DYNAMIC CHARACTERISTICS OF MULTI-STOREYED BUILDINGS WITH AND WITHOUT RIGID FLOOR ROTATION

A DISSERTATION
submitted in partial fulfilment of the
requirements for the award of the Degree
of
MASTER OF ENGINEERING
in
EARTHQUAKE ENGINEERING
with Specialization in Structural Dynamics

By
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Nov, 1975

CERTIFICATE

Certified that the dissertation entitled, "DYNAMIC CHARACTERISTICS OF MULTI-STOREYED BUILDINGS WITH AND WITHOUT RIGID FLOOR ROTATION" which is being submitted by Sri SURENDRA KUMAR AGARWAL in partial fulfilment of the Degree of Master of Engineering in Earthquake Engineering with Specialization in Structural Dynamics of University of Roorkee, Roorkee is a record of his own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of ten months from January 1975 to November, 1975 for preparing this thesis at this University.

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ACKNOWLEDGEMENTS

The author wishes to express his deep sense of gratitude to Dr. A.S.Arya, Professor and Head, School of Research and Training in Earthquake Engineering and Dr.S.P. Sharma, Assistant Director, Structural Engineering Research Centre, Roorkee for their kind help and guidance without which it would have been impossible to complete this work.

The author is grateful to Dr. A.R.Chandrasekaran, Professor, School of Research and Training in Earthquake Engineering for providing the necessary laboratory and other facilities.

Special thanks are due to:

Dr. S.K. Thakkar, Reader in Structural Dynamics, School of Research and Training in Earthquake Engineering and Shri B.K.Goyal, Scientist, Structural Engineering Research Centre, for their valuable help at various stages of this work, and

all those who contributed their own bit in completing this work.

CHAPTER I

INTRODUCTION

1.1 NEED FOR DYNAMIC ANALYSIS

During the last decade, there has been a considerable increase in the construction of multi-storeyed buildings in India with frames to replace the traditional load bearing wall buildings, for both residential and commercial purposes. Structurally a multistoreyed building consists of frames with rigid joints. But to achieve economy in the design of moderately tall buildings under the action of lateral loads due to wind, blast and earthquake forces these are provided with frames having diagonal bracing frames with masonry in-fill, frames with shear walls, coupled shear walls, core units or a combination of these sets of elements.

Seismic studies of multi-storeyed buildings has already assumed greater importance in the country as most of the metropolitan town like Ahmedabad, Bombay, Calcutta and Delhi where these structures are coming up lie in moderate to high seismic zones. The response of any structure during an earthquake is a dynamic phenomenon and is completely defined by its mass, stiffness, damping and load-displacement characteristics. IS-1893 gives two broad approaches for the earthquake analysis of multi-storeyed buildings.

(i) Pseudo Static approach

(ii) Modal analysis approach

Pseudo Static Approach

This method has been recommended for buildings not exceeding 40 m in height. The base shear V_B is calculated based on the seismic coefficients to simulate the effect of earthquake from the formula

$$V_B = C \alpha_h \beta W \qquad (1.1)$$

where

C a flexibility coefficient depending upon the fundamental period T of the structure $= 0.5/T^{1/3}$

(The maximum value of this coefficient is restricted to 1.33 for buildings with load bearing walls and 1.00 for framed buildings. The minimum values are limited to 0.33).

α_h basic horizontal seismic coefficient for the zone

β a coefficient depending upon the soil foundation system, varying from 1.0 to 1.5 for different combinations of soil and foundation type.

W Total dead load + appropriate live load.

The distribution of seismic force along the height of the building is given by

$$Q_i = V_B \frac{W_i h_i^2}{\sum_{i=1}^{i=m} W_i h_i^2} \qquad (1.2)$$

where

W_i Total load coming at the ith floor

Q_i Lateral force at ith floor

h_i Height of the ith floor above the base

m Number of storeyes

Knowing the above force distribution the structure is analysed and designed for these lateral forces as a static problem.

Modal Analysis Approach

This approach considers the earthquake problem as a dynamic one. A multi-storey rigid frame is considered a multi-degree of freedom system in which masses are concentrated at floor levels and the restoring force is mainly provided by columns during vibrations. The equation of motion of such a system subjected to a ground motion can be written in matrix form as

$$M\ddot{Z} + C\dot{Z} + KZ = - M\ddot{Y} \qquad (1.3)$$

where

M diagonal mass matrix

C damping matrix

K square stiffness matrix

Z vector of a relative displacement

$\dot{Z}$ vector of relative velocity

$\ddot{Z}$ vector of relative acceleration

$\ddot{Y}$ ground acceleration

The integration of this equation or solution by mode superposition procedure yields the dynamic response of the structure in terms of frequency and mode shapes.

Knowing the period of the structure and a suitable value of damping, average acceleration S_a is determined from acceleration response spectra. The code has adopted average curves as derived by Housner with appropriate multiplying factors for the various seismic zones. The load Q_i^r acting at the ith floor due to rth mode of vibration is given by the equation

$$Q_i^r = W_i \varphi_i^r C_r \frac{S_a^r}{g} \beta F \qquad (1.4)$$

where

φ_i^r mode shape coefficient for ith floor in rth mode

C_r mode participation factor

$$= \frac{\sum_{i=1}^{i=n} W_i \varphi_i^r}{\sum_{i=1}^{i=n} W_i \left[\varphi_i^r\right]^2} \qquad (1.5)$$

n number of modes considered, usually three are sufficient.

g acceleration due to gravity

F factor depending upon the zone.

Having computed the Q values, shear in any storey (say jth) can be obtained as

$$V_j^r = \sum_{i=j}^{n} Q_i^r \qquad (1.6)$$

The total shear V_j in any storey j could then be assumed to be equal to the root of the sum of square of modal shears in the first three modes of vibration as given below :

$$V_j = \sqrt{(V_j^{(1)})^2 + (V_j^{(2)})^2 + (V_j^{(3)})^2} \qquad (1.7)$$

It is thus now imperative to know the frequency and mode shapes of the multi-storeyed buildings under free vibrations.

1.2 BRIEF REVIEW OF THE PAST WORK

During the last decade the dynamic characteristics of multi-storeyed buildings have drawn the attention of many investigators. The dynamic problem of multistoreyed structure has variously been visualized as a wave propogation problem in a continuous cantilever beam, a shear beam approximation with rigid floors as a two-dimensional structure. for the usual configuration of frame bents with or without shear wall, and also as a three dimensional structure by taking the floor rotations into account . As it is difficult to compare the results of various investigators due to sparseness in size and type of structure, a brief account of the past work on the dynamic analysis of frames with shear walls, coupled shear walls and unsymmetric shear wall buildings has been presented here.

(a) Frames without or with shear walls

Shear beam approximation for the analysis of multi-storeyed frame-buildings is commonly used for elastic dynamic analysis considering floors as rigid. Chandrasekaran (63-65) has suggested cantilever shear beams mathematical model of multistoreyed structure. He has also investigated the effect of joint rotation on the dynamic response and stated that if the beam to column stiffness ratio is of the order of 5 or above, the effect is negligible. But Rubenstein-Hurty(1961) has stated that by ignoring joint rotation the error in the computation of fundamental period of certain buildings may be as high as 100 per cent.

Webster(1966) has presented a method for the elastic analysis of large multi-storey building under lateral loads using stiffness matrix approach. The floors are assumed to be rigid in their own plane but are free to translate and rotate. The stiffness matrix has been extended to allow for the calculation of lowest natural frequency of vibration considering the effects of distributed mass of the structure. However, the author has not exemplified the technique for the dynamic analysis.

Mullick and Sawhney (1970) have presented a method for the dynamic analysis of multi-storeyed buildings with columns and shear walls. The structure is idealised into two distinct systems viz shear wall system and frame system linked to-gether by link beams similar to the one used by Khan and Sbarounis (1964). The important assumptions made by them are :

(i) Material is perfectly elastic

(ii) For beams and columns only flexural deformations are considered while for shear wall, both flexural and shear deformations are considered.

(iii) The floor diaphragms are rigid in their own plane, but have no stiffness normal to this plane i.e. each floor is constrained to translate without rotation and each vertical member is subjected to same displacement.

(iv) Masses of each storey are lumped at floor levels

(v) Damping effects are neglected for finding the natural frequencies and mode shapes.

(vi) The shear wall is replaced by uniform line element and placed along its centroidal axis.

(vii) At each beam level the portion of the shear wall from the edge to its centroidal axis is considered as rigid.

The lateral stiffness matrix of the system is derived which has been directly used with the diagonal mass matrix for finding out frequencies and mode shapes.

The method has been applied to a six storey perspex model. The experimental frequencies are determined by resonance tests of the same model. The theoretical and experimental values of frequencies are reported to match.

Kelkar and Utagikar (1970) have presented a solution for the earthquake analysis for non-symmetric structures where centre of mass does not coincide with the centre of rigidity at various floor levels. A stiffness matrix is derived for two degree of freedom for each floor i.e. one translation and the other floor rotation.

The method has been applied to a two storey building. The frequencies of free vibrations are calculated for two cases one-when floor rotations are considered and second when these are neglected. Also it has been shown that the response spectra developed for one degree of freedom systems can also be used when floor rotations are considered with a small modification in the definition of the participation factors. The authors have finally concluded that the periods of vibration increase when floor rotations are considered. Also the total floor forces amongst the individual frames are unequal depending upon their position from the centre of rigidity. The distribution is quite different when floor rotations are neglected.

Irwin (1971) has incorporated the use of the continuous connection technique in the distribution of the overall lateral loads to various load bearing elements of a complete shear wall buildings. He extended it to the analysis of three dimensional multistorey shear wall buildings subjected to dynamic loads, considering rotational and translational vibrations. The analysis deals with matrices of small order depending upon the number of reference levels ranging from a minimum of two to a maximum equal to number of storeyes in the building. The method has been applied to find the natural frequencies, periods and mode shapes of a typical apartment style shear wall building and using response spectrum and normal mode superposition technique the most probable values of lateral deflections, rotations are calculated. The effect of number of reference levels has been investigated with respect to the computation time and cost.

Sarawat (1973) has used influence coefficient approach for the analysis of frames with shear wall. For shear wall bending and shear deformations are considered while for frame bending and joint rotation effects are considered. Using this approach various parametric studies are carried out like fundamental period, mass distribution and stiffness distribution on the dynamic response of a typical 15-storeyed building.

(b) Coupled Shear Walls

Seismic analysis of coupled shear walls in multi-storeyed buildings has shown the attention of various investigators as late as 1968 onwards. The concept of continuous connection for the static analysis is extended by Mukherjee and Coull (1973) for determining the natural frequencies and mode shapes of free vibration of a coupled shear wall structure. The important assumptions made in the analysis are :

(i) Axial deformations of the connecting beams are neglected i.e. both the walls deflect equally

(ii) The connecting beams deform with a point of contraflexure at some fixed spanwise position, throughout the height of building.

(iii) The discrete set of uniform connecting beams is replaced by an equivalent continuous media such that the moment of inertia (I_c) of connecting beams may be replaced by a uniform equivalent connecting medium of stiffness I_c/h per unit height.

(iv) Effects of vertical inertia forces and the rotatory inertia of the walls are neglected.

The dynamic equation of motion derived for the structure is a 6th order differential equation. Galerkin's method has been used to solve the equation choosing a cosine deflection function to satisfy slope and deflection conditions at base.

The method is applied to 15-storey assymmetrical and 20 storey symmetrical structure, and the resulting fundamental frequencies are compared with the experimental results and the discrepencies hsve been reported as 8 and 5.5 per cent.

As the assumed deflection function fails to satisfy the following boundary conditions at base (x=0) and top (x=h).

$$\frac{d^3y}{dx^3} \neq 0 \quad \text{at } x = 0 \text{ for all time } t$$

$$\frac{d^4y}{dx^4} \neq 0 \quad \text{at } x = h \text{ for all time } t$$

Coull and Mukherjee(1973) have presented a more accurate analysis using more elaborate functions for the solution of the differential equation, once again using Galerkin's Method. The method is once again applied to the same 20 storey structure and it is claimed that with this method a better realistic assessment of the dynamic shear forces and bending moments can be made.

Jennings and Skattum (1973) used the same continuous lamina approach. The equations of motions are derived for the general case using Hamilton's principle.

$$\delta \int_{t_o}^{t_1} (T - u)\, dt = 0 \qquad (1.8)$$

where

T kinetic energy of the system

u total strain energy

δ variational operator

In this analysis both axial and bending deformations of the shear wall and bending shear and axial deformations of beams are included. The eigen value of free vibration of a planar wall with constant, equal properties is solved, both with and without the inclusion of the inertia of vertical motion and it is concluded that neglecting the vertical motion of wall is a poor approximation for actual eigen values, especially the lower values

In the above methods of analysis an assumption is made that the point of contraflexure occurs at the mid point of the connecting beam which implies that the lateral loading on the two shear walls are distributed identically and the magnitudes of the load are in proportion to the stiffness of the pier. But in actual practice, in case of wind pressure distribution on the shear wall of windward side differs from the pressure distribution on the leeward side wall. Also for seismic loading considerations, the inertial loading on the shearwall is proportional to the width of shear wall. Further the stiffness of the pier is proportional to the third power of its width the assumption of lateral loads carried by the shearwalls in proportion to their stiffness does not hold good for coupled shear wall with unequal shear wall.

Considering the above anomolies Tso and Chan (1971) formulated the problem of coupled shear walls with continuous media such that no assumption is made regarding the point of contraflexure at the mid span, in terms of the unknown deflections of two walls. The governing equation 4th order ordinary differential equation, takes the form of a pair of beams under axial loading and situated on elastic foundations.

When inertia effect of walls are considered for free vibration case it turns to be a sixth order coupled equation. The actual numerical evaluation of frequency is performed by trial and error method.

The authors have performed tests on two plexi-glass models, 20 and 15 storeyes with symmetric and antisymmetric shear walls respectively and shown that theoretically predicted fundamental frequency is 5 per cent higher than the experimentally obtained value.

The authors have investigated the effect of depth of connecting beam on the fundamental frequency and concluded that the increase in depth of connecting beams beyond a quarter of the storey height does not provide significant increase in stiffness of the coupled shear wall system.

1.3 SCOPE AND OBJECTIVE OF THE STUDY

A tall building with frames and shear walls connected by rigid floor slabs is a three dimensional complex structural system. Seismic co-efficient approach gives conservative estimates of earthquake forces even for moderately high building and for modal method we need the dynamic characteristics. Existing method of lumped stiffness and mass for finding these characteristics gives erroneous results for frames with shear walls. Another method is the exact approach considering three degrees of freedom (two displacements and one rotation) for each joint but the size of problem is enormous and unmanageable even by moderate size of computers. A simplified approach has been tried to suit the IBM 1620 computer with the following objectives in this thesis .

(i) To present a method for finding out mode shapes and natural frequencies of plane frames having shear walls and incorporated in chapter II.

(ii) To present a method for finding out dynamic characteristics of buildings with rigid floors, for considering the three dimensional effect of the structure and is presented in chapter III.

(iii) To compare the theoretical results from (i) and (ii) with the experimental values, to show the validity of the various simplifying assumptions. Chapter IV.

An eighteen storeyed plane frame with shear wall of variable cross-section and a fifteen storeyed plane coupled shear wall structure are choosen for the study.

An eight storey unsymmetric frame-shear wall building with rigid floors has also been studied to find the effect of rigid floor rotations.

Summary and conclusions derived from the above studies are given in chapter V . The scope of further research in the subject is also discussed.

CHAPTER II

DYNAMIC ANALYSIS OF PLANE FRAMES WITH SHEAR WALLS

Multi-storeyed buildings with orthogonal set of frames and shear walls can be idealised into two systems as suggested by Khan and Sbarounis

(i) Frame system

(ii) Shear wall system

and these two systems are linked together by link beams (Fig. 1).

Stiffness matrix approach has been used for the generation of stiffness matrix for a typical plane frame with shear wall (Fig.2). The various assumption made in the analysis are:

(i) The material is elastic and isotropic.

(ii) Plane sections before bending remain plane after bending, that is, the strain distribution across the section of any member (beam, column or shear wall), is linear.

(iii) Between the centre line and edge of the shear wall the beam is infinitely rigid.

(iv) Lateral loads acting in the plane of the frame are considered and these are assumed to be acting on the end column of the frame only

(v) For finding out the lateral stiffness matrix of the structure only displacements in the plane of the frame are considered.

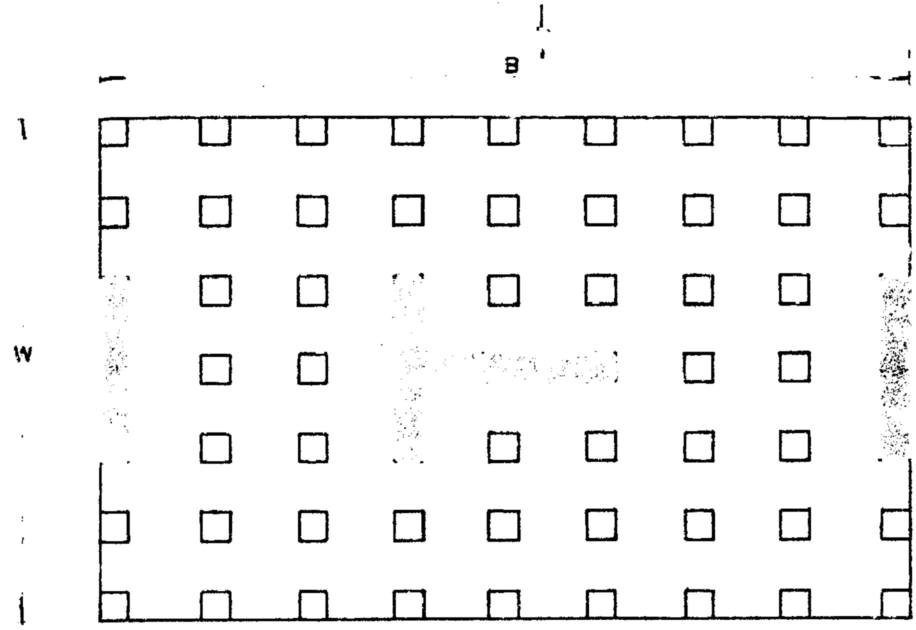

FIG 1a FLOOR PLAN

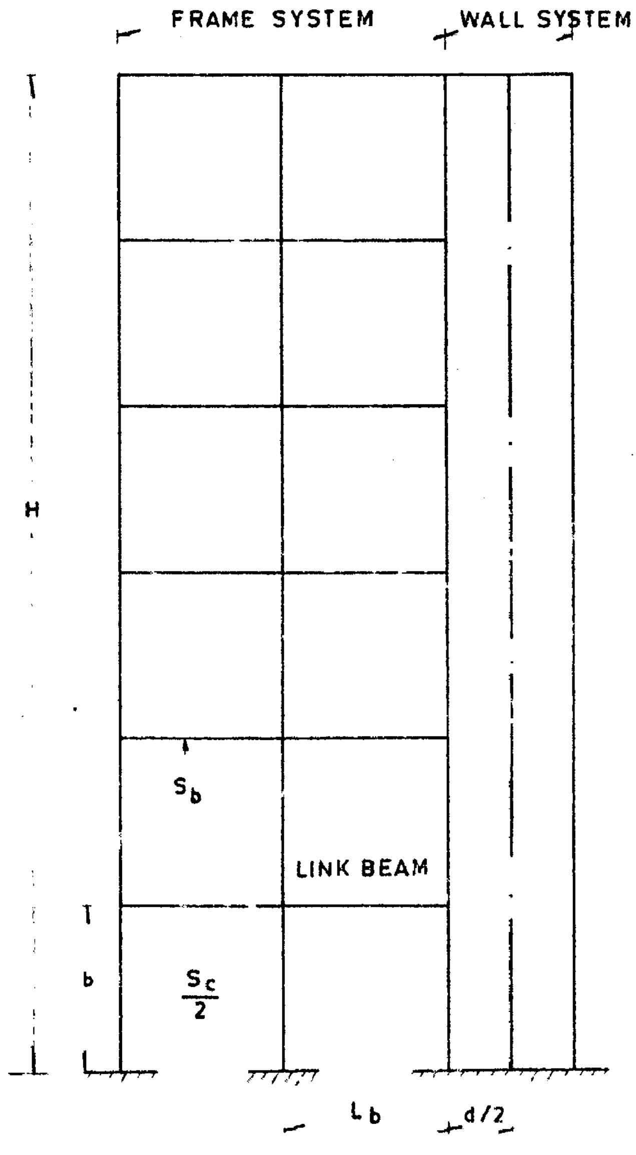

FIG.1b IDIALISED STRUCTURE

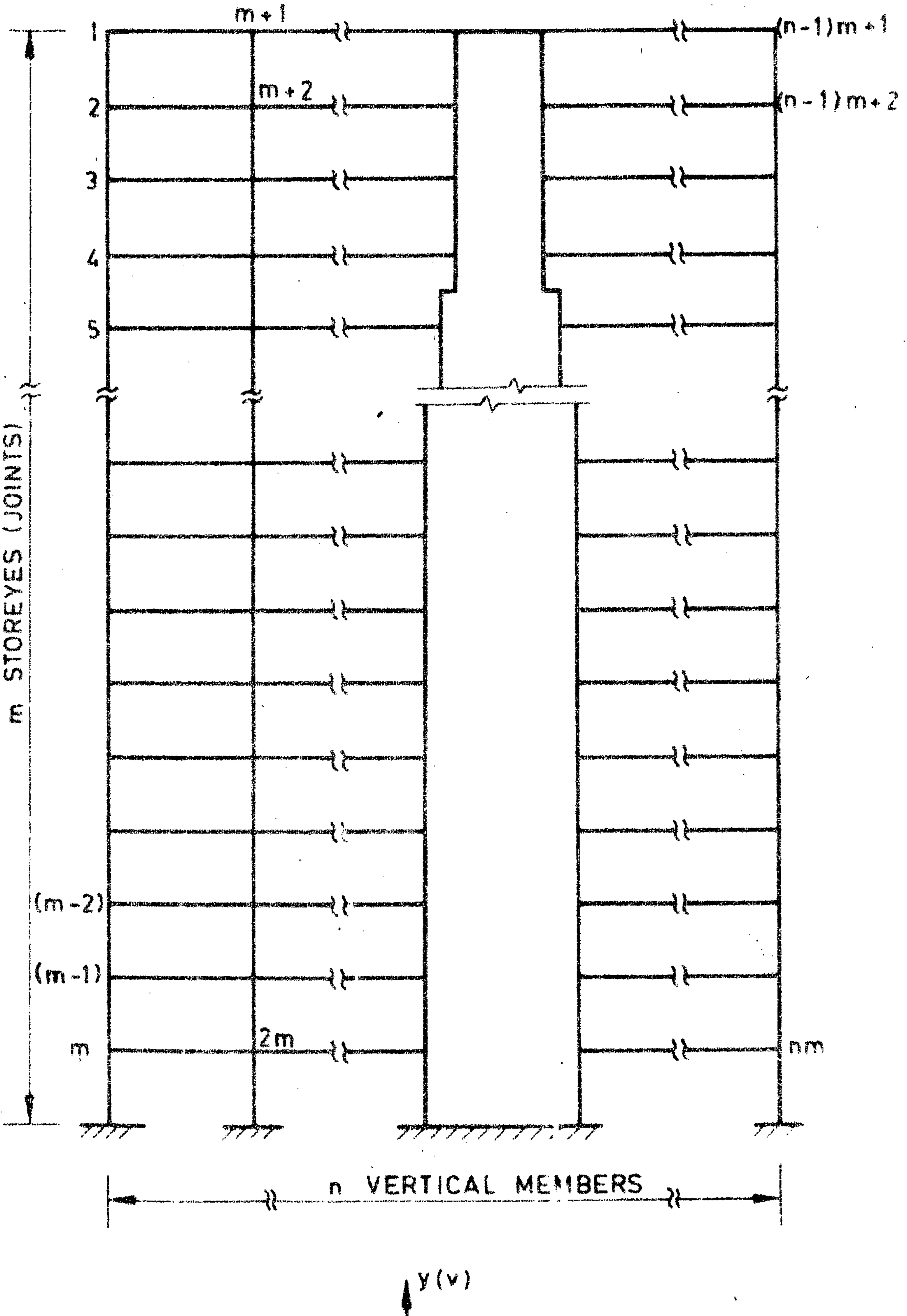

FIG. 2 JOINT NUMBERING AND COORDINATE SYSTEM OF A FRAME.

(vi) The distributed mass of vertical members is replaced by lumps at floor levels.

(vii) Damping effects are neglected in finding out free vibration characteristics.

The stiffness matrix of a prismatic member having finite sizes at the ends can be derived by considering the total energy due to axial, bending and shear forces as under .

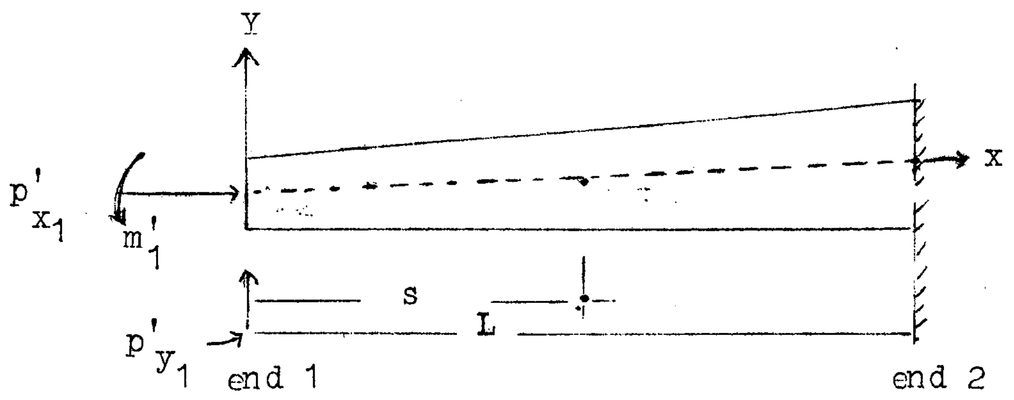

Fig. 3. Local co-ordinate system

Let us consider a member 1-2 (Fig. 3) fixed at 2 and subject to an axial force p'_{x_1} , a vertical shear p'_{y_1} and an anticlockwise moment m'_1 at the free end 1. The total energy of the member is given by

$$U = \int_0^L \left[\frac{p'^2_{x_1}}{2EA} + \frac{(m'_1 - p'_{y_1} s)^2}{2EI} + \eta \frac{p'^2_{y_1}}{2GA} \right] ds \qquad (2.1)$$

where

E Modulus of elasticity

G Modulus of rigidity $= \dfrac{E}{2(1+\nu)}$

ν Poisson's ratio

I Moment of inertia

η Shape factor

The deformations of end 1 are given by

$$u_1 = \frac{\partial U}{\partial p'_{x_1}} \quad , \quad v_1 = \frac{\partial U}{\partial p'_{y_1}} \quad \text{and } \theta_1 = \frac{\partial U}{\partial m'_1}$$

where u_1 and v_1 are displacements along x,y axes and θ_1 is rotation of end 1.

Thus

$$u_1 = p'_{x_1} \int_0^L \frac{ds}{EA} \equiv \alpha_1 \, p'_{x_1}$$

$$v_1 = p'_{y_1} \int_0^L \left(\frac{s^2}{EI} + \frac{\eta}{GA} \right) ds - m'_1 \int_0^L \frac{s}{EI} \, ds$$

$$\equiv \alpha_2 p'_{y_1} + \alpha_3 \, m'_1$$

$$\theta_1 = - p'_{y_1} \int_0^L \frac{s}{EI} \, ds + m'_1 \int_0^L \frac{ds}{EI}$$

$$\equiv \alpha_3 \, p'_{y_1} + \alpha_4 \, m'_1$$

Hence

$$\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \end{Bmatrix} = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & \alpha_3 \\ 0 & \alpha_3 & \alpha_4 \end{bmatrix} \begin{Bmatrix} p'_{x_1} \\ p'_{y_1} \\ m'_1 \end{Bmatrix} \qquad (2.2)$$

$$\text{or,} \quad \delta_1 = F_{11} P_{11} \qquad (2.3)$$

$$\text{or} \quad P_{11} = F_{11}^{-1} \delta_1 \qquad (2.4)$$

where

δ_1 displacement vector of end 1

P_{11} force vector at the end 1 due to deformations at end 1.

F_{11} flexibility matrix and its inverse is $\hat{k}_{11}$ the stiffness matrix

$$\text{or } P_{11} = \hat{k}_{11} \delta_1 \qquad (2.5)$$

Similarly, the force vector P_{12} at the end 1 due to deformations δ_2 at end 2 can be expressed as

$$P_{12} = \hat{k}_{12} \delta_2$$

where $\hat{k}_{12}$ can be derived from $\tilde{k}_{11}$ considering equilibrium of the member.

The total force at the end 1 is given by

$$P_1 = P_{11} + P_{12}$$

where $P_1 = \{p_{x_1} \quad p_{y_1} \quad m_1\}$ in terms of final axial force shear and moment at end 1.

After calculating the values of $\alpha_1, \alpha_2, \alpha_3$ and α_4 for a uniform member k_{11} and k_{12} are written as

$$\hat{k}_{11} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \beta & L\beta/2 \\ 0 & L\beta/2 & \frac{EI}{L} + \frac{L^2\beta}{4} \end{bmatrix}$$

$$\hat{k}_{12} = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 \\ 0 & -\beta & L\beta/2 \\ 0 & -L\beta/2 & \frac{L^2\beta}{4} - \frac{EI}{L} \end{bmatrix}$$

where

$$\frac{1}{\beta} = \frac{L^3}{12EI} + \eta \frac{L}{GA} \qquad (2.6)$$

Considering finite sizes at the ends of the member BC connected to joints A and D by rigid bodies AB and CD as shown in Fig. (4).

Let $\delta_A = \{u_A \quad v_A \quad \theta_A\}$ and $P_A = \{p_{x_A} \quad p_{y_A} \quad m_A\}$ be the displacements and total force vectors at A and δ_B, P_B those at B.

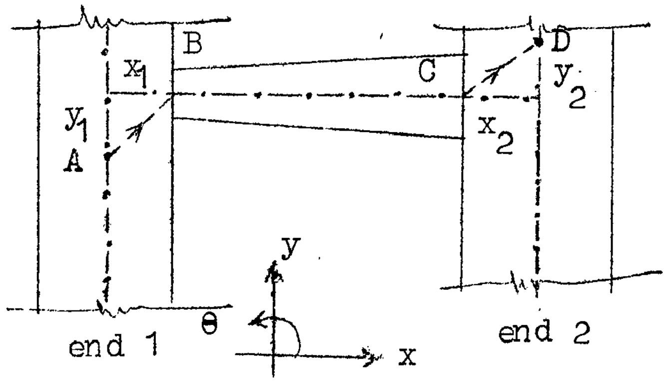

Fig. 4

From equilibrium condition

$$\begin{bmatrix} p_{x_A} \\ p_{y_A} \\ m_A \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -y_1 & x_1 & 1 \end{bmatrix} \begin{bmatrix} p_{x_B} \\ p_{y_B} \\ m_B \end{bmatrix} = 0 \qquad (2.7)$$

Rewriting

$$P_A - H\, P_B = 0 \qquad (2.8)$$

The displacements at Aand B satisfy the rigid body movement condition.

i.e.

$$\begin{bmatrix} u_B \\ v_B \\ \theta_B \end{bmatrix} = \begin{bmatrix} 1 & 0 & -y_1 \\ 0 & 1 & x_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_A \\ v_A \\ \theta_A \end{bmatrix} \qquad (2.9)$$

or $\delta_B = H^T \delta_A$ (2.10)

Also

$$P_A = P_{AA} + P_{AD}$$

$$P_B = P_{BB} + P_{BC}$$

where P_{AA} and P_{BB} is the force vector at A and B due to deformations at A and B respec tively and P_{AD} and P_{BC} are the vectors at A and B due to deformations at D and C respectively.

Let k_{11}^{BC} is the stiffness matrix for BC for end B, then,

$$P_{BB} = k_{11}^{BC} \delta_B$$

Substituting for δ_B

$$P_{BB} = k_{11}^{BC} H^T \delta_A$$

From eqn (2.10) substituting for P_{BB} we get

$$\bar{P}_{AA} = H k_{11}^{BC} H^T \delta_A$$

where $\bar{P}_{AA}$ stands for the force at A due to deformation δ_B.

Hence the stiffness matrix for AD for the joint A is given by

$$k_{11}^{AD} = H k_{11}^{BC} H^T \qquad (2.11)$$

Relation between displacements at C and D is given by

$$\begin{bmatrix} u_c \\ v_c \\ \theta_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & y_2 \\ 0 & 1 & -x_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_D \\ v_D \\ \theta_D \end{bmatrix}$$

or $\delta_c = \bar{H} \delta_D$

Similarly

$$k_{12}^{AD} = H \; k_{12}^{BC} \; \bar{H} \tag{2.12}$$

The stiffness matrix for a member connected to a joint of finite sizes can now be written as

$$\hat{k}_{11} = \begin{bmatrix} \frac{EA}{L} & 0 & -\frac{EAy_1}{L} \\ 0 & \beta & \beta(x_1 + \frac{L}{2}) \\ \frac{-EAy_1}{L} & \beta(x_1+\frac{L}{2}) & \beta\,(x_1+\frac{L}{2}) + \frac{L^2\beta}{4} + E(Ay_1^2 + I)/L \end{bmatrix} \tag{2.13}$$

$$\hat{k}_{12} = \begin{bmatrix} \frac{-EA}{L} & 0 & -\frac{EAy_2}{L} \\ 0 & \beta & \beta\,(x_2 + \frac{L}{2}) \\ \frac{EAy_1}{L} & \beta\,(x_1+\frac{L}{2}) & \beta(x_1+\frac{L}{2})+(x_2+\frac{L}{2}) + \frac{E(Ay_1y_2-I)}{L} \end{bmatrix}$$

From Force -displacement relation we know

$$\hat{P} = \hat{k} \; \hat{\delta}$$

Force displacement relation in absolute coordinates can be written as

$$\hat{P}' = T^T \; \hat{k} \; T \; \hat{\delta}' \tag{2.14}$$

where T is the transformation matrix such that

$$T = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.15}$$

Now using the relations of eqns(14), (13) for the members in absolute coordinates can be written as under:

Hence,

$$k'_{11} = \begin{bmatrix} \frac{EA}{L}\cos^2\varphi + \beta\sin^2\varphi & -(\beta - \frac{EA}{L})\cos\varphi\sin\varphi & -\frac{EA\,y_1}{L}\cos\varphi - \beta(x_1+\frac{L}{2})\sin\varphi \\ & \frac{EA}{L}\sin^2\varphi + \beta\cos^2\varphi & -\frac{EA\,y_1}{L}\sin\varphi + \beta(x_1+\frac{L}{2})\cos\varphi \\ \text{Symmetry} & & \frac{EA}{L}y_1^2 + \beta x_1(L+x_1) + \frac{EI}{L} + \frac{L^2\beta}{4} \end{bmatrix}$$

$$k'_{12} = \begin{bmatrix} -\frac{EA}{L}\cos^2\varphi - \beta\sin^2\varphi & -(\frac{EA}{L} - \beta)\sin\varphi\cos\varphi & -\frac{EAy_2}{L}\cos\varphi - \beta(x_2+\frac{L}{2})\sin\varphi \\ & -\frac{EA}{L}\sin^2\varphi - \beta\cos^2\varphi & -\frac{EAy_2}{L}\sin\varphi + \beta(x_2+\frac{L}{2})\cos\varphi \\ & & \frac{EA}{L}y_1y_2 - \frac{EI}{L} + \beta(x_1+\frac{L}{2})(x_2+\frac{L}{2}) \end{bmatrix} \quad (2.16)$$

The equation of equilibrium for a joint i , connected to n other joints 1 to m and if Q_i be the external load vector at joint can be written as (2.16)

$$\sum_{k=1}^{m} (k^{ik}_{11}\ \delta'_i + k^{ik}_{12}\ \delta'_k) + Q_i = 0 \quad (2.17)$$

where $\bar{k}_{11}^{ik}$ is the stiffness matrix for end 1 due to deformation of end 1 and k_{12}^{ik} is the stiffness matrix for end 1 due to deformation of end 2 for a member ik which can be had using eqn (2.16) δ_i' and δ_k' are the deformation vector for joint i and k respectively in absolute coordinate system.

The stiffness matrix for the frame of Fig. (2) is derived using the assembly technique and to explain the procedure the derivation of the stiffness matrix for the first column only is presented, and others can be treated similarly.

Using the stiffness matrices of equation (2.16) and eqn (2.17) equilibrium 1 to m can be written as shown below. Lateral loads acting in X- direction alone are considered.

$$\begin{bmatrix} \sum \underset{3x3}{[k'_{11}]} & \underset{3x3}{[k'_{12}]} & & & - - - & [k'_{1m+1}]_{3x3} \\ [k'_{21}] & \sum [k'_{22}] & [k'_{23}] & & & [k'_{2m+m}] \\ & [k'_{32}] & \sum [k'_{33}] & [k'_{34}] & & [k'_{3m+3}] \\ & & & [k'_{m-1m}] & \sum [k'_{mm}] & [k'_{m2m}] \end{bmatrix}_{(3mx6m)}$$

$$x \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \\ d_{m+1} \\ d_{2m} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \\ f_{m+1} \\ f_{2m} \end{bmatrix} \qquad (2.18)$$

where

$$d_r = \{u_r \quad \vartheta_r \quad \theta_r\} \qquad \text{and} \qquad f_r = \{p_r \quad 0 \quad 0\}$$

are the deformations and force vectors for a joint r.

Eqn (2.18) can be rewritten in the following form

$$k_{11}\,\delta_{11} + k_{12}\,\delta_2 = F_1$$

such that ,

$$k_{11} = \begin{bmatrix} \sum [k'_{11}] & [k'_{12}] & & & \\ & \sum [k'_{22}] & [k'_{23}] & & \\ & & \sum [k'_{33}] & [k'_{34}] & \\ & & & & [k'_{m-1\ m}] \\ & & & & \sum [k'_{mm}] \end{bmatrix}$$

$$k_{12} = \begin{bmatrix} [k'_{1m+1}] & & & \\ & [k'_{2m+2}] & & \\ & & \ddots & \\ & & & [k'_{m2m}] \end{bmatrix}$$

$$\delta_1 = \{d_1 \quad d_2 \quad \dots \quad d_n\}$$

$$\delta_2 = \{d_{m+1} \quad d_{m+2} \quad \dots \quad d_{2m}\}$$

and

$$F_1 = \{f_1 \quad f_2 \quad f_3 \quad \dots \quad f_{2m}\}$$

It would be noticed that δ_r is the deformation of the rth column /shear wall and F_r the loads acting on it. It is assumed that F_1 to F_{n-1} are zero- all lateral loads act only on the last vertical member and is denoted by F.

$$F = \{ p_{r+1} \quad 0 \quad 0 \quad p_{r+2} \quad 0 \quad 0 \ \ldots \ p_{r+m} \quad 0 \quad 0 \} ,$$

where $r = (n-1)m$

considering the equilibrium for the entire frame we can write

$$
\begin{aligned}
k_{11}\,\delta_1 + k_{12}\,\delta_2 &= 0 \\
k_{21}\,\delta_2 + k_{22}\,\delta_2 + k_{23}\,\delta_3 &= 0 \\
k_{32}\delta_3 + k_{33}\delta_3 + k_{34}\delta_4 &= 0 \\
\ldots & \\
k_{nn-1}\,\delta_{n-1} + k_{nn}\,\delta_n &= F
\end{aligned}
\qquad (2.19a\text{-}n)
$$

From equation (2.19a)

$$\delta_1 = - k_{11}^{-1} \; k_{12}\,\delta_2$$

Substituting δ_1 in eqn (2.19b)

$$\left[k_{22} - k_{21} \; k_{11}^{-1} \; k_{12} \right] \delta_2 + k_{23}\,\delta_3 = 0$$

or

$$\bar{k}_{22}\,\delta_2 + \bar{k}_{23}\,\delta_3 = 0$$

where

$$\bar{k}_{22} = k_{22} - k_{21} k_{11}^{-1} \; k_{12}$$

Proceeding similarly

$$\bar{k}_{33}\,\delta_3 + \bar{k}_{34}\,\delta_4 = 0$$

where

$$\bar{k}_{33} = k_{33} - k_{32}\,\bar{k}_{22}^{-1}\,k_{23}$$

and finally

$$\bar{k}_{n-1\,n-1}\,\delta_{n-1} + k_{n-1\,n}\,\delta_n = 0$$

where

$$\bar{k}_{n-1\,n-1} = k_{n-1\,n-1} - k_{n-1\,n-2}\,\bar{k}^{-1}_{n-2\,n-2}\,k_{n-2\,n-1}$$

or

$$\delta_{n-1} = -\bar{k}^{-1}_{n-1\,n-1}\,k_{n-1\,n}\,\delta_n$$

Substituting for δ_{n-1} in equation (2.19n)

$$\left[k_{nn} - k_{n\,n-1}\,\bar{k}_{n-1\,n-1}\,k_{n-1\,n}\right]\delta_n = F$$

or

$$\bar{k}_{nn}\,\delta_n = F$$

Since F has non zero component in x direction only $\bar{k}_{nn}$, δ_n and F can be rearranged in the form

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} U \\ \varphi \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix} \qquad (2.20)$$

where

$$U = \{u_{r+1}\ u_{r+2}\ \cdots\cdots\ u_{r+m}\}$$

$$\varphi = \{v_{r+1}\ \theta_{r+1}\ v_{r+2}\ \theta_{r+2}\ \cdots\ v_{r+m}\ \theta_{r+m}\}$$

$$P = \{p_{r+1}\ p_{r+2}\ \cdots\cdots\ p_{r+m}\}$$

Partitioning the matrices of equation (2.20) and eliminating for φ it can be shown that

$$\varphi = - A_{22}^{-1} A_{21} U \qquad (2.21)$$

$$AU = P \qquad (2.22)$$

where

$$A = A_{11} - A_{12} A_{22}^{-1} A_{21} \qquad (2.23)$$

$[A]$ is thus the stiffness matrix of the frame for horizontal deformation $\{U\}$. The other two deformations for this vertical member $\{\varphi\}$ are available from eqn (2.21).

2.1 Considering the masses to be lumped at floor levels and connected by weightless springs the equation of motion for free, undamped vibration for the system can be written in matrix form as

$$M\ddot{U} + AU = 0 \qquad (2.24)$$

If the system is vibrating in one of its normal modes we can write

$$\ddot{U} = - p^2 U \qquad (2.25)$$

where p is the angular natural frequency

M is the diagonal mass matrix

Substituting (2.22) in (2.21) we get

$$M\ U\ p^2 = AU$$

Inserting an identity matrix between A and U or

$$M^{1/2} M^{1/2} U p^2 = A M^{-1/2} M^{1/2} U$$

Premultiplying both sides by $M^{-1/2}$ we get

$$M^{1/2} U p^2 = M^{-1/2} A M^{-1/2} M^{1/2} U$$

or $\lambda X = H X$ (2.26)

where $X = M^{1/2} U$

$H = M^{-1/2} A M^{-1/2}$

Equation (26) represents the eigen value problem in which the quantities λ are the eigen values or frequencies and vector X defines the eigen vector or mode shapes. The matrix [A] is available from eqn (23). The mass matrix is calculated by lumping the masses at each floor, for the storey.

2.2 COMPUTATIONAL TECHNIQUE

Tridiagonalisation is used to obtain the condensed stiffness matrix of the frame. This procedure reduced the storage required substantially. Stiffness matrix for one vertical member need only be stored k_{ii}(i = 1....m, size 3m x 3m). Matrix for beams k_{ij} (i = 1...n, j = 1...m) is a diagonal matrix of 3x3 submatrices - one corresponding to each beam. Thus only the submatrices need be stored.

2.3.1 Sequence of computation

Matrix k_{12} can be rewritten in the form

$$k_{12} = \begin{bmatrix} [B_1]_{3x3} & & & \\ & [B_2]_{3x3} & & \\ & & \ddots & \\ & & & [B_n]_{3x3} \end{bmatrix}_{(3n \times 3n)}$$

Let k_{11}^{-1} be defined as given below

$$k_{11}^{-1} = \begin{bmatrix} \dot{k}_{11} & \dot{k}_{12} & \cdots & \dot{k}_{1n} \\ \dot{k}_{21} & \dot{k}_{22} & \cdots & \dot{k}_{2n} \\ \vdots & \vdots & & \vdots \\ \dot{k}_{n1} & \dot{k}_{n2} & \cdots & \dot{k}_{nn} \end{bmatrix}$$

Hence

$$k_{11}^{-1} k_{12} = \begin{bmatrix} \dot{k}_{11}B_1 & \dot{k}_{12}B_2 & \cdots & \dot{k}_{1n}\, B_n \\ \dot{k}_{21}B_1 & \dot{k}_{22}B_2 & \cdots & \dot{k}_{2n}\, B_n \\ \vdots & \vdots & & \vdots \\ \dot{k}_{n1}B_1 & \dot{k}_{n2}B_2 & \cdots & \dot{k}_{nn}B_n \end{bmatrix}$$

where

$$\left[B_r\right] \equiv \left[k'_{r\ m+r}\right]$$

Since this matrix is required for back substitution it is punched out or stored on tape/disc for a subsequent use.

Further,

$$k_{21} = k_{12}^T = \begin{bmatrix} B_1^T & & & & & \\ & B_2^T & & & & \\ & & B_3^T & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & B_n^T \end{bmatrix}$$

$\therefore$ It can be shown easily that

$$k_{21}k_{11}^{-1}k_{12} = \begin{bmatrix} B_1^T \dot{k}_{11} B_1 & \cdots & B_1^T \dot{k}_{12} B_2 & \cdots & B_1^T \dot{k}_{1n} B_n \\ B_2^T \dot{k}_{21} B_1 & \cdots & B_2^T \dot{k}_{22} B_2 & \cdots & B_2^T \dot{k}_{2n} B_n \\ \vdots & & \vdots & & \vdots \\ B_n^T \dot{k}_{n1} B_1 & \cdots & B_n^T \dot{k}_{n2} B_2 & \cdots & B_n^T \dot{k}_{nn} B_n \end{bmatrix} \quad (2.27)$$

Equation (2.27) thus gives an algorithm for calculating the matrix product required for the generation of matrix $[A]$ for equation (2.23).

2.3.2 All the computations involved, starting from the generation of matrix of eqn (2.16) upto the determination of frequencies is carried out in two parts. The first part has been programmed to generate $[A]$ with an intermediate output of $[B_r]$. In the second part the frequencies and mode shapes are computed.

Using the frequencies and mode shapes and the method of mode superposition the most probable displacements of each storey can be calculated. Also we know the stiffness matrix of the structure hence we can calculate the total lateral forces at each storey level. Assuming that the connecting floors are rigid and undergo only a translatory motion thus imparting equal sway to all frames under lateral loads, the load distribution on individual plane frame can be ascertained for which the frames can be analysed using any approach available.

CHAPTER III

DYNAMIC ANALYSIS OF BUILDINGS CONSIDERING RIGID FLOOR ROTATIONS

Multi-storey Buildings are often analysed by dividing them into independent plane frames. A method for this type of analysis has been discussed in the previous chapter. But the danger in using this approach for unsymmetrical structure is great because the floors connecting the frames in this case will have rotation along with two translations.

The previous method has been extended for the analysis of such unsymmetrical buildings and in this the rigid body rotation of floors has been considered. The additional assumptions made in the analysis are :

1. The floors are rigid in their own planes while their rigidity in a direction normal to their planes is negligible.
2. Translation and rotation of the frame are small.
3. Centre of masses of all floors lie on the same vertical line.

The plan and elevation of typical building scheme having (nx 1) columns and m storeyes is given in Fig. (5). The building is treated as two systems of orthogonal frames in x, y plane and connected by rigid floors. Figure (6) shows the rigid body movement of the floors.

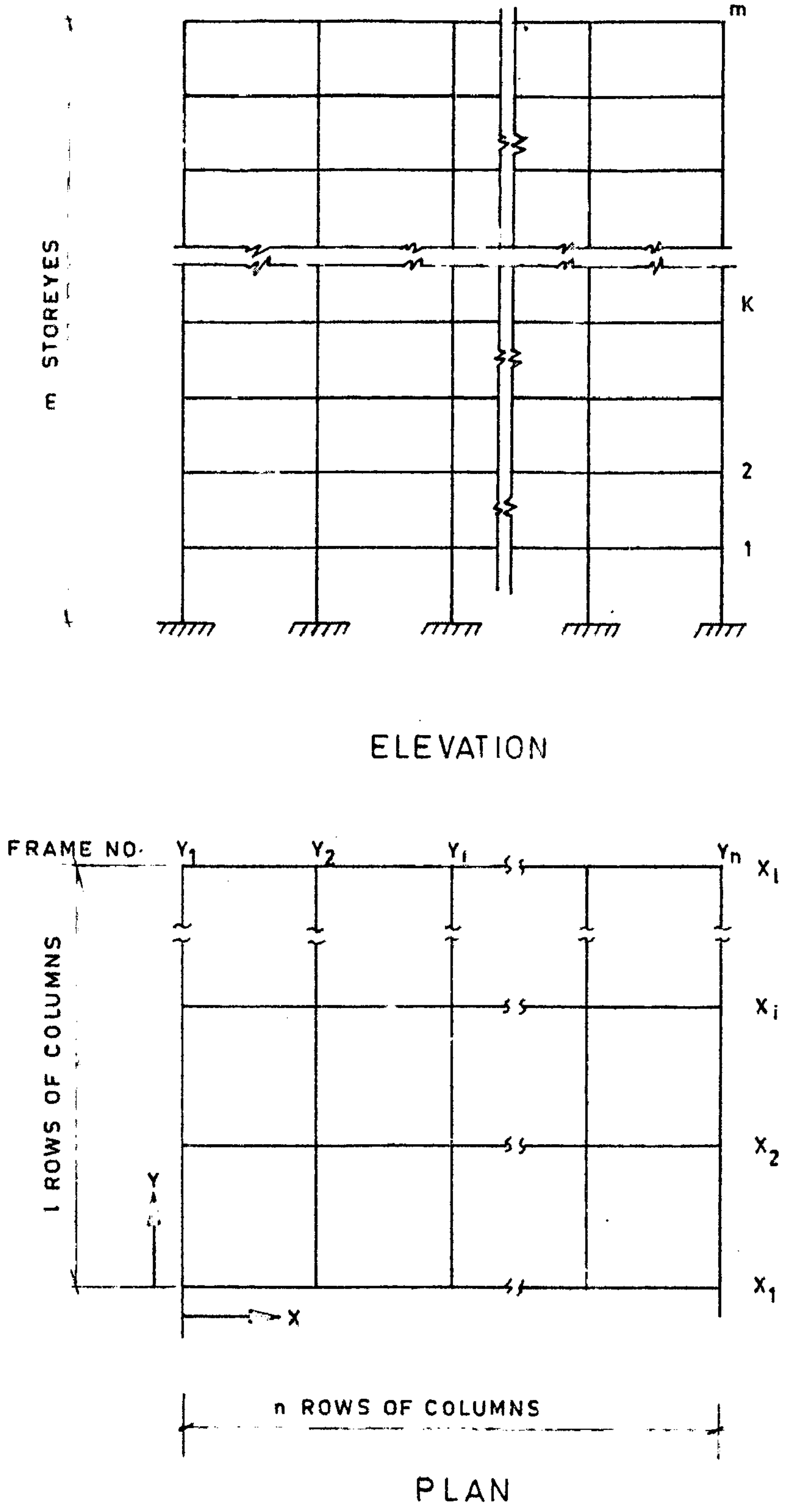

FIG. 5 A TYPICAL BUILDING SCHEME

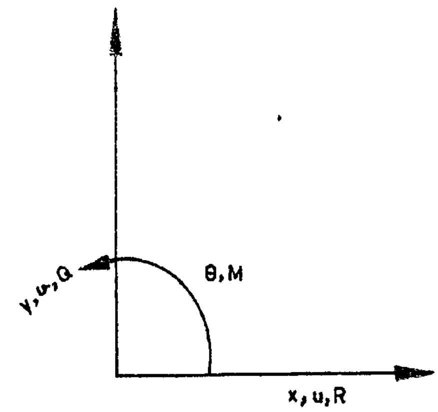

FIG.6a POSITIVE DIRECTIONS OF COORDINATES DISPLACEMENT AND FORCES

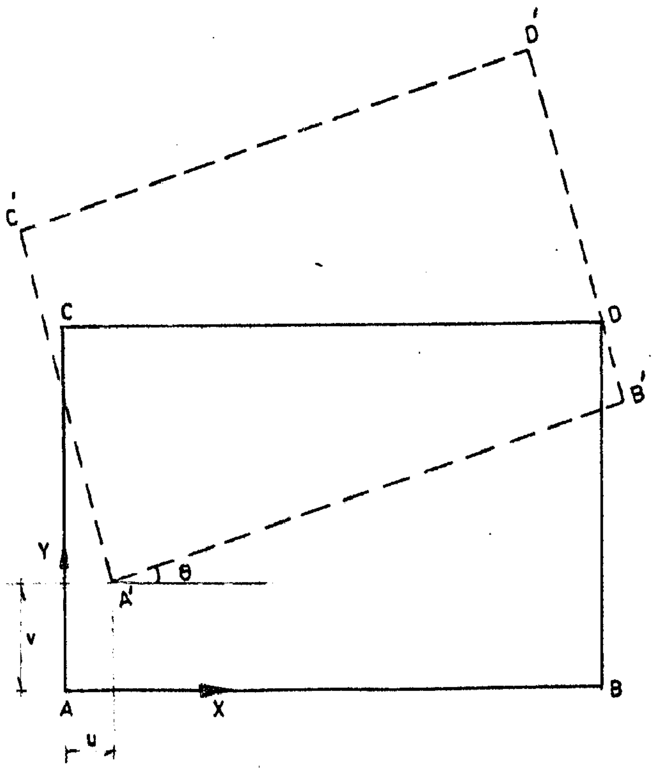

FIG.6b RIGID BODY MOVEMENT OF A FLOOR

and θ_r that of vector θ. Also assuming θ_r as small we can write

$$\delta^i = V + X_i\,\theta \qquad (3.4)$$

and for frame X^j

$$\delta^j = U - Y_j\,\theta \qquad (3.5)$$

Since the floors are in equilibrium under the loads P^i and P^j acting on floor systems Y^i and X^j we can write three equations of equilibrium namely total force in the X and Y directions is zero and the moment of the forces about, say, an origin is zero.

Hence,

$$\sum_{i=1}^{n} P^i = \text{Total external load vector in X-direction}$$
$$= (q_1, q_2 \ldots\ldots q_m) \qquad (3.6)$$
$$= \text{vector } Q$$

$$\sum_{j=1}^{l} P^j = \text{Total external load vector in Y-direction}$$
$$= (r_1, r_2 \ldots\ldots r_m) \qquad (3.7)$$
$$= \text{vector } R$$

Taking moments about the origin we get

$$\sum_{i=1}^{n} X_i P^i - \sum_{j=1}^{l} Y_j\, P^j = M$$ (The total moment vector about the origin due to the external loads Q and R)

(3.8)

Consider a plane frame Y^i of Fig. (5) in x-z plane acted by lateral loads acting on joints 1,2,....m in its own plane at Y = 0 . Proceeding as in the previous chapter, using eqn (2.22) we can write

$$A^i \ \delta^i = P^i \qquad (3.1)$$

where i = 1,2,3,....n

Similarly , for the frame x^j, (j=1,2,3,...l) in the x-direction

$$A^j \delta^j = P^j \qquad (3.2)$$

where

δ^i, δ^j the vectors representing lateral movement of the joints 1,2,....m of the frame

P^i, P^j the lateral load on joints 1,2,...m of the frame.

Since the slabs are assumed to be rigid in their own plane, each of them has three degrees of freedom two in-plane displacements and one rotation.

Let the floor have two displacements U_r and V_r in x and y directions respectively, and a rotation θ_r. Suffix r refers to the rth floor.

The lateral displacement δ_r^i (r = 1,2,...m) for the frame Y^i are given by

$$\delta_r^i = V_r + X_i \sin \theta_r \qquad (3.3)$$

where X_i is the ordinate for frame Y^i and δ_r^i is the rth component of the vector δ^i, V_r that of vector V

Using force displacement relation of eqn (3.1), (3.2), displacement rotation eqns (3.4) , (3. 5) and the three equations (3. 6) , (3. 7), (3. 8) of equilibrium we can write

$$\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} A^i (V+X_i\theta) = Q \qquad (3.9)$$

$$\sum_{j=1}^{l} P_j = \sum_{j=1}^{l} A^j (U - Y_j\theta) = R \qquad (3.10)$$

and

$$\sum_{i=1}^{n} X_i P^i - \sum_{j=1}^{l} Y_i P^j =$$

$$\sum_{i=1}^{n} X_i A^i (V + X_i\theta) - \sum_{j=1}^{l} Y_j A^j (U-Y_j\theta) = M \qquad (3.11)$$

Rewriting eqn (3.9), (3.10) and (3.11)

$$V \sum_{i=1}^{n} A^i + \theta \sum_{i=1}^{n} X_i A^i = Q \qquad (3.12)$$

$$U \sum_{j=1}^{l} A^j - \theta \sum_{j=1}^{l} Y_j A^j = R \qquad (3.13)$$

$$V \sum_{i=1}^{n} X_i A^i - U \sum_{j=1}^{l} Y_j A^j + \theta \left[\sum_{i=1}^{n} X_i^2 A^i + \sum_{j=1}^{l} Y_j^2 A^j \right] = M \qquad (3.14)$$

$$\begin{bmatrix} U \\ V \\ \theta \end{bmatrix} \begin{bmatrix} K \end{bmatrix} = \begin{bmatrix} R \\ Q \\ M \end{bmatrix} \qquad (3.15)$$

$\left[K \right]$, is the stiffness matrix of the system which is written in its expanded form as

$$\left[K \right] = \begin{bmatrix} \alpha_1 & 0 & \alpha_2 \\ 0 & \alpha_3 & \alpha_4 \\ \alpha_2 & \alpha_4 & \alpha_5 \end{bmatrix} \qquad (3.16)$$

where

$$\alpha_1 = \sum_{j=1}^{l} A^j \quad , \alpha_2 = \sum_{j=1}^{l} Y_j A^j \quad , \alpha_3 = \sum_{i=1}^{n} A^i \quad ,$$

$$\alpha_4 = \sum_{i=1}^{n} X_i A^i \quad , \alpha_5 = \sum_{i=1}^{n} X_i^2 A^i + \sum_{j=1}^{l} Y_j^2 A^j$$

3.2 The equation of motion for the rth floor of a shear building can be written as

$$M_r \ddot{U}_r + \sum_{i=1}^{l} K_{r,i}(\delta_{r,i} - \delta_{r-1,i}) - \sum_{i=1}^{l} K_{r+1,i}(\delta_{r+1,i} - \delta_{r,i}) = 0$$

$$M_r \ddot{V}_r + \sum_{j=1}^{n} K_{r,j}(\delta_{r,j} - \delta_{r-1,j}) + \sum_{j=1}^{n} K_{r+1,j}(\delta_{r+1,j} - \delta_{r,j}) = 0$$

$$I_r \ddot{\theta}_r + \sum K_{r,i} \; (\delta_{r,i} - \delta_{r-1,i}) \; X_r^i + \sum_{r=1}^{l} K_{r+1,i}(\delta_{r+1,i} - \delta_{r,i}) \,.$$
$$. X_{r+1}^i + \sum K_{r,j}(\delta_{r,j} - \delta_{r-1,j}) Y_r^j + \sum_{j=1}^{n} K_{r+1,j}(\delta_{r+1,j} - \delta_{r,j}) Y_{r+1}^j$$
$$= 0$$

(3.17)

where I_r is the mass moment of inertia of the rth floor component about its centre of mass.

U_r and V_r are the displacements of the centre of mass of rth floor in X and Y directions respectively

δ^i_r and δ^j_r are displacement of rth floor of ith and jth frame.

X^i_r and Y^j_r are distances of frame i and j from the centre of mass - taken positive to the right of centre of mass.

Equation (3.17) can be rewritten in terms of U,V, θ using relation of eqn (3.) and (3.).

Similar, equations can be written for other floors and all these equations can now be combined into a single matrix equation.

$$\left[M \right] \{Q\} + \left[K \right] \{Q\} = 0 \qquad (3.18)$$

where $\left[K \right]$ is the stiffness matrix given by eqn (3.16)

Q is the vector containing $(u_1, u_2, \ldots u_r \ldots u_n),$ $v_1, v_2 \ldots v_r \ldots v_n, \theta_1, \theta_2, \ldots \theta_r \ldots \theta_M)$ deformations.

$$[M] = \begin{bmatrix} [M_r]_{m \times m} & & \\ & [M_r]_{m \times m} & \\ & & [I_r]_{m \times m} \end{bmatrix}_{3m \times 3m}$$

where

$[M_r]$ is the diagonal lumped mass matrix of all the floors

$[I_r]$ is the diagonal matrix having elements of equivalent mass corresponding to rotation θ and can be calculated as under

Figure (7) shows the plan of a typical floor having a distributed mass ABCD .

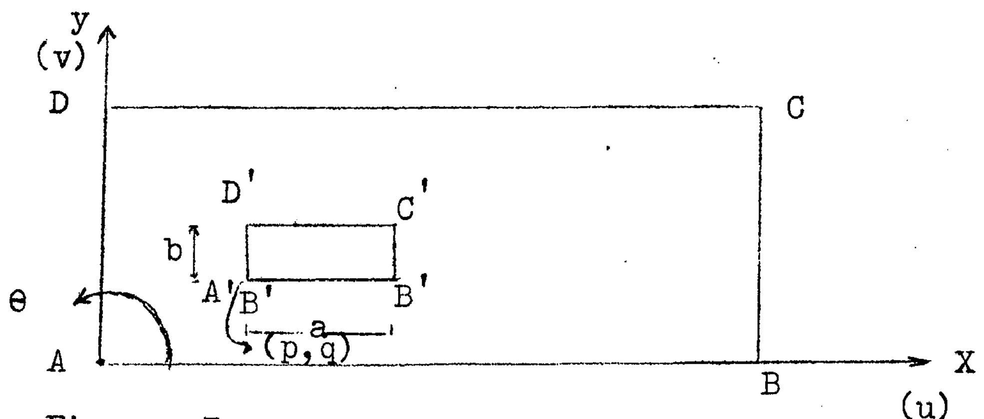

Figure 7.

Let ρ be the mass density.

The moment of an element A'B'C'D' about the origin A = $\rho\, r^2 \ddot{\theta}\, dx\ dy$

where 'r' is the radius vector and ρ the intensity of mass (per unit area). Also

$$r^2 = x^2 + y^2$$

Since θ and $\ddot{\theta}$ are constant in the region total moment

$$= \ddot{\theta} \int_q^{q+b} \int_p^{p+a} r^2 \, dx \cdot dy$$

$$= \ddot{\theta} \rho \int_q^{q+b} \int_p^{p+a} (x^2+y^2) \, dx \, dy$$

$$= \ddot{\theta} \rho \int_q^{q+b} \left| \frac{x^3}{3} + xy^2 \right|_p^{p+a} dy$$

$$= \ddot{\theta} \rho \int_q^{q+b} \left[\frac{1}{3} \left\{ (p+a)^3 - p^3 \right\} + y^2 a \right] dy$$

$$= \ddot{\theta} \rho \left| \frac{y}{3} \left\{ (p+a)^3 - p^3 \right\} + \frac{ay^3}{3} \right|_q^{q+b}$$

$$= \frac{\ddot{\theta} \rho}{3} \left[b(a^3 + 3ap^2 + 3a^2p) + \left\{ (q+b)^3 - q^3 \right\} a \right]$$

$$= \ddot{\theta} \left[\frac{\rho}{3} \left\{ b \, (a^3 + 3ap^2 + 3a^2p) + a \, (b^3 + 3bq^2 + 3b^2q) \right\} \right]$$

$$= \ddot{\theta} \left[\frac{\rho \, ab}{3} (a^2 + 3p^2 + 3ap + b^2 + 3q^2 + 3bq) \right]$$

The equivalent mass to be used in $\left[I_r\right]$ matrix is therefore

$$\frac{\rho\ ab}{3}\ (a^2 + 3ap + 3p^2 + b^2 + 3bq + 3q^2) \qquad (3.19)$$

For a concentrated mass W having radius vector R, the equivalent mass is

$$W\ R^2 \qquad (3.20)$$

Thus the total equivalent mass per floor is calculated using eqn (3.19) and (3.20).

3.3 Equation (3.18) can now be reduced as an eigen value problem similar to eqn (2.26). The frequencies and modes shapes are obtained in the similar manner.

The response of the building in individual modes can be calculated using a known response spectra and the above dynamic characteristics. The response thus obtained by this method will give us the displacements and rotations of each floor with respect to the assumed origin. Knowing this response in a particular mode of all the floors we can obtain the displacements of all the floors. Using eqn (3.4) and (3.5) we can obtain the displacements of individual frames in X and Y directions. The forces at floor level for individual frames in that particular mode can be obtained by multiplying these displacements by the stiffness matrix of the frame.

Such forces are obtained for all the modes, and the most probable forces can then be obtained by modal superposition for which these frames can be analysed and designed.

CHAPTER IV

RESULTS AND DISCUSSION

The validity of assumptions can only be assessed by comparing the theoretical predictions with the experimental results. To illustrate the method discussed in Chapter II free vibration analysis of two examples was carried out

i. A 15-storeyed unsymmetrical coupled shear wall frame for which experimental results were available in the literature.

ii. A 18-storeyed column-shear wall frame having variable cross-section of the shear wall.

Very little experimental data in published literature is available on resonance testing of three dimensional multi-storeyed buildings. Resonance test on a perspex model of an eight-storey building consisting of frame and shear wall was carried out to determine the natural frequency. The same model was analysed using the approach of Chapter III.

4.1 EXAMPLE 1

Tso and Chan have conducted an experimental investigation on 15-storey unsymmetrical coupled shear wall model made out of plexiglass. Figure 8 shows the dimension and properties of the model. Value of fundamental frequency with only inertia forces acting has been reported by the authors. The same frame was analysed for finding out free vibration frequencies and mode shapes using approach of Chapter II. Effect of local wall deformations has been considered as recommended by Michael (1966) which increases the effective span of link beams and is shown in Fig. (8). First three mode shapes and corresponding frequencies using the proposed approach have been given in Fig.(9).

The proposed approach has predicted the value of the fundamental frequency as 63.29 cps against the reported experimental value of 58.4 CPS i.e. the predicted value is 8.5 per cent higher while the theoretical value of Tso and Chan is only 5 per cent higher. It would be seen that the simplifying assumption which resulted in a drastic reduction in storage requirement and computation time has given reasonably accurate results. The higher frequencies were not compared because of lack of the availability of experimental results. The difference may also partly be attributed to the replacement of a distributed mass by discrete masses. This effect would be prominent because of the absence of floor loads. It is expected that in an actual structure carrying

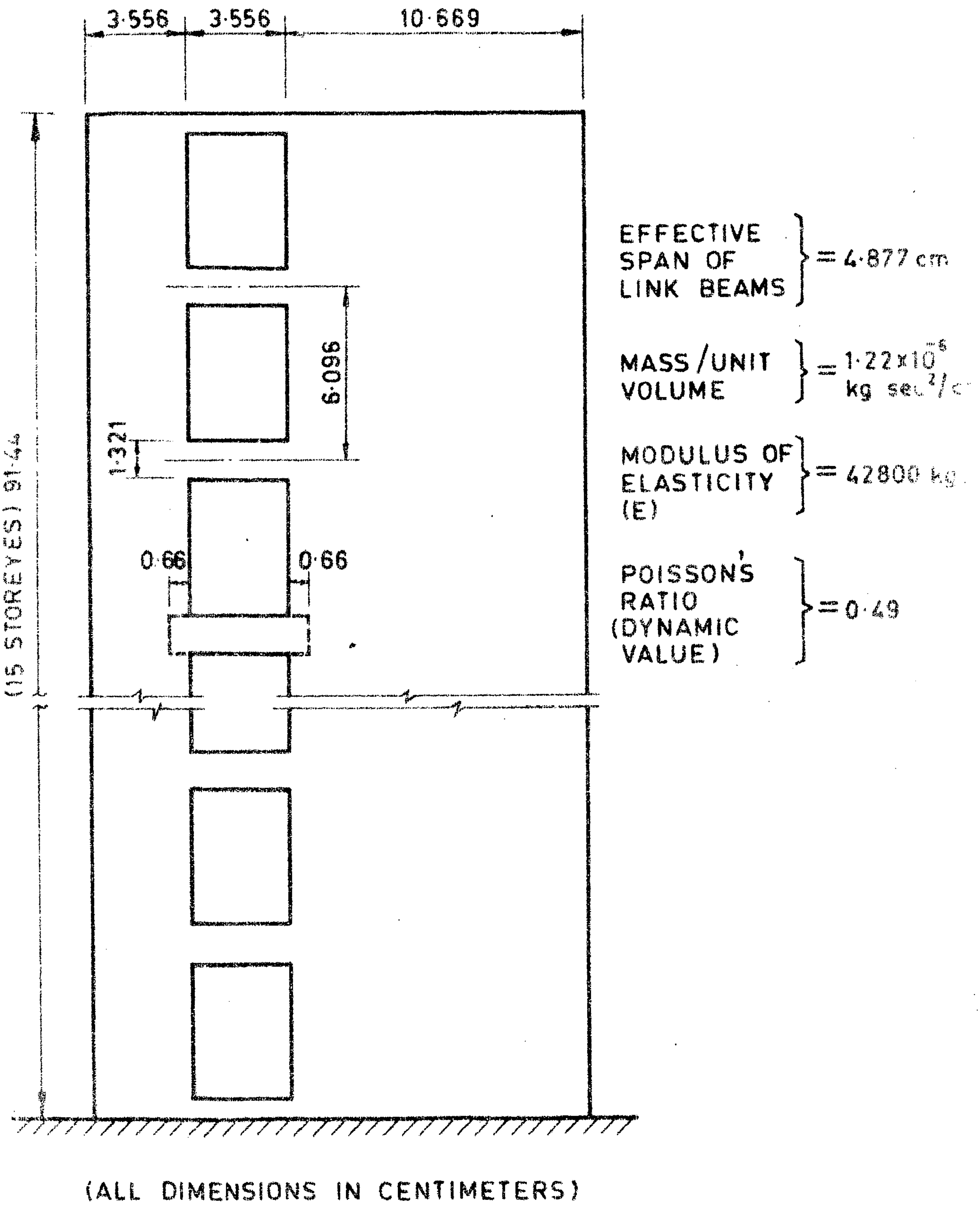

FIG.8 DETAILS OF FRAME SOLVED IN EXAMPLE 1.

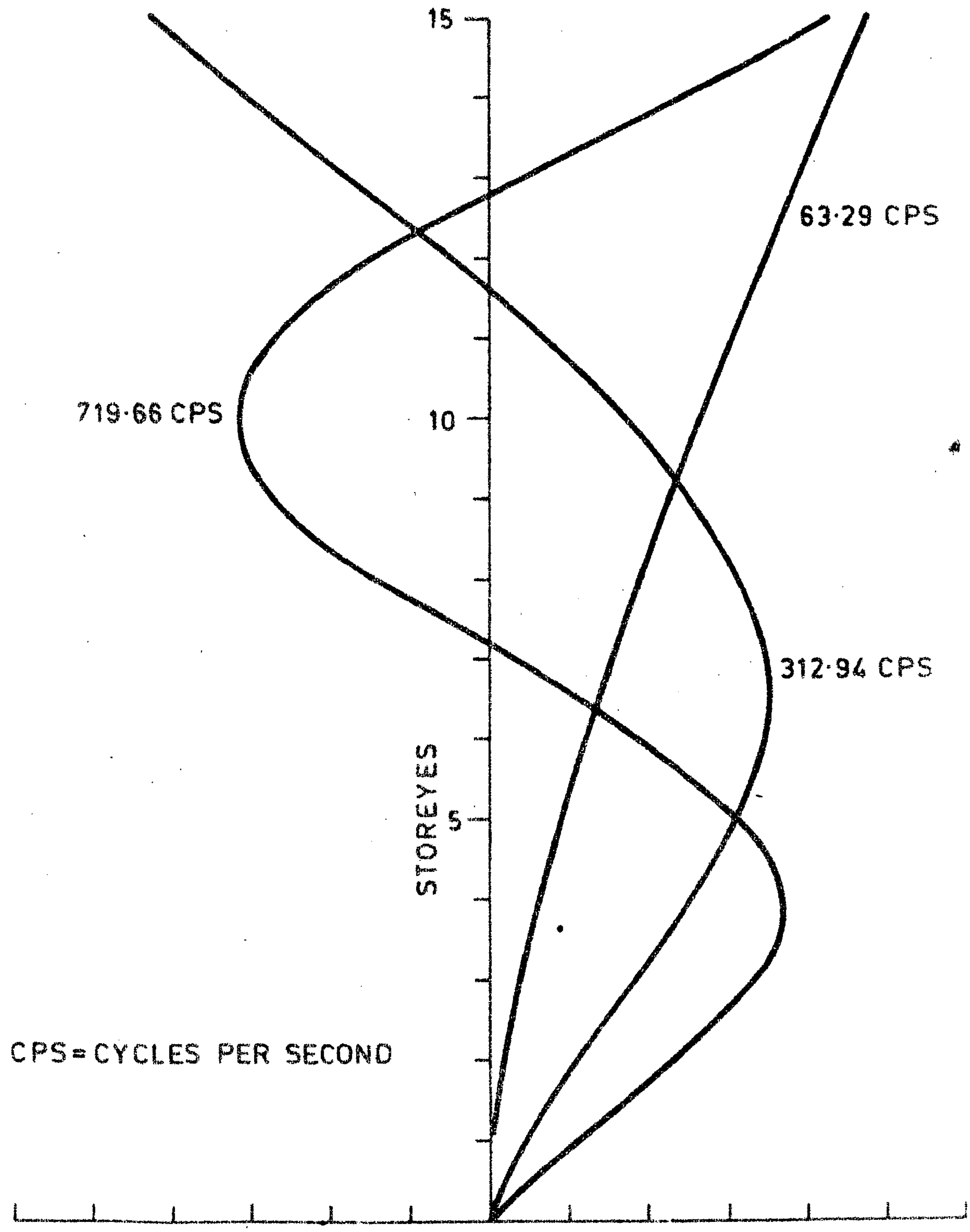

FIG.9 MODE SHAPES AND FREQUENCIES FOR EXAMPLE 1

sizeable floor loads this error would be much smaller.

4.2 EXPAMPLE 2

A typical two-bay 18-storeyed frame having shear wall of variable cross-section and columns-a configuration likely to occur in practice is shown in Fig.(10). The approach of Chapter II has once again been applied for finding the free vibration characteristics of this frame. The first three mode shapes and corresponding frequencies so found have been shown in Fig. (11).

4.3 EXAMPLE 3

A three dimensional 8-storeyed building having two shear walls and one frame consisting of seven columns have been chosen for the study. The plan, elevation, dimensions and properties of the model are given in Fig.12 . The choice of the building was dictated by the availability of this model at the stress Analysis Laboratory of the Structural Engineering Research Centre, Roorkee.

Frequencies and corresponding modes shapes of this structure were calculated for the following three cases, depending on the super-imposed mass, using the approach of Chapter III.

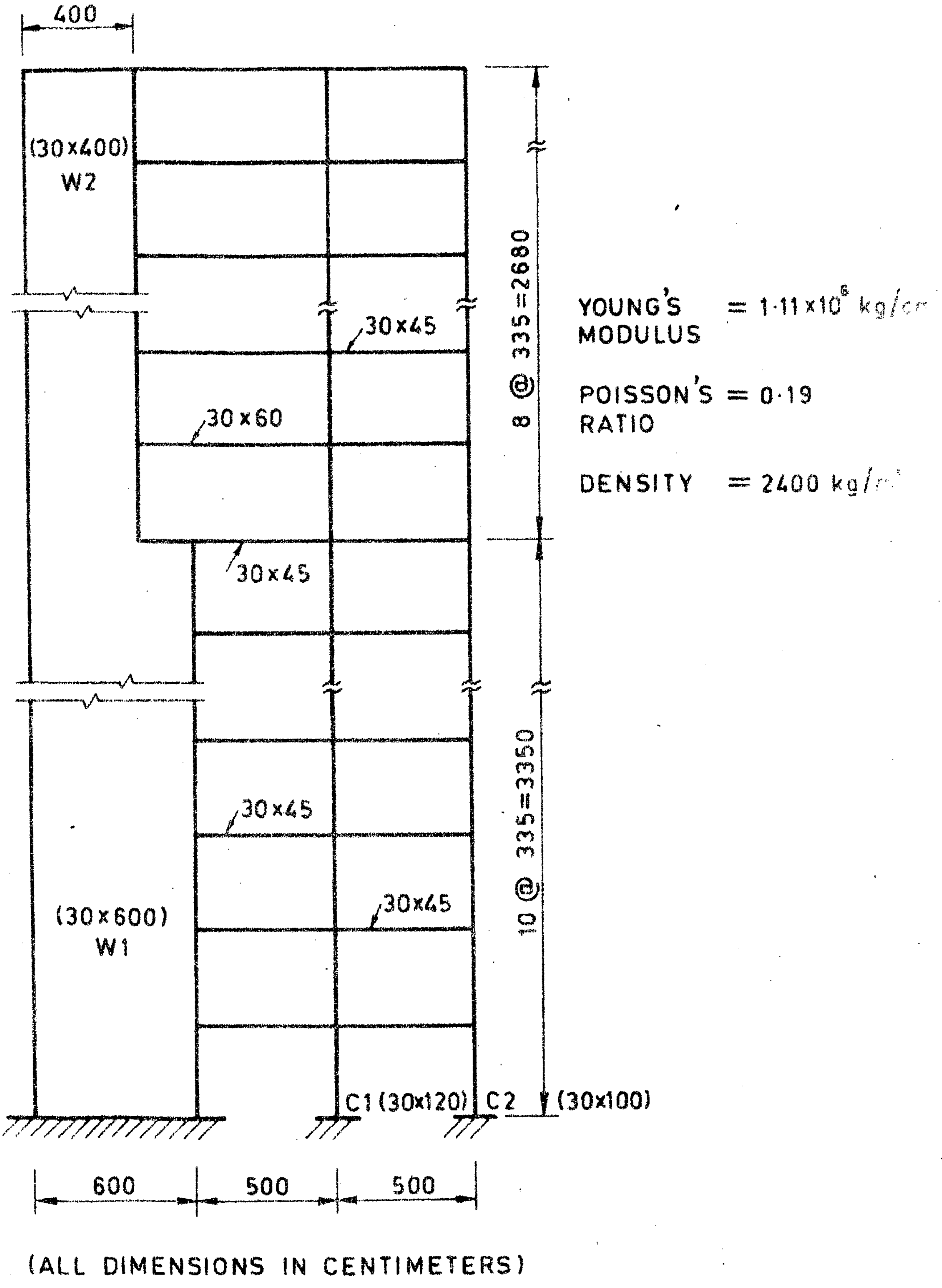

FIG.10 DETAILS OF FRAME SOLVED IN EXAMPLE 2

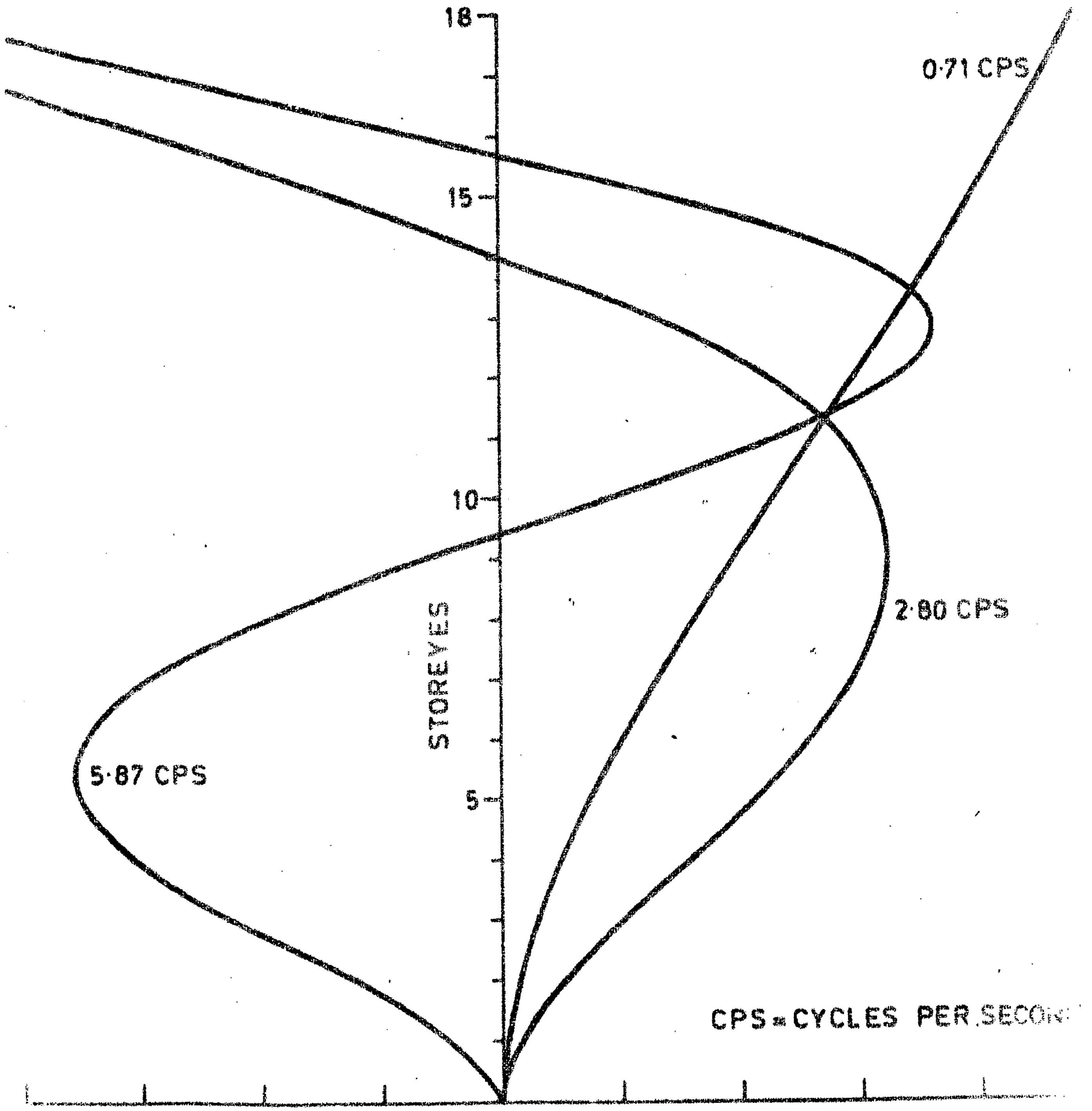

FIG. 11 MODE SHAPES AND FREQUENCIES FOR EXAMPLE 2

Case (i) Structure with self-weight only

Case (ii) Structure with self-weight + two masses (corresponding to the mass of the two acceleration pick ups used in the experimental investigations).

Case (iii) Structure with self-weight + additional masses on all floors except at top glued to the floors Fig. (12).

EXPERIMENTAL TEST

To compare the theoretically determined values of frequencies against the experimental values, experimental investigation was carried out to ascertain the lowest three resonant frequencies. In this method, the system is vibrated under steady state using a mechanical oscillator Fig. (13). Amplitude measurements were taken using acceleration pick ups at the top and also using strain gages pasted on columns and shear walls near the base.

The frequency-amplitude plots for the above three cases were obtained using pick ups and strain gages and also by orienting the model in X and Y directions. A few typical frequency-amplitude plots are given in Fig. (14).

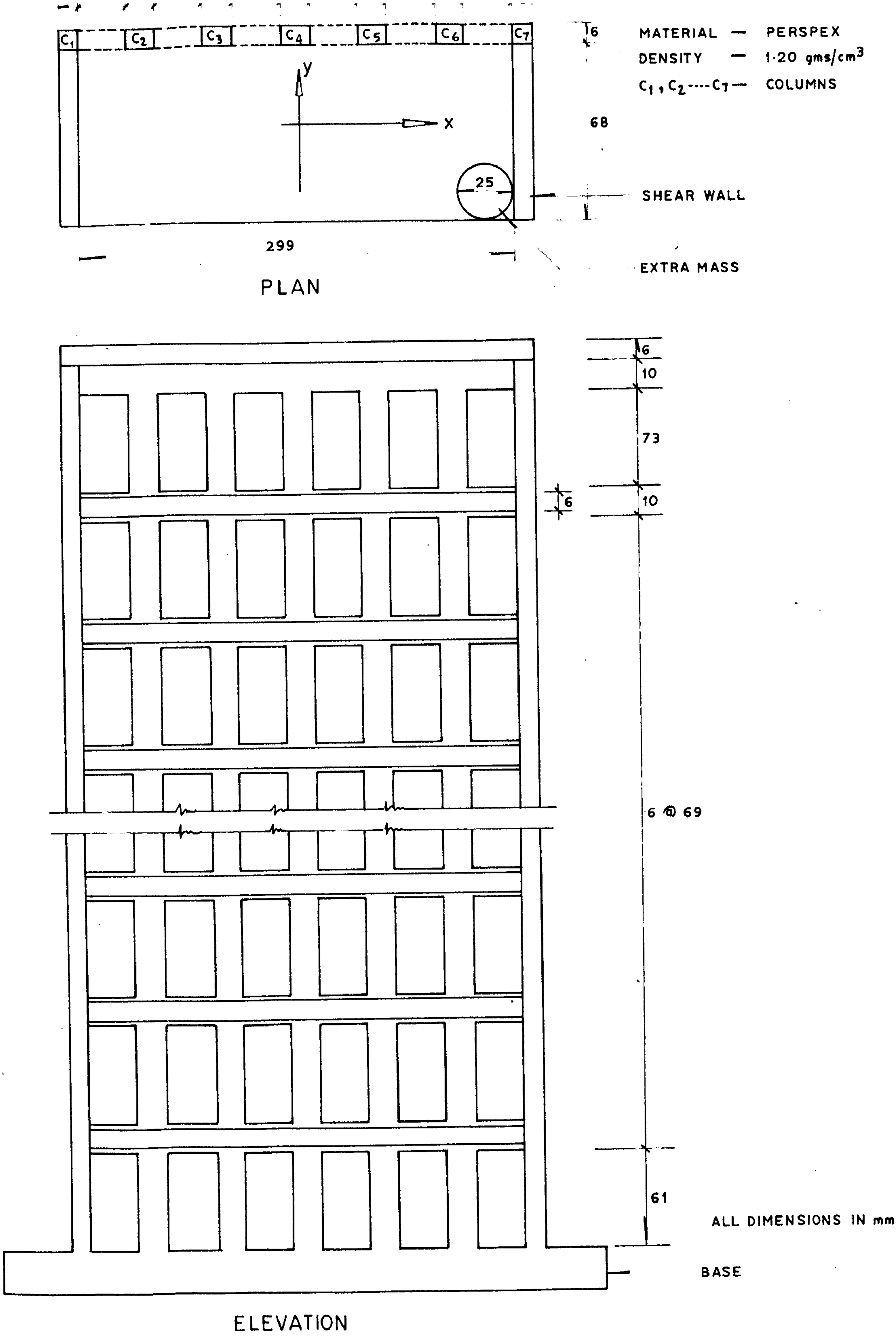

FIG. 12 MODEL FOR EXAMPLE 3

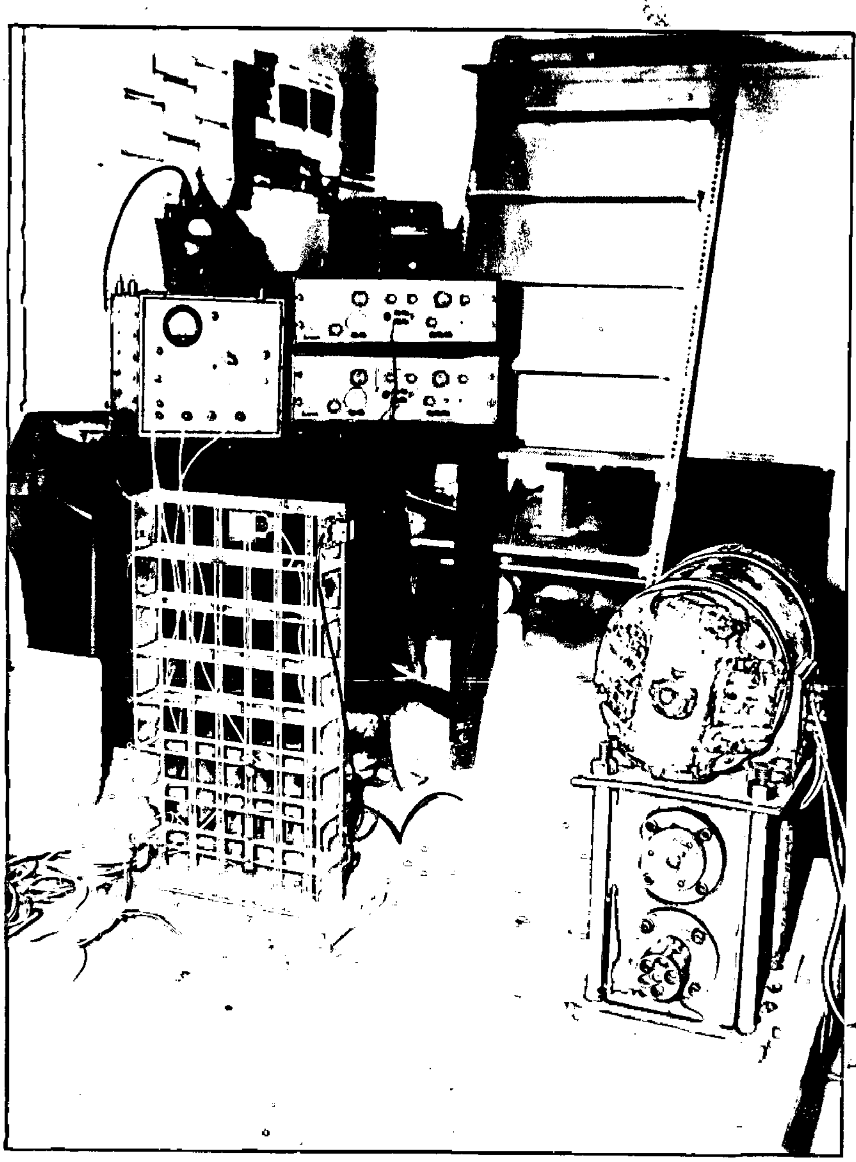

FIG. 13 EXPERIMENTAL SET UP

The free vibration test was carried out for finding out the 1st frequency in both the directions and were practically of the same value (31 CPS).

The computed values of periods and their corresponding experimental values are tabulated below for comparison for all the three cases.

TABLE

Case	PERIODS (Sec.)				
	Theoretical			Experimental	
	Mode 1	Mode 2	Mode 3	Modes 1 and 2	Mode 3
i.	0.033	0.032	0.022	0.03	0.023
ii.	0.038	0.035	0.027	0.036	0.024
iii.	0.05	0.043	0.027	0.042	0.031

In 1st mode of case i only v deformations are prominent, in 2nd mode u -deformations are prominent and a little rotation is available while in 3rd mode u-deformations and rotation both are prominent.

In 1st mode of case iii u and v deformations are prominent and some rotation is available, in 2nd mode u and v deformations are prominent while in 3rd mode u,v and θ deformations are prominent.

It is seen that the agreement between the theoretical and experimental values is excellent. The difference is little prominent in higher modes.

The effect of additional mass is to reduce the frequency and also the agreement between the predicted and experimental values is all the more excellent.

There is larger deviation in case ii which can be attributed to the masses of the pick ups as it could be incorporated in the theoretical analysis only approximately.

This 3-D structure has also been analysed using the approach of Chapter II for case i only. The frequencies and mode shapes of shear walls and frames have been obtained considering these as two dimensional structure and the results are given in Appendix I. The 1st five frequencies obtained by two methods have been compared as below :

2D Analysis	30.78	32.65	97.43	162.5	181.17
3D Analysis	30.79	31.51	44.65	97.65	161.9

The response u_r, v_r, and θ_r (of centre of mass) for the rth floor in 1st three modes for case (i and ii) have been found and plotted in Figs. (15, 16). The frequencies, periods and mode shape values for the

CHAPTER V

C O N C L U S I O N S

The proposed method in Chapter II; can be effectively utilized for finding out the dynamic characteristics of plane frames, with columns, Columns with shear walls and coupled shear walls.

Also the intermediate output of this method can be used for the static analysis of frames for seismic forces calculated above.

The technique reported in Chapter III can be used for finding the dynamic characteristics of multi storey buildings as a whole which means the assymmetry of the building can be considered.

The forces on different frame bents can be calculated. Once again by back substitution the analysis of frames can be carried out without much additional computations.

In short a lot of computational effort and manual labour is saved by the utilization of the reported technique.

three cases are given in Appendix II:

Knowing the above response in each mode, 5 per cent damping and Housner's average spectra curves the most probable values of U,V, and θ for the centre of mass have been calculated and are given in Appendix III for all the three cases. From this response we can obtain the displacements at rth floor for the jth frame using the eqns (3.4 and 3.5).

SCOPE FOR FURTHER RESEARCH

Parametric studies should be carried out on frames having columns and shear wall for the determination of fundamental periods with respect to mass distribution, ratio of shear wall to column stiffness, ratio of column to beam stiffness.

Some more models should be tested for the further verification of approach for the analysis of multistorey buildings with rigid floor rotations.

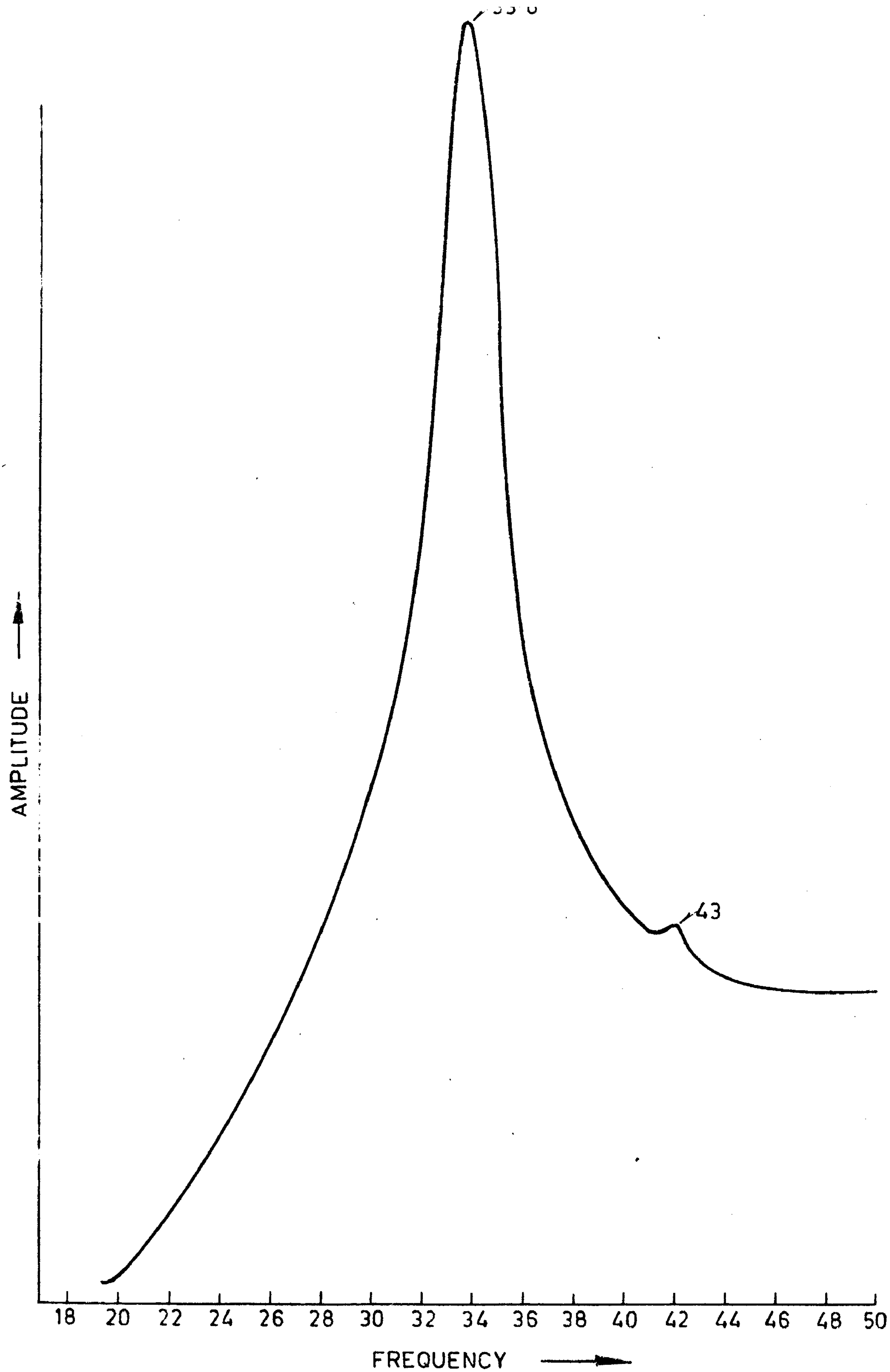

FIG. 14 (a) AMPLITUDE FREQUENCY PLOT FOR Y-DIRECTION CASE I

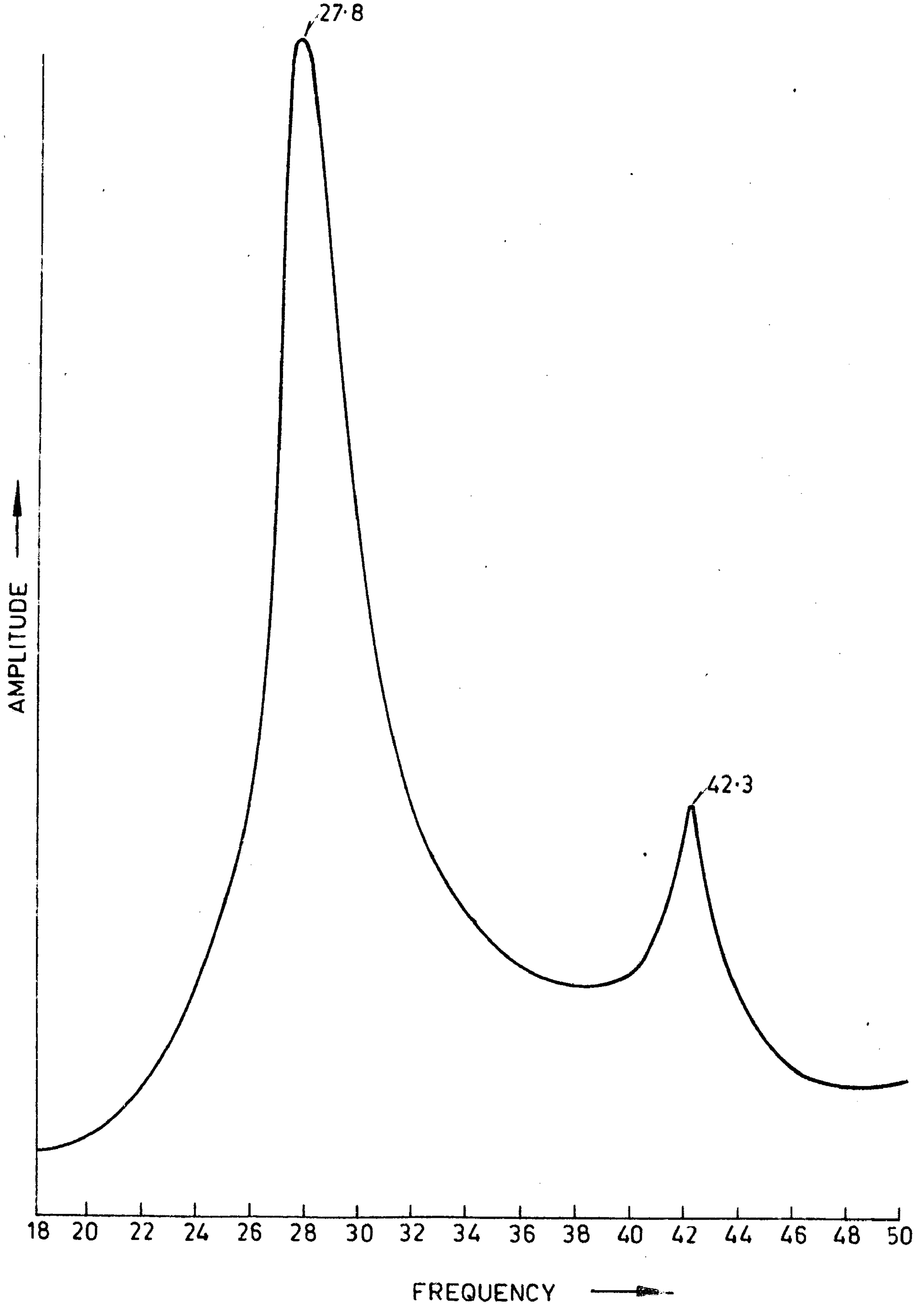

FIG. 14 (b) AMPLITUDE FREQUENCY PLOT FOR Y - DIRECTION CASE II

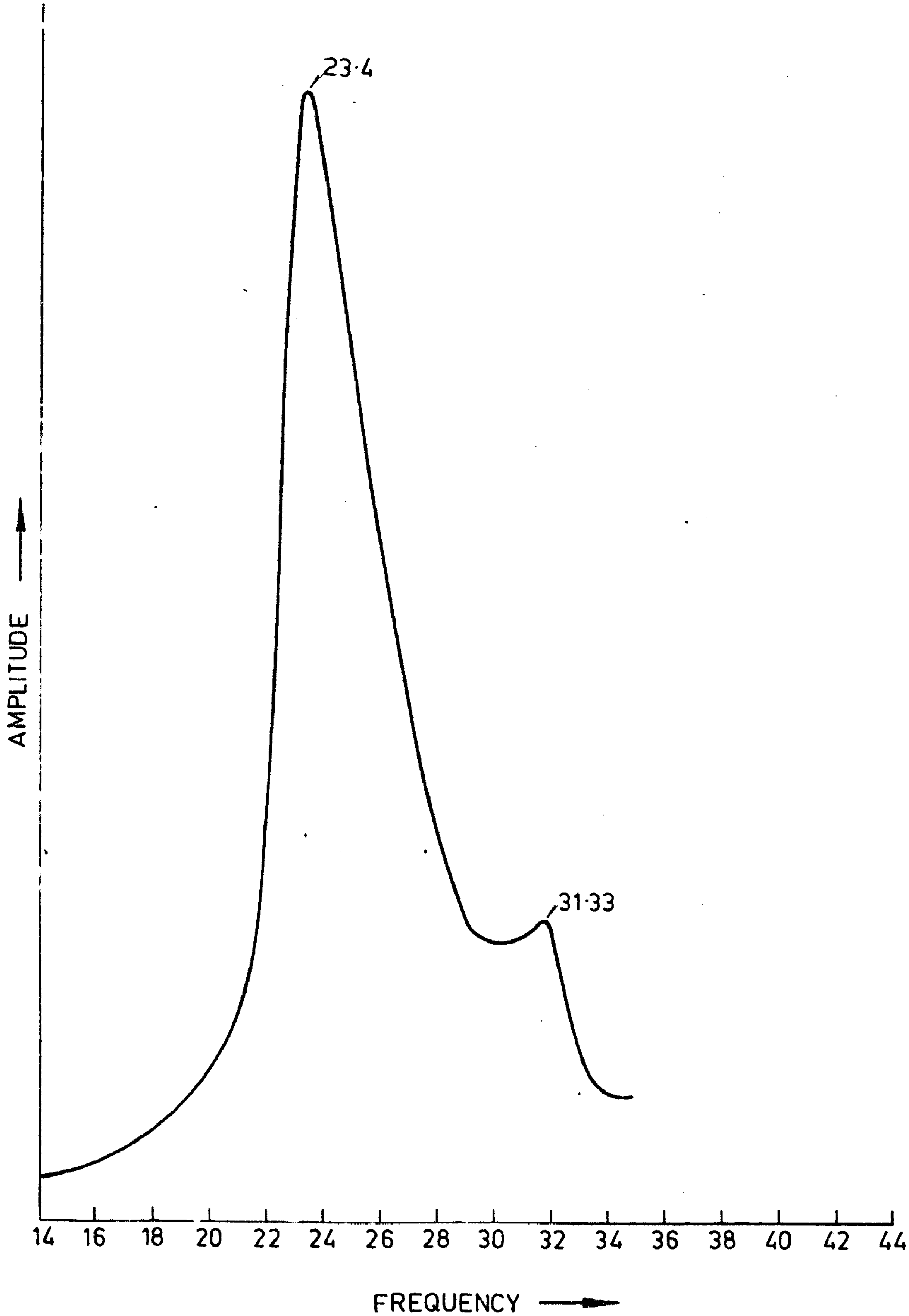

FIG. 14 (c) AMPLITUDE FREQUENCY PLOT FOR Y-DIRECTION CASE III

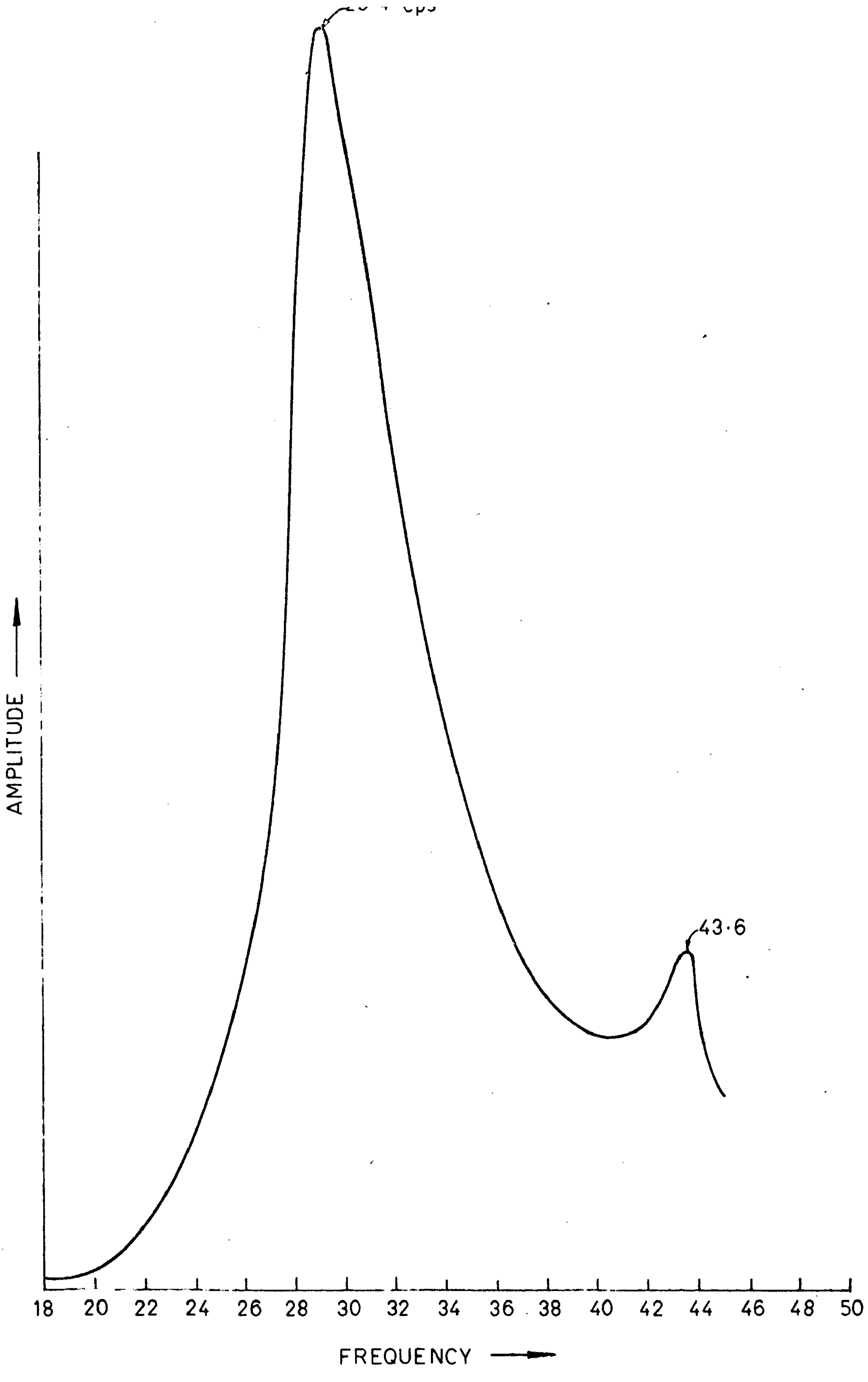

FIG 14(d) AMPLITUDE – FREQUENCY PLOT FOR CASE II X–DIRECTION

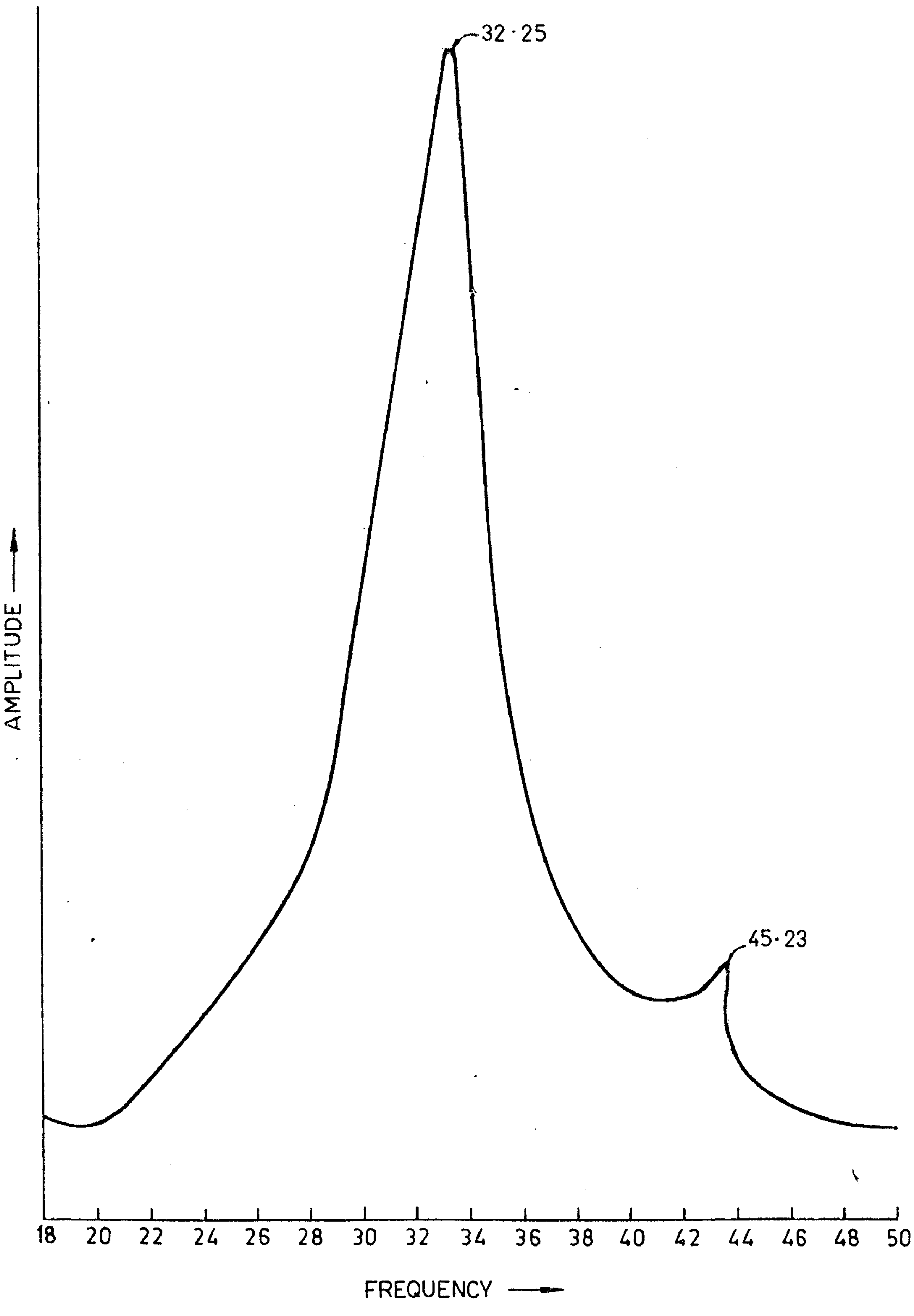

FIG. 14 (e) AMPLITUDE-FREQUENCY PLOT FOR CASE I X-DIRECTION

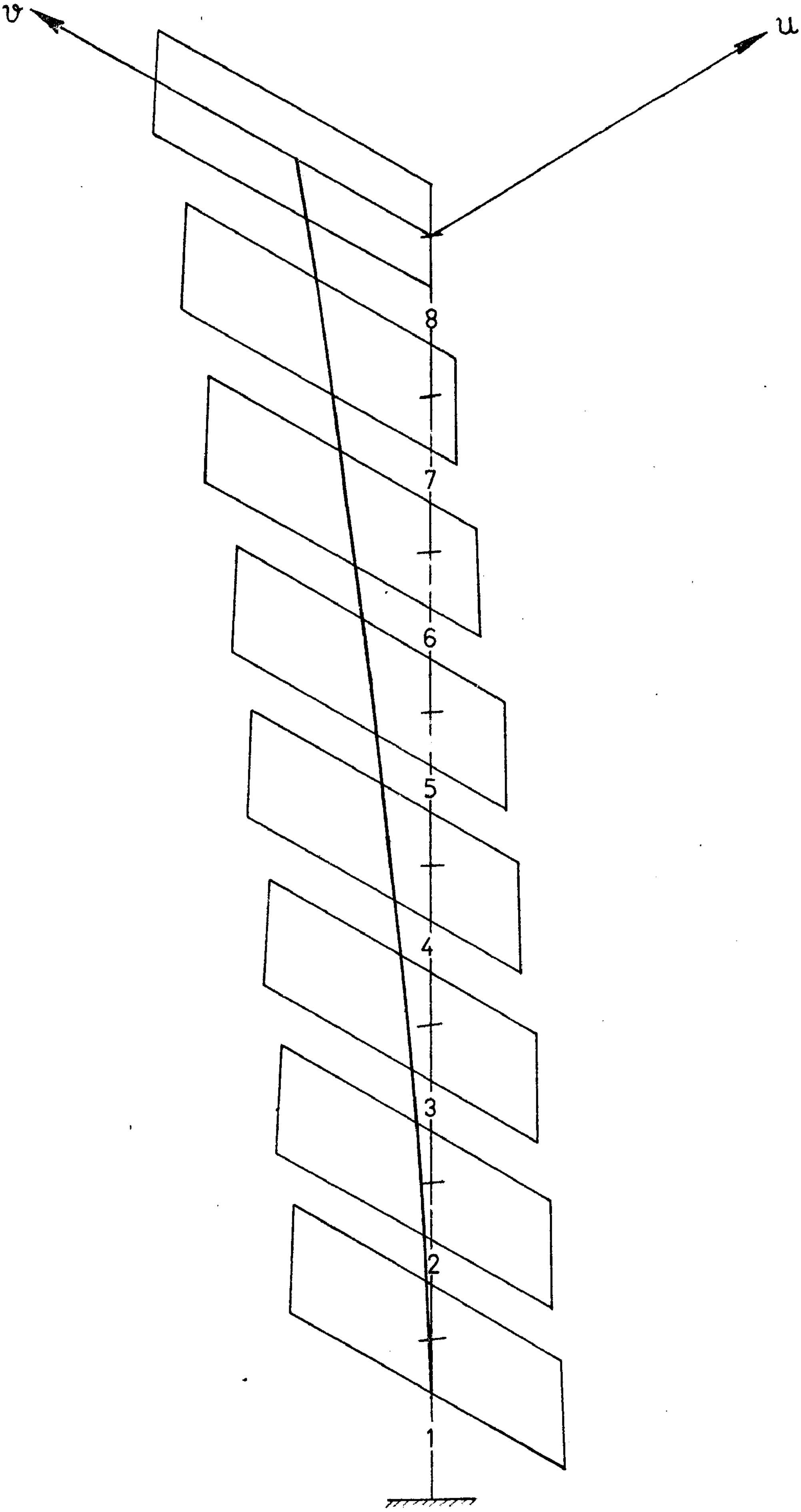

FIG. 15 a. u, v, θ PLOT FOR MODE 1 CASE i

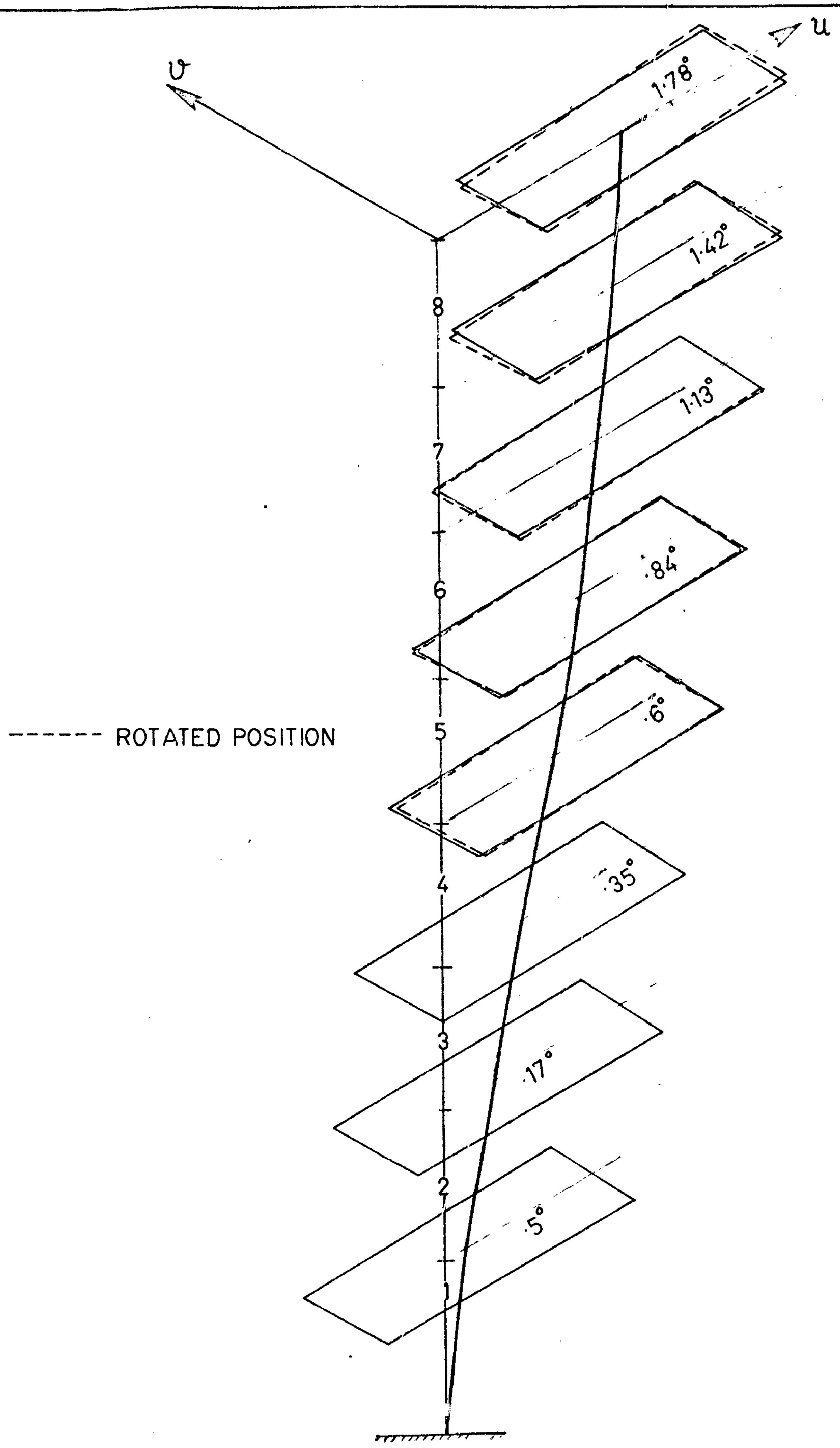

FIG.15b.u, v, θ PLOT FOR MODE 2 CASE I

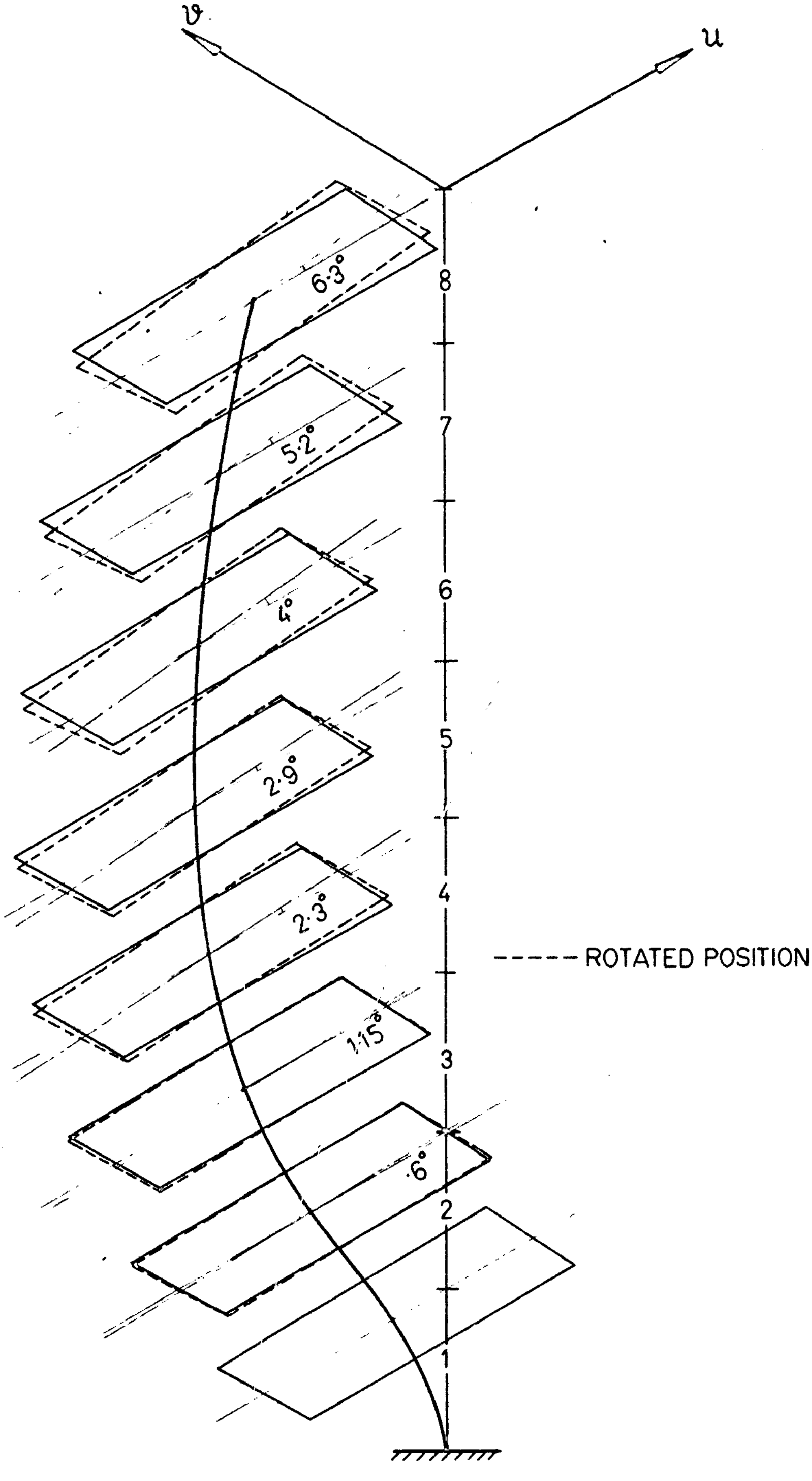

FIG. 15c. u, v, θ PLOT FOR MODE 3 CASE I

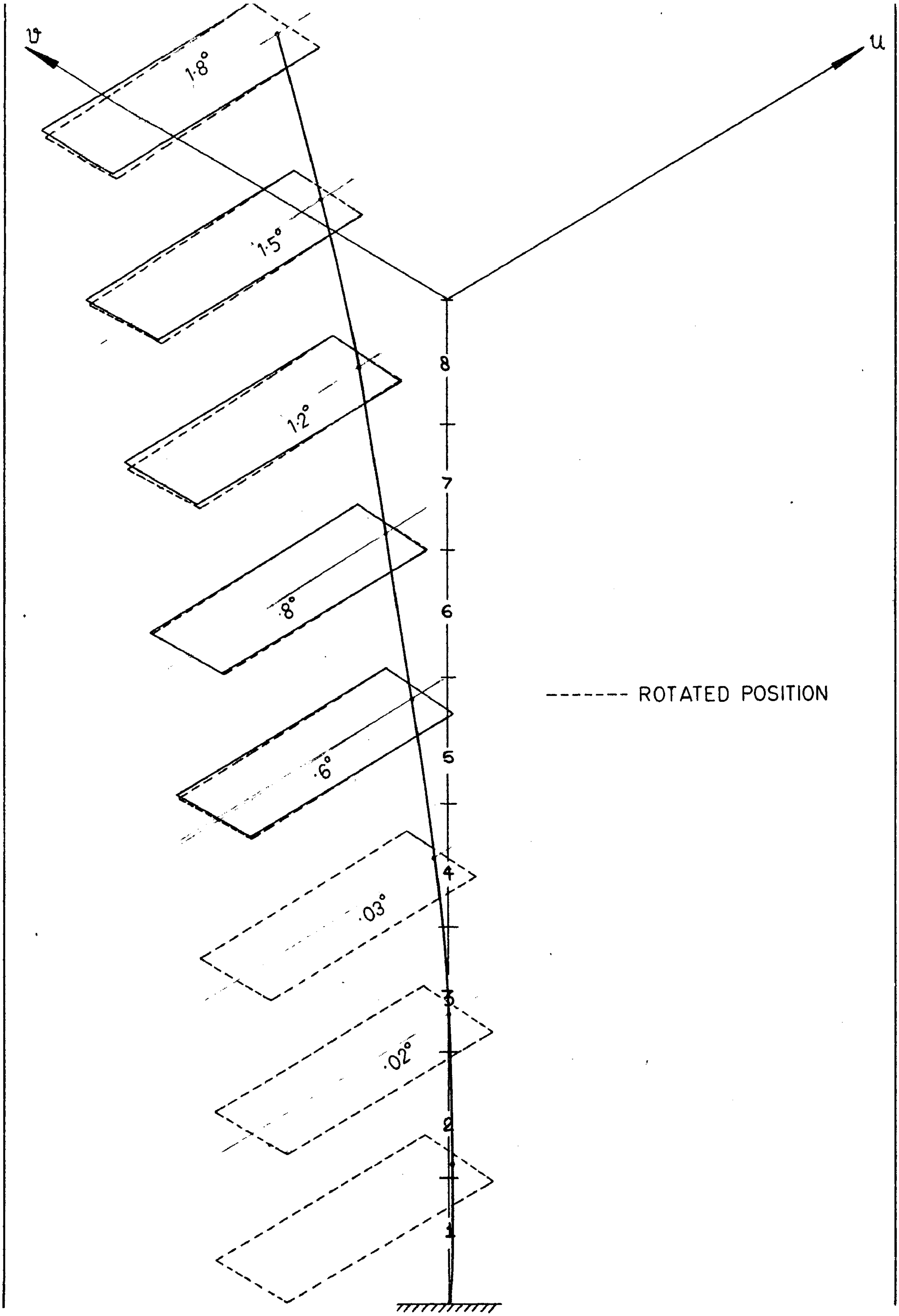

FIG.16a. u, v, θ PLOT FOR MODE 1 CASE iii

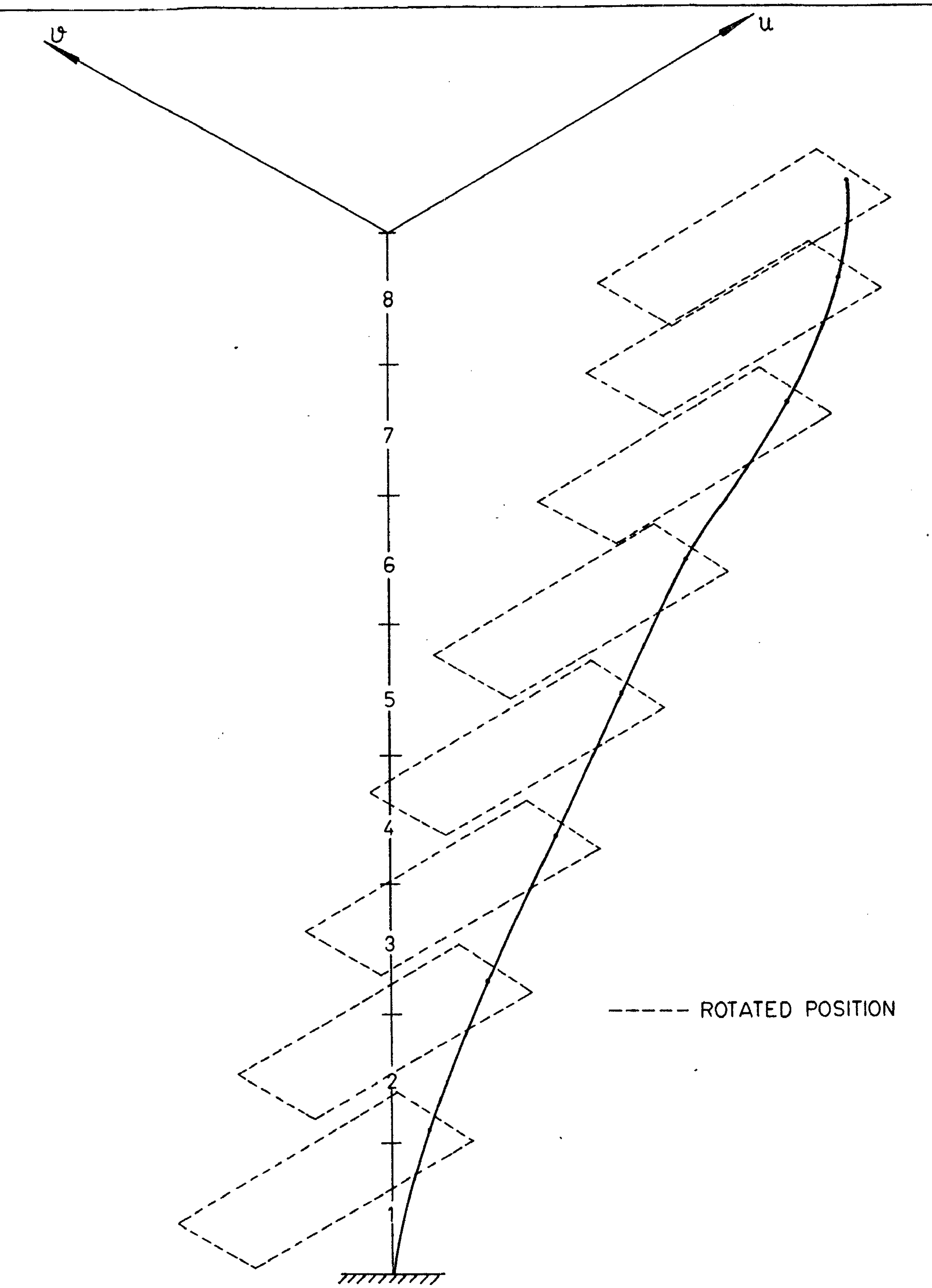

FIG. 16 b. u,v,θ PLOT FOR MODE 2 CASE iii

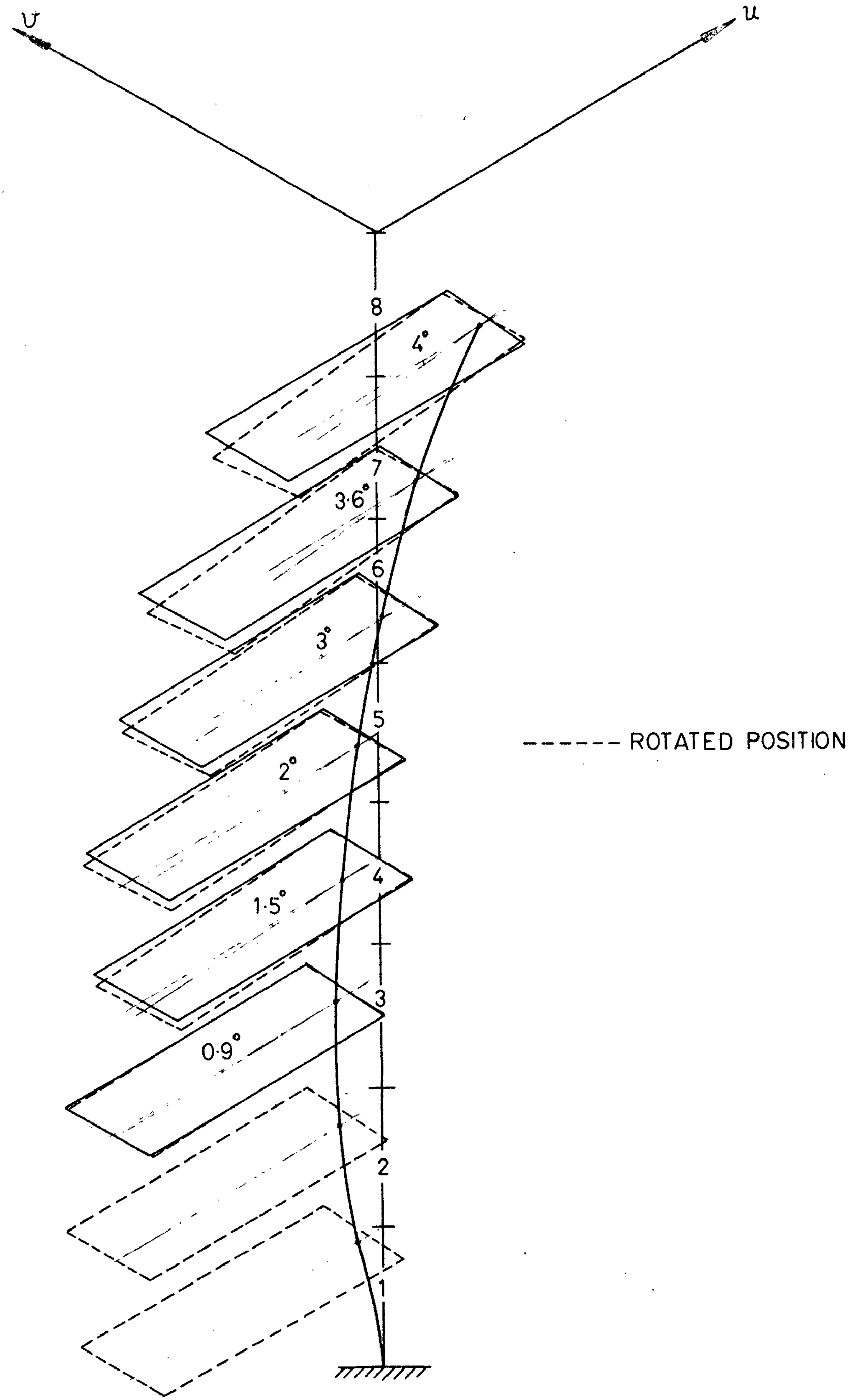

FIG. 16 c. u, v, θ PLOT FOR MODE 3 CASE iii

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APPENDIX I

FREQUENCY AND MODE SHAPES NEGLECTING FLOOR ROTATION

SHEAR WALL ONLY

MODE 1

.17610006E+01 .14204517E+01 .11227915E+01 .83665184E+00 .57236079E+00
.34233193E+00 .16259298E+00 .40075942E-01

MODE 2

.14748699E+01 -.35628396E+00 .50940221E+00 .11030671E+01 .13214709E+01
.11598108E+01 .73762150E+00 .24338030E+00

MODE 3

12012461E+01 -.44205897E+00 -.11758813E+01 -.82185216E+00 .26498681E+00
12156684E+01 .13253696E+01 .61292204E+00

PERIODS

32479708E-01 .55195841E-02 .21358688E-02

FREQUENCIES

30.78 181.17 468.19 829.92 1851.09 1563.30 2041.91 1213.26

FRAME ONLY

MODE 1

63340855E+00 .91562271E+00 .84268441E+00 .73913281E+00 .60853583E+00
56435552E+00 .29302697E+00 .10841097E+00

MODE 2

7723090E+00 -.60838065E+00 -.11046712E+00 .41618284E+00 .80586476E+00
308633E+00 .76115315E+00 .32072018E+00

MODE 3

869994E+00 -.64168993E-01 .72486038E+00 .88364149E+00 .28914945E+00
267165E+00 -.91691687E+00 -.50675191E+00

PERIODS

521830E-01 .10263070E-01 .61531740E-02

FREQUENCIES

.65 97.43 162.51 380.77 411.25 336.19 225.11 283.99

A P P E N D I X II

FREQUENCY, PERIODS AND MODE SHAPES CONSIDERING RIGID FLOOR ROTATIONS

CASE 1

FREQUENCIES

30.79	31.51	44.65	97.21	161.91	181.17	224.72	256.84
283.86	335.93	380.51	410.97	468.19	660.01	829.92	1165.27
1213.26	1563.30	1699.77	1851.09	2041.91	2190.35	2596.64	2867.97

PERIODS

.32478066E-01	.31731681E-01	.22391974E-01	.10286739E-01	.61760480E-02
.55195716E-02	.41498123E-02	.38934571E-02	.35228061E-02	.29767901E-02
.26280453E-02	.24332406E-02	.21358715E-02	.15151122E-02	.12049239E-02
.85816291E-03	.82422156E-03	.63967040E-03	.58831385E-03	.54022000E-03
.48973606E-03	.45654743E-03	.39511271E-03	.34867831E-03	

MODE SHAPES

MODE 1

-.11986890E-03	-.11134992E-03	-.10053322E-03	-.86572757E-04	-.69999044E-04
-.51552287E-04	-.32485443E-04	-.11809090E-04	.12452176E+01	.10044117E+01
.79393280E+00	.59160008E+00	.40471775E+00	.24206390E+00	.11496990E+00
.28337815E-01	-.61702420E-05	-.49857031E-05	-.39487795E-05	-.29485157E[illegible]
-.20215540E-05	-.12112198E-05	-.57626733E-06	-.14234475E-06	

MODE 2

.95058102E+00	.89059646E+00	.80993025E+00	.70238605E+00	.57190764E+00
.42422457E+00	.26930179E+00	.98587337E-01	.16171677E-03	.1304240[illegible]
.10307619E-03	.76798031E-04	.52532391E-04	.31420892E-04	.14925008E-04
.36794155E-05	.30715725E-01	.24823630E-01	.19666471E-01	.14695649E-01
.10086325E-01	.60549228E-02	.28875836E-02	.71575289E-03	

MODE 3

-.16777144E+00	-.20938461E+00	-.23227933E+00	-.23730125E+00	-.22225407E+00
-.18639875E+00	-.13185999E+00	-.52826303E-01	.24904019E-04	.20018855E-04
.15763034E-04	.11712061E-04	.79916455E-05	.47829453E-05	.22765956E-05
.56352258E-06	.10969433E+00	.88438521E-01	.69858718E-01	.52008739E-01
.35540646E-01	.21230111E-01	.10068976E-01	.24768533E-02	

MODE 4

-.97904732E+00	-.60846313E+00	-.11038261E+00	.41487998E+00	.80212394E+00
.92706903E+00	.75463346E+00	.31732051E+00	.28395120E-06	.21210338E-06
.15211283E-06	.10142388E-06	.61057536E-07	.34079293E-07	.15694079E-07
.39233534E-08	.51616744E-02	.61442795E-02	.67988524E-02	.69331594E-02
.62910351E-02	.48304018E-02	.28615432E-02	.89842944E-03	

MODE 5

.95512056E+00	.57871050E-01	-.72609955E+00	-.87842633E+00	-.28190833E+00
.55725154E+00	.91835182E+00	.50682324E+00	-.12674301E-06	-.42163140E[illegible]
.23599951E-07	.68899066E-07	.87215269E-07	.76405788E-07	.4817181[illegible]
.15712550E-07	.48206021E-02	-.94983612E-04	-.35061985E-02	-.50597326E[illegible]
-.45881893E-02	-.28800431E-02	-.11582258E-02	-.15870252E-03	

MODE 6

-.12098613E-06	-.39755659E-07	.70827113E-07	.12046399E-06	.64026989[illegible]
-.37496560E-07	-.90660395E-07	-.53038450E-07	-.10428865E+01	-.2519287[illegible]
.36020214E+00	.77998590E+00	.93442120E+00	.82011278E+00	.52157970[illegible]
.17209697E+00	.37659501E-07	.14012067E-07	-.43622785E-08	-.1679839[illegible]
-.21472688E-07	-.17443120E-07	-.10150669E-07	-.29578590E-08	

MODE 7

-.82811929E+00	.53802360E+00	.83744137E+00	-.15845514E+00	-.91929467E+00
-.29061450E+00	.74857653E+00	.69178433E+00	-.17775871E-07	-.41822191E-09
.12850431E-07	.22519254E-07	.26465445E-07	.25462663E-07	.17867465E-07
.66101763E-08	-.49862109E-02	.32671658E-03	.33151734E-02	.40314911E-02
.36225397E-02	.32316481E-02	.26153326E-02	.11678967E-02	

MODE 8

.76040062E-01	.74896441E-01	-.77007239E-01	-.11904975E+00	.11574097E-01
.64925930E-01	-.42422556E-01	-.80611649E-01	-.22259464E-06	-.49004832E-07
.91543287E-07	.20626049E-06	.26853410E-06	.29003426E-06	.21711777E-06
.83838572E-07	-.96107177E-01	-.23916092E-01	.32089978E-01	.70752475E-01
.85301966E-01	.75127561E-01	.47830743E-01	.15769629E-01	

MODE 9

.62177152E+00	-.84788028E+00	-.13934165E+00	.90152726E+00	-.83915192E-01
-.88478357E+00	.27253926E+00	.90008182E+00	-.10415214E-07	-.36352363E-08
.29910549E-08	.92984337E-08	.13007485E-07	.15728760E-07	.12928681E-07
.55254957E-08	-.67067519E-02	-.27201715E-02	.18988209E-02	.56317052E-02
.63193163E-02	.51647695E-02	.38400215E-02	.17925639E-02	

MODE 10

-.41558657E+00	.85505431E+00	-.61479894E+00	-.30768130E+00	.89673923E+00
-.51786066E+00	-.41774877E+00	.98986844E+00	.58929518E-10	.10389600E-08
.70452777E-09	.33725227E-09	-.23164243E-10	-.98089401E-09	-.10294804E-08
-.21474673E-09	.66235378E-03	.10456575E-02	-.28065404E-03	-.69371503E-03
-.43875246E-03	-.12440941E-02	-.90838569E-03	.30965196E-03	

MODE 11

.25548748E+00	-.67037099E+00	.92919904E+00	-.70672053E+00	.12925818E+0[illegible]
.51645230E+00	-.86158574E+00	.83208421E+00	.90920232E-09	-.56198043E-0[illegible]
-.63920901E-09	-.72364799E-09	.45222376E-09	.14949948E-08	.13650318E-0[illegible]
.90908796E-09	-.20900321E-03	-.65688001E-03	.39681122E-03	-.11713875E-0[illegible]
.52095866E-03	.7*233645E-03	-.57500062E-04	.57844337E-03	

MODE 12

-.11364562E+00	.34025002E+00	-.61600903E+00	.81077902E+00	-.90803111E+00
.89766102E+00	-.73544055E+00	.50938455E+00	-.45471538E-09	.19447368E-09
.12064768E-09	.12748333E-09	-.90317056E-09	-.12870655E-08	-.14354129E-08
-.44668832E-09	.81372337E-04	.29338696E-03	-.27542584E-03	.28214174E-03
-.64032776E-03	.22287424E-03	-.50751480E-03	.21074527E-03	

MODE 13

-.37978533E-08	.20732888E-08	.13666897E-09	.28295359E-08	-.70532057E-10
-.50649192E-09	-.12887724E-08	-.12947116E-08	.84941080E+00	-.31258214E+00
-.83147412E+00	-.58113902E+00	.18737186E+00	.85960611E+00	.93717636E+00
.43340059E+00	.55925526E-08	-.22371516E-08	-.69941015E-08	-.84016596E-08
-.53393989E-08	-.95273456E-09	.17668555E-08	.15641442E-08	

MODE 14

-.23074915E-01	.11240290E-01	.13391297E-01	.99973507E-02	-.19590298E-02
-.11603905E-01	-.10318558E-01	-.38234166E-02	-.53768978E-07	.25201314E-07
.41126727E-07	-.11180832E-07	-.60871115E-07	-.66224574E-07	-.34684472E-07
-.16050042E-08	.80000751E-01	-.27543010E-01	-.75921289E-01	-.53539353[illegible]
.16777031E-01	.78598851E-01	.85935031E-01	.39798586E-01	

MODE 15

.89740852E-09	-.83385087E-09	-.54267492E-09	.25231979E-09	.37169332E-[illegible]9
.45501490E-09	-.23966205E-09	-.52758582E-09	-.64868553E+00	.71260943E[illegible]
.61228076E+00	-.45109042E+00	-.89136171E+00	-.45683500E-01	.90590672E[illegible]
.71860899E+00	-.29982181E-08	.33400879E-08	.32151797E-08	-.1155534[illegible]
-.41127207E-08	-.39181207E-08	-.75949946E-09	.10383078E-08	

MODE 16				
.86781333E-02	-.84050599E-02	-.58089470E-02	.39874616E-02	.81678214E-02
.80367474E-03	-.73293090E-02	-.61503890E-02	.16269056E-07	-.66104285E-08
-.25113160E-07	.23194052E-08	.36209120E-07	-.17120979E-07	-.46100090E-07
-.15645623E-07	-.61961070E-01	.64533272E-01	.55303273E-01	-.41725573E-01
-.81459730E-01	-.32362791E-02	.82878092E-01	.65622105E-01	
MODE 17				
-.38272654E-09	.62814096E-09	-.70280781E-10	-.63579088E-09	.27197413E-09
.49638218E-09	-.19895927E-09	-.65649183E-09	.46076650E+00	-.85861261E+00
.84393531E-01	.90651930E+00	-.20593822E+00	-.85734547E+00	.36498715E+00
.95896594E+00	.19096412E-08	-.35394885E-08	.63546469E-09	.38813912E-08
-.16908737E-08	-.35897427E-08	.17012537E-08	.42705559E-08	
MODE 18				
-.19451786E-09	.46023271E-09	-.40875917E-09	-.13335401E-09	.45701685E-09
-.92749598E-10	-.31784333E-09	.30655214E-09	.30873312E+00	-.79694862E+00
.70729549E+00	.23205502E+00	-.89753725E+00	.52707309E+00	.42543969E+00
-.10231422E+01	.12016253E-08	-.30269109E-08	.34440711E-08	.93897484E-09
-.46907806E-08	.23655575E-09	.31845286E-08	-.12640932E-08	
MODE 19				
.41888621E-02	.67212213E-02	-.65780950E-03	-.61353978E-02	.15543235E-02
.60291280E-02	-.22720130E-02	-.66623726E-02	-.26689466E-07	.54095333E-07
-.17548667E-07	-.41339190E-07	.15314798E-07	.42776362E-07	-.34846594E-07
-.21747318E-07	.43956085E-01	-.77592034E-01	.10002097E-01	.82733858E-01
-.20166136E-01	-.78093479E-01	.34098739E-01	.87542767E-01	
MODE 20				
-.10111121E-09	.30587509E-09	-.47796202E-09	.39556018E-09	-.10773931E-09
-.21667753E-09	.35883290E-09	-.29586612E-09	.19070544E+00	-.60829448E+00
.93364289E+00	-.72277692E+00	.12076778E+00	.55328549E+00	-.89048064E+00
.82443566E+00	.68493965E-09	-.21230017E-08	.42321248E-08	-.31092151E-08
.91671153E-09	.10459578E-08	-.33143106E-08	.20518090E-08	
MODE 21				
.14227199E-09	-.35310661E-09	.42819449E-09	-.31430033E-09	.24545207E-09
-.26513147E-09	.26998792E-09	-.22362634E-09	-.88146857E-01	.31769671E+00
-.62588531E+00	.83637045E+00	-.93165513E+00	.89775329E+00	-.71114164E+00
.46512090E+00	-.15189823E-08	.37743927E-08	-.46334636E-08	.17636208E-08
-.18910029E-08	.20360376E-08	-.25681820E-08	.20889432E-08	
MODE 22				
.22363884E-02	-.50367254E-02	.42678253E-02	.11940300E-02	-.52430386E-02
.32206778E-02	.21676906E-02	-.62152240E-02	.15762096E-07	-.42988524E-07
.41804480E-07	-.6*715509E-08	-.93028619E-08	.26187340E-08	.25600012E-07
-.46411982E-07	-.28970359E-01	.70844404E-01	-.66864473E-01	-.18947786E-01
.81898634E-01	-.49590429E-01	-.38039876E-01	.93931873E-01	
MODE 23				
.12161786E-02	-.34224577E-02	.50772733E-02	-.40362094E-02	.76082798E-03
.29763909E-02	-.49189957E-02	.46757942E-02	.88453686E-08	-.29335481E-07
.43451786E-07	-.30225005E-07	.13284403E-07	.14455364E-07	-.27508737E-07
.29529742E-07	-.17521423E-01	.53000121E-01	-.85701847E-01	.67837191E-01
-.12946037E-01	-.49715086E-01	.81688612E-01	-.76180426E-01	
MODE 24				
.24722344E-02	-.64221272E-02	.96295976E-02	-.76551074E-02	.14429925E-0[illegible]
.56450470E-02	-.93294070E-02	.88681491E-02	.17980767E-07	-.55047047E-[illegible]
.82411009E-07	-.74394462E-07	.25195306E-07	.27416160E-07	-.52173293E-[illegible]
.56006347E-07	-.33112375E-02	.89586555E-02	-.14918776E-01	.11808938E-01
.22536156E-02	-.86542853E-02	.14220161E-01	-.13261309E-01	

CASE 2

FREQUENCIES

26.71	28.65	37.69	91.00	154.48	171.31	210.80	230.72
267.45	324.39	374.45	409.58	447.36	598.31	785.41	1048.57
1154.51	1514.84	1533.46	1823.61	2001.46	2034.50	2401.92	2675.31

PERIODS

.37435642E-01	.34903698E-01	.26530928E-01	.10988055E-01	.64731136E-02
.58370536E-02	.47436255E-02	.43342014E-02	.37389582E-02	.30826733E-02
.26705321E-02	.24415032E-02	.22353111E-02	.16713600E-02	.12732135E-02
.95367722E-03	.86616337E-03	.66013493E-03	.65211812E-03	.54836156E-03
.49963304E-03	.49151908E-03	.41633261E-03	.37378843E-03	

MODE SHAPES

MODE 1

-.31216660E-04	-.28428360E-04	-.25074444E-04	-.21132426E-04	-.16758315E-04
-.12123977E-04	-.75166267E-05	-.26937301E-05	.10841924E+01	.87202064E+00
.68693301E+00	.50993358E+00	.34749084E+00	.20702875E+00	.97946884E-01
.24019513E-01	-.31273046E-05	-.25228576E-05	-.19941668E-05	-.14852312E-05
-.10153709E-05	-.60621568E-06	-.28730033E-06	-.70583316E-07	

MODE 2

.86568937E+00	.80604761E+00	.72402765E+00	.62148156E+00	.50225363E+00
.37072541E+00	.23479697E+00	.85917515E-01	.44683775E-04	.35906925E-04
.28256111E-04	.20955015E-04	.14265945E-04	.84949059E-05	.40176388E-05
.98493075E-06	.26828906E-01	.21621072E-01	.17070574E-01	.12705901E-01
.86842358E-02	.51907573E-02	.24644580E-02	.60728395E-03	

MODE 3

-.19922861E+00	-.21730981E+00	-.21908306E+00	-.20848102E+00	-.18516496E+00
-.14922174E+00	-.10256187E+00	-.40287161E-01	.23687903E-04	.18937901E-04
.14814122E-04	.10924582E-04	.73958007E-05	.43899915E-05	.20718300E-05
.50685997E-06	.93421851E-01	.75200230E-01	.59288367E-01	.44048207E-01
.30039101E-01	.17908650E-01	.84776996E-02	.20804452E-02	

MODE 4

-.80362181E+00	-.40738659E+00	.83965653E-01	.54826376E+00	.85396397E+00
.91483282E+00	.71907443E+00	.29725295E+00	.14876479E-06	.10379716E-06
.67091329E-07	.38054068E-07	.17557417E-07	.70384596E-08	.20854141E-08
.20495926E-09	.23315383E-02	.34036783E-02	.41554120E-02	.44670939E-02
.41642031E-02	.32396898E-02	.19312105E-02	.60869064E-03	

MODE 5

.79041766E+00	-.19729854E+00	-.81922058E+00	-.78333565E+00	-.13640103E+00
.61936652E+00	.89011749E+00	.47115481E+00	-.89853220E-07	-.19004356E-07
.34952723E-07	.70411947E-07	.82351969E-07	.69968812E-07	.43384083E-07
.14001850E-07	.33692423E-02	-.31612134E-03	-.27189276E-02	-.36925436E-02
-.32453623E-02	-.20010088E-02	-.79875016E-03	-.11032384E-03	

MODE 6

-.10092438E-06	-.12117932E-08	.88352179E-07	.10685044E-06	.42628793E-07
-.42518578E-07	-.78943476E-07	-.43318722E-07	-.81697902E+00	-.84505672E-01
.46820130E+00	.82880465E+00	.93713989E+00	.79896645E+00	.49913629E+00
.16234598E+00	.46607291E-07	.17691145E-07	-.47089501E-08	-.19752002E-07
-.25479095E-07	-.21055504E-07	-.12454462E-07	-.37195234E-08	

MODE 7

.67593987E+00	-.74473410E+00	-.54330193E+00	.45775027E+00	.84108831E+00
.88000677E-01	-.75922180E+00	-.60473150E+00	.10968568E-07	-.24208927E-08
-.12114851E-07	-.18775675E-07	-.21058639E-07	-.19497594E-07	-.13294877E-07
-.47688092E-08	.29500047E-02	-.58788340E-03	-.22574457E-02	-.25826494E-02
-.24267065E-02	-.22902291E-02	-.18538221E-02	-.80489505E-03	

MODE 8

.72015841E-01	-.93025191E-03	-.69765333E-01	-.54065571E-01	.88426541E-02
.19346105E-01	-.26453351E-01	-.34964340E-01	-.21934129E-06	.15977491E-07
.19416106E-06	.32253863E-06	.37197554E-06	.36061863E-06	.25183537E-06
.91185110E-07	-.76640366E-01	-.12326859E-01	.36811396E-01	.69596294E-01
.80390132E-01	.69304312E-01	.43584477E-01	.14246127E-01	

MODE 9

.37319004E+00	-.74811527E+00	.41886273E+00	.80809503E+00	-.44840048E+00
-.79252891E+00	.44730722E+00	.84962186E+00	-.23892475E-08	.42919084E-10
.25620019E-08	.46527368E-08	.54058550E-08	.58878197E-08	.47121622E-08
.20114384E-08	-.19615107E-02	-.78049694E-03	.11414221E-02	.22670445E-02
.19624691E-02	.14793301E-02	.13872197E-02	.80946110E-03	

MODE 10

-.17828574E+00	.52346382E+00	-.89596811E+00	.97774316E-01	.82071998E+00
-.72997832E+00	-.26690937E+00	.10288936E+01	-.41068847E-09	.30307657E-09
.27137362E-09	.44855223E-09	.40909554E-09	.97901891E-10	.10732083E-09
.26087641E-09	.23548396E-04	.30889867E-03	-.29987947E-03	.52809749E-04
.26924160E-03	-.43239909E-03	-.21980926E-03	.40701582E-03	

MODE 11

-.91196500E-01	.33598248E+00	-.89718793E+00	.88376564E+00	-.35539594E+00
-.38313435E+00	.86097072E+00	-.90461266E+00	-.49842680E-09	.20054042E-09
.20034760E-09	.34088655E-09	-.44829939E-09	-.92401593E-09	-.74790841E-09
-.54504184E-09	.49896369E-04	.16032262E-03	-.34949324E-03	.28936057E-03
-.22138282E-03	-.24214724E-03	.25701991E-03	-.39630715E-03	

MODE 12

.37251199E-01	-.15508302E+00	.51317390E+00	-.77266545E+00	.92421502E+00
-.94913157E+00	.79513714E+00	-.55851345E+00	.14607474E-09	-.21064433E-10
.91284068E-10	.79269754E-11	.66530180E-09	.74830566E-09	.83997457E-09
.20716948E-09	-.37944096E-04	-.66109032E-04	.20972386E-03	-.24692887E-03
.38474722E-03	-.31896906E-03	.31248816E-03	-.21716929E-03	

MODE 13

-.17924348E-08	.10411546E-08	.31917592E-09	.21534450E-08	-.16554620E-09
-.65371187E-09	-.11132103E-08	-.10678017E-08	.67862282E+00	-.42160774E+00
-.81550292E+00	-.48510700E+00	.27013787E+00	.87420406E+00	.90401597E+00
.40734590E+00	.40446823E-08	-.31883229E-08	-.69088445E-08	-.73271040E-08
-.37663859E-08	.67096908E-09	.29122321E-08	.19624434E-08	

MODE 14

-.12451786E-01	.67720054E-02	.11724795E-01	.75533437E-02	-.23184672E-02
-.99890187E-02	-.82930714E-02	-.28685702E-02	-.47220782E-07	.27901805E-07
.38512956E-07	-.16373000E-07	-.63926598E-07	-.71587415E-07	-.43649181E-07
-.88293748E-08	.64777591E-01	-.32425143E-01	-.70327666E-01	-.44524695E-01
.20614456E-01	.74466336E-01	.78475831E-01	.35698389E-01	

MODE 15

.50013003E-09	-.58171669E-09	-.37278294E-09	.39154036E-09	.30114918E-09
.29881671E-09	-.29629164E-09	-.46597326E-09	-.51022992E+00	.72853477E+00
.38837175E+00	-.60479689E+00	-.81914012E+00	.76982786E-01	.90932169E+00
.66606471E+00	-.27263014E-08	.35940818E-08	.22691193E-08	-.24371830E-08
-.43359799E-08	-.30713803E-08	.31587028E-09	.15091929E-08	

MODE 16

-.47689666E-02	.55397024E-02	.39077214E-02	-.41573131E-02	-.64934048E-02
-.81768150E-05	.61597195E-02	.48535678E-02	-.17043806E-07	.16071897E-07
.24490456E-07	-.91161941E-08	-.38470643E-07	.19226731E-07	.56416115E-07
.27212206E-07	.49998650E-01	-.61736926E-01	-.38343573E-01	.48478912E-01
.71918736E-01	-.37506697E-02	-.77674407E-01	-.58119495E-01	

MODE 17

-.17995753E-09	.37763574E-09	-.21292010E-09	-.49620631E-09	.32527532E-09
.44514099E-09	-.25010582E-09	-.59808512E-09	.31473005E+00	-.71482343E+00
.41660891E+00	.82374089E+00	-.41189880E+00	-.78255567E+00	.47392448E+00
.92154624E+00	.14658920E-08	-.30168313E-08	.21624068E-08	.38128133E-08
-.27837172E-08	-.39174048E-08	.25765937E-08	.49367381E-08	

MODE 18

.85986815E-11	-.10221303E-09	.35055578E-09	-.18500156E-09	-.26843411E-09
.33011509E-09	.12948391E-09	-.51938624E-09	-.17319312E+00	.53757232E+00
-.88791547E+00	.32754139E-01	.84019901E+00	-.66372056E+00	-.31877915E+00
.10446895E+01	.25046074E-09	.12142927E-09	-.33117999E-08	.26383438E-08
.33003783E-08	-.56302817E-08	-.11366969E-08	.52182840E-08	

MODE 19

-.20895506E-02	.40657662E-02	-.18506502E-02	-.47647353E-02	.21815188E-02
.47528192E-02	-.23337664E-02	-.53928706E-02	-.15086431E-07	.31867722E-07
-.55916966E-08	-.43157804E-07	-.36253341E-08	.63783692E-07	-.23644504E-07
-.73115475E-07	.32153362E-01	-.64079796E-01	.30263281E-01	.72445075E-01
-.31786880E-01	-.68399108E-01	.38684461E-01	.79454074E-01	

MODE 20

-.48002478E-10	.16691765E-09	-.41034094E-09	.37574856E-09	-.15299279E-09
-.14130764E-09	.32061860E-09	-.31352614E-09	.91523909E-01	-.35089339E+00
.91107441E+00	-.84867623E+00	.28725572E+00	.45736812E+00	-.89059859E+00
.87654125E+00	.62614747E-09	-.18909529E-08	.47128087E-08	-.36269319E-08
.17501316E-08	.15899297E-08	-.37291734E-08	.31600650E-08	

MODE 21

.95959370E-03	-.26525944E-02	.41449122E-02	.12139927E-04	-.41365358E-02
.31645776E-02	.17470239E-02	-.52155679E-02	.54211663E-08	-.11943711E-0[illegible]
-.14755040E-07	.83363172E-07	-.12876789E-06	.12697908E-06	-.67749009E-07
.47223720E-08	-.18299292E-01	.50016602E-01	-.74210940E-01	-.91192281E-03
.73062134E-01	-.55098160E-01	-.28827539E-01	.89238982E-01	

MODE 22

.63979645E-10	-.15865358E-09	.14777200E-09	.31759705E-09	-.74374923E-09
.60850032E-09	-.17794049E-10	-.42402892E-09	.38524709E-01	-.16740611E+00
.54602480E+00	-.80572492E+00	.94660798E+00	-.94002791E+00	.75821597E+00
-.50166632E+00	-.13409985E-08	.35777993E-08	-.41314732E-08	-.31501252E-08
.11610811E-07	-.89328132E-08	-.89153621E-09	.84542512E-08	

MODE 23

.46530961E-03	-.16167243E-02	.40706939E-02	-.37379892E-02	.11719285E-02
.21212829E-02	-.40374803E-02	.40116180E-02	.54570100E-08	-.2160282[illegible]
.52999099E-07	-.54952554E-07	.23423686E-07	.13728140E-07	-.33796295E-[illegible]
.37779268E-07	-.9*378278E-02	.33264102E-01	-.78782525E-01	.71508437E[illegible]
-.22703437E-01	-.40430004E-01	.76454498E-01	-.74599469E-01	

MODE 24

.76496425E-03	-.25187631E-02	.77205110E-02	-.70895005E-02	.2222689[illegible]
.40232423E-02	-.76575178E-02	.76084671E-02	.89712687E-08	-.3365595[illegible]
.10051832E-06	-.10422345E-06	.44425552E-07	.26036901E-07	-.64098326E[illegible]
.71652465E-07	-.15090538E-02	.45996515E-02	-.12830961E-01	.1164626[illegible]
-.36976165E-02	-.65846560E-02	.12451806E-01	-.12149685E-01	

CASE 3

FREQUENCIES

19.5[illegible]	23.5[illegible]	36.91	71.83	118.92	123.77	166.99	205.5[illegible]
213.88	243.34	277.32	301.74	311.17	541.20	543.49	781.72
948.91	1008.25	1200.59	1332.38	1373.93	1768.38	2100.44	2325.35

PERIODS

.51164839E-01	.42542675E-01	.27091459E-01	.13920085E-01	.84088956E-02
.80792797E-02	.59882803E-02	.48652820E-02	.46753255E-02	.41093416E-02
.36059031E-02	.33140539E-02	.32136680E-02	.18477373E-02	.18399427E-02
.12792264E-02	.10538403E-02	.99181160E-03	.83291881E-03	.75053567E-03
.72783408E-03	.56548640E-03	.47608931E-03	.43004186E-03	

MODE SHAPES

MODE 1

.33016617E+00	.29573644E+00	.25758282E+00	.21387000E+00	.1665361[illegible]
.11783061E+00	.71084475E-01	.24727301E-01	.75278659E+00	.6079834[illegible]
.48106450E+00	.35879909E+00	.24566993E+00	.14705857E+00	.699041[illegible]
.17248574E-01	.31719114E-01	.25625428E-01	.20284313E-01	.1513689[illegible]
.10370665E-01	.62122094E-02	.29552277E-02	.72996278E-03	

MODE 2

.61915626E+00	.58880000E+00	.53914754E+00	.47034566E+00	.385246[illegible]
.28751040E+00	.18367205E+00	.67646685E-01	-.43858657E+00	-.353896[illegible]
-.27969903E+00	-.20831178E+00	-.14238881E+00	-.85069898E-01	-.403508[illegible]
-.99266746E-02	.51417352E-02	.45215289E-02	.36220476E-02	.27431[illegible]
.19118534E-02	.11672992E-02	.56699763E-03	.14405546E-03	

MODE 3

-.63749758E-01	-.11572575E+00	-.14980640E+00	-.16864411E+00	-.16891815E+0[illegible]
-.14870221E+00	-.10891374E+00	-.44689079E-01	-.27813451E+00	-.22521233E+0[illegible]
-.17869509E+00	-.13373322E+00	-.91936296E-01	-.55287034E-01	-.26416417E-0[illegible]
-.65648388E-02	.76700073E-01	.61847020E-01	.48849521E-01	.36354740E-0[illegible]
.24827547E-01	.14817293E-01	.70195262E-02	.17238308E-02	

MODE 4

-.69884020E+00	-.46359517E+00	-.95789397E-01	.29927432E+00	.59191843E+0[illegible]
.68775718E+00	.56021981E+00	.23533732E+00	-.43488121E-01	-.15909554E-0[illegible]
.64131349E-02	.22989772E-01	.30704773E-01	.28587087E-01	.18952540E-0[illegible]
.64877329E-02	.52385162E-02	.64774618E-02	.73684334E-02	.76715258E-0[illegible]
.70641933E-02	.54795858E-02	.32690063E-02	.10324401E-02	

MODE 5

-.52363399E+00	-.56428674E-01	.37520438E+00	.46936399E+00	.146377[illegible]
-.31920082E+00	-.52352793E+00	-.29030394E+00	-.51420241E+00	-.137170[illegible]
.15721135E+00	.35883094E+00	.43180686E+00	.37695873E+00	.237662[illegible]
.77606422E-01	-.20589986E-01	-.49029525E-02	.71265044E-02	.149196[illegible]
.17139967E-01	.14294375E-01	.86050531E-02	.26713451E-02	

MODE 6

.51117804E+00	.11801116E+00	-.32714092E+00	-.47868292E+00	-.207686[illegible]
.23653712E+00	.44896795E+00	.25545284E+00	-.55840152E+00	-.155182[illegible]
.16355628E+00	.39006537E+00	.48289219E+00	.43372798E+00	.28107[illegible]
.94391982E-01	-.16839116E-01	-.57443456E-02	.32268477E-02	.10096[illegible]
.13612935E-01	.13032444E-01	.89053842E-02	.31405915E-02	

MODE 7

-.77950978E+00	.31888963E+00	.61194949E+00	-.73176512E-01	-.65043202E+00
-.22806968E+00	.52466904E+00	.49761258E+00	.18745582E-01	.13638524E-01
.24436755E-03	-.19908959E-01	-.33429094E-01	-.30222766E-01	-.15953393E-01
-.33018301E-02	-.30322134E-02	.10424544E-02	.28705784E-02	.25510298E-02
.15643896E-02	.13070858E-02	.13899380E-02	.78018785E-03	

MODE 8

.56653358E+00	-.56671166E+00	.10703481E-01	.61754930E+00	-.11203742E+00
-.57986038E+00	.21444061E+00	.59579077E+00	-.90947970E-01	-.31853223E-01
.26666783E-01	.71173173E-01	.84309703E-01	.73169739E-01	.50865133E-01
.20252373E-01	.21452606E-01	.47694331E-02	-.65429452E-02	-.13983702E-01
-.17603327E-01	-.12931305E-01	-.96042364E-02	-.26670137E-02	

MODE 9

.27704882E+00	-.15244980E+00	-.74682408E-01	.15817220E+00	-.28513582E-01
-.20119562E+00	.51347126E-01	.19680514E+00	.27387921E+00	.71276286E-01
.83561959E-01	-.18835637E+00	-.22839037E+00	-.20134336E+00	-.12661828E+00
-.40515552E-01	-.65572656E-01	-.18940632E-01	.18362529E-01	.44886797E-01
.55226747E-01	.49096798E-01	.31727908E-01	.10736225E-01	

MODE 10

-.36582328E+00	.52378037E+00	-.55428955E+00	-.90210565E-01	.63245682E+00
-.44676006E+00	-.25277818E+00	.72736226E+00	-.13038562E-01	.27356699E-02
.29680681E-02	.61227226E-02	.70717234E-02	-.18881240E-02	-.25502145E-02
.30581599E-02	.11220215E-02	.14474156E-02	-.53673903E-03	-.12341733E-02
-.13365390E-02	-.22636594E-02	-.15411583E-02	.14243095E-03	

MODE 11

.18859928E+00	-.35477437E+00	.67802735E+00	-.60037610E+00	.19714318E+0[illegible]
.31775656E+00	-.62934621E+00	.63980546E+00	.71744740E-02	-.2735911[illegible]
-.11157995E-02	-.63510860E-02	.86638247E-03	.45874645E-02	.71177626E-03
.50065226E-02	-.30035349E-03	-.64294786E-03	.38452186E-03	-.19756356E-03
.73646669E-03	.94474366E-03	.74111722E-04	.72282806E-03	

MODE 12

.74467573E-01	-.16261441E+00	.40202106E+00	-.57472274E+00	.67674600E+00
-.68846865E+00	.57333578E+00	-.40128406E+00	.56299856E-02	-.19106311E-02
-.22042558E-02	-.56258243E-02	.37289125E-02	.65019985E-03	.68699573E-02
-.18177803E-04	.11742899E-04	-.27793869E-03	.16213169E-03	-.38770627E-03
.72042848E-03	-.12760126E-03	.70297440E-03	-.19787846E-03	

MODE 13

-.35672155E-01	.15743856E-01	.72527166E-02	.16126455E-01	-.53816548E-02
-.99733693E-02	-.11907678E-01	-.26940912E-02	.68360955E+00	-.15723785E+00
-.55268818E+00	-.40956236E+00	.11060314E+00	.58094012E+00	.64428331E+00
.30047913E+00	.23585535E-01	-.55786690E-02	-.19396381E-01	-.14490370E-01
.35589307E-02	.19961081E-01	.22261364E-01	.10408386E-01	

MODE 14

.52771189E-02	-.54282365E-02	-.27668891E-02	.60357300E-02	.76387821E-02
-.16548934E-02	-.93035757E-02	-.68318945E-02	-.62315850E+00	.44266299E+00
.43184617E+00	-.26406199E+00	-.58981070E+00	-.83246789E-01	.52457613[illegible]
.43485997E+00	-.25785805E-02	.11496511E-01	.42983073E-03	-.20058188[illegible]
-.17573634E-01	.12628451E-01	.35519382E-01	.23202690E-01	

MODE 15

.24775524E-01	-.85202538E-02	-.12169387E-01	-.69393051E-02	.3329138[illegible]
.90650353E-02	.62403096E-02	.14988199E-02	.10519293E+00	.5608828[illegible]
-.10319005E+00	-.23480822E+00	-.11638276E+00	.20914363E+00	.4004901[illegible]
.23886828E+00	-.63407990E-01	.18242430E-01	.52142566E-01	.3322885[illegible]
-.14793620E-01	-.50691940E-01	-.50508519E-01	-.21772517E-01	

MODE 16

-.52201558E-02	.57180771E-02	-.11847057E-02	-.59705489E-02	.20194967E-02
.59578674E-02	-.25735916E-02	-.67472718E-02	.41156456E+00	-.53567477E+00
.13645485E+00	.60608724E+00	-.18222460E+00	-.56704015E+00	.26737008E+00
.63854749E+00	.12694188E-01	-.18634461E-01	.44991898E-02	.20303948E-01
-.70170004E-02	-.19516412E-01	.10291661E-01	.22930828E-01	

MODE 17

-.88912239E-02	.55355653E-02	.50662636E-02	-.32205338E-02	-.67941015E-02
-.48284083E-03	.59645769E-02	.46894245E-02	-.22016447E+00	.17471915E+00
.13022521E+00	-.10709651E+00	-.21463145E+00	-.37470034E-01	.23288077E+00
.22101571E+00	.49366006E-01	-.40499117E-01	-.37772285E-01	.27752109E-01
.55154990E-01	.22644519E-02	-.55200668E-01	-.42681521E-01	

MODE 18

-.28975144E-02	.42296149E-02	-.44483901E-02	-.69210464E-03	.49118314E-02
-.36285606E-02	-.21409406E-02	.65501153E-02	.24541218E+00	-.45240771E+00
.55758165E+00	.74258955E-01	-.60706122E+00	.40065184E+00	.26656480E+00
-.69435829E+00	.98113990E-02	-.17789007E-01	.16997091E-01	.32475957E-02
-.18478316E-01	.14623216E-01	.67062946E-02	-.26629289E-01	

MODE 19

-.12726743E-02	.25226184E-02	-.49819033E-02	.44351766E-02	-.12574536E-02
-.28220572E-02	.50141135E-02	-.47479785E-02	.14283895E+00	-.32094471E+00
.64174471E+00	-.54256533E+00	.14559626E+00	.33760975E+00	-.60514286E+00
.58691964E+00	.43204905E-02	-.11085736E-01	.21356421E-01	-.1967455[illegible]
.59342723E-02	.12552729E-01	-.21974355E-01	.19520764E-01	

MODE 20

.40559917E-03	-.10619105E-02	.29826444E-02	-.44534464E-02	.50594901E-02
-.48149893E-02	.39257262E-02	-.28477226E-02	-.64625309E-01	.16009805E+00
-.39773164E+00	.55142576E+00	-.64287830E+00	.64131078E+00	-.51311869E+00
.32912205E+00	-.95085860E-03	.41669457E-02	-.13123290E-01	.21049975E-01
-.23255217E-01	.20866534E-01	-.17073946E-01	.13393695E-01	

MODE 21

-.44040356E-02	.47313346E-02	-.84140794E-03	-.47980655E-02	.12492285E-02
.49806088E-02	-.19712415E-02	-.53664809E-02	-.14737613E+00	.18372361E+00
-.15136609E-01	-.25694270E+00	.78717278E-01	.21138734E+00	-.92293634E-01
-.25697755E+00	.35818484E-01	-.50917670E-01	.96581670E-02	.54347772E-01
-.13976547E-01	-.52171643E-01	.23625940E-01	.58000922E-01	

MODE 22

.22492147E-02	-.34058244E-02	.36710294E-02	.74359196E-03	-.42566951E-02
.27541970E-02	.20199001E-02	-.51079241E-02	.96900597E-01	-.17243109E+00
.19894059E+00	.45262985E-01	-.23616997E+00	.15042703E+00	.10555996E+00
-.27338967E+00	-.22849966E-01	.45026546E-01	-.47333628E-01	-.99693751E-02
.54383150E-01	-.34683591E-01	-.24397888E-01	.63142593E-01	

MODE 23

.11648821E-02	-.22231816E-02	.41537586E-02	-.34404895E-02	.79064[illegible]
.23471390E-02	-.40358439E-02	.38866858E-02	.56534739E-01	-.12531076[illegible]
.24648524E+00	-.20350661E+00	.47277179E-01	.13914287E+00	-.23765693[illegible]
.22458035E+00	-.13253342E-01	.32421800E-01	-.57682583E-01	.4740844[illegible]
-.10986486E-01	-.32100718E-01	.54860706E-01	-.51847597E-01	

MODE 24

.19150542E-02	-.27703330E-02	.57937488E-02	-.47988663E-02	.1104962[illegible]
.32802961E-02	-.56402629E-02	.54318082E-02	.92942534E-01	-.1561512[illegible]
.34380273E+00	-.28385525E+00	.66071862E-01	.19445806E+00	-.332135[illegible]
.31386056E+00	-.19540985E-02	.38169718E-02	-.72500805E-02	.595873[illegible]
-.13808831E-02	-.40347151E-02	.68954009E-02	-.65166855E-02	

APPENDIX III

MOST PROPRABLE VALUES OF U,V,AND THETA FOR CENTRE OF MASS

CASE 1

.21860570E-01	.20818364E-01	.19272625E-01	.17064668E-01	.14228440E-01
.10835448E-01	.70713589E-02	.26583375E-02	.71433903E-02	.57614750E-02
.45542243E-02	.33942798E-02	.23231627E-02	.13906525E-02	.66133880E-03
.16339404E-03	.30748685E-02	.24791227E-02	.19586264E-02	.14587565E-0[illegible]
.99758617E-03	.59659435E-03	.28342735E-03	.69932129E-04	

CASE 2

.27397687E-01	.25980273E-01	.23736331E-01	.20755228E-01	.17116079E-01
.12913078E-01	.83688376E-02	.31308474E-02	.88904270E-02	.71502657E-02
.56328354E-02	.41822234E-02	.28510796E-02	.16997477E-02	.80496080E-03
.19776288E-03	.41613002E-02	.33495460E-02	.26410608E-02	.19627907E-02
.13393835E-02	.79932635E-03	.37895862E-03	.93252673E-04	

CASE 3

.37002843E-01	.33687929E-01	.30009588E-01	.25740592E-01	.2094062[illegible]
.15675062E-01	.10115049E-01	.37845614E-02	.82039909E-01	.66261253[illegible]
.52433267E-01	.3*113915E-01	.26789628E-01	.16044068E-01	.76319649[illegible]
.18857087E-02	.52218030E-02	.42137539E-02	.33313533E-02	.24825010[illegible]
.16983087E-02	.10158166E-02	.48257738E-03	.11904345E-03	