ANALYSIS OF TEST PUMPING DATA FOR UNCONFINED AQUIFERS

A DISSERTATION

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in

HYDROLOGY

By

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CERTIFICATE

This is to certify that the dissertation entitled 'ANALYSIS OF TEST PUMPING DATA FOR UNCONFINED AQUIFERS' being submitted by Mr. A.M. Khaled in partial fulfilment of the requirement for award of the degree of Master of Engineering in Hydrology of the University of Roorkee, is a record of the candidate's own work carried out by him under my supervision and guidance. The material embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is to certify further that Mr. A.M. Khaled has worked for a period of more than four months from 1st July, 1984 for the preparation of this dissertation for Master of Engineering of this University.

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SYNOPSIS

Hydraulic characteristics of aquifers are essential to the understanding and solution of aquifer problems and the proper utilization of ground water resources.

For a reliable determination of these parameters the necessary data is obtained through field tests. Analysis of systematic observation of water level changes and of other test data yield values of the aquifer characteristics. The reliability of these analyses are dependent on several factors amongst the importants, one is the method of analysis. The most widely used method of analysis has been that of type curves which has certain limitations and also inherent subjective bias.

To include the effect of delayed yield (the slow draining behaviour of unconfined aquifers) as well as the effect of partial penetration which is the most encountered case in practice, because of the large number of dimensionless parameters which will come up in the drawdown expression, it would be impossible are at least greately difficult to prepare a set of type curves for the entire range of field application.

In the present work, based on one of the powerful theories of unconfined aquifers, a computer routine is prepared. For the calculation of aquifer parameters a non linear optimization scheme is proposed. The proposed scheme calculates the optimal values of the aquifer parameters based on the minimization of the sum of squares of the differences between the observed and calculated drawdowns.

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NOTATIONS

b	initial saturated thickness of the aquifer, L
b_D	Inverse dimensionless radius equal to b/r.
d	vertical distance between top of perforations and initial position of water table in pumping well, L
d_D	dimensionless d, equal to d/b
h	average head of the vertical section, L
h^o	height of free surface above the horizontal bed rock, L
I	net vertical specific rate of recharge, LT^{-1}
J_o	Bessel's function of the first kind and zeroth order
k_D	ratio of the vertical to horizontal permeability, equal to k_z/k_r
k_h	horizontal permeability, LT^{-1}
k_r	radial permeability, LT^{-1}
k_z	vertical permeability, LT^{-1}
ℓ	vertical distance between bottom of perforations and initial position of water table in pumping well, L
ℓ_D	dimensionless ℓ, equal to ℓ/b
n_r	component of unit outer normal in r direction
n_z	component of unit outer normal in z direction
q	recharge per unit area
Q	pumping rate, L^3T^{-1}
r	radial distance from pumping well, L
r_D	dimensionless r, equal to r/b
s	drawdown, L
S_c	storage coefficient ($= S_s b$)
S_s	specific (elastic) storage L^{-1}

S_{co}	instanteneous yield from storage per unit drawdown per unit horizontal area
S_{c1}	short term delayed yield from storage per unit drawdown per unit horizontal area
S_{c2}	long term delayed yield per unit drawdown per unit horizontal area
S_D	dimensionless drawdown, equal to, $4\pi Ts/Q$
S_n	corrected dradown, L
S^o	free surface drawdown, L
S_T	total effective storage coefficient $(=S_{co} + S_{c1} + S_{c2})$
S_y	specific yield
t	time from start of pumping
T	transmissibility, L^2T^{-1}
t_s	dimensionless time with respect to $S_c(= Tt/r^2S_c)$
t_y	dimensionless time with respect to $S_y(=Tt/r^2S_y)$
u	excess pore water pressure
V(J)	equavalent vertical hydraulic resistance $(=3b/8r^2k_z)$
W	well function $(=S/(Q/4\pi T))$
X	variable of integration
z	vertical distance above bottom of aquifer, L
z_D	dimensionless elevation, equal to z/b
α	reciprocal of Boulton's delay index, T^{-1}
α_1	reciprocal of short term delay index
α_2	reciprocal of long term delay index
β	$k_D\, r^2/b^2$
β_1	$\alpha_1\, S_c b/k_v$
β_2	$\alpha_2\, S_c b/k_v$

η	$(S_c + S_y)/S_c$
η_1	$(S_{c1} + S_{co})/S_{co}$
η_2	$(S_{co} + S_{c1} + S_{c2})/(S_{co} + S_{c1})$
η_T	$\eta_1\ \eta_2$
μ	$\sqrt{k_z/k_h}$
ρ	$\mu\ r_D$
σ	ratio of the storage coefficient to the specific yield $(=S_c/S_y)$
σ_1	$\beta_1\ S_{c1}/S_T$
σ_2	$\beta_2\ S_{c2}/S_T$
τ	$k_z t/S_T b$

CHAPTER 1

INTRODUCTION

1.1 GENERAL

It has long been observed, that, when a well is pumped at steady rate in a homogeneous unconfined aquifer, the water bearing material during the early stages of pumping does not immediately yields up its water. The actual time drawdown curve takes the form of an elongated S, and violates markedly from Theis curve, specially during the intermediate times.

In 1955, Boulton for the first time gave a possible reason and the name of delayed yield from storage to this phenomenon. He presented a semiempirical mathematical model which was capable of reproducing all the three segments of the elongated S-shaped time drawdown curve. According to Boulton, this is the flow from the unsaturated zone above the falling water table which causes the delayed yield phenomenon. To take account of it, he introduced an empirical constant, α (reciprocal of the delay index), the physical meaning of which remained unknown.

As comment on the theory of Boulton, Brutsaert (1970) and Brutsaert et al. (1971) clearly indicated that the unsaturated cone of depression is insignificant and can not explain the phenomenon of delayed yield. Neuman (1972), wrote that Boulton's approach may lead to difficulties in practice, because his coefficient, α, being devoid of any apparent physical meaning and can not be guaranteed to remain constant.

He showed that in the absence of infiltration at the ground surface the entire delay process can be simulated merely by treating the water table as a moving material boundary (or free surface) and at the same time giving due consideration to the effect of elastic storage in the aquifer.

Streltsova (1972), basis her concept on the assumption of leakage when the vertical flow in an unconfined aquifer is taken into account. She notes that in her works, the problem of flow to the well is examined by transforming the two dimensional axisymmetric flow, dependent on the redial distance from the well and the vertical coordinate to a one dimensional statement. Therefore, the reduction to the form of one dimensional axisymmetric flow with the consideration of two heads (free surface and average) leads to the occurrence of a discontinuity of head on the surface of the well.

The water in the vicinity of the well moves downward and causes vertical transfer or leakage at a variable rate that is proportional to the difference between the water table which gradually drops and the average head of the vertical section under consideration. This leakage will therefore be the cause of the diminishing rate of drainage, and the result will be a delay in the transient process of reestablishing equilibrium when a relatively uniform distribution of the head is approached, and the steady Theis theory can be applied with sufficient accuracy.

Boulton (1972) sees more practicability in the Streltsova's theory than in his own. Neuman (1976), on the theory of Streltsova stated that :

> " Streltsoua (1972) partly with the collaboration of Rushton (Streltsova and Rushton, 1973), was able to develop approximate solutions for the fall of water tables, as well as for the average drawdown over the entire aquifer thickness, in response to a fully penetrating well discharging at a constant rate."

Then he tries to make a connection between his own way and that of Steltsova's:

> " Her model has some conceptual simitiarities to ours because the unsaturated zone is neglected (however she included this effect in her model later*), and water is released from storage by compaction of the aquifer material, expansion of water, and gravity drainage at phreatic surface"

A survey of the models presented by Streltsova and Neuman shows that they are following nearly the same path in that, they both include the effect of the vertical gradients and neglect the effect of the unsaturated flow above the falling water table.

However, along with the presentation of models, the possible cause of delayed aquifer response is also stated by each proposer, still Bowuer and Rice (1978) feel that :

> " The physical basis for delayed yield or delayed water table response has received less attention than the mathematics of producing inflection type draw-down curves... ."

They see a possible reason for the delayed aquifer yield in the phenomenon of delayed air entry. They state, that because of the presence of the fine textured layers,

* Author

which will have higher water content than the surroundings or the presence of other saturated to nearly saturated layers in the vedose zone the downward movement of the air is subjected to restrictions. While withdrawing water through a well, the water table drops, but the atmospheric air can not enter the vedose zone to replace water that has drained from the pores at or above the water table, hence the air in the vedose zone will expand and as a result will cause a reduction in the air pressure. This means that the water table (as plane of atmospheric pressure) will drop more than the advance of the dewatered zone. Yielding a lower value of storage coefficient than its full value obtained when the lower boundary of the dewatered zone drops as fast as the water table. The initial storage coefficient continues to be small until the water table has dropped so much that the pressure head of the water in the saturated top layer reached air entry value* of the layer.

As another possible explanation to the phenomenon of delayed yield, Bouwer (1979), presented the theory of soil water hysteresis which is the relation between soil water content and negative pressure head of the soil water. According to him, this produces a lag in the release of pore water from a rewetted soil when the pressure heads are lowered. For an unconfined aquifer this means that the water table must

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* This is the negative value to which the water pressure head in a saturated material must drop before atmospheric air enters the material and displaces the water.

drop some distance in response to pumping a well before pore water is fully released if the water table prior to the pumping has been rising.

With all the above explanations and reasonings still I am feeling to accept Streltsova's statement that the slow draining phenomenon is of complex nature and only the physical properties of the aquifer and the particular conditions of the flow will determine which factor is predominent.

1.2 ON THE TOPIC

Quantitative data on hydraulic characteristics of aquifers are essential to the understanding and solution of aquifer problems and the proper evaluation and utilization of ground water resources. Field tests provide the most reliable method of obtaining these data. Such tests include the removal of water from a well and subsequent observation of the reaction of the aquifer to the change. Analysis of systematic observations of water level changes and of other test data yield values of the aquifer characteristics. The extent and reliability of these analysis are dependent on features of the test including duration of the test, number of observation wells and method of analysis. The most widely used method of the determination of aquifer parameters has been the use of type curves.

In the recent decades, extensive theoretical work has been done in the areas of unconfined aquifers. Various theories have been proposed and based on some of these theories type curves have been prepared. Unfortunately, uptill now in practice, for the analysis of test data of unconfined aquifers the type curves which were prepared by Boulton (1966) only for the fully penetrating wells are in use. However, he himself included in his theory the effect of partial penetration later.

In addition to the fact, that there is a large gap between the theory and practice, the method of analysis through the use of type curves, because of its graphical nature, has inherent subjective bias.

After a deep review, Neuman's theory was chosen as the basis of the work presented hereafter. His model which is based on the physically well defined parameters, accounts for the effect of partial penetration. As per the topic because of the following reasons a digital method of analysis of the test pumping data has been proposed.

1. The relatively large number of dimensionless parameters, in the Neuman's model, which makes it practically impossible to construct a sufficient number of type curves to cover the entire range of values necessary for field application.
2. To take care of the subjectivity which is inherent in the type curve method.
3. The fast growth of the use of digital computers in the area of ground water resources evaluation.
4. The ability of the computer to do the job which is impossible or at least laborious otherwise faster and economical.

Hence in the forth coming pages after the review of literature (Chapter 2) and Neuman's theory (Chapter 3), an algorithm is presented which orients Neuman's model for the computer assisted analysis of test pumping data.

CHAPTER 2

REVIEW OF LITERATURE

2.1 GENERAL VIEW

As early as 1935, Theis derived an equation for drawdown based on its similarity to the heat conduction equation. This equation which had the form :

$$s = \frac{Q}{4\pi T} \int_u^\infty (e^{-x}/x)\,dx \quad ; \quad u = \frac{r^2 S_c}{4Tt} \qquad \ldots(2.1)$$

was used for the analysis of unconfined aquifers; however, he himself pointed out that in the heat conduction equation a specific amount of heat is lost concommitantly and instantaneously with fall in temperature, but in nonartisian aquifers the water from the sediments through which the water table has fallen drains comparatively slowly. He in his derivation neglected this time lag which always caused some error in the analysis.

In 1954, Boulton derived an equation for drawdown in an unconfined aquifer. He started with the continuity equation for incompressible fluids and ended up with the following equation :

$$s = \frac{Q}{2\pi T} V(\rho,\tau) \qquad \ldots(2.2)$$

where

$$V(\rho,\tau) = \int_0^\infty \frac{J_0(\lambda\rho)}{\lambda} [1-\exp(-\tau\,\lambda \tanh\lambda)]\,d\lambda$$

$$\lambda = \beta b \; ; \; \tau = \frac{kt}{S_c b} \text{ and } \rho = r/b$$

He made the following assumptions for the above derivation :

(1) The aquifer is homogeneous, isotropic, infinite in extent and underlain by a horizontal impermeable bed.

(2) The well is unlined and fully penetrating.

(3) The coefficient of storage is constant.

(4) The flow obeys Darcy's law (K = const.)

(5) The water table is initially horizontal.

(6) The well is pumped at constant rate.

In 1955 for the first time Boulton introduced the term delayed yield as the cause of delayed water release from storage in an unconfined aquifer. Based on his 1955 paper, in 1966, he developed type curve procedure for the analysis of test pumping data with the consideration of the delayed yield.

In 1966, Kriz derived a relationship between the parameters of an unconfined aquifer by dimensional analysis. He stated that, when the ratio of drawdown to total hydraulic head in an unconfined aquifer is small, confined aquifer relation may be applied to unconfined aquifer transient flow problems. If this ratio is large, use of the method which does not count for the change in flow thickness about a well in an unconfined aquifer causes inaccuracies in the values determined for aquifer parameters. Hence, he claims that based upon the flow equation of an unconfined homogeneous, isotropic, infinite aquifer, he obtained a more general and less approximate method of the aquifer parameter determination. He started with the equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(hr\frac{\partial h}{\partial r}\right) = \frac{S_y}{k}\cdot\frac{\partial h}{\partial t} \qquad \ldots(2.3)$$

and ended up with the following new equation:

$$\frac{d^2w}{d\psi^2} + \left(\frac{1}{\psi} + \frac{1}{w} \right) \frac{dw}{d\psi} = 0 \qquad \ldots(2.4)$$

where:

$\lambda = h/b$; $w = \lambda^2$; $\psi = r^2/4\gamma t$ and $\gamma = \frac{kb}{S_y}$

The newly obtained equation has the following transformed boundary conditions:

$$\lim_{\psi \to \infty} \sqrt{w}\,(\psi) = 1 \qquad \ldots(2.5)$$

$$\lim_{\psi \to 0} \psi \frac{dw}{d\psi} = \frac{Q}{2\pi k b^2} \qquad \ldots(2.6)$$

Based on his model, he developed a type curve procedure for the test pumping analysis.

In 1969, Taylor and Luthin proposed a computer method for the transient analysis of water table aquifers. They stated that in analyzing drawdown for an unconfined aquifer, some imported parameters which are to be incorporated in the study are the relationships among water content (θ), the aquifer hydraulic conductivity (k), and the capiliary pressure head (H) of the unsaturated portion of the aquifer. The method they have presented takes into account the properties of the unsaturated portion of the aquifer and the contribution of vertical flow. Based on the following equations:

$$\frac{k_o}{r} \frac{\partial \phi}{\partial r} + \frac{\partial}{\partial r} \left(k_o \frac{\partial \phi}{\partial r}\right) + \frac{\partial}{\partial z} \left(k_o \frac{\partial \phi}{\partial z}\right) = 0 \text{ [saturated portion]} \qquad \ldots(2.7)$$

$$\frac{k}{r}\frac{\partial \phi}{\partial r} + \frac{\partial}{\partial r}(k\frac{\partial \phi}{\partial r}) + \frac{\partial}{\partial z}(k\frac{\partial \phi}{\partial z}) = \frac{\partial \theta}{\partial t} \quad \text{[unsaturated portion]} \qquad ...(2.8)$$

They prepared the finite difference schemes for the above equations, which could be solved with the help of the digital computers. The parameters of the above equations are defined as :

$\phi = P/\gamma + z$ (the hydraulic head)

$H = P/\gamma$ (the capiliary head)

θ is the water content which is related to the capiliary head by the relation :

$$\theta = \theta_o/(AH^3+1) \quad ; \quad A = \text{Constant} \qquad ...(2.9)$$

$$k = k_o/(AH^3+1) \qquad ...(2.10)$$

and k_o and k represents the permeabilities of the saturated and unsaturated zones respectively.

In 1971, Boulton extended his theory of delayed yield. In the extension, allowance for delayed yield involves four parameters as compared with two in the original theory. The pump and observations wells may penetrate the aquifer to any depth. The theory assumes that the aquifer and water are incompressible and that the drawdown of the water table is small. In the revised theory, he included the vertical component of the velocity of pore water approaching the well.

In 1972, Streltsova based her analysis of delayed water table response on the leakage owing to vertical hydraulic gradient, contrary to Boulton's analysis which was based on the allowance for delayed yield from storage. She assumed the aquifer to be compressible.

In the same year (1972), Neuman proposed an analytical model for the delayed yield process. He claimed that his approach differs from that of Boulton (1954, 1963, 1970) and Boulton and Pontin(1971)in that itis based only on well defined physical parameters of the aquifer system. Therefore, it provides a possible physical explanation for the mechanism of delayed water table response and eliminates the conceptual difficulties encountered with Boulton's theory of delayed yield from storage above the water table.

In 1973, Boulton published a paper in which he derived equations for the flow to a pumped well in an aquifer having uniform anisotropy and overlain by a low permeability aquitard. The water table is assumed to be located in the aquatard. Drainage from the capiliary zone above the water table is taken into account. Cooley and Case and many others claimed that the drainage from the unsaturated zone above a falling water table has only a minor effect on the flow in the aquifer. But, Boulton showed that the unsaturated and nearly saturated zones above a falling water table may be an important factor. The necessary condition is the existance of a stratum in the vicinity of the water table having much smaller permeability than the main aquifer. For the derivation of the relations the following assumptions were made :

(1) Aquitard is homogeneous and isotropic.
(2) Aquitard and the water contained in it are incompressible.
(3) The main aquifer is compressible and in general anisotropic.
(4) The flow in the aquitard is vertical.

(5) The pumped well completely penetrates the aquifer which is underlain by a horizontal impermeable layer.

(6) The well is pumped at constant rate.

(7) Radius of the well is small.

Starting with :

$$T\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) = S_c \frac{\partial s}{\partial t} - k_u \frac{\partial s_1}{\partial z} \qquad \ldots(2.11)$$

he ended up with the same equation as that of his 1955 paper, which is

$$T\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) = S_c \frac{\partial s}{\partial t} + \alpha\ S_y \int_0^t \frac{\partial s}{\partial \tau} e^{-\alpha(t-\tau)} .\ d\tau \ldots(2.12)$$

He has derived relations for the determination of the thicknesses of the unsaturated and nearly saturated zones above the water table.

In 1975, Neuman published a paper to show how his new theory (1972, 1973, 1974) can be applied for the determination of the hydraulic characteristics of unisotropic, unconfined aquifers from pumping test data. A distinction is made between the case in which the pumping well and the observation well are perforated throughout the saturated thickness of the aquifer and the case in which at least one of these wells is partially penetrating. A mathematical relationship between Boulton's delay index ($1/\alpha$) and other measurable physical parameters was derived. This relationship showed that contrary to the assumption of Boulton, α is not a characteristic constant of the aquifer, but decreases linearly with the logarithm of r, the radial distance from the pumped well.

In 1976, a numerical procedure for the test pumping analysis was proposed by Rushton and Chan. They started with the derivation of the model for confined aquifer and then modified their model to take care of unconfined aquifer, with the inclusion of the delayed yield. Their model which is a discrete space/discrete time model is based upon the following relation:

$$\frac{\partial}{\partial r}\left(bk_r \frac{\partial s}{\partial r}\right) + \frac{b}{r} k_r \frac{\partial s}{\partial r} = S_c \frac{\partial s}{\partial t} + q \qquad \ldots(2.13)$$

Assuming bk_r to be constant and introducing the variable $a = \ln(r)$, they prepared the discrete space/discrete time finite difference scheme for the above equation. In the initial part of derivation, the vertical component of flow was neglected. To orient the model for unconfined aquifer and include the vertical component of flow, proposals are made to add

$$m \frac{\partial v_z}{\partial z} = \frac{s_b - s_a}{V(J)} \; ; \; m = b \qquad \ldots(2.14)$$

which is based on the assumption that the vertical velocity of flow reduces linearly from a maximum at the free surface to zero at the base of the aquifer, to the left side of equation (2.13), and replace its right side by $(S_c + S_y) \frac{\Delta s}{\Delta \tau} + q$. These changes will take care of the vertical velocity as well as the delayed yield.

In the same year (1976), Streltsova published a commentary paper on the role of the flow from unsaturated zone and vertical flow components in draining unconfined formations. She stated, that it has been possible to show that the characteristic delayed drainage term in the general unconfined flow equation may be obtained on the basis of only the consideration of the vertical flow components, i.e., with an assumption that there is no delayed yield. Here she wrote that the slow draining phenomenon is of complex nature.

In 1978, Walton made a comprehensive analysis of water table aquifer test pumping data. After surveying all the work done in the area of unconfined aquifer, he reached the following conclusions:

(1) Specific yield is constant.

(2) Flow above the water table in the capiliary zone plays a negligible role in the response of a water table aquifer to pumping.

(3) Flow of water to a water table aquifer is intimately related to the anisotropy of the aquifer.

(4) Under water table conditions, ground water level initially declines with pumping an accordance with non-leaky artisian aquifer equations, the effective storage coefficient being equal to S_c. At large times, non-leaky artisian aquifer equations again apply, the effective storage coefficient now being equal to $(S_c + S_y)$. In both cases flow is substantially radial. During intermediate times ground water level decline are controlled by vertical component of flow. He accepted the following relations for the practical purpose of the test pumping data analysis:

$$s = \frac{Q}{4\pi T} W(U_A, U_B, \beta) \qquad \ldots(2.15)$$

where:

$U_A = \frac{r^2 S_c}{4Tt}$ (applicable for small values of time)

$U_B = \frac{r^2 S_y}{4Tt}$ (applicable for large values of time) and $$\beta = \frac{r^2 k_v}{b^2 k_h}$$

In same year (1978), Lakshminarayan and Rajagopalan published a paper on the digital model studies of unsteady state radial flow to partially penetrating wells in unconfined, unisotropic aquifers. They claims that their work is an improvement on an earlier numerical solution reported by Streltsova and Rushton (1973) and in which compressibility and anisotropy of the aquifer as well as partial penetration, have also been taken into account. They further claim that attention has been focused on the utility of the digital model for aquifer test data analysis. The basic equation on which they have based their analysis is:

$$k_r\left(\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r}\right) + k_z \frac{\partial^2 h}{\partial z^2} = S_s \frac{\partial h}{\partial t} \qquad \ldots(2.16)$$

They are characterizing different aquifer conditions by particular combinations of:

$$c_1 = k_r/k_z \,(b/r)^2 \quad \text{and} \quad c_2 = S_y/S_s b$$

which are reciprocal of the Neuman's β and σ. As far as the analysis of test pumping data is concerned, they are computing values of the head for the different trial values of S_s, S_y, k_z and k_r. Their termination criteria is the least difference between the observed head at a specific time and calculated head at that time.

In 1980, L. Rolfes presented a numerical method for the calculation of the average drawdown in a fully penetrating observation well in an unconfined aquifer. His numerical method is based on the Neuman's equation for the average drawdown in an observation well.

According to the literatures, there are three powerful theories on the transient unconfined flow with delayed yield: Boulton's, Streltsova's and Neuman's. The first two of these theories are presented in a bit of detail in the remainder portion of this chapter, but Neuman's theory which is the basis of our work is presented as a separate chapter.

2.2 BOULTON'S THEORY

As it was pointed out earlier also, Boulton (1955) for the first time introduced to the technical literature the term 'delayed yield', to explain the slow draining behaviour of water table aquifers. To take account of this phenomenon, he assumed that the drainage to the water table due to a lowering δs of water table between the times τ and $(\tau + \delta\tau)$ since pumping started consists of:

(i) a volume $S_c\,\delta s$ of water instantaneously released from storage per unit horizontal area; and

(ii) a delayed yield from storage per unit horizontal area at any time $t, (t > \tau)$ from the start of pumping

$$\delta s \;\alpha\; S_y\; e^{-\alpha(t-\tau)}$$

where α is an empirical constant. It follows that the total volume of delayed yield per unit drawdown and per unit horizontal area is :

$$\alpha\ S_y \int_t^{\infty} e^{-\alpha(t-\tau)} .\ d\tau = S_y \qquad \ldots(2.17)$$

and thus the total coefficient of storage is $S_c + S_y = \eta . S_c$. He concluded that the appropriate equation for drawdown in an unconfined aquifer is :

$$T\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} \right) = S_c \frac{\partial s}{\partial t} + \alpha\ S_y \int_0^t \frac{\partial s}{\partial t}\ e^{-\alpha(t-\tau)} . d\tau \qquad t = \tau \qquad \ldots(2.18)$$

where the last term on the right side denotes the rate of delayed yield per unit horizontal area at time t. For the special case when $\eta \longrightarrow \infty$, he found the following solution for the above equation

$$s = \frac{Q}{4\pi T} \int_0^{\infty} 2\ J_0[(r/B)x]\ [1 - \frac{1}{x^2+1}\ \exp(-\frac{\alpha t x^2}{x^2+1}) - \varepsilon]\ \frac{dx}{x} \qquad \ldots(2.19)$$

where:

$$\varepsilon = \frac{x^2}{x^2+1}\ \exp\ [-\alpha\ \eta\ t\ (x^2+1)] \qquad \ldots(2.20)$$

and ; $B = \sqrt{T/\alpha\ S_y}$

According to Boulton, α, is an empirical constant, the reverse of which $(1/\alpha)$ he called the delay index. Since he sees the reason for the delay in yield, in the flow of unsaturated zone above the falling water table; hence, there must be a specific time at which this delayed yield should cease to be effective. For the calculation of this time (t_o), he prepared a curve of r/B Vs. values of αt_o (Fig. (2.1). For the known or assumed values of r/B, one enters the curve reads the corresponding value of αt_o, knowing the value of α, t_o is calculated.

Boulton noted that the determination of formation constants through his procedure (type curve prepared on the

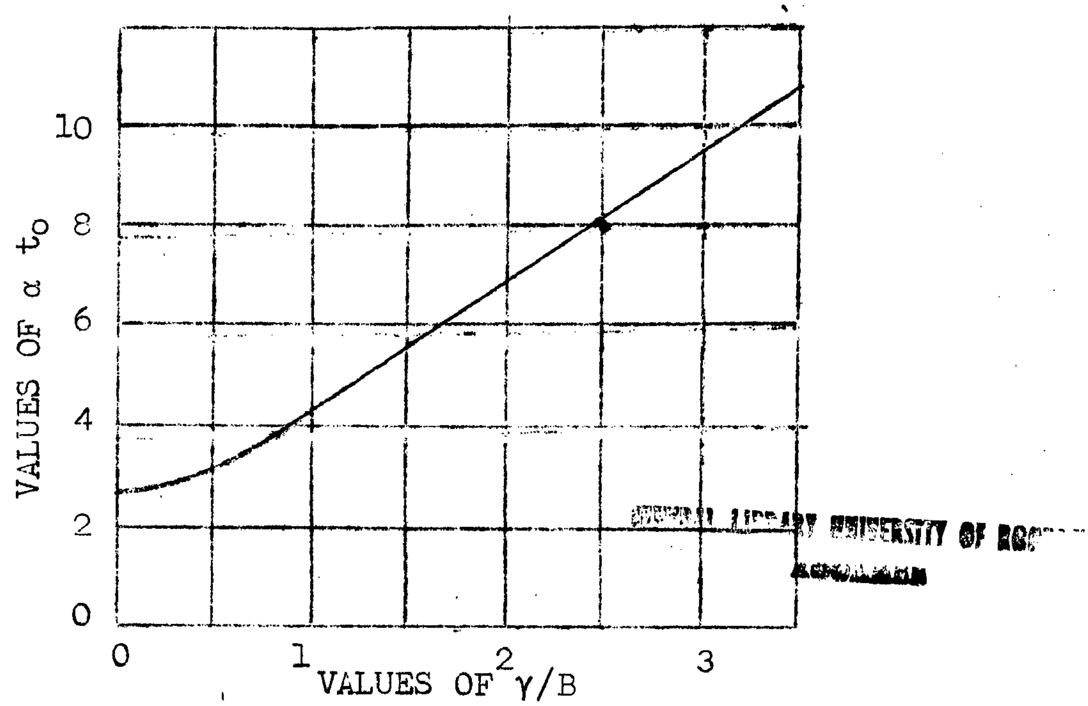


Fig. (2.1): Curve for Estimation Time, t_o, when Delayed Yield Ceases to Influence Drawdown

basis of equation (2.19)) allows for the apparent variation in the coefficient of storage with time during the early part of a pumping test at constant discharge, but does not predict the variation in the coefficient of storage with distance from the well which was observed by Walton and others.

The formarly presented theory which is identical with the Theis nonequilibrium theory, when the effect of delayed yield is negligible, ignores the vertical velocity component of the pore water approaching the well. This limitation may be important particularly in anisotropic aquifers having much greater horizontal than vertical permeability. Later (1971), he extended his theory of delayed yield to overcome the above limitation. His newly developed equation contains four parameters instead of two in the original theory. From laboratory tests carried at the University of Sheffield, U.K., he inferred that in the anisotropic beds of low vertical permeability immediately above the declining water table, the rate of drainage would be much smaller and would occur for a much longer time. Moreover in his extended work, he takes care of partial penetration also. Starting again with the equation of continuity of incompressible fluids

$$\nabla^2(\phi) = \frac{\partial^2\phi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\phi}{\partial\rho} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad \ldots(2.20)$$

where:

$\phi = \frac{u}{b\,\gamma_w} - 1$; and y is the depth of any point below undisturbed water table divided by b, which must be satisfied for

$0 < \rho < \infty$, $0 \leq y \leq 1$; and the following initial and boundary conditions :

$$\lim_{\rho \to 0} \left(\frac{\partial\phi}{\partial\rho} \right) = \frac{Q}{2\pi T b(\ell_D - d_D)} \quad ; \quad d_D \leq y \leq \ell_D$$

$$\frac{\partial\phi}{\partial y} = 0 \; ; \quad y = 1$$

$$\phi \to 0 \quad \text{as } \rho \to \infty \qquad 0 \leq y \leq 1$$

$$\phi = 0 \quad \text{when } t = 0 \text{ and } y = 0 \qquad 0 < \rho < \infty$$

he continues as under.

If V denotes the rate of drainage accorss unit horizontal area of the water table at time t, V is assumed to consist of two parts:

(i) a drainage rate at time t, $V_1 = -b\ S_{co}\ \partial\phi/\partial t$ due to water instantaneously released from storage; and

(ii) a drainage rate V_z due to delayed yield of water from storage. The part of V_z only due to lowering $-h\delta\phi$ of water table between time t' and t'+δt' is:

$$V_z = -b\,\delta\,\phi\ (\alpha_1\ S_{c1}\ e^{-\alpha_1(t-t')} + \alpha_2\ S_{c2}\ e^{-\alpha_2(t-t')}) \qquad \ldots(2.21)$$

where α_1 and α_2 are empirical constants. Assemblage of the above relations, will result in the following time dependent boundary equation :

$$\frac{\partial\phi}{\partial y} = \frac{1}{\eta_T}\frac{\partial\phi}{\partial\tau} + \sigma_1 \int_0^{\tau} \frac{\partial\phi}{\partial\tau}\Big|_{\tau=\tau'} e^{-\beta_1(\tau-\tau')}.d\tau' + \sigma_2 \int_0^{\tau} \frac{\partial\phi}{\partial\tau}\Big|_{\tau=\tau'}.e^{-\beta_2(\tau-\tau')}.d\tau' \qquad \ldots(2.22)$$

when y = 0

For the detailed derivation and solution of equation (2.20), under the above explained boundary and initial conditions, which results in an equation for the drawdown to account for partial penetration as well as the effect of the vertical velocity, the reader is referred to Boulton, 1971.

2.3 STRELTSOVA'S THEORY :

Streltsova is trying to present another possible explanation for the phenomenon of delayed aquifer drainage. This explanation is based on the allowance for leakage owing to vertical hydraulic gradients. These vertical gradients are assumed to be at a rate proportional to the difference of the mean and the free surface heads (drawdowns) of the aquifer. The duration of the existance of such leakage in unconfined aquifer since pumping began represents the transient process of reestablishing the equilibrium, i.e. the process of setting up a relatively uniform distribution of the head in the vertical direction. The isopiezometric surface approximates to vertical cylinderical surfaces and the Theis solution can be applied.

To explain the physical nature of delayed drainage, she states that when a well starts discharging, the elevation of water in the well suddenly drops and at the initial instants results in a discontinuity in head between the falling water table and the level of the water in the well. The water in the vicinity of the well having become suspended, starts moving downward due to gravity gradients. Therefore the flow to a a well, particularly during the early stages of pumping will be strictly three dimensional due to the considerable influence of the vertical gradients. This water moving downward causes leakage at a variable rate, proportional to the difference between the water table which gradually drops and the mean head of considered vertical surface. The leakage will therefore be the cause of the diminishing rate of drainage or the slow draini g of the soil around the well. She concludes that the vertically moving water adds to the horizontal flow and thus decreases the lowering rate of the water table.

The vertical gradients which causes the vertical flow is maximum in the vicinity of the well and as the distance from the well increases and also as the times keeps going on, these gradients and consequently the vertical flow keeps on decreasing and at some time becomes negligible. After that the contribution of the vertical flow ceased to be effective, the flow can essentially be considered horizontal and hence of the nearly uniform distribution of head.

She states that the problem of unsteady radial flow towards a discharging well tapping an unconfined aquifer of infinite extent and finite thickness, requires the solution of the following system of partial differential equations:

$$T\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) = S_c \frac{\partial s}{\partial t} + S_y \frac{\partial s_o}{\partial t} \qquad \ldots(2.23)$$

$$\frac{\partial s_o}{\partial t} = - k_z/S_y \cdot \partial s_o/\partial z \qquad \ldots(2.24)$$

with the initial conditions of :

$$s(r,0) = s_o(r,0) \qquad \ldots(2.25)$$

and boundary condition of :

$$Q = 2\pi kbr \frac{\partial s}{\partial r} \; ; \; (r = r_w \rightarrow 0) \qquad \ldots(2.26)$$

Through the use of finite difference approximation and the Laplace transform, the following generalized form of equation (2.23) was obtained :

$$T\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) = S_c \frac{\partial s}{\partial t} + \alpha\, S_y \int_0^t e^{-\alpha(t-\tau)} \cdot \frac{\partial s}{\partial \tau} \cdot d\tau \qquad \ldots(2.27)$$

where, α which is named as the vertical diffisivity of the aquifer and is the ratio of the specific vertical conductivity (k_z/b_z) and the specific yield (S_y) of the aquifer, characterizes the rate of free surface change.

Equation (2.27) is completely of the same form as the one obtained by Boulton (1955) for the unconfined flow with delayed yield. But it is to be noted that the meaning of the coefficients and parameters is completely different. The parameters is completely different. The parameter α is no more the constant reciprocal of the delay index.

In the above derivation the effect of the unsaturated flow is neglected and the whole delayed drainage process is explained only through the consideration of vertical component of flow. Later (1976), she started giving credit to the unsaturated flow also. She stated that the general differential equation for anisotropic water table aquifers, whose radial flow is augmented from above by an amount V_z per unit area is

$$S_c \frac{\partial s}{\partial t} = T\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) + v_z \qquad \ldots(2.28)$$

She found the general equation for v_z as :

$$v_z = \gamma\, S_y \int_0^t e^{-\gamma(t-\tau)} \left.\frac{\partial s}{\partial t}\right|_{t=\tau} \cdot d\tau \qquad \ldots(2.29)$$

where

$\gamma = \dfrac{\alpha\ \beta_v}{\alpha+\beta_v}$; and β_v is the vertical diffisuvity of the capiliary layer $\left(= \dfrac{k'}{S_y \Delta z}\right)$. The general equation of the unconfined flow with a capiliary surface taken into account is :

$$T\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) = S_c \frac{\partial s}{\partial t} + \gamma\, S_y \int_0^t e^{-\gamma(t-\tau)} \cdot \left.\frac{\partial s}{\partial t}\right|_{t=\tau} \cdot d\tau \qquad \ldots(2.30)$$

To be able to make a concluding comment on the above equation, lets consider the following relation :

$$\frac{1}{\gamma} = \frac{1}{\alpha} + \frac{1}{\beta_v} \qquad \ldots(2.31)$$

The above relation reveals a simple interpretation of the physical mechanism of the contribution of flow of the two zones to the main flow. Equation (2.31) actually represents the sum of the seepage resistances overcome by the flow in the vertical direction to augment the horizontal flow. For the solution of equation (2.30), one can consult Boulton (1971).

CHAPTER 3

NEUMAN'S THEORY

3.1 GENERAL

Walton (1960) observed that

" Three distinct segments of the time drawdown curve may be recognized under water table conditions. Unconfined stratified sediments often react to pumping for a short time after pumping begins, as would an artisian aquifer. Gravity drainage is not immediate but water is released instantaneously from storage by the compaction of the aquifer and its associated beds and by the expansion of the water itself. The second segment of the time drawdown curve represents the intermediate stage in the decline of water levels when the cone of depression slows in its rate of expansion as it is replanished by gravity drainage of sediments. The slope of time drawdown curve decreases as it reflects the presence of recharge in the form of interstitial storage in the vicinity of the pumped well. Test data deviates markedly from the nonequilibrium theory during the second segment which may start from several minutes to several days after pumping starts, depending largely upon aquifer conditions, represents the period during which the time drawdown curves conform closely to the nonequilibrium type curves".

Accepting the above findings of Walton, Neuman comments on the theory proposed by Boulton to explain the unusual behaviour of unconfined aquifers as follows :

Boulton's (1955) semiempirical model which is capable to reproduce all three segments of the time drawdown curve may lead to difficulties in practice, because his coefficient α, being devoid of any physical meaning, can not be guaranteed to remain constant. He (1963) himself conceeded, that although his method allows for the apparent variation in the coefficient of storage with time during the early part of a pumping test, does not predict the variation in the coefficient of storage which has been noted by Walton and other investigators. Neuman explains such inconsistancies between the model and the actual field data by the variable nature of α, which was assumed to be constant by Boulton.

Many investigators seek the reason for delay in yield in the unsaturated flow above the water table. Some of these investigators are : Youngs and Smiles, 1963; Vochand, 1968; Vauchad and Thony, 1969; dos Santos and Youngs, 1969; Youngs, 1969 and Cooley, 1971. These investigators are stressing on the importance of the unsaturated flow and hence claiming that the unsaturated flow plays a predominant role in the phenomenon of delayed yield. In 1971, Baester et al. wrote that many of these models give distorted picture which tends to exaggerate greatly the importance of the unsaturated flow.

Neuman (1972) introduced an analytical approach to flow in unconfined aquifers. This new approach is capable of reproducing all the three segments of the time drawdown curve without recourse to the unsaturated zone. Neuman's approach makes allowance for the vertical gradient and aquifer anisotropy. Since his approach applies to both rise and fall of water table; therefore, he replaced the term 'delayed yield' by the broader term of 'delayed water table response'.

3.2 THEORETICAL DEVELOPMENT

3.2.1 Full Penetration

The following assumptions are made in the theoretical development :

(1) The aquifer is infinite in lateral extent and lies on an impermeable horizontal layer.

(2) The aquifer material is homogeneous but anisotropic.

(3) The principal permeabilities are oriented parallel to the coordinate axes.

(4) Well is fully penetrating and is discharging at a constant rate.

(5) Well is taken as a line sink, which means neglecting the presence of a seepage face and the well storage.

Based on the stated assumptions, Neuman and Witherspoon (1970), found the following governing equation for the flow in an unconfined aquifer :

$$k_r \frac{\partial^2 s}{\partial r^2} + k_r \frac{1}{r} \frac{\partial s}{\partial r} + k_z \frac{\partial^2 s}{\partial z^2} = S_s \frac{\partial s}{\partial t} \qquad \ldots(3.1)$$

$$0 < z < \zeta$$

The boundary and initial conditions are :

$$s(r,z,0) = 0 \qquad \ldots(3.2)$$

$$\zeta(r,0) = b \qquad \ldots(3.3)$$

$$s(\infty,z,t) = 0 \qquad \ldots(3.4)$$

$$\frac{\partial s}{\partial z}(r,0,t)=0 \qquad \ldots(3.5)$$

$$k_r \frac{\partial s}{\partial r} n_r + k_z \frac{\partial s}{\partial z} n_z = (S_y \frac{\partial \zeta}{\partial t} - I)\, n_z \text{ at}(r,\zeta,t) \qquad \ldots(3.6)$$

$$\zeta(r,t) = b - s(r,z,t) \qquad \ldots(3.7)$$

$$\lim_{r \to 0} \int_0^{\zeta} r \frac{\partial s}{\partial r} \cdot dz = -\frac{Q}{2\pi k_r} \qquad \ldots(3.8)$$

Some of the parameters of the above equations are shown in Fig. (3.1).

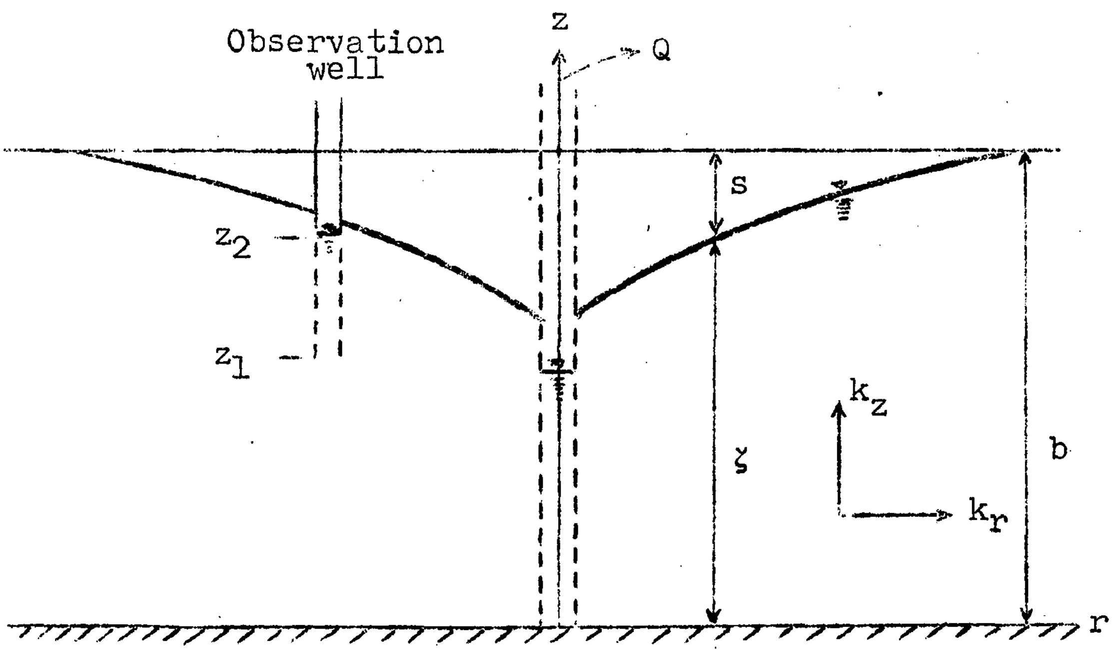


Fig. (3.1) : Schematic diagram of unconfined aquifer with fully penetrating pumped well

Equations ((3.1) - (3.8)) are approximately linearized by simply shifting the boundary condition from the free surface to the horizontal plane, z=b, which causes the elimination of ζ from the above mentioned equations. In this process it is assumed that the aquifer is thick enough and also the drawdown remains much smaller in comparison to ξ. In the absence of infiltration from equations ((3.1)-(3.8)), the following equations are obtained :

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + k_D \frac{\partial^2 s}{\partial z^2} = \frac{1}{\alpha_s}\frac{\partial s}{\partial t} \qquad 0 < z < b \quad ...(3.9)$$

$$s(r,z,0) = 0 \quad ...(3.10)$$

$$s(\infty,z,t) = 0 \quad ...(3.11)$$

$$\frac{\partial s}{\partial z}(r,0,t) = 0 \quad ...(3.12)$$

$$\frac{\partial s}{\partial z}(r,b,t) = -\frac{1}{\alpha_y}\frac{\partial s}{\partial t}(r,b,t) \quad ...(3.13)$$

$$\lim_{r \to 0} \int_0^b r \frac{\partial s}{\partial r}.dz = -\frac{Q}{2\pi k_r} \quad ...(3.14)$$

where

$\alpha_s = k_r/S_c$; and $\alpha_y = \frac{k_z}{S_y}$

After the application of Laplace and Hankel transforms to equations ((3.9)-(3.14)) and carrying on the mathematical simplifications,Neuman obtained the following first order approximation to the original initial boundary value problem in terms of five dimensionless parameters σ, z_D, b_D, k_D and t_s. The obtained solution is :

$$s(r,z,t) = \frac{Q}{4\pi T}\int_0^\infty 4V\, J_o(Vk_D^{1/2})\,[w_o(V)+\sum_{n=1}^{\infty} w_n(V)]\, dV \quad ...(3.15)$$

where

$$W_o(V) = \frac{\{1-\exp[-t_s k_D(V^2-\beta_o^2)]\}\cosh(\beta_o b_D z_D)}{\{V^2+(1+\sigma)\beta_o^2-[(V^2-\beta_o^2)^2 b_D^2/\sigma]\}\cosh(\beta_o b_D)} \quad ...(3.16)$$

$$W_n(V) = \frac{\{1-\exp[-t_s k_D(V^2+\beta_n^2)]\}\cos(\beta_n b_D z_D)}{\{V^2-(1+\sigma)\beta_n^2-[(V^2+\beta_n^2)^2 b_D^2/\sigma]\}\cos(\beta_n b_D)} \quad ...(3.17)$$

and β_o, β_n are the roots of the following equations :

$$\frac{\sigma}{b_D}\beta_o \sinh(\beta_o b_D)-(V^2-\beta_o^2)\cosh(\beta_o b_D]=0 \ ; \quad \beta_o^2 < V^2 \qquad \ldots(3.18)$$

$$\frac{\sigma}{b_D}\beta_n \sin(\beta_n b_D) + (V^2+\beta_n^2) \cos(\beta_n b_D)=0 \ ;$$
$$(n-\tfrac{1}{2})\ \pi < \beta_n\ b_D < n\pi \qquad \ldots(3.19)$$

Average drawdown in an observation well whose perforation extends from elevation z_1 to z_2 (Fig. (3.1)) is simply the average over that vertical distance and is given by

$$s_{z_1,z_2}(r,t) = \frac{1}{z_2-z_1}\int_{z_1}^{z_2} s(r,z,t)\ dz \qquad \ldots(3.20)$$

This drawdown could be calculated by the use of (3.15) and making use of the new expressions for W_o and W_n, which are obtained after the averaging process. For a fully penetrating observation well these expressions are :

$$s(r,z,t) = \frac{Q}{4\pi T}\int_0^\infty 4y\ J_o(y\beta^{1/2})[U_o(y)+\sum_{n=1}^{\infty} U_n(y)]dy \qquad \ldots(3.15a)$$

$$U_o(y) = \frac{\{1-\exp[-t_s\beta(y^2-\gamma_o^2)]\}\ \tanh(\gamma_o)}{\{y^2+(1+\sigma)\ \gamma_o^2-[(y^2-\ \gamma_o^2)/\sigma]\}\gamma_o} \qquad \ldots(3.21)$$

$$U_n(y) = \frac{\{1-\exp[-t_s\beta(y^2+\gamma_n^2)]\}\ \tan\ (\gamma_n)}{\{y^2\ -(1+\sigma)\ \gamma_n^2-[(y^2+\ \gamma_n^2)/\sigma]\}\gamma_n} \qquad \ldots(3.22)$$

where:

$\gamma_o = \beta_o b_D$; $\gamma_n = \beta_n\ b_D$; $\beta = k_D/b_D^2$

$U_o(Y) = 1/b_D^2\ W_o'(V)$; $U_n(y) = 1/b_D^2\ W_n'(V)$

$W_o'(V)$ and $W_n'(V)$ are the new forms of $W_o(V)$ and $W_n(V)$.

According to the above substitutions γ_o and γ_n are the roots of the equations:

$$\sigma\,\gamma_o\,\sinh(\gamma_o) - (y^2-\gamma_o^2)\,\cosh(\gamma_o) = 0\,; \qquad \gamma_o^2 < y^2 \qquad \ldots(3.18a)$$

$$\sigma\,\gamma_n\,\sin(\gamma_n) + (y^2+\gamma_n^2)\,\cos(\gamma_n) = 0\,; \qquad (n-\tfrac{1}{2})\pi < \gamma_n < n\pi \qquad \ldots(3.19a)$$

3.2.2 Partial Penetration

Neuman (1974) extended his formulation of unconfined flow to take into account the effect of partial penetration also. For the theoretical derivation, he started with equations ((3.1)-(3.8)), just with the addition of one more constraint and introducing a slight change in (3.8). If we take into consideration equations ((3.1)-(3.8)) and just rename them as ((3.1')-(3-8'), the new (3-8') and the one extra conztraints are as follows:

$$\lim_{r\to 0} \int_{b-\ell}^{\min(b-d,\xi)} r\cdot\frac{\partial s}{\partial r}\cdot dz = -\frac{Q}{2\pi k_r} \qquad \ldots(3.8')$$

$$\frac{\partial s}{\partial z}(0,z,t) = 0 \qquad 0 \leq z \leq b-\ell\ ;\ b-d \leq z \leq \xi \qquad \ldots(3.9')$$

It is to be noted that the assumptions made for the case of full penetration are considered in the case of partial penetration also. By simply shifting the boundary conditions from the free surface to the horizontal plane, z=b(Fig.(3.2)), equations((3-1')-(3.9')) could be approximated as :

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + k_D\frac{\partial^2 s}{\partial z^2} = \frac{1}{\alpha_s}\frac{\partial s}{\partial z} \quad ; \qquad 0 < z < b \qquad \ldots(3.10')$$

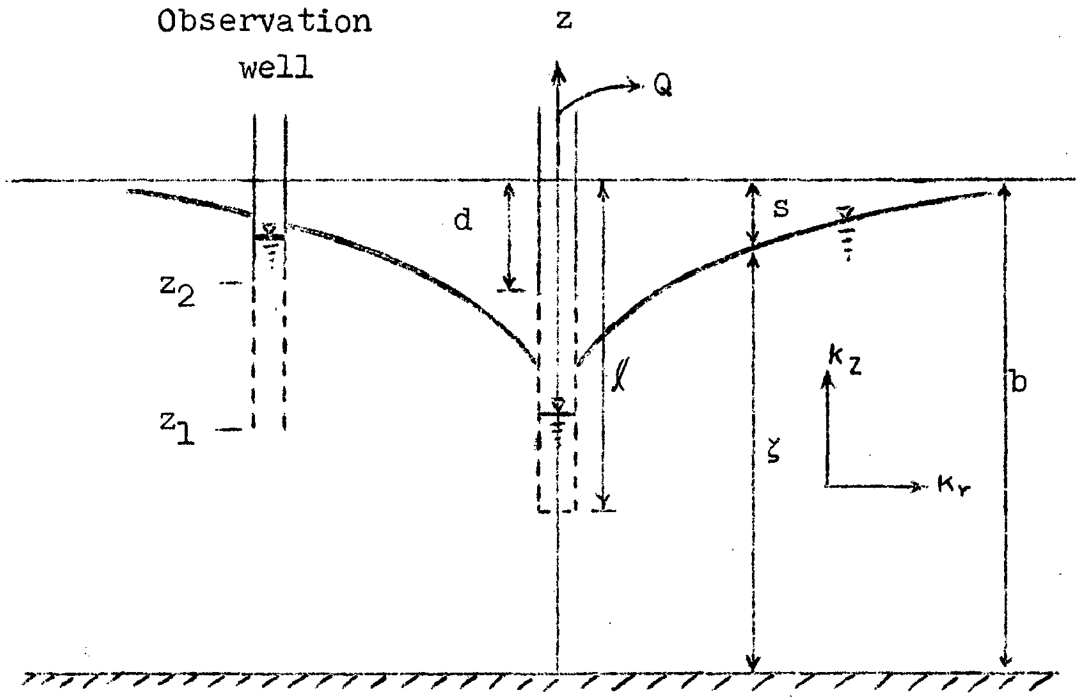


Fig. (3.2) : Schematic diagram of unconfined aquifer with partially penetrating wells

$$s(r,z,0) = 0 \qquad \ldots(3.11')$$

$$s(\infty,z,t) = 0 \qquad \ldots(3.12')$$

$$\frac{\partial s}{\partial z}(r,o,t)=0 \qquad \ldots(3.13')$$

$$\frac{\partial s}{\partial z}(r,b,t) = -\frac{1}{\alpha_y}\frac{\partial s}{\partial t}(r,b,t) \qquad \ldots(3.14')$$

$$\frac{\partial s}{\partial z}(0,z,t) = 0 \quad \text{at} \quad 0 \leq z \leq b-\ell \;;\; b-d \leq z \leq b \qquad \ldots(3.15')$$

a further approximation is introduced by assuming that flux along the perforated section of the well is uniform, and this changes (3.8') to the form :

$$\lim_{r \to o} r\frac{\partial s}{\partial r} = -\frac{Q}{2\pi k_r(\ell-d)} \quad \text{at, } b-\ell < z < b-d \qquad \ldots(3.16)$$

After the application of Laplace and Hankel's transforms to ((3.10')-(3.16')) and carrying on the necessary mathematical operations and simplifications, the following solution in terms of six dimensionless parameters : σ, β, z_D, ℓ_D, d_D, t_s or t_y was obtained as :

$$s(r,z,t) = \frac{Q}{4\pi T} \int_0^\infty 4\, y\, J_o(y\beta^{1/2})[U_o(y) + \sum_{n=1}^{\infty} U_n(y)]dy \quad \ldots(3.17')$$

where:

$$U_o(y) = \frac{\{1-\exp[-t_s\beta(y^2-\gamma_o^2)]\} \cosh(\gamma_o z_D)}{[y^2+(1+\sigma)\gamma_o^2-(y^2-\gamma_o^2)^2/\sigma] \cosh(\gamma_o)} \cdot \frac{\{\sinh[\gamma_o(1-d_D)]-\sinh[\gamma_o(1-\ell_D)]\}}{(\ell_D-d_D) \sinh(\gamma_o)} \quad \ldots(3.18')$$

$$U_n(y) = \frac{\{1-\exp[-t_s\beta(y^2-\gamma_n^2)]\} \cos(\gamma_n z_D)}{[y^2-(1+\sigma)\gamma_n^2 -(y^2+\gamma_n^2)^2/\sigma] \cos(\gamma_n)} \cdot \frac{\{\sin[\gamma_n(1-d_D)]-\sin[\gamma_n(1-\ell_D)]\}}{(\ell_D - d_D) \sin(\gamma_n)} \quad \ldots(3.19')$$

The terms γ_o and γ_n are the roots of the equations (3.18a) and (3.19a).

Average Drawdown in Observation Well

The same as in the case of full penetration, the average drawdown is obtained by simple averaging on the vertical distance of perforation between z_1 and z_2 (Fig.(3.2)). $S_{z1,z2}$ (equation (3.20)) can be calculated from (3.17'), just by redefining (3.18') and (3.19') in the following manner:

$$U_o(y) = \frac{\{1-\exp[-t_s\beta(y^2-\gamma_o^2)]\} [\sinh(\gamma_o z_{2D})-\sinh(\gamma_o z_{1D})]}{[y^2+(1+\sigma)\gamma_o^2-(y^2-\gamma_o^2)^2/\sigma] \cosh(\gamma_o)} \cdot \frac{\{\sinh[\gamma_o(1-d_D)]-\sinh[\gamma_o(1-\ell_D)]\}}{(z_{2D}-z_{1D})\, \gamma_o\, (\ell_D-d_D) \sinh(\gamma_o)} \quad \ldots(3.23)$$

$$U_n(y) = \frac{\left\{1-\exp[-t_s\beta(y^2+\gamma_n^2)]\right\}[\sin(\gamma_n z_{2D})-\sin(\gamma_n z_{1D})]}{[\ y^2-(1+\sigma)\gamma_n^2-(y^2+\gamma_n^2)^2/\sigma]\ \cosh\ (\gamma_n)}$$

$$\frac{\left\{\sin[\gamma_n(1-d_D)]-\sin[\gamma_n(1-\ell_D)]\right\}}{(z_{2D}-z_{1D})\ \gamma_n\ (\ell_D-d_D)\ \sin\ (\gamma_n)} \quad \ldots(3.24)$$

In 1964, Hantush derived the governing equation for flow to a partially penetrating well in a nonleaky artisian aquifer. This equation was later solved by Andrin Viscosky of the Illinois Stat Water Supply Department.

In 1967, Dagan presented a solution for the flow to a partially penetrating well in an incompressible unconfined aquifer. Through a numerical example Neuman compares his own solution in which the effect of delayed **water** table response is taken into the consideration, with that of Hantush (1964) for a confined elastic aquifer and Dagan (1967a,b) for an unconfined rigid aquifer. The result of this comparison in shown in Fig. (3.3).

The time drawdown curve in Fig. (3.3) suggests that water is released from storage in three stages, as discussed by Walton (1960). At the early values of time, the curve approached Hantush's solution and thus indicates that water is released from storage primarily by compaction of the aquifer material and expansion of the water. During the second stage, gravity drainage becomes important and its effect is similar to that of leakage from a nearby source. Neuman in 1972 showed that the smaller the σ, the larger the effect of gravity drainage is and therefore the more pronounced this leakage, is.

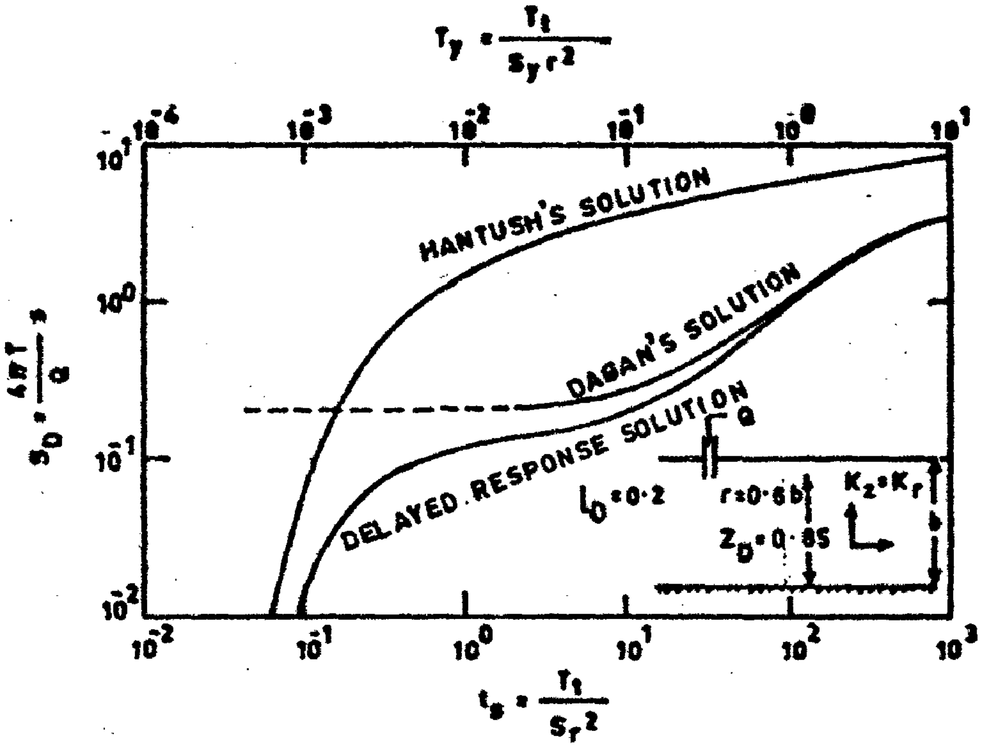


FIG. 3·3 - COMPARISON OF DELAYED RESPONSE SOLUTION WITH THE SOLUTIONS OF HANTUSH AND DAGAN FOR $\sigma = 10^{-2}$, $\beta = 0{\cdot}36$, $l_D = 0{\cdot}2$, $d_D = 0$, AND $z_D = 0{\cdot}85$.

From Fig. (3.3) it can be concluded, that at early stages of time the delayed response solution approaches the Hantush's solution and as the time increases, the effect of elastic storage at a point under consideration dissipates completely, hence, at later times, the delayed response solution approaches that of Dagan's solution. In other words, Hantush's solution becomes envelop at early times and Dagan's solution becomes envelp at later times.

3.3 RELATION BETWEEN BOULTON'S DELAY INDEX (1/α) AND AQUIFER CHARACTERISTICS

Neuman (1975) has developed the following relation between the Boulton's α, and aquifer parameters :

$$\alpha = \frac{k_z}{S_y\ b}\left[3.063 - 0.567 \log\left(\frac{k_D r^2}{b^2}\right)\right] \qquad ...(3.25)$$

He claims that the above relation has a very high correlation coefficient. ($\rho^2 = 0.99$).

3.4 ADJUSTMENT FOR THE DECREASED SATURATED THICKNESS OF THE AQUIFER

Derivation of (3.15) is based on the assumption that the decline of the water table remains small in comparison to the unsaturated thickness of the aquifer. For cases where this is not so, Jacob (1944) recommended that prior to analysis of pumping test data the drawdowns be corrected according to :

$$s_n = s - (s^2/2b) \qquad \ldots(3.26)$$

Equation (3.26) was recommended to be used for such a purpose by Walton(1978) also. But Neuman (1975) is making the following comment on it :

Equation (3.26) was derived by adopting the Dupit assumptions and in particular by assuming that the drawdowns along any vertical are always equal to the drawdown of the water table S_{WT}. It is evident that the Dupit assumptions do not hold in an unconfined aquifer with delayed gravity response as long as the drawdown data do not fall on the late Theis curve. This means that Jacob's correction scheme is strictly applicable only to the late drawdown data and is not applicable to the early and intermediate data. It is therefore recommended that (3.26) be used only in the determination of T and S_y from the late drawdown data and not in the determination of β, T and S from the early and intermediate data.

CHAPTER 4

MODEL'S NUMERICAL ORIENTATION

4.1 GENERAL

As it has been presented in the previous chapter, Neuman (1972) proposed an analytical model for the flow to completely penetrating wells and later (1974), he modified his previous model for the flow to partially penetrating wells in an unconfined aquifer with the consideration of the delayed aquifer response. He derived for both of the cases expressions for the average drawdown in the observation wells. Since full penetration is a special case of partial penetration; hence, the work which will be presented here is based upon his expression for drawdown in the case of partially penetrating wells.

Although the expression for the average drawdown is given in a closed form, but still some difficulties like those stated below arises in its numerical evaluation:

(1) The calculation of an infinite integral

(2) The summation of an infinite series

(3) The instability in some of the expressions; and

(4) The difficulty with the machine because of the much higher values of some of the terms for some critical values of the parameters.

In present chapter, step by step, it is shown that how these difficulties are partially or fully removed.

4.2 NUMERICAL METHOD

Let's once again bring to view the expression for the average drawdown in a partially penetrating observation well:

$$s(r,t) = \frac{Q}{4\pi T} \int_0^\infty 4\, y\, J_o(y\beta^{1/2})[U_o(y) + \sum_{n=1}^{\infty} U_n(y)]\, dy \qquad \ldots(4.1)$$

where

$$U_o(y) = \frac{\left\{1-\exp[-t_s\beta(y^2-\gamma_o^2)]\right\}[\sinh(\gamma_o z_{2D})-\sinh(\gamma_o z_{1D})]}{[y^2 + (1+\sigma)\ \gamma_o^2 - (y^2-\gamma_o^2)^2/\sigma]\ \cosh(\gamma_o)} \cdot \frac{\left\{\sinh[\gamma_o(1-d_D)] - \sinh[\gamma_o(1-\ell_D)]\right\}}{(z_{2D} - z_{1D})\ \gamma_o(\ell_D-d_D)\ \sinh(\gamma_o)} \qquad \ldots(4.2)$$

$$U_n(y) = \frac{\left\{1-\exp[-t_s\beta(y^2+\gamma_n^2)]\right\}[\sin(\gamma_n\ z_{2D})-\sin(\gamma_n z_{1D})]}{[\ y^2 - (1+\sigma)\ \gamma_n^2 - (y^2-\gamma_n^2)^2/\sigma\ ]\ \cos(\gamma_n)} \cdot \frac{\left\{\sin[\gamma_n(1-d_D)] - \sin\ [\gamma_n(1-\ell_D)]\right\}}{(z_{2D}-z_{1D})\ \gamma_n\ (\ell_D- d_D)\ \sin\gamma_n} \qquad \ldots(4.3)$$

and, γ_o and γ_n are the roots of the equations :

$$\sigma\ \gamma_o\ \sinh(\gamma_o)-(y^2-\gamma_o^2)\ \cosh(\gamma_o) = 0\ ; \qquad \gamma_o^2 < y^2 \qquad \ldots(4.4)$$

$$\sigma\ \gamma_n\ \sin\ (\gamma_n)+(y^2+\gamma_n^2)\ \cos\ (\gamma_n) = 0\ ; \quad (2n-1)\pi/2 < \gamma_n < n\pi \qquad \ldots(4.5)$$

To obtain s, (4.1) has to be evaluated. Evaluation of (4.1) for given values of σ, β, t_s, z_{1D}, z_{2D}, ℓ_D and d_D implies the numerical integration of the integrand :

$$4y\ J_o(y)\quad U_o(y) + \sum_{n=1}^{\infty} U_n(y) \qquad \ldots(4.6)$$

over the interval (0, ∞], which in turn implies the evaluation of (4.6) for many values of y , theoretically ranging from zero to infinity. To evaluate (4.6) for specific values of (y) we proceed as follows.

4.2.1 Calculation of γ_o and γ_n :

For a particular value of (y), the values of γ_o and γ_n are to be determined through Newton-Raphson iteration method. Equations to be solved are (4.4) and (4.5).

Iteration schemes are :

$$\gamma_o^{(k+1)} = \gamma_o^{(k)} - \frac{f(\gamma_o^{(k)})}{'f(\gamma_o^{(k)})} \text{;with } \gamma_o^{(o)} = y \qquad \ldots(4.7)$$

and

$$\gamma_n^{(k+1)} = \gamma_n^{(k)} - \frac{f(\gamma_n^{(k)})}{'f(\gamma_n^{(k)})} \text{;with } \gamma_n^{(o)} = (n-\tfrac{1}{2})\pi \qquad \ldots(4.8)$$

<u>OR</u> according to Rolfes (1980) :

$$\gamma_o^{(k+1)} = y - \frac{\sigma\,\gamma_o^{(k)}\ \tanh(\gamma_o^{k})}{y + \gamma_o^{(k)}} \text{ ; with } \gamma_o^{(o)} = y \qquad \ldots(4.9)$$

and

$$\gamma_n^{(k+1)} = (n-\tfrac{1}{2})\pi + \arctan\left(\frac{\sigma\,\gamma_n^{(k)}}{y^2+(\gamma_n^{(k)})^2}\right) \qquad \ldots(4.10)$$

$$\text{with } \gamma_n^{(o)} = (n-\tfrac{1}{2})\,\pi$$

where $k = 0, 1, 2, 3 \ldots\ldots$

As far as the determination of γ_n is concerned both of the methods could safely be used, but in the case of γ_o, there is some difficulty with Newton-Raphson scheme, because for higher values of γ_o (which are unavoidable in the calculations) the terms $\sinh(\gamma_o)$ and $\coh(\gamma_o)$ will blow up beyond the ability of the machine. Hence, in the case of γ_o, Rolfes procedure is recommended. Moreover it was noted that Newton-Raphson method is efficient for larger values of $\sigma(\geq 1)$ and Rolfes method is efficient for smaller values of σ.

4.2.2 New Forms of U_o and U_n :

Application of the transformation $x = \beta^{1/2}y$, to equations (4.2) and (4.3) results in :

$$U_o(y) = \beta\ \hat{U}_o(x) \quad ; \quad \text{and}$$

$$U_n(y) = \beta\ \hat{U}_n(x)$$

where :

$$\hat{U}_o(x) = \frac{\left\{1-\exp[-t_s(x^2-\beta\gamma_o^2)]\right\}[\sinh(\gamma_o z_{2D})-\sinh(\gamma_o z_{1D})]}{[x^2+(1+\sigma)\ \beta\ \gamma_o^2 - (x^2-\beta\ \gamma_o^2)^2/\sigma\beta]\ \cosh(\gamma_o)} \cdot \frac{\left\{\sinh[\gamma_o(1-d_D)] - \sinh[\gamma_o(1-\ell_D)]\right\}}{(z_{2D}-z_{1D})\ \gamma_o\cdot\sinh(\gamma_o)\cdot(\ell_D-d_D)} \quad \ldots(4.11)$$

$$\hat{U}_n(x) = \frac{\left\{1-\exp[-t_s(x^2+\beta\gamma_n^2)]\right\}[\sin(\gamma_n z_{2D})-\sin(\gamma_n z_{1D})]}{[x^2-(1+\sigma)\ \beta\ \gamma_n^2-(x^2+\beta\gamma_n^2)^2/\sigma\beta]\ \cos(\gamma_n)} \cdot \frac{\left\{\sin[\gamma_n(1-d_D)]-\sin[\gamma_n(1-\ell_D)]\right\}}{(z_{2D}-z_{1D})\gamma_n\cdot\sin(\gamma_n)\cdot(\ell_D-d_D)} \quad \ldots(4.12)$$

The above transformation ,changes (4.4) and (4.5) into the forms:

$$\beta\ \sigma\ \gamma_o\ \sinh(\gamma_o)-(x^2-\beta\ \gamma_o^2)\ \cosh(\gamma_o) = 0 \ ; \quad \gamma_o^2 < \frac{x^2}{\beta} \quad \ldots(4.13)$$

and

$$\beta\ \sigma\ \gamma_n\ \sin(\gamma_n) + (x^2+\beta\gamma_n^2)\ \cos(\gamma_n) = 0; \quad (n-\tfrac{1}{2})\pi < \gamma_n < n\pi \ ; \ n\geq 1 \quad \ldots(4.14)$$

4.2.3 Evaluation of $\hat{U}_o(x)$:

In relation (4.11), for small values of t_s, the expression $\exp[-t_s(x^2-\beta\gamma_o^2)]$, will have a value near to 1, hence upon subtraction from 1 computer will commit error. This instability will be removed as follows :

From (4.13) we have :

$$x^2 - \beta\ \gamma_o^2 = \sigma\ \beta\ \gamma_o\ \tanh(\gamma_o)$$

substituting the value of $(x^2-\beta\gamma_o^2)$ in equation (4.11), we will obtain the expression :

$$\hat{U}_o(x) = \frac{\{1-\exp[-t_s(\sigma\beta\ \gamma_o\ \tanh\gamma_o)]\}[\sinh(\gamma_o z_{2D})-\sinh(\gamma_o z_{1D})]}{\{x^2+(1+\sigma)\ \beta\ \gamma_o^2-[\sigma\beta\ \gamma_o\ \tanh(\gamma_o^2)]^2/\sigma\ \beta\}\cosh(\gamma_o)}$$

$$\cdot\ \frac{\{[\sinh[\gamma_o(1-d_D)]-\sinh[\gamma_o(1-\ell_D)]\}}{(z_{2D}-z_{1D})\ \gamma_o\ (\ell_D-d_D)\ \sinh(\gamma_o)} \qquad \ldots(4.11')$$

Now (4.11'), could be written in the following numerically stable form :

$$\hat{U}_o(x) = A(x)\cdot A_1\cdot A_2\cdot A_3 \qquad \ldots(4.11'')$$

where

$$A(x) = 2\sinh\left[\frac{t_s\ \sigma\ \beta\ \gamma_o\ \tanh(\gamma_o)}{2}\right]\cdot\exp\left[\frac{-t_s\sigma\ \beta\ \gamma_o\ \tanh(\gamma_o)}{2}\right]$$

$$/[\ x^2 + (1+\sigma)\ \beta\ \gamma_o^2 - \sigma\ \beta\ \gamma_o^2\ \tanh^2(\gamma_o)] \qquad \ldots(4.11''a)$$

$$A_1 = \frac{\sinh(\gamma_o z_{2D}) - \sinh(\gamma_o z_{1D})}{\cosh(\gamma_o)} \qquad \ldots(4.11''b)$$

$$A_2 = \frac{\sinh[\gamma_o(1-d_D)]-\sinh[\gamma_o(1-\ell_D)]}{\sinh(\gamma_o)} \qquad \ldots(4.11''c)$$

$$A_3 = \frac{1}{(z_{2D}-z_{1D})\ \gamma_o(\ell_D-d_D)} \qquad \ldots(4.11''d)$$

As far as machine is concerned, there are still some difficulties with the evaluation of $U_o(x)$. To remove these difficulties, we follow like under :

1. Approximation of $A(x)$

Let's consider the old form of $A(x)$

$$A(x) = \frac{1-\exp[-t_s\ \sigma\ \beta\ \gamma_o\ \tanh(\gamma_o)]}{x^2+(1+\sigma)\beta\ \gamma_o^2 - \sigma\ \beta\ \gamma_o^2\ \tanh^2(\gamma_o)} \qquad \ldots(4.11''a)$$

and let

$$CX = t_s\ \sigma\ \beta\ \gamma_o\ \tanh(\gamma_o)$$

for smaller values of CX (approx. ≤ 80), relation (4.11''a) can be used, but for larger values of CX, machine is having difficulty with the evaluation of the exp term containing this expression. To remove this problem, for CX > 80, the exponential term could safely be dropped from (4.11''a) and its following form could be used :

$$A(x) = \frac{1}{x^2+(1+\sigma)\ \beta\ \gamma_o^2 - \sigma\ \beta\ \gamma_o^2\ \tanh^2(\gamma_o)} \qquad \ldots(4.15)$$

2. Approximation of A_1, A_2 and A_3

As far as A_3 is concerned, there is no difficulty with its numerical evaluation, but with A_1 and A_2 we have the problem that for higher values of $\gamma_o(>80)$, computer can't handle the values of $\sinh(\gamma_o)$ and $\cosh(\gamma_o)$. To make these parts in the line of calculation for the computer, the following procedure is accepted.

$$A_1 = \frac{(e^{\gamma_o z_{2D}} - e^{-\gamma_o z_{2D}}) - (e^{\gamma_o z_{1D}} - e^{-\gamma_o z_{1D}})}{e^{\gamma_o} - e^{-\gamma_o}}$$

for $\gamma_o > 80$, $e^{-\gamma_o}$ could safely be dropped from the denominator hence :

$$A_1 = e^{\gamma_o(z_{2D}-1)} - e^{-\gamma_o(z_{2D}+1)} - e^{+\gamma_o(z_{1D}-1)} + e^{-\gamma_o(z_{1D}+1)}$$

Again the terms $e^{-\gamma_o(z_{2D}+1)}$ and $e^{-\gamma_o(z_{1D}+1)}$, could be dropped and hence A_1 will become :

$$A_1 = e^{-\gamma_o(1-z_{2D})} - e^{-\gamma_o(1-z_{1D})} \qquad \ldots(4.16)$$

Considering the exponential form of A_2 :

$$A_2 = \frac{(e^{\gamma_o(1-d_D)} - e^{-\gamma_o(1-d_D)}) - (e^{\gamma_o(1-\ell_D)} - e^{-\gamma_o(1-\ell_D)})}{e^{\gamma_o} - e^{-\gamma_o}}$$

and following the same procedure as for A_1, for higher values of γ_o , A_2 could be reduced to :

$$A_2 = e^{-\gamma_o d_D} - e^{-\gamma_o \ell_D} \qquad \ldots(4.17)$$

Multiplication of A_1 and A_2 will result :

$$A_1A_2 = e^{-\gamma_o(1+d_D-z_{2D})} - e^{-\gamma_o(1+d_D-z_{1D})} - e^{-\gamma_o(1+\ell_D-z_{2D})} + e^{-\gamma_o(1+\ell_D-z_{1D})} \qquad \ldots(4.18)$$

Since $z_{2D} > z_{1D}$ and $\ell_D > d_D$, hence $\gamma_o(1+\ell_D-z_{1D})$, will have the largest and $\gamma_o(1+d_D-z_{2D})$ will have the smallest values among the four powers. If we let :

$$C_y = \gamma_o(1 + \ell_D - z_{1D}) \text{ ; and } C_z = \gamma_o(1 + d_D - z_{2D})$$

then :

(i) For $C_y < 80$, (4.15) could be used.

(ii) For $C_y > 80$ and $C_x < 80$, the following approximate form of A_1A_2 is good enough.

$$A_1A_2 = e^{-\gamma_o(1+d_D-z_{2D})} \qquad \ldots(4.19)$$

(iii) For $C_y > 80$ and $C_x > 80$, A_1A_2 could safely be taken as zero.

Since in the evaluation of $\hat{U}_o(x)$, the product of A_1A_2, A_3 and $A(x)$ is involved; hence, to keep some margin for the negative powered values of A_3 and $A(x)$, 60 or even smaller number is to be used instead of 80 as a limit in the approximation of A_1A_2. Moreover, the upper limit, 80, is to be adjusted according to the capacity of the machine available.

4.2.4 Evaluation of $\hat{U}_n(x)$:

There are two difficulties in the numerical evaluation of (4.12).

1. In the expression $1-\exp[-t_s(x^2+\beta\ \gamma_n^2)]$, whenever the value of $t_s(x^2+\beta\ \gamma_n^2)$ gets larger (>80), creates problem with the machine. To remove this problem, the term $\exp[-t_s(x^2+\beta\gamma_n^2)]$ could safely be dropped whenever $t_s(x^2+\beta\gamma_n^2)$ gets targer than even 20.

2. The term, $\cos\gamma_n$, where $\gamma_n \approx (n-\frac{1}{2})\pi$ will cause the difficulty of near zero denominator. To avoid this, we proceed as follows :

From (4.14) we have

$$\cos(\gamma_n) = \frac{\sin(\gamma_n)}{\left(-\frac{x^2 + \beta\,\gamma_n^2}{\beta\,\sigma\,\gamma_n}\right)} \qquad \ldots(4.20)$$

substitution of (4.20) in (4.12) will ensure numerical stability.

4.2.5 Approximation of $\sum_{n=1}^{\infty} \hat{U}_n(x)$:

For the approximation of this series, the method of straight forward addition as was recommended by Neuman (1972) is accepted. The number of terms which are to be used in the process of approximation of this converging series, depends upon the required degree of accuracy.

In our work, we made an attempt to reproduce Neuman's (1975) tabulated values of drawdown which he had calculated for the preparation of type curves, with his given values of σ and β (Chapter 5).

To cut down the computer time, after several trials, with the acceptance of some tolerable error between ours and that of the Neuman's values, we accepted the following truncation criteria.

$$\frac{\hat{U}_n(x)}{\sum_{n=1}^{N} \hat{U}_n(x)} \leq 0.1 \qquad \ldots(4.21)$$

4.2.6 Approximation of $J_o(x)$:

For the numerical calculation of $J_o(x)$ its following polynomial approximation is used.

1. $$J_o(x) = 1-2.2499997(X/3)^2 + 1.2656208(X/3)^4 - 0.3163866(X/3)^6$$
$$+0.0444479(X/3)^8 - 0.0039444(X/3)^{10}$$
$$+0.0002100(X/3)^{12} + \varepsilon ; |\varepsilon| < 5x10^{-8}$$

For $-3 \leq x \leq 3$

2. $$J_o(x) = x^{-1/2} F_o \cos(\theta_o) \text{ for } 3 \leq x \leq \infty$$

where

$$f_o = 0.79788456 - 0.00000077(3/x) - 0.00552740(3/x)^2$$
$$- 0.00009512(3/x)^3 + 0.00137237(3/x)^4$$
$$- 0.00072805(3/x)^5 + 0.00014476(3/x)^6 + \varepsilon$$

$|\varepsilon| < 1.6x10^{-8}$; and

$$\theta_o = X - 0.78539816 - 0.04166397(3/x) - 0.00003954(3/x)^2$$
$$+ 0.00262573(3/x)^3 - 0.00054125(3/x)^4$$
$$- 0.00029333(3/x)^5 + 0.00013558 (3/x)^6 + \varepsilon$$

$|\varepsilon| < 7x10^{-8}$

4.2.7 Partial Sum of the Drawdown Integral :

Let x_n be the value of x corresponding to the nth zero of $J_o(x)$, and let s_N be the partial sum, then

$$s_N = \frac{Q}{4\pi T} \sum_{n=1}^{N} \int_{o}^{\infty} 4x\, J_o(x) \left[\hat{U}_o(x) + \sum_{n=1}^{\infty} \hat{U}_n(x) \right] dx \;; \quad x_o=0 \quad ...(4.22)$$

From which it is clear that :

$$\lim_{N \to \infty} s_N = s$$

The above integral could easily be evaluated with the help of Simpson's Rule. The number of the zeros of the Bessel's function which are to be used in the above approximation depends upon the required accuracy. As per Neuman (1972) the use of 20 zeros are accurate enough. For smaller values of σ ($\leq 10^{-1}$) and larger values of t_s (≥ 1), $3 \leq N \leq 10$ gives reasonable accuracy.

4.4 DETERMINATION OF AQUIFER PARAMETERS

For the determination of the aquifer parameters through a digital computer, an algorithm is accepted in which the sum of the square of residues of the difference of observed and calculated drawdowns in observation wells is minimized.

The formulated objective function and the constraints to which it is subjected are as follows:

$$\text{Min } F = \sum_{i=1}^{NOB} \sum_{j=1}^{ND} (sC_{ij} - sO_{ij})^2 \qquad \ldots(4.23)$$

subjected to

$$S_c \leq S_{c\ max} \qquad \ldots(4.24)$$

$$T \leq T_{max} \qquad \ldots(4.25)$$

$$S_y \leq S_{y\ max} \qquad \ldots(4.26)$$

$$S_c \geq S_{c\ min} \qquad \ldots(4.27)$$

$$T \geq T_{min} \qquad \ldots(4.28)$$

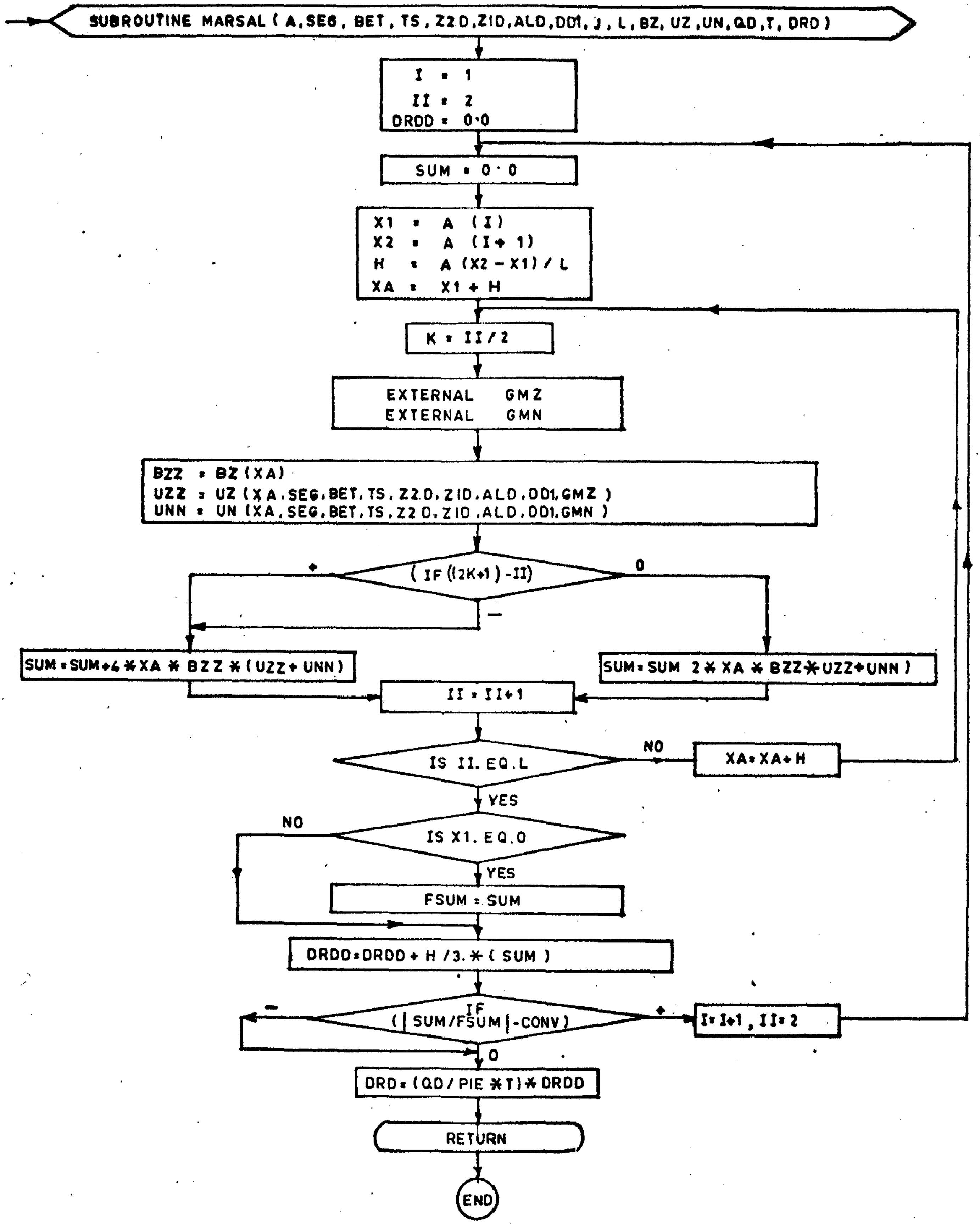
$$S_y \geq S_{y\ min} \qquad \ldots(4.29)$$

where

sC_{ij} are the calculated, sO_{ij} are the observed drawdowns in ith well at jth time, and $S_{c\ max}$, T_{max}, $S_{y\ max}$ are the maximum and $S_{c\ min}$, T_{min} and $S_{y\ min}$ are the minimum values of S_c, T and S_y respectively. To be able to carry on with this minimization two things have been done :

1. Based on the algorithm presented in the previous parts of this chapter, a subroutine has been prepared which is able to calculate values of drawdown at any time and at any radial distance from the pumped well for a known set of values of aquifer parameters.

2. Since the objective function is nonlinear; therefore, for the minimization a nonlinear scheme is used. The scheme chosen here is the Sequential Unconstrained Minimization Technique (SUMT), which is based on the interior penalty function. The prepared subroutine for drawdown calculation, and the subroutine for the above mentioned minimization have been connected by a short main program which provides data to the subroutines, and also calls for the print out of the required results. A flow chart of the drawdown subroutine is presented in Fig. (4.1). A brief explanation of the components of the flow chart are given in Appendix (A).



FIG(4·1): FLOW CHART OF THE SUBROUTINE OF THE CALCULATION OF DRAWDOWN

CHAPTER 5

MODEL'S APPLICATION AND RESULTS

5.1 CHECKING OF THE MODEL

It was the intension to check the developed routine with the help of the real life data. Unfortunately, we didn't find the proper data with which the model could be checked. To show that the routine is working properly, we made an attempt to reproduce the drawdowns calculated by Neuman in 1975, for the preparation of type curves. We fed to the routine whatever values of the dimensionless parameters he had used for the calculations. Those calculations were made for the fully penetrating wells; hence, in our calculations we also assumed the wells to be fully penetrating. From the obtained results which are partly presented in Table (5.1), we came to know, that the Neuman's values could be reproduced if appropriate convergence criterias are used.

By convergence criteria we mean where to terminate the partial sum of the drawdown integral ($\sum_{i=1}^{N} \int_{x_i}^{x_{i+1}} 4xJ_o(x)[\hat{U}_o(x) + \sum_{n=1}^{\infty} \hat{U}_n(x)]\, dx$) and when to truncate the infinite series $\sum_{n=1}^{\infty} \hat{U}_n(x)$. The convergence criteria for the partial sum was named as CON and that for the infinite series as CONV. The smaller the values of CON and CONV, the longer the computer time it takes for the calculation of drawdown.

To find a combination of CON and CONV, which will give reasonably accurate values with the least possible time of the computer, several of their combination were tried. The results of these trials are shown in Table (5.1). Finally with the acceptance of some reasonable difference between our values and that of Neuman's we accepted CON to be 0.01 and CONV to be 0.1. These values could be changed according to degree of accuracy required.

5.2 APPLICATION OF THE MODEL

For an illustration of model's application, because of the lack of real life data, with the help of an arbitrarily assumed set of aquifer parameters a series of drawdowns were generated for an arbitrarily chosen series of times. These drawdowns were generated with the help of the prepared computer routine. The values of the parameters used in this process were : $S_c = 0.003$, $T = 1400.0\ m^2/day$ and $S_y = 0.12$. The discharge was assumed to be 6000.0 m^3/day and the drawdowns were calculated at a radial distance of 30m from the pumping well. The various characteristics of the wells (observation and pumping) were assumed as $z_1 = 50m$, $z_2 = 80m$, $d = 20m$, $\ell = 69.29m$, and $b = 109.29m$.

The generated data was analysed by the commonly used Boulton's method as well as through the proposed nonlinear optimization scheme with the help of the digital computer. From these analysis the following results were obtained :

TABLE 5.1 : COMPARISON OF NEUMAN'S VALUES OF S_D WITH THOSE CALCULATED BY THE PROPOSED METHOD FOR $\beta = 0.01$ AND $\sigma = 10^{-9}$

Trial	CON	CONV	t_s	Values of S_D^* by proposed method	Neuman's values	CPU time (sec.)
			6×10^{-1}	5.65×10^{-1}	6.33×10^{-1}	
			3.5×10^{0}	1.88×10^{0}	1.88×10^{0}	
1	0.1	0.1	1.0×10^{1}	2.61×10^{0}	2.61×10^{0}	0.91
			2.0×10^{2}	3.23×10^{0}	3.45×10^{0}	
			1.0×10^{3}	3.23×10^{0}	3.46×10^{0}	
			6×10^{-1}	5.92×10^{-1}	6.33×10^{-1}	
			3.5×10^{0}	1.93×10^{0}	1.88×10^{0}	
2	0.1	0.01	1.0×10^{1}	2.66×10^{0}	2.61×10^{0}	1.12
			2.0×10^{2}	3.27×10^{0}	3.45×10^{0}	
			1.0×10^{3}	3.27×10^{0}	3.46×10^{0}	
			6×10^{-1}	5.94×10^{-1}	6.33×10^{-1}	
			3.5×10^{0}	1.94×10^{0}	1.88×10^{0}	
3	0.1	0.001	1.0×10^{1}	2.67×10^{0}	2.61×10^{0}	1.82
			2.0×10^{2}	3.28×10^{0}	3.45×10^{0}	
			1.0×10^{3}	3.28×10^{0}	3.46×10^{0}	
			6×10^{-1}	5.94×10^{-1}	6.33×10^{-1}	
			3.5×10^{0}	1.94×10^{0}	1.88×10^{0}	
4	0.1	0.0001	1.0×10^{1}	2.67×10^{0}	2.61×10^{0}	2.92
			2.0×10^{2}	3.28×10^{0}	3.45×10^{0}	
			1.0×10^{3}	3.28×10^{0}	3.46×10^{0}	

Table 5.1 (Contd.)

Trial	CON	CONV	t_s	Values of S_D^* by proposed method	Neuman's values	CPU time (sec.)
			$6x10^{-1}$	$6.24x10^{-1}$	$6.33x10^{-1}$	
			$3.5x10^{0}$	$1.88x10^{0}$	$1.88x10^{0}$	
5	0.01	0.01	$1.0x10^{1}$	$2.62x10^{0}$	$2.61x10^{0}$	3.03
			$2.0x10^{2}$	$3.36x10^{0}$	$3.45x10^{0}$	
			$1.0x10^{3}$	$3.36x10^{0}$	$3.46x10^{0}$	
			$6x10^{-1}$	$6.26x10^{-1}$	$6.33x10^{-1}$	
			$3.5x10^{0}$	$1.89x10^{0}$	$1.88x10^{0}$	
6	0.01	0.001	$1.0x10^{1}$	$2.62x10^{0}$	$2.61x10^{0}$	5.35
			$2.0x10^{2}$	$3.37x10^{0}$	$3.45x10^{0}$	
			$1.0x10^{3}$	$3.37x10^{0}$	$3.46x10^{0}$	
			$6x10^{1}$	$6.27x10^{-1}$	$6.33x10^{-1}$	
			$3.5x10^{0}$	$1.89x10^{0}$	$1.88x10^{0}$	
7	0.01	0.0001	$1.0x10^{1}$	$2.63x10^{0}$	$2.61x10^{0}$	10.71
			$2.0x10^{2}$	$3.37x10^{0}$	$3.45x10^{0}$	
			$1.0x10^{3}$	$3.37x10^{0}$	$3.46x10^{0}$	
			$6x10^{-1}$	$5.95x10^{-1}$	$6.33x10^{-1}$	
			$3.5x10^{0}$	$1.81x10^{0}$	$1.88x10^{0}$	
8	0.01	0.1	$1.0x10^{1}$	$2.57x10^{0}$	$2.61x10^{0}$	1.80
			$2.0x10^{2}$	$3.31x10^{0}$	$3.45x10^{0}$	
			$1.0x10^{3}$	$3.31x10^{0}$	$3.46x10^{0}$	

$* \; S_D = \frac{4\pi T}{Q} S$

(1) The values of the parameters obtained through the proposed scheme are :

$$T = 1398 \ m^2/day \ , \ S_y = 0.123, \text{ and } \ S_c = 0.00264$$

(2) The values of the parameter obtained through Boulton's method are : $S_c = 0.0018$, $T = 1075 \ m^2/day$ and $S_y = 0.089$.

(3) According to Boulton, the reciprocal of the delay index (α) remains constant. Neuman showed that α is not constant, instead, it is linearly related to the logarithm of the redial distance from the pumping well. Applying the Neuman's concept, with the assumed set of parameters:
$T = 1400 \ m^2/day$, $S_y = 0.12$ and $b = 109.29m$
we tried to find the sensitivity of α to the changes in radial distance.

For the above set of parameters we found, that α will have a value of $\alpha_o = 6.0761 \times 10^{-5} \ S^{-1}$ at a radial distance of 1m from the pumping well, and will decrease to 75% of α_o at a radial distance of 15.3 m to 50% of α_o at a radial distance of 234.30 m, to 25% of α_o at radial distance of 3585.93 m, and to zero at a radial distance of 54896.87 m from the pumping well. Boulton's value of α ($=5.70 \times 10^{-5} S^{-1}$) corresponds to a radial distance of 1.96 m from the pumping well (Fig. (5.1)).

Drawdown were also generated for the case of fully penetrating observation and pumping wells for the same series of time as used in the case of partially penetrating wells. The used discharge was 6000.0 m^3/day and the calculations were

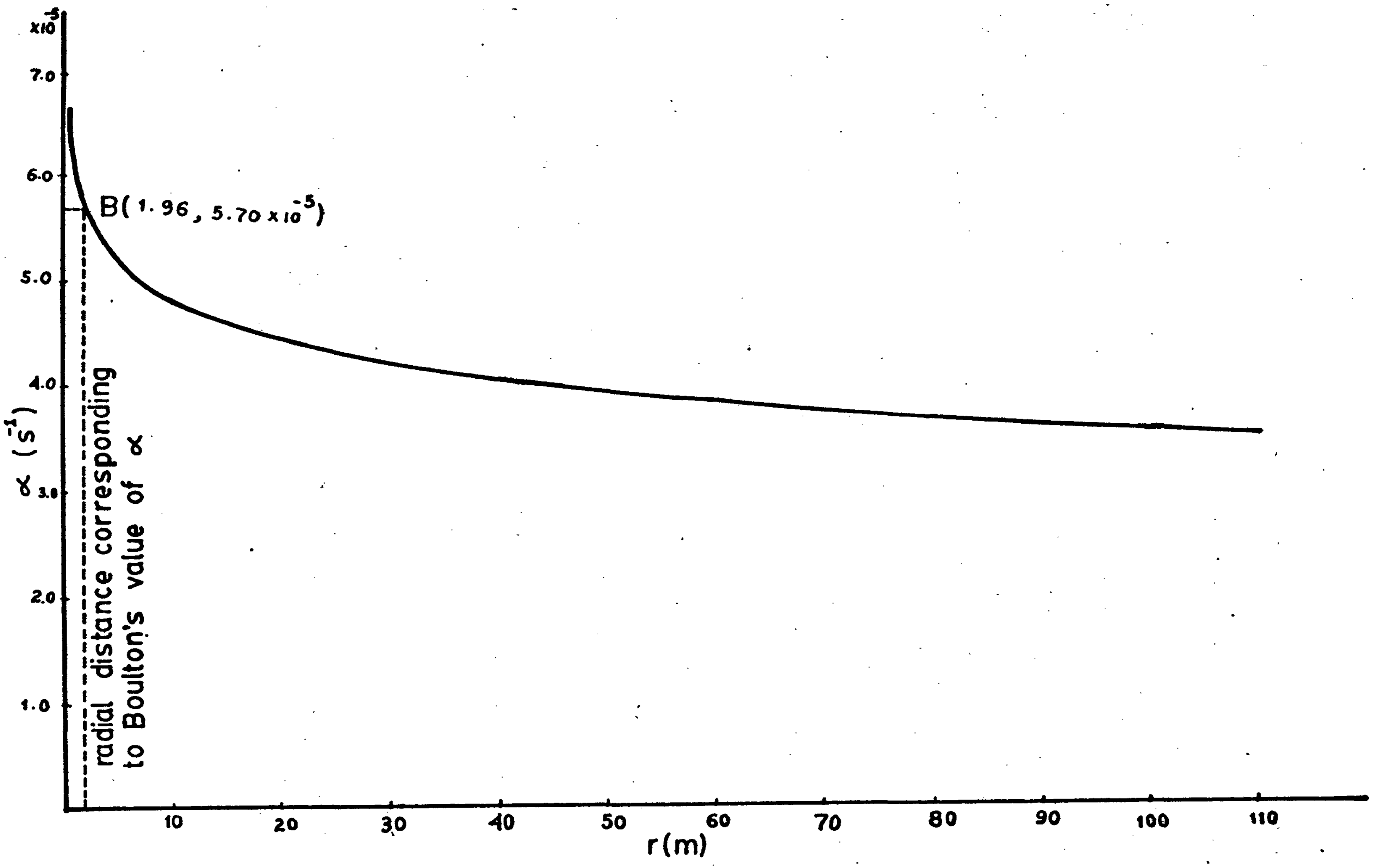


FIG.(5.1): VARIATION OF α WITH RADIAL DISTANCE FROM PUMPING WELL
FOR b=109 29, T=1400 m^2/day, and, S_y=0.12

made at a radial distance of 30.0m from the pumping well. For a comparison, these drawdowns and those for the case of partially penetrating wells are given in Table (5.2). From this table it could clearly be seen that for the same discharge, partial penetration causes greater values of drawdowns.

TABLE 5.2 : GENERATED DRAWDOWNS

Serial No.	Time (min)	Fully Penetrating case (m)	Partially Penetrating case (m)
1	1	0.109	0.209
2	3	0.293	0.468
3	5	0.389	0.573
4	7	0.446	0.616
5	10	0.500	0.662
6	12	0.523	0.682
7	15	0.548	0.704
8	20	0.575	0.727
9	25	0.591	0.752
10	30	0.601	0.761
11	35	0.609	0.768
12	40	0.615	0.774
13	50	0.624	0.782
14	60	0.631	0.789
15	70	0.638	0.796
16	80	0.644	0.802
17	90	0.649	0.808

Table 5.2 (Contd.)

Serial No.	Time (min)	Fully Penetrating case (m)	Partially Penetrating case (m)
18	100	0.656	0.814
19	140	0.679	0.838
20	160	0.690	0.850
21	200	0.714	0.872
22	240	0.737	0.895
23	300	0.770	0.930
24	340	0.792	0.952
25	400	0.825	0.985
26	440	0.846	1.007
27	500	0.868	1.039
28	600	0.919	1.092
29	700	0.969	1.143
30	800	1.018	1.193
31	900	1.065	1.225
32	1000	1.111	1.290

CHAPTER 6

CONCLUSIONS AND SUGGESTIONS

1. In spite of a lot of theoretical work which has been done in the area of unconfined aquifers, still there is a large gap between theory and practice. In the early times, Theis type curves which are basically for the analysis of artisian aquifers were also used for the analysis of the test pumping data of unconfined aquifers. It was 1955, that Boulton included the slow draining behaviour of unconfined aquifers in the formulation of unconfined aquifers radial flow equations, based on which in 1966 he prepared the type curves for the analysis of the test pumping data of unconfined aquifers. From 1966 onward, whatever development has been made in this area, has not come up to the usual practice. And, unfortunately still Boulton's curves are used without taking care of what limitations they have.

2. From the values of the parameters obtained by Boulton's method, it can clearly be seen that analysing field data of partially penetrating wells with the help of Boulton's type curves which are for fully penetrating wells, leads to the under estimation of aquifer parameters.

3. The amount of underestimation will be much more, if the observed data will be collected at small distances from pumping well.

4. From Fig. (5.1), it can be concluded that Boulton's value for α is only one value in the range of variation of α, from maximum in the vicinity of the well to zero at a considerably large distance from the pumping well. According to the above mentioned figure, Boulton's value is not even an average value.

5. The proposed method of the determination of aquifer parameters, takes care of the partial penetration, delayed aquifer response, aquifer anisotropy and the subjectivity which is inherent in the type curve procedure. The results which were obtained for an example through the proposed method, are quite satisfactory.

SUGGESTIONS

1. Partial penetration is a common field practice, because drillers when striking a satisfactory aquifer frequently make no effort to extend well down to the formation. On the belief, that partial penetration has negligible role, often no effort is made to measure the characteristics of observation and pumping wells which are : d, ℓ, b, z_1 and z_2 (Fig. (3.2)). Contrary to the above belief it was shown that partial penetration is an important factor which must be considered in the determination of the unconfined aquifer parameters. Therefore, it is strictly emphasized that during a test pumping process, the previously mentioned wells characteristics, d, ℓ, b, z_1 and z_2, are to be measured and clearly recorded.

2. The various formulae and procedures for the analysis of pumping test data should be used with caution, considering the various assumptions underlying each formula and procedure.

3. According to Neuman (1974), the effect of partial penetration on drawdown in an unconfined aquifer decreases with radial distance from pumping well, and with ratio k_z/k_r. Hence, if still one intend to use Boulton's method of type curves; it is recommended that the observations are to be taken at a larger distance ($\geq b/k_D^{1/2}$) from the pumping well. However, the time factor may still affect the values of the parameters.

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APPENDIX (A)

Description of the Flow Chart

The MARSAL subroutine contains 5 function subprograms. The function subprograms are : BZ, for the calculation of the Bessel's function of the zeroth order and first kind; UZ, for the calculation of the component $\hat{U}_o(x)$ of the drawdown integral; GMZ, for the calculation of γ_o, a parameter of $\hat{U}_o(x)$; UN, for the approximation of the component $\sum_{n=1}^{\infty} \hat{U}_n(x)$ of the drawdown integrals and GMN, for the calculation of γ_n a parameter of $\hat{U}_n(x)$.

Data Requirements

(1) A set of initial feasible values of the aquifer parameters. This could be any feasible set, but better if approximated on the basis of the governing physical conditions.

(2) Upper and lower bounds of the parameters as per the governing physical conditions.

(3) Characteristics of the pumped and observation wells, which are d, ℓ, b, z_1 and z_2 (Fig. (3.2)).

(4) Observed drawdowns and their corresponding times.

(5) Zeros of the Bessel's function.

Operational Details

In the program, the following things could be changed:

(1) The limits of convergence, as per the required degree of accuracy.

The relation for β used in the program does not account for the aquifer's anisotropy. In case, it is desired to include anisotropy, the following relation for β is to be used :

$$\beta = k_z/k_r \cdot r^2/b^2$$

The number of uniform strips in the Simpson's Rule, which is used here to calculate the partial sum of the drawdown integral.