PEAK-TO-AVERAGE POWER RATIO REDUCTION IN OFDM SYSTEMS

A DISSERTATION

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(With Specialization in Communication Systems)

By

NARESH KUMAR JAVVAJI

DEPARTMENT OF ELECTRONICS AND COMPUTER ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE - 247 667 (INDIA)

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report entitled, **"PEAK-To-AVERAGE POWER RATIO REDUCTION IN OFDM SYSTEMS"** towards the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems**, submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from July 2007 to June 2008, under the guidance of **Dr. ARUN KUMAR, Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee.**

I have not submitted the matter embodied in this dissertation for the award of any other Degree or Diploma.

Date: 30-06-08

Place: Roorkee

NARESH KUMAR JAVVAJI

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

30\06\08

Date:

Place: Roorkee

Dr. ARUN KUMAR,
Professor, E&C Department,
IIT Roorkee,
Roorkee – 247 667 (India).

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ABSTRACT

OFDM (Orthogonal Frequency Division Multiplexing) is a promising technique for achieving high data rate and combating multipath fading in wideband data communications. OFDM can be thought as a hybrid of multi-carrier modulation (MCM) and frequency shift keying (FSK) modulation .The transmitted data by dividing into several parallel bit streams and modulating each of these data streams onto individual carriers or subcarriers.

OFDM was exploited for wideband data communications over wireless LAN(IEEE802.11a/g), WIMAX(IEEE802.16a) , High-bit-rate Digital Subscriber Lines (HDSL; 1.6 Mbps),.Asymmetric Digital Subscriber Lines (ADSL; up to 6 Mbps), Very-high-speed Digital Subscriber Lines (VDSL; 100 Mbps), Digital Audio Broadcasting (DAB), and High-Definition Television (HDTV) terrestrial broadcasting.

OFDM has many well-known advantages, such as the robustness to the intersymbol interference (ISI) and multi-path fading, high bandwidth efficiency, and so on. But it has suffered from the high Peak-to-Average Power Ratio (PAPR). The PAPR reduces the power efficiency of the RF high power amplifier in the transmitter and the complexities of analog-to-digital converter (ADC) and digital-to-analog converter (DAC).This issue is especially important for mobile terminals to sustain longer battery life time. Therefore, reducing the PAPR can be regarded as an important issue to realize efficient and affordable mobile communication services.

In current techniques, Partial Transmitted Sequences (PTS) has been proved efficient and distortion less, in which multiple candidate signals are generated to reduce the PAPR but with side effect of high complexity. In general, the complexity of PTS proportional to the number of candidate signals. However, in individual candidate signal, the complexity for combining the phase factor and computing the PAPR still remains high.

The focus of this dissertation is mainly to reduce the complexity of phase factor combining and PAPR computation for an individual candidate signal in Partial Transmit Sequence technique.

In this dissertation, a class of PTS techniques with low complexity are studied and simulated. Complementary Cumulative Distribution Function (CCDF) of the peak-to-average power of an OFDM signal are plotted to measure the PAPR reduction performance of all these techniques. The PAPR reduction performances of each of these suboptimal PTS techniques are compared with the OFDM system without any PAPR reduction technique and also conventional PTS technique.

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Chapter 1

Introduction

The wireless communication technology developed has been divided into generations based on the technology adapted, data rates offered and the user mobility. The first generation of mobile cellular telecommunications systems appeared in early 1980s. The first generation was not the beginning of mobile communication, as there were several mobile radio networks inexistence by then, but they were not cellular systems. The capacity of these early networks was much lower than that of cellular networks.

In mobile cellular networks the coverage area is divided into small cells, which allows the same frequencies to be used in different cells several times without disruptive interference. This increases the system capacity. The first generation used analog transmission techniques for traffic, which was almost entirely voice. The most successful standards were *Nordic Mobile Telephone* (NMT), *Total Access Communication Systems* (TACS), and *Advanced Mobile Phone Service* (AMPS). These protocols were developed during the 70's and 80's. These protocols supported a data transmission rate is between 9.6kbps and 14.4kbps.

The technologies developed during the 90's to 2000 come under the second-generation (2G) mobile services. The second-generation (2G) mobile cellular systems use digital transmission for traffic. The maximum data rate that can be achieved using the 2G protocols is 115kbps. The main advantage of using 2G technologies over the 1G was increase in the performance due to usage of same channel by several users(either by code or time division). By this time the cell phones were used for both voice and data communication. There are four main standards for 2G systems: *Global System for Mobile* (GSM) *communication, Digital AMPS* (D-AMPS), *code division multiple access* (CDMA) ,IS-95 and personal *digital cellular* (PDC) [1] [2].

The emergence of mobile data accessing devices like personal digital assistants (PDA's) and Internet has shifted the focus towards the data communications, which requires high data transmission rates. These have led to the developments of more advanced protocols between

2000 and 2003 and termed as 2.5G protocols. The 2.5G system includes the following technologies: H*igh-Speed Circuit-Switched data* (HSCSD), *General Packet Radio Services* (GPRS), and *Enhanced Data Rates for Global Evolution* (EDGE). The maximum data rates that can be achieved using 2.5G protocols is 144kbps, but this is not enough for enhanced multimedia and high streaming videos transmissions. So *Universal Mobile Telecommunications System* (UMTS), *Wideband Code Division Multiple Access* (WCDMA) and CDMA2000 protocols that also use the Digital Packet Switching are developed to increase the data transmission rate up to 2Mbps. These protocols were developed during 2003 to 2004 and are termed as third generation (3G) protocols.

As the demand for higher data transmission rate and worldwide roaming in cellular devices is increasing, the development of next generation (4G) wireless systems using digital broadband is underway. Therefore, enhancing system capacity as well as achieving a higher bit rate transmission is an important requirement for the 4G system. The main task is to investigate and develop a new broadband air interface which can deal with high data rates of the order of 100 Mbps, high mobility and high capacity. Since the available frequency spectrum is limited, high spectral efficiency is the major task of 4G mobile radio systems. Another important target of the new 4G air interface is the ability to provide efficient support for applications requiring simultaneous transmission of several bits of streams with possibly different Quality of Service (QoS) targets. Among these challenges, channel fading degrades the performance of wireless transmissions significantly, and becomes a bottle-neck for increasing data rates. Channel fading causes performance degradation and renders reliable high data rate transmissions a challenging problem for 4G wireless communications. To combat these situations, orthogonal frequency division multiplex (OFDM) is considered as a promising solution.

Multicarrier Modulation (MCM) is the principle of transmitting data by dividing the input data stream into several lower rate parallel bit streams and using these substreams to modulate several carriers [3]. The first systems using MCM were military HF radio links in 1960s. In a classical MCM system, the total signal frequency band is divided into N nonoverlapping frequency subchannels. Each subchannel is modulated with a separate symbol and then the N subchannels are frequency multiplexed. Spectral overlap of channels is avoided to eliminate interchannel interference. However, this leads to inefficient use of the available spectrum. To cope with the inefficiency, the ideas proposed from the 1960s were to use MCM and Frequency Division Multiplexing (FDM) with overlapping subchannels. Orthogonal Frequency Division Multiplexing (OFDM) is a special case of multicarrier transmission. The word orthogonal indicates that there is a mathematical relationship between the frequencies of carriers in the system. In a normal MCM system, many carriers are spaced apart in such a way that the signals can be received using conventional filters and demodulators. In such receivers, guard bands are introduced between different carriers and in frequency domain which results in reduction of spectrum efficiency. In OFDM, the carriers are arranged such that the frequency spectrum of the individual carriers overlap and the signals are still received without adjacent carrier interference. In order to achieve this, the carriers are chosen to be mathematically orthogonal. The receiver acts as a bank of demodulators, translating each carrier down to DC and integrating the resulting signal over a symbol period to obtain raw data. In 1971, Weinstein and Ebert [2] showed that OFDM waveforms can be generated using a Discrete Fourier Transform (DFT) at the transmitter and receiver for the modulation and demodulation. For a long time, usage of OFDM in practical systems was limited. Main reasons for this limitation were the complexity of real time Fourier Transform and the linearity required in RF power amplifiers.

Orthogonal Frequency Division Multiplexing (OFDM) has become an attractive technique and gained more popularity recently. Many new communication systems have main reasons to use OFDM because its good properties, e.g. it provides high spectral efficiency, robustness to channel fading, immunity to impulse interference, capability of handling very strong echoes (multipath fading). However since 1990s, OFDM is used for wideband data communications over mobile radio FM channels, High-bit-rate Digital Subscriber Lines (HDSL, 1.6Mbps), Asymmetric Digital Subscriber Lines (ADSL, up to 6Mbps), Very-high-speed Digital Subscriber Lines (VDSL, 100Mbps), Digital Audio Broadcasting (DAB), and High Definition Television (HDTV) terrestrial broadcasting and Local Area Network standards IEEE 802.11, IEEE 802.16. In Taiwan, The Directorate General of Telecommunication (DGT) has announced DVB-T (Digital Video Broadcasting for Terrestrial) and Eureka-147 system to be the DTV and DAB transmission standard. These two systems have used the OFDM technique and comparing this technique with the traditional analog technique, the receiver has merit from using lower S/N, also uses emitting power to achieve the same coverage of air waves, and it decreases business investment and equipment cost. The recent interest in this technique is mainly due to the recent advances in digital signal processing technology. International standards making use of OFDM for high-speed wireless communications are already established or being established by IEEE 802.11, IEEE 802.16, IEEE 802.20, and European Telecommunications Standards Institute (ETSI), Broadcast Radio Access Network (BRAN) committees. For wireless applications, an OFDM-based system can be of interest because it provides greater immunity to multipath fading and impulse noise, and eliminates the need for equalizers, while efficient hardware implementation can be realized using fast Fourier transform (FFT) techniques.

OFDM has many advantages over single carrier systems. The implementation complexity of OFDM is significantly lower than that of a single carrier system with equalizer. When the transmission bandwidth exceeds coherence bandwidth of the channel, resultant distortion may cause intersymbol interference (ISI). Single carrier systems solve this problem by using a linear or nonlinear equalization. The problem with this approach is the complexity of effective equalization algorithms. OFDM systems divide available channel bandwidth into a number of subchannels. By selecting the subchannel bandwidth smaller than the coherence bandwidth of the frequency selective channel, the channel appears to be nearly flat and no equalization is needed. Also by inserting a guard time at the beginning of OFDM symbol during which the symbol is cyclically extended, intersymbol interference (ISI) and intercarrier interference (ICI) can be completely eliminated, if the duration of guard period is properly chosen. This property of OFDM makes the single frequency networks possible. In single frequency networks, transmitters simultaneously broadcast at the same frequency, which causes intersymbol interference. Additionally, in relatively slow time varying channels, it is possible to significantly enhance the capacity by adapting the data rate per subcarrier according to the signal-to-noise ratio (SNR) of that particular subcarrier..

Beyond all these advantages, OFDM has some drawbacks compared to single carrier systems. Two of the problems with OFDM are the carrier phase noise and frequency offset. Carrier phase noise is caused by imperfections in the transmitter and receiver oscillators. Frequency offsets are created by differences between oscillators in transmitter and receiver, Doppler shifts, or phase noise introduced by nonlinear channels. The most important disadvantage of OFDM systems is that highly linear RF amplifiers are needed. OFDM is known to suffer from the high peak to average power ratio, it occurs when a number of independently modulated subcarrier adds up coherently. When N signals are added with the same phase, they produce a peak power that is N times the average power, a high PAPR brings disadvantages like an increased complexity of the A/D and D/A converts and reduced efficiency of the RF power amplifier. To reduce the PAPR, it becomes a very popular research topic.

One of the major drawbacks of OFDM is the high peak-to-average power ratio (PAPR) of the transmit signal. If the peak transmit power is limited by either regulatory or application constraints, the effect is to reduce the average power allowed under multicarrier transmission relative to that under constant power modulation techniques. This in turn reduces the range of multicarrier transmission. Moreover, to prevent spectral growth of the multicarrier signal in the form of intermodulation among subcarriers and out-of-band radiation, the transmit power amplifier must be operated in its linear region (i.e., with a large input backoff), where the power conversion is inefficient. This may have a deleterious effect on battery lifetime in mobile applications. In many low-cost applications, the drawback of high PAPR may outweigh all the potential benefits of multicarrier transmission systems.

In literature, there are many solutions to reduce PAPR of the OFDM system. In clipping technique, OFDM signal peaks larger than some threshold are clipped off [5]. Even though this is a simple technique, it introduces in-band distortion and out of band radiation and results in bit error rate (BER) performance degradation. An improvement of clipping method is clipping and filtering to remove the out of band radiation [6]. Coding is distortionless method can reduce more PAPR and coded signal has constant envelope [7, 8]. But, coding method is useful for a small of number subcarriers and low order of constellation. Moreover, required exhaustive search for good codes is an obstacle in practice. Tone Reservation (TR), that is, some carriers are omitted from data transmission and are selected via an algorithmic search (some times in an iterative way between frequency and time domain) [9]. Active Constellation Extension, that is, the signal set is warped such that edge points are allowed to have (any) amplitude larger than the original one [10]. Phase rotation techniques such as Selected Mapping (SLM) and Partial Transmit Sequences (PTS) are efficient to techniques to reduce PAPR [11] [13]. In Selective Mapping (SLM) technique [11], the sequence with the lowest PAPR after making the U different phase changes on the same input data sequence is selected and transmitted. To recover data, the receiver must know the generation process of OFDM signal and phase information. In Partial Transmit Sequence (PTS) [13], data block is divided into number of disjoint sub-blocks, orthogonality is implemented to each sub block and appropriate phase weighting factors are multiplied to each sub-block to reduce PAPR .In current techniques, Partial Transmit Sequences has been proved efficient and distortionless, in which multiple candidate signals are generated to reduce the PAPR but with the side effect of high complexity.

1.1 Statement Of The Problem

This work is aimed at finding effective Low Complexity PTS techniques for PAPR reduction in OFDM systems.

The work is presented as follows:

- Study of OFDM systems, Partial Transmit Sequence methods for Peak-to-Average Power Ratio reduction in OFDM systems with reduced complexity.
- PAPR reduction performances of Partial Transmit Sequences for different approaches to reduce the complexity of phase factor combining and PAPR computation of individual candidate signals respectively.

1.2 Organization of the Report

Chapter 1 gives an overview of the evolution of the wireless systems, a brief description about 4G systems. It summarizes the problem of the statement for this dissertation work. *Chapter 2* reviews the related work done in OFDM systems. Common components for OFDM based systems are explained. Important impairments in OFDM systems are mathematically analyzed. *Chapter 3* discusses Peak to Average Power Ratio of the OFDM signal and different approaches for PAPR reduction are listed and also its effects on the system parameters. We then discussed the factors that influence the selection of a specific PAPR reduction technique chosen. *Chapter 4* presents the implementation details of PTS PAPR reduction methods for OFDM systems. We then give the various steps involved in simulation. Finally, the performance results obtained from the simulations are presented in *Chapter 5.*

Chapter 2

Overview of
Orthogonal Frequency Division Multiplexing

In this chapter, the basic principles of Orthogonal Frequency Division Multiplexing (OFDM) are presented. First history and the general structure of OFDM are mentioned. A basic system model is given, common components for OFDM based systems are explained, and a simple transceiver based on OFDM modulation is presented. Finally different issues in OFDM system are presented.

2.1 Introduction

In Orthogonal Frequency Division Multiplexing (OFDM), the data is divided among large number of closely spaced carriers that are called subcarriers; instead of data is transferred using one carrier by traditional approach. Each subcarrier carried only a small amount of data. That means data is transferred in a parallel way instead of serial way. So, the bit rate of sub-carrier is very lower than bit rate of one carrier. Low bit rate can reduce influence of intersymbol interference (ISI).OFDM signal consists of many subcarriers. Its bandwidth is usually greater than the correlation bandwidth of the fading channel. The majority of the subcarriers should still be adequately received when some of the subcarriers are degraded by multipath fading. OFDM system uses channel coding to interleave data and to add error-correcting code. Symbols of several adjacent subcarriers being completely destroyed when OFDM signal meets burst errors caused by Rayleigh fading, but many symbols are only slightly distorted. Usually, the error of receiving data may be recovered because of multi-carriers, interleaving, and channel coding.

OFDM has many advantages, which make it an attractive modulation scheme for high speed transmission links. However, it has a problem that is high Peak-to-average power ratio (PAPR). These high peaks cause saturation in power amplifier, leading to intermediation products among the subcarrier and disturbing out of band energy. Therefore, it is favorable to study on the methods in overcoming the PAPR issues.

2.1.1 History of OFDM

In 1960, Bell Labs (in America) was the first one to study how to use spread spectrum, the OFDM technique was used in several high-frequency military systems such as KINEPLEX, ANDEFT, and KATHRYN. Much of the research on the high efficient multicarrier transmission scheme based on "orthogonal frequency" carriers. In 1971, Weinstein and Ebert applied the discrete Fourier transform (DFT) to parallel data transmission systems a part of the modulation and demodulation process [1].

In the 1980, OFDM was studied for high speed modems, digital mobile communications, and high density recording. One of the systems realized the OFDM techniques for multiplexed QAM using DFT and by using pilot tone, stabilizing carrier and clock frequency control and implementing trellis coding are also implemented and CCETT (France TELECOM Group Common Research Center for Telecommunications and Television) officially posed it in public that how to use OFDM technique efficiency to transmit digital data in wireless channel environment. After that, it went through DVB (Digital Video Broadcasting) organization to apply ETSI (European Telecommunications Standards Institute) for becoming a formal standard of (EN 300 744), now (EN300 744) becomes an international standard which had been proved from ITU (the International Telecommunication Union). Otherwise, it included 8-VSB from ATSC (Advanced Television Systems Committee) and ISDB (Integrated Services Digital Broadcasting) of OFDM technique [1].

In the 1990s, OFDM was used in broadband data communications over mobile radio FM channels, high bit rate digital subscriber lines, asymmetric digital subscriber lines, very high speed digital subscriber lines, digital audio broadcasting, and high definition television terrestrial broadcasting [1][2].

2.2. General Structure of OFDM

The basic principle of OFDM is to split a high rate input data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. Because the transmission rate is slower in parallel subcarriers, a frequency selective channel appears to be flat to each subcarrier. ISI is eliminated almost completely by adding a guard interval at the beginning of each OFDM symbol. However, instead of using an empty guard time, this interval is filled with a cyclically extended version of the OFDM symbol. This method is used to avoid ICI.

Orthogonal Frequency Division Multiplexing (OFDM) is technique based on multi carrier modulation (MCM) and frequency division multiplexing (FDM). OFDM can be considered as a modulation or multiplexing method. The multi carrier modulation is to divide the signal bandwidth into parallel subcarriers or narrow strips of bandwidth. OFDM uses subcarriers that are mathematically orthogonal; information can be sent on parallel overlapping subcarriers. From which information can be extracted individually. These properties help to reduce interference caused by neighboring carriers and makes OFDM based systems more spectrally efficient.

2.2.1 A Simple System

A block diagram of a basic OFDM system is given in Figure 2.1. Usually raw data is coded and interleaved before modulation. In a multipath fading channel, all subcarriers will have different attenuations. Some subcarriers may even be completely lost because of deep fades. Therefore, the overall BER may be largely dominated by a few subcarriers with the smallest amplitudes. To avoid this problem, channel coding can be used. By using coding, errors can be corrected up to a certain level depending on the code rate and type, and the channel. Interleaving is applied to randomize the occurrence of bit errors. Coded and interleaved data is then be mapped to the constellation points to obtain data symbols. These steps are represented by the first block of Figure 2.1.

The serial data symbols are then converted to parallel and Inverse Fast Fourier Transform (IFFT) is applied to these parallel blocks to obtain the time domain OFDM symbols. Later, these samples are cyclically extended as explained in Section 2.2.2

converted to analog signal and up-converted to the RF frequencies using mixers. The signal is then amplified by using a power amplifier (PA) and transmitted through antennas.

In the receiver side, the received signal is passed through a band-pass noise rejection filter and down-converted to baseband. After frequency and time synchronization, cyclic Prefix is removed and the signal is transformed to the frequency domain using Fast Fourier Transform (FFT) operation. And finally, the symbols are demodulated, deinterleaved and decoded to obtain the transmitted information bits.

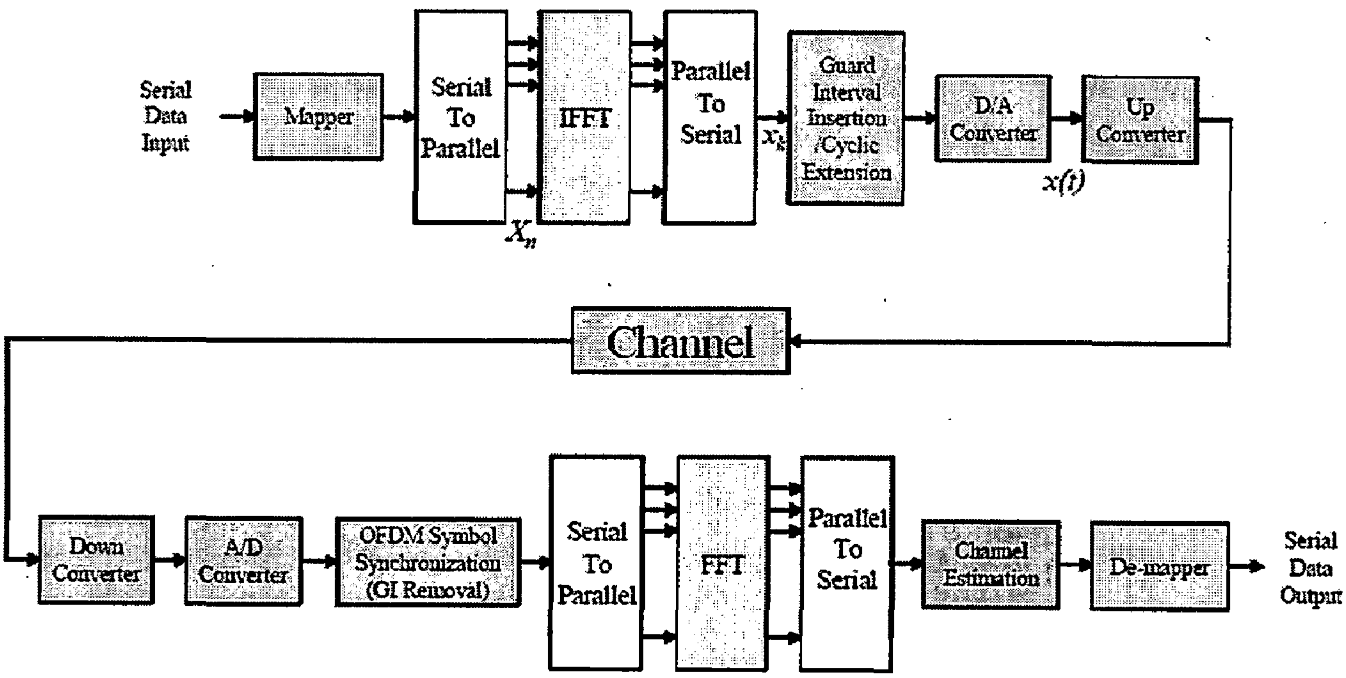


Fig: 2.1. Block Diagram of an OFDM System.

The Discrete Fourier Transform of a (DFT) discrete sequence $f(n)$ of length N, $F(k)$ is defined as ,

$$F(k) = \frac{1}{N}\sum_{n=0}^{N-1} f(n) e^{-j\frac{2\pi kn}{N}} \tag{2.1}$$

and Inverse discrete Fourier Transform $(IDFT)$ as:

$$f(n)=\sum_{k=0}^{N-1}F(k)e^{j\frac{2\pi kn}{N}} \qquad (2.2)$$

An OFDM symbol generated by an N-subcarrier OFDM system, symbol consists of N samples and then the OFDM symbol is [1] [2]

$$x_k=\frac{1}{N}\sum_{n=0}^{N-1}X_n e^{\frac{j2\pi kn}{N}} \quad ,\ 0\le k\le N-1 \qquad (2.3)$$

Where X_n is the symbol transmitted on the n[th] subcarrier and N is the number of subcarriers. Equation (2.3) is equivalent to the N-point inverse discrete Fourier transform (IDFT). Spacing of subcarriers and frequencies are carefully selected to achieve subcarrier orthogonality [1]. Symbols are obtained from the data bits using a M-ary modulation e.g. Binary Phase Shift Keying (BPSK), Quadrature Amplitude Modulation (QAM), etc. Time domain signal is cyclically extended to avoid Inter-symbol Interference (ISI). The symbols X_n are interpreted as frequency domain signals and x_k samples are interpreted as time domain signal. The spectrum of OFDM signals as shown in the figure 2.2. Applying the central limit theorem, assuming that N is sufficiently large, $x(n)$ are modeled as zero-mean complex-valued Gaussian distributed random variables. Sometimes, the sub-carriers at the end sides of the spectrum are set to zero in order to simplify the spectrum shaping requirements at the transmitter, e.g. IEEE 802.11a. These subcarriers are used as frequency guard band and referred as *virtual carriers* in literatures. To avoid difficulties in D/A and A/D converter offsets, and to avoid DC offset, the subcarrier falling at DC is not used as well.

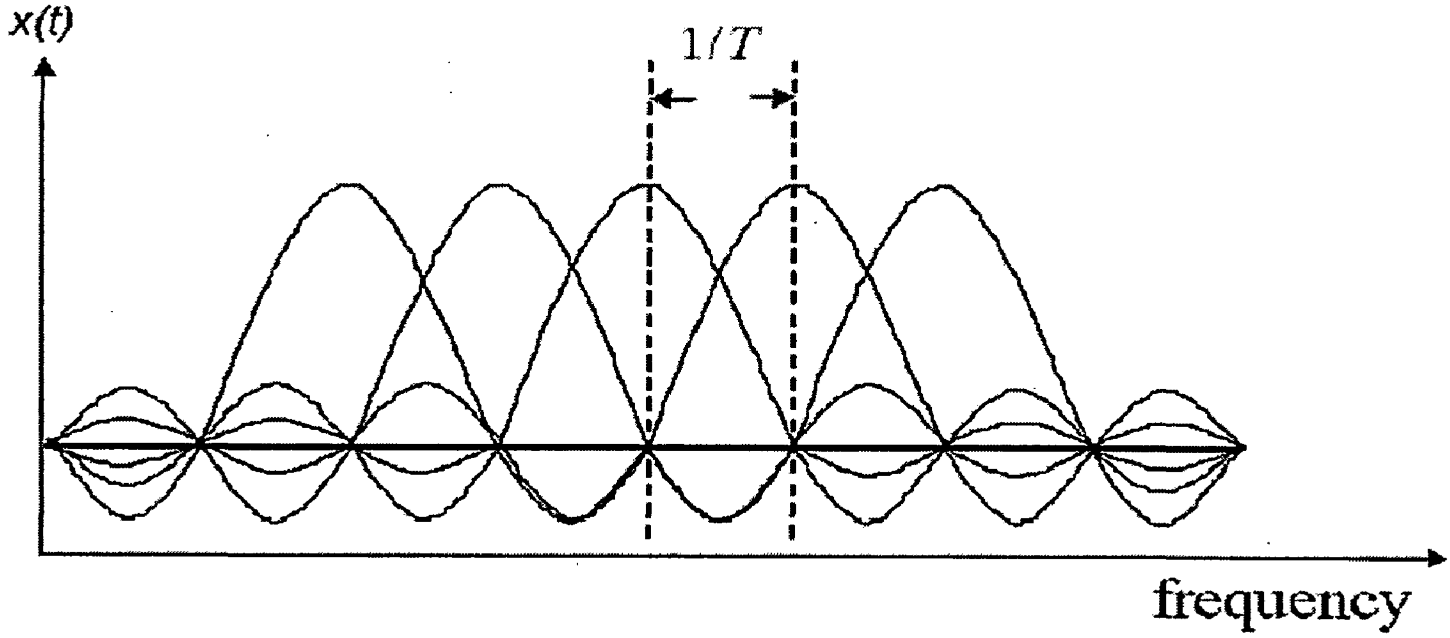


Fig: 2.2. The spectrum of OFDM

After serial to parallel conversion, inverse discrete Fourier Transform (IDFT) is applied to each stream. In practice, this transform can be implemented very efficiently by the inverse fast Fourier Transform (IFFT). The equals transition from frequency domain to time domain. After IFFT, all parallel data is summed and transmitted. Recent advances in very-large-scale integration (VLSI) technology make high-speed, large-size FFT chips commercially affordable. Using this method, both transmitter and receiver are implemented using efficient FFT techniques hat reduce the number of operations from N^2 in DFT down to $N \log_2 N$.

One of the most important features in OFDM system is the division of the frequency selective channel into smaller subchannels. These subchannels can be considers to be equal to coherence bandwidth, in which the channel is behaving like flat fading channel, Whole OFDM symbol experiences frequency selective fading and subcarrier signals flat fading channel.

2.2.2 Guard time and cyclic prefix

The orthogonal of subcarriers can be maintained and individual subcarrier can be completely separated by the FFT at the receiver when there is no intersymbol interference (ISI) and intercarrier interference (ICI) introduced by transmission channel distortion. Since the spectral of an OFDM signal is not strictly band limited, linear distortion such as multipath cause each OFDM symbol to spread energy into the adjacent OFDM symbol and consequently cause intersymbol interference (ISI). To combat intersymbol interference (ISI) a guard time is inserted between consecutive OFDM symbols. Guard time is set to be larger than the delay spread. This way ISI caused by multipath propagation is almost completely removed. As long as the delay spread is smaller than the guard time, there is no limitation in multipath component signal levels. This still leaves interference introduced by copies of the same signal [1][3].

The guard time is denoted as T_g, is chosen larger than delay spread, where T_g is usually less than ($T_{u/4}$), such that multipath components from one symbol cannot interfere with the next symbol. The total symbol duration is $T_{total}=T_g+T_u$, where T_g is the guard interval and T_u is the useful symbol duration. When the guard interval is longer than the channel impulse response, or the multipath delay, the ISI can be eliminated. The guard time is usually implemented with a cyclic extension of the symbol. Part of the signal end is placed in the front of the signal. This effectively extends signal period and still maintains orthogonality of the waveform. In practice guard time is only added to OFDM symbol, not all subcarriers. The basic idea here is to replicate part of the OFDM time-domain symbol from back to the front to create a guard period. This is shown in the Figure 2.3. This figure also shows how cyclic prefix prevents the ISI. As can be seen from the figure, as long as maximum excess delay $(\tau_{\max})$ is smaller than the length of the cyclic extension (T_g), the distorted part of the signal will stay within the guard interval, which will be removed later at the transmitter. Therefore ISI will be prevented.

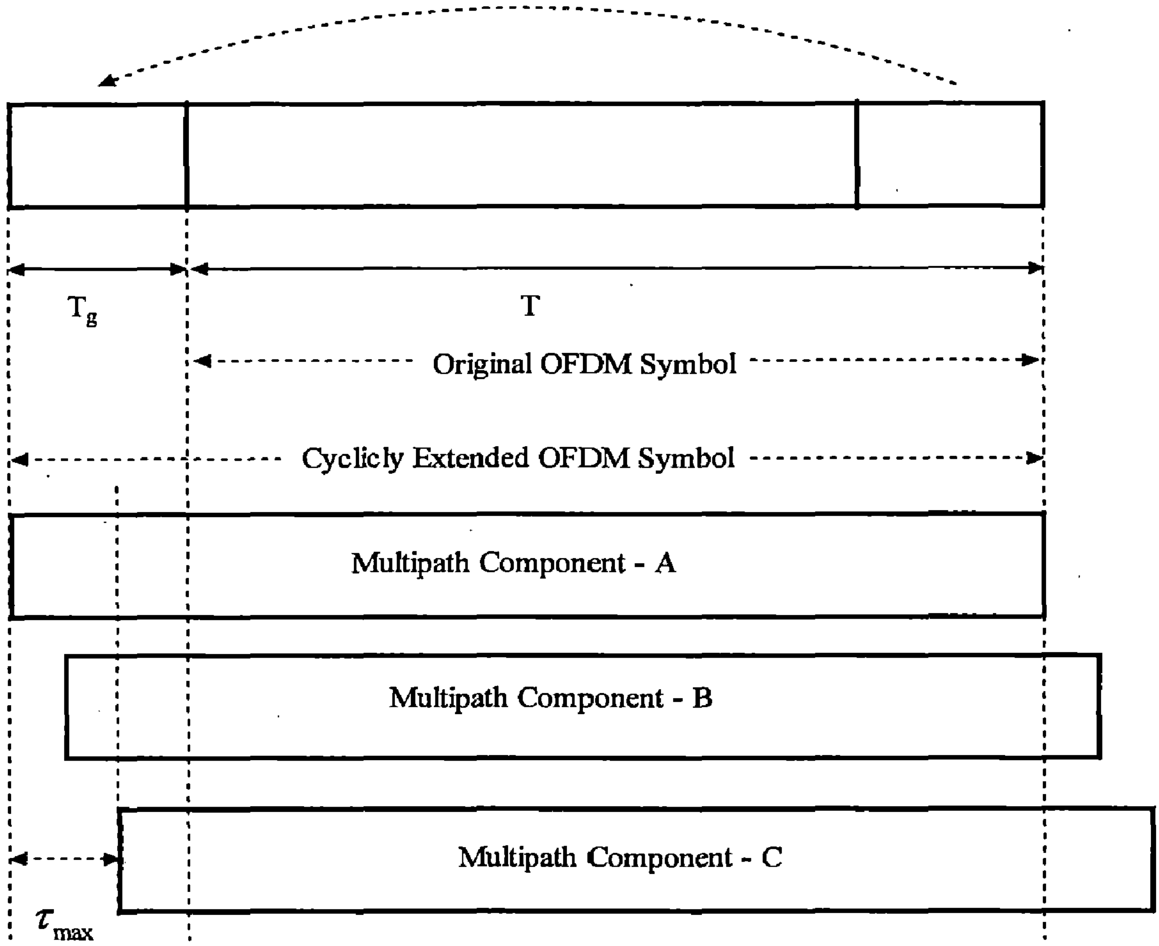


Fig: 2.3. Illustration of Cyclic Prefix Extension

The intercarrier interference (ICI) is due to crosstalk between different subcarriers, which means that the subcarriers are no longer orthogonal. When an OFDM receiver tries to demodulate the first subcarrier, it will encounter some interference from the second subcarrier, because within the FFT interval, there is no integer number of cycles difference between subcarrier 1 and 2. At the same time, there will be crosstalk from the first to the second subcarrier for the same reason [4].

This ensures that delayed replicas of the OFDM carrier always have an integer number of cycles within the FFT interval, as long as the delay is smaller than the guard time. This does not destroy the orthogonal between the subcarriers and introduces a different phase shift for each subcarrier. The orthogonality will be lost if the multipath delay becomes larger than the guard time.

The ratio of the guard interval to the useful symbol duration is application dependent. If this ratio is large, then the overhead will increase causing a decrease in the system throughput. A cyclic prefix is used for the guard time for the following reasons;

1. To maintain the receiver time synchronization; since a long silence can cause synchronization to be lost.
2. To convert the linear convolution of the signal and channel to a circular convolution and thereby causing the DFT of the circularly convolved signal and channel simply be the product of their respective DFTs
3. It is easy to implement in FPGAs.

Cyclic Prefix or Postfix

Postfix is the dual of prefix. In postfix, the beginning of OFDM symbol is copied and appended at the end. If we use prefix only, we need to make sure that the length of cyclic prefix is larger than the maximum excess delay of the channel; if we use both cyclic prefix and postfix, then the sum of the lengths of cyclic prefix and postfix should be larger than the maximum excess delay.

2.3. Different Issues in OFDM

2.3.1 Synchronization

An OFDM receiver needs to perform synchronization operation prior to subcarrier demodulation. Two types of synchronizations are needed. First, symbol boundaries are determined and then proper sampling instants are found out in order to minimize ICI and ISI effects. Second step of synchronization is used to eliminate the carrier frequency offset on the received signal. In an OFDM system, the orthogonality assumption is valid only if the transmitter and the receiver use exactly the same frequency. Any frequency offset results in intercarrier interference (ICI). Additionally, a practical oscillator does not produce a carrier at exactly one frequency; instead it produces a carrier that is phase modulated by random phase jitter. Hence, frequency is never perfectly constant resulting in some ICI in OFDM receiver.

2.3.1.1. Sensitivity to Timing Errors

OFDM is more robust to timing errors relative to phase noise and frequency offsets. The symbol timing offset may vary over an interval equal to the guard time without causing ICI and ISI. Interferences occur only when the FFT interval extends over a symbol boundary or extends over the roll-off region of a system. Therefore, we can state that OFDM demodulation is quite insensitive to timing errors. However, in order to achieve the best possible multipath robustness, there is an optimal timing instant. Deviation from this timing instant increases the sensitivity to delay spread, so the system can handle less delay spread than it is designed for. To minimize this sensitivity, the system should be designed such that the timing error is small compared to the guard interval.

There is several OFDM synchronization techniques used in the literature. The synchronization methods in two categories: *Synchronization Using Cyclic Extension* and *Synchronization Using Special Training Symbols.*

2.3.2 Frequency Offset

Frequency offset is a critical factor in OFDM system design. It results in inter-carrier interference (ICI) and degrades the orthogonality of sub-carriers. Frequency errors will tend to occur from two main sources. These are local oscillator errors and common Doppler spread. Any difference between transmitter and receiver local oscillators will result in a frequency offset. This offset is usually compensated for by using adaptive frequency correction (AFC), however any residual (uncompensated) errors result in a degraded system performance.

2.3.3 Phase noise

Phase noise is introduced by local oscillator in any receiver and can be interpreted as a parasitic phase modulation in the oscillator's signal. Phase noise can be modeled as a zero mean Gaussian random variable. Phase noise has two main effects. First, the phase noise results in attenuation and rotation of the received signal. Second and more important effect is the ICI, because phase noise changes the 1/T separation between subcarriers in the frequency domain.

2.3.4 The PAPR problem of OFDM

An OFDM signal consists of a number of independently modulated sub-carriers, which can give a large peak-to-average power ratio (PAPR) when added up coherently. When N signals are added with the same phase, they produce a peak power that is N times the average power. The situation is plotted in Figure 2.4. Because OFDM signal has a very large PAPR, it is very sensitive to non-linearity of the high power amplifier that is RF power amplifiers should be operated in a very large linear region. Otherwise the signal peaks get into non-linear region of the power amplifier causing signal distortion [1].

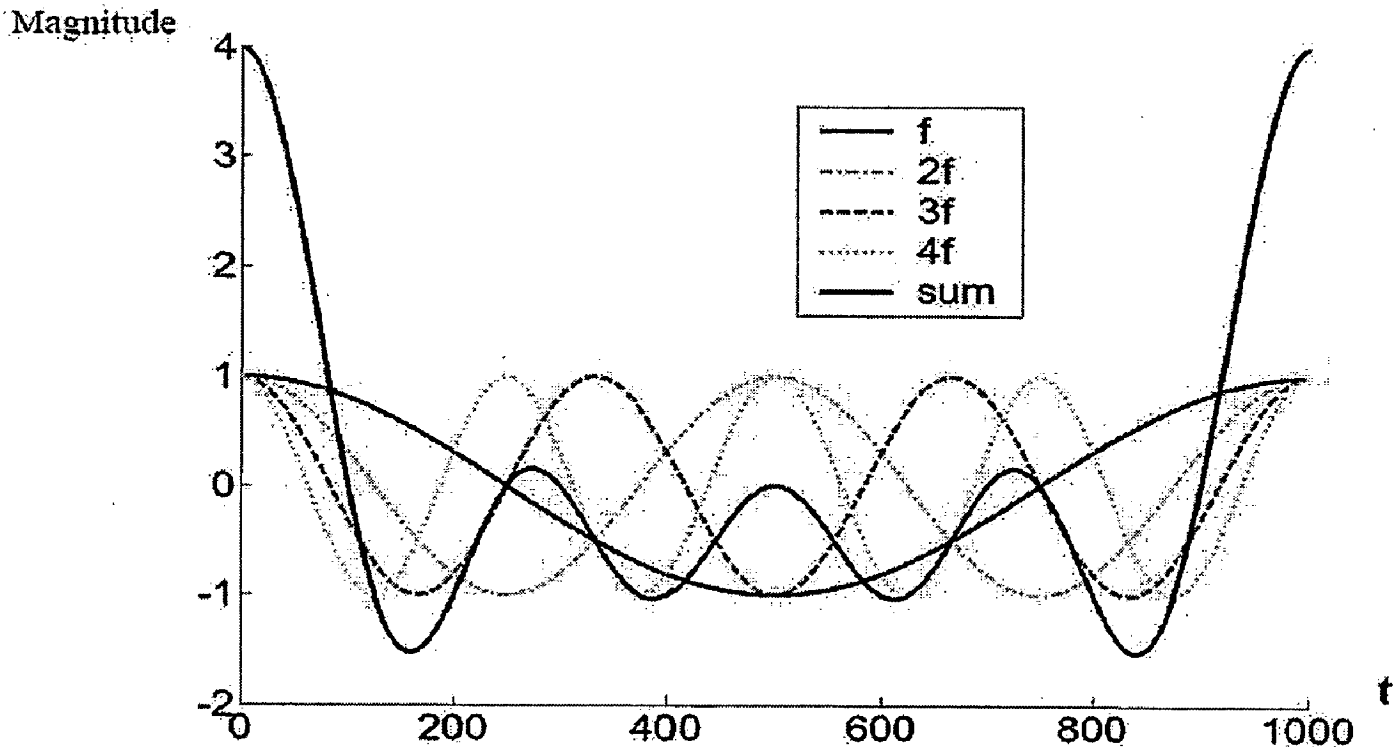


Fig: 2.4. four subcarriers within one OFDM symbol

This distortion introduces intermodulation among the subcarriers and out of band radiation and the high PAPR increases the complexity of analog to digital (A/D) and digital to analog (D/A) converters.

2.3.5 The CCDF of the PAPR [4]

Because PAPR is a random variable, to evaluate the statistical properties of PAPR we consider the cumulative distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold. In [1] a simple approximate expression is derived for the CCDF of the PAPR of a multicarrier signal with Nyquist rate sampling. From the central limit theorem, the real and imaginary parts of the time domain signal samples follow Gaussian distributions, each with a mean of zero and a variance of 0.5 for a multicarrier signal with a large number of subcarriers. Hence, the amplitude of a multicarrier signal has a Rayleigh distribution, while the power distribution becomes a central chi-square distribution with two degrees of freedom.

The probability density function of chi-square distribution with two degrees of freedom is given by

$$p_y(y_n) = \frac{1}{\sigma_x^{\,2}} \exp(-\frac{y_n}{\sigma_x^{\,2}}) \tag{2.5}$$

This distribution has cumulative distribution function (CDF)

$$pr(y_n < y_0) = \int_0^{y_0} p_y(u)du = 1 - \exp({y_0}/{\sigma_x^{\,2}}) \tag{2.6}$$

And the CCDF can be given by

$$pr(y_n > y_0) = \int_{y_0}^{\infty} p_y(u)du = 1 - \left(1 - \exp({y_0}/{\sigma_x^{\,2}})\right) \tag{2.7}$$

The probability that PAPR of randomly generated N-OFDM symbol exceeds the given PAPR threshold PAPR0= ${y_0}/{\sigma_x^{\,2}}$ can approximated by the expression

$$CCDF = \Pr(PAPR > PAPR_0) = 1 - \prod_{n=0}^{N-1} \Pr\left(\frac{y_n}{\sigma_x^2} < \frac{y_0}{\sigma_x^2}\right) = 1 - \{1 - \exp(-PAPR_0)\}^N \qquad (2.8)$$

This expression assumes that the N time domain signal samples are mutually independent and uncorrelated. This is not true, however, when oversampling is applied. Also, this expression is not accurate for a small number of subcarriers since a Gaussian assumption does not hold in this case. Therefore, there have been many attempts to derive more accurate distribution of PAPR.

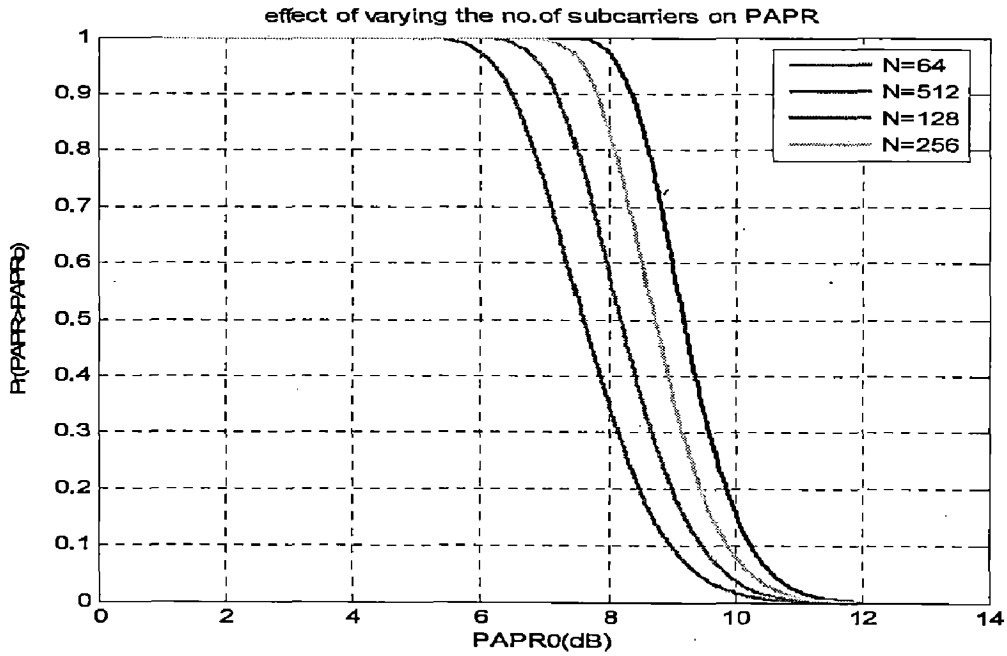


Fig: 2.5. CCDFs of PAPR of an OFDM signal with 64,128,256 and 512 subcarriers

The CCDFs are usually compared in a graph such as Fig. 2.5, which shows the CCDFs of the PAPR of an OFDM signal with 64,128,256 and 512 subcarriers (N = 64,128,256,512) for Quadrature Amplitude Modulation (QAM) modulation and oversampling factor 4 (L = 4).PAPR of an OFDM signal increases with the increase of the number of subcarriers.

PAPR Reduction Approach

In this chapter, we first give a brief review of the Peak-to-Average Power Ratio (PAPR) of the OFDM signal. We then discuss and compare different PAPR reduction methods. We then finally introduced criteria for the selection of the PAPR reduction technique.

3.1 The PAPR of the OFDM signal

As we have seen in the last section the basic cause of a high PAPR in the OFDM signal is the Gaussian signal distribution which arises due to the large number of subchannels and their linear combination due to the IFFT operation.

OFDM signal is the sum of many independent signals modulated onto subchannels of equal bandwidth. Let us denote the collection of all *data symbols* X_n, $n = 0, 1, \ldots, N-1$, as a vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$ that will be termed a *data block.* The complex baseband representation of a multicarrier signal consisting of N subcarriers is given by [4]

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n . e^{j2\pi n \Delta f t}, 0 \le t < NT \qquad (3.1)$$

Where $j = \sqrt{-1}$, Δf is the subcarrier spacing, and NT denotes the useful data block period. In OFDM the subcarriers are chosen to be orthogonal (i.e., $\Delta f = 1/NT$).

The PAPR of the transmit signal is defined as

$$PAPR = \frac{\max\limits_{0 \le t < NT} |x(t)|^2}{1/NT \cdot \int_0^{NT} |x(t)|^2 \, dt} \qquad (3.2)$$

"L-times oversampled" time-domain signal samples are represented as a vector $\mathbf{x} = = [x_0, x_1, \ldots, x_{NL-1}]$

$$x_k = x(kT/L) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n . e^{j2\pi n \Delta f t}, 0 \le t < NT, \qquad (3.3)$$

It can be seen that the sequence $\{x_k\}$ can be interpreted as the inverse discrete Fourier transform

(IDFT) of data block **X** with $(L-1)N$ zero paddings. It is well known that the PAPR of the continuous-time signal cannot be obtained precisely by the use of Nyquist rate sampling, which corresponds to the case of $L = 1$. $L = 4$ can provide sufficiently accurate PAPR results. The PAPR computed from the L times oversampled time domain signal samples is given by

$$PAPR = \frac{\max_{0 \le k < NL-1} |x_k|^2}{E\left[|x_k|^2\right]} \tag{3.4}$$

3.2 PAPR Reduction Methods

The PAPR is considered as one of the major problem in the multicarrier communication systems and a large number of efforts have been put to solve this problem. A number of different PAPR reduction approaches have been developed in the recent years. The different methods which are proposed can be categorized into several classes such as transparent and non transparent or distortion and distortion less and in this section, summarizes different methods how to solve the PAR problem. Before going into details of different PAPR reduction methods we look at the goal of PAR reduction. The goal of a PAPR reduction algorithm is to lower the PAPR as much as possible, while at the same time not disturbing other parts of the system. The complexity of the algorithm should not be too high and it should be easily implement able. We broadly define the two categories of PAR reduction methods.

3.2.1 Transparent Methods

Here the receiver does not require knowledge about the method applied by the transmitter. Similarly, the receiver can use a method unknown to the transmitter. These methods can be easily implemented in existing standards without any changes to existing specifications.

3.2.1.1 Amplitude clipping and Filtering

The simplest technique for PAPR reduction might be amplitude clipping. Figure 3.1 shows the clipping operation [1]. Amplitude clipping limits the peak envelope of the input signal to a predetermined value or otherwise passes the input signal through unperturbed, that is,

$$B(x) = \begin{cases} x, & |x| < A \\ Ae^{j\phi(x)}, & |x| > A \end{cases}, \tag{3.5}$$

Where $\phi(x)$ is the phase of x.

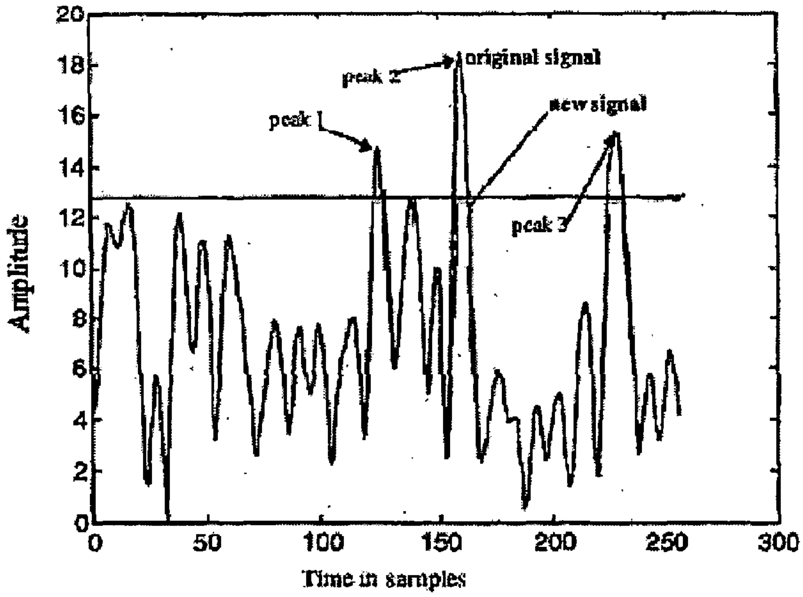


Fig: 3.1. Clipping of OFDM signal

The distortion caused by amplitude clipping can be viewed as another source of noise [5]. The noise caused by amplitude clipping falls both in-band and out of- band. In-band distortion cannot be reduced by filtering and results in error performance degradation, while out-of-band radiation reduces spectral efficiency. Filtering after clipping can reduce out-of-band radiation but may also cause some peak regrowth so that the signal after clipping and filtering will exceed the clipping level at some points [6]. However the filters used in these techniques are complicated and computationally expensive.

3.2.1.2 Peak Windowing

Nee and Prasad [1] mention that to remedy the out-of-band problem of clipping, a different approach is to multiply large signal peaks with a certain nonrectangular window. Figure 3.2 demonstrates how windowing is applied to an OFDM signal. Gaussian shaped window, cosine, Kaiser, and Hamming windows are all suitable. To minimize the out-of-band interference, ideally the window should be as narrowband as possible. On the other hand, the window should not be too long in the time domain; because that implies that many signal samples are affected, which increases the BER.

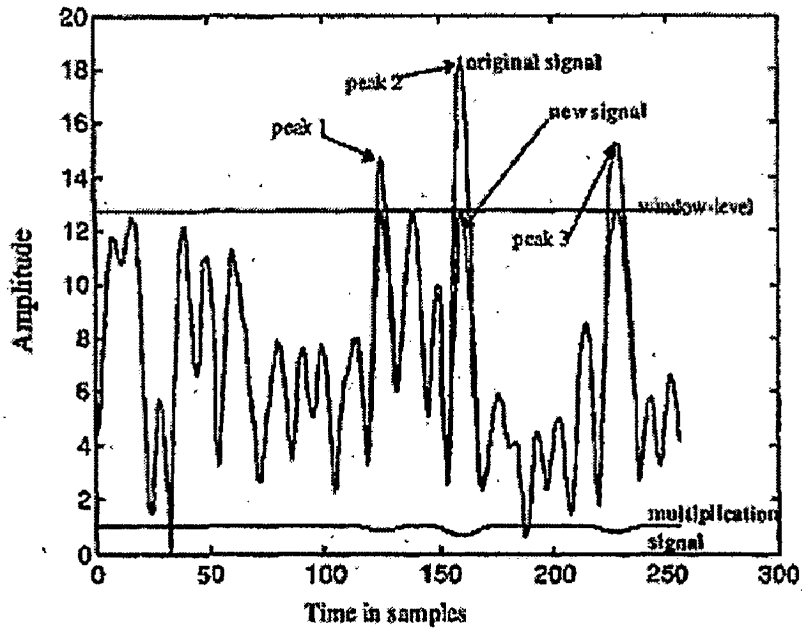


Fig: 3.2. Peak Windowing of OFDM Signal

3.2.1.3 Peak Cancellation

In the case of clipping and peak windowing, this was done by a nonlinear distortion of the OFDM signal, which resulted in a certain amount of out-of-band radiation. This undesirable effect can be avoided by doing a linear peak cancellation technique, whereby a time-shifted and scaled reference function is subtracted from the signal. Such that each subtracted reference function reduces the peak power of at least one signal sample. By selecting an appropriate reference function with approximately the same bandwidth as the transmitted signal, it can be assured that the peak power reduction does not cause any out-of-band interference. A sinc function would be a good choice, but unfortunately it is not time-limited. One way to do this without creating unnecessary out-of-band interference is to multiply it by a windowing function, for instance, a raised cosine window. If the windowing function is the same as used for windowing of the OFDM symbols, then it is assured that the reference function has the same bandwidth as the regular OFDM signal. Hence, peak cancellation will not degrade the out-of-band spectrum properties [1].

3.2.1.4 Tone reservation technique

Tone reservation (TR) and tone interjection (TI), explained below, are two efficient techniques to reduce the PAPR of a multicarrier signal. These methods are based on adding a data-block-dependent time domain signal to the original multicarrier signal to reduce its peaks.

This time domain signal can be easily computed at the transmitter and stripped off at the receiver.

For the TR technique, the transmitter does not send data on a small subset of subcarriers that are optimized for PAPR reduction [9]. The objective is to find the time domain signal to be added to the original time domain signal $\mathbf{x}$ such that the PAPR is reduced. If we add a frequency domain vector $\mathbf{C} = [C_0, C_1, \ldots, C_{N-1}]^T$ to $\mathbf{X}$, the new time domain signal can be represented as $\mathbf{x} + \mathbf{c} = \text{IDFT}\{\mathbf{X} + \mathbf{C}\}$, where $\mathbf{c}$ is the time domain signal due to $\mathbf{C}$. The TR technique restricts the data block $\mathbf{X}$ and peak reduction vector $\mathbf{C}$ to lie in disjoint frequency subspaces (i.e., $X_n = 0$, $n \in \{i_1, i_2, \ldots, i_L\}$ and $C_n = 0$, $n \notin \{i_1, i_2, \ldots, i_L\}$). The L nonzero positions in $\mathbf{C}$ are called peak reduction carriers (PRCs). Since the subcarriers are orthogonal, these additional signals cause no distortion on the data bearing subcarriers. To find the value of C_n, $n \in \{i_1, i_2, \ldots, i_L\}$, we must solve a convex optimization problem that can easily be cast as a linear programming (LP) problem.

In the case of DMT for wireline systems, there are typically subcarriers with SNRs too low for sending any information, so these subcarriers must go unused and are available for PAPR reduction. In wireless systems, however, there is typically no fast reliable channel state feedback to dictate whether some subcarriers should not be used. Instead, a set of subcarriers must be reserved regardless of received SNRs, resulting in a bandwidth sacrifice.

3.2.1.5 The Active Constellation Extension technique

Active constellation extension (ACE) is a PAPR reduction technique similar to TI [10]. In this technique, some of the outer signal constellation points in the data block are dynamically extended toward the outside of the original constellation such that the PAPR of the data block is reduced. The main idea of this scheme is easily explained in the case of a multicarrier signal with QPSK modulation in each subcarrier. In each subcarrier there are four possible constellation points that lie in each quadrant in the complex plane and are equidistant from the real and imaginary axes. Assuming white Gaussian noise, the maximum likelihood decision regions are the four quadrants bounded by the axes; thus, a received data symbol is decided according to the quadrant in which the symbol is observed. any point that is farther from the decision boundaries than the nominal constellation point (in the proper quadrant) will offer increased margin, which guarantees a lower BER. We can therefore allow modification of constellation points within the quarter-plane outside of the nominal constellation point with no degradation in performance. This principle is illustrated in Fig. 3.3, where the shaded region represents the region of increased margin for the data symbol in the first quadrant. If adjusted intelligently, a combination of these additional signals can be used to partially cancel time domain peaks in the transmit signal.

The ACE idea can be applied to other constellations as well, such as QAM and MPSK constellations, because data points that lie on the outer boundaries of the constellations have room for increased margin without degrading the error probability for other data symbols. This scheme simultaneously decreases the BER slightly while substantially reducing the peak magnitude of a data block. Furthermore, there is no loss in data rate and no side information is required. However, these modifications increase the transmit signal power for the data block, and the usefulness of this scheme is rather restricted for a modulation with a large constellation size. It is possible to combine the TR and ACE techniques to make the convergence of TR much faster [9].

3.2.2 Non Transparent Methods

If the transmitter or the receiver incorporates a method to reduce PAPR that requires side information to be transmitted from one side to the other. Majority of the PAPR reduction algorithms are included in this category. This dissertation focuses on a Non transparent method.

3.2.2.1 Coding Techniques

Coding can also be used to reduce the PAPR. A simple idea introduced in [7] is to select those code words that minimize or reduce the PAPR for transmission. In general, the coding techniques have block coding, M Sequences and Golay complementary codes, which are very popular. These techniques do not have self-interference and out-of-band radiation but there exist a large number of code words. The large number of code words results in coding rate degradation. Coding techniques need encoder and decoder to process code words, so the complexity of software or hardware is increased [1].

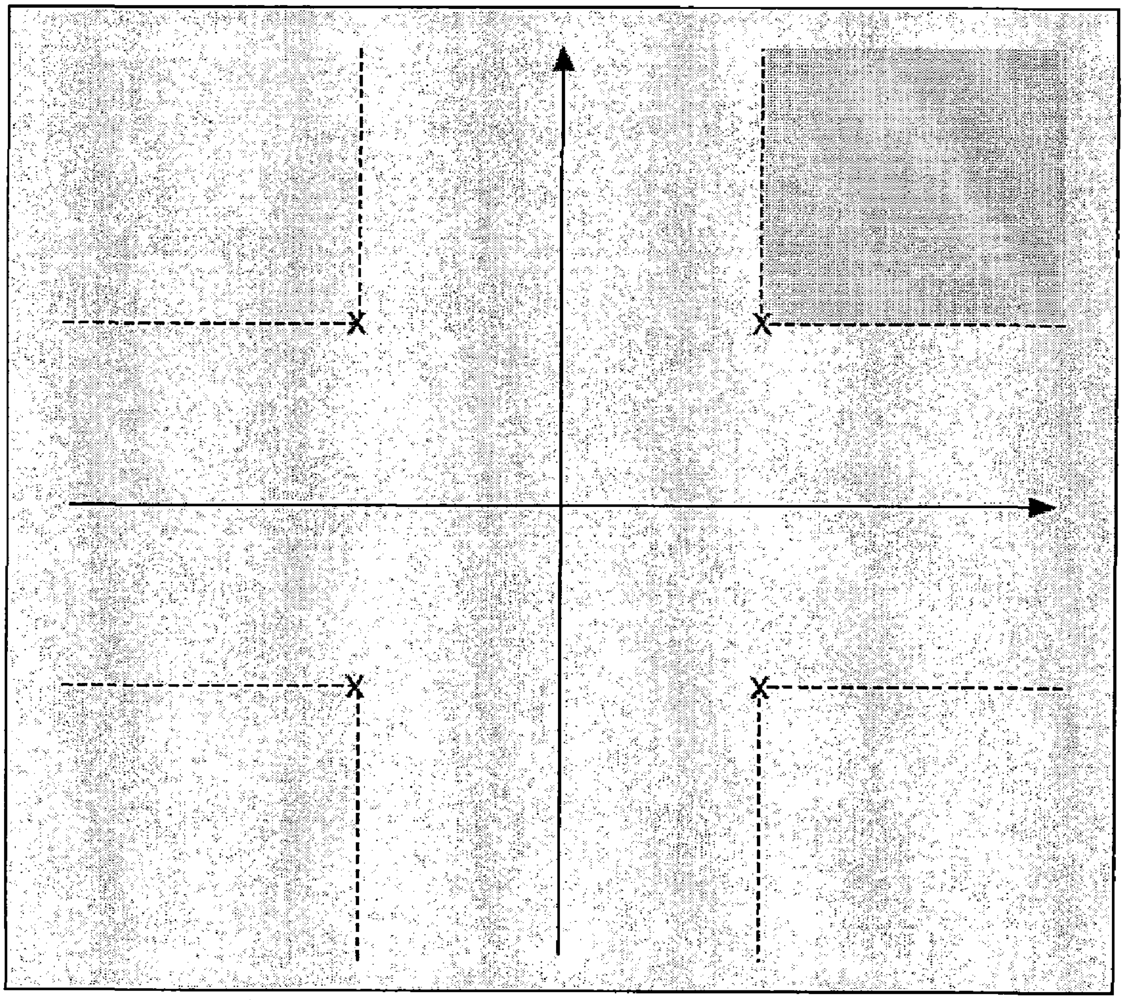


Fig: 3.3 .The ACE technique for QPSK modulation [10].

Block Coding

A block coding scheme for reduction of PAPR is to find code words minimum PAPR from a given set of code words and to map the input data block of these selected code words, thus, it avoids transmitting the code words which generates high peak envelop power, and may provide error correction [7]. But, this reduction of PAPR is at the expense of a decrease in coding rate. For large number of carriers, necessary code sets exist but encoding and decoding is also a difficult task. It is not suitable for higher order bit rates or large number of carriers.

Golay Complementary Codes

Golay complementary codes [8] are sequence pairs for which the sum of autocorrelation function is zero for all delay shifts unequal to zero. The correlation properties of complementary sequences translate into a relatively small PAPR of 3dB when the codes are used to modulate an

OFDM signal. Golay codes may be together with decoding techniques that combined PAPR reduction with good forward-error correction capabilities.

3.2.2.2 Phase Optimization Techniques or Scrambling techniques

It was observed that we will get large PAPR values when symbol phases in the subchannels are lined up in such a fashion that results in a constructive superposition forming a peak in the discrete time signal [1]. By rotating the channel constellations properly the peaks can be reduced. The partial transmit sequence [14] optimization scheme is such method. In partial transmit sequence the data carrying subcarrier blocks is further divided into disjoint carrier subblocks and then phase transformation (phase rotation) is applied for each subblocks. A number of iterations are required to find the optimum. phase rotation factor for different subblocks. Adaptive partial transmit sequence is proposed in [19] to reduce the number of iterations required to find optimum combination of factors for subblocks. Adaptive partial transmit sequence reduces the number of iteration by setting up a desired threshold and trial for different weighing factors until the PAPR drops under the threshold. Complete details of Partial transmit sequence method is discussed in next chapter.

Another method in this category is the selective mapping scheme [11]. In the selective mapping method one single data vector has multiple phase rotations, and the one that minimizes the signal peak is used. Information about which particular data vector and transformation was used is sent as side information to the receiver. In the presence of noise there can be a problem with decoding the signal.

Selected Mapping (SLM)

Assume that an OFDM symbol is scrambled by M different scrambling sequences. Then M statistically independent OFDM symbols represent the same information. If the symbol with the lowest PAPR is selected for transmission, the probability that the PAPR value of the selected symbol ($PAPR_{low}$) exceeds a certain threshold z is given by [11]

$$P\{PAPR_{low} > z\} = (P\{PAPR_{init} > z\})^{M} \tag{3.6}$$

Where $PAPR_{init}$ is the PAPR value of the original symbol. The key idea of *Selected Mapping* (SLM) is to choose one particular signal that exhibits the lowest PAPR value among M candidates, all representing the same information. The block diagram of SLM is shown in Figure

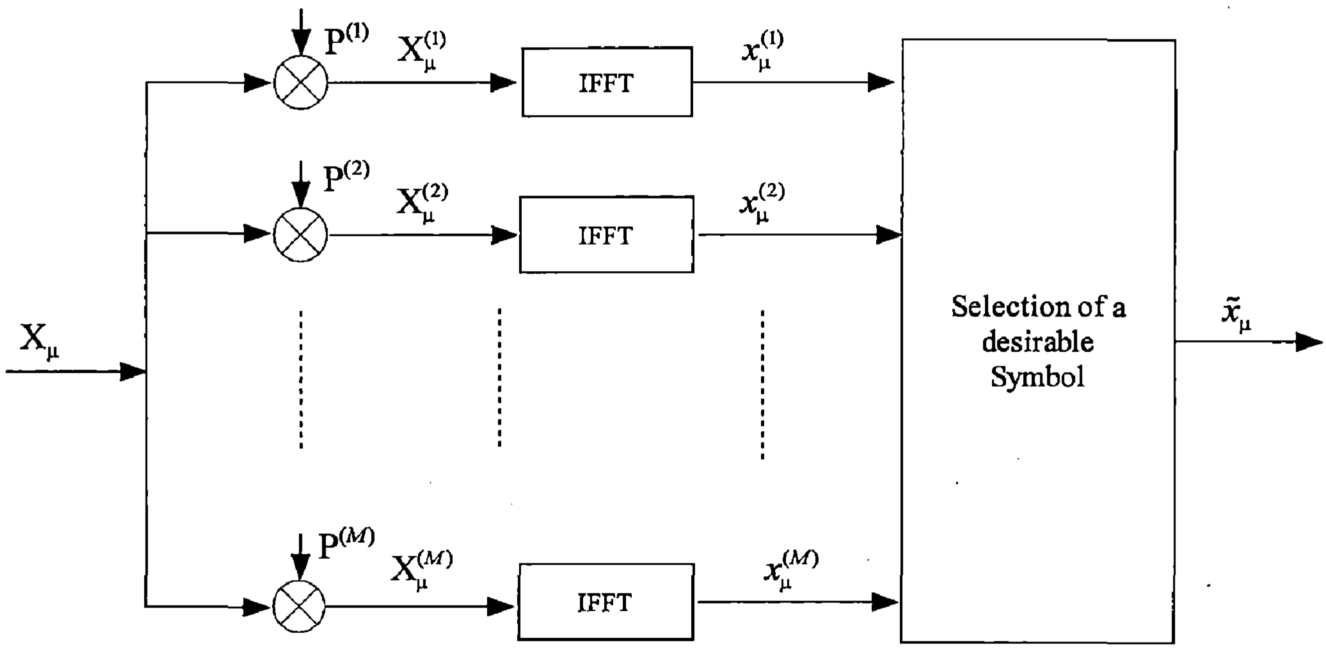


Fig: 3.4. Block diagram of Selected Mapping Method

In Figure 3.4, the M independent OFDM symbols $x_\mu^{(1)}, x_\mu^{(2)}, \ldots, x_\mu^{(m)}$ represent the same information and the symbol $\tilde{x}_\mu$ with the lowest PAPR is selected for transmission. The way of choosing $\mathrm{P}^{(m)}$ vectors is as follows

$$\mathrm{P}^{(m)} = [P_1^{(m)}, \ldots, P_N^{(m)}]^T, \tag{3.7}$$

with $P_\mathrm{v}^{(m)} = e^{j\phi_\mathrm{v}^{(m)}}$ where $\phi_\mathrm{v}^{(m)} \in [0, 2\pi], 1 \le \mathrm{v} \le \mathrm{N}, 1 \le m \le \mathrm{M}$ .After mapping the information to the subcarrier amplitude $\mathrm{X}_{\mu,\mathrm{v}}$, each OFDM symbol is multiplied with the M vector $\mathrm{P}^{(m)}$, resulting in a set of M different OFDM symbols $\mathrm{X}_\mu^{(m)}$ with components represented by [7]

$$X_\mu^{(m)} = X_\mu . e^{j\Phi_\mu^{(m)}}, \ \mu = 1, 2, \ldots, N, \ m = 1, 2, \ldots, M \tag{3.8}$$

All new M OFDM symbols are transformed into time domain defined by

$$x_\mu^{(m)} = \mathrm{IFFT}(X_\mu^{(m)}) \tag{3.9}$$

Finally, the symbol with the lowest PAPR is selected, which is represented as $\tilde{x}_\mu$ in Figure. Notice that, increasing the number distinct vectors used for scrambling (i.e. M), the PAPR reduction amount is increased as shown in figure below.

In order to implement SLM effectively, the components of $P_{v}^{(m)}$ should consist of {±1, ±j} as multiplication with these components can be implemented simply by interchanging, adding and subtracting real and imaginary parts. For the receiver to demodulate the data, it should know which vector is used at the transmitter to scramble the data. The most important disadvantage of SLM appears at this stage, the transmitter has to transfer side information to distinguish the vector used for scrambling. The number of required bits to be transmitted as side information is $\log_2 M$. Side information should be protected carefully; hence it should be sent from a secure channel, which increases system cost.

Simulated the results for 1000 sets of randomly generated data in time domain when the number of subcarriers is 128, oversampling factor L=4, M=4, 8, 16, and 16-QAM in each carrier, and plotted in Figure. Here, vectors $\mathbf{P}^{(m)}$ are generated randomly with $P_{\mu}^{(m)} \in \{\pm 1, \pm j\}$.The advantage of using only phase shifts are multiples of $\pi/2$ is that they can be implemented without any multiplications.

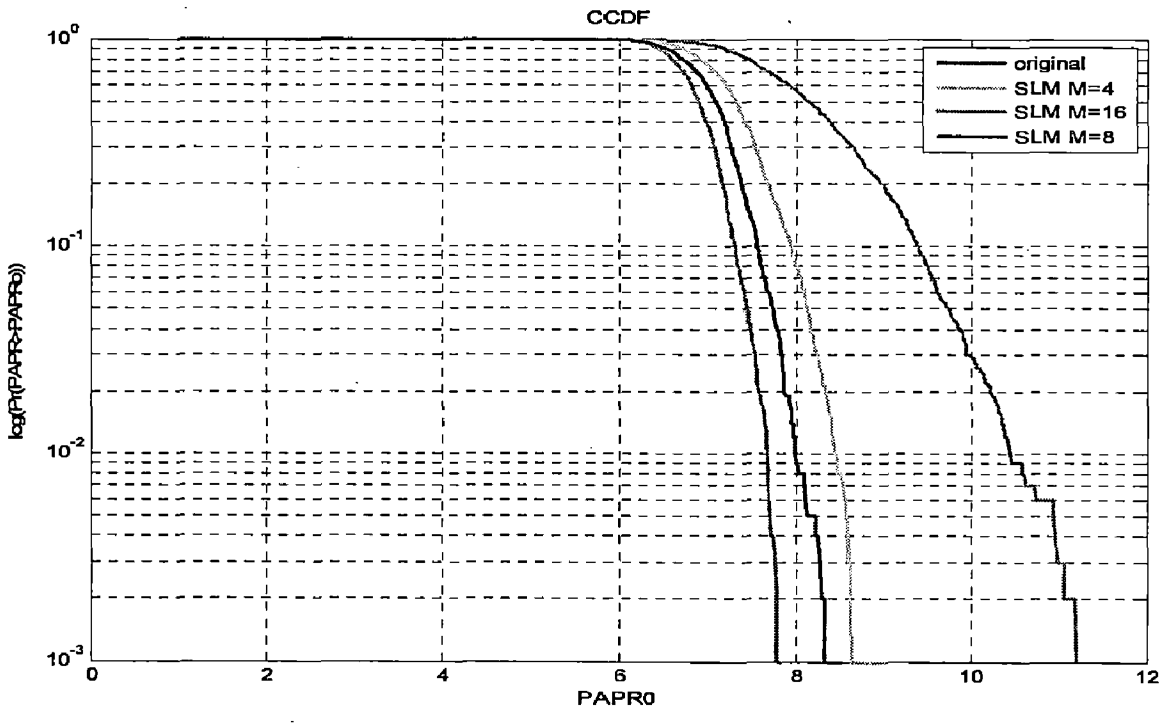


Fig: 3.5. PAPR reduction performance of using SLM

In [12], a method is suggested in order to avoid transmission of side information in a SLM system. In this technique, to generate M different transmit sequences $x^{(m)}$, $1 \le m \le$ M representing the same information word q, labels $b^{(m)}$ are inserted as a prefix to q. The labels are M different binary vectors of length $\log_2 M$. The concatenated word vector of the label and the information word is fed into a scrambler. The labels are hence used to drive the scrambler into one of the M different states before scrambling the information word q itself. Finally, the specific transmit sequence number $\tilde{x}_{\mu}$, which possesses the lowest peak power, is selected for transmission. This method removes the need for side information, however as classical SLM approach, it reduces the probability of the OFDM symbol PAPR to exceed a certain threshold, does not guarantee a certain reduction value.

3.2.2.4 The Interleaving technique

The interleaving technique for PAPR reduction is very similar to the SLM technique. In this approach, a set of interleavers is used to reduce the PAPR of the multicarrier signal instead of a set of phase sequences [13]. An interleaver is a device that operates on a block of N symbols and reorders or permutes them; thus, data block $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$ becomes $\mathbf{X}' = \left[X_{\pi(0)}, X_{\pi(1)}, \ldots, X_{\pi(N-1)}\right]^T$ where $\{n\} \in \{\pi(n)\}$ is a one-to-one mapping $\pi(n) \in \{0, 1, \ldots, N-1\}$ and for all n. To make K modified data blocks, interleavers are used to produce permuted data blocks from the same data block. The PAPR of $(K-1)$ permuted data blocks and that of the original data block are computed using K IDFT operations; the data block with the lowest PAPR is then chosen for transmission. To recover the original data block, the receiver need only know which interleaver is used at the transmitter; thus, the number of required side information bits is $\lfloor \log_2 K \rfloor$. Both the transmitter and receiver store the permutation indices $\{\pi(n)\}$ in memory. Thus, interleaving and deinterleaving can be done simply. The amount of PAPR reduction depends on the number of interleavers $(K-1)$ and the design of the interleavers.

TABLE I we summarizes the PAPR reduction techniques considered [4].

TABLE I

COMPARISON OF *PAPR* REDUCTION TECHNIQUES

	Distortionless	Power increases	Data rate loss	Requires processing at transmitter(Tx) and Receiver(Rx)
Clipping and Filtering	No	No	No	Tx:Amplitude clipping,filtering Rx:None
Coding	Yes	No	Yes	Tx:encoding or table search Rx:Decoding or table search
PTS	Yes	No	Yes	Tx: M IDFTs W^{M-1} complex vector sums Rx: Sideinformation extraction,inverse PTS
SLM	Yes	No	Yes	Tx: U IDFTs Rx: Sideinformation extraction,inverse PTS
Inerleaving	Yes	No	Yes	Tx: K IDFTs,(K-1) interleavings Rx: Sideinformation extraction,inverse interleaving
TR	Yes	Yes	Yes	Tx:IDFTs,find value of PRCs Rx:ignore non data bearing subcarriers
TI	Yes	Yes	No	Tx:IDFTs,search for maximum poin in time ,tones to be modified, value of p and q Rx: Modulo-D-operation
ACE	Yes	Yes	No	Tx: IDFTs,projection onto "shaded areas" Rx:None

3.3 Criteria for selection of PAPR reduction technique [4]

As in everyday life, we must pay some costs for PAPR reduction. There are many factors that should be considered before a specific PAPR reduction technique is chosen. These factors include PAPR reduction capability, power increase in transmit signal, BER increase at the receiver, loss in data rate, computational complexity increase, and so on. Next, we briefly discuss each item.

PAPR reduction capability: Clearly, this is the most important factor in choosing a PAPR reduction technique. Careful attention must be paid to the fact that some techniques result in other harmful effects. For example, the amplitude clipping technique clearly removes the time domain signal peaks, but results in in-band distortion and out-of-band radiation.

Power increase in transmit signal: Some techniques require a power increase in the transmit signal after using PAPR reduction techniques. For example, TR requires more signal power because some of its power must be used for the PRCs. TI uses a set of equivalent constellation points for an original constellation point to reduce PAPR. Since all the equivalent constellation points require more power than the original constellation point, the transmit signal will have more power after applying TI. When the transmit signal power should be equal to or less than that before using a PAPR reduction technique, the transmit signal should be normalized back to the original power level, resulting in BER performance degradation for these techniques.

BER increase at the receiver: This is also an important factor and closely related to the power increase in the transmit signal. Some techniques may have an increase in BER at the receiver if the transmit signal power is fixed or equivalently may require larger transmit signal power to maintain the BER after applying the PAPR reduction technique. For example, the BER after applying ACE will be degraded if the transmit signal power is fixed. In some techniques such as SLM, PTS, and interleaving, the entire data block may be lost if the side information is received in error. This may also increase the BER at the receiver.

Loss in data rate: Some techniques require the data rate to be reduced. As shown in the previous example, the block coding technique requires one out of four information symbols to be dedicated to controlling PAPR. In SLM, PTS, and interleaving, the data rate is reduced due to the side information used to inform the receiver of what has been done in the transmitter. In these techniques the side information may be received in error unless some form of protection such as channel coding is employed. When channel coding is used, the loss in data rate due to side information is increased further.

Computational complexity: Computational complexity is another important consideration in choosing a PAPR reduction technique. Techniques such as PTS find a solution for the PAPR reduced signal by using many iterations. The PAPR reduction capability of the interleaving technique is better for a larger number of interleavers. Generally, more complex techniques have better PAPR reduction capability.

Other considerations: Many of the PAPR reduction techniques do not consider the effect of the components in the transmitter such as the transmit filter, digital-to-analog (D/A) converter, and transmit power amplifier. In practice, PAPR reduction techniques can be used only after careful performance and cost analyses for realistic environments.

PAPR reduction of OFDM signal by use of Partial Transmit Sequences

4.1Introduction

One of the major drawbacks of OFDM is is on its large PAPR of transmitted signal. In order to alleviate the problem, three classes of possible solutions have been propose.In level clipping schemes [5] [6], signals which have larger power than any threshold are forced to be clipped. However, distortion caused by clipping can not be unavoidable in this fashion. The second solution is based on coding. In these methods [7] [8], code rate can be decreased seriously with increase of the number of subbands. To overcome these problems, Partial Transmit Sequences has been proposed as one of the solution [14].

4.2 Partial Transmit Sequence Method

Multiple signal representation (MSR) techniques have been proposed by several researchers to reduce the peak-to-average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals [16]. Use of selected mapping (SLM), partial transmits sequences (PTS) and Random interleaving are some of the schemes which use MSR to reduce the PAR. These are all distortion less PAR reduction techniques. In these schemes, several replicas of the OFDM symbol of a given data frame is formed and the one with the minimum PAR is chosen for transmission. The defects of MSR are a complicated circuit and have to deliver side information which result in coding rate degradation.

In the PTS technique, an input data block of N symbols is partitioned into disjoint subblocks. The subcarriers in each subblock are weighted by a phase factor for that subblock. The phase factors are selected such that the PAPR of the combined signal is minimized. Figure 4.1 shows the block diagram of the PTS technique. In the ordinary PTS technique [15, 16] input data block $\mathbf{X}$ is partitioned into M disjoint subblocks $\mathbf{X}_m = [X_{m,0}, X_{m,1}, \ldots, X_{m,N-1}], m = 1,2,\ldots,M$, such that

$$\sum_{m=1}^{M} \mathbf{X}_m = \mathbf{X}, \tag{4.1}$$

and the subblocks are combined to minimize the PAPR in the time domain. The L-times oversampled time domain signal of $\mathbf{X}_m$, $m=1,2,\ldots,M$, is denoted $\mathbf{x}_m = [x_{m,0}, x_{m,1}, \ldots, x_{m,NL-1}]^T$. $\mathbf{x}_m, m=1,2,\ldots,M$, is obtained by taking an IDFT of length NL on $\mathbf{X}_m$ concatenated with $(L-1)N$ zeros. These are called the partial transmit sequences. Complex phase factors, $b_m = e^{j\phi_m}, m=1, 2, \ldots, M$ are introduced to combine the PTSs. The set of phase factors is denoted as a vector $\mathbf{b}=[b_1, b_2, \ldots, b_M]^T$.

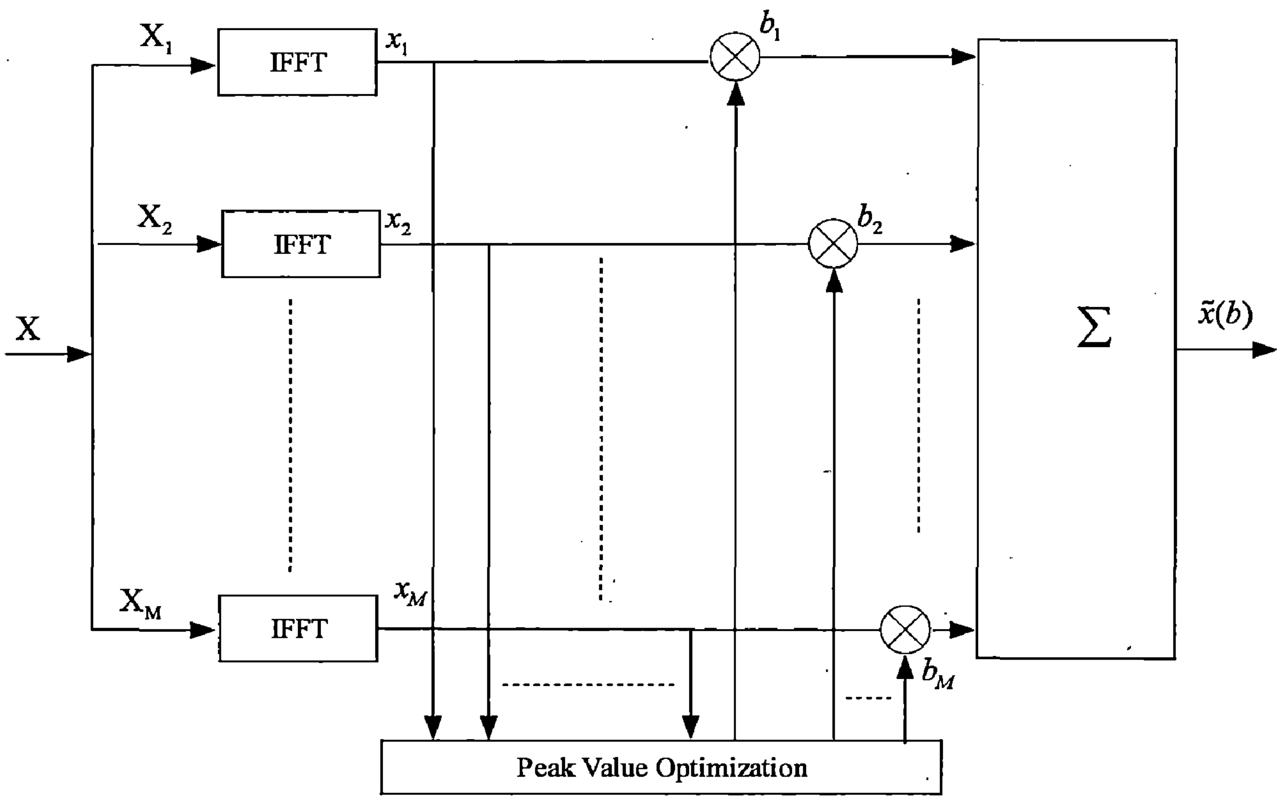


Fig: 4.1. Block diagram of Partial Transmit Sequence approach

In general, the selection of the phase factors is limited to a set with a finite number of elements to reduce the search complexity. The set of allowed phase factors is written as $\phi = \{e^{j\phi_0}, e^{j\phi_1}, \ldots, e^{j\phi_{W-1}}\}$ in which $\phi_l = \frac{2\pi l}{W}, l = 0,1,2,\ldots,W-1$. Where W is the number of allowed phase factors. In addition, we can set $b_1=1$ without any loss of performance. So, we should perform an exhaustive search for $(M-1)$ phase factors. Hence, W^{M-1} sets of phase factors are searched to find the optimum set of phase factors. The search omplexity increases exponentially with the number of subblocks M. PTS needs M $IDFT$ operations for each data block, and the number of required side information bits are $\lfloor \log_2 W^{M-1} \rfloor$, where $\lfloor y \rfloor$ denotes the smallest integer that does not exceed y. The amount of PAPR reduction depends on the number of subblocks M and the number of allowed phase factors W. Another factor that may affect the PAPR reduction performance in PTS is the subblock partitioning [18], which is the method of division of the subcarriers into multiple disjoint subblocks. There are three kinds of subblock partitioning schemes: adjacent, interleaved, and pseudo-random partitioning [18].Subblock partition methods are dicussed in next topic. The PTS technique works with an arbitrary number of subcarriers and any modulation scheme.

The time domain signal after combining is given by

$$\tilde{\mathbf{x}}(b) = \sum_{m=1}^{M} \mathbf{b}_m . IDFT(\mathbf{X}_m)$$

$$= \sum_{m=1}^{M} \mathbf{b}_\mathrm{m} . \mathbf{x}_m \tag{4.2}$$

Where $\tilde{\mathbf{x}}(b) = [\tilde{x}_0(b), \tilde{x}_1(b), ..., \tilde{x}_{NL-1}(b)]^T$. Where $\tilde{x}(b)$ indicates $\tilde{x}$ for a particular phase factor vector b from all the possible phase factor vectors available.

The objective is to find the set of phase factors that minimizes the PAPR. Minimization of PAPR is related to the minimization of

$$\left\{\tilde{b}_1, \tilde{b}_2, ..., \tilde{b}_M\right\} = \underset{\left\{\tilde{b}_1, \tilde{b}_2, ..., \tilde{b}_M\right\}}{\arg\min} \left(\max_{0 \le k \le LN-1} |\tilde{x}_k(b)| \right) \tag{4.3}$$

resulting in the peak power optimized transmit sequence

$$\tilde{\mathbf{x}}(b) = \sum_{m=1}^{M} \tilde{\mathbf{b}}_\mathrm{m} . \mathbf{x}_m \tag{4.4}$$

For the reason that phase factors does not influence the average power of the signals, so the average power of the alternative signals can be written as

$$P_{av} = E\left\{\left|\tilde{x}_k(b)\right|^2\right\} \tag{4.5}$$

After applying PTS, PAPR of the output signal is (consider L=1)

$$PAPR = \frac{\min\limits_{1\le u\le U} \max\limits_{0\le k\le N-1} (|\tilde{x}_k|^2)}{P_{av}} \tag{4.6}$$

Where U is the number of possible candidate signals generated by all the possible combinations of phase factor vectors. To any single candidate signals, its statistic characters won't be changed by phase factors. $\tilde{x}_k(0 \le k < N)$ can be regarded as N independent random variables with the same distribution. Only considering a single candidate signal, the complementary cumulative distribution function (CCDF) of $PAPR$ is [18]

$$\Pr[\max_{0\le k\le N-1} |\tilde{x}_k| > z] = 1-(1-e^{-z})^N \tag{4.7}$$

To PTS, similar to selected mapping (SLM), it should try to ensure the independency of U candidate signals for fully making use of the redundancy caused by them, then

$$\Pr[PAPR > z] = \left(1-(1-e^{-z})^N\right)^U \tag{4.8}$$

It can be regarded as the floor of the ability of PTS restraining $PAPR$.

Subcarriers are divided to *M groups* according to the subblock partition based on

$$x_{m,k} = \begin{cases} x_k, & \text{input data is in this block} \\ 0, & \text{input data is in another blocks} \end{cases}, \tag{4.9}$$

Then we can define $\phi_v, (1 \le v \le M)$ as

$$\begin{cases} \phi_v \cap \phi_u = \Phi \\ \bigcup_{v=1}^{M} \phi_v = \Omega = \{0,1,\ldots,N-1\} \end{cases}, 1 \le v,u \le M, v \ne u \tag{4.10}$$

Where Φ denotes the empty set, Ω is the universal set, that is, all subcarriers. Then equation (4.2) can be rewritten as

$$x_n = IDFT[a_k^u X_k], 1 \le u \le U, 0 \le n < N \tag{4.11}$$

Where X_k is input data of k[th] subcarrier.

If $k \in \phi_v$, then

$$a_k^u = b_v^u \ ,1 \le u \le U, 1 \le v \le M \tag{4.12}$$

Define correlation of any two points between any two candidate signals as

$$R_{il}(x_m^i, x_n^l) = E(x_m^i x_n^{l*})$$

$$= \frac{1}{N} E\left(\sum_{k=0}^{N-1} \sum_{y=0}^{N-1} a_k^i a_y^{l*} X_k X_y^* \ \exp\left(j \frac{2\pi (km - yn)}{N} \right) \right) \ ,1 \le i, l \le U, 1 \le m, n \le N \tag{4.13}$$

Generally, the random variable X_k is independent for different k, but the same static and the average power of X_k is 1.That is to say

$$E(X_k X_y^*) = \delta_{ky} \tag{4.14}$$

Then

$$R_{il}(x_m^i, x_n^l) = \frac{1}{N} \sum_{k=0}^{N-1} a_k^i a_k^{l*} \exp\left(j \frac{2\pi k (m-n)}{N} \right) \tag{4.15}$$

Define m-n= τ , equation (4.15) simplifies as

$$R_{il}(\tau) = \frac{1}{N} \sum_{k=0}^{N-1} a_k^i a_k^{l*} \exp\left(j \frac{2\pi k \tau}{N} \right), -N \le \tau \le N \tag{4.16}$$

Reference equation (4.12), then

$$R_{il}(\tau) = \frac{1}{N} \sum_{v=1}^{V} b_v^i b_v^{l*} \sum_{k \in \phi_v} \exp\left(j \frac{2\pi k \tau}{N} \right), -N \le \tau \le N \tag{4.17}$$

It can be found that the correlation of the several candidate signals is unconcerned with the input signal, while it is dependent on subblock partitions and phase factors. The following section discusses the influence of different subblock partitions. Normally, assume that *N* can be totally divided *M*, and define *P=N/M* presents the number of subcarriers in each subblock.

4.3 Subblock Partition schemes for PTS OFDM

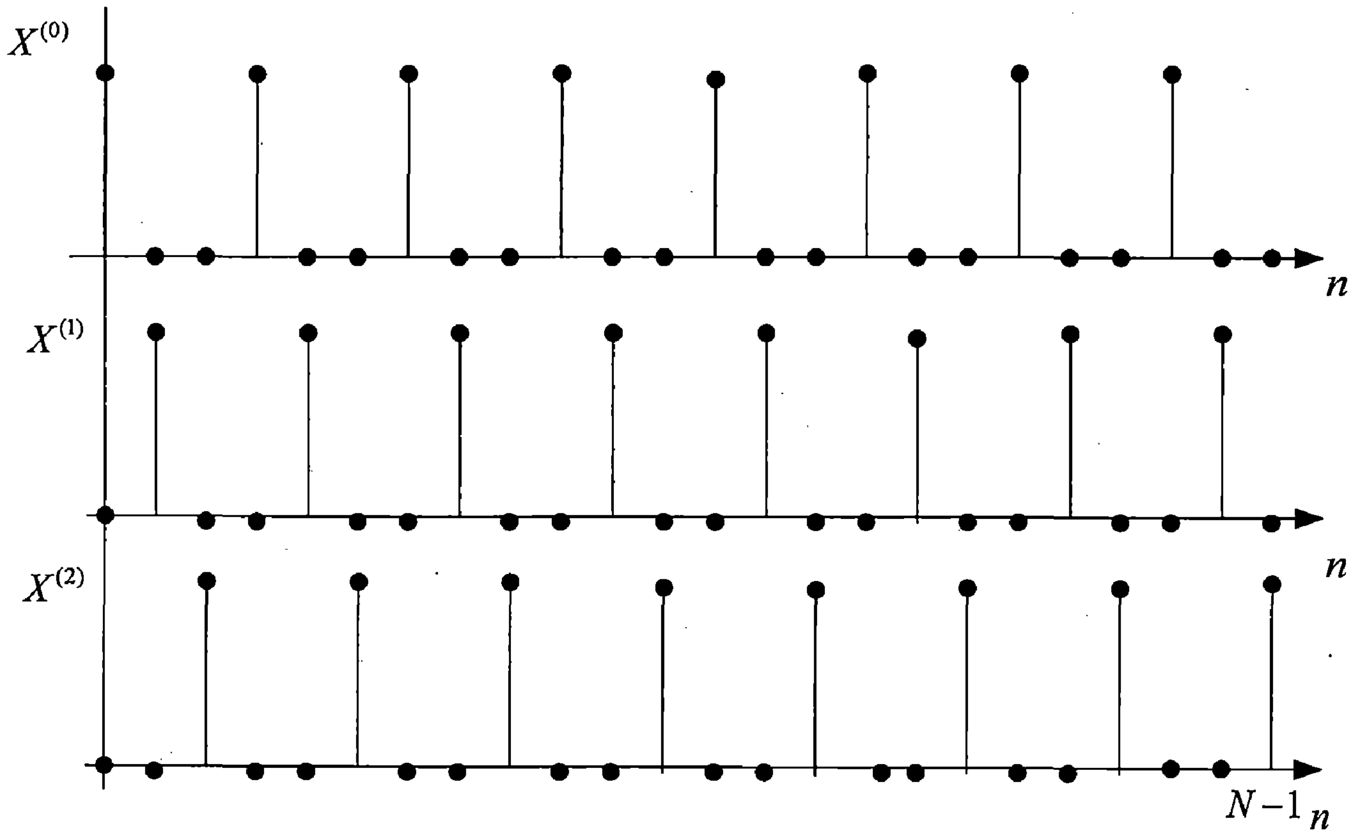
Subblock partition for PTS OFDM is a method of division of subbands into multiple disjoint subblocks [19]. In general, it can be classified into 3 categories; interleaved, adjacent, and pseudo-random SPS [17]. For the interleaved SPS, every subband signal spaced *M* apart is allocated at the same subblock. In the adjacent scheme, *N/M* successive subbands are assigned into the same subblock sequentially. Lastly, each subband signal is assigned into any one of the subblocks randomly in the pseudo-random scheme. As an example, allocation of subband signals with conventional partition schemes are represented in Figure 4.2 , where weighted pulses represent the location of active subbands in each subblock, $X^{(i)}$ for $0 \le i \le M-1$,and M is set to 3.From the Figure, it can be verified that if the k[th] subband in $X^{(0)}$ is active, active, then the kth subbands in other subblocks become inactive states, that is, signals can not be located at the k[th] subhands in $X^{(1)}$ and $X^{(2)}$.

In PTS OFDM, each subblock has to be modulated by IFFT independently. The number of complex multiplications and additions required to modulate a subblock can be given as

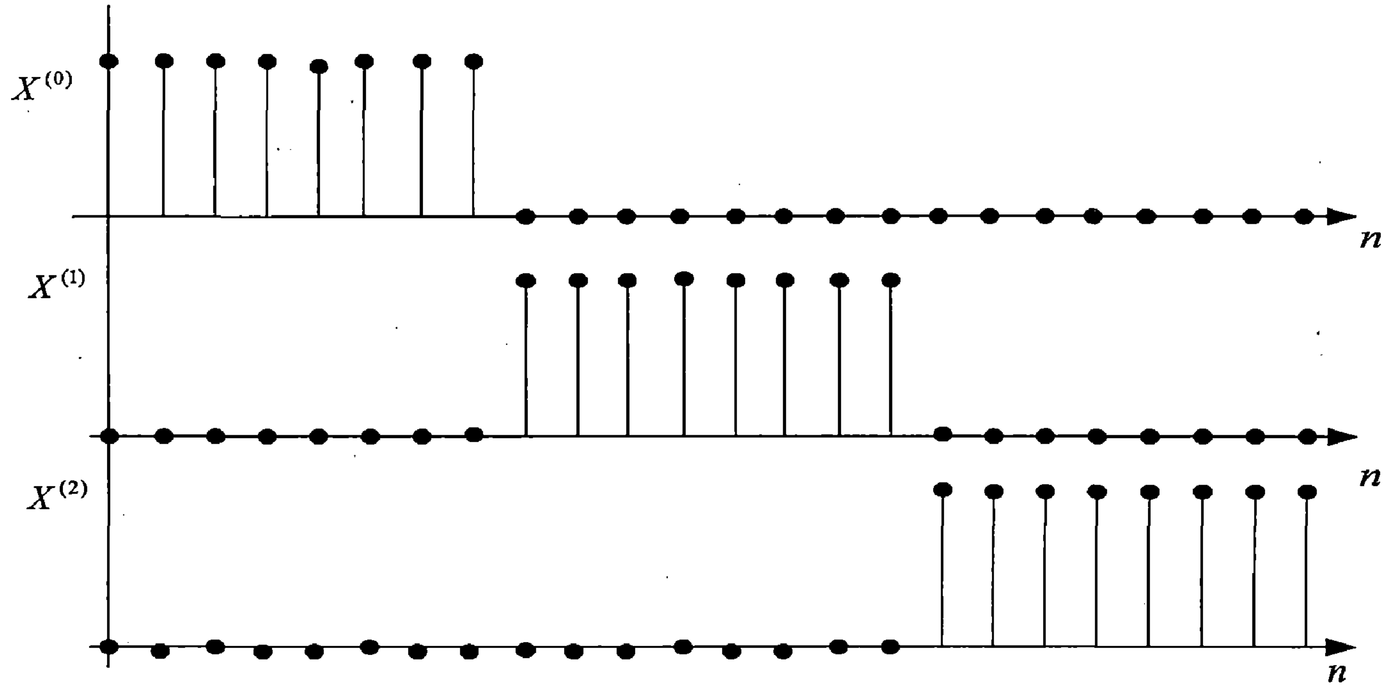
$$n_{mul} = \frac{N}{2}\log_2 N$$

$$n_{add} = N\log_2 N \tag{4.18}$$

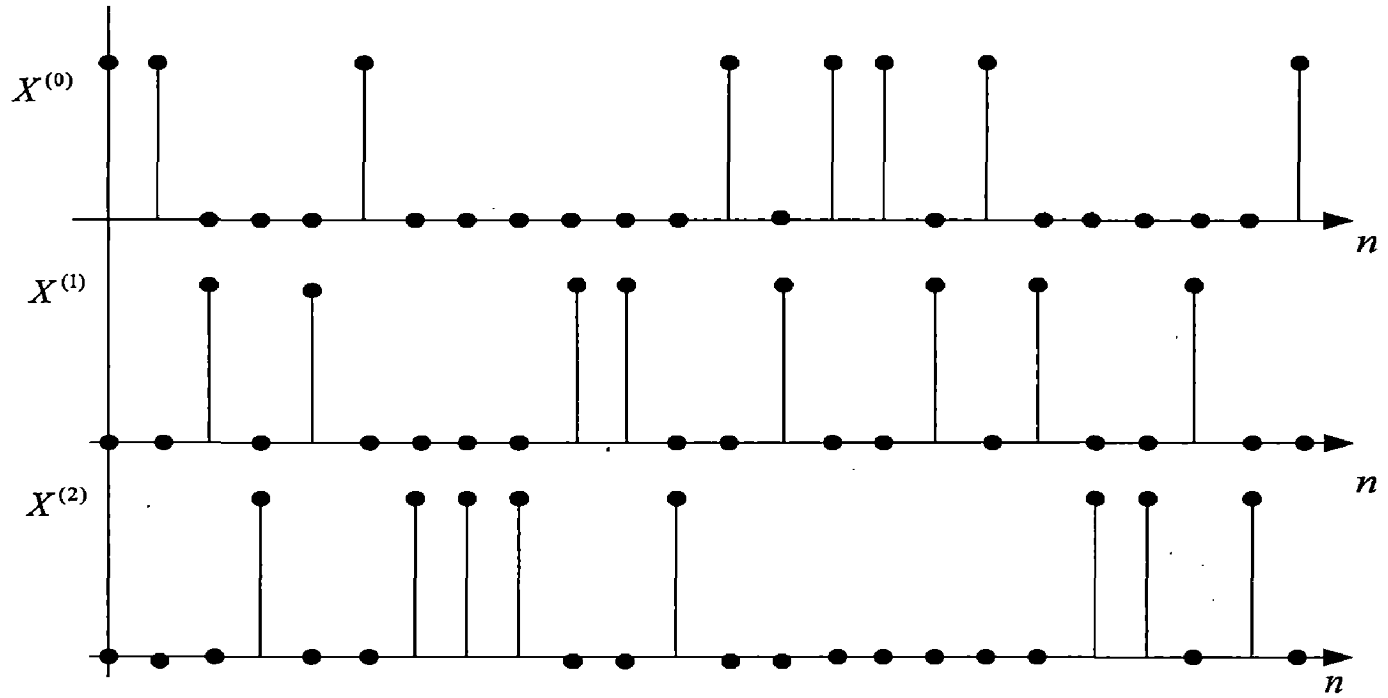
Hence, the total computational complexity to transmit an OFDM symbol can be found by multiplication of (4.9) and the number of subblocks, M. It, therefore, can be verified that the complexity for PTS OFDM has been increased significantly as compared to OFDM system without PTS algorithm.



(a)



(b)



(c)

Fig: 4.2. An example of conventional SPS's

(a) Interleaved. (b) Adjacent. (c) Pseudo-Random.

4.3.1 Performance analysis of subblock partition schemes

a. *Adjacent Subblock partition*

$$\phi_v = \{(v-1)P, (v-1)P+1, ..., vP-1\}, 1 \le v \le M \quad (4.19)$$

Substitute equation (4.19) to equation (4.17), and define correlation function as $R_{il}(\tau)$, then

$$\begin{aligned} R_{c,il}(\tau) &= \frac{1}{N}\sum_{v=1}^{M} b_v^i b_v^{l*} \sum_{k\in\phi_v} \exp\left(j\frac{2\pi k\tau}{N}\right) \\ &= \frac{\sin(\pi\tau P/N)}{N\sin(\pi\tau/N)} \exp\left(j\frac{j\pi\tau(-P-1)}{N}\right)\sum_{v=1}^{M} b_v^i b_v^{l*} \exp\left(j\frac{2\pi v\tau}{M}\right) \end{aligned} \quad (4.20)$$

So

$$|R_{c,il}(\tau)| = \frac{1}{N}\cdot\left|\frac{\sin(\pi\tau P/N)}{\sin(\pi\tau/N)}\right|\cdot\left|\sum_{v=1}^{M} b_v^i b_v^{l*} \exp\left(j\frac{2\pi v\tau}{M}\right)\right|, -N < \tau < N \quad (4.21)$$

Because it is symmetrical around $\tau = 0$, so analyzing 0 < τ < N part is enough. Especially, the correlation value and the maximum correlation value of $\tau = 0$ are

$$\left.\begin{aligned} |R_{c,il}(0)| &= \frac{1}{M}\left|\sum_{v=1}^{M} b_v^i b_v^{l*}\right| \\ \max_{0\le\tau<N}\left(R_{c,il}(\tau)\right) &\le \frac{1}{M}\max_{0\le\tau<N}\left(\left|\sum_{v=1}^{M} b_v^i b_v^{l*} \exp\left(j\frac{2\pi v\tau}{M}\right)\right|\right) \end{aligned}\right\} \quad (4.22)$$

b. *Interleaved Subblock Partition*

$$\phi_v = \{(v-1), (v-1)+V, ..., (P-1)v+(v-1)\}, 1 \le v \le M \quad (4.23)$$

Substitute equation (4.23) to equation (4.17), and define correlation function as $R_{I,il}(\tau)$ then

$$R_{I,il}(\tau) = \frac{1}{N}\sum_{v=1}^{M} b_v^i b_v^{l*} \sum_{k\in\phi_v} \exp\left(j\frac{2\pi k\tau}{N}\right)$$

$$= \frac{1}{N}\exp\left(j\frac{2\pi\tau}{N}\right)\sum_{v=1}^{M} b_v^i b_v^{l*} \exp\left(j\frac{2\pi v\tau}{M}\right)\sum_{h=0}^{P-1}\exp\left(j\frac{2\pi\tau h}{P}\right) \quad (4.24)$$

Because

$$\sum_{h=0}^{P-1}\exp\left(j\frac{2\pi\tau h}{P}\right)=\begin{cases}P, & \tau=mP,\quad m \text{ is integer}\\ 0, & \text{others}\end{cases} \tag{4.25}$$

$$\left|R_{I,il}(\tau)\right|=\frac{P}{N}\left|\sum_{v=1}^{M}b_v^i b_v^{l*}\exp\left(j\frac{2\pi v\tau}{M}\right)\right|\bullet\delta_{|\tau|_P},-N<\tau<N \tag{4.26}$$

$|\tau|_P$ means "τ modulus P", so

$$\left|R_{I,il}(m)\right|=\frac{1}{M}\left|\sum_{v=1}^{M}b_v^i b_v^{l*}\exp\left(j\frac{2\pi vm}{M}\right)\right|,-M<m<M \tag{4.27}$$

There only $2M$ points are nonzero. Because it is symmetrical around m=0, so analyzing $0<m<M$ part is enough.

Especially

$$\begin{gathered}\left|R_{I,il}(0)\right|=\frac{1}{M}\left|\sum_{v=1}^{M}b_v^i b_v^{l*}\right|\\ \max_{0\le\tau<N}\left(R_{I,il}(\tau)\right)\le\frac{1}{M}\max_{0\le m<M}\left(\left|\sum_{v=1}^{M}b_v^i b_v^{l*}\exp\left(j\frac{2\pi vm}{M}\right)\right|\right)\end{gathered} \tag{4.28}$$

c. *Random Subblock Partition*

$$\phi_v=\{P \text{ random independent subcarriers}\},\ 1\le v\le M \tag{4.29}$$

Substitute equation (4.29) to equation (4.17), and define correlation function as $R_{R,il}(\tau)$,then

$$R_{R,il}(\tau)=\frac{1}{N}\left|\sum_{v=1}^{M}b_v^i b_v^{l*}\right|\left|\sum_{k\in\phi_v}\exp\left(j\frac{2\pi k\tau}{N}\right)\right| \tag{4.30}$$

For the fixed $\tau(\tau\neq 0)$, $\exp\left(j2\pi\tau k/N\right)$ appears symmetrical and period change around k, when M=1

$$\sum_{k=0}^{N-1}\exp\left(j2\pi\tau k/N\right)=0,\tau\neq 0 \tag{4.31}$$

Considering the randomicity of subblock partitioning, the value of equation (32) is close to 0 when V is small enough compared with N, then

$$R_{R,il}(\tau) = \begin{cases} 0, & \tau \neq 0 \\ \frac{1}{M} \cdot \left| \sum_{v=1}^{M} b_v^i b_v^{l*} \right|, & \tau = 0 \end{cases} \tag{4.32}$$

Because it is symmetrical around $\tau = 0$, so analyzing $0 \le \tau < N$ part is enough. Then

$$\max\left(R_{R,il}(\tau)\right) = \left|R_{R,il}(0)\right| = \frac{1}{M} \cdot \left| \sum_{v=1}^{M} b_v^i b_v^{l*} \right| \tag{4.33}$$

In all, comparing equations (4.22), (4.28) and (4.33), it's easily to found that with three subblock partitions the correlation value is equal when $\tau = 0$. The maximum correlation peak value achieves the highest when interleaved subblock partition is applied, then the adjacent subblock partition. The random subblock partition can get the smallest maximum correlation peak value. Moreover, with the random subblock partition the values of $R_{R,il}(\tau)$, $\tau = 0$, are close to 0. As a result, correlation of candidate signals is the smallest one with random subblock partition, and its performance of restraining PAPR can be the best. This conclusion is identical with simulation results in next chapter.

4.4 Cimini's Algorithm or Iterative flipping algorithm [15]

In PTS, the transmitted signal is made to have low PAPR by optimally combining signal subblocks. Unfortunately, finding the best weighting factors is a highly complex and nonlinear optimization. Iterative flipping is a suboptimal approach which can provide good performance with lower complexity.

Cimini's weighting factor assignment method is described as ,after dividing the input OFDM block in to M clusters ,an M N-point PTS is formed. $b_l, l = 1,2,...,M$ are the weighting factors and N is the number of sub-carriers. As the first step, assume that $b_l = 1$ for all l and compute the PAP_0 of the combined signal. Next, invert the first weighting factor ($b_l = -1$) and re-compute the resulting PAP. If the new PAP is lower than that in the previous step, retain b_1 as part of the final weighting sequence; otherwise, b_1 reverts to its previous value. The algorithm continues in this fashion until all M possibilities for "flipping" tor flipping the signs have been explored.

The computational complexity is thus reduced to the number of sub-blocks *M*. The weighting factor bit in Cimini's method is calculated only once in order to identify the sub-optimal factor bits. As such, when the current weighting bit combined with a previous bit makes a low *PAP*, an optimum weighting factor can't be found.

Based on the simulation results on next chapter, we showed that these suboptimal strategies, which are less complex and more easily implemented, suffer little performance degradation.

4.5 Adaptive PTS approach for reduction of PAPR of OFDM signal

The main problem of PTS is how to minimize the number of iterations necessary for locating the optimal weighting factors (which increases exponentially with the number of sub-blocks). In the Adaptive PTS approach [21], the iterations are stopped once the PAP drops below a preset threshold. To explain the key idea behind the method, consider an OFDM system with, say, *M* sub-blocks. Assume that the weighting factors are binary. To obtain the optimal weighting factors for each input data frame, 2^{M-1} combinations should be checked in order to obtain the minimum PAPR .Since many input data frames already have low PAPR values, searching all these combinations necessary in most cases. Consequently, the key idea in adaptive PTS is to establish an early terminating threshold. In other words, the search is terminated as soon as the PAP drops below the threshold, rather than after all 2^{M-1} combinations have been searched. Of course, if the threshold is set to a small value, adaptive PTS will be forced to search all the combinations. Similarly, if the threshold is set to a large value, adaptive PTS will search only a fraction of the 2^{M-1} combinations. In this way the threshold affects a tradeoff between PAP reduction and complexity.

4.5.1 Adaptive algorithm for combining the PTS [21]

We only consider binary (i.e. 1 and -1) weighting factors and we divide the input data block into *M* sub-blocks. As a first step, assume that $b_m=1$ for all m and compute PAP of the combined signal(eqn.0).If it is less than a set threshold *L*, then stop the optimization immediately. If not, invert the first phase factor (b_1 = -1) and recomputed the resulting PAP. If it is less than *L,* retain b_1 as part of the final phase sequence and stop the optimization. The algorithm continues in this fashion until the PAP is less than L or all or part (i.e.K) of the 2^{M-1} combinations are searched.

Let **b**=$[b_1,b_2,...,b_m]$. The adaptive algorithm can know be written as followsSet **b**=[1,1,...,1]

1. Set IterCount=1
2. While PAP(X')>L or IterCount<K
3. Change b by one bit
4. IterCount^{++}.

Here, K can be set to 2^{M-1} or a lesser value .The maximum number of iterations of this technique is K, and the minimum is 1.The actual number of iterations varies from one input frame to another. We characterize the complexity of this scheme by the average number of iterations per input frame.

4.6 PAPR Reduction method using Sub-Optimal PTS with threshold

After dividing the input OFDM block into *M* clusters through the sub-block partition method, an *M* *N*-point PTS is formed. The weighting factor assignment method of the proposed method, which consists of M level processes, is depicted in Fig. 3, where l is the number of sub-blocks, i the number of process, and j the fixed optimum weighting factor bit in every processing level [22].

For the first level process, assume that $b_l = 1$ for all l where $b_l, l = 1,2,...,M$, are the weighting factor bits and compute the PAP_o of the combined signal. If it is less than a set threshold *L*, then stop the optimization. If not, invert the first weighting factor $(b_1 = -1)$, re-compute the resulting PAP_{11}, and store it. If it is less than *L*, retain b_1 as part of the final weighting sequence and stop the optimization. If not, make for all and invert the second weighting ($b_2 = -1$), re-compute the resulting, and store it. If it is less than *L*, retain b_2 as part of the final weighting sequence and stop the optimization. The first level processing then continues in this fashion until the last weighting factor b_M. Then the smallest value among the first level processes, $PAP_{i=1}$ is represented as

$$PAP_1 = PAP_{1j} = \min(PAP_{11,} PAP_{12}, ..., PAP_{1m}, ..., PAP_{1M}) \tag{4.34}$$

Where min(.) means the minimum value in the given expression and i is the number of levels, j the position that has the smallest PAP.

Next, invert the weighting factor b_j= -1. If is PAP_1 below, PAP_o change to and proceed to the second level, otherwise, stop the optimization. For the second level process, assume that $b_l = 1$ for all *m,* except for the weighting factor ($b_j = -1$) obtained in the first level processes, and invert the weighting factor ($b_l = -1$) from the first bit b_1 to the last bit b_M in the sequence, as the first level processes In the second level processes the smallest value, is represented as without calculating PAP_{2j}

$$PAP_2 = \min(PAP_{21,} PAP_{22}, ..., PAP_{2m}, ..., PAP_{2(M-1)}) \tag{4.35}$$

If PAP_2 is below PAP_o , change PAP_o to PAP_2 and proceed to the third level, otherwise, stop the optimization.

The third level processing continues in this fashion until the last process. After the last process is finished, the optimum weighting factors for the OFDM frame are given by

$$\{b_{1,} b_{2,}, ..., b_m\} = \min(PAP_{1,} PAP_2, ..., PAP_M) \tag{4.36}$$

There are two methods that can stop the optimization. One is the same PAP condition and the other is a threshold condition. When PAP_i, which is the smallest PAP value in the i^{th} level process, is the same as PAP_{i-1} , which is the smallest value in the $i-1$ th level process, it is regarded as the same PAP condition.

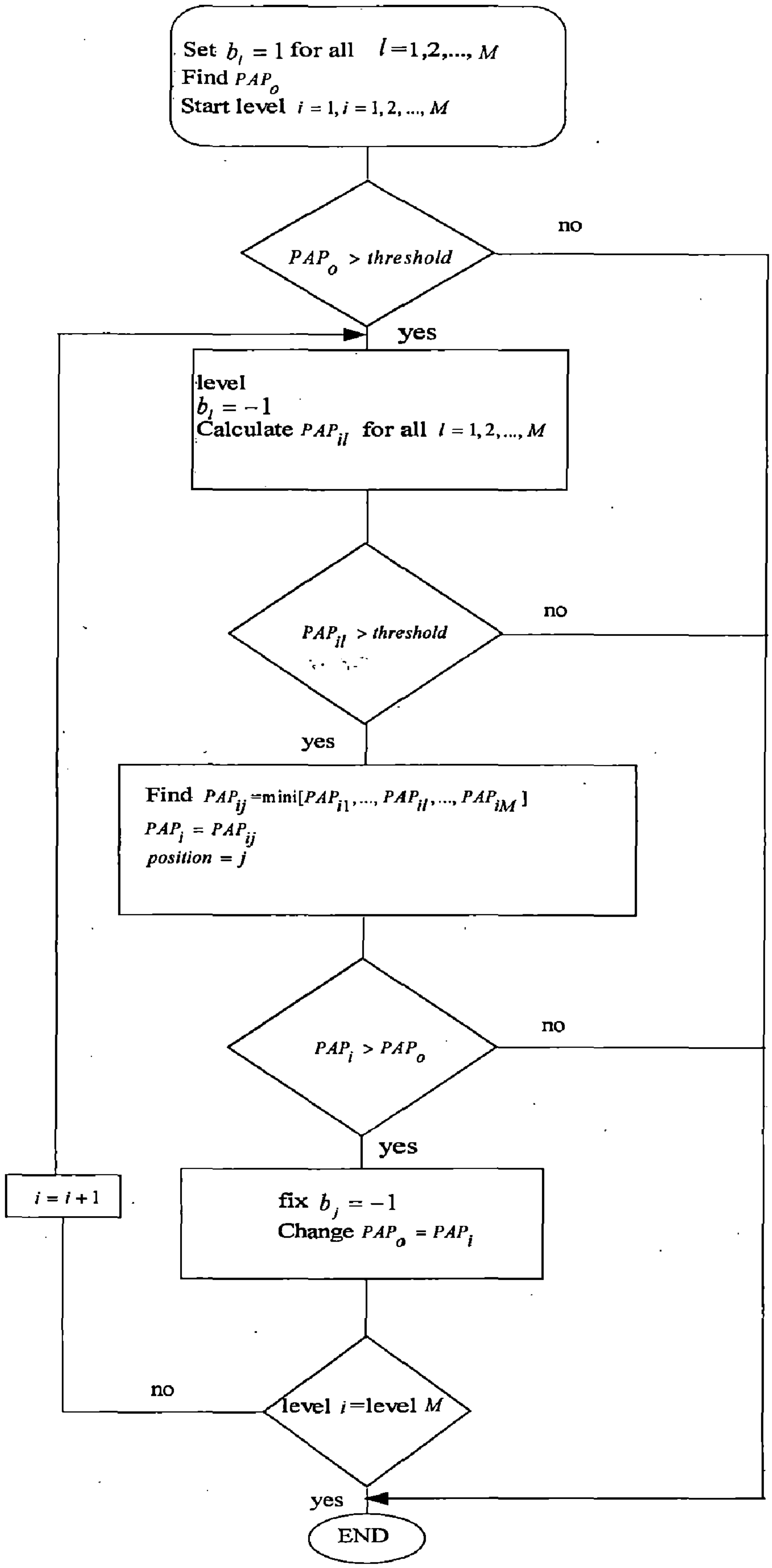


Fig: 4.3. Flow chart for phase factor optimization in Suboptimal PTS with threshold

When the PAP_{il} value is below the threshold L, calculated from the OFDM signal characteristics, the optimization process stops in any iteration. Such a reasonable threshold can reduce the computational complexity without any performance degradation. The two main differences between the suboptimal method and previous cimini's method are the condition that stops the weighting factor optimization and the level processing, explained above.

Table I compares the computational complexities of the PTS, Cimini, and proposed method. For example, the maximum computational complexity with 16 sub-blocks is 32,768 for the PTS method,16 for the Cimini, and $136 = 16 + 15 + 14 + \cdots + 1$ for the suboptimal method. The minimum iteration count can be 1 for the suboptimal method because of the threshold value.

TABLE II

COMPUTATIONAL COMPLEXITY FOR WEIGHTING FACTOR

No.of subblock	PTS's	Cimini's	Suboptimal method
M	2^{M-1}	M	$1 \sim M + (M-1) + (M-2) + \cdots + 1$
4	$2^3 = 8$	4	1~10
8	$2^7 = 128$	8	1~36
16	$2^{15} = 32,768$	16	1~136

4.7 Reduced Complexity PTS technique (RC-PTS)

In this technique, an approach to tackle the PAPR problem to reduce the computational complexity based on the relationship between the weighing factors and the transmitted bit vectors. A preset threshold is considered for further reducing the computational complexity to stop the searching if the minimum PAPR value is below a preset threshold [23].

4.7.1 The Basic Idea of Reduced Complexity Method

Reduced complexity method is based on the property that the weighting factors b_l of sub-blocks are constrained to { -1, 1} when the phase factor $\phi_l = 0$ or π only [23].

Let $\mathbf{b}_1 = [b_{1,1}, b_{1,2}, ..., b_{1,M}]$ be a *M*-dimensional weighting factor vector with transmitted signal $\mathbf{x}_1$ which can be obtained from $\mathbf{b}_1$ by using (4.2). Let consider another weighting factor vector $\mathbf{b}_2 = [b_{2,1}, b_{2,2}, ..., b_{2,M}]$ which is the same as $\mathbf{b}_1$ except at a specific position j . It means that $b_{1,l} = b_{2,l}$ for $l = 1, 2, ..., j-1, j+1, ..., M$ and $b_{1,j} = -b_{2,j}$. Normally, we can get by the use of the (4.2) [15]–[17]. However, we can make use of the relationship between $\mathbf{b}_1$ and $\mathbf{b}_2$ to calculate $\mathbf{x}_2$ from $\mathbf{x}_1$ in order to reduce the computational complexity.

$$\begin{aligned}\mathbf{x}_2 &= \sum_{l=1}^{M} b_{2,l}\mathbf{x}_l = \sum_{l=1, l\neq j}^{M} b_{2,l}\mathbf{x}_l + b_{2,j}\mathbf{x}_j \\ &= \sum_{l=1, l\neq j}^{M} b_{1,l}\mathbf{x}_l + b_{1,j}\mathbf{x}_j - b_{1,j}\mathbf{x}_j + b_{2,j}\mathbf{x}_j \\ &= \sum_{l=1}^{M} b_{1,l}\mathbf{x}_l - 2b_{1,j}\mathbf{x}_j = \mathbf{x}_1 - 2b_{1,j}\mathbf{x}_j \qquad (4.37)\end{aligned}$$

Hence, we can quickly compute $\mathbf{x}_2$ from $\mathbf{x}_1$ by the (4.37) since $b_{1,j} \in \{+1, -1\}$.

4.7.2 Analysis of the Computational Complexity

Like the other PTS methods, RC method needs only several IFFT blocks in parallel for per OFDM symbol. Hence, the main factor of complexity for these PTS methods is the computation for the searching of the optimized weighting factor [23].

The main contribution of RC method is to reduce the computational complexity of the obtaining time domain vector. Here, we compare the computational complexity of obtaining $\mathbf{x}_2$ by the (4.2) used in [15]–[17] and RC method by (4.37), respectively. By using (4.2) [15]–[17], the computational complexity is $(M-1)*N$ additions (where minus is treated the same as the addition operation). If using (4.37) in RC method, the computational complexity reduces to N additions because it is only necessary to calculate the sub-block $\mathbf{x}_j$ given if we already know $\mathbf{x}_1$. Therefore, the computational complexity of RC method is only about the one-(M-1)th of the method by (4.2) in [15]–[17] (For example, if the number of sub-block are 16, then RC method only requires 1/15 computational complexity than by (4.2).

TABLE III

THE COMPUTATIONAL COMPLEXITY OF THE IPTS, OPTS AND RC-PTS

Method	Total computational complexity
IPTS	$M*(M-1)*N$
OPTS	$2^{M-1}*(M-1)*N$
RC PTS with SN	$SN*N$

Table I gives the total computational complexity of the conventional IPTS, OPTS method and the new RC PTS method with the value of search number (SN).

4.7.3 Algorithm of RC Method

We now describe how to recursively use this RC method with (4.37) to find the optimized bit vector $\mathbf{b}_{opt}$ to minimize PAPR [23].

First, it is necessary to choose a starting bit vector $\mathbf{b}_1$. We can set $b_{1,l}=1$ for $l=1,2,...,M$ without loss of generality. Then, we invert the second bit of $\mathbf{b}_1$ to produce $\mathbf{b}_2$ and compute the PAPR value $\mathbf{x}_2$ of obtained by (4.37). By the same token, we invert the third bit of $\mathbf{b}_1$ and $\mathbf{b}_2$ to generate $\mathbf{b}_3$ and $\mathbf{b}_4$, respectively. From $\mathbf{x}_1$ and $\mathbf{x}_2$, we can get $\mathbf{x}_3$ and $\mathbf{x}_4$ using (4.37). By the same manner, the number of weighing factor vectors to be searched are doubled in each iteration and we compute their corresponding PAPR values until all 2^{M-1} possible weighing factor and transmitted bit vectors have been searched. We then choose the bit vector with the minimum PAPR as the optimized transmitted bit vector.

It is obvious that the algorithm is relatively insensitive to the starting weighting factors $\mathbf{b}_1$ when the PAPR of all 2^{M-1} possible bit vectors are calculated. Although this approach can reduce the numbers of computations, the computational complexity of this method is still exponential with the number of subblocks. Therefore, further simplification is required to make this method practical for a moderate or large number of subblocks.

Such simplification can be achieved by systematically reducing the number of weighing factor and transmitted bit vectors retained at each stage of the algorithm described above. The approach is to retain the $N_{\max}$ numbers of weighing factor and transmitted bit vectors with lowest PAPR before inverting the jth bit in next stage. If we select $N_{\max}=1$ then the search number of RC method reduces to the IPTS in [15] and if $N_{\max}=2^{M-1}$ then the search number of RC method as same as the OPTS method. This approach can strike a balance between IPTS and OPTS depending on the value of $N_{\max}$. We now summarize the RC-PTS method for estimating the transmitting weighting factors.

1) Set $\mathbf{b}_1=[1,1,...,1]$ and $i=1$. Compute $\mathbf{x}_1$ by (4.2). Find its PAPR and store $\mathbf{x}_1$;
2) Set $i=i+1$. From each bit vector currently in storage, produce and store a new bit vector by changing the ith bit. Compute the new bit vector using (4.37). Find its PAPR and store the new bit vector.
3) Identify the stored bit vector $b^{(m)}$ with the minimum PAPR.
4) Retain $N_{\max}$ numbers of weighting factor and transmitted bit vectors with lowest PAPR values
5) If $i<M$,go to step 2)

In order to further reduce the number of searching, a preset threshold (P_{th}) is used, where the search is stopped once the minimum PAPR drops below the P_{th} . This RC-PTS method with the application of P_{th} is different from the simple RC-PTS method described above in the step 3 only, which is "3) Identify the stored bit vector with the minimum PAPR. If the minimum PAPR is below the P_{th}, then stop the search and take the $b^{(m)}$ as the output."

4.8 Low Complexity Partial Transmit Sequence technique (LC-PTS)

A class of LC-PTS technique is based on dividing the candidate signals into multiple subsets and utilizing the correlation among candidate signals in each subset so as to reduce the computational complexity [24].

4.8.1 Analysis of the Correlation among Candidate Signals

In PTS, the lth candidate signal $\mathbf{x}'_l$ is generated from the th phase vector. For all L candidate signals, the process of phase factor combination and PAPR computation is highly complicated. For reducing the complexity, the correlation among the candidate signals is considered in this section.

In PTS-OFDM, when N is large, for $\mathbf{x}_m = [x_{m,0}, x_{m,1}, \ldots, x_{m,N-1}]^T$, $x_{m,n}$ is a sequence of mutually independent complex valued $N\left(0, \sigma^2/2\right)$.According to (0), the correlation coefficient between two candidate signals $\mathbf{x}'_u$ and $\mathbf{x}'_v$ can be written as [24]

$$\rho_{u,v} = \frac{\operatorname{cov}(\mathbf{x}'_u, \mathbf{x}'_v)}{\sqrt{D(\mathbf{x}'_u) D(\mathbf{x}'_v)}} = \left(M - \mathrm{Q} + \sum_{q=0}^{\mathrm{Q}-1} S_q \right) \frac{1}{M} \tag{4.38}$$

in which $D(.)$ and cov(.) are the variance and covariance, respectively, and

$$S_q = b^u_{m_q} b^{v*}_{m_q} \tag{4.39}$$

where $m_q (q = 1, 2, \ldots, \mathrm{Q})$ denote the subblocks in which and have different phase factors, i.e.,

$$\begin{cases} b^u_{m_q} \neq b^v_{m_q} & \text{if } m_q = m_1, m_2, \ldots, m_{\mathrm{Q}} \\ b^u_{m_q} = b^v_{m_q} & \text{others} \end{cases} \tag{4.40}$$

$\rho_{u,v}$ stands for the correlation among generated candidate signals. Furthermore, the bound of $\rho_{u,v}$ is given as

$$\rho_{u,v} \geq \frac{M - 2\mathrm{Q}}{M}, \text{ for } M \geq 2\mathrm{Q} \tag{4.41}$$

$$\rho_{u,v} \geq \frac{M-2Q}{M}, \text{ for } M \geq 2Q \tag{4.41}$$

From (4.17), it is shown that the correlation among candidate signals is mainly determined by Q, and the bound is a descending function of Q as $M \geq 2Q$. Thus, the candidate signals are strongly correlated when $Q=1$. For example, for $Q=1$, if M=4, $\rho_{u,v} \geq 0.5$; if M=8, $\rho_{u,v} \geq 0.75$.

4.8.2 LC-PTS approach

Inspired by the correlation among candidate signals, especially for $Q=1$, LC-PTS technique for the complexity reduction. As for all the candidate signals generated in PTS, the complexity of phase factor combing and PAPR computation is reduced [24].

The *LC-PTS* technique is based on a PTS-OFDM system with M subblocks and W phase factors. Assuming $\mathbf{b}^u = \left[e^{j\phi_1}, b_2^u, \ldots, b_{M-1}^u, e^{j\phi_1}\right]$ and $\mathbf{b}^v = \left[e^{j\phi_1}, b_2^u, \ldots, b_{M-1}^u, e^{j\phi_2}\right]$ for generating $\mathbf{x}'_u$ and $\mathbf{x}'_v$, it is shown that $Q=1$. In this case, $\rho_{u,v} \geq \frac{M-2}{M}$.

In general, the PAPR of the candidate signal is determined by the highest-amplitude signal points. Since the positions of highest-amplitude signal points in $\mathbf{x}'_u$ and $\mathbf{x}'_v$ are correlated, it is possible to simplify the computation of $\mathbf{x}'_v$ if $\mathbf{x}'_u$ is already known. The simplified method can be described as below. First, P highest-amplitude positions of are recorded, and then, only computing the recorded points P of $\mathbf{x}'_v$ is necessary rather than all N points in conventional PTS.

In order to reduce the complexity, the L candidate signals are divided into several subsets, and in each subset, the signals are correlated. First, we define prime signals as $\mathbf{x}'_p = \left[\mathbf{x}_1\ \mathbf{x}_2 \ldots \mathbf{x}_M\right]\mathbf{b}^p$, in which the phase factor vector $\mathbf{b}^p$ for generating the prime signals is expressed as

$$\mathbf{b}^p = \left[e^{j\phi_1}, b_2^p, \ldots, b_{M-1}^p, e^{j\phi_1}\right]^T \tag{4.42}$$

In prime signals, the last phase factor is fixed as the first one. Thus, the number of prime signals is W^{M-2}.

$$\mathbf{b}^s = \left[e^{j\phi_1}, b_2^s, \ldots, b_{M-1}^s, e^{j\phi_k}\right]^T, k \neq 1 \tag{4.43}$$

In subordination signals, the first phase factor is fixed to $e^{j\phi_1}$ and the last phase factor is different from that of $\mathbf{b}^p$. As a result, for one prime signal, there are $W-1$ correlative subordination signals. Also, the total number of subordination signals is $(W-1)W^{M-2}$.

Therefore, LC-PTS can be described as the following steps.

1) Partition the input data $\mathbf{X}$ into M subblocks.
2) For each subblock, an IFFT is employed.
3) Introduce the phase factor set.
4) Generate one prime signal; compute its PAPR and record P highest-amplitude positions.
5) Based on the prime signal, generate $(W-1)$ subordination signals, in which only P points are computed for phase factor combination and PAPR calculation.
6) Repeat steps 4) and 5) until all candidate signals are computed.
7) Select the signal with the lowest PAPR for transmission.

In step 4), P positions are selected. To further simplify the computation, a threshold T_h is preset to record the positions where the amplitudes are higher than T_h. So P is an average number and can be estimated by the simulation. In Table IV, the computational complexity of LC-PTS is compared with that of conventional PTS.

It is shown that the complexity of LC-PTS is rather lower. Furthermore, since the values of W, L, M, and N are constants, the complexity is only contributed by P, i.e., by the preset threshold T_h.

TABLE IV

ANALYSIS OF COMPUTATIONAL COMPLEXITY IN LC-PTS

Operation	LC-PTS	Percentage
Complex Add.	$L(M-1)\left(\dfrac{N-P}{W}+P\right)$	$\dfrac{(N-P)/W+P}{N}$
Complex Mul.	$L(M+1)\left(\dfrac{N-P}{W}+P\right)$	$\dfrac{(N-P)/W+P}{N}$
Comp	$L\left(\dfrac{N-1}{W}+P\right)-1$	$\dfrac{L((N-1)/W+P)-1}{LN-1}$

According to (4.2), (4.42), and (4.43), $\mathbf{x}'_s$ can be rewritten as

$$\mathbf{x}'_s = \mathbf{x}'_p + \left(e^{j\phi_k} - e^{j\phi_1}\right)\mathbf{x}_M \tag{4.44}$$

$\mathbf{x}'_s$ denotes a sequence of mutually independent complex valued $N\left(0,\sigma^2/2\right)$. Thus, for $x'_{s,n} \in \mathbf{x}'_s$ $(n = 0,1,2,\ldots,N-1)$, the probability function is $\alpha(A)$ defined as

$$\alpha(A) = \Pr\left[\max_{n=0}^{N-1} \left|x'_{s,n}\right| \le A\right] = \left(1-e^{-A^2/\sigma^2}\right)^N \tag{4.45}$$

Also, $\mathbf{x}_M$ denotes sequence of mutually complex valued $N\left(0,\sigma^2/2\right)$. If adjacent subblock partition style is considered, for $x_{M,n} \in \mathbf{x}_M$ $(n = 0,1,2,\ldots,N-1)$, the probability function $\beta(B)$ is defined as

$$\beta(B) = \Pr\left[\max_{n=0}^{N-1} \left|x'_{m,n}\right| \le B\right] = \left(1-e^{-MB^2/\sigma^2}\right)^{N/M} \tag{4.46}$$

In LC-PTS, for achieving the same PAPR reduction performance as conventional PTS, when $\alpha \to 0, \beta \to 1$, the model is described as selecting the T_h threshold satisfying

$$T_h \le A(\alpha) - \left(e^{j\phi_k} - e^{j\phi_1}\right)B(\beta) \tag{4.47}$$

in which $A(\alpha)$ and $B(\beta)$ are the inverse function of and, i.e.,

$$A(\alpha)=\sigma\sqrt{-\ln\left(1-\alpha^{1/N}\right)} \tag{4.48}$$

and

$$B(\beta)=\sigma\sqrt{-\ln\left(1-\beta^{M/N}\right)/M} \tag{4.49}$$

Furthermore, as $\left|e^{j\phi_k}-e^{j\phi_l}\right|\leq 2$, when M is large, the bound is approximated as

$$T_h \leq A(\alpha) \tag{4.50}$$

A strict theoretical bound of T_h is given in (4.50) without loss of PAPR reduction performance. In fact, for reducing more complexity with moderate performance loss T_h can be selected by simulation. In this case, the PAPR reduction performance will be degraded, i.e., for a given complimentary cumulative distribution function (CCDF) value γ, the PAPR will be added to

$PAPR_o+\varepsilon(\gamma)$ in which $PAPR_o$ is the original PAPR of conventional PTS-OFDM and $\varepsilon(\gamma)$ is the loss of PAPR reduction.

Simulation and Results

In this chapter, we first present the simulation model. We have carried out simulations to evaluate the performance of various suboptimal strategies used in Partial Transmit Sequences to reduce the complexity involved in PAPR reduction. We plot CCDF of Peak-to-Average Power of OFDM signal to measure the PAPR reduction performance of each method discussed in chapter 4. For the simulations in this dissertation, MATLAB was employed with its Communications Toolbox for all data runs. The OFDM transceiver system was implemented as specified in Figure 5.1

5.1 Simulation Model

The OFDM system is explained in chapter 2 and The PTS based PAPR reduction methods are discussed in chapter 4. The following two types of OFDM systems have been considered for comparisons:

1) System1 (Sys.1): Standard OFDM system without PAPR reduction.
2) System 2(Sys.2): OFDM system with PAPR.

The simulation block diagram of the system (sys.2) is shown in Fig.5.1, while sys.1 can be obtained by simply removing "PAPR cancelling (i.e. subblock partitions, one IFFT instead of multiple IFFTs and phase factor combining and optimization) ," and "PAPR decancelling" blocks.

The simulations are performed on additive white Gaussian noise (AWGN) channels and do not consider the effect of guard interval length in our simulation system. The PAPR distribution is observed on the discrete-time samples using oversampling factor of 4. We simulate the system of PAPR reduction performance by using Partial Transmit Sequences techniques. As performance measure we study the probability that the PAPR of an OFDM frame exceeds a given threshold $PAPR_o$. Considering this Complementary Cumulative Distribution Function (CCDF) of $PAPR$

$$CCDF\,(PAPR_o) = \Pr\{PAPR > PAPR_o\} \quad (\Pr\{.\}\text{: probability}).$$

$$CCDF = \frac{1}{R}\sum_{i=1}^{R} m_i,$$

$$\text{Where } m_i = \begin{cases} 1, & when\ PAPR_i > PAPR_o \\ 0, & others \end{cases}$$

Where R is the total no of OFDM symbols are generated and $PAPR_i$ is the $PAPR$ of the i[th] transmitted OFDM symbol.

Computational Cmplexity is defined as

$$CC = \frac{1}{N}\sum_{i=1}^{N} I_i$$

where I_i is the number of iterations per an OFDM symbol i and is the number of OFDM symbols

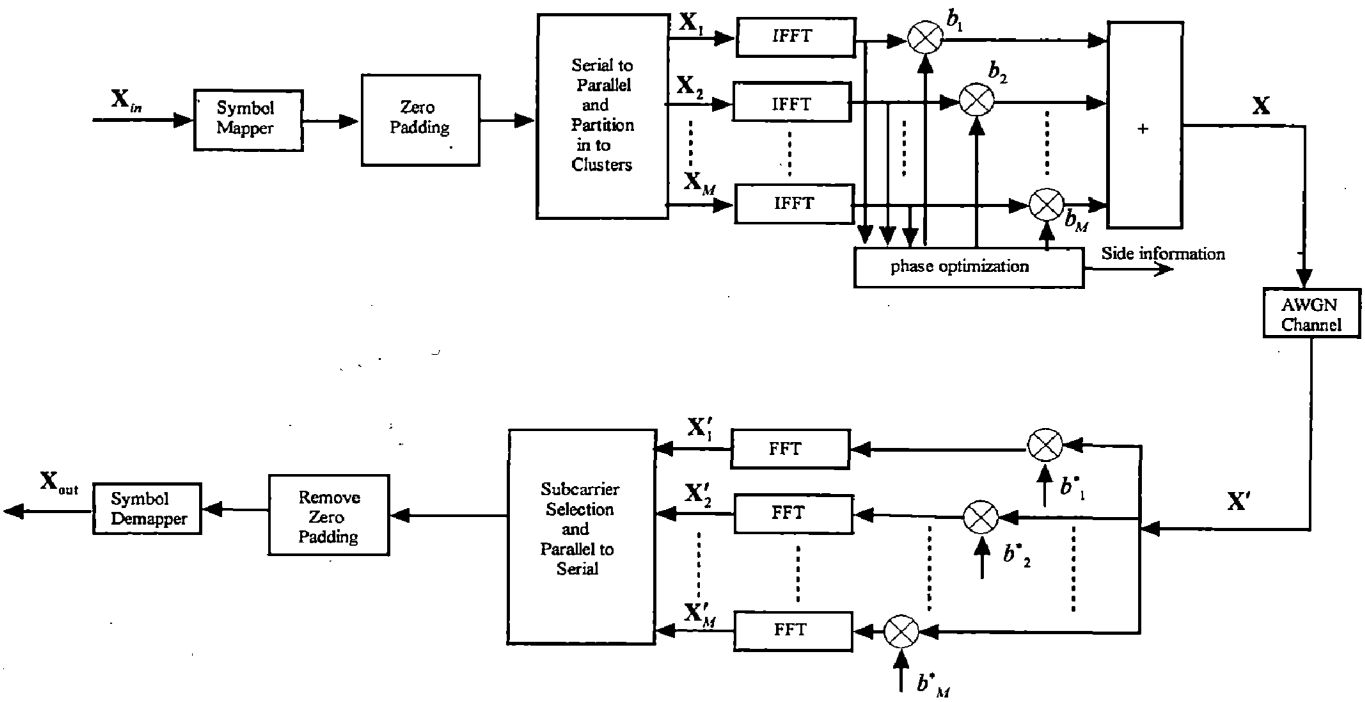


Fig: 5.1. Simulation block diagram of the system.

5.2 PAPR reduction using Ordinary Partial Transmit Sequence method

In the result analysis the complementary cumulative distribution function (CCDF) of the peak-to-average power of an OFDM signal is used. The effect of varying the several simulation parameters is examined. These parameters are the number of clusters and the number of allowed phase rotations for the transmit sequences and the different subblock partition schemes.

For simulation of PTS method in MATLAB environment, we use the following parameters

- No of OFDM data symbols R=1000
- No of OFDM subcarriers N=128
- Oversampling factor $L = 4$
- Modulation scheme for source symbols: 4-QAM
- No of Subblocks M=4,8,or 16
- Subblock partition : adjacent
- Phase factors $b_m = 2$ $(\{+1,-1\})$ or $b_m = 4(\{\pm 1, \pm j\})$

Figure 5.2 shows the CCDF for the Unmodified OFDM signal and the OFDM-PTS signal. In particular, the PAP of an OFDM signal exceeds 11.9 dB for 0.1% of the possible transmitted OFDM blocks. However, by introducing PTS approach with 8 clusters partition with phase factors limited to {+1,-1}, the 0.1% PAP reduces to 7.32 dB. In short, PTS can achieve a reduction of PAPR by approximately 4.5 dB at the 0.1% of PAP.

Figure 5.2 depicts the effects of varying the number of clusters for the weighting factors chosen from a set of {+1,-1}.From Figure 5.2, as expected, the improvement increases as the number of clusters increases [15]. However, with only 16 clusters, it can achieve a reduction of 5.7 dB compared to the unmodified OFDM signal.

Figure 5.3 shows the Comparison by varying size of the possible weighting factors set ,for a fixed number of cluster, M=4.Weighting factors are chosen from set 1, b_m=2 are{+1,-1) and set2, b_m=4 are {+1,-1,+j,-j}. In fig: 5.3, for a fixed number of clusters, the phase factors chosen from a larger set, specifically, {+1,-1, +j,-j}. It is shown that, the added degree of freedom in choosing the combining phase factors provide an additional 0.4 dB reduction, compared to the set size of 2 , {+1,-1}.

For different subblock partitions, probability that PAPR of an OFDM symbol exceeds an arbitrary reference level PAPR0 is depicted in Figure 5.4. It can be observed that probability of very large peak power has been increased significantly if PTS techniques are not used. On the contrary *PAPR0* of PTS OFDM system is decreased about 1.26~3.36[dB] with respect to the partition schemes when Pr(*PAPR*>*PAPR0*)=10^{-3} .That is the probability of *PAPR0*>7.7[dB] is 10^{-3} at the pseudo-Random SPS while it almost reaches 11.07dB at the system without using PTS. The probability of *PAPR*>8.0[dB] is about 2x10^{-1} at the interleaved SPS and 6x10^{-2} at the adjacent PTS, respectively. And it is around 10^{-3} at pseudo-Random SPS. Thus, it can be analyzed that the pseudo-Random SPS shows the best *PAPR* reduction performance as expected.

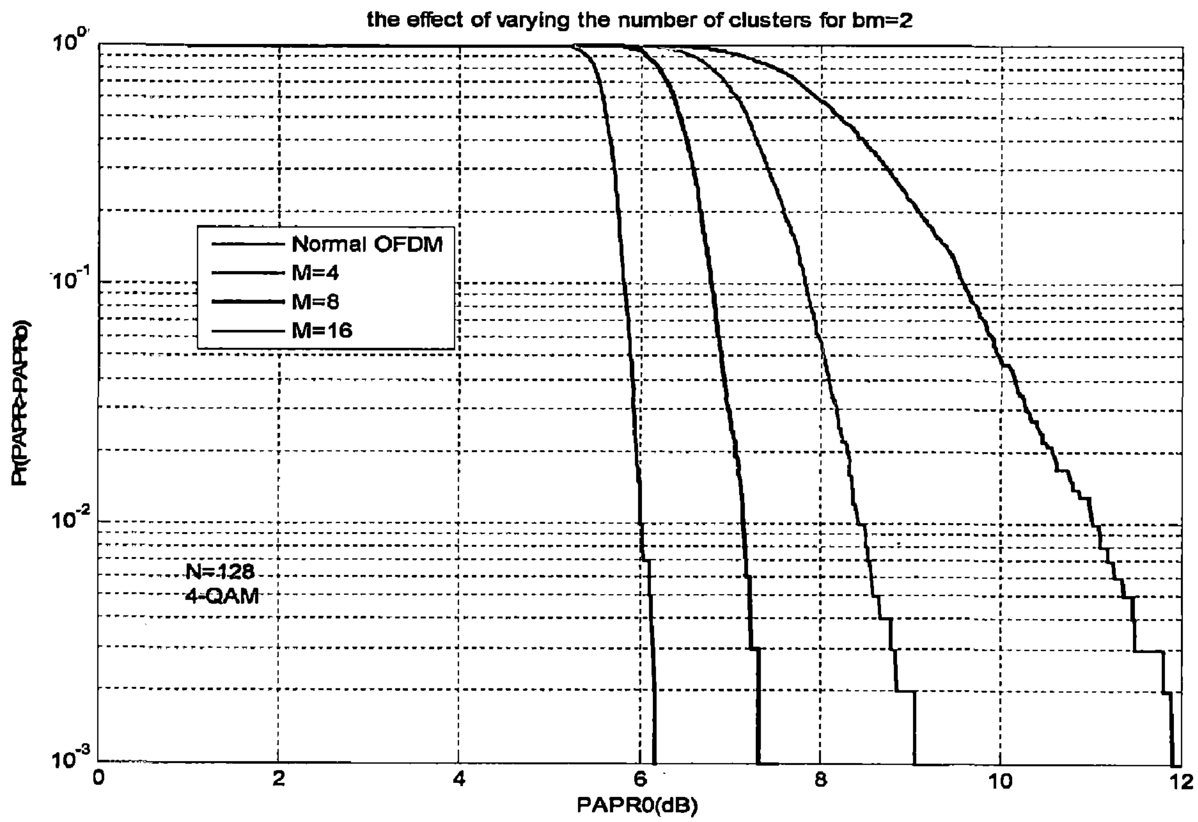


Fig: 5.2. Comparison of unmodified OFDM signal with the OFDM-PTS signal

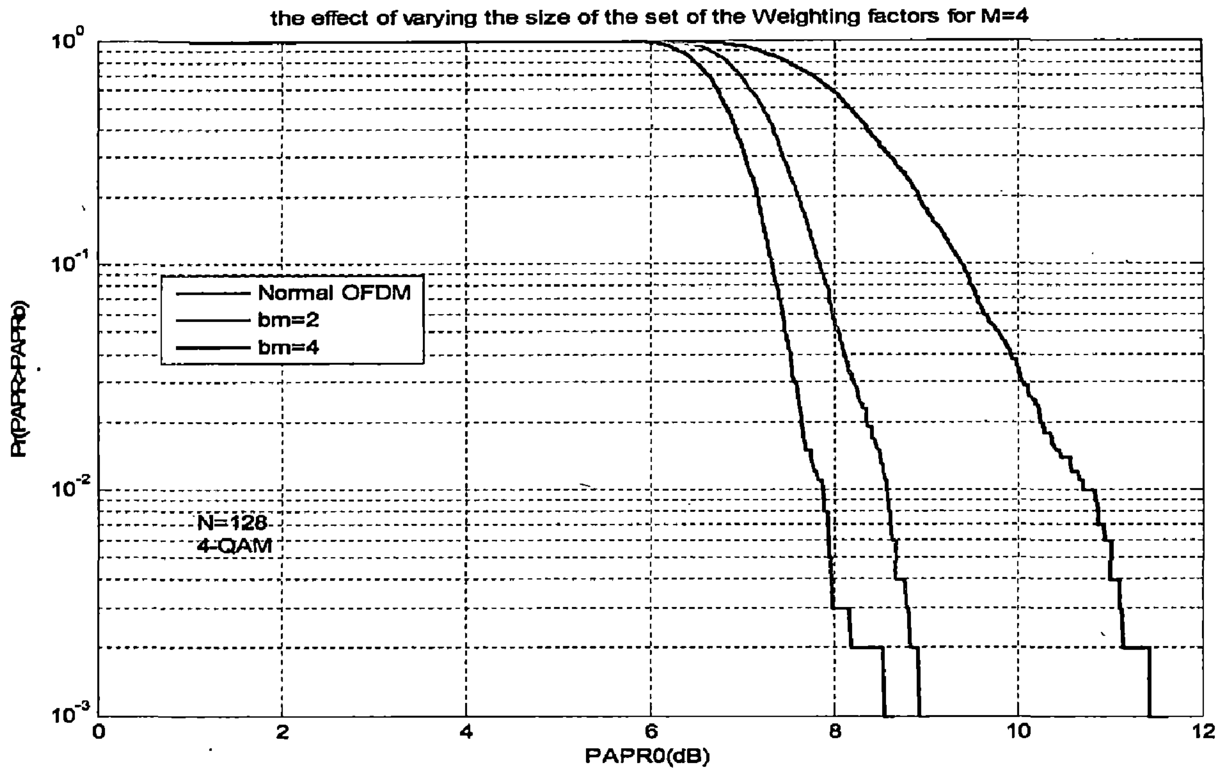


Fig: 5.3. Effect of varying the set of the weighting factors

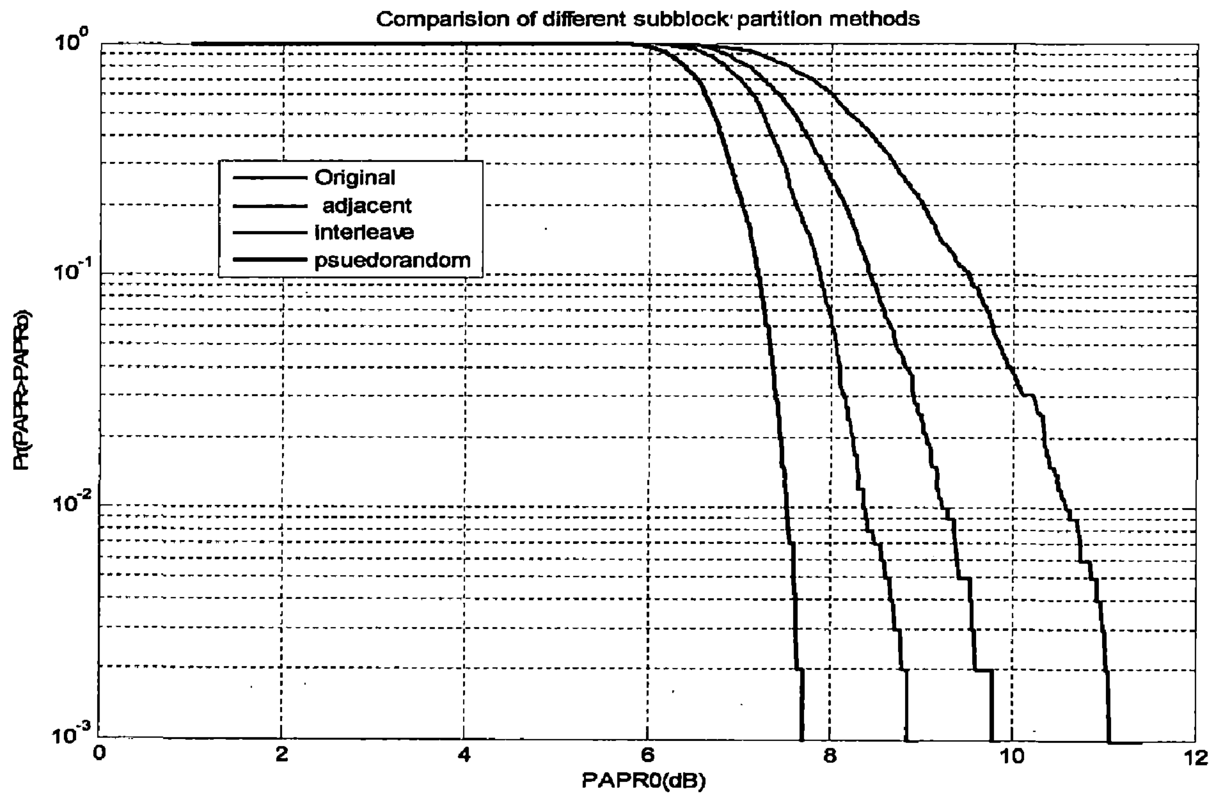


Fig: 5.4. Comparison of different subblock partition methods for PTS OFDM

5.3 PAPR reduction using PTS with Cimini's or iterative flipping algorithm

In Fig.5.5 results are shown for the case of a single OFDM block and 16 clusters each composed of *16* subcarriers. The transmitted signal is oversampled by a factor of 4.Simulations have shown that this is sufficient to capture the peaks. In the results which follow, 1000 random OFDM blocks were generated to obtain the CCDF's. We assume 256 subcarriers and 4-QAM data symbols. The unmodified OFDM signal has a *PAP* which exceeds 10.77 dB for less than 0.1% of the blocks.

By using the ordinary PTS approach with the optimum binary phase sequences for combining, the 0.1%*PAP* reduces to 7.37 dB. In addition, the slope has been improved so that the reduction would be even more significant at lower CCDF values. For the iterative algorithm, the 0.1% *PAP* is 7.94 dB. While degradation of 0.6 dB is encountered, the optimization process has been reduced to 16 sets of 16 additions, a considerable savings over determining over the optimum phase factors.

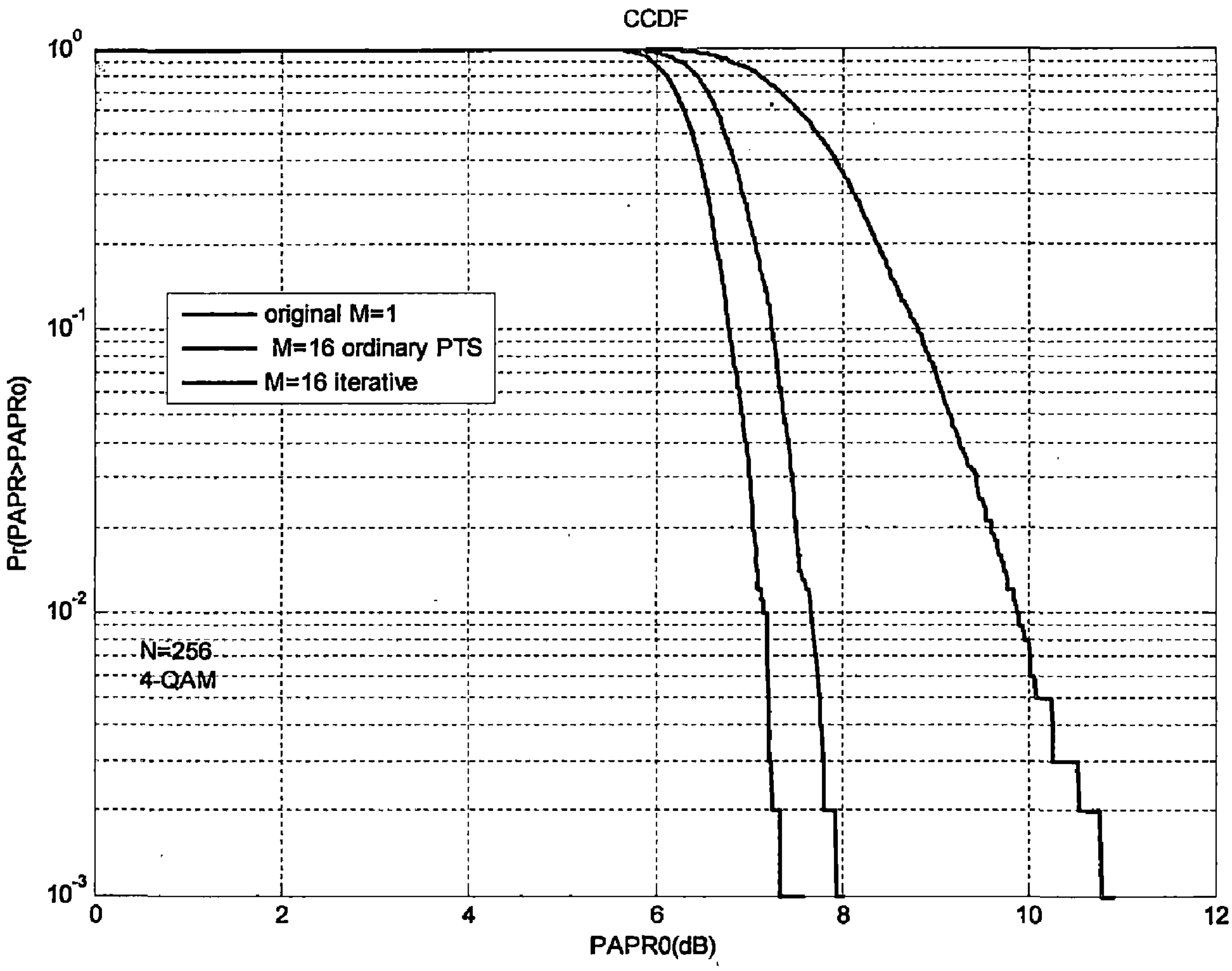


Fig: 5.5. Comparison of the iterative and optimum combining strategies

5.4 PAPR reduction using Adaptive PTS method

For simulation of PTS method in MATLAB environment, we use the following parameters

- No of OFDM data symbols R=10,000
- No of OFDM subcarriers N=256
- Oversampling factor $L = 4$
- Modulation scheme for source symbols:QPSK
- No of Subblocks M=8
- Subblock partition : adjacent
- Phase factors $b_m = 2$ ({+1,-1}) or $b_m = 4(\{\pm 1, \pm j\})$

In the following results, 10^4 random OFDM symbols were generated to obtain CCDFs. Fig. 5.6 shows the Complementary Cumulative Distribution Function (CCDF), the 0.01%PAP of the original OFDM signal was 12.89dB.Ordinary PTS improved it by 3.97dB. Curve (ii) shows the results for APTS with a threshold value (L) of 7.5 dB. In the region CCDF < 10^{-2}, both techniques provided identical performances. Ordinary PTS requires 128 iterations per OFDM symbol while APTS requires only 15 (on average) iterations per OFDM symbol. This amounts to an 87% reduction in complexity.

Results are also shown for APTS with L of 7.4, 7.25 and 7.0 dB. These require 22, 36, 73 iterations (on average) and reduce the complexity up 83, 72 and 43%, respectively. Therefore, it is evident that the complexity of APTS can be greatly reduced by limiting the number of iterations by selecting a suitable threshold value. Lower threshold values yield better performance but result in higher complexity.

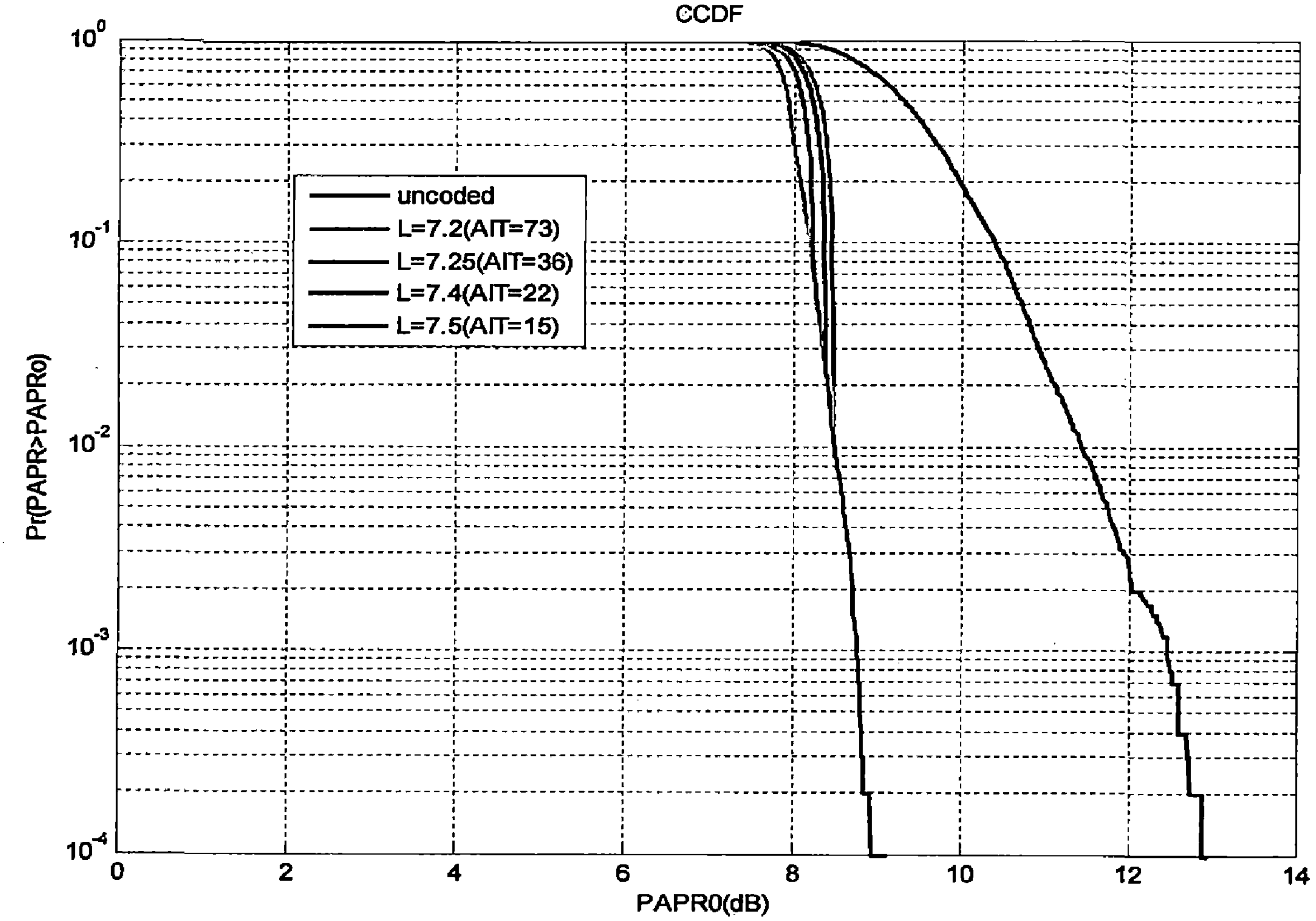


Fig: 5.6. CCDF of OFDM signal with ordinary PTS and APTS

5.5 PAPR reduction using the Suboptimal PTS with threshold

To obtain the CCDF and computational Complexity, 20,000 random OFDM symbols were generated.256 subcarriers assume per OFDM frame. The QPSK data symbols were applied for symbol mapping. 8 subblocks are used for the simulation for system complexity. Weighting factors are {+1.-1}.TABLE V summarizes the compuatational complexity and .Fig:5.7 gives the PAPR reduction performance of sumbptimal PTS method.

TABLE V ITERATION NUMBER OF SUB-OPTIMAL PTS

	No.of Subblock	PTS	Cimini	Suboptimal PTS	
				10.35 dB	0[dB]
Max. iteration	8	128	8	12	37
Average iteration	8	128	8	1.423	22.34

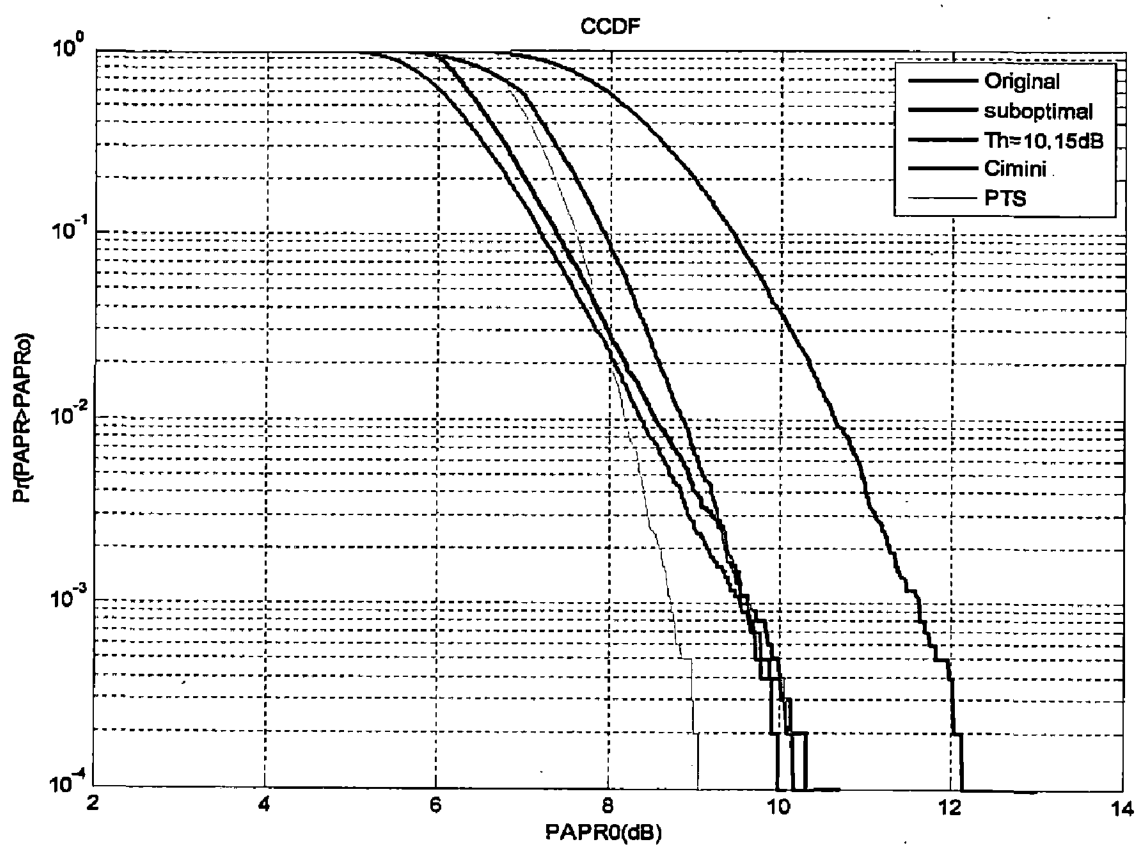


Fig: 5.7 .CCDF of OFDM signal using Suboptimal PTS with threshold

5.6 PAPR reduction using Reduced Complexity PTS (RC-PTS) method

To investigate the PAPR reduction performance of sub-optimal RC-PTS method, the numerical simulation has been performed. To obtain the complementary cumulative density functions (CCDF's) and computational complexity, $R = 10^4$ random OFDM symbols were generated. The 256 sub-carriers $N = 256$ are assumed throughout the simulation and the QPSK data symbol with the energy normalized to unity is used. The weighting factors for optimizing the peak value of the modulated signal are {-1, +1}. The transmitted signal is oversampled by a factor of 4. Because it is shown to have better performance compared with the interleaved and adjacent sub-block partition methods [13], all the simulation results are based on the pseudo-random sub-block partition method with the numbers of subblock $M = 16$.

Fig.5.8 shows the CCDF for different search number using RC-PTS method as well as the IPTS and OPTS method. Initially, we would like to compare the performance of RC-PTS with OPTS. However it is very hard to get the result of OPTS in practice because OPTS needs exhaustive search over all 2^{M-1} combinations. Instead, we compare RC-PTS with 2000 random select trial (RST) which essentially has a comparable performance. For IPTS method, the search number is equal to 16. The search number (SN) 68, 126, 332 and 928 are considered for this method, which implies that the $N_{\max}$ are 5, 10, 30 and 100, respectively.

It is shown in Fig.5.8 that by using the RC-PTS method with the $SN = 68,126,332$ and 928, the PAPR are 7.56 dB, 7.37 dB, 7.15 dB, and 6.74 dB, respectively for $CCDF = 0.1\%$. But for IPTS method, the PAPR is larger than the 8.0 dB, which only achieve relatively poor performance. With $SN = 928$, RC-PTS method attains an asymptotically superior PAPR reduction capability compared with RST=2000. This result indicates that RC-PTS method can achieve PAPR performance comparable to global optimum.

Fig.5.9 compare the relatively computational complexity (RCC) of the RC PTS scheme with the IPTS and OPTS, where different SN is considered.

The RCC defined as

$$RCC = \frac{\text{Total computational complexity with RC PTS}}{\text{Total computational complexity with IPTS or OPTS}}$$

The Fig. 5.9 indicates that when compare with the OPTS, RC-PTS method can obviously reduced the computational complexity. Even with $SN = 928$, the of RCC RC-PTS method is about 3% of the OPTS, but the PAPR performance of RC-PTS method can almost achieve the PAPR performance of the OPTS as shown in Fig. 5.8. On the other hand, Fig. 5.9 also shown that there has comparable computational complexity for RC-PTS method with the IPTS, but RC-PTS method can obviously achieve the better PAPR performance as shown in Fig.5.8.

It is observed that the CCDF performance is improving with the increase of SN. However, this also increases the computational complexity. As such, a preset threshold (P_{Th}) can be used to tradeoff between the computational complexity and the PAPR performance. Fig.5.10 shows the average search number (ASN))for $N_{\max}$ different with P_{Th} the applied to RC-PTS method. ASN is defined as $ASN = \left(\frac{1}{R}\right)\sum_{i=1}^{R} SN_i$, where SN_i is the number of searched weighting factors for the OFDM symbol i and R is the number of OFDM symbols. It can be found from Fig. 5.10 that, with the increase of the $N_{\max}$, the ASN is asymptotically decreased when the P_{Th} is greater than 7 dB. On the other hand, the ASN is rapidly increased with $N_{\max}$ when P_{Th} is lower than 7 dB.

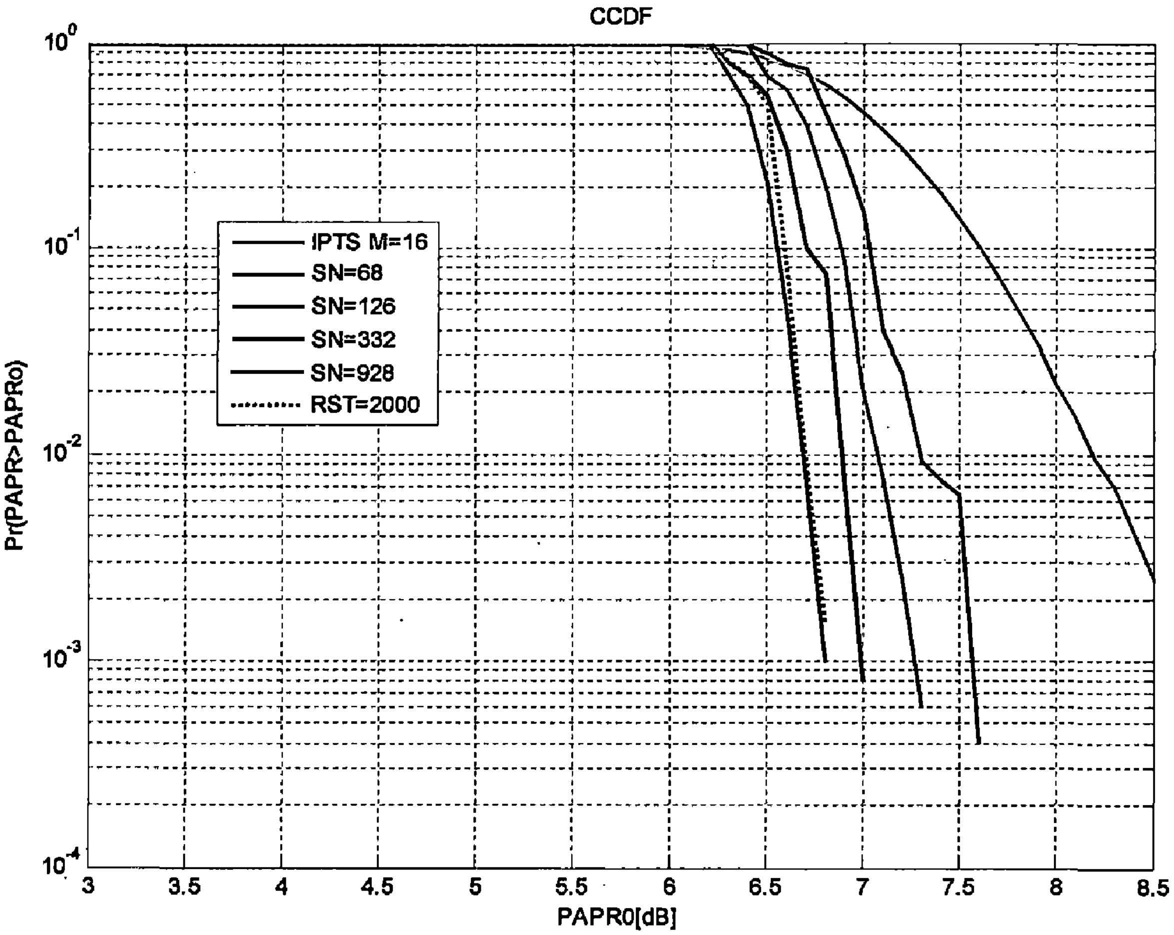


Fig: 5.8. CCDF of IPTS, OPTS and RC-PTS method when M=16

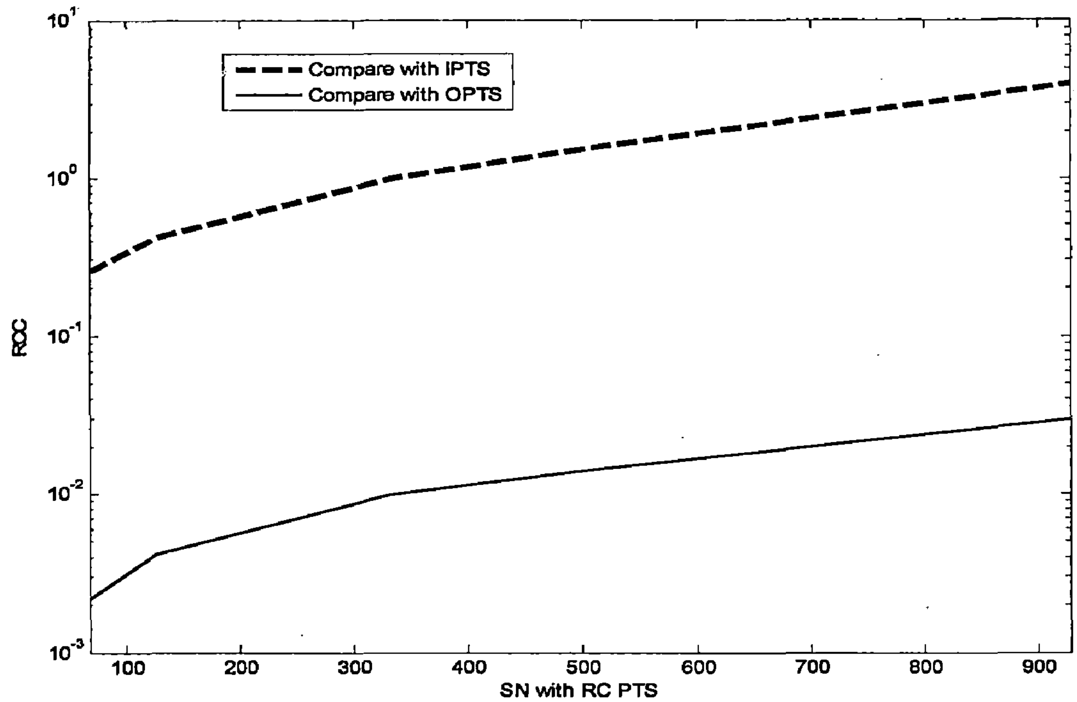


Fig: 5.9. RCC of the RC PTS compare with the IPTS and OPTS.

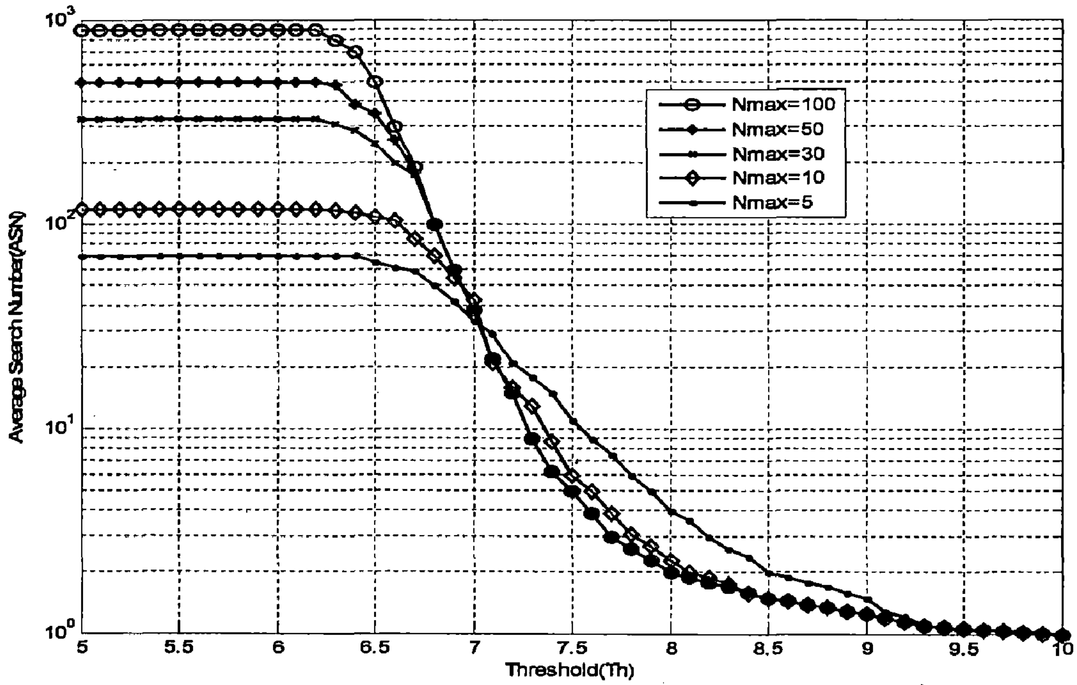


Fig: 5.10 Average search number (ASN) of RC-PTS method against present threshold P_{Th} for different N_{max} when M=16

5.7 PAPR reduction using Low Complexity PTS method

For simulation of PAPR reduction performance of LC-PTS in MATLAB environment, we use the following parameters

- No of OFDM subcarriers N =128 or 256
- No of OFDM data blocks =20,000
- No of Subblocks M=4
- No of phase or weighting factors W=4(i.e. $\{\pm 1, \pm j\}$)
- Oversampling factor L=4
- Modulation Scheme for source symbols: 16QAM

The LC-PTC technique is simulated with different T_h. Tables III and IV give the percentage of the computational complexity of LC-PTS compared to conventional PTS. Figs.5.11and 5.12 shows the PAPR reduction performance of LC-PTS. It is shown that the PAPR reduction performance is degraded with the increase of T_h. As a result, the complexity reduction is achieved at the cost of moderate performance degradation, since in LC-PTS; the PAPR calculation for subordination signal is simplified. Therefore, it is important for LC-PTS to make a tradeoff between computational complexity and PAPR reduction performance by the selection of T_h .

Average Number P =25, 67 for T_h=-8dB,-10dB respectively estimated by the simulation for N=128

TABLE VI

ANALYSIS OF COMPUTATIONAL COMPLEXITY IN LC-PTS FOR N=128

Operation	$T_h = -8\text{dB}$	$T_h = -10\text{dB}$
Complex Add.	40.52%	65.29%
Complex Mul.	40.52%	65.29%
Comp.	45.60%	78.5%

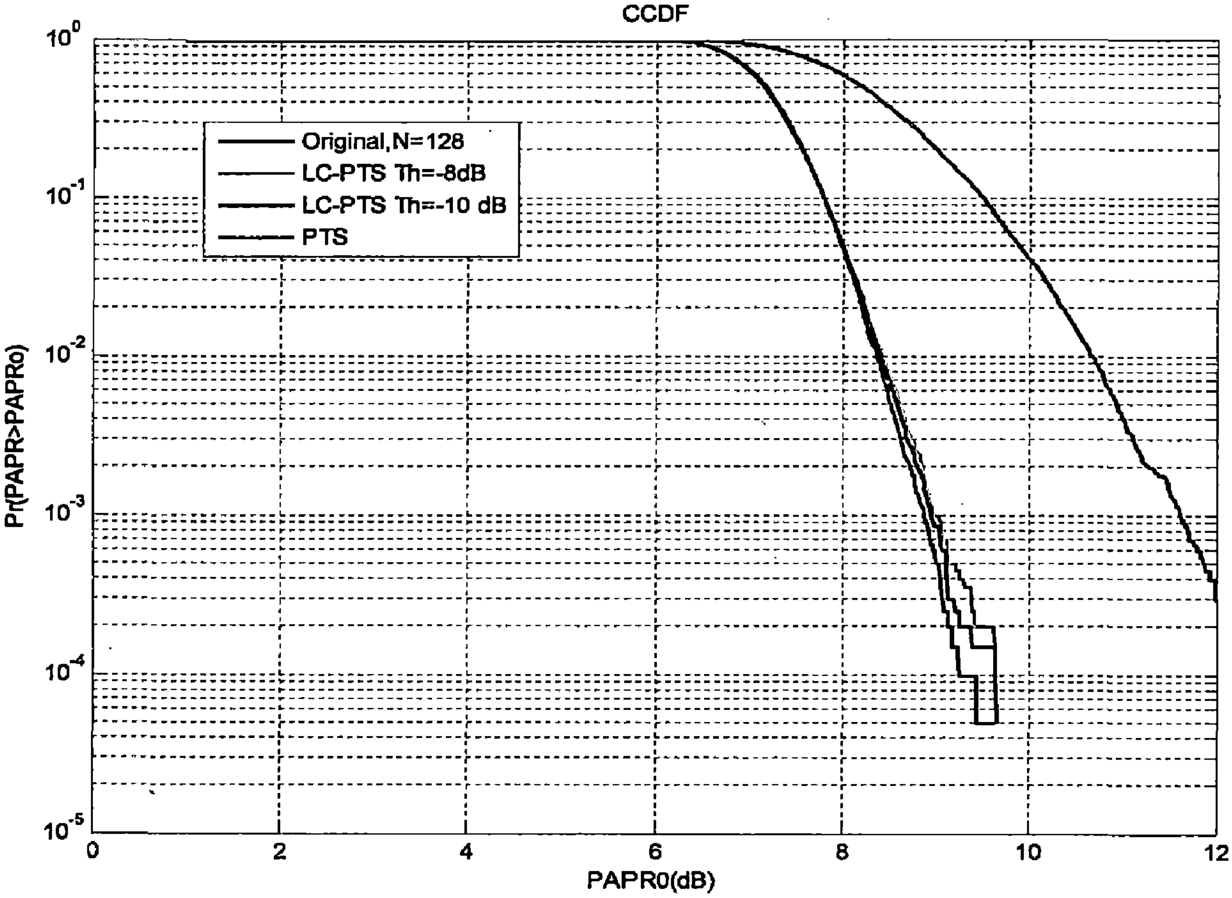


Fig: 5.11. PAPR reduction performance of LC-PTS with N=128

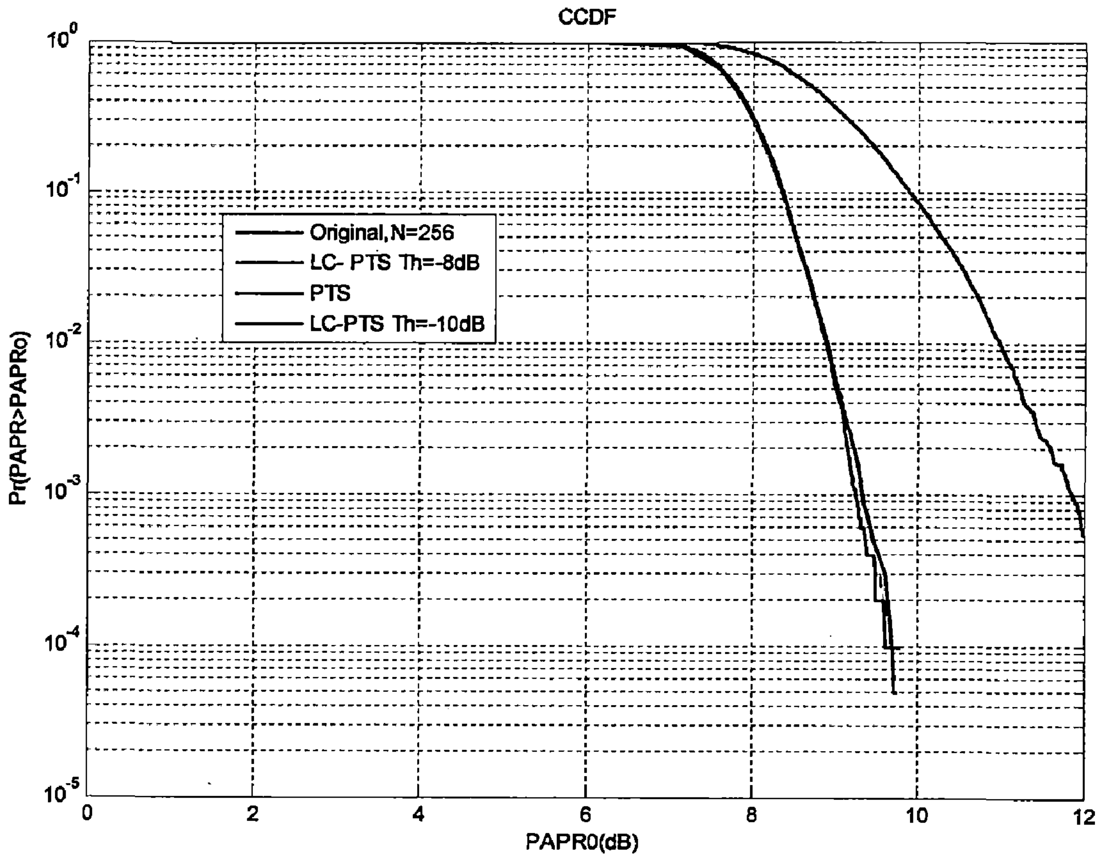


Fig: 5.12. PAPR reduction performance of LC-PTS with N=256

Average Number P =53, 137 for T_h=-8dB,-10dB respectively estimated by the simulation for N=256.

TABLE VII

ANALYSIS OF COMPUTATIONAL COMPLEXITY IN LC-PTS FOR N=256

Operation	$T_h = -8\text{dB}$	$T_h = -10\text{dB}$
Complex Add.	40.52%	65.29%
Complex Mul.	40.52%	65.29%
Comp.	45.60%	78.5%

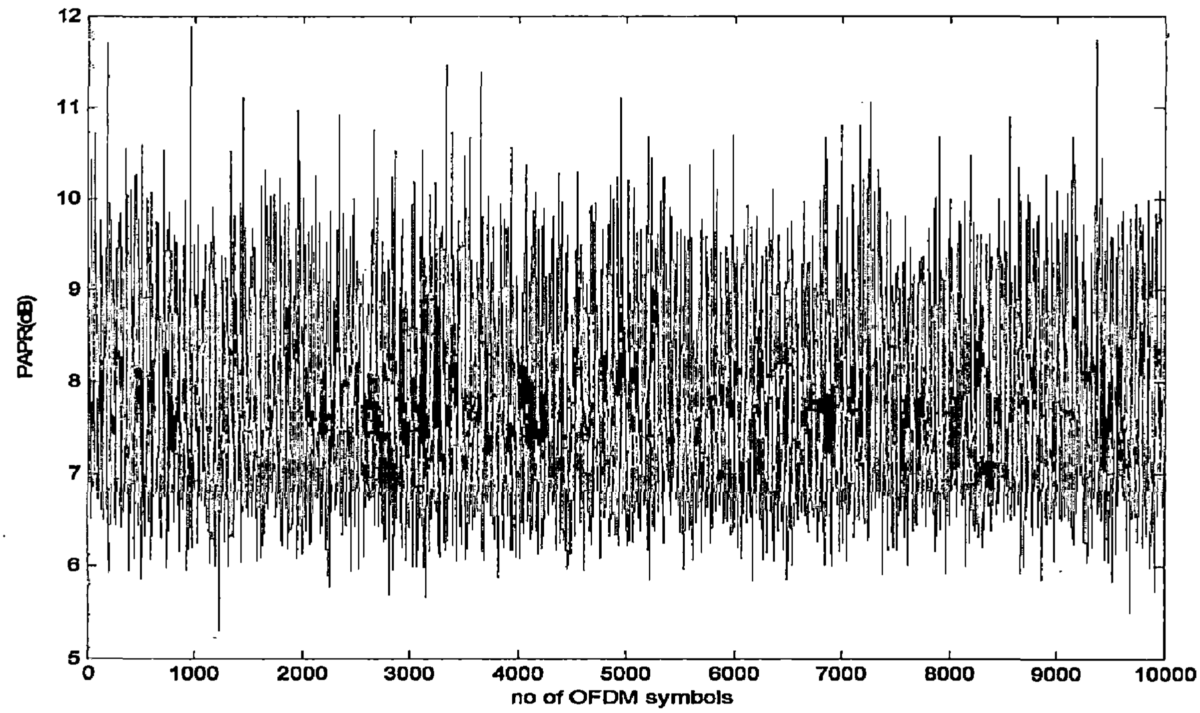


Fig: 5.13. PAPR of 16 QAM OFDM

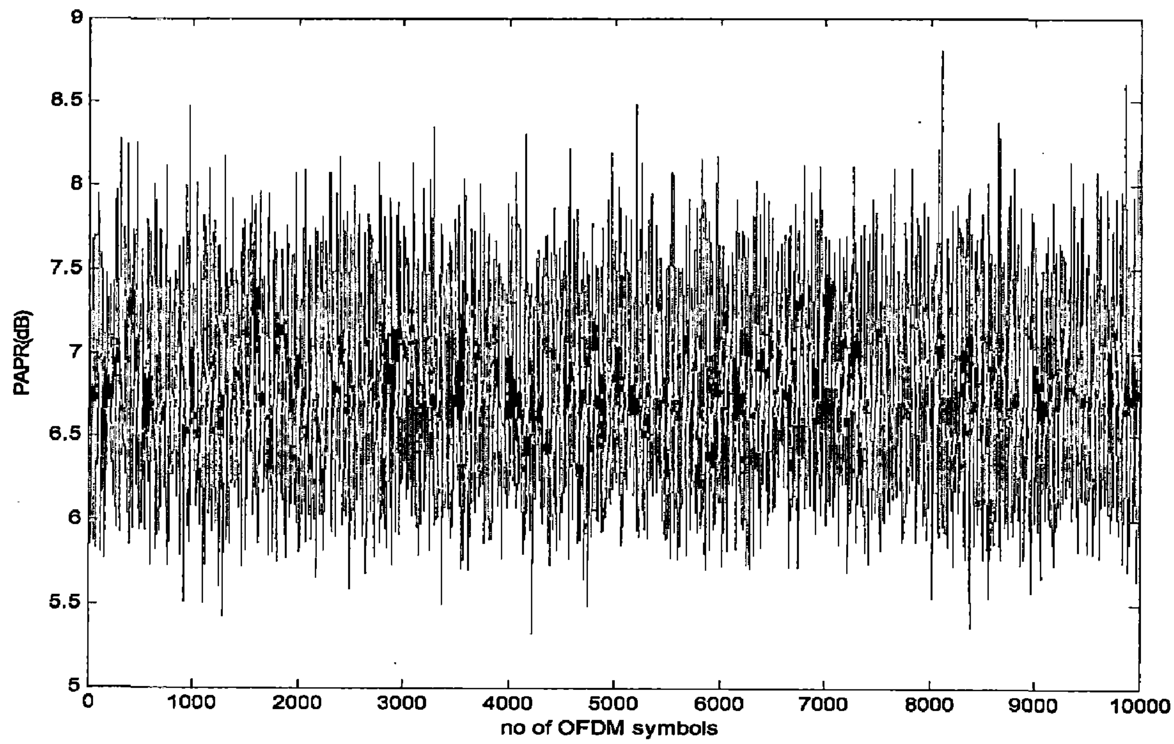


Fig: 5.14. PAPR of 16-QAM OFDM using LC-PTS

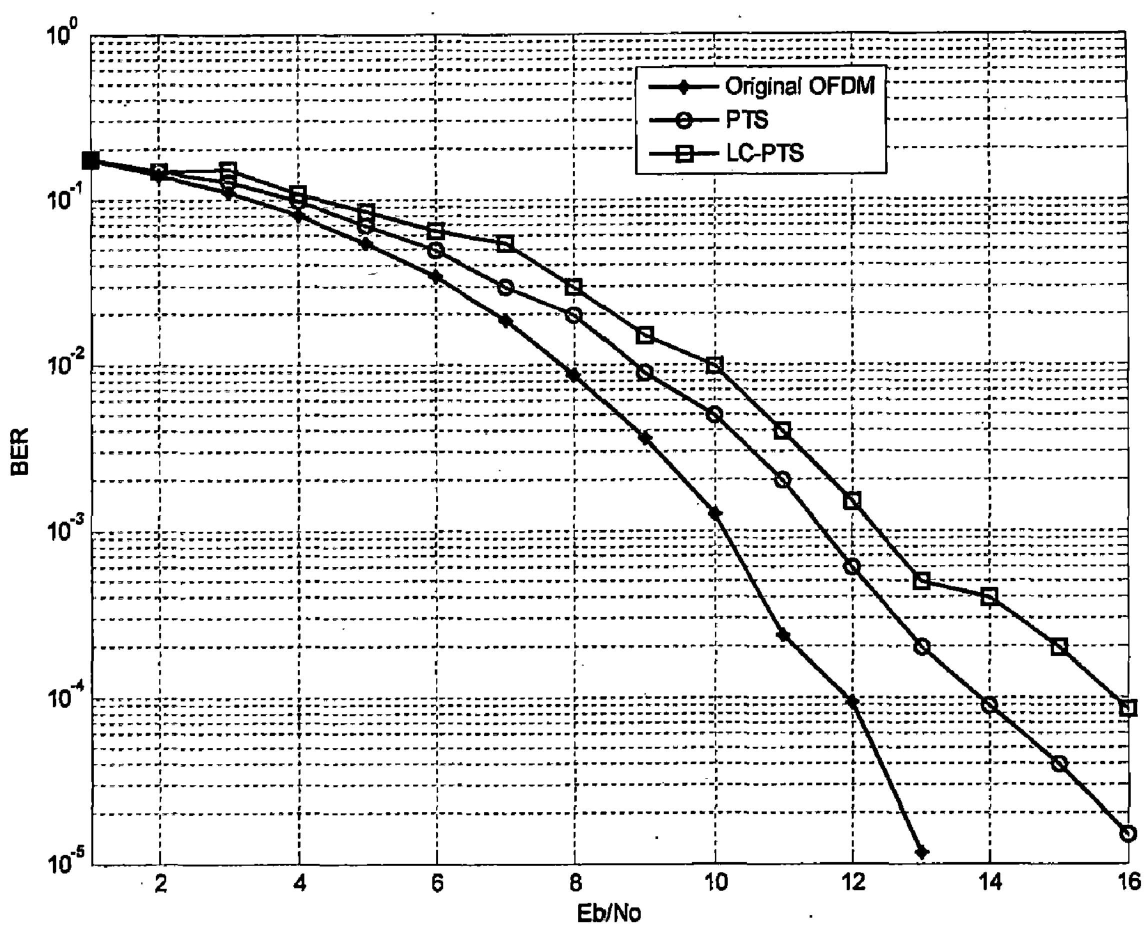


Fig: 5.15 BER Performance of 16-QAM OFDM with LC-PTS

Fig 5.13, Fig 5.14 shows the PAPR of OFDM signal with and without LC-PTS scheme.

Fig 5.15 shows comparison of BER performances of LC-PTS with original OFDM system and conventional PTS OFDM system.BER performance is degraded when using the Partial Transmit Sequences used for PAPR reduction

This dissertation work is aimed at the PAPR reduction performance comparison of different suboptimal strategies in Partial Transmit Sequence (PTS) methods with conventional PTS method and finding an efficient way to reduce complexity of phase factor combining and PAPR computation for an individual candidate signals respectively. The effect of complexity reduction of various PTS techniques is analyzed and simulated. The simulation results are summarized as follows.

- Based on the simulation results, it is proven that the PTS technique to reduce the PAPR of an OFDM signals. Employing Pseudo-Random SPS can reduce further. Increasing the number of subblocks or phase weighting factors PAPR reduction further increases with the side effect of high complexity.
- Iterative flipping algorithm reduces the complexity from exponential variation to linear variation with respect to number of subblocks, but it suffers little performance degradation.
- Adaptive PTS technique eliminating unnecessary iterations that don't contribute significantly to the reduction of the PAPR, both the PAPR and complexity can be reduced simultaneously.
- Sub-optimal PTS, which finds the specific bit in weighting factors that, leads to a smaller PAPR each processing, to reduce the PAPR of the OFDM signal. By using the preset threshold the number of iterations was reduced with only slight performance degradation.
- In reduce complexity PTS, by using the relationship between weighting factors and the transmitted signals the computational complexity is only about one-(M-1)th of the conventional PTS .where M is the number of subblocks. Especially when constraining the number of transmitted signals to be searched at each stage and using a preset threshold, the complexity of this method is rapidly reduced with only slight performance degradation.
- In low complexity PTS, utilizing the correlation among candidate signals in each subset so as to reduce the computational complexity. The selection of threshold is advised for to make good tradeoff between PAPR reduction and complexity.

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