SLIDING AND OVERTURNING OF OBJECTS DURING EARTHQUAKES

A Dissertation
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By

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CERTIFICATE

CERTIFIED that the thesis entitled " SLIDING AND OVER-TURNING OF OBJECTS DURING EARTHQUAKES", which is being submitted by Sri K.C. Mittal in partial fulfilment for the award of Master of Engineering in "EARTHQUAKE ENGINEERING" with specialisation in "STRUCTURAL DYNAMICS" of the university of Roorkee, Roorkee is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for an effective period of 7 months from 15th December 1970 to 15th Nov 1971 for preparing this thesis for Master of Engineering Degree at this University.

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SYNOPSIS

The objective of this thesis is to evaluate the ground motion characteristics such as peak ground acceleration and predominant frequency during an earthquake by simple measurements of objects which slid or overturned during the event. Where accelerograph record is available, the information from sliding and overturning of objects may be used to supplement the accelerogram for working out the attenuation characteristics of the affected area. For achieving the objective stated above, simple objects were shaken on a shake - table and observations of sliding and overturning taken along with the record of table motion. After digitising the table motion, response of friction mounted objects has been calculated by computer programmes made for the purpose. The correlation obtained between the observed and calculated values is quite promising though not entirely satisfactory.

NOTATIONS

A - Amplitude of ground acceleration

C - Constant of integration

D - Ratio of mass moment of inertia to the mass of the object i.e. I/m

I - Mass moment of inertia

M - Mass of the object

N - Amplitude of frictional force

R - Force normal to inclined plane

U - Relative displacement at time t

U_m - The displacement at time t_m

U_o - The relative displacement at time t_o

V - The relative velocity

W - Weight of the object

a - The intercept of the vertical line through C.G. of an object from the nearest edge of its base.

e - Coefficient of restitution

g - Acceleration due to gravity

h - Distance of C.G. of the object from the base.

m - Mass of the object

t - Time

t_m - Time at which the ground velocity equals to the object velocity

Δt - Time interval.

x - Absolute displacement of the object.

$\dot{x}$ - Absolute velocity of the object.

$\ddot{x}$ - Absolute acceleration of the object.

y - Absolute displacement of the ground.

$\dot{y}$ - Absolute velocity of the ground.

$\ddot{y}$ - Absolute acceleration of the ground

V - Absolute velocity of ground.

μ - Coefficient of friction.

α - Angle of inclined plane

θ - Angle of rotation of an object

ϕ_{eq} - Angle of internal friction.

CONTENTS

INTRODUCTION

1.1 Measurement of Ground Acceleration

The forces acting on structures in an earthquake can be worked out from the recorded ground accelerations, which is therefore the most use ful form of ground motion for engineering studies. Ground accelerations near the epi - centre of an earthquake are largest and reduce quickly with distance from epicentre. In order to record stroúng ground accelerations the recording intruments (strong motion accelerographs) must therefore be located close to the epicentre. The seismic zones of the world have their epicentral regions extended over areas large enough to require too large a number of accelerographs to be instaled in any such regions so as to be certain that one of the instruments atleast records an earthquake occuring any where in that region at a resonably close distance from the epicentre. This would mean a closely spaced net-work of these instruments which will be very expensive.

In the absence of such instrumental data, an alternate approximate method is suggested[1, 2, 3, 4] for estimating the maximum ground acceleration at any inhabited place from the observations made on movements of small rigid house-hold objects and analysis of structure-s damaged during the earthquake. Where an earthquake is recorded by an acc-elerograph, this method of study would give additional information to supplement the recorded accelerogram data at several places around the epicentre.

Analysis for the behavious of small rigid objects in 'sliding' and overturning' has been developed in this thesis for correlating these movements to the peak gro-und acceleration. The work is preliminary in that only one component of ground motion has been considered in the analytical procedure with experimental observations and to investigate the effect of various parameters on the results.

The various factors which affect the field observa - tions are also important ultimately in the estimation of maximum ground acceleration at a place. These are discu - ssed in the subsequent paragraphs.

For determing the ground acceleration at a particular location, several observations of movements and non-move - ments are required. In the case of movement of an object the actual force applied by the earthquake must be more than the minimum force required to cause the movement. On the other hand, the force required to move the object whi-ch did not move during the earthquake , will give an upper limit to the earthquake force. The actual force applied by the earthquake must be somewhere between the lower and the upper limit. If these are obtained for a number of objects, the range between the upper and lower limits would be sufficiently narrowed down and an accurate estimation of the actual force can be made.

1.2 Factors Affecting Sliding and Overturning of Objects

Sliding and overturning of rigid object is governed by a number of factors. The earthquake motion is such that it has two mutually perpendicular horizontal components and a vertical component. The overturning of the object will depend upon the horizontal component which best suits the axis of the object (if the object is unsymmatrical in plan) togather with the vertical component. At the instant of the maximum horizontal component, the vertical component may be such as to increase the weight of the object and stablise it or to reduce the weight and add to the overturning effect. The sliding on the other hand generally depends upon the resultant of the two horizontal components together with the vertical component. Also it is known that the component in the vertical direction goes on decreasing at a greater rate than the horizontal component with the distance from the epicentre.

Location of the object is another important factor. Smaller objects would be normally resting on some supports.

Unless the supports are perfectly rigid, their motion will be amplified relative to the ground motion. Objects on upper floor of the house will similarly receive amplified motion.

Besides the ground motion other significant factors are the co-efficient of friction between the contact surface, inclimation of the contact surface if any, orientation of the object and its shape and size.

Last of all and the most important perhaps is the correctness of the information. For reliable data, some clear evidence is desirable. For example, if the object is reported to slide, the fact of sliding is likely to be correct but the extent of sliding would seldom be correctly reported, unless it leaves a clear mark on the support. In the case of overturning of an object chances of incorrect information are expected to be few.

1.3 Assumptions in Theoretical Analysis

For theoritical analysis following assumptions are made :

1) the surface on which the object is placed is exactly plane, homogeneous and horizontal.
2) the coefficient of friction is constant over the range of sliding on the surface and does not vary

with the relative velocity between the surface and the object. That is the dynamic coefficient of friction is the same as its static value.

3) The object has a tendency either to slide only or to overturn only, these two motions being considered uncoupled.

1.4 Outline of Thesis

In the Chapter III, the equations of sliding and overturning of rigid objects have been drived for the numerical analysis. The equations so obtained are used for the objects subjected to the table motions and the theoritical results are compared with the practical results.

Similarly the equations of sliding only, for single as well as for the repeated triangular and rectangular pulses are developed. These equations have been used for checking the accuracy of the computor programme and found satisfactory.

Experimental studies are also conducted. In these studies the amount of sliding and the overturning actions are recorded. During the experiment, different accelerograms are recorded and they have been digitised for the theoritical results. The theoritical and practical results so obtained are compared in Chapter IV.

CHAPTER - II

REVIEW OF LITERATURE

2.1 Chandrashekaran, A.R.[5]

In this paper, the earthquake response of objects, which rest on ground through friction, are investigated. If the object is not firmly tied to the ground, it will have a motion relative to the ground, causing it to slide, rock or overturn.

Two cases are considered - in the first type a dashpot with viscous damping connects the object to the ground. Here the damping force is a linear function of the velocity. The other is a dashpot with coulomb damping in which the damping force is independent of the magnitude of velocity but depends on its phase. It is concluded that relative displacements aways take place irrespevtive of the value of friction. This conclusion is however illogical since the motion must cease to occur when acceleration coefficient becomes less than co-efficient of friction. It is further shown that displacement decreases with increase in friction in case of viscous damping, whereas in case of coulomb damping the displacement pattern, as worked out for four earthquakes, is found to be irregular. The relative displacement is a minimum corresponding to a coulomb friction factor of the order of 0.2 to 0.3. The equation of motion is as follows for the case of coulomb dampin

$$\ddot{V} + \mu g \{ \text{Sin}(V) \} = -\ddot{y} \qquad \ldots (2.1)$$

It is concluded that the relative motion of the mass is not only a function of the maximum acceleration but also of the wave form of the ground motion.

2.2 Goodman, R.E. and Seed, H.B.

They have considered down hill motion of an object on a sloping surface for the purpose of evaluating amount of slip in soil embankments and earthdams. Taking the case of a slope of uniformly dense granular materical subjected through out its, height to constant horizontal acceleration having a magnitude greater than the yeild acceleration, ky, a sliding mass at the surface will advance down the slope like a frictional block on an inclinea plane (Fig. 2.1). Refering to Fig. 2.2 it is seen that no displacement will occur until time t_1, when the induced acceleration reaches the yield acceleration ky_1.

If the yield acceleration is assumed to remain constant throughout the first cycle, the velocity will continue to increase until t_a, when the acceleration again drops below the yield value, the velocity is finally reduces to zero at time t_s and for reversal of acceleration, it may be possible for the sliding mass to move upward. According to these authors this is only possible if the acceleration is greater than 1.0g.

The actual value will infact depend upon the slope as well as the angle of friction. The ~~rate of~~ relative displacement of the sliding mass may then be computed by integration of the velocity vs time diagram.

Mathematical Analysis

Let x be the down ward displacement along the surface and horizontal acceleration induced by earthquake be k(t)g. The angle of friction is Φ_{eq}.

Hence the equation of motion is

$$W \sin\alpha + k(t)\, W \cos\alpha - R \tan\Phi_{eq} = \frac{W}{g} \times \frac{d^2x}{dt^2}$$

and $R = (W \cos\alpha - k(t)\, W \sin\alpha)$ ... (2.2)

substituting the value of R in above equation

$$\frac{1}{g(\sin\alpha \tan\Phi_{eq} + \cos\alpha)} \times \frac{d^2x}{dt^2} = k(t) - \frac{(\cos\alpha \tan\Phi_{eq} - \sin}{(\sin\alpha \tan\Phi_{eq} + \cos}$$

Since $\frac{\cos\alpha \tan\Phi_{eq} - \sin\alpha}{\sin\alpha \tan\Phi_{eq} + \cos\alpha} = \tan(\Phi_{eq} - \alpha) = ky$

Hence $\frac{d^2x}{dt^2} = B(x)\left[k(t) - ky \right]$

Where $B(x) = g(\sin\alpha \tan\Phi_{eq} + \cos\alpha)$

As the accelerations induced during an earthquake are varying in frequencies and amplitudes, the displacements of the sliding mass can be determined by numerical integration.

Velocity at time t_j is

$$V(t_j) = \sum_{i=0}^{j-1} B(x) \left[\frac{k(t_i) + k(t_{i+1})}{2} - ky \right] \Delta t \qquad \ldots \quad (2.3)$$

and the displacement at time t_n is

$$x(t_n) = \sum_{j=0}^{n-1} \frac{V(t_{j+1}) + V(t_j)}{2} \times \Delta t \quad \ldots \ (2.4)$$

this relation is true from t = 0 to that time when the velocity is again zero.

2.3 Newmark[7]

Sliding of a rigid plastic mass :- A simple derivation for a rigid mass in developed to give a quick estimate of the magnitude of the motions to be expected in sliding, when it is subjected to the influence of dynamic forces of an earthquake.

Consider a rigid body having a weight W and a mass M, having a motion* x. The motion of the ground on which the mass rests is designated by y (t), where y is a function of time t. The relative motion of the mass, compared with the ground is designated by U, where U = x-y (Fig. 2.3).

The resistance to motion is proportional to weight W, of magnitude NW. This corresponds to an acceleration of the ground of magnitude Ng that would cause the mass to move relative to the ground. (called the thresh-hold acceleration by the writer). The acceleration considered is a single pulse of magnitude Ag, lasting forr the time interval t_o (Fig 2.4). The velocities are as a funcation of time for both the accelerating force and the resisting force. The maximum velocity for the accelerating force has a magnitude v, given by v = Ag to .... (2.5) After time t_o, the velocity due to accelerating force is constant and velocity due to resisting force is v = Ngt as shown in (Fig. 2.5).

At a time t_m, both velocities are equal and the relative velocity becomes zero, or the body comes to rest relative to the ground.

$$v = Ngt_m$$

$$\text{or} \quad t_m = \frac{v}{Ng}$$

The maximum displacement of the mass relative to the ground U_m is equal to the shaded triangular area of Fig. 2.5.

$$U_m = \frac{1}{2} vt_m - \frac{1}{2} vt_o$$

$$\text{or} \quad U_m = \frac{1}{2} \frac{v^2}{Ng} - \frac{1}{2} \frac{v^2}{Ag}$$

$$\text{hence,} \quad U_m = \frac{1}{2} \frac{v^2}{gN} \left(1 - \frac{N}{A} \right) \qquad \ldots (2.6)$$

The result given by this equation generally overstimates the relative displacement for an earthquake because it does not take into account the pulses in opposite directions and is analogous to the down-hill motion considered in section 2.2. However it should give a reasonable order of magnitude for the relative displacement. ~~It does indicate that the displacement.~~ It does indicate that the displacement is proportional to the square of the maximum ground velocity.

The result derived above is applicable also for a group of pulses, when the resistance in either direction of possible motion is the same. For the situation in which the body has a resistance to motion greater in one direction than in another, one must take into account the comulative effect of the displacements.

2.4 <u>Summary</u> - It is seen that certain minimum ground acceleration is required to start the motion of the object held by friction to the surface on which it rests. In the two previous sections 2.2 and 2.3 the terminology 'yield acceleration' and 'threshold acceleration' are therefore essentially the same and refer to this minimum ground acceleration. The sliding object may come to rest with respect to the ground any moment the relative velocity attains zero value.

It will then continue to move with the ground with equal velocity unless once again the ground acceleration exceeds the threshhold limit. Same concept has been used in writing the equations of motion in this thesis. The equations of motion for overturning of objects has also been developed in Chapter III.

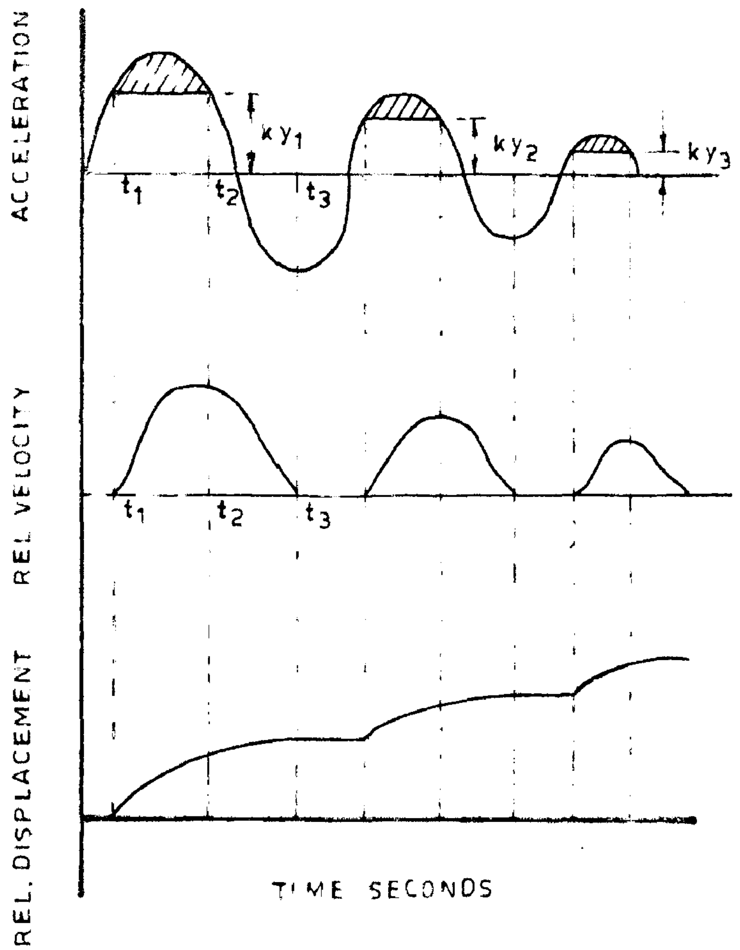


FIG 2.2 _RELATIVE VELOCITY AND DISPLACEMENT FOR DOWN HILL MOTION OF A FRICTION ON BLOCK ON A SLOPING SURFACE

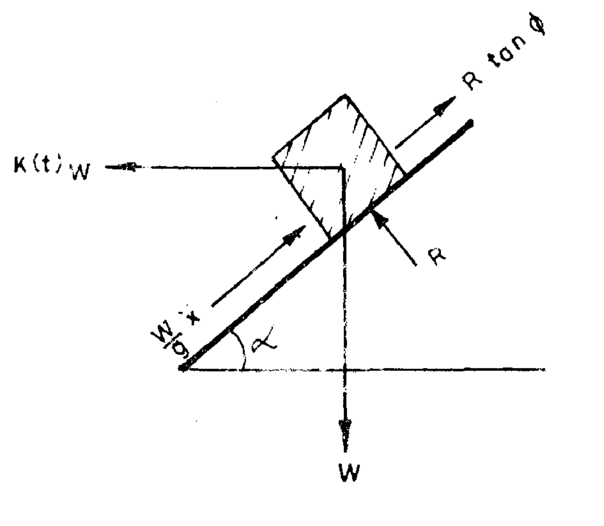


FIG.2.1 _FORCES ON A SLINDING BLOCK RESTING ON AN INCLINED PLANE

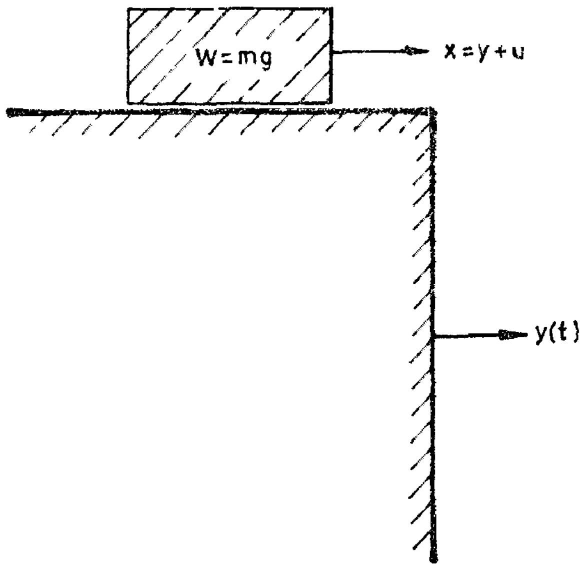


FIG.2.3 _ RIGID BLOCK ON A MOVING SUPPORT

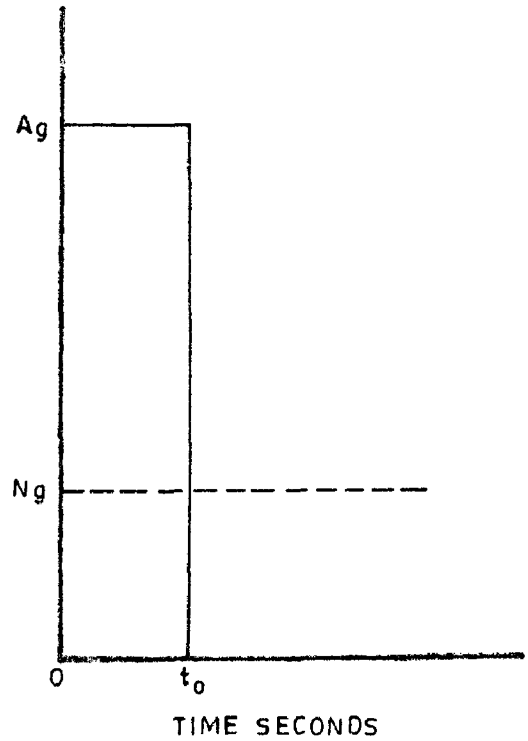


FIG.2.4 _ RECTANGULAR BLOCK ACCELERATION PULSE

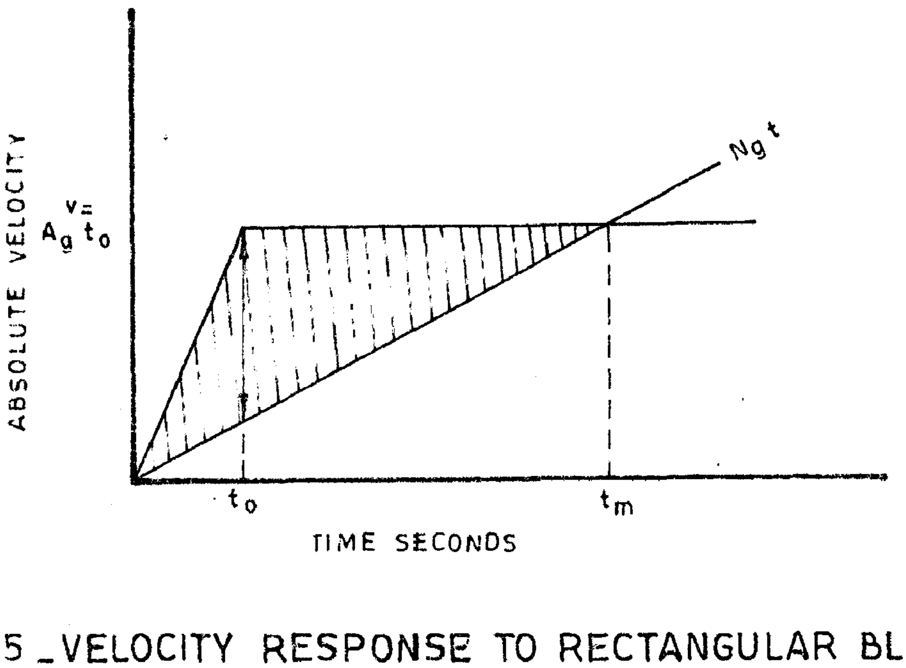


FIG.2.5 _ VELOCITY RESPONSE TO RECTANGULAR BLOCK ACCELERATION

CHAPTER - III

THEORITICAL ANALYSIS

3.1 Introduction

In this Chapter the equations of motion are first written for the sliding of objects on frictional surfaces subjected to simple rectangular and triangular pulses. Closed form solutions are derived for the same and numerical results computed. The general equations of sliding motion is then derived and the general equations of sliding motion is then derived and the method of numerical solution indicated. This numerical solution is programmed for digital computer calculations and the results arrived at for the former simple pulses are verified by the computer solution. Finally the computer solution is used to solve more difficult cases of triangular pulses and the results arrived at have been plotted. Later on the equations of overturning motion are derived.

3.2 Analysis for Response of Rigid Objects on Surfaces Subjected to Rectangular and Triangular pulses

(i) Theoritical Results for Single Rectangular Pulses

Using equations 2.5 and 2.6, the response for single rectangular pulse (Fig. 2.4) are tabulated in the following table 3.1. The area under the pulse is kept constant. As the acceleration increases to infinity and the time to approaches zero the pulse tends to be an impulse and the maximum relative displacement U_m approaches a fixed value of 66 cm.

The curve (Fig. 3.7) between the acceleration and the maximum displacement is plotted and it shows that this curve approaches the absolute maximum displacement value of 66.0 cm approaches the impulse asymtotically.

(ii) For a single triangular pulse the equations are derived below (Fig. 3.3).

For $\frac{N}{A} \frac{t_o}{2} < t < \frac{t_o}{2}$

The equation of motion is

$$m\ddot{x} - mNg = 0$$

$$m\ddot{x} - m\ddot{y} - mNg = -m\ddot{y}$$

$$\dot{V} = Ng - \ddot{y}$$

at $t = \frac{N}{A} \frac{t_o}{2}$, $V = 0$, $\dot{V} = 0$, so that

$$\ddot{y} = Ng$$

$$\frac{dV}{dt} = \ddot{U} = Ng - \ddot{y}$$

Equation of line OA is

$$\ddot{y} = Ag \left(\frac{t}{t_o/2} \right)$$

hence $\frac{dV}{dt} = Ng - Ag \left(\frac{t}{t_o/2} \right)$

$$= Ng - 2Ag \frac{t}{t_o}$$

on integration

$$V = Ngt - \frac{2Ag}{t_o} \times \frac{t^2}{2} + C$$

at $t = \frac{N}{A} \frac{t_o}{2}$, $V = 0$

hence $C = -\frac{N^2 g\, t_o}{4A}$

hence $V = Ngt - \frac{Agt^2}{t_o} - \frac{N^2gt_o}{4A}$

or $V = Ng\left[t - \frac{Nt_o}{4A}\right] - \frac{Agt^2}{t_o}$

at $t = \frac{t_o}{2} = t_1,\ V = V_1$

hence $V_1 = Ng\left[\frac{t_o}{2} - \frac{Nt_o}{4A}\right] - \frac{Agt_o}{4}$

or $$\boxed{V_1 = -\frac{g}{4}\, t_o\,(A-N)\left(1 - \frac{N}{A}\right)} \qquad \ldots (3.1)$$

now $V = \dot{U} = \frac{dU}{dt} = Ng\left[t - \frac{Nt_o}{4A}\right] - \frac{Agt^2}{t_o}$

on integration

$$U = Ng\left[\frac{t^2}{2} - \frac{Nt_o}{4A}\,t\right] - \frac{Agt^3}{3t_o} + C$$

at $t = \frac{t_oN}{2A}$, $U = 0$ and $C = \frac{N^3t_o^2g}{24A^2}$

hence $U = Ng\left[\frac{t^2}{2} - \frac{Nt_o}{4A}\,t\right] - \frac{Agt^3}{3t_o} + \frac{N^3t_o^2g}{24A^2}$

at $t = \frac{t_o}{2},\ U = U_1$

hence $U_1 = Ng\left[\frac{t_o^2}{8} - \frac{N}{4A}\,\frac{t_o^2}{2}\right] - \frac{Agt^3}{24t_o} + \frac{N^3t_o^2g}{24A^2}$

or $$\boxed{U_1 = \frac{gt_o^2}{8}\left[\frac{A}{3} - N\left(1 - \frac{N}{A} + \frac{N^2}{3A^2}\right)\right]} \qquad \ldots (3.2)$$

Now refering Fig. 3.4, for $\frac{t_o}{2} < t < \frac{3t_o}{2}$

the equation of line Ac is

$$y = 2Ag\left[1 - \frac{t}{t_o}\right]$$

Equation of motion is

$$m\ddot{x} - mNg = 0$$

or $\dot{V} = Ng - \ddot{y}$

or $\frac{dV}{dt} = Ng - 2Ag\left[1 - \frac{t}{t_o}\right]$

on integration

$$V = Ngt - 2Ag\left[t - \frac{t^2}{2t_o}\right] + C$$

at $t = \frac{t_o}{2}$, $V = V_1$

hence $C = -\frac{N^2 g t_o}{4A} + \frac{1}{2} Ag\, t_o$

hence $V = Ngt - 2Ag\left[t - \frac{t^2}{2t_o}\right] + \frac{1}{2} Ag\, t_o - \frac{N^2 g t_o}{4A}$

at $t = t_o$, $V = V_o$

hence $\boxed{V_o = - g t_o \quad \frac{A}{2} - N\left(1 - \frac{N}{4A}\right)}$ ... (3.3)

Now $\frac{dU}{dt} = V = Ngt - 2Ag\left[t - \frac{t^2}{2t_o}\right] + \frac{1}{2} Agt_o - \frac{N^2 g t_o}{4A}$

on integration

$$U = \frac{Ngt^2}{2} - 2Ag\left(\frac{t^2}{2} - \frac{t^3}{6t_o}\right) + \left(\frac{1}{2} Agt_o - \frac{N^2 g t_o}{4A}\right) + C$$

at $t = \frac{t_o}{2}$, $U = U_1$

hence $C = -\frac{Agt_o^2}{12} + \frac{N^3 t_o^2 g}{24A^2}$

substituting the balue of C in above equation

$$U = Ng \frac{t^2}{2} - 2Ag\left(\frac{t^2}{2} - \frac{t^3}{6t_o}\right) + \left(\frac{Agt_o}{2} - \frac{N^2gt_o}{4A}\right)t - \frac{Agt_o^2}{12} + \frac{N^3t_o^2 g}{24A^3}$$

at $t = t_o$, $U = U_o$

hence $$U_o = -gt_o^2\left[\frac{A}{4} - \frac{N}{2}\left(1 - \frac{N}{2A} + \frac{N^2}{12A^2}\right)\right] \qquad \ldots (3.4)$$

It is seen from Fig. 3.7 in which the values of the table 3.2 for maximum sliding motion U_m are plotted that for infinite value of A and the same impulse ($A\ t_o$ = 0.2 Sec) the limiting maximum value of U_m is 16.0 cm.

(iii) For a double triangular pulse as shown in Fig. 3.4 the equations 3.3 and 3.4 are further extended as follows :

at $t = 1.5\ t_o$, $U = U^1$, $V = V^1$ (say)

$$U^1 = -gt_o^2\left[\frac{11A}{24} - N\left(\frac{9}{8} - \frac{3}{8}\frac{N}{A} + \frac{1}{24}\frac{N^2}{A^2}\right)\right] \qquad \ldots (3.5)$$

$$V^1 = -\frac{gt_o}{4}\left[N\left(\frac{N}{A} - 6\right) + A\right] \qquad \ldots (3.6)$$

For $1.5\ t_o < t < 2t_o$

Equation of line BD is as

$$\dot{y} = 4Ag\left[\frac{t}{2t_o} - 1\right]$$

Equation of motion is

$$\dot{V} = Ng - \ddot{y}$$

or $$\dot{V} = Ng - 4\,Ag\left[\frac{t}{2t_o} - 1\right]$$

on integration

$$V = Ngt - 4Ag\left[\frac{t^2}{4t_o} - t\right] + C$$

at $t = 1.5\ t_o,\ V = V^1$

$$C = -gt_o\left[4A + \frac{N^2}{4A}\right]$$

and $$U = Ng\left[\frac{t^2}{2} - 4Ag\left(\frac{t^3}{12t_o} - \frac{t^2}{2}\right) - gt_o\left(4A + \frac{N^2}{4A}\right)t + \frac{13}{6}Agt_o^2 + \frac{N^2t_o^2g}{24A^2}\right] \qquad \ldots (3.8)$$

For a double triangular ground accelergtion pulse shown in Fig. 3.4 having $A = 2.0$, $t_o = 0.2$ sec and for $N = 0.3$ the computed values of displacemtns and velocites by above expressions are compared with those obtained numerical integration as given in table 3.3.

3.3 General Equation for Sliding of Rigid Objects on Horizontal Surfaces Subjected to Vibratory Motion

Fig. 3.1 shows a rigid object and the direction of motion for the object and ground to gether with various forces.

Let the relative displacement between mass m and the surface on which it rests be U, then $U = x - y$ and $\dot{U} = \dot{x} - \dot{y} = V$

Where V is the relative velocity, the equation of motion is given as folows :

$$m\ddot{x} + \Phi^{1}(\dot{x}-\dot{y}) = 0$$

or $$m\ddot{x} - m\ddot{y} + \Phi^{1}(\dot{x}-\dot{y}) = -m\ddot{y}$$

or $$\ddot{U} + \Phi(V) = -\ddot{y}$$

$$\dot{V} + \Phi(V) = -\ddot{y} \qquad \ldots (3.9)$$

Where $\Phi(V) = -\mu g\{\text{Sin } V\}$ when $V \neq 0$

and $\Phi(V) = -\ddot{y}$ when $V = 0$

The solution of above equation can be made by numerical integration. For linear acceleration variation of the accelerogram.

$$\ddot{U} = \dot{V} = (\ddot{x}-\ddot{y}) - Ng\{\text{Sin } V\} - \ddot{y}$$

$$\Delta\dot{x} = -\Delta t\, Ng\{\text{Sin } V_{n-1}\}$$

$$\dot{x}_n = \dot{x}_{n-1} + \Delta\dot{x}$$

$$\Delta\dot{y} = \frac{1}{2}\Delta t\,(\ddot{y}_{n-1} + \ddot{y}_n)$$

$$\dot{y}_n = \dot{y}_{n-1} + \Delta\dot{y}$$

$$V_n = V_{n-1} + \Delta\dot{x} - \Delta\dot{y}$$

$$U_n = U_{n-1} + \frac{1}{2}\Delta t\,(\dot{x}_{n-1} + \dot{x}_n) - \dot{y}_{n-1}\Delta t - \frac{\Delta t^2}{6}(2\ddot{y}_{n-1} + \ddot{y}_n)$$

$$\ldots\ldots (3.10)$$

3.4 <u>Numerical Results for Continuous Trapezoidal and Triangular Pulsed</u>

Further computations have been made numerically on the computer by the programme given in Appendix III for continuous repeated triangular and trapezoidal pulses as shown Gig. 3.5 to Fig. 3.6. It is seen from these figures that the rigid object attains a stable position of vibratory motion about a centre of oscillation after two or three cycles of the ground pulse. Since this acceleration pulse is symmetrical, the oscillatory periodic motion is obtained as expected. These results and the computations shown in Table 3.3 confirm the correctness and accuracy of the computer programmes and the maximum time interval for integration that may be used consistant with accuracy of computations.

3.5 Overturning of Rigid Objects on Horizontal Surfaces Subjected to Vibratory Motions

Fig. 3.2 shows a rigid object and its direction of motion. The motion of ground and the various forces acting on the object are also shown therein.

Let the base of the object make an angle θ with the ground. Rotation first begins, when

$$y > \left(\frac{a}{h} \right) g$$

then the equation of motion is given by

$$m\ddot{y} (h \cos \theta + a \sin \theta) + m D \ddot{\theta} \, mg (a \cos \theta - h \sin \theta) = 0$$

or $$\ddot{\theta} + \frac{g}{D} (a \cos \theta - h \sin \theta) = \frac{1}{D} (h \cos \theta + a \sin \theta)\ddot{y} \quad \ldots(3.11)$$

when θ = 0, there is an impact and

$$\dot{\theta}_{(\text{after impact})} = -e\dot{\theta}_{(\text{before impact})} \quad \ldots (3.12)$$

In the above equation (3.11), $D = \frac{I}{m}$, where I is the mass moment of inertia of the object about the point of rotation A or B.

(i) for solid cylinder : $I = \frac{m}{12}(15a^2 + 16h^2)$

and $$D = \frac{I}{m} = \frac{1}{12}(15a^2 + 16h^2) \qquad \ldots \qquad (3.13)$$

(ii) for rectangular solid : $I = \frac{m}{12}(16a^2 + 16h^2)$

and $$D = \frac{I}{m} = \frac{1}{12}(16a^2 + 16h^2) \qquad \ldots \qquad (3.14)$$

Solution of equation (3.11) can be made by numerical integration using Runge-Kutta fourth order procedure as follows :

$$\frac{d\dot{\theta}}{dt} = \ddot{\theta} = -\frac{g}{D}(a \cos\theta - h \sin\theta) + \frac{\ddot{y}}{D}(a \sin\theta + h \cos\theta)$$

$$\theta_{n+1} = \theta_n + \Delta t\, \dot{\theta}_n + \frac{\Delta t}{6}(m_1 + m_2 + m_3)$$

$$\dot{\theta}_{n+1} = \dot{\theta}_n + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)$$

where $$m_1 = \frac{\Delta t}{D}\Big[-g(a \cos\theta_n - h \sin\theta_n) + \ddot{y}(t_n)(a \sin\theta_n + h \cos\theta_n)\Big]$$

$$\theta_1 = \theta_n + \frac{1}{2}\Delta t\, \theta_n$$

$$m_2 = \frac{\Delta t}{D}\Big[-g(a \cos\theta_1 - h \sin\theta_1) + \ddot{y}(t_n + \frac{\Delta t}{2})(a \sin\theta_1 + h \cos\theta_1)\Big]$$

$$\theta_2 = \theta_n + \frac{\Delta t}{2}(\dot{\theta}_n + \frac{1}{2}m_1)$$

$$m_3 = \frac{\Delta t}{D}\Big[-g(a \cos\theta_2 - h \sin\theta_2) + \ddot{y}(t_n + \frac{\Delta t}{2})(a \sin\theta_2 + h \cos\theta_2)\Big]$$

$$\theta_3 = \theta_n + \Delta t \left(\dot{\theta}_n + \frac{1}{2} m_2\right)$$

$$m_4 = \frac{\Delta t}{D}\Big[- g \,(a \operatorname{Cos} \theta_3 - h \operatorname{Sin} \theta_3) + \ddot{y}\,(t_n + \Delta t)$$

$$(a \operatorname{Sin} \theta_3 + h \operatorname{Cos} \theta_3)\Big]$$

When θ_{n+1} becomes negative, the time step Δt_n is reduced to one fifth and linear interpolation then made to fixed the time when $\theta = 0$ i.e., $\Delta t = \left(\frac{\Delta t_n}{\theta_n - \theta_{n+1}}\right)\theta_n$.

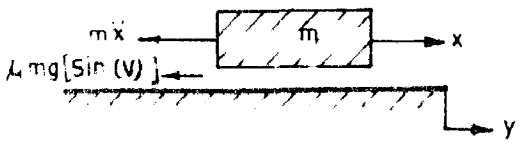


FIG. 3.1 _ FORCES ON OBJECT DURING SLIDING MOTION

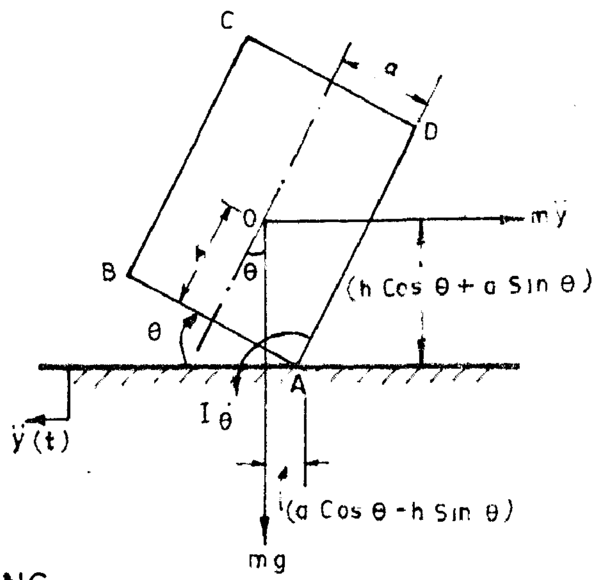


FIG. 3.2 _ FORCES ON OBJECT DURING OVERTURNING MOTION

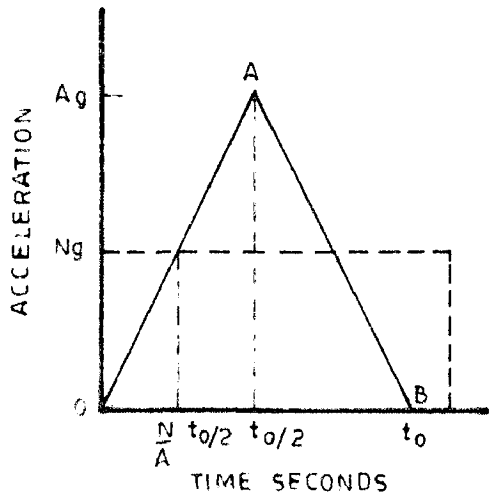


FIG. 3.3 _ TRIANGULAR GROUND ACCELERATION PULSE

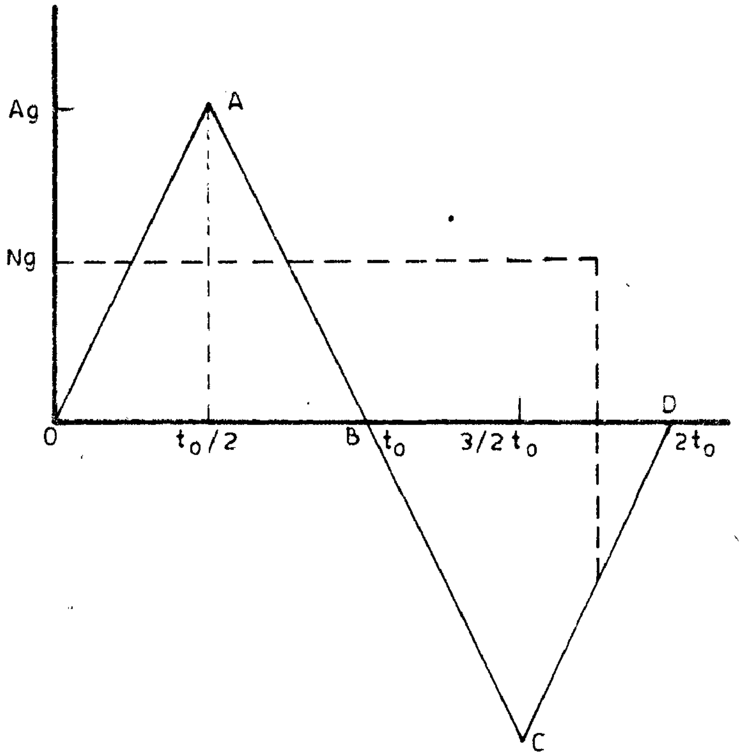
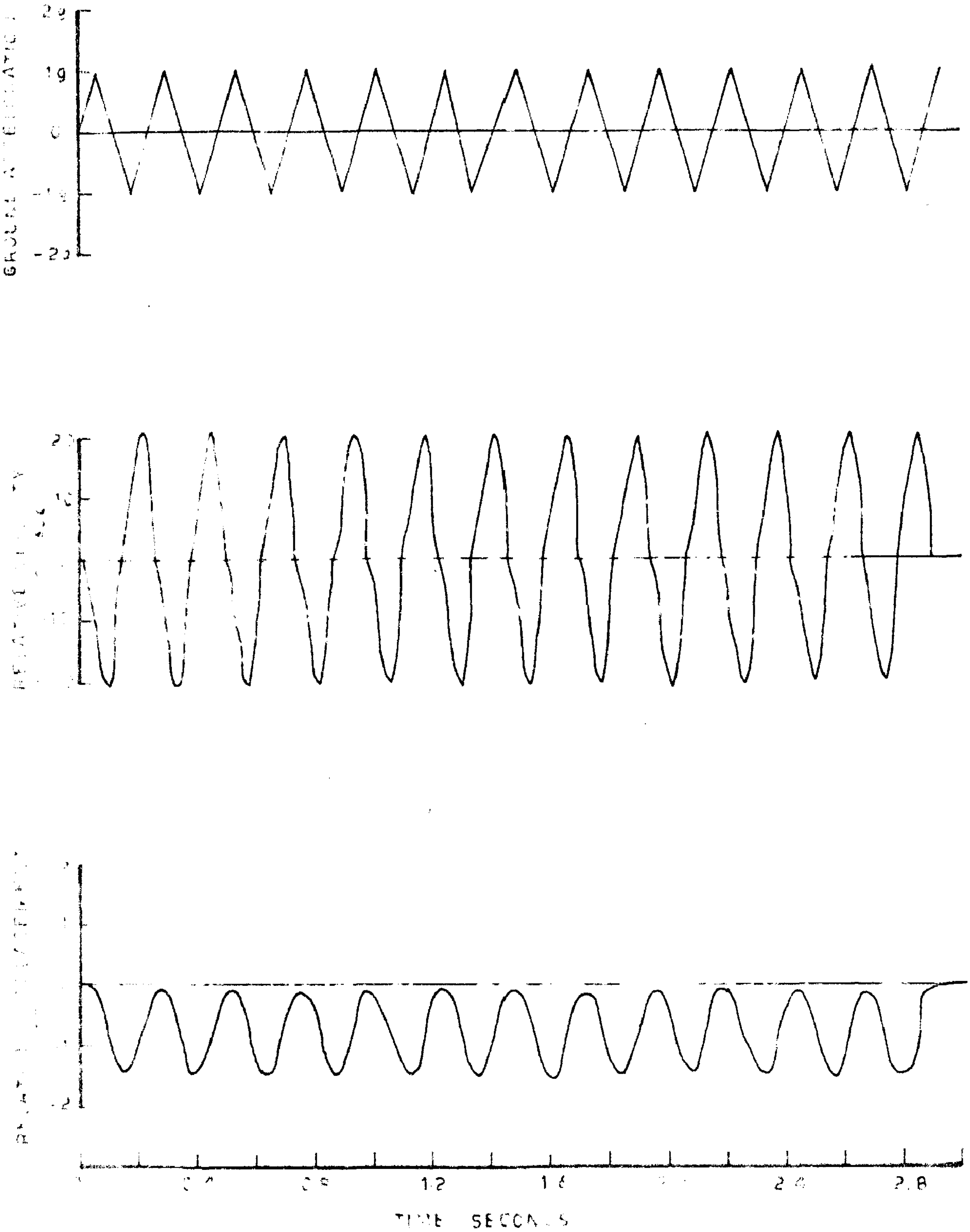


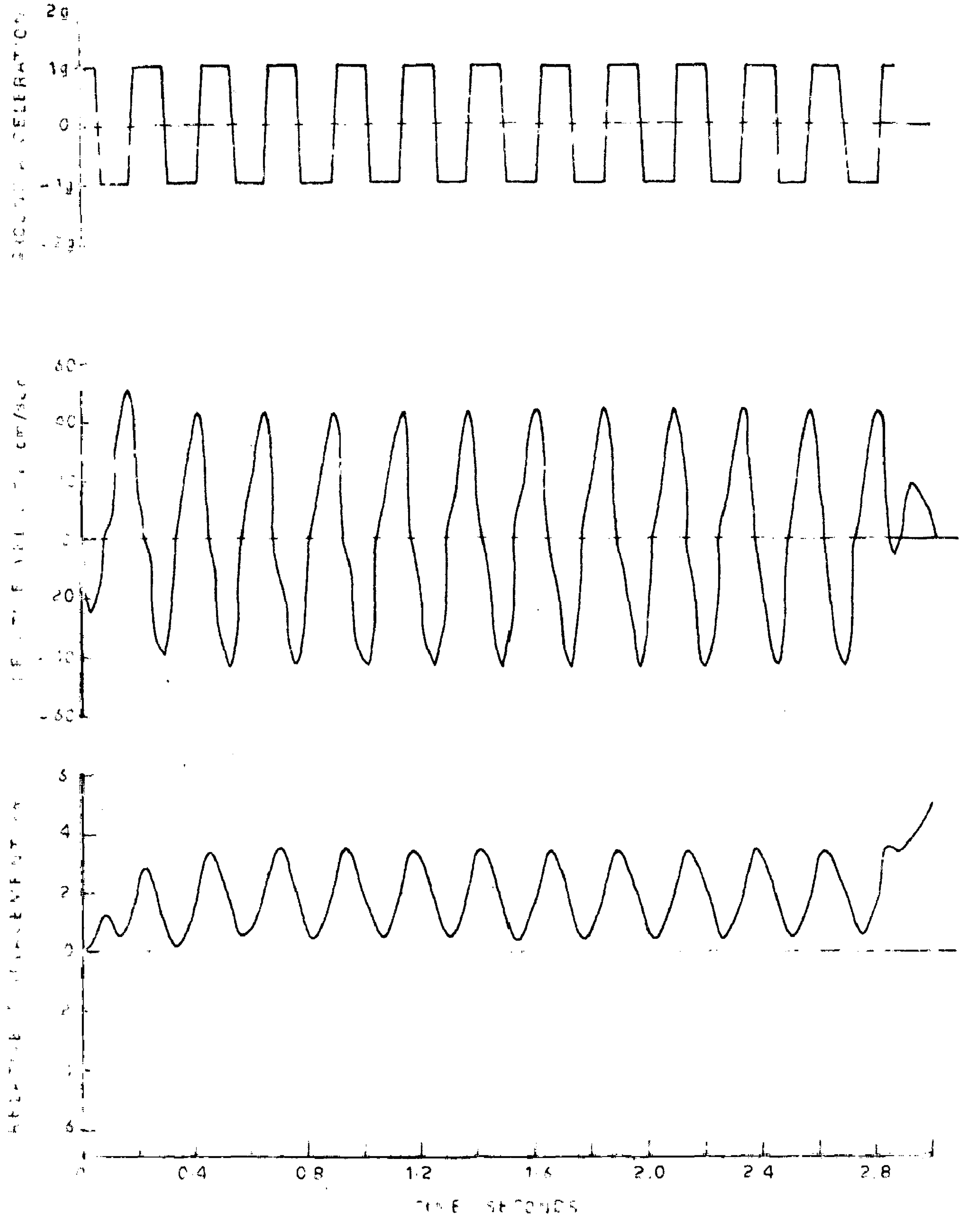
FIG. 3.4 _ DOUBLE TRIANGULAR PULSE



MAXIMUM RELATIVE DISPLACEMENT = 1.48

RESIDUAL RELATIVE DISPLACEMENT = 0.0895 cm

FIG 3.5 _SLIDING RESPONSE FOR A CONTINUOUS TRIANGULAR PULSE (μ = 0.4)



MAXIMUM RELATIVE DISPLACEMENT = 5.009 cm

RESIDUAL RELATIVE DISPLACEMENT = 5.009 cm

FIG. 3.6 – SLIDING RESPONSE FOR A CONTINUOUS TRAPEZOIDAL PULSE ($\mu = 0.4$)

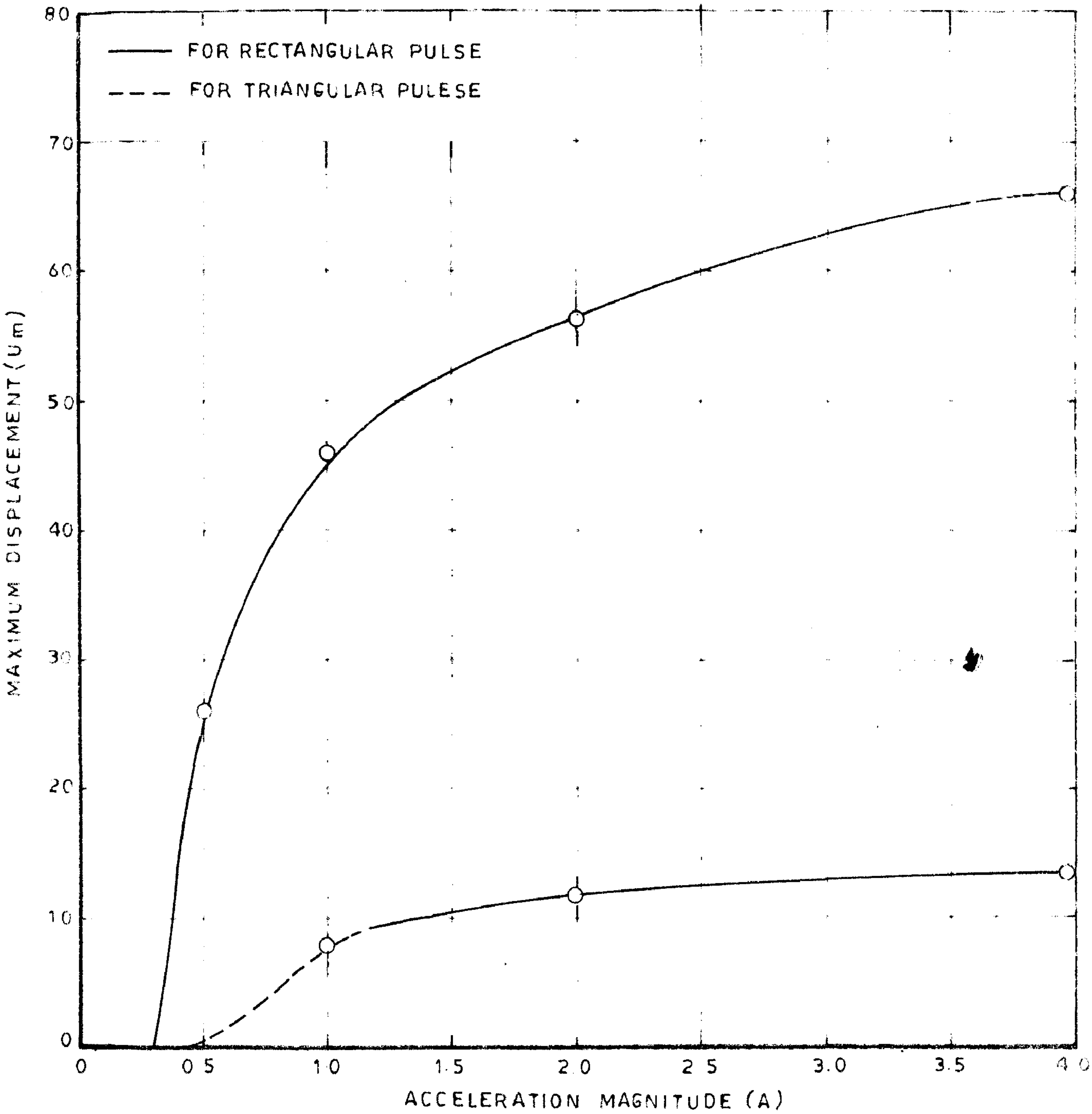


FIG. 3.7 _VARIATION OF MAXIMUM DISPLACEMENT WITH ACCELERATION FOR A CONSTANT PULSE

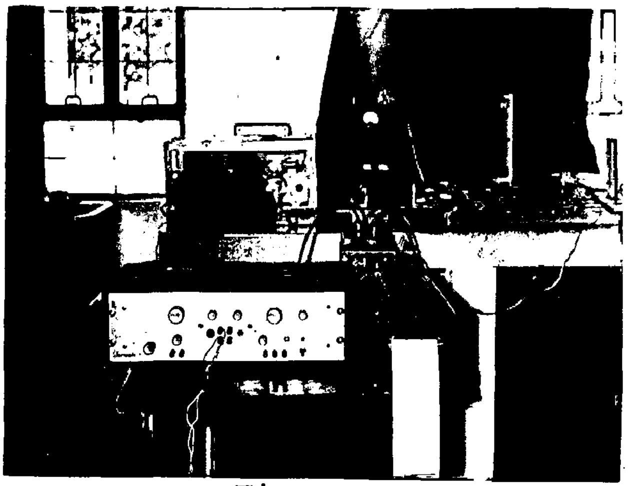


Fig – 4·1

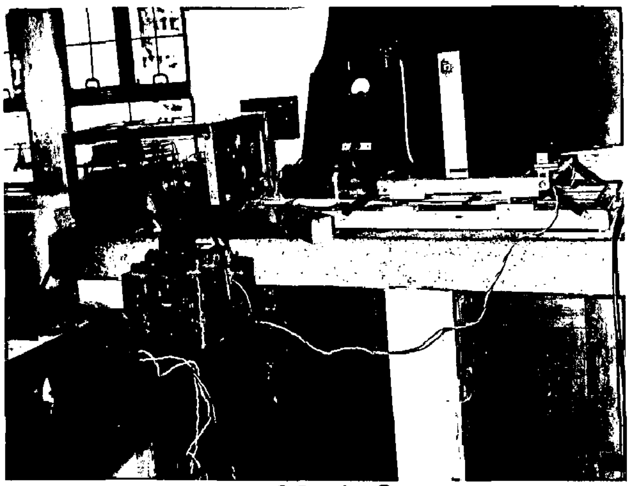


Fig-4·2

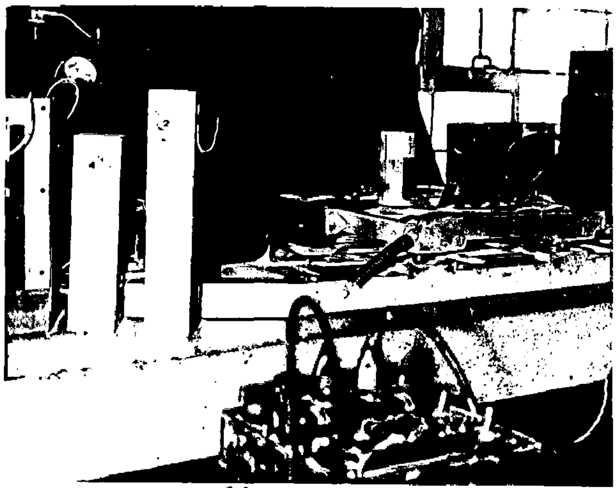


Fig- 4·3

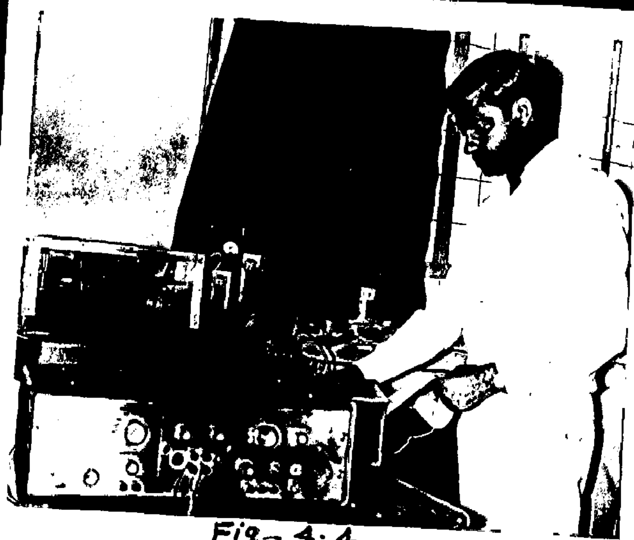


Fig - 4·4

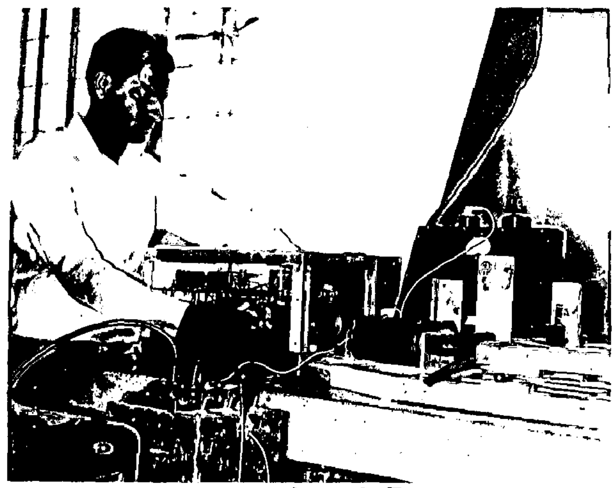


Fig - 4·5

CHAPTER - IV

EXPERIMENTAL STUDIES ON SHAKE-TABLE AND THEORETICAL RESULT

4.1 Experimental Set up

Experimental studies were conducted on a shaking table for evaluating both the sliding and the overturning responses. The shake-table is 38 cm long x 31 cm wide and is supported on steel balls. It is held in position by four springs. An automatic electromagnetic arrangement of giving impacts to the table is provided. A frequency control device is used for regulating the rate of impacts from 1.0 cps to 8 cps. Natural frequency of vibration of the table is 5 cps. The motion of the table can be measured by mounting a vibration pickup on it. The experimental set-up is shown in Fig. 4.1 to 4.5.

4.2 Selection of Objects and the Parameters for Sliding Motion

For the first case of sliding motion, the various values of coefficient of friction could be obtained by trail, by evaluating its value for four different objects and four surfaces. Objects made of wood, cement, allumunium and iron were chosen and the surfaces on which these objects rested were of plywood, straw board and sunmica. The various objects used for sliding motion study are shown in figure 4.6 (a) through (d).

The coefficient of friction was determined by measuring the weight of the object and the horizontal force required to slide it. The latter was measured by a pulley and pan arrangement, by tying a string to the object and stretching it horizontally over a friction less pulley. On the other end of thread a pan was hung, the weights on the pan were increased greadually, till the object just started sliding. This gave the frictional force and when divided by the weight of the object, the coefficient of friction was obtained.

Selection of suitable values of coefficient of friction (μ) over a range from $\mu = 0.21$ to $\mu = 0.55$ was made over a total of 8 values. The surfaces could be mounted easily on the vibration table by screwitn them at four corners.

The acceleration pickup mounted on the vibration table was connected to a pen recorder through an amplifier for recording the table accelerations.

4.3 <u>Device for Recording Sliding Motion</u>

For recording the distance moved by the object a thin and light alluminium strip weighting 5 mgs. was bent at right angles and one arm of it was fixed to the object. The other arm was extended by connecting through a hinge a similar strip to it. At the other end of the hinged extension a hole for pencil lead was drilled. The weight of the extended strip beyong the hinge gave sufficient pressure to hold the pencil point in contact with the paper below for making a record as the object moved on the table. Such strips were used on both the sides of the objects.

Below the pencil point of the recording gauge, paper strips were pasted on the surface and the object was placed between these two strips. Now the table was allowed to vibrate by the impact hammer arrangement provided with the table. The rigid object started moving and the record of its motion was left as a black impression on the paper strip. The accelerations of the table were recorded as stated earlier. Theoritical responses i.e. velocity and the displacement were then worked out from the digitised motion using the particular value of coefficient of friction.

4.4 Experimental study of Overturning Motion

An object of size 25.5 x 5.0 x 2.5 x cm as shown in Fig. 4.6 (e) was studied for overturning motion with its smallest dimension along the direction of motion. It was subjected to four table motions of different frequencies. Each motion caused the object to overturn after some rocking. In the analysis for overturning motion involving rocking of object, the coefficient of restitution was to be determined. This was done experimentally by recording the motion of the rocking object on a tightly stretched paper glued at its two ends to a vertical wooden frame. On one face of the object a small strip of allumunium bent in the form of U and having a pencil lead was fixed. The object was placed in front of the frame that the pencil lead just touched the paper and maintained the contact while in motion. Now the object the was slightly inclined initially by hand and then released. The pencil lead left a clear mark on the paper in the form of an arc of a circle. The angles between the initial and final positions of the pencil mark and the equilibrium positon at rest were measure-d and with the help of equations (3.3) for velocities, the value of coefficient of restitution was determined.

4.5 Recording and Anysing Table Motion

The accelerogram of the table motion so produced, was recorded by the pen recorder instrument. It was digitised by reading the acceleration records on a x, y co-ordinate plotting machine. The theoritical results are obtained by numerical analysis on the computer. For sliding motion a larger time interval of 0.04 sec ($\doteqdot \frac{T}{4}$) was used as the integration is direct and linear variation of acceleration exists over the successive accelerogram points. Whereas the analysis for over-turning motion a time step of $\frac{T}{20}$ or less was found to give accurate results, where T is the predominant time period of the accelerogram. The fourth order Runge-Kutta procedure was used here.

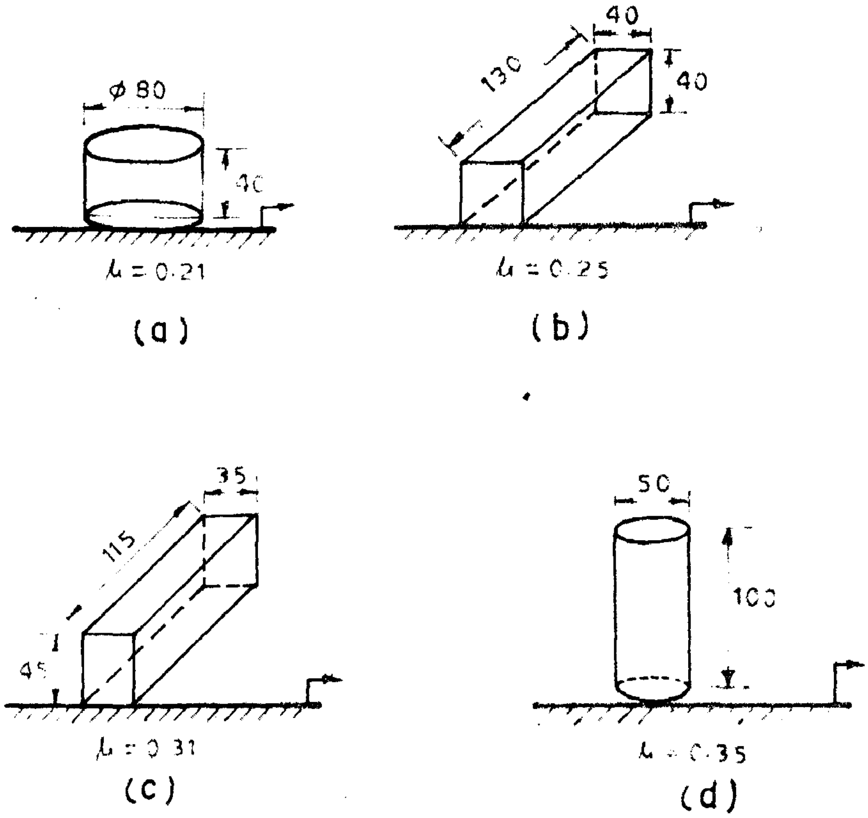
4.6 Comparision of Experimental Results of Sliding with Theory

The results of sliding of objects obtained by experimental sliding and theory are tabulated in table No. 4.1.

Fig. 4.7 to 4.14 show the pattern of sliding of the object, studied experimentally. Fig. 4.10 and 4.12 show the motion in both directions while the rest Figs. show the motion of objects in one direction only. Both directions mean that the object first moves in one direction and then in reverse direction as the velocity changes sign. All these curves show that as soon as the impact is given to the table, the object moves ahead, until the acceleration peak induced in other direction is not greater than the previous acceleration peak.

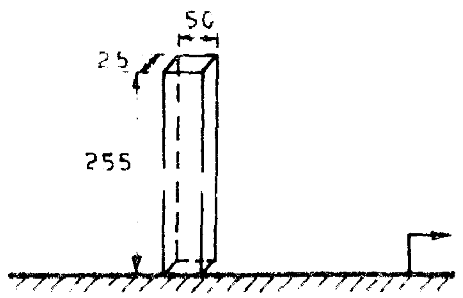
4.7 Comparision of Experimental Results of Overturning with Theory

The results of overturning of the object are plotted in (Fig. 4.15) and (Fig. 4.16). These figures show that experimentally the object overturned at the time marked by star on the accelerogram so obtained, but theoritically it is overturned before approaching to that time.



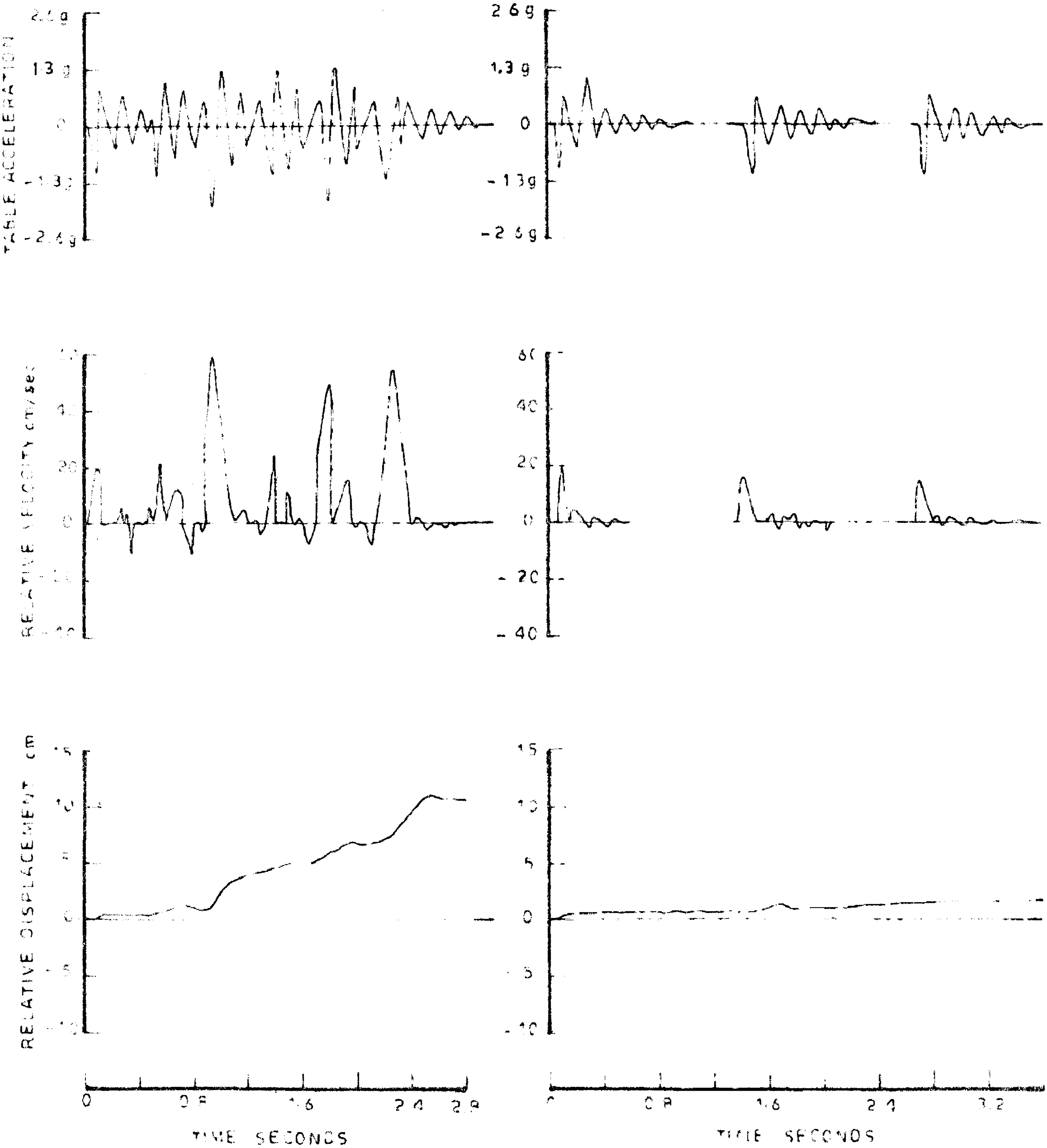
NOTE _ ALL DIMENSIONS ARE IN mm
OBJECT (a) IS OF CONCRETE
(b),(c) AND (d) ARE OF WOOD

FIG. 4.6 _ OBJECTS FOR SLIDING MOTION STUDY



NOTE _ ALL DIMENSIONS ARE IN mm
OBJECT OF WOOD
a = 1.25 cm
h = 12.75 cm

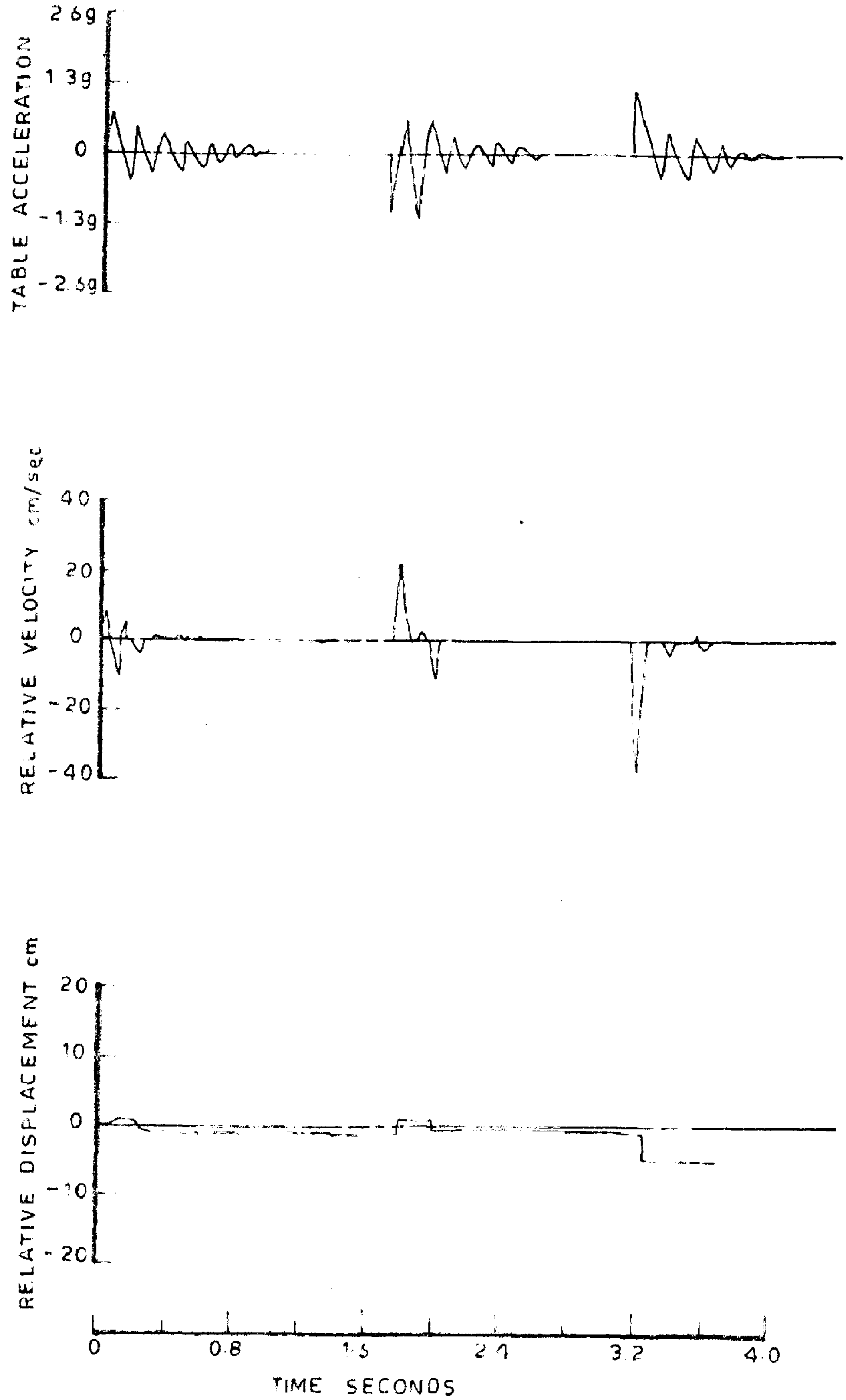
FIG. 4.6(e) _ OBJECT FOR OVERTURNING STUDY



MAXIMUM RELATIVE DISPLACEMENT = 21.86 cm
RESIDUAL RELATIVE DISPLACEMENT = 21.86 cm

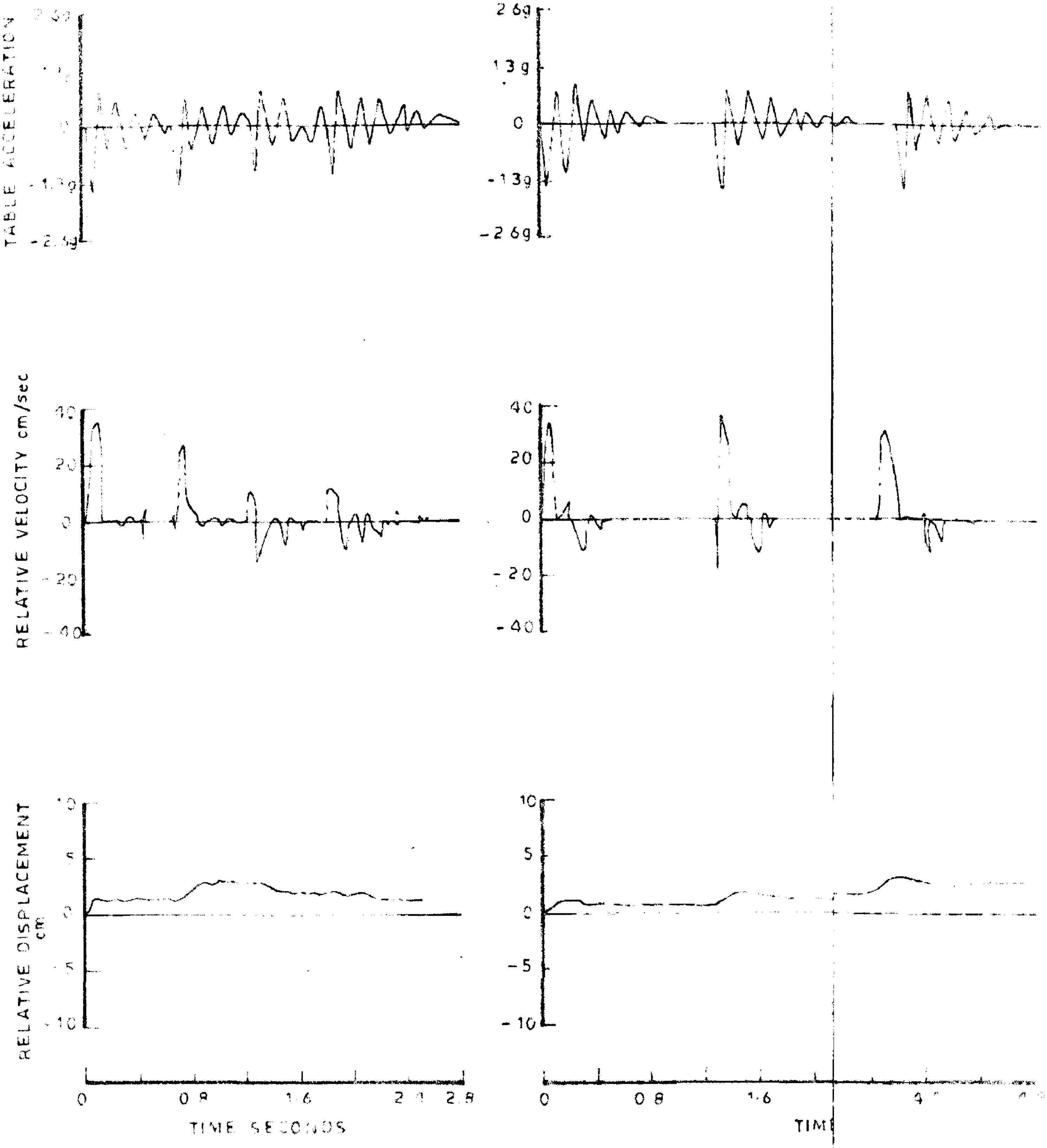
MAXIMUM RELATIVE DISPLACEMENT = 3.235 cm
RESIDUAL RELATIVE DISPLACEMENT = 3.235 cm

FIG. 4.7 _ SLIDING RESPONSE FOR $\mu = 0.21$



MAXIMUM RELATIVE DISPLACEMENT = 4.844 cm
RESIDUAL RELATIVE DISPLACEMENT = 4.844 cm

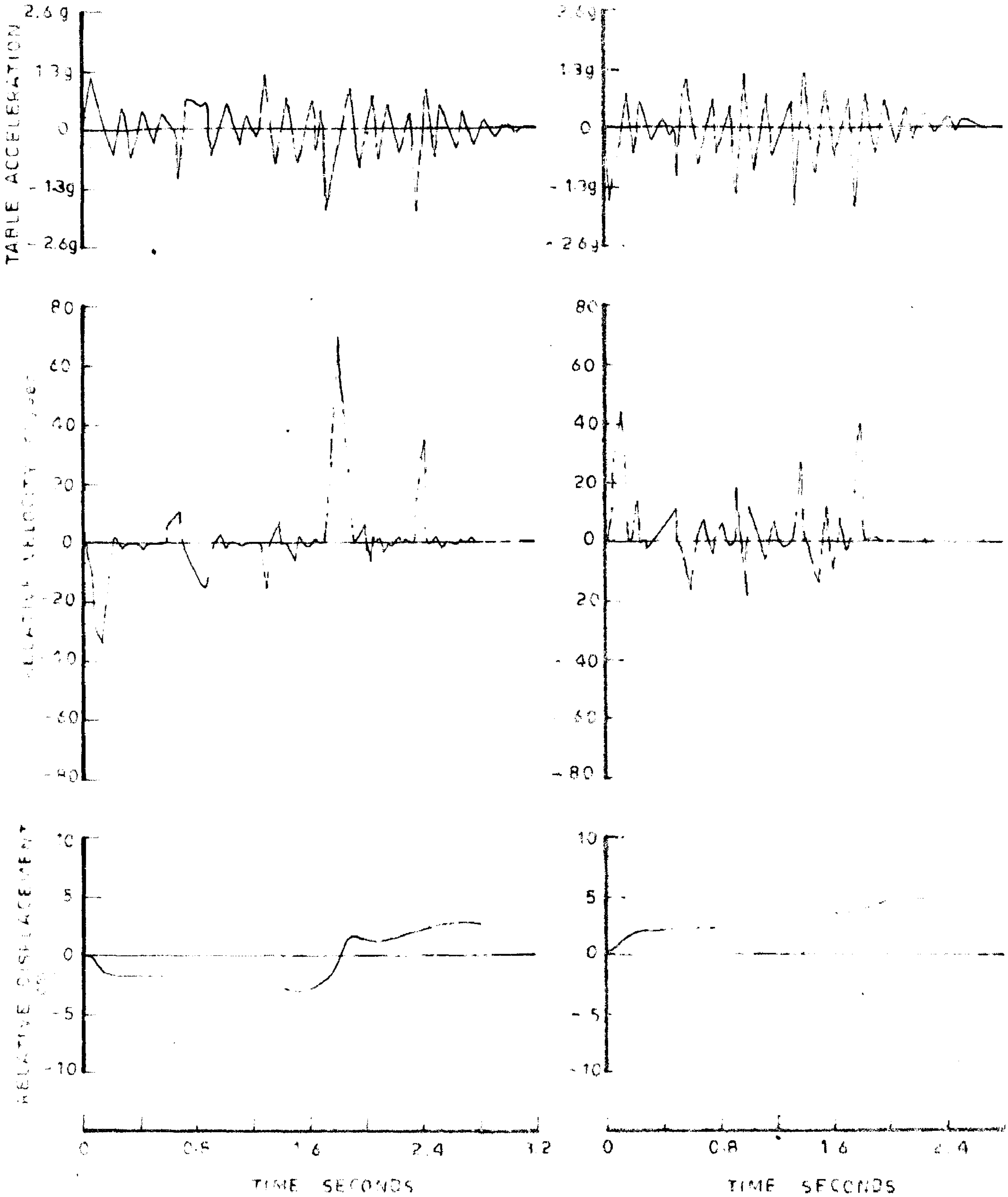
FIG. 4.8 SLIDING RESPONSE FOR $\mu = 0.25$



MAXIMUM RELATIVE DISPLACEMENT = 5.4?3 cm
RESIDUAL RELATIVE DISPLACEMENT = 2.624 cm

MAXIMUM RELATIVE D 6.337 cm
RESIDUAL RELATIVE D 5.485 cm

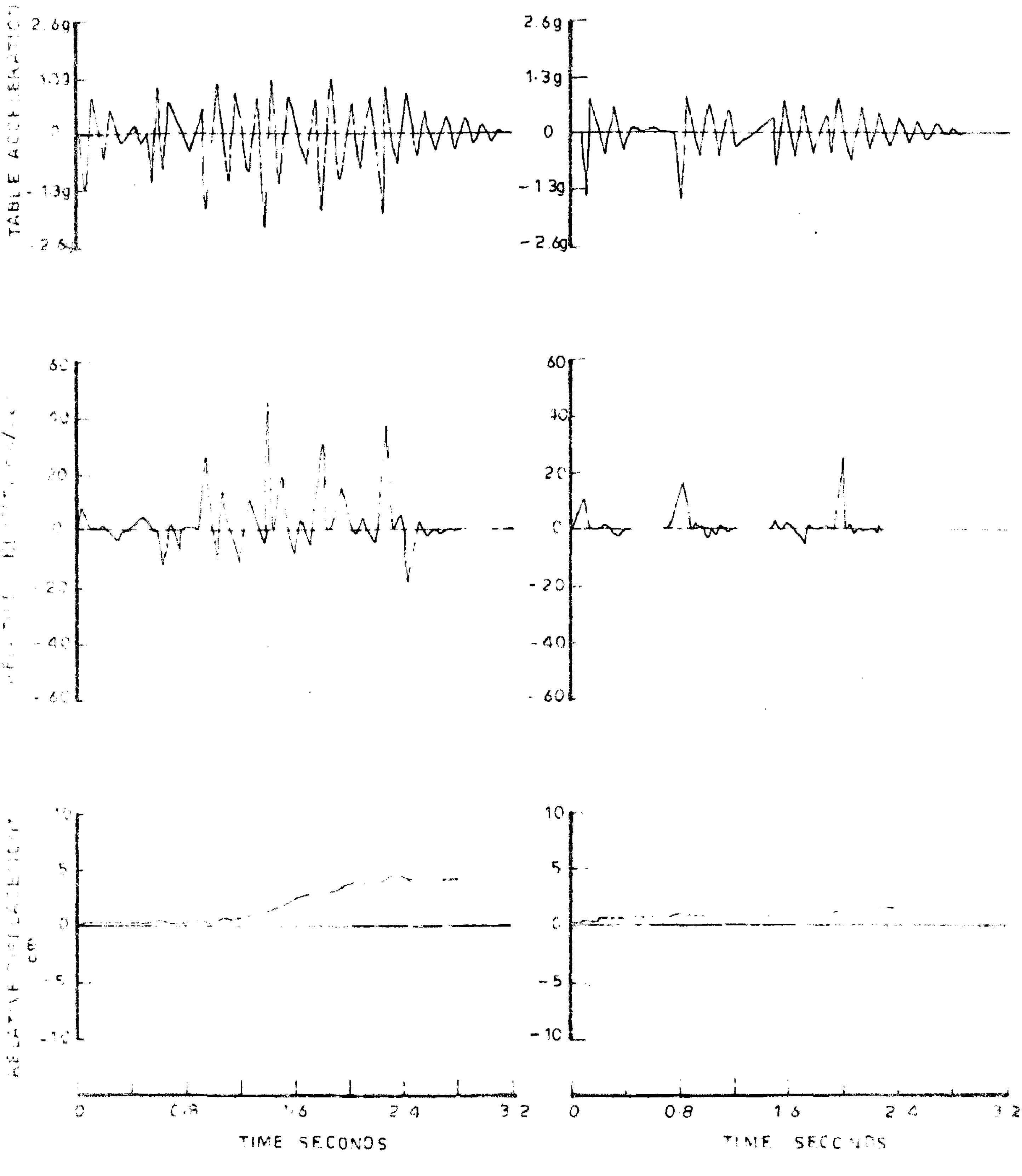
FIG.4.9_ SLIDING RESPONSE FOR $\mu = 0.31$



MAXIMUM RELATIVE DISPLACEMENT = 5.17 cm
RESIDUAL RELATIVE DISPLACEMENT = 5.17cm

MAXIMUM RELATIVE DISPLACEMENT = [illegible]
RESIDUAL RELATIVE DISPLACEMENT = [illegible]

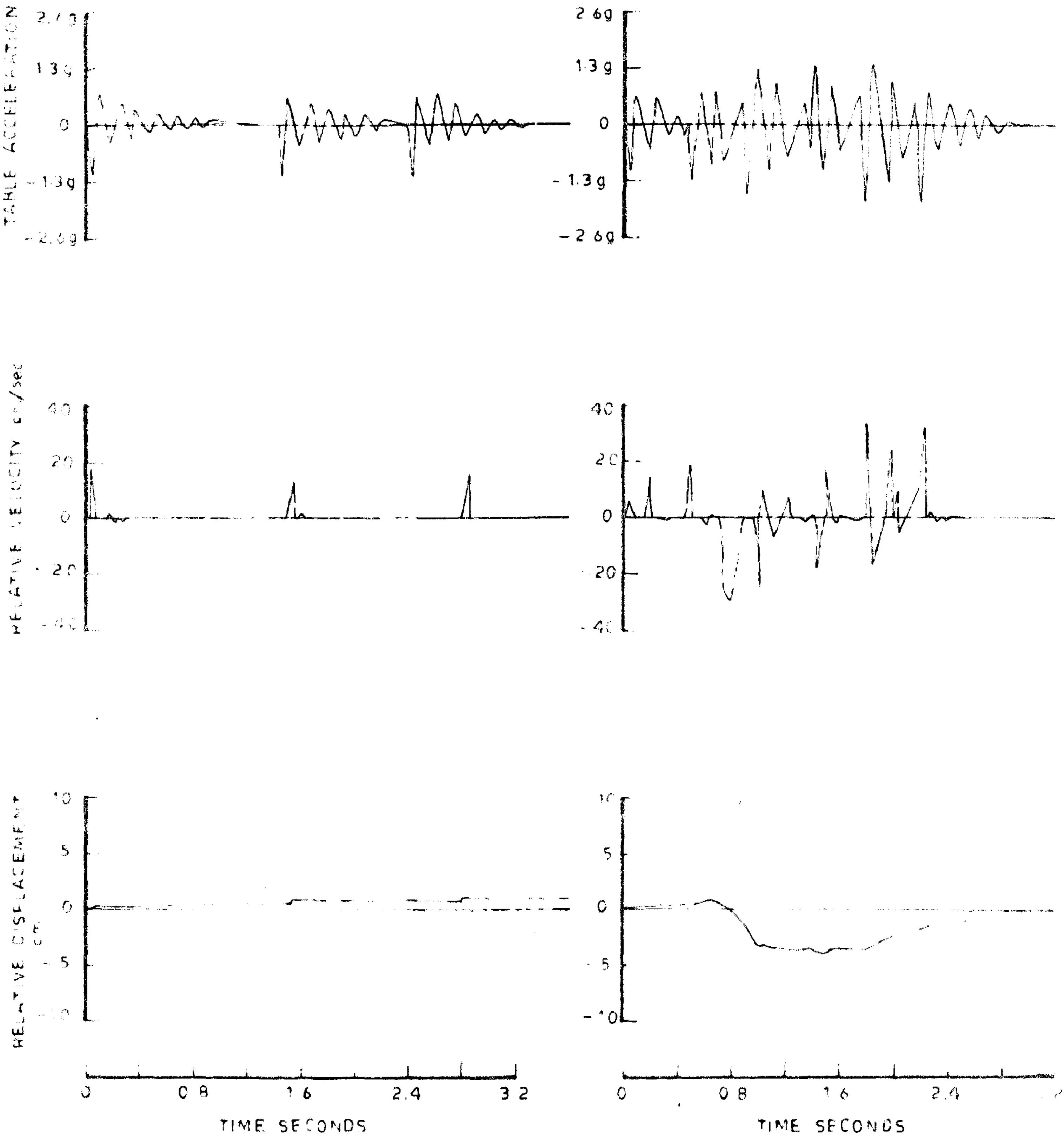
FIG. 4.10 . SLIDING RESPONSE FOR $\mu = 0.35$



MAXIMUM RELATIVE DISPLACEMENT = 9.482 cm
RESIDUAL RELATIVE DISPLACEMENT = 8.356 cm

MAXIMUM RELATIVE DISPLACEMENT [illegible]
RESIDUAL RELATIVE DISPLACEMENT [illegible]

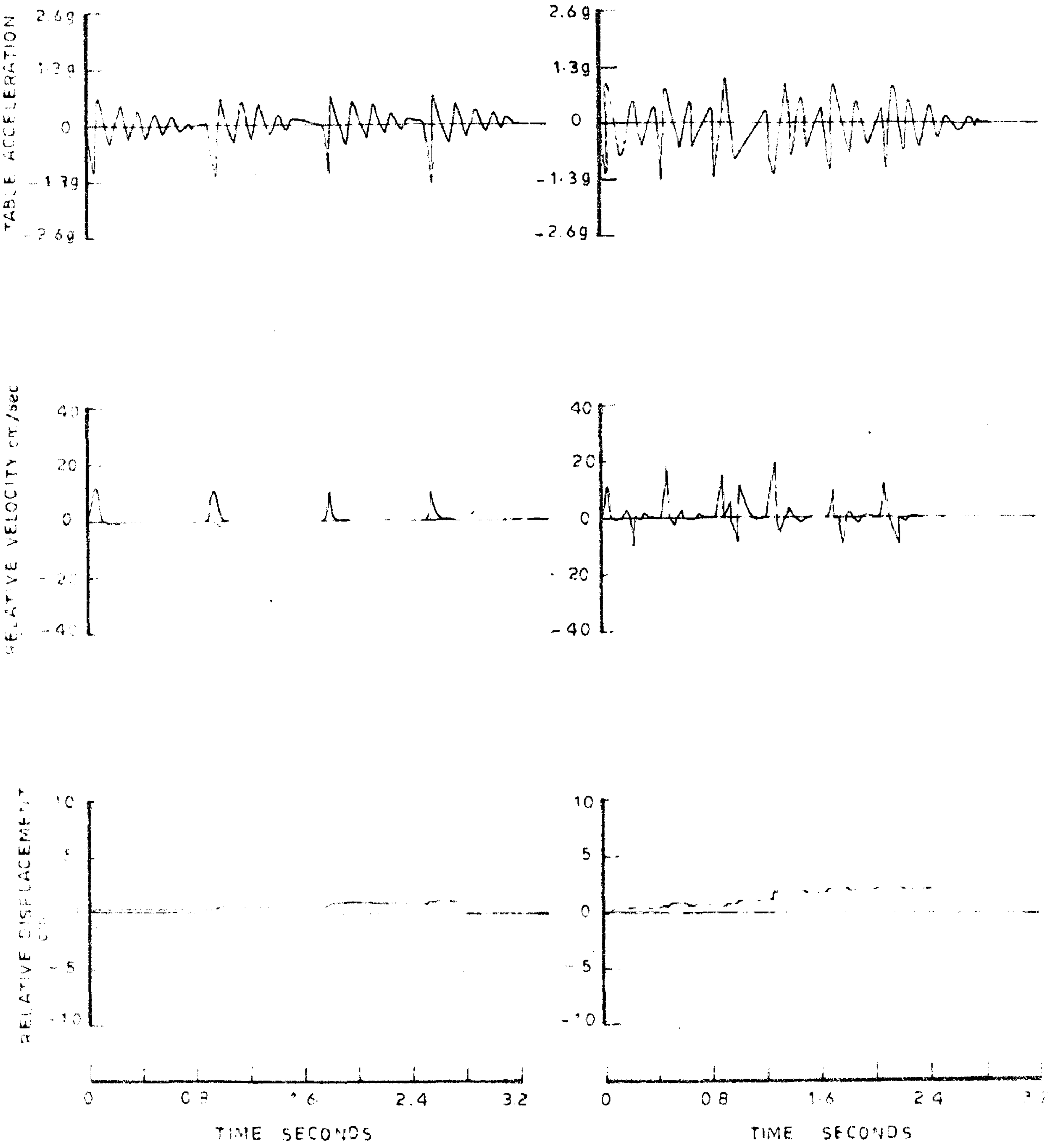
FIG. 4.11 _ SLIDING RESPONSE FOR $\mu = 0.40$



MAXIMUM RELATIVE DISPLACEMENT = 1.524 cm
RESIDUAL RELATIVE DISPLACEMENT = 1.524 cm

MAXIMUM RELATIVE DISPLACEMENT = [illegible]
RESIDUAL RELATIVE DISPLACEMENT = [illegible]

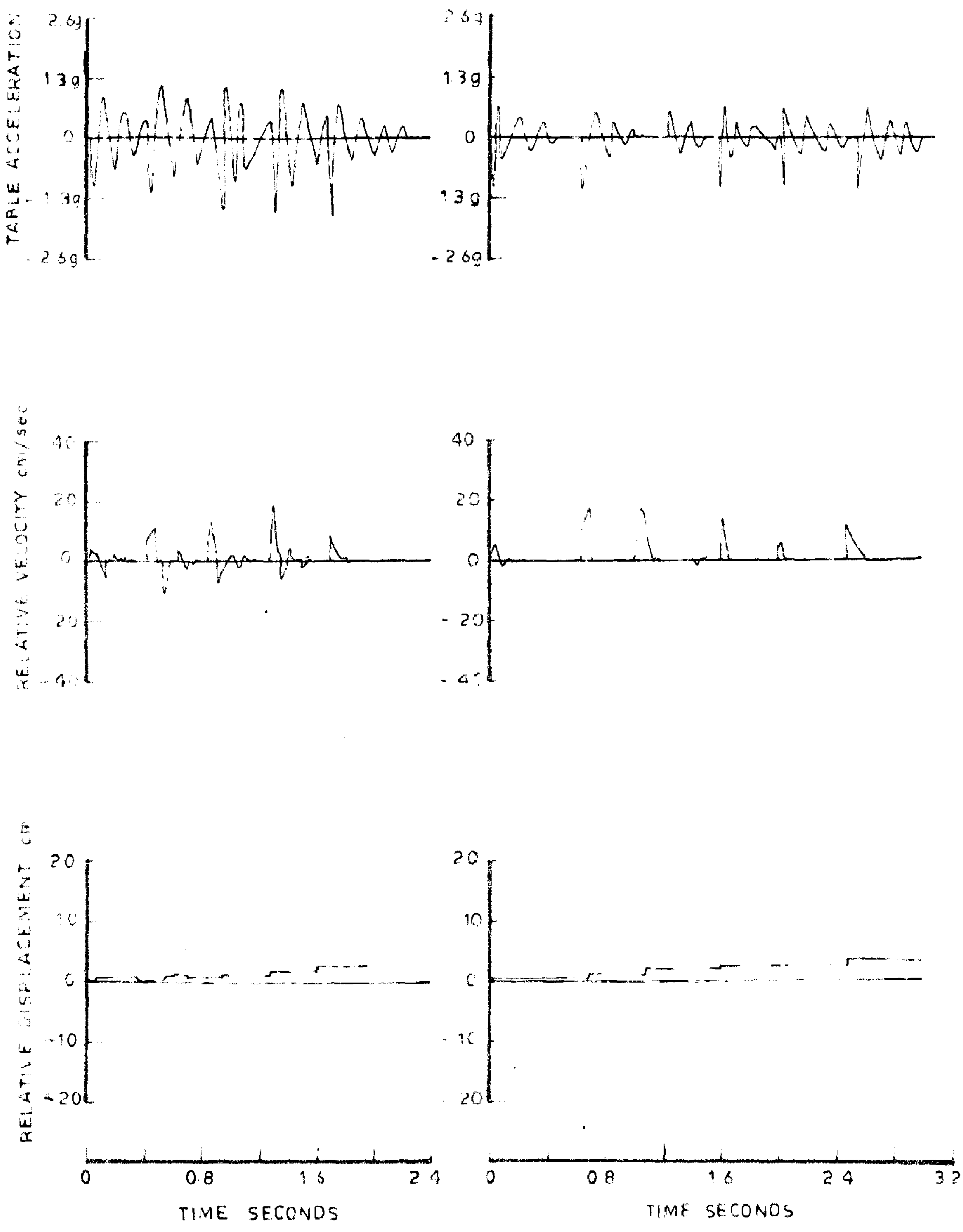
FIG. 4.12 _ SLIDING RESPONSE FOR $\mu = 0.404$



MAXIMUM RELATIVE DISPLACEMENT = 1.93 cm
RESIDUAL RELATIVE DISPLACEMENT = 1.93 cm

MAXIMUM RELATIVE DISPLACEMENT = 4.45 cm
RESIDUAL RELATIVE DISPLACEMENT = 4.406 cm

FIG. 4.13 _ SLIDING RESPONSE FOR $\mu = 0.45$



MAXIMUM RELATIVE DISPLACEMENT = 2.30 cm
RESIDUAL RELATIVE DISPLACEMENT = 2.30 cm

MAXIMUM RELATIVE DISPLACEMENT = 3.24 cm
RESIDUAL RELATIVE DISPLACEMENT = 3.24 cm

FIG. 4.14 _ SLIDING RESPONSE FOR $\mu = 0.55$

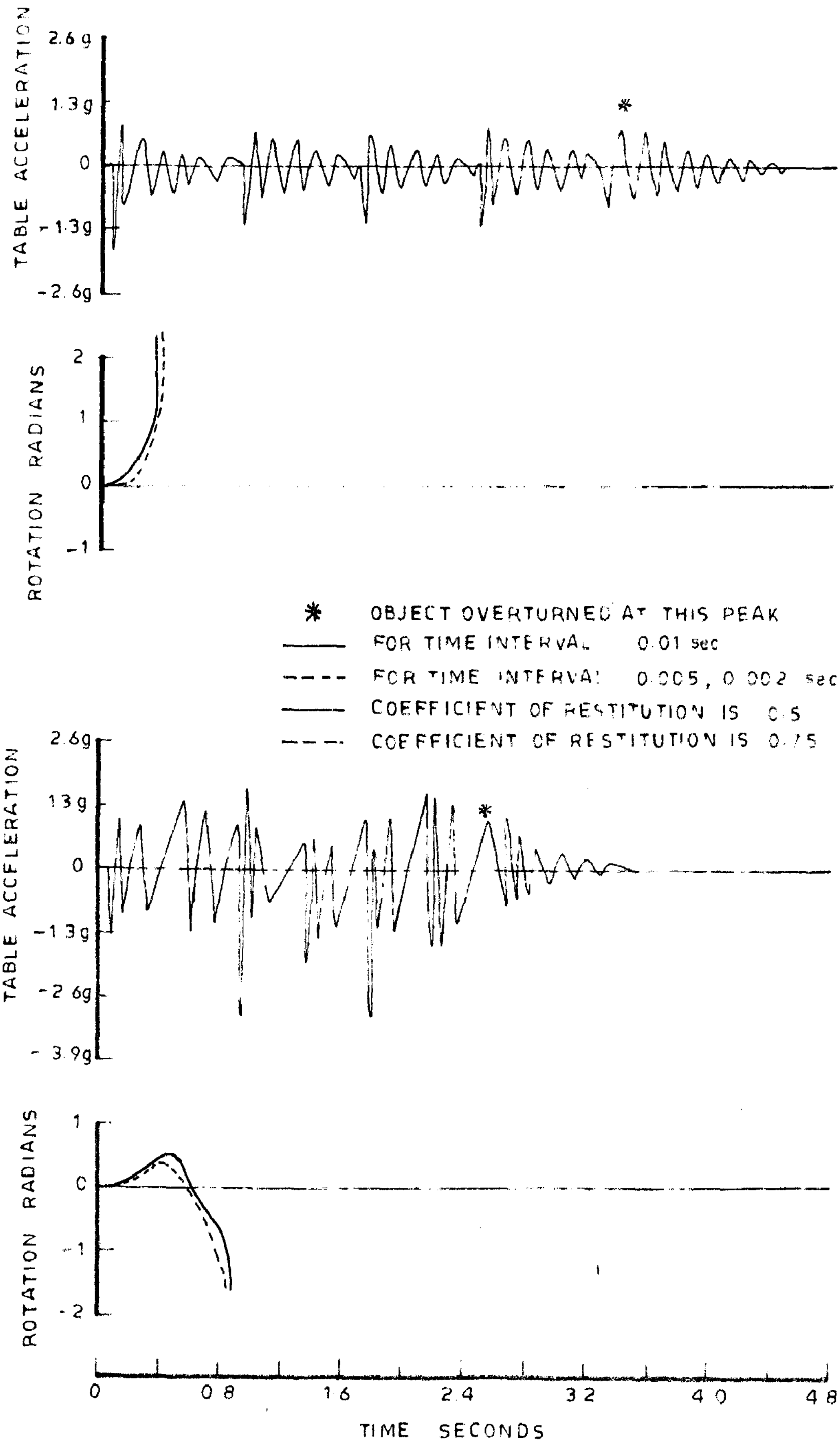


FIG.4.15 _OVERTURNING RESPONSES

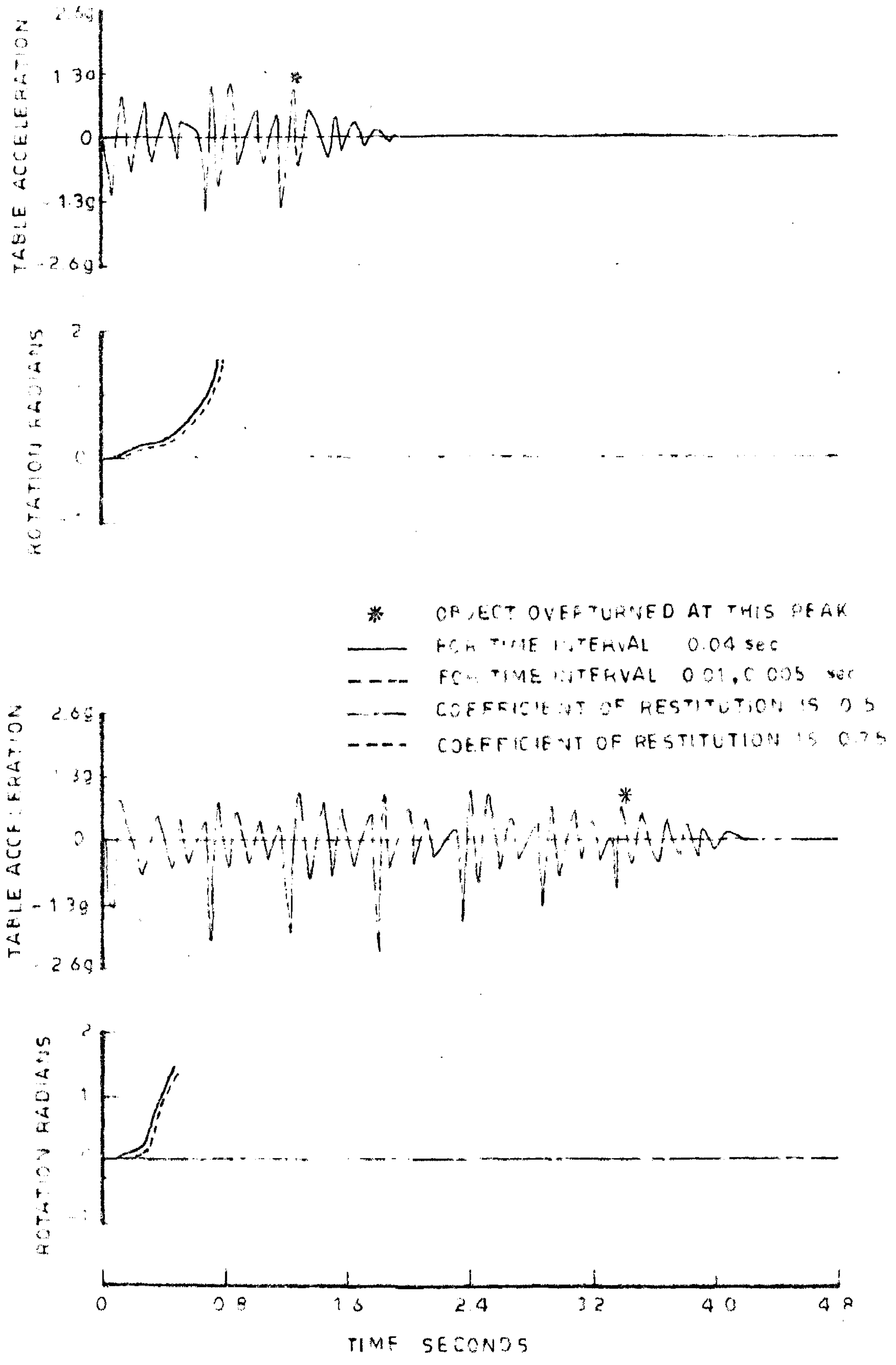


FIG. 4.16 _ OVERTURING RESPONSE

CHAPTER V

MOVEMENT OF OBJECTS DURING KOYNA EARTHQUAKE

5.1 Sliding of Objects - The accelerogram used in analysis is given in (Fig. 5.1) and the computed time wise values of relative displacements and velocities are presented in (Fig. 5.2) to (Fig. 5.7). The Maximum and residual values of displacement are given in table 5.1 for the various coefficients of friction.

Table 5.1 shows that the amount of sliding decreases as the coefficient of friction increases and the displacement of the object is almost negligible for high values of coefficient of friction particularly when it becomes more than 80% of the coefficient of peak ground acceleration. From the figures it is seen that the displacements in some cases continue to grow whereas for other values of μ the displacements first reduces and again increases. In one case of $\mu = 0.1$ the displacement after attaining maximum value reduces to zero, reverses sign and in the end remains very nearly zero. Thus there is no definite trend and it seems that the displacements are very sensitive to the coefficient of friction μ.

5.2 Overturning of Objects

Two small objects A and B of rectangular configration of dimensions 4 x 20 cm and 4 x 12 cm respectively were analysed for overturning motion due to koyna longitudinal component having the direction of ground motion paralel to their shortest dimensions. Af time interval of 0.005 seconds was used in the intergration which corresponds to about one twentieth of 0.1 sec, the predomenent period of the earthquake motion. It is seen that none of the objects overturns and they have rocking motion as shown in Fig. 5.8 and 5.9. The more unstable object A has (a/h) ratio equal to 0.20. Hence a static force of 0.2g acceleration could overturn it, whereas the peak ground acceleration in the accelerogram used is 0.63g occurring at time 3.8518 seconds on the record and having a duration of approximate 0.05 sec.

Angular rotations of the order of the peak value of 0.5^{o} occurs several times, including the instant of peak ground acceleration for this object. For the other object B having $\frac{a}{h} = 0.33$ the peak displacements of maximum order occurs twice, whereas the largest displacement occurs at a time of about 4.325 seconds. When the accelerogram has a peak of 0.45g having a duration of approx. 0.06 sec. Therefore it is evident that the extent of rotation or complete overturning will depend not only on the peak ground acceleration but also the duration for which the peak lasts.

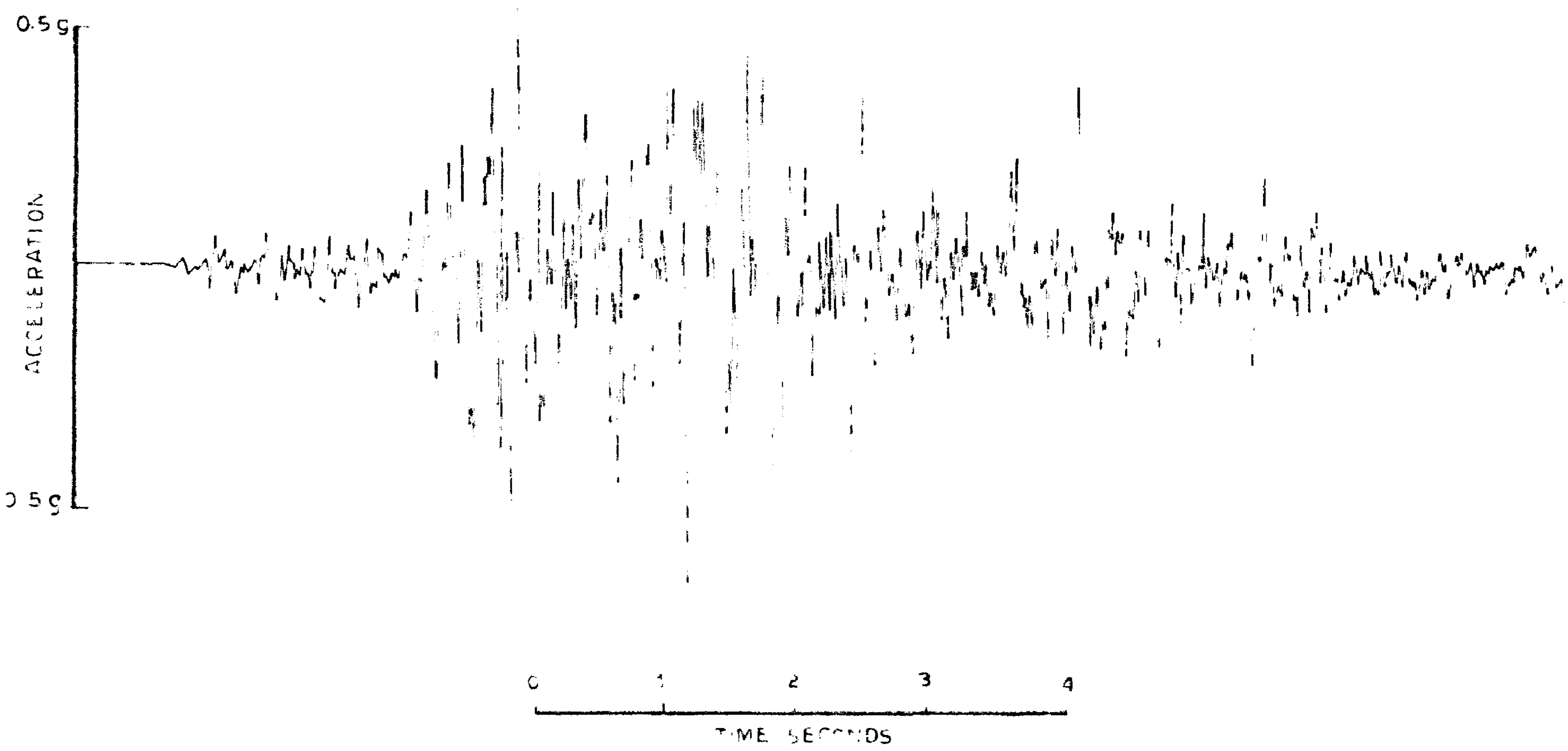


FIG. 5.1 – ACCELEROGRAM FOR KOYNA EARTHQUAKE OF DEC. 11, 1967
LONGITUDINAL COMPONENT

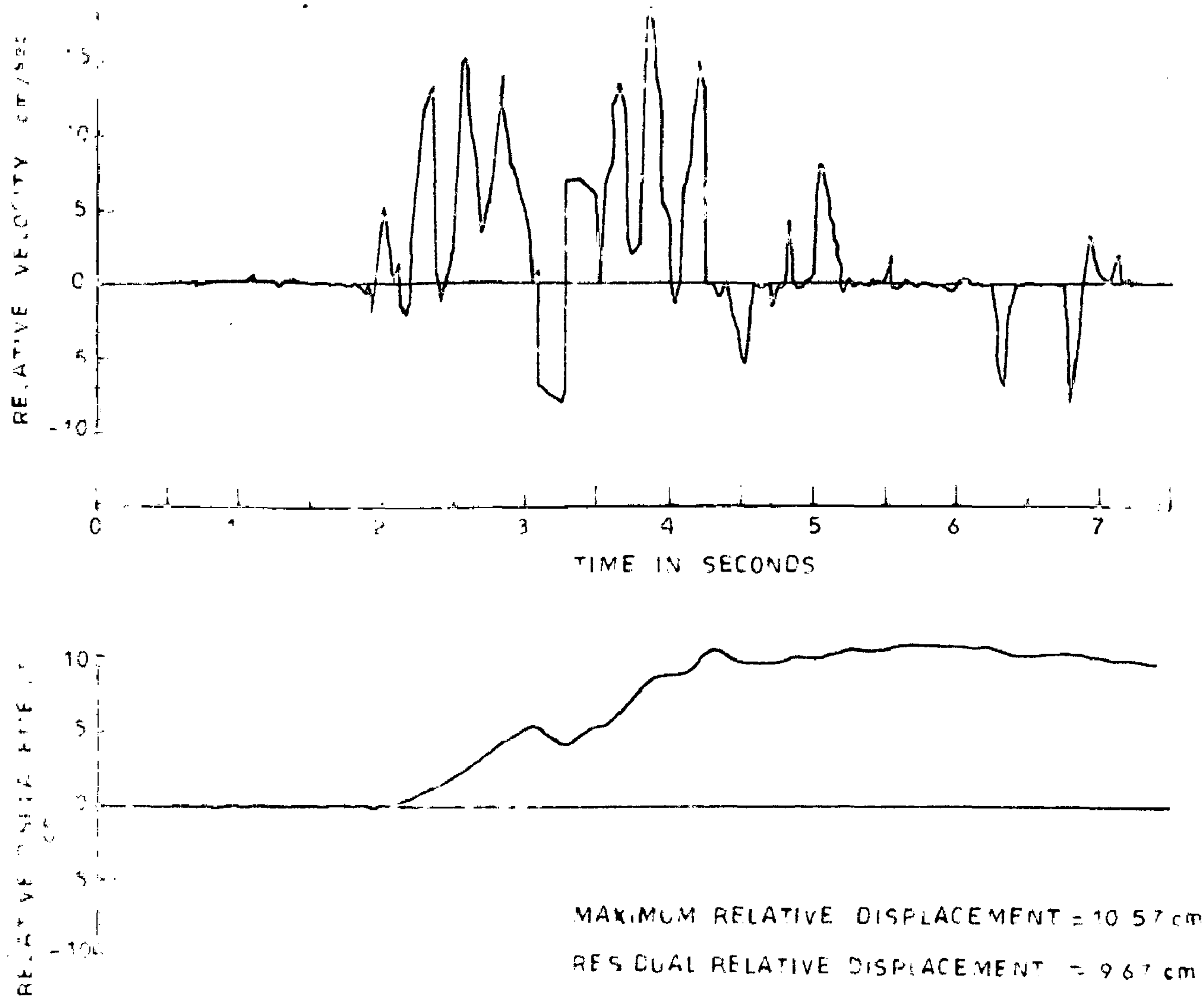


FIG.5.2 _SLIDING RESPONSE FOR $\mu = 0.05$

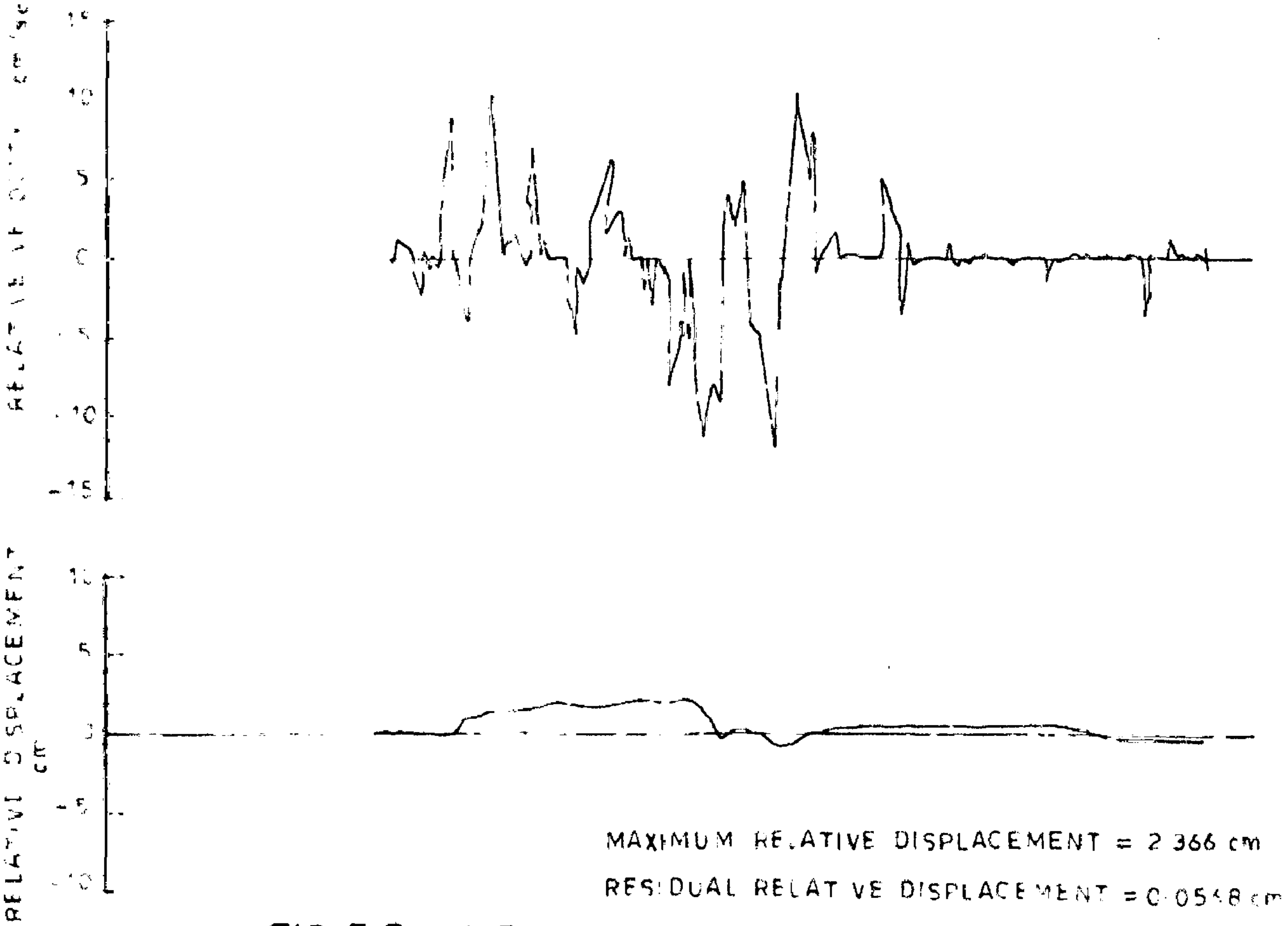


FIG. 5.3 _SLIDING RESPONSE FOR $\mu = 0.1$

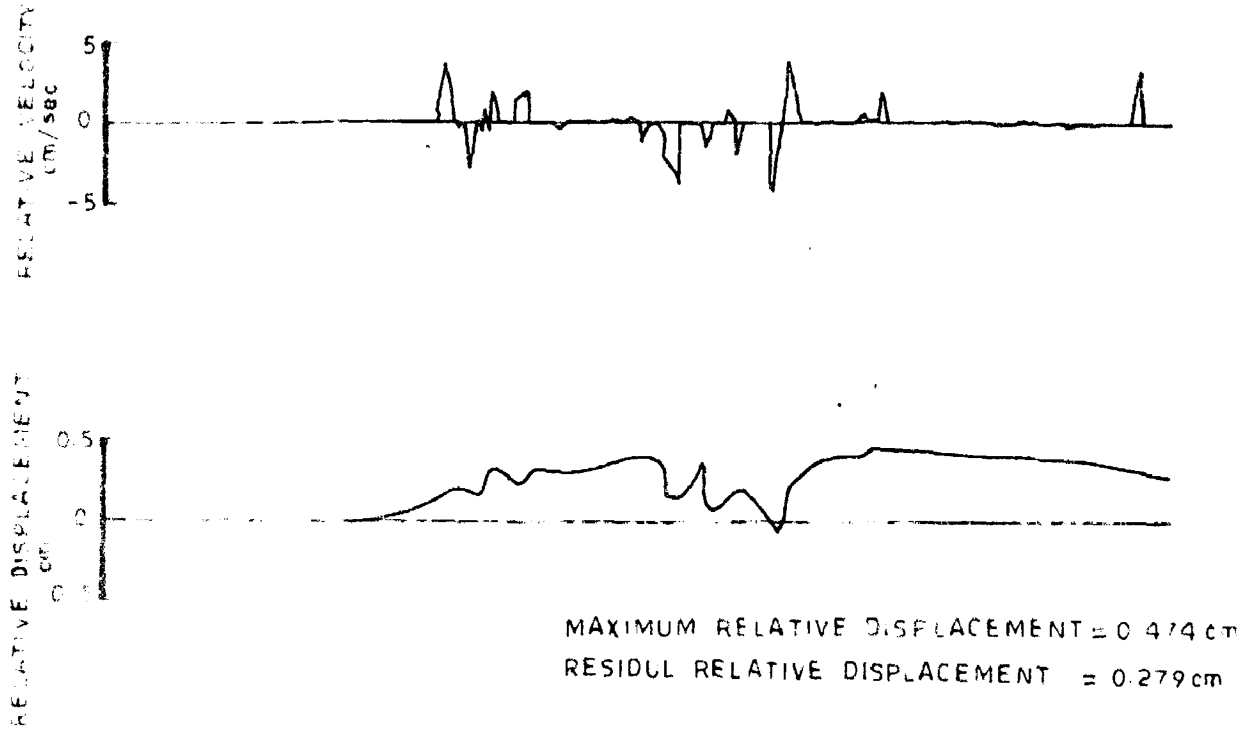


FIG. 5.4 _SLIDING RESPONSE FOR $\mu = 0.2$

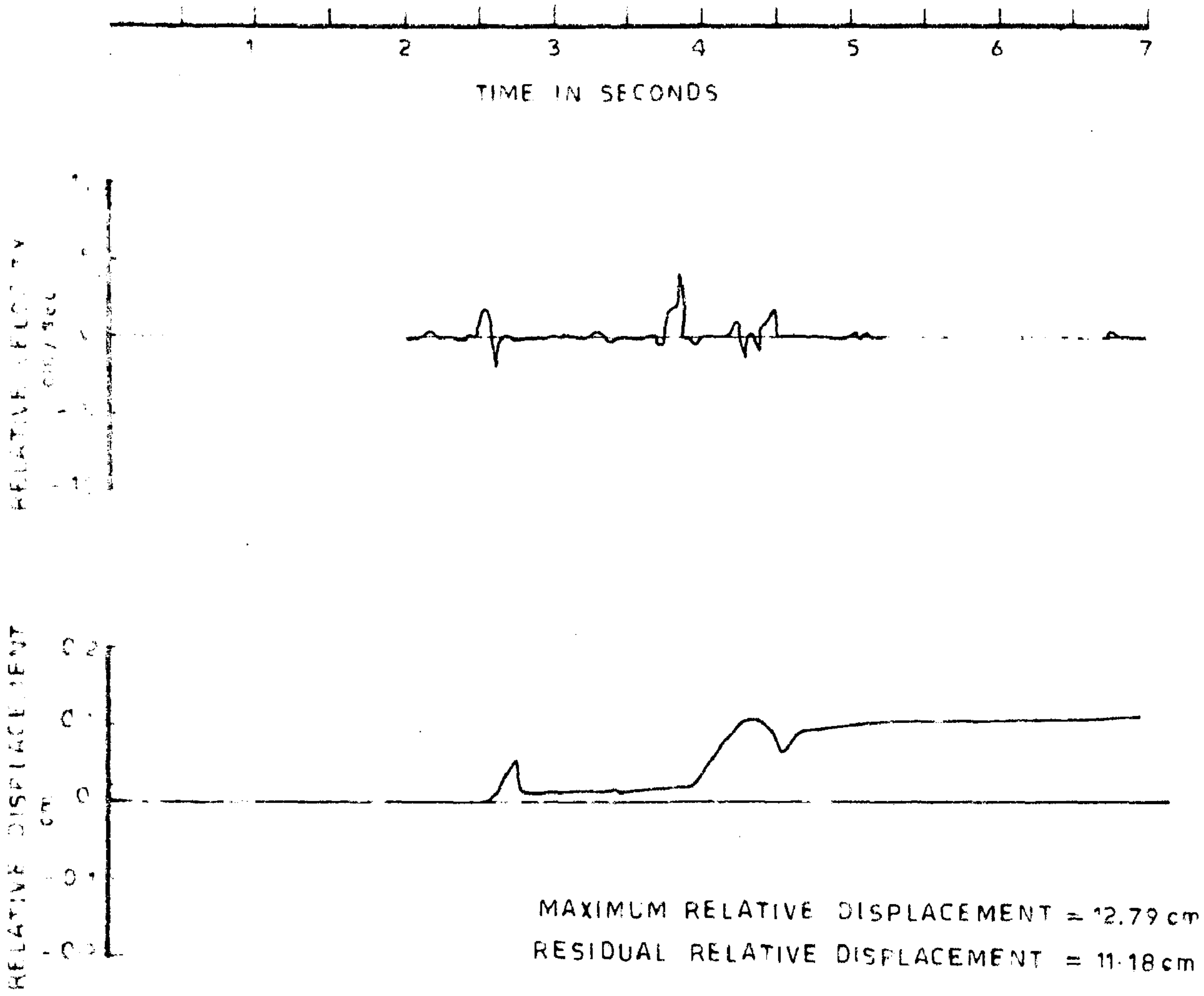


FIG. 5.5 _SLIDING RESPONCE FOR $\mu = 0.3$

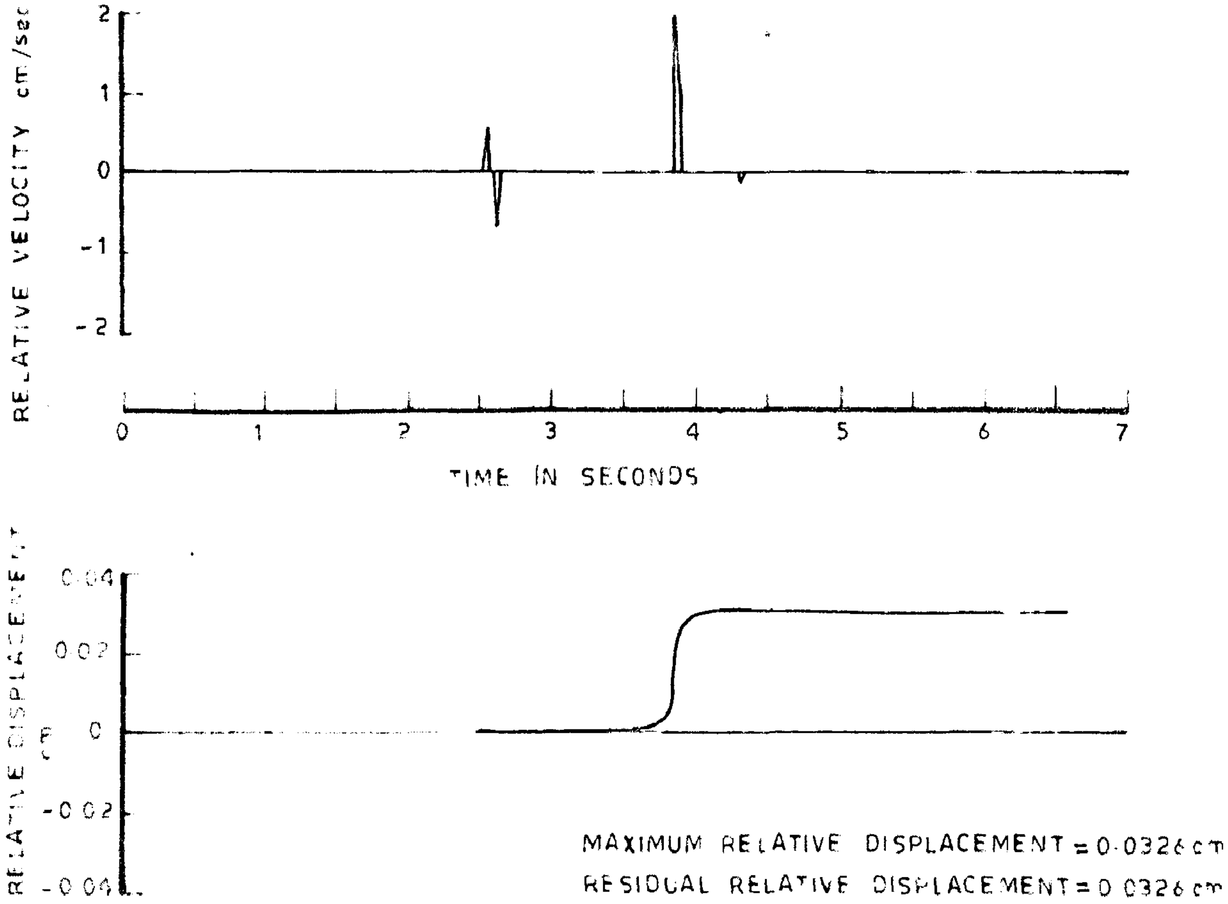


FIG 5.6 _ SLIDING RESPONSE FOR $\mu = 0.4$

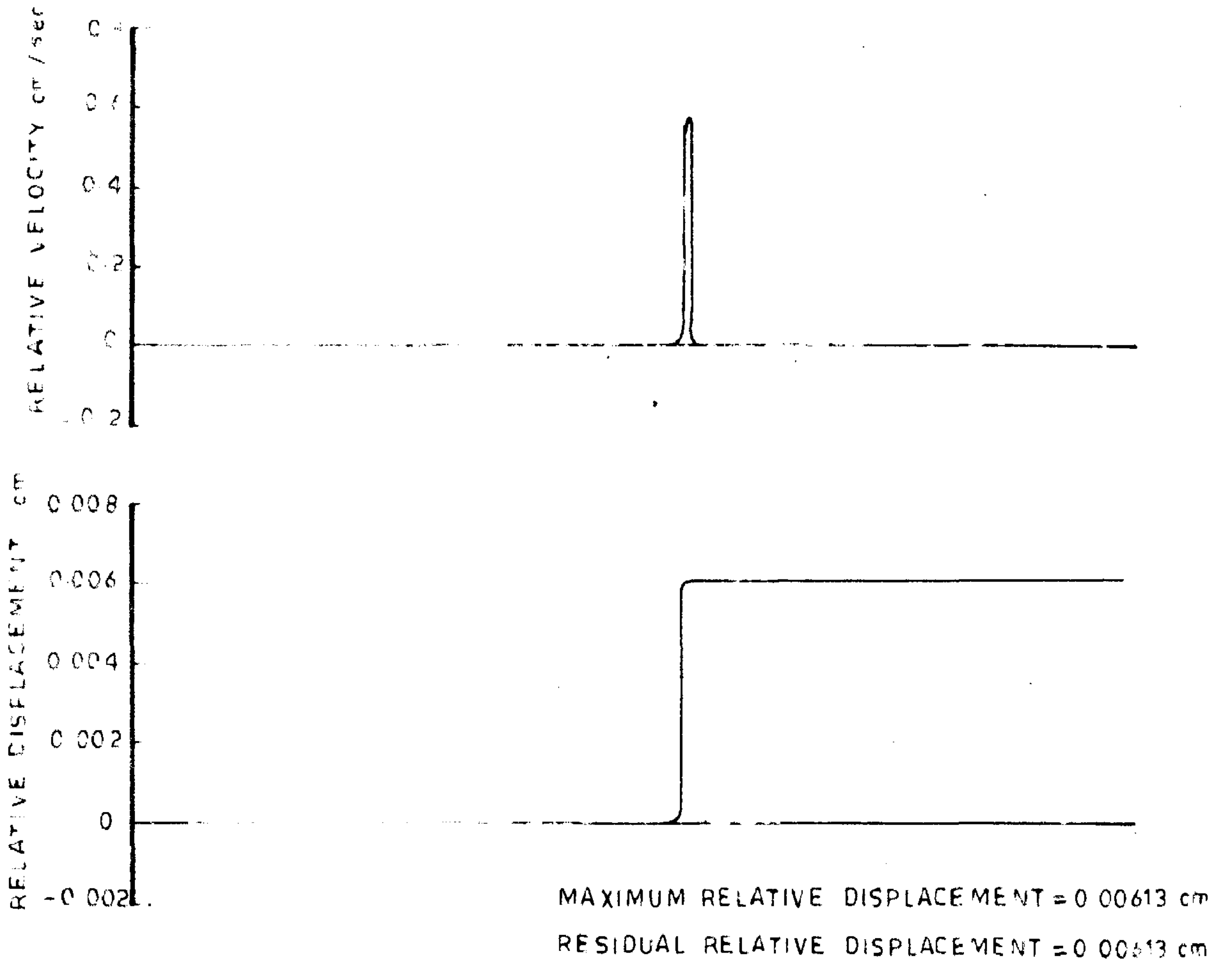


FIG.5.7 _ SLIDING RESPONSE FOR $\mu = 0.5$

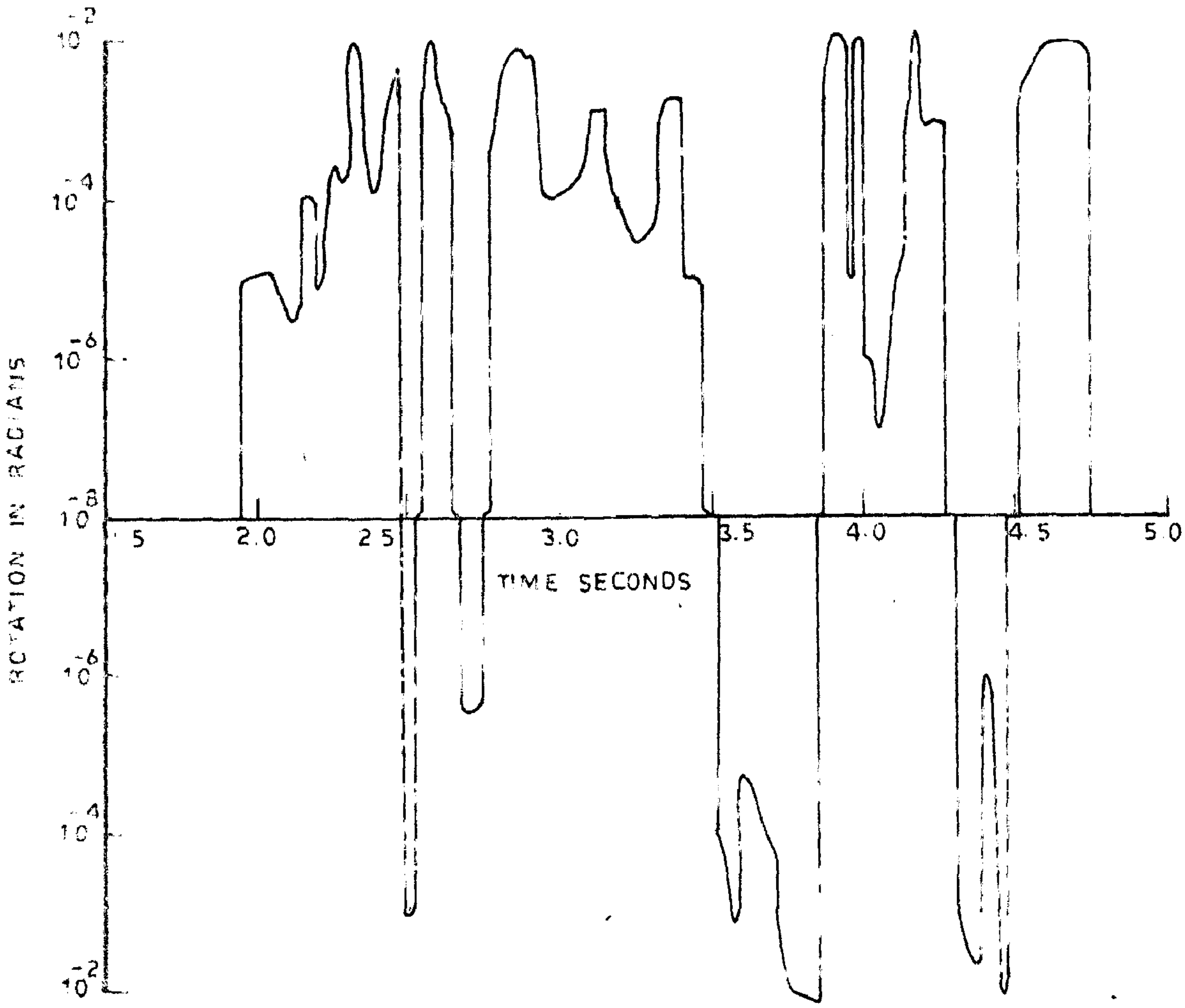


FIG. 5.8 _OVERTURNING OF THE OBJECT OF $(a/h = 0.2)$

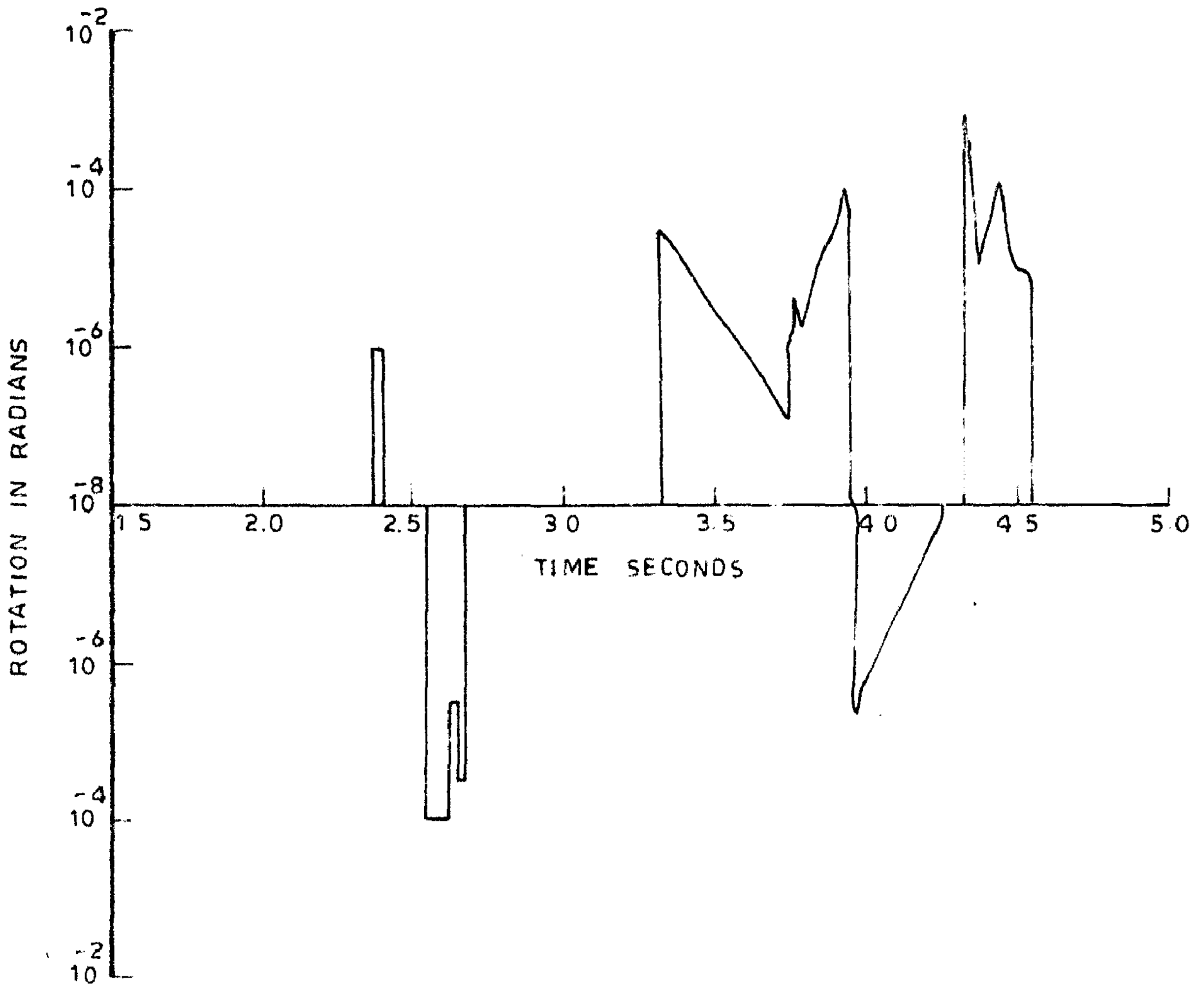


FIG. 5.9 _ OVERTURNING OF OBJECTS ($a/h = 0.33$)

CHAPTER - VI

CONCLUSIONS

1. Experimental studies presented in Chapter IV indicate that the analysis for overturning and sliding are sensitive to the ground motion as well as coefficient of friction. The table motion should therefore be precisely recorded and measured. Perhaps a better precision than used in the work is desirable. The order of the computed displacements for sliding of objects is the same as obtained experimentally although in some cases the correlation is rather poor.

In overturning study, the theoritical analysis indicates overturning much before that observed experimentally. During the experiment, the object was actually seen to rotate about its vertical axis also and the complex combined motion apparantly added to its stability in overturning.

2. Analysis for Koyna earthquake record (Chapter V) reveals that the maximum amount of slidng is rather small and is sensitive to the coefficient of friction. There is no regular trend of the residual sliding with the coefficient of friction. However the maximum amount of sliding increases with decrease in the coefficient of friction.

Overturning of objects does not occur even for an object of low stability having base width to height ratio of 0.20. It is seen that the amount of rotation depends not only on the peak ground acceleration but its frequency content also, that is, the duration of time for which the acceleration peak acts is quite important. Hence the quantity of interest for overturning of objects becomes the area under the acceleration peak or 'the impulse' given to the body.

3. It is also seen that one peak acceleration pulse of the accelerogram alone is not important but the train of peaks in succession are apparantly significant in contributing to the sliding as well as the overturning motion.

4. Only one component of Koyna earthquake motion was considered in the analysis presented in Chapter V. Under the combined action of the two horizontal components of ground motion together with the vertical component, it is prob-able that significantly large displacements and complete overturning of objects may occur. The present analysis is to be extended to include the effect of the vertical component of ground motion.

APPENDIX - I

TABLE - 3.1

(Response for single rectangular pulse)

S.No.	Ag (cm/sec²)	t_o (seconds)	μ or N	U_o (cm)	V_o (cm/sec)	U_m (cm)
1.	2.g	0.1	0.3	8.35	167.0	56.00
2.	1.g	0.2	0.3	13.73	137.3	46.00
3.	0.5g	0.4	0.3	15.70	78.5	26.00

TABLE - 3.2

(Response for Single Triangular Pulse)

S.No.	A	t_o	N	U_o	V_o	U_m
1.	4.00	0.05	0.3	2.1	83.3	14.00
2.	2.00	0.1	0.3	3.53	96.00	11.79
3.	1.00	0.2	0.3	4.78	43.5	8.00
4.	0.50	0.4	0.3	3.53	-1.95	0.0065

TABLE - 3.3

(Comparision of Results for Double Triangular ground) Acceleration)

S.No.	t (sec)	By Eq. 3.1 to Eq. 3.8		By Numerical Integration	
		U (cm)	V (cm/sec)	U (cm)	V (cm/sec)
1.	0.20	14.1656	139.302	14.1644	139.547
2.	0.30	23.3478	12.017	23.3776	12.017
3.	0.30547	23.4150	0.00	23.405	0.00
4.	0.40	19.2003	-59.846	19.210	-59.810

TABLE - 4.1

(Sliding of Objects on Shake Table)

S.No.	Fig. No.	Coeff. of friction (μ)	Displacements (cm)	
			Practical	Theorital
1.	4.7	0.21	2.00	21.862
2.	4.7	0.21	1.80	3.235
3.	4.8	0.25	3.15	-
4.	4.8	0.25	2.50	4.840
5.	4.9	0.31	2.70	2.624
6.	4.9	0.31	2.20	5.488
7.	4.10	0.35	4.65	5.170
8.	4.10	0.35	4.00	9.195
9.	4.11	0.40	5.1	8.346
10.	4.11	0.40	3.10	3.112
11.	4.12	0.404	1.5	1.524
12.	4.12	0.404	4.60	2.206
13.	4.13	0.45	1.80	1.938
14.	4.13	0.45	1.40	4.404
15.	4.14	0.55	3.60	2.302
16.	4.14	0.55	3.50	3.248 1.99 Δ

Δ Table motion corrected for base line.

TABLE - 5.1

(Sliding of Objects due to Koyna Earthquake)

S.No.	μ	Displacements (cm)	
		Maximum value	Residual value
1.	0.05	10.90	10.03
2.	0.10	3.50	0.021
3.	0.20	0.47	0.28
4.	0.30	0.128	0.11
5.	0.40	0.033	0.033
6.	0.50	0.006	0.006
7.	0.60	0.00011	0.00011

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APPENDIX -III

COMPUTER PROGRAMMES

```
C  C  SLIDING  OF  RIGID  OBJECTS  DURING  EARTHQUAKES
      DIMENSION A(600),T(600)
      DIMENSION ZS(40),TJ(40),VS(40)
5     FORMAT(2X,3HFT=F10.2,2X,3HFA=F10.2,2X,2HG=F10.4)
6     FORMAT(2X,2HF=F10.4,2X,3HTD=F10.4)
16    FORMAT(8F10.0)
17    FORMAT(I3,4(F8.4,F9.6))
19    FORMAT(3I3)
20    FORMAT(2X,14HBEGINING TIME=F8.4,4X,6HACCLN=F9.2,4X,6HT(JX)=F8.4)
50    FORMAT(4X,4HTIME,8X,4HDISP,10X,4HVELO,8X,4HTIME,8X,4HDISP,10X,
     14HVELO)
55    FORMAT(2(1X,F10.6,E14.6,E14.6))
      C6=1./6.
23    READ19,NCD
      NO1=4*NCD
      NO=NO1-1
      DO21I=1,NCD
      J1=4*I
      J=J1-3
      READ17,NC,(T(K),A(K),K=J,J1)
21    CONTINUE
      READ16,FT,FA,G
      IT=NO1
      DO22K=1,IT
      T(K)=T(K)*FT
      A(K)=A(K)*FA*G
22    CONTINUE
      READ16,F,TD
      READ19,NK
      PUNCH5,FT,FA,G
      PUNCH6,F,TD
      AJ=-10000.
      AO=F*G
      IX=0
      KK=1
      ZS(1)=0.
      VS(1)=0.
```

```
      TJ(1)=T(IT)
      Z=0.
31    IX=IX+1
      IF(IT-IX)28,29,29
29    AN=ABS(A(IX))
      IF(AN-AO)31,31,32
32    IF(A(IX))33,34,34
33    S=-1.
      GO TO 35
34    S=1.
35    JX=IX-1
      DT=(T(IX)-T(JX))*(S*AO-A(JX))/(A(IX)-A(JX))
      SQ=-1.
         TQ=T(JX)+DT
         AQ=S*AO
      PUNCH20,TQ,AQ,T(JX)
      TJ(KK)=TQ
      V=0.
      VG=0.
      VO=0.
      DO26I=JX,NO
99    SS=-1.
      L=I+1
      IF(SQ)81,82,82
81    TT=T(L)-TQ
      GO TO 83
82    TT=T(L)-T(I)
83    NA=TT/TD
      N=NA+1
      AN=N
      DT=TT/AN
      IF(SQ)84,85,85
84    SL=(A(L)-AQ)/AN
      AK=AQ
      GO TO 86
85      SL=(A(L)-A(I))/AN
      AK=A(I)
86    J=1
38    IF(N-J)27,37,37
37    DVON=DT*S*AO
      VON=VO+DVON
      DVGN=0.5*DT*(2.*AK+SL)
```

```
      VGN=VG+DVGN
      VN=V+DVON-DVGN
      ZN=Z+0.5*DT*(VO+VON)-VG*DT-C6*DT*DT*(3.*AK+SL)
      PVV=VN*V
      IF(PVV)51,25,25
51    IF(SS)52,53,53
52    DT=0.2*DT
      SS=1.
      N=5
      AJ=J-1
      J=1
      SL=0.2*SL
      GO TO 38
53    DDT=V*DT/(V-VN)
      SQ=-1.
      BJ=J-1
      CJ=(AJ*5.+BJ)*DT+DDT
        TQ=T(I)+CJ
      AQ=A(I)+SL*CJ/DT
      KK=KK+1
      VS(KK)=0.
      Z=Z+0.5*V*DDT
      ZS(KK)=Z
      TJ(KK)=TQ
      AN=ABS(AQ)
      V=0.
      VO=0.
      VG=0.
      IF(AN-AO)60,60,98
98    PRD=AQ*SL
      IF(PRD)60,60,97
97    IF(AQ)77,77,78
77    S=-1.
      GO TO 99
78    S=1.
      GO TO 99
60    IX=I
      KK=KK+1
```

```
      ZS(KK)=ZS(KK-1)
      VS(KK)=0.
      GO TO 31
25    V=VN
      Z=ZN
      VO=VON
      VG=VGN
      KK=KK+1
      ZS(KK)=Z
      VS(KK)=VN
      TK=J
      IF(SS)91,92,92
92    TJ(KK)=(AJ*5.+TK)*DT+T(I)
      GO TO 93
91    IF(SQ)94,95,95
94    TJ(KK)=TK*DT+TQ
      GOTO93
95    TJ(KK)=TK*DT+T(I)
93    J=J+1
      AK=AK+SL
      IF(KK-30)38,69,69
69    PUNCH 50
      K1=KK-1
      IF(NK-1)70,70,75
75    AKK=KK/NK
      KQ=AKK-1.
      DO76ID=1,KQ
      IE=ID*NK
      ID1=ID+1
      TJ(ID1)=TJ(IE)
      ZS(ID1)=ZS(IE)
      VS(ID1)=VS(IE)
76    CONTINUE
      K1=ID1
70    PUNCH 55,(TJ(IE),ZS(IE),VS(IE),IE=1,K1)
      PUNCH55,TJ(KK),ZS(KK),VS(KK)
      KK=0
      GO TO 38
27    SQ=1.
26    CONTINUE
28    PUNCH 50
      PUNCH55,(TJ(I),ZS(I),VS(I),I=1,KK)
      GO TO 23
      END
```

```
C  C  OVERTURNING  OF  RIGID  OBJECTS  DURING  EARTHQUAKES
      DIMENSION A(600),T(600),ZS(40),TJ(40)
10    FORMAT(2X,3HFT=F10.2,2X,3HFA=F10.2,2X,2HG=F10.2,2X,3HTD=F10.4)
11    FORMAT(2X,2HR=F10.4,2X,2HH=F10.4,2X,2HC=F10.4,2X,3HUN=F10.4,2X,
     13HVL=F10.4)
16    FORMAT(8F10.0)
17    FORMAT(I3,4(F8.4,F9.6))
19    FORMAT(I3)
20    FORMAT(2X,14HBEGINING TIME=F8.4,4X,6HACCLN=F9.2,4X,6HT(JX)=F8.4)
41    FORMAT(2X,37HOBJECT OVERTURNED.ROTATION=90 DEGREES)
50    FORMAT(6X,2HTM,11X,2HRT,11X,2HTM,11X,2HRT,11X,2HTM,11X,2HRT)
54    FORMAT(1X,31HTIMEATWRONGLYNEGATIVE VELOCITY=F8.4,8X,9HVELOCITY=
     1E16.8)
55    FORMAT(3(1X,F10.6,E15.6))
      C6=1./6.
23    READ19,NCD
      NO1=4*NCD
      NO=NO1-1
      DO21I=1,NCD
      J1=4*I
      J=J1-3
      READ17,NC,(T(K),A(K),K=J,J1)
21    CONTINUE
      READ16,FT,FA,G
      DO22K=1,NO1
      T(K)=T(K)*FT
      A(K)=A(K)*FA*G
22    CONTINUE
      READ16,R,H,C,TD,UN,VL
      READ19,NK
      PUNCH10,FT,FA,G,TD
      PUNCH11,R,H,C,UN,VL
      AJ=-10000.
      AO=R*G/H
      IX=0
      KK=1
      ZS(1)=0.
      TJ(1)=T(NO1)
31    IX=IX+1
      IF(NO1-IX)28,29,29
29    AN=ABS(A(IX))
      IF(AN-AO)31,31,32
32    IF(A(IX))33,34,34
33    S=-1.
      GO TO 35
34    S=1.
```

```
35     JX=IX-1
       DT=(T(IX)-T(JX))*(S*AO-A(JX))/(A(IX)-A(JX))
       SQ=-1.
          TQ=T(JX)+DT
          AQ=S*AO
       PUNCH20,TQ,AQ,T(JX)
       ZS(KK)=0.
       TJ(KK)=TQ
       Z=0.
       V=0.
       DO26I=JX,NO
99     SS=-1.
       L=I+1
       IF(SQ)81,82,82
81     TT=T(L)-TQ
       GO TO 83
82     TT=T(L)-T(I)
83     NA=TT/TD
       N=NA+1
       AN=N
       DT=TT/AN
       IF(SQ)84,85,85
84     SL=(A(L)-AQ)/AN
       AK=AQ
       GO TO 86
85       SL=(A(L)-A(I))/AN
       AK=A(I)
86     D=DT/C
       SL2=SL*0.5
       J=1
38     IF(N-J)27,37,37
37     CO=COS(Z)
       SI=SIN(Z)
       B1=D*(S*AK*(R*SI+H*CO)-G*(R*CO-H*SI))
       ZZ=Z+0.5*DT*V
       CO=COS(ZZ)
       SI=SIN(ZZ)
       B2=D*(S*(AK+SL2)*(R*SI+H*CO)-G*(R*CO-H*SI))
       ZZ=Z+0.5*DT*(V+0.5*B1)
       CO=COS(ZZ)
```

```
      SI=SIN(ZZ)
      B3=D*(S*(AK+SL2)*(R*SI+H*CO)-G*(R*CO-H*SI))
      ZZ=Z+DT*(V+0.5*B2)
      CO=COS(ZZ)
      SI=SIN(ZZ)
      B4=D*(S*(AK+SL)*(R*SI+H*CO)-G*(R*CO-H*SI))
      VN=V+C6*(B1+2.*B2+2.*B3+B4)
      ZN=Z+DT*V+C6*DT*(B1+B2+B3)
      IF(SS)71,72,72
72    IF(J-5)71,73,73
73    IF(ZN)71,74,74
74    EJ=(AJ+1.)*DT*5.
      SQ=-1.
      TQ=T(I)+EJ
      AQ=A(I)+SL*EJ/DT
      GO TO 99
39    PUNCH41
      GO TO 28
71    IF(ZN-1.57)40,40,39
40    IF(ZN)51,25,25
51    IF(SS)52,53,53
52    DT=0.2*DT
      D=0.2*D
      SS=1.
      N=5
      AJ=J-1
      J=1
      SL=0.2*SL
      GO TO 38
53    DDT=Z*DT/(Z-ZN)
      V=V+(VN-V)*DDT/DT
      Z=0.
      V=-V*UN
      SQ=-1.
      S=-1.
      BJ=J-1
      CJ=(AJ*5.+BJ)*DT+DDT
        TQ=T(I)+CJ
      AQ=A(I)+SL*CJ/DT
      KK=KK+1
      ZS(KK)=Z
      TJ(KK)=TQ
      IF(V)56,57,57
56    PUNCH54,TQ,V
      V=-V
57    IF(ABS(V)-VL)58,99,99
58    V=0.
      AN=ABS(AQ)
      IF(AN-AO)60,60,98
98    PRD=AQ*SL
      IF(PRD)60,60,97
97    IF(AQ)77,77,78
77    S=-1.
```

```
      GO TO 99
78    S=1.
      GO TO 99
60    IX=I
      KK=KK+1
      GO TO 31
25    V=VN
      Z=ZN
      KK=KK+1
      ZS(KK)=Z
      TK=J
      IF(SS)91,92,92
92    TJ(KK)=(AJ*5.+TK)*DT+T(I)
      GO TO 93
91    IF(SQ)94,95,95
94    TJ(KK)=TK*DT+TQ
      GOTO93
95    TJ(KK)=TK*DT+T(I)
93    J=J+1
      AK=AK+SL
      IF(KK-30)38,69,69
69    PUNCH50
      K1=KK-1
      IF(NK-1)70,70,75
75    AKK=KK/NK
      KQ=AKK-1.
      DO76ID=1,KQ
      IE=ID*NK
      ID1=ID+1
      TJ(ID1)=TJ(IE)
      ZS(ID1)=ZS(IE)
76    CONTINUE
      K1=ID1
70    PUNCH 55,(TJ(IE),ZS(IE),IE=1,K1)
      PUNCH55,TJ(KK),ZS(KK)
      KK=0
      GO TO 38
27    SQ=1.
26    CONTINUE
28    PUNCH50
      PUNCH55,(TJ(I),ZS(I),I=1,KK)
      GO TO 23
       END
```