BLIND CHANNEL ESTIMATION FOR OFDM SYSTEMS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

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By

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report entitled, **"BLIND CHANNEL ESTIMATION FOR OFDM SYSTEMS"** towards the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems**, submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from July 2007 to June 2008, under the guidance of **Dr. D. K. MEHRA, Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee.**

I have not submitted the matter embodied in this dissertation for the award of any other Degree or Diploma.

Date: 13/06/2008

Place: Roorkee

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is an effective way to increase data rate and simplify equalization problem in wireless communications. OFDM systems require an efficient channel estimation procedure to demodulate the received data coherently.

Pilot based techniques have normally been employed for channel estimation in OFDM systems. The obvious drawback associated with pilot based techniques for channel estimation is bandwidth overhead. Blind channel estimation techniques act as alternative to pilot based techniques to increase the spectral efficiency. Second order statistics (SOS) based blind channel estimation methods overcome the slow and local convergence which persist in HOS methods. Among SOS based blind channel estimation methods, noise subspace method has proved to be effective in terms of convergence speed and estimation accuracy. In this dissertation work, we compare SOS based TXK (Tong ,Xu and Kailath) algorithm and noise subspace algorithm for blind channel identification of FIR channels using Single Input and Multiple Output (SIMO) model.

Conventional OFDM systems are based on sufficient cyclic prefix (CP) to facilitate simple receiver implementation, however, at the cost of significant channel utilization loss that may be the over riding constraint for future high speed services. Thus, there exists increasing interest in OFDM systems with less CP or no CP, for which conventional algorithms assuming sufficient CP may not be appropriate. This dissertation work is focused on study of blind channel estimation techniques for OFDM systems with insufficient CP or no CP. The redundancy introduced by oversampling or by virtual carriers (VCs) is exploited in above blind channel estimation techniques that give the OFDM systems the potential to achieve higher channel utilization.

MIMO-OFDM systems can achieve higher data rates over broadband wireless channels. For the purpose of subspace based blind channel estimation for MIMO OFDM, either cyclic prefix (CP) or zero padding (ZP) has been exploited. In this dissertation work, evaluation of both CP and ZP based techniques for channel estimation in MIMO OFDM is performed.

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CHAPTER 3: BLIND CHANNEL ESTIMATION FOR OFDM SYSTEMS

CHAPTER 4: BLIND CHANNEL ESTIMATION FOR MIMO-OFDM SYSTEMS

Chapter 1
Introduction

Since the introduction of the first mobile phone systems (e.g. NTT system in Japan) in the late 1970's, wireless communication has evolved from analogue system to digital system and from providing single voice service to provision for multiple data services. Currently, the widely deployed second generation wireless systems (for instance, GSM or CDMA systems) are able to provide connections at the data rate up to 28.8kbps to accommodate both voice services and rudimentary data service. Driven by the expectation that demands for data services will outgrow basic voice communications, third generation wireless system standards have been developed to achieve improved data rate – ranging from 9.6Kbps to 2Mbps. However, the increasing popularity of mobile computing and communication devices such as laptops, tablet/pocket PCs and PDAs for an increasingly mobile population accompanied by penetration of internet has created demand for multi-media rich services such as internet browsing and audio/video streaming. Unfortunately, cellular 3G networks have failed to satisfy the rate and quality of service (QoS) requirement of these new types of data services. Therefore, great effort has been invested in research and development of next generation (4G) wireless local and personal area networks (WLANs/WPANs) that are capable of supporting such high data rate services while providing QoS guarantees. The ultimate goal for broad wireless communication is to provide: "*anytime, anywhere and any media, any device,*" broadband services.

4G systems should have the following requirements:

- Generic Architecture: enabling the integration of existing technologies.
- Higher Spectral Efficiency: offering higher data rates in a given spectrum.
- High Scalability: designing different cell configuration (hot spot, ad hoc) for better coverage.
- High Adaptability and Reconfigurability: supporting different standards and technologies.
- Low Cost: it has been proposed that 4G should have a low cost per bit (1/10 of 3G)
- Future proof: opening the door for new technologies.

In order to provide these services, a high data rate and high quality digital communication system is required in a restricted bandwidth. The high data rate requirement motivate research efforts to develop efficient coding and modulation schemes along with sophisticated signal and information processing algorithm to improve the quality and spectral efficiency of wireless communication links. However, these developments must cope with several performance limiting challenges that include channel fading, multiuser interference, limitation of size/power especially at mobile units.

A primary challenge to high data rate in wireless communications is the presence of *multi path fading channel.* Multipath fading results from the fact that radio signal propagates through many paths with different delays from the transmitter to the receiver. For typical narrow band modulation, this gives rise to variations in received signal amplitude (fading); if the delay spread of the various components is a significant fraction of the symbol duration as in frequency selective fading, it also leads to inter symbol interference (ISI).

To combat the adverse effect of ISI, channel equalization is typically employed [1]. In conventional single carrier communication systems, increasing the data rate (equivalently decreasing the symbol duration) for a given multi path channel incurs more severe ISI, implying the need for more complex channel equalization. Thus in single carrier narrow-band modulation, transceiver designs are limited by cost/complexity considerations of feasible equalizer implementations.

1.1 Orthogonal Frequency Division Multiplexing (OFDM):

Multi-carrier modulation (MCM) [2] is an alternative approach to alleviating the impact of frequency selective fading channels. In MCM, high rate data stream is divided into several independent low-rate sub streams that modulate a set of sub carriers and are transmitted parallel over the channel. The increased symbol duration on each sub stream reduces the impact of ISI, thereby increasing the system's immunity to frequency selective fading channels.

Orthogonal Frequency Division Multiplexing (OFDM) ([3]) is the most popular MCM scheme. The set of sub carriers in the OFDM are chosen such that they have minimum spacing while preserving mutual orthogonality of the transmitted time domain signals. A main attraction of OFDM is based on its implementation using cost efficient Fast Fourier Transform (FFT) to implement multiple carrier modulation or demodulation operations. Thus, the robustness to frequency selective fading channels accompanied by the high spectral efficiency and the feasibility of low cost transceiver implementations have led OFDM to being considered as a promising candidate for high rate wireless communications. OFDM has been opted in the following applications [3]:

- European standards: Digital Audio Broadcasting (DAB) with target rates 1.7 Mbps and terrestrial Digital Video Broadcasting (DVB)/T with target rates 20 Mbps.
- Fixed wire applications: Asymmetric Digital Subscriber Lines.
- Broadband Fixed Wireless Access (IEEE 802.16).
- High-speed wireless LANs: IEEE 802.11a with target rates of 6-54 Mbps.

One of the most interesting trends in wireless communication is the proposed use of multiple input multiple output (MIMO) systems. A MIMO system uses multiple transmitter antennas and multiple receiver antennas to break a multipath channel in to several individual spatial channels. The basic idea is to usefully exploit the multipath rather than mitigate it, considering the multipath itself as a source of diversity that allows the parallel transmission of N independent sub streams from the same user. The exploitation of diversity and parallel transmission of several data streams on different propagation paths at the same time and frequency allows for extremely large capacities compared to conventional wireless systems [4]. The prospect of many orders of magnitude improvement in wireless communication performance at no cost of extra spectrum (only hardware and complexity are added) is largely responsible for the success of MIMO as a topic for new research. The combination of the two powerful techniques, MIMO and OFDM, is very attractive, and has become a most promising broadband wireless access scheme.

1.2 Channel estimation techniques for OFDM systems:

For OFDM systems, an efficient and accurate channel estimation procedure is necessary to coherently demodulate received data. Although differential detection could be used to detect the transmitted signal in the absence of channel information, it would result in about 3dB loss in SNR compared to coherent detection. Also, reliable channel estimation is needed for adaptive bit loading [5] for maximizing the data throughput of system.

Channel estimation can be done by either training or pilot based approach (supervised approach) or blind approach (non-supervised approach).

1.2.1 Training or Pilot based channel estimation methods for OFDM systems:

Classical methods for channel estimation are based on the use of training sequence. A known sequence is transmitted for a limited period of time, during which a channel estimate is obtained. Pilot symbols (on pilot subcarriers) are embedded in between the data symbols (on data subcarriers), which provides the channel information at the receiver. These estimated values are interpolated over the data subcarriers and the data symbols are decoded. The pilot spacing in both time and frequency domain plays a significant role as channel characteristics should not change between pilot subcarriers. In order to cope with the Doppler effect due to mobility of wireless systems, reference sequence must be repeated periodically and may result in a significant loss in the useful bit rate. The obvious drawback associated with pilot based techniques for channel estimation is bandwidth overhead.

In, [6], minimum mean square error (MMSE) and least squares (LS) channel estimator are proposed. The MMSE estimator has good performance but high complexity. On the other hand, the LS estimator has low complexity, but its performance is not as good as that of MMSE estimator. In [7], comb type pilot signals, uniformly spaced across subcarriers within each frame, have been used with interpolation for the remaining subcarriers.

1.2.2 Blind channel estimation methods for OFDM systems:

Blind channel estimation is a novel strategy to eliminate the pilot overhead in a communication system. Blind channel identification methods are bandwidth efficient as compared to pilot based methods. But blind channel identification algorithms are highly complex.

Ideally, a blind scheme does not employ any pilot symbols and instead relies only on the information symbol outputs to estimate the channel. It is to be noted, that though the information symbols are individually unknown, one can have statistical knowledge about an ensemble of such symbols. This statistical information provides a viable means to estimate the channel. Theoretically, if such a blind scheme were possible it would eliminate completely the need to transmit pilot symbols and thus would be totally bandwidth efficient.

Various blind channel identification algorithms, based on Higher Order Statistics (HOS) of the received signal, proposed in literature [8,9,10,11] have major drawbacks of slow convergence and local convergence. To overcome these drawbacks, a significant amount of research has been done on Second Order Statistics (SOS) based blind estimation [12]. However problem with SOS based methods is that, under baud rate sampling, SOS loses the phase information of the channel. Tong, Xu, Kailath algorithm [13] was the first one to use SOS of fractionally sampled channel outputs for the blind detection of FIR channel. Moulines *et al* [14] present an algorithm that explicitly exploits the signal and noise subspace separation as well as the special structure of channel matrix. This method works on the principle of orthogonality between signal and noise subspace which leads to the identification of channel. The advantage of subspace methods is that convergence is faster.

In an OFDM system using training, the received signal samples corresponding to the cyclic prefix (CP) are discarded. However, those samples contain useful information that can be exploited for the purpose of channel estimation or channel tracking. Blind channel estimators present in [15] and [16] assume the inherent CP-induced cyclostationarity at the transmitter explicitly or implicitly, while the estimators [17] and [18] belong to the class of deterministic subspace approach. Cai and Akanshu [17] developed a noise subspace algorithm by utilizing the structure of filtering matrix introduced by the CP insertion.

Other than the CP, there exists another resource – *Virtual Carriers* (VC). These VCs are exploited for the purpose of subspace based blind channel estimation in OFDM systems with/without CP [19]. Methods for blind channel identification of OFDM systems without CP are spectral efficient due to the removal of CP. Instead of using VCs, receiver oversampling is used for subspace based blind channel identification of OFDM system without CP [20].

Precoding in OFDM system act as alternative to the CP/VC's for the purpose of subspace based blind channel identification of OFDM systems. The methods in [21, 22] uses a non-redundant linear precoders at the transmitter, and the channel state information (CSI) is contained in each entry of the signal covariance matrix. These methods extract CSI from single column of received covariance matrix. Nallanathan [23] proposed a blind channel estimation method that can overcome the aforementioned shortcomings and extract the CSI from all the columns of received covariance matrix.

Various precoding based blind channel estimation algorithms for MIMO OFDM systems have been proposed in [24, 25, 26]. Zeng and Ng [27] exploits zero padding in OFDM systems for the purpose of subspace based blind identification of MIMO-OFDM channels. Gao and Nallanathan [28] developed a novel subspace algorithm that is suitable for CP based MIMO-OFDM systems. Chenyang Shin *et al.* [29] proposed a method that unifies and generalize the subspace algorithm for SISO-OFDM [19] system to MIMO-OFDM systems with any number of transmit and receive antennas.

1.3 Problem Statement:

This dissertation presents the following work:

1. Study of SOS based blind channel estimation methods for FIR channels using single input multiple output (SIMO) model.
2. Application of Subspace based blind channel identification algorithms to OFDM systems with or without CP.
3. Extension of subspace based blind channel estimation methods for OFDM systems to blind identification of channel in MIMO-OFDM systems.

1.5 Organization of the report:

This report is organized in five chapters:

In *Chapter 1,* we summarize problem statement of the dissertation work and also give an overview of channel estimation problem in OFDM systems.

In *Chapter 2,* we discuss SOS based TXK algorithm and subspace algorithm for blind channel identification of FIR channels using SIMO model, and present simulation results.

In *Chapter 3,* we study the exploitation of VCs for the subspace based blind channel identification methods for OFDM systems with and without CP. Receiver oversampling method for blind channel identification of OFDM systems without CP is presented next. We then discuss semi- blind implementation of subspace methods. We also present simulation results of these algorithms.

In *Chapter 4,* we describe the subspace based blind channel estimation for cyclic prefixed MIMO-OFDM systems. We then discuss subspace based blind channel estimation for zero padded MIMO-OFDM systems. Simulation results of these algorithms are also given.

Chapter 5 concludes the report.

Chapter 2

SOS Based Blind Channel Identification

In this chapter, we first give a brief review of HOS based blind identification methods. We then discuss different SOS based blind identification methods. We then introduce single input multiple discrete channel models (SIMO) for SOS based identification techniques. TXK algorithm for blind identification is described next. We also describe Subspace based method for blind identification of SIMO systems. We finally present simulation results.

2.1 Blind channel equalization and identification:

The innovative idea of self recovering (blind) adaptive equalization was first proposed by Sato [8] and later developed by Godard [9]. These algorithms are the generalized versions of Bussgang's blind equalizers [10]. The Bussgang algorithm performs blind equalization of a linear communication channel by subjecting the received signal to an iterative deconvolution process. Due to the minimization of a nonconvex cost function, there is a likelihood of being trapped in local minima. Also these algorithms have relatively slow rate of convergence.

A family of constant modulus blind equalization algorithms (CMA) proposed by Godard has following important features

- It is more robust than other Bussgang's algorithms with respect to the carrier phase offset due to the fact that the cost function used for its derivation is based solely on the amplitude of the received signal.
- Under steady-state conditions, the Godard algorithm attains a mean-square error that is lower than that of other Bussgang algorithms.

Another blind identification approach was proposed by Hatzinakos and Nikias [11]. In this method they modeled the baud rate sampled received signal as moving- average process. The multipath channel is then identified from the trispectrum of the received signal. The advantage of this method over the adaptive blind equalization methods is that the algorithm will provide exact information of a possibly nonminimum phase channel, whenever the higher order cumulants and the trispectrum of the observation can be estimated accurately. The above algorithm is sensitive to uncertainties associated with timing recovery, unknown phase jitter and frequency offset. Due to the usage of HOS, algorithm has slow convergence than SOS. Finally, the effect of non-Gaussian noise may effect the convergence and the performance of Nikias algorithm.

The major drawbacks of the HOS method are:

- Local Convergence
- Slow Convergence

Local convergence is a possibility for many blind algorithms based on HOS, which typically results from the use of multimodal equations. Slow convergence is due to the fact that HOS needs long streams of output data in order to obtain accurate time averages. This requirement of large data samples can pose a potentially serious obstacle to the application of blind equalization in fast time varying environment. To overcome the above drawbacks, a lot of research has been done on SOS based blind estimation algorithms. However problem with SOS based methods is that, under baud rate sampling, SOS loses the phase information of the channel.

When the input process is nonstationary, the second order statistics of the channel output contains some phase information of the channel so that we can identify channel with both magnitude and phase. For applications in communications, many types of signals exhibit a particular type of nonstationarity called cyclostationarity. The exploitation of cyclostationarity has shown promising results in various applications such as detection and filtering of communication signals, parameter estimation, direction finding etc. Tong, Xu, Kailath algorithm (TXK) [13] exploits the cyclostationarity or the equivalent multichannel nature of fractionally sampled channel outputs for blind detection of FIR channel. Requirement for TXK algorithm is that channel input be uncorrelated, or that its correlation function be known. TXK algorithm does not exploit block Toeplitz structure of the multichannel model of a SIMO system. TXK algorithms success also depends on the receiver ability to estimate the channel noise variance $\hat{\sigma}^2$.

Moulines *et al* [14] present an algorithm that explicitly exploits the signal and noise subspace separation as well as the special structure of channel matrix. By exploiting the special block structure of unknown channel matrix, the subspace method does not require the channel input to be uncorrelate

...d. In fact so long as the input covariance matrix is of full column rank, the subspace method can identify the channel impulse response, when channel matrix is of full column rank. Subspace based blind channel estimation works on the principle of orthogonality between signal and noise subspace which leads to the identification of channel. The advantage of subspace methods is that convergence is faster.

Subspace methods are divided into deterministic subspace methods and statistical subspace methods. Deterministic methods do not assume that the input source has a specific statistical structure. Some of the deterministic methods are Cross relation approach, Noise subspace method, Least squares smoothing techniques etc.

Cross Relation(CR) approach proposed by Hua [32] is very effective for small data samples and applications at high SNR. Hua showed that CR method combined with the ML approach offers performance close to the Cramer Rao bound. Problem with the CR method is that channel order cannot be over estimated.

Least Sqaures Smoothing(LSS) proposed by Tong et al [33] approach is adaptive in implementation. The key idea of LSS rests on the isometric relation between the input and observation spaces. This approach has two attractive features. First, it converts a channel estimation problem to a linear LSS for which there are efficient adaptive implementations using lattice filters. Second, a joint order detection and channel algorithm can be derived that determines the best channel order and channel coefficients to minimize smoothing error.

In statistical subspace approaches, it is assumed that the source is a random sequence with known SOS. The advantage of statistical methods is that they require only upper bound on channel order. Some of the statistical methods are TXK algorithm, Identification via cyclic spectra, Identification via Linear prediction etc.

Identification via Linear Prediction [34] algorithm uses the concept of autoregressive modeling of received signal. Algorithm consists of two steps: 1. Identification of h_0 2. Identification of h_k based on h_0, where h_0 and h_k are the first and k^{th} tap of an FIR channel. It does not require the exact channel order, thus it is robust against over determination of the channel order. Derived from the noiseless model, the linear prediction idea is no longer valid in the presence of noise. The main disadvantage of this algorithm is that it is a two-step approach whose performance depends on identification of h_0.

2.2 Single input multiple output (SIMO) channel models for SOS based identification:

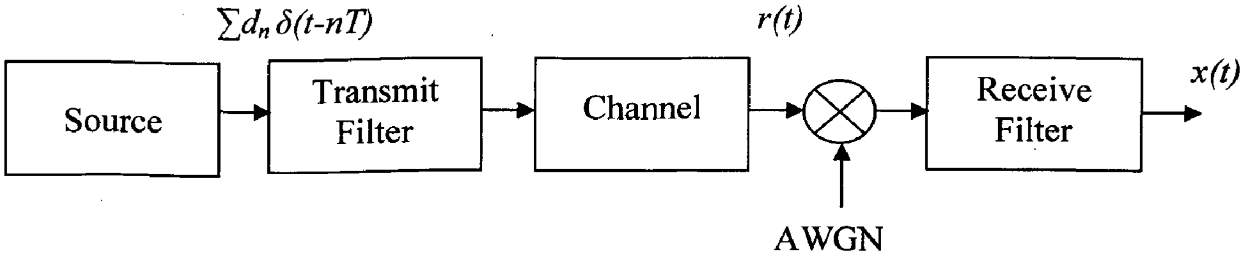


Fig 2.1: Basic elements of a communication system

As shown in Fig 2.1, d_n denote the symbol emitted by the digital source at time *nT*, where T is the symbol duration. This signal is modulated, filtered, sent through the communication channel, filtered, and demodulated. The resulting baseband signal is given by

$$x(t) = \sum_{m=-\infty}^{\infty} d_m h(t - mT) + v(t) \tag{2.1}$$

Where *v(t)* is noise which is band limited complex stationary process, *h(t)* is the encompass of the transmitter filter, channel and receive filter.

We make the following assumptions

- Channel *h(t)* has finite support L.
- At time n, the receiver processes the channel output due to a transmitted signal vector that consists of $(L+N)$ symbols.

Let the oversampling factor be Δ. Then a set of $P = T/\Delta$ sequences can be constructed according to $x_n^{(i)} = x(t_0 + i\Delta + nT)$ for $0 \le i \le P-1$. Each sequence has period *T*. The resulting over sampled signal using (2.1) can be written as

$$x_n^{(i)} = \sum_{m=0}^{L} d_{n-m} h(t_0 + i\Delta + mT) + v_n^{(i)} \tag{2.2}$$

where $v_n^{(i)} = v(t_0 + i\Delta + mT)$ are the samples of *v(t)*.

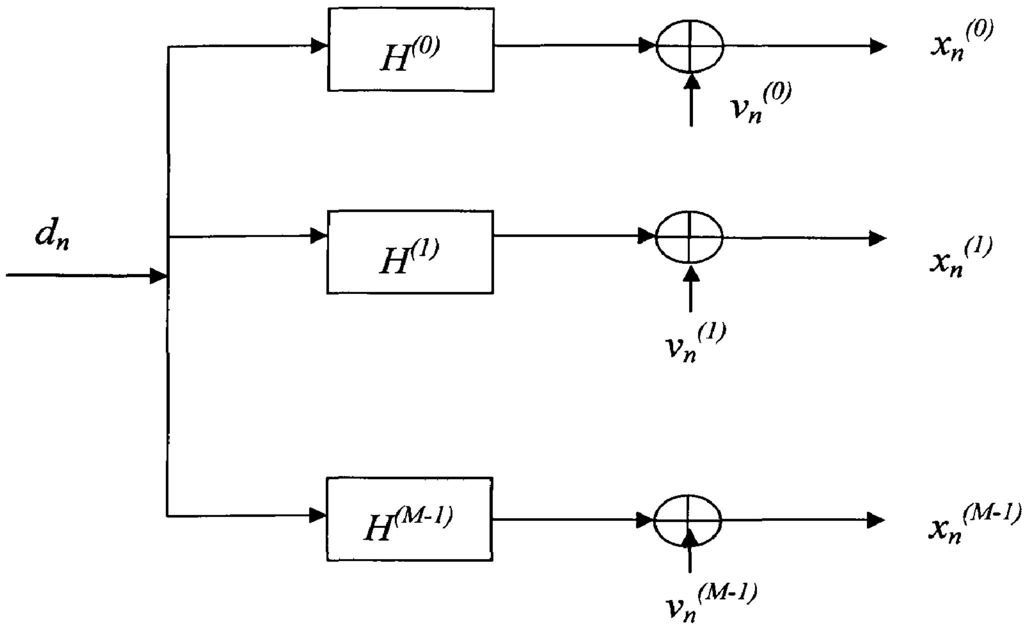


Fig 2.2: Representation of an oversampled channel as a SIMO model

A *single input multiple output* (SIMO) model which consists of M virtual channels is shown in Fig 2.2.Each sequence $x_n^{(i)}$ depends on discrete–time impulse response $H^{(i)}$ characterizing the i th channel, where

$$H^{(i)} \stackrel{def}{=} [h_0^{(i)}, h_1^{(i)} ......, h_L^{(i)}]^T$$

$$\stackrel{def}{=} [h(t_0 + i\Delta), h(t_0 + i\Delta + T), ..., h(t_0 + i\Delta + LT)]^T \quad 0 \le i \le P-1 \tag{2.3}$$

Taking N successive symbols of the received signal sequence as

$$X_n^{(i)} = [x_n^{(i)}, ..., x_{n-N+1}^{(i)}]^T \tag{2.4}$$

Then using (2.2) we obtain

$$X_n^{(i)} = \mathcal{H}_N^{(i)} D_n + V_n^{(i)} \tag{2.5}$$

Where $V_n^{(i)} = [v_n^{(i)},......,v_{n-N+1}^{(i)}]^T$ (dim N x 1) and $D_n = [d_n,\cdots,d_{n-N-L+1}]^T$ (dim $(N + L)$ x 1) and the matrix $\mathcal{H}_N^{(i)}$ is N x $(N+L)$ filtering matrix given by

$$\mathcal{H}_N^{(i)} = \begin{bmatrix} h_0^{(i)} & \cdots & h_L^{(i)} & \cdots & \cdots & 0 & 0 \\ 0 & h_0^{(i)} & \cdots & h_L^{(i)} & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & h_0^{(i)} & & & h_L^{(i)} \end{bmatrix}_{N\times(N+L)} \tag{2.6}$$

By taking such M oversampled signals from (2.5) and grouping them together, we will get

$$\begin{pmatrix} X_n^{(0)} \\ \vdots \\ X_n^{(M-1)} \end{pmatrix} = \begin{pmatrix} \mathcal{H}_N^{(0)} \\ \vdots \\ \mathcal{H}_N^{(M-1)} \end{pmatrix} D_n + \begin{pmatrix} V_n^{(0)} \\ \vdots \\ V_n^{(M-1)} \end{pmatrix} \tag{2.7}$$

(2.7) can be written as

$$X_n = \mathcal{H}_N D_n + V_n \tag{2.8}$$

where $X_n = \begin{pmatrix} X_n^{(0)} \\ \vdots \\ X_n^{(M-1)} \end{pmatrix}_{(MN\times 1)}$ $\quad \mathcal{H}_N = \begin{pmatrix} \mathcal{H}_N^{(0)} \\ \vdots \\ \mathcal{H}_N^{(M-1)} \end{pmatrix}_{(MN\times L+N)}$

$$V_n = \begin{pmatrix} V_n^{(0)} \\ \vdots \\ V_n^{(M-1)} \end{pmatrix}_{(MN\times 1)} \quad D_n = \begin{bmatrix} d_n \\ d_{n-1} \\ \vdots \\ d_{n-N-L+1} \end{bmatrix}$$

In alternate Sylvester matrix representation, the M virtual channel coefficients having the same delay index are all grouped together. Specifically, we write M virtual coefficients as

$$\mathrm{h}'_k = \left[h_k^{(0)}, h_k^{(1)}, \cdots, h_k^{(M-1)}\right]^T, \quad k = 0,1,\cdots,L \tag{2.9}$$

and correspondingly, we define $M \times 1$ received signal vector and noise vector as,

$$\mathrm{x}'_n = \left[x_n^{(0)}, x_n^{(0)}, \cdots, x_n^{(M-1)}\right]^T \text{ and } \mathrm{v}'_n = \left[v_n^{(0)}, v_n^{(0)}, \cdots, v_n^{(M-1)}\right]^T$$

Then using (2.2), NL received samples can be grouped together as

$$X'_n = \begin{bmatrix} \mathrm{x}'_n \\ \mathrm{x}'_{n-1} \\ \vdots \\ \mathrm{x}'_{n-N+1} \end{bmatrix}$$

$$= \mathcal{H}'_N D_n + V'_n \tag{2.10}$$

where $V'_n = \begin{bmatrix} \mathrm{v}'_n \\ \mathrm{v}'_{n-1} \\ \vdots \\ \mathrm{v}'_{n-N+1} \end{bmatrix}$, D_n is given by (2.5) and

$$\mathcal{H}'_N = \begin{bmatrix} \mathrm{h}'_0 & \mathrm{h}'_1 & \cdots & \mathrm{h}'_M & 0 & \cdots & 0 \\ 0 & \mathrm{h}'_0 & \cdots & \mathrm{h}'_{M-1} & \mathrm{h}'_M & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathrm{h}'_0 & \mathrm{h}'_1 & \cdots & \mathrm{h}'_M \end{bmatrix}_{(MN\times(L+N))} \tag{2.11}$$

The block Toeplitz matrix $\mathcal{H}'_N$ is called a Sylvester matrix representation.

The matrices $\mathcal{H}_N$ and $\mathcal{H}'_N$ defined in (2.8) and (2.11) respectively, differ primarily in the way in which their individual rows are arranged. But they contain same information about the channel. More importantly the spaces spanned by the columns of $\mathcal{H}_N$ and $\mathcal{H}'_N$ are canonically equivalent. The multichannel filtering matrix $\mathcal{H}_N$ and $\mathcal{H}'_N$ play a central role in the blind identification problem.

Using (2.8), correlation functions of received signal is given by

$$\boldsymbol{R}_x(\boldsymbol{k}) = E\left(X_n X_{n-k}\right) = \mathcal{H}_N \boldsymbol{R}_d(\boldsymbol{k}) \mathcal{H}_N^H + \boldsymbol{R}_v(\boldsymbol{k})$$

where $\boldsymbol{R}_d(k) = E(D_n D_{n-k}^H)$ and $\boldsymbol{R}_v(k) = E(V_n V_{n-k}^H)$ (2.12)

Here $\boldsymbol{R}_v(k)$ is noise covariance matrix of dimension ($MN \times MN$) and $\boldsymbol{R}_d(k)$ is source covariance matrix of dimension $(N+L) \times (N+L)$.

Our aim is to identify $M(L+1) \times 1$ vector H of channel coefficients

$$H \stackrel{def}{=} [H^{(0)T}, \cdots, H^{(M-1)T}]^T_{(M(L+1)\times 1)}$$

using the second order statistics of the received signal.

2.3 TXK Time domain blind identification algorithm:

One of the best known works on blind channel identification based on second order statistics (SOS) was presented by Tong, Xu and Kailath [13]. They exploit the cyclostationarity present in oversampled received signal for the detection of SIMO channels. With out loss of generality the following assumption can be made:

Assumption: D_n is zero mean stationary process with unit variance. As a result, input signal vector D_n has following autocorrelation function

$$\begin{aligned} \boldsymbol{R}_d(k) &= E(D_n D_{n-k}^H) \\ &= \boldsymbol{J}^k, \ \boldsymbol{k} \geq 0 \\ &= \left(\boldsymbol{J}^H\right)^{|k|}, \ \boldsymbol{k} \leq 0 \ \text{ where } \boldsymbol{J} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}_{d\times d} \ \text{ and } d = L+N. \end{aligned} \tag{2.13}$$

Following theorems discuss necessary and sufficient condition for identification of channel:

Theorem2.1 [13]: Suppose $\mathcal{H}_N$ *is a full column rank and* D_n *satisfies (2.13) then* $\mathcal{H}_N$ *is uniquely determined up to a phase constant by* $\boldsymbol{R}_x(0)$ *and* $\boldsymbol{R}_x(1)$

The following theorem states the conditions for $\mathcal{H}_N$ to full column rank.

Theorem 2.2[14]: Matrix $\mathcal{H}_N$ *is full column rank i.e., rank(*$\mathcal{H}_N$*) = L + N, if*

1) *the polynomials* $H^{(i)}(Z) \stackrel{def}{=} \sum_{j=0}^{L} h_j^{(i)} z^j$ *have no common zero*
2) *N is greater than the maximum degree L of the polynomial* $H^{(i)}(Z)$ *i.e.,* $N \geq L$,
3) *at least one polynomial* $H^{(i)}(Z)$ *has degree L.*

2.3.1 TXK algorithm for noise less situation:

First consider noiseless scenario where $\boldsymbol{R}_v(k)=0$ and using (2.12) and (2.13), we get

$$\boldsymbol{R}_x(0)=\mathcal{H}_N\mathcal{H}_N^H \quad \boldsymbol{R}_x(1)=\mathcal{H}_N\boldsymbol{J}\mathcal{H}_N^H \tag{2.14}$$

Define singular value decomposition (SVD) of $\boldsymbol{R}_x(0)$ as

$$\boldsymbol{U}^H\boldsymbol{R}_x(0)\boldsymbol{U}=diag\left(\sigma_1^2,\cdots,\sigma_d^2,0,\cdots,0\right) \tag{2.15}$$

Let $\mathbf{u}_i$ denote the *i*th column of $\boldsymbol{U}$, and let

$$\boldsymbol{U}_S=\left[\mathbf{u}_1,\cdots,\mathbf{u}_d\right], \tag{2.16}$$

$$\Sigma=diag\left(\sigma_1,\cdots,\sigma_d\right) \tag{2.17}$$

$$\boldsymbol{F}=\Sigma^{-1}\boldsymbol{U}_S{}^H \tag{2.18}$$

From (2.14) and (2.15), we have

$$\mathcal{H}_N=\boldsymbol{U}_S\Sigma\boldsymbol{V} \text{ where } \boldsymbol{V}=\left[v_1,\cdots,v_d\right] \text{is an orthogonal matrix.} \tag{2.19}$$

Thus $\boldsymbol{F}\,\mathcal{H}_N=\Sigma^{-1}\boldsymbol{U}_S{}^H\boldsymbol{U}_S\Sigma\boldsymbol{V}=\boldsymbol{V}$

Using $\boldsymbol{R}_x(1)=\mathcal{H}_N\boldsymbol{J}\mathcal{H}_N^H$, we define

$$\boldsymbol{R}=\boldsymbol{F}\boldsymbol{R}_x(1)\boldsymbol{F}^H=\boldsymbol{F}\,\mathcal{H}_N\boldsymbol{J}\mathcal{H}_N^H\,\boldsymbol{F}^H=\boldsymbol{V}\boldsymbol{J}\boldsymbol{V}^H \tag{2.20}$$

From (2.20), we get Jordan chain of equations,

$$\boldsymbol{R}\boldsymbol{v}_k=\boldsymbol{v}_{k+1}, \quad k=1,\cdots,d-1, \tag{2.21}$$

$$\boldsymbol{R}\boldsymbol{v}_d=0 \tag{2.22}$$

Eq (2.22) shows that $\boldsymbol{v}_d$ is a singular vector of $\boldsymbol{R}$.

Computing $\boldsymbol{R}^H\boldsymbol{R}$, we have

$$\boldsymbol{R}^H\boldsymbol{R}=\boldsymbol{V}\,diag(1,\cdots,1,0)\boldsymbol{V}^H \tag{2.23}$$

It is clear from (2.23) that

1. The matrix $\boldsymbol{R}$ has one and only one singular value equal to 0
2. v_d is a right singular vector of $\boldsymbol{R}$ associated with zero singular vector

Now if $\boldsymbol{R}$ has an SVD i.e., if

$$\left[y_1,\cdots,y_d\right]^H\boldsymbol{R}\left[z_1,\cdots,z_d\right]=diag\left(\gamma_1^2,\cdots,\gamma_d^2\right) \tag{2.24}$$

then there exits a ϕ such that

$$v_d = z_d e^{j\phi} \tag{2.25}$$

Now the problem is to find $v_k \quad k = 1, \cdots, d-1$, using (2.21).

From (2.21),

$$v_i = \left(R^{\dagger}\right)^{d-i} v_d \tag{2.26}$$

So $V = [v_1, \cdots, v_d] = \left[\left(R^{\dagger}\right)^{d-1} v_d, \left(R^{\dagger}\right)^{d-2} v_d, \cdots, v_d\right]$ (2.27)

Using (2.25) in (2.27),

$$V = \left[\left(R^{\dagger}\right)^{d-1} z_d, \left(R^{\dagger}\right)^{d-2} z_d, \cdots, z_d\right] e^{j\phi} \tag{2.28}$$

Substitute (2.28) in (2.19), we will get the final estimate of the channel filtering matrix

$$\hat{\mathcal{H}}_N = U_s \Sigma Q \, e^{j\phi} \tag{2.29}$$

Where $Q = \left[\left(R^{\dagger}\right)^{(d-1)} z_d, \left(R^{\dagger}\right)^{(d-2)} z_d, \cdots, z_d\right]$

2.3.2 TXK algorithm for noisy situation:

Section 2.3.1 provides the essential parts of the proposed blind channel identification algorithm. In this section we consider the noisy case. Under the assumption of white noise with variance σ^2, the noise correlation matrix is given by

$$R_v(k) = E(V_n V_{n-k}^H)$$

which can be written as

$$= \sigma^2 J^{kM}$$

Although neither the noise covariance, nor the signal space dimension d is known a priori, they can be obtained from the $\hat{R}_x(0)$.

SVD of $\hat{R}_x(0)$ has the following form,

$$U^H R_x(0) U = diag\left(\lambda_1 + \sigma^2, \cdots, \lambda_d + \sigma^2, \sigma^2, \cdots, \sigma^2\right) \quad \text{where} \quad \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d \geq 0$$

Therefore, both σ^2 and d can in theory be obtained by determining the most significant singular values of $\hat{\boldsymbol{R}}_x(0)$. In practice, a threshold test can be employed to determine d and then to estimate σ^2 from the singular values of the estimated covariance matrix.

Once the noise covariance σ^2 is determined, obtain noiseless received correlation matrices by subtracting noise correlation matrices from the observation correlation matrices. We can then use the algorithm described in section 2.3.1

Steps involved in implementing TXK Algorithm are as follows:

Step1: Estimate $\hat{\boldsymbol{R}}_x(0), \hat{\boldsymbol{R}}_x(1)$ through time averaging

$$\hat{\boldsymbol{R}}_x(0) = \frac{1}{N_b}\sum_{n=1}^{N_b} X_n X_n^H, \hat{\boldsymbol{R}}_x(1) = \frac{1}{N_b}\sum_{n=1}^{N_b} X_n X_{n-1}^H$$

Step2: From $\hat{\boldsymbol{R}}_x(0)$, estimate noise covariance $\hat{\sigma}$ and the signal dimension d .

Step3: Compute the SVD of $\boldsymbol{R}_0 = \hat{\boldsymbol{R}}_x(0) - \hat{\sigma}^2\boldsymbol{I}$ and form $\boldsymbol{U}_s$ which consists of the singular vectors associated with the d largest singular values, and Σ which consists of positive square root of d largest singular value.

Step4: Compute the SVD of

$$\boldsymbol{R} = \boldsymbol{F}\left(\hat{\boldsymbol{R}}_x(1) - \boldsymbol{R}_n(1)\right)\boldsymbol{F}^H \text{ where } \boldsymbol{F} = \Sigma^{-1}\boldsymbol{U}_s^H, \boldsymbol{R}_n(1) = \hat{\sigma}^2\boldsymbol{J}^M.$$

Get the left and right singular vectors y_d and z_d corresponding to smallest singular value.

Step5: Form an estimate of $\hat{H}$ from

$$\hat{\boldsymbol{H}} = \boldsymbol{U}_s\Sigma\boldsymbol{Q} \text{ where } \boldsymbol{Q} = \left[y_d, \boldsymbol{R}y_d, \cdots, \boldsymbol{R}^{(d-1)}y_d\right]$$

2.4 Subspace based blind channel estimation:

Received correlation matrix given in (2.12) with $\boldsymbol{k} = 0$ can be written as

$$\boldsymbol{R}_x(0) = \mathcal{H}_N\boldsymbol{R}_d(0)\mathcal{H}_N^H + \boldsymbol{R}_v(0) \tag{2.30}$$

where $\boldsymbol{R}_d(0) = E(D_n D_n^H)$ and $\boldsymbol{R}_v(k) = E(V_n V_n^H)$

Noise is assumed to be white with variance σ^2 and is independent of transmitted sequence. (2.30) is rewritten as

$$R_x(0) = \mathcal{H}_N R_d(0) \mathcal{H}_N^H + \sigma^2 I \quad (2.31)$$

By eigenvalue decomposition of (2.31) we will get the *MN* eigenvalues. Let $\lambda_0, \lambda_1, \lambda_2, \cdots \lambda_{MN-1}$ denote the eigenvalues of (2.31). Let the eigenvalues arranged in descending order are $\lambda_0 \geq \lambda_1 \geq \lambda_2, \cdots \lambda_{MN-1}$.

Signal part of the covariance matrix (2.31) has rank $L+N$, hence

$\lambda_i \geq \sigma^2$ for $i = 0, \ldots, L+N-1$

$\lambda_i = \sigma^2$ for $i = L+N, \ldots, MN-1$

Let $[s_0, \cdots, s_{L+N-1}]$ denote the unit norm eigenvectors corresponding to signal subspace associated with eigenvalues $\lambda_0, \cdots, \lambda_{L+N-1}$ and $G_0, \cdots, G_{MN-L-N+1}$ the unit norm eigenvectors corresponding to noise subspace associated with eigenvalues $\lambda_{L+N}, \cdots, \lambda_{MN-1}$.

Let us define two matrices,

$$S = [s_0, \cdots, s_{L+N-1}] \qquad MN \times (L+N) \quad (2.32)$$

$$G = [G_0, \cdots, G_{MN-L-N+1}] \qquad MN \times (MN - L - N)$$

Using (2.31) and (2.32), eigenvalue decomposition of R_x is

$$R_x(0) = S diag(\lambda_0, \cdots, \lambda_{L+N-1}) S^H + \sigma^2 G G^H \quad (2.33)$$

Here S spans the signal subspace and G spans the orthogonal complement, the noise subspace. Columns of the channel matrix $\mathcal{H}_N$ also lie in signal subspace. Since noise and signal subspace are orthogonal, hence columns of filtering matrix are orthogonal to vectors in column subspace.

From above,

$$G_i^H \mathcal{H}_N = 0 \qquad 0 \leq i \leq MN - L - N \quad (2.34)$$

Under appropriate condition stated in *theorem2.3*, the noise subspace can uniquely determines the channel H up to a multiplicative constant.

Theorem 2.3[14]: Assume that i) $N \geq L$ and ii) matrix $\mathcal{H}_{N-1}$ is full column rank (i.e rank($\mathcal{H}_{N-1}$) = $L + N - 1$). Let $\mathcal{H}'_N$ be a nonzero filtering matrix with the same dimensions as

$\mathcal{H}_N$ *.The range of* $\mathcal{H}'_N$ *is included in the range of* $\mathcal{H}_N$ *iff the corresponding vectors H and* H' *are proportional*

In practice, ensemble averages are approximated by time averages. So only estimate of noise eigenvectors $\hat{G}_i$ is available and conditions in (2.34) can be satisfied in the least squares sense. This leads to the minimization of the following quadratic form

$$q(H) \stackrel{def}{=} \sum_{i=0}^{MN-L-N-1} \left|\hat{G}_i^H \mathcal{H}_N\right|^2 \tag{2.35}$$

By exploiting the block Toeplitz structure of $\mathcal{H}_N$, we can put (2.35) in terms of H. It can be shown that

$$G_i^H \mathcal{H}_N = H^H g_i \tag{2.36}$$

where $g_i = \begin{bmatrix} g_i^{(0)} \\ g_i^{(1)} \\ \vdots \\ g_i^{(M-1)} \end{bmatrix}_{M(L+1)\times(L+N)}$ and $\mathcal{H}_N = \begin{pmatrix} \mathcal{H}_N^{(0)} \\ \vdots \\ \mathcal{H}_N^{(M-1)} \end{pmatrix}_{(MN\times(L+N))}$.

$$g_i^{(l)} = \begin{bmatrix} G_{i,0}^{(l)} & \cdots & G_{i,N-1}^{(l)} & \cdots & & \cdots & 0 & 0 \\ 0 & G_{i,0}^{(l)} & \cdots & & G_{i,N-1}^{(l)} & 0 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ \vdots & & & & & & & \vdots \\ 0 & \cdots & 0 & & G_{i,0}^{(l)} & & & G_{i,N-1}^{(l)} \end{bmatrix}_{(L+1)\times(L+N)} \qquad H = \begin{bmatrix} H^{(0)} \\ H^{(1)} \\ \vdots \\ H^{(M-1)} \end{bmatrix}_{(M(L+1)\times 1)}$$

Proof:

Partition G_i as $G_i = \begin{bmatrix} G_i^{(0)} \\ G_i^{(1)} \\ \vdots \\ G_i^{(M-1)} \end{bmatrix}_{MN\times 1}$ where $G_i^{(j)} = \begin{bmatrix} G_{i,0}^{(j)} \\ G_{i,1}^{(j)} \\ \vdots \\ G_{i,N-1}^{(j)} \end{bmatrix}_{N\times 1}$ (2.37)

Using (2.8) in (2.36)

$$G_i^H \mathcal{H}_N = \sum_{l=0}^{M-1} \left(G_i^{(l)}\right)^H \mathcal{H}_N^{(l)} \tag{2.38}$$

Substitute (2.37) in $\left(G_i^{(l)}\right)^H \mathcal{H}_N^{(l)}$ as

$$\left(G_i^{(l)}\right)^H \mathcal{H}_N^{(l)} = \begin{bmatrix} G_{i,0}^{(l)} & G_{i,1}^{(l)} & \cdots & G_{i,N-1}^{(l)} \end{bmatrix} \begin{bmatrix} h_0^{(l)} & \cdots & h_L^{(l)} & \cdots & \ldots & 0 & 0 \\ 0 & h_0^{(l)} & \ldots & h_L^{(l)} & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & h_0^{(l)} & & & h_L^{(i)} \end{bmatrix}$$

$$= \begin{bmatrix} G_{i,0}^{(l)} & G_{i,1}^{(l)} & \cdots & G_{i,N-1}^{(l)} \end{bmatrix} * \begin{bmatrix} h_0^{(l)} & h_1^{(l)} & \cdots & h_L^{(l)} \end{bmatrix} \text{ (Convolution)}$$

$$= \begin{bmatrix} h_0^{(l)} & h_1^{(l)} & \cdots & h_L^{(l)} \end{bmatrix} * \begin{bmatrix} G_{i,0}^{(l)} & G_{i,1}^{(l)} & \cdots & G_{i,N-1}^{(l)} \end{bmatrix} \tag{2.39}$$

Rearranging (2.39) we get

$$\left(G_i^{(l)}\right)^H \mathcal{H}_N^{(l)} = \underbrace{\begin{bmatrix} h_0^l & h_1^l & \cdots & h_L^l \end{bmatrix}}_{\left[H^{(l)}\right]^H} \underbrace{\begin{bmatrix} G_{i,0}^{(l)} & \cdots & G_{i,N-1}^{(l)} & \cdots & \ldots & 0 & 0 \\ 0 & G_{i,0}^{(l)} & \ldots & G_{i,N-1}^{(l)} & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & G_{i,0}^{(l)} & & & G_{i,N-1}^{(l)} \end{bmatrix}}_{g_i^{(l)}}{}_{L+1\times(L+N)}$$

$$= \left[H^{(l)}\right]^H g_i^{(l)} \tag{2.40}$$

Using (2.40) in (2.38), we get

$$G_i^H \mathcal{H}_N = \sum_{l=0}^{M-1} \left(G_i^{(l)}\right)^H \mathcal{H}_N^{(l)} = \sum_{l=0}^{M-1} \left[H^{(l)}\right]^H g_i^{(l)}$$

$$= \underbrace{\begin{bmatrix} H^{(0)H} & H^{(1)H} & \cdots & H^{(M-1)H} \end{bmatrix}}_{H^H} \underbrace{\begin{bmatrix} g_i^{(0)} \\ g_i^{(1)} \\ \vdots \\ g_i^{(M-1)} \end{bmatrix}}_{g_i}$$

$$= H^H g_i$$

Substituting (2.36) in (2.35), we get

$$\left|\hat{G}_i^H \mathcal{H}_N\right|^2 = \hat{G}_i^H \mathcal{H}_N \mathcal{H}_N^H \hat{G}_i = H^H \hat{g}_i \hat{g}_i^H H \qquad (2.41)$$

Using (2.41), (2.35) can be written as

$$q(H) = H^H Q H \qquad \text{where} \qquad Q = \sum_{i=0}^{MN-L-N-1} \hat{g}_i \hat{g}_i^H \qquad (2.42)$$

Estimates of H can be obtained by minimizing $q(H)$ subject to properly chosen constraint avoiding the trivial solution $H = 0$. Here we chose Quadratic constraint subject to $|H| = 1$. The solution is well known that channel estimate is the unit-norm eigenvector associated with the smallest eigenvalue of matrix Q.

Proof:

Condition $|H| = 1$ is equivalent to $H^H H = 1$.

$$\text{Let } J = q(H) + \lambda(1 - H^H H)$$

$$= H^H Q H + \lambda(1 - H^H H) \qquad (2.43)$$

Differentiating (2.43) with respect to H^H and equating it to zero, we will get

$$\frac{\partial J}{\partial H^H} = QH - \lambda H = 0$$

$$QH = \lambda H \text{ (eigenvalue form)} \qquad (2.44)$$

Substitute (2.44) in (2.42)

$$q(H) = H^H Q H$$

$$= H^H \lambda H$$

$$= \lambda H^H H$$

$$= \lambda(1) = \lambda \text{ (Since } H^H H = 1\text{)} \qquad (2.43)$$

From above, H is the eigenvector corresponding to smallest eigenvalue of Q.

2.5 Simulation results for SIMO blind channel identification:

2.5.1 Results for TXK method:

For channel to be identified, we use two ray multipath environment given by

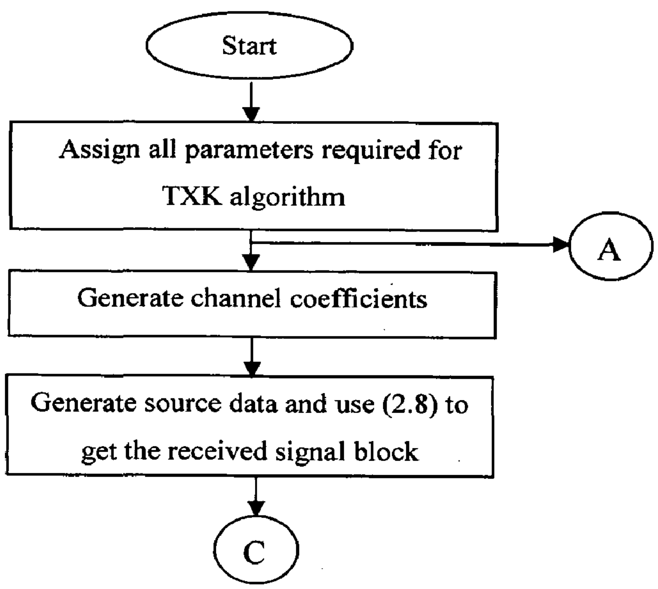
$$h(t) = \left(0.2c(t,0.11) + 0.4c(t-2.5,0.11)\right)W_{6T}(t)$$

Where $c(t,\alpha)$ is a raised cosine filter with α as roll-off factor and $W_{6T}(t)$ is a rectangular window of duration 6 symbol intervals. Discrete channel coefficients are obtained by sampling the above continuous channel.

For simulation of TXK algorithm in MATLAB environment, we use the following parameters

- No of virtual channels: $M = 4$
- Observation interval: $N = 5$ symbol intervals
- Channel length: $L = 5$
- No of Monte Carlo simulation runs: $N_m = 100$
- Modulation scheme for source symbols: 16QAM
- Data window length: N_b=1000 blocks

Fig 2.3 shows the flow chart for simulation of TXK algorithm.



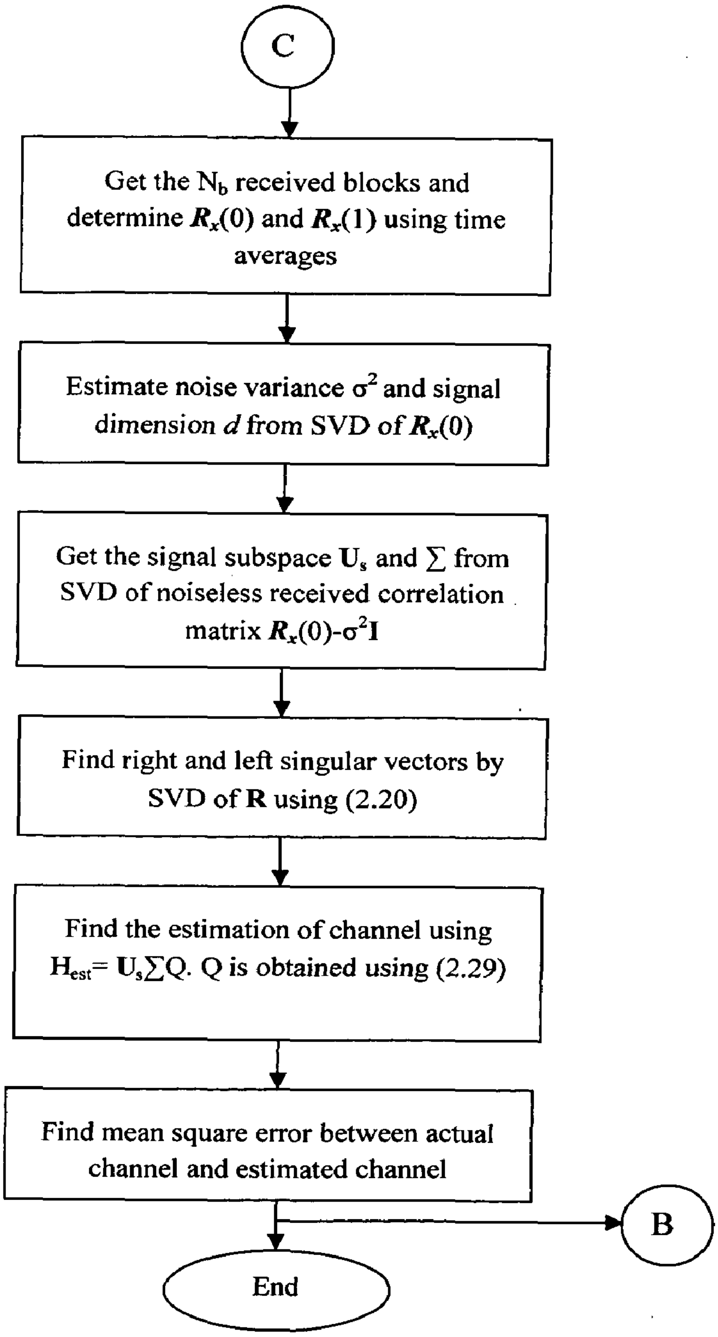


Fig 2.3: Flow chart for simulation of TXK algorithm

All steps from A to B shown in flow chart are repeated for each independent Monte Carlo run. To obtain the performance measure of the channel estimate, we use Normalized Root Mean Square Error (NRMSE) criterion given by

$$NRMSE = \frac{1}{\|H\|}\sqrt{\frac{1}{N_m}\sum_{i=1}^{N_m}\left\|\hat{H}(i) - H\right\|^2} \tag{2.45}$$

where $\hat{H}(i)$ is estimate of the channel for the ith simulation run.

As described in theorems 2.3, there is a complex scalar ambiguity constant associated with the SOS based blind channel estimator. During simulations, amplitude ambiguity is removed by assuming the true channel vector H to have unit norm thus normalizing the estimates. The phase ambiguity is determined from phase of$\left(h_0^{(0)} / \hat{h}_0^{(0)}\right)$, where $h_0^{(0)}$ is the first component of the true channel value and $\hat{h}_0^{(0)}$ is the first component of TXK blind estimate of channel value.

Fig 2.4 shows the estimate of the channel coefficients with different realizations at a SNR=25dB. Fig 2.5 plots the average value of estimated channel coefficients, which are obtained by averaging over 100 independent realizations. For comparison, we have also plotted original channel coefficients. It may be noted that there is a close similarity between the true channel coefficients and averaged channel coefficients identified using TXK method.

Fig 2.6 plots Normalized Root Mean Square Error (NRMSE) at different values of input SNR. As SNR is varied from 0 to 40dB, NRMSE value decreases from 1.13 to 0.2. We may observe that NRMSE value is large. If we increase the SNR beyond 35dB, NRMSE value doesn't show any significant reduction.

We next consider the effect of varying data window length N_b on the performance of SOS estimation using TXK algorithm as shown in Fig 2.7. We have kept SNR = 30dB, it may be noticed that as we increase the data window length N_b from 100 to 3000, NRMSE value decreases from 0.67 to 0.3. We may observe that, beyond window size of 1000 blocks, there is no effect on the performance of the TXK algorithm.

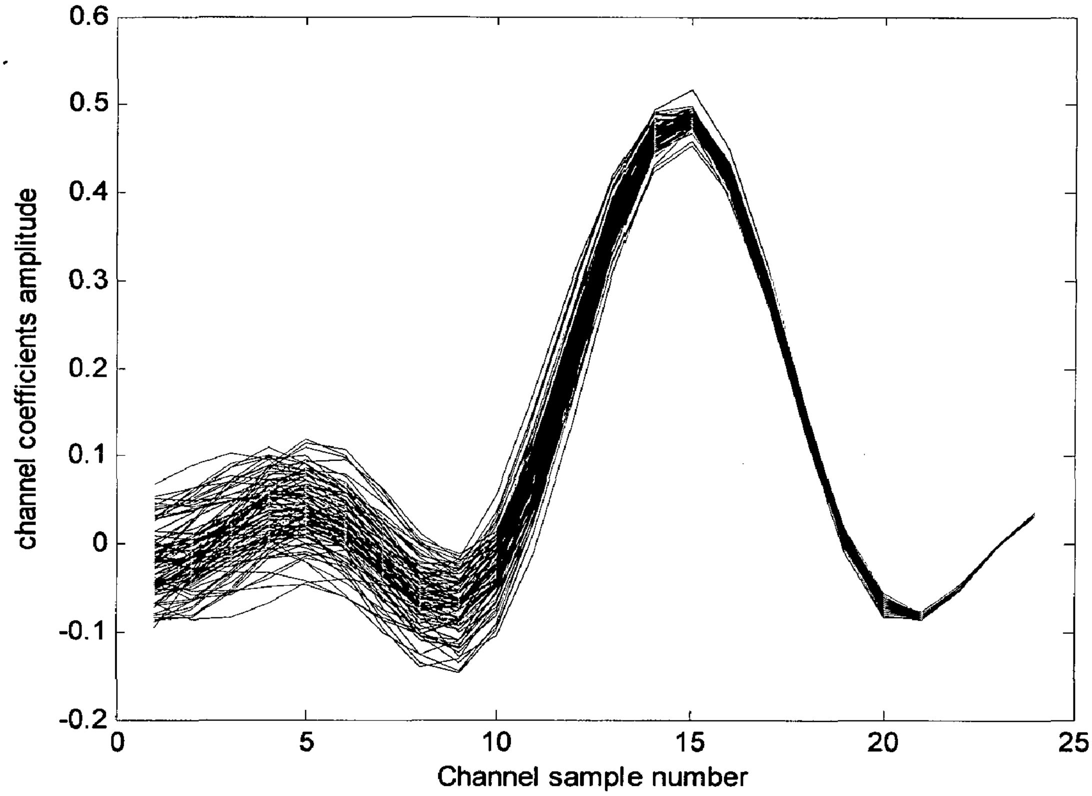


Fig 2.4: 100 independent estimates of the channel using TXK algorithm at SNR = 25dB

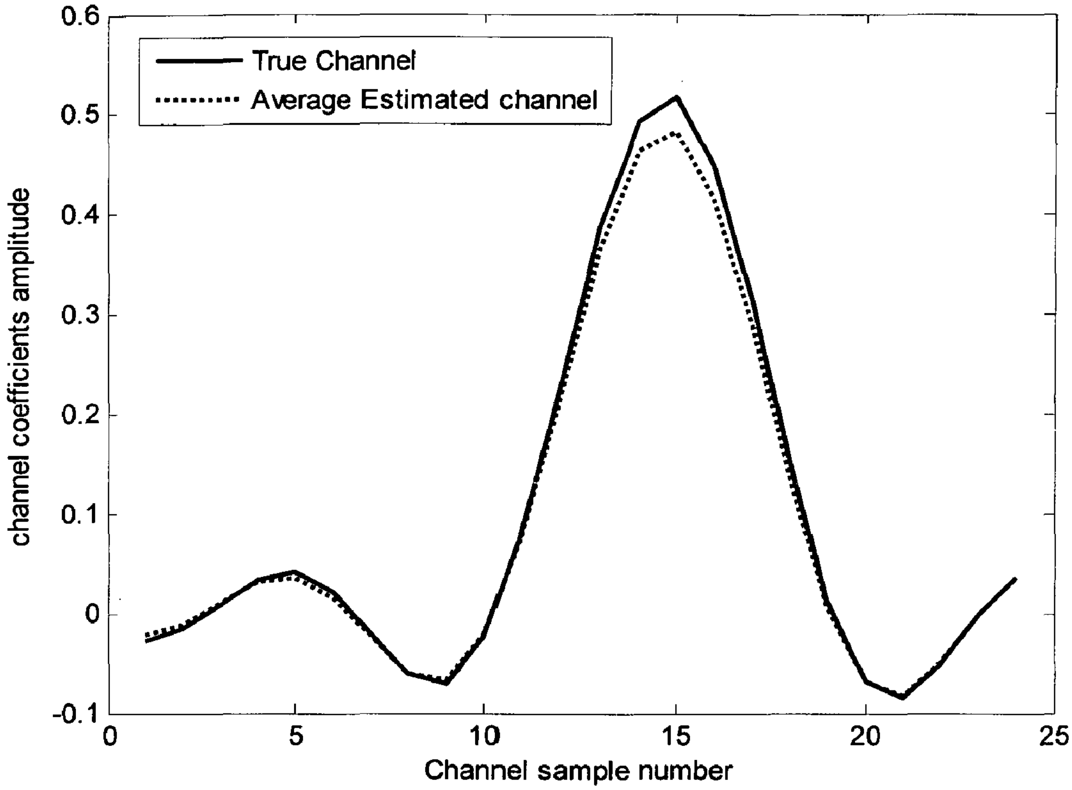


Fig 2.5: Comparison between actual and TXK blind estimate of channel averaged over 100 independent runs at SNR=25dB.

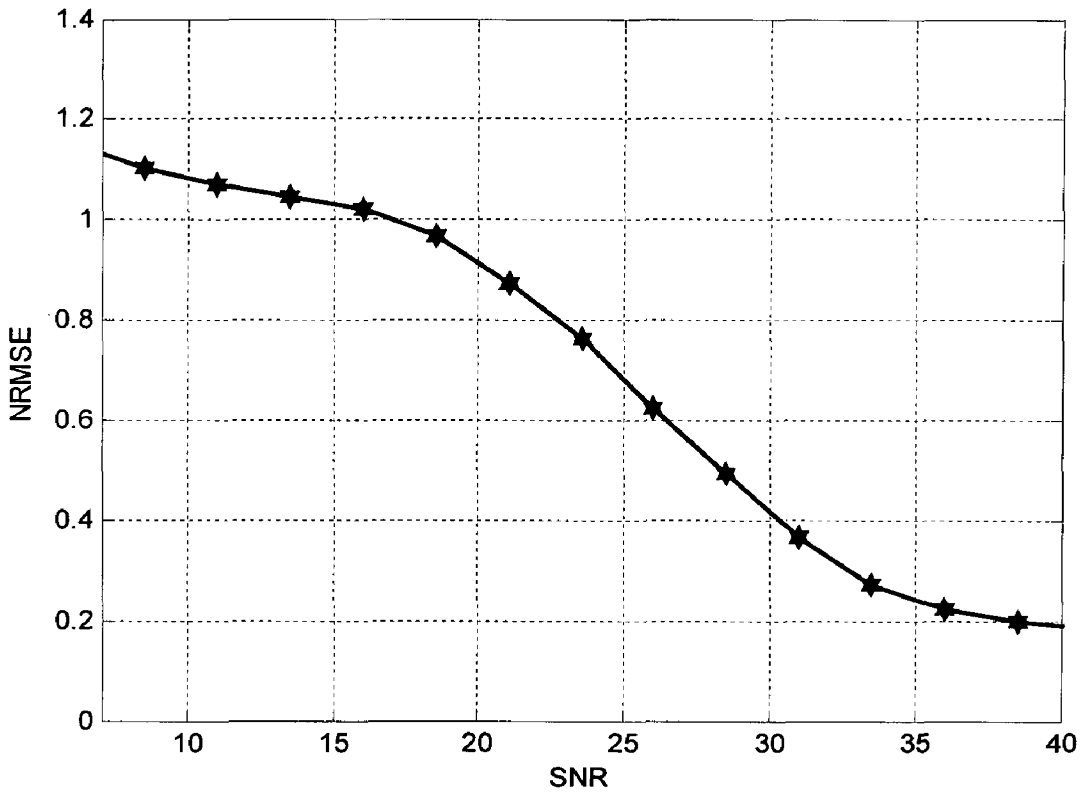


Fig 2.6: Variation of Normalized Root Mean Square Error (NRMSE) with SNR for TXK blind identification method.

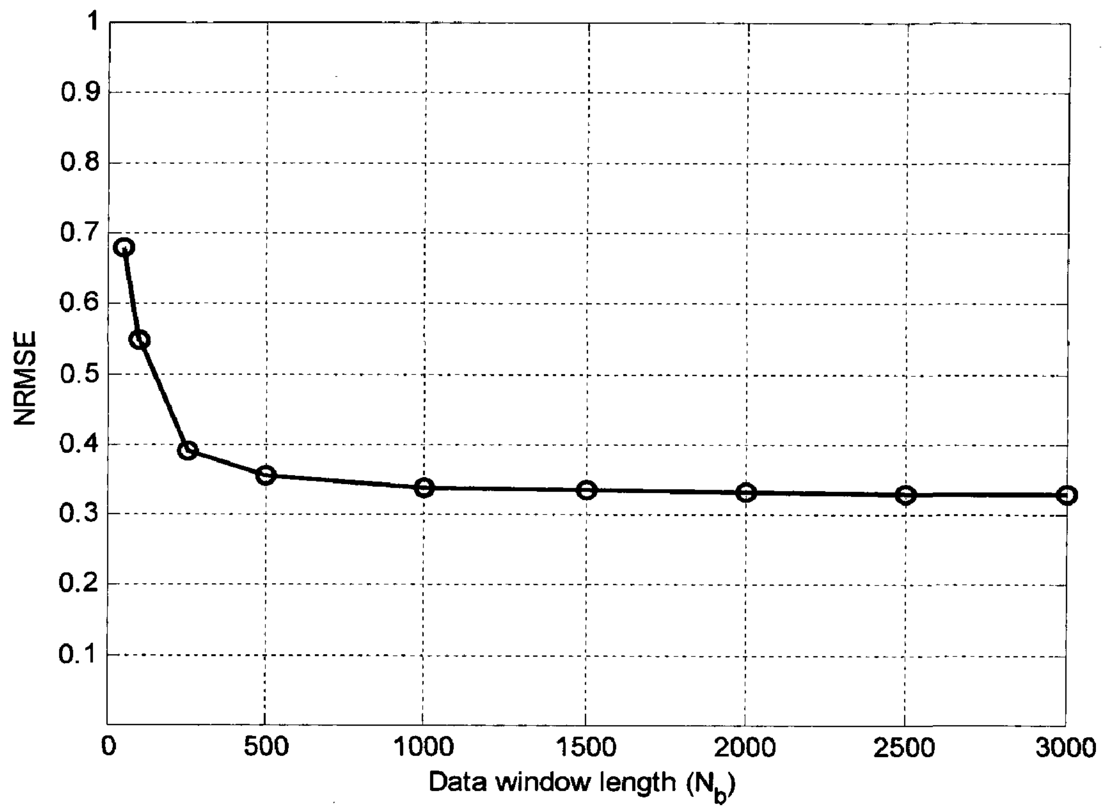


Fig 2.7: Variation of Normalized Root Mean Square Error (NRMSE) for TXK blind identification method with data window length (N_b) for SNR = 30dB

2.5.2 Results for Subspace based method:

For channel to be identified, we use the following complex channel coefficients given in [14]

$$H^{(0)T} = \left[(-0.049, 0.359), (0.482, -0.569), (-0.556, 0.587), (1,1)(-0.171, 0.061)\right]$$

$$H^{(1)T} = \left[(0.443, -0.0364), (1.0, 0.0), (0.921, -0.194), (0.189, -0.208)(-0.087, -0.054)\right]$$

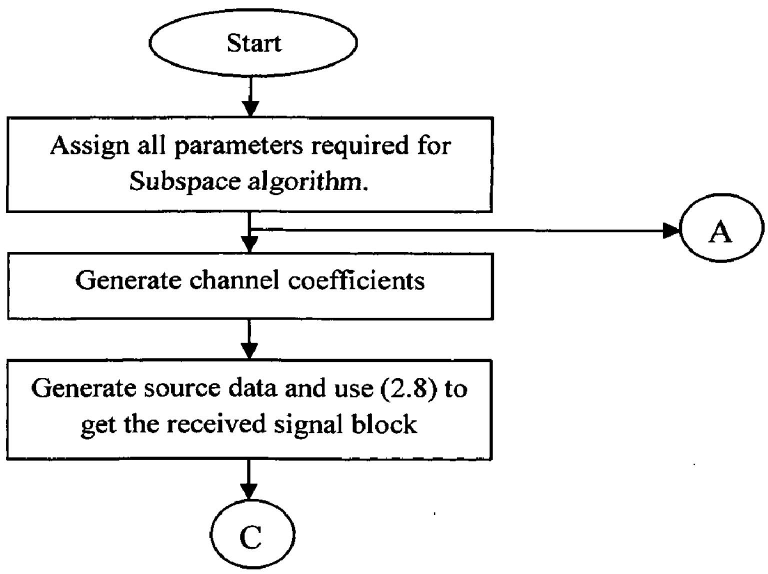
$$H^{(2)T} = \left[(-0.211, -0.322), (-0.199, 0.918), (1.0, 0.0), (-0.284, -0.524)(0.136, -0.19)\right]$$

$$H^{(3)T} = \left[(0.417, 0.030), (1.0, 0.0), (0.873, 0.145), (0.285, 0.309)(-0.049, 0.161)\right]$$

For simulation of Subspace algorithm in MATLAB environment, we use following parameters:

- No of virtual channels: $M = 4$
- Observation interval: $N = 10$ symbol intervals
- Channel length: $L = 4$
- No of Monte Carlo simulations: $N_m = 100$
- Modulation scheme for source symbols: 16QAM
- Data window size: N_b=1000 blocks

Fig 2.8 shows the flow chart for simulation of subspace based blind channel identification algorithm.



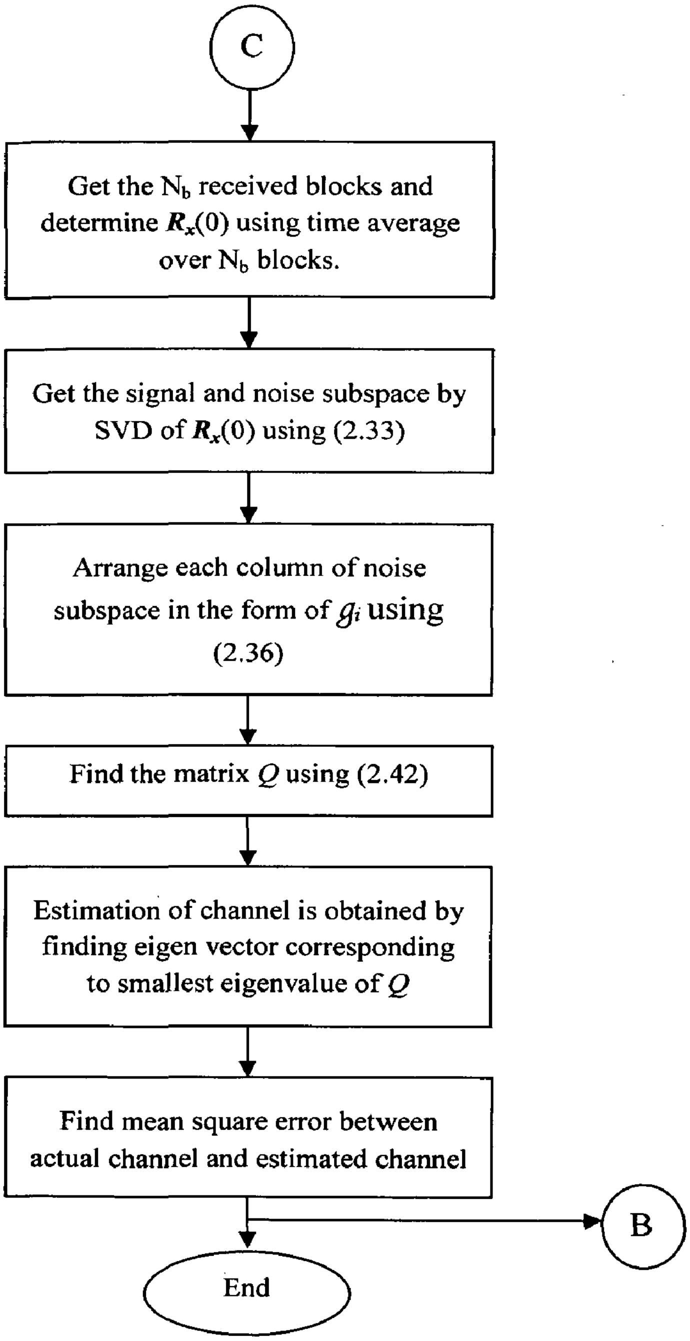


Fig 2.8: Flow chart for simulation of Subspace based blind channel identification algorithm

All steps from A to B shown in flow chart are repeated for each independent Monte Carlo Run. Normalized Root Mean Square Error (NRMSE) is computed as in (2.45). We also simulate TXK algorithm for blind identification of channel specified in this section.

Fig 2.9 shows the Normalized Root Mean Square Error (NRMSE) at different SNR values for both the subspace method and TXK method. As SNR is varied from 5 to 35dB, NRMSE value for subspace method decreases from 0.5 to 0.008, while for TXK algorithm it decreases from 1.1 to 0.2. We may also observe that NRMSE for subspace method is much less than that obtained using the TXK method. Under similar environment, subspace method performs much better than the TXK method.

As shown in Fig 2.10, we compare the variation of NRMSE at different data window lengths (N_b) for subspace method and TXK method. As N_b is varied from 50 to 3000 blocks, NRMSE value for subspace method decreases from 0.1 to 0.008 while for TXK algorithm decreases from 0.7 to 0.35. For a given NRMSE value of 0.3, subspace methods requires less than 50 blocks of received signal, while TXK algorithm requires more than 500 blocks. We may conclude that the convergence of subspace method is faster than that for TXK method. We may also observe that, beyond a window size of 1000 blocks, there is no effect on the performance of the both the algorithms.

Next we consider the effect of reducing the number of noise vectors used in estimation of channel using subspace method. Using (2.36), condition in (2.34) can be written as

$$H^H g_i = 0 \quad 0 \le i \le MN - L - N \tag{2.46}$$

For single noise vector, (2.46) yield a set of $L+N$ linear equations in the $M(L+1)$ unknown channel coefficients of H. If $(L+N) < M(L+1)$, then the system is underdetermined, hence it does not admit a unique solution. Choosing p number of noise vectors so that $p(L+N) > M(L+1)$, the linear system is over determined, and a unique solution will be determined for unknown channel coefficients. Minimum no of noise vectors required for TXK algorithm for subspace based blind channel identification is given by $p_{\min} = \left\lceil \frac{M(L+1)}{L+N} \right\rceil$.

Fig 2.10 and 2.11 plots the variation of NRMSE with respect to SNR and data window length (N_b) for three different scenarios:

I. Full noise subspace (26 noise vectors)
II. Partial noise subspace with 10 noise vectors
III. Partial noise subspace with 4 noise vectorws

We may observe that the use of fewer noise vectors still leads to consistent estimates of the channel coefficients. But there is a performance tradeoff with the usage of less no. of noise vectors. As we decrease the amount of usage of number of noise vectors, NRMSE error increases for both the cases (variation NRMSE with SNR and data window size (N_b)). This is due to the fact that number of constraints for identification of channel decreases. The advantage of using less number of noise vectors is that no of computations required for estimating the channel coefficients decreases.

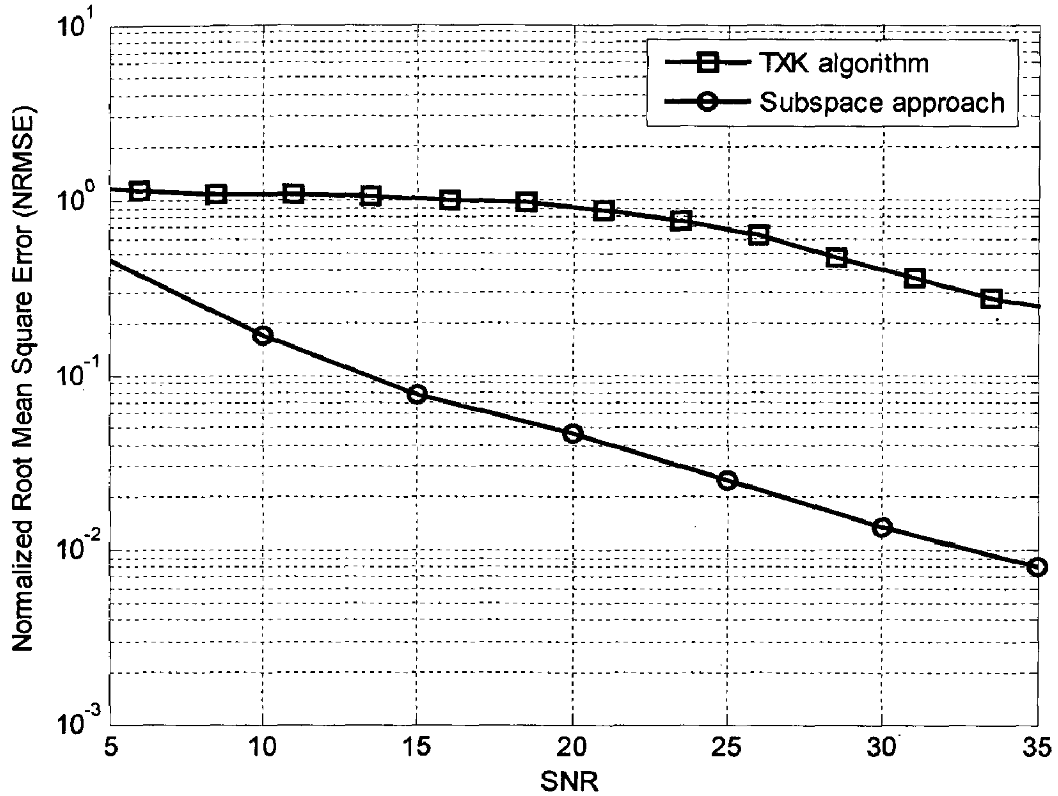


Fig 2.8: Variation of NRME with respect to SNR for Subspace based channel estimation and SOS based channel estimation using TXK algorithm (N_b =1000)

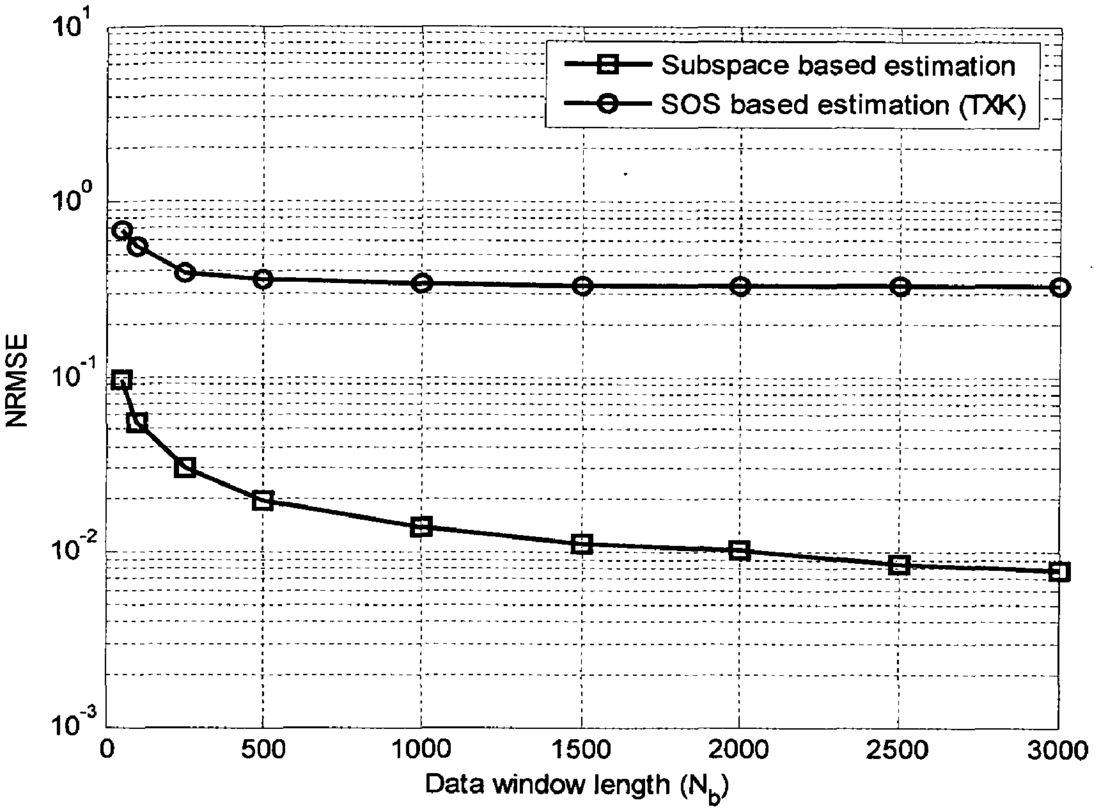


Fig 2.9: Variation of NRME with respect to data window size for Subspace based channel estimation and SOS based channel estimation using TXK algorithm (SNR=30db).

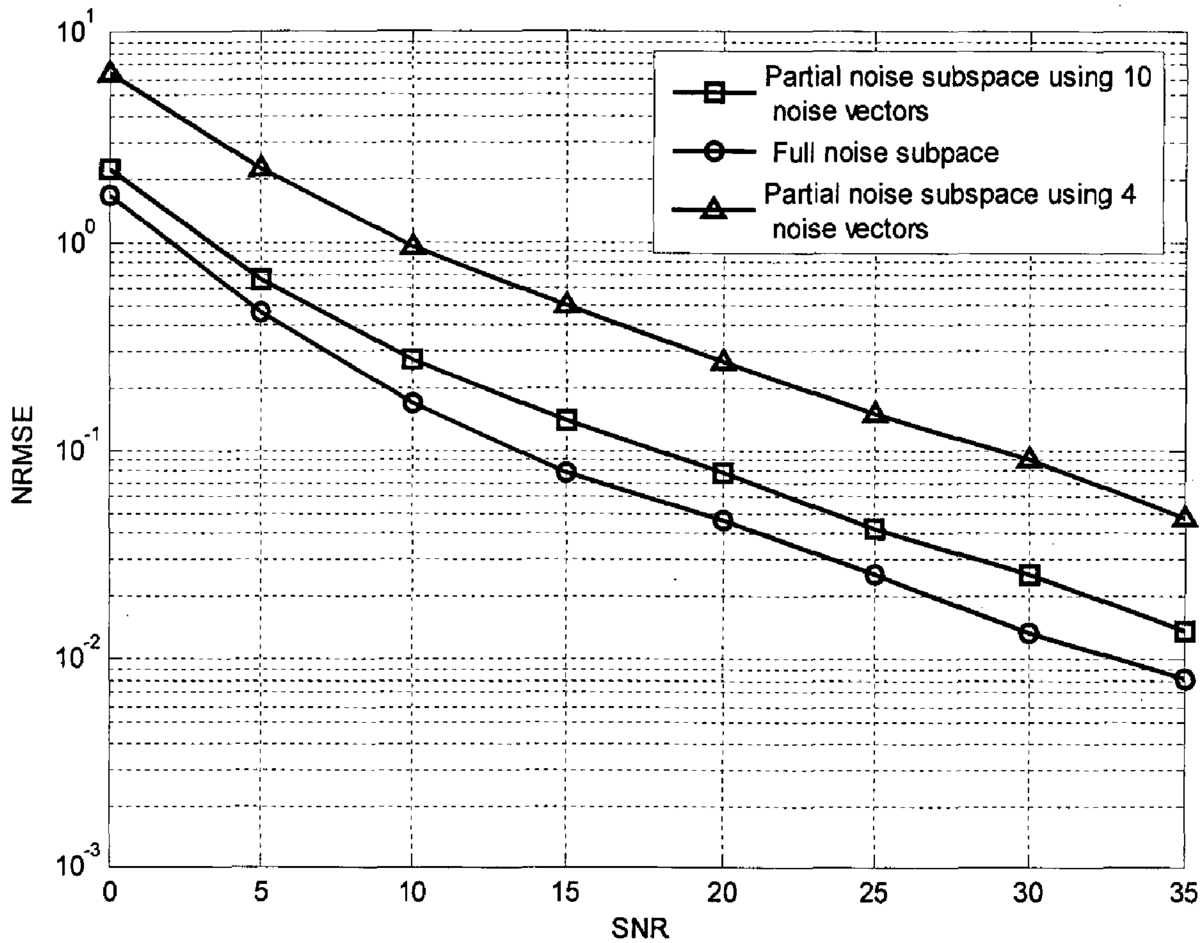


Fig 2.10: variation of NRME vs SNR for full noise subspace, partial noise subspace with 10 noise vectors and 4 noise vectors respectively.

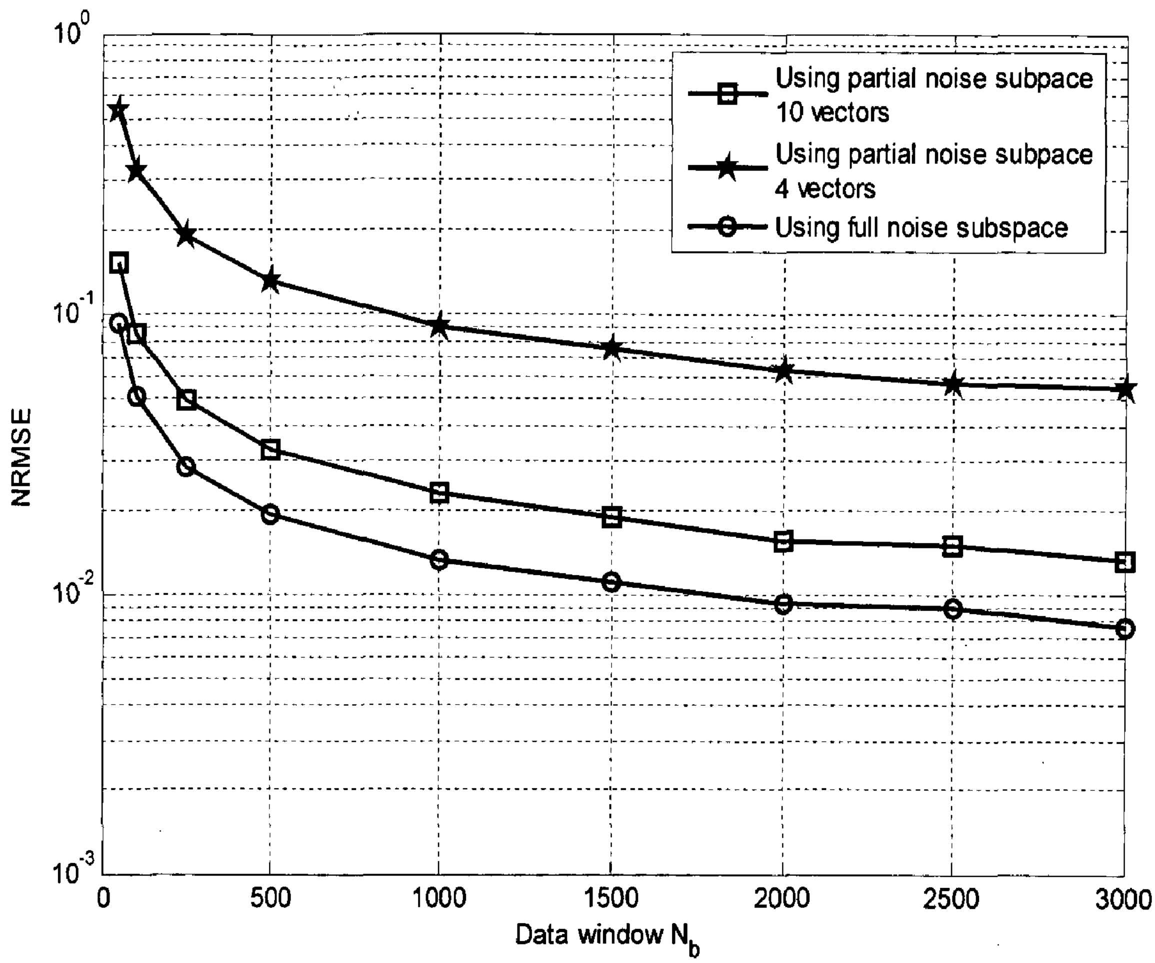


Fig 2.11: variation of NRME vs Data window length (N_b) for full noise subspace, partial noise subspace with 10 noise vectors and 4 noise vectors respectively.

Chapter 3

Blind Channel Estimation for OFDM systems

In this chapter, we first give a brief review of blind channel estimation techniques for OFDM systems. Then system model for OFDM with virtual carriers (VC) and cyclic prefix (CP) is described. Subspace based blind channel estimation method based on exploitation of VCs for OFDM system is described next. We also present subspace method for non-CP OFDM systems using oversampling at the receiver. Semi-blind channel estimation for removing phase ambiguity which is inherent in blind channel estimation techniques is described next. We finally present simulation results.

3.1 Blind channel estimation techniques for OFDM systems:

The presence of the CP has been exploited in the literature for blind and semi blind channel estimation of OFDM system based on second order statistics (SOS) .These methods use channel output sequence prior to the CP removal and subsequent FFT operation. Amongst these, statistically inspired blind estimators in [15] and [16] assume the inherent CP-induced cyclostationarity at the transmitter explicitly or implicitly, while the estimators [17] and [18] belong to the class of deterministic subspace approach. Specifically, Heath and Giannakis *et al.,* [15] propose a blind method based on the cyclostationarity property of the time-varying correlation of the received data samples due to the CP insertion at the transmitter. Advantage of this method is that it can identify channels with equispaced unit circle zeros. However this approach suffers from slow convergence of the estimator. Cai and Akanshu [17] developed a noise subspace algorithm by utilizing the structure of filtering matrix introduced by the CP insertion. It achieves faster convergence for small data records. B.Muquet *et al,* [18] proposed an algorithm that makes use of redundancy introduced by the cyclic prefix to identify a channel based on a subspace approach. This method has following advantages

- It does not require any modification of the classical OFDM transmitter. Thus, it is compatible with existing standard.
- It can be applied to any arbitrary signal constellation.

- It is robust to channel order over determination. Furthermore, it guarantees channel identifiability, regardless of the channel zeros location when the entire noise subspace is considered.

To avoid phase ambiguity which is inherent in blind identification methods, this method uses pilot carriers. Also to increase convergence of the blind methods, this method uses an initial channel estimate which is obtained by using known block of symbols in initial frame. All the methods discussed above are based on CP. The disadvantage of CP based methods is that throughput of the channel decreases.

Roy and Li [20] proposed a subspace based channel estimation for OFDM systems without cyclic prefix (CP). The algorithm is attractive for its potential to increase the systems channel utilization due to the elimination of the CP. This method requires oversampling or receiver diversity, thereby increasing receiver cost/complexity. It performs similar to [17] with regards to estimation accuracy and convergence speed. The disadvantage of this method is that it requires exact channel order estimation.

Other than the CP, there exists another resource that has not been exploited for purposes of channel estimation—the presence of virtual carriers (VC). The sub-carriers in OFDM that are set to zero without any information are referred to as virtual carriers [31]. IEEE 802.11a standard specifies 12 (out of a total of 64 subcarriers) VCs. While they are intended to aid in shaping of the transmit spectrum, the VCs can be exploited for the purposes of channel equalization, channel estimation and frequency offset estimation. Roy and Li [19] propose a subspace based channel estimator for OFDM systems that exploits virtual carriers. It is applicable to OFDM systems with and without CP. For the former case (conventional CP systems), the exploitation of VC brings additional performance gain to the already proposed channel estimators such as in [17]. This method is robust for channel order overestimation.

Other than the CP and VC, precoding technique is used for subspace based blind channel estimation for OFDM. A.P.Petropulu *et al,* [21] proposed a blind method based on precoding. In this method, a nonredundant linear precoder is applied at the transmitter, and the channel state information (CSI) is contained in each entry of the signal covariance matrix. However authors focus on a special design of linear precoders and extract the CSI from only one column of the covariance matrix, which greatly limits the performance accuracy of the algorithm. Similar approach has been proposed in [22], where channel is again, estimated from the single column of the cross correlation matrix of the two consecutive received blocks. This method reduces the effective number of OFDM blocks by half and is, thus less suitable for the case where only a few OFDM blocks are available. Gao and Nallanathan [23] proposed a blind channel estimation method that can overcome the aforementioned shortcomings. This method utilizes a generalized linear nonredundant block precoder and jointly obtains the channel estimation from all the entries of the signal covariance matrix. This joint estimation method performs better than the methods in [21],[22].

3.2 OFDM channel model:

Consider an OFDM system as shown in Fig 3.1 with N subcarriers, of which only D are modulated by the user's data symbols; i.e., the remaining N-D unmodulated carriers constitute VCs. Assume that the subcarriers numbered p_0 to p_0+D-1 are used for data, where p_0 is the index of the first data carrier. Further assume that the length of CP is P. Let the kth block of the "frequency domain" information symbols be

$$\mathbf{d}(k)=\left[d_0(k),d_1(k),\cdots,d_{D-1}(k)\right]^T \tag{3.1}$$

The time domain signal vector is given by

$$\left[s_0(k),s_1(k),\cdots,s_{N-1}(k)\right]^T=\mathbf{Wd}(k) \tag{3.2}$$

where $\mathbf{W}=\begin{bmatrix}\mathbf{W}(0)\\ \mathbf{W}(1)\\ \\ \mathbf{W}(N-1)\end{bmatrix}$ and $\mathbf{W}(i)=\frac{1}{\sqrt{N}}\left[w_N^{ip_0},w_N^{i(p_0+1)},\cdots,w_N^{i(p_0+D-1)}\right]$

By inserting CP i.e, the last $P\,(P>0)$ elements of the (3.2) results in $Q\times 1\,(Q=N+P)$ OFDM symbol vector

$$\begin{aligned}\mathbf{s}(k)&=\left[s_{N-P}(k),\cdots,s_{N-1}(k),s_0(k),s_1(k),\cdots,s_{N-1}(k)\right]^T\\ &=\begin{bmatrix}\mathbf{W}(N-P+1:N,:)\\ \mathbf{W}\end{bmatrix}\mathbf{d}(k)=\bar{\mathbf{W}}\mathbf{d}(k)\end{aligned} \tag{3.3}$$

where $\mathbf{W}(N-P+1:N,:)$ is sub matrix formed by taking rows from $N-P+1$ to N.

To generate the continuous time signal, each element in (3.3) is pulse shaped by $g_{tr}(t)$ and is given by

$$s(t) = \sum_{k=-\infty}^{\infty} \sum_{p=0}^{Q-1} s_p(k) g_{tr}(t-(p+kQ)T) \quad \text{where } T \text{ is period of each element} \tag{3.4}$$

$$s(t) = \sum_{q=-\infty}^{\infty} s_q g_{tr}(t-qT) \quad q = p + kQ \tag{3.5}$$

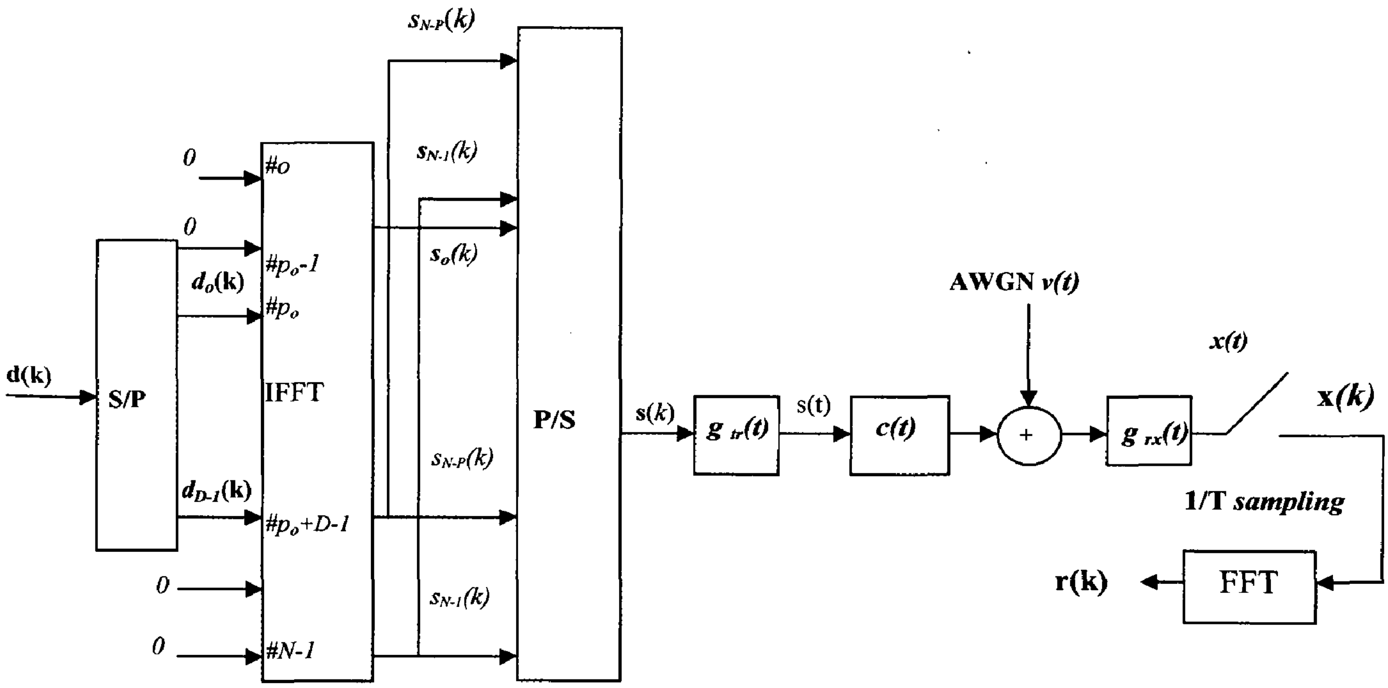


Fig 3.1: Generalized baseband OFDM system model (with both VC and CP)

Defining the composite channel filter from Fig.3.1 $h[t] = g_{tr}[t] * c[t] * g_{rx}[t]$ and the filtered noise $v(t) = n[t] * g_{rx}[t]$ where * denotes linear convolution, the received signal is therefore

$$x(t) = \sum_{q=-\infty}^{\infty} s_q h(t-qT) + v(t) \tag{3.6}$$

Assume the composite channel to have finite support $[0, LT]$ where $L < Q$ (i.e., it is assumed that the channel delay spread does not exceed the OFDM symbol duration); this implies that any inter symbol interference (ISI) is only restricted to the past neighboring symbol as is generally true for OFDM.

3.3 Subspace based blind channel estimation for OFDM system using VC:

In this section we describe the VC's based blind channel estimation for OFDM systems. The advantage of this method is that there is no need of oversampling and we group M OFDM blocks at the receiver.

Sampled received signal $x(t)$ with rate $\frac{1}{T}$ is given by

$$x(i) = x(t_0 + iT) = \sum_{l=0}^{L} s_{i-l} h(t_0 + lT) + v(i) \tag{3.7}$$

Let $h(i) = h(t_0 + iT)$ and $\mathbf{h} = \left[h(L), h(L-1), \cdots, h(0)\right]^T$. (3.8)

Define $(MQ - L) \times MQ$ Toeplitz matrix $\mathcal{H}_M$ as

$$\mathcal{H}_M = \begin{bmatrix} h(L) & \cdots & h(0) & & & \\ & h(L) & \cdots & h(0) & & \\ & & \ddots & & \ddots & \\ & & & h(L) & \cdots & h(0) \end{bmatrix}_{(MQ-L)\times MQ} \tag{3.9}$$

Consider an observation interval over M (considered as smoothing factor) OFDM symbols from $\left(t_0 + ((k - M + 1)Q + L)T\right)$ to $\left(t_0 + ((k+1)Q - 1)T\right)$

$$\mathbf{x}_M(k) = \left[x((k - M + 1)Q + L), \cdots, x((k+1)Q - 1)\right]^T$$

$$= \mathcal{H}_M \begin{bmatrix} \mathbf{s}(k-M+1) \\ \vdots \\ \mathbf{s}(k-1) \\ \mathbf{s}(k) \end{bmatrix} + \underbrace{\begin{bmatrix} v((k-M+1)Q+L) \\ \vdots \\ v((k+1)Q-1) \end{bmatrix}}_{\mathbf{v}(k)}$$

$$= \mathcal{H}_M \underbrace{\left(\mathbf{I}_M \otimes \bar{\mathbf{W}}\right)}_{\tilde{\mathbf{W}}} \bullet \underbrace{\begin{bmatrix} \mathbf{d}(k-M+1) \\ \vdots \\ \mathbf{d}(k-1) \\ \mathbf{d}(k) \end{bmatrix}}_{\mathbf{D}(k)} + \mathbf{v}(k)$$

$$= \underbrace{\mathcal{H}_M \tilde{\mathbf{W}}}_{\mathcal{A}_M} \mathbf{D}(k) + \mathbf{v}(k) \tag{3.10}$$

where $\mathcal{A}_M = \mathcal{H}_M \tilde{\mathbf{W}}$ $((MQ-L)\times MD)$

In order to identify the channel from (3.10) a necessary condition for the subspace method is that $\mathcal{A}_M$ is of full column rank i.e. $M(N+P-D)\geq L$. The presence of virtual carriers implies that $N>D$ and the necessary condition can always be satisfied by choosing appropriate *M*, which is true even for non-CP OFDM systems.

The following *theorem* discusses necessary and sufficient condition for channel identifiability.

Theorem3.1: [19] For $N+P-L\geq D$, $\mathcal{A}_M$ has full column rank (i.e rank($\mathcal{A}_M$) $=MD$) if and only if the channel frequency response has no nulls at any of the data subcarrier frequencies.

Let, the user's transmitted information symbols $d_i(k)$'s are i.i.d. sequences with zero mean and known variance σ_d^2 (σ_d^2=1 without loss of generality). Assume each element of $\mathbf{v}(k)$ in (3.10) is additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2.

Using (3.10), correlation function of received signal is given by

$$\mathbf{R}_{\mathbf{xx}} = E\left(\mathbf{x}_M(k)\mathbf{x}_M(k)^H\right)$$

$$= \mathcal{A}_M \mathbf{R}_{\mathbf{DD}} \mathcal{A}_M{}^H + \mathbf{R}_{\mathbf{vv}}$$

$$= \mathcal{A}_M \mathbf{R}_{\mathbf{DD}} \mathcal{A}_M{}^H + \sigma_n^2 \mathbf{I} \tag{3.11}$$

By using singular value decomposition (SVD) on the received correlation matrix $\mathbf{R}_{\mathbf{xx}}$, we get

$$\mathbf{R}_{\mathbf{xx}} = \mathbf{U}_s \Sigma_s \mathbf{U}_s{}^H + \sigma_n^2 \mathbf{U}_n \mathbf{U}_n{}^H \tag{3.12}$$

Where $\mathbf{U}_s$ contains *MD* columns which span the signal space while $\mathbf{U}_n$ contains $M(N+P-D)-L$ columns which span the noise subspace. $\boldsymbol{\Sigma}_s = diag(\lambda_1, \lambda_2, \cdots, \lambda_{MD})$ is a diagonal matrix which contains singular values corresponding to the signal subspace.

By orthogonality relationship between signal and noise subspace, we get

$$\mathbf{U}_n(i)^H \mathcal{A}_M = 0 \qquad i = 1, \cdots, M(N+P-D)-L \tag{3.13}$$

where $\mathbf{U}_n(i)$ is the *i*th column of $\mathbf{U}_n$.

By using above set of constraints (3.13), we can identify channel vector $\mathbf{h}$.

The following theorem gives sufficient conditions for uniqueness of the channel estimate $\hat{\mathbf{h}}$:

Theorem 3.2: [19] (sufficient condition for identifiablity): Let $\mathbf{h}'$ *and* $\mathbf{h}$ *be distinct L + 1 dimension vectors and* $\mathcal{A}'_M$ *be a matrix constructed using* $\mathbf{h}'$ *as with* $\mathcal{A}_M$ *in (3.10), i.e.,* $\mathcal{A}'_M = \mathcal{H}'_M \tilde{\mathbf{W}}$. *For: 1)* $M \geq 2$; *2)* $N + P - D \geq L$ *and 3)* $\mathbf{h}$ *has no null on any of the data carrier frequencies, it follows that* $\mathbf{h}' = \alpha\mathbf{h}$ *where* α *is a complex scalar if range(*$\mathcal{A}'_M$*) = range(*$\mathcal{A}_M$*).*

Let

$\mathbf{U}_n(i) = [u_i(0), u_i(1), \cdots, u_i(MQ - L - 1)]^T$ where $\mathbf{U}_n(i)$ is ith column of $\mathbf{U}_n$.

Since only estimate of $\hat{\mathbf{U}}_n$ is available, we get the channel estimate as

$$\hat{\mathbf{h}} = \arg\min_{\|\mathbf{h}\|=1} \sum_{i=1}^{M(N+P-D)-Q} \left\| \hat{\mathbf{U}}_n(i)^H \mathcal{A}_M \right\|^2 \tag{3.14}$$

Exploiting the structure of $\mathcal{H}_M$, we get equation similar to (2.36) as

$$\mathbf{U}_n(i)^H \mathcal{H}_M = \mathbf{h}^H g_i \tag{3.15}$$

where g_i is given in (2.36).

Substituting (3.14) in (3.15) we get

$$\left\| \mathbf{U}_n(i)^H \mathcal{A}_M \right\|^2 = \mathbf{U}_n(i)^H \mathcal{H}_M \tilde{\mathbf{W}} \tilde{\mathbf{W}}^H \mathcal{H}_M{}^H \mathbf{U}_n(i)$$

$$= \mathbf{h}^H \underbrace{g_i \tilde{\mathbf{W}}}_{\mathbf{G}_i} \underbrace{\tilde{\mathbf{W}}^H g_i{}^H}_{\mathbf{G}_i^H} \mathbf{h}$$

So, (3.14) can be written as

$$\hat{\mathbf{h}} = \arg\min_{\|\mathbf{h}\|=1} \mathbf{h}^H \mathcal{G}\mathcal{G}^H \mathbf{h} \tag{3.16}$$

where $\mathcal{G} = \left[\mathbf{G}_1, \cdots, \mathbf{G}_{M(N+P-D)-L}\right]$

Is well known that $\hat{\mathbf{h}}$ is the eigen vector corresponding to smallest eigen value of the matrix $\mathcal{G}\mathcal{G}^H$.

<u>Remarks:</u>

1. The presence of the VCs and/or CPs is necessary for the subspace method to work (Theorem 2.1 and 3.1).

2. The condition $N+P-L \geq D$ (theorem3.1) requires that the number of VC and/or CP be greater than the channel memory. It is much stronger than $M(N+P-D) \geq L$ (this is necessary for employment of the subspace method) to assure channel identifiability. This condition can be satisfied in a typical OFDM application scenarios for both CP-OFDM and non-CP OFDM

3. As stated in Theorem 3.2, an amplitude/phase ambiguity exists in the channel estimate – that is inherent to all blind estimation approaches using SOS and cannot resolved without side information. Practical OFDM systems provide pilot tones for tracking the carrier frequency offset which can be exploited to resolve this ambiguity.

Special case: Cai and Akanshu proposed a method for blind channel identification of OFDM systems with CP. It is the special case of VC based blind channel identification algorithm for OFDM systems with $D=N$ (i.e no VC's case). Using (3.1) to (3.16) with the condition $D=N$, we can get the estimate of $\mathbf{h}$ for Cai and Akanshu method.

3.4 Subspace based blind channel estimation for OFDM system without CP:

In this section, instead of exploiting VC's, receiver oversampling is used employing subspace method for blind channel estimation of OFDM system without CP. For the case of OFDM system without CP, we assign $P=0$, $D=N$ and $p_0=0$ in the system as shown in Fig 3.1. Thus for OFDM without CP, we have $\bar{\mathbf{W}}=\mathbf{W}$.

A synchronized rate M/T sampler (i.e., oversampling factor of M compared with information symbol sampling rate $1/T$) after $x(t)$ yields

$$x_i^{(m)} = x\left(t_0 + iT + \frac{mT}{M}\right) = \sum_{l=0}^{L} s_{i-l} h\left(t_0 + lT + \frac{mT}{M}\right) + v_i^{(m)} \quad (3.17)$$

where $v_i^{(m)} = v\left(t_0 + iT + \frac{mT}{M}\right)$, $m = 0, \cdots, M-1$, t_0 is initial timing offset.

Define $h^{(m)}(l) = h\left(t_0 + lT + \frac{mT}{M}\right)$ and $\mathbf{h}^{(m)} = \left[h^{(m)}(0), h^{(m)}(1), \cdots, h^{(m)}(L)\right]^T$ (3.18)

As mentioned in section 3.3, the finite support dispersive channel causes $\mathbf{s}(k-1)$ corresponding to the $(k-1)$th OFDM symbol to partially overlap with the output for the kth symbol $\mathbf{s}(k)$. The ISI affects the beginning ML samples among the MQ samples in the duration from (t_0+kQT) to $\left(t_0+(k+1)QT-\frac{T}{M}\right)$, associated with the symbol $\mathbf{s}(k)$. For $L \ll Q$ (which is plausible in several OFDM applications), the energy of the ISI samples is negligible compared to that of the non-ISI affected samples. Hence we may process only ISI free samples for channel estimation i.e., the $M(Q-L)$ samples over the interval $(t_0+(kQ+L)T)$ to $\left(t_0+(k+1)QT-\frac{T}{M}\right)$; Thus, the received signal for the mth sampling phase corresponding to transmitting symbol $\mathbf{s}(k)$ is given by

$$\mathbf{x}^{(m)}(k)=\left[x_{kQ+L}^{(m)},\cdots,x_{kQ+Q-1}^{(m)}\right]^T$$

$$=\underbrace{\begin{bmatrix} h^{(m)}(L) & \cdots & h^{(m)}(0) & & & \\ & h^{(m)}(L) & \cdots & h^{(m)}(0) & & \\ & & \ddots & & \ddots & \\ & & & h^{(m)}(L) & & h^{(m)}(0) \end{bmatrix}}_{\mathbf{H}^{(m)}}\underbrace{\begin{bmatrix} s_{kQ} \\ \vdots \\ s_{kQ+Q-1} \end{bmatrix}}_{\mathbf{s}(k)}+\underbrace{\begin{bmatrix} v_{kQ+L}^{(m)} \\ \vdots \\ v_{kQ+Q-1}^{(m)} \end{bmatrix}}_{\mathbf{v}^{(m)}(k)}$$

$$=\mathbf{H}^{(m)}\mathbf{s}(k)+\mathbf{v}^{(m)}(k)$$

$$=\mathbf{H}^{(m)}\mathbf{W}\mathbf{d}(k)+\mathbf{v}^{(m)}(k) \tag{3.19}$$

Stacking all M $\mathbf{x}^{(m)}(k)$ ($m=0,\cdots,M-1$) vectors together yields,

$$\mathbf{x}(k)=\left[\mathbf{x}^{(0)T}(k),\mathbf{x}^{(1)T}(k),\cdots,\mathbf{x}^{(M-1)T}(k)\right]^T$$

$$=\underbrace{\underbrace{\begin{bmatrix} \mathbf{H}^{(0)} \\ \vdots \\ \mathbf{H}^{(M-1)} \end{bmatrix}}_{\mathcal{H}}\mathbf{W}}_{\mathcal{A}}\mathbf{d}(k)+\mathbf{v}(k)$$

$$=\mathcal{A}\mathbf{d}(k)+\mathbf{v}(k) \tag{3.20}$$

where $\mathbf{v}(k)=\left[\mathbf{v}^{(0)T}(k),\mathbf{v}^{(1)T}(k),\cdots,\mathbf{v}^{(M-1)T}(k)\right]^{T}$.

The multichannel model (3.20) for OFDM without CP yields an equivalent filtering matrix $\mathcal{A}$ of dimension $M(Q-L)\times Q$. (3.20) is equivalent to (2.8). Thus we can apply directly the technique method presented in chapter2.

A sufficient condition for channel identifiability follows a corollary of Theorem1 and 2 in [14] as

Theorem 3.3[14]: Matrix $\mathcal{H}$ is full column rank i.e., rank($\mathcal{H}$) =Q, if

1) *the polynomials* $H^{(i)}(Z)\overset{def}{=}\sum_{j=0}^{L}h_j^{(i)}z^j$ *have no common zero*
2) $Q\geq L$,
3) *at least one polynomial* $H^{(i)}(Z)$ *has degree L.*

Since $\mathbf{W}$ is unitary, this directly leads to rank($\mathcal{A}$) = rank($\mathcal{H}\mathbf{W}$) = rank($\mathcal{H}$). Therefore, the above conditions for $\mathcal{H}$ to be full column rank also guarantee that $\mathcal{A}$ is full column rank.

The following theorem gives conditions for uniqueness of the channel estimate $\hat{\mathbf{H}}$:

Theorem 3.4: Let $\mathbf{H}=\left[\mathbf{h}^{(0)T},\mathbf{h}^{(1)T},\cdots,\mathbf{h}^{(M-1)T}\right]^{T}$ *formed using (3.18), and* $\mathbf{H}'$ *be a* $M(L+1)\times 1$ *vector distinct from* $\mathbf{H}$; *filtering matrix* $\mathcal{H}$ *and* $\mathcal{H}'$ *are constructed using* $\mathbf{H}$ *and* $\mathbf{H}'$, *respectively. When* $Q\geq L$, *if range* ($\mathcal{H}'$) = *range* ($\mathcal{H}$), *then* $\mathbf{H}'=\alpha\mathbf{H}$ *where* α *is a scalar.*

Using (3.11) to (3.16), we can get the estimate of $\mathbf{H}$.

3.5 Subspace based Semi-blind channel identification for OFDM system:

3.5.1 A mechanism to remove scalar indetermination:

Blind methods always identify the channel up to one scalar indetermination, which has to be removed to allow the received symbols to be equalized. Standards often specify some pilot subcarriers carrying known symbols for phase tracking and channel estimation refinements purposes. Here we discuss a method which exploits pilot subcarriers to remove the blind scalar indetermination.

Let $\hat{\mathbf{h}}_{sub}$ be the channel estimation provided by the subspace based blind channel estimation algorithm.

From theorem 3.2, we have

$$\hat{\mathbf{h}}_{sub} = \alpha \mathbf{h} \tag{3.21}$$

The problem is to find the scalar coefficient α such that $\hat{\mathbf{h}} = \hat{\mathbf{h}}_{sub} / \alpha$ is close in the quadratic sense to the true channel vector $\mathbf{h}$. Let F be the number of pilot subcarriers on which some known symbols are transmitted. Let $d_{pil}(1),\cdots,d_{pil}(F)$ be the known symbols transmitted on the pilot subcarriers and $r_{pil}(1),\cdots,r_{pil}(F)$ be the corresponding FFT-processed received symbols. An estimate of the channel attenuations at the corresponding frequencies is provided by

$$\hat{\tilde{\mathbf{h}}}_{pil} = \left[\frac{r_{pil}(1)}{d_{pil}(1)}, \cdots, \frac{r_{pil}(F)}{d_{pil}(F)} \right]^T \tag{3.22}$$

Let $\mathbf{W}_{pil}$ be the $F \times L$ matrix obtained from the L first columns of matrix $\sqrt{N}\mathbf{W}$ by selecting the rows corresponding to the pilot carriers and by removing the other ones. Another estimation of these coefficients can be inferred for the subspace identification up to α is given by

$$\hat{\tilde{\mathbf{h}}}_{sub,pil} = \mathbf{W}_{pil}\hat{\mathbf{h}}_{sub} = \alpha \mathbf{W}_{pil}\hat{\mathbf{h}} = \alpha \hat{\tilde{\mathbf{h}}}_{pil} \tag{3.23}$$

From (3.22) and (3.23), α can be determined by solving the linear system of equations, $\hat{\tilde{\mathbf{h}}}_{sub,pil} = \alpha \hat{\tilde{\mathbf{h}}}_{pil}$, in the least square sense. However, if the channel estimation $\hat{\mathbf{h}}_{sub} = \alpha \hat{\mathbf{h}}$ obtained using the subspace algorithm is far from the true CIR $\mathbf{h}$, the final channel estimation remains inaccurate, even if α is estimated such that $\left\| \mathbf{h} - \hat{\mathbf{h}}_{sub} / \alpha \right\|$ is minimal. Somehow, no benefit is taken from the knowledge of the channel attenuations on the pilot carriers for the subspace algorithm. This can be overcome by considering the following system of equations:

$$\mathbf{U}_n(i)^H \mathcal{A}_M = \mathbf{U}_n(i)^H \mathcal{H}_M \tilde{\mathbf{W}} = \mathbf{h}^H \mathcal{g}_i \tilde{\mathbf{W}} = \mathbf{h}^H \mathbf{G}_i = 0 \quad \text{(Using (3.13) and (3.15))}$$

or equivalently

$$\mathbf{G}_i^H \mathbf{h} = 0, \; i = 1,\cdots,M(N+P-D)-L \tag{3.24}$$

$$\mathbf{W}_{pil}\mathbf{h} = \tilde{\mathbf{h}}_{pil} \tag{3.25}$$

Since this system of equations holds only approximately, it has to solved in the least squares sense, which amounts to minimizing the quadratic criterion, which is defined as

$$q'(\mathbf{h}) = \sum_{i=1}^{M(N+P-D)-L} \left\|\mathbf{G}_i^H \mathbf{h}\right\|^2 + \beta\left\|\mathbf{W}_{pil}\mathbf{h} - \tilde{\mathbf{h}}_{pil}\right\|^2$$

$$= \mathbf{h}^H Q\,\mathbf{h} + \beta\left(\mathbf{W}_{pil}\mathbf{h} - \tilde{\mathbf{h}}_{pil}\right)^H \left(\mathbf{W}_{pil}\mathbf{h} - \tilde{\mathbf{h}}_{pil}\right) \tag{3.26}$$

where β is weighting factor used to change confidence degree and $Q = \sum_{i=1}^{M(N+P-D)-L} \mathbf{G}_i\mathbf{G}_i^H$.

In order to estimate channel $\hat{\mathbf{h}}$, we have to minimize (3.26).Differentiate with respect to $\mathbf{h}$ and equating to 0, we get

$$Q\,\mathbf{h} + \beta\mathbf{W}_{pil}^H\left(\mathbf{W}_{pil}\mathbf{h} - \tilde{\mathbf{h}}_{pil}\right) = 0$$

$$\left[Q + \beta\mathbf{W}_{pil}^H\mathbf{W}_{pil}\right]\mathbf{h} = \beta\mathbf{W}_{pil}^H\tilde{\mathbf{h}}_{pil}$$

$$\hat{\mathbf{h}} = \left[Q + \beta\mathbf{W}_{pil}^H\mathbf{W}_{pil}\right]^{-1}\left(\beta\mathbf{W}_{pil}^H\tilde{\mathbf{h}}_{pil}\right) \tag{3.27}$$

This above semi-blind subspace algorithm can be seen as a channel-dependent interpolator between the pilot subcarriers.

3.5.2 Training symbol based initialization of the blind algorithm:

An inherent problem in blind channel estimation methods is their slow convergence rate, which often prevents their use in real applications where techniques based on training sequences are preferred. Thus, standards usually specify that, initially a frame of known blocks of symbols transmitted for synchronization and initial channel estimation at the receiver. These pilot symbols can be used to initialize the estimation of the autocorrelation matrix. This enables us to avoid the long convergence period of the blind algorithm and to obtain the same accuracy as the pilot-based estimation. In the following we illustrate this algorithm

1) Obtain an initial channel estimation: $\hat{\mathbf{h}}^{(0)}$ through pilot symbols
2) Using $\hat{\mathbf{h}}^{(0)}$, generate an estimation of matrix $\mathbf{R}_{\mathbf{xx}}^{(0)} = \mathcal{A}_M\left(\hat{\mathbf{h}}^{(0)}\right)\mathbf{R}_{\mathbf{DD}}\mathcal{A}_M\left(\hat{\mathbf{h}}^{(0)}\right)$.
3) Refine iteratively the autocorrelation matrix estimation each time a new block symbol $\mathbf{x}_M(k)$ is received using forgetting factor $\lambda \in [0,1]$

$$\mathbf{R}_{\mathbf{xx}}^{(N)} = \lambda \mathbf{R}_{\mathbf{xx}}^{(N-1)} + (1-\lambda)\mathbf{x}_M(N)\mathbf{x}_M(N)^H$$

4) Perform the subspace algorithm based on $\mathbf{R}_{\mathbf{xx}}^{(N)}$.

3.6 Simulation results for blind channel estimation of OFDM systems:

3.6.1 Results for Oversampling based blind channel estimation of OFDM systems without CP:

For channel to be identified, we have used following channel coefficients given in [20],

$$\mathbf{h}^{(1)} = \left[(-0.1892, 0.4273), (-0.2839, 0.6984), (0.1274, 0.4321), (-0.0451, 0.0912)\right]^T$$

$$\mathbf{h}^{(2)} = \left[(0.360, 0.001), (0.1041, 0.4126), (0.0914, 0.1885), (0.2052, -0.0739)\right]^T$$

For the simulation of above blind channel identification method in MATLAB environment, we use the following parameters:

- Oversampling factor: $M = 2$
- Modulation scheme: BPSK
- SNR = $10 \log\left(\sigma_s^2 / \sigma_n^2\right)$
- Channel length: $L = 3$.
- No of Monte Carlo simulations: $N_m = 100$.
- No of carriers $N = 15$
- Data window length: N_b=300

Steps carried out for simulation of oversampling based blind channel identification of OFDM system without CP are:

1. Obtain randomly generated source data and get the oversampled received signal vector using (3.20) and channel coefficients given above
2. Get N_b blocks of received vector and determine $\mathbf{R}_{\mathbf{xx}}$ using time average over N_b blocks.
3. Get the noise subspace representation from SVD of $\mathbf{R}_{\mathbf{xx}}$.
4. Using noise subspace, determine $\mathcal{G}$ using (3.15) and (3.16).

5. Estimate of channel is obtained by finding eigen vector corresponding to smallest eigenvalue of $\mathcal{G}\mathcal{G}^H$.

All steps from 1 to 5 are repeated for each independent Monte Carlo run. To obtain the performance measure of the channel estimate, we use Normalized Root Mean Square Error (NRMSE) defined as

$$NRMSE = \frac{1}{\|\mathbf{H}\|}\sqrt{\frac{1}{N_m M(L+1)}\sum_{p=1}^{N_m}\left\|\hat{\mathbf{H}}_p - \mathbf{H}\right\|^2} \tag{3.28}$$

where the subscript p refers pth simulation run, $\mathbf{H}$ is true channel given in theorem 3.4 and $\hat{\mathbf{H}}_p$ is estimated channel using oversampling method for pth simulation run.

For comparison purposes, we also simulate Cai and Akanshu method for blind channel identification of OFDM system with the following parameters: sampling rate = $1/T$, cyclic prefix length $P = \frac{N+1}{4} = 4$ and no of data carriers $D = 15$.

Fig 3.2 shows the variation NRMSE as a function of SNR for both Cai and Akanshu method, and oversampling method for blind channel identification of OFDM systems. As SNR is varied from 0 to 40dB, NRMSE value decreases from 0.2 to 0.0015. We may also observe that both Cai and Akanshu method and oversampling method perform similarly with respect to SNR variation. The inherent advantage of oversampling method for blind identification of OFDM channels is that it avoids the CP and therefore leads to higher data throughput.

As shown in Fig 3.3, we compare the variation of NRMSE at different data window lengths (N_b) at SNR=15dB, for both oversampling method and, Cai and Akanshu method for blind channel identification of OFDM system. As N_b is varied from 100 to 2000 blocks, NRMSE decreases from 0.075 to 0.008. We may observe that both the methods (Cai and Akanshu, and Oversamplig method) performs similarly with the variation of data window length.

Variation of NRMSE with OFDM symbol duration (Q):

From (3.13) the number of noise eigen vectors in noise subspace for oversampling method is $(M-1)Q-ML$. Larger Q means larger is the dimension of the noise subspace for oversampling method, yielding more constraints on channel vector and, thus it leads to improvement in channel estimate. The effect of varying Q (meaning longer OFDM symbol duration) on the estimation error for oversampling based blind channel identification of OFDM system without CP is investigated in Fig 3.5 with SNR=40dB, N_b=2000 and Q varying from 15 to 47. We may observe that as Q increases, NRMSE for oversampling method decreases.

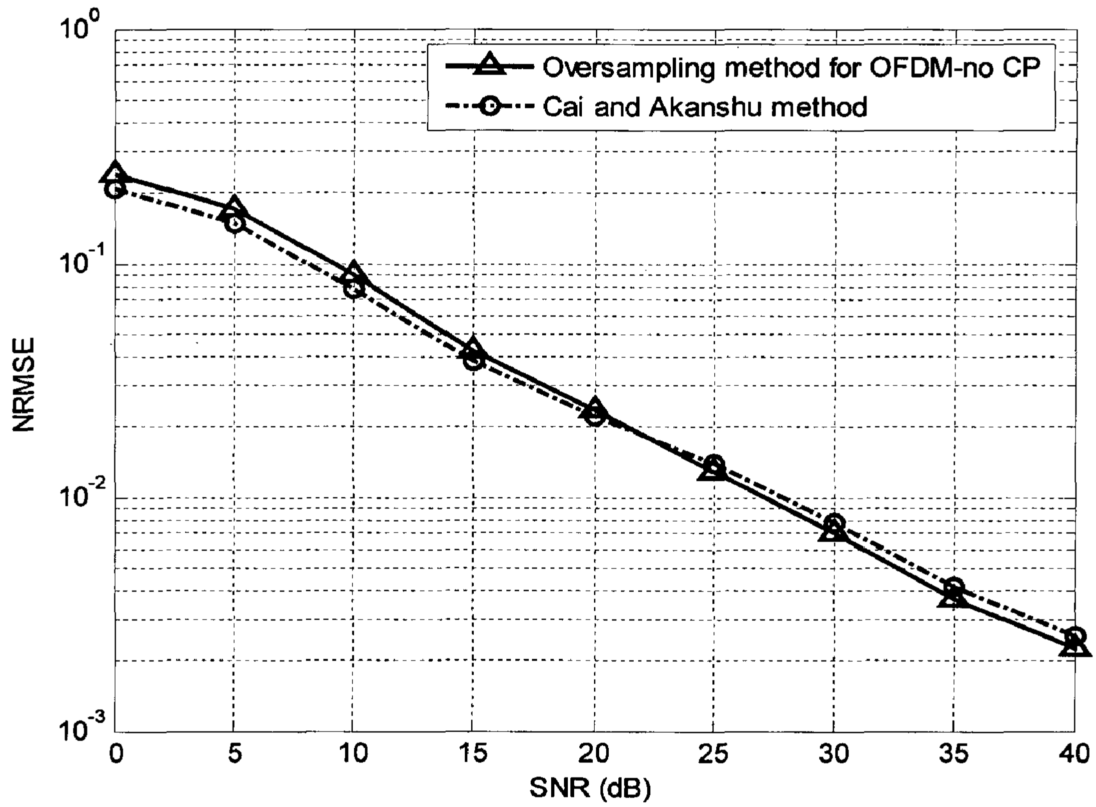


Fig 3.2: Variation of NRMSE with respect to SNR for oversampling based blind channel identification method for OFDM system without CP and, Cai and Akanshu method for OFDM system with CP.

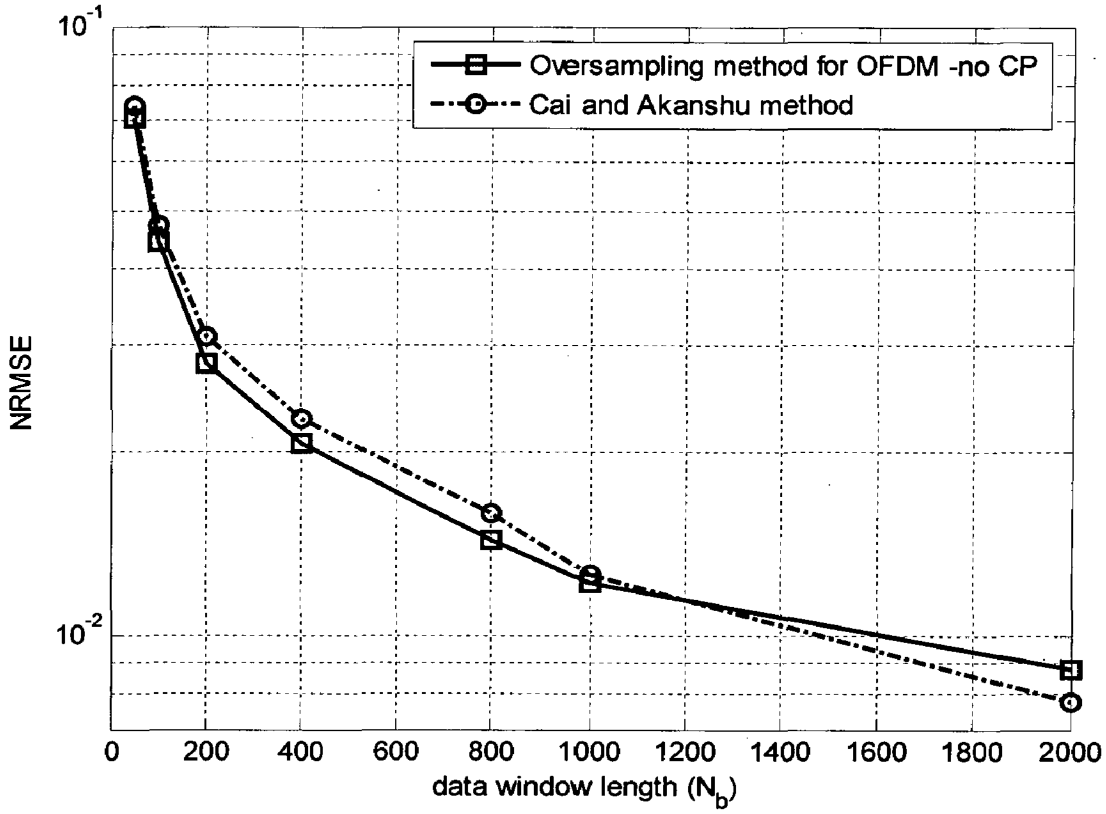


Fig 3.3: Variation of NRMSE with respect to data window size (N_b) for oversampling based blind channel identification method for OFDM system without CP and, Cai and Akanshu method for OFDM with CP. (SNR=15dB).

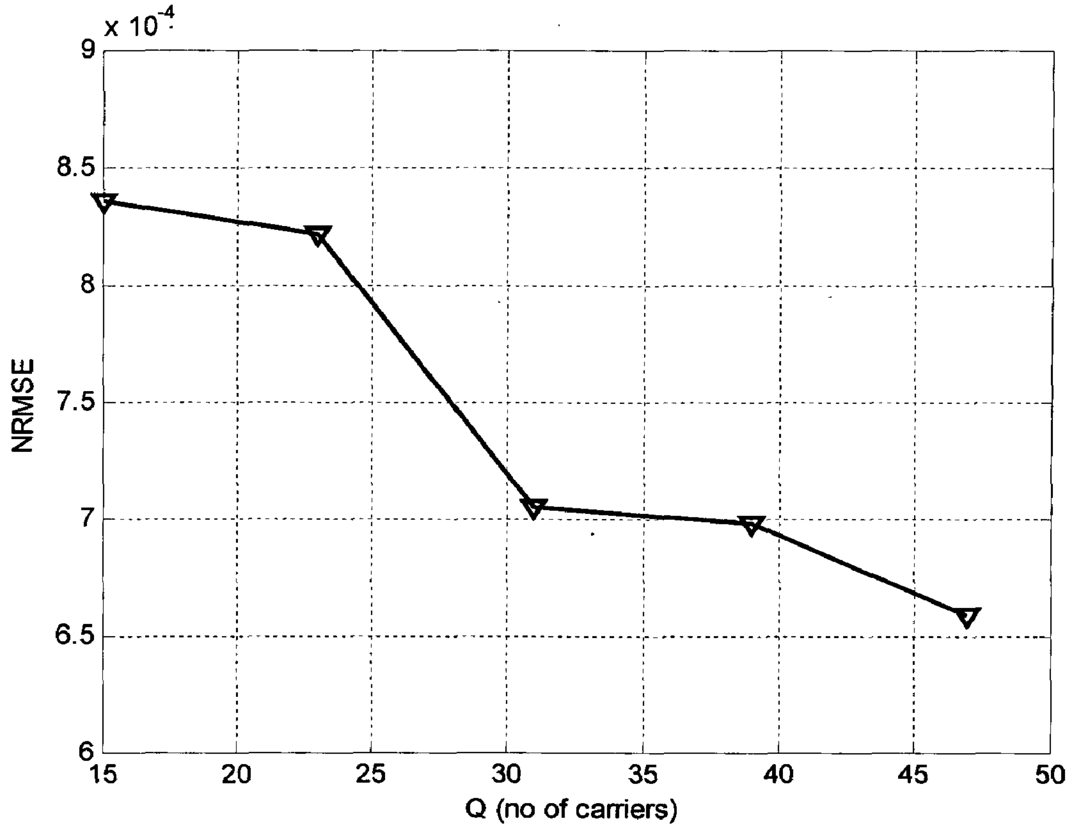


Fig 3.4: Variation of NRMSE with OFDM symbol duration for oversampling based blind channel identification method for OFDM system without CP at SNR=40dB and N_b=2000 OFDM blocks.

3.6.2 Results for VC based blind channel estimation of OFDM systems:

For the simulation of above method in MATLAB environment, we use the following parameters:

- Smoothing factor: $M=2$
- Modulation scheme : BPSK
- SNR = $10\log\left(E_s/E_n\right)$, $E_s=D\sigma_s^2$, $E_n=Q\sigma_n^2$ D is no of data carriers, Q is sum of data carriers and CP.
- No of subcarriers: $N=15$
- No of Monte Carlo runs: $N_m=100$
- Multipath fading channel with order $L=3$ is generated by assuming exponential power delay profile $exp\left(-\frac{\tau}{\tau_{rms}}\right)$, τ stands for path delay and $\tau_{rms}=0.6T$ is rms delay value. (T is symbol period).
- Data window length: N_b=300 OFDM symbol blocks

VC method for blind channel identification of OFDM systems presented in section 3.3 is applicable to systems with or without CP. For simulation for VC method for OFDM system, we tested on different system settings:

1) a system with no CP: $D=11, VC=4$ and $P=0$
2) system with insufficient CP: $P=2$ (less than channel order $L=3$), $VC=2$ and $D=13$
3) system with sufficient CP: $P=4$ (greater than channel order $L=3$), $D=11$ and $VC=4$

For comparison, we also simulate Cai and Akanshu method [17] with the following parameters: $VC=0$, $P=\frac{N+1}{4}=4$ and $D=15$.

Steps carried out for simulation of VC based blind channel identification of OFDM system are:

1. Generate multipath fading channel coefficients of order $L=3$.

2. Obtain randomly generated source data and get the received signal vector using (3.10).
3. Get N_b received OFDM blocks and determine $\mathbf{R}_{xx}$ using time average over N_b blocks.
4. Get the noise subspace from SVD of $\mathbf{R}_{xx}$ as in 3.12.
5. Using noise subspace, determine $\mathcal{G}$ using (3.15) and (3.16).
6. Estimate of channel is obtained by finding eigen vector corresponding to smallest eigenvalue of $\mathcal{G}\mathcal{G}^H$.

All steps from 1 to 6 are repeated for each independent Monte Carlo run. To obtain the performance measure of the channel estimate, we use Normalized Root Mean Square Error (NRMSE) criterion given by

$$NRMSE = \sqrt{\frac{1}{N_m(L+1)}\sum_{p=1}^{N_m}\frac{\left\|\hat{\mathbf{h}}_p - \mathbf{h}_p\right\|^2}{\left\|\hat{\mathbf{h}}_p\right\|^2}} \tag{3.29}$$

$\hat{\mathbf{h}}_p$ is estimated channel by VC method for the pth simulation run and $\mathbf{h}_p$ is true channel generated for pth simulation run.

Fig 3.5 shows the variation of NRMSE as a function of SNR for both Cai and Akanshu method, and VC method with different setting as described above. We can observe that the NRMSE for both the methods decreases with increasing SNR. The advantage of VC based blind channel identification method for OFDM system is that it can be applied to OFDM systems without CP.

Fig 3.6 shows the variation of NRMSE at different data window lengths (Nb) for both Cai and Akanshu method, and VC method with different setting as described above. We can observe that the NRMSE for both the methods decreases with increasing data window lengths (N_b). We may observe that, beyond window size of 1000 blocks, there is no effect on the performance of both the algorithms. We may also observe that the performance of VC method for OFDM systems is same as Cai and Akanshu method for CP based OFDM systems.

Additionally, for a fixed degree of freedom through the combination of VCs and/or CPs, there is a performance gap between the non-CP system ($D = 11, P = 0$), the sufficient

CP system ($D=11, P=4$) and insufficient CP ($D=13, P=2$). We may observe that VC based methods for OFDM system with sufficient CP has less NRMSE than the VC method for OFDM systems without or insufficient CP. This suggests that CP is more advantageous for the noise subspace based estimation than VCs. However, the utilization of VCs provides the receiver an extra source redundancy other than CP and makes VC method feasible for a system with insufficient CP ($P=2$) without increasing smoothing factor M. Note that M must be increased in Cai and Akanshu method, (due to the requirement to satisfy the condition $MP \geq L$), which means a larger observation duration and significant increase of computational complexity.

Fig 3.7 plots the variation of NRMSE at different SNR values for VCs based blind channel identification of OFDM systems with different estimates of channel order. Here we assume OFDM system without CP ($P=0$) scenario. We may observe that NRMSE decreases with increase of SNR for all the cases. We may observe that there is no significant difference in the performance of VC method for blind identification of OFDM channel with exact channel order estimation and order overestimation by 2 or 3. Hence, VCs based blind channel identification of OFDM system is insensitive to order overestimation. Robustness to channel order overestimation is a desirable property, since in practice the accurate information about the channel order is not available and an overestimation is more likely to occur.

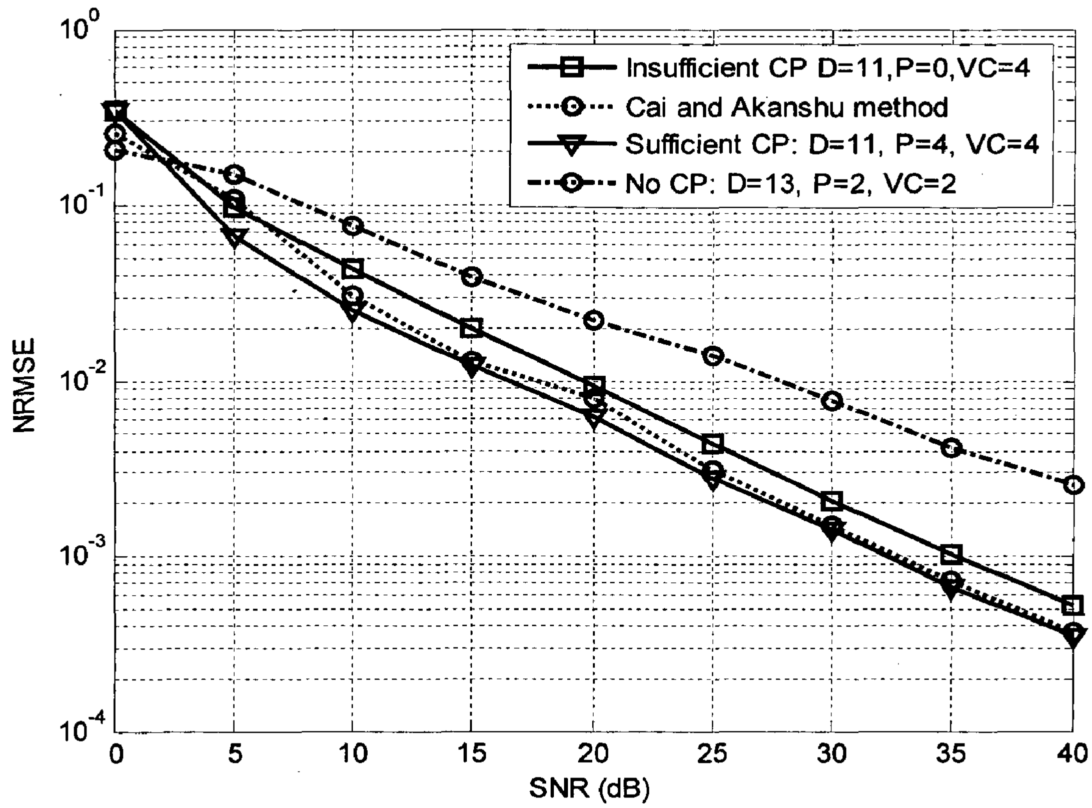


Fig 3.5: Variation of NRMSE with respect to SNR for VC based blind channel identification method for OFDM system with three different scenarios (No CP, Insufficient CP, Sufficient CP) and, Cai and Akanshu method for OFDM with CP.

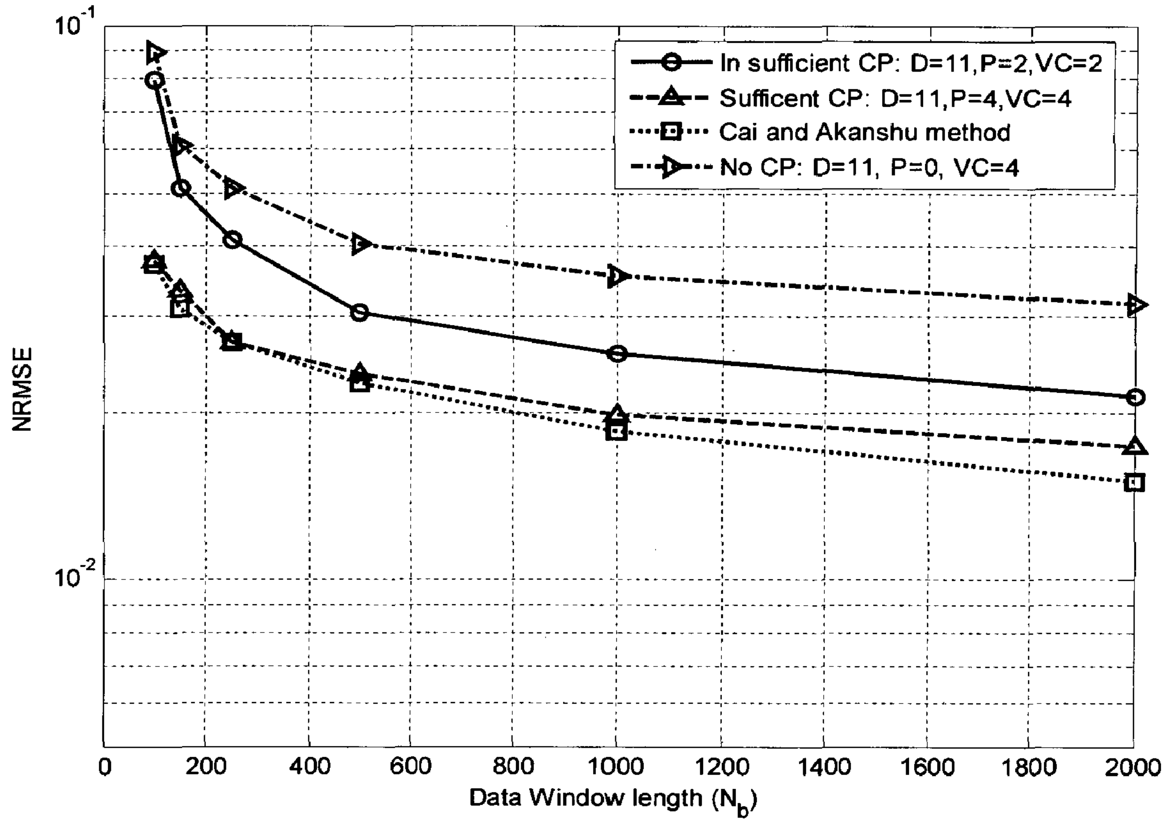


Fig 3.6: Variation of NRMSE with respect to data window size (N_b) for VC based blind channel identification method for OFDM system with three different scenarios (No CP, Insufficient CP, Sufficient CP) and, Cai and Akanshu method for OFDM with CP. (SNR=15dB)

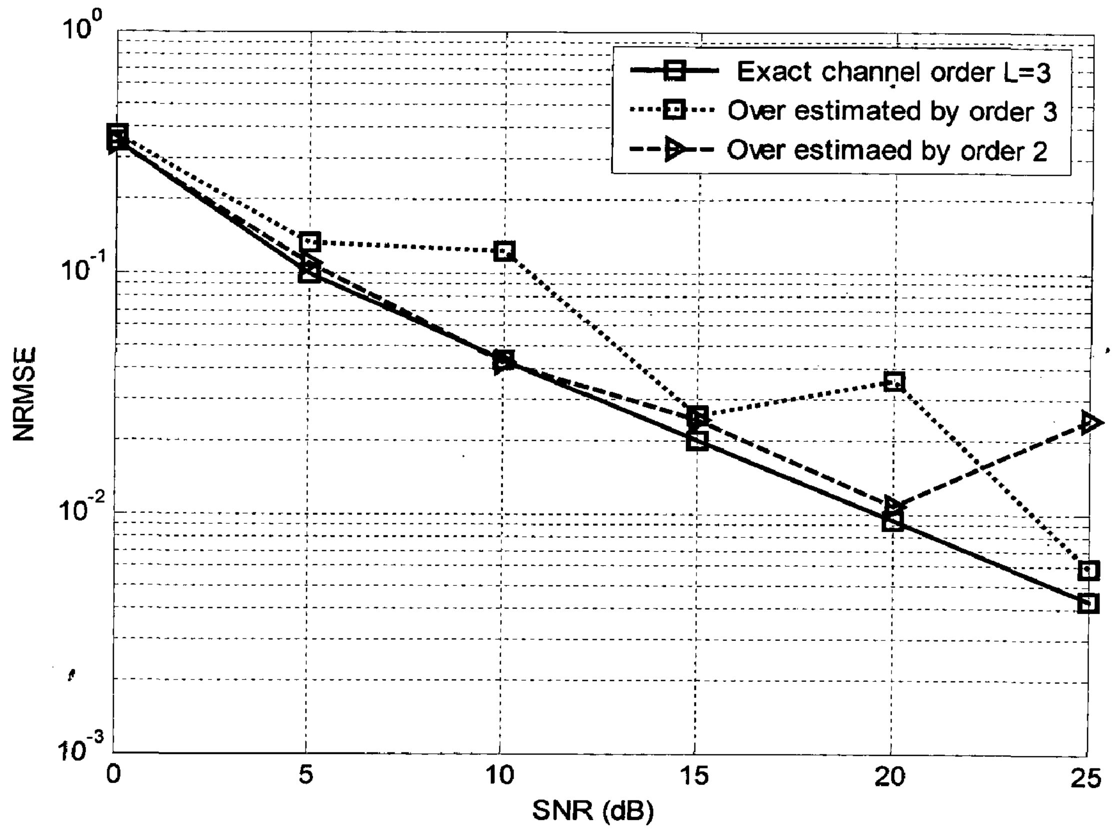


Fig 3.7: NRMSE versus SNR for different estimates of channel order (L) for VCs based blind channel identification of OFDM systems without CP. ($D=11, P=0$)

3.6.3 Results for Subspace based semi blind channel estimation for OFDM systems:

For the simulation of subspace based semi blind channel estimation for OFDM system in MATLAB environment, we use the following parameters:

- No of carriers: $N = 64$
- Virtual carriers: VC=12
- Data carriers: $D = 52$
- Out of 52 data carriers, no of pilot carriers = 4. Pilot position and VCs position is given in the following

$$\left| \underbrace{0\cdots 0}_{6}\underbrace{X\cdots X}_{5}P_1\underbrace{X\cdots X}_{13}P_2\underbrace{X\cdots X}_{6}0\underbrace{X\cdots X}_{6}P_3\underbrace{X\cdots X}_{13}P_4\underbrace{X\cdots X}_{5}\underbrace{0\cdots 0}_{5} \right|$$

- Channel: Multipath fading coefficients for channel order $L=16$ are generated by assuming exponential power delay profile $exp\left(\frac{\tau}{\tau_{rms}}\right)$, ($\tau$ stands for path delay) with rms delay value $\tau_{rms} = 0.6T$ (T is symbol period).
- Forgetting factor: $\lambda = 0.9$.
- Weighting factor: $\beta = 0.9$

Steps for subspace based semi blind channel estimation for OFDM systems:

1. For initial channel estimation for semi blind method, we assume that first two blocks of a frame, $\mathbf{d}(1)$ and $\mathbf{d}(2)$ are known at the receiver. There are no VCs during initial phase. $\mathbf{r}(1)$ and $\mathbf{r}(2)$ are the corresponding FFT processed received data. The receiver forms a initial channel estimate in frequency domain as

$$\hat{h}_i(1) = \frac{1}{2}\left[\frac{r_i(0)}{d_i(0)} + \frac{r_i(1)}{d_i(1)}\right] \text{ for } 1 \le i \le N.$$

2. After getting initial channel estimate, use step 2 of section 3.5.2 to get initial estimate of correlation matrix of received signal vector.

3. Update received correlation matrix after getting each received vector using step 3 of section 3.5.2.

4. After channel initialization, we only know pilot symbols $d_i(k)$ for $i = 12,26,40,54$. One can track the channel transfer function using a running average (over $B = 20$ blocks) on these pilot carriers as follows:

$$\hat{\hat{h}}_{pil}(k+1) = \frac{1}{B}\sum_{l=0}^{B-1}\frac{r_{pil}(k-l)}{d_{pil}(k-l)} \quad \text{for } pil \in \{12,26,40,54\}$$

5. Determine Q using (3.26).

6. Determine the estimate of channel by substituting Q and $\hat{\mathbf{h}}_{pil}$ in (3.27).

Steps from 1 to 6 are repeated for each Monte Carlo run. Normalized Root Mean Square Error (NRMSE) is computed as in (3.28). We also simulate VC based blind channel identification method for OFDM system under same simulation parameters.

Fig 3.8 shows the variation of NRME with data window length (N_b) for semi blind channel estimation method and blind channel method for OFDM systems based on VCs. We have assumed SNR=25dB. As N_b is varied from 100 to 450 OFDM blocks, NRMSE value for semi blind channel estimation method decreases from 0.04 to 0.003, while for blind channel estimation method it decreases from 0.21 to 0.02. For a given NRMSE value of 0.02, semi blind method requires 150 OFDM blocks, while blind VC method requires 450 OFDM blocks. We may conclude that the convergence of semi blind channel estimation method is faster than the blind channel estimation method for OFDM systems.

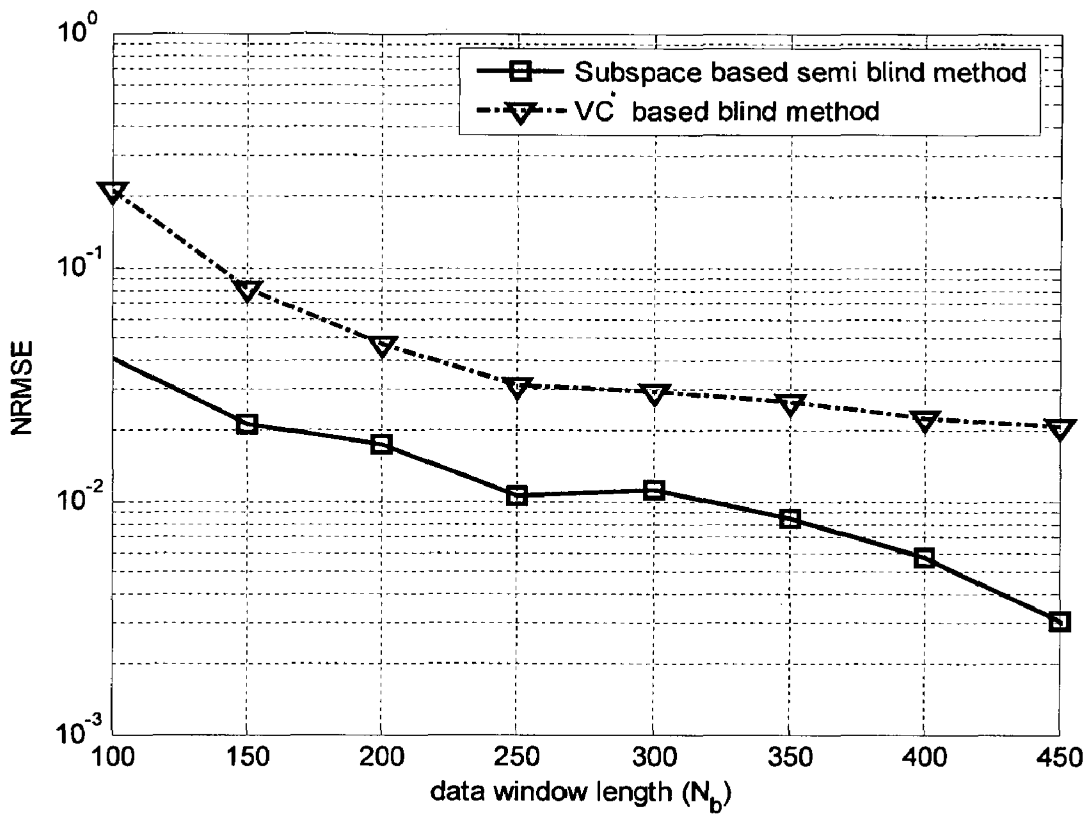


Fig 3.8: Variation of NRMSE with respect to data window size (N_b) for subspace based semi blind and blind channel estimation methods using VC's for OFDM systems. (SNR=25dB)

Chapter 4
Blind Channel Estimation For MIMO-OFDM systems

In this chapter, we first give a brief review of different blind channel estimation methods for MIMO-OFDM. Then system model for MIMO-OFDM is described. We then describe the subspace based blind channel estimation for cyclic prefixed MIMO-OFDM systems. Zero padding (ZP) based blind channel identification method for MIMO-OFDM system is described next. We finally present simulation results.

4.1 Review of blind channel estimation techniques for MIMO-OFDM systems:

Blind channel estimation for MIMO-OFDM systems has been an active area of research in recent years. Zhou *et al.* [24] proposed a subspace based blind channel estimation method for space time coded MIMO-OFDM systems using properly designed redundant linear precoding. Gao and Nallanathan [25] proposed a blind algorithm for MIMO-OFDM systems by utilizing non-redundant linear block precoding. With the assumption that the symbols sent from different transmitters are i.i.d, this method gives the acceptable performance at low SNR region and is applicable to multiple-input single-output (MISO) system. However the method gives an error floor at high SNR.

Bolcskei *et al.* [26] proposed an algorithm for blind channel estimation and equalization for MIMO-OFDM systems using second order cyclostationary statistics induced by employing a periodic nonconstant modulus antenna precoding. The basic idea of this method is to provide each transmit antenna with a different signature in the cyclostationary domain to null out the influence of all but one transmit antenna at a time. This makes a scalar subchannel by subchannel identification of the matrix channel possible. The advantage of this method is that it requires only upper bound on the channel length, it does not impose restrictions on channel zeros and it exhibits low sensitivity to stationary noise. The disadvantage of this method is that algorithm is computationally complex.

Zeng and Ng [27] proposed a subspace technique based on the noise subspace method for estimating the MIMO channels in the uplink of multiuser multiantenna zero padded OFDM system. By making use of property of zero padding, this method no longer needs precise order of the channel and it can accurately estimate the channels subject to scalar ambiguity. Also, a technique is presented to resolve the scalar ambiguity matrix by using one pilot OFDM block. Disadvantage of this method is that it cannot be applied to conventional OFDM systems because they are CP based. Gao *et al.,* [28] develop a novel subspace algorithm that is suitable for CP based MIMO-OFDM systems by applying an appropriate re-modulation on the received signal blocks. It is applicable to the situation when no of transmitting antennas is equal to no of receiving antennas, where the conventional subspace methods cannot be applied.

Chenyang Shin *et al.* [29] proposed a method that unifies and generalize the SISO-OFDM subspace methods [19] to the case of spatial multiplexing MIMO-OFDM systems with any number of transmit and receive antennas. This method exploits the virtual carriers (VC) and cyclic prefix present in OFDM systems. Considering the presence of VCs in OFDM, this method can be applied to MIMO-OFDM systems without CPs, where blind estimation techniques based on CPs cannot be employed, thereby providing the systems to achieve higher channel utilization. Also this method outperforms other techniques when both CP and VC are simultaneously present.

4.2 Subspace based blind channel estimation for cyclic prefixed (CP) MIMO OFDM systems:

4.2.1 CP based MIMO-OFDM system model:

A CP based MIMO-OFDM system with K transmitters and J receivers is shown in Fig 4.1. The information symbols are first divided into K streams and each stream will be grouped into blocks of length N, followed by the normalized inverse discrete Fourier transformation (IDFT). Let the i th information block at the kth transmitter can be written as

$$\mathbf{d}_i^{(k)} = \left[d_i^{(k)}(0), d_i^{(k)}(1), \cdots, d_i^{(k)}(N-1)\right]^T, \quad k = 1,2,\cdots,K,\ i = 0,1,\cdots, \tag{4.1}$$

The normalized IDFT of $\mathbf{d}_i^{(k)}$ is given by

$$\begin{aligned}\mathbf{s}_i^{(k)} &= \left[s_i^{(k)}(0), s_i^{(k)}(1), \cdots, s_i^{(k)}(N-1)\right]^T \\ &= \mathbf{F}_N^H \mathbf{d}_i^{(k)}\end{aligned} \tag{4.2}$$

where $\left(\mathbf{F}_N\right)_{(m,n)} = \frac{1}{\sqrt{N}} e^{j2\pi mn/N}$.

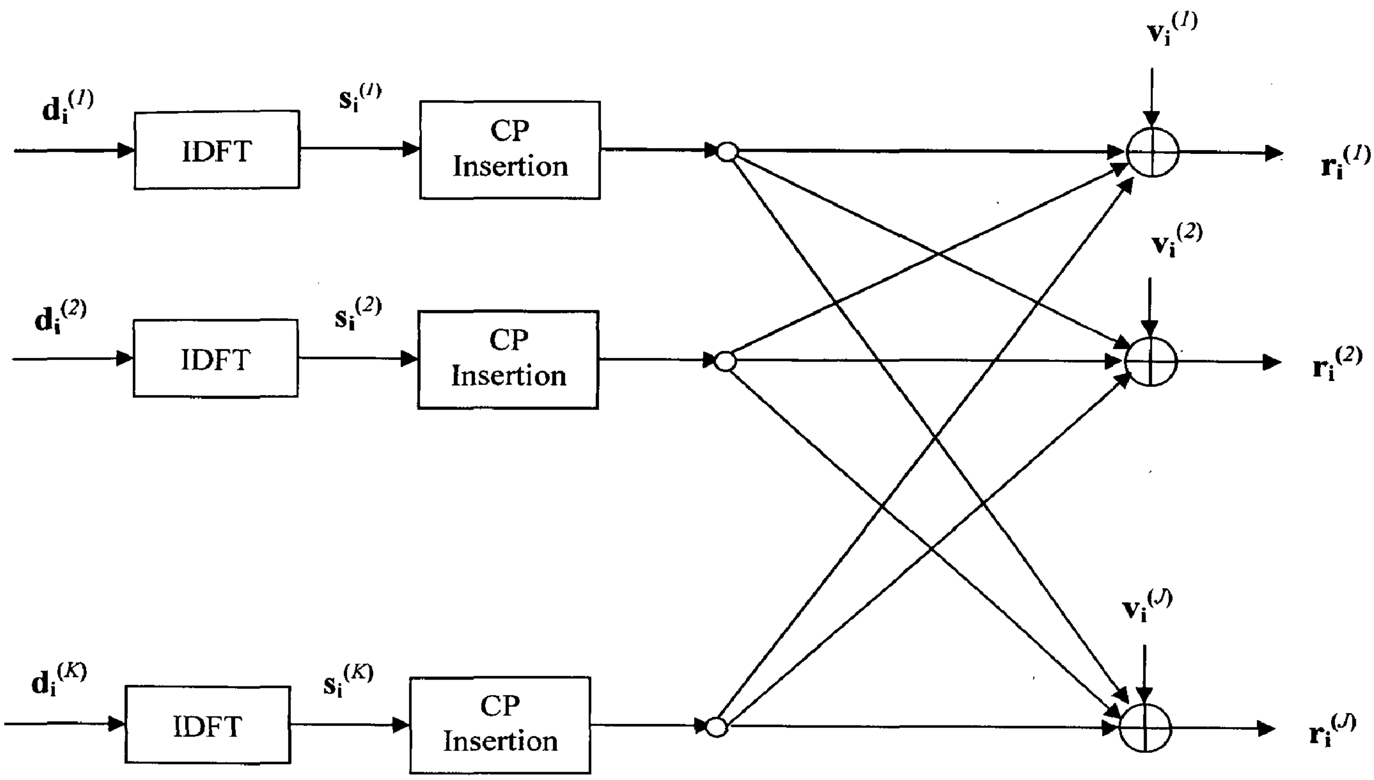


Fig 4.1: CP based MIMO-OFDM System model with K transmitting and J receiving antennas.

After the CP insertion, the overall time domain block from the kth transmitter is

$$\mathbf{t}_i^{(k)} = \begin{bmatrix} \mathbf{s}_{i,L}^{(k)} \\ \mathbf{s}_i^{(k)} \end{bmatrix} \tag{4.3}$$

where $\mathbf{s}_{i,L}^{(k)}$ is the CP that contains last L entries of $\mathbf{s}_i^{(k)}$.

Let

$\mathbf{h}^{(j,k)} = \left[h^{(j,k)}(0), h^{(j,k)}(1), \cdots, h^{(j,k)}\left(L_{(j,k)}\right) \right]^T$ be the channel impulse response (CIR) between transmitter k and receiver j, where $L_{(j,k)}$ is the corresponding channel order and is over bounded by L. For convenience, we pad $L - L_{j,k}$ zeros at the end of $\mathbf{h}^{(j,k)}$ such that all the channels have a length of L. In other words channel order over estimation is taken into account in the model.

Assuming perfect synchronization at the receiver, the received ith block (of length $N+L$) on the jth receiver is then represented by

$$\mathbf{r}_i^{(j)} = \sum_{k=1}^{K} \mathcal{H}\left(\mathbf{h}^{(j,k)}\right) \begin{bmatrix} \mathbf{s}_{i-1,L}^{(k)} \\ \mathbf{s}_i^{(k)} \end{bmatrix} + \mathbf{v}_i^{(j)} \tag{4.4}$$

where
$$\mathcal{H}\left(\mathbf{h}^{(j,k)}\right) = \underbrace{\begin{bmatrix} h^{(j,k)}(L) & \cdots & h^{(j,k)}(0) & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & h^{(j,k)}(L) & \cdots & h^{(j,k)}(0) \end{bmatrix}_{(N+2L)\times(N+L)}}_{N+2L} \tag{4.5}$$

and $\mathbf{v}_i^{(j)} = \left[v_i^j(0), v_i^j(1), \cdots v_i^j(N+L-1)\right]^T$ is the ith noise block on the jth receiver whose elements are zero mean complex Gaussian random variables with the variance σ_n^2 and are both spatially and temporally independent from each other.

Divide $\mathbf{r}_i^{(j)}$ into two parts as

$$\mathbf{r}_i^{(j)} = \left[\left(\mathbf{x}_{i,L}^{(j)}\right)^T \quad \left(\mathbf{x}_i^{(j)}\right)^T\right]^T \tag{4.6}$$

where $\mathbf{x}_{i,L}^{(j)} = \left[x_{i,L}^{(j)}(0), x_{i,L}^{(j)}(1), \cdots, x_{i,L}^{(j)}(L-1)\right]^T = \mathbf{r}_i^{(j)}(1:L)$ and

$$\mathbf{x}_i^{(j)} = \left[x_i^{(j)}(0), x_i^{(j)}(1), \cdots, x_i^{(j)}(N-1)\right]^T = \mathbf{r}_i^{(j)}(L+1:N+L) \tag{4.7}$$

respectively.

Transmitted signals on the same slot from all antennas given by

$$\mathbf{s}_i(n) = \left[s_i^{(1)}(n), s_i^{(2)}(n), \cdots, s_i^{(K)}(n)\right]^T \quad n = 0, \cdots, N-1 \tag{4.8}$$

Grouping all $\mathbf{s}_i(n)$ together, we get

$$\mathbf{s}_i = \left[\mathbf{s}_i^T(0), \mathbf{s}_i^T(1), \cdots, \mathbf{s}_i^T(N-1)\right]^T \tag{4.9}$$

$$\mathbf{s}_{i,L} = \left[\mathbf{s}_i^T(N-L), \mathbf{s}_i^T(N-L+1), \cdots, \mathbf{s}_i^T(N-1)\right]^T \tag{4.10}$$

$$\mathbf{t}_i = \left[\mathbf{s}_{i,L}^T, \mathbf{s}_i^T\right]^T \tag{4.11}$$

Similarly received signal on the same slot from all antennas given by

$$\mathbf{x}_{i,L}(l)=\left[x_{i,L}^{(1)}(l),x_{i,L}^{(2)}(l),\cdots,x_{i,L}^{(J)}(l)\right]^T \quad l=0,\cdots,L-1 \tag{4.12}$$

$$\mathbf{x}_i(n)=\left[x_i^{(1)}(n),x_i^{(2)}(n),\cdots,x_i^{(J)}(n)\right]^T \quad n=0,\cdots,N-1 \tag{4.13}$$

Grouping all $\mathbf{x}_i(n)$ and $\mathbf{x}_{i,L}(l)$ together, we get

$$\mathbf{x}_i=\left[\mathbf{x}_i^T(0),\mathbf{x}_i^T(1),\cdots,\mathbf{x}_i^T(N-1)\right]^T \tag{4.14}$$

$$\mathbf{x}_{i,L}=\left[\mathbf{x}_{i,L}^T(0),\mathbf{x}_{i,L}^T(1),\cdots,\mathbf{x}_{i,L}^T(L-1)\right]^T \tag{4.15}$$

Noise at the receiver is also arranged similarly and is given by

$$\mathbf{v}_i(p)=\left[v_i^{(1)}(p),v_i^{(2)}(p),\cdots,v_i^{(J)}(p)\right]^T \quad p=0,\cdots,N+L-1 \tag{4.16}$$

$$\mathbf{v}_i=\left[\mathbf{v}_i^T(0),\mathbf{v}_i^T(1),\cdots,\mathbf{v}_i^T(N+L-1)\right]^T \tag{4.18}$$

Form lth lag component of all channels from K transmitters to J receivers as

$$\mathbf{H}(l)=\begin{bmatrix} h^{(1,1)}(l) & h^{(1,2)}(l) & \cdots & h^{(1,K)}(l) \\ h^{(2,1)}(l) & h^{(2,2)}(l) & \cdots & h^{(2,K)}(l) \\ \vdots & \vdots & \vdots & \vdots \\ h^{(J,1)}(l) & h^{(J,2)}(l) & \cdots & h^{(J,K)}(l) \end{bmatrix} \quad l=0,\cdots,L \tag{4.19}$$

Taking all lags together, we get

$$\mathbf{H}=\left[\mathbf{H}^T(0),\mathbf{H}^T(1),\cdots,\mathbf{H}^T(L)\right]^T \tag{4.20}$$

The signal blocks from all the J receivers, after proper permutation, can be expressed as

$$\mathbf{r}_i=\begin{bmatrix}\mathbf{x}_{i,L}\\ \mathbf{x}_i\end{bmatrix}=\mathcal{H}(\mathbf{H})\begin{bmatrix}\mathbf{s}_{i-1,L}\\ \mathbf{t}_i\end{bmatrix}+\mathbf{v}_i$$

$$=\mathcal{H}(\mathbf{H})\mathcal{T}_{cp}\begin{bmatrix}\mathbf{s}_{i-1,L}\\ \mathbf{s}_i\end{bmatrix}+\mathbf{v}_i \tag{4.21}$$

$$\text{where } \mathcal{H}(\mathbf{H})=\underbrace{\left.\begin{bmatrix} \mathbf{H}(L) & \cdots & \mathbf{H}(0) & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \mathbf{H}(L) & \cdots & \mathbf{H}(0) \end{bmatrix}\right\}N+L}_{N+2L} \quad {}_{J(N+L)\times K(N+2L)} \tag{4.22}$$

and $\mathcal{T}_{cp} = \begin{bmatrix} \mathbf{I}_{KL} & 0_{KL\times KN} \\ 0_{KL\times KN} & \mathbf{I}_{KL} \\ 0_{KN\times KL} & \mathbf{I}_{KN} \end{bmatrix}_{K(N+2L)\times K(N+L)}$

Theorem 4.1:[28] The identifiability of MIMO channel (4.20) could be guaranteed if

1. *The $J(N+L)\times K(N+L)$ matrix $\mathcal{H}(\mathbf{H})\mathcal{T}_{cp}$ is tall i.e number rows greater than number of columns.*
2. *Matrix $\mathcal{H}(\mathbf{H})\mathcal{T}_{cp}$ is full rank.*

The first condition is satisfied only if $J > K$. Clearly, the direct modeling of the received signals is not applicable to the scenarios with $J = K$, which includes SISO OFDM in IEEE 802.11a and 2×2 MIMO-OFDM in 802.11n.

4.2.2 System remodulation:

By properly remodulating the received signal block, the system model (4.21) could be converted to the one similar to ZPSOS model proposed in [27]. This ensures that the robust property of ZPSOS e.g. applicability to equal transceiver antenna scenario, robustness to channel order overestimation and guarantee of the channel identifiability, could possibly be inherited after the remodulation.

Divide noise vector $\mathbf{v}_i$ into two components as

$$\mathbf{v}_{i1} = \mathbf{v}_i(1:JL) \text{ and } \mathbf{v}_{i2} = \mathbf{v}_i(JL+1:J(N+L)).$$

Construct a new vector $\bar{\mathbf{r}}_i = \begin{bmatrix} \mathbf{x}_{i-1}^T & \mathbf{x}_{i,L}^T \end{bmatrix}^T$, which could be expressed as

$$\bar{\mathbf{r}}_i = \mathcal{H}(\mathbf{H})\begin{bmatrix} \mathbf{t}_{i-1} \\ \mathbf{s}_{i,L} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{(i-1)2} \\ \mathbf{v}_{i1} \end{bmatrix} \tag{4.23}$$

The system remodulation vector is defined as

$$\begin{aligned} \mathbf{z}_i &= \mathbf{r}_i - \bar{\mathbf{r}}_i \\ &= \mathcal{H}(\mathbf{H})\left(\begin{bmatrix} \mathbf{s}_{i-1,L} \\ \mathbf{t}_i \end{bmatrix} - \begin{bmatrix} \mathbf{t}_{i-1} \\ \mathbf{s}_{i,L} \end{bmatrix}\right) + \left(\mathbf{v}_i - \begin{bmatrix} \mathbf{v}_{(i-1)2} \\ \mathbf{v}_{i1} \end{bmatrix}\right) \end{aligned}$$

$$= \mathcal{H}(\mathbf{H})\begin{bmatrix} 0_{JL\times 1} \\ \mathbf{m}_i \\ 0_{JL\times 1} \end{bmatrix} + \eta_i$$

$$= \mathcal{G}\mathbf{m}_i + \eta_i \tag{4.24}$$

where

$$\mathbf{m}_i = \mathbf{t}_i(1:KN) - \mathbf{s}_{i-1} = \left[\mathbf{s}_{i,L}^T, \mathbf{s}_i^T(0), \cdots, \mathbf{s}_i^T(N-L-1)\right]^T - \mathbf{s}_{i-1} \tag{4.25}$$

$$\mathcal{G} = \underbrace{\begin{bmatrix} \mathbf{H}(0) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \mathbf{H}(L) & \ddots & \mathbf{H}(0) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{H}(L) \end{bmatrix}}_{N}{}_{(J(N+L))\times NK} \tag{4.26}$$

$$\eta_i = \left(\mathbf{v}_i - \begin{bmatrix} \mathbf{v}_{(i-1)2} \\ \mathbf{v}_{i1} \end{bmatrix}\right) \tag{4.27}$$

The new noise vector η_i given in (4.27) is colored and has the covariance matrix

$$\mathbf{R}_\eta = E\{\eta_i \eta_i^H\} = \sigma_n^2 \mathbf{R}_w \tag{4.28}$$

where $$\mathbf{R}_w = \begin{bmatrix} 2\mathbf{I}_{JL\times JL} & 0 & -\mathbf{I}_{JL\times JL} \\ 0 & 2\mathbf{I}_{J(N-L)\times J(N-L)} & 0 \\ -\mathbf{I}_{JL\times JL} & 0 & 2\mathbf{I}_{JL\times JL} \end{bmatrix} \tag{4.29}$$

After remodulating, the channel matrix $\mathcal{G}$ is same as the channel filtering matrix in ZP based blind channel estimation for MIMO OFDM systems [27].

The following theorem states the sufficient condition for $\mathcal{G}$ to be full rank.

Theorem 4.2:[27] For $J \geq K$,if there exits a $l \in [0, L]$ such that $\mathbf{H}(l)$ is of full column rank, then $\mathcal{G}$ is of full column rank.

The full column rank property of $\mathbf{H}(l)$ is almost surely guaranteed because signal propagation from each of the K transmitters is most likely independent. In the following we assume that this condition is satisfied.

The standard subspace methods require the noise to be white i.e. covariance of the noise vector to be a scaled identity matrix. But after remodulation, noise become colored whose covariance matrix is given in (4.28). In order to apply subspace method to remodulation case, we need to whiten the noise.

Multiplying (4.24) with $\mathbf{R}_w^{-1/2}$, we obtain

$$\mathbf{y}_i = \underbrace{\mathbf{R}_w^{-1/2}\mathcal{G}}_{\mathcal{A}}\mathbf{m}_i + \tilde{\mathbf{v}}_i$$

$$= \mathcal{A}\mathbf{m}_i + \tilde{\mathbf{v}}_i \tag{4.30}$$

where $\tilde{\mathbf{v}}_i = \mathbf{R}_w^{-1/2}\eta_i$ is the $J(N+L)\times 1$ white noise vector whose entries have variance σ_n^2. In addition, since $\mathbf{R}_w$ is a non-singular matrix, the new channel matrix $\mathcal{A}$ is of full column rank if $J \geq K$.

Obtain covariance matrix of $\mathbf{y}_i$ as

$$\mathbf{R}_{\mathbf{yy}} = \mathbf{E}\{\mathbf{y}_i\mathbf{y}_i^H\}$$

$$= \mathcal{A}\mathbf{R}_{\mathbf{m}}\mathcal{A}^H + \sigma_n^2\mathbf{I}_{J(N+L)} \tag{4.31}$$

where $\mathbf{R}_{\mathbf{m}} = \mathbf{E}\{\mathbf{m}_i\mathbf{m}_i^H\}$ is the source covariance matrix, which should be full rank if no two elements in $\mathbf{m}_i$ are fully correlated. This requirement is normally satisfied, since the two consecutive blocks $\mathbf{s}_i$ and $\mathbf{s}_{i-1}$ in general are not fully correlated.

The covariance matrix $\mathbf{R}_{\mathbf{yy}}$ can be eigen value decomposed as

$$\mathbf{R}_{\mathbf{yy}} = \mathbf{U}_s\Sigma_s\mathbf{U}_s^H + \sigma_n^2\mathbf{U}_n\mathbf{U}_n^H \tag{4.32}$$

where $J(N+L)\times KN$ matrix $\mathbf{U}_s$ spans the signal-subspace of $\mathbf{R}_{\mathbf{yy}}$, and $J(N+L)\times(J(N+L)-KN)$ matrix $\mathbf{U}_n$ spans the noise-subspace of $\mathbf{R}_{\mathbf{yy}}$ and Σ_s is the $KN\times KN$ diagonal matrix whose diagonal elements have eigenvalues greater than σ_n^2.

In the standard subspace method, the matrix $\mathbf{U}_n$ is orthogonal to every column of $\mathcal{A}$.

This can be equivalently expressed as

$$\mathbf{U}_n^H\mathcal{A} = \mathbf{U}_n^H\mathbf{R}_w^{-1/2}\mathcal{G} = 0 \tag{4.33}$$

Alternatively we can write (4.20) as

$$\mathcal{G}^H \mathbf{R}_w^{-H/2}\mathbf{U}_n = 0 \tag{4.34}$$

Let β_i is the ith column of $\mathbf{R}_w^{-H/2}\mathbf{U}_n$, where $i=1,\ldots,\ \big(J(N+L)-KN\big)$.

Condition (4.34) is rewritten as

$$\mathcal{G}^H \beta_i = 0 \quad i=1,\cdots,\big(J(N+L)-KN\big) \tag{4.35}$$

By exploiting the structure of $\mathcal{G}$, (4.35) can be expressed in terms of $\mathbf{H}$.

By dividing the vector β_i into blocks of $J\times 1$ vectors as

$$\beta_i = \left(\beta_i^T(N+L-1), \beta_i^T(N+L-2), \cdots, \beta_i^T(0)\right)^T \tag{4.36}$$

where $\beta_i(m)$ $(m=0,1,\cdots,N+L-1)$ are $J\times 1$ vectors.

Using (4.26) in (4.35), we can get

$$\sum_{l=0}^{L}\mathbf{H}^H(l)\beta_i(n+L-l)=0,\ n=0,1,\cdots,N-1 \tag{4.37}$$

or equivalently

$$\sum_{l=0}^{L}\beta_i^H(n+L-l)\mathbf{H}(l)=0 \tag{4.38}$$

Define matrix $\mathbf{G}_i$ as

$$\mathbf{G}_i = \begin{bmatrix} \beta_i^H(L) & \beta_i^H(L-1) & \cdots & \beta_i^H(0) \\ \beta_i^H(L+1) & \beta_i^H(L) & \cdots & \beta_i^H(1) \\ \vdots & \vdots & & \vdots \\ \beta_i^H(N+L-1) & \beta_i^H(N+L-2) & \cdots & \beta_i^H(N-1) \end{bmatrix} \tag{4.39}$$

Then, using (4.39) and (4.20), (4.38) can be written as

$$\mathbf{G}_i\mathbf{H}=0, \quad i=1,\ldots,\ \big(J(N+L)-KN\big) \tag{4.40}$$

The channel matrix $\mathbf{H}$ can be estimated from

$$\mathbf{G}\mathbf{H}=0 \tag{4.41}$$

where $\mathbf{G} = \begin{bmatrix} \mathbf{G}_0 \\ \mathbf{G}_1 \\ \vdots \\ \mathbf{G}_{(J(N+L)-KN)} \end{bmatrix}$

The estimate of **H**, denoted as $\hat{\mathbf{H}}$, is a basis matrix of the orthogonal complement space of **G**. Therefore, $\hat{\mathbf{H}}$ can be obtained from right singular vectors of **G** and is away from the true **H** by an unknown matrix **B**, namely

$$\hat{\mathbf{H}} = \mathbf{HB} \tag{4.42}$$

The following theorem states that **H** is uniquely determined by span($\mathcal{G}$) subjected to $K \times K$ matrix ambiguity.

Theorem 4.3:[27] Let the matrix $\hat{\mathcal{G}}$ be constructed from $\hat{\mathbf{H}}$ as in (4.26).Let $\mathbf{H}(0)$ and $\hat{\mathbf{H}}(0)$ *be of full column rank. If span($\hat{\mathcal{G}}$) = span($\mathcal{G}$), then there exists $K \times K$ invertible matrix **B** such that*

$$\hat{\mathbf{H}}(l) = \mathbf{H}(l)\mathbf{B}, \quad l = 0,1,\cdots,L \tag{4.43}$$

From theorem4.3, we see that an order overestimation on each $L_{j,k}$ does not effect the channel identifiability of **H** because the estimate $\hat{\mathbf{H}}(l) = \mathbf{H}(l)\mathbf{B} = \mathbf{0}$ for $l = L_{j,k}+1,\cdots,L$.

4.2.3 Pilot based method to determine the ambiguity matrix (B):

Using (4.43), the relation between estimated channel matrix $\hat{\mathcal{G}}$ and actual channel matrix $\mathcal{G}$ is given by

$$\mathcal{G} = \hat{\mathcal{G}}\left(\mathbf{I}_N \otimes \mathbf{B}^{-1}\right) \tag{4.44}$$

where $\otimes$ represents Kronecker product

Substitute (4.44) in (4.24), we get

$$\mathbf{z}_i = \hat{\mathcal{G}}\left(\mathbf{I}_N \otimes \mathbf{B}^{-1}\right)\mathbf{m}_i + \eta_i \tag{4.45}$$

Since $\hat{\mathcal{G}}$ is of full column rank, we can define its pseudo inverse, denoted by $\hat{\mathcal{G}}^{\dagger}$.

Define $\breve{\mathbf{z}}_i = \hat{\mathcal{G}}^{\dagger}\mathbf{z}_i$ and $\breve{\eta}_i = \hat{\mathcal{G}}^{\dagger}\eta_i$. Then

$$\breve{\mathbf{z}}_i = \left(\mathbf{I}_N \otimes \mathbf{B}^{-1}\right)\mathbf{m}_i + \breve{\eta}_i \tag{4.46}$$

By dividing the vectors $\breve{\mathbf{z}}_i$, $\mathbf{m}_i$ and $\breve{\eta}_i$ into blocks of length *K*, we get

$$\breve{\mathbf{z}}_i = \left[\breve{\mathbf{z}}_i^T(0), \breve{\mathbf{z}}_i^T(1), \cdots, \breve{\mathbf{z}}_i^T(N-1)\right]^T \tag{4.47}$$

$$\mathbf{m}_i = \left[\mathbf{m}_i^T(0), \mathbf{m}_i^T(1), \cdots, \mathbf{m}_i^T(N-1)\right]^T \tag{4.48}$$

$$\breve{\eta}_i = \left[\breve{\eta}_i^T(0), \breve{\eta}_i^T(1), \cdots, \breve{\eta}_i^T(N-1)\right]^T \tag{4.49}$$

The relationship (4.46) is now written as

$$\breve{\mathbf{z}}_i(n) = \mathbf{B}^{-1}\mathbf{m}_i(n) + \breve{\eta}_i(n), \quad n = 0, \cdots, N-1 \tag{4.50}$$

Let
$$\breve{\mathbf{Z}}_i = \left[\breve{\mathbf{z}}_i(0), \breve{\mathbf{z}}_i(1), \cdots, \breve{\mathbf{z}}_i(N-1)\right] \tag{4.51}$$

$$\breve{\mathbf{M}} = \left[\mathbf{m}_i(0), \mathbf{m}_i(1), \cdots, \mathbf{m}_i(N-1)\right] \tag{4.52}$$

$$\breve{\Xi}_i = \left[\breve{\eta}_i(0), \breve{\eta}_i(1), \cdots, \breve{\eta}_i(N-1)\right] \tag{4.53}$$

Then

$$\breve{\mathbf{Z}}_i = \mathbf{B}^{-1}\breve{\mathbf{M}} + \breve{\Xi}_i \tag{4.54}$$

If a pilot OFDM block is sent and $N \geq K$, $\breve{\mathbf{M}}$ can be assumed to be full row rank. Then, we can obtain $\mathbf{B}^{-1}$ by

$$\hat{\mathbf{B}}^{-1} \approx \breve{\mathbf{Z}}_i \breve{\mathbf{M}}^H \left(\breve{\mathbf{M}}\breve{\mathbf{M}}^H\right)^{-1} \tag{4.55}$$

Substituting phase ambiguity matrix $\hat{\mathbf{B}}^{-1}$ in (4.43), we get the final estimate of the channel given by

$$\mathbf{H}_{final} = \hat{\mathbf{H}}\hat{\mathbf{B}}^{-1} \tag{4.56}$$

4.3 Subspace based blind channel estimation for zero padded (ZP) MIMO OFDM systems:

A ZP based MIMO-OFDM system with K transmitters and J receivers is shown in Fig 4.2. Output of IDFT is zero padded with L zeros and then transmitted. Then the received ith block at antenna j is

$$r_i^j(n) = \sum_{k=1}^{K}\sum_{l=0}^{L} h^{(j,k)}(l) s_i^{(k)}(n-l) + v_i^j(n), \quad n = 0, 1, \cdots, L+N-1 \tag{4.57}$$

By defining

$$\mathbf{r}_i(n) = \left(r_i^{(1)}(n), r_i^{(2)}(n), \cdots, r_i^{(J)}(n)\right)^T \tag{4.58}$$

$$\mathbf{h}^{(k)}(l) = \left(h^{(1,k)}(l), h^{(2,k)}(l), \cdots, h^{(J,k)}(l)\right)^T \tag{4.59}$$

$$\mathbf{v}_i(n) = \left(v_i^{(1)}(n), v_i^{(2)}(n), \cdots, v_i^{(J)}(n)\right)^T \tag{4.60}$$

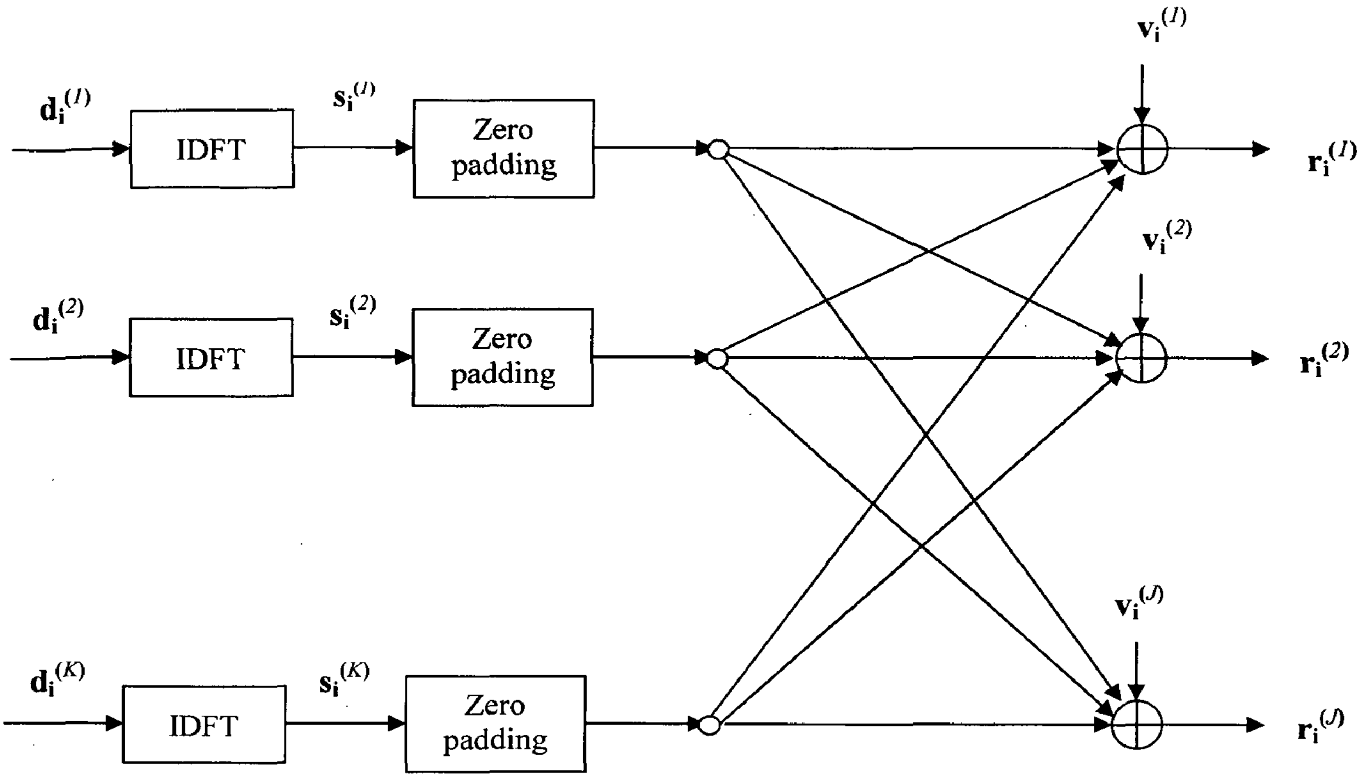


Fig 4.2: ZP based MIMO-OFDM System model with K transmitting and J receiving antennas.

We can express (4.57) into vector form as

$$\mathbf{r}_i(n)=\sum_{k=1}^{K}\sum_{l=0}^{L}\mathbf{h}^{(k)}(l)s_i^{(k)}(n-l)+\mathbf{v}_i(n) \qquad n=0,1,\cdots,L+N-1 \tag{4.61}$$

By changing the order of the summation in (4.61) and defining

$$\mathbf{H}(l)=\left(\mathbf{h}^{(1)}(l),\mathbf{h}^{(2)}(l),\cdots,\mathbf{h}^{(K)}(l)\right) \tag{4.62}$$

$$\mathbf{s}_i(n-l)=\begin{bmatrix} s_i^{(1)}(n-l) \\ s_i^{(2)}(n-l) \\ \vdots \\ s_i^{(K)}(n-l) \end{bmatrix} \tag{4.63}$$

we can express (4.61) into

$$\mathbf{r}_i(n)=\sum_{l=0}^{L}\mathbf{H}(l)\mathbf{s}_i(n-l)+\mathbf{v}_i(n) \quad n=0,1,\cdots,L+N-1 \tag{4.64}$$

Now let

$$\mathbf{s}_i = \begin{bmatrix} \mathbf{s}_i(0) \\ \mathbf{s}_i(1) \\ \vdots \\ \mathbf{s}_i(N-1) \end{bmatrix}, \ \mathbf{r}_i = \begin{bmatrix} \mathbf{r}_i(0) \\ \mathbf{r}_i(1) \\ \vdots \\ \mathbf{r}_i(N+L-1) \end{bmatrix}, \mathbf{v}_i = \begin{bmatrix} \mathbf{v}_i(0) \\ \mathbf{v}_i(1) \\ \vdots \\ \mathbf{v}_i(N+L-1) \end{bmatrix} \text{ and}$$

$$\mathcal{G} = \underbrace{\begin{bmatrix} \mathbf{H}(0) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \mathbf{H}(L) & \ddots & \mathbf{H}(0) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{H}(L) \end{bmatrix}}_{N}{}_{(J(N+L))\times NK}$$

where $\mathcal{G}$ is a $J(N+L)\times KN$ block lower triangular Toeplitz matrix and it is same as (4.26) for CP based MIMO OFDM system.

Then, (4.64) is turned into

$$\mathbf{r}_i = \mathcal{G}\mathbf{s}_i + \mathbf{v}_i, \quad i = 0,1,2,... \tag{4.65}$$

We can observe that (4.65) is similar to (4.24). All conditions for channel identifiability i.e., theorem 4.1 and 4.2 of blind channel identification method for CP MIMO OFDM system, can be directly applied to blind channel identification method for ZP MIMO OFDM system. Using (4.34) to (4.36), we can get the final estimate of $\mathbf{H}$.

4.4 Simulation results for blind channel identification of MIMO-OFDM systems:

For the simulation of blind channel identification of CP based MIMO-OFDM systems in MATLAB environment, we use the following parameters:

- No of transmitters: $K = 2$
- No of receivers: $J = 2$
- Modulation scheme : QPSK
- No of subcarriers: $N = 32$
- No of Monte Carlo simulation runs: $N_m = 100$
- Channel: Multipath fading coefficients of channel order $L = 8$ are generated by assuming exponential power delay profile $exp\left(-\frac{\tau}{\tau_{rms}}\right)$, τ stands for path delay and $\tau_{rms} = 0.6T$ is rms delay. (T is symbol period).

- Data window length: N_b=300 blocks.

Steps carried out in simulation of VC based blind channel identification of OFDM system:

1. Generate channel coefficients of order $L=8$ for all channels between transmitter side and receiver side.
2. Obtain randomly generated source data for each transmitter and get the received signal vector using (4.21).
3. Rearrange the received vector using (4.23) and determine remodulation vector at the receiver using (4.24).
4. After getting N_b remodulated received blocks and determine $\mathbf{R}_{yy}$ using time average over N_b blocks.
5. Get the noise subspace from SVD of $\mathbf{R}_{yy}$ as in (4.32).
6. Using noise subspace, determine $\mathbf{G}$ using (4.41).
7. Estimate of channel $\hat{\mathbf{H}}$ is obtained by finding right singular vector of $\mathbf{G}$.
8. Determine phase ambiguity matrix $\mathbf{B}^{-1}$ using (4.45).
9. Compensate the phase ambiguity present in channel estimate, by post multiplying $\hat{\mathbf{H}}$ with $\mathbf{B}^{-1}$.

All steps from 1 to 9 are repeated for each independent Monte Carlo run. To obtain the performance measure of the channel estimate, we use Normalized Root Mean Square Error (NRMSE) defined as

$$NRMSE = \sqrt{\frac{1}{N_m KJ(L+1)} \sum_{m=1}^{N_m} \sum_{k=1}^{K} \frac{\left\| \mathbf{H}^{(m)}(:,k) - \hat{\mathbf{H}}^{(m)}(:,k) \right\|^2}{\left\| \mathbf{H}^{(m)}(:,k) \right\|^2}} \tag{4.66}$$

where $\mathbf{H}^{(m)}(:,k)$ and $\hat{\mathbf{H}}^{(m)}(:,k)$ is k th column of true channel and estimated channel at the m th simulation run. For comparison purposes, we also simulate Zero Padding (ZP) based blind channel identification method for MIMO-OFDM systems. Here we assume zero padding is of length $L=8$. All remaining parameters are same as in CP method.

Fig 4.3 plots the average value of estimated channel coefficients for channel between 1st transmitter and two receivers ($K = 1, J = 2$) which are obtained by averaging over 100 independent realizations. We have assumed data window size of length N_b=300 blocks and SNR=5dB. For comparison, we have also plotted original channel coefficients. It may be noted that there is a close similarity between the true channel coefficients and averaged channel coefficients identified using CP based blind channel estimation method.

Fig 4.4 shows the NRMSE at different SNR values for both CP based and ZP based blind channel estimation for MIMO OFDM systems. As SNR is varied from 0 to 35dB, NRMSE value decreases from 0.9 to 0.0015. We may also observe that both CP based method and ZP based method perform similarly with respect to SNR variation. The advantage of CP based blind channel estimation method is that it has compatibility with standard OFDM systems such as European digital audio/video broadcasting (DAB,DVB), HIPERLAN, IEEE 802.11a WLAN. On the other hand ZP based OFDM systems has very limited applications.

As shown in Fig 4.5, we compare the variation of NRMSE at different data window lengths (N_b) at SNR=15dB, for both CP based and ZP based blind channel estimation for MIMO OFDM systems. As N_b is varied from 100 to 1000 blocks, NRMSE decreases from 0.04 to 0.005. We may observe that both CP based method and ZP based method perform similarly with respect to data window length (N_b) variation. We may also observe that beyond a window size of 600 blocks, there is no effect on the performance of both the algorithms.

Fig 4.6 plots the variation of NRMSE at different SNR values for CP based blind channel identification of MIMO OFDM systems with different estimates of channel order. We may observe that NRMSE decreases with increase of SNR for all the cases. We may also observe that there is no significant difference in the performance of CP based blind identification of MIMO OFDM channel with exact channel order estimation and order overestimation by 1 or 2. Hence, CP based method is insensitive to order overestimation.

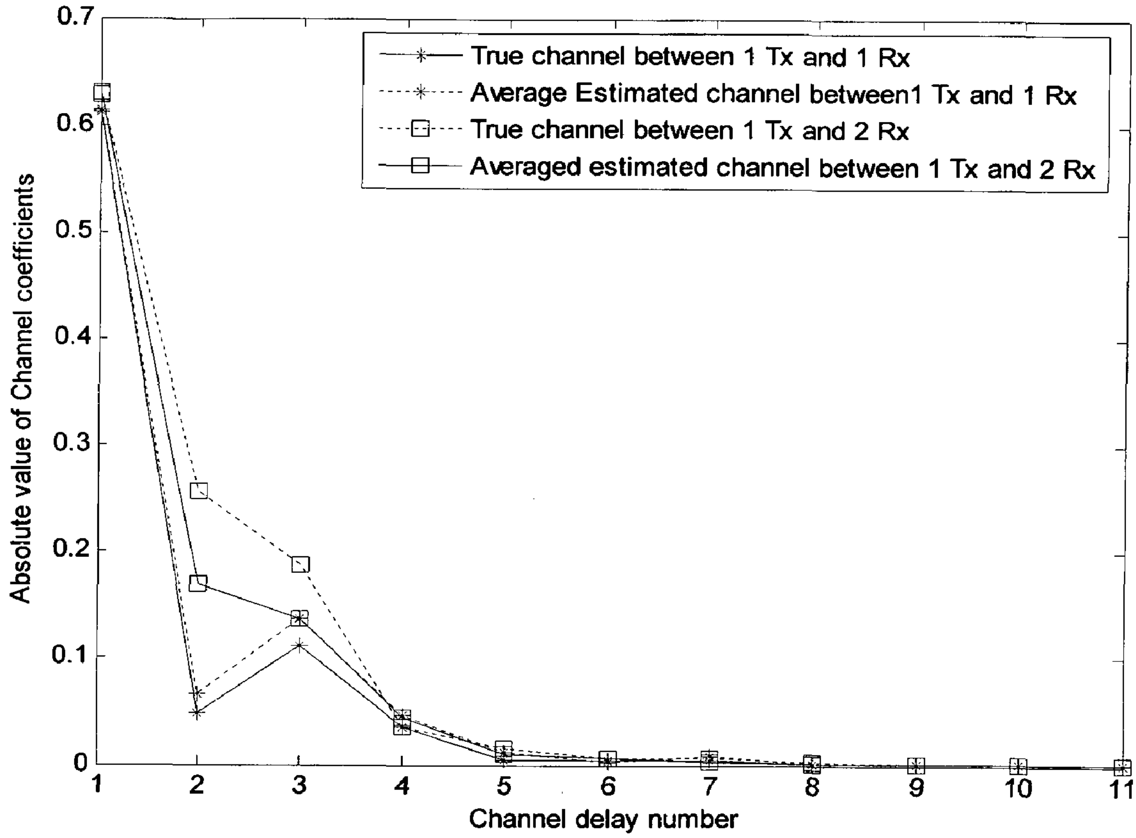


Fig 4.3: Comparison between actual and CP based blind estimate of channels between one transmitter and two receivers, averaged over 100 independent runs at SNR=5dB.

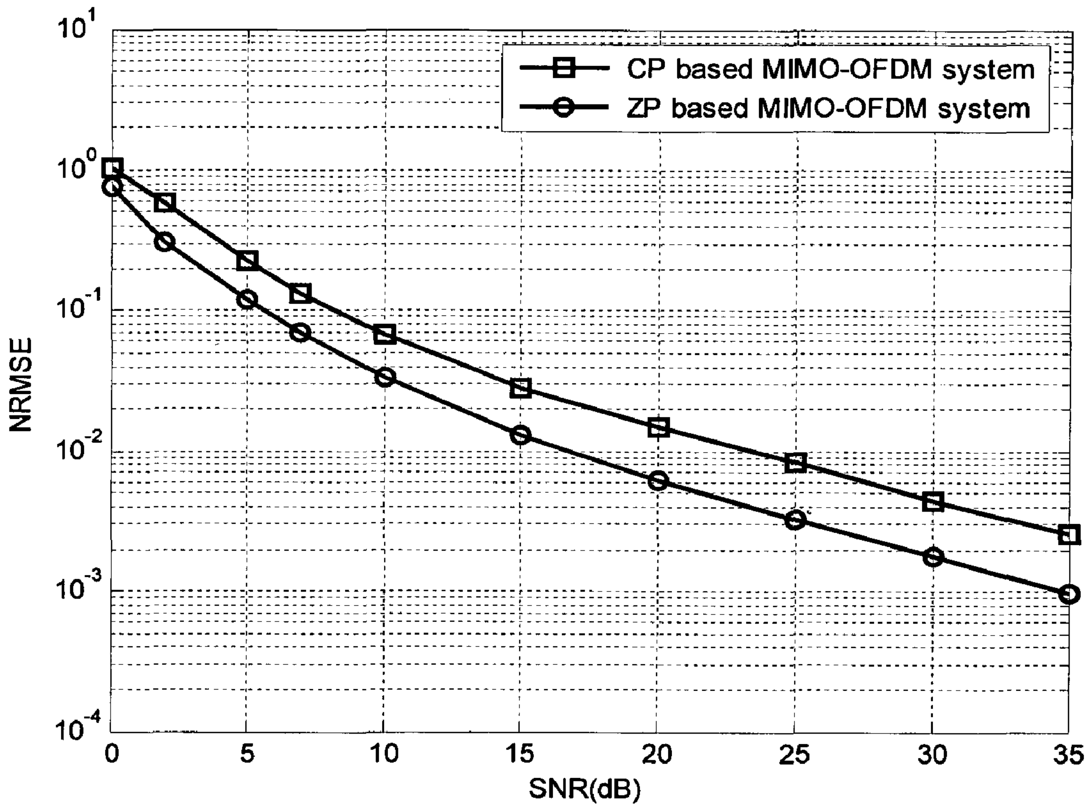


Fig 4.4: Variation of NRMSE with respect to SNR for CP based and ZP based blind channel identification of MIMO OFDM systems

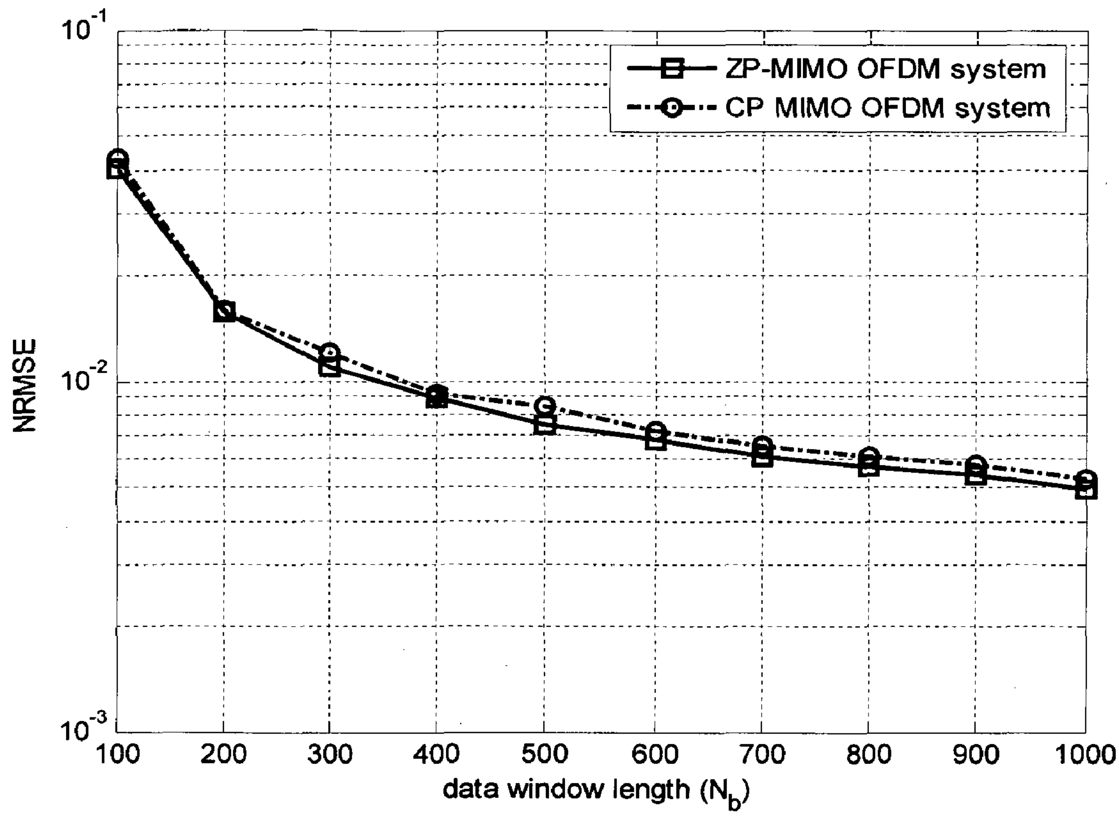


Fig 4.5: Variation of NRMSE with respect to data window length (N_b) for CP based and ZP based blind channel identification of MIMO OFDM systems. (SNR=15dB)

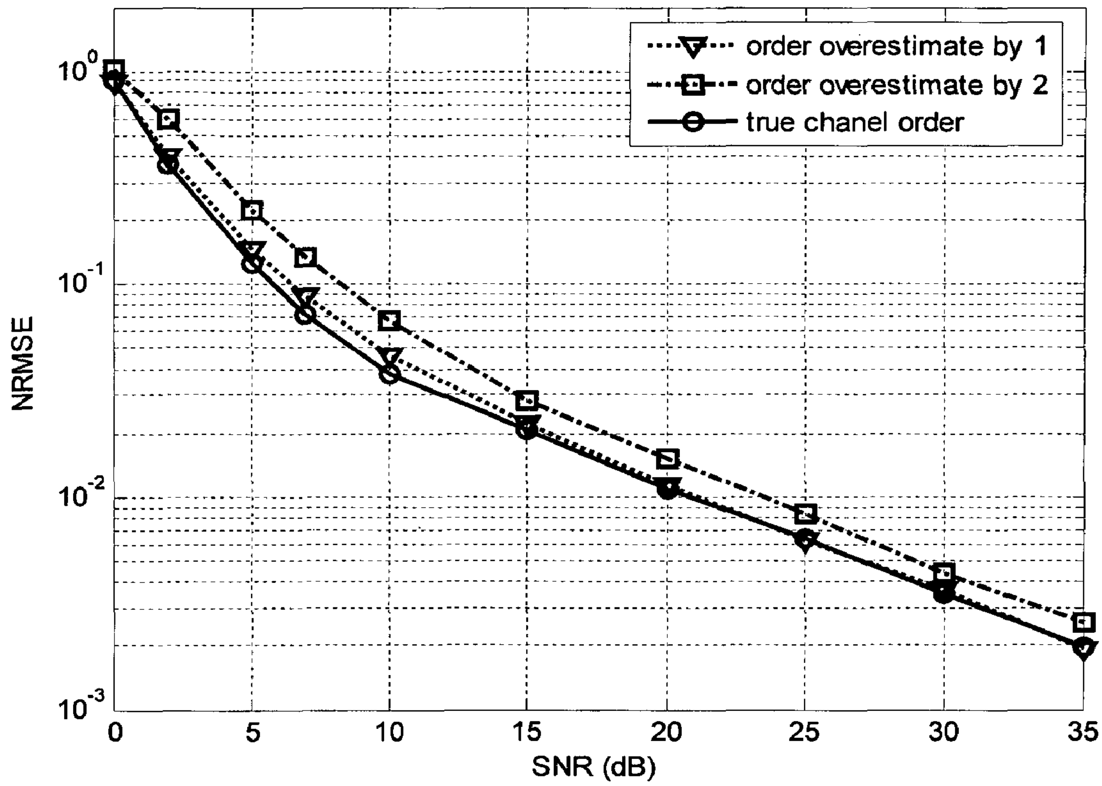


Fig 4.6: NRMSE versus SNR for different estimates of channel order (L) for CP based blind channel identification of MIMO OFDM systems.

Chapter 5

Conclusions

Blind identification techniques are the promising candidates for spectral efficient channel estimation as compared to pilot based channel estimation. Among various blind channel estimation techniques (such as methods based on higher order statistics [10] and second order statistics [12],) noise subspace methods are of significant interest since it involves only second order statistics and uses either Eigenvalue decomposition or Singular value decomposition. This dissertation work is aimed at the performance study of blind channel estimation techniques for FIR channels using Single Input Multiple Output model, OFDM systems and MIMO-OFDM systems.

The simulation performance in terms of NRMSE is calculated at different values of SNR, and data window lengths for the above blind channel estimation methods. The conclusions drawn based on simulation results are as follows:

Blind identification of FIR channels using Single Input Multiple Output model:

TXK algorithm:

By exploiting the non-stationarity of received signal, the TXK algorithm for blind identification of FIR channels using SIMO model is able to identify the channel with only second order statistics that leads to accurate channel estimation with a smaller sample size than the methods using higher order statistics. As simulation results show, the TXK algorithm performs well at high SNR. For the TXK algorithm to be effective at low SNR, a larger number of received blocks are necessary, which limits its effectiveness for rapidly time varying channels.

Subspace based algorithm:

Subspace based algorithm uses orthogonality between signal and noise subspaces to build a quadratic form whose minimization gives the desired channel estimate. Simulation results show that the NRMSE value for subspace method is much smaller than that for TXK method at a given SNR and data window length (N_b). Also subspace methods yield reasonable estimates of the channel coefficients even for shorter data window length which results in the faster convergence of subspace method than TXK method. Hence, subspace based blind channel estimation method is a good candidate for practical digital communication situations. Finally, subspace methods are computationally more efficient as they are able to get consistent estimate of the channel coefficients even if we use less number of noise vectors.

Blind channel estimation for OFDM systems:

Oversampling method for blind channel identification of OFDM systems without CP:

In the absence of CP, blind channel estimators for OFDM based on CP ([15],[16],[17]) will no longer work. However, a source of redundancy introduced by oversampling at the receiver makes it possible for blind channel identification of OFDM systems without CP. Oversampling based blind chnnel estimator achieves similar performance comparable to the Cai and Akanshu method for CP based OFDM systems [17]. This algorithm is attractive for its potential to increase the system's channel utilization due to the elimination of CP.

Virtual carrier(VC) based blind channel estimation for OFDM systems:

Virtual carriers, which are intended to aid in shaping the transmit spectrum, offer an extra source of redundant information other than the CP which can be used to assist in blind channel estimation for OFDM systems. This method distinguish itself from other blind channel estimators [17,20] by making use of the redundant information embedded in both VCs and CP(if any). The advantage of this method is that it is applicable to the conventional OFDM systems with insufficient CP as well as OFDM systems without CP. For the conventional CP based OFDM systems, the exploitation of VC's bring additional performance gain as evidenced by the simulation results. The reduction/elimination of the CP thereby provides the OFDM systems the potential to achieve higher channel utilization. Also the simulation results show that the VC based method is insensitive to channel order overestimation.

Subspace based semi blind channel estimation for OFDM system:

All blind channel estimation methods estimate the channel up to a scalar phase ambiguity. To avoid phase ambiguity pilot carriers are used in semi blind channel estimation. Also to increase convergence of blind channel estimation methods, an initial training sequence is used to estimate the received correlation matrix. Simulation results show that semi blind channel estimation for OFDM systems has faster convergence than the blind channel estimation for OFDM systems.

Blind channel estimation for MIMO OFDM systems:

MIMO-OFDM systems can achieve high data rates over broadband wireless channels. For the purpose of subspace based blind channel estimation for MIMO OFDM, either cyclic prefix (CP) or zero padding (ZP) was exploited. The main constraint in blind channel estimation for MIMO ODFM systems is that algorithm is applicable to scenario where number transmitting antennas is less than or equal to number of receiving antennas. Numerical simulations show that both CP and ZP padded based methods performs similarly with respect to varying SNR and data window lengths. Also they are robust to channel order overestimation. CP based blind channel estimation techniques for MIMO OFDM systems are preferable to ZP based techniques, as they are compatible with many existing standards and the upcoming 4G wireless communication standards.

Future work:

Subspace based methods provide an effective technique for blind estimation of channel. However to remove phase ambiguity and to increase convergence speed we need pilot information which is a part of all standards. A natural idea is to use pilot information in blind techniques to construct a semi blind channel estimator to achieve faster convergence. Some preliminary work on subspace based semi blind channel estimation have been reported in the literature [16,18]. However the above subspace based semi blind channel is not effective in the context of time varying channels as they requires periodic transmission of pilot symbols. Expectation and Maximization (EM) based semi blind channel estimation is very effective for time varying channels as they use channel estimate for previous frame as the initial estimate for the next frame and avoids the need of periodic retransmission. Comparison of EM based techniques and subspace based techniques for channel estimation is a topic of significant interest.

Blind channel estimation for MIMO OFDM systems for a general situation i.e. no restriction on number of transmitting and receiving antennas, through the use of receiver oversampling or VCs is another topic for investigation.

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