MATHEMATICAL MODEL STUDIES OF GROUND WATER BY FINITE DIFFERENCE METHOD FOR DAHA AREA

A DISSERTATION

submitted in partial fulfilment of the requirements

for the award of the degree

of

MASTER OF HYDROLOGY

By

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C E R T I F I C A T E

Certified that the dissertation entitled 'MATHEMATICAL MODEL STUDIES OF GROUND WATER BY FINITE DIFFERENCE METHOD FOR DAHA AREA' which is being submitted by Shri G. G. Dikshit in partial fulfilment of the requirements for the award of the degree of Master of Hydrology of the University of Roorkee, Roorkee is a record of the candidate's own work carried out by him under my supervision and guidance. To the best of my knowledge the matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that Shri G. G. Dikshit has worked for a period of six months from October 1, 1977 to March 31, 1978 in the preparation of this dissertation under my guidance at this University.

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S Y N O P S I S

Water occuring in the subsurface was known tomankind before the beginning of the christian era. As the demand is increasing due to increase in agriculture areas and the improvement in the agriculture practice more and more water is being used. Surface sources are not capable to meet the demand round the year in such a large scale. Hence groundwater sources are being exploited more and more. With the increase in the exploitation of ground water sources a quantitative assessment of the available sources in time and space within the subsurface has become as must. In India this aspect was realised and first attempt to know the precise hydrological parameter of aquifer in Gangetic plains was initiated in 1931. Upto 1967 availability of ground water resources has been followed by some empirical approach. In 1967 first water balance study was done for the arid tracts of Rajasthan. This is lumped nodel which gives only overall assessment of water in a region. Distributed model is to indicate the assessment of water in time and space and is done with mathematical model.

Mathematical model isa computer programme and associated data that can simulate the response of a real event. Two approaches are generally used for mathematical model, (1) Finite difference approach (2) Finite element approach. Finite difference approach got the present shape with progressive development starting from Fayers and Sheldon model (1962), Tyson and Weber model (1964), Fiering model (1964), Pinder and Bredhoeft model (1968), Prickett and Lonnquist model (1968). Finite element approach which is based on concept of variational calculus and minimising equivalent integral originated from the model of Gurtin (1964), Zienkiewicz and Cheung (1967), Newman and Witherspoon (1971).

The main aim of ground water mathematical model is to define yield of a basin in time and space. This can be precisely known if we can know the drawdown of an area due to known pattern of ground water draft and rainfall. This aim is fully achieved through the mathematical modelling of the distributed system of ground water aquifer. A review of different type of finite difference modelling which reduces the boundary condition, initial condition, and the process of solving simultaneous equation to a definite mathematical form have been discussed in greater detail. To make the theory illustrative the problem of Daha area in the Meerut and Muzaffarnagar district of Western U.P. has been considered.

First the hydrological data has been processed and water balance of each period within basin parameter adopted values worked out. The ground water balance as a lump model indicates the recharge due to rainfall is of the order of 4600 hectare metres (22 percent of the rainfall). Where as the ground water draft is of the order of 10,100 hectare metres. So it has become essential to know as to how the aquifer is responding to over draft in time and space in the entire doab. This is possible by considering a distributed model of the ground water system. For the purpose lumped system data has been distributed to work out the input matrix of the computer simulation for the distributed system. One month data with actual initial condition has been simulated to predict the heads at interval of 1014 metre both in x and y direction. The data has been regrouped in the symbolic category statement so that ground water condition is more readily observed like contour diagram. Model error parameters have been changed to different values so as to judge the correctness and the economy in the computer modelling techniques. The exercise has also been repeated with initial condition taken as all zeros so as to work out the drawdown directly from the output matrix of the computer model. Entire data of the water balance period has been processed in a variable pumping rate format so as to run the model for 36 months simulation in a single run. The result of simulation has been found to be more satisfactory in the case when initial conditions are the starting elevations. However, some more model runs are necessary to get clearer picture of the situation and suitability of the model.

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CHAPTER-1

INTRODUCTION

1.1. GENERAL

Increasing pressures throughout the country for better management and higher efficiency in the use of ground water resources is a recognized fact that need not be exmphasized. Also recognized are the detrimental effects of over exploitation of ground water causing diminishing return and possible permanent harm to the aquifer system by land settlement. Knowledge of definite drawdowns andextend of effect related to a known pumping pattern, rainfall pattern and return flow will be very useful to ensure scientific and judicious use of ground water. This knowledge will be used to evaluate the following.

(1) Yield of a ground water basin

(2) Economic level of exploitation

(3) Permissive sustained yield

(4) Maximum sustained yield

(5) Maximum mining yield

(6) Permissive mining yield

(7) Temporal allocation of ground water draft.

1.2. DRAWDOWN COMPUTATION

1.2.1. Drawdown Computation Tools

Four types tools are available to the ground water hydrologists prediction of the aquifer drawdown in response to different kinds of inputs (PRICKETT, 1969).

1. Analytical methods
2. Digital computers
3. Electrical- analog simulators
4. Combined digital analog computers forming the hybrid computer.

Analytical methods apply only to simple aquifer geometries with constant coefficients and generally used in the study of the hydraulics of wells. The remaining tools are generally used in distributed parameter models based on finite-difference approximations to the basic two dimensional partial differential equation describing the flow in a saturated porous medium. So the latter three tools are adopted for regional type of aquifer evaluation and studies.

1.2.2. Digital Methods of Drawdown Computation

For regional type of aquifer evaluation and studies a mathematical model is first developed. A mathematical model is defined as a computer program and associated data that can be used to simulate the response of a real event, in water resources themodel could be of a stream, an estuary, a reservoir, a ground water basin. Any system that can duplicate the response of a ground water reservoir can be termed a model (PRICKETT 1975). As it is related to the hydrology of the aquifer for evaluating the response of a ground water reservoir it is termed as hydrological model also. The results of the mathematical model are the predicted or simulated values of the variables like aquifer elevation or drawdown being modelled.

The mathematical formulations of the problem consists of working with appropriate differential equations and their associated boundary conditions. The approach to solve the partial differential equation is of two type.

(1) Pooled parameter model

(2) Distributed parameter model

In the pooled parameter model which presumes the idealised aquifer condition the system parameter like transmissivity, storage coefficient and height of the saturated thickness of the aquifer are averaged, boundary configurations are simplified, and the response of the system to pumping is determined by application of a standard closed formed solution of an appropriate differential equation. In this type the application of idealized aquifer and mathematical models are restricted to systems that can be highly simplified such that known solutions of differential equations involving homogeneous parameters may be used.

Distributed parameter model can be fitted where adequate reliable and sophisticated data of the aquifer is available. Such elaborate detail study of a groundwater basin is possible in case of experimental basin where the data is specially collected for that purpose.

Quite often in developing countries like ours geologic and hydrologic data are not sufficient to permit a rigorous description of an aquifer required for distributed parameter model. On the other hand, there are cases where economic limitations and very complex aquifer conditions with extensive area prohibit the proposed collection of detailed information required to define an aquifer precisely. Under these type of background conditions, the much needed interpretation and extrapolation is necessary to describe the ground water conditions in quantitative terms. With proper qualification, the use of so-called ' idealized aquifers' with pulled parameters and mathematical models may serve as a means of approximating performances of wells and aquifers even in the face of lack of adequate and accurate basic data.

1.2.3. Steps for Pooled Parameter Model

WALTON and NEILL (1960) had suggested that the analysis of the performance of an aquifer with the use of an idealized aquifer and mathematical model proceeds as follows -

1) Any complex hydrological boundaries of the aquifer, which are evident from areal studies are idealized into elementary geometric forms made up of straight line demarcations.

2) Average geohydrological properties like transmissivity hydraulic conductivity, storage coefficient, saturated thickness of the aquifer and confining layer, if present are selected on the basis of any known test pumping field data. The resulting conceptual model created in these first two steps is called the idealized aquifer.

3) The mathematical model is based on the hydrological properties of an idealized aquifer, the image-well

theory and one appropriate theoretical ground water equation. According to the image well theory hydrologic boundaries may be replaced, for analytical purposes, by imaginary wells which produce the same disturbing effect in the boundaries. Boundary problems are there by simplified to consideration of an infinite aquifer in which real and image wells operate simultaneously.

4) The effect of real and image wells are computed with the principle of superposition and appropriate ground water formula.

5) In the simulation process the record of past pumpage and water levels is used to establish the validity of the idealized aquifer and mathematical model. When the comparison between actual and model simulated response is satisfactory the model is proved and then model provides a means for predicting future aquifer response in a known pumping pattern.

1.3. CASE HISTORY OF DAHA AREA

1.3.1. Location

The Daha area in Uttar Pradesh lies in the two districts of Meerut and Muzaffarnagar between the river Krishni and Hindon (Figure No.1). It comprises of the parts of three blocks of Binauli, Budhana and Kandhala having total 64 villages extended to area of 339.43 square kilometers or 33943 hactares. It is situated within 29°05' North to 29°17' North and 77°19' East to 77°32' East. Thus it is about 21 km. in North South and 24.3 km. in East West direction.

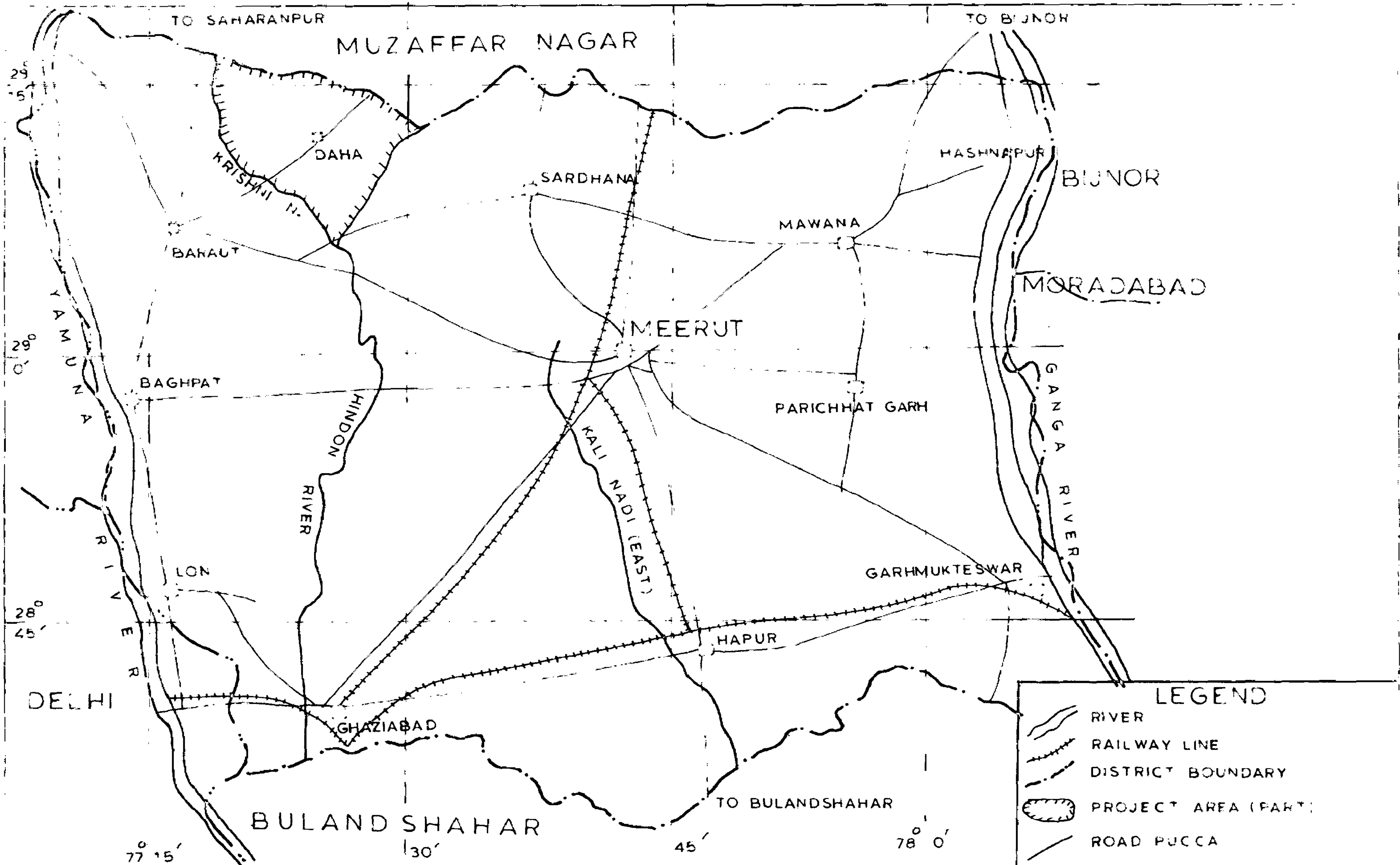


FIG. 1 INDEX MAP OF DAHA AREA

1.3.2. Physiography

The elevation of Daha area varies from 238.203 metres in the North to about 225.055 metres in the south near Barnawa in a total length of 21.12 km. Thus the North to South land slope is about 1/1606. The terrain is in general slope from North to South which is also the flow direction of main drainage.

1.3.3. Hydrogeology of the area

The Daha area is situated in the axial belt of the Indo-Gangetic alluvial plain. This is formed by unconsolidated fluvial deposits comprising sand bed with intermediate lenses of silt, clay and Kankar. In regional scale the aquifers are interconnected, but in localised packets at some places shows semi unconfined and leaky confined condition. Average depth of water table is about 12.5 metres and fluctuation was less than one metres. Average total thickness of the fluvial deposits which has continued all through the pleistocene to recent time is about 2000 metres.

1.3.4. Climate and rainfall

The area experiences extreme type of climate, with very hot summer and cold winters. The annual normal rainfall in the area is 96.5 cms. The rainy season sets in by the middle of June and lasts till the end of September. Most of the precipitation falls in the month of July, August and September. The winter rain is scanty. Potential evapotranspiration and pan evaporation data has been adopted as published by India Meteorological Department for average year.

1.3.5. Irrigation Facilities

The adjoining land in the East of Daha area from the boundary of the Hindon river falls in the command area of upper Ganga Canal and the adjoining land in the west of Daha area from the boundary of the Krishni river falls in the command area of Western Jamuna Canal. As both the canal systems are perenial the adjoining aquifers are getting irrigation recharge throughout the year. The doab in between these two river known as Daha area is having no surface irrigation facility. So the effective precipitation of 96.5 cms. annual average rainfall is supplemented by the use of ground water to meet the evapo-transpiration needs of this fertile land producing bumper sugar cane, cereals and other crops round the year.

The ground water from subsurface sources is being exploited with the help of 121 state tube well 1180 private tube wells, 15 pumping sets, 1130 masonry wells with Rahat and 1890 drinking water wells in an area of 339.43 square km.

1.3.6. Problem

Scientific method of cultivation requires sufficient quantity of water. With the advacement of agriculture practice ground water exploitation is increasing year after year and resulting in increased drawdown in the area. It is presumed that the pumping rate may increase further. As the existing rate of pumping already exceeded the annual natural recharge it is apprehended further increase may create detrimental effects. Extend of possible drawdown in different drafting pattern and effects of a known artificial recharge pattern which is being thought require proper planning.

After simulating the existing hydrological behaviour of the basin the model will be calibrated for further study. From the calibrated mathematical model it will be possible to impose future condition which are likely to develop due to possible increased and redistributed pumping pattern both in time and space, decrease in the return flow due to improvement in irrigation practice, possible change in recharge due to artificial recharge, or any other factor which may change the existi hydrological condition.

The solution of appropriate differential equation representing the ground water hydrology of the area is to be attempted after discussing the various methods of solution so far known to the research World.

The boundary condition, initial condition and the related hydrological behaviour form the typical hydrological characteristics of any basin require careful representation for accurate simulation of the response. These aspects have been discussed in greater detail before arriving actual solution of the problem.

CHAPTER-II

THEORETICAL DEVELOPMENT OF MATHEMATICAL MODEL

2.1. EQUATION OF FLOW

The theory of ground water movement originated in 1856 from the experiments of H. Darcy. The flow condition is defined with most common situation which is technically specified with the following terms.

Heteorogeneous

As opposed to homogeneous a parameter which varies in space is termed as heteorogeneous.

Anisotropic

A parameter which varies in direction is termed as anisotropic as opposed to isotropic condition.

Laminar flow

In laminar flow the water particles appears to move in definite smooth path, or stream lines and infinitesimally thin layers of fluid seen to slide over adjacent layer.

The velocity for two dimensional laminar flow in an anisotropic, non-homogeneous porous medium is described by Darcy's law which is expressed in the matrix form as

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \end{bmatrix} \quad \text{.. (2.1.1.)}$$

or in cartesian notation using Einstein's condition as

$$V_i = - K_{ij} \frac{\partial h}{\partial x_i} \quad \text{..(2.1.2.)}$$

Where V_x and V_y or V_i are the components of the Darcy velocity for flow vector (L/T), K_{xx}, K_{xy}, K_{yx} and K_{yy} or K_{ij} are the components of the permeability tensor (L/T) and are a function of space co-ordinates (x,y) and h is the hydraulic head.(L).

The continuity equation for steady state in two dimension for the above porous medium can be written as

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \qquad ..(2.1.3)$$

Putting the equation (2.1.1.) in (2.1.3.) the equation of flow becomes

$$\frac{\partial}{\partial x}\left[K_{xx}\frac{\partial h}{\partial x} + K_{xy}\frac{\partial h}{\partial y}\right] + \frac{\partial}{\partial y}\left[K_{yx}\frac{\partial h}{\partial x} + K_{yy}\frac{\partial h}{\partial y}\right] = 0 \qquad ..(2.1.4)$$

For convenience the transmissibility tensor may be generated by multiplying permeability tensor by the thickness of the aquifer. The equation (2.1.4.) becomes

$$\frac{\partial}{\partial x}\left[T_{xx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial x}\left[T_{xy}\frac{\partial h}{\partial y}\right] + \frac{\partial}{\partial y}\left[T_{yx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_{yy}\frac{\partial h}{\partial y}\right] = 0 \qquad ..(2.1.5.)$$

Thus the equation (2.1.5) becomes the Laplace equation in steady state for anisotropic and non-homogeneous porous medium. Therefore, the partial differential equation in unsteady state for anisotropic and non-homogeneous porous medium is given by

$$\frac{\partial}{\partial x}\left[T_{xx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial x}\left[T_{xy}\frac{\partial h}{\partial y}\right] + \frac{\partial}{\partial y}\left[T_{yx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_{yy}\frac{\partial h}{\partial y}\right] = S\frac{\partial h}{\partial t} + W(x,y,t) + L_1 + L_2 \qquad ..(2.1.6)$$

Where,

S = Storage coefficient a function of x,y or specific yield in case of unconfined aquifer again a function of x and y.

$W(x,y,t)$ = Net volumetric flux due to recharge or withdrawl per unit surface area (LT^{-1})

L_1 and L_2 = Rate of leakage per unit area of confining layers (LT^{-1})

2.2. GENERAL SOLUTION OF THE EQUATION OF FLOW

No general solution is available to the most general equation described in 2.1.6. After certain simplification a general analytical solution based on pure mathematics approach can be obtained for the equation no.2.1.6. (Bear, 1967). But such solutions will be in the form of infinite series or integrals of complicated function or both which are very difficult and tidius to apply to a specific field problem specially when the boundaries of the flow domain have an irregular shape.
Our interest in this optimum utilisation of the complex ground water system is the specific answers, within specified accuracy and usually at a minimum cost of labour, equipment and time rather than the exact mathematical solution. So although an analytical solution based on rigorous mathematical theory is most satisfactory form of solution and by no means be overlooked by hydrologists, it frequently proves inadequate or impractical for hydrological application. Thus finite difference method which give approximate result is preferred for the solution of above partial differential equation. Equation 2.1.6.

is further simplified to make it amenable to finite difference solution by selecting a new co-ordinate axes which will be called x and y henceforth, chosen in such a manner that the principal components of the transmissibility T_{xx} and T_{yy} are co-linear with the co-ordinate axes x and y (Trescott et al 1976) under this condition the terms T_{xy} and T_{yx} will become zero. Hence the equation 2.1.6 reduces to

$$\frac{\partial}{\partial x}\left[T_{xx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_{yy}\frac{\partial h}{\partial y}\right] = S\frac{\partial h}{\partial t} + W(x,y,t) + L_1 + L_2 \qquad ..(2.2.1.)$$

Although the above equation (2.2.1.) has been obtained from the general assumption of non-homogeneous and anisotropic aquifer a single S (storage coefficient or specific yield) is a defacto assumption of homogeneous and isotropic storage coefficient. Regarding the type of aquifer the general equation 2.2.1. in different special situation represents the aquifer confined, leaky confined or unconfined in nature.

2.3. NATURE OF THE PARTIAL DIFFERENTIAL EQUATION

The most general form of partial differential equation with only two independent variable x and y is given by

$$A\frac{\partial^2 h}{\partial x^2} + 2B\frac{\partial^2 h}{\partial x \partial y} + C\frac{\partial^2 h}{\partial y^2} = D\frac{\partial h}{\partial x} + E\frac{\partial h}{\partial y} + Fh + G \qquad ..(2.3.1.)$$

By letting the parameter H to G which may be function of the independent variable x and y and assuming any value +ve, -ve or 0 magnitude the general equation (2.3.1.) aquires different properties. The relative magnitude of A, B and C determine the basic nature of the equation.

(1) If $AC > B^2$ as in the case of $B = 0$ and A and C are both +ve or both -ve then the equation is Elliptic partial differential equation.

(2) If $AC < B^2$ as in the case of $B = 0$, and either A or C is -ve then the equation is hyperbolic partial differential equation.

(3) If $B^2 = AC$, as in the case of B and either A or C are equal to 0 then the equation is parabolic partial differential equation, considering the partial differential equation of one dimension

$$A \frac{\partial^2 h}{\partial x^2} = K \frac{\partial h}{\partial t} + L \qquad ..(2.3.2)$$

as $\frac{\partial^2}{\partial x \partial y}$ is missing, B = 0 again as $\frac{\partial^2 h}{\partial y^2}$ is missing

C = 0 therefore, $B^2 = AC$ is satisfied as per the criteria (3). Hence the equation is parabolic partial differential equation in one dimension. In the same analogy equation 2.2.1. is a parabolic partial differential equation in two dimension.

The equation 2.2.1. will remain linear parabolic partial differential equation so long h component of T_{xx} and T_{yy} is not variable with respect to x and y. Thus clearly in the case of confined and leaky confined aquifer the equation (2.2.1.) is linear parabolic partial differential equation but in case of unconfined aquifer the equation represent non-linear parabolic partial differential equation.

2.4. WATER BALANCE

The basic concept of mathematical model is aimed at precise determination of the yield of a ground water basin in time and space. In the lumped model which is characterised by the general equation.

Inflow = outflow + change in storage

We get the periodic distribution of input and output parameters for any defined area. The defined area is in the sense of having defined hydrological boundaries. Such information although fairly informative it does not serve the purpose present day finer need in respect yield of a basin in different points with respect to time. In order to achieve that purpose the water balance equation is required to be distributed to different smaller areas called nodal areas. The time variant distribution spread over such a large area with good number of units requires some comprehensive precise treatment. Mathematical model serve that purpose. The basic equation of flow is nothing but the representation of water balance equation.

Considering the equation is

$$T_{xx} \frac{\partial^2 h}{\partial x^2} + T_{yy} \frac{\partial^2 h}{\partial y^2} = S \frac{\partial h}{\partial t} + Q$$

T_{xx} and T_{yy} are the aquifer parameters $\frac{\partial^2 h}{\partial x^2}$ and $\frac{\partial^2 h}{\partial y^2}$ represents the rate of change of head in x and y direction which decide the flow of water in that direction. Thus left hand side of the equation represent the horizontal input to the system and the right hand side terms are change in storage and vertical transfer of water. The equation stands as horizontal flow = change in storage + vertical flow. It is therefore logical that mathematical model is to be fitted within the frame work of water balance equation.

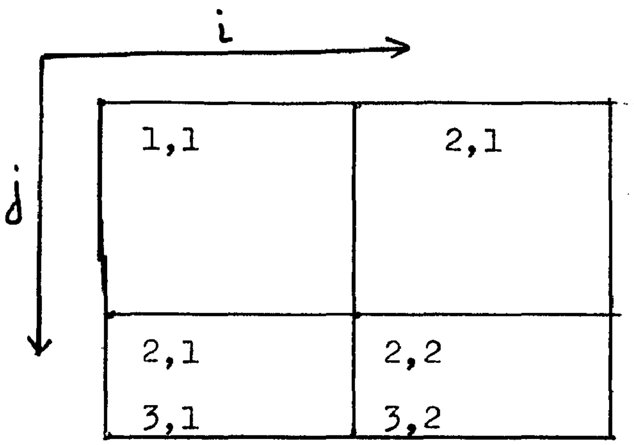
2.5. MATHEMATICAL DERIVATION OF FINITE DIFFERENCE EQUATION

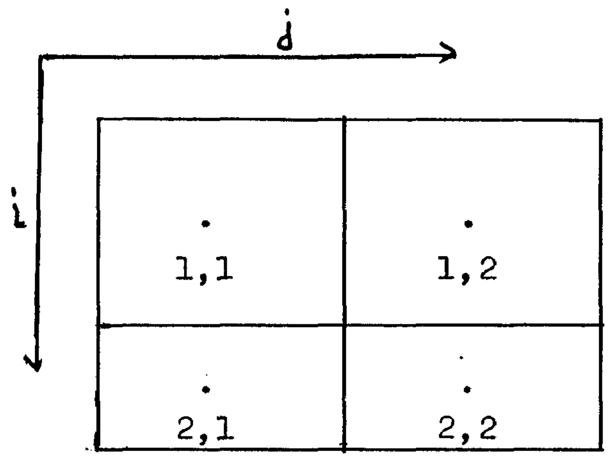
The finite difference equations can be derived in two different ways as follows -

1) Principle of conservation of mass or continuity concept with water balance equation
2) Conventional mathematical treatment

Again, the co-ordinate system within the frame work of co-linear transmissivity tensor the following notations are used.

1. Block corners as grid point
2. Block centre as grid point



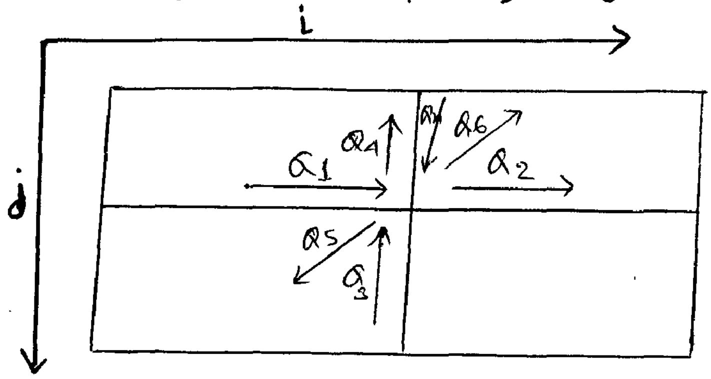


Block corners grid point concept has been utilized by Prickett and Lonnquist (1971) and block centre grid point concept has been utilized by Pinder (1970) and Trescott et al. (1977).

2.5.1. Water Balance Equation Approach

The continuity condition relating to the flow rates entering and leaving the node i, j of given figure requires that the rates be equal as follows.

$$Q_n + Q_1 + Q_3 = Q_2 + Q_4 + Q_5 + Q_6 \qquad ..(2.5.1).$$



Determination of the values of the flow rate terms of the equation 2.5.1. involves three consideration.

(a) Portion of the aquifer represented by flow terms should be defined.

(b) Although the flow rates may takes place in any direction in the aquifer system they are restricted to x and y directions in the finite difference approach.

(c) Since the time is discretized the equation 2.5.1. represent an instantaneous balance at the end of a time increment.

Now

$$Q_1 = T_{i-1,j,2} (h_{i-1,j} - h_{ij}) \frac{\Delta y}{\Delta x}$$

$$Q_2 = T_{i,j,2} (h_{i,j} - h_{i+1,j}) \frac{\Delta y}{\Delta x}$$

$$Q_3 = T_{i,j,1} (h_{i,j+1} - h_{ij}) \frac{\Delta x}{\Delta y}$$

$$Q_4 = T_{i,j-1,1} (h_{i,j} - h_{i,j-1}) \frac{\Delta x}{\Delta y}$$

are the horizontal flows,

where,

$T_{i,j,1}$ = Aquifer transmissivity within the vector volume between node i, j and i, j+1

$T_{i,j,2}$ = Aquifer transmissivity within the vector volume between nodes i, j and i+1, j

h^k_{ij} = calculated heads at the end of time increment measured from arbitrary reference level in the node i, j

k = time ordinate

The vertical flows are

Q_5 = $S\Delta x\, \Delta y\, (h^k_{i,j} - h^{k-1}_{i,j}) / \Delta t$

change in the system node storage

Q_6 = constant withdrawal rate from the vector volume of node i, j

Q_n = special leakage term

when $\Delta x = \Delta y$

After putting the values and simplifying the water balance equation 2.5.1. becomes

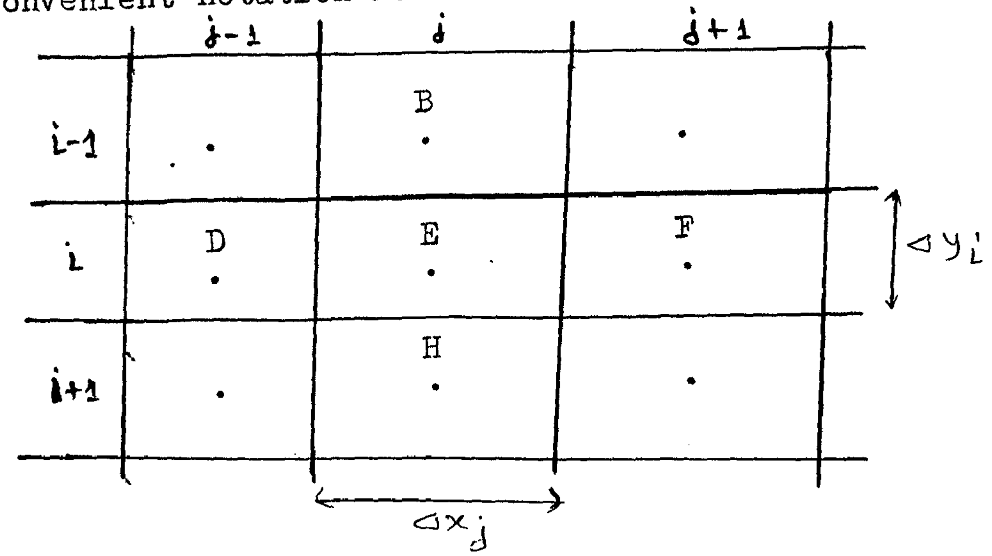
$$T_{i-1,j,2}(h^k_{i-1,j} - h^k_{i,j}) + T_{i,j,2}(h^k_{i+1,j} - h^k_{i,j}) + T_{i,j,1}(h^k_{i,j+1} - h^k_{i,j}) + T_{i,j-1,1}(h^k_{i,j-1} - h^k_{i,j}) = \frac{S\Delta x^2}{\Delta t}(h^k_{i,j} - h^{k-1}_{i,j}) + Q_n^k \quad ..(2.5.2)$$

So this is the flow equation of nonsteady state two dimensional flow. Prickett and Lonnquist followed this approach.

2.5.2. Conventional Mathematical Approach

This approach was first used by Pinder and Bredehoeft (1968). They got the equation 20 similar to 2.5.2.

Trescott et al. (1976) used the same thing with more convenient notation for 3 x 3 node the grid is as follows-



The equation 2.2.1. may be approximated as

$$\frac{1}{\Delta xj}\left[\left(T_{xx}\frac{\partial h}{\partial x}\right)_{i,j+1/2}-\left(T_{xx}\frac{\partial h}{\partial x}\right)_{i,j-1/2}\right]$$

$$+\frac{1}{\Delta y_i}\left[\left(T_{yy}\frac{\partial h}{\partial y}\right)_{i+1/2,j}-\left(T_{yy}\frac{\partial h}{\partial y}\right)_{i-1/2,j}\right]$$

$$=\frac{S_{ij}}{\Delta t}\left(h^k_{i,j}-h^{k-1}_{i,j}\right)+W^k_{i,j}+L^k_1+L^k_2$$

The same equation is again approximated as

$$\frac{1}{\Delta xj}\left[T_{xx(i,j-1/2)}\frac{h^k_{i,j+1}-L^k_{i,j}}{\Delta x_j+1/2}-T_{xx(i,j-1/2)}\frac{h^k_{i,j}-h^k_{i,j-1}}{\Delta x_j-1/2}\right]$$

$$+\frac{1}{\Delta y_i}\left[\left(T_{yy(i+1/2,j)}\frac{h^k_{i+1,j}-h^k_{i,j}}{\Delta y_{i+1/2}}-T_{yy(i-1/2,j)}\frac{h^k_{i,j}-h^k_{i-1,j}}{\Delta y_{i-1/2}}\right]$$

$$=\frac{S_{ij}}{\Delta t}\left(h^k_{i,j}-h^{k-1}_{i,j}\right)+W^k_{i,j}+L^k_1+L^k_2$$

This can now be converted as

$$F_{ij}\left(h^k_{i,j+1}-h^k_{i,j}\right)-D_{i,j}\left(h^k_{i,j}-h^k_{i,j-1}\right)$$

$$+H_{i,j}\left(h^k_{i,+1,j}-h^k_{i,j}\right)-B_{i,j}\left(h^k_{i,j}-h^k_{i-1,j}\right)$$

$$=\frac{S_{ij}}{\Delta t}\left(h^k_{i,j}-h^{k-1}_{i,j}\right)+W^k_{i,j}+L^k_1+L^k_2 \qquad \ldots(2.5.3.)$$

Where, $B_{i,j}=\dfrac{\left[\dfrac{2\,T_{yy(i,j)}\cdot T_{yy(i-1,j)}}{T_{yy(i,j)}\Delta y_{i-1}+T_{yy(i-1,j)}\Delta y_i}\right]}{\Delta y_i}$

The term in the brackets is the harmonic mean of

$$\frac{T_{yy(i,j)}}{\Delta y_i} \quad \text{and} \quad \frac{T_{yy(i-1,j)}}{\Delta y_{i-1}}$$

Similarly,

$$D_{i,j} = \frac{\left[\dfrac{2T_{xx(i,j)}\, T_{xx(i,j-1)}}{T_{xx(i,j)}\, \Delta x_{j-1} + T_{xx(i,j-1)}\, \Delta x_j}\right]}{\Delta x_j}$$

$$F_{ij} = \frac{\left[\dfrac{2T_{xx(i,j)}\, T_{xx(i,j+1)}}{T_{xx(i,j)}\, \Delta x_{j+1} + T_{xx(i,j+1)}\, \Delta x_j}\right]}{\Delta x_j}$$

and

$$H_{i,j} = \frac{\left[\dfrac{2T_{yy(i+1,j)}\, T_{yy(i,j)}}{T_{yy(i,j)}\, \Delta y_{i+1} + T_{yy(i+1,j)}\, \Delta y_i}\right]}{\Delta y_j}$$

The use of harmonic mean ensures continuity accross the cell boundaries at steady state and it make the appropriate coefficient zero at no flow boundaries.

2.6. CONVERGENCE, STABILITY AND ERROR CRITERIA

Suppose that the exact solution of the differential equation is S_1, the exact solution of the finite difference method solution is S_2, and the numerical method solution of the difference equation with the help of computer is S_3, then $| S_1 - S_2 |$ is called the truncation error and $| S_2 - S_3 |$ is called the numerical or round up error (Bear, 1967).

The condition of convergence of the solution is that $|S_1 - S_2| \longrightarrow 0$ every where in the solution domain.

The condition of stability is that every where in the solution domain $| S_2-S_3 | \longrightarrow 0$

The problem is to find S_3 such that over the whole region of interest $| S_1-S_3 |$ is smaller than some error criteria.

As $S_1-S_3 = (S_1-S_2) + (S_2-S_3)$ thetotal error S_1-S_3 is made up of the truncation error and the round off error. The truncation error is due to the arbitrary form selected for the finite difference equation and often the larger part of the total error. In the development of the digital techniques it was soon recognised that unless the time interval Δt is sufficiently small compared to th the net spacing Δx, Δy sometimes radically wrong occurs in the calculating process. In particular any small error made at time't_o' and the round off errors are inevitable in numerical work - is propagated and increases with each successive computations at $t_o + \Delta t$, $t_o + 2\Delta t$, $t_o + 3\Delta t$ etc. After a few such time steps the error terms completely over shadow the desired solution of the problem, making the complete calculation worthless. Such behaviour is termed as the computational instability and is desired to be avoided. As per stability criteria for one dimensional parabolic equation of the form

$\frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t}$ $0 < x < a$, $t > 0$ with constant head boundary condition or Dirichlet conditions prescribed at ends $x = 0$ and $x = a$ and also initial condition at time $t = 0$ is $r = \frac{\Delta t}{\Delta x^2} < 1/2$

The stability criteria for two dimensional parabolic equation of the form under discussion is $r = \Delta t \left(\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} \right) \leq 1/2$.

For example of $\Delta x = \Delta y = 0.1$ then $\Delta t \leq 0.0025$ will satisfy the condition in homogeneous unit.

2.7. EXPLICIT METHOD OF SOLUTION

In this method the value of the unknown at the end of the time step is solely based on the known values at the beginning of the time step. The second derivative with respect to x can be written at the mesh point (i, j) as

$$\frac{\partial^2 h}{dx^2} = \frac{h^k_{i-1,j} - 2\,h^k_{i,j} + h^k_{i+1,j}}{(\Delta x)^2} \qquad ..(2.7.1)$$

Similarly the second derivative with respect to y at (i, j) is

$$\frac{\partial^2 h}{\partial y^2} = \frac{h^k_{i,j-1} - 2\,h^k_{i,j} + h^k_{i,j+1}}{(\Delta y)^2} \qquad ..(2.7.2)$$

The time derivative is written in forward in time scheme or explicit scheme in the form

$$\frac{\partial h}{\partial t} = \frac{h^{k+1}_{i,j} - h^k_{i,j}}{\Delta t} \qquad ..(2.7.3.)$$

Hence the solution of the general equation in two dimension with standard boundary condition in mathematical equation

$$\frac{\partial^2 h}{dx^2} + \frac{\partial^2 h}{dy^2} = \frac{\partial h}{\partial t} \qquad ..(2.7.4.)$$

$0 < x < a, \quad 0 < y < b, \quad t > 0$

Dirichlet condition or constant head condition prescribed in the boundaries

$x = 0, \quad x = a$

$y = 0, \quad y = b$

isgiven by substituting (2.7.1), (2.7.2) and (2.7.3) in (2.7.4)

as
$$h^{k+1}_{i,j} = h^{k}_{i,j} + \Delta t \left[\frac{h^{k}_{i-1,j} - 2h^{k}_{i,j} + h^{k}_{i+1,j}}{\Delta x^2} + \frac{h^{k}_{i,j-1} - 2h^{k}_{i,j} + h^{k}_{i,j+1}}{\Delta y^2} \right] \quad .. (2.7.5)$$

Giving $h^{k+1}_{i,j}$ in terms of known values explicitly.

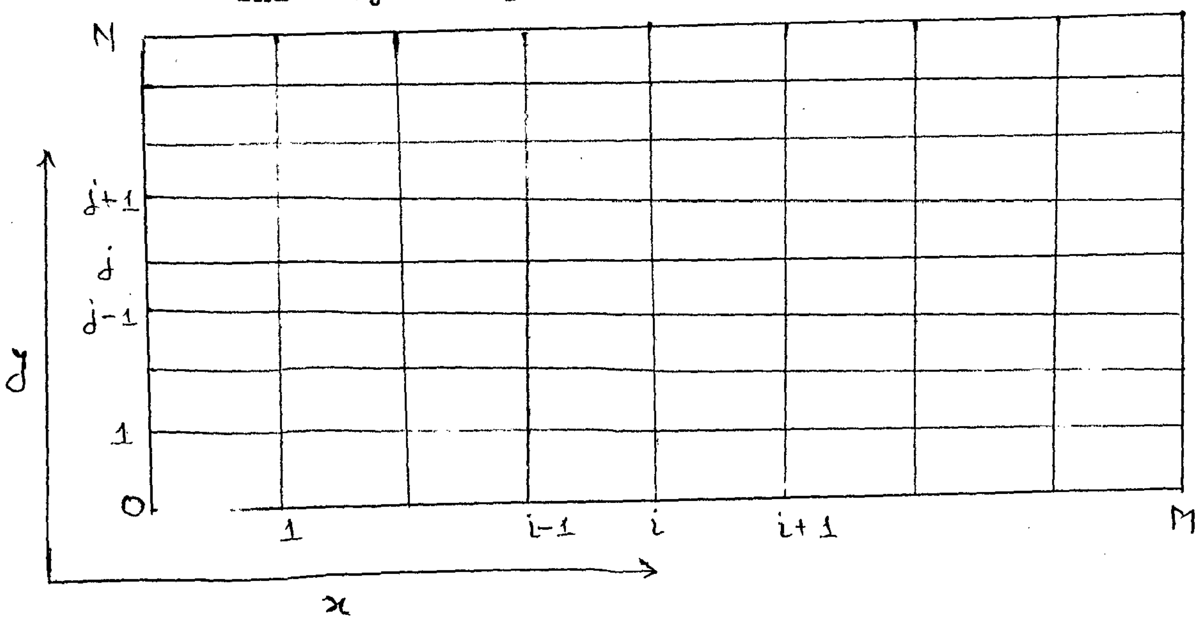
The demerit of the explicit method is that for convergence criteria we have to continue the process of evaluation for a long time wasting valuable computer time labour and money.

2.8. IMPLICIT METHOD OF SOLUTION

In this method the value of any unknown at the end of the time step is a function of the (1) known values at the beginning of the time step and also the remaining (2) unknown values at the end of the time step.

The region $0 \leq x \leq a$ and $0 \leq y \leq b$ is subdivided into M intervals of each width Δx along x-axis and into N interval each width Δy along y axis such that $M \Delta x = a$ and $N \Delta y = b$.

This may be represented in figure as follows.



The time co-ordinate is denoted along the perpendicular to x-y plane. Two dimensional table showing h_{ij} is to be written corresponding to the above figure for each time step Δt for recording calculations. Choosing K as suffix corresponding to t such that K^{th} time level is given by $K\Delta t$ and assuming that the solution has been obtained upto K^{th} level the value is to be known for (K+1) th level.

The differential equation of (2.7.4) at the mesh point (i,j, k + 1/2) can be written by a backward in time scheme or implicit scheme as

$$\frac{h^{k+1}_{i,j} - h^{k}_{i,j}}{\Delta t} = \frac{1}{\Delta x^2}\left[\frac{h^{k+1}_{i-1,j}+h^{k}_{i-1,j}}{2} - 2\left(\frac{h^{k+1}_{i,j}+h^{k}_{i,j}}{2}\right) + \frac{h^{k+1}_{i+1,j}+h^{k}_{i+1,j}}{2}\right] + \frac{1}{\Delta y^2}\left[\frac{h^{k+1}_{i,j-1}+h^{k}_{i,j-1}}{2} - 2\,\frac{h^{k+1}_{i,j}+h^{k}_{i,j}}{2} + \frac{h^{k+1}_{i,j+1}+h^{k}_{i,j+1}}{2}\right] \quad \text{.. (2.8.1)}$$

After simplification the equation can be written as

$$a\, h^{k+1}_{i-1,j} + b\, h^{k+1}_{i,j} + c\, h^{k+1}_{i+1,j} + d\, h^{k+1}_{i,j-1} + e\, h^{k+1}_{i,j} = f^{k}_{i,j} \quad \text{..(2.8.2)}$$

Where a, b,c,d and e are constants and $f^{k}_{i,j}$ is a known function of the values at the k^{th} level. In total (M-1) (N-1) equations similar to (2.8.2) for i = 1, 2... M-1 and j = 1,2-- N-1 will be obtained. Since the boundary values are known there are (M-1) x (N-1) unknown which can be solved simultaneously solving the equations. This procedure is stable. This procedure had been developed by O' Brien Hymen and Kaplan and Weber (1966) applied this method for unconfined aquifer modelling.

2.8.1. Average difference Implicit Procedure

This procedure uses an average of the approximation at the beginning and end of the time step. This procedure was developed by Crank and Nicolson. Knowles, Claborn and Wells used this method for solution of the unconfined aquifer in 1972. As per average difference procedure

$$\frac{\partial h}{\partial x} = \frac{h^{k+1}_{m,n} + h^{k}_{m,n} - h^{k+1}_{i,j} - h^{k}_{i,j}}{2 L_{m,n}}$$

Where $L_{m,n}$ is the distance along the direction of flow between the nodes m,n and i,j. They have used water balance approach for deriving the final equation using gradient of the Darcy's law by the above equation. The change in water table elevation with respect to time has been approximated by $\frac{\partial h_{i,j}}{\partial t} = \frac{h^{k+1}_{i,j} - h_{i,j}}{\Delta t}$

substituting the values in the water balance equation of the type (2.5.1.) for unconfined case (T variable) they got the final equation

$$\frac{1}{16} \sum_{m,n} (K_{i,j} + h_{m,n}) \frac{W_{m,n}}{L_{m,n}} (h^{k}_{i,j} + h^{k+1}_{i,j} - 2BL_{i,j} + h^{k}_{m,n} + h^{k+1}_{m,n} - 2 BL_{m,n})(h^{k}_{m,n} + h^{k+1}_{m,n} - h^{k}_{i,j} - h^{k+1}_{i,j}) + A_{i,j} Q_{net\, i,j} = \frac{A_{i,j} S_{i,j}}{\Delta t} (h^{k+1}_{i,j} - h^{k}_{i,j})$$

..2.8.3.

Where,

$W_{m,n}$	=	Width of the face shared by the node i,j and any adjoining node m,n.
$BL_{i,j}$	=	Elevation of the bottom of the node,i,j
$A_{i,j}$	=	Cross section area for the node i,j
$K_{i,j}$	=	Permeability of node i,j

From the equation (2.8.3) $h_{i,j}^{k+1}$ is estimated. This system is valid for linear system of all spaces and time increments.

2.8.2. <u>Alternative Direction Implicit Procedure</u>

Peaceman and Rechford (1955) first proposed A.D.I. method which is more efficient than either the explicit or implicit schemes, when the problem is more than one dimension. A.D.I. method of Peaceman and Rachford (also called A.D.P.) is carried in two stage. For going to (k+1) th level from kth level they replaced one of the second derivatives at the kth level and the other at (k+1)th level. Thus the equation (2.7.4) becomes

$$\frac{h_{i-1,j}^{k} - 2\,h_{i,j}^{k} + h_{(k+1),j}^{k}}{(\Delta x)^2} + \frac{h_{i,j-1}^{k+1} - 2h_{i,j}^{k+1} + h_{i,j+1}^{k+1}}{(\Delta y)^2} = \frac{h_{i,j}^{k+1} - h_{i,j}^{k}}{\Delta t} \qquad ..(2.8.4)$$

Next stage when they go (k+1)th to (k+2)th level they reversed the process and followed implicit scheme. So in the second case the equation is -

$$\frac{h^{k+2}_{i-1,j} - 2h^{k+2}_{i,j} + h^{k+2}_{i+1,j}}{(\Delta x)^2} + \frac{h^{k+1}_{i,j-1} - 2h^{k+1}_{i,j} + h^{k+1}_{i,j+1}}{(\Delta y)^2} = \frac{h^{k+2}_{i,j} - h^{k+1}_{i,j}}{\Delta t} \quad ..(2.8.5)$$

Only one of the two terms in the left hand side of each of these two equations includes unknown while the other at old time. Thus (N-1) equations in N-1 unknown since $h^{k+1}_{i,0}$ and h^{k+1}_{iN} are known because of the boundary condition can be framed for the first column with 2.8.4. Similarly other column are computed. In the second stage with the equation 2.8.5. the calculation is for rows. Similarly there will be M-1 equations for M-1 unknown. These are solved for first row. Same thing is repeated for other rows. The process is again reversed from (k+2) to (k+3) level and so on. Stage 1 and 2 together make the thing stable unconditionally. The advantage of this method over Gaussian elemination is that the computer storage necessary is greatly reduced. Eshett and Langenbaugh (1965) used this method.

Originally Peaceman and Rachford discussed the above procedure for the solution of steady state Laplace equation. It was extended by Pinder and Bredehoeft (1968) to transient case. They calculated the head values for each node in the matrix by solving equation implicitly for rows at half time interval i.e. solving for $k - \frac{\Delta t}{2}$ with reference to $k-\Delta t$. The equation for columns are then solved implicitly for further time interval. It is stated by this way the equation converges to true solution. Both the original and transient case discussed so far in non-iterative method. The iterative alternative direction implicit (I.A.D.I.) was first suggested by Prickett and Lonnquist (1971). This modification of A.D.I. involves first for a given time increment, reducing a large set of simultaneous equations down to a number of small sets. This is done by solving the node equations by Gauss elemination of individual column of the model while all terms related to the nodes in the adjacent column are held constant. According to Peaceman and Rachford the set of column equations is then implicit in the direction along the column and explicit in the direction orthogonal to the column alignment. The process is repeated to improve the solution so it is iterative procedure.

Knowles (1974) modified I.A.D.I. to suit the compared aquifer of artesian and water table condition at the same program. This has been written in Fortran-V language and published by Texas Water Board. Trescott et al (1976) have modified the non-iterative A.D.I of Pinder and Bredehoeft (1968) and accepted some features of Prickett and Lonnquist (1971) and then published a comprehensive program for iterative A.D.I. While in the same organisation Konikow (1974) tried the Tresscott et al program for some U.S. unconfined aquifer and found some difficulty in convergence with I.A.D.I. approach (Trescott et al 1976).

Upto 1970 A.D.I. was the only efficient option availabl for numerical solution. For many of the field problems A.D.I.

is convergent and competitive in terms of the computational work required, with respect to other iterative techniques available. However, it may be difficult, to obtain a solution for some field problems with A.D.I. For example steady state simulations involving extreme variable coefficient case.

2.9. LINE SUCCESSIVE OVER-RELAXATION METHOD (LSOR)

L.S.O.R. method improves the head values of one row (or column) at a time. Whether the solution is oriented along rows or column is generally immaterial for isotropic problem but has a significant effect on the convergence rate in anisotropic condition (Trescott et al 1976). The solution should be oriented in the direction of the larger coefficients. The differences in the magnitude of the coefficients may result from anistropic transmissivity or from a large difference of grid spacing between the x and y directions. Considering equation 2.5.3 with the solution oriented along rows, an intermediate value is computed by the line Gauss- Seidel iteration formula. If h^+ is the intermediate head value at node (i,j) the equation (2.5.3) is written as

$$Dh^+_{j-1} + Eh^+ + Fh^+_{j+1} = Q\lambda$$

$$j = 1,2 \ldots\ldots\ N_x \qquad (2.9.1)$$

and $Q_\lambda = W - Bh^n_{i-1} - Hh^{n-1}_{i+1} - \frac{S}{\Delta t}\ h^{k-1}$

In the matrix form the equation is written as $\bar{\bar{A}}_\lambda\ \bar{h}^+ = \bar{Q}_\lambda$

(2.9.2)

In order to reduce rounding error the equation (2.9.2) is written in the residual form by adding and subtracting

$\bar{\bar{A}}_{\lambda} \; \bar{h}^{\,n-1}$ to the right hand side of the equation

$$\therefore \quad \bar{\bar{A}}_{\lambda} \; \bar{\pi}^{\,+} = R_{\lambda}^{\,n-1} \qquad (2.9.3)$$

where $\bar{\pi}^{\,+} = \bar{h}^{\,+} - \bar{h}^{\,n-1}$

and $\bar{R}_{\lambda}^{\,n-1} = \bar{Q}_{\lambda} - \bar{\bar{A}}_{\lambda} - \bar{\bar{A}}_{\lambda} \; \bar{h}^{\,n-1}$

Equation (2.9.3) is the L.S.O.R. residual formulation and expanded has the following form for a 3x3 problem.

$$\begin{bmatrix} E_1 & F_1 & & & & & & & \\ D_2 & E_2 & F_2 & & & & & & \\ & D_3 & E_3 & 0 & & & & & \\ & & 0 & E_4 & F_4 & & & & \\ & & & D_5 & E_5 & F_5 & & & \\ & & & & D_6 & E_6 & 0 & & \\ & & & & & 0 & E_7 & F_7 & \\ & & & & & & D_8 & E_8 & F_8 \\ & & & & & & & D_9 & E_9 \end{bmatrix} \begin{bmatrix} \pi_1^{+} \\ \pi_2^{+} \\ \pi_3^{+} \\ \pi_4^{+} \\ \pi_5^{+} \\ \pi_6^{+} \\ \pi_7^{+} \\ \pi_8^{+} \\ \pi_9^{+} \end{bmatrix} = \begin{bmatrix} R_{\lambda 1}^{n-1} \\ R_{\lambda 2}^{n-1} \\ R_{\lambda 3}^{n-1} \\ R_{\lambda 4}^{n-1} \\ R_{\lambda 5}^{n-1} \\ R_{\lambda 6}^{n-1} \\ R_{\lambda 7}^{n-1} \\ R_{\lambda 8}^{n-1} \\ R_{\lambda 9}^{n-1} \end{bmatrix}$$

The first is solved by the Thomas algorithm for simultaneous equation with a tridiegonal coefficient metric. In certain problems, the rate of convergence of LSOR can be improved by applying a one dimensional correction (1DC) procedure introduced by Watts (1971) or the extended two dimensional correctior (2DC) method described by Aziz and Settari (1972).

2.10. STRONGLY IMPLICIT PROCEDURE (SIP)

The set of equations corresponding to 2.5.3 for a 3 x 3 problem may be expressed in the matrix form as

$$\bar{\bar{A}}\,\bar{h} = \bar{Q} \quad \ldots \qquad (2.10.1)$$

Direct solution of the equation by Gaussian elimination usually requires more works and computer storage than iterative methods for problems of practical size because $\bar{\bar{A}}$ decomposes into a lower triangular matrix with non-zero elements from B to E in each row and an upper triangular matrix with non-zero elements from E to H in each row. All of these intermediate coefficients must be computed during Gaussian elimination, and the coefficients in upper triangular matrix must be saved for backward substitution. To reduce the computational time and storage requirement of direct Gaussian elimination, Stone (1968) developed the iterative method using approximate factorization. In this approach a modifying matrix $\bar{\bar{B}}$ is added to $\bar{\bar{A}}$ forming $(\overline{\overline{A + B}})$ so that the equation (2.10.1) becomes

$$(\overline{\overline{A + B}})\ \bar{h} = \bar{Q} + \bar{\bar{B}}\,\bar{h} \qquad (2.10.2)$$

$(\overline{\overline{A + B}})$ can be made close to $\bar{A}$ but can be factored into the product of a lower triangular matrix $\bar{\bar{L}}$ and an upper triangular matrix $\bar{\bar{U}}$, each of which has no more than three non-zero elements in each row regardless of the size of N_x and N_y. Therefore, if the right hand side of equation (2.10.2) is known, simple recursion formulas can be derived, resulting in a considerable savings of computer time and storage. This leads to the iteration scheme -

$$(\overline{\overline{A + B}})\ \bar{h}^{n} = \bar{Q} + \bar{\bar{B}}\ \bar{h}^{n-1} \qquad (2.10.3)$$

Writting 2.10.3 in residual form

$$(\overline{\overline{A + B}})\ \bar{\pi}^{n} = \bar{R}^{n-1} \qquad (2.10.4)$$

in which $\bar{\pi}^{n} = \bar{h}^{n} - \bar{h}^{n-1}$ and $\bar{R}^{n-1} = \bar{Q} - \bar{\bar{A}}\ \bar{h}^{n-1}$

The iterative scheme defined by equation (2.10.3) and (2.10.4) is closer to direct methods of solution (more implicit) than A.D.I. (hence the term strongly implicit procedure or SIP) The SIP algorithm requires (1) relationships among elements of $\bar{\bar{L}}$ $\bar{\bar{U}}$ and ($\overline{\overline{A + B}}$) defined by rules of matrix multiplication for the equation

$$\bar{\bar{L}}\ \bar{\bar{U}} = (\overline{\overline{A + B}}) \qquad (2.10.5)$$

and (2) relationships among the elements of $\bar{\bar{A}}$ and ($\overline{\overline{A + B}}$) $\bar{\bar{L}}$ and $\bar{\bar{U}}$ have the following form for a general 3 x 3 problem.

$$\bar{\bar{L}} = \begin{bmatrix} \theta_1 \\ \beta_2 & \theta_2 \\ 0 & \beta_3 & \theta_3 \\ \alpha_4 & 0 & \beta_4 & \theta_4 \\ & \alpha_5 & 0 & \beta_5 & \theta_5 \\ & & \alpha_6 & 0 & \beta_6 & \theta_6 \\ & & & \alpha_7 & 0 & \beta_7 & \theta_7 \\ & & & & \alpha_8 & 0 & \beta_8 & \theta_8 \\ & & & & & \alpha_9 & 0 & \beta_9 & \theta_9 \end{bmatrix}$$

$$\bar{\bar{U}} = \begin{bmatrix} 1 & \delta_1 & 0 & n_1 & & & & & \\ & 1 & \delta_2 & 0 & n_2 & & & & \\ & & 1 & \delta_3 & 0 & n_3 & & & \\ & & & 1 & \delta_4 & 0 & n_4 & & \\ & & & & 1 & \delta_5 & 0 & n_5 & \\ & & & & & 1 & \delta_6 & 0 & n_6 \\ & & & & & & 1 & \delta_7 & 0 \\ & & & & & & & 1 & \delta_8 \end{bmatrix}$$

The product $\bar{\bar{L}} \; \bar{\bar{U}} = (\overline{\overline{A + B}})$ is

$$= \begin{bmatrix} \hat{E}_1 & \hat{F}_1 & 0 & \hat{H}_1 & & & & & \\ \hat{D}_2 & \hat{E}_2 & \hat{F}_2 & \hat{G}_2 & \hat{H}_2 & & & & \\ 0 & \hat{D}_3 & \hat{E}_3 & \boxed{\hat{F}_3} & \hat{G}_3 & \hat{H}_3 & & & \\ \hat{B}_4 & \hat{C}_4 & \boxed{\hat{D}_4} & \hat{E}_4 & \hat{F}_4 & \boxed{\hat{G}_4} & \hat{H}_4 & & \\ & \hat{B}_5 & \hat{C}_5 & \hat{D}_5 & \hat{E}_5 & \hat{F}_5 & \hat{G}_5 & \hat{H}_5 & \\ & & \hat{B}_6 & \boxed{\hat{C}_6} & \hat{D}_6 & \hat{E}_6 & \boxed{\hat{F}_6} & \hat{G}_6 & \hat{H}_6 \\ & & & \hat{B}_7 & \hat{C}_7 & \boxed{\hat{D}_7} & \hat{E}_7 & \hat{F}_7 & \boxed{\hat{G}_7} \\ & & & & \hat{B}_8 & \hat{C}_8 & \hat{D}_8 & \hat{E}_8 & \hat{F}_8 \\ & & & & & \hat{B}_9 & \boxed{\hat{C}_9} & \hat{D}_9 & \hat{E}_9 \end{bmatrix}$$

CHAPTER-III

SELECTION AND DESIGN OF MODEL

3.1. SELECTION OF APPROPRIATE MODEL

As already discussed digital computer is the main tool for regional type of aquifer evaluation and studies. So computer facility available is the main deciding criteria to decide about the type of solution procedure to be followed. The nearest computer centre is the Structural Engineering Research Centre, Roorkee having a I.B.M. 1620 computer with memory strength of 40 K. The secondary storage with magnetic tapes or magnetic disks or magnetic drums is also not available with this computer Centre. So except the consideration of small problems like well hydraulic no regional type of aquifer evaluation should be considered with this computer.

The India Meteorological Department headquarters in New Delhi we have I.B.M. 360 model 44 level 4 type computer. The computer centre has not added any standard library subroutine generally used for modern programming. So this computer can be used for limited purpose with 128 K memory space available.

The Delhi University we have I.B.M. 360 model 44 level 5 type computer. This centre is having library subroutines and memory space 128 K. Apart from this this computer has got some facility for secondary storage with magnetic tape.

Second deciding factor is the convergence of the model solution. From the history of past modelling examples it was found that Prickett's and Lannquist's I.A.D.I. is most suitable to our situation within the limit of available computer facility. Strongly implicit procedure is known to the most efficient computer program but this program with all allied features requires 228 K memory storage. So Prickett's program of I.A.D.I. or M.A.D.I. is accepted for computer simulation at these circumstances.

3.2. DECISION ABOUT CO-ORDINATE AXES

From the contour of the ground water table it was observed that in natural state flow takes place from the North to the South and in the case of pumping the same situation prevails. Apart from this due to excess pumping in this area ground water inflow in taking place from the East and the West. Hence as discussed in the theory, to keep the transmissibility vector co-linear with the coordinate axes, the axes were drawn parallel to the latitude and longitude lines passing through the project area.

3.3. DECISION ABOUT THE GRID SIZE

Ruston (1977) has performed large number of computer model and left his idea regarding the permissible mesh spacing in aquifer problems solved by the finite difference techniques. His experience is as follows.

An increase in the mesh spacing is an alternative means of economy. The choice of the mesh size is usually a matter of expediency. In general it is considered that the aquifer should be modelled by approximately 1000 nodes with minimum of 250 nodes. If there is insufficient detail in the vicinity of a well or river, then the correction can be made. The approximation

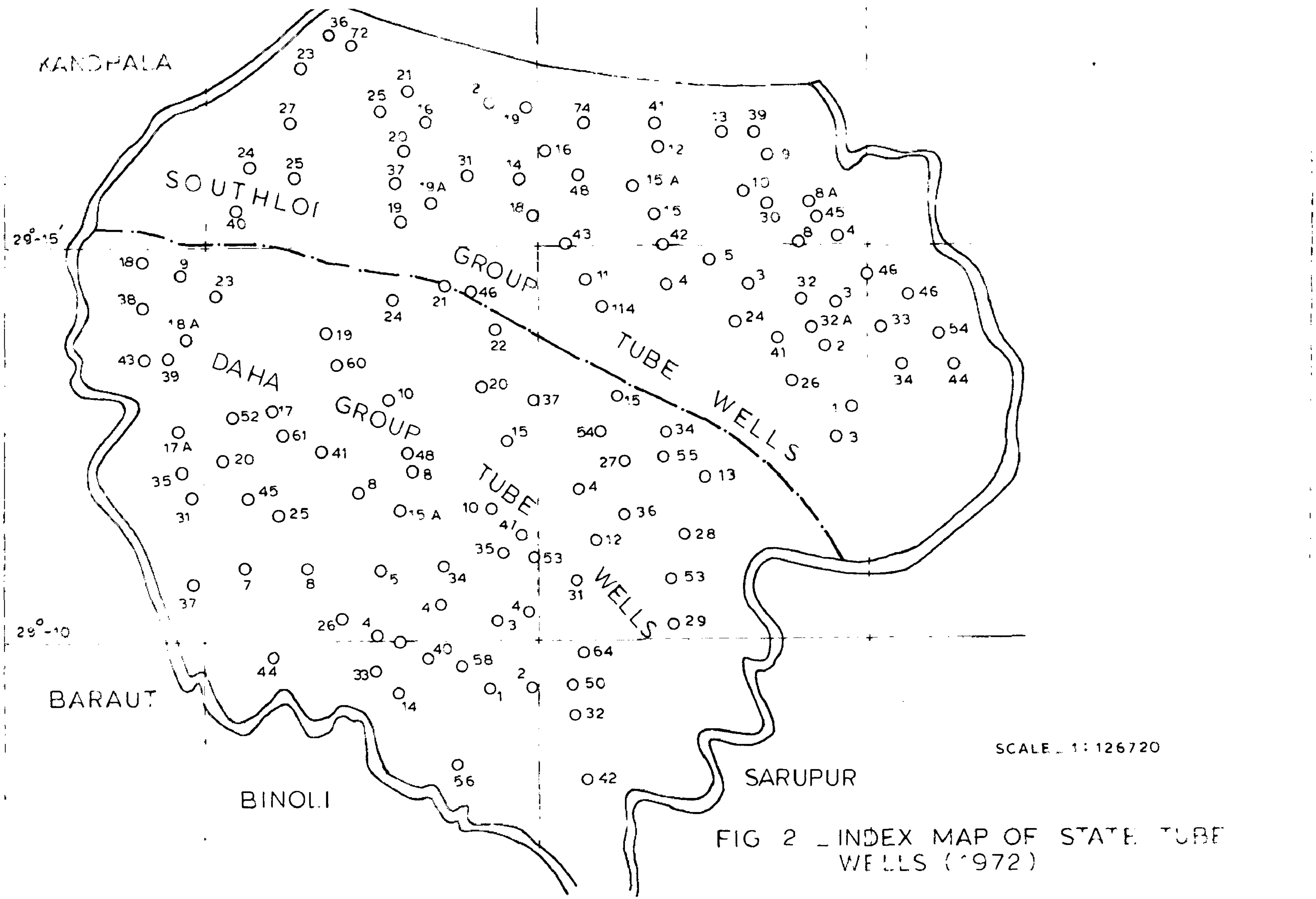


FIG 2 _ INDEX MAP OF STATE TUBE WELLS (1972)

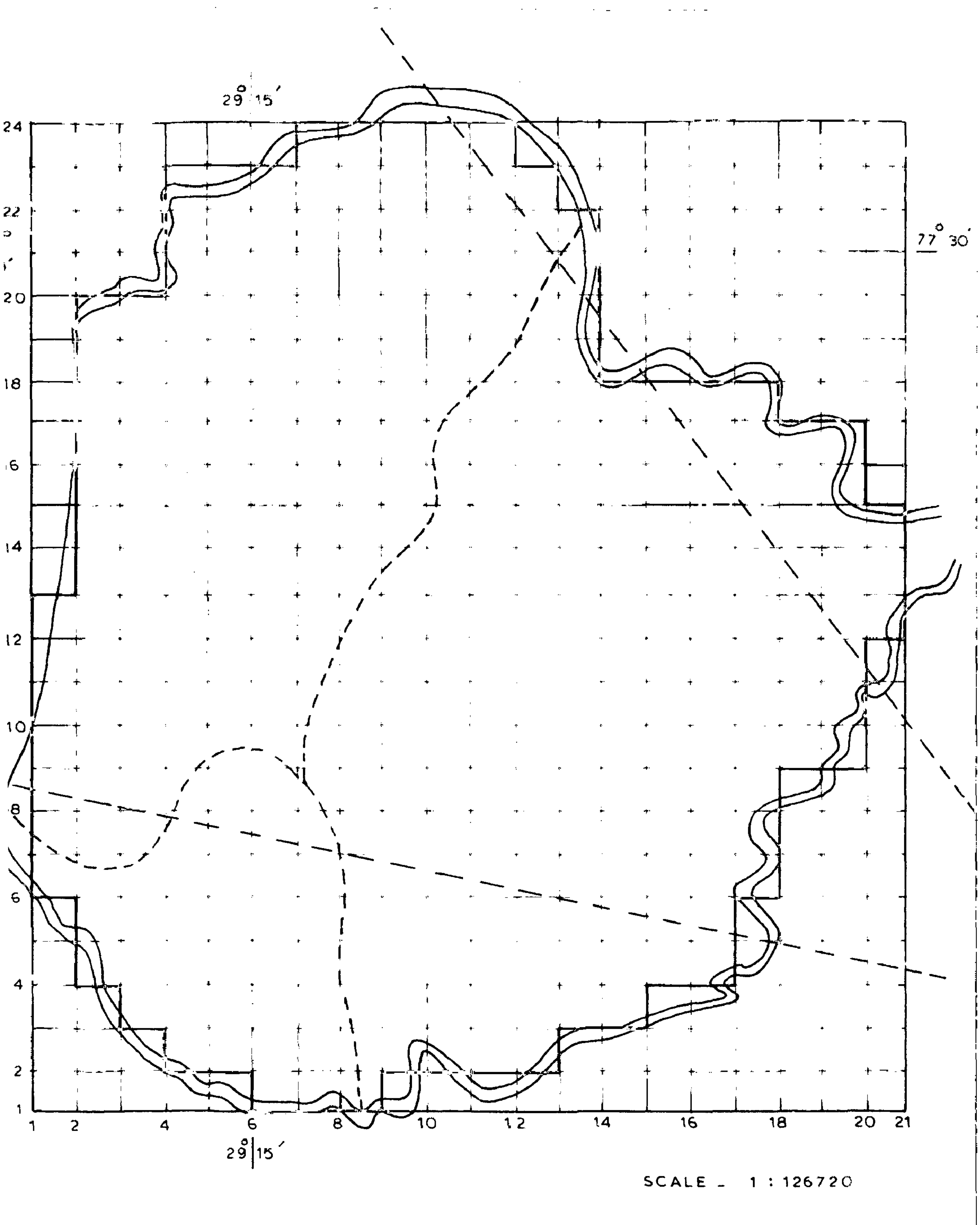


FIG 3 FINITE DIFFERENCE GRID SUPERIMPOSED ON PROJECT AREA

inherent in the finite different can be expressed as a truncation error which consists of the higher order terms of the Taylor's expansion. However, the actual magnitude of these higher order terms can only be calculated for the simplest of the problems. Nevertheless, the first term of the truncation error does show that the errors are proportional to the square of the mesh interval. Thus the errors usually increase significantly with the larger mesh intervals. Further, the truncation error indicates that if the surface to be represented is different from a smooth parabolic shape then inaccuracies are certain to occur. Since no general rules can be made about the permissible size of the mesh interval the aim of the above paper of Ruston was to explore the effects of changing mesh spacing for a number of typical examples.

Considering all these aspect the mesh spacing was kept at 1014 metres square shape. As the complete project area was having large number of withdrawl points through state tubewells, private tube wells, masonary wells with Rahets and drinking water well there was no scope of flexible grid design in point of interests as usually the situation in the Western countries (Vide figure no.2). Thus the entire area was covered with 24 columns and 21 rows having 374 as internal nodes and 230 external nodes of uniform grid size. The grid superimposed on project area marked as figure no.3.

3.4. DECISION ABOUT TIME STEP

Indian agriculture is monsoon oriented and from time immemorial timing of agriculture is governed by the moon's position in the constellation known as Nakshatras. Again this relation of position takes place on monthly basis. So keeping our national need this model was designed to give head prediction once in 30 days. This major time step was divided into 11 minor time steps with non-uniform time increments. Prickett (1971) was of the opinion that small time increment are needed for accuracy when the water levels are fluctuating rapidly (as when a well first starts pumping), the small time increment becomes less and less important as time goes on because water level fluctuations slow in their rate of decline. Thus, a rapid fluctuation in water level dissipate, it is desirable to attempt to use larger time increments because the total number of time steps (N steps) and program iteration (Iter) can then be reduced for a more efficient program. Trescott et al (1977) expressed similar view. So 1.2 was kept as the factor of increment and 0.933 day was as initial delta. In the eleventh time step by increment of 1.2 in constant rate will become 30 days.

Regarding the second aspect of the time step it is related with the stability and convergence criteria. Although the method adopted was unconditionally stable this factor was also taken into account so that the same data can be used uniformly in different type of model. Ruston was of the opinion that frequently the size of the time step can not be increased because of stability or convergence criteria (Ruston et al 1977). This principle applies not only to explicit approximations but also to certain central difference approximations (Ruston -1973).

3.5. REPRESENTATION OF BOUNDARY CONDITION

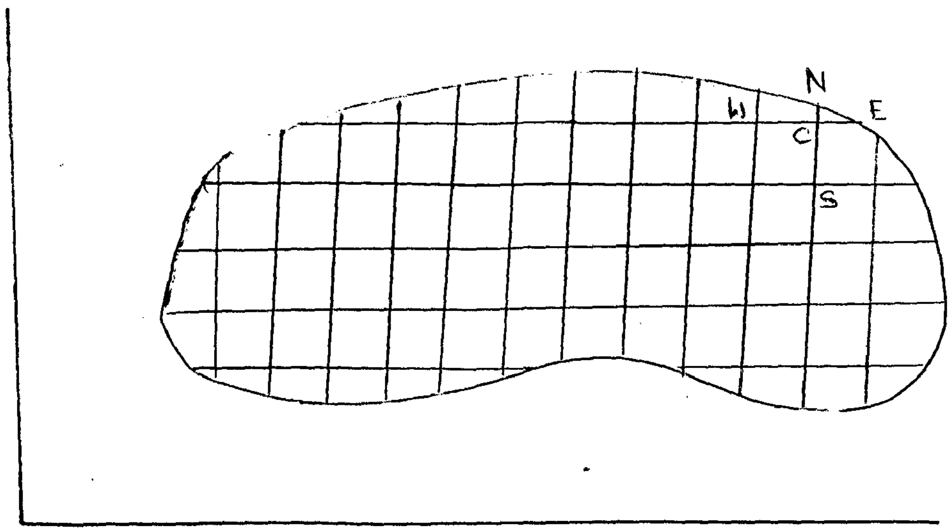
In solving a specific physical problem, such a flow of a liquid through a specified porous medium domain, it is necessary to choose from the infinite number of possible solutions only that one that satisfies certain additional conditions imposed by the physical situation at the boundaries of the considered domain. These are called boundary conditions and the problem referred to as a boundary value problem. If the problem is one in which the dependent variables are also time dependent (i.e. unsteady flow), the boundary conditions must be specified for all times $t \geq 0$. In general any specification of boundary conditions for the second order partial differential equation considered here should include (a) the geometric shape of the boundary and (b) a statement of how the dependent variable 'h' (or ϕ), and/or its derivatives vary on the boundary.

In the theory of partial differential equation a problem in which the value of the function is prescribed or the value of the head 'h' is known then the condition encountered is known as constant head boundary or Dirichlet condition boundary. Constant head boundary is specified by assigning a negative storage coefficient to the nodes that defines the constant head boundary. This indicates to the program that these nodes are to be skipped in the computation.

In the theory of partial differential equation a problem in which the specified value of the normal derivatives is known meaning thereby along the boundary, the flux normal to the boundary surface (or curve in two dimensional flow) is prescribed for all points of the boundary as a function of (x,y,t) then it is the case of constant flux boundary or Neumann condition boundary. A constant flux may be zero (impermeable boundaries) or have a finite value. A zero flux boundary is treated by assigning a value of zero transmissibility to nodes outside the boundary. The harmonic mean of the transmissivity at the cell boundary is zero, and consequently, the flux accross the boundary is zero. A no flow boundary is inserted around the border of the model as a computational expediency and constant head or finite flux boundaries are placed inside the border. A finite flux boundary is treated by assigning recharge (or discharge) wells to the appropriate nodes.

When a combination of function values and its derivatives is known or prescribed such condition in the theory of partial differential equation is known as a mixed condition.

In actual practice such as a river always irregular in shape. The question may arise how in exact mathematics the situation can be accomodated.



Considering the most general case when both the derivatives are not possible to be replaced as N and E are nodal points.

Let $CE = \alpha \Delta x$ and $CN = \beta \Delta y$

Using Taylor's expansion for h_E and h_W about h_C we have

$$h_E = h(x + \alpha\Delta x, y)$$
$$= h_C + \alpha\Delta x \frac{\partial h}{\partial x} + \frac{(\alpha\Delta x)^2}{2} \frac{\partial^2 h}{\partial x^2} + \ldots\ldots\ldots$$

$$h_W = h(x - \Delta x, y)$$
$$= h_C - \Delta x \frac{\partial h}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 h}{\partial x^2} - \ldots\ldots\ldots$$

Eliminating $\partial h/\partial x$ from the above two equations we get

$$h_E + \alpha h_W = h_C (1+\alpha) + \frac{\alpha(1+\alpha)}{2} \Delta x^2 \frac{\partial^2 h}{\partial x^2}$$

$$\therefore \frac{\partial^2 h}{\partial x^2} = \frac{2}{\alpha(1+\alpha)} \frac{h_E + \alpha h_W - (1+\alpha) h_C}{\Delta x^2} \qquad (3.5.1.)$$

This equation (3.5.1.) is required for finite difference replacement for the second derivative in the direction of x. When α is equal to 1 we get out old equation.

Similarly

$$\frac{\partial^2 h}{\partial y^2} = \frac{2}{\beta(1+\beta)} \frac{h_N + \beta h_S - (1+\beta) h_C}{(\Delta y)^2} \qquad (3.5.2.)$$

So if h_E and h_N is known as a position in the boundary the situation can be solved.

Initially it was presumed that the two rivers in the east and west of the doab are forming the ground water divide.

But from the ground water contour it has been proved that ground water inflow is taking place from the east and the west apart from the North due to natural gradient. So to simulate such boundary situation recharge well node equivalent to total inflow to the system has been considered. Due to natural gradient some ground water outflow is taking place from southern boundary. That situation has been simulated by keeping southern nodes as equivalent discharge wells. Thus considering the field condition the representation of the boundary river was not required in this model and the effects of the river was represented by the net recharge boundary.

3.6. REPRESENTATION OF INITIAL CONDITION

The initial condition as expressed by Tresscott et al (1977) has actually prevailing in the project area. Tresscott writes - ' If the initial conditions are specified so that the transient flow is occuring in the system at start of the simulation, it should be recognized that water level will change during the simulation not only in response to the new pumping stress, but also due to the initial conditions. So the program accommodate this feature of the design criteria. Some time in the simulation process, the important results are not the computed head but the changes in the head caused by this stress such as pumping in heads. Prickett was much interested with this aspect of the design criteria. So apart from initial condition design Prickett's idea was also tested.

Ruston and Wedderburn (1973) had experience of aquifer simulation in different aquifer starting condition. The gist of their observation are as follows. When analysing the time variant aquifer problems it is important that the initial conditions at the start of the calculation are correct. In most aquifer problems, seasonal variations cause the inputs to the aquifer to vary with time. Thus the aquifer is in a dynamic state with water both flowing from one part of the aquifer to another and being taken into or released from storage. Therefore, at the instant chosen to be the starting point of the solution, the dynamic state of the aquifer must be represented adequately.

There are four starting conditions that are frequently used.

(a) The heads within the aquifer are all zero.

(b) The heads correspond to a steady state solution due to the inflow and other conditions which apply at the starting point of the calculation.

(c) The heads resulting from a steady state solution with average values of the inflows and outflows are used.

(d) The heads are in a state of dynamic balance.

(1) This has been observed that for most aquifers (d) is the only satisfactory condition from which to start the analysis. So if possible the aquifer should be in state of dynamic balance before the calculation commences.

(2) Alternatively the calculation should start from an average steady state solution with a period of time of $\frac{t\,T}{L^2 S} > 1.0$ preceeding the period under investigation.

Where

T = Transmissibility

S = Storage coefficient

L = Length of one dimensional aquifer

t = time

(3) If it is necessary to start from zero head then the period of time of $t\,T/L^2 S > 2.5$ should preceede the period under investigation.

(4) For aquifer having different properties over certain area, the term $L^2 S/T$ should be replaced by $\Sigma\, L^2 S/T$.

These recommendations assume that each aquifer can be idealized as a one-dimensional problem in which the water leaves the aquifer at one of its boundaries. If the aquifer does not conform to this idealization, then the length L, should be taken as the longest flow path of water within the aquifer.

3.7. CONVERGENCE TEST, ERROR

All computer programmes for aquifer evaluations should include an internal check on the errors that are inherently present in solving the finite difference equations. These are four following types of error checks possible as follows, any one of which is applied depending on the type of problem.

(1) To keep the running check of the water balance of the system.

(2) Fixing upper limit on the maximum change in water level between iteration in any node.

(3) Subsituting the drawdown or heads back into finite difference equation and evaluating error that exists.

(4) Controlling the sum of the changes in heads in all nodes during the iteration over the entire model.

The fourth type works well with aquifer problems concerning the regional analysis. (Prickett - 1968).

A rule of thumb for choosing the initial value of Error which had been found useful for both uniform and non-uniform time increments, is given by the following empirical equation

$$ERROR = Q \times DELTA/10 \times SF_2$$

Where

Q = total net withdrawl rate of model

DELTA = Initial time increments in days

SF_2 = Average storage factor

This concept was tested and the model was run with various values of the error term.

CHAPTER-IV

HYDROLOGICAL DATA AND PROCESSING

4.1. TEST PUMPING EXPERIMENT DATA

The knowledge of the hydraulic properties of the aquifer is of prime requirement to decide about the type of aquifer so that appropriate differential equation may be used. Test pumping experiment was conducted on 21-10-75 to 28-10-75. Elaborate processing and study has been done on this data to decide about the type of aquifer and value of the aquifer parameters. On the basis of the study the aquifer has been declared as unconfined aquifer and the accepted values of the aquifer parameters are as follows.

4.1.1. Transmissivity

It is the product of the average permeability of the aquifer (k) and the thickness of the aquifer (h). It is designated as T and it has the dimension $L^2 T^{-1}$. In the homogeneous unit metre square per day the accepted value of T was taken as 1771.2 m^3/day/metre.

4.1.2. Storage Coefficient

This is the volume of water released or stored per unit surface area of the aquifer per unit change in the component of head normal to the surface. It is dimensionless and designated as S and the value of S was found as 0.118.

4.2. WATER REQUIREMENT OF CROPS

The water requirement of crop in any area in any crop period depends on the actual rainfall and the effective part of the rainfall for the evaportranspiration need of the crops.

4.2.1. Rainfall data and processing

Table no.1 gives the monthly rainfall pattern in the Daha area for the three effective raingauges situated within the area or the nearby area. Apart from the three raingauges at Budhana Kandla and Sardhana the nearest Bagpat raingauge was also considered to draw Thiessen polygon. The Thiessen areas have been demarcated in dotted line in the figure no.3. As the Bagpat raingauge was not found to be effective in the area it was not required for processing and calculation. The area was thus divided to three homogeneous rainfall areas for the calculation of crop water requirements.

4.2.2. Estimation of consumptive use

The consumptive use can be estimated by various methods but the present day practice is with pan evaporation method which is based on the average meteorological data of the area. Class A pan evaporation data was known from the publication of India Meteorological Department for the nearby city of Meerut. The data has been given in the Table no.2. The consumptive use has been obtained by multiplying the pan evaporation (E_p) with crop consumptive use coefficient k. Standard table is available from which the value of k for a percentage of growing season is calculated for each group of crops.
The cropping pattern from the three blocks are available as given in the table no.3. The blocks are further sub-divided into the homogeneous rainfall zones. Thus the total area has

(All figures are in Millimeter)

Raingauge Station	Year	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Tc
BUDHANA	1972	2.5	40.2	4.7	4.3	0.0	19.0	249.4	249.7	70.4	16.0	0.0	2.6	643.
	1973	3.0	5.2	1.2	0.0	12.0	84.0	290.2	168.4	27.8	93.0	0.0	3.0	687.
	1974	0.0	0.0	0.0	0.0	28.0	19.0	132.0	176.2	16.2	0.0	0.0	8.0	379.
	1975	24.0	20.0	7.0	0.0	0.0	40.7	141.8	154.2	38.2	11.1	0.0	0.0	436.
KANDLA	1972	0.0	34.0	9.0	28.0	0.0	39.0	275.0	190.4	172.1	41.1	25.0	2.0	815.
	1973	4.0	7.0	12.0	0.0	0.0	8.0	230.8	206.4	55.0	16.3	0.0	0.0	539.
	1974	5.5	3.2	0.0	0.0	12.0	22.0	156.0	113.0	135.0	5.0	0.0	7.0	456.
	1975	20.0	18.2	1.2	0.0	0.0	46.0	121.0	186.0	374.0	16.0	0.0	0.0	782.
SARDHANA	1972	3.2	83.0	13.2	8.0	0.0	28.8	244.6	188.0	101.2	6.8	11.4	5.6	693.
	1973	11.6	5.2	0.0	1.0	11.2	99.0	152.6	69.4	44.6	41.2	0.0	0.0	435.
	1974	0.0	0.0	0.0	0.0	4.8	16.2	187.06	186.78	34.4	5.2	0.0	8.8	443.
	1975	32.0	2.8	8.2	0.0	0.0	79.0	302.06	282.44	253.0	52.2	0.0	0.0	1112.

TABLE NO. 2 - NORMAL PAN EVAPORATION (CLASS A PAN) AND POTENTIAL EVAPOTRANSPIRATION OF DAHA AREA

(All figures are in millimeters)

Sl. No.	Month	Pan Evaporation	Potential Evapotranspiration
1.	January	61.0	53.1
2.	February	100.0	75.1
3.	March	173.0	127.1
4.	April	259.1	174.7
5.	May	322.0	222.2
6.	June	273.1	225.3
7.	July	180.2	163.0
8.	August	152.0	142.1
9.	September	135.0	142.2
10.	October	120.0	111.3
11.	November	82.0	65.9
12.	December	57.0	43.4
	Total	1914.0	1545.4

TABLE NO. 3 - CROPPING PATTERN OF DAHA AREA

(All areas are in Hectares)

Area	Wheat	Sugar cane	Rice	Other Kharif	Other Rabi	Total	Forest & Orchard
KANDHLA							
A	1989	1238	133	1213	595	5168	Nil
C	1683	1159	39	922	438	4241	6
Total	3672	2397	172	2135	1033	9409	6
BUDHANA							
B	5788	3604	387	3529	1731	15039	114
D	6894	4766	161	3796	1803	17420	160
Total	12682	8370	548	7325	3534	32459	274
SARDHANA							
E	1082	739	29	600	287	2737	452
Grand Total	17436	11506	749	10060	4854	44605	732

been divided into 5 homogeneous zones, A,B,C,D and E.

Out of the consumptive use of the different crops a portion is met by the effective rainfall. Effective rainfall is a function of total rainfall, consumptive use rate and net water applications. The standard table of normal effective rainfall as related to normal monthly rainfall and average monthly consumptive use was used to calculate the effective rainfall in different period.

The difference between the consumptive use and effective rainfall gives the net water requirement (NIR). The net water requirement was divided by 0.65 to calculate field water requirement (FWR) by taking 65 percent as irrigation efficiency. Again field water requirement was divided by 0.90 to calculate gross irrigation require (GIR) by taking 90 percent as delivery efficiency.

4.3. GROUND WATER BALANCE OF THE AREA AS A LUMPED SYSTEM

As ground water is the only source of supplying water to the crops to supplement effective rainfall a ground water balance study of the area became a must. The equation of the hydrologic equillibrium provides a quantitative statement of this balance. The periodic hydrological ground water balance of the area is written as

$$R_r + R_c + R_I + I_g + S_I = S_E + O_g + E_T + T_P + \Delta S$$

Where,

R_r = Recharge due to rainfall

R_c = Recharge from canal seepage

R_I = Recharge from Irrigation water

I_g	=	ground water inflow to the basin
S_I	=	Influent seepage from streams
S_E	=	Effluent seepage to the stream
O_g	=	ground water outflow from the basin
E_T	=	Evaporation and transpiration from ground water
T_P	=	Draft from ground water
ΔS	=	Change in storage of the aquifer.

The parameter of the water balance equation were calculated as follows -

4.3.1. Recharge due to rainfall (R_r)

Following empirical formulas are available for this purpose -

1. Chaturvedi formula $R = 2.0\ (P-15)^{2/5}$
2. I.R.I. formula $R = 1.35\ (P-14)^{1/2}$
3. Amritsar formula $R = 2.5\ (P-16)^{1/2}$

None of the formula is satisfactory to take care of all possible hydrological condition. So this factor was calculated from water balance equation by balancing inputs and output in different percentage of recharge (22 percent in rainy period 14 percent in other period).

4.3.2. Recharge from canal seepage (R_c)

There is no surface water irrigation canals in the area. Ground water irrigation channels has been taken into account in the factor of calculating recharge due to irrigation. There is no small stream within the area. Hence R_c is 0.

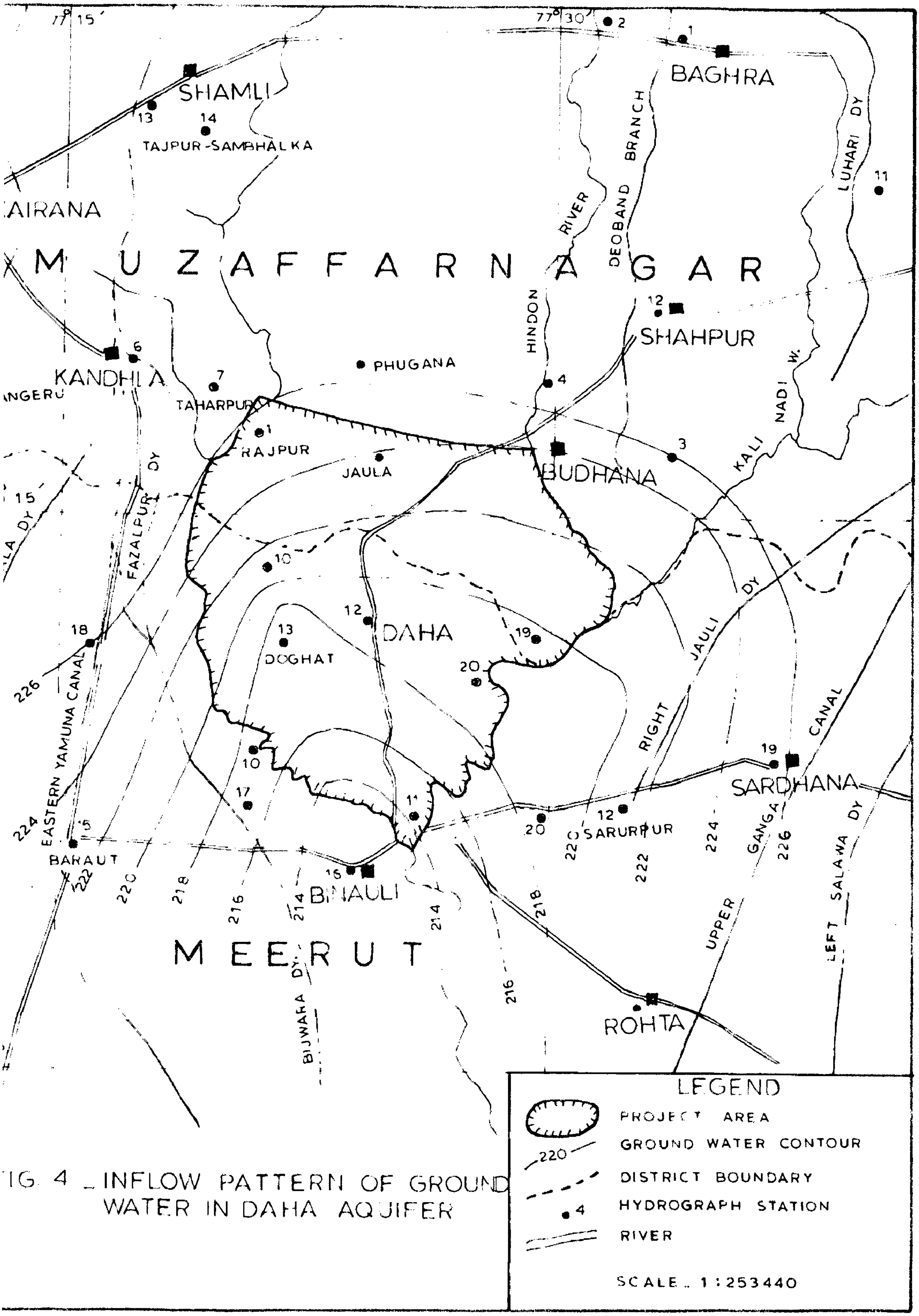


FIG. 4 _ INFLOW PATTERN OF GROUND WATER IN DAHA AQUIFER

4.3.3. Recharge from Irrigation water (R_I)

As per practice adopted in the calculation of crop water requirement by taking 65 percent as irrigation efficiency and 90 percent as delivery efficiency the net efficiency of the irrigation method comes to 59.5 percent and, hence 40.5 percent is the loss to the ground water as return flow. Thus 40.5 percent was taken as the recharge due to irrigation.

4.3.4. Ground Water Inflow to the basin (I_g)

This factor was calculated from the ground water contour map prepared from the ground water table observation data. The working formula is $Q = TIL$

Where Q = Inflow, T = Transmissivity, I = average gradient and L = average length across which the inflow is taking place. However this process was used only for inflow from North. Inflow from the East and the West across the river was calculated by different method.

4.3.5. Influent seepage (S_I)

Ground water contour map of the adjoining area of the Ganga canal command area in the East and the Jamuna Canal command area in the East of Daha area was also drawn along with the contour map of study area. Fig.4 will show that the Daha area is receiving water from the adjoining area throughout the year, Thus the influent seepage from the river has also been included in the above inflow calculation.

4.3.5. Effluent Seepage (S_E)

As the area is always receiving water there is no effluent seepage accept in the southern portion which has been included in the ground water outflow.

4.3.6. Ground water outflow (O_g)

Ground water outflow was calculated in the similar way as ground water inflow.

4.3.7. Evaporation and Transpiration from ground water (E_T)

Total area of forest and big trees were given as described in the table no.3 and the potential evaporatin transpired by the trees for different months were collected from India Meteorological Department publication for the nearest meteorological station. Thus E_T values were computed.

4.3.8. Ground Water withdrawal (T_P)

For each state tubewell of the area running hour and the discharges were collected regularly. A sample survey was conducted for calculating the draft through private tube wells, pumping sets and Rahets. From the sample survey average running hour and discharges of private tube well, pumping sets and masonary well with Rahet were calculated. Regarding drinking water draft there are drinking water wells in each village at the rate of 100 users per well. So the draft was taken as 5 metre cube per day per well.

Item	-75	
	4 to 31.10.74	1.11.74 to 31.5.75
T_P	.4393	6501.2740
E_T	.1950	557.7874
O_g	.0196	38.1740
ΔS_g	.8128	-2492.1788
$T_P+E_T+O_g+\Delta S_g$	.8411	4605.0566
R_I	.3184	2047.9013
R_C	0	0.0
I_g(North)	.8759	278.0911
I_g(East and West)	.2892	2279.0642
$R_1+R_C+I_g(N)+I_g(E+W)$	.4835	4605.0566
R_d	3576	0.0
Percentage rainfall		0

4.3.9. Change in storage (ΔS)

The change in the ground water storage was determined by drawing the water table contours in the doab at the specified time interval. The using these contours and the mean elevation and effective porosity or S change in storage ΔS was determined.

4.3.10. Ground water balance discussion

The total draft calculated from draft figures was not found comparable with the gross irrigation water requirement by consumptive use concept. So detail season wise ground water balance was worked out for the entire period. The ground water inflow from the East and the West was then again scrutimized by further analysis to test the correctness of the water balance equation data. Ground water balance has been given in the table no.4 which is self explanatory.

4.4. PROCESSING OF DATA FOR DISTRIBUTED MODEL

So far the data of seasonal water balance equation is representing the lumped system model. Now the same data is to be distributed to the area so as to represent the approximate distributed model of the area.

4.4.1. Distribution of boundary inflows

From the average gradient of water level contours along the boundaries it was seen that 64.5 percent of the flow takes place in the western boundary and 35.5 percent of the flow takes along the Eastern boundary. Thus the inflow figures were divided between the two boundaries in the proportion 0.645 to 0.355. The seasonal inflows from the water balance equation were used to calculate the cumulative inflow

TABLE-4

GROUND WATER BALANCE OF DAHA AREA

Item	1972-73		1973-74		1974-75	
	1.6.72 to 31.10.72	1.11.72 to 31.5.73	1.6.73 to 31.10.73	1.11.73 to 31.5.74	1.6.74 to 31.10.74	1.11.74 to 31.5.75
T_P	5858.2713	10099.4163	3944.3192	8300.3450	4372.4393	6501.2740
E_T	574.1950	557.7874	574.1950	557.7874	574.1950	557.7874
O_g	24.8887	30.7019	17.2426	32.8268	30.0196	38.1740
ΔS_g	211.7935	-1714.6273	2377.4557	-2701.5167	-927.8128	-2492.1788
$T_P+E_T+O_g+\Delta S_g$	6669.1485	8973.2783	6913.2125	6189.4425	4048.8411	4605.0566
R_I	1845.3554	3181.3160	1242.4605	2614.6087	1377.3184	2047.9013
R_C	0.0	0.0	0.0	0.0	0.0	0.0
I_g(North)	189.6019	256.7676	153.6412	170.2837	152.8759	278.0911
I_g(East and West)	58.4536	5510.1312	939.5593	3404.5501	287.2892	2279.0642
$R_1+R_C+I_g(N)$ $+ I_g(E+W)$	2093.4109	8948.2148	2335.6610	6189.4425	1817.4835	4605.0566
R_d	4575.7376	25.0635	4577.5515	0.0	2231. 3576	0.0
Percentage of rainfall	22	14	22	0	18	0

Note - All units are hectare meters.

4.3.9. Change in storage (ΔS)

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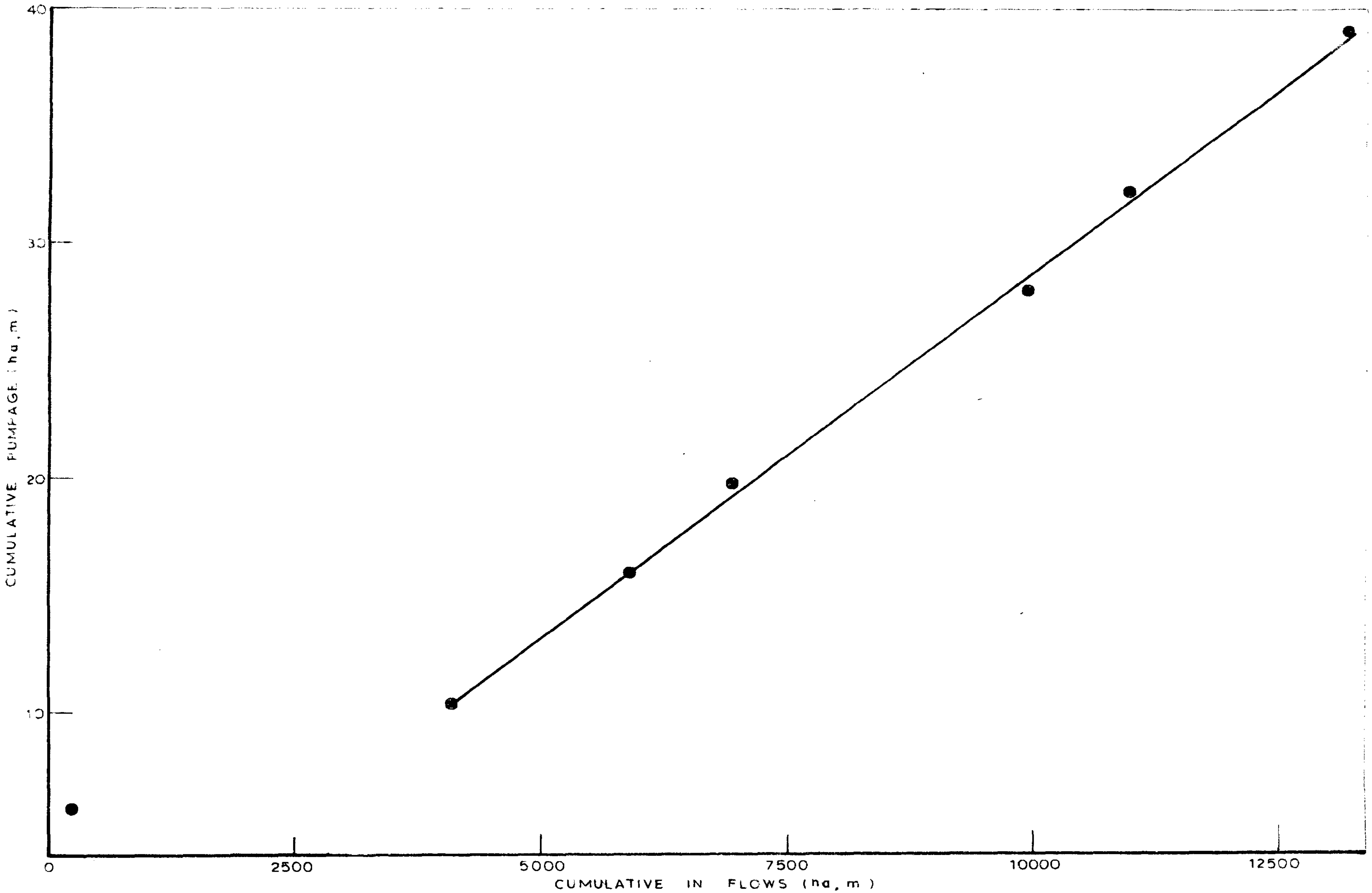

FIG. 5 _ CUMULATIVE IN FLOWS VS CUMULATIVE PUMPAGE IN DAHA AREA

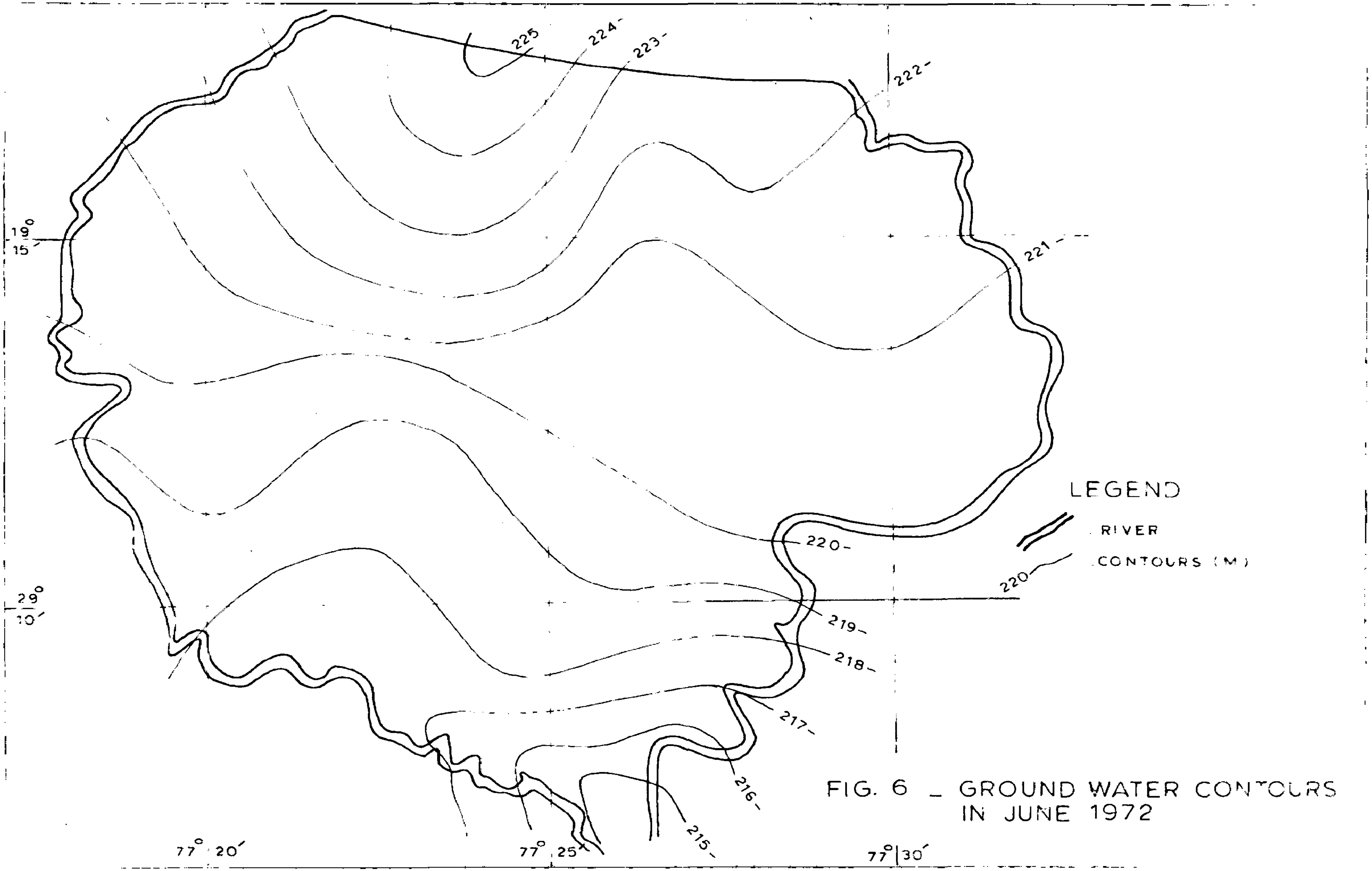

FIG. 6 – GROUND WATER CONTOURS IN JUNE 1972

figures which were plotted against cumulative pumpage figures (Figure 5). Except for the first season which is perhaps due to truncation effect a definite trend was observed. This graph was therefore utilised to work out monthly distribution of inflows across the East and the Western boundaries. Inflows from the North and outflow from the South boundary were known from the monthly water level contours. Now among the nodes of the four boundaries the pulled values were divided equally so as to represent the distributed feature of the lumped system. Apart from the boundary inflow or outflow the boundary nodes represent 1/2 the common internal node net draft.

4.4.2. Computation of Initial condition

Out of 36 monthly observation of ground water table June 1972 represents the first month for which water balance has been worked out. Again June is low water table position. So June 1972 water table data was considered for initial condition. Out of 21 observation points of water table elevation 18 observation points were within the doab. Ground water contour drawn as shown in the figure 6. This figure was used to interpolate the intermediate nodal point heads all 374 internal nodes. This gives the distributed feature of the initial condition. Appendix-3 nodal values indicate the computed figures of the initial condition.

4.4.3. Computation stress matrix

From the blockwise cropping pattern data and Theissen plygon areas the entire Daha area was divided into 5 homogeneous pumping areas A,B,C,D and E. Ground water draft due to net pumpage in these homogeneous area nodes are kept equal in each node of a single area. The net pumpage is the actual draft adjusted by return flow, rainfall recharge and evapotranspiration due to big trees. Monthly net draft was distributed from the seasonal net draft figures in the proportion of the crop water requirement calculated in the consumptive use concept for each month. Next as per the number of nodes in each sub area the nodal values were computed. Boundary nodes got 1/2 weightage. Appendix-3 nodal values indicate the final computed figures. As draft was taken as +ve, recharge has been taken as -ve for mathematical treatment of the data.

CHAPTER-V

PROGRAMMING AND RESULTS

5.1. SPECIAL FEATURE OF PROGRAM TO REPRESENT WATER TABLE CONDITION

Water table condition has been illustrated in figure-7. It shows a well pumping from an quifer that is unconfined on the top and water is being released from the storage by gravity drainage of the inter stices in the portion of the aquifer being dewatered. Gravity drainage of the interstices decreases the saturated thickness of the aquifer and therefore the aquifer transmissivity.

In designing the computer model the aquifer is discretized in the usual manner, and values of hydraulic conductivity $PERM_{i,j}$ are assigned to individual vector volume representing different portion of the aquifer. In addition, values of the aquifer bottom elevation $BOT_{i,j}$ was given as 200 metres to the individual nodes of the model.

Bottom figures in the above figure-7 shows a typical aquifer vector volume of the model in which the flow of water is passing through a vector volume that is wedge shape. The equivalent transmissivity of the wedge shape vector volume between the node points i,j and i+1, j is approximated by the formula (Prickett - 1968).

$$T_{i,j,2} = PERM_{i,j,2}\sqrt{(h_{i,j}-BOT_{i,j})\,(h_{i+1,j}-BOT_{i+1,j})}$$

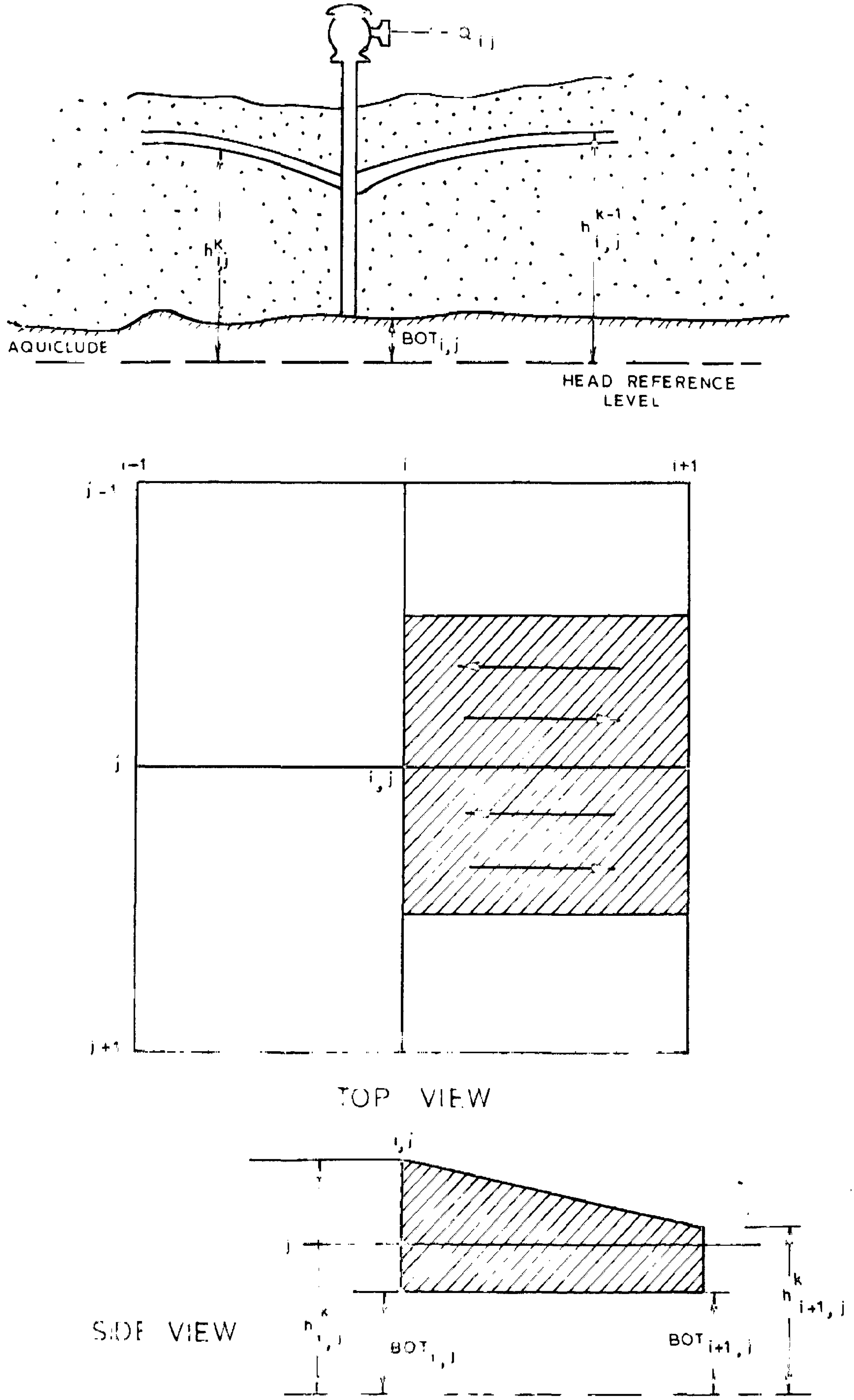

FIG. 7 SIMULATION IN WATER TABLE CONDITIONS

Where

$T_{i,j,2}$ = aquifer transmissivity of the vector volume between i, j and i+1, j.

Similarly, the equivalent aquifer transmissivity of the vector volume between node points i, j and i, j+1 is given by

$$T_{i,j,1} = PERM_{i,j,1} \sqrt{(h_{i,j} - BOT_{i,j})(h_{i,j+1} - BOT_{i,j+1})}$$

Transmissivity calculated with the above two equations represent geometric means which are more accurate than the values computed between nodes, especially when dealing with steep gradient near pumping centres. The flow chart of the computer program has been added in the Appendix-2.

5.2. PROGRAM WITH ACTUAL INITIAL CONDITION

The 374 node points were represented with actual water level recorded in the month of June 1972. The highest water level recorded in the node (12,1) was 225.10 metres and the lowest water level recorded in the node (15,21) was 214.55 metres. In the first successful attempt of the computer solution the ERROR term was kept as 100000. For this strength of data the total memory requirement was 009F34 bytes. The storage requirement in HICORE was 011 DCF and in LOCORE was 004200 for one month computation in 12 minor time steps. The number of iteration in the computation of each minor time step was one only. The error recorded was 11226 in the first minor time step which reduced progressively in different time step and finally to 734 in the 11th time step for 30 days pumping. I.B.M. 360 model 44 with magnetic tape input arrangement took 1 minute 57 seconds for one month computation.

Encouraged by the first attempt the error parameter was reduced to 12000 looking at the maximum error recorded in the first attempt. It was apprehended that more computation time may be consumed due to increased number of iteration for this reduced error parameter. So the program was modified by keeping 11 iteration as the maximum desired for each time step for refinement of heads apart from the general time limit of the computer run as 15 minutes. In punched card input system IBM 360/44 computer required 4 minute 8 seconds. The total memory requirement was 004200 bytes. The storage requirement in HICORE was 01158 F and in LOCORE was 004200. The error recorded was 11225 in the fourth 733 in the 11th time step but 2171 in the 12th. Thus the fluctuation is sinusoidal type with amplitude reducing in the form a damping curve. The number of iteration was only one in each minor time step.

Simulated values were categorised by modification of the program. The heads were divided into 13 classes at an interval of 1 metres each group represented with separate symbol.

For further refinement of heads the error term was again reduced to 8000. The total memory requirement in this attempt was OOAIBO bytes. HICORE storage was 012047 and LOCORE storage requirement was 004200. The error recorded was 7619 in Ist time step continuously decreased then increased finally to 606 and 2048 in the last two time steps. The fluctuation is sinusoidal type with ampltitude reducing in the form of a damping curve. Thus it shows a converging trend. I.B.M. 360/44 with magnetic tape input arrangement took 2 minute 35 second for one month computation.

The simulated values were found to be satisfactory except the east and the west boundary points. A table of the simulated values compared with the actual observed values is given in the table no.5. From the accepted values of the model parameters and the net pumping rates Q_{max} = 50.27 m^3/day Delta/10 = 0.0933 and SF2 = 0.118. So design error suggested should be 5538, one attempt was made with this designed error but the computer remained engaged in the Do loop even one minor time step could not be calculated, with the time of 15 minutes computation time.

5.3. <u>PROGRAM WITH INITIAL CONDITION ALL ZERO</u>

In the major program it may be that actual head is not of any interest but the actual drawdown from a known initial condition is of prime interest for a different type of pumping patterns. In such situation the program is modified. So as per this objective the initial head of all the 374 internal nodes were kept at zero. Remaining 230 external nodes were kept at 200 metre head as the bottom level of the aquifer (like first set of experimental model runs). The error parameter was kept as 100000. The time consumed was 3 minute 54 second in IBM/44 punch card input system. The total memory requirement was 009F38 bytes. The storage requirement in HICORE was 011347 and in LOCORE was 004200. The number of iteration was 11 in the Ist iteration and rest all one. The maximum error recorded was 32208 and

INITIAL CONDITION

Sl. No.	Simulated drawdown (July 1972) (initial 0 level condition)
1.	1.6754
2.	7.4066
3.	0.9130
4.	1.8035
5.	4.2828
6.	1.1354
7.	- 0.1988
8.	1.0657
9.	19.2277
10.	2.2469
11.	-0.4600
12.	1.4308
13	9.4521
14	7.7623
15	0.4719
16	8.7468
17	37.0159
18	14.5976
19	4.1316
20	27.3392
21	20.6211
22	2.0773
23	1.8077
24	7.0082
25	24.1241
26	58.9052
27	41.8322
28	8.5530

TABLE-5

SAMPLE OUTPUT OF SIMULATED HEAD ACTUALLY OBSERVED HEAD IN THE TWO TYPE OF INITIAL CONDITION
(All Units are in metres)

Sl. No.	Node No.	Observed level in June 1972	Observed level in July 1972	Simulated head in July 1972 (initial condition)	Simulated drawdown (July 1972) (initial 0 level condition)
			INTERIOR NODES		
1.	8,2	223.10	223.50	223.0274	1.6754
2.	11,2	224.80	225.20	224.1090	7.4066
3.	6,4	222.00	222.30	221.8619	0.9130
4.	11,4	224.70	225.00	222.5120	1.8035
5.	16,4	222.35	222.65	222.0049	4.2828
6.	5,7	221.00	221.30	220.8628	1.1354
7.	11,7	220.25	220.65	220.1064	- 0.1988
8.	16,7	221.50	221.85	221.1097	1.0657
9.	21,7	221.55	221.95	221.0458	19.2277
10.	5,10	220.00	220.40	219.7654	2.2469
11.	11,10	219.40	219.65	218.7825	-0.4600
12.	16,10	220.60	221.00	220.1537	1.4308
13	21,10	221.25	221.65	220.7029	9.4521
14	5,13	218.80	219.20	218.8867	7.7623
15	11,13	218,55	218.80	217.8221	0.4719
16	16,13	220.05	220.45	219.1408	8.7468
17	20,13	220.55	220.85	219.4984	37.0159
18	8,16	217.65	217.95	216.9478	14.5976
19	12,16	217.20	217.60	216.4982	4.1316
20	16,16	217.55	217.85	216.8246	27.3392
21	13,18	216.00	216.35	215.4393	20.6211
			BOUNDARY NODES		
22	8,1	223.40	223.70	223.3777	2.0773
23	4,2	222.00	222.30	222.0446	1.8077
24	16,2	222.80	222.95	221.7561	7.0082
25	3,15	219.20	219.40	216.9177	24.1241
26	18,15	219.40	221.65	217.0161	58.9052
27	17,20	215.15	215.45	214.2748	41.8322
28	12,21	215.35	215.70	214.9588	8.5530

progressively reduced to 7463 in the 11th time step. Thus a converging trend was noticed.

Encouraged by the above attempt the program was again revised. The error parameter was reduced to 10,000 and category statement of drawdown was attempted below actual drawdown figs. The computer time consumed was 4 minute 14 seconds in the same computer. Maximum recorded error was 3331 in the Ist time step which progressively increased to 9153 in the 12th time step. So converging trend was not noticed in this case. The memory requirement was 0115 F^F. Simulated drawdown has been given in the Table-5.

In the next attempt further modification was thought and only drawdown measurement expressed in category statement was desired. In this attempt the time consumed by the computer was 3 minute 17 seconds. The error recorded was 2950 progressively increased to 9153 in the 12th time step. The total memory requirement was 00A168 bytes. The storage requirement in HICORE was 011577 and in LOCORE was 004200.

5.4. PROGRAM WITH INITIAL CONDITION AND 36 MONTHS VARIABLE NET PUMPING

Among the first two sets of attempts initial condition programs were more converging in nature. In this initial condition program 36 months continuous variable pumping stress matrix was applied to the aquifer model. The program was accepted by the computer. Some model running is being continued to get the simulation of the entire period.

CHAPTER-VI

CONCLUSIONS

1. The lumped system model indicated generally 22 percent of the rainfall recharges in the ground in the doab. This percentage is not same in all the years and during drought year recharge due to rainfall decreases substantially. Thus during 1974-75 when the rainfall was much less than the normal rainfal, the recharge due to rainfall was 18 percent of the rainfall.

2. The annual recharge into the aquifer is of the order of 4600 hectare metres against annual maximum draft of about 16000 hecature metres in 1972. Such a huge overdraft must have been provided by the ground water boundary on the west and the east of the aquifer.

3. Due to overdraft the inflow of ground water has not changed much from the north and outflow to the south has remained more or less same. But inflow from the east and the west was effected much with the draft. This inflow increased with the increase cumulative draft linearly with cumulative pumpage.

4. The distributed model designed by Prickett and Lonnquist was considered to be most suitable for the present problem. Because this method is fairly efficient and competitive to other method. Strongly Implicit Method is known to be most efficient method but it requires memory space more than the capacity of IBM 360/44 and could not be tested.

5. Among the two designs of model analysis, the initial condition programme indicated in most cases the difference between

the simulated head and the observed head within the maximum of one metre. Where as the initial condition program with all zero head indicated wider difference. Such situation has also indicated by the nonconvergence of error terms.

6. More refinement of head is possible by decreasing the error term and thereby increasing the number of iteration. This has been tested by decreasing the error from 100000 to 8000 in different steps.

7. South and North boundary nodes behaved better way than the east and west boundary nodes in the simulation process. This is perhaps due to the artificial representation of boundary condition. However interior nodes remained uneffected.

8. For simulating the river in the boundary more data for river bed characteristics, influent and effluent zone with period should be known. In the absence of this information the revised technique to avoid the river was only a possible working arrangement.

9. Excessive recharge is being allowed in the east and the west boundary nodes. It is suggested that as such point recharge which is an approximation technique should be tried in distance artificial boundary so that actual boundary can be properly simulated. In the absence of data outside the doab this aspect was not tested.

10. Within the limit of the short duration of six month all aspects of distributed model could not be tested. More model run is to be required to prove the model and to make it suitable to improve model behaviour and to impose future condition.

APPENDIX-1

ILLUSTRATIVE SOLUTION OF I.A.D.I

Considering the equation derived from water balance concept to be solved for the required solution, the equation is rearranged to facilitate node equation solving by columns and rows.

Thus

$$h^k_{i,j}\left(T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + \frac{S\Delta x^2}{\Delta t}\right)$$

$$- T_{i-1,j^2,}\ h^k_{i-1,j} - T_{i,j,2}\ h^k_{i+1,j} - T_{i,j,1}\ h^k_{i,j+1}$$

$$- T_{i,j-1,}\ h^k_{i,j-1} = \frac{S\Delta x^2}{\Delta t}\ h^{k-1}_{i,j} + Q^k_{i,j} + Q^k_n \quad (1)$$

<u>Note</u> (W has L has been replaced with Q)

For calculation by columns the equation (1) is rearranged as followed.

$$- T_{i,j-1,1} h^k_{i,j-1} + h^k_{i,j}\left(T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + \frac{S\Delta x^2}{\Delta t}\right)$$

$$- T_{i,j,1}\ h^k_{i,j+1} = \left(\frac{S\Delta x^2}{\Delta t}\right) h^{k-1}_{i,j} + Q^k_{i,j} + T_{i-1,j2}\ h_{i-1,j}$$

$$+ T_{i,j2}\ h^k_{i+1,j} + Q^k_n \quad (2)$$

The equation (2) is of the form

$$AA_j\ h^k_{i,j-1} + BB_j\ h^k_{ij} + CC_j\ h_{i,j+1} = DD_j \quad (3)$$

Where $AA_j = - T_{i,j-1,2}$

$$BB_j = T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + \frac{S\Delta x^2}{\Delta t}$$

$$CC_j = - T_{i,j,1}$$

$$DD_j = \frac{S\Delta x^2}{\Delta t} h^{k-1}_{i,j} + Q^k_{i,j} + T_{i-1,j,2}\, h^k_{i-1,j} + T_{i,j,2}\, h^k_{i+1,j} + Q^k_n$$

Similarly for the calculation by rows equation ~~xx~~ (1) is re-arranged as

$$-T_{i-1,j,2}\, h^k_{i-1,j} + h^k_{i,j}\left(T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + \frac{S\Delta x^2}{\Delta t}\right)$$

$$- T_{i,j,2} h^k_{i+1,j} = \left(\frac{S\Delta x^2}{\Delta t} \right) h^{k-1}_{i,j} + Q_{i,j} + T_{i,j-1,1}\, h^k_{i,j-1}$$

$$+ T_{i,j,1}\, h_{i,j+1} + Q_n \qquad (4)$$

Which is of the form

$$AA_i\, h^k_{i-1,j} + BB_i\, h^k_{i,j} + CC_i\, h_{i+1,j} = DD_i \qquad (5)$$

Where

$$AA_i = - T_{i-1,j,2}$$

$$BB_i = T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + \frac{S\Delta x^2}{\Delta t}$$

$$CC_i = - T_{i,j,2}$$

$$DD_i = \frac{S\Delta x^2}{\Delta t} h^{k-1}_{i,j} + Q^k_{i,j} + T_{i,j-1,1}\, h^k_{i,j-1}$$

$$+ T_{i,j,1}\, h_{i,j+1} + Q^k_n$$

There are three head unknown in each equation written for each node along a column (3) or row (5). In standard matrix form notation a set of equations defined by the equation (3) or (5) forms what is termed a tri-diagonal matrix. The solution of a set of column or row head equation is accomplished by Gauss elimination incorporating what Peaceman and Rachford term G and B arrays applied to tri-diagonal matrices.

The complete illustration is simplified by the presumption that the grid is consisting of four node row as shown in the given figure although method can be applied for any number of nodes.

i =	1	2	3	4
j-1	0	0	0	0
j	0	0	0	0
j+1	0	0	0	0

The heads at the nodes 1,2,3 and 4 along jth row can be calculated by first writing the flow equations (2x2.5) for each node going in order to increasing column number (i). Secondly, the resulting equations are arranged in such a manner that the head at the node of interest $h^k_{i,j}$ is a function of known parameter and the head at only the node $h^k_{i+1,j}$. When this is done, the head at the last node of the row h_{4i} will be a function of known parameters. Finally, all other heads can then be calculated in order of decreasing number of column. Hence the first jth row i.e. i = 1 the equation (5) is

$$AA_1\ h^k_{o,j} + BB_1\ h^k_{1,j} + CC_1\ h^k_{2,j} = DD_1 \qquad (6)$$

Since no node with co-ordinates o,j exists, AA_1 is set equal to zero and the equation (6) becomes

$$BB_1\ h^k_{1,j} + CC_1\ h^k_{2,j} = DD_1 \qquad (7)$$

$$\text{i.e. } h^k_{1,j} = \frac{DD_1}{DD_1} - \left(\frac{CC_1}{BB_1}\right) h^k_{2,j}$$

$$\text{or, } h^k_{1,j} = G_1 - B_1\ h^k_{2,j} \qquad (8)$$

Where $G_1 = \frac{DD_1}{BB_1}$ and $B_1 = \frac{CC_1}{BB_1}$

Now the head at the node of interest $h^k_{1,j}$ is a function of the known parameters G_1 and B_1 and the head at only the node $h^k_{i+1,j}$ or $h^k_{2,j}$.

Proceeding to the next jth row node of the above figure where i = 2, the equation of flow is written as

$$AA_2 \, h_{1,j} + BB_2 \, h^k_{2,j} + CC_2 \, h^k_{3,j} = DD_2 \qquad (9)$$

Solving for $BB_2 \, h^k_{2,j}$ from equation (9) yields

$$BB_2 \, h^k_{2,j} = DD_2 - CC_2 \, h^k_{3,j} - AA_2 \, h^k_{1,j} \qquad (10)$$

Substituting the equation (8) in (10)

$$BB_2 \, h^k_{2,j} = DD_2 - CC_2 \, h^k_{3,j} - AA_2 \, (G_1 - B_1 \, h^k_{2,j})$$

Rearranging the term

$$h^k_{2,j} = \frac{DD_2 - AA_2 G_1}{BB_2 - AA_2 B_1} - \left[\frac{CC_2}{BB_2 - AA_2 B_1} \right] h^k_{3,j}$$

i.e. $$h^k_{2,j} = G_2 - B_2 \, h^k_3 \qquad (11)$$

Where

$$G_2 = \frac{DD_2 - AA_2 \, G_1}{BB_2 - AA_2 \, B_1} \quad \text{and} \quad B_2 = \frac{CC_2}{BB_2 - AA_2 \, B_1}$$

Thus again the head of interest $h^k_{2,j}$ is function of the known parameters G_2 and B_2 and the head at only the node $h_{i+1,j}$ or $h_{3,j}$. Proceeding to the next jth row node where i =3 the flow equation can be written as

$$AA_3 \, h_{2,j} + BB_2 h_{3,j} + CC_3 h_{4,j} = DD_3 \qquad (12)$$

Solving for the terms $BB_3\ h_{3,j}$ yields

$$BB_3\ h^k_{3,j} = DD_3 - CC_3\ h^k_{4,j} - AA_3\ h^k_{2,j} \qquad (13)$$

Substituting equation (11) in (13) gives

$$BB_3\ h^k_{3,j} = DD_3 - CC_3\ h^k_{4,j} - AA_3\ (G_2 - B_2 h_{3,j})$$

Rearranging terms

$$h^k_{3,j} = \frac{DD_3 - AA_3\ G_2}{BB_3 - AA_3\ B_2} - \left[\frac{CC_3}{BB_3 - AA_3 B_2}\right] h^k_{4,j}$$

i.e.

$$h^k_{3,j} = G_3 - B_3\ h^k_{4,j} \qquad (14)$$

Where, $$G_3 = \frac{DD_3 - AA_3\ G_2}{BB_3 - AA_3\ B_2}$$

$$B_3 = \frac{CC_3}{BB_3 - AA_3\ B_2}$$

Finally, the flow equation is written for the last jth row node as

$$AA_4\ h^k_{3,j} + BB_4\ h^k_{4,j} + CC_4\ h^k_{5,j} = DD_4 \qquad (15)$$

Since ter is no node at $h^k_{5,j}$ the term CC_4 is set equal to zero and the equation (15) becomes

$$AA_4\ h^k_{3,j} + BB_4\ h^k_{4,j} = DD_4 \qquad (16)$$

Rearranging terms it gives

$$BB_4\ h^k_{4,j} = DD_4 - AA_4\ h^k_{3,j} \qquad (17)$$

Substituting (14) in (17) it gives

$$BB_4 h^k_{4,j} = DD_4 - AA_4\ (\ G_3 - B_3 h^k_{4,j}\ )$$

$$\therefore\ h_{4,j} = G_4 \qquad (18)$$

Where $G_4 = \frac{DD_4 - AA_4\ G_3}{BB_4 - AA_4\ B_3}$

Since the head $h^k_{4,j}$ is now known, substituting its value in (14) allowes calculation of the head at node $h^k_{3,j}$. The head $h^k_{3,j}$ is then substituting in (11) allowes calculation of head $h^k_{2,j}$. Finally $h^k_{2,j}$ is substituted into equation (8) giving the value $h^k_{1,j}$. Thus all heads in the jth row have been determined.

A study of G and B terms given in the above four series of calculations reveals their general form as

$$G_N = \frac{DD_N - AA_N\ G_{N-1}}{BB_N - AA_N\ BN_{-1}} \qquad (19)$$

and $$B_M = \frac{CC_N}{BB_N - AA_N\ B_{N-1}} \qquad (20)$$

where N = i for row calculations and, by inference, N = j for column calculations. In addition, AA_N is set equal to zero for the first node of a row or column, CC_N is set equal to zero for the last node of the row or column.

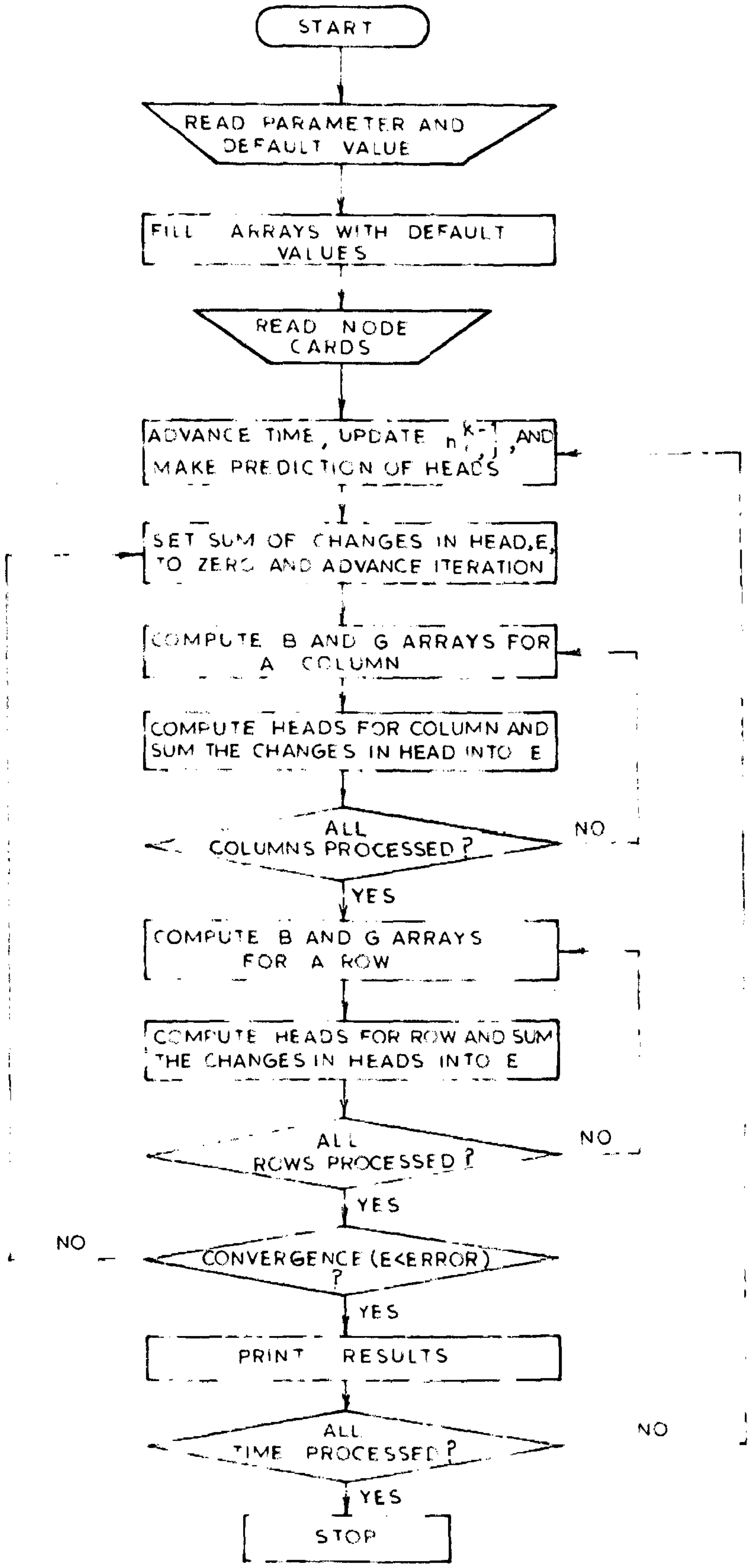

FIG A2 _ FLOW CHART

****

PARAMETER VALUES

****

NSTEPS DELTA [illegible]
[illegible] .933 7 [illegible].

****

DEFAULT VALUES

****

[illegible] *	[illegible] *	TT *	HH *	QQ *	S2 *	PP *	BOTT
24	21	[illegible]	200	0.0	121270	88.56	200

****

CATEGORIES OF HEADS

****

[illegible]	*SHEAD*[illegible]	*SHEAD*	*SHEAD*	*SHEAD*	*SHEAD*	*SHEAD*	*SHEAD*	*SHEAD*	*SHEAD*	*SHEAD*	*SHEAD*
13A	[illegible] 213.C	214.D	215.E	216.F	217.G	218.H	219.R	220.S	221.T	222.U	223.V 224.

****

NODAL NET WITHDRAWAL AND AQUIFER PARAMETER VALUES

****

I*	J*	T1 *	T2 *	H *	Q *	SF2 *	P1 *	P2 *	BOTT *
6	1	1771.2	1771.2	222.80	-581.22	121270.	88.56	88.56	200.
7	1	1771.2	1771.2	223.15	-581.22	121270.	88.56	88.56	200.
8	1	1771.2	1771.2	223.40	-581.22	121270.	88.56	88.56	200.
9	1	1771.2	1771.2	223.75	-4.64	121270.	88.56	88.56	200.
10	1	1717.2	1717.2	224.60	-4.64	121270.	88.56	88.56	200.
11	1	1771.2	1771.2	225.30	-4.64	121270.	88.56	88.56	200.
12	1	1771.2	1771.2	225.10	-4.64	121270.	88.56	88.56	200.
13	1	1771.2	1771.2	224.60	-4.64	121270.	88.56	88.56	200.
4	2	1771.2	1771.2	222.00	-2.80	121270.	88.56	88.56	200.
5	2	1771.2	1771.2	222.40	-2.80	121270.	88.56	88.56	200.
6	2	1771.2	1771.2	222.55	-2.[illegible]	121270.	88.56	88.56	200.
7	2	1771.2	1771.2	222.80	386.60	121270	88.56	88.56	200.
8	2	1771.2	1771.2	223.15	386.60	121270.	88.56	88.56	200.
9	2	1771.2	1771.2	223.55	1539.77	121270.	88.55	88.56	200.
10	2	1771.2	1771.2	224.20	1539.77	121270	88.56	88.56	200.
11	2	1771.2	1771.2	224.80	1539.77	121270.	88.56	88.56	200.
12	2	1771.2	1771.2	224.72	1539.77	121270.	88.56	88.56	200.
13	2	1771.2	1771.2	224.20	-4.64	121270.	88.56	88.56	200.
14	2	1771.2	1771.2	223.70	-4.64	121270.	88.55	88.56	200.
15	2	1771.2	1771.2	223.55	-4.64	121270.	88.56	88.56	200.
16	2	1771.2	1771.2	222.80	-4.64	121270.	88.56	88.56	200.
17	2	1771.2	1771.2	222.65	-4.64	121270.	88.56	88.56	200.
18	2	1771.2	1771.2	222.60	-4.64	121270.	88.56	88.56	200.
19	2	1771.2	1771.2	222.50	-4.64	121270.	88.56	88.56	200.
20	2	1771.2	1771.2	222.40	-4.64	121270.	88.56	88.56	200.
3	3	1771.2	1771.2	221.30	-2.[illegible]	121270.	88.56	88.56	200.
4	3	1771.2	1771.2	221.75	-2.80	121270.	88.56	88.56	200.
5	3	1771.2	1771.2	222.00	[illegible].60	121270.	88.56	88.56	200.
6	3	1771.2	1771.2	222.30	[illegible].60	121270.	88.56	88.56	200.
7	3	1771.2	1771.2	222.40	[illegible].60	121270.	88.56	88.56	200.
[illegible]	3	1771.2	1771.2	[illegible]	[illegible].60	121270.	88.56	88.56	200.
[illegible]	3	1771.2	1771.2	[illegible]	[illegible].77	121270.	88.56	88.56	200.
[illegible]	3	1771.	[illegible]	[illegible]	[illegible].77	121270.	88.56	88.56	200.
11	3	1771.[illegible]	[illegible]	[illegible]	[illegible].77	121270.	88.56	88.56	200.
12	3	1771.2	1771.2	223.55	[illegible].77	121270.	88.56	88.56	200.
13	3	1771.2	1771.[illegible]	[illegible].70	[illegible].77	121270.	88.56	88.56	200.
14	3	1771.2	1771.2	[illegible].15	[illegible].77	121270.	88.56	88.56	200.
[illegible]	3	1771.2	1771.2	222.7[illegible]	[illegible].77	121270.	88.56	88.56	200.
[illegible]	[illegible]	1771.[illegible]	1771.[illegible]	[illegible]	1539.71	121270.	88.56	88.56	200.
17	[illegible]	1771.[illegible]	1771.[illegible]	[illegible]	[illegible].77	121270.	88.56	88.56	200.
[illegible]	[illegible]	1771.2	1771.2	[illegible]	[illegible].77	121270.	88.56	88.56	200.
[illegible]	[illegible]	1771.[illegible]	1771.[illegible]	[illegible].40	[illegible].77	121270.	88.56	88.56	200.

		1771.2	1771.2	222.0	-1539.71	121270.	88.56	88.56	200.
2		1771.2	1771.2	221.	-2.8	121270.	88.56	88.56	200.
		1771.2	1771.2	221.0	-2.8	121270.	88.56	88.56	200.
4		1771.2	1771.2	221.0	366.60	121270.	88.56	88.56	200.
		1771.2	1771.2	221.75	366.60	121270.	88.56	88.56	200.
6		1771.2	1771.2	222.0	366.60	121270.	88.56	88.56	200.
7		1771.2	1771.2	222.1	366.60	121270.	88.56	88.56	200.
		1771.2	1771.2	222.0	1539.77	121270.	88.56	88.56	200.
		1771.2	1771.2	222.0	1539.77	121270.	88.56	88.56	200.
10		1771.2	1771.2	222.60	1539.77	121270.	88.56	88.56	200.
11		1771.2	1771.2	222.70	1539.77	121270.	88.56	88.56	200.
12		1771.2	1771.2	222.0	1539.77	121270.	88.56	88.56	200.
13		1771.2	1771.2	222.60	1539.77	121270.	88.56	88.56	200.
14		1771.2	1771.2	222.65	1539.77	121270.	88.56	88.56	200.
15		1771.2	1771.2	222.45	1539.77	121270.	88.56	88.56	200.
16		1771.2	1771.2	222.5	1539.77	121270.	88.56	88.56	200.
17		1771.2	1771.2	222.40	1539.77	121270.	88.56	88.56	200.
18	4	1771.2	1771.2	222.35	1539.77	121270.	88.56	88.56	200.
19		1771.4	1771.4	222.25	2539.77	121270.	88.56	88.56	200.
20	4	1771.2	1771.2	222.25	-640.71	121270.	88.56	88.56	200.
1		1771.2	1771.2	222.0	-640.71	121270.	88.56	88.56	200.
2	4	1771.2	1771.2	222.00	-640.71	121270.	88.56	88.56	200.
3	4	1771.2	1771.2	221.75	-640.71	121270.	88.56	88.56	200.
4	5	1771.2	1771.2	220.85	-2.80	121270.	88.56	88.56	200.
	5	1771.2	1771.2	221.0	366.60	121270.	88.56	88.56	200.
4	5	1771.2	1771.2	221.25	366.60	121270.	88.56	88.56	200.
	5	1771.2	1771.2	221.50	366.60	121270.	88.56	88.56	200.
6		1771.2	1771.2	221.70	366.60	121270.	88.56	88.56	200.
7	5	1771.2	1771.2	221.90	366.60	121270.	88.56	88.56	200.
8	5	1771.2	1771.2	221.05	1539.77	121270.	88.56	88.56	200.
9	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
1	5	1771.2	1771.2	222.25	1539.77	121270.	88.56	88.56	200.
2	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
4	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
5	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
6	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
7	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
	5	1771.2	1771.2	222.15	1539.77	121270.	88.56	88.56	200.
	5	1771.2	1771.2	222.05	1539.77	121270.	88.56	88.56	200.
	5	1771.2	1771.2	221.95	1539.77	121270.	88.56	88.56	200.
	5	1771.2	1771.2	221.90	1539.77	121270.	88.56	88.56	200.
	5	1771.2	1771.2	221.60	-641.71	121270.	88.56	88.56	200.
	6	1771.2	1771.2	220.85	-2.8	121270.	88.56	88.56	200.
	6	1771.2	1771.2	220.70	-2.8	121270.	88.56	88.56	200.
	6	1771.2	1771.2	220.85	366.60	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.00	366.60	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.25	366.60	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.40	366.60	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.65	366.60	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.45	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.0	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.40	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.30	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.	221.25	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.0	1539.30	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.55	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.75	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.60	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.50	1539.77	121270.	88.56	88.56	200.
	6	1771.2	1771.2	221.50	1539.77	121270.	88.56	88.56	200.

19	6	1771.2	1771.2	221.5	1539.77	121270.	88.56	88.56	200.
20	6	1771.2	1771.2	221.65	1539.77	121270.	88.56	88.56	200.
21	6	1771.2	1771.2	221.45	1539.77	121270.	88.56	88.56	200.
22	6	1771.2	1771.2	221.40	1539.77	121270.	88.56	88.56	200.
23	6	1771.2	1771.2	221.20	-840.71	121270.	88.56	88.56	200.
1	7	1771.2	1771.2	220.90	-2.8	121270.	88.56	88.56	200.
2	7	1771.2	1771.2	220.55	386.60	121270.	88.56	88.56	200.
3	7	1771.2	1771.2	220.75	386.60	121270.	88.56	88.56	200.
4	7	1771.2	1771.2	220.80	386.60	121270.	88.56	88.56	200.
5	7	1771.2	1771.2	221.00	386.60	121270.	88.56	88.56	200.
6	7	1771.2	1771.2	221.10	386.60	121270.	88.56	88.56	200.
7	7	1771.2	1771.2	221.15	386.60	121270.	88.56	88.56	200.
8	7	1771.2	1771.2	221.00	1539.77	121270.	88.56	88.56	200.
9	7	1771.2	1771.2	220.70	1539.77	121270.	88.56	88.56	200.
10	7	1771.2	1771.2	220.95	1539.77	121270.	88.56	88.56	200.
11	7	1771.2	1771.2	221.25	1539.77	121270.	88.56	88.56	200.
12	7	1771.2	1771.2	221.35	1539.77	121270.	88.56	88.56	200.
13	7	1771.2	1771.2	220.55	1539.77	121270.	88.56	88.56	200.
14	7	1771.2	1771.2	221.00	1539.77	121270.	88.56	88.56	200.
15	7	1771.2	1771.2	221.35	1539.77	121270.	88.56	88.56	200.
16	7	1771.2	1771.2	221.50	1539.77	121270.	88.56	88.56	200.
17	7	1771.2	1771.2	221.70	1539.77	121270.	88.56	88.56	200.
18	7	1771.2	1771.2	221.65	1539.77	121270.	88.56	88.56	200.
19	7	1771.2	1771.2	221.75	1539.77	121270.	88.56	88.56	200.
20	7	1771.2	1771.2	221.75	1539.77	121270.	88.56	88.56	200.
21	7	1771.2	1771.2	221.55	1539.77	121270.	88.56	88.56	200.
22	7	1771.2	1771.2	221.20	1539.77	121270.	88.56	88.56	200.
23	7	1771.2	1771.2	221.15	-840.71	121270.	88.56	88.56	200.
24	7	1771.2	1771.2	220.85	-840.71	121270.	88.56	88.56	200.
1	8	1771.2	1771.2	220.35	-2.8	121270.	88.56	88.56	200.
2	8	1771.2	1771.2	220.40	386.60	121270.	88.56	88.56	200.
3	8	1771.2	1771.2	220.60	386.60	121270.	88.56	88.56	200.
4	8	1771.2	1771.2	220.65	386.60	121270.	88.56	88.56	200.
5	8	1771.2	1771.2	220.65	386.60	121270.	88.56	88.56	200.
6	8	1771.2	1771.2	220.75	386.60	121270.	88.56	88.56	200.
7	8	1771.2	1771.2	220.60	386.60	121270.	88.56	88.56	200.
8	8	1771.2	1771.2	220.25	2936.10	121270.	88.56	88.56	200.
9	8	1771.2	1771.2	220.05	2936.10	121270.	88.56	88.56	200.
10	8	1771.2	1771.2	219.90	2936.10	121270.	88.56	88.56	200.
11	8	1771.2	1771.2	219.85	2936.10	121270.	88.56	88.56	200.
12	8	1771.2	1771.2	219.90	2936.10	121270.	88.56	88.56	200.
13	8	1771.2	1771.2	219.95	1539.77	121270.	88.56	88.56	200.
14	8	1771.2	1771.2	220.40	1539.77	121270.	88.56	88.56	200.
15	8	1771.2	1771.2	220.90	1539.77	121270.	88.56	88.56	200.
16	8	1771.2	1771.2	221.15	1539.77	121270.	88.56	88.56	200.
17	8	1771.2	1771.2	221.45	1539.77	121270.	88.56	88.56	200.
18	8	1771.2	1771.2	221.45	1539.77	121270.	88.56	88.56	200.
19	8	1771.2	1771.2	221.55	1539.77	121270.	88.56	88.56	200.
20	8	1771.2	1771.2	221.60	1539.77	121270.	88.56	88.56	200.
21	8	1771.2	1771.2	221.50	1539.77	121270.	88.56	88.56	200.
22	8	1771.2	1771.2	221.20	1539.77	121270.	88.56	88.56	200.
23	8	1771.2	1771.2	221.10	1539.77	121270.	88.56	88.56	200.
24	8	1771.2	1771.2	220.70	-840.71	121270.	88.56	88.56	200.
1	9	1771.2	1771.2	220.25	252.15	121270.	88.56	88.56	200.
2	9	1771.2	1771.2	220.25	252.15	121270.	88.56	88.56	200.
3	9	1771.2	1771.2	220.45	598.50	121270.	88.56	88.56	200.
4	9	1771.2	1771.2	220.45	553.50	121270.	88.56	88.56	200.
5	9	1771.2	1771.2	220.55	596.10	121270.	88.56	88.56	200.
6	9	1771.2	1771.2	220.30	658.50	121270.	88.56	88.56	200.
7	9	1771.2	1771.2	220.10	2936.10	121270.	88.56	88.56	200.
8	9	1771.2	1771.2	219.85	2936.10	121270.	88.56	88.56	200.
9	9	1771.2	1771.2	219.60	2936.10	121270.	88.56	88.56	200.

10	9	1771.2	1771.2	219.75	2956.10	121270.	88.56	88.56	200.
11	9	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
12	9	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
13	9	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
14	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
15	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
16	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
17	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
18	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
19	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
20	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
21	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
22	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
23	9	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
24	9	1771.2	1771.2	[illegible]	-640.71	121270.	88.56	88.56	200.
2	10	1771.2	1771.2	[illegible]	252.15	121270.	88.56	88.56	200.
3	10	1771.2	1771.2	[illegible]	696.50	121270.	88.56	88.56	200.
4	10	1771.2	1771.2	[illegible]	696.50	121270.	88.56	88.56	200.
5	10	1771.2	1771.2	[illegible]	696.50	121270.	88.56	88.56	200.
6	10	1771.2	1771.2	[illegible]	696.50	121270.	88.56	88.56	200.
7	10	1771.2	1771.2	[illegible]	2956.1	121270.	88.56	88.56	200.
8	10	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
9	10	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
10	10	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
11	10	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
12	10	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
13	10	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
14	10	1771.2	1771.2	[illegible]	2956.1	121270.	88.56	88.56	200.
15	10	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
16	10	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
17	10	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
18	10	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
19	10	1771.2	1771.2	221.15	1559.77	121270.	88.56	88.56	200.
20	10	1771.2	1771.2	221.20	1559.77	121270.	88.56	88.56	200.
21	10	1771.2	1771.2	[illegible]	1559.77	121270.	88.56	88.56	200.
22	10	1771.2	1771.2	221.10	1559.77	121270.	88.56	88.56	200.
23	10	1771.2	1771.2	221.10	1559.77	121270.	88.56	88.56	200.
24	10	1771.2	1771.2	[illegible]	-640.71	121270.	88.56	88.56	200.
2	11	1771.2	1771.2	[illegible]	252.15	121270.	88.56	88.56	200.
3	11	1771.2	1771.2	[illegible]	696.50	121270.	88.56	88.56	200.
4	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
5	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
6	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
7	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
8	11	1771.2	1771.2	219.65	2956.10	121270.	88.56	88.56	200.
9	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
10	11	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
11	11	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
12	11	1771.2	1771.2	[illegible]	2956.10	121270.	88.56	88.56	200.
13	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
14	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
15	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
16	11	1771.2	1771.2	220.35	[illegible]	121270.	88.56	88.56	200.
17	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
18	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
19	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
20	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
21	11	1771.2	1771.2	221.10	[illegible]	121270.	88.56	88.56	200.
22	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
23	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
24	11	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
2	12	1771.2	1771.2	[illegible]	[illegible]	121270.	88.56	88.56	200.
3	12	1771.2	1771.2	219.75	[illegible]	121270.	88.56	88.56	200.

4	12	1771.2	1771.2	219.80	696.50	121270.	88.56	88.56	200.
5	12	1771.2	1771.2	219.50	696.50	121270.	88.56	88.56	200.
6	12	1771.2	1771.2	219.50	696.50	121270.	88.56	88.56	200.
7	12	1771.2	1771.2	220.00	2936.10	121270.	88.56	88.56	200.
8	12	1771.2	1771.2	218.80	2936.10	121270.	88.56	88.56	200.
9	12	1771.2	1771.2	218.70	2936.10	121270.	88.56	88.56	200.
10	12	1771.2	1771.2	218.80	2936.10	121270.	88.56	88.56	200.
11	12	1771.2	1771.2	219.00	2936.10	121270.	88.56	88.56	200.
12	12	1771.2	1771.2	219.15	2936.10	121270.	88.56	88.56	200.
13	12	1771.2	1771.2	219.20	2936.10	121270.	88.56	88.56	200.
14	12	1771.2	1771.2	219.35	2936.10	121270.	88.56	88.56	200.
15	12	1771.2	1771.2	219.60	2936.10	121270.	88.56	88.56	200.
16	12	1771.2	1771.2	220.25	2936.10	121270.	88.56	88.56	200.
17	12	1771.2	1771.2	220.45	2936.10	121270.	88.56	88.56	200.
18	12	1771.2	1771.2	220.55	2936.10	121270.	88.56	88.56	200.
19	12	1771.2	1771.2	220.75	2936.10	121270.	88.56	88.56	200.
20	12	1771.2	1771.2	220.75	1539.77	121270.	88.56	88.56	200.
21	12	1771.2	1771.2	220.75	1539.77	121270.	88.56	88.56	200.
22	12	1771.2	1771.2	220.65	1539.77	121270.	88.56	88.56	200.
23	12	1771.2	1771.2	220.20	1423.60	121270.	88.56	88.56	200.
24	12	1771.2	1771.2	220.30	1423.60	121270.	88.56	88.56	200.
2	13	1771.2	1771.2	219.65	252.15	121270.	88.56	88.56	200.
3	13	1771.2	1771.2	219.60	252.15	121270.	88.56	88.56	200.
4	13	1771.2	1771.2	219.45	696.50	121270.	88.56	88.56	200.
5	13	1771.2	1771.2	219.25	696.50	121270.	88.56	88.56	200.
6	13	1771.2	1771.2	218.80	696.50	121270.	88.56	88.56	200.
7	13	1771.2	1771.2	218.50	2936.10	121270.	88.56	88.56	200.
8	13	1771.2	1771.2	218.50	2936.10	121270.	88.56	88.56	200.
9	13	1771.2	1771.2	218.35	2936.10	121270.	88.56	88.56	200.
10	13	1771.2	1771.2	218.40	2936.10	121270.	88.56	88.56	200.
11	13	1771.2	1771.2	218.55	2936.10	121270.	88.56	88.56	200.
12	13	1771.2	1771.2	218.70	2936.10	121270.	88.56	88.56	200.
13	13	1771.2	1771.2	219.00	2936.10	121270.	88.56	88.56	200.
14	13	1771.2	1771.2	219.15	2936.10	121270.	88.56	88.56	200.
15	13	1771.2	1771.2	219.50	2936.10	121270.	88.56	88.56	200.
16	13	1771.2	1771.2	220.05	2936.10	121270.	88.56	88.56	200.
17	13	1771.2	1771.2	220.20	2936.10	121270.	88.56	88.56	200.
18	13	1771.2	1771.2	220.35	2936.10	121270.	88.56	88.56	200.
19	13	1771.2	1771.2	220.45	2936.10	121270.	88.56	88.56	200.
20	13	1771.2	1771.2	220.35	3093.60	121270.	88.56	88.56	200.
21	13	1771.2	1771.2	220.35	1423.60	121270.	88.56	88.56	200.
22	13	1771.2	1771.2	220.35	1423.60	121270.	88.56	88.56	200.
3	14	1771.2	1771.2	219.40	252.15	121270.	88.56	88.56	200.
4	14	1771.2	1771.2	219.20	696.50	121270.	88.56	88.56	200.
5	14	1771.2	1771.2	219.00	696.50	121270.	88.56	88.56	200.
6	14	1771.2	1771.2	218.65	2936.10	121270.	88.56	88.56	200.
7	14	1771.2	1771.2	218.55	2936.10	121270.	88.56	88.56	200.
8	14	1771.2	1771.2	218.20	2936.10	121270.	88.56	88.56	200.
9	14	1771.2	1771.2	218.00	2936.10	121270.	88.56	88.56	200.
10	14	1771.2	1771.2	218.00	2936.10	121270.	88.56	88.56	200.
11	14	1771.2	1771.2	218.05	2936.10	121270.	88.56	88.56	200.
12	14	1771.2	1771.2	218.15	2936.10	121270.	88.56	88.56	200.
13	14	1771.2	1771.2	218.35	2936.10	121270.	88.56	88.56	200.
14	14	1771.2	1771.2	218.75	2936.10	121270.	88.56	88.56	200.
15	14	1771.2	1771.2	218.90	2936.10	121270.	88.56	88.56	200.
16	14	1771.2	1771.2	219.55	2936.10	121270.	88.56	88.56	200.
17	14	1771.2	1771.2	220.00	2936.10	121270.	88.56	88.56	200.
18	14	1771.2	1771.2	220.10	1544.35	121270.	88.56	88.56	200.
19	14	1771.2	1771.2	220.25	1544.05	121270.	88.56	88.56	200.
20	14	1771.2	1771.2	220.30	1423.60	121270.	88.56	88.56	200.
21	14	1771.2	1771.2	220.15	1423.60	121270.	88.56	88.56	200.
22	14	1771.2	1771.2	220.10	1423.60	121270.	88.56	88.56	200.

3	15	1771.2	1771.2	215.20	252.15	121270.	88.56	88.56	200.
4	15	1771.2	1771.2	215.10	252.15	121270.	88.56	88.56	200.
5	15	1771.2	1771.2	215.35	1955.55	121270.	88.56	81.56	200.
6	15	1771.2	1771.2	216.50	2936.10	121270.	88.56	88.56	200.
7	15	1771.2	1771.2	217.00	2936.10	121270.	88.56	88.56	200.
8	15	1771.2	1771.2	217.50	2936.10	121270.	88.56	88.56	200.
9	15	1771.2	1771.2	217.00	2936.10	121270.	88.56	88.56	200.
10	15	1771.2	1771.2	217.7	2936.10	121270.	88.56	88.56	200.
11	15	1771.2	1771.2	217.35	2936.10	121270.	88.56	88.56	200.
12	15	1771.2	1771.2	217.65	2936.10	121270.	88.56	88.56	200.
13	15	1771.2	1771.2	217.70	2936.10	121270.	88.56	88.56	200.
14	15	1771.2	1771.2	217.15	2936.10	121270.	88.56	88.56	200.
15	15	1771.2	1771.2	216.25	2936.10	121270.	88.56	88.56	200.
16	15	1771.2	1771.2	216.50	2936.10	121270.	88.56	88.55	200.
17	15	1771.2	1771.2	215.	2936.10	121270.	88.56	88.56	200.
18	15	1771.2	1771.2	215.40	1425.60	121270.	88.56	88.56	200.
4	16	1771.2	1771.2	216.70	252.15	121270.	88.56	88.56	200.
5	16	1771.2	1771.2	215.35	590.50	121270.	88.56	88.56	200.
6	16	1771.2	1771.2	217.00	2936.10	121270.	88.56	88.56	200.
7	16	1771.2	1771.2	217.70	2936.10	121270.	88.56	88.56	200.
8	16	1771.2	1771.2	217.65	2936.10	121270.	88.56	88.56	200.
9	16	1771.2	1771.2	217.55	2936.10	121270.	88.56	88.56	200.
10	16	1771.2	1771.2	217.50	2936.10	121270.	88.56	88.56	200.
11	16	1771.2	1771.2	217.30	2936.10	121270.	88.56	88.56	200.
12	16	1771.2	1771.2	217.20	2936.10	121270.	88.56	88.56	200.
13	16	1771.2	1771.2	216.55	2936.10	121270.	88.56	88.56	200.
14	16	1771.2	1771.2	217.00	2936.10	121270.	88.56	88.56	200.
15	16	1771.2	1771.2	217.25	2936.10	121270.	88.56	88.56	200.
16	16	1771.2	1771.2	217.50	2936.10	121270.	88.56	88.56	200.
17	16	1771.2	1771.2	217.05	3095.60	121270.	88.56	88.56	200.
18	16	1771.2	1771.2	216.40	1425.60	121270.	88.56	88.56	200.
4	17	1771.2	1771.2	216.40	252.15	121270.	88.56	88.56	200.
5	17	1771.2	1771.2	216.00	252.15	121270.	88.56	88.56	200.
6	17	1771.2	1771.2	217.85	1271.95	121270.	88.56	88.56	200.
7	17	1771.2	1771.2	217.50	2936.10	121270.	88.56	88.56	200.
8	17	1771.2	1771.2	217.50	2936.10	121270.	88.56	88.56	200.
9	17	1771.2	1771.2	217.45	2936.10	121270.	88.56	88.56	200.
10	17	1771.2	1771.2	217.25	2936.10	121270.	88.56	88.56	200.
11	17	1771.2	1771.2	217.10	2936.10	121270.	88.56	88.56	200.
12	17	1771.2	1771.2	216.85	2936.10	121270.	88.56	88.56	200.
13	17	1771.2	1771.2	216.50	2936.10	121270.	88.56	88.56	200.
14	17	1771.2	1771.2	216.55	2936.10	121270.	88.56	88.56	200.
15	17	1771.2	1771.2	216.80	2936.10	121270.	88.56	88.56	200.
16	17	1771.2	1771.2	216.15	3095.60	121270.	88.56	88.56	200.
17	17	1771.2	1771.2	216.85	3095.60	121270.	88.56	88.56	200.
18	17	1771.2	1771.2	217.00	1425.60	121270.	88.56	88.56	200.
6	18	1771.2	1771.2	217.85	1271.95	121270.	88.56	88.56	200.
7	18	1771.2	1771.2	217.55	1271.95	121270.	88.56	88.56	200.
8	18	1771.2	177.2	217.45	1271.95	121270.	88.56	88.56	200.
9	18	1771.2	1771.2	217.35	1271.95	121270.	88.55	85.56	200.
10	18	1771.2	1771.2	217.20	2936.10	121270.	88.56	88.56	200.
11	18	1771.2	1771.2	217.05	2936.10	121270.	88.56	88.56	200.
12	18	1771.2	1771.2	216.75	2936.10	121270.	88.56	88.56	200.
13	18	1771.2	1771.2	216.00	2936.10	121270.	88.56	88.56	200.
14	18	1771.2	1771.2	215.90	2936.10	121270.	88.56	88.56	200.
15	18	1771.2	1771.2	216.00	3095.60	121270.	88.55	88.56	200.
16	18	1771.2	1771.2	216.20	3095.60	121270.	88.55	88.56	200.
17	18	1771.2	1771.2	216.35	1425.60	121270.	88.56	88.56	200.
18	18	1771.2	1771.2	216.00	1425.60	121270.	88.55	88.56	200.
[illegible]	19	1771.2	1771.2	217.25	1271.95	121270.	88.55	88.56	200.
[illegible]	19	1771.2	1771.2	217.15	2936.10	121270.	88.56	88.56	200.
[illegible]	19	1771.2	1771.2	21[illegible]	2936.10	121270.	88.56	88.56	200.

12	[illegible]	177[illegible]	1771.2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	200.
12	[illegible]	177[illegible]	1771.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	177[illegible]	1771.2	215.4[illegible]	[illegible]	121270.	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	177[illegible]	177[illegible]	[illegible]	[illegible]	121271.	[illegible]	[illegible]	200.
[illegible]	[illegible]	177[illegible]	1771.2	[illegible]	[illegible]	121270.	[illegible]	[illegible]	[illegible]
17	[illegible]	177[illegible]	1771.[illegible]	[illegible]	142[illegible]	121270.	[illegible]	[illegible].56	200.
[illegible]	[illegible]	177[illegible]	1771.[illegible]	217.[illegible]	1271.[illegible]	121270.	[illegible]	[illegible].56	200.
[illegible]	[illegible]	177[illegible]	1771.2	217.[illegible]	1271.[illegible]	121270.	[illegible]	[illegible].56	200.
[illegible]	[illegible]	[illegible]	1771.2	[illegible]	1271.[illegible]	121270.	[illegible]	[illegible].56	200.
12	[illegible]	177[illegible]	1771.2	[illegible]	1271.[illegible]	121270.	[illegible]	[illegible].56	200.
[illegible]	[illegible]	177[illegible]	177[illegible]	[illegible]	[illegible].60	121271.	[illegible]	[illegible].56	200.
[illegible]	[illegible]	177[illegible]	[illegible]	[illegible]	[illegible]	12127[illegible]	[illegible]	[illegible]	200.
[illegible]		177[illegible]	1771.[illegible]	[illegible]	142[illegible]	121270.	[illegible].56	[illegible].56	200.
[illegible]	[illegible]	177[illegible]	177[illegible]	[illegible]	1425.60	121270.	[illegible].56	[illegible].56	200.
17	[illegible]	177[illegible]	[illegible]	[illegible]	14[illegible].60	121270.	[illegible]	[illegible].56	200.
12	[illegible]	177[illegible]	1771.[illegible]	[illegible]	[illegible].40	121270.	80.56	[illegible].56	200.
1[illegible]	21	177[illegible]	[illegible]	21[illegible]	19[illegible].40	121270.	[illegible]	[illegible].56	200.
14	21	177[illegible]	177[illegible]	[illegible]	[illegible].40	121270.	[illegible].56	[illegible].56	200.
[illegible]	21	177[illegible]	177[illegible]	214.[illegible]	[illegible].40	121270.	[illegible]	[illegible].56	200.

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