CONCEPTUAL MODELLING
OF
EXCESS-RAINFALL DIRECT RUNOFF PROCESS

A Dissertation
submitted in partial fulfilment of
the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
HYDROLOGY

By

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DATED: 5th APRIL, 1976

C E R T I F I C A T E

This is to certify that the dissertation entitled " CONCEPTUAL MODELLING OF EXCESS-RAINFALL DIRECT RUNOFF PROCESS" being submitted by Sri Pulin Chandra Buragohain in partial fulfilment of the requirements for award of the degree of Master of Engineering in Hydrology of the University of Roorkee, is a record of the candidate's own work carried out by him under my supervision and guidance. The material embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that Sri Buragohain has worked for a period of six months since October 1975 to March 1976 in the preparation of this dissertation under my guidance, at this University.

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S Y N O P S I S

Direct runoff hydrograph from a natural watershed is simulated using a conceptual model instead of the traditional unit hydrograph approach. The Nash model is taken as the basis of this study. Two approaches have been considered for this purpose. In the first approach the Nash model is used with nonuniform areal distribution of rainfall considering the catchment as one unit. In the second approach, the catchment is divided into three subareas on the basis of tributary drainage boundaries. Each subarea is then represented by the Nash model. Then the combined flow from the two tributary subareas are taken through a linear channel to join the flow from the third subarea and the direct runoff hydrograph is reconstructed. The performance of the two approaches is evaluated using rainfall runoff data from catchment Bridge No.566 of Indian Railways. The model efficiency for both the cases are found to be quite promising, though the subarea type model gave better performance in dealing with the rainfall data with nonuniform areal distribution of rainfall.

The sensitivity of the subarea type model to time distribution of excess rainfall is tested with assumed values of model parameters and typical rainfall excess distributions. The results show good response sensitivity as indicated by the variation of hydrograph peak.

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CHAPTER - 1

INTRODUCTION

CHAPTER-1

INTRODUCTION

1.1. SIGNIFICANCE AND NEED FOR MODELLING STUDIES:

Hydrologic investigations involving small water sheds require greater attention in view of growing development ofwater resources, construction activities and conservation of soil and water in small water sheds. The determination of runoff from rainfall which is the basic problem of hydrologic investigations occupies a central place in applied hydrology. This will not only help in prediction of floods but also in determination of possible effects of development activity in the water shed.

The type of hydrologic analysis made on small watershed will depend largely on the availability of hydrologic data. It becomes uneconomical to provide gauging for small catchments and hence hydrologic datas are generally not available for small water sheds. These are likely to be available on a regional basis and so the methods developed for the analysis of rainfall runoff process are done in regional basis. Though the emperical formulas and synthetic approaches are available for predicting runoff from rainfall all these have limited applications. The results obtained with these formulas may vary considerably according to the selection of the coefficients in these formulas when these are actually applied to practical problems.

The unit hydrograph approach proposed by Sherman to stream flow prediction has developed into one of the most powerful tools of applied hydrology. It has, however, retained an emperical character and no general theoretical basis for the method has been evolved. The absence of a general theory of the unit hydrograph to produce accurate forecast for specific catchments, leads to the development of conceptual models for prediction of runoff from rainfall. Looking from different angles on the complex problem of rainfall and runoff, mathematical representation of the water shed responses is found to be better suited to simulate the transformation of input rainfall to produce the runoff hydrograph. Development of numerical techniques and high speed computers has made sufficient advancement in this direction and as a result of it, a good number of conceptual models have been formulated in the recent past. The recent experiment with conceptual models of the runoff process has shown promise of considerable progress.

1.2. RAINFALL RUNOFF PROCESS :

The rainfall runoff process in a catchment is a complex and complicated phenomenon governed by a large number of known and unknown climatic and physiographic factors that vary both in space and time. The rain falling on the catchment undergoes number of transformations under the influence of these factors before it emerges as a runoff at the catchment outlet. In this process two stages can be distinguished, (a) the process whereby rainfall results in generation of an amount of excess water in surface, subsurface and ground water zones and (b) the way in which the excess water flowing as surface runoff, subsurface runoff and ground water runoff appears as total runoff at catchment outlet.

To approximate a hydrologic system by a linear model in analysing the rainfall runoff relationship, one must use rainfall excess as input instead of direct observed rainfall data, remove baseflow from the total runoff to obtain the output runoff and then use the modifidd input and output to obtain a unit hydrograph. In other words hydrologist try to remove the factors contributing to the nonlinearity of the rainfall runoff relationship.

In the formation of flood hydrograph three factors play a definite and effective role. These factors as stated by Chebotarev (3), are (1) time of travel (2) amount of losses and (3) rainfall intensity. Since those factors have an important role and if the differences in the geomorphological and infiltration properties of the water shed are appreciable, then it will not be enough to consider only the rainfall intensity effecting the shape of the flood wave. However, the traditional method of analysing a hydrologic system is the unit hydrograph approach where in the catchment system is assumed to be a lumped, linear, time-invarient and deterministic one and thus it does not consider the areal distribution and variability of rainfall input. The weakness of this method naturally lies in these assumptions.

1.3. DEVELOPMENT OF CONCEPTUAL MODELS:

Numerous methods based on conceptual models have been proposed for predicting the runoff hydrograph resulting from the application of real or hypothetical storms to a water shed. Because of complexities of the runoff processes almost all of the methods developed for use on ungauged water shed are based on simplified relationships between average conditions within the water sheds, the applied storm and the resulting hydrograph. The computational effort required to describe the detailed dynamics of the various phenomenon within the water shed boundaries, has any other approach impractical. Most of the conceptual models have been proposed for either determination of unit hydrograph or for instantaneous unit hydrograph. The basic components of the models are linear reservoir, linear channel and time area diagram. There have also been studies considering the nonlinear and distributed nature of the catchment system. Nash (16) has put forward an important hypothesis in which the catchment is represented by a series of linear reservoir of equal storage coefficient. He has suggested that the instantaneous unit hydrograph could be derived by routing the instantaneous rainfall through a series of linear reservoirs of equal delay times. In order to incorporate the effect of translation of flow into the unit hydrograph analysis Dooge (19) proposed to use the concept of linear channel for the first time and to represent the basin system by a series of alternating linear channels and linear reservoirs. Laurenson (14) was the first to introduce the nonlinear reservoirs in conceptual models. However most of the models mentioned above have not taken into account the nonuniform areal distribution of rainfall data.

1.4. PROBLEM AND SCOPE OF PRESENT STUDY :

The problem of the present study is to simulate the direct runoff hydrograph from a natural water shed using a lumped system model instead of conventional unit hydrograph approach. The Nash model is taken as the basis of this study. Thetime distribution of rainfall excess and indirectly the effect of a distributed system are induced in this study.

A lumped system model intended for the simulation of an entire water shed may be used to represent individually many subareas of the water shed in such a manner that the simulation of the water shed becomes a distributed system. For this purpose the water shed area may be divided into number of subareas and each subarea is represented by a lumped system model. By routing the flow space wise through all the lumped system models representing the subarea, the total simulation of the entire water shed becomes a distributed system. This type of simulation was termed by Chow (6) as distributed system of lumped system model. So two approaches have been put forward for this purpose. In the first approach, the Nash model has been used to simulate the direct runoff hydrograph with time distribution of rainfall considering the whole catchment as one unit. In the second approach the principle of distributed system of lumped system is applied to simulate the direct runoff hydrograph from the water shed. For this purpose, instead of dividing the catchment arbitrarily, the catchment is divided into subareas on the basis of internal water shed boundaries. This type of subdivision of catchment was also suggested by Delleur (2). More recently this type of division of catchment for simulation of rainfall runoff process was used by Porter (21) who modified Laurenson model. Then this subarea type model (or distributed system of lumped system model) is used to simulate the direct runoff hydrograph from the water shed. For this present study the performance of Nash model and subarea type model is evaluated using data from the catchment Bridge No.566 of Indian Railways. In order to study the sensitivity of the subarea type model to different types of time distribution of rainfall excess, a study is made with assumed values of model parameters and typical rainfall excess distribution.

Some typical linear and non-linear models used in simulating the rainfall runoff process are also reviewed in order to study the present trends in the deterministic approach of rainfall runoff relationships.

CHAPTER - 2

SYSTEM CONCEPT IN HYDROLOGY

CHAPTER - 2

SYSTEM CONCEPT IN HYDROLOGY

2.1. INTRODUCTION:

A system is an aggregation or assemblage of objects united by some form of regular interaction or independence. The system is said to be dynamic if there is a process taking place in it. If the process is considered probabilistic or stochastic, the system is said to be stochastic. Otherwise, it is a deterministic system. Furthermore, the system is called sequential if it consists of input, output, and some working fluid (matter, energy or information) known as through put passing through the system. The hydrologic cycle on a drainage basin is a sequential, dynamic system in which water is a major throughput. The systems approach seems to be one of the most powerful techniques introduced into the science, engineering and technology today. A system is considered to consist of various components which contribute to the relationship between input and output of the system. To a hydrologic system, rainfall and runoff may be input and output respectively. Surface storage, channel storage, soil water storage and ground water storage may be considered to be components of the hydrologic system. The hydrologic and hydraulic processes, such as evaporation, infiltration, surface runoff, channel inflow, capillary rise, transpiration, deep percolation and ground water flow etc. determine the interaction among the components of the system. It is a natural law that a physical process tends to achieve and maintain an equilibrium state.

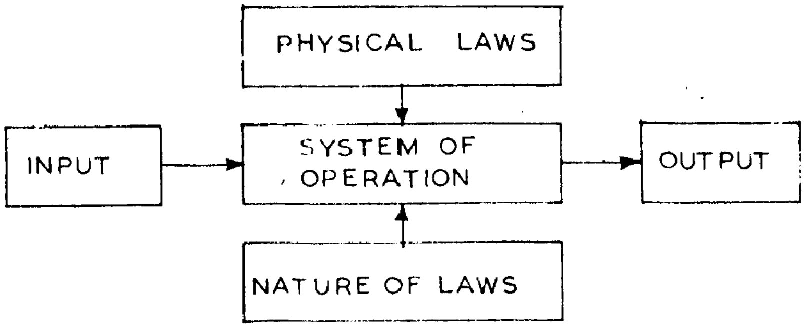


FIG. 2·1

The role of the system in generating output from input or in interrelating input and output, is its essential feature. The output from any system depends on the nature of the input, the physical laws involved and the nature of the system itself - both the nature of the components and the structure of the system according to which they are connected. In physical hydrology, as in other branches of applied physics, all three are taken into account in predicting the output. In the system approach, however, the overall operation of the system is examined without taking into account all the complex details of the system or all the complex physical laws involved. Concentration is on the system operations which depend on the physical laws and the nature of the system, however, the nature of this dependence may not be known and may be ignored in this approach to the problem. This is represented by the horizontal components in the Fig.2.1 Thus in unit hydrograph studies, once the unit hydrograph has been derived from the records of input and output, it can be used as a prediction tool without reference to the nature of the catchment or the physical laws involved. If, however, we wish to derive a synthetic unit hydrograph or to examine the validity of the unit hydrograph procedure, it is necessary to examine the connection between unit hydrograph, the characteristics of the watershed and the physical laws governing its behaviour. This relationship is represented by vertical components in Fig.2.1.

If we concentrate on the relationship between the three elements involved - input, system and output then the problems which arise can be conveniently classified as under :

(i) the problem of predicting an unknown output

(ii) the problem of identifying an unknown system operations.

(iii) the problem of detecting an unknown input signal

In the study of hydrological systems we are frequently required to solve the problem of synthesis involved in the simulation of a system for which records of input and output are available. In this case we have to device a system (either an abstract mathematical system or real model) which will enable us to simulate the system; i.e. to produce for the given input which corresponds to the given output within the required degree of accuracy.

2.2. HYDROLOGICAL CYCLE AS SYSTEM :

As already discussed the hydrological cycle is treated as a hydrological system for mathematical representation of a natural watershed. The different components of hydrological cycle viz., precipitation evaporation, interception, infiltration, runoff etc. are considered as components of the hydrological system. The hydrological system is treated as a ' Black box ' in which the transformation of input precipitation is explained through ' system function or transfer function. Therefore, in hydrological system studies, overall effects of various factors effecting the runoff process are taken into account by the system function, but in no case independent identify of any of these factors is permitted to exist or operate. The components of the basin hydrological cycle is shown schematically in Fig.2.2

2.3. CATCHMENT AS A SYSTEM:

Though the entire hydrological cycle is treated as a hydrological system, but in practice, the hydrologist confines his attention to individual basins or catchment areas.

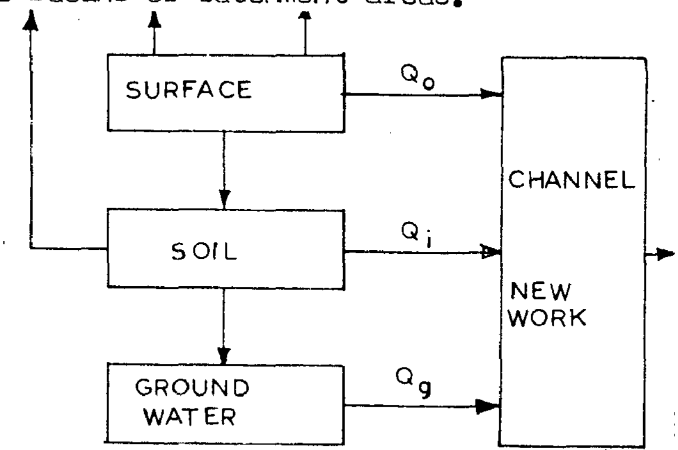


FIG. 2.3

Thus, he leaves problems of the atmosphere to the meteorologist, those of the lithosophere to the geologist and those of the sea to the oceanographer. This narrows his concern to the particular subsystem of the total hydrological cycle. In isolating this subsystem from the larger system represented by the whole system, it is necessary to cut across certain lines of transport of moisture from one part of the cycle to the other. The figure 2.3 thus represent either inputs or outputs from the subsystem representing the catchment area. The catchment system, therefore, is not a closed system and can only be treated as such if a record is available of all the inputs and outs.

Though classical hydrology describes the hydrological cycle in terms of surface runoff, inflow and ground waterflow, in practice quantitative hydrology usually ignors this three fold division and considers the hydrograph being made up of a direct storm response and a baseflow.Thus in the analysis of the relationship between storm rainfall and flood runff, the system analysed by the practical hydrologist corresponds closely to that indicated in Fig.2.4

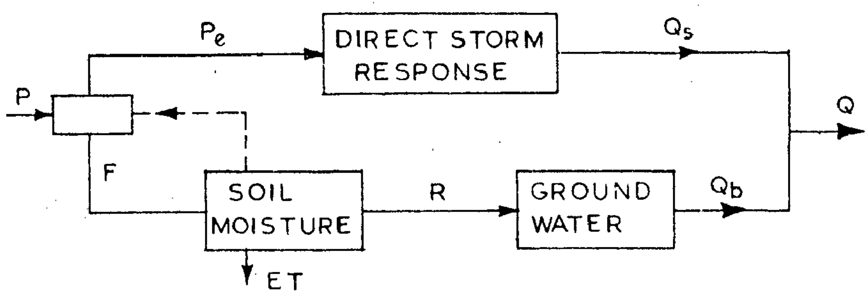


FIG.2.4 SIMPLIFIED CATCHMENT MODEL

The system shown in Fig.2.4 is seen to consist of three subsystems: subsystems involving direct storm response, the subsystem involving ground water response and the subsystem involving the soil phase which has a feed back loop to the separation of precipitation into precipitation excess and infiltration. In the transition from classical hydrology to system hydrology, the main emphasis has been on the subsystem involving direct storm response, which has been studied by unit hydrograph procedures.

2.4. CONCEPT OF MODELS :

Models are representations of reality. If they were as complex and difficult to control as reality, there would be no advantage in their use. Fortunately we can usually construct models that are much simpler than reality and still be able to use them to predict and explain phenomenon with a high degree of accuracy. The reason is that although a very large number of variables may be required to predict a phenomenon with perfect accuracy, only a small number of variables usually accounts for most of it. The trick of course is to find the right variables and the correct relationship between them.

2.5. HYDROLOGIC MODELS :

Hydrologic models are mathematical formulations to simulate natural hydrologic phenomenon which are considered as processes or as systems. Most natural hydrologic phenomenon are so complex that they are beyond human comprehension, or that exact laws governing such phenomena have not yet been fully discovered. Before such laws can ever be found, complicated hydrologic phenomenon can only be approximated by modelling.

2.6. TYPES OF HYDROLOGIC MODELS AND THEIR CLASSIFICATION :

Hydrologic models can be divided into two basic catagories: models that possesses certain physical properties of their prototypes and models that have only an abstract form. The former catagory or the physical models can be divided into scale models, analog models and simulation models.

A scale model that looks like the prototype is the simplest type. For example the ordinary hydraulic models of rivers and structures that are investigated in many hydraulic laboratories and whose scales are based on geometric and force considerations are scale models.

An analog model replaces prototype properties with quantities that bear the same relations to each other as do those of the prototype, but they are easier to measure or visualise. For example, the Hele-Shaw(7) model shows the movement of a viscous liquid between two closely spaced parallel plates is analogous to seepage flow in a two dimensional cross-section of an aquifer. Many electronic analog models for surface and ground water flows are built on the principle of analogy between the flow of water and the flow of electric current.

A simulation model retains the essence of the prototype without actually attaining reality itself. It reproduces the behaviour of a hydrologic phenomenon in every important detail but does not reproduce the phenomenon itself. In a broad sense, it is common used to include the Scale and Analog models but the definition adopted here refers specially to the simulation on digital computers. In hydrology, the Stanford watershed model may be therefore, described as a simulation model. This model simulates the land phase of the hydrologic cycle in a watershed on a digital computer.

Abstract models, or the second basic catagory are generally referred to as theoretical or mathematical models since they attempt to represent prototype theoretically in a mathematical form. These models neither resemble nor initate prototype physically but replace the relevant features of the system by a mathematical relationship. According to certainty or uncertainty of such relationship on a priority basis, the model can be further divided into deterministic and stochastic or probabilistic. A differentiation between deterministic and stochastic or probabilistic models can be assisted by relating them to the concept of certainty and uncertainty. Certainty implies that no matter how many times a hydrologic phenomenon is processed under a given set of invarient conditions the same outcome is assumed to result always. On the other hand, uncertainty implies that every time a phenomenon is produced it may be different. Theoretically, certainties may be forecasted while the risk aspect of uncertainties can be predicted with an element of probability. In these sense, therefore, deterministic models make forecastings while stochastic or probabilistic models make predictions.

Abstract models are the product of modern age, since these quantitative models must depend on adequate mathematical tools which have now become available for practical applications. Such models to be useful must inevitably be complex yet at the same time be workable. Major types of these models as well as the physical models are classified diagrammatically in Fig.2.5

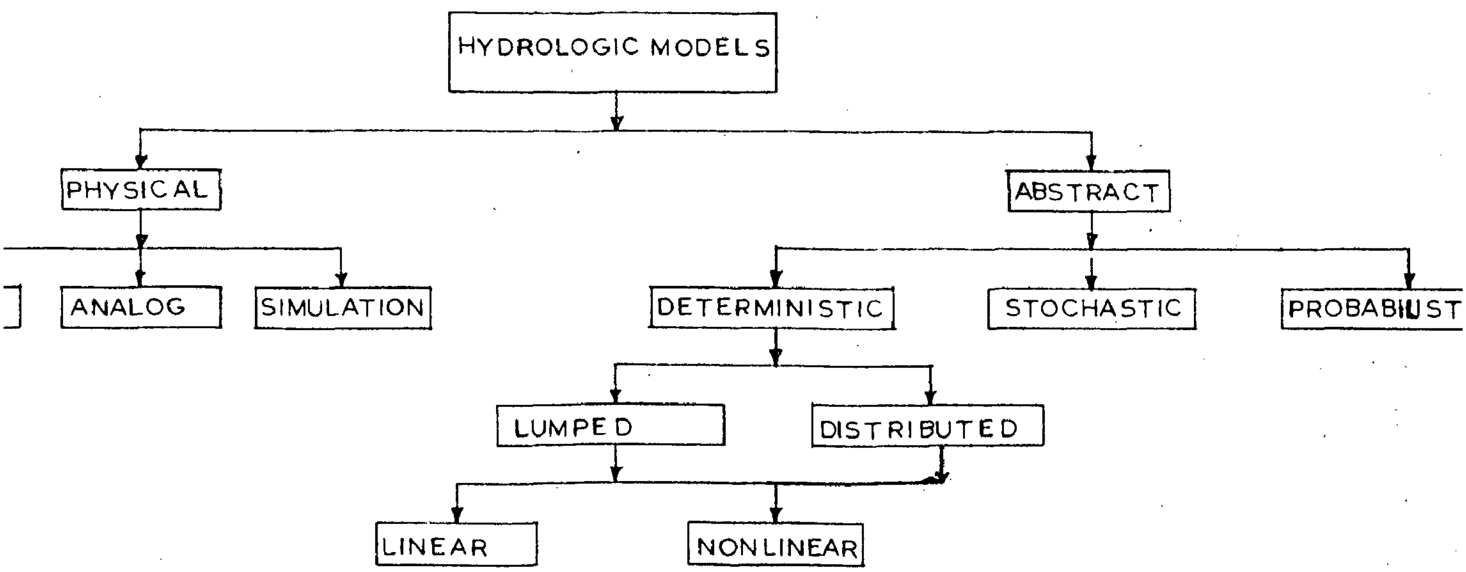


FIG.2·5 CLASSIFICATION OF HYDRLOGIC MODELS

In abstract modelling, hydrologic phenomenon are treated as systems. By this so called system concept, the hydrologic system is considered to consist of an input and output and some working medium known as throughput, such as the water passing through the system. For example, a water shed can be analysed as a system. For this system, the input is the rainfall and ground water inflow, the output is the evaportranspiration, infiltration and runoff and the throughput is the water moving over the watershed. By system concept, a hydrologic phenomenon can be readily interpreted by modern system analysis techniques and then modelled mathematically and solved on computers.

Mathematically, the input and output relationship of a hydrologic system may be represented by

$$Q = \phi I \qquad (2.1)$$

in which Q is the output, I is the input and ϕ is the transfer function or system function which represents the operation performed by the system on the input to transform it to output. For example, the unit hydrograph is a transfer function of the water shed system. It should be noted that input I and output Q are time functions and can be expressed as I (t) and Q(t) respectively with t denoting time. Then the equation (2.1) will become

$$Q(t) = \phi I (t) \qquad (2.2.)$$

The objective of the modelling is essentially to derive a mathematical formulation for the transfer function of the system.

2.6.1. Deterministic models:

In deterministic modelling, a hydrologic system is often treated either as lumped or as distributed although this treatment is equally applicable to stochastic or probabilistic modelling. A lumped system model is a gross representation of the hydrologic system as determined from the input and output data pertaining to the system; thus the system is regarded as a single point in space without dimensions. In contrast to this is the distributed system model which considers the hydrologic processes that are taking place within various distributed points or areas within the internal space of the system. If the internal space is divided into a number of small units spaces and each unit space is modelled as a lumped system, then the distributed system model becomes simply a conglomerationof lumped system models.

2.6.1.1. Lumped and Distributed System :

The hydrological system model whose input function does not involve spatial coordinates may be termed as lumped system model. Therefore a lumped system can be located at any single point in the working space. A lumped system model can be represented mathematically by ordinary differential equations as shown in Fig.2.6

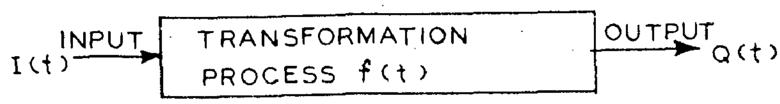


FIG. 2.6 LUMPED SYSTEM

Mathematical equations representing a distributed system involve spatial coordinates. As shown in Fig. 2.7 below input to such a system is distributed and therefore it can not be located at a single point. The distributed system can only be described with partial differential equations and therefore, theoretical solution to such system models requires complete knowledge of boundary conditions.

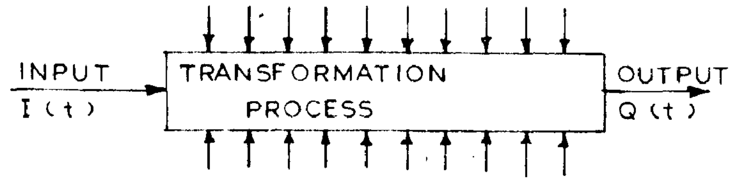


FIG. 2.7 DISTRIBUTED SYSTEM

2.6.1.2. Linear and non-linear hydrological system:

A hydrological system model is said to perform a linear operation if a step input to the system produces an output response which is directly proportional to the input at any time. A linear system model can be described by a linear differential equation.

In general, the hydrological system may be defined by a differential equation of following type:

$$f(\phi) = a_n \frac{d^n \phi}{dt^n} + a_{n-1} \frac{d^{n-1} \phi}{dt^{n-1}} + \ldots\ldots + a_o \phi \qquad (2.3)$$

The system is said to be linear and time invarient only if all the coefficient a_o, a_1, a_n etc. are constant. The system is linear but time varient if one or more of these coefficients is a function of the independent variable t but are not the function of ϕ However, a system would be non-linear if one or more of these coeff. are the function of ϕ .

Time invariant linear systems are to work with, as principles of super position and homogeneity hold good. The principle of super position may be stated as $f(\phi_1) + f(\phi_2) + \ldots\ldots\ldots$

$$+ f(\phi_n) = f(\phi_1 + \phi_2 + \ldots\ldots\ldots + \phi_n) \qquad (2.4)$$

where as homogeneity of the system assures.

$$f(\alpha, \phi_n) = \alpha \cdot f(\phi_n) \qquad (2.5)$$

2.6.2. Stochastic and Probabilistic Models :

Indeterministic behaviour of hydrologic phenomenon may be described in many ways. One tengible approach is to hypothesise the risk in uncertainty as definable by an element of probability. In fact it does not imply that all uncertainties can be measured in terms of probability. On the basis of this understanding, a simple modelling concept can be taken by assuming that hydrologic events are purely random variables. In this way, hydrologic data have been analysed by many mathematical models of probability distribution. Among the commonly used such probability models are log-normal distribution, Gumbel's extreme distribution and the log- Pearson Type III distribution. (7)

However, the concept of prediction implied in these models is more than one of pure randomness since the occurance of hydrologic events may be affected by its antecedent event or events. In fact, it has been discovered that the variability of groups of recorded stream flows in their natural order of occurance is actually larger than if the same flows occurred in random sequences. By assuming hydrologic events as pure random variables is simple to ignore the effect of sequence since the variables may occur in different sequences but in a random fashion.In order to cope with this situation the hydrologic process may be treated as stochastic process. For example, the record of a hydrologic phenomenon may be analysed as a time series and thus available mathematical models of time series can be used as a stochastic models to represent the hydrologic process involved.

In a way it can be seen that the deterministic process and the pure probabilistic process are only two special cases of the stochastic process. When the probability or certainty of the random variable is one, the stochastic process simply reduces to deterministic. When the probability is independent of any parameter index, time or space and the family of random variables belongs to same population, the stochastic process becomes purely probabilistic in which no deterministic component exists. On a scale of probability 0 to 1, the purely probabilistic and the deterministic cases occupy respectively the two extremeties, while the stochastic process may occur any where between them. For instance, the simple Ist order Markov Chain model which is a stochasticmodel. This model consists of two terms, namely the trend term and the noise term.

For the special cases, the noise term may be zero thus producing a deterministic model or the trend term may be zero, then resulting in a probabilistic model.

Today stochastic modelling is at the highest level of hydrologic modelling, although it has not been well developed in view of many practical difficiulities yet to overcome. By stochastic modelling, all components of a hydrologic system can be theoretically described by stochastic processes. In the system, the input, the output and the transformation of input to output in theform of through put passing through the system may be therefore represented mathematically by time series since these component processes, in general change with time and are function of time. The transformation of input to output is characterised by the physical features and hydrologic behaviour of the system. All the processes are assumed to be governed by mathematically simulated stochastic laws. Thus the input stochastic process is denoted by $\left[X_t ; t \in T\right]$, where X_t is the input stochastic variable, the output stochastic process by $\left[Y_t ; t \in T\right]$, where Y_t is the output stochastic variable; and the through put stochastic process, representing the transformation of input to output by $\left[Z_t, t \in T\right]$ where Z_t is the throughput stochastic variable. These stochastic processes can be simply denoted by $\left[X_t\right]$, $\left[Y_t\right]$ and $\left[Z_t\right]$ respectively.

The time parameter t in the stochastic process may be either continuous or discrete. For practical and analytical purposes and for a possible solution of mathematically simulated model by digital computers, the stochastic processes may be taken as a discrete time functions. The index set T represents a length of time long enough to discribe the hydrologic phenomenon under consideration. Units of time parameter t can be choosen in convenient time intervals, so that for the interval values of $t = 1, 2, \ldots, T$, the stochastic variables define the respective processes in satisfactory details. It should be noted that the time interval to be choosen for the discrete time parameter will affect the simulated stochastic laws of the process. In general, smaller time intervals will make the stochastic laws more complicated as the magnitude and extend of dependence among the stochastic variables based on the historical hydrologic data of a process will be greater and in more detail.

The input and output relationship of a stochastic hydrologic system may be represented mathematically by a system equation as 2.1.

$$[Y_t] = \phi\{[X_t] \quad , [Z_t]\} \qquad (2.6)$$

where $\phi\{[X_t] \quad , [Z_t]\}$ is the transfer function that represents the operation performed by the system on the input and the throughput in order to transfer them into output.

In most cases, the input, output and througput of a hydrologic system are amounts of water, although in certain cases they can be taken as energy or other forms of medium. By the basic principle of system continuity, the output is equal to the input minus the throughput which is the amount of flow in the system. Thus, a single transfer function may be written as

$$\phi\{[X_t] , \quad [Z_t]\} \quad = [X_t] - [Z_t] \qquad (2.7)$$

Hence from equation 2.6 and 2.7 the hydrologic system equation becomes

$$[Y_t] \quad = [X_t] \quad - \quad [Z_t] \qquad (2.8)$$

For the t^{th} time interval or the time interval from t to t + 1 equation(2.8)may be written as

$$Y_t \quad = \quad X_t \quad - Z_t \qquad (2.9)$$

Where X_t is the input in the t^{th} time interval

Y_t is the output in the t^{th} time interval

Z_t is the change of throughput in the t^{th} time interval.

2.7. TIME INVARIANT SYSTEM AND TIME VARIANT SYSTEM :

In all application of system theory, attention is first concentrated on the properties of linear time invariant systems and these have been assumed wherever possible. The assumption of time invariancce allows us to predict the output for a given input exactly if the input has occurred in the past and the corresponding output has been recorded.We would still not be able to predict the output for an input not contained in our records of observed measurements. If, however, we make the further assumption of linearity, it is possible to decompose all of the inputs into sets of characteristic signals and obtain the output by the superposition of the outputs due to the separate characteristics signals. This enables us to allow for the change in form of the input. The assumption of linearity and time invariance were implicit in the unit hydrograph method from the very beginning.

For a linear time invarient system we have the algebraic relationship

$$Y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \qquad (2.10)$$

Where $x(t)$ is the input and $Y(t)$ is the output. The system operation is characterised by the function $h(t)$ which can be variously described as the impulse response, the impulse function or the IUH of the system. Since the system we deal within hydrology are causal, the upper limit of the integration can be placed equal to t rather than infinity and for the case of an isolated input, the lower limit can be set equal to zero, thus giving the system equation in the form

$$Y(t) = \int_{0}^{t} X(\tau)\, h(t-\tau)\, d\tau \qquad (2.11)$$

For the case of periodic input the equation would take form

$$Y(t+nT) = \int_{t-T}^{t} X(\tau + nT)\, h(t-\tau)\, d\tau \qquad (2.12)$$

while for the block input so often used in regard to rainfall we would have,

$$y(t) = \sum_{\sigma=0}^{\sigma=t} x(\sigma)\, h_D(t-\sigma) \qquad (2.13)$$

where $x(\sigma)$ represents the successive volumes of the rainfall histogram and $h_D(t)$ the finite period of unit hydrograph.

Once the linearity and time invariance have been assumed the problem of prediction reduces itself to the computation of $Y(t)$ when $x(t)$ and $h(t)$ are known. In hydrological terms it means the prediction of the direct storm runoff knowing the design values effective precipitation and unit hydrograph.

Methods of analysing time varying linear differential systems have been considerably advanced, but little seems to have been developed for analysing time varying non linear systems, or a time varying non-differential linear system.

2.8. HYDROLOGIC SYSTEM INVESTIGATIONS:

System investigations include the study of hydrologic systems for the explicit purpose of establishing quantitative relationship between precipitation and runoff which can be used for reconstruction or prediction of flood sequences and water shed yields.

2.8.1. Methods of System investigation:

The method of system investigation fall into two principal catagories :

1. Paramatric hydrology
2. Stochastic hydrology

2.8.1.1. Paramatric or Deterministic Hydrology:

Paramatric hydrology is the development of relationships among physical parameters involved in hydrologic events and the use of these relationships to generate or synthesise nonrecorded hydrologic sequences. The principal current methods of parametric hydrology include correlation analysis, partial system synthesis with linear analysis and general system synthesis and general non-linear analysis.

Although the various methods of system investigations have certain basic differences, they share two characteristics of prime importance.

(a) Their dependence on historical records of the values of the parameters.

(b) The assumption of stationarity or time invariance of the historical records.

Both of these characteristics place definite theoretical limitations on the generality of the solutions. By and large, the most important parameters used in the system investigations are precipitation which constitutes the principal system input and runoff which is the principal system output.

The first characteristics (historical dependence) means that to the extent that historical records of the input and output are affected by systematic and random errors of measurement, by inhomogeneity and by lack of completeness, the results of parametric or stochastic hydrology are affected also.

The second characteristic (time invariance) requires that hydrologic systems must not change with time relative to their behaviour during the recorded past. It is evident that if the system change due to natural or artificial causes, or if the historical records do not cover episodd involving certain extreme behaviour patterns, the application of these techniques for prediction of future events or reconstructions of past events has limited reliability.

A brief discussion of someof the methods in paramatric methods are given in the following sections.

2.8.1.2. Method of Correlation analysis:

In correlation analysis, measured input rainfall is correlated to observed runoff hydrograph through a linearly operating working model. The models are evolved:

(a) by correlating rainfall to different hydrograph parameters which may or may hot be related to catchment geomorphology, and

(b) by defining the system response solely in terms of different geomorphological and meterological parameters.

A good number of emperical approaches have been developed in the recent past. Many of them are based on 'genetic runoff principle' which is widely practised in the USSR and in the Eastern Europe. The genetic principle is a modified version of time area concept. A model proposed by Rostomov is discussed below.

Rostomov (15) has suggested some emperical relations for storm runoff computations of small drainage basins. The approach correlates morphometric elements, soil and vegetation condition, meterological factors etc. to the runoff process. Maximum runoffhas been reportedly produced from a storm duration which equals the time of concentration. The latter is estimated on the basis of two length parameters which the runoff volume is assumed to travel; i.e. along the valley slope and the bed slope with different velocities. Further, the discharge rate relationship includes many additional parameters which account for basin shape, its prominent topographic features and non uniformity of rainfall distributions etc.

2.8.1.3. Partial System Synthesis with Linear Analysis:

In system analysis, the relationship between measured input and output is established by a mathematical process without any attempt to describe the internal mechanisms of the system in explicit form. This relationship has the form of a " unique function " which is made to operate on the input in order to produce the output. In general this function is not required to have a physical meaning or to possess parameters fulfilling conditions of dimensional consistency.

In system synthesis, on the other hand, the investigator attempts to describe the operation of the system by a linkage or combination of components, whose presence is presumed to exist in the system and whose functions are known and predictable. The linkage of components must be made in such a manner that the correct output is produced whenever a specified input is applied. In general, the process of synthesis does not yield a unique model of the unknown system.

Pure synthesis or analysis can be performed on a system independently or a combination of both can be employed. A method of partial synthesis with linear analysis is described first, because it is the basis of the classical unit hydrograph procedure.

The basic operations involved in the unit hydrograph procedure are represented in the flow diagram in the Fig. (2.8) .

The sequence of partial system synthesis with linear analysis is described through three sub-systems. Sub-system (1) performs the operation of subtracting the values of infiltration function from the recorded input. This is determined by emperical procedures such as the antecedent precipitation index, by judgement or by iteration. Sub-system (3) separates the so called hydrograph components from the recorded output function. Sub-system (2) is a linear convolutional process, sub-system (2) is analysed by any of the numerical methods available on the subject. Unit hydrograph is one of the popular time invarient system function of sub-system (2) whose derivation comprises of the following three steps:

(i) Rainfall input function is modified by the infiltration index of the basin system to result in rainfall excess.

(ii) Water shed surface function is operated upon modified input to produce surface runoff.

(iii) Output runoff is produced by modifying surface runoff function through the base flow.

The three steps referred above are related to the three sub-systems, identified earlier. Therefore, analysis of partial system is performed on the modified inputs and outputs where as system operation totally depends upon the validity of assumptions regarding the modification of input. A few important approaches of partial system synthesis are discussed below.

Bernard (15) correlated basin characteristics to the parameter of a 'distribution graph '. The distribution graph is a unit hydrograph in which time scale is expressed in days, from the beginning of storm and the flow scale is given as percentage of the contributing area. The effective percentage of the area, day from the beginning of storm, water shed parameter U, all were graphically correlated. The water shed parameter U was found to be function of shape of the basin, flow condition in the main channel, length of the longest water travel (ft) and the contour fall (ft).

Derivation of the 'distribution graph ' as well as ' Unit graph ' suggested by Sherman (15) utilised the properties of the recorded input precipitation and runoff data. Therefore the two approaches could not be extended to ungauged catchments. Snyder (15) correlated basin characteristics to unit hydrograph parameters, and thus synthetic U.H. were developed for ungauged catchments. Some of the relations proposed for development of unit hydrograph are given below :

$$t_p = C_t \, (L L_c)^{0.3}$$

$$T = 3 + 3 \, t_{p/24} \qquad (2.14)$$

$$q_p = 640 \, \frac{C_p}{t_p}$$

Where
L = Length of the longest water course
L_c = distance along main channel from the gauge to the centroid of drainage area
t_p = time to peak
C_t = Coeff.
C_p = Coeff. ranging from 0.5 to 0.69

In some cases, Snyder's coeff. were found to produce unrealistic unit hydrographs as the basin slope was not taken into account. Taylor and Schwarz defined the Coeff. $C_t = \frac{0.6}{\sqrt{S_{st}}}$ where S_{st} = average basin slope. Basin lag (t_p) and peak discharge q_p were defined as exponential functions.

$$\left.\begin{aligned} t_p &= C_t \, e^{m'} \, t_R \\ q_p &= 1800^{m'} \, e^{(.121 \, S_{st.}^{\;.142} - 0.05 - m') \, t_R} \\ T &= t \left(T_p + \frac{t_R}{2} \right) \end{aligned}\right\} \quad (2.15)$$

Where $m' = 0.212 / (LL_c)^{0.36}$

t_R = effective rain duration

T = time base

Interestingly the analysis suggests effects of basin physiography on IUH ($t_R = 0$ and $t_p = C_t$) . So IUH peak is inversely proportional to the catchment length while its base is inversely proportional to average channel slope S_{st} .

2.8.1.4. General System Synthesis:

The basis for the construction of synthetic models in hydrology is a statement of continuity, which can be expressed as an equation of state of the form

$$I = Q + \Delta S \quad t_1 \rightarrow t_2 \qquad (2.16)$$

Where I is the total inflow, Q is the total outflow and ΔS is the change in internal storage, all referred to the time interval t_1 to t_2. In differential form this equation becomes

$$i(t) = q(t) + \left(\frac{ds}{dt}\right) \qquad (2.17)$$

Where i(t) is the rate of total inflow to the system q(t) is the rate of total outflow and $\frac{ds}{dt}$ rate of change in internal storage.

The process of general synthesis ordinarily begins with the postulation of a more or less complex model whose structure is based on quantitative and semiquantitative knowledge of the phenomenon involved in the hydrologic cycle. This model contains elements defined by explicit functions which describe operation affected on various portions of the input and storage. Such a model is shown in Fig.(2.9) to illustrate one way in which the process of synthesis can be accomplished.

The recorded input is processed through the model and the resulting output is compared with the recorded output of the natural system. If an acceptable agreement is not found, one or more of the functions of the component subsystems are modified and adjusted and the process is repeated in a systematic way until there is adequate correspondence between the synthetic and the recorded outputs.

A number of synthetic models of hydrologic units have been proposed over the last few years. The results obtained so far through these models are encouraging

but the performance of the models are not sufficiently reliable, so that complete confidence can be placed in extended reconstruction of runoff histories. This lack of reliability is due to probably number of causes the most significance perhaps being,

(a) Errors in recorded data which form the time series.

(b) effects of the areal distribution of the parameters

(c) Imperfections of the model structures

(d) Non uniqueness of the process of synthesis

Since errors in the data are always present, they can affect any of the system investigation methods. However, in synthetic models where the recorded input and output play a basic role on the adjustment of the system function of the component subsystem, their effects may be exaggerated or compounded out of reasonable properties.

Variable distributions of the precipitation over a basin cause which have similar effects to other types of data error but their magnitude may be considerably larger. The division of hydrologic unit into sub units with various input rates to account for areal changes has been used in the past in connection with the unit hydrograph method. Similar procedures are currently being investigated in connection with general synthetic models.

The imperfections of the model structure stem from the fact that considerable simplification must be introduced in the synthetic system for practical reasons. It is necessary, not only because the mathematical and analog manipulation of data becomes extremely difficult beyond a certain level of complexity, but primarily because sufficient physical data on the natural system are very hard to collect and evaluate.

Finally, the non uniqueness of the processes of synthesis has a bearing on the reliability of a model; while various synthetic assemblies of components may produce equivalent outputs for the same input within the range of available data, the outputs may diverge strongly outside this range. Since there is no assurance that any model is a faithful image of the natural system, the uncertainties regarding its out of range performance must always exist.

2.8.2. Stochastic Hydrology :

Stochastic hydrology is the use of statistical characteristic of hydrologic variables to solve hydrologic problems. This often involves the generation of non-historic sequences to which certain level of probability can be attached. The problems relating to extended forecasts of the water yields of river basins differ conceptually from the processes discussed above. In extended hydrologic forecasts the investigator normally seeks to establish the level of probability with which various sequences of output may occur in the future, while in analysis and sythesis the reconstruction and prediction seek to establish the input output relationship with certainty.

The stochastic problem can be stated briefly as follows provided the actual records of the output are available; given a historical sequence of events, what inferences can be derived from their statistical distribution so that the probabilities of future sequences can be assessed. A number of stochastic models of hydrologic process have been proposed for this purpose [Thomas and Fiering (1)] Two typical approaches are Monte Carlo (1) and theory of Markov process(1).

The main steps involved in the application of the methods of stochastic hydrology are indicated diagramatically in the Fig.2.10.

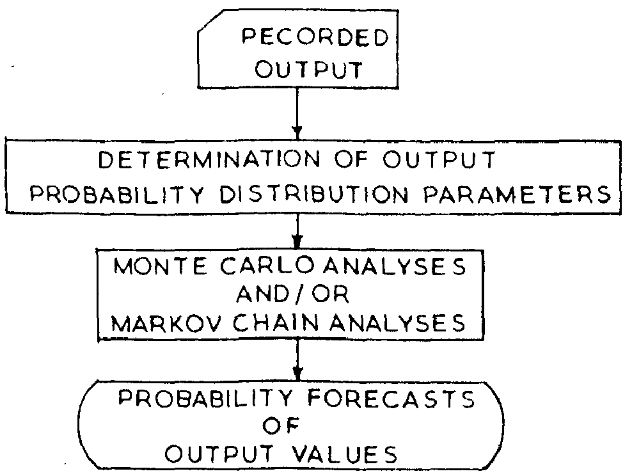


FIG. 2·10 DIAGRAM OF STOCHASTIC HYDROLOGY METHODS.

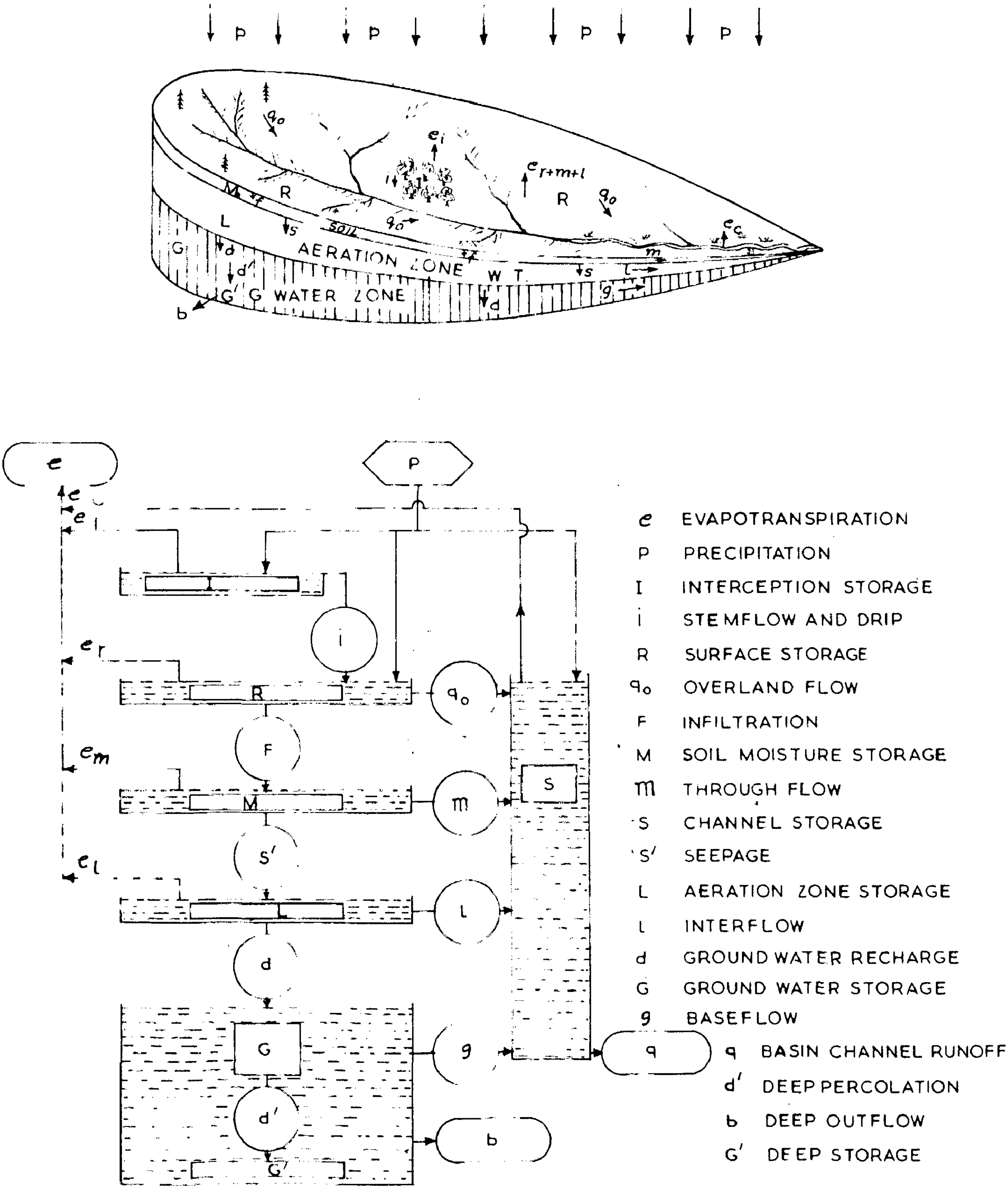
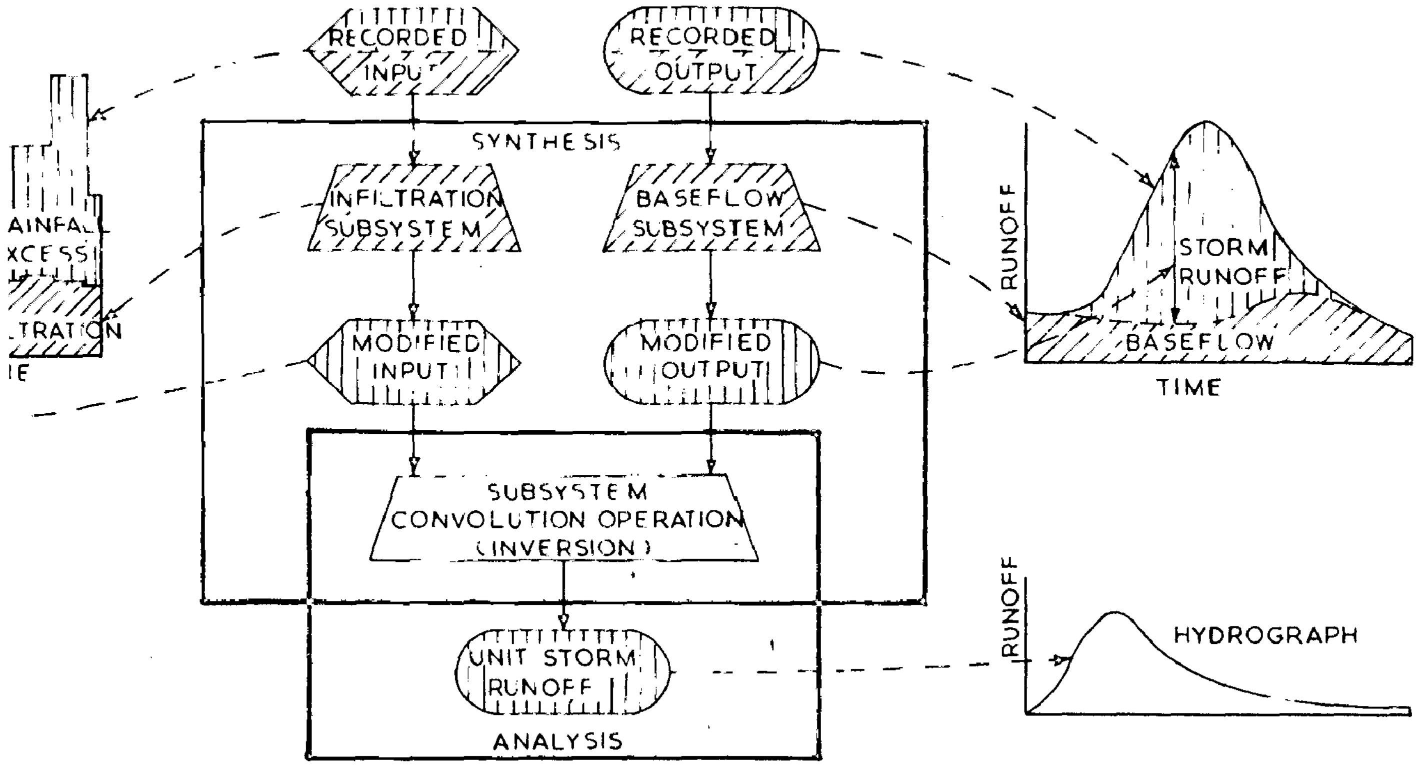


FIG.2.2 COMPONENT OF BASIN HYDROLOGICAL CYCLE.



2·8 FLOW CHART FOR PARTIAL SYSTEM SYNTHESIS.

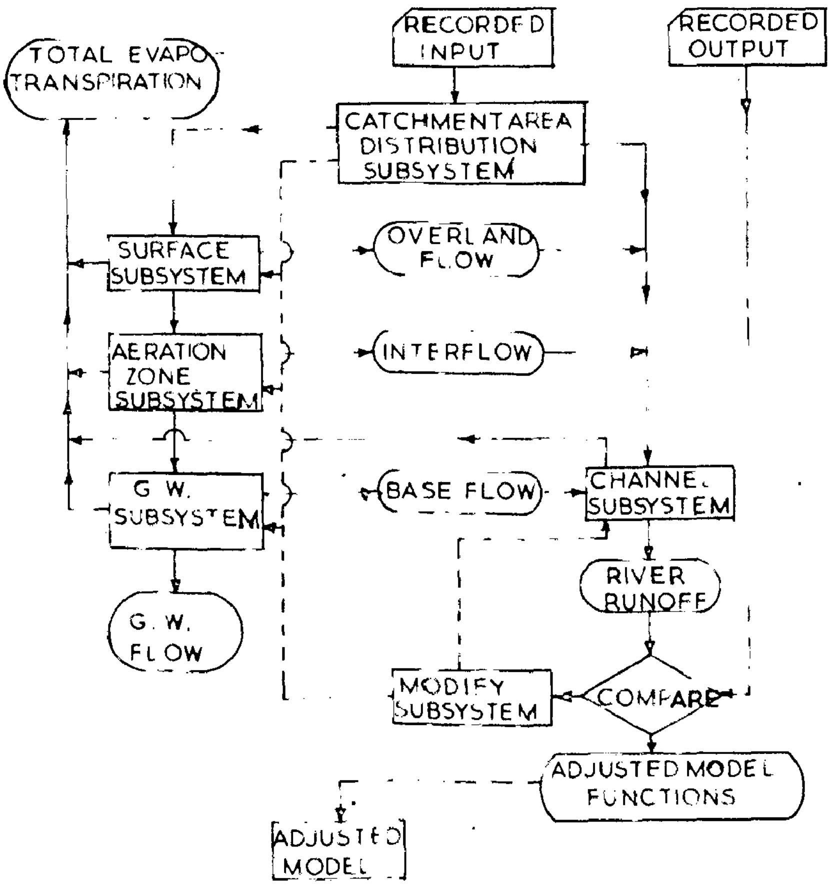


FIG 2·9 FLOW CHART OF A TYPICAL SYNTHESIS OPERATION

CHAPTER - 3

CONCEPTUAL CATCHMENT MODELS-A REVIEW

CHAPTER-3

CONCEPTUAL CATCHMENT MODELS - A REVIEW

3.1. INTRODUCTION

In recent years, a number of mathematically sophisticated methods of hydrologic analysis have been developed. All these methods and those proposed in the earlier years for hydrologic design are essentially techniques of hydrologic modelling. The complicated hydrologic phenomenon can be better approximated by modelling only. A brief description of the modelling approaches of rainfall runoff process and some popular concepts used in hydrologic models are being discussed in this Chapter. Also some well known linear and non-linear conceptual models used in simulating the runoff hydrograph in a natural water shed are reviewed.

3.2. PRINCIPLE OF CONCEPTUAL MODELS

The process of modelling a physical system can be divided into the following three phases :

1. Model formulation
2. Model calibration and
3. Model verification

Execution of the modelling process is part art and part science. The designer must combine existing knowledge of the physical processes with conceptual representations of unknown principles underlying the process being modeled.

3.2.1. Requirements in a model:

If a model were required solely to forecast the flow from a particular basin, it would probably be adequate to specify the models form and parametric values such that the computed output was a sufficiently close reproduction of the observed output. If the model is also to help us to understand the process of converting rainfall into discharge and the relative importance of different elements in this process, and particularly if it is hoped eventually to use the model for basins without records by establishing relation between the model parameters and basin characteristics, it is essential to obtain some guide to the relative signififance of model parts and the accuracy of parametric values. Methods of measuring significance and accuracy of determination must be found which are applicableto complex non-linear models.

Although simplification of the operation of a basin is necessary, especially in terms of variability over the area, it is desirable that the model should reflect the physical reality as closely as possible. If it is hoped to transfer the model to an ungauged basin the parametric values can be determined only by measuring the physical characteristics of the basin. Therefore the further the operation of the model departs from known physical laws the more tenuous is likely to be the relationship between model parameters and the basin characteristics. On the other hand if the model parameters are to be fixed by optimisation on comparison of computed and observed outputs, the more detailed and complex the model the more difficult it becomes to establish the values of the parameters, particularly if these are interdependent. This conflict cannot be resolved entirely, but there should be no unnecessary large number of parameters to be optimised and model parts with similar elements should not be combined.

The requirement of versatility should be added to those of simplicity and lack of duplication. Each additional part of the model must substantially extend the range of application of the whole model. In other words we are prepared to accept additional parts and hence greater difficulty in determining parametric values only if the increased versatility of the model makes it much more likely to obtain a good fit between observed and computed output.

3.2.2. Fitting the Model:

To remove subjectivity in fitting the model to the data on in determining the parametric values, O'Donnell (19) suggested automatic optimisation. This involves successive changes of parameter values according to some preconceived rule on pattern of increments which takes into account the results of previous steps and in particular whether or not a change improved the fitting.

Clearly optimisation needs an index of agreement or disagreement between the observed and computed discharges. Linear regression analysis suggests a sum of squares criteria such as :

$$F^2 = \quad (q'-q)^2 \qquad \ldots(3.1)$$

where F^2 is the index of disagreement and q and q' are the observed and computed discharges at corresponding times. The sum be taken over all q's at intervals t, or at preselected times such as peaks or troughs in the hydrographs. F^2 is analogous to the residual variance of a regression analysis. If $F^2 = 0$, then the model has taken account for all the model parameters.

The initial variance F_o^2 is defined by
$F_o^2 = (q-\bar{q})^2$ (3.2) , where $\bar{q}$ is the mean of the observed q's and the sum is taken as before, may also be defined as the " no model " value of F^2. This enables the efficiency of a model to be defined by R^2 (analogous to the coefficient of determination) as the proportion of the initial variance accounted for by that model.

$$R^2 = \frac{F_o^2 - F^2}{F_o^2} \qquad (3.3)$$

The efficiency of a separable model part may be judged by the change in R^2 which follows insertion of the part or by the proportion of the residual variance accounted for by its insertion

$$r^2 = \frac{F_1^2 - F_2^2}{F_1^2} = \frac{R_2^2 - R_1^2}{1-R_1^2} \qquad (3.4)$$

where the suffixes 1 and 2 denote before and after insertion of the model part under consideration.

The quantity F^2 is a function of the parameter space and, of course, of the input and output. Optimisation involves finding the value of the parameters which minimise F^2. This may be done by a 'Steepest descent' method or a search can be conducted in the super space by moving parallel to the parameter axes.

3.2.3 <u>Progressive modification:</u>

If one accepts that it is desirable to have a simple rather than a complex model, and this is certainly true if it is hoped to obtain stable values of the optimised parameters, then it would seem that a systematic procedure would be as follows :-

(1) Assume a simple model, but one which can be elaborated further.

(2) Optimise the parameters and study their stability.

(3) Measure the efficiency R^2

(4) Modify the model - if possible by the introduction of new part - repeat (2) and (3), measure r^2 and decide on acceptance or rejection of the modification.

(5) Choose the next modification. A comparative plotting of computed and observed discharge hydrographs may indicate what modification is desirable.

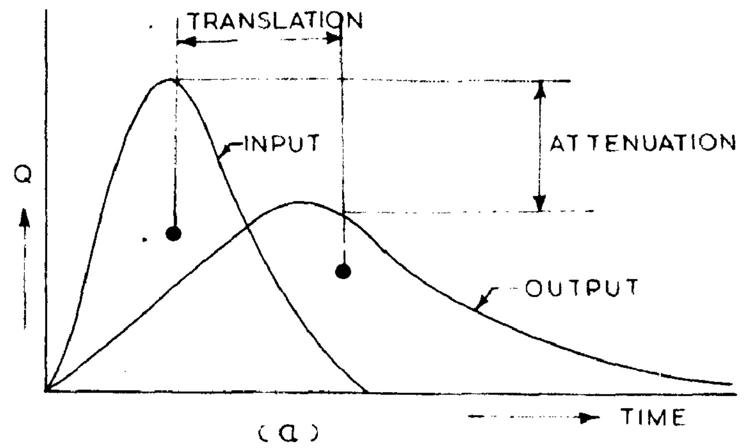
(6) Because all models can not be arranged in increasing order of complexity it may be necessary to compare two or more models of similar complexity. This may be done by comparing R^2.

3.3. COMPONENTS OF CONCEPTUAL MODELS

A large number of conceptual models have been proposed in recent years for mathematical simulation of a drainage basin or system.

The various components of these models are discussed in the following subsections.

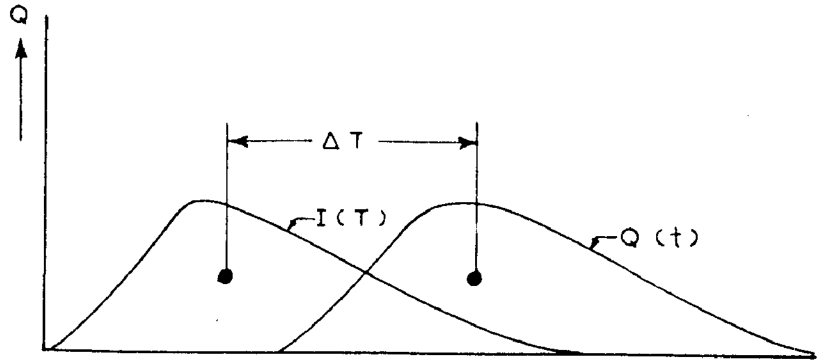
3.3.1. Catchment action :



CATCHMENT ACTION FIG. 3.1

As a result of catchment action, output response gets distributed over a large time period which not only attenuates the input hydrograph peak but also shifts it in time. The translation and attenuation of input hydrograph is due to storage action of the basin system. The catchment action is illustrated in the above figure 3.1(a). In different approaches varied explanation for the storage actions of the basin system have been given. In some recent developments pure mathematical functions were used as system functions but in most of the conceptual models, the basin action is represented through conceptual identities i.e. the pure translation as represented by the linear channels and storage effec by linear or non-linear reservoirs.

3.3.2. Concept of Pure Translation:



(b)
PURE TRANSITION

FIG. 3.1

Pure translation is a physical concept which signifies the 'time lag '. Therefore, as shown in figure 3.1(b) pure translatory functions affect only the time parameter i.e. if an input function I(t) at any time t is translated through such a system, an output response Q(t) identical to the input function I(t) but delayed by time Δt, is received at the outlet, where t is the translation time of the system. Mathematically pure translation may be expressed as

$$Q(t) = I(t - \Delta t). \tag{3.5}$$

3.3.3. Concept of Linear Channel

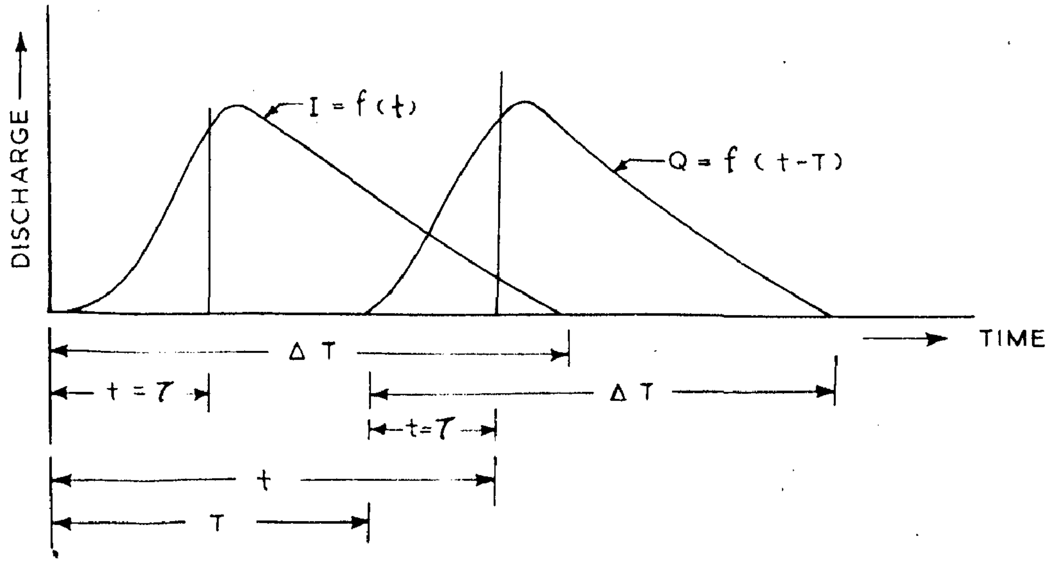


FIG. 3.2 PURE TRANSLATION EFFECT IN A LINEAR CHANNEL

A linear channel conceptually represents pure translatory effects of a system and therefore, it may be defined as:

" A conceptual channel in which the time (T) required to translate a discharge (Q) of any magnitude through a channel reach of given length (x), is always constant. Thus, when an inflow hydrograph is routed through the channel, its shapeis not affected i.e. if $I=f(t)$ be the inflow function to a linear channel, after routing, the outflow function $Q(t)$ would be identical to the inflow function except for a time lag which is introduced by the system and whose magnitude is given by the translation time (Δt) of the linear channel ". Fig.3.2

Therefore $$Q(t) = f(t - \Delta t) \tag{3.6}$$

The cross-sectional area at every point of the channel has a linear relationship to the channel discharge at that point, i.e. $A = CQ$. Consequently the velocity is also same for all discharges at any section of the channel and is equal to $\frac{1}{C}$, where C is the translation coefficient.

3.3.4. Reservoir action - Linear and non-linear reservoirs:

The catchment action on its input precipitation is analogus to the reservoir action on its inflow hydrograph. A reservoir too, translates and attenuates the inflow hydrograph by regulating its outflowing discharge over a desired time period. The analogy suggests that a drainage basin system could perhaps be analytically represented by the reservoir concept.

Depending upon the mode of its operation, a reservoir may be classified as a linear or non-linear reservoir. A linear reservoir is a conceptual identity in which the storage S is directly proportional to the outflow discharge Q i.e.

$$S \propto Q$$

$$\text{or} \quad S = K\,Q \qquad (3.9)$$

where K = reservoir constant/or storage coefficient and has the dimension of time and is equal to the average delay time, imposed on its inflow by the reservoir model.

From the equation of continuity we have,

$$I - Q = \frac{ds}{dt} \qquad (3.10)$$

$$\text{or } (I-Q)\,dt = ds$$

From equation 3.9 we get,

$$\frac{ds}{dt} = K\,\frac{dQ}{dt} \qquad (3.11)$$

Substituting equation 3.11 in equation 3.10 we get,

$$(I-Q) = K\,\frac{dQ}{dt}$$

$$\text{or} \quad \frac{dt}{K} = \frac{dQ}{(I-Q)}$$

Let $(I-Q) = x$

$\therefore$ $-dQ = dx$, (inflow rate is not changing with time)

$$\text{or} \quad \frac{dt}{K} = -\frac{dx}{x}$$

$$\text{or} \quad \frac{1}{K} \, dt = - \frac{dx}{x}$$

$$\text{or} \quad \frac{t}{K} = - (\log_e x - \log_e C)$$

$$\text{or} \quad \frac{x}{C} = e^{-t/K}$$

$$\therefore \quad (I-Q) = Ce^{-t/K} \qquad (3.12)$$

when $t = 0, \; Q = 0$

$$\therefore \quad C = I$$

$$\therefore \quad (I-Q) = I \, e^{-t/K}$$

$$\therefore \quad Q = I \, (1 - e^{-t/K})$$

when $t = \infty$, $Q = I$, which means that the outflow approaches an equilibrium condition, becoming equal to inflow.

If inflow stops after sometime and let at that time, $t = t_0$, $Q = Q_0$, and $I = 0$, then from $I-Q = Ce^{-t/K}$ we have,

$$0 - Q_0 = Ce^{-to/K}$$

$$\text{or} \quad C = - Q_0 \, e^{to/K}$$

$$\therefore (0-Q) = - Q_0 \, e^{to/K} \cdot e^{-t/K}$$

$$\text{or } Q = Q_0 \, e^{-\frac{1}{K}(t - t_0)}$$

Let $t - t_0 = \tau$

$$\text{Then } Q = Q_0 e^{-\tau/K} \qquad (3.13)$$

For an instantaneous inflow which fills the reservoir storage S in time $t_o = 0$, equation 3.9 gives $Q_o = \frac{S}{K}$ and equation 3.13 gives

$$Q = \frac{S}{K} e^{-t/K}$$

For an unit input or S = 1, this becomes the equation of IUH of the linear reservoir, i.e.

$$u(t) = \frac{1}{K} e^{-t/K} \tag{3.14}$$

The functional relationship between the storage and discharge of a non-linear reservoir may be written as

$$S = KQ^{B} \tag{3.15}$$

Where K and B are constant which represent the two characteristic parameters ofa a non-linear conceptual model.

3.3.5. Concept of Time-area Diagram:

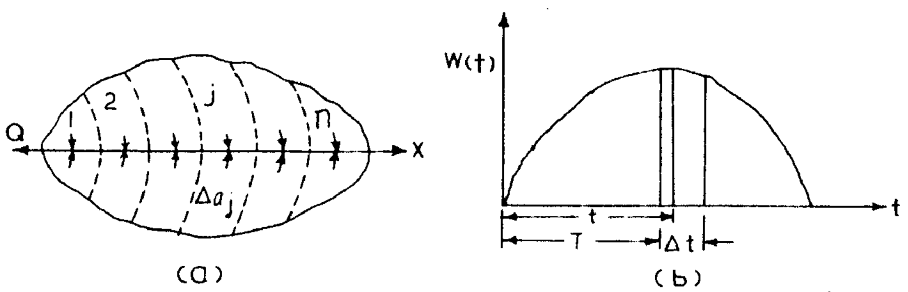


FIG. 3.3 DEVELOPMENT OF AREA TIME DIAGRAM.(a) DRAINAGE BASIN SIMULATED BY A LINEAR CHANNEL,(b) AREA TIME DIAGRAM.

A catchment area can be considered analogous to a linear channel containing spatially varied flow. The area of the catchment is subdivided into number of n sub areas of size Δa_j by isochrones (contours of equal time of travel Δt) with $j = 1, 2 \ldots\ldots\ldots\ldots n$ as shown in the Fig.3.3

The inflow from sub-areas j is equal to $i_j \Delta a_j$, where i_j is the rate of effective rainfall. The outflow at the outlet for this flow is therefore

$$Q = i_j \Delta a_j \quad \delta(t-T, \Delta t) \qquad (3.7)$$

where $T = (j-1)\Delta t$

When Q is divided by 'a' and plotted against t the diagram as shown in the Fig. (3.3) is produced being represented by

$$w(t) = \sum_{j=1}^{n} \frac{i_j \Delta a_j}{a} \quad \delta(t-T, \Delta t) \qquad (3.8)$$

When $\Delta t \rightarrow 0$ and i_j is constant, the diagram is represented by curve with its ordinate directly proportional to the shape of the catchment area projected into the channel and is called a time area diagram.

3.4. NASH MODEL

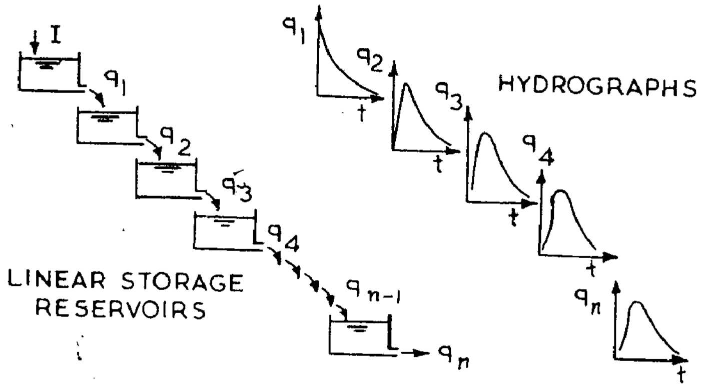


FIG. 3.4 ROUTING OF INSTANTANEOUS INFLOW THROUGH A SERIES OF LINEAR STORAGE RESERVOIRS (NASH'S MODEL)

Nash (16) proposed a conceptual model by considering a drainage basin as n identical linear reservoirs in series. He has shown that a cascade of equal linear storages results in the gamma density function. The governing relation would be the continuity or conservation of mass equation ,

$$i(t) - Q(t) = \frac{ds}{dt} \qquad (3.16)$$

Where S is the volumes of surface storage on the water shed at time t, that would eventually become runoff.

For an input of 1" of water for an instantaneous duration, the input would be $\delta(0)$. The storage-flow relation for each cascade would be

$$S = KQ \qquad (3.17)$$

The output for an instantaneous unit input is the IUH denoted by u(0,t). For the first linear storage equation (3.16) becomes

$$\delta(0) - u_1(0,t) = K \frac{d\, u_1(0,t)}{dt} \qquad (3.18)$$

and the solution of equation 3.18 is

$$u_1(0,t) = \frac{\delta(0)}{\left[D + \frac{1}{K}\right]} \qquad (3.19)$$

Where D is the linear operator $\frac{d}{dt}$

$u_1(t)$ becomes input into the second reservoir, and the process is repeated to complete the n reservoir.

In general, the solution of the output from the n^{th} reservoir is

$$u(0,t) = \frac{\delta(0)}{\left[D + \frac{1}{K}\right]^n} \qquad (3.20)$$

Nash (16) has shown that the solution of equation 3.20 is in the form of gamma function.

$$u(0,t) = \frac{1}{K\,\Gamma n}\,(t/K)^{n-1}\,e^{-t/K} \qquad (3.21)$$

This can be shown as under -

The instantaneous unit hydrograph for the single linear reservoir models mentioned above would be

$$u(0,t) = 1/K\,e^{-t/k} \qquad \text{(Ref. eq.3.14)}$$

This IUH has no peak and the time of peak of the 1UH can not exceed the duration of the rain excess.

In routing the flow, this outflow is considered as the inflow to the second reservoir. So using $u(t) = \frac{1}{K} e^{-t/K}$ as input function with $(t-\tau)$ being the variable instead of t and using the Kernal function with $(t-\tau)$ being the variable we get the ordinate of the DRH at time t,

$$Q(t) = \int_0^{t' \leq t_0} u(t-\tau)\, I(\tau)\, d\tau$$

But $u(\tau) = I(\tau) = \frac{1}{k} e^{-t/k}$ and $u(t-\tau) = \frac{1}{K} e^{-(t-\tau)/K}$,

Therefore we get the outflow from the second reservoir,

$$q_2 = \int_0^t \frac{1}{k} e^{-t/k} \frac{1}{K} e^{-(t-\tau)/K} d\tau$$

$$= \frac{t}{K^2} e^{-t/k}$$

This outflow is then used as inflow to the third reservoir and routed through the latter.

Thus we get the outflow from the third reservoir is

$$q_3 = \int_0^t \left[\frac{\tau}{K^2} e^{-\tau/k}\right] d\tau \left[\frac{1}{K} e^{-(t-\tau)/K}\right]$$

$$= \int_0^t \left[\frac{\tau}{K^2} e^{-\tau/K}\right] \left[1/K \; e^{-t/k} \frac{1}{K} e^{\tau/K}\right] d\tau$$

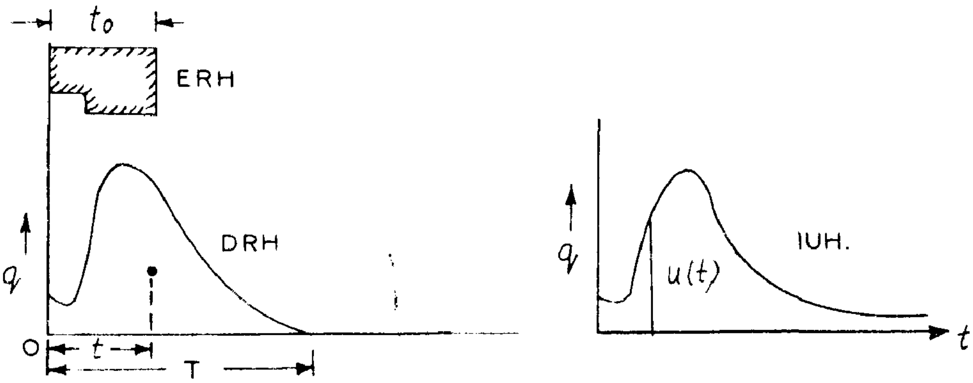
$$= \frac{t^2}{2K^3} e^{-t/K}$$

continuing this routing procedure we get the outflow q_n from the n^{th} reservoir as

$$u(t) = \frac{1}{K\Gamma(n)} \left(\frac{t}{K}\right)^{n-1} e^{-t/K} \qquad (3.22)$$

which is the IUH of the simulated drainage basin. If n is an integer $\sqrt{n} = \lfloor n-1$.

The Nash Model Fig.3.4 is linear since the storage coefficient K is constant. It may be non-linear if K is variable or a function of Q. Also this model does not involve the concept of translation of flow.



The values of the parameters of Nash model viz. K and n can be determined by the method of moments.

Taking moments of DRH about its origin, we have,

$$MDRH_1 = \frac{\int_0^T t.\ q(t)\ dt}{\int_0^T q(t)\ dt}$$

nd

$$MDRH_2 = \frac{\int_0^T t^2\ q(t)\ dt}{\int_0^T q(t)\ dt}$$

Similarly taking moments of ERH about its origin,

$$MERH_1 = \frac{\int_0^{t_o} t.\ i(t)\ dt.}{\int_0^{t_o} i(t)\ dt}$$

$$MERH_2 = \frac{\int_0^{t_0} t^2 . i(t)dt}{\int_0^{t_0} i(t)\, dt}$$

First moment M_1 of IUH about its origin, $t = 0$ is

$$M_1 = \frac{\int_0^{\infty} t . u(t)dt}{\int_0^{\infty} u(t)\, dt}$$

and second moment M_2 of IUH about its origin, $t=0$, is

$$M_2 = \frac{\int_0^{\infty} t^2\, u(t)\, dt}{\int_0^{\infty} u(t)\, dt}$$

Relation between M_1 , M_2 and n, K :

$$M_1 = n\,K \text{ (lag time of centroid of IUH)} \qquad (3.23)$$

and $$M_2 = n\,(n+1)\,K^2 \qquad (3.24)$$

Also $$MDRH_1 - MERH_1 = n\,K \qquad (3.25)$$

i.e. the time difference between the centroids of ERH and DRH should be equal to M_1.

It can be further proved that

$$MDRH_2 - MERH_2 = n\,(n+1)\,K^2 + 2nK\, MERH_1 \qquad (3.26)$$

The first and the second moments of ERH and DRH can be computed from the given ERH and DRH and the parameters n and K defining the IUH can be evaluated by using the above equations.

Then using this value of n and K the ordinates of instantaneous unit hydrograph can be found out from the relation,

$$u(t) = \frac{1}{K\,\Gamma(n)}\,(t/k)^{n-1}\, e^{-t/k}$$

3.4.1. Relationships between unit hydrograph parameters and catchment Characteristics:

The application of the principle of dimensional analysis to obtain the relationship between characteristics of the unit hydrograph and topographic properties of a water shed is not possible unless careful consideration is given to the selection of variables. It is seen that in small water sheds drainage area size A, length of the main stream L and length to the centre of area L_{Ca} are highly correlated.

Unit hydrograph synthesis for ungaged basins is based on emperical. expressions which relate pertinent physical characteristics of the water shed to geometric aspects of the unit graph. These relationships are predicted on the basis that the unit hydrograph of an area represents the integrated effect of all the sensibly constant basin factors and their modifying influence on the translation and storage of a runoff volume from a uniform excess rain occuring during a unit period of time.

Dimensional analysis [Murphy (12)] has proved to be a useful tool in engineering fields in establishing relationship within a system of variables. Strahler (12) discussed the application of these principles to the field of geomorphology in connection with fluvially eroded landforms. These studies give rise to the thoughtthat the principles of dimensional analysis may be used to derive the desired relationships between watershed characteristics and unit graph properties required for hydrograph synthesis.

The field of quantitative geomorphology has received considerable attention in recent years. These studies were stimulated by the work of Horton (12) who suggested that the development of morphological characteristics depends the three main factors :

1. Surface resistivity to sheet errosion
2. Runoff intensity
3. Ground slope

Since these factors vary with soil and bedrock conditions, watër shed in different regions wouldbe expected to exhibit wide differences in the degree of development of their drainage systems.

3.4.1.1. Nash's approach:

It would be extremely useful if the UH of a catchment could be predicted from considerations of the physical properties of the catchment. This ability would be useful in assessing the flood potentialities of a catchment in the absence of records of discharge, and also in predicting the effect of artificial changes in the catchment characteristics. The more reliable the relationship established between the catchment characteristics and the UH, the more reliable can be effects of a proposed change be assessed. But unfortunately a very modest success has been achieved in the establishment of such relations.

Empirical relations:

The usual procedure in seeking empirical relations has been to derive unit hydrographs of a given short period (1 hr.) for as many catchments as possible and to seek significant correlations between various measures or parameters of the unit hydrographs and the characteristics of the catchments. This implies expressing the unit hydrograph parameters and the catchment characteristics numerically. The chosen unit hydrograph parameters must be such that the full unit hydrograph can be reconstructed from the given paramatric values. For example, if a relation is obtained between the magnitude of the instantaneous unit hydrograph peak and some catchment characteristics such as area, slope, shape etc., use of the relations will provide an estimate of the instantaneous unit hydrograph peak for a catchment for which no records are available.

A more general correlation was attempted by Nash (18) using the records of some 30 British catchments. The unit hydrograph parameters were obtained, as far as possible, from floods caused by short intense rainfall reasonably well distributed over the catchment and isolated in time from adjacent storms. The base flow was separated by a straight line on the hydrograph from the point of beginning of the rise to a point on the recession, such that the time elapsed between the end of effective rainfall and the end of storm runoff was three times the time lag between the centre of area of storm runoff and the effective rainfall.

The distribution of rainfall losses during the storm was made as follows : Any rainfall which fell at the beginning of the storm and which did not appear to have caused storm runoff was taken as lost. The remainder of the losses were taken as occuring at a constant rate (inches per hour) throughout the duration of the storm.

This provided input and output graphs and it remained to choose and measure parameters of the unit hydrograph. The first parameter chosen was the first moment of the instantaneous unit hydrograph about a vertical through its origin, the second third etc. as the second, third etc. moments of the instantaneous unit hydrograph about a vertical, through its centre of area. Evaluation procedure of these moments are given in section 3.4.

The following quantities were defined as the unit hydrograph parameters and used as the dependent variables in the subsequent regression analysis:

$$\left.\begin{aligned} m_1 &= u_1' \\ m_2 &= u_2/(u_1')^2 \\ m_3 &= u_3/(u_1')^3 \end{aligned}\right\} \quad (3.27)$$

Obviously m_2, m_3 etc. are dimensionless measures of the unit hydrograph shape. m_1 is the mean delay suffered by a particle of water between falling on the catchment and passing the gauging station, and has unit in hours. m_2 is a coefficient of variation of delay time and m_3 is a coefficient of skewness.

Correlations were sought between the m 's and the catchment characteristics. A variety of catchment characteristics was tried, but mainly due to the high correlations existing between many of them it was not possible to isolate the effect on the instantaneous unit hydrographs of these individually. The best prediction equations obtained were,

$$
\left.\begin{aligned}
m_1 &= 27.6\ A^{0.3}\ \ OLS^{-0.3} \\
m_2 &= 0.41\ L^{-0.1}
\end{aligned}\right\} \quad (3.28)
$$

were m_1 is in hours, A is the catchment area in square miles, L is the length of the main channel from the gauging site to the boundary in miles, and OLS is the overland slope of the catchment expressed in parts per ten thousand which was calculated as follows:

A grid of rectangular mesh was drawn on the map of the catchment, the mesh being such that about 100 inter-sections occured within the catchment boundary. At each intersection the minimum distance in feet between adjacent 25' contours was measured and the slope at each point taken as 25 ft. in this distance. This provided a set of slope values of which the mean was calculated and taken as OLS. When an intersection occurred at a point between two contours of the same value the slope was taken as zero if the point was in a valley and as indeterminate if on a hill. The latter was neglected in calculating the mean.

The coefficient of multiple correlation r and the standard error of estimate expressed as a percentage error associated with equations 3.24 & 3.27 were found to be highly significant, through the second i.e. for m_2 was much less so. Since m_2 is second order quantity and more variable between UH's for the same catchment and thus more difficult to measure. This partial failure in finding a correlation for m_2 discouraged attempts to relate m_3 to the catchment characteristics. So a general instantaneous unit hydrograph unit equation of two parameters is chosen. The eqqation chosen was -

$$u(0,t) = \frac{1}{K\Gamma(n)} e^{-t/k} (t/k)^{n-1} \qquad (3.29)$$

which is the equation of the discharge from a series of n linear, reservoirs (S = KQ) each discharging into the next, when the inflow is an instantaneous unit volume. The significance of this equation is that its acceptance as a description of the operation of a highly damped, time invariant linear system is almost as general as the assumption that the output is related to the input by a linear differential equation with constant coefficients.

$$i(t) = a \frac{d^n}{dt^n} q(t) + b \frac{d^{n-1}}{dt^{n-1}} q(t) + \qquad (3.30)$$

where a,b,c etc. are constants.

Further evidence of the suitability of this equation was provided by plotting $m_1 : m_2$ for the general catchments involved, and on the same plane plotting the line corresponding to equation 3.29 This line appeared to be as good an approximation to the scatter of points as was possible.

The relations between m_1 and m_2 and the parameters K and n of equation 3.29 were easily shown to be

$$\left.\begin{aligned} m_1 &= n K \\ m_2 &= 1/n \end{aligned}\right\} \qquad (3.31)$$

These equations can be combined with equations 3.28 to give n and K directly in terms of the catchment characteristics,

$$\left.\begin{aligned} K &= 11\, A^{0.3}\, OLS^{-0.3}\, L^{-0.1} \\ n &= 2.4\, L^{0.1} \end{aligned}\right\} \qquad (3.32)$$

Using equation 3.29 as the general instantaneous unit hydrograph equation we obtain the equations for the unit hydrograph of any finite duration as follows :-

$$u\ (0,t) = \frac{1}{K\,\Gamma n}\ e^{-t/K}\ (t/K)^{n-1}$$

The equation of S-curve is given by

$$S(t) = \int_0^t u\ (o,t)\ dt$$

From equation (6) we have,

$$S(t) = \frac{1}{\Gamma n}\int_0^{t/k} e^{-t/k}\,(t/K)^{n-1}\ d\ (t/k)$$

$$= I\ (n,\ t/k)$$

Where $I\ (n,\ t/k) = \frac{1}{\Gamma n}\int_0^{t/k} e^{-x}\ x^{n-1}\ dx$ is the incomplete gamma function of order n at (t/k).

The unit hydrograph of period T is given by

$$u(T,t) = \frac{1}{T}\left[I\ (n,t/k) - I(n,\ \frac{t-T}{K}\ )\right] \qquad (3.33)$$

which is the general equation of the unit hydrograph of period T.

Tables of $I\ (n,t)$ are available which enable us to calculate the ordinates of $u\ (T,t)$.

3.5. BRIEF REVIEW OF SOME WELL KNOWN MODELS:

Some of the conceptual hydrograph models (linear and non-linear) for routing rainfall excess through a cascade of linear reservoirs have been reviewed. Because of linearity of the models, convolution of the instantaneous unit hydrograph is mathematically identical to synthetic hydrographs obtained by direct routing. The review of these models gives an impression how far these are successful in predicting the response of a water shed considering the catchment system as linear or non -linear as the case may be. The simulation techniques used in the models linear or non-linear are deterministic in nature.

3.5.1. Dooge's Model:

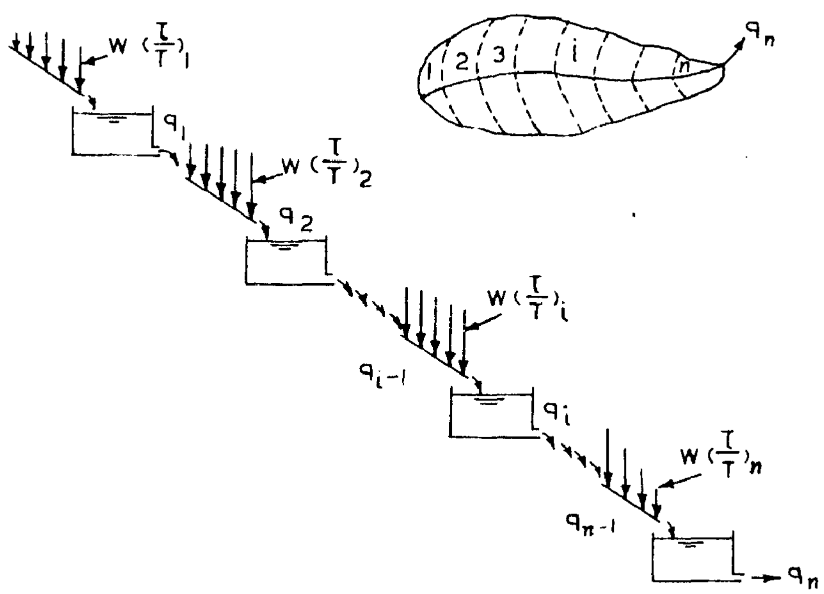


FIG.3·5

The Nash model (16) does not involve the concept of translation of flow. In order to incorporate this concept into the analysis, Dooge (10) proposed to use the concept of a linear channel for the first time and to represent the basin system by a series of alternating linear channels and linear reservoirs. In Dooge's model, Fig. 3.5 which is linear, the drainage area is divided by isochrones into n number of subareas. The idea of isochrones is believed to have been first proposed by Ross (5) in 1921. Each sub area is represented by a linear channel in series with a linear reservoir. The outflow from the linear channel is represented by a time area diagram which, together with outflow from the preceding subarea, serves as the inflow to the linear reservoir. The instantaneous unit hydrograph of the simulated drainage area was shown as

$$u(t) = \frac{S}{T} \int_0^{t' \leq T} \left[\frac{\delta(t-\tau)}{\prod_{i=1}^{i(\tau)} (1 + K_i D)} \right] \omega\left(\frac{\tau}{T}\right) d\tau \qquad (3.34)$$

Where S is the input volume taken as unity,

T is the total Translation time of the basin from which isochrones are constructed. = LC,

where L = total channel length of the basin

C = constant translation coefficient for all linear channels.

i is the order of reservoirs equal to 1,2 ..... counted downstream to the basin outlet.

$i(\tau)$ is the function of τ representing an integer equal to the order number of the subarea where τ is considered,

K_i is a storage coefficient

D is the differential operator d/dt

$(t - \tau)$ is the Dirac delta function where t is the time elapsed.

τ is the translation time between the element in the subarea and the outlet.

$w(\tau/T)$ is the ordinate of a dimension less time area diagram.

Although the Dooge model takes into account the translation effect of flow in a drainage basin, the equation for instantaneous unit hydrograph can not be easily solved for practical applications.

3.5.2. K.P. Singh Model:

To overcome the difficulties of the Dooge's model for practical applications, Singh (23) developed a model which consists of a linear channel of translation coefficient C and two linear reservoirs of different storage coefficients K_1 and K_2 in series. The theory is developed using a non-linear approach which accounts for the apparent variations in instantaneous unit hydrograph derived from different storms overa given drainage basin. The transformation of rainfall excess having a non uniform areal and time distribution to a direct surface runoff hydrograph at the basin outlet is performed, giving consideration to the influence of both overland and channel flows. The characteristics of overland and channel flows vary from place to place in any drainage basin and their effects on the instantaneous unit hydrographs are considered in forms of the translation and storage factors of these flows over the drainage basin.

Assumptions : The following assumptions are made for successful application of the model.

1. Channel storage Vs discharge curves for a given runoff event can be approximated by a linear storage discharge relation at least over the part associated with considerable direct surface runoff rates.

2. The accuracy in the liniarization of the overland flow component and the magnitude of K are important only for small drainage areas.

3. The drainage basin is relatively homogeneous with respect to the physiography and topography of the sub areas, sufficient to permit computation of a satisfactory concentration times diagram.

4. The reduction of hyetographs to effective hyeteograph and total hydrographs to direct surface runoff hydrographs is sufficiently accurate. Adequate knowledge of areal and time distribution of losses and abstractions and the baseflow hydrograph is necessary for data reduction.

The proposed nonlinear theory for instantaneous unit hydrograph accounts for the variability of instantaneous unit hydrographs derived from different storms over a drainage basin in terms of three physically significant parameters and a functional parameter $w(\tau)$. The non-uniform areal distribution of rainfall excess from a given storm is accounted for in the concentration time diagram. The effects caused by duration and non-uniform time distribution of average rainfall excess over a drainage basin are condensed into a single characteristic Re, the equivalent instantaneous rainfall excess. The instantaneous unit hydrographs for the model is given by

$$u(t) = \frac{1}{K_2 - K_1} \int_0^{t_1} \left[e^{-(t-\tau)/K_2} - e^{-(t-\tau)/K_1} \right] w(\tau) d\tau \qquad (3.35)$$

in which, $u(t)$ = instantaneous unit hydrograph ordinate in time t after occurence of instantaneous unit rainfall excess, in inches/hour.

K_2 = the channel storage coefficient in hours.

K_1 = the overland storage coefficient in hours.

$w(\tau)$ = the ordinates of the concentration time diagram with base equal to T or the time of concentration in hours $(hr.)^{-1}$

τ corresponds to a variable time of travel

$t_1 = t$ for $t \leq T$

and $t_1 = T$ for $t \geq T$

Parameters T, K_2 and K_1 depend not only on more or less permanent basin characteristics but also on storm characteristics.

The linear channl is used to produce a time area diagram for the whole drainage basin with variable areal distribution of the instantaneous effective rainfall. In applying this model to actual data, K_1 is assumed to have a constant average value of about 0.25 very small in comparison to K_2 and the time area diagram to be one of a number of basic geometric forms such as a rectangle, triangle, trapezoid and sine curve, instead ofthe actual diagfams.

3.5.3. Kulandaiswamy's Five Parameter Model:

Kulandaiswamy (13) considered the rainfall and runoff relationship by system analysis and proposed a general equation of storage for non-linear reservoir as follows :

$$S = \sum_{n=0}^{N} a_n (Q, I) \frac{d^n Q}{dt^n} + \sum_{m=0}^{M} b_m (Q, I) \frac{d^m I}{dt^m} \qquad (3.36)$$

Where a_n and b_m are function of outflow Q or inflow I or both and also their derivatives.

As a simplification and approximation, the coefficients are assumed to be functions of the average values $\bar{Q}$ and $\bar{I}$ instead of Q and I. Substituting the above modified equation in the continuity equation and dropping the insignificant terms in the differential equation after being tested by actual data, the resulting differential equation is ofthe form

$$a_2 \frac{d^3Q}{dt^3} + a_1 \frac{d^2Q}{dt^2} + a_0 \frac{dQ}{dt} + Q = - b_1 \frac{d^2I}{dt^2} - b_0 \frac{dI}{dt} + 1 \qquad (3.37)$$

The outflow by this equation can be written as :

$$Q(t) = \frac{-b_1 D^2 - b_0 D+1}{a_2 D^3 + a_1 D^2 + a_0 D+1} \; I\,(t) \qquad (3.38)$$

Where D is the differential operator $\frac{d}{dt}$. To compensate for the terms being dropped from the general equation, which would take the translation effect into consideration, it is desirable to replace I (t) by $I(t - \tau_o)$ where τ_o is the translation time which is equal to the difference in time between the beginning of the effective rainfall and direct runoff and may be determined from the actual data. When I(t) or $I\,(t - \tau_o)$ is a unit instantaneous input of $\delta(t - \tau_o)$, equation 3.38 represents the function of an instantaneous unit hydrograph.

Equation 3.38 is a polynomial of degree three and has three roots. Assuming that the system is stable, the following four cases describing different mathematical models are possible, depending upon the nature of the roots.

Let the roots of the equation be m, n and P.

Case I: Where all the roots are real and unequal

$$m \neq n \neq P$$

IUH for this case is given by the following equation

$$u(t) = - m\, n\, P \left[A\, e^{mt} + B\, e^{nt} + C e^{pt} \right] \qquad (3.39)$$

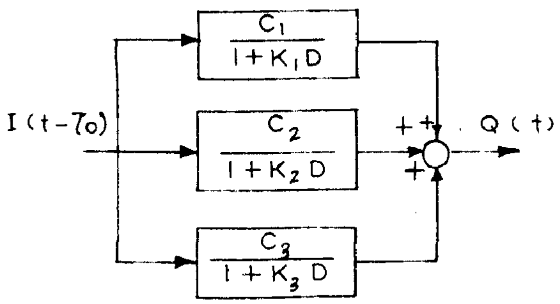
Where A, B, C are as follows:

$$A = \frac{-b_1 m^2 - b_o m + 1}{m^2 - m(n+p) + nP}$$

$$B = \frac{- b_1 n^2 - b_o n + 1}{- n^2 - n(m+p) + mp}$$

$$C = \frac{-b_1 p^2 - b_o p + 1}{p^2 - p(m+n) + mn}$$

This case is equivalent to 3 linear reservoirs in parallel.



$$Q(t) = \left[\frac{C_1}{1+K_1 D} + \frac{C_2}{1+K_2\, D} + \frac{C_3}{1+K_3 D} \right] I\,(t - \tau_o) \qquad (3.40)$$

Where C_1, C_2, C_3 are constants in terms of a_o, a_1, a_2, b_o and b_1.

Case 2:

When all the roots are real and two ofthem are equal.

Let $n = p$

IUH for this case is given by

$$u(t) = mn^2 \left[A_e^{m+} + B_e \, n+ \; + C \, t \, e^{nt} \right] \qquad (3.41)$$

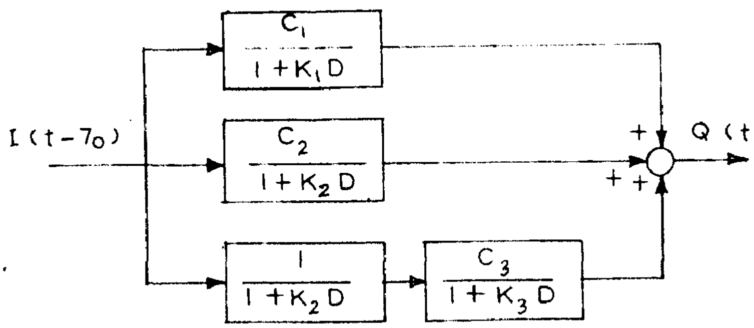
Where

$$C = \frac{- b_1 \, n^2 \; - b_o \; n+1}{n-m}$$

$$B = \frac{b_{on} + C \; (n-2m) - 2}{n^2 - mn}$$

$$A = - bo - B$$

This is equivalent to 4 operational blocks.



$$Q(t) = \left[\frac{C_1}{1+K_1 D} + \frac{C_2}{1+ K_2 D} + \frac{C_3}{(1+K_2 D)^2} \right] I \; (t - \tau_o) \qquad (3.42)$$

Case 3:

When all the roots are real and equal

($m=n=p$)

Equation of IUH is given by

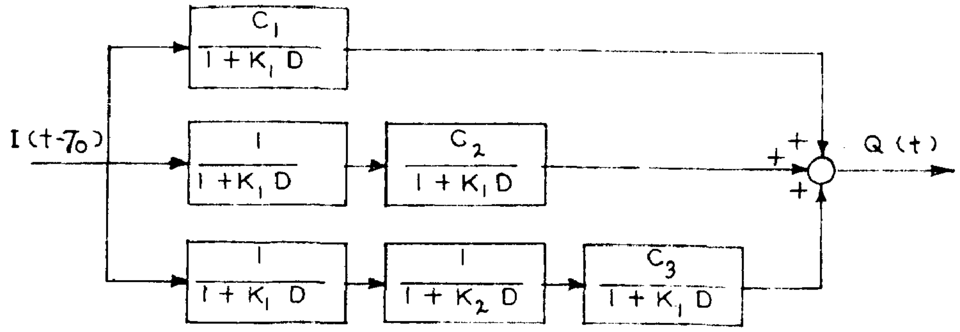
$$u(t) = \left[A + Bt + Ct^2 \right] e^{mt} \qquad (3.43)$$

Where $A = - b_1 \, m^3$

$$B = - b_o \, m^3 - 2 \, b_1 \, m^4$$

$$C = 1/2 \left[m^3 - b_o \, m^4 - b_1 \, m^5 \right]$$

This is equivalent to,



$$Q(t) = \left[\frac{C_1}{1+K_1 D} + \frac{C_2}{(1+K_1 D)^2} + \frac{C_3}{(1+K_1 D)^3} \right] I(t - \tau_o) \quad (3.44)$$

Case 4:

When one root is real and two roots are complex conjugates:

Let $n = r + jw$ $\qquad j = \sqrt{-1}$

$p = r - jw$

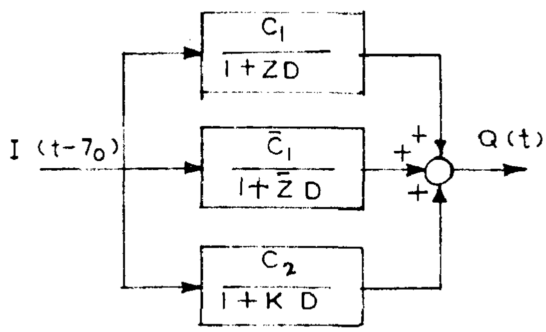
IUH is given by , $u(t) = -m(r^2 + w^2) \left[A_e^{mt} + e^{rt} \left\{ B \cos wt + \frac{C+Br}{w} \sin W t \right\} \right]$

Where $A = \dfrac{-b_1 m^2 - b_o m + 1}{m^2 - 2m + (r^2 + w^2)}$ (3.45)

$B = -b_1 - A$

$C = -b_o - b_{1m} + A(2r - m)$

This is equivalent to,



$$Q(t) = \left[\frac{C_1}{1+ZD} + \frac{\bar{C}_1}{1+\bar{Z}D} + \frac{C_2}{1+KD} \right] I(t - \tau_o) \quad (3.46)$$

Where C_1, $\bar{C}_1$, and Z, $\bar{Z}$ are complex numbers in terms of a_2, a_1, a_o, b_1 and b_o.

General Expression for response model :

A general mathematical model for the storage of non-linear reservoirs as proposed by Kulandaiswamy is

$$S = \sum_{n=0}^{N} a_n (Q, I) \frac{d^n Q}{dt^n} + \sum_{m=0}^{M} bm (Q, I) \frac{d^m I}{dt^m} \qquad (3.36)$$

Where a_n and b_m are functionsof outflow Q or inflow I or both and also their derivatives, can not be solved as a general case. Three cases has been put forwarded for solving this general equation.

Case 1: Assumption : Coefficient of higher order are neglected.

$$S = a_{oq} + a_1 \frac{dq}{dt} + b_o i \qquad (3.47)$$

Where a_o is functiom of q and a_1, b_o are constants for the storm under consideration.

$$\frac{ds}{dt} = q \frac{da_o}{dt} + a_o \frac{dq}{dt} + a_1 \frac{d^2q}{dt^2} + b_o \frac{d_i}{dt}$$

$$= q \frac{da_o(q)}{dq} \frac{dq}{dt} + a_o (q) \frac{dq}{dt} + a_1 \frac{d^2q}{dt^2} + b_o \frac{d_i}{dt}$$

$$= \frac{dq}{dt} \left[q \frac{da_o(q)}{dq} + a_o (q) \right] + a_1 \frac{d^2q}{dt^2} + b_o \frac{di}{dt}$$

$$= \phi(q) \frac{dq}{dt} + a_1 \frac{d^2q}{dt^2} + b_o \frac{di}{dt}$$

Where $\phi (q) = q \frac{da_o(q)}{dq} + a_o(q)$ nonlinearity is directly considered in this equation.

From continuity equation, $I - q = \frac{ds}{dt}$, we get

$$q(t) = \frac{1 - b_o D}{a_1 D^2 + \phi (q) D + 1} i(t) \qquad (3.48)$$

This equation can not be solved in general. The author has solved it by Runga Kutta method. After solution it is found that ϕ (q) follows a definite relation with q but does not show any definite relation with a_1 and b_o so he assumed these values to be constant. This equation can be used only qualitatively, in many cases the results failed to give coincidence result with the observed values.

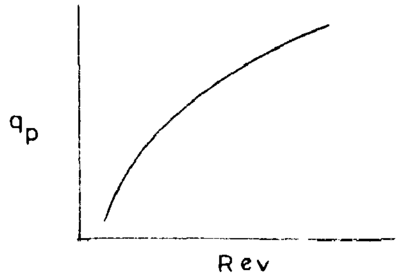
Case 2: In this case the nonlinearity is considered indirectly.

$$a_{oq} + a_1 \frac{dq}{dt} + a_2 \frac{d^2q}{dt^2} + b_{oi} + b_1 \frac{di}{dt} \qquad (3.49)$$

Here a_o, a_1, a_2, b_o and b_1 are not function of i and q. These are assumed to be constant for a particular storm but vary from storm to storm. So, the nonlinearity is considered indirectly. The instantaneous unit hydrograph for the catchment changes with storm to storm. If this is the case, it can be assumed that the catchment is behaving nonlinearly. The model itself is a lumped model but nonlinear with respect to time

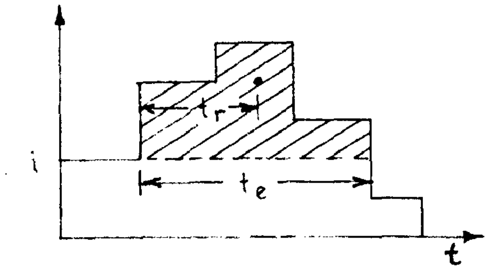
$$q(t) = \frac{1 - b_o D - b_1 D^2}{a_2 D^3 + a_1 D^2 + a_o D + 1} i(t) \qquad (3.50)$$

This case represents the Kulandaiswamy's Five parameter model, the solution of which has already been discussed.

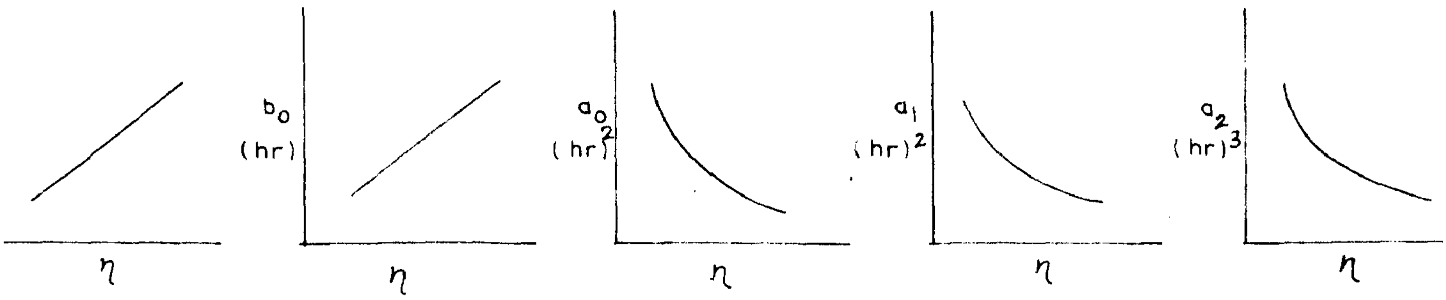


Here a plot of q_p Vs R_{ev} (Peak value of runoff Vs total volume of runoff excess) is considered and a_o, a_1, a_2, b_o, b_1 all are plotted against q_p for a number of storms and found that a_0, a_1, a_2 vary with q_p and b_0, b_1 are constant for the catchment. Now knowing q_p, a_0, a_1, a_2 can be found out from the plot.

It has been found that the volume of rainfall excess, R_{ev}, alone is not sufficient for correlating the coefficient of surface runoff is dependent on volume of rainfall excess, time and spatial distribution of rainfall excess. Since the system is considered as lumped, only time distribution of rainfall excess is considered.



From the hyetrograph volume of rainfall excess, R_{ev}, duration of rainfall excess t_e, and time to the centre of area of rainfall excess t_r are determined. These three parameters are combined to form a single parameter η where, $\eta = Rev \frac{tr}{t_e}$. Depending on the particular cases these parameters will take into account the time distribution of rainfall. For a given drainage basin he gave the plots of these parameters as shown below:



When we consider a_0, a_1, a_2 the slopes are found to be same which shows that when we consider 3 parameter model the result obtained is same as that of 5 parameter model and for higher values of η all the coefficient becomes independent of η.

Case 3: This is a simplified version of 5 parameter model where only a_0, a_1, b_0 has been considered and hence known as 3 parameter model.

$$S = a_0 q + a_1 \frac{dq}{dt} + b_0 i \qquad (3.51)$$

$$q(t) = \frac{1 - b_0 D}{a_1 D^2 + a_0 D + 1} \, i(t) \qquad (3.52)$$

Coefficients a_0, a_1, a_2 in case of 5 parameter model and a_0, a_1, in case of 3 parameter model decrease with increased value of η. Coefficient b_0 and b_1 (b_0 in case of 3 parameter model) increase linearly with increased value of η. This shows that the coefficients vary from storm to storm which will take into account the nonlinear behaviour of the catchment.

While Kulandaiswamy's model was found to produce better fit to observed data than many other lumped-system models tested in the investigation, a simplification with loss of superior fitness but gain in true linearity was proposed by Prasad by retaining only two terms of the general non-linear storage equation.

3.5.4. Ramanand Prasad's model: A non-linear hydrologic system Response model:

A drainage basin transform rainfall excess into direct runoff by means of storage constituents. The storage action is evidenced by delay modulation and attenuation of the input (rainfall excess) as compared with the output (direct runoff hydrograph). All the storage effects in the basin might be considered combinable into a single conceptual reservoir. A linear systems requiring that the storage S be a linear function of the outflow Q.

$$S = KQ^{m} + K' \; ; \quad m = 1, \quad K' = 0 \qquad (3.53)$$

Where K = Storage constant.

An approximate physical representation of a single linear reservoir as defined by equation 3.53 is a reservoir with vertical walls and a proportional weir type outlet. For a reservoir with vertical walls,

$$S = AY^{n} \quad , \quad n = 1 \qquad (3.54)$$

and for a proportional weir type outlet,

$$Q = BY^{p}, \quad p = 1 \qquad (3.55)$$

Where A and B are constants, Y = stage or water level. Eliminating Y from equation 3.54 and 3.55 we get

$$S = \left(\frac{A}{B} \right) Q$$

or $S = K\,Q \, , \quad \frac{A}{B} = K$ which is the equation (3.53).

The characteristics of a vertical walled reservoirs are illustrated in Fig.3.7(a) and those of a proportional weir in Fig.3.7(b). The Fig.3.7(c) shows the combined characteristics curve of these two features which is the storage outflow relation defined by equation 3.53. Considering the complex nature of the natural drainage basin storage, it appears unlikely that storage effects could be truely linear ; for actual physical reservoir and outlets, the exponent of Y in equation 3.54 and equation 3.55 or the exponent of Q in equation 3.55 rarely equals one.

Assumption of a time invarient reservoir :

All the effective storage components in a watershed may be conceived as being equivalent to a single lumped storage: Let the lumped storage (S_1) be represented by the power function of Y as,

$$S_1 = D' Y^g \qquad (3.56)$$

in which D' and g are constants.

From this point onward the subscript 1 will be dropped from S_1 keeping in mind that all the storage will now be thought of as being for a lumpede equivalent (or conceptual system). The storage stage relationship given by equation 3.56 hold good for same natural reservoirs. Equation 3.56 is illustrated in Fig.3.7(d). Let it be further assumed that the lumped storage system has an outlet control as shown in Fig.3.7(e) for which the storage relation is given by

$$Q = C Y^M \qquad (3.57)$$

where C,M are constants. The value of M in equation 3.57 for various geometric channels outlets are 1.5,2, 2.5 for rectangular, hyperbolic and triangular sections (Ref. V.T. Chow : Open Channel Hydraulics) respectively. This range is believed to cover the characteristics for the gage control of some natural channels. The storage discharge relationship for a lumped system with an outlet control as shown in Fig.3.7(f) may be found by elemating Y from equation 3.56 and 3.57.

$$S = \frac{D'}{C^{g/M}} \cdot Q^{g/M} \qquad (3.58)$$

If D', C, g, and M are constants for the system, assumed as time invarient, equation 3.58 may be written as

$$S = K_1 Q^N \qquad (3.59)$$

where $N = \frac{g}{M}$, $K_1 = \frac{D'}{C^N}$

Thus equation 3.59 describes a conceptual non-linear storage reservoir with an outlet control for which the values of K_1 and N depend on the characteristics of the conceptual reservoir.

Consideration of Dynamic effect in the conceptual model:

Unsteady effects are assumed to be negligible in the development of equation 3.59 so that it describes only the dotted curve of Fig.3.7 (f) and not the actual loop which is observed in natural channels. To allow for unsteady flow effects, the storage discharge relation of equation 3.59 modifies to

$$S = K_1 Q^N + K_2 \frac{dQ}{dt} \qquad (3.60)$$

in which K_2 may be a complicated function of several variables effecting the wedge storage as well as the storage discharge relationship, but it is assumed to be constant for a particular hydrograph. If the input into the system is represented by the storage S and the output from the system by Q all being a function of time t then the equation of constinuity leads to the differential equation.

$$\frac{ds}{dt} = R - Q \tag{3.61}$$

Differentiating equation 3.60 and eliminating $\frac{ds}{dt}$ from equation 3.61

we get,

$$K_2 \frac{d^2Q}{dt^2} + K_1 NQ^{N-1} \frac{dQ}{dt} + Q = R \tag{3.62}$$

which is a second order non linear differential equation describing the response of a time invarient, lumped non linear system. The equation 3.62 is the Prasad's non linear hydrologic system response model describing the relationship between the input R and the output Q from a basin. This equation may be written in terms of differential operator D as

$$Q = \frac{1}{K_2 D^2 + K_1 NQ^{N-1} D + 1} \cdot R \tag{3.63}$$

in which

$$\frac{1}{K_2 D^2 + K_1 NQ^{N-1} D + 1}$$

is the non linear operator which transforms the input R into output Q

<u>Solution of the system Response equation :</u>

Solution of the system response equation by Numerical method leads to the following three equations which forms the system.

$$\ddot{Q}_i = \frac{(R - K_1 NQ^{N-1} \dot{Q} - Q)}{K_2} \tag{3.64}$$

$$\dot{Q}_{i+1} = \dot{Q}_i + (\ddot{Q}_i + \ddot{Q}_{i+1}) \quad \frac{h}{2} \qquad (3.64)$$

$$Q_{i+1} + Q_i + \dot{Q}_i\, h + \ddot{Q}_i \quad \frac{h}{3}^2 + \ddot{Q}_{i+1} \frac{h^2}{6} \qquad (3.66)$$

where $\dot{Q} = \frac{dQ}{dt}$ and $\ddot{Q} = \frac{d^2Q}{dt^2}$

These three equations have three unknows

$\ddot{Q}_{i+1}$, $\dot{Q}_{i+1}$ Q_{i+1} which can be found for given values of N, K_1 and K_2 by the following iterative procedure :-

i) obtain $\ddot{Q}_i$ from equation 3.64 for which $\dot{Q}_i$ and Q_i will be known eitherfrom a previous step or from initial conditions.

ii) Assume $\ddot{Q}_{i+1} = \ddot{Q}_i$ as a first approximation

iii) Get an approximate value of $\dot{Q}_{i+1}$ and Q_{i+1} from equation 3.65 & 3.66 and of the assumed value of $\ddot{Q}_{i+1}$.

iv) From the approximate value of $\dot{Q}_{i+1}$ and Q_{i+1} in step (iii) recalculate the new value of $\ddot{Q}_{i+1}$ using equation 3.64.

v) If this new value of $\ddot{Q}_{i+1}$ is different from the previously assumed value, repeat step (iii) to step (v) with this new value of $\ddot{Q}_{i+1}$. If this new value of $\ddot{Q}_{i+1}$ is close to previously assumed value repeat the whole procedure for another i.

Step (i) to (v) will give the solution of the system response equation for a given value of N, K_1, K_2.

3.5.5. Laurenson's model :

An approach on non linear reservoir in simulating rainfall runoff process:

For simulating rainfall runoff process Laurenson(14) used distributed input to 3 non linearreservoirs in series. The catchment is divided by isochrones and the subareas are represented by non linear storage reservoirs. Then the inflow is routed through the reservoirs to get the required outflow. In the light of general observation and requirement, analysis of hydrologic records have shown that, in order to represent real situations accurately, a runoff routing procedure should have provision for :

i) Time variation in rainfall excess

ii) Areal variation in rainfall excess

iii) The fact that different elements passed through different amount of storage.

iv) The fact that storage in the catchment is distributed not lumped.

v) The fact that the storage Vs discharge relationship is non linear.

It consists of multiple routing through a series of concentrated storages the output from one becoming the inflow to the next. The total area of the catchment is divided into sub areas with lumped parameter. The routing method is similar to Muskingum routing method.

Procedure :

i) Hyetograph of rainfall for farthest of upstream of area is determined with shape given by nearest recording rain gauge and scale the maximum ordinate equal to average rainfall for the sub areas.

(2) Losses are subtracted to give rainfall excess.

(3) Find out the inflow hydrograph for rainfall by converting the hydrograph by relation,

$$Q = i A$$

Where

i = intensity of rainfal

A = Sub area

(4) The inflow hydrograph is routed through storage for sub areas by non linear routing method.

(5) Similarly next sub area rainfall hydrograph is developed and added with time is shifted to outflow hydrograph from upstream. The combined hydrograph is routed through appropriate storage

$$S = K(q)\, q \qquad (3.67)$$

Where K is a function of q.

$$(I-Q) = \frac{ds}{dt}$$

$$(i_1 + i_2)\frac{\Delta t}{2} - (q_1 + q_2)\frac{\Delta t}{2} = S_2 - S_1 \qquad (3.68)$$

Writting, $S_2 = K_2(q_2)q_2$ and $S_1 = K(q_1)q_1$ and substituting in (3.68) we get

$$q_2 = C_0 i_2 + C_1 i_1 + C_2 q_1 \qquad (3.69)$$

Where,

$$C_0 = C_1 = \frac{\Delta t}{2 K_2 + \Delta t}$$

$$C_2 = \frac{2K_1 - \Delta t}{2K_2 + \Delta t}$$

where 1 and 2 represents start and end of Δt respectively.

Since the values of the coefficients C_0, C_1 and C_2 depends on K_2 and K_2 depends on q_2, this equation is solved by iteration method by assuming $K_2 = K_1$ and find q_2. Redetermine K_2 knowing q_2 and find 2nd value of q_2 by iteration. This q_2 becomes q_1 for next routing period. From this q_1 corresponding value of K_1 is determined.

A number of significant conclusions has arisen from this runoff routing study.

Provision for item (ii) and (v) already stated represents a considerable potential advantage over unit graph procedures for hydrograph determination. Allowances for item (ii) and (iii) is made by subdividing the catchment into subareas, and a simple and satisfactory way of subdividing the catchment into areas of equal storage delay time from the outlet has been presented.

It is considered that one of the main difficulties in runoff routing to date has been in determining (or even defining) what is catchment storage. Consequently one of the major results of this investigation has been to show that the catchment storage is a distributed storage with an average delay time equal to the lag of the catchment.

3.5.6. The Nash - Sutcliffe Layer Model :

Nash and Sutcliffe (8) describe the operation of their model as follows:-

The model assumes that the basin is analogous to a vertical stack of horizontal soil layers, each of which can contain a certain amount of water at field capacity. Evaporation from the top layer takes place at the potential rate and from the second layer only on exhaustion of the first, and then at the potential rate multiplied by a parameter C, the value of which is less than unity. On exhaustion of the second layer, evaporation from the third layer occurs at the potential rate multiplied by C^2 and so on. A constant evaporation potential applied to the basin would reduce the soil moisture storage in a roughly exponential manner.

When rainfall exceeds evaporation, a function (h) of the excess contributes to generated runoff, and of the remainder anything in excess of a thresh hold value (f) also contributes to generated runoff. The remaining rainfall excess is used to restore the storages in the several layers to field capacity, beginning with the first and proceeding downwards until the rainfall is exhausted or all the layers are at field capacity. Any final excess also contributes to generated runoff.

The potential evaporation is calculated from Penman's formula with an albedo 0.25; to allow for systematic error, the potential evaporation is multiplied by a factor (a) before it is compared with rainfall. The capacity of each soil layer (except the lowest) is taken as 1", and the number of layers m is a parameter to be estimated.

To allow for functional values of m, as the estimation procedure requires, a related parameter (Z) is specified for use instead of m ; this is defined as the total storage at field capacity, and m is redefined as Z rounded upwards to a whole number. Thus if the estimation procedure sets Z to 3.1, m becomes 4 and the capacities of the four soil layers become 1,1,1 and 0.1 inches respectively.

The model therefore contains five parameters, C,Z,a,f,h. The schematic diagram of the model is shown in Fig.3.6(b).

3.5.7. The Dawdy - O'Donnel Model:

A general review of mathematical models of catchment behaviour was given by Dawdy O' Donnel.[8] They divide the mathematical model in two categories:

1. The comprehensive simulation of catchment behaviour which treats the catchment components in lumped form.

2. The complete specification of each component.

They used a model of the first category shown diagramatically in Fig.3.6(a).

This model represents a river basin by means of four interconnected reservoirs with volumes at any instant denoted by R,S,M,G as shown in the Fig. 3.6 (a)

The surface storage, R, is augmented by rainfall P; and depleted by evaporation, ER, infiltration, F and, when R exceeds a threshold R^*, channel inflow Q_1.

The channel storage, S, is augmented by channel inflow, Q_1; and depleted by surface runoff at the gauging station, Q_s.

The soil moisture storage, M, is augmented by infiltration, F, and capillary rise, C; and depleted by transpiration, EM, and when M exceeds a threshold, M*, by percolation, D

The ground water storage G, is augmented by deep percolation, D, depleted by capillary rise, C, and baseflow at gauging station B; and if and while G exceeds G*, M is absorbed into G, C and D no longer operate, but EM and F now acts on G.

Thereare nine parameters that control the functioning of the model. At the beginning of each interval, the volume in R lies between zero and R*, the Ist parameter; P is added to R; and ER, if any, is given first call on the sum. Next, F is calculated according to certain criteria based on Horton type equation, considering the rate of supply available from surface storage and the potential rate of infiltration at the start of the interval. This involves maximum and minimum infiltration rates, fo and fe and an exponential die away exponent, K (three more parameters). In preparation for the next interval, a potential rate of infiltration fi is calculated for the end of the current interval. Then Q_1 is determined by the excess, if any and R* left in surface storage, after E_R and F have been abstracted.

The channel storage, S, is assumed to be a linear storage having a storage constant K_s, the fifth parameter. Then, Q_s is a function of the volume in S at the beginning of the interval, of the inflow Q_1 and of Ks. A simple budget yields the volume left in S ready for the start of the next interval.

At the beginning of an interval, M lies between Zero and M^*, the sixth parameter. Either E_M is removed or F is added, for one of the two will be zero depending on whether or not E_R satisfied E_p, the potential evapotranspiration. One of the several alternatives is now followed depending on whether or not G, at the start of the interval, is greater than G^*, the seventh parameter, and if not, whether or not the quantity in M is now greater than M^*. If G is less than G^*, D is set equal to the excess, if any, over M^* now in M; C is zero if D exists, otherwise it is determined as a function of demand in M, of supply in G, and of a maximum rate of rise, C_{max}, the eighth parameter.

Also, M is left at M^* if D exists or if augmented by C, if not. If G, at the beginning of the interval, is greater than G^*, F, if any, acts on G directly in place of D and C similarly in place of E_M. In this alternative, M remains at M^*.

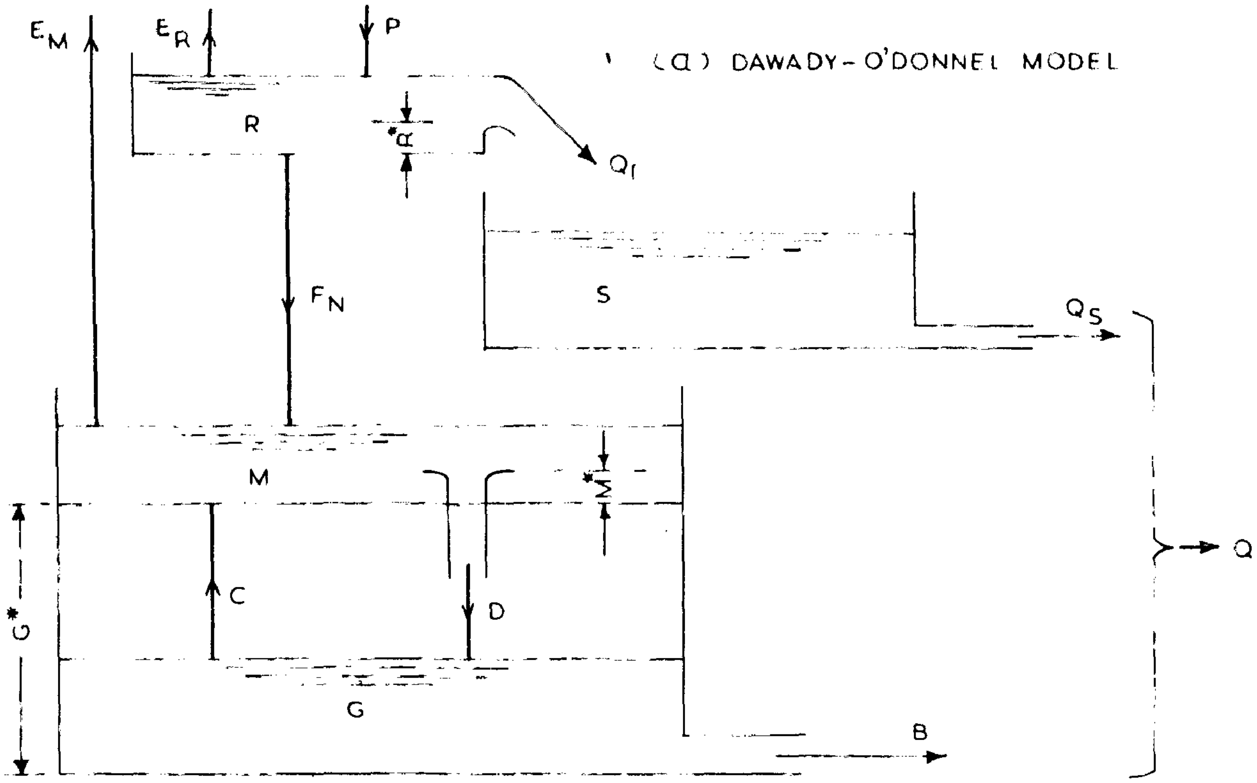
Then, G is assumed to be a linear storage having a storage constant K_G, the ninth parameter; B is then a function of the volume in G, at the start of the interval, of the inflow D or abstraction C and of Ks. Again, a budget yields the volume left in G ready for the start of the next interval.

In addition to the nine parameters listed above, the initial volume in each of the four reservoirs must be specified. To estimate these quantities would increase the number of parameters from 9 to 9+4 = 13. To avoid this complication, users of the model have postulated a long period with no rainfall and no streamflow prior to the start of a rainfall-runoff synthesis; it was then reasonable to set all four initial storages to zero and to assume that the starting potential infiltration rate had recovered to the maximum value of fo. This reduces the number of parameters to be estimated to nine.

3.5.8. Summary:

Satisfactory prediction of basin response is a destination towards which hydrologists have been long striving. There has been an increasing attempt to simulate physical basin system by analytical method, over the past decade or two. So far, a completely satisfactory theory has not been found. These lead to the development of linear conceptual models through which the basin response can be better represented. More recently, nonlinearities of the catchment system have been quantitatively studied on the basis of observed departures of the results of the linear models from actual basin behaviour. These and other studies have confirmed that the non-linear effects are significant enough to warrant undertaking a systematic approach towards development of a non-linear hydrologic system response model.

The models reviewed are good representation of various technique for deterministic simulation of hydrologic behaviour of watershed. They provide a full indication of the recent advance in the field of deterministic hydrology. The nonlinearity of the rainfall runoff relationship has been concerned only in recent years after the traditional methods of linearity. The non-linearity of the hydrologic behaviour mostly concerned from the theoretical point of view. In practice, the concept of non-linearity and its method of analysis are still very limited and has little application in the practical field of engineering.



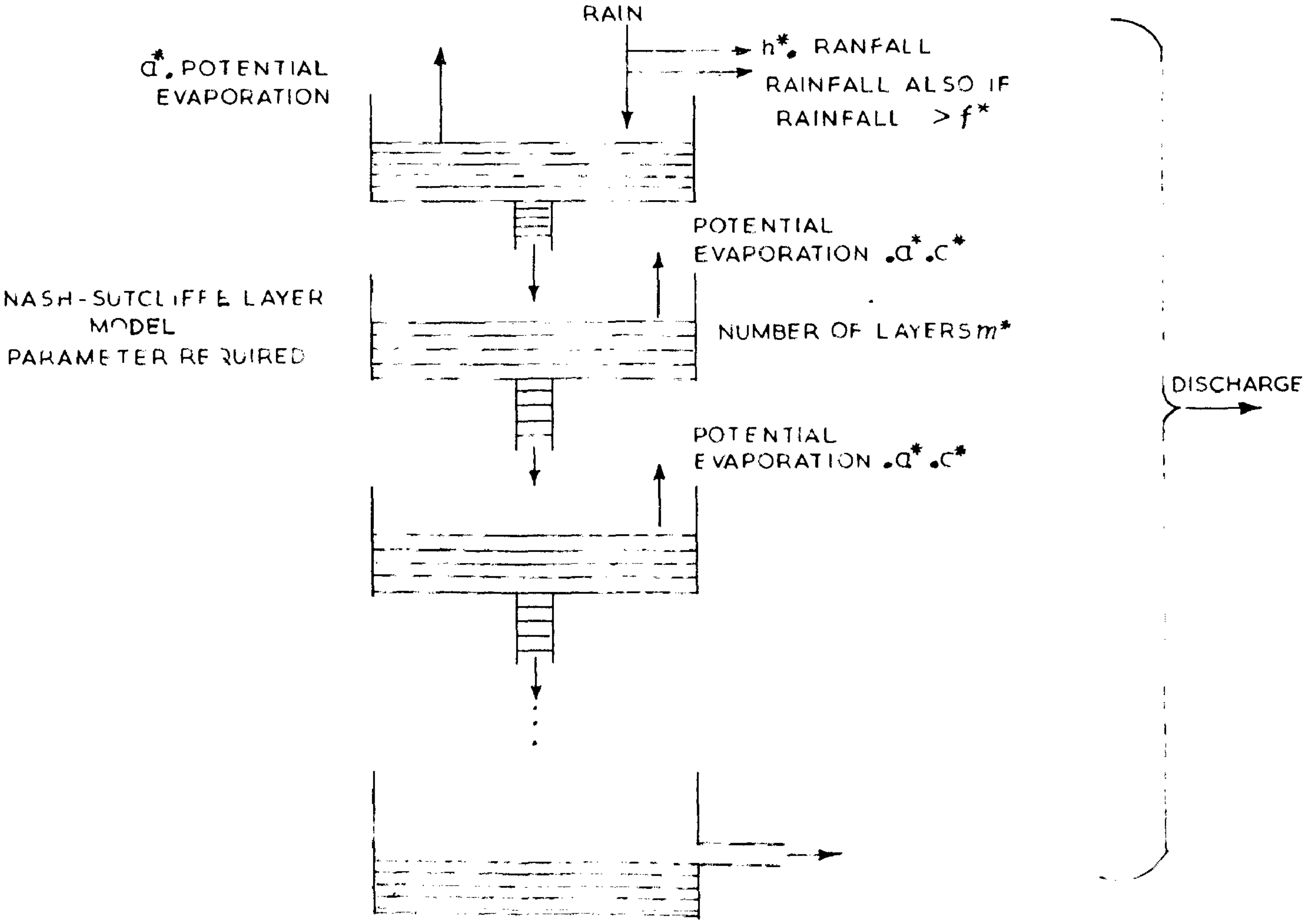


FIG.3.6(a) DAWADY-O DONNEL MODEL (b) NASH SUTCLIFFE LAYER MODEL

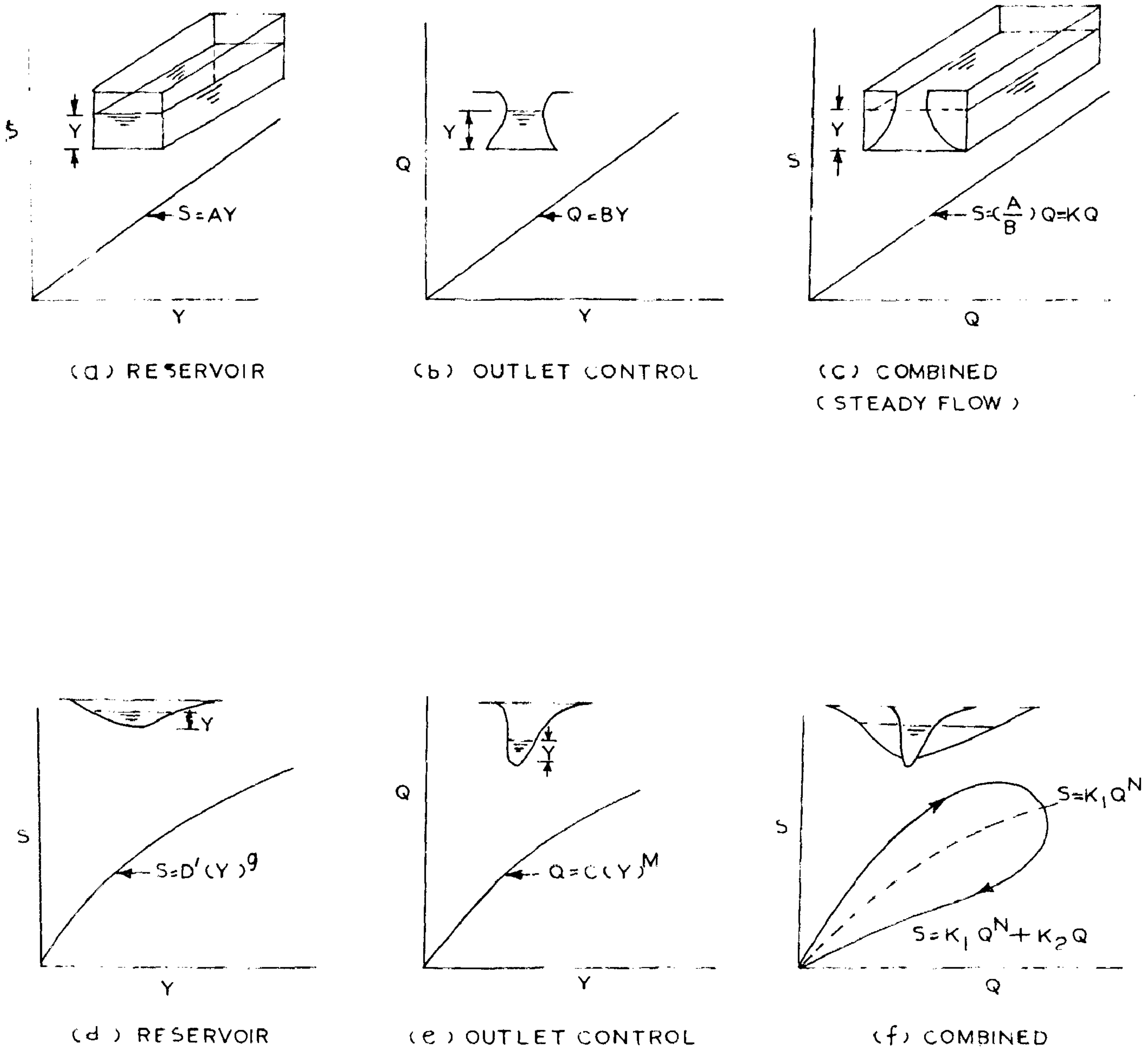


FIG. 3.7 PHYSICAL CONCEPT OF NONLINEAR SYSTEM.

CHAPTER - 4

THE PROPOSED MODEL AND THE DATA

CHAPTER - 4

THE PROPOSED MODEL AND THE DATA

4.1. INTRODUCTION

As a matter of general knowledge, water-shed response is non-linear. Likewise the storage flow relation when characterised as a series of linear reservoirs should theoretically be incorrect, but it gives reasonably accurate simulations. In choosing a model for simulation of water shed hydrograph assumptions are made which can not possibly be varified physically. It is assumed that what is observed as rainfall bears a consistant relation with true rainfall and that the rainfall excess calculation in the model is correct. During the process of modelling on selecting a model for simulation of rainfall runoff process, simplicity and completeness of the model are the two important considerations to be accounted for. A good model should involve minimum assumptions and approximations. These considerations are tried to taken into account to the possible extent in selecting and formulating the model for the present study. The performance of model was tested using data from catchment bridge No.566.

4.2. CATCHMENT BRIDGE NO.566 :

The location of the catchment Bridge No.566, whose hydrologic data have been used in the present investigation is shown in Fig.4.1. The catchment lies in between latitude 20°N / 24° N and longitude 76°E / 80° E. It has got one main river and one major tributary. It is a natural catchment of 53 sq.miles in lower Godawari basin in India. It is situated in Batul-Katol section of Indian railways.

Fig.4.2 shows the details of the catchment area.

4.3. RAINFALL RUNOFF DATA:

The rainfall and runoff data on the catchment are collected by the Ministry of Indian Railways since 1958. Five numbers of non-recording rain gauges were installed at the selected sites within the catchment whose locations are indicated in Fig.4.2. The stage discharge relationship was established for the stream at the outlet. During flood producing storms, the rainfall records were maintained at hourly interval and even half hourly interval during high intensity rainfall. In day timethe flood discharges were measured directly at hourly intervals. The stages were also recorded simultaneously and the computed runoff from stage discharge relationships were supplied along with measured discharges. During the period of poor visibility or no visibility only the river stages were observed and therefore only the computed discharges are available from the stage discharge relationship. The rainfall is measured in inches and the discharges are given in cusecs.

For the purpose of analysis two hydrographs resulting from strom of 16.8.62 and 21.8.61 were selected. The rainfall data for storm No.1 (strom of 16.8.62) were recorded at hourly intervals while for the storm No.2 (storm of 21.8.61) were recorded at half hourly intervals. Both the hydrographs have well defined rising limb culminating in a single peak followed by gradual recession. Both the storms exhibited non uniform areal distribution over the catchment as is evident from recorded rainfall amounts for the rain gauge stations given in Table 4.1. (a)& (b)

These two storms provided a good data record for evaluating the performance of Nash model (16) and the distributed system of lumped-system model with nonuniform areal rainfall distribution, in simulating the direct runoff hydrograph from the watershed.

4.4. METHOD OF ANALYSIS :

The procedure adopted in this study to separate the baseflow from the total runoff to obtain direct runoff hydrograph is based on the assumption that the catchment runoff is mostly resulted from surface runoff and base flow could be taken as nearly at a constant rate. The excess rainfall (effective rainfall) was estimated by using ϕ- index (infiltration index) approach where in the abstractions are assumed to occur at a constant rate so as to give the excess rainfall volume equal to direct runoff volume. The average rainfall for the catchment was computed using Thiessen Polygon technique.

4.4.1. FIRST APPROACH - WHOLE CATCHMENT AS ONE UNIT :

In this approach the whole catchment is considered as one unit for modelling the excess rainfall direct runoff process by applying the Nash model. Nash (16) modelled the catchment as a cascade of n linear reservoirs each with storage coefficient K. The storage effect of the cascade is represented by n^{th} order differential equation. The solution of this equation leads to the expression for instantaneous unit hydrograph u (t) as a two parameter gamma distribution as given below :

$$u(t) = \frac{1}{K\Gamma(n)} \left(\frac{t}{K} \right)^{n-1} e^{-t/k} \qquad (4.1)$$

Where $\Gamma(.)$ is the gamma function.

The values of K and n in Nash's model can be evaluated by the method of moments using the following relationship as suggested by Nash (18)

$$M_1 = nK = M_{DRH_1} - M_{ERH_1} \quad (4.2)$$

$$M_2 = n(n+1)\,K^2 = M_{DRH_2} - M_{ERH_2} - 2nK\,M_{ERH_1} \quad (4.3)$$

Where M_1, M_{ERH_1}, M_{DRH_1} are the first moments and M_2, M_{ERH_2} and M_{DRH_2} are the second moments of IUH, excess rainfall hyetograph and direct runoff hydrograph respectively about the time origin.

Using the values of n and K derived from excess rainfall direct runoff data from a storm, the direct runoff hydrograph can be reconstructed in order to evaluate the performance of the model for the storm whose data has been used to derive these constants. This can be achieved by driving unit hydrograph of any specified duration using incomplete gamma function tables. This approach has been explained by Nash (18) in his study on British catchments. The unit hydrograph thus derived can also be used with the excess rainfall data of other storms to evaluate the performance of the model in reconstructing direct runoff hydrograph.

4.4.2. SECOND APPROACH - CATCHMENT DIVIDED INTO SUB AREAS:

In the present study, the main emphasis is given in evaluating the performance of distributed system of lumped system models in simulating excess rainfall direct runoff process of a catchment with non-uniform areal rainfall distribution. Hence in the second approach the catchment was divided into subareas which were defined by water shed boundaries of tributary drains. Each subarea (tributary water shed) is then simulated by means of cascade of n linear reservoirs of equal storage coefficient K as suggested by Nash. The catchment area under study was divided into three subareas. The subareas A_1 and A_2 correspond to the two tributary streams and the subarea A_3 corresponds to the main stem of the stream from the confluence of tributaries to the outlet. The inflow to each of the three cascades of linear reservoirs consists of average excess rainfall over the subarea represented by the particular cascade. The outflow from two cascades representing subareas A_1 and A_2 i.e. Q_1 and Q_2 are then combined together and let through a linear channel to the outlet where it combines with the outflow Q_3 from the third cascade representing portion of the main stream draining subarea A_3. The linear channel simulates the time lag for the outflow of the two tributary streams from their confluence to the outlet while passing through subarea A_3. This time lag is due to channel flow and hence it is different from that experienced by input excess rainfall over subarea A_3. The structure of this model has been shown schematically in Fig.4.3.

4.4.2.1. Relationship of n and K with catchment characteristics:

In order to evaluate parameters representing number of reservoirs and storage coefficients for the three subareas A_1, A_2 and A_3, the relationships proposed by Nash (18) relating n and K with catchment characteristics were adopted. For the British catchments these relationships were as follows :

$$nK = C_1 \; A^{0.3} \, . OLS^{-0.3} \qquad (4.4)$$

$$\frac{1}{n} = C_2 \; L^{-0.1} \qquad (4.5)$$

Where A is the area of catchment in square miles, OLS is the overland slope in parts per ten thousand, L is the length of the main channel from catchment outlet to the extreme boundary, and C_1 and C_2 are constants. For British catchments as proposed by Nash (18)

$$C_1 = 27.6 \text{ and } C_2 = 0.41$$

These above relationships eq.4.4 and 4.5 were adopted for the catchment under study and the constants C_1 and C_2 were evaluated using n and K values obtained in the first approach and the catchment characteristics area A, length L and overland slope OLS for the whole catchment. The modified relationships were then used to derive values of n_1, K_1 for subarea A_1; n_2, K_2 for subarea A_2 and n_3, K_3 for subarea A_3 from their catchment characteristics. The time constant of linear channel was evaluated by trial and error so as to give a good reconstruction of observed ddirect runoff hydrograph, but it was not allowed to exceed in value the product n_3K_3 which is the lag time for excess rainfall input to subarea A_3.

4.5. EVALUATION CRITERIA FOR PERFORMANCE :

For evaluating the results of these approaches sum of squares criteria F^2 was computed which is the sum of the squares of the deviations between the observed and the reconstructed (calculated) direct runoff hydrograph ordinates.

$$F^2 = \sum_{I=1}^{M} \left[Q_o(I) - Q_c(I) \right]^2 \qquad (4.6)$$

Where $Q_o(I)$ and $Q_c(I)$ are the I^{th} ordinates of observed and calculated direct runoff hydrograph respectively and M is the total number of ordinates. In order to compare performance of the approaches using data of different storms model efficiency R^2 suggested by Nash and Sutcliffe (19) was used.

$$R^2 = \frac{F_o^2 - F^2}{F_o^2} \qquad (4.7)$$

Where F_o^2 represents the initial variance of the observed data.

$$F_o^2 = \sum_{I=1}^{M} \left[Q_o(I) - \bar{Q}_o\right]^2 \qquad (4.8)$$

Where $\bar{Q}_o$ represents mean of the observed direct runoff values.

4.6. SUBAREA TYPE MODEL WITH ASSUMED DATA:

The time distribution of rainfall excess plays an important role in the formation of direct runoff hydrograph. In order to study the sensitivity of the subarea type model to different types of time distribution of rainfall excess a study was made using this model with assumed values of model parameters and typical rainfall excess distribution.

The division of the catchment area in subareas along the internal water shed boundaries of tributary drains was assumed to give subarea A_1, A_2 and A_3. The subarea A_1 was represented by n_1 and K_1, subarea A_2 by n_2 and k_2 and subarea A_3 by n_3 and K_3 where n_1, n_2 and n_3 are number of linear reservoirs and k_1, k_2, k_3 are storage coefficients, for the three subareas A_1, A_2, A_3 respectively. The translation coefficient of linear channel for the subareas A_1 and A_2 was assumed equal to product $n_3\ k_3$.

The input rainfall excess was considered to be equal to 1 inch. The duration of rainfall excess was taken as 8 hours for the sake of convenience. First the rainfall excess was assumed to be uniformly distributed for the duration of 8 hours over all the three subareas and the direct runoff hydrograph was evaluated applying the subarea model. Then the rainfall distribution was made nonuniform in time. Keeping the duration same such that the total excess rainfall, Rev, remained constant i.e, 1 inch. The types of rainfall distribution included those skew to the right, those skew to the left and symmetrical distribution (Fig.4.4). Some statistical rainfall distribution such as Normal, Binomial and Poissons distribution were also tried (Fig.4.5).

From the hyetrograph volume of rainfall excess, Rev, duration of rainfall excess, te, and time to centre of area of rainfall excess, tr_1, i.e. first moment ($MERH_1$) and second moment of rainfall excess, t_{r2} - ($MERH_2$), were determined for each type of excess rainfall distribution. These parameters were combined to form parameters η and η^*, where $\eta = Rev \dfrac{t_{r1}}{t_e}$ and $\eta^* = Rev \dfrac{t_{r2}}{(t_e)^2}$. Depending upon the particular cases these parameters η and η^* will give a measure of the time distribution of rainfall excess. For the different cases the magnitude of peak of hydrograph and time to peak were studied to evaluate sensitivity of the model to changes in time distribution of rainfall excess. Kulandaiswamy has used the parameter η to represent time distribution of excess rainfall in his studies of rainfall runoff process.

4.7. The performance of a lumped system model such as Nash model and the performance of subarea type model are studied using the performance criteria outlined in this Chapter. The data of two representative storms on Bridge No.566 catchment provided a good data sample having non-uniform areal distribution of rainfall. The results of this study are described and discussed in Chapter-5.

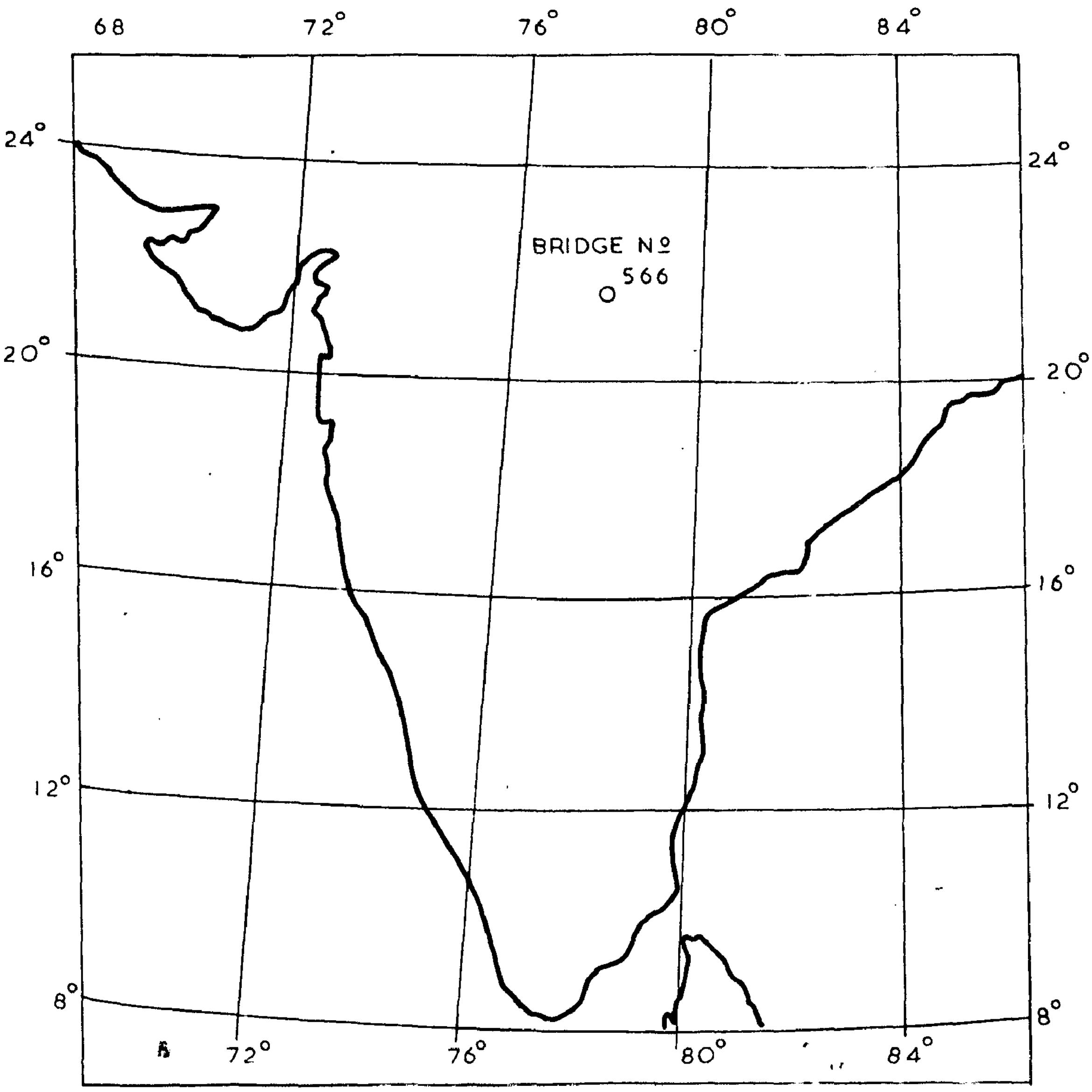


FIG.4·1 LOCATION MAP OF CATCHMENT FOR BRIDGE No 566

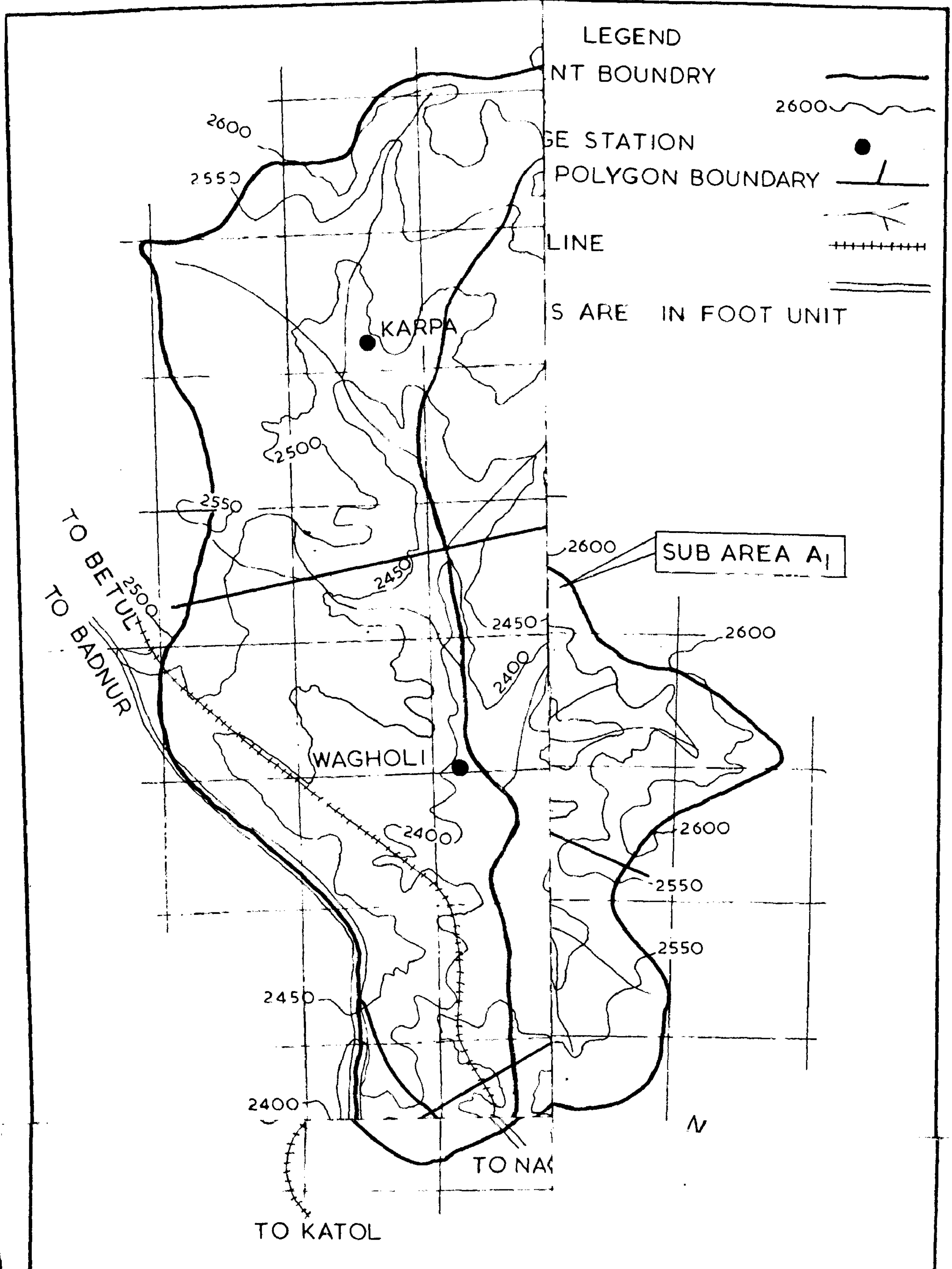


FIG.4·2 CATCHMENT DIVIDED ... IDS.

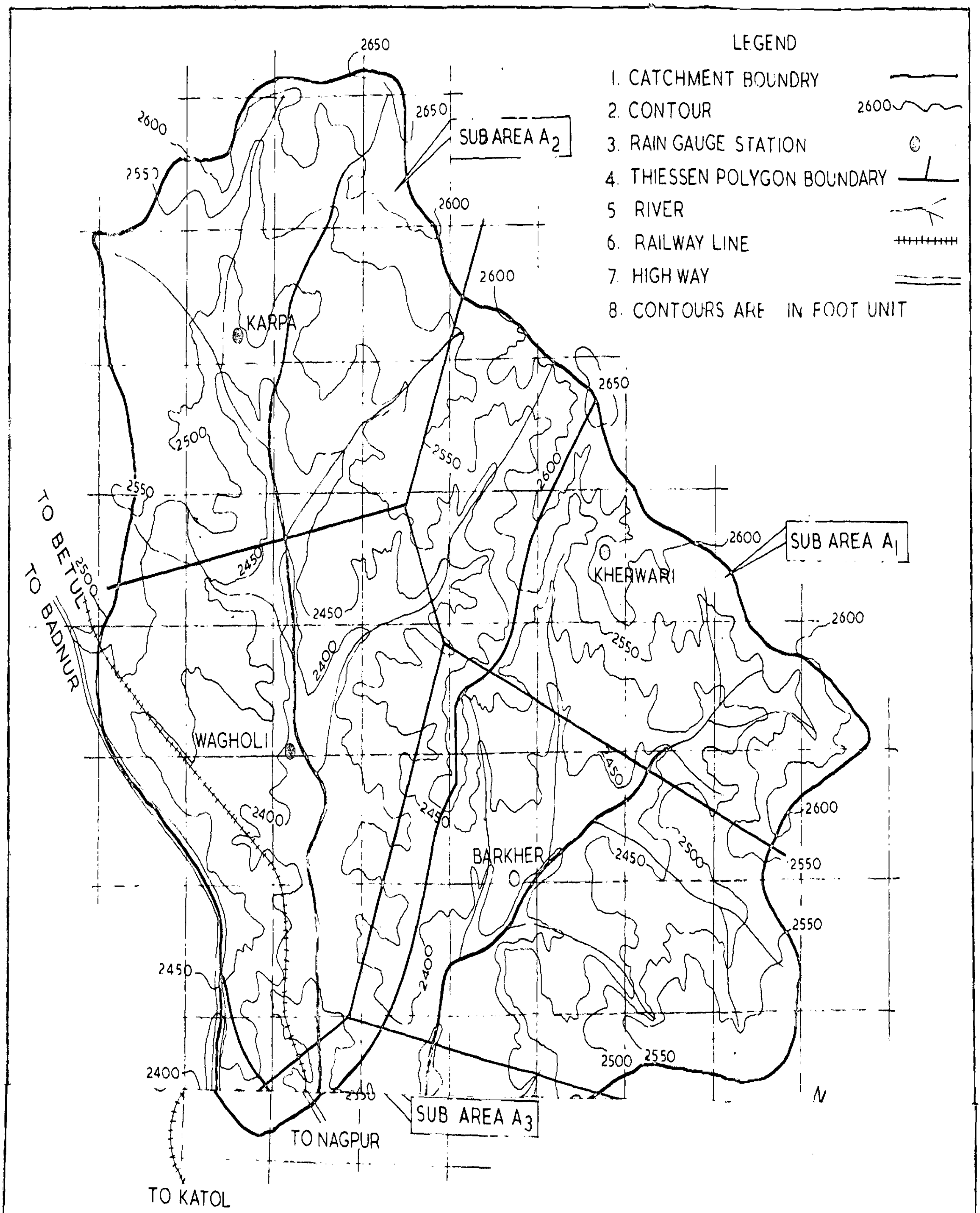
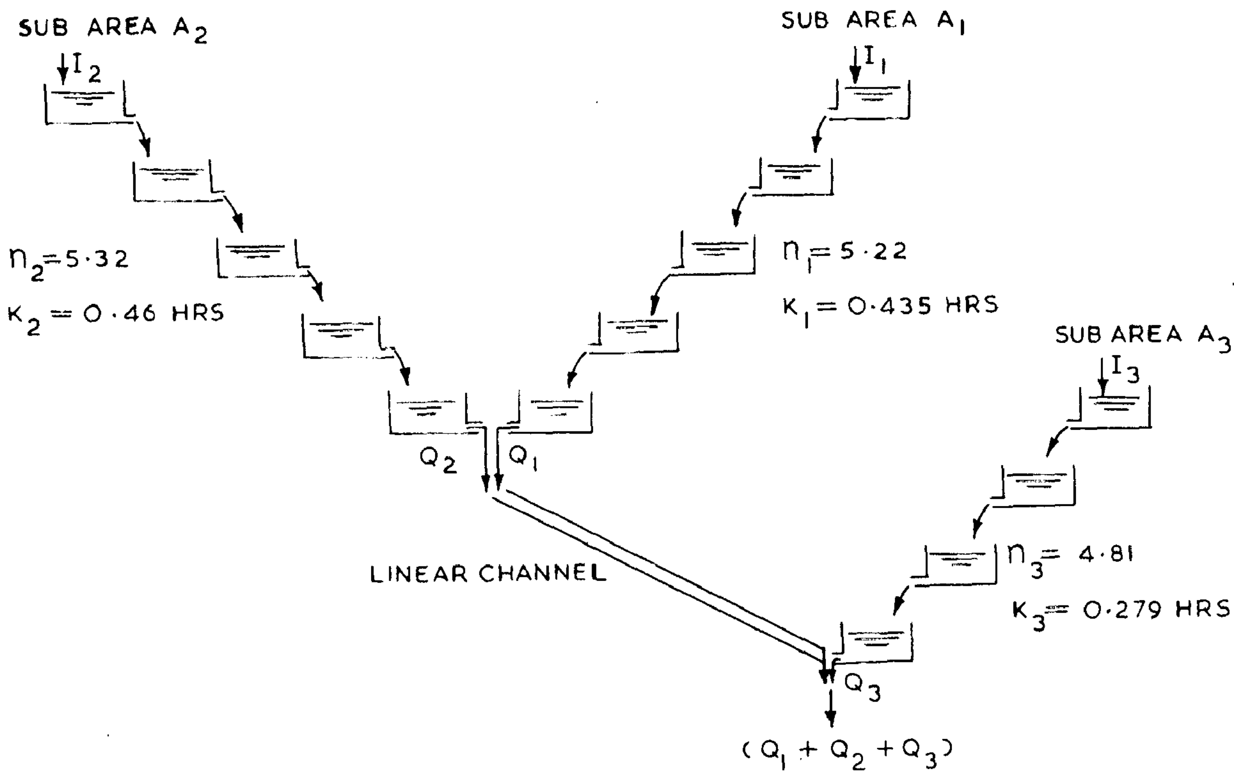
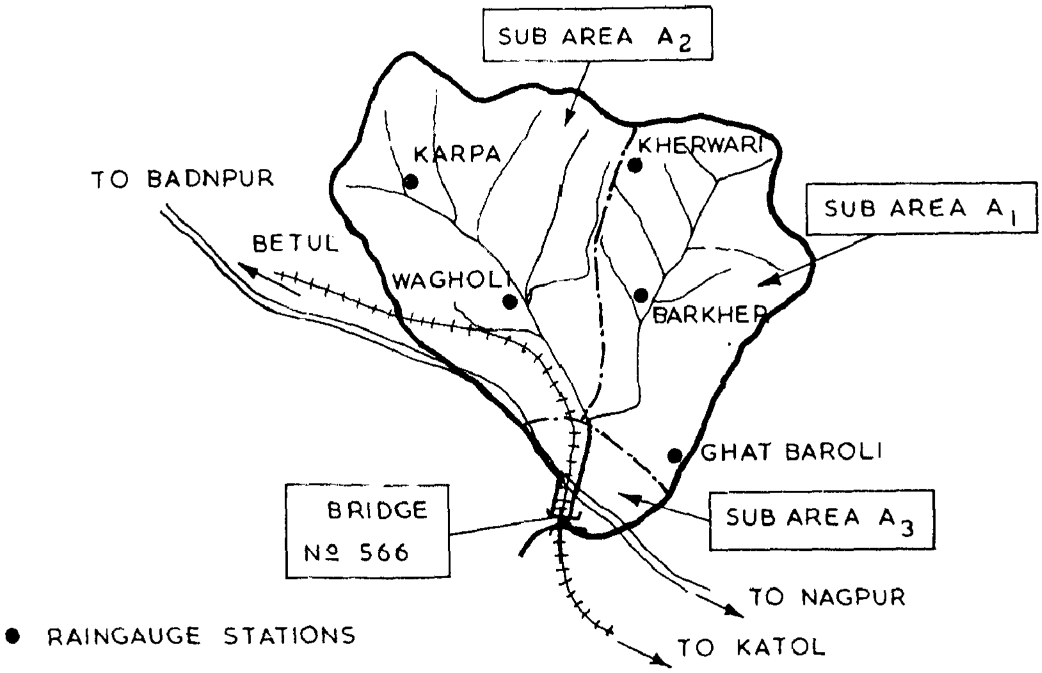


FIG.4·2 CATCHMENT DIVIDED INTO THIESSEN POLYGON, SUBAREAS AND GRIDS.



MODEL DIAGRAM



CATCHMENT

FIG. 4·3 SCHEMATIC DIAGRAM OF MODEL

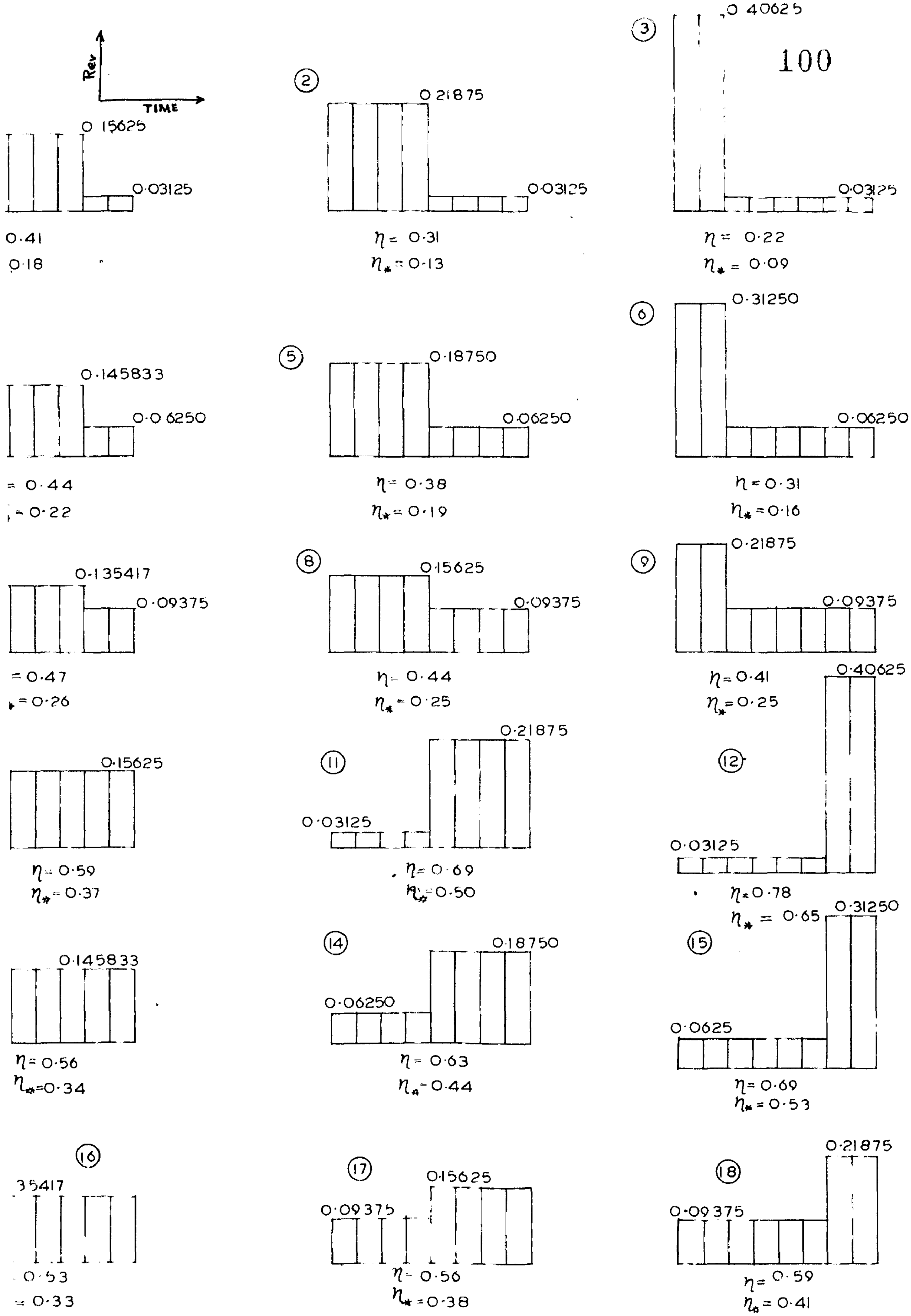


FIG 4·4(a) ASSUMED TYPICAL RAINFALL DISTRIBUTION

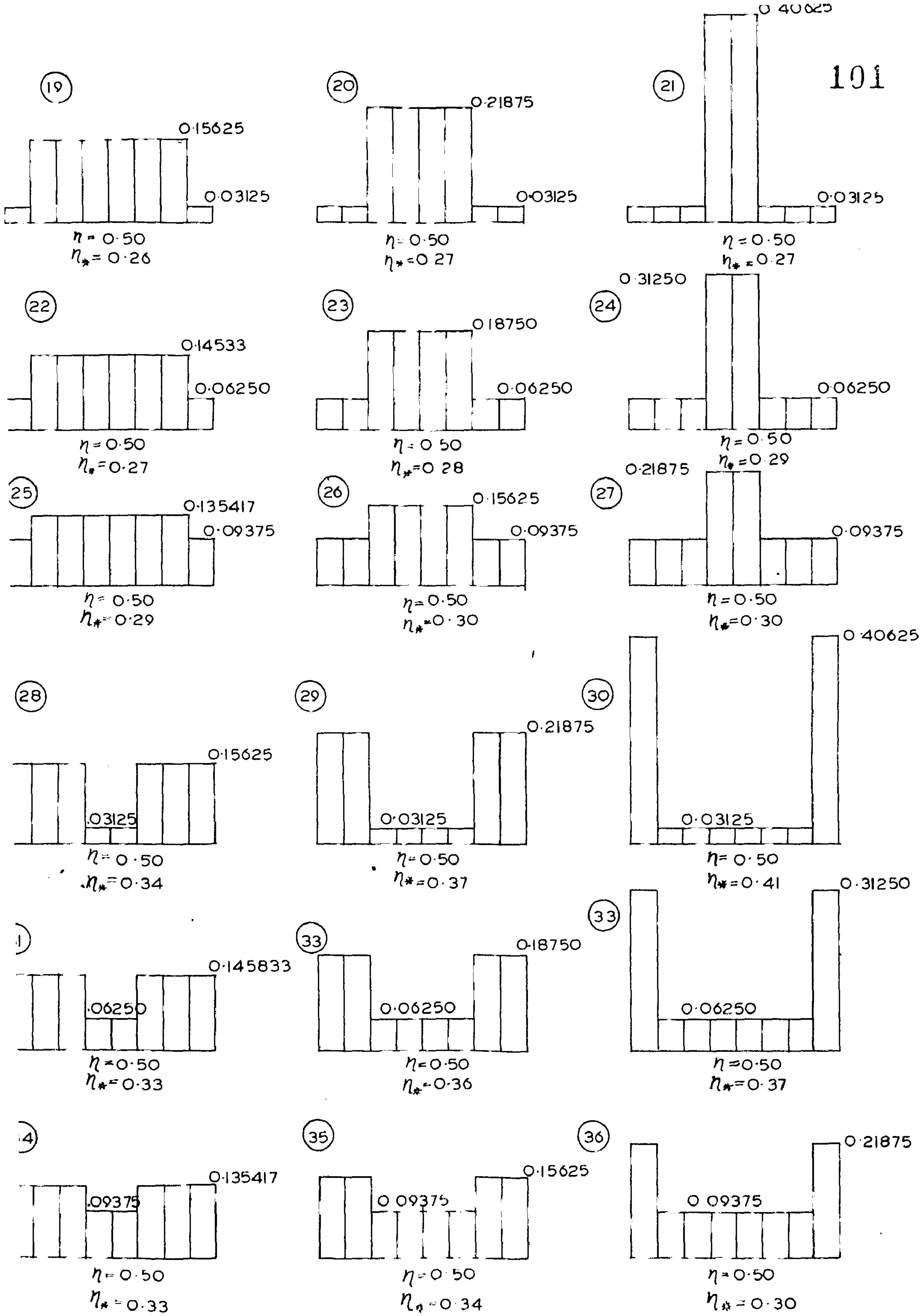


FIG. 4·4 (b) ASSUMED TYPICAL RAINFALL DISTRIBUTION

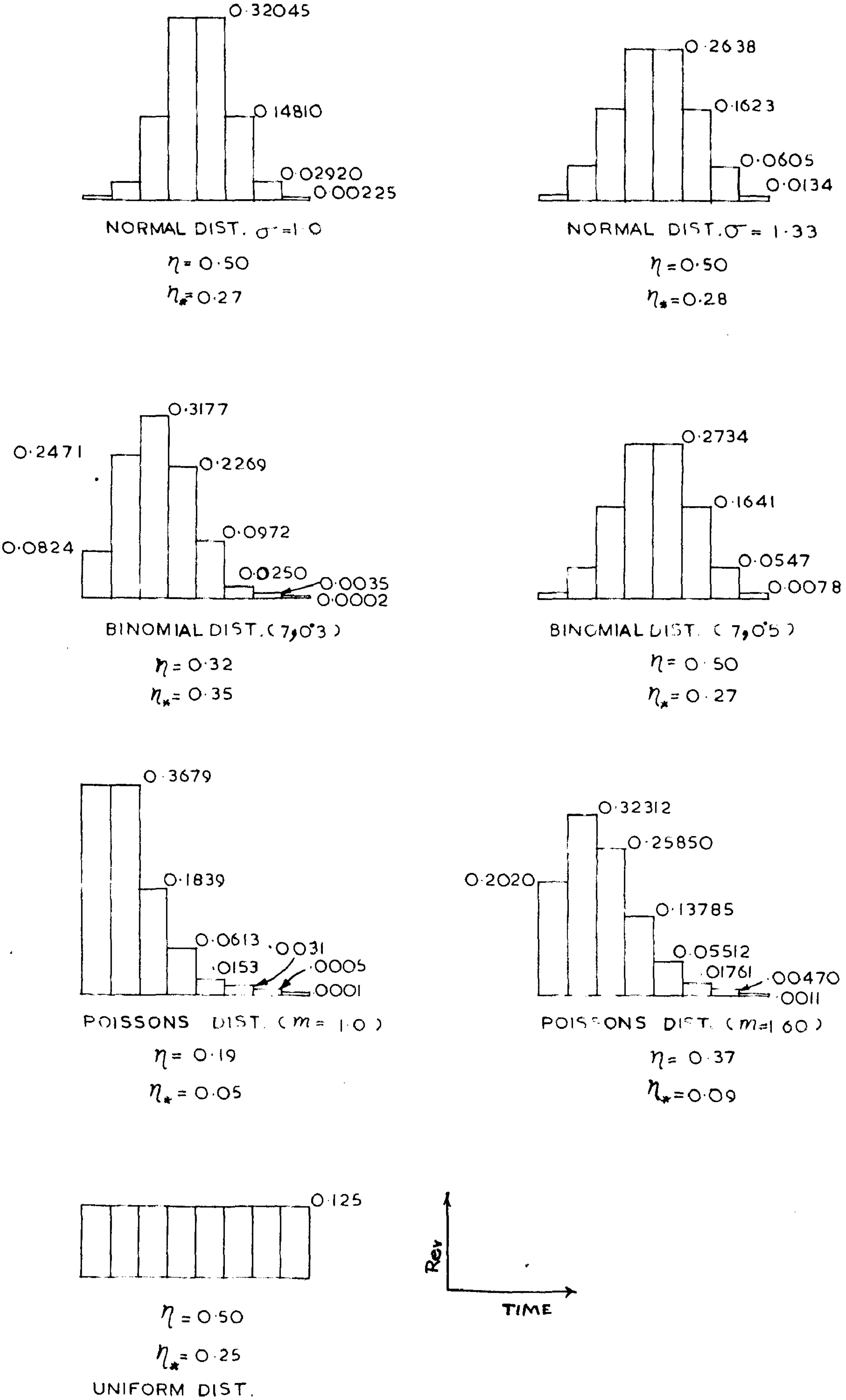


FIG. 4·5 STATISTICAL RAINFALL DISTRIBUTION

CHAPTER - 5

MODEL EVALUATION AND DISCUSSION OF RESULTS

CHAPTER - 5

MODEL EVALUATION AND DISCUSSION OF RESULTS

5.1. INTRODUCTION:

The data of catchment Bridge No.566 was used to evaluate relative performance of conceptual models based on two approaches. In the first approach the whole catchment was considered as one unit and the Nash model (16) was used. In the second approach the catchment was divided into three subareas on the basis of internal water shed boundaries and the subarea type model (or distributed system of lumped system model - Chow (6)) was used. The average rainfall for the catchment was computed using Thiessen polygon technique. The thiessen weights of the different rain gauge stations are given in Table 5.1. The time distribution of average excess rainfall intensity and average total rainfall intensity over the catchment as a whole as well as over the subareas considered is given in Fig.5.1.

5.2. EVALUATION OF TWO APPROACHES:

An analysis was made of the rainfall data of storm No.1 of date 16.8.62. For this purpose the centroid of the whole catchment, each subareas and the thiessen polygons was found out. For finding out the centroid, the boundary of the whole catchment area, the subareas and the thiessen polygons were marked in a piece of uniform hard board. First the board was act along the boundary of the whole catchment and the centroid of the piece was found out by hanging the piece by a thread at three different points alternately. The point of intersection of the three lines is the centroid of the piece. Then the board was cut along the boundary of the three subareas and the centroid of each subarea was found out as above. Similarly the centroid of the each thiessen polygon was found out.

Using this centroids of area of thiessen polygon and hourly values of total rainfall for storm 1 of 16.8.62, the centroids of hourly rainfall volume were computed. Similarly considering the centroids of area of each of three subareas and average hourly rainfall over them, the centroids of hourly rainfall volume were computed. The distances of these centroids of hourly rainfall volumes from the catchment outlet for the two cases i.e. thiessen polygon and subareas are given in Table 5.2 along with the distance of centroid of the whole catchment from the outlet. These distances were evaluated in order to compare the performance of Nash model which considers lumped rainfall input and the subarea type model. The tabulated values clearly show that the centroids of hourly rainfall values for subareas are nearly similar to those given by thiessen polygons. Where as a lumped model of Nash type takes all the hourly rainfall volumes as concentrated at the centroid of the catchment area. This analysis suggests a better performance of subarea type model in dealing with non-uniform areal distribution of rainfall as compared to lumped model. This will be examined in thefollowing analysis.

The detail analysis and evaluation of the two approaches are described in the following sub-sections.

5.2.1. Whole catchment as one unit : First approach :

For the whole catchment considered as one unit represented by n identical linear reservoirs with equal storage coefficient K, in series the following cases were analysed.

5.2.1.1. Case 1 A :

In this case storm No.1 of 16.8.62 andthe resulting runoff hydrograph were taken for analysis. Using a constant value of infiltration index ϕ = 0.11975 inch/hour, the values of n and K were calculated by the method of moments which were found to be n = 5.5 and K = 0.54 hours. With this value of n and K one hour unit hydrograph is calculated by using Incomplete gamma function table 2. The unit hydrograph is given in Table 5.3. Then with the help of this unit hydrograph and excess rainfall volume the direct runoff hydrograph is reconstructed. The reconstructed (calculated) and the observed direct runoff hydrograph compared reasonably well giving efficiency R^2 = 83.7 percent (Fig.5.2) and Table 5.4. However, the peak of calculated direct runoff hydrograph could not match with that for observed hydrograph which was higher. This seems to be mainly due to non uniform rainfall distribution over the catchment being considered as a single lumped input instead of distributed one and also due to assumption of constant value of infiltration index ϕ throughout the storm and every where in the catchment i.e. assumption of constant value of ϕ in time and space.

5.2.1.2. Case 1 B :

So, in order to get an idea of the effect of change of infiltration index ϕ with time for this lumped cascade type model, the value of ϕ was changed arbitrarily for each hour of the four hour excess rainfall duration for storm No.1 by a factor P = 0.90, in a manner so as to obtain same value of abstractions.

$$\phi_2 = P\phi_1$$
$$\phi_3 = P^2\phi_1$$
$$\phi_4 = P^3\phi_1$$

and $\phi_1 + \phi_2 + \phi_3 + \phi_4 = 4\phi$

Where $\phi_1, \phi_2, \phi_3, \phi_4$ are the values of infiltration index in inch/hour for the first, second, third, and fourth hour of the excess rainfall respectively.

The change of ϕ with time changed the moments of excess rainfall hyetograph ($MERH_1$ and $MERH_2'$) only slightly and hence the n and K were taken same as in case 1 A, for the reconstruction of direct runoff hydrograph. The model efficiency R^2 obtained for this case was 83.6 percent and the peak of the calculated direct runoff hydrograph was somewhat below that for case 1A. (Fig.5.3 and Table 5.4)

5.2.1.3. Case 2 A :

For this case the storm No.2 which occurred on 21.8.61 and recorded at half hourly interval was considered. For the reconstruction of direct runoff hydrograph for this storm half hour unit hydrograph was calculated considering the n and K values derived from storm 1 and a constant infiltration index ϕ = 0.18434 inch/hour (Table 5.5). The model efficiency R^2 was obtained as 82.3 percent through the calculated peak remained below the observed peak and also the time to peak of calculated hydrograph was half an hour earlier than that for the observed hydrograph (Fig.5.4 and Table 5.6).

5.2.2. Catchment divided into sub-areas: (second approach)

For the second approach the catchment was divided into subareas A_1, A_2 and A_3 and the subarea type model (Fig.4.3) was used for reconstructing the direct runoff hydrograph. The catchment characteristics viz. Area A, overland slope OLS and the length L for the whole catchment as well as for the three subareas were obtained by measurement from the map of the catchment plotted to scale of 1 inch = 1 mile. For obtaining overland slope OLS, the method suggested by Nash (16) on the study of British catchments was adopted as given below.

A grid of rectangular mesh of 1 sq. inch was drawn on 1 inch to a mile map of the catchment boundary Fig.4.2. At each intersection point the minimum distance in feet between adjacent 50 ft. contours was measured and the slope at each point taken as 50 ft. in this distance. This provided a set of slope values of which the mean was calculated and taken as overland slope OLS. when an intersection occurred at a point between two contours of the same value the slope was taken as zero if the point was in a valley and as indeterminate if on a hill. The later was neglected in calculating the mean.

The values of the parameters representing the catchment characteristics for the whole catchment and for the subareas are given in Table 5.7. Using the values of A, L, and OLS for the whole catchment together with the values of parameters n and K derived for the whole catchment, the relationship for first and the second moments were modified by computing the constants C_1 and C_2 in Eq.4.1 and Eq.4.2

The modified equations which are used for the catchment under study are given below

$$nK = 4.58 \quad A^{0.3} \quad OLS^{-0.3} \qquad (5.1)$$

$$\frac{1}{n} = 0.233 \quad L^{-0.1} \qquad (5.2)$$

Where A is the area in sq.miles, OLS is the overland slope in parts per ten thousand and L is the length of stream from catchment (sub area) outlet to the extreme boundary in miles.

Using these relationships (eq.5.1 and eq.5.2) and the catchment characteristics for the three subareas the values of parameters n_1 and k_1 for subarea A_1, n_2 and K_2 for subarea A_2, and n_3 and K_3 for subarea A_3 were computed as given in Table 5.7. The following cases were studied using the sub-area type model.

5.2.2.1. <u>Case 1 C</u> :

In this case the value of infiltration index ϕ =0.1278 inch/hour was assumed for all subareas throughout the storm 1 of 16.8.62. Unit hydrograph for the three subareas were calculated using the parameters n_1, and K_1 for subarea A_1, n_2 and K_2 for subarea A_2 and n_3 and K_3 for subarea A_3 with the help of incomplete gamma function Tables (2) and are given in Table 5.5. Then with the help of these unit hydrographs and the excess rainfall from each subareas the direct runoff hydrographs were calculated. Finally, assuming the translation coefficient of linear channel as zero hours and applying the subarea type moddl the direct runoff hydrograph for the whole catchment was reconstructed. The model efficiency R^2 was obtained as 72.2 percent, Fig.5.5

5.2.2.2. Case 1 D :

Keeping all other conditions same as in case 1 C, except that the translation coefficient of linear channel was taken as one hour, the model efficiency was obtained as 82.3 percent. Thus the translation time through linear channel improves model efficiency direct runoff hydrographs do not occur at the same time. The magnitude of peak of calculated hydrograph is higher for subarea type model than in the case of first approach (Fig.5.2).

5.2.2.3. Case 2 B :

Subarea type model was then used with storm No.2 which occurred on 21.8.61 taking constant value of infiltration index ϕ =0.19070 inch/hour for all subareas throughout the storm. The translation coefficient of linear channel was taken as one hour. The half hour unit hydrograph(Table 5.5) for the three subareas are constructed using the parameters n_1, k_1, n_2, k_2 & n_3, k_3 direct runoff hydrograph for whole catchment was reconstructed as in the case 1 C. Table 5.6 The model efficiency R^2 was obtained as 90 per cent for this storm which was not used in deriving any parameters in the model. The peaks of calculated and observed hydrograph occurred at the same time and the calculated peak was higher than that for case 2 A using the whole catchment as one unit. Fig.5.4. However, calculated and observed peak magnitudes still remains different.

5.2.2.4. Case 1 E :

The effect of spatial variation of infiltration index on this subarea type model is examined in this case. Hence the values of ϕ were taken different for all the three subareas in an arbitrary manner. For this one assumption that rainfall excess volume from subarea A_1 was nearly equal to that of subarea A_2 was made. As a result the value of ϕ changed to ϕ_1 = 0.078995inch/hour for subarea A_1, ϕ_2 = 0.1631125 inch/hour for subarea A_2 and ϕ_3 =0.1278 inch/hour for subarea A_3. The model efficiency R^2 in this case was 84 percent keeping all other conditions same as in case I D. The spatial variation of ϕ has improved the model efficiency as well as the peak of the calculated direct runoff hydrograph than that found in case 1 A, Fig.5.5.

5.2.2.5. Case I F:

The effect of variation of ϕ with time on the model efficiency, adopting a similar relationship to that for case 1 B using p = 0.90, was examined in the case of subarea type model. All other conditions were kept same as those for case 1 D. The model efficiency R was obtained as 80 percent, Fig.5.2.

5.3. DISCUSSION OF RESULTS (FOR REAL CATCHMENT DATA) :

The results of different cases studied using the whole catchment as one unit and subarea type model are given in Table 5.8. The results clearly show that performance of Nash model is quite promising even when the rainfall data has non-uniform areal rainfall distribution. The subarea type model proposed in this study gave satisfactory results even when the relationships (eq.4.1 and 4.2) derived for British catchments were used to evaluate the parameters of the model. This is because of the fact that it has indirectly taken into account the effect of distributed system model. The subarea type model is better suited for accounting areal variations of rainfall also. The better performance of subarea type model is comparison to Nash model in dealing with non uniform areal distribution of rainfall supports the results expected after evaluation of distances of centroids of hourly rainfall volumes from catchment outlet described in section 5.2.

Sometimes the point rainfall recorded at any rain gauge may not be true representative value of the distributed input rainfall over its thiessen area. Particularly when area assigned to a rain gauge is very large, the possibilities of rainfall not occurring over the entire area are more. Therefore, the more the number of rain gauges, smaller would be the area assigned to them and correspondingly, better would be the representation of unevenness of nonuniformly distributed storms.

The results of this study also compare favourably with those of a study by Mathur (15) using a pure translation approach. For the data of the catchment understudy, the model efficiency, R^2 obtained by him ranges from 76 percent to 90 percent for three storms studied. However, the peak magnitude and timings of calculated and observed hydrographs were matching better than in the present study. In the study by Mathur (15) catchment was divided by means of mean delay time contours based on ordinates of one hour unit hydrograph derived from a storm assuming uniform areal distribution of rainfall.

5.4. PERFORMANCE OF SUBAREA TYPE MODEL WITH ASSUMED RAINFALL DATA:

The subarea type model has thus been shown to be better suited for accounting for areal variation of rainfall on the basis of analysis using real catchment data. Since the formation of direct runoff hydrograph is also dependent on time distribution of rainfall excess, a study was made to find the sensitivity of the subarea type model to different types of time distribution of rainfall excess with assumed values of parameters and typical rainfall excess distribution.

The analysis procedure was as follows :

First the catchment area was divided into subareas A_1, A_2 and A_3 such that 40 percent of A_1, 40 percent of A_2 and 20 percent of A_3 gave total catchment area. Also the division was done on the basis of water shed boundaries of tributary drains. Then $n_1 = 2$ and $k_1 = 4$ hours for subarea A_1, $n_2 = 2$ and $k_2 = 4$ hours for subarea A_2 and $n_3 = 3$ and $k_3 = 4$ hours for subarea A_3 was assumed arbitrarily. The translation coefficient of linear channel for subarea A_1 and A_2 was taken as 12 which is equal to the product of $n_3 K_3$. Though it is different from the time lag experienced by input rainfall over subarea A_3. The input volume of rainfall excess was taken as 1 inch uniformly distributed over the three subareas for a duration of eight hours. Subsequently this uniform distribution of rainfall excess was changed to non-uniform in time. Keeping the volume and duration of rainfall excess same as before. Some statistical distribution such as Normal (2), Binomial (2) and Poissons distribution (2) were also tried. The types of rainfall distribution was shown in Fig.4.4(a), (b) and Fig.4.5. A computer programme was prepared to evaluate the direct runoff hydrograph applying this subarea type model with this different types of nonuniform rainfall distribution. The parameter η and η^* were calculated for each type of rainfall excess distribution.

5.4.1. Discussion of Results :

The results obtained with various types of rainfall excess distribution are given in Table 5.9 and Table 5.10. The results show clearly the sensitivity of the subarea type model to changes in time distribution of rainfall excess. The effect of variations is indicated in magnitude of the peak of the hydrograph and to a lesser extent in time to peak for the particular combination of assumed catchment characteristics.

This study with assumed data also shows the desirability of evaluating first and second moments of excess rainfall distribution in order to distinguish between the distributions. Both these moments are also considered in evaluation of parameters n and K for Nash' s model and hence this derivation of n and K effectively takes into account the time variations of rainfall. This is quite in contrast to approaches based on unit hydrograph theory where in the excess rainfall is assumed to occur at an uniform rate though actually it may be somewhat nonuniform.

5.5. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH:

Although the rainfall runoff process is non linear, it can be approximated by a linear model such as Nash's. This model can be justified by the fact that its shape is similar to a hydrograph and that there are many storage processes within a water shed. For example, there is detention storage, overland flow storage, channel storage ground water storage and others. For simplification, Nash assumed that all of the storage processes took place at the outlet of the water shed through n linear reservoirs placed in series. Nash ignored the variation in translation time over catchment, since he assumed that all points have the same translation time.

On the other hand the representation of the rainfall runoff process in standard unit hydrograph theory is an undesirable simplification to make when modelling catchments of any great size. The development of a model which takes realistic accuracy of time distribution of rainfall and catchment characteristics continues to be an important objective. The subarea type model developed in this study has indirectly accounted for the distributed nature of catchment.

This limited study has shown that in dealing with rainfall data with non-uniform areal distribution, the performance of Nash model is quite promising. This can be further improved by considering a subarea type model in which the catchment is divided into subareas on the basis of tributary drainage boundaries and each subarea be represented by a cascade of linear reservoirs. Thus the simulation of the entire catchment becomes a distributed one. The more the division of subareas the better will be the performance of the model. The division of subareas should be such that each subarea contains only one well defined channel. This type of division of area for simulation of rainfall runoff process from a water shed has got more logic than the division of subarea by means of isochrones or mean delay time contours. The more versatile arrangements of subareas may also be of value in modelling large catchment. Combination of subareas possible when the subareas are defined in this way are more likely to conform closely with regions of comparative hydrologic homogenity. In this respect it is worth mentioned that the correlation between the parameters of the model and the measurements of the physical characteristics of the catchment would be valuable assistance in the synthesis of model parameters for ungauged catchments.

The accuracy of prediction by the model increases with the intensity of raingauges in the catchment. It has been reasoned that higher the intensity of raingauges, greater would be the accuracy with which the spatial distribution of rainfall can be defined and accounted for by the model.

However, further investigation is necessary on these lines considering data from different catchments and suitable regional relationships for parameter n and K. Also optimization study using computers should be undertaken to evaluate performance of this approach.

TABLE 4.1.(a)

OBSERVED RAINFALL DATA WITH WEIGHTED AVERAGE RAINFALL

Storm No.1 Dated: 16.8.62

Time in hours	Observed rainfall in inches at station					Total weighted average rainfall in inches			
	Ghatbaroli	Wagholi	Barkher	Kherwari	Karpa	A	A_1	A_2	A_3
12 00	-	-	-	0.10	-	0.0196	0.0338	0.013	0
13 00	0.20	-	-	0.40	-	0.1042	0.1636	0.055	0.1856
14 00	0.15	0.40	0.50	0.25	0.42	0.3654	0.3658	0.38965	0.1680
15 00	0.15	0.55	1.02	0.36	0.23	0.49326	0.67338	0.40605	0.1788
16 00	0.15	1.04	0.37	0.25	-	0.3803	0.2982	0.4570	0.21408
17 00	0.45	0.60	-	-	-	0.19005	0.0639	0.24075	0.4608
18 00	0.15	-	-	-	-	0.0193	0.0213	0.00225	0.1392

OBSERVED RAINFALL DATA WITH WEIGHTED AVERAGE RAINFALL

Storm No.2 : Date: 21.8.61

Time in hours	Observed rainfall in inches at station					Total weighted average rainfall in inches			
	Ghatbaroli	Wagholi	Barkher	Kherwari	Karpa	A	A_1	A_2	A_3
2 30	-	-	-	0.35	-	0.0686	0.1183	0.0455	0
3 00	-	0.12	-	0.30	-	0.0852	0.1014	0.0858	0.00864
3.30	-	0.27	-	0.25	0.20	0.1544	0.0845	0.2218	0.01944
4 00	-	0.38	0.70	0.20	0.44	0.3815	0.4316	0.3905	0.02736
4 30	0.60	0.13	-	0.40	0.45	0.2879	0.2204	0.3007	0.56616
5 00	0.40	0.28	0.53	0.30	0.30	0.36025	0.4338	0.30405	0.39136
5 30	0.50	0.33	0.47	0.15	0.25	0.32975	0.3661	0.28185	0.48776
6 00	0.30	-	0.15	0.15	0.20	0.14785	0.1713	0.11475	0.27840
6 30	0.20	0.26	0.25	0.10	0.16	0.19565	0.1922	0.19585	0.20432
7 00	0.25	0.24	0.20	-	-	0.13005	0.1395	0.10635	0.24928
7 30	-	0.10	-	-	-	0.0220	-	0.03900	0.00720
8 00	-	0.14	-	-	-	0.0308	-	0.05460	0.01008
8 30									

TABLE 5.1

THIESSEN WEIGHT OF RAINGAUGE STATIONS

Catchment or Subarea		Thiessen weight of raingauge station(%)				
Name	Area in sq.mile	Ghatbaroli	Wagholi	Barkher	Kherwari	Karpa
A	53.00	12.9%	22%	22.5%	19.6%	23%
A_1	20.24	14.2%	0%	52%	33.8%	0%
A_2	28.58	1.5%	39%	4.5%	13%	42%
A_3	4.18	92.8%	7.2%	0%	0%	0%

TABLE 5.2.

DISTANCE OF CENTROIDS OF HOURLY RAINFALL VOLUMES FROM CATCHMENT OUTLET FOR STORM NO.1

Time (hrs.)	Distance based on five Thiessen polygon (miles)	Distanced based on three subareas (miles)	Distance based on whole catchment (miles)
1200	7.22	5.87	5.20
1300	5.94	5.24	5.20
1400	5.74	5.75	5.20
1500	5.43	5.75	5.20
1600	4.80	5.75	5.20
1700	3.62	5.16	5.20
1800	2.08	3.35	5.20

TABLE 5.3.

ONE HOUR UNIT HYDROGRAPH USING NASH'S CASCADE MODEL

Time in hour	Catchment A $n=5.5, K=0.54$ hr.	Sub area A_1 $n_1=5.22, K_1=0.435$ hr.	Subarea A_2 $n_2=5.32$, $K_2=0.46$ hr.	Subarea A_3 $n_3=4.81$ $K_3=0.279$ hr.
0	0	0	0	0
1	760	905	935	872
2	7250	4940	6150	1450
3	11300	4480	6420	350
4	8300	1960	3310	26
5	4150	605	1230	3
6	1630	143	251	0
7	525	31	77	
8	200	6	17	
9	50	0	3	
10	12		0	
11	3			
12	0			

TABLE 5.4.

RESULTS OF DIFFERENT CASE STUDIES :

Storm No.1
Date: 16.8.62

Time in hrs.	Observed direct runoff Hydrograph cusecs	Calculated direct runoff hydrograph(cusecs)					
		Case 1A	Case 1B	Case 1C	Case 1D	Case 1E	Case-1F
12 00				0	0	0	0
13 00	0	0	0	83	50	50	19
14 00	780	187	172	756	152	196	64
15 00	1662	2064	1919	3825	760	1012	586
16 00	3034	5688	5423	7854	3867	4125	3605
17 00	12032	8203	8057	8997	8123	8248	7973
18 00	7416	7585	7671	6557	9086	9077	9205
19 00	3880	4915	5123	3057	6169	5921	6438
20 00	2200	2407	2565	1071	2935	2673	3126
21 00	948	962	1041	293	1059	927	1138
22 00	393	337	369	68	292	243	318
23 00	200	110	120	17	68	57	74
24 00	0	32	35	3	17	15	19
01 00		8	9	0	3	2	4
02 00		2	2		0	0	1
03 00		0	0				0
	$F_o^2=145.36 \times 10^6$	$F^2=23.49 \times 10^6$	$F^2=23.65 \times 10^6$	$F^2=40.38 \times 10^6$	$F^2=25.82 \times 10^6$	$F^2=23.44 \times 10^6$	$F^2=29.21 \times 10^6$
		$R^2=83.7\%$	$R^2=83.6\%$	$R^2=72.2\%$	$R^2=82.3\%$	$R^2=84\%$	$R^2=80\%$

TABLE 5.5.

HALF HOUR UNIT HYDROGRAPH USING NASH'S CASCADE MODEL

Time in hr.	Whole catchment area A, n=5.5, K=0.54 hr.	Sub area A_1 n_1=5.32 K_1=0.435 hr.	Sub area A_2 n_2=5.32 k_2=0.46 hr.	Subarea A_3 n_3 = 4.81 k_3 = 0.279 hr.
00	0	0	0	0
50	72	130	122	250
00	1450	1680	1750	1500
50	5220	4280	5040	1785
00	9250	5590	7260	1110
50	11540	5000	7050	495
00	11140	3820	5810	204
50	9500	2480	4060	36
00	7100	1430	2570	17
50	5030	802	1500	5
00	3270	408	962	1
50	2115	194	285	0
00	1220	93	217	
50	720	43	105	
00	330	19	50	
50	276	8	24	
00	120	4	11	
50	63	1	4	
00	32	0	2	
50	18		1	
00	7		0	
50	5			
00	1			
50	1			
00	0			

TABLE 5.6

RESULTS OF DIFFERENT CASE STUDIES

Storm No.2 : Date: 21.8.61

Time in hrs.	Observed direct runoff hydrograph cusecs	Calculated direct runoff hydrograph cusecs	
		Case 2 A	Case 2 B
2.30			
3 00	0	0	0
3 30	10	5	3
4 00	20	111	39
4 30	30	758	242
5 00	50	2386	1235
5 30	290	4822	3293
6 00	4118	7593	6288
6 30	7282	9894	9119
7 00	12869	11124	11241
7 30	18720	11074	12212
8 00	12470	10056	11435
8 30	10262	8396	9622
9 00	6690	6467	7494
9 30	3810	4672	5277
10 00	2672	3168	3402
10 30	1649	2038	2070
11 00	974	1280	1165
11 30	770	756	604
12 00	610	440	334
12 30	500	259	146
13 00	440	133	74
13 30	330	77	35
14 00	270	41	16
14 30	210	22	7
15 00	200	10	3
15 30	140	5	1
16 00	80	2	0
16 30	70	1	
17 00	10	0	
17 30	0		
	$F_o^2 = 667.50 \times 10^6$	$F^2 = 118.03 \times 10^6$	$F^2 = 67.10 \times 10^6$
		$R^2 = 82.3\%$	$R^2 = 90\%$

TABLE 5.7

CATCHMENT CHARACTERISTICS

Type	Area, A (sq.mile)	Length of main channel L (miles)	Overland slope, OLS (parts per 10,000)	No. of linear reservoirs (n)	Storage coefficient, K (hours)
(a) Whole catchment	53.00	11.6875	225	5.50	0.540
(b) Subarea A_1	20.24	7.0000	209	5.22	0.435
(c) Subarea A_2	28.58	8.5625	234	5.34	0.460
(d) Subarea A_3	4.18	3.1250	249	4.81	0.279

SUMMARY OF RESULTS

Case type	Storm No.	Observed peak (cusecs)	Calculated peak (Cusecs)	Difference between time to peak (hour)	Translation coeff. of linear channel for subarea model (hour)	Model efficiency R^2 (%)
Case 1 A: Whole catchment	1	12,032	8203	0	—	83.7
Case 1 B: Whole catchment	1	12,032	8057	0	—	83.7
Case 1 C: Subarea model	1	12,032	8997	0	Zero	72.2
Case 1 D: Subareas model	1	12,032	9086	One	One	82.3
Case 1 E: Subareas Model	1	12,032	9077	One	One	84.0
Case 1 F: Subareas model	1	12,032	9205	one	one	80.0
Case 2 A: Whole catchment model	2	18,720	11124	half	—	82.3
Case 2 B: Subareas model	2	18,720	12212	0	One	90.0

TABLE 5.9

RESULTS WITH ASSUMED RAINFALL DISTRIBUTION

Fig.No.	Rainfall excess Rev. inch	Peak discharge in inch depth	Time to peak (hr)	1st moment ($MERH_1$) (hour)	2nd moment ($MERH_2$) $(hour)^2$	n (inch)	n^* (inch)
1	1	0.081	22	3.25	11.50	0.41	0.18
2	1	0.083	21	2.50	8.00	0.31	0.13
3	1	0.083	20	1.75	5.50	0.22	0.09
4	1	0.079	23	3.50	14.00	0.44	0.22
5	1	0.079	22	3.00	12.00	0.38	0.19
6	1	0.077	21	2.50	10.00	0.31	0.16
7	1	0.078	23	3.75	16.50	0.47	0.26
8	1	0.077	23	3.50	16.00	0.44	0.25
9	1	0.075	22	3.25	15.75	0.41	0.25
10	1	0.082	24	4.75	23.50	0.59	0.37
11	1	0.084	24	5.50	32.00	0.69	0.50
12	1	0.086	25	6.25	41.50	0.78	0.65
13	1	0.080	24	4.50	22.00	0.56	0.34
14	1	0.082	24	5.00	28.00	0.63	0.44
15	1	0.081	25	5.50	34.00	0.69	0.53
16	1	0.078	23	4.25	20.50	0.53	0.32
17	1	0.079	24	4.50	24.00	0.56	0.38
18	1	0.078	24	4.75	26.50	0.59	0.41
19	1	0.082	23	4.00	16.77	0.50	0.26
20	1	0.085	23	4.00	17.13	0.50	0.27
21	1	0.088	23	4.00	17.17	0.50	0.27
22	1	0.080	23	4.00	17.53	0.50	0.27
23	1	0.083	23	4.00	18.25	0.50	0.29
24	1	0.084	23	4.00	18.34	0.50	0.29
25	1	0.079	23	4.00	18.30	0.50	0.29
26	1	0.080	23	4.00	19.38	0.50	0.30
27	1	0.081	23	4.00	19.52	0.50	0.30
28	1	0.074	23	4.00	21.86	0.50	0.34
29	1	0.070	24	4.00	23.88	0.50	0.37
30	1	0.066	24	4.00	25.95	0.50	0.41
31	1	0.075	23	4.00	21.16	0.50	0.33
32	1	0.072	24	4.00	22.75	0.50	0.36
33	1	0.069	24	4.00	23.67	0.50	0.37
34	1	0.076	23	4.00	21.08	0.50	0.33
35	1	0.074	23	4.00	21.63	0.50	0.34
36	1	0.073	24	4.00	19.25	0.50	0.30

TABLE 5.10

RESULTS WITH STATISTICAL RAINFALL DISTRIBUTION

Types of statistical distribution	Rainfall excess Rev.inch	Peak discharge in inch depth	Time to peak hour	Ist moment $MERH_1$ (hour)	2nd moment $MERH_2$ $(hour)^2$	n (inch)	n^* (inch)
Normal dist. σ = 1.33	1	0.086	23	4.00	17.95	0.5	0.28
Normal dist. σ = 1.00	1	0.088	23	4.00	17.25	0.5	0.27
Binomial dist. (7,0.5)	1	0.087	23	4.00	17.33	0.50	0.27
Binomial (7,0.3)	1	0.088	21	2.60	8.22	0.32	0.13
Poison's dist. m= 1.6	1	0.089	20	1.50	3.06	0.19	0.05
Poisons Dist. m= 1.6	1	0.087	21	2.97	5.99	0.37	0.09
Uniform Dist.	1	0.077	23	4.00	16.00	0.50	0.25

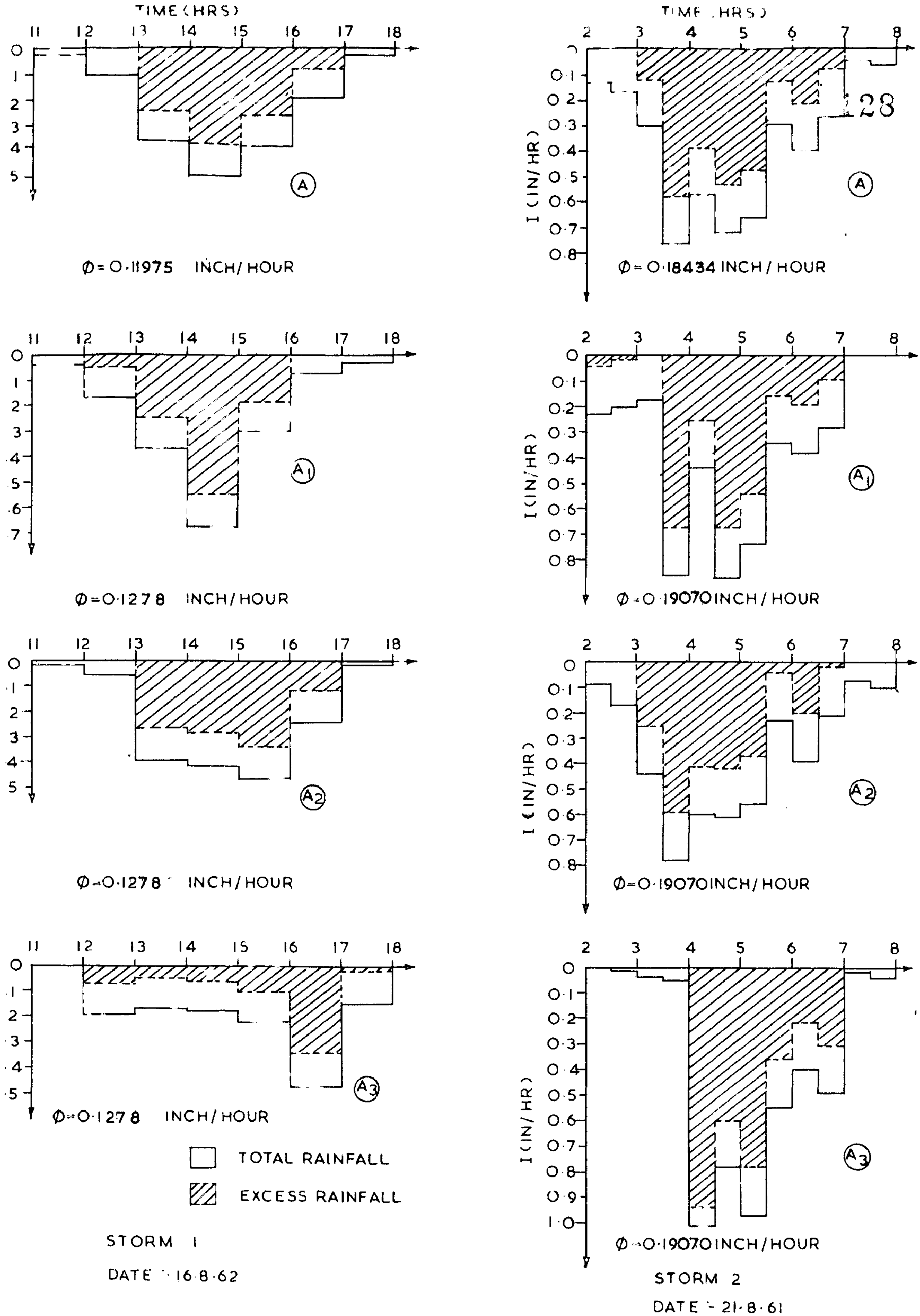


FIG. 5·1 TIME DISTRIBUTION OF AVERAGE RAINFALL OVER AREA A, A_1, A_2, A_3 FOR STORM 1 AND 2.

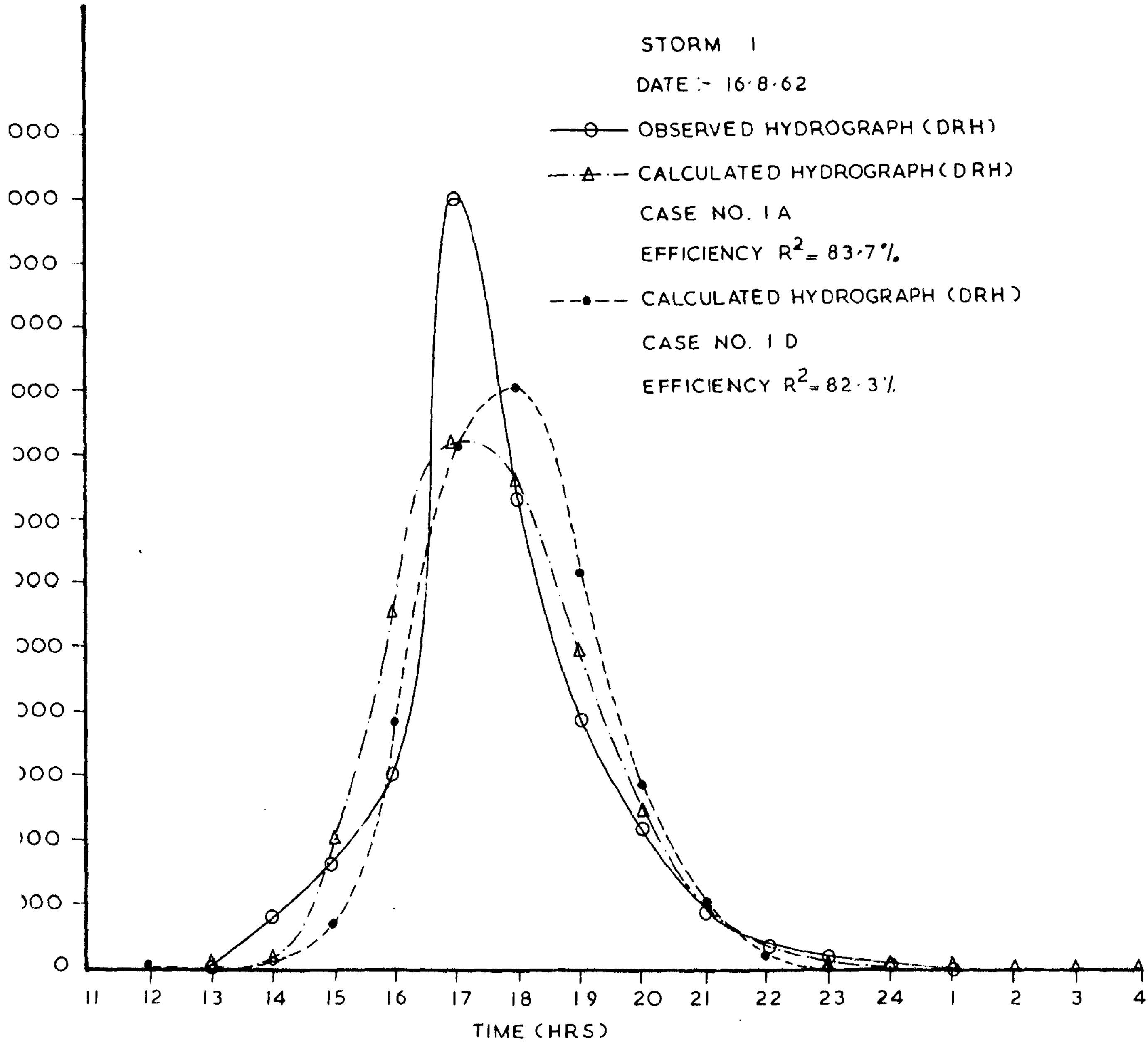


FIG. 5·2 OBSERVED AND CALCULATED DIRECT RUNOFF HYDROGRAPH

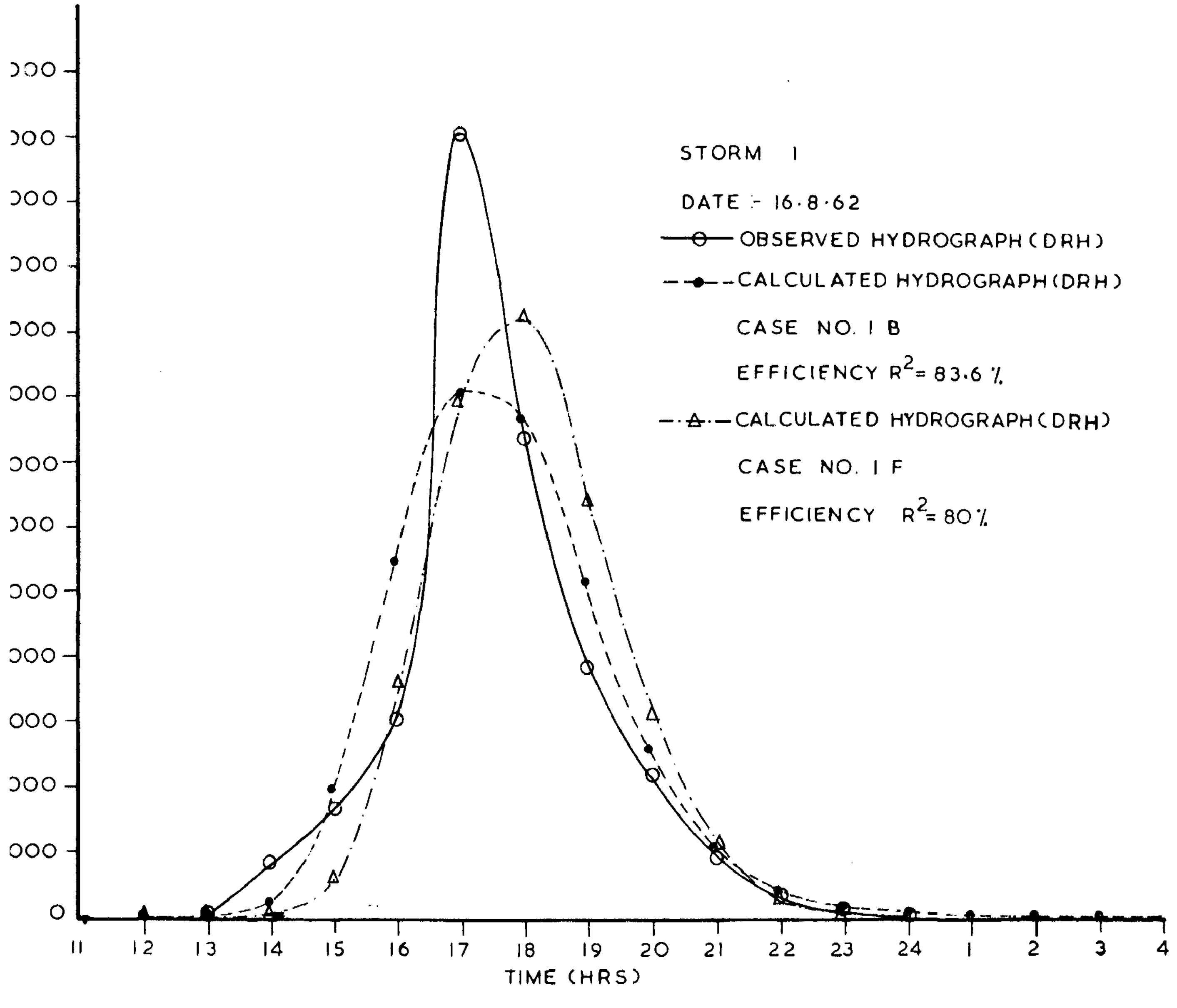


FIG. 5·3 OBSERVED AND CALCULATED DIRECT RUNOFF HYDROGRAPH

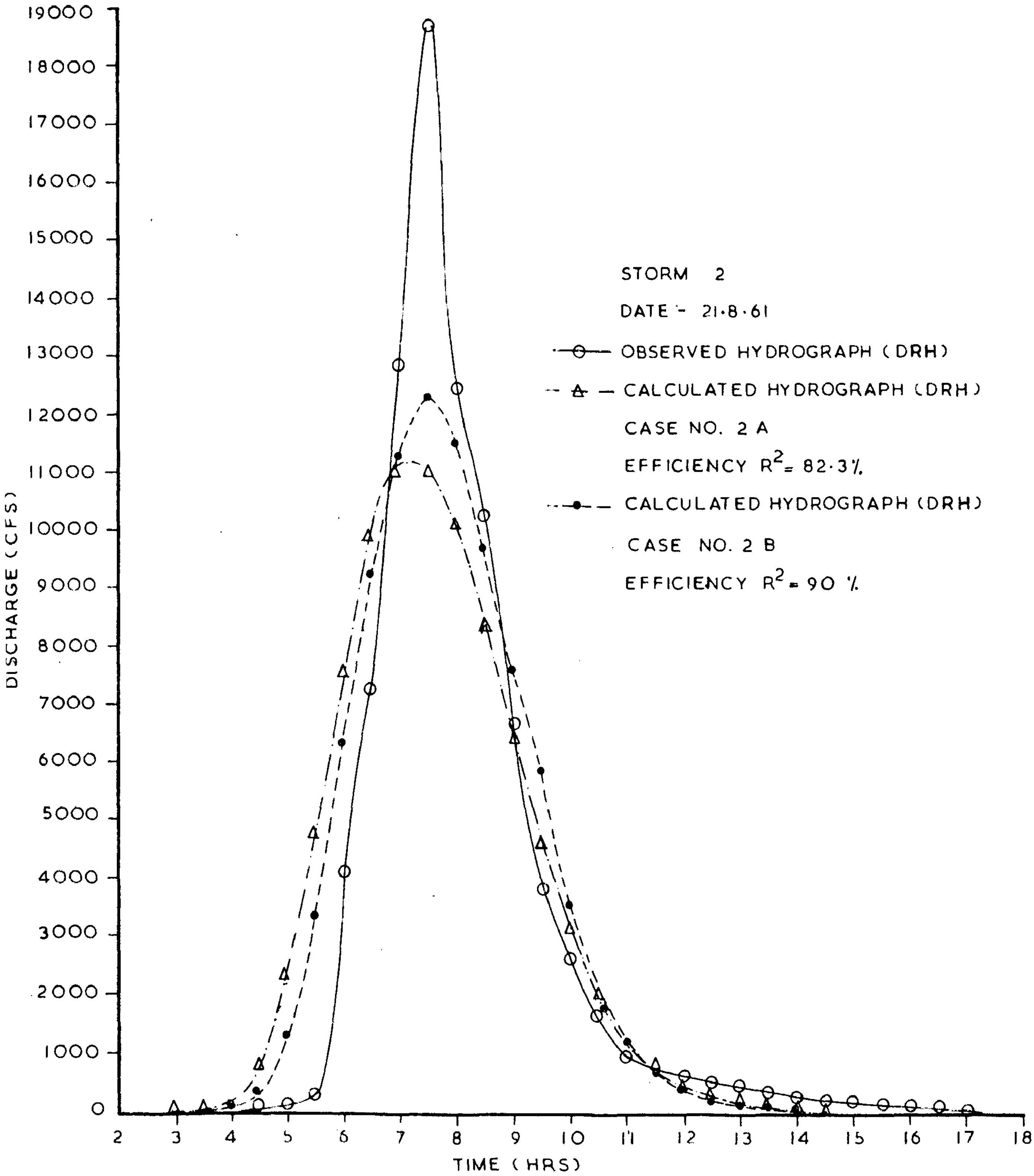


FIG. 5·4 OBSERVED AND CALCULATED DIRECT RUNOFF HYDROGRAPH

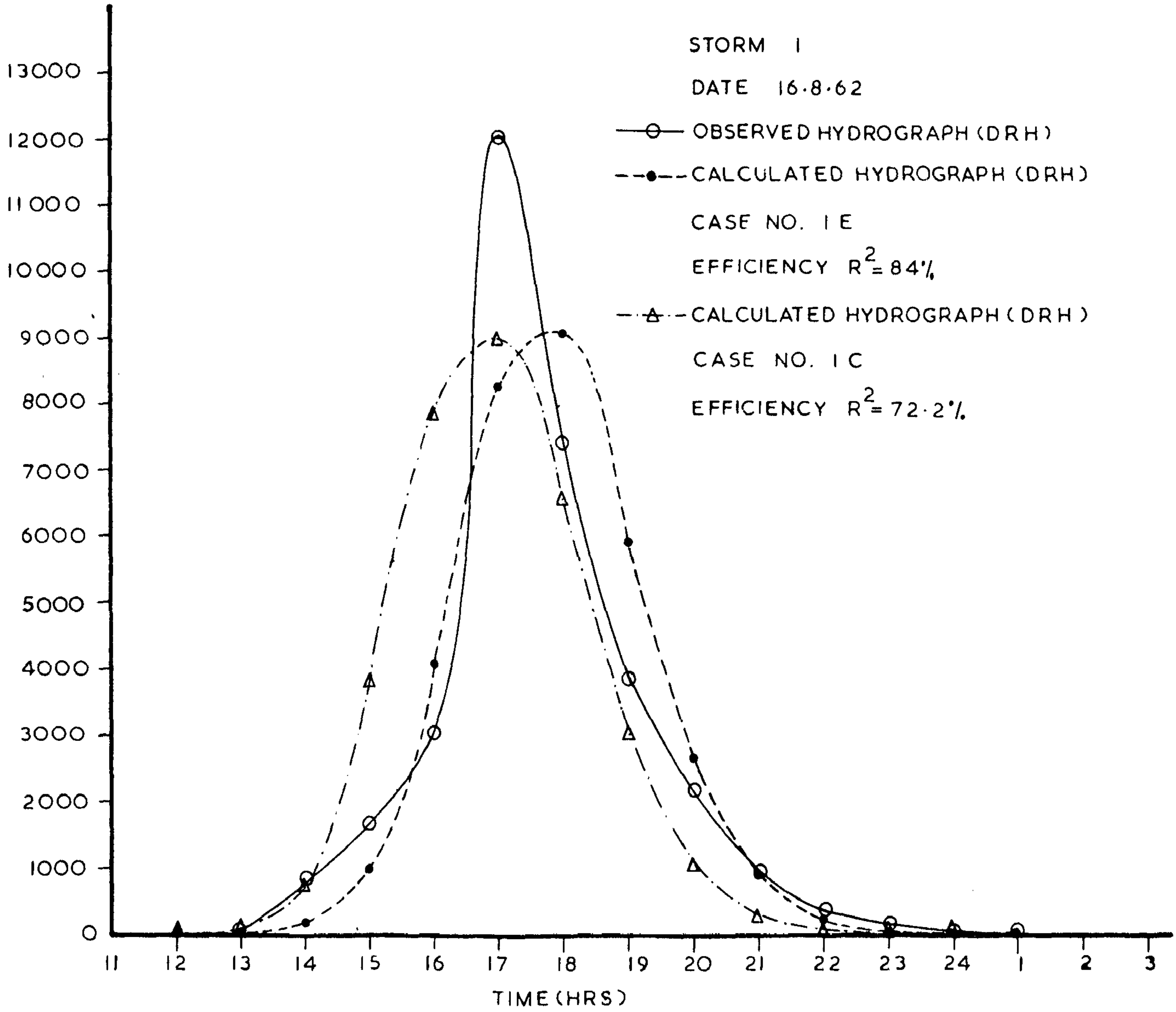


FIG. 5·5 OBSERVED AND CALCULATED DIRECT RUNOFF HYDROGRAPH

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APPENDIX

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C  C  DESERTATION/BURAGOHAIN/M.E.HYDROLOGY
C      CONCEPTUAL MODELLING OF EXCESSRAINFALLDIRECTRUNOFF PROCESS
      DIMENSION CQ(200),ACQ(200),AQ1(200),AQ2(200),AQ(200),AQ3(200)
      READ13,K1,LK1,K2,LK2,K3,LK3
13     FORMAT(6I5)
      READ 90, CA1,CA2,CA3
90     FORMAT(3F9.6)
      READ14,KT1,KT2,N,NUM
14    FORMAT(4I5)
      DO 300 I=1,200
      ACQ(I)=0.0
      AQ1(I)=0.0
      AQ2(I)=0.0
      AQ3(I)=0.0
300   CQ(I)=0.0
       NT=0
       INUM=0
       AKC=K1
       LKC=LK1
       KT=KT1
30     IF(INUM-1)70,71,71
70     READ72,M
72     FORMAT(I5)
      READ15,(CQ(I),I=1,M)
15     FORMAT(8F9.6)
       JM=M+1
       DO 16I=JM,N
      CQ(I)=0.0
16     CONTINUE
71     NT=NT+1
       DO 20 I=1,N
20     ACQ(I)=CQ(I)
       JK=0
       EX=1.0-(1.0/EXPF(1.0/AKC))
19     S1=0.0
       JK=JK+1
       SUM=0.0
       DO 17 I=1,N
      QV=(S1*EX)+ACQ(I)*(1.0-AKC*EX)
       S2=S1+ACQ(I)-QV
       S1=S2
       ACQ(I)=QV
       SUM=SUM+ACQ(I)
17     CONTINUE
      PUNCH18,NT,JK,S1,SUM
18     FORMAT(2I8,2E16.8)
       IF(JK-LKC)19,24,24
24     CONTINUE
       IF(NT-2)41,41,50
41     NC=0
       NK=N+KT
       DO 25 I=1,NK
       NC=NC+1
       IF(NC-KT)31,31,32
31     AQ(I)=0.0
       GO TO 25
```

```
   32   KIT=I-KT
        AQ(I)=ACQ(KIT)
 25      CONTINUE
         IF(NT-1)52,51,52
  51     DO 100 I=1,NK
 100    AQ1(I)=AQ(I)
 52      IF(NT-2)53,54,54
 53      AKG=K2
         LKG=LK2
         KT=KT2
         INUM=NUM
         GO TO 30
 54      DO 101 I=1,NK
 101     AQ2(I)=AQ(I)
        AKG=K3
         LKG=LK3
         INUM=NUM
         GO TO 30
 50      CONTINUE
         DO91I=1,N
 91      AQ3(I)=ACQ(I)
         DO 60 I=1,NK
         ACQ(I)=CA1*AQ1(I)+CA2*AQ2(I)+CA3*AQ3(I)
 60      CONTINUE
        PUNCH80,(ACQ(I),I=1,NK)
  80    FORMAT(6F12.6)
         STOP
         END
     2     4     2     4     4     3
  0.4       0.4        0.2
    12    12   100     1
     8
```