ANALYSIS OF FLOW TO MULTIAQUIFER WELLS

A DISSERTATION

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requirements for the award of the degree
of
MASTER OF ENGINEERING
in
HYDROLOGY

By

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled " Analysis of Flow to Multiaquifer Wells" in partial fulfilment of the requirement for the award of the degree of Master of Engineering in Hydrology, submitted in the School of Hydrology of the University of Roorkee,Roorkee is an authentic record of my work carried out during the period October 1978 to August 1985 under the supervision of Dr.G.C. Mishra, Reader, School of Hydrology, University of Roorkee,Roorkee.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge. The candidate stayed at the University of Roorkee,Roorkee for more than three months during the above mentioned period for the thesis work.

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SYNOPSIS

Several water bearing strata separated by aquicludes or aquitards may be encountered during subsurface exploration at a locality. If the aquifer can provide dependable yield, the aquifer may be tapped to meet the required demand. Tapping lower aquifers need economic considerations. In the present study a technoeconomic factor i.e. saving in energy by simultaneous tapping of top two aquifers has been studied. It is found from the study that for shallow aquifer when depth to piezometric surface is upto 5 m the saving in energy is considerable. For $T_1/T_2 = 1$ and $\phi_1/\phi_2 = 1$ the saving in energy is of the order of 12% and when $T_1/T_2 = 5/2$ the corresponding value is about 7%. When piezometric depth exceeds 20 m, the saving in energy is negligible. Well storage has no significant influence in saving in energy.

NOTATIONS

The following notations have been used in the thesis. Notations in the literature review are as per the original text.

Notations	Description	Unit
C	Time Step,	t
$K_i(R)$	Unit impulse kernel which is defined as the drawdown at time 't' when unit impulse quantity of water is withdrawn from the aquifer 'i' at t = 0,	ℓ
m, n	Time steps,	t
Q_p	Constant well discharge,	t^3 t^{-1}
$Q_p(n)$	Well discharge during time step 'n',	ℓ^3 t^{-1}
$Q_1(n)$	Discharge contribution of aquifer 1 to pumping during time step n,	ℓ^3 t^{-1}
$Q_2(n)$	Discharge contribution of aquifer 2 to pumping during time step n,	ℓ^3 t^{-1}
$Q_w(n)$	Discharge from well storage to pumping during time step 'n'	ℓ^3 t^{-1}

Notations	Description	Unit
$Q_w(\gamma)$	Withdrawal from well storage or replenishment at the time step 'γ'	$\ell^3\ t^{-1}$
r	Distance of the observation well from the pumped well,	ℓ
r_c	Radius of large diameter dugwell,	ℓ
r_w	Radius of bore well,	ℓ
$S_i(r,t)$	Drawdown in the piezometric surface of aquifer 'i' at a distance 'n' from the pumping well at time 't' after the onset of pumping,	ℓ
$S_w(n)$	Drawdown at well face at the end of time step 'n'	ℓ
T_i	Transmissivity of i^{th} aquifer	$\ell^2\ t^{-1}$
t	Time	t
β_i	Hydraulic diffusivity of i^{th} aquifer (T_i/ϕ_i)	$\ell^2\ t^{-1}$
ϕ_i	Storage coefficient of i^{th} aquifer defined as the volume of water instantaneously released from the aquifer storage per unit drawdown per unit horizontal area	

Notation	Description	Unit
$\delta_{r,i}(m)$	Discrete kernel coefficient defined as the drawdown at the end of m^{th} timestep at distance 'r' from the pumping well in response to **withdrawal** of unit quantity of water from the storage of i^{th} aquifer during the 1^{st} time period	

CONTENTS

CHAPTER 1

INTRODUCTION

While considerable work has been done on wells tapping a single aquifer, little study has been made on wells drawing from several aquifers. Most of the mathematical solutions, developed so far to determine drawdown and contributions of individual aquifers during unsteady flow to a multiaquifer well are intractable. However, recently developed semi-analytical solution to unsteady flow into a well opened to several aquifers using a discrete kernel approach are found to be tractable. No study has however been made on the saving in energy requirement when more than one aquifer is tapped to meet a specified demand. In this dissertation a comparative study of energy requirement in case of a well opening to two aquifers separated by a aquiclude has been made to that of a single aquifer well.

Nautiyal provided solution for unsteady flow in case of continuous pumping from wells.

In this thesis the comperative study has been made after developing equations for intermittant pumping. Well storage has also been taken into consideration in the analysis.

In case centrifugal pump is used the pumping rate will depend upon drawdown. Hence maintenance of uniform rate of pumping is not possible. In the present analysis, the unsteady flow to multiaquifer well bas been studied when the pumping rate is a linear function of drawdown.

The scheme of presentation of the thesis is as follows :

Chapter 2 covers literature study on solution to unsteady flow to wells. Emphasis has been given to research works on multiaquifer wells when discrete kernel approach bas been used. In chapter 3 flow to well open to two aquifers separated by aquiclude due to intermittant pumping has been analysed. Results and discussions have been dealt in Chapter 4.

CHAPTER 2

REVIEW OF LITERATURE

In this chapter brief reference has been made to research works on single and multiaquifer wells. Diserete kernel approach being the method of solution in this thesis, literature on diserete kernel solutions are dealt with more extensively.

SINGLE AQUIFER WELLS

Theis (1935) was the pioneer in analysing unsteady flow towards a well. Research workers prior to Theis dealt with the steady state flow. Theis based his solution on the equation developed by Carslaw and Jaeger(1959) for an analogous problem of conduction of heat through solids. His solution is given by the following equation

$$S(r,t) = \frac{Q}{4 \pi T} \int_{\frac{r^2}{4\beta t}}^{\infty} \frac{e^{-x}}{x} \, dx \qquad ..(2.1)$$

where,

S(r,t) = drawdown at any time 't' after the onset of pumping and at a distance of 'r' from the pumping well,

Q = rate of pumping,

r = radial distance of observation point from the well,

T = transmissivity,

t = time interval between the onset of pumping and observation of drawdown,

β = diffusivity = $\frac{T}{\phi}$, and

ϕ = storage coefficient.

Following assumptions were made in in Theis equation:

1) The aquifer is infinite, homogeneous, isotropic and of uniform thickness over the area of influence of pumping.
2) Pumping is continued at a constant rate.
3) Prior to pumping the water level is nearly horizontal over the area influenced by pumping.
4) The well fully penetrates the aquifer and receives water from the entire thickness of the aquifer by horizontal flow.
5) The well is of infitesimal diameter.
6) The aquifer is confined and release of water from storage is instantaneous.

While Theis confined his work to constant discharge, Abuzied and Scott(1963) and Hontush (1964) provided solutions to unsteady flow due to variable discharge from well. Jacob and Lohman (1952) on the other hand evolved unsteady flow equation for constant head discharge.

MULTIAQUIFER WELL

Using Theis equation Sokol (1963) derived equation for steady state flow to a well open to several aquifers. He has related water level fluctuations in a non-pumping multiaquifer well to head change in any one of the aquifers penetrated by the well. He concludes that the ratios of the water level fluctuation in the well to the head change is equal to the ratio of transmissivity of the aquifer in which the head change occurs to the sum of transmissivities of all the aquifers punctured by the well.

Thiem equation gives

$$Q' = \frac{2 \pi T (h - h_w)}{\log_e (r/r_w)}$$

in which

Q' = flow rate,

T = transmissivity of aquifer,

h = height of potentiametric surface at distance r, and

h_w = height of water level in the discharging well of radius r_w.

Sokol equation reads as follows :

$$\Delta h_w = \frac{T_J \ \Delta h_J}{\sum_{i=1}^{M} T_i}$$

where

Δh_w = water level fluctuation in the well,

Δh_J = head change in J^{th} aquifer,

T_J = transmissivity of J^{th} aquifer,

T_i = transmissivity of i^{th} aquifer, and

M = number of aquifers penetrated by the well.

Papadopulos (1966) used integral transform technique to obtain solution for unsteady flow to a well tapping two confined aquifers of infinite areal extent and having different potentiometric surfaces. Though his solution was exact to obtain head distribution, it was intractable for numerical calculation. He then developed asymptotic solution for both head and discharge distribution which were amenable to computation and were accurate enough for practical application. However no numerical result was presented by him. Also the solution can not be used to determine formation constants of individual aquifers. Besides, the number of parameters involved in the solution has to be tried in different combinations to obtain the type curve matching the observed response curve. This procedure is cumbersome and does not give a practical solution. His procedure is given below :

The solutions for $t \leq t_o$ are

$$H_1 - h_1 = \frac{H_1 - H_2}{1 + \partial} A(\tau/\epsilon^2, \rho/\epsilon)$$

$$H_2 - h_2 = \frac{-\partial (H_1 - H_2)}{1 + \partial} A(\tau/\epsilon^2, \rho/\epsilon)$$

$$Q_1(t) = 2 \pi T_1 (H_1 - H_2) G(\tau/\epsilon^2)/(1 + \partial)$$

$$Q_2(t) = - Q_1(t)$$

These solutions are for the boundary value problem where the aquifers remain unpumped for a period t_o during which flow occurs from one aquifer to the other through the well screen owing to the difference in initial heads in upper and lower aquifers.

For t > to

$$H_1 - h_1 = \frac{H_1 - H_2}{1 + \partial} A(\tau/\theta^2, \gamma/\theta)$$

$$+ \frac{Q}{4 \pi T_1 (1 + \partial)} [W(\gamma^2/4 \tau^*)$$

$$- \frac{1}{(1 + \partial)} (\ell_n \alpha^2) A(\tau^*/\theta^2, \gamma/\theta)]$$

$$H_2 - h_2 = - \frac{\partial(H_1 - H_2)}{1 + \partial} A(\tau/\theta^2, \alpha\gamma/\theta)$$

$$+ \frac{Q}{4 \pi T_2 (1 + \partial)} [W(\alpha^2 \gamma^2/4 \tau^*)$$

$$+ \frac{1}{(1 + \partial)} (\ell_n \alpha^2) A(\tau^*/\theta^2, \alpha \gamma/\theta)]$$

$$Q_1(t) = \frac{2\pi T_1 (H_1 - H_2) G(\tau/\theta^2)}{1 + \partial}$$

$$+ \frac{Q\partial}{2(1 + \partial)} [2 e^{-(1/4)\tau^*} - \frac{1}{1 + \partial}$$

$$(\ell_n \alpha^2) G(\tau^*/\theta^2)] \qquad ...(2.2)$$

$$Q_2(t) = Q - Q_1(t) \qquad ...(2.3)$$

where

$$G(x) = \frac{4x}{\pi} \int_0^\infty e^{-x u^2} [\frac{\pi}{2} + \tan^{-1} \frac{Y_o(u)}{J_o(u)}] u \, du,$$

h_1, h_2 = heads at any distance 'r' and time 't',

T_1, T_2 = transmissivities of upper and lower aquifers,

H_1, H_2 = initial heads in upper and lower aquifers,

$Q_1(t), Q_2(t)$ = discharges from upper and lower aquifers at time 't',

J_o, Y_o = zero order Bessel functions of 1st and second kind respectively,

t = time since the well is completed,

t_o = time at which pumping started,

r = radial distance to any point from the axis of the well,

r_w = radius of the well,

ν_1, ν_2 = hydraulic diffusivities of upper and lower aquifers ($= T/\phi$),

α = $\sqrt{1/2}$,

∂ = T_1/T_2,

ϵ = $\alpha^{[\partial/(1 + \partial)]}$,

ρ = r/r_w,

τ = t_1/r_w^2,

τ^* = $(t_1 - t_o)/r_w^2$,

Q = constant discharge from the well,

$$A(x,y) = 1 - \frac{2}{\pi} \int_0^\infty \frac{e^{-xu^2}}{u} \cdot \frac{J_o(u)\, Y_o(u.y) - Y_o(u)\, J_o(u.y)}{J_o^2(u) + Y_o^2(u)}, \text{ and}$$

$$W(x) = \int_x^\infty \frac{e^{-u}}{u}\, du, \text{ an exponential integral.}$$

The same technique of integral transform (Schapery's (1962) approximate method of inversion of Laplace transforms) was used by Khader and Verankutty (1975) to obtain solution to unsteady flow into multiaquifer well open to two aquifers. They have presented numerical results determing contribution of individual aquifer to the total discharge of well.

DISCRETE KERNEL SOLUTION

Using linear system theory Maddock developed functional relationship between excitation and response and obtained the following expression for drawdown at a point due to pumping of a number of wells.

$$S(K,n) = \sum_{J=1}^{M} \sum_{i=1}^{n} \partial\,[K,J,(n-i+1)]\,[q(J,i)]$$

where,

$S(K,n)$ = the drawdown at K^{th} well at n^{th} time period,

M = total number of wells,

$q(J,i)$ = discharge from J^{th} well in i^{th} time period, and

$\partial(K,J,i)$ = coefficients known as algebraic technological function.

The above expression has been derived with the assumption that the aquifer had no previous development i.e. $S(x,y,o) = 0$

Morel-Seytoux (1975) also relied on the functional relationship between excitation and response and used the carslow and Jeager's equation on heat conduction to solve ground water problems in homogenous isotropic aquifer of infinite areal extent.

Morel-Seytoux considered discrete kernel approach superior to finite difference methods. According to him it is possible to solve problems of optimal management with the use of discrete kernel approach through the efficient technique of mathematical programming rather than through the use of successive trial and error required in simulation The mathematical programming problem is considerably reduced in size compared to a formulation that incorporates the finite difference equation of the hydrological model. Morel-Seytoux method has been explained in detail in the Appendix.

Nautiyal (1985) used discrete kernel approach to find out solution to unsteady flow to a well tapping two aquifers separated by an aquiclude. In his work potentiometric surfaces prior to pumping were assumed equal in all the aquifers. A solution was obtained to determine contribution of each aquifer to the total discharge of well at various times. The solution for $Q_1(n)$ and $Q_2(n)$ is given by the following equation :

$$\begin{vmatrix} Q_1(n) \\ Q_2(n) \end{vmatrix} \begin{vmatrix} 1, & 1 \\ \partial_{rw1} - \partial_{rw2}(1) \end{vmatrix} \cdot \begin{vmatrix} Q_P \\ \sum_{\gamma=1}^{n-1} Q_2(\gamma)\partial_{rw2}(n-\gamma+1) - \\ \sum_{\gamma=1}^{n-1} Q_1(\gamma)\partial_{rw1}(n-\gamma+1) \end{vmatrix}$$

where,

$Q_1(n)$ and $Q_2(n)$ are the contribution of I^{st} and 2^{nd} aquifer.

Following conclusions were arrived at by Nautiyal :

1) When all the aquifers tapped have equal diffusivity values, their contributions are proportional to the respective transmissivity and are independent of time.

2) The aquifer having the lowest diffusivity contributes the entire quantity of water pumped from the well in the beginning of pumping. Other aquifers start contributing with the progress of pumping. After a large time interval when steady state has reached the contributions of each aquifer is found to be proportional to its transmmisivity values.

Solution to unsteady flow into a well opened to several aquifers intervened by aquicludes when initial piezometric surfaces are at different levels has been analysed by Mishra (1985) by discrete kernel approach. The solution is tractable and is amenable to numerical computation. Results for a well opened to three confined aquifers having different initial hydraulic heads and separated by aquicludes have been presented. Exchange of flow among the aquifers prior to and after pumping have been evaluated . Contribution of each aquifer to pumping has been found. Besides, variation of composite hydraulic head with time at the well point has been studied. Following observations have been made by Mishra

1) When the aquifers have equal hydraulic diffusivity, the composite hydraulic head at the well point attains a near steady state value very quickly.

2) In a three aquifer system, when two aquifers have equal initial hydraulic head and same hydraulic diffusivity, the flow they receive from the aquifer having highest initial hydraulic head are proportional to their respective transmissivity values.

3) The aquifer which has the highest initial hydraulic head always loses water to the aquifer with the lowest hydraulic head. The aquifer with intermediate initial hydraulic head at any time would either loose or gain water depending on whether the composite hydraulic head at the well point at that time is less or more than the intermediate initial hydraulic head.

In matrix form solution given by Mishra is

$$\begin{vmatrix} 1, & 1, & 1, & \ldots & 1, & 1 \\ -\partial_{rw1}(1), & \partial_{rw2}(1), & 0, & \ldots & 0, & 0 \\ -\partial_{rw1}(1), & 0, & \partial_{rw3}(1), & \ldots & 0, & 0 \\ \vdots & & & & & \\ -\partial_{rw1}(1), & 0, & 0, & \ldots\ldots & \partial_{rwM}(1), & 0 \\ -\partial_{rw1}(1), & 0, & 0, & \ldots\ldots & 0, & \frac{1}{\pi r_w^2} \end{vmatrix}$$

$$
\times \begin{vmatrix} Q_1(I) \\ Q_2(I) \\ Q_3(I) \\ \vdots \\ Q_M(I) \\ Q_W(I) \end{vmatrix} =
$$

$$
\begin{vmatrix}
H_2-H_1 + \sum_{\gamma=1}^{I-1} Q_1(\gamma)\partial_{rw1}(I-\gamma+1) - \sum_{\gamma=1}^{I-1} Q_2(\gamma)\partial_{rw2}(I-\gamma+1) \\
H_3-H_1 + \sum_{\gamma=1}^{I-1} Q_1(\gamma)\partial_{rw1}(I-\gamma+1) - \sum_{\gamma=1}^{I-1} Q_3(\gamma)\partial_{rw3}(I-\gamma+1) \\
\vdots \\
H_M-H_1 + \sum_{\gamma=1}^{I-1} Q_1(\gamma)\partial_{rw1}(I-\gamma+1) - \sum_{\gamma=1}^{I-1} Q_M(\gamma)\partial_{rwM}(I-\gamma+1) \\
H_{max}-H_1 + \sum_{\gamma=1}^{I-1} Q_1(\gamma)\partial_{rw1}(I-\gamma+1) - \frac{1}{\pi r_w^2} \sum_{\gamma=1}^{I-1} Q_W(\gamma)
\end{vmatrix}
$$

ANALYSIS OF FLOW TO A WELL WHEN PUMPING RATE IS A FUNCTION OF DRAWDOWN

Due to very significant effect of the well storage on drawdown, Theis equation is not suitable for large-diameter wells. Analytical solution of unsteady flow to a well considering well storage has been developed by several research workers. Papadopulos and Cooper (1967) analysed the flow to large diameter well using integral transform technique. The solution given by Papadopulos and Cooper is for a constant abstraction rate. Therefore when constant abstraction rate can not be maintained, which is often the case, when centrifugal pump is used, the solution can not be used to predict the drawdown. The problem of variable abstraction rate has been analysed by Lai and Su (1974) who have given an equation for the drawdown in and around a well of large diameter in a leaky artesian aquifer induced by an arbitrary time-dependent pumping rate using Laplace transform technique. The effect of the storage capacity of the well on the drawdown is found to be significant when the time of pumping is not large or the ratio of the transmissivity in the aquifer to its storage coefficient is small. The analysis of Lai et al., does not allow for the effect of linearly and exponentially variable abstraction rate that actually occurs in practice. Evaluation of drawdown in their method requires numerical integration of improper integral involving Bessel's functions. The numerical integration, therefore, involves large computations.

Rushton and Holt (1981) have presented an elegant numerical solution for analysis of pumping test data from large diameter well both during abstraction as well as recovery phases. The existence of the seepage face in the abstraction well, variable abstraction rate and well losses can also be included in the numerical model. The model simulates the water levels in a confined aquifer quite accurately.

Rushton and Singh (1983) have analysed flow to large diameter well for variable abstraction rate using numerical method. The abstraction rate is linear function of the drawdown. Mishra and Chachadi (1984) have analysed the same problem by discrete kernel approach.

CONCLUSION

From the literature study it is observed that energy consideration has not been given due importance. Energy requirement depend upon drawdown. When a multiaquifer system is tapped, there will be contribution to pumping by all the aquifers which are opened to the pump. In the present study it is aimed to find out the saving in energy component when two aquifers are tapped, taking well storage into consideration.

CHAPTER 3

UNSTEADY FLOW TO A WELL OPENED TO TWO AQUIFERS DUE TO INTERMITTANT PUMPING

INTRODUCTION

Water wells are generally constructed tapping more than one aquifer in order to have dependable yield. When centrifugal pump is used for abstraction the pumping rate decreases with the increase in drawdown and as such constant pumping rate can not be maintained in practice. Analysis of unsteady flow to a well tapping two aquifers separated by an aquiclude has been carried out by Papadopulos (1966) and Khader and Veerankuty (1975) who have used integral transform technique. The unsteady flow to a well tapping two aquifers separated by an aquiclude has been analysed by Mishra et al (1985) using a diserete kernel approach for continuous pumping. Mishra and Chachadi have analysed unsteady flow to a large diameter well when the pumping rate is a function of drawdown. In the present work, unsteady flow to a multiaquifer well has been analysed for the case when the pumping rate depends on the drawdown. The analysis has been done using diserete kernel approach for intermittant pumping. The energy spent in lifting the water has been evaluated.

STATEMENT OF THE PROBLEM

A schematic cross section of a well tapping two confined aquifers is shown in fig.3.1.The aquifers are separated by an aquiclude. Therefore no exchange of flow takes place between the two aquifers through the intervening layer. Each of the aquifers is homogeneous,isotropic, infinite in areal extent and is of uniform thickness. Drawdown in the piezometric surfaces are caused by discharge from aquifers. It is required to find the following

i) Contribution of each aquifer to pumping which is linearly related to drawdowns in the piezometric surfaces at the well face

ii) The energy consumption in intermittant pumping

iii) The conservation of energy due to tapping of the lower aquifer and due to provision of well storage in comparison to tapping of the top aquifer

ANALYSIS

Following assumptions have been made in the analysis.

a) Both the aquifers are initially at rest condition prior to pumping.

b) The well discharges at a rate which is linearly proportional to the drawdown.

c) At any time the drawdown at both the aquifers at the well face are same but vary with time.

d) The time parameter is discrete within each time step, the abstraction rates of water derived from each of the aquifers and well storage are separate constants.

The differential equation that describes the exially symmetric, radial, unsteady flow in each aquifer is given by

$$\frac{\partial^2 S_i}{\partial r^2} + \frac{1}{r}\frac{\partial S_i}{\partial r} = \frac{\phi_i}{T_i}\frac{\partial S_i}{\partial t} \quad - (1)$$

where

S_i = drawdown in piezometric surface in the i^{th} aquifer,

r = radial distance,

t = time,

ϕ_i = storage coefficient, and

T_i = transmissivity in the i^{th} aquifer.

The above differential equation is to be solved for the following boundary conditions :

$$S_i(\infty, t) = 0 \qquad \ldots.(2)$$

$$S_i(r_w, t) = S_W(t), \; i = 1,2 \qquad \ldots.(3)$$

$$\sum_{i=1}^{2} 2\pi\, r_w\, T_i \frac{\partial S}{\partial r}(r_w, t) = Q_p(t) - \pi r_c^2 \frac{\partial S_W}{\partial t} \qquad \ldots.(4)$$

Besides the initial condition to be satisfied is

$$S(r, 0) = 0 \qquad \ldots.(5)$$

Had the aquifers been tapped separately for the initial condition $S_i(r,0) = 0$, and boundary condition $S_i(\infty, t) = 0$, solution to differential equation (1), when unit impulse quantity of water is withdrawn from the aquifer 'i', is (Carlslaw and Jaeger, 1959)

$$S_i(r,t) = \frac{1}{4\pi T_i} \frac{e^{-\frac{r^2}{4\beta_i t}}}{t} \; ; \; \beta_i = \frac{T_i}{Q_i} \qquad \ldots(6)$$

Defining the unit impulse kernel

$$k_i(t) = \frac{e^{-\frac{r^2}{4\beta_i t}}}{4\pi T_i t} \qquad \ldots(7)$$

drawdown for variable withdrawal from the aquifer 'i' can be written in the form

$$S_i(r,t) = \int_0^t Q_i\,(c)\; k_i(t-c)\; dc \qquad \ldots(8)$$

where $Q_i(c)$ is variable discharge rate from the i^{th} aquifer at time 'c'. Dividing the time span by discrete time steps and assuming that the aquifer discharge Q(c) is constant within each time step but varies from step to step, the drawdown at the end of time step 'n' can be written as (Morel - Seytoux, 1975)

$$S_i(r,n) = \sum_{\gamma=1}^{n} \delta_{r,i}\,(n-\gamma+1)\; Q_i(\gamma) \qquad \ldots(9)$$

When the discrete kernel coefficient $\delta_{r,i}(m)$ is defined as

$$\delta_{r,i}(m) = \int_0^1 k_i\,(m-c)\; dc$$

$$= \frac{1}{4\pi T_i}\left[E_1\left(\frac{r^2}{4\beta_i m}\right) - E_1\left(\frac{r^2}{4\beta_i (m-1)}\right)\right] \qquad \ldots(10)$$

in which $E_1(X)$ is an exponential integral (Abramowitz and Stegun 1970)

$$E_1(X) = \int_X^\infty \frac{e^{-u}}{u}\; du$$

The discrete kernel coefficient $\delta_{r,i}(m)$ is the drawdown at the end of m^{th} time step at distance 'r' from the pumping well in response to withdrawal of unit quantity of water from the storage of i^{th} aquifer during the first time period. A unit step may be 0.1 day, 1 day or 1 week etc.

When the two aquifers are tapped by a single well and the well is pumped, there is contribution from each aquifer to the pumping through the respective well screens besides from the well storage.

Let $Q_1(n)$ and $Q_2(n)$ be the contribution of aquifer 1 and 2 respectively and $Q_w(n)$ be the contribution from well storage at time step n. In discrete system the boundary condition (equation 4) to be satisfied at the well face at each timestep can be written as

$$Q_1(n) + Q_2(n) + Q_w(n) = Q_P(n) \qquad \ldots.(11)$$

When the abstraction rate $Q_P(n)$ decreases with increase in drawdown and relation between abstraction rate and drawdown is linear the following relationship holds good:

$$Q_P(n) = \left[1 - \frac{S_w(n)}{S_F} \right] Q_I \text{ during pumping} \qquad \ldots.(12)$$

and $Q_P(n) = 0$ when there is no pumping

in which

$S_w(n)$ is the drawdown at the well face at the end of time step 'n',

S_F and Q_I are the pump characteristics as explained in figure 3.2

Drawdown $S_w(n)$ at the well face at the end of time step n is given by

$$S_w(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_w(\gamma) \qquad \ldots.(13)$$

When $Q_w(\gamma)$ represents rate of withdrawal from well storage or replenishment at time step γ. $Q_w(\gamma)$ values are unknown a priori. A negative value of $Q_w(\gamma)$ means there is replenishment of well storage which occurs during recovery period. Making use of equations 11, 12, 13 the following expression is obtained

$$Q_1(n) + Q_2(n) + Q_w(n) = [1 - \frac{1}{S_F \pi \gamma_c^2} \sum_{\gamma=1}^{n} Q_w] Q_I \qquad \ldots.(14)$$

or

$$Q_1(n) + Q_2(n) + Q_w(n) \; (1 + \frac{Q_I}{S_F \pi r_c^2})$$

$$= [\, 1 - \frac{1}{S_F \pi r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma)] \; Q_I \qquad \ldots.(15)$$

The drawdown at the well face in aquifer 1 at the end of time step n due to abstraction from the first aquifer storage is given by

$$S_{1W}(n) = \sum_{\gamma=1}^{n} Q_1(\gamma) \; \delta_{r_w 1}(n - \gamma + 1) \qquad \ldots.(16)$$

where

$$\delta_{r_w 1}(m) = \frac{1}{4 \pi T_1} [\, E_1(\frac{r_w^2}{4 \beta_1 m}) - E_1(\frac{r_w^2}{4 \beta_1 (n-1)})] \qquad \ldots.(17)$$

Similarly the drawdown at the well face at the end of time step n in aquifer 2 due to abstraction from 2nd aquifer storage is given by

$$S_{2W}(n) = \sum_{\gamma=1}^{n} Q_2(\gamma) \quad \partial_{r_{w2}}(n - \gamma + 1) \qquad \ldots(18)$$

where

$$\partial_{r_w 2}(m) = \frac{1}{4 \pi T_2} \left[E_1 \left(\frac{r_w^2}{4 \beta_2 m} \right) - E_1 \left(\frac{r_w^2}{4\beta_2 (m-1)} \right) \right] \qquad \ldots(19)$$

Since

$$S_{1W}(n) = S_{2W}(n), \text{ therefore}$$

$$\sum_{\gamma=1}^{n} Q_1(\gamma) \quad \partial_{r_w 1}(n - \gamma + 1) = \sum_{\gamma=1}^{n} Q_2(\gamma) \quad \partial_{r_w 2}(n - \gamma + 1) \qquad \ldots(20)$$

Rearranging

$$Q_1(n) \quad \partial_{rw1}(1) - Q_2(n) \quad \partial_{rw_2}(1) = \sum_{\gamma=1}^{n-1} Q_2(\gamma) \ \partial_{rw_2} - \sum_{\gamma=1}^{n-1} Q_1(\gamma) \quad \partial_{rw_1} (n - \gamma + 1) \qquad \ldots(21)$$

Therefore,

$$\sum_{\gamma=1}^{n} Q_1(\gamma) \quad \partial_{rw1}(n - \gamma + 1) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_W(\gamma) \qquad \ldots(22)$$

or

$$Q_1(n) \quad \partial_{rw_1}(1) - \frac{1}{\pi r_c^2} \quad Q_W(n)$$

$$= \frac{1}{\pi r_c} \sum_{\gamma=1}^{n-1} Q_W(\gamma) - \sum_{\gamma=1}^{n-1} Q_1(\gamma) \quad \partial_{rw_1}(n-\gamma+1) \quad ..(23)$$

Equations 15, 21 and 23 can be written in the following matrix form

$$\begin{bmatrix} 1 & , 1 & ,(1+\dfrac{Q_I}{S_F \ \pi \ r_c^2}) \\ \partial_{rw1}(1), & -\partial_{rw2}(1), & 0 \\ \partial_{rw1}(1), & 0 \ , & -\dfrac{1}{\pi \ r_c^2} \end{bmatrix} \cdot \begin{bmatrix} Q_1(n) \\ Q_2(n) \\ Q_w(n) \end{bmatrix}$$

$$= \begin{bmatrix} [1 - \dfrac{1}{S_F \ \pi \ r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma)] \ Q_I \\ \sum_{\gamma=1}^{n-1} Q_2(\gamma) \ \partial_{rw2}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_1(\gamma) \ \partial_{rw1}(n-\gamma+1) \\ \dfrac{1}{\pi \ r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma) - \sum_{\gamma=1}^{n-1} Q_1(\gamma) \ \partial_{rw}(n-\gamma+1) \end{bmatrix} \qquad ..(24)$$

Therefore

$$\begin{bmatrix} Q_1(n) \\ Q_2(n) \\ Q_w(n) \end{bmatrix} = \begin{bmatrix} 1 & ,1 & , 1 + \dfrac{Q_I}{S_F \ \pi \ r_c^2} \\ \partial_{rw1}(1), & -\partial_{rw2}(1), & 0 \\ \partial_{rw1}(1), & 0 \ , & -\dfrac{1}{\pi \ r_c^2} \end{bmatrix}^{-1}$$

$$\cdot \begin{bmatrix} [1 - \dfrac{1}{S_F \pi r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma)] \ Q_I \\ \sum_{\gamma=1}^{n-1} (Q_2(\gamma) \ \partial_{rw2}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_1(\gamma) \ \partial_{rw1}(n-\gamma+1) \\ \dfrac{1}{\pi \ r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma) - \sum_{\gamma=1}^{n-1} Q_1(\gamma) \ \partial_{rw1}(n-\gamma+1) \end{bmatrix} \qquad ..(25)$$

Thus $Q_I(n)$, $Q_2(n)$ and $Q_w(n)$ can be solved from equation (25) when there is pumping.

In particular for time step 1

$$\begin{bmatrix} Q_1(1) \\ Q_2(1) \\ Q_w(1) \end{bmatrix} = \begin{bmatrix} 1 & ,1 & ,1 + \frac{Q_I}{S_F \pi r_c^2} \\ \partial_{rw1}(1), & -\partial_{rw2}(1), & 0 \\ \partial_{rw1}(1), & 0 & , -\frac{1}{\pi r_c^2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} Q_I \\ 0 \\ 0 \end{bmatrix} \qquad ..(26)$$

When there is no pumping

$$Q_1(n) + Q_2(n) + Q_w(n) = 0 \qquad ...(27)$$

During no pumping period equations 27, 21 and 23 can be written in the following matrix form

$$\begin{bmatrix} 1 & , 1 & , 1 \\ \partial_{rw1}(1), & -\partial_{rw2}(1), & 0 \\ \partial_{rw1}(1), & 0 & , -\frac{1}{\pi r_c^2} \end{bmatrix} \cdot \begin{bmatrix} Q_1(n) \\ Q_2(n) \\ Q_w(n) \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{\gamma=1}^{n-1} Q_2(\gamma)\, \partial_{rw2}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_1(\gamma)\, \partial_{rw1}(n-\gamma+1) \\ \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma) - \sum_{\gamma=1}^{n-1} Q_1(\gamma)\, \partial_{rw}(n-\gamma+1) \end{bmatrix} \qquad ..(28$$

Therefore

$$\begin{bmatrix} Q_1(n) \\ Q_2(n) \\ Q_w(n) \end{bmatrix} = \begin{bmatrix} 1 & , 1 & , 1 \\ \partial_{rw1}(1), & -\partial_{rw2}(1), & 0 \\ \partial_{rw1}(1), & 0 & , -\frac{1}{\pi r_c^2} \end{bmatrix}^{-1}$$

$$0$$

$$\cdot \begin{vmatrix} \sum_{\gamma=1}^{n-1} Q_2(\gamma) \quad \partial_{rw2}(n-\gamma+1) & -\sum_{\gamma=1}^{n-1} Q_1(\gamma) \quad \partial_{rw1}(n-\gamma+1) \\ \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma) & -\sum_{\gamma=1}^{n-1} Q_1(\gamma) \quad \partial_{rw1}(n-\gamma+1) \end{vmatrix} \qquad ..29$$

Thus $Q_1(n)$, $Q_2(n)$ and $Q_w(n)$ can be solved in succession starting from timestep 1. When there is pumping equation 25 is to be used and when there is no pumping equation 29 is to be used.

When the rate of abstraction is independent of draw-down the matrix takes the following form

$$\begin{vmatrix} Q_1(n) \\ Q_2(n) \\ Q_w(n) \end{vmatrix} = \begin{vmatrix} 1 & , 1 & , 1 \\ \partial_{rw1}(1), & -\partial_{rw2}(1), & 0 \\ \partial_{rw1}(1), & 0 & , -\frac{1}{\pi r_c^2} \end{vmatrix}$$

$$Q_p(n)$$

$$\cdot \begin{vmatrix} \sum_{\gamma=1}^{n-1} Q_2(\gamma) \quad \partial_{rw2}(n-\gamma+1) & -\sum_{\gamma=1}^{n-1}(\gamma) \quad \partial_{rw1}(n-\gamma+1) \\ \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma) & -\sum_{\gamma=1}^{n-1} Q_1(\gamma) \quad \partial_{rw1}(n-\gamma+1) \end{vmatrix}$$

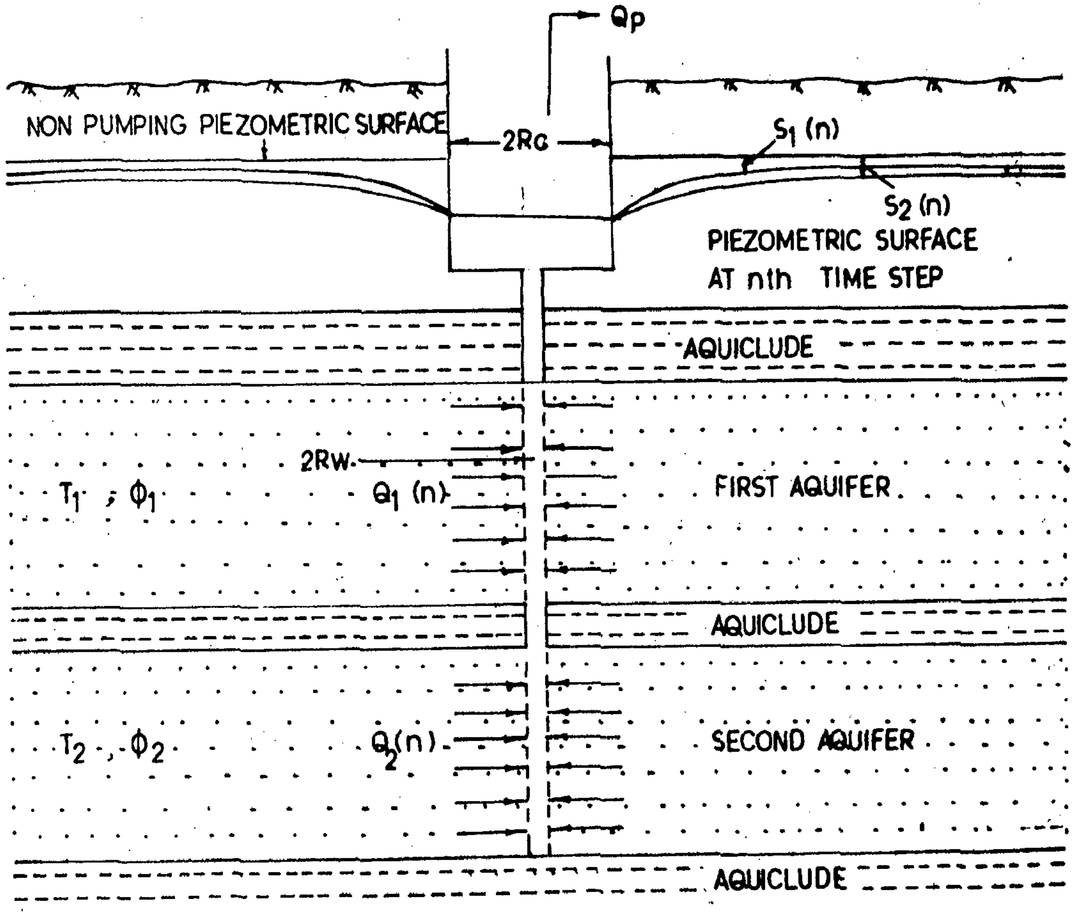

FIG.3.1 SCHEMATIC SECTION OF A WELL TAPPING TWO CONFINED AQUIFERS SEPARATED BY AN AQUICLUDE

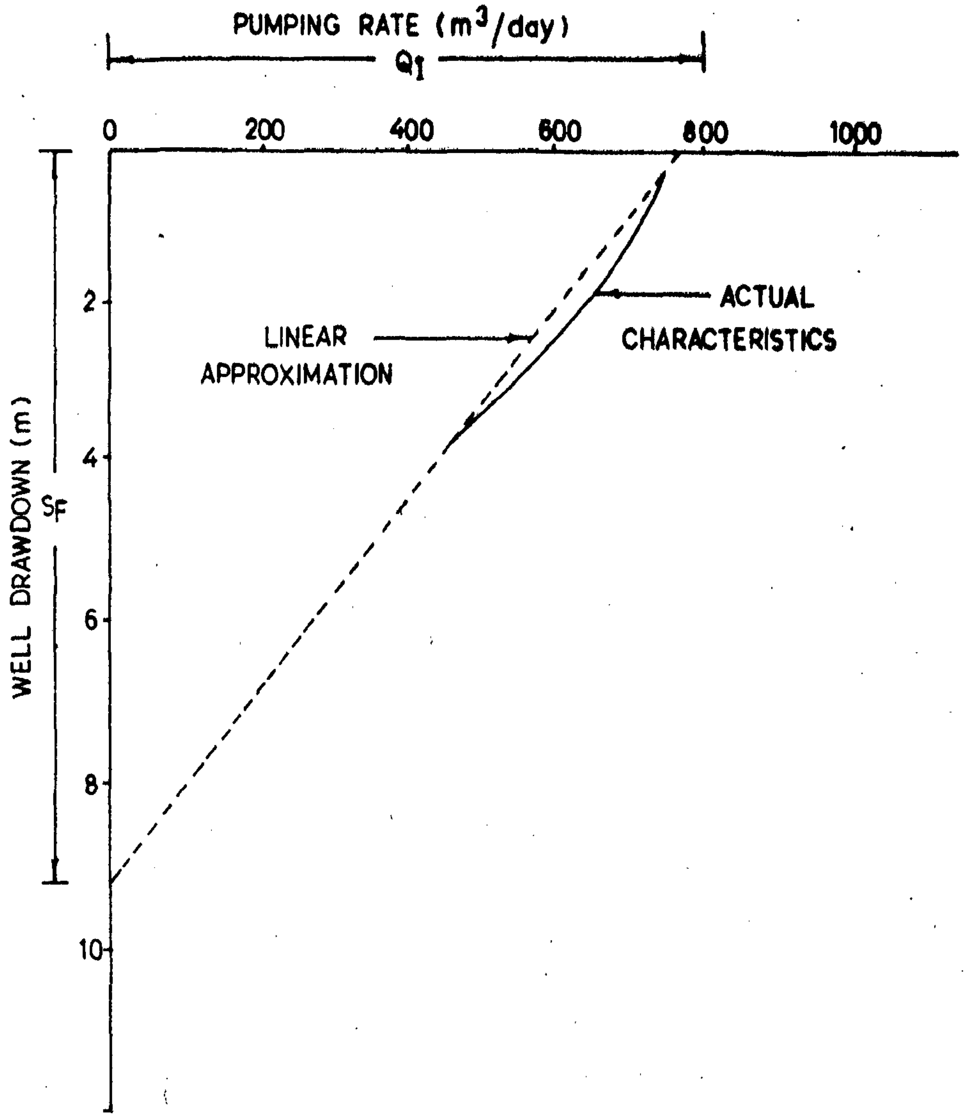

FIG 3·2 VARIATION OF PUMPING RATE OF A CENTRIFUGAL PUMP WITH DRAWDOWN

CHAPTER 4

RESULTS AND DISCUSSIONS

While presenting the results it has been aimed to find out saving in energy when a multiaquifer is tapped. Two types of wells have been considered - wells with storage and wells for which storage can be neglected. Making use of the rational approximation (Stegun and Abramowitz) the discrete kernel coefficients have been generated for each aquifer for known values of transmissivities, storage coefficients and radi of well screens. It has been assumed that the pumping is intermittant, that is every period of pumping is followed by same period of rest. Quantities of flow contributed by each aquifer and by the well storage are solved in succession starting from time step 1 for assumed pumping pattern for those cases where the withdrawal rate is independent of drawdown. In case of centrifugal pump there is no control over the pumping rate as the pumping rate depends upon the drawdown at well face which varies with time. A linear relationship has been assumed to hold good between the abstraction rate and the drawdown at the well face. For such problem the pumping rates during different time steps have been found for assumed initial pumping rate Q_I and the pump characteristic S_F.

In fugures 4.1(a) through 4.1(c) typical variation of $Q_1(n)/Q_I$ with $\frac{r_w^2}{4\beta_1 n}$, $\frac{Q_2(n)}{Q_I}$ with $\frac{r_w^2}{4\beta_2 n}$ and $\frac{Q_w(n)}{Q_I}$ with $\frac{r_w^2}{4\beta_1 n}$ have been presented. These results correspond to the case when pumping rate is dependent upon the drawdown and well storage is significant. Thus at any time $Q_p(n)$ is given by $Q_p(n)=Q_1(n) + Q_2(n) + Q_w(n)$. These results have been presented by in non-dimensional form as the pumping rate is linear function of drawdown. The ratios $Q_1(n)/Q_I$, $Q_2(n)/Q_I$ and $Q_w(n)/Q_I$ are independent of Q_I. When well storage is negligible the variation of $Q_1(n)/Q_I$ with $\frac{r_w^2}{4\beta_1 n}$ is presented in figure 4.2 for the same value of S_F and Q_I. For these assumed values of S_F and the pumping schedule the pumping rate fluctuates around 472 m^3/unit time against the initial pumping rate of 500.0 m^3/unit time. Since $Q_1(n) + Q_2(n) = Q_p(n)$, the variation of $Q_2(n)$ can be known by substracting $Q_1(n)$ from $Q_p(n)$.

The variation of $S_w(n)/(\frac{Q_I}{4\pi T_1})$ $\frac{r_w^2 \phi_i}{4 T_1 n}$ for $T_1/T_2 = 2.0$, $\frac{\phi_1}{\phi_2} = 10.0$, $R_c/R_w = 1.0$, $Q_I = 500$ m^2/unit time and $S_F = 15.0$ m has been shown in figure 4.3(a). The saw-tooth shape of the graph is due to the periodic pumping. However, it can be seen from the figure that minimum and maximum drawdown increase at a slow rate as the intermittant pattern of pumping continues. In figure 4.3(b) variation of $S_w(n)/(Q_I/4\pi T_1)$ with $\frac{r_w^2}{4\beta_1 n}$ has been shown for a well having storage. The drawdowns at the well point with and without storage are compared in Table No. 1. As can be seen from the Table, the drawdown at the end of first hour after pumping is about 18 percent less than that when there is no well storage. At the end of 12 hours of pumping there is no effect of well storage on drawdown. During recovery, well without storage recovers faster. One hour after the stopage of pumping the recovery at the well with storage is about 50 percent less than that of the well without storage. However, after 12 hours of stoppage of pumping there is not much difference in drawdown at the well with and without storage.

If only the top aquifer is tapped and pumping rate at any time is $Q_P(t)$ the work done E_S in lifting the water from the well to the ground surface in time span t_p can be written as

$$E_S = C \int_0^{t_p} Q_P(t) \, (S_w(t) + G_L) \, dt$$

where the constant C takes into account the pump efficiency In discrete system the above equation can be written as

$$E_S = C \sum_{i=1}^{n} Q_P(i) \, (S_w(i) + G_L)$$

$$= C \sum_{i=1}^{n} Q_P(i) \, \left(\sum_{\gamma=1}^{i} [\delta_1(i - \gamma + 1) . Q_P(\gamma) + G_L] \right) \quad ..(A-1)$$

This will be the total energy consumption upto the end of time step 'n'. When both the aquifers are tapped the drawdown at the well face for the same pumping pattern will however be less than that of the drawdown that would have resulted had the top aquifer been pumped alone. The energy consumption resulting from pumping of a multiaquifer well can be written as

$$E_m = C \sum_{i=1}^{n} Q_p(i) \left(\sum_{\gamma=1}^{i} [\partial_1(i - \gamma + 1).Q_1(\gamma)+G_L] \right) \quad ..(4.2)$$

The ratio $\frac{E_m}{E_S}$ has been calculated for different values of

$\frac{T_1}{T_2}$, $\frac{\phi_1}{\phi_2}$ and $\frac{R_c}{R_w}$ and are presented in figures 4.4(a) through 4.4(ℓ) for different values of ground level. As seen from the figures, as the ground level increases the advantage of tapping the multiple aquifer for energy consideration is reduced and is negligible at G_L = 20 m. However for smaller values of G_L that is for G_L = 5 m the saving in energy is 12% , for $T_1/T_2 = 1$. When $T_1/T_2 = \frac{5}{4}$, $\frac{5}{3}$, $\frac{5}{2}$ and 5 the corresponding saving in energy is 10.83%, 9.23%, 7.16%, 4.33% , respectively. The ratio $\frac{E_m}{E_s}$ is a nonlinear function of $Q_p(n)$. Therefore saving in energy is a nonlinear function of rate of withdrawal. Results presented in figures 4.4(c) through 4.4(ℓ) correspond to the case when the pumping rate is independent of drawdown. The figures 4.4(a) and 4.4(b) represents the energy saving when pumping rate is a linear function of drawdown.

The effect of well storage on energy saving has been studied. The influence of well storage on the saving in the energy expended in lifting the water are compared for different well storage i.e. for $\frac{R_c}{R_w} = 1, 10, 20$. As seen from the tables No. 2 well storage has not much influence on saving in energy.

Though the ratio of saving in energy is small, magnitude of energy saved will be considerable in view of large consumption, for example in a period of 1 year if only one aquifer will be pumped total consumption of energy is of the order 603670 Tm for $T_1/T_2 = 1$, $\phi_1/\phi_2 = 1$, $Q_p(2n+1) = 500\ m^2/day$ for n = 1, 2, 3 ..... 182. If two aquifers are tapped the corresponding energy expended will be 530582 Tm for $T_1/T_2 = 1$, $\phi_1/\phi_2 = 1$. Thus saving in energy would be 73088 Tm.

TABLE 1

DRAWDOWN AT THE WELL FACE
(WELL WITH AND WITHOUT STORAGE)

Sl. No.	$S_w(n)$ $R_w = .1$, $R_c = .1$	$S_w(n)$ $R_w = .1$, $R_c = .2$
1	0.7191665E+00	0.5912478E+00
2	0.7529367E+00	0.7192386E+00
3	0.7724162E+00	0.7590399E+00
4	0.7861955E+00	0.7784013E+00
5	0.7968609E+00	0.7912281E+00
6	0.80556I6E+00	0.8010654E+00
7	I.8129062E+00	0.8091356E+00
8	0.8192621E+00	0.8160036E+00
9	0.8284629E+00	0.8219892E+00
10	0.8298684E+00	0.8272958E+00
11	0.8343930E+00	0.8320627E+00
12	0.8385208E+00	0.8363900E+00
13	0.1293508E+00	0.2593257E+00
14	0.9787393E+0	0.1321900E+00
15	0.8094712E+0	0.3987136E+0
16	0.6972435E+01	0.7666366E+01
17	0.[illegible]155114E+01	0.6619532E+01
18	0.5525656E+01	0.5872662E+01
19	0.5022332E+01	0.5296908E+01
20	0.4809745E+01	0.4833675E+01
21	0.4261759E+01	0.4450484E+01
22	0.3965829E+01	0.4127047E+01
23	0.3710039E+01	0.3849731E+01

Sl. No.	$S_w(n)$ $R_w = .1$ $R_c = .1$	$S_w(n)$ $R_w = .1$ $R_c = .2$
24	0.3486474E+01	0.3608929E+01
25	0.7504815E+00	0.6238847E+00
26	0.7825071E+00	0.7497882E+00
27	0.8004515E+00	0.7879115E+00
28	0.8128573E+00	0.8058075E+00
29	0.8222833E+00	0.8173229E+00
30	0.8298579E+00	0.8259746E+00
31	0.8361762E+00	0.8329657E+00
32	0.8415900E+00	0.8388468E+00
33	0.8463235E+00	0.8439257E+00
34	0.8505279E+00	0.8483962E+00
35	0.8543099E+00	0.8523894E+00
36	0.8577472E+00	0.8559985E+00
37	0.1488694E+00	0.2790431E+00
38	0.1168070E+00	0.1514331E+00
39	0.9931368E+01	0.1125572E+00
40	0.8755221E+01	0.9480009E+01
41	0.7886852E+01	0.8380699E+01
42	0.7209063E+01	0.7584053E+01
43	0.6659960E+01	0.6961139E+01
44	0.6202962E+01	0.6453202E+01
45	0.5814786E+01	0.6027609E+01
46	0.5479721E+01	0.5663909E+01

TABLE 2.1

VARIATION OF $\frac{E_m}{E_s}$ WITH INITIAL DEPTH TO PIEZOMETRIC SURFACE

$T1 = 500.0000\ m^2/day$

$T2 = 100.0000\ m^2/day$

$\phi_1 = 0.0100$

$\phi_2 = 0.0020$

$R_W = 0.1000$

$QP(1) = 500.000\ m^3/unit\ time$

$R = 0.10\ m$

SUM1	SUM2	RATIO	GLEVEL
.8546856E+05	0.7955541E+05	0.9308149E+00	0.2000000E+01
.1104686E+06	0.1045554E+06	0.9464721E+00	0.3000000E+01
.1354686E+06	0.1295554E+06	0.9563504E+00	0.4000000E+01
.1604686E+06	0.1545554E+06	0.9631507E+00	0.5000000E+01
.1854686E+06	0.1795554E+06	0.9681177E+00	0.6000000E+01
.2104686E+06	0.2045554E+06	0.9719048E+00	0.7000000E+01
.2354686E+06	0.2295554E+06	0.9748877E+00	0.8000000E+01
.2604686E+06	0.2545554E+06	0.9772980E+00	0.9000000E+01
.2854686E+06	0.2795554E+06	0.9792861E+00	0.1000000E+02
.3104686E+06	0.3045554E+06	0.9809541E+00	0.1100000E+02
.3354686E+06	0.3295554E+06	0.9823734E+00	0.1200000E+02
.3604686E+06	0.3545554E+06	0.9835959E+00	0.1300000E+02
.3854686E+06	0.3795554E+06	0.9846598E+00	0.1400000E+02
.4104686E+06	0.4045554E+06	0.9855941E+00	0.1500000E+02
.4354686E+06	0.4295554E+06	0.9864212E+00	0.1600000E+02
.4604686E+06	0.4545554E+06	0.9871584E+00	0.1700000E+02
.4854686E+06	0.4795554E+06	0.9879197E+00	0.1800000E+02
.5104686E+06	0.5045554E+06	0.9884162E+00	0.1900000E+02
.5354686E+06	0.5295554E+06	0.9889571E+00	0.2000000E+02

TABLE 2.2

$T1 = 500.000\ m^2/day$

$T2 = 100.000\ m^2/day$

$\phi_1 = 0.01000$

$\phi_2 = 0.00200$

$R_w = 0.100\ m$

$QP(1) = 500.000\ m^3/day$

$R_e = 1.0\ m$

SUM1	SUM2	RATIO	GLEVEL
.8546856E+05	0.7938659E+05	0.9288396E+00	0.2000000E+01
.1104686E+06	0.1043866E+06	0.9449438E+00	0.3000000E+01
.1354686E+06	0.1293866E+06	0.9551041E+00	0.4000000E+01
.1604686E+06	0.1543866E+06	0.9620986E+00	0.5000000E+01
.1854686E+06	0.1793866E+06	0.9672075E+00	0.6000000E+01
.2104686E+06	0.2043866E+06	0.9711027E+00	0.7000000E+01
.2354686E+06	0.2293866E+06	0.9741708E+00	0.8000000E+01
.2604686E+06	0.2543866E+06	0.9766499E+00	0.9000000E+01
.2854686E+06	0.2793866E+06	0.9786948E+00	0.1000000E+02
.3104686E+06	0.3043866E+06	0.9804103E+00	0.1100000E+02
.3354686E+06	0.3293866E+06	0.9818702E+00	0.1200000E+02
.3604686E+06	0.3543966E+06	0.9831276E+00	0.1300000E+02
.3854686E+06	0.3793866E+06	0.9842219E+00	0.1400000E+02
.4104686E+06	0.4043866E+06	0.9851828E+00	0.1500000E+02
.4354686E+06	0.4293866E+06	0.9860335E+00	0.1600000E+02
.4604686E+06	0.4543866E+06	0.9867918E+00	0.1700000E+02
.4854686E+06	0.4793866E+06	0.9874719E+00	0.1800000E+02
.5104686E+06	0.5043866E+06	0.9880855E+00	0.1900000E+02
.5354686E+06	0.5293866E+06	0.9886418E+00	0.2000000E+02

TABLE 2.3

T1 = 500.0 m^2/day

T2 = 100.0000 m^2/day

ϕ_1 = 0.010

ϕ_2 = 0.002

R_w 0.10

QP(1) = 500.0 m^3/day

Rc = 2.000 m

SUM1	SUM2	RATIO	GLEVEL
.8546856E+05	0.7890017E+05	0.9231484E+00	0.2000000E+01
.1104686E+06	0.1039002E+06	0.9405406E+00	0.3000000E+01
.1354686E+06	0.1289002E+06	0.9515135E+00	0.4000000E+01
.1604686E+06	0.1539002E+06	0.9590674E+00	0.5000000E+01
.1854686E+06	0.1789002E+06	0.9645848E+00	0.6000000E+01
.2104686E+06	0.2039002E+06	0.9687916E+00	0.7000000E+01
.2354686E+06	0.2289002E+06	0.9721050E+00	0.8000000E+01
.2604686E+06	0.2539002E+06	0.9747824E+00	0.9000000E+01
.2854686E+06	0.2789002E+06	0.9769908E+00	0.1000000E+02
.3104686E+06	0.3039002E+06	0.9788436E+00	0.1100000E+02
.3354686E+06	0.3289002E+06	0.9804202E+00	0.1200000E+02
.3604686E+06	0.3539002E+06	0.9817782E+00	0.1300000E+02
.3854686E+06	0.3789002E+06	0.9829600E+00	0.1400000E+02
.4104686E+06	0.4039002E+06	0.9839978E+00	0.1500000E+02
.4354686E+06	0.4289002E+06	0.9849165E+00	0.1600000E+02
.4604686E+06	0.4539002E+06	0.9857354E+00	0.1700000E+02
.4854686E+06	0.4789002E+06	0.9864700E+00	0.1800000E+02
.5104686E+06	0.5039002E+06	0.9871326E+00	0.1900000E+02
.5354686E+06	0.5289002E+06	0.9877334E+00	0.2000000E+02

TABLE 2.4

T1 = 500.0 m^2/day

T2 = 250.0 m^2/day

ϕ_1 = 0.010

ϕ_2 = 0.002

R_w = 0.10 m

QP(1)= 500.0 m^3/day

R_e = 2.0 m

SUM1	SUM2	RATIO	GLEVEL
.8546856E+05	0.7359696E+05	0.8610998E+00	0.2000000E+01
.1104686E+06	0.9859697E+05	0.8925342E+00	0.3000000E+01
.1354686E+06	0.1235970E+06	0.9123664E+00	0.4000000E+01
.1604686E+06	0.1485970E+06	0.9260192E+00	0.5000000E+01
.1854686E+06	0.1735970E+06	0.9359913E+00	0.6000000E+01
.2104686E+06	0.1985970E+06	0.9435944E+00	0.7000000E+01
.2354686E+06	0.2235970E+06	0.9495831E+00	0.8000000E+01
.2604686E+06	0.2485970E+06	0.9544222E+00	0.9000000E+01
.2854686E+06	0.2735970E+06	0.9584136E+00	0.1000000E+02
.3104686E+06	0.2985970E+06	0.9617623E+00	0.1100000E+02
.3354686E+06	0.3235970E+06	0.9646119E+00	0.1200000E+02
.3604686E+06	0.3485970E+06	0.9670662E+00	0.1300000E+02
.3854686E+06	0.3735970E+06	0.9692022E+00	0.1400000E+02
.4104686E+06	0.3985970E+06	0.9710779E+00	0.1500000E+02
.4354686E+06	0.4235970E+06	0.9727383E+00	0.1600000E+02
.4604686E+06	0.4485970E+06	0.9742184E+00	0.1700000E+02
.4854686E+06	0.4735970E+06	0.9755461E+00	0.1800000E+02
.5104686E+06	0.4985970E+06	0.9767437E+00	0.1900000E+02
.5354686E+06	0.5235970E+06	0.9778295E+00	0.2000000E+02

TABLE 2.5

$T1 = 500.0000\ m^2/day$

$T2 = 250.0000\ m^2/day$

$\phi_1 = 0.0100$

$\phi_2 = 0.0020$

$R_w = 0.10\ m$

$QP(1) = 500.0000\ m^3/day$

$R_e = 1.0000\ m$

SUM1	SUM2	RATIO	GLEVEL
.8546856E+05	0.7392371E+05	0.8649227E+00	0.2000000E+01
.1104686E+06	0.9892370E+05	0.8954919E+00	0.3000000E+01
.1354686E+06	0.1239237E+06	0.9147783E+00	0.4000000E+01
.1604686E+06	0.1489237E+06	0.9280553E+00	0.5000000E+01
.1854686E+06	0.1739237E+06	0.9377530E+00	0.6000000E+01
.2104686E+06	0.1989237E+06	0.9451469E+00	0.7000000E+01
.2354686E+06	0.2239237E+06	0.9509707E+00	0.8000000E+01
.2604686E+06	0.2489237E+06	0.9556766E+00	0.9000000E+01
.2854686E+06	0.2739237E+06	0.9595582E+00	0.1000000E+02
.3104686E+06	0.2989237E+06	0.9628147E+00	0.1100000E+02
.3354686E+06	0.3239237E+06	0.9655858E+00	0.1200000E+02
.3604686E+06	0.3489237E+06	0.9679726E+00	0.1300000E+02
.3854686E+06	0.3739237E+06	0.9700498E+00	0.1400000E+02
.4104686E+06	0.3989237E+06	0.9718739E+00	0.1500000E+02
.4354686E+06	0.4239237E+06	0.9734887E+00	0.1600000E+02
.4604686E+06	0.4489237E+06	0.9749280E+00	0.1700000E+02
.4854686E+06	0.4739237E+06	0.9762191E+00	0.1800000E+02
.5104686E+06	0.4989237E+06	0.9773838E+00	0.1900000E+02
.5354686E+06	0.5239237E+06	0.9784397E+00	0.2000000E+02

TABLE 2.6

T1 = 500.0 m^2/day

T2 = 250.0 m^2/day

ϕ_1 = 0.01

ϕ_2 = 0.002

R_w = 0.10 m

QP(1) = 500.0 m^3/day

RC = 0.10 m

SUM1	SUM2	RATIO	GLEVEL
.8546856E+05	0.7403607E+05	0.8662375E+00	0.2000000E+01
.1104686E+06	0.9903607E+05	0.8965091E+00	0.3000000E+01
.1354686E+06	0.1240361E+06	0.9156078E+00	0.4000000E+01
.1604686E+06	0.1490361E+06	0.9287556E+00	0.5000000E+01
.1854686E+06	0.1740361E+06	0.9383589E+00	0.6000000E+01
.2104686E+06	0.1990361E+06	0.9456808E+00	0.7000000E+01
.2354686E+06	0.2240361E+06	0.9514479E+00	0.8000000E+01
.2604686E+06	0.2490361E+06	0.9561080E+00	0.9000000E+01
.2854686E+06	0.2740361E+06	0.9599518E+00	0.1000000E+02
.3104686E+06	0.2990361E+06	0.9631767E+00	0.1100000E+02
.3354686E+06	0.3240361E+06	0.9659208E+00	0.1200000E+02
.3604686E+06	0.3490361E+06	0.9682844E+00	0.1300000E+02
.3854686E+06	0.3740361E+06	0.9703413E+00	0.1400000E+02
.4104686E+06	0.3990361E+06	0.9721477E+00	0.1500000E+02
.4354686E+06	0.4240361E+06	0.9737467E+00	0.1600000E+02
.4604686E+06	0.4490361E+06	0.9751721E+00	0.1700000E+02
.4854686E+06	0.4740361E+06	0.9764506E+00	0.1800000E+02
.5104686E+06	0.4990361E+06	0.9776039E+00	0.1900000E+02
.5354686E+06	0.5240361E+06	0.9786495E+00	0.2000000E+02

TABLE 2.7

$T1 = 300.00000 m^2/day$

$T2 = 150.00000 m^2/day$

$\phi_1 = 0.01000$

$\phi_2 = 0.00200$

$R_w = 0.10000m$

$QP(1) = 250.00000 m^3/day$

$R_c = 1.00000m$

SUM1	SUM2	RATIO	GLEVEL
.3935513E+05	0.3465961E+05	0.8806886E+00	0.2000000E+01
.5185513E+05	0.4715961E+05	0.9094493E+00	0.3000000E+01
.6435513E+05	0.5965961E+05	0.9270374E+00	0.4000000E+01
.7685513E+05	0.7215961E+05	0.9389043E+00	0.5000000E+01
.8935513E+05	0.8465961E+05	0.9474511E+00	0.6000000E+01
.1018551E+06	0.9715961E+05	0.9539001E+00	0.7000000E+01
.1143551E+06	0.1096596E+06	0.9589392E+00	0.8000000E+01
.1268551E+06	0.1221596E+06	0.9629852E+00	0.9000000E+01
.1393551E+06	0.1346596E+06	0.9663054E+00	0.1000000E+02
.1518551E+06	0.1471596E+06	0.9690790E+00	0.1100000E+02
.1643551E+06	0.1596596E+06	0.9714307E+00	0.1200000E+02
.1768551E+06	0.1721596E+06	0.9734500E+00	0.1300000E+02
.1893551E+06	0.1846596E+06	0.9752026E+00	0.1400000E+02
.2018551E+06	0.1971596E+06	0.9767382E+00	0.1500000E+02
.2143551E+06	0.2096596E+06	0.9780947E+00	0.1600000E+02
.2268551E+06	0.2221596E+06	0.9793017E+00	0.1700000E+02
.2393551E+06	0.2346596E+06	0.9803827E+00	0.1800000E+02
.2518551E+06	0.2471596E+06	0.9813563E+00	0.1900000E+02
.2643551E+06	0.2596596E+06	0.9822379E+00	0.2000000E+02

TABLE 2.8

T1 = 300.0 m^2/day

T2 = 150.0 m^2/day

ϕ_1 = 0.010

ϕ_2 = 0.002

R_w = 0.10 m

QP(1) = 250.0 m^3/day

R_e = 0.10 m

SUM1	SUM2	RATIO	GLEVEL
.3935513E+05	0.3473230E+05	0.8825356E+00	0.2000000E+01
.5185513E+05	0.4723230E+05	0.9108511E+00	0.3000000E+01
.6435513E+05	0.5973230E+05	0.9281670E+00	0.4000000E+01
.7685513E+05	0.7223230E+05	0.9398501E+00	0.5000000E+01
.8935513E+05	0.8473231E+05	0.9482646E+00	0.6000000E+01
.1018551E+06	0.9723231E+05	0.9546138E+00	0.7000000E+01
.1143551E+06	0.1097323E+06	0.9595749E+00	0.8000000E+01
.1268551E+06	0.1222323E+06	0.9635583E+00	0.9000000E+01
.1393551E+06	0.1347323E+06	0.9668270E+00	0.1000000E+02
.1518551E+06	0.1472323E+06	0.9695577E+00	0.1100000E+02
.1643551E+06	0.1597323E+06	0.9718729E+00	0.1200000E+02
.1768551E+06	0.1722323E+06	0.9738609E+00	0.1300000E+02
.1893551E+06	0.1847323E+06	0.9755865E+00	0.1400000E+02
.2018551E+06	0.1972323E+06	0.9770983E+00	0.1500000E+02
.2143551E+06	0.2097323E+06	0.9784338E+00	0.1600000E+02
.2268551E+06	0.2222323E+06	0.9796221E+00	0.1700000E+02
.2393551E+06	0.2347323E+06	0.9806863E+00	0.1800000E+02
.2518551E+06	0.2472323E+06	0.9816449E+00	0.1900000E+02
.2643551E+06	0.2597323E+06	0.9825128E+00	0.2000000E+02

TABLE 2.9

T1 = 300.0 m^2/day

T2 = 150.0 m^2/day

ϕ_1 = 0.010

ϕ_2 = 0.002

R_W = 0.10 m

QP(1) = 250.0 m^3/day

R_c = 2.0 m

SUM1	SUM2	RATIO	GLEVEL
.3935513E+05	0.3445337E+05	0.8754479E+00	0.2000000E+01
.5185513E+05	0.4695337E+05	0.9054720E+00	0.3000000E+01
.6435513E+05	0.5945336E+05	0.9238326E+00	0.4000000E+01
.7685513E+05	0.7195337E+05	0.9362208E+00	0.5000000E+01
.8935513E+05	0.8445337E+05	0.9451429E+00	0.6000000E+01
.1018551E+06	0.9695337E+05	0.9518752E+00	0.7000000E+01
.1143551E+06	0.1094534E+06	0.9571356E+00	0.8000000E+01
.1268551E+06	0.1219534E+06	0.9613594E+00	0.9000000E+01
.1393551E+06	0.1344534E+06	0.9648254E+00	0.1000000E+02
.1518551E+06	0.1469534E+06	0.9677208E+00	0.1100000E+02
.1643551E+06	0.1594534E+06	0.9701758E+00	0.1200000E+02
.1768551E+06	0.1719534E+06	0.9722837E+00	0.1300000E+02
.1893551E+06	0.1844534E+06	0.9741134E+00	0.1400000E+02
.2018551E+06	0.1969534E+06	0.9757164E+00	0.1500000E+02
.2143551E+06	0.2094534E+06	0.9771325E+00	0.1600000E+02
.2268551E+06	0.2219534E+06	0.9783925E+00	0.1700000E+02
.2393551E+06	0.2344534E+06	0.9795210E+00	0.1800000E+02
.2518551E+06	0.2469534E+06	0.9805374E+00	0.1900000E+02
.2643551E+06	0.2594534E+06	0.9814577E+00	0.2000000E+02

TABLE 2.10

T1 = 300.0 m^2/day

T2 = 60.00 m^2/day

ϕ_1 = 0.010

ϕ_2 = 0.002

R_W = 0.100 m

QP((1)) = 250.0 m^3/day

R_e = 2.0 m

SUM1	SUM2	RATIO	GLEVEL
.3935513E+05	0.3654789E+05	0.9286690E+00	0.2000000E+01
.5185513E+05	0.4904789E+05	0.9458637E+00	0.3000000E+01
.6435513E+05	0.6154789E+05	0.9563789E+00	0.4000000E+01
.7685513E+05	0.7404789E+05	0.9634736E+00	0.5000000E+01
.8935513E+05	0.8654789E+05	0.9685833E+00	0.6000000E+01
.1018551E+06	0.9904789E+05	0.9724389E+00	0.7000000E+01
.1143551E+06	0.1115479E+06	0.9754516E+00	0.8000000E+01
.1268551E+06	0.1240479E+06	0.9778705E+00	0.9000000E+01
.1393551E+06	0.1365479E+06	0.9798555E+00	0.1000000E+02
.1518551E+06	0.1490479E+06	0.9815137E+00	0.1100000E+02
.1643551E+06	0.1615479E+06	0.9829196E+00	0.1200000E+02
.1768551E+06	0.1740479E+06	0.9841269E+00	0.1300000E+02
.1893551E+06	0.1865479E+06	0.9851747E+00	0.1400000E+02
.2018551E+06	0.1990479E+06	0.9860928E+00	0.1500000E+02
.2143551E+06	0.2115479E+06	0.9869038E+00	0.1600000E+02
.2268551E+06	0.2240479E+06	0.9876254E+00	0.1700000E+02
.2393551E+06	0.2365479E+06	0.9882716E+00	0.1800000E+02
.2518551E+06	0.2490479E+06	0.9888537E+00	0.1900000E+02
.2643551E+06	0.2615479E+06	0.9893808E+00	0.2000000E+02

TABLE 2.11

T1 = 300.0 m^2/day

T2 = 60.0 m^2/day

ϕ_1 = 0.010

ϕ_2 = 0.002

R_W = 0.10 m

QP(1)= 250.0 m^3/unit time

R_c = 1.0 m

SUM1	SUM2	RATIO	GLEVEL
.3935513E+05	0.3685256E+05	0.9364106E+00	0.2000000E+01
.5185513E+05	0.4935256E+05	0.9517392E+00	0.3000000E+01
.6435513E+05	0.6185256E+05	0.9611131E+00	0.4000000E+01
.7685513E+05	0.7435256E+05	0.9674378E+00	0.5000000E+01
.8935513E+05	0.8685256E+05	0.9719929E+00	0.6000000E+01
.1018551E+06	0.9935255E+05	0.9754300E+00	0.7000000E+01
.1143551E+06	0.1118526E+06	0.9781158E+00	0.8000000E+01
.1268551E+06	0.1243526E+06	0.9802722E+00	0.9000000E+01
.1393551E+06	0.1368526E+06	0.9820417E+00	0.1000000E+02
.1518551E+06	0.1493526E+06	0.9835200E+00	0.1100000E+02
.1643551E+06	0.1618526E+06	0.9847734E+00	0.1200000E+02
.1768551E+06	0.1743526E+06	0.9858496E+00	0.1300000E+02
.1893551E+06	0.1868526E+06	0.9867838E+00	0.1400000E+02
.2018551E+06	0.1993526E+06	0.9876022E+00	.1500000E+02
.2143551E+06	0.2118526E+06	0.9883251E+00	0.1600000E+02
.2268551E+06	0.2243526E+06	0.9889684E+00	0.1700000E+02
.2393551E+06	0.2368526E+06	0.9895446E+00	0.1800000E+02
.2518551E+06	0.2493526E+06	0.9900635E+00	0.1900000E+02
.2643551E+06	0.2618526E+06	0.9905333E+00	0.2000000E+02

TABLE 2.12

T1 = 300.0000 m^2/day

T2 = 60.0000 m^2/day

ϕ_1 = 0.0100

ϕ_2 = 0.0020

R_w = 0.1000 m

QF(1) = 250.0000 m^3/day

R_g = 0.1000 m

SUM1	SUM2	RATIO	GLEVEL
.3935513E+05	0.3696148E+05	0.9391783E+00	0.2000000E+01
.5185513E+05	0.4946148E+05	0.9538397E+00	0.3000000E+01
.6435513E+05	0.6196148E+05	0.9628057E+00	0.4000000E+01
.7685513E+05	0.7446148E+05	0.9688551E+00	0.5000000E+01
.8935513E+05	0.8696149E+05	0.9732120E+00	0.6000000E+01
.1018551E+06	0.9946149E+05	0.9764995E+00	0.7000000E+01
.1143551E+06	0.1119615E+06	0.9790683E+00	0.8000000E+01
.1268551E+06	0.1244615E+06	0.9811309E+00	0.9000000E+01
.1393551E+06	0.1369615E+06	0.9828234E+00	0.1000000E+02
.1518551E+06	0.1494615E+06	0.9842373E+00	0.1100000E+02
.1643551E+06	0.1619615E+06	0.9854361E+00	0.1200000E+02
.1768551E+06	0.1744615E+06	0.9864655E+00	0.1300000E+02
.1893551E+06	0.1869615E+06	0.9873590E+00	0.1400000E+02
.2018551E+06	0.1994615E+06	0.9881418E+00	0.1500000E+02
.2143551E+06	0.2119615E+06	0.9888333E+00	0.1600000E+02
.2268551E+06	0.2244615E+06	0.9894486E+00	0.1700000E+02
.2393551E+06	0.2369615E+06	0.9899996E+00	0.1800000E+02
.2518551E+06	0.2494615E+06	0.9904960E+00	0.1900000E+02
.2643551E+06	0.2619615E+06	0.9909454E+00	0.2000000E+02

TABLE 2.13

T1 = 500.0 m^2/day

T2 = 500.0 m^2/day

ϕ_1 = 0.002

ϕ_2 = 0.001

QP(1) = 500.0 m^3/day

R_w = 0.10 m

R_e = 0.10 m

SUM1	SUM2	RATIO	GLEVEL
0.2376707E+06	0.1658225E+06	0.6976986E+00	0.1000000E+01
0.3291707E+06	0.2573225E+06	0.7817296E+00	0.2000000E+01
0.4206707E+06	0.3488226E+06	0.8292059E+00	0.3000000E+01
0.5121707E+06	0.4403227E+06	0.8597185E+00	0.4000000E+01
0.6036706E+06	0.5318225E+06	0.8809813E+00	0.5000000E+01
0.6951704E+06	0.6233226E+06	0.8966471E+00	0.6000000E+01
0.7866706E+06	0.7148226E+06	0.9086682E+00	0.7000000E+01
0.8781707E+06	0.8063226E+06	0.9181844E+00	0.8000000E+01
0.9696707E+06	0.8978227E+06	0.9259048E+00	0.9000000E+01
0.1061171E+07	0.9893228E+06	0.9322939E+00	0.1000000E+02
0.1152670E+07	0.1080823E+07	0.9276686E+00	0.1100000E+02
0.1244171E+07	0.1172323E+07	0.9422520E+00	0.1200000E+02
0.1335671E+07	0.1263822E+07	0.9462078E+00	0.1300000E+02
0.1427171E+07	0.1355322E+07	0.9496564E+00	0.1400000E+02
0.1518671E+07	0.1446822E+07	0.9526896E+00	0.1500000E+02
0.1610171E+07	0.1538322E+07	0.9553782E+00	0.1600000E+02
0.1701671E+07	0.1629822E+07	0.9577776E+00	0.1700000E+02
0.1793171E+07	0.1721322E+07	0.9599321E+00	0.1800000E+02
0.1884671E+07	0.1812822E+07	0.9618773E+00	0.1900000E+02
0.1976171E+07	0.1904322E+07	0.9636424E+00	0.2000000E+02

TABLE 2.14

T1 = 500.0 m^2/day

T2 = 100.0 m^2/day

ϕ_1 0.002

ϕ_2 = 0.002

R_w = 0.10 m

QP(1) = 500.0 m^3/day

R_c = 0.10 m

SUM1	SUM2	RATIO	GLEVEL
0.2376707E+06	0.2115550E+06	0.8901181E+00	0.1000000E+01
0.3291707E+06	0.3030549E+06	0.9206619E+00	0.2000000E+01
0.4206707E+06	0.3945551E+06	0.9379191E+00	0.3000000E+01
0.5121707E+06	0.4860551E+06	0.9490098E+00	0.4000000E+01
0.6036706E+06	0.5775551E+06	0.9567388E+00	0.5000000E+01
0.6951704E+06	0.6690551E+06	0.9624332E+00	0.6000000E+01
0.7866706E+06	0.7605553E+06	0.9668027E+00	0.7000000E+01
0.8781707E+06	0.8520553E+06	0.9702616E+00	0.8000000E+01
0.9696707E+06	0.9435551E+06	0.9730676E+00	0.9000000E+01
0.1061171E+07	0.1035055E+07	0.9753901E+00	0.1000000E+02
0.1152670E+07	0.1126555E+07	0.9773437E+00	0.1100000E+02
0.1244171E+07	0.1218055E+07	0.9790094E+00	0.1200000E+02
0.1335671E+07	0.1309555E+07	0.9804474E+00	0.1300000E+02
0.1427171E+07	0.1401055E+07	0.9817010E+00	0.1400000E+02
0.1518671E+07	0.1492555E+07	0.9828034E+00	0.1500000E+02
0.1610171E+07	0.1584055E+07	0.9837809E+00	0.1600000E+02
0.1701671E+07	0.1675555E+07	0.9846529E+00	0.1700000E+02
0.1793171E+07	0.1767055E+07	0.9854361E+00	0.1800000E+02
0.1884671E+07	0.1858555E+07	0.9861431E+00	0.1900000E+02
0.1976171E+07	0.1950055E+07	0.9867847E+00	0.2000000E+02

TABLE 2.15

T1 = 500.0 m^2/day

T2 = 200.0 m^2/day

ϕ_1 = 0.002

ϕ_2 = 0.002

R_x = 0.10 m

QP(1)= 500.0 m^3/day

R_c = 0.10 m

SUM1	SUM2	RATIO	GLEVEL
0.2376707E+06	0.1944939E+06	0.8183336E+00	0.1000000E+01
0.3291707E+06	0.2859938E+06	0.8688313E+00	0.2000000E+01
0.4206707E+06	0.3774938E+06	0.8973619E+00	0.3000000E+01
0.5121707E+06	0.4689938E+06	0.9156981E+00	0.4000000E+01
0.6036706E+06	0.5604938E+06	0.9284761E+00	0.5000000E+01
0.6951704E+06	0.6519939E+06	0.9378907E+00	0.6000000E+01
0.7866706E+06	0.7434938E+06	0.9451145E+00	0.7000000E+01
0.87817[illegible]7E+06	0.8349938E+06	0.9508331E+00	0.8000000E+01
0.9696707E+06	0.9264939E+06	0.9554728E+00	0.9000000E+01
0.1061171E+07	0.1017994E+07	0.9593123E+00	0.1000000E+02
0.1152670E+07	0.1109494E+07	0.9625423E+00	0.1100000E+02
0.1244171E+07	0.1200994E+07	0.9652964E+00	0.1200000E+02
0.1335671E+07	0.1292494E+07	0.9676739E+00	0.1300000E+02
0.1427171E+07	0.1383994E+07	0.9697464E+00	0.1400000E+02
0.1518671E+07	0.1475494E+07	0.9715691E+00	0.1500000E+02
0.1610171E+07	0.1566994E+07	0.9731848E+00	0.1600000E+02
0.1701671E+07	0.1658494E+07	0.9746267E+00	0.1700000E+02
0.1793171E+07	0.1749994E+07	0.9759215E+00	0.1800000E+02
0.1884671E+07	0.1841494E+07	0.9770905E+00	0.1900000E+02
0.1976171E+07	0.1932994E+07	0.9781512E+00	0.2000000E+02

TABLE 2.16

T1 = 500.0 m^2/day

T2 = 300.0 m^2/day

ϕ_1 = 0.002

ϕ_2 = 0.002

R_w = 0.10 m

QP(1)= 500.0 m^3/day

R_c = 0.10 m

SUM1	SUM2	RATIO	GLEVEL
0.2376707E+06	0.1819663E+06	0.7656239E+00	0.1000000E+01
0.3291707E+06	0.2734663E+06	0.3807734E+00	0.2000000E+01
0.4206707E+06	0.3649663E+06	0.8675821E+00	0.3000000E+01
0.5121707E+06	0.4564663E+06	0.8912385E+00	0.4000000E+01
0.6036706E+06	0.5479664E+06	0.9077241E+00	0.5000000E+01
0.6951704E+06	0.6394664F+06	0.9198699E+00	0.6000000E+01
0.4766706E+06	0.7309663E+06	0.9291898E+00	0.7000000E+01
0.8781707E+06	0.8224663E+06	0.9365677E+00	0.8000000E+01
0.9696707E+06	0.9139663E+06	0.9425533E+00	0.9000000E+01
0.1061171E+07	0.1005466E+07	0.9475069E+00	0.1000000E+02
0.1152670E+07	0.1096966E+07	0.9516739E+00	0.1100000E+02
0.1244171E+07	0.1188466E+07	0.9552273E+00	0.1200000E+02
0.1335671E+07	0.1279966E+07	0.9582946E+00	0.1300000E+02
0.1427171E+07	0.1371466E+07	0.9609683E+00	0.1400000E+02
0.1518671E+07	0.1462966E+07	0.9633201E+00	0.1500000E+02
0.1610171E+07	0.1554467E+07	0.9654047E+00	0.1600000E+02
0.1701671E+07	0.1645967E+07	0.9672648E+00	0.1700000E+02
0.1793171E+07	0.1737467E+07	0.6989352E+00	0.1800000E+02
0.1884671E+07	0.1828967E+07	0.9704434E+00	0.1900000E+02
0.1976171E+07	0.1920467E+07	0.9718119E+00	0.2000000E+02

TABLE 2.17

T1 = 500.00000 m^2/day

T2 = 400.00000 m^2/day

ϕ_1 = 0.002[illegible]

ϕ_2 = 0.002[illegible]

R_w = 0.10000 m

QP(1)= 500.00000 m^3/day

R_c = 0.10000 m

SUM1	SUM2	RATIO	GLEVEL
0.2376707E+06	0.1722988E+06	0.7249478E+00	0.1000000E+01
0.3291707E+06	0.2637989E+06	0.8014044E+00	0.2000000E+01
0.4206707E+06	0.3552989E+06	0.8446010E+00	0.3000000E+01
0.5121707E+06	0.4467988E+06	0.8723631E+00	0.4000000E+01
0.6036706E+06	0.5382988E+06	0.8917094E+00	0.5000000E+01
0.6951704E+06	0.6297988E+06	0.9059631E+00	0.6000000E+01
0.7866706E+06	0.7212988E+06	0.9169006E+00	0.7000000E+01
0.8781707E+06	0.8127988E+06	0.9255590E+00	0.8000000E+01
0.9696707E+06	0.9042989E+06	0.9325935E+00	0.9000000E+01
0.1061171E+07	0.9957989E+07	0.9383967E+00	0.1000000E+02
0.1152670E+07	0.1087299E+07	0.9432870E+00	0.1100000E+02
0.1244171E+07	0.1178799E+07	0.9474572E+00	0.1200000E+02
0.1335671E+07	0.1270299E+07	0.9510568E+00	0.1300000E+02
0.1427171E+07	0.1361799E+07	0.1541945E+00	0.1400000E+02
0.1518671E+07	0.1543299E+07	0.9569453E+00	0.1500000E+02
0.1610171E+07	0.1544799E+07	0.9694006E+00	0.1600000E+02
0.1701671E+07	0.1636299E+07	0.9615836E+00	0.1700000E+02
0.1793171E+07	0.1727799E+07	0.9635439E+00	0.1800000E+02
0.1884671E+07	0.1819299E+07	0.9653139E+00	0.1900000E+02
0.1976171E+07	0.1910799E+07	0.9669197E+00	0.2000000E+02

TABLE 2.18

T1 = 500.0 m^2/day

T2 = 500.0 m^2/day

ϕ_1 = 0.002

ϕ_2 = 0.002 m^3/day

R_w = 0.10 m

QP(1)= 500.0 m

R_c = 0.10

SUM1	SUM2	RATIO	GLEVEL
0.2376707E+06	0.1645826E+66	0.6924816E+00	0.1000000E+01
0.3291707E+06	0.2560826E+06	0.7779627E+00	0.2000000E+01
0.4206707E+06	0.3475826E+06	0.8262583E+00	0.3000060E+01
0.5121707E+06	0.4390827E+06	0.8572975E+00	0.4000000E+01
0.6036706E+06	0.5305827E+06	0.8789275E+00	0.5000000E+01
0.6951704E+06	0.6220824E+06	0.8948631E+00	0.6000000E+01
0.7866706E+06	0.7135824E+06	0.9070917E+00	0.7000000E+01
0.8781707E+06	0.8050823E+06	0.9167720E+00	0.8000000E+01
0.9696707E+06	0.8965824E+06	0.9246256E+00	0.9000000E+01
0.1061171E+06	0.9880823E+06	0.9311249E+00	0.1000000E+02
0.1152670E+07	0.1079582E+07	0.9365925E+00	0.1100000E+02
0.1244171E+07	0.1171082E+07	0.9412551E+00	0.1200000E+02
0.1335671E+07	0.1262583E+07	0.9452798E+00	0.1300000E+02
0.1427171E+07	0.1354083E+07	0.9487880E+00	0.1400000E+02
0.1518671E+07	0.1445583E+07	0.9518735E+00	0.1500000E+02
0.1610171E+07	0.1537083E+07	0.9546086E+00	0.1600000E+02
0.1701671E+07	0.1628583E+07	0.9570492E+00	0.1700000E+02
0.1793171E+07	0.1720083E+07	0.9592406E+00	0.1800000E+02
0.1884671E+07	0.1811583E+07	0.9612197E+00	0.1900000E+02
0.1976171E+07	0.1903083E+07	0.9630153E+00	0.2000000E+02

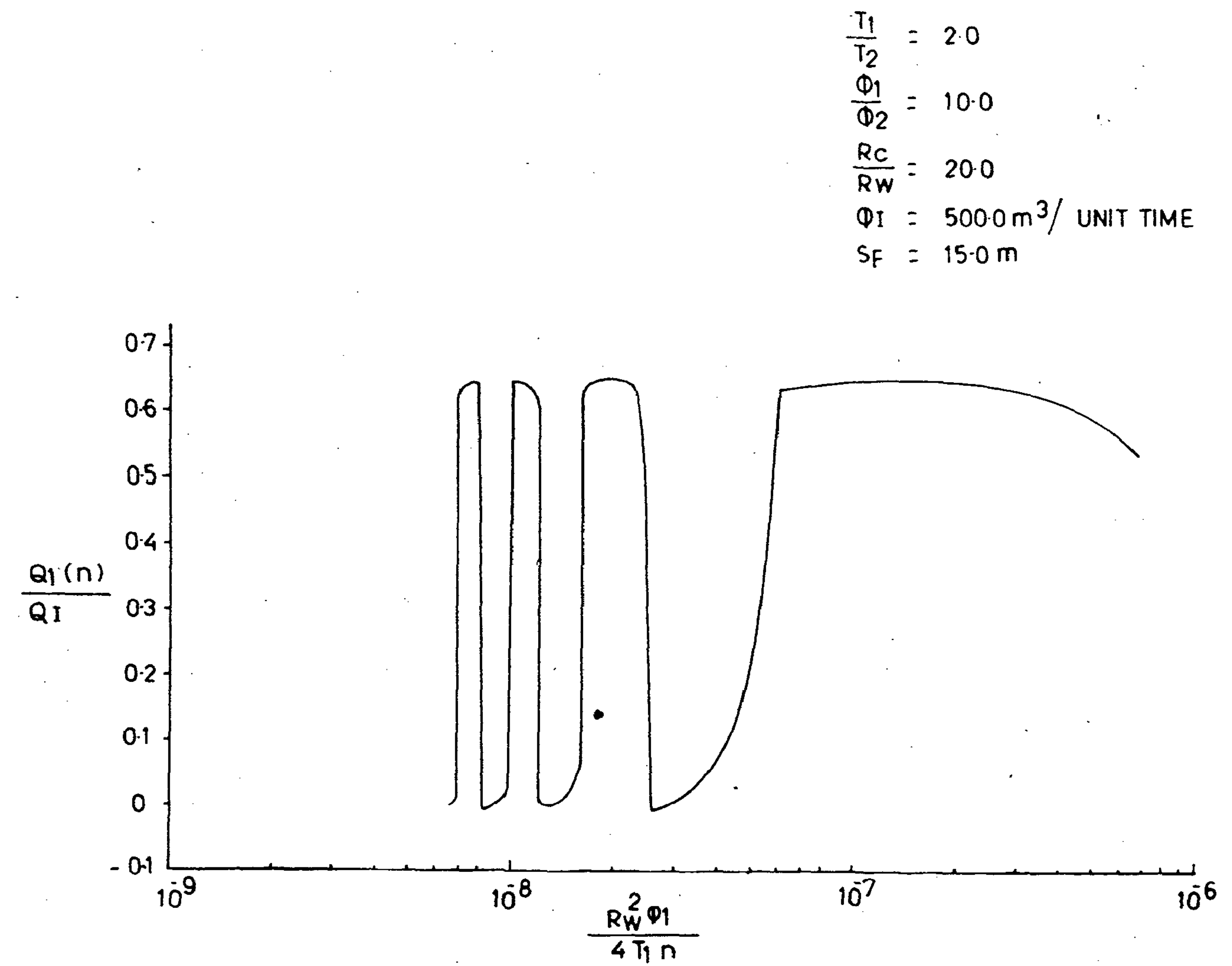

FIG. 4·1(a) VARIATION OF $Q_1(n)/Q_I$ WITH $\frac{R_W^2 \Phi_1}{4 T_1 n}$

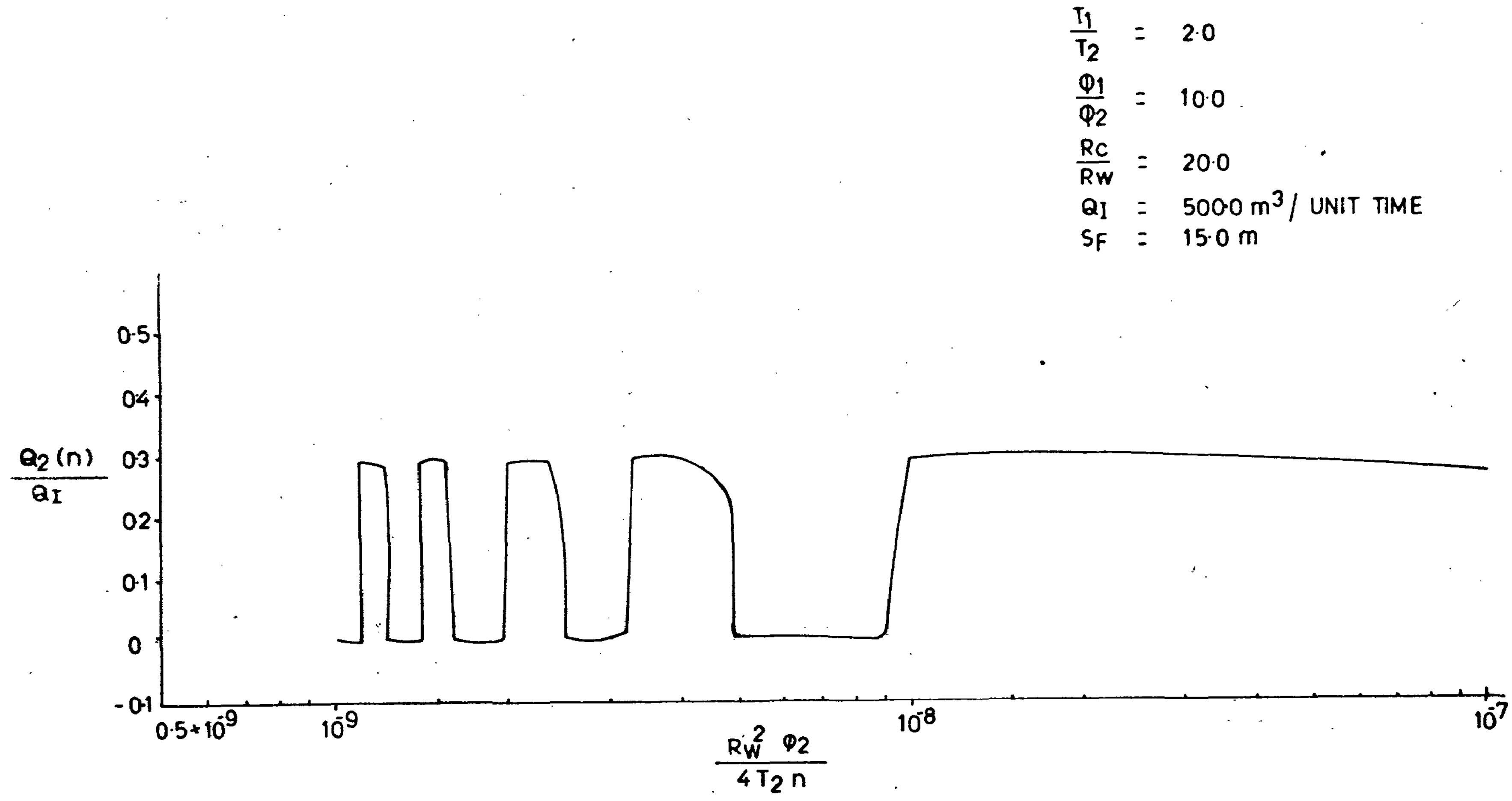

FIG 4.1(b) VARIATION OF $\frac{Q_2(n)}{Q_I}$ WITH $\frac{R_W^2 \ \Phi_2}{4 T_2 n}$

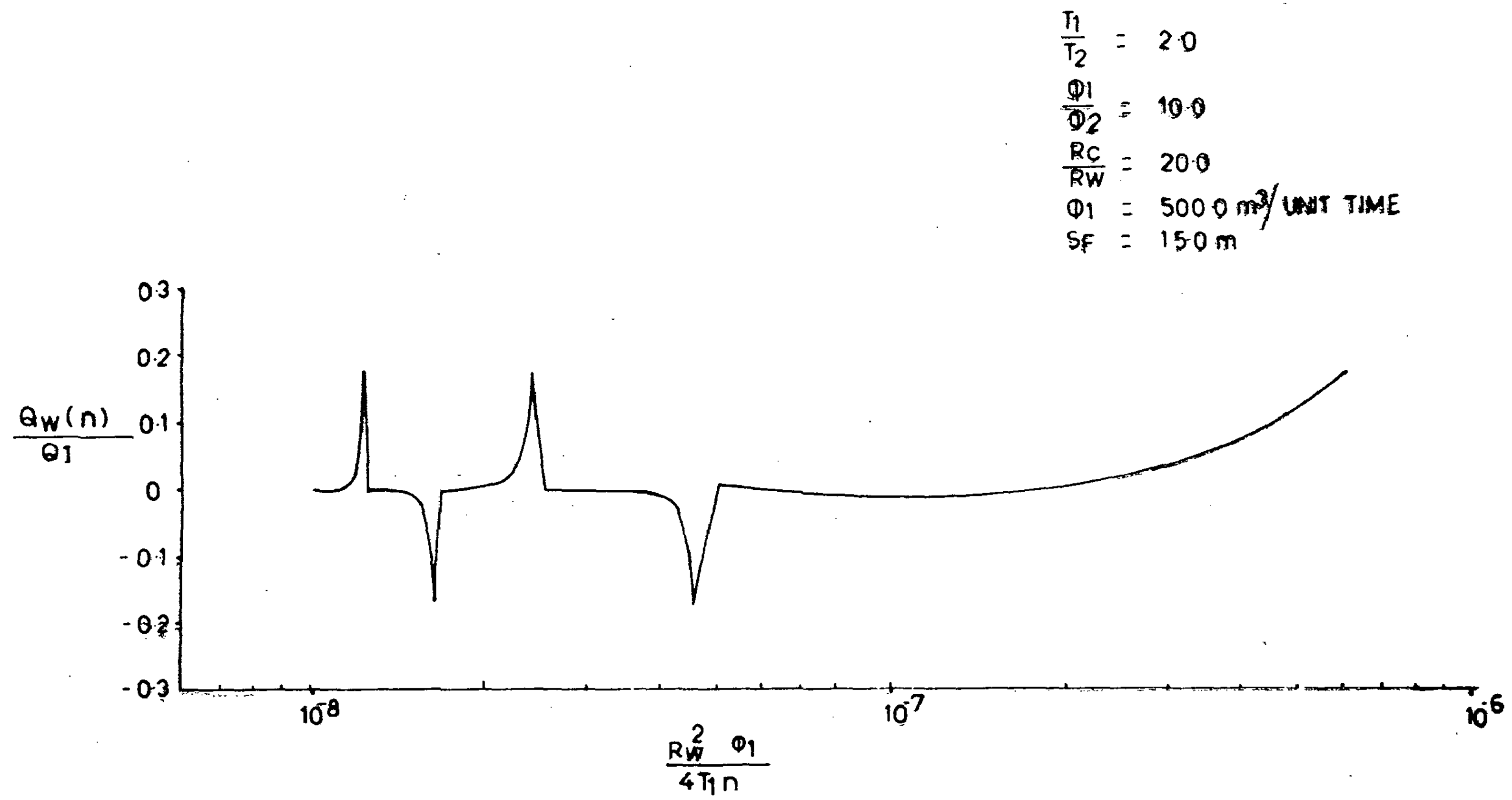

FIG 4·1(c) VARIATION OF $\frac{Q_W(n)}{Q_I}$ WITH $\frac{R_W^2 \; \Phi_1}{4 T_1 n}$

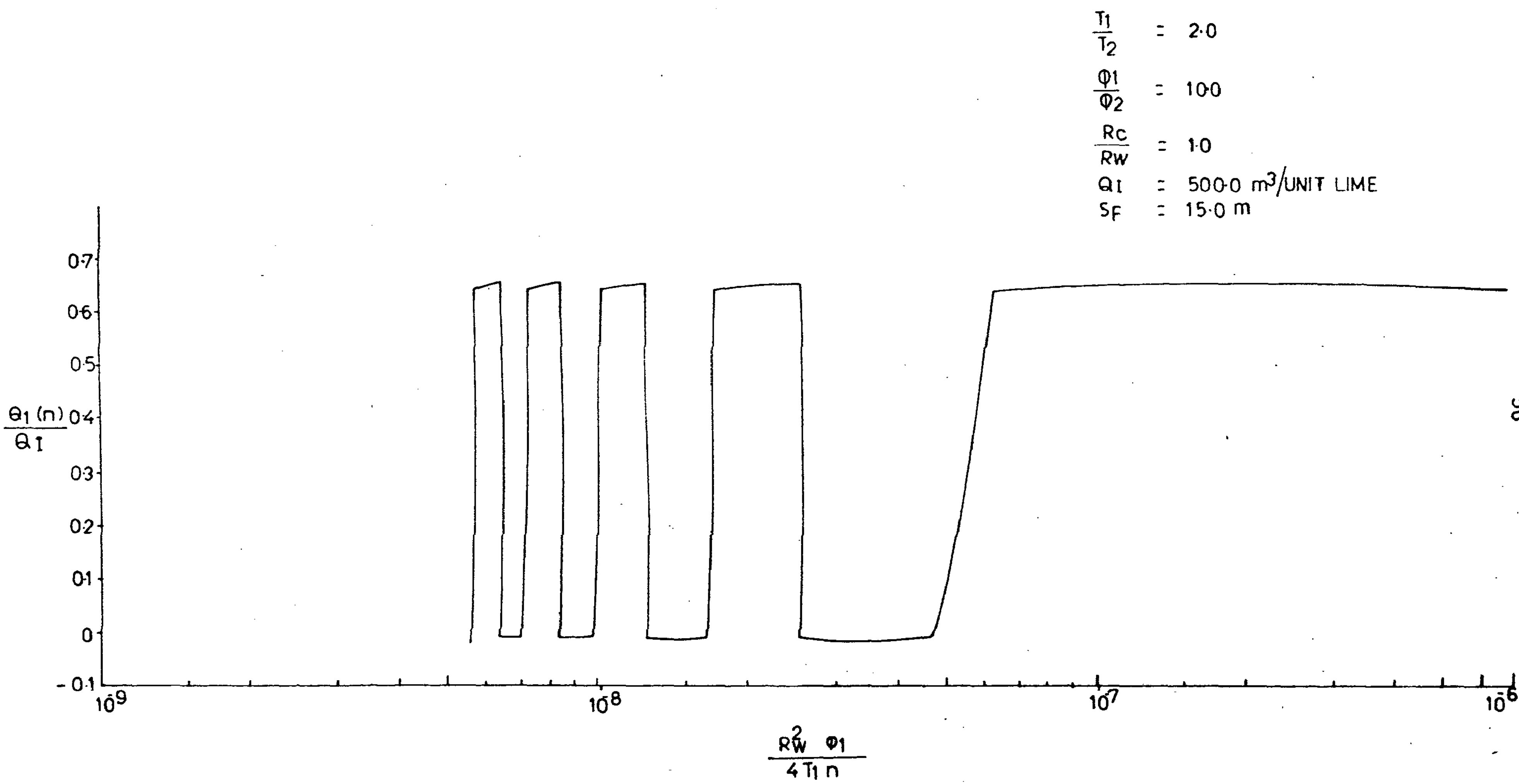

FIG. 4·2 VARIATION OF $\frac{Q_2(n)}{Q_I}$ WITH $\frac{R_w^2 \Phi_2}{4 T_2 n}$

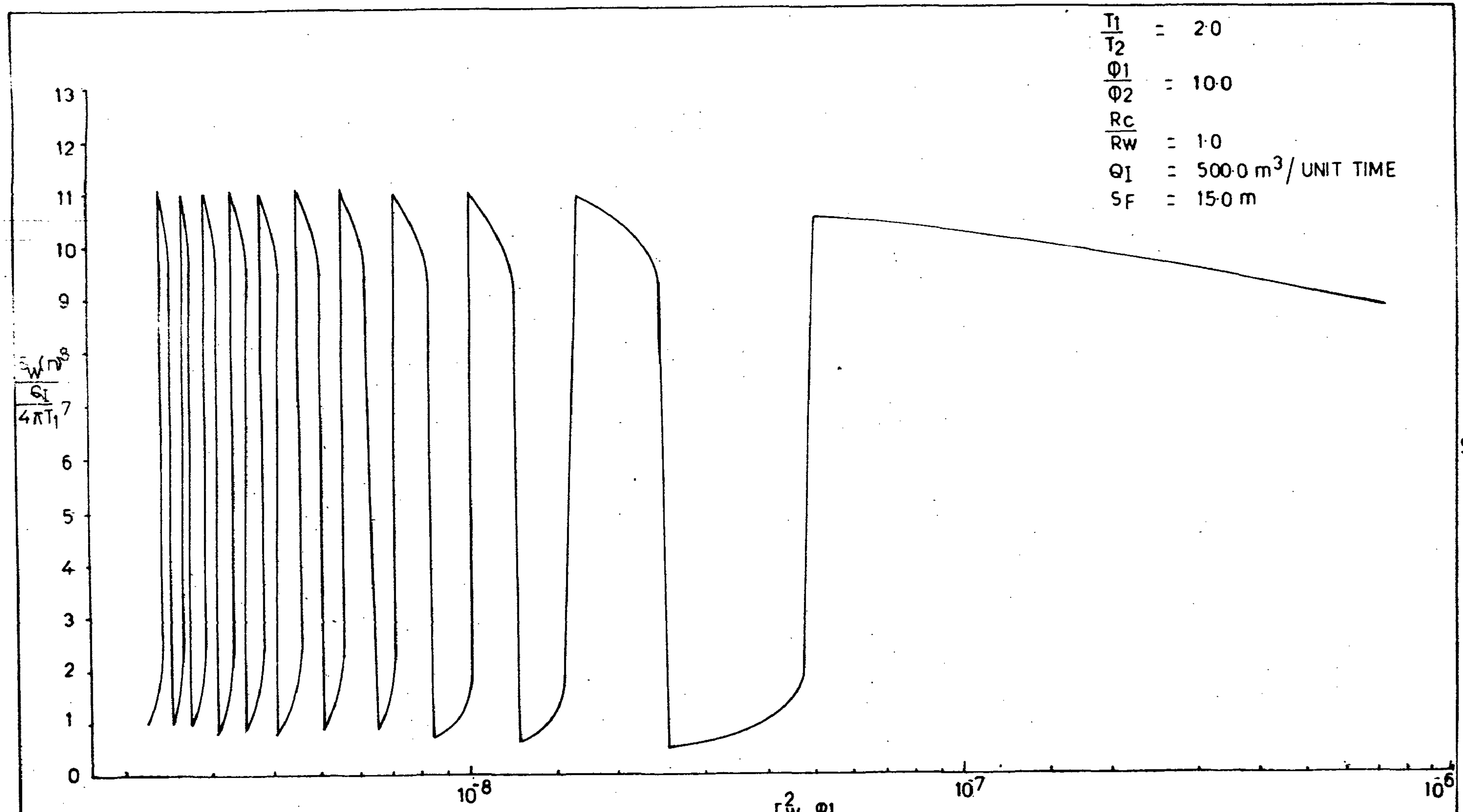

FIG 4.3(a) VARIATION OF $\frac{S_w(n)}{\frac{Q_I}{4\pi T_1}}$ WITH $\frac{R_w^2 \Phi_1}{4 T_1 n}$

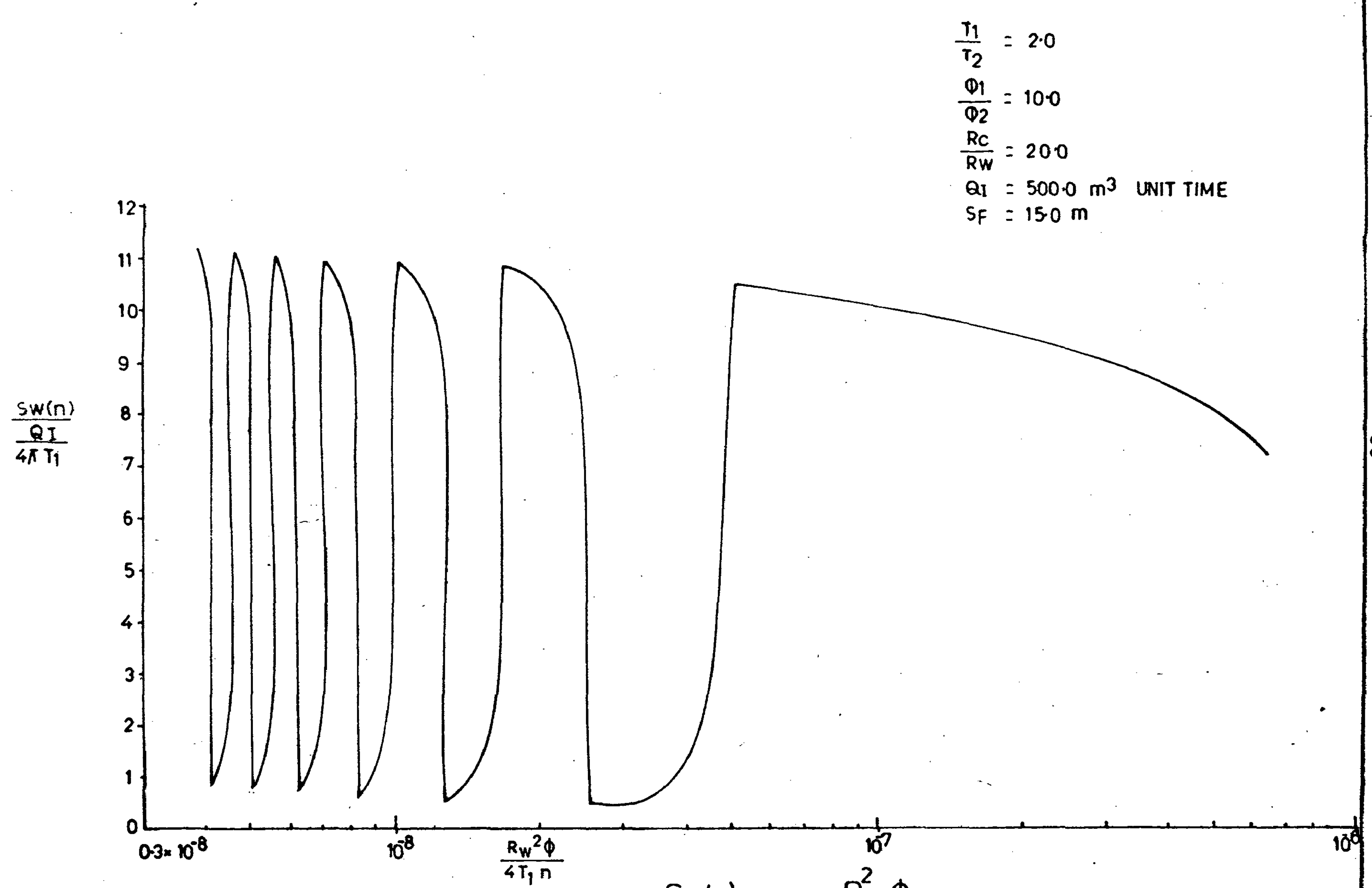

FIG 4·3(b) VARIATION OF $\frac{Sw(n)}{\frac{Q_I}{4\pi T_1}}$ WITH $\frac{R_W^2 \Phi_1}{4 T_1 n}$

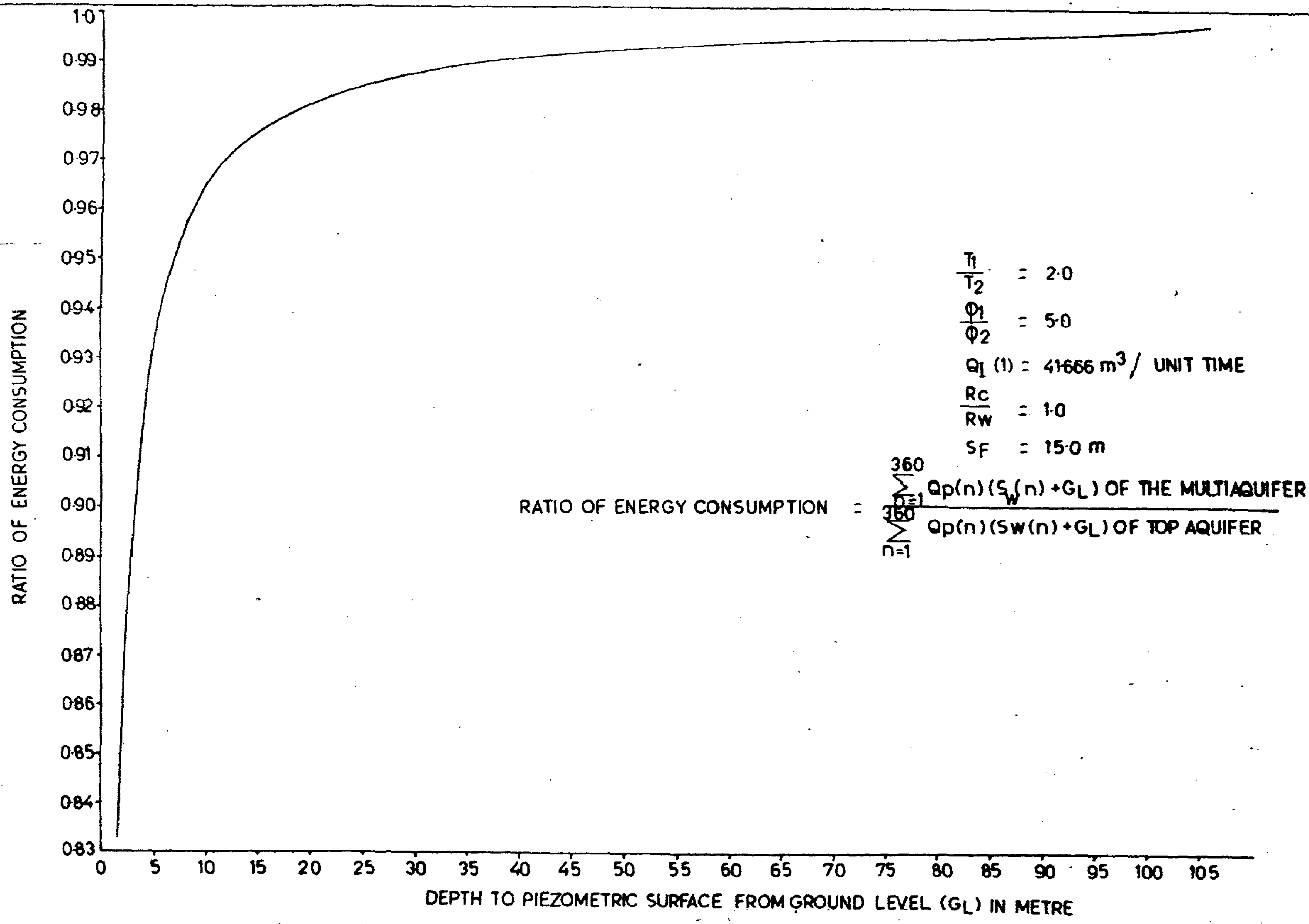

FIG 4.4(a) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

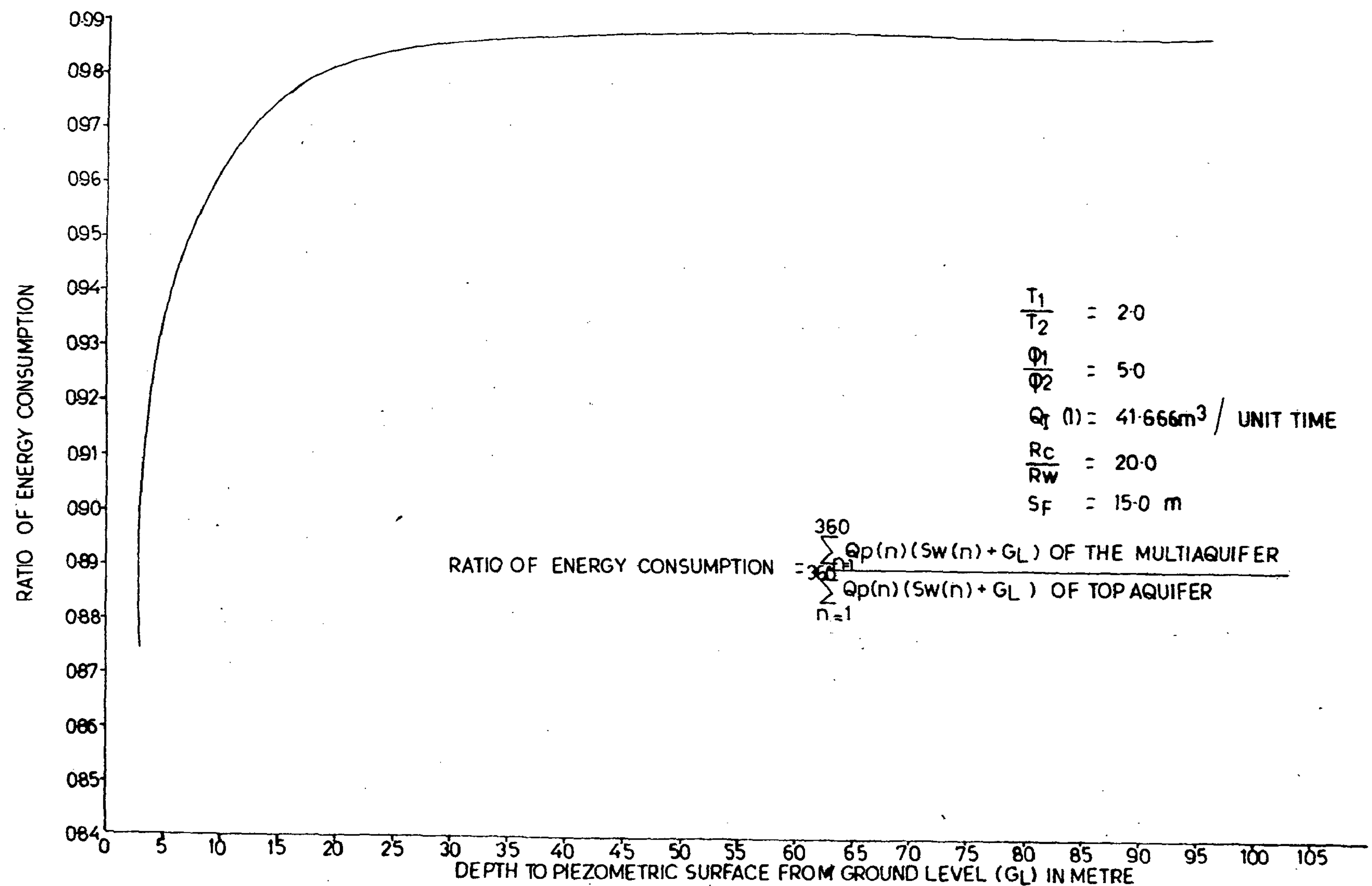

FIG44(b) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

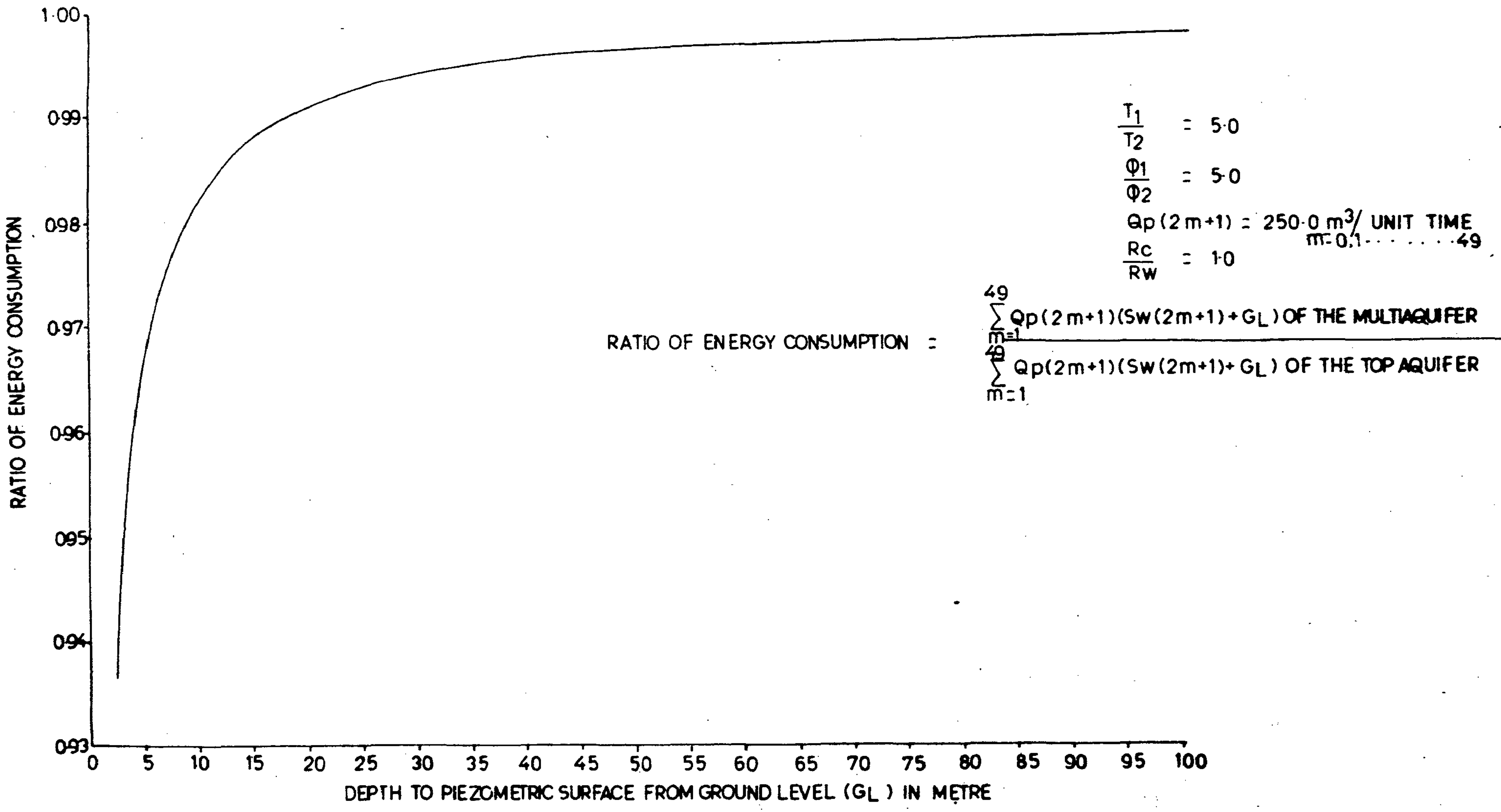

FIG 4·4(c) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

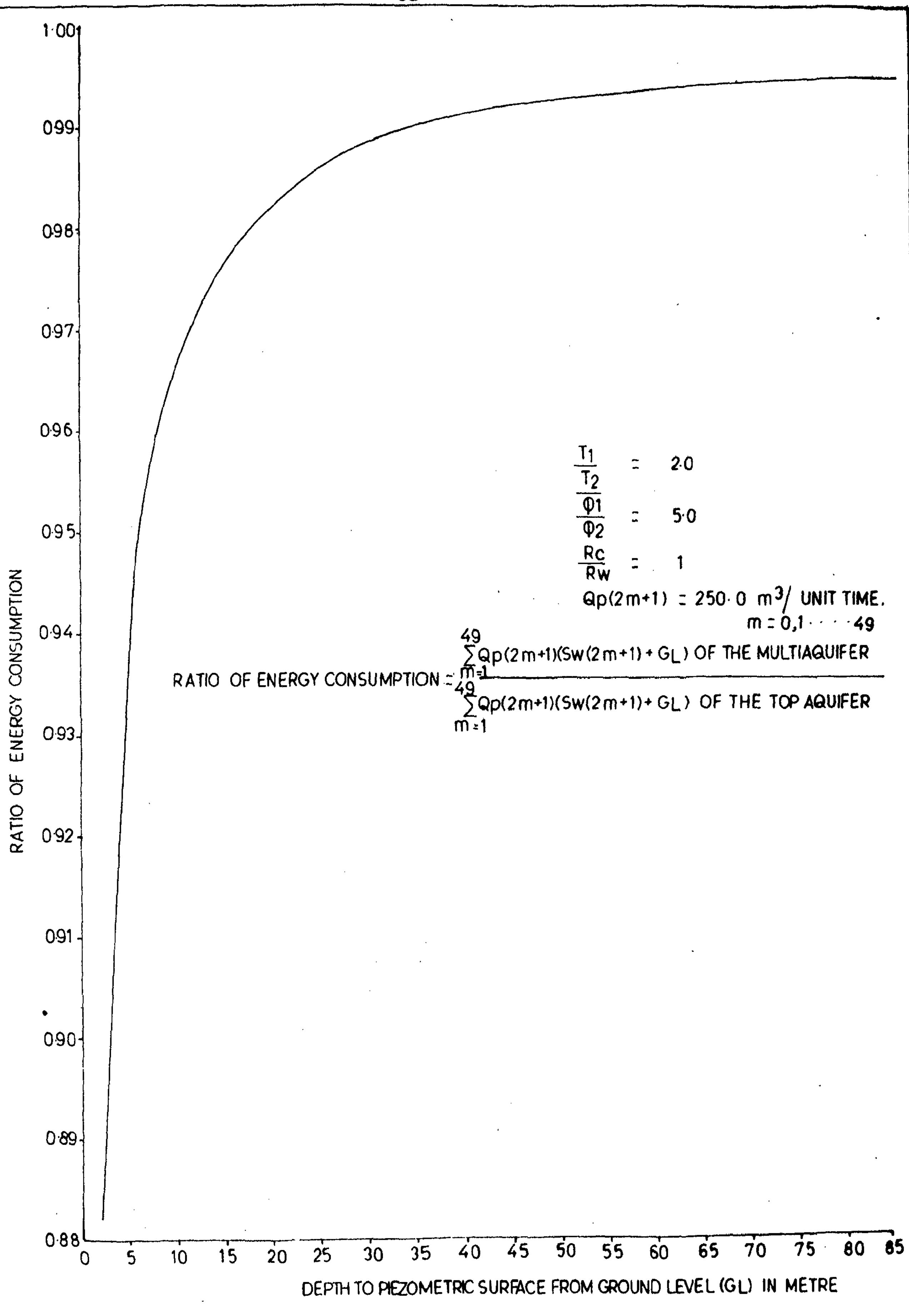

FIG. 4·4(d) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

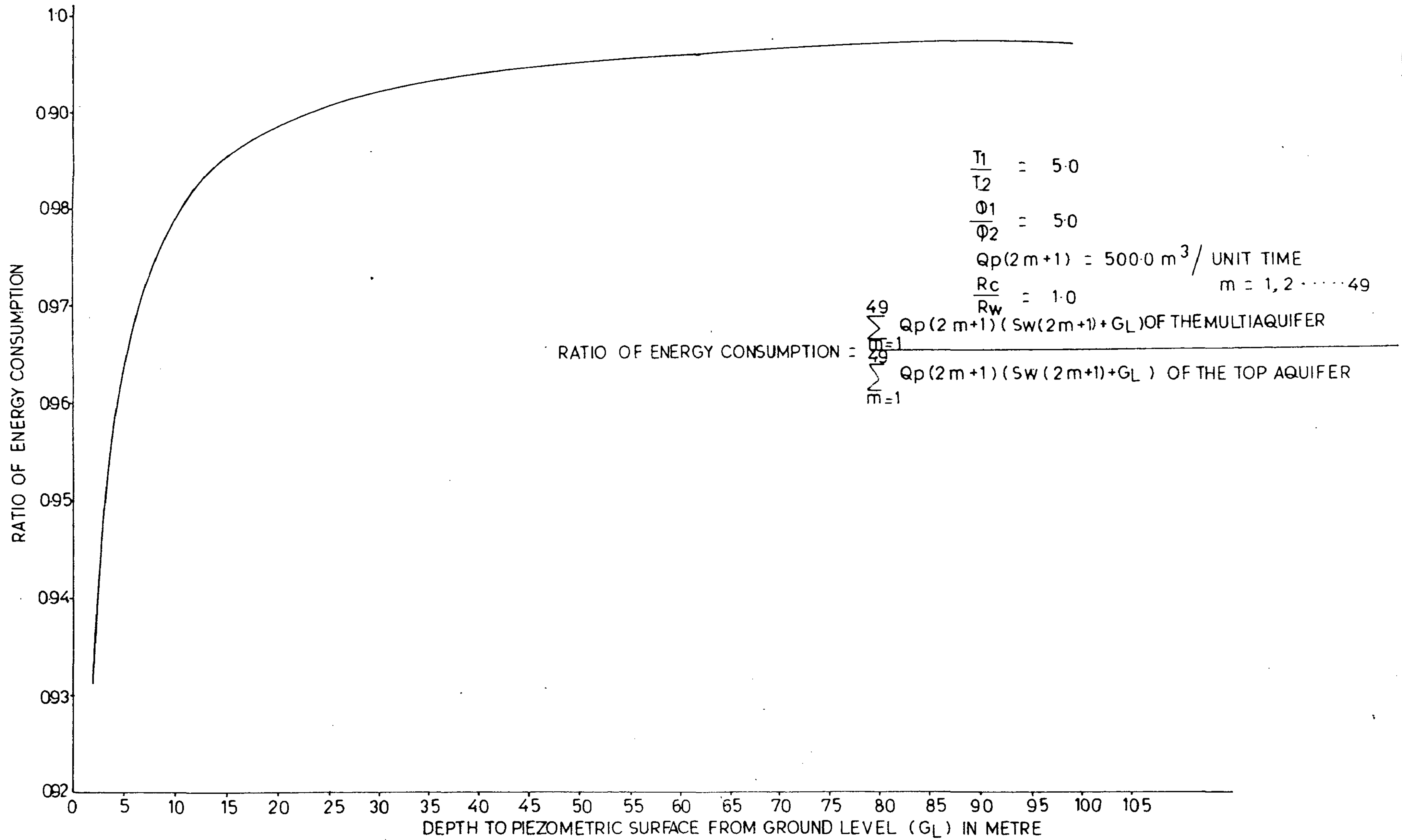

FIG. 4·4(e) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

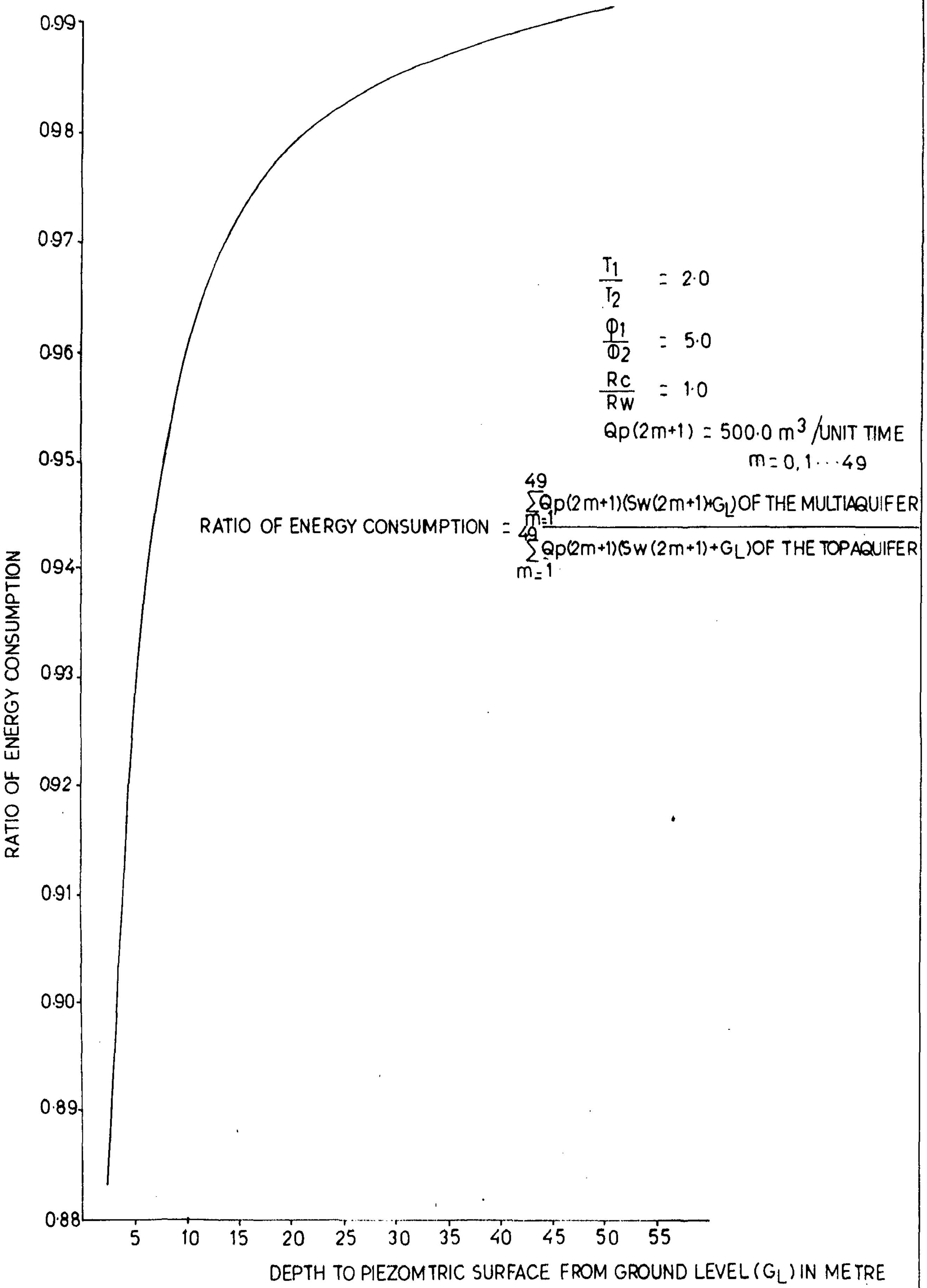

FIG 44(f) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

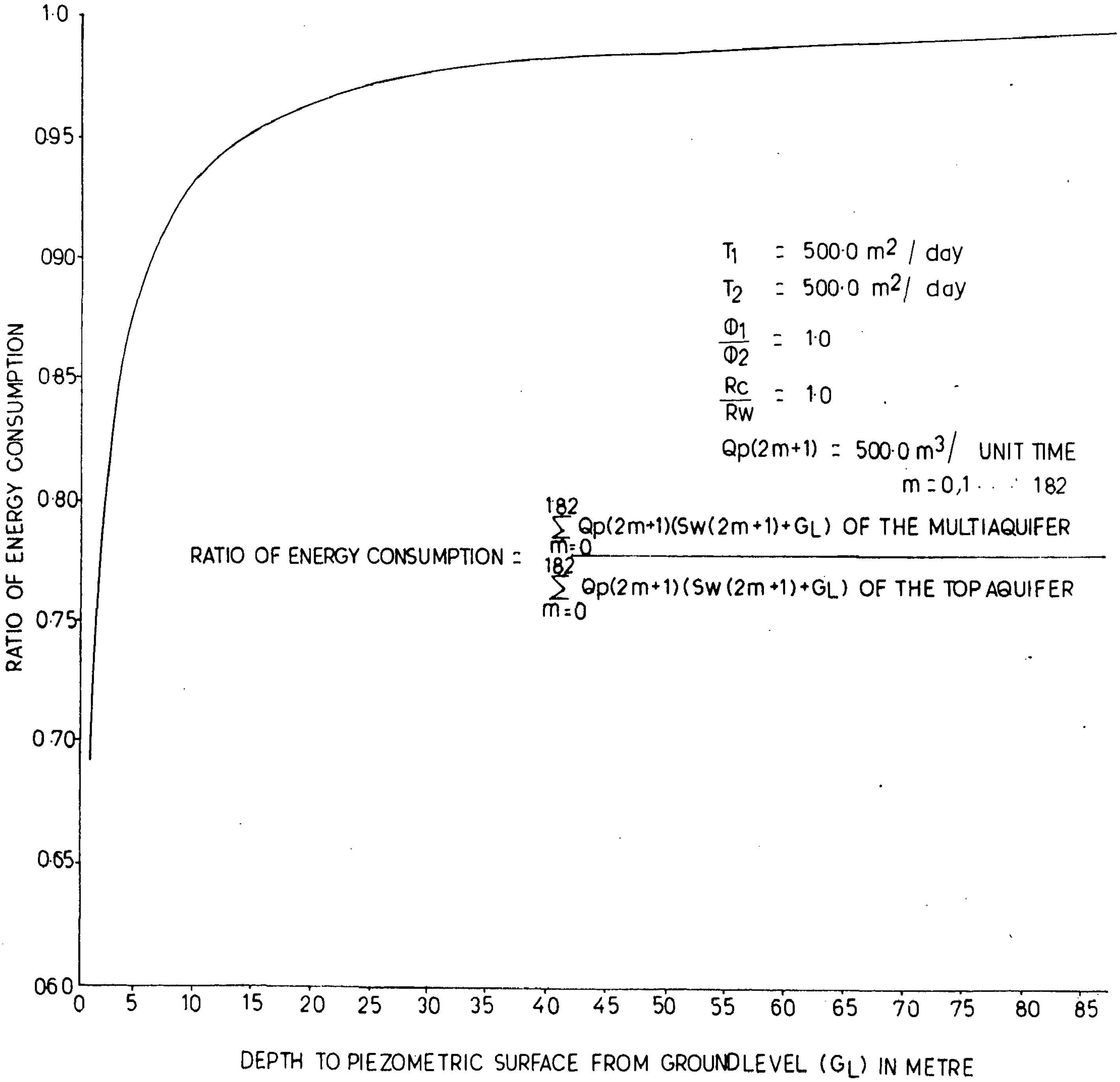

FIG 4·4(g) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

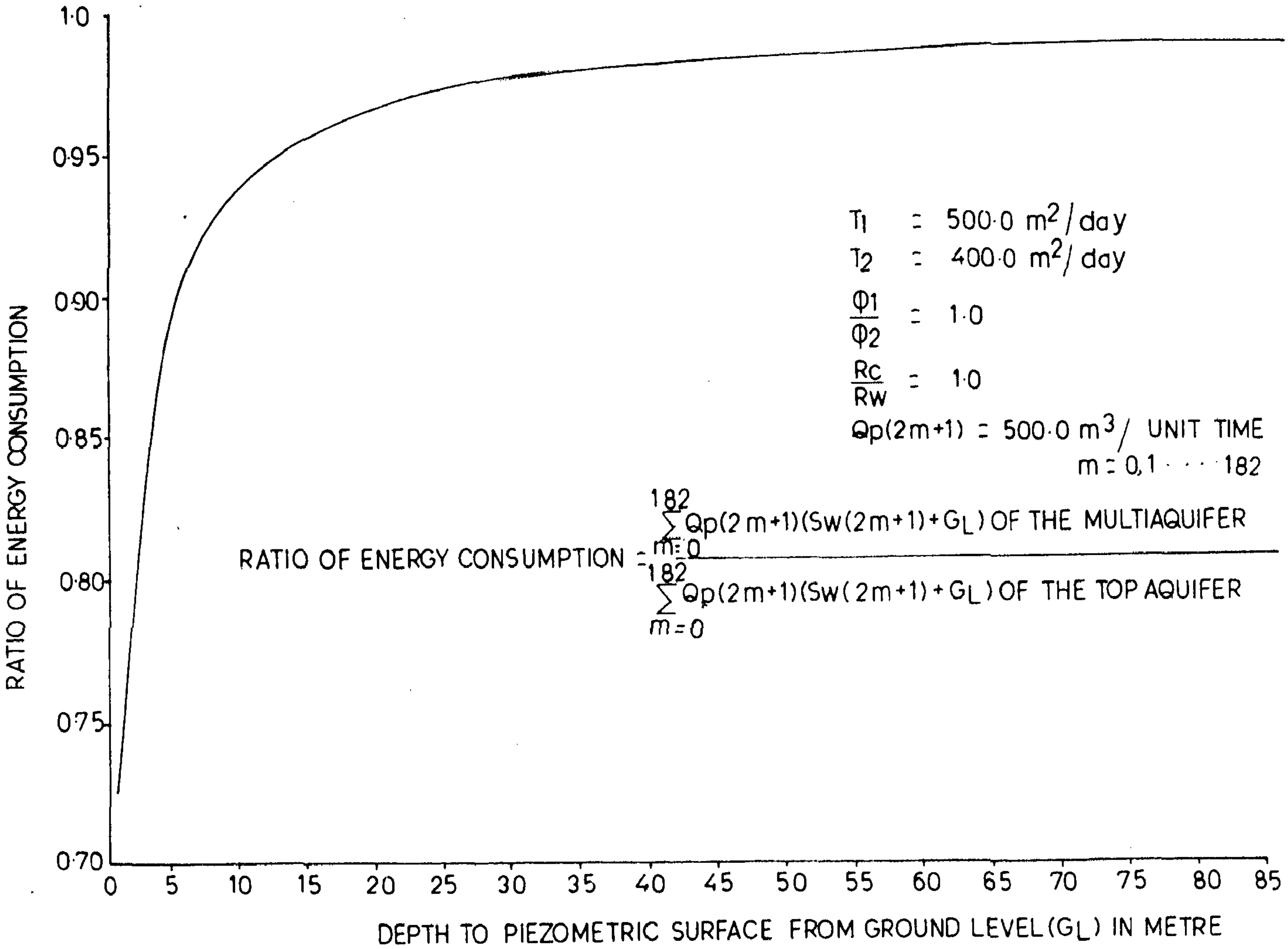

FIG 4·4(h) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

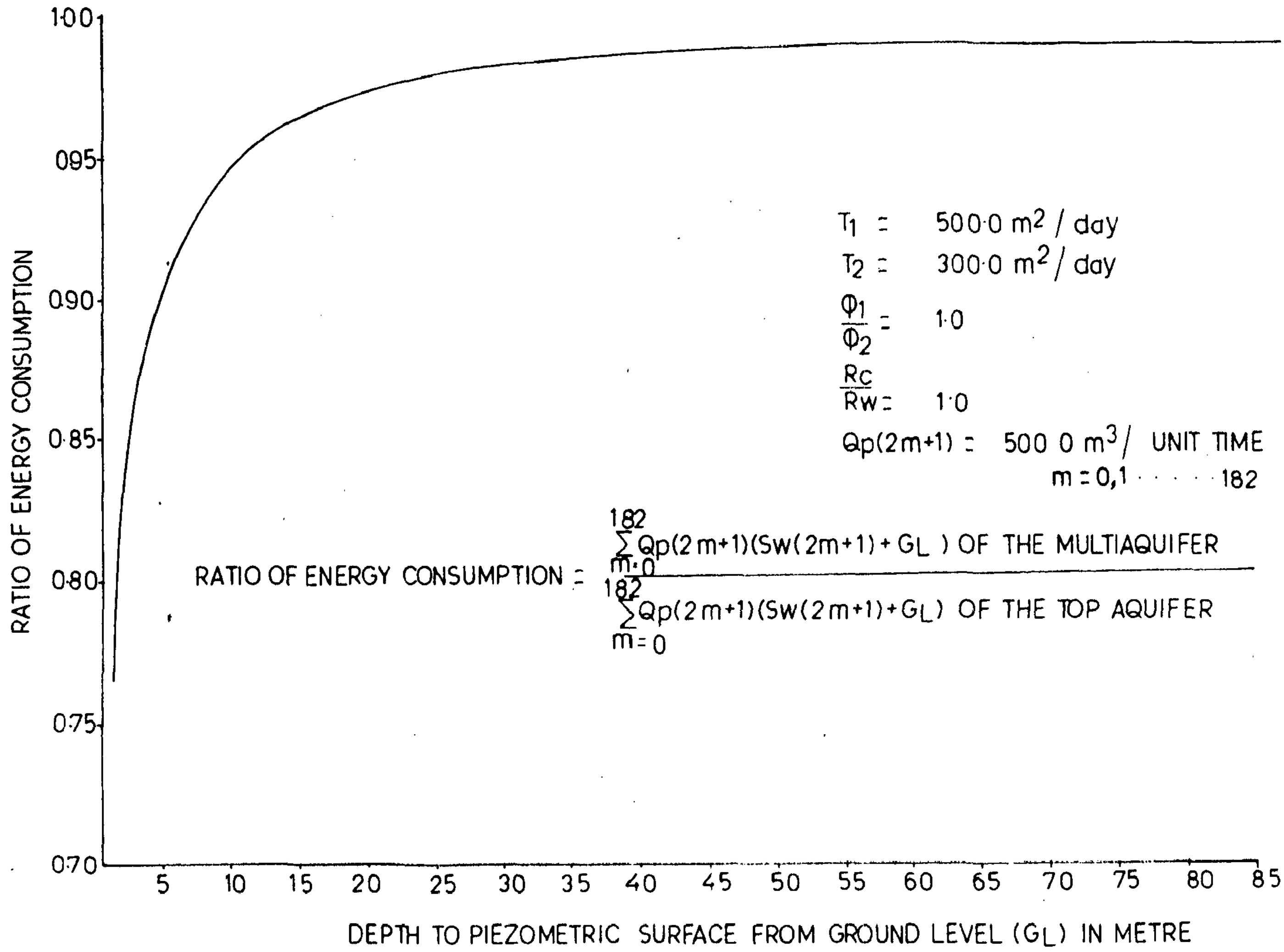

FIG 4·4(i) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

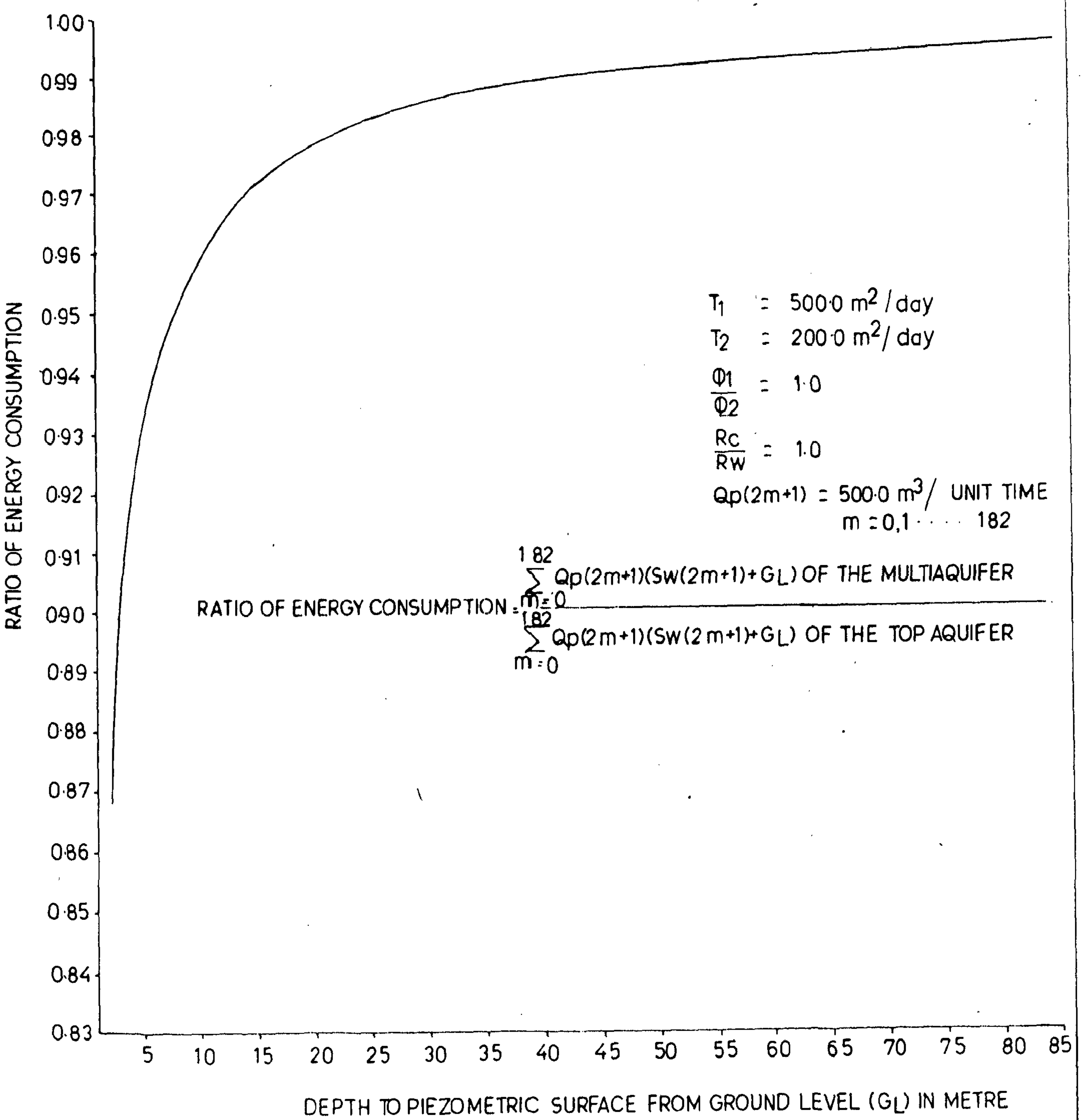

FIG 4.4(j) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

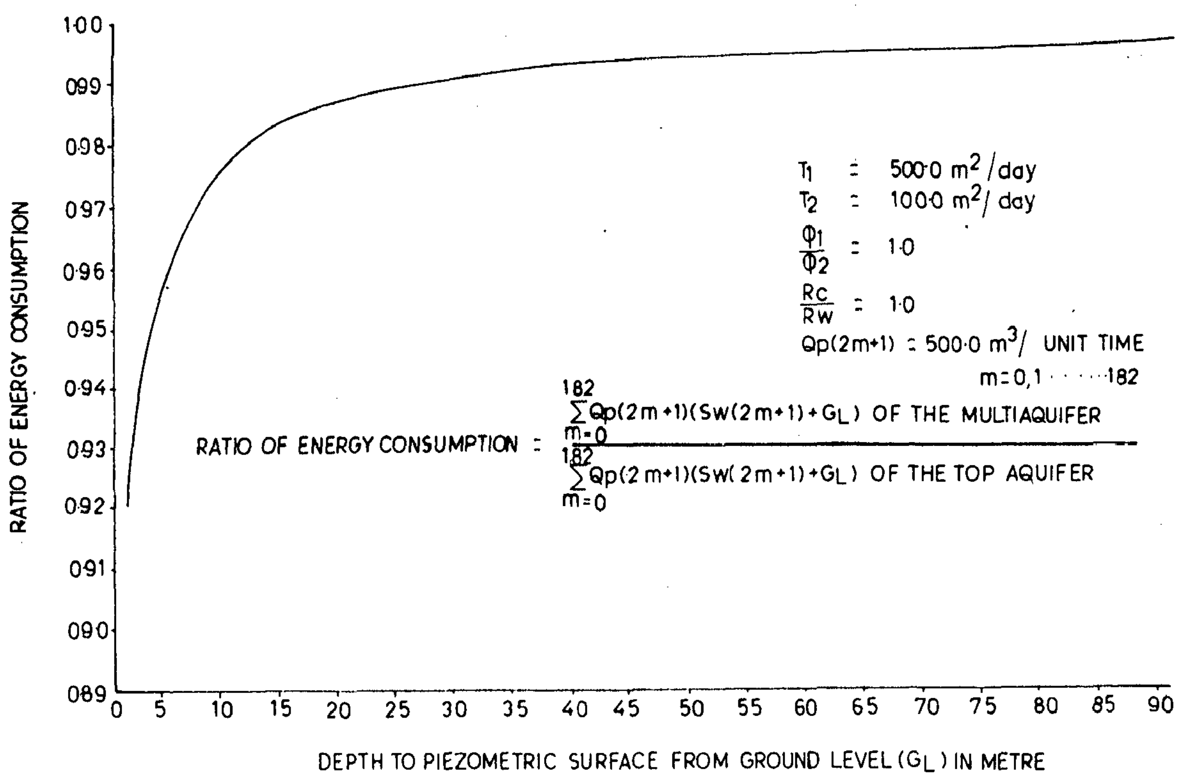

FIG 44(k) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

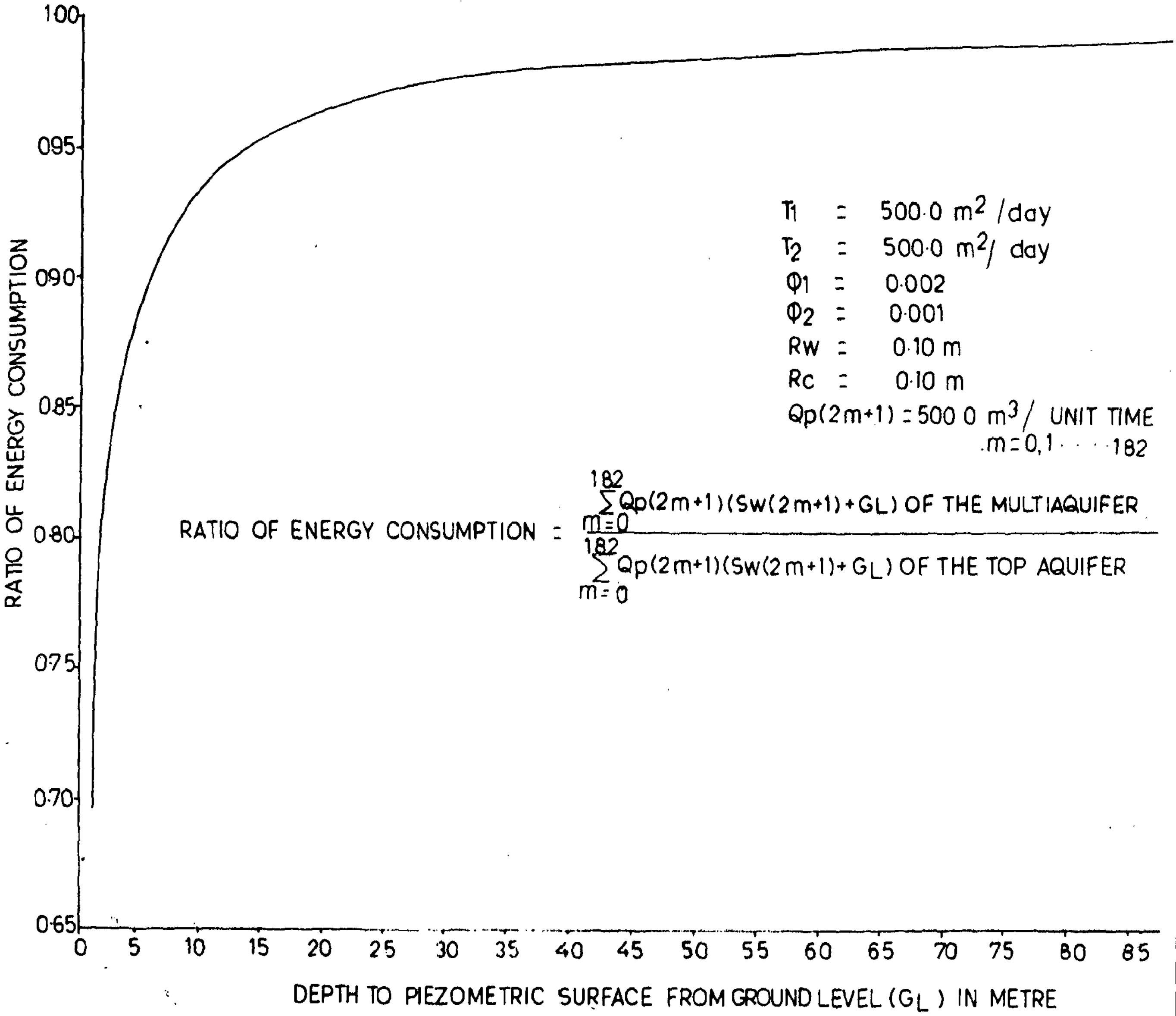

FIG 4.4(I) VARIATION OF RATIO OF ENERGY CONSUMPTION WITH GROUND LEVEL

CHAPTER 5

CONCLUSION

In the present study using discrete kernel approach flow to multiaquifer wells has been analysed. Two cases have been considered -- well with storage and well without storage. Based on the study the following conclusions have been arrived :

1. For shallow aquifer i.e. when depth of piezometric surface is less than or equal to 5 m, with T_1/T_2 upto 5/2 the saving in energy is more than 7%.

2. When the piezometric surface is at a depth greater than 20 m from ground level the advantage with respect to saving of energy is negligible.

3. Well storage has no significant influence on energy saving.

REFERENCES

Abramowitz,M. and I.A. Stegun. 1970. Handbook of Mathematical Functions. Dover Publications,Inc.,New York.

Abu-Zied, M.A. and V.H. Scott (1963), 'Nonsteady flow for wells with decreasing discharge', Proc. Am. Soc. Civil Engrs., 89(Hy3), pp. 119 - 132.

Carslaw, H.S. and J.C. Jaeger. 1959. Conduction of Heat in Solids. Oxford University Press, New York.

Hantush, M.S. 1964, 'Drawdown around wells of variable discharge', J.Geophys.Res., 69 , pp. 4221-4235.

Jacob, C.E. and S.W. Lohman 1952, 'Nonsteady flow to a well of constant drawdown in an extensive aquifer', Am.Geophys. Union, 33, pp.559 - 569.

Khader,A. and M.K. Veerankutty. 1975. 'Transient well flow in an unconfined - confined aquifer system'. Journ. Hydrology., Vol.26, pp. 123 - 140.

Lai, R.Y.S. and Cheh-Wu Su., 1974, 'Nonsteady flow to a large well in a leaky aquifer', Journal of hydrology, Vol.22, pp. 333 - 345.

Mishra, G.C. 1985. 'Unsteady flow to a multiaquifer flowing well', Report No.TN-9,National Institute of Hydrology,Roorkee.

Mishra,G.C. and A.G. Chachadi. 1985. 'Analysis of Unsteady flow to a large-diameter well'.Proceedings, International Workshop on Rural Hydrogeology and Hydraulics in Fissured Basement Zones,University of Roorkee.

Nautiyal,M.D. 1984, Flow to a well in multiaquifer system Ph.D. Thesis submitted to University of Roorkee, (UP), India.

Morel-Seytoux, H.J. 1975a. Optimal legal conjunctive operation of surface and ground water. Proc. Second World Congress, International Water Resources Assoc., New Delhi. Vol.4, pp. 119 - 129.

Morel-Seytoux, H.J. and C.J. Daly. 1975. A discrete kernel generator for stream aquifer s tudies. Water Resources Res., Vol.11, pp. 253 - 260.

Papadopulos, I.S. 1966. Non-steady flow to multiaquifer well. Journ. Geophys. Res., Vol.71, pp.4791-4767.

Papadopulos, I.S. and H.H. Cooper, Jr.1967. Drawdown in a well of large diameter. Water Resources Res., Vol.3, pp. 241 - 244.

Rushton, K.R. and S.M. Holt, 1981. Estimating aquifer parameters for large-diameter well. Ground Water Vol.19, pp. 505 - 516.

Rushton, K.R. and V.S. Singh. 1983. Drawdown in large-diameter wells due to decreasing abstraction rates. Ground Water, Vol.21, pp. 670-679.

Schapery, R.A. 1962. Approximate methods of transform inversion for viscoelastic stress analysis. Proc. 4th U.S. Nat. Congr. Appl. Mech., pp. 1075-1085.

Sokol, D. 1963. Position of fluctuation of water level in wells perforated in more than one aquifer. Journ. Geophys. Res., Vol.68, pp. 1079-1080.

Theis, C.V. 1935. The relation between the lowering of piezometric surface and rate and duration of discharge of a well using ground water storage. Am. Geophys. Union Trans., Vol.16, pp. 519-524.

APPENDIX - I

DISCRETE KERNELS FOR DRAWDOWNS AND ITS APPLICATION FOR VARIOUS PUMPING

The differential equation which describes the axially symmetrical, radial unsteady flow in an aquifer is

$$\frac{\partial^2 S_i}{\partial r^2} + \frac{1}{\gamma} \frac{\partial S_i}{\partial r^2} = \frac{\phi_i}{T_i} \frac{\partial S_i}{\partial t} \qquad ..(A-1)$$

where,

S_i = drawdown in piezometric surface in the i^{th} aquifer,

γ^2 = radial distance,

t = time,

ϕ_i = storage coefficient of i^{th} aquifer, and

T_i = transmissivity of i^{th} aquifer.

When unit impulse quantity of water is withdrawn from the i^{th} aquifer at time $t = 0$, the drawdown at time 't' at a radial distance 'n' from the well is given by

$$S_i(r,t) = \frac{1}{4\pi T_i} \frac{e^{-\frac{\phi n^2}{4\pi T_i t}}}{t - (A - 2)} \qquad ..(A-2)$$

for the initial condition

$$S_i(r,0) = 0,$$

and boundary condition

$$S_i(\infty, t) = 0,$$

The term $\frac{1}{4\pi T_i t} e^{-\frac{\phi_i r^2}{4 T_i t}}$ is designated by Morel-Seytoux as unit impulse kernel and is denoted by the coefficient $K_i(r,t)$. For an unit pulse excitation the drawdown after time 'm' at a distance 'r' is given by

$$\partial_i(m) = \int_0^1 \frac{1}{4 \pi T_i(m - c)} e^{-\frac{r^2}{4\pi T_i(m-c)}} dc$$

$$= \int_0^1 \frac{1}{4 \pi T_i(m - c)} e^{-\frac{r^2}{4 \beta (m-c)}} dc \quad ..(A-3)$$

where $\beta_i = \frac{T_i}{\phi_i}$ = diffusivity of the i^{th} aquifer

Let $\frac{r^2}{4\beta(m-c)} = x$

when $c = 0 \quad x = \frac{r^2}{4\beta m}$

$c = 1 \quad x = \frac{r^2}{4\beta(m-1)}$

$$dc = -\frac{r^2}{4\beta x^2} dx$$

Substituting for c and dc in equation (A-3)

$$\partial_i(m) = \int_{\frac{r^2}{4 \beta_i m}}^{\frac{r^2}{4\beta(m-1)}} \frac{1}{4\pi T_i} \frac{e^{-x}}{x} dx$$

$$= \frac{1}{4\pi T} \left[\int_{\frac{r^2}{4\beta m}}^{\infty} \frac{e^{-x}}{x} dx - \int_{\frac{r^2}{4\beta(m-1)}}^{\infty} \frac{e^{-x}}{x} dx \right]$$

$$= \frac{1}{4\pi T_i} \left[E_1 \left(\frac{r^2}{4\beta_i m}\right) - E_1 \left(\frac{r^2}{4\beta_i(m-1)}\right) \right] \quad ..(A-4)$$

Where $E_1(x)$ is exponential integral (Abramouitz and Stegun 1970) defined as

$$E_1(X) = \int_X^{\infty} \frac{e^{-X}}{X} dx$$

$\partial(m)$ has been designated by Morel Seytoux as discrete kernel coefficient which is the drawdown after the end of time step m at a distance r due to unit pulse excitation.

Drawdown at time 't' at a distance 'r' due to variable pumping is given by the following equation

$$S_i(r,t) = \int_0^1 Q_i(t) \, \partial c * \frac{1}{4\pi T_i(t-c)} e^{-\frac{r^2 \phi_i}{4T_i(t-c)}} \quad \text{--(A-5)}$$

Dividing time span into unit discrete time steps and assuming that the aquifer discharge is constant within each timestep, the drawdown at the end of n^{th} time step has been derived by Morel Seytoux (1975) as

$$S_i(r,n) = Q(1) \int_0^1 K_i(r,t-c)\, dc$$
$$+ Q(2) \int_1^2 K_i(r,t-c)\, dc + \ldots\ldots$$
$$+ Q(\gamma) \int_{\gamma-1}^{\gamma} K_i(r,t-c)\, dc + \ldots\ldots$$
$$+ Q(n) \int_{n-1}^{n} K_i(r,t-c)\, dc$$

Thus

$$S_i(r,n) = \sum_{\gamma=1}^{n} Q(\gamma) \int_{\gamma-1}^{\gamma} \frac{e^{-\frac{\gamma^2}{4\beta_i(n-c)}}}{4\pi\ T(n - c)}\ dc \qquad -(A-6)$$

with the substitution

$$c - \gamma + 1 = X$$

the equation (A-6) is simplified to

$$S_i(\imath,n) = \sum_{\gamma=1}^{n} Q\ (\gamma) \int_{0}^{1} \frac{e^{-\frac{r^2}{4\beta_i(n-X-\gamma+1)}}}{4\pi\ T_i(n-X-\gamma+1)}\ dx$$

$$= \sum_{\gamma=1}^{n} Q\ (\gamma)\ \ \partial_i\ (n - \gamma + 1)$$

APPENDIX - II

```
C   PROGRAM FOR FINDING ENERGY CONSUMPTION IN PUMPING
C          A SINGLE AQUIFER AND A MULTIAQUIFER

    DIMENSION DRW1(400),DRW2(400),QW(400),QP(400),A(10,10),B(10),
    1Q1(400),Q2(400),S1(400),S2(400),S3(400),DAQUI(400),
    2WELF1(400), U1(400),WELF2(400),U2(400),UP1(400),UP2(400)
    OPEN(UNIT=5,DEVICE='DSK',FILE='REK4,DAT')
    OPEN(UNIT=6,DEVICE=DSK,FILE=REK4,OUT)
    READ (5,77),T1,T2,PHI1, PHI2,RW,QP(1),MPUMPT,MFINAL,GLEVEL
    READ (5,78),RC
77  FORMAT(6F10.5,2I4,F10.5)
78  FORMAT(F10.5)
    WRITE (6,56)
56  FORMAT (5X,'T1',9X,'PHI1', 5X, 'T2',7X,'PHI2',7X,'RW',7X,
    1 QP(1),6X,MPUMPT,2X,MFINAL)
67  FORMAT (2X,6F10.5,2I8)
    WRITE(6,67)T1,PHI1,T2,PHI2,RW,QP(1),MPUMPT,MFINAL
    PAI=3.1415926
    DO 2 N=1,MFINAL
    AM=N
    CALL DPQ (AM,T1,PHI1,RW,DM)
2   DRW1(N)=DM
    CALL DPQ (AM,T2,PHI2,RW,DM)
    DRW2(N)=DM
    A(1,1)=DRW1(1)
    A(1,2)=-DRW2(1)
    A(1,3)=0.
    A(2,1)=1.
    A(2,2)=1.
    A(2,3)=1.
    A(3,1)=-DRW1(1)
    A(3,2)=0.
    A(3,3)=1./(PAI*RC*RC)
    N=3
    CALL MATIN (A,N)
C   TYPE*,'MATRIX INVERSION'
```

```
C  TYPE*,((A(J,I),I=1,3),J=1,3)
   B(1)=0.
   B(2)=QP(1)
   B(3)=0.
   Q1(1)=A(1,1)*B(1)+A(1,2)*QP(1)+A(1,3)*B(3)
   Q2(1)=A(2,I)*B(1)+A(2,2)*QP(1)+A(2,3)*B(3)
   QW(1)=A(3,1)*B(1)+A(3,2)*QP(1)+A(3,3)*B(3)
   DO 33 N=2,MFINAL
33 QP(N)=QP(1)
   DO 44 N=2,MFINAL,2
44 QP(N)=0.
   DO 55 N=2,MFINAL
   SUM1=0.
   SUM2=0.
   SUM3=0.
   JJ=N-1
   DO 66 JP=1, JJ
   SUM1=SUM1+Q1(JP)*DRW1(N-JP+1)
   SUM2=SUM2+Q2(JP)*DRW2(N-JP+1)
66 SUM3=SUM3+QW(JP)/(PAI*RC*RC)
   B(1)=SUM2-SUM1
   B(2)=QP(N)
   B(3)=SUM1-SUM3
   Q1(N)=A(1,1)*B(1)+A(1,2)*QP(N)+A(1,3)*B(3)
55 Q2(N)=A(2,1)*B(1)+A(2,2)*QP(N)+A(2,3)*B(3)
   QW(N)=A(3,1)*B(1)+A(3,2)*QP(N)+A(3,3)*B(3)
   DO 8 J=1, MFINAL
   SUM0=0.
   DO 9 JJ=1,J
9  SUM=SUM+Q1(JJ)*DRW1(J-JJ+1)
8  S1(J)=SUM
   DO 10 J=1, MFINAL
   SUM=0.
   DO 11 JJ=1,J
11 SUM=SUM+Q2(JJ)*DRW2(J-JJ+1)
10 S2(J)=SUM
```

```
      DO 20 J=1,MFINAL
      SUM=0.
      DO 21 JJ=1,J
 21   SUM=SUM+QW(JJ)/(PAI*RW*RW)
 20   S3(J)=SUM
      DO 12 J=1, MFINAL
      WELF1(J)=S1(J)*4.*PAI*T1/QP(1)
      WELF2(J)=S2(J)*4.*PAI*T2/QP(1)
      AJ=J
      U1(J)=WR*RW/(4.*T1/PHI1*AJ)
      U2(J)=RW*RW/(4.*T2/PHI2*AJ)
      UP1(J)=1./U1(J)
      UP2(J)=1./U2(J)
 12   CONTINUE
      WRITE (6,200)
200   FORMAT (2X,'Q1(N)',6X,'Q2(N)',6X,'QW(N)',6X,'S1(J)',
     16X,'S2(J)',6X,'S3(J)',6X,'WELF1(J)',3X,'U1(J),6X,
     2'UP1(J)',5X,'WELF2 (J)',3X,'U2(J)',6X,'UP2(J)')
      DO 34 N=1,MFINAL
      WRITE(6,7)Q1(N),Q2(N),QW(N),S1(N),S2(N),S3(N),WELF1(N),
     1U1(N),UP1(N),WELF2(N),U2(N),UP2(N)
 34   CONTINUE
 7    FORMAT(1X,12E11.4)
      DO 166 J=1,MFINAL
      SUM=0.
      DO 167 JJ=1,J
167   SUM=SUM+QP(JJ)*DRW1(J-JJ+1)
166   DAQU1(J)=SUM
      WRITE(6,172)
171   CONTINUE
      SUM1=0.
      SUM2=0.0
      DO 168 N=1,MFINAL
      SUM1=SUM1+QP(N)*(DAQU1(N)+GLEVEL)
168   SUM2=SUM2+QP(N)*(S1(N)+GLEVEL)
```

```
      RATIO=SUM2/SUM1
  172 FORMAT(10X,'SUM1',10X,'SUM2',10X,'RATIO',10X,'GLEVEL')
      WRITE(6,169)SUM1,SUM2,RATIO,GLEVEL
  169 FORMAT(4E16.7)
      GLEVEL=GLEVEL+1.
      IF(GLEVEL.LT.100.)GO TO 171
      STOP
      END

C     SUBROUTINE DPQ(AM,T,PHI,RW,DM)
      PAI=3.1415926
      CAPA=T/PHI
      X=RW*RW/(4.0*CAPA*AM)
      CALL EXI(X,EXFN)
      AA=EXFN
      IF(ABS(AM-1.0)-0.001)1,1,2
    2 X=RW*RW/(4.0*CAPA*(AM-1.0))
      CALL EXI(X,EXFN)
      DM=(AA-EXFN)/(4.0*PAI*T)
      GO TO 3
    1 EXFN=0.0
      DM=AA/(4.0*T*PAI)
    3 CONTINUE
      RETURN
      END
      SUBROUTINE EXI(X,EXFN)
      IF(X-1.0)1,1,22
    1 EXFN=-ALOG(X)-0.57721566+0.99999193*X-0.24991055*X**2+0.055199
     1X**3-0.00976004*X**4+0.00107857*X**5
      GO TO 3
   22 CONTINUE
      IF(X-81.)5,4,4
    5 CONTINUE
    2 EXFN=(((X**4+8.5733287*X**3+18.059017*X**2+8.6347608*X+0.26777
     1/(X**4+9.5733223*X**3+25.632956*X**2+21.099653*X+3.9584969))/
     2(X*EXP(X)))
```

```
      GO  TO 3
4     EXFN=0.
3     CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE MATIN(AUX,N)
      DIMENSION AUX(10,10),B(10),C(10)
      NN=N-1
      AUX(1,1)=1./AUX(1,1)
      DO 8 M=1, NN
      K=M+1
      DO  3 I=1,M
      B(I)=0.0
      DO  3 J=1,M
3     B(I)=B(I)+AUX(I,J)*AUX(J,K)
      D=0.0
      DO 4 I=1,M
4     D=D+AUX(K,I)*B(I)
      D=-D+AUX(K,K)
      AUX(K,K)=1./D
      DO 5 I=1,M
5     AUX(I,K)=-B(I)*AUX(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
6     C(J)=C(J)+AUX(K,I)*AUX(I,J)
      DO 7  J=1,M
7     AUX(K,J)=-C(J)*AUX(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
8     AUX(I,J)=AUX(I,J)-B(I)*AUX(K,J)
      RETURN
      END
```