DISTRIBUTED SPACE-FREQUENCY CODING IN WIRELESS RELAY NETWORKS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

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By

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report entitled, **"DISTRIBUTED SPACE-FREQUENCY CODING IN WIRELESS RELAY NETWORKS "** towards the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems,** submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from July 2009 to June 2010, under the guidance of **Dr.ANSHUL TYAGI, Assistant Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee.**

I have not submitted the matter embodied in this dissertation for the award of any other Degree or Diploma.

Date: 29/06/2010

Place: Roorkee

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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ABSTRACT

Recently, there has been much interest in modulation techniques to achieve transmit diversity motivated by the increased capacity of multiple-input multiple-output (MIMO) channels. To achieve transmit diversity the transmitter needs to be equipped with more than one antenna. The antennas should be well separated to have uncorrelated fading among the different antennas; hence, higher diversity orders and higher coding gains are achievable. It is affordable to equip base stations with more than one antenna, but it is difficult to equip the small mobile units with more than one antenna with uncorrelated fading. In such a case, transmit diversity can only be achieved through user cooperation leading to what is known as cooperative diversity. Cooperative diversity provides a new dimension over which higher diversity orders can be achieved.

An ad-hoc network with a sender, a destination and a third station acting as a relay is analyzed. Different combining methods and diversity protocols are compared. The simulation results have shown that amplify and forward protocol gives a better performance than decode and forward protocol. To combine the incoming signals the channel quality should be estimated as well as possible. Information about the average quality shows nice benefits, and a rough approximation about the variation of the channel quality increases the performance even more. Whatever combination of diversity protocol and combining method is used second level diversity is observed.

The problem with the multi-node decode-and-forward protocol and the multi-node amplify-and-forward protocol is the loss in the data rate as the number of relay nodes increases. The use of orthogonal subchannels for the relay nodes transmissions, either through Time-Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA), results in a high loss of the system spectral efficiency. This leads to the use of what is known as distributed space-frequency coding, where relay nodes are allowed to simultaneously transmit over the same channel by emulating a space-frequency code. The term distributed comes from the fact that the virtual multi-antenna transmitter is distributed between randomly located relay nodes. Relay nodes

are used to form a virtual multi-antenna transmitter to achieve diversity through the use of distributed space-frequency codes.

The design of distributed space frequency codes (DSFCs) for wireless relay networks employing the amplify and forward (AAF) protocol is considered. DSFCs are designed to achieve the multipath (frequency) and cooperative diversities of the wireless relay channels. The DSFC can achieve full diversity of order LN, where L is the number of paths of the channel and N is the number of relay nodes. Simulation results are also presented.

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INTRODUCTION

1.1 Overview

In a wireless transmission the signal quality suffers occasionally severely from a bad channel quality due to effects like fading caused by multi-path propagation. To reduce such effects diversity can be used to transfer the different samples of the same signal over essentially independent channels. In this thesis diversity is realized by using a third station as a relay.

In such a system combinations of several relaying protocols and different combining methods are examined to see their effects on the performance. The transmission protocols used in this thesis are Amplify and Forward and Decode and Forward. In the simulation these can both be seen to achieve full diversity as was proved in [1]. Basically three different types of combining methods are examined which differs in the knowledge of the channel quality they need to work.

One combination that achieves a good performance is then used to see the effect on the performance depending on the location of the relay. This information is crucial to decide the worth of a mobile relay.

The main problem with the multi-node decode-and-forward (DAF) protocol and the multi-node amplify-and-forward (AAF) protocol is the loss in the data rate as the number of relay nodes increases. The use of orthogonal subchannels for the relay node transmissions, results in a high loss of the system spectral efficiency. This leads to the use of what is known as distributed space-frequency coding (DSFC), where relay nodes are allowed to simultaneously transmit over the same channel by emulating a space-frequency code. The term distributed comes from the fact that the virtual multi-antenna transmitter is distributed between randomly located relay nodes. Employing DSFCs reduces the data rate loss due to relay nodes transmissions without sacrificing the system diversity order. [2,3,4].

1.2 Objectives

The dissertation has two objectives. The first one was performance analysis of an ad-hoc network with a sender, a destination and a third station acting as a relay with the single link channel. The channels are modelled containing thermal noise, Rayleigh fading and path loss. The performance for different combining methods and diversity protocols are compared. Diversity protocols used are amplify and forward (AAF) and decode and forward (DAF). The diversity is realized by building an ad-hoc network using a third station as a relay. Second objective of the project was to assess the design of distributed space-frequency coding (DSFC) for broadband multipath fading channels with relay networks to exploit the frequency diversity of the channel. In this part sufficient condition for the code structures at the source and relay nodes were derived to achieve full diversity of order NL, where N is the number of relay nodes and L is the number of paths per channel.

1.3 Structure of this Thesis

The heart of this thesis is in the following four chapters:

Chapter 1 Introduction and the objectives of the dissertation work is summarized.

Chapter 2 explains the model of a single link channel. Two different modulation types are introduced (BPSK, QPSK) and the channel model (fading, path loss, noise) is explained.

Chapter 3 explains the arrangement of the diversity system used in this thesis. Two relay protocols are described and various combining methods are introduced. It presents the results of the simulation. In a first part the performance of different combinations of diversity protocols and combination methods are shown. In a second part the effect of the location of the relay station are presented.

Chapter 4 the design of distributed space-frequency codes (DSFCs) for wireless relay networks is considered. The use of DSFCs with the amplify-and-forward (AAF) protocols is considered. The code design criteria to achieve full diversity, based on the pairwise error probability (PEP) analysis, are derived. Simulation results also presented.

Chapter 5 gives the conclusion of the thesis work.

Chapter 2

SINGLE LINK TRANSMISSION

In this chapter the system model for a single link transmission as illustrated in Fig. 2.1 is presented. This thesis considers the modulator, channel and demodulator block which are described below.

2.1 Signal Model and Modulation

The transferred data is a random bipolar bit sequence which is either modulated with Binary Phase Shift Keying (BPSK) or Quadrature Phase Shift Keying (QPSK). As illustrated in Fig. 2.2, QPSK in fact consists of two independent (orthogonal) BPSK systems and therefore has double bandwidth compared to BPSK. Without any loss of generality the simulations are done in the baseband.

2.2 Channel Model

In a wireless network, the data which is transferred from a sender to a receiver has to propagate through the air. During propagation several phenomena will distort the signal. Within this thesis, thermal noise, path loss and Rayleigh fading are considered, as illustrated in Fig. 2.3. Path loss and fading are multiplicative, noise is additive.

$$y_d[n] = \underbrace{h_{s,d}[n]}_{attenuation}.x_s[n] + z_{s,d}[n] = \underbrace{d_{s,d}}_{pathloss}.\underbrace{a_{s,d}[n]}_{fading}.x_s[n] + \underbrace{z_{s,d}[n]}_{noise} \tag{2.1}$$

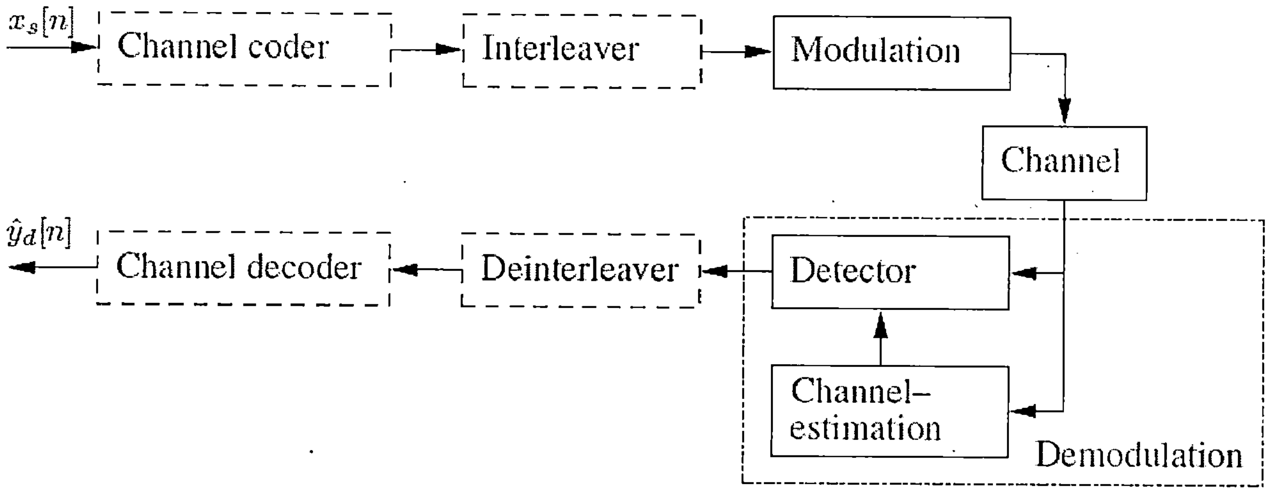

Figure 2.1: modulation, channel and demodulation block.

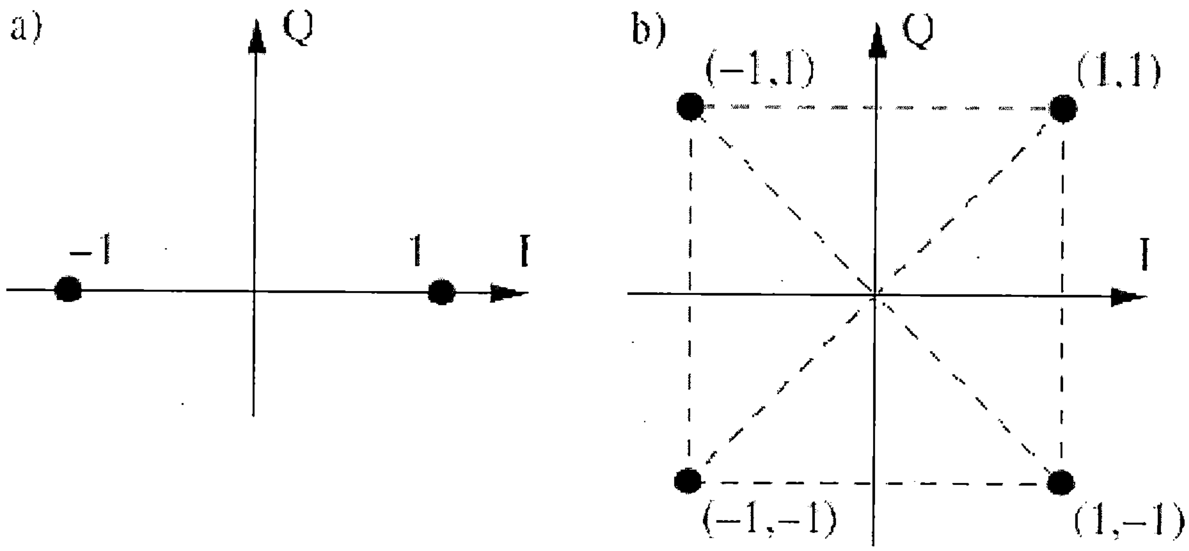

Figure 2.2: a) BPSK, b) QPSK, I denotes the In phase channel, and Q the Quadrature channel.

In (2.1) s,d denote the sender respective the destination, $x_s[n]$ is the transmitted symbol and $y_d[n]$ the received symbol.

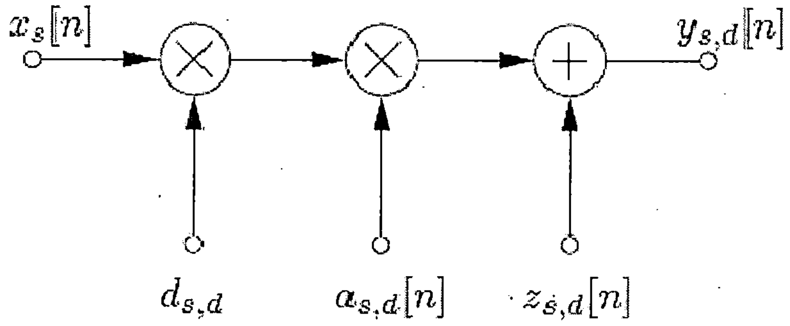

Figure 2.3: Channel model: path loss $d_{s,d}$, fading $a_{s,d}[n]$ and noise $z_{s,d}[n]$.

2.2.1 Noise

The main sources of noise in a wireless network are interference and electronic components like amplifiers. If the latter dominates, thermal noise can be assumed, which can be characterized as additive complex Gaussian noise. The scalar $z_{s,d}[n]$ can then be simulated as the sum of a real and a imaginary noise vector, both Gaussian distributed, mutually independent and zero mean with variance σ_n^2. The total noise power will be $N_0 = 2\sigma_n^2$.

2.2.2 Signal to Noise Ratio

The signal-to-noise ratio (SNR) is a widely used value to indicate the signal quality at the destination.

$$SNR = \left(\frac{S}{N_0}\right) = \frac{|h_{s,r}|^2 \cdot \xi}{N_0} \tag{2.2}$$

In (2.2) $\xi = E\left[|x_s|^2\right]$ denotes the energy of the transmitted signal and N_0 the total power of the noise.

2.2.3 Path Loss and Fading

The signal is attenuated mainly by the effects of free-space path loss and fading, both included in $h_{s,d} = d_{s,d}.a_{s,d}$.

The path loss $d_{s,d}$ (assuming a plane-earth model) is proportional to $\frac{1}{R^2}$. As long as the distance between the sender and receiver does not change too much, it can be assumed to be constant for the whole transmission. The power of the received signal is attenuated proportional to $\frac{1}{R^4}$.

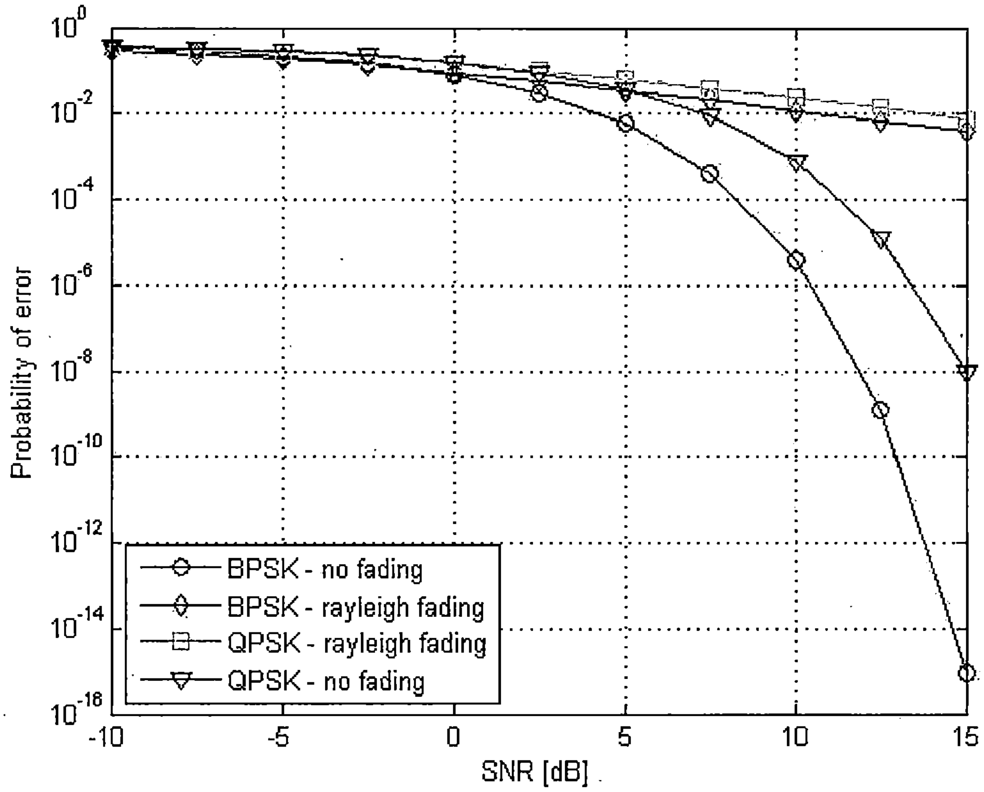

Figure 2.4: The severe effect of a Rayleigh faded channel, compared to the nonfaded channel.

In a wireless network it occurs quite often that the line-of-sight link is blocked. Instead of this direct connection, the signal will propagate to the sender on many different ways. This occurs especially in an urban environment, where buildings prevent a line-of-sight link but enable various deferent ways for indirect connection by rejecting the propagating signal. This effect is referred to as multi-path propagation.

Only small changes in the whole system might change the characteristic of the channel and therefore the signal quality considerably. This effect, known as fading, will alter the signal by attenuating it and adding a phase shift to it. The fading coefficient $a_{s,d}$ can be modeled as a zero mean, complex Gaussian random variable with variances $\sigma_{s,d}^2$. This means that the angle $\angle a_{s,d}$ is uniformly distributed on $[0, 2\pi)$ and the magnitude $|a_{s,d}|$ is Rayleigh distributed [1]. 'This Rayleigh distributed magnitude can have a bad effect on the signal quality at the receiver, as illustrated in Fig. 2.4. Even a system with a high SNR might experience significant errors due to fading.

Block Fading

In a fast fading channel, the channel characteristic changes within one burst of data. The block fading channel model pays attention to this effect. The burst is broken up into smaller pieces, blocks, which can then be assumed to have a constant channel characteristic. The block length has to be long enough, to allow the channel characteristic to be estimated perfectly. The magnitude and the angle of the fading coefficient $a_{s,d}$ of the block are known by the receiver.

In a block fading channel, there is a high possibility that burst errors occur, i.e. that there are a lot of errors within one block. Such bursts of errors are very difficult to correct with an error correcting code. To prevent them occurring, the signal can be interleaved to get the errors distributed uniformly over the whole signal, as illustrated in Fig. 2.1. The interleaving and the coding block are not simulated but it is assumed that they exist. Therefore, if you simulate such a transmission, it does not matter how the errors are distributed over the whole signal. The only thing that is of interest is the average bit error ratio (BER). To get an accurate result the signal should be transferred over as many different channel characteristics as possible. Without loss of generality the block length within the simulation can be assumed to be one. This significantly reduces the computing time.

2.3 Receiver Model

The receiver detects the received signal symbol by symbol. In the case of a BPSK modulated signal the symbol/bit is detected as

$$\hat{y}_d[n] = \begin{cases} +1 & (\mathrm{Re}\{y_d[n]\} \geq 0) \\ -1 & (\mathrm{Re}\{y_d[n]\} < 0) \end{cases} \tag{2.3}$$

For a QPSK modulated signal there are two bits transferred per symbol, which are detected as

$$\hat{y}_d[n] = \begin{cases} [+1,+1] & (0^\circ \leq \angle y_d[n] < 90^\circ) \\ [-1,+1] & (90^\circ \leq \angle y_d[n] < 180^\circ) \\ [+1,-1] & (-90^\circ \leq \angle y_d[n] < 0^\circ) \\ [-1,-1] & (-180^\circ \leq \angle y_d[n] < -90^\circ) \end{cases} \tag{2.4}$$

2.4 BER of a Single Link Transmission

The signal quality received at the destination depends on the SNR of the channel and the way the signal is modulated. The theoretical probability of a bit error is derived in [5] and is summarized in Tab. 2.1.

Modulation Type	no Fading	Rayleigh Fading
BPSK	$P_b = Q\left(\sqrt{\frac{\xi}{\sigma^2}}\right)$	$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right)$
QPSK	$P_b = Q\left(\sqrt{\frac{\xi}{2\sigma^2}}\right)$	$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}}\right)$

Table 2.1: Theoretical BER for a single link transmission. $\bar{\gamma}_b$ denotes the average signal-to-noise ratio, defined as $\bar{\gamma}_b = \frac{\xi}{2\sigma^2}E(a^2)$, where $E(a^2) = a^2$.

In this section the model of a single link transmission has been presented. The signal is modulated using Binary Phase Shift Keying (BPSK) or Quadrature Phase Shift Keying (QPSK). The channel consists of path loss and Rayleigh fading which are multiplicative components and thermal noise which is additive.

In the next chapter this simple single link model is enhanced to a diversity arrangement using one direct link and one multi-hop link.

Chapter 3
MULTI HOP RELAY NETWORKS

There are several approaches to implement diversity in a wireless transmission. Multiple antennas can be used to achieve space and/or frequency diversity. But multiple antennas are not always available or the destination is just too far away to get good signal quality. To get diversity, an interesting approach might be to build an ad-hoc network using another mobile station as a relay. The model of such a system is illustrated in Fig. 3.1. The sender S, sends the data to the destination D, while the relay station R is listening to this transmission. The relay sends this received data burst after processing to the destination as well, where the two received signals are combined. As proposed in [1], orthogonal channels are used for the two transmissions. Without loss of generality, this can be achieved using time divided channels, which is done in all the simulations in this thesis.

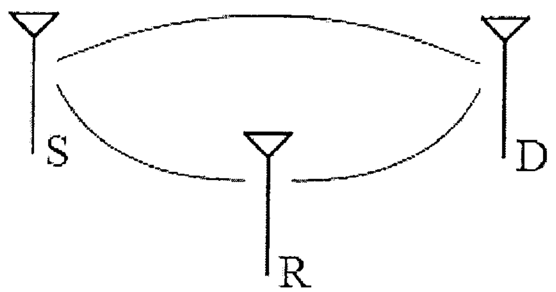

Figure 3.1: The data is transmitted on one hand directly to the destination, and on the other hand the data is sent to the receiver via the relay.

3.1 Cooperative Transmission Protocols

The cooperative transmission protocols used in the relay station are either Amplify and Forward (AAF) or Decode and Forward (DAF). These protocols describe how the received data is processed at the relay station before the data is sent to the destination.

3.1.1 Amplify and Forward

This method is often used when the relay has only limited computing time/power available or the time delay, caused by the relay to de- and encodes the message, has to be minimized. Of course when an analogue signal is transmitted a DAF protocol cannot be used.

The idea behind the AAF protocol is simple. The signal received by the relay was attenuated and needs to be amplified before it can be sent again. In doing so the noise in the signal is amplified as well, this is the main downfall of this protocol.

The incoming signal is amplified block wise. Assuming that the channel characteristic can be estimated perfectly, the gain for the amplification can be calculated as follows.

The power of the incoming signal (2.1) is given by

$$E\left[\left|y_r^2\right|\right] = E\left[\left|h_{s,r}\right|^2\right] E\left[\left|x_s\right|^2\right] + E\left[\left|z_{s,r}\right|^2\right] = \left|h_{s,r}\right|^2 \xi + 2\sigma_{s,r}^2$$

Where s denotes the sender and r the relay. To send the data with the same power the sender did, the relay has to use a gain of

$$\beta = \sqrt{\frac{\xi}{\left|h_{s,r}\right|^2 \xi + 2\sigma_{s,r}^2}} \qquad (3.1)$$

This term has to be calculated for every block and therefore the channel characteristic of every single block needs to be estimated.

3.1.2 Decode and Forward

Nowadays a wireless transmission is very seldom analogue and the relay has enough computing power, so DAF is most often the preferred method to process the data in the relay. The received signal is first decoded and then re-encoded. So there is no amplified noise in the sent signal, as is the case using a AAF protocol. There are two main implementations of such a system.

The relay can decode the original message completely. This requires a lot of computing time, but has numerous advantages. If the source message contains an error correcting code, received bit errors might be corrected at the relay station. Or if there is no such code implemented a checksum allows the relay to detect if the received signal contains errors. Depending on the implementation an erroneous message might not be sent to the destination.

But it is not always possible to fully decode the source message. The additional delay caused to fully decode and process the message is not acceptable, the relay might not have enough computing capacity or the source message could be coded to protect sensitive data. In such a case, the incoming signal is just decoded and re-encoded symbol by symbol.

3.2 Combining Type [6]

As soon as there is more than one incoming transmission with the same burst of data, the incoming signals have to be combined before they will be compared as indicated in (2.3) and (2.4).

3.2.1 Equal Ratio Combining (ERC)

If computing time is a crucial point, or the channel quality could not be estimated, all the received signals can just be added up. This is the easiest way to combine the signals, but the performance will not be that good in return.

$$y_d[n] = \sum_{i=1}^{k} y_{i,d}[n]$$

Within this thesis one relay station is used, so the equation simplifies to

$$y_d[n] = y_{s,d}[n] + y_{r,d}[n], \tag{3.2}$$

Where $y_{s,d}$ denotes the received signal from the sender and $y_{r,d}$ the one from the relay.

3.2.2 Fixed Ratio Combining (FRC)

A much better performance can be achieved, when fixed ratio combining is used. Instead of just adding up the incoming signals, they are weighted with a constant ratio, which will not change a lot during the whole communication. The ratio should represent the average channel quality and therefore should not take account of temporary influences on the channel due to fading or other effects. But influences on the channel, which change the average channel quality, such as the distance between the different stations, should be considered. The ratio will change only gently and therefore needs only a little amount computing time. The FRC can be expressed as

$$y_d[n] = \sum_{i=1}^{k} d_{i,d} \cdot y_{i,d}[n],$$

Where $d_{i,d}$ denotes weighting of the incoming signal $y_{i,d}$. Using one relay station, the equation simplifies to

$$y_d[n] = d_{s,d} \cdot y_{s,d}[n] + d_{s,r,d} \cdot y_{r,d}[n] \tag{3.3}$$

Where $d_{s,d}$ denotes the weight of the direct link and $d_{s,r,d}$ the one of the multi-hop link. Within this thesis only the best achievable performance of a FRC system is of interest. So the best ratio is approximated by comparing different possible values. This ratio is then used to compare with the other combining methods.

3.2.3 Signal to Noise Ratio Combining (SNRC)

A much better performance can be achieved, if the incoming signals are weighted on an intelligent way. An often used value to characterise the quality of a link is the SNR, which can be used to weight the received signals.

$$y_d[n] = \sum_{i=1}^{k} SNR_i \cdot y_{i,d}[n]$$

Using one relay, the equation can be written as

$$y_d[n] = SNR_{s,d} \cdot y_{s,d}[n] + SNR_{s,r,d} \cdot y_{r,d}[n], \tag{3.4}$$

Where $SNR_{s,d}$ denotes the SNR of the direct link and $SNR_{s,r,d}$ the one over the whole multi-hop channel.

The estimation of the SNR of a multi-hop link using AAF or a direct link can be performed by sending a known symbol sequence in every block. If the multi-hop link is using a DAF protocol the receiver can only see the channel quality of the last hop. It is assumed that the relay sends some additional information about the quality of the unseen hops to the destination, so the SNR of the multi-hop link can be estimated as well. Whatever protocol is used, an additional sequence needs to be sent to estimate the channel quality. This results in a certain loss of bandwidth.

Estimate SNR using AAF

Using AAF, the received signal from the relay is

$$y_{r,d} = h_{r,d}x_r + z_{r,d} = h_{r,d}\beta(h_{s,r}x_s + z_{s,r}).$$

The received power will then be

$$E\left[\left|y_{r,d}\right|^2\right] = \beta^2 \left|h_{r,d}\right|^2 (\left|h_{s,r}\right|^2 \xi + 2\sigma_{s,r}^2) + 2\sigma_{r,d}^2,$$

So the SNR of the one relay multi-hop link can be estimated as

$$SNR = \frac{\beta^2 \left|h_{s,r}\right|^2 \left|h_{r,d}\right|^2 \xi}{\beta^2 \left|h_{r,d}\right|^2 2\sigma_{s,r}^2 + 2\sigma_{r,d}^2} \tag{3.5}$$

Estimate SNR using DAF

To calculate the SNR of a multi-hop link using DAF, first the BER of the link is calculated which can then be translated to an equivalent SNR.

The BER of a single link is given in Tab. 2.1. The BER over a one relay multi-hop link can then be calculated as

$$BER_{s,r,d} = BER_{s,r}(1 - BER_{r,d}) + (1 - BER_{s,r})BER_{r,d}$$

To calculate the SNR, the inverse functions of those in Tab. 2.1 are used. For a BSPK modulated Rayleigh faded signal this will be

$$SNR = \frac{1}{2}\left[Q^{-1}(BER)\right]^2 \tag{3.6}$$

For a QPSK modulated signal this will change to

$$SNR = \left[Q^{-1}(BER)\right]^2 \tag{3.7}$$

3.2.4 Maximum Ratio Combining (MRC)

The Maximum Ratio Combiner (MRC) achieves the best possible performance by multiplying each input signal with its corresponding conjugated channel gain. This assumes that the channels' phase shift and attenuation is perfectly known by the receiver.

$$y_d[n] = \sum_{i=1}^{k} h_{i,d}^{*}[n] \cdot y_{i,d}[n]$$

Using a one relay system, this equation can be rewritten as

$$y_d[n] = h_{s,d}^{*}[n] \cdot y_{s,d}[n] + h_{r,d}^{*}[n] \cdot y_{r,d}[n] \tag{3.8}$$

By looking at this equation a little bit closer, the big disadvantage of this combining method in a multi-hop environment can be seen. The MRC only considers the last hop (i.e. the last channel) of a multi-hop link. So in this thesis the MRC should only be used in combination with a DAF protocol. There is still the problem that the relay might send incorrectly detected symbols, which will have severe effects on the performance. So the use of MRC is only recommended if an error correcting code is used. This can be simulated by using a magic genie as described in 3.1.2.

3.2.5 Enhanced Signal to Noise Combining (ESNRC)

Another plausible combining method is to ignore an incoming signal when the data from the other incoming channels have a much better quality [6]. If the channels have more or less the same channel quality the incoming signals are rationed equally. In the system used in this thesis this can be expressed as

$$y_d[n] = \begin{cases} y_{s,d}[n] & (SNR_{s,d}/SNR_{s,r,d} > 10 \\ y_{s,d}[n] + y_{s,r,d}[n] & (0.1 \le SNR_{s,d}/SNR_{s,r,d} \le 10 \\ y_{s,r,d}[n] & (SNR_{s,d}/SNR_{s,r,d} < 0.1 \end{cases} \tag{3.9}$$

Using this combining method, the receiver does not have to know the channel characteristic exactly. An approximation of the channel quality is enough to combine the signals. As a further benefit, the equal ratio combining does not need a lot of computing power. All the figures presented in the next chapter are labeled using the same abbreviations.

There are two popular implementations to transmit over a wireless network. One is the simple direct link which sends the data only once. The other is the two sender arrangement which sends the data twice over different antennas. These two standard implementations put the performance of the arrangements used in this thesis into perspective.

The diversity arrangement has to send the data twice and therefore requires twice the bandwidth of the single link transmission. To compensate for this effect, the single link channel is modulated using BPSK and the diversity arrangement uses QPSK. As QPSK has twice the bandwidth of BPSK both arrangements have the same overall bandwidth. Notice that the relay causes a certain time delay for the diversity arrangement.

The performance of a two sender transmission with MRC at the receiver can be expressed [5] as

$$P_b = \frac{1}{4}(1-\mu)^2(2+\mu) \qquad \mu = \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}},$$

Where $\bar{\gamma}_b$ denotes the average signal-to-noise ratio, defined as $\bar{\gamma}_b = \frac{\xi}{2\sigma^2}E(a^2)$, where $E(a^2) = a^2$

In this chapter the different aspects of a multi-hop and a diversity arrangement have been presented. First two different transmission protocols Amplify and Forward (AAF) and Decode and Forward (DAF) have been described. When the destination receives different samples of the same data, these samples need to be combined. The Equal Ratio Combining (ERC) just adds up the different received signal while the Fixed Ratio Combining (FRC) is weighting the incoming signals with a fixed ratio. When the channel quality is estimated precisely, more powerful combining methods as Maximum Ratio Combining (MRC), Signal-to-Noise Ratio Combining (SNRC) or Enhanced Signal-to-Noise Combining (ESNRC) can be used.

3.3 Key results

The performance of different combinations of the methods described in the last chapter are analyzed to illustrate their potential benefits. In the first part, it is assumed that the three stations (sender, relay and destination) have an equal distance from each other and therefore the same path loss and average signal-to-noise ratio is assumed. With this equidistant arrangement the different combining and amplifying types are compared to see their advantages and disadvantages. In the second part, the location of the relay station is varied to see the effect on the performance for different locations of the relay.

In this section it is assumed, that the three stations are arranged at the edges of a triangle with a length of one. This means that all the channels will have the same path loss and therefore the same average signal-to-noise ratio.

3.3.1 Equidistant Arrangement

In this section it is assumed, that the three stations are arranged at the edges of a triangle with a length of one. This means that all the channels will have the same path loss and therefore the same average signal-to-noise ratio.

3.3.1.1 Amplify and Forward

To compare the benefits of the different combining method, the optimal ratio for the FRC needs to be evaluated first. Fig. 3.2 illustrates the effects of the different weighting. As seen, a much better performance is achieved using FRC instead of ERC simply by assuming that the direct link has in general a better quality than the multi-hop link. This is obvious in an equidistant arrangement, where the signal over the multi-hop has to propagate over twice the distance than over the direct link. The result of the simulation illustrated in Fig. 3.2 shows that the best performance using FRC is achieved with a ratio of 2:1. FRC with this ratio is now used to compare performances with one of the other combining types.

In Fig. 3.3 the effect on the performance of the different combining types using a AAF protocol can be seen. The BPSK single link transmission should demonstrate if there is any benefit at all using diversity, while the QPSK two senders link indicates a lower bound for the transmission. Using the equidistant arrangement, the aim is to get as close to the latter curve as possible or to get an even better performance.

The first pleasant result is that whatever combining type is used, the AAF diversity protocol achieves a benefit compared to the direct link. Even the equal ratio combining shows advantages. But compared to the fixed ratio combining, the performance looks quite poor. Otherwise you should call to mind that the equal ratio combining does not need any channel information, except the phase shift, to perform the combining. The fixed ratio combining on the other hand needs some channel information to calculate the appropriate weighting.

The signal-to-noise ratio combining (SNRC) and the enhanced signal-to-noise ratio combining (ESNRC) show roughly the same performance, which is much better than the one using FRC/ERC. This is not surprising considering that the former two combining methods are using much more detailed channel information than the latter two. Actually the big surprise is that the performance of the combining methods, which have precise information about every single block, is just about one decibel better than the one using FRC which has just average knowledge of the channel quality.

The other unexpected thing is that the SNRC shows approximately the same performance than the ESNRC. Remember that for the ESNRC a roughly estimated channel quality for every single block is sufficient. This is in contrast to the SNRC, which needs exact information of the channel quality for every single block. This means that the transferred signal in an AAF system contains some information that allows correcting of a small difference in the channel quality.

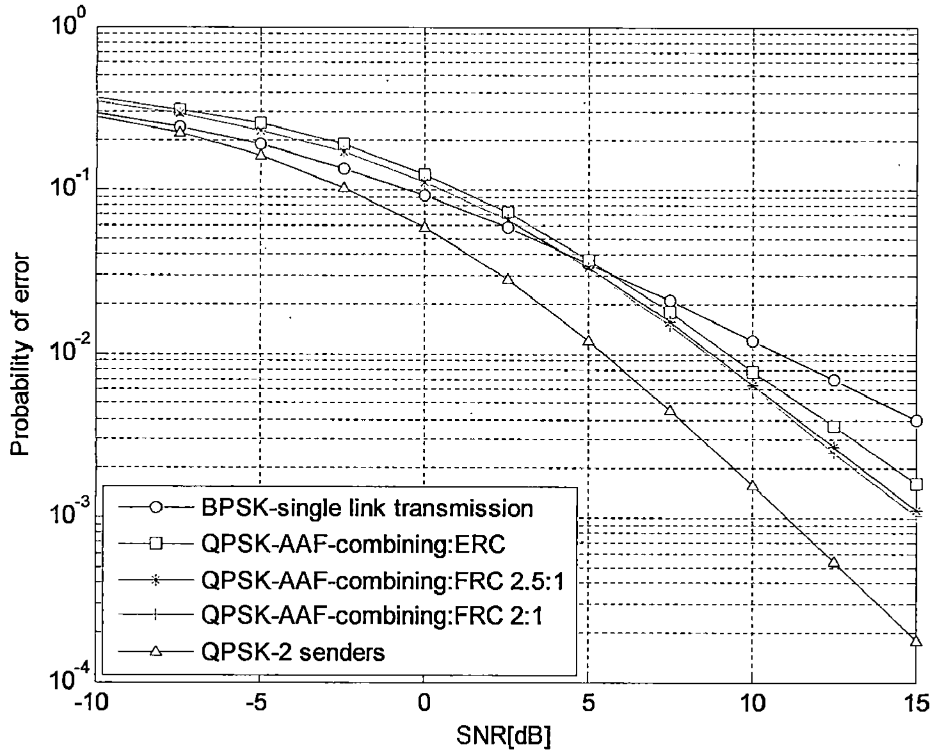

Figure 3.2: To estimate the best ratio for FRC different ratios are plotted. The ratio 2:1 gives a good result.

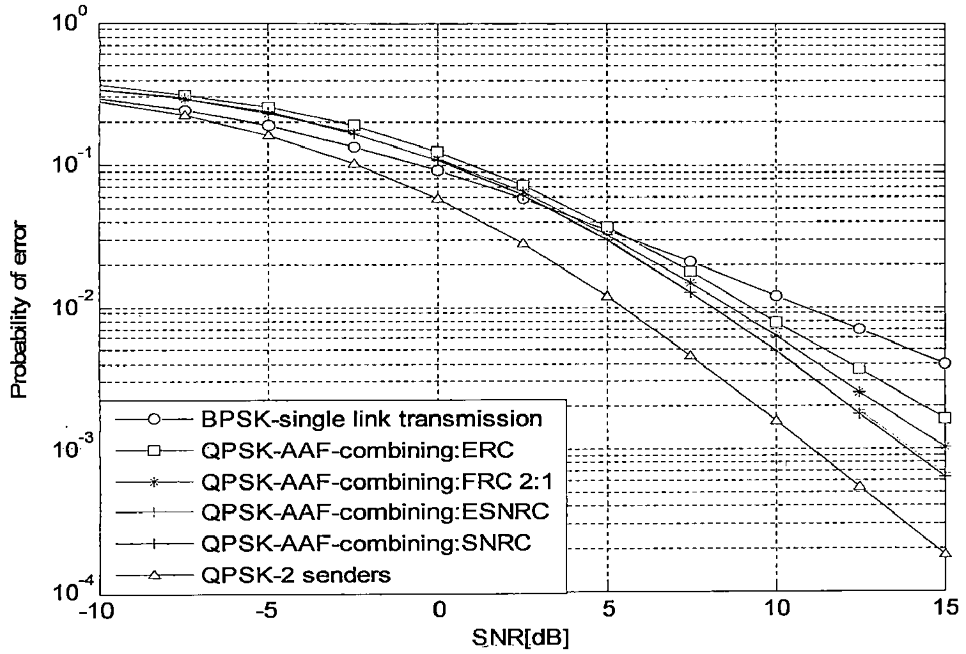

Figure 3.3: The different combining types are compared with each other. The best performance results when using SNRC/ESNRC.

Using the AAF protocol, there is no point in wasting a lot of computing power and bandwidth to get some exact channel information. And even if the channel quality could not be estimated at all (and therefore ERC is used), there is still a benefit using diversity.

3.3.1.2 Decode and Forward

To compare the benefits of the different combining method, the optimal ratio for the FRC needs to be evaluated first, which is done exactly in the same way as before. The FRC is simulated with different weighting to estimate the ratio that results in the best performance. The simulations, illustrated in Fig. 3.4, show the best performance when a ratio of 3:1 is used. It is quite surprising that this ratio differs that much from the ratio using AAF. The reason for that is discussed in Sec. 3.3.1.3. The FRC with a ratio of 3:1 can now be used to compare with the other combining methods.

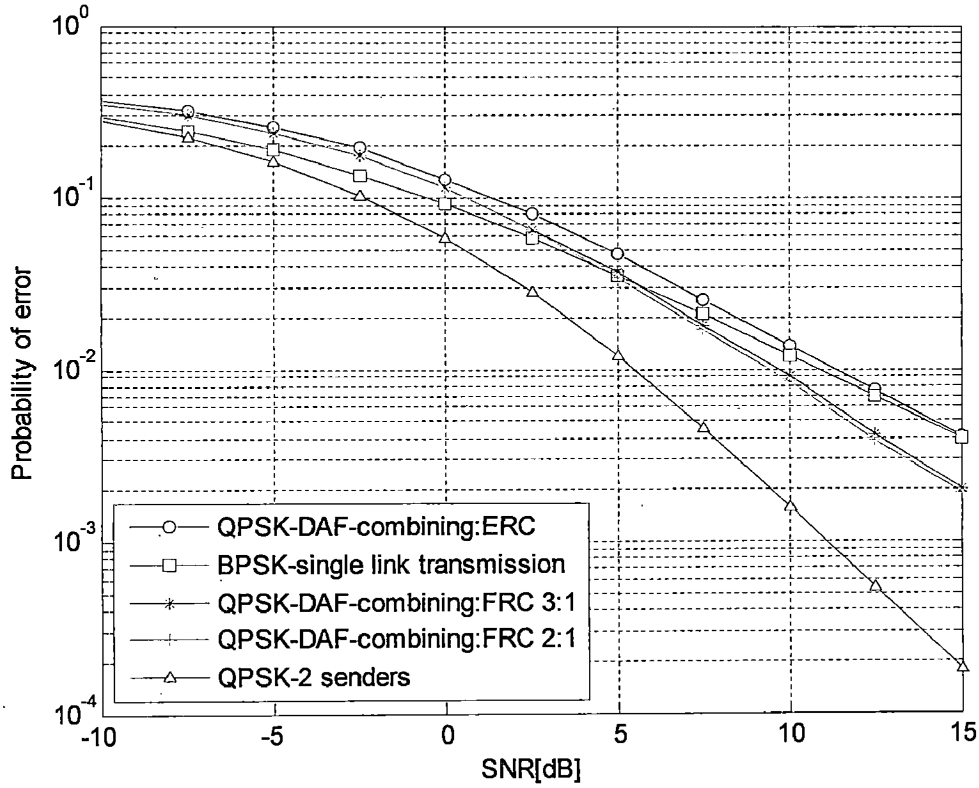

Figure 3.4: To estimate the best ratio for FRC different ratios are plotted. The ratio 2:1 results in the best performance.

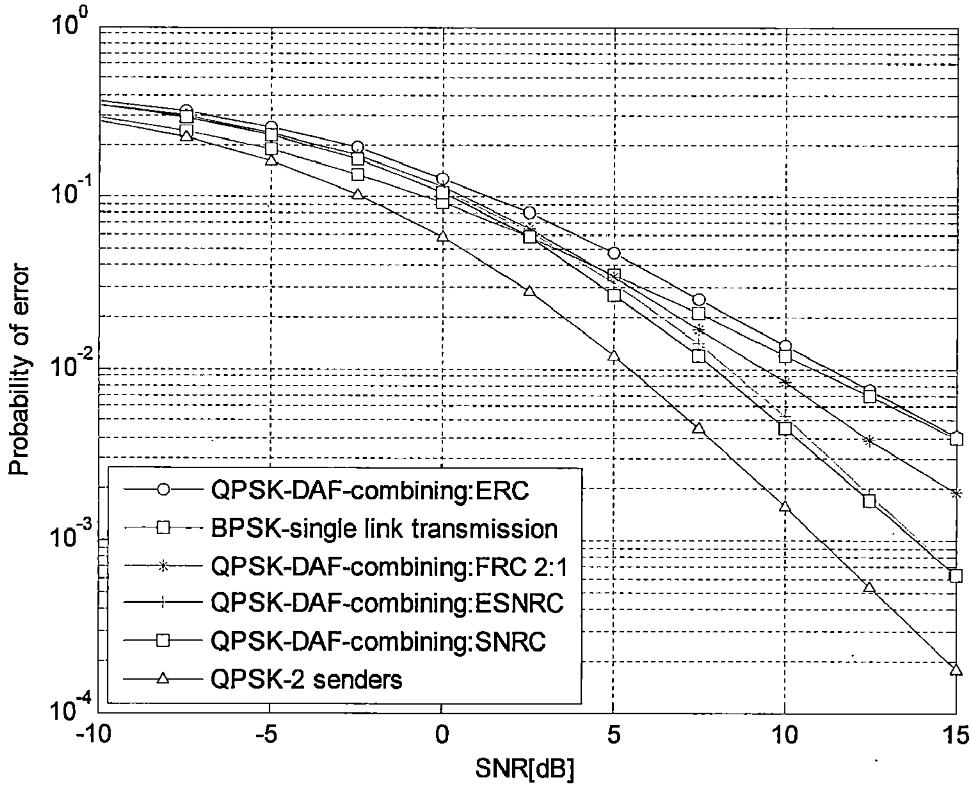

Figure 3.5: The different combining types are compared with each other. The best performance results when using SNRC.

The different combining methods using the DAF protocol are illustrated in Fig. 3.5. The first thing that attracts attention is the bad performance of the equal ratio combining. Especially for a small SNR the performance is significantly worse than the one of the BPSK single link transmission and therefore should not be used at all.

The fixed ratio combing shows obviously a much better performance than the BPSK single link transmission. To achieve a BER of about 10^{-2} the required SNR for the FRC is about 2.5 dB less than the one for the single link transmission. That is quite a remarkable benefit.

In contrast to the AAF protocol, a big benefit results using one of the block analysing combining methods (SNRC/ESNRC). Using the DAF protocol shows now the benefit estimating every single block separately and hence using more computing power.

There is now an additional benefit, to estimate every block precisely when using SNRC, instead of just approximating the channel quality combining the signals with ESNRC. But considering that the achievable benefit is about half a decibel it might not be worth wasting the additional computing power and bandwidth which is required to get precise channel estimation. If AAF is used, there is no benefit at all, using the SNRC instead of ESNRC. From now on, the focus will be laid on ESNRC, FRC and ERC.

3.3.1.3 Amplify and Forward versus Decode and Forward

Fig. 4.5 illustrates the performance of the AAF diversity protocol compared with the DAF protocol. The surprising result is that the AAF diversity protocol always results in a better performance than the DAF protocol whatever combining type is used.

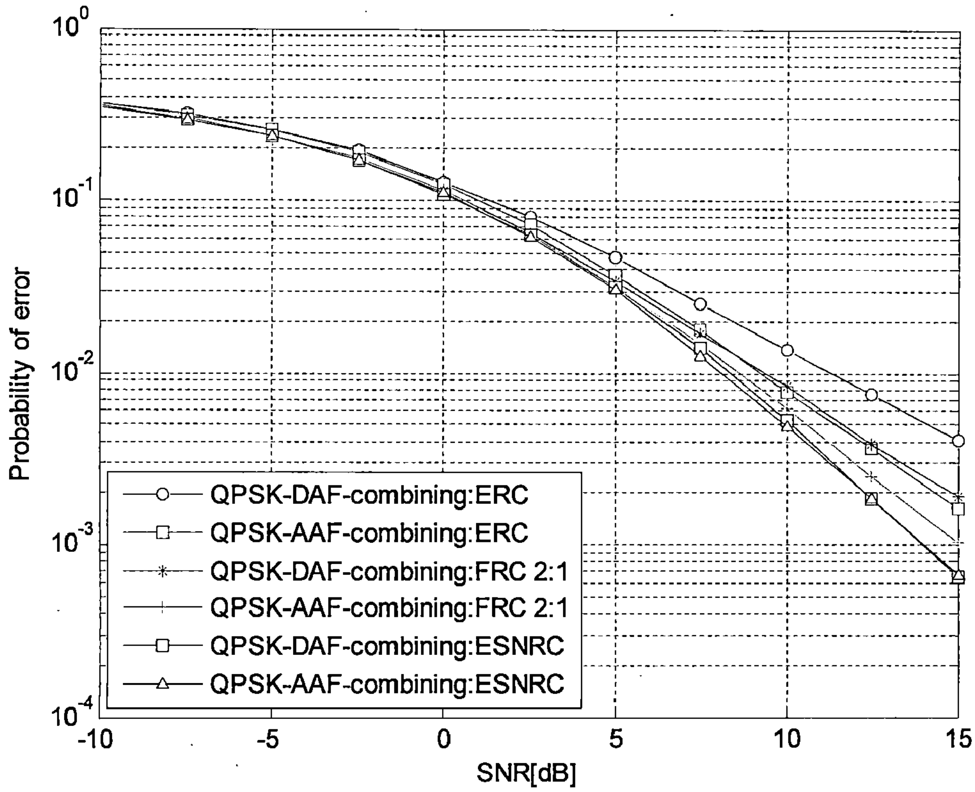

Figure 3.6: The two diversity protocols (AAF and DAF) are compared with each other. Independent of the combining type, the AAF always results in the better performance.

Using equal ratio combining results in a big difference between the two protocols. While the one using AAF shows a quite good performance already, the one using DAF shows no improvement at all. The reason is that a wrong detected symbol at the relay station is really difficult to correct at the destination, where the two incoming signals are combined. The incorrectly detected symbol is sent by the relay with the same power as the correct symbol over the direct link. This means that, when the two signals are combined at the destination, it is equally likely that the symbol is both correctly and incorrectly detected. So an incorrectly detected symbol at the relay station will have a fifty percent probability of also being incorrectly detected at the destination.

This stands in contrast with the equal ratio combining in a system using AAF. Instead of detecting the symbol at the relay, it is amplified and transmitted to the sender. Normally a symbol that would have been detected wrongly is just 'a little bit' wrong. When this symbol is amplified before sent to the destination, it has on average much less energy than the correct symbol coming directly from the sender. There is now a high probability that the incorrect symbol will be corrected by the signal from the direct link, when combined at the destination. This is of course only the case when the symbol over the direct link did not suffer too much from a bad channel.

It is obvious now, why the fixed ratio combining shows such a good performance. The direct link has on average the better quality than the multi-hop link, so it is sensible to weigh the direct link more by assuming that the multi-hop link is more susceptible to errors than the direct link. It also explains why the optimal ratio in the system using DAF is higher than the one using AAF. The DAF relay sends the wrongly detected symbols with the full power, so it takes much more to correct this wrong powerful symbol.

The ESNRC shows roughly the same performance in a AAF or DAF system. The DAF using system benefits a lot by analysing every single block. Using this combining method the big disadvantage of the wrongly detected symbol at the relay can be reduced. In the majority of the cases, when a symbol is wrongly detected by the relay, the multi-hop has a much poorer channel quality than the direct link, and therefore will not be considered at all.

It might be sensible to ask now, what the purpose is of making the effort at the relay station to decode and re-encode the data, when there is no benefit at all doing that. As mentioned in Sec. 3.1.2 there are mainly two different types of how a DAF system can be implemented. Within this thesis there is no error correcting code added to the data, so there is no chance to correct any wrongly detected bits at the receiver. This is, as seen before, crucial to get a good performance in a DAF system.

3.3.2 Moving the Relay

So far, the three stations were positioned equidistantly and therefore the three channels had all the same average signal-to-noise ratio. In this section the effect is shown when the relay station is moved. For the following simulations the AAF diversity protocol is used and the incoming signals at the destination are combined using ESNRC. As seen in the Sec. 3.3.1.3 this is the combination that results in the best possible performance.

The x-axis in the figures shows the average signal-to-noise ratio, for a channel of length one. This was the case for all three channels in the equidistant arrangement in the last section. In this section, the relay is moved, so the distance from the relay to the sender/destination will change. But in all the simulations, the distance between the sender and the destination is set to one, and therefore the signal-to-noise ratio shown in the x-axis is only valid for the direct link.

3.3.2.1 Relay between Sender and Destination

The propagation over the multi-hop link does not need to make any detour, when the relay is situated between the sender and the destination. This is the optimal scenario and should result in the best possible performance.

If the relay is situated very close to the sender, the whole arrangement corresponds approximately to a two sender system. The effect on the signal quality when moving the relay between the two other stations is shown in Fig. 3.7. With this optimal configuration, the resulting benefit is huge and much better than the one for the two sender system. The best performance is achieved, when the relay is situated in the middle between the sender and the destination, or slightly closer to the sending station.

The resulting performance is not symmetrical at all. The preferred position of the relay is in the middle between the sender and the destination. When this is not possible the relay should be closer to the sender than to the destination. Recall minding how the AAF protocol works, this is obvious. The noise received in the relay station is amplified with the signal. So on one hand it is desirable, that the received noise at the relay station does not has much energy. On the other hand, the closer the relay comes to the sender, the further away is the destination and therefore the worse is the channel quality of the second hop. The quality of the first hop is more important for the overall channel quality than the second hop, so the performance is not symmetrical.

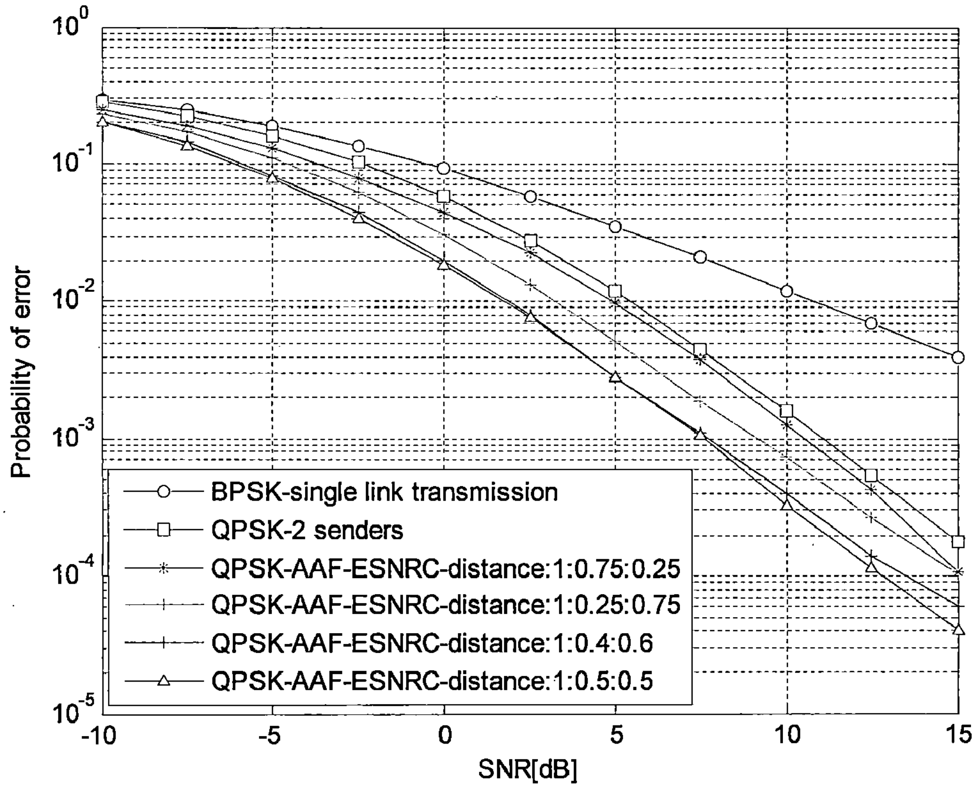

Figure 3.7: A big benefit results when the relay is located between the sender and the destination.

Another point that should be paid attention to is the huge benefit compared to the BSPK direct link. To achieve a BER of about 10^{-2} the SNR is up to eight decibels less than using only a direct link transmission.

3.3.2.2 Equidistant Position of the Relay

Normally there is no relay station available just between the sender and the destination. To see the effect the length of the multi-hop link has on the system performance, the relay is moved away gently from the optimum position between the sender and the destination. This is illustrated in Fig. 3.8.

The first thing that attracts attention is how fast the performance gets worse when the distance of the relay increases. By increasing the distance by fifty percent, the resulting performance is roughly the same as the one for a two sender system, which is about three decibels less than the one of the optimal position. The position of the relay, where all three stations are equidistant, results in another 2.5 decibel loss in the system performance. This equidistant arrangement still shows an advantage compared to the BPSK single link transmission.

This changes pretty fast, when the distance of the relay is increased further. Another fifty percent, result in a situation, where there is no useful advantage anymore using the relay link. But the higher diversity level can still be recognised.

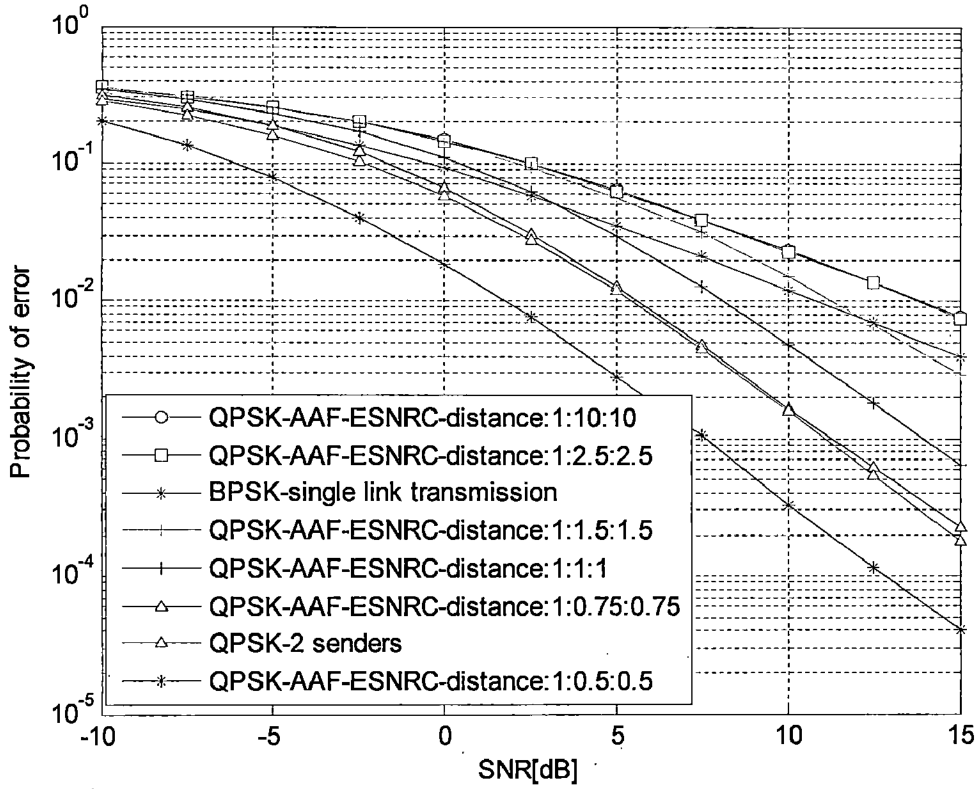

Figure 3.8: Shows the effect of increasing the distance of the relay to the sender and the destination.

When the relay is situated in the double distance of the equidistant arrangement, there is no benefit at all using the relay link. The resulting performance is roughly the same as the one of the QPSK single link transmission. This means, that the relay link, does not contain any useful information anymore. There is now just too much noise in the signal to get any benefit.

3.3.2.3 Moving the Relay Close to the Sender/Destination

In Fig. 3.9 the arrangement is illustrated where the relay is much closer to either the sender or the destination. In contrast to the situation where the relay was situated between the two other stations, the arrangement shows now much more symmetry. The reason for that is that the direct link contains the better signal quality and therefore is mainly responsible for the performance.

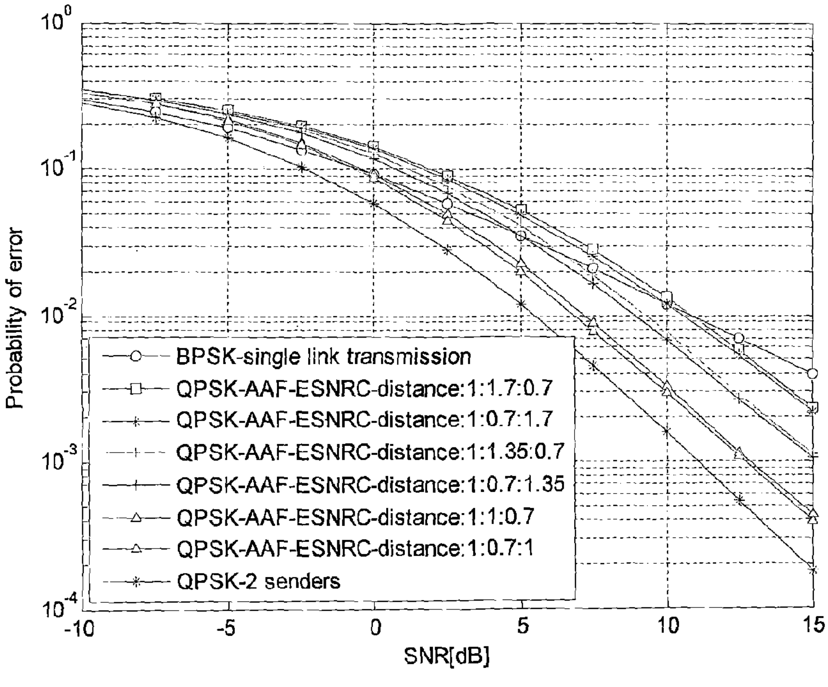

Figure 3.9: The relay is moved close to the sender/destination.

The main interest is now to determine where a mobile station can be located so that there is some benefit from using it as a relay station. Looking at Fig. 3.8 and Fig. 3.9 you can get the basic idea. If the relay is located close to the sender or the destination, the distance to the other station can be about forty percent longer than the one to the direct link. When the relay is roughly the same distance from both stations, this distance should not be much longer than the direct link to get a benefit. This results roughly in an elliptical region between base and mobile, where a second mobile station has to be situated to make it an attractive candidate as a relay.

Chapter 4
Distributed Space-Frequency Coding (DSFC)

The main problem with the multi-node decode-and-forward protocol and the multi-node amplify and-forward protocol is the loss in the data rate as the number of relay nodes increases. The use of orthogonal subchannels for the relay node transmissions, either through TDMA or FDMA, results in a high loss of the system spectral efficiency. This leads to the use of what is known as distributed space-time coding, where relay nodes are allowed to simultaneously transmit over the same channel by emulating a space-time code. Several works have considered the application of the existing space-time codes in a distributed fashion for the wireless relay network [7,8,9].

For the case of multi-path fading channels, the design of distributed space-frequency codes (DSFCs) is crucial to exploit the multi-path (frequency) diversity of the channel. The presence of multi-paths provides another mean for achieving diversity across the frequency axis. Design for distributed space-frequency codes (DSFCs) over relay channels that can exploit the multi-path diversity of wireless channels. We prove that the proposed design of DSFC can achieve full diversity of order LN, where L is the number of multi-paths per channel and N is the number of relay nodes. We prove the previous result for any number of relays N and for the cases of $L = 1$ (flat fading channel) and $L = 2$ (two-ray fading channel).

In this chapter, we will consider the design of distributed space-frequency coding (DSFC) for broadband multipath fading channels to exploit the frequency diversity of the channel. The presence of multipath in broadband channels provides another means for achieving diversity across the frequency axis. Exploiting the frequency axis diversity can highly improve the system performance by achieving higher diversity orders. The main problem for the wireless relay network is how to design space-frequency codes distributed among spatially separated relay nodes while guaranteeing to achieve full diversity at the destination node. The spatial separation of the relay nodes presents other challenges for the design of DSFCs such as time synchronization and carrier offset synchronization.

In this section, we will present some structures for distributed space-frequency codes (DSFCs) over wireless broadband relay networks. The presented DSFCs are designed to achieve the frequency and cooperative diversities of the wireless relay channels. The use of DSFCs with the amplify-and-forward (AAF) protocols is considered. The code design criteria to achieve full diversity, based on the pair wise error probability (PEP) analysis, are derived. For the case of DSFC with the AAF protocol, a structure for distributed space- frequency coding will be presented and sufficient conditions for that structure to achieve full diversity will then be derived.

4.1 DSFC with the AAF Protocol

In this section, the design and performance analysis for DSFCs with the AAF protocol are presented. A structure is proposed and sufficient conditions for the proposed structure to achieve full diversity are then derived for some special cases.

4.1.1 System Model

In this section, the system model for the distributed space-frequency coding is presented. We consider a two-hop relay channel model where there is no direct link between the source and the destination nodes. A simplified system model is depicted in Fig. 4.1. The system is based on orthogonal frequency division multiplexing (OFDM) modulation with K subcarriers. The channel between the source and the n-th relay node is modeled as multi-path fading channel with L paths as

$$H_{s,r_n}(\tau)=\sum_{l=1}^{L}\alpha_{s,r_n}(l)\delta(\tau-\tau_l),$$

(4.1)

Where τ_l is the delay of the l-th path, and $\alpha_{s,r_n}(l)$ is the complex amplitude of the l-th path. The $\alpha_{s,r_n}(l)$'s are modeled as zero-mean complex Gaussian random variables with variance $E\left[\left|\alpha_{s,r_n}(l)\right|^2\right]=\sigma^2(l)$, where we assume symmetry between the relay nodes to simplify the analysis. The channels are normalized such that $\sum_{l=1}^{L}\sigma^2(l)=1$. A cyclic prefix introduced to convert the multi-path frequency-selective fading channels to flat fading sub-channels on the subcarriers.

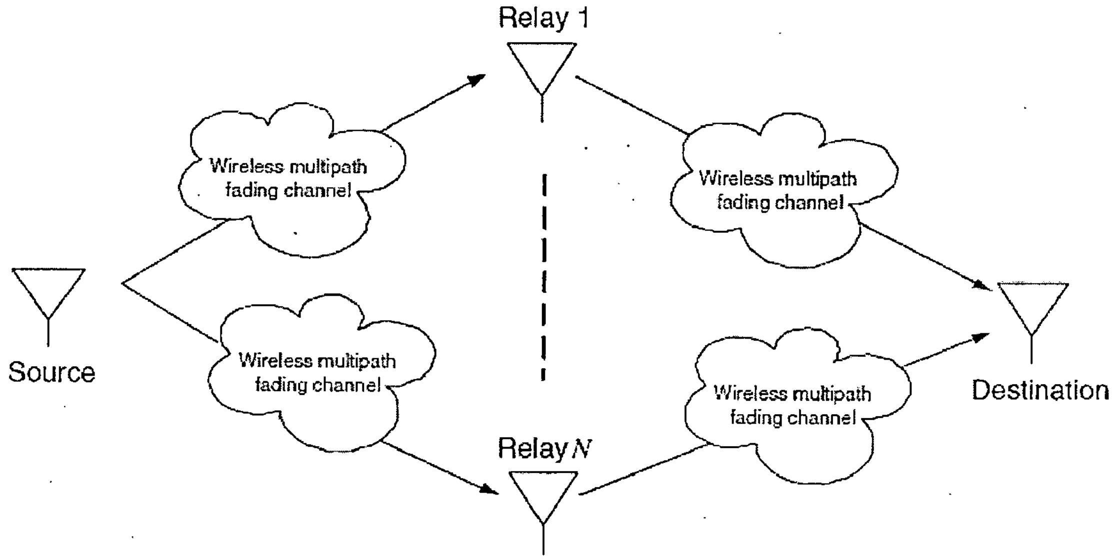

Fig. 4.1. Simplified system model for the distributed space-frequency codes.

The system has two phases as follows. In phase 1, if N relays are assigned for helping the source, the source broadcasts the information to the N relays. The received signal in the frequency domain on the k-th subcarrier at the n-th relay node is given by

$$y_{s,r_n}(k)=\sqrt{P_1}H_{s,r_n}(k)s(k)+\eta_{s,r_n}(k),\ \mathrm{k}=1,\cdots,\mathrm{K}, \tag{4.2}$$

where P_1 is the transmitted source power, $H_{s,r_n}(k)$ is the attenuation of the source to the n-th relay channel on the k- th subcarrier, *s(k)* is the transmitted source symbol on the k-th subcarrier, and $\eta_{s,r_n}(k)$ is the n-th relay additive white Gaussian noise on the k-th subcarrier. $\eta_{s,r_n}(k)$ is modeled as circularly symmetric complex Gaussian random variable with variance $N_0/2$ per dimension. The relay noise terms on the subcarriers can be easily seen to be statistically independent assuming that the noise terms at the input of the FFT at the relay nodes are independent.

The channel attenuation in the frequency domain, $H_{s,r_n}(k)$, is given by

$$H_{s,r_n}(k)=\sum_{l=1}^{L}\alpha_{s,r_n}(l)e^{-j2\pi(k-1)\Delta f\tau_l}, \tag{4.3}$$

where $\Delta \mathrm{f} = 1/\mathrm{T}$ is the subcarrier frequency separation, and T is the OFDM symbol duration. We assume perfect channel state information at any receiving node but no channel information at transmitting nodes. We assume that all the noise terms are independent for different receiving nodes.

The proposed DSFC is described as follows. The transmitted data from the source node is parsed into subblocks of size $NL \times 1$. Let $P = \lfloor K/NL \rfloor$ denote the number of subblocks in the transmitted OFDM block. The transmitted $NL \times 1$ source codeword is given by

$$s = [s(1), s(2), \cdots, s(K)]^T = [\mathbf{B}_1^T, \mathbf{B}_2^T, \cdots, \mathbf{B}_P^T, \mathbf{0}_{K-PLN}^T]^T,$$

Where $\mathbf{B}_i$ is the i-th subblock of dimension $\mathrm{NL} \times 1$. Zeros are padded if K is not an integer multiple of NL. For each subblock, $\mathbf{B}_i$, the n-th relay only forwards the data on L subcarriers. For example, relay 1 will only forward $[\mathbf{B}_i(1), \cdots, \mathbf{B}_i(L)]$ for all i's and send zeros on the remaining set of subcarriers. In general, the n-th relay will only forward $[\mathbf{B}_i((n-1)L+1), \cdots, \mathbf{B}_i((n-1)L+L)]$ for all i's.

At the relay nodes, each node will normalize the received signal on the subcarriers that it will forward before retransmission and send zeros on the remaining set of subcarriers. If the k-th subcarrier is to be forwarded by the n-th relay, the relay will normalize the received signal on that subcarrier by the factor $\beta(k) = \sqrt{\dfrac{1}{P_1 |H_{s,r_n}(k)|^2 + N_0}}$ [1]. The relay nodes will use OFDM modulation for transmission to the destination node. At the destination node, the received signal on the k-th subcarrier, assuming it was forwarded by the n-th relay, is given by

$$y(k) = H_{r_n,d}(k)\sqrt{P_2}\left(\sqrt{\frac{1}{P_1 |H_{s,r_n}(k)|^2 + N_0}}(\sqrt{P_1} H_{s,r_n}(k) s(k) + \eta_{s,r_n}(k))\right) + \eta_{r_n,d}(k), \tag{4.4}$$

Where P_2 is the relay node power, $H_{r_n,d}(k)$ is the attenuation of the channel between the n-th relay node and the destination node on the k-th subcarrier, and $\eta_{s,r_n}(k)$ is the destination noise on the k-th subcarrier. The $\eta_{r_n,d}(k)$'s are modeled as zero mean, circularly symmetric complex Gaussian random variables with a variance of $N_0/2$ per dimension.

4.1.2 Performance Analysis

In this section, the PEP of the DSFC with the AAF protocol is presented. Based on the PEP analysis, code design criteria are derived. The received signal at destination on the k-th subcarrier given by (4.4) can be rewritten as

$$y(k) = H_{r_n,d}(k)\sqrt{P_2}\left(\sqrt{\frac{1}{P_1\left|H_{s,r_n}(k)\right|^2 + N_0}}\sqrt{P_1}H_{s,r_n}(k)s(k)\right) + z_{r_n,d}(k), \tag{4.5}$$

Where $z_{r_n,d}(k)$ accounts for the noise propagating from the relay node as well as the destination noise. $z_{r_n,d}(k)$ follows a circularly symmetric complex Gaussian random variable with a variance $\delta_z^2(k)$ of $\left(\frac{P_2\left|H_{r_n,d}(k)\right|^2}{P_2\left|H_{r_n,d}(k)\right|^2 + N_0} + 1\right)N_0$. The probability density function of $z_{r_n,d}(k)$ given the channel state information (CSI) is given by

$$p\left(z_{r_n,d}(k)/CSI\right) = \frac{1}{\pi\delta_z^2(k)}\exp\left(-\frac{1}{\delta_z^2(k)}\left|z_{r_n,d}(k)\right|^2\right). \tag{4.6}$$

The receiver applies a Maximum Likelihood (ML) detector to the received signal, which is given as

$$\hat{s} = \arg\min_{s}\sum_{k=1}^{K}\frac{1}{\delta_z^2(k)}\left|y(k) - \frac{\sqrt{P_1P_2}H_{s,r_n}(k)H_{r_n,d}(k)}{\sqrt{P_1\left|H_{s,r_n}(k)\right|^2 + N_0}}s(k)\right|^2, \tag{4.7}$$

Where the n index (which is the index of the relay node) is adjusted according to the k index (which is the index of the subcarrier).

Now, sufficient conditions for the proposed code structure to achieve full diversity are derived. The pdf of a received vector $y = [y(1), y(2), \cdots, y(K)]^T$ given that the codeword s was transmitted is given by

$$p\left(y/s, CSI\right) = \left(\prod_{k=1}^{K}\frac{1}{\pi\delta_z^2(k)}\right)\exp\left(\sum_{k=1}^{K} -\frac{1}{\delta_z^2(k)}\left|y(k) - \frac{\sqrt{P_1P_2}H_{s,r_n}(k)H_{r_n,d}(k)}{\sqrt{P_1\left|H_{s,r_n}(k)\right|^2 + N_0}}s(k)\right|^2\right) \tag{4.8}$$

The PEP of mistaking s by $\tilde{s}$ can be upper-bounded as [16]

$$PEP(s \to \tilde{s}) \le E\left\{\exp\left(\lambda\left[\ln p(y/\tilde{s}) - \ln p(y/s)\right]\right)\right\}, \tag{4.9}$$

and the relation applies for any λ, which can selected to get the tightest bound. Any two distinct codeword's s and $\tilde{s} = \left[\tilde{\mathbf{B}}_1, \tilde{\mathbf{B}}_2, \cdots, \tilde{\mathbf{B}}_p\right]^T$ will have at least one index p_0 such that $\tilde{\mathbf{B}}_{p_0} \ne \mathbf{B}_{p_0}$. We will assume that s and s̃ will have only one index p_0 such that $\tilde{\mathbf{B}}_{p_0} \ne \mathbf{B}_{p_0}$, which corresponds to the worst case PEP.

Averaging the PEP expression in (4.9) over the noise distribution given in (4.6) we get

$$PEP(s \to \tilde{s}) \le E\left\{\exp\left(-\lambda(1-\lambda)\sum_{n=1}^{N}\sum_{l=1}^{L}\left(\frac{P_1\left|H_{s,r_n}(J+(n-1)L+l)\right|^2 P_2\left|H_{r_n,d}(J+(n-1)L+l)\right|^2}{\left(P_1\left|H_{s,r_n}(J+(n-1)L+l)\right|^2 + P_2\left|H_{r_n,d}(J+(n-1)L+l)\right|^2 + N_0\right)N_0}\right)\right.\right.$$
$$\left.\left.\times\left|\mathbf{B}_{p_0}((n-1)L+l) - \tilde{\mathbf{B}}_{p_0}((n-1)L+l)\right|^2\right)\right\} \tag{4.10}$$

Where $J = (p_0 - 1)NL$. Take $\lambda = 1/2$ to minimize the upper-bound in (4.10), hence, we get

$$PEP(s \to \tilde{s}) \le E\left\{\exp\left(-\frac{1}{4}\sum_{n=1}^{N}\sum_{l=1}^{L}\left(\frac{P_1\left|H_{s,r_n}(J+(n-1)L+l)\right|^2 P_2\left|H_{r_n,d}(J+(n-1)L+l)\right|^2}{\left(P_1\left|H_{s,r_n}(J+(n-1)L+l)\right|^2 + P_2\left|H_{r_n,d}(J+(n-1)L+l)\right|^2 + N_0\right)N_0}\right)\right.\right.$$
$$\left.\left.\times\left|\mathbf{B}_{p_0}((n-1)L+l) - \tilde{\mathbf{B}}_{p_0}((n-1)L+l)\right|^2\right)\right\} \tag{4.11}$$

At high SNR, the term $\frac{P_1\left|H_{s,r_n}(k)\right|^2 P_2\left|H_{r_n,d}(J+(k)\right|^2}{\left(P_1\left|H_{s,r_n}(k)\right|^2 + P_2\left|H_{r_n,d}(k)\right|^2 + N_0\right)N_0}$ can be approximated by $\frac{P_1\left|H_{s,r_n}(k)\right|^2 P_2\left|H_{r_n,d}(J+(k)\right|^2}{\left(P_1\left|H_{s,r_n}(k)\right|^2 + P_2\left|H_{r_n,d}(k)\right|^2\right)N_0}$ [10], which is the scaled harmonic mean of the source-relay and relay-destination SNRs on the k-th subcarrier. The scaled harmonic mean of two nonnegative numbers, a_1 and a_2, can be upper- and lower- bounded as

$$\frac{1}{2}\min\left(a_1,a_2\right) \le \frac{a_1 a_2}{a_1+a_2} \le \min\left(a_1,a_2\right) \tag{4.12}$$

Using the lower-bound in (5.12) the PEP in (5.11) can be further upper-bounded as

$$\begin{aligned} PEP(s \to \tilde{s}) \le E\Bigg\{ \exp\Bigg(-\frac{1}{8}\sum_{n=1}^{N}\sum_{l=1}^{L} \min\Bigg(\frac{P_1}{N_0}\left|H_{s,r_n}((p_0-1)NL+(n-1)L+l)\right|^2, \\ \frac{P_2}{N_0}\left|H_{r_n,d}((p_0-1)NL+(n-1)L+l)\right|^2 \Bigg) \left|\mathbf{B}_{p_0}((n-1)L+l)-\tilde{\mathbf{B}}_{p_0}((n-1)L+l)\right|^2 \Bigg) \Bigg\} \end{aligned} \tag{4.13}$$

If $P_2 = P_1$ and SNR is defined as $\frac{P_1}{N_0}$, then the PEP is now upper-bounded as

$$\begin{aligned} PEP(s \to \tilde{s}) \le E\Bigg\{ \exp\Bigg(-\frac{1}{8}\sum_{n=1}^{N}\sum_{l=1}^{L} \min\Big(SNR\left|H_{s,r_n}((p_0-1)NL+(n-1)L+l)\right|^2, \\ SNR\left|H_{r_n,d}((p_0-1)NL+(n-1)L+l)\right|^2 \Big) \left|\mathbf{B}_{p_0}((n-1)L+l)-\tilde{\mathbf{B}}_{p_0}((n-1)L+l)\right|^2 \Bigg) \Bigg\} \end{aligned} \tag{4.14}$$

4.1.3 PEP Analysis for L=1

The case of L equal to 1 corresponds to a flat, frequency nonselective fading channel. The PEP in (5.14) is now given by

$$\begin{aligned} PEP(s \to \tilde{s}) \le E\Bigg\{ \exp\Bigg(-\frac{1}{8}\sum_{n=1}^{N} \min\Big(SNR\left|H_{s,r_n}((p_0-1)NL+(n-1)L+l)\right|^2, \\ SNR\left|H_{r_n,d}((p_0-1)NL+(n-1)L+l)\right|^2 \Big) \left|\mathbf{B}_{p_0}((n-1)L+l)-\tilde{\mathbf{B}}_{p_0}((n-1)L+l)\right|^2 \Bigg) \Bigg\} \end{aligned} \tag{4.15}$$

It can be shown that the random variables $SNR\left|H_{s,r_n}(k)\right|^2$ and $SNR\left|H_{r_n,d}(k)\right|^2$ follow an exponential distribution with rate $1/\mathrm{SNR}$ for all k. The minimum of two exponential random variables is an exponential random variable with rate that is the sum of the two random variables rates. Hence, $\min\left(SNR\left|H_{s,r_n}(k)\right|^2, SNR\left|H_{r_n,d}(k)\right|^2\right)$ follows an exponential distribution with rate $2/\mathrm{SNR}$.

The PEP upper-bound is now given by

$$PEP(s \rightarrow \tilde{s}) \leq \prod_{n=1}^{N} \frac{1}{1+\frac{1}{16} SNR \left| \mathbf{B}_{p_0}((n-1)L+l) - \tilde{\mathbf{B}}_{p_0}((n-1)L+l) \right|^2} \tag{4.16}$$

At high SNR, we neglect the 1 term in the denominator of (4.16). Hence, the PEP can now be upper-bounded as

$$PEP(s \rightarrow \tilde{s}) \leq \left(\frac{1}{16} SNR \right)^{-N} \left(\prod_{n=1}^{N} \left| \mathbf{B}_{p_0}((n-1)L+l) - \tilde{\mathbf{B}}_{p_0}((n-1)L+l) \right|^2 \right)^{-1} \tag{4.17}$$

The result in (4.17) is under the assumption that the product

$$\prod_{n=1}^{N} \left| \mathbf{B}_{p_0}((n-1)L+l) - \tilde{\mathbf{B}}_{p_0}((n-1)L+l) \right|^2$$

is non-zero. Clearly, if that product is non-zero, then the system will achieve a diversity of order NL, where L is equal to 1 in this case. From the expression in (4.14) the coding gain of the space-frequency code is maximized when the product $\min_{s \neq \tilde{s}} \prod_{n=1}^{N} \left| \mathbf{B}_{p_0}((n-1)L+1) - \tilde{\mathbf{B}}_{p_0}((n-1)L+1) \right|^2$ is maximized. This product is known as the minimum product distance [11].

4.1.4 PEP Analysis for L=2

The PEP in (5.14) can now be given as

$$\begin{aligned} PEP(s \rightarrow \tilde{s}) \leq E\Bigg\{ \exp\Bigg(-\frac{1}{8} \sum_{n=1}^{N} \sum_{l=1}^{2} \min\Big(& SNR \left| H_{s,r_n}((p_0-1)NL+(n-1)L+l) \right|^2, \\ & SNR \left| H_{r_n,d}((p_0-1)NL+(n-1)L+l) \right|^2 \Big) \left| \mathbf{B}_{p_0}((n-1)L+l) - \tilde{\mathbf{B}}_{p_0}((n-1)L+l) \right|^2 \Bigg) \Bigg\} \end{aligned} \tag{4.18}$$

Where L = 2. The analysis in this case is more involved since the random variables appearing in (4.18) are correlated. Signals transmitted from the same relay node on different subcarriers will experience correlated channel attenuations. As a first step in deriving the code design criterion, we prove that the channel attenuations, $|H_{s,r_n}(k_1)|^2$ and $|H_{s,r_n}(k_2)|^2$ for any $k_1 \neq k_2$, have a bivariate Gamma distribution as their joint pdf [12]. The same applies for $\left|H_{r_n,d}(k_1)\right|^2$ and $\left|H_{r_n,d}(k_2)\right|^2$ for any $k_1 \neq k_2$.

To evaluate the expectation in (4.15) we need the expression for the joint pdf of the two random variables $M_1 = \min\left(SNR\left|H_{s,r_n}(k_1)\right|^2, SNR\left|H_{r_n,d}(k_1)\right|^2\right)$ and $M_1 = \min\left(SNR\left|H_{s,r_n}(k_2)\right|^2, SNR\left|H_{r_n,d}(k_2)\right|^2\right)$ for some $k_1 \neq k_2$. Although M_1 and M_2 can be easily seen to be marginally exponential random variables, they are not jointly Gamma distributed. Define the random variables $X_1 = SNR\left|H_{s,r_n}(k_1)\right|^2, X_2 = SNR\left|H_{s,r_n}(k_2)\right|^2, Y_1 = SNR\left|H_{r_n,d}(k_1)\right|^2$, $Y_2 = SNR\left|H_{r_n,d}(k_2)\right|^2$. All of these random variables are marginally exponential with rate $1/\mathrm{SNR}$. Under the assumptions of our channel model, the pairs (X1;X2) and (Y1; Y2) are independent. Hence, the joint pdf of $(\mathrm{X}_1, \mathrm{X}_2, \mathrm{Y}_1, \mathrm{Y}_2)$, using the result in the Appendix, is given by

$$\begin{aligned}
& f_{X_1,X_2,Y_1,Y_2}(x_1,x_2,y_1,y_2) \\
& = f_{X_1,X_2}(x_1,x_2) f_{Y_1,Y_2}(y_1,y_2) \\
& = \frac{1}{SNR^2(1-\rho_{x_1x_2})(1-\rho_{y_1y_2})} \exp\left(-\frac{x_1+x_2}{SNR(1-\rho_{x_1x_2})}\right) I_0\left(\frac{2\sqrt{\rho_{x_1x_2}}}{SNR(1-\rho_{x_1x_2})}\sqrt{x_1x_2}\right) \\
& \times \exp\left(-\frac{y_1+y_2}{SNR(1-\rho_{y_1y_2})}\right) I_0\left(\frac{2\sqrt{\rho_{y_1y_2}}}{SNR(1-\rho_{y_1y_2})}\sqrt{y_1y_2}\right) U(x_1)U(x_2)U(y_1)U(y_2)
\end{aligned} \tag{4.19}$$

Where. $I_0(\cdot)$ the modified Bessel function of the first kind of order is zero and $U(\cdot)$ is the Heaviside unit step function [13]. $\rho_{x_1x_2}$ is the correlation coefficient between X_1 and X_2 and similarly, $\rho_{y_1y_2}$ is the correlation coefficient between Y_1 and Y_2. The joint cumulative distribution function (cdf) of the pair $(\mathrm{M}_1, \mathrm{M}_2)$ can be computed as

$$\begin{aligned}
& F_{M_1,M_2}(m_1,m_2) \\
& \square \Pr\left[M_1 \le m_1, M_2 \le m_2\right] \\
& = \Pr\left[\min(X_1,Y_1) \le m_1, \min(X_2,Y_2) \le m_2\right] \\
& = 2\int_{y_1=0}^{m_1}\int_{x_1=y_1}^{\infty}\int_{y_2=0}^{m_2}\int_{x_2=y_2}^{\infty} f_{X_1,X_2}(x_1,x_2) f_{Y_1,Y_2}(y_1,y_2)\, dy_1 dx_1 dy_2 dx_2,
\end{aligned} \tag{4.20}$$

Where we have used the symmetry assumption of the source-relay and relay-destination channels. The joint pdf of (M_1, M_2) can now be given as

$$f_{M_1,M_2}(m_1,m_2) = \frac{\partial^2}{\partial m_1 \partial m_2} F_{M_1,M_2}(m_1,m_2)$$
$$= 2 f_{Y_1,Y_2}(m_1,m_2) \int_{x_1=m_1}^{\infty} \int_{x_2=m_2}^{\infty} f_{X_1,X_2}(x_1,x_2) dx_1 dx_2 \tag{4.21}$$
$$+2 \int_{x_1=m_1}^{\infty} \int_{y_2=m_2}^{\infty} f_{X_1,X_2}(x_1,m_2) f_{Y_1,Y_2}(m_1,y_2) dx_1 dx_2.$$

To get the PEP upper-bound in (4.18) we need to calculate the expectation

$$E\left\{\exp\left(-\frac{1}{8}\left(M_1 \left|\mathbf{B}(k_1)-\tilde{\mathbf{B}}(k_1)\right|^2 + M_2 \left|\mathbf{B}(k_2)-\tilde{\mathbf{B}}(k_2)\right|^2\right)\right)\right\}$$
$$= \int_{m_1=0}^{\infty} \int_{m_2=0}^{\infty} \exp\left(-\frac{1}{8}\left(m_1 \left|\mathbf{B}(k_1)-\tilde{\mathbf{B}}(k_1)\right|^2 + m_2 \left|\mathbf{B}(k_2)-\tilde{\mathbf{B}}(k_2)\right|^2\right)\right) \tag{4.22}$$
$$f_{M_1,M_2}(m_1,m_2) dm_1 dm_2.$$

At high enough SNR $I_0\left(\frac{2\sqrt{\rho_{x_1x_2}}}{SNR(1-\rho_{x_1x_2})}\sqrt{x_1x_2}\right)$ can be approximated to be 1 [13]. Using this approximation, the PEP upper-bound can be approximated at high SNR as

$$PEP(s \to \tilde{s}) \le \left(\prod_{m=1}^{2N} \left|\mathbf{B}_{p_0}(m) - \tilde{\mathbf{B}}_{p_0}(m)\right|^2\right)^{-1} \left(\frac{1}{16}(1-\rho)SNR\right)^{-2N}, \tag{4.23}$$

Where $\rho = \rho_{x_1x_2} = \rho_{y_1y_2}$. Again, full diversity is achieved when the product $\prod_{m=1}^{2N} \left|\mathbf{B}_{p_0}(m) - \tilde{\mathbf{B}}_{p_0}(m)\right|^2$ is non-zero. The coding gain of the space-frequency code is maximized when the product $\min_{s \neq \tilde{s}} \prod_{m=1}^{2N} \left|\mathbf{B}_{p_0}(m) - \tilde{\mathbf{B}}_{p_0}(m)\right|^2$ is maximized.

The analysis becomes highly involved for any $L \geq 3$. It is very difficult to get closed-form expressions in this case due to the correlation among the summed terms in (4.14) for which no closed-form pdf expressions, similar to (4.19), are known [14].

4.2. SIMULATION RESULTS

In this section, we present some simulations for the proposed distributed space-frequency code. Fig. 4.2 shows the case of a simple two-ray, L = 2, channel model with a delay of $\tau=$ 5μsec between the two rays. The two rays have equal powers, i.e., $\sigma^2(1)=\sigma^2(2)$. The number of subcarriers is K = 128 with a system bandwidth of 1 MHz. Fig. 2 shows the symbol error rate (SER) of the proposed DSFCs versus the SNR defined as SNR = $\frac{P_S+P_R}{N_0}$, and we use P_S = P_R, i.e., equal power allocation between the source and relay nodes.

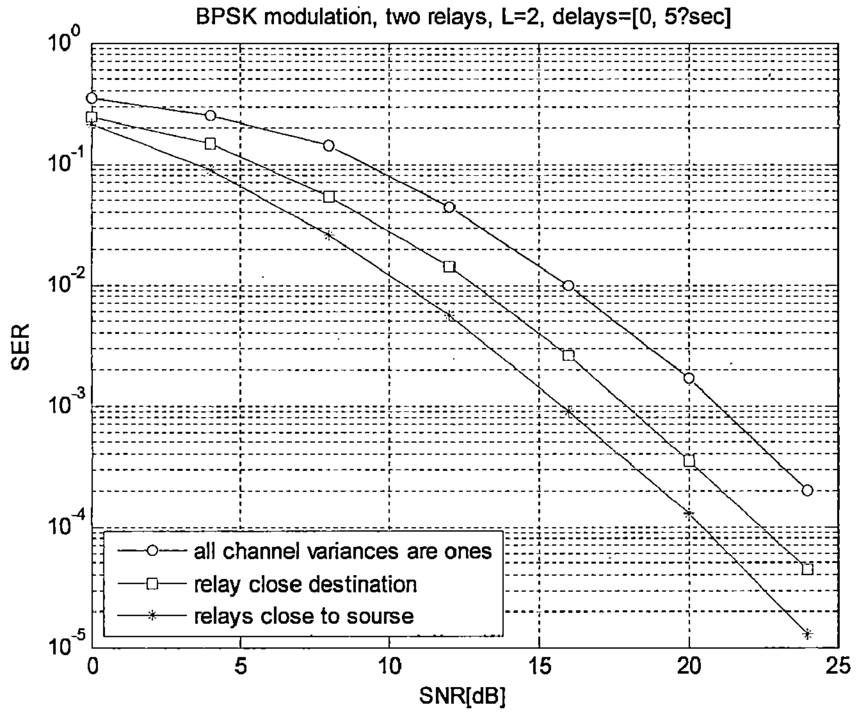

Fig.4. 2. SER for DSFCs for BPSK modulation, L=2, and delay= [0, 5μsec] versus SNR.

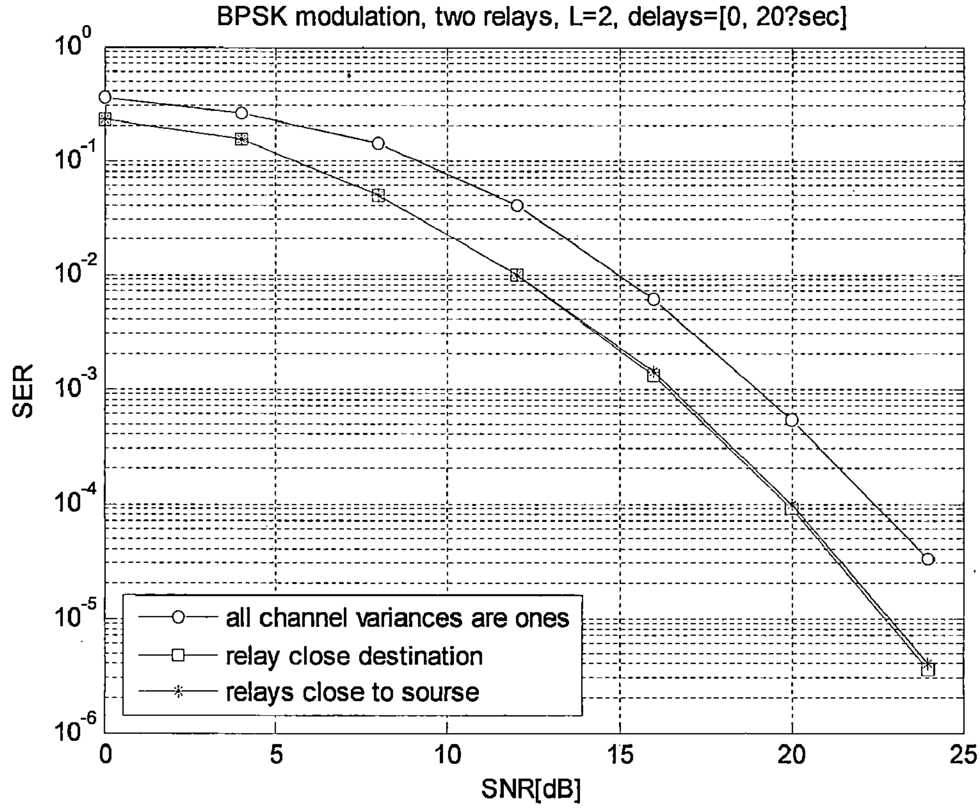

Fig. 3. SER for DSFCs for BPSK modulation, L=2, and delay=[0, 20μsec] versus SNR.

We simulated three cases: all channel variances are ones, relays close to source, and relays close to destination. For the case of relays close to source, the variance of any source-relay channel is taken to be 10 and the variance of any relay-destination channel is taken to be 1. For the case of relays close to destination, the variance of any source-relay channel is taken to be 1 and the variance of any relay-destination channel is taken to be 10. Fig. 4.3 shows the case of a simple two-ray, L = 2, with a delay of $\tau = 20\mu sec$ between the two rays. The simulation setup is the same as that used in Fig.4. 2.

Chapter 5

CONCLUSIONS

This thesis has shown the possible benefits of a wireless transmission using cooperative diversity to increase the performance. The diversity is realized by building an ad-hoc network using a third station as a relay. The data is sent directly from the base to the mobile or via the relay station. Such a system has been simulated to see the performance of different diversity protocols and various combining methods.

The AAF protocol has shown a better performance than the DAF protocol whatever combining method was used at the receiver. The choice of combining method has a big effect on the error rate at the receiver. When AAF is used at the relay station the easy to implement Equal Ratio Combining (ERC) shows some benefits compared to the single link transmission. If possible the Fixed Ratio Combining (FRC) should be used. This only need knowledge of the average channel quality, and shows a much better performance than the ERC. If knowledge of the current state of the channel quality is available more sophisticated combining methods can be used. The Enhanced Signal-to-Noise Ratio Combining (ESNRC) has shown a very good performance considering that a rough approximation of the channel quality is sufficient.

A design of distributed space-frequency codes was discussed for the wireless relay network employing the amplify-and-forward protocol. We derive sufficient conditions for the code structure, based on the PEP analysis, to achieve full diversity and maximum coding gain. We analysed the PEP analysis for L=1 and L=2. We prove that the proposed codes can achieve full diversity of order LN, promised by the multi-path and cooperative diversities of the wireless relay channel, for the special cases of L = 1 and L = 2.

The location of the relay is crucial to the performance. The best performance was achieved when the relay is at equal distance from the sender and the destination or slightly closer to the former. In general the relay should not be to far from the line between the two stations.

Future work

During a wireless communication the involved stations might moving around. Sometimes there is a well placed mobile station available that can be used as a relay. But most of the time the mobile station is not located optimally or is too far away to be useful as a relay at all. It would be very interesting to see the overall performance of this more complicated system.

Another way to enhance this project would be to use more relays. Such a system should show higher levels of diversity and might have a lot of potential.

The construction for the design of DSFCs can be easily generalized to the case of multi-antenna nodes, where any node may have more than one antenna. Each antenna can be treated as a separate relay node and the analysis presented before directly applies.

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