# ACTIVE VIBRATION CONTROL OF PLATE WITH INTEGRATED SENSORS AND ACTUATORS

**A DISSERTATION**

***Submitted in partial fulfillment of the requirements for the award of the degree***

***of***

**MASTER OF TECHNOLOGY**

***in***

**MECHANICAL ENGINEERING**

**(With Specialization in Machine Design Engineering)**

***By***


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# CANDIDATE'S DECLARATION

I hereby declare that the work, which is being presented in this dissertation titled "**ACTIVE VIBRATION CONTROL OF PLATE INTEGRATED WITH SENSORS AND ACTUATORS**" in the partial fulfillment of the requirements for the award of the degree of **Master of Technology** with the specialization in **Machine Design,** submitted in the Department of **Mechanical & Industrial Engineering, Indian Institute of Technology Roorkee, India,** is an authentic record of my own work carried out during the period from June 2005 to June 2006, under the supervision and guidance of **Dr. S.C. Jain,** Professor, Department of Mechanical & Industrial Engineering, IIT Roorkee, India.

I have not submitted the matter embodied in this dissertation for the award of any other degree.

**Date:** 29/06/06

**Place: Roorkee** **(Hipparkar Dattatraya Rama)**

## CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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# ABSTRACT

Active vibration control using piezoelectric sensors and actuators have recently emerged as a practical and promising technology. Efficient and accurate modeling of these structures bonded to or embedded with actuators and sensors is needed for efficient design of smart structures. This dissertation addresses the modeling of smart plate.

A finite element model of piezolaminated composite plate based on first order shear deformation theory and linear piezoelectric theory is presented. Finite element has five mechanical degree of freedom per node and one electrical degree of freedom per piezoelectric layer. Finite element has n-host structure layer and two piezoelectric layers. In deriving the finite element model of piezolaminated plate first displacement equation is given followed by strain displacement relationship, constitutive equation of piezoelectric, force and bending moment relation, strain energy equation, electrical energy equation, work done by external forces and electrical charges, kinetic energy equation. Governing equations are derived using Hamilton's principle. Constant gain negative velocity feedback controller is used for vibration control. Genetic algorithm is used to obtain optimum voltage in oder to get desired shape of plate. A code is developed in MATLAB for making numerical studies. Code is validated for static and dynamic analysis with the available literature.

Numerical studies is carried out in reference of layered composite plate for the effect of ply orientation angle on deflection, Effect of piezolayer thickness on the natural frequency, Variation of natural frequency with orientation angle of plate, Variation of Fundamental natural frequencies with patch coverage area, Effect of feedback control gain on transient response, Effect of sensor/actuator pairs position on the plate response, Mode shapes of plate, Shape control of composite plate and optimum voltage in order to get desired shape using genetic algorithm. Finally, it is observed that piezoelectric actuator can be used to control the shape of the plate. And a combination of sensor-actuator-controller can be used to suppress the vibration in composite plate.

# CONTENTS

## NOMENCLATURE

A = relative to surface area, $m^2$

V = volume, $m^3$

C= structural damping

Q = elastic constants, $N/m^2$

D = electric displacement vector, $C/m^2$

F = force, N

M = bending moment, Nm

G = shear modulus of material, $N/m^2$

h = thickness, m

$K_{uu}$ = stiffness matrix

$M_{uu}$ = mass matrix

$K_{u\Phi}$ = elastic electric coupling matrix

$K_{\Phi\Phi}$ = electric stiffness matrix

$G_c$ = constant gain

q = displacement field vector

$q_i$ = nodal displacement field, m

K = kinetic energy, J

P = Potential energy, J

n = number of layers

t = time, sec

u = displacement field in x direction, m

v = displacement field in y direction, m

w = displacement field in z direction, m

W = work, J

T = transformation matrix

E = electric field matrix

$\sigma_q$ = surface charge density

**Greek Symbols**

$\varepsilon$ = strain field

$\sigma$ = stress, N/m$^2$

$\nu$ = Poisson ratio

$\Phi$ = electric potential, Volts

$\omega$ = frequency, rad/s

$\rho$ = material density, kg/m$^2$

**Subscripts**

b = relative to bending

m = refers to membrane or mid plane

s = refers to shear

p = relative to point force

t = relative to traction force

b = relative to body force

c = relative to electric charge

e = relative to electrical force

s = relative to the sensor

a = relative to actuator

x = relative to x direction

y= relative to y direction

z= relative to z direction

**Superscripts**

s = relative to constant strain

T = matrix transpose

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## 1.1 Motivation

All the mechanical systems are subjected to various conditions that may results in vibrational motion; hence vibration is significance factor to be considered in the design of lighter mechanical systems, systems working at high speeds and the systems like micro sensing, micro actuation, space structures etc. where accuracy finds a great importance. Vibrations may lead to fatigue in the material, damage to the structure, deterioration of system performance, increased noise level and increase in the difficulty of predicting the behavior of the structure. Active vibration methods can be used to eliminate the undesired vibrations. The use of smart structures is experiencing a tremendous growth in actively controlling the vibration.

Advanced composite materials are finding increasing application in aircraft, automobiles, marine and submarine vehicles besides other engineering applications. The fiber reinforced composite posses two desirable features: one is their high stiffness-weight ratio and the other is their anisotropic material property that can be tailored through variation of the fiber orientation and stacking sequence- a feature which gives the designer an added degree of flexibility. In this thesis a finite element model for a composite plate has been developed using the first order shear deformation theory for laminated plates to analyze the behavior of piezoelectric material over a plate structure.

The field of smart structure has been an emerging area of research for the last few decades. Smart structures or intelligent structures can be defined as structures that are capable of sensing and actuating in a controlled manner in response to an input. The ability of the piezoelectric materials to convert electrical to mechanical energy and vice versa makes them to be employed as actuators and sensors. If these are bonded properly onto a structure, structural deformations can be induced by applying a voltage to the materials, employing them as actuators. Similarly, these piezoelectric materials can also be employed as sensors since deformations of a structure would cause the deformed piezoelectric materials to produce an electric charge. The extent of structural deformations can be observed by measuring the electrical voltage the sensors produce. This voltage is multiplied by some gain according to the control law implemented and is feedback to the actuators. The actuator made of smart materials react to the voltage and generate mechanical changes. Using the changes from the actuators, the vibration or other dynamic characteristic can be closely controlled.

The advantage of incorporating this special type of material into the structure is that the resulting sensing and actuating mechanism becomes part of the structure itself. This is possible due to the direct and inverse piezoelectric effects: when a mechanical force is applied to a piezoelectric material, an electrical voltage is generated (direct) and, conversely, when an electric field is applied, a mechanical force is induced (inverse). With the recent advances in piezoelectricity technology, it has been shown that piezoelectric actuators based on the converse piezoelectric effect can offer excellent potential for active vibration control techniques, especially for vibration suppression or isolation. Smart materials and structures have wide range of application due to potential advantage in wide range of application. Such as aeronautical engineering, aerospace engineering, civil engg., automobile engg., precision instruments and machines.

For a complex structure, it is very expensive to implement smart materials over the entire surface. Hence the sensors and actuators are usually discretely distributed over the structure. One of the limitations of the piezoelectric actuators is the amount of the force it can exert. Hence it is important that the actuators are placed at optimal locations so that the required control effort is minimum. Thus optimization of the placement of these sensors and actuators over the structure becomes an important task in suppressing the vibration of the structure. The problem becomes critical as the number of sensors/actuators to be used over the structure increases and as the mode shape becomes complicated. Therefore optimization techniques have to be used in such cases to find a good set of sensors/actuators positions. The significance in using such algorithms is not only the solution to problems but also, drastically cost savings in both experimental and time expenses. Genetic algorithms produce a global optimum and can be applied to complicated problems with relative ease. It is more flexible and provides more accurate solution.

This work deals with the modeling of a composite plate using finite element method assuming the first order shear deformation theory for plates. The dynamic characteristics of the structure are studied and piezoelectric patches are used to control its vibration. The optimum voltage of the piezoelectric actuators is approximated using genetic algorithms. The objective function for the genetic algorithm is taken as the error which is difference between actual shape and desired shape of the composite plate. Mode shapes of a composite plate are plotted.

### 1.2 Preamble

The primary objectives of this study is to develop a simple finite element for multilayered composites plates. The element contains five degrees of freedom, three displacements and two slopes (i.e. shear rotations) per node. Element also contains one piezoelectric degree of freedom per piezoelectric layer. The accuracy of the element is demonstrated through the problems. Objective of this thesis is to study the dynamic characteristics of the structure with and without piezoelectric patches over the plate. Genetic algorithm is used to obtain the optimum voltage in order to get desired shape of the plate.

### 1.3 Organization of the thesis

Chapter 2 addresses some theoretical background needed for the work. The strategies in the active vibration control have been discussed. Some perspectives in smart structures and piezoelectric materials have been given. It also introduces genetic algorithms and discusses their advantages.

Chapter 3 contains a brief discussion regarding the previous work that has been done in this field. Summarized details of work carried out by different authors, their objectives and conclusions are given.

Chapter 4 details the development of the finite element model using the first order shear deformation theory, the derivation of the equations is given in detail along with constant gain negative velocity feedback control technique.

Chapter 5 discusses the optimization of the actuators voltage over the plate. The ıse of genetic algorithms is discussed. The objective function is derived in this section vhich is then used in the algorithm for optimizing.

Chapter 6 presents solution to various problems considered. The results are onsidered with help of graphs and figures.

This chapter covers the theoretical background of smart materials especially piezoelectric materials and a brief description of active control strategy of vibration suppression.

## 2.1 Perspectives in smart structures

The field of smart structures has been an emerging area of research for the last few decades. Smart structures are the structures that are capable of sensing and actuating in a controlled manner in response to a stimulus. The development of this field is supported by the development in the field of materials science and in the field of control. In material science, new smart materials are developed that allow them to be used for sensing and actuation in an efficient and controlled manner. These smart materials are to be integrated with the structures so that they can be employed as actuators and sensors effectively.

There are following smart materials which can be used as sensor/actuator application.

- **Shape memory alloys**

The term shape memory alloys (SMA) is applied to a group of metallic materials that can return to a previously defined shape when subjected to an appropriate thermal procedure. Generally, these materials can be plastically deformed at some relatively low temperature, and upon exposure to some higher temperature will return to their shape prior to the deformation. SMAs allow one to recover up to 5% strain from the phase change induced by temperature. SMAs are best for one way tasks such as deployment. SMAs are little used in vibration control. Examples Ni-Ti alloys, Cu-Al-Ni, Fe-Mn, and Fe-Mn-Si, etc.

- **Piezoelectric materials**

Piezoelectricity is the ability of material to develop an electrical charge when subjected to a mechanical strain conversely. They have a recoverable strain of 0.1%

ınder electric field. They can be used as actuators as well as sensors. Examples PZT, ?VDF etc.

➢ **Magnetostrictive materials**

As a magnetostrictive material is magnetized, there is a change in length. Conversely, if an external force produces a strain in magnetostrictive materials; its nagnetic state will change. Magnetostrictive materials have a recoverable strain 0.15% ınder magnetic field. The maximum response is obtained when the material is subjected o compressive loads. They can be used in high precision applications. An example is Terfenol-D.

➢ **Electrostrictive materials**

These are quite similar to piezoelectric materials with slightly better strain capability, but very sensitive to temperature. The conceptual difference between piezoceramics and electrostrictors is their response upon reversing the electric field. Piezoceramics can be elongated and compressed, while electrostrictors only exhibit an elongation, independent of the direction of the applied electric field. This effect is found n all materials, though in very small quantities 10-5 to 10-7 %

➢ ***Ferromagnetic shape memory alloys***

Ferromagnetic shape memory alloys (FSMA) are a recently discovered class of ıctuator materials, whose salient features are magnetically driven actuation and large strains (around 6%) e.g. NiMn-Ga ternary alloy. As the name suggests FSMAs are ferromagnetic alloys that also support the shape memory effect.

➢ **Electrorheological and magneto rheological fluids**

When an external electric field is applied to an ER fluid, the viscosity of the fluid ncreases remarkably, and when the electric field is taken away, the viscosity of the fluid goes back to the original state. The phenomenon is called ER effect. These fluids can change from a thick fluid (similar to motor oil) to nearly a solid substance within a span of a millisecond when exposed to an electric field; the effect can be completely reversed ust as quickly when the field is removed. Examples of MR fluid is tiny iron particles uspended in oil and that of ER fluid are milk chocolate and oil.

- **Fiber optics**

Fiber optics is becoming popular as sensors because they can be easily embedded n composite structures with little effect on the structural integrity. They are widely used n structural health monitoring equipments.

For the vibration suppression of thin structure, piezoelectric materials are generally recommended for the following reasons.

- Low weight
- Ease of mounting on the structure
- Fast response
- Dual nature (sensing and actuating)
- Low energy consumption
- High efficiency and compactness

A detailed description of piezoelectric material is given below.

### .2 Piezoelectric materials:

In 1880, Pierre and Paul-Jacques curie discovered the direct piezoelectric effect n various crystals such as Tourmaline, Rouchelle salt and Quartz. The crystals generate lectrical charges on their surfaces when they were mechanically strained in certain lirections. In the following year, they also discovered the converse effect that the shape f crystals would change when an electric field was applied to them.

The ability of the piezoelectric materials to exchange electrical and mechanical nergy opens up the possibility of employing them as actuators and sensors. If iezoelectric materials are bonded properly to the structure, structural deformations can e induced by applying a voltage to the materials, employing them as actuators. On the ther hand, they can be employed as sensors since deformations of a structure would ause the deformed piezoelectric materials to produce an electric charge. The extent of lectrical deformation can be observed by measuring the electrical voltage the materials produce. Unfortunately, the piezoelectric effect in natural crystals is rather weak so they cannot be used effectively as actuators and sensors.

However, recent developments in the field of materials science have provided piezoelectric materials that have sufficient coupling between electrical and mechanical domains. Two of the commonly used piezoelectric materials are Polyvinyllidene fluoride (PVDF), a semi crystalline polymer film, and lead zirconate titanate (PZT), a piezoelectric ceramic material. PZT has larger electromechanical coupling coefficients than PVDF so PZT can induce larger forces or moments on structures. However, PZT is relatively brittle while PVDF is flexible and can be easily cut into any desired shape. PVDF also has good sensing properties so it is commonly used for sensors. In this thesis, we concentrate on using as a transducer for vibration control of flexible structures.

There are of two broad classes of piezoelectric materials used in vibration control: ceramics and polymers. The piezopolymers are mostly used as sensors, because they require high voltages as well as light weight and flexible so they are not effective as actuators on stiff structures. The best known is the Polyvinylidene Fluoride (PVDF). Piezoceramic are used extensively as actuators, for a wide range of frequency including ultrasonic applications. The best-known Piezoceramic is lead Zirconate Titanate (PZT).

Piezoelectric materials have some limitations also like voltage that can be applied is limited in the range of -500V to 1500V, the piezo materials cannot be used above their curie temperature, which is 200°C to 300°C due to possibility of depolarization.

### 2.2.1 Classification of piezoelectrics

**Pyroelectrics:** Materials in which electric field generates as a result of application of heat and degree of polarization depends on the temperature.

**Ferroelectrics:** Materials in which spontaneous polarization can be induced by an electric field. Reversing external electric field can change their polarization direction. Examples are PZT and PVDF.

**Ferroelastic:** Materials in which spontaneous polarization can be induced due to mechanical load.

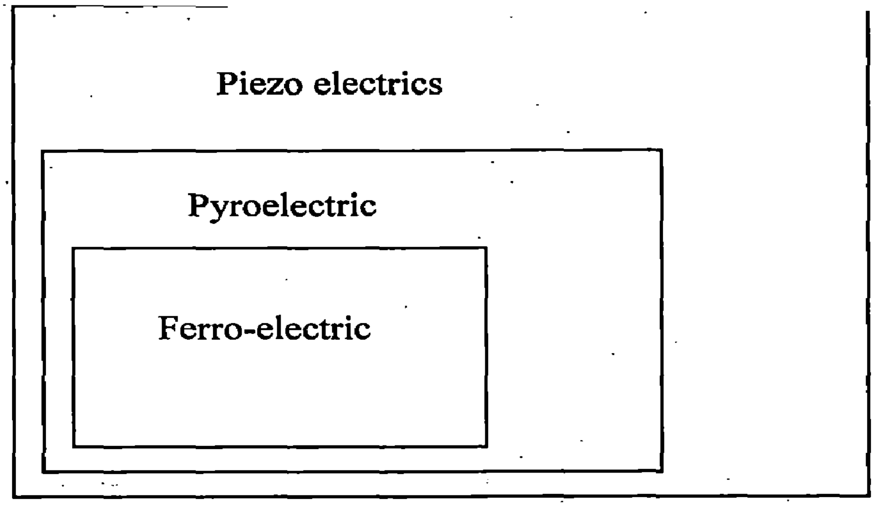



Figure 2.1 Classifications of Piezoelectric Materials

**2.2.2 Piezoelectric constitutive relations**

Piezoceramic materials are assumed to be linear and the actuation strain is modeled like thermal strain. Piezoceramic can be idealized as an orthotropic material such as unidirectional laminated composite. The constitutive relations are based on the assumption that the total strain in the actuator is the sum of the mechanical strain induced by the stress, the thermal strain due to temperature and the controllable actuation strain due to electric voltage.

The axes are identified by numerals: 1 corresponding to x-axis, 2 corresponding to y-axis and 3 corresponding to z-axis. A piezoelectric material produces strains when an electric field is applied along its poling direction, which is generally along the 3-direction for a monolithic-type material conversely, it generates electric displacement when it is strained. While the former property is used in actuation, the latter is used in sensing. Figure 2.3 shows the Axis Configuration of Piezoelectric Patch. The constitutive relations governing these properties are as follows:

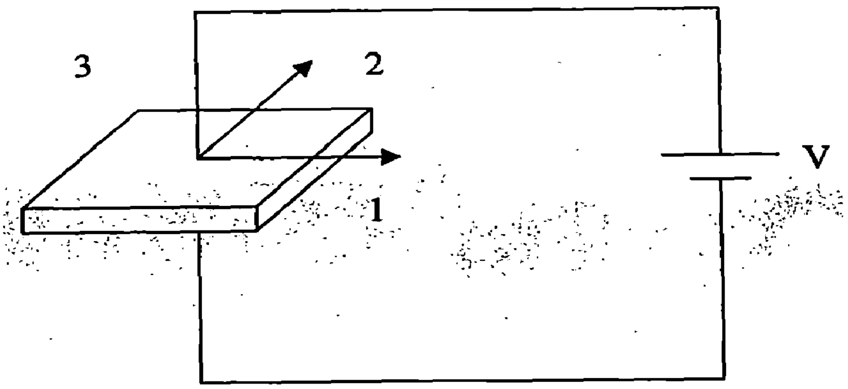



Figure 2.2. Axis Configuration of Piezoelectric Patch

Coupled electromechanical constitutive relations are:

$$\{\sigma\}=[Q]\{\varepsilon\}-[e]^T\{E\} \tag{2.1}$$

$$\{D\}=[e]\{\varepsilon\}+[s]\{E\} \tag{2.2}$$

Where

$\{\sigma\}$ = Stress vector

$\{\varepsilon\}$ = Strain vector

$[Q]$ = Elasticity constant matrix

$\{E\}$ = Electric field

$\{D\}$ = Electric displacement

$[e]$ = Piezoelectric constant stress matrix

$[s]$ = Dielectric constant matrix

Rewriting above equation

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{Bmatrix} + \begin{bmatrix} s_{11} & 0 & 0 \\ 0 & s_{22} & 0 \\ 0 & 0 & s_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \tag{2.3}$$

Where $d_{33}$, $d_{31}$ and $d_{15}$ are called piezoelectric strain coefficient of a mechanical-free piezo element.

- $d_{31}$ Characterizes strain in the 1 and 2 directions due to an electric field $E_3$ in the 3 direction
- $d_{33}$ Characterizes strain in the 3 direction due to field in the 3 direction.
- $d_{15}$ Relates shear strains in 2-3 and 3-1 planes due to field respectively.

Thus, if an electric field $E_3$ is applied to a free piezo-element, it causes longitudinal strains $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$. If an electric field $E_1$ or $E_2$ is applied, the material eacts with shear strain $\gamma_{31}$ and $\gamma_{23}$ respectively.

If a compressive force is applied in the polarization direction (axis 3), or tensile orce is applied in the plane perpendicular to polarization direction (axis 2 or 1), it will esult in a voltage that has the same polarity as the original poling direction.

During the manufacture of a piezoceramic, a large (greater than 1 kV/mm) field is pplied across the ceramic to create polarization. This is called coercive field during ubsequent testing, if the field greater than coercive field, is applied opposite to the olarization direction, the ceramic will lose its piezoelectric properties. This phenomenon s called depoling. However, it is possible to repole the material. If an applied electric ield is aligned with the initial polarization direction, there is no depoling sufficient high oltage can cause arcing or a brittle fracture. Poling is also possible if high temperature or arge stress is applied.

**.3 Active control strategy for vibration suppression**

A control is considered to be any system that exists for the purpose of regulating r controlling the flow of energy, information, money, or other quantities in some desired ashion. A control system is an interconnection of many components or functional units n such a way as to produce a desired result. In a closed loop or active control system, here is typically some model of the system to be controlled, a control law, and some ensors that make measurements to carry to the control figure 2.1 shows a block diagram of a typical closed loop control system. In this control system, the control U(t) is modified by information obtained about the system output, Y(t).

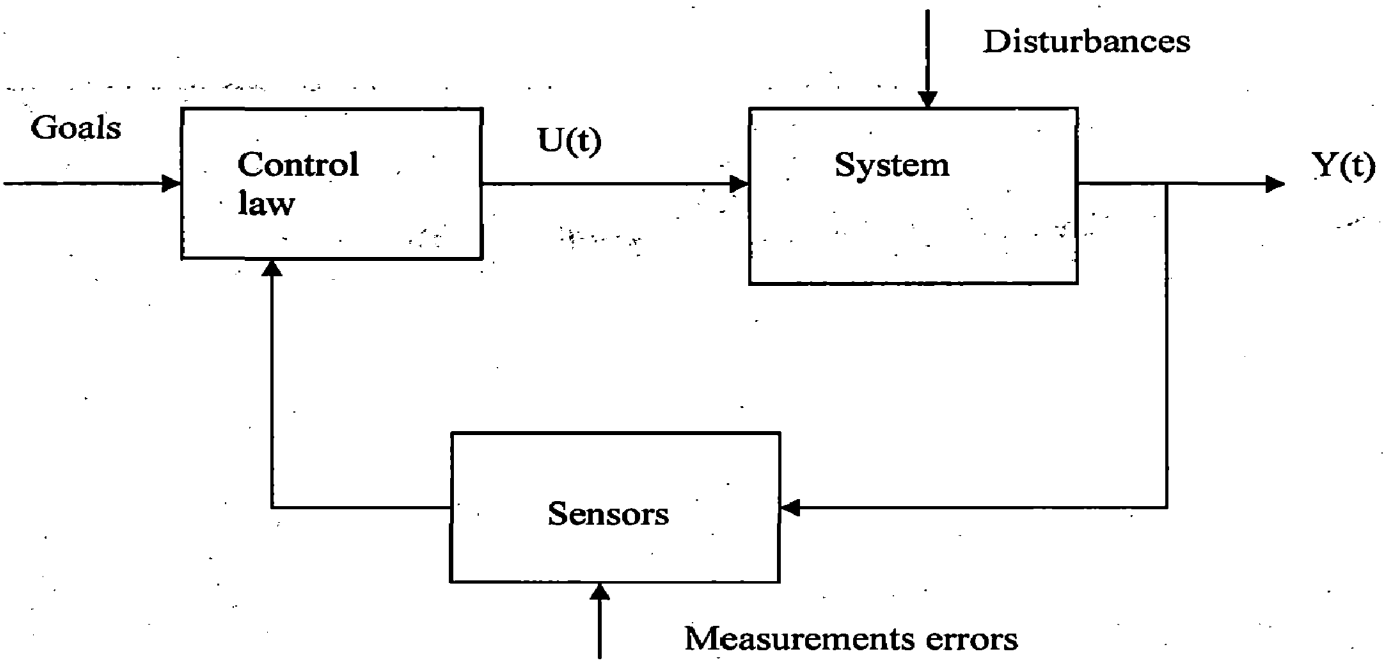



Figure 2.3. A Closed loop control system

The next chapter discusses about the previous work that is carried by different authors in the field of vibration control using the smart materials. It includes the brief introduction of their work done and conclusions derived by them.

Figure 2.3. A Closed loop control system

Smart structures, incorporating piezoelectric materials, offer an efficient method for implementing active control technologies and as a consequence the topic has received considerable interest in recent years. Piezoelectric materials have been widely used for sensors and actuators due to their attractive properties, such as low weight and rapid response. Below, some relevant work in the area of vibration control using the smart structures is reviewed.

**Crawley and De Luis (1987)** developed analytical models to predict the static and dynamic response of intelligent systems to an applied voltage. These models were applicable to systems with segmented piezoelectric actuators that are either bonded to an elastic member or embedded in a laminated composite structure. They also consider perfect bonding conditions between the actuators and the structures as well as bonds of finite thickness and stiffness. The model was used to select an optimal location for actuators. With a PZT consisting of two Piezoceramic actuator devices bonded at equal but opposite distances from the neutral axis of the beam, three experimental systems were constructed; an aluminum beam with two surface-mounted PZTs, a glass-epoxy beam with two PZTs and a graphite-epoxy beam with one PZT. The PZTs were used to excite steady-state resonant vibrations in the beams and the experimental responses were seen to agree with the analytical models. Static tests performed on the glass-epoxy laminated material with embedded PZTs showed a reduction in ultimate strength of only 20% and no significant change in the global elastic modulus of the composite laminate.

**Collins et al. [1990]** provided a brief but informative history of piezoelectricity and some of the basic physical properties and mathematical relationship used in the study of piezoelectric materials. The fact, that other noncrystaline materials, such as wood, bone and some polymers, exhibit piezoelectric behavior is discussed. In particular, the discovery of the piezoelectric effect in polyvinylidene fluoride (PVDF) is presented. In its unpolarized form, PVDF is clear, lightweight and tough yet highly flexible film that is often used as a passive protective coating for many surfaces due to its high resistance to various chemicals and ultraviolet light. After suitable processing consisting of stretching at high temperatures, exposure to high electric fields during cooling (poling) and surface metallization, PVDF can be used as a sensor and/or actuator for active structural control. After the outstanding introduction to the history, manufacturing and application of piezopolymers, Coolins et al. proceeds to model and experimentally verify the use of PVDF as model sensors and spatial filters for various mechanical systems.

**Hagood, Chung and Von Flotow [1990]** presented a more detailed consideration of the interaction between an elastic structure and piezoelectric actuators used for active structural control. Also, they developed state space models for voltage and current-driven piezoelectric devices and assessed how the dynamic of the actuator and passive electrical network influence the overall system dynamics. These models are used to predict the behavior of a cantilevered beam with surface-mounted Piezoceramic devices. Lastly, open and closed loop control experiments were performed to verify the analytical models, thus showing significant effects of the electrical circuits on the beam dynamics.

**Cox and Linder [1991]** discussed in their paper, the use of a modal domain optical fiber sensor (MD sensor) as a component in an active control system to suppress vibrations in a flexible beam. An MD sensor consists of a laser source, an optical fiber, and detection electronics. They have shown that the output of the MD sensor is proportional to the integral of the axial strain along the optical fiber.

**Kulkarni and Hanagud [1991]** developed an electromechanical model for a combined generic three-dimensional, isotropic, linearly elastic body and piezoelectric body undergoing small or large deformations using a variational formulation. The model is then used to develop a two-dimensional finite element formulation to predict the static and dynamic response of the system to applied voltages that vary with time and spatial distribution on the surface of the patch. A detailed treatment of the electromechanical coupling between the active element and the elastic body is presented. The stresses and strains predicted by the static analytical model were verified using a cantilevered aluminum beam with Piezoceramic patches bonded to the top and the bottom surfaces.

The same beam was used to verify the dynamic response of the system to various types of applied voltages. It was found that electric fields applied to the actuators in a non-uniform manner, such as a linear, cosine or exponential variation along its length, result in different types of bending moment distributions being applied to the beam.

**Hilton E. et.al., [1995]**, dealt with the free vibration analysis of prismatic folded plate and shell structures supported on diaphragms at two opposite edges with the other two edges arbitrarily restrained. The analysis was carried out by using curved /variable thickness finite strips based on Mindlin_Reissner shell theory, which allows for transverse shear deformation and rotatory inertia effects. The accuracy and relative performance of a family of C0 strips were examined. Results are presented for a series of problems including plates, cylindrical shells and box girders. In a companion paper these accurate and inexpensive finite strips were used for structural optimization.

**Banks, Smith and Wang [1995]** examined the interaction between piezoelectric actuators on such structures as beams, plates and more complicated shell structures such as right circular cylinder. The changes in mechanical stiffness of the structure due to the attachments were studied in addition to the ability of the patches to apply and moments, which were found to depend on the geometry and placement of the patch in addition to the applied voltage. The influence of internal forces and moments due to the structure and the actuator as well as those due to actuating the patch are then related to the time-dependent structural equations of motion. With that, these models can be applied directly to controlling the vibration of such structures, particularly those of a curved nature. Since the structure at the focus of this research was a toroidal shell, the result from Banks, Smith and Wang will contribute greatly when control of the entire inflated member is considered.

**Langley [1995]** claimed in his paper that the equations for natural frequencies developed by Rayleigh are invalid. Since the point mobility of a membrane has an infinite imaginary component, any inertial force created by an attached point mass would lead to an infinite displacement at that point. Clearly this is not the case; however, it was found that large deformations do take place around the attachment that can only be represented y multi-term Rayleigh-Ritz method. With this approach, Langley presents solutions for the free vibration of a circular membrane with a point mass attachment as well as the forced response of a square membrane with a point mass attachment using the modal summation method. The results are used to understand the physical limitations of linear membrane theory with point mass attachments. In order to obtain more accurate results, he suggests the use of a nonlinear theory, considering the attachment to have a finite area or taking into account the finite bending stiffness of the membrane.

**Niekerk and Tongue [1995]** investigates ways to actively reduce the transient noise transmission through a membrane covering a circular duct. Once the nature of sound has been identified, only a few millisecond are available to determine the control signal and actuate the structure. Therefore, piezopolymer actuators were used because they offer the ability to operate at high frequencies. Their experiment used a speaker to impart sound waves into a duct whose cross-section is covered by an elastic membrane fitted with discrete piezopolymer actuators. The sound pressure imparted to the film was measured immediately before the membrane by a microphone. The resulting velocity of the centre of the membrane was measured with a laser vibrometer, whose output was used as a feedback signal for the control loop. The output of this control loop was given to the PVDF actuators attached to the membrane. The resulting sound pressure level was measured at a microphone on the side of the membrane opposite the speaker. Three control schemes were investigated analytically, and then experimentally, namely optimal control, sliding control and velocity feedback control. It was found that a reduction in transient noise transmission through the membrane was possible. Experimental results confirmed this analytical prediction. The PVDF actuators performed fast enough to control the membrane and velocity feedback was found to be the most stable and easily implemented control method.

**Suleman and Venkayya [1995]** developed finite element formulations for composite plates with laminated piezoelectric layers. They developed 24 degrees of freedom piezoelectric plate elements with one electrical degree of freedom per surface.

They made assumptions that the electrical degrees of freedom were constant along the plane and vary linearly through the thickness of the piezoelectric layers. An advantages of their methodology is that the analysis eliminates problems associated with modeling thin plate elements with isoparametric solid elements, which have excessive shear strain energies and higher stiffness in the thickness directions.

**Masad [1996]** studied a rectangular membrane with uniform tension and thickness, but with linearly varying density along one dimension of the membrane. Solutions to natural frequencies and mode shapes were obtained using both a numerically accurate analytical techniques and approximation methods for the inhomogeneous membrane. The resulting frequencies and mode shapes were compared to those of an equivalent, homogeneous membrane. As expected, some variation was seen between the two types of membranes and the approximate method was able to closely predict the natural frequencies.

A square membrane under uniform tension with a centrally located circular area of finite radius and either continuously or discontinuously varying density was considered by **Bambill, et al. [1997]**. He determines the natural frequencies using both the optimized Rayleigh-Ritz method and an approximate conformal mapping approach. Reasonable agreement is seen between the two methods for the discontinuously varying density. Likewise, acceptable results are obtained for the square membrane with circular centre section of continually varying density.

**Saravanos, D.A. [1997]**, developed finite elements that enable the formal analysis of piezoelectric composite shell. His methodology was based on what is called a "Mixed Laminate Theory". This theory utilizes unique approximations for displacements and electric potentials. The first order shear deformation theory was assumed on the mechanical displacements, while discrete layer approximations are assumed on the electrical potentials. The advantages of this mixed laminate theory are they

1. accurately and efficiently model thin and/ or moderately thick laminated piezoelectric shell with arbitrary laminations and electric configurations, and they

2. captured the through the thickness electric heterogeneity induced by the embedded piezoelectric layers.

Finite element methods were compared to exact solutions. The analysis was justified with excellent convergence and agreement with fundamental frequencies and through the thickness electromechanical modes of moderately thin plates.

**Jinsong et al. [1998]** used a FOS and ER fluid actuator for vibration monitoring of smart composite structures. They found same sensitivity in FOS as piezoelectric materials with lower cost. They found change in structural damping and natural frequencies with varying electric field and hence can be monitored by using FOS and ER fluid.

**G. R. Liu, X.Q. Peng and K.Y. Lam** developed a model based on the classical laminated plate theory to model the static and dynamic response of laminated composite plates integrated with piezoelectric sensors and actuators. A Finite element model is developed to study the vibration control and active vibration suppression of laminated composite plates with integrated piezoelectric layers. The formulations are derived using Hamilton's principle. They have taken into account the mass and stiffness of the piezolayes and a simple negative velocity feedback control algorithms is used to couple direct effect of the piezoelectric for active control the dynamic response of the integrated structure.

**Pronsato et al. [1999]** considers a rectangular membrane under uniform tension with a rectangular center section having a constant density different than the rest of the structure. Again, approximation techniques are used to find the frequency coefficients, from which natural frequencies can be extracted. The approximation was made possible by using a truncated Fourier series consisting of a linear combination of sine waves to approximate the displaced shape of the vibrating membrane. The results are tabulated for different ratios of density between the outer and inner areas as well as different dimensions of the inner rectangular section. Finally, good correlation is seen when the results are compared to a previous paper in which only one polynomial expression was used to represent the displacement of the membrane. It can be shown that results attained by Pronsato are even accurate than those of the previous paper.

**Payman Afshari and G. E. O. Widera, [2000]**, developed a series of plate elements, based on the modified complementary energy principal, to study the free undamped vibration response of laminated composite plates. They selected Mindlin thin plate theory to govern the general characteristic and behavior of these plate elements. A series of in-plane strain functions were assumed from which the corresponding in-plane strains and corresponding stresses for each lamina were determined. The transverse stresses were then computed by satisfying the equations of the equilibrium. Eight-noded isoparametric elements were utilized to describe the displacement field. These hybrid plate elements are used to form the stiffness and the consistent mass matrices. The fundamental natural frequencies were then computed by solving the generalized eigenvalue problem and their application demonstrated via a number of examples.

**Wang and Chen [2000]** performed a modal analysis of a simply supported plate using only PZT patches as actuators and PVDF patches as sensors. Their work included a theoretical development of the interaction between the smart actuators and sensors with the steel plate, generation of a column in the frequency response function matrix, generation of the plate's mode shapes, and extraction of the plate's modal parameters. They acknowledged that smart materials have a major advantage over the conventional structural testing. Piezoceramic transducers can be integrated into the structure, and that the idea of using smart materials for system identification is also important to other applications such as structural vibration and acoustic control.

**Sze and Yao [2000]** prepared a number of finite element models for modeling of smart structures with segmented piezoelectric patches. These included eight-node solid shell element for modeling homogenous and laminated host structures as well as an eight-node solid shell and a four-node piezoelectric membrane element for modeling surface bonded piezoelectric sensing and actuating patches. They studied number of problems with these models and found results agreeing with experimental results.

**Clayton L. Smith [2001]** demonstrated that multiple layer modeling is achievable by single layer equivalent modeling using equivalent material properties. He derived finite element methods for modeling piezoelectric structures, which account for mechanical and electrical characteristic of the structure and validated the linear theory of piezoelectricity with ABAQUS models using piezoelectric elements. He also demonstrated equivalent single layer techniques for modeling piezoelectric laminated structures and determined equivalent loading techniques for modeling piezoelectric structures and piezoelectric laminated structures subject to electrical loading conditions. He simplified the analysis of piezoelectric laminated structures such that computational models can be developed to investigate the static and dynamic response using equivalent representations of the structures.

**Makhecha D. P. et al. [2002]**, studied the effect of higher order theory, that accounts for the realistic variation of in-plane and transverse displacements through the thickness, on the modal loss factors and natural frequencies of thick composite laminated/sandwich plates. They presented a displacement based C0 continuous isoparametric, eight noded quadrilateral plate element, based on realistic higher order theory. The accuracy and effectiveness of the present model over the first and other higher order theories for vibration and damping characteristic were demonstrated considering thick laminated/sandwich plates. They suggested that, the higher order terms such as stretching term in the transverse displacement field, slope discontinuity in thickness direction for in-plane displacements, and various other high order terms are important in evaluating the damping and forced response characteristics of sandwich laminates. This mainly depends on the ply-angle, lamination scheme, aspect ratio and core thickness.

**Yan Y. J. and Yam L.H. [2002]** presented in their paper the optimal design methodology of number and locations of actuators in active vibration control of a space truss using multiple piezoelectric ceramic stack actuators. They applied eigenvalue distribution of the energy correlative matrix of the control input force, to determine the optimal number of actuators required, and genetic algorithms (GAs) were adopted to search for the optimal locations of actuators. Their results showed that the disturbance acting on a structure is s key factor in determining the optimal number and locations of actuators in active structural vibration control, and a global and efficient optimization solution of multiple actuator locations can be obtained using the GAs.

**Liew K.M. et al [2003]** suggested that in conventional analyses of composite laminates, the assumption of perfect bonding of adjoining layers is well accepted, although this is an oversimplification of the reality. It is possible that the bond strength may be less than that of the laminate. Thus, the study of weak bonding is an interesting focus area. In their study, an elastic bonding model based on three dimensional theory of elasticity in a layer wise framework is used to study composite laminates. The differential quadratue (DQ) discretization is used to analyze the layer wise model. The present model enables the simulation of actual bonding stress continuity as well as the kinematics continuity conditions were satisfied through the inclusion of the elastic bonding layer. Their model was employed to investigate the free vibration of thick rectangular cross-ply laminates of different boundary conditions and lamination schemes.

**Peng F et al. [2003]** investigated the actuator placement on a plate structure and vibration control of the structure. They optimized the location of actuators based on maximizing the controllability grammian. It was implemented using structuring analysis in ANSYS and genetic algorithms. Further they used a filtered-x LMS based multichannel adaptive control to suppress the vibrations. They also performed the numerical simulations in suppressing tri sinusoidal response at three points of the plate. Their results demonstrated that the genetic algorithm is an effective method to reduce the energy required for achieving significant vibration reduction.

**Shimpi R. P. and Ainapure [2004]** extended variationally consistent layer-wise trigonometric shear deformation theory to deal with free vibration of two layered laminated cross-ply plates. They derived governing differential equations by making use of a displacement field which allows a sinusoidal variation of the in-plane displacements through the laminates thickness. And concluded that, in their displacement based theory, constitutive relations between shear-stress and shear-strains are satisfied in both the layers, and, therefore, shear correction factor is not required. Compatibility at the layer interface in respect of in-plane displacement and compatibility in respect of transverse shear-stress is satisfied, yet present theory contains fewer unknown variables than that of the first order shear deformation theory. Effects of rotary inertia and other inertias are also included. They also suggested that their theory will be convenient for finite element modeling and finite element based on the present theory will be from shear locking. From the illustrative example, it is seen that the present theory give the accurate results. Efficiency of the theory was demonstrated through illustrative examples.

- The formulation is restricted to linear elastic material behavior (small displacement and strains).
- This formulation uses the Mindlin assumption (thin plate) in which the normality condition is removed resulting in a constant state of transverse shear strain through thickness and zero transverse normal strain.

$$\varepsilon_z = 0 \qquad \varepsilon_{yz} = \varepsilon_{yz}(x, y) \qquad \varepsilon_{xz} = \varepsilon_{xz}(x, y) \tag{4.1}$$

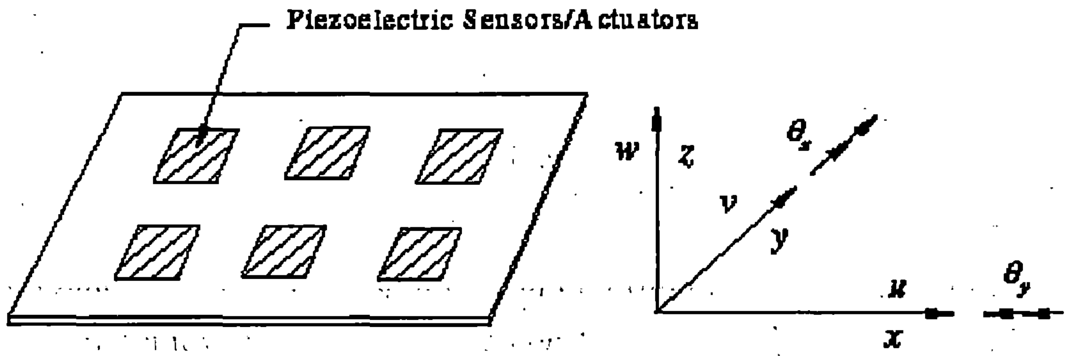



Fig.4.1. Coordinate system of laminated finite element with integrated piezoelectric material

## 4.2 Boundary conditions

### 4.2.1 Simply supported plate

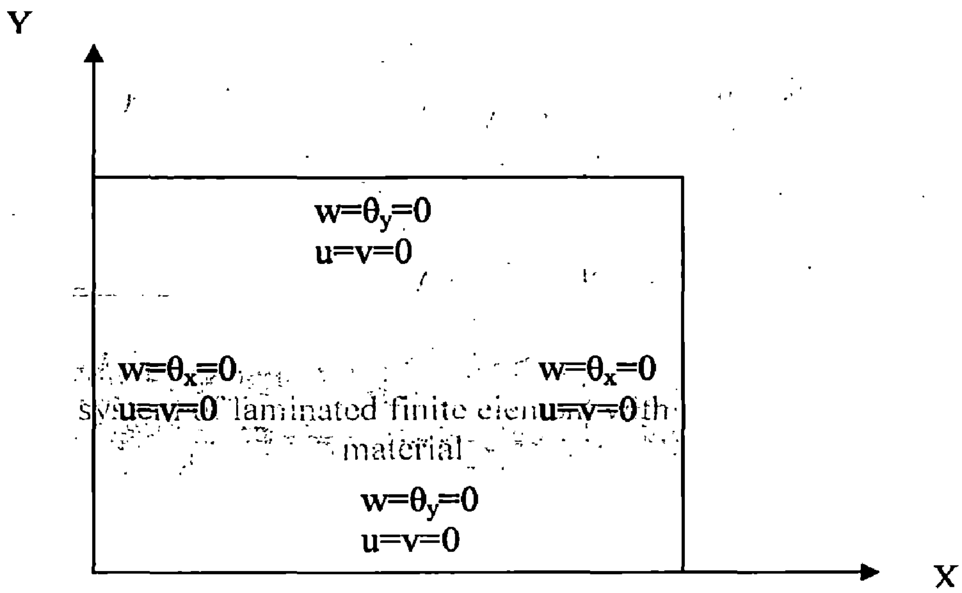



Figure 4.2. Boundary conditions for simply supported plate

## .2.2 Cantilever plate

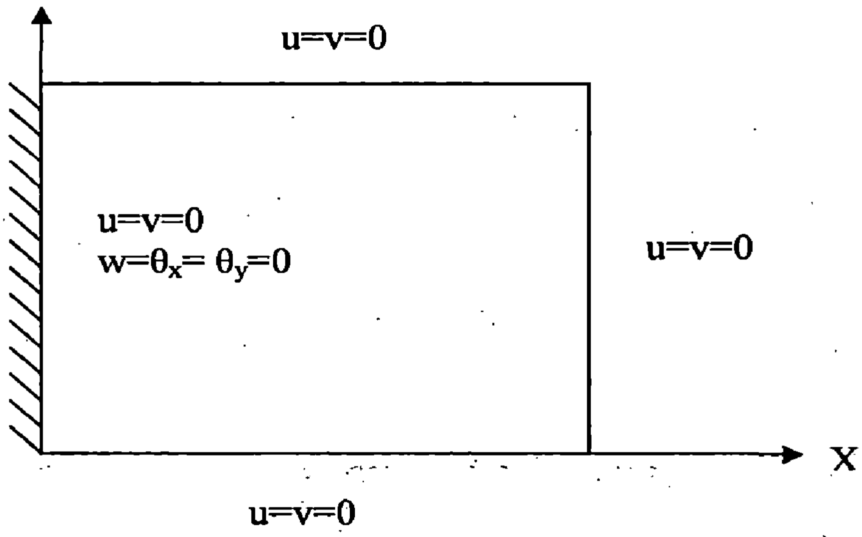



Figure 4.3, Boundary conditions for cantilever plate

## .3 Basic Equations

### .3.1 Displacement functions

Based on the first-order shear theory of small strains, the displacemen omponents u,v and w at (x,y,z) from median surface are expressed as functions o nidplane displacements $u_0$,$v_0$,$w_0$ and independent rotations $\theta_y$ and $\theta_x$ of the normals in y: nd xz planes, respectively of a piezoelectric composite plate takes following form:

$$u(x, y, z, t) = u_{0i}(x,y,t) + z\theta_{xi}(x,y,t)$$
$$v(x,y,z,t) = v_{0i}(x,y,t) + z\theta_{yi}(x,y,t)$$
Figure 4.3: Boundary conditions for cantilever plate
$$w(x,y,z,t) = w_{0i}(x,y,t) \tag{4.2}$$

simple 4 noded quadratic element is considered.

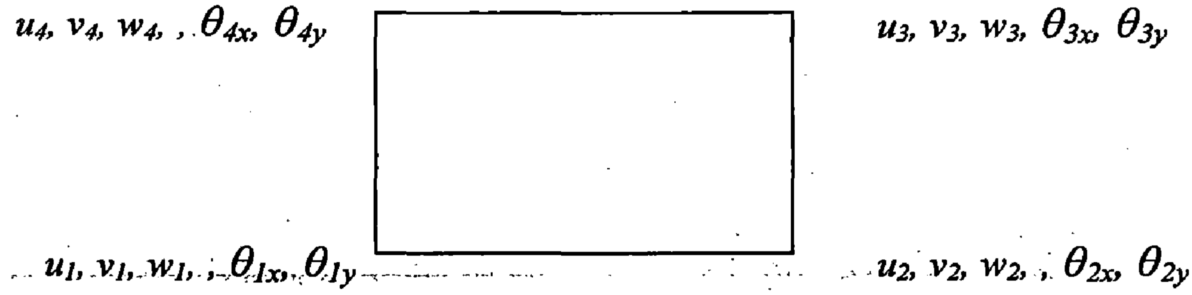



Figure 4.4. Four noded quadratic element

Two-dimensional interpolation (shape) functions are used to define the geometry ield at any point in the element cross-section. These shape functions relate the urvilinear coordinates in the nodal cartesian coordinate system to the element coordinate ystem.

These shape functions and their derivatives are

$$N_i = \tfrac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \tag{4.3}$$

$$\frac{\partial N_i}{\partial \xi} = \tfrac{1}{4}\xi_i(1+\eta\eta_i) \qquad \frac{\partial N_i}{\partial \eta} = \tfrac{1}{4}\eta_i(1+\xi\xi_i)$$

Shape functions can be used to express the element deformations in terms of iodal displacements

$$u = \sum_{i=1}^{4} N_i u_{0i} + N_i z\theta_{yi} \qquad v = \sum_{i=1}^{4} N_i v_{0i} + N_i z\theta_{xi} \qquad w = \sum_{i=1}^{4} N_i w_{0i} \tag{4.4}$$

It can be written as in matrix form

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} N_i & 0 & 0 & zN_i & 0 & 0 \\ 0 & N_i & 0 & 0 & zN_i & 0 \\ 0 & 0 & N_i & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_{0i} \\ v_{0i} \\ w_{0i} \\ \theta_{yi} \\ \theta_{xi} \end{Bmatrix}$$

$$\{u\} = [N_d]\{q\} \tag{4.5}$$

Where $\{u\} = \{u \quad v \quad w\}^T$ and $\{q\} = \{u_{0i} \quad v_{0i} \quad w_{0i} \quad \theta_{yi} \quad \theta_{xi}\}^T$ (4.6) and $$[N_d]=\sum_{i=1}^{4}\begin{bmatrix} N_i & 0 & 0 & zN_i & 0 & 0 \\ 0 & N_i & 0 & 0 & zN_i & 0 \\ 0 & 0 & N_i & 0 & 0 & 0 \end{bmatrix} \tag{4.7}$$

### 4.3.2 Strain-Displacement relations

The displacement relations are given in an element as below

$$\varepsilon_x=\frac{\partial u}{\partial x}=\sum_{i=1}^{4}\left\{N_i\left(\frac{\partial u_{0i}}{\partial x}+z\frac{\partial\theta_{yi}}{\partial x}\right)\right\}$$
$$\varepsilon_y=\frac{\partial v}{\partial y}=\sum_{i=1}^{4}\left\{N_i\left(\frac{\partial v_{0i}}{\partial y}+z\frac{\partial\theta_{xi}}{\partial y}\right)\right\}$$
$$\varepsilon_z=0$$
$$\varepsilon_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\sum_{i=1}^{4}\left\{N_i\left(\frac{\partial u_{0i}}{\partial y}+\frac{\partial v_{0i}}{\partial x}+z\frac{\partial\theta_{yi}}{\partial y}+\frac{\partial\theta_{xi}}{\partial x}\right)\right\}$$
$$\varepsilon_{yz}=\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}=\sum_{i=1}^{4}\left\{N_i\left(\frac{\partial w_{0i}}{\partial y}+\theta_{xi}\right)\right\}$$
$$\varepsilon_{xz}=\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}=\sum_{i=1}^{4}\left\{N_i\left(\frac{\partial w_{0i}}{\partial x}+\theta_{yi}\right)\right\}$$

The above equations are arranged in a matrix form given below

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_m\\0\end{Bmatrix}+\begin{Bmatrix}z\varepsilon_b\\\varepsilon_s\end{Bmatrix} \tag{4.8}$$

where

$$\{\varepsilon_m\}=[B_m]\{q\} \tag{4.9}$$

$$\{\varepsilon_b\}=[B_b]\{q\} \tag{4.10}$$

$$\{\varepsilon_s\}=[B_s]\{q\} \tag{4.11}$$

$$[B_m]=\sum_{i=1}^{4}\left\{N_i\begin{bmatrix}\frac{\partial}{\partial x} & 0 & 0 & 0 & 0\\ 0 & \frac{\partial}{\partial y} & 0 & 0 & 0\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & 0 & 0\end{bmatrix}\right\}$$

$$[B_b]=\sum_{i=1}^{4}\left\{N_i\begin{bmatrix}0 & 0 & 0 & \frac{\partial}{\partial x} & 0\\ 0 & 0 & 0 & 0 & \frac{\partial}{\partial y}\\ 0 & 0 & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x}\end{bmatrix}\right\}$$

$$[B_s]=\sum_{i=1}^{4}\left\{N_i\begin{bmatrix}0 & 0 & \frac{\partial}{\partial y} & 0 & 1\\ 0 & 0 & \frac{\partial}{\partial x} & 1 & 0\end{bmatrix}\right\}$$

### 4.3.3 Electric Field

The present element has five degrees of freedom $u_{0i}$, $v_{0i}$, $w_{0i}$, $\theta_{xi}, \theta_{yi}$ per node and one electrical degree of freedom, $\phi$ per piezoelectric layer. Thus assuming electric field is constant over the actuator and sensor thickness and varying linearly through the thickness of the piezoelectric layer. The electric field $\phi$ is applied in thickness direction only. The electric field strength considering the piezoelectric sensor and actuator layer is given by

$$\{E\}=-\begin{bmatrix}0 & 0\\ 0 & 0\\ 1/t_s & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1/t_a\end{bmatrix}\begin{Bmatrix}\phi_s\\ \phi_a\end{Bmatrix}=-[B_\phi]\{\phi\} \tag{4.12}$$

$t_s$ = thickness of sensor layer

$t_a =$ thickness of actuator layer

### 4.3.4 Constitutive Equations of Piezoelectric Materials

The linear equations coupling elastic field and electric field in piezoelectric medium are expressed by the direct and converse piezoelectric equations, respectively. These equations for plate shape sensor and actuator are written as

$$\{\sigma\} = [\bar{Q}]\{\varepsilon\} - [\bar{e}]^T\{E\} \tag{4.13}$$

$$\{D\} = [\bar{e}]\{\varepsilon\} + [\varepsilon\ ]\{E\} \tag{4.14}$$

where

$\{\sigma\}$ - stress tensor

$\{D\}$ - electric displacement vector

$\{\varepsilon\}$ - strain tensor

$\{E\}$ - electric field

$[\bar{Q}]$ - transformed elastic constants at constant electric field

$[\bar{e}]$ - transformed piezoelectric stress coefficient

$[\varepsilon\ ]$ dielectric tensor

where $\quad [\bar{Q}] = [T^T][Q][T]$

$$[\bar{e}] = [T^T][e]$$

The elastic constant matrix and the transformation matrix is given as

In this chapter, a finite element model of piezolaminated layered plate is eveloped based on first order shear deformation theory and linear piezoelectricity. A rief of negative velocity feedback and vibration control is outlined. In deriving the finite lement model first first order shear deformation theory is outlined followed by boundary onditions, basic equations (displacement function, strain displacement relations, stress train relations, force and bending moment), energy formulations, governing equations re obtained by using Hamilton's principle.

## .1 First order shear deformation theory

Classical plate theory is based on the assumptions that a straight line erpendicular to the plane of plate is (1) inextensible, (2) remains straight and (3) rotates uch that it remains perpendicular to the tangent to the deformed surface. i.e. the ansverse normal and shear stresses are neglected. Thus it unpredicts the deflections and ver predicts the frequencies as well as buckling loads of plates. That is why it is ecessary to use some other theory.

The first order shear deformation theory extends the kinematics of the classical late theory by relaxing the normality restriction and allowing for arbitrary but constant otation of transverse normals. It means the condition (3) is removed. The more ignificant difference between the CPT and FSDT is the effect including the shear eflections on the predicted deflections, frequencies and buckling loads. So the primary bjective in developing analytical solution for the rectangular plates using FSDT is to ring out the effects of shear deformations on deflection, stresses, frequencies and uckling loads.

In the present formulation, the following assumptions (Reddy, 1999) are onsidered:

- The piezoelectric layers are perfectly bonded together.
- Straight line perpendicular to plane of plate is remain straight.

$$[Q]=\begin{bmatrix} \frac{E_{11}}{(1-\upsilon_{12}\upsilon_{21})} & \frac{\upsilon_{12}E_{22}}{(1-\upsilon_{12}\upsilon_{21})} & 0 & 0 & 0 \\ \frac{\upsilon_{21}E_{22}}{(1-\upsilon_{12}\upsilon_{21})} & \frac{E_{22}}{(1-\upsilon_{12}\upsilon_{21})} & 0 & 0 & 0 \\ 0 & 0 & 2G_{12} & 0 & 0 \\ 0 & 0 & 0 & 2G_{23} & 0 \\ 0 & 0 & 0 & 0 & 2G_{13} \end{bmatrix}$$

$$[T]=\begin{bmatrix} c^2 & s^2 & 0 & 0 & cs \\ s^2 & c^2 & 0 & 0 & -cs \\ 0 & 0 & c & -s & 0 \\ 0 & 0 & s & c & 0 \\ -2cs & 2cs & 0 & 0 & c^2-s^2 \end{bmatrix}$$

### 4.3.5 Force and Bending Moment

Bending force, shear force and bending moment are given by

$$F_b=\sum_{k=1}^{n}\int_{h_{k-1}}^{h_k}\left\{[Q_k^b]\{\varepsilon_m+z\varepsilon_b\}-[e_i]^T\{E_i\}\right\}dz \tag{4.15}$$

$$F_s=\sum_{k=1}^{n}\int_{h_{k-1}}^{h_k}[Q_k^s]\{\varepsilon_s\}dz \tag{4.16}$$

$$M=\sum_{k=1}^{n}\int_{h_{k-1}}^{h_k}\left\{[Q_k^b]\{\varepsilon_m+z\varepsilon_b\}-[e_i]^T\{E_i\}\right\}zdz \tag{4.17}$$

Where

$$[Q_k^b]=\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \qquad [Q_k^s]=\begin{bmatrix} G_{23} & 0 \\ 0 & G_{13} \end{bmatrix}$$

Using equations (4.9), (4.10) and (4.11) the above equations can be written as

$$F_b=[A_T]\{\varepsilon_m\}+[B_T]\{\varepsilon_b\}-[A_e]\ \{E\} \tag{4.18}$$

$$F_s=[C_T]\{\varepsilon_s\} \tag{4.19}$$

$$M = [B_T]\{\varepsilon_m\} + [D_T]\{\varepsilon_b\} - [A_{e^2}]\{E\} \tag{4.20}$$

Where

$$[A_T] = \sum_{k=1}^{n} \begin{bmatrix} Q_{11k}(h_k - h_{k-1}) & Q_{12k}(h_k - h_{k-1}) & 0 \\ Q_{12k}(h_k - h_{k-1}) & Q_{22k}(h_k - h_{k-1}) & 0 \\ 0 & 0 & G_{12k}(h_k - h_{k-1}) \end{bmatrix}$$

$$[B_T] = \sum_{k=1}^{n} \frac{1}{2} \begin{bmatrix} Q_{11k}(h_k^2 - h_{k-1}^2) & Q_{12k}(h_k^2 - h_{k-1}^2) & 0 \\ Q_{12k}(h_k^2 - h_{k-1}^2) & Q_{22k}(h_k^2 - h_{k-1}^2) & 0 \\ 0 & 0 & G_{12k}(h_k^2 - h_{k-1}^2) \end{bmatrix}$$

$$[C_T] = \sum_{k=1}^{n} \begin{bmatrix} G_{23k}(h_k - h_{k-1}) & 0 \\ 0 & G_{13k}(h_k - h_{k-1}) \end{bmatrix}$$

$$[D_T] = \sum_{k=1}^{n} \frac{1}{3} \begin{bmatrix} Q_{11k}(h_k^3 - h_{k-1}^3) & Q_{12k}(h_k^3 - h_{k-1}^3) & 0 \\ Q_{21k}(h_k^3 - h_{k-1}^3) & Q_{22k}(h_k^3 - h_{k-1}^3) & 0 \\ 0 & 0 & G_{12k}(h_k^3 - h_{k-1}^3) \end{bmatrix}$$

$$[A_e] = [e]^T (h_{k_s} - h_{k_s-1}) + [e]^T (h_{k_a} - h_{k_a-1})$$

$$[A_{e^2}] = \frac{1}{2}[e]^T (h_{k_s}^2 - h_{k_s-1}^2) + [e]^T (h_{k_a}^2 - h_{k_a-1}^2)$$

## 4.2 Energy Formulations

### 4.2.1 Kinetic energy

The element kinetic energy is given by

$$T = \frac{1}{2}\{\dot{q}\}^T [M_{uu}]\{\dot{q}\} \tag{4.21}$$

where

$$[M_{uu}] = \iiint_V [N_d]^T \begin{bmatrix} \rho & \rho & \rho & z^2\rho & z^2\rho \end{bmatrix} [N_d]\, dV \tag{4.22}$$

$$\{\dot{q}\} = \{\dot{u}_{0i} \quad \dot{v}_{0i} \quad \dot{w}_{0i} \quad \dot{\theta}_{yi} \quad \dot{\theta}_{xi}\}^T$$

$$\rho = \sum_{k=1}^{n} \rho_k \tag{4.23}$$

### 4.4.2 Electrical work done

The electrical energy is given by

$$W_e = \frac{1}{2}\int_V \{E\}^T \{D\} dV \tag{4.24}$$

Using equations (4.9), (4.10), (4.11), (4.14), (4.12) equation(4.23) can be written in reduced form

$$W_e = -\frac{1}{2}\{\phi\}^T [K_{\phi u}]\{q\} - \frac{1}{2}\{\phi\}^T [K_{\phi\phi}]\{\phi\} \tag{4.25}$$

where

$$[K_{\phi u}] = \int_V [B_\phi]^T [e]\{[B_m] + z[B_b]\} dV \tag{4.26}$$

$$[K_{\phi\phi}] = \int_V [B_\phi]^T [\varepsilon^s][B_\phi] dV \tag{4.27}$$

### 4.4.3 Work done by external forces

a) Work done by point force

$$W_p = \{u_0\}^T \{F_p\}$$

$$W_p = \{q\}^T [N_d]^T \{F_p\} \tag{4.28}$$

b) Work done by traction force (load)

$$W_t = \int_A \{u_0\}^T \{F_t\} dA$$

$$W_t = \int_A \{q\}^T [N_d]^T \{F_t\} \, dA \tag{4.29}$$

C) Work done by body force

$$W_b = \int_V \{q\}^T [N_d]^T \{F_b\} \, dV \tag{4.30}$$

### 4.4.4 Work done by electric charge

$$W_c = -\int_A [E]^T \{\sigma_q\} dA$$

$$\{E\} = -[B_\phi]\{\phi\}$$

$$W_c = \int_A \{\phi\}^T [B_\phi]^T \{\sigma_q\} \, dA \tag{4.31}$$

### 4.4.5 Potential Energy

$$U = \frac{1}{2}\int_A (\{\varepsilon_m\}^T F_b + \{\varepsilon_b\}^T M + \{\varepsilon_s\}^T F_s) dA \tag{4.32}$$

Putting equations (6) (17) (18) (21) in equation (4.32), we get

$$U = \frac{1}{2}\int_A \{q\}\{^T [B_m]^T ([A_T][B_m]\{q\} + [B_T][B_b]\{q\}) + \{q\}^T [B_b]^T ([B_T][B_m]\{q\} + [D_T][B_b]\{q\}) \cdots$$

$$\cdots + \{q\}^T [B_s]^T ([C_T][B_s]\{q\}) - \{q\}^T [B_m]^T [e]^T \{E\} (h_k - h_{k-1}) - \{q\}^T [B_b]^T [e]^T \{E\} \frac{1}{2}(h_k^2 - h_{k-1}^2)\} \, dA$$

$$= \frac{1}{2}\{q\}^T [K_{uu}]\{q\} + \frac{1}{2}\{q\}^T [K_{u\phi}]\{\phi\} \tag{4.33}$$

$$[K_{uu}] = \int_A \{[B_m]^T [A_T][B_m] + [B_m]^T [B_T][B_b] + [B_b]^T [B_T][B_m] \cdots$$

$$\cdots + [B_b]^T [D_T][B_b] + [B_s]^T [C_T][B_s]\} \, dA \tag{4.34}$$

$$\left[K_{u\phi}\right]=\iint_A\left\{\left[B_m\right]^T[e]^T\left[B_\phi\right]\left(h_k-h_{k-1}\right)+\left[B_b\right]^T[e]^T\left[B_\phi\right]\frac{1}{2}\left(h_k^2-h_{k-1}^2\right)\right\}dA \tag{4.35}$$

## 4.5 Equations of motion

Hamilton's principle is employed here to derive finite element equations and given as below

$$\int_{t_1}^{t_2}\left\{\delta\left(K-P+W_e-W_m+W_{ext}\right)\right\}dt=0 \tag{4.36}$$

Putting equations (23) (26) (29) (30)(31) (33) into (36) we get,

$$[M_{uu}]\{\ddot{q}\}+[K_{uu}]\{q\}+\left[K_{u\phi}\right]\{\phi\}=\{F_m\} \tag{4.37}$$

$$\left[K_{\phi u}\right]\{q\}+\left[K_{\phi\phi}\right]\{\phi\}=\{F_q\} \tag{4.38}$$

where

Mechanical forces are given as

$$\{F_m\}=[N_d]^T\{F_p\}+\iint_A[N_d]^T\{F_t\}dA+\iint_V[N_d]^T\{F_b\}dV \tag{4.39}$$

Electrical forces are given as

$$\{F_q\}=\iint_A\left[B_\phi\right]^T\{\sigma_q\}dA \tag{4.40}$$

There are two layers one is sensor and other one is actuator layer. The above equations (4.37) and (4.38) can be written as

$$[M_{uu}]\{\ddot{q}\}+[K_{uu}]\{q\}+\left[K_{u\phi_s}\right]\{\phi_s\}+\left[K_{u\phi_a}\right]\{\phi_a\}=\{F_m\} \tag{4.41}$$

$$\left[K_{\phi_a\phi}\right]\{\phi_a\}+\left[K_{\phi_a u}\right]\{q\}=\{F_{q_a}\}$$

$$\left[K_{\phi_s\phi}\right]\{\phi_s\}+\left[K_{\phi_s u}\right]\{q\}=\{F_{q_s}\}$$

External applied charge on sensor layer is zero $\{F_{q_s}\}=0$, then the voltage developed on sensor layer is given as

$$\{\phi_s\} = -\left[K_{\phi_s u}\right]\{q\}\left[K_{\phi_s\phi}\right]^{-1} \tag{4.42}$$

Put $\{\phi_s\}$ into above equation (4.41), we get

$$[M_{uu}]\{\ddot{q}\} + [K_{uu}]\{q\} - \left[K_{\phi_s u}\right]\{q\}\left[K_{\phi_s\phi}\right]^{-1}\left[K_{u\phi_s}\right]\{q\} + \left[K_{u\phi_a}\right]\{\phi_a\} = \{F_m\} \tag{4.43}$$

In general, all the structures are lightly damped. Hence adding the artificial linear viscous damping the global equations of motion can be obtained by assembling the elemental equations and is given by,

$$[M_{uu}]\{\ddot{q}\} + [C_{uu}]\{\dot{q}\} + [[K_{uu}] - [K_{u\phi}][K_{\phi\phi}]^{-1}[K_{\phi u}]]\{q\} = \{F_m\} - [K_{u\phi}][K_{\phi\phi}]^{-1}\{F_q\} \tag{4.44}$$

Where $[M_{uu}]$, $[K_{uu}]$, $([K_{u\phi}] = [K_{\phi u}])$, $[K_{\phi\phi}]$ and $\{F_s\}$ are the corresponding global quantities.

$\{\phi_a\}$ is the actuator voltage vector and $[C_{uu}]$ is the structural damping included via Rayleigh damping which is given by,

$$[C_{uu}] = \alpha[M] + \beta[K] \tag{4.45}$$

where $\alpha$ and $\beta$ are the Rayleigh's constant and $[M]$, $[K]$ effective mass and stiffness matrix.

### .6 Negative velocity feedback and vibration control

In order to provide proper velocity information to the piezoelectric actuators, the voltage induced in the sensor layer is differentiated and fed back. Accordingly, a feedback control gain is used to enhance the sensor signal and also to change its sign before the voltage is injected into the piezoelectric actuators [8].

*onstant-gain negative velocity feedback*

In this method of control, the sensor signal is differentiated so that strain rate (related to velocity) information is obtained and the actuator voltage is given by

$$\phi_a(t) = -G_c\dot{\phi}_s(t) \tag{4.46}$$

The velocity feedback can enhance the system damping and therefore effectively control the oscillation amplitude decays the feedback voltage also decreases. This will reduce the effectiveness at low vibration levels for a given voltage unit. In order to provide proper velocity information to the piezoelectric actuators, the voltage induced in the sensor layer is differentiated and fed back. Accordingly, a feedback control gain is used to enhance the sensor signal and also to change its sign before the voltage is injected into the piezoelectric actuators [18].

Once the governing equations are derived, the genetic algorithm are used to optimize the location of actuators. The genetic algorithm requires an objective function or optimization. The minimizing of dissipation of energy explained in the previous hapter is taken as the objective function for this optimizing technique. This chapter xplains the development of genetic algorithms and the various variables used in detail in is chapter.

### .1 Description

Genetic algorithms were developed by Holland in 1975. Although these lgorithms emerged simultaneously with two other streams known as evolution strategies ES) and evolutionary programming (EP), GAs are today the most widely known type of volutionary algorithms. Differing from conventional search technique, the common eatures of this algorithm is to simulate the search process of natural evolution and take dvantage of the Darwinian survival of the fittest principle. In short, evolutionary lgorithms start with an arbitrarily initialized population of coded individuals, each of hich represents a search point in the space of potential solution. The goodness of each ndividual is evaluated by a fitness function which is defined from the objective function f optimization problem.

Then, the population evolves toward increasingly better regions of search space y means of both random and probabilistic (or deterministic in some algorithms) iological operations.

The basic operators used in GAs consists of selection (selection of parent for reeding), crossover (the exchange of parental information to create children) and utation (the changing of an individual). In addition, following the Darwinian Theory, an itism operator (protection of best individuals) is found in more elaborated GAs.

Note however here that the ergodicity of the biological operators used in GAs akes them potentially effective at performing global search (in probability) . also, GAs ve the attribute of a probabilistic evolutionary search (although it is most commonly referred to as a randomized search), and are neither bound to assumptions regarding continuity nor limited by required prerequisites.

The GA technique has been theoretically and empirically proven to provide robust searches in complex spaces. Much of the early work of GAs used a universal internal representation involving fixed-length binary chromosomes with binary genetic operators.

Consequently, most of the binary developed (which could fill several volumes) is based on binary coding. In developing the fundamental theorem of GAs, Holland (1975) focused on modeling ideal simple GAs to better understand and predict GA behavior with above –average fitness receive exponentially increasing trials in subsequent generations. Many properties in terms of the binary genetic operator's effectiveness were concluded from this theorem. However, it is pointed out that these properties give some limited insight into the GA behavior. Mitchell believes that a more useful approach to understanding to understanding and predicting GA behavior would be analogous to that of statistical mechanics in physics whose traditional goal is to describe the laws of physical systems in terms of macroscopic quantities, such as pressure and temperature, rather than in terms of the macroscopic particles (molecules) making up the system. Such an approach will aim at laws of GA behavior described by more macroscopic statistics such as "mean fitness in the population" or "mean degree of symmetry in the chromosomes" rather than keeping track of the huge number of individual components in the system (e.g. the exact genetic composition of each population). Regarding theoretical guidelines about which GA to apply, the real problems encountered by GAs usually compel tailoring the GA at hand as the use of different encoding and operator variants could provide different solutions.

One realizes that there are therefore no rigorous guidelines for predicting which variants and more particularly, which encoding, works the best. By addressing the binary/floating point debate, the work it is confirmed that there is no best approach and that the best representation depends on the problems at hand.

As one can understand, there are many controversies in the GA community over the approaches used, revealing the GA theory is by no means a closed book (indeed, there are more open questions that solved ones). One final point worth mentioning about the

GA theory is that many of today's algorithms show enormous differences to the original SGA.

**5.2 Fundamentals**

The purpose here is not to give a thorough theoretical analysis of the GAs mechanism, as there are excellent introductory tutorials in the literature. Instead, the objective of this section is to provide some answers to explicit questions one may have about GA. In the following, the structure of a simple GA will be presented along with a general overview of the main techniques/variants that are employed in the GA process. Then, the most important features which differentiate GAs from conventional optimization techniques are described. Eventually, the strengths and weaknesses of GAs are outlined and the type of problems for which the use of these algorithms is pertinent is ndicated.

**5.3 What are GAs ?**

Like all evolutionary algorithms, a GA is a search procedure modeled on the nechanics of natural selection rather than a simulated reasoning process. These lgorithms were originally used for the study of artificial systems. Since their inception GAs have been subject to growing interest as an optimization techniques in nearly all inds of engineering applications. Today, there are so many different GAs that it turns ut, there is no rigorous definition of GAs accepted by all in the evolutionary omputation community that differentiate GAs from other evolutionary computation ıethods. Indeed, some currently used GAs can be very far from Holland's original onception. However, it can be said that most methods called "GAs" have at least the ollowing elements in common: populations of individuals, selection according to the dividual's fitness, crossover to produce new individuals, random mutation of new dividuals, and replacement of the populations. These elements are illustrated next, in e description of how a simple GA works. A typical GA flowchart appears in figure 5.1.

**ow do GAs works?**

GAs are based on the collective learning process within a population of dividuals (trial solutions called chromosomes), each of which represents a search point in the space of potential solutions to given problem. The chromosomes code a set of parameters (called genes). The population (of size ns) is generally randomly initialized (at generation ng=0) in the parametric search space. The individuals are evaluated and ranked in terms of a fitness function. Then, the population evolves towards fitter regions of the search space by means of the sequential application of genetic operators.

The basic operators of a simple GA consist of selection, crossover and mutation. Following the Darwinian theory of survival of fittest, an elitism operator is usually found in the generational replacement. A generation is accomplished when the sequence defined by the application of all operators to the individual parents is performed, as illustrated in figure 5.1. The GA produces as many generations as necessary until the convergence criterion is reached. The goal, throughout this process of simulated evolution, is to obtain the best chromosome in the final population to be a highly evolved solution to the problem.

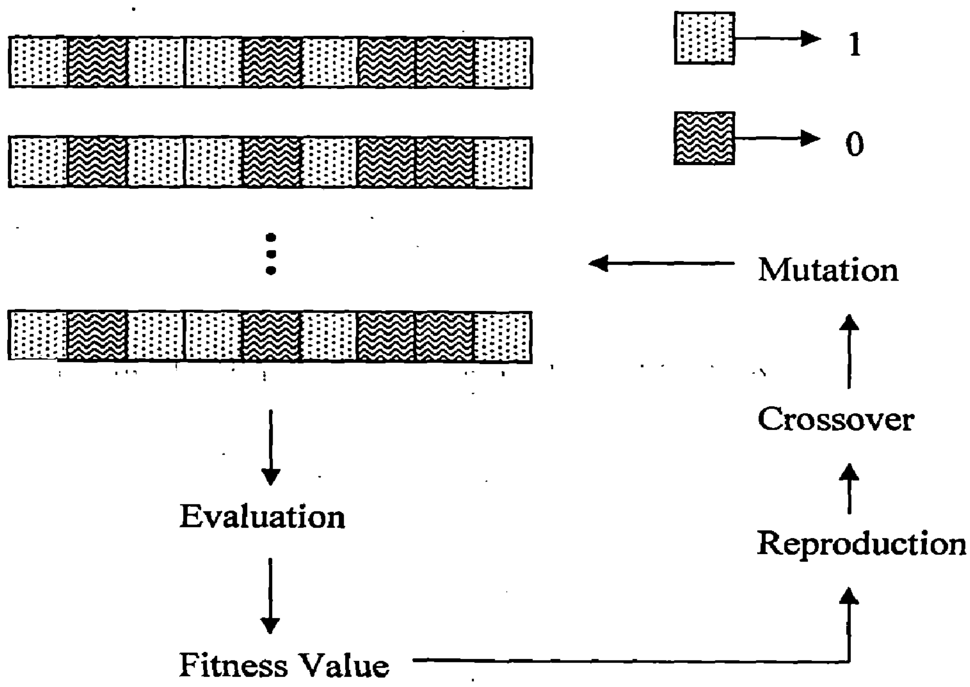



Figure 5.1. Typical Genetic Algorithms

## 5.4 Basic steps in genetic algorithms

### 5.4.1 Encoding scheme

To enhance the performance of GA, a chromosome representation that stores problem specific information is desired. Although GAs were developed to work on chromosomes encoded as binary strings, it is today common knowledge that for numerical optimization problems, one should use a GA with floating point representation. One important point that may, however, not be obvious when one start to use GAs is that the crossover variants used should be appropriate to the encoding used. There indeed exist both conventional (binary) and arithmetical crossover techniques to fit the two different representations. Note that when using the real representation, a chromosome is a vector of *np* genes for the *np* parameters. It should be emphasized here that because much of the early work of GAs used a universal coding involving abstract binary chromosomes (that needed to be decoded), research on GAs has been slow to spread from computer to engineering, and very little theory exist in the literature on real-valued encoding.

### 5.4.2 Fitness function

The fitness plays the role of the environment in which the chromosomes are to be valuated. This is thus a crucial link between the GA and the system. This function can be simply taken as the objective function to optimize or as a transformation of it. It is assumed that the fitness function to be optimized is positive. In cases where the objective function happens to be negative, the fitness function will be transformation of the objective function.

### 5.4.3 Parent selection

Basically, the selection operator determines which of the individuals in the current population will be pass to their genetic material to the next generation. Using the GA language, one says that it builds up the mating pool by selecting ns individuals from the current population. There are many ways t achieves effective selection, including proportionate, ranking and tournament schemes. The key assumption is to give preference to fitter individuals. Using fitness proportionate selection, the number of times individual is expected to reproduce is equal to its fitness divided by the average of fitnesses in the population. The most popular and easiest mechanism is the Roulette wheel selection where each chromosome in the current population has a roulette wheel slot sized in proportion to its fitness. However, depending on the environment proportionate and ranking selection schemes may lead to premature convergence or on the contrary, to a slow finishing. Those are well-known severe technical problems of GAs. However, both problems can be avoided if scaled fitness values are used instead of the original values. Another way to circumvent these technical problems is to use a more adequate selection operator. In many applications, tournament selection has proved to yield superior results to fitness rank selection. In the simplest form, the so called binary selection, two chromosomes are selected randomly from the current population but only the one with higher fitness value is inserted into the mating pool with a probability pt. One interesting feature about this selection scheme is that one can adjust the selection pressure directly from the tournament probability pt (typically larger than 0.5). Regardless of which selection technique is used, the selection operator produces an intermediate population, the mating pool, which consists only of individuals that are members of the current population. The following two operators, crossover and mutation, are then applied to this mating pool in order to generate children.

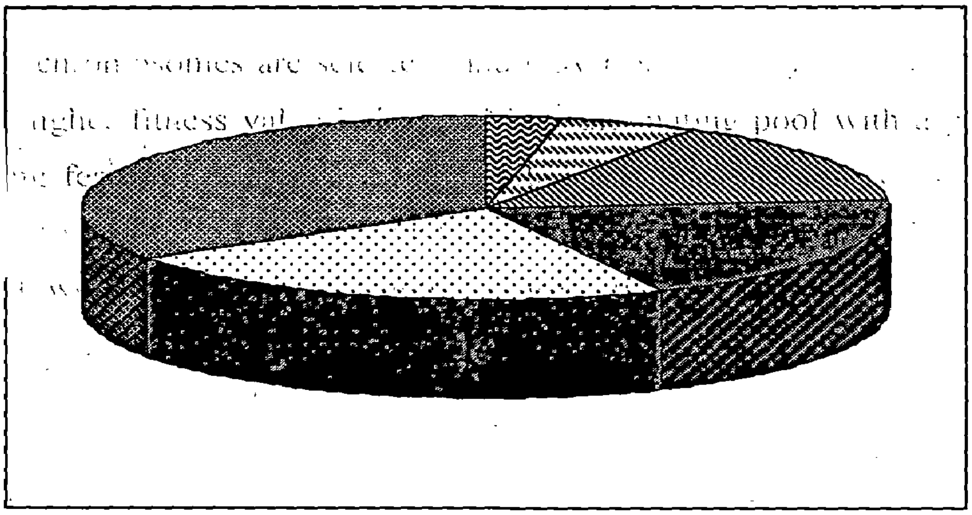


Figure 5.2. Roulette Wheel

### 5.4.4 Crossover

The crossover operator is the key operator to generate new individuals in the population.. In addition, it has been shown in the literature that so-called "deceptive" problems can be made "easy" by the use of an appropriate definition of the crossover function. This operator is applied to each pair of the mating pool with a crossover probability pc, usually taken from [0.6, 1], to produce one or two children. With probability 1-pc, no changes are made to the parents (they are simply cloned), but with probability pc, genetic material is exchanged between the two parents. In the simplest crossover, the single point crossover, a crossover point is randomly selected and the portions of the two chromosomes beyond this point are exchanged. Multipoint crossover is similar except that multiple cross points are chosen at random with no duplication. Uniform crossover generalizes the scheme by making every gene a potential crossover point. Single, multipoint and uniform crossovers are generally considered conventional binary techniques; and when real encoding is used, arithmetic crossovers are the most suited.

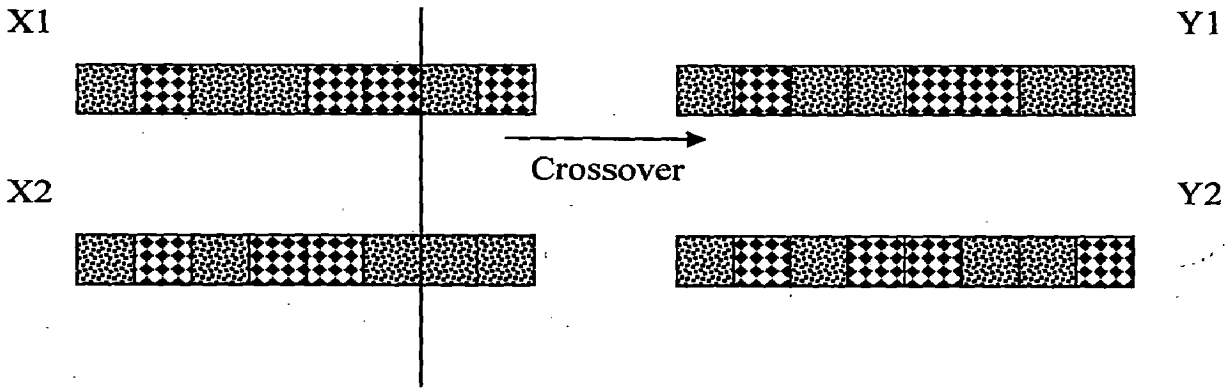



Figure 5.3 Crossover

### 5.4.5 Mutation

This operator should allow a GA the finding of solutions which contains genes that are nonexistent in the initial population. It can also prevent the GA from loosing some genetic material without any chance of adopting it again. Often viewed as a background operator, mutation modifies genes values according to a mutation probability. Using binary encoding, this simply means changing a 1 to a 0 and vice versa with a small probability. Using real encoding, when a global modification called jump mutation is applied, each gene in any chromosome is replaced with a random value (from the entire parametric search space) with probability *pmj*. A "mutation-based" operator can also be applied locally with the creep variant (not a pure mutation operator in the sense of GAs) which consists in the addition or subtraction with probability *pmc* of a small value to the gene (1% of actual gene value).

Whereas the crossover reduces the diversity in the population, the mutation operator increases it again. The higher the mutation probability, the smaller is the degree of premature convergence. A high mutation probability will however transform a GA into some kind of random search algorithm, which is of course not the intention of the algorithm! Mutation probabilities are usually small (so as not to interfere with the combination of the best features of parents made by the crossover operation), and range from 0.001 to 0.10, the higher values being typically applied with real encoding.

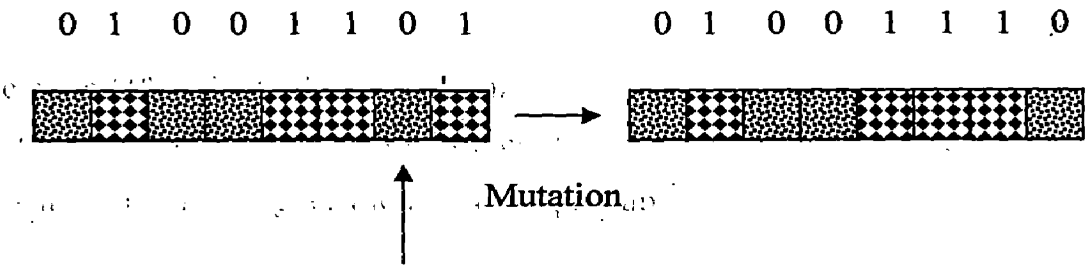



Figure 5.4. Mutation

### 4.6 Replacement strategies

In the simplest form of GAs, when the operation of selection, crossover and utation are completed on the ns individuals of the current population, this entire pulation is replaced with the children created. This is the traditional generational placement. Variations where not all individuals are replaced in each generation exist. e simplest case of such a strategy is the elitist strategy where the individual with the hest fitness (according to the Darwinian theory of survival of fittest) is directly transferred from the old to the new generation and only the other *ns*-1 children are generated by the application of genetic operators.

Generational replacement with probability *pr* is often used in which *ns*x*pr* parents are replaced with children while the *ns*x(1-*pr*) best parents are kept. An alternative to replacing an entire population at once is to replace one organism in the population whenever a new organism is created. This variant is known as a steady-state GA.

### 5.4.7 Convergence criterion

The most widely used stopping conditions are either that a given number of generations have been done already, or that the population has become uniform. When the first condition is chosen, GAs are typically iterated for anywhere from 10 to 500 or more iterations. User defined convergence criterion that are better suited to the problem being solved should be preferred (although most of the studies do not address this problem). It is however not easy to define such a criterion, as it will be shown in this work.

### 5.4.8 Performance criteria

What does it mean for a GA to perform well or properly? Some performance criteria can provide answers to this question. The best fitness reached (best-so-far) is a typical one. One criterion for computational cost is the number of the function evaluations. Indeed, in almost all GA applications, the time to perform a function evaluation vastly exceeds the time required to execute other parts of the algorithms which are thus considered to take negligible time). Note that because randomness plays a large in the each run (two number with different random number seeds will generally produce different output), often GA researches report statistics (about the best fitness for instance) averaged over many different runs of the GA on the same problem.

Besides the genetic operators presented here, there exist a number of different operators (inversion, recording), in addition to advanced features (diploid, dominant and recessive genes, sharing fitness function) which are used in different applications but not yet widely. GAs is still far from maturity.

### 5.4.9 Method of solution for shape control

The shape of plate is described by the shape of midplane of the plate, which in turn is described by the transverse deflection of each segment. The desired shape $w_d(x)$ of the midplane is given a priori.

The input voltage to the actuators required to bend the plate to the desired shape are to be obtained by minimizing the error between the desired shape and the achieved shape. Here the achieved shape represents the shape formed from the effect of the actuators. A simple way of defining an error function is by taking the square of the difference between the desired shape and achieved transverse displacement and integrated throughout the length. A error will be a function of applied voltages.

$$J = \sum_{i=1}^{N} (w_{d_i} - w_{c_i})^2 \tag{5.1}$$

Where N= number of piezoelectric layers

$w_{d_i}$ = deflection of desired

$w_{c_i}$ = deflection of calculated shape

The forgoing error function is, however, independent of the extension of the niddle plane.

In practical applications, there will be a lower and upper bound constraint on the nput voltages to be supplied to the actuators. This may be due to saturation of the ctuator for voltages of higher magnitudes or due to the limitations of the voltage supply. 'herefore these bounds on the magnitude of the voltages should be considered for the nalysis. The constraints are given by equation (5.2).

$$\begin{aligned} &\bar{V}_{min}\} - \{\bar{V}\} \le 0 \\ &\bar{V}\} - \{\bar{V}_{min}\} \le 0 \end{aligned} \tag{5.2}$$

/here $\{\bar{V}_{min}\} = \{0, v_{min,1}, 0, v_{min,2}, \ldots\ldots, v_{min,N}, 0\}$

$\{\bar{V}_{max}\} = \{0, v_{max,1}, 0, v_{max,2}, \ldots\ldots, v_{max,N}, 0\}$

$v_{min,i}$ is the lower bound of the *ith* actuator

$v_{max,i}$ is the upper bound of the *ith* actuator

A MATLAB code is written to perform numerical studies and has been used to solution for various static and dynamic problems. The variation of maximum static deflection with various parameters of the laminated plate has been studied. Free vibration analysis of isotropic and orthotropic is carried out using the code to obtain the natural frequencies and mode shapes. The program is executed for active vibration control of piezolaminated composite plate for various boundary conditions. First the model is validated with the results available in the literature. Then the code is used to solve different problems for static and dynamic solutions. The influence of feedback control gain, ply orientation and the sensor/actuators position on the response of the plate is analyzed. At the end, desired shape of plate is obtained by applying a suitable voltage on piezoelectric actuator. Optimum voltage on the patch is decided with the help of GA.

### 5.1 Validation of the model

The theoretical formulation of the plate model is first validated by comparing the 'esults obtained by Liu, Peng and Lam [10].

#### ı.1.1 Static Validation

The piezoelectric square laminated composite plate integrated with piezoelectric ensors and actuators is considered. The plate dimensions considered are 400mm x 00mm x 1mm. The plate is constructed of four layers unidirectional graphite/epoxy omposites. Piezoceramic layers having thickness of 0.1 mm are bonded symmetrically n both surfaces of the plate. The material properties of the plate T300/976 and ezoelectric material PZT (G1195N) are shown in Table 1. [-30/30/30/-30] angle of ply ientation is considered. The plate is discritised into identical 8x8 plate elements. Simply pported boundary condition is considered.

| Properties | T300/976 | PZT(G1195N) |
|---|---|---|
| $E_{11}$(Gpa) | 150 | 63 |
| $E_{22}$(Gpa) | 9 | 63 |
| $\upsilon_{12}$ | 0.3 | 0.29 |
| $G_{12}$(Gpa) | 7.1 | 24.8 |
| $G_{23}$(Gpa) | 2.5 | - |
| $G_{13}$(Gpa) | 7.2 | - |
| $\rho$(Kg/m$^3$) | 1600 | 7600 |
| $d_{31}$(pm/V) | - | -166 |
| $d_{32}$(pm/V) | - | -166 |

Table 6.1. Material properties of the plate and piezoelectric material

In this analysis all the Piezoceramic are act as a actuators. The Piezoceramic are polarized by applying voltage across the thickness of the piezoelectric layers. Due to this the plate gets deflected transversely. The comparison of the result given by [10] and present finite element model are plotted graphically as shown in figure 6.1.

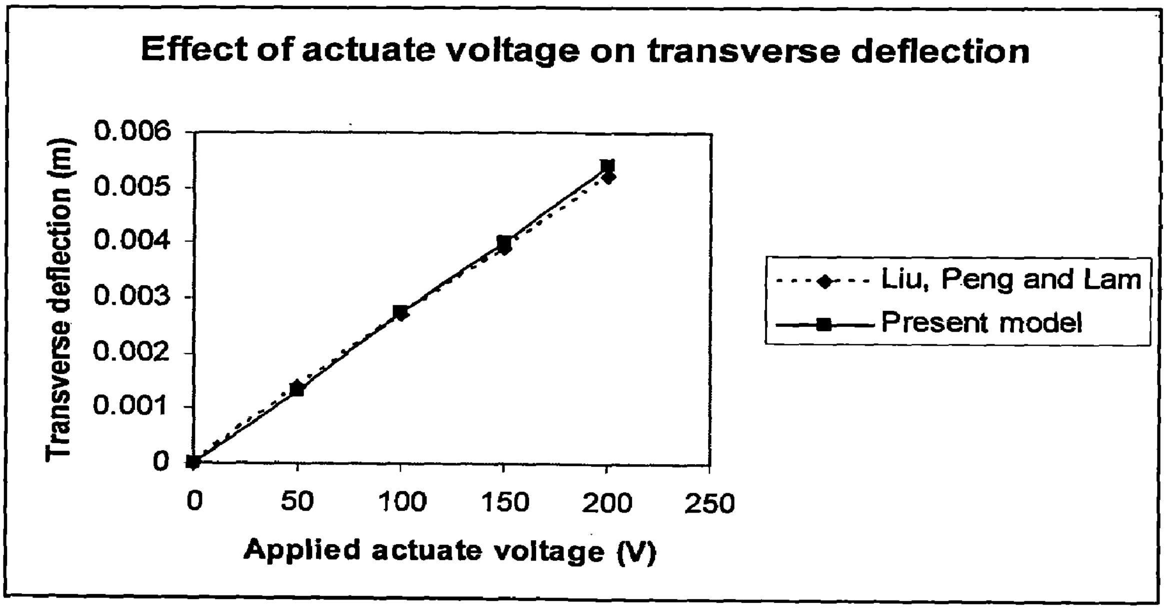



Figure 6.1. Comparison of effect of actuate voltage on the transverse deflection

### 6.1.2 Dynamic Validation

Table 6.2 shows the first six natural frequencies of the composite plate under [-30/30/30/-30] symmetric ply orientation. The natural frequencies calculated by present model are agreed very well to the result given in the literature.

| Mode No. | Natural Frequency (rad/sec) | |
|---|---|---|
| | By Liu, Peng and Lam | By present model |
| 1 | 174.007 | 175.15 |
| 2 | 383.470 | 388.23 |
| 3 | 464.894 | 470.35 |
| 4 | 658.804 | 676.22 |
| 5 | 754.18 | 768.28 |
| 6 | 936.055 | 950.29 |

Table 6.2. Comparison of the natural frequency of present model

## 6.2 Static Analysis

### 6.2.1 Effect of symmetric ply orientation on the transverse deflection with applied loads

The program was executed to obtain the results using different fiber orientation angles for different boundary conditions.

**Material and Geometric properties**

$E_1 = 1 \times 10^6$ N/m$^2$

$E_2 = 30 \times 10^6$ N/m$^2$

$\nu = 0.3$

$G_{12} = E_1 / (1 + \nu) = G_{13} = G_{23}$

$\rho = 2752.3$ Kg/m$^3$

Plate dimensions 10m x 10m x 0.1m

Load =500 N

The material properties and geometric properties of the plate are stated above. The Table 6.3 gives the deflections for different orientations angles in a composite plate for a simply supported and a cantilevered plate.

| Sl. No. | Orientation angle of fibers (degrees) | Maximum deflection for simply supported plate (mm) | Maximum deflection for Cantilever plate (mm) |
|---|---|---|---|
| 1 | 0 | 0.5606 | 5.5126 |
| 2 | 15 | 0.5245 | 5.2145 |
| 3 | 30 | 0.4646 | 4.6158 |
| 4 | 45 | 0.4394 | 4.5125 |
| 5 | 60 | 0.4646 | 4.4849 |
| 6 | 75 | 0.5606 | 4.1226 |
| 7 | 90 | 0.5606 | 3.5126 |
| 8 | 105 | 0.5245 | 4.1312 |
| 9 | 120 | 0.4646 | 4.5016 |
| 10 | 150 | 0.4646 | 4.7012 |
| 11 | 180 | 0.5606 | 5.2541 |

Table 6.3. Deflection for different orientation angle of the fibers for a simply supported and cantilever plate

It was observed that for simply supported plate the deflection was found to be maximum for orientation angles 0, 45 and 90 degrees, which means that the plate offers minimum stiffness at these orientation angles. Where as the deflection is minimum for angles 22.5 and 67.5 degrees. It means the plate offers maximum stiffness when the fibers are oriented at these angles. The following figure 6.2 and figure 6.3 shows the change in deflection with respect to the change in orientation angle of the fibers graphically for simply supported plate and cantilever plate respectively.

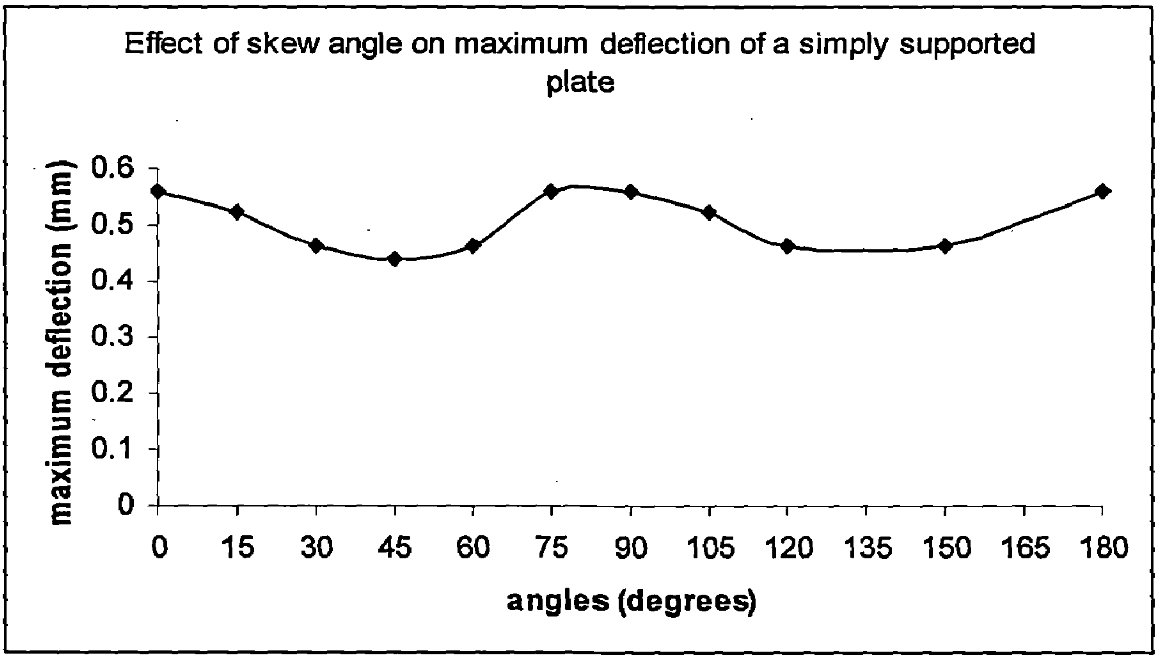



Figure 6.2 Variation of maximum deflection with the orientation angle for simply supported plate

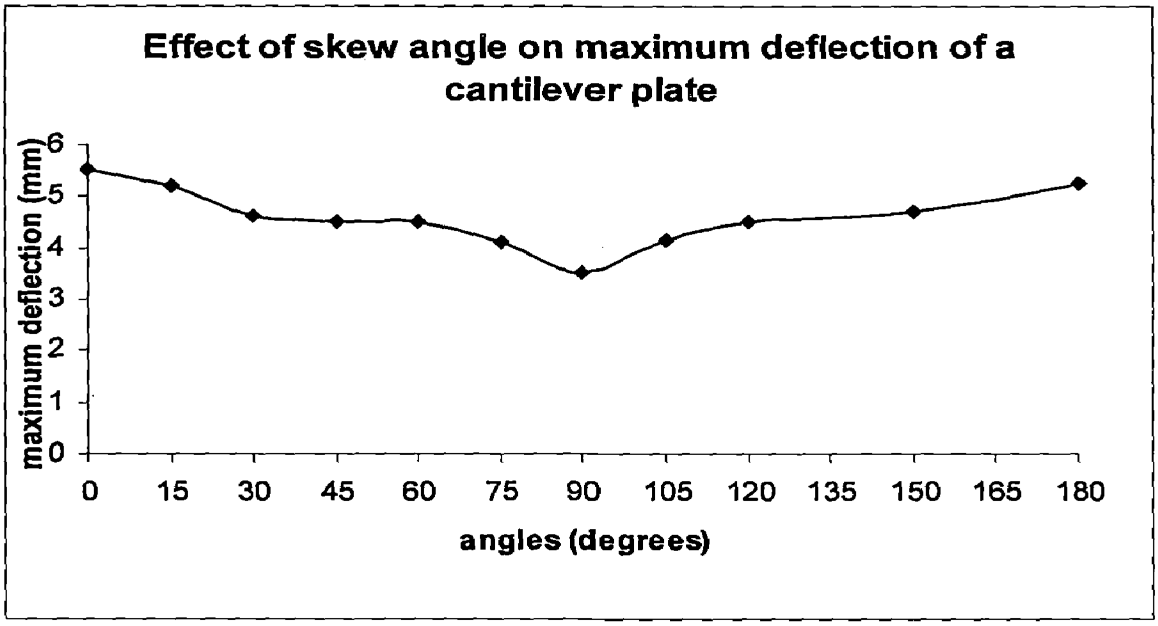



Figure 6.3 Variation of maximum deflection with the orientation angle for cantilever plate

### 6.2.2 Transverse deflection of plate with uniformly distributed loading

Geometry of plate and piezomaterial are as below

Length = 400 mm

Width = 400 mm

Thickness = 0.8 mm

Thickness of piezolayer = 0.1 mm

The plate (T300/976) and PZT (G1195N) properties are shown in following table

| Properties | T300/976 | PZT(G1195N) |
|---|---|---|
| $E_{11}$(Gpa) | 150 | 63 |
| $E_{22}$(Gpa) | 9 | 63 |
| $\upsilon_{12}$ | 0.3 | 0.29 |
| $G_{12}$(Gpa) | 7.1 | 24.8 |
| $G_{23}$(Gpa) | 2.5 | - |
| $G_{13}$(Gpa) | 7.2 | - |
| $\rho$(Kg/m$^3$) | 1600 | 7600 |
| $d_{31}$(pm/V) | - | -166 |
| $d_{32}$(pm/V) | - | -166 |

Table 6.4. Properties of plate and piezomaterial

The program was executed for a plate with piezolayer for given material properties and geometric properties. The 4x4 finite element model has been made for given plate dimensions. Simply supported boundary condition is to be considered. The load applied on the plate is uniformly distributed load having magnitudes of 500, 1000 and 2000 N/m$^2$. The following Table 6.5 shows the transverse deflection of plate for various longitudinal distances (centerline) for various uniformly distributed loads.

| Longitudinal distance (mm) | Transverse deflection (mm) for udl=500 N/m$^2$ | Transverse deflection (mm) for udl=1000 N/m$^2$ | Transverse deflection (mm) for udl=2000 N/m$^2$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 50 | 3.2 | 4.6 | 7.2 |
| 100 | 4.1 | 5.8 | 9.5 |
| 150 | 4.9 | 7.1 | 11.7 |
| 200 | 5.5 | 8.3 | 13.2 |
| 250 | 4.9 | 7.1 | 11.7 |
| 300 | 4.1 | 5.8 | 9.5 |
| 350 | 3.2 | 4.6 | 7.2 |
| 400 | 0 | 0 | 0 |

Table 6.5. Transverse deflection of plate for various longitudinal distances (centerline) for various uniformly distributed loads

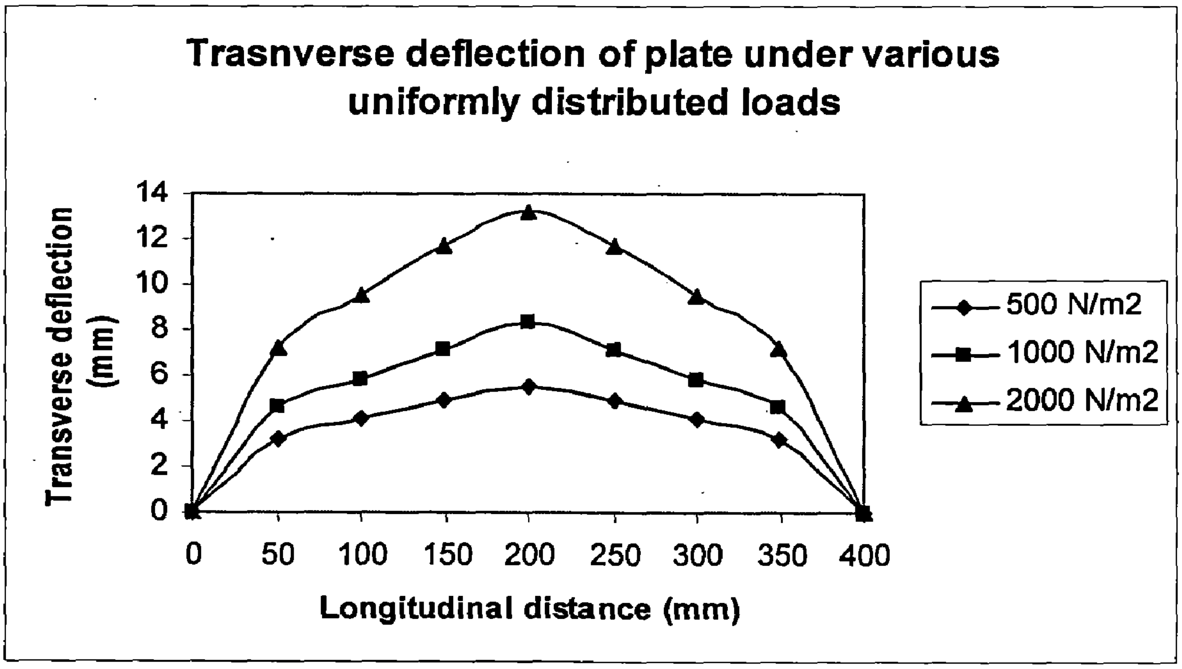



Figure 6.4 Variation of transverse deflection of plate for various longitudinal distances (centerline) for various uniformly distributed loads

### 6.3 Dynamic analysis

A numerical example is presented to demonstrate the use of the code for simulating the response of a laminated composite plate with integrated piezoelectric sensors and actuators in active vibration control.

Geometry of plate and piezomaterial:

Length = 400 mm

Width = 400 mm

Thickness = 0.8 mm

Thickness of piezolayer = 0.1 mm

The plate is constructed of four layers of T300/976 unidirectional graphite/epoxy composites and piezoelectric PZT G1195 Piezoceramic, are symmetrically bonded on the upper and lower surfaces of the plate. It is assumed that the upper layer behaves like sensor and lower layer behaves like actuator. This Piezoceramic layers are modeled as two additional layers. The material properties of plate and Piezoceramic are shown in following Table 6.6.

| Properties | T300/976 | PZT(G1195N) |
|---|---|---|
| $E_{11}$(Gpa) | 150 | 63 |
| $E_{22}$(Gpa) | 9 | 63 |
| $\upsilon_{12}$ | 0.3 | 0.29 |
| $G_{12}$(Gpa) | 7.1 | 24.8 |
| $G_{23}$(Gpa) | 2.5 | - |
| $G_{13}$(Gpa) | 7.2 | - |
| $\rho$(Kg/m$^3$) | 1600 | 7600 |
| $d_{31}$(pm/V) | - | -166 |
| $d_{32}$(pm/V) | - | -166 |

Table 6.6. Properties of plate and piezomaterial

### 6.3.1 Natural frequency of plate

The program was executed for natural frequency of laminated plate having various ply orientation angles. First the program is executed for symmetric laminate ply orientation and then for antisymmetric ply orientation of the plate. For calculation of the natural frequency simply supported boundary condition is taken. The conditions for simply supported boundary condition are stated in chapter 4. These natural frequencies are shown in following Tables 6.7 and 6.8.

1. **Symmetric Laminate ply orientation**

| Mode | -15/15/15/-15 | -30/30/30/-30 | -45/45/45/-45 |
|---|---|---|---|
| 1 | 167.75 | 175.15 | 179.20 |
| 2 | 359.80 | 388.23 | 406.70 |
| 3 | 487.70 | 470.35 | 457.70 |
| 4 | 657.00 | 676.22 | 687.5 |
| 5 | 701.60 | 768.28 | 839.4 |
| 6 | 977.88 | 950.29 | 871.7 |

Table 6.7. Calculated natural frequencies of plate for symmetric ply orientation

2. **Antisymmetric laminate ply orientation**

| Mode | -15/15/-15/15 | -30/30/-30/30 | -45/45/-45/45 |
|---|---|---|---|
| 1 | 168.4 | 178.9 | 183.9 |
| 2 | 363.2 | 401.1 | 441.5 |
| 3 | 488.7 | 471.8 | 441.5 |
| 4 | 673.7 | 715.8 | 735.9 |
| 5 | 695.5 | 760.9 | 855.3 |
| 6 | 995.9 | 953.9 | 855.3 |

Table 6.8. Calculated natural frequencies of plate for antisymmetric ply orientation

The same plate is then fixed in cantilevered position and natural frequencies foı various symmetric ply orientations are calculated. The following Table 6.9 shows the natural frequencies of composite plate for cantilever boundary condition.

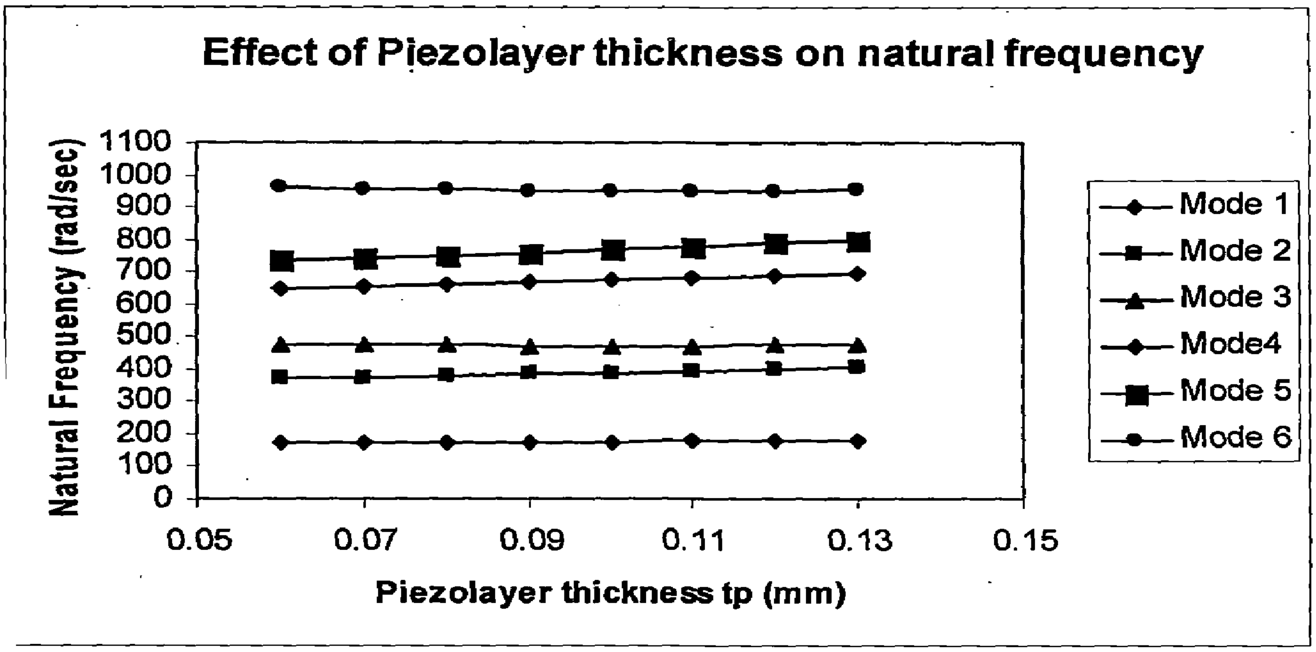
| Mode | -15/15/15/-15 | -30/30/30/-30 | -45/45/45/-45 |
|---|---|---|---|
| 1 | 37.48 | 32.51 | 27.58 |
| 2 | 73.84 | 78.45 | 78.81 |
| 3 | 193.18 | 192.29 | 167.69 |
| 4 | 237.86 | 220.17 | 225.49 |
| 5 | 293.36 | 287.62 | 280.18 |
| 6 | 442.43 | 457.10 | 465.57 |

Table 6.9. Calculated natural frequencies of cantilever plate

### 6.3.2 Effect of piezolayer thickness on the natural frequency

The above composite plate is considered to study how natural frequency varies with increase in piezolayer thickness. The material and geometrical properties of plate are :aken as stated above. The thickness of the piezolayer is varies from 0.06 to 0.13 and ıatural frequency of the composite plate are calculated. The program was executed by aking simply supported boundary condition of a plate. A symmetric [-30/30/30/-30] ply ›rientation for layered plate is considered. A plot showing variation of piezolayer hickness with natural frequency is shown in figure 6.5.

From figure 6.5 it is observed that increase in piezolayer thickness slightly ıcreases the natural frequency of the composite plate. This is due to increase in the tiffness of plate as the Piezoceramic is getting bonded with the plate surface. The ıcrease in mass of the Piezoceramic has negligible effect.




gure 6.5. Effect of piezolayer thickness on the natural frequency of plate

**iation of natural frequency with orientation angle of plate**

e above plate is considered to study the how natural frequency varies ı angle of the fiber in composite material changes. The material properti‹ properties are taken from above example. The plate is fixed as cant ınd the only one layer of plate is considered. In this case study the orien e fiber are varied from 0 degree to 90 degrees. The program was execut‹ entation angles of plate and corresponding natural frequencies are noted. ' natural frequencies are then plotted graphically as shown in figure 6.6.

; observed that as we increase the orientation angle of a fiber in comp plate, the natural frequency increases and then decreases. The nε s minimum when the orientation angle is 0 degrees. It increases with inc ɜ up to 45 degrees, and then decreases in the same manner till 90 deg result it is concluded that the plate offers minimum stiffness when angle of the fiber is 0 degrees or 90 degrees that is when they are alon; ɜxes, and maximum stiffness when the fibers are oriented at an angl‹

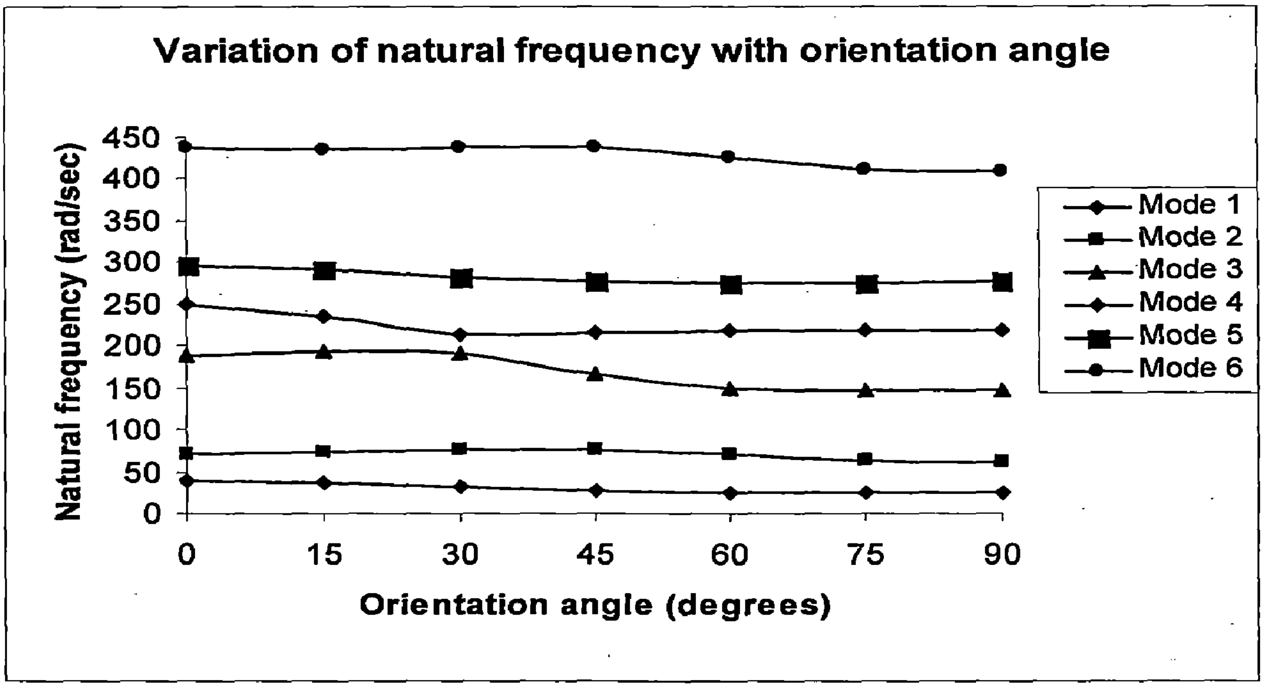



Figure 6.6. Variation of natural frequency with orientation angle for a plate

### 6.3.4 Variation of Fundamental natural frequencies with patch coverage area

In this problem, the piezoelectric patches are placed over the surface and the variation of the natural frequency is studied. Firstly the natural frequency of the bare composite plate is calculated. Then the patch is covered all over the plate and the coverage area is gradually reduced to observe the changes in the natural frequency of the plate.

| Mode no. | Natural frequency of bare plate | Natural frequency of plate with patch coverage | | | | |
|---|---|---|---|---|---|---|
| | | 100% | 56.25% | 25% | 6.25% | 0% |
| 1 | 175.2 | 163.73 | 169.2 | 172.69 | 174.44 | 175.2 |
| 2 | 388.2 | 356.58 | 374.9 | 376.1 | 385.39 | 388.2 |
| 3 | 470.4 | 422.8 | 437.2 | 445.6 | 460.3 | 470.4 |
| 4 | 676.2 | 610.3 | 634.9 | 655.1 | 670.5 | 676.2 |
| 5 | 767.3 | 701.6 | 720.5 | 738.5 | 757.9 | 767.3 |
| 6 | 950.3 | 875 | 895.3 | 929.7 | 947.1 | 950.3 |

Table 6.10. Natural frequencies for a simply supported plate for different coverage area of the piezoelectric patch

However the results obtained do not recommend the optimum coverage area of he patch over the plate, but these results can be used as guidelines for selecting the patch :overage area depending upon the application and the natural frequencies of the system.

The Table 6.10 gives the natural frequencies for the bare plate and plate with iezoelectric patch over it. The readings are taken for coverage area 6.25%, 25%, 6.25%, and 100% of the plate. As expected, it was observed that the natural frequency is ıaximum for the bare plate and as we go on covering the plate with the patch natural ·equency goes on decreasing. The decrease in natural frequency is minimum for 5% ɔverage area and it is maximum for coverage area 100% i.e. patch all over the plate. his reduction in natural frequency is due to the increase in the mass. The stiffness has ss effect compared to the effect of increase in mass. This variation in natural frequency ith coverage area is plotted graphically as shown in figure 6.7.

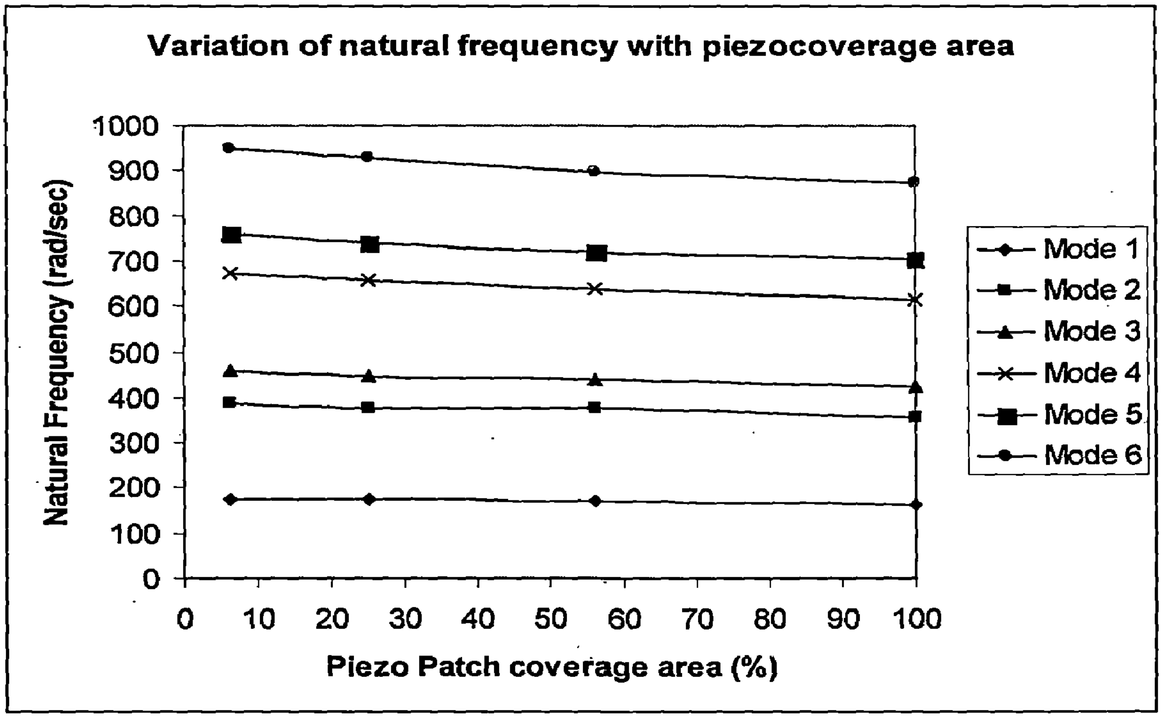



Figure 6.7. The variation of natural frequency with piezoelectric patch coverage area for a simply supported plate

### .3.5 Effect of feedback control gain on transient response

The same configuration of laminated composite plate material and geometrical roperties given in above problem is considered again for active vibration control. The minated composite plate is integrated with sensors and actuators on the surface. In ctive vibration control, the upper piezoceramics are served as sensors and lower one rved as actuators. The composite laminated plate is discritized into 64 elements and a mply supported boundary condition is used.

To control the free vibration of plate, the collocated sensors and actuators should coupled into sensors/actuator pair through closed control loops. In this active control a nple negative velocity feedback control algorithms is used. It is assumed that the plate vibrating freely due to initial disturbance. The initial displacement for each degree of edom is given for dynamic analysis. Newmark-$\beta$ direct integration method is used to culate the transient response of the plate. The parameters $\gamma$ and $\beta$ are taken as 0.5 1 0.25 respectively.

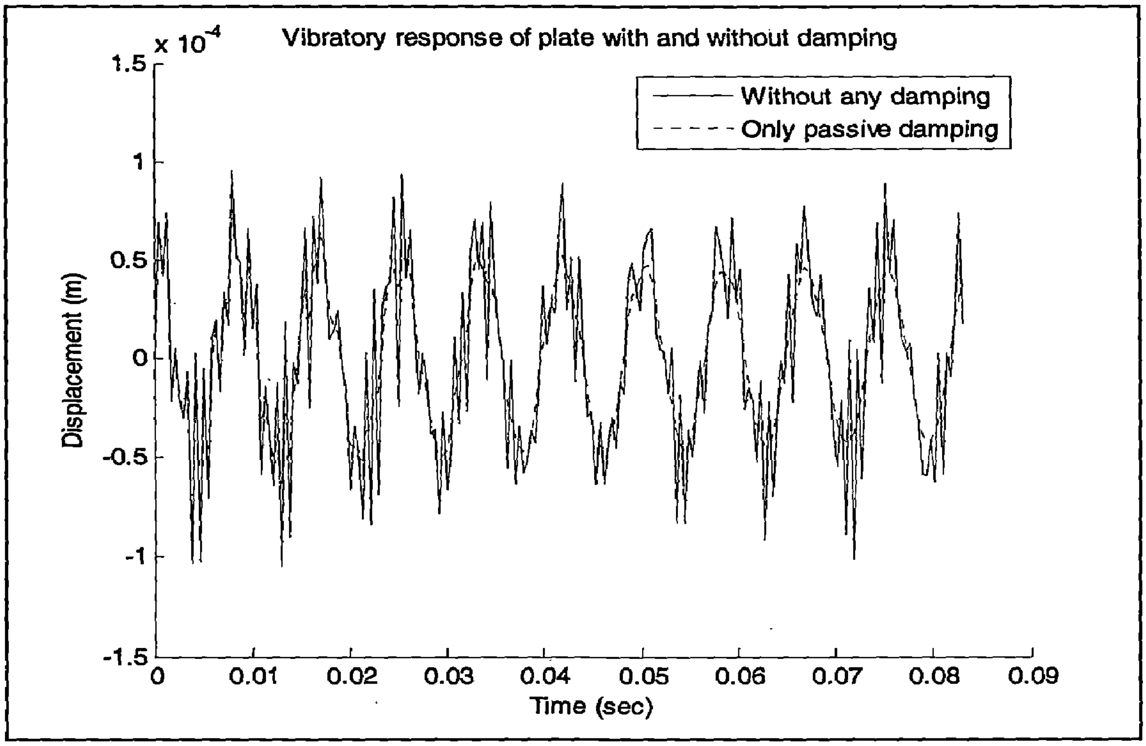



Figure 6.8. Vibratory response of plate with and without damping

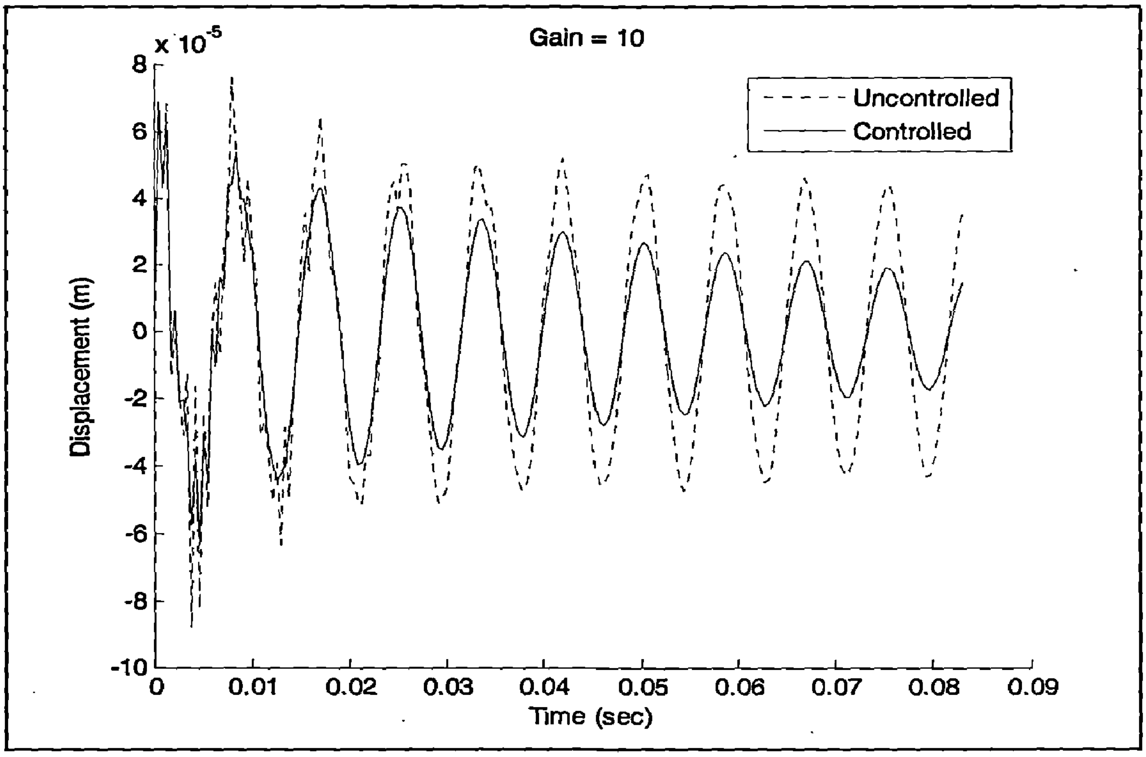



Figure 6.9. The effect of feedback control gain (G=10) on the plates response

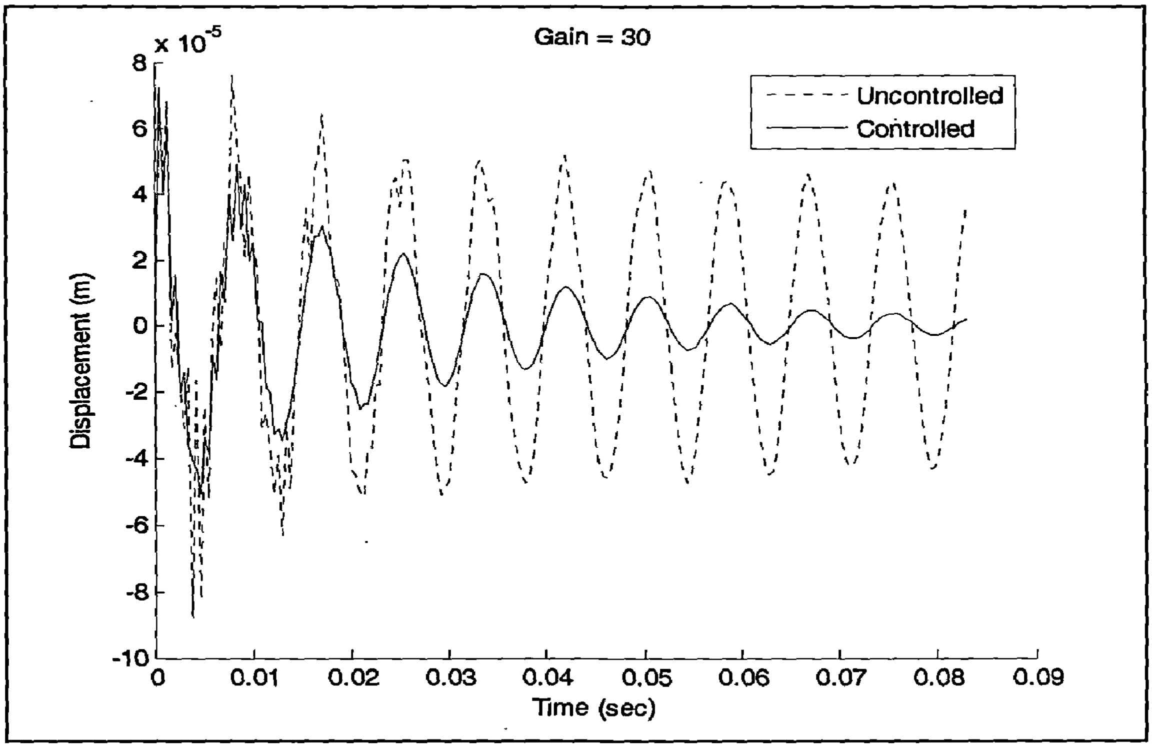



Figure 6.10. The effect of feedback control gain (G=30) on the plates response

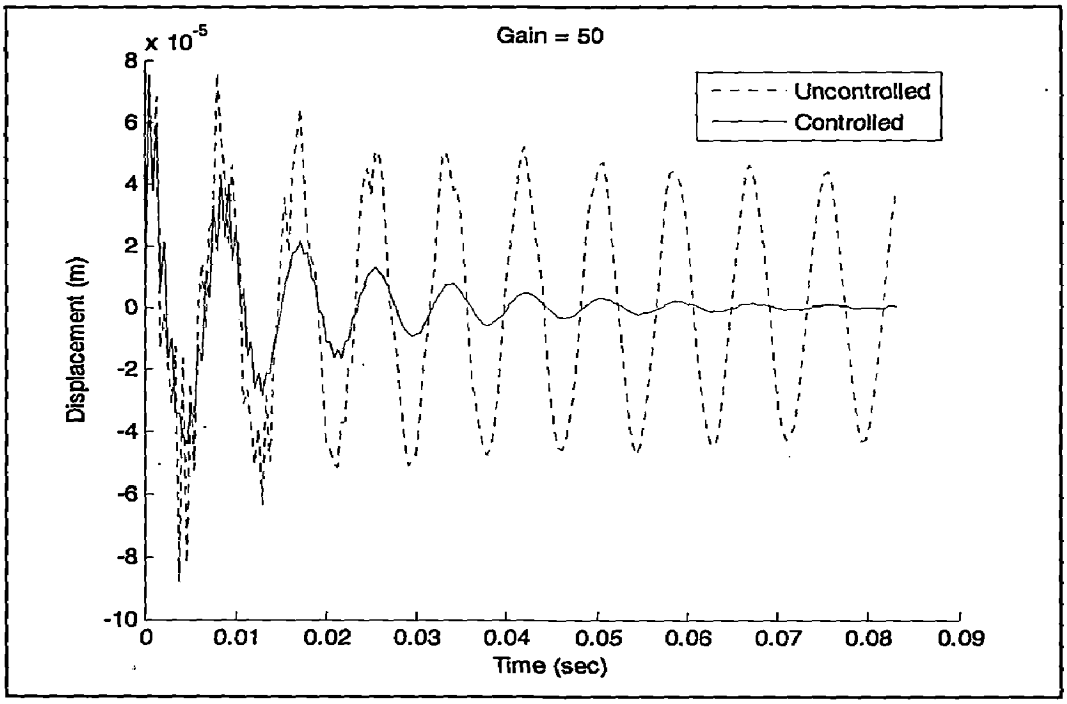



Figure 6.11. The effect of feedback control gain (G=50) on the plates response

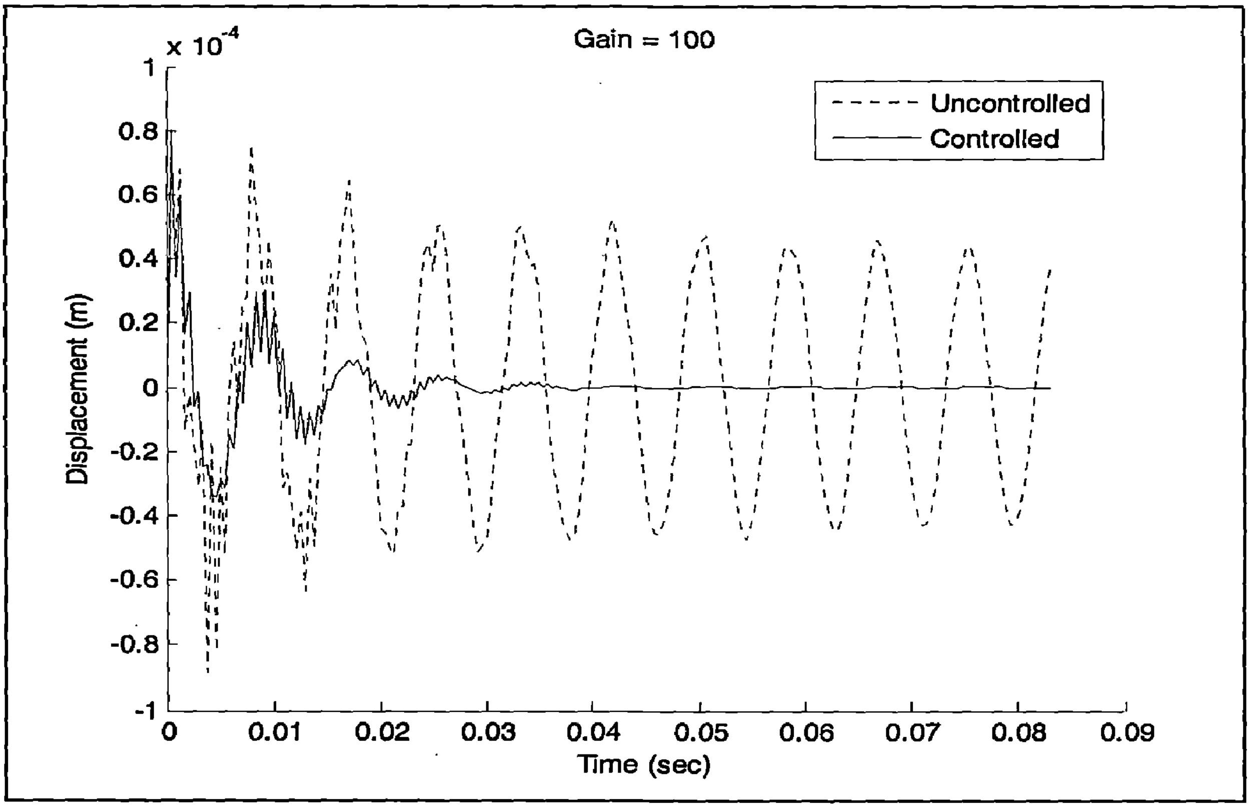
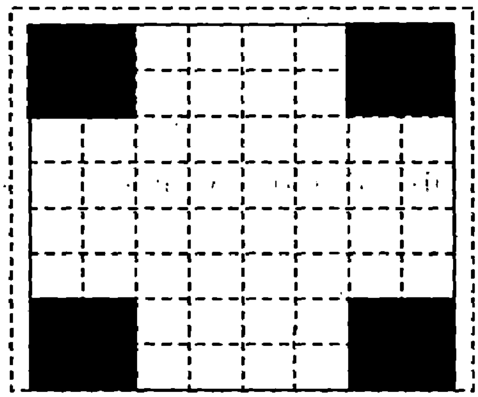


Figure 6.12. The effect of feedback control gain (G=100) on the plates response

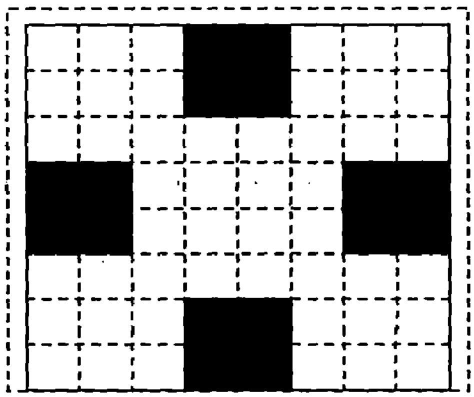
Figures 6.8 to 6.12 shows the effect of feedback control gain on the transient esponse of the plate. It can be seen that with higher control gain, the vibration of plate is amped out more quickly.

### 6.3.6 Effect of sensor/actuator pairs position on the plate response

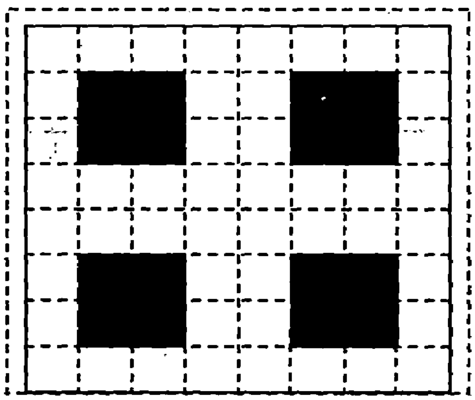
The effect of the sensor/actuator pair's position on the response of the simply supported plate is investigated. Four pairs of piezoelectric sensors/actuators are bonded on different positions of the upper and lower surfaces of the plate as shown in figure 6.13. Each sensor actuator patch has dimensions of 100 mm x 100 mm. it can be seen from figure 6.14 (a) (b) (c) (d) that when the sensor/actuator pairs are bonded on the center of the plate, the control effect is best.



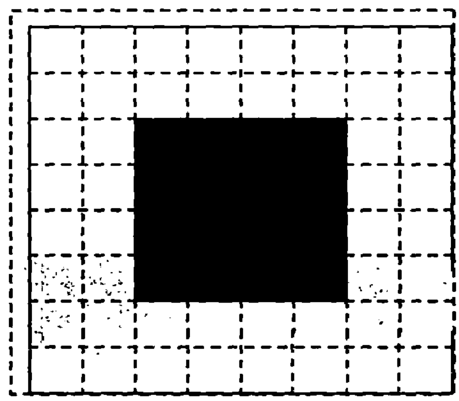
Position A



Position B

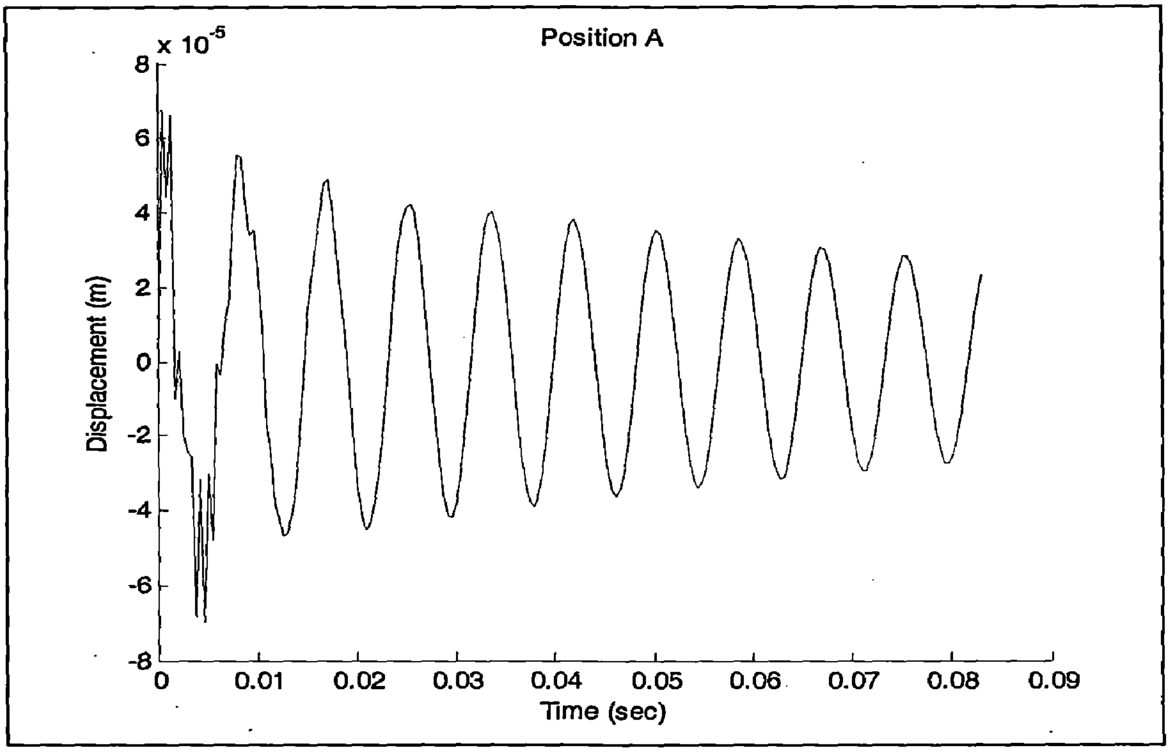


Position C

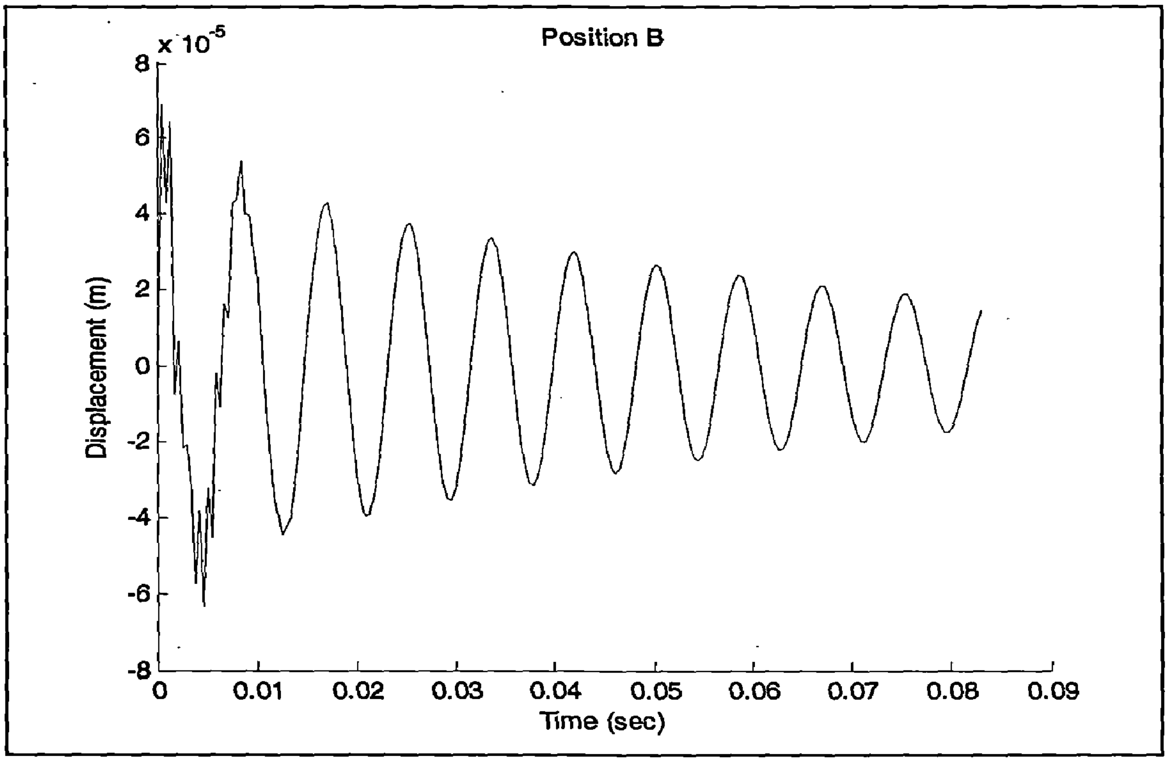


Position D

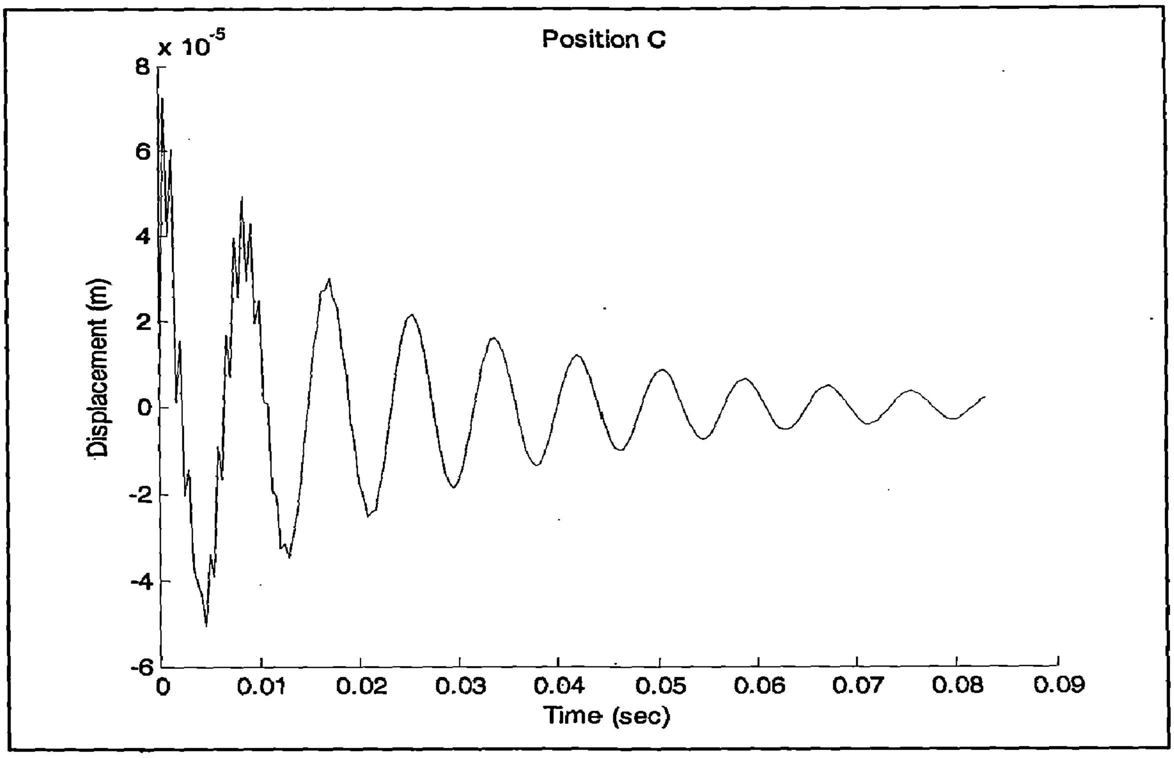
Figure 6.13. The positions of sensor/actuator pairs


Position A
x 10^-5
8
6
4
2
0
-2
-4
-6
-8
Displacement (m)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
Time (sec)

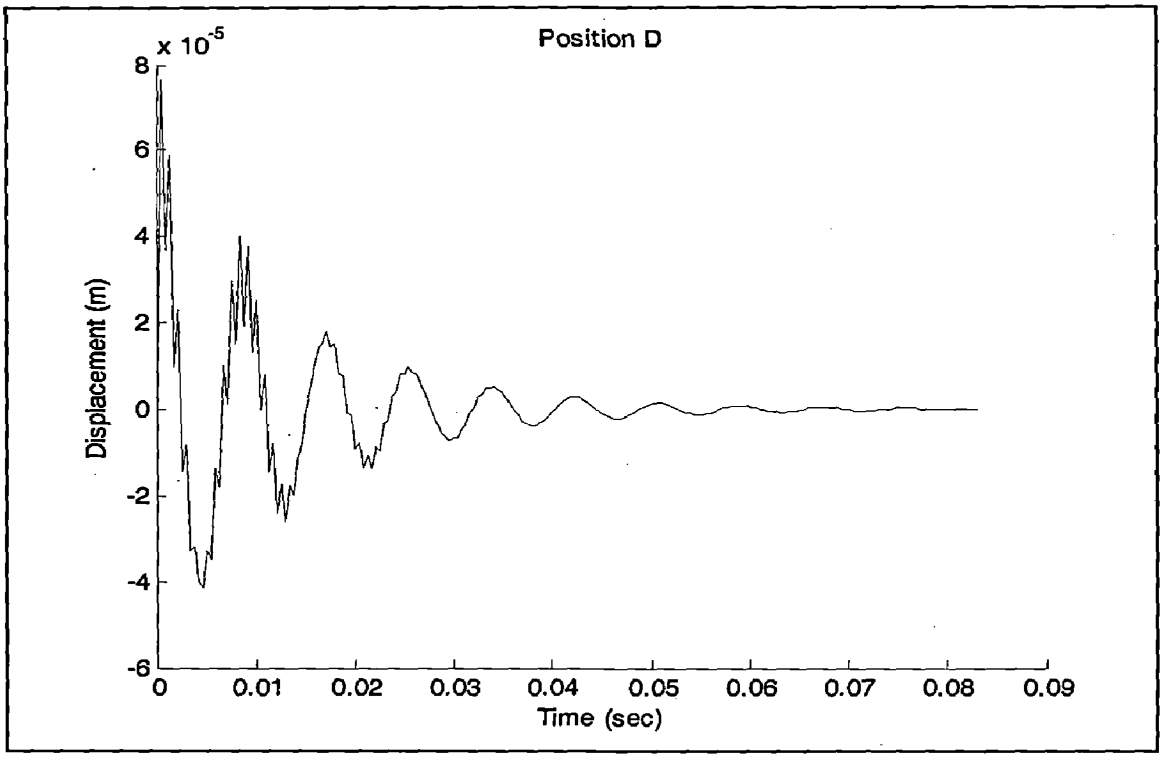
(a)


Position B
x 10^-5
8
6
4
2
0
-2
-4
-6
-8
Displacement (m)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
Time (sec)

(b)




(c)




(d)

Figure 6.14. The effect of sensor/actuator pairs position on the plate response (G=20)

### 6.3.7 Mode shapes

First four mode shapes of composite plate are shown below for simply supported and cantilever plate.

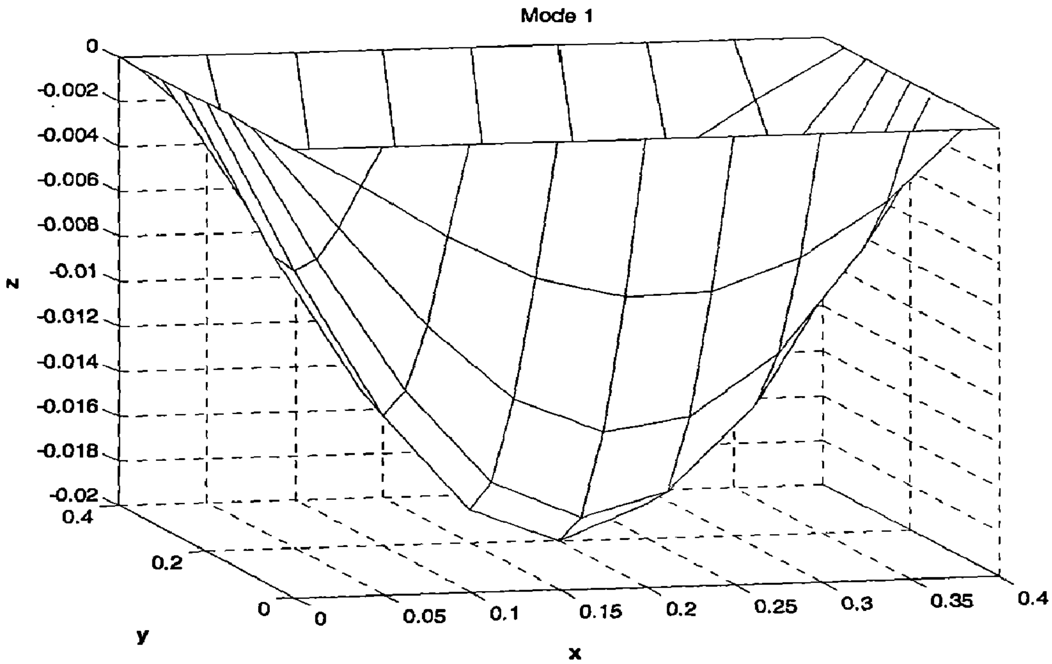



Figure 6.15. 1st Mode shape of simply supported piezolaminated plate

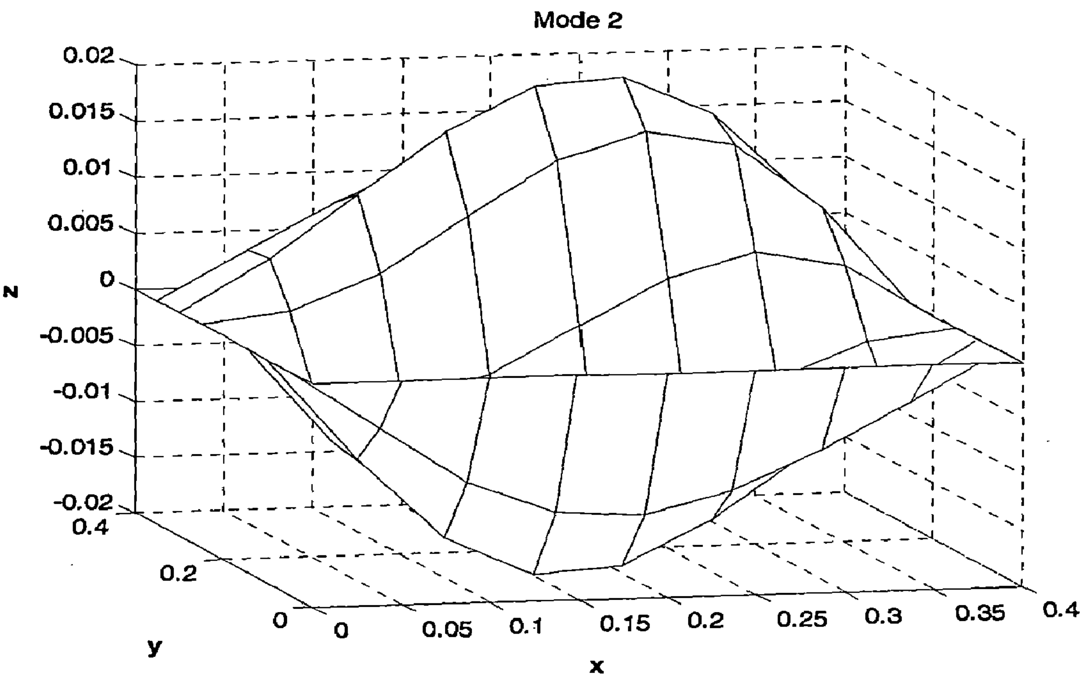



Figure 6.16. 2nd Mode shape of simply supported piezolaminated plate

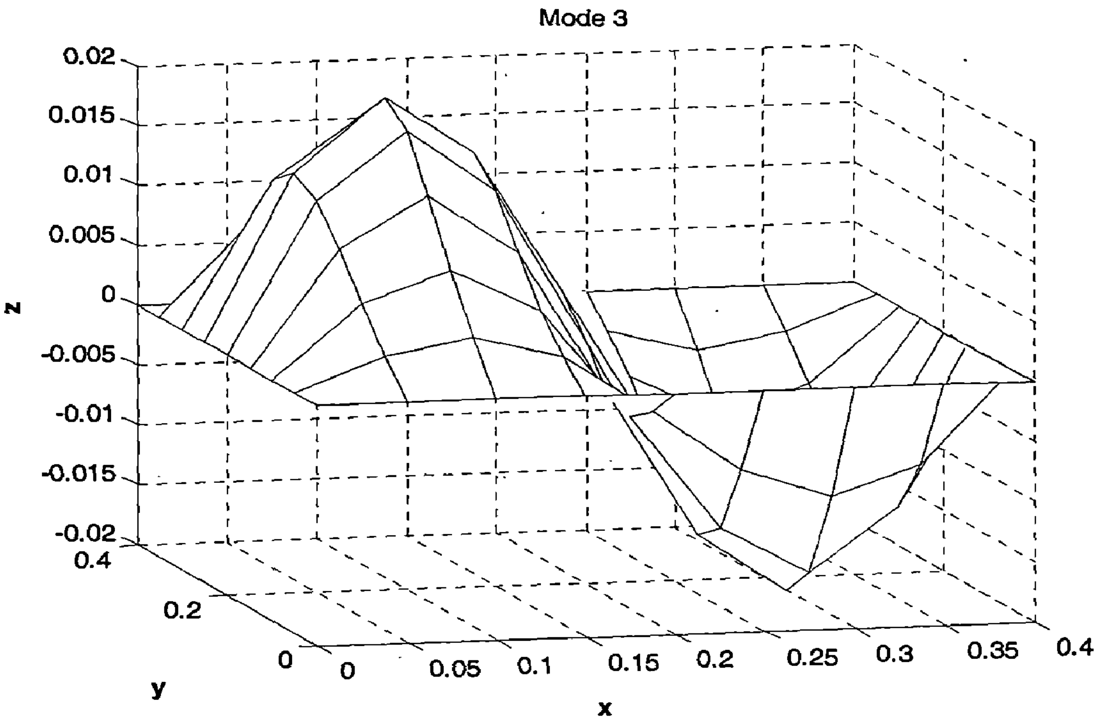



Figure 6.17. 3rd Mode shape of simply supported piezolaminated plate

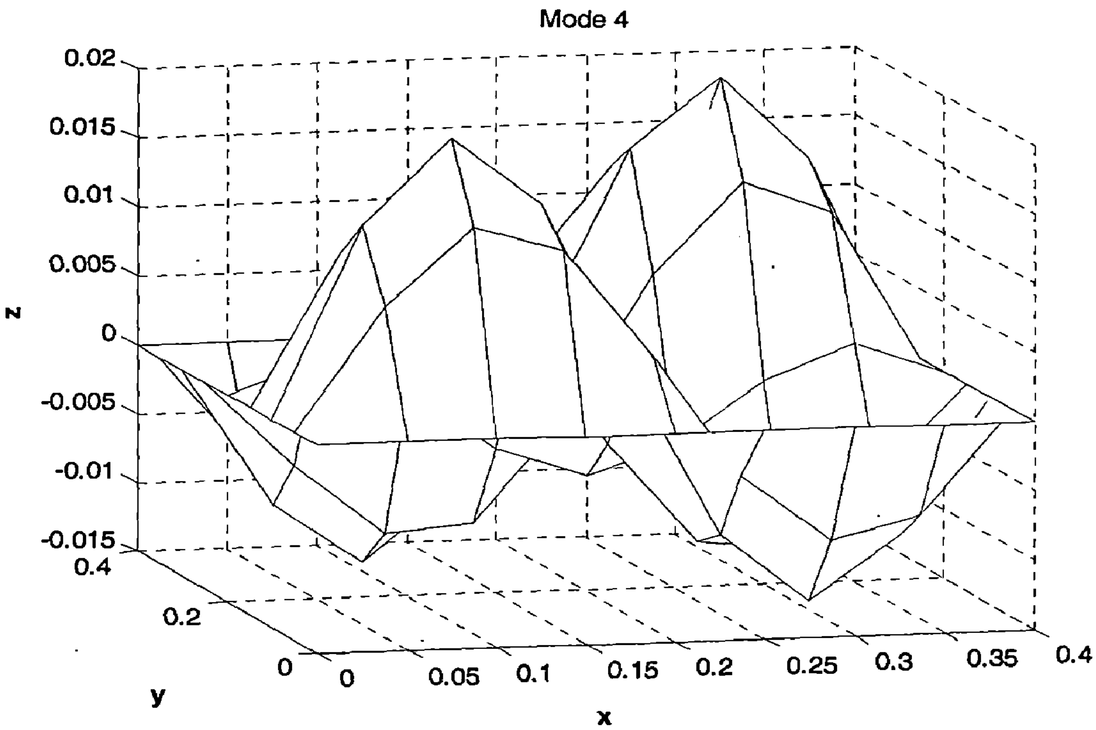



Figure 6.18. 4th Mode shape of simply supported piezolaminated plate

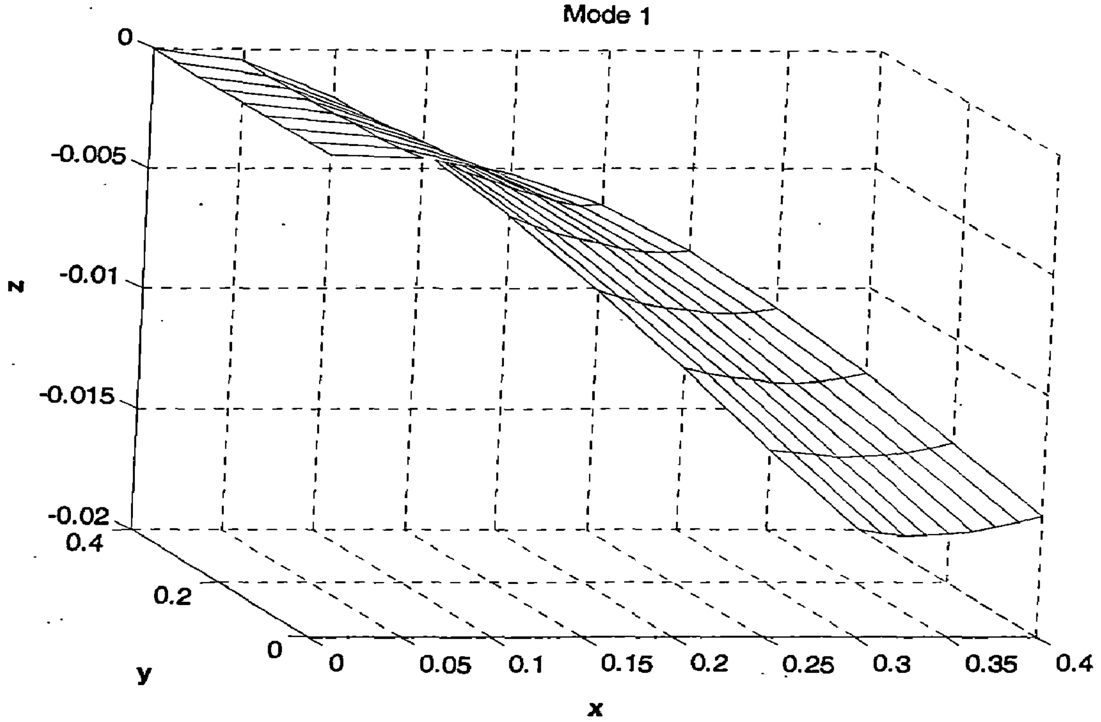



Figure 6.19. 1st Mode shape of cantilevered piezolaminated plate

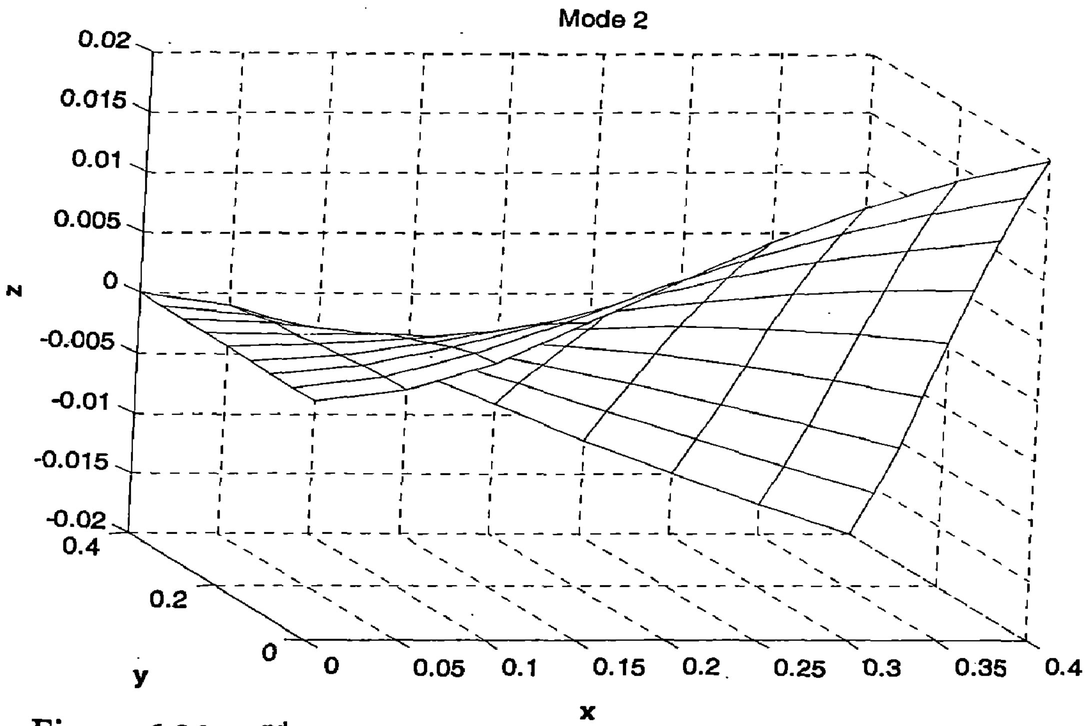



Figure 6.20. 2nd Mode shape of cantilevered piezolaminated plate

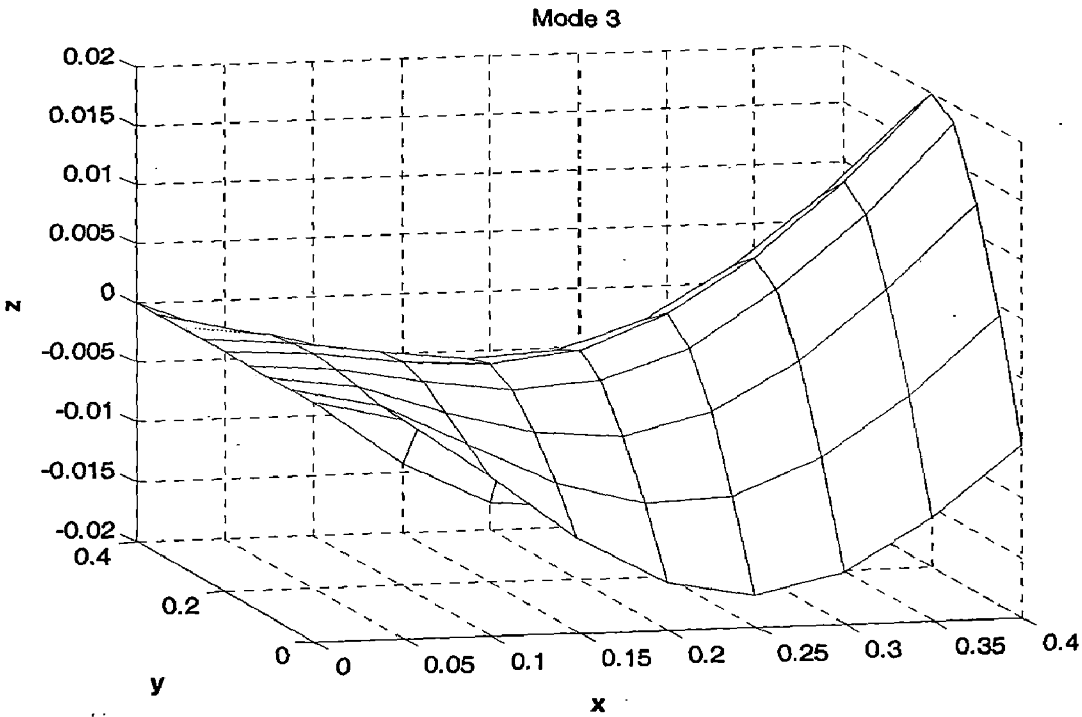



Figure 6.21. 3rd Mode shape of cantilevered piezolaminated plate

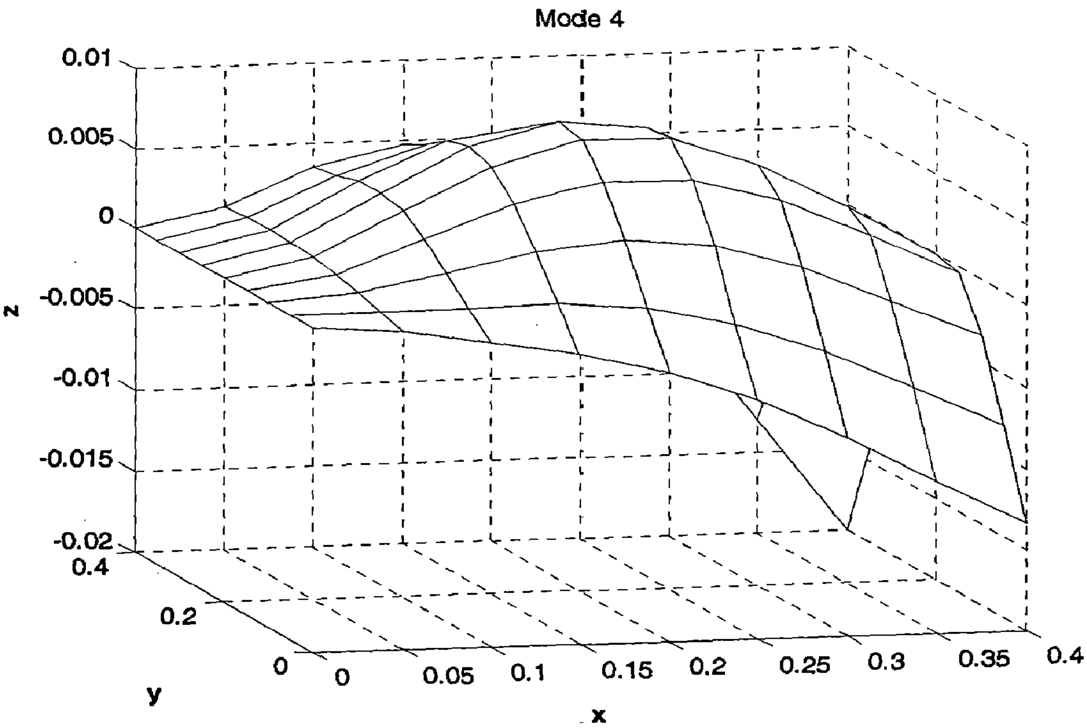



Figure 6.22. 4th Mode shape of cantilevered piezolaminated plate

## 6.4 Shape control of composite plate

### 6.4.1 Plate completely covered by piezolayer

The piezolaminated plate is considered for how shape varies with application of voltage at actuator layer. The plate is integrated with piezomaterial on both surfaces and both layers upper and lower act as actuators. The properties of plate and piezoelectric material are stated in previous problem. The boundary condition applied to the plate is simply supported.

First, a uniformly distributed load of 50 N/m$^2$ is applied on the composite plate. The plate will get displaced from its original position due to application of load. Then by keeping same load on the plate apply voltage on actuator. Increase the voltage of the actuator so that the plate can achieve the original shape. Readings are taken for various applied actuator voltages and plotted graphically as shown in figure 6.23.

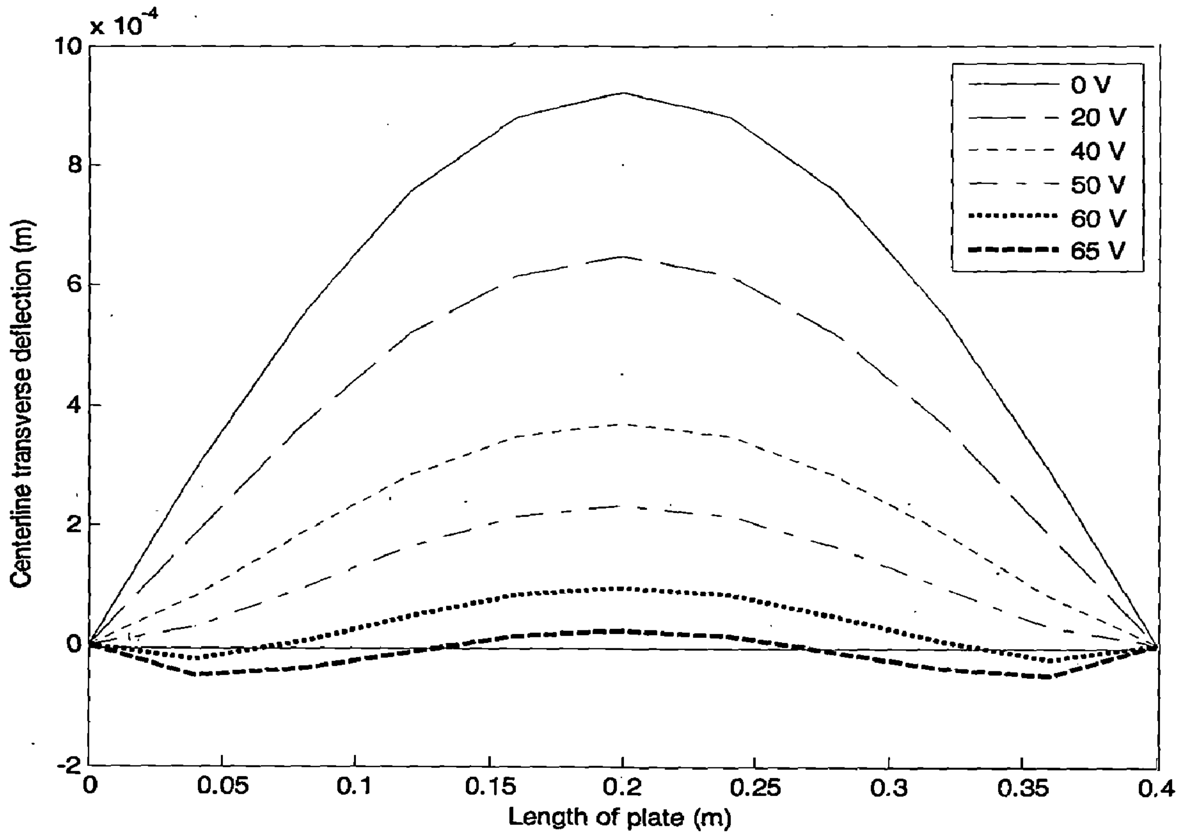



Figure 6.23. Shape control of plate completely covered by piezolayer

From figure 6.23 it is seen that as increase in the actuator voltage the transverse deflection of the plate decreases. It means that the disturbed shape of composite plate by a uniformly distributed load gets its original position by applying a voltage on the actuator. From figure 6.23 it is studied that for voltage (V) = 65V the plate gets its original position.

### 6.4.2 Plate having piezopatches with uniform voltage

The same plate is then bonded with actuators at different places on the upper and lower surfaces instead of completely piezolayer on the surface. Figure 6.24 shows the placement of the actuators onto the plate. The whole plate is modeled as 10x10 equal parts. Figure 6.24 shows the placement of actuators on the composite plate. The load on the plate is kept constant (udl=50 N/$m^2$). The boundary condition applied to the plate is simply supported.

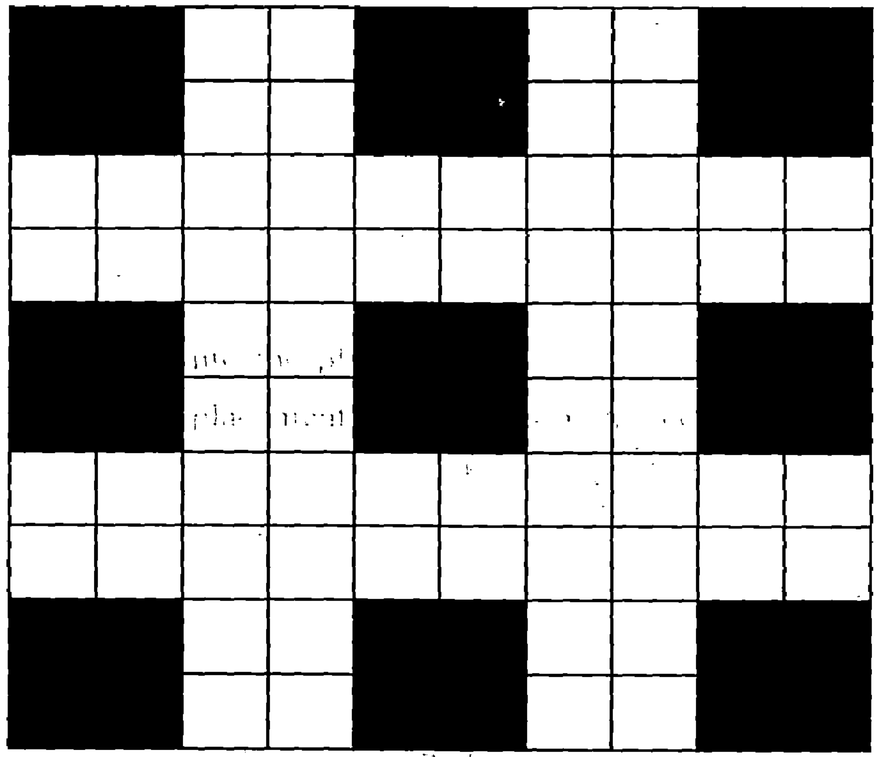


Figure 6.24. Position of actuators on the composite plate

Then apply a uniform voltage to the actuators so that the strain develops in the actuators and plate gets its original position. The voltage on the actuators goes on increasing upto the extent the plate completely gets its original position. The figure 6.25 shows the shape control of composite plate for different voltages. From figure 6.25 it is seen that for voltage = 200V the distorted shape of plate gets its original position. The voltage required in this case is more than the voltage required for a plate which is completely covered by piezolayer.

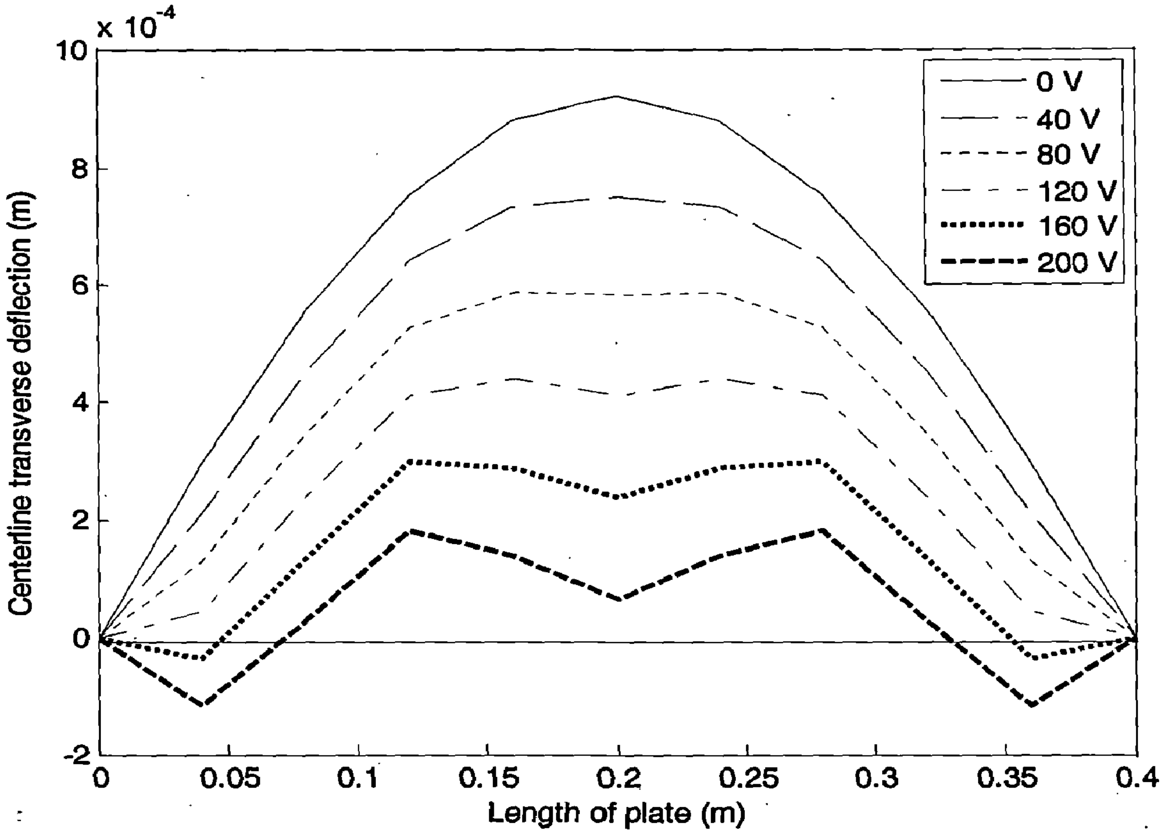



Figure 6.25. Shape control of plate containing piezopatches

### .4.3 Optimum shape control of plate by Genetic Algorithms

The optimum voltage required to get the desired shape is obtained by GA and the ariation of the centerline deflection with length of plate is shown in the figure 6.26. The enetic algorithm parameters are set as: The number of individuals in the population is 30, Maximum number of generation is 100, Crossover probability is 0.8 and Mutation obability is 0.02.

The optimum voltage required for actuators by genetic algorithms are given in Table 6.11.

| | | |
|---|---|---|
| 61.842 | 98.198 | 61.842 |
| 98.198 | 145.62 | 98.198 |
| 61.842 | 98.198 | 61.842 |

Table 6.11. Voltage given by GA for optimum shape of plate

From Table 6.11 it is seen that the voltage required to the actuators is less. In previous case the total voltage required for nine actuators is 1800V but in this case the total voltage required is 1571.56. The percentage of voltage saving is upto 12.69% and the maximum voltage is applied on the actuator which is bonded on the center of plate.

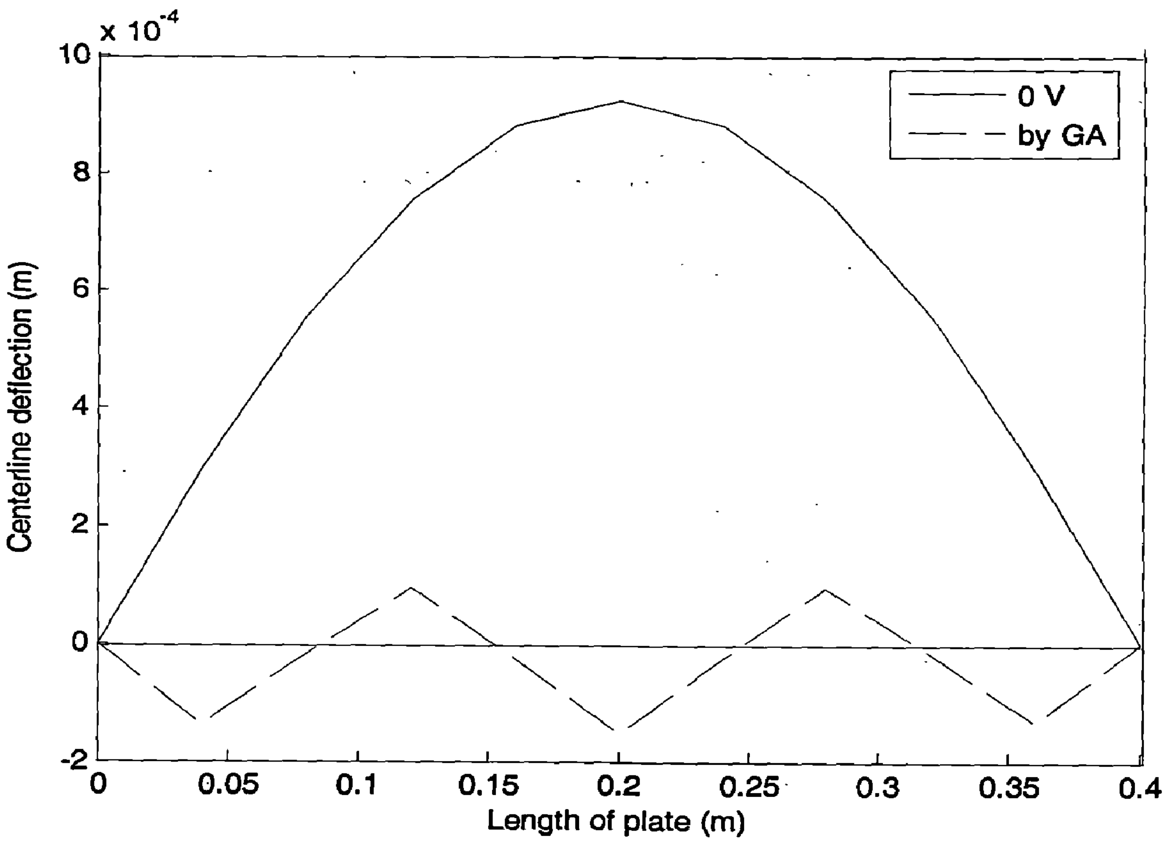



Figure 6.26. Shape control of plate by GA

## 7.1 Conclusions

In this work a finite element model of piezolaminated composite plate based on first order shear deformation theory and linear piezoelectric theory is presented. Finite element has five mechanical degrees of freedom per node and one electrical degree of freedom per piezoelectric layer. Finite element has n number of host structure layers and two piezoelectric layers.

In deriving the finite element mode of piezolaminated composite plate first displacement relation is given followed by strain displacement relation, strain energy equation, electrical energy equation, work done by electrical forces and electrical charges, kinetic energy equation and governing equation of motion has been derived using Hamilton's principle.

Finite element model is validated for static and dynamic analysis for with the results available in the literature. Genetic algorithms is used to find out the optimum voltage in order to obtain desired shape of plate. Based on numerical study following conclusions are drawn

- For simply supported plate the transverse deflection was found to be maximum for symmetric ply orientation angle 0°, 45°, and 90° whereas the minimum deflection was found at symmetric ply orientation angle 22.5° and 67.5°. In case of cantilever plate, the deflection decreases as the symmetric ply orientation angle increases upto 90° and beyond that it starts to increase.
- Natural frequency of piezolaminated plate increases as the ply orientation angle increases both in symmetric laminate ply and antisymmetric laminate ply orientation. However increasing rate is more in case of antisymmetric laminate ply orientation.
- Increase in piezolayer thickness, slightly increases the natural frequency of composite plate because of slightly increases in stiffness/mass ratio.

- Natural frequency of piezolaminated plate decreases as the percentage of coverage area of the piezoelectric patch increases because of slightly increases in stiffness/mass ratio.
- It is observed that with higher control gain, the vibration of plate is damped out more quickly because of increase in effective damping.
- It is also observed that the constant gain negative velocity feedback controller is asymptotically stable because amplitude of vibration decreases with time.
- The effect of the sensor/actuator pairs position on the response of the simply supported plate is investigated and it is observed that when the sensor/actuator pair are bonded on the center of the plate, the control effect is found best.
- Genetic algorithm is used to obtain the optimum voltage in order to get desired shape. It is observed that, on applying the suitable voltage on a suitable positioned actuator desired shape can be obtained at less energy supplied.

## 7.2 Future scope

1. The developed finite element model of layered plate can be validated with experimental results.
2. Developed finite element model of layered plate can be extended to account the viscoelastic effect between the two adjacent layers.
3. Developed finite element model of layered plate can be extended to account the temperature effect in order to include pyroelectric effect and thermal stress effect.
4. Classical controller can be replaced with non-classical controller to control the non-linear vibration.

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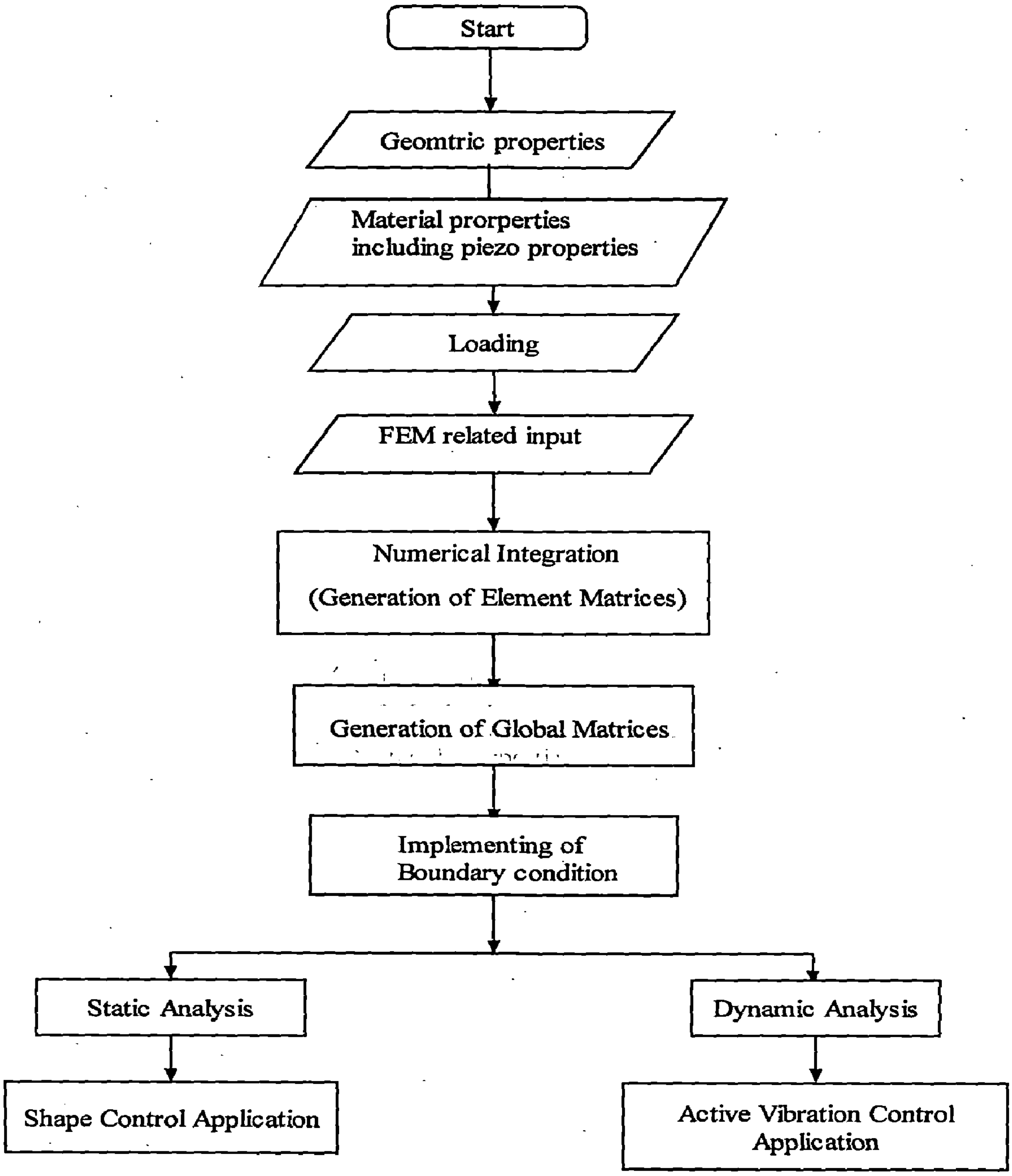
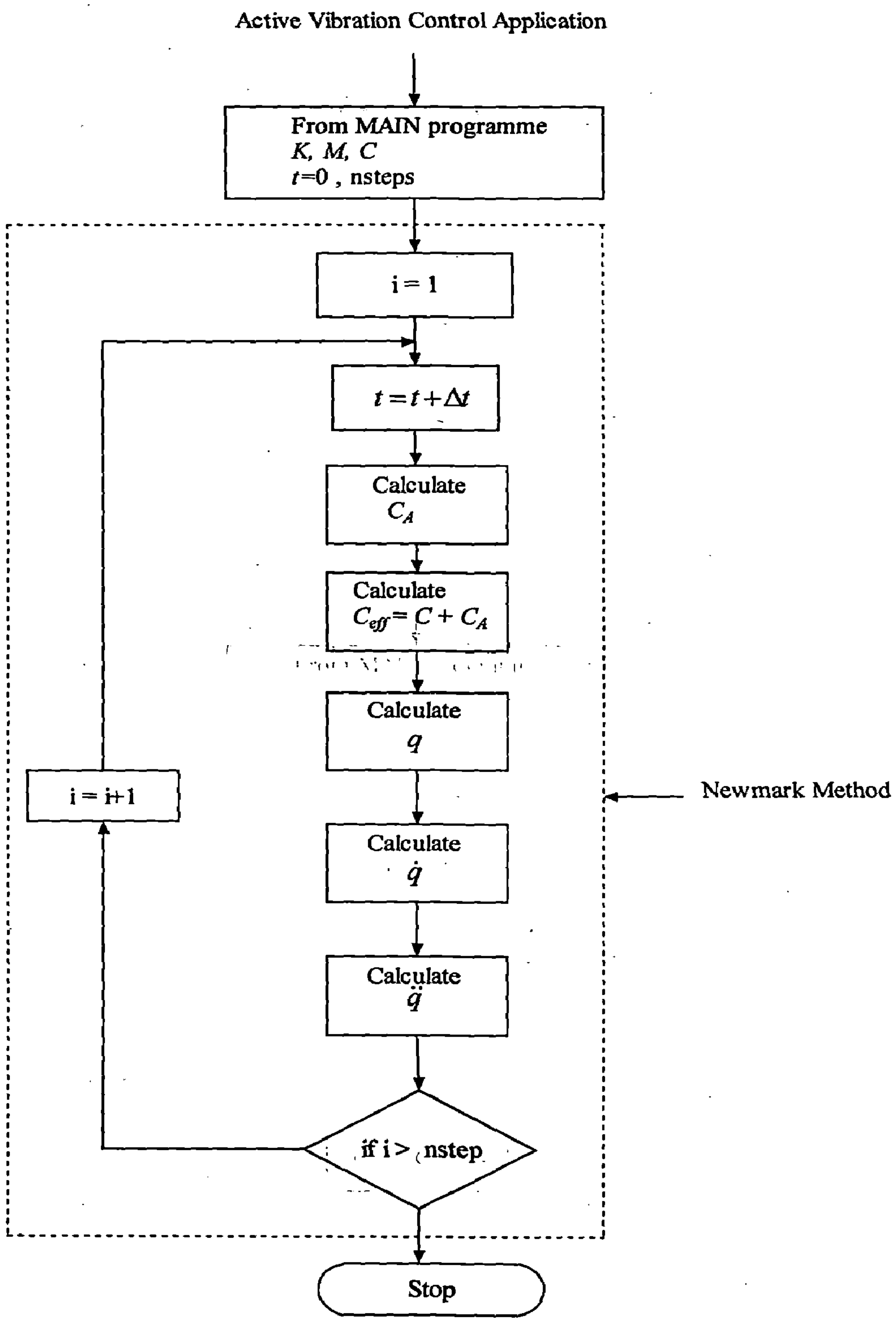
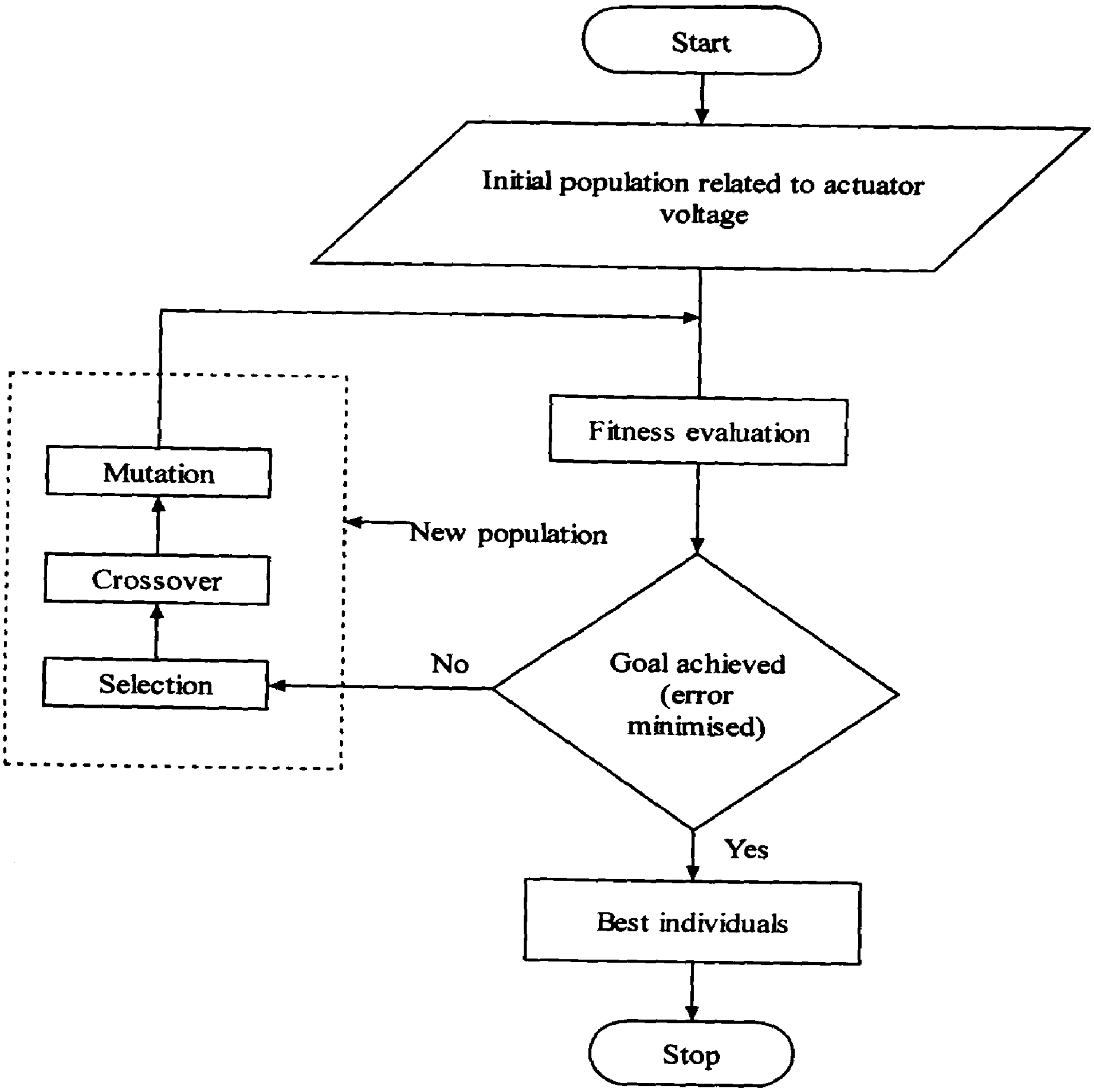



Flow chart of MAIN Programme




Flow chart of active vibration control




Flow chart of Genetic algorithms