ICI CANCELLATION IN OFDM SYSTEMS IN PRESENCE OF PHASE NOISE AND TIME VARYING CHANNELS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

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ELECTRONICS AND COMMUNICATION ENGINEERING
(With Specialization in Communication Systems)

By

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report entitled, "**ICI CANCELLATION IN OFDM SYSTEMS IN PRESENCE OF PHASE NOISE AND TIME VARYING CHANNELS**" towards the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems**, submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from June 2007 to June 2008, under the guidance of **Dr. S. K. VARMA, Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee.**

I have not submitted the matter embodied in this dissertation for the award of any other Degree or Diploma.

Date: 27/6/08

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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ABSTRACT

The ever-increasing demand for very high-rate wireless data transmission calls for technologies which make use of the available electromagnetic resource in most intelligent way. Key objectives are spectrum efficiency, robustness against multipath propagation, range, power consumption, and implementation complexity. These objectives are often conflicting, so techniques and implementations are sought which offer the best possible tradeoff between them. Two of the most effective means of closing the gap between the achieved performance and channel capacity are advanced channel coding and Orthogonal Frequency Division Multiplexing (OFDM). In conjunction with advanced forward error correction coding, advanced OFDM is the modulation of choice when it comes to improving robustness against multipath fading at reasonable cost of implementation. Therefore, the OFDM has been adopted as a standard in terrestrial digital broadcasting and wireless local area networks. OFDM still faces challenges like Inter-carrier Interference (ICI), and Peak to Average Power Ratio (PAPR).

In this thesis work, causes of ICI, different ICI cancellation methods are discussed. Performance evaluation of ICI self cancellation methods is done. In case of multipath channels low complexity ICI matrix estimation and equalization technique is implemented, and its performance evaluation is carried out.

CONTENTS

Chapter 1

Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is one of the multi carrier modulation techniques that transmit signals through multiple carriers. These subcarriers have different frequencies and they are orthogonal to each other. Orthogonal frequency division multiplexing techniques have been applied in both wired and wireless communications, such as the Asymmetric Digital Subscriber Line (ADSL) and the IEEE 802.11 standard. Due to the recent technological developments, wireless communications is gradually becoming an affordable alternative to wire line communications.

The wireless communication technology developed has been divided into generations based on the technology adapted, data rates offered and the user mobility. The first generation of mobile cellular telecommunications systems appeared in early 1980s. The first generation was not the beginning of mobile communication, as there were several mobile radio networks in existence by then, but they were not cellular systems. The capacity of these early networks was much lower than that of cellular networks.

In mobile cellular networks the coverage area is divided into small cells, which allows the same frequencies to be used in different cells several times without disruptive interference. This increases the system capacity. The first generation used analog transmission techniques for traffic, which was almost entirely voice. The most successful standards were Nordic Mobile Telephone (NMT), Total Access Communication Systems (TACS), and Advanced Mobile Phone Service (AMPS). These protocols were developed during the 70's and 80's. These protocols supported a data transmission rate is between 9.6kbps and 14.4kbps.

The technologies developed during the 90's to 2000 come under the second-generation (2G) mobile services. The second-generation (2G) mobile cellular systems use digital transmission for traffic. The maximum data rate that can be achieved using the 2G protocols is 115kbps.

The main advantage of using 2G technologies over the 1G was, increase in the performance due to usage of same channel by several users(either by code or time division). By this time the cell phones were used for both voice and data communication. There are four main standards for 2G systems: Global System for Mobile (GSM) communication, Digital AMPS (D-AMPS), Code Division Multiple Access (CDMA) IS-95 [1], and Personal Digital Cellular (PDC).

The emergence of mobile data accessing devices like Personal Digital Assistants (PDA's) and Internet has shifted the focus towards the data communications, which requires high data transmission rates. These have led to the developments of more advanced protocols between 2000 and 2003 and termed as 2.5G protocols. The 2.5G system includes the following technologies: High-Speed Circuit-Switched data (HSCSD), General Packet Radio Services (GPRS), and Enhanced Data Rates for Global Evolution (EDGE). The maximum data rates that can be achieved using 2.5G protocols is 144kbps, but this is not enough for enhanced multimedia and high streaming videos transmissions. So Universal Mobile Telecommunications System (UMTS), Wideband Code Division Multiple Access (WCDMA) and CDMA2000 protocols that also use the Digital Packet Switching are developed to increase the data transmission rate up to 2Mbps. These protocols were developed during 2003 to 2004 and are termed as third generation (3G) protocols.

As the demand for higher data transmission rate and worldwide roaming in cellular devices is increasing, the development of next generation (4G) wireless systems using digital broadband is underway. Therefore, enhancing system capacity as well as achieving a higher bit rate transmission is an important requirement for the 4G system. The main task is to investigate and develop a new broadband air interface which can deal with high data rates of the order of 100 Mbit/s, high mobility and high capacity. Since the available frequency spectrum is limited, high spectral efficiency is the major task of 4G mobile radio systems. Another important target of the new 4G air interface is the ability to provide efficient support for applications requiring simultaneous transmission of several bits of streams with possibly different Quality of Service (QoS) targets.

These developments must cope up with several performance limiting challenges that include channel fading, multi-user interference, limitations of size/power especially at mobile units. Among these challenges, channel fading [3] degrades the performance of wireless transmissions significantly, and becomes a bottle-neck for increasing data rates. As each path has a different attenuation, time delay & phase shift, the signals from different paths add constructively or destructively, resulting in signal strength fluctuations. This phenomenon is known as multi-path fading. Channel fading causes performance degradation and renders reliable high data rate transmissions; a challenging problem for 4G wireless communications. To combat these situations, Orthogonal Frequency Division Multiplex (OFDM) is considered as a promising solution.

As wireless communication systems are usually interference limited, new technologies should be able to handle the interference successfully. Interference can be from other users, e.g. Co-channel Interference (CCI) and Adjacent Channel Interference (ACI), or it can be due to users own signal (self-interference), e.g. Inter-Symbol Interference (ISI). ISI is one of the major problems for high data rate communications which is treated with equalizers in conventional single-carrier systems. However, for high data rate transmission, complexity of equalizers become very high due to the smaller symbol time and large number of taps needed for equalization. This problem is especially important for channels with large delay spreads.

Multi-carrier modulation [2] is one of the transmission schemes which are less sensitive to time dispersion (frequency selectivity) of the channel. A basic multi-carrier transmitter diagram is shown in Figure 1.1.

In multi-carrier systems, the transmission bandwidth is divided into several narrow sub-channels and data is transmitted parallel in these sub-channels. Data in each sub-channel is modulated at a relatively low rate so that the delay spread of the channel does not cause any degradation as each of the sub-channels will experience a flat response in frequency.

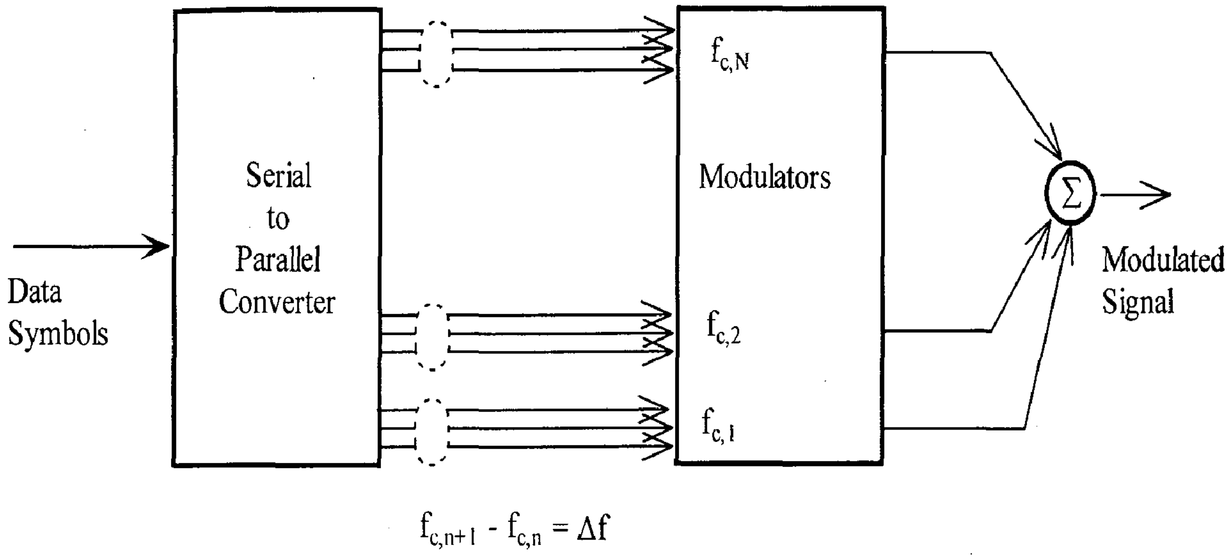


Figure1.1 Basic Multi-Carrier Transmitter

Although, the principles are known since early sixties, multi-carrier modulation techniques, especially Orthogonal Frequency Division Multiplexing (OFDM), gained more attention in the last ten years due to the increased power of digital signal processors.

OFDM is a multi-carrier modulation technique that can overcome many problems that arise with high bit rate communication, the biggest of which is the time dispersion. In OFDM, the carrier frequencies are chosen in such a way that there is no influence of other carriers in the detection of the information in a particular carrier when the orthogonality of the carriers is maintained. The data bearing symbol stream is split into several lower rate streams and these streams are transmitted on different carriers. ISI can be removed by cyclically extending the OFDM symbol. The length of the cyclic extension should be at least as long as the maximum excess delay of the channel.

Although OFDM has proved itself as a powerful modulation technique, it has its own challenges. Sensitivity to frequency offsets caused when a receiver's oscillator does not run at exactly the same frequency of transmitter's oscillator is one of the major problems. This offset perturbs the orthogonality [4] of the sub-carriers, reducing the performance. Another problem is the large Peak-to-average Power Ratio (PAPR) of the OFDM signal, which requires power amplifiers with large linear ranges. Hence, power amplifiers require more back-off, which, in turn, reduces the power efficiency. Some other problems include phase distortion, time-varying channel and time synchronization.

Most standards employing OFDM do not utilize the available resources effectively. Most of the time, systems are designed for the worst case scenarios. The length of the cyclic prefix, for example, is chosen in such a way that it is larger than the maximum expected delay of the channel, which introduces a considerable amount of overhead to the system. However, it can be changed adaptively depending on the channel conditions, instead of setting it according to the worst case scenario, if the maximum excess delay of the channel is known. The information about the frequency selectivity of the channel can also be very useful for improving the performance of the wireless radio receivers through transmitter and receiver adaptation. OFDM symbol duration, subcarrier bandwidth, number of sub-carriers etc. can be changed adaptively, if the frequency selectivity is estimated.

While OFDM solves the ISI problem by using cyclic prefix, it has another self-interference problem: Inter-Carrier Interference (ICI), or crosstalk among different sub-carriers, caused by loss of orthogonality due to frequency instabilities, timing offset or phase noise[5][6]. ISI and ICI are dual of each other occurring in different domains; one in time-domain and the other in frequency-domain. ICI is a major problem in multi-carrier systems and needs to be taken into account when designing systems. Several methods are used to mitigate ICI, like frequency domain equalization, time domain windowing, and ICI self cancellation schemes [7][8][9], Polynomial Cancellation Coding (PCC).

1.1 Statement of the Problem

This work is aimed at finding an effective ICI Cancellation method for OFDM systems. The work is presented as follows:

- Study of OFDM systems, Intercarrier Interference Self-Cancellation Scheme for OFDM systems.

- Performance evaluation of ICI self cancellation schemes for different cases.
- Study of ICI Coefficient Estimation and Equalization technique, which reduces ICI in presence of multipath channels.

1.2 Organization of the Report

Chapter one gives an overview of the evolution of the wireless systems, a brief description about 4G systems. It summarizes the problem statement for this thesis work. Chapter two deals with the characteristics of wireless channels. Chapter 3 describes the basic ideas of OFDM systems. Chapter4 discusses impairments that cause ICI and some important ICI reduction schemes are explained.Chapter5 presents the simulation results. In chapter 6 conclusion of this dissertation is given.

Chapter 2

Propagation Characteristics of Wireless Channels

In an ideal channel, the received signal would be exact replica of the transmitted signal. However in a real channel, the signal is modified during transmission through the channel. The received signal consists of a combination of attenuated, reflected, and refracted, replicas of the transmitted signal. On top of all this, the channel adds noise to the signal and can cause a shift in the carrier frequency if the transmitter or receiver is moving (Doppler Phenomenon). Understanding of these effects on the signal is important because the performance of a radio system is dependent on the radio channel characteristics.

2.1 Attenuation

Attenuation is the drop in the signal power when transmitting from one point to another. It can be caused by the transmission path length, obstructions in the signal path, and multipath effects. Figure 2.1 shows some of the radio propagation effects that cause attenuation. Any object which obstructs the line of sight signal from the transmitter to the receiver can cause attenuation. Shadowing of the signal can occur whenever there is an obstruction between the transmitter and receiver. It is generally caused by buildings and hills, and is the most important environmental attenuation factor.

Shadowing is most severe in heavily built up areas, due to the shadowing from buildings. However, hills can cause a large problem due to the large shadow they produce. Radio signals diffract off the boundaries of obstructions, thus preventing total shadowing of the signals behind hills and buildings. However, the amount of diffraction is dependent on the radio frequency used, with low frequencies diffracting more then high frequency signals. Thus high frequency signals, especially, Ultra High Frequencies (UHF), and microwave signals require line of sight for adequate signal strength. To over come the problem of shadowing, transmitters are usually elevated as high as possible to minimize the number of obstructions.

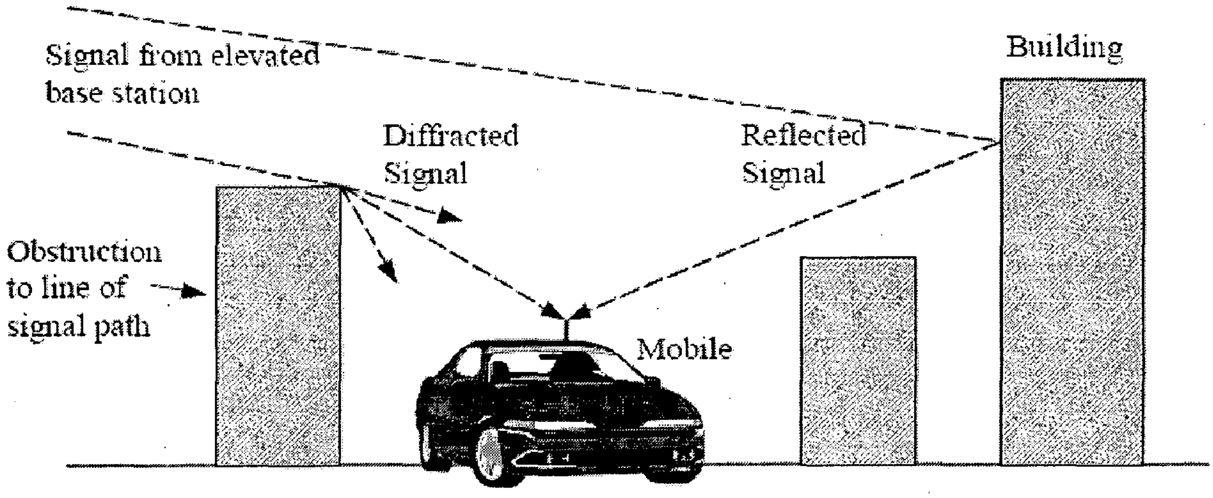


Figure 2.1: Most of Signal Propagation Effects.

2.2 Multipath Effects

The transmission characteristics are not determined by attenuation alone. The loss or attenuation of the signal may also fluctuate with distance and time. When a signal leaves the transmission antenna, it gets reflected, or refracted by the various structures in its path. Signal loss arises from the presence of various obstacles in the channel. The transmission loss fluctuates around a mean or median value.

Multipath fading [3] can be flat or frequency selective. If the mobile radio channel has a constant gain and linear phase response over a band which is greater than the bandwidth of the transmitted signal, then the received signal will undergo flat fading. In flat fading, the multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. However, the strength of the signal changes with time, due to fluctuation in the gain of the channel caused by the multipath. If the channel possesses a constant gain and linear phase response over a bandwidth which is smaller than the bandwidth of the transmitted signal, then the channel creates frequency selective fading.

2.2.1 Rayleigh Fading

In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. It is well known that the envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution. The Rayleigh distribution has a probability density function (pdf) given by

$$P(r) = \begin{cases} \dfrac{r}{\sigma^2}\exp\left(-\dfrac{r^2}{2\sigma^2}\right) & 0 \le r \le \infty \\ 0 & r < 0 \end{cases}$$

where σ is the rms value of the received voltage signal before envelope detection, and σ^2 is the time average power of the received signal before envelope detection. Figure 2.2 shows the Rayleigh distribution signal envelope of one of the paths as a function of time.

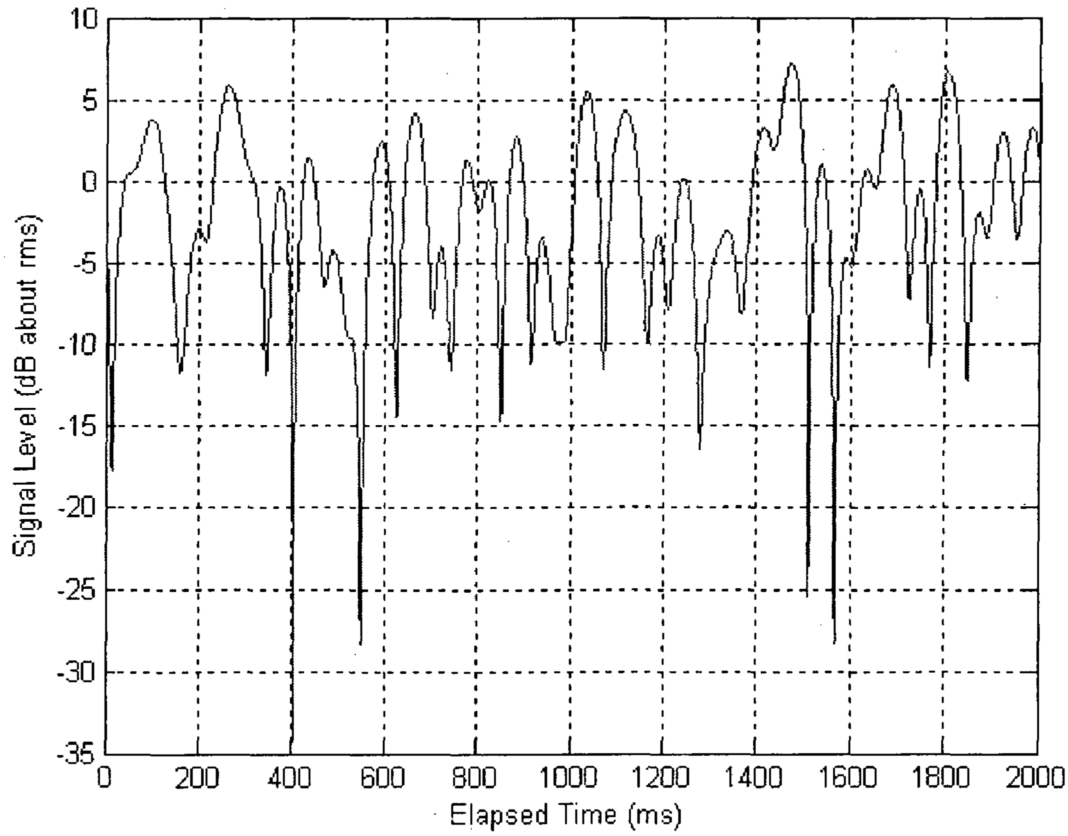


Figure 2.2 Typical Rayleigh Distributed Fading Envelope as a Function of Time

2.2.2 Frequency Selective Fading

If the channel possesses a constant gain and linear phase response over a bandwidth that is smaller than the bandwidth of transmitted signal, then the channel creates frequency selective fading on the received signal. Under such conditions, the channel impulse response has a multipath delay spread which is greater than the reciprocal bandwidth of the transmitted message waveform. When this occurs, the received signal includes multiple versions of the transmitted waveform which are attenuated (faded) and delayed in time, and hence the received signal is distorted. Frequency selective fading is due to time dispersion of the transmitted symbols with in the channel. Thus the channel induces intersymbol interference (ISI).Viewed in the frequency domain, certain frequency components in the received signal spectrum have greater gains than others. For frequency selective fading, the spectrum of the transmitted signal has a bandwidth which is greater than the coherence bandwidth of the channel. Frequency selective fading is caused by multipath delays which approach or exceed the symbol period of the transmitted symbol. Frequency selective fading channels are also known as wideband channels since the bandwidth of the signal s(t) is wider than the bandwidth of the channel impulse response.

2.2.3 Flat Fading

Flat fading is historically the most common type of fading described in the technical literature. In flat fading the multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. How ever the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multipath. In a flat fading channel, the reciprocal bandwidth of the channel is much larger than the multipath time delay spread of the channel. Flat fading channels are also known as amplitude varying channels and are sometimes referred as narrowband channels.

2.2.4 Fast Fading

Depending on how rapidly the transmitted baseband signal changes as compared to the rate of change of channel, a channel may be classified either as a fast fading or slow fading channel. In a fast fading channel, the channel impulse response changes rapidly within the stmbol duration. That is, the coherence time of the channel is smaller than the symbol period of the transmitted signal. This causes frequency dispersion(time selective fading) due to Doppler spreading. Which leads to signal distortion. Viewed in the frequency domain, signal distortion due to fast fading increases with increasing Doppler spread relative to the bandwidth of the transmitted signal. Therefore, a signal undergoes fast fading if $T_S > T_C$ .It should be noted that when a channel is specified as a fast or slow fading channel, it does not specify whether the channel is flat fading or frequency selective in nature. In the case of the flat fading channel, we can approximate the impulse response to be simply a delta function. Hence, flat fading, fast fading channel is a channel in which the amplitude of the delta function varies faster than the rate of change of the transmitted baseband signal.

2.2.5 Slow Fading

In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal. In this case, the channel may be assumed to be static over one or several reciprocal bandwidth intervals. In frequency domain this implies that the Doppler spread of the channel is much less than the bandwidth of the baseband signal. Therefore, a signal undergoes slow fading if $T_S << T_C$.

2.2.6 Delay Spread

The received radio signal from a transmitter consists of typically a direct signal, plus reflections of object such as buildings, mountings, and other structures. The reflected signals arrive at a later time than the direct signal because of the extra path length, giving rise to a slightly different arrival time of the transmitted pulse, thus spreading the received energy. Delay spread is the time spread between the arrival of the first and last significant multipath signal seen by the receiver. In a digital system, the delay spread can lead to inter-symbol interference. This is due to the delayed multipath signal overlapping with the following symbols. This can cause significant errors in high bit rate systems, especially when using time division multiplexing (TDMA). Figure 2.3 shows the effect of inter-symbol interference due to delay spread on the received signal. As the transmitted bit rate is increased the amount of inter-symbol interference also increases.

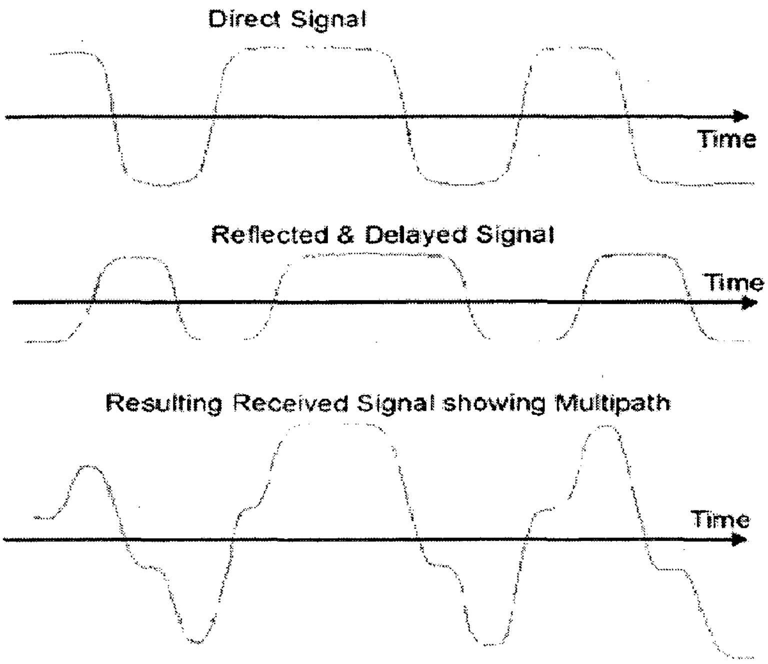


Figure 2.3 Multipath Delay Spread

2.3 Doppler Shift

Doppler spread [3] is caused by the relative motion of transmitter and receiver. For example, in an urban environment in the city center, the vehicles are always moving; the moving pedestrians are also changing their locations continuously, thus their movements affect the transmission medium. A high Doppler can be experienced when a user is located in a fast moving car or in a speedy train, because the relative motion will be higher when either transmitter or receiver is moving very fast. This relative motion of transmitter and receiver changes the received signal from the original transmitted signal. When they are moving towards each other, the frequency of the received signal is higher than the source and when they are moving away from each other, the received frequency decreases. When the relative speed is higher, then Doppler shift can be very high, and thus the receiver may become unable to detect the transmitted signal frequency. Even at lower relative motion when the Doppler shift is usually very little, if the transmission and reception technique is very sensitive to carrier frequency offset, then the system may fail.

Chapter 3
Orthogonal Frequency Division Multiplexing: An Overview

In this chapter, the basic principles of Orthogonal Frequency Division Multiplexing (OFDM) are presented. A basic system model is given, common components for OFDM based systems are explained, and a simple transceiver based on OFDM modulation is presented.

3.1 System Model

A block diagram of a basic OFDM system is given in Figure 3.1. Usually raw data is coded and interleaved before modulation. In a multipath fading channel, all subcarriers will have different attenuations. Some subcarriers may even be completely lost because of deep fades. Therefore, the overall BER may be largely dominated by a few subcarriers with the smallest amplitudes. To avoid this problem, channel coding can be used. By using coding, errors can be corrected up to a certain level depending on the code rate and type, and the channel. Interleaving is applied to randomize the occurrence of bit errors. Coded and interleaved data is then mapped to the constellation points to obtain data symbols. These steps are represented by the first block of Figure 3.1.

The serial data symbols are then converted to parallel and Inverse Fast Fourier Transform (IFFT) is applied to these parallel blocks to obtain the time domain OFDM symbols. Later, these samples are cyclically extended as explained in Section 3.2 converted to analog signal and up-converted to the RF frequencies using mixers. The signal is then amplified by using a power amplifier (PA) and transmitted through antennas.

In the receiver side, the received signal is passed through a band-pass noise rejection filter and down-converted to baseband. After frequency and time synchronization, cyclic Prefix is removed and the signal is transformed to the frequency domain using Fast Fourier Transform (FFT) operation. And finally, the symbols are demodulated, deinterleaved and decoded to obtain the transmitted information bits.

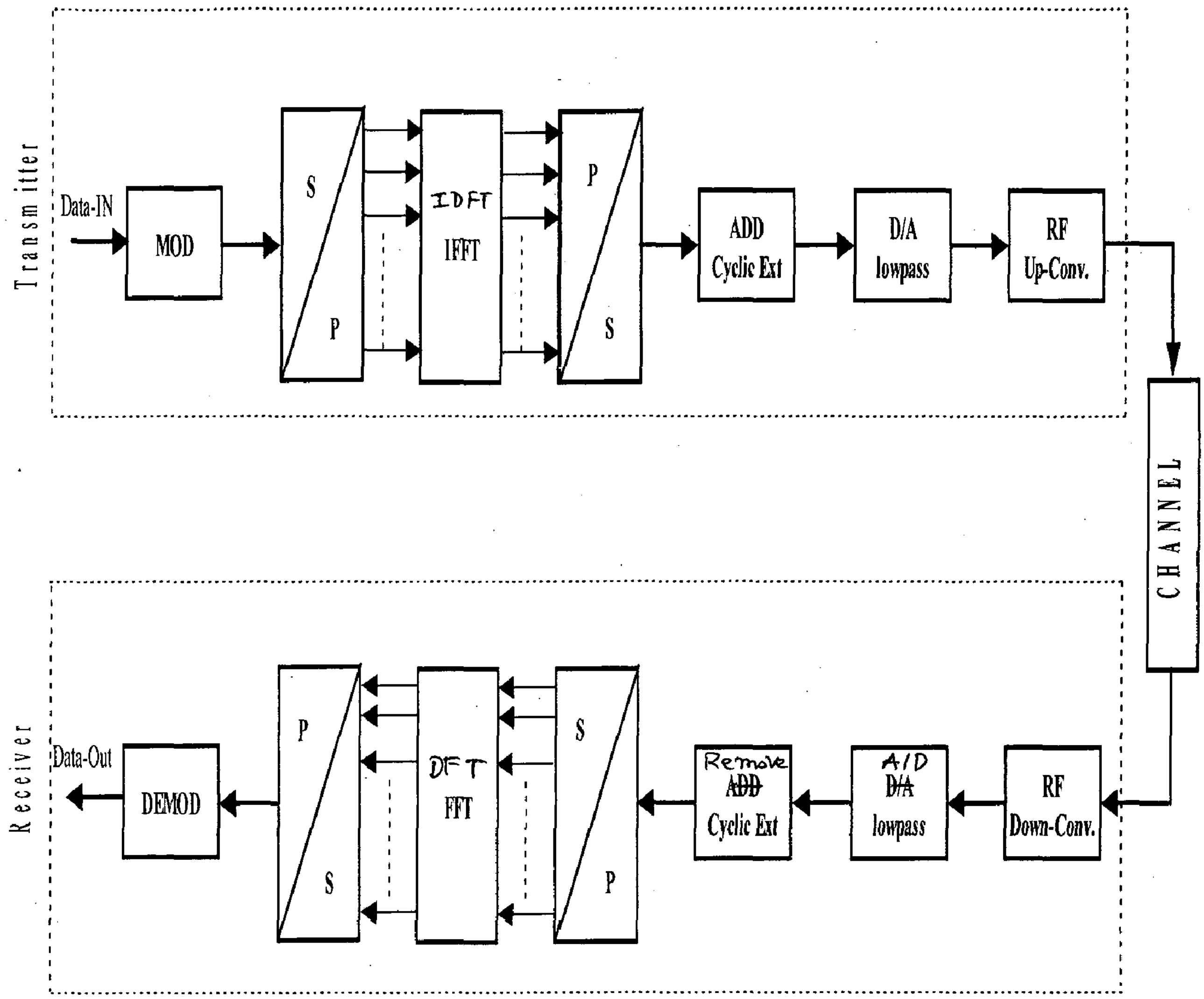


Figure 3.1 Block Diagram of an OFDM Transceiver.

The Discrete Fourier Transform of a (DFT) discrete sequence $f(n)$ of length N, $F(k)$, is defined as ,

$$F(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} f(n)e^{-j\frac{2\pi kn}{N}} \qquad k = 0 \cdots N-1 \qquad (3.1)$$

and Inverse Discrete Fourier Transform $(IDFT)$ as:

$$f(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} F(k)e^{j\frac{2\pi kn}{N}} \qquad n = 0 \cdots N-1 \qquad (3.2)$$

OFDM converts serial data stream into parallel blocks of size N, and uses (*IDFT*) to obtain *OFDM* signal. Time domain samples, then, can be calculated as

$$x(n) = IDFT\{X(k)\}$$
$$= \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} \qquad 0 \le n \le N-1 \tag{3.3}$$

where $X(k)$ is the symbol transmitted on the k^{th} subcarrier and N is the number of subcarriers. Symbols are obtained from the data bits using a M-ary modulation e.g. Binary Phase Shift Keying (BPSK), Quadrature Amplitude Modulation (QAM), etc. Time domain signal is cyclically extended to avoid Inter-Symbol Interference (ISI). The symbols $X(k)$ are interpreted as frequency domain signals and samples $x(n)$ are interpreted as time domain signal.

3.2 Cyclic Extension of OFDM Symbol

Time domain OFDM signal is cyclically extended to mitigate the effect of time dispersion. The length of cyclic prefix has to exceed the maximum excess delay of the channel in order to avoid ISI. The basic idea here is to replicate part of the OFDM time-domain symbol from back to the front to create a guard period. This is shown in the Figure 3.2. This figure also shows how cyclic prefix prevents the ISI. As can be seen from the figure, as long as maximum excess delay $(\tau_{\max})$ is smaller than the length of the cyclic extension (T_g), the distorted part of the signal will stay within the guard interval, which will be removed later at the receiver. Therefore ISI will be prevented.

Postfix is the dual of prefix. In postfix, the beginning of OFDM symbol is copied and appended at the end. If we use prefix only, we need to make sure that the length of cyclic prefix is larger than the maximum excess delay of the channel; if we use both cyclic prefix and postfix, then the sum of the lengths of cyclic prefix and postfix should be larger than the maximum excess delay.

The ratio of the guard interval to the useful symbol duration is application dependent. If this ratio is large, then the overhead will increase causing a decrease in the system throughput.

A cyclic prefix is used for the guard time for the following reasons;

1. To maintain the receiver time synchronization; since a long silence can cause Synchronization to be lost.
2. To convert the linear convolution of the signal and channel to a circular convolution and thereby causing the DFT of the circularly convolved signal and channel simply be the product of their respective DFTs.

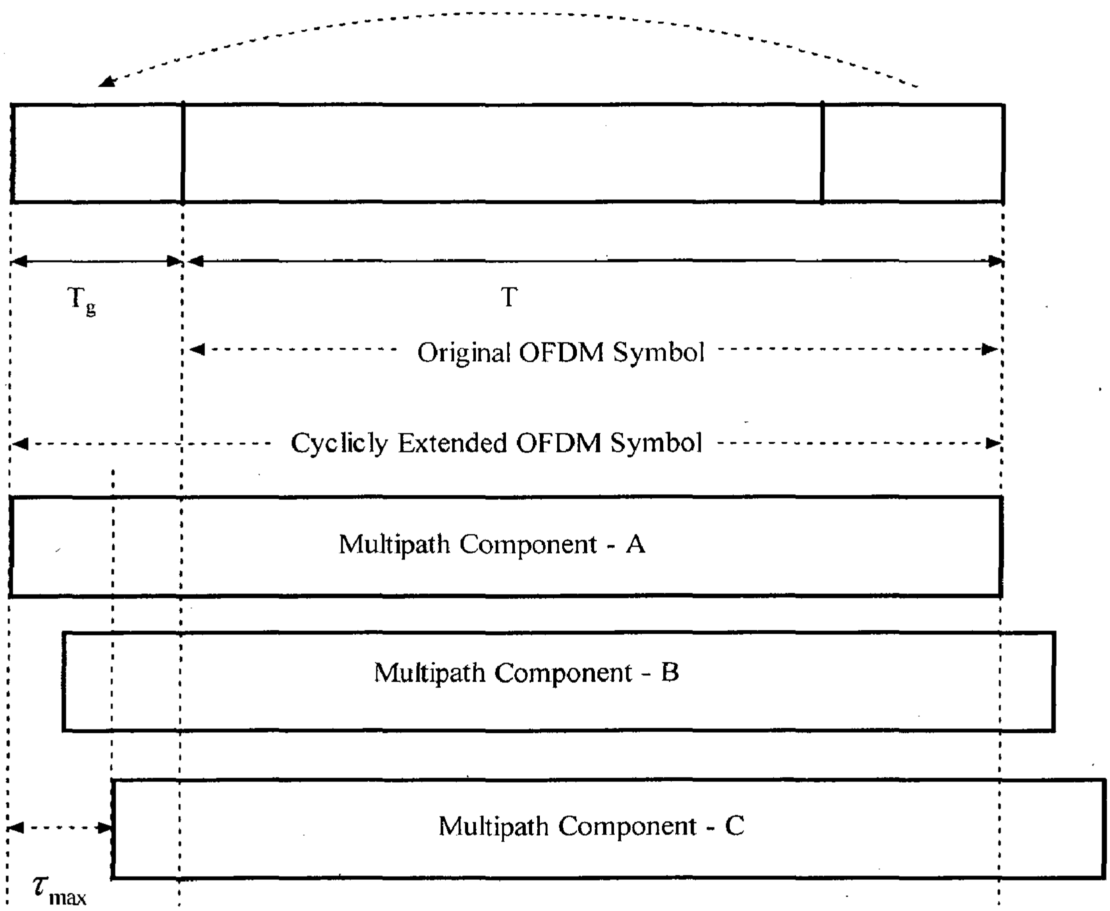


Figure3.2: Illustration of Cyclic Prefix Extension

3.3 Filtering

Filtering is applied both at the receiver and at the transmitter. At the transmitter, it is used to reduce the effect of side lobes of the *sinc* shape in the OFDM symbol. This effectively band pass filters the signal, removing some of the OFDM side-lobes. The amount of side-lobe removal depends on the sharpness of the filters used. In general digital filtering provides a much greater flexibility, accuracy and cut off rate than analog filters making them especially useful for band limiting of an OFDM signal. Some commonly used filters are rectangular pulse (sinc filter), root raised cosine filter, Chebyshev and Butterworth filter. In the receiver side, a matched filter is used to reject the noise and Adjacent Channel Interference (ACI).

3.4 OFDM Impairments

OFDM faces problems like Frequency Offset, Peak to Average Power Ratio (PAPR), Phase Noise, and Inter-carrier Interference.

3.4.1 Frequency Offset

Frequency offset is a critical factor in OFDM system design. It results in inter-carrier interference (ICI) and degrades the orthogonality of sub-carriers. Frequency errors will tend to occur from two main sources. These are local oscillator errors and common Doppler spread. Any difference between transmitter and receiver local oscillators will result in a frequency offset. This offset is usually compensated for by using Adaptive Frequency Correction (AFC), however any residual (uncompensated) errors result in a degraded system performance. The characteristics of ICI are similar to Gaussian noise, hence it leads to degradation of the SNR. The amount of degradation is proportional to the fractional frequency offset which is equal to the ratio of frequency offset to the carrier spacing.

Assume that we have the symbols X(k) to be transmitted using an OFDM system. These symbols are transformed to the time domain using IDFT. This baseband signal (OFDM symbol) is then up-converted to RF frequencies and transmitted over the wireless channel. In the receiver, the received signal is down-converted to baseband. But, due to the frequency mismatch between the transmitter and receiver, the received signal has a frequency offset. This signal is denoted as y(n). The frequency offset is added to the OFDM symbol in the receiver. Finally, to recover the data symbols, DFT is applied to the OFDM symbol taking the signal back to frequency domain. Let Y (k) denotes the recovered data symbols. This process is shown below.

$$X(k) \xrightarrow{\text{IDFT}} x(n) \xrightarrow{\textit{frequency offset}} y(n) \xrightarrow{\text{DFT}} Y(k)$$

Let us apply the above operations to $X(k)$ in order to get $Y(k)$. From Eq.(3.2)

$$x(n) = IDFT\{X(k)\} = \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}}$$

The effect of Frequency Offset on $x(n)$ will be a phase shift of $2\pi \in n/N$, where $\in$ is the normalized frequency offset. Therefore

$$y(n) = x(n) \times e^{\frac{j2\pi \in n}{N}} \tag{3.4}$$

After taking DFT to get received symbols $Y(k)$ and some basic manipulations we will get

$$\begin{aligned} Y(k) &= \sum_{m=0}^{N-1} X(m) S(m-k) \\ &= X(k)S(0) + \sum_{\substack{m=0 \\ m \neq k}}^{N-1} X(m) S(m-k) \end{aligned} \tag{3.5}$$

where $S(m-k) = \frac{\sin(\pi(m-k+\in))}{\pi(m-k+\in)} e^{j\pi(m-k+\in)}$ (3.6)

In Eq.(3.5) the second term shows ICI caused by frequency offset[7].

3.4.2 Phase Noise

The Phase Noise is the difference between the phase of the carrier and the phase of the local oscillator. The PN can be classified [6] based on the ratio of its bandwidth with respect to the OFDM subcarrier spacing. If we denote this ratio $\rho = \frac{B_\theta}{B_S}$, where B_S is subcarrier spacing, B_θ is bandwidth of the Phase noise $\theta(t)$, when $\rho << 1$, $\theta(t)$ is constant or slowly varying during an OFDM symbol. On the contrary, when $\rho >> 1$, $\theta(t)$ varies fastly during an OFDM symbol. When $\rho << 1$, we call it as slow Phase noise, when $\rho >> 1$ we speak of Fast Phase noise, and when $\rho \approx 1$ we speak of moderate Phase noise. Since $B_S = \frac{B}{N}$, B is the total bandwidth of the OFDM system, increasing N increases the rate of the Phase Noise.

3.5 Applications

After the IFFT/FFT technique was introduced, the implementation of OFDM became more convenient. Generally speaking, the OFDM applications may be divided into two categorieswired and wireless technologies. In wired systems such as Asymmetric Digital Subscriber Line (ADSL) and high speed DSL, OFDM modulation may also be referred as Discrete Multitone Modulation (DMT). In addition, wireless OFDM applications may be shown in numerous standards such as IEEE 802.11

OFDM was also applied for the development of Digital Video Broadcasting (DVB) in Europe. In the DVB standards, the number of subcarriers can be more than 8,000, and the data rate could go up as high as 15Mbps.

Chapter 4

Inter-Carrier Interference Cancellation in OFDM

In Orthogonal Frequency Division Multiplexing (OFDM) based systems, the loss of orthogonality among subcarriers causes Inter-carrier Interference (ICI). ICI affects detection of the OFDM symbols. If not compensated for, ICI will result in an error floor. In this chapter, the impairments causing ICI will be analyzed and commonly used ICI reduction methods will be given.

4.1 Factors Inducing ICI

ICI is different from the co-channel interference in MIMO systems. The co-channel interference is caused by reused channels in other cells, while ICI results from the other sub-channels in the same data block of the same user. Even if only one user is in communication, ICI might occur, yet the co-channel interference will not happen. There are different factors that cause the ICI, namely frequency offset, phase noise, and Doppler shift. Here we will present the ICI reduction methods in presence of phase noise and time varying channels.

4.1.1 Doppler Effect

The relative motion between receiver and transmitter, or mobile medium among them, would result in the Doppler Effect, a frequency shift in narrow-band communications. For example, the Doppler Effect would influence the quality of a cell phone conversation in a moving car. In general, the Doppler frequency shift can be formulated as a function of the relative velocity, the angle between the velocity direction and the communication link.

The value of Doppler shift [3] could be written as

$$f_d = \frac{v}{\lambda}\cos(\theta)$$

Where θ is the angle between the velocity and the communication link, which is generally modeled as a uniform distribution between 0 and 2π, v is the receiver velocity, and the λ is the carrier wavelength.

4.1.2 Synchronization Error

It can be assumed that most of the wireless receivers cannot make perfect frequency synchronization. In fact, practical oscillators for synchronization are usually unstable, which introduce frequency offset. Although this small offset is negligible in traditional communication systems, it is a severe problem in the OFDM systems. This will also cause the phase noise effect.

4.2 ICI Self Cancellation Methods

Currently a few different approaches for reducing ICI have been developed. These approaches include frequency-domain equalization, time-domain windowing, and the ICI self-cancellation scheme. Here we will discuss ICI self cancellation methods and ICI estimation and equalization technique.

The data conversion method [8], in which data pair (a,-a) is modulated onto two adjacent subcarriers $(l, l+1)$, where a is a complex data, to reduce ICI, as ICI signals generated by the subcarrier will be cancelled out significantly by the ICI generated by subcarrier $l+1$, is the basic ICI cancellation method.

4.2.1 Data Conjugate Method

In this method [9] the high speed information data passes through the serial to parallel converter and become parallel data streams of N/2 branch. Then they are converted into N branch parallel data by the data conjugate method. After serial to parallel converter, parallel data streams are remapped as the form of $X'_{2k} = X_k$ and $X'_{2k+1} = -X_k^*$. Here X_k is the information data to the k-th branch before data-conjugate conversion, and X'_{2k} is the information data to the -$2k$th branch after ICI cancellation mapping. Likewise, every information data is mapped into a pair of adjacent sub-carriers by data-conjugate method, so the $N/2$ branch data are extended to map onto the N sub-carriers.

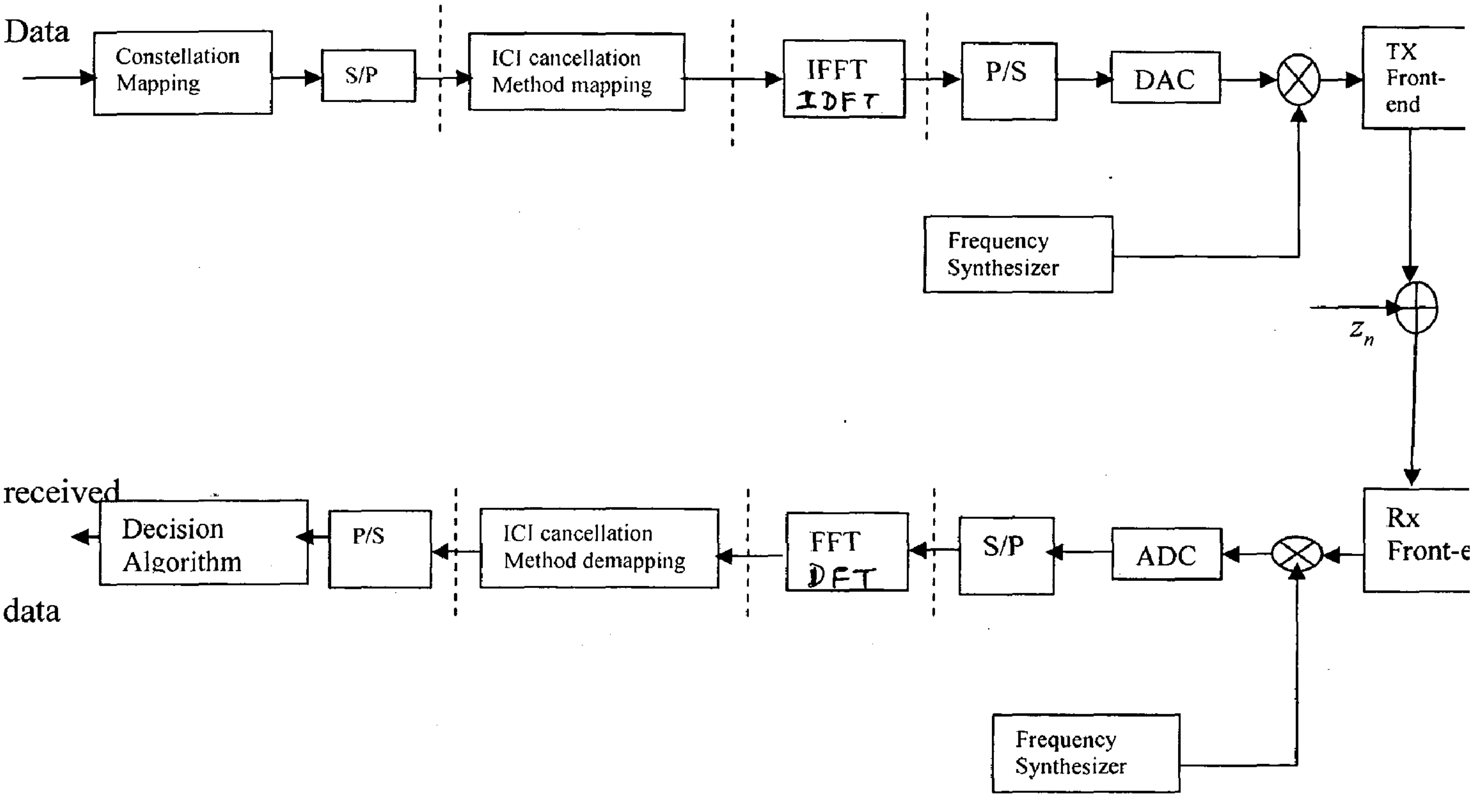


Fig 4.1 OFDM System with Data Conjugate method

The original data can be recovered from the simple relation of $Z'_k = 0.5(Y_{2k} - Y^*_{2k+1})$.Here, Y_{2k} is the 2k-th sub-carrier data, Z'_k is the k-th branch information data after de-mapping. Finally, the information data can be found through the detection process.

4.2.2 Symmetric Data-conjugate Method

The ICI could be effectively reduced by ICI self-cancellation of data-conversion method [5], in which one data symbol to be transmitted, is mapped onto a pair of sub-carriers with opposite sign. The restriction on the use of data conversion method in multi path channels is that the channel response of the two adjacent sub-carriers is nearly the same or the coherence bandwidth of the channel is larger than the sub-carrier spacing. A novel ICI self cancellation of symmetric data-conjugate method [10][11], with frequency diversity is discussed and compared with previous methods.

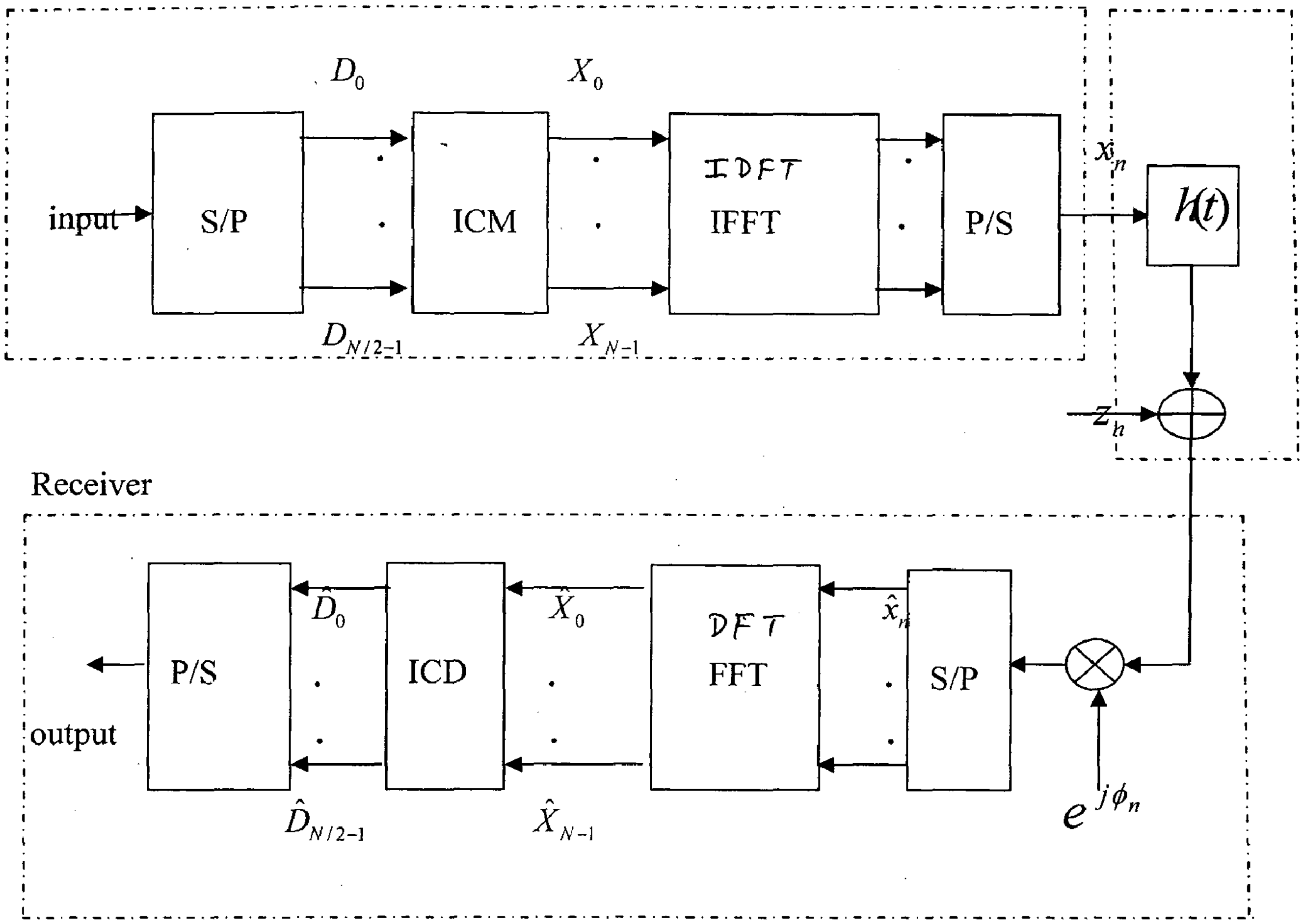


Fig 4.2 OFDM System with Symmetric Data Conjugate Method.

Denote X_k (k=0,1,.... , N-1) as the modulated symbols on the k-th sub-carrier of the OFDM symbol, which are assumed independent, zero-mean random variables, with average power of σ_X^2 .The complex base-band OFDM signal after the inverse fast Fourier transform (IFFT) is expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}} \tag{4.1}$$

Where N is the total number of subcarriers and x_n has duration of T seconds. Experiencing multipath fading and with AWGN adding on, the received signal contaminated by PHN can be written as

$$\hat{x}_n = (x_n \otimes h_n)e^{j\phi_n} + z_n \tag{4.2}$$

Where h_n, ϕ_n and z_n represent channel impulse response, the phase noise and the AWGN, respectively, while $\otimes$ denotes the circular convolution. The multiplication of two signals in time domain is equivalent to the convolution on the spectra of the corresponding signals in the frequency domain, these samples, x_n are then processed by fast Fourier transform (FFT) and the frequency domain signal becomes

$$\begin{aligned}\hat{X}_k &= \sum_{l=0}^{N-1}(H_l X_l) Q_{k-l} + Z_k \\ &= Q_0 H_k X_k + \sum_{l=0, l\neq k}^{N-1} Q_{k-l} H_l X_l + Z_k\end{aligned} \tag{4.3}$$

Where H_k is the channel frequency response and Z_k denotes the frequency domain expression of z_n.The term Q_k is the discrete Fourier transform (DFT) of the PHN divided by $\sqrt{N}$ and is given by

$$Q_k = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\phi_n} e^{-j2\pi nk/N}$$

Which has period of N. The received data $\hat{X}_k$ is the circular convolution of the transmitted data X_k and Q_k when the channel is flat ($H_l \equiv 1$).So Q_k can be considered as a weighting function on the transmitted frequency domain symbols. The term Q_0, which is the time average of the PHN process, introduces a rotation of the entire constellation and is usually known as the common phase error(CPE). The second term in the equation (4.3) is the ICI. It is seen that ICI weighting function is symmetric conjugate as follows.

$$\begin{aligned}Q_k + Q^*_{-k} &= \frac{1}{N}\sum_{n=0}^{N-1}(e^{j\phi_n} + e^{-j\phi_n})e^{-j2\pi nk/N} \\ &= \frac{2}{N}\sum_{n=0}^{N-1}(\cos\phi_n)e^{-j2\pi nk/N} \\ &\approx \frac{2}{N}\sum_{n=0}^{N-1} e^{-j2\pi nk/N}\end{aligned}$$

$$\approx \frac{1-e^{j2\pi nk}}{1-e^{j2\pi nk/N}} (k \neq 0)$$
$$\approx 0$$

In which $\cos\phi_n$ is approximated to 1 because the phase noise ϕ_n is zero mean with very small standard deviation (STD).From above equations, the symmetric conjugate property of the ICI weighting function Q_k is expressed as follows

$$\left.\begin{array}{l} Q_k + Q_{-k}^* \approx 0, k \neq 0 \\ Q_0 + Q_0^* \approx 2 \end{array}\right\} \tag{4.4}$$

4.2.2.1 ICI cancelling Modulation:

In the ICI self-cancellation scheme using symmetric data–conjugate method, the relationship of the transmitted symbols D_k (k=0,…,N/2-1) before ICM and X_k $(k = 0,....N-1)$ after ICM is $X_k = D_k$, $X_{N-1-k} = -D_k^*$, $k = 0,....N/2-1$

So the transmitted time domain signal can be expressed as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{\frac{N}{2}-1} \left[D_k e^{\frac{j2\pi kn}{N}} - D_k^* e^{\frac{-j2\pi n(N-1-k)}{N}} \right]$$

Then the frequency domain receiving signal can be written as

$$\begin{aligned} \hat{X}_k &= \sum_{l=0}^{N-1} (H_l X_l) Q_{k-l} + Z_k \\ &= \sum_{l=0}^{N/2-1} D_l H_l Q_{k-l} + \sum_{l=N/2}^{N-1} -D_{N-1-l}^* Q_{k-l} H_l + Z_k \\ &= \sum_{l=0}^{N/2-1} D_l H_l Q_{k-l} + \sum_{m=N/2-1}^{0} -D_m^* Q_{k-N+1+m} H_{N-1-m} + Z_k \\ &= \sum_{l=0}^{\frac{N}{2}-1} \left(D_l H_l Q_{k-l} - D_l^* H_{N-1-l} Q_{-(N-1-l-k)} \right) + Z_k \end{aligned} \tag{4.5}$$

$$\hat{X}_{N-1-k}^* = \sum_{l=0}^{\frac{N}{2}-1} \left(D_l^* H_l^* Q_{N-1-k-l}^* - D_l H_{N-1-l}^* Q_{l-k}^* \right) + Z_{N-1-k}^* \tag{4.6}$$

4.2.2.2 ICI Cancelling Demodulation:

In the present method, since the transmitted symbol is mapped onto the symmetric pair of subcarriers instead of the adjacent pair and the channel response of H_k and H_{N-1-k} may not suffer from fading simultaneously, the received symbols $\hat{X}_k$ and $\hat{X}_{N-1-k}$ can be combined to achieve the frequency diversity. So in the symmetric data-conjugate method the decision variable is expressed as

$$\hat{D} = \frac{1}{2}\left(H_k^* \hat{X}_k - H_{N-1-k} \hat{X}_{N-1-k}^*\right)$$

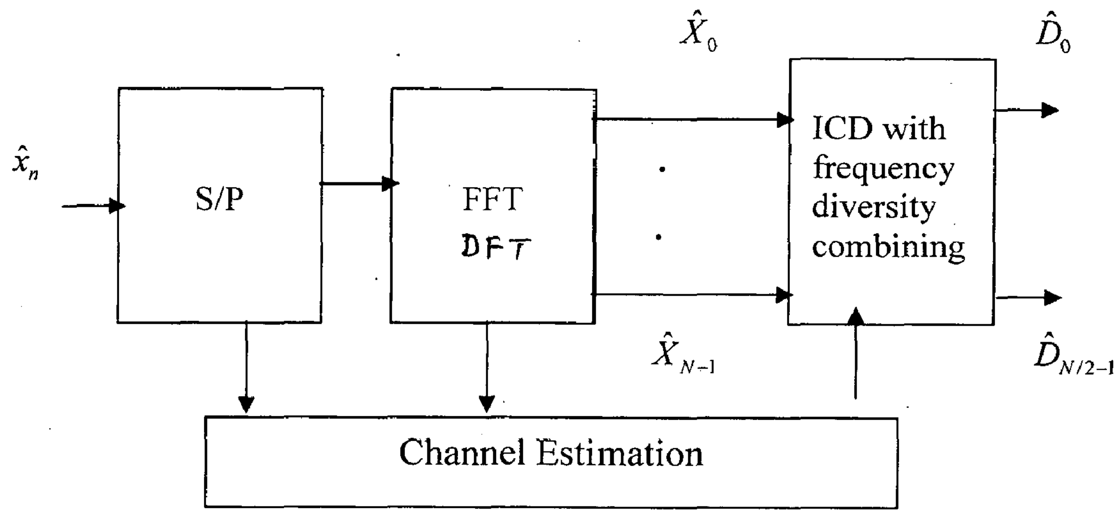


Fig 4.3 The receiver structure with frequency diversity combining

In the present method, since the transmitted symbol is mapped onto the symmetric pair of subcarriers instead of the adjacent pair and the channel response of H_k and H_{N-1-k} may not suffer from fading simultaneously, the received symbols $\hat{X}_k$ and $\hat{X}_{N-1-k}$ can be combined to achieve the frequency diversity. So in the symmetric data-conjugate method the decision variable is expressed as

$$\hat{D}_k = \frac{1}{2}\left(H_k^* \hat{X}_k - H_{N-1-k} \hat{X}_{N-1-k}^*\right)$$

$$= \frac{1}{2}\left(|H_k|^2 + |H_{N-1-k}|^2\right) D_k + I_k - J_k + W_k$$

where

$$I_k = \frac{1}{2}\sum_{l=0, l\neq k}^{N/2-1}\left[H_k^* H_l Q_{k-l} + H_{N-1-k} H_{N-1-l}^* Q_{l-k}^*\right] D_l$$

$$J_k = \frac{1}{2}\sum_{l=0, l\neq k}^{N/2-1}(H_k^* H_{N-1-l} Q_{-(N-1-k-l)} + H_{N-1-k} H_l^* Q_{N-1-l-k}^*) D_l^*$$

$$W_k = \frac{1}{2}\left(H_k^* Z_k - H_{N-1-k} Z_{N-1-k}^*\right)$$

The first term in expression of $\hat{D}_k$ is desired signal , "$I_k - J_k$" is the ICI signal, and W_k is the AWGN .When the channel is flat, the frequency response of channel H_k always equals to 1. At the receiver the decision variable $\hat{D}_k$ of the k th subcarrier becomes

$$\hat{D}_k = \frac{1}{2}\left(\hat{X}_k - \hat{X}_{N-1-k}^*\right)$$
$$= D_k + \frac{1}{2}\sum_{l=0, l\neq k}^{N/2-1}\left[Q_{k-l} + Q_{-(k-l)}^*\right] D_l - \frac{1}{2}\sum_{l=0, l\neq k}^{N/2-1}\left[Q_{-(N-1-k-l)} + Q_{N-1-l-k}^*\right] D_l^* + \frac{1}{2}\left(Z_k - Z_{N-1-k}^*\right) \tag{4.7}$$

As seen in above, CPE is completely removed. In equation (4.7) the decision variable $\hat{D}_k$ for this method is similar in form to that of the optimal two-branch maximal ratio combining(MRC) receiver diversity system[12].Assuming D_k is independent, zero mean random variable, the carrier to interference ratio (CIR) at the 0 th subcarrier is calculated as

$$CIR = \frac{4}{\sum_{l=1}^{N/2-1}\left|Q_{-l} + Q_l^*\right|^2 + \sum_{l=0}^{N/2-1}\left|Q_{-(N-1-l)} + Q_{N-1-l}^*\right|^2} \tag{4.8}$$

For Adjacent Data conjugate method[9] CIR is given by

$$CIR_2 = \frac{4}{\sum_{l=1}^{\frac{N}{2}-1}\left(\left|Q + Q_{2l}^*\right|^2 + \left|Q_{2l+1} + Q_{2l-1}^*\right|^2\right)} \tag{4.9}$$

For conventional OFDM system suffering from phase noise

$$CIR_3 = \frac{\left|Q_0\right|^2}{\sum_{l=1}^{N-1}\left|Q_l\right|^2} = \frac{1}{\sum_{l=1}^{N-1}\left|Q_l\right|^2} \tag{4.10}$$

Considering symmetric conjugate–negative property of the ICI weighting function, the denominator in CIR expression approximates to zero and CIR can be very large when the standard deviation of PHN is small, which means that ICI is significantly reduced.

4.3 ICI Estimation and Equalization

Due to its spectral efficiency and robustness over multipath channels, orthogonal frequency-division multiplexing (OFDM) has served as one of the major modulation schemes for high-speed communication systems. In presence of time varying channels[3] and Doppler shift one of the effective schemes to mitigate ICI involves an equalizer. The employment of traditional minimum mean square error (MMSE) and zero forcing (ZF) equalizers for OFDM systems in [19], normally requires a large matrix inversion. As number of subcarriers in OFDM symbol increases, any direct implementation of a traditional MMSE or ZF equalizer should be avoided. It has been shown in [20] that ICI power is concentrated only on a few adjacent subcarriers. In other words, the subject subcarrier would be interfered with only by a few neighbors. Accordingly, the computational complexity of the ICI equalizer can be reduced significantly without much sacrifice in performance[27].

4.3.1 System Model:

In OFDM system, the usable bandwidth is devided into N spectrally equispaced subcarriers. Thus, N transmitted signals are modulated onto each subcarrier independently. An OFDM signal consists of N subcarriers with a frequency spacing Δf.

Let $X_{i,k}$ denote the complex valued transmitted information data for the kth subcarrier in the ith OFDM symbol block. Since the samples in the guard interval are discarded at the receiver, we need to consider the signals transmitted in the period of T_f only. Thus with in the ith OFDM symbol duration, the discrete time transmitted signal $x_{i,n}$ for the nth time sample in the ith OFDM symbol block is represented as

$$x_{i,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{i,k} e^{\frac{j2\pi kn}{N}} \tag{4.11}$$

OFDM system performance is determined by the channel over which signals are transmitted. In mobile wireless communications, there exist multiple propagation paths between the transmitter and the receiver in reality. We assume that the channel consists of L delays, and the corresponding CIR can be written as

$$h_{l,n} = \sum_{l=0}^{L-1} \gamma_{l,n} \delta(n-\tau_l) \tag{4.12}$$

Where L is the number of delay paths, $\gamma_{l,n}$ is the time varying attenuation factor coefficient for the l th delay path, and τ_l is the delay spread between the transmitted and received signals. Based on the WSSUS assumption [21], the channel correlation function is simplified as

$$\Phi_h(t_1,t_2,l_1,l_2) = \phi_\tau(\tau_l)\phi_t(\Delta t)\delta(\tau_{l_1} - \tau_{l_2}) \tag{4.13}$$

Where $\Delta t = (n_1 - n_2)T_s$, T_s is the sampling period, $\phi_\tau(\tau_l)$ is the multipath intensity profile of the channel and the Fourier transform of the spaca time correlation function is the Doppler power spectrum. In this analysis Jakes channel model [25] is used, for which the space time correlation function is given by

$$\Phi_t(\Delta t) = J_0(2\pi f_d \Delta t) \tag{4.14}$$

Where $J_0(.)$ is the zeroth order Bessel function of the first kind, and f_d is the maximum Doppler frequency shift. In mobile communication systems, as the mobile subscriber moves across the area covered by several base stations, the differences in the base station locations, signal propagation paths, and signal incident angles with respect to the direction of the subscriber's movement result in the distinct Doppler frequency shifts among propagation paths. Every incident waveform on the mobile subscriber undergoes a Doppler frequency shift due to motion. The Doppler frequency shift for the l th path (in hertz) is given by

$$f_l = \frac{v}{\lambda}\cos\theta_l = f_D \cos\theta_l \tag{4.15}$$

Where λ is the incident wavelength, v is the mobile station's moving speed, and θ_l is the incident angle for the l th incident wave. It is assumed that the incident angles θ_l constitute a random process uniformly distributed in the interval between $-\pi$ and π. Consequently we obtain the probability density function of the Doppler frequency shift, which is denoted P_{f_l} and given as

$$P_{f_l} = \begin{cases} \dfrac{1}{\pi f_D \sqrt{1-(f_l/f_D)^2}} & |f_l| \le f_D \\ 0 & \text{otherwise} \end{cases} \tag{4.16}$$

It is assumed that the doppler frequency shift for each delay path has the identical probability density function as given above.

At the OFDM receiver, in the absence of time synchronization errors, the discrete-time received signal $y_{i,n}$ in the i th OFDM symbol block is given by

$$y_{i,n} = \sum_{l=0}^{L-1} \gamma_{l,n} x_{i,n-l} e^{j2\pi f_l (n-\tau_l)} + w_n \tag{4.17}$$

Where w_n is additive white gaussian noise (AWGN).The received signal can be separated into N orthogonal subcarrier signals by a discrete Fourier transform(DFT).The OFDM demodulated symbol $Y_{i,m}$, which is on the m th subcarrier in the i th OFDM symbol block , can be obtained as

$$\begin{aligned} Y_m &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y_n e^{-j2\pi nm/N} + W_m \\ &= \sum_{k=0}^{N-1} \sum_{l=0}^{L-1} X_k e^{-j2\pi l(k+\epsilon_l)/N} H_l^{(m-k)} + W_m \\ &= \alpha_m X_m + \sum_{\substack{k=0 \\ k \ne m}}^{N-1} X_k C_{m,k} + W_m \\ &= \alpha_m X_m + I_m + W_m \end{aligned} \tag{4.18}$$

Where W_m denotes the DFT of Gaussian noise w_n, ϵ_l is the OFDM symbol normalized Doppler frequency for the l th delay path, $\alpha_m \equiv \sum_{l=0}^{L-1} H_l^0 e^{-j2\pi(m+\epsilon_l)l/N}$ represents the multiplicative channel distortion for the subject subcarrier m, $I_m \equiv \sum_{\substack{k=0 \\ k \ne m}}^{N-1} X_k C_{m,k}$ represents ICI, and $C_{m,k} = \sum_{l=0}^{L-1} e^{-j2\pi l(k+\epsilon_l)l/N} H_l^{m-k}$ is the ICI coefficient specifying interference from subcarrier k to subcarrier m .Besides, H_l^{m-k} denotes the DFT of the time varying multipath channel, which is given by

$$H_l^{m-k} = \frac{1}{N}\sum_{n=0}^{N-1}\gamma_{l,n} e^{\frac{-j2\pi(m-k-\epsilon_l)n}{N}} \tag{4.19}$$

Furthermore, in matrix form received signal can be written as

$$\vec{Y} = \tilde{C}\vec{X} + \vec{W} \tag{4.20}$$

Where $\vec{Y} \equiv [Y_0 Y_1 .... Y_{N-1}]^T, \vec{X} \equiv [X_0 X_1 .... X_{N-1}]^T, \vec{W} \equiv [W_0 W_1 ... W_{N-1}]^T$, and matrix $\tilde{C} \in C^{N \times N}$ is called the ICI coefficient matrix. The off diagonal elements in $\tilde{C}$ represents ICI caused by the time varying nature of the channel in presence of multiple Doppler frequency shifts.

Time variations of the channel within one OFDM symbol block in the presence of multiple Doppler frequency shifts destroy the subcarrier's orthogonality [4], result in ICI, and hence, lead to an irreducible error probability floor when only a one-tap equalizer is employed. The performance degradation due to ICI becomes significant as the carrier frequency, OFDM symbol duration, and vehicular velocity increase. An equalizer is effective for ICI mitigation. However, the traditional MMSE and ZF equalizers for OFDM systems process N subcarrier signals simultaneously using N available received signals in one OFDM symbol block, and therefore, these schemes require a large matrix inversion. As the ICI coefficient matrix C becomes very large when N is large, any intuitive implementation of a traditional MMSE or ZF equalizer should be avoided. It has also been shown that ICI power is concentrated only on a few adjacent subcarriers. In other words, the subject subcarrier would be interfered with only by a few neighbors. Accordingly, the computational complexity of the ICI equalizer can be significantly reduced without much sacrifice in performance. In this approach using the Q-tap equalizer integrated with an MMSE ICI coefficient estimator is proposed for frequency-domain ICI equalization and channel estimation. New ICI mitigation algorithm includes two steps. At the receiver channel is estimated, and then, a Q-tap equalizer is employed to suppress ICI thereafter. Only the ICI coefficients from a few neighboring subcarriers are needed in this algorithm as it is assumed that most of the interference arises from transmitted signals from neighboring subcarriers. Furthermore, this assumption greatly reduces computational complexity, since we do not need to estimate each ICI coefficient in the ICI matrix.

4.3.2 Q-tap Equalizer:

To design Q-tap equalizer, instead of employing all subcarriers only a few subcarriers around the subject subcarrier are used to reduce the complexity. A simplified MMSE equalizer with only Q-taps is employed to reduce ICI [27]. For example, if Q is set as 3, two neighboring subcarriers are employed in the simplified equalizer. If the subject subcarrier is X_2, then the received signals Y_1, Y_2, Y_3 are all used in equalizer for better symbol detection of X_2.The derivation of a Q-tap MMSE equalizer is similar to the traditional MMSE equalizer. Rather than a huge equalization matrix involving all subcarriers, individual equalizer for each subcarrier is implemented.

Assume Q=2q+1, the Q-tap equalizer coefficients for the m th subcarrier are defined as

$$\vec{g} \equiv \left[g_{-q,m} g_{-q+1,m} \cdots g_{0,m} \cdots g_{q,m} \right]^T \tag{4.21}$$

Which minimize the mean square error(MSE)

$$E\left\{ \left| X_m - \hat{X}_m \right|^2 \right\} \tag{4.22}$$

Where $\hat{X}_m = (\vec{g}_m^Q)^H \vec{Y}_m^Q$, $\vec{Y}_m^Q$ is defined as

$$\vec{Y}_m^Q \equiv \left[Y_{(m-q)_N} Y_{(m-q+1)_N} \cdots\cdots Y_{(m)_N} \cdots Y_{(m+q)_N} \right]^T \tag{4.23}$$

and $(.)_N$ denotes the modulo operation with modulus N. $\vec{Y}_m^Q$

$$\vec{Y}_m^Q = \tilde{C}_m^Q \vec{X}_m^Q + W_m^Q \tag{4.24}$$

Where $\vec{X}_m^Q \equiv \left[X_{(m-2q)_N} X_{(m-2q+1)_N} \cdots\cdots X_{(m)_N} \cdots X_{(m+2q)_N} \right]^T_{4q+1\times 1}$

$$\vec{W}_m^Q \equiv \left[W_{(m-q)_N} W_{(m-q+1)_N} \cdots\cdots W_{(m)_N} \cdots W_{(m+q)_N} \right]^T$$

and $\tilde{C}_m^Q$ is defined as

$$\tilde{C}_m^Q \equiv \begin{bmatrix} C_{(m-q)(m-2q)} & \cdots & C_{(m-q),(m-q)} & \cdots & C_{(m-q),m} & 0 & 0 & \cdots & 0 \\ & \ddots & & \ddots & & & \ddots & & \\ 0 & \cdots & C_{m,(m-q)} & \cdots & C_{m,m} & \cdots & C_{m,(m+q)} & 0 & \cdots \\ & \ddots & & \ddots & & & \ddots & & \\ 0 & 0 & \cdots & 0 & C_{(m+q),m} & \cdots & C_{(m+q),(m+q)} & \cdots & C_{(m+q),(m+2q)} \end{bmatrix}_{Q\times 4q+1}$$

The Q-tap MMSE equalizer solution for the m th subcarrier is given by

$$\vec{g}_m^Q = (\tilde{R}_{\vec{Y}_m^Q \vec{Y}_m^Q})^{-1} \vec{\eta}_{X_m \vec{Y}_m^Q} \tag{4.25}$$

where

$$\tilde{R}_{\vec{Y}_m^Q \vec{Y}_m^Q} = E\left\{\vec{Y}_m^Q (\vec{Y}_m^Q)^H\right\} \qquad \text{and}$$

$$\vec{\eta}_{X_m \vec{Y}_m^Q} \equiv E\left\{\vec{Y}_m^Q X_m^*\right\}$$

Which can be calculated as

$$\begin{aligned} \vec{\eta}_{X_m \vec{Y}_m^Q} &\equiv E\left\{\vec{Y}_m^Q X_m^*\right\} \\ &= E\left\{\tilde{C}_m^Q \vec{X}_m^Q X_m^* + W_m^Q X_m^*\right\} \\ &= \sigma_X^2 \vec{v}_m^Q \end{aligned} \tag{4.26}$$

Where $\sigma_X^2 = E\left\{|X_m|^2\right\}$, and $\vec{v}_m^Q$ is related to m th column of the matrix $\tilde{C}_m^Q$ i.e

$$\vec{v}_m^Q = \left[C_{(m-q)_N,m} C_{(m-q+1)_N,m} \ldots . C_{m,m} \ldots . C_{(m+q)_N,m}\right]^T.$$

To calculate $\tilde{R}_{\vec{Y}_m^Q \vec{Y}_m^Q}$

$$\begin{aligned} \tilde{R}_{\vec{Y}_m^Q \vec{Y}_m^Q} &\equiv E\left\{\vec{Y}_m^Q (\vec{Y}_m^Q)^H\right\} \\ &= E\left\{\left(\tilde{C}_m^Q \vec{X}_m^Q + W_m^Q\right)\left(\tilde{C}_m^Q \vec{X}_m^Q + W_m^Q\right)^H\right\} \\ &= \tilde{C}_m^Q E\left\{\vec{X}_m^Q \left(\vec{X}_m^Q\right)^H\right\}\left(\tilde{C}_m^Q\right)^H + \sigma_w^2 \tilde{I}_{Q\times Q} \\ &= \sigma_X^2 \tilde{C}_m^Q \left(\tilde{C}_m^Q\right)^H + \sigma_w^2 \tilde{I}_{Q\times Q} \end{aligned} \tag{4.27}$$

Where $\tilde{I}_{Q\times Q}$ is the $Q\times Q$ identity matrix. Complete Q-tap MMSE equalizer solution can be obtained as

$$\vec{g}_m^Q = \left(\tilde{C}_m^Q \left(\tilde{C}_m^Q\right)^H + \frac{\sigma_w^2}{\sigma_X^2} \tilde{I}_{Q\times Q}\right)^{-1} \vec{v}_m^Q \tag{4.28}$$

This Q-tap MMSE equalizer involves only a $Q \times Q$ matrix inverse. If we set Q as small as possible without sacrificing much quality of service, computational complexity can be reduced. The full-tap (traditional) MMSE equalizer requires an $N \times N$ matrix inversion with N^3 complex multiplications, while a Q-tap MMSE equalizer needs N sets of $Q \times Q$ matrix inversions in total with only NQ^3 complex multiplications. Therefore, with the help of the Q-tap equalizer, the computational complexity in terms of the number of complex multiplications is reduced by $\left[1-\left(Q^3 / N^2\right)\right]\%$ .In the design of equalizer $\tilde{C}_m^Q$ matrix is required, so that needs to be estimated.

4.3.3 ICI Coefficient Estimation:

To get the equalizer coefficients, it requires ICI coefficient matrix at the receiver. The estimation of ICI matrix is as follows. Since the channel is fast time –varying, the estimation of ICI coefficients can depend only the currently received OFDM symbol block. For this purpose, a linear MMSE estimator for each element $C_{m,k}$ of $\tilde{C}$ is designed. The estimation of $C_{m,k}$ is achieved by using a linear combination of the currently received signal $\vec{Y}$ such that

$$\hat{C}_{m,k} = \vec{\beta}_{m,k}^H \vec{Y} \tag{4.29}$$

Where $\hat{C}_{m,k}$ denotes the estimation of $C_{m,k}$, and $\vec{\beta}_{m,k}$ is obtained by minimizing the MSE between $\hat{C}_{m,k}$ and $C_{m,k}$ .The solution to $\vec{\beta}_{m,k}$ is given by

$$\vec{\beta}_{m,k} = \tilde{R}_{\vec{Y}\vec{Y}}^{-1} \vec{\mu}_{C_{m,k}\vec{Y}} \tag{4.30}$$

Where $\tilde{R}_{\vec{Y}\vec{Y}}$ is the autocorrelation matrix of the received signal $\vec{Y}$, $\mu_{C_{m,k}\vec{Y}}$ is the cross-correlation vector between the received signal $\vec{Y}$ and $C_{m,k}$ i.e , $\tilde{R}_{\vec{Y}\vec{Y}} \equiv E\left\{\vec{Y}\vec{Y}^H\right\}$ and $\vec{\mu}_{C_{m,k}\vec{Y}} \equiv E\left\{\vec{Y}C_{m,k}^*\right\}$ .Let $R_{m_1 m_2}$, $0 \le m_1, m_2 \le N-1$ denote an element in the autocorrelation matrix $\tilde{R}_{\vec{Y}\vec{Y}}$ .Thus, $R_{m_1 m_2}$ can be written as

$$
\begin{aligned}
R_{m_1 m_2} &\equiv E\left\{Y_{m_1} Y_{m_2}^*\right\} \\
&= E\left\{\sum_{k_1=0}^{N-1} X_{k_1} C_{m_1,k_1} \left(\sum_{k_2=0}^{N-1} X_{k_2} C_{m_2,k_2}\right)^*\right\} + E\left\{W_{m_1} W_{m_2}^*\right\} \\
&= E\left\{\sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} X_{k_1} X_{k_2}^* C_{m_1,k_1} C_{m_2,k_2}^*\right\} + \sigma_w^2 \delta(m_1 - m_2) \\
&= \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} E\left\{X_{k_1} X_{k_2}^*\right\} E\left\{C_{m_1,k_1}, C_{m_2,k_2}^*\right\} + \sigma_w^2 \delta(m_1 - m_2)
\end{aligned}
\tag{4.31}
$$

The autocorrelation $E\left(X_{k_1} X_{k_2}^*\right)$ can be calculated follows. Assume the set P consists of N_p subcarrier indices on which the pilot symbols are transmitted. Thus, X_k is a pilot signal if $k \in P$. Therefore

$$
E\left\{X_{k_1} X_{k_2}^*\right\} = \begin{cases} E_s \delta(k_1 - k_2), & if\ k_1 \notin P \cap k_2 \notin P \\ X_{k_1} X_{k_2}^*, & if\ k_1 \in P \cap k_2 \in P \\ 0, & otherwise \end{cases}
\tag{4.32}
$$

Now to calculate the cross correlation between ICI coefficients C_{m_1,k_1} and C_{m_2,k_2}, by using properties of the WSSUS it can be seen as

$$
\begin{aligned}
E\left\{C_{m_1,k_1} C_{m_2,k_2}^*\right\} &= \frac{1}{N^2} \sum_{l=0}^{L-1} \sum_{l'=0}^{L-1} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E\left\{ e^{\frac{-j2\pi(k_1+\zeta_l)l}{N}} e^{\frac{j2\pi(k_1+\zeta_{l'})l'}{N}} \gamma_{l,n} \gamma_{l',n'}^* e^{\frac{-j2\pi(m_1-k_1-\zeta_l)n}{N}} e^{\frac{j2\pi(m_2-k_2-\zeta_{l'})n'}{N}} \right\} \\
&= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} \left\{ \phi_h(n-n') e^{\frac{-j2\pi(m_1-k_1)n + j2\pi(m_2-k_2)n'}{N}} \sum_{l=0}^{L-1} \phi_\tau(\tau_l) e^{\frac{-j2\pi(k_1-k_2)l}{N}} E\left\{ e^{\frac{j2\pi(n-n')\varepsilon_l}{N}} \right\} \right\} \\
&= \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} \phi_h(n-n') e^{\frac{-j2\pi(m_1-k_1)n + j2\pi(m_2-k_2)n'}{N}} J_0\left(\frac{2\pi(n-n')\varepsilon_D}{N}\right) \sum_{l=0}^{L-1} \phi_\tau(\tau_l) e^{\frac{-j2\pi(k_1-k_2)l}{N}}
\end{aligned}
$$

To find the cross correlation between the received signal and the ICI coefficient, let ρ_{m_1}, for $0 \le m_1 \le N-1$, represent an element in the cross-correlation vector $\bar{\mu}_{C_{m,k}\bar{Y}}$.

$$
\begin{aligned}
\rho_{m_1} &\equiv E\left\{C_{m,k} Y_{m_1}^*\right\} \\
&= E\left\{C_{m,k}\left(\sum_{k_1=0}^{N-1} X_{k_1} C_{m_1,k_1} + W_{m_1}\right)^*\right\} \\
&= \sum_{k_1=0}^{N-1} E\left\{X_{k_1}^*\right\} E\left\{C_{m,k} C_{m_1,k_1}^*\right\} + E\left\{C_{m,k} W_{m_1}^*\right\}
\end{aligned}
\tag{4.33}
$$

It is assumed that ICI coefficient $C_{m,k}$ and AWGN $W_{m_1}^*$ are uncorrelated, and $E\left\{C_{m,k} W_{m_1}^*\right\}$ in above equation becomes zero. If the transmitted signal X_k is not a pilot, $E\left(X_k^*\right)=0$.Thus, the cross correlation ρ_{m_1} can be simplified as

$$
\rho_{m_1} = \sum_{\substack{k_1=0 \\ k_1 \in P}}^{N-1} X_{k_1}^* E\left\{C_{m,k} C_{m_1,k_1}^*\right\} \tag{4.34}
$$

A Q-tap equalizer would reduce the computational complexity by only sacrificing performance degradation slightly. Therefore, instead of trying to estimate every element in the matrix $\tilde{C}$, we only need to estimate certain elements in the $\tilde{C}$ matrix, which are the elements specified by Eq.(4.24).

Chapter 5

Simulation and Results

In this chapter, we present the system structure and simulation parameters. Different simulations are carried out to show performances of OFDM system with and with out ICI cancellation methods. To compare the performance of ICI cancellation method, we plot the SNR vs BER curves for each system. For all simulations MATLAB was used.

The OFDM system is explained in chapter 3 and ICI cancellation methods are discussed in chapter 4.The following systems are considered for simulations.

1) OFDM system with Phase noise and with out ICI cancellation method

2) OFDM system with Phase noise and Data conjugate method

3) OFDM system with Phase noise and Symmetric data conjugate method

4) OFDM system with Doppler frequency shift and a Q-tap equalizer

5.1 Simulation of Standard OFDM System:

System parameters:

Modulation	: 64 QAM
No of subcarriers	: 64
Standard Deviation of PHN	: 3 Degrees
Band width	: 20 MHz

Step 1: Random data is generated first by the randint() function.

Step 2: Then the generated data is modulated using Qammod() function.

If 16-QAM is employed as the modulation technique then the incoming binary bits are grouped in to 4 bits each and then each group is mapped to the constellation plane corresponding to 16-QAM. The constellation plane of 16-QAM is shown in Figure 5.1.

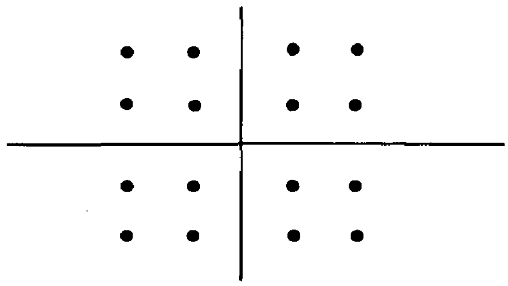


Figure 5.1: Constellation Plane for 16-QAM

Step 3: After serial to parallel conversion take IFFT to get the time domain symbols.

Step4 : Pass the time domain data through AWGN channel. In presence of Phase noise, generate the PHN as a Gaussian samples and add phase noise after passing through AWGN channel.

Step 5: At the receiver take FFT of the received symbols and after parallel to serial conversion Demodulate the symbols using Qamdemod() function. The flow chart is given in Fig 5.2.Fig 5.3

Shows the flow chart for the simulation of OFDM system with self cancellation methods in presence of Phase noise. Here ICM is the ICI cancelling modulator, ICD is the ICI cancelling demodulator, are the extra blocks included in the simulation to implement self cancellation methods.

5.2 Performance Comparison of the Conventional OFDM system and OFDM with ICI Cancellation methods:

The flow charts in figures 5.2, 5.3 give idea of simulation process. In each simulation number of subcarriers, symbol sizes are initialized and data is generated. Simulations are carried out for different SNR values and SNR vs BER plots are made.In Fig 5.4 Carrier to interference ratio of different ICI cancellation methods are compared, with conventional OFDM system and it is observed that Symmetric data conjugate method provides better performance compared to Data conjugate method and Conventional OFDM.

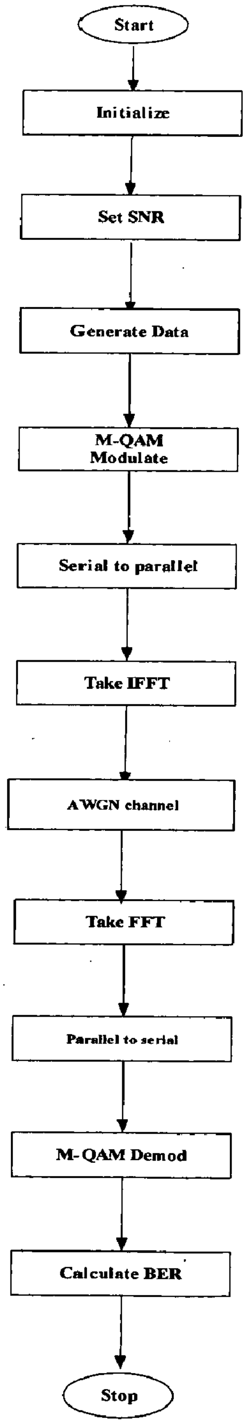
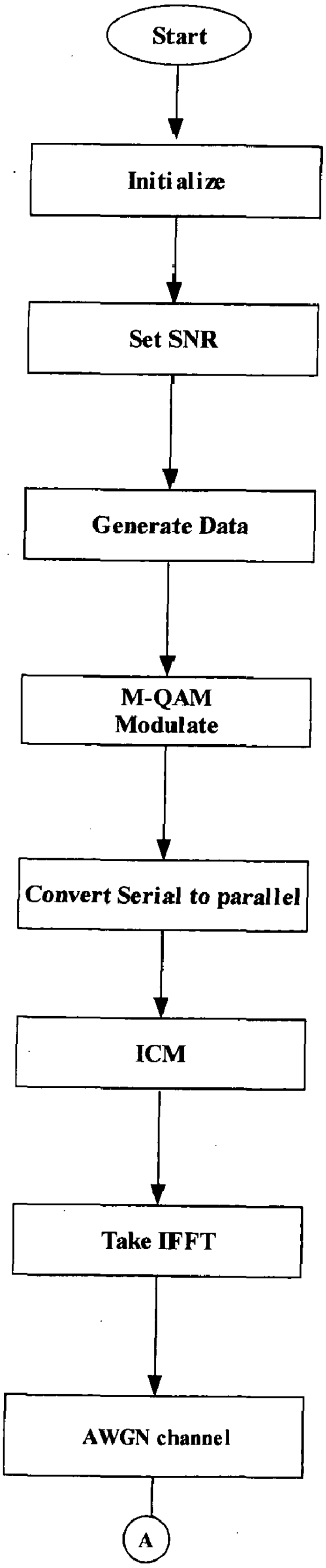


Fig 5.2 Flow Chart for Simulation of Standard OFDM System


Start
Initialize
Set SNR
Generate Data
M-QAM
Modulate
Convert Serial to parallel
ICM
Take IFFT
AWGN channel
A

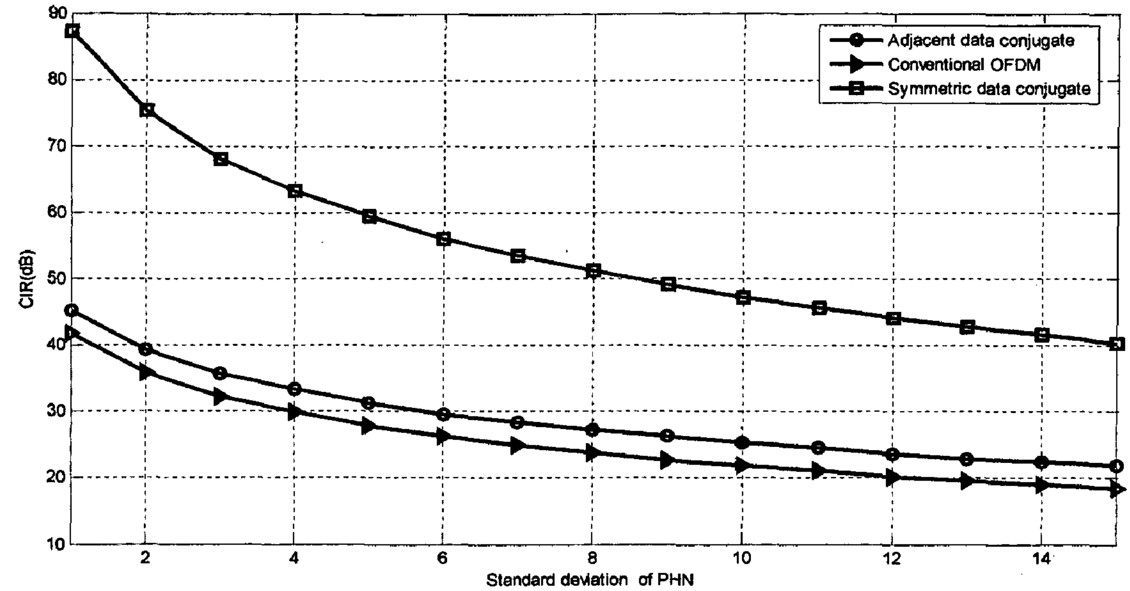


Fig 5.4 Carrier to Interference ratio comparison of self cancellation methods

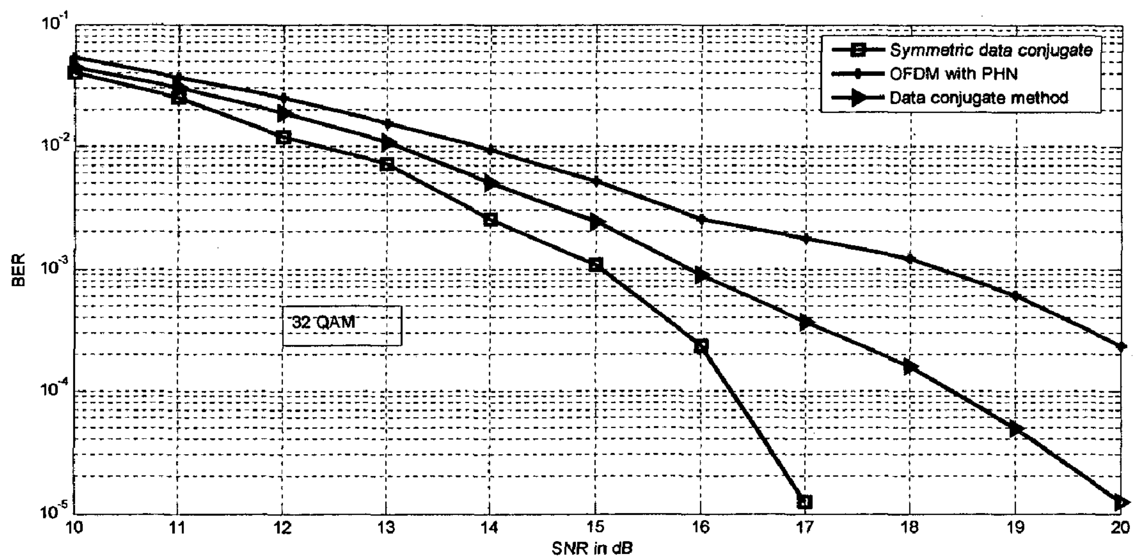


Fig 5.5 BER Comparison of Symmetric Data Conjugate Method, Data Conjugate Method, Conventional OFDM for 32QAM.

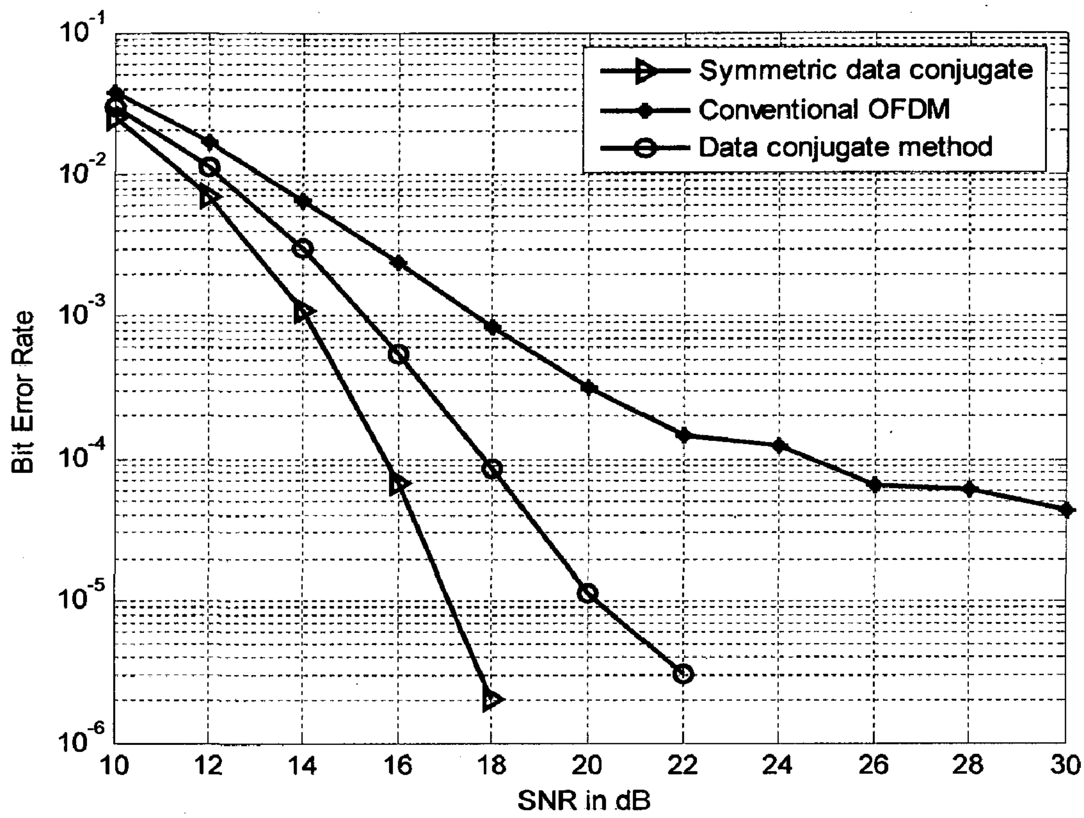


Fig 5.6 BER Comparison for 64QAM

Figures 5.5, 5.6 presents the BER comparison of the OFDM system with ICI cancellation methods and with out ICI cancellation methods. In fig 5.5 32QAM is used for modulating data, and Phase noise standard deviation in both figures 5.5,5.6 is taken as 3 degrees. Where as in Fig 5.6 Data is modulated using 64QAM. From both these figures it is evident that symmetric data conjugate method is better than Conventional OFDM and Data conjugate method.

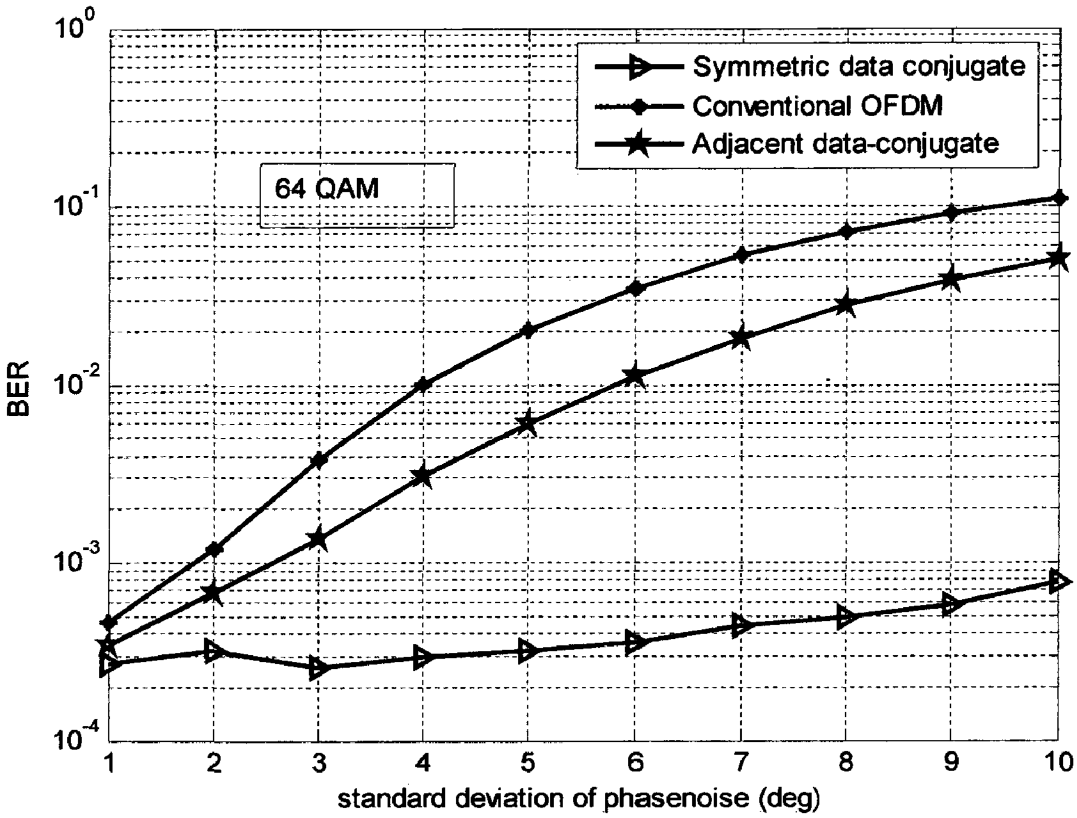


Fig 5.7 BER Comparison for Variation of PHN Standard Deviation.

In Fig 5.7 the BER comparison of Symmetric data conjugate method, Data conjugate method, Conventional OFDM for 64QAM modulated data are shown, as the standard deviation of Phase noise varies from 1 to 10 degrees. As the standard deviation of the PHN increases the BER also increases due to increase in noise power. In figure 5.8 BER comparisons for the 32QAM modulated data, as the Phase noise standard deviation varies from 1 to 10 degrees is shown. And from Fig 5.7,5.8 it can be seen that Symmetric data conjugate method gives better performance in terms of BER. BER of Symmetric data conjugate method increases slowly when compared to conventional OFDM and Data conjugate method, as the Phase noise standard deviation increases.

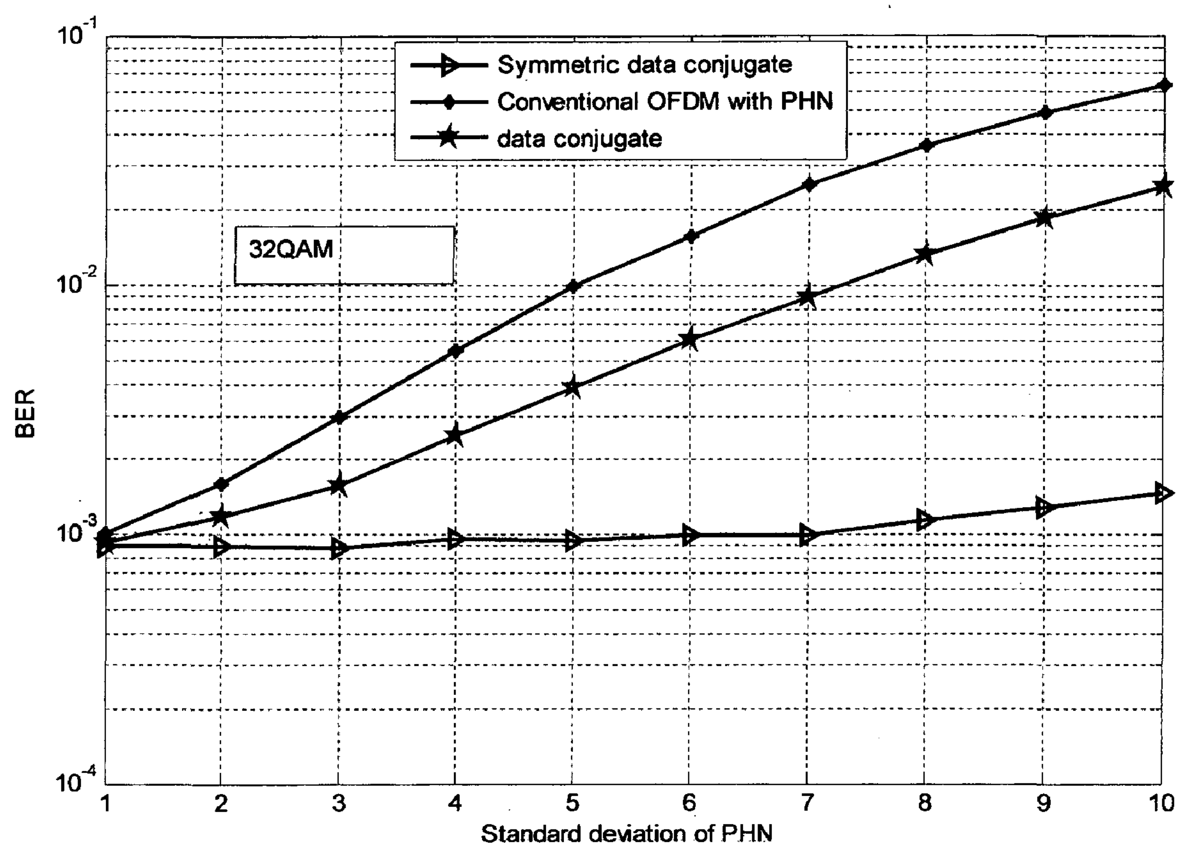


Fig 5.8 BER Comparison with PHN Standard Deviation Variation.

Fig 5.9 shows the comparison of BER for two different Phase noise levels. In this simulation 64QAM is employed.The BER simulations for Phase noise standard deviations of 4 degrees and 7 degrees is plotted and compared. From this simulations we can see that Symmetric data conjugate method's advantage is more in presence of more Phase noise.The BER advantage of Symmetric data conjugate method at Phase noise level 7degrees is more, when compared to Phase noise level at 4degrees. We also observe that Data conjugate method also provides better performance when compared to conventional OFDM system in presence of Phase noise.

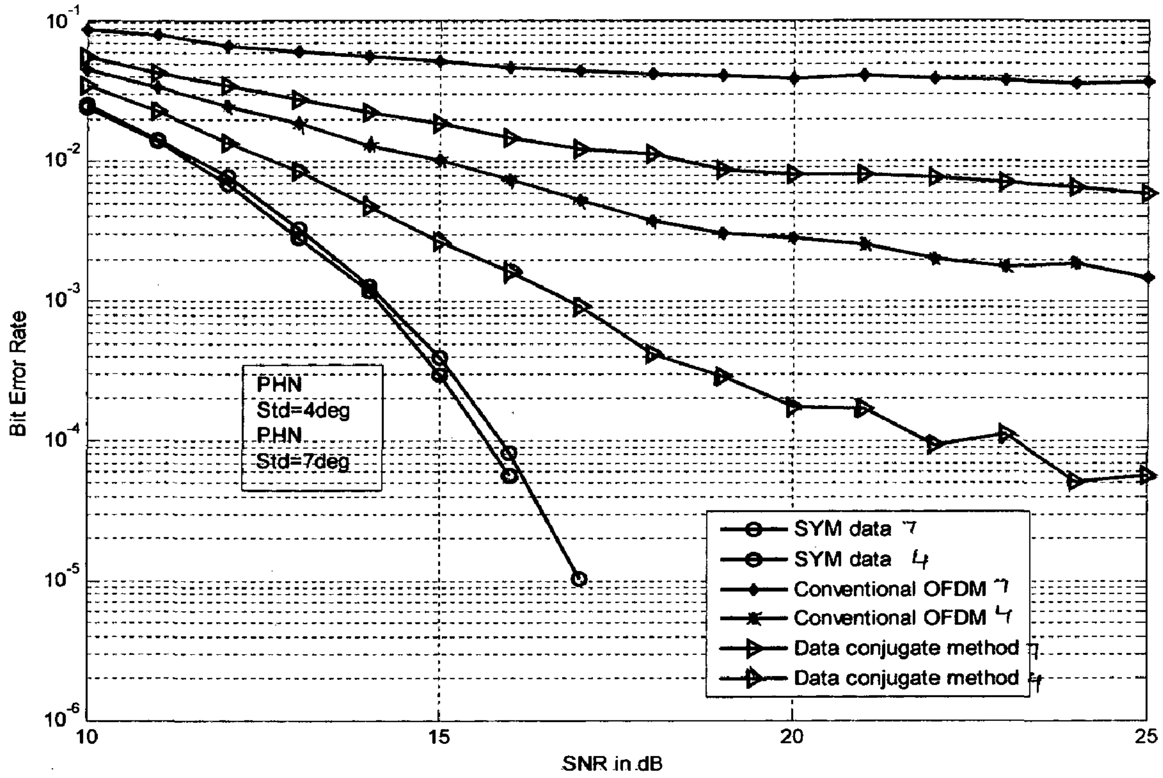


Fig 5.9 BER Comparison for PHN levels of 4degrees, 7 degrees.

Fig 5.10 gives the simulation results for BER comparison of OFDM with Symmetric data conjugate method. In this simulation OFDM with 8QAM, and Symmetric data conjugate method with 16 QAM are considered, so that the data rate of both systems are equal. Here we see that Symmetric data conjugate method gives better performance than conventional OFDM system even for the same data rate.

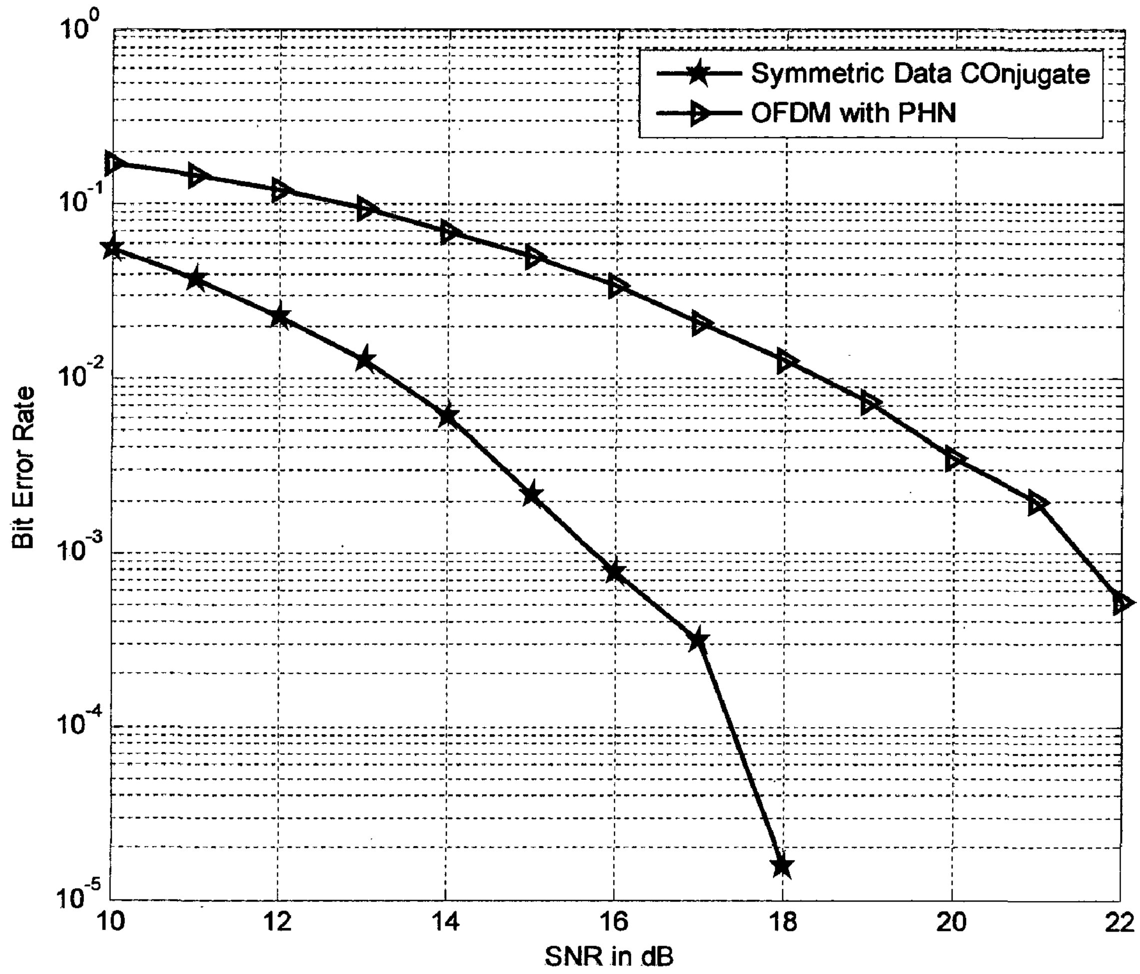


Fig 5.10 BER Comparison for OFDM with 8QAM, Symmetric data conjugate with16 QAM.

Fig 5.11 gives the BER comparions of different methods in presence of two path channel. Here a two path channel is generated using Rayleighchan() function and the data is passed through this channel. We observe that even in presence of multipath channel the Symmetric data conjugate method gives better performance when compared to Data conjugate method and conventional OFDM system with phase noise.

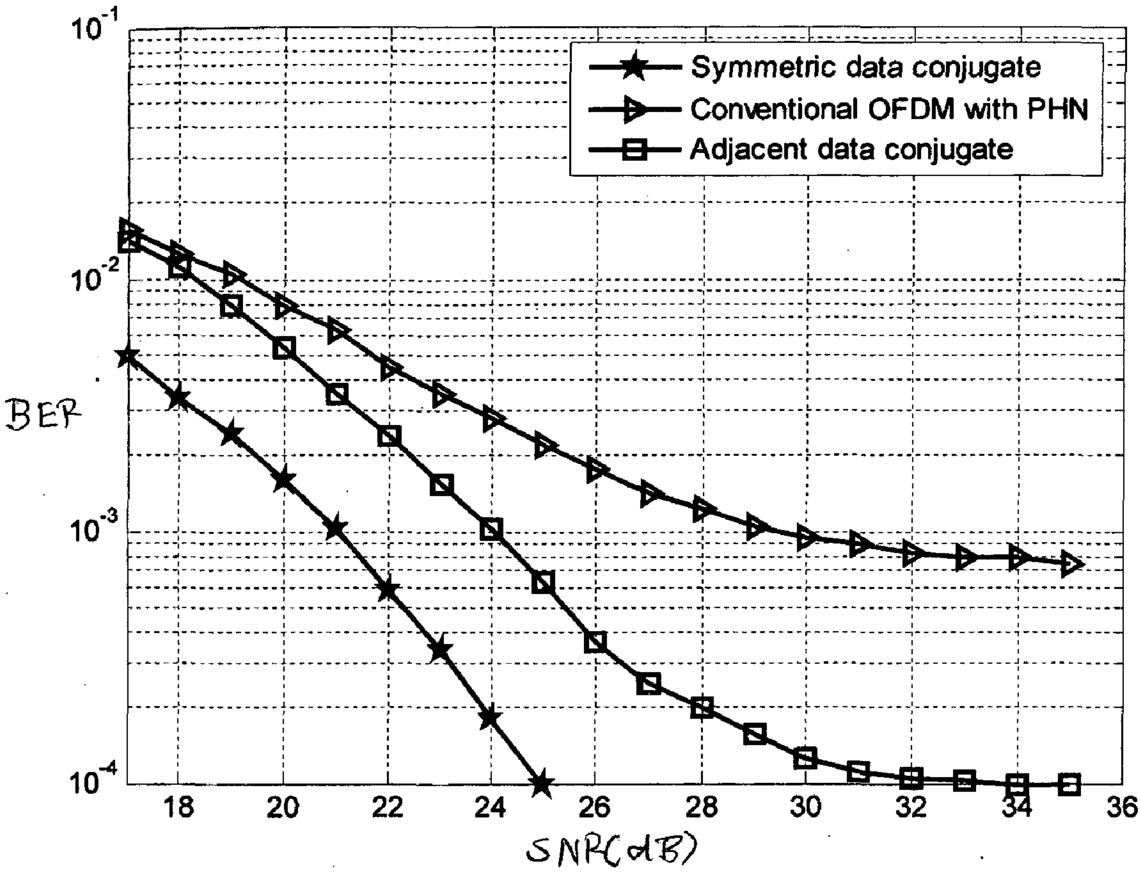


Fig 5.11 BER Performance in a Two Path Channel

Simulation results of Q-tap equalizer are shown in Fig 5.12. In this simulation symbol error rate is compared for Q=1,Q=3. 256 sub carriers are used with Np/N=1/8, $\epsilon_D = 0.8$. We observe from the simulation that as the number taps Q increases, SER decreases. In this simulation COST 207 channel and QPSK modulation are used.

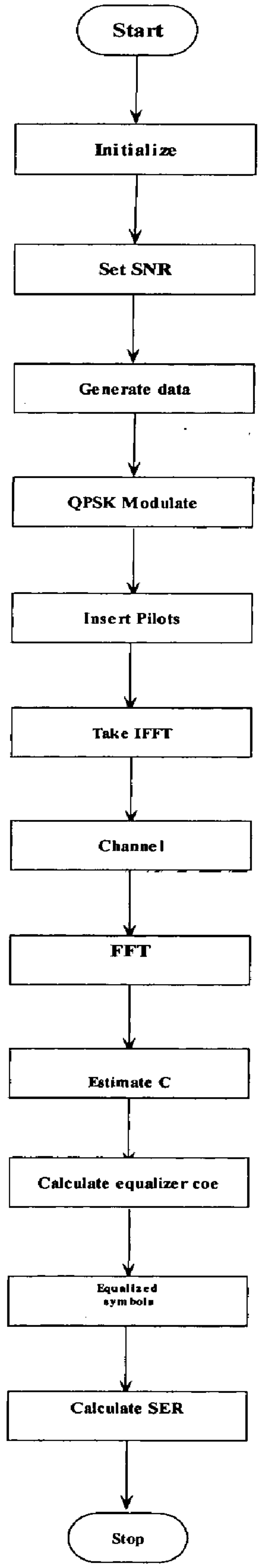


Fig 5.12 Flow Chart for the Simulation of SER with Q-tap equalizer