A MATHEMATICAL MODEL FOR STREAM - AQUIFER INTERACTION

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

HYDROLOGY

By

KAILASH YADAV

DEPARTMENT OF HYDROLOGY
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE - 247 667 (INDIA)

JUNE, 2006

CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this dissertation entitled **"A Mathematical Model for Stream-Aquifer Interaction"** in partial fulfillment of the requirement for the award of the degree of **Master of Technology in Hydrology,** submitted in the Department of Hydrology of Indian Institute of Technology Roorkee, India is an authentic record of my work carried out during the period from August, 2005 to June, 2006 under the supervision of **Dr. M. Perumal,** Associate Professor, Department of Hydrology and **Prof. G.C. Mishra**, Emeritus Fellow, Department of W.R.D.& M., I.I.T.Roorkee.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree.

Kayadav

I.I.T.Roorkee **(KAILASH YADAV)**

Date: 29/06/06 Candidate's Signature

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

G. C. Mishra

(G. C. Mishra)

Emeritus Fellow

Department of W. R. D. & M.

Indian Institute of Technology

Roorkee (Roorkee-247667)

(INDIA)

(M. Perumal)

Associate Professor

Department Of Hydrology

Indian Institute of Technology

Roorkee (Roorkee-247667)

(INDIA)

ACKNOWLEDGEMENT

With all my respect and honor, I express my heartily gratitude to **Dr. M. Perumal,** Assoc. Professor, Department of Hydrology, I.I.T. Roorkee and **Prof. G. C. Mishra,** Emeritus Fellow, Department of W.R.D. &.M, I.I.T. Roorkee, for their inspiration and guidance.

I would like to thank Professor and Head, Department of Hydrology and all the faculty members for their encouragement in carrying out my dissertation successfully.

Also I would like to express my thanks to my friends for their invaluable cooperation to finalize this dissertation successfully.

I would like to thank to Mr. Jay Kumar, Mr. Bhabagrahi Sahoo, and Mrs. Tarun Yadav, Mrs. Seema Yadav, Mr. Munendra Yadav, Mr. Ranjan Sinha, Mr. Venu Gopal Nair, Mr.Satyveer Singh and Mr. H.V.Trivedi for their invaluable help in my dissertation.

My gratefulness to my parents, who has been a perennial source of inspiration and beacon light of faith and hope for me, is infinite and inexpressible.

Lastly, I would like to thank my family (Mr. Sheelesh, Munesh, Reetesh, Mr. Dinesh, Vipnesh, Tarun, Prabhat, Prasant, Beena, Geeta, Anita, Sunita) my grandmother, my uncle, my aunty, Pramod Kumar Sharma, Nidhish Mishra, Ashok, Gaurav Varshney for their moral support.

June, 2006 **(KAILASH YADAV)**

I.I.T Roorkee

ABSTRACT:

A stream, forming a boundary is often encountered in regional groundwater flow modeling. In case of a partially penetrating stream with considerable stream discharge, besides treating the stream as a prescribed head boundary, the exchange of flow between the stream and aquifer has to be introduced through the boundary nodes while modeling the groundwater flow. The recharge from a stream to an aquifer is proportional to the head difference in the level of water in the stream and in the aquifer in the vicinity of the stream. The coefficient of proportionality, recognized as reach transmissivity, depends upon aquifer characteristics and the shape of the stream cross- section. The water level in the aquifer depends on the abstractions and recharges including recharge from the stream. Such an implicit and complex stream-aquifer interaction problem has been analyzed by Morel-Seytoux and Daly (1975) who have used reach transmissivity and discrete kernel theory for finding an expression for recharge.

The actual magnitude of exchange of flow depends on the local geology, particularly the hydraulic conductivity of the interface between the stream and the aquifer. Really the process of interaction over spatially varied channel boundaries is difficult to examine. The most common scenario of the interaction of groundwater is that of the interaction of streams with contiguous alluvial aquifer. This type of system has been the focus of study for more than 100 years, beginning from the work of Boussinesq (1877) to the present.

In this study, it is proposed to use the variable parameter Muskingum method advocated by Perumal (1994) for routing the discharge hydrograph in stream reach. For studying stream - aquifer interaction, the stream having hydraulic connection with the underlying aquifer can be subdivided into a number of sub- reaches. Each of the sub-reach

can be considered as a rectangular recharging reach. The stage in each of the sub-reaches varies with time due to unsteady nature of flood wave and / or due to interaction of stream with aquifer or vice versa, including the mutual interaction between the recharging sub-reaches. It is required to estimate the water level variations at different locations of the considered stream reach taking into account the stream-aquifer interaction. Further, it is required to estimate the recharge details in the interactive reaches considering (i) only a small stretch of the stream reach is interactive, and (ii) a long stretch of the stream reach is interactive. It is proposed to study the stream-aquifer interaction using the approximate analytical solution developed by Hantush (1967) by estimating the rise and fall of the water-table in an infinite unconfined aquifer in response to uniform percolation from rectangular recharging sub-reaches.

Based on the study it is concluded that the use of Hantush solution may be considered as an alternative approach for studying stream-aquifer interaction. Further, the study reveals that the effect of mutual influence of interacting sub-reaches on the stream-aquifer interaction is not significant.

CONTENTS

List of Figures

NOTATIONS

Q_L	=	a volumetric flow between two given sections of stream in units of volume per time;
k	=	hydraulic conductivity of streambed sediment in units of length per time;
W_p	=	wetted perimeter of the stream in units of length;
L	=	length of the reach under study;
h_S	=	head in the stream determined by adding stream depth to the elevation of the streambed in units of length;
M	=	average thickness of the streambed deposits extending from the top to the bottom of the streambed in units of length;
Q	=	stream flow;
A	=	cross-sectional area;
h_A	=	head of the aquifer beneath the stream head in units of length;
t	=	time;
v	=	mean cross-sectional velocity;
Q_R	=	leakage through the stream's bed and banks;
S_o	=	bed slope;
S_f	=	friction slope;
y_M	=	flow depth at the middle reach Δx;
Q_3	=	normal discharge corresponding to the mid-section
y_u	=	inlet stage;
y_d	=	outlet stage;
w.t.	=	water table;
h	=	hour;
m	=	meter;
V.P.M.	=	variable parameter Muskingum;
a	=	length of the recharging zone in Hantush solution;
b	=	width of the recharging zone in Hantush solution;

$Q(i-1,j)$ = inflow to the i^{th} reach at time $j\Delta t$;

$Q_R(i,j)$ = lateral flow in the i^{th} reach at time $j\Delta t$;

R = hydraulic radius of the stream;

v_x = downstream component of lateral inflow velocity (Henderson 1966);

W = width of the stream;

u = the inlet of the reach of the stream;

d = the outlet of the reach of the stream ;

S = the storage;

i = reach no. ;

j = no of time interval;

δ = discrete kernel;

U = unit step;

h_o = initial stage of the stream;

H_O = depth to water table from the stream bed;

T = transmissivity;

e = height of the stream bed from the standard datum;

ϕ = refers to the storage coefficient;

x = direction along stream;

Y = direction perpendicular to stream;

y = Refers to the stream stage;

s = rise in watertable

CHAPTER-I
INTRODUCTION

1.1 GENERAL

A stream, forming a boundary is often encountered in regional groundwater flow modeling. In case of a partially penetrating stream with considerable stream discharge, besides treating the stream as a prescribed head boundary, the exchange of flow between the stream and the aquifer has to be introduced through the boundary nodes while modeling the groundwater flow (Ruston and Redshaw, 1978). The recharge from a stream to an aquifer is proportional to a head difference in the level of water in the stream and in the aquifer in the vicinity of the stream (Bouwer, 1969). The coefficient of proportionality, recognized as reach transmissivity, depends upon aquifer characteristics and the shape of stream cross-section (Morel-Seytoux, 1964; Bouwer, 1969). The water level in the aquifer depends on the abstractions and recharges including recharge from the stream. Such an implicit and complex stream-aquifer interaction problem has been analyzed by Morel-Seytoux and Daly (1975) who have used reach transmissivity and discrete kernel theory for finding an expression for recharge.

Few studies have been carried out with the computation of rise in water-table due to recharge from water bodies. Hantush (1967) has derived an expression for rise in water-table height due to recharge from a basin of finite length and width. If the dimension of length is increased to a very large value, the solution will correspond to rise in water-table due to recharge from a stream. However, the solution involves the numerical integration.

It has been often assumed for a stream, which is hydraulically connected with an aquifer, that exchange flow rate is linearly dependent to the potential difference between the prevailing aquifer and stream heads (Morel-Seytoux 1975). There has been evidence that this process can be very non -linear (Dillon, 1983, 1984; Rushton and Redshaw 1978). As it is difficult to determine the exact nonlinear relationship, the linear relationship is still in use.

1.2 NECESSITY FOR STUDYING STREAM - AQUIFER INTERACTION PROBLEM

In recent years, studies of the interaction of the groundwater and surface water have expanded in scope including studies of head water streams, lakes, wetlands. Interest in the relation of groundwater to headwater streams increased greatly in past twenty years because of concern related to acid precipitation.

To evaluate the interaction of stream and aquifer in all environments, at all scales, analytical and numerical methods need to be continually improved, For example, to effectively manage the resources, it will be necessary to simulate system as realistic as possible, i.e., consideration of realistic system geometries and transient conditions. This is a need to evaluate the effects of complex aerial and temporal distribution of recharge on the interaction of stream and aquifer.

The decline of groundwater levels around pumping wells near a surface-water body creates gradients that capture some of the ambient groundwater flow that would have, without pumping, discharged as base flow to the surface water. At sufficiently large pumping rates, these declines induce flow out of the body of surface water into the aquifer, a process known as induced infiltration, or induced recharge. The sum of these two effects leads to stream flow depletion. Quantifying the amount of induced infiltration, which is a function of many factors, is an important consideration in conjunctive water use as water demand increases and the reliability of surface water supplies is threatened by stream flow depletion. Stream–aquifer interactions are also important in situations of groundwater contamination by polluted surface water, and in situations of degradation of surface water by discharge of saline or other low-quality groundwater. Because of the potential for pollution of both groundwater and surface water from varied sources and by varied pollutant species, quantifying the amount of induced infiltration is also an important factor in evaluating the reliability of well-water quality.

The topics of water-resource depletion, GW–SW interactions, and water-resource sustainability were recently re- examined by Sophocleous (1997, 1998, 2000a, 2000b). To understand this depletion, a thorough knowledge of the hydrologic principles, concisely stated by Theis (1940), is required. Under natural conditions, prior to development by wells, aquifers approach a state of dynamic equilibrium: over hundreds of years, wet years, when recharge exceeds discharge, are offset by dry years, when discharge exceeds recharge. Discharge from wells upsets this equilibrium by producing a loss from aquifer storage; a new state of dynamic equilibrium is approached when there is no further loss or minimal loss from storage. This state is accomplished either by an increase in recharge, a decrease in natural discharge, or a combination of the two.

Consider a stream–aquifer system such as an alluvial aquifer discharging into a stream, where the term "stream" is used in the broadest sense of the word to include streams, lakes, ponds, and wetlands. A new well drilled at some distance from the stream and pumping the alluvial aquifer forms a cone of depression. The cone grows as water is taken from storage in the aquifer. Eventually, however, the periphery of the cone arrives at the stream. At this point, discharge from the aquifer to the stream appreciably diminishes or ceases, or water starts to flow from the stream into the aquifer. The cone continues to expand with continued pumping of the well until a new equilibrium is reached, in which induced recharge from the stream balances the pumping.

Studies of stream-aquifers are really very important for the following reasons:

1. We can optimize the cost of discharge measurement at every reach.
2. Helpful in watershed management.
3. Surplus water in the stream can be controlled by just passing the surplus stream water through highly hydraulic conductive areas, if the areas are under recharging condition.

1.3 SCOPE OF THE STUDY

Previous studies dealing with stream–aquifer interaction use constant parameter discharge hydrograph routing method. But in real practice, it is not appropriate to consider routing parameters as constant because of transient conditions. So study can be made more realistic by using routing methods which are capable of varying the parameters.

Further, the required variable for estimating stream-aquifer interaction is stream stage and, therefore, it should be directly estimated by the discharge routing method, rather than subsequent conversion of discharge to stage using stage-discharge relationship.

The available studies on stream-aquifer interaction consider the hydraulic heads of the stream reach and that of the contiguous aquifer only but not the mutual effect of other interacting reaches, which are related to one another through aquifer.

To overcome the deficiencies of the existing discharge routing method in using constant parameters, a variable parameter Muskingum routing method may be used as a component-model of the mathematical model for describing stream - aquifer interaction.

1.4 OBJECTIVE

The following is the broad objective of the study:

Given the flood hydrograph at a location in the stream, it is aimed to estimate the downstream hydrograph considering the interaction of the river /stream with the hydraulically connected aquifer.

1.5 CASES FOR THE PROPOSED STUDY

The following stream-aquifer interaction scenarios as shown in the diagram are considered for the proposed study:

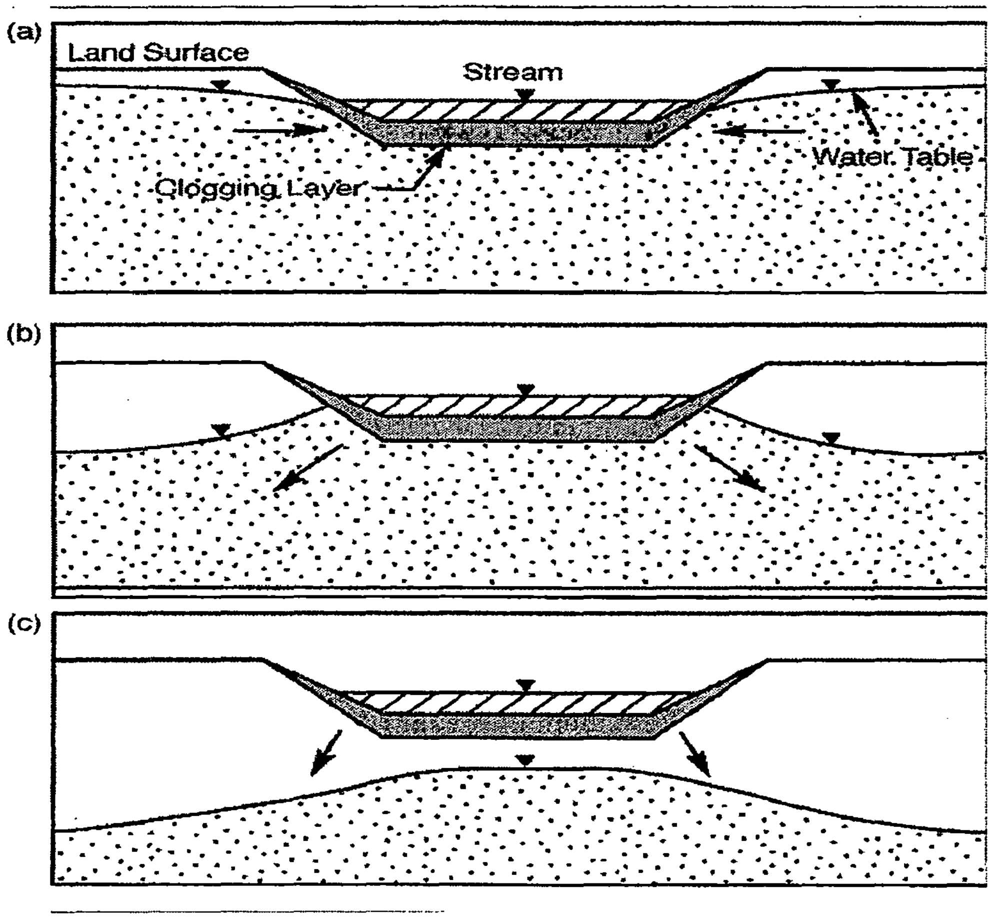


Fig. 1 Stream- aquifer interaction a) connected gaining stream b) connected loosing stream c) disconnected stream with a shallow water table (adapted from Sophocleous, 2002)

CHAPTER-II
REVIEW OF LITERATURE

2.0 INTRODUCTION

Groundwater and surface water are not isolated components of the hydrologic system, but instead interact in a variety of physiographic and climatic landscapes. Thus, development or contamination of one commonly affects the other. Therefore, an understanding of basic principles of interactions between groundwater and surface water is needed for effective management of water resources. Interest in the relationship of groundwater to headwater streams increased greatly in the past two decades because of concerns of acid rain. Interest in the relationship of groundwater to wetlands and to coastal areas has increased in the past 20 years as these ecosystems are lost to development. Recently, attention has been focused on exchanges between near-channel and in-channel water which are key to evaluating ecological structure of stream systems and are critical to stream restoration and riparian management effects (Sophocleous, 2002). This Chapter attempts to compile literature available on the mathematical approaches followed by different investigators for studying stream-aquifer interaction. But before presenting these works, it is worthwhile to under stand the physical mechanism of the stream-aquifer interaction.

2.1 STREAM-AQUIFER INTERACTION PROCESS

The word stream-aquifer interaction means the transportation of water from aquifer to stream or vice versa depending on the prevailing hydraulic conditions.

The hydraulic exchange of groundwater and surface water along a stream channel is controlled by (1) the distribution and magnitude of hydraulic conductivities, both within the channel and the associated alluvial plain sediments; (2) the relation of stream stage to the adjacent groundwater level; (3) the geometry and position of stream channel within the alluvial plain. The direction of the exchange process varies with hydraulic head, whereas flow (volume/unit time) depends on sediment hydraulic conductivity. Precipitation events and seasonal patterns alter the hydraulic head and, thereby, induce changes in flow direction. One can explain the stream-aquifer interaction phenomenon with the help of two terms:

Influent seepage

Effluent seepage

2.1.1 Influent Seepage

When the water in the stream is flowing at a higher hydraulic head than that of aquifer, then the water moves towards aquifer through the pores of the stream bed and joins the water-table through percolation process. This process is termed as influent seepage.

2.1.2 Effluent Seepage

When the hydraulic head of the stream is lower than that of the aquifer, then the water moves towards stream/river and joins the stream through percolation process and this process is termed as effluent seepage.

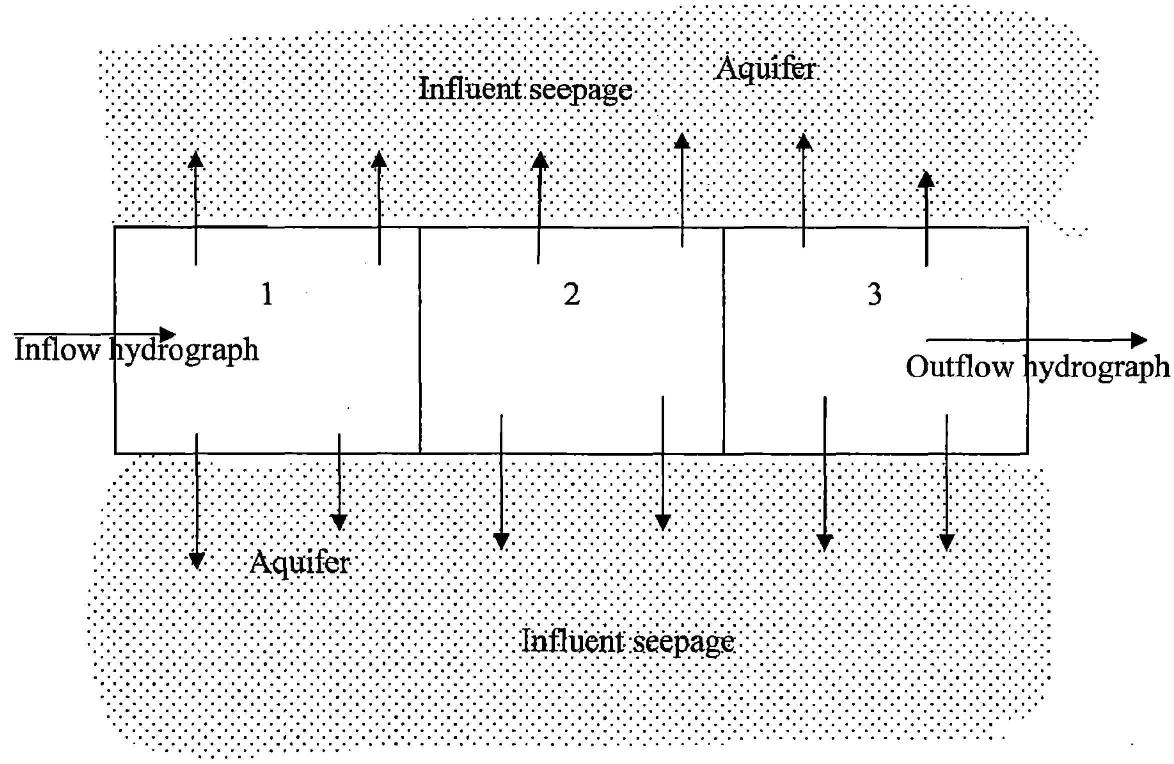


Fig .2 Recharge from a stream when stream stage is higher than the water level in the aquifer

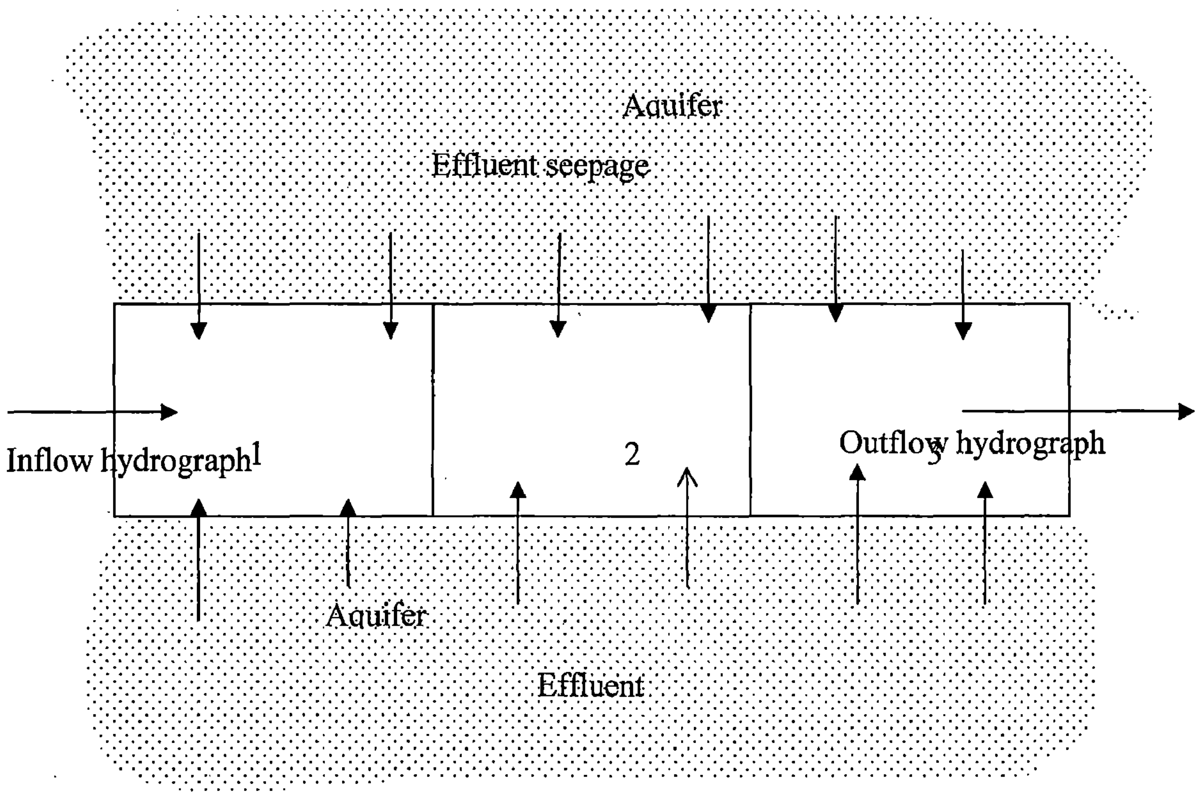


Fig .3 Effluent seepage during recession of a flood wave

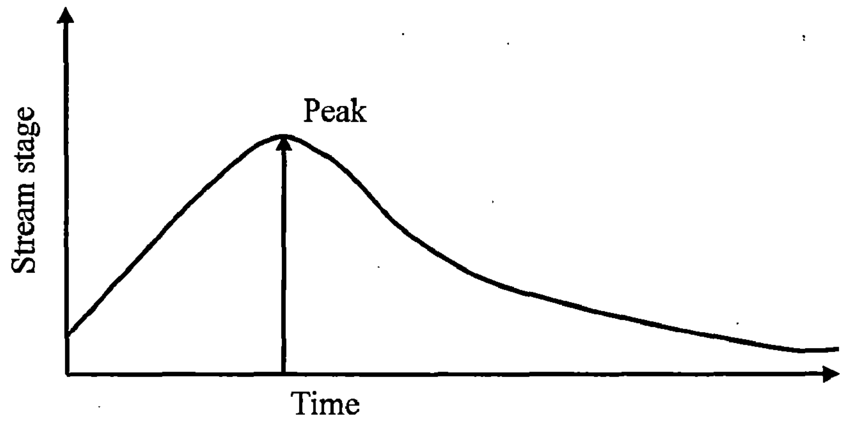


Fig .4 Stage hydrograph at the inlet point of the stream reach

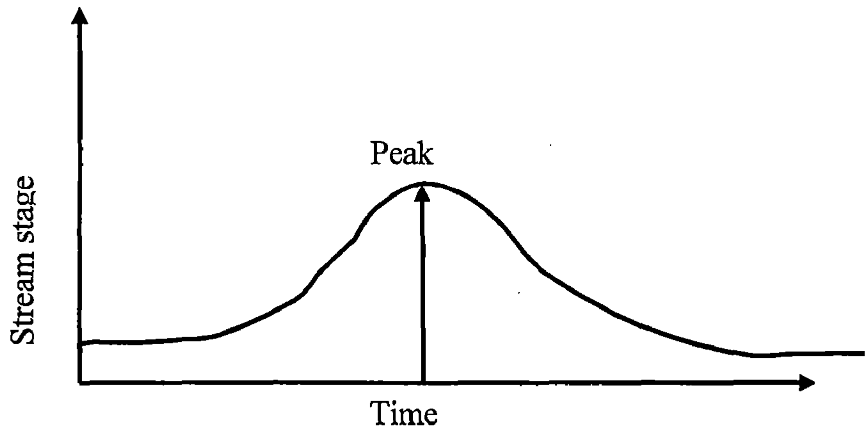


Fig.5 Stage hydrograph at a down stream location

2.1.3 Mechanism of Stream-Aquifer Interaction

The process of influent or effluent seepage is a linear process and is proportional to the difference between the prevailing hydraulic heads of stream and that of the aquifer adjacent to it. It may be estimated for a stream-aquifer system shown in Fig.6 as:

$$q = \left(\frac{K'W_P L}{M}\right)(h_S - h_A)$$

where q is the leakage through the stream's bed and banks; K' is the hydraulic conductivity of the bed; W_P is the wetted perimeter of the bed; L is the length of the stream; M is the thickness of the streambed; h_A is the aquifer head; and h_S is the stream head.

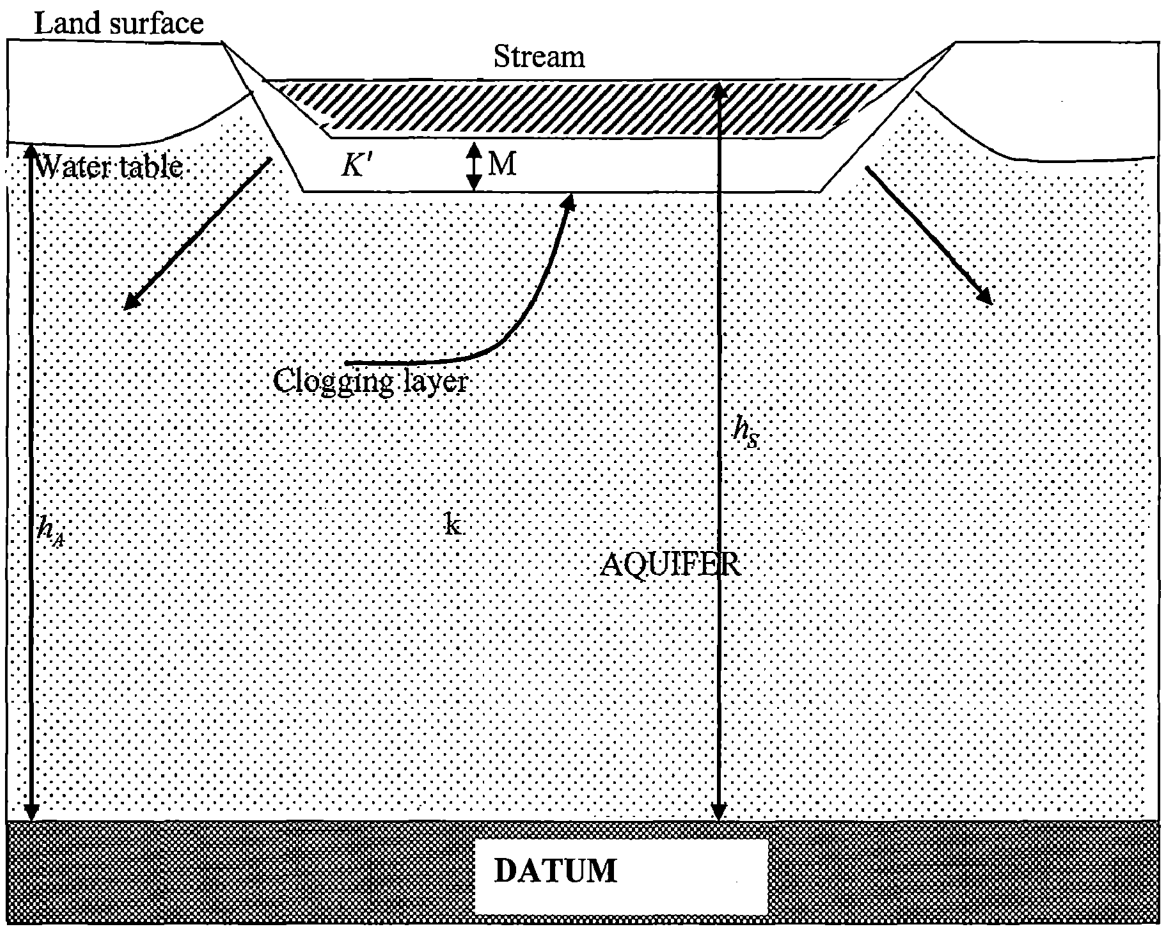


Fig.6 Stream-aquifer interaction

In this formulation, transient leakage across the streambed could change depending on the hydraulic heads of the stream and the aquifer over a computational time interval.

Thus, due to the above process of influent and effluent seepage, it is clear that there is always a movement of water towards aquifer or reverse until both hydraulic heads coincides. So this process of flow exchange transport affects the response of the stream flow at every sub-reach of the stream, i.e., when the hydraulic head of stream is higher than the hydraulic head of the aquifer adjacent to it, then this interaction process reduces the input to the next reach, while on depletion it increases the input to the same reach.

The actual magnitude of flow depends on the local geology, particularly the hydraulic conductivity of the interface boundary layer at the bottom of the stream. Really the process of interaction over spatially varied channel boundaries are difficult to examine.

The most common scenario of the interaction of groundwater is that of the interaction of streams with contiguous alluvial aquifer. This type of system has been the focus of study for more than 100 years, from the work of Boussinesq (1877) to the present, and stream –aquifer interaction continues to be the most common topic of papers discussing the interaction of groundwater and surface water.

Brunke and Gonser (1997) comprehensively summarize the interactions between streams and groundwater. Under conditions of low precipitation, base flow in many streams constitutes the discharge for most of the year (effluent condition). In contrast, under conditions of high precipitation, surface runoff and interflow gradually increase, leading to higher hydraulic pressures in the lower stream reaches, which cause the stream to change from effluent to influent condition, infiltrating its banks and recharging the aquifer. During flooding, the stream loses water to bank infiltration, which reduces the flood level and recharges the aquifer. The volume of this bank storage depends on the duration, height, and shape of the flood hydrograph, as well as on the transmissivity and storage capacity of the aquifer. During a dry season, the release of stored water compensates for a decrease in stream discharge. In some stream reaches, the water released to the stream from bank storage originating from flood runoff exceeds groundwater discharge under baseflow conditions. Thus, successive discharge and recharge of the aquifer has a buffering effect on the runoff regimes of streams.

2.2 ASSESSMENT OF WATER RECHARGED FROM A RECTANGULAR REACH

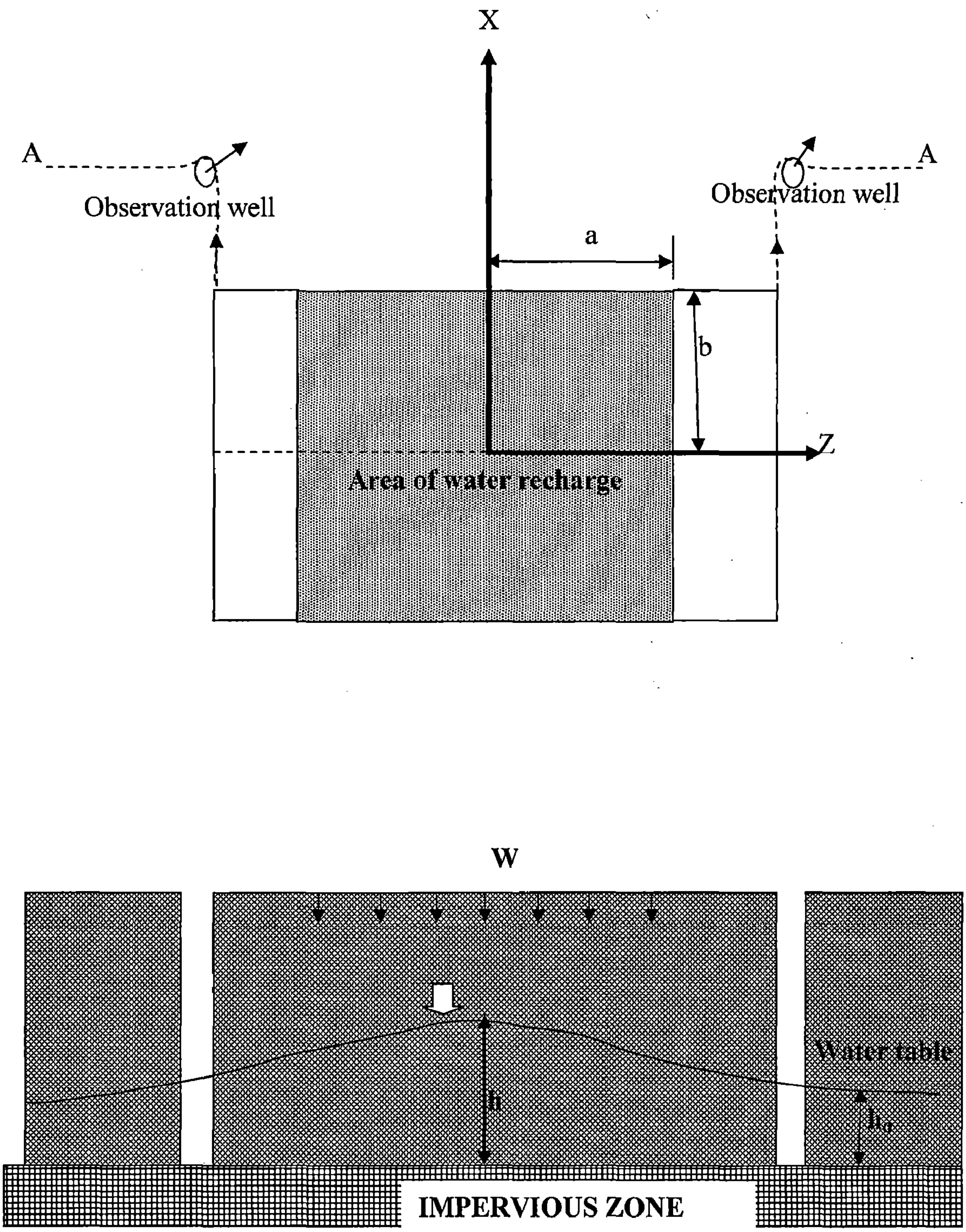


Fig. 7 Schematic diagram showing plan and section-AA of the spreading basin

Figure 7 shows a Schematic section and plan view of a recharging rectangular reach, water is recharged through the reach during a certain period of time. Continuous monitoring of groundwater level is done at an observation well. It is required to find out the quantity of groundwater recharged through the reach and its distribution in space and time using groundwater level data.

Hantush (1967) developed the following approximate analytical expression for the rise and fall of the water table in an infinite unconfined aquifer in response to uniform percolation from a rectangular spreading basin in the absence of any other source of withdrawal or recharging.

$$h^2 = h^2{}_o + \left(\frac{w\bar{h}t}{2\phi}\right)\left[\begin{array}{l} F\left\{\dfrac{a+X}{2\sqrt{\left(\dfrac{k\bar{h}t}{\phi}\right)}};\dfrac{b+Z}{2\sqrt{\left(\dfrac{k\bar{h}t}{\phi}\right)}}\right\} + F\left\{\dfrac{a-X}{2\sqrt{\left(\dfrac{k\bar{h}t}{\phi}\right)}};\dfrac{b+Z}{2\sqrt{\left(\dfrac{k\bar{h}t}{\phi}\right)}}\right\} \\ + F\left\{\dfrac{a+X}{2\sqrt{\left(\dfrac{k\bar{h}t}{\phi}\right)}};\dfrac{b-Z}{2\sqrt{\left(\dfrac{k\bar{h}t}{\phi}\right)}}\right\} + F\left\{\dfrac{a-X}{2\sqrt{\left(\dfrac{k\bar{h}t}{\phi}\right)}};\dfrac{b-Z}{2\sqrt{\left(\dfrac{k\bar{h}t}{\phi}\right)}}\right\} \end{array}\right] \qquad \ldots (2.1)$$

$$F(p,q) = \int_0^1 erf\left(\frac{p}{\sqrt{z}}\right) erf\left(\frac{q}{\sqrt{z}}\right) dz$$

$$= \int_{-1}^{1} erf\left(\frac{p}{\sqrt{0.5+0.5V}}\right) erf\left(\frac{q}{\sqrt{0.5+0.5V}}\right) 0.5dV$$

$$erf(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-u^2} du$$

Equation (2.1) can be re-written as:

$$h^2 = h^2_o + \left(\frac{w\bar{h}t}{2\phi}\right)[f(X,Z,t)] \qquad \ldots (2.2)$$

The recharge function f(X,Z,t) is expressed as:

$$f(X,Z,t) =$$

$$\int_{-1}^{1} erf\left(\frac{a+X}{2\sqrt{\left(\frac{k\bar{h}t}{\phi}\right)(0.5+0.5V)}}\right) erf\left(\frac{b+Z}{2\sqrt{\left(\frac{k\bar{h}t}{\phi}\right)(0.5+0.5V)}}\right)\frac{dV}{2} + \int_{-1}^{1} erf\left(\frac{a-X}{2\sqrt{\left(\frac{k\bar{h}t}{\phi}\right)(0.5+0.5V)}}\right) erf\left(\frac{b+Z}{2\sqrt{\left(\frac{k\bar{h}t}{\phi}\right)(0.5+0.5V)}}\right)\frac{dV}{2}$$

$$+ \int_{-1}^{1} erf\left(\frac{a+X}{2\sqrt{\left(\frac{k\bar{h}t}{\phi}\right)(0.5+0.5V)}}\right) erf\left(\frac{b-Z}{2\sqrt{\left(\frac{k\bar{h}t}{\phi}\right)(0.5+0.5V)}}\right)\frac{dV}{2} + \int_{-1}^{1} erf\left(\frac{a-X}{2\sqrt{\left(\frac{k\bar{h}t}{\phi}\right)(0.5+0.5V)}}\right) erf\left(\frac{b-Z}{2\sqrt{\left(\frac{k\bar{h}t}{\phi}\right)(0.5+0.5V)}}\right)\frac{dV}{2}$$

$$\ldots (2.3)$$

in which

$\bar{h}$ = Weighted mean of the depth of saturation during the period of flow,

w = Constant rate of percolation,

ϕ = Storage coefficient of the aquifer,

t = Time measured since the onset of recharge i.e. the percolated water joins the water table.

k = Coefficient of permeability,

$2a,2b$ = Dimension of the rectangular strip in x and Y direction,
The rise in water table height, s(X, Z, t) is given by:

$$s(X,Z,t) = h - h_o \qquad \text{... (2.4)}$$

and the average water level height is given by

$$\bar{h} = \frac{h + h_o}{2} \qquad \text{... (2.5)}$$

Substitution of $\bar{h}$ into equation (2.2) gives:

$$h^2 - h^2{}_o = \left(\frac{w(h + h_o)t}{4\phi}\right) f(X,Z,t) \qquad \text{... (2.6)}$$

$$h - h_o = \left(\frac{wt}{4\phi}\right) f(X,Z,t) \qquad \text{... (2.7)}$$

$$s(X,Z,t) = \left(\frac{wt}{4\phi}\right) f(X,Z,t) \qquad \text{... (2.8)}$$

Equation (2.8) gives the rise of water table due to recharge at w unit rate.

For continuous recharge at a unit rate (w=1 unit), the water table rise is given by:

$$s(X,Z,t) = \left(\frac{t}{4\phi}\right) f(X,Z,t) = U(X,Z,t) \qquad \text{... (2.9)}$$

If recharge takes place for one unit time, and no recharge after that, the rise at the end of n^{th} unit time step at j^{th} stream reach i.e. at (X_j, Z_j) due to recharge at i^{th} stream reach, i.e., at (X_i, Z_i) is given by:

$$\delta_R(i,j,n) = U(i,j,n) - U(i,j,n-1) \qquad \text{... (2.10)}$$

For first unit time step (n=1)

$$\delta_R(i,j,1) = U(i,j,1)$$

If recharge varies with time, the rise of water table at j^{th} reach due to recharge from all reaches at the end of n^{th} unit time step is given by:

$$s_R(j,n) = \sum_{i=1}^{R}\sum_{\gamma=1}^{n} Q_R(i,\gamma)\delta_R(i,j,n-\gamma+1) \quad \text{... (2.11)}$$

where:

$Q_R(i,\gamma)$ = Recharge rate at i^{th} stream reach during γ^{th} unit time step.

$\delta_R(i,j,m)$ = Discrete Kernel coefficient of rise/fall at j^{th} reach due to unit pulse recharge at i^{th} reaches during m^{th} unit time step.

2.3 EXPRESSION FOR THE TRANSMISSIVITY COEFFICIENT (REACH TRANSMISSIVITY)

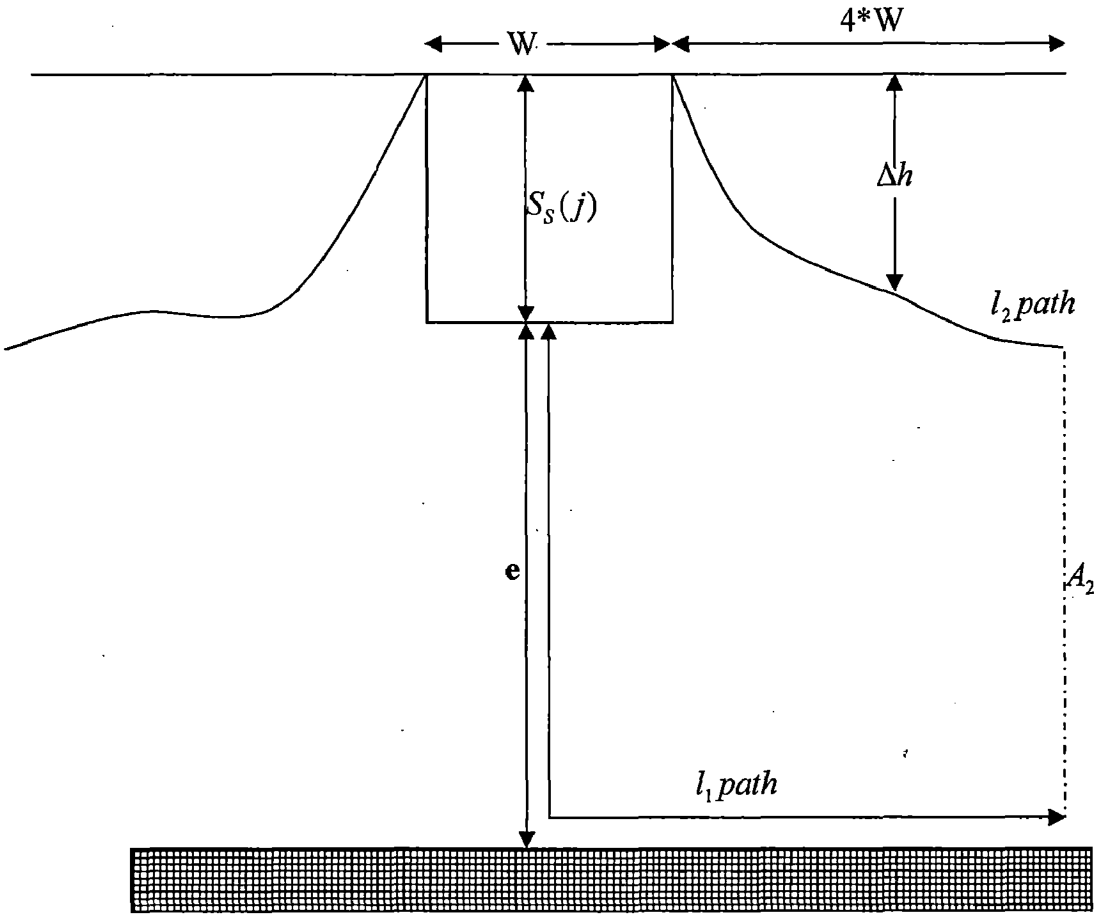


Fig. 8 Flow path and influence area considered for estimating reach-transmissivity coefficient

Let us consider the stage at jth time step as $s_s(j)$ and the height of streambed from the impervious datum as e. Other variables are shown as in Fig.-8.

Then the path l_1 and l_2 can be expressed as:

$$\left.\begin{aligned} & l_1 = e + \frac{W}{2} + 4W \\ & l_2 = 4W \\ & Thus \\ & \bar{l} = \frac{l_1 + l_2}{2} = 4W + \frac{W}{4} + \frac{e}{2} \end{aligned}\right\} \quad \ldots (2.12)$$

Where $\bar{l}$ is the average length of the path

Now for average area

$$A_1 = 2 * S_S(j) + W$$
$$And$$
$$A_2 = 2 * [S_S(j) + e - \Delta h]$$

Neglecting the Δh term

$$\bar{A} = 2 * S_S(j) + e + \frac{W}{2} \quad \ldots. (2.13)$$

Using Equation (2.12) the average Darcy's velocity is given as:

$$velocity = k \frac{\Delta h}{4.25 * W + 0.5 * e} \quad \ldots (2.14)$$

Thus discharge across the stream boundary is expressed as: (using Equation (2.13))

$$Q = k \frac{\Delta h}{4.25 * W + 0.5 * e}\left[2 * S_S(j) + e + \frac{W}{2}\right] \quad \ldots (2.15)$$

Hence, the rate of recharge/unit reach length, for unit fall of head, known as reach transmissivity is expressed as:

$$\Gamma_R(j) \approx k\frac{\left[2*S_S(j)+e+\frac{W}{2}\right]}{4.25*W+0.5*e} = \frac{T}{e}\frac{\left[2*S_S(j)+e+\frac{W}{2}\right]}{4.25*W+0.5*e} \quad \text{... (2.16)}$$

Thus recharge for the entire reach of length Δx, is expressed as:

$$\Gamma_R(j) \approx k\frac{\left[2*S_S(j-1)+e+\frac{W}{2}\right]}{4.25*W+0.5*e}*\Delta x = \frac{T}{e}\frac{\left[2*S_S(j-1)+e+\frac{W}{2}\right]}{4.25*W+0.5*e}*\Delta x$$

where $\Gamma_R(j)$ represents the transmissivity coefficient for the j^{th} time of the interacting stream reach.

2.4 AVAILABLE SOLUTIONS FOR STREAM-AQUIFER INTERACTION PROBLEM

Stream-aquifer interaction problem started gaining attention in hydrologic literature from late sixties. Zitta and Wiggert (1971) have given a numerical solution for flood routing in channels with bank seepage using the continuity equation (Stoker, 1957), and the Boussinesq equation governing one dimensional unsteady flow in unconfined aquifer. Perkins and Koussis (1996) used the USGS MODFLOW for solving the stream-aquifer interaction problem. They replaced STREAM module of USGS-MODFLOW by the Muskingum-Cunge diffusive wave routing scheme advocated by Koussis (1978) considering routing parameters remains constant during the routing process. Birkhead and James (2002) modified the Muskingum method to explicitly account for the interaction between channel flow and bank storage in stream with permeable stream banks of varying hydraulic conductivity.

Hantush, et al. (2002) provided a solution using Muskingum routing method in conjuction with one dimensional Boussinesq equation. Recently Gunduz and Aral (2005) advocated an approach for the simultaneous solution for surface and groundwater equation using matrix method. For solving the stream-aquifer interaction problem, szilagi (2004) employed the Kalinin-Milyukov method (Kalinin and Nilyukov, 1957) one dimensional Boussinessq equation given by Hantush (2002).

Among these methods, none of the method is capable of handling the two-dimensional groundwater flow as they consider only the one dimensional flow for studying stream -aquifer interaction problem. Further the routing methods used for solving this problem, be it Muskingum routing method or the K-method, have considered the routing parameters as constant.

Knowing the deficiencies of the existing solution methods it is proposed to develop a method which is capable for handling the two-dimensional groundwater flow including mutual interaction of interactive reaches of the stream and employ a routing method which is capable of realistically modeling the stream flow by varying the routing parameters in contrast to the available methods.

Moreover, previous methods assumed that the entire given stream reach is interactive with the aquifer, which is hardly true in nature. Therefore, the proposed method would be capable to handle both interactive and non interactive portions of the stream reach, simultaneously.

2.5 CONCLUSIONS

In this chapter, the past studies related to the stream-aquifer interaction problem were reviewed and it is inferred that they deal with one dimensional stream-aquifer interaction. These studies limited to fully penetrating stream and streams having considerable interface between stream and aquifer. These studies are not concern with streams contiguous with aquifer.

CHAPTER-III
ANALYSIS

3.1 STATEMENT OF THE PROBLEM

For studying the problem of stream - aquifer interaction, the stream having hydraulic connection with the underlying aquifer can be subdivided into a number of sub- reaches. Each of the sub-reach can be considered as a rectangular stream reach. The stage in each of the sub-reaches varies with time due to unsteady nature of flood wave and /or due to interaction of stream with aquifer or vice versa. A schematic diagram illustrating the case of single sub- reach stream-aquifer problem is shown in Fig. 9.

It is required to estimate the water level variations at different locations of the considered stream reach and in the aquifer adjoining to the stream due to stream-aquifer interaction and unsteady nature of flow.

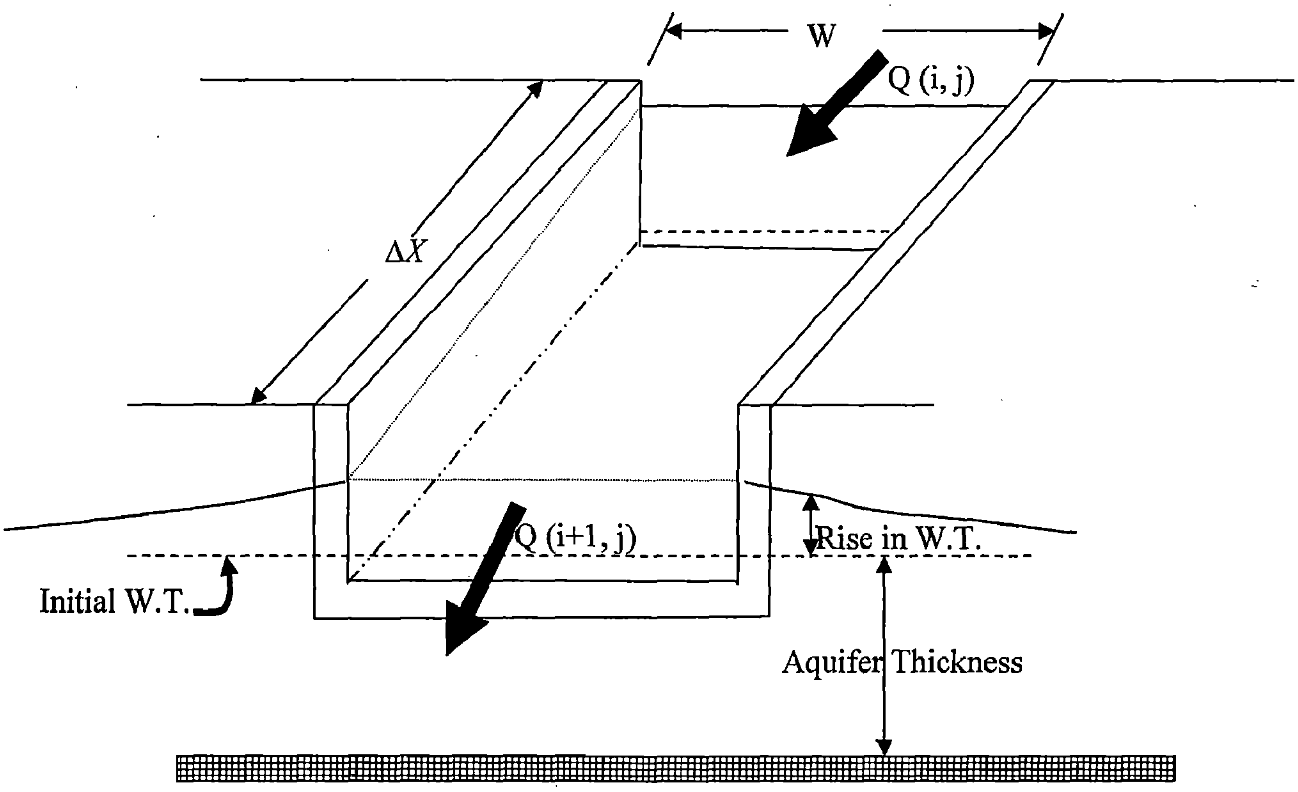


Fig. 9 Schematic diagram of stream-aquifer interaction

3.2 APPROACHES FOR SOLVING THE PROBLEM

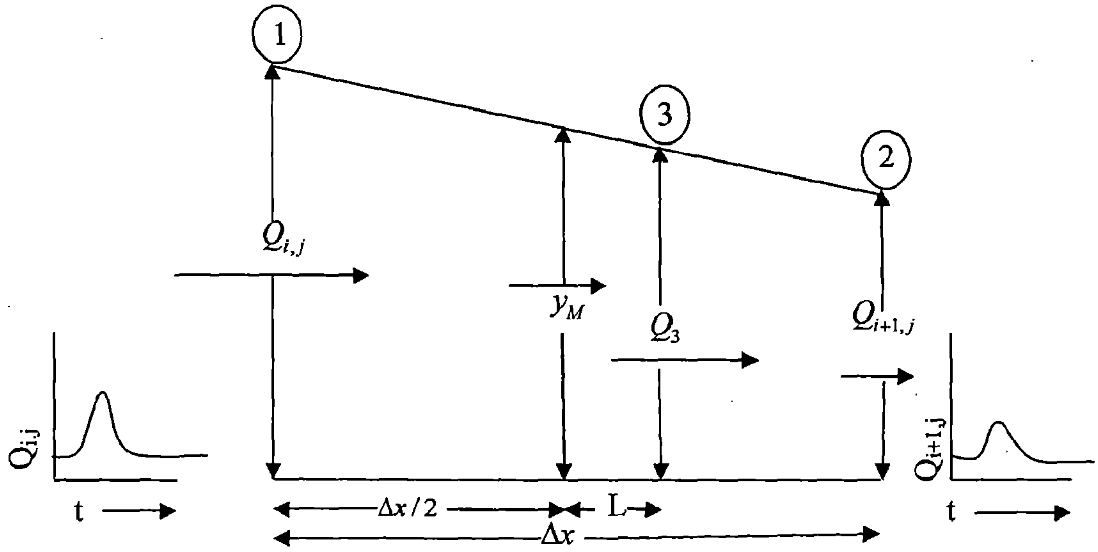


Fig. 10 A snapshot view of unsteady flow in the Muskingum sub- reach

Depending on the situation of flow prorogation in channel reaches, the following two types of analysis are required:

a) Analysis for those reaches where stream-aquifer interaction does not exist

b) Analysis for those reaches where stream-aquifer interaction exists

3.2.1 Analysis for Those Reaches Where Stream-Aquifer Interaction Does Not Exist

Routing floods in stream reaches with and without considering stream-aquifer interaction is carried out using the variable Parameter Muskingum (VPM) method advocated by Perumal (1994 a). For the sake of avoiding frequent reference to Chapter-II, it is considered appropriate to describe this method in this Chapter only.

Flood routing in channels is often carried out on the assumption that the flood wave movement is one-dimensional and governed by the St. Venant equations. For gradually varied unsteady flow in rigid bed channels without considering lateral flow, these equations are written as (Henderson, 1966):

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \text{.... (3.1)}$$

where Q=stream flow, A=cross-sectional area.

The momentum equation is given by

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t} \qquad \text{... (3.2)}$$

where S_0= bed slope; v = mean cross-sectional velocity; S_f = friction slope; x = stream length; y = stream depth; t = time; g=acceleration due to gravity.

The magnitudes of the various other terms in equation (3.2) are small in comparison with S_0 (Henderson, 1966; NERC, 1975) and, therefore, quite often some of them can be eliminated or approximated by some procedure when studying many flood routing problems.

Assumptions:

The proposed method is developed based on the following assumptions:

a) A prismatic channel having any shape of cross-section is assumed ;

b) The slope of the water surface $\frac{\partial y}{\partial x}$, the slope due to local acceleration $\frac{1}{g}(\frac{\partial v}{\partial t})$, and the slope due to convective acceleration $\frac{v}{g}\frac{\partial v}{\partial x}$ all remain constant at any instant of time in a given routing reach;

c) The magnitudes of the multiples of the derivatives of flow and section variables with respect to both time and distance are negligible.

d) At any instant of time during unsteady flow, the steady flow relationship is applicable between the stage at the middle of the reach and the discharge passing somewhere downstream of it. The same assumption is employed in the Kalinin-Milyukov (Appollov et al., 1964; Miller & Cunge, 1975), method of flood routing.

e) The storage in the Muskingum sub-reach and, hence, the water level is unaffected by the stream-aquifer interaction.

Friction slope approximation:

Figure 10 shows a channel reach of length Δx. According to assumption (d), the stage at the middle of the reach corresponds to the normal depth of that discharge which is passing at the same instant of time at an unspecified distance L downstream from the middle of the reach. Let this discharge be denoted as Q_3, and the inflow and outflow sections are represented as section-1 and section-2 respectively.

The discharge at any section of the reach may be expressed as:

$$Q = Av \qquad \ldots (3.3)$$

The velocity v can be expressed by Manning's or Chezy's friction law as:

$$v = C_f R^m S_f^{\frac{1}{2}} \qquad \ldots (3.4)$$

where C_f is the friction coefficient (C_f = C for Chezy's friction law, and C_f = 1/n for Manning's friction law); R is hydraulic radius (A/P); P is the wetted perimeter; m is an exponent which depends on the friction law use (for example, m = 2/3 for Manning's friction law, and m = ½ for Chezy's friction law).

Equation (3.3) is re-written using equation (3.4) as:

$$Q = AC_f R^m S_f^{\frac{1}{2}} \qquad \text{... (3.5)}$$

Differentiating equation (3.5) with respect to x and invoking assumption (b) that S_f is constant over x gives:

$$\frac{\partial Q}{\partial x} = \left\{\frac{\partial A}{\partial y} + mP\frac{\partial R}{\partial y}\right\} v \frac{\partial y}{\partial x} \qquad \text{... (3.6)}$$

The celerity of the flood wave can be arrived at from equation (3.6) as:

$$c = \frac{\partial Q}{\partial A} = \left[1 + m\left\{\frac{P\partial R/\partial y}{\partial A/\partial y}\right\}\right] v \qquad \text{... (3.7)}$$

Unlike the kinematic wave which has unique celerity for a given discharge, the flood wave governed by constant water surface slope does not result in unique celerity for the same discharge occurring in the rising and falling limbs of the hydrograph.

Differentiating equation (3.6) with respect to x gives:

$$\frac{\partial^2 Q}{\partial x^2} = \left[\frac{\partial A}{\partial y} + mP\frac{\partial R}{\partial y}\right]\frac{\partial v}{\partial x}\frac{\partial y}{\partial x} + v\left[\frac{\partial A}{\partial y} + mP\frac{\partial R}{\partial y}\right]\frac{\partial^2 y}{\partial x^2}$$

$$+ v\left[\frac{\partial^2 A}{\partial x \partial y} + m\frac{\partial P}{\partial x}\frac{\partial R}{\partial y} + mP\frac{\partial^2 R}{\partial x \partial y}\right]\frac{\partial y}{\partial x} \qquad \text{... (3.8)}$$

Using assumption (b) and (c), equation (3.8) reduces to:

$$\frac{\partial^2 Q}{\partial x^2} = 0 \qquad \text{... (3.9)}$$

Equation (3.9) implies that the discharge is also varying linearly over the reach considered.

Approximate expression of friction slope:

Using equation (3.1), (3.2) and (3.6), and assumptions, the friction slope S_f can be expressed as:

$$S_f = S_0\left\{1-\frac{1}{S_0}\frac{\partial y}{\partial x}\left[1-\left[mF\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)\right]^2\right]\right\} \qquad \text{... (3.10)}$$

in which F is the Froude the number defined as:

$$F = \left[\frac{v^2\partial A/\partial y}{gA}\right]^{1/2} \qquad \text{... (3.11)}$$

Location of weighted discharge section:

Using equation (3.5) and (3.10), the discharge at the middle of the reach is expressed as:

$$Q_M = A_M C_f R_M^m S_0^{1/2}\left\{1-\frac{1}{S_0}\frac{\partial y}{\partial x}\bigg|_M\left[1-m^2F_M^2\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]\right\}^{1/2} \qquad \text{... (3.12)}$$

where the subscript M denotes the mid-section of the reach.

The normal discharge Q_3 corresponding to y_M occurs at section-3, as shown in Fig. (10) Which is located at a distance L downstream of the middle of the reach, and it is expressed as:

$$Q_3 = A_M C_f R_M^m S_0^{\frac{1}{2}} \qquad \text{... (3.13)}$$

Equation (3.12) is modified using equation (3.13) as:

$$Q_M = Q_3\left\{1-\frac{1}{S_0}\frac{\partial y}{\partial x}\bigg|_M\left[1-m^2F_M^2\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]\right\}^{1/2} \quad \text{... (3.14)}$$

For the sake of brevity, let

$$\frac{1}{S_0}\frac{\partial y}{\partial x}\bigg|_M\left[1-m^2F_M^2\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]=r \quad \text{... (3.15)}$$

Based on the typical values of S_0 and $\partial y/\partial x$ in natural streams (Henderson, 1966), it may be considered that $|r|<1$. Under such a condition, expanding equation (3.14) in a binomial series and then neglecting the higher order terms of r leads to:

$$Q_M = Q_3 - \frac{Q_3}{2S_0}\left[1-m^2F_M^2\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]\frac{\partial y}{\partial x}\bigg|_M \quad \text{... (3.16)}$$

Since $\partial y/\partial x$ is constant at any instant of time over the routing reach:

$$\frac{\partial y}{\partial x}\bigg|_M = \frac{\partial y}{\partial x}\bigg|_3 \quad \text{... (3.17)}$$

Where $\frac{\partial y}{\partial x}\Big|_3$ is the water surface slope at section- 3.

Equation (3.16) may be re-written using equation (3.6) and (3.17) as:

$$Q_M = Q_3 - \frac{Q_3}{2S_0}\frac{\left[1-m^2F_M^2\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]}{\frac{\partial A}{\partial y}\Big|_3\left[1+m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right]v_3}\frac{\partial Q}{\partial x}\bigg|_3 \quad \text{... (3.18)}$$

Since the discharge also varies linearly, the term adjunct to $\frac{\partial Q}{\partial x}\Big|_3$ represents the distance L between the mid-section and that downstream section, where the normal discharge corresponding to the depth at the mid-section passes at the same instant of time, i.e., L is expressed as:

$$L = \frac{Q_3\left[1 - m^2 F_M^2 \left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]}{2S_0 \left.\frac{\partial A}{\partial y}\right|_3 \left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right] v_3} \quad \text{.... (3.19)}$$

Derivation of storage- weighted discharge relationship:

Using equation (3.1), (3.3) and (3.5) and assumption (b), the following expression is arrived at:

$$\frac{\partial Q}{\partial t} + \left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)\right] v \frac{\partial Q}{\partial x} = 0 \quad \text{... (3.20)}$$

Applying equation (3.20) at section -3 and rearranging the term yields:

$$\left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)\right] v_3 \left.\frac{\partial Q}{\partial x}\right|_3 = -\left.\frac{\partial Q}{\partial t}\right|_3 \quad \text{... (3.21)}$$

Due to the linear variation of discharge over the routing reach, $\partial Q/\partial x|_3$ may be approximated as:

$$\left.\frac{\partial Q}{\partial x}\right|_3 = \left.\frac{\partial Q}{\partial x}\right|_2 = \frac{O - I}{\Delta x} \quad \text{... (3.22)}$$

Where, I and O denote the inflow and out flow at section-1 and section-2 respectively, and Δx is the reach length. Due to the linear variation of discharge as depicted in Fig. (10), Q_3 may be expressed as:

$$Q_3 = O + \left(\frac{1}{2} - \frac{L}{\Delta x}\right)(I - O) \quad \text{... (3.23)}$$

Substitution of equations (3.22) and (3.23) in equation (3.21) gives:

$$I - O = \frac{\Delta x}{\left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right] v_3} \frac{\partial}{\partial t}\left[O + \left(\frac{1}{2} - \frac{L}{\Delta x}\right)(I - O)\right] \quad \text{... (3.24)}$$

Let the weighted parameter be:

$$\theta = \left(\frac{1}{2} - \frac{L}{\Delta x}\right) \quad \text{... (3.25)}$$

Equation (3.24) is same as the differential equation governing the Muskingum method with travel time K expressed as:

$$K = \frac{\Delta x}{\left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right] v_3} \quad \text{... (3.26)}$$

and the weighting parameter θ after substitution of L from equation (3.19) is expressed as:

$$\theta = \frac{1}{2} - \frac{Q_3\left[1 - m^2 F_M^2 \left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_M^2\right]}{2S_0 \left.\frac{\partial A}{\partial y}\right|_3 \left[1 + m\left(\frac{P\partial R/\partial y}{\partial A/\partial y}\right)_3\right] v_3 \Delta x} \quad \text{... (3.27)}$$

The parameter relationships given above enable one to reduce equation (3.24) to the form of the conventional Muskingum differential equation as:

$$I - O = \frac{\partial}{\partial t}[K(\theta I + (1 - \theta)O)] \quad \text{... (3.28)}$$

with the storage in the reach expressed as:

$$S = K[\theta I + (1 - \theta)O] \quad \text{... (3.29)}$$

When a constant discharge is used as the reference discharge, Q_O the generalized expressions for variable K and θ reduce to:

$$K = \frac{\Delta x}{\left[1 + m\left(\frac{P\partial R / \partial y}{\partial A / \partial y}\right)_0\right] v_0} \qquad \text{... (3.30)}$$

and

$$\theta = \frac{1}{2} - \frac{Q_0\left[1 - m^2 F_M^2\left(\frac{P\partial R / \partial y}{\partial A / \partial y}\right)_0^2\right]}{2S_0 \left.\frac{\partial A}{\partial y}\right|_0 \left[1 + m\left(\frac{P\partial R / \partial y}{\partial A / \partial y}\right)_0\right] v_0 \Delta x} \qquad \text{... (3.31)}$$

where the suffices 0 represents the reference level.

But the effect of the Froude number is not very significant (Perumal, 1994a) and, therefore it can be neglected. The expressions of K and θ valid for Manning's friction law and applicable for a uniform rectangular cross-section channel reach are arrived at as:

$$K = \frac{\Delta x}{\left(1 + \frac{2 * W}{3(W + 2 * y_3)}\right) v_3} \qquad \text{... (3.32)}$$

$$\theta = \frac{1}{2} - \frac{Q_3}{2 * S_0 * W\left(1 + \frac{2 * W}{3(W + 2 * y_3)}\right) v_3 * \Delta x} \qquad \text{... (3.33)}$$

When the variables in these expressions are fixed about a reference discharge, and a wide rectangular cross-section is assumed, these expressions reduce to those derived by Cunge (1969) and Dooge et al. (1982).

Stage hydrograph computation:

The flow depth y_d corresponding to outflow O is estimated using equation (3.6) as:

$$y_d = y_M + \frac{Q_d - Q_M}{W\left(1+\frac{2*W}{3*(W+2*y_M)}\right)v_M} \qquad \text{.... (3.34)}$$

In which y_M is estimated iteratively from the normal discharge relationship given by equation (3.13). Using the computed flow depths y_d and y_M in the first sub-reach, the upstream flow depth corresponding to the inflow discharge can be estimated using the assumption of a linear variation of water surface.

3.2.2 Analysis For Those Reaches Where Stream-Aquifer Interaction Exists

For the sake of developing the computer code used for solving the stated problem which requires the simultaneous solution of coupled stream flow and the stream-aquifer interaction equations at number of predefined nodes of the stream reach, it is more convenient to change the notations of the inflow and outflow variables of a sub-reach from I and Q, respectively, to $Q(i-1,j)$ and $Q(i,j)$: the notation $Q(i-1,j)$ denotes the inflow to the ith reach at time $j\Delta t$ and $Q(i,j)$ denotes the outflow from ith reach at time $j\Delta t$ or inflow to the (i=1)th reach. Similarly, S(i, i+1, j) denotes the storage in the ith reach at time $j\Delta t$, and $Q_R(i,j)$ denotes the lateral flow in the ith reach at time $j\Delta t$

Single Reach Interaction Analysis

It is proposed to study unsteady flow movement in a given reach of length L km which consists of a small length of interactive stream reach. It is assumed that the stream reach is sub-divided into a number of sub-reaches of length equal to that of the interactive stream reach.

The continuity equation applicable for the i[th] stream reach which corresponds to the interactive reach can be expressed as:

Inflow – *Outflow* – *Lateralflow* = Change in storage

$$\overline{Q}(i-1,j)-\overline{Q}(i,j)-\overline{Q}_R(j)=\frac{\Delta S(i,i+1,j)}{\Delta t} \qquad ...(3.35)$$

where, $\overline{Q}(i-1,j)$ is the average inflow at the section-i between the time $j\Delta t$ and $(j-1)\Delta t$. The notation i, j denote the location of the node and the temporal node. It is expressed as:

$$\overline{Q}(i-1,j)=\frac{Q(i-1,j)+Q(i-1,j-1)}{2} \qquad ...(3.35a)$$

Similarly $\overline{Q}(i,j)$ is the average outflow at the section-i between the time $j\Delta t$ and $(j-1)\Delta t$. It is expressed as:

$$\overline{Q}(i,j)=\frac{Q(i,j)+Q(i,j-1)}{2} \qquad ...(3.35b)$$

The variable $\overline{Q}_R(j)$ denotes the average rate of lateral flow between the time $j\Delta t$ and $(j-1)\Delta t$.

$$\overline{Q}_R(j)=\frac{Q_R(j)+Q_R(j-1)}{2} \qquad ...(3.35c)$$

According to the Muskingum equation

$$S=K\big(\theta * Q(i-1,j)+(1-\theta)Q(i,j)\big) \qquad ...(3.36)$$

and $\Delta S = K[\theta\big(Q(i-1,j)-Q(i-1,j-1)\big)+(1-\theta)\big(Q(i,j)-Q(i,j-1)\big)]$ ... (3.37)

Incorporating equation (3.35), (3.35a), (3.35b), (3.35c), (3.37) we have:

$$\frac{Q(i-1,j)+Q(i-1,j-1)}{2}-\frac{Q_R(j)+Q_R(j-1)}{2}-\frac{Q(i,j)+Q(i,j-1)}{2} = \left(\frac{K[\theta\{Q(i-1,j)-Q(i-1,j-1)\}+(1-\theta)\{Q(i,j)-Q(i,j-1)\}]}{\Delta t}\right) \quad \text{... (3.38)}$$

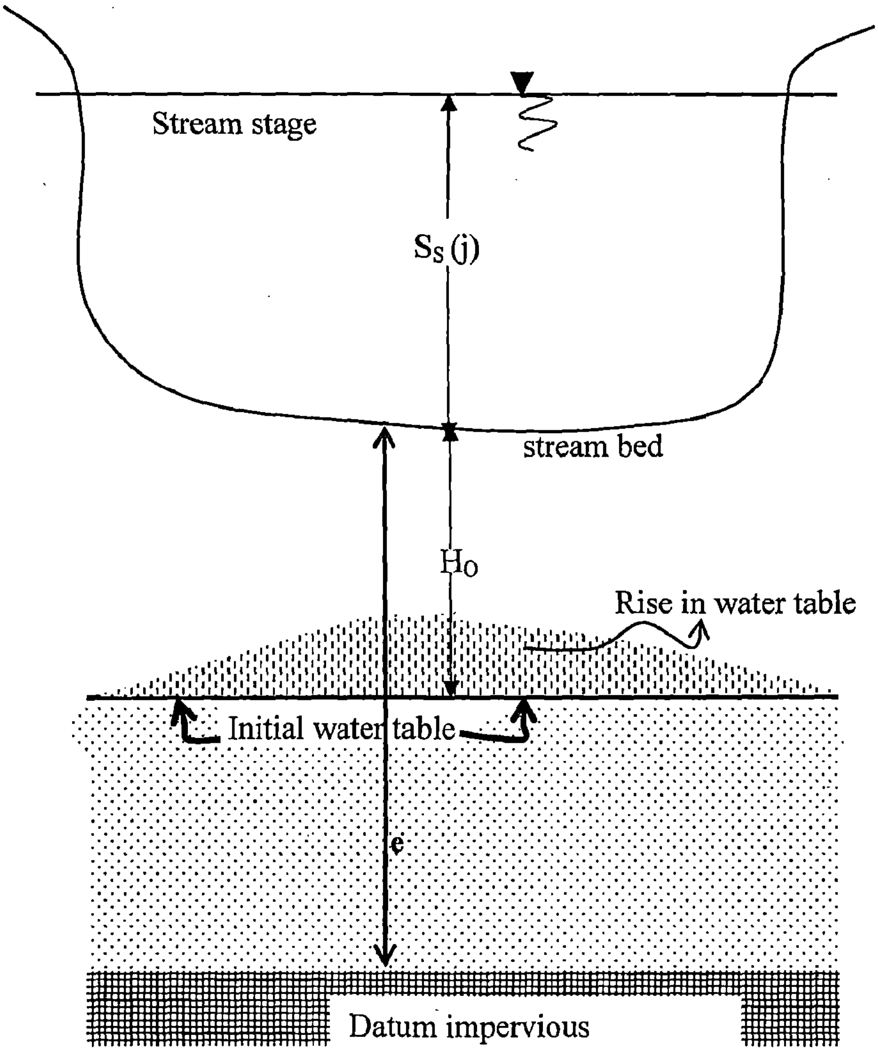


Fig. 11.Defination sketch of stream-aquifer interaction scenario

The influent seepage from the interactive sub-reach is estimated using equations (2.11) and (2.16) as:

$$Q_R(j)=\Gamma_R\left(S_S(j)+H_O-\sum_{\gamma=1}^{j}Q_R(\gamma)\delta(j-\gamma+1)\right) \quad \text{... (3.39)}$$

$$(1+\Gamma_R\delta(1))Q_R(j)=\Gamma_R\left[S_S(j)+H_O-\sum_{\gamma=1}^{j-1}Q_R(\gamma)\delta(j-\gamma+1)\right] \quad \text{... (3.40)}$$

where the stream stage is given by

$$S_S(j) = \frac{K\left(\theta Q(i-1,j) + (1-\theta)Q(i,j)\right)}{W\Delta X} - \frac{KQ(0,0)}{W\Delta X} + h_o \qquad \text{... (3.41)}$$

$$Q_R(j) = \frac{\Gamma_R}{(1+\Gamma_R\delta(1))}\left[\begin{array}{l} \frac{K\left(\theta Q(i-1,j) + (1-\theta)Q(i,j)\right)}{W\Delta X} - \frac{KQ(0,0)}{W\Delta X} + h_o \\ + H_O - \sum_{\gamma=1}^{j-1} Q_R(\gamma)\delta(j-\gamma+1) \end{array}\right] \qquad \text{... (3.42)}$$

Equation (3.38) can be re-arranged by bringing known terms to the right hand side and the unknown terms to the left hand side as:

$$\left.\begin{array}{l} \frac{2K(1-\theta)+\Delta t}{\Delta t} Q(i,j) + Q_R(j) \\ = Q(i-1,j) + Q(i-1,j-1) - Q_R(j-1) - Q(i,j-1) \\ -\frac{2K}{\Delta t}[\theta\{Q(i-1,j) - Q(i-1,j-1)\} - (1-\theta)Q(i,j-1)] \end{array}\right\} \qquad \text{... (3.43)}$$

Equation (3.42) can be re-written as:

$$\left.\begin{array}{l} Q_R(j) - \frac{\Gamma_R}{(1+\Gamma_R\delta(1))} * \frac{K(1-\theta)Q(i,j)}{W\Delta X} = \\ \frac{\Gamma_R}{(1+\Gamma_R\delta(1))}\left[\frac{K\theta Q(i-1,j)}{W\Delta X} - \frac{KQ(0,0)}{W\Delta X} + h_o + H_O - \sum_{\gamma=1}^{j-1} Q_R(\gamma)\delta(j-\gamma+1)\right] \end{array}\right\} \qquad \text{... (3.44)}$$

Equation (3.43) and (3.44) can be expressed in matrix form as:

$$\begin{bmatrix} -\dfrac{\Gamma_R}{(1+\Gamma_R\delta(1))} * \dfrac{K(1-\theta)}{W\Delta X} & 1 \\ \dfrac{2K(1-\theta)+\Delta t}{\Delta t} & 1 \end{bmatrix} \begin{bmatrix} Q(i,j) \\ Q_R(j) \end{bmatrix}$$

$$= \begin{bmatrix} \left\langle \dfrac{\Gamma_R}{(1+\Gamma_R\delta(1))} \left(\dfrac{K\theta Q(i-1,j)}{W\Delta X} - \dfrac{KQ(0,0)}{W\Delta X} + h_o + H_O - \sum_{\gamma=1}^{j-1} Q_R(\gamma)\delta(j-\gamma+1) \right) \right\rangle \\ \left\langle \begin{array}{l} Q(i-1,j)+Q(i-1,j-1)-Q_R(j-1)-Q(i,j-1) \\ -\dfrac{2K}{\Delta t}[\theta\{Q(i-1,j)-Q(i-1,j-1)\}-(1-\theta)Q(i,j-1)] \end{array} \right\rangle \end{bmatrix}$$

which represent the two equations in two unknowns those can be solved by matrix inversion method.

When the length of the interactive stream reach cannot be considered as a single Muskingum sub-reach due to violation of assumptions of the method for routing flood waves, it becomes necessary to sub-divide the interactive reach into many sub-reaches. Under this situation, the estimation of water level at any location of the interactive stream reach or in the adjacent aquifer would involve the cumulative effect of the recharge process taking place in these sub-reaches, including the effect of mutual interaction between these sub-reaches. To illustrate this aspect, analysis for estimating water level variation and influent or effluent seepage analysis stream-aquifer interaction of a long stream reach is presented it is assumed that this reach may be divided into three equal sub-reaches. A schematic description of three mutually interactive stream sub-reaches is shown in Fig.12. The term reach specifies the reach number.

Analysis for three reaches interaction:

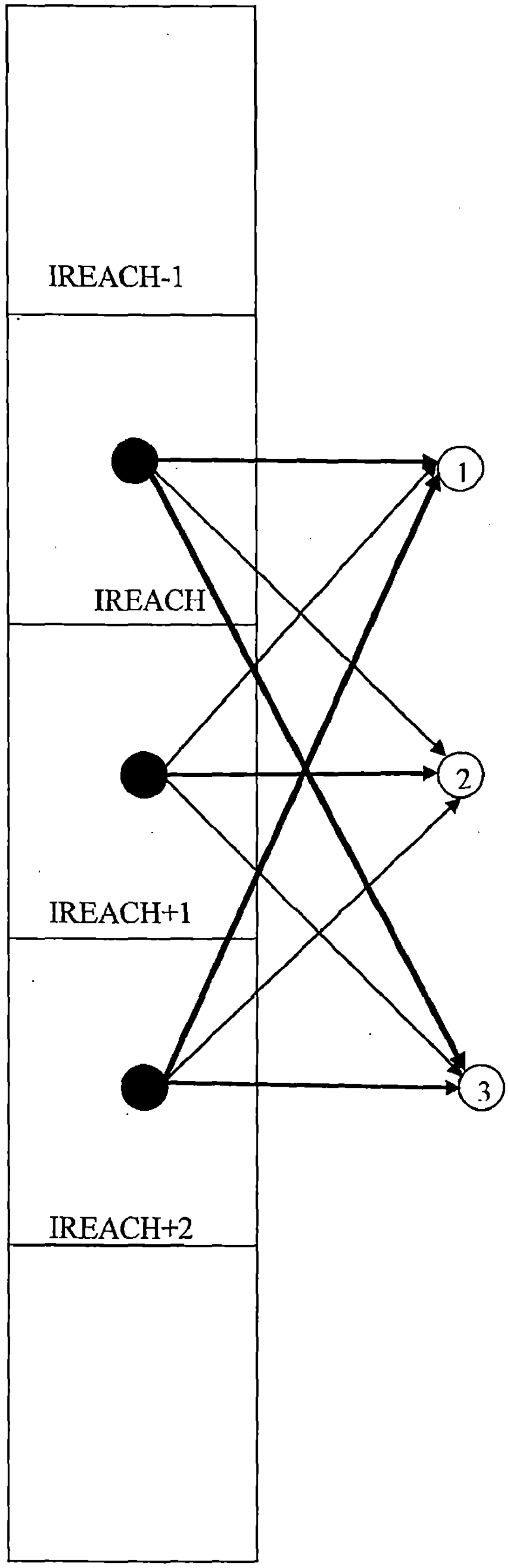


Fig.12. Three reach interaction

Multi-reach interaction analysis:

Using the lumped continuity equation applicable for the i[th] reach, which is interactive the unknown terms $Q(i,j) and Q_R(i,j)$ can be expressed in terms of known values as:

$$\left.\begin{aligned}&\frac{2K(1-\theta)+\Delta t}{\Delta t}Q(i,j)+Q_R(i,j)=Q(i-1,j)+Q(i-1,j-1)-Q_R(i,j-1)\\&-Q(i,j-1)-\frac{2K}{\Delta t}[\theta\{Q(i-1,j)-Q(i-1,j-1)\}-(1-\theta)Q(i,j-1)]\end{aligned}\right\} \quad \dots (3.45)$$

For the (i+1)[th] reach, which is interactive the unknown terms $Q(i,j), Q(i+1,j) and Q_R(i,j), Q_R(i+1,j)$ can be expressed in terms of known values as:

$$\left.\begin{aligned}&\frac{2K(1-\theta)+\Delta t}{\Delta t}Q(i+1,j)+Q_R(i+1,j)=Q(i,j)+Q(i,j-1)-Q_R(i+1,j-1)\\&-Q(i+1,j-1)-\frac{2K}{\Delta t}[\theta\{Q(i,j)-Q(i,j-1)\}-(1-\theta)Q(i+1,j-1)]\end{aligned}\right\} \quad \dots (3.46)$$

or

$$\left.\begin{aligned}&\frac{2K(1-\theta)+\Delta t}{\Delta t}Q(i+1,j)+Q_R(i+1,j)-Q(i,j)+\frac{2K}{\Delta t}\theta Q(i,j)\\&=Q(i,j-1)-Q_R(i+1,j-1)-Q(i+1,j-1)-\frac{2K}{\Delta t}[\theta\{-Q(i,j-1)\}-(1-\theta)Q(i+1,j-1)]\end{aligned}\right\}$$

... (3.46a)

For the (i+2)[th] reach, which is interactive the unknown terms $Q(i,j), Q(i+1,j), Q(i+2,j) and Q_R(i,j), Q_R(i+1,j), Q_R(i+2,j)$ can be expressed in terms of known values as:

$$\left.\begin{aligned}&\frac{2K(1-\theta)+\Delta t}{\Delta t}Q(i+2,j)+Q_R(i+2,j)\\&=Q(i+1,j)+Q(i+1,j-1)-Q_R(i+2,j-1)-Q(i+2,j-1)\\&-\frac{2K}{\Delta t}[\theta\{Q(i+1,j)-Q(i+1,j-1)\}-(1-\theta)Q(i+2,j-1)]\end{aligned}\right\} \qquad \text{... (3.47)}$$

or it can be re-written as:

$$\left.\begin{aligned}&\frac{2K(1-\theta)+\Delta t}{\Delta t}Q(i+2,j)+Q_R(i+2,j)-Q(i+1,j)+\frac{2K}{\Delta t}\theta Q(i+1,j)\\&=Q(i+1,j-1)-Q_R(i+2,j-1)-Q(i+2,j-1)-\frac{2K}{\Delta t}[\theta\{-Q(i+1,j-1)\}-(1-\theta)Q(i+2,j-1)]\end{aligned}\right\} \qquad \text{... (3.47a)}$$

The influent seepage for the i[th] reach can be expressed as:

$$Q_R(i,j)=\Gamma_R(i,j-1)\begin{pmatrix}\dfrac{K[\theta Q(i-1,j)+(1-\theta)Q(i,j)]}{W\Delta X}-\dfrac{KQ(0,0)}{W\Delta X}+h_o+H_O\\-\sum_{\gamma=1}^{j}Q_R(i,\gamma)\delta(i,i,j-\gamma+1)\\-\sum_{\gamma=1}^{j}Q_R(i+1,\gamma)\delta(i+1,i,j-\gamma+1)\\-\sum_{\gamma=1}^{j}Q_R(i+2,\gamma)\delta(i+2,i,j-\gamma+1)\end{pmatrix} \qquad \text{... (3.48)}$$

or one can re-write

$$\begin{bmatrix}\left[-\dfrac{K(1-\theta)}{B\Delta X}\right]Q(i,j)\\+\left[\dfrac{1}{\Gamma_R(i,j-1)}+\delta(i,i,1)\right]Q_R(i,j)\\+[\delta(i+1,i,1)]Q_R(i+1,j)\\+[\delta(i+2,i,1)]Q_R(i+2,j)\end{bmatrix}=\begin{pmatrix}\dfrac{K*\theta*Q(i-1,j)}{W\Delta X}-\dfrac{KQ(0,0)}{W\Delta X}+h_o+H_O\\-\sum_{\gamma=1}^{j-1}Q_R(i,\gamma)\delta(i,i,j-\gamma+1)\\-\sum_{\gamma=1}^{j-1}Q_R(i+1,\gamma)\delta(i+1,i,j-\gamma+1)\\-\sum_{\gamma=1}^{j-1}Q_R(i+2,\gamma)\delta(i+2,i,j-\gamma+1)\end{pmatrix} \qquad \text{... (3.48a)}$$

The influent seepage for the $(i+1)^{th}$ reach can be expressed as:

$$Q_R(i+1,j)=\Gamma_R(i+1,j-1)\begin{pmatrix} \dfrac{K[\theta * Q(i,j)+(1-\theta)Q(i+1,j)]}{W\Delta X}-\dfrac{KQ(0,0)}{W\Delta X}+h_o+H_O \\ -\sum_{\gamma=1}^{j}Q_R(i,\gamma)\delta(i,i+1,j-\gamma+1) \\ -\sum_{\gamma=1}^{j}Q_R(i+1,\gamma)\delta(i+1,i+1,j-\gamma+1) \\ -\sum_{\gamma=1}^{j}Q_R(i+2,\gamma)\delta(i+2,i+1,j-\gamma+1) \end{pmatrix} \quad \ldots (3.49)$$

or one can re-write

$$\begin{bmatrix} \left[-\dfrac{K\theta}{W\Delta X}\right]Q(i,j)+\left[-\dfrac{K(1-\theta)}{W\Delta X}\right]Q(i+1,j) \\ +[\delta(i,i+1,1)]Q_R(i,j) \\ \left[\dfrac{1}{\Gamma_R(i+1,j-1)}+\delta(i+1,i+1,1)\right]Q_R(i+1,j) \\ [\delta(i+2,i+1,1)]Q_R(i+2,j) \end{bmatrix}=\begin{pmatrix} H_O-\dfrac{KQ(0,0)}{W\Delta X}+h_o \\ -\sum_{\gamma=1}^{j-1}Q_R(i,\gamma)\delta(i,i+1,j-\gamma+1) \\ -\sum_{\gamma=1}^{j-1}Q_R(i+1,\gamma)\delta(i+1,i+1,j-\gamma+1) \\ -\sum_{\gamma=1}^{j-1}Q_R(i+2,\gamma)\delta(i+2,i+1,j-\gamma+1) \end{pmatrix}$$

... (3.49a)

The influent seepage for the $(i+2)^{th}$ reach can be expressed as:

$$Q_R(i+2,j)=\Gamma_R(i+2,j-1)\begin{pmatrix} \dfrac{K[\theta Q(i+1,j)+(1-\theta)Q(i+2,j)]}{W\Delta X}-\dfrac{KQ(0,0)}{W\Delta X}+h_o+H_O \\ -\sum_{\gamma=1}^{j}Q_R(i,\gamma)\delta(i,i+2,j-\gamma+1) \\ -\sum_{\gamma=1}^{j}Q_R(i+1,\gamma)\delta(i+1,i+2,j-\gamma+1) \\ -\sum_{\gamma=1}^{j}Q_R(i+2,\gamma)\delta(i+2,i+2,j-\gamma+1) \end{pmatrix}$$

... (3.50)

or one can re-write

$$\begin{bmatrix} \left[-\dfrac{K\theta}{W\Delta X}\right]Q(i+1,j) \\ +\left[-\dfrac{K(1-\theta)}{W\Delta X}\right]Q(i+2,j) \\ +\left[\delta(i,i+2,1)\right]Q_R(i,j) \\ +\left[\delta(i+1,i+2,1)\right]Q_R(i+1,j) \\ +\left[\dfrac{1}{\Gamma_R(i+2,j-1)}+\delta(i+2,i+2,1)\right]Q_R(i+2,j) \end{bmatrix} = \begin{pmatrix} H_O-\dfrac{KQ(0,0)}{W\Delta X}+h_o \\ -\sum\limits_{\gamma=1}^{j-1}Q_R(i,\gamma)\delta(i,i+2,j-\gamma+1) \\ -\sum\limits_{\gamma=1}^{j-1}Q_R(i+1,\gamma)\delta(i+1,i+2,j-\gamma+1) \\ -\sum\limits_{\gamma=1}^{j-1}Q_R(i+2,\gamma)\delta(i+2,i+2,j-\gamma+1) \end{pmatrix}$$

... (3.50a)

Incorporating equation (3.45), (3.46a), (3.47a), (3.48a), (3.49a), (3.50a) we have a matrix:

$$
\begin{bmatrix}
1 & 0 & 0 & \frac{2K(1-\theta)+\Delta t}{\Delta t} & 0 & 0 \\
0 & 1 & 0 & \frac{2K}{\Delta t}\theta-1 & \frac{2K(1-\theta)+\Delta t}{\Delta t} & 0 \\
0 & 0 & 1 & 0 & \frac{2K}{\Delta t}\theta-1 & \frac{2K(1-\theta)+\Delta t}{\Delta t} \\
\left[\frac{1}{\Gamma_R(1,j-1)} + \delta(1,1,1)\right] & [\delta(2,1,1)] & [\delta(3,1,1)] & \left[-\frac{K(1-\theta)}{W\Delta X}\right] & 0 & 0 \\
[\delta(1,2,1)] & \left[\frac{1}{\Gamma_R(2,j-1)} + \delta(2,2,1)\right] & [\delta(3,2,1)] & \left[-\frac{K\theta}{W\Delta X}\right] & \left[-\frac{K(1-\theta)}{W\Delta X}\right] & 0 \\
[\delta(1,3,1)] & [\delta(2,3,1)] & \frac{1}{\Gamma_R(3,j-1)}+\delta(3,3,1) & 0 & \left[-\frac{K\theta}{W\Delta X}\right] & \left[-\frac{K(1-\theta)}{W\Delta X}\right]
\end{bmatrix}
\begin{bmatrix}
Q_R(i,j) \\ Q_R(i+1,j) \\ Q_R(i+2,j) \\ Q(i,j) \\ Q(i+1,j) \\ Q(i+2,j)
\end{bmatrix}
=
$$

$$\begin{bmatrix} \begin{pmatrix} Q(i-1,j)+Q(i-1,j-1)-Q_R(i,j-1)-Q(i,j-1) \\ -\frac{2K}{\Delta t}[\theta\{Q(i-1,j)-Q(i-1,j-1)\}-(1-\theta)Q(i,j-1)] \end{pmatrix} \\ \left(Q(i,j-1)-Q_R(i+1,j-1)-Q(i+1,j-1)-\frac{2K}{\Delta t}[\theta\{-Q(i,j-1)\}-(1-\theta)Q(i+1,j-1)]\right) \\ \left(Q(i+1,j-1)-Q_R(i+2,j-1)-Q(i+2,j-1)-\frac{2K}{\Delta t}[\theta\{-Q(i+1,j-1)\}-(1-\theta)Q(i+2,j-1)]\right) \\ \begin{pmatrix} \frac{K*\theta*Q(i-1,j)}{W\Delta X}-\frac{KQ(0,0)}{W\Delta X}+h_o+H_O-\sum_{\gamma=1}^{j-1}Q_R(i,\gamma)\delta(1,1,j-\gamma+1) \\ -\sum_{\gamma=1}^{j-1}Q_R(i+1,\gamma)\delta(2,1,j-\gamma+1)-\sum_{\gamma=1}^{j-1}Q_R(i+2,\gamma)\delta(3,1,j-\gamma+1) \end{pmatrix} \\ \begin{pmatrix} H_O-\frac{KQ(0,0)}{W\Delta X}+h_o-\sum_{\gamma=1}^{j-1}Q_R(i,\gamma)\delta(1,2,j-\gamma+1) \\ -\sum_{\gamma=1}^{j-1}Q_R(i+1,\gamma)\delta(2,2,j-\gamma+1)-\sum_{\gamma=1}^{j-1}Q_R(i+2,\gamma)\delta(3,2,j-\gamma+1) \end{pmatrix} \\ \begin{pmatrix} H_O-\frac{KQ(0,0)}{W\Delta X}+h_o-\sum_{\gamma=1}^{j-1}Q_R(i,\gamma)\delta(1,3,j-\gamma+1) \\ -\sum_{\gamma=1}^{j-1}Q_R(i+1,\gamma)\delta(2,3,j-\gamma+1)-\sum_{\gamma=1}^{j-1}Q_R(i+2,\gamma)\delta(3,3,j-\gamma+1) \end{pmatrix} \end{bmatrix}$$

Flow chart for single reach interaction:

START

Routing step J=0
I=0

Enter value of discharge Q (0, J), and Q (I, 0), bed slope (S_0), width (W), total length, no. of reaches

Estimate Y_M using Newton Raphson from

$$F(Y_M) = Q(0,0) - \frac{\sqrt{S_0} * W^{5/3} * Y_M^{\ 5/3}}{n*(W+2*Y_M)^{2/3}}$$

Using Y_M=Y_0(initial stage)estimate

$$K = \frac{\Delta X}{\left(1+\dfrac{2*W}{3(W+2*Y_0}\right)v_0}, \theta = \frac{1}{2} - \frac{Q(0,0)}{2*S_0*W\left(1+\dfrac{2*W}{3(W+2*y_o}\right)v_o*\Delta X}$$

B → J=J+1

C → I=I+1

ITERATION =1

D → ESTIMATE C_1, C_2, C_3

$$C_1 = \frac{-K\theta + \Delta t/2}{K(1-\theta)+\Delta t/2} \qquad C_2 = \frac{K\theta + \Delta t/2}{K(1-\theta)+\Delta t/2}$$

$$C_3 = \frac{K(1-\theta) - \Delta t/2}{K(1-\theta)+\Delta t/2}$$

$$Q(I,J) = C_1 * Q(I-1,J) + C_2 * Q(I-1,J-1) + C_3 * Q(I,J-1)$$

AND

$$Q_3 = \theta * Q(I-1,J) + (1-\theta) * Q(I,J)$$

Estimate Y_M using Newton Raphson from

$$F(Y_M) = Q_3 - \frac{\sqrt{S_0} * W^{5/3} * Y_M^{\ 5/3}}{n*(W+2*Y_M)^{2/3}}$$

A

A

Estimate
$$Q_M = (Q(I,J) + Q(I-1,J))/2$$

Estimate $Y_3 = Y_M + \dfrac{Q_3 - Q_M}{W\left(1 + \dfrac{2*W}{3*(W+2*Y_M)}\right)v_M}$

Estimate A_3 corresponding to Y_3

Estimate $v_3 = Q_3/A_3$

$$K = \frac{\Delta X}{\left(1+\dfrac{2*W}{3(W+2*Y_3)}\right)v_3}, \theta = \frac{1}{2} - \frac{Q_3}{2*S_0*W\left(1+\dfrac{2*W}{3(W+2*y_3)}\right)v_3*\Delta X}$$

ITERATION = ITERATION +1

IS ITERATION>2 — NO → D; YES ↓

IS I>=INTERACTION REACH-1 — NO → C; YES ↓

ITERATION =1

D¹ →

$$\begin{bmatrix} -\dfrac{\Gamma_R}{(1+\Gamma_R\delta(1))}*\dfrac{K(1-\theta)}{W\Delta X} & 1 \\ \dfrac{2K(1-\theta)+\Delta t}{\Delta t} & 1 \end{bmatrix}\begin{bmatrix} Q(I,J) \\ Q_R(J) \end{bmatrix} = \begin{bmatrix} \dfrac{\Gamma_R}{(1+\Gamma_R\delta(1))}\left(\dfrac{K\theta Q(I-1,J)}{W\Delta X} + H_O - \sum_{\gamma=1}^{J-1} Q_R(\gamma)\delta(J-\gamma+1)\right) \\ Q(I-1,J) + Q(I-1,J-1) - Q_R(J-1) - Q(I,J-1) \\ -\dfrac{2K}{\Delta t}[\theta\{Q(I-1,J) - Q(I-1,J-1)\} - (1-\theta)Q(I,J-1)] \end{bmatrix}$$

FIND OUT THE VALUE OF Q(I,J) AND $Q_R(J)$ CONSIDERING $Q_R(0)=0$

FIND OUT
$$Q_3 = \theta * Q(I-1,J) + (1-\theta) * Q(I,J)$$

A¹

A1

Estimate Y_M using Newton Raphson from

$$F(Y_M) = Q_3 - \frac{\sqrt{S_0} * W^{5/3} * Y_M^{\ 5/3}}{n * (W + 2 * Y_M)^{2/3}}$$

Estimate

$$Q_M = (Q(I,J) + Q(I-1,J))/2$$

$$\text{Estimate } Y_3 = Y_M + \frac{Q_3 - Q_M}{W\left(1 + \frac{2 * W}{3 * (W + 2 * Y_M)}\right) v_M}$$

Estimate A_3corresponding to Y_3

Estimate $v_3 = Q_3/A_3$

$$K = \frac{\Delta X}{\left(1 + \frac{2 * W}{3(W + 2 * Y_3}\right) v_3}, \theta = \frac{1}{2} - \frac{Q_3}{2 * S_0 * W\left(1 + \frac{2 * W}{3(W + 2 * y_3}\right) v_3 * \Delta X}$$

ITERATION = ITERATION +1

IS ITERATION>2

NO → D1

YES

I=INTERACTION REACH+1

C1 → ITERATION =1

D2 → ESTIMATE C_1, C_2, C_3

$$C_1 = \frac{-K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2} \qquad C_2 = \frac{K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2}$$

$$C_3 = \frac{K(1-\theta) - \Delta t/2}{K(1-\theta) + \Delta t/2}$$

A2

A^2

$$Q(I,J) = C_1 * Q(I-1,J) + C_2 * Q(I-1,J-1) + C_3 * Q(I,J-1)$$

AND

$$Q_3 = \theta * Q(I-1,J) + (1-\theta) * Q(I,J)$$

Estimate Y_M using Newton Raphson from

$$F(Y_M) = Q_3 - \frac{\sqrt{S_0} * W^{5/3} * Y_M^{\ 5/3}}{n * (W + 2 * Y_M)^{2/3}}$$

Estimate

$$Q_M = (Q(I,J) + Q(I-1,J))/2$$

Estimate $Y_3 = Y_M + \dfrac{Q_3 - Q_M}{W\left(1 + \dfrac{2*W}{3*(W + 2*Y_M)}\right)v_M}$

Estimate A_3corresponding to Y_3

Estimate v_3=Q_3/A_3

$$K = \frac{\Delta X}{\left(1 + \dfrac{2*W}{3(W + 2*Y_3}\right)v_3}, \theta = \frac{1}{2} - \frac{Q_3}{2*S_0*W\left(1 + \dfrac{2*W}{3(W + 2*y_3}\right)v_3 * \Delta X}$$

ITERATION = ITERATION +1

IS ITERATION>2 — NO → D^2; YES ↓

I=I+1

IS I>NO.OF REACH — NO → C^1; YES ↓

IS J>=TOTAL TIME STEPS — NO → B; YES ↓

STOP

Flow chart for three reaches interaction:

START

Routing step J=0
I=0

Enter value of discharge Q (0, J), and Q (I, 0), bed slope (S_0), width (W), total length, no. of reaches

Estimate Y_M using Newton Raphson from

$$F(Y_M) = Q(0,0) - \frac{\sqrt{S_0} * W^{5/3} * Y_M^{\ 5/3}}{n * (W + 2 * Y_M)^{2/3}}$$

Using Y_M=Y_0(initial stage)estimate

$$K = \frac{\Delta X}{\left(1 + \frac{2*W}{3(W + 2*Y_0)}\right) v_0}, \theta = \frac{1}{2} - \frac{Q(0,0)}{2 * S_0 * W\left(1 + \frac{2*W}{3(W + 2*y_o)}\right) v_o * \Delta X}$$

B → J=J+1

C → I=I+1

ITERATION =1

D → ESTIMATE C_1, C_2, C_3

$$C_1 = \frac{-K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2} \qquad C_2 = \frac{K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2}$$

$$C_3 = \frac{K(1-\theta) - \Delta t/2}{K(1-\theta) + \Delta t/2}$$

A

A

$$Q(I,J) = C_1 * Q(I-1,J) + C_2 * Q(I-1,J-1) + C_3 * Q(I,J-1)$$

AND

$$Q_3 = \theta * Q(I-1,J) + (1-\theta) * Q(I,J)$$

Estimate Y_M using Newton Raphson from

$$F(Y_M) = Q_3 - \frac{\sqrt{S_0} * W^{5/3} * Y_M^{\ 5/3}}{n * (W + 2 * Y_M)^{2/3}}$$

Estimate

$$Q_M = (Q(I,J) + Q(I-1,J))/2$$

Estimate $Y_3 = Y_M + \dfrac{Q_3 - Q_M}{W\left(1 + \dfrac{2 * W}{3 * (W + 2 * Y_M)}\right) v_M}$

Estimate A_3corresponding to Y_3

Estimate $v_3=Q_3/A_3$

$$K = \frac{\Delta X}{\left(1 + \dfrac{2 * W}{3(W + 2 * Y_3)}\right) v_3}, \theta = \frac{1}{2} - \frac{Q_3}{2 * S_0 * W\left(1 + \dfrac{2 * W}{3(W + 2 * y_3)}\right) v_3 * \Delta X}$$

ITERATION = ITERATION +1

IS ITERATION>2 — NO → D

YES

IS I>=INTERACTION REACH-1 — NO → C

YES

$$\theta = \theta_1 = \theta_2 = \theta_3$$

ITERATION =1

A^1

A1

$$\begin{bmatrix} 1 & 0 & 0 & \frac{2K_1(1-\theta_1)+\Delta t}{\Delta t} & 0 & 0 \\ 0 & 1 & 0 & \frac{2K_2}{\Delta t}\theta_2 - 1 & \frac{2K_2(1-\theta_2)+\Delta t}{\Delta t} & 0 \\ 0 & 0 & 1 & 0 & \frac{2K_3}{\Delta t}\theta_3 - 1 & \frac{2K_3(1-\theta_3)+\Delta t}{\Delta t} \\ \left[\frac{1}{\Gamma_R(1,j-1)} + \delta(1,1,1)\right] & [\delta(2,1,1)] & [\delta(3,1,1)] & \left[-\frac{K_1(1-\theta_1)}{W\Delta X}\right] & 0 & 0 \\ [\delta(1,2,1)] & \left[\frac{1}{\Gamma_R(2,j-1)} + \delta(2,2,1)\right] & [\delta(3,2,1)] & \left[-\frac{K_2\theta}{W\Delta X}\right] & \left[-\frac{K_2(1-\theta_2)}{W\Delta X}\right] & 0 \\ [\delta(1,3,1)] & [\delta(2,3,1)] & \frac{1}{\Gamma_R(3,j-1)} + \delta(3,3,1) & 0 & \left[-\frac{K_3\theta_3}{W\Delta X}\right] & \left[-\frac{K_3(1-\theta_3)}{W\Delta X}\right] \end{bmatrix} \begin{bmatrix} Q_R(ireach,j) \\ Q_R(ireach+1,j) \\ Q_R(ireach+1,j) \\ Q(ireach,j) \\ Q(ireach+1,j) \\ Q(ireach+2,j) \end{bmatrix} =$$

D1

$$\begin{bmatrix} \begin{pmatrix} Q(ireach-1,j) + Q(ireach-1,j-1) - Q_R(ireach,j-1) - Q(ireach,j-1) \\ -\frac{2K_1}{\Delta t}[\theta_1\{Q(ireach-1,j) - Q(ireach-1,j-1)\} - (1-\theta_1)Q(ireach,j-1)] \end{pmatrix} \\ \left(Q(ireach,j-1) - Q_R(ireach+1,j-1) - Q(ireach+1,j-1) - \frac{2K_2}{\Delta t}[\theta_2\{-Q(ireach,j-1)\} - (1-\theta_2)Q(ireach+1,j-1)]\right) \\ \left(Q(ireach+1,j-1) - Q_R(ireach+2,j-1) - Q(ireach+2,j-1) - \frac{2K_3}{\Delta t}[\theta_3\{-Q(ireach+1,j-1)\} - (1-\theta_3)Q(ireach+2,j-1)]\right) \\ \begin{pmatrix} \frac{K_1 * \theta_1 * Q(ireach-1,j)}{W\Delta X} + H_O - \sum_{\gamma=1}^{j-1} Q_R(ireach,\gamma)\delta(1,1,j-\gamma+1) \\ -\sum_{\gamma=1}^{j-1} Q_R(ireach+1,\gamma)\delta(2,1,j-\gamma+1) - \sum_{\gamma=1}^{j-1} Q_R(ireach+2,\gamma)\delta(3,1,j-\gamma+1) \end{pmatrix} \\ \left(H_O - \sum_{\gamma=1}^{j-1} Q_R(ireach,\gamma)\delta(1,2,j-\gamma+1) - \sum_{\gamma=1}^{j-1} Q_R(ireach+1,\gamma)\delta(2,2,j-\gamma+1) - \sum_{\gamma=1}^{j-1} Q_R(ireach+2,\gamma)\delta(3,2,j-\gamma+1)\right) \\ \left(H_O - \sum_{\gamma=1}^{j-1} Q_R(ireach,\gamma)\delta(1,3,j-\gamma+1) - \sum_{\gamma=1}^{j-1} Q_R(ireach+1,\gamma)\delta(2,3,j-\gamma+1) - \sum_{\gamma=1}^{j-1} Q_R(ireach+2,\gamma)\delta(3,3,j-\gamma+1)\right) \end{bmatrix}$$

A2

Solving above equation of matrix form considering initial lateral flow as zero

FIND OUT

$$Q_{31} = \theta_1 * Q(IREACH-1,J) + (1-\theta_1) * Q(IREACH,J)$$

$$Q_{32} = \theta_2 * Q(IREACH,J) + (1-\theta_2) * Q(IREACH+1,J)$$

$$Q_{33} = \theta_3 * Q(IREACH+1,J) + (1-\theta_3) * Q(IREACH+2,J)$$

Estimate Y_M using Newton Raphson from

$$F(Y_M) = Q_3 - \frac{\sqrt{S_0} * W^{5/3} * Y_M^{\;5/3}}{n * (W + 2 * Y_M)^{2/3}}$$ FOR ALL Q_{31}, Q_{32}, Q_{33}

Estimate

$$Q_{M1} = (Q(IREACH-1,J) + Q(IREACH,J))/2$$

$$Q_{M2} = (Q(IREACH,J) + Q(IREACH+1,J))/2$$

$$Q_{M3} = (Q(IREACH+1,J) + Q(IREACH+2,J))/2$$

Estimate $Y_3 = Y_M + \dfrac{Q_3 - Q_M}{W\left(1 + \dfrac{2*W}{3*(W+2*Y_M)}\right)v_M}$ for all 1, 2 3, interaction reaches

Estimate A_3 corresponding to Y_3

Estimate $v_3 = Q_3/A_3$

$K = \dfrac{\Delta X}{\left(1 + \dfrac{2*W}{3(W+2*Y_3)}\right)v_3}, \theta = \dfrac{1}{2} - \dfrac{Q_3}{2*S_0*W\left(1 + \dfrac{2*W}{3(W+2*y_3)}\right)v_3 * \Delta X}$ for all 1, 2, 3, reaches i.e. $\theta_1, \theta_2, \theta_3$ and then put $\theta = \theta_3$

A3

A^3

ITERATION = ITERATION +1

IS ITERATION>2

NO → D^1

YES

I=INTERACTION REACH+3

C^1 →

ITERATION =1

D^2 →

ESTIMATE C_1, C_2, C_3

$$C_1 = \frac{-K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2} \qquad C_2 = \frac{K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2}$$

$$C_3 = \frac{K(1-\theta) - \Delta t/2}{K(1-\theta) + \Delta t/2}$$

$$Q(I,J) = C_1 * Q(I-1,J) + C_2 * Q(I-1,J-1) + C_3 * Q(I,J-1)$$

AND

$$Q_3 = \theta * Q(I-1,J) + (1-\theta) * Q(I,J)$$

Estimate Y_M using Newton Raphson from

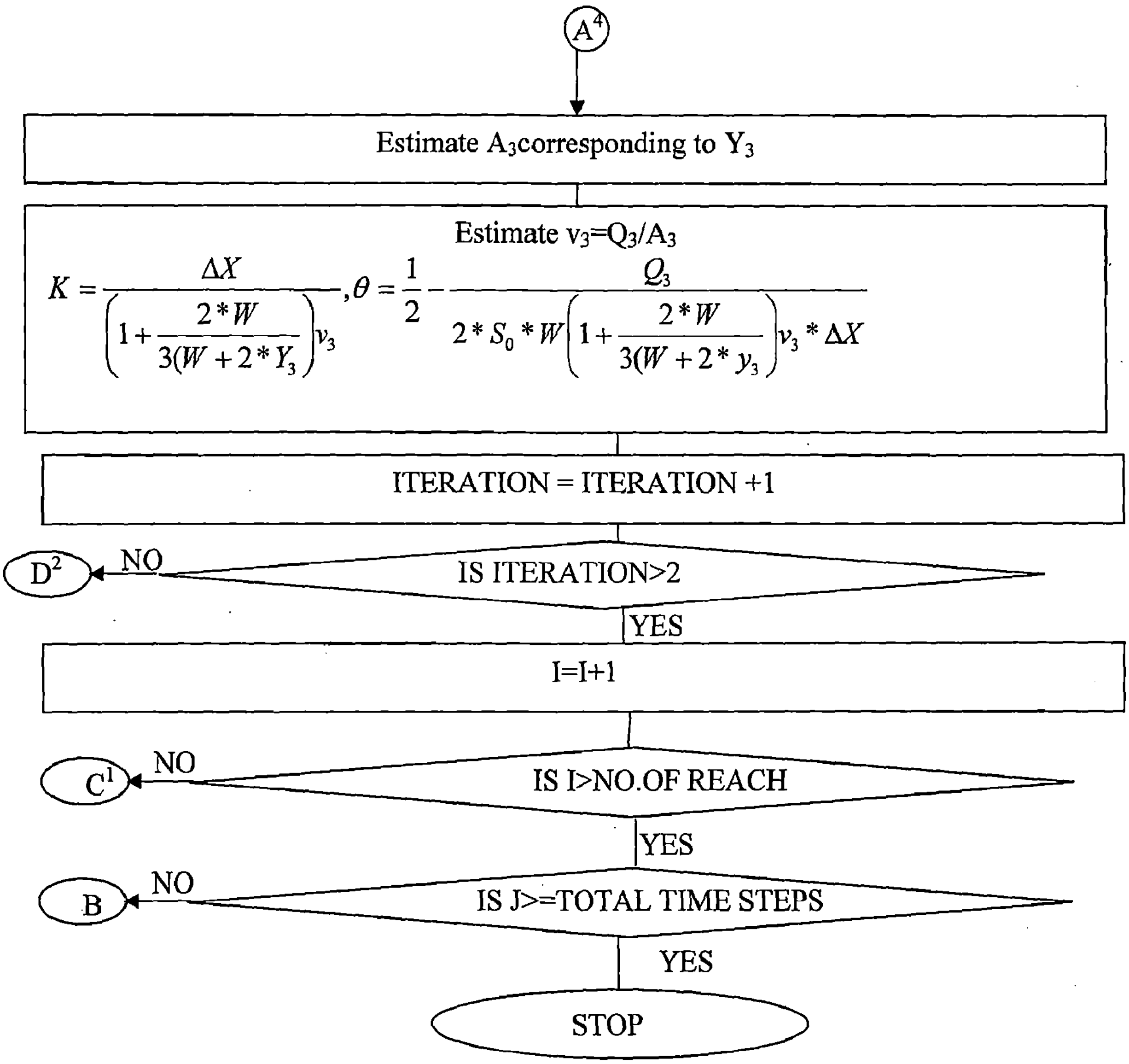
$$F(Y_M) = Q_3 - \frac{\sqrt{S_0} * W^{5/3} * Y_M^{\ 5/3}}{n * (W + 2 * Y_M)^{2/3}}$$

Estimate

$$Q_M = (Q(I,J) + Q(I-1,J))/2$$

Estimate $Y_3 = Y_M + \dfrac{Q_3 - Q_M}{W\left(1 + \dfrac{2*W}{3*(W+2*Y_M)}\right)v_M}$

A^4


A4
Estimate A_3 corresponding to Y_3
Estimate $v_3=Q_3/A_3$
$K=\frac{\Delta X}{\left(1+\frac{2*W}{3(W+2*Y_3)}\right)v_3}, \theta=\frac{1}{2}-\frac{Q_3}{2*S_0*W\left(1+\frac{2*W}{3(W+2*y_3)}\right)v_3*\Delta X}$
ITERATION = ITERATION +1
IS ITERATION>2
NO
D2
YES
I=I+1
IS I>NO.OF REACH
NO
C1
YES
IS J>=TOTAL TIME STEPS
NO
B
YES
STOP

3.3 CONCLUSIONS

In this chapter the solution analysis for the stream-aquifer interaction process has been explained by considering a small interactive stretch and a long interactive stretch in the stream. The solution involves the system of two and six simultaneous equations, respectively, for the case of small and of longer interactive reach, which is divided into three sub-reaches. The equations are given in matrix form which can solved by matrix inversion method. The flow charts for both cases of solutions have also been presented.

CHAPTER-IV
RESULTS AND DISCUSSION

4.1 GENERAL

Based on the analysis presented in chapter-III, different scenarios of stream-aquifer interaction have been studied for a better understanding of this phenomenon. These scenarios correspond to two cases of stream reaches: 1) a long impervious stream reach intervened by a small length of interactive reach, and 2) a longer impervious stream reach intervened by a relatively long interactive reach which needs to be sub-divided for the application of the VPM routing method. A general approach adopted for the study is as follows:

A given inflow hydrograph is routed using the VPM method in a rectangular channel reach which may be considered as impervious in the upstream reaches before it propagates over a short interactive reach, which is hydraulically connected with the adjoining aquifer. The routing in the interactive reach involves the accounting of lateral flow in the form of influent or effluent seepage (henceforth, termed as seepage) which in turn depends on the hydraulic gradient formed by the average transient flow depth of the interactive reach and the level of water-table in the adjoining aquifer. The solution algorithm for estimating the seepage from the interactive reach and the outflow at its outlet was described in Section-3.2.2.

The presence of a long interactive stream reach requires the sub-division of that reach into multiple sub-reaches. In this study, a six km length of such interactive reach was considered and it was sub-divided into three equal sub-reaches for the present analysis. Due to mutual dependence among the six unknown variables (two variables for each sub-reach corresponding to seepage and outflow) of the three interactive sub-reaches, they need to be estimated simultaneously using the coupled solution of the governing equations. The coupled solution algorithm for estimating the seepage and the associated outflow of the three sub-reaches was described in Section-3.2.2.

4.2 APPLICATION

The solutions developed in Chapter-III, for two cases of stream-aquifer interaction problem were studied by routing a hypothetical inflow hydrograph in a rectangular channel of 20 km reach length. The bed slope, S_O of the considered channel is 0.0002; and the width of the channel is 50m. The channel is characterized by a uniform Manning's roughness coefficient, n=0.02; the initial flow depth in the channel reach corresponds to a discharge of Q_O =100 m^3/s.

It is considered that the aquifer which is contiguous with the interactive stream reach is characterized by a reach transmissivity of T = 100 m^2/h and a storage coefficient of S = 0.02. For routing the given hydrograph using the VPM method, the 20 km channel reach is divided into 10 equal sub-reaches of 2 km each. Corresponding to the first case of stream-aquifer interaction, it is considered that only 2 km stretch of the stream is subjected to stream-aquifer interaction, which corresponds to the considered sub-reach length of 2 km. The hypothetical inflow hydrograph is given by the four-parameter Pearson type-III distribution expressed as:

$$I(t) = I_b + (I_p - I_b)\left(\frac{t}{t_p}\right)^{\frac{1}{(\gamma-1)}} \exp\left[\frac{\left(1-\frac{t}{t_p}\right)}{(\gamma-1)}\right] \qquad \ldots (4.1)$$

where, I_b is the initial steady flow (100 m^3/s) in the reach; I_p is the peak flow (1000 m^3/s); t_p is the time to peak (10h) and γ is the skewness factor (1.15) which decides the shape of the hydrograph.

It may be noted that the discharge variable is expressed in units of m^3/h rather than in m^3/s in order to be consistent with the discharge unit of seepage.

4.3 STUDY OF SINGLE INTERACTIVE REACH CASE

Three scenarios have been studied in this case depending on the initial position of the water-table with reference to streambed as depicted in Fig. 1.

In the first scenario, the initial water-table is positioned at a height 3 m above the stream bed which is higher than the initial flow depth in the channel reach corresponding to 100 m^3/s (i.e. 360000 m^3/h).

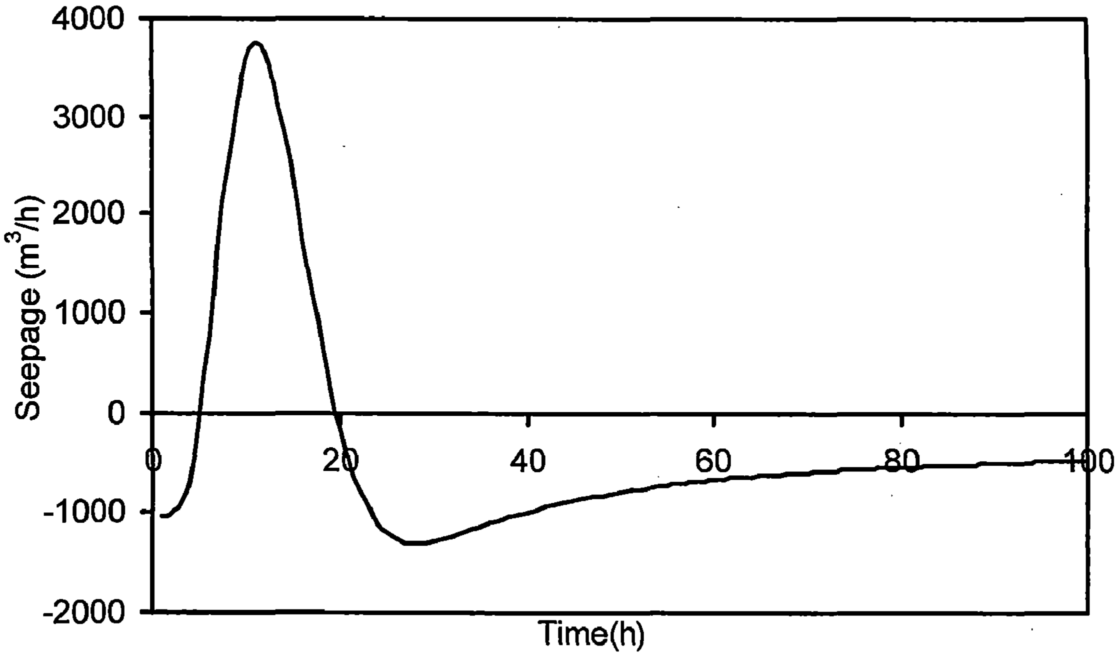


Fig. 13 Seepage hydrograph (when H_o = -3m)

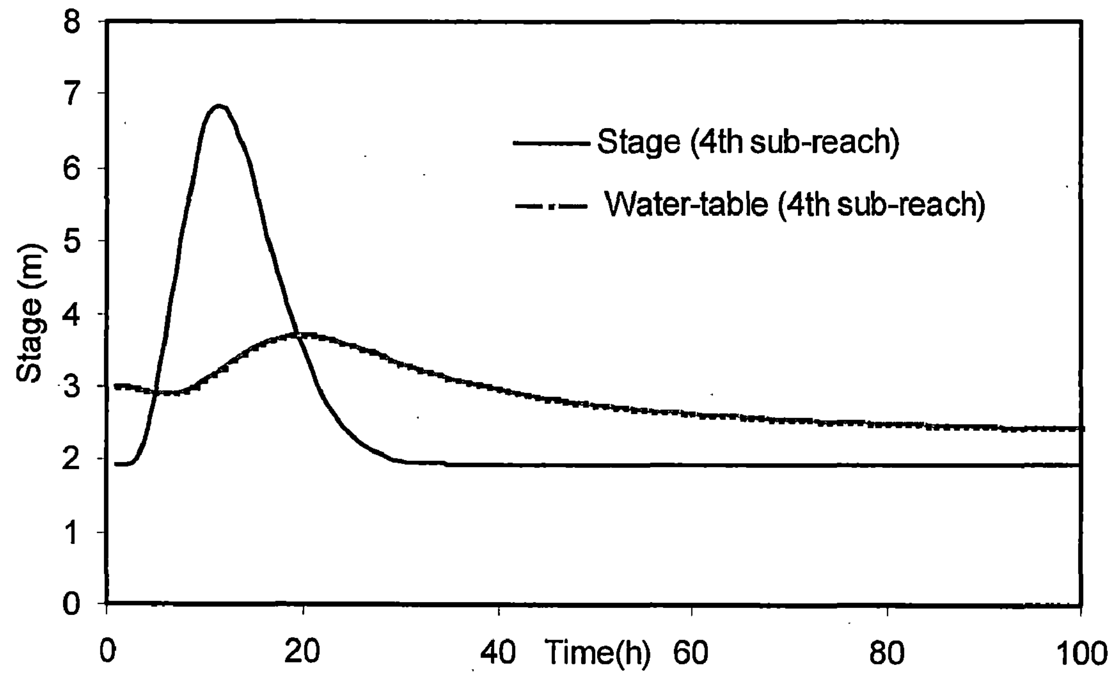


Fig. 14 Average stage-hydrograph and the variation of Water-table of the 4th interactive sub-reach (when H_o = -3m)

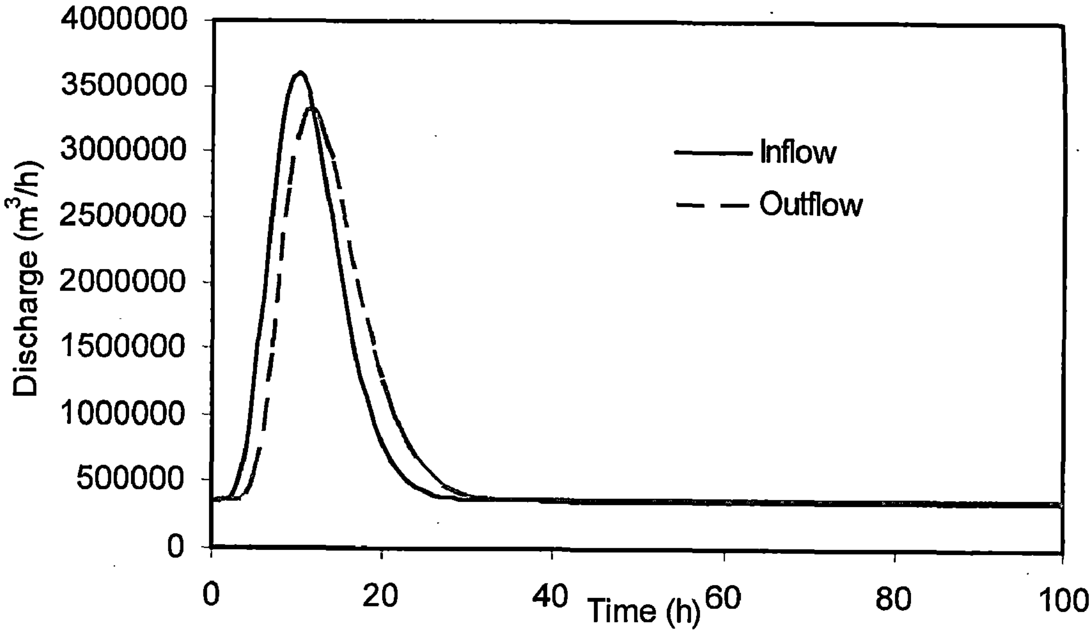


Fig. 15 Inflow outflow hydrograph (when H_o = -3m)

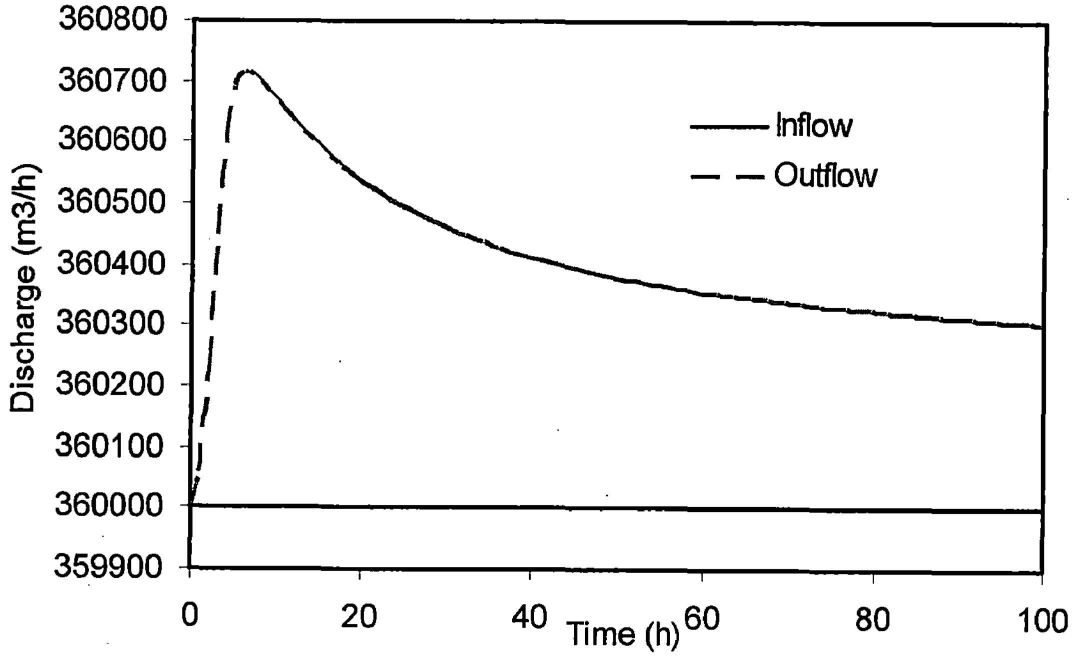


Fig. 16 Inflow and outflow hydrograph of the 4th sub-reach (when H_o = -3)

Figures 13 and 14, respectively, illustrate the time variation of seepage of the interactive sub-reach, and the consequent water-table variation in the aquifer and the variation of average stage hydrograph of the interactive reach. It is inferred from Figure 13 that prior to the arrival of flood, the seepage contribution is towards the stream due to the initial position of the water-table being above that of the initial stage of the stream. It may be noted that seepage from aquifer to stream is designated as negative seepage. However, when the stream stage starts raising at the inlet of the interactive sub-reach, but still the level of water-table in the aquifer is higher than the stream water level, the contribution towards stream decreases and eventually the stream starts contributing to the aquifer when the direction of hydraulic gradient changes, with seepage becoming positive(influent seepage). Integration of seepage hydrograph reveals that the net seepage contribution is towards stream. Alternatively, if the initial flow of 100 m^3/s is sustained at the inlet for a long time, i.e. when the steady flow persists, then the net seepage contribution is always towards the stream and it decreases after reaching a peak contribution as depicted by the hydrograph at the outlet of the interactive sub-reach (Figure 16)

In the second scenario under this case, the initial position of the water-table is assumed to be located at 1 m above the stream bed level, which is slightly lower than the initial flow depth (1.922m) corresponding to 100 m^3/s.

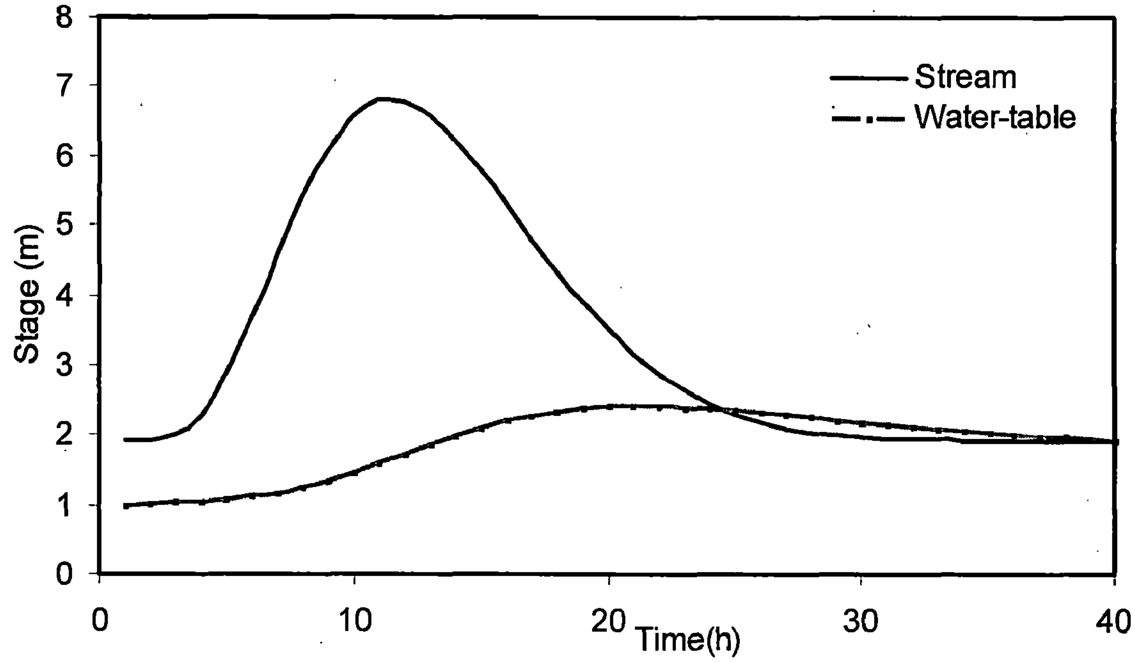


Fig. 17 Average stage-hydrograph and the variation of Water-table (when H_o = -1m)

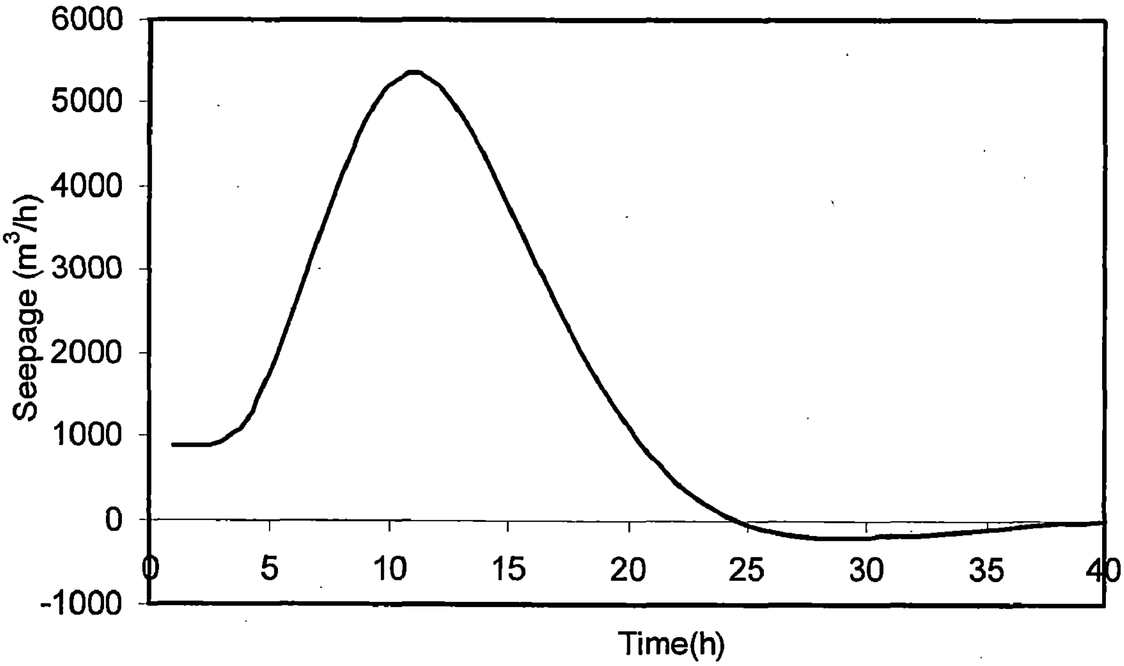


Fig. 18 Seepage hydrograph (when H_o = -1m)

In this scenario, the results indicate that the net contribution is towards aquifer, reaching a peak contribution and eventually the contribution ceases. Average stage-hydrograph of the interactive reach and the corresponding water-table variation in the adjacent aquifer are shown in Figure 17. The corresponding seepage hydrograph is shown in Figure 18 with major contribution being from stream to aquifer, and a minor contribution of aquifer to stream at a much later period which eventually leads to the water-table position same as that of stream flow depth corresponding to $100m^3/s$.

In the third scenario, the initial water-table is considered to be at 7 m below that of the level of stream bed, i.e., in the beginning of unsteady flow movement in the stream, the initial flow of 100 m^3/s contributes towards recharging the aquifer. When the flood wave passes the interactive reach, the seepage contribution to aquifer also increases consistent with the variation of stage in the stream. When the unsteady flow ceases and the initial flow of $100m^3/s$ is attained, the water-table continues to rise, but slowly, which will eventually bring the water-table level corresponding to the level of steady streamflow. Figure 19 illustrates the variability of seepage due to the movement of flood wave in the interactive sub-reach. Figure 20 illustrates the average stage-hydrograph of the interactive sub-reach, and the variation of water-table level in the aquifer adjacent to it.

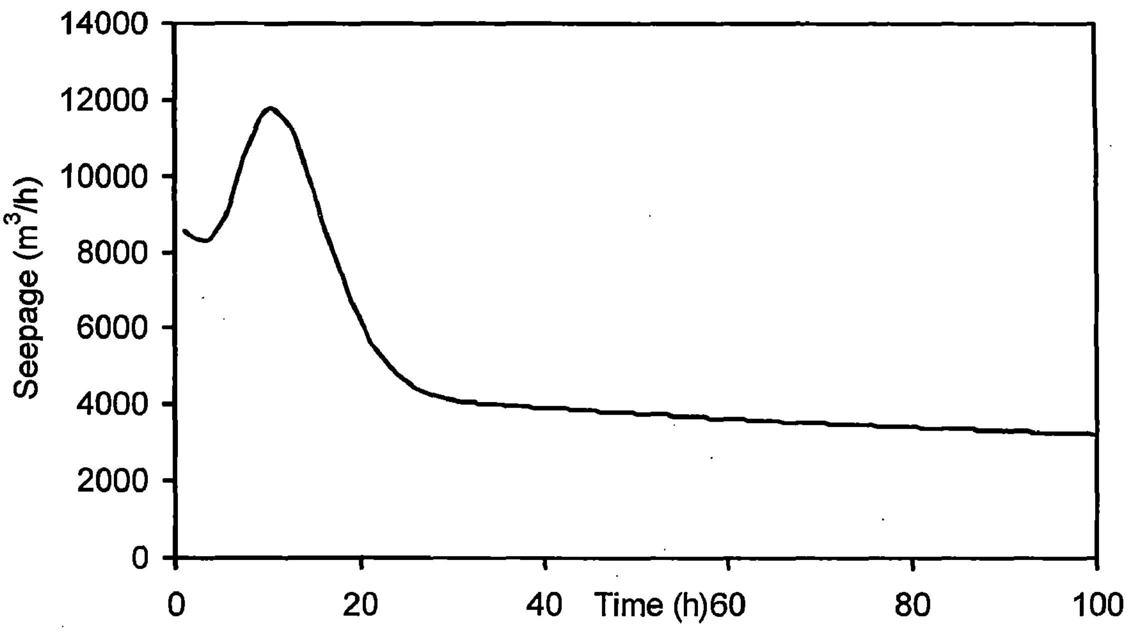


Fig. 19 Seepage hydrograph (when H_o = 7m)

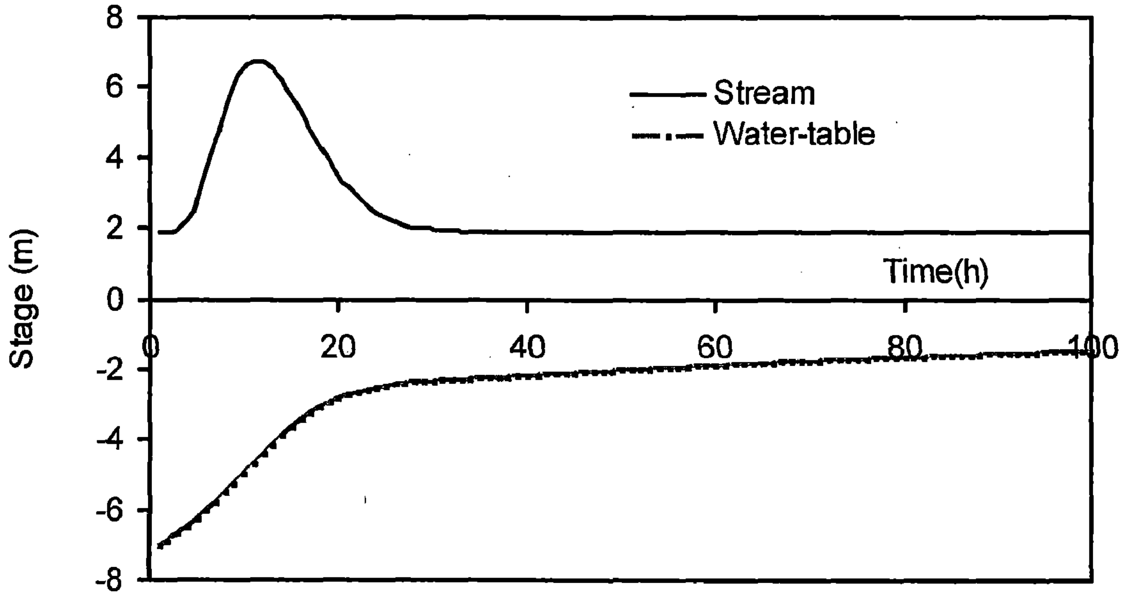


Fig. 20 Average stage-hydrograph and the variation of Water-table (when H_o = 7m)

4.4 STUDY OF THE THREE INTERACTIVE REACHES CASE

Two scenarios have been studied in this case, depending on the initial position of the water-table with reference to stream bed. Also a comparison of the interaction behaviour of a small interactive reach is made when the same interactive reach form a sub-reach of a longer interactive reach.

In the first scenario, before the arrival of flood hydrograph, the initial water-table is positioned at a height of 3 m above the stream bed, which is higher than the initial flow depth in the channel reach corresponding to 100 m^3/s.

It may be inferred from Figure 21 that at the outlet of the stream at 20km, the effect of the presence of small interactive stretch or a longer interactive stretch has not made significant differences in the routed hydrograph. This is due to the reason that the magnitude of stream discharge is very high in comparison to that of seepage rate.

Figure 22 illustrates the variation of the level of water-table with time in the aquifer and the average stage hydrograph of the interactive reach. Figure 23 shows the seepage hydrograph of the 4^{th} interactive sub-reach when it is a small reach, and a part of a longer interactive reach. It is inferred from Figure 23 that prior to the arrival of flood, the seepage contribution is towards the stream due to the initial position of the water-table above that of the initial stage of the stream.

However, when the stream stage starts raising at the inlet of the interactive sub-reach, but still the level of water-table in the aquifer is higher than the stream water level, the contribution towards aquifer decreases and eventually the stream starts contributing to the aquifer when the direction of hydraulic gradient changes, with exchange of flow towards aquifer. Integration of seepage hydrograph reveals that the net seepage contribution is towards stream.

It may be inferred from Figure 23 that the seepage hydrograph shows a higher peak in the case of small reach interaction than that corresponding to the same sub-reach, but when it is part of a longer interactive reach.

Fig. 24 depicts the seepage hydrograph of each of the three consecutive sub-reaches having interaction with aquifer and among these sub-reaches. It is inferred from this Figure that the effect of mutual interaction is not significant at all.

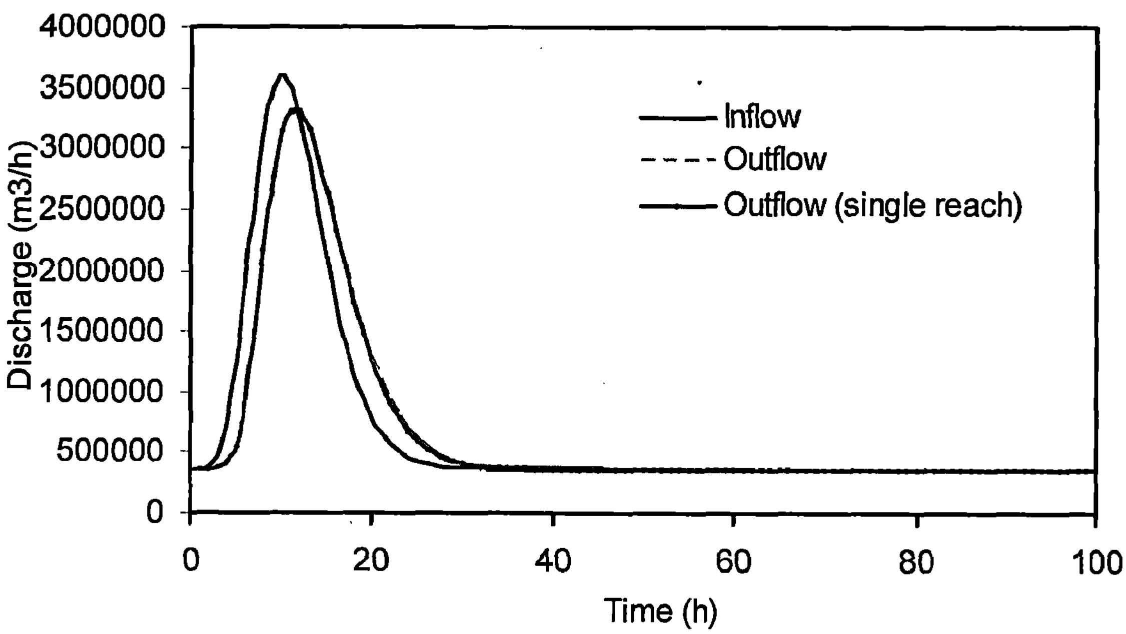


Fig. 21 Routed hydrograph at 20 km of the two channels with small and loner interactive reaches (when H_o = -3m)

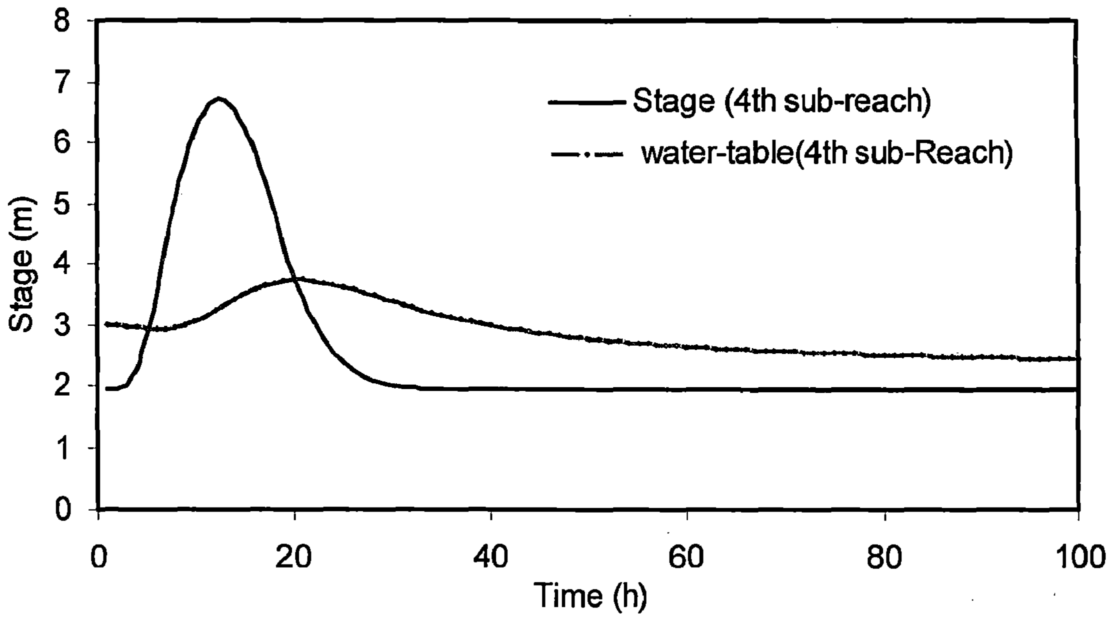


Fig. 22 Average stage-hydrograph and the variation of Water-table for the 4^{th} sub –reach of the longer interactive reach (when H_o = -3m)

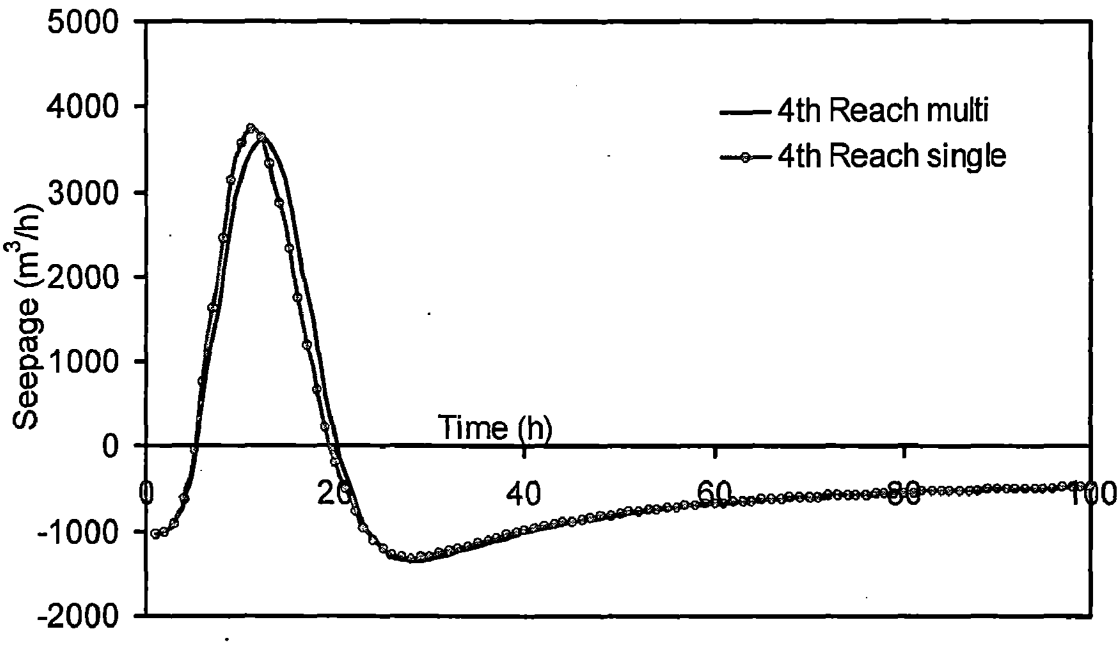


Fig. 23 Seepage hydrograph of the 4^{th} sub-reach corresponding to small and longer interactive reaches (when H_o = -3m)

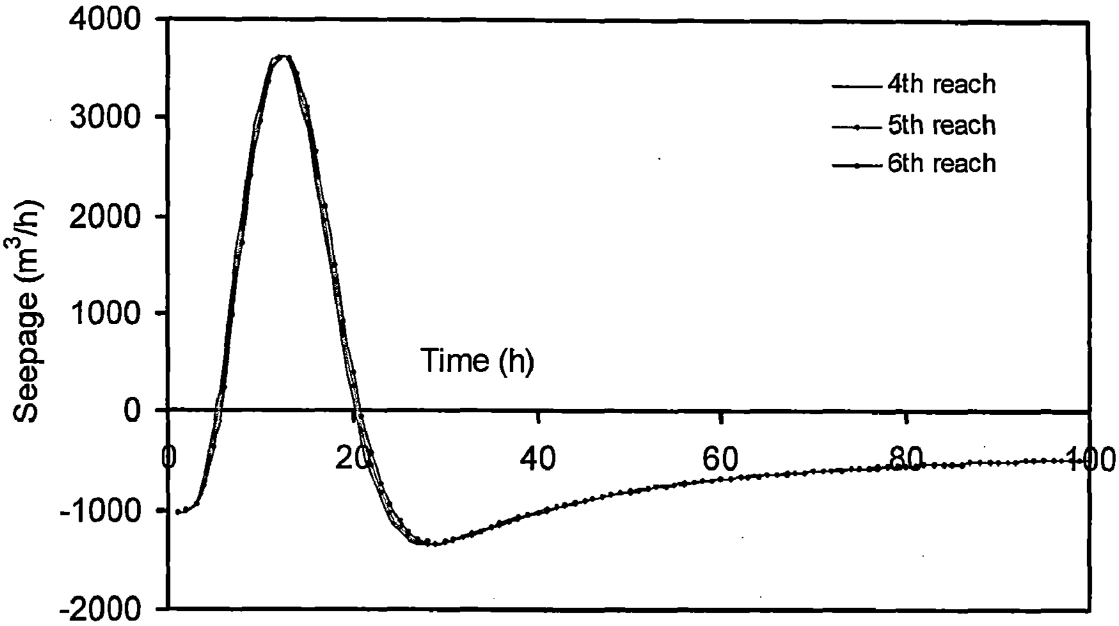


Fig. 24 Seepage hydrograph of the three sub-reaches corresponding to the longer interactive reaches (when H_o = -3m)

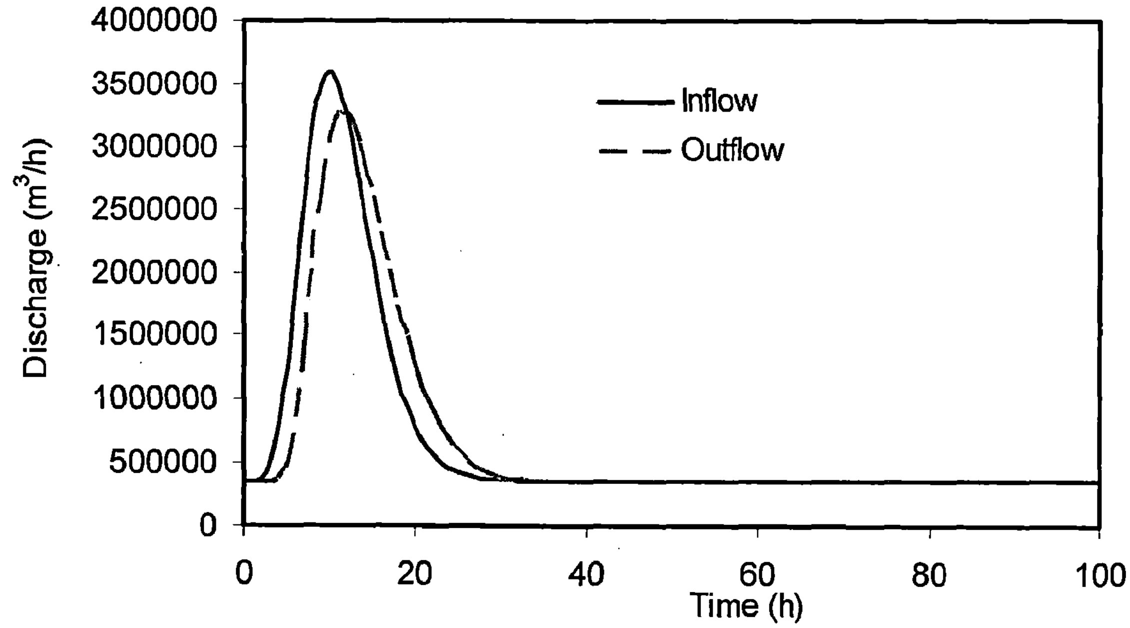


Fig. 25 Routed hydrograph at 20 km of the channel with longer interactive reach (when H_o = 7m)

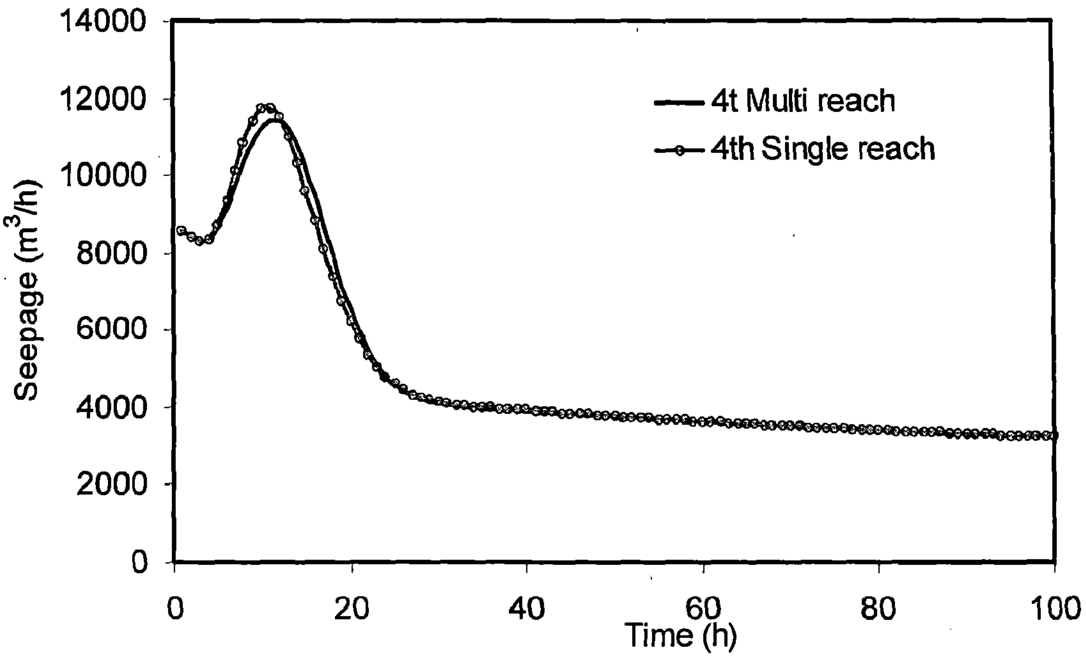


Fig. 26 Seepage hydrograph of the 4^{th} sub-reach corresponding to small and longer interactive reaches (when H_o = 7m)

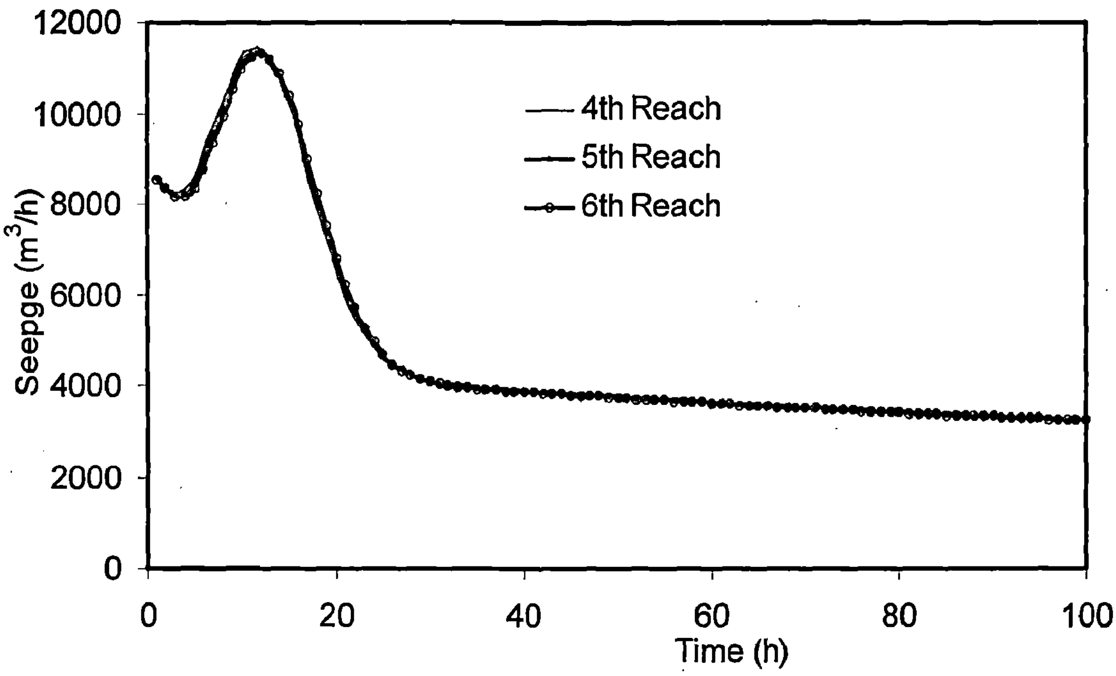


Fig. 27 Seepage hydrograph of the three sub-reaches corresponding to the longer interactive reach (when H_o =7m)

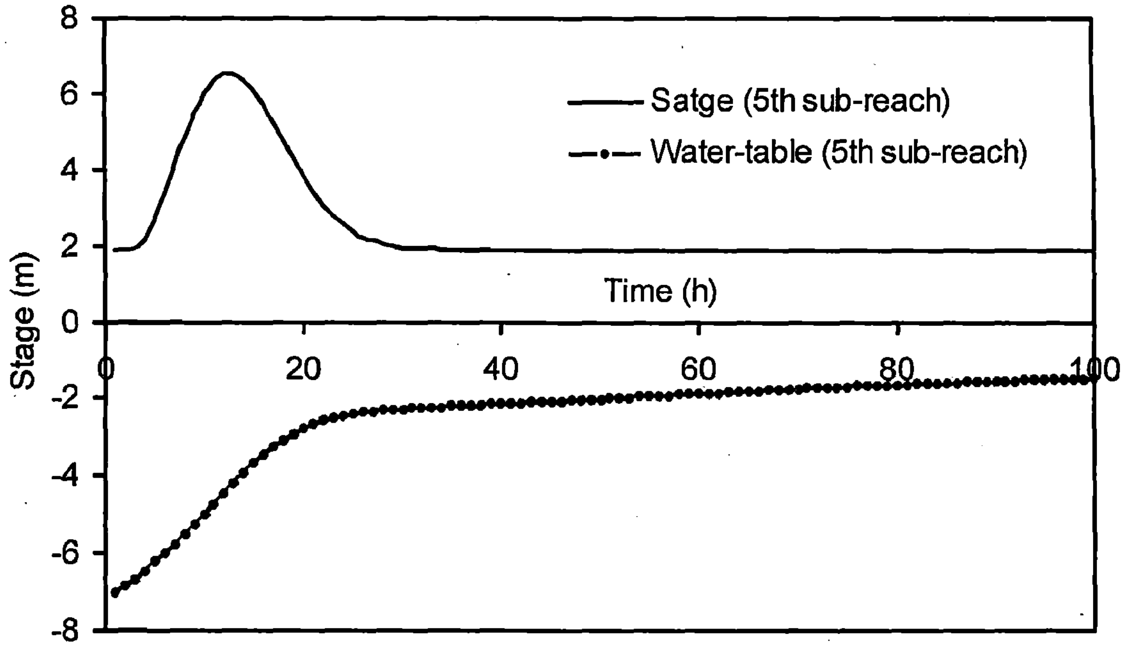


Fig. 28 Average stage-hydrograph and the variation of Water-table for the 5th sub –reach of the longer interactive reach (when H_o =7m)

In the second scenario, the initial water-table is considered to be at a level 7 m below that of the stream bed, i.e., in the beginning of unsteady flow movement in the stream, the initial flow of 100 m^3/s contributes towards recharging the aquifer. When the flood wave passes the interactive reach, seepage to aquifer also increases consistent with the variation of stage in the stream. When the unsteady flow ceases and the initial flow of 100m^3/s is sustained, the water-table continues to rise, but slowly, which will eventually bring the water-table level corresponding to the level of steady stream flow.

Fig. 25 depicts the inflow hydrograph and the routed outflow hydrograph at the reach outlet of the stream of 20 km and indicates the peak attenuation because of time delay and the aquifer contribution. Fig. 26 illustrates the variation of seepage of single reach interaction and multi-reach interaction corresponding to the 4th sub-reach of both cases of channels. It may be inferred from this Figure that the seepage hydrograph shows a higher peak in single reach interaction than that corresponding to the same sub-reach, but when it is part of a longer interactive reach.

Figure 27 illustrates the variability of seepage due to the movement of flood wave in the interactive sub-reaches and it inferred that the effect of interaction between the sub-reaches is negligible. Figure 28 illustrates the average-stage hydrograph at the location of middle of the interactive sub-reaches, (i.e., 5^{th} sub-reach) and the variation of water-table level in the aquifer adjacent to it.

4.5 EFFECT OF INTERACTION AMONGTHE REACHES

The effect of interaction among the reaches was assessed for the two cases of stream-aquifer interaction studied in Section 4.2 considering 1) the 4^{th} sub-reach as a small interactive reach, and 2) the same sub-reach as the first sub-reach of a 6 km interactive reach. Two scenarios were considered for each of these two cases with reference to the position of the initial water-table corresponding to H_0 = - 3m (i.e., 3m above the streambed) and H_0 = 7m (i.e., 7m below the streambed).

It was found that corresponding to the first scenario in which the initial stream stage is lower than the water table, the net seepage of the first sub-reach of the longer interactive reach is higher by about 3 percent when comparison with the net seepage from the same reach, when it is the small interactive reach. In this case the stream-aquifer interaction was estimated for duration of 100 hours. However, for the second scenario (H_o = 7m) , the net seepage difference between the two cases work out to be about 0.05 percent, thus, indicating insignificant difference.

4.6 EFFECT OF WIDTH OF THE STREAM ON THE STREAM-AQUIFER INTERACTION

To quantify the effect of the size of the stream channel on the stream-aquifer interaction process, routing of inflow hydrograph given by equation (4.1) was studied in each of the two different channels with W = 25m and W = 50m.

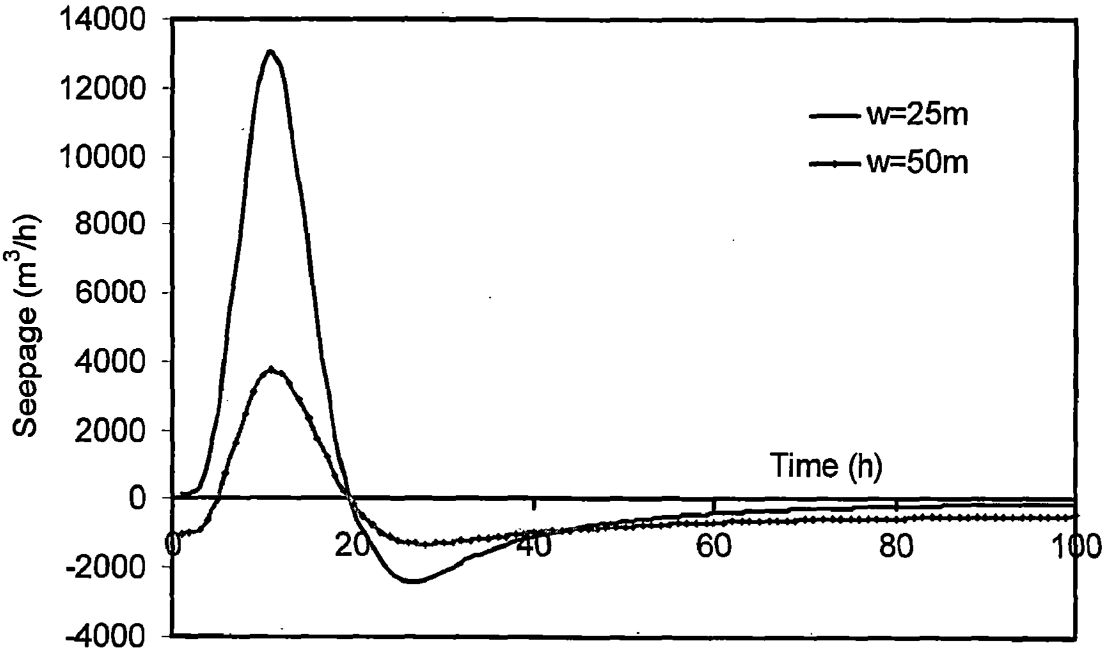


Fig. 29 Seepage hydrograph of the 4[th] sub-reach corresponding to different channel widths (when H_o = -3m)

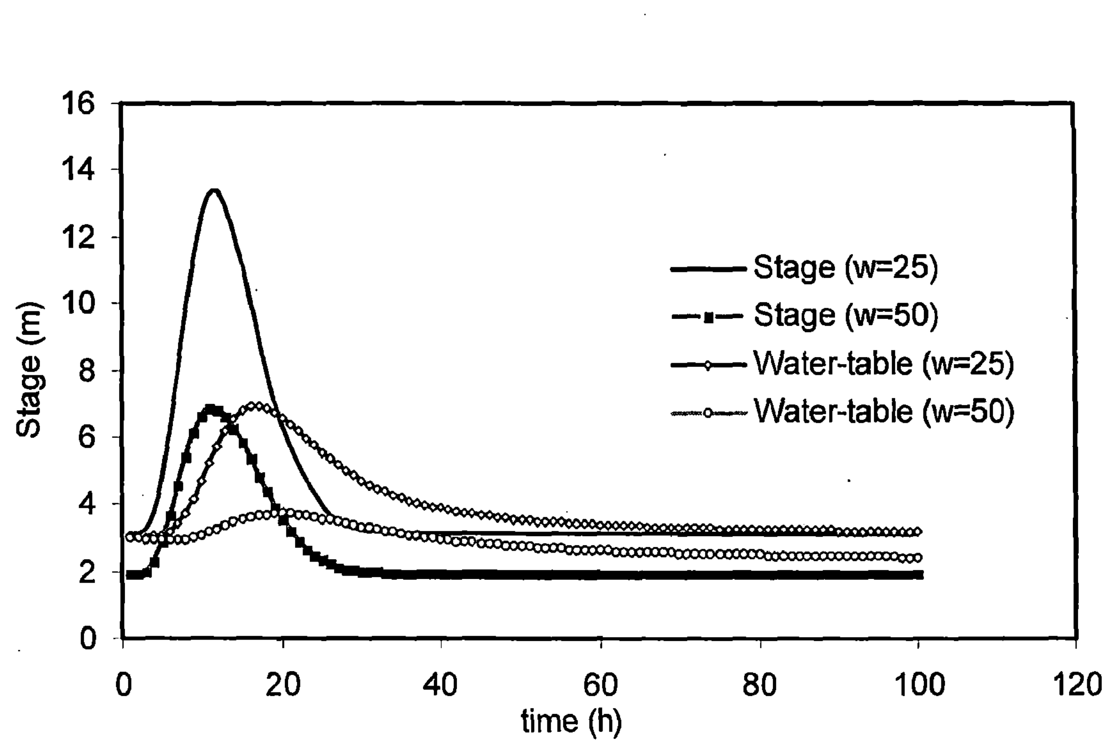


Fig. 30 Average stage-hydrograph and the variation of Water-table for streams with different channel width (when H_0 =-3m)

Each of these channels belongs to the first case of channel reach, i.e., long impervious reach intervened by a small interactive reach of 2km length. In each of these channels, routing was performed for the scenario of initial water-table positioned at a level of 3m above the streambed. Figure (29) illustrates the seepage hydrograph of the interactive sub-reach for these two channel reaches. Figure (30) illustrates the average stage-hydrograph of the interactive reach and the consequent water-table variation in the aquifer.

Figure 29 brings out the effect of change of stream width on the seepage hydrograph. When the width decreases the seepage contribution increases strongly and the stream contributes towards aquifer in a large amount. Seepage varies significantly in comparison to the case corresponding to larger stream width, and attains no seepage condition faster than the larger width case.

4.7 SENSITIVITY WITH RESPECT TO OTHER PARAMETERS

The sensitivity of stream aquifer-interaction process to the variation of parameters like transmissivity, storativity, slope of stream bed and roughness coefficient was studied. It was found that found that if transmissivity and storativity are large then interaction is also large. This aspect was studied for transmissivity value of T = $20 m^2/h$ and $100\ m^2/h$ when there is a constant flow in the stream reach. It was found that for the case of T = $100\ m^2/h$, the net seepage was 244 per cent more than that corresponding to the case of T= $20\ m^2/h$. Similarly for the storage coefficient of S= 0.02, the net seepage was 18 per cent more than that for the case of S= 0.01.

The stream aquifer interaction is also sensitive to variation of bed slope S_o and Manning's roughness coefficient as revealed by the following results. :

1. If the initial water-table is higher than the initial stream stage, then with the increase in S_o, the interaction increases; while with increase in roughness coefficient, the interaction decreases. For example, for the case of initial water-table positioned at a height of 3 m above the stream bed and with initial flow depth corresponding to $100 m^3/s$ at the inlet of the interactive reach, the following results were obtained while routing the inflow hydrograph as discussed in Section-4.2

   a) When S_o = 0.001, the net seepage was estimated to be 150 per cent more than that estimated for the case S_o = 0.0002.

b) For the case of variation in roughness coefficient, it was inferred that for n=0.02 the net contribution was towards the stream, while for n = 0 .035 it was reverse. But this was not the situation in case of steady flow in the stream, and for this case the flow was towards stream throughout. but with increase in slope contribution was more.

2. If the level of initial water-table is lower than that of streambed, then with increase in slope interaction decreases; while with increase in roughness the interaction increases. For the case of water-table at a depth of 7 m below the streambed and with the initial stream flow of 100m^3/s at the inlet of the interactive reach the following results were obtained while routing the inflow hydrograph as discussed in section 4.2 :
    a. When S_O = 0.0002, the net seepage was 12 percent more than that corresponding to S_O = 0.001.
    b. When n=0.035, the interaction was more by 13 percent than that corresponding to n=0.02.

4.8 STUDY OF EVOLUTION OF WATER-TABLE

Two scenarios have been studied in this case depending on the initial position of the water-table with reference to streambed, when there is a sustained constant inflow in the stream:

In the first scenario, the initial water-table is positioned at a height 3 m above the stream bed which is higher than the initial flow depth in the channel reach corresponding to 100 m^3/s. Fig. 31 illustrates the evolution of water-table during a 10h period showing the depletion of aquifer. The depletion occurs at a faster rate near the stream bank and it decreases with increase of distance from the stream bank.

In the second scenario, the initial water-table is positioned at a height 7 m below the stream bed which is lower than the initial flow depth in the channel reach corresponding to the constant inflow of 100 m^3/s in the stream.Fig. 32 illustrates the evolution of water-table during a 10h period showing the formation of mound beneath the stream reach. The water-table rises very soon and the rise increases slowly as distance increases from the centre of the recharging reach.

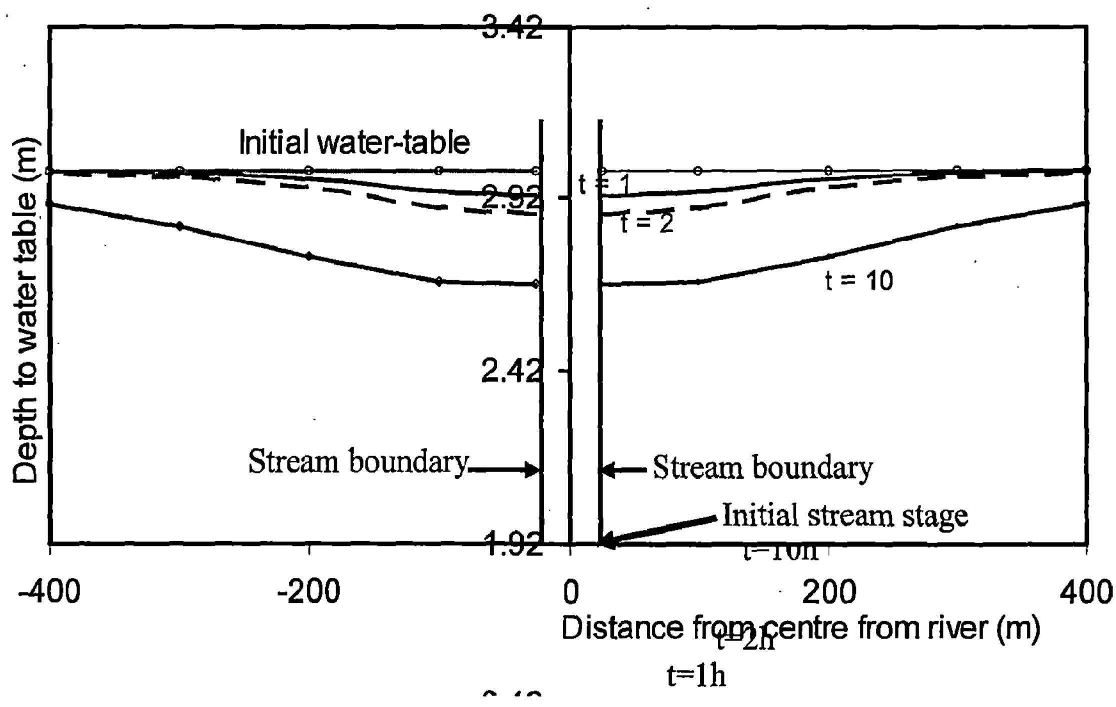


Fig. 31 Evolution of water-table of the aquifer adjacent to 4^{th} sub-reach (when H_0 = -3m)

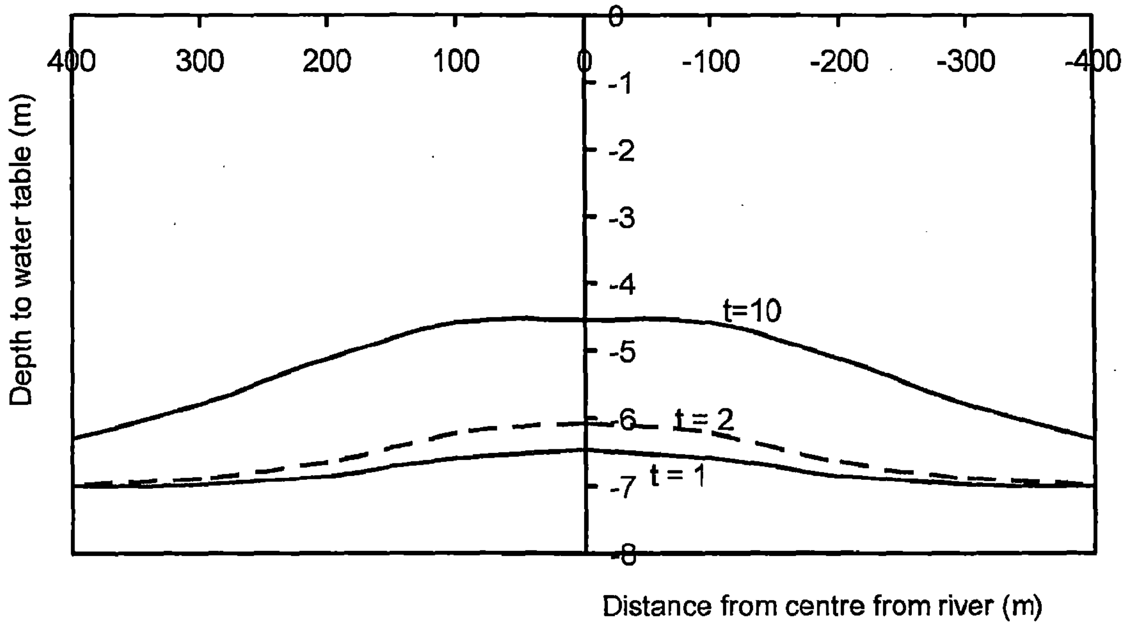


Fig. 32 Evolution of water-table of the aquifer adjacent to 4th sub-reach (when H_o = 7m)

CHAPTER-V
CONCLUSIONS

5.1 CONCLUSIONS AND RECOMMENDATIONS

Based on the solutions presented in Chapter-III, for two cases of stream of stream-aquifer interaction in a stream reach; a) a long impervious reach intervened by a small interactive reach, and 2) a long impervious stream reach intervened by a relatively long interactive reach, different scenarios of stream-aquifer interaction process are studied by routing a flood event for different initial water table positions. Based on this study, the following conclusions are drawn:

1) The magnitude of seepage is not significant in comparison to stream flow during a flood. Therefore, there is no much difference in the routed hydrographs, whether there is interaction or not. One may infer from this statement, that routing can be performed independent of stream-aquifer interaction process.
2) The effect of interaction among the reaches on the stream-aquifer interaction is negligible. Therefore, each sub-reach of a long interactive reach can be studied independent of interaction in other sub-reaches.
3) Stream-aquifer interaction is more significant for narrow channel in comparison to that of the large width stream, while routing the same flood wave.
4) For a given discharge, seepage is higher in streams characterized by steep slope in comparison to seepage in streams characterized by mild slope, when the initial level of water-table is higher than the stream stage.

5) For a given discharge, seepage is higher in streams characterized by small Manning's n value, in comparison to seepage in streams characterized by large n value, when the initial water-table is higher than the stream stage.
6) A water table mound is developed under the stream bed during the passage of a flood wave, and it dissipates to merge with the level of flow in the stream.

The above conclusions have been arrived at based on this study of routing a given flood wave. To give more credibility to these conclusions, a number of different flood hydrographs need to be routed in different configurations of channels characterized by varying roughness values and bed slopes, and for different initial levels of water-table. Field verification of this study may be conducted.

REFERENCES

Apollov BA, Kalinin GP, Kormarov VD, (1964), "Hydrological forecasting" (translated from Russian), Israel Program for Scientific Translation, Jerusalem.

Boussinesq, J., (1877), "Essai sur la theorie des eaux courantes". Mem. Acad. Sci. Inst. Fr. 23, 1, 252–260.

Bouwer. H. (1969) "Groundwater Hydrology" McGraw Hill Book Company, New York.

Birkhead A. L. and James C.S. (2002). "Muskingum River routing with dynamic storage", Journal of Hydrology 264 , 113-132.

Brunke M, Gonser T (1997), "The ecological significance of exchange processes between rivers and ground-water", Fresh- water Biol 37:1–33

Cunge, J.A. (1969)"ON the subject of a flood propagation computation method (Muskingum Method)".J.Hydr.Res.,7(2),205-230.

Dooge, J. C. I. 1982. "Parametrization of hydrologic processes." In Land surface processes, 243-288. See Eagleson 1982b.

Gunduz O. and Aral M. M., (2005) "River networks and groundwater flow: a simultaneous solution of a coupled system", Journal of Hydrology, 301, 216-234.

Hantush, M. M., Harada, M. and Marino, M. A. (2002) "Hydraulics of stream flow routing with bank storage". Journal of Hydraulic Engineering, 7(1), 76-89.

Hantush. M. S. (1967), "Growth and decay of groundwater mounds in response to uniform percolation" Water Resour. Research, 8, 29-36.

Henderson, F. M. (1966) Open Channel Flow. MacMillan & Co. New York, USA.

Kalinin, G. P., and Milyukov, P. I. (1957). "Raschete neustanovivshegosya dvizheniya vody v otkrytykh ruslakh (On the computation of unsteady flow in open channels)." Met. i Gydrologia Zhurnal, 10, 10–18 (Leningrad).

Koussis, AD, (1978), "Theoretical Estimation of Flood Routing Parameters", Journal of the Hydraulics Division, ASCE, Vol. 104, No. HY1, January 1978, 109-115.

Miller,W.A. and Cunge, J.A. 1975."Simplified equations of unsteady flow," unsteady flow in open channels, vol I, K. Mahmood and V. Yevjevich, ed. Pp 183-249.

Morel-Seytoux, H.J., Daly C.J. (1975 A) "discrete kernel generator for stream-aquifer studies". Water Resour. Research, 11: 253-260.

Morel-Seytoux, H.J., (1975) "A combined model of water table and river stages evolution". Water Resour. Research, 10(5/6), 963-972.

NERC, (1975) Flood Studies Report, Institute of Hydrology, Wallingford, Oxford.

Perkins, P. and Koussis, D. (1996) "Stream–aquifer interaction model with diffusive wave approximation", Journal of Hydraulic Engineering, 122(4), 210-218.

Perumal, M. (1994a) "Hydrodynamic derivation of a variable parameter Muskingum method", Hydrological Sc. Journal-des sciences hydrologiques, 39(5), 431-458.

Ruston K.R. and Redshow S.C. (1978) "Seepage and Groundwater Flow", Prentice Hall India.

Sophocleous MA (1997) Managing water resources systems: why safe yield is not sustainable. Ground Water 35(4):561

Sophocleous MA (ed) (1998) Perspectives on sustainable develop- ment of water resources in Kansas. Bull 239, Kansas Geologi- cal Survey, Lawrence, Kansas.

Sophocleous MA (2000a) From safe yield to sustainable develop- ment of water resources – the Kansas experience. J Hydrol235:27–43

Sophocleous MA (2000b) The origin and evolution of safe yield policies in the Kansas Groundwater Management Districts. Nat Resour Res 9(2):99–110.

Sophocleous, M.A. (2002) "The interaction between groundwater and surface water". Hydrogeology Journal, 10, 52-67.

Stoker , J. J., (1957) "Water Waves". , Interscience, New York.

Szilagyi, J. (2004), "Vadose zone influences on aquifer parameter estimates of saturated-zone hydraulic theory", Journal of Hydrology 286(1-4),78-86.

Theis CV (1940) The source of water derived from wells: essential factors controlling the response of an aquifer to development. Civ Eng 10(5):277–280.

Zitta, V.L. and Wiggert, J.M. (1971) "Flood routing in channels with bank seepage". Water Resour. Res., 7(5), 1341-1345.

APPENDIX-1

COMPUTER PROGRAMME

FOR SINGLE REACH INTERACTION

```
      DIMENSION Q(0:100,0:100),QR(0:100),
     1STAGEA(0:50,0:100),STAGER(50,100),
     2GW(96),GX(96),HDKER(100),USTEP(100),
     3AAA(2,2),CCC(2),AA(2,2)
      open(UNIT=2,FILE='YADAV.OUT',STATUS='UNKNOWN')
      open(3,status='old',file='GAUSS.DAT')
      read(3,*) (gw(i),i=1,96)
      read(3,*) (gx(i),i=1,96)
      PAI=3.14159265
C     GENERATION OF KERNEL COEFFICIENTS
C     UNIT TIME IS ONE HOUR
      TRANS=100.
      PHI=0.02

c     H0=5.0

      E=105.

      DELT=1.

      NTIME=100

      WIDTH=50.

      DELX=2000.

      WRITE(2,*)'TRANSMISSIVITY=',TRANS,
     1'VALUE OF PHI =',PHI,'
     2DEPTH OF DATUM FROM BED=',E,
     3'TIME STEP DURATION=',DELT,
     4'WIDTH OF RIVER =',WIDTH,
```

```
     5'LENGTH OF STREAM=',DELX

c         COMPUTATION OF KERNEL COEFFICIENT

          A=WIDTH/2

          B=DELX/2.

C         XX=0.+4.5*WIDTH

          XX=25.

          YY=0.

      DO N=1,NTIME

          AN=N

          TIME=AN*DELT

          CALL HANTUSH(TRANS,PHI,GW,GX,XX,YY,TIME,A,B,RES)

          USTEP(N)=RES

          END DO

          HDKER(1)=USTEP(1)/(DELT*4.*A*B)

          DO N=2,NTIME

          HDKER(N)=(USTEP(N)-USTEP(N-1))/DELT

          HDKER(N)=HDKER(N)/(4.*A*B)

          END DO


          CONV=0.
          QR(0)=0.

C         COMPUTATION OF SURFACE FLOW TILL INFLUENT REACH
          NREACH=10

          IREACH=5
```

```
      S0=0.0002

      CMAN=0.02

      WRITE(2,*)'TOTAL NO OF REACH=',NREACH,
      1'FIRST INTERACTION REACH=',IREACH,
   2'VALUE OF BED SLOPE=',S0,
   3'MANNINGS ROUGHNESS COEFICIENT=',CMAN

      Q0=100.

      PEAKQ=1000.

C     Q0 and PEAKQ are in m**3/sec
C     TP is in hour

      TP=10.

      GAMA=1.15

      PI=PEAKQ*3600.

      Q3=Q0*3600.

      BI0=Q3

C     INITIAL CONDITION
      DO I=0,NREACH

      Q(I,0)=Q3

      END DO

C     BOUNDARY CONDITION

      DO N=1,NTIME

      TIME=N*DELT

      Q(0,N)=Q3

      END DO
```

```
C       DO N=1,NTIME

C       TIME=N*DELT

C       Q(0,N)=Q3+(PI-Q3)*(TIME/TP)**(1./(GAMA-1.))
C       1* EXP((1.-TIME/TP)/(GAMA-1.))

C       END DO

C       INITIAL GUESS OF CMK, THETA

        CALL STAGEYM(Q3,WIDTH,CMAN,S0,YM)

        HBASE=YM

        H0=-3.

        WRITE(2,*)'DEPTH OF THE INITIAL WATER TABLE FROM RIVER BED=',H0

C       MUSKINGUM PARAMETERS ARE NOW PREDICTED

        Y3=YM

        V3 = Q3/(WIDTH*Y3)

        THETA=0.5-0.5*Q3/( S0*WIDTH*
        1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

        CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

C       GAMAR=(TRANS/E)*(2*HBASE+E+0.5*WIDTH)/(4.25*WIDTH+.5*E)*DELX

C       QR(0)=GAMAR*(HBASE+H0)

        IF(IREACH.LE.NREACH) THEN

        DO 100 N=1,NTIME

C       XXXXXXXXXXXXXX
        DO 200 I=1,IREACH-1

C        XXXXXXXXXXXXX

        DO 300 ITER=1,2
```

```
      C =CMK*(1.-THETA) + 0.5*DELT

      C1 = (-CMK*THETA + 0.5*DELT)/C

      C2 = (CMK*THETA+ 0.5*DELT)/C

      C3 = (CMK*(1.- THETA) - 0.5*DELT)/C

      Q(I,N)=C1*Q(I-1,N)+C2*Q(I-1,N-1)+C3*Q(I,N-1)

      Q3=THETA*Q(I-1,N)+(1-THETA)*Q(I,N)

      CALL STAGEYM(Q3,WIDTH,CMAN,S0,YM)

C     WRITE(2,*)'THE VALUE OF YM = ',YM
      QM=(Q(I-1,N)+Q(I,N))/2.0

      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

      A3=WIDTH*Y3

      V3=Q3/A3

      THETA=0.5-0.5*Q3/( S0*WIDTH*
      1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

C     COMPUTATION UP TO I-1TH REACH

C     COMPUTATION OF DISCHARGE AND STAGE

300   CONTINUE

C     XXXXXXXXX
C     WRITE(2,*)'RESULT AT SECTIONS PRIOR TO RECHAREGE'
C     WRITE(2,*)I,Q(I,N),N

200   CONTINUE

C     COMPUTATION OF INFLUENT SEEPGE AND FLOW
C     TERM1= PREVIOUS STREAM STAGE

      DO 301 ITER=1,2
```

```
      TERM1=CMK*(THETA*Q(IREACH-1,N-1)+
      1(1.-THETA)*Q(IREACH,N-1)-BI0)/(WIDTH*DELX)+HBASE

      GAMAR=(TRANS/E)*(2*TERM1+E+0.5*WIDTH)/(4.25*WIDTH+.5*E)*DELX

      TERM2=GAMAR/(1+GAMAR*HDKER(1))

      MMM=2

      AAA(1,1)=-TERM2*(CMK*(1.-THETA)/(WIDTH*DELX))

      AAA(1,2)=1

      AAA(2,1)=(2*CMK*(1-THETA)+DELT)/DELT

      AAA(2,2)=1

C     WRITE(2,*)'MATRIX ELEMENT'
C     WRITE(2,*)AAA(1,1),AAA(1,2)
C     WRITE(2,*)AAA(2,1),AAA(2,2)

C     DO KK=1,2
C     DO JJ=1,2
C     AA(KK,JJ)=AAA(KK,JJ)
C     END DO
C     END DO

      CALL MATIN(AAA,MMM)

C     WRITE(2,*)'INVERSE MATRIX ELEMENT'
C     WRITE(2,*)AAA(1,1),AAA(1,2)
C     WRITE(2,*)AAA(2,1),AAA(2,2)

C     TERM11=AA(1,1)*AAA(1,1)+AA(1,2)*AAA(2,1)
C     TERM12=AA(1,1)*AAA(1,2)+AA(1,2)*AAA(2,2)
C     TERM21=AA(2,1)*AAA(1,1)+AA(2,2)*AAA(2,1)
C     TERM22=AA(2,1)*AAA(1,2)+AA(2,2)*AAA(2,2)

C     WRITE(2,*)'IDENTITY MATRIX'
C     WRITE(2,*)TERM11,TERM12
C     WRITE(2,*)TERM21,TERM22
      IF (N-1)13,13,14
13    CONV=0.
      GO TO 15
14    CONTINUE
```

```
      CONV=0.

      DO NGAMA=1,N-1

      CONV=CONV+QR(NGAMA)*HDKER(N-NGAMA+1)

C     WRITE(2,*)CONV
      END DO

15    CONTINUE

      CCC(1)=TERM2*((CMK*(THETA*Q(IREACH-1,N)-BI0)/(WIDTH*DELX))
      1+H0+HBASE-CONV)

      CCC(2)=Q(IREACH-1,N)+Q(IREACH-1,N-1)-QR(N-1)-Q(IREACH,N-1)
      1-(2*CMK/DELT)*(THETA*(Q(IREACH-1,N)-Q(IREACH,N-1))
     2-(1-THETA)*Q(IREACH,N-1))

      SUM1=0.
      SUM2=0.

      DO M=1,MMM

      SUM1=SUM1+AAA(1,M)*CCC(M)

      SUM2=SUM2+AAA(2,M)*CCC(M)

      END DO

      Q(IREACH,N)=SUM1

      QR(N)=SUM2

      Q3=THETA*Q(I-1,N)+(1-THETA)*Q(I,N)

      CALL STAGEYM(Q3,WIDTH,CMAN,S0,YM)

C     WRITE(2,*)'THE VALUE OF YM = ',YM
      QM=(Q(I-1,N)+Q(I,N))/2.0

      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))
```

```
      A3=WIDTH*Y3

      V3=Q3/A3

      THETA=0.5-0.5*Q3/( S0*WIDTH*
      1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

301   CONTINUE

C     WRITE (2,*)'VALUE OF STAGE='

      STAGER(IREACH,N)=((CMK*(THETA*Q(IREACH-1,N)+(1-
THETA)*Q(IREACH,N)
      1-BI0)/(WIDTH*DELX))+HBASE)

      STAGEA(IREACH,N)=(H0-CONV)

C     WRITE (2,*)'VALUE OF AQUIFER STAGE =',(H0-CONV)
C     WRITE(2,*)STAGEA(IREACH,N)

C     WRITE(2,*)'FLOW AT INFLUENT SECTION',N,Q(IREACH,N),QR(N)
C     TERM1=AA(1,1)-AA(2,1)
C     TERM2=CCC(1)-CCC(2)
C     CHECK1=TERM2/TERM1
C     CHECK2=CCC(1)-CHECK1*AA(1,1)
C     WRITE(2,*)'CHECK1=',CHECK1,'CHECK2=',CHECK2
C     COMPUTATION BEYOND I= IREACH

      DO 20 I=IREACH+1,NREACH

C     XXXXXXXX

      DO 30 ITER=1,2

      C  =CMK*(1.-THETA) + 0.5*DELT

      C1 = (-CMK*THETA + 0.5*DELT)/C

      C2 = (CMK*THETA+ 0.5*DELT)/C

      C3 = (CMK*(1.- THETA) - 0.5*DELT)/C

      Q(I,N)=C1*Q(I-1,N)+C2*Q(I-1,N-1)+C3*Q(I,N-1)
```

```
      Q3=THETA*Q(I-1,N)+(1-THETA)*Q(I,N)

      CALL STAGEYM(Q3,WIDTH,CMAN,S0,YM)

C     WRITE(2,*)'THE VALUE OF YM = ',YM

      QM=(Q(I-1,N)+Q(I,N))/2.0

      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

      A3=WIDTH*Y3

      V3=Q3/A3

      THETA=0.5-0.5*Q3/( S0*WIDTH*
      1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

30    CONTINUE


20    CONTINUE

100   CONTINUE



      DO N=0,NTIME
C     WRITE(2,556)N,QR(N),Q(IREACH,N)
C556  FORMAT(1I5,5X,1F10.2,5X,1F10.2)

      WRITE(2,555)N,(Q(I,N),I=0,NREACH,2)

555  FORMAT(1I5,3X,8F15.2)

      END DO

      DO N=1,NTIME

      WRITE(2,556)N,QR(N),STAGEA(IREACH,N),STAGER(IREACH,N)

556   FORMAT(1I5,5X,1F15.2,5X,1F8.4,5X,1F8.4)
```

```
      END DO
      ELSE

      IF(IREACH.GT.NREACH)THEN

      DO 111 N=1,NTIME

C     XXXXXXXXXXXXXX
      DO 211 I=1,IREACH-1

C      XXXXXXXXXXXXX

      DO 311 ITER=1,2

      C  =CMK*(1.-THETA) + 0.5*DELT

      C1 = (-CMK*THETA + 0.5*DELT)/C

      C2 = (CMK*THETA+ 0.5*DELT)/C

      C3 = (CMK*(1.- THETA) - 0.5*DELT)/C

      Q(I,N)=C1*Q(I-1,N)+C2*Q(I-1,N-1)+C3*Q(I,N-1)

      Q3=THETA*Q(I-1,N)+(1-THETA)*Q(I,N)

      CALL STAGEYM(Q3,WIDTH,CMAN,S0,YM)

C     WRITE(2,*)'THE VALUE OF YM = ',YM
      QM=(Q(I-1,N)+Q(I,N))/2.0

      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

      A3=WIDTH*Y3

      V3=Q3/A3

      THETA=0.5-0.5*Q3/( S0*WIDTH*
      1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

C      COMPUTATION UP TO I-1TH REACH
```

```
C       COMPUTATION OF DISCHARGE AND STAGE

311     CONTINUE

C       XXXXXXXX
C       WRITE(2,*)'RESULT AT SECTIONS PRIOR TO RECHAREGE'
C       WRITE(2,*)I,Q(I,N),N

211   CONTINUE

111     CONTINUE

        DO N=0,NTIME
C       WRITE(2,556)N,QR(N),Q(IREACH,N)
C556    FORMAT(1I5,5X,1F10.2,5X,1F10.2)

        WRITE(2,511)N,(Q(I,N),I=0,NREACH,2)

511   FORMAT(1I5,3X,8F15.2)

        END DO

        ELSE
        WRITE(*,*)'INCORRECT DATA'
        END IF
        END IF


        STOP
        END


C       PROGRAMME FOR COMPUTATION OF STAGE AT MID SECTION

        SUBROUTINE STAGEYM(Q3,WIDTH,CMAN,S0,YM)

C       H=INITIAL GUESS OF  STAGE
C     Q3=DISCHARGE AT MID SECTION (m**3/sec)
C       RIVER WIDTH (m)
C       AN=n; Manning's Roughness Coefficient
C       S0=Bed Slope
C       YM=Stage at mid section (m)

        Q3=Q3/3600.
```

```
      YOLD=1.0
200   CONTINUE
      FUNC=Q3-SQRT(S0)*(WIDTH*YOLD)**(5./3.)
      1/(CMAN*(WIDTH+2.*YOLD)**(2./3.))
      DFUN=-SQRT(S0)*WIDTH**(5./3.)/CMAN
      1*(5.*WIDTH*YOLD**(2./3.)+6.*YOLD**(5./3.))
      1/(3.*(WIDTH+2.*YOLD)**(5./3.))
      YNEW=YOLD-FUNC/DFUN
      IF (ABS(YOLD-YNEW).LE.0.0001) GO TO 100
      YOLD=YNEW
      GO TO 200
100   CONTINUE
      YM=YNEW
      Q3=Q3*3600.
      RETURN
      END
C     PROGRAMME FOR ERROR FUNCTION
      SUBROUTINE ERF(X,ERFX)
      XINDEX=X
      X1=X
      IF(X)4,5,5
4     X1=-X
5     CONTINUE
      IF(X1-15.)1,2,2
1     CONTINUE
```

```
      T=1.0/(1.0+0.3275911*X1)

      ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.42141374*T**3-1.
     1 45315202*T**4+1.06140542*T**5)*EXP(-X1**2)

      GO TO3

2     ERFX=1.

3     CONTINUE

      IF(XINDEX)6,7,7

6     ERFX=-ERFX

7     CONTINUE

C       WRITE(2,52)X,ERFX
C52     FORMAT(2F10.5)

      RETURN
      END

C       PROGRAMME FOR COMPUTING THE RISE IN WATER TABLE

        SUBROUTINE HANTUSH(T,PHI,GW,GX,XX,YY,TIME,A,B,RES)

        DIMENSION GW(96),GX(96)

        TERM5=2.*(T*TIME/PHI)**0.5

        TERM1=A+XX

        TERM2=B+YY

        TERM3=A-XX

        TERM4=B-YY

        TERM11=TERM1/TERM5

        TERM22=TERM2/TERM5

        TERM33=TERM3/TERM5
```

```
    TERM44=TERM4/TERM5

    SUM1=0.
    SUM2=0.
    SUM3=0.
    SUM4=0.

  DO 10 I=1,96

    V=GX(I)

    TERM=(0.5+0.5*V)**0.5

    X=TERM11/TERM

    CALL ERF(X,ERFX)

     ERFX1=ERFX

    X=TERM22/TERM

   CALL ERF(X,ERFX)

  ERFX2=ERFX

    X=TERM33/TERM

    CALL ERF(X,ERFX)

  ERFX3=ERFX

    X=TERM44/TERM

    CALL ERF(X,ERFX)

  ERFX4=ERFX

  ATERM1=ERFX1*ERFX2

  ATERM2=ERFX3*ERFX2

  ATERM3=ERFX1*ERFX4

  ATERM4=ERFX3*ERFX4

    SUM1=SUM1+0.5*ATERM1*GW(I)
```

```
      SUM2=SUM2+0.5*ATERM2*GW(I)
      SUM3=SUM3+0.5*ATERM3*GW(I)
      SUM4=SUM4+0.5*ATERM4*GW(I)
 10   CONTINUE
      RES=(SUM1+SUM2+SUM3+SUM4)*TIME*0.25/PHI
      RETURN
      END

      SUBROUTINE MATIN (AAA,MMM)
      DIMENSION AAA(2,2),B(3),C(3)
      NN=MMM-1
      AAA(1,1)=1./AAA(1,1)
      DO 8 M=1,NN
      K=M+1
      DO 3 I=1,M
      B(I)=0.0
      DO 3 J=1,M
3     B(I)=B(I)+AAA(I,J)*AAA(J,K)
      D=0.0
      DO 4 I=1,M
4     D=D+AAA(K,I)*B(I)
      D=-D+AAA(K,K)
```

```
      AAA(K,K)=1./D
      DO 5 I=1,M
5     AAA(I,K)=-B(I)*AAA(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
6     C(J)=C(J)+AAA(K,I)*AAA(I,J)
      DO 7 J=1,M
7     AAA(K,J)=-C(J)*AAA(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
8     AAA(I,J)=AAA(I,J)-B(I)*AAA(K,J)
      RETURN
      END
```

MULTI REACH INTERACTION

```
DIMENSION Q(0:100,0:100),QR(50,0:100),
      1STAGEA(0:50,0:100),STAGER(0:50,0:100),
      2GW(96),GX(96),HDKER(3,3,100),USTEP(100),CONVS(3),
   3AAA(6,6),CCC(6),AA(6,6),SSUM(6),CONV(3)
      open(UNIT=2,FILE='ASTREAMA2.OUT',STATUS='UNKNOWN')
      open(3,status='old',file='GAUSS.DAT')
      read(3,*) (gw(i),i=1,96)
      read(3,*) (gx(i),i=1,96)
      PAI=3.14159265
```

```
C       GENERATION OF KERNEL COEFFICIENTS
C       UNIT TIME IS ONE HOUR

        TRANS=100.

        PHI=0.02

        E=105.

        DELT=1.

        NTIME=100

        WIDTH=50.

        DELX=2000.

        WRITE(2,*)'TRANSMISSIVITY=',TRANS,'VALUE OF PHI =',PHI,'DEPTH OF
        1DATUM FROM BED=',E,'TIME STEP DURATION=',DELT,'WIDTH OF RIVER
=',
   2WIDTH,'LENGTH OF STREAM=',DELX


C       COMPUTATION OF KERNEL COEFFICIENT
        A=WIDTH/2

        B=DELX/2.

        XX=0.+4.5*WIDTH

C       XX=0.

        YY=0.

    DO N=1,NTIME

        AN=N

        TIME=AN*DELT

        CALL HANTUSH(TRANS,PHI,GW,GX,XX,YY,TIME,A,B,RES)

        USTEP(N)=RES

        END DO
```

```
      HDKER(1,1,1)=USTEP(1)/(DELT*4.*A*B)

      HDKER(2,2,1)=HDKER(1,1,1)

      HDKER(3,3,1)=HDKER(1,1,1)

      DO N=2,NTIME

      HDKER(1,1,N)=(USTEP(N)-USTEP(N-1))/DELT

      HDKER(1,1,N)=HDKER(1,1,N)/(4.*A*B)

      HDKER(2,2,N)=HDKER(1,1,N)

      HDKER(3,3,N)=HDKER(1,1,N)
C     WRITE(2,*)HDKER(3,3,N)
      END DO

      YY=DELX

      DO N=1,NTIME

      AN=N

      TIME=AN*DELT

      CALL HANTUSH(TRANS,PHI,GW,GX,XX,YY,TIME,A,B,RES)

      USTEP(N)=RES

      END DO

      HDKER(1,2,1)=USTEP(1)/(DELT*4.*A*B)

      HDKER(2,1,1)=HDKER(1,2,1)

      HDKER(2,3,1)=HDKER(1,2,1)

      HDKER(3,2,1)=HDKER(2,3,1)

      DO N=2,NTIME

      HDKER(1,2,N)=(USTEP(N)-USTEP(N-1))/DELT
C     WRITE(2,*)'CHECK1=',HDKER(1,2,N)
      HDKER(1,2,N)=HDKER(1,1,N)/(4.*A*B)
```

```
      HDKER(2,1,N)=HDKER(1,2,N)

      HDKER(2,3,N)=HDKER(1,2,N)

      HDKER(3,2,N)=HDKER(2,3,N)
C     WRITE(2,*)'3,2=',HDKER(3,2,N)

      END DO


      YY=DELX*2.

      DO N=1,NTIME

      AN=N

      TIME=AN*DELT

      CALL HANTUSH(TRANS,PHI,GW,GX,XX,YY,TIME,A,B,RES)

      USTEP(N)=RES

      END DO


      HDKER(1,3,1)=USTEP(1)/(DELT*4.*A*B)

      HDKER(3,1,1)=HDKER(1,3,1)

      DO N=2,NTIME

      HDKER(1,3,N)=(USTEP(N)-USTEP(N-1))/DELT

      HDKER(1,3,N)=HDKER(1;3,N)/(4.*A*B)

      HDKER(3,1,N)=HDKER(1,3,N)
C     WRITE(2,*)'3,1=',HDKER(3,1,N)
      END DO

C     COMPUTATION OF SURFACE FLOW TILL INFLUENT REACH
      NREACH=10

      IREACH=4

      DO N=0,NTIME
```

```
      DO I=1,NREACH

      QR(I,N)=0.

      END  DO
      END DO

      S0=0.0002

      CMAN=0.02

      Q0=100.

      PEAKQ=1000.

C     Q0 and PEAKQ are in m**3/sec
C     TP is in hour
      TP=10.

      GAMA=1.15

C     Q3 AND PEAKQ ARE CONVERTED INTO m**3 per hour

      PI=PEAKQ*3600.

      Q3=Q0*3600.

      BI0=Q3
C     INITIAL CONDITION
      DO I=0,NREACH

      Q(I,0)=Q3

      END DO
C     BOUNDARY CONDITION

      DO N=1,NTIME

      TIME=N*DELT

      Q(0,N)=Q3+(PI-Q3)*(TIME/TP)**(1./(GAMA-1.))
      1* EXP((1.-TIME/TP)/(GAMA-1.))

C     Q(0,N)=Q3
      END DO
```

```
C       INITIAL GUESS OF CMK, THETA
        QQ3=Q3
        CALL STAGEYM(QQ3,WIDTH,CMAN,S0,YM)
C
C
C
        HBASE=YM

        H0=7

        WRITE(2,*)'TOTAL NO OF REACH=',NREACH,'FIRST INTERACTION
REACH=',
        1IREACH,'VALUE OF BED SLOPE=',S0,'MANNINGS ROUGHNESS
COEFICIENT=',
    2CMAN,' DEPTH OF INITIAL WATER TABLE=',H0
C
C
C

C       MUSKINGUM PARAMETERS ARE NOW PREDICTED

        Y3=YM

        V3 = Q3/(WIDTH*Y3)

        THETA=0.5-0.5*Q3/( S0*WIDTH*
        1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

        CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)
C
C
C
C       GAMAR=(TRANS/E)*(2*HBASE+E+0.5*WIDTH)/(4.25*WIDTH+.5*E)*DELX

C       QR1=GAMAR*(HBASE+H0)


C       DO I=1,NREACH

C       QR(I,0)=QR1

C       END   DO


C
```

```
C
      IF(IREACH.LE.NREACH-2) THEN

      DO 100 N=1,NTIME

C     XXXXXXXXXXXXXX
      DO 200 I=1,IREACH-1

C      XXXXXXXXXXXXX

      DO 300 ITER=1,2

      C  =CMK*(1.-THETA) + 0.5*DELT

      C1 = (-CMK*THETA + 0.5*DELT)/C

      C2 = (CMK*THETA+ 0.5*DELT)/C

      C3 = (CMK*(1.- THETA) - 0.5*DELT)/C

      Q(I,N)=C1*Q(I-1,N)+C2*Q(I-1,N-1)+C3*Q(I,N-1)

      Q3=THETA*Q(I-1,N)+(1-THETA)*Q(I,N)
      QQ3=Q3

      CALL STAGEYM(QQ3,WIDTH,CMAN,S0,YM)

C     WRITE(2,*)'THE VALUE OF YM = ',YM

      QM=(Q(I-1,N)+Q(I,N))/2.0

      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

      A3=WIDTH*Y3

      V3=Q3/A3

      THETA=0.5-0.5*Q3/( S0*WIDTH*
     1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

C      COMPUTATION UP TO I-1TH REACH
C     COMPUTATION OF DISCHARGE AND STAGE
```

```
300     CONTINUE

C       XXXXXXXX
C       WRITE(2,*)'RESULT AT SECTIONS PRIOR TO RECHAREGE'
C       WRITE(2,*)I,Q(I,N),N
200   CONTINUE

C       COMPUTATION OF INFLUENT SEEPGE AND FLOW
C       TERM1= PREVIOUS STREAM STAGE

        THETA1=THETA

        THETA2=THETA

        THETA3=THETA

        CMK1=CMK

        CMK2=CMK

        CMK3=CMK

C       DO 303 ITER=1,2

        MMM=6

        DO I=1,6

        DO J=1,6

        AAA(I,J)=0

        END DO
        END DO


        AAA(1,1)=1.

        AAA(1,4)=1.+2.*CMK1*(1.-THETA1)/DELT

        AAA(2,2)=1.

        AAA(2,4)=2.*CMK2*THETA2/DELT-1.

        AAA(2,5)=1.+2.*CMK2*(1.-THETA2)/DELT
```

```
      AAA(3,3)=1.

      AAA(3,5)=2.*CMK3*THETA3/DELT-1.

      AAA(3,6)=1.+2.*CMK3*(1.-THETA3)/DELT

      TERM1=CMK1*(THETA1*Q(IREACH-1,N-1)+
      1(1.-THETA1)*Q(IREACH,N-1)-BI0)/(WIDTH*DELX)+HBASE

      GAMAR1=(TRANS/E)*(2*TERM1+E+0.5*WIDTH)/(4.25*WIDTH+.5*E)*DELX


      TERM2=CMK2*(THETA2*Q(IREACH,N-1)+
      1(1.-THETA2)*Q(IREACH+1,N-1)-BI0)/(WIDTH*DELX)+HBASE

      GAMAR2=(TRANS/E)*(2*TERM2+E+0.5*WIDTH)/(4.25*WIDTH+.5*E)*DELX


      TERM3=CMK3*(THETA3*Q(IREACH+1,N-1)+
      1(1.-THETA3)*Q(IREACH+2,N-1)-BI0)/(WIDTH*DELX)+HBASE

      GAMAR3=(TRANS/E)*(2*TERM3+E+0.5*WIDTH)/(4.25*WIDTH+.5*E)*DELX

C     GAMAR1=0.001
C     GAMAR2=0.001
C     GAMAR3=0.001
C     WRITE(2,*)'GAMA1=GAMA2=GAMA2=',GAMAR1


      AAA(4,1)=HDKER(1,1,1)+1./GAMAR1

      AAA(4,2)=HDKER(2,1,1)

      AAA(4,3)=HDKER(3,1,1)

      AAA(4,4)=-CMK1*(1.-THETA1)/(WIDTH*DELX)

      AAA(5,1)=HDKER(1,2,1)

      AAA(5,2)=HDKER(2,2,1)+1./GAMAR2

      AAA(5,3)=HDKER(3,2,1)

      AAA(5,4)=-CMK2*THETA2/(WIDTH*DELX)
```

```
      AAA(5,5)=-CMK2*(1.-THETA2)/(WIDTH*DELX)

      AAA(6,1)=HDKER(1,3,1)

      AAA(6,2)=HDKER(2,3,1)

      AAA(6,3)=HDKER(3,3,1)+1./GAMAR3

      AAA(6,5)=-CMK3*THETA3/(WIDTH*DELX)

      AAA(6,6)=-CMK3*(1.-THETA3)/(WIDTH*DELX)

C     WRITE(2,*)'MATRIX ELEMENT'
C
C     DO I=1,6
C     WRITE(2,222)(AAA(I,J),J=1,6)
C222  FORMAT(6F10.5)
C     END DO

C     DO I=1,6
C     DO J=1,6
C     AA(I,J)=AAA(I,J)
C     END DO
C     END DO
C
      CALL MATIN(AAA,MMM)
C     WRITE(2,*)'CHECK'
C     TERM11=AA(1,1)*AAA(1,1)+AA(1,2)*AAA(2,1)+AA(1,3)*AAA(3,1)
C     1+AA(1,4)*AAA(4,1)+AA(1,5)*AAA(5,1)+AA(1,6)*AAA(6,1)
C     WRITE(2,*)TERM11

      IF (N-1)13,13,14

13    CONTINUE

      CONVS(1)=0.

      CONVS(2)=0.

      CONVS(3)=0.

      GO TO 15

14    CONTINUE
```

```
      DO JOB=1,3

      CONVS(JOB)=0.

      DO IPER=1,3

      DO NGAMA=1,N-1

      CONVS(JOB)=CONVS(JOB)+QR(IREACH-1+IPER,NGAMA)
      1*HDKER(IPER,JOB,N-NGAMA+1)
      END DO
      END DO
      END DO

15    CONTINUE

      CCC(1)=Q(IREACH-1,N)+Q(IREACH-1,N-1)-QR(IREACH,N-1)-Q(IREACH,N-1)
      1-(2*CMK1/DELT)*(THETA1*(Q(IREACH-1,N)-Q(IREACH-1,N-1))-(1-
THETA1)*
      2Q(IREACH,N-1))

      CCC(2)=Q(IREACH,N-1)-QR(IREACH+1,N-1)-Q(IREACH+1,N-1)
      1-(2*CMK2/DELT)*(THETA2*(-Q(IREACH,N-1))-(1-THETA2)*
      2Q(IREACH+1,N-1))


      CCC(3)=Q(IREACH+1,N-1)-QR(IREACH+2,N-1)-Q(IREACH+2,N-1)
      1-(2*CMK3/DELT)*(THETA3*(-Q(IREACH+1,N-1))-(1-THETA3)*
      2Q(IREACH+2,N-1))

      CCC(4)=CMK1*(THETA1*Q(IREACH-1,N)-BI0)/(WIDTH*DELX)+H0-
CONVS(1)
      1+HBASE

      CCC(5)=HBASE-CMK2*BI0/(WIDTH*DELX)+H0-CONVS(2)

      CCC(6)=HBASE-CMK3*BI0/(WIDTH*DELX)+H0-CONVS(3)


      DO II=1,6

      SSUM(II)=0.

      DO M=1,6
```

```
      SSUM(II)=SSUM(II)+AAA(II,M)*CCC(M)

      END DO
      END DO

      QR(IREACH,N)=SSUM(1)

      QR(IREACH+1,N)=SSUM(2)

      QR(IREACH+2,N)=SSUM(3)

      Q(IREACH,N)=SSUM(4)

      Q(IREACH+1,N)=SSUM(5)

      Q(IREACH+2,N)=SSUM(6)

      Q3=THETA1*Q(IREACH-1,N)+(1-THETA1)*Q(IREACH,N)
      QQ3=Q3

      CALL STAGEYM(QQ3,WIDTH,CMAN,S0,YM)

C     WRITE(2,*)'THE VALUE OF YM = ',YM
      QM=(Q(IREACH-1,N)+Q(IREACH,N))/2.0

      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

      A3=WIDTH*Y3
      V3=Q3/A3

      THETA1=0.5-0.5*Q3/( S0*WIDTH*
      1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK1=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)
C     WRITE(2,*)'CMK=',CMK, 'THETA=', THETA

      Q3=THETA2*Q(IREACH,N)+(1-THETA2)*Q(IREACH+1,N)
      QQ3=Q3

      CALL STAGEYM(QQ3,WIDTH,CMAN,S0,YM)
C     WRITE(2,*)'THE VALUE OF YM = ',YM

      QM=(Q(IREACH,N)+Q(IREACH+1,N))/2.0
```

```
      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

      A3=WIDTH*Y3

      V3=Q3/A3

      THETA2=0.5-0.5*Q3/( S0*WIDTH*
     1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK2=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

      Q3=THETA3*Q(IREACH+1,N)+(1-THETA3)*Q(IREACH+2,N)
      QQ3=Q3

      CALL STAGEYM(QQ3,WIDTH,CMAN,S0,YM)
C     WRITE(2,*)'THE VALUE OF YM = ',YM

      QM=(Q(IREACH+1,N)+Q(IREACH+2,N))/2.0

      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

      A3=WIDTH*Y3

      V3=Q3/A3

      THETA3=0.5-0.5*Q3/( S0*WIDTH*
     1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK3=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

C303  CONTINUE

      DO JOB=1,3

      CONV(JOB)=0.

      DO IPER=1,3

      DO NGAMA=1,N

      CONV(JOB)=CONV(JOB)+QR(IREACH-1+IPER,NGAMA)
```

```
        1*HDKER(IPER,JOB,N-NGAMA+1)
        END DO
        END DO
        END DO

        STAGER(IREACH,N)=CMK1*(THETA1*Q(IREACH-1,N)+(1-
THETA1)*Q(IREACH,
        1N)-BI0)/(WIDTH*DELX)+HBASE

        STAGEA(IREACH,N)=(H0-CONV(1))


        STAGER(IREACH+1,N)=CMK2*(THETA2*Q(IREACH,N)+(1-THETA2)
        1*Q(IREACH+1,N)-BI0)/(WIDTH*DELX)+HBASE

        STAGEA(IREACH+1,N)=(H0-CONV(2))


        STAGER(IREACH+2,N)=CMK3*(THETA3*Q(IREACH+1,N)+(1-THETA3)
        1*Q(IREACH+2,N)-BI0)/(WIDTH*DELX)+HBASE

        STAGEA(IREACH+2,N)=(H0-CONV(3))



        THETA=THETA3
        CMK=CMK3

C       WRITE(2,*)'FLOW AT INFLUENT SECTION',N,Q(IREACH,N),QR(N)
C       TERM1=AA(1,1)-AA(2,1)
C       TERM2=CCC(1)-CCC(2)
C       CHECK1=TERM2/TERM1
C       CHECK2=CCC(1)-CHECK1*AA(1,1)
C       WRITE(2,*)'CHECK1=',CHECK1,'CHECK2=',CHECK2
C       COMPUTATION BEYOND I= IREACH
        DO 20 I=IREACH+3,NREACH
C       XXXXXXXX
        DO 30 ITER=1,2

        C  =CMK*(1.-THETA) + 0.5*DELT

        C1 = (-CMK*THETA + 0.5*DELT)/C

        C2 = (CMK*THETA+ 0.5*DELT)/C
```

```
      C3 = (CMK*(1.- THETA) - 0.5*DELT)/C

      Q(I,N)=C1*Q(I-1,N)+C2*Q(I-1,N-1)+C3*Q(I,N-1)

      Q3=THETA*Q(I-1,N)+(1-THETA)*Q(I,N)
      QQ3=Q3
      CALL STAGEYM(QQ3,WIDTH,CMAN,S0,YM)
C     WRITE(2,*)'THE VALUE OF YM = ',YM
      QM=(Q(I-1,N)+Q(I,N))/2.0

      VM=(QM/(WIDTH*YM))

      Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

      A3=WIDTH*Y3

      V3=Q3/A3

      THETA=0.5-0.5*Q3/( S0*WIDTH*
      1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

      CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

30    CONTINUE

20    CONTINUE

100   CONTINUE


      DO N=0,NTIME
C     WRITE(2,556)N,QR(IREACH,N),Q(IREACH,N)
C556  FORMAT(1I5,5X,1F10.2,5X,1F10.2)

      WRITE(2,555)N,(Q(I,N),I=0,NREACH,2 )

555  FORMAT(1I5,5X,8F15.2)

      END DO


      DO N=0,NTIME

      WRITE(2,556)N,QR(IREACH,N),QR(IREACH+1,N),QR(IREACH+2,N),Q(IREA
CH,
      1N),Q(IREACH+1,N),Q(IREACH+2,N)
```

```
556     FORMAT(1I5,5X,1F10.2,5X,1F10.2,5X,1F10.2,5X,F10.2,5X,F10.2,5X,
    1F10.2)

        END DO


        DO N=0,NTIME

        WRITE(2,111)N,STAGER(IREACH,N),STAGER(IREACH+1,N),
        1STAGER(IREACH+2,N),STAGEA(IREACH,N),STAGEA(IREACH+1,N),
    2STAGEA(IREACH+2,N)

111     FORMAT(1I5,5X,1F10.5,5X,1F10.5,5X,1F10.5,5X,F10.5,5X,F10.5,5X,
    1F10.5)

        END DO



        ELSE
        IF (IREACH.GT.NREACH) THEN
        DO 101 N=1,NTIME

C       XXXXXXXXXXXXXX
        DO 201 I=1,IREACH-1

C        XXXXXXXXXXXXX

        DO 301 ITER=1,2

        C  =CMK*(1.-THETA) + 0.5*DELT

        C1 = (-CMK*THETA + 0.5*DELT)/C

        C2 = (CMK*THETA+ 0.5*DELT)/C

        C3 = (CMK*(1.- THETA) - 0.5*DELT)/C

        Q(I,N)=C1*Q(I-1,N)+C2*Q(I-1,N-1)+C3*Q(I,N-1)

        Q3=THETA*Q(I-1,N)+(1-THETA)*Q(I,N)
        QQ3=Q3

        CALL STAGEYM(QQ3,WIDTH,CMAN,S0,YM)
```

```
C       WRITE(2,*)'THE VALUE OF YM = ',YM

        QM=(Q(I-1,N)+Q(I,N))/2.0

        VM=(QM/(WIDTH*YM))

        Y3=YM+((Q3-QM)/(WIDTH*(1.+(((2./3.)*WIDTH)/(WIDTH+2.0*YM)))*VM))

        A3=WIDTH*Y3

        V3=Q3/A3

        THETA=0.5-0.5*Q3/( S0*WIDTH*
        1(1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3*DELX )

        CMK=DELX/((1.+2.*WIDTH/(3.*(WIDTH+2.*Y3)))*V3)

C        COMPUTATION UP TO I-1TH REACH
C       COMPUTATION OF DISCHARGE AND STAGE
301     CONTINUE

C       XXXXXXXX
C       WRITE(2,*)'RESULT AT SECTIONS PRIOR TO RECHAREGE'
C       WRITE(2,*)I,Q(I,N),N
201   CONTINUE
101     CONTINUE

        DO N=0,NTIME
C       WRITE(2,556)N,QR(IREACH,N),Q(IREACH,N)
C556    FORMAT(1I5,5X,1F10.2,5X,1F10.2)

        WRITE(2,511)N,(Q(I,N),I=0,NREACH,2 )

511   FORMAT(1I5,5X,8F15.2)

        END DO



        ELSE
        WRITE(*,*)'CHECK UR INTERACTION REACH NO.'
        END IF
        END IF

        STOP
        END
```