available. The average load at each 132/33 K.V. substation is taken 60 MVA, except at substation $S_4$, where the load is assumed to be 120 MVA, It is assumed that circuites of 120 MVA capacity would be used on the proposed network.

The estimated cost of one circuit along each path is shown in Table (1).

| Path | Cost per circuit | Path | Cost per circuit |
|---|---|---|---|
| | $\times 10^4$ R.S. | | $\times 10^4$ R.S. |
| $P_{12}$ | 312 | $P_{24}$ | 391 |
| $P_{13}$ | 352 | $P_{25}$ | 220 |
| $P_{14}$ | 193 | $P_{34}$ | 204 |
| $P_{15}$ | 299 | $P_{35}$ | 317 |
| $P_{23}$ | 315 | $P_{45}$ | 370 |

<u>Solution</u>

Let each group of one, two, three and four substations must have $h_1$, $h_2$, $h_3$, $h_4$ circuits respectively connected into it

$h = N1 + M1$   $S$ = maximum rating of circuits on proposed network.

$$\therefore \quad N_1 = \left\{\frac{S_L}{S}\right\}, \left\{\frac{S_L}{S}\right\} = \text{Smallest integer greater than or equal to } S_L/S$$

$$M_1 = \text{Small integer (for firm supply)}$$

$$\therefore \quad h_1 = \left\{\frac{60}{120}\right\} + M_1$$

TAKING $M_1 = 1$

$$h_2 = \left\{\frac{60 \times 2}{120}\right\} + 1 = 2$$

$$h_3 = \left\{\frac{60 \times 3}{120}\right\} + 1 = 2 + 1 = 3$$

$$h_4 = \left\{\frac{60 \times 3 + 120}{120}\right\} + 1 = 3 + 1 = 4$$

Hence design equations (constraints) satisfying the group security are given as :

$$P_{12} + P_{32} + P_{42} + P_{52} \geq 2 \qquad \ldots (1)$$

$$P_{13} + P_{23} + P_{43} + P_{53} \geq 2 \qquad \ldots (2)$$

$$P_{14} + P_{24} + P_{34} + P_{54} \geq 2 \qquad \ldots (3)$$

$$P_{15} + P_{25} + P_{35} + P_{45} \geq 2 \qquad \ldots (4)$$

Security constraints for each load substation

$$P_{12} + P_{42} + P_{52} + P_{13} + P_{43} + P_{53} \geq 2 \quad \ldots (5)$$

$$P_{12} + P_{32} + P_{52} + P_{14} + P_{34} + P_{54} \geq 2 \quad \ldots (6)$$

$$P_{12} + P_{32} + P_{42} + P_{15} + P_{35} + P_{45} \geq 2 \quad \ldots (7)$$

$$P_{13} + P_{23} + P_{53} + P_{14} + P_{24} + P_{54} \geq 2 \quad \ldots (8)$$

$$P_{13} + P_{23} + P_{43} + P_{15} + P_{25} + P_{45} \geq 2 \quad \ldots (9)$$

$$P_{14} + P_{24} + P_{34} + P_{15} + P_{25} + P_{35} \geq 2 \quad \ldots (10)$$

Security constraints for each group of two load substation

$$P_{12} + P_{52} + P_{13} + P_{53} + P_{14} + P_{54} \geq 3 \quad \ldots (11)$$

$$P_{12} + P_{42} + P_{13} + P_{43} + P_{15} + P_{45} \geq 3 \quad \ldots (12)$$

$$P_{12} + P_{32} + P_{14} + P_{34} + P_{15} + P_{35} \geq 3 \quad \ldots (13)$$

$$P_{13} + P_{23} + P_{14} + P_{24} + P_{15} + P_{25} \geq 3 \quad \ldots (14)$$

Security constraints for each group of 3 load substations.

$$P_{12} + P_{13} + P_{14} + P_{15} \geq 4 \qquad \ldots (15)$$

Security constraint for the group of 4 load substations.

The objective function to be minimised is

$$312\ P_{12} + 352\ P_{13} + 193\ P_{14} + 299\ P_{15} + 315\ P_{23} + 391\ P_{24}$$
$$+ 220\ P_{25} + 204\ P_{34} + 317\ P_{35} + 370\ P_{45} = Z$$

The above equations are solved by Lexicographical method and bounded variable problem method (Circuits along a path are restricted to be 3 at most). The result obtained is given in TABLE 2.

Integer Solution

| Path | No. of circuits | Path | No. of circuits |
|---|---|---|---|
| $P_{12}$ | 1 | $P_{15}$ | 1 |
| $P_{13}$ | 1 | $P_{25}$ | 1 |
| $P_{14}$ | 1 | $P_{34}$ | 1 |
| All other $P_{ij}$ = 0 | | | |

Cost = Z = Rs 1580 x $10^4$

Example 2

The value of h in Ist and 4th equation of Ist example are raised from 2 to 4 showing the increasing demand at load substations $S_2$ and $S_5$

Integer Solution

| Path | No. of circuits | Path | No. of circuits |
|---|---|---|---|
| $P_{12}$ | 1 | $P_{15}$ | 1 |
| $P_{13}$ | 1 | $P_{25}$ | 3 |
| $P_{14}$ | 1 | $P_{34}$ | 1 |
| All other $P_{ij}$ | | | |

Cost = Z = Rs. 2020 x $10^4$

Example 3

In in example (1) only 2 load substations $S_2$, $S_3$ are to be connected to generating station (1). The constraints will be

$$P_{12} + P_{32} \geqslant 2 \quad \ldots \quad (1)$$
$$P_{13} + P_{23} \geqslant 2 \quad \ldots \quad (2)$$
$$P_{12} + P_{13} \geqslant 2 \quad \ldots \quad (3)$$

Objective function is $Z = 312\ P_{12} + 352\ P_{13} + 193\ P_{23}$ ...(4)

Integer Solution

Above equations are solved by Enumeration method and result is

| Path | No. of Circuits |
|---|---|
| $P_{12}$ | 1 |
| $P_{13}$ | 1 |
| $P_{23}$ | 1 |

Example 4

The value of h in first equation of example (3) is raised from 2 to 4

Integer Solution

| Path | No. of circuits |
|---|---|
| $P_{12}$ | 2 |
| $P_{13}$ | 0 |
| $P_{23}$ | 2 |

Cost = Z = Rs. $1010 \times 10^4$

# CHAPTER - V

# CONCLUSION

Formulations of optimal network expansion problem given in II Chapter from Section 1 - 6 give the integer solution to the problem and use circuits along the permitted paths only when writing the network equations. In section (7) of Chapter (II) a mixed integer programming formulation has been given for the design and expansion of the electrical network in which design constraints such as network security and the cost of energy losses are taken into account and facilitates extension to time phased problems. In Section (8) of Chapter II, it has been suggested to obtain an optimal design considering the reliability constraints with the security constraints. The programme can be developed to test the reliability of the network by solving yearly load flows for over loads with any one combination of circuits out. Overloads are defined by thermal limits of the transmission circuit components or by the permissible power angle across the line as determined from system stability studies. In Section (9) of Chapter II a diakoptic approach to the problem is discussed to reduce the storage requirement of the computer.

In Chapter III different solution techniques has been discussed to solve the linear programming problems. Algorithm with flow charts are given. In Chapter IV examples are solved by Lexicographical method, bounded variable problem method and enumeration method. Enumeration method is not so efficient as other two methods. But programmes presented with Lexicographical approach are useful to solve large size problems.

A critical problem faced by a Planning Engineer is as follows :

Ensuring that selection of network expansion patterns for economic studies include true optimum or a near optimum pattern.

Currently such works are in progress taking practical limitations into account that how many alternatives can be formulated and analyzed. Still sincere efforts are required to develop more powerful planning tools that apply to this type of problem.

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# APPENDIX 1

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:  LEXICOGRAPHICAL  METHOD OF INTEGER  PROGRAMMING R.S.CHAUDHARY
       N  NO.  OF  VARIABLES
       M  NO.  OF  CONSTRAINTS
       A(I,J)  COEFF.  ON L.H.SIDE OF CONSTRAINTS
       A(1,J)  COEFF.  ON  L.H.SIDE  OF  OBJECTIVE  FUNCTION
       A(I,1)  R.H.SIDE OF CONSTRAINTS*(-1)
    READ 1,N,M
  1 FORMAT (10I5)
    N1=N+1
    M1=M+N+1
    NB=N+2
    DIMENSION A(85,22),X(22),NR(22),A1(85),D(22),U(22),A2(85),RT(22),
   1P(22),P1(22)
    READ 2,(A(1,J),J=2,N1)
    READ 3,((A(I,J),J=2,N1),I=NB,M1)
    READ 3,(A(I,1),I=NB,M1)
  2 FORMAT(8F10.1)
  3 FORMAT(16F5.1)
    DO 115  I=2,N1
    DO 115  J=2,N1
    A(I,J)=0.
 15 A(I,I)=1.
    DO 115  I=1,N1
 16 A(I,1)=0.
    ITERATION FOR OPTIMALITY  TEST  STARTS
 10 DO 5 I=2,M1
    IF(A(I,1))6,5,5
    SELECTION OF NONOBJ.ROW(I1)WITH SMALLESTELEMENT IN FIRSTCOLUMN
  6 I=2
 13 AM=A(I,1)
    I1=I
 18 IF(I-M1)9,10,10
  9 I=I+1
    IF(AM-A(I,1))11,11,12
 12 GO TO 13
 11 GO TO 18
 10 DO 15 J=2,M1
    IF(A(I1,J))15,15,17
 15 CONTINUE
    PROBLEM  HAS  NO  SOLUTION
    STOP
 17 DO 23 J=2,N1
    IF(A(I1,J))24,24,25
    SELECTION OF INDEX NR(J)  IN ROW  (I1)
 24 NR(J)=0
    GO TO 23
 25 NR(J)=J
 23 CONTINUE
    J=2
 30 IF(NR(J))26,26,27
 26 J=J+1
```

```
      GO TO 30
   27 DO 28 I=1,M1
   28 A1(I)=A(I,J)
      J1=J
   39 IF(J-N1)29,35,35
   29 J=J+1
   33 IF(NR(J))31,31,32
   31 J=J+1
      IF (J-N1)34,35,35
   34 GO TO 33
   32 I=1
   40 IF(A1(I)-A(I,J))36,37,38
   36 GO TO 39
   37 I=I+1
      GO TO 40
   38 GO TO 27
   35 LMIN=J1
      DETERMINATION  OF CONSTANTS  D(J)
      DO 51 J=2,N1
      IF(NR(J))52,52,53
   52 D(J)=0.
      GO TO 51
   53 IF (J-LMIN)54,55,54
   55 U(J)=1.
      D(J)=A(I1,J)
      GO TO 51
   54 I=1
   64 IF(A(I,J))56,57,56
   57 I=I+1
      IF(I-M1)62,62,56
   62 GO TO 64
   56 IZ=I-1
      I=1
   65 IF(A(I,LMIN)-0.)58,59,58
   59 I=I+1
      IF(I-M1)63,63,58
   63 GO TO 65
   58 IZM=I-1
      IF(IZ-IZM)60,61,60
   60 D(J)=0.
      GO TO 51
   61 I3=IZM+1
      I4=IZ+1
      RT(J)=A(I4,J)/A(I3,LMIN)
      T=1.
   66 IF(T-RT(J))67,68,69
   67 T=T+1.
      GO TO 66
   69 U(J)=T-1.
      D(J)=A(I1,J)/U(J)
      GO TO 51
   68 I=1
   75 A2(I)=A(I,J)-RT(J)*A(I,LMIN)
      IF(A2(I))70,71,72
   71 I=I+1
      IF(I-M1)73,73,70
```

```
 73 GO TO 75
 72 U(J)=RT(J)
    D(J)=A(I1,J)/U(J)
    GO TO 51
 70 U(J)=RT(J)-1.
    D(J)=A(I1,J)/U(J)
 51 CONTINUE
    DETERMINATION  OF  MAX. D(J)=D1
    J=2
    DL=D(J)
 80 IF (J-N1)76,77,77
 76 J=J+1
    IF(DL-D(J))78,79,79
 79 GO TO 80
 78 GO TO 84
 77 D1=DL
    DO 81 J=2,N1
    T1=1.
    P(J)=A(I1,J)/D1
    IF(P(J))85,86,87
 87 IF(T1-P(J))82,83,83
 82 T1=T1+1.
    GO TO 87
 83 P1(J)=T1
    GO TO 81
 86 P1(J)=0.
    GO TO 81
 85 IF(T1+P(J))38,89,90
 88 T1=T1+1.
    GO TO 85
 89 P1(J)=-T1
    GO TO 81
 90 P1(J)=-(T1-1.)
 81 CONTINUE
    Q=-A(I1,1)/D1
    DETERMINATION  OF CONSTANT  Q
    T2=1.
 93 IF(T2-Q)94,91,91
 94 T2=T2+1.
    GO TO 93
 91 Q=T2
    MODIFICATION  OF  OLD  MATRIX A(I,J)
    DO 101 I=1,M1
    A(I,1)=A(I,1)+Q*A(I,LMIN)
.01 A(I,LMIN)=A(I,LMIN)
    DO 102 J=2,N1
    IF(J-LMIN)103,102,103
.03 DO 105 I=1,M1
.05 A(I,J)=A(I,J)-P1(J)*A(I,LMIN)
.02 CONTINUE
    GO TO 110
  5 CONTINUE
    PROGRAMME  CONVERGE  TO  GIVE SOLUTION
    PUNCH 2,A(1,1)
    PUNCH 2,(A(I,1),I=2,N1)
    STOP
    END
```

**************************************************************

## APPENDIX 2

```
*************************************************************************
C  C  BOUNDED VARIABLE PROBLEM  R. S. CHAUDHARY
      READ 1,N,M
C        N  NO.  OF  VARIABLES
C        M  NO. OF CONSTRAINTS
C        A(1,J)  COEFF.  ON  L.H.SIDE  OF  OBJECTIVE  FUNCTION
C        A(I,J)  COEFF.  ON L.H.SIDE OF CONSTRAINTS
C        A(I,1)  R.H.SIDE OF CONSTRAINTS*(-1)
1     FORMAT(8I10)
      N1=N+1
      M1=M+N+1
      NB=N+2
      DIMENSION A(7,4),NR(4),A1(7),D(4),U(4),A2(7),RT(4),P(4),P1(4),
     1AM(4),ANEW(4)
      READ 2,(A(1,J),J=2,N1)
      READ 3,((A(I,J),J=2,N1),I=NB,M1)
      READ 3,(A(I,1),I=NB,M1)
      READ 3,(AM(I),I=2,N1)
    2 FORMAT(8F10.1)
    3 FORMAT(16F5.1)
      DO 115 I=2,N1
      DO 115 J=2,N1
      A(I,J)=0.
  115 A(I,I)=1.
      DO 116 I=1,N1
  116 A(I,1)=0.
C     ITERATION FOR OPTIMALITY  TEST  STARTS
110   DO 117 I=2,M1
      IF(A(I,1))118,117,117
117   CONTINUE
      DO 5 I=2,N1
      IF(AM(I)-A(I,1))6,5,5
118   DO 120 I=2,N1
      IF(AM(I)-A(I,1))6,120,120
120   CONTINUE
C     SELECTION OF NONOBJ.ROW(I1)WITH SMALLESTELEMENT IN FIRSTCOLUMN
      I=2
13    AM2=A(I,1)
      I1=I
11    IF(I-M1)9,10,10
9     I=I+1
      IF(AM2-A(I,1))11,11,13
10    ANEW(1)=-A(I1,1)
      DO 140 J=2,N1
140   ANEW(J)=A(I1,J)
      GO TO 141
6     I2=2
128   AM1=A(I2,1)-AM(I2)
      I1=I2
129   IF(I2-N1)123,124,124
123   I2=I2+1
      IF(AM1-A(I2,1))128,129,129
124   ANEW(1)=A(I1,1)-AM(I1)
      DO 130 J=2,N1
130   ANEW(J)=-A(I1,J)
141   DO 15 J=2,N1
```

```
      IF(ANEW(J))15,15,17
15    CONTINUE
C     PROBLEM  HAS  NO  SOLUTION
      STOP
C     SELECTION OF INDEX NR(J)  IN ROW  (I1)
   17 DO 23 J=2,N1
      IF(ANEW(J))24,24,25
24    NR(J)=0
      GO TO 23
   25 NR(J)=J
   23 CONTINUE
      J=2
   30 IF(NR(J))26,26,27
   26 J=J+1
      GO TO 30
   27 DO 28 I=1,M1
   28 A1(I)=A(I,J)
      J1=J
   39 IF(J-N1)29,35,35
   29 J=J+1
   33 IF(NR(J))31,31,32
   31 J=J+1
      IF (J-N1)34,35,35
   34 GO TO 33
   32 I=1
   40 IF(A1(I)-A(I,J))36,37,38
   36 GO TO 39
   37 I=I+1
      GO TO 40
   38 GO TO 27
   35 LMIN=J1
C     DETERMINATION  OF CONSTANTS  D(J)
      DO 51 J=2,N1
      IF(NR(J))52,52,53
   52 D(J)=0.
      GO TO 51
   53 IF (J-LMIN)54,55,54
   55 U(J)=1.
      D(J)=ANEW(J)
      GO TO 51
   54 I=1
   64 IF(A(I,J))56,57,56
   57 I=I+1
      IF(I-M1)62,62,56
   62 GO TO 64
   56 IZ=I-1
      I=1
   65 IF(A(I,LMIN)-0.)58,59,58
   59 I=I+1
      IF(I-M1)63,63,58
   63 GO TO 65
   58 IZM=I-1
      IF(IZ-IZM)60,61,60
   60 D(J)=0.
      GO TO 51
   61 I3=IZM+1
      I4=IZ+1
      RT(J)=A(I4,J)/A(I3,LMIN)
      T=1.
   66 IF(T-RT(J))67,68,69
   67 T=T+1.
      GO TO 66
   69 U(J)=T-1.
```

```
      GO TC 51
   68 I=1
   75 A2(I)=A(I,J)-RT(J)*A(I,LMIN)
      IF(A2(I))70,71,72
   71 I=I+1
      IF(I-M1)73,73,70
   73 GO TC 75
   72 U(J)=RT(J)
      D(J)=ANEW(J)/U(J)
      GO TO 51
   70 U(J)=RT(J)-1.
      D(J)=ANEW(J)/U(J)
   51 CONTINUE
C     DETERMINATION OF MAXM. D(J)   =D1
      J=2
   84 DL=D(J)
   80 IF (J-N1)76,77,77
   76 J=J+1
      IF(DL-D(J))78,79,79
   79 GO TO 80
   78 GO TO 84
   77 D1=DL
      DO 81 J=2,N1
      T1=1.
      P(J)=ANEW(J)/D1
      IF(P(J))85,86,87
   87 IF(T1-P(J))82,83,83
   82 T1=T1+1.
      GO TO 87
   83 P1(J)=T1
      GO TO 81
   86 P1(J)=0.
      GO TO 81
   85 IF(T1+P(J))88,89,90
   88 T1=T1+1.
      GO TC 85
   89 P1(J)=-T1
      GO TC 81
   90 P1(J)=-(T1-1.)
   81 CONTINUE
      Q=ANEW(1)/D1
C     DETERMINATION  OF CONSTANT  Q
      T2=1.
   93 IF(T2-Q)94,91,91
   94 T2=T2+1.
      GO TC 93
   91 Q=T2
      DO 101 I=1,M1
      A(I,1)=A(I,1)+Q*A(I,LMIN)
  101 A(I,LMIN)=A(I,LMIN)
      DO 102 J=2,N1
      IF(J-LMIN)103,102,103
  103 DO 105 I=1,M1
  105 A(I,J)=A(I,J)-P1(J)*A(I,LMIN)
  102 CONTINUE
      GO TO 110
    5 CONTINUE
C     PROGRAMME  CONVERGE  TO  GIVE SOLUTION
      PUNCH 2,A(1,1)
      PUNCH 2,(A(I,1),I=2,N1)
      STOP
      END
```

# APPENDIX 3

```
*******************************************************************
C  C  ENUMERATION  METHOD OF  INTEGER  PROGRAMMING  R.S.CHAUDHARY
C        N  NO.  OF  SARIABLES
C        M  NO.  OF  CONSTRAINTS
C        A(I,J)  COEFF.  ON L.H.SIDE OF CONSTRAINTS
      READ 1,N,M
    1 FORMAT(8I10)
      DIMENSION A(7,400),B(7),C(400),NR(400),IX(400)
      READ 2,(C(J),J=1,N)
      READ 2,((A(I,J),J=1,N),I=1,M)
      READ 2,(B(I),I=1,M)
    2 FORMAT (8F10.1)
      DO 3 J=1,N
    3 IX(J)=0
      IT=0
      J=1
    6 CA=C(J)
      JM=J
    7 IF(J-N)8,9,9
    8 J=J+1
      IF(CA-C(J))10,10,11
   10 GO TO 7
   11 GO TO 6
    9 CM=CA
      JM1=JM
      DO 12 J=1,N
   12 NR(J)=J
      NF=0
      NF1=1
      NS=N
   50 DO 15 I=1,M
      IF(A(I,JM)-B(I))16,15,15
   16 NR(JM)=0
      IT=IT+1
      IT1=IT-1
      NT=N*IT
      NT1=N*IT1
      J=JM1
      DO 17 K=JM,NS
      J1=NT+K-JM+JM1
      C(J1)=C(J)+C(K)
      NR(J1)=J
      DO 17 I=1,M
   17 A(I,J1)=A(I,J)+A(I,K)
      IF(JM1-1)18,19,18
   18 NTO=NT+1
      N1=NT+JM1-1
      DO 20 J=NTO,N1
      C(J)=0.
      NR(J)=0
      DO 20 I=1,M
   20 A(I,J)=0.
```

```
19 DO 21 K=NF1,NS
   J1=NT+K-NF
   JM2=J1-JM
   IN=1
41 JM3=IN*N
   IF(JM2-JM3)22,23,42
42 IN=IN+1
   GO TO 41
23 IX(J1)=IX(K)+1
   GO TO 21
22 IX(J1)=IX(K)
21 CONTINUE
   J=1
   NT2=NT+N
24 IF(NR(J))25,26,25
26 J=J+1
   GO TO 24
25 CA1=C(J)
   JM=J
27 IF(J-NT2)28,29,29
28 J=J+1
30 IF(NR(J))32,32,33
32 J=J+1
   IF(J-NT2)34,29,29
34 GO TO 30
33 IF(CA1-C(J))35,35,36
35 GO TO 27
36 GO TO 25
29 CM=CA1
   ITN=1
40 NS=N*ITN
   IF(NS-JM)38,39,39
38 ITN=ITN+1
   GO TO 40
39 ITN1=ITN-1
   NF=ITN1*N
   JM1=JM-NF
   NF1=NF+1
   GO TO 50
15 CONTINUE
   IX(JM)=IX(JM)+1
   PUNCH 2,(C(J),J=NF1,NS)
   PUNCH 2,((A(I,J),J=NF1,NS),I=1,M)
   PUNCH 1,(IX(J),J=NF1,NS)
   PUNCH 2,C(JM)
   STOP
   END
```

*******************************************************************