EQUALIZATION OF OFDM OVER DOUBLY SELECTIVE CHANNELS

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By

DILEEP .T

DEPARTMENT OF ELECTRONICS AND COMPUTER ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE - 247 667 (INDIA)

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report entitled, "**EQUALIZATION OF OFDM OVER DOUBLY SELECTIVE CHANNELS**" towards the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with specialization in **Communication Systems**, submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee, Roorkee (India) is an authentic record of my own work carried out during the period from July 2006 to June 2007, under the guidance of **Dr. D. K. MEHRA, Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology Roorkee.**

I have not submitted the matter embodied in this dissertation for the award of any other Degree or Diploma.

Date: 29-06-2007

Place: Roorkee

Dileep

DILEEP .T

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

Date: 29.6.07

Place: Roorkee

DMehra

Dr. D. K. MEHRA,
Professor, E&C Department,
IIT Roorkee,
Roorkee – 247 667 (India).

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DILEEP .T

ABSTRACT

Orthogonal Frequency-Division Multiplexing (OFDM) is now regarded as a feasible alternative to the conventional single carrier modulation techniques for high data rate communication systems, mainly because of its inherent equalization simplicity. It is well known fact that orthogonal frequency division multiplexing (OFDM) provides a practical solution to counter the inter symbol interference problem imposed by the frequency selective fading channel. The application of OFDM in fast frequency selective fading multipath channels is fraught with several difficulties many of which have not been practically treated in the literature. Current trends in broadband wireless communication system dictate that channel time variation will soon play an important role in OFDM system. The primary advantage of cyclic prefix OFDM in time invariant frequency selective fading channels i.e. The absence of inter symbol interference and inter carrier interference do not carry over to time and frequency selective i.e. doubly selective channels. As a result the standard zero forcing and minimum mean square error equalization techniques will become prohibitively complex. Block linear equalizers and serial linear equalizers are normally designed to combat doubly selective fading. The implementation of block linear equalizers, which collect and process all the available data in the received frame in blocks, leads to an unsustainable computational complexity. On the other hand, serial linear time varying (LTV) equalizers, which process few data at a time, exhibit a good tradeoff between complexity and performance. In this dissertation work the performance of block linear and serial linear equalizers are evaluated for a single input multi output (SIMO) and multi input multi output (MIMO) systems for block transmission over doubly selective channels. These approaches are extended for the equalization of OFDM system.

CONTENTS

CHAPTER ONE

INTRODUCTION

Access speeds ranging from few hundreds of kilo bits per sec to few megabits per sec are required in many wireless applications such as high speed internet access, networking, digital audio and video broad casting. Orthogonal frequency division multiplexing (OFDM) has emerged as one of the practical technique for such applications. The increasing need to provide either high data rate services for low mobility users or low data rate services for high mobility users give rise to frequency selective propagation and time selectivity.

Successful communication over a wireless link entails overcoming two main hurdles. The first is posed by the effects of multi-path propagation [1, 2]. In simple terms, reflections from physical objects produce multiple versions of the same signal at the receiver. The cumulative effect at the receiver is a signal composed of various time and phase delayed echoes. When the delay spread of this received signal exceeds the duration of transmitted signals, energy from each transmitted symbol spills over and contaminates neighboring symbols. This effect is commonly known as inter-symbol interference (ISI). Such channels are referred to as frequency selective channels, since the channel response is not uniform over the bandwidth of the transmitted signal. The second hurdle to be overcome in a wireless communication system is induced by mobility of the communicating devices. As a result of motion, time variations are introduced in the wireless communication channel. Motion results in Doppler shifts in frequency of the transmitted signal at the receiver. For instance, when a pure tone is transmitted through a time varying channel, the observed signal at the receiver is composed of a band of frequencies. Such a channel is termed time selective, since its characteristics vary with time. Channels that are frequency as well as time selective are referred to as "doubly selective". There is a growing demand for higher data rate systems that can function in a highly mobile environment. This mandates the design of communication systems that can function in doubly selective channels.

An accurate description of the doubly selective channel is required for the design of the low complexity detection strategies for the wireless communication system received signal is distorted. Frequency selective fading is due to time dispersion of the transmitted symbols within the channel. Thus channel induces inter symbol interference. Viewed in frequency domain certain frequency components in the received signal spectrum have greater gains than the other.

Depending on how rapidly the transmitted base band signal changes as compared to the rate of change of the channel a wireless channel may be classified either as a fast fading channel or slow fading channel. In a fast fading channel the channel impulse response changes rapidly within the symbol duration. This causes frequency dispersion (also called time selective fading) due to Doppler spreading which leads to signal distortion. In a slow fading channel the channel impulse response changes at a rate much slower than the transmitted base band signal. In this case the channel may be assumed to be static over one or several reciprocal bandwidth intervals. In frequency domain this implies that the Doppler spread of the channel is much less than the bandwidth of the base band signal.

These four types of fading allows us to partition a Cartesian plane having time dispersion as abscissa and Doppler spread as ordinate into four rectangular regions corresponding to four different physical situations as shown in Figure 1.1.If a channel is fast fading and frequency selective it is known as a doubly selective channel.

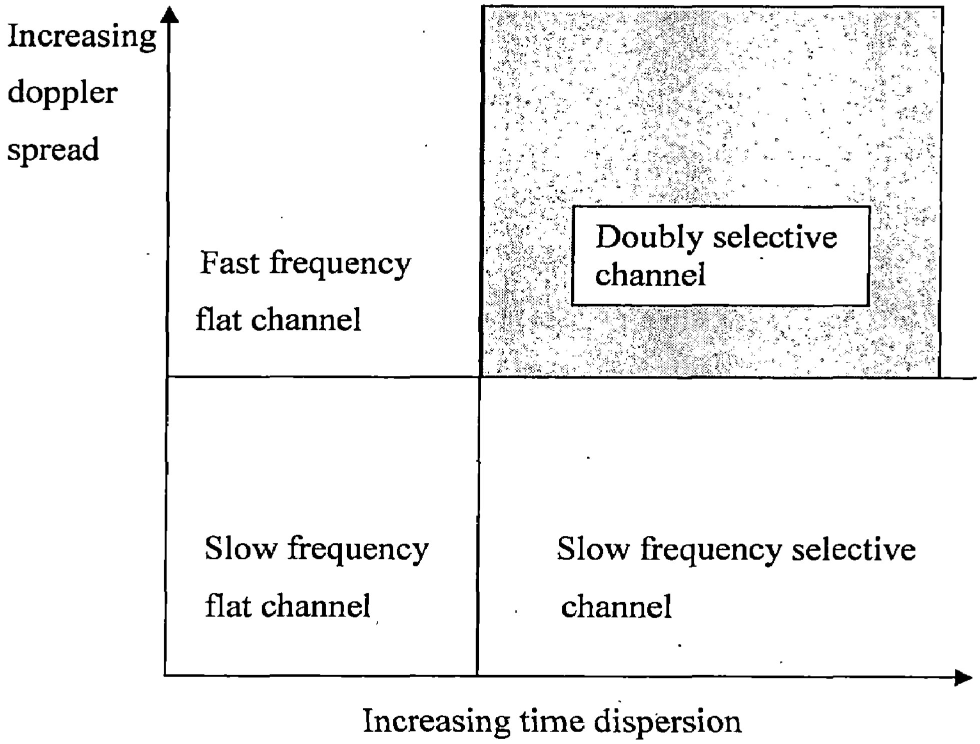


Figure 1.1 Four types of wireless channels

1.2 OFDM over doubly selective channel

Orthogonal frequency-division multiplexing (OFDM) has attracted a lot of attention, due to its simple implementation and robustness against frequency-selective channels. In doubly selective channels, the time variation of the channel over an OFDM block destroys the orthogonality between subcarriers and so induces intercarrier interference (ICI). In addition to this, interblock interference (IBI) arises when the channel delay spread is larger than the cyclic prefix (CP), which again results in ICI. An emerging application that uses OFDM as a transmission technique is digital video broadcasting (DVB).DVB encounters long-delay multipath channels. Using a CP of length equal to the channel order, results in a significant decrease in throughput. On the other hand, applying DVB over mobile channels for high speed terminals induces ICI which has been shown to decrease performance significantly. Hence, equalization techniques are required to restore the orthogonality and so to eliminate ICI/IBI. In this report we will discuss a TV-FIR time domain equalizer to convert the doubly selective channel whose delay-spread is larger than the CP into a purely frequency-selective channel with a delay spread that fits within the CP. By doing this, we restore the orthogonality between subcarriers and eliminate both ICI and IBI.

1.3 A brief history of Equalizers for doubly selective fading

Block linear Equalizers (BLE) and serial linear equalizers (SLE) are normally designed to combat doubly selective fading. As their name suggests, SLEs apply the same filters to every received symbol. Though serial equalizers can be used in block transmission systems, they do not fully exploit the structure of the received blocks. On the other hand, BLE*s* apply different filters to symbols of the received block and can result in improved BER performance. BLEs are usually far more complex to design and implement.

Block linear equalizers (BLEs) for frequency-selective channels require only a single receive antenna for the zero-forcing (ZF) solution to exist [5]. A frequency-selective channel can be diagonalized by means of the discrete Fourier transform (DFT), the design and implementation complexity can be reduced, at the cost of a slight decrease in performance. On the other hand, serial linear equalizers (SLEs), more specifically, finite impulse response (FIR) equalizers, for frequency-selective channels generally require at least two receive antennas for the ZF solution to exist, but allow for a flexible tradeoff between complexity and performance [6, 7]. BLEs are preferred over SLEs for frequency selective channels because of their superior BER performance and lesser complexity.

Recently, equalizers have also been developed for doubly selective channels [8]. Authors used a TIV FIR serial equalizer to equalize a BEM FIR channel, but this requires many receive antennas for the linear zero-forcing solution to exist .As for frequency-selective channels, BLEs for doubly selective channels only require a single receive antenna for the ZF solution to exist. However, since a doubly selective channel cannot be diagonalized by means of a channel-independent transformation (such as the DFT), they cannot be simplified and hence are always complex to design and implement. Barhumi, Leus and Moonen used a BEM FIR serial equalizer to equalize a BEM FIR channel and it require only two receive antennas for the zero-forcing solution to exist [9, 10].

Decision Feedback Equalizers (DFEs) have been previously proposed in literature for the case of frequency-selective channels, some based on block processing [11 ,12] and others based on serial processing by means of finite impulse response (FIR) filters [13]. Barhumi, Leus and Moonen extended their work of time varying SLE and designed serial decision feed back equalizer (SDFE) for doubly selective channel [14]. The proposed TV FIR DFE consists of a TV FIR feed forward filter, a TV FIR feedback filter, and a decision device. Basis expansion model (BEM) is used to model the TV FIR feed forward and feedback filters.

1.4 Statement of the problem

The present work encompasses a study of different equalizers for the block transmission over doubly selective channels. Channel is modeled using basis expansion model and the channel coefficients of BEM are assumed to be known at the receiver. In this report we will discuss about Block linear equalizers(BLE) and Time varying FIR serial linear equalizers (TV FIR SLE).The aim of the work is to compare the performance and complexity of these equalizers for different channel conditions and for different number of transmit and receive antennas. At the end performance of TV FIR SLE and BLE are compared for an OFDM system.

1.5 Organization of the report

The work embodied in this dissertation has been arranged as detailed below.

Chapter 2: Doubly selective channel

In this chapter first we will present characterization of fading multipath channel and discuss the conditions for it to become doubly selective channel .Then we will discuss about random model and basis expansion model for the doubly selective channel.

Chapter 3: Equalization of doubly selective channel

In this chapter first we will discuss about block linear equalizers and time varying FIR serial linear equalizer (TV FIR SLE) for doubly selective channel. Based on the channel model and the equalizer structure the zero forcing and minimum mean square error solutions are considered. The condition for the existence of zero forcing solution is considered for both equalizers. At the end we will see a comparison between BLE and TV FIR SLE in terms of complexity.

Chapter 4: Equalization of OFDM over doubly selective channel

This chapter introduces a simple OFDM system, and then Time Domain Equalizer model (TEQ) for the equalization of the doubly selective channel is discussed. At the end we will discuss how a TV FIR SLE can be used as a time domain equalizer.

Chapter 5.Simulation results and Conclusions

In this chapter we will compare the performance of block linear equalizer and TV FIR serial linear equalizer for SISO, SIMO and MIMO systems. The performances of the equalizers are tested under ZF and MMSE conditions. The variations in the performance of these equalizers for different Doppler spreads are also studied. In the second part of this chapter we will compare the performance block linear equalizer and time varying FIR serial linear equalizer for SIMO and MIMO OFDM systems.

CHAPTER TWO

DOUBLY SELECTIVE CHANNEL

The performance of block transmission over doubly selective channel depends on the knowledge of the mobile radio channel. Research in this area has been going on for some years and has concentrated mainly on two areas (i) collection of results from extensive measurement campaigns (ii) From these measurements channel models are derived which should fulfill the following two criteria.

- They must be simple enough to allow an analytical computation of basic system performance.
- They must be very close to physical reality.

These requirements are contradictory so models of different complexity and accuracy have been developed

Due to the increasing demand on mobile multimedia services with high data rates for users with high velocities, new propagation models have to be derived for broadband mobile radio channels. These models can efficiently be used in computer simulations in order to analyze the performance of broadband mobile communication systems. Previously defined propagation models, such as GSM-models by COST 207 , have been developed for bandwidths lower than 1 MHz. Hence, these models are not well suited for systems with larger bandwidth like UMTS or DVB-T having a bandwidth of 5 MHz and supporting data rates up to 2 Mbit/sec per user, depending on the service environment and the mobility characteristics.

Because of the movement of the mobile station and, hence, the changing channel characteristics, the channel has to be treated as a time-variant, multipath fading channel. This can be characterized by statistical channel models such as WSSUS (Wide Sense Stationary Uncorrelated Scattering)-models. WSSUS-models [15] require only two sets of parameters to characterize fading and multipath effects: the *Power Delay Profiles (PDP)* and the *Doppler Power Spectra (DPS)*. Both parameter sets can be described by a single function, which is called *scattering function.* Before discussing models for time varying multipath fading channel we will be presenting fading multipath channel and its characterization using correlation functions and power spectra.

2.1 Characterization of fading multipath channel [1]

The three most important effects of multipath fading are:

- Rapid changes in signal strength over a small traveled distance or time interval,
- Random frequency modulation due to varying Doppler shifts on different multipath signals, and
- Time dispersion (echoes) caused by multipath propagation delays.

Let the transmitted signal be

$$S(t) = \mathrm{Re}\left[S_l(t)e^{j2\pi f_c t}\right] \tag{2.1}$$

Where $S_l(t)$ is the low pass representation of the transmitted signal $S(t)$

The received band pass signal can be expressed in a form

$$y(t) = \sum_n \alpha_n(t)S(t - \tau_n(t)) \tag{2.2}$$

Where $\alpha_n(t)$the attenuation factor for the signal received on the n'th path and $\tau_n(t)$ is the propagation delay for the n'th path. Substitution for S(t) from (2.1) into (2.2) yields the result

$$y(t) = \mathrm{Re}\left\{\left[\sum_n \alpha_n e^{-j2\pi f_c \tau_n(t)} S_l(t - \tau_n(t))\right] e^{j2\pi f_c t}\right\} \tag{2.3}$$

Equivalent low pass received signal is given by

$$y_l(t) = \sum_n \alpha_n(t)e^{-j2\pi f_c \tau_n(t)} S_l(t - \tau_n(t)) \tag{2.4}$$

Thus the equivalent low pass channel is described by the time variant impulse response as

$$h(t;\tau) = \sum_n \alpha_n(t)e^{-j2\pi f_c \tau_n(t)} \delta(\tau - \tau_n(t)) \tag{2.5}$$

Where $h(t;\tau)$ represents the response of the channel at time 't' due to an impulse applied at time $t-\tau$. Let $S_l(t)=1$ for all τ and hence the received signal for the case of discrete multipath equation reduces to

$$y_l(t) = \sum_n \alpha_n(t)e^{-j2\pi f_c \tau_n(t)} = \sum_n \alpha_n(t)e^{-j\theta_n(t)} \tag{2.6}$$

Where $\theta_n(t) = 2\pi f_c \tau_n(t)$. Thus the received signal consists of the sum of a number of time variant vectors having amplitudes $\alpha_n(t)$ and phases $\theta_n(t)$. Large dynamic changes in the medium are required for $\alpha_n(t)$ to change sufficiently to cause a significant change in the received signal. $\theta_n(t)$ changes by 2π radians whenever τ_n change by $1/f_c$ and it is a small number. Hence θ_n can change by 2π radians with relatively small motions of the medium. Since $\tau_n(t)$ is random $y_l(t)$ in (2.6) can be modeled as a random process. When there are large number of paths, the central limit theorem can be applied that is $y_l(t)$ may be modeled as a complex valued Gaussian random process.

2.1.1 Channel correlation functions and power spectra.

The characteristic of fading multipath channel can be defined in terms of correlation functions and power spectral density functions. When the equivalent low pass channel impulse response h(t;τ) is characterized as a complex value random process in the t variable and it is assumed that h(t;τ) is wide sense stationary (WSS), then autocorrelation function (ACF) of h(t;τ) can be defined as

$$\phi_h(\Delta t, \tau_1, \tau_2) = \frac{1}{2} E[h^*(t;\tau_1) h(t+\Delta t;\tau_2)] \tag{2.7}$$

In most radio transmission media the attenuation and phase shift of the channel associated with the path delay τ_1 is uncorrelated with the attenuation and phase shift associated with τ_2. This is usually called uncorrelated scattering. Applying this assumption in (2.7) gives

$$\frac{1}{2} E[h^*(t,\tau_1) h(t+\Delta t,\tau_2)] = \phi_h(\Delta t;\tau_1)\ \delta(\tau_1 - \tau_2) \tag{2.8}$$

If Δt=0 the resulting ACF $\phi_h(0,\tau) = \phi_h(\tau)$ is the average power output of the channel as a function of time delay τ. For this reason $\Phi_h(\tau)$ is called the multipath intensity profile or delay power spectrum of the channel. The range of values of τ over which $\phi_h(\tau)$ is essentially nonzero is called the multipath spread of the channel and it is denoted by T_m.

Analogous characterization in frequency domain is obtained by taking fourier transform of h(t;τ) Thus

$$h(t;f)=\int_{-\infty}^{\infty} h(t;\tau)e^{-j2\pi f\tau}d\tau \qquad (2.9)$$

Under the assumption that the channel is WSS the ACF

$$\phi_h(\Delta t,f_1,f_2)=\frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[E\left(h^*(t;\tau_1)h(t+\Delta t;\tau_2)e^{j2\pi(f_1\tau_1-f_2\tau_2)}d\tau_1 d\tau_2\right)\right]$$

$$=\int_{-\infty}^{\infty}\phi_h(\Delta t;\tau_1)e^{-j2\pi\Delta f\tau_1}d\tau_1=\phi_h(\Delta t;\Delta f) \qquad (2.10)$$

$\phi_h(\Delta t;\Delta f)$ is the fourier transform of the multipath intensity profile known as spaced frequency spaced time correlation function of the channel. The multipath spread is a measure of the coherence bandwidth of the channel.

$$\Delta f_c \approx \frac{1}{T_m} \qquad (2.11)$$

Where Δf_c denote the coherence bandwidth of the channel. If Δf_c is small compared to the bandwidth of the transmitted signal, the channel is said to be frequency selective. In this case signal is severely distorted by the channel. If $(\Delta f)_c$ is large in comparison with the band width of the transmitted signal, the channel is said to be frequency non selective.

Fourier transform of $\phi_h(\Delta t;\Delta f)$ with respect to the variable Δt is the function $S_h(\lambda;\Delta f)$ given by

$$S_h(\lambda;\Delta f)=\int_{-\infty}^{\infty}\phi_h(\Delta t;\Delta f)e^{-j2\pi\lambda\Delta t}d\Delta t \qquad (2.12)$$

The function $S_h(\lambda)$ is a power spectrum that gives the signal intensity as a function of the Doppler frequency 'λ'. $S_h(\lambda)$ is known as the doppler power spectrum of the channel. The range of values of 'λ' over which $S_h(\lambda)$ is essentially nonzero is called the Doppler spread f_d of the channel. Since $S_h(\lambda)$ is related to $\phi_h(\Delta t)$ by the fourier transform, the reciprocal of f_d is a measure of the coherence time of the channel.

$$(\Delta t)_c=\frac{1}{f_d} \qquad (2.13)$$

Where $(\Delta t)_c$ denotes the coherence time. Slowly changing channel has a large coherence time or a small Doppler spread. Similarly,

$$S(\lambda;\tau)=\int_{-\infty}^{\infty}\phi_h(\Delta t;\tau)e^{-j2\pi\lambda\Delta t}d\Delta t \tag{2.14}$$

$$S(\lambda;\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi_h(\Delta t;\Delta f)e^{-j2\pi\lambda\Delta t}e^{j2\pi\tau\Delta f}d\Delta t d\Delta f \tag{2.15}$$

$S(\lambda;\tau)$ is called the scattering function of the channel. It provides a measure of the average power output of the channel as a function of the time delay 'τ' and the doppler frequency 'λ'.Relations between channel correlation functions and power spectra are given in the figure 2.1.

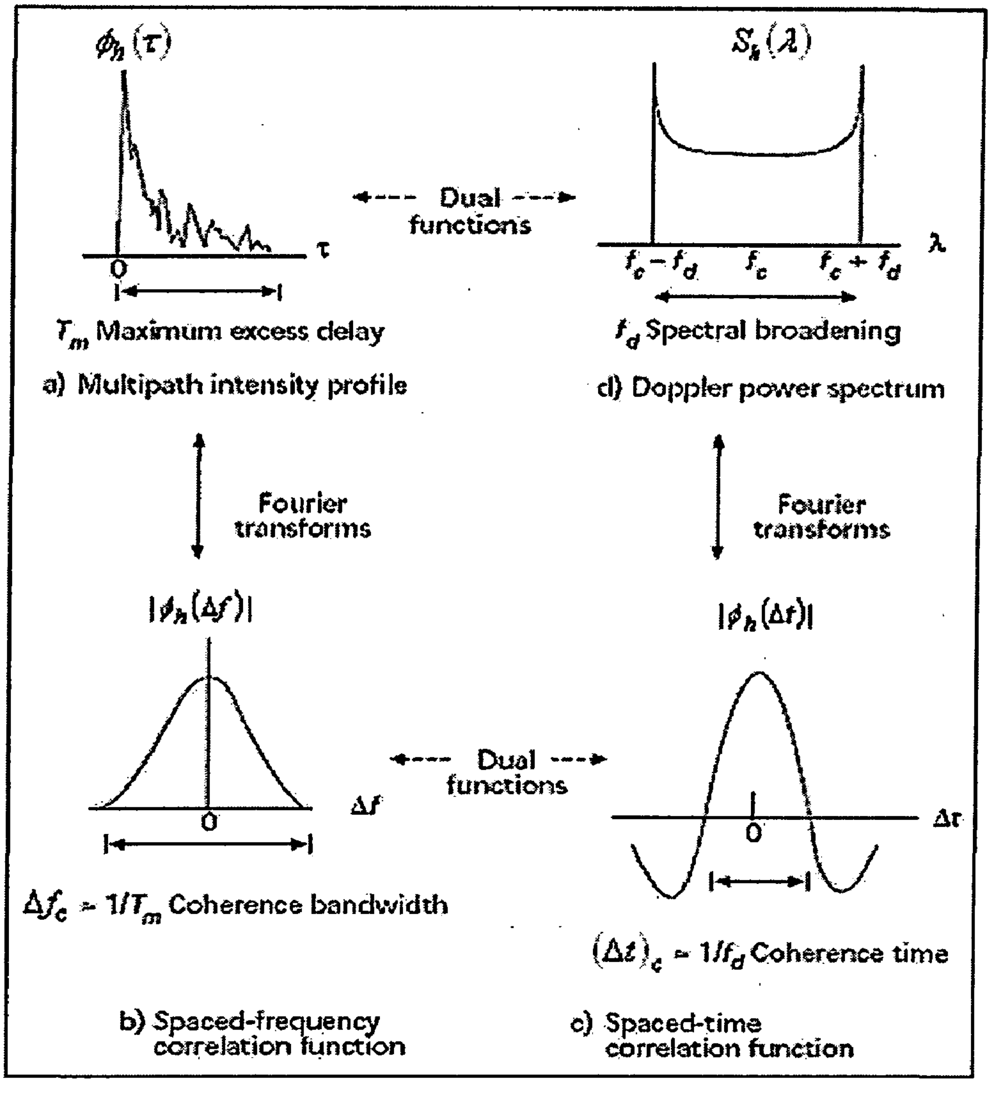


Figure 2.1 Relationships among channel correlation functions and power spectra

A widely used model for the doppler power spectrum of a mobile radio channel is Jakes model [1,16]. The autocorrelation of the time variant transfer function $h(t;f)$ is given as

$$\begin{aligned}\phi_h(\Delta t) &= \frac{1}{2}[E[h^*(t;f)h(t+\Delta t;f)]] \\ &= J_0(2\pi f_m \Delta t)\end{aligned} \tag{2.16}$$

Where $J_0(\ )$ is the zero order Bessel function of the first kind and f_m is the maximum doppler frequency. Doppler power spectrum is given by

$$S_h(\lambda) = \int_{-\infty}^{\infty} \phi_h(\Delta t) e^{-j2\pi\lambda\Delta t}\, d\Delta t = \begin{cases} \dfrac{1}{\pi f_m} \dfrac{1}{\sqrt{1-(f/f_m)^2}} & (|f| \le f_m) \\ 0 & (|f| > f_m) \end{cases} \tag{2.17}$$

Thus from the above discussion we can say that a doubly selective channel has the characteristics

- Delay spread > symbol time
- Coherence bandwidth < symbol rate
- Channel fading rate > symbol rate
- Coherence time < symbol time

2.2 Doubly selective channel models

From the above discussions we know that the time varying multipath channel can be characterized in time domain by impulse response $h(t;\tau)$. An analogous representation in frequency domain is obtained by taking fourier transform and represented as $h(t;f)$ as given in (2.9). Similarly we can define $h(\lambda,\tau)$ and $h(\lambda;f)$.These functions can be sampled at every T_s time instants to get the functions

$$\begin{aligned} h(n;l) &= h\ (nT_s;lT_s) \\ h(n;k) &= h\ (nT_s;kf_s) \\ h(v;l) &= h\ (vf_s;lT_s) \\ h(v;k) &= h\ (vf_s;kf_s) \end{aligned} \tag{2.18}$$

Where $t = nT_s, \tau = lT_s, f = kf_s$, $\lambda = vf_s$ and f_s is the sampling frequency. Using the above charectistics a wireless channel can be represented in sampled domain for block transmission using any of the models discussed below.

- Time lag model

  The time lag model, better known as the impulse response of the LTV channel can be denoted by h(n,l) , $n \in Z, l = 0,1,...,N_h - 1$ where N_h is the discrete delay spread. Here h(n,l) specifies the output of the channel at time index n to a kronecker delta applied at time index (n-l).In block transmission system we impose an additional constraint n = 0,1...N-1.

- Time-frequency model

  This is a time varying frequency response parameterization of the LTV channel. We denote it in sampled time varying frequency response *h(n,k)* where k = 0,1...N-1 as

$$h(n,k) = \frac{1}{\sqrt{N}} \sum_{l} h(n,l) e^{-j2\Pi kl/N} \tag{2.19}$$

- Doppler-lag model

  This form of system parameterization is of particular interest for channels which are both delay and Doppler spread limited as it enables a sparse yet complete representation of the channel. We can parameterize the system in terms of its sampled Doppler h(v,l) where v =0,1 ... N-1

$$h(v,l) = \frac{1}{\sqrt{N}} \sum_{n} h(n,l) e^{-j2\Pi vn/N} \tag{2.20}$$

- Doppler-frequency model

  This model extensively used for characterize the channel response at the OFDM receiver [17].we can represent sampled Doppler frequency response *h(v,k)* where *v,k=0,1...N-1* as

$$h(v,k) = \frac{1}{N} \sum_{l} \sum_{n} h(n,l) e^{-j2\Pi kn/N} e^{-j2\Pi vl/N} \tag{2.21}$$

All four sampled transmission functions defined above are N-periodic in the Doppler and frequency indices '*v*' and '*k*'.

Most explicit models of TV communication channels treat the TV taps as uncorrelated stationary random process which are assumed to be low pass Gaussian with zero mean (Rayleigh fading) or non zero mean (rician fading) depending on whether line of sight prorogation is absent or present [1,18] .Correlations of unknown taps capture average channel characteristics and are used to track the channels time evolution using kalman filtering estimators. But recently Basis expansion models have gained popularity for cellular radio applications, especially when the multipath is caused by a few strong reflectors and path delays exhibit variations due to the kinematics of the mobiles. The time varying (TV) channel is expressed as a superposition of TV bases with time invariant (TI) coefficients. By assigning time variations to the bases, rapidly fading channels with coherence time as small as few tens of symbols can be captured. Such finitely parameterized expansions render TV channel estimation tractable. In this section first we present most general model for doubly selective channel known as random model and then we will see that deterministic basis expansion model is nothing but a simplified version of random model.

2.2.1 Random models

Consider the fading communication system model of figure 2.2.Its equivalent model is shown in Fig 2.3 and we can write

$$y_c(t) = \sum_{l=\infty}^{\infty} S(l) h_c(t; t - lT_s) + V_c(t) \qquad (2.22)$$

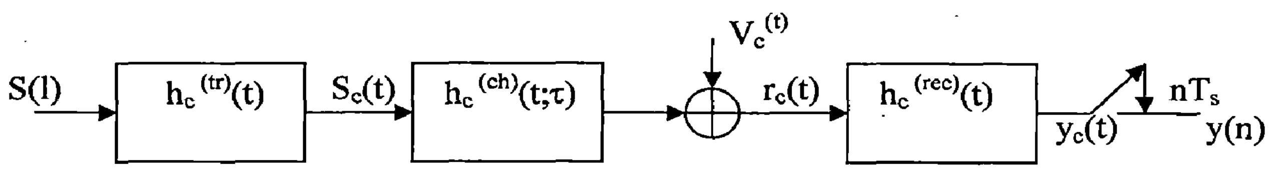


Figure 2.2.Continuous time varying communication system

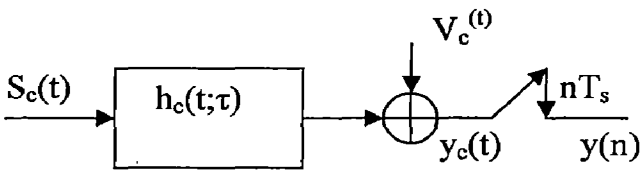


Figure 2.3. Equivalent model for Continuous time varying communication system

In Fig 2.2 subscript 'c' denotes continuous in time. $h_c(t;\tau)$ in Fig 2.3 is the convolution of the spectral pulse $h_c^{(t_r)}(t)$,the TV impulse response $h_c^{(ch)}(t;\tau)$,the receiver filter $h_c^{(rec)}(t)$. $S(l)$ is the sequence of input symbols and $V_c(t)$ is the noise process. The output of the sampler y(n) can be expressed as

$$y(n) = \sum_{l=0}^{L} h(n;l)S(n-l) + V(n) \tag{2.23}$$

For n= {0, 1……………….., N-1}

Where $h(n;l) = h_c(nT_s;lT_s)$ is truncated to an order 'L' If $h(n;l) = h(l)\forall n$ then (2.23) yields a time invariant frequency selective channel and the I/O relationship becomes

$$y_{(n)} = \sum_{l=0}^{L} h(l)S(n-l) + V(n) \tag{2.24}$$

If L =0 (2.23) yields time selective channel with an I/O relationship

$$y(n) = h(n)S(n) + V(n) \tag{2.25}$$

Equation (2.23) provides a generic input output relationship. The vector process h(n) can be modeled as a multi channel AR process (autoregressive) with respect to time i.e. un correlated with each other. Then the channel vector can be written as

$$h_{(n)} = \sum_{i=1}^{P} A_{(i)}h_{(n-i)} + u_{(n)} \tag{2.26}$$

Where $h_{(n)} = [h_{(n,0)},\ldots\ldots h_{(n,L)}]^T$ and $u(n)$ is an independently, identically distributed (iid) circular complex gaussain vector process whose components are uncorrelated with each other. The coefficient matrices $\{A_{(i)}\}_{i=1}^{P}$ are estimated using the multichannel Yule –walker equations [19].

$$R_{hh}(\tau) = \sum_{i=1}^{P} A_{(i)}\, R_{hh}(\tau - i) + \sigma_u^2 \delta(\tau) I \tag{2.27}$$

Where $R_{hh}(\tau)$ is the channel correlation matrix and σ_u^2 is the mean square value of the process u(n). The channel correlation matrix entries are estimated from output statistics conditioned on input. With $R_{hh}(\tau)$ available we can solve $\hat{A}(i)$ using (2.27). Kalman filter is employed to track the channel coefficients after casting AR model in a state space form. The channel space vector h_{ss} and output vector y(n) can be written as

$$h_{ss}(n+1) = Ah_{ss} + \tau\, u(n) \tag{2.28}$$

$$y(n) = [S(n).........S(n-L)]h_{ss}(n) + V(n) \tag{2.29}$$

Where $h_{ss}(n) = \left[h^T(n)........h^T(n-P+1)\right]^T$ is the channel state vector, A is a constant matrix consisting of the AR parameter matrices $\{A_{(i)}\}_{i=1}^{P}$ in its first block row,P-1 identity matrices in its first sub block diagonal and zero elsewhere, and τ=[I 0….0].

2.2.2 Exponential Basis expansion model [3]

The variation in the impulse response can be captured by means of a basis expansion model given by

$$h(n;l) = \sum_{q=1}^{Q} \bar{h}_q(l)\; b_q(n) \tag{2.30}$$

Consider continuous time signal shown in Fig.2.2.It may be written as

$$S_c(t) = R_e\left\{e^{jwct} \sum_{l} S(l) h_c^{(tr)}(t - lT_s)\right\} \tag{2.31}$$

It is transmitted through a TV multipath channel, where the channel transfer function is

$$h_c^{ch}(t;\tau) = \sum_{q=1}^{Q} A_q(t)\; \delta(\tau - d_q(t)) \tag{2.32}$$

Where $S(l)$ are the input symbols, Q is the number of paths and $A_q(t), d_q(t)$ denote each paths TV attenuation and delay respectively. The received base band signal $r_c(t)$ is given by

$$r_c(t) = \sum_{q=1}^{Q} A_q(t) S_c(t - d_q(t)) + V_c(t) \tag{2.33}$$

Receive filter suppresses the AWGN and we obtain

$$y_c(t)=\sum_l\left\{S_{(l)}\left[\sum_{q=1}^{Q}\int_{(l-1)T_s}^{lT_s}A_q(\tau)h_c^{(tr)}(\tau-lT_s-d_q(\tau))h_c^{(rec)}(t-\tau)e^{jw_cd_q(\tau)}d\tau\right]\right.$$

$$\left.+\int_{(l-1)T_s}^{lT_s}V_c(\tau)h_c^{(rec)}(t-\tau)d\tau\right\} \tag{2.34}$$

Let $h_2(t)=h_c^{(tr)}*h_c^{(rec)}$ where $*$ denotes convolution and assume

Assumption A 2.1: constant attenuation and delay over a symbol.

Assumption A 2.2: linearly varying delays across sthe symbol. That is $d_q(l)=V_ql+\varepsilon_q$ where V_q is proportional to the path velocity and the sampling time T_s.

Under the assumption A2.1: from equation (2.34) we can write the channel response as

$$h_c(t;t-lT_s)=\sum_{q=1}^{Q}A_q(l)h_2(t-lT_s-d_q(l))e^{jw_cd_q(l)} \tag{2.35}$$

After sampling the output at the symbol rate $1/T_s$ we get

$$h_c(nT_s;lT_s)=\sum_{q=1}^{Q}A_q(n-l)h_2(nT_s-V_q(n-l)-\varepsilon_q)\exp^{(jw_cv_q(n-l)+\varepsilon_q)} \tag{2.36}$$

Assume $h_q(l)=A_q(n-l)h_2(nT_s-V_q(n-l)-\varepsilon_q)e^{(jw_c\varepsilon_q)}$ (2.37)

and $$h(n;l)=\sum_{q=1}^{Q}h_q(l)e^{jw_q(n-l)} \tag{2.38}$$

With $h(n;l)=h_c(nT_s;lT_s)$ and $w_q=w_cv_q$ .If $\bar{h}_q(l)=h_q(l)e^{-jw_ql}$ equation (2.38) reduces to (2.30) .Therefore the output of the doubly selective channel can be expressed as

$$y(n)=\sum_{q=1}^{Q}\left[\sum_{l=0}^{L}\bar{h}_q(l)b_q(n)S(n-l)\right]+V_{(n)} \tag{2.39}$$

Where $b_q(n)=e^{jw_qn}$

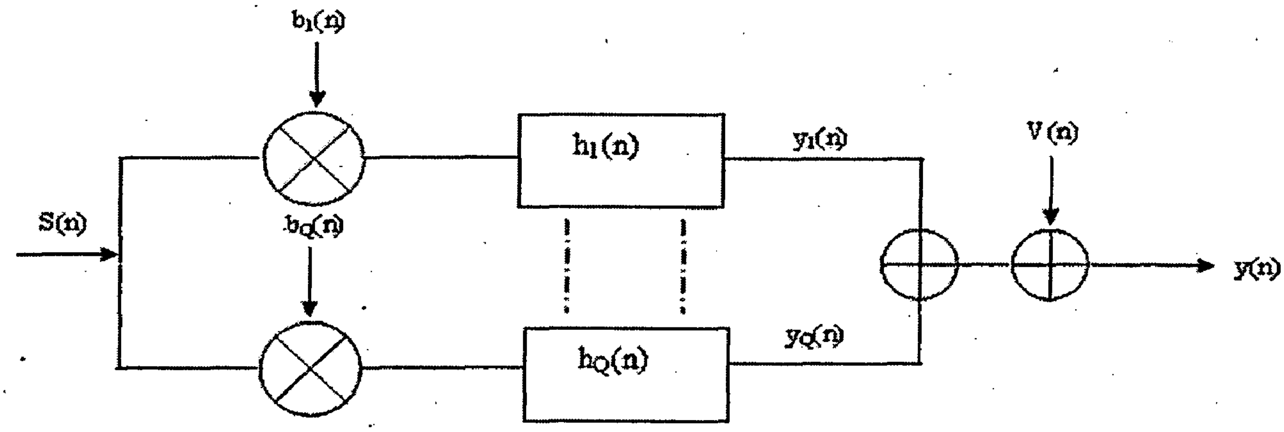


Figure 2.4 Multichannel discrete time equivalent of a TV basis expansion model

The block diagram of the exponential basis expansion model is given by Fig 2.4 The complex exponentials can be viewed as each path's doppler arising due to motion. Since the same input is modulated by Q different complex exponentials in Fig.2.4, some redundancy is introduced at the output which is called channel or doppler diversity.

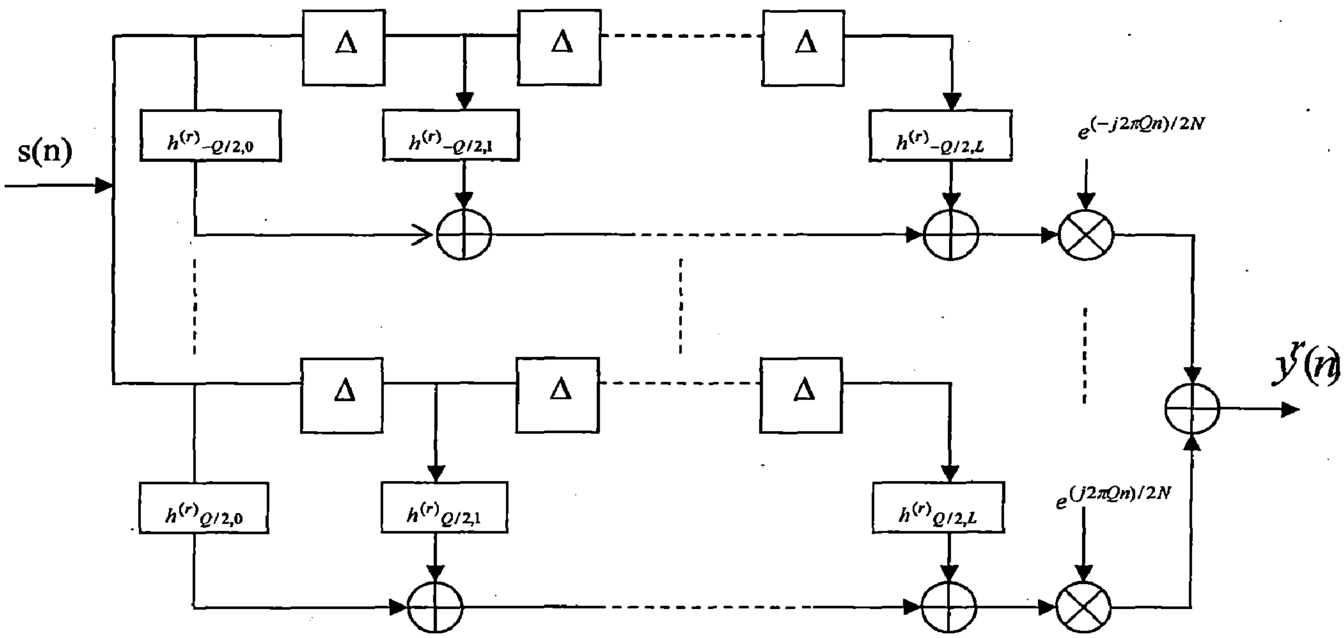


Figure 2.5 Basis Expansion model for doubly selective channel

Consider a SIMO system with N_r receive antennas. The received signal at the r'th receive antennas as shown in the Fig. 2.5 is given by

$$y^{(r)}(n)=\sum_{v=-\infty}^{\infty} h^{(r)}(n;v)s(n-v)+\eta^{(r)}(n) \tag{2.40}$$

For $n \in \{0,1.....N-1\}$ Where $\eta^{(r)}(n)$ is the additive noise at r'th receive antenna $h^{(r)}(n;v)$ the doubly selective channel for r'th receive antenna and s(n) is the transmitted symbol.

Using BEM and modeling channel as a TV FIR filter under the assumptions.

Assumption A2.3: the delay spread is bounded by $\tau_{\max}$.

Assumption A2.4: The Doppler spread is bounded by $f_{\max}$.

The channel response can be represented as

$$h^{(r)}(n;v)=\sum_{l=0}^{L} \delta_{v-l} \sum_{q=\frac{-Q}{2}}^{Q} e^{\frac{j2\pi qn}{N}} h_{q,l}^{(r)} \tag{2.41}$$

Where L and Q satisfy the following conditions.

Condition C2.1: LT $\geq \tau_{\max}$

Condition C2.2: $Q/NT \geq 2f_{\max}$.

Where 'T' is the symbol period. In the expansion model 'L' represents discrete delay spread and $\frac{Q}{2}$ represents discrete doppler spread. The coefficients $h_{q,l}^{(r)}$ remain invariant for $n \in \{0,.................N-1\}$.

CHAPTER THREE

EQUALIZATION OF DOUBLY SELECTIVE CHANNELS

The goal of channel equalization is to remove the effects of the channel on the transmitted symbol sequence s(n). This can be done either by inverse filtering (e.g. Linear- (LE) or Decision-Feedback- Equalization (DFE)) or by applying sequential detection (e.g. Viterbi algorithm).An equalizer-filter can be optimized according to three different cost functions:

- Zero forcing criterion: invert the channel impulse response
- MMSE criterion: minimize the mean-squared-error
- Minimum Bit-Error-Rate (BER) criterion

In the following discussions on different equalizers only the first two criteria are considered.

In a block transmission system, the information symbols are arranged in the form of blocks separated by known symbols. Such a system is suitable for communication over doubly dispersive channels. It is based on the assumption that the channel is constant during the transmission of a sufficiently short message. This implies that the information symbols are transmitted in the form of blocks of sufficiently short duration. Channel estimation is performed for each block with the help of known transmitted symbols that separate the information blocks. Moreover decision symbols are then used to reestimate the channel and the new estimate is in turn used to obtain a new detection. The process of channel estimation and symbol detection is thus repeated until a reliable symbol detection is reached.

Block linear equalizers (BLE) and serial linear equalizers (SLE) are used to equalize doubly selective channel effects. The block linear equalizers have the ability to obtain perfect linear equalization for a larger class of signal sets. It also has a reduced effective delay through the feedback equalizer for the decision feed back equalizer structures. On the other hand serial linear equalizers equalizer is not able to perfectly equalize the channel, it also introduces more delay compared to BLE's for decision feed back equalizers. But it outperforms BLE's in design and implementation complexities.

We will discuss about zero forcing and minimum mean square error solutions for SLE's and BLE's in this chapter.

Notations: Superscripts *, T, and ^{H} represents conjugate, transpose and hermitian respectively. We denote convolution by $*$, Dirac delta by δ(t) and 1- and 2-dimensional kronecker delta as δ[n] and δ[n,m] respectively. We denote N x N identity matrix as I_N and M x N all zero matrix as O_{MxN}, diag {x} denotes the diagonal matrix with 'x' on the diagonal.

3.1 Data Model

Consider a single-input multiple-output (SIMO) system, where N_r receive antennas are used. Focusing on a base band-equivalent description, when transmitting a symbol sequence s(n) at rate 1/T and sampling each receive antenna at the same rate 1/T, the received sample sequence at the r'th receive antenna can be written as:

$$y^{(r)}(n) = h^{(r)}(n;v) * s(n) + \eta^{(r)}(n) \tag{3.1}$$

Where $y^{(r)}(n)$ is the symbol received by the r'th receive antenna, $s(n)$ is the transmitted symbol, $\eta^{(r)}(n)$ is the additive noise at the r'th receive antenna and $h^{(r)}(n;v)$ is doubly selective channel impulse response corresponding to r'th receive antenna. Expression for doubly selective channel can be derived using BEM as given in previous chapter. From (2.41) we know that

$$h^{(r)}(n;v) = \sum_{l=0}^{L} \delta_{v-l} \sum_{q=-\frac{Q}{2}}^{Q/2} e^{j2\pi\frac{qn}{N}} h_{q,l}^{(r)} \tag{3.2}$$

Where 'L' represents discrete delay spread, $\frac{Q}{2}$ is the discrete doppler spread. The coefficients $h_{q,l}^{(r)}$ are assumed to remain invariant for $n \in \{0,.................N-1\}$.Substituting for doubly selective channel impulse response $h^{(r)}(n;v)$ from (3.2) into (3.1) gives

$$y^{(r)}(n)=\sum_{l=0}^{L}\sum_{q=-\frac{Q}{2}}^{\frac{Q}{2}} e^{j\frac{2\pi qn}{N}} h_{q,l}^{(r)} s(n-l)+\eta^{(r)}(n) \qquad (3.3)$$

for $n \in \{0,1 \ldots\ldots\ldots. N-1\}$.

A cyclic prefix based transmission scheme is discussed in this section. A zero padding based transmission scheme is discussed in [9,20].By cyclic prefix based transmission scheme assumption $s(n)=s(N+n)$ for $n \in \{-L,\ldots\ldots,-1\}$.Let the symbol block be represented by $s=[s(0) \; \ldots\ldots. s(N-1) \;]^T$, received symbol block at r'th receive antenna by $y^{(r)}=[y^{(r)}(0)\ldots\ldots\ldots. y^{(r)}(N-1)]^T$ and the corresponding additive noise at the r'th receive antenna by $\eta^{(r)}=[\eta^{(r)}(0),\ldots\ldots.\eta^{(r)}(N-1)]^T$. Equation (3.3) can be written as

$$\begin{bmatrix} y^{(r)}(0) \\ y^{(r)}(1) \\ \\ y^{(r)}(N-1) \end{bmatrix} = \sum_{l=0}^{L}\sum_{q=-\frac{Q}{2}}^{\frac{Q}{2}} h_{q,l}^{(r)} \begin{bmatrix} 1 & & & \\ & e^{(j2\pi q)/N} & & \\ & & \ddots & \\ & & & e^{(j2\pi q(N-1))/N} \end{bmatrix} \begin{bmatrix} s(-l) \\ s(1-l) \\ \\ s(N-1-l) \end{bmatrix} + \begin{bmatrix} \eta^{(r)}(0) \\ \eta^{(r)}(1) \\ \\ \eta^{(r)}(N-1) \end{bmatrix} \qquad (3.4)$$

The column vector $\begin{bmatrix} s(-l) \\ s(1-l) \\ \\ s(N-1-l) \end{bmatrix}$ can be further simplified using the cyclic prefix

assumption ie we can substitute $s(l) \; = s(l) \;\; if \;\; l \geq 0$ and $s(l) \; = s(N-l) \;\; otherwise$

$$\begin{bmatrix} s(-l) \\ s(1-l) \\ \\ s(N-1-l) \end{bmatrix} = \begin{bmatrix} s(N-l) \\ s(N+1-l) \\ \\ s(N-1-l) \end{bmatrix} \qquad (3.5)$$

The new column vector (RHS of (3.5)) obtained after simplification is nothing but the transmitted symbol block cyclically shifted down the row wise by 'l' positions. This can be obtained by multiplying symbol block by an identity matrix shifted cyclically down row wise 'l' times. I.e.

$$\begin{bmatrix} s(-l) \\ s(1-l) \\ \\ s(N-1-l) \end{bmatrix} = Z_l \begin{bmatrix} s(0) \\ s(1) \\ \\ s(N-1) \end{bmatrix} \quad (3.6)$$

Where Z_l is an identity matrix shifted cyclically down by 'l' rows. Substituting (3.6) in (3.4) gives

$$y^{(r)} = H^{(r)} s + \eta^{(r)} \quad (3.7)$$

Where $H^{(r)}$ is a NxN matrix given by

$$H^{(r)} = \sum_{l=0}^{L} \sum_{q=\frac{-Q}{2}}^{\frac{Q}{2}} h_{q,l}^{(r)} D_q Z_l \quad (3.8)$$

Where D_q = diag $\left\{ \left[1, \dots\dots\dots e^{j2\pi q(N-1)/N} \right]^T \right\}$ and Z_l is the NxN circulant matrix with $[Z_l]_{n,n'} = \delta_{(n-n'-l) \bmod N}$ Substituting (3.8) into (3.7) we can write the output sample block as

$$y^{(r)} = \sum_{l=0}^{L} \sum_{q=\frac{-Q}{2}}^{Q/2} h_{q,l}^{(r)} D_q Z_l s + \eta^{(r)} \quad (3.9)$$

After stacking all the sample blocks

$$\begin{aligned} y &= \left[y^{(1)T}, \dots\dots\dots\dots y^{(N_r)T} \right]^T \\ \eta &= \left[\eta^{(1)T}, \dots\dots\dots\dots \eta^{(N_r)T} \right]^T \\ H &= \left[H^{(1)T}, \dots\dots\dots\dots H^{(N_r)T} \right]^T \end{aligned} \quad (3.10)$$

Using (3.7) and (3.10) we can prove that the received sample block is given by

$$y = Hs + \eta \quad (3.11)$$

In the following we assume that perfect channel knowledge is available.

3.2 Block Linear Equalizer (BLE)

The traditional way to equalize the BEM channel is by using N x N_r N block equalizer. Let $G^{(r)}$ be the NxN block linear equalizer that operates on the r'th receive antenna. $G = \left[\ G^{(1)} \ldots\ldots G^{(N_r)} \right]$.

$$\hat{S} = \sum_{r=1}^{N_r} G^{(r)} y^{(r)} \tag{3.12}$$

Hence an estimate of 's' is then computed by substituting (3.7) into (3.12)

$$\hat{s} = \sum_{r=1}^{N_r} G^{(r)} y^{(r)} = \left(\sum_{r=1}^{N_r} G^{(r)} H^{(r)} \right) s + \sum_{r=1}^{N_r r} G^{(r)} \eta^{(r)} \tag{3.13}$$

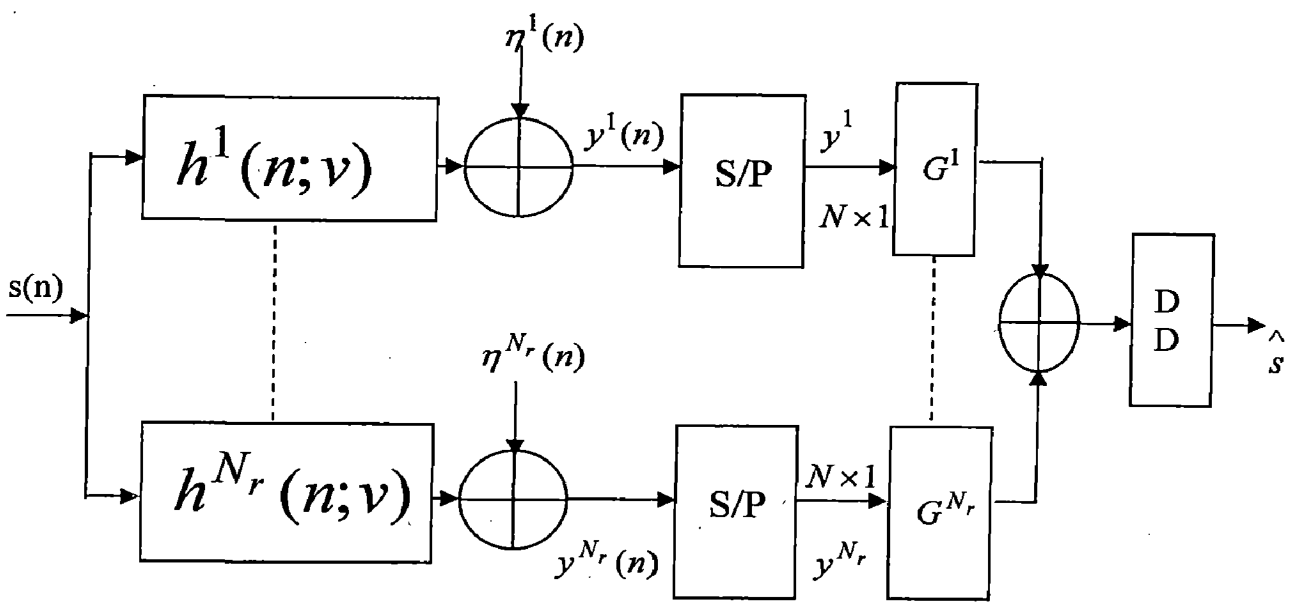


Figure 3.1 Block Linear Equalization

Simplifying (3.13) using (3.10) gives

$$\hat{s} = GY = GHs + G\eta \tag{3.14}$$

3.2.1 Minimum Mean Square Error Block Linear Equalizer (MMSE BLE)

The MMSE BLE is determined as

$$G_{MMSE} = \arg \min_{G} E\left\{ \| \hat{s} - s \|^2 \right\} \tag{3.15}$$

From (3.14) we can write

$$E\left\{\|\hat{s}-s\|^2\right\} = E\left\{ [GHs+G\eta-s][GHs+G\eta-s]^H \right\}$$

$$= E\left\{GHs(GHs)^H - GHss^H + ss^H + G\eta\eta^H G^H - ss^H H^H G^H\right\} \quad (3.16)$$

Assuming noise and signal are uncorrelated and putting $\frac{\partial}{\partial G^*}E\left(ee^H\right)=0$ where $e=\hat{s}-s$ is the estimation error vector, gives

$$GHR_sH^H + GR_\eta - R_sH^H = 0 \quad (3.17)$$

Where $R_s = E\left\{ss^H\right\}$ is the symbol covariance matrix, and $R_\eta = E\{\eta\eta^H\}$ is the noise covariance matrix. From (3.17)

$$G_{MMSE} = R_sH^H\left(HR_sH^H + R_\eta\right)^{-1} \quad (3.18)$$

Using matrix inversion lemma

$$(A+BCD)^{-1} = A^{-1} - A^{-1}B\left(DA^{-1}B + C^{-1}\right)^{-1}DA^{-1} \quad (3.19)$$

We can derive a useful relation given by

$$\left[A^{-1} + B^HC^{-1}B\right]^{-1}B^HC^{-1} = AB^H\left[BAB^H + C\right]^{-1} \quad (3.20)$$

Using the above relation we can further simplify (3.18) in to the form

$$G_{MMSE} = \left(H^HR_\eta^{-1}H + R_s^{-1}\right)^{-1}H^HR_\eta^{-1} \quad (3.21)$$

For the white data and noise with variances σ_s^2 and σ_η^2 respectively $R_S = \sigma_s^2 I_N$ $R_\eta = \sigma_\eta^2 I_N$ .Then the MMSE equalizer reduces to

$$G_{MMSE} = \left(H^H + \frac{\sigma_\eta^2}{\sigma_s^2}I_N\right)^{-1}H^H \quad (3.22)$$

3.2.2 ZF Block Linear Equalizer (ZF BLE)

An unbiased ZF solution is obtained if

$$GH = I_N \quad (3.23)$$

Many ZF solutions satisfy (3.23). A simple ZF solution is obtained as

$$G_{ZF} = \left(H^HH\right)^{-1}H^H \quad (3.24)$$

However (3.24) does not necessarily lead to the minimum norm ZF solution. Minimum norm ZF solution is obtained by minimizing the quadratic cost function $E\{(GY)^H (GY)\}$ subject to (3.23) or equivalently by setting the signal power to infinity in the MMSE solution (3.22). Then we will get the ZF solution as

$$G_{ZF} = \left(H^H R_\eta^{-1} H\right)^{-1} H^H R_\eta^{-1} \tag{3.25}$$

It can be noted that if noise is white $R_\eta = \sigma_\eta^2 I_N \quad R_\eta^{-1} = \frac{1}{\sigma_\eta^2} I_N$ Equation (3.25) reduces to (3.24)

The block linear equalization technique presented above can be easily applied to MIMO systems. Consider a MIMO system with N_t transmit antennas and N_r receive antennas. The transmitted symbol block can be represented as $s = \left[s^{(1)T}, \ldots\ldots\ldots s^{(N_r)T}\right]^T$ where $s^{(r)}$ is the symbol block transmitted by the r'th transmit antenna. The channel matrix can be represented as follows

$$H = \begin{bmatrix} H^{(1,1)} & \ldots\ldots\ldots H^{(N_t,1)} \\ . & \\ H^{(1,N_r)} & \ldots\ldots\ldots H^{(N_t,N_r)} \end{bmatrix}$$

where $H^{(p,q)}$ is the channel matrix from p'th transmit antenna to q'th receive antenna. $\eta^p = \left[\eta^{(p1)T}, \ldots\ldots\ldots \eta^{(pN_r)T}\right]^T$ where $\eta^{(pq)}$ is the noise block from p'th transmit antenna to q'th receive antenna and let $\eta = \sum_{p=1}^{N_t} \eta^p$

Then the received symbol block can be represented as $y = Hs + \eta$ where y has the same meaning as that of (3.11).The MMSE and ZF equalizers can be obtained using the expressions (3.21) and (3.25).

3.3 Time Varying FIR Serial Linear Equalizer (TV FIR SLE)

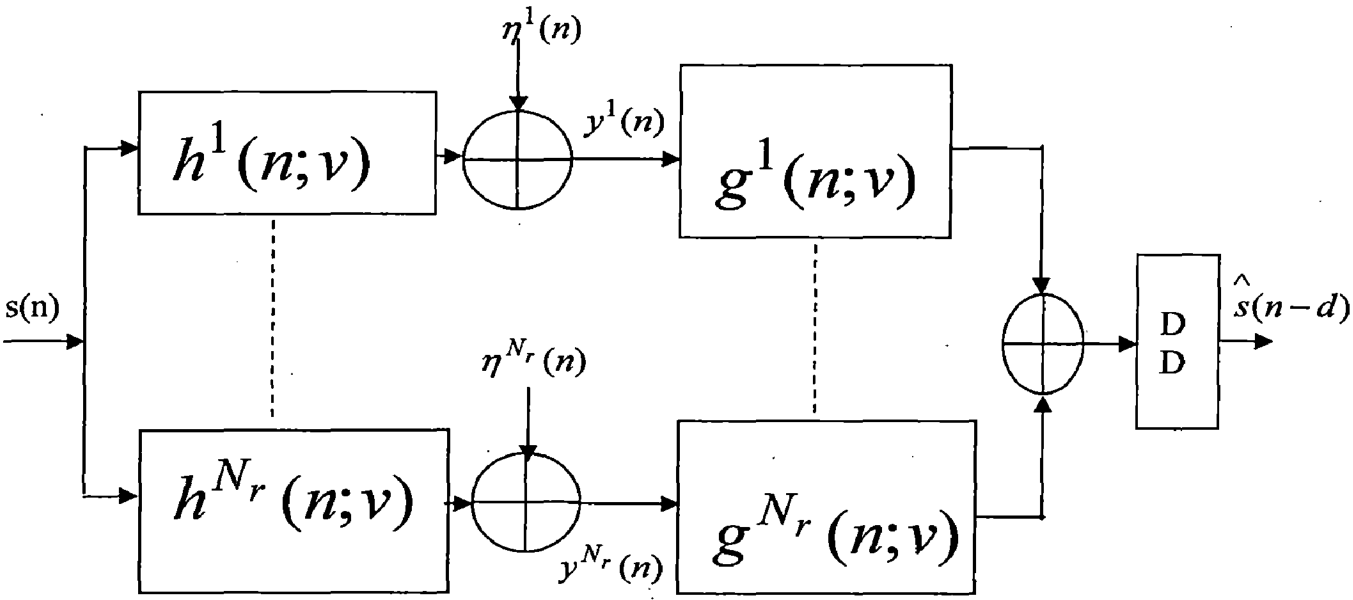


Figure 3.2 Block diagram for TV FIR serial linear equalizer

In the following, we will consider MMSE and ZF TV FIR Serial linear Equalizer $g^{(r)}(n;v)$, for the r'th receive antenna as shown in Fig 3.2. An estimate of $s(n)$ is calculated for $n \in \{0,1,......N-1\}$ as

$$\hat{s}(n) = \sum_{r=1}^{N_r} \sum_{v=-\infty}^{\infty} g^{(r)}(n;v) y^{(r)}(n-v) \tag{3.26}$$

Since the doubly selective channel $h^{(r)}(n\ ;v)$ is described by the BEM, it is also convenient to design TV FIR equalizer $g^{(r)}(n\ ;v)$ using the BEM. This will make equalizer design problem contain only the BEM coefficients of both doubly selective channel and TV FIR equalizer. TV FIR equalizers $g^{(r)}(n\ ;v)$ is designed for $n \in \{0,.....N-1\}$ to have $L' + 1$ taps, where the time variation of each tap is modeled by $Q' + 1$ complex exponential basis functions with frequencies on the same DFT grid as for the channel. The equalizer expression for the r'th channel can be written as

$$g^{(r)}(n\ ;v) = \sum_{l'=-d}^{L'-d} \delta_{v-l'} \sum_{q'=\frac{-Q'}{2}}^{\frac{Q'}{2}} e^{\frac{j2\pi q' n}{N}} g_{q',l'}^{(r)} \tag{3.27}$$

Where d is the synchronization delay of the TV FIR equalizer. TV FIR equalizer in (3.27) has the same structure as that of channel given in (3.2). Using (3.26) and (3.27) we can arrive at an estimate given by

$$\hat{s}(n) = \sum_{l'=-d}^{L'-d} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} e^{j2\pi q' n/N} g_{q',l'}^{(r)} y^{(r)}(n-l') \tag{3.28}$$

Assuming $y^{(r)}(n) = y^{(r)}(n+N)$ for $n \in \{-L'+d, \ldots\ldots, d-1\}$. In other words, for each receive antenna virtually insert a cyclic prefix at the receiver, in order to obtain a circulant convolution on each branch of TV FIR equalizer. Considering block level, $N \times N$ matrix $G^{(r)}$ which is the equalizer for the r'th receive antenna in terms of BEM is given by

$$G^{(r)} = \sum_{l'=-d}^{L'-d} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} g_{q',l'}^{(r)} D_{q'} Z_{l'} \tag{3.29}$$

Where $D_{q'} = \text{diag}\left\{\left[1, \ldots\ldots e^{j2\pi \frac{q'(N-1)}{N}}\right]^T\right\}$ and $Z_{l'}$ is the NxN circulant matrix as defined earlier.

Let $s = [s(0), s(1) \ldots\ldots\ldots s(N-1)]^T$ $\quad Y = [y^{(1)T}, \ldots\ldots\ldots y^{(N_r)T}]^T$

$$\eta = [n^{(1)T}, \ldots\ldots\ldots\ldots \eta^{(N_r)T}]^T \quad H = [H^{(1)T}, \ldots\ldots\ldots\ldots H^{(N_r)T}]^T \tag{3.30}$$

Using (3.29), (3.30) and (3.9) we can write

$$\hat{s} = \sum_{r=1}^{N_r} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} \sum_{l'=-d}^{L'-d} g_{q'l'}^{(r)} D_{q'} Z_{l'} \sum_{q=\frac{Q}{2}}^{Q/2} \sum_{l=0}^{L} h_{q,l}^{(r)} D_q Z_l s + \sum_{r=1}^{N_r} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} \sum_{l'=-d}^{L'-d} g_{q',l'}^{(r)} D_{q'} Z_{l'} \eta^{(r)} \tag{3.31}$$

$$\hat{s} = \sum_{r=1}^{N_r} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} \sum_{l'=-d}^{L'-d} \sum_{l=0}^{L} \sum_{q=\frac{Q}{2}}^{Q/2} g_{q',l'}^{(r)} h_{q,l}^{(r)} D_{q'} Z_{l'} D_q Z_l s + \sum_{r=1}^{N_r} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} \sum_{l'=-d}^{L'=d} g_{q',l'}^{(r)} D_{q'} Z_{l'} \eta^{(r)} \tag{3.32}$$

Defining $p = q + q'$ $k = l + l'$ and using the property $Z_{l'} D_q = e^{-j2\pi\frac{ql'}{N}} D_q Z_{l'}$. The estimate given in (3.31) can be simplified as

$$\hat{s} = \sum_{r=1}^{N_r} \sum_{P=-\left(\frac{Q+Q'}{2}\right)}^{Q+Q'/2} \sum_{k=-d}^{L+L'-d} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} \sum_{L'=-d}^{L'-d} e^{-j2\frac{\pi(p-q')l'}{N}} g_{q',l'}^{(r)} h_{p-q',k-l'}^{(r)} D_p Z_k s$$

$$+ \sum_{r=1}^{N_r} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} \sum_{l'=-d}^{L'-d} g_{q',l'}^{(r)} D_{q'} Z_{l'} \eta^{(r)} \qquad (3.33)$$

Define $f_{p,k} = \sum_{r=1}^{N_r} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} \sum_{L'=-d}^{L'-d} e^{-j2\frac{\pi(p-q')l'}{N}} g_{q',l'}^{(r)} h_{p-q',k-l'}^{(r)}$

and the substitution in (3.31) yields

$$\hat{S} = \sum_{P=-\left(\frac{Q+Q'}{2}\right)}^{Q+Q'/2} \sum_{k=-d}^{L+L'-d} f_{pk} D_p Z_k S + \sum_{r=1}^{N_r} \sum_{q'=\frac{-Q'}{2}}^{Q'/2} \sum_{l'=-d}^{L'-d} g_{q',l'}^{(r)} D_{q'} Z_{l'} \eta^{(r)} \qquad (3.34)$$

Let f, $g^{(r)}$, A, $B^{(r)}$ be the column vectors defined as

$$f^T = \left[f_{-\left(\frac{Q+Q'}{2}\right),-d}, \ldots\ldots f_{-\left(\frac{Q+Q'}{2}\right),L+L'-d}, \ldots\ldots f_{\frac{Q+Q'}{2},L+L'-d} \right]$$

$$g^{(r)T} = \left[g_{-\frac{Q'}{2},-d}^{(r)}, \ldots\ldots g_{-\frac{Q'}{2},L'-d}^{(r)}, \ldots\ldots g_{\frac{Q'}{2},L'-d}^{(r)} \right]$$

$$A = \begin{bmatrix} D_{-\frac{(Q+Q')}{2}} & Z_{-d} \\ \vdots & \\ D_{-\frac{Q+Q'}{2}} & Z_{L+L'-d} \\ \vdots & \\ D_{\frac{Q+Q'}{2}} & Z_{L+L'-d} \end{bmatrix} \qquad B^{(r)} = \begin{bmatrix} D_{-\frac{Q'}{2}} & Z_{-d} \\ \vdots & \\ D_{-\frac{Q'}{2}} & Z_{L'-d} \\ \vdots & \\ D_{\frac{Q'}{2}} & Z_{L'-d} \end{bmatrix}$$

If X is a $N \times M$ matrix and Z is a $K \times L$ matrix, the kronecker product of X and Z is defined to be the $NK \times ML$ matrix as given below

$$X \otimes Z = \begin{bmatrix} x_{11}Z & x_{12}Z & \dots & x_{1M}Z \\ x_{21}Z & x_{22}Z & \dots & x_{2M}Z \\ \vdots & & & \\ x_{N1}Z & x_{N2}Z & \dots & x_{NM}Z \end{bmatrix}$$ Where x_{mn} =X(m,n).

We know that

$$f^T \otimes I_N = \left[f_{-\left(\frac{Q+Q'}{2}\right),-d} I_N, \dots\dots f_{-\left(\frac{Q+Q'}{2}\right),L+L'-d} I_N, \dots f_{\frac{Q+Q'}{2},L+L'-d} I_N \right] \tag{3.35}$$

$$\left(f^T \otimes I_N\right)A = f_{-\left(\frac{Q+Q'}{2}\right),-d} D_{\frac{(Q+Q')}{2}} \; Z_{-d} + \dots + f_{-\left(\frac{Q+Q'}{2}\right),L+L'-d} D_{-\frac{Q+Q'}{2}} \; Z_{L+L'-d} + \dots + f_{\frac{Q+Q'}{2},L+L'-d} D_{\frac{Q+Q'}{2}} \; Z_{L+L'-d} \tag{3.36}$$

Similarly

$$\left(g^{(r)T} \otimes I_N\right)B^{(r)} = g^{(r)}_{-\frac{Q'}{2},-d} D_{-\frac{Q'}{2}} \; Z_{-d} + \dots\dots + g^{(r)}_{-\frac{Q'}{2},L'-d} D_{-\frac{Q'}{2}} \; Z_{L'-d} + \dots\dots\dots + g^{(r)}_{\frac{Q'}{2},L'-d} D_{\frac{Q'}{2}} \; Z_{L'-d} \tag{3.37}$$

Then equation (3.34) can be written as

$$\hat{s} = \left(f^T \otimes I_N\right)As + \sum_{r=1}^{N_r} \left(g^{(r)T} \otimes I_N\right)B^{(r)}\eta^{(r)} \tag{3.38}$$

$B^{(r)}$ does not depend on 'r' in our case. However if different equalizer orders and delays are selected for the different receive antennas, $B^{(r)}$ will depend on r. (3.38) can be further rewritten as

$$\hat{s} = \left(f^T \otimes I_N\right)As + \left(g^T \otimes I_N\right)\mathbf{B}\eta \tag{3.39}$$

Where g is a column vector and B is a diagonal matrix defined as

$$g = \left[g^{(1)T}, \ldots\ldots, g^{(N_r)T}\right]^T \text{ and } B = \begin{bmatrix} B^{(1)} & & \\ & \ddots & \\ & & B^{(N_r)} \end{bmatrix}$$

The term f_{pk} corresponding to the r'th receive antenna is related to a 2-D convolution of the BEM coefficients of the doubly selective channel for the r'th receive antenna and the BEM coefficients of the TV FIR equalizer for the r'th receive antenna. Consider a $(L'+1)\times(L'+L+1)$ block Toeplitz matrix.

$$T_{l,L'+1}\left(h_{q,l}^{(r)}\right) = \begin{bmatrix} h_{q,0}^{(r)} \ldots\ldots h_{q,L}^{(r)} & & 0 \\ \ddots & \ddots & \\ 0 \ldots\ldots h_{q,0}^{(r)} & \ldots\ldots & h_{q,L}^{(r)} \end{bmatrix}$$

We can define $\psi_q^{(r)} = \Omega_q T_{l,L'+1}\left(h_{q,l}^{(r)}\right)$ (3.40)

$$\text{Where } \Omega_q = diag\left\{\left[e^{-j2\frac{\pi q(-d)}{N}}, \ldots\ldots e^{-j2\pi q\frac{(-d+L')}{N}}\right]^T\right\}$$

$$\text{Similarly } T_{q,Q'+1}\left(\psi_q^{(r)}\right) = \begin{bmatrix} \psi_{-\frac{Q}{2}}^{(r)} \ldots\ldots\ldots \psi_{Q/2}^{(r)} & & 0 \\ \ddots & & \ddots \\ 0 & \psi_{-\frac{Q}{2}}^{(r)} \ldots\ldots\ldots & \psi_{\frac{Q}{2}}^{(r)} \end{bmatrix} \tag{3.41}$$

Let $\psi^{(r)} = T_{q,Q'+1}\left(\psi_q^{(r)}\right)$ and Ψ is the column vector defined as $\psi = \left[\psi^{(1)T} \ldots\ldots\ldots \psi^{(N_r)T}\right]^T$. Then using (3.41) and f_{pk} expression we can write [4]

$$f^T = g^T \psi \tag{3.42}$$

Substituting (3.42) into (3.39) finally leads to

$$\hat{s} = \left(g^T \psi \otimes I_N\right) As + \left(g^T \otimes I_N\right) B\eta \tag{3.43}$$

3.3.1 MMSE TV FIR Equalizer

The BEM coefficients of the MMSE TV FIR are determined as

$$g_{MMSE} = \arg \min_{g} E\left\{\| \hat{s} - s \|^2\right\} \tag{3.44}$$

Substituting the expression of the estimate from (3.43)

$$E\left[\left\{(g^T\psi \otimes I_N)As + (g^T \otimes I_N)B\eta - s\right\}\left\{(g^T\psi \otimes I_N)As + (g^T \otimes I_N)B\eta - s\right\}^H \right]$$

$$\begin{aligned} &= \text{trace } \left\{(g^T\psi \otimes I_N)AR_sA^H(\psi^H g^* \otimes I_N)\right\} \\ &+ \text{trace} \left\{(g^T \otimes I_N)B\ R_\eta B^H(g^* \otimes I_N)\right\} \\ &- 2R\left\{trace\left\{(g^T\psi \otimes I_N)AR_s\right\}\right\} + \text{trace}\left\{R_s\right\} \end{aligned} \tag{3.45}$$

Using the following properties

P4.1) trace $\left\{(X^T \otimes I_N)v\right\} = X^T subtr\left\{v\right\}$

P4.2) trace $\left\{x^T \otimes I_N\right\}Z(x^* \otimes I_N) = x^T subtr\left\{Z\right\}x^*$

(3.45) can be further simplified to get the MSE value as

$$E\left\{\| \hat{S} - S \|^2\right\} = trace\ \left\{R_s\right\} + g^T\psi\ R_A\ \psi^H g^* + g^T R_B g^* - 2R\left\{g^T\psi\gamma_A\right\} \tag{3.46}$$

For an arbitrary $k \times 1$ vector x, a $kN \times N$ matrix x .The subtr{.} operation splits the matrix up into $N \times N$ submatrices and replaces each submatrix by its trace. Let A be the $pN \times qN$ matrix as given below.

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1q} \\ \vdots & \cdots & \vdots \\ A_{p1} & \cdots & A_{pq} \end{bmatrix}$$

Where A_{ij} is the (i,j)th $N \times N$ submatrix of A .Then $p \times q$ matrix subtr{A}is given by

$$subtr\{A\} = \begin{bmatrix} tr\{A_{11}\} & \cdots & tr\{A_{1q}\} \\ \vdots & \cdots & \vdots \\ tr\{A_{p1}\} & \cdots & tr\{A_{pq}\} \end{bmatrix}.$$

Hence subtr{.} reduces the row and column dimension by a factor N .In (3.39) $\gamma_A = subtr\{AR_s\}$ $R_A = subtr\{AR_s A^H\}$ and $R_B = subtr\{B\ R_\eta B^H\}$. The BEM coefficients of the MMSE TV FIR are now obtained by solving $\partial E\{\| \hat{S} - S \|^2\} / \partial g^* = 0$ .ie differentiating (3.46) gives

$$g^T \psi R_A \psi^H + g^T R_B - \frac{\partial}{\partial g}\left[g^T \psi \gamma_A + \left(g^T \psi \gamma_A\right)^*\right] = 0 \tag{3.47}$$

That is $g^T_{MMSE} = \gamma_A^H \psi^H \left(\psi R_A \psi^H + R_B\right)^{-1}$ (3.48)

Multiplying with $R_A^{-1} R_A$, we will get

$$g^T_{MMSE} = \gamma_A^H R_A^{-1} \left\{R_A \psi^H \left(HR_A \psi^H + R_B\right)^{-1}\right\} \tag{3.49}$$

Applying matrix inversion lemma

$$g^T_{MMSE} = \gamma_A^H R_A^{-1} \left(\psi^H R_B^{-1} \psi + R_A^{-1}\right)^{-1} \psi^H R_B^{-1} \tag{3.50}$$

$$= e_d^T \left(\psi^H R_B^{-1} \psi + R_A^{-1}\right)^{-1} \psi^H R_B^{-1} \tag{3.51}$$

Where $\gamma_A^H R_A^{-1} = e_d^T$,where e_d is a *(Q+Q'+1)(L+L'+1) X 1* unit vector with the one in the *(d(Q+Q'+1)+ (Q+Q')/2+1)*'st position

3.3.2 ZF TV FIR Equalizer

From (3.43) an unbiased ZF solution is obtained if

$$\left(g^T \psi \otimes I_N\right) A = I_N \tag{3.52}$$

Many ZF solutions satisfy the above equation. A simple solution would be

$$g^T_{ZF} = e_d^T \left(\psi^H \psi\right)^{-1} \psi^H \tag{3.53}$$

Minimum norm ZF solution is obtained by minimizing the quadratic cost function $E\left\{\left[\left(g^T \otimes I_N\right) By\right]^H \left(g^T \otimes I_N\right) By\right\}$ subject to (3.52). This is equivalent to setting the signal power equal to infinity in the MMSE solution (3.51).

This leads to $g^T_{ZF} = e_d^T \left(\psi^H R_B^{-1} \psi\right)^{-1} \psi^H R_B^{-1}$ (3.54)

Here also (3.54) reduces to (3.53) in the white noise case.

The serial linear equalization technique presented above can also be easily applied to MIMO systems. Consider a MIMO system with N_t transmit antennas and N_r receive antennas. Same block of data is transmitted through all the transmit antennas. All channels are assumed to have same maximum Doppler spread and maximum delay spread, therefore discrete Doppler spread (Q/2) and discrete delay spread (L) has same values for all the channels then the received symbol can be represented as

$$\hat{s}=\sum_{r=1}^{N_r}\sum_{q'=\frac{-Q'}{2}}^{Q'/2}\sum_{l'=-d}^{L'-d} g_{q'l'}^{(r)}D_{q'}Z_{l'}\sum_{t=1}^{N_t}\sum_{q=-\frac{Q}{2}}^{Q/2}\sum_{l=0}^{L} h_{q,l}^{(t,r)}D_qZ_ls+$$

$$\sum_{t=1}^{N_t}\sum_{r=1}^{N_r}\sum_{q'=\frac{-Q'}{2}}^{Q'/2}\sum_{l'=-d}^{L'-d} g_{q',l'}^{(r)}D_{q'}Z_{l'}\eta^{(t,r)} \tag{3.55}$$

Defining $p=q+q'$ $k=l+l'$ and using the property $Z_{l'}D_q=e^{-j2\pi\frac{ql'}{N}}D_qZ_{l'}$. The estimate given in (3.55) can be simplified as

$$\hat{s}=\sum_{t=1}^{N_t}\sum_{r=1}^{N_r}\sum_{q'=\frac{-Q'}{2}}^{Q'/2}\sum_{l'=-d}^{L'-d}\sum_{l=0}^{L}\sum_{q=\frac{Q}{2}}^{Q/2} g_{q',l'}^{(r)}D_{q'}Z_{l'}h_{q,l}^{(t,r)}D_qZ_ls$$
$$+\sum_{t=1}^{N_t}\sum_{r=1}^{N_r}\sum_{q'=\frac{-Q'}{2}}^{Q'/2}\sum_{l'=-d}^{L'=d} g_{q',l'}^{(r)}D_{q'}Z_{l'}\eta^{(t,r)} \tag{3.56}$$

Define $f_{p,k}=\sum_{t=1}^{N_t}\sum_{r=1}^{N_t}\sum_{q'=\frac{-Q'}{2}}^{Q'/2}\sum_{L'=-d}^{L'-d} e^{-j2\frac{\pi(p-q')l'}{N}} g_{q',l'}^{(r)}h_{p-q',k-l'}^{(t,r)}$ and the substitution in (3.56)

yields $$\hat{s}=\sum_{P=-\left(\frac{Q+Q'}{2}\right)}^{Q+Q'/2}\sum_{k=-d}^{L+L'-d} f_{pk}D_pZ_ks+\sum_{r=1}^{N_r}\sum_{q'=\frac{-Q'}{2}}^{Q'/2}\sum_{l'=-d}^{L'-d} g_{q',l'}^{(r)}D_{q'}Z_{l'}\eta^{(r)} \tag{3.57}$$

Using same method as that of SIMO equalizer we can generate $\psi=\left[\psi^{(1)T}.........\psi^{(N_r)T}\right]^T$ where $\psi^{(m)}=\sum_{t=1}^{N_t}\psi^{(t,m)}$.. similarly noise vector can be represented as $\eta=\left[\eta^{(1)T},..............\eta^{(N_r)T}\right]^T$ where $\eta^{(m)}=\sum_{t=1}^{N_t}\eta^{(t,m)}$..

Estimate of the transmitted symbol block can be obtained as (3.43).The MMSE and ZF equalizers can be obtained using the expressions (3.50) and (3.53).

The structure of time varying FIR serial linear equalizer for SIMO system with N_r receive antennas is depicted in Fig 3.3. Each antenna has its on equalizers and it has the same structure as that of basis expansion model. A delay of $d\Delta$ is encountered for synchronization, where Δ is the sampling time at the receiver. Each equalizer $g^{(r)}(n;v)$ in the structure has $L^{'}+1$ taps. The time variation of each tap is modeled by $(Q'+1)$ complex exponential basis functions with frequencies on the same FFT grid as the FFT grid for the channel. The values of $L^{'}$ and Q' are selected in such a way that their values are always greater than the discrete delay spread and discrete Doppler spread of the channel respectively. This structure utilizes Doppler diversity in conjunction with multipath diversity in an optimal manner. Another advantage of this structure is that it can have different values for $L^{'}$ and Q' for different receivers depending on the maximum delay spread and maximum Doppler spread encountered in the transmission media. The tap coefficients $g_{q,l}^{(r)}$ are calculated in terms of channel tap coefficients using either ZF or MMSE criteria. These tap coefficients are considered to be constant throughout a block. All the equalizer outputs are summed and a decision is made about the transmitted symbol block.

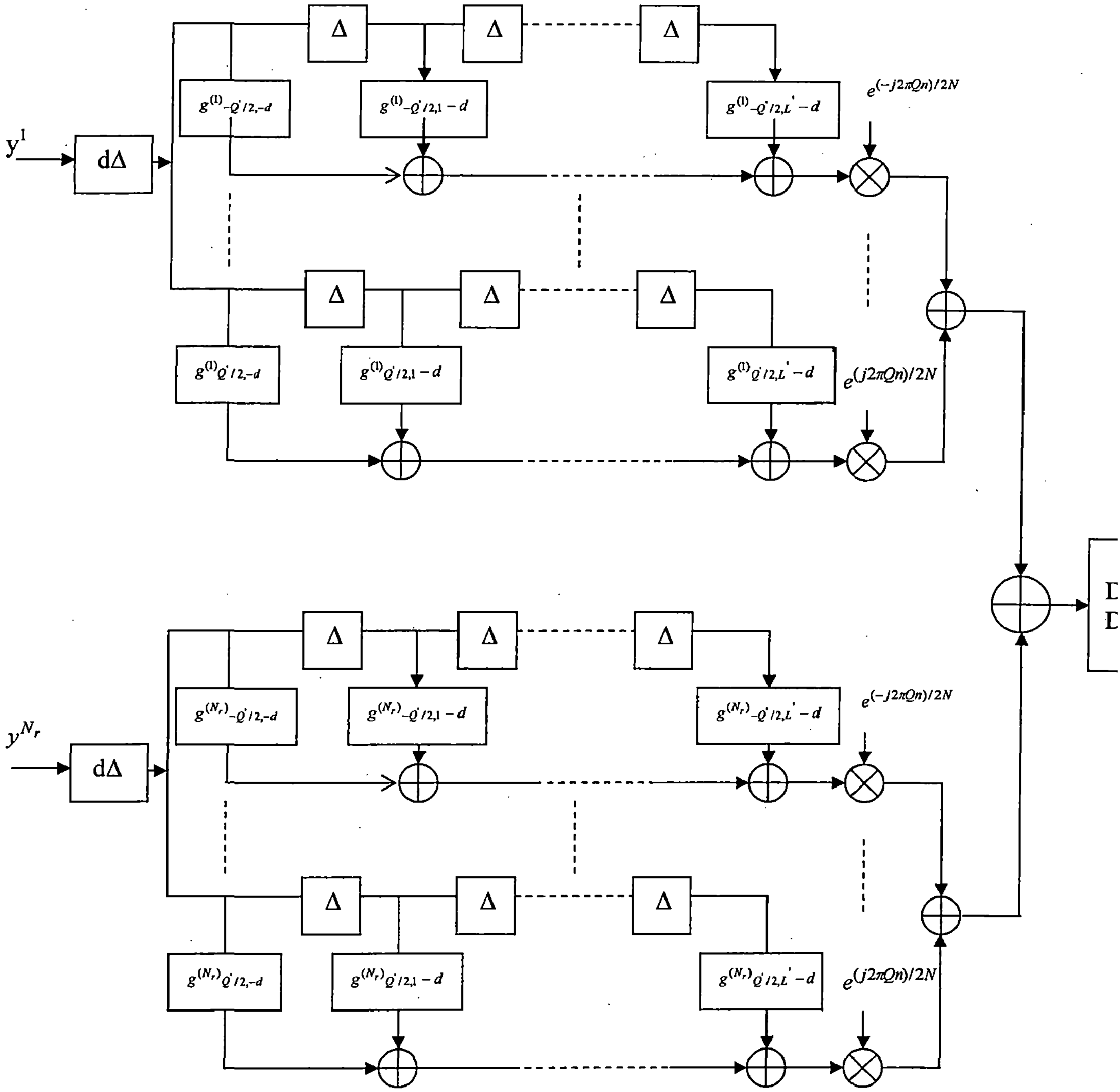


Figure 3.3 Structure of TV FIR SLE

3.4 Existence of ZF solution

The existence of the ZF BLE in (3.24) or (3.25) requires that H has a full column rank Since H is a $N * N_r \times N$ matrix, it has full column rank for all values of 'Nr'. On the other hand the existence of the ZF TV FIR equalizer requires that ψ has a full column rank. This happens as ψ has at least as many rows as columns. That means $N_r(Q'+1)(L'+1) \geq (Q+Q'+1)(L+L'+1)$, this can always be obtained with sufficiently large Q' and L' if $N_r \geq 2$ .Solving the above condition

(i) for a fixed 'L' $$Q' \geq \frac{Q(L+L'+1)}{(N_r-1)(L'+1)-L} - 1 \quad Q' \geq 0 \qquad (3.58)$$

Equation (3.58) implies the following necessary conditions

(i) $N_r \geq 2$ (ii) $L'+1 \geq \left(\frac{L}{N_r-1}\right)$

(ii) Solving for L' for a fixed Q' gives

$$L' \geq \frac{L(Q+Q'+1)}{(N_r-1)(Q'+1)-Q} - 1 \qquad L' \geq 0 \qquad (3.59)$$

Equation (3.59) implies the following necessary conditions

(i) $N_r \geq 2$ (ii) $Q'+1 > \left(\frac{Q}{N_r-1}\right)$

3.5 Design Complexity

The design complexity is the complexity associated with computing the equalizer. To design a BLE we have to compute the inverse of an NxN matrix. This requires O (N^3) flops. On the other hand for the design of the TV FIR equalizer inverse of a KxK matrix should be calculated. Where $K = (Q+Q'+1)(L+L'+1)$. This requires O(K^3) flops. Assuming that the channel is far under spread and L' and Q' are not much larger than L and Q, it is possible to make K less than N. Thus the design complexity of TV FIR equalizer is less than that of BLE.

3.6 Implementation complexity

Implementation complexity is the run time complexity calculated in terms of number of multiply-add operations required to estimate the transmitted block. The estimation of the transmitted block requires N^2 multiply add (MA) operations per receive antenna for the BLE. TV FIR equalizer requires $N(Q'+1)(L'+1)$ MA operations per receive antenna. If the channel is far under spread and L' and Q' are not much larger than L and Q, we may assume that $(Q'+1)(L'+1)$ is less than block size N. Thus the implementation complexity of the TV FIR equalizer is less than the implementation complexity of BLE.

3.7 Data model for zero padding based transmission scheme

A zero padding based transmission scheme can also be used instead of cycle prefix based transmission scheme discussed in this section. From (3.3) we know that the received sample sequence at the r'th receive antenna. For $n \in \{0,1,.....,N-1\}$ can be written as

$$y_{(n)}^{(r)} = \sum_{l=0}^{L} \sum_{q=-\frac{Q}{2}}^{Q/2} e^{j2\frac{\pi qn}{N}} h_{q,l}^{(r)} s(n-l) + \eta^{(r)}(n) \tag{3.60}$$

Suppose we want to transmit a symbol burst $s(n)$ where $s(n) \neq 0$ for $n \in \{0,1......M-1\}$ Where $M = N - L_{ZP}$ and $L_{ZP} \geq L$. Here subscript ZP stands for zero padding and L_{ZP} is the length of zero padding. Let Mx1 symbol block be $s = [s_{(0)}, s_{(1)} \ldots s_{(M-1)}]^T$ then the received sample block at the r'th receive antenna $y^{(r)} = [y_{(0)}^{(r)}, \ldots\ldots y_{(N-1)}^{(r)}]^T$ can be written as

$$y^{(r)} = H^{(r)} T_{ZP} s + \eta^{(r)} \tag{3.61}$$

Where $\eta^{(r)} = \left[\eta_{(0)}^{(r)},.......\eta_{(N-1)}^{(r)}\right]^T$ is the noise vector, $T_{ZP} = \left[I_M, O_{M\times L_{ZP}}\right]^T$ and $H^{(r)}$ is an N xN lower triangular matrix. Using (3.60) $H^{(r)}$ can be written as

$$H^{(r)} = \sum_{l=0}^{L} \sum_{q=-\frac{Q}{2}}^{Q/2} h_{q,l}^{(r)} D_q Z_l \tag{3.62}$$

$D_q = diag\left\{\left[1.........e^{j2\frac{\pi q(n-1)}{N}}\right]^T\right\}$ and Z_l is an N x N lower triangular Toeplitz matrix with first column $\left[O_{1\times l} 1, O_{1\times(N-l-1)}\right]^T$ substituting (3.62) in (3.61) the Nx1 received sample block at the r'th receive antenna can be written as

$$y^{(r)} = \sum_{l=0}^{L} \sum_{q=-\frac{Q}{2}}^{Q/2} h_{q,L}^{(r)} D_q Z_l T_{ZP} s + \eta^{(r)} \tag{3.63}$$

Stacking the Nr received, channel and noise sample blocks as $y = \left[y^{(1)T}......y^{(N_r)T}\right]^T$ $H = \left[H^{(1)T},.....H^{(N_r)T}\right]^T$ and $\eta = \left[\eta^{(1)T},............\eta^{(N_r)T}\right]^T$ We obtain

$$y = HT_{ZP}s + \eta \tag{3.64}$$

Both MMSE and ZF solutions can also obtained from the above data model using same methods discussed for cyclic prefix based transmission scheme. Block Decision feedback equalizers and time varying FIR decision feed back equalizers can also be used to equalize doubly selective effects. The proposed TV FIR DFE [14] consists of a TV FIR feed forward filter, a TV FIR feedback filter, and a decision device. Basis expansion model (BEM) is used to model the TV FIR feed forward and feedback filters.

CHAPTER FOUR

EQUALIZATION OF OFDM OVER DOUBLY SELECTIVE CHANNELS

Orthogonal frequency division multiplexing (OFDM) has been used for wireless applications, like digital video broadcasting, digital audio broad casting, digital television and mobile radio communication .The OFDM communication system divides a serial data stream into several parallel streams, which operate at a symbol rate lower than the original stream, and then modulates the parallel streams with orthogonal carriers. This technique divides the wideband channel into several different subbands.For the receiver and transmitter functions, OFDM uses discrete Fourier transform (DFT) and inverse DFT (IDFT) techniques, eliminating the necessity for a bank of mixers. A cyclic prefix (CP) is inserted between successive OFDM symbols in the transmission and removed prior to demodulation .If the channel impulse response (CIR) is shorter than the length of the cyclic prefix, the cyclic technique makes the linear convolution imparted by the channel appear as circular convolution to the DFT process at the receiver, so as to achieve sub carrier isolation.

Theoretically, one-tap equalizers can effectively cancel any type of time-invariant multipath distortion in the frequency domain as long as the channel echo duration is smaller than the guard interval. Digital video broadcasting (DVB), one of the main applications of OFDM encounters long-delay multipath channels. Using a cyclic prefix of length equal to the channel order, results in a significant decrease in throughput. On the other hand, applying DVB over mobile channels for high speed terminals induces ICI which decreases performance significantly. In time- and frequency-selective fading, the orthogonality of OFDM is lost, leading to sub carrier interference that greatly complicates optimal data detection. OFDM has been applied to scenarios in which time selectivity can be effectively ignored, but future wireless applications are expected to operate at high transmit-frequencies, at high levels of mobility, and at high capacities, resulting in fading that is doubly selective. Thus, the primary advantage of classical OFDM which is interference-free operation will not carry over to important future applications.

The following arguments more clearly explain the potential for doubly selective channels in future OFDM applications. (i) As communication systems are implemented in higher frequency bands, they employ smaller wavelengths, implying that their sensitivity to physical movement grows proportionally. (ii) Increasing either the efficiency or the bandwidth of OFDM systems will increase their sensitivity to channel variation.

Different approaches for reducing ICI have been proposed, including frequency-domain equalization and/or time-domain windowing. In [21], [22] the authors propose matched-filter, least-squares (LS) and minimum mean-square error (MMSE) receivers incorporating all sub carriers. For multiple-input multiple-output (MIMO) OFDM over doubly selective channels, a frequency-domain ICI mitigation technique is proposed in [23]. A time-domain windowing (linear preprocessing) approach to restrict ICI support in conjunction with iterative MMSE estimation is presented in [17]. ICI self-cancellation schemes are proposed in [24] and [25]. There, redundancy is added to enable self-cancellation, which implies a substantial reduction in bandwidth efficiency. To avoid this rate loss, partial response encoding in conjunction with maximum-likelihood sequence detection to mitigate ICI in OFDM systems is studied in [26]. However, all of the above-mentioned literatures assume the channel delay spread fits within the CP, and hence, no IBI is present. In [27] it is assumed that the channel delay spread is larger than the CP, and the TV channel is approximated by the basis expansion model (BEM). The BEM coefficients are then used to design the equalizer to equalize the true channel.

In this chapter we will start with analysis of inter carrier interference in doubly selective channel and then we will discuss about a time domain equalizer which equalizes both IBI and ICI in an OFDM system.

4.1 Inter carrier interference mechanism [17]

Due to the time variation of the channel the orthogonality between the sub carriers is destroyed and hence the ICI is introduced.ICI is the amount of energy on a specific sub carrier leaked from neighboring sub carriers. Consider an OFDM system as shown in Fig 4.1 ,each block of data contains information from N-frequency domain symbols $\{s_k^{(i)}\}_{k=0}^{N-1}$ drawn from a M-QAM alphabet .Here 'i' denotes the OFDM block index and M denotes the size of QAM alphabet. The k'th frequency domain symbol is used to modulate a cyclically prepended inverse Fourier transform (IDFT) basis vector, so that a sampled version of the time domain transmitted signal $\{ x^{(i)}(n) \}_{n=-N_p}^{N-1}$ for the i'th block is given by

$$x^{(i)}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k^{(i)} e^{\frac{j2\pi nk}{N}} \qquad (4.1)$$

Where $N_p (N_p \le N)$ denotes the length of the cyclic prefix .To prevent inter block interference at the receiver and to ensure a circular convolution with the channel response. The time domain blocks are prepended with prefix length N_p of at least the channel impulse response length .The time domain blocks are then serially transmitted through a doubly selective channel.

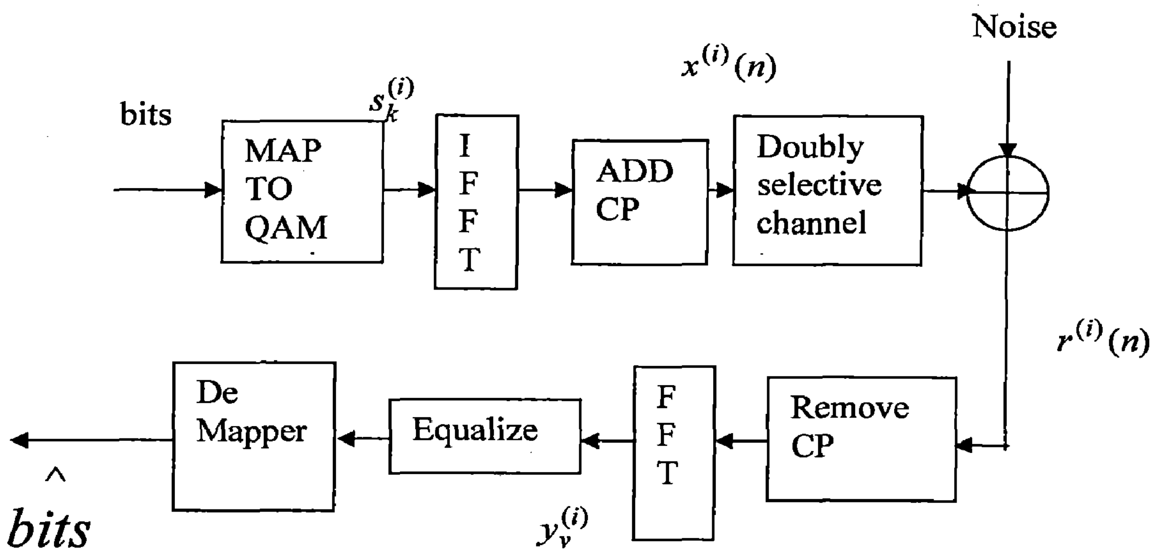


Fig 4.1 OFDM system model

Assume that the time domain received signal $r^{(i)}(n)$ is a noise corrupted and linearly distorted version of $x^{(i)}(n)$ as a consequence of a LTV channel with impulse response $h^{(i)}(n;l)$ .Here $h^{(i)}(n;l)$ specifies the output of the system at the time index n within the i'th block to a kronecker delta applied at time index (n-l) .From chapter 2 (2.18) we know that

$h(n;l) = h\ (nT_s;lT_s)$

$$h(n;k) = h\ (nT_s;kf_s)$$

$$h(v;l) = h\ (vf_s;lT_s)$$

$$h(v;k) = h\ (vf_s;kf_s)$$

The channel is assumed to change continuously within a block as well as between the block. The channel response during the i'th OFDM symbol interval is defined by

$$h^{(i)}(n;l) = h(iN + iN_p + n;l) \qquad -N_p \le n < N \tag{4.2}$$

If the channel is causal with a maximum impulse response duration L

Where $L \le N_p \le N$ then

$$r^{(i)}(n) = \sum_{m=0}^{L-1} h^{(i)}(n,l)x^{(i)}(n-l) + w_n^{(i)} \qquad n \in \{0,\dots\dots\dots\dots N-1\} \tag{4.3}$$

Where $w_n^{(i)}$ are zero mean white Gaussian noise samples with variance σ_w^2.At each block 'i' ,the receiver drops the sample corresponding to the cyclic prefix and applies $\{\ r^{(i)}(n)\ \}_{n=0}^{N-1}$ to a DFT yielding $\{\ y_v^{(i)}\ \}_{v=0}^{N-1}$.

$$y_v^{(i)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r^{(i)}(n) e^{-j\frac{2\Pi v}{N}n} \tag{4.4}$$

Using the system equations (4.1) (4.3) and (4.4)

$$y_v^{(i)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \left(\sum_{l=0}^{L-1} h^{(i)}(n;l) \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k^{(i)} e^{j\frac{2\Pi(n-l)}{N}k} + w_n^{(i)} \right) e^{-j\frac{2\Pi v}{N}n}$$

$$= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} w_n^{(i)} e^{-j\frac{2\Pi v}{N}n} + \sum_{k=0}^{N-1} \left(\frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} \left(\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} h^{(i)}(n;l) e^{-j\frac{2\Pi(v-k)}{N}n} \right) e^{-j\frac{2\Pi k}{N}l} \right) s_k^{(i)}$$

$$= w_v^{(i)} + \sum_{k=0}^{N-1} \left(\frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} \left(\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} h^{(i)}(v-k;l) e^{-j\frac{2\Pi k}{N}l} \right) \right) s_k^{(i)}$$

$$= w_v^{(i)} + \sum_{k=0}^{N-1} h^{(i)}(v-k;k) s_k^{(i)} \tag{4.5}$$

The term $h^{(i)}(v-k;k)$ can be interpreted as the frequency domain response at sub carrier v to a frequency domain impulse centered at k.The above equation can be written in vector form as

$$\begin{bmatrix} y_0^{(i)} \\ y_1^{(i)} \\ \\ y_{N-1}^{(i)} \end{bmatrix} = \begin{bmatrix} h^{(i)}(0,0) & h^{(i)}(-1,1) \ldots\ldots\ldots\ldots h^{(i)}(1-N,N-1) \\ h^{(i)}(1,0) & h^{(i)}(0,1) \ldots\ldots\ldots\ldots h^{(i)}(2-N,N-1) \\ \vdots & \vdots \\ h^{(i)}(N-1,0) & h^{(i)}(N-2,1) \ldots\ldots\ldots h^{(i)}(0,N-1) \end{bmatrix} \begin{bmatrix} s_0^{(i)} \\ s_1^{(i)} \\ \\ s_{N-1}^{(i)} \end{bmatrix} + \begin{bmatrix} w_0^{(i)} \\ w_1^{(i)} \\ \\ w_{N-1}^{(i)} \end{bmatrix} \quad (4.6)$$

Due to orthogonality of the DFT basis vectors the frequency domain noise samples $w_v^{(i)}$ are statistically equivalent to their time domain counterparts namely zero mean white Gaussian with variance σ_w^2.From (4.5) it is evident that each $y_v^{(i)}$ contains contributions from all the symbols $s_k^{(i)}$ transmitted in the i'th OFDM block. Equation (4.6) can be written in matrix form as

$$Y_v^{(i)} = H_{v,k} S^{(i)} + W_v^{(i)} \quad (4.7)$$

Where $S^{(i)}$ is the transmitted symbol block, $Y_v^{(i)}$ is the received symbol block, $W_v^{(i)}$ is the noise vector, $H_{v,k}$ is known as sub carrier coupling matrix. We can see that non diagonal coupling matrix introduces ICI.Variance of the sub carrier coupling coefficients $h^{(i)}(v-k;k)$ gives a better idea about properties of ICI.Applying WSSUS assumption we can get the autocorrelation function of the channel as

$$E\{h(n;l)h^*(n-q;l-m)\} = r_t(q)\sigma_l^2\delta(m) \quad (4.8)$$

Where $r_t(q)$ denotes the normalized autocorrelation and σ_l^2 is the variance of the l'th tap. Let $d = v-k$ .Then $h^{(i)}(d;k) = h^{(i)}(v-k;k)$ .From (2.21) we know that

$$h(d,k) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{N-1} h^{(i)}(n;l) e^{-j\frac{2\Pi}{N}(lk+dn)} \quad (4.9)$$

Defining the N point rectangular window as

$$u_n = \begin{cases} 1 & 0 \le n < N \\ 0 & else \end{cases}$$

Then variance of the sub carrier coupling coefficients is given by

$$E\left\{ |h(d,k)|^2 \right\} = \frac{1}{N^2} \sum_{n,l,m,p} u_n u_m E\left\{h(n;l) h^*(m;p)\right\} e^{j\frac{2\Pi}{N}(pk-lk+md-nd)}$$

$$= \frac{1}{N^2} \sum_l \sigma_l^2 \sum_{n,m} u_n u_m r_t(n-m) e^{j\frac{2\Pi}{N}(md-nd)}$$

$$= \frac{1}{N^2} \sum_l \sigma_l^2 \sum_q \left(\sum_n u_n u_{n-q} \right) r_t(q) e^{-j\frac{2\Pi}{N}qd} \qquad (4.10)$$

From (4.10) we can see that $E\left\{ |h(d,k)|^2 \right\}$ is not a function of 'k', so we can represent it as $E\left\{ |h(d,.)|^2 \right\}$.Consider a (2N-1) point triangular window given by

$$v_q = \begin{cases} N - |q| & -N < q < N \\ 0 & else \end{cases} \qquad (4.11)$$

From (4.10) and (4.11) we can write

$$E\left\{ |h(d,.)|^2 \right\} = \frac{1}{N^2} \sum_l \sigma_l^2 \sum_q v_q r_t(q) e^{-j\frac{2\Pi}{N}qd}$$

$$= \left(S(\phi) * v(\phi)\right)\Big|_{\phi=\frac{2\Pi}{N}d} \sum_l \sigma_l^2 \qquad (4.12)$$

In (4.12) $S(\phi)$ denotes the Doppler spectrum given by $S(\phi) = \sum_q r_t(q) e^{-j\phi q}$

where $\phi \in \Re$ and $v(\phi)$ the DTFT of $v_n N^{-2}$ given by

$$v(\phi) = \frac{1}{N^2} \sum_q v_q e^{-j\phi q} = \left(\frac{\sin\left(\frac{\varphi N}{2}\right)}{N \sin\left(\frac{\phi}{2}\right)} \right)^2 \qquad (4.13)$$

The RHS of (4.13) commonly known as dirichlet sinc.Equation (4.12) gives an interpretation of the ICI generating mechanism. Essentially the Doppler spectrum $S(\phi)$ is convolved with the Dirichlet sinc $v(\phi)$ and then sampled on the regular grid at $\phi = \frac{2\Pi}{N} d$.For a linear time invariant (LTI) channel ie zero Doppler spread the nulls of $(S(\phi) * v(\phi))$ fall on the grid implying

$$E\{ |h(d,.)|^2 \} = \delta(\langle d \rangle_N) \sum_l \sigma_l^2 \qquad (4.14)$$

Where $\langle . \rangle_N$ denotes the modulo N operation.

But for a linear time varying channel (LTV) ie. Nonzero Doppler spread the nulls of $(S(\phi) * v(\phi))$ no longer fall on the grid; as a result ICI is produced.

4.2 Time-Domain Equalization (TEQ)

Channel shortening first became an issue for reduced-state sequence estimation (RSSE) in the 1970's, and then reappeared in the 1990's in the context of multicarrier modulation. Channel shortening has also recently been proposed for use in multiuser detection. In a multicarrier receiver, a time-domain equalizer (TEQ) is needed to mitigate the distortion due to the transmission channel. This typically takes the form of a filter that is designed such that the delay spread of the channel-TEQ combination has a much shorter delay spread than that of the channel alone. In this section we will discuss about channel shortening method using a time domain equalizer for OFDM system.

Consider a single-input multiple-output (SIMO) OFDM System as shown in Fig 4.2 with N_r receive antennas. At the transmitter, the Conventional OFDM modulation is applied, i.e., the incoming bit sequence is parsed into blocks of frequency-domain QAM symbols. Each block is then transformed into a time-domain sequence using an N-point inverse discrete Fourier transform (IDFT). A CP of length N_p is inserted at the head of each block. The time-domain blocks are then serially transmitted over a multipath fading channel. The channel is assumed to be time varying. Focusing only on the base band-equivalent description, the received signal at the r'th receive antenna at time is given by

$$y^{(r)}(t) = \sum_{n=-\infty}^{\infty} h^{(r)}(t; t - nT) x(n) + \eta^{(r)}(t) \qquad (4.15)$$

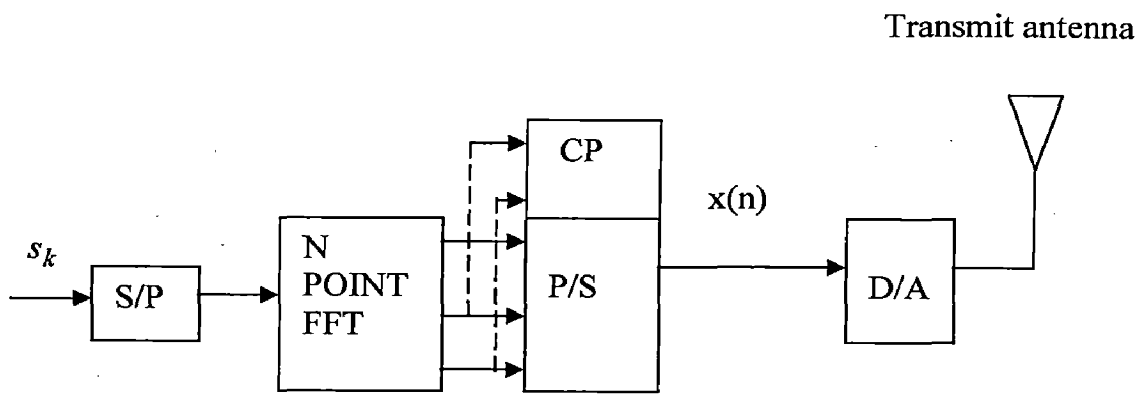


Figure 4.2(a) OFDM Transmitter

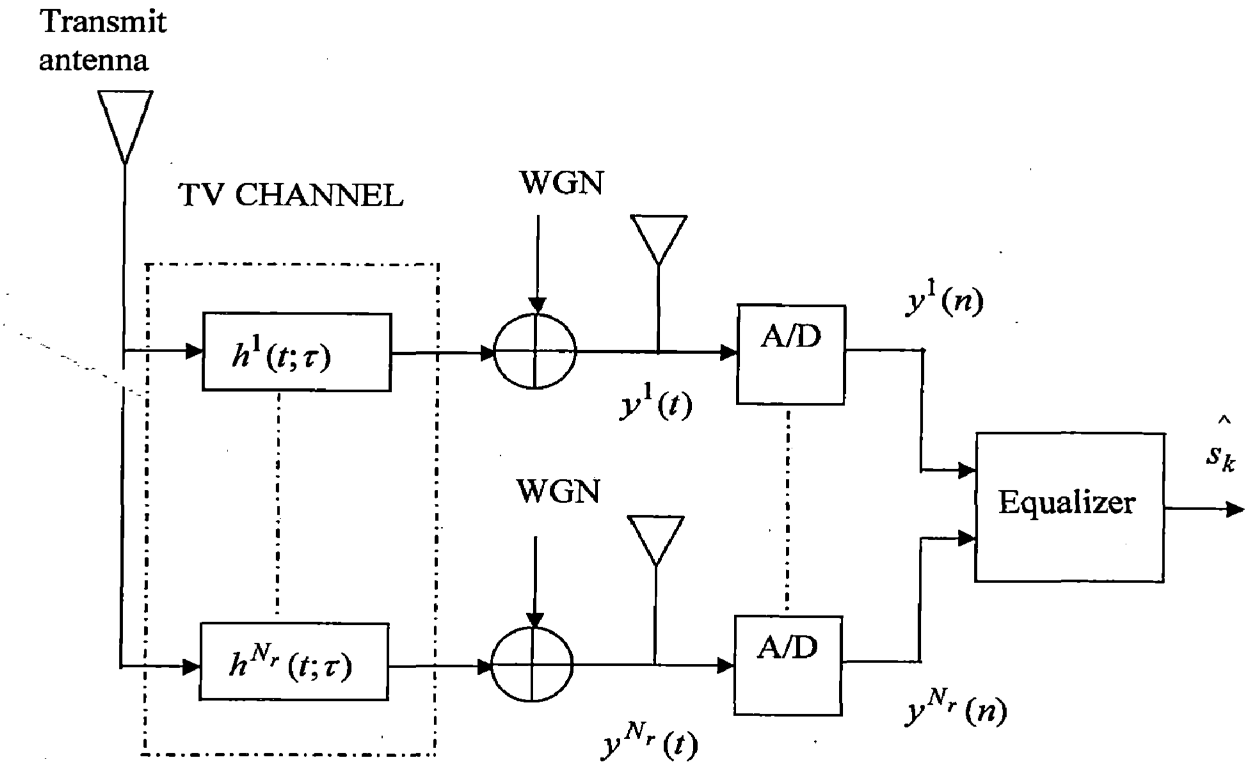


Figure 4.2(b) SIMO OFDM receiver

Where $h^{(r)}(t;\tau)$ is the base band-equivalent of the doubly selective channel from the transmitter to the r'th receive antenna, $\eta^{(r)}(t)$ is the base band-equivalent filtered additive noise at the r'th receive antenna and x(n) is the discrete time-domain sequence transmitted at a rate of $1/T$ symbols per second. Assuming $s_k^{(i)}$ is the QAM symbol transmitted on the k'th sub carrier of the i'th OFDM block. Where $k \in \{0....N-1\}$ and N is the total number of sub carriers in the OFDM block. x(n) can be written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k^{(i)} e^{\frac{j2\pi(m-N_p)k}{N}} \tag{4.16}$$

Where $i = \lfloor n/(N+N_p) \rfloor$ and $m = n - i(N+N_p)$, where N_p is the cyclic prefix length.

Using the BEM we can write the channel response for the r'th receive antenna as given in (3.2).

$$h^{(r)}(n;\theta) = \sum_{l=0}^{L} \delta_{\theta-l} \sum_{q=-\frac{Q}{2}}^{Q/2} e^{j2\pi\frac{qn}{K}} h_{q,l}^{(r)}(i) \tag{4.17}$$

The channel coefficients $h_{q,l}^{(r)}(i)$ remain invariant over a period of length $(N+L')T$ but may change from block to block. Received sample sequence at the r'th antenna can be written as

$$y^{(r)}(n) = \sum_{l=0}^{L} \sum_{q=-\frac{Q}{2}}^{\frac{Q}{2}} e^{j\frac{2\pi qn}{K}} h_{q,l}^{(r)}(i) x(n-l) + \eta^{(r)}(n) \tag{4.18}$$

TV-FIR TEQ denoted by $g^{(r)}(n;\theta)$ is applied at every receive antennas. The purpose of the TEQ is to convert the doubly selective channel into a frequency-selective channel with a delay spread that fits within the CP.That is TEQ will convert the doubly selective channel of order $L > N_p$ and $f_{max} \neq 0$ into a target impulse response (TIR) $b(\theta)$ that is purely frequency selective with order $L'' \leq N_p$ and $f_{max}=0$. The purpose of the TV-FIR TEQ is thus to mitigate both IBI and ICI. As shown in Fig. 4.2,a TEQ $g(i)$,a TIR

$b(i)$ and a synchronization delay 'd' should be designed such that the difference of the outputs of the upper branch and the lower branch is minimized.

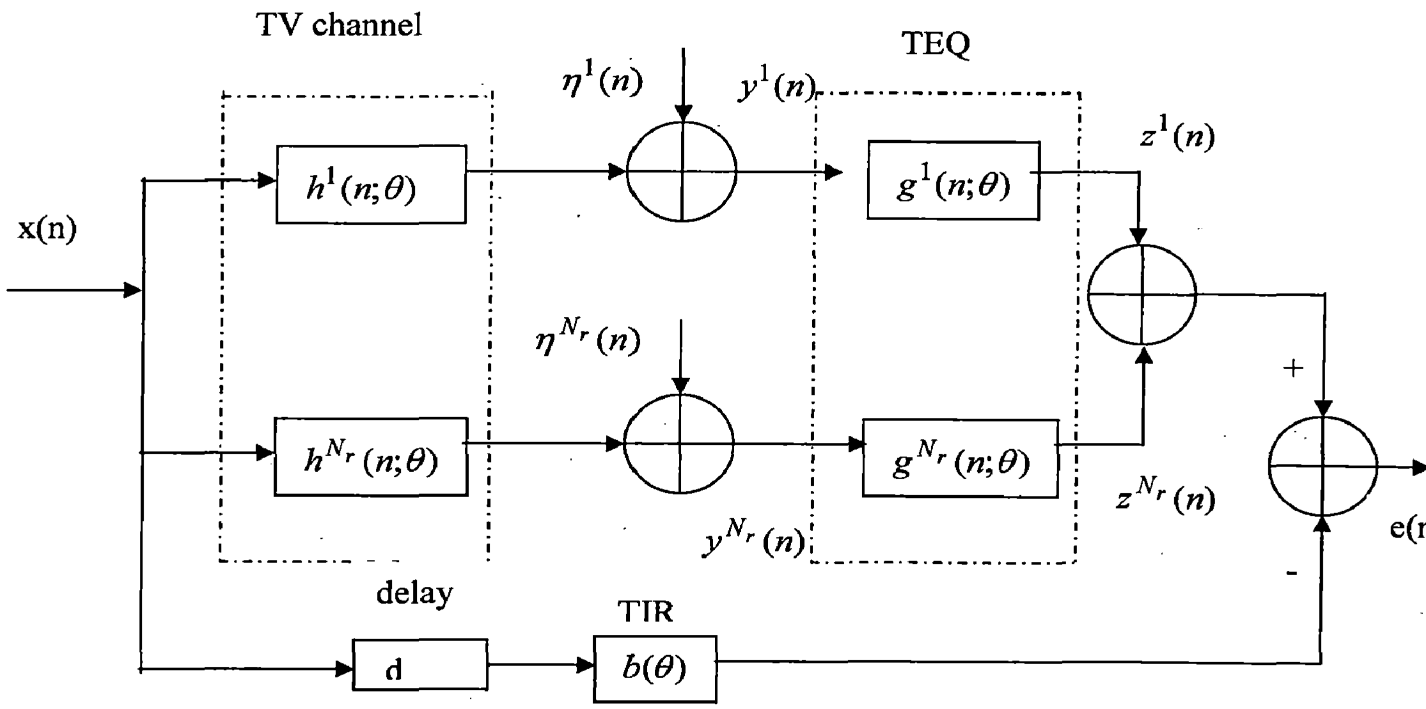


Figure 4.3 Block diagram of TEQ

Non linearity constraints need to be added to avoid the trivial solution for $g^{(i)}(n;\theta)$ *and* $b(\theta)$. A unit norm constraint and the unit energy constraint are generally used due to their superior performance compared to other constraints. A detailed derivation for the TEQ is given in [27].Let the TIR be given by

$$\text{b}(\theta)=\sum_{l''=0}^{L''}\delta(\theta-l'')b_{l''}(i) \tag{4.19}$$

In vector form $b(i)$ can be represented as $b(i)=[b_{(0)}(i),...,b_{L''}(i)]^T$.From (3.43) we know that the output of TV FIR SLE is given by

$$\hat{S}=\left(g^T\psi\otimes I_N\right)As+\left(g^T\otimes I_N\right)B\eta \tag{4.20}$$

Where all variables has the same meaning as that of chapter 3.Defining error vector e(i) as $e[i]=[e(i(N+N_h)),...,e(i(N+N_h)+N-1)]^T$.Using (4.20) error vector can be represented as the difference between upper and lower arm of the TEQ as given below

$$e(i)=\left(f^T(i)\otimes I_N\right)A_{(i)}x(i)+\left(g^T(i)\otimes I_N\right)\left(I_{Nr}\otimes \mathbf{B}(i)\right)\eta(i)-(\tilde{b}(i)\otimes I_N)A(i)x(i) \tag{4.21}$$

Where the argument $\tilde{b}(\mathrm{i}) = \mathrm{C}\ b(i)$ with the matrix C is given by

$$\mathrm{C} = \begin{bmatrix} 0_{((Q+Q')(L+L'+1)/2+d)\times(L''+1)} \\ I_{L''+1} \\ 0_{((Q+Q')(L+L'+1)/2-L''-d-1)\times(L''+1)} \end{bmatrix}$$

Hence we can write the cost function as

$$\begin{aligned} \rho(i) &= E\{e^{H}(i)e(i)\} \\ &= trace\{(f^{T}(i)\otimes I_{N})A(i)R_{x}A^{H}(i)(f^{*}(i)\otimes I_{N})\} \\ &\quad + trace\{(g^{T}(i)\otimes I_{N})\tilde{B}(i)R_{\eta}\tilde{B}^{H}(i)(g^{*}(i)\otimes I_{N})\} \\ &\quad + trace\{(\tilde{b}^{T}(i)\otimes I_{N})A(i)R_{x}A^{H}(i)(\tilde{b}^{*}(i)\otimes I_{N})\} \\ &\quad - 2trace\{R\{(f^{T}(i)\otimes I_{N})A(i)R_{x}A^{H}(i)(\tilde{b}^{*}(i)\otimes I_{N})\}\} \end{aligned} \tag{4.22}$$

Where $\tilde{B}(i) = (I_{Nr}\otimes \mathbf{B}(i))$

Then using unit norm constraint ie.

$$\min_{g(i),b(i)} \rho(i) \text{ Such that } ||\mathrm{b(i)}||^{2}=1 \tag{4.23}$$

we can get the equalizer and TIR filter coefficients as given below.

$$\begin{aligned} g^{T}(i) &= \tilde{b}^{T}(i)(\psi^{H}(i)R_{\tilde{B}}^{-1}(i)\psi(i) + R_{A}^{-1}(i))^{-1}\psi^{H}(i)R_{\tilde{B}}^{-1}(i) \\ b(i) &= eig_{\min}(R^{\perp}(i)) \\ where R^{\perp}(i) &= C^{T}(\psi^{H}(i)R_{\tilde{B}}^{-1}(i)\psi(i) + R_{A}^{-1}(i))^{-1}C \end{aligned} \tag{4.24}$$

Here $eig_{\min}(A)$ is the eigen vector corresponding to the minimum eigen value of the matrix A. Similarly it is possible to get the equalizer and TIR filter coefficients by using unit energy constraint.ie.

$$\min_{g(i),b(i)} \rho(i) \text{ such that } \tilde{b}^{H}(i)R_{A}(i)\tilde{b}(i) = 1 \tag{4.25}$$

The solution to this optimization problem is given by [27]

$$g^T(i) = \tilde{b}^T(i)(\psi^H(i)R^{-1}_{\tilde{B}}(i)\psi(i) + R^{-1}_A(i))^{-1}\psi^H(i)R^{-1}_{\tilde{B}}(i)$$

$$b(i) = eig_{\max}(\tilde{R}^{\perp}(i)) \qquad (4.26)$$

$$where\, \tilde{R}^{\perp}(i) = C^T(\psi^H(i)R^{-1}_{\tilde{B}}(i)\psi(i) + R^{-1}_A(i))^{-1}\psi^H R^{-1}_{\tilde{B}}(i)\psi R_A(i)C$$

Here $eig_{\max}(A)$ is the eigen vector corresponding to the maximum eigen value of the matrix

The structure of time domain equalizer for SIMO OFDM system with N_r receive antennas is depicted in Fig 4.4. Each antenna has its own equalizers and it has the same structure as that of basis expansion model. Here Δ is the sampling time at the receiver. Each equalizer $g^{(r)}(n;\theta)$ in the structure has $L'+1$ taps. The time variation of each tap is modeled by $(Q'+1)$ complex exponential basis functions with frequencies on the same FFT grid as the FFT grid for the channel. The values of L' and Q' are selected in such a way that their values are always greater than the discrete delay spread and discrete Doppler spread of the channel respectively. The time domain equalizer is used to convert the doubly selective channel in to a frequency selective channel with the help of a target impulse response (TIR) $b(\theta)$ .the structure of the TIR can be represented by the tap delay line model as shown in Fig 4.4.The basis expansion model (BEM) coefficients $g_{q',l'}$ of the time domain equalizer is calculated in terms of the BEM coefficient of the channel and they are designed in such that the value of the error $e(n)$ is minimum. A. TEQ optimizes the performance on all sub carriers in a joint fashion. An optimal frequency-domain per tone equalizer (PTEQ) [28], [29] can be obtained by transferring the TEQ operations to the frequency domain. PTEQ optimizes the performance on each sub carrier separately leading to improved performance but it is more complex compared to TEQ to design and implement.

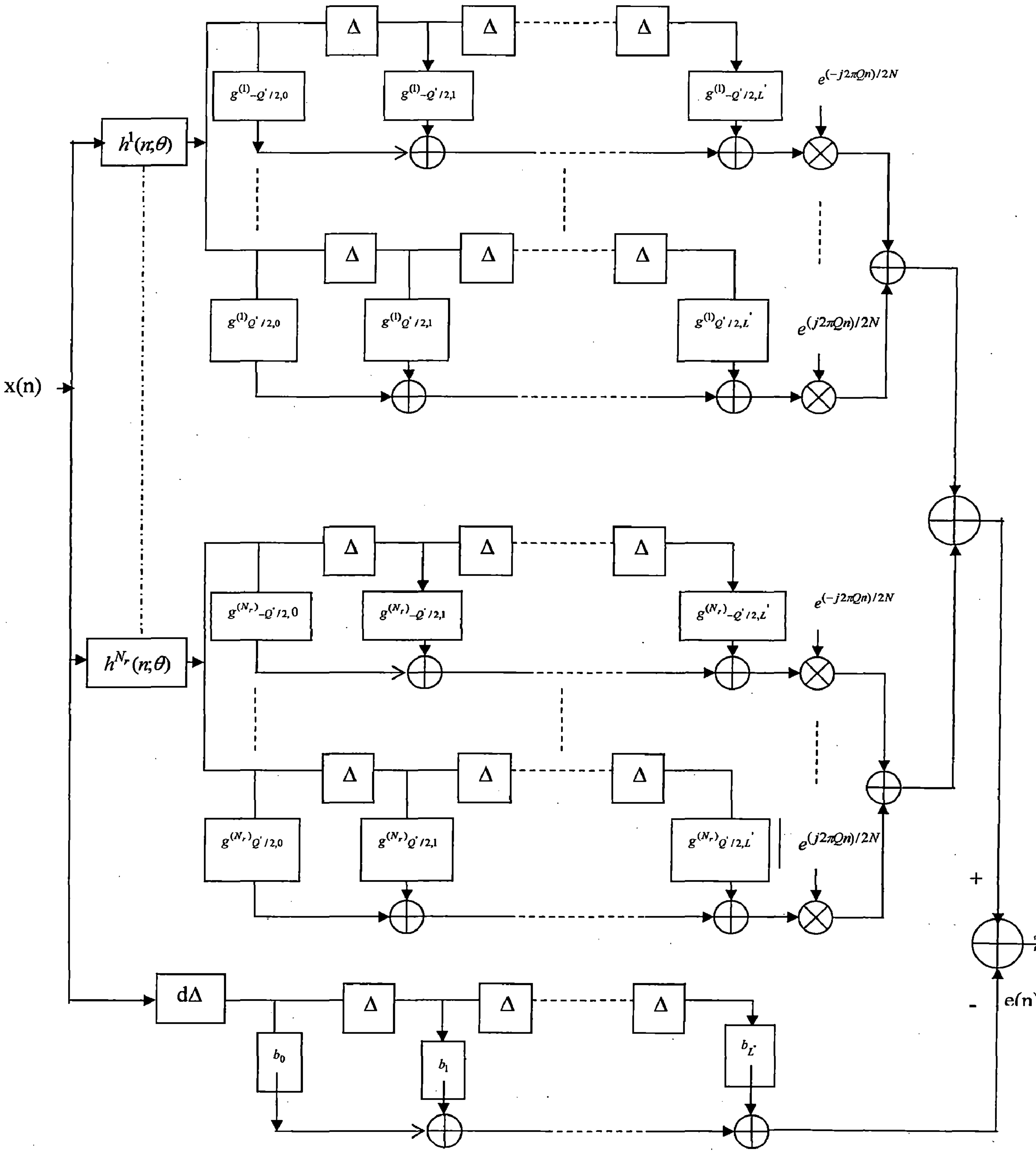


Figure 4.4 Structure of time domain equalizer

CHAPTER FIVE

SIMULATION RESULTS AND CONCLUSIONS

In order to provide a comparison of various equalization techniques discussed in this work, extensive simulations have been performed and results are presented in this chapter. We first present the simulation techniques for the block linear equalizers. We next consider time varying FIR serial linear equalizers. We finally use these approaches for the equalization of an OFDM transmission over doubly selective channels.

5.1 Performance evaluation of block and time varying linear equalizers

In the following simulations the performance of ZF\MMSE solutions of block linear equalizers and time varying serial linear equalizers are compared for a SIMO system with receive antennas $N_r =$ 2 and 4 and for MIMO system with transmit antennas $N_t = 2$ and receive antennas $N_r =$ 4.The parameters are chosen as given below for both simulations.

- Sample period $T = 25$ µs.
- Block size N=256.
- Maximum delay spread $\tau_{\max}$=75 µs.
- Maximum Doppler spread $f_{\max} = 300$ Hz.
- Discrete Doppler spread Q/2 =$\lceil f_{\max} NT \rceil = 2$.
- Discrete delay spread L=$\lceil \tau_{\max} / T \rceil$=3.

5.1.1 Simulation techniques for block linear equalizers

Following steps are performed to obtain BER Vs SNR plots of block linear equalizers.

Step (1): At the transmitter side random data is first generated through library function rand (*l*) which generates data uniformly distributed between 0 and 1.This data is then converted into binary data according to the following decision rule: if the generated random value is less than or equal to 0.5, then it is converted to 0 or else it is made 1.The binary data is then mapped in to QPSK constellation. In QPSK technique the incoming binary bits are grouped in to 2 bits each and then each group is mapped to the constellation plane corresponding to QPSK.The constellation plane for QPSK is shown in Fig 5.1.

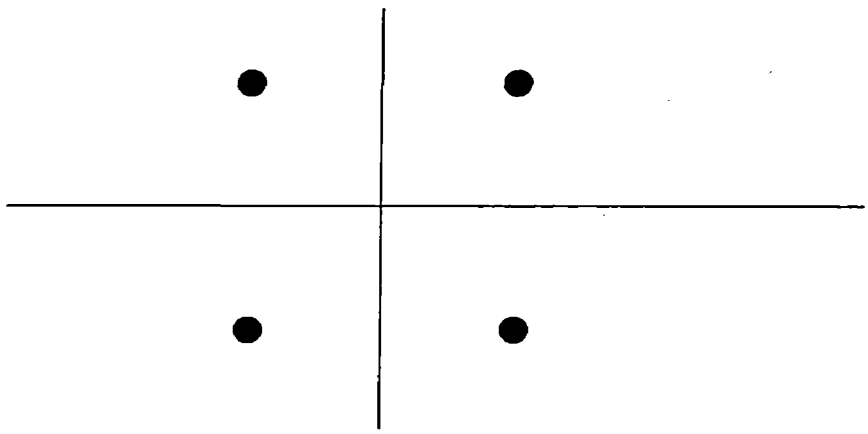


Figure 5.1 Constellation plane for QPSK

The symbols obtained after mapping the binary bit stream will be in any one of the constellation plane. 'N' such symbols are grouped together and that will be our transmitted block's'.

Step (2): The second step in simulation is to model the channel and to obtain the channel matrix corresponding to the doubly selective channel. As discussed earlier we use exponential basis expansion model (BEM) for modeling doubly selective channel. The first task in modeling is to generate BEM coefficients $h_{q,l}^{(r)}$,which are assumed to be known at the receiver. The channel taps are correlated in time with a correlation function according to the Jakes model $R_{hh}(\tau) = J_0(2\pi f_m \tau)$, where $J_0(\ )$ is the zero order Bessel function of the first kind and f_m is the maximum Doppler frequency. The channel is assumed to follow AR model.

$$h_{(n)} = \sum_{i=1}^{P} A_{(i)} h_{(n-i)} + u_{(n)} \tag{5.1}$$

Where $h_{(n)} = \left[h_{(n,0)},\ldots\ldots h_{(n,L)}\right]^T$ and $u(n)$ is an independently, identically distributed (iid) circular complex gaussain vector process whose components are uncorrelated with each other.

The auto correlation function of the channel obeys the equation

$$R_{hh}(\tau) = \sum_{i=1}^{P} A_{(i)} \, R_{hh}(\tau - i) + \sigma_u^2 \delta(\tau) I_{L+1} \tag{5.2}$$

For all the simulations 'P' is selected as 'L+1'. Here σ_u^2 is the variance of iid Gaussian random variable u(n).The variance is assumed as unity and u(n) are generated using matlab command randn( ).Putting values $0, T_s \ldots \mathrm{L}\, T_s$ to Jakes spectrum we will get L+1 equations as given below.

$$R_{hh}(0) = \sum_{i=1}^{P} A_{(i)} \, R_{hh}(-i) + \sigma_u^2$$

$$R_{hh}(1) = \sum_{i=1}^{P} A_{(i)} \, R_{hh}(1 - i) + \sigma_u^2 \tag{5.3}$$

$$R_{hh}(L) = \sum_{i=1}^{P} A_{(i)} \, R_{hh}(L - i) + \sigma_u^2$$

Solving these equations we will get the values of $A_{(i)}$.The channel taps are calculated from the equation

$$\begin{bmatrix} h(n;0) \\ h(n;1) \\ \\ h(n;L) \end{bmatrix} = \sum_{i=1}^{P} A_{(i)} \begin{bmatrix} h(n-i;0) \\ h(n-i;1) \\ \\ h(n-i;L) \end{bmatrix} + \begin{bmatrix} u(n;0) \\ u(n;1) \\ \\ u(n;L) \end{bmatrix} \tag{5.4}$$

Similarly we can calculate all *h(n;l)* from the previous tap values for n=0…N-1.For the *l*'th tap of the r'th BEM channel, the BEM coefficient vector $h_l^{(r)} = \left[h_{-Q/2,l}^{(r)}, \ldots, h_{Q/2,l}^{(r)} \right]^T$ is obtained by

$$h_l^{(r)} = \rho^H h_{jakes}^{(r)}(l) \tag{5.5}$$

Where $h_{jakes}^{(r)}(l) = \left[h^{(r)}(0;l), \ldots, h^{(r)}(N-1;l) \right]^T$ is the l'th tap of the r'th time varying channel obtained from (5.4) over N symbol periods and ρ is an $N \times (Q+1)$ matrix with the (q +Q/2 +1)'th column is given by $\left[1, \ldots\ldots\ldots e^{j2\pi q(N-1)/N}\right]^T$.

The channel matrix between t'th transmit and r'th receive antenna is calculated using

$$H^{(t,r)} = \sum_{l=0}^{L} \sum_{q=\frac{-Q}{2}}^{\frac{Q}{2}} h_{q,l}^{(t,r)} D_q Z_l \tag{5.6}$$

Where D_q = diag $\left\{ \left[1, \ldots\ldots\ldots e^{j2\pi q(N-1)/N}\right]^T \right\}$ and Z_l is the NxN circulant matrix with $[Z_l]_{n,n'} = \delta_{(n-n'-l) \bmod N}$. The channel matrix is obtained as

$$H = \begin{bmatrix} H^{(1,1)} & \ldots\ldots\ldots H^{(N_t,1)} \\ . & \\ H^{(1,N_r)} & \ldots\ldots\ldots H^{(N_t,N_r)} \end{bmatrix} \tag{5.7}$$

where $H^{(p,q)}$ is the channel matrix from p'th transmit antenna to q'th receive antenna. The noise vector $\eta = \left[\eta^{(1)T}, \ldots\ldots\ldots\ldots \eta^{(Nr)T}\right]^T$ is generated using the box muller method, where $\eta^{(r)}$ are the noise vectors for the r'th receiving antenna.

Step (3): Third step in simulation is performed at the receiver section. The symbol covariance matrix $R_S = E\{SS^H\}$ and noise covariance matrix $R_\eta = E\{\eta\eta^H\}$ are formed. The symbol covariance matrix used in the simulation will be in the form $\sigma_s^2 I_N$, where σ_s^2 is the variance of the symbols and I_N is $N \times N$ identity matrix. Symbols with unity variance are selected for our simulations resulting in a covariance matrix of I_N. Noise covariance matrix also has the form $\sigma_\eta^2 I_N$ where σ_η^2 is the variance of the noise samples. From SNR of the channel the two sided power spectral density $N_0/2$ is calculated using the assumption average signal power is unity. The variance of the noise samples are chosen as $N_0/2$. The block equalizer 'G' is calculated using either ZF or MMSE solutions are obtained using the equations (5.8) or (5.9).

$$G_{MMSE} = \left(H^H R_\eta^{-1} H + R_s^{-1}\right)^{-1} H^H R_\eta^{-1} \tag{5.8}$$

$$G_{ZF} = \left(H^H R_\eta^{-1} H\right)^{-1} H^H R_\eta^{-1} \tag{5.9}$$

An estimate of the transmitted block is obtained using $\hat{s} = Gy$.The symbols are converted back to bit format using a maximum likelihood decoding. Error in transmission is calculated by comparing transmitted and finally decoded block of bits.

5.1.2 Simulation techniques for TV FIR serial linear equalizer

We next consider the simulation techniques for time varying serial linear equalizer. Parameters of the channel are selected same as that of the block linear equalizer. The transmitter side for the TV FIR SLE is same as that of BLE .Since TV FIR SLE is having same structure as that of the channel, the no of taps for TV FIR SLE $L'+1$ and no of complex exponential basis functions $Q'+1$ should be selected such that the complexity of the equalizer is minimum. The synchronization delay is chosen as $d = \lfloor L + L'/2 \rfloor + 1$.Other design parameters are same as that of the block linear equalizer.

The basis expansion model coefficients are calculated in the same way as discussed in section 5.1.1.Using these coefficients a $(L'+1)\times(L'+L+1)$ Toeplitz matrix is formed as given below.

$$T_{l,L'+1}\left(h_{q,l}^{(r)}\right) = \begin{bmatrix} h_{q,0}^{(r)} \ldots\ldots h_{q,L}^{(r)} & & 0 \\ \ddots & \ddots & \\ 0 \ldots\ldots h_{q,0}^{(r)} & \ldots\ldots & h_{q,L}^{(r)} \end{bmatrix} \tag{5.10}$$

A diagonal matrix $\Omega_q = diag\left\{\left[e^{-j2\frac{\pi q(-d)}{N}}, \ldots\ldots e^{-j2\pi q\frac{(-d+L')}{N}}\right]^T\right\}$ is formed and multiplied with Toeplitz matrix given in (5.10) to obtain $\psi_q^{(r)} = \Omega_q T_{l,L'+1}\left(h_{q,l}^{(r)}\right)$.Using these $\psi_q^{(r)}$ matrices a block Toeplitz channel matrix $\psi^{(r)} = T_{q,Q'+1}\left(\psi_q^{(r)}\right)$ is formed as given below.

$$T_{q,Q'+1}\left(\psi_q^{(r)}\right) = \begin{bmatrix} \psi_{-\frac{Q}{2}}^{(r)} \cdots\cdots\cdots \psi_{Q/2}^{(r)} & & 0 \\ & \ddots & & \ddots \\ 0 & \psi_{-\frac{Q}{2}}^{(r)} \cdots\cdots\cdots & & \psi_{\frac{Q}{2}}^{(r)} \end{bmatrix} \tag{5.11}$$

These channel matrices for all receive antennas are stacked together to form a block column vector. $\psi = \left[\psi^{(1)T} \cdots\cdots \psi^{(N_r)T}\right]^T$. The dimension of each block in the column vector will be $(Q'+1)(L'+1) \times (Q+Q'+1)(L+L'+1)$. The matrix ψ will be the channel matrix.

The matrices A, $B^{(r)}$ and e_d are formed as shown below. e_d is a (Q+Q'+1)(L+L'+1) X 1 unit vector with the one in the (d(Q+Q'+1)+ (Q+Q')/2+1)'st position

$$A = \begin{bmatrix} D_{-\frac{(Q+Q')}{2}} & Z_{-d} \\ \vdots & \\ D_{-\frac{Q+Q'}{2}} & Z_{L+L'-d} \\ \vdots & \\ D_{\frac{Q+Q'}{2}} & Z_{L+L'-d} \end{bmatrix} \quad B^{(r)} = \begin{bmatrix} D_{-\frac{Q'}{2}} & Z_{-d} \\ \vdots & \\ D_{-\frac{Q'}{2}} & Z_{L'-d} \\ \vdots & \\ D_{\frac{Q'}{2}} & Z_{L'-d} \end{bmatrix} \tag{5.12}$$

In all simulations the no of taps for TV FIR SLE $L'+1$ and no of complex exponential basis functions $Q'+1$ are kept same for all receive antennas making $B^{(r)}$ independent of 'r'. $B^{(r)}$ for different receivers are arranged diagonally to form matrix B.

$$B = \begin{bmatrix} B^{(1)} & & \\ & \ddots & \\ & & \ddots \\ & & & B^{(N_r)} \end{bmatrix} \tag{5.13}$$

The symbol covariance matrix $R_S = E\{SS^H\}$ and noise covariance matrix $R_\eta = E\{\eta\eta^H\}$ are formed as explained in section 5.1.1.Using subtr{.} operation explained in chapter 3, the matrices $R_A = subtr\{AR_S A^H\}$ and $R_B = subtr\{B\ R_\eta B^H\}$ are calculated. Finally the equalizer taps calculated using either ZF or MMSE are obtained using following equations

$$g_{MMSE}^T = \gamma_A^H R_A^{-1}\left(\psi^H R_B^{-1}\psi + R_A^{-1}\right)^{-1}\psi^H R_B^{-1} \tag{5.14}$$

$$g_{ZF}^T = e_d^T\left(\psi^H R_B^{-1}\psi\right)^{-1}\psi^H R_B^{-1} \tag{5.15}$$

An estimate of the transmitted symbols are made using equation (5.16)

$$\hat{s} = \left(g^T\psi \otimes I_N\right)As + \left(g^T \otimes I_N\right)B\eta \tag{5.16}$$

The symbols are converted back to bit format using a maximum likelihood decoding. Error in transmission is calculated by comparing transmitted and finally decoded block of bits.

5.1.3 Results

The plots for signal to noise ratio (SNR) versus bit error rate (BER) for different antenna configurations are shown in figures from 5.2 to 5.7.In all the examples channel is modeled using basis expansion model with discrete delay spread L=3 and discrete Doppler spread (Q/2) =2.A block of 256 (i.e. N=256) symbols are transmitted in all the cases. Minimum of 100 blocks are transmitted for all simulations. Design complexity of the block linear equalizer depends only on block size and it is calculated as N^3 flops. Design complexity of time varying equalizer depends on the design parameters L' and Q' and it is calculated as K^3 flops, where $K = \left(Q + Q' + 1\right)\left(L + L' + 1\right)$. Percentage reduction in design complexity is calculated using (5.17)

$$\text{Percentage reduction in design complexity} = \frac{N^3 - K^3}{N^3} \times 100 \tag{5.17}$$

Implementation of block linear equalizers requires N^2 multiply add (MA) operations per receive antenna. Time varying equalizer requires TV FIR equalizer requires

$P = N(Q' + 1)(L' + 1)$ MA operations per receive antenna. Percentage reduction in implementation complexity is calculated using (5.18)

$$\text{Percentage reduction in implementation complexity} = \frac{N^2 - P}{N^2} \times 100 \qquad (5.18)$$

Example 1.

In example one we compare performance of ZF block linear equalizer (ZF BLE) and zero forcing time varying FIR serial linear equalizer (ZF TV FIR SLE) for a SIMO system with two receive antennas. The time varying equalizer is modeled using basis expansion model with L' =10 and Q'=10.From figure 5.2 we can see that ZF TV FIR SLE is out performed by ZF BLE.We can notice a SNR loss of 5dB for TV FIR SLE compared to ZF BLE at a BER of 10^{-2}.At lower values of BER's block linear equalizers perform much better than time varying equalizers. Comparing design complexities of both the equalizers we can see that there is 44.80 percentage reduction in the case of the time varying equalizer. Time varying equalizer has also got the advantage of 52.73 reduction in implementation complexity. It is possible to improve the performance of the time varying equalizer by increasing the values of L' and Q',but it will also increase the design and implementation complexities of the equalizer.

Example 2.

In example two we consider ZF solutions for a SIMO system with four receive antennas. The design parameters of time varying equalizer are L' =6 and Q'=6.Comparing figure 5.3 and figure 5.2 we can see that for four receive antennas performance of ZF TV FIR SLE comes much closer to the performance of ZF BLE .At a BER of 10^{-2} we can notice a 2dB SNR loss for TV FIR SLE.But it is compensated by 80.85 percentage reduction in implementation complexity and 92.06 percentage reduction in design complexity. Comparing figure 5.3 and 5.2 we can see that performance of both equalizers is improved as the no of receive antennas are increased. Performance of time varying equalizer comes closer to performance of block linear equalizer at higher number of receive antennas.

Example 3.

In example three we compare the performance of MMSE solutions of block linear equalizer and time varying FIR serial equalizer for a SISO system. Time varying equalizer is modeled using the design parameters $L'=10$ and $Q'=10$. We can see from figure 5.4 that the performance loss of TV FIR SLE is about 3 dB at a BER of 10^{-2} .The performance of time varying equalizer saturates at higher SNR's,there fore block linear equalizers performs much better than serial linear equalizer at higher SNR's. Time varying equalizer has the advantage of 52.73 percentage reduction in implementation complexity and 44.80 percentage reduction in design complexity

Example 4.

In example four given in figure 5.5, we compare the performance of MMSE solutions of block linear equalizer and time varying FIR serial equalizer for SIMO systems with two receive antennas. Time varying equalizer is modeled using the design parameters $L'=6$ and $Q'=6$. The figure 5.5 shows that for two receive antennas performance of MMSE TV FIR SLE almost coincides with that of MMSE BLE at BER of 10^{-2} .Throughout the measuring range performance of both equalizers are almost same. But in such cases TV FIR SLE's are preferred because of 80.85 percentage reduction in implementation complexity and 92.06 percentage reduction in design complexity compared to block linear equalizers. Comparing figures 5.5 and 5.2 we can see that performance of MMSE equalizers are better than that of ZF equalizers. At a BER of 10^{-2} MMSE time varying equalizer shows an improvement of 2dB over ZF time varying equalizer.

Example 5.

In example five we compare performance of ZF block linear equalizer (ZF BLE) and zero forcing time varying FIR serial linear equalizer (ZF TV FIR SLE) for MIMO system with two transmit and four receive antennas. Comparing figure 5.6 with figure 5.3 we can see that the performance of MIMO systems with two transmit and four receive antennas are much better compared to SIMO system with four receive antennas. The design parameters of time varying equalizer are selected as L' =4 and Q' =6. Here ZF TV FIR SLE has a performance loss of 2dB at a BER of 10^{-2} compared to block linear equalizer. But it has the advantage of 86.32 percentage reduction in implementation complexity and 95.93 percentage reduction in design complexity

Example 6.

Finally in figure 5.7 we compare the performance of MMSE solutions of block linear equalizer and time varying equalizer for MIMO system with two transmit and four receive antennas. The design parameter for time varying equalizer are chosen as L'=4 and Q'=4 .From the figure 5.7 we can see that performance of MMSE TV FIR SLE compares well with MMSE BLE throughout the measuring range. Here TV FIR SLE offers 90.23 percentage reduction in implementation complexity and 97.77 percentage reduction in design complexity, We can conclude that MMSE TV FIR SLE outperforms MMSE BLE.Comparing the figures 5.7 and 5.6 we can see that MMSE MIMO equalizers performs better than ZF MIMO equalizers. At a BER of 10^{-2} MMSE BLE has 2dB improvement over ZF BLE and MMSE time varying equalizer has 4 dB improvement over ZF time varying equalizer.

Example 7

In example seven we compare the performance of ZF solution of block linear equalizer for two different Doppler spreads 300 Hz and 450 Hz. Discrete delay spreads for the channels in both the cases are assumed to be the same. For channel with maximum Doppler spread of 300 Hz discrete Doppler spread (Q/2) is chosen as two and the discrete delay spread is kept at L=3.For channel with maximum Doppler spread 450 Hz, the discrete Doppler spread is chosen as (Q/2) =3 and discrete delay spread is kept at L=3.The basis expansion model coefficients are calculated using the same method as given in section 5.1.1 .We consider a SIMO system with two receive antennas and block of 256 symbols transmitted through the channel. From figure 5.8 we can see that the performance of both equalizers are nearly same through out the measuring range. We can notice that there is a degradation of 1dB for Doppler spread 450 Hz at a BER of 10^{-3} .

Example 8.

In example eight we consider the performance of time varying equalizer against two different Doppler spreads 300 Hz and 450 Hz. A SIMO system with two receive antennas are considered in this example. Channel conditions are kept as same as that of example seven. A time varying equalizer is modeled with design parameters $L'=10$ and $Q'=10$.From figure 5.9 we can see that there is a degradation of 1dB for Doppler spread 450 Hz at a BER of 10^{-3} for time varying equalizer. but generally speaking the performance of equalizer at 300 Hz is almost same as that of performance of the equalizer at 450 Hz. We can conclude that time varying equalizer performs well against Doppler spreading.

Example 1

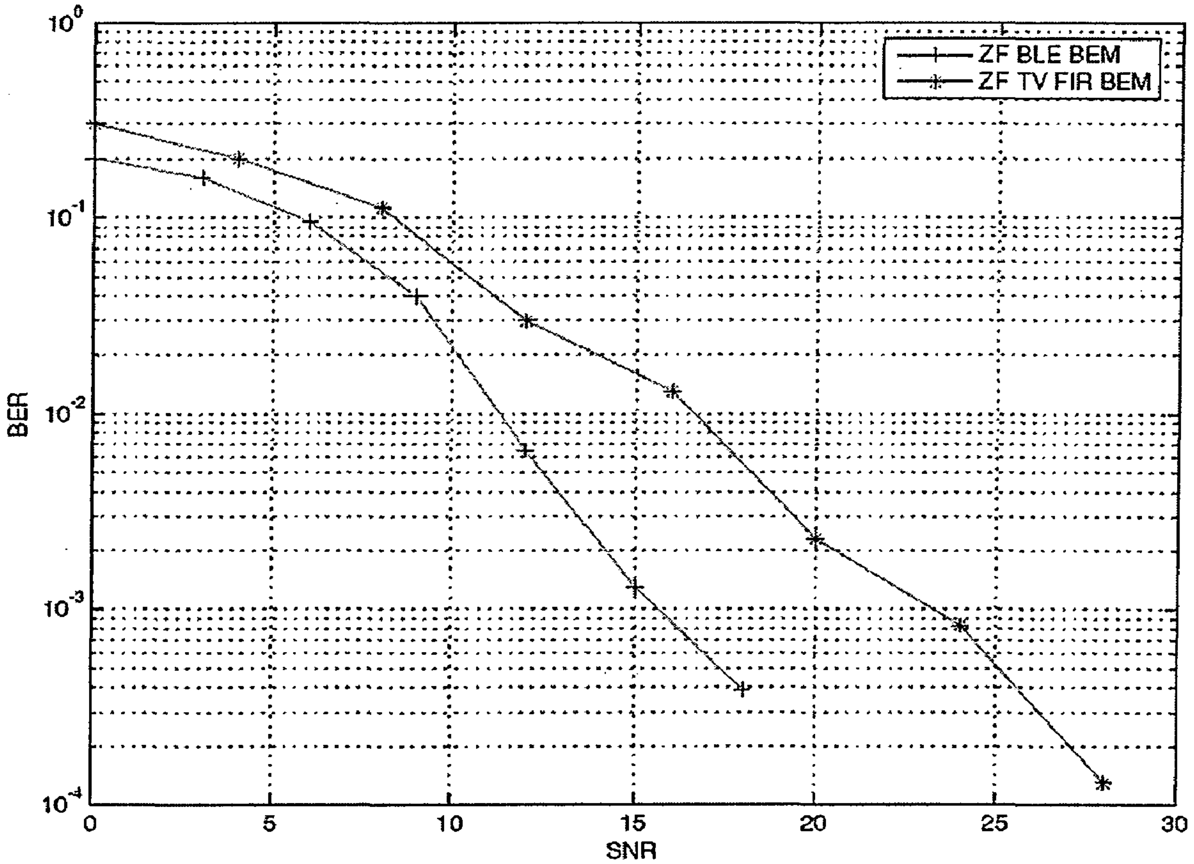


Figure 5.2 BER Vs SNR for ZF SIMO systems with two receive antennas

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial equalizer	9261000	30976

Example 2

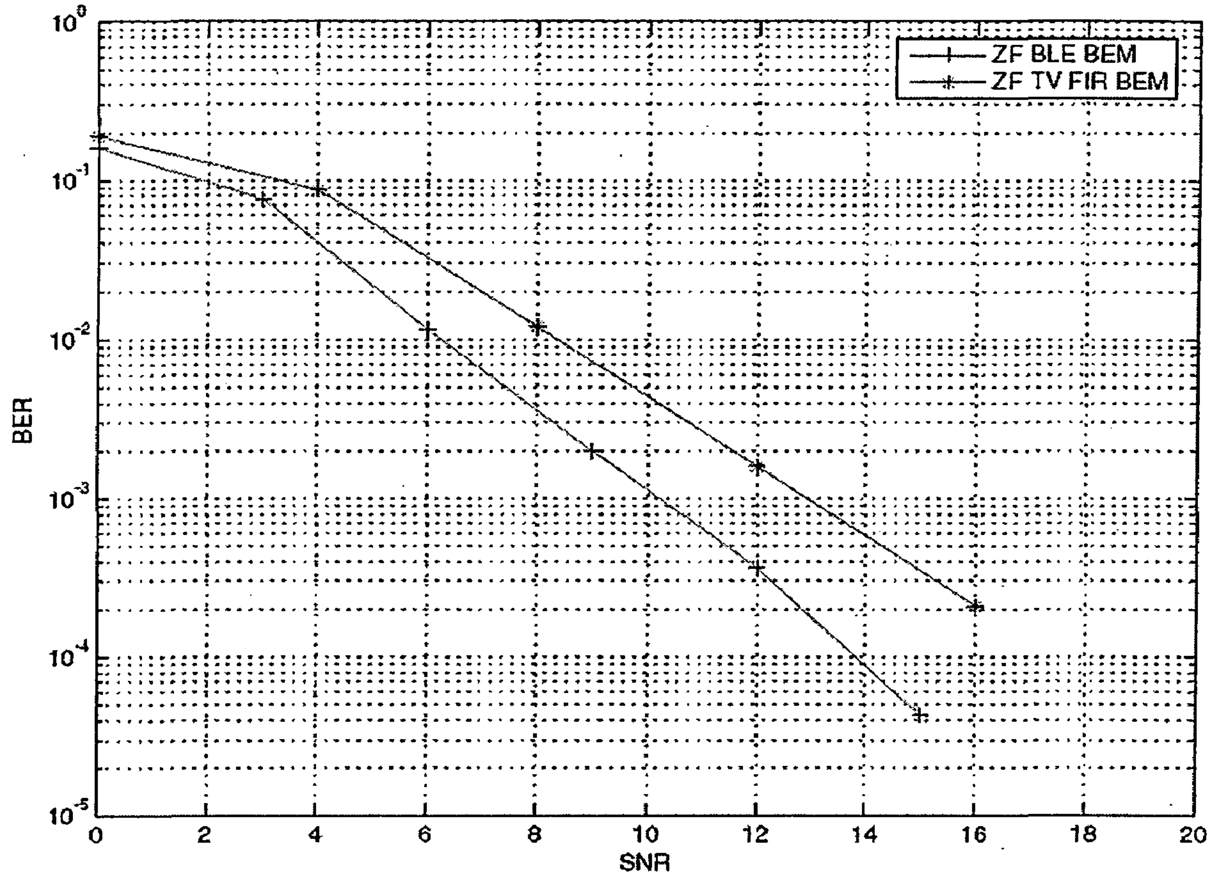


Figure 5.3 BER Vs SNR for ZF SIMO systems with four receive antennas

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial equalizer	1331000	12544

Example 3

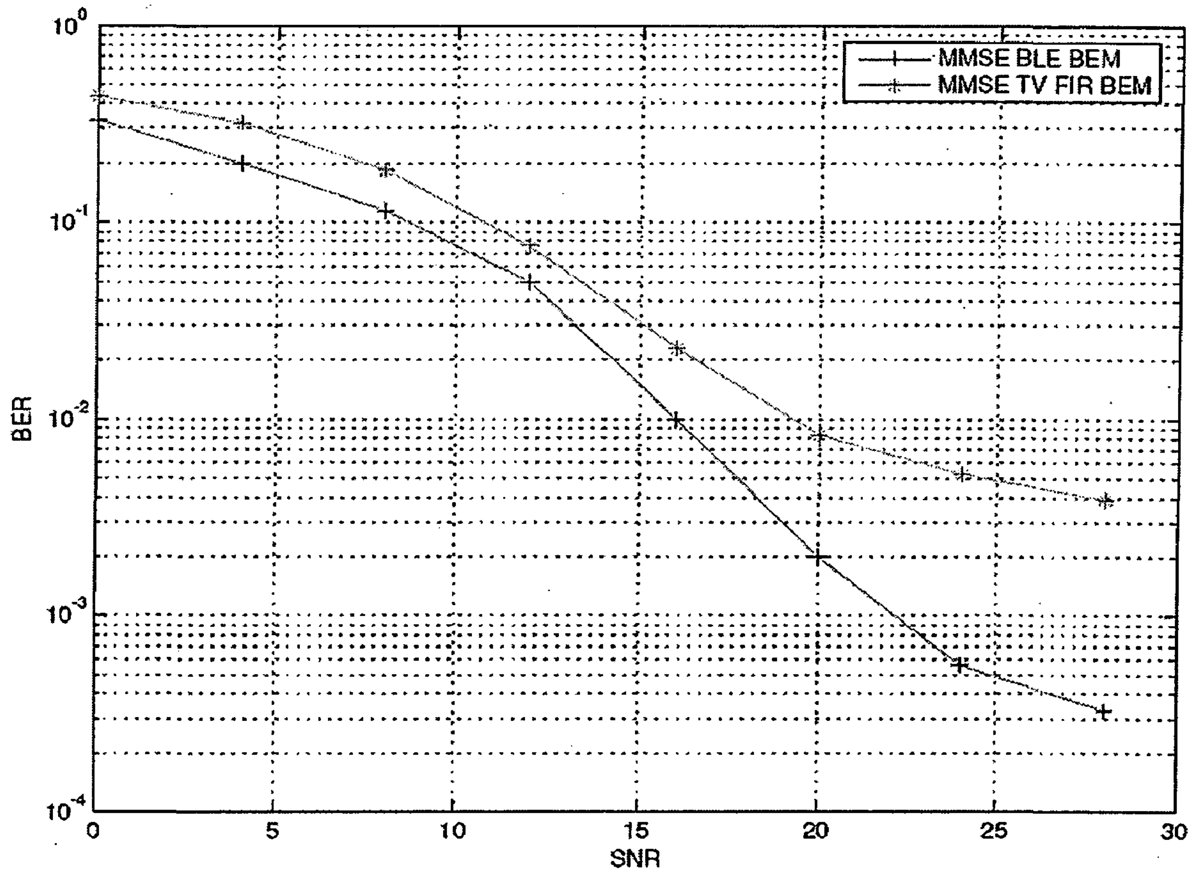


Figure 5.4 BER Vs SNR for MMSE SISO systems

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial equalizer	9261000	30976

Example 4

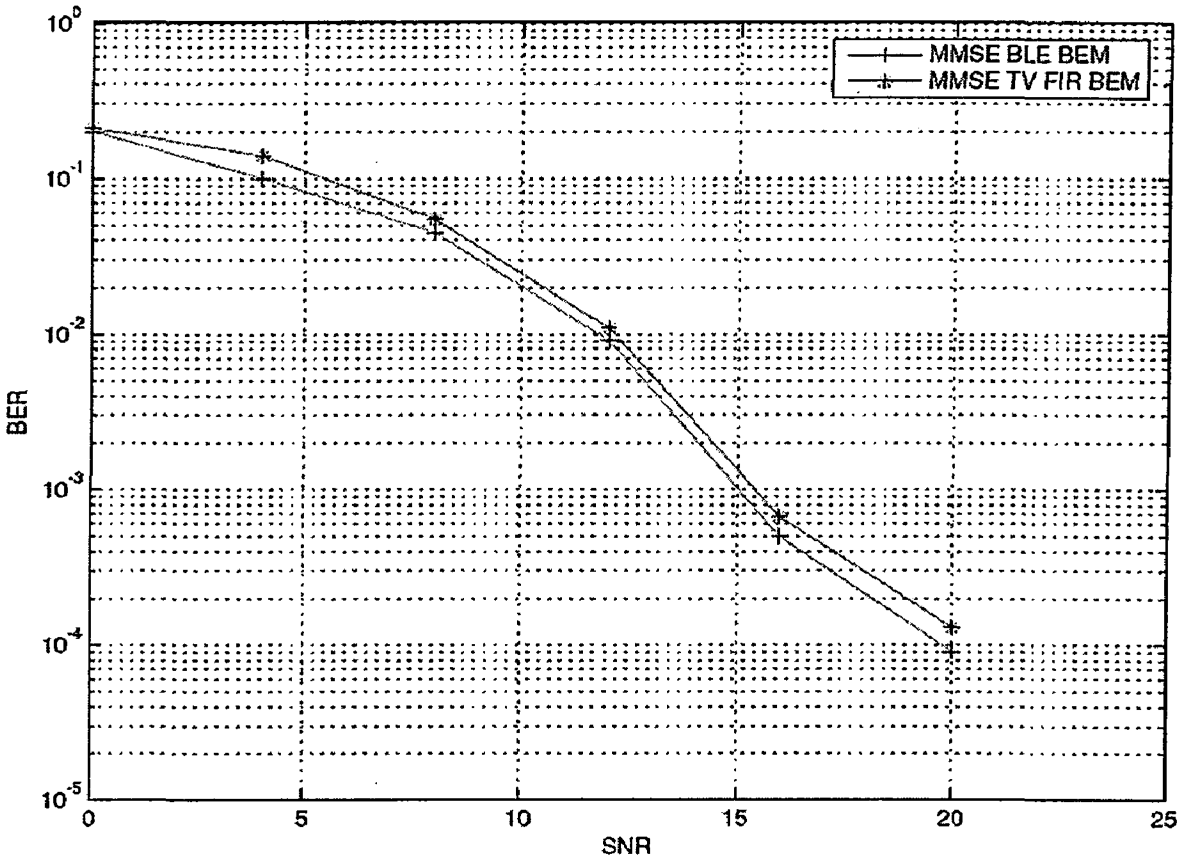


Figure 5.5 BER Vs SNR for MMSE SIMO systems with two receive antennas

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial equalizer	1331000	12544

Example 5

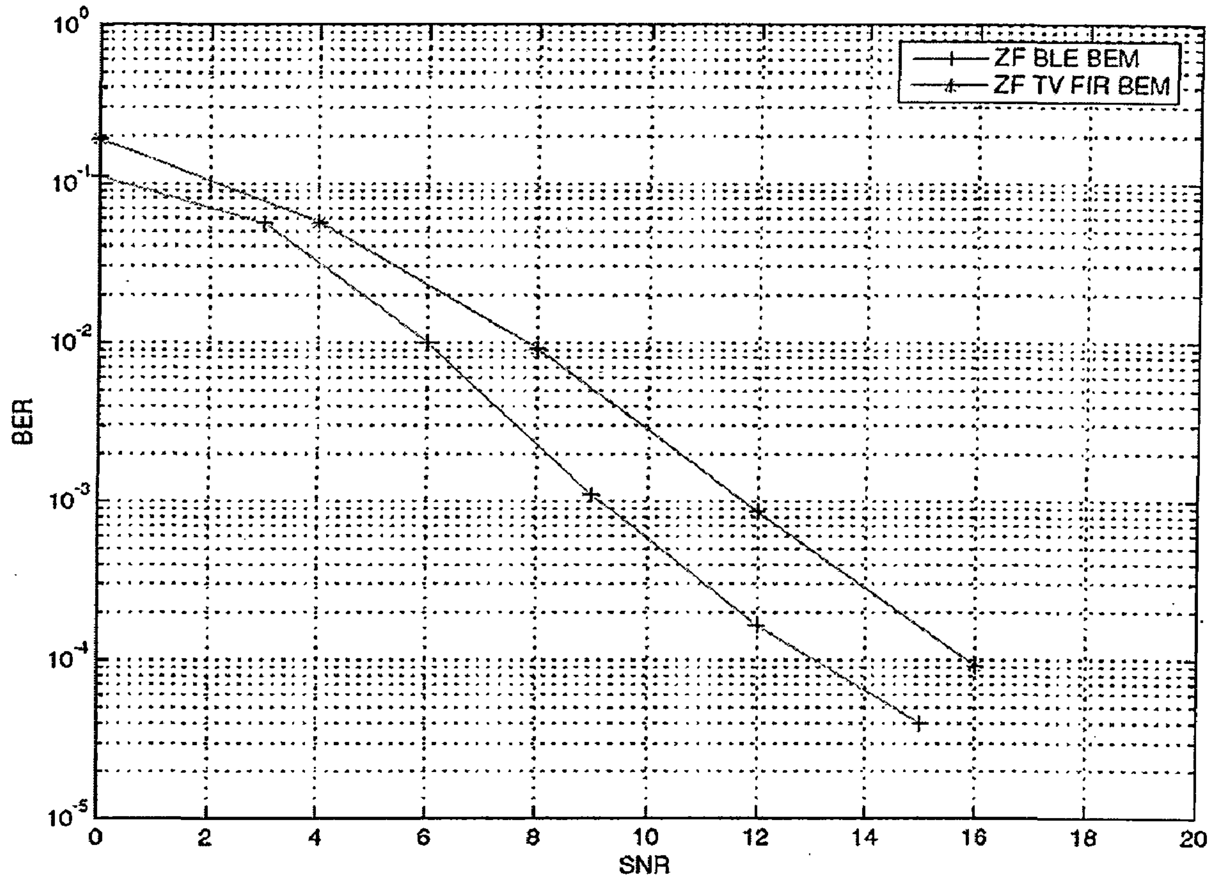


Figure 5.6 BER Vs SNR for ZF MIMO systems with two transmit and four receive antennas

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial equalizer	681472	8960

Example 6

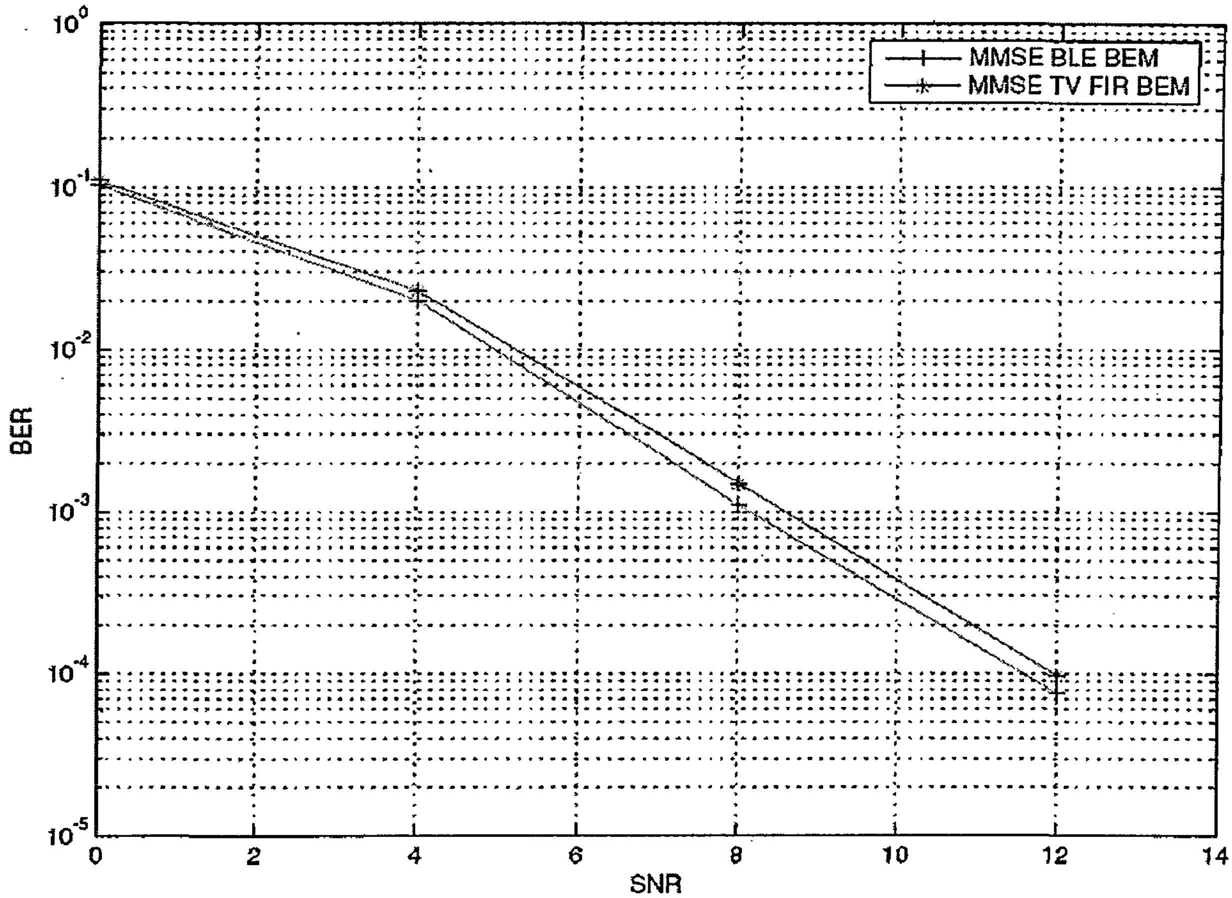


Figure 5.7 BER Vs SNR for MMSE MIMO systems with two transmit and four receive antennas

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial equalizer	373248	6400

Example 7

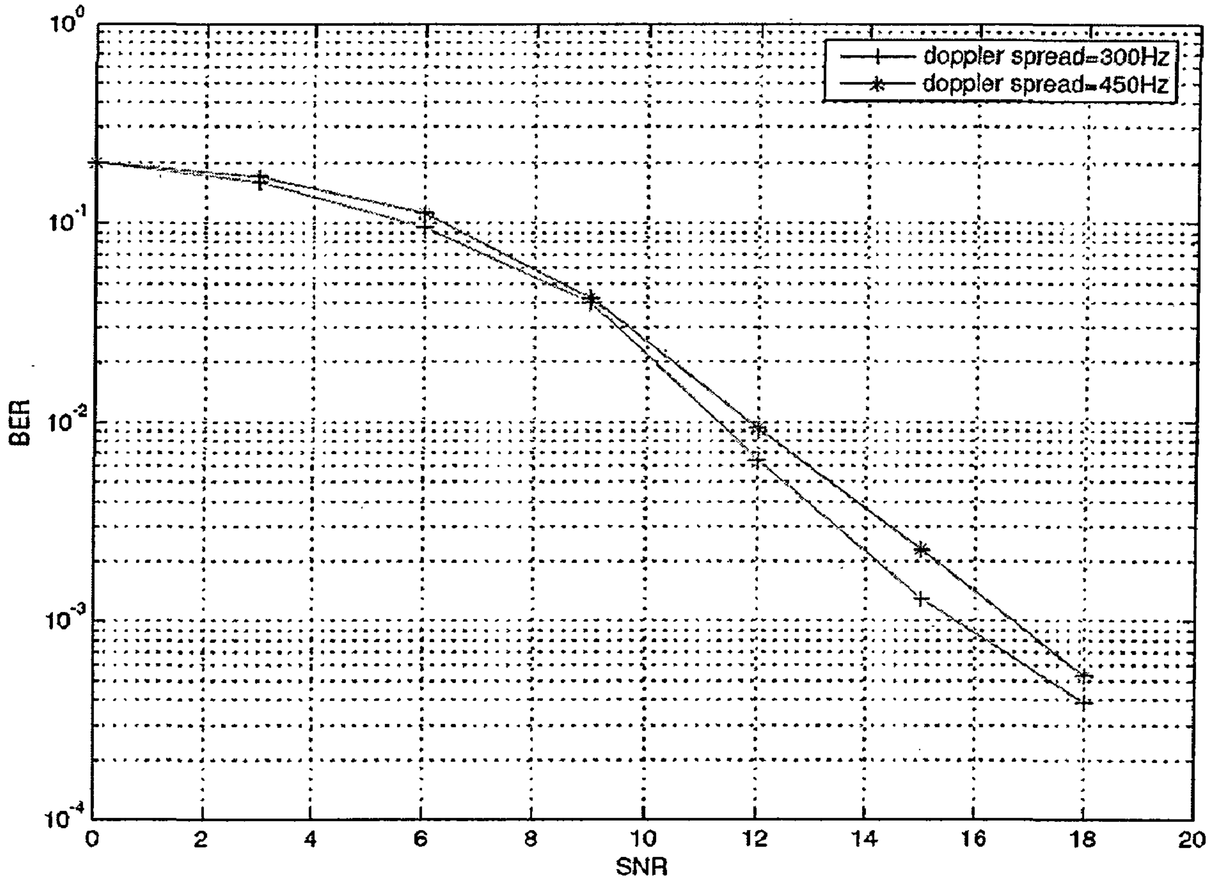


Figure 5.8 BER Vs SNR for ZF BLE receiver with two receive antennas for different Doppler spreads

Example 8

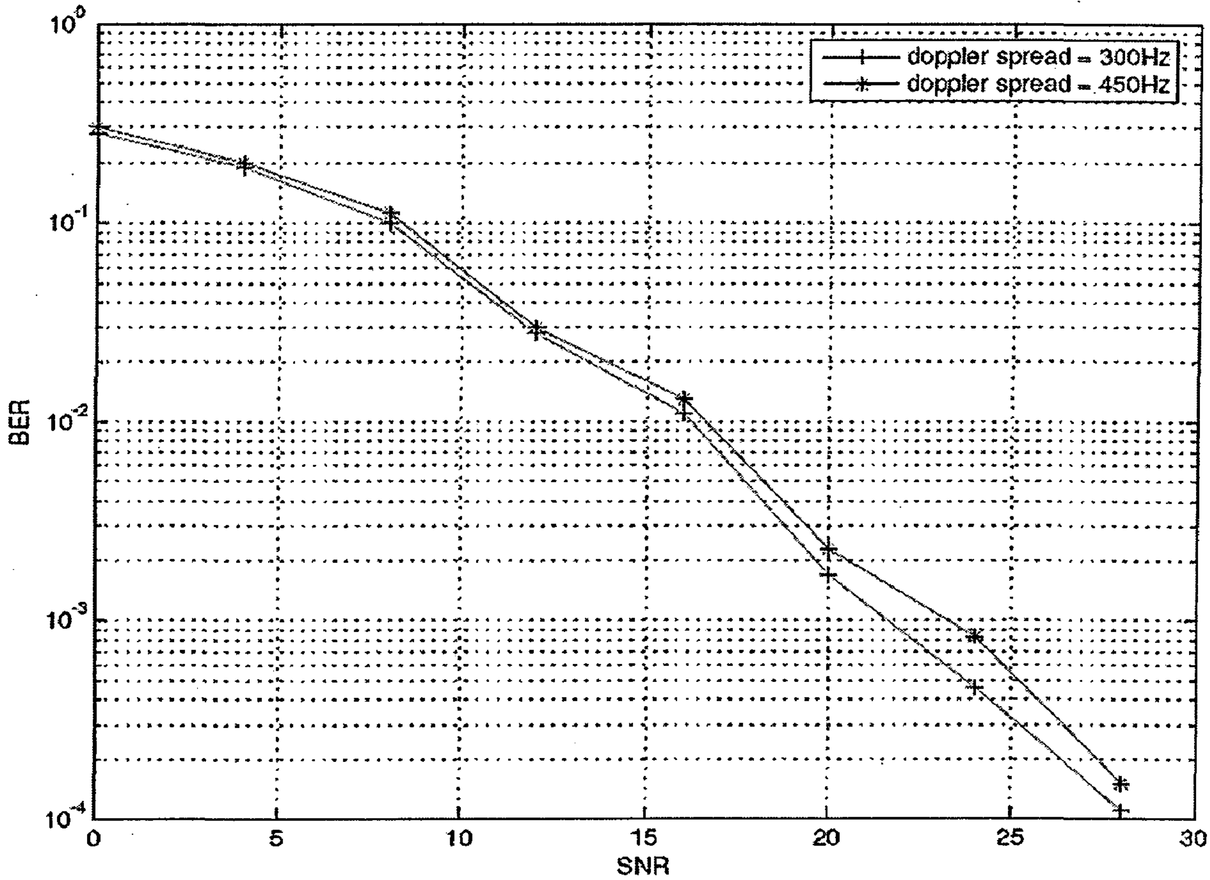


Figure 5.9 BER Vs SNR for ZF TV FIR receiver with two receive antennas for different Doppler spreads

5.2 Performance evaluation of BLE and TV FIR equalizer for equalization of OFDM

The simulation techniques presented in the previous sections are applicable to all cyclic prefix based block transmission schemes; this makes it also applicable to OFDM system. Flow chart for simulating time varying equalization for OFDM is given in Fig 5.10.

At the transmitter side random bits are generated and this binary data is then mapped in to QPSK constellation. The symbols obtained after mapping the binary bit stream will be in any one of the constellation plane of QPSK. A block of 'N' symbols are formed.. IFFT operation is performed on this block and a cyclic prefix of length v is inserted at the head of each block. This block is transmitted through the doubly selective channel. The channel matrix is calculated using the same methods given in section 5.1.1 and 5.1.2. the channel parameters are chosen same as that of section 5.1.An estimate of the transmitted signal block is obtained by using a block linear equalizer or a time varying serial linear equalizer. FFT operation is performed on the estimate block to get back the symbol block; this block is further converted into bits using shortest distance decoding. Minimum of 100 blocks are transmitted and error in transmission is calculated and BER Vs SNR curves are plotted. Design and implementation complexities are calculated using the methods mentioned in section 5.1.3.

Example 9

In example nine we compare performance of ZF block linear equalizer (ZF BLE) and zero forcing time varying FIR serial linear equalizer (ZF TV FIR SLE) for a SIMO OFDM system with two receive antennas. The time varying equalizer is modeled using basis expansion model with $L^{'}=10$ and $Q^{'}=10$. From figure 5.11 we can notice a SNR loss of 3dB for TV FIR SLE compared to ZF BLE at a BER of 10^{-2}.Comparing complexities of both the equalizers we can see that there is 44.80 percentage reduction in design complexity and 52.73 reduction in implementation complexity in the case of the time varying equalizer.

Example 10

In example ten we compare performance of MMSE block linear equalizer and MMSE time varying FIR serial linear equalizer for a SIMO OFDM system with two receive antennas. Time varying equalizer is modeled using the design parameters $L'=6$ and $Q'=6$. From figure 5.12 we can notice a SNR loss of 3dB for MMSE TV FIR SLE compared to MMSE BLE at a BER of 10^{-2}. But in such cases TV FIR SLE's have 80.85 percentage reduction in implementation complexity and 92.06 percentage reduction in design complexity compared to block linear equalizers. Comparing figure 5.11 and 5.12 we can see that MMSE equalizers perform better than ZF equalizers. Both BLE and time varying MMSE equalizers offers 1dB gain over ZF equalizers.

Example 11.

In example eleven we compare performance of ZF block linear equalizer and ZF time varying FIR serial linear equalizer for a MIMO OFDM system with two transmit and four receive antennas. The design parameters of time varying equalizer are selected as $L'=4$ and $Q'=6$. From the figure 5.13 we can see that performance of ZF TV FIR SLE coincides with that of ZF BLE throughout the measuring range. Here TV FIR SLE offers 86.32 percentage reduction in implementation complexity and 95.93 percentage reduction in design complexity.Comparing 5.13 and 5.11 we can notice that performance improvement and complexity reduction are obtained in the case of MIMO equalizers compared to that of SIMO equalizers. We can observe a 5dB improvement for MIMO BLE and 8dB improvement for time varying MIMO equalizer compared to SIMO system in example 10 at a BER of 10^{-2}.

Example 12.

Finally in figure 5.14 we compare the performance of MMSE solutions of block linear equalizer and time varying equalizer for MIMO OFDM system with two transmit and four receive antennas. The design parameter for time varying equalizer are chosen as $L'=4$ and $Q'=4$ .From the figure 5.14 we can see that performance of MMSE TV FIR SLE is similar to that of MMSE BLE throughout the measuring range. Time varying

MMSE MIMO equalizers show almost equal performance of time varying ZF MIMO equalizers but they are less complex. The performance of these equalizers can be improved by increasing design parameters L' and Q'. Here TV FIR SLE offers 90.23 percentage reduction in implementation complexity and 97.77 percentage reduction in design complexity, we can conclude that time varying MIMO equalizers are superior to block linear MIMO equalizers .

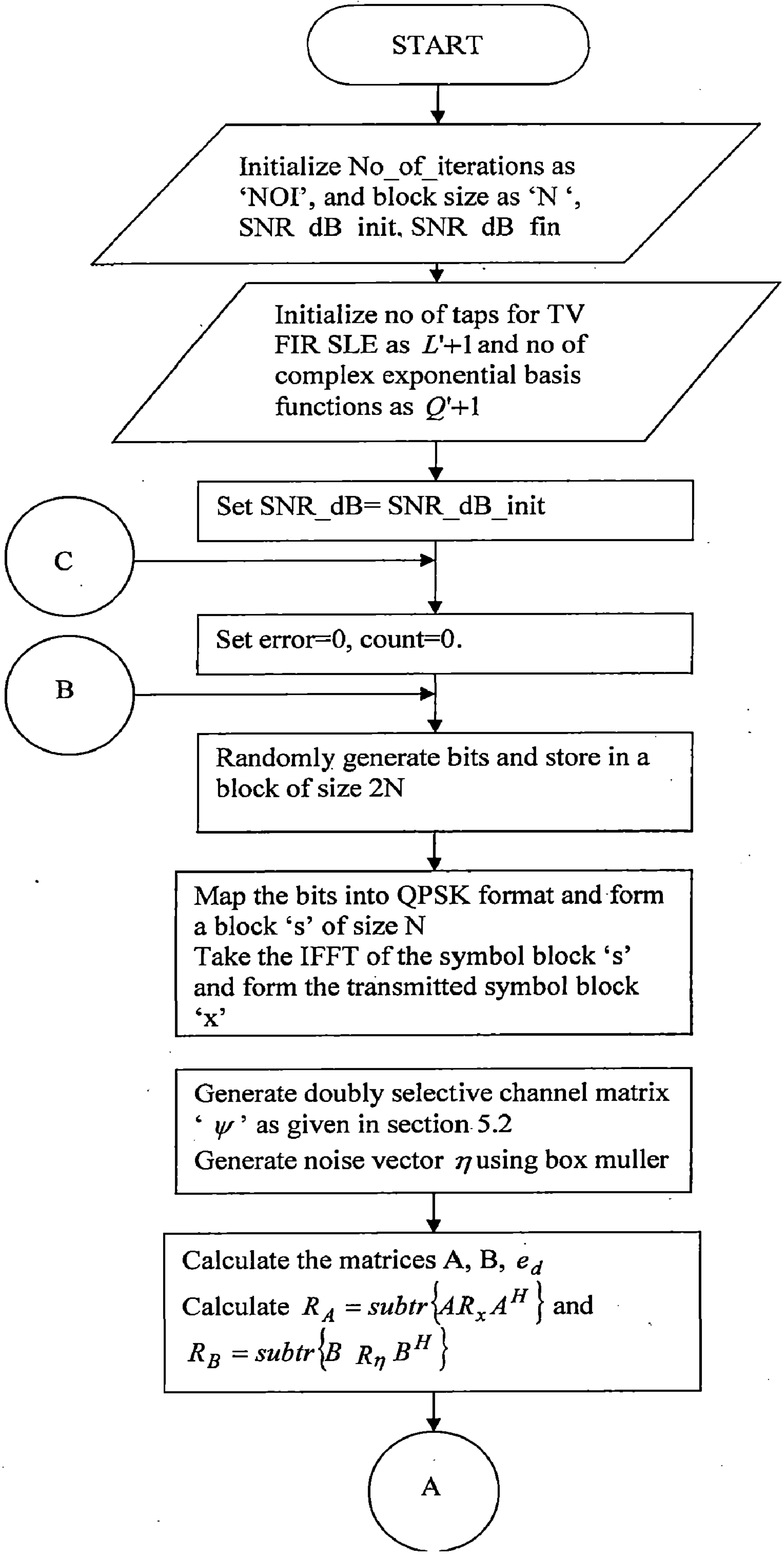


Figure 5.10 (a) Flow chart for time varying equalization of OFDM

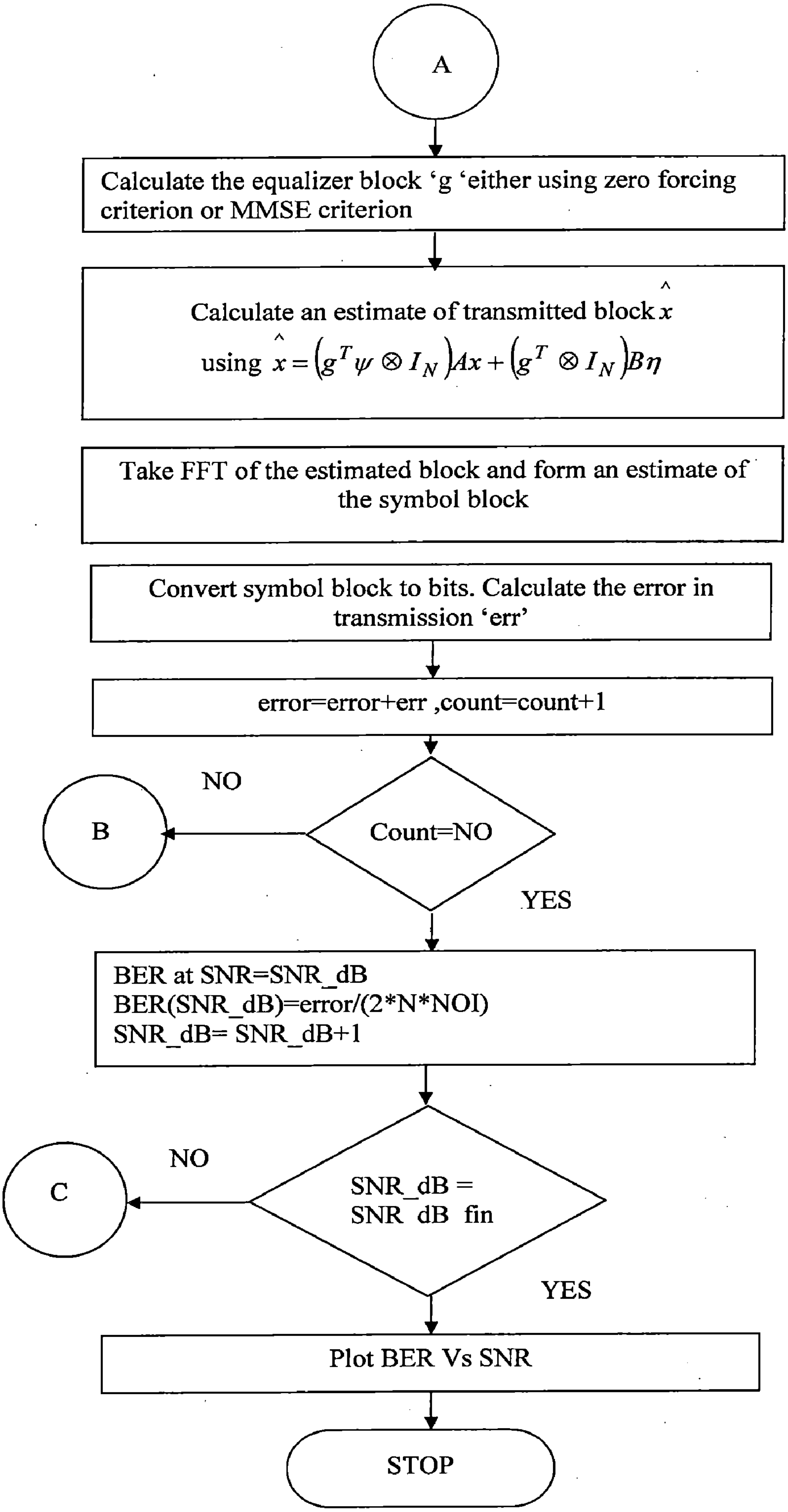


Figure 5.10 (b) Flow chart for time varying equalization of OFDM

Example 9

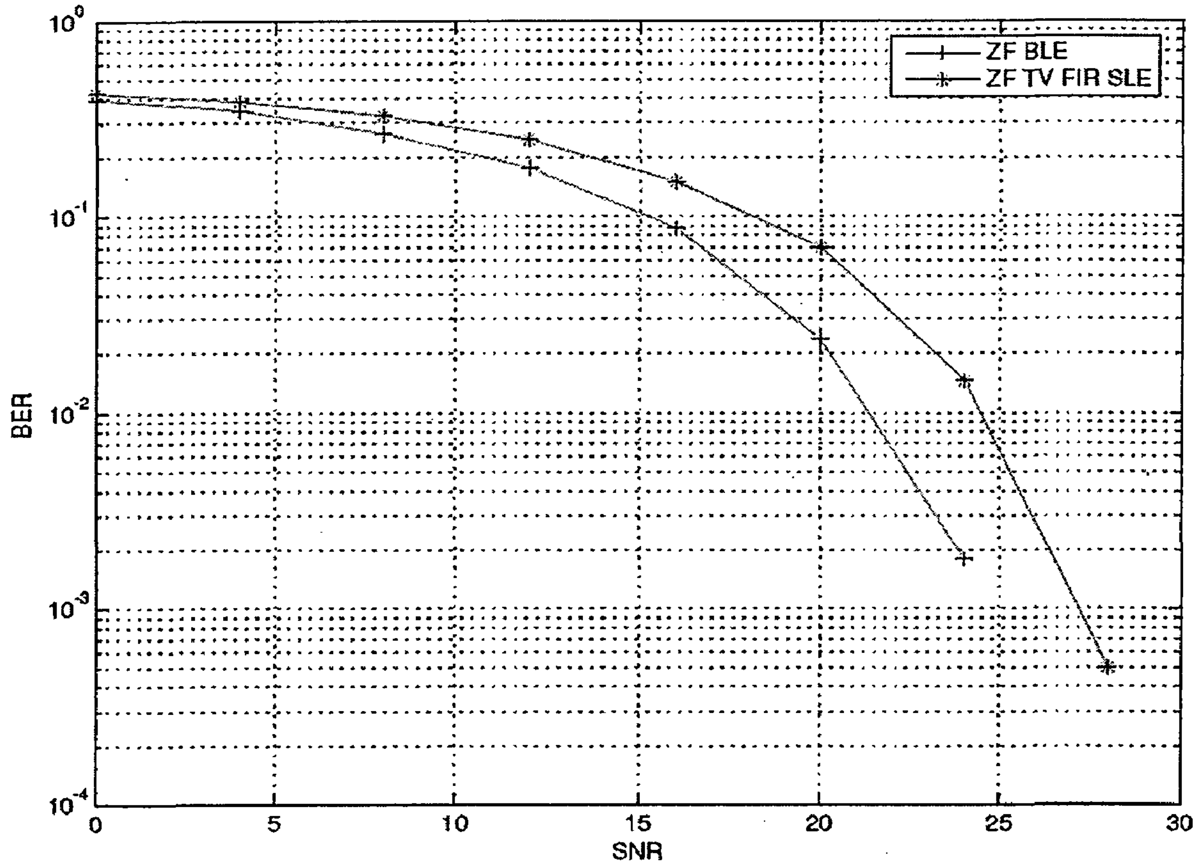
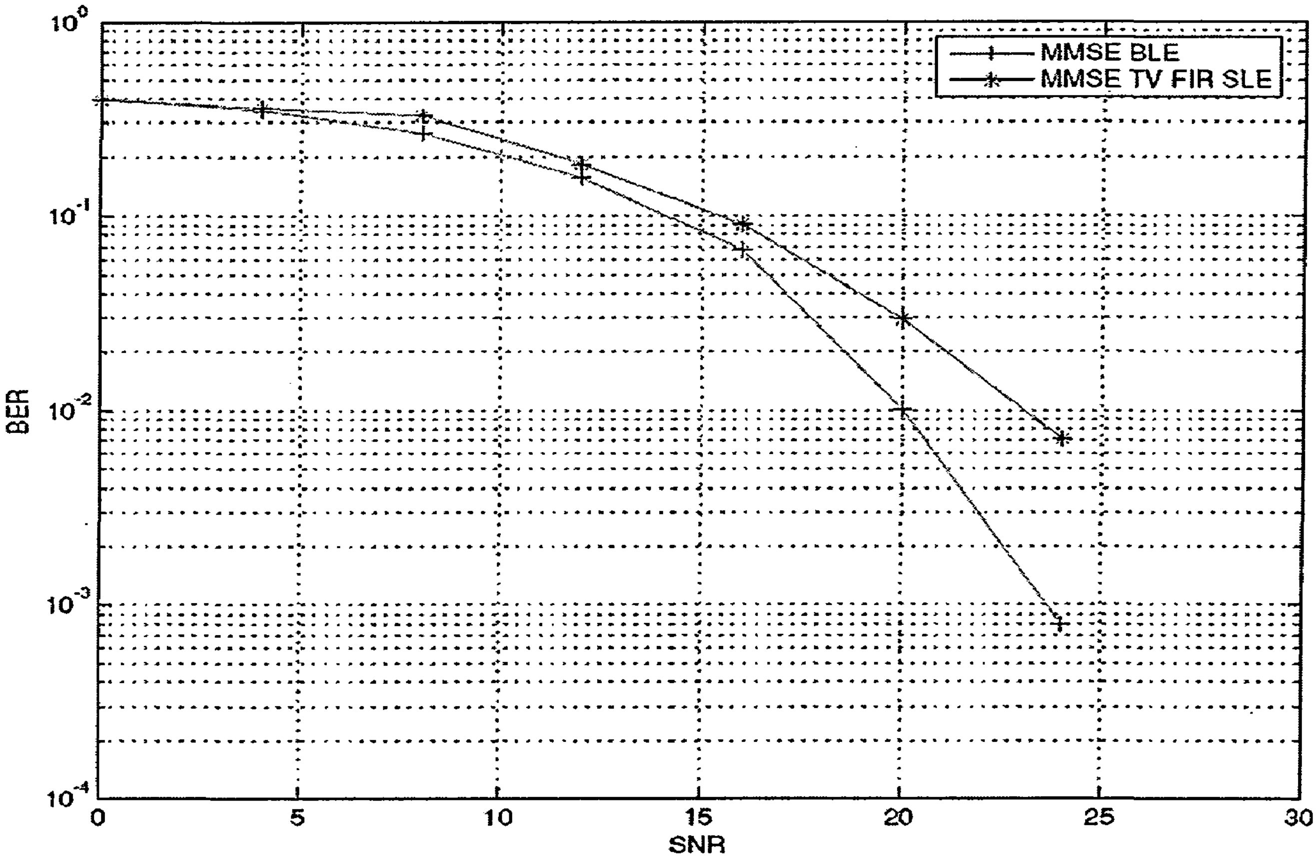


Figure 5.11 BER Vs SNR for ZF SIMO OFDM systems with two receive antennas

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial linear equalizer	9261000	30976

Example 10



'igure 5.12 BER Vs SNR for MMSE SIMO OFDM systems with two receive antennas

:qualizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
3lock linear equalizer	16777216	65536
Time varying FIR serial :qualizer	1331000	12544

Example 11

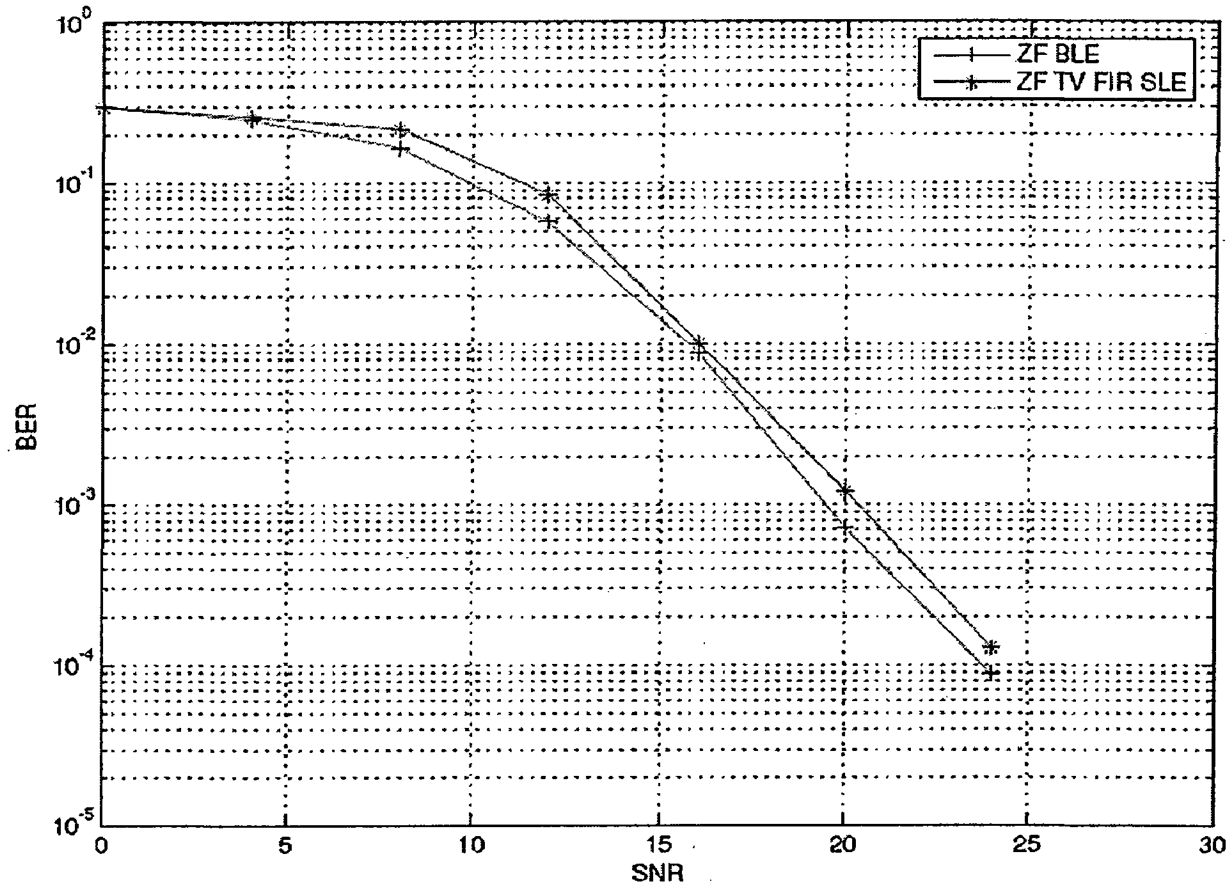


Figure 5.13 BER Vs SNR for ZF MIMO OFDM systems with two transmit and four receive antennas

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial equalizer	681472	8960

Example 12

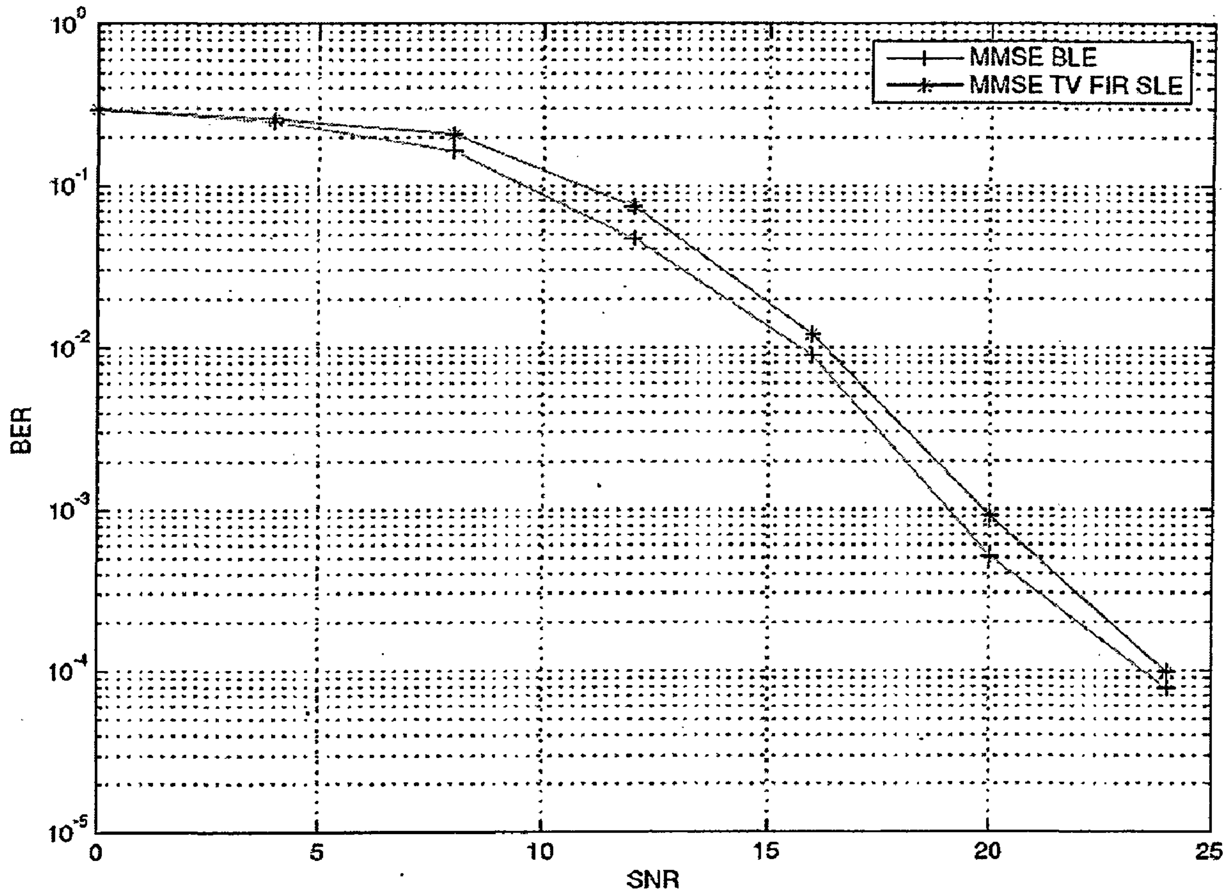


Figure 5.14 BER Vs SNR for MMSE MIMO OFDM systems with two transmit and four receive antennas

Equalizer type	Design complexity (in flops)	Implementation complexity (in multiply-add operations per receive antenna)
Block linear equalizer	16777216	65536
Time varying FIR serial equalizer	373248	6400

5.3 Conclusions

This dissertation work was aimed at studying equalization schemes for block transmission systems like OFDM over doubly selective channel. The traditional way of equalization of doubly selective channel is by using block linear equalizers or time varying serial linear equalizers. We first modeled doubly selective channel using basis expansion model. We then considered zero forcing and MMSE solutions for block linear equalizer and time varying equalizer. The time varying equalizer is modeled using the structure of basis expansion model. In this dissertation work, we first considered block linear equalization and time varying serial linear equalization for a block of QPSK modulated symbols for SIMO and MIMO systems. Based on the simulation results, our conclusions are:

1) Time varying serial linear equalizer requires at least two receive antennas for the existence of the zero forcing solution. Zero forcing solution is possible for block linear equalizer even with single receive antenna. MMSE solutions are always better than ZF solutions.
2) Performance of block linear equalizer is much better than the serial linear equalizer for a SIMO system with two receive antennas. Time varying equalizer provides tremendous reduction in complexity. As the no of receive antennas increases performance of time varying equalizer comes closer to the performance of block linear equalizer .If the number of receive antennas are more than two then time varying equalizer gives similar performance as that of block linear equalizer along with much lower complexity.
3) The simulation results shows that performance of block linear and time varying equalizers for 2×4 MIMO systems are much better than the performance of 1×4 SIMO systems. Time varying equalizer provides remarkable reduction in complexity at a performance comparable to that of block linear equalizers. We can conclude that time varying equalizers out performs block linear equalizers for MIMO systems.
4) From the simulations we have seen that performance of block linear equalizers and time varying serial linear equalizers degrades marginally for changes in Doppler spreads.

We then evaluated the performance of equalizers for OFDM system under various antenna configurations. The conclusions drawn based on the simulation results are as follows:

1) The performance of time varying equalizer are similar to the performance of block linear equalizer for SIMO and MIMO systems. Due to remarkable reduction in complexity provided by time varying equalizers at a comparable performance, they are preferred over block linear equalizer for SIMO and MIMO OFDM systems.

The scope of the report is limited to equalization of the doubly selective channels using linear equalizer structures .The approach presented in this report can be used for decision feed back equalization of block transmission over doubly selective channel. The time varying equalizer considered in this report is used as a time domain equalizer (TEQ) in OFDM and discrete multi tone systems. An optimal frequency-domain per tone equalizer (PTEQ) can be obtained by transferring these TEQ operations to the frequency domain. PTEQ optimizes the performance on each sub carrier separately and provides better performance than time domain equalizer.

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APPENDIX: MATLAB Source Code

Program for block linear equalizer for OFDM system

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Main program for plotting BER Vs SNR
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all;
clc;
ber=zeros(1,7);%for storing BER for different SNR's
k=1;
for snrdb_1=0:4:24
error1=0;
for u=1:200
    error_1=ofdm_system1(error1,snrdb_1);%function of ofdm
system
    error1=error_1;
end
ber(k)=error1;
k=k+1;
end
ber_1=ber/102400;
snrdb_1=0:4:24;
figure;
semilogy(snrdb_1,ber_1,'-+r');
xlabel('SNR');ylabel('BER');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function ofdm
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function err=ofdm_system1(error,snrdb1)
N=256;     %block length
SNRdb=snrdb1;
input=zeros(1,512);
input=rand(1,512);
for i=1:512
    if(input(i)>.5)
        input(i)=1;
    else
        input(i)=0;%bits are randomly generated
    end
end
i=1;
modulated_array=zeros(1,256);
```

```
ifft_op=zeros(1,256);
txd_signal=zeros(256,1);
rxd_signal=zeros(256,1);
eta=zeros(256,1);
eqlzr=zeros(256);
equalized_signal=zeros(256,1);
s_cap=zeros(1,256);
fft_op=zeros(1,256);
while i<512    %Mapping to QPSK constellation
    if ((input(i)==0)&&(input(i+1)==0))
        modulated_array((i+1)/2)=-0.7071+0.7071i;
    elseif((input(i)==0)&&(input(i+1)==1))
        modulated_array((i+1)/2)=0.7071+0.7071i;
    elseif((input(i)==1)&&(input(i+1)==0))
        modulated_array((i+1)/2)=-0.7071-0.7071i;
    else
        modulated_array((i+1)/2)=0.7071-0.7071i;
    end
    i=i+2;
end
% ifft_op=modulated_array;
ifft_op=ifft(modulated_array,256);
txd_signal=(ifft_op)';
L=3;Q=4;Nr=2;
H=get_H(L,Q,Nr,N);  %The function returns channel matrix
eta=noise(N,Nr,SNRdb,ifft_op); %returns noise vector
rxd_signal=(H*txd_signal)+eta;
eqlzr=pinv(H);
%pseudo inverse of the channel matrix for zero forcing
%equalization
%mmse solution can be obtained by changing equation
equalized_signal=(eqlzr*rxd_signal);
s_cap=(equalized_signal)';
% fft_op=s_cap;
fft_op=fft(s_cap,256);
output=zeros(1,512);
i=1;j1=1;
for i=1:256 %de mapping
    if ((real(fft_op(i))<0)&&(imag(fft_op(i))>0))
        output(j1)=0;output(j1+1)=0;
    elseif((real(fft_op(i))>0)&&(imag(fft_op(i))>0))
        output(j1)=0;output(j1+1)=1;
    elseif((real(fft_op(i))<0)&&(imag(fft_op(i))<0))
        output(j1)=1;output(j1+1)=0;
    else
        output(j1)=1;output(j1+1)=1;
    end
```

```
    j1=j1+2;
end
for i=1:512
    if (input(1,i)~=output(1,i))
        error=error+1;
    end
end
err=error;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function for generating noise samples
%Box muller method is used for generating noise vector
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function awgn=noise(N,Nr,SNRdb,ifft_op)
m=Nr*N;
awgn1=zeros(1,m);
awgn_1=zeros(1,m);
snr = 10^(SNRdb/10);
Ps=0;
for i=1:N
    Ps=Ps+abs(ifft_op(i));
end
Ps=Ps/N;
No = Ps/snr;
std_dev = sqrt(No/2);
for i=1:m
        sq=1;
        while (sq>=1)
                r1=rand(1);
                r2=rand(1);
                sq = r1*r1+r2*r2;
         end
        as1=2*r1-1;
        as2=2*r2-1;
        wc=as1*as1+as2*as2;
        Y=sqrt(-2*log(wc)/wc);
        awgn_1(i)=as1*Y+as2*Y*j;
        awgn1(i) = std_dev*awgn_1(i);
end
awgn=(awgn1)';
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function for generating channel matrix
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function H=get_H(L,Q,Nr,N)
for i=1:Nr
```

```
    for m=1:N
        for n=1:N
            H1_t=get_H_r_t(L,Q,Nr,N);
%function call for individual channel matrices
            H(((i-1)*N)+m,n)=H1_t(m,n);
        end
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function for generating basis expansion model coefficients
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function h=get_hql(Q,L,N)
f_max=300;
T=0.000025;
for l=0:L
    Z=2*pi*f_max*T*l;
    r_hh(l+1)=besselj(0,Z);
end
r=zeros(L,L);
for i=1:L
    for j=1:L
        r(i,j)=r_hh(abs(i-j)+1);
    end
end
A=zeros(L,1);
B=zeros(L,1);
for l=1:L
    B(l,1)=r_hh(1,l+1);
end
A=inv(r)*B;
u=randn(N,L)+i*randn(N,L);
for n=1:N
    for i=1:L
        if ((n-i)>0)
            h(n,l)=u(n,l)+A(i,1)*h((n-i),l);
        end
    end
end
row=zeros(N,Q+1);
for l=1:L+1
    for q=1:Q+1
    for n=1:N
        t(i,l)=h(n,l);
        row(n,q+(Q/2)+1)=exp((j*2*pi*(q-(Q/2)-1)*(n-1))/N);
    end
    row_h=conj(row)';
```

```
     h_l=row_h*t;
     for j=1:Q+1
         h(j,l)=h_l(j;1);
     end
     end
end
```

%%

function for generating channel matrix for each recievers

%%

```
function H_r_t=get_H_r_t(L,Q,Nr,N)
h=zeros((Q+1),(L+1));
h=get_hql(Q,L,N);
D1=zeros(N);
Z1=zeros(N);
DZ=zeros(N);
H=zeros(N*Nr,N);
H1=zeros(N);
for q=1:Q+1
    for l=1:L+1
        for i=1:N
            for k=1:N
                if (i==k)
                  D1(i,k)=exp((j*2*pi*(q-(Q/2)-1)*(k-))/N);
                end
                if(mod((i-k-l+1),N)==0)
                    Z1(i,k)=1;
                end
                if(mod((i-k-l+1),N)~=0)
                    Z1(i,k)=0;
                end
            end
        end
        DZ=D1*Z1;
        H1=H1+(h(q,l)*DZ);
    end
end
H_r_t=(H1)';
for i=1:Nr
    for m=1:N
        for n=1:N
            H(((i-1)*N)+m,n)=H1_t(m,n);
        end
    end
end
```

Program for time varying SLE for OFDM system

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
main program for plotting BER Vs SNR
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all;
clc;
ber=zeros(1,7);
k=1;
for snrdb_1=0:4:24
error1=0;
for u=1:400
    error_1=ofdm_system1(error1,snrdb_1);
    error1=error_1;
end
ber(k)=error1;
k=k+1;
end
ber_1=ber/51200;
snrdb_1=0:4:24;
semilogy(snrdb_1,ber_1,'-+');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function for OFDM system
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function err=ofdm_system1(error,snrdb1)
N=64;
SNRdb=snrdb1;
input=zeros(1,128);
input=rand(1,128);
for i=1:128
    if(input(i)>.5)
        input(i)=1;
    else
        input(i)=0;
    end
end
i=1;
L=3;Q=4;Nr=2;
modulated_array=zeros(1,64);
ifft_op=zeros(1,64);
txd_signal=zeros(64,1);
rxd_signal=zeros(64,1);
eta=zeros((N*Nr),1);
fft_op=zeros(1,64);
```

```
while i<128
    if ((input(i)==0)&&(input(i+1)==0))
        modulated_array((i+1)/2)=-0.7071+0.7071i;
    elseif((input(i)==0)&&(input(i+1)==1))
        modulated_array((i+1)/2)=0.7071+0.7071i;
    elseif((input(i)==1)&&(input(i+1)==0))
        modulated_array((i+1)/2)=-0.7071-0.7071i;
    else
        modulated_array((i+1)/2)=0.7071-0.7071i;
    end
    i=i+2;
end
% ifft_op=modulated_array;
ifft_op=ifft(modulated_array,64);
s=(ifft_op)';
Nr=2;
L=3;
Q=4;
L1=20;
SNRdb=snrdb1;
Q1=20;
d=12;
A=get_a(L,Q,L1,Q1,d,N);
B=get_b(L1,Q1,d,N,Nr);
zye=get_zye(L,Q,L1,Q1,d,N,Nr);
awgn=noise(N,Nr,SNRdb,ifft_op);
ed=zeros((Q+Q1+1)*(L+L1+1),1);
m=((Q+Q1)*(L+L1+1))/2+d+1;
ed_t=zeros(1,(Q+Q1+1)*(L+L1+1));ed(m,1)=1;
ed_t=(ed)';
g_t_zf=ed_t*pinv(zye);
I=eye(N);
s_cap=kron((ed_t),I)*A*s+kron((g_t_zf),I)*B*awgn;
% fft_op=(s_cap)';
fft_op=fft(s_cap',64);
output=zeros(1,128);
i=1;j1=1;
for i=1:64
    if ((real(fft_op(i))<0)&&(imag(fft_op(i))>0))
        output(j1)=0;output(j1+1)=0;
    elseif((real(fft_op(i))>0)&&(imag(fft_op(i))>0))
        output(j1)=0;output(j1+1)=1;
    elseif((real(fft_op(i))<0)&&(imag(fft_op(i))<0))
        output(j1)=1;output(j1+1)=0;
    else
        output(j1)=1;output(j1+1)=1;
    end
```

```
    j1=j1+2;
end
for i=1:128
    if (input(1,i)~=output(1,i))
        error=error+1;
    end
end
err=error;
```

%%%

function to get the matrices D_q and Z_1

%%%

```
function [D Z]=get_dq(l1,q1,N1)
l=l1;
q=q1;
N=N1;
D1=zeros(N);
Z1=zeros(N);
for i=1:N
    for k=1:N
        if (i==k)
            D1(i,k)=exp((j*2*pi*q*(k-1))/N);
        end
        if(mod((i-k-1),N)==0)
            Z1(i,k)=1;
        end
        if(mod((i-k-1),N)~=0)
            Z1(i,k)=0;
        end
    end
end
D=D1;
Z=Z1;
```

%%%

function for generating channel matrix

%%%

```
function zye=get_zye(L,Q,L1,Q1,d,N,Nr)
h=get_hql(Q,L,N):
T=zeros((L1+1),(L1+L+1));
zye_q_r=zeros((L1+1),(L1+L+1));
zye_r=zeros((Q1+1)*(L1+1),(Q+Q1+1)*(L+L1+1));
zye_r_t=zeros((Q+Q1+1)*(L+L1+1),(Q1+1)*(L1+1));
zye=zeros(Nr*(Q1+1)*(L1+1),(Q+Q1+1)*(L+L1+1));
zye_t=zeros((Q+Q1+1)*(L+L1+1),Nr*(Q1+1)*(L1+1));
omega=zeros(L1+1);
count=0;
for q=1:Q+1
    for l=1:L+1
        T(1,l)=h(q,l);%formation of the toeplitz matrix
```

```
            T1(1,1)=h1(q,1);
        end
        for i=2:L1+1
            for k=2:L1+L+1
                T(i,k)=T(i-1,k-1);
                T1(i,k)=T1(i-1,k-1);
            end
        end
        for i=1:L1+1
            omega1(1,i)=exp((-j*2*pi*(q-(Q/2)-1)*(-d+i-1))/N);
        end
        omega=diag((omega1)');
        zye_q_r=omega*T;
        zye_q_r1=omega*T1;
        for i=1:L1+1
            for k=1+count:L1+L+1+count
                zye_r(i,k)=zye_q_r(i,k-count);
                zye_r1(i,k)=zye_q_r1(i,k-count);
            end
        end
        count=count+L1+L+1;
    end
    for i=L1+2:(Q1+1)*(L1+1)
        for k=L1+L+2:(Q+Q1+1)*(L+L1+1)
            zye_r(i,k)=zye_r((i-L1-1),(k-L1-L-1));
            zye_r1(i,k)=zye_r1((i-L1-1),(k-L1-L-1));
        end
    end
        zye_r_t_1=(zye_r)';
        zye_r_t_2=(zye_r1)';
    for m=1:(Q+Q1+1)*(L+L1+1)
        for n=1:(Q1+1)*(L1+1)
            zye_t(m,n)=zye_r_t_1(m,n);
            zye_t(m,n+((Q1+1)*(L1+1)))=zye_r_t_2(m,n);
        end
    end
    zye=(zye_t)';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function to calculate matrix A
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function A=get_a(L,Q,L1,Q1,d,N)
A1=zeros((Q+Q1+1)*(L+L1+1)*N,N);
DZ2=zeros(N);
count=0;
for q=-(Q+Q1)/2:(Q+Q1)/2
```

```
    for l=-d:L+L1-d
        [D Z]=get_dq(l,q,N);
        DZ2=D*Z;
        for m=count+1:count+N
            for n=1:N
                A1(m,n)=DZ2(m-count,n);
            end
        end
        count=count+N;
    end
end
A=A1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function to calculate matrix B
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function B=get_b(L1,Q1,B_r,N,Nr)
B=zeros(Nr*(Q1+1)*(L1+1)*N,Nr*N);
for i=1:Nr
    for m=1:(Q1+1)*(L1+1)*N
        for n=1:N
            B((m+((Q1+1)*(L1+1)*N*(i-1))),(n+(N*(i-
1))))=B_r(m,n);
        end
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function to calculate matrix B_r
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function B_r=get_b_r(L1,Q1,d,N,Nr)
B_r_1=zeros((Q1+1)*(L1+1)*N,N);
DZ2=zeros(N);
count=0;
for q=-Q1/2:Q1/2
    for l=-d:L1-d
        [D Z]=get_dq(l,q,N);
        DZ2=D*Z;
        for m=count+1:count+N
            for n=1:N
                B_r(m,n)=DZ2(m-count,n);
            end
        end
        count=count+N;
    end
end
B_r=B_r_1;
% B_r for all receive antennas are assumed to be same
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function for subtr{.} operation
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function subtr_a=get_subtr(A,N)
[m n]=size(A);
p=m/N;
q=n/N;
subtr_a=zeros(p,q);
for i=1:p
    for j=1:q
        sum=0;
        for k=1:N
            sum=sum+A((i-1)*N+k,(j-1)*N+k);
        end
        subtr_a(i,j)=sum;
    end
end
%The subtr{.} operation mentioned in chapter 3
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function for getting Rb
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function R_B=get_R_B(B,SNRdb,N)
snr = 10^(SNRdb/10);
Ps=1;
No = Ps/snr;
std_dev = sqrt(No/2);
B_h=(conj(B))';
R_B=get_subtr(((No/2)*B*B_h),N);
% The noise standard deviation calculated as sqrt(No/2)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function for getting Ra
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function R_A=get_R_A(A,N)
B_h=(conj(A))';
R_B=get_subtr((A*A_h),N);
%The symbols are assumed to have unity variance
```