CARRIER FREQUENCY OFFSET ESTIMATION FOR OFDM SYSTEMS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

ELECTRONICS & COMMUNICATION ENGINEERING

(With Specialization in Communication Systems)

By

MURALI KUNDE

DEPARTMENT OF ELECTRONICS AND COMPUTER ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE -247 667 (INDIA)

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report entitled, "**CARRIER FREQUENCY OFFSET ESTIMATION FOR OFDM SYSTEMS**" towards the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems**, submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from July 2009 to May 2010, under the guidance of **DR. ANSHUL TYAGI, Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee.**

I have not submitted the matter embodied in this dissertation for the award of any other Degree or Diploma.

Date: 26|05|2010

Place: Roorkee

K. Murali

MURALI KUNDE

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

Date: 26/05/2010

Place: Roorkee

Tyagi

DR. ANSHUL TYAGI,

Professor, E&C Department,

IIT Roorkee,

Roorkee – 247 667 (India).

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K. Murali

MURALI KUNDE

ABSTRACT

Synchronization is an essential task for any digital communication system. Without a proper and accurate synchronization method, it is not possible to reliably receive the transmitted data. Synchronization is the first and most important task that must be performed at the receiver. So, whole receiver architecture depends on the synchronization method that is used.

Orthogonal frequency division multiplexing (OFDM) is one of the most promising techniques for achieving high speed wireless data communication. OFDM is a multicarrier transmission technique, which divides the single wideband channel into a number of narrowband channels called sub-channels; each subcarrier in each sub-channel is being modulated by a low rate data stream and sub-carriers are transmitted in parallel over the channel. The increased symbol duration reduces the impact of ISI.

When the synchronization in an OFDM system is not perfect, the orthogonality among different subcarriers is destroyed and the ICI will be introduced. Therefore in OFDM system CFO estimation is an important issue that needs to be considered.

In this dissertation work, we have used repeated data symbols to estimate the carrier frequency offset for OFDM system, which is one of the earliest CFO estimation schemes for OFDM system. Following that we will discuss a Numerical technique for estimation of CFO for OFDM system. The technique is a blind maximum likelihood (ML) estimate of frequency offset using the Newton - Raphson method. Reduced complexity CFO estimation technique for OFDM systems which use null subcarriers is also been exploited. For simulation MATLAB is used and it is demonstrated through simulation results that the performance of CFO estimation using null subcarriers approach is close to the Cramer Rao bound.

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LIST OF ABBREVATIONS

Additive White Gaussian Noise..AWGN

Bit Error Rate..BER

Carrier Frequency Offset..CFO

Cramer-Rao Lower Bound...CRLB

Cramer-Rao Bound...CRB

Cyclic Prefix..CP

Channel Impulse Response...CIR

Discrete Fourier Transform...DFT

Digital Audio Broadcasting...DAB

Digital Video Broadcasting...DVB

Digital Signal Processing..DSP

Frequency Division Multiplexing..FDM

Forward Error Correction..FEC

Fast Fourier Transform...FFT

Global systems for Mobile Telecommunications..GSM

Inter- Symbol Interference...ISI

Inverse Discrete Fourier Transform..IDFT

Inverse Fast Fourier Transform..IFFT

Inter Carrier Interference...ICI

Maximum Likelihood..ML

Normalized Mean Square Error...MSE

Multiple input multiple output..MIMO

Orthogonal Frequency Division Multiplexing...OFDM

Personel Digital Cellular..PDC

Signal to Noise Ratio..SNR

Virtual Subcarrier..VSC

Very Large Scale Integration.. VLSI

CONTENTS

Chapter 1

INTRODUCTION

There has been a paradigm shift in mobile communication systems every decade. The first generation (1G) systems introduced in the 1980s were based on analog technologies, and second generation (2G) systems in 1990s, such as Global systems for Mobile Telecommunications (GSM), Personel Digital Cellular (PDC) and interim standard (IS)-95, on digital technologies for mixed voice-oriented data traffic. The third generation (3G) systems are also based on digital technologies for mixed voice, data and multimedia traffic and mixed-circuit and packet-switched network [1].

As the demand for higher data transmission rate and worldwide roaming in cellular devices increasing, the development of next generation (4G) wireless systems using digital broadband is underway. Therefore, enhancing system capacity as well as achieving a higher bit rate transmission is an important requirement for 4G systems. Fourth generation (4G) aims to provide variable rate multimedia services to the user (which include text, voice, data, audio, image or video), over broadband connections in a seamless manner. Together with an ever increasing quest for high data rates, poses a challenge to develop efficient coding/modulation techniques and signal processing algorithms, so that wireless links may be utilized as efficiently as possible [2].

These developments must cope up with several performance limiting challenges that include channel fading, multi-user interference, limitations of size/power especially at mobile units. A primary challenge to high rate in wireless communications is the presence of multi path fading channel. Multipath fading results from the fact that radio signal propagates trough many paths with different delays from the transmitter to receiver. For typical narrow band modulation, this gives rise to variations in received signal amplitude (fading); if the delay spread of the various components is significant fraction of the symbol duration as in frequency selective fading, it also leads to inter symbol interference (ISI).

Multicarrier modulation (MCM) [3] is an alternative approach to alleviating the impact of frequency selective fading channels. Orthogonal Frequency Division Multiplexing (OFDM) is a widely known multicarrier modulation scheme in which a serial data stream is split into parallel streams that modulate a group of orthogonal subcarriers. OFDM is a promising technology in broadband wireless communications due to its ability in mitigating multipath effects. Hence, it has been adopted as the key technology for standards such as DVB (Digital Video Broadcasting, DAB (Digital Audio Broadcasting) and it has been selected as the basis for the air interface for several new high-speed wireless local area network (WLAN) also known as Wi-Fi standards including IEEE 802.11a, IEEE 802.11g, and HIPERLAN. One of the main reasons to use OFDM is to increase the robustness against frequency selective fading and narrowband interference. In a single carrier system, a single fade or interfere can cause the entire link to fail, but in a multicarrier system, only a small percentage of subcarriers will be affected. Error correction coding can be used to correct the few erroneous subcarriers. A single carrier system suffers from trivial inter symbol interference (ISI) problem when data rate is extremely high [4].

OFDM is an effective way to increase data rate and simplify the equalization in wireless communications. It splits entire bandwidth into number of overlapping narrow band subchannels requiring lower symbol rates. Hence, OFDM symbol has much longer symbol interval and suffers from much less inter symbol interference (ISI) than single carrier transmission. Furthermore, the ISI can be easily eliminated by inserting a cyclic prefix (longer than the length of the channel impulse response) in front of each transmitted block and it is removed at the receiver block. OFDM is inherently robust against frequency selective fading channel, since the total bandwidth divided for multiple sub bands, the bandwidth of each subcarrier becomes small compared with the coherence bandwidth of the channel, i.e., the individual subcarrier experiences flat fading, which just requires a complex multiplication on each subcarrier data for equalization. The high spectral efficiency in OFDM is achieved using orthogonal signals allowing spectrum in each subchannel to overlap another without interfering. OFDM has an additional advantage of being computationally efficient because the fast Fourier transform (FFT) technique can be used to implement the modulation and demodulation functions. Combined with the progress in digital signal processing (DSP) and very large scale integration (VLSI) technologies OFDM has become a technologically practical and commercially affordable [4].

One of the most interesting trends in wireless communication is the proposed use of multiple input multiple output (MIMO) systems [5]. A MIMO system uses multiple transmitter antennas and multiple receiver antennas to break a multipath channel into several individual several spatial channels. The basic idea is to usually exploit the multipath rather than mitigate it, considering the multipath itself as a source of diversity that allows the parallel transmission of N independent sub streams from the same user. The exploitation of diversity and parallel transmission of several data streams on different propagation paths at the same time and frequency allows for extremely large capacities compared to conventional wireless systems. The prospect of many orders of magnitude improvement in wireless communication performance at no cost of extra spectrum (only hardware and complexity are added) is largely responsible for success of MIMO as a topic for new research. The combination of the two powerful techniques, MIMO and OFDM, is very attractive and, has become a most promising wireless access scheme.

However OFDM is very sensitive to carrier drifts. A carrier offset at the receiver can cause loss of subcarrier orthogonality, and thus can introduce inter-carrier interference (ICI). In addition, the frequency offset occurs due to a Doppler shift which results from a relative movement between transmitter and receiver in mobile radio environment. In digital communications estimating the frequency offset is essential for reliable performance of the receiver. Otherwise, the desirable properties of this type of transmission are lost [6]. In the literature, two categories of approaches have been put forward to mitigate the effects of the CFO. In the first category, the OFDM system can be made robust to ICI using techniques mentioned in the next paragraph. In the second category, CFO can be estimated at the receiver and then compensated.

To make OFDM robust to ICI two approaches can be employed: windowing and self intercarrier interference cancellation based approaches [7], [8], [9]. The self intercarrier interference cancellation based scheme can be regarded as a coding scheme, where only the code words with low ICI are used. Hence, spectrum efficiency is reduced since the coding rate is less than one [7]. The windowing scheme is achieved by shaping signals at the output of the IDFT by a window. This scheme normally results in SNR loss and ICI in case of no carrier frequency offset [8].

For the approaches in the second category, different CFO estimation schemes have been proposed in the literature. Typical CFO estimators have been developed using CP [10], training symbols [11], [12], virtual (null) subcarriers [13], [14], [15] channel information [16] and blind approaches [17], [18], [19]. The CP based approaches uses the redundant information contained within the CP [10]. In this sense the bandwidth efficiency of the system is affected since the extra CP acts like a pilot signal. When the training signals are periodic, the CFO estimation based ML criterion can be achieved using correlation operations [11], [12]. As a result, the training symbols are not required to be known and can be used to transmit system configuration information. In [11], two identical OFDM symbols are used with an estimation range of one subcarrier spacing. In [12], one training OFDM symbol with two identical parts, where the estimation range is two subcarrier spacing. In [20], an enhanced CFO estimation scheme was proposed to extend the estimation range to M subcarrier spacings using one OFDM symbol with M identical parts. The complexity of the scheme in [20] is approximately proportional to M.

CFO estimation can also be achieved by the exploitation of the inherent structure of OFDM signals. This generally referred to as a blind approach. This approach provides solution to the carrier offset estimation problem without using reference symbols, pilot carriers, or excess CP. Blind methods have attracted increasing interest recently because of their high accuracy and bandwidth efficiency. In [21] and [22], the periodic structure of the guard interval in OFDM systems is exploited and CFO is estimated based upon the ML criterion. However, the performance deteriorates significantly when the length of the channel impulse response (CIR) is large. Channel side information can be available for CFO estimation in wireless systems; a lot of schemes have been proposed to take advantage of channel side information. Recently, virtual subcarriers and numerical techniques are using in CFO estimation. The virtual subcarrier (VSC) based algorithms transmit null symbols known to the receiver. These VSCs do not waste any power.

1.1 Statement of Problem

This work is aimed at performance study of carrier frequency offset estimators in OFDM systems.

The dissertation presents the following work

- Effect of CFO on the performance of OFDM system
- CFO estimation technique for OFDM system using training sequences and their performance analysis.
- CFO estimation technique for OFDM system using Numerical technique based on the Newton-Raphson method and their performance analysis.
- CFO estimation technique for OFDM system using Null subcarriers and their performance analysis.

1.2 Organization of the Report

This report is organized in five chapters:

In chapter 1, we summarize problem statement of the dissertation work and also given an overview of carrier frequency offset estimation problem in OFDM systems.

In chapter 2, gives brief introduction to the OFDM systems and the effect of CFO on the performance of the OFDM systems. Then the CFO estimation using two identical training sequences is presented. Simulation results are also given.

In chapter 3, the numerical approach to estimate the CFO for OFDM systems and simulation results are presented.

In chapter 4, discusses reduced complexity CFO estimation technique for OFDM systems which use null subcarriers is presented. Simulation results are also given.

Chapter 5, gives the conclusion of the dissertation work.

Chapter 2

CFO Estimation using Training Sequences for OFDM Systems

In this chapter OFDM system and system model in presence of CFO are described first. The impact of CFO on OFDM systems presented next. Maximum likelihood (ML) estimate of frequency offset using the repeated data symbols is discussed. Finally simulation results are presented.

2.1. OFDM System

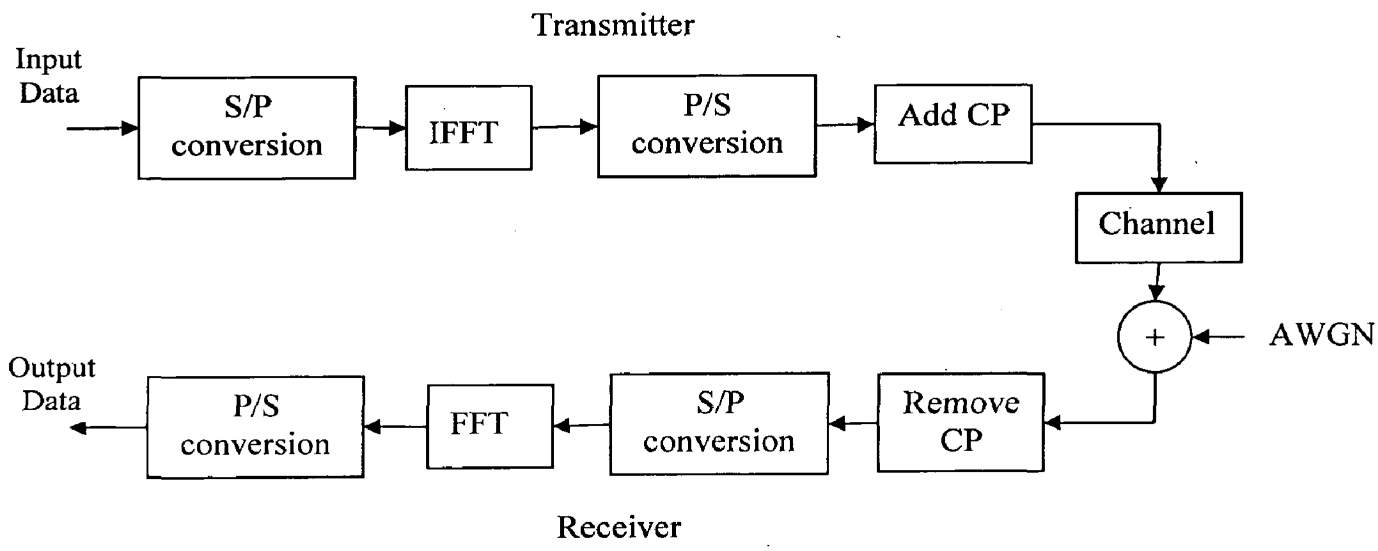


Figure 1: Block diagram of an OFDM system

The schematic diagram of Figure.1 is a baseband equivalent representation of an OFDM system. The input binary data is first fed into a serial to parallel (S/P) converter. Each data stream then modulates the corresponding sub-carrier by MPSK or MQAM. The modulated data symbols are then transformed by the Inverse Fast Fourier Transform (IFFT). The parallel data are converted back to a serial data stream before being transmitted over the frequency selective channel. The received data corrupted by multipath fading and AWGN are converted back to parallel data after discarding the prefix, and applying Fast Fourier Transform (FFT) and demodulation. These parallel data again converted to serial data using parallel to serial (P/S) converter.

OFDM modulation is accomplished by taking N-point IFFT of the symbol vector

$\mathbf{d} = [d_0, d_1, d_2, ..., d_{N-1}]^T$ as

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k e^{j2\pi nk/N}, \qquad n = -Ng, ..., N-1 \tag{2.1}$$

Where N_g is the length of the guard interval,

And d_k is the data symbol at the k^{th} subcarrier.

To avoid inter-symbol interference due to multipath effect, we insert cyclic prefix (a replica of the last several symbols of the block) in the beginning of the serial sequence after the parallel to serial conversion. This compensates the lost data due to multipath effect and simplifies the equalization at the receiver.

At the receiver after removing CP, the received symbol corrupted by fading channel and AWGN is given by,

$$x(n) = \sum_{l=0}^{L-1} s(n-l)h(l) + z(n), \qquad n = 0, 1, ..., N-1 \tag{2.2}$$

Where $h(l)$ is the gain of the l^{th} tap in the Tapped Delay Model of the channel impulse response (CIR),

And $z(n)$ is an additive white Gaussian noise with zero mean and variance σ^2.

After taking the N-point FFT of the received vector given by

$\mathbf{X} = [X(0), X(1), ..., X(N-1)]^T$

Where $X(0), X(1), ..., X(N-1)$ are the FFT of $x(0), x(1), ..., x(N-1)$.

Therefore in the time domain the received vector is represented as

$\mathbf{x} = [x(0), x(1), ..., x(N-1)]^T$

In the frequency domain the received OFDM symbol can be expressed as

$$X(k) = d_k H(k) + Z(k) \quad k = 0, 1, ..., N-1 \tag{2.3}$$

Where $H(k)$ is the channel frequency response at subcarrier k and it is given by

$$H(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}h(n)e^{-\frac{j2\pi nk}{N}} \tag{2.4}$$

And $Z(k)$ is the frequency response of the noise on the k^{th} subcarrier is given by

$$Z(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}z(n)e^{-\frac{j2\pi nk}{N}} \tag{2.5}$$

Then the received signal $X(k)$ is expressed in the time domain as follows:

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}d_k H(k)e^{\frac{j2\pi nk}{N}}+z(n), \qquad n=0,1,........,N-1 \tag{2.6}$$

2.2. OFDM model in the presence of CFO

The discrete time OFDM signal model is

$$s(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}d_k e^{j2\pi nk/N}, \qquad n=0,........,N-1$$

From equation (2.6) the OFDM signal at the receiver can be written as

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}d_k H(k)e^{\frac{j2\pi nk}{N}}e^{\frac{j2\pi n\Delta fT}{N}}+z(n), \qquad n=0,........,N-1$$

The above equation may also be written as

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}d_k H(k)e^{j(2\pi k/N+2\pi\phi)n}+z(n) \tag{2.7}$$

Where $\phi=\Delta f.T_s$ is the normalized CFO,

T_s is the sampling interval and $T_s=\dfrac{T}{N}$

The above signal model can be written in a vector form, we have

$$\mathbf{x}=\mathbf{PWHd}+\mathbf{Z} \tag{2.8}$$

Where $\mathbf{P}$ is phase shift due to the frequency offset and it is defined as

$\mathbf{P}=$ diag $[1,e^{j2\pi\phi},.....,e^{j2\pi(N-1)\phi}]$, And we can notice from above equation $\mathbf{P}\mathbf{P}^H = \mathbf{P}^H\mathbf{P} = \mathbf{I}$.

$\mathbf{H}$ is a $M \times M$ diagonal matrix with diagonal elements being $H(k)$,

$\mathbf{W}$ is a $N \times M$ matrix and it is defined from $\mathbf{U}$. Where $\mathbf{U}$ is a $N \times N$ IFFT matrix with partition

$\mathbf{U} = [\mathbf{W} \mid \mathbf{V}]$

Where $\mathbf{V}$ is a $N \times (N-M)$ matrix.

$$\mathbf{U} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{j\frac{2\pi}{N}} & \cdots & e^{j\frac{2\pi(N-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j\frac{2\pi(N-1)}{N}} & \cdots & e^{j2\pi\frac{(N-1)^2}{N}} \end{bmatrix}_{N\times N}$$

$$\mathbf{W} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{j\frac{2\pi}{N}} & \cdots & e^{j\frac{2\pi(M-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j\frac{2\pi(N-1)}{N}} & \cdots & e^{j2\pi\frac{(N-1)(M-1)}{N}} \end{bmatrix}_{N\times M}$$

$$\mathbf{V} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\frac{2\pi M}{N}} & e^{j\frac{2\pi(M+1)}{N}} & \cdots & e^{j\frac{2\pi(N-1)}{N}} \\ \vdots & \vdots & & \vdots \\ e^{j\frac{2\pi(N-1)M}{N}} & e^{j\frac{2\pi(N-1)(M+1)}{N}} & \cdots & e^{j2\pi\frac{(N-1)^2}{N}} \end{bmatrix}_{N\times(N-M)}$$

We can notice that $\mathbf{U}$ is a unitary matrix hence $\mathbf{W}^H\mathbf{V} = 0$ and $\mathbf{W}\mathbf{W}^H + \mathbf{V}\mathbf{V}^H = \mathbf{I}$

Where $(.)^H$ denotes conjugate transpose of a matrix

Put $\tilde{\mathbf{d}} = \mathbf{H}\mathbf{d}$ in equation (2.8), we get

$$\mathbf{x} = \mathbf{P}\mathbf{W}\tilde{\mathbf{d}} + \mathbf{Z} \tag{2.9}$$

2.3. Impact of carrier frequency offset on OFDM system performance

The amount of frequency mismatch between the received signal carrier and the local oscillator frequencies is called Frequency Offset. The principle disadvantage of OFDM is its sensitivity to frequency offset caused by the oscillator instabilities and/or Doppler shifts due to the movement of the mobile terminals. The presence of a CFO destroys the orthogonality among subcarriers, and the resulting inter-carrier interference (ICI) degrades the bit error rate (BER) severely. A very small amount of frequency offset can lead to significant degradation in system performance. To maintain signal to interference ratios of 20dB or greater for the OFDM carriers, offset must be limited to 4% or less of the inter-carrier spacing [11].

There are two effects caused by frequency offset in OFDM systems;

- Reduction of signal amplitude in the output of the filters matched to each of the carriers.
- Introduction of ICI from the other carriers which can cause loss of subcarrier orthogonality.

From equation (2.6) the received signal in the presence of CFO can be expressed from

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k H(k) e^{\frac{j2\pi nk}{N}} e^{\frac{j2\pi n\Delta fT}{N}} + z(n) \qquad (2.10)$$

Where ΔfT is the carrier frequency offset,

T is the OFDM symbol period.

The data sequence $X(k)$ recovered by applying FFT to the received signal is given by

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi nk}{N}}, \qquad k = 0,1,........,N-1 \qquad (2.11)$$

Substituting equation (2.10) in (2.11), we get

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \frac{1}{\sqrt{N}} \sum_{k'=0}^{N-1} d_{k'} H(k') e^{\frac{j2\pi nk'}{N}} e^{\frac{j2\pi n\Delta fT}{N}} e^{-\frac{j2\pi nk}{N}} + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} z(n) e^{-\frac{j2\pi nk}{N}}$$

$$= \sum_{k'=0}^{N-1} d_{k'} H(k') \frac{1}{N} \sum_{n=0}^{N-1} e^{\frac{j2\pi}{N}(k'+\Delta fT-k)n} + Z(k) \tag{2.12}$$

At $k' = k$ the date sequence is given

$$X(k) = d_k H(k) \frac{1}{N} \sum_{n=0}^{N-1} e^{\frac{j2\pi}{N}(\Delta fT)n} + Z(k) \tag{2.13}$$

From equation (2.13), $X(k)$ may also be written as

$$X(k) = d_k H(k) \frac{1}{N} \sum_{n=0}^{N-1} e^{\frac{j2\pi}{N}(\Delta fT)n} + \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} d_{k'} H(k') \frac{1}{N} \sum_{n=0}^{N-1} e^{\frac{j2\pi}{N}(k'+\Delta fT-k)n} + Z(k) \tag{2.14}$$

Simplifying the first term from above equation

$$d_k H(k) \frac{1}{N} \sum_{n=0}^{N-1} e^{\frac{j2\pi n\Delta fT}{N}} = d_k H(k) \frac{1}{N} \frac{1 - e^{j2\pi\Delta fT}}{1 - e^{\frac{j2\pi\Delta fT}{N}}}$$

$$= d_k H(k) \frac{1}{N} \frac{e^{\frac{j2\pi\Delta fT}{2}} (e^{-\frac{j2\pi\Delta fT}{2}} - e^{\frac{j2\pi\Delta fT}{2}})}{e^{\frac{j2\pi\Delta fT}{2N}} (e^{-\frac{j2\pi\Delta fT}{2N}} - e^{\frac{j2\pi\Delta fT}{2N}})}$$

$$= d_k H(k) \frac{\sin(\pi\Delta fT)}{N \sin(\frac{\pi\Delta fT}{N})} e^{j\pi\Delta fT(\frac{N-1}{N})} \tag{2.15}$$

Simplifying the second term from equation (2.14)

$$\sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} d_{k'} H(k') \frac{1}{N} \sum_{n=0}^{N-1} e^{\frac{j2\pi}{N}(k'+\Delta fT-k)n} = \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} d_{k'} H(k') \frac{1}{N} \frac{(1 - e^{2\pi(k'+\Delta fT-k)})}{(1 - e^{\frac{2\pi(k'+\Delta fT-k)}{N}})}$$

$$= \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} d_{k'} H(k') \frac{1}{N} \frac{e^{\frac{2\pi(k'+\Delta fT-k)}{2}} (e^{\frac{2\pi(k'+\Delta fT-k)}{2}} - e^{-\frac{2\pi(k'+\Delta fT-k)}{2}})}{e^{\frac{2\pi(k'+\Delta fT-k)}{2N}} (e^{\frac{2\pi(k'+\Delta fT-k)}{2N}} - e^{-\frac{2\pi(k'+\Delta fT-k)}{2N}})}$$

$$= \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} d_{k'} H(k') \frac{1}{N} \frac{\sin(\pi(k' + \Delta fT - k))}{\sin(\frac{\pi}{N}(k' + \Delta fT - k))} e^{j\pi \frac{(N-1)}{N}(k'+\Delta fT-k)} \tag{2.16}$$

Substituting equations (2.15) and (2.16) in (2.14), we get

$$X(k) = d_k H(k) \frac{\sin(\pi \Delta fT)}{N \sin(\frac{\pi \Delta fT}{N})} e^{j\pi\Delta fT(\frac{N-1}{N})} + \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} d_{k'} H(k') \frac{1}{N} \frac{\sin(\pi(k' + \Delta fT - k))}{\sin(\frac{\pi}{N}(k' + \Delta fT - k))} e^{j\pi \frac{(N-1)}{N}(k'+\Delta fT-k)} + Z(k) \tag{2.17}$$

This may be written as

$$X(k) = d_k H(k) \frac{\sin(\pi \Delta fT)}{N \sin(\frac{\pi \Delta fT}{N})} e^{j\pi\Delta fT(\frac{N-1}{N})} + I_k + Z(k) \tag{2.18}$$

Where, $I_k = \sum_{\substack{k'=0 \\ k' \neq k}}^{N-1} d_{k'} H(k') \frac{1}{N} \frac{\sin(\pi(k' + \Delta fT - k))}{\sin(\frac{\pi}{N}(k' + \Delta fT - k))} e^{j\pi \frac{(N-1)}{N}(k'+\Delta fT-k)}$

In equation (2.18)

- The first component is the modulation value d_k modified by the channel transfer function. This component experiences an amplitude reduction by the factor $\frac{\sin(\pi \Delta fT)}{N \sin(\frac{\pi \Delta fT}{N})}$ and phase shift due to the frequency offset.
- The second term I_k is the ICI caused by the frequency offset.

We can see that recovered data symbols will be equal to the actual data symbols, if and only if the CFO Δf is zero.

In order to evaluate the statistical properties of the ICI, some further assumptions are necessary. It is assumed that the modulation values have zero mean and are uncorrelated (i.e. $E[d_k]=0$ and $E[d_k d_{k'}^*]=|d|^2\,\delta_{k',k}$), with this provision $E[I_k]=0$, and

$$E\left[|I_k|^2\right]=|d|^2\sum_{\substack{k'=0\\k'\neq k}}^{N-1}E\left(|H(k')|^2\right).\left\{\frac{(\sin(\pi\Delta fT))^2}{\left(N.\sin\left((\pi/N).(k'-k+\Delta fT)\right)\right)^2}\right\} \qquad (2.19)$$

The average channel gain, $E\left(|H(k')|^2\right)=|H|^2$, is constant so above equation can be written as

$$E\left[|I_k|^2\right]=|d|^2.|H|^2.(\sin(\pi\Delta fT))^2.\sum_{\substack{p=-k\\p\neq 0}}^{N-k-1}\left\{\frac{1}{\left(N.\sin\left((\pi/N).(p+\Delta fT)\right)\right)^2}\right\} \qquad (2.20)$$

The sum in above equation can be bounded for $\Delta fT=0$. It consists of $N-1$ positive terms. The interval of the sum is contained within the longer interval $-(N-1)\leq p\leq(N-1)$, its location dependent on k. Also note the following; the argument of the sum is periodic with period N, it is an even function of p, and it is even about $p=N/2$. Thus the $N-1$ terms of the sum are a subset of the N terms in the intervals $-N/2\leq p\leq 1$ and $1\leq p\leq N/2$ for every k. Consequently,

$$\sum_{\substack{p=-k\\p\neq 0}}^{N-k-1}\left\{\frac{1}{\left(N.\sin(\pi p/N)\right)^2}\right\}<2\sum_{p=1}^{N/2}\left\{\frac{1}{\left(N.\sin(\pi p/N)\right)^2}\right\} \qquad (2.21)$$

Observe that $(\sin(\pi p/N))^2\geq(2p/N)^2$ for $|p|\leq N/2$. Therefore,

$$2\sum_{p=1}^{N/2}\frac{1}{\left(N\sin(\pi p/N)\right)^2}<2\sum_{p=1}^{N/2}\frac{1}{(2p)^2}<\frac{1}{2}\sum_{p=1}^{\infty}\frac{1}{p^2}=\pi^2/12=0.882$$

upper bounds the sum for small ΔfT. Numerically, we have determined that the sum in Eq.(2.20) is bounded by 0.5947 for $\Delta fT<0.5$ so that

$$E\left[|I_k|^2\right]\leq 0.5947|d|^2|H|^2.(\sin\pi\Delta fT)^2; \qquad |\varepsilon|\leq 0.5 \qquad (2.22)$$

upper bounds the variance of the inter-carrier interference for values of carrier frequency offset up to plus or minus one half the carrier spacing.

Equation (2.22) may be used to give a lower bound for the SNR at the output of the DFT for the OFDM carriers in a channel with AWGN and frequency offset. Thus,

$$SNR \geq \frac{|d|^2 |H|^2 \left\{ \sin(\pi \Delta f T) \big/ \pi \Delta f T \right\}^2}{\left\{ 0.5947 |d|^2 |H|^2 .(\sin \pi \Delta f T)^2 + E\left[|Z(k)|^2 \right] \right\}} \tag{2.23}$$

It is easily established that $|d|^2 |H|^2 \big/ E\left[|Z(k)|^2 \right] = E_c / N_0$ where, E_c is the average received energy of the individual carriers and $N_o / 2$ is the power spectral density of AWGN in the band pass transmission channel. Therefore, equation (2.23) may be more conveniently expressed as

$$SNR \geq \{E_c / N_0\}.\{\sin(\pi \Delta f T) / \pi \Delta f T\}^2 \big/ \left\{ 1 + 0.5947 (E_c / N_0).(\sin \pi \Delta f T)^2 \right\} \tag{2.24}$$

The simulation of the basic OFDM system performance with different values of CFO's at different values of SNR in AWGN channel is performed. SNR value of the subcarrier at the output of the DFT is calculated and its degradation effect due to increase in frequency offset value has been observed. Equation (2.24) which will give the lower bound for the SNR at the output of the DFT for the OFDM subcarrier in a channel with AWGN and CFO is plotted and compared with the practical OFDM system simulated. The simulated results are shown in section 2.5.

2.4 CFO estimation using training sequences

In [11], Moose proposed one of the earliest schemes to estimate the CFO. This method is a maximum likelihood (ML) estimate of frequency offset using the repeated data symbols (training sequence). This scheme was proposed by assuming that the frequency offset as well as the channel impulse response be constant for a period of two symbols.

The method uses two identical successive OFDM symbols to estimate the CFO. Here, the first OFDM symbol, or training symbol, is produced by doing FFT on a data sequence, d_k with $k = 0,1,...,N-1$, and the second one by d_k with $k = N, N+1,....,2N-1$. Both sequences are identical, that is $d_k = d_{k+N}$ for $k = 0,1,...,N-1$.

From equation (2.1) the first and second OFDM symbols are defined as

$$s_1(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k e^{j2\pi nk/N}, \qquad n = 0,1,........,N-1 \tag{2.25}$$

$$s_2(n) = \frac{1}{\sqrt{N}} \sum_{k=N}^{2N-1} d_k e^{j2\pi nk/N}, \qquad n = N, N+1,........,2N-1$$

Above equation can be written as

$$s_2(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_{k+N} e^{j2\pi n(k+N)/N}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k e^{j2\pi nk/N}, \qquad n = N, N+1,........,2N-1 \qquad (\because d_k = d_{k+N}) \tag{2.26}$$

From equations (2.25) and (2.26) the $2N$ point OFDM symbol can be expressed as

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k e^{j2\pi nk/N}, \qquad n = 0,1,........,2N-1 \tag{2.27}$$

From equation (2.7) the OFDM signal at the receiver with $2N$ point sequence and in the absence of noise, can be written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k H(k) e^{\frac{j2\pi nk}{N}} e^{\frac{j2\pi n\Delta fT}{N}}, \qquad n = 0,1,..........2N-1 \tag{2.28}$$

The first and second OFDM received signals in the presence of CFO can be expressed as

$$x_1(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k H(k) e^{\frac{j2\pi nk}{N}} e^{\frac{j2\pi n\Delta fT}{N}},$$

$$x_2(n)=\frac{1}{\sqrt{N}}\sum_{k=N}^{2N-1} d_k H(k) e^{\frac{j2\pi nk}{N}} e^{\frac{j2\pi n\Delta fT}{N}}$$

For simplicity assume $h(n)=\delta(n)$ for AWGN channel. Then above equations can be written as

$$x_1(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} d_k e^{\frac{j2\pi nk}{N}} e^{\frac{j2\pi n\Delta fT}{N}}, n=0,1,........,N-1 \quad (2.29)$$

$$x_2(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} d_k e^{\frac{j2\pi nk}{N}} e^{\frac{j2\pi n\Delta fT}{N}}, \qquad n=N,N-1,........,2N-1 \quad (2.30)$$

Then, at the receiver, after FFT, the first and second recovered sequences become

$$X_1(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\frac{1}{\sqrt{N}}\sum_{k'=0}^{N-1} d_{k'} e^{\frac{j2\pi nk'}{N}} e^{\frac{j2\pi n\Delta fT}{N}} e^{-\frac{j2\pi nk}{N}} \quad (2.31)$$

$$X_2(k)=\frac{1}{\sqrt{N}}\sum_{n=N}^{2N-1}\frac{1}{\sqrt{N}}\sum_{k'=0}^{N-1} d_{k'} e^{\frac{j2\pi nk'}{N}} e^{\frac{j2\pi n\Delta fT}{N}} e^{-\frac{j2\pi nk}{N}}$$

The above equation can be written as

$$X_2(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\frac{1}{\sqrt{N}}\sum_{k'=0}^{N-1} d_{k'} e^{\frac{j2\pi(n+N)k'}{N}} e^{\frac{j2\pi(n+N)\Delta fT}{N}} e^{-\frac{j2\pi(n+N)k}{N}}$$

$$=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\frac{1}{\sqrt{N}}\sum_{k'=0}^{N-1} d_{k'} e^{\frac{j2\pi nk'}{N}} e^{\frac{j2\pi n\Delta fT}{N}} e^{-\frac{j2\pi nk}{N}} e^{j2\pi\Delta fT}, \quad (2.32)$$

From equation (2.31) and (2.32), we have

$$X_2(k)=X_1(k)e^{j2\pi\Delta f} \quad (2.33)$$

$$e^{j2\pi\Delta fT}=\frac{X_2(k)}{X_1(k)}=\frac{X_2(k)X^*_1(k)}{|X_1(k)|^2}$$

By using maximum likelihood estimation technique the estimation of the CFO is defined as

$$\Delta\tilde{f}=\frac{1}{2\pi T}\tan^{-1}\left[\frac{\sum_{k=0}^{N-1}\mathrm{Im}(X_2(k)X^*_1(k))}{\sum_{k=0}^{N-1}\mathrm{Re}(X_2(k)X^*_1(k))}\right] \quad (2.34)$$

$$\tilde{\phi} = \frac{1}{2\pi N}\tan^{-1}\left[\frac{\sum_{k=0}^{N-1}\mathrm{Im}(X_2(k)X^*_1(k))}{\sum_{k=0}^{N-1}\mathrm{Re}(X_2(k)X^*_1(k))}\right] \qquad (\because \phi = \Delta f.\frac{T}{N}) \tag{2.35}$$

From equation (2.33), we can notice that between the first and second FFT's, both the ICI and the sequence are altered in exactly the same way, by a phase shift proportional to frequency offset. Therefore it is possible to obtain the accurate estimates from equation (2.35) even when the offset is too large.

2.5 Simulation Results

For the simulation of effect of CFO on OFDM systems in the MATLAB environment, the following parameters are used.

- FFT size : $N = 256$
- Modulation scheme: 8-PSK
- Channel: Flat Channel
- Noise: AWGN
- SNR: 11, 17, 23, and 29 dB
- Frequency offset: 0 to 0.5

Figure 2.2 shows the plot of SNR values at the output of the DFT for the OFDM subcarriers in a channel with AWGN and relative frequency offset. Figure also contains the theoretical lower bound values of SNR at the output of the DFT for the OFDM subcarriers with relative frequency offsets, which is given by equation (2.24). The graph is plotted for different SNR values of 11, 17, 23 and 29 dB varying the relative frequency offset value from 0 to 0.5. It can be observed from the plot that the practically obtained SNR values are above the theoretical lower bound SNR values.

Figure 2.3 shows the simulation results for the estimate of relative frequency offset (i.e. $\hat{\varepsilon}$) obtained using two identical training sequences i.e. equation (2.35) verses actual value of frequency offset ε for E_c/N_0 values of 5 and 17 dB. The simulation parameters that are considered while performing simulations in MATLAB environment are as follows;

- FFT size : $N = 256$
- Modulation scheme used: 8-PSK
- Channel: Flat Channel
- Noise: AWGN

We observe from the plot that curve plotted using $E_c/N_0 = 5$ dB is varying more than the one plotted using $E_c/N_0 = 17$ dB. This tells us that as the E_c/N_0 value increases the accuracy of the estimation will be increased.

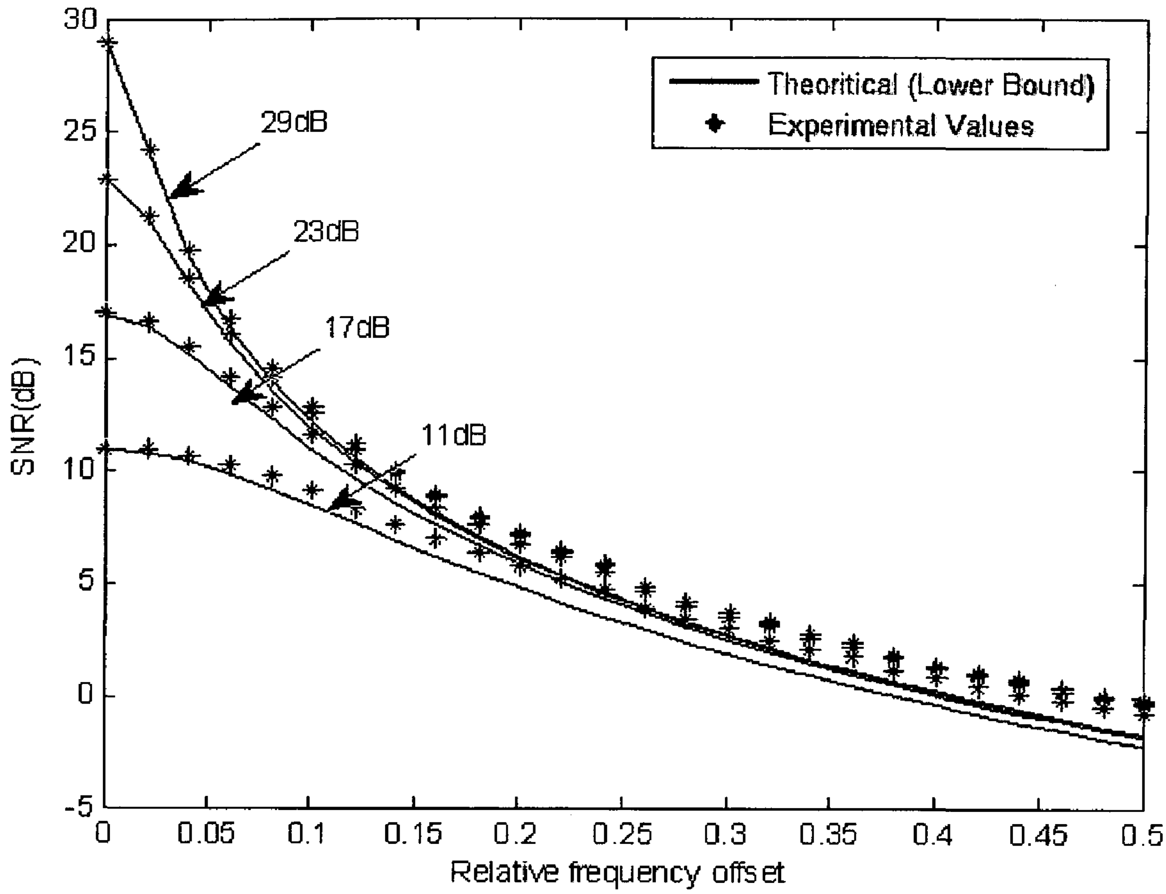


Figure 2.2. SNR versus relative frequency offset.

Chapter 3

CFO Estimation using Numerical Technique for OFDM Systems

This method is a blind maximum likelihood (ML) estimate of frequency offset using the numerical technique based on the Newton - Raphson method [23]. The scheme is characterized by low complexity and fast convergence while maintaining the estimation accuracy.

The method uses an OFDM system with N subcarriers, with M of them carrying data and the rest $N-M$ set to zero (virtual carriers).

From equation (2.9) the OFDM signal at the receiver can be written as

$$\mathbf{x} = \mathbf{PW\tilde{d}} + \mathbf{Z}$$

The likelihood function for ϕ and $\mathbf{\tilde{d}}$ is then given by

$$L(\phi,\mathbf{\tilde{d}}) = \frac{1}{(\pi\sigma^2)^N}\exp\{-\frac{1}{\sigma^2}\times(\mathbf{x}-\mathbf{PW\tilde{d}})^H(\mathbf{x}-\mathbf{PW\tilde{d}})\} \qquad (3.1)$$

To maximize the likelihood function, we are equivalently to minimize the score function

$$S(\phi,\mathbf{\tilde{d}}) = (\mathbf{x}-\mathbf{PW\tilde{d}})^H(\mathbf{x}-\mathbf{PW\tilde{d}}) \qquad (3.2)$$

For minimizing above equation take the gradient of $S(\phi,\mathbf{\tilde{d}})$ with respect to $\mathbf{\tilde{d}}$ and setting to zero, we get

$$S(\phi,\mathbf{\tilde{d}}_{ML}) = \mathbf{x}^H(\mathbf{I}-\mathbf{PWW}^H\mathbf{P}^H)\mathbf{x} \qquad (3.3)$$

From equations (3.1) and (3.3) the likelihood function can be written as

$$L'(\phi) = \frac{1}{(\pi\sigma^2)^N}\exp\{-\frac{1}{\sigma^2}\times\mathbf{x}^H(\mathbf{I}-\mathbf{PWW}^H\mathbf{P}^H)\mathbf{x}\} \qquad (3.4)$$

3.1 Newton - Raphson method

Newton - Raphson method is the best known method for finding successively better approximations to the roots of a real-valued function. The method can often converge remarkably quickly, especially if the iteration begins sufficiently near the desired root [24].

Given a function $f(x)$ and its derivative $f'(x)$, starting from an arbitrary initial guess value x_0, a better approximation of root is given by

$$x_1 = x_0 - \frac{f(x)}{f'(x)} \tag{3.5}$$

The process is repeated until a sufficiently accurate value is reached. The estimation at (k+1)th iteration step is

$$x_{k+1} = x_k - \frac{f(x)}{f'(x)} \tag{3.6}$$

Using the above equation we can achieve the ML estimation of ϕ, it is given by

$$\hat{\phi}_{(k+1)} = \hat{\phi}_k - \left[\frac{\partial^2 \ln L'(\phi)}{\partial \phi^2}\right]^{-1} \times \frac{\partial \ln L'(\phi)}{\partial \phi} \tag{3.7}$$

Where ϕ_0 is initial guess value,

ϕ_k estimation at k th iteration and

ϕ_{k+1} estimation at $(k+1)$ th iteration.

The first derivative of log-likelihood function is

$$\frac{\partial \ln L'(\phi)}{\partial \phi} = \frac{1}{\sigma^2} \cdot \frac{\partial Z}{\partial \phi}$$

Where $\mathbf{Z} = \mathbf{x}^H \mathbf{P} \mathbf{W} \mathbf{W}^H \mathbf{P}^H \mathbf{x}$ and it may also be written as

$$\mathbf{Z} = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} x^*_m \cdot x_n \cdot Q_{mn} \cdot e^{j(m-n)\phi} \tag{3.8}$$

Where Q_{mn} is the value of the m th row and n th column of matrix and it is given by

$$\mathbf{Q} = \mathbf{W}\mathbf{W}^H$$

The first derivative of log-likelihood function is

$$\frac{\partial \ln L'(\phi)}{\partial \phi} = \frac{j}{\sigma^2} \cdot \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} (m-n) \cdot x_m^* \cdot x_n \cdot Q_{mn} \cdot e^{j(m-n)\phi}$$

The second derivatives of the log-likelihood function is

$$\frac{\partial^2 \ln L'(\phi)}{\partial^2 \phi} = \frac{-1}{\sigma^2} . \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} (m-n)^2 x_m^* x_n . Q_{mn} e^{j(m-n)\phi}$$

$$= \frac{-1}{\sigma^2} . \mathbf{x}^H \mathbf{P} \mathbf{Q}^{(2)} \mathbf{P}^H \mathbf{x} \qquad (3.10)$$

Where $\mathbf{Q}^{(1)}$ and $\mathbf{Q}^{(2)}$ are calculated from matrix $\mathbf{Q}$

$$[\mathbf{Q}^{(1)}]_{mn} = (m-n).[\mathbf{Q}]_{mn}$$

And

$$[\mathbf{Q}^{(2)}]_{mn} = (m-n)^2.[\mathbf{Q}]_{mn} \; .$$

Substituting equation (3.9) and (3.10) in equation (3.7), we get estimation of CFO

$$\hat{\phi}^{(k+1)} = \hat{\phi}^{(k)} - \frac{\frac{j}{\sigma^2} . \mathbf{x}^H \mathbf{P} \mathbf{Q}^{(1)} \mathbf{P}^H \mathbf{x}}{\frac{-1}{\sigma^2} . \mathbf{x}^H \mathbf{P} \mathbf{Q}^{(2)} \mathbf{P}^H \mathbf{x}}$$

$$= \hat{\phi}^{(k)} + j . \frac{\mathbf{x}^H \mathbf{P} \mathbf{Q}^{(1)} \mathbf{P}^H \mathbf{x}}{\mathbf{x}^H \mathbf{P} \mathbf{Q}^{(2)} \mathbf{P}^H \mathbf{x}} \qquad (3.11)$$

From equation (3.11), we can notice that the complexity of the iteration procedure is very low, considering that vector $\mathbf{x}, \mathbf{Q}^{(1)}$ and $\mathbf{Q}^{(2)}$ are constant for all the iteration steps.

Results obtained from the Newton-Raphson method may oscillate about the local maximum or minimum without converging on a root but converging on the local maximum or minimum. To overcome the problem we try multiple initial points, spanning the whole range of possible CFO values.

Example: Considering the normalized CFO may range from 0 to 1, one possible choice is the set {0.1, 0.3, 0.5, 0.7, 0.9}.

Starting from the set of initial points, the algorithm iteratively calculates estimate values. The iteration procedure may results in two possible estimates. The likelihood of each estimate is then evaluated using equation (3.4) and the estimate with the maximum value of likelihood function is selected.

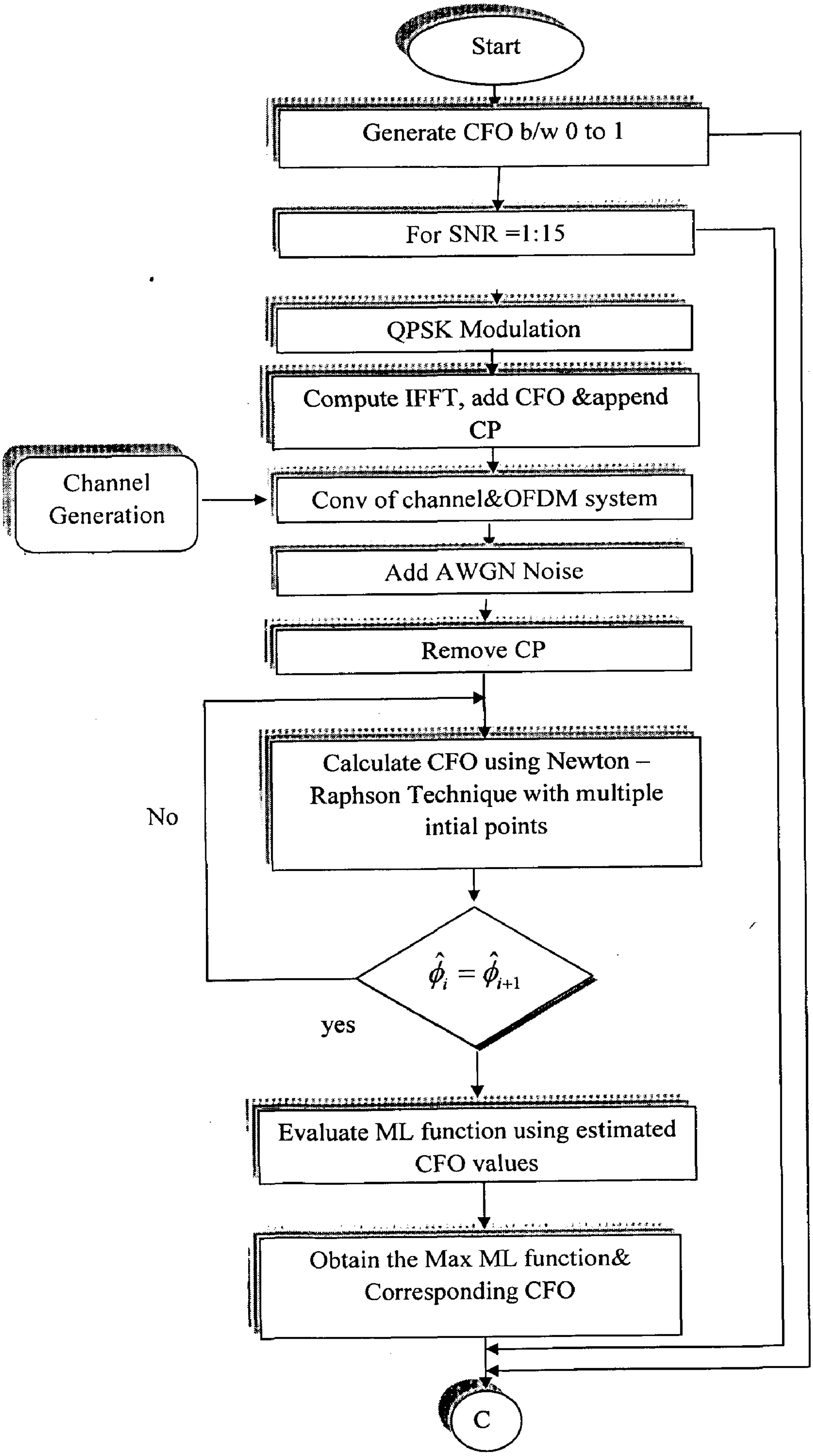
Example: Consider normalized CFO 0.66 and SNR 10 dB. Table 1 shows the steps of iteration procedure. From the table we get 0.12475 as the local minima and 0.60473 as the estimated value of CFO.

Table 1

Iteration Procedure

Iteration (k)	$\hat{\phi}_1^{(k)}$	$\hat{\phi}_2^{(k)}$	$\hat{\phi}_3^{(k)}$	$\hat{\phi}_4^{(k)}$	$\hat{\phi}_5^{(k)}$
0	0.10000	0.30000	0.50000	0.70000	0.90000
1	0.12639	0.11236	0.69280	0.62223	0.66285
2	0.12476	0.12512	0.62021	0.60561	0.61255
3	0.12475	0.12475	0.60543	0.60473	0.60491
4	0.12475	0.12475	0.60473	0.60473	0.60473
5	0.12475	0.12475	**0.60473**	**0.60473**	**0.60473**
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
Likelihood	35.63580	35.63580	**35.85420**	**35.85420**	**35.85420**

Figure 3.1 shows the flow chart for simulation of Newton-Raphson method for estimation of CFO for OFDM system.


Start
Generate CFO b/w 0 to 1
For SNR =1:15
QPSK Modulation
Compute IFFT, add CFO &append CP
Channel Generation
Conv of channel&OFDM system
Add AWGN Noise
Remove CP
Calculate CFO using Newton – Raphson Technique with multiple intial points
No
$\hat{\phi}_i = \hat{\phi}_{i+1}$
yes
Evaluate ML function using estimated CFO values
Obtain the Max ML function& Corresponding CFO
C

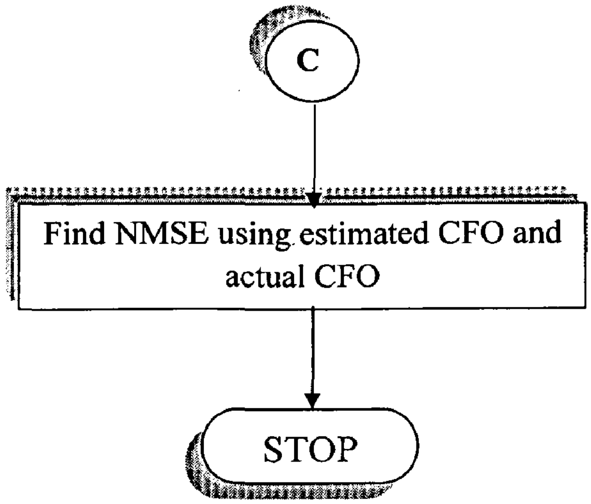


Figure 3.1 Flow chart for simulation of Newton-Raphson method.

3.2 Simulation Results

For the simulation of Newton-Raphson method in the MATLAB environment, the following parameters are used.

- Number of subcarriers (FFT size) N=64
- Number of data carrying subcarriers P=52
- Number of null subcarriers N-P=12
- Number of Monte Carlo runs=10000
- Noise: AWGN
- Channel $\mathbf{h} = [0.227 \ \ 0.46 \ \ 0.688 \ \ 0.46 \ \ 0.227]^T$

Figure 3.2 shows the achieved normalized mean square error (MSE) using proposed numerical technique and the conventional ML method [25].

$$\textbf{NMSE} = \frac{N}{N_t}\sum_{t=1}^{N_t}(\hat{\phi}_t - \phi)^2$$

Where N_t is the number of Monte Carlo trials,

ϕ is the actual normalized CFO,

And $\hat{\phi}_t$ is the estimated normalized CFO at the tth trial.

For each SNR value, 10^4 values of CFO have been randomly selected in the range [0,1] and corresponding MSEs have been calculated. The technique preserves low complexity and fast convergence while maintaining the high estimation accuracy..

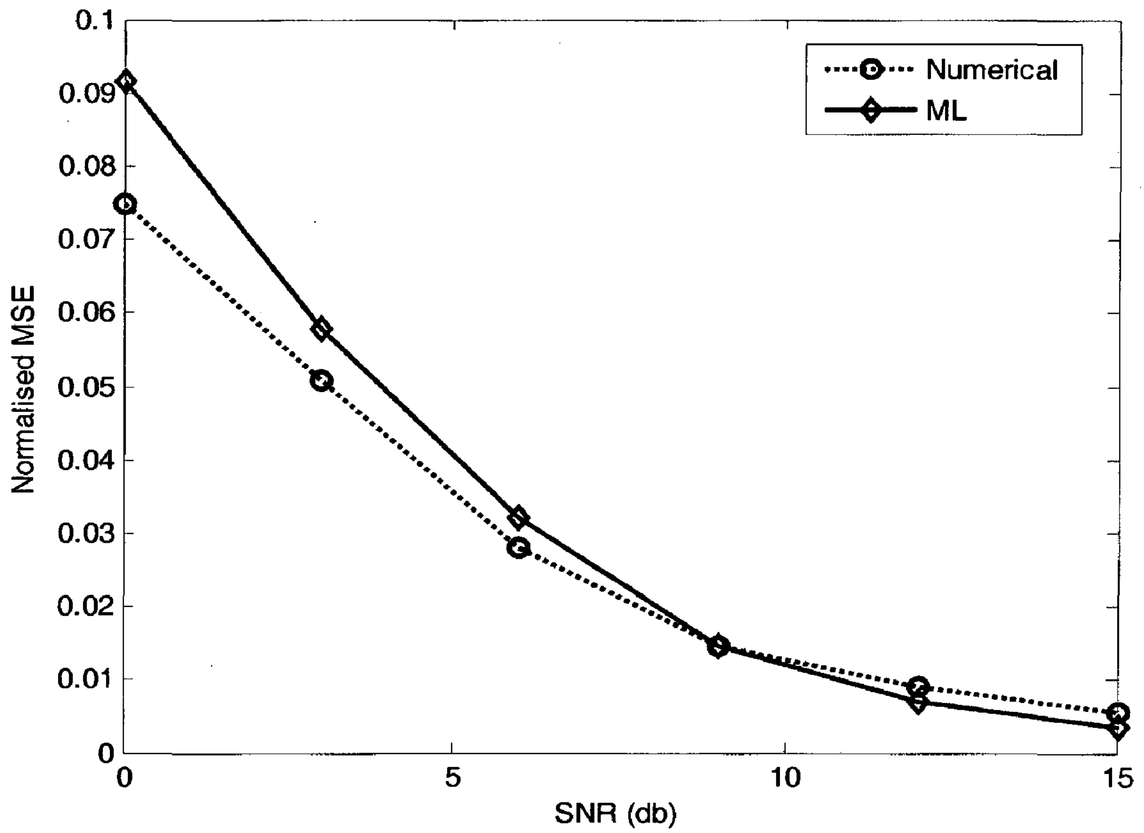


Figure 3.2 Normalized MSE versus SNR

Chapter 4

CFO Estimation using virtual (null) subcarriers for OFDM Systems

In this chapter CFO estimation using null subcarrier for OFDM system presented. Null subcarrier allocation for the reduced complexity CFO estimation is discussed next. Finally simulation results are presented.

In OFDM system with N subcarriers, N information symbols are used to construct one OFDM symbol. Each of the N symbols is used to modulate a subcarrier and the N modulated subcarriers are added together to form an OFDM symbol. In the presence of virtual carriers (subcarriers with zero transmitted), only M out of N subcarriers are used to modulate information symbols (we assume that the first M carriers are used to modulate information symbol, while the last $N-M$ carriers are virtual carriers). CFO estimation can also be obtained using virtual subcarriers [14].

The discrete time OFDM signal model is

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{M-1} d_k e^{j2\pi nk/N}, \qquad n = 0, \ldots\ldots, N-1 \tag{4.1}$$

From equation (2.8) the OFDM signal at the receiver can be written as

$$\mathbf{x} = \mathbf{PWHd} + \mathbf{Z} \tag{4.2}$$

The unknown parameters in (4.1) are ϕ and $\tilde{\mathbf{d}}$. Assume that the covariance matrix of z is σ^2 I, where σ^2 is the noise variance and I denotes the identity matrix.

The likelihood function for ϕ and $\tilde{\mathbf{d}}$ is then given by

$$L(\phi, \tilde{\mathbf{d}}) = \frac{1}{(\pi\sigma^2)^N} \exp\{-\frac{1}{\sigma^2} \times (\mathbf{x} - \mathbf{PW}\tilde{\mathbf{d}})^H (\mathbf{x} - \mathbf{PW}\tilde{\mathbf{d}})\} \tag{4.3}$$

Thus the ML estimate for ϕ and $\tilde{\mathbf{d}}$ are given by

$$(\phi_{ML}, \tilde{\mathbf{d}}_{ML}) = \arg \max_{\phi, \tilde{\mathbf{d}}} L(\phi, \tilde{\mathbf{d}}) \tag{4.4}$$

To maximize the likelihood function, we are equivalently to minimize

$$S(\phi, \tilde{\mathbf{d}}) = (\mathbf{x} - \mathbf{PW}\tilde{\mathbf{d}})^H (\mathbf{x} - \mathbf{PW}\tilde{\mathbf{d}}) \tag{4.5}$$

For minimizing above equation take the gradient of $S(\phi,\tilde{\mathbf{d}})$ with respect to $\tilde{\mathbf{d}}$ and setting to zero, we get [26]

$$\Delta_d S(\phi,\tilde{\mathbf{d}}) = \mathbf{W}^H\mathbf{P}^H\,(\mathbf{x}-\mathbf{P}\mathbf{W}\tilde{\mathbf{d}}) = \mathbf{0}$$

From above we can solve for $\tilde{\mathbf{d}}_{ML}$

$$\tilde{\mathbf{d}}_{ML} = \mathbf{W}^H\mathbf{P}^H\mathbf{x} \tag{4.6}$$

Substituting equation (4.6) in equation (4.5), we get

$$\begin{aligned}
S(\phi,\tilde{\mathbf{d}}_{ML}) &= (\mathbf{x}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H\mathbf{x})^H(\mathbf{x}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H\mathbf{x})\\
&= ((\mathbf{I}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)\mathbf{x})^H(\mathbf{I}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)\mathbf{x}\\
&= \mathbf{x}^H(\mathbf{I}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)^H(\mathbf{I}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)\mathbf{x}\\
&= \mathbf{x}^H(\mathbf{I}-(\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)^H-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H+(\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)^H\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)\mathbf{x}
\end{aligned} \tag{4.7}$$

Simplifying the term $(\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)^H$

$$\begin{aligned}
(\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)^H &= (\mathbf{W}^H\mathbf{P}^H)^H(\mathbf{P}\mathbf{W})^H\\
&= \mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H
\end{aligned}$$

Therefore equation (4.7) becomes

$$\begin{aligned}
S(\phi,\tilde{\mathbf{d}}_{ML}) &= \mathbf{x}^H(\mathbf{I}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H+\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)\mathbf{x}\\
&= \mathbf{x}^H(\mathbf{I}-2\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H+\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{W}\mathbf{W}^H\mathbf{P}^H)\mathbf{x} \qquad (\because \mathbf{P}\mathbf{P}^H = \mathbf{P}^H\mathbf{P} = \mathbf{I})\\
&= \mathbf{x}^H(\mathbf{I}-2\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H+\mathbf{P}\mathbf{W}\mathbf{W}^H(\mathbf{I}-\mathbf{V}\mathbf{V}^H)\mathbf{P}^H)\mathbf{x} \qquad (\because \mathbf{W}\mathbf{W}^H+\mathbf{V}\mathbf{V}^H = \mathbf{I})\\
&= \mathbf{x}^H(\mathbf{I}-2\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H+\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{V}\mathbf{V}^H\mathbf{P}^H)\mathbf{x}\\
S(\phi,\tilde{\mathbf{d}}_{ML}) &= \mathbf{x}^H(\mathbf{I}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)\mathbf{x} \qquad (\because \mathbf{W}^H\mathbf{V} = 0)
\end{aligned} \tag{4.8}$$

Since $\mathbf{P}\mathbf{P}^H = \mathbf{P}^H\mathbf{P} = \mathbf{I}$, we can simplify equation (4.8)

$$S(\phi,\tilde{\mathbf{d}}_{ML}) = \mathbf{x}^H\mathbf{P}\mathbf{P}^H(\mathbf{I}-\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H)\mathbf{P}\mathbf{P}^H\mathbf{x}$$

$$= \mathbf{x}^H\mathbf{P}(\mathbf{P}^H\mathbf{P}-\mathbf{P}^H\mathbf{P}\mathbf{W}\mathbf{W}^H\mathbf{P}^H\mathbf{P})\mathbf{P}^H\mathbf{x}$$

$$= (\mathbf{P}^H\mathbf{x})^H(\mathbf{I}-\mathbf{W}\mathbf{W}^H)\mathbf{P}^H\mathbf{x} \qquad (\therefore \mathbf{P}\mathbf{P}^H = \mathbf{P}^H\mathbf{P} = \mathbf{I})$$

$$= (\mathbf{P}^H\mathbf{x})^H\mathbf{V}\mathbf{V}^H\mathbf{P}^H\mathbf{x} \qquad (\therefore (\because \mathbf{W}\mathbf{W}^H + \mathbf{V}\mathbf{V}^H = \mathbf{I}))$$

$$= (\mathbf{x}^H\mathbf{P}\mathbf{V})(\mathbf{x}^H\mathbf{P}\mathbf{V})^H$$

Above equation can be written as

$$S(\phi,\tilde{\mathbf{d}}_{ML}) = \sum_{k=M}^{N-1}\left\|\mathbf{x}^H\mathbf{P}\mathbf{u}_k\right\|^2 \qquad (4.9)$$

Equation (4.9) gives the cost function to estimate the CFO for an OFDM system.

4.1 Reduced complexity CFO estimation using null subcarriers

The scheme uses only one training OFDM symbol with null subcarriers for CFO estimation and all odd subcarriers in the training OFDM symbol are imposed as null subcarriers. As a result, the training OFDM symbol consists of two identical components [14].

- The fractional part of the CFO (within the range of subcarriers spacing), which causes the loss of orthogonality among the subcarriers, to be estimated using simple correlation operation.
- The integer part of the CFO, which results in a shift of the subcarrier indexes, is estimated using subcarrier at even positions.

Consider an OFDM system with N subcarriers. In the technique, CFO estimation is achieved using one OFDM symbol, where there are N_z null subcarriers and N_p pilot tones with

$$N_z + N_p = N \qquad (4.10)$$

The set contains all the null subcarrier indexes is denoted by

$$\Gamma_z = \{a_1, a_2, \ldots\ldots, a_{N_z}\} \text{ with } a_1 < a_2 < \ldots\ldots\ldots < a_{N_z}$$

The set that contains all the pilot-tone indexes denoted by

$\Gamma_p = \{b_1, b_2, ..., b_{N_p}\}$ with $b_1 < b_2 < ...... < b_{N_p}$

The training OFDM symbol in the time domain is given by

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k \in \Gamma_p} d_k e^{J2\pi nk/N}, \qquad n = -Ng, ..., N-1 \tag{4.11}$$

d_k is the pilot symbol at the kth subcarrier.

After sampling and removing the guard interval, the received signal is then given by

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k \in \Gamma_p} d_k H(k) e^{j(2\pi k/N + 2\pi\phi)n} + z(n), \qquad n = 0, 1, ..., N-1$$

Vector Form

$\mathbf{P} = \text{diag}(1, e^{j2\pi\phi}, ..., e^{j2\pi(N-1)\phi})$,

$\mathbf{d} = [d_{b_1}, d_{b_2}, ..., d_{b_{N_p}}]^T$,

$\mathbf{Z} = [z(0), z(1), ..., z(N-1)]^T$,

$\mathbf{H} = \text{diag}(H(b_1), H(b_2), ..., H(b_{N_p}))$.

Now the received signal can be written into the following vector

$$\mathbf{x} = [x(0), x(1), ..., x(N-1)]^T$$

$$= \mathbf{PW\tilde{d}} + \mathbf{Z} \tag{4.12}$$

Where W is an $N \times N_p$ matrix with $[W]_{n,k} = \frac{1}{\sqrt{N}} e^{j2\pi(n-1)b_k/N}$

And $\mathbf{\tilde{d}} = [\tilde{d}_{b_1}, \tilde{d}_{b_2}, ..., \tilde{d}_{b_{N_p}}]^T = \mathbf{Hd}$.

Assume that the covariance matrix of z is σ^2I, where σ^2 is the noise variance and I denotes the identity matrix.

The likelihood function for ϕ and $\tilde{d}$ is then given by

$$L(\phi, \mathbf{\tilde{d}}) = \frac{1}{(\pi\sigma^2)^N} \exp\{-\frac{1}{\sigma^2} \times (\mathbf{x} - \mathbf{PW\tilde{d}})^H (\mathbf{x} - \mathbf{PW\tilde{d}})\} \tag{4.13}$$

To maximize the likelihood function, we are equivalently to minimize

$$S(\phi,\tilde{\mathbf{d}}) = (\mathbf{x}-\mathbf{PW\tilde{d}})^H(\mathbf{x}-\mathbf{PW\tilde{d}}) \tag{4.14}$$

Taking the gradient of $S(\phi,\tilde{\mathbf{d}})$ with respect to $\tilde{\mathbf{d}}$ and setting to zero, we get

$$\tilde{\mathbf{d}}_{ML} = \mathbf{W}^H\mathbf{P}^H\mathbf{x} \tag{4.15}$$

Substituting (4.15) in (4.14), we get estimation of ϕ as

$$\begin{aligned} S(\hat{\phi}) &= (\hat{\mathbf{P}}^H\mathbf{x})^H(\mathbf{I}-\mathbf{WW}^H)\hat{\mathbf{P}}^H\mathbf{x} \\ &= (\hat{\mathbf{P}}^H\mathbf{x})^H(\sum_{i\in\Gamma_z}\mathbf{v}_i\mathbf{v}_i^H)\hat{P}^H\mathbf{x} \\ &= \sum_{i\in\Gamma_z}\left|\mathbf{v}_i^H\,\hat{\mathbf{P}}^H\,\mathbf{x}\right|^2 \end{aligned} \tag{4.16}$$

Where $\hat{\mathbf{P}} = \text{diag}(\ 1, e^{j2\pi\hat{\phi}}, \ldots\ldots, e^{j2\pi(N-1)\hat{\phi}})$

And $\mathbf{V}_i$ is $N\times 1$ vector with the nth element given by $(\frac{1}{\sqrt{N}})e^{j2\pi(n-1)i/N}$.

Let all odd subcarriers are null subcarriers, and even subcarriers are pilot tones, we have,

$$\sum_{i\in\Gamma_2}\mathbf{v}_i\mathbf{v}_i^H = \frac{1}{2}\begin{bmatrix} I & -I \\ -I & I \end{bmatrix} \tag{4.17}$$

Where Γ_2={1,3,........, $N-1$} and N is assumed to be an even number.

For convenience define the X vector as

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \tag{4.18}$$

Where $\mathbf{x}_1$ and $\mathbf{x}_2$ are $(N/2)\times 1$ vectors denoting half and the second-half parts of the received training OFDM symbol, respectively.

Furthermore define $\hat{\mathbf{P}}$ as

$$\hat{\mathbf{P}} = \begin{bmatrix} \hat{\mathbf{P}}_{temp} & 0 \\ 0 & e^{j\pi N\hat{\phi}}\hat{\mathbf{P}}_{temp} \end{bmatrix} \tag{4.19}$$

Where $\hat{\mathbf{P}}_{temp} = \text{diag}(1, e^{j2\pi\hat{\phi}}, \ldots\ldots, e^{j2\pi(N-2)\hat{\phi}})$.

Substituting equation (4.17) and (4.19) in (4.16), we get

$$S_2\left(\hat{\phi}\right) = \left(\hat{\mathbf{P}}^H \mathbf{x}\right)^H \left(\sum_{i\in\Gamma_2} \mathbf{v}_i \mathbf{v}_i^H\right) \hat{\mathbf{P}}^H \mathbf{x}$$

$$= \frac{1}{2}\left(\mathbf{x}_1^H \mathbf{x}_1 + \mathbf{x}_2^H \mathbf{x}_2 - e^{j\pi N\hat{\phi}} \mathbf{x}_1^H \mathbf{x}_2 - e^{j\pi N\hat{\phi}} \mathbf{x}_2^H \mathbf{x}_1\right) \tag{4.20}$$

The closed form of solution can be achieved by minimizing equation (4.20) i.e., derivate equation (4.20) with respect to $\hat{\phi}$

$$\frac{dS_2(\hat{\phi})}{d\hat{\phi}} = j\frac{\pi N}{2} e^{-J\pi N\hat{\phi}} \mathbf{x}_1^H \mathbf{x}_2 - j\frac{\pi N}{2} e^{J\pi N\hat{\phi}} \mathbf{x}_2^H \mathbf{x}_1$$

By setting $\frac{dS_2(\hat{\phi})}{d\hat{\phi}} = 0$, $\hat{\phi}$ is then obtained as

$$\hat{\phi} = \frac{1}{\pi N} \arg\left(\mathbf{x}_1^H \mathbf{x}_2\right) + \frac{k}{N}, \quad k = 0, \pm 1, \pm 2, \ldots\ldots \tag{4.21}$$

When k is an odd number, equation (4.21) is maximized rather than minimized. Hence, when $\hat{\phi}$ is limited in the range of [0,1), the closed-form solution to minimizing equation (4.21) is given by

$$\hat{\phi}_k = \frac{1}{\pi N} \arg\left(\mathbf{x}_1^H \mathbf{x}_2\right) + \frac{2k}{N}, \quad k = 0, \ldots, \frac{N}{2} \tag{4.22}$$

The CFO estimation is very simple with all odd subcarriers are zeros. However, we must identify k to extend the estimation range. This can be achieved by using a reduced-complexity ML CFO estimation scheme, where no extra symbol is required, and k is identified using the even null subcarriers in the training OFDM symbols.

In the scheme all odd subcarriers and some of the even subcarriers are null subcarriers, we have

$$\sum_{i\in\Gamma_2} \mathbf{v}_i\mathbf{v}_i^H = \sum_{i\in\Gamma_2} \mathbf{v}_i\mathbf{v}_i^H + \sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2}} \mathbf{v}_i\mathbf{v}_i^H \tag{4.23}$$

Substituting equation (4.23) in (4.16), we get

$$S_2\left(\hat{\phi}\right) = \left(\hat{\mathbf{P}}^H\mathbf{x}\right)^H \left(\sum_{i\in\Gamma_2} \mathbf{v}_i\mathbf{v}_i^H\right)\hat{\mathbf{P}}^H x + \left(\hat{\mathbf{P}}^H\mathbf{x}\right)^H \left(\sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2}} \mathbf{v}_i\mathbf{v}_i^H\right)\hat{\mathbf{P}}^H\mathbf{x}. \tag{4.24}$$

When there is no noise, and given that the tentative normalized CFO is the actual normalized CFO, the two terms in (4.24) are simultaneously equal to zero (thus, reaching the minima). We can minimize the two terms separately and the CFO estimation task is divided into two steps.

- The fractional normalized CFO is identified by minimizing the first term of (4.24).
- The integer normalized CFO is obtained by minimizing the second term of (4.24).

As a result, in the first step, the task is equivalent to minimizing the metric given by (4.20). Hence, the solution to the fractional normalized CFO $\hat{\phi}_0$ is given by (4.22).

For convenience, define

$$\hat{\mathbf{P}}_k \equiv diag\left(1, e^{j2\pi\hat{\phi}_k}, \ldots, e^{j2\pi(N-1)\hat{\phi}_k}\right)$$

$$= \sqrt{N} diag(v_{2k}\hat{\mathbf{P}}_0) \tag{4.25}$$

Where $\hat{\mathbf{P}}_0 \equiv diag\left(1, e^{j2\pi\hat{\phi}_0}, ..., e^{j2\pi(N-1)\hat{\phi}_0}\right)$

By replacing $\hat{\mathbf{P}}$ in the second term of (4.24) with $\hat{\mathbf{P}}_k$, we have

$$E(k) = \left(\hat{\mathbf{P}}_k^H \mathbf{x}\right)^H \left(\sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2}} \mathbf{v}_i \mathbf{v}_i^H \right) \hat{\mathbf{P}}_k^H \mathbf{x}$$

$$= \sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2}} \left| \mathbf{v}_{i+2k}^H \hat{\mathbf{P}}_k^H \mathbf{x} \right|^2 \quad (4.26)$$

For convenience, we define E_i as follows

$$E_i = \left| \mathbf{v}_i^H \hat{\mathbf{P}}_0^H \mathbf{x} \right|^2 \quad (4.27)$$

In the second step, the integer normalized CFO is found using the following criterion

$$\hat{k} = \arg_k \min \sum_{\substack{i\in\Gamma_2 \\ i\notin\Gamma_2}} E_{i+2k} \quad (4.28)$$

The fractional normalized CFO is first compensated, and E_i is then obtained using an FFT. Since both the fractional CFO compensation and FFT processing are available in the OFDM system, it follows that only extra adders are required in the second step to estimate the integer normalized CFO, which are trivial compared with multipliers and FFT. Consequently, the complexity of this scheme is very less and the estimation range is much larger. In this method the fractional part and integer part of normalized CFO are obtained separately, so this method is suboptimal. Furthermore, since only one training OFDM symbol is required, the transmission efficiency is high.

4.2 Even Null Subcarrier Allocation

For the reduced-complexity CFO estimation scheme even null subcarrier allocations at the even positions are critical to system performance. In the following, we propose a way to allocate them.

Assume that the actual normalized CFO is $\phi = \phi_0 + \frac{2\bar{k}}{N}$, where ϕ_0 is the fractional part of the normalized CFO with $-\frac{1}{N} \le \phi_0 < \frac{1}{N}$, and $\bar{k}$ is an integer with $0 \le \bar{k} \le \frac{N}{2}$. Suppose that ϕ_0 is perfectly obtained in the first of CFO estimation. Then, $\hat{\phi} = \phi_0$ and

$$\mathbf{v}_i{}^H \hat{\mathbf{P}}_0^H \mathbf{x} = \mathbf{v}_i{}^H \hat{\mathbf{P}}_0^H \mathbf{P}\mathbf{W}\tilde{\mathbf{d}} + \mathbf{v}_i{}^H \hat{\mathbf{P}}_0^H \mathbf{z}$$

$$= \mathbf{v}_{i-2\bar{k}}^H \mathbf{W}\tilde{\mathbf{d}} + n_i$$

$$= \begin{cases} \tilde{d}_{i-2\bar{k}} + n_i, & i-2\bar{k} \in \Gamma_p \\ n_i, & i-2\bar{k} \in \Gamma_z \end{cases} \tag{4.29}$$

Where $n_i = \mathbf{v}_i{}^H \hat{\mathbf{P}}_0^H \mathbf{z}$ is the additive white Gaussian noise term with zero mean and variance σ^2. As a result, (4.27) can be written as follows

$$E_i = \begin{cases} \left|\tilde{d}_{i-2\bar{k}}\right|^2 + |n_i|^2 + \tilde{d}^*_{i-2\bar{k}} n_i + \tilde{d}_{i-2\bar{k}} n^*_i, & i-2\bar{k} \in \Gamma_p \\ |n_i|^2, & i-2\bar{k} \in \Gamma_z \end{cases} \tag{4.30}$$

Where $k = \bar{k}$, we have

$$\sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2'}} E_{i+2k} = \sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2'}} E_{i+2\bar{k}} = \sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2'}} \left|n_{i+2\bar{k}}\right|^2 \tag{4.31}$$

When $k \ne \bar{k}$, we have

$$\sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2'}} E_{i+2k} = \sum_{\substack{i+2k-2\bar{k}\in\Gamma_z \\ i\in\Gamma_z \\ i\notin\Gamma_2'}} \left|n_{i+2k}\right|^2 + \sum_{\substack{i+2k-2\bar{k}\in\Gamma_p \\ i\in\Gamma_z \\ i\notin\Gamma_2'}} \left(\left|\tilde{d}_{i+2k-2\bar{k}}\right|^2 + \left|n_{i+2k}\right|^2 + \tilde{d}^*_{i+2k-2\bar{k}} n_{i+2k} + \tilde{d}_{i+2k-2\bar{k}} n^*_{i+2k}\right)$$

$$= \sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2}} |n_{i+2k}|^2 + \sum_{\substack{i+2k-2\bar{k}\in\Gamma_p \\ i\in\Gamma_z \\ i\notin\Gamma_2}} (|\tilde{d}_{i+2k-2\bar{k}}|^2 + \tilde{d}^*_{i+2k-2\bar{k}} n_{i+2k} + \tilde{d}_{i+2k-2\bar{k}} n^*_{i+2k}) \tag{4.32}$$

For the integer part of the normalized CFO estimation, if $k \neq \bar{k}$ is selected and from (4.28), (4.31) must be smaller than (4.32). Hence, we have

$$\sum_{\substack{i+2k-2\bar{k}\in\Gamma_p \\ i\in\Gamma_z \\ i\notin\Gamma_2}} (|\tilde{d}_{i+2k-2\bar{k}}|^2 < - \sum_{\substack{i+2k-2\bar{k}\in\Gamma_p \\ i\in\Gamma_z \\ i\notin\Gamma_2}} (\tilde{d}^*_{i+2k-2\bar{k}} n_{i+2k} + \tilde{d}_{i+2k-2\bar{k}} n^*_{i+2k}) + \sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2}} (|n_{i+2\bar{k}}|^2 - |n_{i+2k}|^2)$$

(4.33)

When the SNR is high, $\sum_{\substack{i\in\Gamma_z \\ i\notin\Gamma_2}} (|n_{i+2\bar{k}}|^2 - |n_{i+2k}|^2)$ is negligible and (4.33) is reduced to the following

$$\eta(k,\bar{k}) < - \sum_{\substack{i+2k-2\bar{k}\in\Gamma_p \\ i\in\Gamma_z \\ i\notin\Gamma_2}} (\tilde{d}^*_{i+2k-2\bar{k}} n_{i+2k} + \tilde{d}_{i+2k-2\bar{k}} n^*_{i+2k}) \tag{4.34}$$

Where

$$\eta(k,\bar{k}) = \sum_{\substack{i+2k-2\bar{k}\in\Gamma_p \\ i\in\Gamma_z \\ i\notin\Gamma_2}} |\tilde{d}^*_{i+2k-2\bar{k}}|^2 \tag{4.35}$$

.

The right part of (4.34) is a random variable with zero mean complex Gaussian distribution. Assume that the imaginary part and real part of the noise term are independent and identically distributed. The variance of the right part of (4.34) is then as follows:

$$\sigma_{all}{}^2(k,\bar{k}) = 2\sigma^2 \sum_{\substack{i+2k-2\bar{k}\in\Gamma_p \\ i\in\Gamma_z \\ i\notin\Gamma_2}} |\tilde{d}^*_{i-2k-2\bar{k}}|^2 = 2\sigma^2\eta(k,\bar{k}). \tag{4.36}$$

From (4.34), the pair-wise error probability (k rather than $\bar{k}$ is selected) is as follows

$$p_e(\bar{k} \to k) = Q\left(\frac{\eta(k,\bar{k})}{\sigma_{all}^2(k,\bar{k})}\right) \le e^{-\frac{(\eta(k,\bar{k}))^2}{2\sigma_{all}^2(k,\bar{k})}} = e^{-\frac{\eta(k,\bar{k})}{4\sigma^2}} \qquad (4.37)$$

Where $Q(x) = \frac{1}{2\pi}\int_0^\infty e^{-\frac{t^2}{2}}dt$.

In the following, we try to find the best even null sub-carrier allocation criterion according to the following rule ,

$$\min \max_{k \ne \bar{k}} p_e(\bar{k} \to k). \qquad (4.38)$$

From (4.38), it can be seen that the above rule is equivalent to the following

$$\max \min_{k \ne \bar{k}} \eta(\bar{k} \to k). \qquad (4.39)$$

Note that $\eta(\bar{k},k)$ does not depend on the specific value of k and $\bar{k}$. Instead, it depends on the difference $k-\bar{k}$. For convenience, we define

$\eta(k) = \eta(k,0)$.

Equation (4.37) indicates that the performance of the integer part of the normalized CFO estimation is mainly determined by small values of $\eta(k)$ among all possible $k \ne 0$. From (4.35), it can also be seen that $\eta(k)$ relates to the number of summation terms and product of the channel frequency response and the training data. Since channel is not known, all we can control is the number of summation terms. As a result, we should try to make the number of summation terms in equation (4.35) as large as possible. To achieve this task, we can use a binary sequence $\mathbf{c} = [c_0, c_1, \ldots\ldots, c_{K-1}]^T$ to represent the null-subcarrier allocations at the even positions. Where K is the length of the sequence. In the binary sequence, '1' is used to denote a position where there is a pilot tone and '0' for null subcarrier. To make small $\eta(k)$ as large as possible, the binary sequence should be designed according to the following rules [14]:

- The set of Hamming distances between the binary sequence and all possible cyclic shift versions constitutes the distance distribution of sequence. We should find a sequence with good distance distribution in the sense that the cyclic-shift versions with a small distance to the original binary one as few as possible.
- The zeros in the binary sequence should be as far apart as possible to ensure frequency-domain diversity.

We define the autocorrelation function of the binary sequence $\mathbf{c}$ as

$$R_a(\tau) = \sum_{k=0}^{K-1} (-1)^{c_k + c_{\mathrm{mod}(k+\tau,K)}} \tag{4.40}$$

Where $0 \le \tau < K-1$ is an integer number and $\mathrm{mod}(i, j)$ is the remainder of i divided by j.

When $\tau \ne 0$, $R_a(\tau)$ is referred to as the out-of-face autocorrelation function. The Hamming distance between the sequence and its τ cyclic shift versions is as follows:

$$d(\tau) = \frac{K - R_a(\tau)}{2} \tag{4.41}$$

As a result for rule 1, the task is to find a binary sequence with the out-of-phase autocorrelation function as small as possible. For rule 2, the positions of zeros in the sequence with a small out-of-phase autocorrelation function are normally very random, which can be used to take advantage of frequency domain diversity. Hence, in the following, we only consider using rule 1 to find a proper sequence, which determines the way of null subcarrier allocations.

For convenience, we define the minimum Hamming distance of a binary sequence by the minimum Hamming distance between the sequence and all its cyclic shift versions. When K is an odd number, the maximal minimum Hamming distance among all the binary sequences with length K is less than or equal to $(K+1)/2$. When K is an even number, the maximal minimum Hamming distance among all the binary sequences with length K is less than or equal to $K/2$ with the exception of $K=2$. For example, the m-sequence [27] is optimal in the sense that it can achieve the maximal minimum Hamming distance. However, such sequence only exists for $K = 2^m - 1$, where m is a natural number. For practical OFDM systems with $K = 2^m$, there are no such optimal sequences in general. To deal with this issue, we append a zero at the end of a specific m-sequence. Since there are $2^m - 1$ cyclic shift versions of an m-sequence, we can generate $2^m - 1$ such sequences with length 2^m. Among the $2^m - 1$ sequences, we select the best one in the sense of distribution of the Hamming distance, which is referred as the extended m-sequence.

For $m = 5$ ($K = 32$), by computer search, we get the following extended m-sequence (in hexadecimal)

D215D8F8.

The minuimum Hamming distance of the above sequence is 14, which is close to $K/2 = 16$.

When $m = 6$, the extended m-sequence is given by

A4E2F28C20FD59BA.

The minuimum Hamming distance of the above sequence is 30, which is close to $K/2 = 32$.

Beside the proposed extended m-sequence, the almost autocorrelation sequence [28] proposed in can also be used.

For $K = 64$, the almost-perfect autocorrelation sequence is given by

0C6A01B2F3957E4D.

The minimum Hamming distance of above sequence is 32.

In the above discussion, we assume that the actual CFO value is in the range of the inverse of the sampling duration. As a result, the length of the binary sequence is required to be $N/2$. When the oscillators at the transmitter and the receiver both have relatively high precision, the actual CFO may in the limited range. In this case, we only need to find a shorter sequence to achieve the CFO estimation task.

Conventionally, we can use one training OFDM symbol with M identical parts to estimate a CFO within the range of M subcarriers. For this case and using optimal ML CFO estimation, it is shown in [29] that the Cramer-Rao bound is given by

$$\mathrm{var}(\phi - \hat{\phi}) = \frac{3(SNR)^{-1}}{2\pi^2 N^3 (1 - 1/M^2)} \tag{4.42}$$

Where SNR is defined by

$$SNR = \frac{\tilde{\mathbf{d}}^H \tilde{\mathbf{d}}}{N\sigma^2}. \tag{4.43}$$

In this case, when M is even, the null subcarriers in even positions can also be represented by a binary sequence.

For example, for $N = 64$ and $M = 32$, the binary sequence ($K = 32$) is given by

80008000.

By using the reduced complexity CFO estimation scheme, we can also divide the sub-carriers in the training OFDM symbol into N/M identical parts (in the sense of null subcarrier allocations). For each part, we allocate the even null sub-carriers based upon the same extended m-sequence or almost perfect autocorrelation sequence.

For example, for $N = 64$ and $M = 32$, there are two identical parts, and for each part, we can use the following almost perfect autocorrelation sequence ($K = 16$)

20D7.

Then, the overall even null subcarrier allocations for training OFDM symbol are as follows:

20D720D7.

4.3 Simulation results

In this section, we simulate an OFDM system with 64 subcarriers. In the simulations, the performance measure is the normalized mean-square error (NMSE), which is defined by

$$\mathbf{NMSE} = \frac{N}{N_t}\sum_{t=1}^{N_t}(\hat{\phi}_t - \phi)^2 \tag{4.44}$$

Where N_t is the number of Monte Carlo trials,

ϕ is the actual normalized CFO,

And $\hat{\phi}_t$ is the estimated normalized CFO at the tth trial.

The CIR is given by

$$h(k) = \sum_{i=0}^{5} A_i g_T(kT_s - \tau_i - t_0) \tag{4.45}$$

Where $\{A_i\}$ and $\{\tau_i\}$ are attenuation and delays of the paths,

t_0 is a timing phase which is chosen equal to $3T_s$ and

$g_T(t)$ is the impulse response of the raised-cosine rolloff filter with a rolloff factor of 0.5 is given by

$$g_T(t) = \sin c(\frac{t}{T})\frac{\cos(\frac{\Pi\beta t}{T})}{1-\frac{4\beta^2 t^2}{T^2}} \tag{4.46}$$

The normalized delays $\{\tau_i / T\}$ are chosen equal to $\{0,\ 0.054,\ 0.135,\ 0.621,\ 1.135\}$, and $\{A_i\}$ are independent and Gaussian random variable with zero mean and variances (in decibels) $\{-3, 0, -2, -6, -8, -10\}$.

The SNR is defined as the received instantaneous signal power divided by the noise variance. However, the use of null subcarriers means that some sub channels may not be excited by the transmitted signal. Therefore, when the transmitted signal power is fixed, low instantaneous received signal power may not mean a poor instantaneous channel realization. Using the SNR definition given by (4.43), we cannot know whether a poor received signal power is induced by poor channel conditions or by poor subcarrier allocation. To make the SNR independent of the null subcarrier allocation, we assume that the transmitted signal power is fixed. That is

$$|d_{b_i}|^2 = \frac{N}{N_p} \qquad i = 1, 2, ..., N_p \tag{4.47}$$

In the simulations, the SNR is defined as

$$\mathbf{SNR} = \frac{\sum_{k=0}^{N-1} |H(k)|^2}{N\sigma^2} \tag{4.48}$$

We take the Cramer-Rao bound given by equation (4.42) as the baseline. The result of equation (4.42) with M=2 is denoted by Cramer Rao bound 1, which is the best performance achieved by the optmail ML CFO estimator using one training OFDM symbol with two identical parts. The result of equation (4.42) with M=N is denoted by Cramer Rao bound 2, which is the best performance that can be achieved by an optimal ML CFO estimator using one training OFDM symbol with any number of identical parts.

Figure (4.1) shows the NMSE performance of the reduced complexity CFO estimator using periodic training OFDM symbol. The normalized CFOs in the simulations are 0, 0.01, 0.1, 0.2 and 0.4 respectively. From figure (4.1), it can be seen that when M=N=64 (one pilot tone is used), the performance is poor, due to the fact that pilot tone may be in deep fading. When more pilot tones are used, frequency-domain diversity can be achieved, resulting in good performance. When M=N/2=32 (two pilot tones are used), it can be seen that the performance is quite good at high SNR values (figure (4.2)). However there is performance degradation at low SNR values. When M=N/4=16 (four pilot tones are used), the performance approaches the CRB1 for almost all SNR values (figure (4.3)).

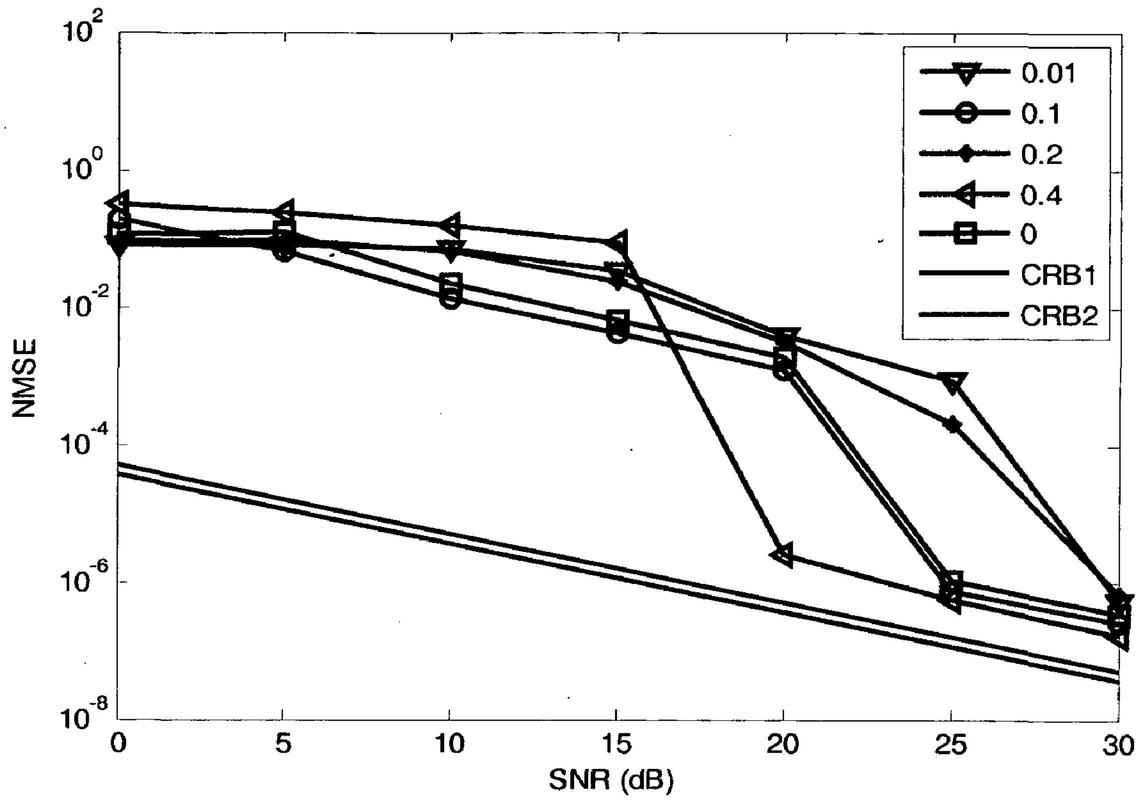


Figure 4.1 NMSE performance of the reduced-complexity CFO estimation using one training OFDM symbol with identical components (M=64).

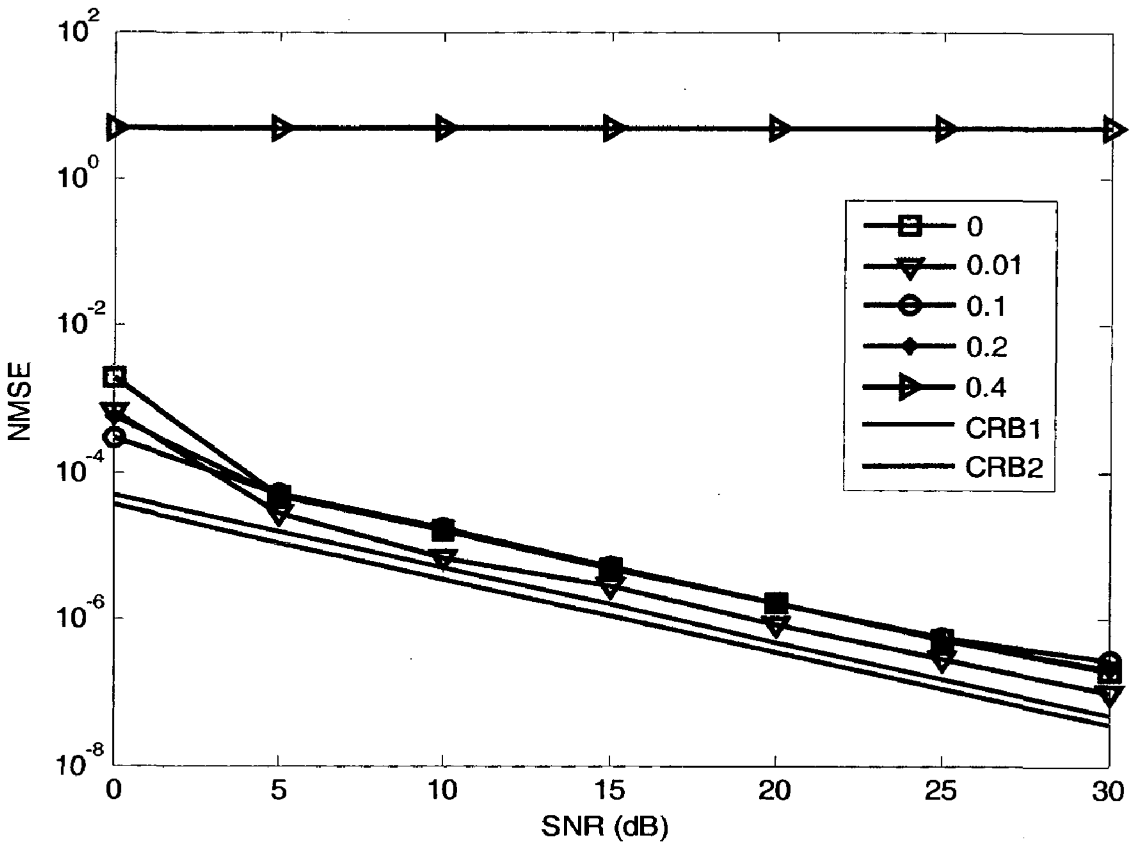


Figure 4.2 NMSE performance of the reduced-complexity CFO estimation using one training OFDM symbol with identical components (M=32).

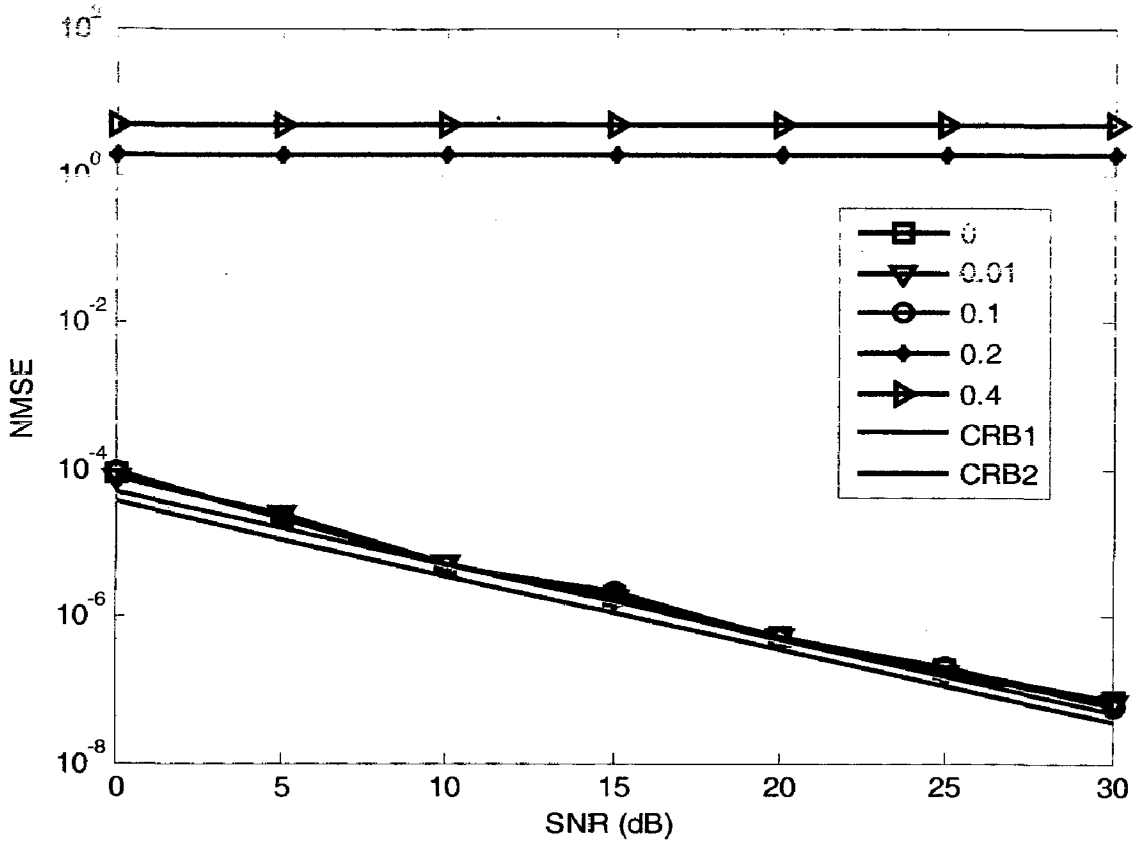


Figure 4.3 NMSE performance of the reduced-complexity CFO estimation using one training OFDM symbol with identical components (M=16).

Chapter 5
CONCLUSIONS

The basic and most essential task that is to be performed in any digital communication system is synchronization, without which a reliable reception of transmitted data is quite impossible. Synchronization for any digital communication system can be viewed in two parts: Carrier frequency synchronization and Symbol timing synchronization. This dissertation work is aimed at performance study of Carrier Frequency Offset (CFO) estimation methods for OFDM systems using training sequences, numerical method and virtual subcarriers. The conclusions that are drawn from the previous discussions and simulation results are as follows

- We have started with the introduction of OFDM system and need for estimation of carrier frequency offset in OFDM. The CFO causes serious problem in OFDM which destroys the orthogonality among subcarriers, thus resulting in inter carrier interference (ICI). A very small amount of frequency offset can lead to significant degradation in system performance. The CFO estimation is therefore a crucial point in the design of an OFDM system.
- An algorithm for maximum likelihood estimate (MLE) of frequency offset using the DFT values of a repeated data symbol has been presented. Both the signal values and the ICI contribute coherently to the estimate the CFO therefore it is possible to obtain the accurate estimates even when the offset is too large.
- We have investigated a numerical technique for blind ML estimation of CFO in OFDM systems based on Newton - Raphson method. The scheme preserves low complexity and fast convergence while maintaining the estimation accuracy.
- We have also looked at a reduced-complexity CFO estimator for OFDM through exploitation of null subcarriers in one training OFDM symbol. By imposing all odd subcarriers as null subcarriers, the fractional normalized CFO can be estimated by simple correlation operations. The integer normalized CFO is achieved through the exploitation of even null subcarriers, which are allocated based on some specific sequences, such as the extended m-sequence and almost-perfect auto-correlation sequence.

Scope of Future Work

- In chapter 4, the estimation of integer CFO and fractional CFO are performed independently. If it is possible to combine the two processes together, it will simplify the system complexity and improve system efficiency.
- CFO estimation using numerical techniques are also a topic of significant interest.

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