LINEAR DISTRIBUTED PARAMETER HYDROLOGIC MODELS FOR THE UPPER CAUVERY SUB-BASIN IN KARNATAKA

A DISSERTATION
submitted in partial fulfilment of
the requirements for the award of the Degree
of
MASTER OF ENGINEERING
in
HYDROLOGY

by
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CERTIFICATE

Certified that the dissertation entitled 'LINEAR DISTRIBUTED PARAMETER HYDROLOGIC MODELS FOR THE UPPER CAUVERY SUB-BASIN IN KARNATAKA' which is being submitted by Sri L.Rangaswamappa in partial fulfilment of the requirements for the award of the degree of Master of Engineering in Hydrology of the University of Roorkee, Roorkee is a record of the candidate's own bonafide work carried out by him under our supervision and guidance. To the best of our knowledge the matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that Sri L.Rangaswamappa has worked for a period of six months from October 1976 to March 1977 in the preparation of this dissertation under our guidance, at this University.

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SYNOPSIS

In this dissertation, three linear distributed parameter hydrologic models have been evolved. The input to the system has been considered as distributed. It is computed by assuming the abstractions at a constant rate during the storm duration.

The catchment is split up into sub-systems in accordance with the meteorological homogenity and drainage properties to account for the distributive nature of the input. In the first model, unit hydrographs have been developed for each sub-system with the help of a conceptual model to represent the transfer function. The differential responses from each sub-system are subjected to pure-translations by introducing the linear channels at the outlet of each subsystem to the gauge. Appropriate time lags of these differential responses have been established. The total response of the system is established by superimposing them.

In the second model, each subsystem so developed is conceptualised by a cascade of linear reservoirs and accordingly the transfer function has been established. Net response at the outlet of the catchment is obtained by establishing the time-lags for each linear channels.

Both the proposed models have been applied on to the Upper Cauvery Sub-basin. Three storms have been tested, computed and observed floods have been compared. The two match satisfactorily well.

In practice, the natural flows from each sub-system have to pass through the natural channels and thus are subjected to dampening effects of the channel. To incorporate these effects, in the third model a cascade of linear reservoirs has been conceptualised to represent the channel action. The analog simulation model of the above has been proposed for the Upper Cauvery Sub-basin. The same could not be tested for the observed storms owing to the non-availability of the components in AC-20 Analog Computer available for work.

However, the proposal of simulation has been tested for the uniform input of unit duration and the conceptual identities of the model have been established by establishing the unit hydrograph for the sub-basin. A case study of four extreme distributions is carried out for one day storm duration. The responses for these inputs have been established and the same are compared with the responses obtained by using the Unit Hydrograph Approach i.e. the proposal No.1.

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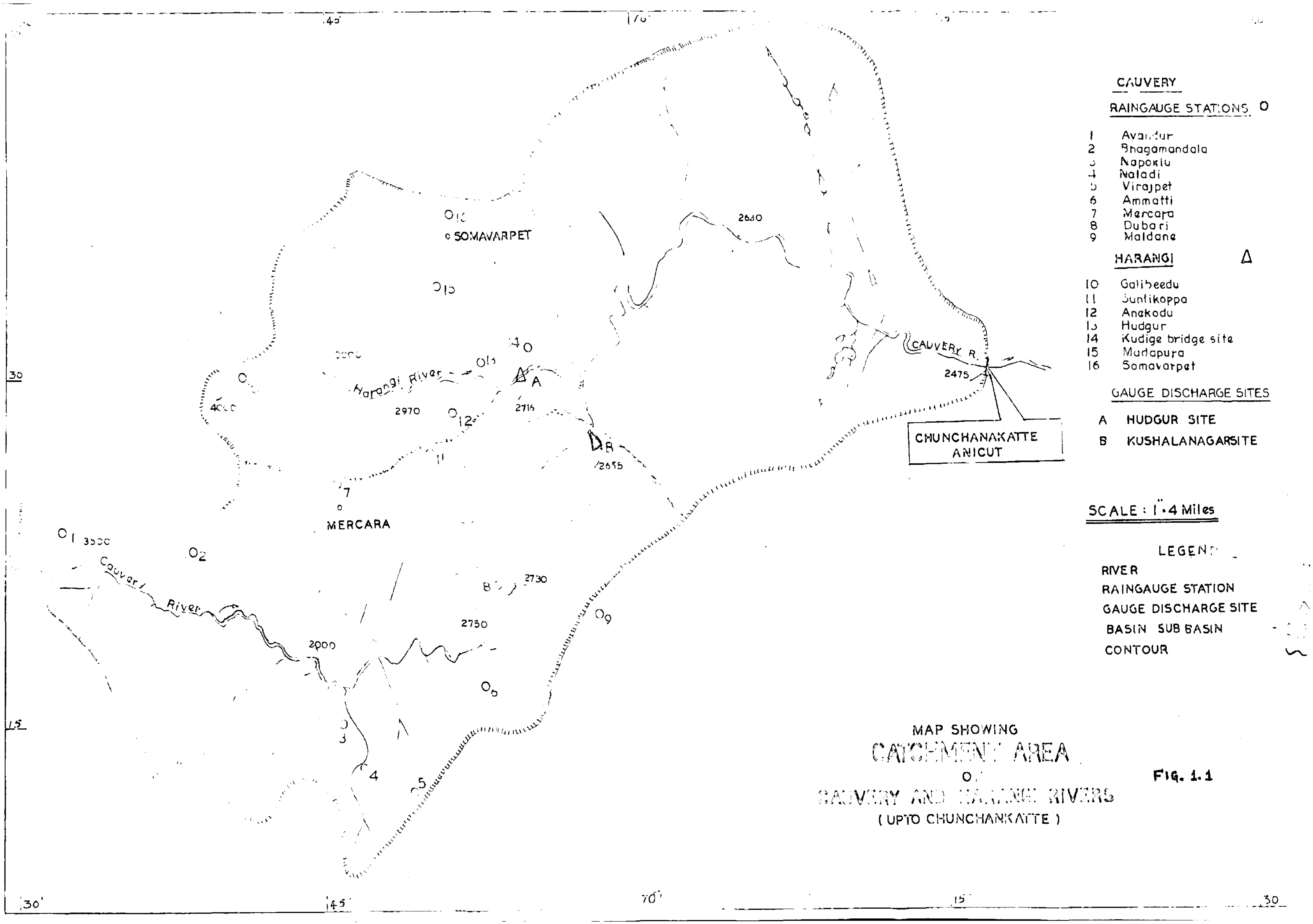
(i) General Features of the Upper Cauvery Sub Basin

(ii) Statement of the Problem

1.1 General Features of the Upper Cauvery Sub Basin:

The river Cauvery rises in the Coorg District of Karnataka highup amid the Western Ghat, it flows generally south-east direction across the pleatu of Karnataka and finally pours itself into Bay of Bengal in Tamil Nadu. The river rises at an elevation of about 1235 metres and within a distance of about 20 kilometres it falls into an elevation of about 885 metres. Thereafter the river has an average bed slope of about 1/1000.

An index map showing the upper Cauvery sub-basin upto Chunchankatte Anicut in which catchment upto Kushalnagar brdge site is demarketed with all the relevant features is shown in Fig.1.1. The upper Cauvery sub basin considered for the present study lies in Coorg district of Karnataka. The catchment is mainly in Western Ghat region and extends in size upto 1200 sq. kms. (upto Kushalnagar bridge site). The major contribution to the catchment is from monsoon rainfall which starts in May extends upto October. The catchment usually experiences complex storms. It receives heavy to very heavy rainfall during monsoon. The rainfall over the catchment continues for a longer period thus making it impossible to select or isolate a storm and its responses. Since the catchment is located in the Western Ghat region there is too much variation in spatial and time distribution



MAP SHOWING
CATCHMENT AREA
OF
CAUVERY AND HARANGI RIVERS
(UPTO CHUNCHANKATTE)

FIG. 1.1

of rainfall. As seen from the previous records of the rainfall over the catchment, the storm centres over the middle centroid of the catchment and then gradually extends to the upper and lower centroids. The seasonal monsoon rainfall varies from 250 cms. in the upper reaches to 100 cms. in the lower reaches.

The course of river Cauvery in Coorg is tortous; its bed is rocky; its banks are high and covered with luxuriant vegetation. During rains it swells into a torrent of 6 to 10 metres deep. The area falling in the Upper Cauvery sub - basin in Karnataka mainly comprises of igneous rocks and metamorphic rocks of Pre-Cambrian age, either exposed at the surface or covered by a thin mantle of residual and transported soils. The main component rock types are the Dharwar Schists, Granites and Granitic Gneiss and Charkonites.

The Igneous and Metamorphic rocks weathered to depth upto 30 metres in places depending upon the rock types, topography and climate. The weathered material is predominantly gravelly or sandy in varying amount of clay.

There are about 8 non-recording raingauge stations operating in and nearby the catchment. The point rainfall observations are being taken at an interval of one day. The discharge observations are being made at Kushalnagar bridge site which is located on the main stream exactly at the outlet of the subsystem considered for the study. The current meter observations are being taken during monsoon period from the year 1969 and onwards. During the non-monsoon period the daily discharges are read from the stage-discharge relationship established at Kushalnagar bridge site.

As discussed above, owing to the complexities in physiography and meteorology the common approaches in hydrology i.e, the unit hydrograph theory etc., did not give very satisfactory results. Therefore, this study is proposed to evolve at suitable hydrologic models to suit the requirements of the Upper Cauvery Sub-basin.

1.2 Statement of the Problem:

Hydrologic investigations involving large drainage basins require greater attention in view of growing development of water resources. Also there is a greater need for the proper prediction of magnitude of floods which are pre-requisites for the proper design of hydro-structures. This includes not only the knowledge of the peak flood but also the time distribution of discharges throughout the period of flows. The flood peak and time distribution of runoff from a drainage basin during a storm depend upon the meteorological conditions and also on the physiographical characteristics of the basin. To find a suitable hydrologic solution to such a problem with reasonable accuracy is a task which has drawn the attention of hydrologists from time to time.

As discussed in section 1.1, in the hydrologic analysis of the upper Cauvery sub-basin, the hydrologists are normally confronted with the following typical hydrological features.

(1) There is a considerable variation of precipitation over the entire sub-basin in time and as well as in space.

(2) The storms are complex. Once the rainy season sets in, the wet spell continues over a number of days making it impossible to select or isolate a storm and its responses to suit the requirements of the existing techniques like theory of unit hydrograph etc..

(3) Before the onset of the monsoon the discharges are of very low order and they are picked up to very high orders over a short period.

Looking at the complexities in determining the rainfall runoff relationship, it is proposed in this study to develop a hydrologic model which would have the following characteristics.

(1) It must be capable of accounting for the time distribution of precipitation.

(2) Highly uneven rainfalls in space must be taken care off by the model.

1.3 Present Approach:

It is proposed to develop a linear distributed parameter hydrologic model to suit the requirements of the upper Cauvery sub-basin. For this purpose the input to the hydrologic system will be treated as distributed. The transformation process of transforming the rainfall excess into the runoff is considered as linear. The same has been taken care of by the linear conceptual identities for eg. Linear Reservoir, Linear Channel along with the help of derived identities like I.U.H. and U.H.

The basic definitions and the concepts to be used in the development of the hydrologic model has been discussed in chapter No.2.

CHAPTER 2

(i) Definitions and Concepts

(ii) Brief Summary of Linear Conceptual Models developed for Rainfall-Runoff Transformation.

2.1 Definitions and Concepts:

2.1.1 Introduction:

The Transformation process of rainfall excess generated over a natural catchment during a storm into runoff at its outlet is a complex process. It is very difficult to study the same by direct application of the physical laws. The mathematical experimentation of the drainage basin's response is considered to be more scientific to study this process.

The complicated hydrologic phenomenon can be better understood by modelling. In the recent years a number of mathematical models have been developed. Most of them treat the hydrologic system to be a black box. Balck box is one of the manalytical methods used to investigate the physical systems when details about the components influencing the system are unknown or too complex to be expressed analytically. In this method the solution is not the details of the components and their interaction inside a system rather it only gives an equivalent 'device' which transforms input into output.

A brief review of the hydrologic system and the conceptual identities that are being used to analyse the system is presented in the following sub sections.

2.1.2 The Hydrologic System:

For mathematical representation of a drainage basin, the hydrological cycle is treated as a hydrologic system. The hydrologic system is treated as a 'black-box' in which the transformation of input precipitation is explained through a 'system function' or transfer function. In the hydrological system studies, overall effects of various factors affecting the runoff process are taken into account by the system function, but in no case independent identity of any of these factors is permitted to exist or operate.

2.1.3 System Function or Transfer Function:

The transformation process of a hydrological system which transforms the input precipitation $I(t)$, to produce the output runoff $Q(t)$ is normally defined as

$$Q(t) = \phi \cdot I(t) \quad \ldots \quad \ldots \quad \ldots \quad 2.1$$

Where ϕ is a system function or transfer function which may be linear or non - linear function. Thus depending upon the nature of the transfer function process the hydrological system may be classified as a Linear or Non-Linear System.

2.1.4 Linear and Non-Linear Hydrological Systems:

A hydrological system is said to perform a linear operation of a step input to the system produces an output responses which is directly proportional to the input at any time. A linear system can be described by linear equation.

In general the hydrological system may be defined by differential equations of the following type.

$$f(\psi) = a_n \frac{d^n \psi}{dt^n} + a_{n-1} \frac{d^{n-1} \psi}{dt^{n+1}} + \dots$$

$$+ a_o \psi \quad \dots \quad \dots (2.2)$$

The system is said to be linear and time invariant only if all the coefficients $a_o, a_1 \dots a_n$ etc. are constants. The system is said to linear but time variant, if one or more of these coefficients are function of the independent variable t but are not the function of ψ . However, a system would be non-linear if one or more of these coefficients are the function of ψ .

Time invariant linear systems are easy to work with, as principles of super position and homogenity hold good. The principle of super position may be stated as

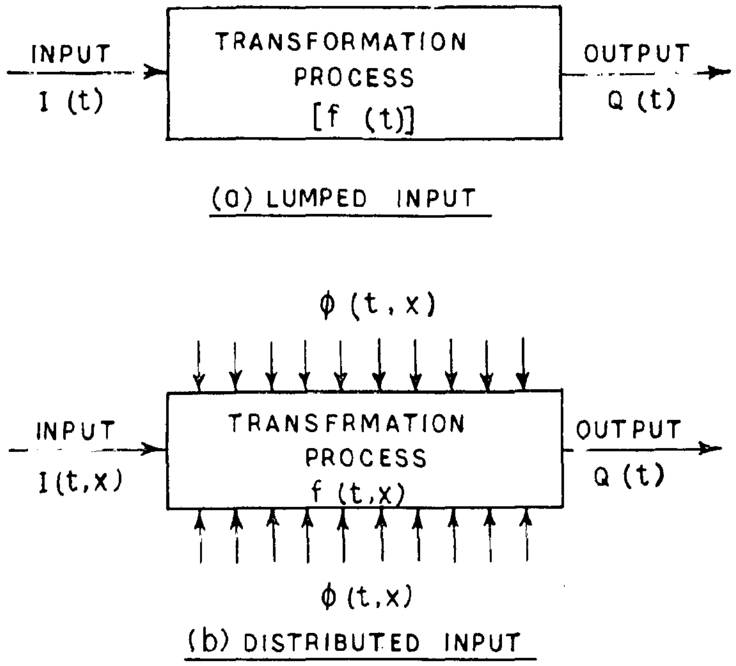


FIG. 2·1_ INPUT TO A SYSTEM

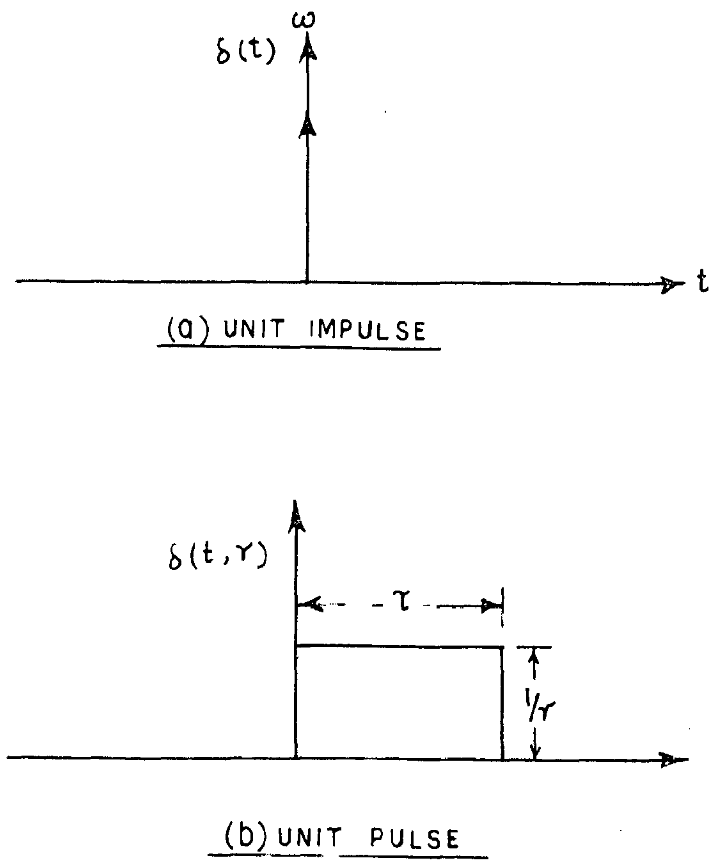


FIG. 2.3.

$$f(\psi_1) + f(\psi_2) + \dots\dots + f(\psi_n) =$$

$$f(\psi_1 + \psi_2 + \dots \psi_n) \quad \dots \quad 2.3$$

where as homogenity of the system assures

$$f(\alpha \cdot \psi_n) = \alpha \cdot f(\psi_n) \quad \dots \quad 2.4$$

However depending upon the nature of the input function a linear or non-linear hydrologic system may be further be classified into a lumped or distributed system.

2.1.5 <u>Lumped and Distributed Systems</u>

The hydrologic system may be defined as 'lumped' if the input function does not involve spatial co-ordinates. Therefore, a lumped system can be located at any single point in the working space. As shown in Fig. 2.1(a) a lumped system can be represented mathematically by ordinary differential equations.

Mathematical equations representing a 'distributed system involve spatial co-ordinates as shown in Fig. 2.1(b), input to such a system is distributed and therefore, it cannot be located at a single point. The distributed system can only be described by partial differential equations and, therefore, theoretical solution to such system requires complete knowledge of the boundry conditions.

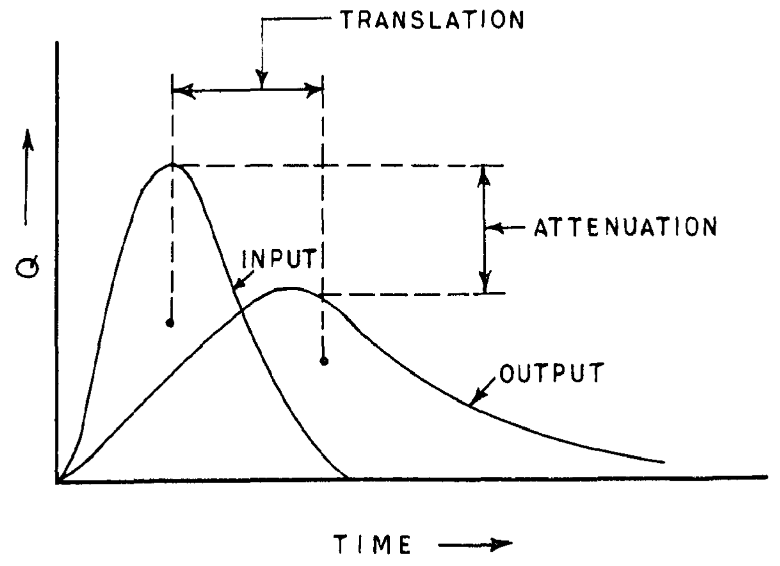


FIG. 2·2 _(a) THE CATCHMENT ACTION

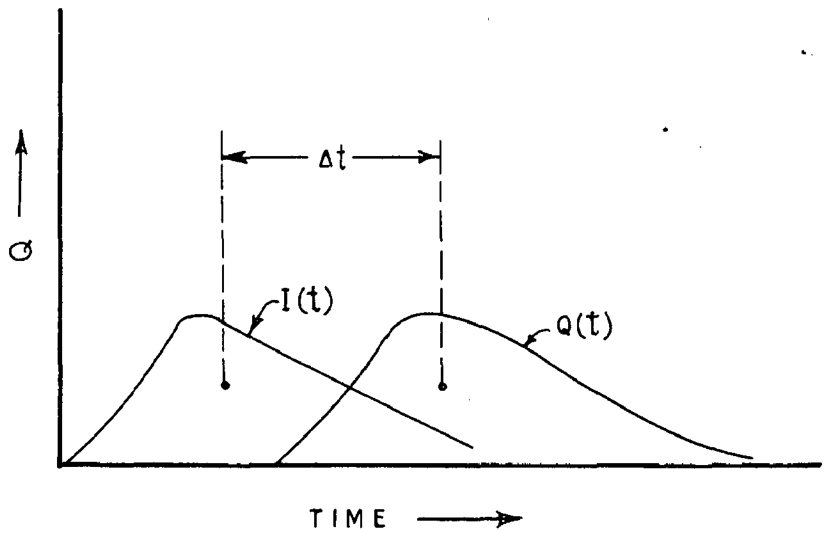


FIG. 2·2(b) PURE TRANSLATION

2.1.6 <u>The Catchment Action:</u>

As shown in Fig. 2.2 (a), as a result of the catchment action, output response gets distributed over a larger time period which not only attenuates the input hydrograph peak but also shifts it in time. The translation and attenuation of input hydrograph is due to storage actions of the basin system. In most of the conceptual models, the catchment action is represented through conceptual identities such as Linear Channels and linear or non-linear reservoirs.

2.1.7 <u>Pure Translation:</u>

It is a physical concept which signifies the 'time lag'. As illustrated in Fig.2.2(b) pure translatory functions affect only the time parameter. It is introduced in a system by a conceptual identity defined as a linear channel.

2.1.8 <u>The Linear Channel:</u>

A linear channel conceptually represents the pure translatory effects of a system and, therefor can be defined as:

" A conceptual channel on which the time (T) required to translate a discharge (Q) of any magnitude through a channel reach of given length (x), is always constant. Thus when an inflow hydrograph is routed through the channel, its shape is not affected.

If I = f(t) be the inflow function to a linear channel, after routing the outflow function Q(t) would be identical to the inflow function except for a time lag which is introduced by the system and whose magnitude is given by the translation time (Δt) of the linear channel''. Hence

$$Q(t) = f(t - \Delta t) \quad \ldots \quad 2.5$$

2.1.9 The Reservoir Action - Linear and Non-Linear Reservoirs:

The catchment action on its input precipitation is analogous to the reservoir action on its inflow hydrograph. A reservoir too, translates and attenuates the inflow hydrograph by regulating its outflow over a desired period of time. This analogy suggests that a drainage basin system could perhaps be analytically represented by the reservoir concept.

A reservoir may be classified as a linear or non-linear depending upon its mode of operation. A linear reservoir is a conceptual identity in which the storage S is directly proportional to the outflow discharge Q.

$$S \propto Q$$

$$S = KQ \quad \ldots \quad 2.6$$

where the constant K has the dimension of time and equal to the average delay-time, imposed on its flow by the reservoir model.

The functional relationship between the storage and discharge of a non-linear reservoir may be written as

$$S = KQ^{B} \quad \ldots\ldots 2.7$$

where K and B are dimensional constants which represents the two characteristic parameters of a non-linear conceptual model.

2.1.10 Derived Identities:

These are identities which may be derived with the help of conceptual identities or from available record of input and response functions to represent the transformation process of a system.

In linear hydrologic modelling popularly used derived indentities are (i) the unit hydrograph (ii) the instantaneous unit hydrograph.

2.1.11 Unit Hydrograph (U.H.):

As per the definitions given by Sherman (Sherman L.K.) ''The discharge time relationship resulting from a study effective rainfall of unit duration uniformly distributed over a catchment.''

2.1.12 Instantaneous Unit Hydrograph (I.U.H.):

If the duration of the effective precipitation becomes infinitersimally small, the resulting unit-hydrograph is called an I.U.H. which is expressed by u (o,t)

2.1.13 Responses:

Response is the output of a subsystem due to certain type of inputs. According to the nature of the input to the system, the responses may further be classified as (a) unit pulse response (b) unit impulse response.

2.1.14 Unit Pulse Response:

A unit pulse function is defined as follows

$$d(t, \tau) = 1/\tau \quad 0 < t \leq \tau$$
$$= 0 \quad \text{otherwise}$$

The term pulse used here refers to the rectangular pulses.

Unit pulse response is the output of a system, that is initially at rest, to a unit pulse input.

2.1.15 Unit Impulse Response:

A Unit impulse function $\Delta(t)$ is defined as follows

$$\Delta(t) = 0 \quad t \neq 0$$

$$\int_{-\infty}^{+\infty} \Delta(t)\, dt = 1$$

Unit impulse response is the output of the system that is initially at rest to a unit impulse input.

The unit pulse and unit impulse action is as shown in Fig. 2.3 (a) and Fig. 2.3(b)

Utilising the properties and concepts discussed in this section different hydrologists have proposed various hydrologic models which have got a direct relevance in the development of the proposed model. A brief summary of these hydrologic approaches is presented in the next section.

2.2 A Brief Summary of Linear Conceptual Models Developed for Rainfall-Runoff Transformation Process:

The committee on floods, in the year 1930 suggested that the hydrograph due to an instantaneous storm could provide a good indication of watershed response. It was considered that peak of such a hydrograph would reflect the width of the watershed and the velocity of flow, all other factors being constant. This idea was not adopted by other investigators at that time probably because of crudeness of the available data and the difficulties inherent in the derivation of IUH by numerical or graphical differentiation of the S - curve based on unit hydrograph of finite duration.

The most notable of the early attempts to derive a relationship between rainfall and runoff was in a series of articles by Zoch (Zoch, R.T. 1934-37) in which it was assumed that the x runoff from each elementary area of the watershed was related to the storage S over the elementary area by a linear relationship $S = KQ$

Clark (Clark C.O., 1945) introduced the first instantaneous unit hydrograph theory. Acknowledging the introduction of the instantaneous storm by the Boston Society and the introduction of unit hydrograph by Sherman (Sherman L.K. 1932) he combined the two to from instantaneous unit hydrograph. He further considered the IUH to be the result of time area concentration diagram routed through a single reservoir at the gauging station and proceeded to estimate the reservoir constant K and the base length T of the concentration diagram, for the runoff hydrograph. It should be noted that the time area curve used by Clark is that due to instantaneous rainfall and thus differs from other such curves used previously which included rainfall of finite duration.

The early theories of the unit hydrograph assumed the principle of super-position as the only way in which the runoff due to rainfall at recessive time intervals could be added. Sherman voiced his belief that there is the summation process of nature and other investigators accepted this view without qualifications. The first doubt about the validity of the principle of superposition and hence of the linearity of rainfall runoff process appeared to have originated from flood routing studies.

Kelly (Kelly, J.J.O. 1955) showed that a logical extension of the procedure of which unit hydrographs of unit period could be derived by means of time-shift of the S-curve is the reduction to values approaching zero of the unit period and time shift. This leads to the concept of the Instantaneous Unit Hydrograph. This unit hydrograph corresponding to a rainfall of unit volume inan instant has special properties. Its ordinates are the slope of the S-curve and conversely, the S-curve is its integral. A two parameter model for the unit hydrograph based on the routing of time area diagram through a reservoir was proposed.

An assumption that the operation performed by the catchment on an instantaneous rainfall is equivalent to a succession of routing through linear storage, was made by Nash (Nash, J.E. 1957) in deriving a general equation of the I.U.H. containing two parameters and of sufficient flexibility to permit the close approximation of any empirically derived instantaneous unit hydrograph.

In the discussion of the above paper by Nash Dooge (Dooge J.C.I. 1957) suggested the assumption of a hydrograph produced by successive equal storages was not attractive from a physical view point. The writer prefered the assumption of a triangular inflow routed through a linear storage, claiming it to be more reasonable, empirically satisfactory and mathematically more convenient.

A year later Nash (Nash, J.E. 1958) brought another paper where he discussed the historical development of various methods for determining the rainfall runoff relationship and showed them to be the particular cases of general unit hydrograph. The methods he discussed were :-

(i) Rational method

(ii) Tangent method

(iii) Time-Area method

(iv) Unit Hydrograph theory

In the year 1959 Dooge (Dooge C.J.I. 1959) introduced the concept of linear channels and linear reservoirs, such that the translation effects were due solely to linear channels and the storage effects solely to linear reservoirs.

In the same year Nash (Nash J.E. 1959) showed that the number of degrees of freedom which are useful to maintain in the form of instantaneous unit hydrograph were limited by the number of significant independent correlations with the catchment characteristics. The moments of the instantaneous unit hydrograph were suggested as a series of parameters of the response for which correlations could be sought.

Again in the year 1960 Nash (Nash J.E. 1960) correlated the moments of the instantaneous unit hydrograph with topographical characteristics for a large number of British catchments and a general equation for the I.U.H. was chosen. The use of the correlation to predict the hydrograph for catchments where sufficient data is not available was also explained with example.

Though recognising the hydrologic transformations to be non-linear, it was suggested that linear systems theory provides a first order approximation to changes in out-put statistics for planned or unplanned modifications of the hydrologic environment. The changes in input-output statistics may be used as measure of linearity of watershed response. Theory of random functions is employed to predict the out-put statistics of a variety of transformation processes as rainfall to runoff etc. This was stated by Chester C.Kisiel (Chester C.Kisel 1967).

Use of functionals and statistics is involved in the general methods and procedures for determining the rainfall-runoff relationship of 'Block Box' analysis in a paper by Chao-Lin Chiu (Chao Lin Chiu.1967) possible approaches were presented for solving the problems concerning the non-linearity and non-stationarity of hydrologic systems.

Yet another 'Block-Box' approach based on Automatic control techniques was introduced in the paper by Capella Viscaino and Sanches Bribiesca (Capella Viscaino A., and San-Ches Bribiesca J.L., 1967) as a syntehsis system obtained from observed inputs (rainfall) and outputs (runoff). The model could take into consideration rainfall distribution in a basin area, once a weighing function has been established for each input from a set of orthogonal functions.

The spatial impulse response was employed in a linearisation of the catchment dynamics to obtain a simple super position relation for estimating the effect of real variability in rainfall distribution upon the peak surface runoff. A paper to this effect was introduced by Eagleson (Peter S.Eagleson 1967).

In the year 1972 Mathur (Mathur B.S. 1972) developed a simpler approach based on linear channel concept to account for the unevenness of rainfall distribution. The response model developed by him for Indian catchments is based on series of arrangement of linear channels. Each linear channel of the series network receives its distributed input from a sub watershed area which is assigned to it. This study concluded that different sub areas of drainage basin system are directly correlated to the basin's response function through the linear channel concept. Further it was shown that rainfall input on different sub area need not be the same and thus spatial non-uniformity of rainfall is taken into account. The Response model developed by Mathur has lead to the classical methodology which is capable of taking the spatial variation in rainfall over the natural catchment. The model is capable of identifying the parts of the sub area responsible for flood peaks, thus enabling the flood forecasting programmes.

Keeping inview the efforts and limitations of the above linear conceptual models, a proposal for the development of a linear distributed parameter hydrologic model has been discussed in the next chapter.

CHAPTER 3

(i) Formulation of the Hydrologic Model

(ii) Input Function

(iii) Proposed Models 1 and 2

3.1.0 Formulation of the Hydrologic Model:

As discussed in section 1.2 of Chapter No.1 it is proposed to develop a hydrologic model capable of taking into account the following typical features.

(i) Uneven distribution of rainfall in time

(ii) Unevenness of precipitation in space

For the purpose a linear distributed parameter model is preferred. The proposed linearity of the system enables to apply the principles of homogenity and superposition which considerably simplify the computations.

The distributed nature of input is accounted for the entire system by splitting it up into sub-systems. The following three different proposals have been studied to evaluate the response of the system.

3.1.1. Proposal No. 1:

The transformation process of each sub-system is taken into account by a Unit Hydrograph developed with the help of a conceptual model. Linear channels have been introduced to the response of each sub-system to compute the net response at the out let.

3.1.2. Proposal No. 2:

The transformation process is taken care of by a cascade of linear reservoirs developed for each sub-system and linear channels are introduced to account for the response at the gauge.

3.1.3. Proposal No. 3:

A particular configuration of linear reservoirs is adopted to account for the storage effects of the catchment. The channel effects have been taken care of by the linear reservoir in series.

3.2.0 The Input Functions

For all the three proposals the following procedure is adopted to evaluate the distributed input to the system. Normally the input to the system i.e. the rainfall data is available over the different raingauges spreadout over the entire basin. This procedure is adopted when no information is available about the variations in the infiltration rates for the different sub systems which are considered for the analysis. Therefore a constant rate of abstraction in the form of ϕ-index is evaluated for the entire system and the same is adopted for the different sub systems in evaluating the precipitation excess.

The mean gross depth of precipitation over the entire (catchment) system is evaluated by using the Thiessen Weight equations.

$$D_a = \sum_{i=1}^{n} a_i D_i \qquad i = 1, n \qquad \ldots (3.1)$$

where

D_a = Mean depth of gross rainfall over the system

a_i = Thiessen Weight of raingauge station i=1, n

D_i = Observed point rainfall of the raingauges i = 1, n

The constant rate of abstraction i.e. the ϕ-index is evaluated using the observed hydrograph at the out let (a suitable time variation in base flow is adopted) and the corresponding hydrograph. The effective precipitation over the catchment for each time unit will thus be given by

$$P_e = (P_a - \phi) \qquad \ldots \ldots (3.2)$$

where

P_e = Weighted effective mean depth of rainfall input over the system

P_a = Mean depth of gross rainfall over the system

ϕ = Phi-index

A weight factor for each day's of precipitation is computed by the relation

$$P_{wj} = P_{ai} / P_{ai} \quad i=1 \text{ to } n \quad j=1,n \text{ days} \qquad \ldots \ldots (3.3)$$

The mean depth of gross rainfall over each sub system is computed using the Thiessen weight equation

$$P_{ai} = \sum_{t=1}^{n} c_{ai} P_{ci} \quad i = 1, n \qquad \ldots \ldots (3.4)$$

where

P_{ai} = Mean depth of gross rainfall over the sub system $i = 1,n$

c_{ai} = Thiessen Weight of raingauge station $i=1,n$ influencing the sub system $t=1,n$

P_{ci} = Observed point rainfall of raingauge stations $i = 1, n$ influencing the sub system $i = 1,n$

The effective mean depth of precipitation is obtained by the relationship

$$P_{esi} = F_j * P_{ai}$$
$$i = 1, n \qquad \ldots \ldots (3.5)$$
$$j = 1, n \text{ days}$$

where

P_{esi} = Mean depth of effective rainfall over the syb system i = 1, n for the duration considered.

F_j = Factor evaluated for the duration j = 1, n days

P_{ai} = Mean depth of gross rainfall over the sub system i = 1, n

3.3.0 Proposal No. 1

3.3.1 Introduction:

A three parameter linear distributed hydrologic model is proposed to evaluate the response of a natural catchment. In order to account for the uneven-ness in spatial distribution of rainfall over the catchment, it is proposed to split up the catchment system into sub systems. This can be done on the following basis.

(i) Keeping inview the spatial uniformity of the precipitation over the sub system.

(ii) On the basis of the drainage properties of the catchment ((i) area drained by each large tributary may be treated as a sub system. (ii) the area assigned to the sub system must drain with in the sub system.

The transfer function for each sub system is taken care by (i) the Unit Hydrograph which will be derived with the help of existing rainfall records and its corresponding runoff using one of the conceptual models.

(ii) a linear channel - to account for the time lag of the differential responses in reaching the out let.

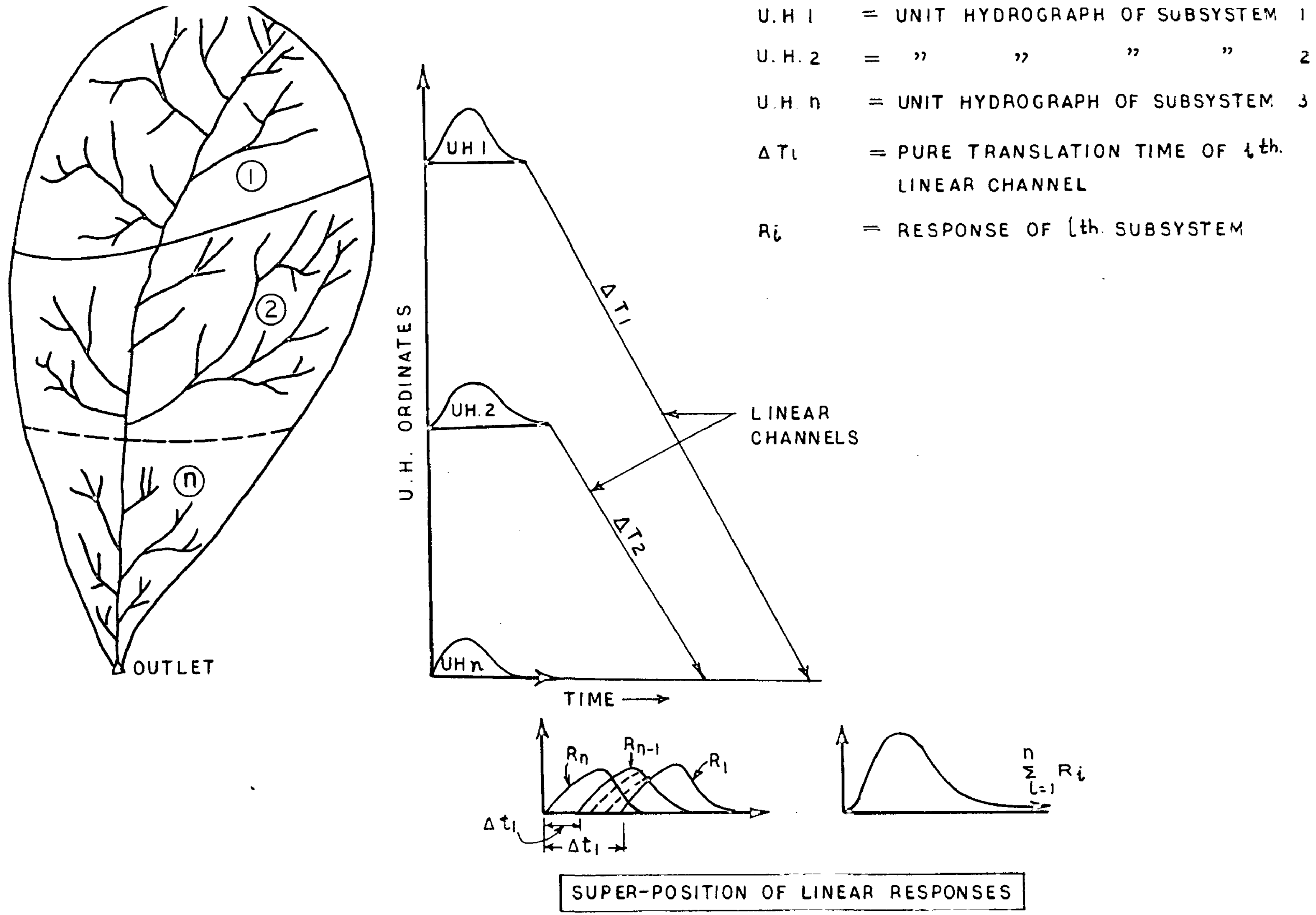


FIG. 3·0. STRUCTURE OF THE PROPOSED MODEL.

As the runoff is not measured for each sub system, a conceptual model whose parameters can empherically be defined interms of basin characteristics will suit the requirements. The structure of the proposed model is illustrated in Fig. No.3.0

3.3.2 Development of Unit Hydrograph for Sub Systems:

In this study a conceptual model proposed by Nash (J.E. Nash 1957) is used to develop the Unit Hydrograph for each sub system of the basin. The two parameters of the model i.e. (i) 'n' - the number of linear reservoirs in series (ii) K - the storage coefficient which is constant for the linear reservoirs are given by Nash (J.E. Nash 1960) in terms of the catchment characteristics.

$$K = C_1 \, A^{0.25} \times OLS^{-0.3} \times L^{-0.035} \qquad \ldots \ldots (3..6)$$

$$n = C_2 \, L^{0.035} \qquad \ldots \ldots (3.7)$$

Where C_1 and C_2 are the constants derived for the catchment, knowing the values of n and K from the observed excess rainfall hyetograph and its corresponding hydrograph

and

A = Area in sq. kms.

OLS = Weighted Overland Slope in parts per 10,000

L = Length of main channel in kms.

The constants C_1 and C_2 have to be established for the system under consideration. The first and second moments of (M_1 and M_2) of an instantaneous Unit Hydrograph have been defined as under.

$$M_1 = nK = M_{DRH1} - M_{ERH1} \quad \ldots \ldots (3.8)$$

$$M_2 = n(n+1)\,K^2 = M_{DRH2} - M_{ERH2} - 2nK\,M_{ERH1} \quad \ldots \ldots (3.9)$$

where

M_{ERH1} = First moment of excess rainfall hyetograph

M_{ERH2} = Second moment " " "

M_{DRH1} = First moment of direct runoff hydrograph

M_{DRH2} = Second moment " " "

The established values of C_1 and C_2 are considered to be representative for the entire system and used to derive n and K for sub systems using the equation 3.6 and 3.7. These values of n and K are considered to be the established values of n and K parameters for the sub system. Using these established values of n and K in mathematical model given by Nash (J.E. Nash 1957)

$$u\,(0,t) = 1/K\,\Gamma n\; e^{-t/K} \,.\, (t/K)^{\,n-1} \quad \ldots \ldots (3.10)$$

an instantaneous unit hydrograph for the sub system is derived. Since the responses for input functions of finite duration are required, the unit hydrograph of unit duration T is derived as follows

The equation of the S-curve is given by

$$S(t) = \int_0^t u(0,t)\, dt \qquad \ldots \ldots (3.11)$$

From equation 3.10 we have

$$S(t) = \frac{1}{\Gamma n} \int_0^{t/K} e^{-t/K} (t/K)^{n-1}\, d\,(t/K)$$

$$= f\,(n,\ t/K)$$

where

$$f\,(n, t/K) = \frac{1}{\Gamma n} \int_0^{t/K} e^{-x}\, x^{n-1}\, dx$$

is an incomplete gamma function of order n at (t/K).

The unit hydrograph of period T is given by

$$u\,(T,t) = 1/T \;\left|\, f(n, t/K) - f\left(n,\ \frac{t-T}{K}\right) \right| \qquad \ldots \ldots (3.12)$$

3.3.3. Establishment of the Linear Channels of the System:

For a particular storm input, differential out puts of each sub system can be computed by using the unit hydrograph. The linear channel is used to give the appropriate time lag for routing these differential outputs. As indicated in Fig. No. 3 the uppermost sub system will have the largest value of the time-lag and its value would reduce for the following sub systems. The time lag for the lowest sub system is practically zero as the sub system is contributing to the-out let directly.

The values of the time lags (i.e. pure translation time of each linear channel) can be arrived at by trial and error, considering a particular observed record of storm precipitation and the runoff at the out let.

3.3.4 Testing of the Model:

The input function (excess rainfall hyetograph) can be arrived at by using the procedure described vide section 3.2.0. For a observed rainfall runoff record the model can be established. Knowing the rainfall excesses and the transfor function (unit hydrograph) of the sub systems, the differential responses of each of these can be computed. Then these responses are subjected to pure translation. Super position is to be done to match the computed direct runoff with the observed direct run - off. These pure translations are represented by the linear channel which is associated with the conceptual representation of each sub-system.

Now the adopted base flows may be added to the computed runoff to obtain the total runoff at the gauge. The same may be compared with the observed runoff at the gauge.

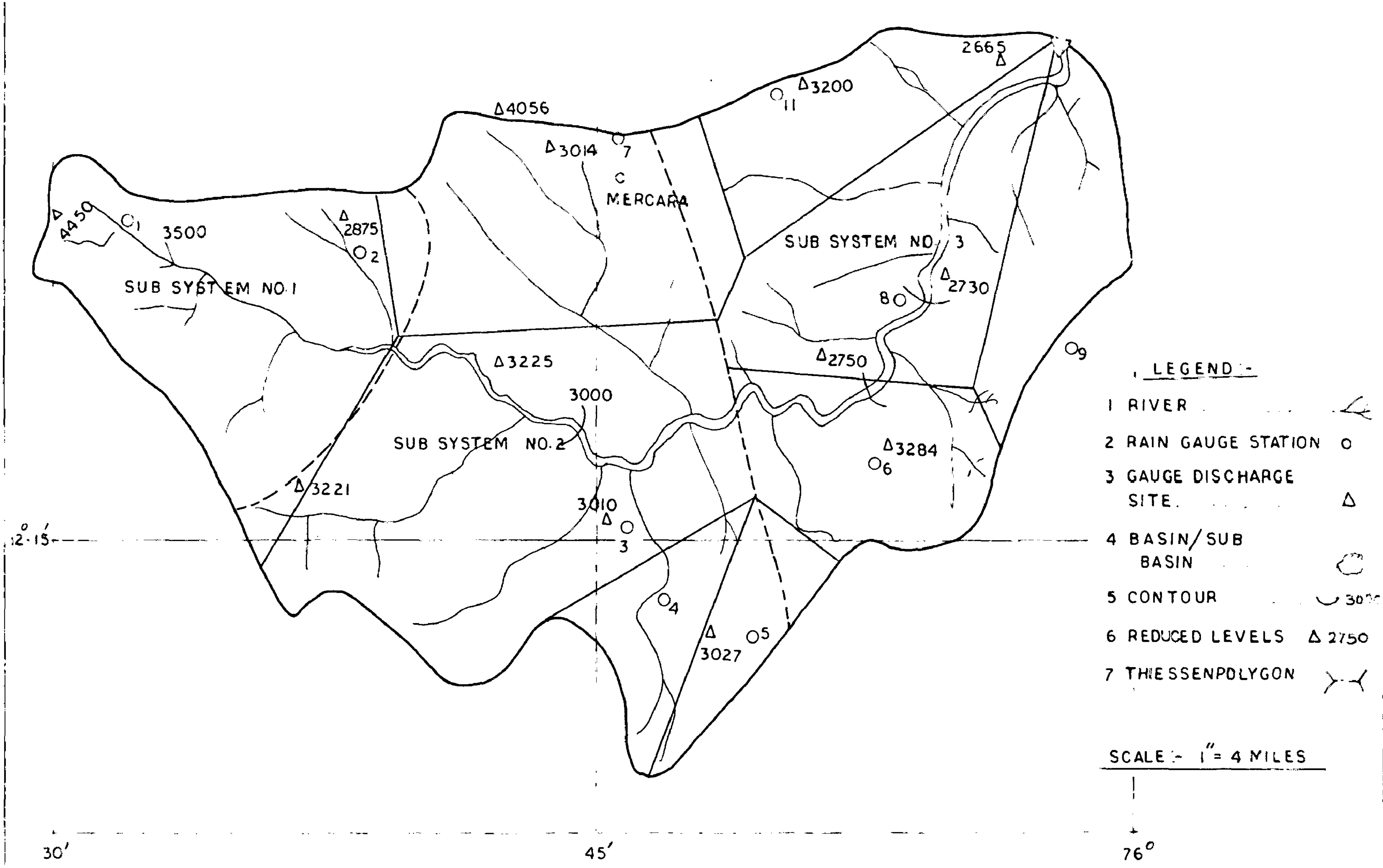


FIG. 3·1. RELEVENT FEATURES OF CATCHMENT

3.4.0 Application of the Proposed Model to the Upper Cauvery Sub-Basin in Karnataka:

3.4.1. Introduction:

It is proposed to develop the model as per the procedure described in section 3.3 to the Upper Cauvery Sub basin in Karnataka. The location details of the catchment with all relevant features such as location of raingauges, gauge-discharge site etc., are shown in Fig. No.3.1.

3.4.2. Availability of the Data:

(i) Rainfall Data: There are 9 non recording type of raingauge stations located in the catchment. The point rainfall observations are being taken at an interval of a day. The observations are being taken in mm. The rainfall data is being collected by (i) The Bureau of Economics and Statistics (ii) Water Resources Development Organisation of Karnataka.

3.4.3. (ii) Runoff Data:

The discharge observations are being made at Kushalnagar Bridge Site. The current meter observations are being taken during monsoon periods. During non-monsoon period the discharges are being computed from the established stage discharge relationship for the Kushalnagar Bridge Site. The daily runoff data is available from 1969 onwards.

3.4.4. Storms Selected for the Analysis:

Four storms that have occurred over the catchment on dates (1) 8.7.70 (2) 7.7.72 (3) 15.7.72 (4) 4.7.73 have been considered for the analysis. The details of storm and the direct runoff hydrograph ordinates are shown in Appendix 4 and 5.

3.4.5(a) Development of the Proposed Model:

As discussed in section 1.2 of Chapter No.1, the Upper Cauvery sub basin is experiencing uneven distribution of rainfall in time and space. In order to account for the above variations, the distributed input to the system is to be considered. For this purpose, the Upper Cauvery sub basin system is split up into 3 sub systems on the basis of the meteorological observations made in the past. Formation of the sub systems with all the relevant features such as Thiessen Polygons etc., are shown in Fig. No.3.1. The phisiographic features and theissen weightages are given in Appendix Nos. 1 and 2 respectively.

3.4.5 (b) Conceptual Representation of the Upper Cauvery Sub Basin:

Each sub system is conceptually represented by (1) a Unit Hydrograph (2) a Linear Channel as shown in Fig. No.3.2.

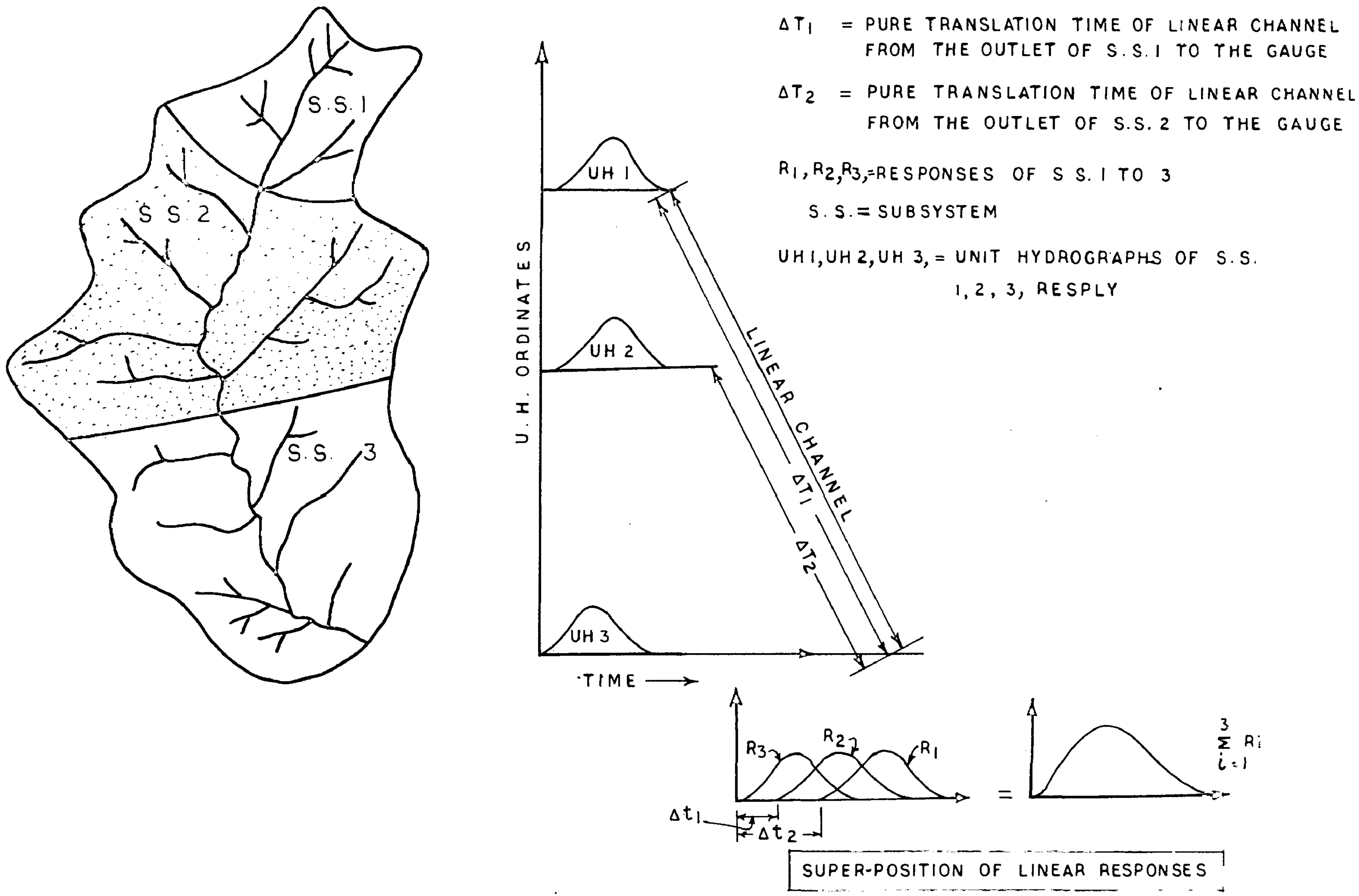


FIG. 3·2_STRUCTURE OF THE PROPOSED MODEL FOR UPPER CAUVERY SUB-BASIN.

3.4.5 (c) Development of U.H. for Sub Systems:

As discussed in Section 3.3.2 of Chapter No.3, the conceptual model proposed by Nash (J.E. Nash, 1957) is used to develop the unit hydrograph for each sub system. The parameters of this conceptual model are (1) n- the number of linear reservoirs in series (2) K - the storage coefficient which is assumed to be constant for a cascade. The value of n and K are obtained as follows.

Storm No. (2), (3), (4) and their corresponding direct runoff hydrographs are considered for establishing the values of C_1 and C_2. The storms selected are having different type of distribution hence the mean value of the coefficients can be taken as representative of the catchment. Three values of n and K for the entire catchment are obtained on the basis storms stated above. Mean values of n and K are computed. Using these mean values (of n and K) with catchment characteristics in the equations 3.6 and 3.7, the values of C_1 and C_2 are computed. Subsequently using these values of C_1 and C_2 and physiographic characteristics of the sub systems the values of n and K for each are found out. The detailed computation is shown in Appendix No.3.

The identified values of n and K of each sub system are used to develop the unit hydrograph as per the procedure detailed in Section 3.3.2. The unit hydrographs developed for each sub systems are given below in Table No.3.1.

TABLE NO. 3.1

TRANSFER FUNCTIONS

Time in Days →	U.H. Ordinates in Cumecs					
Sub System No. ↓	1	2	3	4	5	6
1	10.16	12.26	2.40	0.48	-	-
2	21.08	28.17	8.88	2.32	0.55	-
3	19.38	26.38	9.00	2.48	0.62	0.13

3.4.5 (d) Establishment of the Linear Channels of the Model:

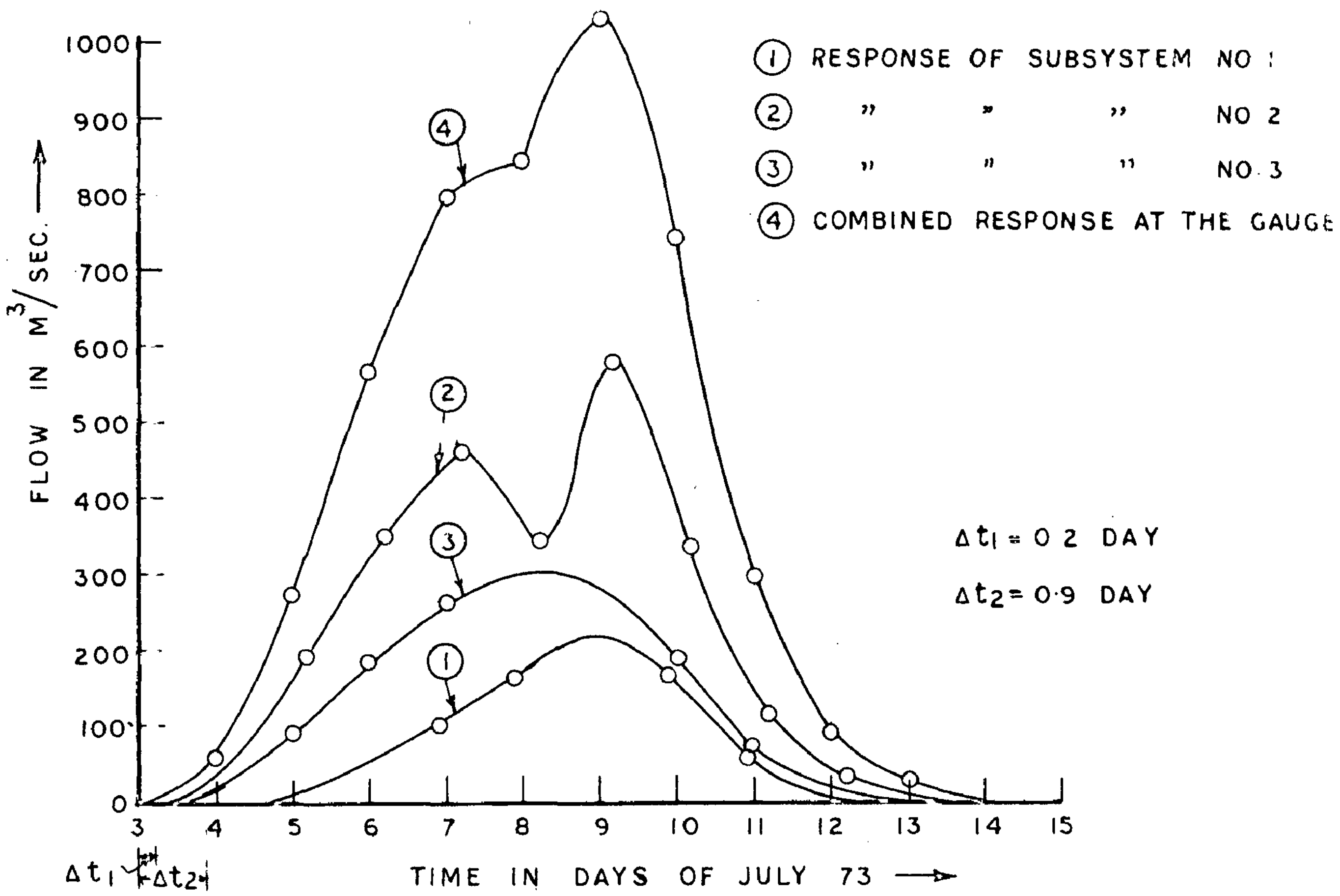
Storm No. (4) is considered for establishing Linear Channels. The rainfall excess for the above storm which is under investigation have been computed and given in Appendix No.6.

Differential responses for the storm No.4 used in the identification of the translation times are given in Table No.3.2.

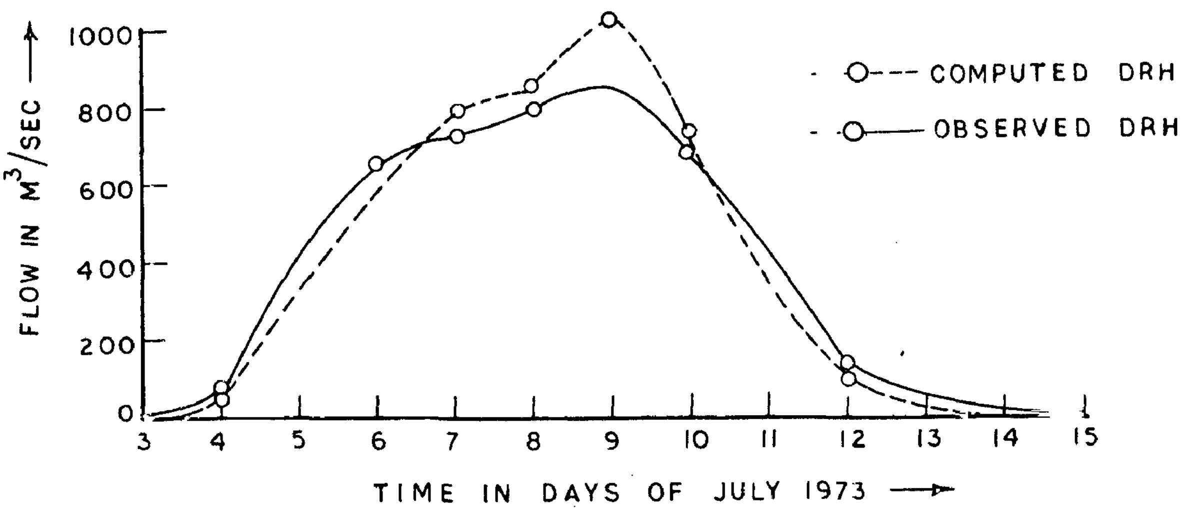
The superposition of the differential responses of the 3-sub systems at the outlet has been done by adjusting time lags of sub system 1 and sub system 2 by trial and error till the computed response agrees well with the observed response. The results indicated that the translation times of linear channel are $\Delta t_1 = 0.9$ day (the time lag for the differential output of sub system No.1 to reach the out let).

$\Delta t_2 = 0.2$ day (time lag for the differential output of sub system No.2 to reach the out let.)

Super-position of differential outputs of storm No. 4 and comparision of the observed and computed direct runoff hydrographs are shown in Fig. No.3.3.



SUPER-POSITION OF DIFFERENTIAL RESPONSES



COMPARISION OF DIRECT RUN-OFF HYDROGRAPHS OF STORM NO. 4 DTD. 3.7.73

FIG. 3.3. ESTABLISHMENT OF LINEAR CHANNEL.

3.4.6 Testing of the Proposed Model:

The model developed has been tested for the storms Nos. 1, 2 and 3 shown in Appendix No.5.

3.4.6 (a) Input Function:

The input function of the above 3 storms selecte-d for testing the model are computed as per the procedure described in Section 3.2. They are shown in Appendix No.6.

3.4.6 (b) Transfer Function:

The transfer function i.e. the unit hydrograph has been established vide section 3.4.5 (c) and shown in Table No.3.1.

3.4.6 (c) Computation of Differential Responses:

The input function of the above 3 storms are applied on to unit hydrographs, the differential responses of each sub system due to each storm are computed and they are shown in Table No.3.2

3.4.7 Computation of the Runoff Hydrograph at the Out Let of the System:

The differential outputs of all the 3-sub systems due to each storm are superimposed with the time lags Δt_1 = 0.9 day and Δt_2 = 0.2 day respectively and the

[illegible]	7	8	9	10	11	12
1	10.52	2.51	0.20			
	97.95	0.42	0.93			
8.7.73	21.17	5.95	1.43	0.23		
2						
7.7.73	0.21					
3	0.[illegible]					
	2.11	0.31				
15.7.73	1.45	0.19				
4	65.75	15.92	2.55	0.23		
	237.73	115.65	30.37	5.09	0.53	
4.7.73	191.77	75.90	21.55	5.29	1.45	0.10

combined response at the outlet is obtained. Base flows are added to the computed responses of each storm. The computed responses are compared with the observed responses.

The computational scheme of responses at the outlet of the system and comparision of observed and computed responses are shown in Fig. Nos. 3.4, 3.5 and 3.6 respectively.

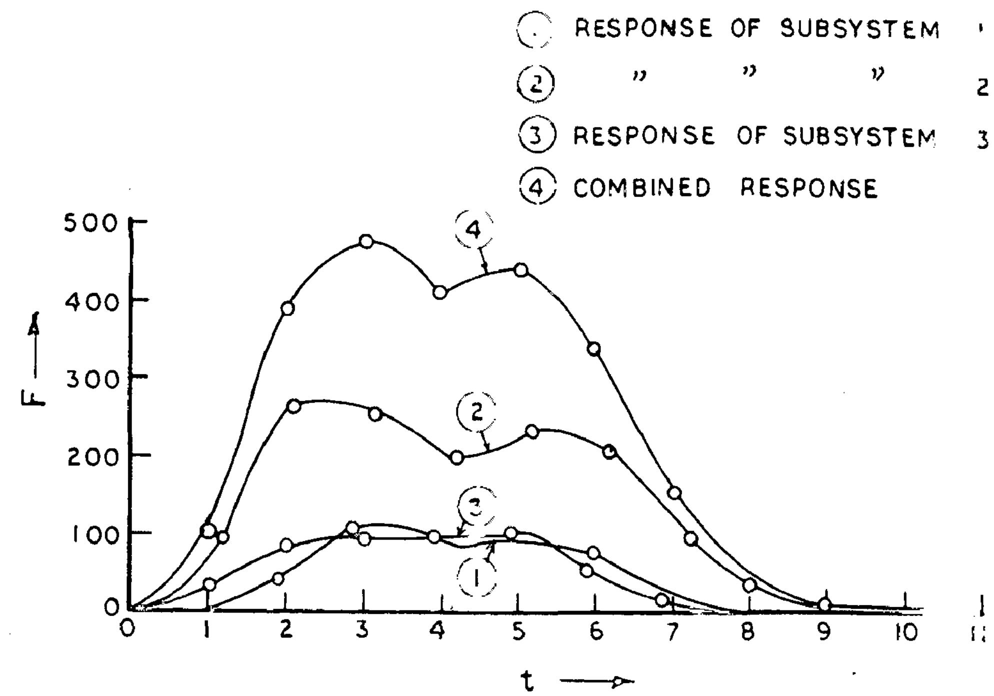
TABLE NO. 3.2

RESPONSES OF SUB-SYSTEMS AT THEIR OUTLETS

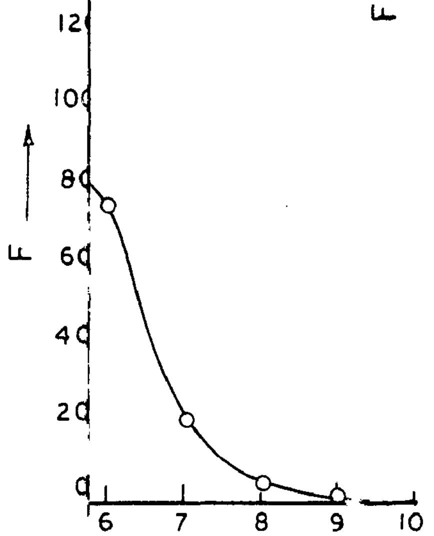
STORM NO	Time in days → / Sub System No. ↓	RUN OFF HYDROGRAPH ORDINATES IN CUMECS 1	2	3	4	5	6	7	8	9	10	11	12
1	1	43.68	106.06	96.09	99.75	105.42	53.46	16.52	2.51	0.28			
	2	97.60	268.71	262.79	200.71	239.14	205.91	97.95	6.42	0.93			
8.7.70	3	31.00	82.70	93.56	94.43	90.92	77.95	21.17	5.95	1.48	0.28		
2	1	16.97	24.33	8.82	1.72	0.18							
	2	65.35	205.38	185.27	56.92	19.82	3.10						
7.7.72	3	20.35	58.70	51.60	17.00	4.62	1.13	0.21					
3	1	24.51	38.38	47.56	41.48	10.03	2.07	0.14					
	2	45.11	122.68	132.73	83.82	36.90	10.30	2.11	0.31				
15.7.72	3	19.96	52.75	69.67	52.77	20.96	5.84	1.44	0.19				
4	1	6.70	58.18	109.42	153.93	218.51	171.03	65.75	19.92	2.99	0.36		
	2	44.05	190.62	344.33	465.10	348.03	583.78	337.78	119.65	30.37	5.83	0.53	
4.7.73	3	22.09	93.44	184.11	268.78	304.98	283.46	191.77	75.98	21.56	5.23	1.04	0.10

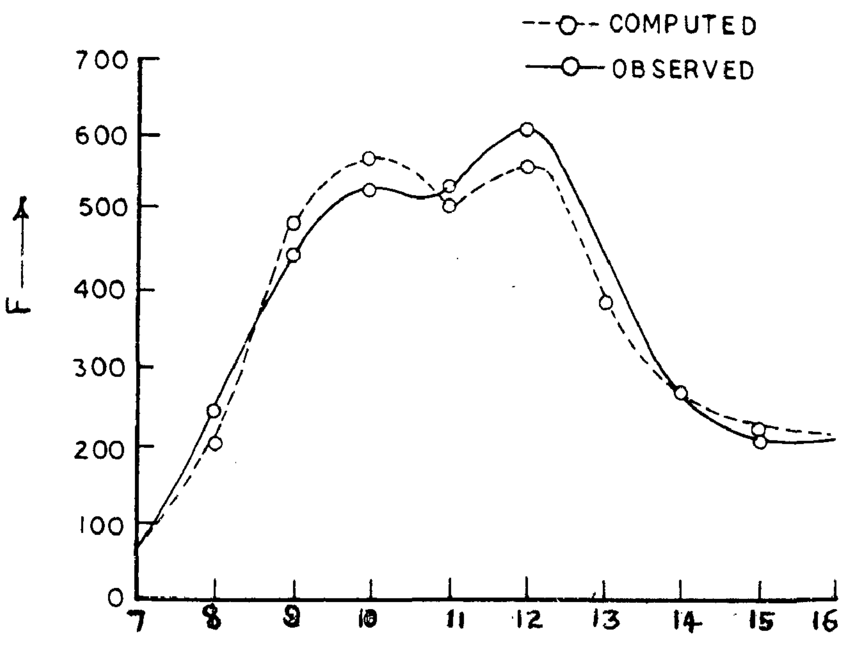
combined response at the outlet is obtained. Base flows are added to the computed responses of each storm. The computed responses are compared with the observed responses.

The computational scheme of responses at the out - let of the system and comparision of observed and computed responses are shown in Fig. Nos. 3.4, 3.5 and 3.6 respectively.



(d) SUPER POSITION OF 3 RESPONSES WITH TIME LAGS





(e) COMPARISION OF THE OBSERVED AND COMPUTED HYDROGRAPHS

INDEX -

I = RAINFALL INTENSITY IN cms/DAY

t = TIME IN DAYS

F = FLOW IN M^3/SEC

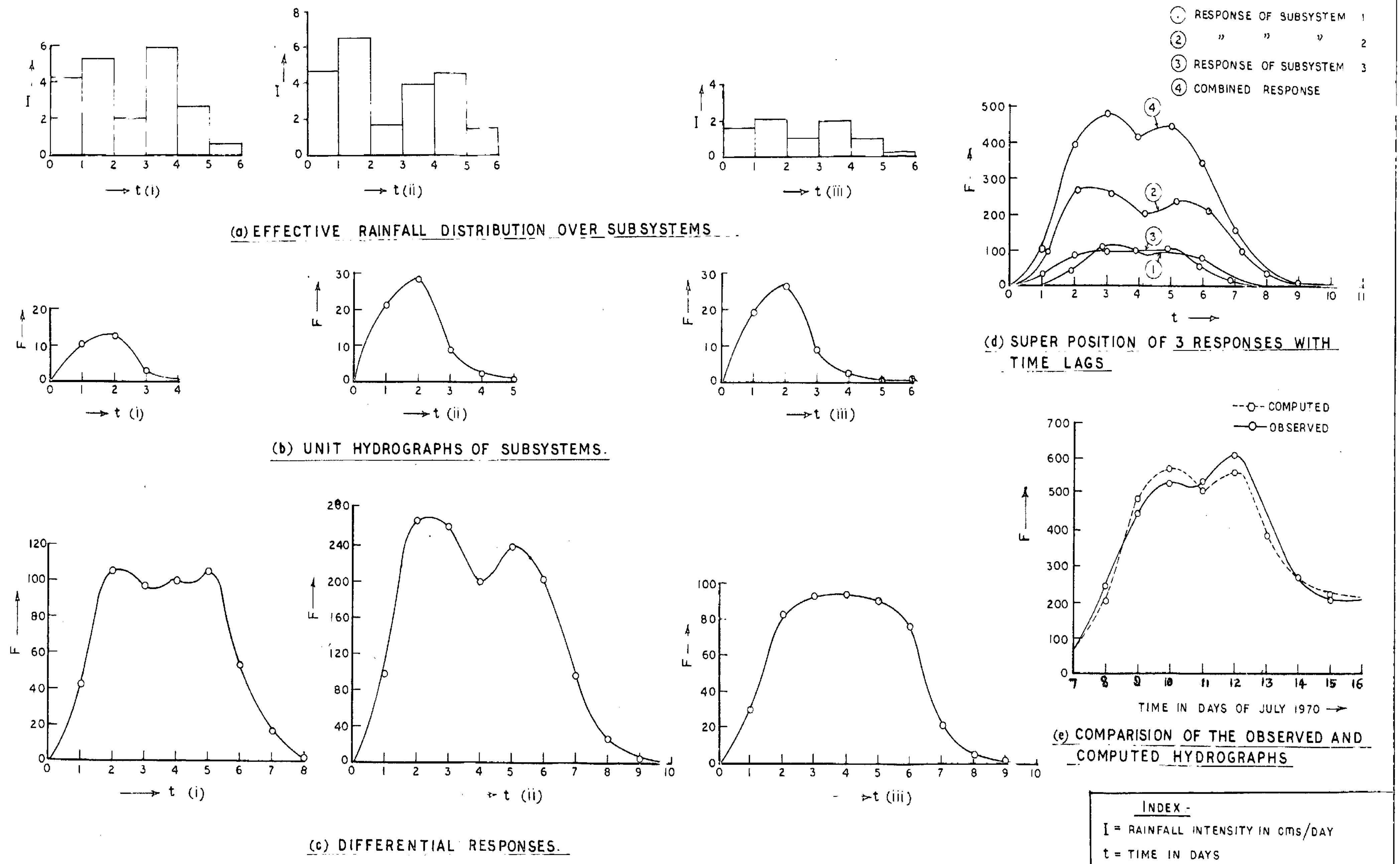


FIG. 3·4_ ANALYSIS OF STORM NO. 1 DTD 8.7.70.

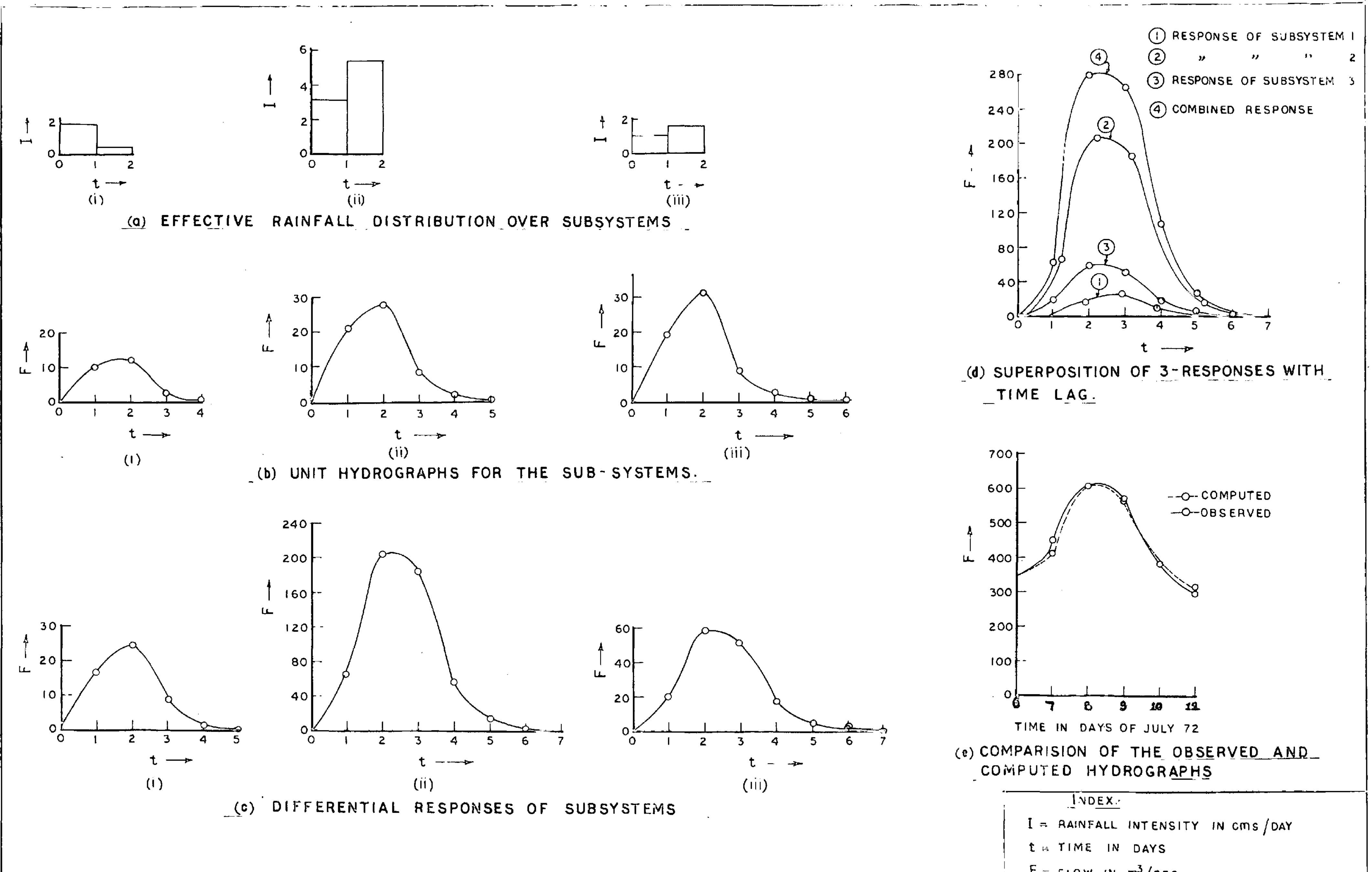


FIG.3·5 ANALYSIS OF STORM NO. 3 DTD. 7. 7. 72

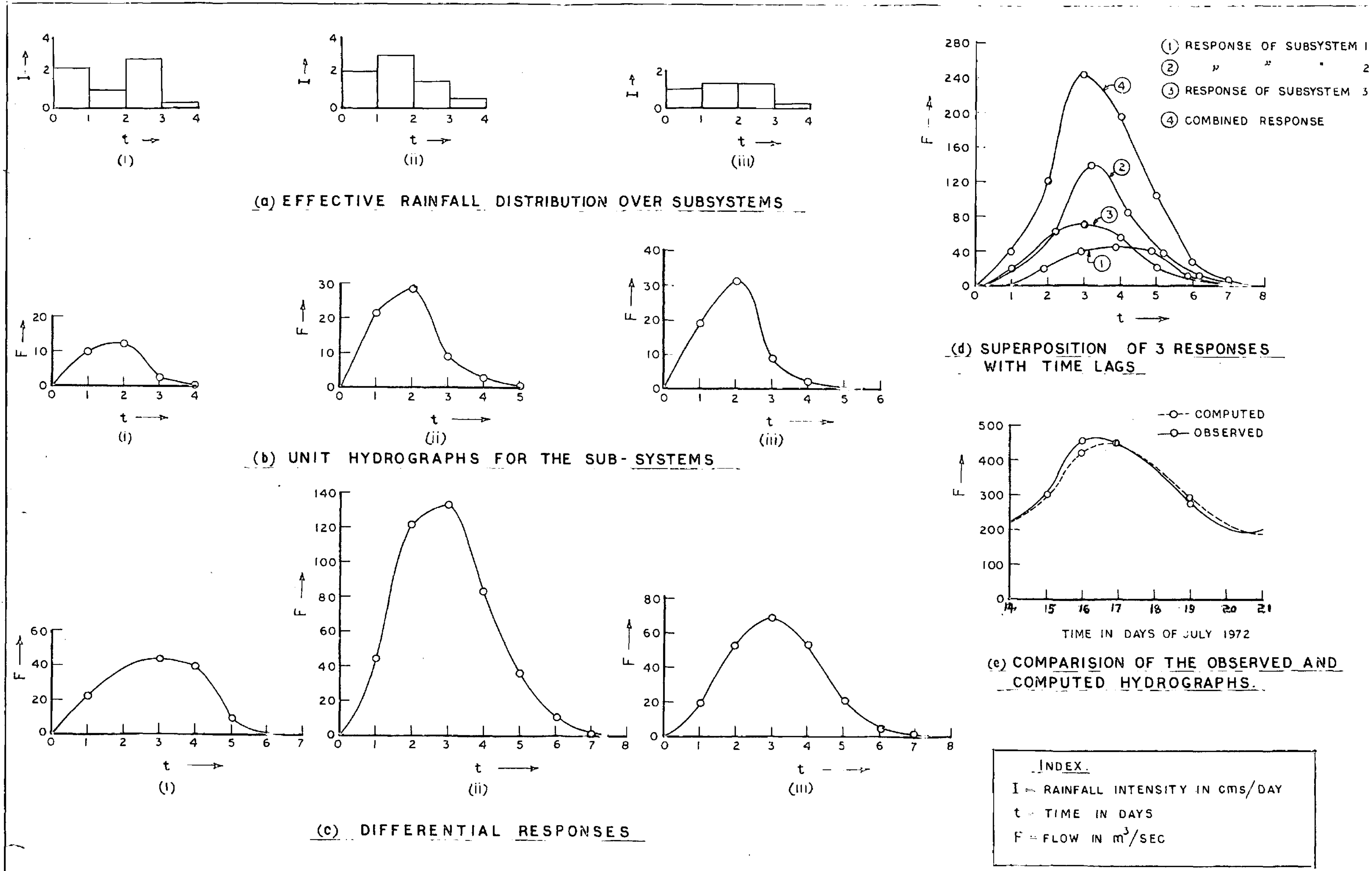


FIG. 3·6 ANALYSIS OF STORM NO. 3. DTD. 15. 7. 72.

3.5.0 Proposal No. 2

3.5.1 Introduction:

In the Proposal No.1, Unit Hydrographs for different sub systems are developed to represent the transfer function. Though the results are satisfactory, yet the approach suffers from the limitation of the unit hydrograph theory. In order to improve, in the present Proposal No.2, the transfer function has been proposed to be taken care of by means of mathematical equations developed with the help of cascade of linear reservoirs which conceptually represent each sub system. Two types of transfer functions are to be developed for each sub system to evaluate its response due to certain input for the following cases.

(i) the period for which the rainfall excess input of a unit duration exists over the sub system.

(ii) periods for which the rainfall excess input stated in (i) does not exists over the sub system.

The differential outputs of the sub systems are subjected to pure translation which is represented by the linear channels. The linearity of the model will enable to apply the principle of superposition in arriving at the combined responses at the outlet of the system.

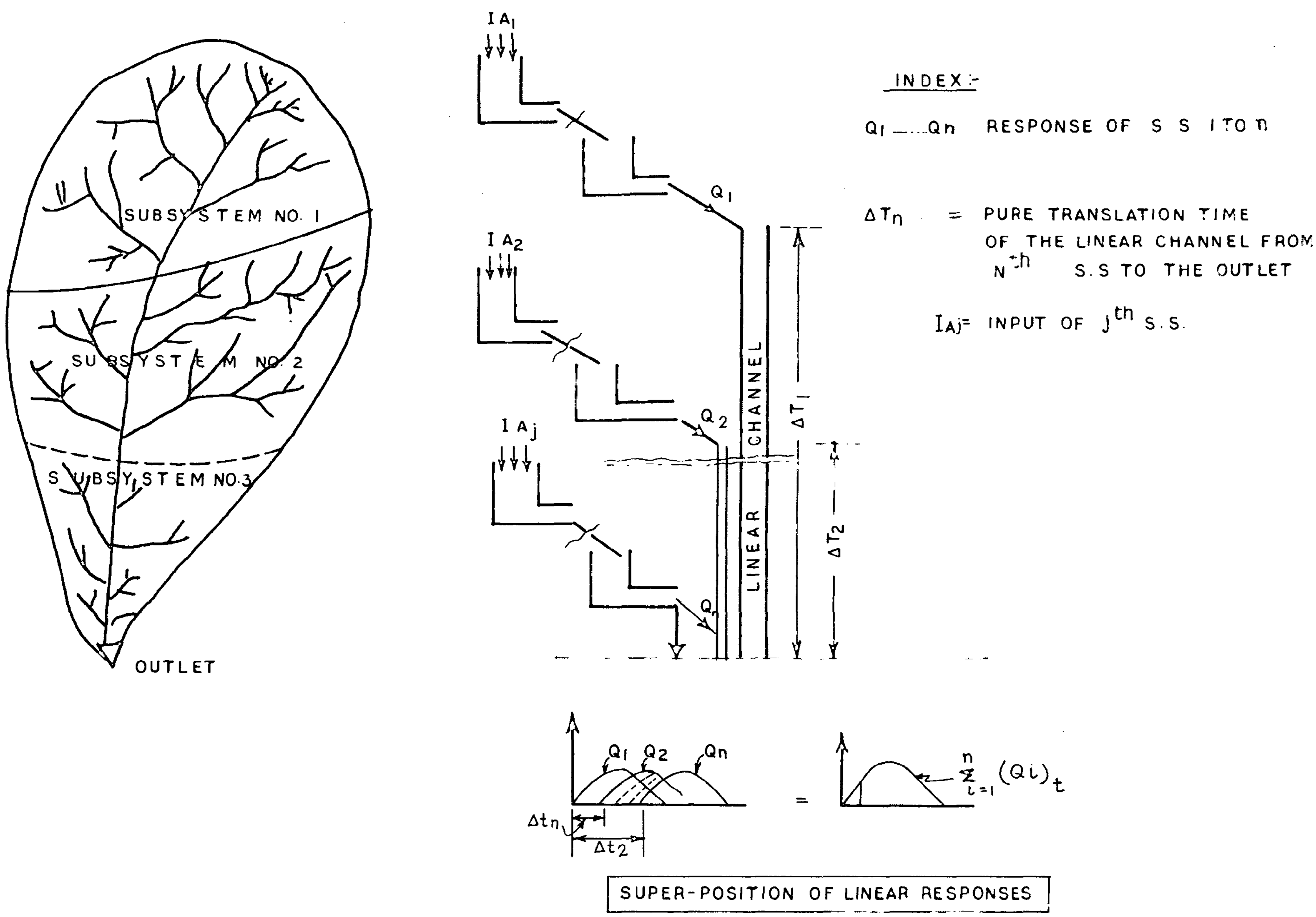


FIG. 3·51. STRUCTURE OF THE PROPOSED MODEL

3.5.2 Development of the Model:

The model is proposed to be developed in the following lines.

(a) Formation of Sub Systems:

Formation of the sub-systems has to be done as described in Section 3.3.1.

(b) Identification of the Parameter n and K:

The parameters n and K of each sub system has to be identified as per the procedure discussed in Section 3.3.2. The conceptual representation of the model is given in Fig. No.3.5.1.

3.5.3 Input Function:

Input function of each sub system of a distributed hydrologic system is to be evaluated as per the methodology described in Section 3.2.0.

3.5.4 The Transfer Function:

It is assumed that the transformation process is linear. Hence a linear transfer function which is capable of representing the transformation process can be developed as follows.

The parameters n and K of the cascade of linear reservoirs are utilised to develop the transfer function.

The above parameters can be identified as described in Section 3.3.2. Integer values of 'n' are considered and the mathematical equation which represent the transformation process during

(i) the period for which the rainfall excess of unit duration exists over the subsystem

(ii) the periods for which the rainfall excess stated in(i) ceases over the subsystem.

(a) <u>Development of the transfer function when the rainfall excess of unit duration exists:</u>

Let I be the rainfall excess in cms/day over the sybsystem of area A sq. kms. Q_1 be the surface outflow in cumecs and S be the storage of the linear reservoir.

From the continuity equation of flow it is known that

Rate of change of storage = Input–Output

$$\therefore \quad \frac{ds}{dt} = IA - Q_1 \qquad \ldots\ldots (3.5.1)$$

Assumption of linearity of the reservoir will leads to

$$\frac{ds}{dt} = K\,\frac{dQ_1}{dt} \qquad \ldots\ldots (3.5.2)$$

Equating 3.5.1 and 3.5.2 it takes the form

$$\frac{dQ_1}{dt} + \frac{Q_1}{K} = \frac{IA}{K} \qquad \ldots\ldots (3.5.3)$$

and this is a linear differential equation of order 1 and the solution of this for the initial conditions at $t_1 = 0$, $Q_1 = 0$ is

$$Q_{1(t_1)} = IA \quad (1 - e^{-t/K}) \qquad \ldots \quad (3.5.4)$$

and it governs the discharge from the linear reservoir No. 1 for the period t_1 for which rainfall excess exists.

Now for the second linear reservoir input will be equal to the output of the first linear reservoir of the cascade.

Input $= Q_{1(t_1)}$

Output $= Q_2 \; (t_1)$

combination of the continuity equation of flow and the linearity of the reservoir will leads to

$$dQ_2 \,/\, dt + Q_2/K = Q_1/K \quad \ldots (3.5.5)$$

This is is a linear differential equation of order one and its solution for the initial conditions $t_1 = 0$, $Q_2 = 0$ will be

$$Q_{2(t_1)} = IA \left[1 - \{ t_1/K + 1 \} e^{-t_1/K} \right] \quad \ldots (3.5.6)$$

and this governs the discharge from second linear reservoir of the cascade for the period t_1 during which rainfall excess exists.

Similarly the discharge at the end of n^{th} linear reservoir of the cascade for the period t_1 during which

~~rainfall~~ excess exists over the sub-system can be shown to be of the general form

$$Q_{n(t_1)} = IA \left[1 - \left\{ \sum_{p=0}^{n-1} \frac{(t_1/K)^{n-1-p}}{(n-1-p)!} \right\} e^{-t_1/K} \right]$$

. . . . (3.5.7)

where

$Q_{n(t_1)}$ = the discharge at the end of the n^{th} reservoir for the period t_1

n = number of linear reservoirs in series

K = storage coefficient

(b) Development of the Transfer Function for the periods greater than the period of rainfall excess

At the end of t_1-days the rainfall input to the system ceases and output at the outlet of the cascade of linear reservoirs is to be evaluated for the periods greater than rainfall excess.

(i) Analysis of First Linear Reservoir of the Cascade

Rainfall → Input = 0.0

Let Output = Q_1^1 for $t > t_1$

combination of the continuity equation of flow with the linearity of the reservoir will leads to

$dQ_1^1 / dt + Q_1^1 / K = 0$ . . (3.5.8). This is a first order linear differential equation and its solution for the initial conditions at $t = t_1$, $Q^1_{1(t)} = Q_{1(t_1)}$ will be

$$Q^1_{1(t)} = IA \left(e^{\frac{t_1}{K}} - 1 \right) e^{-t/K} \quad \ldots (3.5.9)$$

(ii) Analysis of Second Linear Reservoir of the Cascade

Input $= Q^1_{1(t)}$

let Output $= Q^1_{2(t)}$

combination of the continuity equation of flow with the linearity of the reservoir will leads to

$$dQ^1_2 / dt + Q^1_2 / K = Q^1_1 / K \quad \ldots (3.5.10)$$

and its solution for the initial conditions at $t = t_1$, $Q^1_{2(t)} = Q_{2(t_1)}$ will be

$$Q^1_{2(t)} = IA \left[t/K\, e^{t_1/K} - t/K + e^{t_1/K} - t_1/K\, e^{t_1/K} - 1 \right] * e^{-t/K} \quad \ldots (3.5.11)$$

Similarly the discharge at the end of the n^{th} linear reservoir is given by the general form of the equation

$$Q^1_{n,t} = IA \left[\left\{ \sum_{p=0}^{n-1} \left(\frac{t-t_1}{K}\right)^{n-1-p} / (n-1-p)! \right\} e^{t_1/K} * e^{-t/K} - \left\{ \sum_{p=0}^{n-1} \frac{(t/K)^{n-1-p}}{(n-1-p)!} \right\} e^{-t/K} \right] \quad \ldots (3.5.12)$$

where

$Q^1_{n,t}$ = discharge at the end of n^{th} reservoir at time t

n = number of linear reservoirs of the cascade

K = storage coefficient

The responses of each subsystem for the periods (i) when the rainfall excess of unit duration exists (ii) when the rainfall excess specified in (i) ceases can be computed using the equation 3.5.7 and 3.5.12 respectively. These differential responses are subjected to pure translation by using linear channel.

3.5.5 Establishment of the Linear Channels of the System:

The linear channels are used to give the appropriate time lags for routing the differential outputs of a distributed system. The linear channels has to be established as per the description given in Section 3.3.3.

3.5.6. Testing of the Model:

The model developed on the lines discussed above has to be tested to evaluate its performance. Testing of the model has to be done as per the procedure described in Section 3.3.4

3.6.0 Application of the Proposed Model No.2 to the Upper Cauvery Sub-Basin in Karnataka:

It is proposed to develop the model as per procedure described in Section 3.5.0 to the Upper Cauvery sub-basin and then apply to test its performance.

3.6.1 Development of the Model:

The model is proposed to be developed on the following lines.

(a) Formation of Sub-systems:

As discussed in Section 3.5.1 the Upper Cauvery Sub Basin is splitup into 3-subsystems to account for the uneven distribution of rainfall in time and space. Then each sub-system is conceptualised by means of a cascade of linear reservoirs.

(b) Identification of the parameters n and K for each Sub-system:

The parameters n and K for each sub-system are identified as per the procedure described in appendix No.3. Integer values of the parameter 'n' are considered for the model. The values of identified n and K are given in the following table.

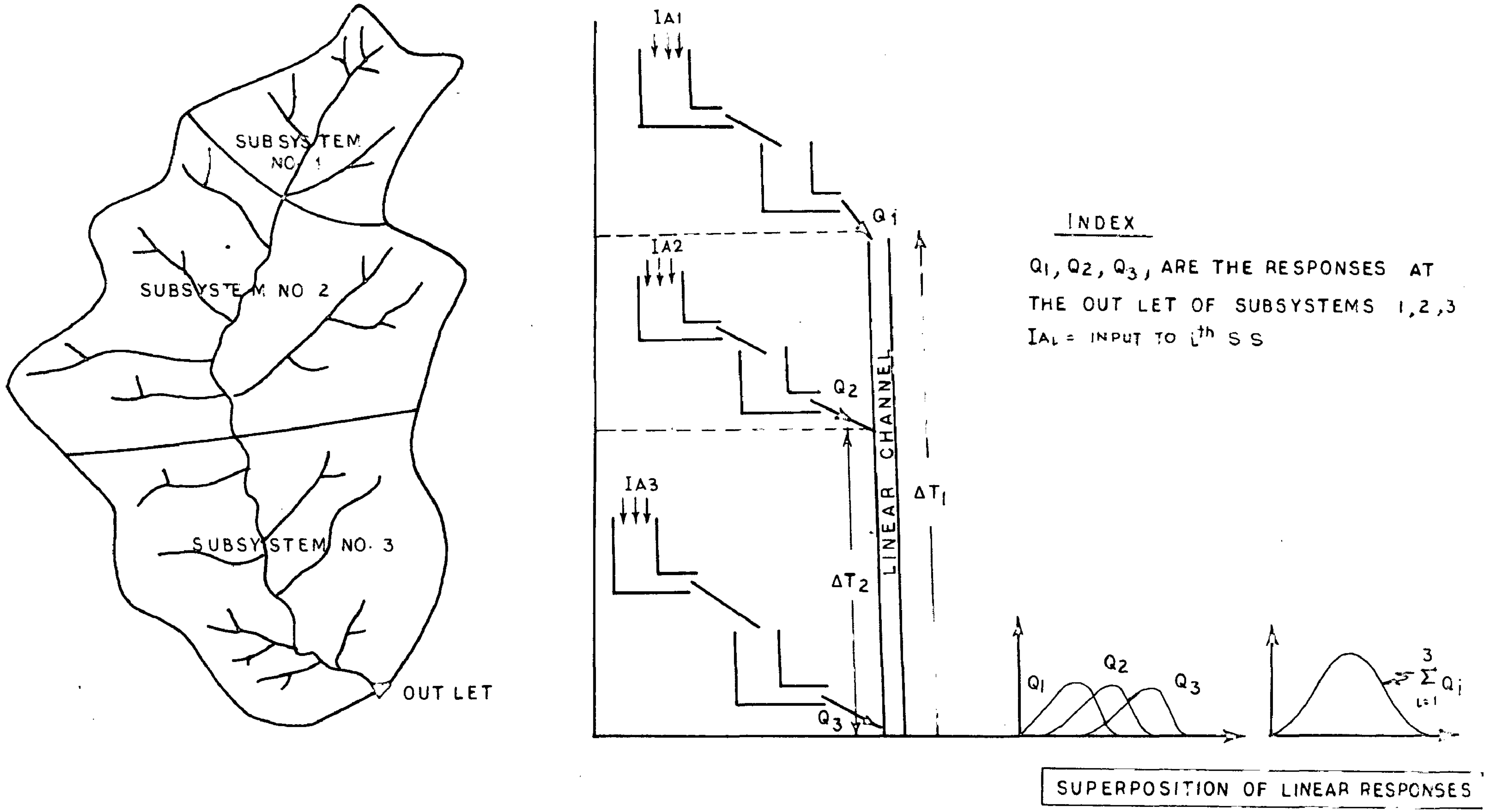


FIG. 3·6·1. STRUCTURE OF THE PROPOSED MODEL FOR UPPER CAUVERY SUB-BASIN

Sub-System No.	n	K
1	2	0.45
2	2	0.58
3	2	0.61

(c) <u>Establishment of the Linear Channel:</u>

The linear channels are introduced from the outlet of the sub-system No.1 and 2. Since the sub-system 3 is contributing to the gauge directly there is no necessity of establishing linear channel to it. The time lags of the linear channels are identified as described in Section 3.4.8 and the same values of time lags i.e. Δt_1 = 0.9 day (time lag for linear channel from the outlet of subsystem No.1 to the gauge) and Δt_2 = 0.2 day (time lag for linear channel from the outlet of the subsystem No.2 to the gauge) are adopted.

The structure of the Model is shown in Fig. No.3.6.1.

3.6.2. Testing the Model:

Three storms of Nos. (1), (2), (3) are selected for testing the performance of the model. The details of storms are shown in Appendix Nos.3 and 4. The input functions of these storms are developed as per the procedure described in Section 3.2.0 and they shown in Appendix No.6.

The input functions are applied on the transfer functions developed in Section 3.5.4 and the differential responses at the outlet of each subsystem are computed and tabulated in Table No.3.6.1. These differential outputs are superimposed with the established values of time lags i.e. Δt_1 = 0.9 day and Δt_2 =0.2 day and the responses of each storm are obtained at the outlet of the system. Adopted base flows are added to these responses. Computed responses are compared with the observed responses of the storm Nos. (1), (2), (3) respectively. The comparision is shown in Fig. No.3.6.2.

RESPONSES OF SUBSYSTEMS AT THEIR OUTLETS

Storm No.	Time in Days → / Sub-System No. ↓	1	2	3	4	5	6	7	8	9	10
1	1	70.09	116.15	77.27	118.22	87.95	37.11	9.32	1.46	0.13	
	2	144.92	302.33	222.76	209.84	247.94	179.61	73.03	20.16	4.86	1.[illegible]
	3	45.31	91.69	88.50	99.49	83.55	44.69	16.32	4.62	1.17	0.[illegible]
2	1	27.27	18.01	5.15	0.94	0.08					
	2	97.03	240.22	137.78	41.98	10.24	2.44	0.58			
	3	29.74	66.74	39.68	12.97	3.60	0.91	0.19			
3	1	37.72	33.32	57.29	26.78	6.40	1.05	0.06			
	2	66.98	137.48	121.20	71.01	27.04	7.43	1.76	0.36		
	3.	29.17	58.30	71.23	42.91	15.73	4.55	1.15	0.25		

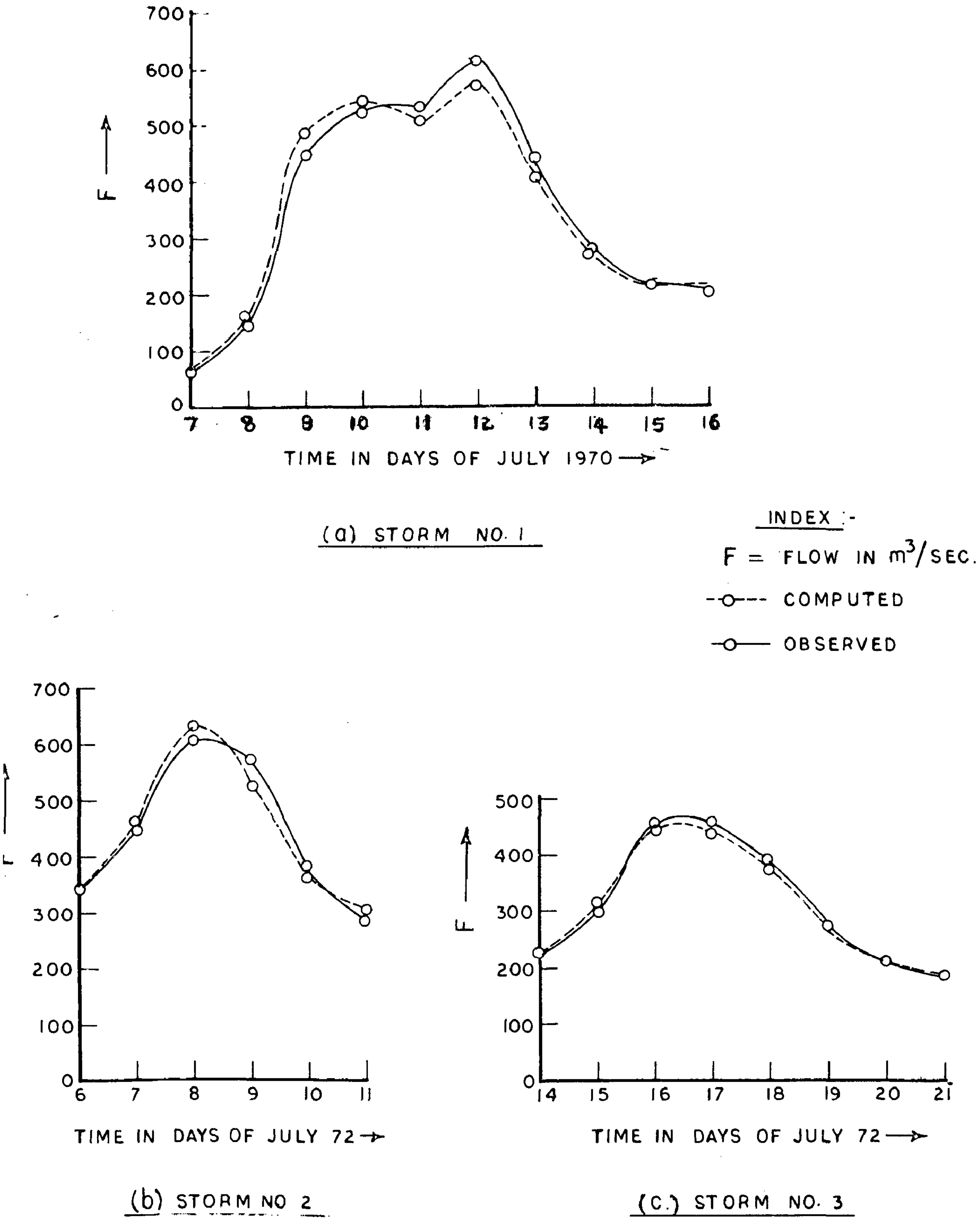


FIG. 3·6·2 COMPARISION OF OBSERVED AND COMPUTED HYDROGRAPHS.

CHAPTER 4

Analog Simulation of the Upper Cauvery Sub-Basin

4.0 Analog Simulation of the Upper Cauvery Sub-Basin in Karnataka:

4.1 Introduction:

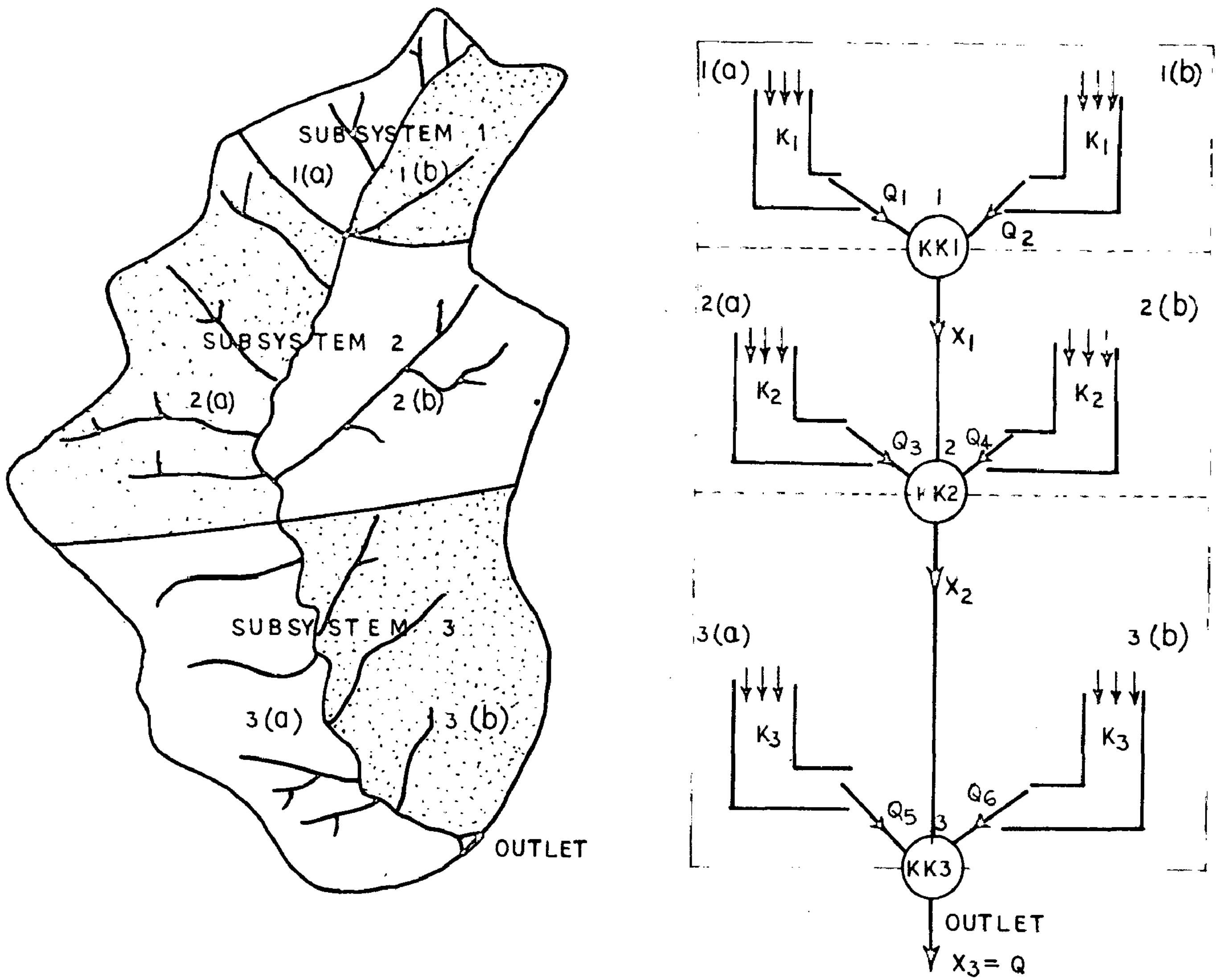
In the previous Chapter, linear channels have been introduced to account for the time lags of the differential responses from the outlets of subsystems to the gauge. In practice this flow has to pass through the natural channels which will have their storage effects. Linear channelscan take care of pure-translations of the differential responses but attenuation in peaks cannot be introduced. In order to over come the above limitation, in this proposed approach the channel action is attemptedly represented by introducing another linear reservoir. The outputs of each subsystem are routed through these linear reservoirs in series as illustrated in Fig.4.1 which shows the configuration of the conceptual representation of the physical system proposed for the Upper Cauvery Sub-basin.

4.2 Principle of Analysis - Mathematical Model:

The physical system shown in fig. 4.1 can be analysed mathematically in the following ways.

Consider the Fig. 4.2. Application of the continuity equation of flow to the Fig.4.2 leads to

$$dS/dt = F-Q \quad \ldots \quad \ldots \quad (4.1)$$



INDEX :-

K_1, K_2, K_3, = STORAGE COEFFICIENTS OF LINNEAR RESERVOIRS OF SUBSYSTEM 1 TO 3

KK1, KK2, KK3 = STORAGE COEFFICIENT OF CONCAPTUAL LINEAR RESERVOIR

X_1 X_2 X_3 = RESPONSES AT THE OUTLET OF SUBSYSTEM 1 TO 3

FIG. 4·1 _ CONCEPTUAL REPRSENTATION OF THE UPPER CAUVERY SUB-BASIN

where

F = Input

Q = Output

S = Storage

Linearity assumption of the reservoir leads to

$$S = KQ \qquad \ldots \ldots (4.2)$$

and $$dS/dt = K\, dQ/dt \qquad \ldots \ldots (4.3)$$

where

K = storage coefficient

Combining equation (4.1) and (4.3), the mathematical model for Fig. 4.2 becomes

$$K \frac{dQ}{dt} + Q = F \qquad \ldots \ldots (4.4)$$

Fig. 4.1 consists of nine such sections as described in Fig.4.2 thus the mathematial model for Fig. 4.1 consist of nine first order linear differential equations namely

$$K_1 \frac{dQ_1}{dt} + Q_1 = F_1 \qquad \ldots \ldots (4.5)$$

$$K_1 \frac{dQ_2}{dt} + Q_2 = F_2 \qquad \ldots \ldots (4.6)$$

$$KK1 \frac{dX_1}{dt} + X_1 = Q_1 + Q_2 \qquad \ldots \ldots (4.7)$$

$$K_2 \frac{dQ_3}{dt} + Q_3 = F_3 \qquad \ldots \ldots (4.8)$$

FIG. 4·2

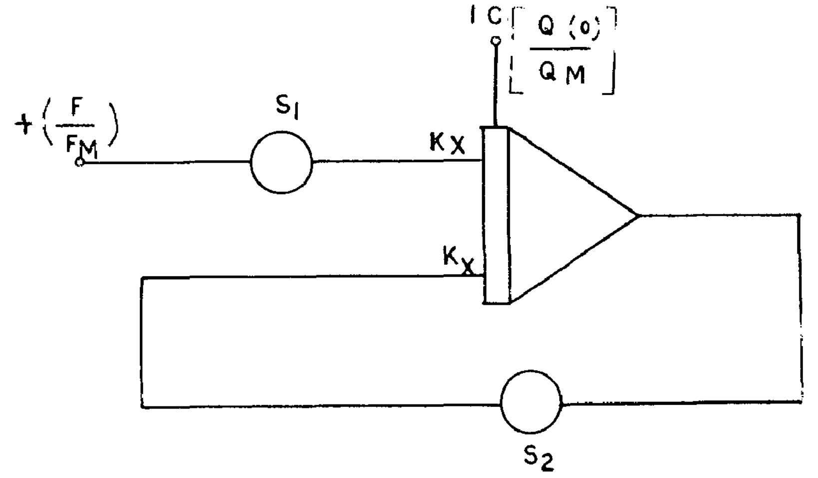


FIG. 4·3

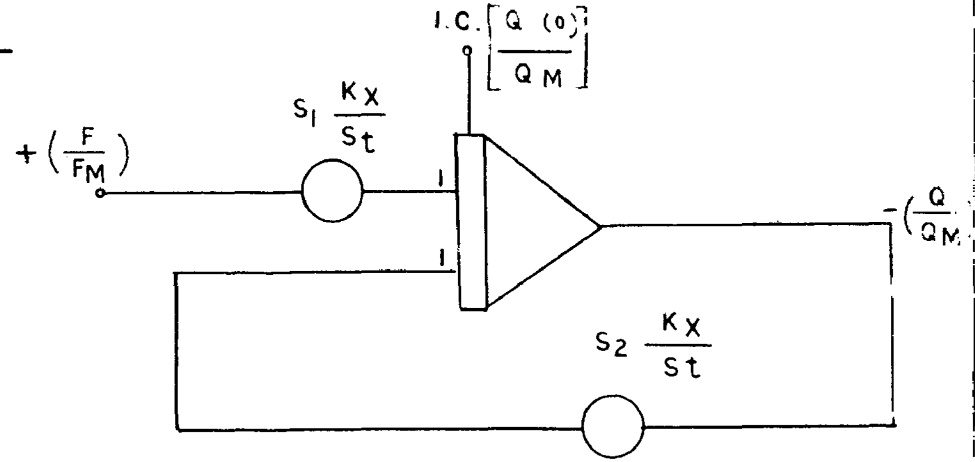


FIG. 4·4

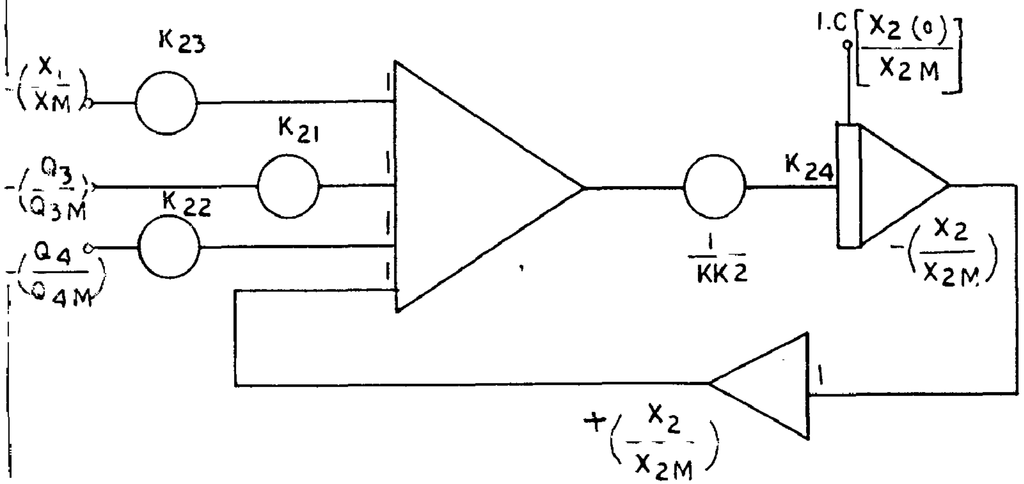


FIG. 4·5

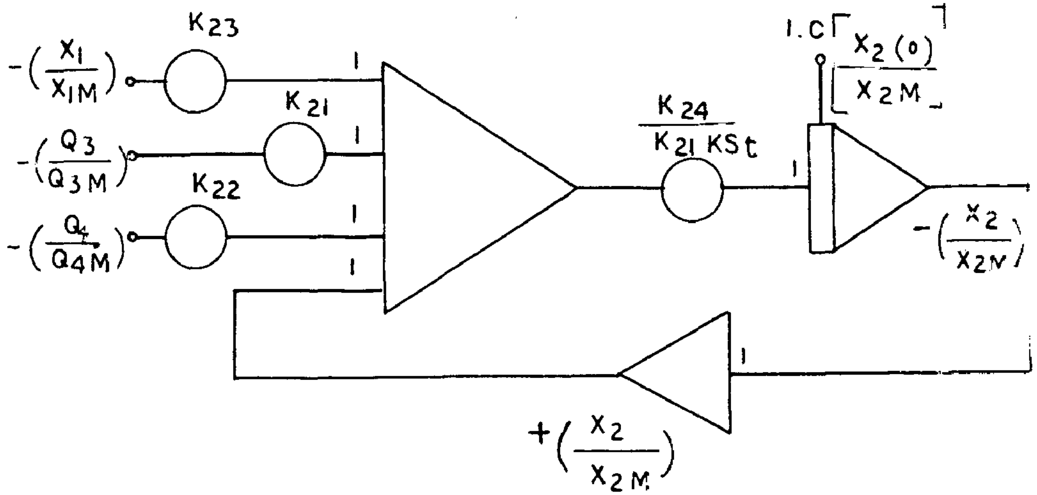


FIG. 4·6

$$K_2 \frac{dQ_4}{dt} + Q_4 = I_4 \qquad \cdots \quad (4.9)$$

$$KK2 \frac{d\,X_2}{dt} + X_2 = Q_3 + Q_4 + X_1 \qquad \cdots \quad (4.10)$$

$$K_3 \frac{dQ_5}{dt} + Q_5 = I_5 \qquad \cdots \quad (4.11)$$

$$K_3 \frac{dQ_6}{dt} + Q_6 = I_6 \qquad \cdots \quad (4.12)$$

$$KK3 \frac{dX_3}{dt} + X_3 = Q_5 + Q_6 + X_2 \qquad \cdots \quad (4.13)$$

where

K_1, K_2, K_3 are the storage coefficients of the linear reservoirs respresenting the subsystem 1, 2, 3 respectively

KK1, KK2, KK3 are the storage coefficients of the linear reservoirs, which are considered at the outlet subsystem 1,2,3 respectively

Q_1, Q_2 . . . . . . Q_6 are the outflows from subsystem 1(a) through 3(b)

X_1, X_2, X_3 are the outflows from the conceptual linear reservoirs 1,2,3

4.3 Analog Simulation of the Basin:

Both magnitude and time scaling are necessary before the above mathematical model is simulated on the analog computer. 'Normalised Variable Approach' is used for

where

F_M, Q_M, $\mathring{Q}_M$ are the possible maximum values of F, Q_M, $\mathring{Q}_M$ respectively

$$\frac{\mathring{Q}}{\mathring{Q}_M} = S_1(F/F_M) - S_2(Q/Q_M) \qquad \ldots (4.17)$$

where

$$S_1 = F_M / K\mathring{Q}_M \qquad \ldots (4.18)$$

and

$$S_2 = Q_M/K\mathring{Q}_M \qquad \ldots (4.19)$$

Analog simulation of the equation (4.17) is shown in Fig. 4.3 where

$$K_X = \frac{\mathring{Q}_M}{Q_M} \qquad \ldots (4.20)$$

There is only one integrator in Fig.4.3 and the gain of this is to be modified from K_X to K_X/S_t in order to introduce the time scale factor S_t so that

$$\tau = S_t . t \qquad (4.14)$$

The modified dagram is shown in Fig.4.4 in which the fact that integrators have gains of 1 and 10 only have also been taken care of

Equations (4.7), (4.10), (4.13) are of similar nature. Considering the equation 4.10, it is normalised to give magnitude scaling and 'Fictitious Integrator Gain Method' (time scaling on the computer) is used for time scaling. In these approaches it is first assumed that there is one to one correspondance between the computer time and the problem time ($\tau = t$), and the equations are rearranged to represent the per unit quantities of the variables. Then the analog simulation diagram is drawn for the normalised equations. Thereafter the gains of the integrators in the simulated diagram are divided by time scale factor 'S_t' so that

$$\tau = S_t \cdot t \qquad \ldots \quad (4.14)$$

where

t = problem time

τ = computer time

S_t = time scale factor

In the mathematical model the equations 4.5, 4.6, 4.8, 4.9, 4.11, 4.12 are of similar nature and they can be represented in the general form

$$K\dot{Q} + Q = F \qquad (4.15)$$ and this is normalised to give

$$\frac{\dot{Q}}{\dot{Q}_M} = \frac{F_M}{K\dot{Q}_M} (F/F_M) - Q_M/K\dot{Q}_M \times (Q/Q_M) \qquad \ldots \quad \ldots (4.16)$$

where

F_M, Q_M, $\mathring{Q}_M$ are the possible maximum values of F, Q_M, $\mathring{Q}_M$ respectively

$$\frac{\dot{Q}_o}{\mathring{Q}_M} = S_1(F/F_M) - S_2(Q/Q_M) \qquad .. \quad (4.17)$$

where

$$S_1 = F_M / K\mathring{Q}_M \qquad .. \quad (4.18)$$

and

$$S_2 = Q_M/K\mathring{Q}_M \qquad .. \quad (4.19)$$

Analog simulation of the equation (4.17) is shown in Fig. 4.3 where

$$K_I = \frac{\dot{Q}_M}{Q_M} \qquad .. \quad (4.20)$$

There is only one integrator in Fig.4.3 and the gain of this is to be modified from K_I to K_I/S_t in order to introduce the time scale factor S_t so that

$$\tau = S_t . t \qquad (4.14)$$

The modified dagram is shown in Fig.4.4 in which the fact that integrators have gains of 1 and 10 only have also been taken care of

Equations (4.7), (4.10), (4.13) are of similar nature. Considering the equation 4.10, it is normalised to give

$$\frac{\overset{\circ}{X}_2}{\overset{\circ}{X}_{2M}} = \left[\frac{Q_{3M}}{\overset{\circ}{X}_{2M}} (Q_3/Q_{3M}) + \frac{Q_4}{\overset{\circ}{X}_{4M}} (Q_4/Q_{4M}) + \frac{X_{1M}}{\overset{\circ}{X}_{2M}} (X_1/X_{1M}) - \frac{X_{2M}}{\overset{\circ}{X}_{2M}} (X_2/X_{2M}) \right] \cdot \frac{1}{KK2} \quad \ldots \ldots (4.21)$$

where

Q_{3M}, Q_{4M}, X_{1M}, X_{2M}, $\overset{\circ}{X}_{2M}$ are the possible maximum values of Q_3, Q_4, X_1, X_2 and $\overset{\circ}{X}_2$

Equation 4.21 can be further reduced to

$$\frac{\overset{\circ}{X}_2}{\overset{\circ}{X}_{2M}} = \left[K21\ (Q_3/Q_{3M}) + K22\ (Q_4/Q_{4M}) + K23\ (X_1/X_{1M}) - KX2\ (X_2/X_{2M}) \right] \cdot \frac{1}{KK2} \quad \ldots \ldots (4.22)$$

where

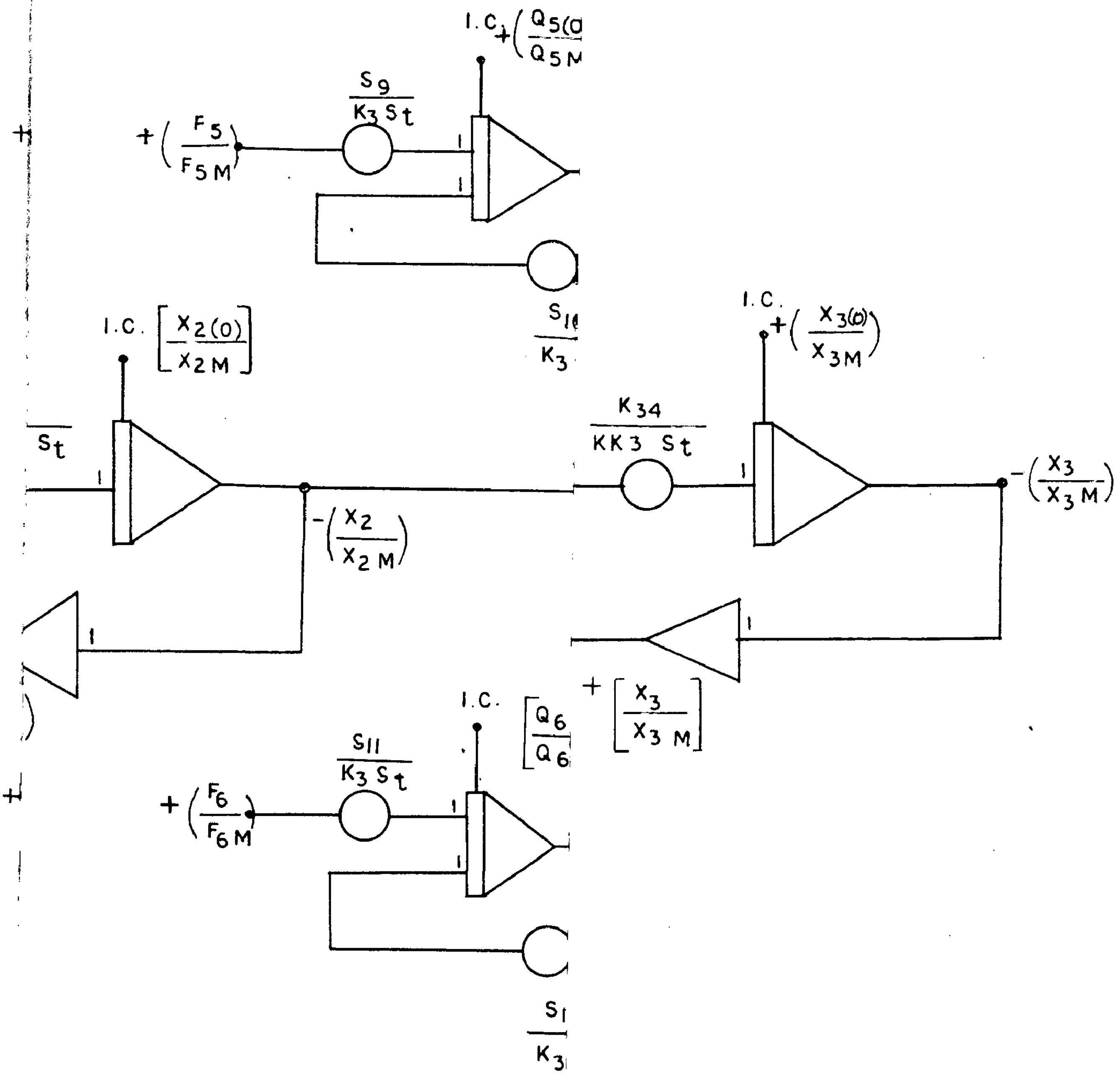
$$K21 = Q_{3M} / \overset{\circ}{X}_{2M} \quad \ldots \ldots (4.23)$$

$$K22 = Q_{4M} / \overset{\circ}{X}_{2M} \quad \ldots \ldots (4.24)$$

$$K23 = X_{1M} / \overset{\circ}{X}_{2M} \quad \ldots \ldots (4.25)$$

$$KX1 = X_2 / \overset{\circ}{X}_{2M} \quad \ldots \ldots (4.26)$$

Equation 4.22 can be can be simulated on analog computer as shown in Fig. 4.5 where $K24 = \dfrac{\overset{\circ}{X}_{2M}}{X_{2M}}$ . . . (4.27)



UVERY SUB-BASIN.

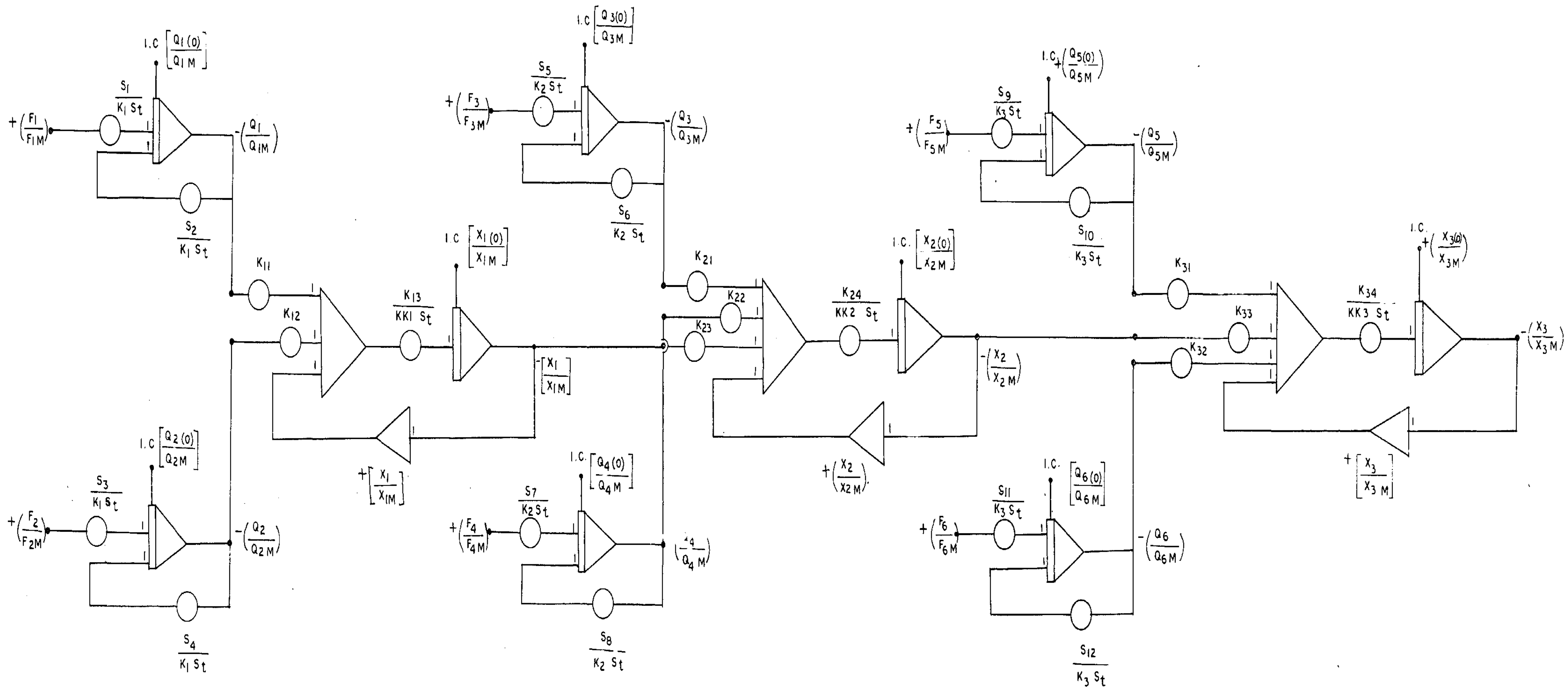


FIG. 4·7. ANALOG SIMULATION OF THE UPPER CAUVERY SUB-BASIN.

the gain of the integrator in Fig.4.5 is modified from K24 to $K24/S_t$ in order to introduce time scale factor S_t. The modified diagram is shown in Fig.4.6.

Analog simulation of the entire physical system i.e. the Upper Cauvery Sub-Basin is shown in Fig. 4.7.

4.4 <u>Salient Features of AC - 20 Analog Computer:</u>

The AC-20 is a 10 volt computer with 20 chopper stabilised operational amplifiers and matching non-linear units. It employs solid state circutary with built in electronic protection engineered and tested for reliable performance over a wide range of temperature. It has got a centralised problem board wired with computing components, individual amplifier overload indicators and push button control for mode selection. It has POTSET, RESET, HOLD and COMPUTE modes of operation for precision computing. It can also be operated on REPETITIVE mode for fast problem checking and preliminary parameter optimisation with 1,5, 20 computations per second. To facilitate the modular expansion of the computing capacity SLAVE mode is included and this allows the control of a number of AC-20s from any of the units.

The basic modules / components of AC-20 are

(i) Twenty chopper stabilised operational amplifiers with individual overload indicators. Out of the above ten can be connected as summer or integrator and the other 10 as Summer only with five inputs.

(ii) Four Electronic Quarter Square Multipliers.

(iii) One Diode Function Generator

(iv) Thirty, ten turn helical coefficient potentiometers for accurate coefficient setting

(v) One Reference potentiometer of 0.1 percent linearity for precision parameter setting.

The following are the general requirements of the AC-20

(i)	Operating temperature range . . .	$20^{\circ}C$ to $40^{\circ}C$
(ii)	Humidity	upto 60 percent
(iii)	Power requirements	230 V, 50HZ, single phase

The specifications for reference power supply are

Nominal voltage	+10V and −10V
Line Regulation	0.001 percent
Load Regulation	0.01 percent
Output Current	200 mA

4.5 Conclusions:

The components required for analog simulation of the basin are more than the available in the AC-20 analog computer. Hence the problem could not be simulated. However another approach has been considered where the transformation process of the distributed system is simulated by considering the unit hydrograph approach. The same has been discussed in the next chapter.

CHAPTER 5

Analog Simulation of the Upper Cauvery Sub-Basin by Unit Hydrograph Approach

5.0 Analog Simulation of the Upper Cauvery Sub-Basin by Unit Hydrograph Approach:

5.1 Introduction:

In this Chapter a case study is made for the computation of synthetic responses for assumed precipitation over the system. As discussed in the previous chapter the simulation of the Upper Cauvery Sub-basin could not be taken up because of the limited components in the AC-20 Analog Computer. However, using the unit hydrograph developed for the Upper Cauvery sub-basin, the identities of the conceptual representation for the sub-basin shown in Fig.4.1 have been identified for the uniform precipitation excess over the entire system. This has been achieved by giving the trial values for the storage coefficients of the cascade of linear reservoirs represented by KK1, KK2 and KK3. The trials were continued till the response of the analog simulation matched well with the computed Unit Hydrograph. The computed unit hydrograph has been arrived at by the super position of the 3 Unit Hydrographs keeping in view the time-lags given by the respective linear channels. This has been given in table No.5.1.

TABLE NO: 5.1

UNIT HYDROGRAPH (COMPUTED)

Time in days	1	2	3	4	5	6
U.H. ordinates in m^3/sec	30.03	64.63	37.50	7.03	1.82	0.33

5.2 Principle of Analysis-Mathematical Model:

The physical system shown in Fig.4.1 is analysed mathematically and the mathematical model developed in Section 4.2 of Chapter 4 will hold good for the present study.

In the mathematical model the equation 4.5, 4.6, 4.8, 4.9, 4.11, 4.12 are of similar nature and they can be represented in the general form

$$K\dot{Q} + Q = P \quad \ldots (5.1)$$

and this is normalised to give

$$\frac{\dot{Q}}{\dot{Q}_m} = \frac{P}{K\dot{Q}_m} - \frac{Q_m}{K\dot{Q}_m}\,(Q/Q_m) \quad \ldots (5.2)$$

where

Q_m and $\dot{Q}_m$ are the possible maximum values of Q and $\dot{Q}$ respectively

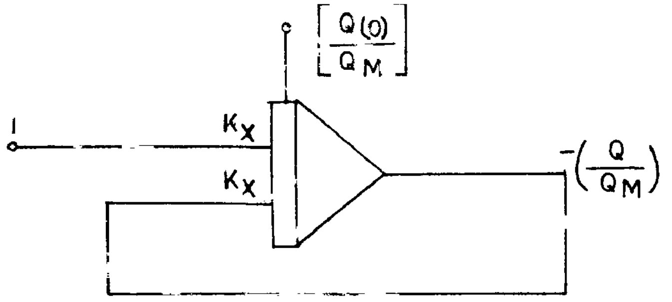


FIG. 5·1

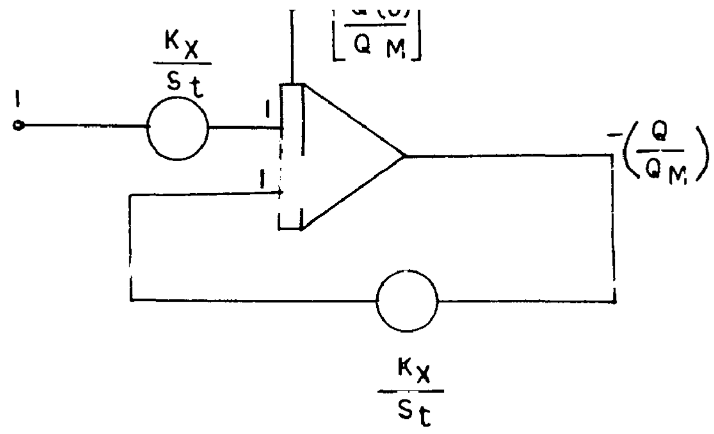


FIG. 5·2

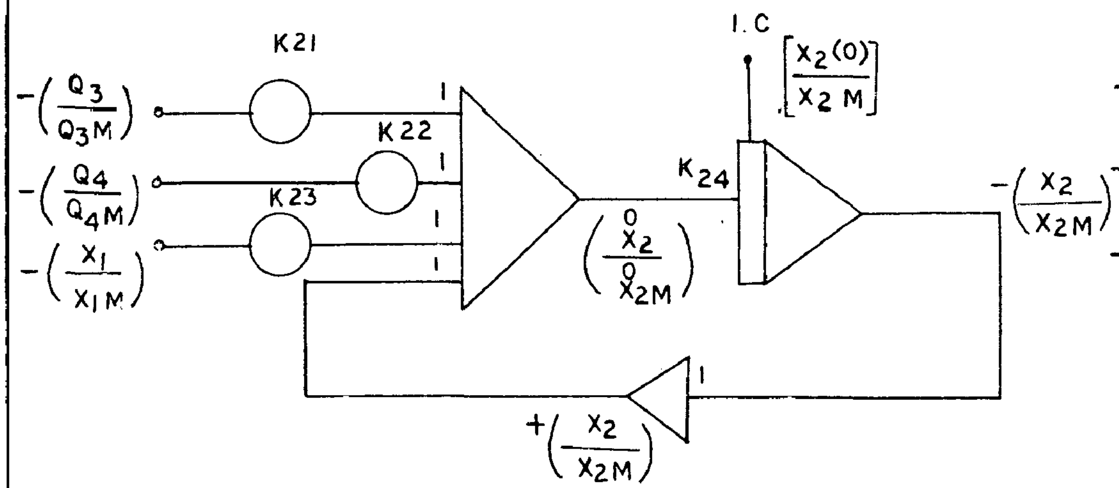


FIG 5·3

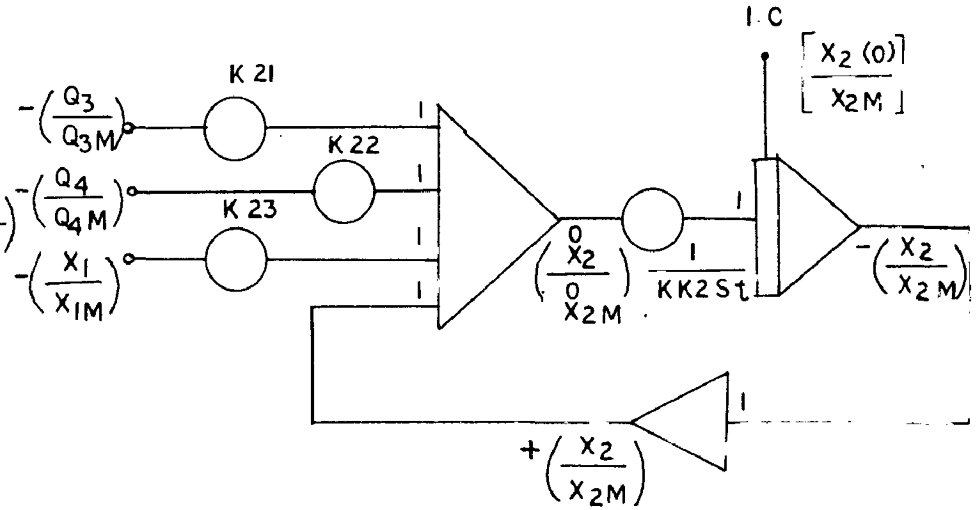


FIG. 5·4

$$K_{24} = \frac{\overset{o}{X}_{2M}}{X_{2M}} = \frac{1}{KK_2} \left[\text{Evns} \quad 5\cdot18 \ \& \ 5\cdot19 \right]$$

Solution of Equation (5.1) is

$$Q = F\,(1-e^{-t/K}) \quad \ldots \quad (5.3)$$

$$\therefore \quad Q_M = F \quad \ldots \quad (5.4)$$

From equation (5.3), $\dot{Q} = \frac{F}{K}\, e^{-t/K}$ . . (5.5)

$$\therefore \quad \dot{Q}_M = \frac{F}{K} \quad \ldots\ldots (5.6)$$

Substituting the values of Q and Q_M in Equation (5.2),

$$\frac{\dot{Q}}{\dot{Q}_M} = 1 - (Q/Q_M) \quad \ldots \quad (5.7)$$

Analog simulation of Equation (5.7) is shown in Fig.5.1 where $K_X = \frac{\dot{Q}}{\dot{Q}_M} = \frac{1}{K}$ . . . (5.8). In Fig. 5.1 there is only one integrator and the gain of this is to be modified from K_X to $\frac{K_X}{S_0}$ in order to introduce the time scale factor S_0 so that $\tau = S_0\, t$ . . . . (5.9). The modified diagram is shown in Fig. 5.2 in which the fact that integrators have gains of 1 and 10 only have also been taken care of

Equation 4.7, 4.10 and 4.13 are of similar nature. Considering the equation 4.10

$$KK2\,\dot{X}_2 + X_2 = Q_3 + Q_4 + X_1 \quad \ldots \quad (5.10)$$

The normalised form of the equation (5.10) is

$$KX2 = \frac{X_{2M}}{KK2\ (\mathring{X}_{2M})} \qquad ..(5.16)$$

Solution of Equation (5.10) assuming Q_3, Q_4, and X_1 to be constants is

$$X_2 = (Q_3 + Q_4 + X_1)\ (1-e^{-t/KK2}) \qquad ..(5.17)$$

$\therefore$ The possible maximum values of X_2 and $\mathring{X}_2$ are

$$X_{2M} = Q_3 + Q_4 + X_1 \qquad ..(5.18)$$

$$\mathring{X}_{2M} = \frac{Q_3+Q_4+X_1}{KK2} \qquad ..(5.19)$$

A similar analysis gives the approximate maximum value of X_{1M} as

$$X_{1M} = Q_{1M} + Q_{2M} \qquad ..(5.20)$$

Substituting the value of X_{1M}, X_{2M}, $\mathring{X}_{2M}$ [illegible], in equns.(5.13), through (5.16) the values of the constants of the equation (5.12) will be

$$K21 = \frac{Q_{3M}}{\sum_{i=1}^{4} Q_{iM}} \qquad ..(5.21)$$

$$K22 = \frac{Q_{4M}}{\sum_{i=1}^{4} Q_{iM}} \qquad ..(5.22)$$

$$\frac{\dot{X}_2}{\dot{X}_{2M}} = \left[\frac{Q_{3M}}{KK2(\dot{X}_{2M})} (Q_3/Q_{3M}) + \frac{Q_{4M}}{KK2(\dot{X}_{2M})} (Q_4/Q_{4M}) + \frac{X_{1M}}{KK2\ (\dot{X}_{2M})} (X_1 / X_{1M}) - \frac{X_{2M}}{KK2\ (\dot{X}_{2M})} (X_2/X_{2M}) \right] \quad \ldots (5.11)$$

where

Q_{3M}, Q_{4M}, X_{1M}, X_{2M} and $\dot{X}_{2M}$ are the possible maximum values of Q_3, Q_4, X_1, X_2 and $\dot{X}_2$ respectively.

$$\frac{\dot{X}_2}{\dot{X}_{2M}} = \left[K21 \left(\frac{Q_3}{Q_{3M}}\right) + K22\ (Q_4/Q_{4M})\ \ (X_1/X_{1M})\ - KX2\ (X_2/X_{2M}) \right] \quad \ldots (5.12)$$

where

$$K21 = \frac{Q_{3M}}{KK2\ (\dot{X}_{2M})} \quad \ldots (5.13)$$

$$K22 = \frac{Q_4}{KK2(\dot{X}_{2M})} \quad \ldots (5.14)$$

$$K23 = \frac{X_{1M}}{KK2(\dot{X}_{2M})} \quad \ldots (5.15)$$

$$KX2 = \frac{X_{2M}}{KK2\ (\overset{\circ}{X}_{2M})} \qquad ..(5.16)$$

Solution of Equation (5.10) assuming Q_3, Q_4, and X_1 to be constants is

$$X_2 = (Q_3 + Q_4 + X_1)\ (1-e^{-t/KK2}) \qquad ..(5.17)$$

$\therefore$ The possible maximum values of X_2 and $\overset{\circ}{X}_2$ are

$$X_{2M} = Q_3 + Q_4 + X_1 \qquad ..(5.18)$$

$$\overset{\circ}{X}_{2M} = \frac{Q_3+Q_4+X_1}{KK2} \qquad ..(5.19)$$

A similar analysis gives the approximate maximum value of X_{1M} as

$$X_{1M} = Q_{1M} + Q_{2M} \qquad ..(5.20)$$

Substituting the value of X_{1M}, X_{2M}, $\overset{\circ}{X}_{2M}$ ~~[illegible]~~, in equns.(5.13), through (5.16) the values of the constants of the equation (5.12) will be

$$K21 = \frac{Q_{1M}}{\sum_{i=1}^{4} Q_{iM}} \qquad ..(5.21)$$

$$K22 = \frac{Q_{4M}}{\sum_{i=1}^{4} Q_{iM}} \qquad ..(5.22)$$

$$K23 = \frac{\sum_{i=1}^{8} C_{in}}{\sum_{i=1}^{4} C_{in}} \qquad \ldots (5.23)$$

$$KK2 = \frac{\sum_{i=1}^{4} C_{in}}{\sum_{i=1}^{4} C_{in}} \qquad \ldots (5.24)$$

Analog simulation of the equation (5.12) is shown in Fig.5.3 where

$$K24 = \frac{\overset{\circ}{X}_{21}}{X_{21}} = \frac{1}{KK2} \qquad \ldots (5.25)$$

The gain of the integrator in Fig.5.3 is modified from K24 to $\frac{K24}{a_0}$ in order to introduce the time scale factor a_0. The modified diagram is shown in Fig.5.4.

Analog simulation of the physical system i.e. Upper Cauvery Sub-basin is shown in Fig.5.5

5.3 Experimental Details

The simulation model shown in Fig.5.5 is patched up on the problem board of the AC-20 analog computer. In order to facilitate the easy checking of the connections,

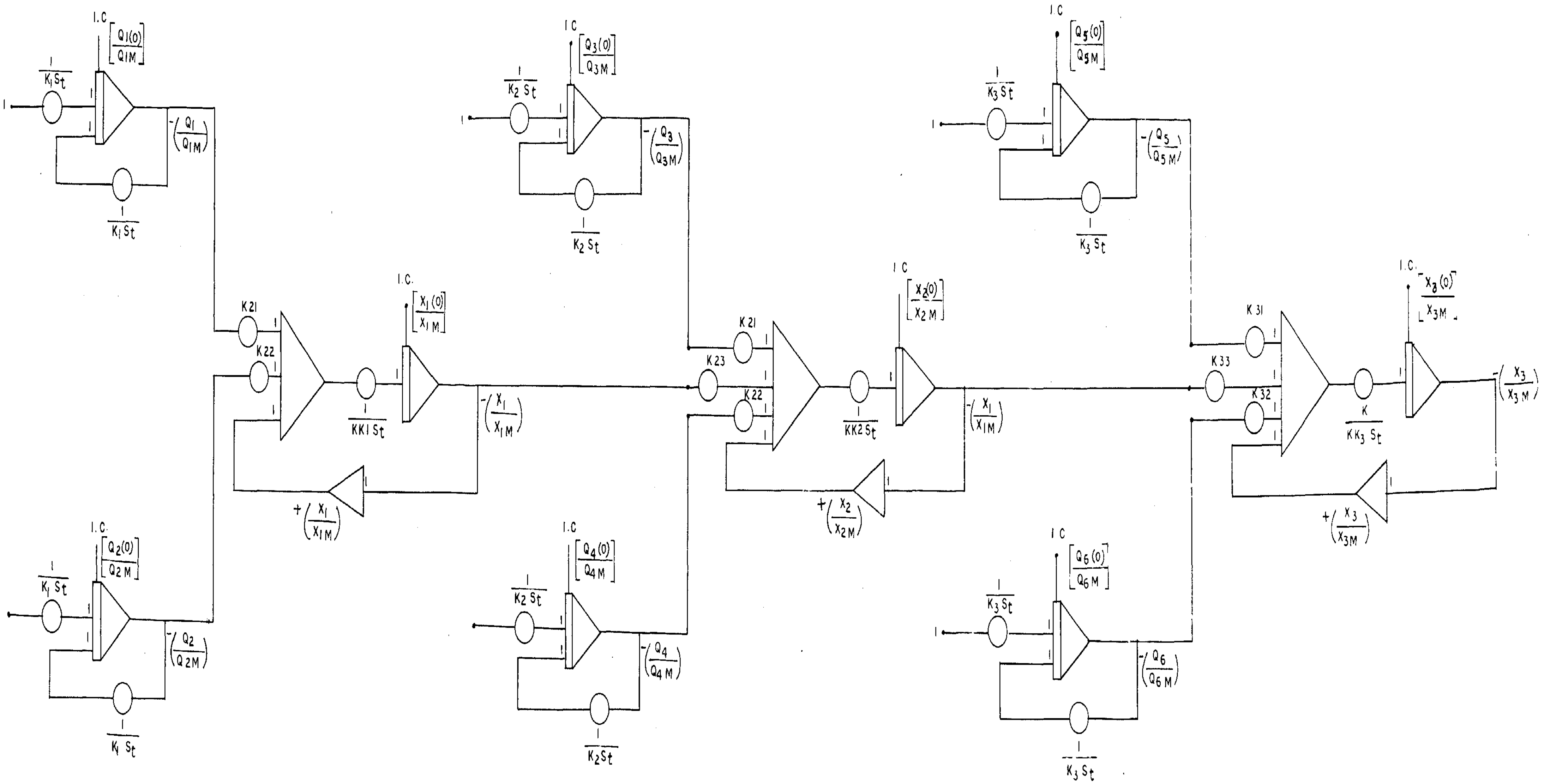


FIG.5·5. ANALOG SIMULATION OF THE UPPER CAUVERY SUB-BASIN

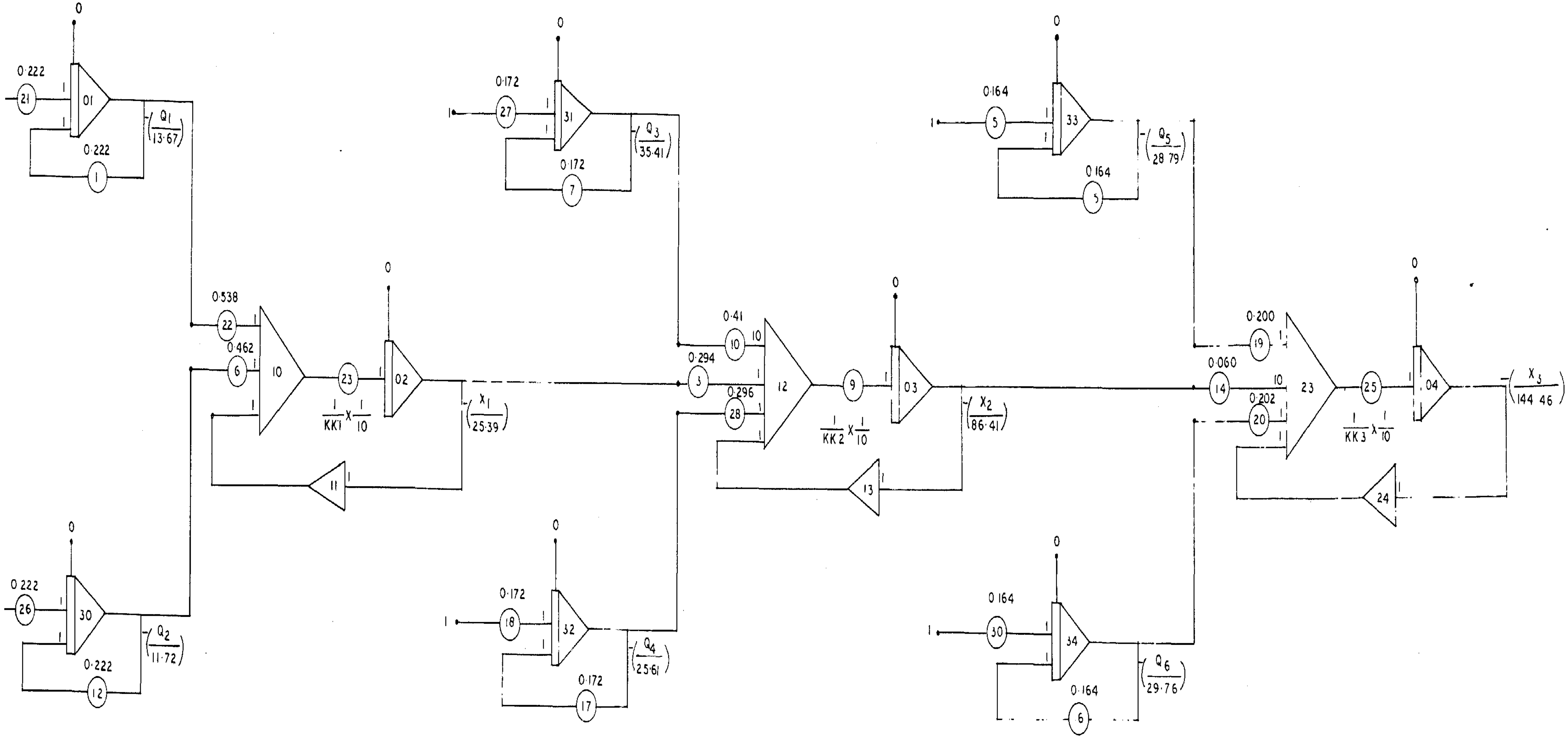


FIG.5·6. ANALOG SIMULATION OF THE UPPER CAUVERY SUB-BASIN the modules are numbered on the diagram. Coefficients of all the potentiometers used in the circuit are calculated and noted in the diagram along with the gains of the Integrators, Summers and Inverters. The same is shown in Fig. 5.6. The detailed calculations of the constants are shown in Table 5.2.

Computer is switched on. It is placed in the pot set state by pressing the Pot-set push button; in this state all the coefficient potentiometers are set using the reference potentiometer and null balancing arrangement. Thereafter, the output indicator is set to the integrator numbered '04' at which the output is required.

Then the computer is placed in the RESET Mode by pressing the RESET push button. The RESET mode operation is required for giving the initial conditions to the system, but in the present case all the initial conditions are zero. From RESET mode the computer is placed in the COMPUTE Mode by pressing the respective push button and simultaneously a stop-watch is started, at the end of ten seconds the computer is placed in 'Hold' mode and the output of the Integrator 04 is noted from the Voltmeter. Keeping the computer still in 'Hold' state all the 'one per unit' input to the system are removed immediately, since one day input is considered for simulating the unit hydrograph. Thereafter the computer is placed in the COMPUTE mode for ten seconds at the end of which it is placed in 'Hold'mode state and the output of Integrator 04 is noted down. The process is continued till the required member of readings are taken. The readings are tabulated and corresponding responses are calculated and the same is compared with the computed unit-hydrograph.

The experiment is repeated for various combinations of variables KK1, KK2 and KK3 till the simulated unit hydrograph matches well with the computed Unit Hydrograph. Thus the values of the above variables are identified.

The readings of the various trials are shown in table No.5.3.

Comparision of the simulated and computed unit hydrographs are shown in Fig. No.5.7.

Using the identified values of KK1, KK2 and KK3, four different cases shown in Table 5.4 has been studied.

(Note: If no t. . all the detailed equations are der .
In the esponse of the system ten seconds of computer ti

Name of Variable	or the constant	Value of the constant
$\frac{Q_{1M}}{Q_{2M}}$	M	0.538
X_{1M}	M	0.462
Q_{3M}	1M	0.410
Q_{4M} X_{2M}	M	0.296
Q_{5M} Q_{6M}	M / M	0.294
X_{3M}	1M	0.200
	Q_{1M}	0.202
	$\frac{Q_{1M}}{Q_{1M}}$	0.598

TABLE NO: 5.2

DETAILS OF CONSTANTS

(Note: If no time scaling is used, one second of computer time is equal to one day of the problem time $\therefore$ all the detailed equations are derived on 1 day basis.
In the table given below a time scale factor of $S_t = 10$ is selected so that $\tau = 10t$ $\therefore$ in the response of the system ten seconds of computer time is equal to one day of the problem time)

Name of Variable	Expression for the Variable	Value of the Variable
Q_{1M}	$= IA_1$	13.67
Q_{2M}	$= IA_2$	11.72
X_{1M}	$= Q_{1M} + Q_{2M}$	25.39
Q_{3M}	$= IA_3$	35.41
Q_{4M}	$= IA_4$	25.61
X_{2M}	$= \sum_{i=1}^{4} Q_{iM}$	86.41
Q_{5M}	$= IA_5$	23.79
Q_{6M}	$= IA_5$	29.26
X_{3M}	$= \sum_{i=1}^{6} Q_{iM}$	144.46

Name of Constant	Expression for the constant	Value of the constant
K11	$\frac{Q_1}{\sum_{i=1}^{2} Q_{iM}}$	0.538
K12	$\frac{Q_2}{\sum_{i=1}^{2} Q_{iM}}$	0.462
K21	$\frac{Q_{3M}}{\sum_{i=1}^{4} Q_{iM}}$	0.410
K22	$\frac{Q_{4M}}{\sum_{i=1}^{4} Q_{iM}}$	0.296
K23	$\frac{\sum_{i=1}^{2} Q_{iM}}{\sum_{i=1}^{4} Q_{iM}}$	0.294
K31	$\frac{Q_5}{\sum_{i=1}^{6} Q_{iM}}$	0.200
K32	$\frac{Q_6}{\sum_{i=1}^{6} Q_{iM}}$	0.202
K33	$\frac{\sum_{i=1}^{4} Q_{iM}}{\sum_{i=1}^{6} Q_{iM}}$	0.598

TABLE NO: 5.3

OUTPUT RESPONSE OF INTEGRATOR - 04

TRIAL NO:1

Combination of variables	Time in days	1	1·6	2	3	4	5	6
KK1=1	Output of 04	0.19	0.30	0.28	0.20	0.12	0.07	0.04
KK2=1 KK3=1	U.H.Ordinates in cumecs	27.45	43.34	40.45	28.89	17.33	10.11	5.78

TRIAL NO: 2

Combination of variables	Time in days	1	1·6	2	3	4	5	6
KK1=1	Output of 04	.33	.42	.37	0.16	0.08	0.03	0.015
KK2=0.5 KK3=0.5	U.H.Ordinates in cumecs.	47.67	60.67	53.45	23.11	11.56	4.33	2.17

TRIAL NO: 3

Combination of variables	Time in days	1	(1.6)	2	3	4	5	[illegible]
KK1=0.5	Output of 04	0.355	0.460	0.400	0.180	0.060	0.020	0.
KK2=0.5 KK3=0.5	U.H.ordinates in cumecs	51.28	66.45	57.78	26.00	8.67	2.89	1.

INDEX.

TRIAL NO.	COMBINATIONS OF VARIABLES		
	KK1	KK2	KK3
1	1	1	1
2	1	0·5	0·5
3	0·5	0·5	0·5

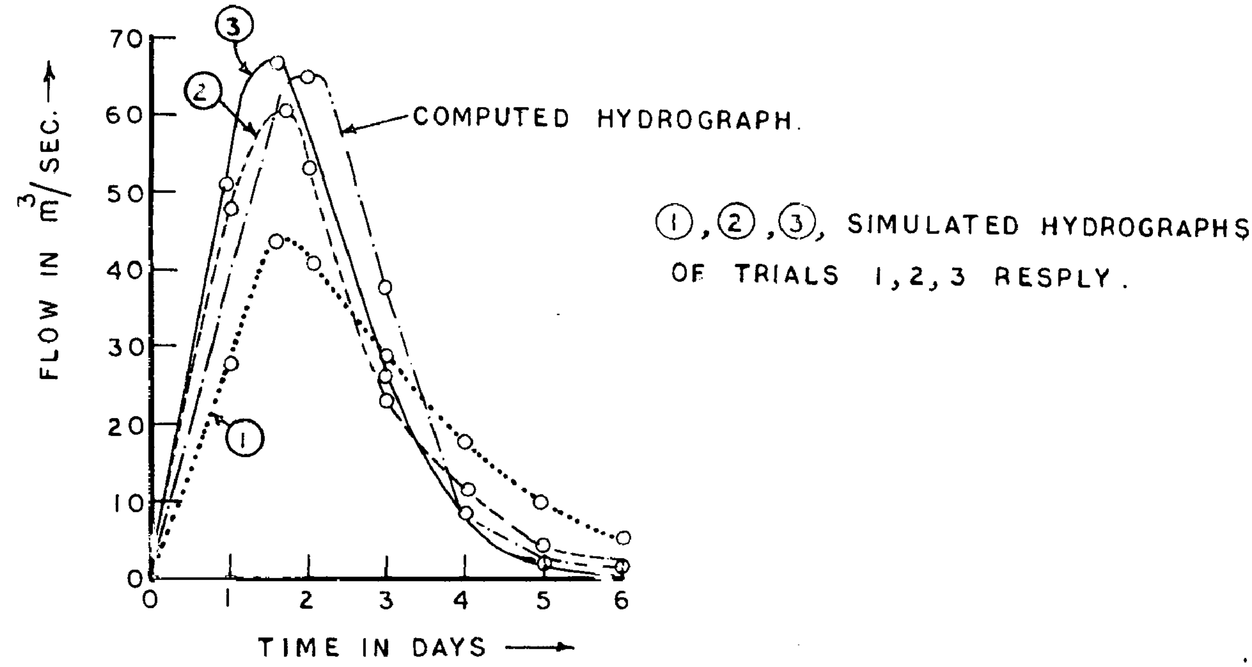


FIG. 5·7. COMPARISION OF THE COMPUTED AND SIMULATED UNIT HYDROGRAPHS.

TABLE NO: 5.4

Case	Rainfall Excess in cms/day over		
	S.S.1	S.S.2	S.S.3
(i)	0.5	1	0.4
(ii)	1.0	0.5	1.0
(iii)	0.8	0.3	0.05
(iv)	0.05	0.3	0.8

The simulated responses of the above four cases are compared with the computed responses and the same is shown in Fig.5.8.

5.4 Conclusions:

The results of this study showed a reasonable accuracy. The study has indicated that storage effects of the natural channels can be accounted in the modelling of the catchment by the conceptual consideration of the linear reservoirs.

INPUT DETAILS

CASE	RAINFALL EXCESS IN CMS OVER		
	S.S.1	S.S.2	S.S.3
i	0.5	1.0	0.40
ii	1.0	0.5	1.0
iii	0.8	0.3	0.05
iv	0.05	0.3	0.8

INDEX.

F = FLOW IN m^3/SEC

t = TIME IN DAYS

S.S. = SUBSYSTEM

—O— COMPUTED DRH

-O--- SIMULATED DRH

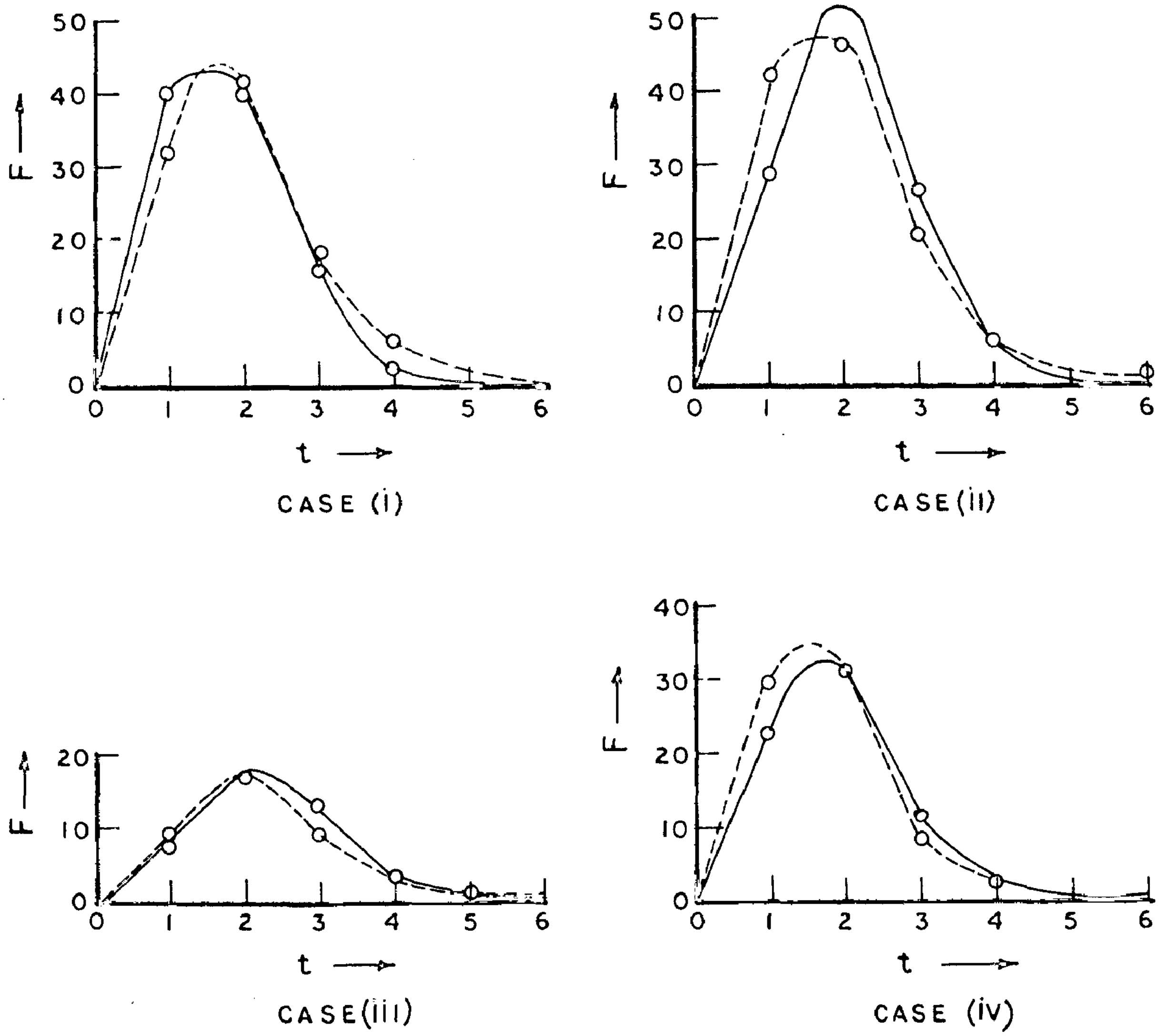


FIG. 5.8_COMPARISION OF COMPUTED AND SIMULATED RUN-OFF HYDROGRAPHS.

CHAPTER 6

Summary of Proposals, Conclusions

Discussion of Results and Proposals for Future Work

6.0 Summary of Proposals, Conclusions, Discussion of Results and Proposals For Future Work

In this dissertation, three linear distributed parameter hydrologic models are evolved and the same are applied to the Upper Cauvery Sub-basin in Karnataka. Each model has been tested for a set of storms.

The distributed input i.e. the precipitation excess is computed by using thiessen polygon method. It would have been more appropriate to use the isohyetal procedure for a catchment like the Upper Cauvery Sub-basin where the landscape is hilly. However, the same could not be done as the procedure involves too much labour it is time consuming also and the results are likely to be subjected to personnel errors. It was considered appropriate to investigate a weight factor between the thiessen approach and the isohyetal approach to modify the input. The factor worked out to be 6 to 7 percent on either side of the mean values adopted and no definite trend could be established.

A constant ∅-index is adopted to evaluate the precipitation excess, it would have been appropriate to consider the ∅-indicies for each subsystem but due to non-availability of the data the same could not be adopted. However, refinement is sought by introducing a weight factor which is computed on each time unit basis. In all the three proposals, the input function is evaluated on the lines discussed above.

In the proposal No.1, three unit hydrographs are developed for the three subsystems of the distributed system by using a conceptual model. Linear channels, considered from the outlet of each subsystem to the gauge enabled in getting the appropriate time lags of the differential responses. The linearity assumption of the model has enabled the proper superposition of the differential responses in order to arrive at the combined response of the system at its outlet.

Eventhough the results of the proposal No.1 are reasonably accurate the approach suffers from the limitation of unit hydrograph theory. Hence the Proposal No.2 is considered for developing the linear transfer function which is capable of representing the transformation process during the period for which the rainfall excess over the system exists and for the periods greater than the period of rainfall excess till the flow becomes negligible. The same has been achieved by representing each subsystem by a cascade of linear reservoirs. This has enabled to consider the distributive nature of input effectively.

In both the proposals linear channels have been introduced to account for the time lags of the differential responses from the outlets of the subsystems to the gauge. In practice this flow has to pass through the natural channels which will have their storage effects. Linear channels can take care of differential responses but attenuation in peaks cannot be introduced. In order to overcome the above limitation in the Proposal No.3 the channel action is attemptedly represented by introducing another linear reservoir. The storage coefficients of these cascade of linear reservoirs are proposed to be identified by simulating the problem on the AC-20 analog computer. The analog simulation is shown in Fig.No. 4.7 The components available in AC-20 where not sufficient to simulate the problem. However the unit hydrograph approach has been adopted for arriving at the storage coefficient values of the conceptual reservoirs. The results of this study has indicated reasonable accuracy in the simulation of the distributed system.

The fact "that exact simulation of the catchment action is possible only when the distributive nature of input is considered" has been revealed by the present study. A suitable simplified procedure has been developed to account for the distributed input to the system.

Proposal No.1 has indicated that the combination of unit hydrographs along with linear channels are capable of modelling the catchment.

Proposal No.2 has indicated that the conceptual representation of subsystems of a distributed system by means of a cascade of linear reservoirs and linear channels is capable of modelling the hydrologic system.

Analog simulation of the catchment has revealed that the linear reservoirs which are considered to represent the channel action are capable of accounting the storage effects of the channels.

The precipitation excess has been considered as proportionate to the gross rainfall registered over the storm duration. This needs further investigation. Exact relationship can only be obtained when the output responses of different subsystems are known.

The ϕ-index involves many assumptions. If the infiltration capacity curves for the soil types are known, the accuracy of the precipitation excess computation can be improved upon.

For an ideal distributed system the catchment has to be divided into subsystems of infinitesimaly small dimensions i.e. point representation of the drainage basin. But the same is of theoretical interest and does not find any useful application in practice. In the models proposed, the subsystems have been worked out on the basis of meteorological homogenity and drainage properties.

However, it would be more appropriate to divide the catchment into subsystem on tributary-wise if the precipitation observations are made extensively over the catchment.

Straight proportion practice has been adopted for seperating the baseflows. However, it will be more appropriate to have an inside into the time distribution of the baseflows between the point of rise and the end point of a particular flood wave.

APPENDICES

APPENDIX NO: 1

CATCHMENT CHARACTERISTICS

S.No.	Name of system or sub system	Area in Sq.km.	Length of Main channel in kms.	Over Land Slope in parts per 10,000
1.	System	1204.00	70.15	206
2.	Sub-system No.1	2116.60	18.02	218
3.	Sub-system No.2	508.50	21.56	225
4.	Sub-system No.3	483.76	30.57	151

THIESSEN WEIGHT OF RAINGAUGE STATIONS

Name of System or Sub System	Area in Sq.kms.	Thiessen Weight of Raingauge Station (%)								
		Avandur	Napoklu	Naladi	Virajpet	Ammati	Mercara	Dubari	Meldane	Suntikoppa
Whole Catchment	1204.00	17.30	23.20	4.10	3.80	8.20	14.50	13.20	6.70	9.00
Sub System No.1	211.60	93.15	-	-	-	-	6.85	-	-	-
Sub System No.2	508.50	1.80	55.20	9.80	6.50	-	26.70	-	-	-
Sub System No.3	483.76	-	-	-	2.57	20.48	5.13	32.76	16.62	22.44

APPENDIX NO: 3

Identification of the Parameters 'n' and 'K'

For the above purpose 3 recorded storm events which are having different distributions of Excess Rainfall Hyetograph (ERH) and Direct Runoff Hydrograph (DRH) are selected. The ERH and DRH of the storms selected are given in Table No. A 3.2 and A 3.3.

Using the DRH and ERH of a storm, the values of 'n' and 'K' are calculated as shown below.

$$M_{DRH1} - M_{ERH1} = nK \qquad \ldots\ldots (1)$$

$$M_{DRH2} - M_{ERH2} = n(n+1)K^2 + 2nKM_{ERH1} \qquad \ldots (2)$$

where M_{DRH1}, M_{ERH1} are the first moments of DRH and ERH about the time origin.

M_{DRH2}, M_{ERH2} are the second moments of DRH and ERH about the time origin.

The values of M_{ERH1}, M_{ERH2}, M_{DRH1}, M_{DRH2} are calculated and substituted in the above equations (1) and (2) and the equations are solved to get the values of n and K. The mean values of n and K are calculated, using these values of n and K along with the physiographic characteristics (that are given in Appendix No.1) in the following equations.

$$K = C_1 \; A^{0.25} \cdot OLS^{-0.3} \cdot L^{-0.085} \quad \ldots \ldots (3)$$

$$n = C_2 \; L^{0.085} \quad \ldots \ldots (4)$$

the values of the constants C_1 and C_2 are obtained. These values are assumed as the representative values for the catchment. The details of the above calculations are given in the following table A 3.1. (Page No. 82)

Subsequently these established values of C_1 and C_2 are used in the equations (3) and (4) along with the physiographic characteristics of the subsystems to evaluate the values of n and K for the subsystems. These values of n and K are the identified valued of the parameters and they are shown below.

Sub System No.	Values of	
	n	K
1	2.18	0.45
2	2.20	0.58
3	2.27	0.61

Storm No.	M_{RRH1}	M_{RRH2}	M_{DRH1}	M_{DRH2}	n	K	Mean values of n	Mean values of K	Identified values of C_1	Identified values of C_2
2	1.13	1.51	2.[illegible]	6.82	2.19	0.59	2.36	0.72	0.777	1.696
3	1.71	3.86	2.99	10.71	2.10	0.61				
4	3.55	14.83	6.20	43.12	2.78	0.95				

TABLE NO: A 3.2

EXCESS RAINFALL HYETOGRAPHS

(a)

Storm No.2	Date	Days of July 1972 6	7
	E.R.H. ordinates in cms.	2.0	3.4

(b)

Storm No.3	Date	Days of July 1972			
		15	46	17	18
	E.R.H. ordinates in cms	1.70	2.00	1.80	1.60

(c)

Storm No. 4	Date	Days of July 1973						
		4	5	6	7	8	9	10
	E.R.H. ordinates in cms.	1.47	4.81	5.48	7.82	8.54	5.13	0.85

Storm No.2	Date	Days of July 1972					
		6	7	8	9	10	11
1	D.R.H. ordinates in cumecs	0	118.72	285.52	264.51	80.05	0

(b)

Storm No.3	Date	Days of July 1972							
		14	15	16	17	18	19	20	21
	D.R.H. ordinates in cumecs	0	80.29	242.00	245.93	182.27	81.41	18.95	0

(c)

Storm No.4	Date	Days of July 1973													
		2	3	4	5	6	7	8	9	10	11	12	13	14	15
	D.R.H. ordinates in cumecs.	0	4.92	71.28	425.22	651.59	708.17	744.24	763.36	685.49	473.41	130.91	54.70	36.81	0

APPENDIX NO. 4

OBSERVED RAINFALL DATA IN MM

STORM NO	DATE	AVANDUR	NAPOKLU	MALADE	VIRAJPET	AMMATI	MERCARA	SUDARI	MALDARE	SUNTIKOPPA
		NAME OF RAINGAUGE STATION								
	8.7.70	65.5	55.9	240.0	40.0	53.3	39.4	20.3	19.1	-
	9.7.70	72.5	88.9	165.0	96.0	52.8	66.0	12.7	26.7	36.8
1	10.7.70	45.2	28.2	80.0	63.0	19.8	33.2	27.7	10.7	36.1
	11.7.70	86.0	36.1	112.0	30.0	34.3	88.2	15.5	25.4	9.7
	12.7.70	40.0	83.1	70.0	30.0	38.1	64.0	3.8	11.4	23.4
	13.7.70	31.5	127.0	78.0	54.0	13.2	27.6	-	5.1	24.4
2	7.7.72	57.0	133.0	128.5	41.0	34.7	43.6	39.4	15.2	47.0
	8.7.72	54.5	177.0	131.7	95.0	68.1	66.0	30.5	25.4	7.1
	15.7.72	40.5	14.0	120.5	9.0	3.1	68.0	17.8	11.4	30.5
3	16.7.72	14.2	34.0	118.3	22.0	18.3	77.6	25.4	8.9	24.1
	17.7.72	52.6	4.0	112.5	23.0	9.7	60.0	50.0	7.3	6.1
	18.7.72	12.1	26.0	63.5	11.0	8.1	39.0	16.5	1.3	5.1
	3.7.73	16.5	12.0	84.0	14.0	10.9	10.4	10.7	-	13.2
	4.7.73	14.6	50.1	124.0	31.0	18.5	41.6	33.0	9.4	40.6
4	5.7.73	70.2	90.0	161.4	33.0	52.1	88.6	42.7	34.3	41.9
	6.7.73	62.5	100.2	178.0	76.0	75.7	86.6	76.2	47.5	77.5
	7.7.73	105.4	133.0	157.2	89.0	76.2	102.0	71.1	53.3	73.7
	8.7.73	125.1	177.0	150.3	116.0	80.9	138.2	9.1	24.6	89.2
	9.7.73	30.2	87.0	160.0	90.0	110.5	77.0	22.9	52.6	17.8
	10.7.73	26.3	20.0	56.0	39.0	30.2	33.8	7.6	22.9	11.7

APPENDIX NO: 5

DIRECT RUNOFF DEPTH COMPUTATIONS

Storm No.	Date	observed runoff in cumecs	Base flow in cumecs	Direct runoff in cumecs	Direct runoff depth in cms
1	2	3	4	5	6
	7.7.70	62.34	62.34	0	
(1)	8.7.70	244.22	77.87	166.35	
	9.7.70	448.44	93.40	355.04	$D = \frac{2238.8 \times 86400}{1204 \times 10^6}$ x
	10.7.70	525.81	108.93	416.88	10
	11.7.70	532.40	124.46	410.94	= 16.07
	12.7.70	613.54	139.99	473.55	
	13.7.70	439.15	155.52	283.63	
14.7.70		272.34	171.06	100.63	
	15.7.70	218.70	186.57	32.13	
	16.7.70	202.14	202.14	0	
	6.7.72	342.15	342.15	0	
	7.7.72	450.51	331.77	118.74	D =
	8.7.72	607.91	321.39	286.52	$\frac{749.82 \times 86400}{1204 \times 10^6}$ X
(2)	9.7.72	575.51	311.00	264.51	
	10.7.72	380.58	300.53	80.05	= 5.38
	11.7.72	290.22	290.22	0	

	2	3	4	5	6
	14.7.72	225.41	225.41	0	
3)	15.7.72	299.64	219.35	80.29	
	16.7.72	455.29	213.29	242.00	$D = \frac{850.15 \times 86400}{120 \times 10^6} \times 10$
	17.7.72	452.46	207.23	245.23	$= 6.10$
	18.7.72	383.44	201.17	182.27	
	19.7.72	276.52	195.11	81.41	
	20.7.72	208.00	189.05	18.95	
	21.7.72	183.00	183.00	0	
	2.7.73	66.73	66.73	0	
4)	3.7.73	79.52	74.60	4.92	
	4.7.73	153.75	82.47	71.28	
	5.7.73	515.56	90.34	425.22	
	6.7.73	750.00	98.21	651.59	$D = \frac{4750.1 \times 86400}{1204 \times 10^6} \times 100$
	7.7.73	814.25	106.08	708.17	
	8.7.73	858.19	113.95	744.24	$= 34.1$
	9.7.73	885.18	121.82	763.36	
	10.7.73	815.18	129.69	685.49	
	11.7.73	611.98	137.57	473.41	
	12.7.73	276.34	145.44	130.91	
	13.7.73	208.00	153.31	54.70	
	14.7.73	198.00	161.18	36.81	
	15.7.73	169.09	169.09	0	

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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	[illegible]	[illegible]	[illegible]	[illegible]
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	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	0.22
[illegible]	[illegible]	3.10	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]
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	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	0.10
	[illegible]	[illegible]	[illegible]	[illegible]
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๑	๙.๗.๒๐	๖.๐๗	๔.๔๔	๐.๗๓๒๑	๗.๒๐	๕.๒๕	๕.๘๐	๖.๕๓	๒.๘๓	๒.๘๙
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