ESTIMATING TRANSIENT RESPONSE OF AN A.C. MACHINE FROM LIAPUNOV STABILITY CRITERIA

By

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ABSTRACT

The dissertation embodies the solution of the problem of finding the estimate of the transient response of a stable power system consisting of a synchronous generator connected to an infinite bus, involving governor and regulator action.

The swing equation with damping and saliency effects is formulated and then it is trans- -formed into a set of first order differential equations, as needed for state space approach. A Liapunov function is framed using Cartwright's method. Eigen values of a number of matrices are found out, which serve as the estimate of upper and lower bounds of the time constants associated with the system transient response. The concept of these estimates is based on finding the maximum and minimum of $\left[-\dot{V}(X)/V(X)\right]$. Where $V(X)$ is the Liapunov function and $\dot{V}(X)$ is its derivative with respect to time.

These are compared with the actual transient response, obtained by numerical method on 1620 IBM digital computer.

The problem continues to be more complex, when velocity governor, angle regulator and their combined action is investigated. The order of the nonlinear differential equation increases to that of fourth degree, beyond which, it becomes difficult to construct Liapunov functions.

Finally a new approach is attempted to estimate the time constants directly from $\underset{\mathrm{Max.}}{\mathrm{Min.}}\left[-\dot{V}(x)/V(x)\right]$ by Monte Carlo method.

CERTIFICATE

Certified that the dissertation entitled 'Estimating Transient Response of an A.C. Machine from Liapunov Stability Criteria' which is being submitted by Shri H.C. Agarwal in partial fulfil--ment for the award of the degree of Master of Engineering in Advanced Electrical Machines at University of Roorkee, is a record of candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissert--ation has not been submitted for the award of any other degree or diploma.

This is to further certify that he has worked for a period of 6 months from January to June'70 for preparing this dissertation, at this university.

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LIST OF PRINCIPAL SYMBOLS

Symbol	Description
$V(X)$	Liapunov Function
$\dot{V}(x)$	Derivative of Liapunov Function
A	System Coefficient Matrix
V, V'	Real Symmetric Positive Definite matrices.
R, R'	Real Symmetric Positive Definite or Semi-definite matrices.
A^T	Transpose of Matrix A
V^{-1}	Inverse of Matrix V
β, λ, λ'	Eigen Values of A, RV^{-1} & $R'V'^{-1}$ matrices
δ	Rotor angle with respect to a synchronously rotating reference axis
P_1	Mechanical Power Input
Pe, P_d	Electrical Power Output and Damping Power
K_d	Damping Coefficient
δs	Rotor angle at Stable Equilibrium
$g(x)$	Nonlinear Function of State Variable x
$\xi max., \xi min.$	Max. & Min. of $[-\dot{V}(X)/V(X)]$
X_o	System Initial Condition
t_o	Stabting Time
X, X^T	State Vector and its Transpose
Xd, Xd', Xd''	Direct Axis Reactance, Transient and Sub-transient reactances.

Xq, Xq', Xq"	Quadrature Axis reactance, Transient and Sub-transient reactances.
Tdo'	Direct Axis Transient O.C. Time Constant
Tdo", Tqo"	Direct & Quad. Axis Sub-Transient O.C. Time Constants
f	Supply Frequency
H	Intrtia Constant
G	Rated Appatent Power of the machine
G1	Velocity Governor Gain
T1 , T2	Governor Servomechanism & Turbine Time Constants
w_o	Angular Frequency of the System
p , p'	Differential Operators d/dt and d/dT
V1	Voltage of the Infinite Bus
Eq'	Voltage behind direct axis transient reactance
Efd, Efdo	Field Circuit Voltage, with and without regulator action
K3, K4, K5	Angle Regulator constants
T	Dimensionless time variable

CHAPTER I

INTRODUCTION

1.1 INTRODUCTION

The least information, needed to be known about a system is stability. Where stability is defined as that attribute of the system, or part of the system, which enables it to develop restoring forces between the elements thereof, equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements.* The study of stability has assumed enormous importance since the development of more complex power systems connected to large size synchronous machines operating through long distance transmission lines. To ensure reliability of service to the consumers, it is necessary to maintain synchronism between the machines, during the steady state and the transient disturbances as well. Practically, no power system remains in the steady state due to ever occouring disturbances by load changing, switching operations,

* American Standard Definitions of Electrical Terms, ASA-C42-1941.

faults and loss of excitation. The region of stability can be determined for any nonlinear system, if a suit--able Liapunov function can be found out. If the initial point lies within this region, the system is asymptotically stable. This is Direct Method of Liapunov, which obviates the need of integrating the nonlinear differential equations.

Whereas the maximum information which is essential from a system is its transient response. The determination of transient response entails integration of nonlinear differential equations, which is only possible by well known numerical methods suitable for high speed digital computers.

The practical design area lies some where in between these two expremes. This requires a knowledge of system behavior which is less than the complete time response, besides finding the stability limit of the system.

1.2 STATEMENT OF THE PROBLEM

In view of the practical design requirements, mentioned above, the problem centers on first ascertaining the stability of the system and then devising methods to estimate the upper and lower bounds of the time constant of the system transient response, the Direct Method of Liapunov is used.

The system is described by a set of first order differential equations through the use of a single higher order differential equation, derived for a synchronous machine connected to an infinite bus. The saliency and damping effects are included.

The system stability is ensured by the negative real parts of all the eigen values of the coefficient matrix obtained from its differential equations, and also selecting a suitable Liapunov function $V(X)$, which is positive definite ~~or semi-definite~~ , such that its derivative $\dot{V}(X)$ is atleast negative semidefinite. This is verified with the help of the matix equation

$$A^T V + VA = -2\ R \qquad ..(1.11)$$

where A^T is the transpose of the coefficient matrix A

V is the matrix from the Liapunov function $X^T V X$

R is a Real SymmetricPositive Definite Matrix(AII.5) or Semidefinite Matrix (AII.5)

The complete transient response xof the system is obtained by step by step integration of differential equations by Runge-Kutta-Gill method on digital computer.

Kalman&Bertram(41) suggested that if a Liapunov function V(X) is considered as the measure of the distance of any point on the trajectory of the system from the origin, an idea of the speed with which the system approaches its steady state is obtained from $\left[-\dot{V}(X)/V(X)\right]$. They meant that the maximum and minimum of $\left[-\dot{V}(X)/V(X)\right]$ can give the upper and lower boundary of the region, within which the transient response of the system is expected to exist.

Later Vogt(43) discoussed the relations amongst maximum and minimum of $\left[-\dot{V}(X)/V(X)\right]$ and maximum and minimum eigen values of matrices A and RV^{-1}. He modified the approach so as to make it applicable even for certain class of nonlinear systems.

A Liapunov function is constructed by Cartwright's method constraining $\dot{V}(X)$ to be negative semidefinite. Higher order terms are neglected so as to make it a quadratic function. The matrix R is calculated from eq.(1.11). Then matrix RV^{-1} and its eigen values are determined.

Further a new matria R' is now choosen and the matrix equation (1.11) is solved for n(n+1)/2 unknown elements of V' matrix , which is assumed to be symmetrical. Where n is the order of the matrix. Again $R'V'^{-1}$ and its eigen values are calculated.

Upper and lower bounds of the transient response are plotted with the help of minimum and maximum eigen values of A, RV^{-1} and $R'V'^{-1}$ along with the actual transient response, as mentioned above.

A new approach is tried to find the extreme values of the ratio $\left[-\dot{V}(X)/V(X)\right]$ directly, by Monte Carlo method, and the different estimates are compared.

The same procedure is adopted, with more complex differential equations, obtained by including governor, angle regulator and their combined effect. The difficulty is experienced in constructing Liapunov functions of higher order systems.

CHAPTER II

REVIEW

2.1 INTRODUCTION

The modern trends in the design of big generating units, having higher transient reactance and lower inertia constant have considerably narr--owed down the margins of stability. It has become essential to predict stability accurately and quick-ly, due to the increase in complexity of modern power systems. The stability study is generally divided into two categories namely, (i) Steady State Stability (ii) Transient Stability. Whereas this distinct division is hardly realistic. The studies associated with small disturbances are covered by steady state stability and those with large disturb--ances are categorized under transient stability.

2.2 REVIEW

Steady State Stability

The power systems with several generating units are inherently nonlinear. The steady state stability criteria is applied after linearizing the involved nonlinear differential equations by small displacement theory. The changes in the dependant variables are assumed to be very small. A number of methods have been suggested by various authors. Some of the popular techniques are detailed below.

1. Routh Hurwitz's Criteria

Concordia (1,2,3) in the years 1944,'48 and'50 obtained the characteristic equation from the coe--fficients of the linearized equations, considering voltage regulator, angle regulator and buck boost voltage regulator action respectively. The information about the absolute stability was obtained by applying Routh Hurwitz's Criteria. This tells the position of its roots with respect to the imaginary axis. If all the roots lie to the left of the axis, the system is stable. He investigated the gain in the stability limit for various operating angle and amplification factors.

Yu & Vongsuria(4) in his paper of the year 1966, considered saliency, short circuit ratio, tieline resistance and reactance in a system contain--ing a synchronous machine connected to an infinite bus with continuously acting voltage regulator and governor.

2. Niquist Criteria

The previous method gave the information about absolute stability and no clue is available as to its degree. Where Niquist criteria has the advantage of predicting both, along with furnishing an idea to improve upon it.

Messerle & Bruck (5), used this technique in the year 1956, for investigating the stability, when control of prime mover torque and field excit-

-ation is affected by governors, controllers, voltage and angle regulators.

Aldred & Shackshaft(6) in 1960, obtained a basic closed loop pattern for a synchronous machine including voltage regulator effect, and could interpret the results from Niquist plots.

Jacovides & Adkins(7), in the year 1966, studied the effect of proportional, integrator and derivative type of voltage regulator feedbacks and compared the results with the help of Néquist Looi.

3. Root Locus Technique

Root Locus involves the plot of the poles and zeros of the open loop transfer function, when the gain is varied from 0 to ∞ . It gives an idea of the range within which the parameter should lie to maintain the system to be stable. For variation of other parameters other than gain, root contours are drawn..

The work of Stapleton(8) in 1964 is creditable in this direction. He used the root locus plots to study the variation in performance when parameters such as gain, exciter time constant and derivative circuit of regulating system were varied.

4. D-Partition Method

A curve is plotted while varying the angular frequency from $-\infty$ to ∞ in a plane of two variable parameters. This divides the region into stable and unstable portions.

Venikov & Litkins(9) in 1956, investigated the effects of voltage and angle operated regulator on the stability limit.

Yu & Vongsuria (4) in the year 1966, used the D-Partition method for evaluating the steady state stability of a synchronous machine with voltage regulator and speed governor.

Stroev & Sreedharan (10) in 1967, applied this method to choose the best combination of regulator parameters in view of stability. Effect of variation of two parameters at a time was investigated and the allowable range of these parameters was determined.

5. Geometrical Method

Walker (11) in 1953 and later on Gove(45) in 1965 used the capability charts to find the limit of stable operation. Gove obtained modified charts, when automatic voltage regulator action was considered. Effect of damping coefficient and exciter time constant was investigated.

6. State Space Approach

This method requires the system to be expressed in the form of a set of first order differential equations, which facilitates the application of modern control theory.

Laughton (12) in the year 1966, used matrix algebra in calculating a set of general coefficients and introduced them in the state space equations. If all the eigen values of the coeffic--ient matrix had negative real parts, it ensured system stability. The explicit solution in time could also be obtained by convolution integral in matrix form.

7. Liapunov's Method

The Direct Method of Liapunov is an entirely new approach, by which a system stability can be studied without the knowledge of the explicit time solution of the differential equations. The method involves finding of a suitable function.

Undrill (13) in 1967, obtained a model of synchronous machine with 3-phase , tee-form transm-ission system constraints, including a simple voltage regulator. He later in other paper used the Liapunov equation to determine the optimum settings of governor and voltage regulator parameters.

8. Analogue Computer Method

Analogue representation eases the job of evaluating the performance of complex system invol--ving large number of variable parameters. The stability limit is determined by increasing the load in small steps.

Aldred & Shackshaft (14) in 1958, predict--ed the stability limit of the system with voltage regulator, by solving the system equations on electronic analogue computer. Ideal characteristic was simulated by subsidiary feedback and series ne-tworks. New voltage-excitation characteristics were introduced to predetermine power angle curves.

Miles (15) in the year 1962, analysed the effect of flux variation, governor and regulator action by solving the multimachine system equations on analogue computer.

9. Digital Computer Method

With the development of hgih speed digital computers, it has been possible to tackle the complicated problems most effectively, accurately and at a faster rate, if they could be coded in digital form.

Messerle & Bruck (5) and Jacovides & Adkins(7) used digital computers for plotting the points of Niquist Loci.

Stroev & Sreedhran (10) compiled the program for stability investigation by D-Partition curve.

Yu & Vongsuria (4), calculated the roots of the characteristic equations.

Laughton (12), determined the eigen values of the coefficient matrix of system state space equations.

Aldred & Shackshaft (14) used the digital computer also for solving the differential equation, to study the stability.

Ewart & DeMello (16) in the year 1967, evolved a digital computer programme for finding dynamic stability limits of a single mabhine connected to an infinite bus through a transmission line, having excitation and prime mover controls. The effect of terminal voltage, transmission line reactance, and machine inertia on the stability limits was investigated.

TRANSIENT STABILITY

When the system is aperiodically disturbed in such a way, that it comes to equilibrium condition before the occourance of the next, the maximum power delivered without losing the synchronism between the generating units is termed as the transient stability limit.

Due to abrupt change in the conditions, the former approach of small perturbations cannot hold good. Therefore other methods are adopted for the study of transient stability.

1. Step by Step Solution

The earlier methods as stated by Crary(17) and Kimbark (18) involve hand calculation of the change in angular position of the rotor using step by step method. The accelerating power is assumed to be constant from the middle of the preceding interval to the middle of the interval considered, and the angular velocity remains constant through--out the interval at the value calculated for the middle of the interval.

2. Equal Area Criteria

Crary (17) and Kimbark (18) adopted this method for single machine systems. The load can be increased to a limit where the areas A_1 and A_2, determined from the power angle curve, become equal. This procedure enables the determination of transeint stability limit without the necessity of solving the equation. Critical switching angle can also be calculated at which the fault should be cleared before the system goes unstable.

3. Energy Integral Criteria

Aylett (19) in the year 1958, used the concept of energy integrals and singular points for transient stability study. He integrated the differential equation to obtain an expression containing kinetic energy and potential energy terms. The total energy of the system is equated to a constant, to give a curve defining the stable regions. If total energy is less than this constant, the system is within the stable region. Formulae for calculation of critical switching time and angle were obtained.

4. Phase Plane Method

Phase plane technique which owes its origin to control theory, makes use of the nonlinear differential equation, transformed into two variable parameter equation. This is then utilized for plotting the phase trajectory.

Dharam Rao (20) in 1962, adopted this method to determine critical angle and time by plotting trajectories during and after fault.

Rao & Rao (21) in the year 1963, plotted transient stability limits, taking constant voltage behind transient reactance, constant flux linkage and field decrement into consideration.

5. Direct Method of Liapunov

This method came to be known after the publication of the famous Liapunov's memoire in the Russian Journal in the year 1892. It assumed wide importance, specially in the control theory, in the Soviet Union, as the principal tool for tackling linear and nonlinear stability problems. Now a lot of work is being done, during the past few years in the western countries. This method defines stability in the large, instead of confin--ing the study close to the equilibrium points, and eliminates any need of determining the solution of nonlinear differential equations. The method embodies the selection of a suitable Liapunov function $V(X)$ which is positive definite, such that its derivative $\dot{V}(X)$ is negative definite. Then the solution will reach the steady state asymptotically.

Gless (22) was the first to apply this method for power system stability, in the year 1966, He could guess a suitable Liapunov function for a single machine represented by a constant voltage at the back of its transient reactance, neglecting damping torque and governor action. Flux linkage in the rotor circuit was assumed to be constant. He compared this method with equal area criteria and phase plane technique. Finally a Liapunov function for a 3-machine system was also guessed and the stability was predicted, when the machine velocities and angles were known at the time of final system disturbance.

Many methods of constructing Liapunov functions are available in literature of controlsystems, but are less used in the power system. The most popular amongst them are

i. Variable Gradient Method (23)

ii. Ingwerson's Method (24)

iii. Zubov's Method (25)

iv. Aizerman's Method (26)

v. Cartwright's Method (27)

El-Abiad & Nagappan (28) in 1966, obtained the transient stability limit through a digital computer program for a multimachine system. If the condition of the system after fault fell on the boundry of region of asymptotic stability, it could give the critical switching time.

Zaslavskaya, Putilov & Tarirov (29) in the year 1967, published a paper, including a general expression for Liapunov function for a multimachine system. They explained procedure for defining the region of asymptotic stability and then finding out the critical switching time.

Yu & Vongsuria (30) in the same year, applied Zubov's method to construct a Liapunov function. The differential equation was expressed in the form of first order state space equations, with the steady state condition of the post fault system transferred to the origin. They utilized the damping coefficient to obtain a series of functions by this method and could prove its effect--iveness even with a truncated series function.

Williams (31) in the year 1968, extended his efforts in getting a Liapunov function for a single machine system connected to an infinite bus, including saliency, damping torque and governor action. He could obtain better estimate of the stability region as compared to earlier methods of energy integral criteria.

Dharam Rao (32) in 1969, varified Routh Hurwitz conditions through the Direct Method of Liapunov. He generated several Liapunov functions by Cartwright's method and Aizerman's method. The domain of stability given by these functions was compared with actual one obtained by digital computers. Governor action, pole saliency and damping effects were also considered.

6. Analogue Methods

Higher order systems with large number of variable parameters can be studied in respect of effects of various disturbances occouring in the system by analogue methods without resorting to tedious mathemetical calculations. Names of many investigators in the field appear in the literature. The important papers are due to Boast & Rector (33) in 1951, Van Ness (34) in 1954, Casson (35) in 1958 and Aldred (36) in the year 1962.

7. Digital Computer Methods

Johnson and Ward (37) in the year 1957, used digital computers for calculation of transient stability characteristics of power systems. They preferred Runge Kutta method, as it proved to be accurate and self starting. The results were compared with step by step calculations.

Lane, Long & Powers (38) in the year 1958, described a method of automatic calculation of transient stability data without any manual intervention. The program contained Runge-Kutta-Gill method for integration of differential equations. Faults and switching operations were automatically included in the program.

Humpage & Stott (39) in 1965, analysed transient stability problem by another numerical method known as Predictor and Corrector method. They claimed saving in computing time and compared the results with those from RungeKutta method.

ESTIMATE OF TRANSIENT RESPONSE

Chetaev (40) was the first to introduce the idea of finding the estimate of the transient response behaviour from the Liapunov function V(X). Afterwards Bedelbaev(40) and Razumchin(40) introduced modification to this concept.

Kalman & Bertram (41) in the year 1960, discussed it for linear systems and maintained that the transient response can be viewed as the rate with which the value of the Liapunov function reaches zero.

Popov (42) in the same year, proposed a method of estimating the quality of transient response by obtaining an integrated square output from the frequency response of the linear part.

Vogt(43) in 1965, devised an improved procedure of estimating the upper and lower bounds of transient response time constants even for nonlinear systems by linear approximation of the system. He outlined the method of choosing the Liapunov function such that the estimates may reach closer to actual response. He could relate the estimates with the eigen values of the matrix RV^{-1} obtained from the Liapunov Stability Equation

$$A^T V + VA = -2R$$

and with those of the coefficient matrix A.

Recently, Bhaumik & Mahalanbis (44) in the year 1969, ponted out that the improvements in results could be brought about, by including the approximation of nonlinear parts in the analysis. They considered Lure's type of nonlinear system as an illustration and utilized Lure's-Postnikov form of Liapunov funtion.

CHAPTER III

TRANSIENT RESPONSE ESTIMATE OF SINGLE MACHINE CONNECTED TO AN INFINITE BUS

3.1 INTRODUCTION

The system is assumed to consist of a synchronous machine connected directly to an infinite bus. (Fig.1). The swing equation with saliency and damping effects is considered. A Liapunov function by Cartwright's method is constructed and the eigen values of matrices A , RV^{-1}, and $R'V'^{-1}$ are calculated. Upper and lower bounds of the system transient response are estimated from these eigen values and by Monte Carlo Technique.

3.2 SWING EQUATION

The dynamical behaviour of a synchronous machine can be mathematically expressed by swing equation. The order of this nonlinear differential equation may vary from two to four depending on the details incorporated in the machine represent--ation and its control systems.

A swing equation for a synchronous machine connected to an infinite bus , as derived in Appendix I, can be given by eq.(AI.38)

$$M \frac{d^2\delta}{dt^2} + Kd \frac{d\delta}{dt} = Pi - Pm1 \sin\delta + Pm2 \sin 2\delta \qquad ..(2.11)$$

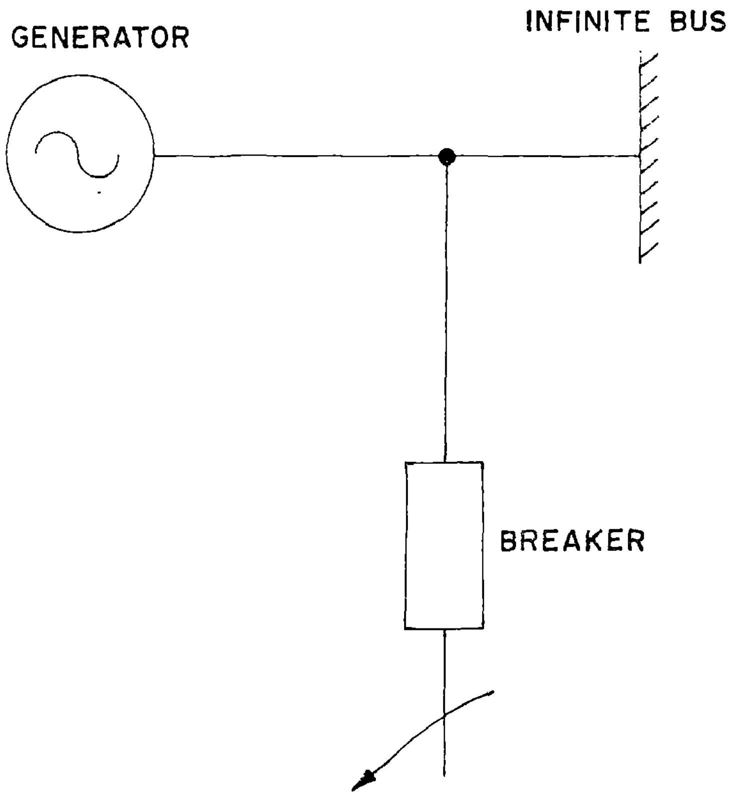

FIG. I. ONE MACHINE CONNECTED TO AN INFINITE BUS.

where

δ = Rotor angle with respect to a synchronously rotating reference axis.

Pi = Mechanical Power Input corrected for rotational losses.

$$Pm1 = \frac{Eq'V_1}{Xd'}$$

$$Pm2 = \left[\frac{Xq - Xd'}{2XqXd'}\right]V_1^2$$

$$Kd = \frac{2}{\pi}\int_0^{\pi/2}\left\{\frac{(Xd'-Xd'')}{Xd'^2}Tdo''\sin^2\delta + \frac{(Xq'-Xq'')}{Xq'^2}Tqo''\cos^2\delta\right\}V_1^2 d\delta$$

Introducing a new dimensionless variable 'T', such that,

$$T = t\sqrt{\frac{Pm1}{M}}$$

the swing equation becomes,

$$\frac{d^2\delta}{dT^2} + Kd1\frac{d\delta}{dT} = P1 - \sin\delta + P2\sin 2\delta \qquad ..(2.12)$$

where

$Kd1 = Kd/\sqrt{Pm1\ M}$

$P1 = Pi/Pm1$

$P2 = Pm2/Pm1$

The following assumptions are made.

1. Mechanical Power input is constant.
2. Voltage at the back of the transient reactance is constant.

3. The speed change during a transient is assumed to have negligible effect on stator voltages.
4. The armature resistance is neglected.
5. Pole saliency and damping effects are considered.

Let δs is the stable singularity, obtained by Newton Raphson's method (AfII.1), on digital computer. Transferring this singularity to the origin by the assumption

$$x1 + \delta s = \delta \qquad ..(2.13)$$

The differential equation can be represent-ed in state variable notation, if,

$$\frac{dx1}{dT} = \dot{x}1 = x2 \qquad ..(2.14)$$

$$\frac{d2x1}{dT^2} = \dot{x}2 = P1 - Sin(x1+\delta s)+P2 Sin\ 2(x1+\delta s)-Kd1x2 \qquad ..(2.15)$$

where δs = Stable point

These two first order differential equations can be represented as

$$\dot{X} = f\ (X) \qquad ..(2.16)$$

where $f(0) = 0$

$$f(X) = \begin{bmatrix} f1(X) \\ f2(X) \end{bmatrix}$$

$$f1(X) = x2$$

$$f2(X) = -Kd1x2-Sin(x1+\delta s)+P2 Sin2(x1+\delta s)+P1$$

$$\dot{X} = \begin{bmatrix} \dot{x}1 \\ \dot{x}2 \end{bmatrix}$$

3.3 DIRECT METHOD OF LIAPUNOV

The trajectory of the system will reach the origin i.e. steady state condition, asymptot--ically (AII.2), if there exists a Liapunov function V(X), such that the following conditions are satisfied in some vicinity of the origin.

1. $V(0) = 0$
2. $V(X) > 0$ for $X \neq 0$
3. Grad $V(X)$ is continuous
4. $\dot{V}(X) \leq 0$ for $X \neq 0$

..(2.17)

3.4. CARTWRIGHT'S METHOD

This method is capable of generating Liapunov functions satisfactorily, for systems upto the fourth order. This technique generalizes from the linear system to the nonlinear one.

Rearranging the equations (2.14) & (2.15),

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -k_1 x_2 - g(x_1) \qquad ..(2.18)$$

where

$$Kd1 = k_1$$

$$g(x_1) = \text{Sin}(x_1 + \delta s) - P_2 \text{Sin}\, 2(x_1 + \delta s) - P_1$$

..

which is apparently a nonlinear function. The above system can be linearized, if a linear term $k_2 x_1$ is substituted for $g(x_1)$.

Thus

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -k_1x_2 - k_2x_1 \qquad ..(2.19)$$

A Liapunov function in the quadratic form is taken as

$$V = \tfrac{1}{2}a_1x_1^2 + \tfrac{1}{2}a_2x_2^2 + \tfrac{1}{2}a_3x_1x_2 \qquad ..(2.20)$$

where

a_1, a_2 and a_3 are constant coefficients.

Differentiating the eq.(2.20) w.r.t. T,

$$\dot{V} = a_1x_1\dot{x}_1 + a_2x_2\dot{x}_2 + \tfrac{1}{2}a_3(x_1\dot{x}_2 + \dot{x}_1x_2) \qquad ..(2.21)$$

Substituting eq.(2.19) in eq.(2.21),

$$\dot{V} = a_1x_1x_2 + a_2x_2(-k_1x_2 - k_2x_1) + \tfrac{1}{2}a_3\left\{x_1(-k_1x_2 - k_2x_1) + x_2^2\right\}$$
$$= (-a_2k_1 + \tfrac{1}{2}a_3)x_2^2 - \tfrac{1}{2}a_3k_2x_1^2 + (a_1 - a_2k_2 - \tfrac{1}{2}a_3k_1)x_1x_2 \qquad ..(2.22)$$

As per the conditions mentioned in eq.(2.17), $\dot{V}(X)$ can be constrained to be negative definite (AII.3) or negative semi-definite (AII.4) function, by choosing the suitable values for a_1, a_2 & a_3. For convenience in higher order systems, it is made negative semi-definite function of state variable x_2, with the following values of the coefficients of V.

$$a_3 = 0$$
$$a_2 = 1 \qquad ..(2.23)$$
$$a_1 = k_2$$

Putting eq.(2.23) in eq.(2.22),

$$\dot{V} = -k_1x_2^2 \qquad ..(2.24)$$

Which is negative for any value of x_2 and x_1

except at the origin, and is zero at the origin and for any value of x_1, when x_2 is zero. This shows that eq. (2.24) is constrained to be negative semi--definite.

Substituting the values of a_1, a_2 & a_3 from eq.(2.23) in eq. (2.20),

$$V = \tfrac{1}{2}k_2 x_1^2 + \tfrac{1}{2}x_2^2 \qquad ..(2.25)$$

Which is positive definite (AII.3) except at the origin, where it is zero, provided

$$k_2 > 0$$

In the linearized system of eqs.(2.19), $k_2 x_1$ appeared for the nonlinear function $g(x_1)$. Therefore in order to change over to the nonlinearity again, $\tfrac{1}{2}k_2 x_1^2$ in eq. (2.25) can be replaced by

$$\int g(x_1)\, dx_1$$

Whence eq. (2.25) becomes,

$$V = \tfrac{1}{2} x_2^2 + \int_0^{x_1} g(v)\,dv \qquad ..(2.26)$$

or,

$$V = \tfrac{1}{2}x_2^2 + \int_0^{x_1} \left\{ \sin(v+\delta_s) - P_2 \sin 2(v+\delta_s) - P_1 \right\} dv \qquad ..(2.27)$$

This is the same form, as appeard in the past literature (22)

2.5 REGION OF STABILITY

If a linear system is stable, it is stable (AII.1) in the entire state space, whereas the nonlinear system stability is confined to an enclosed region due to the presence of the integral terms in the expressions for the Liapunov function. This clearly shows that the Liapunov function cannot be positive definite in the entire space. There will be a specific limit to the value of the function beyond which, it does'nt represent closed surfaces, which is an essential requirement for the asymptotic stability of the system.

Therefore,

$V = b$

where b = constant

will represent closed surfaces, if $b < b_{max}$

where b_{max} is the limiting value of V

b_{max} can be obtained by equating the elements of gradient (V) to zero, and substituting the nontrivial values of the state space variables, thus obtained in the expression of Liapunov function.

Taking the Liapunov function of eq.(2.27) for example,

$$\nabla V = \begin{Bmatrix} \sin(x_1+\delta_s)-P_2\sin 2(x_1+\delta_s)-P_1 \\ x_2 \end{Bmatrix} \quad ..(2.28)$$

$= 0$

or,

$$\sin(x_1+\delta_s) - P_2\sin 2(x_1+\delta_s) - P_1 = 0$$
$$x_2 = 0 \qquad ..(2.29)$$

The solution of eqs. (2.29) gives two values, namely corresponding to stable focus and saddle sing--ularities. The nontrivial solution will be the saddle point singularity, when the stable equilibrium is transferred to the origin. These two points can be determined by Newton Raphson method(AIII.1) on digital computer.

3.6 METHOD OF ESTIMATING THE TRANSIENT RESPONSE

The Liapunov function can be considered as a measure of distance between the equilibrium point and the point on the trajectory in the state space. With this concept, Let

$$\xi = \frac{-\dot{V}(X)}{V(X)} , \qquad ..(2.30)$$

valid in the region of asymptotic stability. Eq. (2.30) gives an idea of the rate with which the system reaches its steady state.

Integrating eq. (2.30),

$$V(X) = V(X_o) \; e^{-\int_{t_o}^{t} \xi \, dt} \qquad ..(2.31)$$

where

$V(X_o)$ is the value of V at the starting time t_o.

If

$$\xi_{min} = \text{Min.} \left[\frac{-\dot{V}(X)}{V(X)} \right] \qquad ..(2.32)$$

and

$$\xi_{max.} = \text{Max.} \left[\frac{-\dot{V}(X)}{V(X)} \right] \qquad ..(2.33)$$

Then,

$$V(X) \leq V(X_0)\ e^{-\xi_{min}(t-t_0)} \qquad ..(2.34)$$

$$\geq V(X_0)\ e^{-\xi_{max}(t-t_0)} \qquad ..(2.35)$$

Eqs. (2.34) & (2.35) define the boundary between which the actual response lies. The estimates can be brought as closer to the actual response as desired by judicious choice of the Liapunov function. This technique is useful in designing the systems based on the concept of improving this estimate by changing the set of variable system parameters.

Vogt(43) proved a relation amongst the real parts of the eigen values of the coefficient matrix of the linearized system, matrix RV^{-1}, as obtained from the Liapunov equation, and the estimates (ξ), for a certain class of linear and nonlinear system and Liapunov functions.

The Liapunov function can be reduced to a quadratic form, by elimination of third and higher order terms from the series expansion of nonlinear factors, if it can be expressed as

$$V(X) = V_1(X) + V_2(X) \qquad ..(2.36)$$

where

$V_1(X)$: Factor containg quadratic terms.

$V_2(X)$: Factor containing third and higher degree terms.

Eliminating $V_2(X)$,

$$V(X) = V_1(X)$$

$$= X^T V X \text{ (say)} \qquad ..(2.37)$$

where

X = n-dimensional state vector

X^T = Transpose of the state vector

V = n x n real symmetric positive definite matrix (AII.5)

The time derivative of $V(X)$ is

$$\dot{V}(X) = \dot{X}^T V X + X^T V \dot{X} \qquad ..(2.38)$$

Similarly, the system is alo linearized by neglecting higher order terms.

Thus initially a system expressed as

$$\dot{X} = h\ (X) \qquad ..(2.39)$$

can be shown as

$$\dot{X} = AX + h_1(X) \qquad ..(2.40)$$

where

$h_1(X)$ contains higher degree terms.

Neglecting $h_1(X)$,

$$\dot{X} = A\,X \qquad ..(2.41)$$

Substituting eq.(2.41) in eq.(2.38),

$$\begin{aligned}\dot{V}(X) &= X^T A^T V X + X^T V A X \\ &= X^T (A^T V + VA)\, X \\ &= -X^T (2R)\, X \qquad \text{(Say)}\end{aligned} \qquad ..(2.42)$$

Where

R = Real Symmetric positive definite or semi-definite n x n matrix, if the system is asymptotically stable.

Therefore from eq.(2.42)

$$A^T V + VA = -\,2\,R \qquad ..(2.43)$$

Which is known as Liapunov Stability Equation. Substituting $V(X)$ and $\dot{V}(X)$ from eqs.(2.37)&(2.42) respectively in eqs. (2.32) and (2.33),

$$\xi_{min} = \text{Min} \left[\frac{X^T (2R) X}{X^T V X} \right] \qquad ..(2.44)$$

$$\xi_{max} = \text{Max} \left[\frac{X^T (2R) X}{X^T V X} \right] \qquad ..(2.45)$$

This division is permissible as V is always positive definite except at the origin, when the numerator is also zero. As the relative shape and size of V(X) and $\dot{V}(X)$ remains same through the space, the ratios may be considered for a specific constant value of V(X).

For convenience:

$$V(X) = X^T V X = 1 \text{ (say)}$$

Then from eqs.(2.44) & (2.45)

$$\xi \begin{matrix}\text{min.or}\\ \text{max.}\end{matrix} = \begin{matrix}\text{min.or}\\ \text{max.}\end{matrix} \left[X^T(2R)X\right] \qquad ..(2.46)$$

Using Lagrange Multiplier Technique, for optimization,

$$\xi \begin{matrix}\text{min.or}\\ \text{max.}\end{matrix} = \begin{matrix}\text{min.or}\\ \text{max.}\end{matrix} \left[X^T(2R)X - \lambda\, X^T V X\right] \qquad ..(2.47)$$

where λ is such that

$$X^T V X = 1$$

Then for extremal values of $\left[X^T(2R)X - \lambda X^T V X\right]$ with respect to X, we get

$$(2R - \lambda V)\, X = 0 \qquad ..(2.48)$$

or,

$$(2R)\, X = \lambda\, VX$$

or,

$$X^T(2R)X = \lambda X^T V X = \lambda \qquad ..(2.49)$$

where

$$X^T V X = 1 \text{ (Assumed)}$$

Therefore $X^T(2R)X$ is minimum or maximum, depending on, when λ is minimum or maximum respect--ively.

Where λ is the eigen value of RV^{-1}, as evident from eq. (2.48).

Vogt(43) in his paper concluded that

$$\xi_{max.} \geq 2\lambda_{max}(RV^{-1}) \geq -2Re\,\beta_{max}(A) \qquad ..(2.50)$$

$$\xi_{min.} \leq 2\lambda_{min}(RV^{-1}) \leq -2Re\,\beta_{min}(A) \qquad ..(2.51)$$

where

λ_{max} & λ_{min} are the maximum and minimum eigen values respectively of RV^{-1} matrix

and

β_{max} & β_{min} are the maximum and minimum eigen vlaues respectively of A matrix

A check, that all the real parts of the eigen values of the coefficient matrix A of the linearized system , are negative, ensures the stability of the system in the small neighbourhood of the null solution.

3.7 MONTE CARLO METHOD

This method requires a source of generation of random numbers, which are not repeated even after the generation of several million numbers. A subroutine in machine language, for use on IBM 1620 digital computer, is prepared for this purpose. A program in accordance with the flow chart(Fig.14) is written to calculate directly the value of ξ_{min} and ξ_{max} from $\left[-\dot{V}(X)/V(X)\right]$.

3.8 TRANSIENT RESPONSE

The transient response of the state variables is determined by numerical integration of system first order eqations(2.16), using Runge-Kutta-Gill method.(AIII.7)

3.9 EXAMPLE

A salient-pole synchronous generator having the following constants

$x_d=1.15$ $x_d''=0.24$ $x_q' = 0.75$

$x_d'=0.37$ $x_q=0.75$ $x_q'' = 0.34$

$T_{do}'=5.0$ $T_{do}''=0.035$ $T_{qo}''= 0.035$

f = 50c/s Inertia Constant H = 2.5 Kw Sec./Kva. is delivering current of 1.00 per unit at 0.91 p.f. lagging through a circuit breaker to an infinite bus having a voltage of 1.00 per unit. A three phase short circuit occouring at the terminals of the generator is cleared without disconnecting the generator from the bus.

The swing equation for the system shown in Fig.2 can be written as (Refer AI.38)

$$M \frac{d2\delta}{dt^2} + Kd \frac{d\delta}{dt} = P_i - P_{m1}\sin\delta + P_{m2}\sin 2\delta \quad ..(AI.38)$$

Where

$$M = GH/\pi f$$

$$= 1 \times 2.5/\pi \times 50$$

$$= 1.59 \times 10^{-2} \quad ..(2.52)$$

$$Kd = \frac{2}{\pi}\int_0^{\pi/2} V_i^2\left\{\frac{(X_d' - X_d'')}{X_d'^2} T_{do}'' \sin^2_\delta + \frac{(X_q' - X_q'')}{X_q'^2} T_{qo}'' \cos^2\delta\right\} d\delta$$

$$= \frac{.035}{\pi}\int_0^{\pi/2} (1.68-0.22\cos 2\delta)\, d\delta$$

$$= .0294 \quad ..(2.53)$$

$$P_{m1} = E_q' V/ X_d'$$

Given that

$V_1 = 1.00 \angle 0^0$ $\qquad$ $I = 1.00 \angle -\cos^{-1} 0.91$

$= 1.00 \angle -24.5^0$

Refering to the Vector Diagram of Fig.9 ,

$$E_q = V_1 + j\,X_q\,I$$

$$= 1.00\angle 0^0 + \angle 90^0 \times 0.75 \times 1.00 \angle -24.5^0$$

$$= 1.31 + j0.68$$

$$= 1.48 \angle 27.5^0$$

$$I_d = I \,\mathrm{Sin}\,(27.5 + 24.5)$$

$$= I \sin 52^0$$

$$= 0.788$$

$$E_q' = E_q - (X_q - X_d')\,I_d$$

$$= 1.48 - (0.75 - 0.37) \times 0.788$$

$$= 1.48 - 0.38 \times 0.788$$

$$= 1.18$$

Therefore,

$$P_{m1} = \frac{1.18 \times 1.00}{0.37}$$

$$= 3.18 \qquad ..(2.54)$$

$$P_{m2} = V_1^2 \left[\frac{X_q - X_d'}{2\,X_d'\,X_q}\right]$$

$$= (0.75 - 0.37)/(2 \times 0.75 \times 0.37)$$

$$= 0.685 \qquad ..(2.55)$$

Substituting these values in eq.(AI.38),

$$1.59 \times 10^{-2} \frac{d^2\delta}{dt^2} + .0294 \frac{d\delta}{dt} = 0.91 - 3.18 \sin\delta + 0.685 \sin 2\delta \qquad ..(2.56)$$

Defining,

$$T = t\sqrt{\frac{P_{m1}}{M}}$$

$$= t\sqrt{\frac{3.18}{1.59 \times 10^{-2}}}$$

$$= 14.14\ t$$

$$Kd_1 = \frac{K_d}{\sqrt{P_{m1} \times M}}$$

$$= \frac{.0294}{\sqrt{3.18 \times 1.59 \times 10^{-2}}}$$

$$= 0.131$$

$$P_1 = P_i / P_{m1} = 0.286$$

$$P_2 = P_{m2}/P_{m1} = 0.216$$

The swing equation (2.56) is modified as

$$\frac{d^2\delta}{dT^2} + 0.131 \frac{d\delta}{dT} = 0.286 - \sin\delta + .216 \sin 2\delta \qquad ..(2.57)$$

$$= f(\delta) \qquad \text{(say)}$$

The singularities of this system are obtained by solving $f(\delta) = 0$ by the Newton Raphson's method on digital computer.(AIII.1)

The results are:

Stable focus: 0.48 radians or 27.6^{o}

Saddle Point: 2.94 radians or 168.4^{o}

Transferring the stable focus to the origin,

$x_1 = \delta - \delta_s$

where δ_s is the Stable focus.

The swing equation is now given by,

$$\frac{d2x_1}{dT^2} + \frac{dx_1}{dT}(.131) = .286 - \mathrm{Sin}(x_1+27.5^{o}) + .216\,\mathrm{Sin}(2x_1+55^{o}) \quad ..(2.58)$$

MATRIX A

Expressing eq. (2.58) in state variable form,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -.131x_2 - \left\{\mathrm{Sin}(x_1+27.5^{o}) - .216\,\mathrm{Sin}(2x_1+55^{o}) - .286\right\} \quad ..(2.59)$$

Linearizing the above system,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -.131x_2 - x_1 + .432x_1 \quad ..(2.60)$$

or,

$$\dot{x}_2 = -.131x_2 - .568\,x_1$$

In, matrix form it can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -.568 & -.131 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad ..(2.61)$$

Therefore, the Coefficient Matrix is

$$A = \begin{bmatrix} 0 & 1.0 \\ -.568 & -.131 \end{bmatrix} \quad ..(2.62)$$

The characteristic equation for the above matrix is determined by digital computer, and is given by

$$\beta^2 + .131\,\beta + .568 = 0 \qquad ..(2.63)$$

The eigen values of the above equation (2.63) are calculated by Newton's Method (AIII.3)

Whence,

$$\beta_1 = -.066 + j\ .751$$
$$\beta_2 = -.066 - j\ .751 \qquad ..(2.64)$$

The negative sign of the real parts shows that the system is atleast stable in the small neighbourhood of the origin.

MATRIX RV^{-1}

The Liapunov Function from eq. (2.27) by Cartwright's Method is,

$$V(X) = \frac{x_2^2}{2} + \int_0^{x_1} \left\{ \text{Sin}(u+.48) - .216\,\text{Sin}(2u+.96) - .286 \right\} du \qquad ..(2.65)$$

Choosing only those terms in the integral which give second order terms , after integration,

$$V(X) = \frac{x_2^2}{2} + \int_0^{x_1} (u - .432u)\ du$$

$$= \frac{x_2^2}{2} + \int_0^{x_1} (.568\ u)\ du$$

$$= 0.5\ x_2^2 + .284\ x_1^2$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} .284 & 0 \\ 0 & .5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad ..(2.66)$$

$= X^T V X$ (say)

Therefore,

$$V = \begin{bmatrix} .284 & 0 \\ 0 & .5 \end{bmatrix} \quad ..(2.67)$$

The Liapunov Stability equation(2.43),

$A^T V + VA = -2\ R$

is solved for R and then RV^{-1} is evaluated by a common programme (AIII.5) written, as per flow chart of Fig.13.

The results obtained are,

$$R = \begin{bmatrix} 0 & 0 \\ 0 & .065 \end{bmatrix} \quad ..(2.68)$$

$$RV^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & .131 \end{bmatrix} \quad ..(2.69)$$

The characteristic equation and its eigen values by the method explained earlier, are

$$\lambda^2 - .131\lambda + 0 = 0 \quad ..(2.70)$$

$$\lambda_1 = 0$$
$$\lambda_2 = .131 \quad ..(2.71)$$

MATRIX $R'V'^{-1}$

Now a realsymmetric positive definite matrix R' is assumed.

Let

$$R' = \frac{1}{100}\begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix} \qquad ..(2.72)$$

and the Liapunov stability equation (2.43) is solved for unknown matrix V'. This involves $n(n+1)/2$ equations to be solved, as V' is symmetric. Where n is the order of the system.

These equation can be written as (AII.6)

$$\begin{bmatrix} 2a_{11} & 2a_{21} & 0 \\ a_{12} & (a_{11}+a_{22}) & a_{21} \\ 0 & 2a_{12} & 2a_{22} \end{bmatrix} \begin{bmatrix} v'_{11} \\ v'_{12} \\ v'_{22} \end{bmatrix} = \begin{bmatrix} -2R'_{11} \\ -2R'_{12} \\ -2R'_{22} \end{bmatrix} \qquad ..(2.73)$$

where

$$a_{ij} \quad \begin{cases} i=1,2 \\ j=1,2 \end{cases}$$

are the elements of A matrix

and $$v'_{ij} \quad \begin{cases} i=1,2 \\ j=1,2 \end{cases}$$

are the unknown elements of V' matrix.

Substituting the known values from (2.62) and (2.72) in eq.(2.73),

$$\begin{bmatrix} 0 & -1.136 & 0 \\ 1 & -0.131 & -.568 \\ 0 & 2.0 & -.262 \end{bmatrix} \begin{bmatrix} v'_{11} \\ v'_{12} \\ v'_{22} \end{bmatrix} = \begin{bmatrix} -0.02 \\ -0.02 \\ -0.1 \end{bmatrix} \qquad ..(2.74)$$

The results are obtained by digital computer (AIII.6)

These are,

$$v'_{11} = 0.275$$

$$v'_{12} = .017 \qquad ..(2.75)$$

$$v'_{22} = .516$$

Therefore,

$$V' = \begin{bmatrix} 0.275 & 0.017 \\ 0.017 & 0.516 \end{bmatrix} \qquad ..(2.76)$$

Then from eqs. (2.72) & (2.76),

$$R'V'^{-1} = \begin{bmatrix} 0.033 & 0.018 \\ 0.030 & 0.097 \end{bmatrix} \qquad ..(2.77)$$

The characteristic equation for eq.(2.77) is evaluated as

$$\lambda'^2 - .130\ \lambda' + .003 = 0 \qquad ..(2.78)$$

The eigen values are

$$\lambda'_1 = .03$$

$$\lambda'_2 = .1 \qquad ..(2.79)$$

MONTE CARLO TECHNIQUE

The stability region is defined by

$$V = b_{max}$$

where

b_{max} can be calculated by substituting the saddle point with respect to the new origin, transferred at stable focus, in the eq.(2.66)

$$V = .284\ x_1^2 + 0.5\ x_2^2 \qquad ..(2.66)$$

The saddle point is given by

$$x_1 = 2.94 - .48$$
$$= 2.46 \qquad ..(2.80)$$
$$x_2 = 0$$

Therefore

$$b_{max} = 0.284 \times (\ 2.46\ )^2$$
$$= 1.74 \qquad ..(2.81)$$

The range for the state variable x_1, beyond which the system is unstable, can be shown as

$$0 \leq x_1 \leq 2.46 \qquad ..(2,82)$$

The range for x_2 is determined from eq. (2.66) by equating it to b_{max} and solving for x_2, when $x_1 = 0$

Therefore,

$$0.5\ x_2^2 = 1.74$$

or,

$$x_2 = 1.865$$

Hence,

$$0 \leq x_2 \leq 1.865 \qquad ..(2.83)$$

From eq. (2.42) and (2.68),

$$\dot{V}(X) = -X^T (2R) X$$

$$= -.130\ x_2^2 \qquad ..(2.84)$$

Therefore,

$$\left[-\frac{\dot{V}(X)}{V(X)} \right] = \frac{.13\ x_2^2}{.284 x_1^2 + .5\ x_2^2} \qquad ..(2.85)$$

The maxima and minima of the eq.(2.85) is determined by the program(AIV.7) written as per the flow chart of Fig.(14).

The results are:

$$\xi_{max.} = .26151$$
$$\xi_{min.} = .00004 \qquad ..(2.86)$$

TRANSIENT RESPONSE

The transient response in respect of state variables and the system Liapunov function(AIII.7) is determined by Runge-Kutta-Gill method.

The computing step is .05 second and for every .25 second , the transient response is printed. The initial condition in the state space is choosen as

$$x_{1o} = 1.0$$

$$x_{2o} = 0.5$$

REMARKS

The upper and lower bounds of the transient response as estimated from the maximum and minimum values of $\beta\,[A]$, $\lambda[RV^{-1}]$ and $\lambda'[R'V'^{-1}]$, are plotted along with the transient response obtained by Runge-Kutta-Gill method (Fig.2)

On comparing the plots, it is concluded that estimates from the matrix $[RV^{-1}]$ and those from the Monte Carlo method are almost similar.

Whereas the lower and upper estimates from the eigen values of A are coincident and are running very close to the actual response.

But the boundries of the region obtained, when a real symmetric positive definite matrix R' is assumed and matrix V' is calculated, prove to be the best estimate, as they are closest to the actual time response.

RESULTS RUNGE KUTTA GILL METHOD

WITHOUT GOVERNOR OR REGULATOR

TIME	X1	X2	V
0.000			
	1.00000	.50000	.40900
.250			
	1.10100	.31200	.39294
.500			
	1.15500	.12200	.38631
.750			
	1.16300	-.06400	.38618
1.000			
	1.12400	-.24300	.38832
1.250			
	1.04200	-.41000	.39241
1.500			
	.92000	-.56000	.39718
1.750			
	.76400	-.68500	.40038
2.000			
	.58000	-.77800	.39818
2.250			
	.37800	-.83400	.38836
2.500			
	.16600	-.85200	.37078

2.750			
	-.04400	-.83400	.34833
3.000			
	-.24700	-.78400	.32465
3.250			
	-.43400	-.70800	.30413
3.500			
	-.59900	-.60900	.28734
3.750			
	-.73700	-.49200	.27529
4.000			
	-.84400	-.36100	.26746
4.250			
	-.91700	-.22100	.26323
4.500			
	-.95500	-.07600	.26190
4.750			
	-.95600	.06500	.26167
5.000			
	-.92200	.20100	.26162
5.250			
	-.85600	.32400	.26059
5.500			
	-.76100	.43200	.25778
5.750			
	-.64200	.52000	.25225

6.000

-.50300 .58600 .24355

6.250

-.35000 .63000 .23324

6.500

-.18900 .65100 .22205

6.750

-.02700 .64700 .20951

7.000

.13100 .61800 .19584

7.250

.27900 .56400 .18115

7.500

.41100 .48800 .16705

7.750

.52200 .39300 .15461

8.000

.60700 .28400 .14497

8.250

.66300 .16500 .13845

8.500

.69000 .04400 .13618

8.750

.68600 -.07500 .13646

9.000

.65300 -.18700 .13858

9.250			
	.59300	-.28800	.14134
9.500			
	.51000	-.37300	.14343
9.750			
	.40800	-.43800	.14320
10.000			
	.29200	-.48300	.14086
10.250			
	.16800	-.50500	.13553
10.500			
	.04100	-.50500	.12799
10.750			
	-.08200	-.48500	.11952
11.000			
	-.19900	-.44700	.11115
11.250			
	-.30500	-.39500	.10443
11.500			
	-.39600	-.32900	.09866
11.750			
	-.46900	-.25500	.09498
12.000			
	-.52300	-.17400	.09282
12.250			
	-.55600	-.08900	.09176

12.500			
	-.56800	-.00400	.09163
12.750			
	-.55800	.07700	.09139
13.000			
	-.52900	.15400	.09133
13.250			
	-.48200	.22300	.09084
13.500			
	-.41800	.28300	.08967
13.750			
	-.34100	.33000	.08747
14.000			
	-.25400	.36400	.08457
14.250			
	-.16000	.38400	.08100
14.500			
	-.06300	.38900	.07679
14.750			
	.03300	.37900	.07213
15.000			
	.12500	.35400	.06710

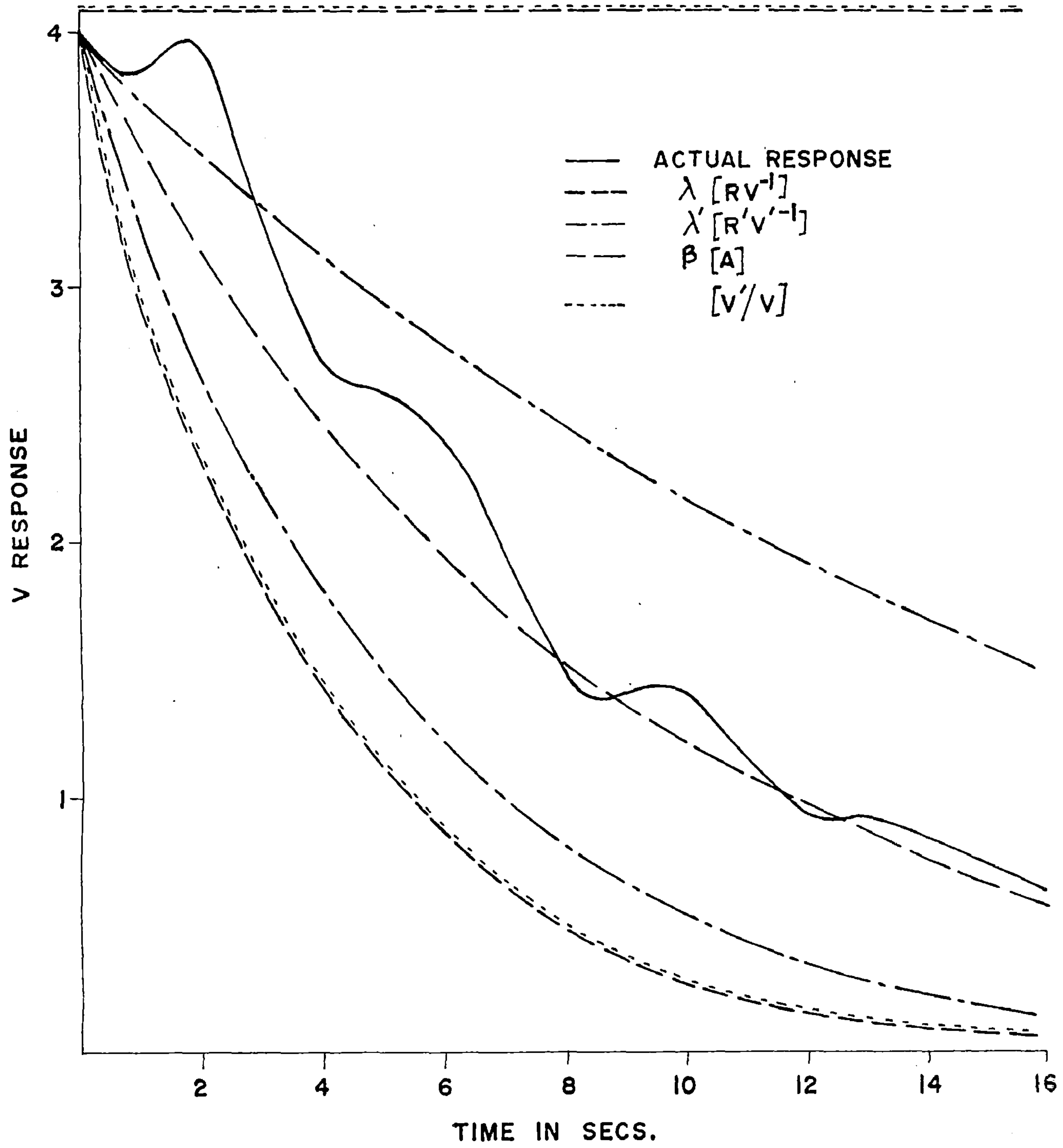

FIG. 2. MACHINE CONNECTED TO AN INFINITE BUS.

CHAPTER IV

TRANSIENT RESPONSE ESTIMATE OF A SINGLE MACHINE CONNECTED TO AN INFINITE BUS WITH GOVERNOR ACTION

4.1 INTRODUCTION

The system of the previous chaper, having a synchronous mabhine connected to an infinite bus is now incorporated with governor action.(Fig.3) Therefore the assumption of constant mechanical power input, made so far, is changed to that of variable mechanical power input and a transfer function representing prime mover and governor is described. The swing equation is modified to include the effect of input power control. A Liapunov function for the third order system is constructed by Cartwright's method and the estimates of transient response are again determined.

4.2 SWING EQUATION

Let the variation ΔP_1 in mechanical power input due to the velocity governor action be given by

$$\Delta P_1 = \frac{G_1}{(1+T_1 p)\ (1+T_2 p)\ \omega_0} \cdot \frac{d\delta}{dt} \qquad ..(4.11)$$

where

G_1 = Velocity governor gain

w_0 = System rated angular frequency

T_1 = Servomechanism time constant, sec.

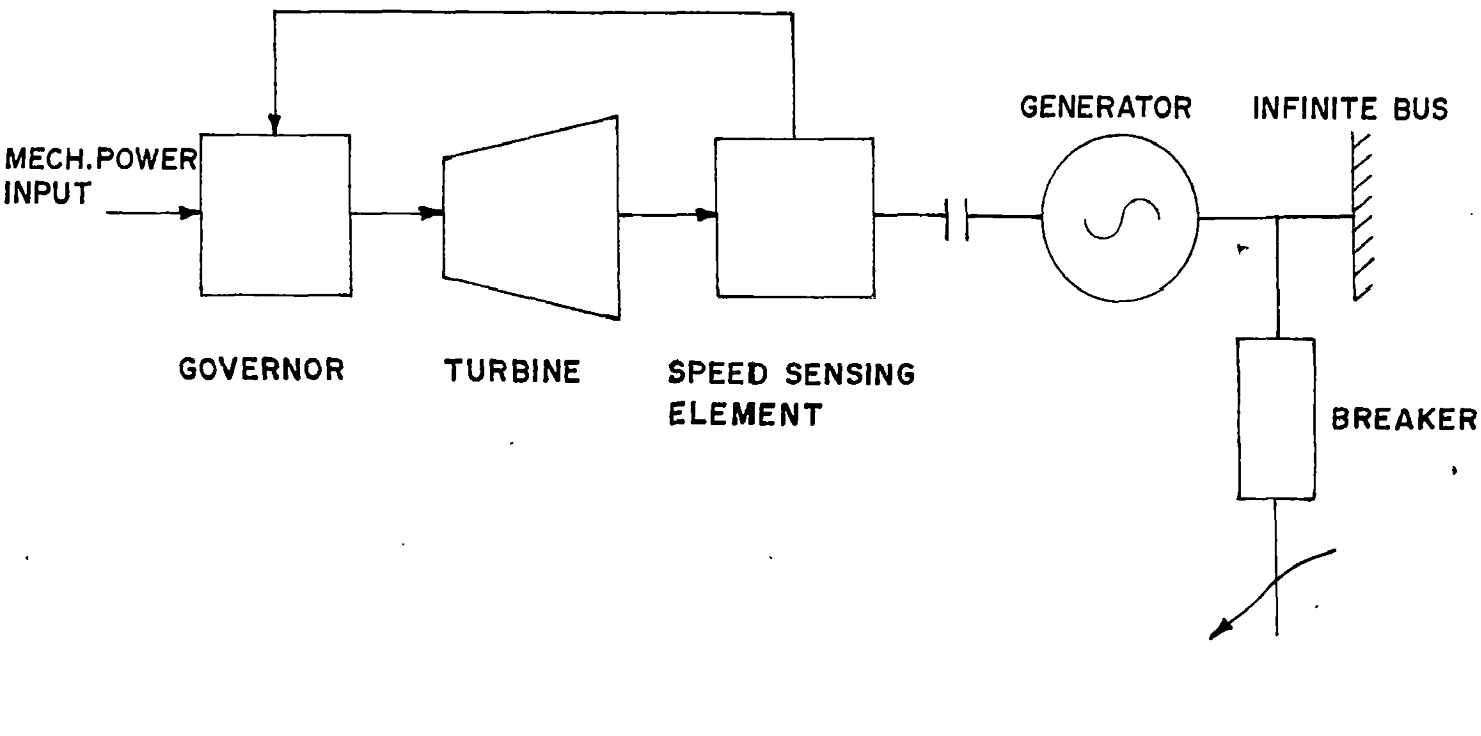

FIG. 3. ONE MACHINE CONNECTED TO AN INFINITE BUS WITH VELOCITY GOVERNOR.

T_2 = Prime movertime constant, sec.

In order to limit the order of the system to three, the eq. (4.11) is simplified by neglecting the prime mover constant T_2.

Thus,

$$\Delta P_i = \frac{G_1}{(1+T_1 p) w_o} \cdot \frac{d\delta}{dt} \qquad ..(4.12)$$

Defining the new variable,

$$T = t \sqrt{\frac{P_{mi}}{M}}$$

so that

$$\frac{dT}{dt} = \sqrt{\frac{P_{mi}}{M}} = C \qquad ..(4.13)$$

so that

Eq. (4.12) can be written as

$$\Delta P_i = \frac{G_1}{\left(1+ T_1 p' \cdot \frac{dT}{dt}\right) w_o} \cdot \frac{d\delta}{dT} \cdot \frac{dT}{dt}$$

$$= \frac{G_1}{\left(\frac{dt}{dT} + T_1 p'\right) w_o} \cdot \frac{d\delta}{dT}$$

$$= \frac{G_1}{(c + T_1 p') w_o} \cdot \frac{d\delta}{dT} \qquad ..(4.14)$$

where $p' = d/dT$

The swing equation (2.12) without governor action is

$$\frac{d^2\delta}{dT^2} + Kd1\frac{d\delta}{dT} = P_1 - \sin\delta + P_2\sin 2\delta$$

$$= P_1 - \sin\delta \qquad (2.12)$$

Sin2δ term is neglected for further simplification.

Introducing change in mechanical power input proportional to the velocity, the equation (2.12) becomes

$$\frac{d^2\delta}{dT^2} + Kd1\frac{d\delta}{dT} = P_1 - \Delta P_i - \sin\delta \qquad ..(4.15)$$

Substituting eq. (4.14) in eq. (4.15),

$$\frac{d^2\delta}{dT^2} + Kd1\frac{d\delta}{dT} = P_1 - \frac{G_1}{(c+T_1 p')w_o} \cdot \frac{d\delta}{dT} - \sin\delta \qquad ..(4.16)$$

Rearranging ,

$$T_1\frac{d^3\delta}{dT^3} + (c+T_1Kd1)\frac{d^2\delta}{dT^2} + \binom{c.Kd1+G_1/w_o}{+T_1\cos\delta}\frac{d\delta}{dT}$$

$$= cP_1 - c.\sin\delta \quad ..(4.17)$$

4.3 LIAPUNOV FUNCTION

The third order system can be expressed as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -k_1x_3-k_2x2 - g(x_1) \quad ..(4.18)$$

where

$g(x_1)$ is the nonlinear factor in state variable x_1.

Considering $g(x_1)= k_3x_1$

The system equations (4.18) become

$$\dot{x}_1= x_2$$
$$\dot{x}_2= x_3 \quad ..(4.19)$$
$$\dot{x}_3= -k_1x_3-k_2x_2-k_3x_1$$

where k_1 ,k_2,k_3 are constants.

Assume that the Liapunov function in quadratic form be given as

$$V= \tfrac{1}{2}a_1x_1^2+\tfrac{1}{2}a_2x_2^2+\tfrac{1}{2}a_3x_3^2 +\tfrac{1}{2}a_4x_1x_2+\tfrac{1}{2}a_5x_1x_3+\tfrac{1}{2}a_6x_2x_3 \quad ..(4.20)$$

Differentiating with respect to time,

$$\dot{V} = a_1x_1\dot{x}_1+a_2x_2\dot{x}_2+a_3x_3\dot{x}_3+a_4/2(\dot{x}_1x_2+x_1\dot{x}_2)+a_5/2(\dot{x}_1x_3+ +\dot{x}_3x_1)+a_6/2(\dot{x}_2x_3+x_2\dot{x}_3) \quad ..(4.21)$$

Substituting eq. (4.19) in eq.(4.21),

$$\dot{V} = a_1x_1x_2+a_2x_2x_3+a_3x_3(-k_1x_3-k_2x_2-k_3x_1)+a_4/2(x_2^2+ +x_1x_3)+a_5/2(x_2x_3+x_1(-k_1x_3-k_2x_2-k_3x_1))+a_6/2 (x_3^2+x_2(-k_1x_3-k_2x_2-k_3x_1))$$

Rearranging,

$$\dot{V} = -\tfrac{1}{2}k_3 a_5 x_1^2 - (\tfrac{1}{2}k_2 a_6 - \tfrac{1}{2}a_4)x_2^2 - (a_3 k_1 - \tfrac{1}{2}a_6)x_3^2 + (a_1 - \tfrac{1}{2}k_2 a_5 -$$

$$- \tfrac{1}{2}k_3 a_6)x_1 x_2 + (\tfrac{1}{2}a_4 - \tfrac{1}{2}k_1 a_5 - a_3 k_3)x_1 x_3 + (a_2 - a_3 k_2 + \tfrac{1}{2}a_5 -$$

$$- \tfrac{1}{2}k_1 a_6)x_2 x_3 \qquad ..(4.22)$$

Constraining eq.(4.22) to the negative semi-definite in x_2,

$$a_5 = 0$$

$$a_3 k_1 - \tfrac{1}{2}a_6 = 0$$

$$a_1 - \tfrac{1}{2}k_2 a_5 - \tfrac{1}{2}k_3 a_6 = 0 \qquad ..(4.23)$$

$$\tfrac{1}{2}a_4 - \tfrac{1}{2}k_1 a_5 - a_3 k_3 = 0$$

$$a_2 - a_3 k_2 + \tfrac{1}{2}a_5 - \tfrac{1}{2}k_1 a_6 = 0$$

Choosing $a_3 = 1$ arbitrarily, the solution of eq.(4.23) is

$$a_1 = k_1 k_3$$

$$a_2 = k_2 + k_1^2$$

$$a_4 = 2k_3 \qquad ..(4.24)$$

$$a_5 = 0$$

$$a_6 = 2k_1$$

Substituting eq.(4.24) in eq.(4.22),

$$\dot{V} = -(k_1 k_2 - k_3)\, x_2^2 \qquad ..(4.25)$$

and equation (4.20) becomes

$$V = \tfrac{1}{2}k_1 k_3 x_1^2 + \tfrac{1}{2}(k_2 + k_1^2)x_2^2 + \tfrac{1}{2}x_3^2 + k_3 x_1 x_2 + k_1 x_2 x_3$$

or,

$$V = \tfrac{1}{2}(k_1 x_2 + x_3)^2 + \tfrac{1}{2}k_2 x_2^2 + \tfrac{1}{2}k_1 k_3 x_1^2 + k_3 x_1 x_2 \qquad ..(4.26)$$

As the nonlinear function g(x1) was replaced by the linear term $k_3 x_1$ in eq.(4.18), $\int g(x_1)\,dx_1$ can be substituted for $\tfrac{1}{2}k_3 x_1^2$ in eq. (4.26)

Thus,

$$V = \tfrac{1}{2}(k_1 x_2 + x_3)^2 + \tfrac{1}{2}k_2 x_2^2 + g(x_1)x_2 + k_1 \int_0^{x_1} g(u)\,du \qquad ..(4.27)$$

4.4 EXAMPLE

Considering the same system of chaper III (2.9), and introducing the control of mechanical power input by velocity governor, as shown in Fig.3, with the values of the constants given as:

$G| = 20$

and $T_1 = 0.1$ Sec.

The swing equation (4.17)

$$T_1 \frac{d^3\delta}{dT^3} + (c + T_1 Kd1)\frac{d^2\delta}{dT^2} + (c.Kd1 + \frac{G|}{w_0} + T_1 \cos\delta)\frac{d\delta}{dT} = c.P_1 - c\sin\delta \qquad ..(4.17)$$

can be expressed as

$$\frac{d^3\delta}{dT^3} + (\frac{c}{T_1} + Kd1)\frac{d^2\delta}{dT^2} + (\frac{c.Kd1}{T_1} + \frac{G|}{w_0 T_1} + \cos\delta)\frac{d\delta}{dT} = \frac{c.P_1}{T_1} - \frac{c}{T_1}\sin\delta \qquad ..(4.28)$$

where from (2.57) ,

$c = 1/14.14$

$Kd1 = .131$

$P_1 = .286$

$w_0 = 100\,\pi$

Substituting these values in eq.(4.28) we get,

$$\frac{d3\delta}{dT^3} + \left(\frac{1}{1.414} + .131\right)\frac{d2\delta}{dT^2} + \left(\frac{.131}{1.414} + \frac{1}{.5\,\pi} + \text{Cos}\,\delta\right)\frac{d\delta}{dT}$$

$$= \frac{.286}{1.414} - \frac{1}{1.414}\,\text{Sin}\,\delta$$

or,

$$\frac{d3\delta}{dT^3} + .837\frac{d2\delta}{dT^2} + (.728 + \text{Cos}\,\delta)\frac{d\delta}{dT} = .202 - .706\,\text{Sin}\,\delta \quad ..(4.29)$$

SINGULARITIES

Stable focus and saddle point singularities are determined from the equation

$$.202 - .706\,\text{Sin}\,\delta = 0$$

Therefore,

Stable Focus: .20 radians or 16.6°

Saddle Point: 2.85 radians or 163.4°

Transferring the origin of the state space on the stable focus, the eq.(4.29) can be written as,

$$\frac{d3x_1}{dT^3} + .837\,\frac{d2x_1}{dT^2} + (.728 + \text{Cos}(x_1 + .20))\frac{dx_1}{dT}$$

$$= .202 - .706\,\text{Sin}(x_1 + .20) \quad ..(4.30)$$

where $(x_1 + .20) = \delta$

MATRIX A

Expressing eq.(4.30) in state variable form,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -.837x_3-(.728+\text{Cos}(x_1+.29))x_2-(.706\text{Sin}(x_1+ +.29)-.202) \qquad ..(4.31)$$

Neglecting higher degree terms from eq. (4.31),

$$\dot{x}^1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -.837\ x_3-1.728x_2-.706x_1 \qquad ..(4.32)$$

Expressing eq.(4.32) in matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -.706 & -1.728 & -.837 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad ..(4.33)$$

Therefore the coefficient matrix is given by,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -.706 & -1.728 & -.837 \end{bmatrix} \qquad ..(4.34)$$

The characteristic equation for the matrix (4.34) can be expressed as,

$$p^3 + .837\ p^2 + 1.728p + .706 = 0$$

and the eigen values are

$$\beta_1 = -.191 + j1.232$$
$$\beta_2 = -.191 - j1.232 \qquad ..(4.36)$$
$$\beta_3 = -.454$$

The negative real parts of these values predict that the system is stable in the immediate neighbourhood of the equilibrium condition.

MATRIX RV^{-1}

From eqs.(4.32) and (4.26), the Liapunov function V(X) can be shown by

$$V = \tfrac{1}{2}(.7x_2^2+x_3^2+1.674x_2x_3)+\tfrac{1}{2}(1.728x_2^2)+(.706x_1x_2)$$
$$+\tfrac{1}{2}(.837\times0.706)x_1^2$$
$$= .296x_1^2+1.214x_2^2+0.5x_3^2+.706x_1x_2+.837x_2x_3 \qquad ..(4.37)$$

Equation(4.37) in matrix form will be,

$$V(X) = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} .296 & .353 & .0 \\ .353 & 1.214 & .418 \\ 0 & .418 & .5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad ..(4.38)$$

$$= X^TVX$$

Therefore matrix V is given by,

$$V = \begin{bmatrix} .296 & .353 & .0 \\ .353 & 1.214 & .418 \\ 0 & .418 & .5 \end{bmatrix} \qquad ..(4.39)$$

Substituting eqs.(4.34) and (4.39) in the Liapunov Stability Equation

$$A^TV + VA = -2R$$

the matrix R is determined as

$$R = \begin{bmatrix} .0 & .0 & .0 \\ .0 & .869 & .0 \\ .0 & .0 & .0 \end{bmatrix} \qquad ..(4.40)$$

and from eqs. (4.40) and (4.39),

$$RV^{-1} = \begin{bmatrix} .001 & -.001 & .0 \\ -.995 & .833 & -.697 \\ .001 & -.001 & .001 \end{bmatrix} \qquad ..(4.41)$$

The characteristic equation obtained for eq.(4.41) is

$$\lambda^3 - .835\lambda^2 + 0\lambda + 0 = 0 \qquad ..(4.42)$$

The eigen values are

$$\lambda_1 = 0$$

$$\lambda_2 = 0 \qquad ..(4.43)$$

$$\lambda_3 = .835$$

MATRIX $R'V'^{-1}$

Now a real symmetric positive definite matrix R' is choosen.

Let,

$$R' = \frac{1}{10}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 2 \end{bmatrix} \qquad ..(4.44)$$

A set of six equations (Aii.6) is obtained from Liapunov Stability Equation in terms of the unknown elements of the matrix V'.

This set is

$$\begin{bmatrix} 2a_{11} & 2a_{21} & 2a_{31} & 0 & 0 & 0 \\ a_{12} & (a_{22}+a_{11}) & a_{32} & a_{21} & a_{31} & 0 \\ a_{13} & a_{23} & (a_{11}+a_{33}) & 0 & a_{21} & a_{31} \\ 0 & 2a_{12} & 0 & 2a_{22} & 2a_{32} & 0 \\ 0 & a_{13} & a_{12} & a_{23} & (a_{22}+a_{33}) & a_{32} \\ 0 & 0 & 2a_{13} & 0 & 2a_{23} & 2a_{33} \end{bmatrix} \begin{bmatrix} v'_{11} \\ v'_{12} \\ v'_{13} \\ v'_{22} \\ v'_{23} \\ v'_{33} \end{bmatrix} = \begin{bmatrix} -2\,R'_{11} \\ -2\,R'_{12} \\ -2\,R'_{13} \\ -2\,R'_{22} \\ -2\,R'_{23} \\ -2\,R'_{33} \end{bmatrix} \quad ..(4.45)$$

where

$$v'_{ij} \quad \begin{cases} i = 1,2,3 \\ j = 1,2,3 \end{cases}$$

are the unknown elements of V' matrix.

Substituting eqs. (4.34) and (4.44) in eq.(4.45),

$$\begin{bmatrix} 0 & 0 & -1.412 & 0 & 0 & 0 \\ 1 & 0 & -1.728 & 0 & -.706 & 0 \\ 0 & 1 & -0.837 & 0 & 0 & -.706 \\ 0 & 2 & 0 & 0 & -3.456 & 0 \\ 0 & 0 & 1 & 1 & -0.837 & -1.728 \\ 0 & 0 & 0 & 0 & 2 & -1.674 \end{bmatrix} \begin{bmatrix} v'_{11} \\ v'_{12} \\ v'_{13} \\ v'_{22} \\ v'_{23} \\ v'_{33} \end{bmatrix} = \begin{bmatrix} -0.4 \\ -0.2 \\ -0.2 \\ -0.8 \\ -0.2 \\ -0.4 \end{bmatrix} \quad ..(4.46)$$

The solution of the above equations (4.46) is

$v'_{11} = .773$ $\quad v'_{22} = 1.916$

$v'_{12} = .783$ $\quad v'_{23} = .684$ ..(4.47)

$v'_{13} = .283$ $\quad v'_{33} = 1.057$

Therefore the matrix V' will be

$$V' = \begin{bmatrix} .773 & .783 & .283 \\ .783 & 1.916 & .684 \\ .283 & .684 & 1.057 \end{bmatrix} \quad ..(4.48)$$

From eqs. (4.44) and (4.48),

$$R'V'^{-1} = \begin{bmatrix} .350 & -.119 & .077 \\ -.139 & .283 & -.051 \\ .129 & -.072 & .202 \end{bmatrix} \quad ..(4.49)$$

The characteristic equation for the above matrix of eq.(4.49) , is determined as,

$$\lambda'^3 - .835\lambda'^2 + .197\lambda' - .014 = 0$$

..(4.50)

and its eigen values are

$$\lambda'_1 = .198$$

$$\lambda'_2 = .148 \quad ..(4.51)$$

$$\lambda'_3 = .494$$

MONTE CARLO TECHNIQUE

The Liapunov function in quadratic form from eq. (4.37) is

$$V(X) = .296\,x_1^2 + 1.214x_2^2 + 0.5x_3^2 + .706x_1x_2 + .637x_2x_3$$

and from eq. (4.40),

$$\dot{V}(X) = X^T(-2R)X$$
$$= -.738\,x_2^2 \qquad ..(4.52)$$

The saddle point as referred to new origin can be shown as

$$x_1 = 2.85 - .29$$
$$= 2.56$$
$$x_2 = 0 \qquad ..(4.53)$$
$$x_3 = 0$$

Hence b_{max} is obtained by substituting (4.53) in eq. (4.37),

$$b_{max} = .296 \times (2.56)^2$$
$$= 1.94 \qquad ..(4.54)$$

Therefore, the region in which the state variable x_1 can vary , is

$$0 \leq x_1 \leq 2.56 \qquad ..(4.55)$$

and region for x_2 can be obtained from

$$1.214\,x_2^2 = b_{max}$$
$$= 1.94$$

Therefore, $x_2 = 1.267$

Hence,

$$0 \leq x_2 \leq 1.207 \qquad ..(4.56)$$

Region for x_3 is determined from

$$0.5\, x_3^2 = 1.94$$

or, $$x_3 = 1.97$$

Therefore,

$$0 \leq x_3 \leq 1.97 \qquad ..(4.57)$$

Random numbers are generated for state variables x_1, x_2 and x_3 within the ranges specified in eqs. (4.55), (4.56) and (4.57), and ξ_{max} & ξ_{min} are calculated from

$$\xi \begin{pmatrix} max \\ or \\ min \end{pmatrix} = \underset{Min}{\overset{Max}{or}} \left[\frac{-\dot{V}(X)}{V(X)} \right]$$

$$= \underset{Min}{\overset{Max}{or}} \left[\frac{.738\, x_2^2}{\left(.296x_1^2 + 1.214x_2^2 + .5x_3^2 + .706x_1 x_2 + .837\, x_2 x_3 \right)} \right] \qquad ..(4.58)$$

The results are

$$\xi_{max} = .57727$$

$$\xi_{min} = .00000 \qquad ..(4.59)$$

<u>TRANSIENT RESPONSE</u>

The system transient response is obtained by numerical integration of the set of first order

differential equations (4.31),using Runge-Kutta-
-Gill method.

The initial condition of the system is given by

$$x_{10} = .1$$

$$x_{20} = .1$$

$$x_{30} = .1$$

REMARKS

The upper and lower bounds obtained from the eigen values of $R'V'^{-1}$ seem to be again reasonable , keeping in veiw the actual transient response. The upper boundary from the matrix A, runs , well below the transient response curve, initially and then along with the curve afterwards.

The region defined by matrix RV^{-1} is very wide. Whereas the estimation of lower limit by M onte Carlo method is best. (See Fig.4)

RESULTS RUNGE KUTTA GILL METHOD

WITH GOVERNOR

TIME	X1	X2	X3	V
0.000				
	.10000	.10000	.10000	.03553
.250				
	.12700	.11500	.02300	.03362
.500				
	.15600	.11200	-.04400	.03161
.750				
	.18200	.09400	-.09900	.02972
1.000				
	.20200	.06400	-.13800	.02831
1.250				
	.21300	.02600	-.15800	.02720
1.500				
	.21500	-.01300	-.16200	.02680
1.750				
	.20600	-.05300	-.15000	.02617
2.000				
	.18800	-.08800	-.12400	.02500
2.250				
	.16300	-.11500	-.08900	.02321
2.500				
	.13200	-.13200	-.04800	.02046
2.750				

	.09800	-.13900	-.00500	.01728
3.000				
	.06300	-.13500	.03500	.01395
3.250				
	.03100	-.12200	.07000	.01099
3.500				
	.00300	-.10100	.09600	.00867
3.750				
	-.01900	-.07400	.11300	.00713
4.000				
	-.03400	-.04500	.11900	.00648
4.250				
	-.04100	-.01500	.11300	.00617
4.500				
	-.04200	.01000	.09900	.00608
4.750				
	-.03600	.03300	.07800	.00606
5.000				
	-.02600	.04900	.05200	.00570
5.250				
	-.01200	.05900	.02400	.00524
5.500				
	.00200	.06100	-.00200	.00450
5.750				
	.01700	.05800	-.02600	.00394
6.000				

	.03100	.04800	-.04500	.00334
6.250				
	.04200	.03500	-.05800	.00303
6.500				
	.04900	.01900	-.06500	.00289
6.750				
	.05100	.00300	-.06400	.00278
7.000				
	.05000	-.01100	-.05800	.00271
7.250				
	.04600	-.02500	-.04700	.00266
7.500				
	.03800	-.03500	-.03300	.00249
7.750				
	.02800	-.04100	-.01700	.00219
8.000				
	.01700	-.04400	-.00100	.00195
8.250				
	.00600	-.04200	.01300	.00160
8.500				
	-.00300	-.03700	.02600	.00128
8.750				
	-.01100	-.02900	.03500	.00104
9.000				
	-.01700	-.02000	.04000	.00094
9.250				

	-.02100	-.00900	.04100	.00089
9.500				
	-.02200	0.00000	.03800	.00087
9.750				
	-.02100	.00900	.03300	.00089
10.000				
	-.01800	.01600	.02400	.00081
10.250				
	-.01300	.02100	.01500	.00077
10.500				
	-.00700	.02400	.00500	.00071
10.750				
	-.00100	.02400	-.00400	.00061
11.000				
	.00400	.02200	-.01200	.00051
11.250				
	.00900	.01800	-.01800	.00042
11.500				
	.01300	.01200	-.02200	.00036
11.750				
	.01500	.00600	-.02400	.00034
12.000				
	.01600	0.00000	-.02300	.00034
12.250				
	.01600	-.00400	-.02000	.00032
12.500				

	.01400	-.00900	-.01600	.00032
12.750				
	.01100	-.01300	-.01000	.00030
13.000				
	.00700	-.01500	-.00400	.00027
13.250				
	.00400	-.01500	0.00000	.00024
13.500				
	0.00000	-.01400	.00600	.00019
13.750				
	-.00300	-.01200	.01000	.00015
14.000				
	-.00500	-.00900	.01300	.00012
14.250				
	-.00700	-.00500	.01400	.00011
14.500				
	-.00800	-.00200	.01400	.00011
14.750				
	-.00800	.00100	.01300	.00011
15.000				
	-.00800	.00400	.01100	.00011

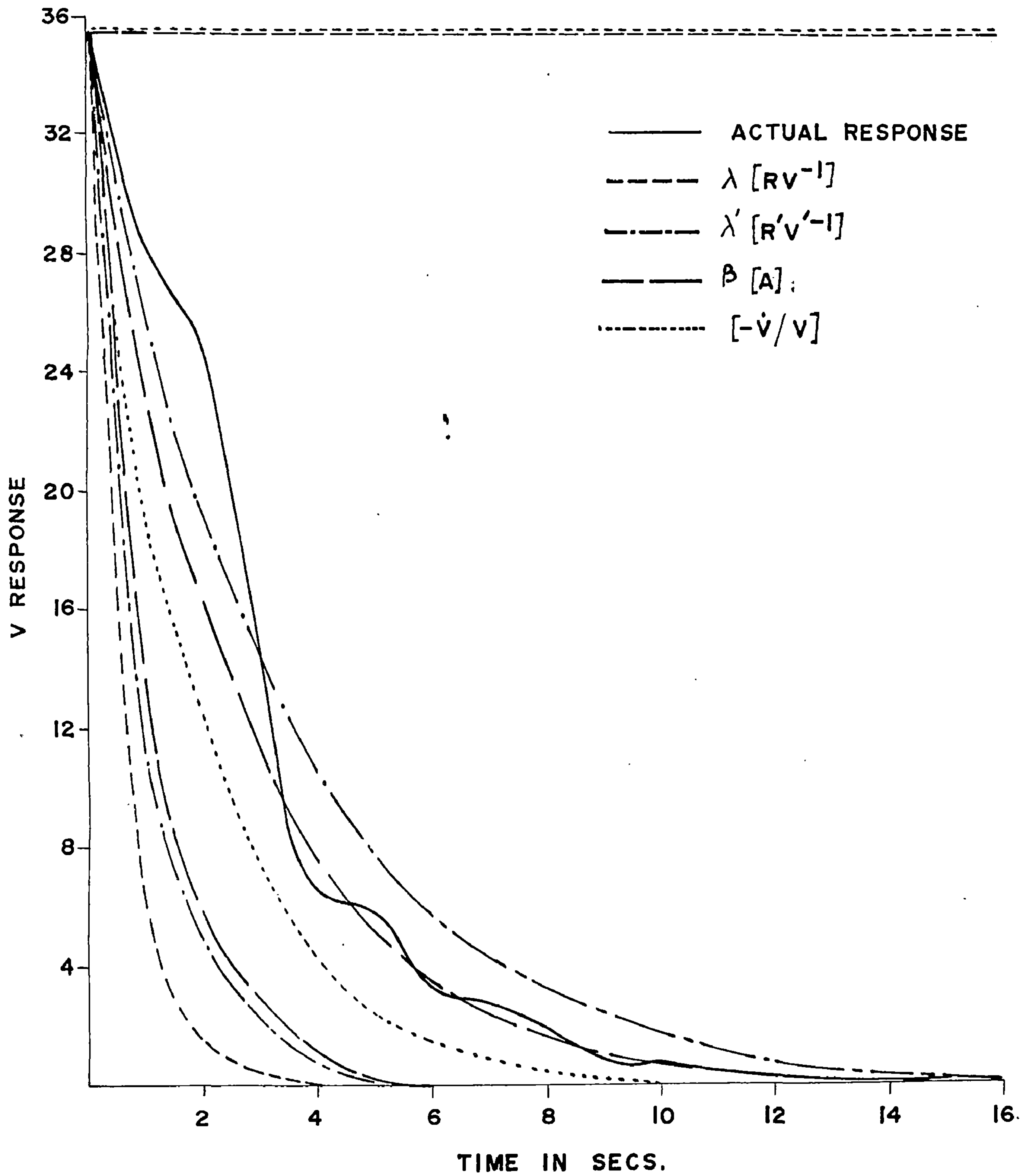

FIG. 4. MACHINE WITH VELOCITY GOVERNOR ACTION.

CHAPTER V

TRANSIENT RESPONSE ESTIMATE OF SINGLE MACHINE CONNECTED TO AN INFINITE BUS WITH ANGLE CONTROL

5.1 INTRODUCTION

The power system consisting of one synchronous machine supplying power against an infinite bus includes a feedback loop to control the angle of the rotor with respect to a rotating reference axis at synchronous speed.

The Angle Regulator is represented by a simple network, feeding back signals proportional to angle, velocity and acceleration, as shown in Fig. 5. An expression of swing equation for the system with angle regulator action is derived. The matrices A, RV^{-1} and $R'V'^{-1}$ are determined, while the Liapunov function (4.27)of the Chapter IV is used. The estimates of the transient resp-onse from the maximum and minimum eigen values of the above matrices, and by Monte Carlo method are obtained.

5.2 SWING EQUATION

Based on the two axis model of synchronous machine, the voltage equation for the field circuit

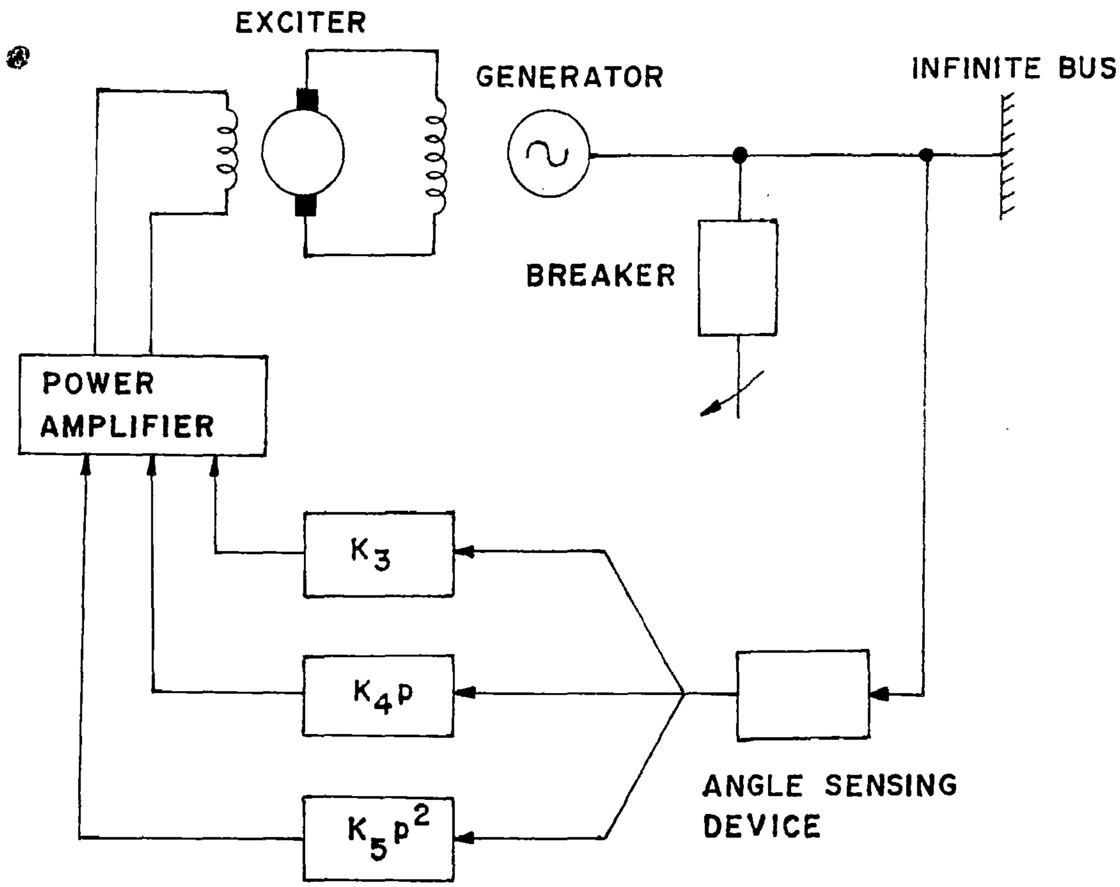

FIG. 5. ONE MACHINE CONNECTED TO AN INFINITE BUS WITH ANGLE REGULATOR.

can be written as

$$v_{fd} = p\psi_{fd} + r_{fd} i_{fd} \qquad ..(5.11)$$

where

v_{fd} = Voltage across the terminals of the field circuit

r_{fd} = Field circuit resistance

i_{fd} = Field current

ψ_{fd} = Flux linkage with the field circuit

Defining,

$$E_q' = \frac{w X_{ad}}{X_{ffd}} \psi_{fd}$$

$$E_{fd} = \frac{w X_{ad}}{r_{fd}} v_{fd} \qquad ..(5.12)$$

$$E = w X_{ad} \cdot i_{fd}$$

$$T_{do}' = \frac{X_{ffd}}{r_{fd}}$$

where

X_{ad} = Armature reaction ~~due to~~ direct axis reactance

X_{ffd} = Field winding reactance

E_q' = Voltage behind direct axis transient reactance

T_{do}' = Direct axis o.c. transient time constant

Multiplying (5.11) by $\frac{w\,X_{ad}}{X_{ffd}}$ on both sides,

$$\frac{w\,X_{ad}}{X_{ffd}}\cdot v_{fd} = \frac{p\,w\,X_{ad}}{X_{ffd}}\cdot\psi_{fd} + \frac{w\,X_{ad}}{X_{ffd}}\cdot r_{fd}\cdot i_{fd} \quad ..(5.14)$$

or,

$$\frac{r_{fd}}{X_{ffd}}\cdot\frac{wX_{ad}}{r_{fd}}\cdot v_{fd} = p\left(\frac{wX_{ad}\cdot\psi_{fd}}{X_{ffd}}\right) + \frac{r_{fd}}{X_{ffd}}\cdot w\,X_{ad}\cdot i_{fd}$$

Therefore,

$$\frac{1}{T'_{do}}\cdot E_{fd} = p\,E'_q + \frac{E}{T'_{do}} \quad ..(5.15)$$

or,

$$E'_q = \frac{E_{fd} - E}{T'_{do}\,p} \quad ..(5.16)$$

From the vector diagram of Fig. 9,

$$E = E'_q + I_d(X_d - X'_d) \quad ..(5.17)$$

$$E'_q = V_1 \cos\delta + I_d X'_d \quad ..(5.18)$$

From eq. (5.18),

$$I_d = \frac{E_q' - V_1 \cos\delta}{X_d'} \quad ..(5.19)$$

Substituting eq. (5.19) in eq.(5.17),

$$E = E_q' + \frac{E_q' - V_1 \cos\delta}{X_d'}(X_d - X'_d)$$

$$E = E_q' \left[\frac{X_d}{X_d'}\right] - \left[\frac{X_d - X_d'}{X_d'}\right] V_1 \cos\delta \qquad ..(5.20)$$

From eq. (5.16) ,

$$T_{do}'\, p\, E_q' = E_{fd} - E \qquad ..(5.21)$$

Substituting eq.(5.20) in eq. (5.21),

$$T_{do}'p\, E_q' = E_{fd} - E_q'\left[\frac{X_d}{X_d'}\right] + \left[\frac{X_d - X_d'}{X_d'}\right] V_1 \cos\delta \qquad ..(5.22)$$

Therefore,

$$E_q' = \frac{E_{fd} + ((X_d - X_d')/X_d')\, V_1 \cos\delta}{(T_{do}'p + X_d/X_d')} \qquad ..(5.23)$$

The angle regulator action can be expressed mathematically as,(Fig. 6)

$$E_{fd} = E_{fdo} + K_3\delta + K_4 p\delta + K_5 p^2\delta \qquad ..(5.24)$$

where

E_{fdo} = Value of E_{fd} without the angle regulator action

K_3, K_4 & K_5 = Gain constants of the angle regulator

p = Differential operator

Considering the swing equation without governor or regulator action (AI.32)

$$M.\frac{d^2\delta}{dt^2} + Kd\frac{d\delta}{dt} = P_i - P_{m1}\sin\delta$$

Sin 2δ term is neglected.

where

$$P_{m1} = \frac{E_q' \, V_1}{X_d'} \qquad ..(5.25)$$

Substituting eq. (5.24) in eq. (5.25),

$$E_q' = \frac{E_{fdo} + K_3\delta + K_4 p\delta + K_5 p^2\delta + \left(\frac{X_d - Xd'}{X_d'}\right) V_1 \cos\delta}{\left(\frac{X_d}{X_d'} + T_{do}' \, p\right)} \qquad ..(5.26)$$

Substituting this value of E_q' in eq.(5.25),

$$P_{m1} = \frac{\left\{E_{fdo} + K_3\delta + K_4 p\delta + K_5 p^2\delta + \left(\frac{X_d - X_d'}{X_d'}\right) V_1 \cos\delta\right\} V_1}{X_d'\left(\frac{X_d}{X_d'} + T_{do}' \, p\right)} \qquad ..(5.27)$$

Replacing P_{m1} in swing equation (AI.36) by eq.(5.27), the swing equation with angle regulator action can be written as

$$M.\frac{d^2\delta}{dt^2} + K_d\frac{d\delta}{dt} = P_1 - \frac{\left[E_{fdo} + K_3\delta + K_4 p\delta + K_5 p^2\delta + \left(\frac{X_d - X_d'}{X_d'}\right) V_1 \cos\delta\right] V_1 \sin\delta}{X_d'\left(\frac{X_d}{X_d'} + T_{do}' p\right)} \qquad ..(5.28)$$

Simplifying and rearranging,

$$M X_d' T_{do}' \frac{d^3\delta}{dt^3} + (M X_d + Kd T_{do}'.X_d' + K_5 V_1 \sin\delta) \frac{d^2\delta}{dt^2}$$

$$+ (K_d X_d + K_4 V_1 \sin\delta) \frac{d\delta}{dt} = P_1 X_d - (E_{fdo} + K_3\delta + \frac{X_d - X_d'}{X_d'} V_1 \cos\delta) V_1 \sin\delta$$

$$= P_1 X_d - (K_3\delta + E_{fdo}) V_1 \sin\delta - \frac{X_d - X_d'}{X_d'} \frac{V_1^2}{2} \sin 2\delta$$

$$= P_1 X_d - (K_3\delta + E_{fdo}) V_1 \sin\delta \quad ..(5.29)$$

Sin 2δ term is neglected for simplification.

5.3 LIAPUNOV FUNCTION

The swing equation with angle regulator action (5.29) is a third degree equation. Therefore the expression (4.26) for Liapunov function, derived in Chapter IV is used here.

i.e.,

$$V(X) = \tfrac{1}{2}(k_1 x_2 + x_3)^2 + \tfrac{1}{2}k_2 x_2^2 + k_3 x_1 x_2 + \tfrac{1}{2}k_1 k_3 x_1^2 \quad ..(4.26)$$

5.4 EXAMPLE

The same example of Chapter III is considered, with the constants of the angle regulator circuit, as given below.

$K_3 = 5$ $\qquad K_4 = 4$ $\qquad K_5 = 0.2$

Substituting the given values for all the constants in eq.(5.29), the swing equation becomes,

$$2.95 \times 10^{-2} \frac{d^3\delta}{dt^3} + (7.3+30\sin\delta)10^{-2} \cdot \frac{d^2\delta}{dt^2} + (.0338+$$

$$+4\sin\delta)\frac{d\delta}{dt} = 1.05 - (5\delta + E_{fdo}) \sin\delta \qquad ..(5.30)$$

Changing variable t to 'T' as defined in earlier chapters by the relation,

$T = 14.14\ t$

the swing equation (5.30) can be shown as

$$83.5 \frac{d^3\delta}{dT^3} + (14.6 + 40\sin\delta)\frac{d^2\delta}{dT^2} + (.478+56.5\sin\delta)\frac{d\delta}{dT}$$

$$= 1.05 - (5\delta + E_{fdo}) \sin\delta \qquad ..(5.31)$$

Where

$E_{fdo} = E$ in the steady state

$= V_1 \cos\delta + I_d X_d$ (Fig. 9)

$= 0.886 + .788 \times 1.15$

$= 1.792$ p.u.

Therefore the eq.(5.31) becomes

$$83.5 \frac{d^3\delta}{dT^3} + (14.6+40 \sin\delta) \frac{d^2\delta}{dT^2} + (.478 + 56.5\sin\delta)\frac{d\delta}{dT}$$

$$= 1.05 - (5\delta + 1.792) \sin\delta \qquad ..(5.32)$$

or,

$$\frac{d^3\delta}{dT^3} + (.175 + .48\sin\delta)\frac{d^2\delta}{dT^2} + (.0057 + .677\sin\delta)\frac{d\delta}{dT}$$

$$= .0126 - (.06\delta + .0215) \sin\delta \qquad ..(5.33)$$

SINGULARITIES

The expression

$$0.0126 - (.06\delta + .0215)\ \mathrm{Sin}\delta = 0 \qquad ..(5.34)$$

is solved for stable focus and saddle point singularities by Newton Raphson method. These points are:

Stable Point: .317 radian or 18.2°

Saddle Point: 3.08 radian or 176.5° ..(5.35)

MATRIX A

The stable focus is transferred to the origin of the state space and the swing equation (5.33) is expressed in the form of state space first order differential equations.

Thus the system is represented by

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -(.175+.48\mathrm{Sin}(x_1+.317))x_3-(.0057+.677\ \mathrm{Sin}(x_1+.317))x_2-((.06(x_1+.317)+.0215)\ \mathrm{Sin}(x_1+.317)-.0126) \qquad ..(5.36)$$

Neglecting higher order terms in the series expansion of transcendental functions, eq.(5.36) can be expressed as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -(.175+0.48 \times .317)x_3-(.0057+.677 \times .317)x_2$$

$$-(.06 \times 2 \times .317 + .0215)x_1$$

$$= -.327\ x_3 - .221\ x_2 - .0595\ x_1 \qquad ..(5.37)$$

Expressing eq.(5.37) in matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -.0595 & -.221 & -.327 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad ..(5.38)$$

The coefficient matrix A is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -.0595 & -.221 & -.327 \end{bmatrix} \qquad ..(5.39)$$

The characterisitic equation for the matrix of eq. (5.39) is

$$\beta^3 + .327\ \beta^2 + .221\ \beta + .059 = 0 \qquad ..(5.40)$$

The eigen values are calculated as

$$\beta_1 = -.022 + j\ .456$$

$$\beta_2 = -.022 - j\ .456$$

$$\beta_3 = -.283$$

All the negative real parts of these eigen values show that the system is stable in the neighbourhood of the origin, which is the stable equilibrium point.

MATRIXRV^{-1}

The Liapunov function V(X), from eq.(4.26)is,

$$V(X) = \tfrac{1}{2}(k_1x_2+x_3)^2+\tfrac{1}{2}k_2x_2^2+k_3x_1x_2+\tfrac{1}{2}k_1k_3x_1^2$$

where from eq. (5.37),

$$\begin{aligned} k_1 &= .327 \\ k_2 &= .221 \\ k_3 &= .0595 \end{aligned} \qquad ..(5.42)$$

Therefore, on substitution of these values of the constants in expression for Liapunov function, we get,

$$\begin{aligned} V(X) &= \tfrac{1}{2}(.327x_2+x_3)^2+\tfrac{1}{2}\times.221\ x_2^2+.0595\ x_1x_2+\tfrac{1}{2}\times.327\times .0595\ x_1^2 \\ &= .0097\ x_1^2+.163x_2^2+.5x_3^2+.0595x_1x_2+.327x_2x_3 \end{aligned} \qquad ..(5.43)$$

When expressed in matrix form

$$V(X) = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} .0097 & .03 & .0 \\ .03 & .163 & .163 \\ .0 & .163 & .5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad ..(5.44)$$

Therefore, matrix V can be shown as

$$V = \begin{bmatrix} .0097 & .03 & .0 \\ .03 & .163 & .163 \\ .0 & .163 & .5 \end{bmatrix} \qquad ..(5.45)$$

Substituting eqs.(5.39)&(5.45) in the Liapunov stability equation

$$A^T V + VA = -2\,R$$

and solving for the matrix R , we get

$$R = \begin{bmatrix} .0 & .0 & .0 \\ .0 & .006 & .0 \\ .0 & .0 & .0 \end{bmatrix} \qquad ..(5.46)$$

Then calculating the value of RV^{-1},

$$RV^{-1} = \begin{bmatrix} .016 & -.005 & .001 \\ -.905 & .326 & -.105 \\ -.174 & .049 & -.015 \end{bmatrix} \qquad ..(5.47)$$

The characteristic equation for the above matrix (5.47) is

$$\lambda^3 - .326\,\lambda^2 = 0 \qquad ..(5.48)$$

Therefore its eigen values are

$$\lambda_1 = 0$$

$$\lambda_2 = 0 \qquad ..(5.49)$$

$$\lambda_3 = .326$$

MATRIX $R'V'^{-1}$

A real symmetric positive definite matrix R' is assumed.

$$R' = \frac{1}{100}\begin{bmatrix} .5 & .5 & .2 \\ .5 & 1.0 & .5 \\ .2 & .5 & .5 \end{bmatrix} \qquad ..(5.50)$$

After substitution of the matrix R$ from eq. (5.50) in the Liapunov Stability equation

$$A^T V' + V'A = -2R'$$

the solution for the matrix V' involves the solution of the six equations given below

$$\begin{bmatrix} .000 & .000 & -.118 & .000 & .000 & .000 \\ 1.000 & .000 & -.221 & .000 & -.059 & .000 \\ .000 & 1.000 & -.327 & .000 & .000 & -.059 \\ .000 & 2.000 & .000 & .000 & -.442 & .000 \\ .000 & .000 & 1.000 & 1.000 & -.327 & -.221 \\ .000 & .000 & .000 & .000 & 2.000 & -.654 \end{bmatrix} \begin{bmatrix} v'_{11} \\ v'_{12} \\ v'_{13} \\ v'_{22} \\ v'_{23} \\ v'_{33} \end{bmatrix} = \begin{bmatrix} -.01 \\ -.01 \\ -.004 \\ -.02 \\ -.01 \\ -.01 \end{bmatrix}$$

..(5.51)

The equations (5.51) are solved and the unknown elements are given by

$$v_{11}' = .059 \qquad v_{22}' = .764$$

$$v_{12}' = .178 \qquad v_{23}' = .853 \qquad ..(5.52)$$

$$v_{13}' = .084 \qquad v_{33}' = 2.624$$

Therefore, the matrix V' can be written as

$$V' = \begin{bmatrix} .059 & .178 & .084 \\ .178 & .764 & .853 \\ .084 & .853 & 2.624 \end{bmatrix} \qquad ..(5.53)$$

From eqs. (5.50) and (5.53),

$$R'V'^{-1} = \begin{bmatrix} .369 & -.105 & .023 \\ .213 & -.048 & .01 \\ .084 & -.018 & .005 \end{bmatrix} \qquad ..(5.54)$$

The characteristic equation for the above matrix (5.54) is

$$\lambda'^3 - .326\lambda'^2 + .005\lambda' = 0 \qquad ..(5.55)$$

Hence the eigen values for this matrix are

$$\lambda'_1 = 0$$

$$\lambda'_2 = .019 \qquad (5.56)$$

$$\lambda'_3 = .309$$

MONTE CARLO TECHNIQUE

The Liapunov function from theeq. (5.43), is

$$V(X) = .0097x_1^2 + .163x_2^2 + .5x_3^2 + .0595x_1x_2 + .327x_2x_3$$

The saddle point (5.35), referred to the stable focus as the new origin of the state space, can be given by

$$x_1 = 3.08 - .32$$

$$= 2.76$$

$$x_2 = 0 \qquad ..(5.57)$$

$$x_3 = 0$$

The region of stability is defined by b_{max}, where it can be obtained by substituting the saddle point from the equation (5.57) in eq. (5.43).

Therefore,

$$b_{max} = .0097 \times (2.76)^2$$
$$= .073 \qquad (5.58)$$

The range for the state space variable x_1 can be specified as

$$0 \leq x_1 \leq 2.76 \qquad ..(5.59)$$

The range for the state variable x_2, can be given by equating (5.43) to b_{max} and solving for x_2, while $x_1 = x_3 = 0$.

$$x_2 = \sqrt{\frac{.073}{.163}} = .67$$

Hence,

$$0 \leq x_2 \leq .67 \qquad ..(5.60)$$

Similarly,

$$.5\, x_3^2 = b_{max} = .073$$

where

$$x_1 = x_2 = 0$$

or,

$$x_3 = \sqrt{\frac{.073}{.5}} = .382$$

Therefore the range for x_3 will be

$$0 \leq x_3 \leq .382 \qquad ..(5.61)$$

From eq. (5.46) ,

$$\dot{V}(X) = X^T (-2R) X$$

$$= .012 \; x_2^2$$

Random numbers are generated between the ranges specified for x_1 , x_2 and x_3 by eqs.(5.59), (5.60) and (5.61) respectively. ξ_{max} & ξ_{min} are evaluated from

$$\xi_{\begin{pmatrix} max \\ or \\ min \end{pmatrix}} = \begin{matrix} Max \\ or \\ Min \end{matrix} \left[\frac{-\dot{V}(X)}{V(X)} \right]$$

$$= \begin{matrix} Max \\ or \\ Min \end{matrix} \left[\frac{.012 \; x_2^2}{.0097x_1^2 + .163x_2^2 + .5x_3^2 + .0595x_1x_2 + .327 \; x_2x_3} \right] \quad ..(5.62)$$

The results are

$$\xi_{max} = .00893 \qquad ..(5.63)$$

$$\xi_{min} = .00000$$

TRANSIENT RESPONSE

The system transient response is obtained by numerical integration of the set of first order differential equations (5.36), using Runge-Kutta--Gill method.

The initial condition of the system is given by

$x_{10} = .1$

$x_{20} = -.05$

$x_{30} = .05$

REMARKS

The upper and lower bounds corresponding to matrices A , RV^{-1} and $R'V'^{-1}$ are sketched. The transient response determined by Runge-Kutta-Gill method is compared with these estimates along with the one obtained directly from $\underset{\text{min.}}{\overset{\text{max.}}{\text{or}}} \left[-\dot{V}(X)/V(X)\right]$ by Monte Carlo method.

The transient response estimates from the eigen values of matrices RV^{-1} and $R'V'^{-1}$ are almost similar.

The upper boundary obtained from the coefficient matrix A , reaches very close to the transient response and its lower boundary is same as those of the previous two matrices.

Whereas the lower boundary defined by Monte Carlo method is crossed by the transient response at a number of places, the upper bound- - aries from Monte Carlo Technique, matrices RV^{-1} and $R'V'^{-1}$ are exactly the same. (Fig.6)

RESULTS RUNGE KUTTA GILL METHOD

WITH REGULATOR

TIME	X1	X2	X3	V
0.000				
	.10000	-.05000	.05000	
.250				
	.08903	-.03785	.04711	.00063
.500				
	.08100	-.02647	.04384	.00063
.750				
	.07572	-.01595	.04026	.00062
1.000				
	.07295	-.00636	.03642	.00061
1.250				
	.07245	.00224	.03237	.00060
1.500				
	.07398	.00980	.02814	.00059
1.750				
	.07727	.01630	.02379	.00058
2.000				
	.08204	.02169	.01934	.00057
2.250				
	.08803	.02597	.01486	.00055
2.500				
	.09494	.02913	.01039	.00054
2.750				

	.10258	.03117	.00599	.00052
3.000				
	.11044	.03213	.00171	.00051
3.250				
	.11848	.03205	-.00238	.00050
3.500				
	.12638	.03096	-.00622	.00050
3.750				
	.13389	.02896	-.00977	.00049
4.000				
	.14079	.02610	-.01296	.00049
4.250				
	.14688	.02251	-.01575	.00049
4.500				
	.15199	.01826	-.01809	.00049
4.750				
	.15597	.01350	-.01996	.00050
5.000				
	.15871	.00832	-.02134	.00050
5.250				
	.16011	.00286	-.02222	.00050
5.500				
	.16013	-.00274	-.02261	.00049
5.750				
	.15874	-.00839	-.02253	.00048
6.000				

	.15594	-.01397	-.02201	.00047
6.250				
	.15176	-.01937	-.02108	.00046
6.500				
	.14627	-.02448	-.01979	.00044
6.750				
	.13955	-.02924	-.01818	.00042
7.000				
	.13169	-.03356	-.01632	.00039
7.250				
	.12281	-.03738	-.01425	.00037
7.500				
	.11304	-.04067	-.01203	.00034
7.750				
	.10252	-.04339	-.00971	.00032
8.000				
	.09139	-.04553	-.00734	.00030
8.250				
	.07980	-.04707	-.00497	.00028
8.500				
	.06790	-.04802	-.00262	.00026
8.750				
	.05584	-.04839	-.00034	.00025
9.000				
	.04375	-.04820	.00183	.00024
9.250				

	.03178	-.04748	.00391	.00023
9.500				
	.02005	-.04625	.00584	.00022
9.750				
	.00869	-.04456	.00764	.00021
10.000				
	-.00218	-.04245	.00927	.00021
10.250				
	-.01249	-.03994	.01075	.00020
10.500				
	-.02213	-.03708	.01206	.00020
10.750				
	-.03101	-.03392	.01321	.00020
11.000				
	-.03907	-.03049	.01420	.00019
11.250				
	-.04624	-.02683	.01502	.00019
11.500				
	-.05247	-.02299	.01568	.00019
11.750				
	-.05772	-.01900	.01618	.00018
12.000				
	-.06196	-.01491	.01652	.00018
12.250				
	-.06517	-.01075	.01670	.00018
12.500				

	-.06734	-.00657	.01674	.00018
12.750	-.06846	-.00240	.01661	.00018
13.000	-.06855	.00171	.01633	.00018
13.250	-.06761	.00575	.01589	.00018
13.500	-.06568	.00965	.01530	.00018
13.750	-.06280	.01339	.01455	.00018
14.000	-.05900	.01692	.01365	.00018
14.250	-.05436	.02020	.01259	.00019
14.500	-.04892	.02320	.01139	.00019
14.750	-.04278	.02589	.01005	.00019
15.000	-.03601	.02822	.00858	.00019

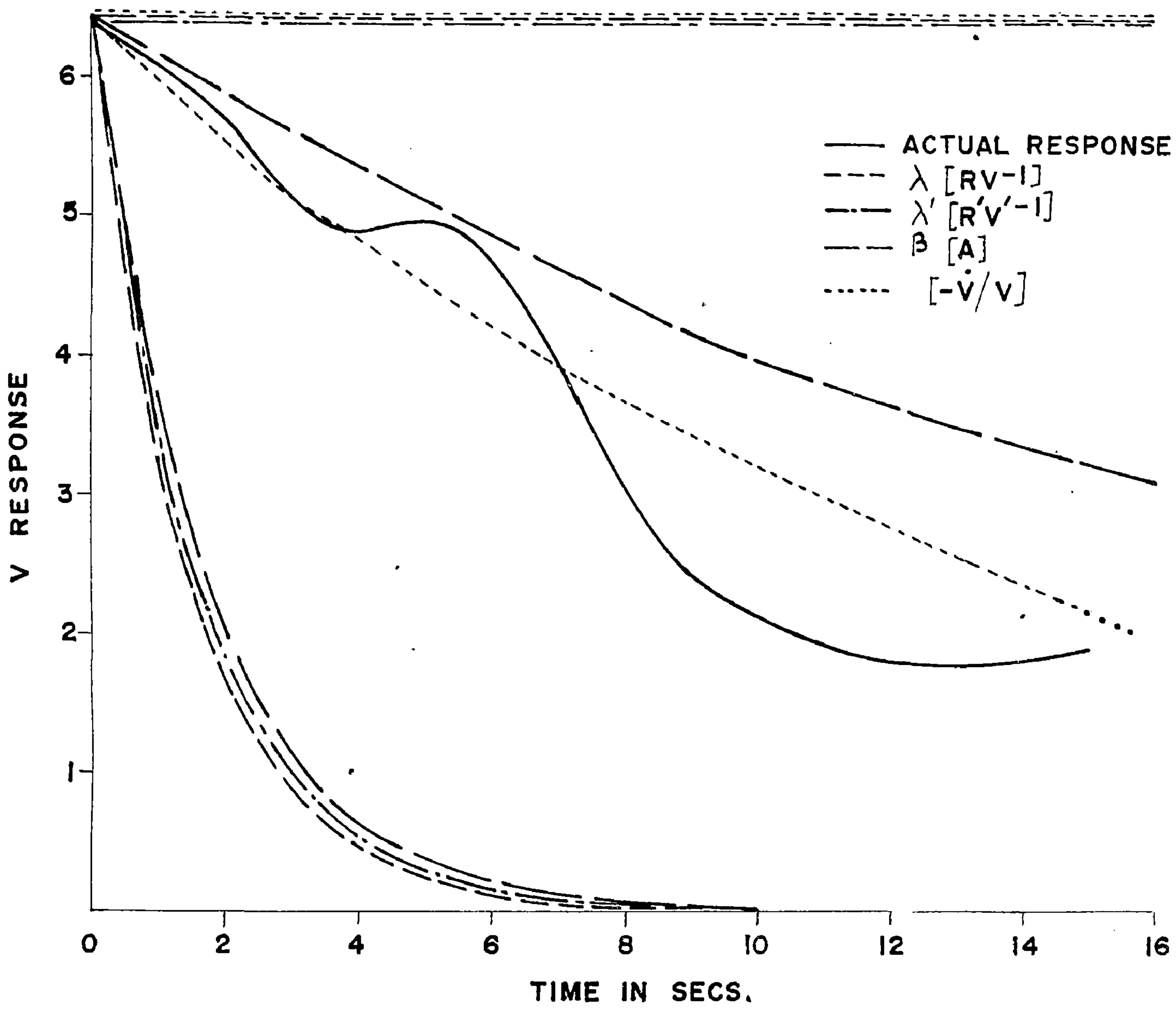

FIG. 6. MACHINE WITH ANGLE REGULATOR ACTION.

CHAPTER VI

TRANSIENT RESPONSE ESTIMATE OF A SINGLE MACHINE CONNECTED TO AN INFINITE BUS WITH GOVERNOR AND ANGLE REGULATOR ACTION

6.1 INTRODUCTION

This chapter deals with a system of one synchronous machine connected to an infinite bus, incorporating a combined effect of governor and angle regulator action.(Fig.7). A swing equation of fourth order is formulated. The method of Cartwright, for constructing a Liapunov function is extended to this fourth order system. Coeffi--cient matrix A ,matrices HV^{-1} and $H'V'^{-1}$ are determined. The transient response by Runge-Kutta Gill method and its estimates from the above stated matrices is plotted. Finally , the upper and lower boundaries are directly calculated from $\left[-\dot{V}(X)/V(X)\right]$ by Monte Carlo Method.

6.2 SWING EQUATION

The swing equation for a system with angle regulator action(5.29) is

$$M\, X_d' T_{do}' \frac{d^3\delta}{dt^3} + (MX_d + K_d T_{do}' X_d' + K_5 V_1 \sin\delta)\frac{d^2\delta}{dt^2}$$

$$+ (K_d X_d + K_4 V_1 \sin\delta)\frac{d\delta}{dt} = P_i X_d - (K_3\delta + E_{fdo})V_1 \sin\delta \qquad ..(5.29)$$

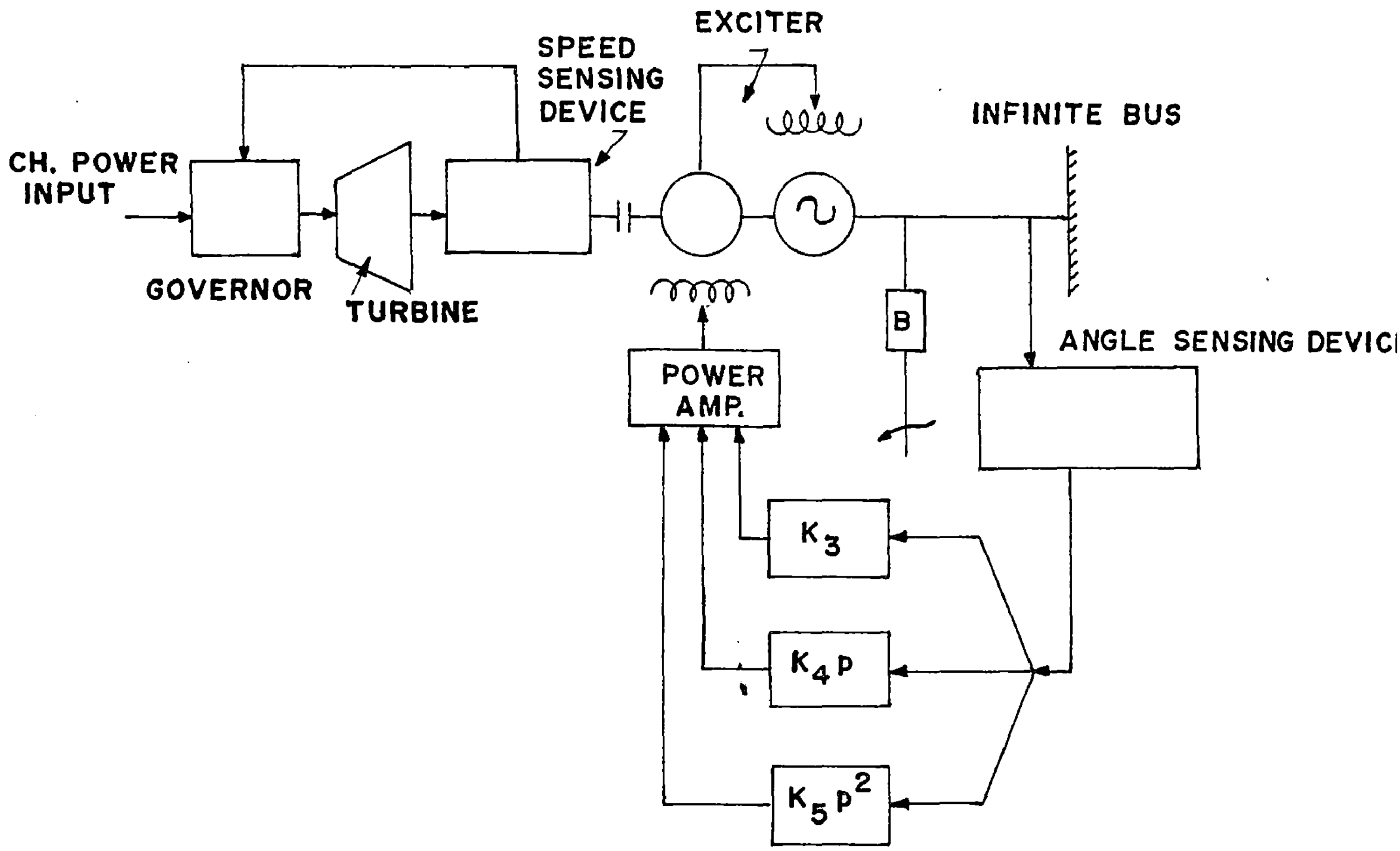

FIG. 7. ONE MACHINE CONNECTED TO AN INFINITE BUS WITH VELOCITY GOVERNOR AND ANGLE REGULATOR.

The governor action (4.13) can be expressed by

$$\Delta P_i = \frac{G_1}{w_o(1+T_1 p)} \cdot \frac{d\delta}{dt} \qquad ..(4.12)$$

For inclusion of governor action, replace P_i by $(P_i - \Delta P_i)$ in (5.29).

Hence,

$$M.X_d'.T_{do}'.\frac{d^3\delta}{dt^3} + (MX_d + K_d T_{do}' X_d' + K_5 V_1 \sin\delta)\frac{d^2\delta}{dt^2}$$

$$+(K_d X_d + K_4 V_1 \sin\delta)\frac{d\delta}{dt} = X_d\left\{P_i - \frac{G_1}{w_o(1+T_1 p)} \cdot \frac{d\delta}{dt}\right\}$$

$$-\left(K_3\delta + Efdo\right)V_1 \sin\delta$$

..(6.11)

Multiplying eq. (6.11) by (1+Tip) on both sides, and simplyfying ,

$$M\,X_d' T_{do}' T_1 \cdot \frac{d^4\delta}{dt^4} + (M.X_d' T_{do}' + T_1(M.X_d + K_d T_{do}' X_d'$$

$$+K_5 V_1 \sin\delta)\frac{d^3\delta}{dt^3} + ((M.X_d + K_d.T_{do}' X_d' + T_1 K_d X_d) + (K_4 T_1 +$$

$$+K_5)V_1 \sin\delta + T_1 K_5 V_1 \cos\delta \frac{d\delta}{dt})\frac{d^2\delta}{dt^2} + ((K_d X_d + \frac{G_1 X_d}{w_o}) +$$

$$(K_4 + T_1 K_3)\ V_1 \sin\delta + T_1(K_3\delta + Efdo + K_4 \frac{d\delta}{dt})\ V_1 \cos\delta)\frac{d\delta}{dt}$$

$$= P_i X_d - (K_3\delta + Efdo)\ V_1 \sin\delta$$

..(6.13)

6.3 LIAPUNOV FUNCTION

The Liapunov function for the fourth order system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3 \qquad ..(6.14)$$
$$\dot{x}_3 = x_4$$
$$x_4 = -k_1x_4 - k_2x_3 - k_3x_2 - g(x_1)$$

where

$g(x_1)$ is anonlinear function of variable x_1.

K_1, k_2 and k_3 are constants.

can be determined in the following way;

Assume

$$g(xi) = k_4x_1$$

where k_4 is a constant.

Therefore the state space equations become

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3 \qquad ..(6.15)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = -k_1x_4 - k_2x_3 - k_3x_2 - k_4x_1$$

Let the Liapunov function be expressed by

$$2V(X) = a_1(x_4 + Ax_3 + Bx_2 + Cx_1)^2 + a_2(x_3 + Dx_2 + Ex_1)^2 + a_3(x_2 + Fx_1)^2 + a_4x_1^2 \qquad ..(6.16)$$

Where

a_1, a_2, a_3 & a_4 are constants.

Differentiating eq.(6.16) w.r.t time 't', and rearranging the terms,

$$\dot{V}(X) = -a_1(x_4+Ax_3+Bx_2+Cx_1)((k_1-A)x_4+(k_2-B)x_3+(k_3-C)x_2+$$
$$+k_4x_1)+a_2(x_3+Dx_2+Ex_1)(x_4+Dx_3+Ex_2) + a_3(x_2+$$
$$+Fx_1)(x_3+Fx_2) + a_4x_1x_2 \qquad ..(6.17)$$

$\dot{V}(X)$ of eq.(6.17) is constrained to be negative semidefinite in the state variable x_2.

Thus equating the coefficients of terms, other than that of x_2^2 to zero,

$$-a_1(k_1-A) = 0$$
$$-a_1A(k_2-B)+a_2D = 0$$
$$-a_1C\,k_4 = 0$$
$$-a_1(k_2-B)+a_2 = 0$$
$$-a_1k_3+a_2D = 0$$
$$-a_1k_4+a_2E = 0$$
$$-a_1(k_1k_3+B(k_2-B))+a_2(E+D^2)+a_3 = 0 \qquad ..(6.18)$$
$$-a_1k_1k_4+a_2ED+a_3F = 0$$
$$-a_1Bk_4+a_2E^2+a_3F^2+a_4 = 0$$

Solving the above equations(6.18),

$A = k_1$ $\qquad$ $B = k_2-k_3/k_1$

$C = 0$ $\qquad\qquad E = k_1 k_4 / k_3$

$D = k_1$ $\qquad\qquad F = 0$

$$a_1/k_1 = a_2/k_3$$

$$a_3 = \frac{a_1(k_1 k_2 k_3 - k_3^2 - k_1^2 k_4)}{k_1^2}$$

$$a_4 = \frac{a_1 k_4 (k_1 k_2 k_3 - k_3^2 - k_1^2 k_4)}{k_1 k_3}$$

..(6.19)

Assume that

$$a_1 = k_1^2 k_3$$

Substituting the values of eq.(6.19) in the expression (6.18) of Liapunov function,

$$2V = k_1^2 k_3 (x_4 + k_1 x_3 + (k_2 - k_3/k_1) x_2)^2 + k_3^2 k_1 (x_3 + k_1 x_2 + k_1 k_4 x_1 / k_3)^2$$
$$+ k_3 (k_1 k_2 k_3 - k_3^2 - k_1^2 k_4) x_2^2 + k_1 k_4 (k_1 k_2 k_3 - k_3^2 - k_1^2 k_4) x_1^2$$

..(6.20)

In equation (6.15), the nonlinear term $g(x_1)$ was replaced by a linear term $k_4 x_1$.

Therefore in order to include nonlinearity in the Liapunov function of eq. (6.20), replace

k_4 by $\frac{d}{dx_1}(g(x_1)) = g'(x_1)$

$k_4 x_1$ by $g(x_1)$

$k_4 x_1^2/2$ by $\int_0^{x_1} g(u)\,du$

Hence the Liapunov function for the nonlinear system can be expressed as

$$2V = k_1^2 k_3 (x_4 + k_1 x_3 + (k_2 - k_3/k_1) x_2)^2 + k_3^2 k_1 (x_3 + k_1 x_2 + k_1 g(x_1)/k_3)^2 + k_3 (k_1 k_2 k_3 - k_3^2 - k_1^2 \; g(x_1)') x_2^2 + 2k_1 k_3 (k_1 k_2 - k_3) \int_0^{x_1} g(u) \, du - k_1^3 g^2 (x_1) \quad ..(6.21)$$

6.4 EXAMPLE

The example (3.9) of Chapter III is taken along with the follwoing data for velocity governor and angle regulator circuits.

Velocity Governor:

$G_1 = 20$

$T_1 = 0.1$

Angle Regulator

$K_3 = 5$

$K_4 = 4$

$K_5 = 1.2$

Substituting the different values in swing equation (6.13) and expressing it in terms of the new time variable T , we get

$$\frac{d^4\delta}{dT^4} + (.882 + 2.892 \sin\delta) \frac{d^3\delta}{dT^3} + (.13 + 2.72 \sin\delta + 2.89 \cos\delta \frac{d\delta}{dT}) \frac{d^2\delta}{dT^2}$$

$$+ (.012 + .54 \sin\delta + (.06\delta + .021) \cos\delta + .68 \cos\delta \frac{d\delta}{dT}) \frac{d\delta}{dT}$$

$$= .009 - (.0425\delta + .0182) \sin\delta \quad .. \quad (6.22)$$

SINGULARITIES

The stable focus and saddle point singularities can be obtained by solving the equation

$$(.0425\delta + .0152)\,\mathrm{Sin}\,\delta - .009 = 0 \qquad ..(6.33)$$

These points are given by

Stable focus: .219 radians or 12.55°

Saddle Point: 3.071 radians or 175.96°

..(6.24)

MATRIX A

Origin of the State Space is transferred to the stable focus and the swing equation(6.22) is expressed in the form of state space first order differential equations.

Thus,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -(.882 + 2.892\,\mathrm{Sin}(x_1 + .219))x_4 - (.13 + 2.72\,\mathrm{Sin}(x_1 + .219) + 2.89\,\mathrm{Cos}(x_1 + .219)\,x_2^2)\,x_3 - (.012 + .54\,\mathrm{Sin}(x_1 + .219) + (.08(x_1 + .219) + .68x_2 + .021)\,\mathrm{Cos}(x_1 + .219))x_2 - (.0425(x_1 + .219) + .0152)\,\mathrm{Sin}(x_1 + .219) + .009$$

..(6.25)

Linearizing eq. (6.25),

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -(.882+.2892 \times .219)x_4-(.13+2.72 \times .219)x_3$$

$$-(.058 +.6 \times .219)x_2-(.0332+.085 \times .219)x_1$$

$$= -1.509x_4-.72x_3-.189x_2-.0518x_1$$

..(6.26)

When expressed in matrix form, it can be given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} .000 & 1.000 & .000 & .000 \\ .000 & .000 & 1.000 & .000 \\ .000 & .000 & .000 & 1.000 \\ -.0518 & -.189 & -.72 & -1.509 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

..(6.27)

Therefore, the coefficient matrix A can be expressed as

$$A = \begin{bmatrix} .000 & 1.000 & .000 & .000 \\ .000 & .000 & 1.000 & .000 \\ .000 & .000 & .000 & 1.000 \\ -.0518 & -.189 & -.72 & -1.509 \end{bmatrix}$$

..(6.28)

The evaluated characteristic equation for the above matrix (6.28) is

$$\beta^4 + 1.509\,\beta^3 + .720\,\beta^2 + .189\,\beta + .052 = 0$$

The eigen values are

$\beta_1 = -.041 + j.320$

$\beta_2 = -.041 - j.320$

$\beta_3 = -.616 + j\ 0$ ..(6.29)

$\beta_4 = -.812 + j\ 0$

MATRIX RV^{-1}

Consider the expression (6.20) for the Liapunov function, which can be written as shown below, while substituting the value of constants from eq.(6.28),

$$V(X) = .215(x_4+1.51x_3+.595x_2)^2+.027(x_3+1.51x_2+.414x_1)^2$$
$$+.0045(.205-.0356-.118)x_2^2 + .039(.205-.0356-.118) .x_1^2$$
$$= .0064x_1^2 + .1421x_2^2 + .514\ x_3^2 + .215\ x_4^2 + .0337x_1x_2$$
$$+.0224x_1x_3+.4675x_2x_3+.256x_2x_4+.65x_3x_4$$

..(6.30)

In matrix form, the Liapunov function is

$$V(X) = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} \begin{bmatrix} .0064 & .0168 & .0112 & .000 \\ .0168 & .1421 & .2337 & .128 \\ .0112 & .2337 & .514 & .325 \\ .000 & .128 & .325 & .215 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

..(6.31)

Therefore the matrix V can be written as

$$V = \begin{bmatrix} .0064 & .0168 & .0112 & .000 \\ .0168 & .1431 & .2337 & .128 \\ .0112 & .2337 & .514 & .325 \\ .000 & .128 & .325 & .215 \end{bmatrix} \quad ..(6.34)$$

Substituting matrix V (6.34) and matrix A (6.28) in the Liapunov Stability equation

$$A^T V + VA = -2R$$

and evaluating the matrix R, we get

$$R = \begin{bmatrix} .000 & .000 & .000 & .000 \\ .000 & .007 & .000 & .000 \\ .000 & .000 & .000 & .000 \\ .000 & .000 & .000 & .000 \end{bmatrix} \quad ..(6.35)$$

Then the matrix RV^{-1} is found out from eqs. (6.35) and (6.34)

Thus,

$$RV^{-1} = \begin{bmatrix} -.089 & -.057 & .138 & -.174 \\ -1.445 & -.414 & 1.459 & -1.959 \\ -.209 & -.486 & .861 & -1.005 \\ .338 & .509 & -.964 & 1.152 \end{bmatrix} \quad ..(6.36)$$

The characteristic equation of the matrix (6.36) is

$$\lambda^4 - 1.51\lambda^3 + .759\lambda^2 - .002\lambda = 0 \quad ..(6.37)$$

The eigen values can be calculated as

$\lambda_1 = 0$

$$\lambda_2 = .0026$$

$$\lambda_3 = .753 + j.432 \qquad ..(6.38)$$

$$\lambda_4 = .753 - j.432$$

MATRIX $R'V'^{-1}$

A real symmetric positive Semi-definite matrix R' is assumed such that

$$R' = \begin{bmatrix} .01 & .01 & .01 & .01 \\ .01 & .05 & .01 & .01 \\ .01 & .01 & .01 & .01 \\ .01 & .01 & .01 & .01 \end{bmatrix} \qquad ..(6.39)$$

The Liapunov stability equation is solved for unknown matrix V'. The solution involves the following equations, which are obtained from the set (AII.27) by substituting the values of the elements of matrices A and R' from eqs. (6.28) and (6.39).

$$-.1036\, v_{14}' = -.02$$

$$v_{11}' - .189 v_{14}' - .0518 v_{24}' = -.02$$

$$v_{12}' - .72 v_{14}' - .0518 v_{34}' = -.02$$

$$v_{13}' - 1.509 v_{14}' - .0518 v_{44}' = -.02$$

$$2v_{12}' - .378 v_{24}' = -.1$$

$$v_{13}' - .189 v_{34}' + v_{22}' - .72 v_{24}' = -.02$$

$$v_{14}' - .189 v_{44}' + v_{23}' - 1.509\, v_{24}' = -.02$$

$$2v_{23}' - 1.44\, v_{34}' = -.02$$

$$v_{24}'-.72v_{44}'+v_{33}'-1.509\ v_{34}' = -.02$$

$$2v_{34}'-3.016v_{44}' = -.02 \qquad ..(6.41)$$

The solution of the eqns.(6.41)is

$v_{11}' = .153$ $v_{12}' = .449$ $v_{13}' = .490$ $v_{14}' = .192$

$v_{22}' = 2.597$ $v_{23}' = 4.576$ $v_{24}' = 2.664$ $v_{33}' = 9.993$

$v_{34}' = 6.370$ $v_{44}' = 4.228$

Therefore the matrix V' is given by

$$V' = \begin{bmatrix} .153 & .449 & .490 & .192 \\ .449 & 2.597 & 4.576 & 2,644 \\ .490 & 4.576 & 9.993 & 6.370 \\ .192 & 2.644 & 6.370 & 4.228 \end{bmatrix} \qquad ..(6.42)$$

Evaluating $R'V'^{-1}$ from equations (6.39) and (6.42), we get

$$R'V'^{-1} = \begin{bmatrix} .223 & .082 & -.247 & .314 \\ -.482 & 1.166 & -1.513 & 1.575 \\ .236 & .069 & -.233 & .300 \\ .210 & .110 & -.265 & .353 \end{bmatrix} \qquad ..(6.43)$$

The characteristic for the above matrix (6.43) is given by

$$\lambda'^4 - 1.508\ \lambda'^3 + .392\ \lambda'^2 = 0 \qquad ..(6.44)$$

The eigen values are

$$\lambda_1' = 0,\ \lambda_2' = 0,\ \lambda_3' = 1.174,\ \lambda_4' = .333 \qquad ..(6.45)$$

MONTE CARLO TECHNIQUE

The saddle point (6.34), referred to the stable focus as the new origin of the state space, can be given by

$$x_1 = 3.071 - .219 = 2.852$$

$$x_2 = 0 \quad ..(6.46)$$

$$x_3 = 0$$

The region of stability, defined by b_{max} is obtained by substituting the saddle point from eq.(6.46) in eq.(6.30).

Thus,

$$b_{max} = .0064 \times (2.852)^2 = .052 \quad ..(6.47)$$

The range of the state variable x_1 can be written as

$$0 \leq x_1 \leq 2.852 \quad ..(6.48)$$

The range for the state variable x_2 is obtained by equating (6.30) to b_{max} and solving for x_2, while $x_1 = x_3 = x_4 = 0$

Hence

$$x_2 = \sqrt{.052/.1421} = .605$$

Therefore the range for x_2 will be

$$0 \leq x_2 \leq .605 \quad ..(6.49)$$

Similarly,

$$x_3 = \sqrt{.052 / .514} = .318$$

where $x_1 = x_2 = x_4 = 0$

Therefore the range for x_3 is

$$0 \leq x_3 \leq .318 \qquad ..(6.50)$$

and

$$x_4 = \sqrt{.052/.215} = .492$$

where $x_1 = x_2 = x_3 = 0$

Thus the range for x_4 is

$$0 \leq x_4 \leq .492 \qquad ..(6.51)$$

From eq.(6.35),

$$\dot{V}(X) = -.014\ x_2^{\ 2} \qquad ..(6.52)$$

Random numbers are generated between the ranges specified for x_1, x_2, x_3 and x_4 by eqs.(6.48), (6.49),(6.50) and (6.51) respectively. ξ_{max} and ξ_{min} are evaluated from

$$\xi_{\substack{max\\or\\min}} = \underset{or\ Min}{Max} \left(-\dot{V}(X)/V(X) \right)$$

$$= \underset{\substack{or\\Min}}{Max} \left[\frac{.014\ x_2^{\ 2}}{(.0064\ x_1 + .0337x_2 + .0224x_3)x_1 + (.1421\ x_2 + .4675x_3 + .286x_4)x_2 + (.514x_3 + .65x_4)x_3 + .215\ x_4^{\ 2})} \right] \qquad ..(6.53)$$

The results are

$$\xi_{max} = .09348 \qquad ..(6.54)$$

$$\xi_{min} = .00000$$

TRANSIENT RESPONSE

The systemtransient response is again determined by numerical integration of the set of state space equations (6.25), using Runge-Kutta-Gill method on digital computer.

The initial conditions are:

$x_{10} = .1$ $\qquad$ $x_{2o} = -.05$

$x_{30} = .05$ $\qquad$ $x_{4o} = -.05$

REMARKS

The boundaries of the estimates with the help of the eigen values from eqs. (6.29), (6.38) and (6.45) are drawn along with the transient response obtained by Runge Kutta Gill method.

It is concluded that the estimates from RV^{-1} and $R'V'^{-1}$ are almost similar.

The upper boundary obtained from matrix A is closer to the transient response curve and the lower one lies in between that of RV^{-1} and $R'V'^{-1}$.

Monte Carlo technique gives the upper boundary estimate exactly similar to those of

RV^{-1} and $R'V'^{-1}$, and the lower estimate is shifted a bit upward from the transient response.(Fig.8)

RESULTS RUNGE KUTTA GILL METHOD

WITH GOVERNOR AND REGULATOR

TIME	X1	X2	X3	X4	V
0.000	.1000000	-.0500000	.0500000	-.0500000	.0000315
.250	.0889402	-.0389366	.0389740	-.0387741	.0000258
.500	.0803290	-.0303113	.0303763	-.0304021	.0000226
.750	.0736257	-.0235966	.0236059	-.0240426	.0000208
1.000	.0684091	-.0183920	.0182344	-.0191358	.0000197
1.250	.0643297	-.0143891	.0139492	-.0152993	.0000190
1.500	.0611306	-.0113464	.0105180	-.0122658	.0000184
1.750	.0585924	-.0090736	.0077653	-.0098445	.0000180
2.000	.0565424	-.0074185	.0055564	-.0078964	.0000177
2.250	.0548419	-.0062589	.0037863	-.0063185	.0000173
2.500	.0533799	-.0054957	.0023728	-.0050333	.0000170
2.750					

.0520678	-.0050483	.0012502	-.0039816	.0000167

3.000

.0508350	-.0048507	.0003663	-.0031177	.0000164

3.250

.0496261	-.0048487	-.0003212	-.0024059	.0000162

3.500

.0483981	-.0049978	-.0008469	-.0018178	.0000159

3.750

.0471177	-.0052611	-.0012386	-.0013311	.0000156

4.000

.0457606	-.0056079	-.0015194	-.0009277	.0000153

4.250

.0443089	-.0060131	-.0017082	-.0005931	.0000150

4.500

.0427509	-.0064557	-.0018207	-.0003156	.0000147

4.750

.0410794	-.0069182	-.0018699	-.0000854	.0000144

5.000

.0392913	-.0073863	-.0018667	.0001050	.0000141

5.250

.0373868	-.0078480	-.0018202	.0002624	.0000138

5.500

.0353687	-.0082934	-.0017378	.0003919	.0000136

5.750

.0332421	-.0087145	-.0016261	.0004980	.0000133

6.000

.0310140 -.0091045 -.0014905 .0005842 .0000130

6.250

.0286929 -.0094581 -.0013354 .0006535 .0000127

6.500

.0262884 -.0097709 -.0011649 .0007083 .0000125

6.750

.0238111 -.0100396 -.0009823 .0007508 .0000122

7.000

.0212725 -.0102613 -.0007904 .0007826 .0000119

7.250

.0186845 -.0104342 -.0005917 .0008052 .0000116

7.500

.0160596 -.0105568 -.0003884 .0008196 .0000114

7.750

.0134104 -.0106282 -.0001825 .0008268 .0000111

8.000

.0107498 -.0106480 .0000244 .0008277 .0000108

8.250

.0080907 -.0106160 .0002308 .0008229 .0000105

8.500

.0054460 -.0105327 .0004354 .0008130 .0000102

8.750

.0028286 -.0103986 .0006369 .0007983 .0000099

9.000

.0002509 -.0102146 .0008342 .0007792 .0000096

9.250

-.0022746	-.0099819	.0010262	.0007561	.0000094

9.500

-.0047361	-.0097020	.0012120	.0007291	.0000091

9.750

-.0071218	-.0093765	.0013905	.0006985	.0000088

10.000

-.0094207	-.0090074	.0015609	.0006644	.0000085

10.250

-.0116221	-.0085967	.0017224	.0006269	.0000083

10.500

-.0137158	-.0081470	.0018742	.0005863	.0000081

10.750

-.0156925	-.0076605	.0020154	.0005426	.0000078

11.000

-.0175433	-.0071402	.0021453	.0004960	.0000076

11.250

-.0192600	-.0065889	.0022631	.0004465	.0000075

11.500

-.0208354	-.0060097	.0023683	.0003944	.0000073

11.750

-.0222628	-.0054058	.0024601	.0003397	.0000071

12.000

-.0235366	-.0047807	.0025380	.0002828	.0000070

12.250

-.0246517	-.0041380	.0026014	.0002237	.0000069

12.500

	-.0256044	-.0034813	.0026497	.0001629	.0000068
12.750	-.0263915	-.0028144	.0026827	.0001005	.0000068
13.000	-.0270111	-.0021412	.0026999	.0000369	.0000067
13.250	-.0274619	-.0014657	.0027011	-.0000275	.0000067
13.500	-.0277441	-.0007920	.0026861	-.0000923	.0000067
13.750	-.0278584	-.0001240	.0026549	-.0001571	.0000066
14.000	-.0278070	.0005340	.0026076	-.0002214	.0000066
14.250	-.0275926	.0011784	.0025443	-.0002846	.0000066
14.500	-.0272192	.0018049	.0024654	-.0003463	.0000066
14.750	-.0266919	.0024098	.0023713	-.0004059	.0000065
15.000	-.0260164	.0029894	.0022626	-.0004627	.0000065

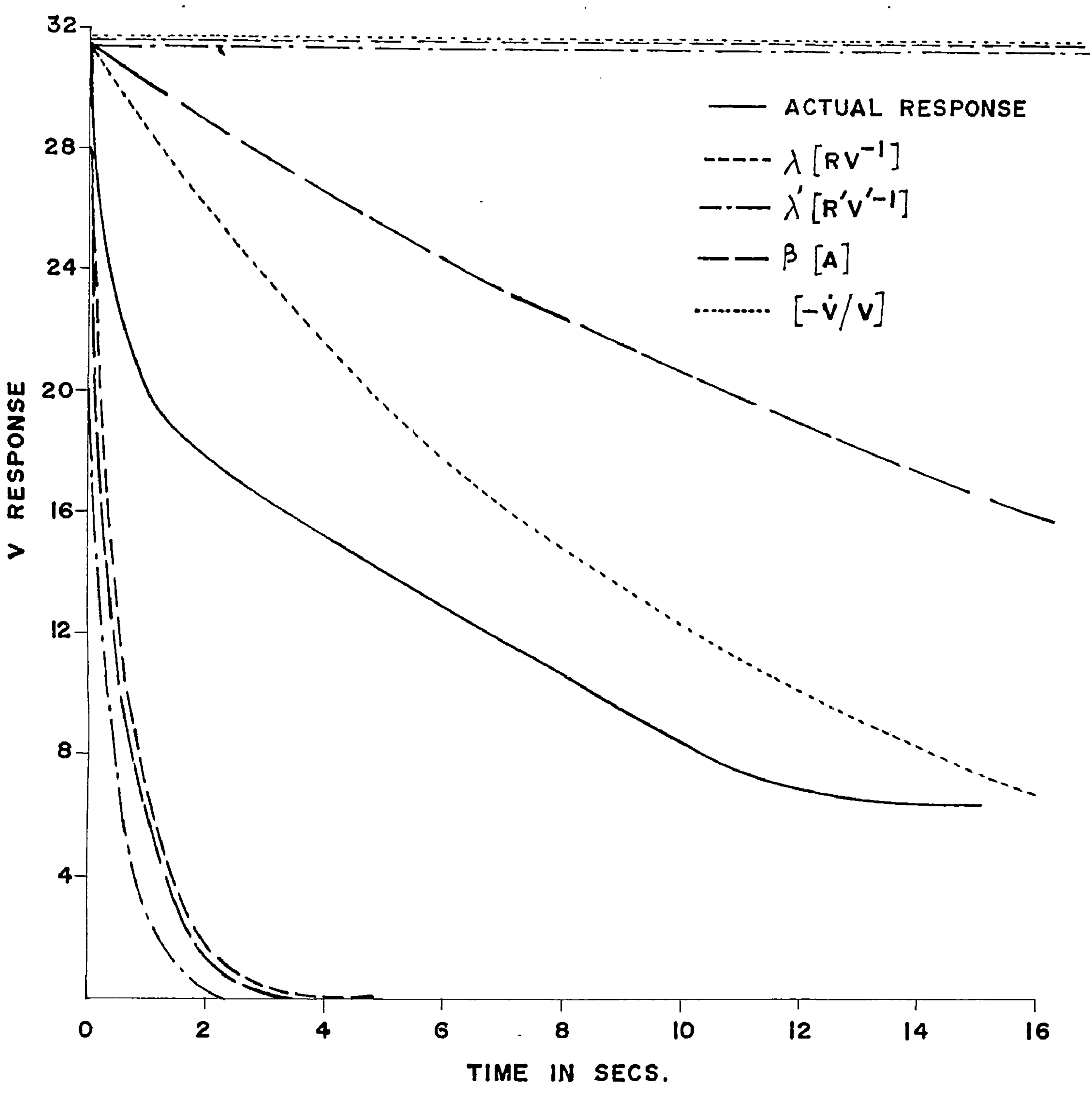

FIG. 8. MACHINE WITH REGULATOR & GOVERNOR.

CHAPTER VII

CONCLUSIONS

7.1 SUMMARY OF CONCLUSIONS

The Liapunov functions and the Direct Method of Liapunov have been made use of for ascertaining the stability of the systems. Recently efforts are being made to correlate the Liapunov function of a stable system with its transient response. This concept has helped in designing the stable systems by varying the system parameters, to bring the output response within the desired limit, without the necessity of evaluating the transient response by integration of system diff--erential equations.

The method is applied to stable power systems consisting of one synchronous machine supplying power against an infinite bus. This configuration is choosen , as any complex system can be reduced to this form , so as to enable study in a particular region of interest. The parts of the systems other than those of interest can be assumed equivalent to an infinite bus. The cases , without any control, with prime mover control, with excitation control and with both the controls working together, are analysed for estimating the transient response. The non-linearities of the system are considered by including the first terms of their expansions in series form.

The Liapunov functions are constructed by Cartwright's Method, and the procedure is further adopted to develop a Liapunov Function for a fourth order system, wherein a combined action of governor and angle regulator is sought.

The upper and lower boundaries of the estimates are plotted from the eigen values of matrices A , RV^{-1} and $R'V'^{-1}$. Further a new method is devised to find the maximum and minimum values of the time constants directly from $(-\dot{V}(X)/V(X))$ by Monte Carlo Technique. The estimates by this approach are reasonably in correspondence with those from RV_{-1} and $R'V'^{-1}$ matrices.

Runge-Kutta-Gill Method is used to evaluate the system response for comparing it with the results obtained. A number of numerical methods for matrix manipulations, suitable for use on IBM 1620 computer , are utilized.

7.2 SCOPE FOR FURTHER WORK

While this work is confined to the problem of finding the transient response estimates of stable systems with given fixed parameters, it can be further extended to design the systems, satisfying certain requirements in respect of transient response overshoot, settling time and performance index etc.

Although it has been possible to construct Liapunov functions for simple nonlinear systems, ingenuity is still needed in evolving new methods to find Liapunov funtions for complex systems. There is no method available as yet to find Liapunov functions, which can define the actual stability region of a system. High speed digital computers may probably give some solution in the near future.

Once, an appropriate Liapunov function is found out, it will prove a versatile tool in predicting precisely the system stability and its transient behaviour.

APPENDIX I

AI.1 SWING EQUATION (17,18)

The equation of motion, neglecting damping may be written as

$$I \cdot \frac{d^2 \theta}{dt^2} = Ta \qquad ..(AI.11)$$

where

I = Moment of Inertia of the rotating part

θ = Total electrical angular displacement from a fixed reference axis.

Ta = Accelerating Torque

The accelerating torque is the net torque or the algebric sum of the shaft torque , torque due to rotational losses and the electromagnetic torque.

Therefore

$$Ta = Ti - Te \qquad ..(AI.12)$$

where

Ti = Shaft torque corrected for rotational losses

Te = Electromagnetic Torque

It is convenient to measure the angular position and the angular velocity with respect to a synchronously rotating reference axis. Hence,

$$\theta = \delta + wo\ t \qquad ..(AI.13)$$

where

δ = Angle with reference to the rotating axis

wo = Rated normal synchronous speed.

Substituting eqs. (AI.12) and (AI.13) in (AI.11), the swing equation modifies to

$$I.\frac{d^2\delta}{dt^2} = Ti - Te \qquad ..(AI.14)$$

Multiplying by w on both the sides,

$$M.\frac{d^2\delta}{dt^2} = Pi - Pe \qquad ..(AI.15)$$

where

M = Iw = Angular momentum

Pi = Tiw = Shaft power input corrected for rotational losses

Pe = Tew = Electrical Power output

but

$$GH = \frac{1}{2} M w \qquad ..(AI.16)$$

where

H : Inertia constant in Kw sec/Kva capacity of the machine

G : Rated apparent power of the machine in KVA.

Hence,

$$M = GH/\pi f \qquad ..(AI.17)$$

AI.2 <u>ELECTRIC POWER</u> (Pe)

From the vector diagram of Fig.9,

$$Pe = V_1 I \cos \theta \qquad ..(AI.18)$$

Expressing it in terms of d and q axis components of V_1 and I,

$$Pe = Id\ Vd + Iq\ Vq \qquad ..(AI.19)$$

where

$$Vd = V_1 \sin \delta \qquad ..(AI.20)$$

and $$Vq = V_1 \cos \delta \qquad ..(AI.21)$$

$$Id = (Eq' - Vq) / Xd' \qquad ..(AI.22)$$

and

$$Iq = Vd / Xq \qquad ..(AI.23)$$

Substituting eqs. (AI.20), (AI.21), (AI.22) and (AI.23) in eq. (AI.19),

$$Pe = \frac{Eq' - Vq}{Xd'} . Vd + \frac{Vd}{Xq} . Vq$$

$$= \frac{Eq'\ V_1}{Xd'} \sin \delta - V_1^2 . \frac{(Xq - Xd')}{2\,Xd'\,Xq} \sin 2\delta \qquad ..(AI.24)$$

Therefore, the swing equation can now be expressed as

$$M \frac{d^2\delta}{dt^2} = Pi - \frac{Eq'\ V_1}{Xd'} \sin \delta + V_1^2 \frac{(Xq - Xd')}{2\,Xd'\,Xq} \sin 2\delta$$

$$= Pi - Pm1 \sin \delta + Pm2 \sin 2\delta \qquad ..(AI.25)$$

where

$$Pm1 = \frac{Eq'\ V_1}{Xd'}$$

$$Pm2 = \left[\frac{Xq - Xd'}{2\ Xd'\ Xq} \right] V_1^2$$

AI.3 DAMPING POWER

The damper winding or the solid rotor sturucture of the synchronous machine develops high damping power, which cannot be neglected for transient stability study.

In order t o derive the expression of the damping power , an equivalent circuit as shown in Fig.10 is drawn, based on induction theory. A symmetrical rotor is assumed initially.

The armature, field and damper circuits are inductively coupled. Whereas in the equivalent T circuit , the identity of armature and damper circuit is preserved, which is quite suitable to find the damper current Ik. Xd' is the transient reactance, as seen from the armature terminals with damper circuit open and field short circuited. Just like induction motor , the damper branch contains the resistance Rkd/s , where Rkd is the damper resistance and s is the slip. The value of the damper leakage reactance XRkd , is such that at large slips, the impedance as seen from the armature side is equal to the sub-transient reactange Xd" , when the damper circuit is short circuited.

$$Xd'' = XRkd \cdot Xd' / (Xd' + XRkd) \qquad ..(AI.26)$$

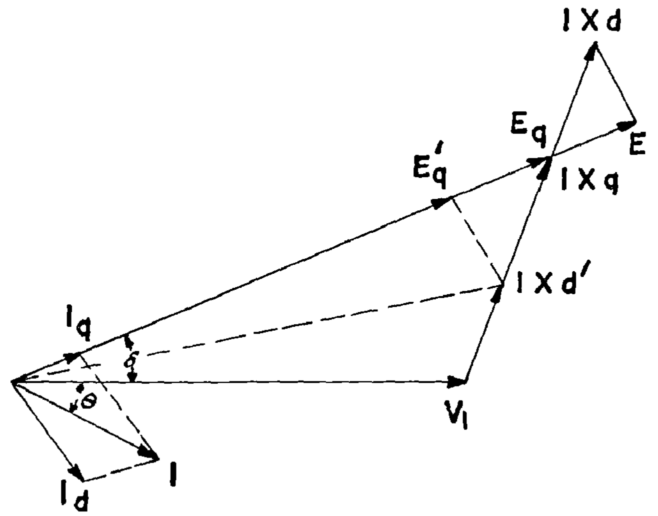

FIG.9. VECTOR DIAGRAM FOR SALIENT-POLE SYNCHRONOUS MACHINE.

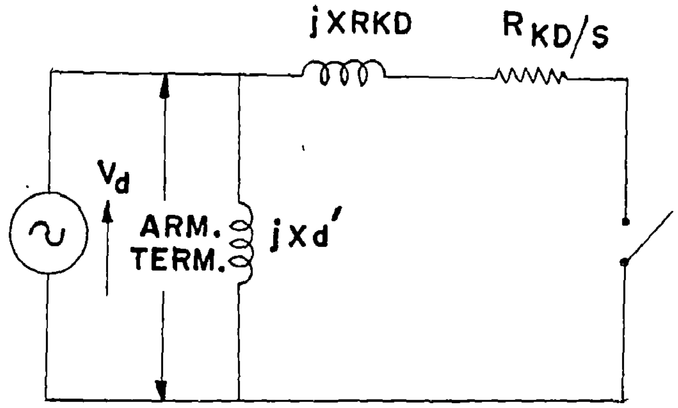

FIG.10 EQUIVALENT CIRCUIT OF SYNCHRONOUS MACHINE WITH DAMPER.

The term Rkd/s is neglected at large values of slip.

Hence,

$$XRkd = Xd'\ Xd''/(Xd'-Xd'') \qquad ..(AI.27)$$

Stator current is given by

$$Is = \frac{V_1(\ Rkd/s + j\ (XRkd + Xd'))}{(Rkd/s + j\ XRkd)\ (j\ Xd')} \qquad ..(AI.28)$$

Which for small slips can be expressed as

$$Is = V_1/\ j\ Xd' \qquad ..(AI.29)$$

and the rotor current is

$$Ir = Is\ \frac{j\ Xd'}{Rkd/s + j(XRkd+Xd')}$$

$$= Is\ \frac{j\ Xd'}{Rkd/s} \qquad ..(AI.30)$$

Putting the value of Is from (AI.29) in (AI.30),

$$Er = V_1/\ \frac{Rkd}{s} \qquad ..(AI.31)$$

The damping power can be shown by

$$Pd = I\ r^2\ Rkd\ \ (1-s)\ /s$$

$$= I\ r^2\ Rkd\ /s \quad \text{neglecting } (1-s)$$

$$= V_1^2 s/\ Rkd \qquad ..(AI.32)$$

The equivalent reactance as seen from the damper side with armature circuit open is XRkd+Xd'

From eq. (AI.26)

$$XRkd + Xd' = XRkd \cdot Xd'/Xd''$$

and from eq. (AI.27)

$$XRkd + Xd' = \frac{(Xd')^2}{(Xd' - Xd'')} \qquad ..(AI.33)$$

Therefore by the definition of the direct axis sub-transient open circuit time constant Tdo", it can be shown as

$$Tdo'' = \frac{XRkd + Xd'}{w\ (Rkd)} = \frac{(Xd')^2}{w(Xd' - Xd'')\ Rkd}$$

Hence,

$$Rkd = \frac{(Xd')^2}{w(Xd' - Xd'')\ Tdo''} \qquad ..(AI.34)$$

Substituting the value of eq.(AI.34) in (AI.32),

$$Pd = \frac{V_1^2 (Xd' - Xd'')\ Tdo''}{(Xd')^2} \quad (sw) \qquad ..(AI.35)$$

When the rotor is not symmetrical, the value of the damping power fluctuates between the above and the value obtained by replacing the direct axis constants by quadrature axis constants.

Therefore the average damping power can be obtained by substituting Vd for V_1 in eq. (AI.35) and Vq in the corresponding quadrature axis expression where $V_d = V_1 \sin\delta$

$V_q = V_1 \cos\delta$ ..(AI.36)

Thus,

$$P_d = \frac{2}{\Pi}\int_0^{\Pi/2} V_1^2\left[\left\{\frac{X_d' - X_d''}{X_d'^2}\right\}T_{do}''\ \sin^2\delta + \left\{\frac{X_q' - X_q''}{X_q'^2}\right\}T_{qo}''\ \cos^2\delta\, d\delta\right]\frac{d\delta}{dt}$$

$$= K_d \frac{d\delta}{dt} \qquad ..(AI.37)$$

Where

K_d = Damping Coefficient

$$= \frac{2}{\Pi}\int_0^{\Pi/2} V_1^2\left\{\frac{X_d' - X_d''}{X_d'^2}\,T_{do}''\sin^2\delta + \frac{X_q' - X_q''}{X_q'^2}\,T_{qo}''\ \cos^2\delta\right\}d\delta$$

Including the damping power developed, in the swing equation (AI.25) , we get,

$$M\frac{d^2\delta}{dt^2} + K_d\frac{d\delta}{dt} = P_i - P_{m1}\sin\delta + P_{m2}\sin 2\delta$$

..(AI.38)

APPENDIX II

AII.1 STABILITY

Let a system be defined by

$$\dot{X} = f(X,t) \qquad ..(AII.11)$$

Where

X is a state vector

and

f(X,t) is a state vector whose elements are function of state variables x1,x2,...

An equilibrium state Xe (Fig.11) of the system is stable if for each real number $\epsilon > 0$, there is a real number $\delta(\epsilon, t0) > 0$ such that

$$\| X_0 - X_e \| \leq \delta \qquad ..(AII.12)$$

implies

$$\| \phi(t; Xo,to) - Xe \| \leq \epsilon \text{ for all } t \geq to \qquad ..(AII.13)$$

where

$\phi(t;Xo,to)$ is the solution of eq.(AII.11)

Fig. 11 shows a stable equilibrium Xe of a second order system and the trajectory is starting from XO.

AII.2 ASYMPTOTIC STABILITY

An equilibrium state Xe of the system defined by (AII.11) is asymptotically stable, if it is stable and if every solution starting at a state Xo sufficiently near Xe converges to Xe as t increases indefinitely. Namely, given two real numbers $\delta > 0$ and $\mu > 0$, there are real numbers $\epsilon > 0$ and $T(\mu, \delta, to)$ such that

$$\| Xo - Xe \| \leq \delta \qquad ..(AII.14)$$

$$\| \phi(t; X0, to) - Xe \| \leq \epsilon \text{ for all } t \geq to \qquad ..(AII.15)$$

and

$$\| \phi(t; X0, to) - Xe \| \leq \mu \text{ for all } t \geq to + T(\mu, \delta, to) \qquad ..(AII.16)$$

Figure.12 shows an asymptotically stable equilibrium state Xe of a second order system with a trajectory starting from X0.

AII.3 POSITIVE(NEGATIVE) DEFINITE

A scalar function V(X) is positive (negative) definite if at all nonzero points X in the spherical region $\| X \| \leq K$ the value of V(X) are positive (negative), that is $V(X) > 0$ (< 0), and if $V(0) = 0$. ..(AII.17)

AII.4 POSITIVE(NEGATIVE) SEMIDEFINITE

A scalar function V(X) is positive (negative) semidefinite if for all X, such that $\| X \| \leq K$, $V(X) \geq 0$ (≤ 0) and if $V(0) = 0$

..(AII.18)

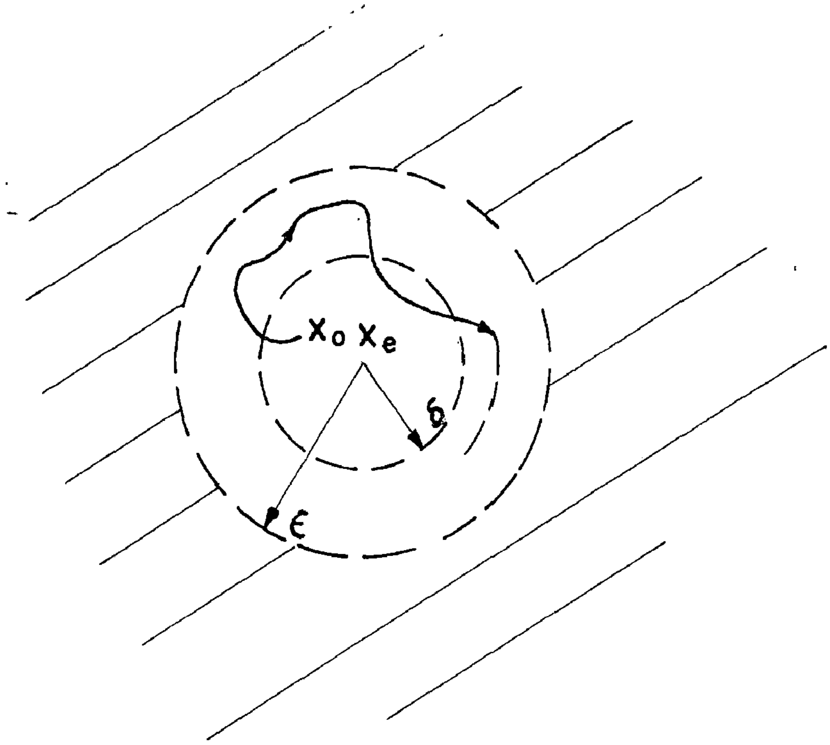

FIG. II GEOMETRIC INTERPRETATION OF THE DEFINITION OF STABILITY.

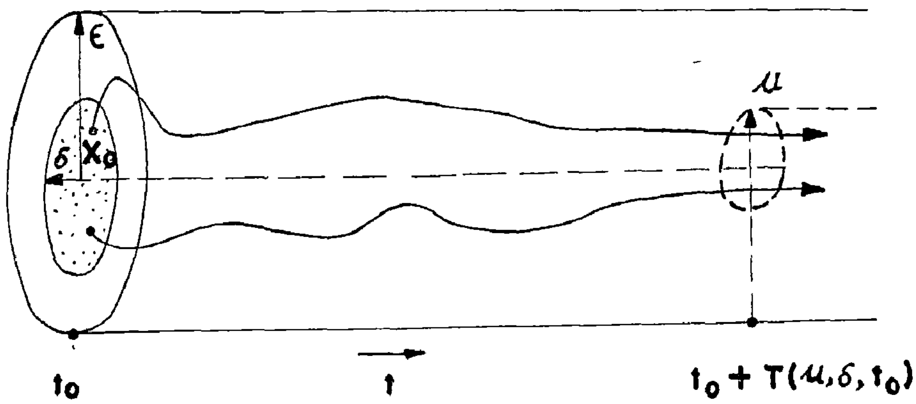

FIG. 12. GEOMETRIC INTERPRETATION OF THE DEFINITION OF ASYMPTOTIC STABILITY.

AII.5 SYLVESTER'S THEOREM

In order that a quadratic form

$$V(X) = X^T V X \qquad ..(AII.19)$$

Where V is a constant matrix

be positive definite , it is necessary and sufficient that each of the quantities

$$\det \begin{vmatrix} v11 \end{vmatrix} , \det \begin{vmatrix} v11 & v12 \\ v21 & v22 \end{vmatrix} , \det \begin{vmatrix} v11 & v12 & v13 \\ v21 & v22 & v23 \\ v31 & v32 & v33 \end{vmatrix} ... \det \begin{vmatrix} V \end{vmatrix}$$

be positive. ..(AII.20)

If any of the above determinants fail to be positive by being zero, the function is only semidefinite. The matrix V is negative definite or semidefinite if the matrix – V is positive definite or semidefinite.

When the matrix V has real elements and is symmetric about its diagonal & i.e. $vij=vji$ $i \neq j$. The above definitions are real symmetric positive (negative) definite or semidefinite respectively.

AII.6 MATRIX V' EQUATIONS

The Liapunov stability equation as derived in Chapter III is

$$A^T V' + V'A = - 2 R' \qquad ..(AII.21)$$

where V' and R' are real symmetric positive definite matrices. Matrices A and R' are known for a fourth order system.

A set of ten equations in terms of ten unknown elements of matrix V' is to be determined , so they may be solved to obtain the matrix V'.

Le the matrices A , V' and R' are defined by

$$A = \begin{bmatrix} a11 & a12 & a13 & a14 \\ a21 & a22 & a23 & a24 \\ a31 & a32 & a33 & a34 \\ a41 & a42 & a43 & a44 \end{bmatrix} \quad .(AII.22)$$

$$V' = \begin{bmatrix} v11' & v12' & v13' & v14' \\ v12' & v22' & v23' & v24' \\ v13' & v23' & v33' & v34' \\ v14' & v24' & v34' & v44' \end{bmatrix} \quad ..(AII.23)$$

$$R' = \begin{bmatrix} R11' & R12' & R13' & R14' \\ R12' & R22' & R23' & R24' \\ R13' & R23' & R33' & R34' \\ R14' & R24' & R34' & R44' \end{bmatrix} \quad ..(AII.24)$$

$$A^T V' = \begin{bmatrix} a11 & a21 & a31 & a41 \\ a12 & a22 & a32 & a42 \\ a13 & a23 & a33 & a43 \\ a14 & a24 & a34 & a44 \end{bmatrix} \begin{bmatrix} v11' & v12' & v13' & v14' \\ v12' & v22' & v23' & v24' \\ v13' & v23' & v33' & v34' \\ v14' & v24' & v34' & v44' \end{bmatrix}$$

..(AII.25)

$$V'A = \begin{bmatrix} v_{11}' & v_{12}' & v_{13}' & v_{14}' \\ v_{12}' & v_{22}' & v_{23}' & v_{24}' \\ v_{13}' & v_{23}' & v_{33}' & v_{34}' \\ v_{14}' & v_{24}' & v_{34}' & v_{44}' \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad .(AII.26)$$

Substituting (AII.25) , (AII.26) & (AII.24) in eq. (AII.21) , and by equating the elements of the L.H.S. to the corresponding elements of the R.H.S., a set of nonsimilar equations can be written as

$$\begin{bmatrix} 2a_{11} & 2a_{21} & 2a_{31} & 2a_{41} & .000 & .000 & .000 & .000 & .000 & .000 \\ a_{12} & a_{11}+a_{22} & a_{32} & a_{42} & a_{21} & a_{31} & a_{41} & .000 & .000 & .000 \\ a_{13} & a_{23} & a_{11}+a_{33} & a_{43} & .000 & a_{21} & .000 & a_{31} & a_{41} & .000 \\ a_{14} & a_{24} & a_{34} & a_{11}+a_{44} & 0 & .000 & a_{21} & .000 & a_{31} & a_{41} \\ 0 & 2a_{12} & 0 & 0 & 2a_{22} & 2a_{32} & 2a_{42} & 0 & 0 & 0 \\ 0 & a_{13} & a_{12} & 0 & a_{23} & a_{33}+a_{22} & a_{43} & a_{32} & a_{42} & 0 \\ 0 & a_{14} & 0 & a_{12} & a_{24} & a_{34} & a_{22}+a_{44} & 0 & a_{32} & a_{42} \\ 0 & 0 & 2a_{13} & 0 & 0 & 2a_{23} & 0 & 2a_{33} & 2a_{43} & 0 \\ 0 & 0 & a_{14} & a_{13} & 0 & a_{24} & a_{23} & a_{34} & a_{33}+a_{44} & a_{43} \\ 0 & 0 & 0 & 2a_{14} & 0 & 0 & 2a_{24} & 0 & 2a_{34} & 2a_{44} \end{bmatrix} \times$$

$$
\times \begin{bmatrix} v11' \\ v12' \\ v13' \\ v14' \\ v22' \\ v23' \\ v24' \\ v33' \\ v34' \\ v44' \end{bmatrix} = -2 \begin{bmatrix} R11' \\ R12' \\ R13' \\ R14' \\ R22' \\ R23' \\ R34' \\ R33' \\ R34' \\ R44' \end{bmatrix} \qquad ..(AII.27)
$$

The set of equations for lower order systems can be determined directly from (AII.27) by eliminating the unwanted rows and coloumns.

APPENDIX III

AIII.1 NEWTON RAPHSON METHOD (46)

This method is used for evaluation of real roots of the transcendental equation

$$h(x) = 0 \qquad ..(\text{AIII}.11)$$

The roots are obtained through a number of iterations by the exression

$$x^{(k+1)} = x^k - \frac{h(x^k)}{h'(x^k)} \qquad ..(\text{AIII}.12)$$

until the required accuracy is achieved

$x^{(k+1)}$: Value of the root on (k+1)th iteration

x^k : Value of the root on k th iteration

$h(x^k)$: Value of the function h for $x=x^k$

$h'(x^k)$: Value of the differential of the function w.r.t x for $x=x^k$

The transendental equations encountered in this work, have two roots. One is near zero radian and the other lies near 3.14159 radians. Therefore these are taken as the initial values to start with iteration for more accurate results.

AIII.2 CHARACTERISTIC EQUATION (47)

The Leverrier - Faddeev method is used for finding the characteristic equation of a matrix.

The following procedure is adopted.

Let

$$A1 = A \qquad \text{and } m1 = \text{tr } A$$

$$A2 = A(A1-m1I) \qquad \text{and } m2 = \tfrac{1}{2} \text{ tr } A2$$

$$A3 = A(A2-m2I) \qquad \text{and } m3 = \tfrac{1}{3} \text{ tr } A3 \qquad ..(AIII.13)$$

.

.

.

$$An = A(A_{n-1}-m_{n-1}I) \text{ and } mn = \frac{1}{n} \text{ tr } An$$

The characteristic equation will then be given by

$$\lambda^n - m1\lambda^{n-1} - m_2\lambda^{n-2} -\ldots-m_n=0$$

..(AIII.14)

AIII.3 EIGEN VALUES (46)

The eigen values are the roots evaluated from the characteristic equation of section AIII.2.

The program utlises synthetic substitution and Newton Raphson method for evaluation of all real and complex roots of the algebric equations with real or complex coefficients.

AIII.4 INVERSE OF A MATRIX (47)

The inverse of a matrix is obtained by elimination method .

Let A is any n x n matrix, of which the inverse has to kbe found out . This matrix can be associated with a set of linear equation, as

$$\begin{bmatrix} a11 & a12 & \cdots & a1n \\ a21 & a22 & \cdots & a2n \\ \vdots & & & \\ an1 & an2 & \cdots & ann \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ \vdots \\ xn \end{bmatrix} = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix} \begin{bmatrix} y1 \\ y2 \\ \vdots \\ yn \end{bmatrix}$$

..(AIII.15)

or

$$AX = Y$$

If by elimination process, A is reduced to a unit matrix i.e.

$$\begin{bmatrix} 1 & & & 0 \\ & \ddots & & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ \vdots \\ xn \end{bmatrix} = \begin{bmatrix} b11 & b12 & \cdots & b1n \\ b21 & b22 & \cdots & b2n \\ \vdots & & & \\ bn1 & bn2 & \cdots & bnn \end{bmatrix} \begin{bmatrix} y1 \\ y2 \\ \vdots \\ yn \end{bmatrix}$$

..(AIII.16)

or,

$$X = B Y$$

Then $B = A^{-1}$ ..(AIII.17)

AIII.5 MATRIX RV^{-1}

When the matrices V & A are known, the Liapunov stability equation (2.43)

$$A^{T}V + VA = -2R$$

can be solved for the unknown matrix R. Then the inverse of matrix V is calculated and the product

RV_{-1} is determined. The program is written in accordance with the flow chart of Fig. 13.

AIII .6 MATRIX V' SOLUTION (47)

This program solves a set of n(n+1)/2 linear equations by elimination method. An appropriate multiple of the first equation is added to each of the other equations, so as to eliminate the coefficients of the x1 term from $\frac{n(n+1)}{2} - 1$ equations. (The first equation should have x1 term, or it may exchanged for another one having it) Then an appropriate multiple of the next equation is added to the remaining terms, so as to eliminate x2 term coefficient from them. (If the second equation does not contain x2 term, it should be interchanged with another one having it) . The process is repeated until a set of pivotal equation, as shown below, is found out.

$$
\begin{array}{l}
a11\ x1 + a12\ x2 + \cdots + a1n\ xn = b1 \\
\qquad a22\ x2 + a23x3 + a2n\ xn = b2 \\
\qquad\qquad a33x3 + a3n\ xn = b3 \\
\qquad\qquad\qquad \vdots \\
\qquad\qquad\qquad ann\ xn = bn
\end{array}
\qquad ..(AIII.18)
$$

From the last equation: xn = bn/ann

On substituting the result in the last but one equation x_{n-1} is found out . The process is repeated upto the first equation, when x1 is known.

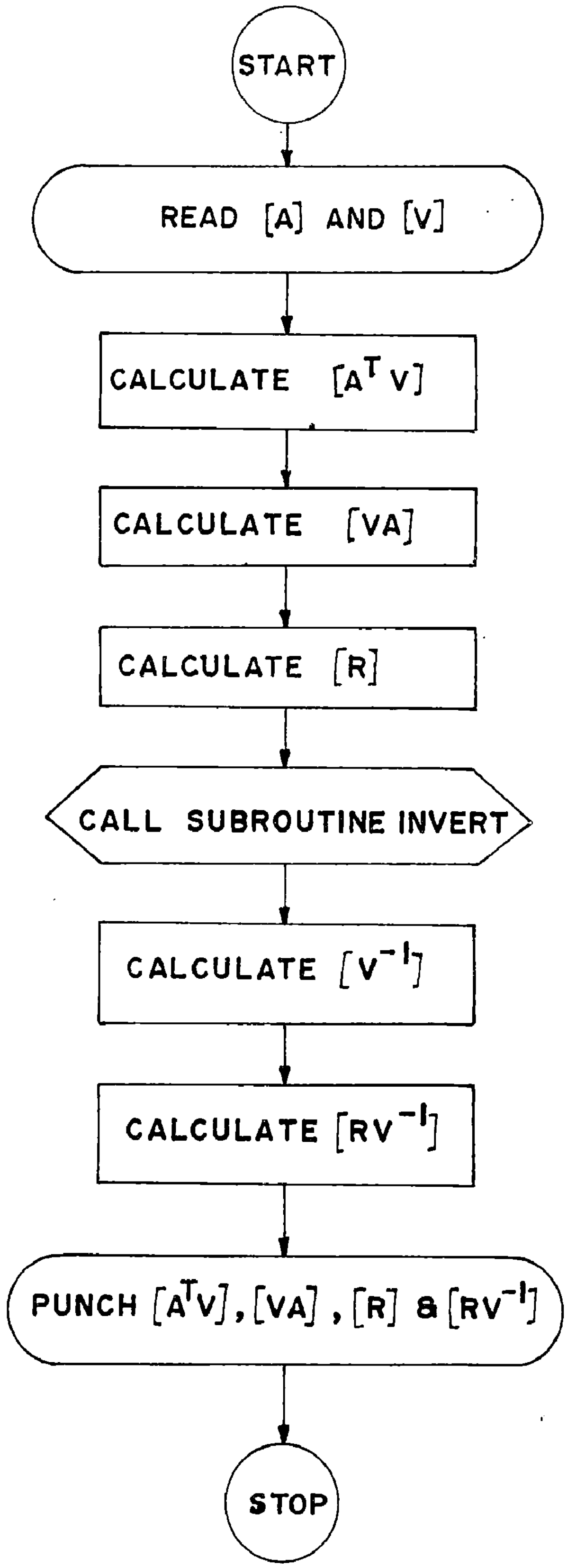

FIG. 13. FLOW CHART FOR FINDING $[RV^{-1}]$

AIII.7 RUNGE KUTTA GILL METHOD (38)

Runge Kutta Gill method is a modification of the Runge Kutta method for numerically integrating the n first order differential equations. It saves the memory space of the digital computer, while possessing all the advantages of the original one.

Let the N- first order differential equation are represented by

$$x_i'(t) = f_i(t, x1, x2, \ldots xN) \qquad ..(AIII.19)$$

where

$$i = 1, 2 \ldots\ldots N$$

The initial values are given as

$$x_{i,o}$$

The method involves iteration in four steps for each interval of time. The scheme is given below.

$$x_{i,j}' = fi\ (t, x1, j-1\ ,\ \ldots\ xN,\ j-1)$$

$$x_{i,j} = x_{i,j-1} + h(\ a_j(x_{i,j}' - b_j q_{i,j-1}))$$

$$q_{i,j} = q_{i,j-1} + 3(a_j(x_{ij}' - b_j q_{i,j-1})) - c_j x_{i,j}' \qquad ..(AIII.20)$$

where $j = 1, 2, 3, 4$

$h =$ Integration step length

$a1 = \frac{1}{2}$ $a2 = 1-\sqrt{\frac{1}{2}}$, $a3 = 1+\sqrt{\frac{1}{2}}$, $a4 = 1/6$

$b1 = 2$ $b2 = 1$ $b3 = 1$ $b4 = 2$

$c1 = a1$ $c2 = a2$ $c3 = 1+\sqrt{\frac{1}{2}}$ $c4 = \frac{1}{2}$

$q_{i,o} = 0$ initially, and thereafter in advancing the solution, $q_{i,o}$ for the next step is equated to $q_{i,4}$ of the preceeding step.

AIII.8 MONTE CARLO METHOD (48,49)

The method requires a subroutine to generate nonrepeatable random numbers between 0 and 1, at a very high speed, by digital computers. A subroutine (AIV.7) in machine language is written for this purpose. The program to determine maximum and minimum values of $\left[-\dot{V}(X) / V(X)\right]$ is written according to the flow chart of Fig. 14. It is arranged to iterate for five hundred times and to print the results after every twenty five iterations.

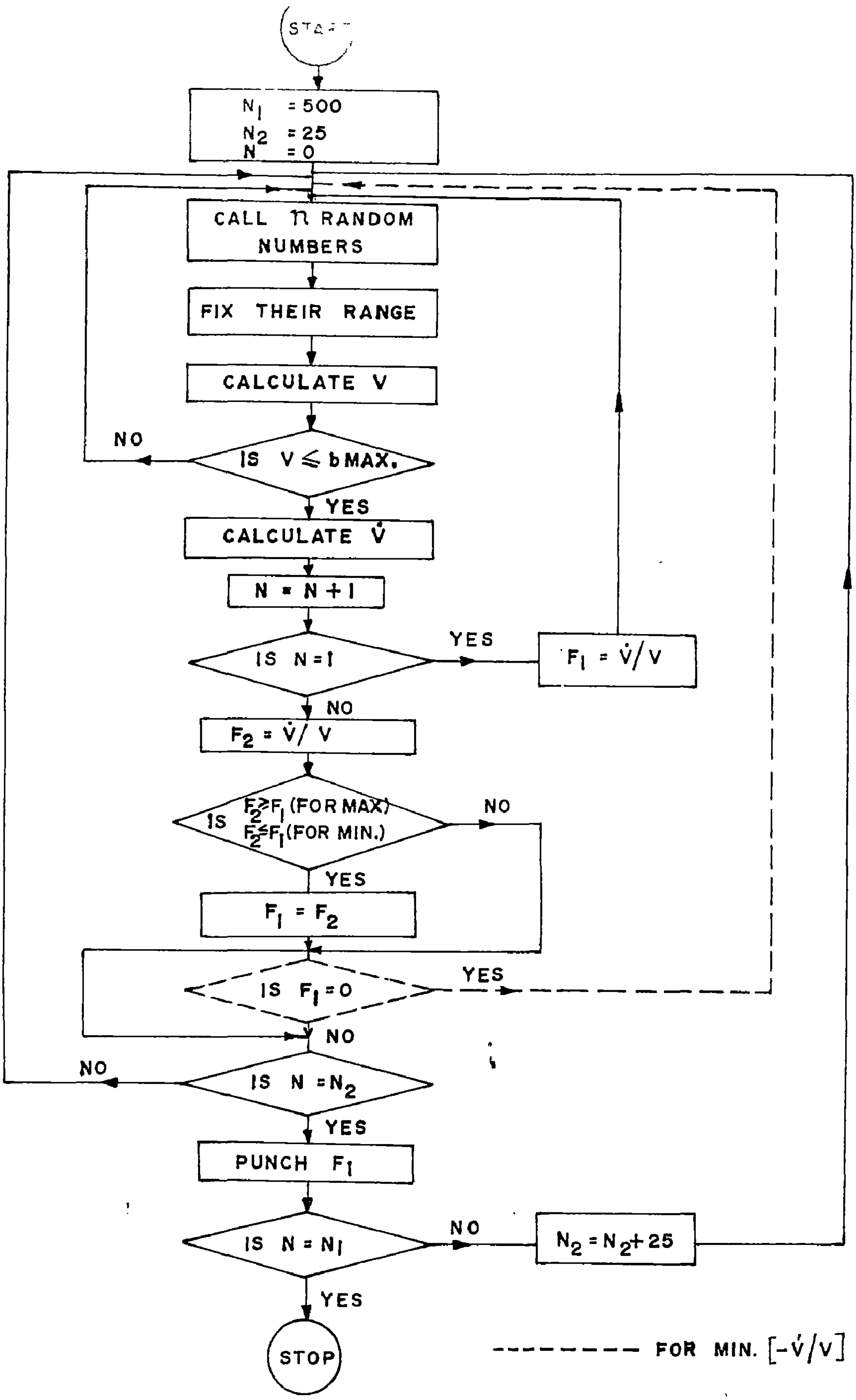

FIG. 14. FLOW CHART FOR FINDING $\underset{\text{MIN.}}{\text{MAX.}}\left[-\dot{V}(X)/V(X)\right]$ BY MONTE CARLO METHOD.

APPENDIX AIV.1

```
C   C  NEWTON RAPHSON METHOD HC AGARWAL EED 21304
       DO 100 I=1,2
       READ1,XO
     1 FORMAT(F10.5)
       XN=XO
     4 A=
       B=
       XD=A/B
       X=ABSF(XD)
       XN1=XN-XD
       IF(X-.001)2,2,3
     3 XN=XN1
       GO TO 4
     2 DEG=XN1*180./3.14159
       PUNCH1,XN1
       PUNCH1,DEG
   100 CONTINUE
       STOP
       END
    0.00000
    3.14159
```

APPENDIX AIV.2

```
C   C  PROGRAM RV INVERSE HC AGARWAL EED 21304
       DIMENSION A(4,4),C(4,4),B(4,4),V(4,4),D(4,4)
       DO 15 L=1,3
       READ 4,N
       READ1,((A(I,J),J=1,N),I=1,N)
       READ1,((V(I,J),J=1,N),I=1,N)
     4 FORMAT(I1)
    10 FORMAT(29X,F8.3)
     1 FORMAT(4F8.3)
       PUNCH 11
    11 FORMAT(29X,17HMATRIX A(TRANS)*V/)
       DO2 I=1,N
       DO2 J=1,N
     2 B(I,J)=A(J,I)
       DO3 I=1,N
       DO3 J=1,N
       SUM=0.
       DO5 K=1,N
     5 SUM=B(I,K)*V(K,J)+SUM
       C(I,J)=SUM
     3 PUNCH 10,C(I,J)
       DO6 I=1,N
       DO6 J=1,N
       SUM=0.
       DO7 K=1,N
```

```
 7 SUM=V(I,K)*A(K,J)+SUM
   D(I,J)=SUM
   B(I,J)=C(I,J)+D(I,J)
   B(I,J)=-0.5*B(I,J)
 6 CONTINUE
   PUNCH 12
12 FORMAT(29X,10HMATRIX V*A/)
   PUNCH10,((D(I,J),J=1,N),I=1,N)
   PUNCH 13
13 FORMAT(29X,8HMATRIX R/)
   PUNCH 10,((B(I,J),J=1,N),I=1,N)
   CALL INVERT(V,N)
   PUNCH 14
14 FORMAT(29X,18HMATRIX R*V(INVERS)/)
   DO8 I=1,N
   DO8 J=1,N
   SUM=0.
   DO9 K=1,N
 9 SUM=B(I,K)*V(K,J)+SUM
   C(I,J)=SUM
 8 PUNCH10,C(I,J)
15 CONTINUE
   STOP
   END
```

```
      SUBROUTINE INVERT(V,N)
      DIMENSION V(4,4),A(4,8),ID(4)
      NN=N+1
      N2=2*N
      DO 200 I=1,N
      DO 200 J=1,N
  200 A(I,J)=V(I,J)
      K=1
      DO1 I=1,N
      DO1 J=NN,N2
      A(I,J)=0.
    1 CONTINUE
      DO21 I=1,N
      A(I,N+I)=1.
   21 ID(I)=I
    2 CONTINUE
      KK=K+1
      IS=K
      IT=K
      B=ABSF(A(K,K))
      DO3 I=K,N
      DO 3 J=K,N
      IF(ABSF(A(I,J))-B)3,3,31
   31 IS=I
      IT=J
      B=ABSF(A(I,J))
```

```
  3 CONTINUE
    IF(IS-K)4,4,41
 41 DO 42 J=K,N2
    C=A(IS,J)
    A(IS,J)=A(K,J)
 42 A(K,J)=C
  4 CONTINUE
    IF(IT-K)5,5,51
 51 IC=ID(K)
    ID(K)=ID(IT)
    ID(IT)=IC
    DO 52 I=1,N
    C=A(I,IT)
    A(I,IT)=A(I,K)
 52 A(I,K)=C
  5 CONTINUE
    IF(A(K,K))6,120,6
  6 CONTINUE
    DO 7 J=KK,N2
    A(K,J)=A(K,J)/A(K,K)
    DO 7 I=KK,N
    W=A(I,K)*A(K,J)
    A(I,J)=A(I,J)-W
    IF(ABSF(A(I,J))-.0001*ABSF(W))71,7,7
 71 A(I,J)=0.
  7 CONTINUE
```

```
      K=KK
      IF(K-N)2,81,120
   81 IF(A(N,N))8,120,8
    8 CONTINUE
      DO9 J=NN,N2
      A(N,J)=A(N,J)/A(N,N)
    9 CONTINUE
      N1=N-1
      DO 10 M=1,N1
      I=N-M
      II=I+1
      DO 10K=II,N
      DO10 J=NN,N2
      A(I,J)=A(I,J)-A(I,K)*A(K,J)
   10 CONTINUE
      DO 11 I=1,N
      IF(ID(J)-I)11,111,11
  111 DO 112 K=NN,N2
  112 V(I,K-N)=A(J,K)
   11 CONTINUE
      RETURN
  120 PUNCH 1000
      RETURN
 1000 FORMAT(19H MATRIX IS SINGULAR)
      END
```

APPENDIX AIV.3

```
C  C  CHARACTERISTIC EOATION HC AGARWAL EED 21304
      DIMENSION A(4,4),B(4,4),C(5),D(4)
      DO 12 M=1,3
      READ11,N
   11 FORMAT(I2)
      READ10,((A(I,J),J=1,N),I=1,N)
   10 FORMAT(4F8.3)
      C(1)=1.
      K=1
      DO2 I=1,N
      DO2 J=1,N
    2 B(I,J)=A(I,J)
    1 CONTINUE
      C(K+1)=0.
      DO3 I=1,N
      C(K+1)=C(K+1)+B(I,I)
    3 CONTINUE
      FK=K
      C(K+1)=-C(K+1)/FK
      DO4 I=1,N
      B(I,I)=B(I,I)+C(K+1)
    4 CONTINUE
      IF(K-N+1)5,6,5
    5 DO7 J=1,N
      DO8 I=1,N
```

```
      D(I)=B(I,J)
    8 CONTINUE
      DO7 I=1,N
      B(I,J)=0.
      DO7 IS=1,N
      B(I,J)=B(I,J)+A(I,IS)*D(IS)
    7 CONTINUE
      K=K+1
      GO TO 1
    6 C(N+1)=0.
      DO9 J=1,N
      C(N+1)=C(N+1)-A(1,J)*B(J,1)
    9 CONTINUE
      L=N+1
      PUNCH10,(C(I),I=1,L)
   12 CONTINUE
      STOP
      END
```

APPENDIX AIV.4

```
C  C  EIGEN VALUES HC AGARWAL EED 21304
      DIMENSION CR(5),CI(5),DR(5),DI(5),ER(5),EI(5)
      DO 1 J=1,9
      READ 2,N,ACC
    2 FORMAT(I5,E10.3)
    3 FORMAT(4F8.3)
   11 FORMAT(16X,I3,2F13.3)
      N1=N+1
      N2=N
      READ 3,(CR(I),I=1,N1)
      DO 4 I=1,N1
    4 CI(I)=0.0
      DR(1)=CR(1)
      DI(1)=CI(1)
      ER(1)=CR(1)
      EI(1)=CI(1)
      DO 5 NROOT=1,N
      X=0.
      Y=1.
    6 DO 7 I=2,N1
      DR(I)=CR(I)+DR(I-1)*X-DI(I-1)*Y
    7 DI(I)=CI(I)+DR(I-1)*Y+DI(I-1)*X
      DO 8 I=2,N2
      ER(I)=DR(I)+ER(I-1)*X-EI(I-1)*Y
    8 EI(I)=DI(I)+ER(I-1)*Y+EI(I-1)*X
```

```
      DENO= ER(N2)**2+EI(N2)**2
      X=X-(DR(N1)*ER(N2)+DI(N1)*EI(N2))/DENO
      Y=Y+(DR(N1)*EI(N2)-DI(N1)*ER(N2))/DENO
      DIFF=DR(N1)**2+DI(N1)**2
      IF(DIFF-ACC) 9,9,6
    9 N1=N1-1
      N2=N2-1
      DO 10 I=2,N1
      CR(I)=DR(I)
   10 CI(I)=DI(I)
    5 PUNCH 11,NROOT,X,Y
    1 CONTINUE
      STOP
      END
```

APPENDIX AIV.5

```
C   C   V MATRIX DETERMINATION HC AGARWAL EED 21304
      DIMENSION AA(16,17),A(16,17),Y(16),X(16),ID(16)
      READ1,N
    1 FORMAT(I2)
      NN=N+1
      READ2,((AA(I,J),J=1,NN),I=1,N)
    2 FORMAT(4F8.3)
      DO 3 I=1,N
      DO 3 J=1,NN
    3 A(I,J)=AA(I,J)
      K=1
    4 CONTINUE
      DO 5 I=1,N
    5 ID(I)=I
    6 CONTINUE
      KK=K+1
      IS=K
      IT=K
      B=ABSF(A(K,K))
      DO 7 I=K,N
      DO 7 J=K,N
      IF (ABSF(A(I,J))-B) 7,7,8
    8 IS=I
      IT=J
      B=ABSF(A(I,J))
```

```
  7 CONTINUE
    IF(IS-K)9,9,10
 10 DO 11 J=K,NN
    C=A(IS,J)
    A(IS,J)=A(K,J)
 11 A(K,J)=C
  9 CONTINUE
    IF(IT-K) 12,12,13
 13 IC=ID(K)
    ID(K)=ID(IT)
    ID(IT)=IC
    DO 14 I=1,N
    C=A(I,IT)
    A(I,IT)=A(I,K)
 14 A(I,K)=C
 12 CONTINUE
    IF(A(K,K)) 15,16,15
 15 CONTINUE
    DO 17 J=KK,NN
    A(K,J)=A(K,J)/A(K,K)
    DO 17 I=KK,N
    W=A(I,K)*A(K,J)
    A(I,J)=A(I,J)-W
    IF(ABSF(A(I,J))-.0001*ABSF(W)) 18,17,17
 18 A(I,J)=0.
 17 CONTINUE
```

```
      K=KK
      IF(K-N) 6,19,16
   19 IF(A(N,N)) 20,16,20
   20 CONTINUE
      Y(N)=A(N,NN)/A(N,N)
      NM=N-1
      DO 21 I=1,NM
      K=N-I
      KK=K+1
      Y(K)=A(K,NN)
      DO 21 J=KK,N
      Y(K)=Y(K)-A(K,J)*Y(J)
   21 CONTINUE
      DO 22 I=1,N
      DO 22 J=1,N
      IF(ID(J)-I) 22,23,22
   23 X(I)=Y(J)
   22 CONTINUE
      PUNCH2,(X(I),I=1,N)
      GO TO 24
   16 PUNCH 25
   25 FORMAT(19H NO UNIQUE SOLUTION)
   24 STOP
      END
```

APPENDIX AIV.6

```
C  C  RUNGE KUTTA GILL METHOD HC AGARWAL EED 21304
  900 FORMAT(3F10.3)
  903 FORMAT(F10.3,I2)
  904 FORMAT(20X,5F10.7)
   17 FORMAT(13X,F10.3)
      DIMENSION YO(4),YN(4),Q(4),V(4),C(4),F(4),D(4),W(4)
    1 READ 900,HPR,XEND,H
      READ 903,XO,N
      PUNCH 17, XO
      DO2 I=1,N
      READ 904,YO(I)
    2 CONTINUE
      PUNCH904,(YO(I),I=1,N)
      XPR=HPR
      X=XO
      DO 3 I=1,N
      YN(I)=YO(I)
    3 Q(I)=0.0
 1112 U=X
      DO 4 I=1,N
    4 V(I)=YN(I)
      I1=1
      GO TO 100
    5 DO 6 I=1,N
      C(I)=H*F(I)
```

```
      D(I)=.5*(C(I)-2.0*Q(I))
      W(I)=YN(I)+D(I)
      Q(I)=Q(I)+3.0*D(I)-.5*C(I)
    6 V(I)=W(I)
      U=X+0.5*H
      I1=2
      GO TO 100
    8 DO 9 I=1,N
      C(I)=H*F(I)
      D(I)=0.29289325*(C(I)-Q(I))
      W(I)=W(I)+D(I)
      Q(I)=Q(I)+3.0*D(I)-.29289325*C(I)
    9 V(I)=W(I)
      I1=3
      GO TO 100
   11 DO 12 I=1,N
      C(I)=H*F(I)
      D(I)=1.7071067*(C(I)-Q(I))
      W(I)=W(I)+D(I)
      Q(I)=Q(I)+3.0*D(I)-1.7071067*C(I)
   12 V(I)=W(I)
      U=X+H
      I1=4
      GO TO 100
   14 DO 15 I=1,N
      C(I)=H*F(I)
```

```
      D(I)=.16666667*(C(I)-2.0*Q(I))
      YN(I)=W(I)+D(I)
   15 Q(I)=Q(I)+3.0*D(I)-.5*C(I)
      X=X+H
      IF(X-XPR)16,200,200
   16 GO TO 1112
  200 PUNCH17,X
      Z=
  208 PUNCH904,(YN(I),I=1,N),Z
  207 IF(X-XEND) 203,202,202
  202 GO TO 101
  203 XPR=XPR+HPR
      GO TO 1112
  100 F(1)=
      F(2)=
      F(3)=
      F(4)=
      GO TO (5,8,11,14),I1
  101 STOP
      END
```

APPENDIX AIV.7

```
C  C  TRANSIENT ESTIMATE MONTE CARLO METHOD HC AGARWAL EED 21304
      DIMENSION A(4)
    9 FORMAT(F10.5)
      COMMON X
      N1=800
      N2=25
      N=0
    3 DO 1 I=1,4
      CALL RANDOM
      A(I)=X
    1 CONTINUE
      A(1)=
      A(2)=
      A(3)=
      A(4)=
      V=
      IF(V-     )2,2,3
    2 VD=
      N=N+1
      IF(N-1)4,4,5
    4 F1=VD/V
      GO TO 3
    5 F2=VD/V
      IF(F2-F1)6,6,7
    7 F1=F2
```

```
 6 IF(N-N2)3,8,8
 8 PUNCH 9,F1
   IF(N-N1)10,11,11
10 N2=N2+25
   GO TO 3
11 STOP
    END
```

```
                    SUBROUTINE RANDOM
N9415544565400000000805000000000                          Z0000000000000001
J3J0241   K332    92     22    99    91K6J0241   991659999    -0Z   J0252J0312-00002
M3J0396   92J1J0323   -1L2J032L     J5J040K       J2J0402     -1Z   J0312J0372-00003
1259999   -1M9J0312      26N9997   99J6J0402N9997J6J0323-    92Z   J0372J0432-00004
42            Z                                                     J0432J0444-00005
J2345678Z                                                       1   J0234J0242-00006
```

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