SUSPENSION SYSTEM'S VIBRATION CONTROL: A PARAMETRIC UNCERTAINTIES PROBLEM IN CONTROL ENGINEERING

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

ELECTRONICS AND COMPUTER ENGINEERING
(With Specialization in Control and Guidance)

By

GULSHAN KUMAR DUBEY

M.G. CENTRAL LIBRARY
ACC No. G 20243
Date 8/11/10
I.I.T. ROORKEE

DEPARTMENT OF ELECTRONICS AND COMPUTER ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE-247 667 (INDIA)

JUNE, 2010

CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report, titled " **Suspension System's Vibration Control: A Parametric Uncertainties Problem in Control Engineering"**, being submitted in partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Control and Guidance**, in the Department of Electronics and Computer Engineering, Indian Institute of Technology, Roorkee is an authentic record of my own work carried out from July 2009 to June 2010, under guidance and supervision of **Dr. R. MITRA**, Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology, Roorkee.

The results embodied in this dissertation have not submitted for the award of any other Degree or Diploma.

Date : 29 Julye 2010 (29/06/10)

Place: Roorkee

GULSHAN KUMAR DUBEY

CERTIFICATE

This is to certify that the statement made by the candidate is correct to the best of my knowledge and belief.

(Dr. R. MITRA)

Professor, E&C Department,

Indian Institute of Technology, Roorkee

Roorkee – 247 667, (INDIA)

ACKNOWLEDGEMENT

With great sense of pleasure and privilege, I take this opportunity to express my deepest sense of gratitude towards Dr. Ranajit Mitra for his valuable suggestions, sagacious guidance, scholarly advice and insightful comments and constructive suggestions to improve the quality of the present work. His professionalism, accurate advice, suggestions and his ways of thinking inspired me, and this inspiration guides me in every moment of my life.

I would like to thank all my Professors for their help to clear my fundamentals in different subjects.

I would like to thank all my friends specially Vivek Singh and Vaibhav Gupta for their help and motivation for work. I would like to thank Dereje Shiferaw, Research Scholar, IITR and Mukesh D Patil, Research Scholar, IITB for their help for providing me study materials.

I would also like to thank the Lab staff of Control Sytems Lab for their continuous support and well maintained Lab.

Here, I would like to thank persons, whose presence gives me courage and motivation to keep working. Finally, I would like to extend my gratitude to all those persons who directly or indirectly contributed towards this work.

Last but not the least, I am highly indebted to my parents and family members, whose sincere prayers, best wishes, moral support and encouragement have a constant source of assurance, guidance, strength, and inspiration to me.

GULSHAN KUMAR DUBEY

ABSTRACT

In this dissertation work, QFT and H-infinity controllers were designed for a vehicle suspension system and their performance is compared and also with a standard PID controller. A suspension system works in uncertainties, which implies variation in plant parameters. Hence in designing controller for this system, sensitivity, which can be defined as variation in response affected due to change in parameters, and robustness analysis are primary issues. The performance objective for the controller design is to keep the error between the controlled output and the set-point as small as possible and sensitivity function reshaping is the main design tool utilized.

In the Quantitative Feedback Theory (QFT) controller, which is a classical approach to design a controller suited for systems having large uncertainties by using feedback of measurable plant outputs to generate an acceptable response from a system with disturbance signals and plant uncertainties, measured displacement of the suspension system is used as a feedback. The QFT controller is based on reshaping of the loop-transmission function (product of plant and controller transfer function) on which sensitivity of the system depends. The QFT controller is implemented in MATLAB.

The H-infinity (H_∞) controller, which is also a frequency response approach used to design a robust controller that can reshape the sensitivity function directly to achieve the desire performance objectives, is done by selection of suitable weight function. The controller is also implemented in MATLAB and it realizes the sensitivity reshaping.

The QFT and H-infinity controllers outperform the trial and error method of PID. Simulation results have been included for all.

CONTENTS

Candidate's Declaration and Certificate i

Acknowledgement ii

Abstract iii

1. **Introduction** 1
   - 1.1.History 2
   - 1.2.Problem Statement 3
   - 1.3.Organization of the dissertation work 4
2. **Suspension System**
   - 2.1.Introduction 5
   - 2.2.Quarter Car Suspension System 5
   - 2.3.Part Description 8
   - 2.4.Advanced Development in Suspension System Industry 9
3. **Analysis of Suspension System**
   - 3.1.Introduction 16
   - 3.2.Dynamics of Quarter Car Suspension System 16
   - 3.3.Time Response 20
   - 3.4.Frequency Response 22
   - 3.5.Observations 23
   - 3.6.Problems 23
4. **Control Strategies**
   - 4.1.Introduction 24
   - 4.2.Control System Design Basics 24
   - 4.3.PID Controller 25
   - 4.4.Quantitative Feedback Theory 29
   - 4.5.H-infinity 34
5. **Simulation Results**
   - 5.1.Introduction 40
   - 5.2.Passive System Simulation 40
     - 5.2.1. Step Response 41

5.2.2. Frequency Response 43
5.2.3. Effect of Parametric Variation 44
5.2.4. Observations 48
5.2.5. Problems 48
5.3.PID Controller Simulation Results 49
5.3.1. Step Response 50
5.3.2. Effect of Parametric Variation 51
5.3.3. Observations 52
5.3.4. Problems 52
5.4.QFT Controller Simulation Results 53
5.4.1. QFT Design Technique 53
5.4.2. Step Response 56
5.4.3. Effect of Parametric Variation 57
5.4.4. Observations 59
5.4.5. Problems 59
5.5.Comparison between PID and QFT techniques 60
5.5.1. Conclusion of Comparative study 62
5.6.H-infinity Controller Simulation Results 63
5.6.1. H-infinity Controller 63
5.6.2. Step Response 64
5.6.3. Effect of Parametric Variation 65
5.6.4. Observations 66
5.6.5. Problems 68
5.7.Comparison between QFT and H-infinity techniques 69
5.7.1. Conclusion of Comparative study 71
6. Conclusion and Future work
6.1.Conclusion 72
6.2.Future Scope 72
References **73**

Vibration Control is one of the major problems in control engineering. Vibrations can occur in modern control application such as automobile, air-planes, robotics, magnetic levitation and many more. Vibration can cause damage to parts and components used in automobiles, planes, machines etc.

Performance of machines greatly suffer due to vibrations specially vibrations in automobiles is major cause of component damage and discomfort to driver discomfort and fatigue [1]. Suspension system plays a beneficial role in such cases. It is responsible for drive comfort and drive safety.

Suspension systems are widely used to overcome the problem of vibration in automobiles. Control of Suspension system is a mature and fruitful area of research, development and manufacturing. The reduction in vehicle body displacement and time taken to return in its ideal position are the two main characteristics of Vibration control. The Idea of active suspension system is generated to achieve desire performance of the suspension system [2]. The design of active suspension systems requires a clear concept, related not only to the mechanics of the system but also to automatic control system.

The dynamic equations of motion for a suspension system are non-linear. The system operates in unstructured environment and is always subjected to external disturbance from road unevenness. PID [8] controller is the most basic controller and widely used in industries. The PID controllers have some short comings and short comings are overcome by using QFT and H-infinity controllers.

Parameters variation is the biggest problem in the control of the system. Quantitative Feedback Theory (QFT) is widely used to design robust controllers for the systems having parameters variation [10]. QFT is having some advantages over PID. Loop-shaping of controller is a problem associated with QFT method. H-infinity is also widely used to design a robust controller [17]. H-infinity control method is compared with QFT.

Here we concentrate on general studies of the theory of vibration isolation including active control forces and their robustness with the variation in the system parameters.

1.1. History

Lots of research and development in the field of control of suspension system and its performance is done in last few years. The basic reason behind this research is highly developed sensors and microcontrollers at low cost are available. However, the automotive engineers persevered to improve both ride and handling.

Early motors cars used solid axle beams suspended from chassis rails with springs. These derived from horse-drawn carriage designs and were a function of what could be manufactured and maintain by black-smiths. One characteristic, their inter-leaf friction, was both a blessing and a burden: it helped the low-frequency behavior by providing a crude form of damping, but it created an uncomfortable response to sharp road inputs [2].

By 1960 random vibration had become a research topic primarily to problems associated with rough burning of rocket engines and fatigue of aircrafts parts due to turbulence excited vibrations. At about this time, optimal control techniques were being developed and applied and there began to be a clear convergence of ideas about techniques to optimize the dynamics of control systems and mathematical vibration systems.

During the decade of the 1960s the ideas of active control for vibration isolation began to be widely proposed but the widespread practical implementation of these concepts has only later become practical.

In 1980's, there was a considerable time lag between the early active suspension systems using no electronics and the surge of prototype and limited production systems which began to appear.

It was apparent by the same time that electronic sensors and computers had reached a state such that sophisticated suspension systems were at least possible but the question of which type of actuator to use was difficult [3].

Automotive suspensions deal with large forces, velocities, and deflections and there are questions about how to generate forces efficiently, reliably, and at acceptable financial and energy costs. By the 1990's there were commercially available automotive active suspensions. Such systems had some clear advantages over passive suspensions but also some disadvantages.

In the late 90's and early 2000, the optimization theories were well developed and the application of these techniques was used to optimize the suspension system. It was almost universally assumed in the vibration community that only passive devices such as spring, dampers and extra masses would be used to design isolation systems while control engineers almost always thought of active actuating devices responding to sensed variables for their systems [4].

Suspension design engineers optimize the values of mass, spring, and damper values to achieve the isolation in vibration while control engineers work for the control techniques for the actuating devices to achieve the isolation in vibration.

There are many attempts in recent years to sort out the essential and realizable functions which active suspensions with actuating devices can be expected to perform and there are a number of proposed versions of active suspensions which use different hardware configurations to reduce power, increase reliability and reduce cost. So it is not possible here to review the many hundreds of contribution to active vibration isolation over the past 50 years.

1.2. Problem Statement

Suspension systems work in unstructured environment and are subjected to external disturbance (i.e. road disturbance). The prime objective of this dissertation work will focus on the development simple and optimal control technique, which can handle the variations in the parameters very effectively.

The objectives of this work can be summarized as follows:

1. To analysis of the suspension system dynamics and study of effect of parameter uncertainty.
2. To design an optimal controller for the suspension system to achieve the desired performance.
3. To implementation the developed controller over the suspension system and evaluate of its robustness.
4. To provide comparisons of the developed controller to obtain the optimal control.

1.3. Organization of the dissertation work

The report has been organized into 6 chapters. In this chapter, an introduction, historical perspective of suspension system, problem statement is presented. Chapter 2 contains the introduction about the suspension system, types, and its parts. Also, it includes the advanced development in the field of suspension systems. Chapter 3 contains passive suspension system dynamics and effect of the variation in its parameters. Chapter 4 consists of different control strategies for controlling the suspension system. These control strategies are PID, QFT and H-infinity. Chapter 5 presents the MATLAB simulation results for the passive system and for the active system with different control strategies. Chapter 6 presents the conclusion of the study and suggestions are given for further study of this subject.

2.1. Introduction

This chapter includes the description of the Suspension system and its types, its necessity in the automobile or vehicle. This chapter contains the description of its important parts such as spring, damper and sensors, and also includes advanced development in the field of suspension systems.

2.2. Quarter Car Suspension System

2.2.1 Structure

The single mass-spring-damper model described is single degree of freedom because we assumed that the mass moves up and down. But if we are talking about the suspension system which also consists of mass-spring and damper then we say that the suspension system is more complex. The system involves more masses, springs and dampers. The number of springs and dampers depend upon their arrangement in the system.

Here, we consider system with two masses-springs and one damper. This suspension system is with two degrees of freedom (2DOF) as two masses are involved and their movement can be in more than one direction.

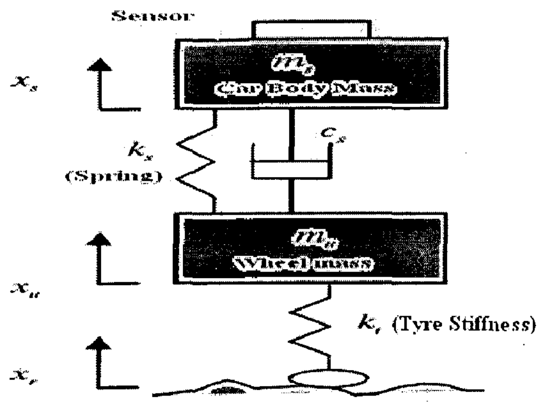

Fig.2.1. A Suspension System Model

The two different masses called as the sprung mass and the un-sprung mass. The sprung mass represents the mass of the car body and the un-sprung mass represents the mass of the vehicle wheel assembly.

A spring and a damper placed in between the car body and the wheel represent a passive spring and a shock absorber. The second spring represents stiffness or compressibility of the pneumatic tyre.

2.2.2. Necessity of the system

The vehicle suspension system is responsible for driving comfort and safety as the suspension carries the vehicle body and transmits all force between the body and the road. The Driving Comfort and Safety can be explained here [1]:

Driving Comfort- Driving comfort results from keeping the physiological stress that the vehicle occupants are subjected to by vibrations and noise down to as low as possible.

The displacement/velocity/acceleration of the body is an obvious quantity for the motion and vibration of the car body and can be used for determining a quantitative value for driving comfort.

Driving Safety- Driving safety is the result of a harmonious suspension design in terms of wheel suspension, springing, steering, and braking, and is reflected in an optimal dynamic behavior of the vehicle.

Tyre load variation is an indicator for the road contact and can be used for determining a quantitative value for safety.

So the main tasks of the suspension system are:

1. To maximize the friction between the tyre and the road surface,
2. To provide steering stability and safety with good handling,
3. To ensure the comfort of the passengers.

2.2.3. Suspension's Types

1. Passive Suspension System

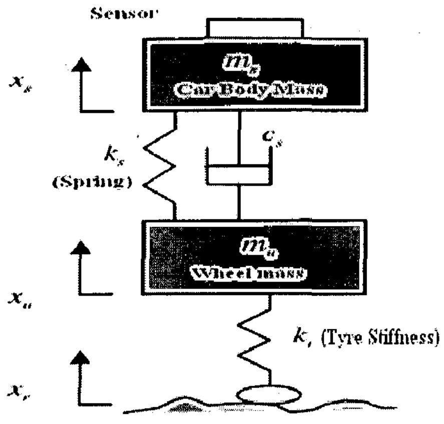

Fig.2.2. A Passive Suspension System Model

The passive suspension system is shown in Fig. 2.2. The performances of the passive systems are highly system dependent as they are unable to adapt or re-tune to changing disturbances or structural characteristics over time. The fixed setting of a passive suspension system is always a compromise between comfort and safety for any given input set of road conditions and a specific stress. The semi-active or active suspension systems are used to solve this conflict [1].

2. Active Suspension System

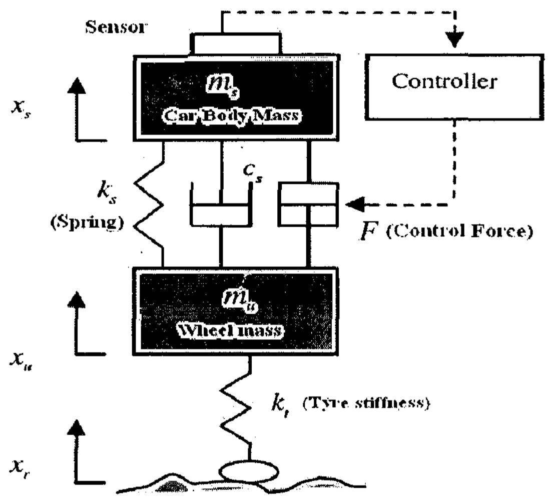

Fig.2.3. Active Suspension System Model

The structure of active suspension system is shown in Fig. 2.3. Recently active system with various control methods and actuators technology has become a popular topic in vibration control and applied on many systems such as suspension system, precision machine platform, building structures, etc. The idea of active system is that desirable performance characteristics can be achieved through cooperating sensors, actuators and control techniques within mechanical structures.

There is a little different semi-active and active suspension system that the mechanism of semi-active suspension systems is the adaptation of the damping and/or stiffness of the spring to the actual demands while active suspension systems provide an extra force input in addition to possible existing passive systems and therefore it needs more energy than semi-active systems. Also they are expensive and complex.

The design methodology for the active systems involves actuation concept, alternate control strategies, approach to solving the problem, controller hardware and software and sensors to be integrated effectively in the systems. The control strategies are to be discussed in the next chapters.

2.3. Part Description

2.3.1. Spring

The spring carries the body mass and isolates the body from road disturbances and thus contributes to drive comfort.

The springs mainly are of four types – Coil spring, Leaf Spring, Torsion Bar and Air spring. Coil spring is used commonly in today's suspension systems.

Springs are great at absorbing energy, but not so good at dissipating it. That's why we need other structures like dampers.

2.3.2. Dampers: Shock Absorbers

Dampers are the devices which convert the kinetic energy of suspension into heat energy that can be dissipated. This conversion of energy is used to reduce the vibration of the suspension.

The damper contributes to both driving safety and comfort. Its task is the damping body and wheel oscillations, where the avoidance of wheel oscillations directly refers to drive safety, as a non-bouncing wheel is the condition for transferring road-contact forces.

The Dampers are of different types such as Hydraulic, Pneumatic, Electric and Magneto Rheological fluid shock absorber. The working of a hydraulic damper is explained here

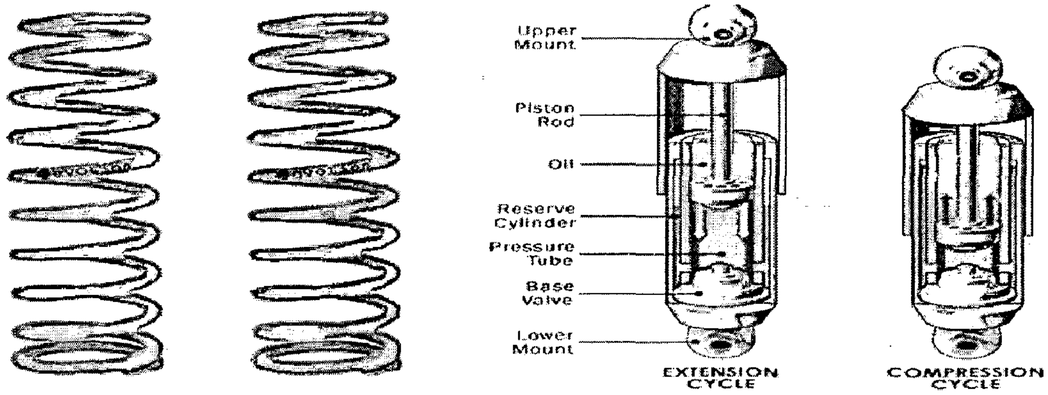

Fig.2.4. Coil Springs and Hydraulic Damper

It works in two cycles – The Compression cycle - It occurs as the piston moves downward, compressing the hydraulic fluid in the chamber below the piston. This cycle controls the motion of the vehicle's un-sprung mass. The Extension cycle – It occurs as the piston moves toward the top of the pressure tube, compressing the fluid in the chamber above the piston. This cycle controls the sprung mass.

Now, the Modern shock absorbers are velocity-sensitive. The faster the suspension moves, the more resistance the shock absorber provides. This enables system to control all the unwanted motions that can occur in a moving vehicle, including bounce , acceleration etc.

2.3.3. Sensors

Sensors are the important part of the suspension system. Slight changes in vibration serve as a leading indicator of worn bearings, misaligned mechanical components, and other issues in machinery, including industrial equipment. Very small accelerometers with very wide bandwidth are ideal for monitoring vibration in motors, fans, and compressors.

2.4. Advanced Development in Suspension System Industry

In the world of Automobile, suspension system is very useful and important. The Researchers and scientists always think about the advanced development in the suspension system and its parts. Here is some development in this field:

2.4.1. Sensors : MEMS Accelerometer

An MEMS accelerometer is an instrument for measuring acceleration, detecting and measuring vibrations, or for measuring acceleration due to gravity (inclination). Accelerometers can be used to measure vibration on vehicles, machines, process control systems and safety installations. They can also be used to measure seismic activity, inclination, machine vibration, dynamic distance and speed with or without the influence of gravity.

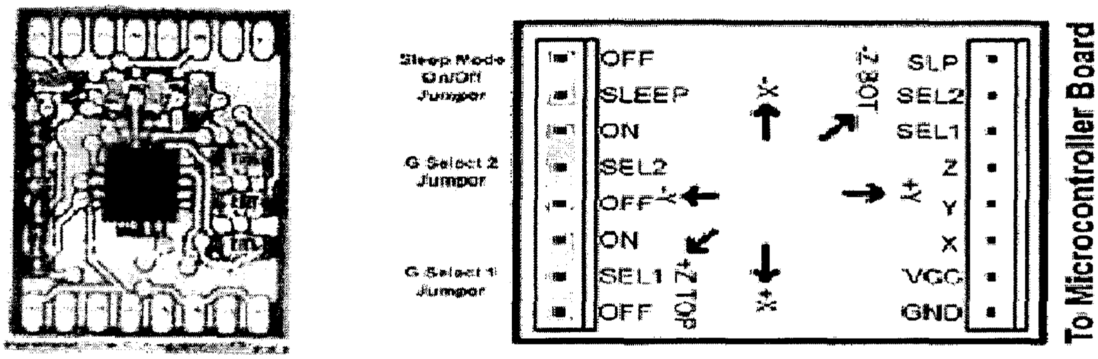

Fig.2.5. A Robokits MEMS Accelerometer a chip image and its Layout

MEMS Technology

MEMS stands for Microelectromechanical systems, a manufacturing technology that enables the development of electromechanical systems using batch fabrication techniques similar to those used in integrated circuit (IC) design. MEMS integrate mechanical elements, sensors, actuators and electronics on a silicon substrate using a process technology called micro fabrication.

This combination of silicon-based microelectronics and micromachining technology allows the system to gather and process information, decide on a course of action, as well as control the surrounding environment, which in turn increases the affordability, functionality and performance of products using the system. Due to this increase in value, MEMS are expected to drive the development of "smart" products within the automobile, scientific, consumer goods, defense and medical industries.

How MEMS work:

The sensors gather information by measuring mechanical, thermal, biological, chemical, magnetic and optical signals from the environment. The microelectronic ICs act as the decision-making piece of the system, by processing the information given by the sensors. Finally, the actuators help the system respond by moving, pumping, filtering or somehow controlling the surrounding environment to achieve its purpose.

Working Principle of MEMS accelerometers

There are many different ways to make an accelerometer. Some way to do it is by sensing changes in capacitance. Capacitive interfaces have several attractive features. In most micromachining technologies no or minimal additional processing is needed. They have excellent sensitivity and the transduction mechanism is intrinsically insensitive to temperature. Capacitive sensing is independent of the base material and relies on the variation of capacitance when the geometry of a capacitor is changing. Neglecting the fringing effect near the edges, the parallel-plate capacitance is:

$$C_o = \epsilon_o \epsilon \frac{A}{d} = \epsilon_A \frac{1}{d} \qquad (2.1)$$

Where, A is the area of the electrodes, d the distance between them and the permittivity of the material separating them.

A change in any of these parameters will be measured as a change of capacitance and variation of each of the three variables has been used in MEMS sensing. Typical MEMS accelerometer is composed of movable proof mass with plates that is attached through a mechanical suspension system to a reference frame, as shown in Fig.2.6.

Movable plates and fixed outer plates represent capacitors. The deflection of proof mass is measured using the capacitance difference. The free-space (air) capacitances between the movable plate and two stationary outer plates C1 and C2 are functions of the corresponding displacements x1 and x2:

$$C_1 = \epsilon_A \frac{1}{x_1} = \epsilon_A \frac{1}{d+x} = C_0 - \Delta C, C_2 = \epsilon_A \frac{1}{x_2} = \epsilon_A \frac{1}{d-x} = C_0 + \Delta C \qquad (2.2)$$

If the acceleration is zero, the capacitances C_1 and C_2 are equal because $x_1 = x_2$. The proof mass displacement x results due to acceleration. If $x \neq 0$, the capacitance difference is found to be

$$C_2 - C_1 = 2\Delta C = 2\epsilon_A \frac{1}{d^2 - x^2} \quad (2.3)$$

Measuring ΔC, one finds the displacement x by solving the nonlinear algebraic equation

$$\Delta C x^2 + x\epsilon_A - \Delta C d^2 = 0 \quad (2.4)$$

This equation can be simplified. For small displacements, the term Cx^2 is negligible. Thus, $\Delta C x^2$ can be omitted. Then, from

$$x \approx \frac{d^2}{\epsilon_a}\Delta C = d\frac{\Delta C}{C_0} \quad (2.5)$$

one concludes that the displacement is approximately proportional to the capacitance difference ΔC.

As one can see in the Fig.2.6, every sensor has a lot of capacitor sets.

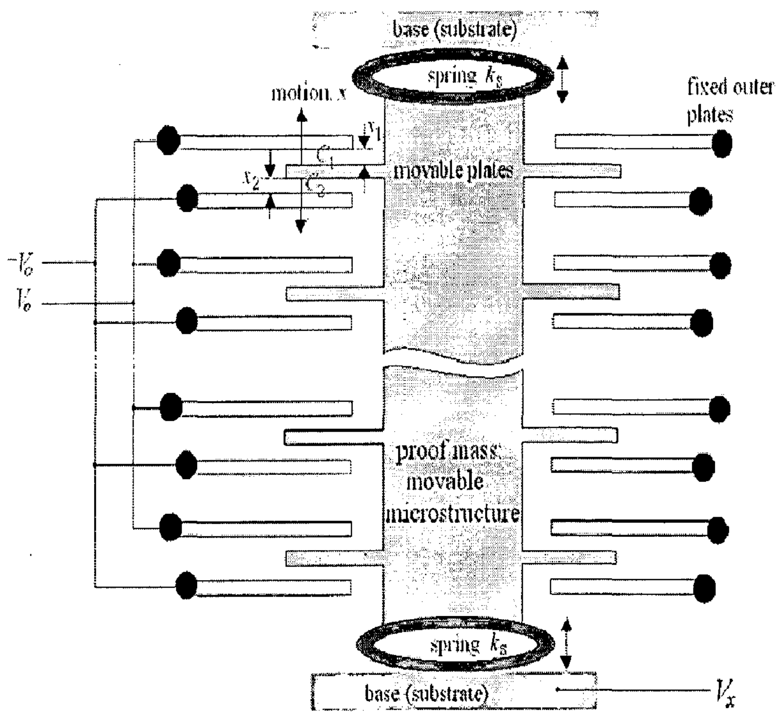

Fig.2.6. Accelerometer structure

All upper capacitors are wired parallel for an overall capacitance C_1 and likewise all lower ones for overall capacitance C_2, otherwise capacitance difference would be negligible to detect. Equation 2.5 now doesn't hold true just for one pair of capacitors, but for all system. Sensor's fixed plates are driven by 1MHz square waves with voltage amplitude V_0 coming out of oscillator. Phases of the square waves that drives upper and lower fixed plates differs for 180. One can picture to himself this hole system as a simple voltage divider whose output goes forward through buffer and demodulator. First of all we are interested in voltage output V_x, which is actually the voltage of the proof mass. It holds true that

$$(V_x + V_0)C_1 + (V_x - V_0)C_2 = 0 \tag{2.6}$$

and if we use equations 2.2 and 2.5 we get for voltage output

$$V_x = V_0 \frac{C_2 - C_1}{C_2 + C_1} = \frac{x}{d} V_0 \tag{2.7}$$

V_x is square wave with the right amplitude proportional to acceleration. We also can't just simply use this output signal, because it is weak and noisy. If we accelerate the sensor ($a_1 > 0$), the voltage output V_x changes proportional to alternating voltage input V_0 (equation 2.7). To avoid signal attenuation, we read V_x with voltage follower (buffer), therefore signal V_y is actually Vx multiplied by 1. If we inverse the acceleration , signals V_x and V_y get negative sign. Demodulator then gives us the sign of the acceleration, because it multiplies the input signal V_y with the square waves V_0 coming from oscillator.

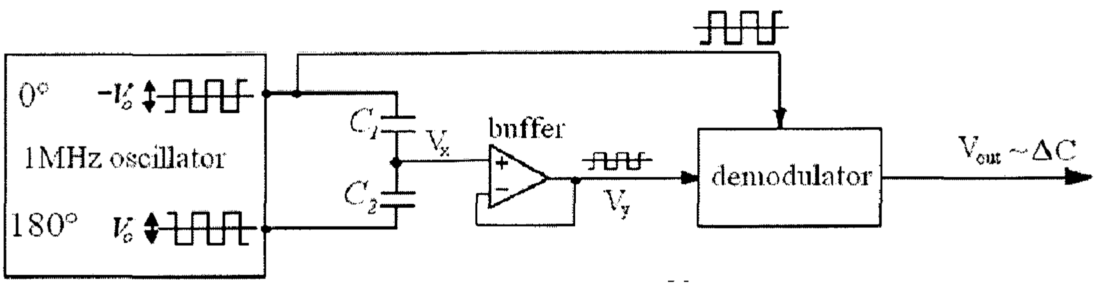

Fig.2.7. Electric circuit that measures acceleration through capacitor changes

For an ideal spring, according to Hook's law, the spring exhibit a restoring force F_S which is proportional to the displacement x. Thus, $F_S = k_S x$, where k_S is the spring constant. From Newton's second law of motion, neglecting the air friction (which is negligibly small), the following differential equation results $ma = md^2x/dt^2 = k_S x$. Thus, the acceleration, as a function of the displacement, is

$$a = \frac{k_s}{m} x \tag{2.8}$$

Then, making use of equation 2.7, the acceleration is found to be proportional to voltage output

$$a = \frac{k_s d}{m V_o} V_x \tag{2.9}$$

2.4.2. Damper

Dampers are evolved over the time. Previously Pneumatic and Hydraulic dampers used in the suspension systems. Currently Electrical and MR dampers are used widely in the suspension systems. But in future Electronics dampers will take places of all its previous kinds.

BMW is well known in the field of automobile has developed a damper based on Electronics Control. This Damper is called Electronic Damper Control (EDC). EDC is a processor – controlled wheel suspension system that adjusts the shock absorbers to changing road or driving conditions. EDC regulates damper forces electronically, adapting to road , load and driving conditions.

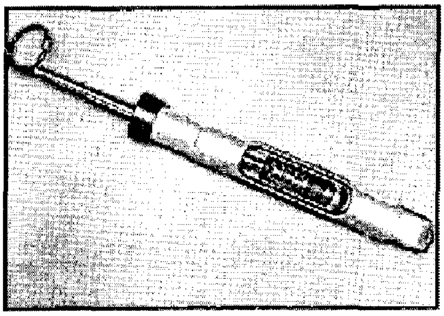

Fig. 2.8. A BMW EDC

When driving on good roads, the dampers are automatically set to "soft" and provide maximum ride comfort. when the car suddenly passes over a bump, the system automatically switches to "medium" or "hard" to insure optimum road-holding.

The parameters considered when adjusting the dampers are road speed, load, transverse acceleration, acceleration in the direction of travel, and vertical acceleration of the vehicle. The system uses the following sensors for this purpose:

- steering angle sensor
- speed sensor on the final drive
- vertical acceleration sensor on the front and rear axles

The signal processor, control logics, hardware monitor, and power terminals for feeding an electric signal to the dampers are integrated in the control unit. Sensitive sensors constantly monitor all factors influencing the vehicle's behavior and occupants' comfort, including road conditions, load changes and vehicle speed. In a fraction of a second, the signals are analyzed by the EDC microprocessor and orders are sent to the actuators on the shock absorbers, which, with the help of magnetic valves, are variably adjusted to provide optimal suspension.

Electronic Damper Control (EDC) reduces variations in wheel load, ensures tyres have excellent traction and counteracts bodyshell movement regardless of the weight your automobile may be carrying - and regardless of the state of the road's surface. Electronic Damper Control (EDC) can even help shorten braking distances.

3.1. Introduction

In this chapter, an analysis is done to study the suspension system. Its equation of motion, transfer function and State-space model are derived. The Step responses and Frequency response are studied also studied in the chapter with simulating command in the MATLAB environment. This study also includes the effects of variation in its parameters.

3.2. Dynamics of Quarter Car Suspension System (QCS)

In this paper, we are considering a quarter car model with 2 DOF MIMO, as shown in Fig. 3.1. The sprung mass, m_s, represents the car chassis, while the un-sprung mass, m_u, and represents the wheel assembly.

The spring, k_s, and damper, c_s, represent a passive spring and shock absorber that are placed between the car body and the wheel assembly, while the spring, k_t, serves to model the compressibility of the pneumatic tyre.

The variables x_s, x_u, and x_r are the car body deflection, the wheel deflection, and the road disturbance respectively. The force F, kN, applied between the sprung and un-sprung masses, is controlled by feedback and represents the active component of the suspension system.

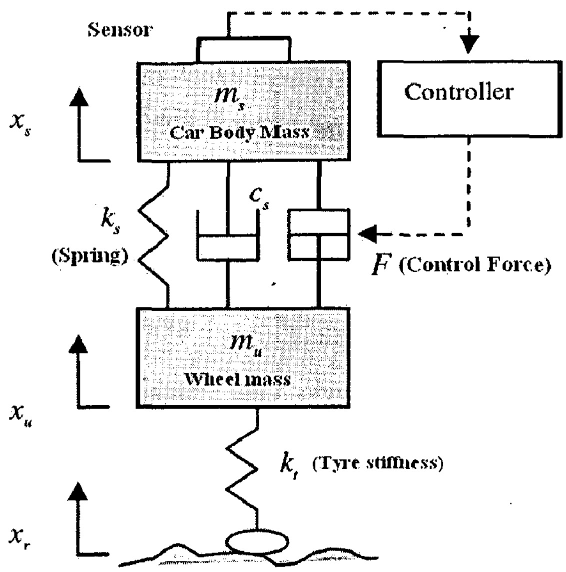

Fig.3.1. Active Suspension System with a Controller

3.2.1. Equations of motion

According to Newton's second law of motion:

Inertia force of mass = Forces acting on the mass

There are two masses so the equations are as follows:

Motion's equation of sprung mass, m_s:

$$m_s\ddot{x}_s = F - k_s(x_s - x_u) - c_s(\dot{x}_s - \dot{x}_u) \tag{3.1}$$

Motion's equation of un-sprung mass, m_u:

$$m_u\ddot{x}_u = -F + k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u) - k_t(x_u - x_r) \tag{3.2}$$

3.2.2. Transfer function of MIMO system

The system is having 2 inputs called as the control force, F, and the road disturbance, x_r. The car body deflection or vibrations of the system get affected when any of the inputs get changes. So this system is known as Multi Input Multi Output (MIMO). Where outputs of the system are used to be the car body deflection, x_s, the wheel deflection, x_u or the suspension deflection, x_s - x_u.

The transfer function of the system can be derived after taking Laplace of the equations of motion, converting equations into frequency domain. The derivation of transfer function is easy to determine. Here I skipped few steps of derivation.

The transfer function matrix can be shown here:

$$\begin{bmatrix} x_s \\ x_u \end{bmatrix} = \frac{1}{\nabla}\begin{bmatrix} (m_u + k_t) & k_t(c_s s + k_s) \\ -m_s s^2 & k_t(m_s s^2 + c_s s + k_s) \end{bmatrix}\begin{bmatrix} F \\ x_r \end{bmatrix} \tag{3.3}$$

Where

$$\nabla = m_s m_u s^4 + (m_s c_s + m_u c_s)s^3 + (m_s(k_s + k_t) + k_s m_u)s^2 + c_s k_t s + k_s k_t$$

The matrix of four transfer functions is shown as follows:

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{3.4}$$

G_{11} and G_{12} are the transfer function of the car body deflection due to control input and road disturbance respectively and G_{21} and G_{22} are the transfer function of the wheel body deflection due to control input and road disturbance respectively.

Now, only deflection in the car body is considered. So the required transfer functions are:

$$G_{11} = \frac{x_s}{F} = \left[\frac{m_u s^2 + k_t}{\nabla}\right] \; keeping \; x_r = 0 \tag{3.5}$$

$$G_{12} = \frac{x_s}{x_r} = \left[\frac{k_t(c_s s + k_s)}{\nabla}\right] keeping \; F = 0 \tag{3.6}$$

In the suspension system, the stiffness of the tire/wheel is very high. It could be high as much as k_t = 190000 N/m [7] and even more. Because of high stiffness the movement in the wheel body is equal to the road disturbance. So in the calculation of the transfer function, k_t might be neglected to keep the things easy. Hence after the neglecting the stiffness of tire, the transfer function can be reduced in the given form:

$$G_{11} = \frac{x_s}{F} = \left[\frac{1}{m_s s^2 + c_s s + k_s}\right] \; keeping \; x_r = 0 \tag{3.7}$$

$$G_{12} = \frac{x_s}{x_r} = \left[\frac{(c_s s + k_s)}{m_s s^2 + c_s s + k_s}\right] keeping \; F = 0 \tag{3.8}$$

Here from equation 3.8, we can say that the vibration of the car body due to road disturbances is dependent upon the three parameters such as sprung mass, m_s , spring constant, k_s and damper coefficient , c_s .But damper coefficient does not change frequently. Hence, effectively variation occurs in remaining two parameters sprung mass, m_s, and spring constant, k_s only.

the variation ranges of these two parameters are assumed [10] as:

$$k_s \in [9600 \quad 22400] \tag{3.9}$$

and

$$m_s \in [240 \quad 450] \tag{3.10}$$

3.2.3. State-Space Model

To transform the motion equation of the quarter car suspension model into state space model, the following variables are considered:

$$x_1 = x_s; \qquad x_2 = \dot{x}_s; \qquad x_3 = x_u; \qquad x_4 = \dot{x}_u;$$

A linear, time-invariant model of the quarter car suspension model is constructed from the equations of motion and parameter values. The inputs to the model are the road disturbance and actuator force respectively and the outputs are the car body deflection, acceleration and suspension deflection. Then the motion equations of the quarter car model for the active suspension can be written in state space form as follows:

$$\dot{x} = A.x + B.U \tag{3.11}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{k_s}{m_s} & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & -\frac{c_s}{m_{us}} & \frac{k_s - k_t}{m_{us}} & -\frac{c_s}{m_{us}} \end{bmatrix} \tag{3.12}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m_s} \\ 0 & 0 \\ \frac{k_t}{m_u} & \frac{-1}{m_u} \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} x_r \\ F \end{bmatrix} \tag{3.13}$$

3.3. Time Response

The system is having two inputs, the control force F and the road disturbance xr which are responsible for the vibrations in the system. Fig. 3.2 and Fig. 3.3 show the responses in the car body i.e. sprung mass.

The MATLAB simulation software is used to show the open-loop performance (without any controller). MATLAB step command is run to see the response of unit step actuated control force input and unit step road disturbance input.

M.G. CENTRAL LIBRARY
ACC No. G 20243
Date....................
I.I.T. ROORKEE

3.3.1. Step Response due to Force

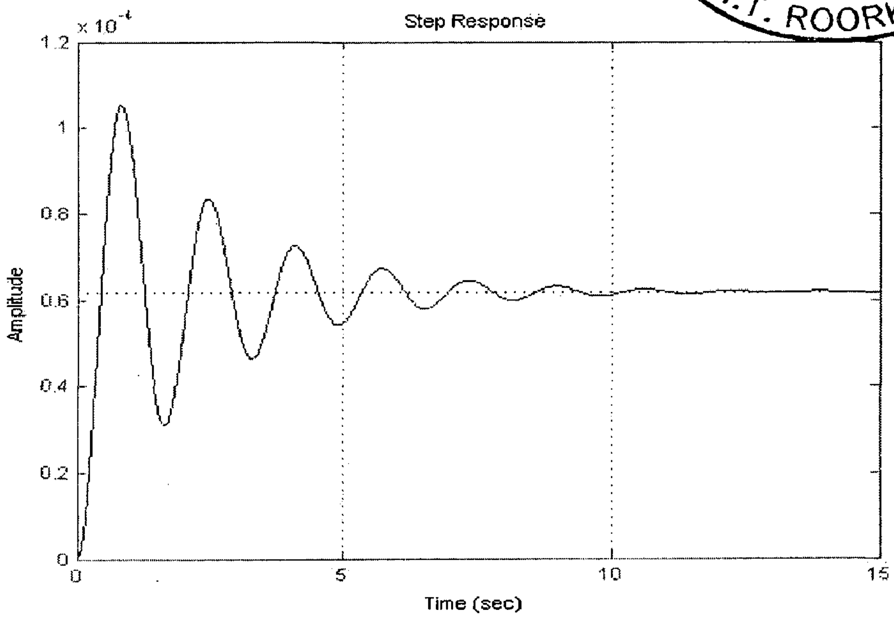

Fig.3.2. Step Response of sprung mass due to Actuator force

In Fig. 3.2, shows the response in the car body, sprung mass, due to the actuating control force, F. It clear shown that the deflection of the car body is not high due to unit force applied. So the passenger of the vehicle feels fewer amounts of deflection. But the settling time for these oscillations is much as ac. So the system takes very much time to reach its steady state.

3.3.2. Step Response due to Road disturbance input

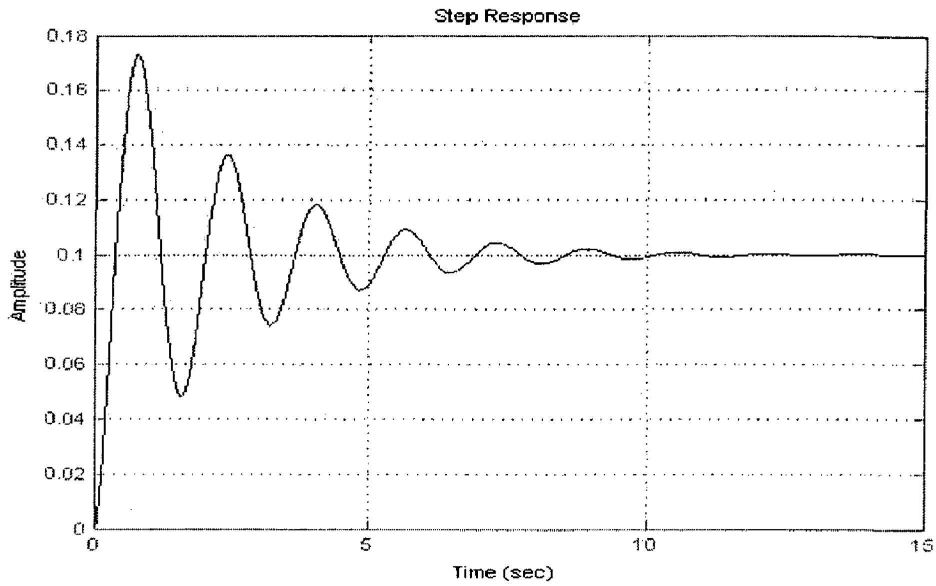

Fig.3.3. Step Response of sprung mass due to road disturbance

In Fig. 3.3, the deflection of the car body due to road disturbances is shown here. Let the road disturbance input is 10 cm (0.1 m) step input, then we can see that the car body oscillates for the long time (more than 10 secs) and the maximum deflection of the body is about 17.5 cm (0.175 m) which 75 % of the road disturbance.

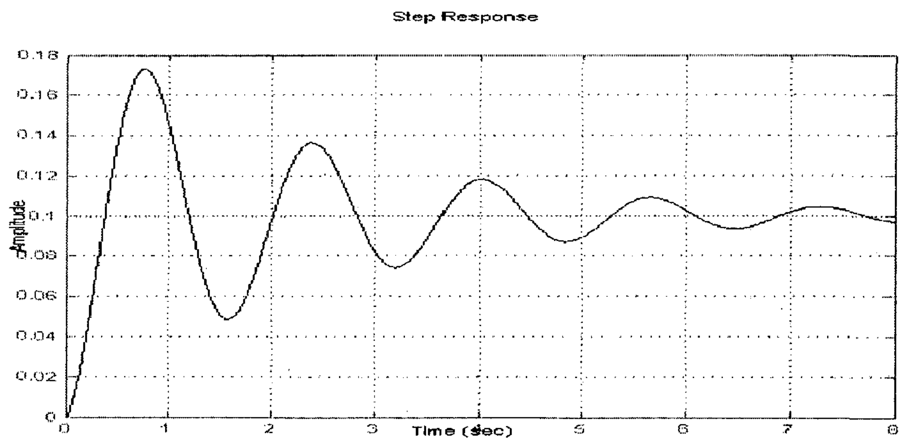

Fig.3.4. Step Response with close view

The passenger of the vehicle will not feel good having this much oscillation in the system. The maximum overshoot and more settling time can be the factors of damages to the suspension system.

3.4. Frequency Response

The frequency response of the system is Fig. 3.5. with input road disturbances and the control force, respectively.

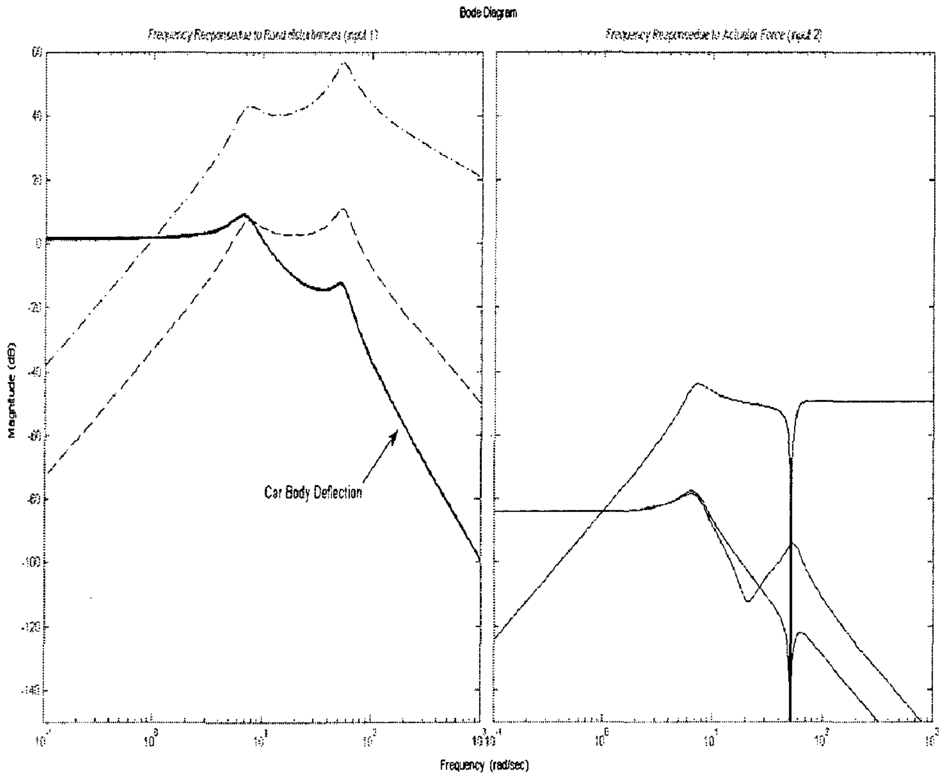

Fig.3.5. Frequency Response of sprung mass due to both inputs

The frequency response of the suspension system due to the road disturbance input is shown in the left of Fig. 3.5. The solid blue line shows the frequency response of the deflection of the car body. There are two peaks points i.e. resonance points, shown in the figure. The first point is due natural frequency of the car body, the sprung mass, vibrations and the second is due to natural frequency of the wheel mass, the un-sprung mass, vibrations.

3.5. Observations

There are some observations based on the analysis of the suspension system, which are as follows:

- The suspension system's equation has no roots with positive real parts, hence, all poles in the LHS of s-plane. These Left Half Plane poles make the system open-loop stable.
- The system is having two natural frequencies of sprung mass and un-sprung mass.
- There is uncertainty in the parameters of the suspension system.
- These uncertainties in the parameters of the suspension system could affect the comfort and the safety of the vehicle.
- It may be necessary for the designer to retune the controller to achieve the desire performance regardless the parametric uncertainties.

3.6. Problem Statements

The following problems are associated with the system for a Controller Design Engineer to be solved are:

- ➢ Design a Robust Controller for the system to achieve the desire performance regardless the parametric uncertainties.
- ➢ Design a Robust Controller which reduces the peak amplitude at the resonance frequency.

4.1. Introduction

There are different controlling methods which can be applied to a system for achieving desire output. This chapter consists of different control strategies for controlling the suspension system. The PID, QFT and H-infinity are described in this chapter. Their basic concepts, working methods are given in this chapter.

4.2. Control System Design Basics

An active vibration control is a method that relies on the use of an external power source called actuator (e.g. a hydraulic piston, a piezoelectric device or an electric motor). The actuator will provide a force or displacement to the system based on the measurement of the response of the system using control systems. Let see the basics of Control system, in Fig. 4.1 shows a feedback control system

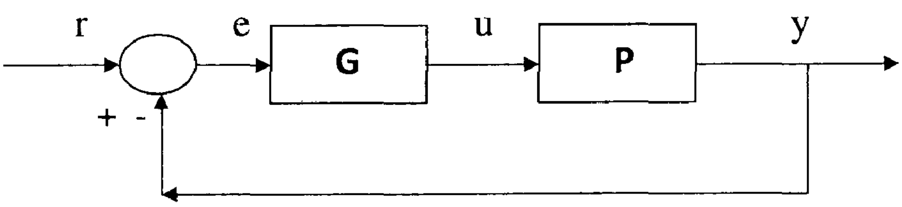

Fig.4.1. Control System for the Plant

The reference signals are denoted by r, the input to the controlled system P is u, the output of the system is y. The loop is closed by feeding back the tracking error e = r - y to the controller G.

There are different closed-loop transfer functions can be defined which play an important role in a controller design process. They are as follows:

1) The Loop-transmission gain function, L:

$$L = GP \tag{4.1}$$

2) The Sensitivity function, S:

$$S = (1 + GP)^{-1} = (1 + L)^{-1} \tag{4.2}$$

3) The Complementary sensitivity function, **T**:

$$T = GP(1 + GP)^{-1} = L(1 + L)^{-1} \quad (4.3)$$

Now, we can see the role of these transfer functions with the help of following equations

$$y = GP(r - y)$$

$$= (1 + GP)^{-1}GP\, r$$

$$y = Tr \quad (4.4)$$

and

$$e = r - y$$

$$= r - Tr = (1 - T)r$$

$$e = Sr \quad (4.5)$$

Equations 4.4 and 4.5 show that S and T are related to output of the system and error of between the signals.

4.3. Proportional-Integral-Derivative (PID) Controller

The PID name comprises the first letters of the 3 terms which make up with controller: P stands for the Proportional term in the controller, I stand for Integral term for the controller and D stands for Derivative term. PID controller's algorithms are mostly used in feedback loops. PID or 3-terms controllers are widely used in industry. PID controllers can be implemented in many forms.

It is interesting to note that more than half of the industrial controllers in use today utilize PID control schemes. Below is a simple diagram of illustrating the schematic of the PID controller. Such set up is known as non interacting form or parallel form.

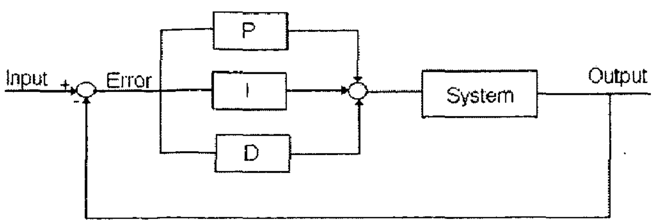

Fig.4.2. PID controller Schematic

Proportional control

$$P_{term} = K_p \times Error \tag{4.6}$$

It uses proportion of the system error to control the system. In this action an offset is introduced in the system.

Integral control

$$I_{term} = K_i \times \int Error.dt \tag{4.7}$$

It is proportional to the amount of error in the system. In this action, the I-action will introduce a lag in the system. This will eliminate the offset that was introduced earlier on by the P-action.

Derivative control

$$D_{term} = K_d \times (d(Error))/dt \tag{4.8}$$

It is proportional to the rate of change of the error. The D-action will introduce phase lead in the system. This will eliminate the lag in the system that was introduced by the I-action earlier on.

4.3.1. PID control

The three controllers when combined together can be represented by the following transfer function.

$$G_c(s) = K_p + K_d s + K_i/s) \tag{4.9}$$

The above equation 4.9 can be expressed in the form:

$$G_c(s) = K_d \left(\frac{(s+a_1)(s+a_2)}{s}\right) \tag{4.10}$$

which indicates that PID controller is similar to lag lead compensator, with one absent pole. It is not possible to realize the PID controller by passive RC network because of the pure integration term and one more zero than the pole in its transfer function . However, it may be realized by active electronic components, Hydraulic and Pneumatic components.

Since, the PID controller is similar to lag lead compensator; its design may be undertaken by analytical methods when the mathematical model of the plant is available. If mathematical model of a plant can be derived then it is possible to apply various design techniques for

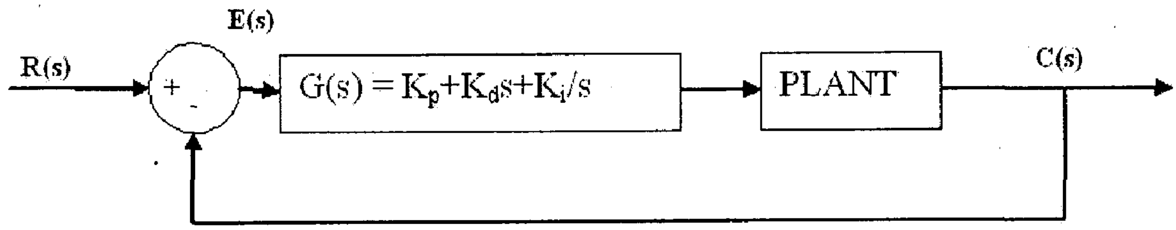

Fig.4.3. Block Diagram of PID Controller controlling Plant

determining parameters of controller that will meet the transient & steady-state specifications of closed loop system. However if the plant is so complicated that its mathematical model cannot be easily obtained, then an analytical approach to the design of PID control is not possible. Then we must resort to the experimental approach to the design of PID controllers.

4.3.2. Selecting the PID controller coefficients

The process of selecting controller parameters to meet given performance specifications is known as controller tuning.

A) Manual Tuning: Manual tuning of PID control is surprisingly common. Basically, the manual tuning is a trial and error process. But there is a procedure for tuning systems when we have some knowledge of system models. A systematic design procedure is as follows:

Step 1) Determine whether the priority for the closed-loop system is reference tracking or disturbance rejection.

Step 2) Determine whether steady state accuracy is essential to the control systems performance.

Step 3) Proportional control tuning: Introduce proportional action by increasing the value of proportional gain, K_p , until the speed of response is acceptable.

Step 4) Integral control tuning: If steady state accuracy is considered important then introduce integral action into the controller by increasing the gain K_i . The integral gain should be increased so that an acceptable settle time is achieved.

Step 5) Balancing the controller terms: Increase K_i may increase the overshoot, to compensate, decrease K_p also introduce K_d term. A little fine tuning will be necessary to achieve acceptable time responses.

B) Ziegler-Nichols (Z-N) tuning methods: Ziegler-Nichols proposed two tuning method First and Second. First method is associated with the over-damped response system and second method is associated with the under-damped response system. Here, the second method is discussed.

For the system under study, Ziegler-Nichols tuning second method will be used. In this method, $K_i = 0$ (i.e. the integral time T_i will be set to infinity) and $K_d = 0$ (i.e. the derivative time T_d to zero). This is used to get the initial PID setting of the system.

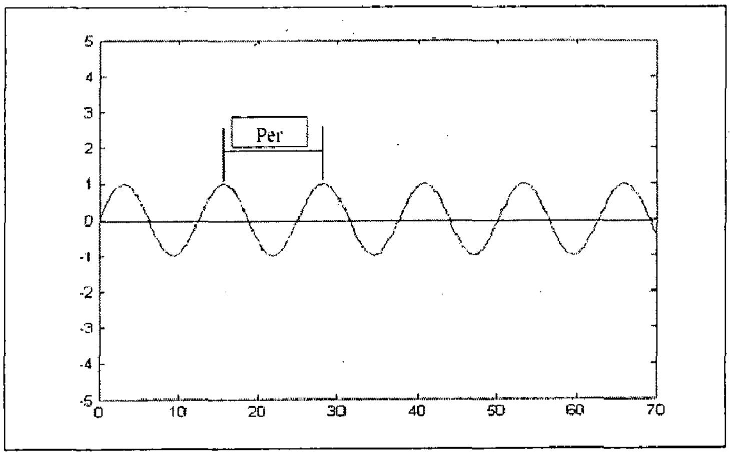

Fig.4.4. A Sustained oscillation and its Period

In this method, only the proportional control action will be used. The K_p will be increased to a critical value K_{cr} at which the system output will exhibit sustained oscillations. In this method, if the system output does not exhibit the sustained oscillations hence this method does not apply.

The PID controller transfer function also can be written as:

$$G_c(s) = K(1 + \frac{1}{sT_i} + sT_d) \tag{4.11}$$

From Ziegler-Nichol's second method, the table suggesting tuning rule according to the formula is shown. From these we are able to estimate the parameters of K_p, T_i and T_d.

Table 4.1: Z-N PID Parameters

Type of controller	K_p	T_i	T_d.
P	$0.5\ K_{cr}$	∞	0
PI	$0.45\ K_{cr}$	$(1/1.2)\ P_{er}$	0
PID	$0.6\ K_{cr}$	$0.5P_{er}$	$0.125\ P_{er}$

Where P_{er} is the period of oscillation at critical value K_{cr}.

4.4. Quantitative Feedback Theory (QFT)

4.4.1. A Brief Introduction

QFT is a classical approach to design a controller and suited for the systems having large uncertainties. It uses feedback of measurable plant outputs to generate an acceptable response from a system in the face of disturbance signals and plant modeling uncertainty.

The plant uncertainty can be represented as either parametric (or structured) uncertainty, which implies specific knowledge about the variation in plant parameters, or non-parametric (unstructured) where only information about the variation in the plant's gain is known. It can also be represented by a mixture of these two uncertainties, describing the gain and parameter variations in the plant model.

The QFT is a practical method for designing control systems by quantitatively mapping the design specifications to constraints on the loop transmission gain-phase shape. So this method takes into account quantitative information on the plant's variability, the robust performance requirements, control performance specifications, the expected disturbance amplitude, and attenuation requirements. The QFT technique is based on the classical idea of frequency-domain shaping of the open-loop transfer function.

As shown in Fig. 4.5, it uses unity feedback, a cascade compensator G(s), to reduce the variations of the plant output due to plant parameter variations and disturbances.

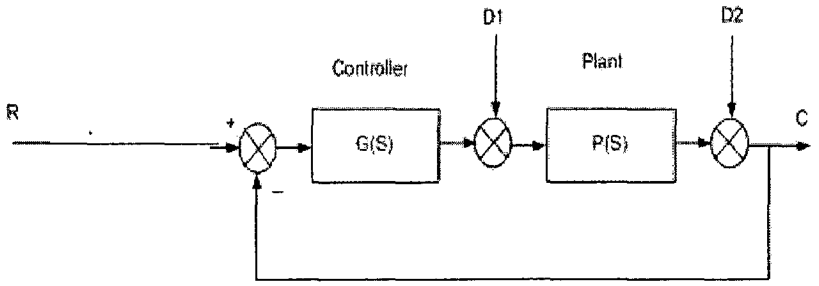

Fig.4.5. Feedback Control System

The feedback loop compensator is designed to ensure that the robustness and disturbance-rejection requirements can be met. So QFT is a transparent frequency-domain design technique in that the tradeoff between compensator complexity and performance are readily visualized, while at the same time parametric uncertainty is also addressed.

Usually a system plant is represented by its Transfer Function, after a process of system modelling and identification. As a result of experimental measurements the values of coefficients in the Transfer Function have a range of uncertainty. Therefore, in QFT every parameter of this function is included into an interval of possible values, and the system may be represented by a family of plants rather than by a standalone expression.

$$P(s) = \left\{ \frac{\prod_i (s+z_i)}{\prod_j (s+p_i)}, \forall z_i \in [z_{i,min}, z_{i,max}], p_i \in [p_{j,min}, p_{j,max}] \right\} \quad (4.12)$$

where z_i and p_i are zeros and poles of the plant respectively. Hence these QFT design are undertaken using a Nichols chart (NC). Because a whole set of plants rather than a single plant is considered, the magnitude and phase of the plants, at each frequency, yields a set of points on the Nichols chart, instead of a single point.

A frequency analysis is performed for a finite number of representative frequencies and a set of 'templates' are obtained in the NC diagram which encloses the behavior of the open loop system at each frequency. Plant uncertainty is represented by a magnitude-phase surface called the 'plant template', whose size and shape at a given frequency is determined by the set of all possible values of the transfer function coefficients. Larger templates indicate greater uncertainty; to avoid unnecessary conservatism, the template is usually chosen to be the smallest convex polygon enclosing all of the points.

In this, the uncertainty in the plant gain and phase is represented as a template on the Nichols chart. These templates are then used to define regions called bounds in the frequency domain, where the open loop frequency response must lie, in order to satisfy the performance and stability specifications, and the disturbance-rejection requirements for the entire plant set.

These regions or bounds are as following:

1. The Stability bounds are calculated using these templates and the phase margin.
2. The performance bounds are derived using the templates and upper and lower limits on the frequency-domain response.
3. The disturbance bounds are based on the templates and the upper limit only.

The compensator is determined through a loop-shaping process using a Nichols chart that displays the phase and gain margins, stability bounds, performance bounds, and disturbance-rejection bounds. The disturbance-rejection and tracking action of the compensator is based on keeping the loop-transfer gain above the disturbance and tracking bounds on the Nichols chart.

During the loop shaping process, modification of the poles and zeros of the compensator produces transparent results, enabling the designer to examine the trade-off between compensator complexity (order) and system performance.

Therefore, this QFT can be considered as a natural extension of classical frequency- domain design, which offers a formal approach to handle the plant uncertainties. The test of any good control theory is that it links the amount of feedback with the amount of uncertainty in a quantitative fashion. For example, if no uncertainties or disturbances are present, the technique should automatically yield the result that no feedback is required at all.

4.4.2. Basics of QFT

To understand the Basics of the QFT technique, The basic concept of controller should be understood. The Basic concept of the controller is explained here [15]:

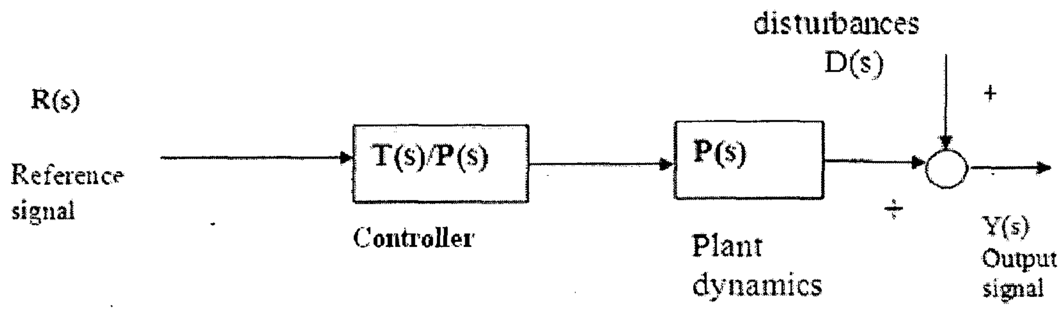

Fig.4.6. A Basic Controller Concept

For a desired input reference there should be a desired output. Hence the desired transfer function is

$$T(s) = \frac{Y(s)}{R(s)} \tag{4.13}$$

As the sensitivity function is responsible for variation in response with the variation in the parameters. The sensitivity function is defined as:

$$S = \frac{1}{1+G(s)P(s)} = \frac{1}{1+L(s)} \tag{4.14}$$

So the sensitivity function is inversely proportional to the loop-transmission gain. The Loop-shaping of the Loop-Transmission function is being done in the QFT technique to obtain the desire output from the plant. where the Loop-Transmission Function is given as

$$L(s) = G(s)P(s) \tag{4.15}$$

4.4.3. Design Technique Procedure

The QFT design approach consists of a number of distinct steps, which can be illustrated as follows [12]:

1. Design specifications: The specification can be defined in the time-domain using familiar figures such as the rise time t_r , settling time t_s and maximum peak overshoot M_p, or directly in the frequency domain. Simple poles and zeros can be added to represent the specifications more closely. Specifications on the tolerance of the closed-loop system response, and on the disturbance-attenuation requirements, can also be given in various forms, such as gain and phase margins, rejection of disturbances at different points, tracking bandwidth, etc.

2. Choosing the frequency array: A frequency array must be chosen prior to the detailed design stage. This consists of different frequency points, at which the templates and various bounds are computed. There is no strict criterion for choosing the frequency array. These chosen frequency points are known as the trial frequencies.

3. Representation of plant uncertainty and template computation: Typically, parametric uncertainty models are used to represent uncertainty in the low-to-medium frequency range, and non-parametric uncertainty in the high-frequency range. The plant uncertainty is represented by templates on the Nichols Chart, showing the variation of the system frequency response (gain and phase) over its operating range at each chosen frequency.

4. Selection of the nominal plant model: The QFT method is based on point wise design at the trial frequencies. It is necessary to define one of the representative functions in the uncertainty set to be the nominal transfer function.

5. Generation and integration of bounds: The specifications from Step 1 and the templates from Step 3 are used to generate bounds at the trial frequencies in the frequency-domain. The open-loop transfer-function in Fig. 4.5. is given by:

$$L(s) = G(s)P(s) \tag{4.16}$$

where P(s) denotes the nominal transfer-function and G(s) the controller. The transfer function L(w) must lie on or above the bounds at each of the trial frequencies. Satisfaction of the bounds for the nominal plant ensures satisfaction of the specification for all plants described by the uncertainty. In general following specification are considered in QFT:

- Robust stability margin

$$\left|\frac{L(j\omega)}{1+L(j\omega)}\right| \leq W_s \tag{4.17}$$

- Robust tracking performance

$$|T_L(j\omega)| \leq \left|\frac{L(j\omega)}{1+L(j\omega)}\right| \leq |T_U(j\omega)| \tag{4.18}$$

6. Loop shaping: The controller is designed via a loop-shaping process, in the Nichols plane. The composite bounds, evaluated at the trial frequencies and the characteristic of the nominal open-loop transfer function are plotted together. The design is performed by adding gains or dynamic elements to the nominal plant frequency response to change the shape of the open-loop transfer function i.e. the boundaries are satisfied at each of the trial frequencies. The Final controller is then simply the aggregate of these gain and dynamic elements. Loop shaping is carried out in the frequency domain, on the Nichols diagram, by utilizing classical control design techniques. One of the principal benefits of adopting this approach is that it is very transparent, and the compensation can be built up gradually so that the changes to the controller are clearly evident at each step.

4.5. H-infinity method

4.5.1. H-infinity Basics

H-infinity optimization of control systems deals with the minimization of the peak value of certain closed-loop frequency response functions [19]. To clarify this, consider by way of example the basic SISO feedback system of Fig. 4.7

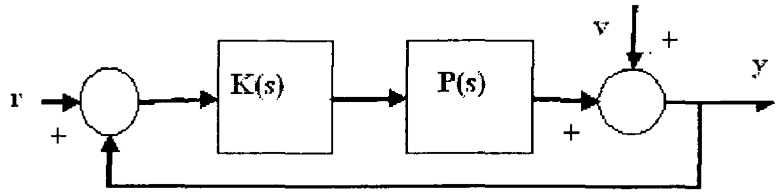

Fig.4.7. SISO Feedback loop

The plant has transfer function P(s) and the compensator has the transfer function K(s). The signal v represents disturbance acting on the system and z is the control system output. Then

$$S = \frac{1}{1+PK} \tag{4.19}$$

the sensitivity function of the feedback system. As the name implies, the sensitivity function characterizes the sensitivity of the control system output to disturbances. For a system, ideally error should be equal to zero, so according to equation 4.5, it should be equal to zero.

The problem is that of finding a compensator K that makes the closed-loop system stable and minimizes the peak value of the sensitivity function. This peak value is defined as

$$||S||_{\infty} = max_{\omega \epsilon R}|S(j\omega)| \tag{4.20}$$

where R denotes the set of real numbers. Because for some functions the peak value may not be assumed for any finite frequency, we replace the maximum here and in the following by the supermum or least upper bound, so that

$$||S||_{\infty} = sup_{\omega \epsilon R}|S(j\omega)| \tag{4.21}$$

The justification is that if the peak value $||S||_{\infty}$ of the sensitivity function S is small, then the magnitude of S necessarily is small for all frequencies, so that disturbances are uniformly attenuated over all frequencies. Minimization of $||S||_{\infty}$ is worst-case optimization, because it amounts to minimizing the effect on the output of the worst disturbance (namely, a harmonic disturbance at the frequency where $|S|$ has its peak value).

A little contemplation reveals that minimization of $||S||_{\infty}$ as it stands is not a useful design tool. The frequency response function of every physical plant and compensator decreases at high frequencies. This means that often the sensitivity S can be made small at low frequencies but eventually reaches the asymptotic value one for high frequencies. Just how small S is at low frequencies is not reflected in the peak value but is of paramount importance for the control system performance. For this reason, it is customary to introduce a frequency dependent weighting function W and consider the minimization of

$$\|WS\|_\infty = sup_{\omega \epsilon R} |W(j\omega)S(j\omega)| \tag{4.22}$$

Characteristically, W is large at low frequencies but decreases at high frequencies.

4.5.2. Design Procedure

There is a general augmented plant model, which comprises the physical system model and the weighting functions that define the performance specification. The particular weighting functions are explained for different control design approaches in the following subsections [21].

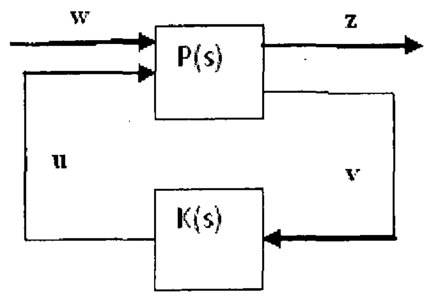

Fig.4.8. General control configuration

The H_∞ design problem is cast into a general control configuration (Fig.4.8) and solved. P is the generalized plant and K is the controller. The signal w contains all external inputs, including disturbances, sensor noise and commands; the output z is an error signal; v equals to the vector of the system-measured outputs y and u is the control input.

The stabilizing controller can be found by minimizing the H_∞ norm from the vector of inputs w to the vector of outputs z of the augmented plant P. The input–output mapping of the control system is

$$\begin{bmatrix} z \\ v \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{4.23}$$

By substituting $u = K(s)v$ in equation , the closed-loop transfer function is given by the lower linear fractional transformation $F_1(P, K)$ as

$$T_{zw} = F_1(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \tag{4.24}$$

The H_∞control problem is formed by finding an admissible controller K,

$$\|T_{zw}\|_\infty = \|F_1(P, K)\|_\infty = max_\omega(F_1(P, K)(j\omega)) \leq \gamma \tag{4.25}$$

where γ equals to an H-infinity norm $\|T_{zw}\|_\infty$ and for H-infinity controller synthesis [19]

$$\gamma \leq 1; \tag{4.26}$$

γ is a constant and maximum stability margin.

$\|F_1(P, K)\|$ is the transfer function between the error signals and external inputs. In H_∞ control problems the objective is to minimize $\|F_1(P, K)\|_\infty$

The minimization of this function minimizes the sensitivity function S and complementary sensitivity function T. In further discussion, we will see how the minimization of this function minimizes the sensitivity (S) and complementary sensitivity (T) of the system.

Here, the above procedure is described in more general. Let P is a Plant to be controlled and K is a controller. W_1 and W_2 are the weight functions.

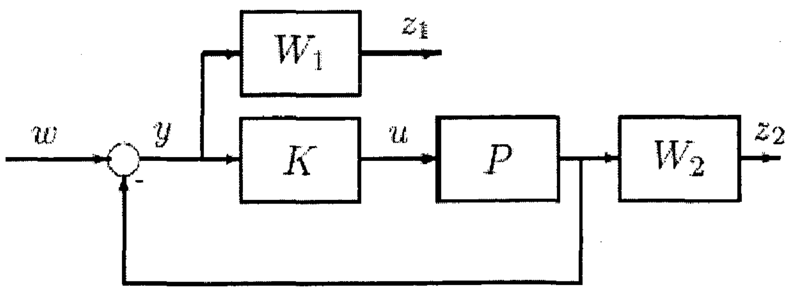

Fig.4.9. Controller using H-infinity control theory

Now, error signal outputs are:

$$z_1 = W_1(w - Pu) \tag{4.27}$$

$$z_2 = W_2 P u \tag{4.28}$$

$$y = w - Pu \tag{4.29}$$

The input–output mapping of the control system is

$$\begin{bmatrix} z_1 \\ z_2 \\ y \end{bmatrix} = \begin{bmatrix} W_1 & -W_1 P \\ 0 & W_2 P \\ 1 & -P \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{4.30}$$

The closed-loop transfer function is given by the lower linear fractional transformation $F_1(P, K)$ as

$$F_1(P, K) = \begin{pmatrix} W_1 \\ 0 \end{pmatrix} + \begin{pmatrix} -W_1 P \\ W_2 P \end{pmatrix} K(1 + PK)^{-1} = \begin{pmatrix} W_1 S \\ W_2 T \end{pmatrix} \tag{4.31}$$

In controller design process, the objective is to minimize $\|F_1(P, K)\|_\infty$

$$\|F_1(P, K)\|_\infty = \left\| \begin{pmatrix} W_1 S \\ W_2 T \end{pmatrix} \right\|_\infty \leq 1 \tag{4.32}$$

Where S and T are sensitivity and complementary sensitivity function respectively.

4.5.3. Selection of the Weight functions

The performance of the H-Infinity controller is dependent basically on the selection of the weight function. There are different approaches to select these weight functions. One approach [20] is given as:

1) The performance weight function W_1 is in the given form:

$$W_1 = \frac{s + M\omega_B}{M(s + \emptyset\omega_B)} \tag{4.33}$$

Where

1.1) Steady-state offset less than $\emptyset$;

1.2) Closed-loop bandwidth higher than ω_B;

1.3) Amplification of high-frequency noise less than a factor M;

2) The second strategy to determining performance weight function is given here:

$$W_1 = \frac{\eta s+\lambda}{(s+\xi\lambda)} \quad (4.34)$$

The parameter η , which is limited to $0 \leq \eta < 1$, is the high-frequency limited value of W_1 , serves to constrain the maximum resonance peak in the ideal frequency response. The parameter $\lambda > 0$ gives the 0 dB crossing of W_1 when ξ is small. The parameter λ is suggested to be related to the steady-state error of the system.

There is a different approach to determine the weight functions is suggested by [22]

A) For performance weight function

$$W_1 = \frac{s+a}{(bs+0.02)} \quad (4.35)$$

Where

a) a influence the singular value characteristics of S (sensitivity function) and T (complementary sensitivity function).
b) b influence the singular value characteristics of S.

B) For robustness weight function

$$W_2 = \frac{(Ls+1)(0.5Ls+1)}{2} \quad (4.36)$$

Where

a) L is a constant and influence system complementary sensitivity function.

5.1. Introduction

In this chapter, Simulations results are shown. There are different Simulation results applied with Passive system, PID controller system, QFT controller system and H-Infinity controller system. The Problem with the system and control theory is discussed based on the simulation results. All simulations are done with the help of latest version of MATLAB simulation software.

5.2. Passive System Simulation

The Passive system consists of different parts such as sprung mass, un-sprung mass, body spring, wheel spring and damper. The Passive system simulation is done with the parameters in Table 5.1:

Table 5.1. Passive system's Parameters values

m_s	body mass (sprung)	290 kg
m_u	wheel mass (unsprung)	59 kg
k_s	spring constant (body)	16 182 N/m
k_t	spring constant (wheel)	190 000 N/m
F	control force	(in kN)
c_s	damping ratio of the damper	1000 Ns/m
r	Road disturbance	(in m)
x_s x_u	body and wheel travel	(in m)

The simulation model of the quarter car suspension system is drawn with the help of its motion's equations.

The simulation file is shown below:

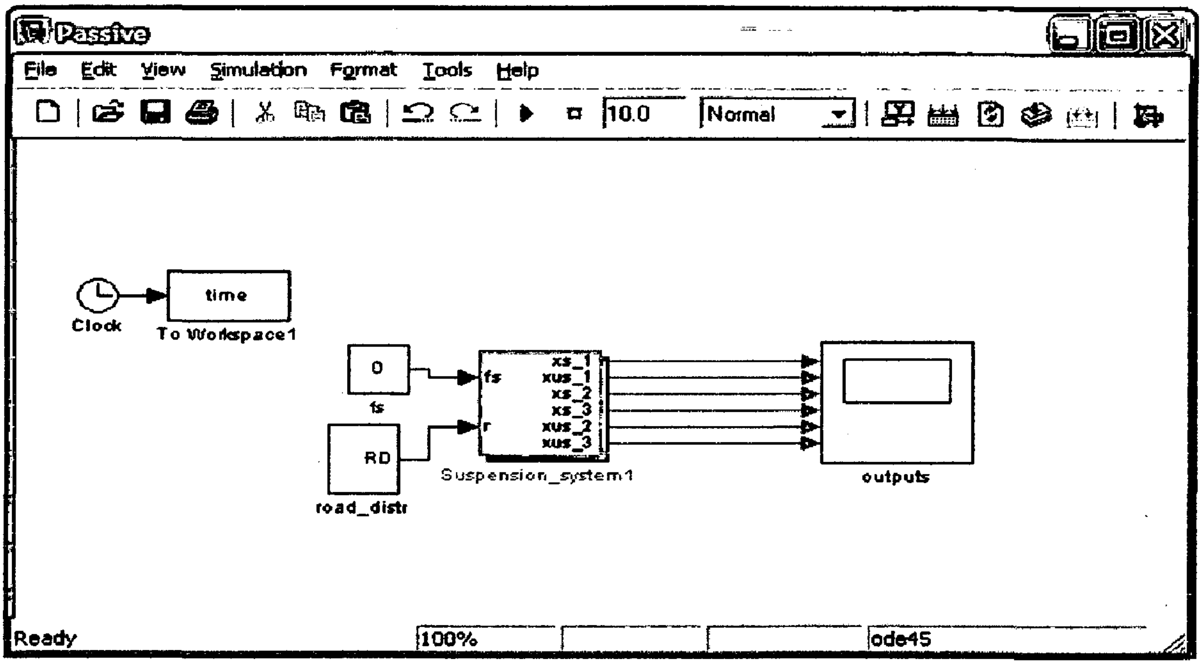

Fig.5.1. Simulation Block

There is clearly shown that the system is having two inputs such as Road disturbance and control force. Also there are different outputs taken to fulfill the simulation purpose they are such as sprung mass (car body mass) displacement and its velocity and acceleration and un-sprung mass (wheel body mass) displacement and its velocity and its acceleration. The dissertation work results are discussed only with displacement of sprung mass.

5.2.1. Step Response

It can be seen from Fig.5.1. that the system response is affected due to two inputs i.e. actuating control force and road disturbance. Let see the effect of these two inputs in the system response

Fig. 5.2 and Fig 5.3 show the system response due to both control force and road disturbance, respectively,

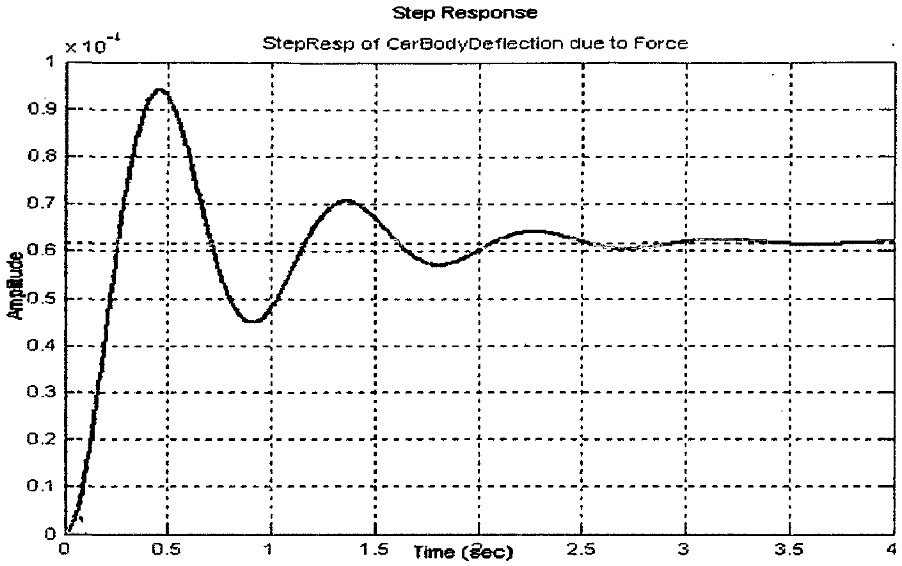

Fig.5.2. Step Response of the system due to control force

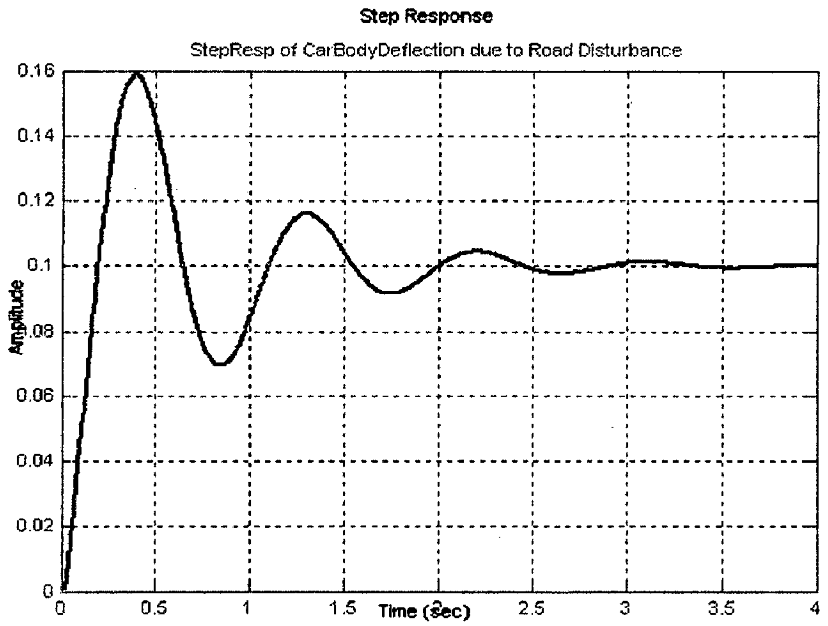

Fig. 5.3. Step response of the system due to road disturbance

Now, Fig. 5.2 shows that the system response due to force is required gain where the system response due to road disturbances is under-damped and its posses a overshoot and oscillation in its response.

This Overshoot and oscillation are responsible for making ride uncomfortable and these are to be controlled for better ride and safety of the vehicle.

5.2.2. Frequency Response

The frequency response of the system due to road disturbance is taken with *bode* MATLAB command. This response is shown in the below figure:

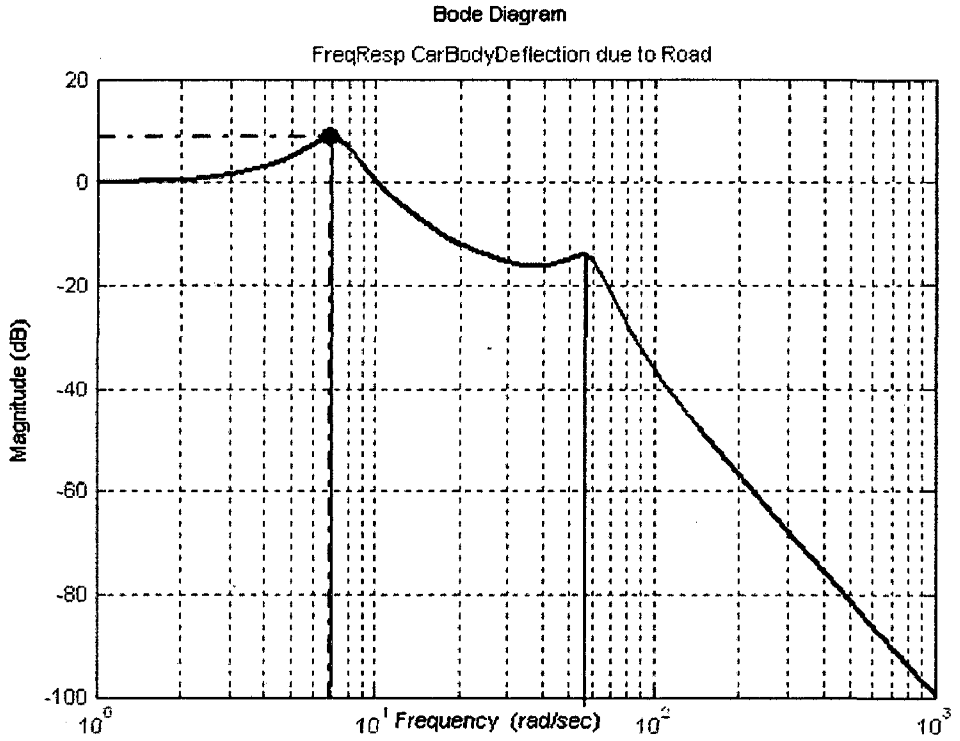

Fig.5.4. Frequency response (Bode plot) due to road disturbance inputs

As shown in Fig. 5.4, There are two peak points i.e. resonance points, shown in the figure. The first point is due natural frequency of the car body, the sprung mass, vibrations and the second is due to natural frequency of the wheel mass, the un-sprung mass, vibrations.

If resonance occurs in the system it can be very harmful – leading to eventual failure of the system. Consequently, one of the major step for vibration control is to predict when this type of resonance may occur and then to determine what steps to take to prevent it from occurring. As the amplitude plot shows, changing damping can significantly reduce the magnitude of the vibration. Also, the magnitude can be reduced if the natural frequency can be shifted away from the forcing frequency by changing the stiffness or mass of the system.

5.2.3. Effect of Parametric variation

As we consider the variation in the parameters of the system, then there is effect of this variation on the response of the system. Let see the variation in the responses.

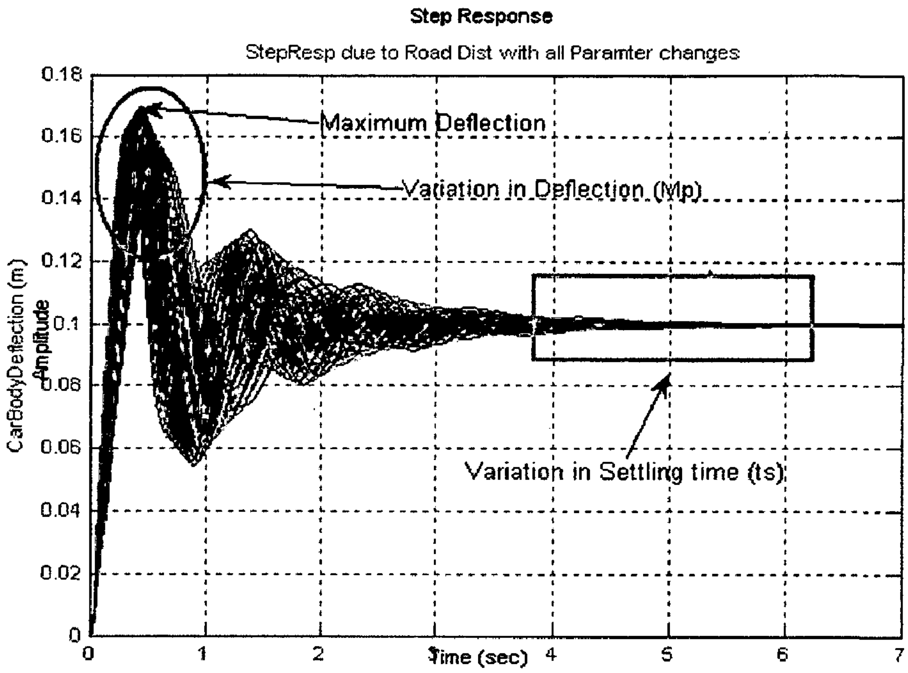

Fig.5.5. Variation in Step Response of the system

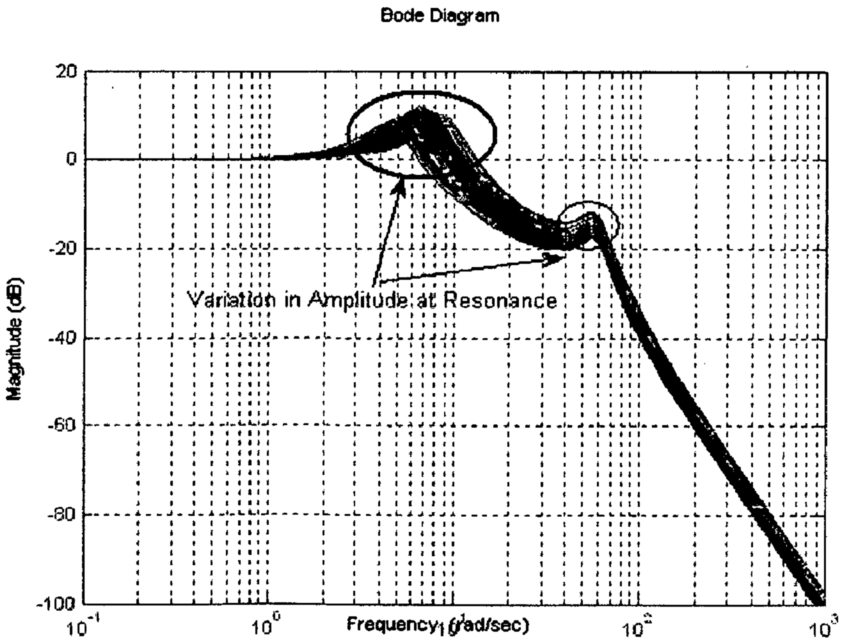

Fig.5.6. Variation in Frequency response of the system

The Fig. 5.5. shows that there is a variation in the oscillation of the car body (sprung mass) due to its parametric uncertainties which are present in the spring constant and sprung mass. The Maximum Deflection and settling time both get affected due to this variation. As sprung mass increases the maximum deflection and settling time increases. Also, as spring constant increases the maximum deflection and settling time increases. In further section, the effect of individual parameter is discussed.

Fig. 5.6 , shows that the parametric uncertainties are also responsible for the variation in the frequency response. Due to this variation, the maximum amplitude at the resonance frequency is varying accordingly.

Variation in sprung mass ms keeping body spring constant

We have seen the variation in both parameters simultaneously on the system response. Now let see the effect of uncertainties in individual parameter. So that we can conclude that which parameter variation is more affective for the system response.

To see this effect on the system, we keep one parameter constant and vary the other parameter.

Let start with the body spring constant, ks, which keep constant and change the value of sprung mass, ms. The following figures are showing mass variation effect for different values of spring constant.

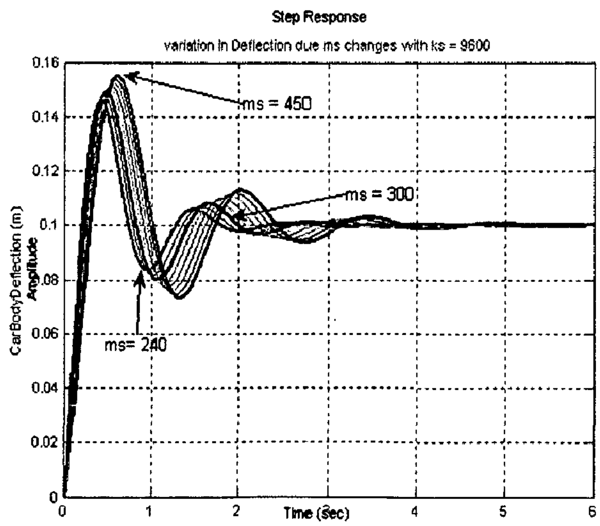

Fig.5.7. Variation in response due to changing mass ms with spring ks = 9600

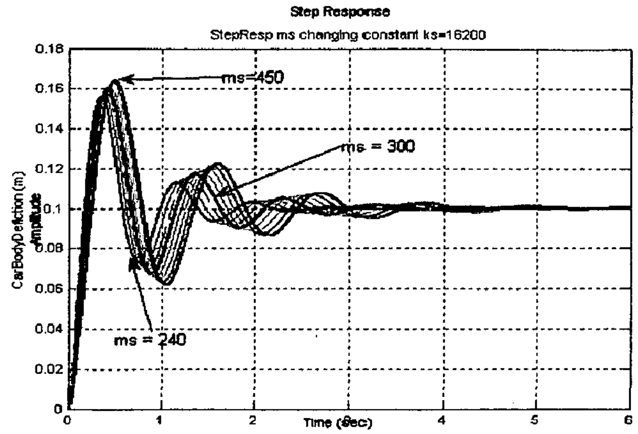

Fig.5.8. Variation in response due to changing mass ms with spring ks = 16200

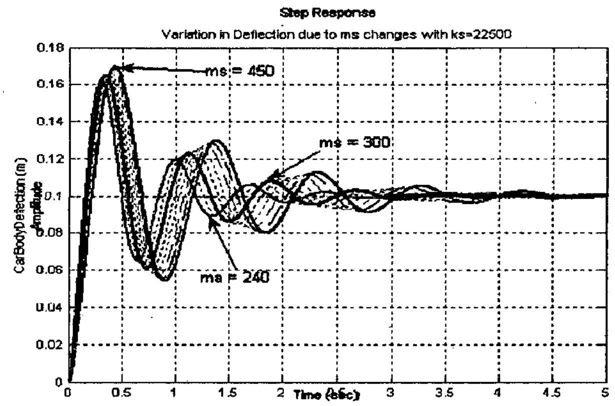

Fig.5.9. Variation in response due to changing mass ms with spring ks = 22500

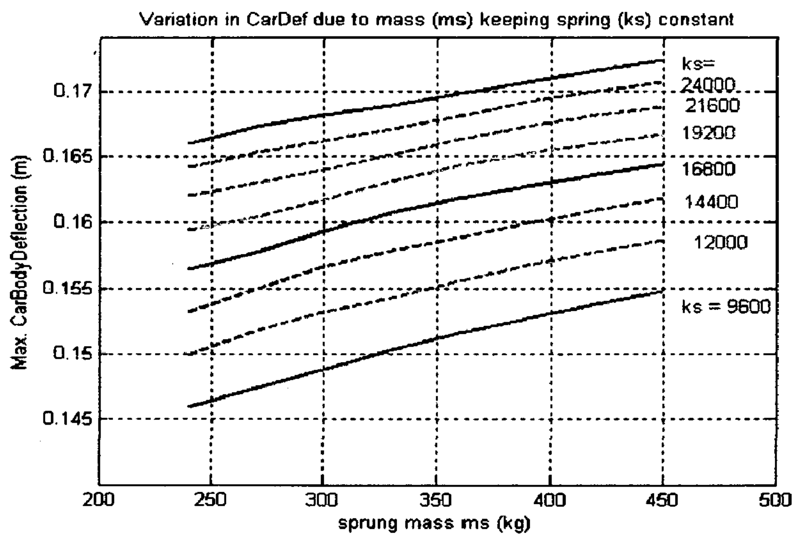

Fig.5.10. Maximum Deflection variation in mass ms with different spring ks

Variation in spring constant ks keeping sprung mass constant

Now we change the sprung mass ms constant and vary the body spring ks constant. Let see the effect of variation in this parameter.

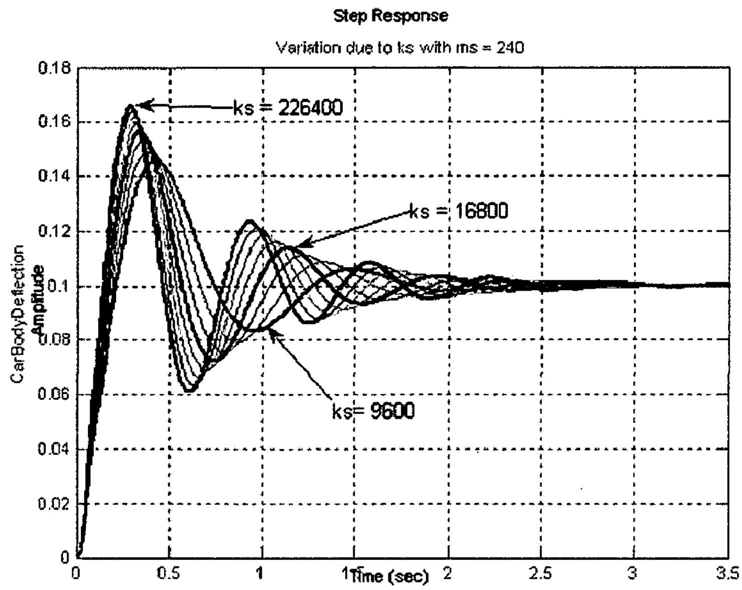

Fig.5.11. Response of variation in spring ks with sprung mass ms = 240

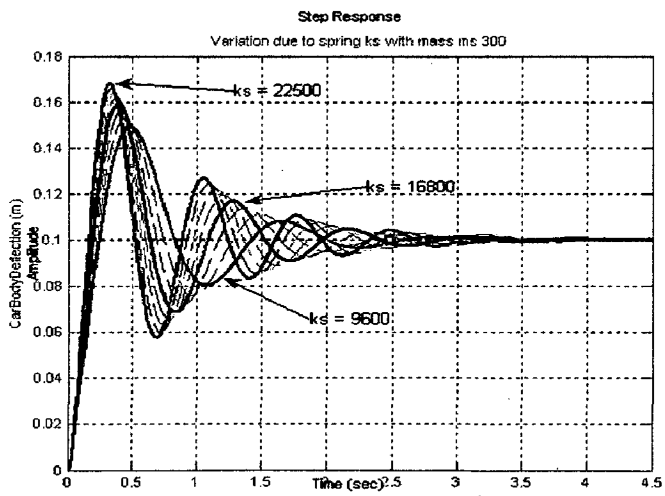

Fig.5.12. Response with variation in spring ks and sprung mass ms = 300

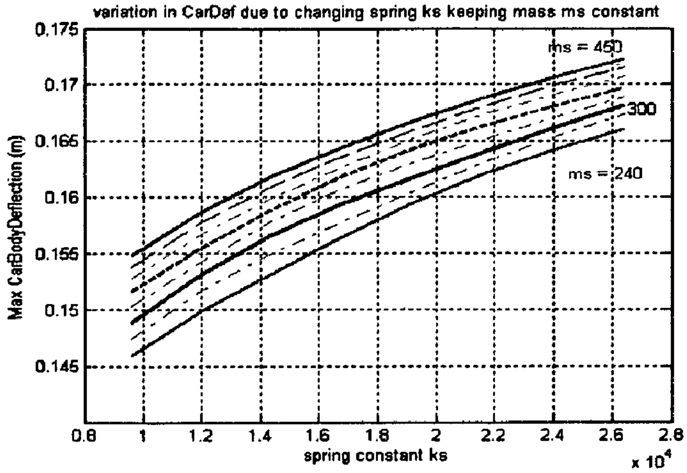

Fig.5.13. Max. Deflection with variation in spring ks and sprung mass ms

5.2.4. Observations

There are some observations based on the simulations of the suspension system. These are:

1) Sprung mass and spring constant both parameters are equally affect the system dynamics, as changes in these parameters result in varying the damping of the system.
2) The maximum deflection and oscillation are directly proportional to the sprung mass. As mass increases the maximum deflection of the car body i.e. sprung mass increases.
3) Again, the maximum deflection and oscillation are directly proportional to the spring constant. As spring constant increases the maximum deflection increases.
4) For the system with higher values of spring constant, the effect of variation in sprung mass is less. But the maximum deflection and oscillation are more as compare to the system with lower values of spring constant
5) As both parameters change, the maximum deflection and oscillation changes. So the Drive comfort and drive safety are also affected due to this variation. As oscillation increases the drive comfort decrease.

5.2.5. Problems

The performance of the passive system is dependent on the system as they are unable to adapt or re-tune to changing parametric uncertainties over time. So we aim to design a robust controller which would give a desired performance and drive comfort as well.

5.3. PID Controller Simulation Results

The Block diagram for the system with controller is shown below:

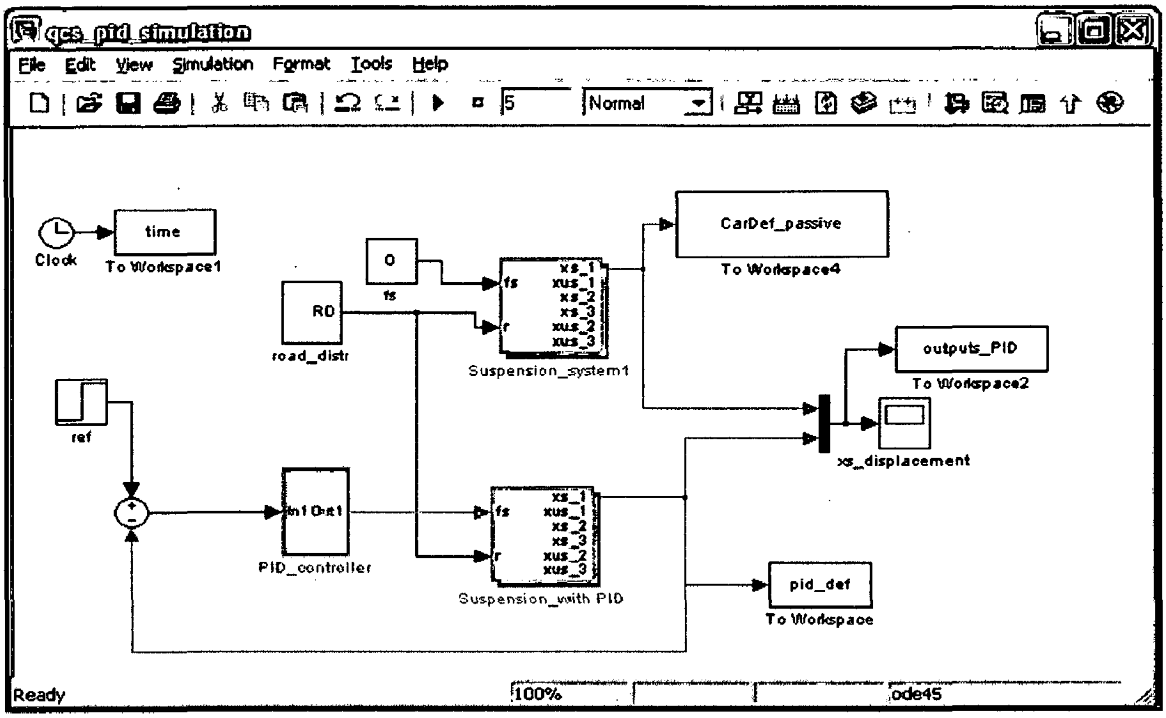

Fig.5.14. Simulink Block diagram for PID controller

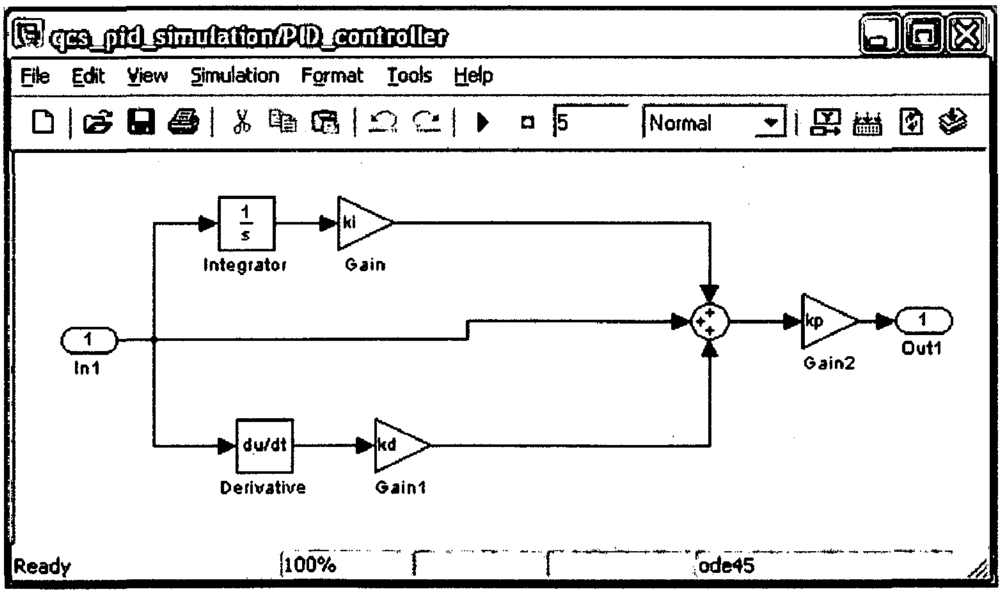

Fig.5.15. Simulink block of PID configuration (ideal)

5.3.1. Step Response

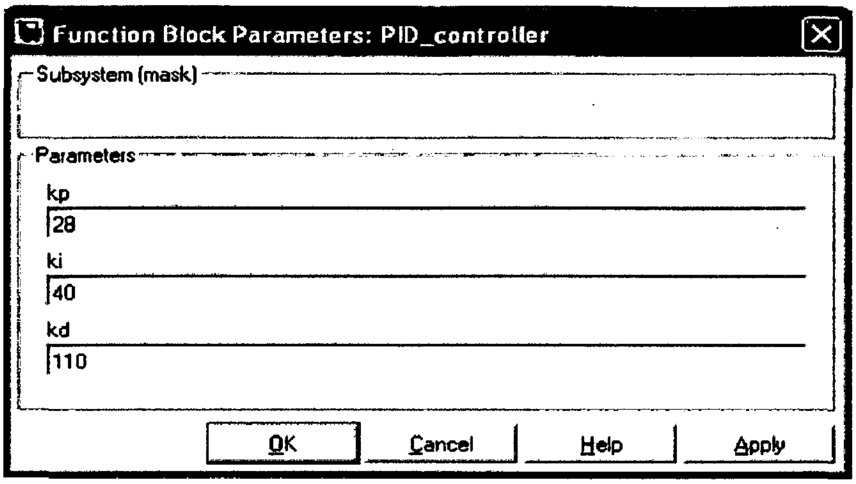

Fig.5.16. PID parameters values obtained manually

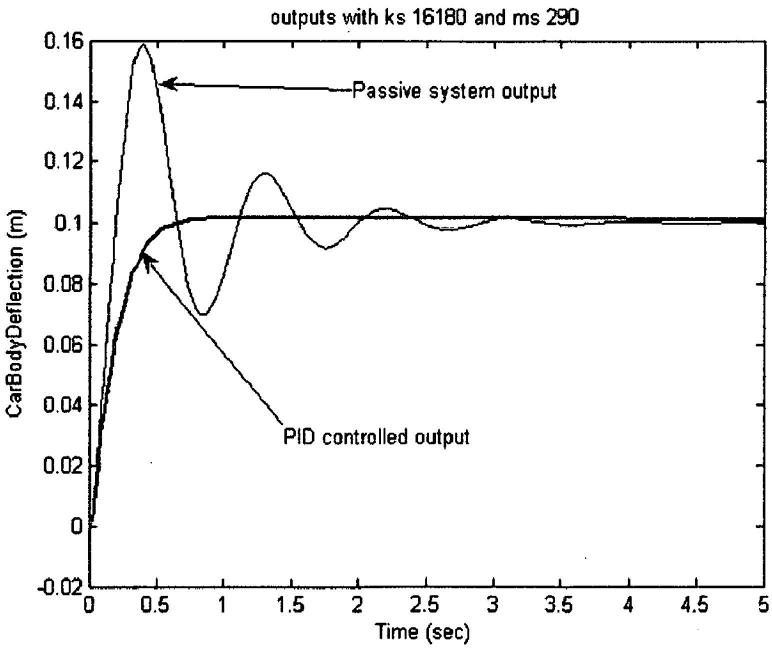

Fig.5.17. Suspension system output with PID controller

A PID controller is design with the above mention parameters values, these values are obtained by manually tuning of the parameters of PID controller. The Ideal configuration is used to implement it as shown in Fig.5.16.

5.3.2. Effect of Parametric Variation

Now Let see the effect of variation of the parameter on PID controller. The following figures show the output of the passive system and the PID controlled system with least and highest values of parametric uncertainty.

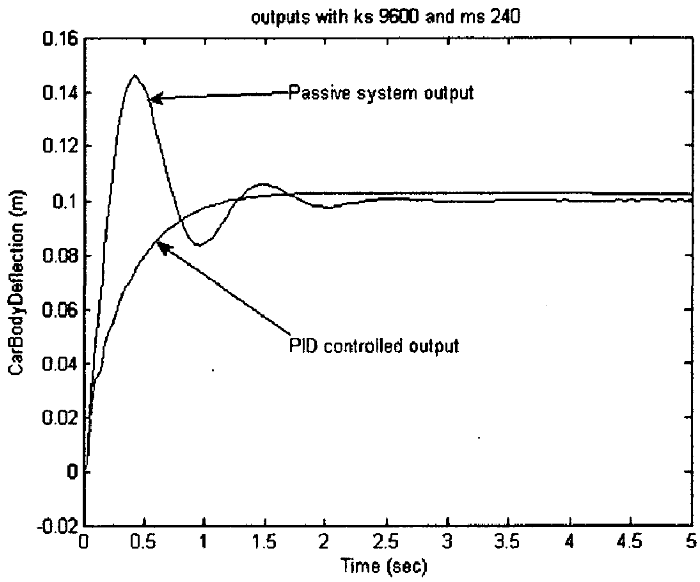

Fig.5.18. Step response with PID for ms 240 and ks 9600

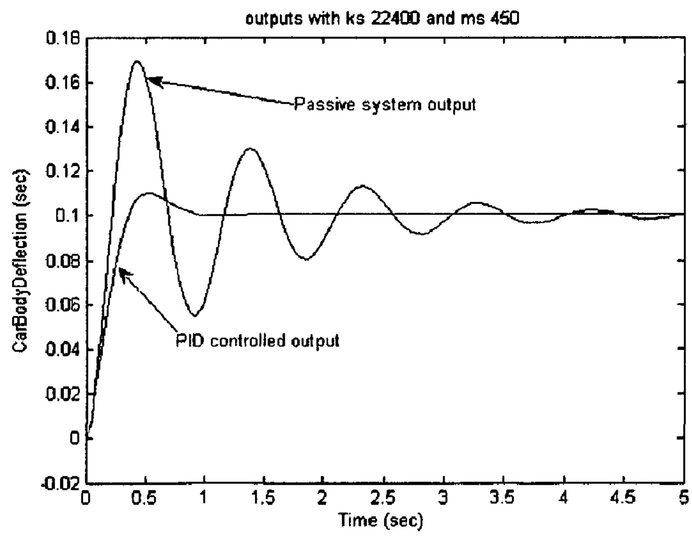

Fig.5.19. Step response with PID output for ms 450 and ks 22400

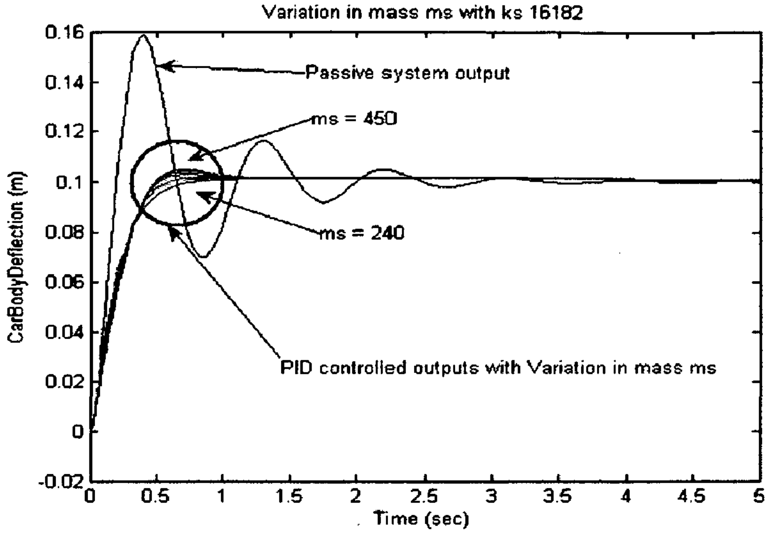

Fig.5.20. PID controlled output with ks 16182 and varying ms

The above figures show the output of PID controller with different values of sprung mass ms and spring constant k_s which show that there are no big changes in the response. but for low values of k_s its shows a certain steady state error while for high values of k_s , it works fine.

Fig. 5.20 shows output variation with the changes in sprung mass ms keeping spring constant ks. There is no big sprung mass variation on the system and the mass variation effect is suppressed by PID controller. That means the robustness of PID controller is exists.

5.3.3. Observations

The some observations and Problem based on the implementation of PID are as follows:

1) P increases overshoot, D decreases overshoot and increase rise time, I increases settling time and overshoot.
2) Different combination values of PID parameter do not show the robustness behavior.
3) PID control design involves only one requirement at a time either reference tracking or disturbance rejection.

5.3.4. Problems

The problem in the design of PID controller is that the manual tuning of PID parameters requires more time.

5.4. QFT Controller Simulation Results

5.4.1. QFT Design Technique

There are simulations results shown as per the step of QFT technique discussed in Chapter 4. The simulation results of those steps are as follows:

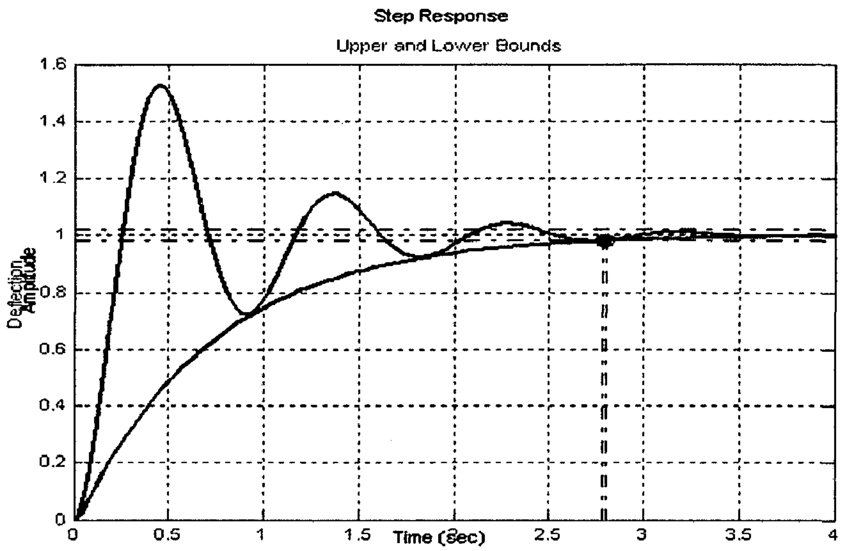

Fig.5.21. Upper and Lower Bounds Generation

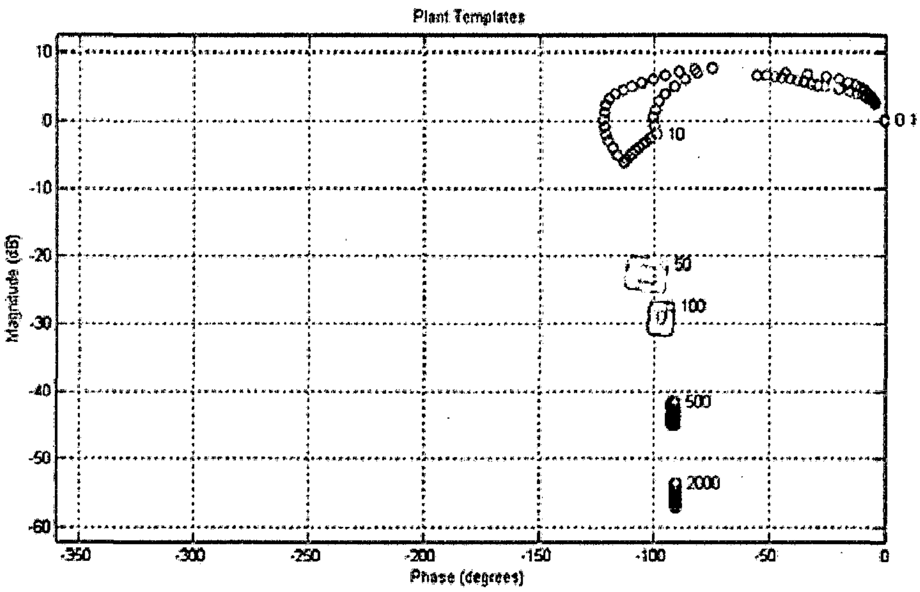

Fig.5.22. Plant Templates Generation

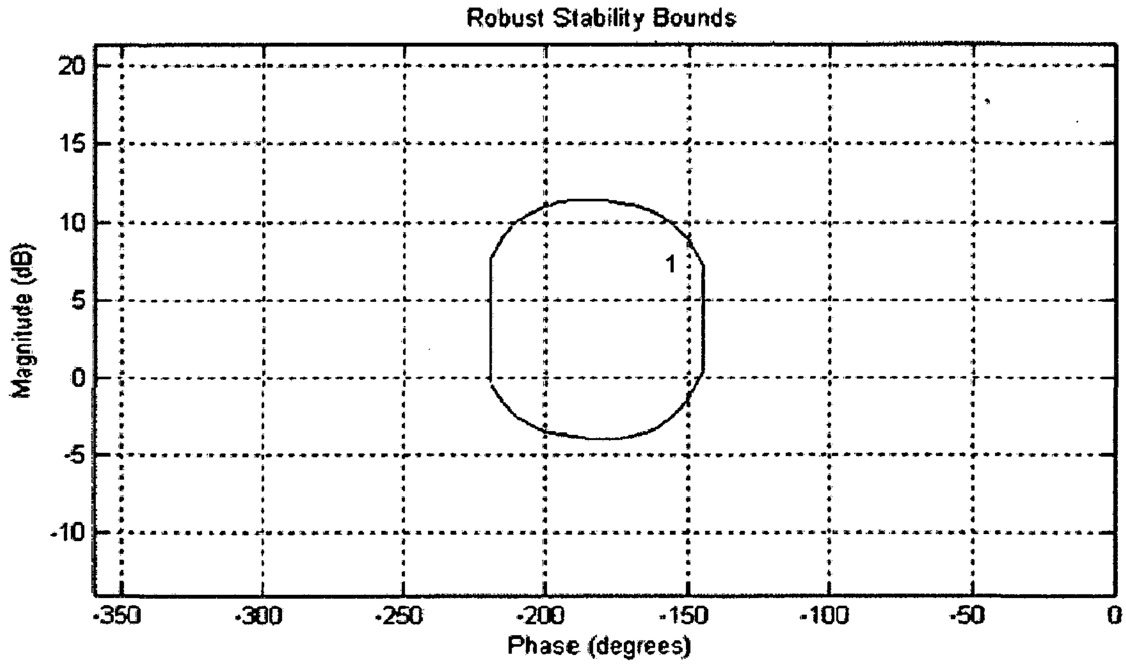

Fig.5.23. Robust Stability Bounds

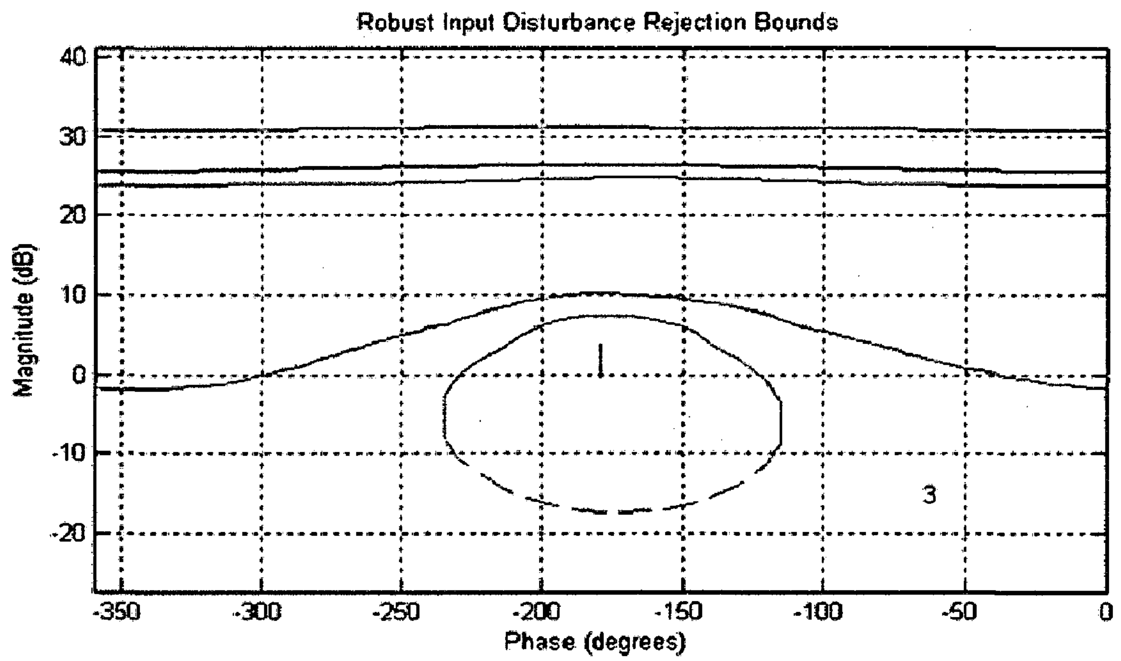

Fig.5.24. Robust Input Disturbance Rejection Bounds

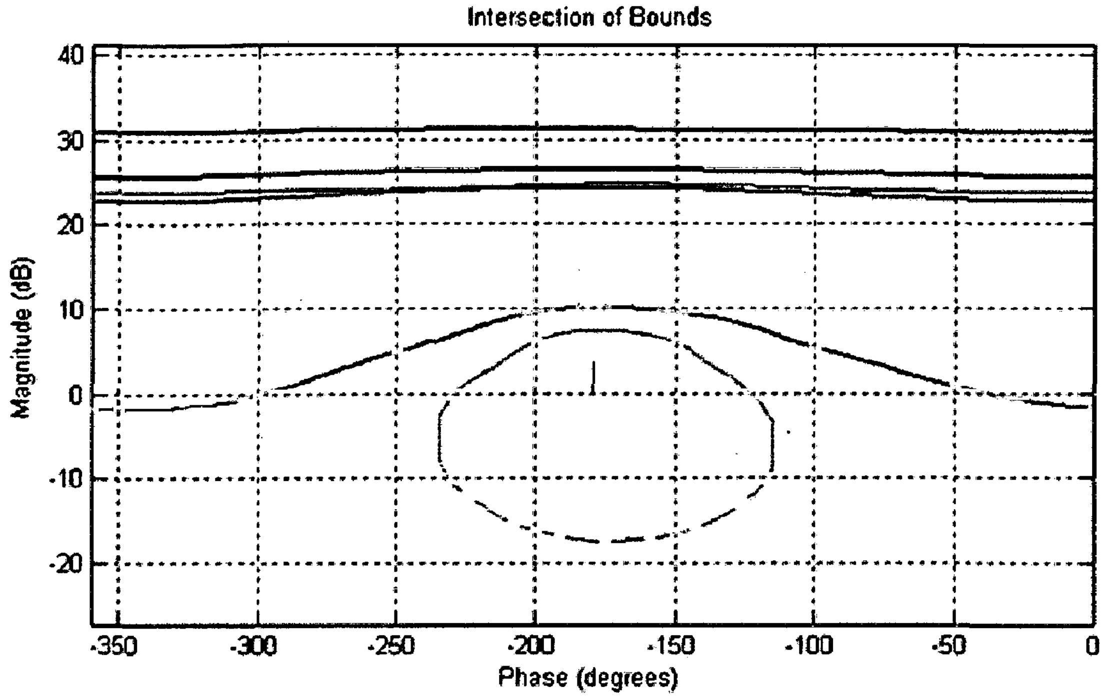

Fig.5.25. Intersection of QFT Bounds

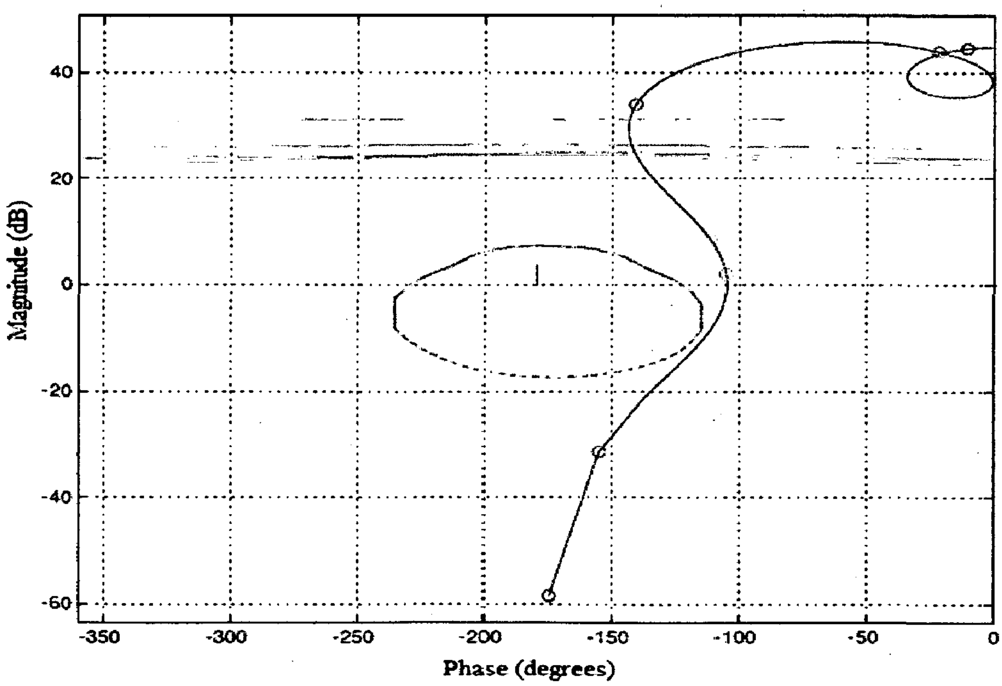

Fig.5.26. Loop-shaping Response

Now, with the step by step simulation the QFT controller is generated.

The controller design specifications are assumed as follows:

Maximum overshoot in %: Mp < 30 %

Settling time, sec: Ts < 2 secs

The QFT controller is obtained as:

$$G(s) = (48250(s + 105)(s + 0.01))/((s + 48)(s + 40))$$

5.4.2. Step Response

Fig. 5.27 shows that the QFT controller achieves the desired response with the specific parameters of the suspension system.

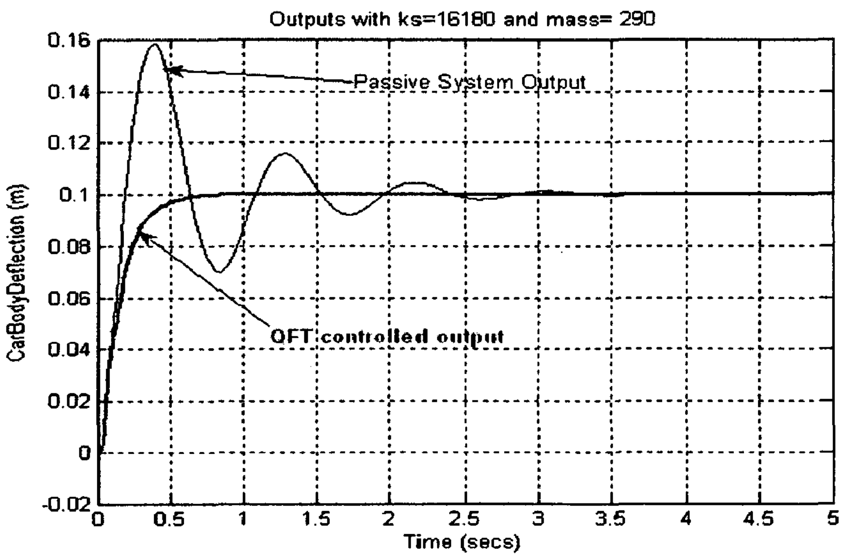

Fig.5.27. QFT controlled Step Response

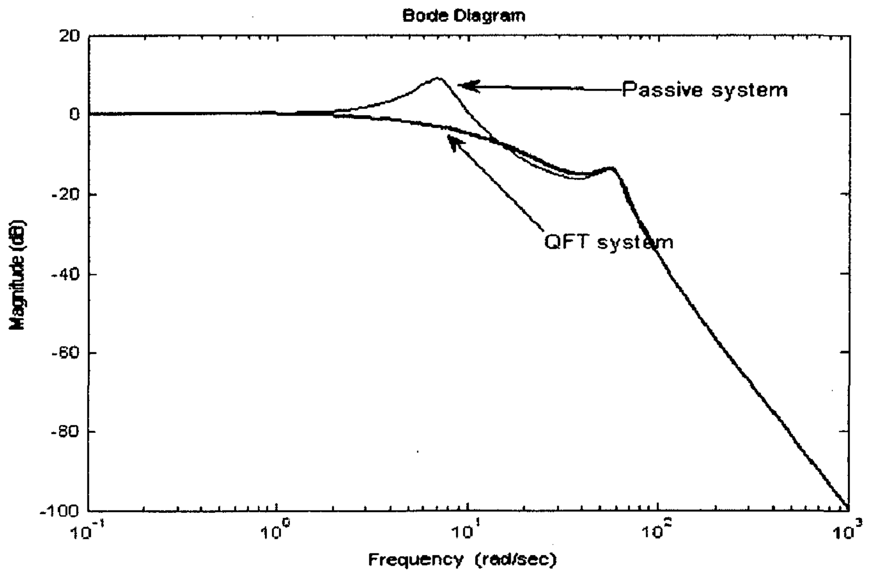

Fig.5.28. QFT controller Frequency Response

QFT controller also reduce the amplitude at resonance frequency as shown in Fig.5.28. The Robustness of the controller is to be checked when the parameters uncertainties occur in the system.

5.4.3. Effect of Parametric Variation

The parameters values are changed in the simulink block. The different results are obtained as such :

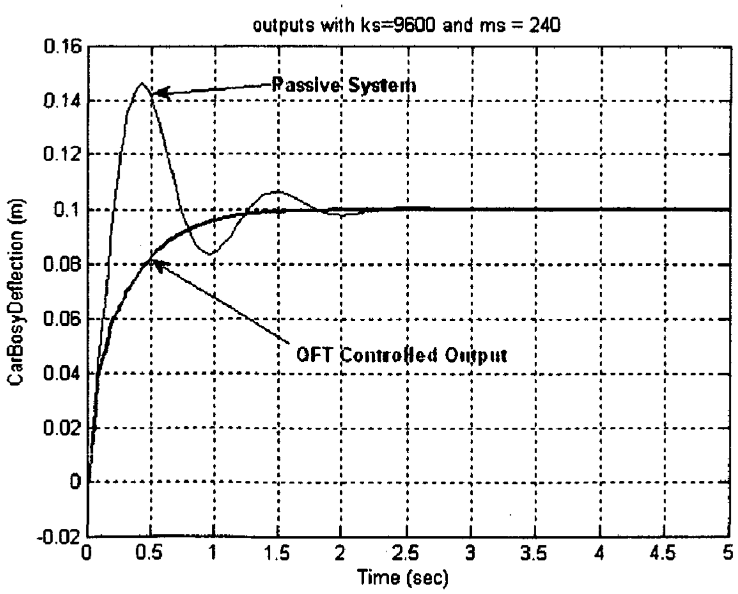

Fig.5.29. Step Response with ms=240 and ks=9600

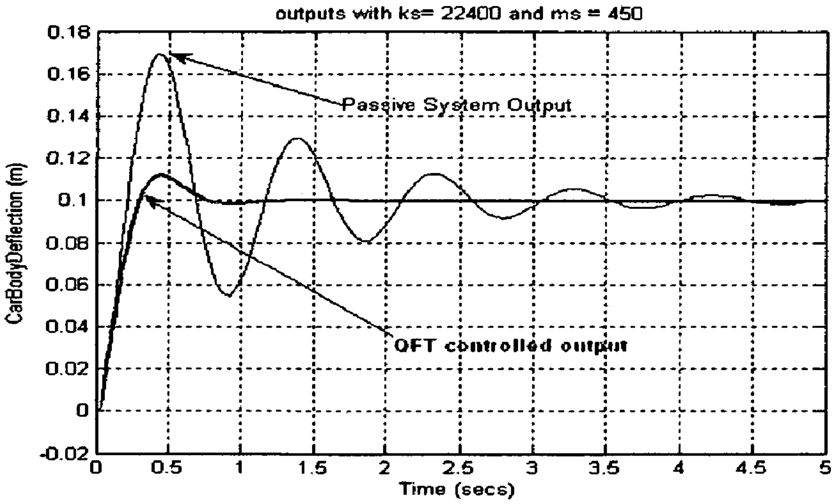

Fig.5.30. Response with ms=450 and ks=22400

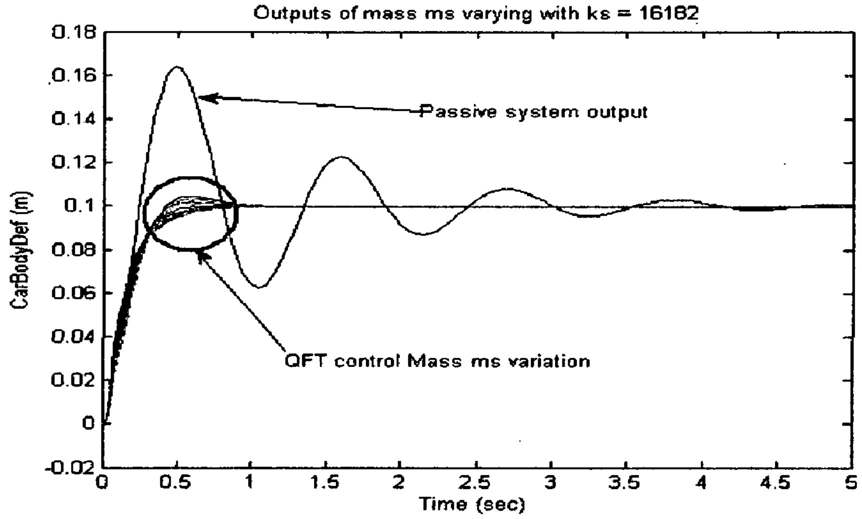

Fig.5.31. Response varying with sprung mass ms and ks = 16182

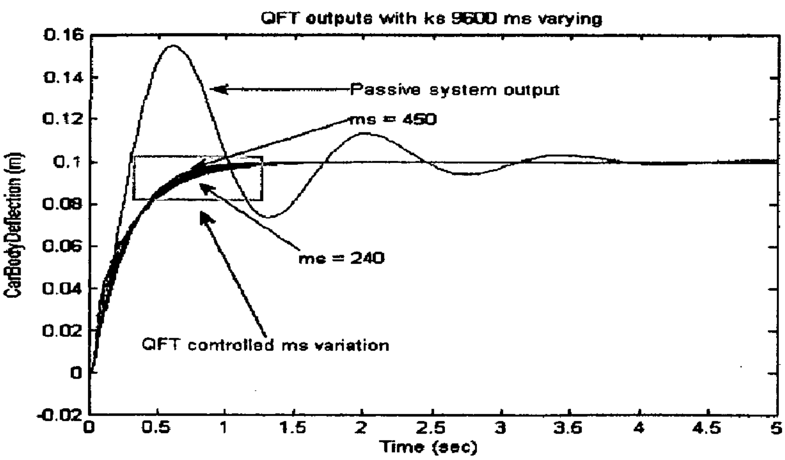

Fig.5.32. Response varying with sprung mass ms and ks = 9600

QFT controller is also robust controller and it performs well for the parametric uncertainties problem such as discussed problem. Fig. 5.31 and Fig. 5.32 show that the variation in sprung mass ms and for different spring constant ks 16182 and 9600, respectively. The sprung mass variation is suppressed well by QFT controller. The next section will discuss the comparison between PID controller and QFT controller.

5.4.4. Observations

The some observations and Problem based on the implementation of QFT are as follows:

1. As we decrease the gain of QFT controller, produces the less control force because of this control strategy is less effective while as we increase the gain causes more oscillation in response and also un-stability in the system.
2. As value of zero approaches to origin, the system behavior improves. While increase in value of poles causes the increase the overshoot of the response.

5.4.5. Problems

The problem with the implementation of QFT control is that it involves a good approach to choose the values of zeros-poles for the loop-shaping to achieve the desired performance.

5.5. Comparison between PID and QFT techniques

The comparison between these two techniques is discussed in this section. The comparison is based on the technique methodology and Robustness.

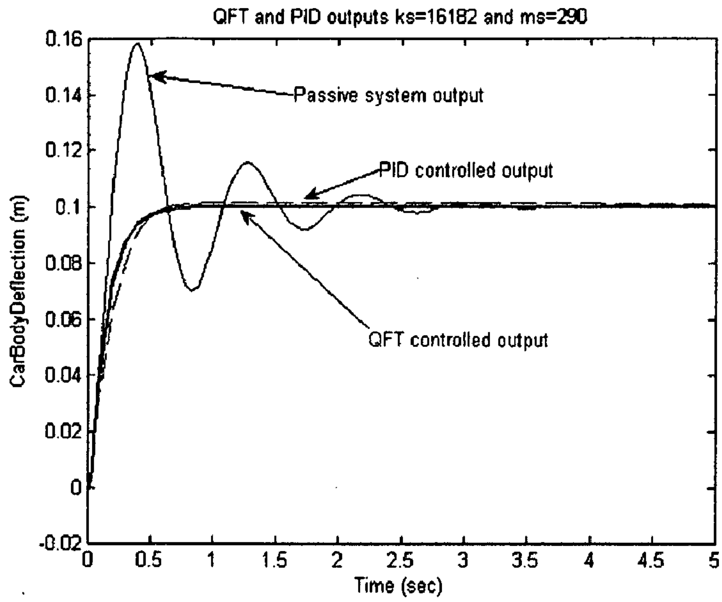

Fig.5.33. PID and QFT controlled step response

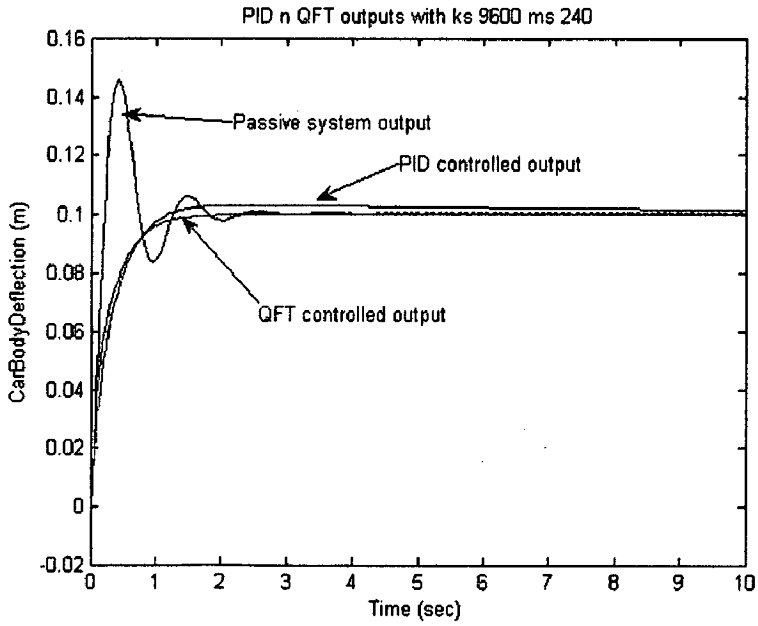

Fig.5.34. PID and QFT step response with ms = 240 and ks = 9600

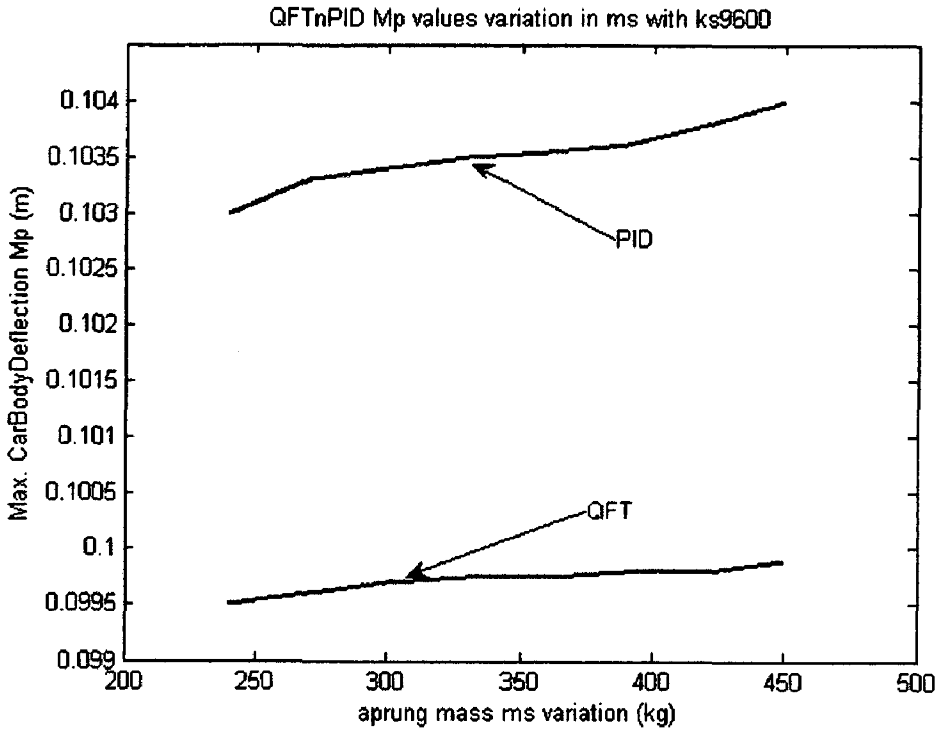

Fig.5.35. Varying Max. deflection with sprung mass and ks = 9600

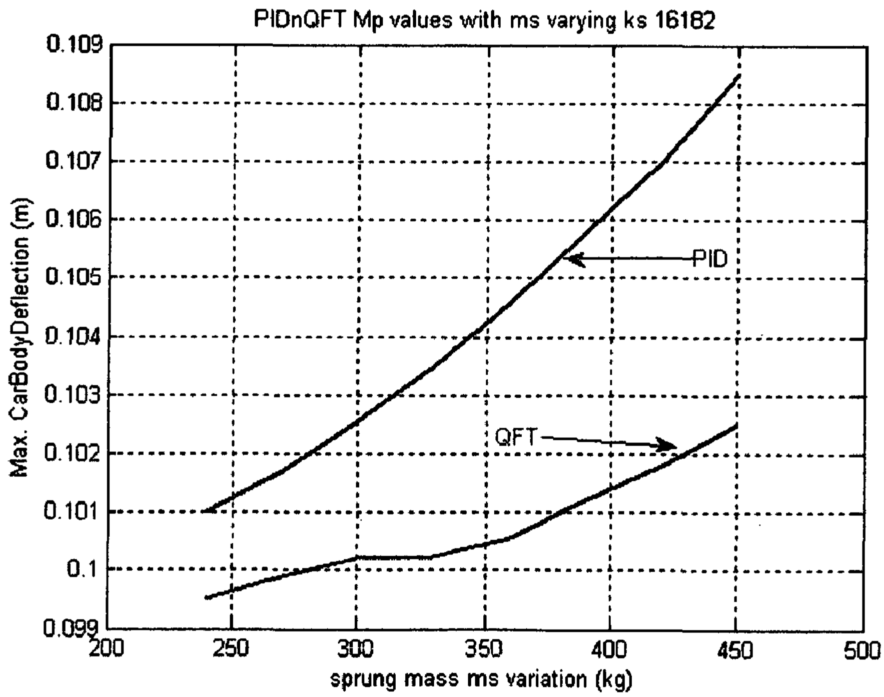

Fig.5.36. Varying Max. deflection with sprung mass and ks = 16182

Fig. 5.33 and Fig. 5.34 shows, the output of both controllers at different values of sprung mass as well spring constant. The outputs of both controllers are desirable. Fig. 5.35 and Fig. 5.36 shows, the mass affect for the particular value of spring constant on the maximum deflection of sprung body. The variation effect of sprung mass is faster in PID controller as compare to QFT controller.

5.5.1. Conclusion of Comparative study:

The study of PID and QFT controller is based on the simulation results. The following observations are:

a) PID is a time-domain based controller design technique while QFT is a frequency-domain based controller design technique.
b) PID controller introduces one pole and two zeros in the forward path while in QFT controller technique, design trade-offs at each frequency are transparent between stability, performance, and controller complexity (order).
c) In PID technique, the time taken to design controller is based on tuning of Proportional, Derivative and Integral Gains while QFT technique, the time taken for controller design can be reduced, and controllers can be redesigned to cope up with any changes in specifications and uncertainties.
d) In PID technique, control design involves only one requirement at a time either reference tracking or disturbance rejection. while in QFT controller can meet many performance requirements simultaneously.
e) QFT controller is more robust to a wide range of plant parameter variations than PID controller.

5.6. H-infinity Controller Simulation Results

5.6.1. H-infinity controller

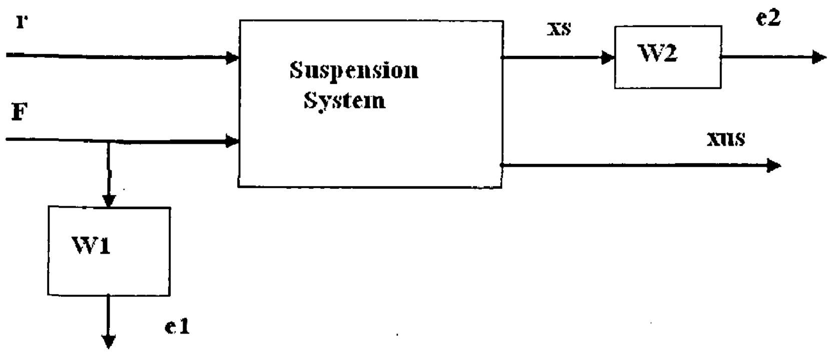

Fig.5.37. Block Diagram for H-infinity synthesis

After providing the suitable weight function MATLAB command *hinfsyn* synthesis H-infinity controller. The generated controller's numerator and denominator are given here

Numerator = [-35.35 -9.50e4 - 1.32e7 -6.66e8 - 1.7e10 s - 1.834e11]

Denominator = [1 2.234e5 2.496e7 9.578e8 2.158e10 2.508e11 3.14e10]

Once the H-infinity controller obtained, it can be placed with the system as shown in the following figure

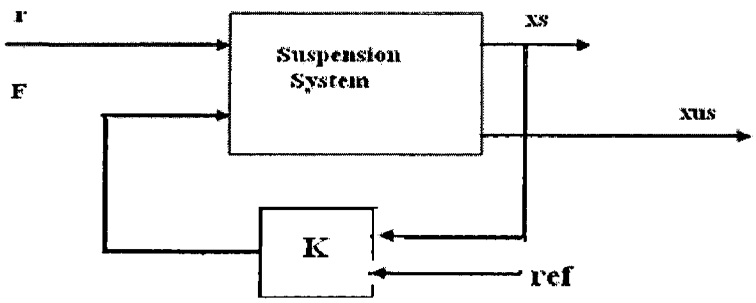

Fig.5.38. H-infinity controlled suspension system

5.6.2. Step Response

The H-infinity output is shown in the following figure. The desired output is achieved with this controller. Fig 5.39 shows the suspension system with h-infinity respond very smoothly. Fig. 5.40 shows that H-infinity attenuates the resonance frequency amplitude effectively.

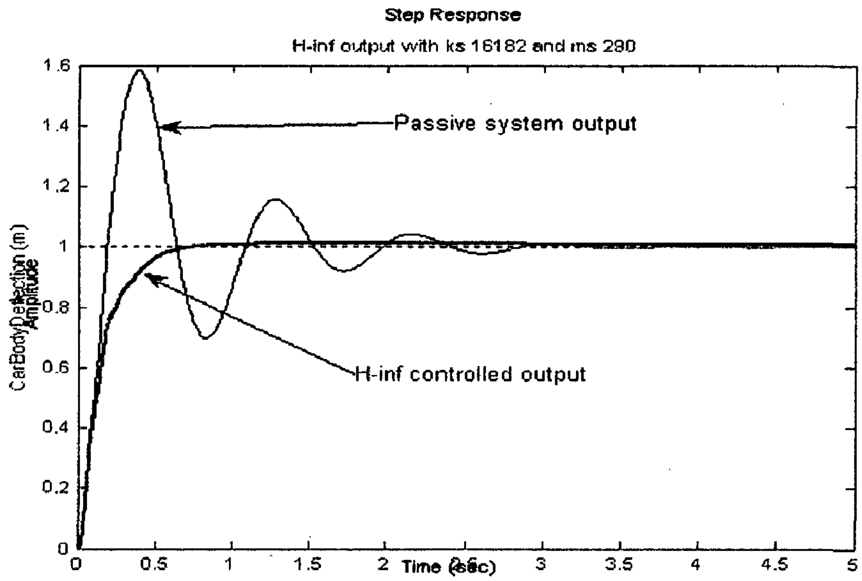

Fig.5.39. H-infinity controlled output for the system

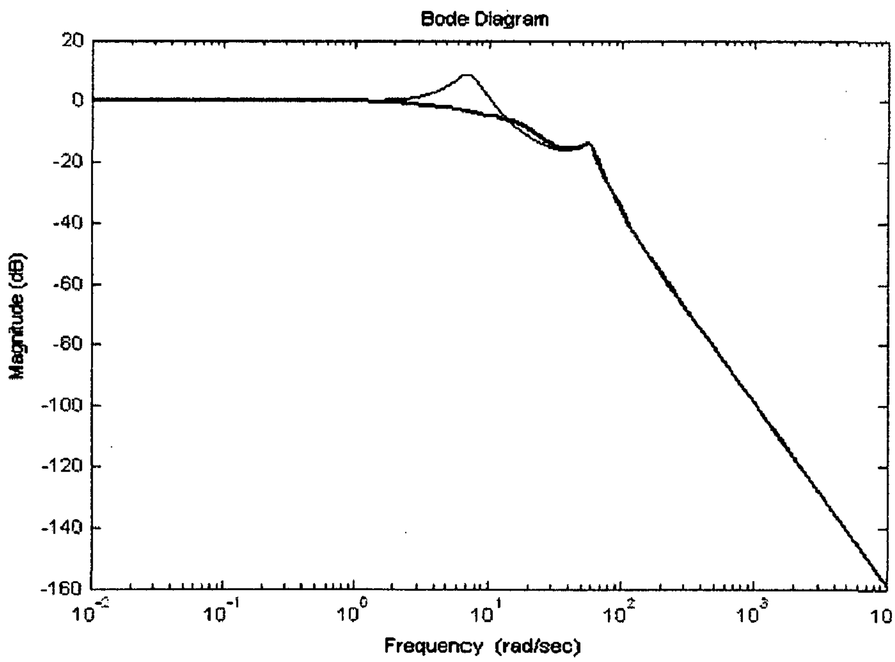

Fig.5.40. H-infinity controlled frequency response for the system

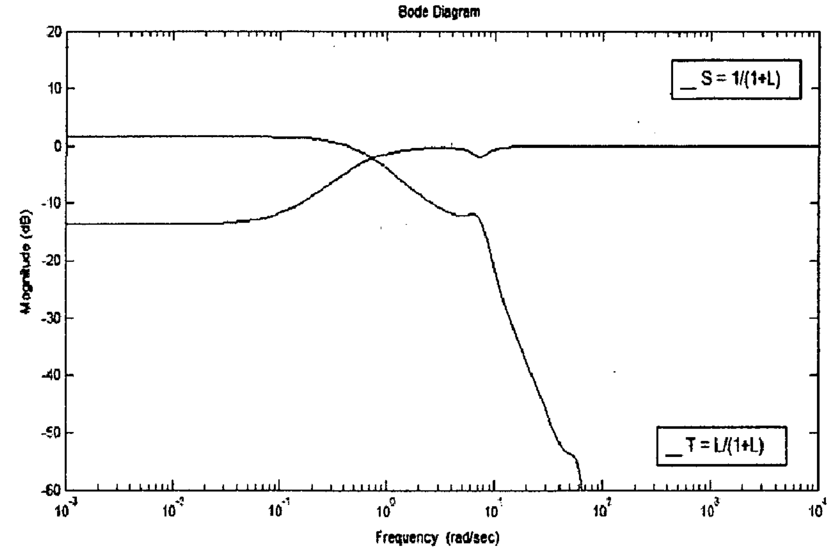

Fig.5.41. Sensitivity and Complementary Sensitivity Function Plot

The frequency response of the sensitivity function and complementary sensitivity function is shown in Fig.5.41. The frequency response of the sensitivity plot suggests that the sensitivity S can be made small at low frequency but when frequency increases the value of S reaches to the highest value. Which also implies that the frequency response of the system and controller decrease at high frequencies.

5.6.3. Effect of Parametric Variation

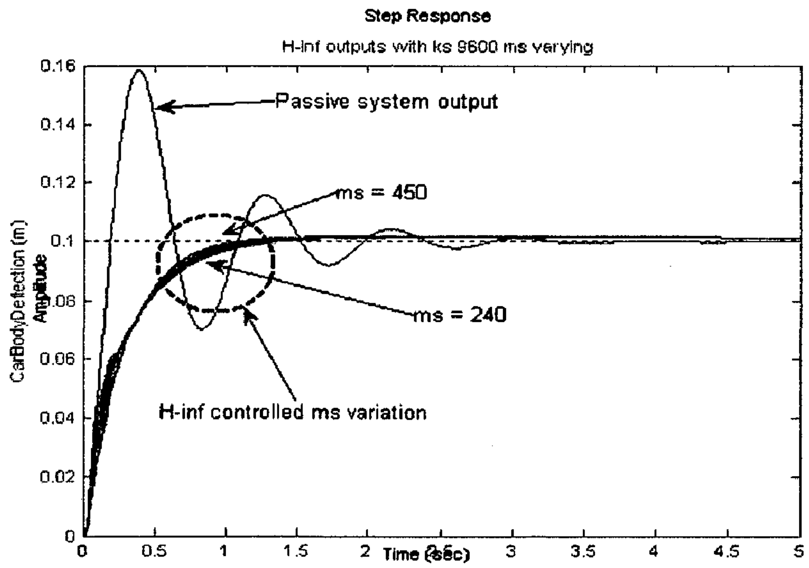

Fig. 5.42. Response with sprung mass variation for spring constant ks = 9600

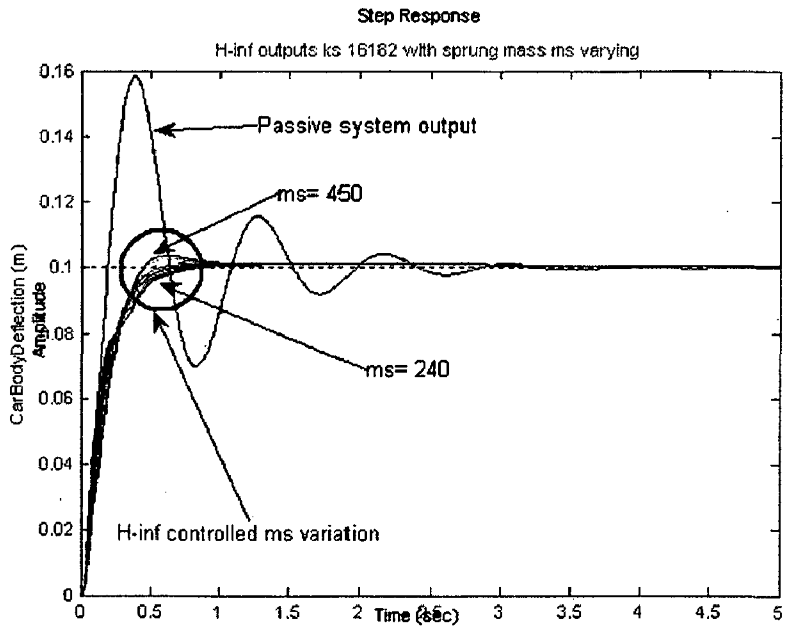

Fig 5.43. Sprung mass variation effect for spring constant ks = 16182

5.6.4. Observations

The some observations based on the implementation of H-infinity are as follows:

1) The higher value of λ increase the rise time of the response.
2) The higher value of ξ causes the increment in the steady-state error.
3) The lower the sensitivity function S, the better the performance
4) The small value of T, uncertainties can be handled effectively.

It is found from the previous work that there are no rules are given to choose the appropriate weight function for the system. Due to this finding an algorithm in H-infinity control design process is difficult. Trial and error method is used to find out the appropriate weight functions and to synthesize controller. So a systematic algorithm is proposed to this trial and error process of the weight function selection on the basis of these observations of the implementation:

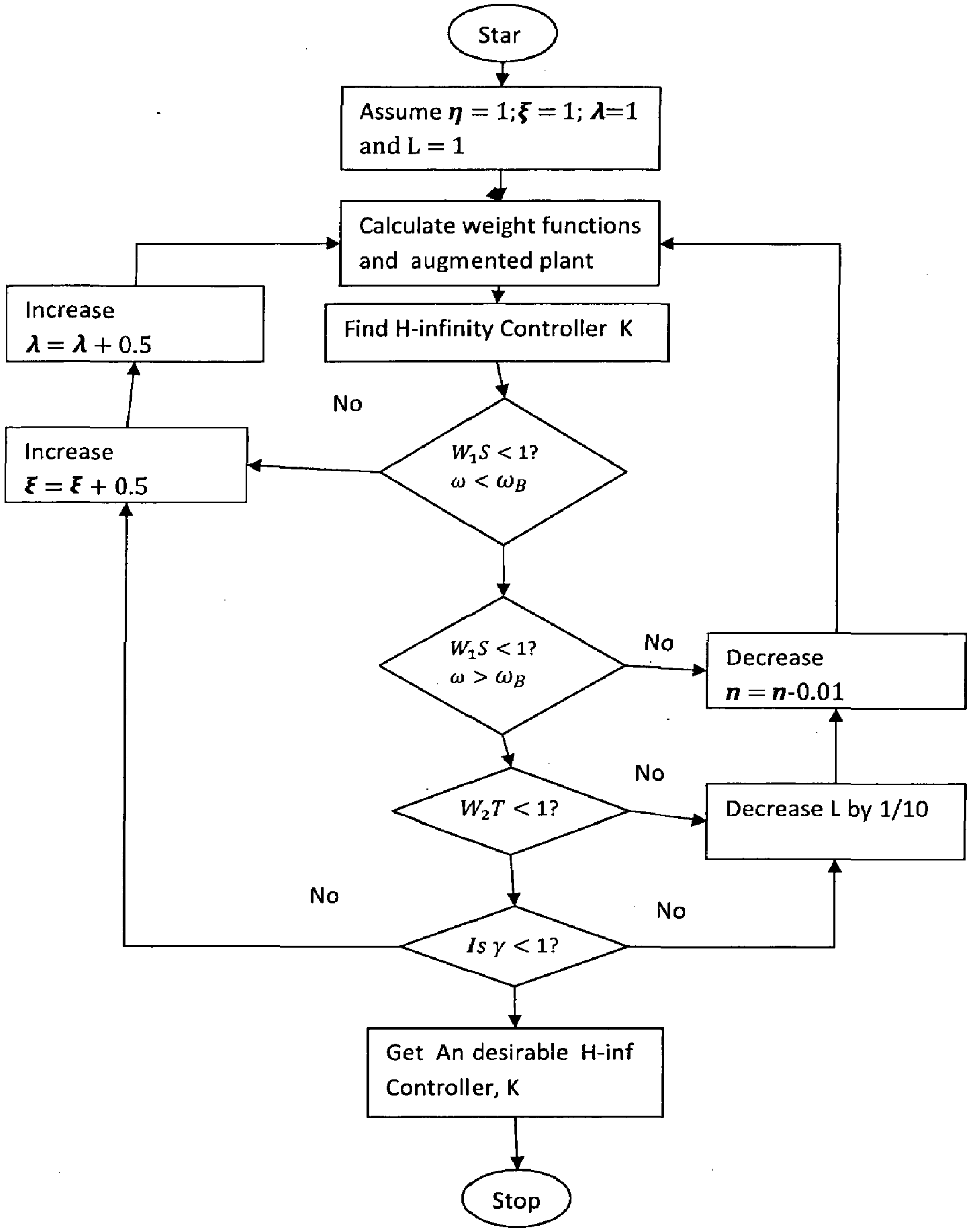

Fig.5.44. A Designed algorithm for selection of Weight functions

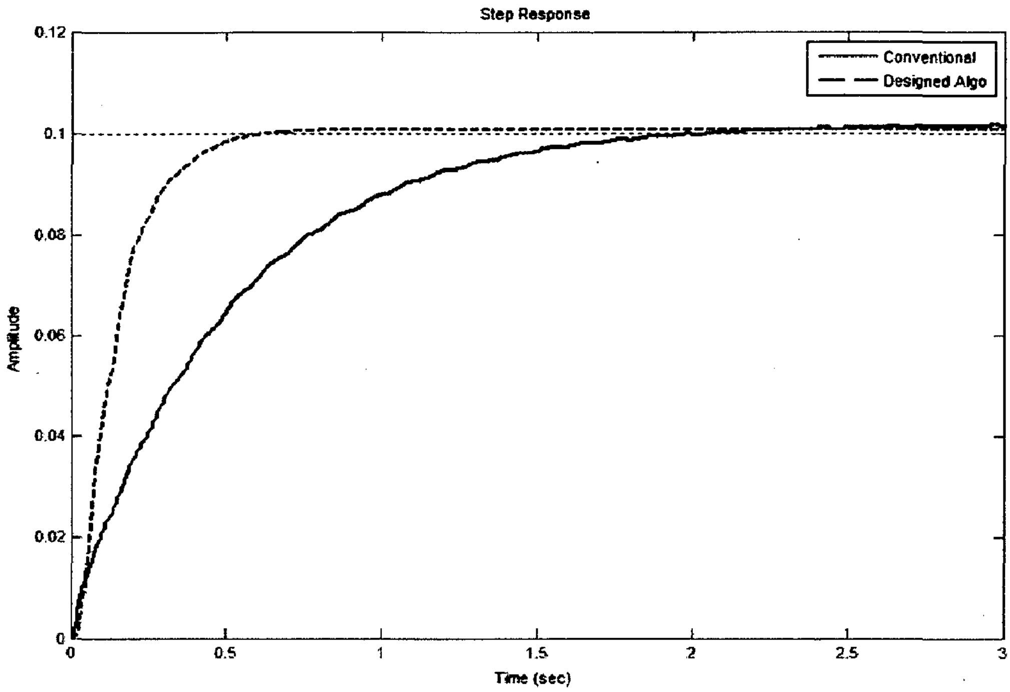

Fig.5.45. Step Response from the Designed Algorithm

Fig. 5.45. shows that the different weight functions are responsible for the different outputs. The performance of the controller depends upon the process how appropriate weight functions are chosen. In the above figure, the output is obtained from the designed algorithm is compared to the response obtain from the work [18]. The performance parameter ISE obtained is 0.0139 and 0.0287 respectively.

5.6.5. Problems

The some problems based on the implementation of H-infinity are as follows:

1) There can be different combinations of weight functions occur. These different combinations produce different optimal controllers. So it is difficult to decide criteria to obtain optimal controller.
2) As we know, S + T = 1; so there is a trade-off between a small sensitivity function S for good performance and a small complementary sensitivity function T for robustness. It is difficult to make both functions small simultaneously at the same frequency.

5.7. Comparison between H-infinity and QFT control techniques

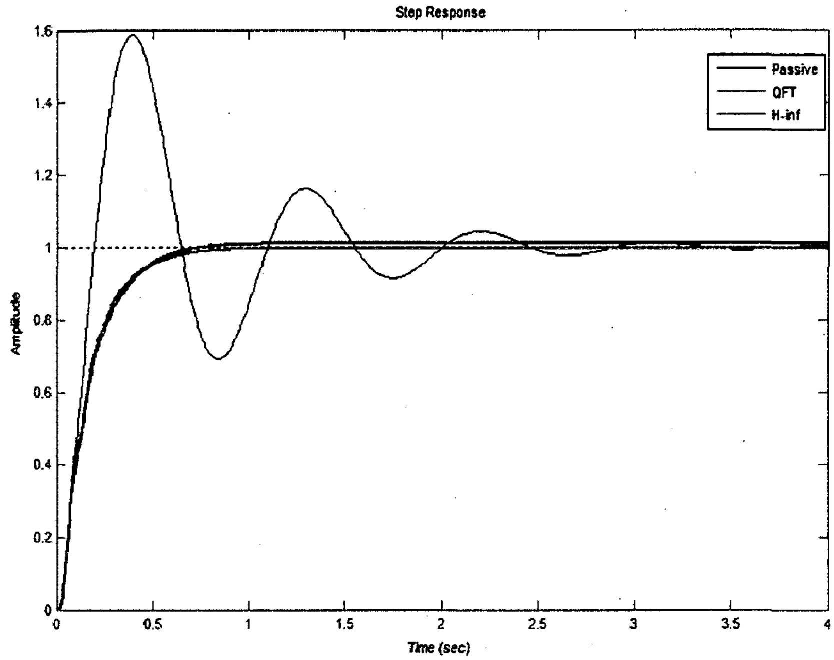

Fig.5.46. Step Response of system with QFT and H-inf controller

Table 5.2. ISE values for QFT and H-inf controllers

Spring Constant (ks) /Sprung Mass (ms)		ISE		
		240	300	450
QFT	9600	0.0174	0.0182	0.0202
	16182	0.0136	0.0144	0.0166
	22400	0.0123	0.0131	0.0154
H-inf	9600	0.0166	0.0175	0.0198
	16182	0.0132	0.0141	0.0165
	22400	0.0120	0.0129	0.0154

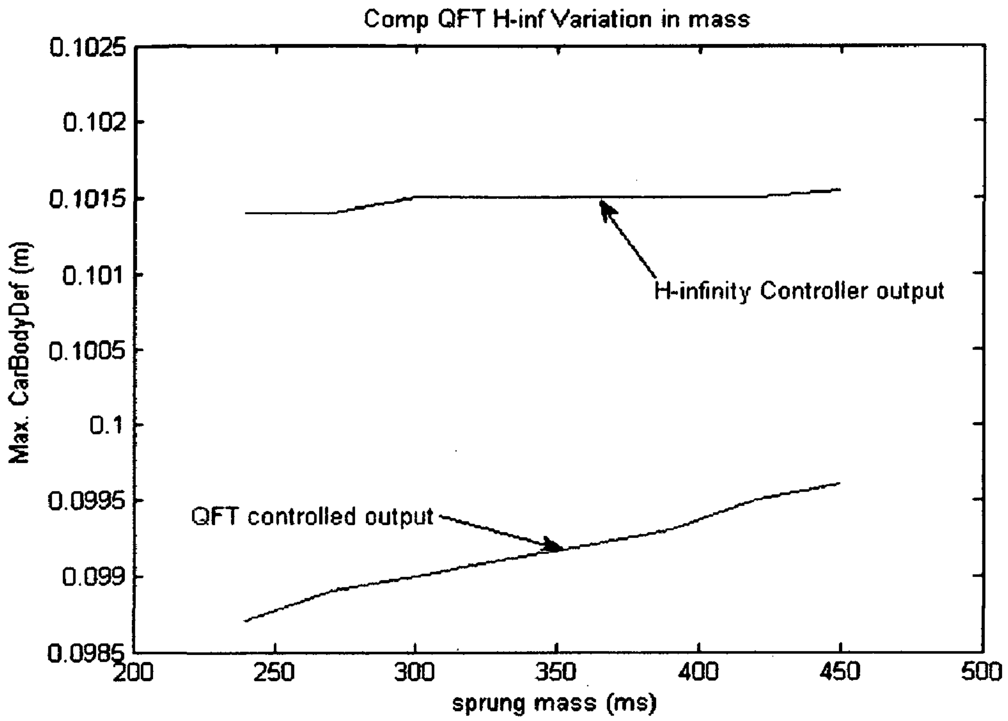

Fig.5.47. Comparison of Max Deflection with sprung mass ks = 9600

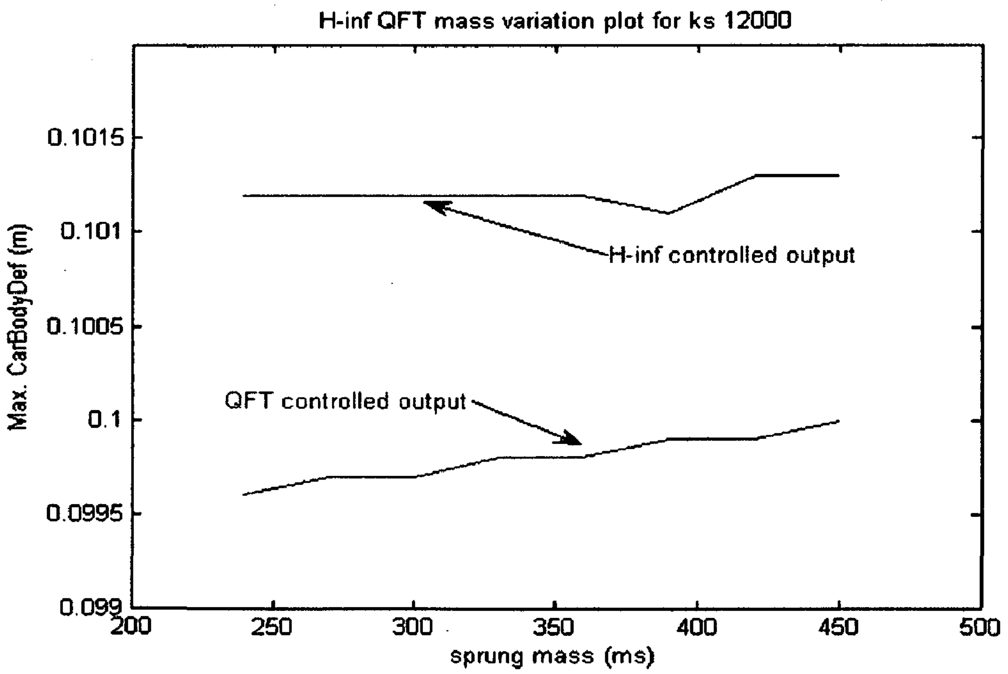

Fig.5.48. Comparison of Max Deflection with sprung mass ks = 12000

H-infinity controller and QFT controller both are robust controller with parameters variation. Fig.5.47 and Fig.5.48 shows that maximum deflection of the car body with respect to changes in sprung mass for the different values of spring constants. The plots show that the both controllers produce output within less error even in parameters variations. But the deflection amplitude is more than provided input (0.1) in case of H-infinity control.

Also the ISE table 5.2 shows the obtained ISE value for different sprung mass and spring constant for QFT and H-infinity controllers. ISE values for H-infinity controllers at different combinations of sprung mass and spring constant are almost equal to ISE values for QFT controllers. Similarly, ISE values change for QFT with respect to the change in sprung mass for different spring constant nearly equals to ISE values change for H-infinity. This means H-infinity designed controller handles uncertainties little better than QFT designed controller. QFT designed controller for the studied system is performs little better than H-infinity designed controller in terms of ride comfort.

5.7.1. Conclusion of Comparative study

The study of the QFT controller and H-Infinity controller is based on the simulation results. The following observations are:

1) Both techniques are basically frequency domain techniques. Although, QFT requires whole plant set information in terms of its magnitude and phase along with parameter ranges. While in H-infinity requires only nominal plant and the uncertainties.
2) In QFT, the design specifications are converted to the upper and lower bounds for the loop-transmission gain, while H-infinity, the design specification converted as its norms on the closed loop gain.
3) H-infinity designed controllers are generally higher order compensators. Hence implementation is difficult and cost is high. While QFT designed controllers can be obtained with low order, which can be implemented easily and also cost is low.
4) Both techniques involve difficulties in their methodology such as QFT involves the loop-shaping of loop-transmission gain by changing the values of zeros-poles. While H-infinity involves the selection of weight functions to achieve the desired performance and robustness in controller.

6.1. Conclusion

Suspension system deals with road disturbance and variation in its parameters. The variation in the parameters of the system affects the comfort and the safety of the vehicle. It is necessary to reduce the effect of this variation.

Sprung mass and spring constant are the two factors which affect the vehicle comfort and as sprung mass increases the amplitude of vibration also increase. Again, the maximum deflection and oscillation are directly proportional to the spring constant. As spring constant increases the maximum deflection increases.

PID performs well for the system but it has difficulty with the proper tuning of its parameters. Also PID is not a robust controller for a wide range of variation. QFT is robust controller design technique for parametric uncertainties. On comparing between two controller design techniques, the simulation results with QFT prove more effective than PID. The difficulty with QFT is that it involves loop-shaping process of the loop-transmission function.

H-infinity control design technique is also a robust controller design technique. It involves the minimization of sensitivity function and complementary sensitivity function directly. On comparison with QFT, the simulation results of H-infinity showed that both controllers can work effectively with the parameter variations. Moreover, the selection of weight functions is like a process of loop-shaping.

Hence, the performance of the controller with QFT and H-infinity design techniques can be same. The choice is depends upon the control engineer.

6.2 Future Scope

Application of various new optimization techniques opens a doorway to improve the performances. The performance of H-infinity controller can be further improved by the Application of new optimization techniques such as Genetic algorithm, Pattern search, in selecting the weight functions.

The future research would include the controller complexity reduction using optimizing techniques. This will help in reducing the controller order while keeping intact the behavior of original controller.

References

1. A Book on "*Theory of Ground Vehicles*" by J. Y. Wong, Prof. Carleton University, Canada, 4th Edition, John Willey & Sons Inc., NY
2. M. Appleyard, P. E. Wellstead, "*Active Suspension : some background* ", IEEE Proceedings Control theory Application, March 1995, vol. 42, no.2, pages:123-128
3. D. Karnopp, "*Active and Semi Active vibration isolation*", Current Advances in Mechanical Design and Production, 6th Cairo International MDP conference, Jan 1996, pages: 409-423
4. Chi Z., He Y., Naterer G. F., "*Design Optimization of vehicle suspensions with a quarter vehicle model*", Transactions of the CSME, June 2008, vol. 32 no. 2, pages:297-312
5. Matej Andrejasic, "*MEMS Accelerometers*", seminar on MEMS, University of Ljubljana, Dept. of Physics, March 2008
6. A Book on "*Modelling MEMS and NEMS*" by John A. Pelesko, David H. Bernstein, Chapman & Hall Co.
7. K. N. Anakwa, R. Dion, S. C. Jones, "*Development and Control of a Prototype Pneumatic Active Suspension system*", IEEE transactions on Education, Feb 2002, vol. 45, no.1, pages:43-49
8. S. Molueeswaran, "*Development of Active Suspension systems for automobile using PID controller*", Proceedings of the World Congress on Engineering, July 2008, vol. 2, pages:3-7
9. S. Molueeswaran, S. Vijayarangan, "*Analytical and Experimental studies on active suspension system of light passanger vehicle to improve ride comfort*", ISSN, MECHANIKA, March 2007, vol. 2, pages:34-41
10. Ali K. Sedigh, M. J. Yazadanpanah, "*A QFT approach to robust control of automobile active suspension*", 5th ASIAN Control Conference, IEEE, Jan 2004, pages:604-610
11. K. Takayuki, I. Jun, M. Konishi, "*Positioning control of one link arm with parametric uncertainty using QFT method*", SICE Annual Conference, Japan, August 2008, pages:3268-3271
12. S. F. Wu, M. J. Grimble, S. G. Breslin, "*Introduction to QFT for lateral robust flight control systems design*", Elsevier Science Ltd., Control Engineering Practice 6, 1998, pages:805-828

13. Yu Jinying, K. Zhao, Jian Cao, "*Application of one-degree of freedom QFT tracking controller to Hydraulic simulator design*". IEEE conference, Japan, 2006, pages:545-548
14. P. S. V. Nataraj and Mukesh D. Patil, "*Robust control design for non-linear Magnetic Levitation System using QFT*", IEEE conference, 2008
15. A Book on "*Quantitative Feedback Theory, Fundamental and Applications*", by Constantine H. Houpis, Steven J. Rosmussen, 2nd Edition, Marcel Dekker Inc, NY
16. D. Wang, F. Wang, H Bai, "*Design and Performance of QFT-H-infinity controller for Magnetic Bearing of High Speed motors*", ICIEA IEEE conference 2009, pages:2624-2629
17. A. Kruczek, A. Stribrsky, H. Martin, "*H-infinity controlled actuators in automotive active suspension system*", Proceeding of 9th Biennial ASME Conference on Engineering Systems Design and Analysis, Israel, July 7-9, 2008, pages:1-5
18. C. Poussot, O. Sename, L. Dugard, "*Multi-objective qLPV H-infinity control of a half vehicle*", Mini conference on vehicle system dynamics, identification and anomalies, VSDIA, Hungry, HAL journals, October 2007
19. K. Huibert, "*Robust Control and H-infinity Optimization – Tutorial Paper*", Automatica, vol. 29, no.2, pages:255-273
20. R.W. Beaven, M.T. Wright and D.R. Seaward, "*Weighting Functions Selection in the H-infinity Design*", Control Eng. Practice, Elseiver Science Ltd., 1996, vol. 4, no. 5, pages:625-623
21. P. Rafal, M. Katja, A. Smirnov, "*H-infinity control of active magnetic suspension*", Mechanical Systems and Signal Processing 24, Elsevier Science Ltd., 2010, pages:995-1006
22. G. Cao, S. Fan, G. Xu, "*The Characterstics analysis of Magnetic Bearing based on H-infinity controller*", Proceedings of the 5th World Congress on Intelligent control and Automation, June 2004, pages:752-756