ADAPTIVE FILTERING TECHNIQUES FOR CP BASED CHANNEL ESTIMATION IN OFDM SYSTEMS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree
of
MASTER OF TECHNOLOGY
in
ELECTRONICS AND COMMUNICATION ENGINEERING
(With Specialization in Communication Systems)

By

KRANTI KUMAR BALAGA

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DEPARTMENT OF ELECTRONICS AND COMPUTER ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE -247 667 (INDIA)
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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report entitled, **"ADAPTIVE FILTERING TECHNIQUES FOR CP BASED CHANNEL ESTIMATION IN OFDM SYSTEMS"** towards the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems**, submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from July 2008 to June 2009, under the guidance of **Dr.D.K.MEHRA, Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee.**

I have not submitted the matter embodied in this dissertation for the award of any other Degree or Diploma.

Date: 29-06-09

Place: Roorkee

B. Kranthi Kumar

KRANTI KUMAR BALAGA

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

Date: 29-06-09

Place: Roorkee

Dmehra

Dr. D. K. MEHRA,

Professor, E&C Department,

IIT Roorkee,

Roorkee – 247 667 (India).

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B. Kranthi Kumar

KRANTI KUMAR BALAGA

ABSTRACT

The use of Orthogonal Frequency Division Multiplexing (OFDM) for high rate data transmission over fading dispersive channels has been of wide interest in recent years. OFDM, which already forms a part of several wireless broadcasting standards, is being viewed as a potential candidate for design of upcoming 4G systems. When the intervening channel is doubly selective, the transmitted signal undergoes impairments due to multipath fading and Doppler spreads. In order to facilitate the use of coherent modulation techniques, a receiver has to employ efficient channel estimation schemes.

Channel can be estimated by exploiting the Cyclic prefix (CP) as a training sequence in OFDM systems. Although it is appended to ease the equalizer design in the OFDM systems and is normally discarded at the receiver. The channel can be modeled as autoregressive model of order 2 (AR-2) and it can be estimated using Kalman filter which provides an optimal solution when the model parameters, noise characteristics are known a priori and Gaussian. For practical filtering applications, noise may not be Gaussian and its statistics are not known in advance. In such situations, H-infinity filters provide a recursive estimation of the channel in case of unknown noise statistics and it requires the knowledge of AR parameters. Dual filtering techniques are used to estimate the fading channel as well as its AR parameters recursively.

In this dissertation work, we have used the state space model approach for deriving the different adaptive filtering algorithms namely Kalman, H-infinity, Dual-Kalman and Dual-H-infinity. The CP based channel estimation is carried out using these adaptive filtering techniques in OFDM systems.

For simulation MATLAB is used and it is demonstrated through simulation results that the performance of different adaptive filtering algorithms (H-infinity, Dual-Kalman and Dual-H-infinity) approaches to optimal Kalman filtering algorithm with known parameters.

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LIST OF ABBREVIATIONS

AMPS	Advanced Mobile Phone Service
AR	Auto Regressive
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
B-LMS	Block LMS
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
CP	Cyclic Prefix
CSI	Channel State Information
DAB	Digital Audio Broadcasting
DFT	Discrete Fourier Transform
DPSK	Differential Phase Shift Modulation
DSL	Digital Subscriber Line
DVB	Digital Video Broadcasting
EDGE	Enhanced Data Rates for GSM Evolution
FFT	Fast Fourier Transform
GPRS	Global Packet Radio Service
GSM	Global System for Mobile communications
HF	H-infinity Filter
HSCSD	High- Speed Circuit Switched Data
i.i.d.	independent and identically distributed
ICI	Inter-carrier Interference
IDFT	Inverse Discrete Fourier Transform

IEEE	Institute of Electrical and Electronics Engineering
IFFT	Inverse Fast Fourier Transform
ISI	Inter-symbol Interference
KF	Kalman Filter
LMS	Least Mean Squares
LS	Least Squares
MCM	Multi-Carrier Modulation
MIMO	Multiple-Input Multiple-Output
MMSE	Minimum Mean Square Error
MSE	Mean Square Error
OFDM	Orthogonal Frequency Division Multiplexing
QAM	Quadrature Amplitude Multiplexing
QOS	Quality of Service
QPSK	Quadrature Phase Shift Keying
RLS	Recursive Least Squares
SISO	Single Input Single Output
SNR	Signal to Noise Ratio
TDL	Time Delay Line
TIV	Time Invariant
TV	Time Varying
UMTS	Universal Mobile Telephone System
WCDMA	Wideband Code Division Multiple Access
WLAN	Wireless Local Area Network
WSS	Wide Sense Stationary

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Chapter 1

Introduction

Wireless Communication has undergone a phenomenal growth in the last three decades. Wireless systems have evolved through three generations and fourth generation is an open area of research. The first generation (1G) systems were based on analog transmission techniques for mostly voice. The most successful standards were Nordic Mobile Telephone (NMT), Total Access Communication Systems (TACS), and Advanced Mobile Phone Service (AMPS). These protocols were developed during the 70's and 80's. These protocols supported a data transmission rate between 9.6kbps and 14.4kbps. The technologies developed during the 90's to 2000 come under the second-generation (2G) mobile services. The second-generation (2G) mobile cellular systems were based on digital transmission. The maximum data rate that can be achieved using the 2G protocols is 115kbps. The main advantage of using 2G technologies over the 1G was, increase in the performance due to usage of same channel by several users (either by code or time division multiple access). By this time the cell phones were used for both voice and data communication. There are four main standards for 2G systems: Global System for Mobile (GSM) communication, Digital AMPS (D-AMPS), Code Division Multiple Access (CDMA) IS-95, and Personal Digital Cellular (PDC) [1].

The emergence of mobile data accessing devices like Personal Digital Assistants (PDA's) and internet based data communications, which requires high data transmission rates, have led to the developments of more advanced protocols between 2000 and 2003 and termed as 2.5G protocols. The 2.5G system includes the following technologies: High-Speed Circuit-Switched data (HSCSD), General Packet Radio Services (GPRS), and Enhanced Data Rates for Global Evolution (EDGE) [1]. The maximum data rates that can be achieved using 2.5G protocols is 144kbps, but this is not enough for enhanced multimedia and high streaming videos transmissions. Universal Mobile Telecommunications System (UMTS), Wideband Code Division Multiple Access (WCDMA) and CDMA2000 protocols that also use the Digital Packet Switching, and are developed to increase the data transmission rate up to 2Mbps. These protocols were developed during 2003 to 2004 and are termed as third generation (3G) protocols.

The new IEEE and High Performance Radio Local Area Network (HIPERLAN) standards specify bit rates up to 54Mbit/s, although 24Mbit/s will be the typical rate used in most applications. Such high data rates impose large bandwidths, thus pushing carrier frequencies for values higher than the UHF band.

As the demand for higher data transmission rate and worldwide roaming in cellular devices is increasing, the development of next generation (4G) wireless systems using digital broadband is underway. Therefore, enhancing system capacity as well as achieving a higher bit rate transmission is an important requirement for the 4G system. Fourth generation (4G) aims to provide variable rate multimedia services to the user (which include text, voice, data, audio, images or video), over broadband connections in a seamless manner. The constraints on power and the paucity of spectrum, together with an ever increasing quest for high data rates, poses a challenge to develop efficient coding/modulation techniques and signal processing algorithms, so that wireless links may be utilized as efficiently as possible. The main task is to investigate and develop a new broadband air interface which can deal with high data rates of the order of 100 Mbit/s, high mobility and high capacity. Since the available frequency spectrum is limited, high spectral efficiency is the major task of 4G mobile radio systems. Another important target of the new 4G air interface is the ability to provide efficient support for applications requiring simultaneous transmission of several bits of streams with possibly different Quality of Service (QoS) targets [2].

These developments must cope up with several performance limiting challenges that include channel fading, multi-user interference, limitations of size/power especially at mobile units. Among these challenges, channel fading degrades the performance of wireless transmissions significantly, and becomes a bottle-neck for increasing data rates. As each path has a different attenuation, time delay & phase shift, the signals from different paths add constructively or destructively, resulting in signal strength fluctuations. This phenomenon is known as multi-path fading. Channel fading causes performance degradation and renders reliable high data rate transmissions; a challenging problem for 4G wireless communications. To combat these situations, Orthogonal Frequency Division Multiplex (OFDM), a form of Multi-Carrier Modulation (MCM), has recently been used as a transmission technique owing to its robustness to frequency selective fading and ease of implementation. It carries out simultaneous transmission over multiple subcarriers and simplifies the equalization problem to the design of a single tap filter. When used over rapidly time varying environments, OFDM is prone to errors since

Doppler spread causes a loss of subcarrier orthogonality and introduces inter carrier interference (ICI). However OFDM is a promising technique to achieve high rate data communication over dispersive channels for prospective 4G systems. Coherent demodulation and decoding for OFDM requires the availability of channel state information at the receiver. The physical layer challenges in the implementation of broadband OFDM systems are being addressed recently. These primarily include the design of suitable channel estimation techniques over fading channels for OFDM systems.

1.1. Cyclic Prefix Based Channel Estimation

Bingham [3], was among the first to explore multicarrier modulation (MCM) as an effective technique which uses several interleaved bit streams to modulate a set of carriers in parallel. Water filling algorithm for adaptive loading of bits on the subcarriers is explored; besides this, equalization in presence of channel impairments and phase jitter; and the use of trellis codes to exploit additional coding gain has been studied. The key concept behind such systems is the use of multiple complex exponentials as information bearing carriers that retain orthogonality when propagating over linear dispersive media. The scope for MCM techniques, in particular OFDM, has widened and their potential in providing high data rate communications.

Estimation of fading channel is a critical task for the implementation of wireless OFDM transceivers, and has been challenging research problem. The use of adaptive filters and their variants for efficient estimation of time varying plants has been under research for almost two decades. Davidov in [4] propose a modification to the ordinary LS algorithm, to track deterministic rapidly time-varying systems. Boroujeny and Gazor in [5] have analyzed a family of LMS based adaptive filters, namely the conventional LMS, transform domain normalized LMS (TD-LMS), and LMS-Newton algorithm. Their performance in tracking a time varying plant is compared in terms of steady state excess MSEE.

The conventional method of channel estimation involves transmission of known pilot symbols and LS estimation at pilot tones, followed by suitable filtering/processing at the remaining tones. In [6], Beek analyze channel estimation for OFDM, based on time domain channel statistics. Performance-complexity tradeoff exhibited by the conventional MSEE and LS estimators in demonstrated. In [7], Chen and Zhang study a Kalman filter

(KF) based channel estimator in time-frequency-selective fading environment. Based on Autoregressive (AR) model of the dynamic channel, a vector Kalman filter of dimension equal to the number of subcarriers can be used to track the channel. However, its complexity becomes prohibitive as the number of OFDM subcarriers increases. A state space model for the fading channel based on Jakes' model is proposed, and a scalar Kalman filter is employed at each subcarrier to estimate the channel gain, making use of the time-domain correlation of each subchannel. The channel estimate is further refined with an MMSE combiner, which explores the frequency-domain correlations between the subchannels. This scheme involves two-step processing but offers a performance comparable to the high complex vector Kalman estimator [7].

Wang and Liu [8],[9] have proposed an RLS based adaptive method for estimating digital subscriber line (DSL) channel in an MCM system. The method initializes with a pilot based MMSE channel estimate, and subsequently makes use of cyclic prefix (CP) part of the received symbols for channel estimation. In [10], a CP based frame work for estimation of channel in OFDM systems has also been considered. The multipath channel is modelled as a tapped delay line filter, while the time varying nature of each tap is governed by an autoregressive process of order 2 (AR-2). On the CP based model for doubly selective channel, a variety of adaptive filtering algorithms have been used, namely, block LMS (B-LMS), modified LMS (M-LMS), least squares (LS) and Kalman filter (KF). The Kalman filtering approach gives the optimal solution when the noise characteristics are known a priori and the estimation problem is Gaussian [11]. But in many situations noise characteristics may not be Gaussian, one of the techniques for handling such situations uses H_∞ filtering approach [12]. It is a minimization approach where the maximum "energy" of the estimation of overall disturbances is minimized. A robust H_∞ channel estimation algorithm can be used to estimate the channel fading in the time domain. As in the case of Kalman filtering, the H-infinity filtering also uses the state-space model for estimation of the fading channel

1.2. Statement of Problem

The work reported aims to develop an Adaptive channel estimation algorithm by exploiting the cyclic prefix (CP) in the OFDM system. Cyclic prefix (CP) is a repeated part of the transmit data (known) which can be used for the initial estimation of the channel by using state variable model for OFDM system. The channel is assumed to be

2^{nd} order AR model where the parameters of the AR model are not known. By using Dual H-infinity and Dual-Kalman filtering, we can estimate the channel as well as its AR parameters. The main objectives of this Dissertation are

1. Study of the different Adaptive filtering algorithms
2. Developing the state variable model for OFDM systems using CP interval.
3. Using CP based framework, study of a variety of adaptive filters for estimation of fading channel with low and moderate Doppler spreads in SISO-OFDM systems.

1.3 Organization of the Thesis

Chapter one gives an overview of the evolution of wireless systems through 2G, 3G and 4G systems and a brief introduction to CP based estimation of fading channels for OFDM system.

Chapter two reviews the wireless channel characteristics and basics of OFDM. To estimate the channel information directly from the transmitted data, a CP based channel estimation method is considered.

Chapter three discusses the mathematical models of four different (Kalman, Dual-Kalman, H-infinity and Dual-H-infinity) algorithms for estimation of AR process of order 2 and simulation results are presented.

Chapter four provides a review of SISO-OFDM systems and techniques studied for channel estimation. A state-space model based on CP is developed and channel estimation is accomplished using adaptive filtering. Thus the CP, whose purpose is to render the OFDM subcarriers independent, is used to carry out estimation of fading channel in the low and moderate Doppler spreads. The method is compared with several variants of adaptive filters namely, Kalman, H-infinity, Dual-Kalman and Dual-H-infinity. Their steady state mean square estimation error (MSEE) and as well as bit error rate (BER), are studied, for different Doppler spreads. Finally simulation results are presented.

Chapter five concludes the thesis which also discusses future work.

Chapter 2

Channel Estimation for OFDM Systems

2.1 Introduction

In recent years, the use of OFDM for data transmission over fading wireless channels has been widely studied. It has been adopted in several standards like Digital Audio Broadcasting (DAB), Terrestrial Digital Video Broadcasting (DVB-T), IEEE 802.11a wireless LAN and IEEE 802.16a. Its popularity stems from its efficient bandwidth usage, its ability to reduce the inter-symbol interference (ISI) caused by multipath fading, and easy implementation through FFT algorithm. When operating over a doubly selective fading environment, the channel impairments necessitate equalization of the received symbols prior to demodulation. The channel information plays an important role in the implementation of multi carrier modulation (MCM) systems. It is essential for bit and power allocations and signal detection. Without the knowledge of channel parameters, the MCM system either cannot work or may incur significant performance loss. Some techniques, such as differential phase-shift keying (DPSK) modulation, can be used to eliminate the need for channel information at the receiver, but it incurs a 3-4-dB performance loss. On the contrary, coherent modulation necessitates the use of suitable channel estimation algorithm.

There are several channel estimation methods for OFDM systems broadly categorized as training based and iterative estimation methods [13]. Training based methods require the transmission of explicit pilot sequences followed by suitable filtering / decision feedback operations. This chapter focuses on estimation of fading wireless channels for OFDM, using the ideas of Cyclic Prefix (CP) based estimation and adaptive filtering. Doubly selective environment with slow to moderate time variations is assumed, which facilitates a quasi-static channel model. There are a number of adaptive filtering algorithms for tracking time-varying systems namely Least Mean Square (LMS) and its variants, the method of Least Square (LS) and Kalman filter (KF) etc.

In the following section wireless channel characteristics and basics of OFDM are presented. In the next section the channel estimation for OFDM using pilot method is presented. To estimate the channel information directly from the transmitted data, a CP based channel estimation method is discussed.

2.2 Wireless Channel Characteristics and OFDM

In mobile radio communication systems, the signal transmitted on a wireless channel is subjected to a number of impairments, notable reflections, attenuations and scattering of power. Wireless channels are thus characterized by multipath transmission of the signal, i.e. the received signal results from summation of different replicas of the original signal. Each replica has its own particular amplitude attenuation and, delay which varies with time. This leads to time varying signal strength and signal fading [14].

Suppose that the signal $s(t)$ is transmitted on the time variant multipath channel, where

$$s(t) = \mathrm{Re}[s_l(t)e^{j2\pi f_c t}] \tag{2.1}$$

Then the received band pass signal will be

$$x(t) = \sum_n a_n(t)s(t-\tau_n(t)) \tag{2.2}$$

Where $a_n(t)$ is the attenuation factor for the signal received on the n^{th} path and $\tau_n(t)$ is the propagation delay for the n^{th} path. Substitution from (2.1) into (2.2) gives

$$x(t) = \mathrm{Re}\left\{[\sum_n a_n(t)e^{-j2\pi f_c\tau_n(t)}s_l(t-\tau_n(t))]e^{j2\pi f_c t}\right\} \tag{2.3}$$

From equation (2.3) equivalent low pass received signal can be written as

$$r_l(t) = \left[\sum_n a_n(t)e^{-j2\pi f_c\tau_n(t)}s_l(t-\tau_n(t))\right] \tag{2.4}$$

Since $r_l(t)$ is the response of an equivalent low pass channel to the equivalent low pass signal $s_l(t)$, equivalent low pass impulse response of the time variant multipath channel is given by

$$h(t,\tau) = \left[\sum_n a_n(t)e^{-j2\pi f_c\tau_n(t)}\delta(t-\tau_n(t))\right] \tag{2.5}$$

Assume that $h(t,\tau)$ is a wide-sense stationary uncorrelated scattering (WSSUS), a WSS process has a time-invariant mean; and the autocorrelation function is a function of only the time lag. Similarly, uncorrelated scattering (US) implies that the scatter contributing to the different multipath components are mutually uncorrelated. The equivalent continuous-time low-pass channel impulse response (CIR), $h(t,\tau)$, modelled as a complex valued random process, denotes the channel response at time t to the impulse applied at instant t-τ. Under the WSS assumption, the autocorrelation function of $h(t,\tau)$ is given by [14]

$$\phi_h(\tau_1,\tau_2;\Delta t)=\frac{1}{2}E\left[h^*(\tau_1;t)h(\tau_2;t+\Delta t)\right] \tag{2.6}$$

In most of the radio transmission medium, the attenuation and phase shift of the channel associated with path delay τ_1 is uncorrelated with the attenuation and phase shift associated with path delay τ_2 , from the US assumption and Eq.(2.6) we obtain

$$\frac{1}{2}E\left[h^*(\tau_1;t)h(\tau_2;t+\Delta t)\right]=\phi_h(\tau_1;\Delta t)\delta(\tau_1-\tau_2) \tag{2.7}$$

By putting Δt=0 in equation (2.7), we obtain the autocorrelation function $\phi_h(\tau;0)=\phi_h(\tau)$, it is called as multipath intensity profile or the delay power spectrum of the channel. The range of values of τ over which $\phi_h(\tau)$ is essentially nonzero is called as the multipath spread of the channel and is denoted by T_m .

An analogous characterization of the time-variant multipath channel can be done in frequency domain. By taking the Fourier transform of $h(t,\tau)$ we obtain time-variant transfer function $H(f;t)$ where f is the frequency variable.

$$H(f;t)=\int_{-\infty}^{\infty} h(t,\tau)e^{-j2\pi f_c\tau}d\tau \tag{2.8}$$

$H(f;t)$ has the same statistics as the $h(t,\tau)$. Assuming that channel is wide sense stationary, the autocorrelation function of $H(f;t)$ is given by

$$\phi_H(f_1,f_2;\Delta t)=\frac{1}{2}E\left[H^*(f_1;t)H(f_2;t+\Delta t)\right] \tag{2.9}$$

By substituting Eq. (2.8) into Eq. (2.9) we obtain the relation

$$\phi_H(f_1,f_2;\Delta t)=\int_{-\infty}^{\infty}\phi_h(\tau_1;\Delta t)e^{-j2\pi\Delta f\tau_1}d\tau_1=\phi_H(\Delta f;\Delta t) \tag{2.10}$$

Where, $\Delta f=f_2-f_1$. $\phi_H(\Delta f;\Delta t)$ is the Fourier transform of the multipath intensity profile, and is called as the spaced-frequency, spaced-time correlation function of the channel or doubly selective channel. By taking Δt=0 in Eq. (2.10) we get $\phi_H(\Delta f;0)=\phi_H(\Delta f)$ and $\phi_h(\tau;0)=\phi_h(\tau)$, the relationship is simply

$$\phi_H(\Delta f)=\int_{-\infty}^{\infty}\phi_h(\tau)e^{-j2\pi\Delta f\tau}d\tau \tag{2.11}$$

$\phi_H(\Delta f)$ is an autocorrelation function in the frequency variable, it provides with a measure of the frequency coherence of the channel. As a result of the Fourier transform relationship between $\phi_H(\Delta f)$ and $\phi_h(\tau)$, the reciprocal of the multipath spread is a measure of the coherence bandwidth of the channel.

$$(\Delta f)_c\approx\frac{1}{T_m} \tag{2.12}$$

Where $(\Delta f)_c$ is the coherence bandwidth of the channel. When the bandwidth of the transmitted signal is larger than the $(\Delta f)_c$, then the channel is said to be frequency selective. In this case, the signal is distorted by the channel. On the other hand, if the bandwidth of the transmitted signal is smaller than the $(\Delta f)_c$, then the channel is said to be frequency non selective.

Time variation of the channel is measured by the parameter Δt in $\phi_H(\Delta f;\Delta t)$. The time variations in the channel are seen as Doppler broadening and as Doppler shift of a spectral line. In order to relate Doppler effects to the time variations of the channel function $s_H(\Delta f,\lambda)$ is defined as

$$s_H(\Delta f,\lambda)=\int_{-\infty}^{\infty}\phi_H(\Delta f;\Delta t)e^{-j2\pi\lambda\Delta t}d\Delta t \tag{2.13}$$

When Δf is set to zero the above relation becomes

$$s_H(\lambda) = \int_{-\infty}^{\infty} \phi_H(\Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t \tag{2.14}$$

The function $s_H(\lambda)$ is a power spectrum that gives the signal intensity as a function of Doppler frequency λ. Hence it is called Doppler power spectrum of the channel. The range of values of λ over which $s_H(\lambda)$ is essentially nonzero is called Doppler spread B_d of the channel. Since $s_H(\lambda)$ is related to $\phi_H(\lambda)$ by the Fourier transform, the reciprocal of B_d is a measure of coherence time of the channel. That is

$$(\Delta t)_c \approx \frac{1}{B_d} \tag{2.15}$$

Where $(\Delta t)_c$ denotes the coherence time. A slowly changing channel has a large coherence time or a small Doppler spread.

2.2.1 OFDM

Orthogonal frequency division multiplexing(OFDM) is a parallel transmission scheme, where a high-rate serial data stream is split up into a set of low-rate sub streams, each of which is modulated on a separate sub-carrier (frequency division multiplexing). Thereby, the bandwidth of the sub-carriers becomes small compared with the coherent bandwidth of the channel, i.e., the individual sub-carriers experience flat fading, which allows for simple equalization. This implies that the symbol period of the sub-streams is made long compared to the delay spread of the time-dispersive radio channel. That improves the robustness of OFDM to channel delay spread. Selecting a special set of (orthogonal) carrier frequencies, high spectra of the sub-carriers overlap, while mutual influence among the sub-carriers can be avoided as shown in Fig.2.1.

At one sub-carrier centre frequency, all other spectra are zero demonstrating the sub-carrier orthogonality. One of the most important properties of OFDM transmission is its robustness against multipath delay. This is especially true if the signal's sub-carriers are to retain their orthogonality through the transmission process. The addition of guard period between transmitted symbols can be used to accomplish this. The guard period allows time for multipath signals from previous symbol to dissipate before information

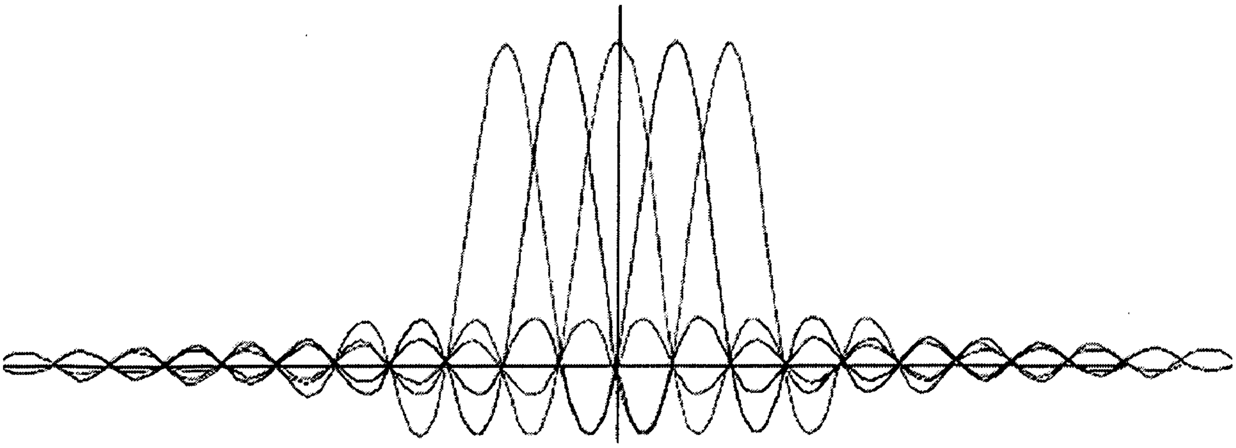

Fig.2.1.The overlapping spectra(sinc functions) of OFDM sub-carriers.

from the current symbol is recorded. The guard time is chosen to be larger than the delay spread such that multipath components from one symbol cannot interfere with next symbol (thus preventing ISI).

OFDM is thus a multicarrier transmission technique [3], where the spacing between two adjacent carriers is identical to the inverse of the symbol period. To generate OFDM successfully the relationship between all the carriers must be carefully controlled to maintain the orthogonality of the carriers. For this reason, OFDM is generated by firstly choosing the spectrum required based on the input data. Each carrier to be produced is assigned some data to transmit. The required amplitude and phase of the carrier is then calculated based on the modulation scheme typically differential BPSK, QPSK, or QAM). The required spectrum is then converted back to its time domain signal using an Inverse Fourier Transform. In most applications, an Inverse Fast Fourier Transform (IFFT) is used. The IFFT performs the transformation very efficiently, and provides a simple way of ensuring that the carrier signals produced are orthogonal. The Fast Fourier Transform (FFT) transforms a cyclic time domain signal into its equivalent frequency spectrum. Finding the equivalent waveform, generated by a sum of orthogonal sinusoidal components, does this. The amplitude and phase of the sinusoidal components represent the frequency spectrum of the time domain signal. OFDM has several advantages like less inter symbol interference, simplicity of channel equalization, efficient use of spectrum, etc.

2.3 Channel Estimation for OFDM

Radio channels in mobile radio systems are usually multipath fading channels, which causes inter symbol interference (ISI) in the received signal. Equalizers can remove ISI from the signal, but requires channel impulse response (CIR). Adaptive channel equalizers utilize channel estimates to overcome the effects of inter symbol interference. Diversity techniques (for e.g. the IS-95 Rake receiver) utilize the channel estimate to implement a matched filter such that the receiver is optimally matched to the received signal instead of the transmitted one. Maximum likelihood detectors utilize channel estimates to minimize the error probability. Accurate channel estimation can be used in OFDM systems to improve their performance by allowing for coherent demodulation. Furthermore, for systems with receiver diversity, optimum combining can be obtained by means of channel estimators. Channel statistics can be determined by using pilot sequence method or Blind method.

2.3.1 Pilot –assisted channel estimation:

The most preferred method to estimate the channel and the offset in frequency is to use pilot symbols. Pilot symbols are symbols that are known to the transmitter and receiver in advance. The basic idea with pilot symbols is that there is a strong correlation between the pilot symbol fading and the fading of information data symbols that are sent close to the pilot symbol in time and sub-carrier. If the channel is static then directly use the pilots at once and estimate the channel at the receiver, if it is time varying continuous transmission of pilot sequences are needed. For that different channel estimation methods are used to estimate the channel using pilots. The pilot symbols and the information data symbols are typically placed in some kind of pattern on the different sub-carriers and over time. Different possibilities exist for allocating pilots in the time-frequency domain of an OFDM system. Pilot tone placement has a great impact on the performance of channel estimation [15]. For time-invariant frequency-selective channels, the pilot tones should minimize the effects of frequency selectivity and equally spaced pilots should be optimum on the FFT grid. It is better to transmit a few pilots in each OFDM symbol rather than clump them together in one symbol. This allows for better tracking of the channel variations. The method in [15] uses the pilots from a block of OFDM symbols to estimate the time-varying channel and alleviates the need for interpolating the channel frequency response.

The relation between transmitted and received signal at the q^{th} pilot symbol for the k^{th} OFDM symbol is [16]

$$Y_k(q) = X_k(q)H_k(q) + Z_k(q)$$

The received pilot signals $\{Y_k(q)\}$ are extracted from $\{Y(k)\}$, the channel transfer function $\{H(k)\}$ can be obtained from the information carried by $\{H_k(q)\}$ and with the knowledge of the known pilot symbols $\{X_k(q)\}$ we can calculate the channel estimation $\{\hat{H}_k(q)\}$ at pilots is

$$\hat{H}_k(q) = \frac{Y_k(q)}{X_k(q)} + \frac{Z_k(q)}{X_k(q)} = H_k(q) + Z'_k(q)$$

$Z_k(q)$ is the noise contribution at the q^{th} pilot sub-carrier, $Z'_k(q)$ is a scaled noise contribution at that sub-carrier. Different methods can then be applied to estimate the channel over all sub-carrier frequencies using pilots. Channel estimation in OFDM is a two-dimensional (2-D) problem i.e., channel needs to be estimated in time-frequency domain. Due to the computational complexity of 2-D estimators, the scope of channel estimators can be limited to one-dimensional (1-D). The idea behind 1-D estimators is to estimate the channel in one dimension (may be frequency) and later estimate the channel in the second dimension (may be time), thus obtaining a 2-D channel estimate. Different approaches for channel estimation are based on minimum mean-squared error (MMSE) estimate of pilot signals. Because of the robustness of the MMSE estimator, the AWGN and the ICI components are reduced significantly in fast or slow-fading noisy radio channel environments. The computational complexity of the MMSE estimator can be reduced by using a simplified linear minimum mean-squared error (LMMSE) estimator with low-rank approximation by singular value decomposition (SVD) [17]. After the estimation of the channel transfer functions at pilot tones, the channel responses of data tones can be interpolated according to adjacent pilot tones. If the channel is continuously times varying continuous transmission of pilot sequences are needed but it is not efficient and one of the obvious drawbacks is that it is wasteful of bandwidth. For fast fading channels this might not be adequate since the coherence time of the channel might be shorter than the symbol time.

In digital communication systems, channel equalization and channel estimation are essential for successful data transmission. While channel equalization and estimation are usually done by pilot assisted-based methods as discussed above, blind methods have also been developed which do not require use of pilot symbols and possess desirable advantages such as better bandwidth efficiency. But they still have their own drawbacks. These methods are extremely computationally intensive and hence are impractical to implement in real-time systems. Techniques for blind channel estimation of OFDM systems using redundant precoding, subspace-based blind and semi-blind channel estimation for OFDM systems have been explored [18].

The pilot based method for channel estimation in OFDM is effectively used when the channel is quasi-static. For the fast varying channels above channel estimation methods are not efficient, so an adaptive channel estimation algorithm by exploiting the cyclic prefix in the MCM system is proposed in [8]. The cyclic prefix used in MCM systems is originally designed to reduce ISI. However, it is nothing but a repeated part of the transmit data which can be used for channel estimation. Based on this observation, we discuss an adaptive channel estimation algorithm to estimate the channel, adaptively exploiting the information in cyclic prefix. The algorithm uses decision directed samples, and hence, no extra training is needed.

2.3.2 Fading Channel Characterization for OFDM Systems

For band limited channel that causes ISI, it is convenient to develop an equivalent discrete time model for the continuous time system. The cascade of analog pulse shaping filter at the transmitter $h^{tr}(\tau)$, assumed to be time-invariant, the time-varying channel impulse response $h(t,\tau)$ from Eq. (2.5), and the matched filter at the receiver with impulse response $h^{rec}(\tau)$, together with the sampler may be represented by an equivalent discrete time transversal filter with coefficients $h_n(l)$. We assume a symbol spaced sampling of the continuous time channel impulse response (CIR), hence sampling of the impulse response is carried out every T_s seconds. The useful OFDM symbol duration $T = NT_s$, where N is the number of data subcarriers. In practice, the noise sequence at the matched filter output of the discrete time model is correlated, and it becomes necessary to cascade a noise-whitening with the sampler. The result usually referred to as equivalent discrete time white noise filter model [10].

$$h_n(l) = \left[h^{tr}(\tau) * h(t,\tau) * h^{rec}(\tau) \right]\Big|_{t=nT, \tau=lT_s} \tag{2.16}$$

here $*$ denotes the convolution operator, and T_s is the signalling interval.

For an under spread channel whose delay spread is bounded by τ_{max}, the number of discrete channel taps L is determined by $\left\lfloor \tau_{max}/T_s \right\rfloor$, since T_s is the delay resolution of the model. Here we consider a low to moderate Doppler environment, which allows for a block fading (quasi-static) channel assumption. This implies that the channel tap variations within an OFDM symbol duration are negligible, and hence we may define an $L \times 1$ channel tap vector for each OFDM symbol as

$$\mathbf{h}_n = \left[h_n(0) h_n(1) \ldots h_n(L-1) \right]^T \tag{2.17}$$

where $h_n(l)$ is the l^{th} channel tap for the n^{th} OFDM symbol. However, the channel tap values change randomly from one OFDM symbol to other.

The classical time autocorrelation function, according to Jake's model, at a time lag Δt is

$$\phi_t(\Delta t) = J_0(2\pi f_d \Delta t) \tag{2.18}$$

where $J_0(.)$ is the zeroth-order Bessel's function of the first kind, and f_d is the maximum Doppler frequency. The classical Doppler spectrum is obtained through Fourier transform as

$$s(f) = \begin{cases} \left(\pi \sqrt{f_d^2 - f^2} \right)^{-1} & \text{for } f \le f_d \\ 0 & \text{otherwise} \end{cases} \tag{2.19}$$

In actual practice, the aforesaid autocorrelation function is difficult to achieve. An autoregressive (AR) process of order p may be used to approximate the Bessel's autocorrelation [12] [19]. This implies solving a system of p linear Yule-Walker equations. It has been found that for an order as low as 2, a good autocorrelation matching is achieved for time lag upto 20. Thus the classical Doppler spectrum for each of the L channel taps is approximated by an independent AR-2 process [19].

For the l^{th} channel tap at n^{th} OFDM symbol, we have

$$h_n(l) = -a_1 h_{n-1}(l) - a_2 h_{n-2}(l) + v_n(l) \tag{2.20}$$

where a_1 and a_2 are the AR-2 coefficients and $v_n(l)$ is the modelling noise for l^{th} tap at symbol n.

Equating the autocorrelation functions of the Jake's model and the AR-2 model (at a discrete lag of m symbol intervals), we have

$$\phi(m) = E\left[h_n(l)h^*_{n-m}(l)\right] = J_0(2\pi f_d mT) \tag{2.21}$$

The poles of the transfer function for Eq. (2.20) are located at [10]

$$p = \left(1 - \frac{\omega_d}{\pi}\right)e^{\pm j0.7\omega_d} \tag{2.22}$$

The AR-2 coefficients a_1 and a_2 are found as

$$a_1 = -2r_d \cos(0.7\omega_d) \tag{2.23}$$

and

$$a_2 = r_d^2 \tag{2.24}$$

where

$$r_d = \left(1 - \frac{\omega_d}{\pi}\right) \text{ and } \omega_d = 2\pi f_d T \tag{2.25}$$

For a close approximation between the AR-2 and Jake's model, the pole radius r_d should be close to unity.

2.4 CP Based Channel Estimation Method

Wang and Liu ([8], [9]) have proposed an RLS based adaptive method for estimating digital subscriber line (DSL) channel in MCM systems. The method initially finds a pilot based MMSE channel estimate, and subsequently makes use of cyclic prefix (CP) part of the received symbols for channel estimation. The CP is inherently appended to the symbols in MCM systems, for eliminating ISI, and is normally discarded at the receiver and it can be viewed as a constantly sent training sequence for channel estimation. Hence we can use it to adaptively estimate the channel without additional training sequences.

2.4.1 MCM System Using Cyclic Prefix

Fig.2.2 shows an OFDM system using cyclic prefix with adaptive channel estimation [9]. The system has N data subcarriers. Input data are buffered, converted to a parallel stream and modulated to i.i.d. equi-probable symbols $X_n(k)$, where $X_n(k)$ denotes the k^{th} symbol of the n^{th} OFDM symbol. These symbols are drawn from a complex constellation, depending upon the underlying modulation scheme, e.g. BPSK, QPSK, 16-QAM, 64-QAM ect. OFDM modulation is accomplished by taking N-point IDFT of the symbol vector

$$\mathbf{X}_n = \left[X_n(0)\, X_n(1) \ldots X_n(N-1)\right]^T \tag{2.26}$$

The modulated time-domain signal is

$$x_n(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_n(k) e^{j2\pi mk/N}, \quad 0 \le \mathrm{m} \le \mathrm{N-1} \tag{2.27}$$

A cyclic prefix of length gi i.e. $\mathbf{x}_n{}^f = \left[x_n(0) x_n(1) \ldots x_n(gi-1)\right]^T$ is constructed by

$x_n(m) = x_n(N+m),\ 0 \le \mathrm{m} \le gi-1$. The CP is appended to form the transmitted vector as

$$\mathbf{x}'_n = \left[x_n(0) x_n(1) \ldots x_n(gi-1) \vdots x_n(gi) x_n(gi+1) \ldots x_n(gi+N-1)\right]^T \tag{2.28}$$

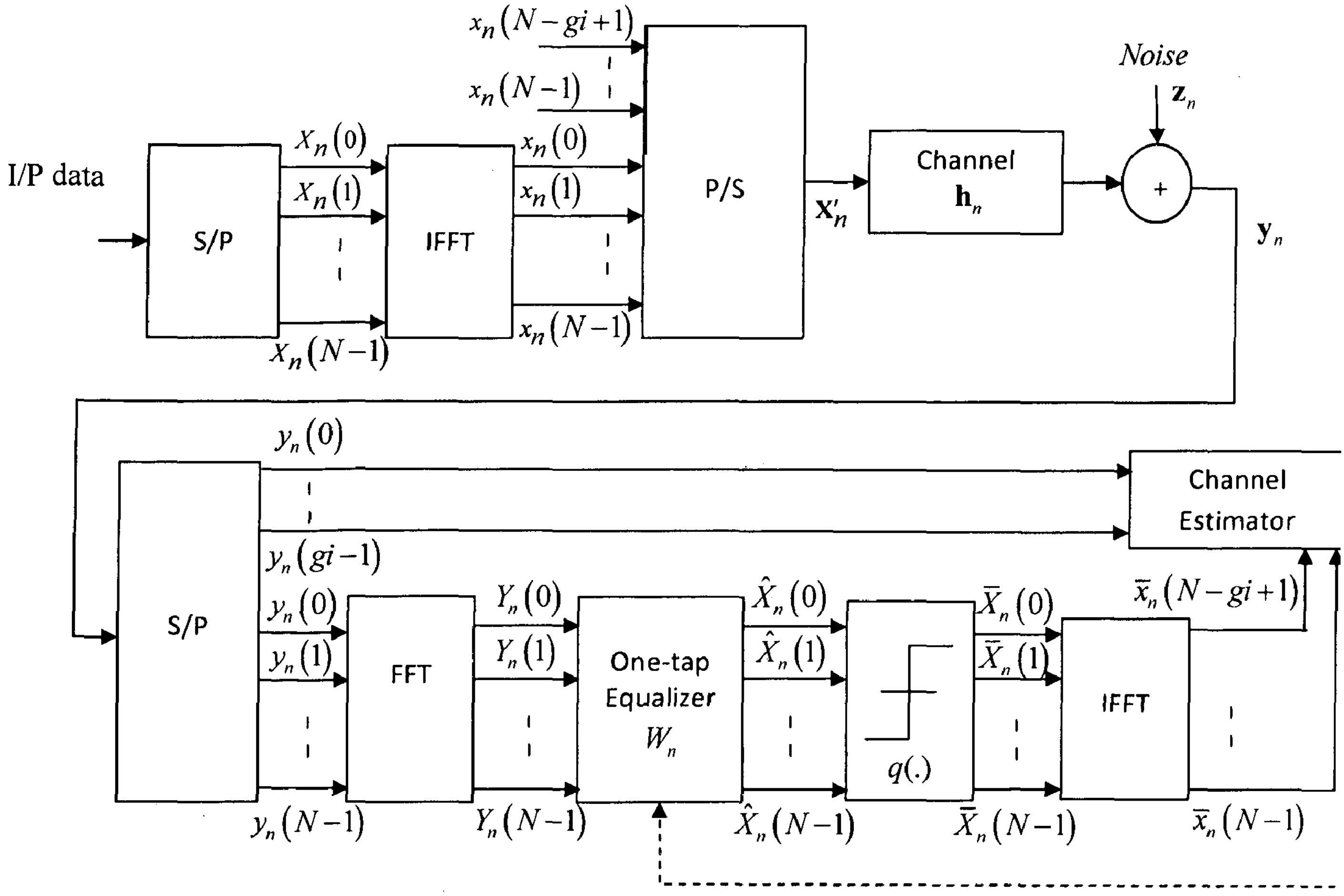

Fig. 2.2. OFDM system with cyclic prefix and adaptive channel estimation

The channel we may define as an L tap vector for each OFDM symbol as in Eq. (2.17) i.e.,

$$\mathbf{h}_n = [h_n(0) h_n(1) \ldots h_n(L-1)]^T$$

where $h_n(l)$ is the l^{th} channel tap of the n^{th} OFDM symbol.

The channel noise $z_n(m)$ is assumed to be independent identically distributed (i.i.d.) real Gaussian distribution with zero mean and variance σ^2 at instant m in the n^{th} OFDM symbol.

The received symbol corrupted by fading channel and AWGN becomes

$$y_n(m) = \sum_{l=0}^{L-1} h_n(l) x_n(m-l) + z_n(m), \quad 0 \le \mathrm{m} \le N + gi + L - 1 \tag{2.29}$$

At the receiver, the prefix part $\mathbf{y}_n^f = [y_n(0) y_n(1) \ldots y_n(gi-1)]^T$ is discarded. The demodulation is performed only on $\mathbf{y}_n = [y_n(0) y_n(1) \ldots y_n(N-1)]^T$ by the N-point DFT operation. The demodulated received vector is

$$\mathbf{Y}_n = [Y_n(0)\ Y_n(1) \ldots Y_n(N-1)]^T \tag{2.30}$$

It may be seen from the Eq. (2.29) that there is no interference from the previous blocks in the received signal $\mathbf{y}_n$. It shows that the cyclic prefix reduces ISI between $\mathbf{X}_n$'s and the subchannels can be viewed as independent with each other, i.e., [9]

$$Y_n(k) = X_n(k) H_n(k) + Z_n(k) \tag{2.31}$$

where $H_n(k)$ is the channel frequency response at the subcarrier k given by

$$H_n(k) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} h_n(l) e^{-j2\pi lk/N}, \quad 0 \le k \le N-1 \tag{2.32}$$

and $Z_n(k)$ is the noise on k^{th} subcarrier of n^{th} OFDM symbol i.e.,

$$Z_n(k) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} z_n(m) e^{-j2\pi mk/N}, \quad 0 \le k \le N-1 \tag{2.33}$$

For the independent subchannel of Eq. (2.31), only a one-tap equalizer $W_n(k)$ as shown in Fig. 2.2 is needed to get the estimation of $X_n(k)$ from $Y_n(k)$, i.e., [9]

$$\hat{X}_n(k) = Y_n(k).W_n(k) \tag{2.34}$$

where

$$W_n(k) = \frac{1}{H_n(k)} \tag{2.35}$$

Then the decision is made upon $\hat{X}_n(k)$, resulting in $\bar{X}_n(k) = q(\hat{X}_n(k))$, where $q(.)$ is some type of quantization function (decision operation). Then the decoding and de-interleaving are done based on $\bar{X}_n(k)$.

2.4.2 Adaptive Filtering for Channel Estimation

In this section we show that by using cyclic prefix, retraining is not necessary in the MCM systems to track the channel variations. Let's first consider the prefix part $\mathbf{y}_n^f$ which is originally discarded. The relation between $\mathbf{y}_n^f$ and the transmitted signal is [8]

$$\mathbf{y}_n^f = \mathbf{A}_n \mathbf{h}_n + \mathbf{z}_n^f \tag{2.36}$$

Where $\mathbf{A}_n = \begin{bmatrix} x_n(0) & x_{n-1}(N+gi-1) & \cdots & x_{n-1}(N+gi-L+1) \\ x_n(1) & x_n(0) & x_{n-1}(N+gi-1) & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ x_n(gi-1) & x_n(gi-2) & \cdots & x_n(gi-L) \end{bmatrix}$ and

$\mathbf{z}_n^f = [z_n(0)\ z_n(1) \ldots z_n(gi-1)]^T$

The lower triangle part of matrix $\mathbf{A}_n$ is composed by $\mathbf{x}_n^f$, while the upper triangle part is composed by last $(gi-1)$ samples of $\mathbf{x}_{n-1}$ (previous symbol). However, the last $(gi-1)$ samples of $\mathbf{x}_{n-1}$ are also the elements of the prefix $\mathbf{x}_{n-1}^f$. Hence, if all cyclic prefix parts concatenate together to form a pair of sequence

$\{x_m^f\} = \{\ldots x_{n-1}(0)\, x_{n-1}(1) \ldots x_{n-1}(gi-1)\, x_n(0) \ldots x_n(gi-1) \ldots\}$ and

$\{y_m^f\} = \{\ldots y_{n-1}(0)\, y_{n-1}(1) \ldots y_{n-1}(gi-1)\, y_n(0) \ldots y_n(gi-1) \ldots\}$, the relation between these two satisfies Eq. (2.36), based on this equation, an adaptive channel estimation algorithm is needed to estimate the channel $\hat{\mathbf{h}}_n$ by solving Eq. (2.36).

The receiver operates in training and decision directed modes. During training, the CP of the transmitted symbols $\mathbf{x}_n^f$ is assumed to be known at the receiver. The known transmitted CP and the CP part of the received OFDM symbol $\mathbf{y}_n^f$ form input to the channel estimation block. This gives the estimated channel vector ($\mathbf{h}_n$) for the OFDM symbol under consideration, which is used to equalize and demodulate the received symbol. In the decision directed mode, the receiver uses the estimated channel vector from the previous OFDM symbol ($\hat{\mathbf{h}}_{n-1}$) to demodulate the received symbol and generate an estimate of the transmitted CP. This estimated CP, together with the CP of received

OFDM symbol, helps the channel estimator to provide an improved channel estimate. Using the CP-based model (as discussed above) for channel estimation in OFDM systems, a variety of adaptive filtering algorithms, namely Kalman filter (KF) and H-infinity filter (HF) are considered in chapter (4).

Chapter 3

Adaptive Filtering Algorithms for Autoregressive Model Parameter Estimation

3.1 Introduction

In numerous applications of signal processing and communication we are faced with the necessity to remove noise and distortion from signals. These phenomena are due to time-varying physical processes, which sometimes are unknown. One of these situations is during the transmission of the signal (message) from one point to another. The medium which is unknown, introduces noise and distortion due to the variations of its properties. These variations may be slow or fast varying. Since most of the time the variations are unknown, it is the use of the "adaptive filtering" that diminishes and sometimes completely reduces the signal distortion.

An adaptive filter is a filter that self-adjusts its transfer function according to an optimizing algorithm. Because of the complexity of optimizing algorithms, most adaptive filters are digital filters that perform digital signal processing and adapt their performance based on the input signal. For some applications, adaptive coefficients are required since some parameters of the desired processing operation (for instance, the properties of some noise signal) are not known in advance. In these situations it is common to employ an adaptive filter, which uses feedback to refine the values of the filter coefficients and hence its frequency response (Fig 3.1) [12].

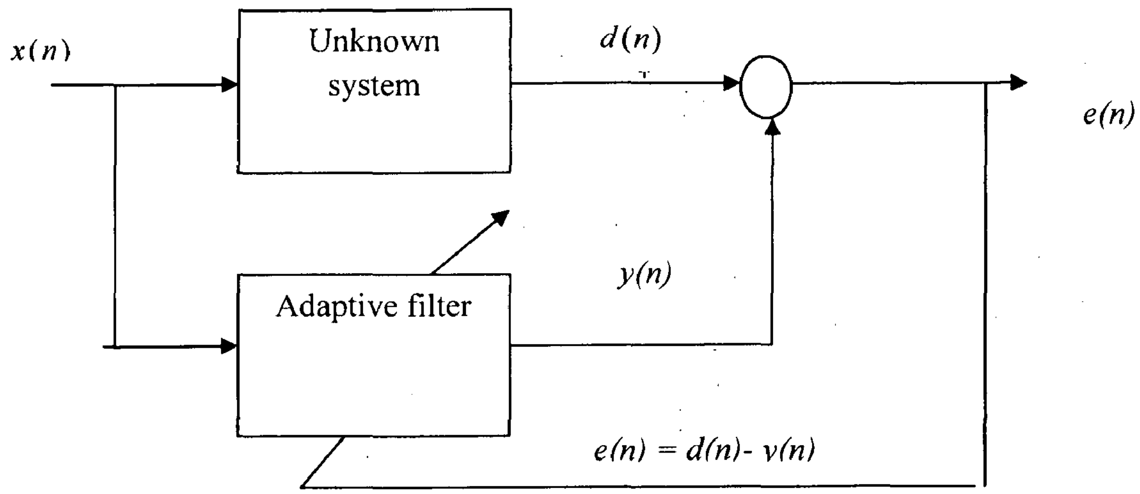

Fig. 3.1. A generic adaptive filter

Generally speaking, the adapting process involves the use of a cost function, which is a criterion for optimum performance of the filter (for example, minimizing the noise component of the input or error), to feed an algorithm, which determines how to modify the filter coefficients to minimize the cost on the next iteration. We need a filter that could handle modeling errors and noise uncertainty, estimators that can tolerate such uncertainty (robustness) gives optimal state estimation, for that we require adaptive filtering algorithms.

In this chapter we first present the Autoregressive model (AR) and then we consider the mathematical models of four different (Kalman, Dula-Kalman, H-infinity and Dual H-infinity) algorithms for estimation of AR process of order 2. We discuss the comparative study of different parameters for different algorithms. Finally simulation results are presented.

3.2 Autoregressive Model

In statistics and signal processing, an autoregressive (AR) model is often used to predict various types of natural phenomena. It is one of a group of linear prediction formulas that attempt to predict a state of the system based on the previous states. In general the time domain description of the input-output relation for the stochastic model can be described as:

(Present value of the input) + (linear combination of past values of model output)

= (linear combination of present and past values of model input)

In the AR modeling, the past values of the model input are not considered and so the model can be defined by the difference equation [12]

$$s(k)+a_1 s(k-1)+a_2 s(k-2)+\ldots+a_p s(k-p)=u(k)$$

Where the time series $s(k), s(k-1), \ldots\ldots s(k-p)$ represents the realization of an autoregressive process (AR) of order 'p'([12],[19]). $a_1, a_2, \ldots\ldots a_p$ are constants called the AR parameters and $u(k)$ is a white noise which is excitation to the system. To specify an AR model the parameters to be specified are the filter coefficients and the variance of the excitation signal. For a given set of signal samples, these weights (AR parameters) can be evaluated by solving the Yule-Walker set of equations (derived from the autocorrelation of the process) [12].

This AR model enables us to predict the tap gain process independently of the data based estimator. The system equation from this model and the observation equation jointly form a state-space representation of the dynamics of the tap-gain process. This state-space representation of the overall process is used to formulate the parameters of a Kalman filtering or H_∞ filtering as discussed below. This model is used in a broad range of applications. It plays a key role in speech processing such as analysis, coding and enhancement [22]. In the framework of biomedical engineering, AR spectral analysis is a suitable technique.

3.3 Kalman Filter

Kalman filter is a linear filter and it can be applied to stationary and non-stationary environments without any modification and its solution can be computed recursively. In particular, each updated estimate of the state is computed from the previous estimate and the new input data, so only previous estimate requires storage. In addition, to eliminate the need for storing the entire past observed data, the Kalman filter is computationally more efficient than computing the estimate directly from all of those past data at each step of the filtering process. A distinctive use of Kalman filter is that its mathematical formulation is described in terms of state space concepts i.e., a state space model based on the auto-regressive (AR) parameters model.

The signal $s(k)$ can be modeled by a pth order AR process defined by [12]

$$s(k) = -\sum_{i=1}^{p} a_i s(k-i) + u(k) \tag{3.1}$$

where the so-called driving processing $u(k)$ is assumed zero-mean white Gaussian with variance σ_u^2. The 'a_i' are the AR parameters.

However, in real cases, the signal $s(k)$ is often corrupted by an additive noise $v(k)$ white or coloured, yielding the following noisy observations

$$y(k) = s(k) + v(k) \tag{3.2}$$

Our primary aim is to estimate the signal s(k) modeled by a p^{th} order AR process. For this we can first have the state space representation of the above equations (3.1) & (3.2) as [21]

$$\mathbf{x}(k) = \phi(k)\mathbf{x}(k-1) + \Gamma u(k) \tag{3.3}$$
$$y(k) = \boldsymbol{H}\mathbf{x}(k) + v(k)$$

where $\mathbf{x}(k)$ is the state vector defined as:

$$\mathbf{x}(k) = [s(k) \quad s(k\text{-}1) \quad \ldots s(k\text{-}p\text{+}1)]^{\mathrm{T}}$$

u(k) corresponds to the error during the estimation of the AR parameters i.e. the process noise and *v(k)* is the observation noise.

$\phi(k)$ is the transition matrix which is constructed from the AR parameters as

$$\phi(k) = \begin{bmatrix} -a_1 & \cdots & \cdots & -a_p \\ 1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\boldsymbol{H}$ and $\boldsymbol{\Gamma}$ are the observation vector and the input vector respectively and are defined as follows

$\boldsymbol{H} = \boldsymbol{\Gamma}^{\mathrm{T}} = [1\ 0\ 0 \ldots 0]$ (**H** has dimensions [1x p])

The solution of the state space model by means of Kalman filtering algorithm gives us the estimate of the signal as

$$\hat{s}(k/k) = \boldsymbol{H}\,\hat{\mathbf{x}}(k/k) \tag{3.4}$$

In the algorithm we use the following values in the intermediate steps:

$\alpha(k)$: the innovation sequence or more generally the noise present in the input.

C(k) : the covariance matrix of *v(k)*.

P(k/k-1) : the priori error covariance matrix

P(k/k) : the current error covariance matrix.

$K(k)$: the Kalman gain.

The posterior estimate of the state vector can be written as [20],[21]

$$\hat{\mathbf{x}}(k \mid k) = \phi(k)\hat{\mathbf{x}}(k-1/k-1) + \boldsymbol{K}(k)\alpha(k) \tag{3.5}$$

This is the actual estimate of the filtered signal that we make from the noisy signal input.

The innovation sequence is defined as

$$\alpha(k) = y(k) - \boldsymbol{H}\phi(k)\hat{\mathbf{x}}(k-1 \mid k-1) \tag{3.6}$$

its covariance matrix is defined as:

$$\boldsymbol{C}(k) = \boldsymbol{H}\boldsymbol{P}(k \mid k-1)\boldsymbol{H}^T + \sigma_v^2 \tag{3.7}$$

The Kalman gain is defined as:

$$\boldsymbol{K}(k) = \boldsymbol{P}(k/k-1)\boldsymbol{H}^T\boldsymbol{C}(k)^{-1} \tag{3.8}$$

The priori error covariance matrix ***P(k/k-1)*** is defined as:

$$\boldsymbol{P}(k \mid k-1) = \boldsymbol{\phi}(k)\boldsymbol{P}(k-1 \mid k-1)\boldsymbol{\phi}(k)^T + \Gamma\sigma_u^2\Gamma^T \tag{3.9}$$

and the current error covariance matrix ***P(k/k-1)*** is defined as:

$$\boldsymbol{P}(k \mid k) = [\mathbf{I}_p - \boldsymbol{K}(k)\boldsymbol{H}]\boldsymbol{P}(k \mid k-1) \tag{3.10}$$

Kalman filters give a very accurate estimate of the original signal from the noisy data provided we have the proper AR parameters available with us. But in real cases we cannot have them before we have the signal. But this signal is generally corrupted by noise. So it becomes important that we use methods to obtain the AR parameters from the noisy signal and then use them to obtain the estimate of original signal. This means that the original problem of signal estimation now becomes the problem of signal and parameter estimation. This leads to joint parameter and signal estimation problem, also referred to the dual-estimation problem.

3.4 Dual Kalman Filter

Kalman Filter gives the best linear estimate of a process under Minimum Mean Square Error (MMSE) criterion. But it requires the knowledge of AR parameters of signal beforehand. But it is not possible practically, to always have apriori knowledge of AR parameters. For the AR parameter estimation, we can use another Kalman Filter to estimate it from the noisy signal [21]. The underlying principle behind using the dual Kalman filters instead of Kalman filtering revolves around the dependence of the autoregressive parameters on the original signal samples.

Dual-Kalman filter involves two interacting Kalman filters working in parallel. Indeed, each time a new observation is available, the signal is estimated using the latest estimated value of the parameters, and conversely the parameters are estimated using the latest a posterior signal estimate. In case of AR modeling, the AR parameters actually define as $\boldsymbol{\theta} = [a_1 \;\; a_2 \ldots\ldots a_p]^T$ from the noisy observations $s(k)$. Also from equations (3.4) and (3.5) we can represent the signal estimate as a function of $\boldsymbol{\theta}$ (AR parameter vector).

Here $\hat{s}(k/k) = \boldsymbol{H}\hat{\mathbf{x}}(k/k)$

where $\hat{\mathbf{x}}(k \mid k) = \phi(k)\hat{\mathbf{x}}(k-1/k-1) + \boldsymbol{K}(k)\alpha(k)$

we get

$$\hat{s}(k \mid k) = \boldsymbol{H}(k)[\phi(k)\hat{\mathbf{x}}(k-1 \mid k-1) + \boldsymbol{K}(k)\alpha(k)]$$

$$= -\hat{\mathbf{x}}(k-1 \mid k-1)^T \boldsymbol{\theta} + \boldsymbol{HK}(k)\alpha(k)$$

$$= -\hat{\mathbf{x}}(k-1 \mid k-1)^T \boldsymbol{\theta} + v_\theta(k) \qquad (3.11)$$

When the signal is assumed to be stationary, the AR parameters are time invariant and satisfy the following relationship:

$$\boldsymbol{\theta}(k) = \boldsymbol{\theta}(k-1) \qquad (3.12)$$

Therefore using a second Kalman filter in parallel, denoted KF2 in Fig. 3.2 makes it possible to estimate the AR parameters from the noisy observations. The corresponding state-space representation is based on Eq. (3.11) & Eq. (3.12) and is given by [21]

$$\begin{aligned} \theta(k) &= \theta(k-1) \\ \hat{s}(k \mid k) &= \boldsymbol{H}_\theta(k)\theta(k) + v_\theta(k) \end{aligned} \tag{3.13}$$

where $v_\theta(k) = \boldsymbol{HK}(k)\alpha(k)$

and $\quad \boldsymbol{H}_\theta(k) = -\hat{\mathbf{x}}(k-1 \mid k-1)^T$ (3.14)

Using the covariance matrix of $\alpha(k)$ and the expression of $v_\theta(k)$, we can write the variance for the $v_\theta(k)$ as

$$\begin{aligned} \boldsymbol{R}_\theta &= E[v_\theta(k)v_\theta^{\,T}(k)] \\ &= E[\boldsymbol{HK}(k)\alpha(k)\alpha^T(k)\boldsymbol{K}^T(k)\boldsymbol{H}^T] \\ &= \boldsymbol{HK}(k)E[\alpha(k)\alpha^T(k)]\boldsymbol{K}^T(k)\boldsymbol{H}^T \end{aligned}$$

$$\boldsymbol{R}_\theta = \boldsymbol{HK}(k)\boldsymbol{C}(k)\boldsymbol{K}(k)^T\boldsymbol{H}^T \tag{3.15}$$

where

$$\boldsymbol{C}(k) = E[\alpha(k)\alpha^T(k)]$$

The implementation of Dual-Kalman filter can be represented as a flowchart in Fig. 3.2.

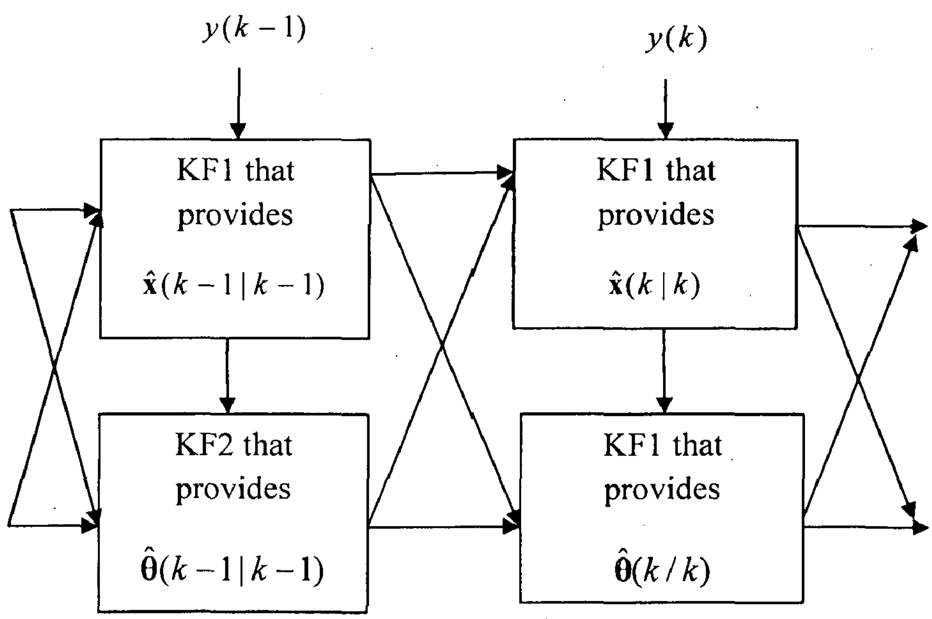

Fig. 3.2. Dual Kalman Filter

Apart from estimating the AR parameters, we also need to estimate the noise parameters i.e. variance of $u(k)$ and $v(k)$. This can be done by using the error covariance matrices. From Eq. (3.9) and Eq. (3.10) we can write:

$$\boldsymbol{P}(k\,|\,k)=[\boldsymbol{\phi}(k)\boldsymbol{P}(k-1\,|\,k-1)\boldsymbol{\phi}(k)^{T}+\Gamma\sigma_{u}^{2}\Gamma^{T}-\boldsymbol{K}(k)\boldsymbol{H}\boldsymbol{\phi}(k)\boldsymbol{P}(k-1\,|\,k-1)\boldsymbol{\phi}(k)^{T}-\boldsymbol{K}(k)\boldsymbol{H}\Gamma\sigma_{u}^{2}\Gamma^{T}]$$

Since the innovation variance $\boldsymbol{C}(k)$ is a scalar and the priori covariance matrix $\boldsymbol{P}(k\,|\,k-1)$ is real and symmetric, Eq. (3.8) can be rewritten in the following form:

$$\boldsymbol{HP}(k\,|\,k-1)=\boldsymbol{C}(k)\boldsymbol{K}(k)^{T}$$

From the above two equations $\boldsymbol{P(k/k)}$ can be written as

$$\boldsymbol{P}(k\,|\,k)=[\boldsymbol{\phi}(k)\boldsymbol{P}(k-1\,|\,k-1)\boldsymbol{\phi}(k)^{T}+\Gamma\sigma_{u}^{2}\Gamma^{T}-\boldsymbol{K}(k)\boldsymbol{HP}(k\,|\,k-1)]$$

$$[\boldsymbol{P}(k\,|\,k)-\boldsymbol{\phi}(k)\boldsymbol{P}(k-1\,|\,k-1)\boldsymbol{\phi}(k)^{T}+\boldsymbol{K}(k)\boldsymbol{HP}(k\,|\,k-1)]=\Gamma\sigma_{u}^{2}\Gamma^{T}$$

$$\Gamma^{-1}[\boldsymbol{P}(k\,|\,k)-\boldsymbol{\phi}(k)\boldsymbol{P}(k-1\,|\,k-1)\boldsymbol{\phi}(k)^{T}+\boldsymbol{K}(k)\boldsymbol{HP}(k\,|\,k-1)](\Gamma^{-1})^{T}=\sigma_{u}^{2}$$

Therefore, from the above equation, one can express variance of process noise [u(k)] can be written as

$$\sigma_{u}^{2}=\boldsymbol{D}\{\boldsymbol{P}(k\,|\,k)-\boldsymbol{\phi}(k)\boldsymbol{P}(k-1\,|\,k-1)\boldsymbol{\phi}(k)^{T}+\boldsymbol{K}(k)\boldsymbol{C}(k)\boldsymbol{K}(k)^{T}\}\boldsymbol{D}^{T} \qquad (3.16)$$

where $\boldsymbol{D}=[\Gamma^{T}\Gamma]^{-1}\Gamma^{T}$ is the pseudo inverse of Γ.

Using this, we can write a recursive equation to get an estimate of variance σ_{u}^{2} as follows

$$\hat{\sigma}_{u}^{2}(k)=\frac{k-1}{k}\hat{\sigma}_{u}^{2}(k-1)+\frac{1}{k}\boldsymbol{DL}(k)\boldsymbol{D}^{T} \qquad (3.17)$$

where $\boldsymbol{L}(k)=\boldsymbol{P}(k/k)-\boldsymbol{\phi}(k)\boldsymbol{P}(k-1/k-1)\boldsymbol{\phi}(k)^{T}+\boldsymbol{K}(k)\alpha^{2}(k)\boldsymbol{K}(k)^{T}$

Similarly variance of $v(k)$ can be estimated using Eq. (3.7) as

$$\boldsymbol{C}(k)=\boldsymbol{HP}(k\,|\,k-1)\boldsymbol{H}^{T}+\sigma_{v}^{2}$$

$$\sigma_{v}^{2}=\boldsymbol{C}(k)-\boldsymbol{HP}(k\,|\,k-1)\boldsymbol{H}^{T}$$

where the variance of the innovation process $C(k)$ is replaced by its instantaneous value $\alpha^2(k)$,by using this, we can write a recursive equation to get an estimate of variance σ_v^2 as follows

$$\hat{\sigma}_v^2(k) = \frac{k-1}{k}\hat{\sigma}_v^2(k-1) + \frac{1}{k}M(k) \tag{3.18}$$

where $M(k) = \alpha^2(k) - \boldsymbol{HP}(k/k-1)\boldsymbol{H}^T$

The equations of the "Dual-Kalman Filtering" algorithm are summarized as follows [21]

1. Signal Estimation

$$\hat{\mathbf{x}}(k \mid k-1) = \phi(k)\hat{\mathbf{x}}(k-1/k-1)$$

$$\boldsymbol{P}(k \mid k-1) = \phi(k)\boldsymbol{P}(k-1 \mid k-1)\phi(k)^T + \Gamma\sigma_u^2\Gamma^T$$

$$\alpha(k) = y(k) - \boldsymbol{H}\phi(k)\hat{\mathbf{x}}(k-1 \mid k-1)$$

$$\boldsymbol{C}(k) = \boldsymbol{HP}(k \mid k-1)\boldsymbol{H}^T + \sigma_v^2$$

$$\boldsymbol{K}(k) = \boldsymbol{P}(k/k-1)\boldsymbol{H}^T\boldsymbol{C}(k)^{-1}$$

$$\hat{\mathbf{x}}(k \mid k) = \phi(k)\hat{\mathbf{x}}(k-1/k-1) + \boldsymbol{K}(k)\alpha(k)$$

$$\boldsymbol{P}(k \mid k) = [\mathbf{I}_p - \boldsymbol{K}(k)\boldsymbol{H}]\boldsymbol{P}(k \mid k-1)$$

2. Auto regressive model parameter estimation

$$\boldsymbol{H}_\theta(k) = -\hat{\mathbf{x}}(k-1 \mid k-1)^T$$

$$v_\theta(k) = \boldsymbol{HK}(k)\alpha(k)$$

$$\boldsymbol{R}_\theta = \boldsymbol{HK}(k)\boldsymbol{C}(k)\boldsymbol{K}(k)^T\boldsymbol{H}^T$$

$$\boldsymbol{C}_\theta(k) = [\boldsymbol{H}_\theta\boldsymbol{P}_\theta(k-1/k-1)\boldsymbol{H}_\theta^T + \boldsymbol{R}_\theta(k)]^{-1}$$

$$\boldsymbol{K}_\theta(k) = \boldsymbol{P}_\theta(k-1/k-1)\boldsymbol{H}_\theta^T\boldsymbol{C}_\theta(k)^{-1}$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K_\theta(k)v_\theta(k)$$

$$P_\theta(k/k) = [\mathbf{I}_P - K_\theta(k)H_\theta]P_\theta(k/k-1)$$

3. Noise parameters estimation

$$L(k) = P(k/k) - \phi(k)P(k-1/k-1)\phi(k)^T + K(k)\alpha^2(k)K(k)^T$$

$$D = [\Gamma^T\Gamma]^{-1}\Gamma^T = [1\ 0 \ldots.\ 0]$$

$$\hat{\sigma}_u^2(k) = \frac{k-1}{k}\hat{\sigma}_u^2(k-1) + \frac{1}{k}DL(k)D^T$$

$$M(k) = \alpha^2(k) - HP(k/k-1)H^T$$

$$\hat{\sigma}_v^2(k) = \frac{k-1}{k}\hat{\sigma}_v^2(k-1) + \frac{1}{k}M(k)$$

The Kalman filtering approach gives the optimal solution when the noise characteristics are known a priori and the estimation problem is Gaussian. But in many situations noise may not be Gaussian, one of the techniques for handling such situations uses H_∞ filtering approach.

3.5 H-infinity Filter

The objective of H_∞ filtering estimators is to minimize the maximum energy gain from the disturbances to the estimation errors. This will guarantee that if the disturbances are small (in energy) then the estimation errors will be as small as possible (in energy), no matter what the disturbances are. In other words the maximum energy gain is minimized over all possible disturbances. Since they make no assumption about the disturbances, they have to accommodate for all conceivable disturbances, and are thus over-conservative. Let $T(f)$ denotes the transfer operator that maps the disturbances at the input of the recursive estimation strategy to estimation errors at the output [12]

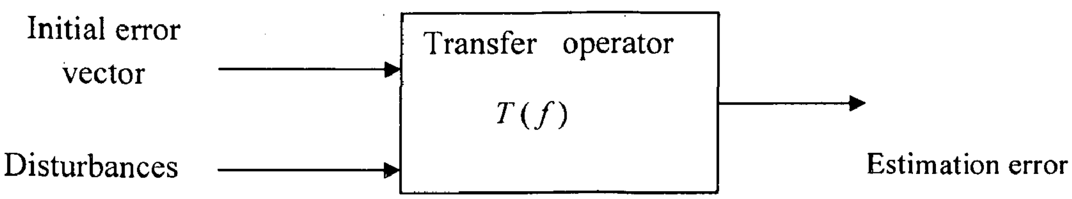

Fig.3.3. Transfer operator from disturbances to output prediction error.

We may then define the energy gain of the estimator as the ratio of the error energy at the output to the total disturbance energy at the input. Clearly, the ratio depends on the particular choice of input disturbances. To remove this dependence, we consider the largest energy gain over all conceivable disturbance sequence. In so doing we will discuss the H_∞ norm of the transfer operator $T(f)$ and formulate the optimal H_∞ estimation problem as a causal estimator that minimizes the H_∞ norm of $T(f)$, where $T(f)$ is a transfer operator that maps the disturbances to the estimation error. H_∞ optimal estimator that minimizes $\|T(f)\|_\infty$ is

$$J = \|T(f)\|_\infty^2$$

3.5.1 H_∞ Estimation Algorithm

As in the case of Kalman filtering, the H-infinity filtering also uses the state-space model based on the p^{th} order AR process. Here apart from estimating the signal *s(k)* we also use the current estimated value of the state to improve the previous values of the estimate.

From Eq. 3.1& Eq. 3.2 the state of the system for H-infinity filtering can be defined as:

$$\mathbf{x}(k) = \phi(k)\mathbf{x}(k-1) + \Gamma u(k) \tag{3.19}$$

$$y(k) = \boldsymbol{H}\mathbf{x}(k) + v(k)$$

We shall not make any assumptions on the disturbances, the driving process noise [*u(k)*] and the measurement noise [*v(k)*] except that they have finite energy and they may have zero mean. The finite energy assumptions is reasonable since in any practical system, both *u(k)* and *v(k)* are samples of band limited noise process.

Unlike Kalman filter, the H-infinity filter not only deals with the estimation of the state vector **x***(k)*, but also makes it possible to focus on the estimation of a specific linear combination of the state vector components:

$$z(k) = \boldsymbol{H}\mathbf{x}(k) \tag{3.20}$$

where $\boldsymbol{H}$ is a $1 \times p$ liner transformation operator. Here, as we aim at estimating the signal s*(k)*, this operator is defined as ***H***=[1 0 . . .0]. The Eq. (3.19), Eq. (3.20) and Fig. 3.4, the

H-infinity filter provides an estimation of the signal $\hat{s}(k/k) = H\hat{\mathbf{x}}(k/k)$, by minimizing the H-infinity norm of the transfer operator T(f). This operator maps the discrete-time noise disturbances *u(k)* and *v(k)* and the unknown initial state error vector

$e_0 = (\mathbf{x}_0 - \hat{\mathbf{x}}_0)$ to the estimation error $e(k) = H\mathbf{x}(k) - H\hat{\mathbf{x}}(k)$.

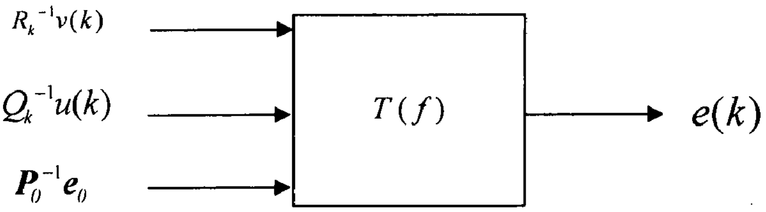

Fig. 3.4. Transfer operator T(f)

For this purpose the standard H-infinity norm (or) objective functions (or) cost function used is [20] ,[22].

$$\|J\|_\infty \stackrel{\Delta}{=} \sup_{v(k),u(k),\mathbf{x}_0} J \tag{3.21}$$

where

$$J \stackrel{\Delta}{=} \frac{\sum_{k=0}^{N-1} \|e(k)\|^2}{e^H{}_0 P_0^{-1} e_0 + \sum_{k=0}^{N-1} \left\{ R_k^{-1} \|v(k)\|^2 + Q_k^{-1} \|u_k\|^2 \right\}} \tag{3.22}$$

with N the number of available samples. In addition, $Q_k > 0 \, and \, R_k > 0$ are weighting parameters, which often correspond to the instantaneous power of the sequences $u(k)$ and $v(k)$, respectively. In practical systems, the values of Q_k and R_k play the role of variances of the noises in the excitation process and the measurement process respectively. Furthermore, $P_0 > 0$ denotes a positive define matrix that reflects a priori knowledge on how small the estimation error $e_0 = (\mathbf{x}_0 - \hat{\mathbf{x}}_0)$ is. These weighting parameters are usually tuned by the designer to achieve performance requirements.

The direct minimization of J is not tractable, so instead a performance bound is choosen and seek an estimation strategy that satisfies the threshold. The following suboptimal design strategy is usually considered [20]

$$\|J\|_{\infty} \stackrel{\Delta}{=} \sup_{v(k),u(k),x_0} J \leq \gamma \tag{3.23}$$

where sup stands for supremum (least upper bound) and γ (>0) is a prescribed level of noise attenuation. From Eqn. (3.16) H_{∞} optimal estimator guarantees the smallest estimation error energy over all possible disturbances with finite energy. H_{∞} Optimal estimator so found is of a minimax nature. The performance criterion can be represented as

$$\min_{\hat{x}(k)} \max_{v_k, u_k, (x_0)} J = \min_{\hat{x}(k)} \left[\max_{v(k),u(k),(x_0)} \left\{ -\frac{1}{2}\gamma^{-1} e^{H}{}_{0} P_0^{-1} e_0 + \frac{1}{2}\sum_{k=0}^{N-1}\left[\|e(k)\|^2 - \gamma^{-1}\left(Q_k^{-1}\|u_k\|^2 + R_k^{-1}\|v_k\|^2 \right)\right]\right\}\right]$$

It is proved in [25] that there exists an H-infinity estimator $\hat{s}(k/k)$ for a given γ >0 if there exists a stabilizing symmetric positive definite solution $P(k)$ to the following discrete-time Riccati type equation [20], [23]

$$P(k) = \phi(k)P(k-1)C(K)^{-1}\phi(k)^T + \Gamma Q_k \Gamma^T \tag{3.24}$$

where

$$C(k) = [\mathbf{I}_p - \gamma^{-1} H^T H P(k-1) + H^T R_k^{-1} H P(k-1)] \tag{3.25}$$

and the innovation process and H-infinity gains are respectively given by

$$\alpha(k) = y(k) - H\phi(k)\hat{x}(k-1 \mid k-1) \tag{3.26}$$

$$K(k) = P(k)C(k)^{-1} H^T R_k^{-1} \tag{3.27}$$

The necessary and sufficient condition for the existence of the H_{∞} estimator is that

$$P(k)C(k)^{-1} > 0 \tag{3.28}$$

If the condition in Eqn. (3.28) if fulfilled, the H-infinity estimator exists and is defined by

$$\hat{\mathbf{x}}(k \mid k) = \phi(k)\hat{\mathbf{x}}(k-1/k-1) + \boldsymbol{K}(k)\alpha(k) \tag{3.29}$$

$$\hat{s}(k/k) = \boldsymbol{H}\hat{\mathbf{x}}(k/k) \tag{3.30}$$

The equations of the "H-infinity Filtering" algorithm are summarized as follows [20], [23]

$$\hat{\mathbf{x}}(k \mid k-1) = \phi(k)\hat{\mathbf{x}}(k-1/k-1)$$

$$\alpha(k) = y(k) - \boldsymbol{H}\phi(k)\hat{\mathbf{x}}(k-1 \mid k-1)$$

$$\boldsymbol{C(k)} = [\mathbf{I}_p - \gamma^{-1}\boldsymbol{H}^T\boldsymbol{HP}(k-1) + \boldsymbol{H}^T R_k^{-1}\boldsymbol{HP}(k-1)]$$

$$\boldsymbol{K}(k) = \boldsymbol{P}(k-1)\boldsymbol{C}(k)^{-1}\boldsymbol{H}^T R_k^{-1}$$

$$\hat{\mathbf{x}}(k \mid k) = \phi(k)\hat{\mathbf{x}}(k-1/k-1) + \boldsymbol{K}(k)\alpha(k)$$

$$\hat{s}(k/k) = \boldsymbol{H}\hat{\mathbf{x}}(k/k)$$

$$\boldsymbol{P(k)} = \phi(k)\boldsymbol{P}(k-1)\boldsymbol{C(K)}^{-1}\phi(k)^T + \Gamma Q_k \Gamma^T \qquad [\boldsymbol{P}(0) = \boldsymbol{P}_0 = \mathbf{I}_P]$$

H-infinity filter is the robust estimation criterion that minimizes the worst possible disturbances of the estimation error.

3.6 Dual H-infinity Filter

For the joint estimation of both signal and its AR parameters, we require another H-infinity filter. For this joint estimation coupling filter based approaches can be considered. This method makes it possible to provide robust estimation of the signal and its AR parameters. To estimate the AR parameters $\boldsymbol{\theta}(k)=[a_1\ a_2\ .\ .\ .\ a_P]^T$ from the estimated signal $\hat{s}(k \mid k)$ we use Eq. (3.29) & Eq. (3.30) to express $\hat{s}(k \mid k)$ as a function of AR parameters as

$$\boldsymbol{\theta}(k) = \boldsymbol{\theta}(k-1)$$

$$\hat{s}(k \mid k) = \boldsymbol{H}_\theta(k)\boldsymbol{\theta}(k) + v_\theta(k)$$

where

$$v_\theta(k) = \boldsymbol{HK}(k)\alpha(k)$$

and $\boldsymbol{H}_\theta(k) = -\hat{\mathbf{x}}(k-1 \mid k-1)^T$

By defining the Autoregressive parameters estimation error as $\boldsymbol{e}_\theta(k) = \hat{\mathbf{x}}(k-1 \mid k-1)\boldsymbol{\theta}(k) - \hat{\mathbf{x}}(k-1 \mid k-1)\hat{\boldsymbol{\theta}}(k)$, a second H-infinity can be used to recursively estimated $\boldsymbol{\theta}(k)$ as follows:

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \boldsymbol{K}_\theta(k)\alpha_\theta(k) \tag{3.31}$$

where

$$\alpha_\theta(k) = \hat{s}(k \mid k) - \hat{\mathbf{x}}(k-1 \mid k-1)\hat{\boldsymbol{\theta}}(k-1) \tag{3.32}$$

$$\boldsymbol{C}_\theta(k) = [\mathbf{I}_P - \gamma^{-1}\boldsymbol{H}_\theta{}^T\boldsymbol{H}_\theta\boldsymbol{P}_\theta(k-1) + \boldsymbol{H}_\theta{}^T\boldsymbol{R}_{v_\theta}{}^{-1}\boldsymbol{H}_\theta\boldsymbol{P}_\theta(k-1)] \tag{3.33}$$

$$\boldsymbol{K}_\theta(k) = \boldsymbol{P}_\theta(k-1)\boldsymbol{C}_\theta(k)^{-1}\boldsymbol{H}_\theta{}^T\boldsymbol{R}_{v_\theta}{}^{-1} \tag{3.34}$$

$$\boldsymbol{P}_\theta(k) = \boldsymbol{P}_\theta(k-1)\boldsymbol{C}_\theta(k)^{-1} \tag{3.35}$$

Apart from estimating the AR parameters, we also need to estimate the noise parameters. This can be done by using the Riccati equation From Eq. (3.24) we can write:

$$P(k) = \phi(k)P(k-1)C(k)^{-1}\phi(k)^T + \Gamma Q_k \Gamma^T$$

$$P(k) - \phi(k)P(k-1)C(k)^{-1}\phi(k)^T = \Gamma Q_k \Gamma^T$$

$$(\Gamma)^{-1}[P(k) - \phi(k)P(k-1)C(k)^{-1}\phi(k)^T](\Gamma^{-1})^T = Q_k$$

Therefore, from the above equation, we can write a recursive equation to get an estimate of variance Q_k as follows

$$\hat{Q}_k(k) = \frac{k-1}{k}\hat{Q}_k(k-1) + \frac{1}{k}DL(k)D^T \tag{3.36}$$

where

$$L(k) = [P(k) - \phi(k)P(k-1)C(k)^{-1}\phi(k)^T]$$

and $D = [\Gamma^T\Gamma]^{-1}\Gamma^T$ is the pseudo inverse of Γ.

Similarly variance of $v_\theta(k)$ can be estimated as:

$$R_{v_\theta} = HK(k)\alpha^2(k)K(k)^T H^T \tag{3.37}$$

we can write a recursive equation to get an estimate of R_{v_θ} as follows

$$\hat{R}_{v_\theta}(k) = \frac{k-1}{k}\hat{R}_{v_\theta}(k-1) + \frac{1}{k}DM(k)D^T \tag{3.38}$$

where

$$M(k) = HK(k)\alpha^2(k)K(k)^T H^T$$

The equations of the "Dual-H-infinity Filtering" algorithm are summarized as follows [23], [24]

1. Signal Estimation

$$\hat{x}(k|k-1) = \phi(k)\hat{x}(k-1/k-1)$$

$$\alpha(k) = y(k) - H\phi(k)\hat{\mathbf{x}}(k-1 \mid k-1)$$

$$C(k) = [\mathbf{I}_P - \gamma^{-1} H^T HP(k-1) + H^T R_k^{-1} HP(k-1)]$$

$$K(k) = P(k-1)C(k)^{-1} H^T R_k^{-1}$$

$$\hat{\mathbf{x}}(k \mid k) = \phi(k)\hat{\mathbf{x}}(k-1/k-1) + K(k)\alpha(k)$$

$$P(k) = \phi(k)P(k-1)C(K)^{-1}\phi(k)^T + \Gamma Q_k \Gamma^T \qquad [P(0) = P_0 = \mathbf{I}_P]$$

2. Auto regressive model parameter estimation

$$H_\theta(k) = -\hat{\mathbf{x}}(k-1 \mid k-1)^T$$

$$v_\theta(k) = HK(k)\alpha(k)$$

$$\alpha_\theta(k) = \hat{s}(k \mid k) - \hat{\mathbf{x}}(k-1 \mid k-1)\hat{\mathbf{\theta}}(k-1)$$

$$C_\theta(k) = [\mathbf{I}_P - \gamma^{-1} H_\theta^T H_\theta P_\theta(k-1) + H_\theta^T R_{v_\theta}^{-1} H_\theta P_\theta(k-1)]$$

$$K_\theta(k) = P_\theta(k-1)C_\theta(k)^{-1} H_\theta^T R_{v_\theta}^{-1}$$

$$\hat{\mathbf{\theta}}(k) = \hat{\mathbf{\theta}}(k-1) + K_\theta(k)\alpha_\theta(k) , \qquad [\hat{\mathbf{\theta}}(0) = \mathbf{0}]$$

$$P_\theta(k) = P_\theta(k-1)C_\theta(k)^{-1} \qquad [P_\theta(0) = \mathbf{I}_P]$$

3. Noise parameters estimation

$$L(k) = [P(k) - \phi(k)P(k-1)C(k)^{-1}\phi(k)^T]$$

$$D = [\Gamma^T\Gamma]^{-1}\Gamma^T = [1 \ \ 0 \ldots. \ 0]$$

$$\hat{Q}_k(k) = \frac{k-1}{k}\hat{Q}_k(k-1) + \frac{1}{k} DL(k)D^T$$

$$M(k) = HK(k)\alpha^2(k)K(k)^T H^T$$

$$\hat{R}_{v_\theta}(k) = \frac{k-1}{k}\hat{R}_{v_\theta}(k-1) + \frac{1}{k} DM(k)D^T$$

where $R_{v_0} > 0$, $P_\theta(0) > 0$ are the weighting parameters , the values of Q_k and R_k play the role of variances of the noises and γ is the prescribed noise attenuation level. Remaining symbols have the same meaning as those in the Dual-Kalman filter.

3.7 Simulation Results

In this section, we present simulation results for estimation of the signal using different adaptive filtering algorithms (Kalman,Dual-Kalman,H-infinity and Dual-H-infinity) in MATLAB environment. To study the performance of the system, we have generated an Auto-Regressive process of order 2 with known parameters which remain stationery over the entire signal duration and we have studied the filter's convergence properties, i.e. how fast the filter adapts to the true values of the AR process. A second order AR process can be generated using (3.1) with order, p = 2 [19].

$$s(k) = a_1 s(k-1) + a_2 s(k-2) + u(k)$$

where the AR parameters a_1 and a_2 and the variance of the process noise, σ_u^2 is chosen to make $s(k)$ a process with unit variance. We have generated an AR process of order 2, where a_1 and a_2 are -0.975 and 0.95 respectively with σ_u^2 = 0.0731. These values of the parameters were also considered in [12]. The zero mean complex Gaussian noise $v(k)$ with variance σ_v^2 is added to signal to get observations.

As discussed earlier, Kalman filter gives the best linear estimate under the Minimum Mean Square Error (MMSE) criterion. But it requires apriori knowledge of the signal parameters. But in practical situations it is difficult to estimate the signal parameters. In H-infinity case whatever may be the noise characteristics it can estimate the system, but it require the knowledge of AR parameters. Dual filtering is used to analyze the convergence of weight vector with an AR process of order 2. The true weights in this case are -0.975 and 0.95. The vector is initialized to all zeros in the beginning of each experiment. It has been observed that on an average, it takes around 500 iterations for the algorithm to estimate the weight vector from the noisy signal.

In Table 3.1, we have given mean square error (MSE) computed as

$$MSE = \frac{1}{N}\sum_{i=1}^{N} |e(k)|^2 \qquad (3.39)$$

where $e(k)$ is the estimated error and can be written as

$$e(k) = s(k) - \hat{s}(k/k)$$

where N is the number of iterations.

From Table.3.1, it may be observed that the estimated values converge to the true values at high SNR and we conclude that the Dual Algorithms successfully estimate the AR parameters which are unknown. The estimation error plots of Kalman, H-infinity, Dual Kalman and Dual H-infinity algorithms are shown in Fig. 3.5.

Fig. 3.5 a) shows the MSE performance for Kalman and Dual Kalman filters for AR-2 process. It is seen that at low SNR's (below 10 dB) Kalman filter performance is better and for high SNR's both algorithms have similar performance. Fig. 3.5 b) shows the MSE performance for Kalman and H-infinity filters. Here true values of the AR parameters are known, for Kalman filter the noise statistics are also assumed to be known. For SNR of 5 dB, H-infinity filter gives MSE=0.2654 where as Kalman filter give MSE=0.2179. As expected Kalman filter gives improved performance at low SNR.

Fig. 3.5 c) compares the MSE performance for H-infinity and Dual-H-infinity filters for AR-2 process. It is seen that at low SNR's (below 10dB) performance of H-infinity filter is better than dual-H-infinity filter. For SNR of 5 dB, H-infinity filter gives MSE=0.2485 where as Dual-H-infinity filter give MSE=0.2899. Fig. 3.5 d) shows the MSE performance for Dual-Kalman and Dual-H-infinity filters. Here the AR parameters as well as noise statistics are estimated. The MSE performances of both filters are mostly similar.

Fig. 3.5 e) shows the MSE performance of all the four filters for AR-2 process. The results are as expected and Kalman filter gives the best performance, since the knowledge of true parameters will lead to better filtering. But in practical applications, we do not have access to signal from which the true parameters can be extracted. Algorithms like Dual Kalman, Dual H-infinity have the ability to estimate the parameters of the signal from its noisy version and are suitable for real-time applications. According to Table.3.1 and Fig. 3.5 d) Dual H-infinity approach provides better estimates as compared to Dual Kalman filter.

SNR	10 dB	20 dB	30 dB	40 dB
Kalman Filter with known Parameters	MSE = 0.0837	MSE = 0.0093	MSE = 0.001	MSE = 0.0001
Dual Kalman Filter	MSE = 0.0907 $\hat{a}_1$ = -0.534 $\hat{a}_2$ = 0.335 $\hat{\sigma}_u^2$ = 0.3226	MSE = 0.0139 $\hat{a}_1$ = -0.8984 $\hat{a}_2$ = 0.787 $\hat{\sigma}_u^2$ = 0.0442	MSE = 0.0018 $\hat{a}_1$ = -0.946 $\hat{a}_2$ = 0.892 $\hat{\sigma}_u^2$ = 0.0578	MSE = 0.0005 $\hat{a}_1$ = -0.9607 $\hat{a}_2$ = 0.9214 $\hat{\sigma}_u^2$ = 0.0623
H-infinity Filter	MSE = 0.0886	MSE = 0.0096	MSE = 0.001	MSE = 0.0001
Dual H-infinity Filter	MSE = 0.0896 $\hat{a}_1$ = -0.508 $\hat{a}_2$ = 0.2103 $\hat{\sigma}_u^2$ = 0.2489	MSE = 0.0104 $\hat{a}_1$ = -0.9032 $\hat{a}_2$ = 0.8246 $\hat{\sigma}_u^2$ = 0.0386	MSE = 0.001 $\hat{a}_1$ = -0.9632 $\hat{a}_2$ = 0.927 $\hat{\sigma}_u^2$ = 0.0642	MSE = 0.0001 $\hat{a}_1$ = -0.9708 $\hat{a}_2$ = 0.9421 $\hat{\sigma}_u^2$ = 0.07024

Table 3.1. Comparison of different adaptive filtering algorithms for AR (2) parameters and driving process estimates based on 500 realizations. The true values are a_1=-0.975, a_2=0.95 and σ_u^2 =0.0178 [12].

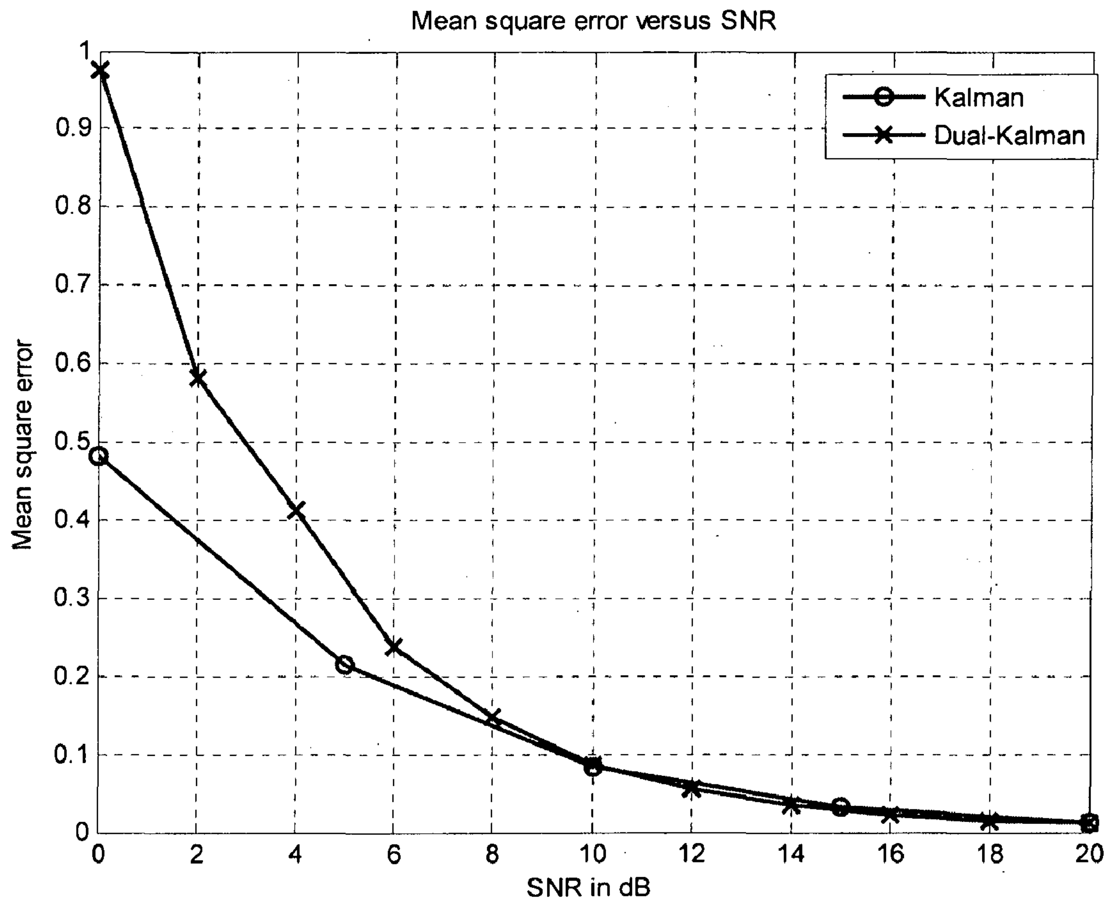

Fig. 3.5 a) MSE performance of Dual Kalman and Kalman Filtering Algorithms

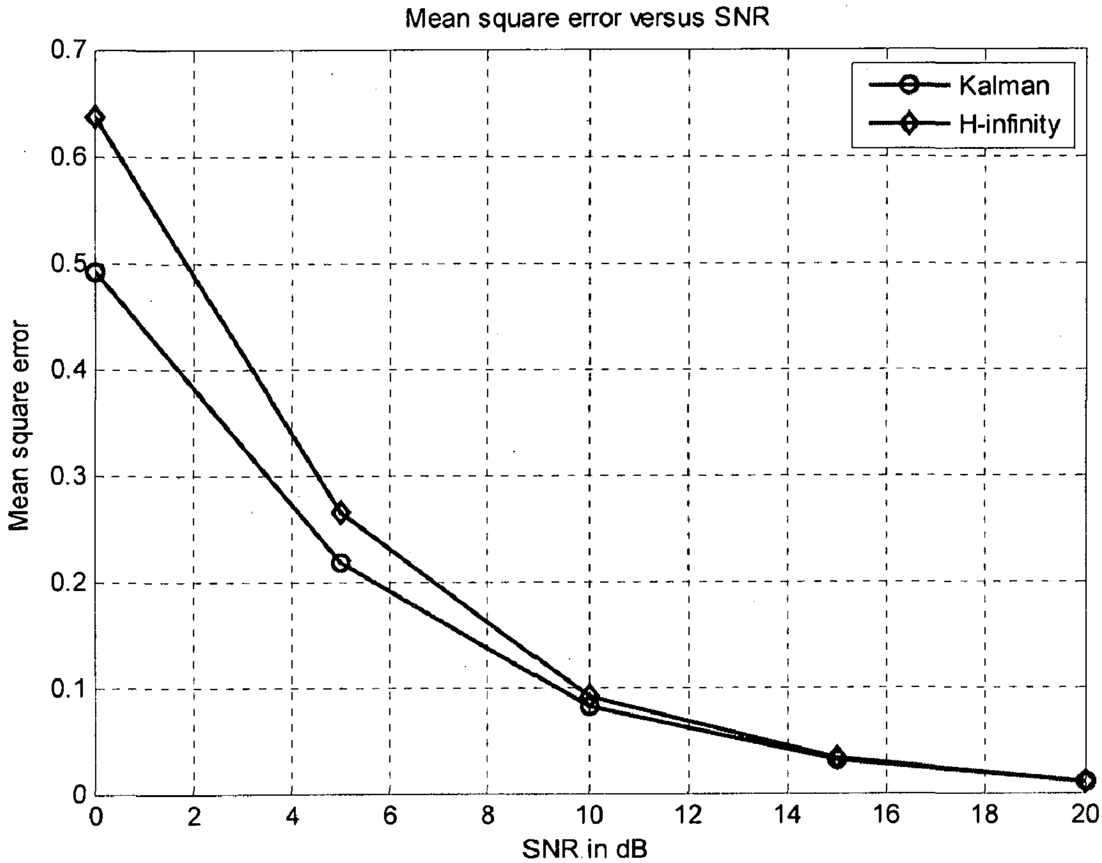

Fig. 3.5 b) MSE performance of Kalman and H-infinity estimation algorithms

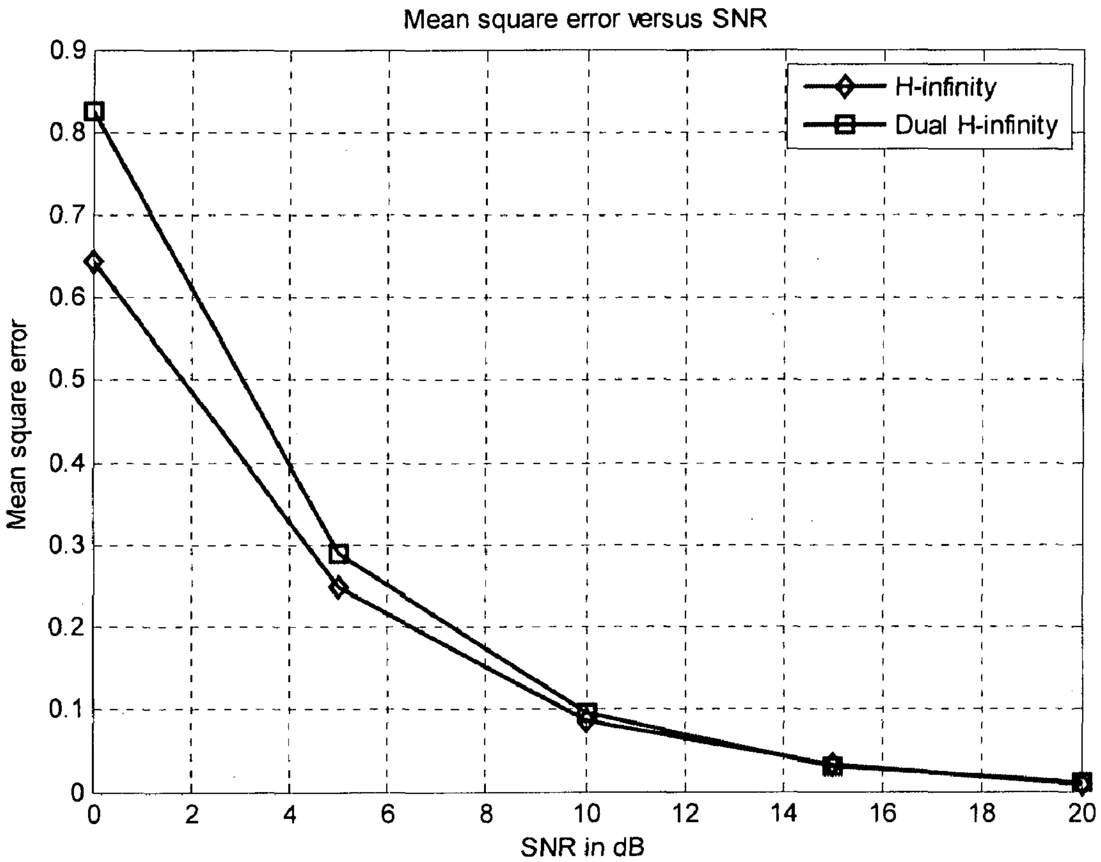

Fig. 3.5 c) MSE performance of H-infinity and Dual H-infinity estimation algorithms

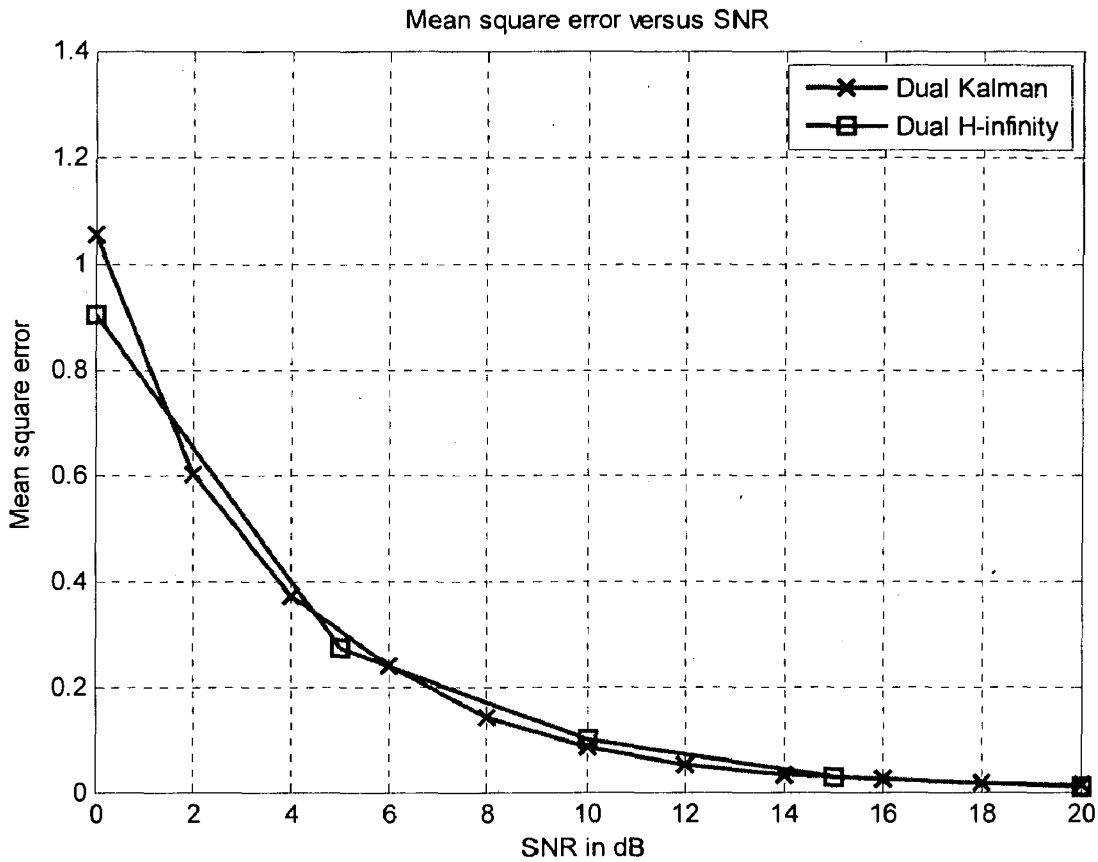

Fig. 3.5 d) MSE performance of Dual Kalman and Dual H-infinity estimation algorithms

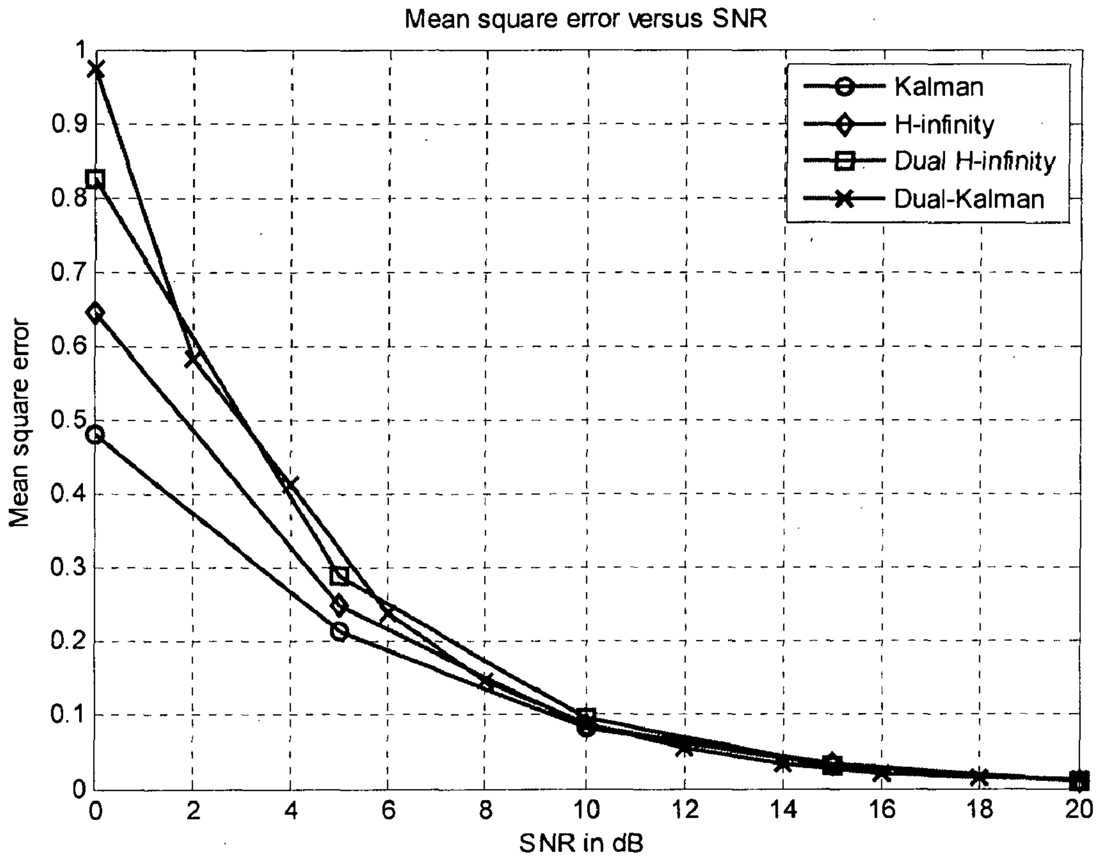

Fig.3.5.e) MSE performance of different estimation algorithms

Chapter 4

Applications of Adaptive Filtering Algorithms for Channel Estimation in OFDM

4.1 Introduction

In OFDM systems, due to user mobility, each carrier is subject to Doppler shifts resulting in time-varying fading. Thus, the estimation of the fading process over each carrier is essential to achieve coherent symbol detection at the receiver. In that case, training sequence/pilot aided techniques and blind techniques are two basic families for channel estimation. Training based methods require the transmission of explicit pilot sequences followed by suitable filtering. This chapter focuses on estimation of fading wireless channels for OFDM, using the ideas of Cyclic Prefix (CP) based estimation and adaptive filtering.

The time-varying fading channels are usually modelled as zero-mean wide-sense stationary circular complex Gaussian processes, whose stochastic properties depend on the maximum Doppler frequency denoted by f_d. According to the Jakes model [26], the theoretical Power Spectrum Density (PSD) of the fading process, is band-limited. Moreover, it exhibits twin peaks at $\pm f_d$. The fading wireless channel statistics can be directly estimated by means of the Least Mean Square (LMS) and the Recursive Least Square (RLS) algorithms as in [27]. Alternatively, Kalman filtering algorithm combined with an Autoregressive (AR) model to describe the time evolution of the fading processes and it provides superior performance over the LMS and RLS based channel estimators in [28]. In addition, when the AR model parameters are unknown, dual filtering algorithms are used to estimate the fading channels.

In this chapter, for the channel estimation of OFDM, a system model and architecture over fading channels are presented. In the next section a CP based model and the different channel estimation algorithms (Kalman, Dual-Kalman, H-infinity and Dual-H-infinity) are discussed. The performance results are discussed in the next section, finally simulation results are presented.

4.2 System Model

In the following, we consider a low to moderate Doppler environment, which allows for a block fading (quasi-static) channel assumption. This implies that the channel tap variations within an OFDM symbol duration are negligible, and hence we may define an $L\times1$ channel tap vector for each OFDM symbol as

$$\mathbf{h}_n = [h_n(0)h_n(1)\ldots h_n(L-1)]^T \tag{4.1}$$

where $h_n(l)$ is the l^{th} channel tap for the n^{th} OFDM symbol.

The classical Doppler spectrum for each of the L channel taps is approximated by an independent AR-2 process [19].

For the l^{th} channel tap at n^{th} OFDM symbol, we have

$$h_n(l) = -a_1 h_{n-1}(l) - a_2 h_{n-2}(l) + v_n(l) \tag{4.2}$$

where a_1 and a_2 are the AR-2 coefficients as defined in the second chapter and $v_n(l)$ is the modelling noise for l^{th} tap at symbol n.

4.2.1 OFDM architecture over fading channel

We consider an OFDM system as in Fig. 4.1 with N data subcarriers. Input data are buffered, converted to a parallel stream and modulated to i.i.d. equi-probable symbols $X_n(k)$, where $X_n(k)$ denotes the k^{th} symbol of the n^{th} OFDM symbol. Each symbol mapped to some complex constellation points, $X_n(k)$, k=0,1,. . .,N-1 at each n. The modulation is implemented by N-point inverse discrete Fourier transform (IDFT) for the symbol vector

$$\mathbf{X}_n = [X_n(0)\ X_n(1)\ldots X_n(N-1)]^T \tag{4.3}$$

is

$$x_n(m+gi) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_n(k)e^{j2\pi mk/N},\quad 0 \le \mathrm{m} \le N-1 \tag{4.4}$$

CP of length gi is appended to form the transmitted vector as

$$\mathbf{x}'_n = [x_n(0)x_n(1)\dots x_n(gi-1) \vdots x_n(gi)x_n(gi+1)\dots x_n(gi+N-1)]^T \quad (4.5)$$

where

$$x_n(m) = x_n(N+m), \quad 0 \le m \le gi-1$$

The received symbol corrupted by fading channel and AWGN becomes

$$y_n(m) = \sum_{l=0}^{L-1} h_n(l)x_n(m-l) + z_n(m), \; 0 \le \mathrm{m} \le N+gi+L-1 \quad (4.6)$$

where n is the OFDM symbol index,

$z_n(m)$ is an AWGN sample with zero mean and variance σ^2 at instant m in the n^{th} OFDM symbol.

Demodulation involves removing the cyclic prefix and taking N-point DFT of the received vector to get

$$\mathbf{Y}_n = [Y_n(0)\ \mathrm{Y}_n(1)\dots \mathrm{Y}_n(N-1)]^T \quad (4.7)$$

In frequency domain, we have over each subcarrier

$$Y_n(k) = X_n(k)H_n(k) + Z_n(k) \quad (4.8)$$

where $H_n(k)$ is the channel frequency response at subcarrier k given by

$$H_n(k) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} h_n(l)e^{-j2\pi lk/N}, \quad 0 \le k \le N-1 \quad (4.9)$$

and $Z_n(k)$ is the noise on k^{th} subcarrier of n^{th} OFDM symbol i.e.,

$$Z_n(k) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} z_n(m)e^{-j2\pi mk/N}, \; 0 \le k \le N-1 \quad (4.10)$$

At the receiver, the channel estimator is followed by frequency domain equalizer. A description of channel estimation techniques is given in section 4.3. After equalization, the estimated symbol at the k^{th} symbol becomes [10]

$$\hat{X}_n(k) = \frac{Y_n(k)}{\hat{H}_n(k)} = \frac{X_n(k)H_n(k)}{\hat{H}_n(k)} + \frac{Z_n(k)}{\hat{H}_n(k)} \quad (4.11)$$

where $\hat{H}_n(k)$ is the estimate of $H_n(k)$ defined in Eq. (4.9). The estimated symbols $\hat{X}_n(k)$ are then demapped to output bits.

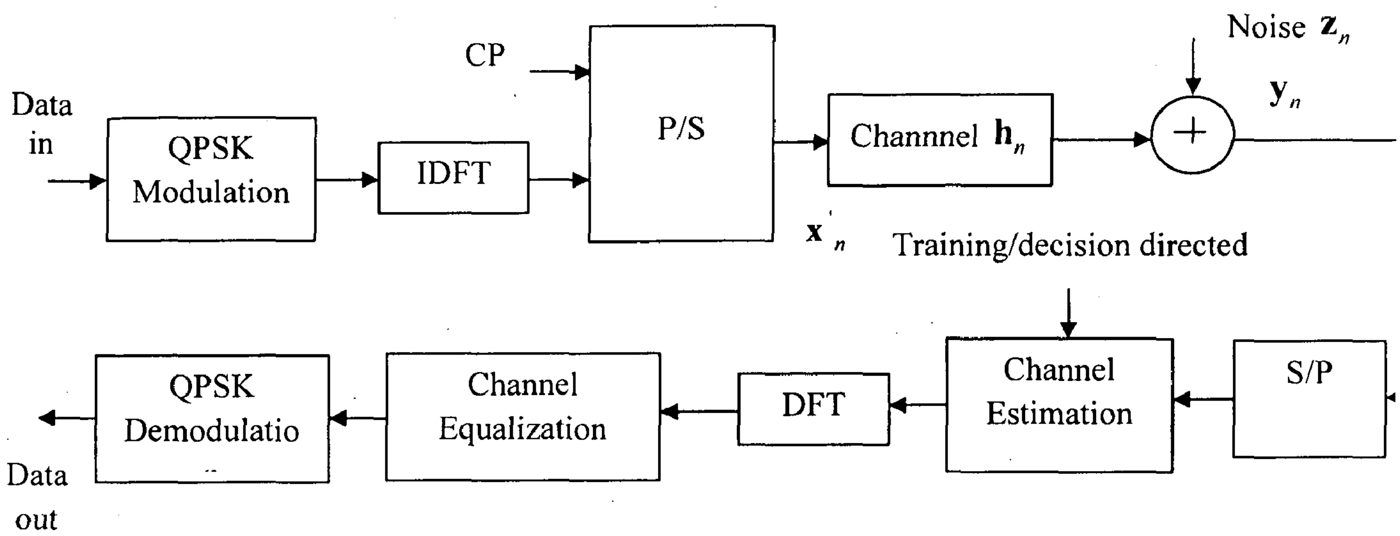

Fig. 4.1. Block schematic of the OFDM system.

4.3 CP Based Channel Estimation Techniques

This section describes the use of various adaptive filtering algorithms in CP based frame work for channel estimation in OFDM systems as discussed in chapter. 2. From Eq. (4.6), we know that

$$y_n(m) = h_n(0)x(m) + h_n(1)x(m-1) + ... + h_n(L-1)x(m-L+1) + z_n(m) \quad (4.12)$$

Gathering the received samples of the n^{th} received OFDM symbol for time instants $0 \le m \le gi-1$, we obtain a $gi \times 1$ vector

$$\mathbf{y}_{n,CP} = [y_n(0) y_n(1) y_n(2) ... y_n(gi-1)]^T, \quad (4.13)$$

which is the CP of the received OFDM symbol, and

$$\mathbf{z}_{n,CP} = [z_n(0) z_n(1) z_n(2) ... z_n(gi-1)]^T \quad (4.14)$$

is the $gi \times 1$ vector of AWGN samples affecting the CP part of the n^{th} received OFDM symbol.

4.3.1 Kalman Filtering (KF) algorithm

When operating in a non-stationary environment, Kalman filter [29] is known to yield an optimal solution to the linear filter problem. This subsection describes the application of KF to the channel estimation problem in OFDM. For this purpose, the system is formulated as a state-space model, with unknown channel taps comprising the state of the system. We assume that the state $\mathbf{s}_n$, to be estimated at OFDM symbol index n, comprises of channel taps at two consecutive OFDM symbols [10]

$$\mathbf{s}_n = \left[\mathbf{h}_{n-1}\ \mathbf{h}_n\right]^T_{2L\times 1} \tag{4.15}$$

From Eq. (4.1) and Eq. (4.2) we have

$$\mathbf{h}_n = \left[h_n(0)h_n(1)\ldots h_n(L-1)\right]^T_{L\times 1}$$

$$\mathbf{h}_{n-1} = \left[h_{n-1}(0)h_{n-1}(1)\ldots h_{n-1}(L-1)\right]^T_{L\times 1}$$

and

$$\mathbf{h}_n = a_1\mathbf{h}_{n-1} + a_2\mathbf{h}_{n-2} + \mathbf{v}_n \tag{4.16}$$

From above equations we get

$$\begin{bmatrix}\mathbf{h}_{n-1}\\ \mathbf{h}_n\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{L\times L} & \mathbf{I}_L\\ a_2\mathbf{I}_L & a_I\mathbf{I}_L\end{bmatrix}\begin{bmatrix}\mathbf{h}_{n-2}\\ \mathbf{h}_{n-1}\end{bmatrix} + \mathbf{v}_n \tag{4.17}$$

We observe that Eq. (4.17) provides the basis for forming the process equation as

$$\mathbf{s}_n = \mathbf{B}\mathbf{s}_{n-1} + \mathbf{v}_n \tag{4.18}$$

Here, transition matrix

$$\mathbf{B} = \begin{bmatrix}\mathbf{0}_{L\times L} & \mathbf{I}_L\\ a_2\mathbf{I}_L & a_I\mathbf{I}_L\end{bmatrix}_{2L\times 2L} \tag{4.19}$$

$\mathbf{0}_{L\times L}$ denotes the $L\times L$ matrix of all zeros and $\mathbf{I}_L$ is the $L\times L$ identity matrix.

Process noise vector

$$\mathbf{v}_n = \left[\mathbf{0}_{1\times L} \vdots v_n(0)v_n(1)\ldots v_n(L-1)\right]^T_{2L\times 1} \tag{4.20}$$

where $v_n(l)$ is the modelling noise as in (4.2)

From Eq. (4.12), we have

$$y_n(m) = h_n(0)x(m) + h_n(1)x(m-1) + \ldots + h_n(L-1)x(m-L+1) + z_n(m)$$

$$\begin{bmatrix} y_n(0) \\ y_n(1) \\ \vdots \\ y_n(gi-1) \end{bmatrix} = \begin{bmatrix} x_n(0) & x_{n-1}(N+gi-1) & \ldots & x_{n-1}(N+gi-L+1) \\ x_n(1) & x_n(0) & x_{n-1}(N+gi-1) & \ldots \\ \vdots & \vdots & \vdots & \vdots \\ x_n(gi-1) & x_n(gi-2) & \ldots & x_n(gi-L) \end{bmatrix} \begin{bmatrix} h_n(0) \\ h_n(1) \\ \vdots \\ h_n(L-1) \end{bmatrix} + z_n(m)$$

where $0 \le m \le gi-1$

We observe from above that following provides the basis for forming measurement equation as

$$\mathbf{y}_{n,CP} = \overline{\mathbf{A}}_n \mathbf{s}_n + \mathbf{z}_{n,CP} \tag{4.21}$$

where the measurement matrix $\overline{\mathbf{A}}_n$ in Eq. (4.21) is formed from the matrix $\mathbf{A}_n$ by augmenting it with a null matrix as

$$\overline{\mathbf{A}}_n = \left[\mathbf{0}_{gi\times L} \;\vdots\; \mathbf{A}_n\right]_{gi\times 2L} \tag{4.22}$$

Here $\mathbf{A}_n$ is a $gi \times L$ matrix of transmitted symbols that determine the CP of the received OFDM symbol.

$$\mathbf{A}_n = \begin{bmatrix} x_n(0) & x_{n-1}(N+gi-1) & \ldots & x_{n-1}(N+gi-L+1) \\ x_n(1) & x_n(0) & x_{n-1}(N+gi-1) & \ldots \\ \vdots & \vdots & \vdots & \vdots \\ x_n(gi-1) & x_n(gi-2) & \ldots & x_n(gi-L) \end{bmatrix}_{gi\times L}$$

Considering that the CP appended to an OFDM symbol is a replication of the last gi values of that symbol, we may write $\mathbf{A}_n$ in terms of transmitted CP value as,

$$\mathbf{A}_n = \begin{bmatrix} x_n(0) & x_{n-1}(gi-1) & x_{n-1}(gi-2) & \dots & x_{n-1}(gi-L+1) \\ x_n(1) & x_n(0) & x_{n-1}(gi-1) & \dots & x_{n-1}(gi-L+2) \\ \vdots & \vdots & \vdots & & \vdots \\ x_n(gi-1) & x_n(gi-2) & \cdots & \dots & x_n(gi-L) \end{bmatrix}_{gi\times L} \tag{4.23}$$

$\mathbf{A}_n$ has gi rows corresponding to gi consecutive time instants of the CP. The L elements of each row are the transmitted symbol values affecting the received CP value at that instant. This matrix structure assumes that the CP length is at least equal to the number of taps in the channel impulse response, i.e. no inter block interference.

The measurement noise vector $\mathbf{z}_{n,CP}$, in Eq. (4.21), comprises the $gi \times 1$ vector of AWGN samples affecting the cyclic prefix part of the OFDM symbol.

We observe that Eq. (4.18) and Eq. (4.21) provide the basis for forming the process equation and measurement equation, respectively for the state space model, as follows

$$\mathbf{s}_n = \mathbf{B}\mathbf{s}_{n-1} + \mathbf{v}_n$$

$$\mathbf{y}_{n,CP} = \overline{\mathbf{A}}_n \mathbf{s}_n + \mathbf{z}_{n,CP} \tag{4.24}$$

A Kalman filter is employed to estimate the unknown state of the system. Cyclic prefix of the received OFDM symbol $\mathbf{y}_{n,CP}$ is given as input observation to Kalman algorithm, the following estimation equations (as discussed in chapter. 3) are given by [12] [21].

$$[\boldsymbol{P}_{n|n-1}]_{2L\times 2L} = \mathbf{B}\boldsymbol{P}_{n-1|n-1}\mathbf{B}^H + \mathbf{Q}_1 \tag{4.25}$$

$$[\boldsymbol{\alpha}_n]_{gi\times 1} = \left[\mathbf{y}_{n,CP} - \overline{\mathbf{A}}_n \hat{\mathbf{s}}_{n-1}\right] \tag{4.26}$$

$$[\mathbf{C}_n]_{gi\times gi} = \overline{\mathbf{A}}_n \boldsymbol{P}_{n|n-1} \overline{\mathbf{A}}_n^H + \mathbf{Q}_2 \tag{4.27}$$

$$[\mathbf{K}_n]_{2L\times gi} = \boldsymbol{P}_{n|n-1} \overline{\mathbf{A}}_n^H \mathbf{C}_n^{-1} \tag{4.28}$$

$$[\hat{\mathbf{s}}_n]_{2L\times 1} = \mathbf{B}\hat{\mathbf{s}}_{n-1} + \mathbf{K}_n \boldsymbol{\alpha}_n \tag{4.29}$$

$$[\hat{\mathbf{h}}_n]_{L\times 1} = R\hat{\mathbf{s}}_n \quad , \quad R = [\mathbf{0}_{L\times L} \ \mathbf{I}_L]_{L\times 2L} \tag{4.30}$$

$$[\boldsymbol{P}_n]_{2L\times 2L} = [\mathbf{I}_{2L} - \mathbf{K}_n \overline{\mathbf{A}}_n]\boldsymbol{P}_{n|n-1} \tag{4.31}$$

where $\mathbf{K}_n$ is the $2L \times gi$ Kalman gain matrix , $\hat{\mathbf{s}}_n$ is the state estimate at the n^{th} OFDM symbol, $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are the covariance matrices of $\mathbf{v}_n$ and $\mathbf{z}_{n,CP}$ respectively, $\boldsymbol{P}_{n|n-1}$ is the priori covariance matrix of estimation error , and $\boldsymbol{P}_n$ is the current covariance matrix of estimation error. When the channel taps are modelled as a zero mean random process, the algorithm is initialized with an all-zero state vector. Besides this, the assumption of uncorrelated scattering (US) causes the different channel taps to be i.i.d., and the error covariance matrix is initialized as an identity matrix.

$$\mathbf{s}_0 = \hat{\mathbf{s}}_0 = \mathbf{0}_{2L\times 1}$$

$$\boldsymbol{P}_0 = E\left[(\mathbf{s}_0 - \hat{\mathbf{s}}_0)(\mathbf{s}_0 - \hat{\mathbf{s}}_0)^H\right] = \mathbf{I}_n$$

The receiver operates in training and decision directed modes. In training mode the known transmitted CP ($\mathbf{x}_{n,CP}$) and CP part of the received OFDM symbol ($\mathbf{y}_{n,CP}$) form the input to the above Kalman filter algorithm, and get the channel estimation $H_n(k)$, we get

$$\hat{X}_n(k) = \frac{Y_n(k)}{\hat{H}_n(k)} \tag{4.32}$$

In decision directed mode the receiver uses the estimated channel vector from the previous OFDM symbol to demodulate the received symbol and generate an estimate of transmitted CP ($\hat{X}_{n,CP}(k)$). Here the transmitted CP part can be estimated by previous estimated channel i.e.,

$$\hat{X}_{n,CP}(k) = \frac{Y_{n,CP}(k)}{\hat{H}_{n-1}(k)} \tag{4.33}$$

This estimated CP and CP of the received OFDM symbol ($\mathbf{y}_{n,CP}$) helps to estimate the channel.

The equations from (4.25) to (4.31) can be carried out by providing the AR parameters that are involved in the transition matrix **B** and the driving process variances are available. In case, these are unknown, for estimating these parameters Dual-Kalman filtering technique is used.

4.3.2 Dual-Kalman Filtering algorithm

To estimate the AR parameters $\boldsymbol{\theta}_n$ from the estimated fading process $\hat{\mathbf{h}}_n$, Eq. (4.30) is firstly represented as an AR-2 model to express the estimated fading process as a function of $\boldsymbol{\theta}_n$ (AR parameter vector).

$$\hat{\mathbf{h}}_n = \begin{bmatrix} \hat{\mathbf{h}}_{n-1} & \hat{\mathbf{h}}_{n-2} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \mathbf{w}_n \tag{4.34}$$

$$\begin{bmatrix} \hat{h}_n(0) \\ \hat{h}_n(1) \\ \vdots \\ \hat{h}_n(L-1) \end{bmatrix}_{L\times 1} = \begin{bmatrix} \hat{h}_{n-1}(0) & \hat{h}_{n-2}(0) \\ \hat{h}_{n-1}(1) & \hat{h}_{n-2}(1) \\ \vdots & \vdots \\ \hat{h}_{n-1}(L-1) & \hat{h}_{n-2}(L-1) \end{bmatrix}_{L\times 2} \begin{bmatrix} a_1 \\ \\ a_2 \end{bmatrix}_{2\times 1} + \mathbf{w}_n$$

$$\mathbf{r}_n = \mathbf{H}\boldsymbol{\theta}_n + \mathbf{w}_n \tag{4.35}$$

where $\mathbf{r}_n$ is the estimated channel vector, $\boldsymbol{\theta}_n$ is the AR parameter vector defines as

$\boldsymbol{\theta}_n = [a_1 \ a_2]^T$ and $\mathbf{w}_n$ is the $L\times 1$ noise vector as in Eq. (4.20).

When the channel is assumed to be stationary, the AR parameters are time-invariant and satisfy the following relationship

$$\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1} \tag{4.36}$$

As Eq. (4.35) and Eq. (4.36) define a state-space representation for the estimation of the AR parameters, a second Kalman filter can be used to recursively estimate $\boldsymbol{\theta}_n$ as follows [21]

$$[\boldsymbol{P}_{\theta_{n/n-1}}]_{2\times 2} = \boldsymbol{P}_{\theta_{n-1/n-1}} \tag{4.37}$$

$$[\boldsymbol{\alpha}_{\theta_n}]_{L\times 1} = [\mathbf{r}_n - \mathbf{H}\hat{\boldsymbol{\theta}}_{n-1}] \tag{4.38}$$

$$[\mathbf{C}_{\theta_n}]_{L\times L} = \mathbf{H}\boldsymbol{P}_{\theta_{n/n-1}}\mathbf{H}^H + \mathbf{Q}_3 \tag{4.39}$$

$$[\mathbf{K}_{\theta_n}]_{2\times L} = \boldsymbol{P}_{\theta_{n/n-1}}\mathbf{H}^H\mathbf{C}_{\theta_n}^{-1} \tag{4.40}$$

$$[\hat{\boldsymbol{\theta}}_n]_{2\times 1} = \hat{\boldsymbol{\theta}}_{n-1} + \mathbf{K}_{\theta_n}\boldsymbol{\alpha}_{\theta_n}\,, \tag{4.41}$$

$$[\boldsymbol{P}_{\theta_{n/n}}]_{2\times 2} = [\mathbf{I}_2 - \mathbf{K}_{\theta_n}\mathbf{H}]\boldsymbol{P}_{\theta_{n|n-1}} \tag{4.42}$$

where $\mathbf{Q_3}$ is the covariance matrix of the $\mathbf{w}_n$, the error covariance matrix and the initial AR parameter vector are defined as

$$\hat{\boldsymbol{\theta}}_0 = \boldsymbol{\theta}_0 = \mathbf{0}_{2\times 1}\,,$$

$$\boldsymbol{P}_{\theta_{0/0}} = E\left[(\boldsymbol{\theta}_0 - \hat{\boldsymbol{\theta}}_0)(\boldsymbol{\theta}_0 - \hat{\boldsymbol{\theta}}_0)^H\right] = \mathbf{I}_2$$

Noise parameters estimation

Apart from estimating the AR parameters, we also need to estimate the noise parameters for the fading channel environment i.e., variance of $\mathbf{v}_n$ and $\mathbf{z}_{n,CP}$. This can be done by using the error covariance matrices. From Eq. (4.25) and Eq. (4.31) we can write the noise variances recursively as derived in the chapter.3.

$$[\boldsymbol{L}_n]_{2L\times 2L} = \boldsymbol{P}_n - \mathbf{B}\boldsymbol{P}_{n-1|n-1}\mathbf{B}^H + \mathbf{K}_n\boldsymbol{\alpha}_n\boldsymbol{\alpha}_n^H\mathbf{K}_n^H \tag{4.43}$$

$$\hat{Q}_1(n) = \frac{n-1}{n}\hat{Q}_1(n-1) + \frac{1}{n}\boldsymbol{D}\boldsymbol{L}_n\boldsymbol{D}^T$$

$$\boldsymbol{D} = \left[1\,0\,.\,.\,.\,.\,.\,0\right]_{1\times 2L} \tag{4.44}$$

$$[\mathbf{M}_n]_{gi\times gi} = \boldsymbol{\alpha}_n\boldsymbol{\alpha}_n^H - \overline{\mathbf{A}}_n\boldsymbol{P}_{n|n-1}\overline{\mathbf{A}}_n^H \tag{4.45}$$

$$\hat{Q}_2(n) = \frac{n-1}{n}\hat{Q}_2(n-1) + \frac{1}{n}\boldsymbol{D}_1\mathbf{M}_n\boldsymbol{D}_1^T \tag{4.46}$$

$$\boldsymbol{D}_1 = \left[1\,0\,.\,.\,.\,.\,.\,0\right]_{1\times gi}\,,$$

where $\hat{Q}_1(n)$ and $\hat{Q}_2(n)$ are the estimated variances of the process noise $\mathbf{v}_n$ and modelling noise $\mathbf{z}_{n,CP}$ respectively.

4.3.3 H-infinity Filtering (HF) algorithm

H-infinity filtering (HF) algorithm is employed to estimate the unknown state of the system. Cyclic prefix of the received OFDM symbol $\mathbf{y}_{n,CP}$ is given as input observation to H-infinity filter and following estimation equations are used as discussed in the chapter. 3 [20]

$$[\boldsymbol{\alpha}_n]_{gi\times 1} = \left[\mathbf{y}_{n,CP} - \overline{\mathbf{A}}_n\hat{\mathbf{s}}_{n-1}\right] \tag{4.47}$$

$$[\mathbf{C}_n]_{2L\times 2L} = [\mathbf{I}_{2L} - \gamma^{-1}\overline{\mathbf{A}}_n^H\overline{\mathbf{A}}_n\boldsymbol{P}_{n-1} + \overline{\mathbf{A}}_n^H\mathbf{Q}_2\overline{\mathbf{A}}_n\boldsymbol{P}_{n-1}] \tag{4.48}$$

$$[\mathbf{K}_n]_{2L\times gi} = \boldsymbol{P}_{n-1}\mathbf{C}_n^{-1}\overline{\mathbf{A}}_n^H\mathbf{Q}_2 \tag{4.49}$$

$$[\hat{\mathbf{s}}_n]_{2L\times 1} = \mathbf{B}\hat{\mathbf{s}}_{n-1} + \mathbf{K}_n\boldsymbol{\alpha}_n \tag{4.50}$$

$$[\hat{\mathbf{h}}_n]_{L\times 1} = R\hat{\mathbf{s}}_n \;, \qquad R = [\mathbf{0}_{L\times L}\ \mathbf{I}_L]_{L\times 2L} \tag{4.51}$$

$$[\boldsymbol{P}_n]_{2L\times 2L} = \mathbf{B}\boldsymbol{P}_{n-1}\mathbf{C}_n^{-1}\mathbf{B}^H + \mathbf{Q}_1 \;, \tag{4.52}$$

where $\mathbf{K}_n$ is the $2L\times gi$ Kalman gain matrix , $\hat{\mathbf{s}}_n$ is the state estimate at the n^{th} OFDM symbol, $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are the weighting parameters , γ (>0) is the prescribed level of attenuation and $\boldsymbol{P}_n$ is the $2L\times 2L$ positive definite matrix, initialized as identity matrix.

$$\boldsymbol{P}_0 = \mathbf{I}_{2L}$$

and the algorithm is initialized with an all-zero state vector.

$$\mathbf{s}_0 = \hat{\mathbf{s}}_0 = \mathbf{0}_{2L\times 1}$$

4.3.4 Dual H-infinity Filtering algorithm

For the joint estimation of both channel and its AR parameter, we require another H-infinity filter. This method makes it possible to provide robust estimation of the fading channel and its AR parameters. For the channel estimation requires state space model in

Eq. (4.24). To estimate the AR parameters $\boldsymbol{\theta}_n$= [a₁ a₂]^T from the estimated channel $\hat{\mathbf{h}}_n$ requires an AR-2 model as a function of AR parameter vector $\boldsymbol{\theta}_n$ i.e.,

$$\hat{\mathbf{h}}_n = a_1\hat{\mathbf{h}}_{n-1} + a_2\hat{\mathbf{h}}_{n-2} + \mathbf{w}_n$$

$$\hat{\mathbf{h}}_n = \begin{bmatrix} \hat{\mathbf{h}}_{n-1} & \hat{\mathbf{h}}_{n-2} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \mathbf{w}_n$$

$$\mathbf{r}_n = \mathbf{H}\boldsymbol{\theta}_n + \mathbf{w}_n$$

where $\mathbf{r}_n$ is the estimated channel vector, $\boldsymbol{\theta}_n$ is the AR parameter vector defines as

$$\mathbf{r}_n = [\hat{h}_n(0)\hat{h}_n(1)...\hat{h}_n(L-1)]^T_{L\times l},$$

$\boldsymbol{\theta}_n = [a_1 \ \ a_2]^T$ and $\mathbf{w}_n$is the $L \times 1$ noise vector as in Eq. (4.23).

When the channel is assumed to be stationary, the AR parameters are time-invariant and satisfy the following relationship

$$\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1}$$

A state-space representation for the estimation of the AR parameters is

$$\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1}$$

$$\mathbf{r}_n = \mathbf{H}\boldsymbol{\theta}_n + \mathbf{w}_n \tag{4.53}$$

The above state-space representation for the estimation of the AR parameters, a second H-infinity filter can be used to recursively estimate of $\boldsymbol{\theta}_n$ as follows [23]

$$[\boldsymbol{\alpha}_{\theta_n}]_{L\times 1} = [\mathbf{r}_n - \mathbf{H}\hat{\boldsymbol{\theta}}_{n-1}] \tag{4.54}$$

$$[\mathbf{C}_{\theta_n}]_{2\times 2} = [\mathbf{I}_2 - \gamma^{-1}H^H H P_{\theta_{n-1}} + H^H \mathbf{Q}_3 H P_{\theta_{n-1}}] \tag{4.55}$$

$$[\mathbf{K}_{\theta_n}]_{2\times L} = P_{\theta_{n-1}} \mathbf{C}_{\theta_n}{}^{-1} H^H \mathbf{Q}_3 \tag{4.56}$$

$$[\hat{\boldsymbol{\theta}}_n]_{2\times 1} = \hat{\boldsymbol{\theta}}_{n-1} + \mathbf{K}_{\theta_n}\boldsymbol{\alpha}_{\theta_n}, \qquad [\hat{\boldsymbol{\theta}}_0 = \mathbf{0}] \tag{4.57}$$

$$[P_{\theta_n}]_{2\times 2} = P_{\theta_{n-1}}\mathbf{C}_{\theta_n}^{-1} \tag{4.58}$$

where $\mathbf{Q_3}$ is the weighting parameter, γ (> 0) is the prescribed level of attenuation and $P_{\theta n}$ is the positive definite matrix, initialized as identity matrix.

$$P_{\theta_0} = \mathbf{I}_2,$$

and the initial AR parameter vector are defined as

$$\hat{\boldsymbol{\theta}}_0 = \boldsymbol{\theta}_0 = \mathbf{0}_{2\times 1}$$

Noise parameters estimation

Apart from estimating the AR parameters, we also need to estimate the noise parameters. This can be done by using the Riccati equation From Eq. (4.52) as derived in previous chapter, we can write [23]

$$[L_n]_{2L\times 2L} = [P_n - \mathbf{B}P_{n-1}\mathbf{C}_n^{-1}\mathbf{B}^H] \tag{4.59}$$

$$\hat{Q}_1(n) = \frac{n-1}{n}\hat{Q}_1(n-1) + \frac{1}{n}DL_nD^T \tag{4.60}$$

$$D = [1\,0\ldots\ldots 0]_{1\times 2L}$$

$$[\mathbf{M}_n]_{gi\times gi} = \boldsymbol{\alpha}_n\boldsymbol{\alpha}_n^H - \overline{\mathbf{A}}_n P_{n|}\overline{\mathbf{A}}_n^H \tag{4.61}$$

$$\hat{Q}_2(n) = \frac{n-1}{n}\hat{Q}_2(n-1) + \frac{1}{n}D_1\mathbf{M}_nD_1^T \tag{4.62}$$

$$D_1 = [1\,0\ldots\ldots 0]_{1\times gi}$$

where $\hat{Q}_1(n)$ and $\hat{Q}_2(n)$ are the estimated variances of the process noise $\mathbf{v}_n$ and modelling noise $\mathbf{z}_{n,CP}$ respectively.

4.4. Results and Discussion

4.4.1 Simulation Environment

The scheme detailed in the previous section is tested through MATLAB simulations. For simulation, 4-QPSK is used as the underlying modulation scheme with zero mean, unit variance, equi-probable modulation symbols, drawn from the constellation. An OFDM symbol with 128 data subcarriers is assumed. The useful OFDM symbol duration $T(=NT_s)$ is 0.1 ms. Slow to moderate time variations are considered, with mobile velocities 4.5 km/hr to 45 km/hr. These correspond to Doppler frequencies of 10 Hz and 100 Hz, i.e., normalized Doppler spreads $f_d T = 0.001$ and $f_d T = 0.01$. An L-tap channel is generated where each tap is a complex Gaussian random variable, with zero mean and unit variance. The time varying nature of each tap is independently governed by an AR-2 process and updated at each OFDM symbol (i.e. every T seconds) in accordance with Eq. 4.2. The noise is assumed to be zero mean, complex additive white and Gaussian, with variance σ^2.

As described in section 4.3, the transmission proceeds in training and decision directed modes. The training mode uses the CP part of the OFDM symbol as a known training sequence. We use two training patterns in simulation, for low and moderate Doppler spreads. The first scheme partitions the transmission into blocks of 100 OFDM symbols each, it uses the CP of 10 initial OFDM symbols as training, and operates in decision directed mode for the subsequent 90 symbols. The process is repeated after each block of 100 symbols. The pattern hereafter denoted as (10, 90). For the system under simulation, we have data subcarriers N=128, and the CP length gi =N/8=16. The channel estimation algorithm detailed in section 4.3 endeavours to estimate the channel tap vector $\mathbf{h}_n$ for each OFDM symbol. In Dual-Kalman and Dual H-infinity algorithms the initial AR parameter vector is taken as zeros and initial noise variances are taken as 0.1. For H-infinity and dual H-infinity algorithms the prescribed level of attenuation $\gamma(>0)$ is taken as 10 within the symbol duration.

4.4.2 Estimation Error Performance

Fig. 4.2 and Fig. 4.3 shows the MSEE performance of different adaptive filtering algorithms in CP based channel estimation for OFDM systems. Here the fading coefficients are modeled as AR-2 processes. We consider a 4-tap channel with low and moderate Doppler spreads, (10,90) transmission pattern and a CP length of 16. 100 independent realizations of the channel are generated for each value of received SNR.

Fig. 4.2 a) shows the MSEE performance for Kalman and H-infinity filters. Here we consider a normalized Doppler spread $f_d T = 0.001$ and the true values of the AR parameters are assumed to be known aprior. For the comparison, the MSEE for the Kalman at 20 dB is 0.00057 where as H-infinity gives 0.0017. The MSEE shows a steady fall from 10^{-2} to 10^{-4}, as the received SNR increases from 10 dB to 30 dB for both the filters. As SNR increases the performance of both gets similar.

Fig. 4.2 b) shows the MSEE performance for Dual-Kalman and Dual-H-infinity algorithms when the AR parameters are unknown. We observe that Dual-H-infinity filter gives slightly better performance as compared to Dual-Kalman filter, the MSEE for the Dual-H-infinity filter at SNR= 25 dB is 0.00093 where as Dual-Kalman filter MSEE=0.0018.

Fig. 4.2 c) shows the MSEE performance for four different algorithms. It is found that all the schemes perform reasonably well in tracking a low Doppler spread channel. It is observed that Dual algorithms successfully estimate the channel when the AR parameters are not known and at high SNR's they get closer to Kalman filtering algorithm with known parameters. For MSEE=10^{-3}, dual filtering algorithms performs with in 3 dB of the Kalman filtering curve.

Fig. 4.3 a) shows the MSEE performance for Kalman and H-infinity filters. Here we consider a normalized Doppler spread $f_d T = 0.01$ and the true values of the AR parameters are assumed to be known aprior. Both the algorithms saturates at an MSEE of 0.0058. It is seen that from SNR of 10dB-20dB, Kalman filter gives better performance when compared to H-infinity filter. For SNR of 20 dB, Kalman filter gives MSEE of 0.0245 where as H-infinity filter gives MMSE of 0.0356.

Fig. 4.3 b) shows the MSEE performance for Dual-Kalman and Dual-H-infinity algorithms when the AR parameters are unknown. Both algorithms nearly saturates at MSEE of 0.0182. It is seen that from SNR of 10dB-15dB both performance of both filters are comparable. For SNR of 15 dB, Dual-H-infinity filter gives MMSE= 0.092 where as Dual Kalman filter gives MMSE= 0.12. For high SNR's both algorithms have similar performance.

Fig. 4.3 c) shows the MSEE performance for four different algorithms for moderate Doppler spreads. At low SNR(below 10 dB), the Dual filtering algorithm estimate the channel and performs within 4-5 dB to the Kalman filter where channel parameters are known, but as the SNR increases, it saturates.

4.4.3 BER Performance Comparison of Different Estimators

Fig. 4.4 and Fig. 4.5 shows the BER performance of different adaptive filtering algorithm in CP based channel estimation for OFDM systems. We consider a 4-tap channel, at low and moderate Doppler spreads. A CP length of $g_i = N/8 = 16$ with a (10, 90) training pattern is considered.

Fig. 4.4 a) shows the BER performance for Kalman and H-infinity filters with known AR parameters and normalized Doppler spread $f_d T = 0.001$. The BER shows a steady fall from 10 dB to 30 dB for both the filters. It may be seen that the performance of H-infinity filter is very close to the performance of Kalman filter where noise statistics are known.

Fig. 4.4 b) shows BER performance for the channel estimation in OFDM using Dual-Kalman and Dual-H-infinity algorithms, when the channel AR parameters are unknown and $f_d T = 0.001$, it may be observed that the Dual-H-infinity filter gives better estimate of AR parameters as well as noise statistics compared to Dual-Kalman filter. For instance at SNR of 20 dB Dual-H-infinity filter gives BER of 0.0026 where as the Dual-Kalman filter gives 0.0042. At high values of SNR's both filters give similar performance.

Fig. 4.4 c) shows the BER performances of four adaptive filters in a CP based channel estimation in OFDM systems for low Doppler spread of $f_d T = 0.001$. As the SNR increases the BER performance of all the filtering algorithms discussed above are similar. For BER=10^{-3}, there is a performance advantage of 5 dB for Kalman filter where AR parameters and noise statistic are known i.e., the Dual filtering algorithm tracks the channel and performs within 5 dB to the Kalman filter.

Fig. 4.5 a) show the BER performance of Kalman and H-infinity filtering algorithms in OFDM systems with fading coefficients modeled as AR-2 process and channel statistics known with $f_d T = 0.01$. Both the filters saturate at a BER 0.0084. When SNR's changes from 15 dB-25 dB Kalman filter gives better BER because of the noise statistics (variances of the state and observation noises) are known. It may be seen that for SNR=20 dB, Kalman filter gives the BER of 0.0107 while the H-infinity with unknown noise statistics gives BER of 0.016.

Fig. 4.5 b) shows BER performance of Dual-Kalman and Dual-H-infinity filtering algorithms in OFDM systems where fading coefficients of an AR-2 process are assumed to be unknown. Both the filters saturates at a BER 0.0215. For SNR's below 25 dB the graph shows that Dual H-infinity gives better BER performance.

Fig. 4.5 c) shows the BER performance of all the above filters for CP based channel estimation in OFDM systems for moderate Doppler spreads of $f_d T = 0.01$. It may be seen that there is a close similarity between the Kalman filter and H-infinity filter, both the filters saturates a BER 0.0084. The Dual-H-infinity filtering technique provides almost 2 dB better performance than Dual-Kalman filtering technique.

The results demonstrate that the Dual filtering techniques perform well at low Doppler spreads and the performances are close to the Kalman filter where the AR model parameters are known.

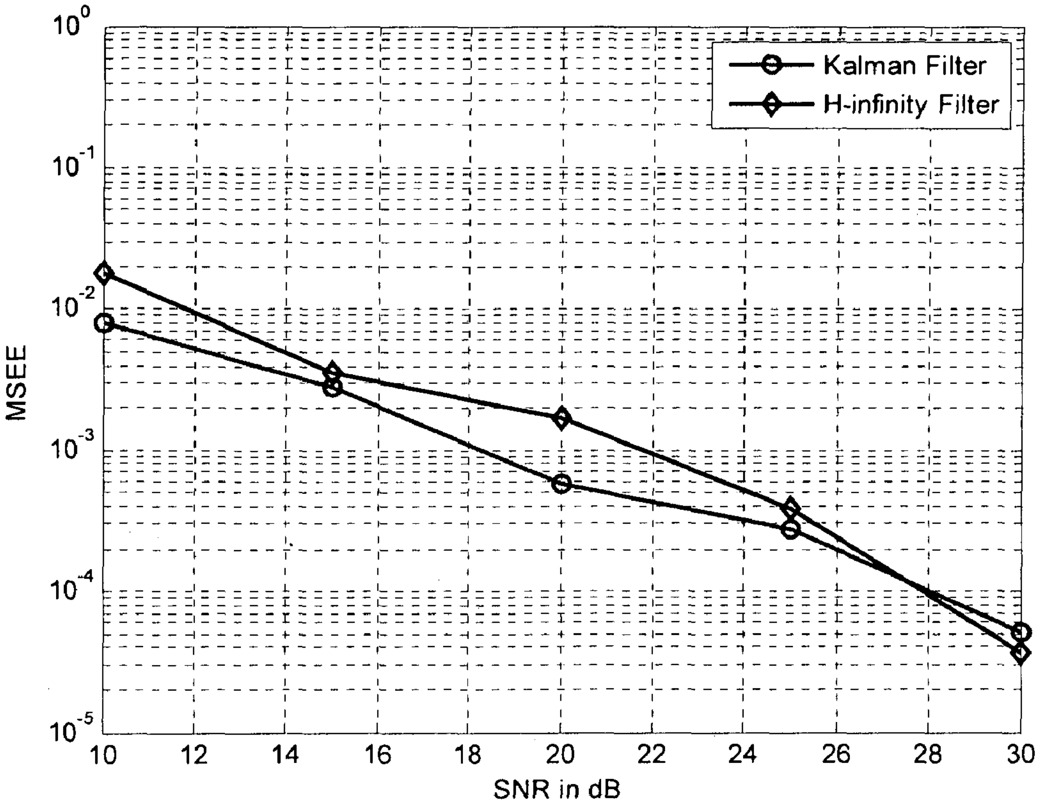

Fig. 4.2 a) MSEE performance of Kalman and H-infinity estimation algorithms,

L=4, $f_dT = 0.001$.

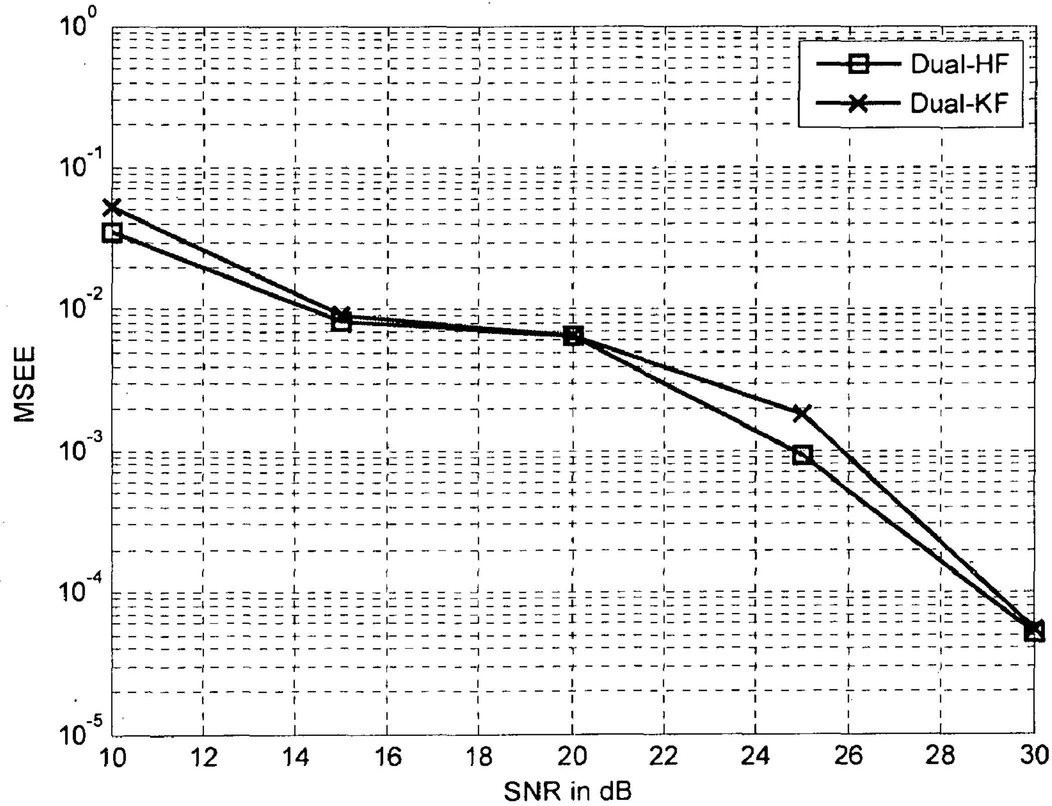

Fig. 4.2 b) MSEE performance of Dual-Kalman and Dual-H-infinity estimation algorithms,

L=4, $f_d T = 0.001$.

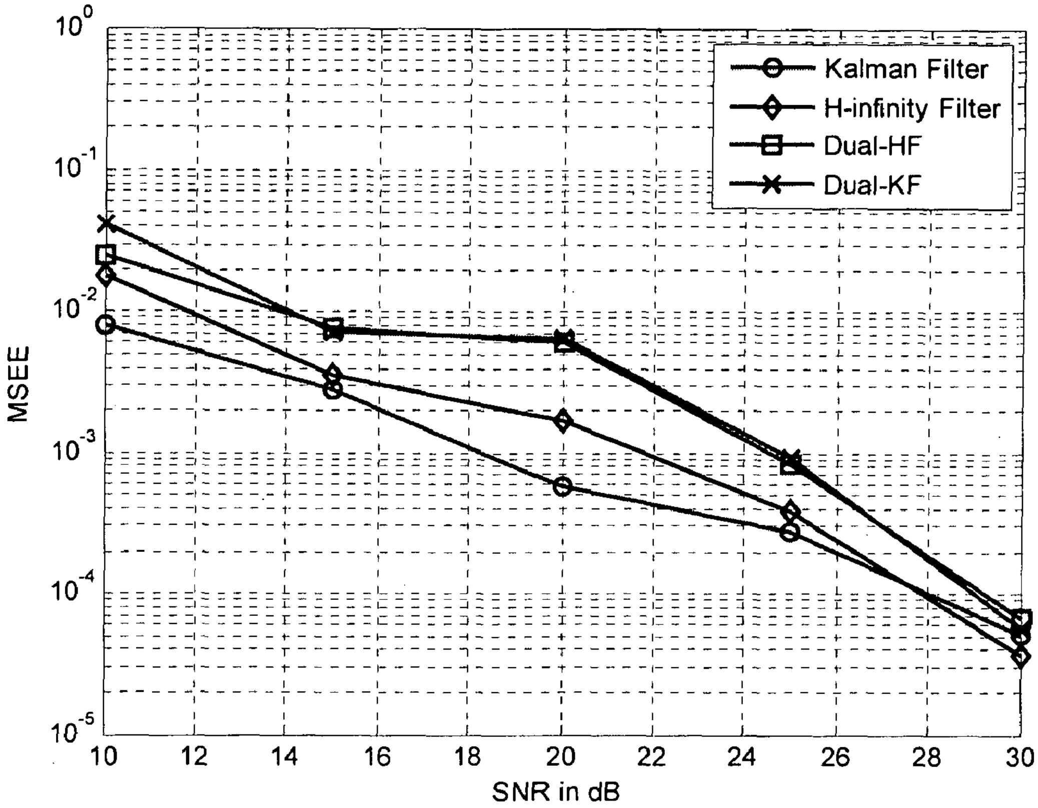

Fig. 4.2 c) MSEE performance of different estimation algorithms,

L=4, $f_d T = 0.001$.

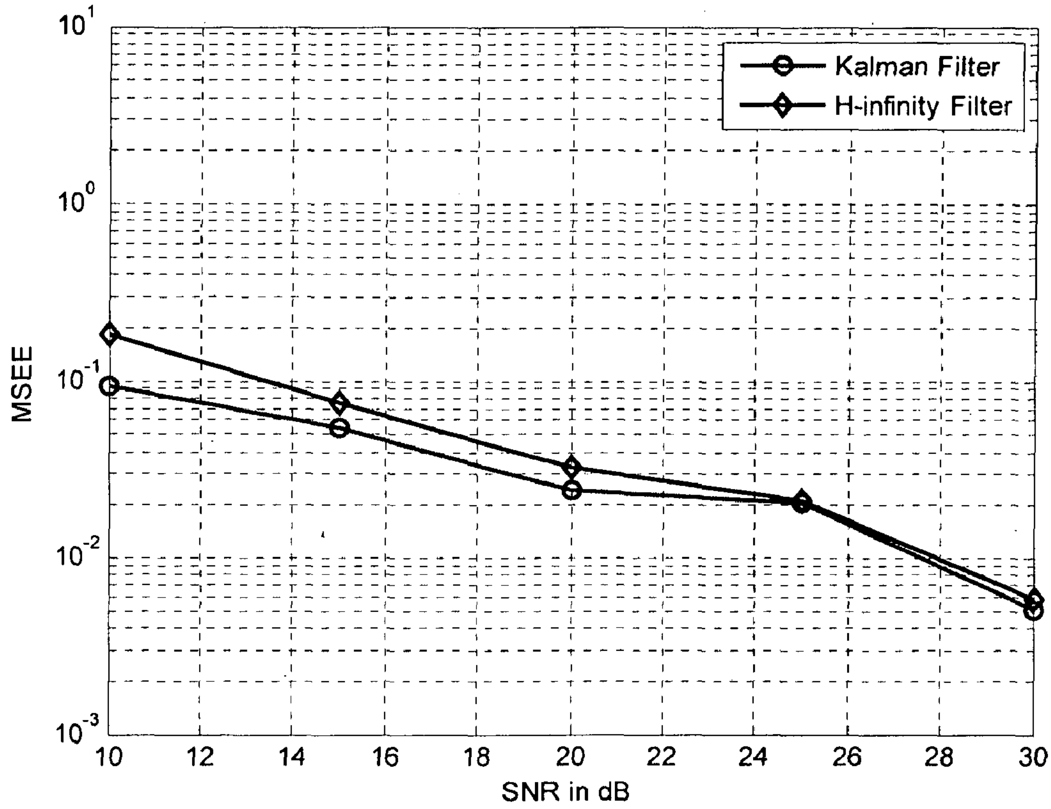

Fig. 4.3 a) MSEE performance of Kalman and H-infinity estimation algorithms,

L=4, $f_dT = 0.01$.

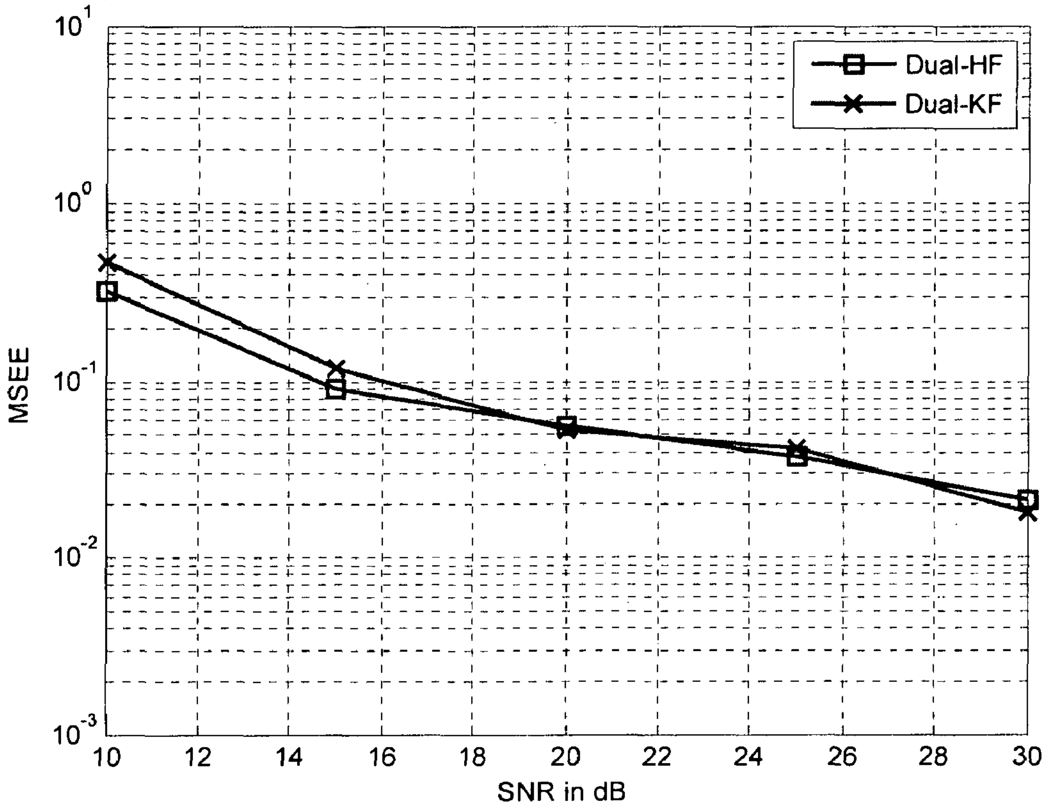

Fig. 4.3 b) MSEE performance of Dual-Kalman and Dual-H-infinity estimation algorithms,

L=4, $f_d T = 0.01$.

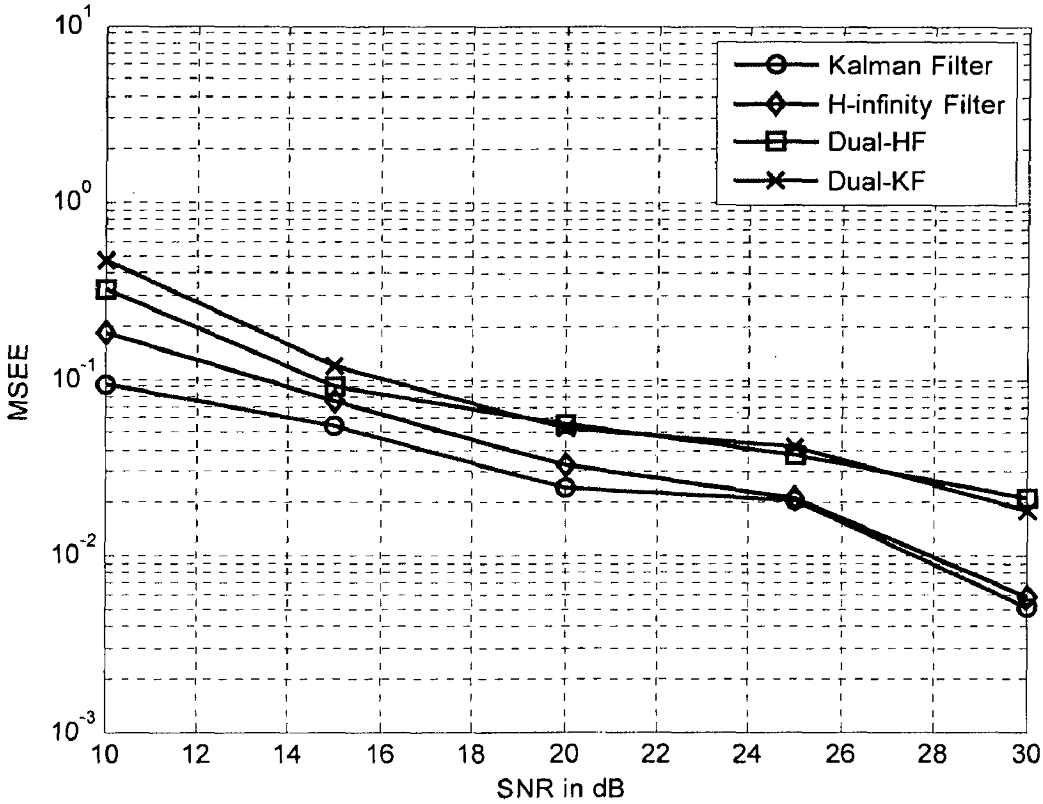

Fig. 4.3 c) MSEE performance of different estimation algorithms,

L=4, $f_d T = 0.01$.

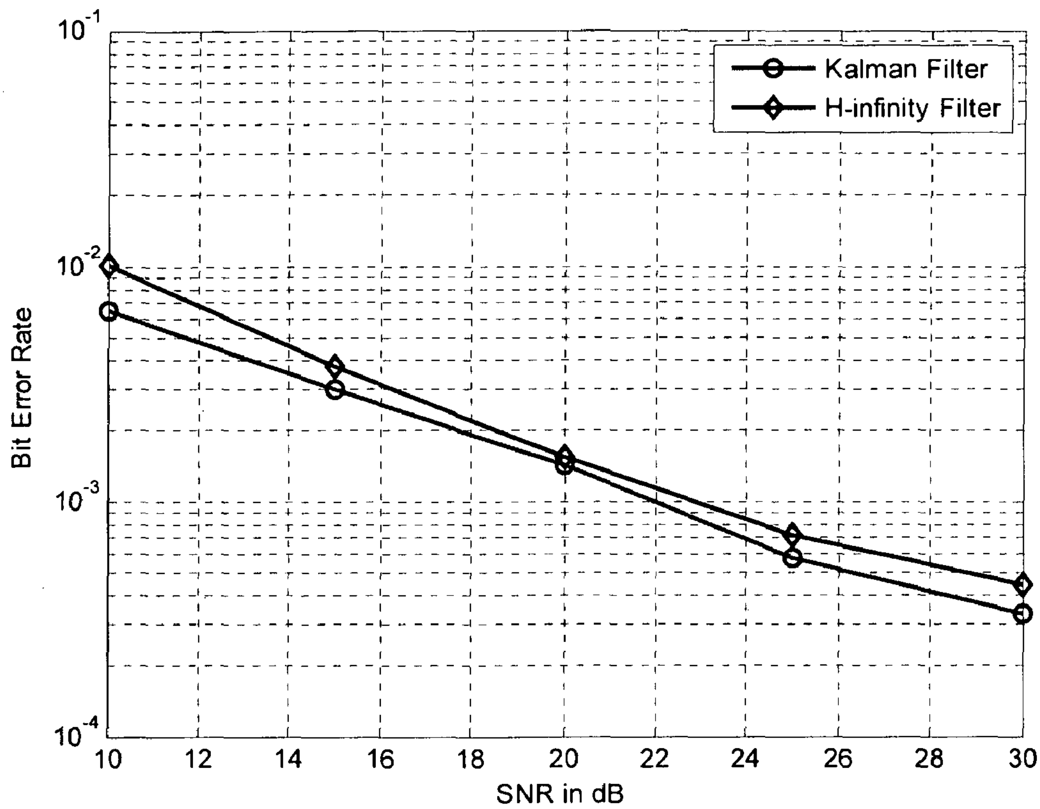

Fig. 4.4 a) BER performance of Kalman and H-infinity estimation algorithms,

L=4, $f_d T = 0.001$.

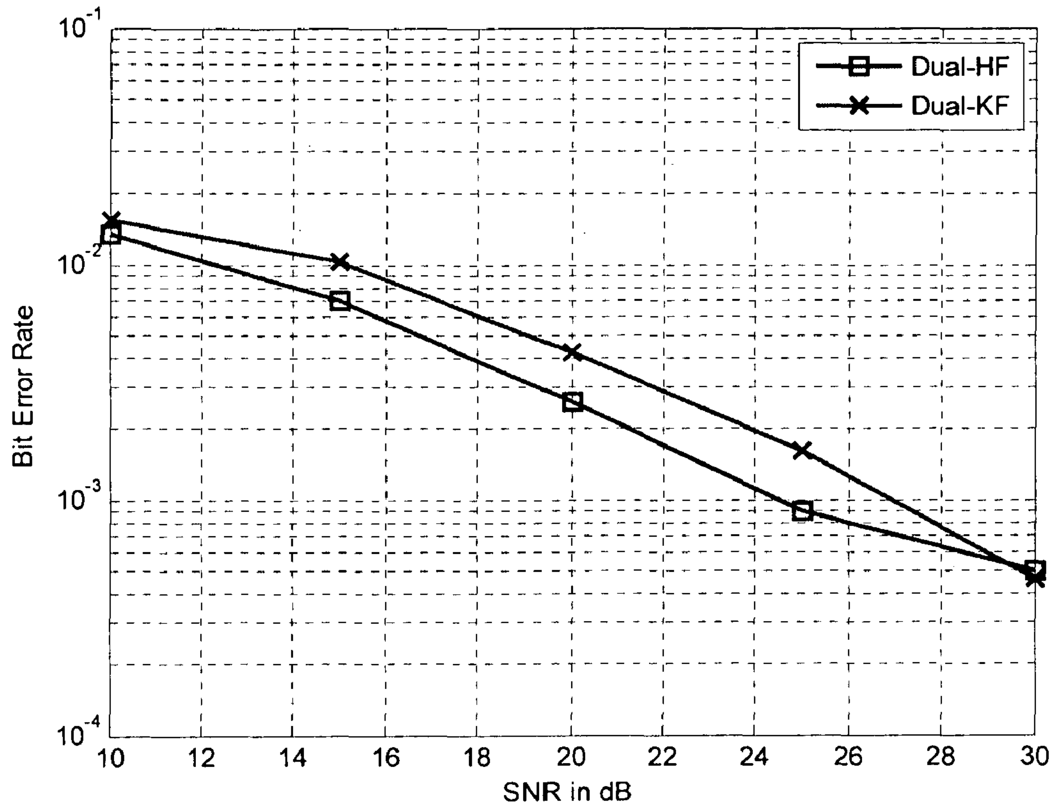

Fig. 4.4 b) BER performance of Dual-Kalman and Dual-H-infinity estimation algorithms,

L=4, $f_d T = 0.001$.

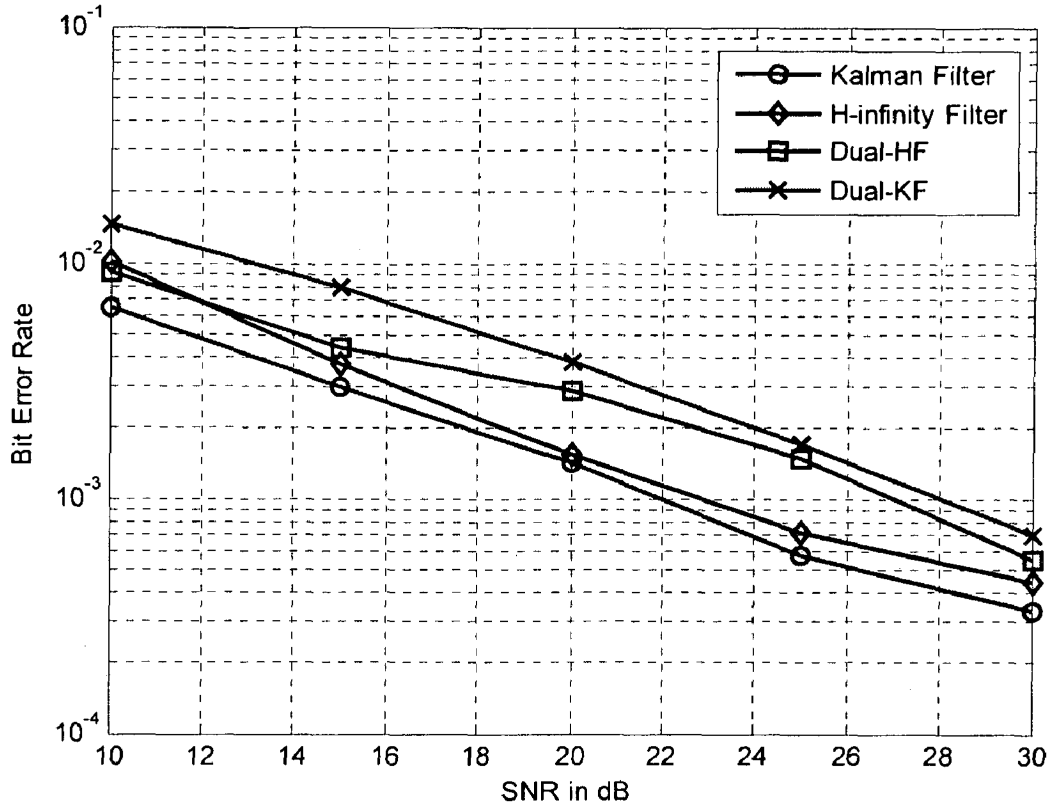

Fig. 4.4 c) BER performance of different estimation algorithms,

L=4, $f_dT = 0.001$.

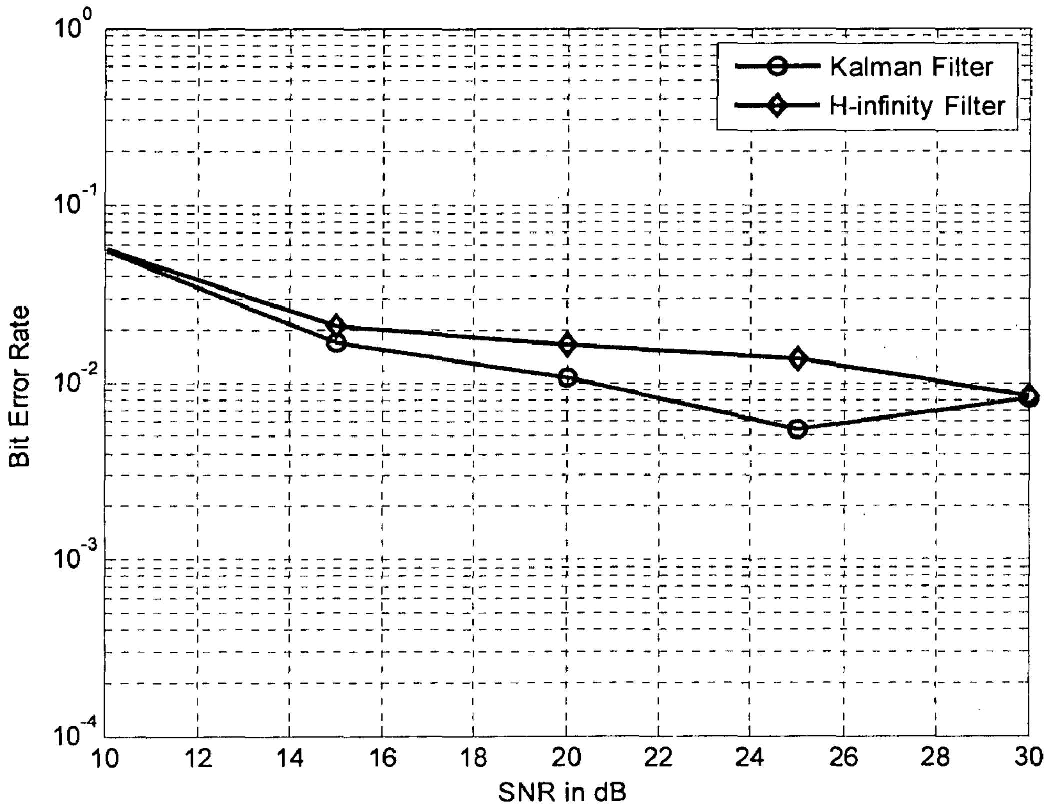

Fig. 4.5 a) BER performance of Kalman and H-infinity estimation algorithms,

L=4, $f_d T = 0.01$.

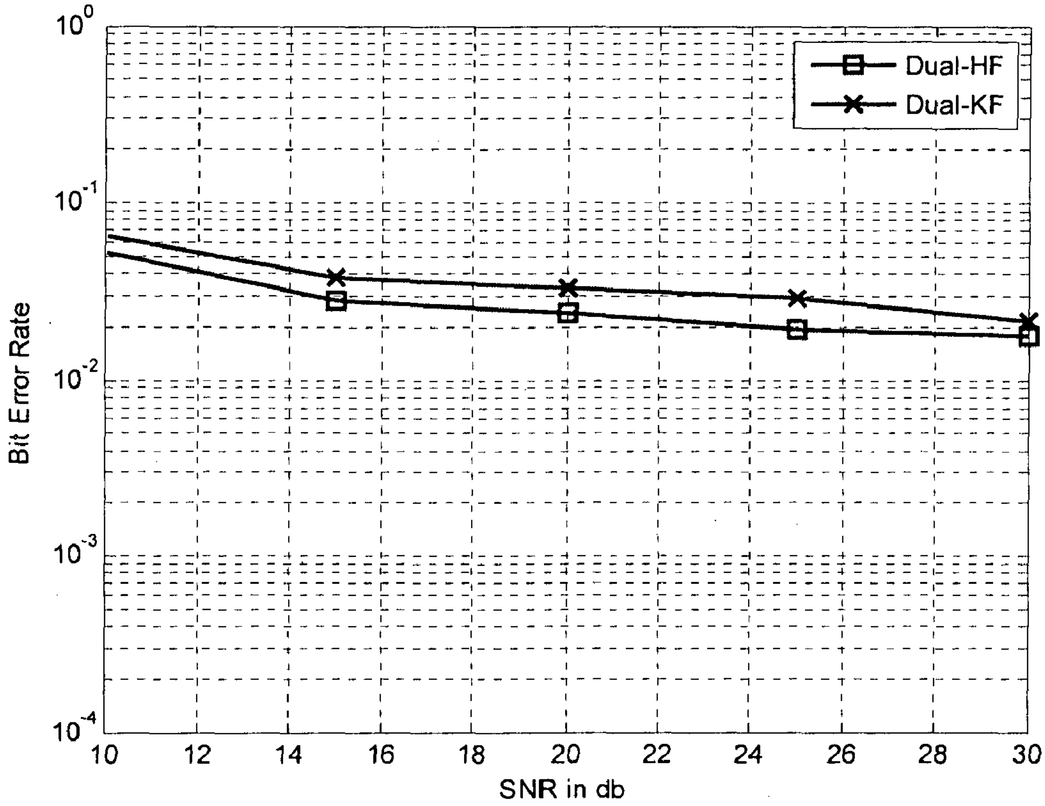

Fig. 4.5 b) BER performance of Dual-Kalman and Dual-H-infinity estimation algorithms,

L=4, $f_d T = 0.01$.

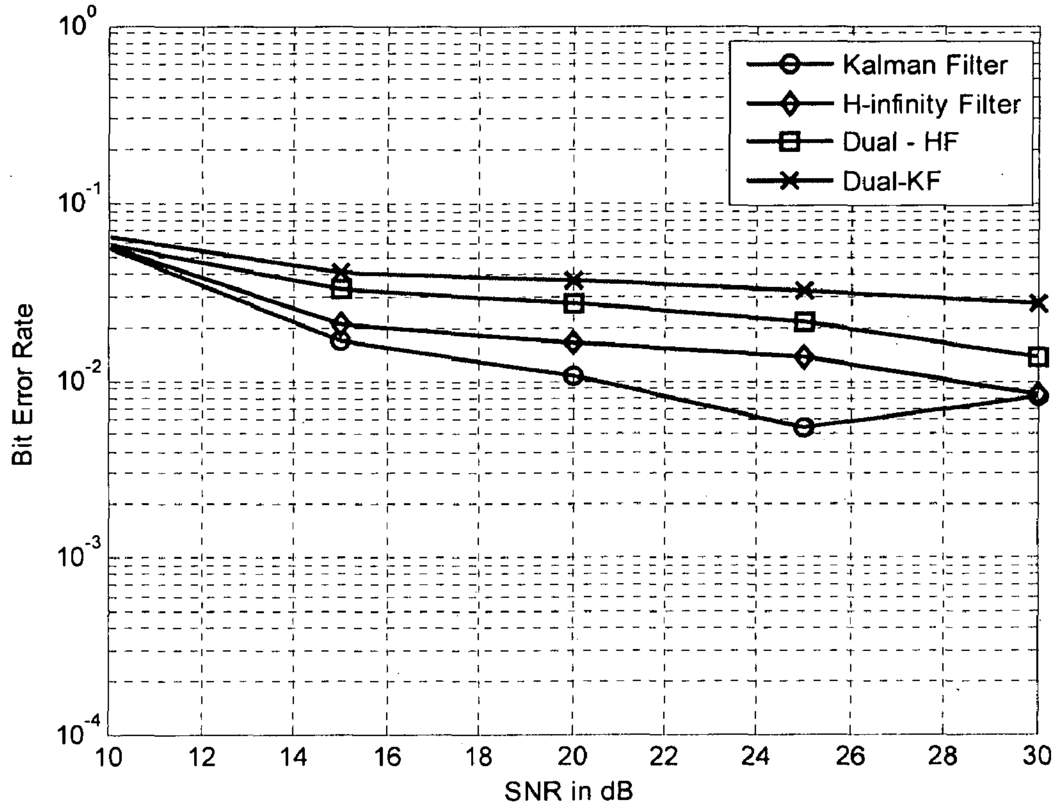

Fig. 4.5 c) BER performance of different estimation algorithms,

L=4, $f_d T = 0.01$.

Chapter 5

Conclusions

In numerous applications of signal processing and communication we are faced with the necessity to remove noise and distortion from signal. These phenomena are due to time varying processes and most of these time variations are unknown. To estimate the signal from its noisy observations a variety of adaptive state estimation filters namely Kalman and H-infinity and have found a variety of applications such as speech enhancement, linear equalization and channel estimation for wireless communication systems.

The work reported here is aimed at the application of different adaptive filtering techniques (Kalman, H-infinity, Dual-Kalman and Dual-H-infinity) for state estimation in simple AR-2 model and channel estimation in CP based OFDM systems. The received CP of initial OFDM symbols is used as training, and subsequently in the remaining interval it operates in decision directed mode. State space formulation of the problem facilitates the use of adaptive filters, which helps to find the optimal solution to a linear problem. A CP based framework is used to simplify receiver architecture and avoid the need for frequent retraining. The conclusion drawn based on the simulation results are as follows:

Adaptive Filtering Algorithms for AR Model Parameter Estimation

We have used the Autoregressive (AR) model of order 2 for estimating the state of the system from noisy observations using different adaptive filtering algorithms. As simulation results show, at high SNR the mean square error (MSE) performance of Dual filtering and H-infinity filters are close to the optimal Kalman filter. At low SNR, the Dual filtering algorithms perform within 2.5 dB of the Kalman filter. It is seen that H-infinity filter performs within 1 dB of the Kalman filter for achieving MSE of 0.2. Dual filtering algorithms exhibit very close performance. At low SNR (below 5 dB) Dual H-infinity filter shows an improvement in performance of 1 dB compared to Dual-Kalman filter. It has been observed that the estimated AR parameters values converge to the true values at high SNR and we conclude that the Dual algorithms successfully estimate the state, AR parameters and driving process variances.

Adaptive Filtering Algorithms for Channel Estimation in OFDM

We have used the state variable model for OFDM systems in CP interval for estimating the channel using different filtering algorithms. The results demonstrate that the adaptive filtering techniques perform well at low Doppler spread. As the simulation show, the MSEE and BER performance of the Dual filtering algorithms and H-infinity filtering is close to the optimal Kalman filter. Typically, for $f_d T = 0.001$ the MSEE performance of the H-infinity filter is within 1.5 dB of the Kalman filter. The MSEE of both the Dual filtering algorithms has similar performance. At high SNR values the MSEE saturates below 10^{-4} for all the filters. It may be seen that the performance of H-infinity filter is very close (0.5 to 1 dB) to the Kalman filter. For BER=10^{-3}, there is a performance advantage of 2 dB for Dual-H-infinity filter compared to the Dual - Kalman filter. The performance of dual filtering is within 5 dB to the Kalman filter for BER of 10^{-3}.

We have observed that, for moderate SNR ($f_d T = 0.01$) the MSEE of both the H-infinity and Kalman filters has similar performance. For SNR of 15 dB, Dual-H-infinity filter gives MSEE of 0.092 where as Dual-Kalman filter gives MSEE=0.12. At higher SNR the BER saturates below 10^{-2}. The Dual-H-infinity filtering technique provides 2 dB improvement in BER performance than Dual-Kalman filtering technique.

Scope of Future work

This thesis studies channel estimation for SISO-OFDM systems using different adaptive filtering algorithms (Kalman, H-infinity, Dual-Kalman and Dual-H-infinity). It is possible to extend the channel estimation method to MIMO-OFDM systems by altering the state space model suitably, which is a possible line of future work. CP based channel estimation for OFDM using adaptive filtering algorithms perform well at low Doppler spreads. However, at moderate Doppler spreads, the methods prove to be inadequate and exhibit an error floor, which calls for an improvement. The study of different efficient adaptive filtering techniques for channel estimation in OFDM systems over moderate to high Doppler spread is a significant topic of interest.

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