STRESS ANALYSIS OF CONCRETE GRAVITY DAMS

A Dissertation
submitted in partial fulfilment
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WITH SPECIALIZATION IN STRUCTURAL DYNAMICS

By
M. K. DIXIT

DEPARTMENT OF EARTHQUAKE ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE (INDIA)
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C E R T I F I C A T E

Certified that the thesis entitled "STRESS ANALYSIS OF CONCRETE GRAVITY DAMS" which is being submitted by Sri Mahendra Kishore Dixit in partial fulfilment for the award of the degree of Master of Engineering in 'STRUCTURAL DYNAMICS' Earthquake Engineering of the University of Roorkee, Roorkee is a record of Students' own work carried out by him under our supervision and guidance. The matter embodied in this thesis has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of 9 months from January 1971 to October 1971 for preparing this thesis for Master of Engineering degree at this university.

S. S. Saini

(S.S. Saini)
Lecturer in Structural dynamics, Department of Earthquake Engineering, University of Roorkee, Roorkee.

(A.R. Chandrasekaran)
Professor in Structural dynamics, Department of Earthquake Engineering, University of Roorkee, Roorkee.

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SYNOPSIS

Several methods are available for stress analysis of concrete gravity dams. In this dissertation a comparative study of a few of these methods, namely, (a) Davis method (1) and (b) USBR method (2), has been made. The stresses as obtained by these methods are also compared with theory of elasticity solution of Zienkiewicz(3) for a triangular dam section and with the finite element stresses for Koyna dam(15) under static forces.

Static and dynamic forces have been considered in this study. The dynamic forces are treated as "equivalent static forces". Three different variations of the seismic coefficient along the height of the dam have been studied with a view to investigate the most appropriate variation to be considered in stress analysis of concrete gravity dams. Dams of different heights and different upstream and down-stream slopes have been analysed.

The study herein indicates that the stress distributions as obtained by the Davis and the USBR methods differ considerably from both the theory of elasticity(3) and the finite element analysis (15) for

the regions at and near the base of the dam where the width of the dam is large. For hydrostatic pressure, the Davis and the USBR methods give different stresses as compared to those obtained by the finite element analysis (15). This difference is more significant at and near the base of the dam. Under static forces the theory of elasticity and the finite element method indicate a definite tension at the U/s face on the base. This fact is not revealed by the Davis and the USBR methods. The Davis and the USBR methods differ significantly from each other only at and near the base of the dam.

The varying seismic coefficient is found suitable for stress analysis. A linear variation of seismic coefficient may be used for preliminary design purposes. Under lateral forces a stress concentration is observed at the elevations of the dam where the slope of the dam face changes abruptly.

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CHAPTER I

INTRODUCTION

Design and stability analysis of concrete gravity dams require the knowledge of the stress distribution within the dam. The state of stress at any point within the dam is completely defined if the three components of the stress namely, normal vertical stress (σy), normal horizontal stress (σx) and shear stress (τxy) are known.

Several methods are available for computation of these stress components (σy, σx and τxy). It is aimed in this thesis to carry out a comparative study of a few of these methods namely, Davis method (1) and USBR method (2). The static stresses as obtained by these methods are compared with the theory of elasticity solution (3) and the finite element solution (15) for triangular dam and Koyna dam respectively. The theory of elasticity method for stress analysis is expected to give exact results as it satisfies the compatibility of stress and strain besides satisfying the boundary conditions everywhere within the dam and the foundation zone. But the effort involved in it is too much for each dam section. And therefore, this method is generally not used for stress analysis and design purposes. The finite element method is expected to give results fairly close to the theory of elasticity solution. This method can account for any general variation in geometry and material properties. Too much effort is involved in developing a general digital computer programme for this method.

Dams of different heights have been included in the present study. The stress analysis is carried out for static and dynamic forces. The dynamic forces are considered as equivalent static forces. The static forces consist of the self weight, the hydrostatic pressure and the uplift pressure. The dynamic forces consist of the hydrodynamic pressure and the inertial force due to the acceleration of the mass of the dam. Three different variations of seismic coefficient along the height of the dam have been studied.

Chapter II of this thesis deals with the methods of stress analysis. The Davis, the USBR and the finite element methods are discussed in detail while theory of elasticity approach is briefly discussed.

Chapter III deals with the evaluation of loads. Sectional geometry of the dams analysed is also given.

Results are discussed in Chapter IV and finally the conclusions are given in Chapter V.

The study herein indicates that the Davis and the USBR methods give significantly different stress distributions from those obtained by the theory of elasticity method (3) and the finite element analysis (16) for the regions at and near the base of the dam. For regions near the top of the dam all the methods of stress analysis give similar stress distributions. For the hydrostatic pressure, the stresses as obtained by the Davis and the USBR methods are much different from the finite element stresses (16) particularly at and near the base of the dam. Further, under static forces the finite element analysis and the theory of elasticity method indicate a definite tension at the U/S face on the base. This fact is not revealed by the Davis and the USBR methods. The Davis and the USBR methods give similar stress distribution except for the regions at and near the base of the dam.

The varying seismic coefficient based on the dynamic analysis is found suitable for stress analysis. However, for preliminary stress analysis a linear variation of seismic coefficient may be used. Under lateral forces a concentration of stresses is observed at the elevations of the dam where the slope of the dam face changes abruptly.

CHAPTER II

METHODS OF STRESS ANALYSIS

2.1 DAVIS METHOD (1)

This method presents a step by step computation of the three components of stress, namely normal vertical stress σy, shear stress (τxy) and normal horizontal stress σx.

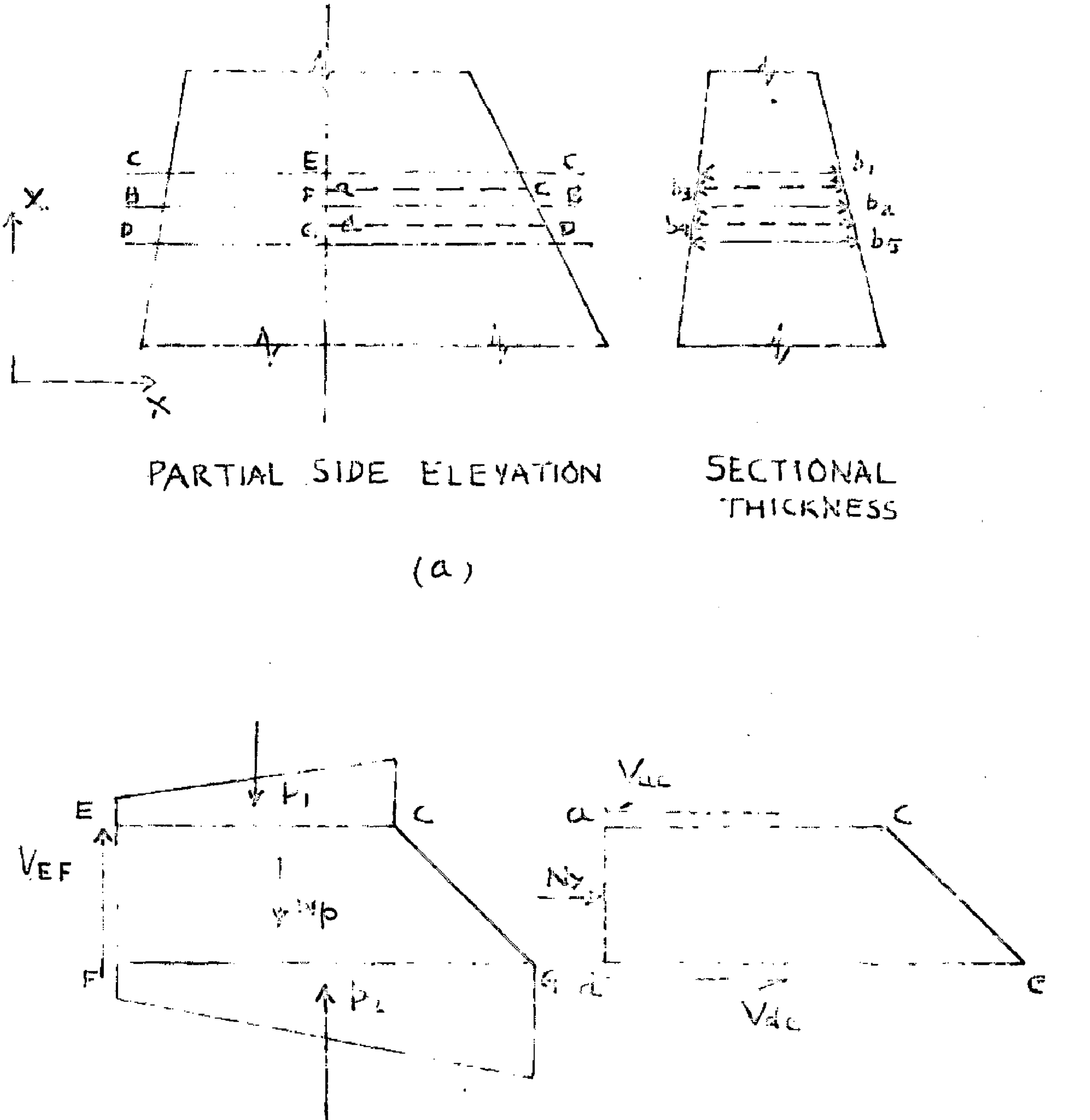
The concrete gravity dam is considered as a vertical cantilever element of unit thickness (varying thickness in case of Buttress dams). This method assumes linear variation of normal stress on any horizontal plane of the dam.

Based on this assumption the normal vertical stress (σy), under direct force and a bending moment combined, is given by

$$\sigma y_{1,\ 2} = \frac{W}{A} \pm \frac{M}{I} y \qquad \ldots \qquad \ldots \qquad (2.1.1)$$

where,

σy = Normal stress on any horizontal plane of the dam. Suffixes 1, 2 refer to the end points on U/S and d/s faces on that plane.



(b) VERTICAL EQUILIBRIUM OF BLOCK ECFG

(c) HORIZONTAL EQUILIBRIUM OF BLOCK acde

FIGURE _ 1

W = Total vertical force above the plane under consideration.

A = Area of the plane under consideration

I = Moment of Inertia of the horizontal plane (section) about its c.g.

M = Moment of all the forces acting above the plane under consideration and about the c.g. of this plane.

y = Distance of most remote fibre from the c.g. of the plane under consideration

In order to compute shear stresses and normal horizontal stresses, small prismoidal elements are separated from the dam by dividing the section with vertical and horizontal planes. Then the equilibrium of these elements is maintained under unknown stress functions. Equilibrium conditions enable to determine the shear stress τ_{xy} and normal horizontal stress σ_x.

Let the stresses are required at point F in plane BB of the dam shown in fig. 1. Draw planes CC and DD above and below the plane BB at equal distances. Next construct a vertical plane cutting these horizontal planes in E, F and G. The elementary prism ECFB may now be visualised as separated from the dam and to be held in equilibrium by forces acting on it. Normal stress σ_y can be calculated for planes CC, BB and DD by equation (2.1.1).

Consider the vertical equilibrium of the elementary prism ECFB. The vertical forces acting on it are,

1. Total normal stress on face EC $= p_1$ (say)
2. Total normal stress on face FB $= p_2$ (say)
3. Weight of block ECFB $= W_p$ (say)
4. Total shear across plane EF $= V_{EF}$ (say)

Total stresses p_1 and p_2 are given by,

$$p_1 = \sum_{C}^{E} b_1 \sigma_y \Delta_x$$

$$p_2 = \sum_{B}^{F} b_2 \sigma_y \Delta_x$$

The summation can be easily done, as the variation of σ_y is linear. The equilibrium of forces demand that,

$$V_{EF} = p_1 + W_p - p_2 \quad \ldots \quad \ldots \quad (2.1.2)$$

∴ Average intensity of shear on plane EF

$$V_1 = \frac{V_{EF}}{(b_3 \times EF)}$$

Similarly equilibrium of prism FBDG will give average shear intensity on plane FG, as

$$V_2 = \frac{V_{FG}}{(b_4 \times FG)}$$

Now shear stress at the point F in plane BB will be given as

$$\tau_{xy} = \frac{V_1 + V_2}{2} \quad \ldots \quad \ldots \quad (2.1.3)$$

b_1, b_2, b_3, b_4 are sectional thicknesses as illustrated in fig. 1.

When the point F falls on the d/s face of the dam, only the equilibrium of the lower block is considered (prism FBDG) as the upper block at that point is absent. And therefore the stress (τ_{xy}) will be obtained by averaging over a single prismoidal element.

For computations of normal horizontal stress (σ_x), the prism acde (fig. 1) is separated from the dam and is maintained in horizontal equilibrium. As the horizontal and vertical shear stresses are equal, Total shear acting on plane ac = V_{ac}

$$= \sum_{c}^{a} b_3 \, \tau_{xy} \, \Delta_x$$

and, total shear acting on plane de = V_{de}

$$= \sum_{e}^{d} b_4 \, \tau_{xy} \, \Delta_x$$

The shear stresses on planes ac and de can be computed for all points as outlined earlier. Horizontal equilibrium demands that if N_x be the total horizontal force normal to ad and acting on (ad), then

$$N_x = V_{ac} - V_{de} \qquad \ldots \quad (2.1.4)$$

∴ Average intensity of horizontal stress on ad,

$$\sigma_x = \frac{N_x}{\text{sectional area between a and d}} \qquad \ldots \ (2.1.5)$$

For computations of stresses on base, the dam is assumed to extend in the foundation and normal vertical stress for two planes is calculated by eqn. (2.1.1). While the shear stresses and normal horizontal stresses are computed as outlined above.

PRINCIPAL STRESSES

Once the three components of stress σ_y, σ_x and τ_{xy} are known, the principal stresses can be computed by the formulae

$$\sigma_{1,2} = \frac{\sigma_y + \sigma_x}{2} \pm 1/2 \left[(\sigma_x - \sigma_y)^2 + 4 \tau_{xy}^2 \right]^{1/2} \qquad \ldots \quad (2.1.6)$$

and

$$\tan 2\alpha = \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \qquad \ldots \quad (2.1.7)$$

where,

σ_1 = Major principal stress

σ_2 = Minor principal stress

α = angle of principal stress from vertical measured clockwise.

Shape of the dam profile and the size of the elements (Dimension EF in fig. 1) influence the accuracy of this method. In the present study, the size of the block is kept as 1 foot vertical. The horizontal points on a plane are taken at a interval of 10' to 20'. A computer programme is written in FORTRAN language. Listing of this programme is given in Appendix B.

2.2 USBR METHOD OF STRESS ANALYSIS (2)

This method assumes a linear variation of normal stress and a parabolic distribution of shear stress on a horizontal section of the dam straight in plan. Derivation of stress equations is briefly discussed here. The notations and sign convention in this section are same as used in USBR Tech. Memorandum 607. Typical loading conditions are shown in fig. 2.1 and fig. 22.

DERIVATION OF STRESS EQUATIONS:

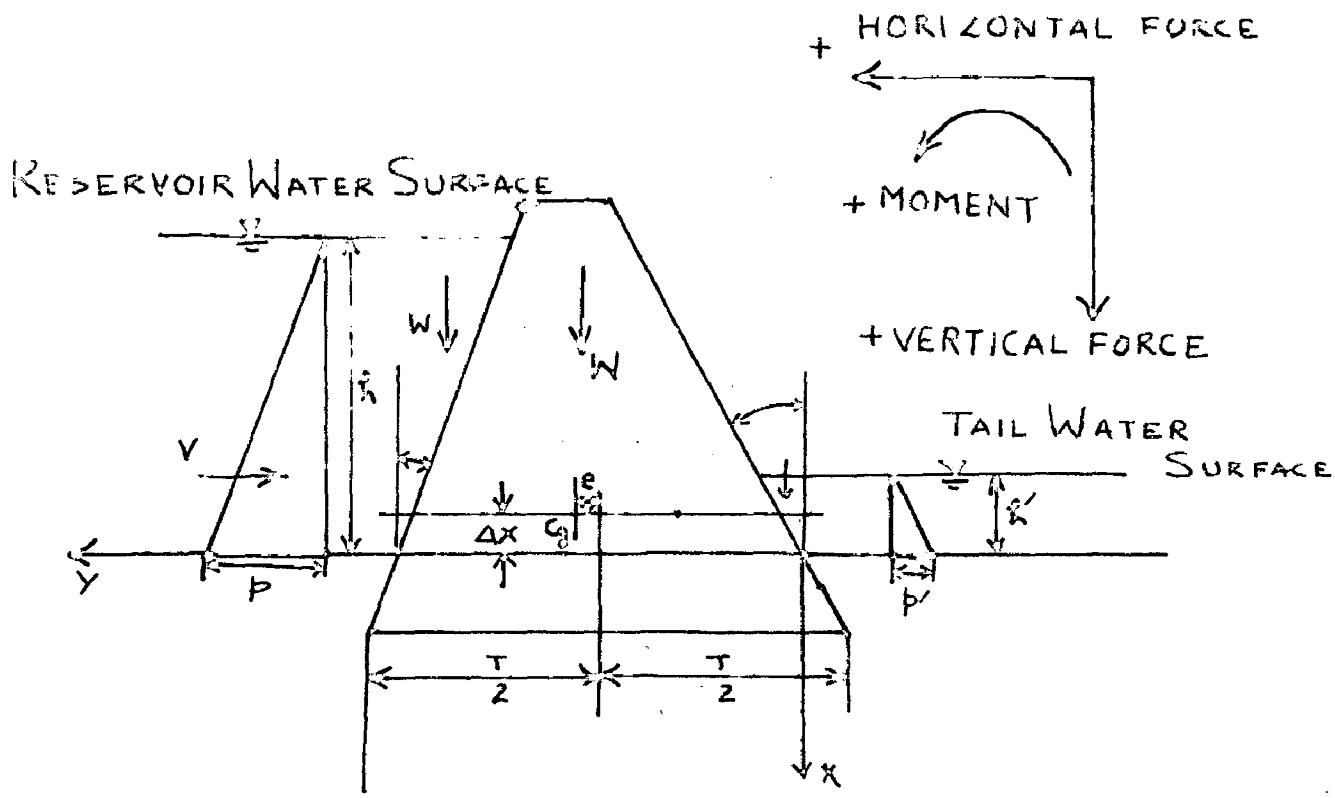
Normal Stress on Horizontal Plane -

Assumption of linear variation of normal stress on horizontal plane, yields in stresses due to bending and direct compression combined, as given by

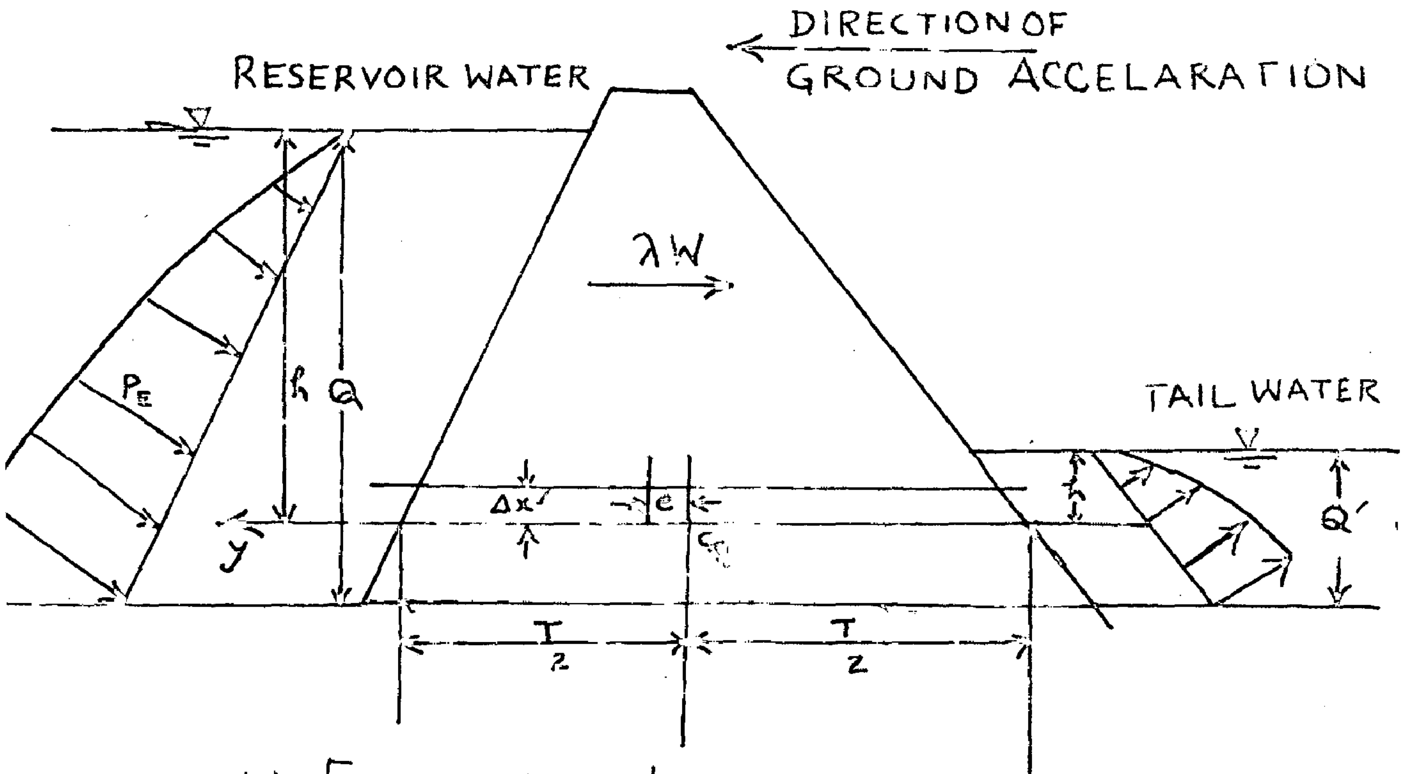
$$\sigma_x = \frac{\Sigma W}{A} \pm \frac{\Sigma M}{I} y \quad \ldots \quad (2.2.1)$$

where,

- σ_x = Normal stress on hors. section of dam.
- ΣW = Total vertical load, above section
- ΣM = Moment of all the forces acting above the section and about the c.g. of the section.
- I = Moment of Inertia of the section about its c.g.



(a) NORMAL LOADING CONDITION



(b) EARTHQUAKE LOADING CONDITION

FIGURE_2.1

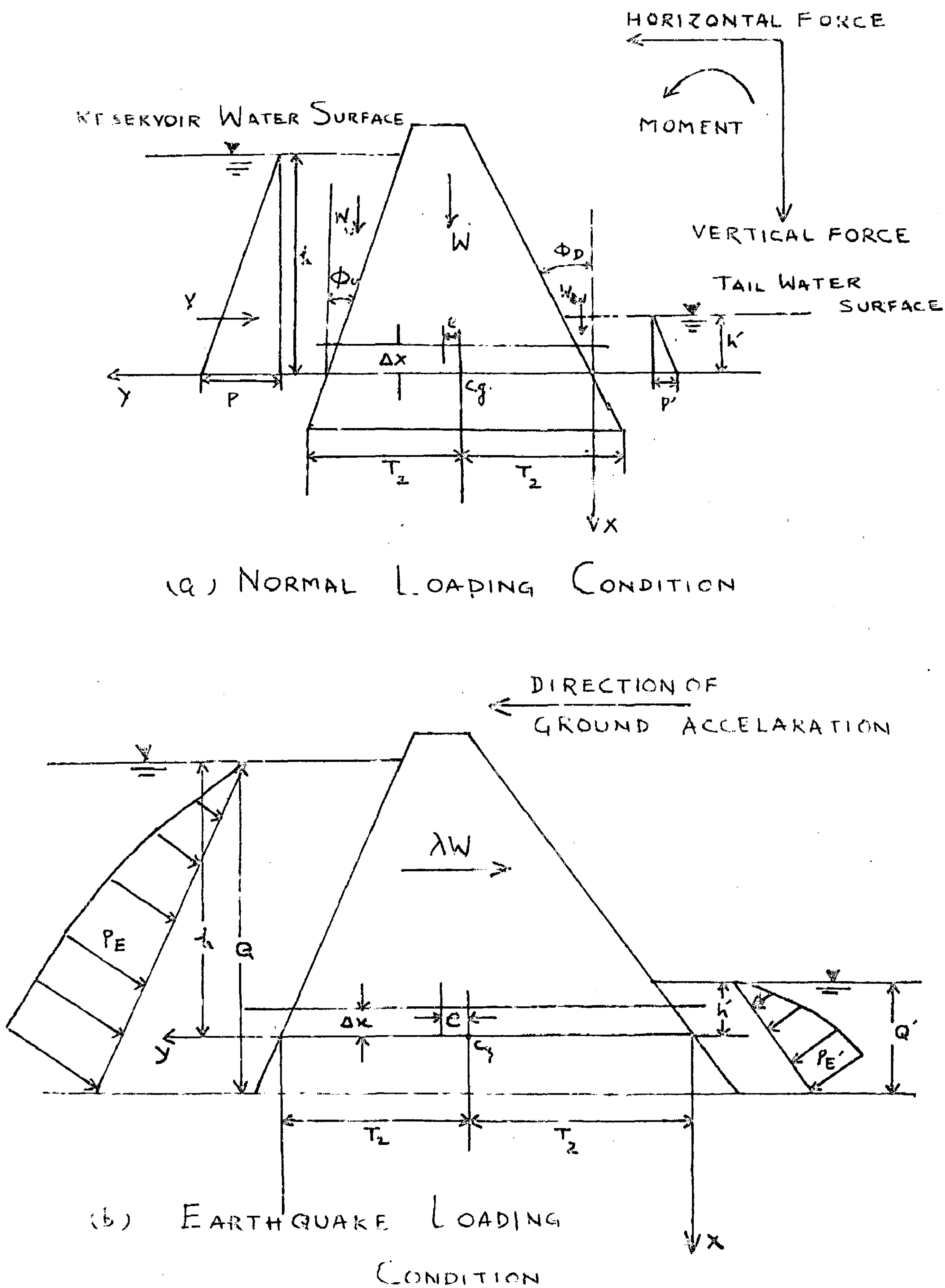
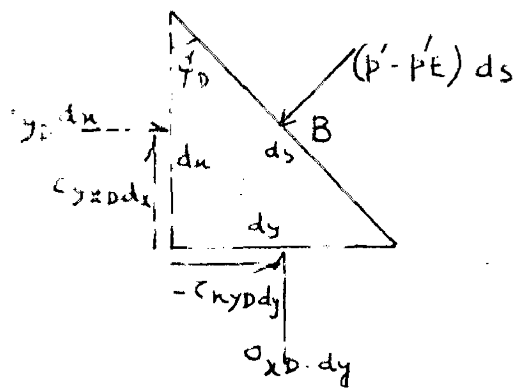
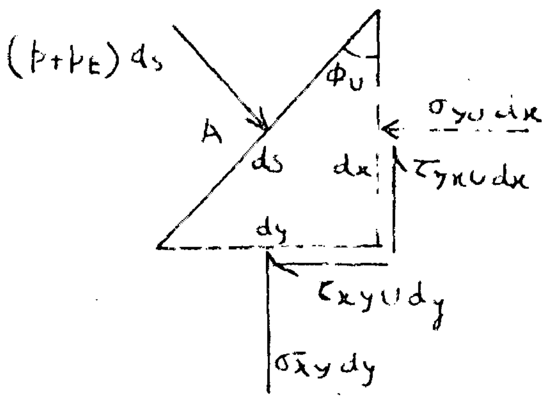

HORIZONTAL FORCE
MOMENT
VERTICAL FORCE
RESERVOIR WATER SURFACE
TAIL WATER SURFACE
W
Φu
ΦD
ΔX
c.g.
T2
T2
P
P'
h
h'
Y
X
(a) NORMAL LOADING CONDITION
DIRECTION OF GROUND ACCELARATION
λW
PE
PE'
Q
Q'
C
(b) EARTHQUAKE LOADING CONDITION

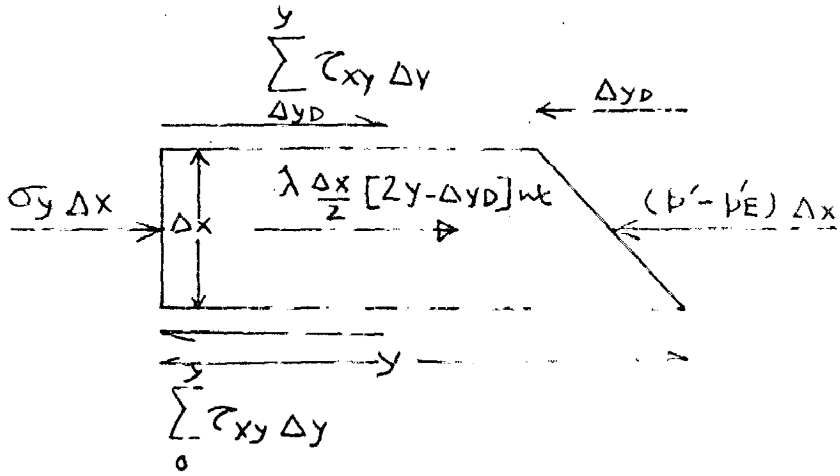
FIGURE _ 2.2



(a) DIFFERENTIAL ELEMENT AT D/S END



(b) DIFFERENTIAL ELEMENT AT U/S END



(c) ELEMENT BETWEEN TWO HORIZONTAL PLANE

FIGURE _ 3

y = Distance of most remote fibre from c.g. of the sections.

A = Sectional area

At D/S Face

$$\sigma_{xD} = \frac{\Sigma W}{A} - \frac{6\,\Sigma M}{T^2}$$

At U/S Face

$$\sigma_{x_U} = \frac{\Sigma W}{A} + \frac{6\,\Sigma M}{T^2}$$

∴ Stress at any interior point,

$$\sigma_x = \sigma_{x_D} + \frac{\sigma_{x_U} - \sigma_{x_D}}{T}\, y$$

$$= \sigma_{x_D} + \frac{12\,\Sigma M}{T^3}\, y$$

or σ_x = a + by ... ... (2.2.2)

where,

$$a = \sigma_{x_D}$$

$$b = \frac{12\,\Sigma M}{T^3}$$

Shear Stresses :

As the distribution is assumed to be parabolic, unit shear stress may be expressed by,

$$\tau_{xy} = a_1 + b_1 y + C_1\, y^2 \qquad \ldots \; (2.2.3)$$

When y = 0

Unit shear stress at D/S face

$$\tau_{xyD} = a_1$$

When y = T

Unit shear stress at U/S face

$$\tau_{xy_U} = a_1 + b_1 T + C_1 T^2$$

And total shear stress is given by

$\tau_{xy}\ dy$ = Resultant horizontal force above section.

$$= - \Sigma V.$$

These conditions yield,

$$a_1 = \tau_{xy_D}$$

$$b_1 = - \frac{1}{T}\left[\frac{6\Sigma V}{T} + 2\tau_{xy_U} + 4\tau_{xY_D}\right]$$

$$C_1 = + \frac{1}{T^2}\left[\frac{6\Sigma V}{T} + 3\tau_{xy_U} + 3\tau_{xy_D}\right]$$

To compute shear stresses at D/S and U/S face (τ_{xyU} and τ_{xyD}), consider the equilibrium of an element of the dam, separated from the corresponding face of the dam, as shown in fig. 3.

At U/S

$$\tau_{xyU} = \tau_{yxU}$$

Vertical equilibrium of the element, demands:

$$\tau_{yxU} = \tau_{xyU} = -(\sigma_{xU} - p - PE) \tan \phi U. \quad \ldots (2.2.4)$$

At D/S

Rotational equbm. about B, demands,

$$\tau_{xyD} = \tau_{yxD}$$

Vertical Eqbm. demands,

$$\tau_{xyD} = \left(\sigma_{xD} - p' + p'E\right) \tan \phi D \quad \ldots (2.2.5)$$

Normal Stresses on a Vertical Plane

Consider eqbm. of an element between two horizontal planes Δx distance apart as shown in fig. 3.

Horizontal Equilibrium demands

$$\sigma_y \; dx = \sum_{o}^{y} \tau_{xy} \, \Delta y - \lambda WC \frac{\Delta x}{2}(2y - \Delta yD) + (p'-p'E)\Delta x - \sum_{\Delta yD}^{y} \tau_{xy} \, \Delta y^{*}$$

* for functions for a horizontal plane Δx above the horizontal section under consideration. Substituting for xy; replacing (* functions) by (function - Δ functions) and neglecting differentials of 2nd and higher orders and simplifying we get

$$\sigma_y = a_2 + b_2 y + c_2 y^2 + d_2 y^3 \quad .. \quad (22.6)$$

where,

$$a_2 = (a_1 \tan \phi_D + p' - p'E)$$

$$b_2 = b_1 \tan \phi_D + \frac{\Delta a_1}{\Delta x} - \lambda W_c$$

$$c_2 = c_1 \tan \phi_D + 1/2 \quad \frac{\Delta b_1}{\Delta x}$$

$$d_2 = \frac{\Delta c_1}{3\Delta x}$$

Using $\frac{\partial}{\partial x} = \lim_{x \to 0} \frac{\Delta}{\Delta x}$

These constants can be expressed in terms of partial differentials.

Partial differentials are evaluated as (2)

1. $$\frac{\partial a_1}{\partial x} = \frac{\partial \tau_{xyD}}{x}$$

$$= \tan \phi D \left(\frac{\partial \sigma_{xD}}{\partial x} - W^* \pm^{\ddagger} \frac{\partial p'E}{\partial x} \right)$$

$$+ \frac{\partial \tan \phi D}{\partial x} \left(\sigma x_D - p' \pm^{\ddagger} p'E \right) \quad \ldots (2.2.7)$$

W^* - to be omitted if tail water is absent.

2. $$\frac{\partial \sigma_{xD}}{\partial x} = W_c + \tan \phi_U \left(\frac{12\Sigma M}{T^3} + \frac{2\Sigma W}{T^2} - \frac{2p}{T} \pm^{\ddagger} \frac{2pE}{T} \right)$$

$$+ \tan \phi_D \left(\frac{12\Sigma M}{T^3} - \frac{4\Sigma W}{T^2} + \frac{4p'}{T} \pm^{\ddagger} \frac{4p'E}{T} \right)$$

$$- \frac{6\Sigma V}{T^2} \quad \ldots (2.2.8)$$

3. $$\frac{\partial p'E}{\partial x} = \frac{p'E - p'E^*}{\Delta x} \quad \ldots \quad \ldots (2.2.9)$$

4. $$\frac{\partial \tan \phi D}{\partial x} = \frac{\tan \phi_D - \tan \phi^{x}D}{\Delta x} \quad \ldots \quad (2.2.10)$$

5. $$\frac{\partial b_1}{\partial x} = -\frac{1}{T^2}\left[\frac{6\,\partial\,\Sigma V}{\partial x} - \frac{\partial T}{\partial x}\left(\frac{12\Sigma V}{T} + 2\tau_{xyU} + 4\tau_{xyD}\right)\right]$$

$$- \frac{1}{T}\left[2\,\frac{\partial \tau_{xyU}}{\partial x} + 4\,\frac{\partial \tau_{xyD}}{\partial x}\right]$$

$$\ldots (2.2.11)$$

6. $$\frac{\partial\,\Sigma V}{\partial x} = -\left(p - p' \overset{\ddagger}{\pm} \lambda W_c T \overset{\ddagger}{\pm} pE + \overset{\ddagger}{p'}E\right) \ldots (2.2.12)$$

7. $$\frac{\partial T}{\partial x} = \tan \phi_U \mp \tan \phi_D \quad \ldots \quad (2.2.13)$$

8. $$\frac{\partial \tau_{xyU}}{\partial x} = \tan \phi_U \left[W^{*} - \frac{\partial \sigma_{xU}}{\partial x} \overset{\ddagger}{\pm} \frac{\partial pE}{\partial x}\right]$$

$$+ \frac{\partial \tan \phi U}{\partial x}\left(p \overset{\ddagger}{\pm} pE - \sigma_{xU}\right) \ldots (2.2.14)$$

ω^* to be omitted if resr. water is absent.

9. $$\frac{\partial \sigma_{xU}}{\partial x} = W_c + \tan \phi_U \left(\frac{4p}{T} \overset{\ddagger}{\pm} \frac{4pE}{T} - \frac{4\Sigma W}{T^2} - \frac{12\Sigma M}{T^3}\right)$$

$$+ \tan \phi_D \left(\frac{2\Sigma W}{T^2} \overset{\ddagger}{\pm} \frac{2p'E}{T} - \frac{2p'}{T} - \frac{12\Sigma M}{T^3}\right)$$

$$+ \frac{6\Sigma V}{T^2} \ldots (2.2.15)$$

10. $$\frac{\partial pE}{\partial x} = \frac{pE - pE^{*}}{\Delta x} \quad \ldots \quad \ldots (2.2.16)$$

11. $$\frac{\partial \tan \phi_U}{\partial x} = \frac{\tan \phi_U - \tan \phi^{*}_U}{\Delta x} \quad \ldots (2.2.17)$$

12. $$\frac{\partial C_1}{\partial x} = \frac{1}{T^3}\left[\frac{6\,\partial\,\Sigma U}{\partial x} - \frac{\partial T}{\partial x}\left(\frac{18\Sigma V}{T} + 6\,\zeta_{xyU} + 6\,\zeta_{xyD}\right)\right] + \frac{1}{T^2}\left[3\,\frac{\partial \zeta_{xyU}}{\partial x} + 3\left(\frac{\partial \zeta_{xyD}}{\partial x}\right)\right] \quad \ldots \quad (2.2.18)$$

In all terms preceeded by ± sign ; + sign is to be taken if the direction of horz. shock is U/S and vice-versa.

The effect of earthquake shock can be neglected by dropping all the terms preceeded by ± sign.

A computer programme in FORTRAN language has been written. Listing of the programme is given in Appendix 'B'.

Notations used

w_c	=	wt density of concrete
w	=	wt density of water
p	=	water pressure at section due to normal loading condition
λ	=	Horizontal seismic coeff.
p_E	=	change in water pressure due to Earthquake.
W	=	Dead wt of concrete above section
M	=	Moment of W about cg of section
W_w	=	Vertical component of water load

M_w	=	Moment due to W_w
W_{wE}	=	†Vertical component of water load change due to Earthquake.
M_{wE}	=	Moment of W_{wE}
V	=	Horizontal component of water load
M_p	=	Moment of V
V_E	=	Horizontal inertial force of concrete
M_E	=	Moment of V_E
V_{pE}	=	Change in horizontal component of water load due to Earthquake.
M_{pE}	=	Moment of V_{pE}
ΣM	=	$M + M_w + M'_w + M_p + M'_p \pm M_E \pm M_{wE} \pm M'_{wE} + M_{pE} \pm M'_{pE}$
ΣV	=	$V + V' \pm V_E \pm V_{pE} + V'_{pE}$
ΣW	=	$W + W_w + W'_w \pm W_{wE} \pm W'_{wE}$

† Take (+) if Horizontal accelaration is in u/s direction.

‡ Take (-) if Horizontal accelaration is in u/s direction.

(') represents the corrosponding quantities for tail water.

2.3 FINITE ELEMENT METHOD (6, 7, 8, 9, 10, 11, 12)

Finite element method is a powerful method of structural analysis wherein a continuous system is considered to be an assembage of discrete elements interconnected at a finite number of nodal points. Two dimensional plane stress behaviour of concrete gravity dams is assumed.

Basic assumptions

Continuity between adjacent elements of the system is maintained by requiring that within each element, lines initially straight remain straight in their displaced position. This requirement is satisfied if the strains ϵx, ϵy and γ are assumed to be constant within each element. Therefore the stresses that act on the edges of each element are also constant for triangular plate elements. These stresses are in turn replaced by stress resultants that act at the corners of elements.

The finite element method in general has three basic phases :

(1) Structural Idealisation :

This is merely subdivision of original system into segments of various sizes and shapes. In this dissertation triangular plate elements have been discussed. Forces acting on actual structure are replaced by equivalent static concentrated forces at the nodal points of the finite element systems.

2. Evaluation of Element properties :

This is usually the most complex phase of finite element analysis. Nodal forces and nodal stresses are expressed in terms of nodal deflections and these relationships are expressed in matrix form and are known as element stiffness matrix and element stress matrix respectively. This will be discussed later in this chapter.

3. Structural Analysis :

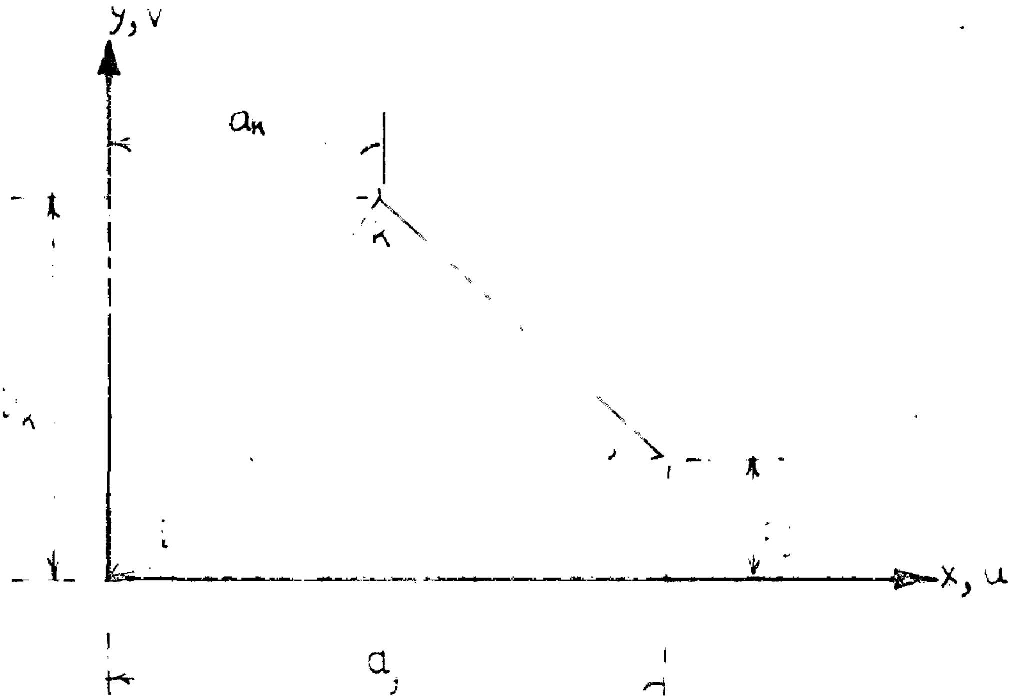
This includes determination of stiffness matrix and stress matrix for the entire assemblage. This is done by systematic superposition of element stiffness and stress matrices respectively.

Derivation of element Stiffness matrix and element stress matrix :

The derivations given here are based on Wilson's approach (6).

Strain - Displacement Relationship -

The three components of strains within each element are expressed in terms of six corner displacements.



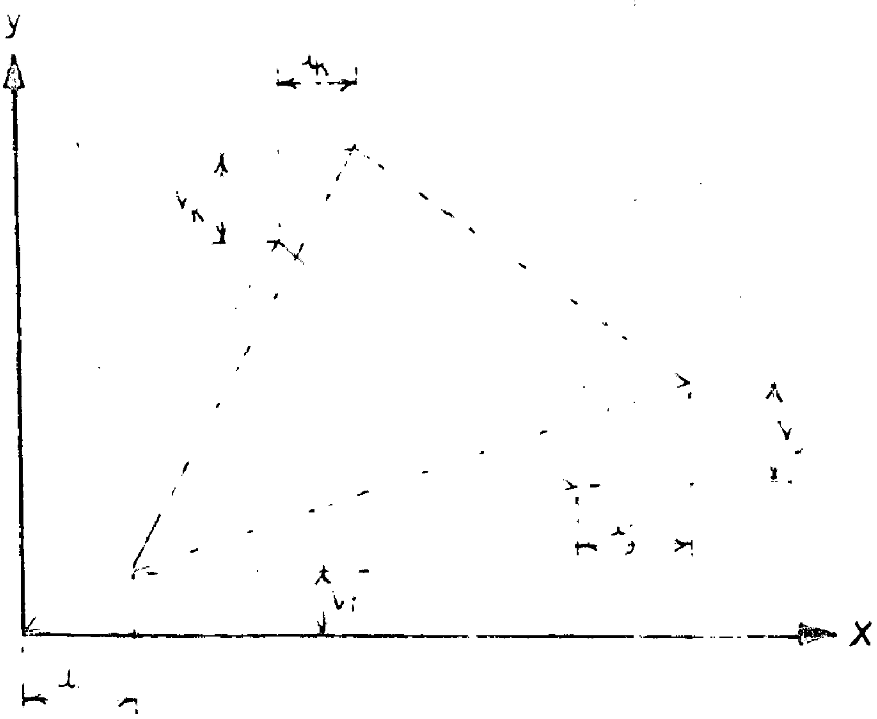


FIG. 4 – ELEMENT DIMENSIONS AND ASSUMED DISPLACEMENT PATTERN

Geometry of a typical element and assumed displacement field are illustrated in fig. (4). Linear displacement field is expressed as

$$\left.\begin{aligned} U &= U_i + C_1 x + C_2 y \\ V &= V_i + C_3 x + C_4 y \end{aligned}\right\} \quad \ldots (2.4.1)$$

where U and V are the deflections in x and y directions respectively. C_1, C_2, C_3, C_4 are arbitrary constants.

These constants C_1, C_2, C_3 and C_4 are expressed in terms of corner displacements and geometry of the element, as under

$$\begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \frac{1}{(a_j b_k - a_k b_j)} \begin{bmatrix} b_j - b_k & 0 & b_k & 0 & -b_j & 0 \\ a_k - a_j & 0 & -a_k & 0 & a_j & 0 \\ 0 & b_j - b_k & 0 & b_k & 0 & -b_j \\ 0 & a_k - a_j & 0 & -a_k & 0 & a_j \end{bmatrix} \begin{Bmatrix} U_i \\ V_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix}$$

...(2.4.2)

Strains within the element can be expressed in terms of displacements at nodes, by eqn. (2.4.1)

$$\begin{aligned} \epsilon_x &= \frac{\partial U}{\partial x} = C_1 \\ \epsilon_y &= \frac{\partial V}{\partial y} = C_4 \\ \gamma_{xy} &= \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} = C_2 + C_3 \end{aligned} \quad \ldots (2.4.3)$$

From equations (2.4.2) and (2.4.3), strains can be rewritten as

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{a_j b_k - a_k b_j} \begin{bmatrix} b_j - b_k & 0 & b_k & 0 & -b_j & 0 \\ 0 & a_k - a_j & 0 & -a_k & 0 & a_j \\ a_k - a_j & b_j - b_k & -a_k & b_k & a_j & -b_j \end{bmatrix} \begin{Bmatrix} U_i \\ V_i \\ U_j \\ V_j \\ U_k \\ V_k \end{Bmatrix}$$

... (2.4.4)

or in symbolic form

$$\{\epsilon\} = [A]\{U\}$$

Stress Strain Relationship

For elastic isotropic material the stress strain relationship for plane stress case is expressed as (4)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

or in symbolic form

$$\{\sigma\} = [C]\{\epsilon\} \qquad \text{... (2.4.5)}$$

where

E = modulus of elasticity of material

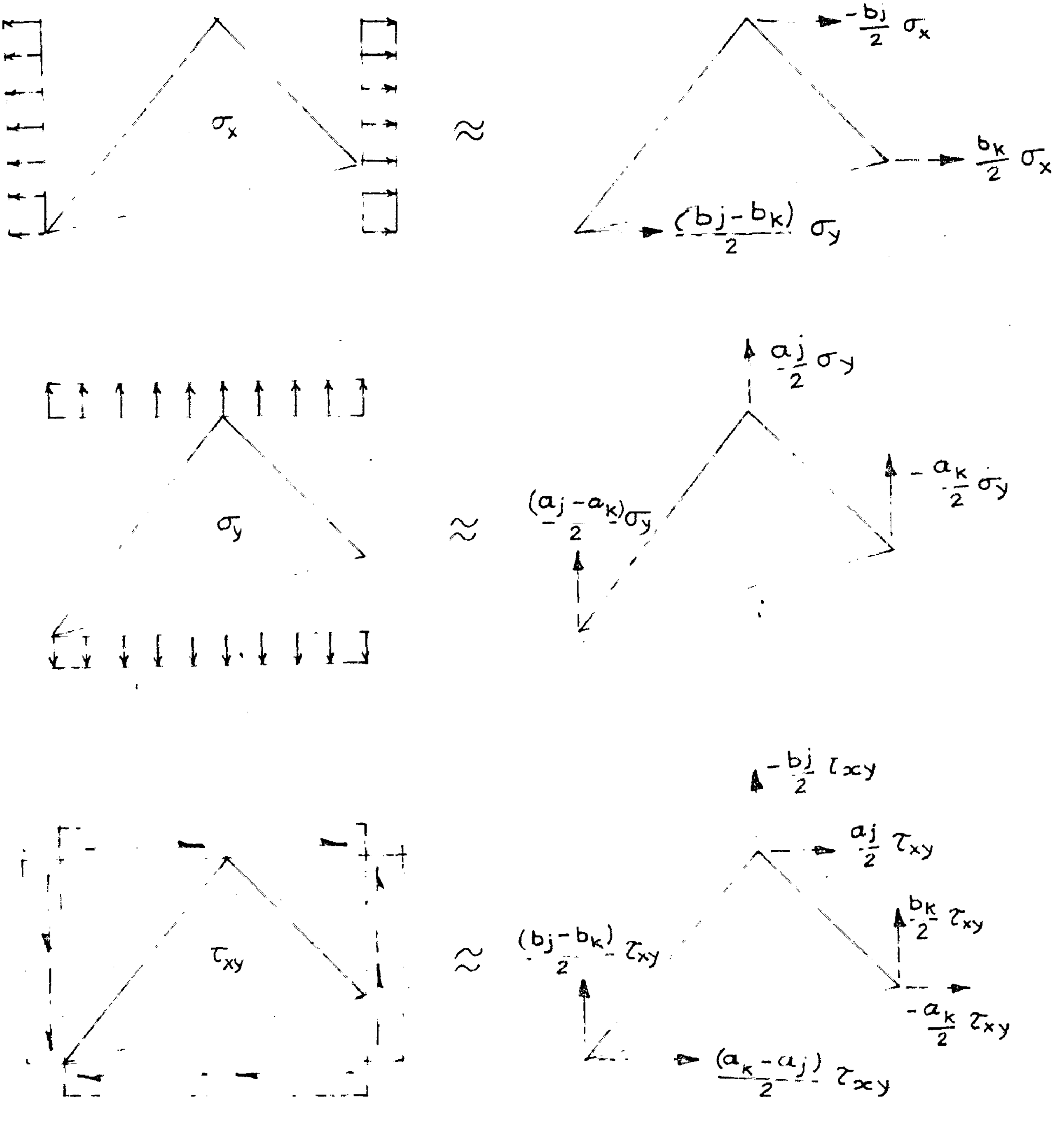


FIG. 5 _ STRESS RESULTANTS

ν = poissoins ratio

$\{\sigma\}$ = stress vector

$\{\epsilon\}$ = strain vector

$[C]$ = connecting matrix

Nodal Force and Stress Relationship :

Uniform strains along the edges of elements result in uniform stresses along the edges of element. These stresses are concentrated as stress resultants at the nodal points. Fig. 5 shows the stress resultants.

Stress resultants are expressed in terms of stresses for unit thickness of elements as,

$$\begin{Bmatrix} Sx^{(i)} \\ Sy^{(i)} \\ Sx^{j} \\ Sy^{j} \\ Sx^{k} \\ Sy^{k} \end{Bmatrix} = 1/2 \begin{bmatrix} bj-bk & 0 & ak-aj \\ 0 & ak-aj & bj-bk \\ bk & 0 & -ak \\ 0 & -ak & bk \\ -bj & 0 & aj \\ 0 & aj & -bj \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad \ldots (2.4.6)$$

or in symbolic form

$$\{S\} = [B]\{\sigma\}$$

where, S represents the stress resultant. The subscripts refer to the direction in which it is acting and superscripts refer to the nodal point at which it is acting.

It may be noted that the matrices A and B of eqns. (2.4.4) and (2.4.6) are transposed of each other, if the constants of each matrix are not considered.

Element stresses can be expressed in terms of nodal displacement by substituting equation (2.4.5) into equation (2.4.4), i.e.

$$\{\sigma\} = [C][A]\{U\}$$

$$\text{or} \quad \{\sigma\} = [S_e]\{U\} \qquad \ldots (2.4.7)$$

where,

$[S_e]$ = Element stress matrix

= $[C]\ [A]$

$\{U\}$ = nodal displacement vector

$\{\sigma\}$ = element stress vector

Substituting eqn. (2.4.7) into (2.4.6), we get a relation between nodal forces and nodal displacements, which is,

$$\{S\} = [B][C][A]\{U\} \qquad \ldots (2.4.8)$$

or

$$\{S\} = [K_e]\{U\}$$

where, $[K_e]$ = $[B][C][A]$ ... (2.4.9)

= stiffness matrix for an element.

Element stiffness matrix is a (6 x 6) matrix. The element stress matrix is a (3 x 6) matrix. Since the element stress matrix relates stresses within the element to the corner displacements, the nodal stresses can be expressed in terms of nodal displacements by a (9 x 6) nodal stress matrix. This nodal stress matrix is obtained from element stress matrix. It may be noted that the rows corresponding to each nodal points are identical.

Equilibrium equations for complete structure :

Nodal point loads can be expressed in terms of nodal point displacement for the entire assemblage, as

$$\{F\} = [K] \{U\} \quad \ldots \quad (2.4.10)$$

where,

$\{F\}$ = Nodal point load vector

$[K]$ = Stiffness for entire assemblage

$\{U\}$ = Nodal point displacement vector

The stiffness matrix for the entire assemblage is obtained by systematic superposition of element stiffness matrices. This addition can be illustrated if eqn. (2.4.8) is rewritten in terms of a typical element q

$$\begin{Bmatrix} S_i^{(q)} \\ S_j^{(q)} \\ S_k^{(q)} \end{Bmatrix} = \begin{bmatrix} K_{ii}^{(q)} & K_{ij}^{(q)} & K_{iK}^{(q)} \\ K_{ji}^{(q)} & K_{jj}^{(q)} & K_{jK}^{(q)} \\ K_{ki}^{(q)} & K_{kj}^{(q)} & K_{kk}^{(q)} \end{bmatrix} \begin{Bmatrix} U_i \\ U_j \\ U_k \end{Bmatrix} \quad \ldots (2.4.11)$$

where in terms of arbitrary nodal points l and m, $S_l^{(q)}$ and U_m are the vectors of the form

$$S_l^{(q)} = \begin{Bmatrix} S_x \\ S_y \end{Bmatrix}_l^{(q)} \quad \ldots (2.4.12)$$

$$U_m^{(q)} = \begin{Bmatrix} U_x \\ U_y \end{Bmatrix}_l^{(q)} \quad \ldots (2.4.13)$$

and the stiffness coefficient $K_{lm}^{(q)}$ is a (2 x 2) submatrix of the form

$$K_{lm}^{(q)} = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}_{lm}^{(q)} \quad \ldots (2.4.14)$$

This term $K_{lm}^{(q)}$ represents the forces developed on the element 'q' at nodal point 'l' due to unit displacement at nodal point 'm'. Therefore, the general term K_{lm} for the complete structure is given as

$$K_{lm} = \sum_{q} k_{lm}^{(q)} \quad \ldots \quad (2.4.15)$$

It may be pointed out that K_{lm} exists only if l equals m, or if l and m are adjacent nodal points in the physical system. Again the stiffness matrix for the entire assemblage is a sparse matrix.

<u>Boundary conditions</u>

For concrete gravity dams, the base is assumed to be fixed and therefore the displacements of the nodal points that lie on the base, are zero. These displacement restrains at nodal points on the base of the dam, are introduced in the total stiffness matrix. This is done by eliminating the rows and the columns corresponding to the restrained nodal points.

The modified stiffness matrix is a symmetric sparse matrix.

<u>Solution of Equilibrium Equations</u>

The force displacement relationship for the entire assemblage is given as,

$$[K]\{U\} = \{F\} \quad \ldots \quad (2.4.10)$$

Eqn. (2.4.10) represents a set of linear simultaneous equations. The order of these equations is usually very large. Several methods (13) are available to solve such a large system of equations. Iteration methods (6, 13, 16, 17, 18) have been discussed here.

When Ganss siedel technique (13) is applied to equation (2.4.10), it involves repeated calculation of new displacements from the equation (13, 6)

$$U_i^{(S+1)} = \frac{1}{K_{ii}} \left[Fi - \sum_{j=1}^{i} Kij\ Uj^{(S+1)} - \sum_{j=i+1}^{n} Kij\ Uj^{(S)} \right] \quad \ldots (2.4.16)$$

where, Ui refers to the i^{th} displacement, n is the number of unknowns and S refers to the cycle of iteration.

It may be pointed out that the modified stiffness matrix is real, symmetric and positive definite and hence the convergence of iteration techniques is guaranteed (13, 6). Also since the stiffness matrix is a sparse matrix the effort in solving such a system of large equations is reduced.

Since the stiffness matrix is a diagonally dominant matrix i.e., the diagonal terms are considerably greater than other terms, the initial gness for any displacement may be obtained as,

$$U_i = \frac{Fi}{K_{ii}} \quad \ldots \quad \ldots \quad (2.4.17)$$

A scheme for the acceleration of convergence, of the solution of the system of equations may also be incorporated. This scheme for fast convergence is applied after every few cycles of iteration.

Aietkin's and Luisterkin's schemes for accelerated convergence (13, 18) are discussed here.

Aietkin's Convergence Scheme (13)

This requires the information about the displacements during any three successive iterations. The accelerated value of any displacement is calculated as, (13)

$$U_{ti}^{(S)} = U_i^{(S)} - \frac{(U_i^{(S+1)} - U_i^{(S)})^2}{U_i^{(S+2)} - 2Ui^{(S+1)} + Ui^{(S)}}$$

$$\ldots \quad (2.4.18)$$

where superscfipts refer to the number of iteration cycles and subscripts refer to the i^{th} displacement. $Uti^{(S)}$ is the true value of U_i in S^{th} iteration.

Luiesterkin's Convergence Scheme (13, 18)

The accelerated value of any unknown (displacement) is calculated by (13),

$$Ut_i^{(S)} = Ui^{(S)} + \frac{Uj^{(S+1)} - Ui^{(S)}}{1 - \lambda} \quad \ldots (2.4.19)$$

where,

$$\lambda = \frac{1}{n} \sum_{j=1,n} \frac{Uj^{(S+1)} - Uj^{(S)}}{Uj^{(S)} - Uj^{(S-1)}}$$

n = total number of unknowns

S = number of iteration cycles.

Nodal point stresses

Nodal point stresses may be calculated by averaging the element stresses of all the elements connected to the nodal point. This direct averaging gives similar stresses, as obtained by taking a weighted average (10, 12). Accuracy of these stresses depend upon the fineness of the mesh considered in structural idealization (6, 12).

2.4 BRIEF OUTLINE OF THE METHOD BASED ON THEORY OF ELASTICITY (3)

This method assumes two-dimensional plane stress behaviour of the concrete gravity dam. In the Davis and the USBR methods of stress analysis, linear variation of normal stress on horizontal planes of the dam is assumed. This assumption is found incompatible with the theory of elasticity for the regions at and near the base of the dam (3). Therefore, a stress function (airy's stress function (4)) is so assumed that it satisfies the compatibility of stresses and strains and also the boundary conditions at the infinite limits of the plane of the foundation. .

The method essentially consists in evaluating the aforesaid stress function which is obtained in terms of partial differential equations (3). These equations are solved by the finite difference approximation (3,13). Details of this method are given in Appendix 'A'.

The method is expected to give exact results as it satisfies the compatibility, the equilibrium and the boundary requirements at all points within the dam and the foundation zone. However, the method is very tedious and involves much effort for each dam section. And therefore, it is generally not used for the stress analysis.

CHAPTER III

EVALUATION OF LOADS

The following static and dynamic forces have been considered in this study. The dynamic forces have been treated as equivalent static forces. Stresses have been computed for each of the loads separately. The total stresses under combined loading situation may be obtained by linear superposition of these stresses under individual loads.

3.1 STATIC FORCES

3.1.1 Self Weight

3.1.2 Hydrostatic pressure

The reservoir is considered full and no tail water is assumed. In finite element technique this force is concentrated only at the nodal points that lie on the upstream face of the dam. The horizontal and vertical components of this force have been considered.

3.1.3 Uplift pressure

Due to seepage of water through the foundation, uplift forces act on the dam. In the present study only one drainage gallery is considered. Half of the water pressure intensity at U/S face is assumed to be released at drainage gallery line.

3.2 DYNAMIC FORCES

3.2.1 Hydrodynamic pressure

I.S. Code (14) recommendation which is based on Zanger's pressure distribution, has been followed to calculate this force. This force is concentrated at the nodal points that lie on the upstream face of the dam in the finite element analysis.

Due to horizontal earthquake there is an instantaneous hydrodynamic pressure exerted on the dam in addition to hydrostatic pressure. Assuming water to be incompressible the hydrodynamic pressure at depth y, below the reservoir surface is given by (14)

$$P_e = C \quad \alpha_h \cdot w \cdot h \quad \ldots \quad (3.1)$$

where,

P_e = hydrodynamic pressure at depth y (kg/m^2)

Cq = coefficient that varies with the shape of the dam face and depth y.

w = density of water (kg/m^3)

h = depth of reservoir (m)

α_h = horizontal seismic coefficient assumed unity in present analysis

For vertical U/S slope or constant U/S slope, the constant C is expressed as (14)

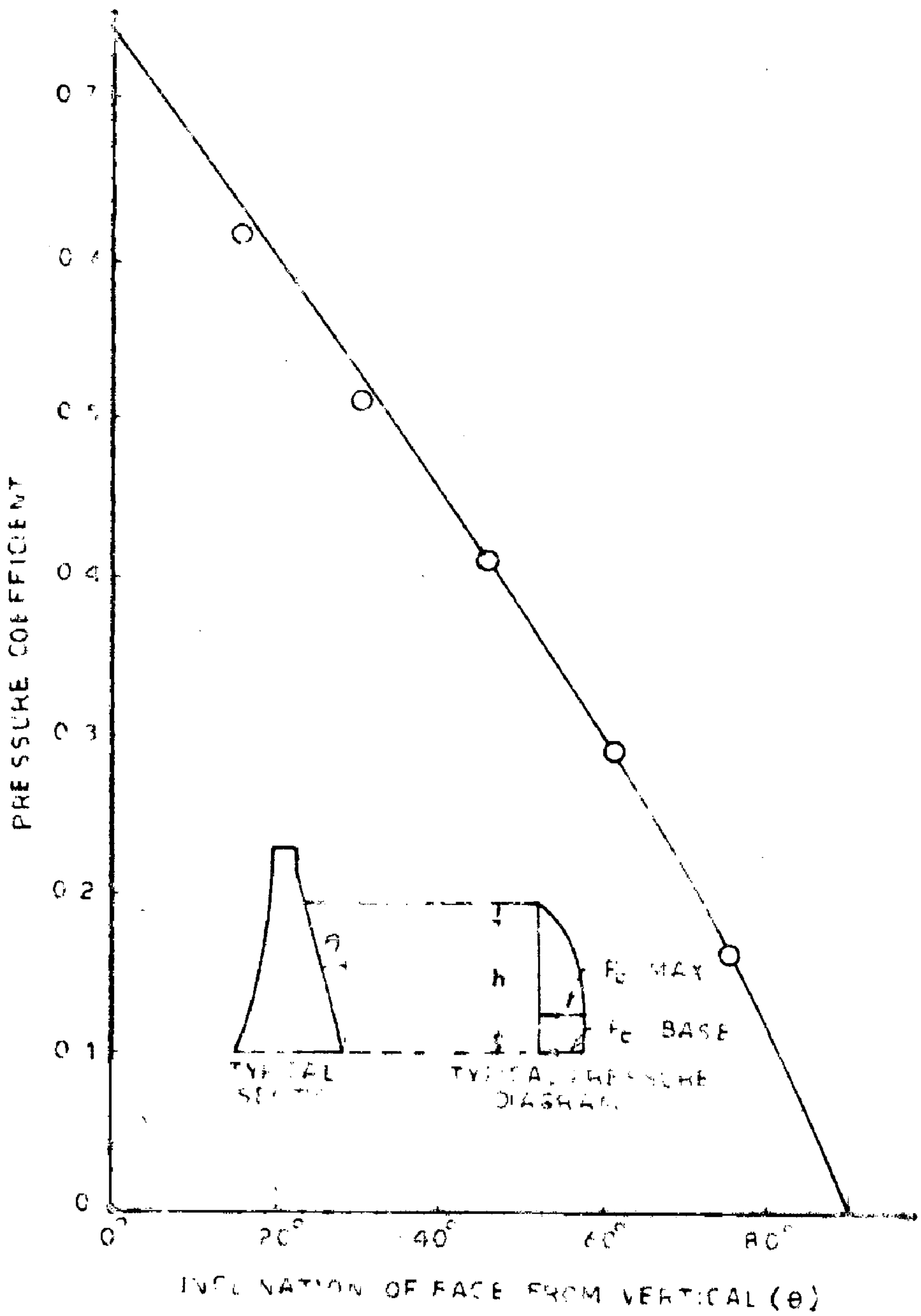


FIG. 6 _MAXIMUM VALUE OF THE PRESSURE COEFFICIENT(cm) FOR CONSTANT SLOPING FACES

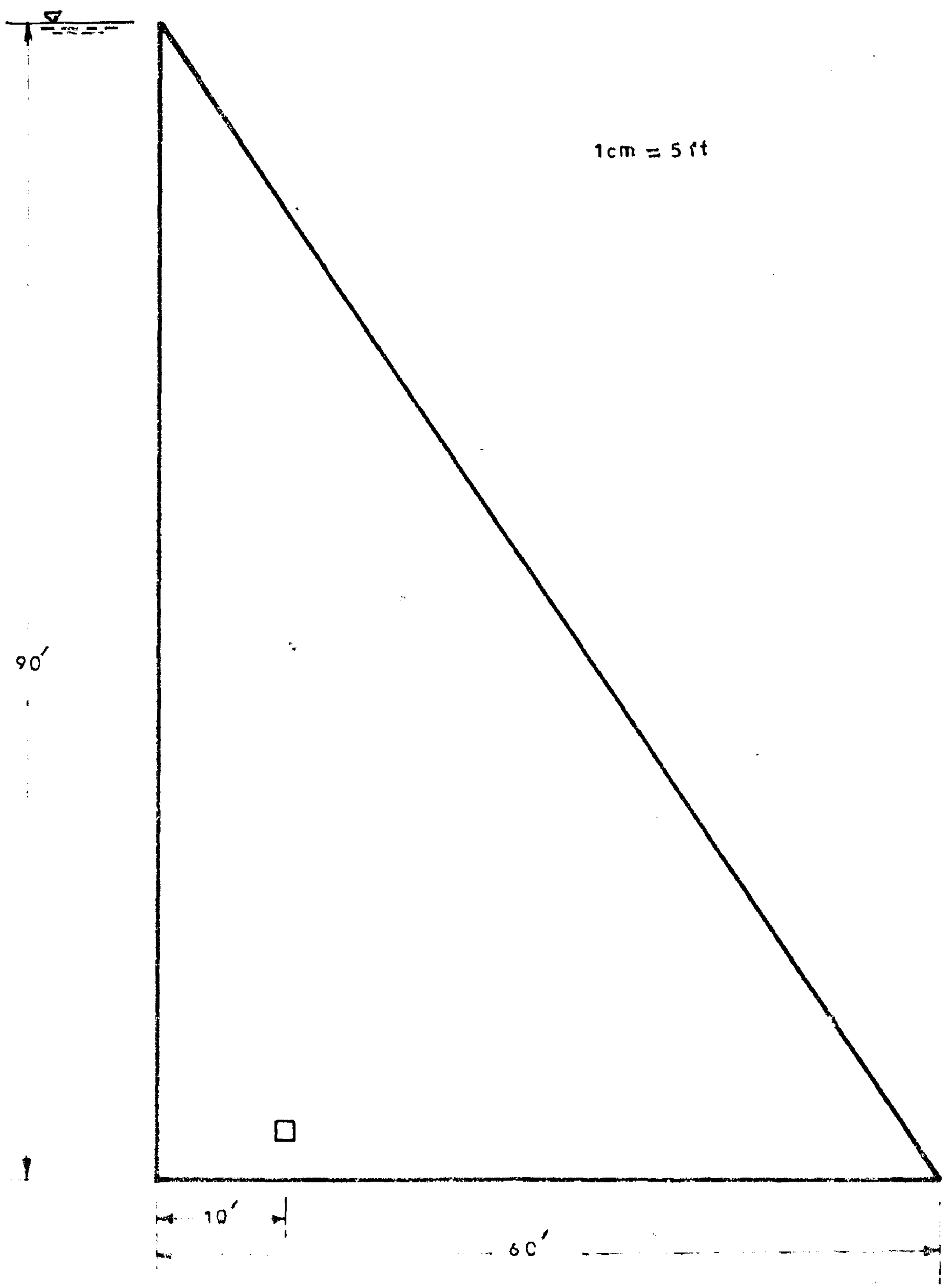


FIG. 6.1 _ TRIANGULAR DAM

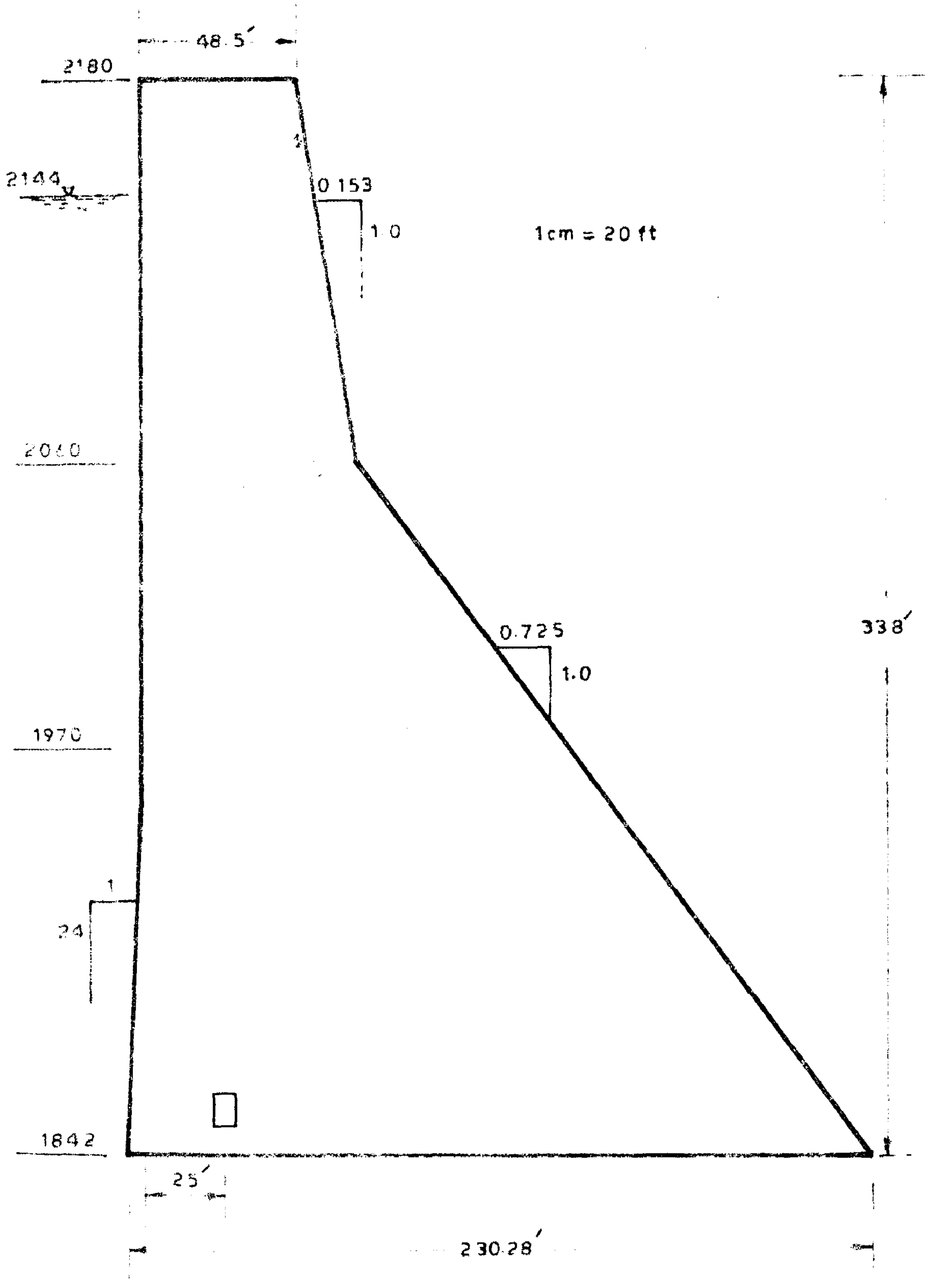


FIG.6.2_ KOYNA DAM

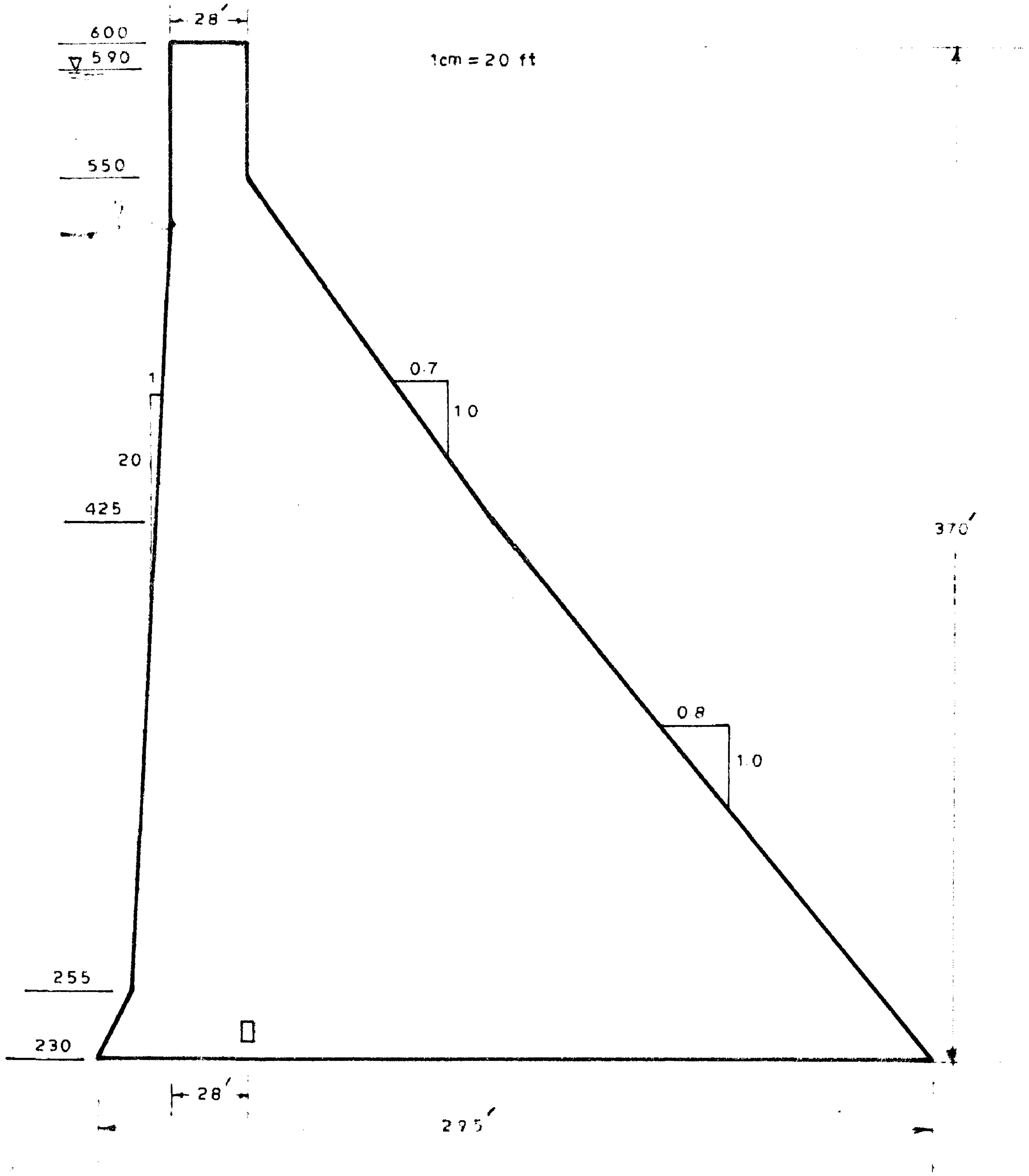


FIG.6.3 _ NAGARJUNASAGAR DAM

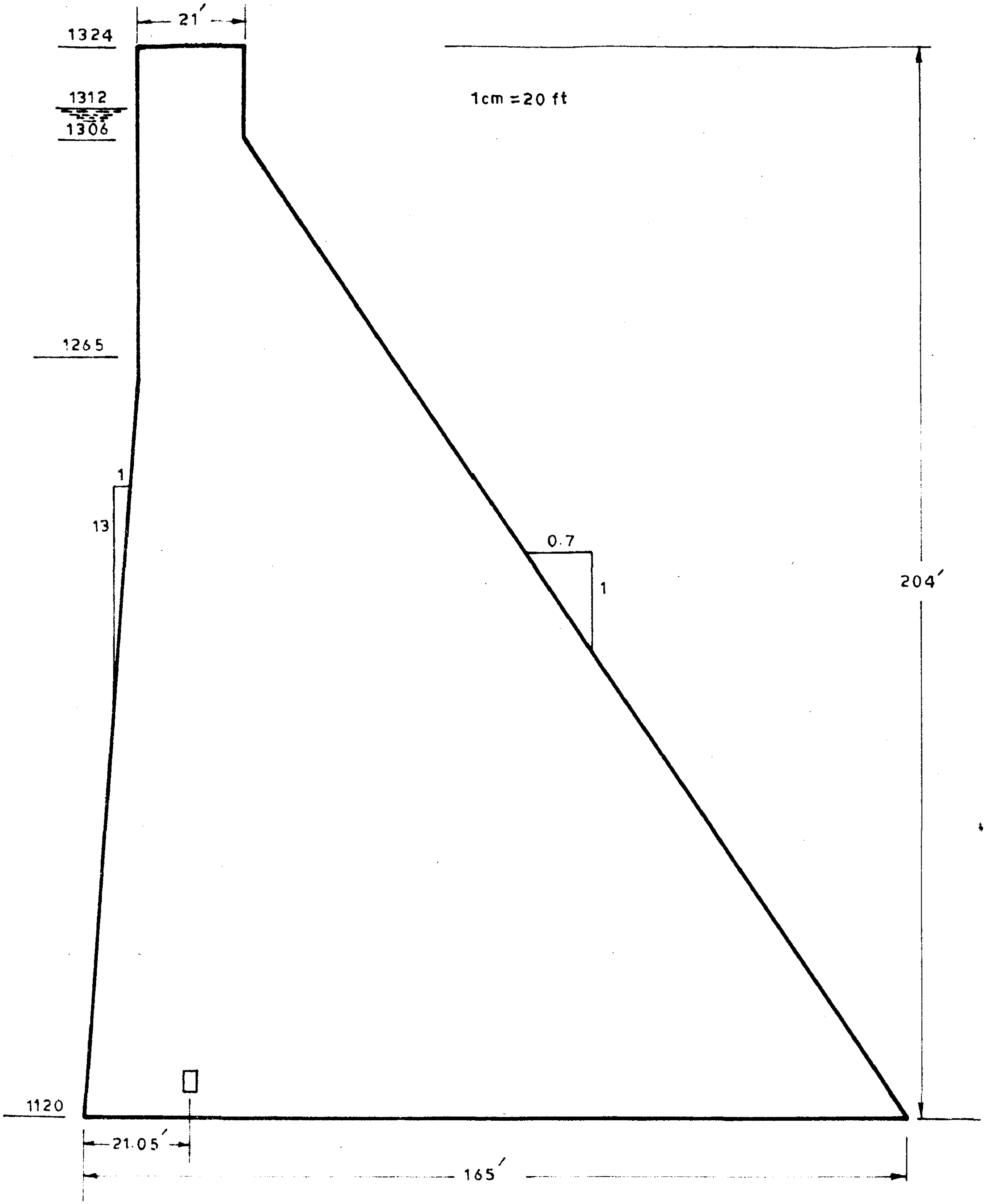


FIG. 6.4_ GANDHISAGAR DAM

$$C = \frac{Cm}{2}\left[-\frac{y}{h}\left(2 - \frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2 - \frac{y}{h}\right)}\right] \quad \ldots \quad (3.2)$$

where, Cm is the maximum value of constant C, that can be obtained from fig. 6.

For dams with a combination of vertical and sloping upstream face, the equivalent slope for determination of value of coefficient 'C' is chosen as per following criteria.

(a) If the height of the vertical portion of the water face of the gravity dam is equal to or greater than one half of the total height of the dam, equivalent slope is taken as vertical.

(b) If the height of the vertical portion of the waterface of the gravity dam is less than one half of the total height of the dam, equivalent slope line is obtained by joining the intersection of the reservoir surface with the dam face, to the intersection of ground surface with the extreme point of the dam.

Approximate total horizontal shear on a section is given by (14)

$$V_e = 0.726\ P_e\ y \quad (Kg/m) \quad \ldots \quad (3.3)$$

Since the hydrodynamic pressure acts normal to the dam face, which may be sloping, therefore there shall be a vertical component of this load. Magnitude of this vertical component at any horizontal section is determined by (14),

$$W = (V_2 - V_1) \tan \phi \quad \ldots \quad (3.4)$$

where,

W = Increase or decrease in vertical component of water pressure

V_2 = Total shear due to horizontal component of hydrodynamic pressure at the elevation of section under consideration

V_1 = Total shear due to horizontal component of hydrodynamic pressure at the elevation of the dam at which the slope of the face commences.

ϕ = angle of dam face with the vertical

3.2.2 <u>Inertial force</u>

Due to earthquake excited ground motion, a lateral force is exerted on the dam because of the inertia of the mass of the dam. This force is in proportion with the acceleration of the mass of the dam.

In the present study this force has been considered as equivalent static force. This equivalent static force is assumed to act in down stream direction. This force depends upon the variation of acceleration of the mass of the dam along the height and is obtained at any elevation of the dam by multiplying the weight of the dam above that elevation with the corresponding seismic coefficient. Three different variations of seismic coefficient along the height of the dam have been considered in this study. These variations are discussed below :

(a) A uniform seismic coefficient along the height of the dam with unit ordinate is considered. I.S. Code (14) (1966) recommends a similar variation.

(b) A linear variation of seismic coefficient along the height of the dam with top ordinate as unity and base ordinate as zero. This is similar to the recommendation of I.S. Code (14) revised version (1971).

(c) A varying seismic coefficient along the height of the dam as shown in fig. 7. This is based on the dynamic analysis of few dams (18).

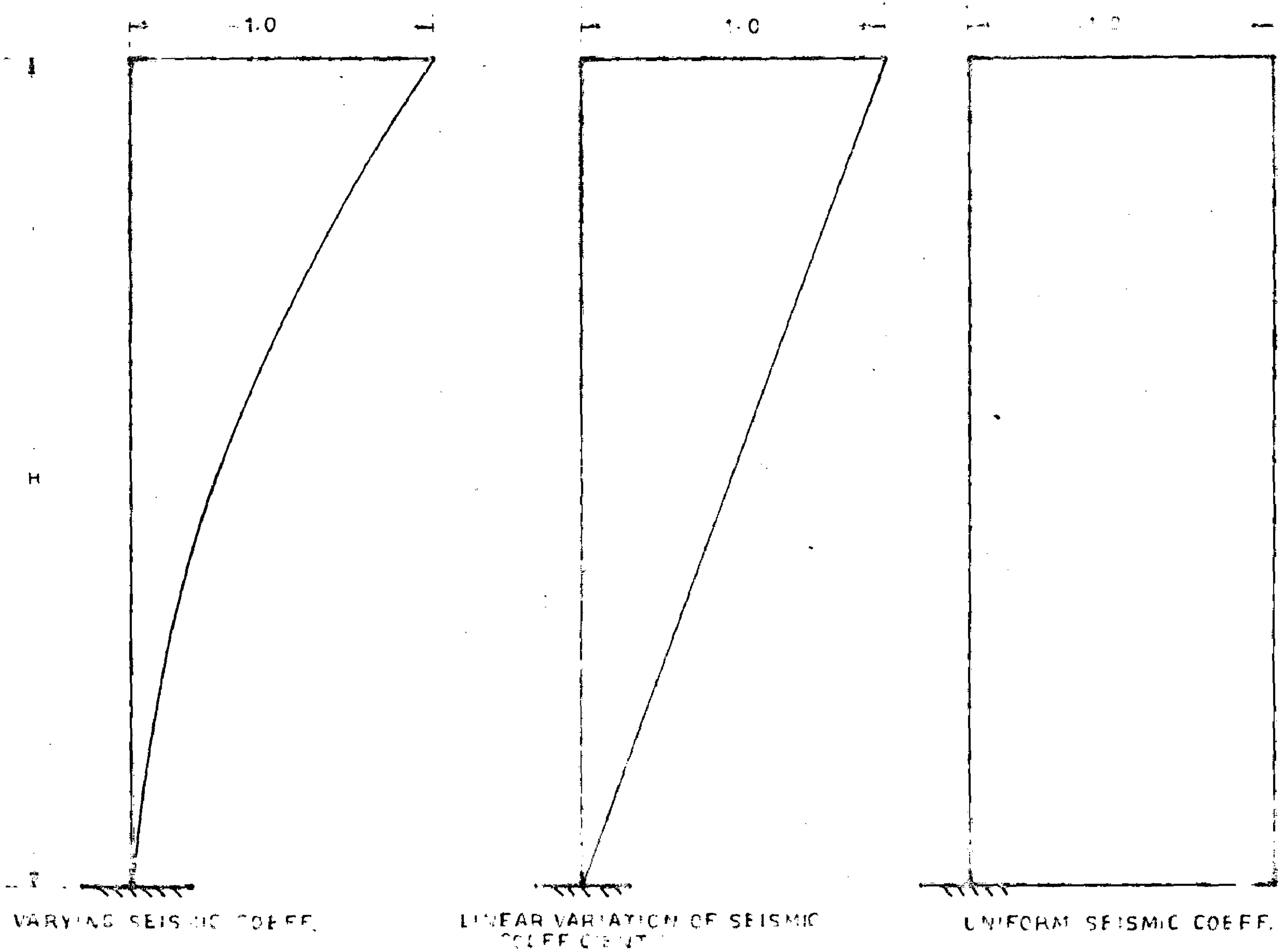


FIG. 7 _ DIFFERENT VARIATIONS OF SEISMIC COEFFICIENT ALONG THE HEIGHT OF THE DAM

The top ordinate in each case has been kept as unity. It is expected that the varying seismic coefficient will give more realistic stress distribution as compared to linear or uniform seismic coefficient. Further it may be noted that the linear variation is a modification of uniform seismic coefficient.

3.3 PROPERTIES OF THE DAMS ANALYSED

Stress analysis has been carried out for the following dam sections.

1. Triangular dam
2. Koyana dam
3. Nagarjuna Sagar dam
4. Gandhi Sagar dam

The geometry for each of these dam sections is shown in figs. 6·1, 6·2, 6·3 and 6·4 . The typical dimensions are also presented in a tabular form for each of these dam sections.

Dam Section	Height in ft.	Base width in ft.	Gallery distance from vertical U/S face in ft	Value of 'Cm' in eqn. (3.2)
Triangular	90.0	60.0	10.0	0.735
Gandhi Sagar	204.0	165.0	10.0	0.67
Koyna	338.0	230.28	25.0	0.735
Nagarjuna Sagar	370.0	295.0	28.0	0.69

Following properties have been assumed,

Poissoins ratio for concrete	=	0.2
Weight density of concrete for Triangular dam	=	140.625 lb/Cft
Weight density of concrete for other sections	=	165.0 lb/Cft.
Weight density of water	=	62.5 lb/Cft.

The weight density of concrete in case of triangular dam has been taken different from the value for other dam section because it corresponds to the value used in the theory of elasticity solution of Zienkiewicz (3).

CHAPTER IV

DISCUSSION OF RESULTS

Comparison of different methods of stress analysis under different loading situations :

4.1 COMPARISON WITH THEORY OF ELASTICITY SOLUTION (3)

Figs. 8 and 9 show the stress distribution as obtained by the Davis and the USBR methods for a triangular dam. The loads considered are self weight and hydrostatic pressure only. The theory of elasticity solution of ZienkiewicZ (3) is also plotted.

The Davis and the USBR methods give linear distribution of normal vertical stress (σ_y) on all horizontal planes of the dam. The theory of elasticity analysis indicates that the variation is not linear particularly at and near the base of the dam where the width is large. Further the theory of elasticity analysis indicates certain tension at the U/S point on the base. While Davis and USBR methods do not indicate this fact. Thus it may be concluded that the assumption of linear variation of normal vertical stress (σ_y) in the Davis and the USBR methods is not justified for the regions at and near the base.

The Davis method gives fairly linear distribution of shear stress τ_{xy} (fig. 8) on all horizontal planes. The USBR method gives parabolic shear stress, but for the top regions of the dam the shear stress varies almost linearly across horizontal planes. The theory of elasticity solution differs considerably from both the Davis and the USBR methods for the regions at and near the base. The maximum difference being at the base near U/S and d/S faces of the dam.

Normal horizontal stress (σ_x) (fig. 9) as obtained by the USBR and the Davis methods also differ significantly from the theory of elasticity solution (3) for the regions at and near the base of the dam. The maximum difference in stress values is observed near the U/S face of the dam but not at the U/S face.

It may be seen from fig. 8 and 9 that for top 2/3rd height of the dam all the three methods of stress analysis namely, the Davis, the USBR and the theory of elasticity; give similar stress distribution. But for the bottom 1/3rd height the theory of elasticity method gives a stress distribution considerably different from that obtained by the Davis and the USBR methods. Thus it may be concluded that the Davis and the USBR methods give relatively approximate stress distributions at and near the base of the dam.

The Davis and the USBR methods give identically same distribution of normal vertical stress σ_y (fig. 8). But the two methods give different shear stresses and normal horizontal stresses. This differencé is significant only at and near the base of the dam. The USBR method slightly overestimates the shear stress as compared to the stresses obtained by the Davis method (fig. 8).

4.2 Comparison of the Davis and the USBR methods with the finite element method

Only static stresses as obtained by the Davis and the USBR methods are compared with the finite element solution (15). Figures 10, 11÷ 12 show the stresses for Koyna dam under dead weight; hydrostatic pressure and uplift pressure separately.

For dead weight alone (fig. 10), a fairly good agreement in the stresses as obtained by different approaches, namely, the Davis, the USBR and the finite element methods, is observed. However, the finite element method does not give linear distribution of normal vertical stress σ_y at and near the base of thé dam where the width of the dam is large. Further the minimum direct compression (major principal stress) is under-estimated by the Davis and the USBR methods. The maximum direct compression (minor principal stress) is underestimated by the Davis and the USBR methods at U/S face but the same is overestimated towards the d/s face on the base. Shear stresses (τ_{xy}) as obtained by Davis and the USBR methods for dead weight only are also shown in fig. 13. The Davis and the USBR methods give zero shear stress at the vertical U/S face but the finite element method indicates a definite shear stress value. The finite element stresses differ considerably from the other two methods at and near the base. The Davis and the USBR methods underestimate shear stress τ_{xy} at the U/S face and they overestimate shear stress at d/s face of the dam. The Davis and the USBR methods also give different shear stresses. This difference is more significant at the base of the dam.

Fig. 11 shows the stresses due to hydrostatic pressure alone for Koyna dam. The finite element analysis gives different stress distribution (15) as compared to the stresses obtained by the Davis and the USBR methods. This difference is maximum at the base of the dam where the width of the dam is large. The finite element analysis (15) indicates that the bending stresses (σ_y) under hydrostatic pressure are not linear particularly at and near the base of the dam. This fact can not be revealed by the Davis and the USBR methods. Fig. 13 shows the shear stresses for hydrostatic pressure alone for the

Koyna dam. It may be seen that the shear stresses as obtained by the finite element analysis are considerably different from those obtained by the Davis and the USBR methods. This difference is also maximum at the base. The Davis and the USBR methods give almost similar stress distribution except for the difference at and near the base of the dam. For the hydrostatic pressure alone, the maximum direct tension (σ_1, fig. 11) is underestimated by the Davis and the USBR methods. The maximum difference being at the U/S face on the base of the dam. The maximum direct compression (σ_2, fig. 11) is overestimated by the Davis and the USBR methods.

It may be concluded that for hydrostatic pressure alone, the Davis and the USBR methods give fairly accurate results for the regions near the top of the dam. But for the regions at and near the base of the dam, the Davis and USBR methods give highly inaccurate results.

Fig. 12 compares the finite element stresses (15) with the stresses obtained by the Davis and the USBR methods for the case of uplift pressure alone for Koyna dam. The three methods of stress analysis give similar distribution of normal vertical stress σ_y. The finite element analysis (15) indicates certain minimum tension on the base (minor principal stress σ_2, fig. 12) while the Davis and the USBR methods give small compression on the base.

It may be concluded that for dead weight and for uplift pressure, the stress distribution as obtained by the Davis, the USBR and the finite element methods is similar except for the small difference at the base of the dam. However, for hydrostatic pressure there is a significant difference in the stresses as obtained by the Davis or the USBR methods from those obtained by the finite element analysis (15). Further it may be noted from figs. 10, 11 and 12 that the finite element analysis gives certain tension (σ_y) at the U/S face on the base of the dam under combined Dead weight, uplift pressure and hydrostatic pressure. This fact is not revealed by the Davis and the USBR methods. It may also be pointed out that the theory of elasticity method also indicates certain tension (σ_y, fig. 8) at the U/S face on the base of the dam where the width is large.

4.3 COMPARISON OF THE DAVIS AND THE USBR METHODS FOR OTHER LOADS

Figs. 14 and 15 show the static stresses under dead weight only for the Gandhi Sagar and the Nagarjuna Sagar dams. The normal vertical stress σ_y is identically same for the two methods. This is because the two methods involve the same assumption for the calculation of this stress component. The maximum direct compression (minor principal stress fig. 15) is almost linear on all the horizontal planes of the dam. Further the Davis and the USBR methods give almost similar stress distribution, except for the small difference at and near the base. The Davis method gives slightly less stress values. The difference of the stresses in the two methods is due to the fact that the USBR method assumes parabolic shear stress distribution while the Davis method calculates shear stress based on actual distribution. The shear stresses as obtained by the two methods are compared in figs. 8 and 13 for Triangular dam and the Koyna dam. As may be seen from figs 8 and 13, the Davis and the USBR methods give almost similar stress distribution except for the regions at and near the base of the dam.

Fig. 16 shows the stresses under hydrostatic pressure alone for the Gandhi Sagar dam. Only major and minor principal sttesses have been presented. This figure also confirms that the stress distribution as obtained by the two methods, i.e., the Davis and the USBR, is similar except for a small difference which is significant only at the base. The USBR method gives slightly higher values.

DYNAMIC STRESSES

Stresses for the dynamic forces have been computed by treating dynamic forces as equivalent static forces.

Figs. 17, 18, 19 and 20 show the stresses due to hydrodynamic pressure alone for Nagarjuna Sagar, Gandhi Sagar, Koyna and triangular dam sections respectively. The two principal stresses have been compared (fig. 17) by the Davis and the USBR methods. The USBR method gives slightly higher stress. For Nagarjuna Sagar dam (fig. 17) at base, no compression is indicated by USBR method for regions at some distance from U/S face on the base. While the Davis method indicates certain compression. The difference in the stresses as obtained by the Davis and the USBR methods is insignificant at and near the top of the dam.

For other dams, namely, Koyna, Gandhi Sagar and Triangular dams, the two methods give similar stress distribution except for the little difference at and near the base of the dam.

Fig. 23 to 30 show the stresses under inertial force with three different seismic coefficient namely, a uniform seismic coefficient, a linear seismic coefficient, and a varying seismic coefficient.

Stresses as obtained by the Davis and the USBR methods compare in a manner similar to that for hydrodynamic and hydrostatic pressure loading cases. The two methods give similar stress distributions near the top of the dam. Near the base of the dam, the two methods give different stress distributions but the variation of stresses is not very much different. The USBR method gives stresses slightly higher than those obtained by the Davis method.

COMPARISON OF STRESSES OBTAINED FOR THREE DIFFERENT SEISMIC COEFFICIENTS

Fig. 31 shows the variation of normal vertical stress (σy) along the height of the dam for different dams. The force is only inertial force with three different variations of seismic coefficient along the height of the dam as discussed in Chapter III.

It may be seen from fig. 31, that the three variations of seismic coefficient give considerably different stresses near the base of the dam. If the base stresses as obtained for the three different variations of seismic coefficient are kept equal to those obtained for varying seismic coefficient, the uniform seismic coefficient underestimates the stresses near the top of the dam.

In order to have a comparative idea of the stresses for the different variations of seismic coefficient, the ratio of stress (σ_y) as obtained for uniform seismic coefficient to the stress for linear seismic coefficient is shown in fig. 32. The ratio of the stress (σ_y) for uniform seismic coefficient to the stress (σ_y) for varying seismic coefficient is also shown in fig. 32.

It may be seen from fig. 32, that if the base stresses as obtained for the three variations of seismic coefficient are kept equal to the stress as obtained for varying seismic coefficient, the stresses near the top of the dam, as obtained for uniform seismic coefficient are about half the stresses obtained for varying seismic coefficient. The linear variation of seismic coefficient gives stresses almost 2/3rd of the stresses obtained for varying seismic coefficient. It may be pointed out that the IS Code 1893 (1966 & 1971) (14) recommends that the ratio of stresses obtained for uniform seismic coefficient to the stresses obtained for linear seismic coefficient is 1.5. The same ratio is found as 2.0 in the present study.

As discussed above, if the criterion for comparison of stresses obtained for the three different variations of seismic coefficient is chosen as the equal base stress (or equal base moment), the uniform seismic coefficient underestimates the stresses near the top of the dam. Therefore, the uniform seismic coefficient should not be used for design purposes. A varying seismic coefficient should be adopted. Further the ratio of the stresses as obtained for uniform seismic coefficient to those obtained for linear seismic coefficient is not 1.5 (as suggested by IS Code (14)) instead it is found as 2.0.

It may be concluded that the linear variation of seismic coefficient gives stress distribution close to the one obtained by varying seismic coefficient, as compared to the stresses due to uniform seismic coefficient. And therefore, a linear seismic coefficient should be used in preliminary design purposes. However, for detailed stress analysis, a seismic coefficient based on the dynamic analysis should be used.

CONCENTRATION OF STRESSES AT NECKS

In fig. 31, variation of normal vertical stress (σ_y) is plotted along the height of the dam for Koyna, Gandhi Sagar and Nagarjuna Sagar dams. The force is only inertial force with different seismic coefficient. For Koyna dam and Nagarjuna Sagar dam, concentration of stress occurs at the elevations of the dam where the slope of the dam face changes abruptly. This concentration is even more significant for the linear and varying seismic coefficients. It may be concluded that during preliminary design of the concrete gravity dam, this concentration of stresses should be checked for lateral forces. And proper care should be taken for the same.

CHAPTER V

CONCLUSIONS

The Davis and the USBR methods give linear distribution of normal vertical stress (σy) on all horizontal planes of the dam. The theory of elasticity approach (3) indicates that the variation is not linear for the regions at and near the base of the dam where the width of the dam is large. Other stress components, namely, normal horizontal stress (σx) and shear stress (τxy), as obtained by the Davis and the USBR methods also differ considerably from the theory of elasticity solution at and near the base of the dam.

For the dead weight and uplift pressure the finite element stresses (15) are in good agreement with the Davis and the USBR stresses, except for the small difference at and near the base. However, for the hydrostatic pressure the finite element analysis indicates that the bending stress (σy) is not linear particularly at and near the base of the dam where the width of the dam is large. The Davis and the USBR methods do not reveal this fact.

For static loads, the theory of elasticity method and the finite element method indicate a definite tension (σy) at the U/S face on the base of the dam.

This fact is not revealed by the Davis and the USBR methods of stress analysis.

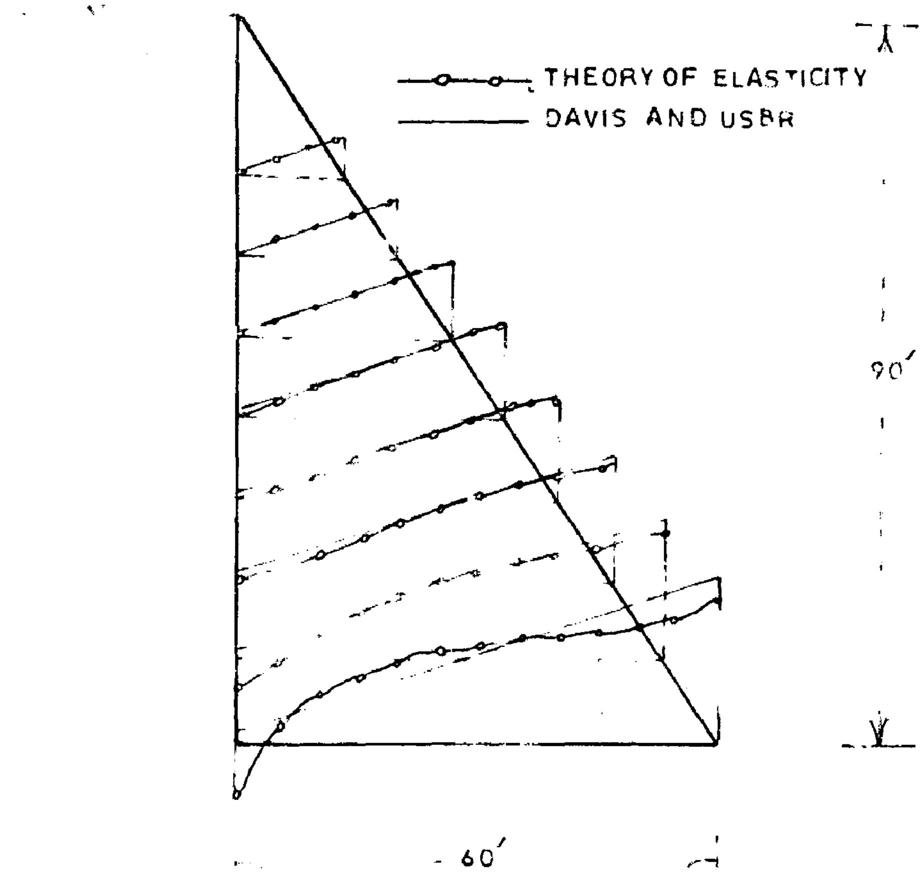
The Davis and the USBR methods give identically similar distribution of normal vertical stress (σy) on all horizontal planes of the dam. But the two methods give different shear stresses. This difference is significant at and near the base of the dam. Further both of these methods give relatively approximate stress distributions for the regions at and near the base of the dam where the width of the dam is large. For preliminary design purposes either of the Davis and the USBR methods may be used. However, for detailed stress analysis the finite element analysis should be used.

For calculation of the lateral inertial force, a varying seismic coefficient along the height of the dam, based on the dynamic analysis is found suitable. However, for the preliminary design purposes a linear variation of seismic coefficient along the height of the dam may be used. For detailed stress analysis, the dynamic stress analysis using finite element technique is recommended.

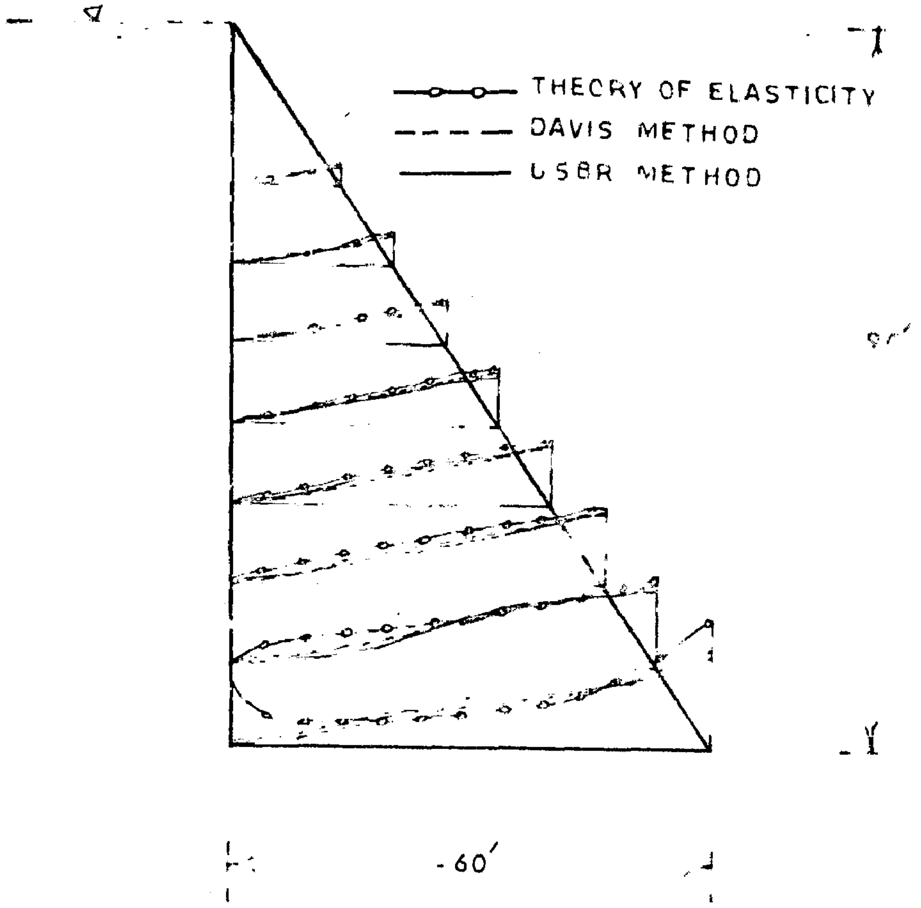
Due to lateral inertial forces a stress concentration occurs at the elevations of the dam where the slope of the dam face changes abruptly. This stress concentration should be properly checked during preliminary stress analysis of concrete gravity dams.

FIGURES

1 cm = 43 $ psi



NORMAL VERTICAL STRESS σ_y



SHEAR STRESS τ_{xy}

FIG. 8 _COMPARISON OF STRESSES AS OBTAINED BY DAVIS AND USBR METHODS WITH THEORY OF ELASTICITY SOLUTION FOR A TRIANGULAR DAM

LOADS ARE SELF WEIGHT AND HYDROSTATIC PRESSURE ONLY

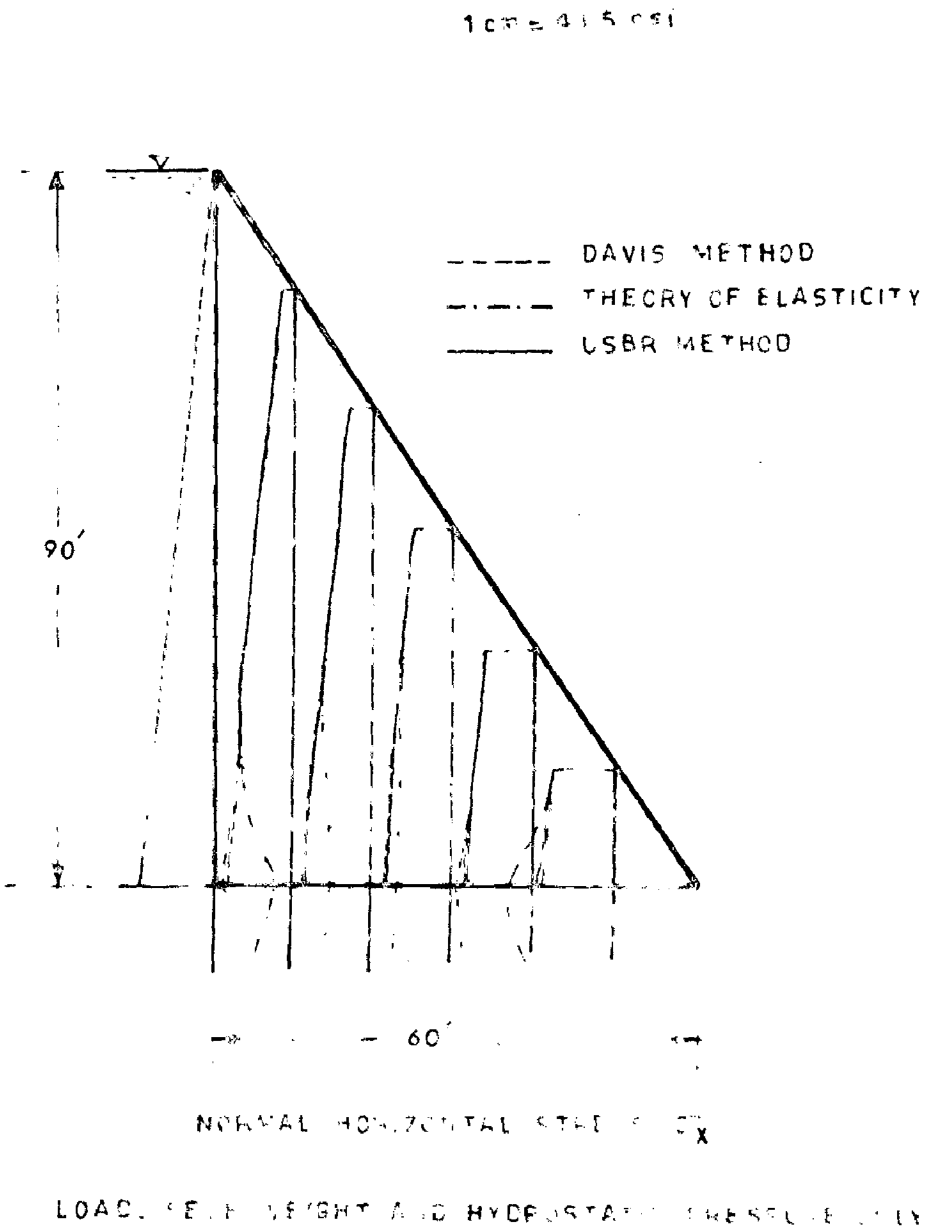


FIG. 9 _ COMPARISION WITH THEORY OF ELASTICITY

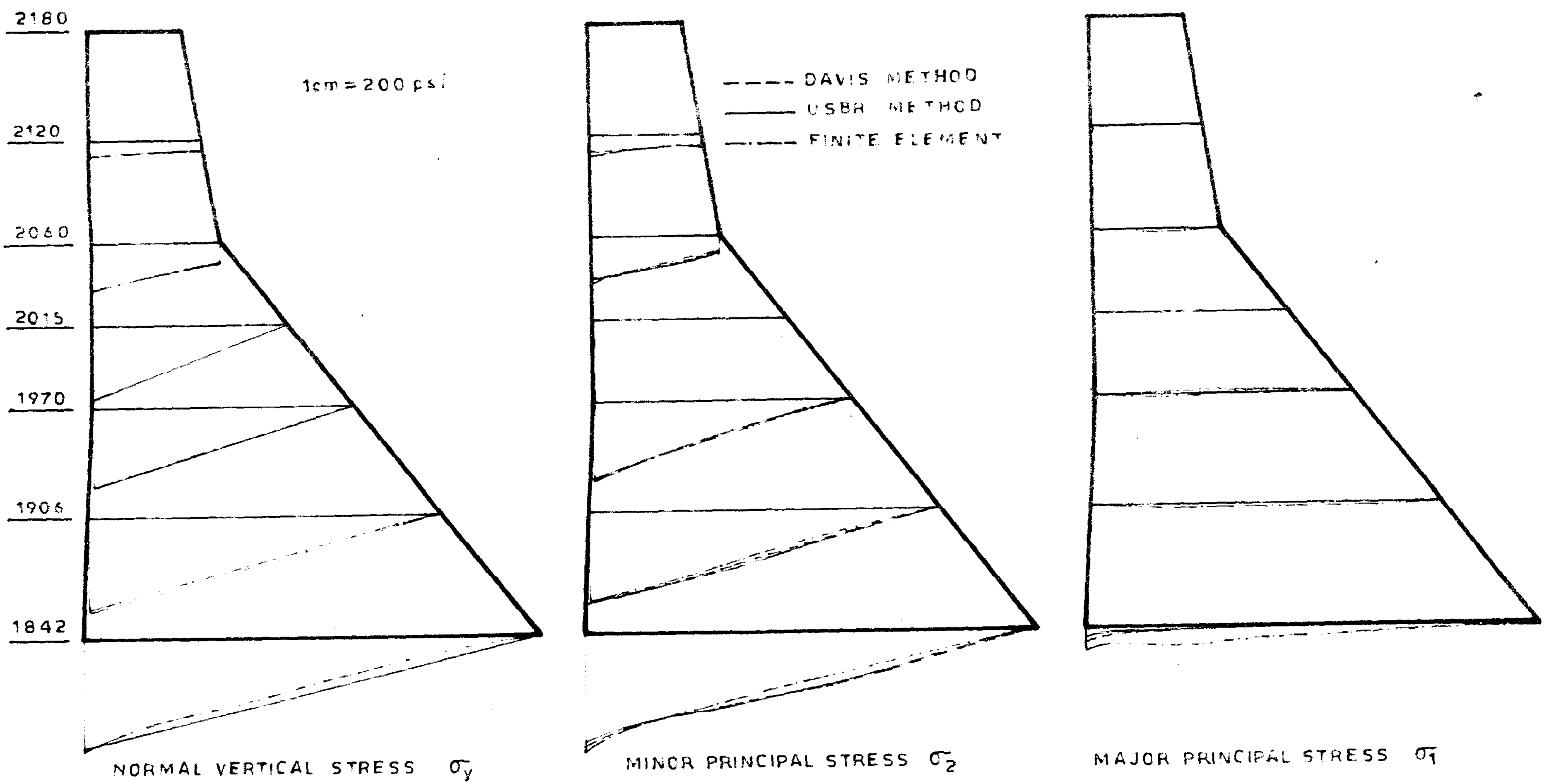


FIG.10_STRESSES UNDER DEAD WEIGHT ONLY (KOYNA DAM)

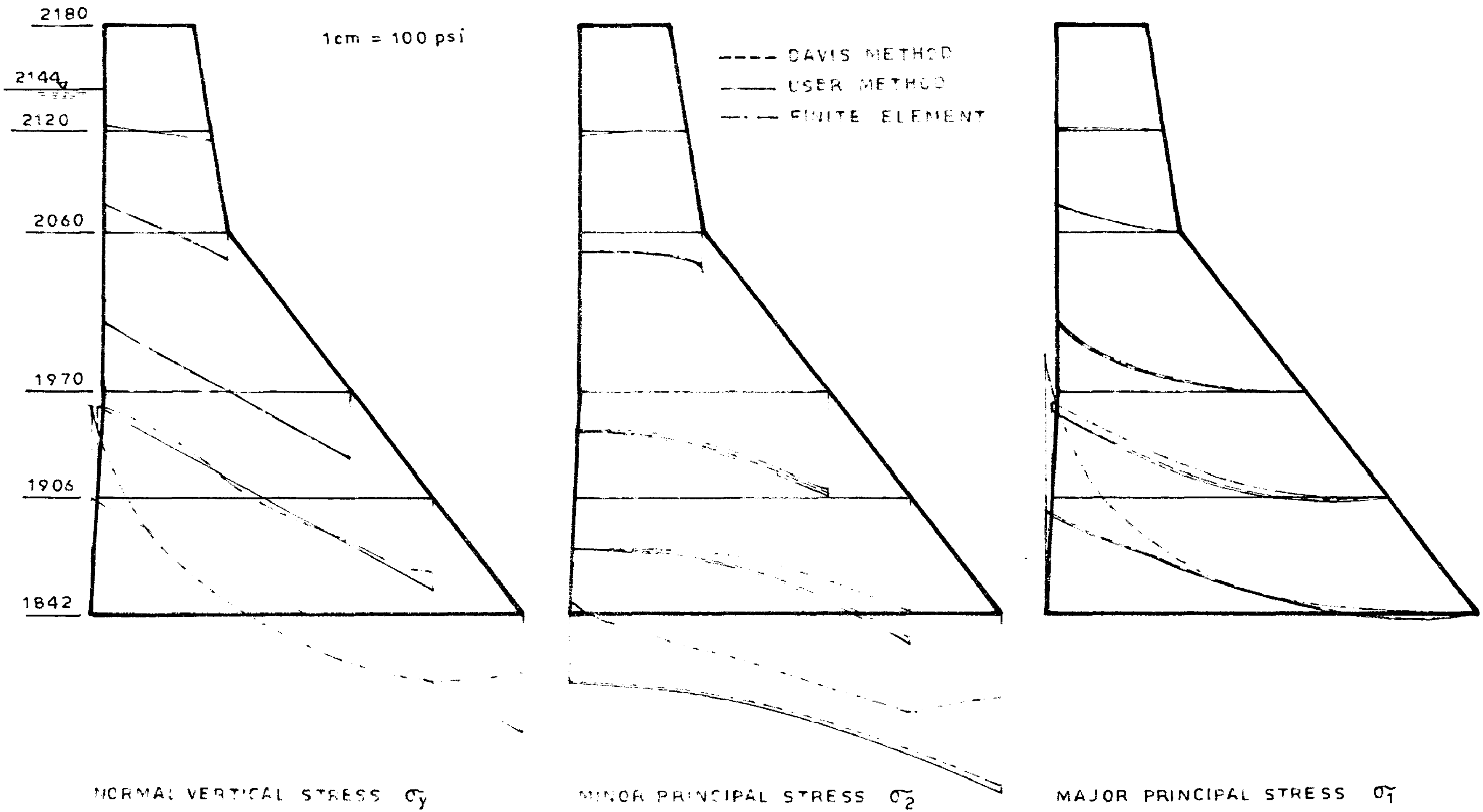


FIG. 11 _ STRESSES DUE TO HYDROSTATIC PRESSURE ONLY (KOYNA DAM)

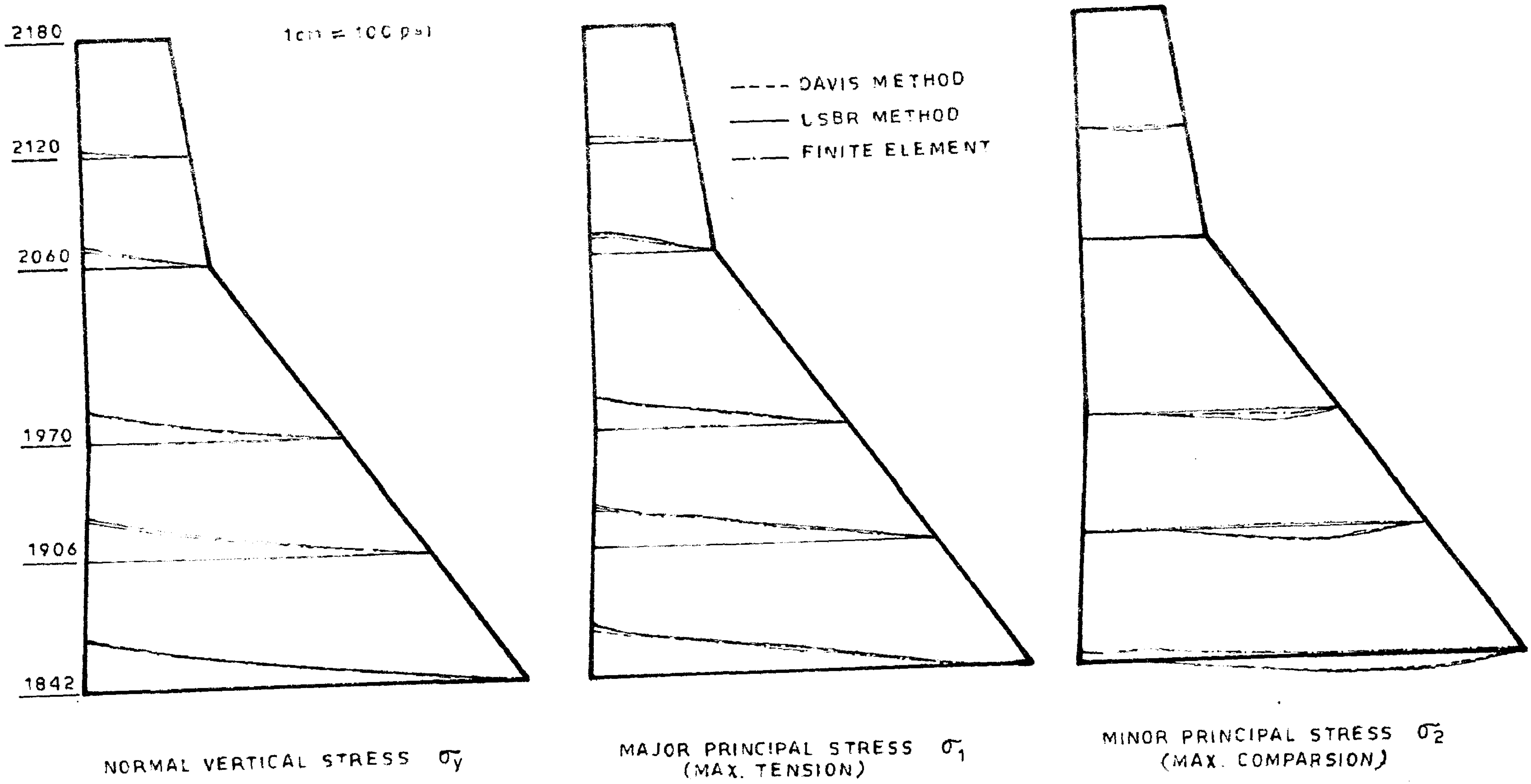


FIG.12 – STRESSES UNDER UPLIFT, PRESSURE ALONE – DRAINAGE GALLERY AT 25′ FROM VERTICAL U/S FACE

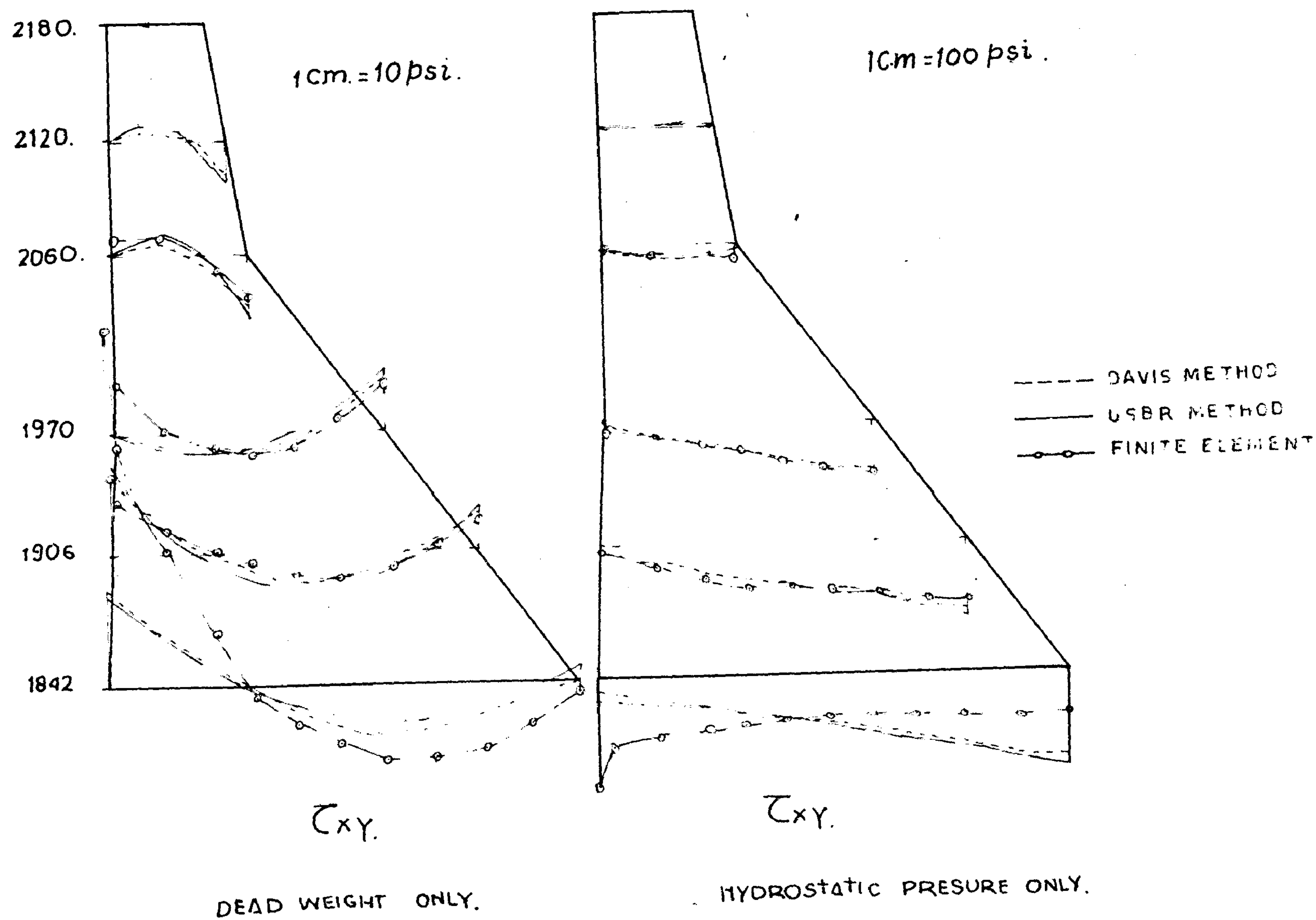


FIG. 13 _COMPARISON OF SHEAR STRESSES FOR KOYNA DAM

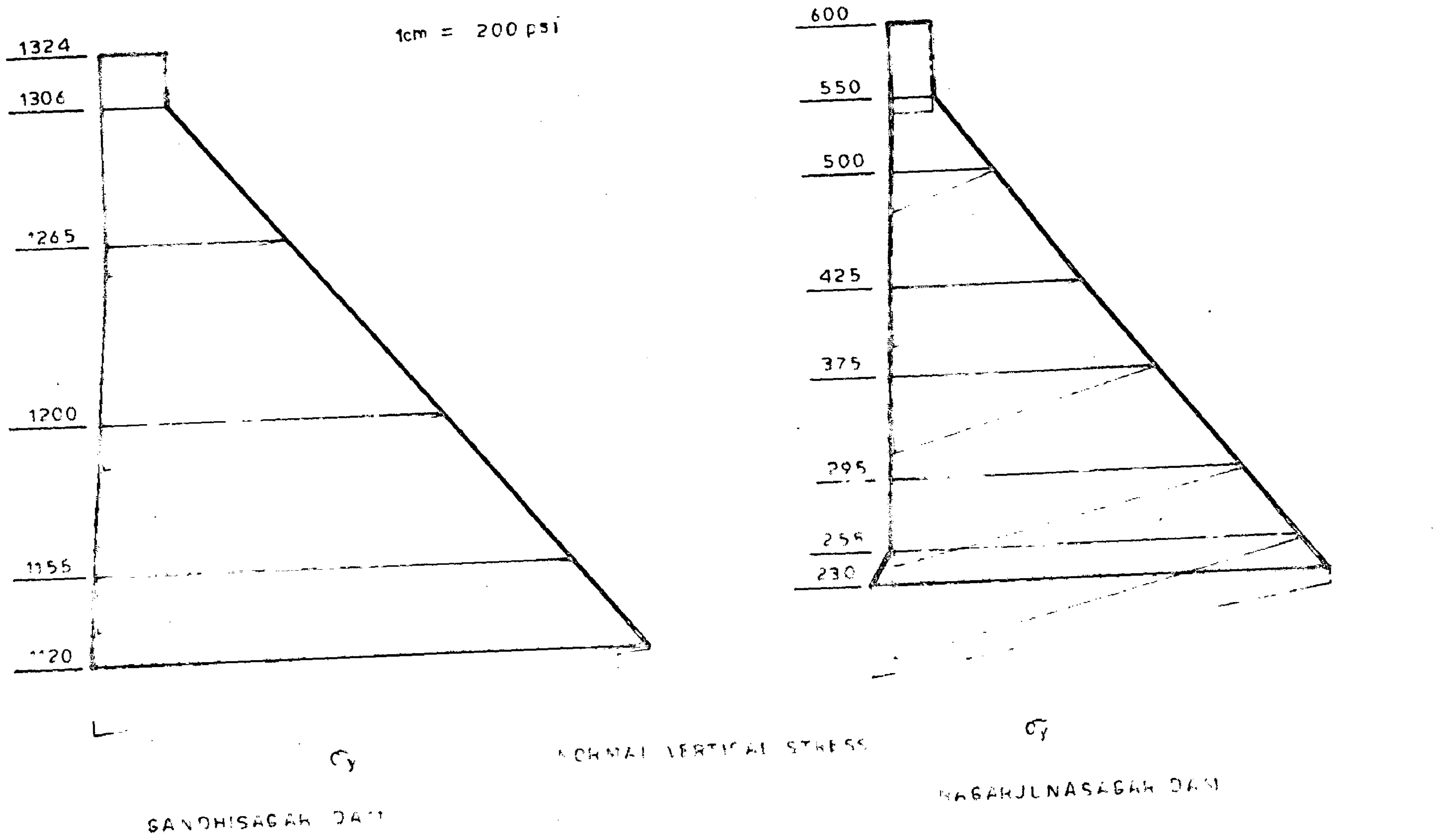


FIG.14 _ NORMAL VERTICAL STRESS (σ_y) UNDER DEAD WEIGHT ONLY

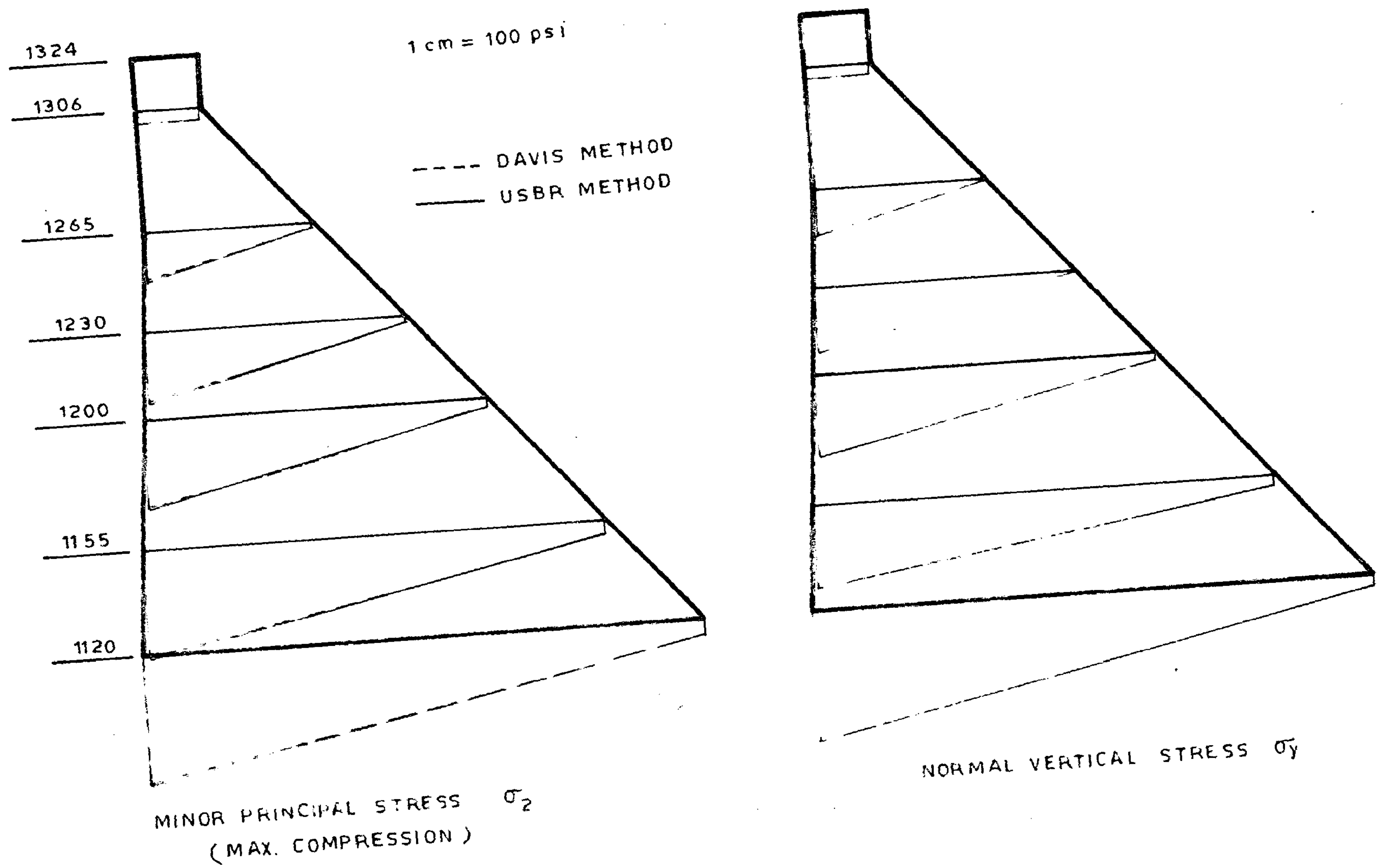


FIG.15 _ STATIC STRESSES DUE TO DEAD WEIGHT ONLY
GANDHISAGAR DAM

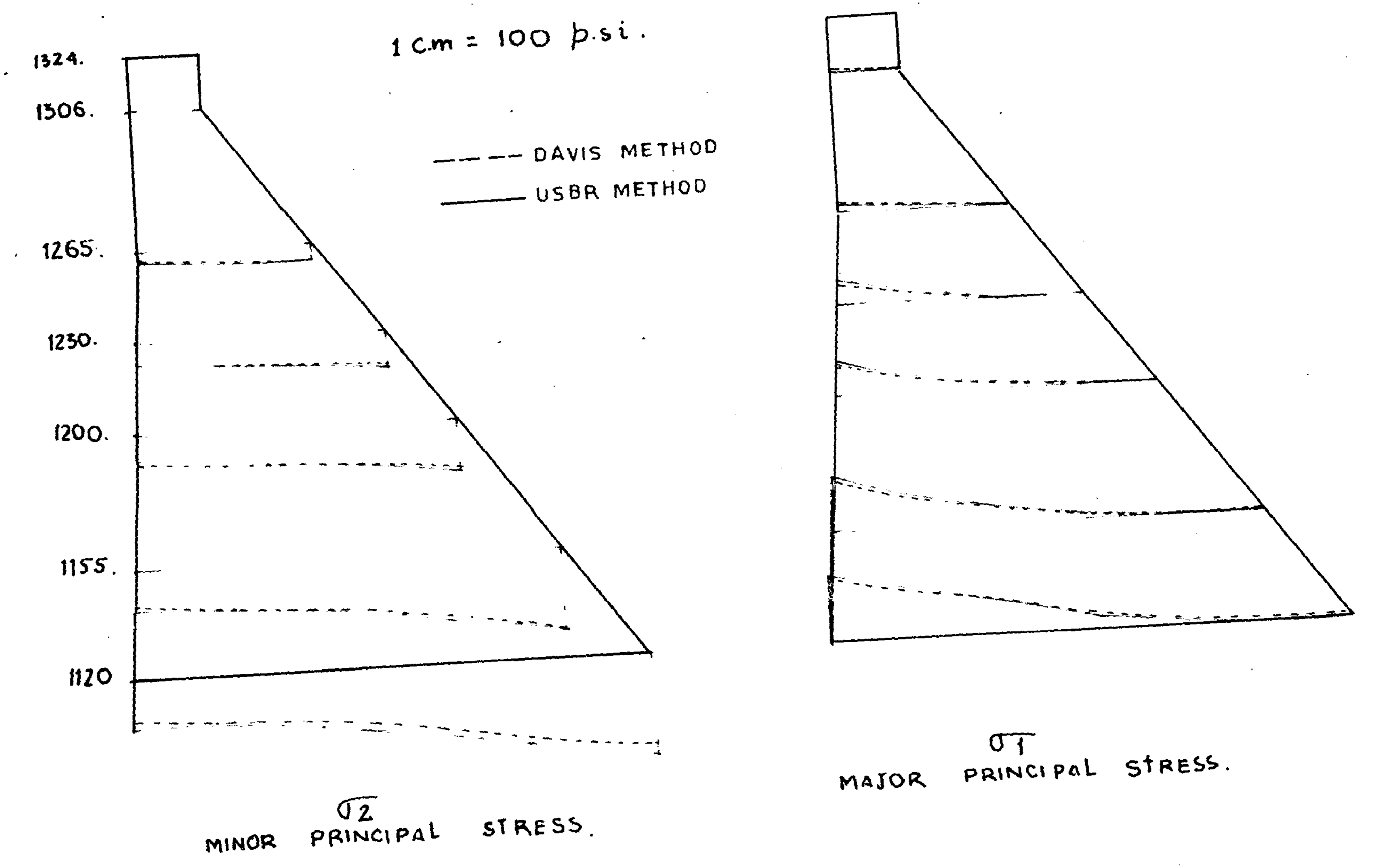


FIG. 16 – STRESSES DUE TO HYDROSTATIC PRESSURE ALONE
(GANDHISAGAR DAM)

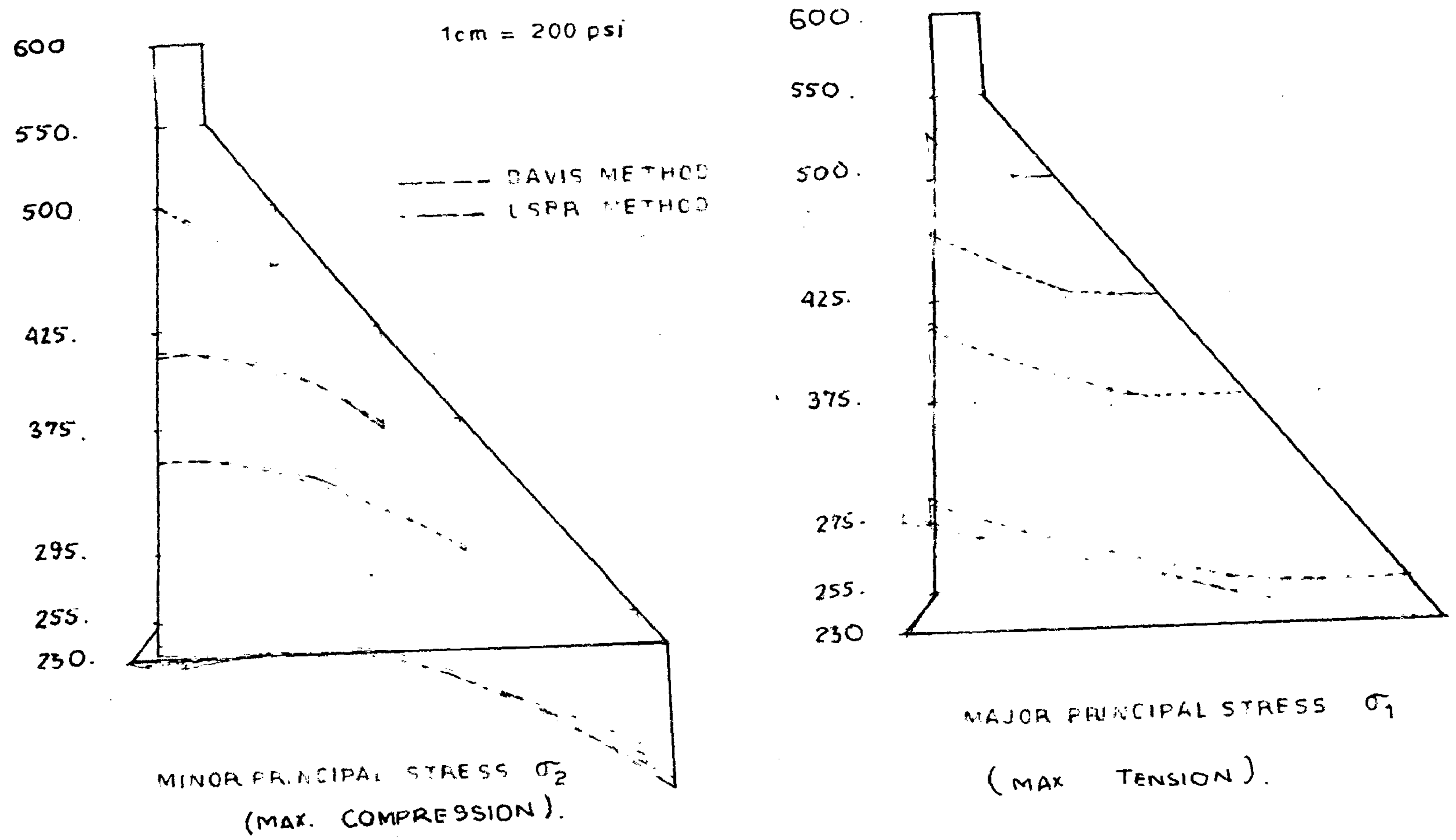


FIG. 17 _ STRESSES UNDER HYDRODYNAMIC PRESSURE ONLY
(NAGARJUNASAGAR DAM)

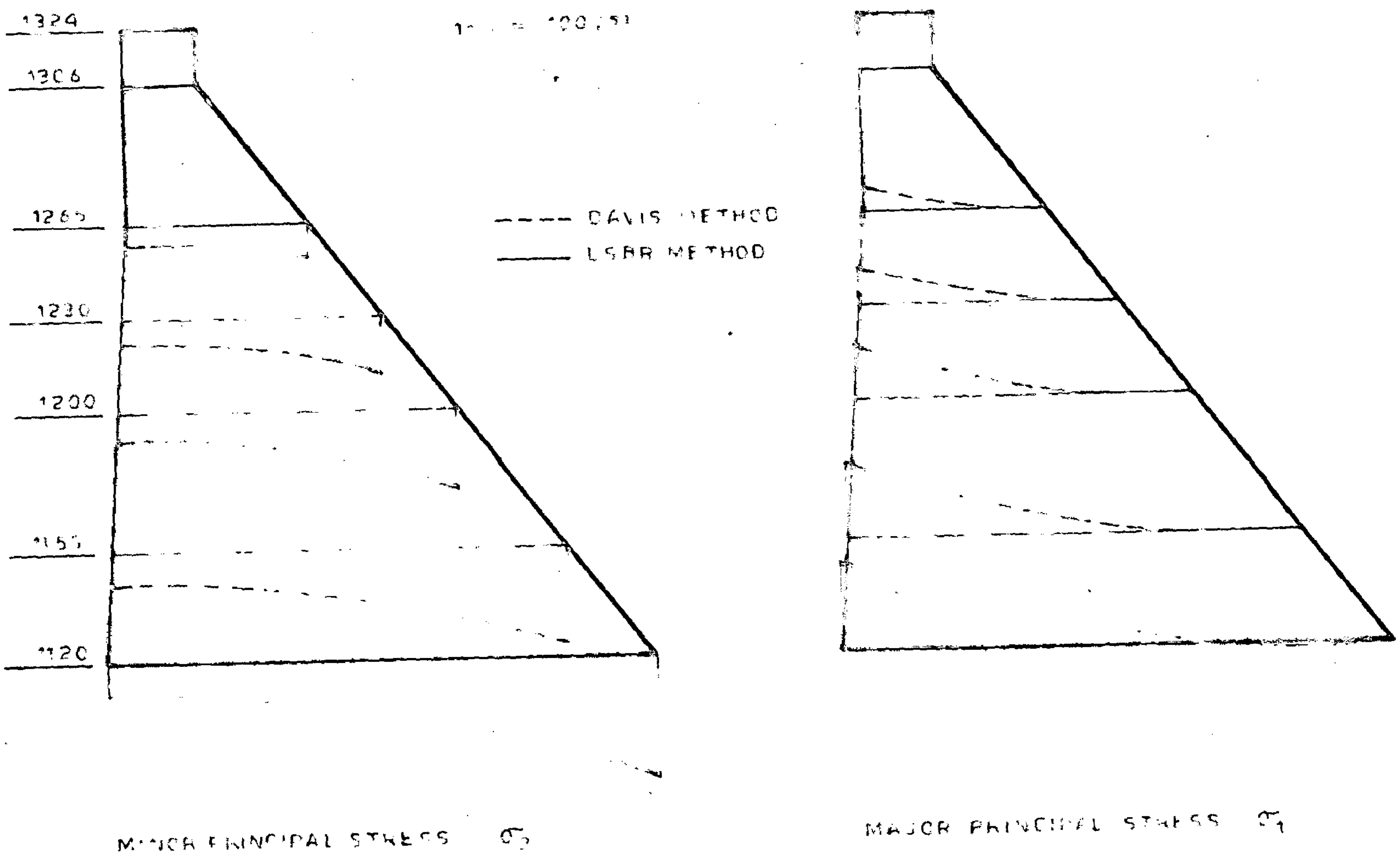


FIG.18 _STRESSES DUE TO HYDRODYNAMIC PRESSURE ONLY
(GANDHISAGAR DAM)

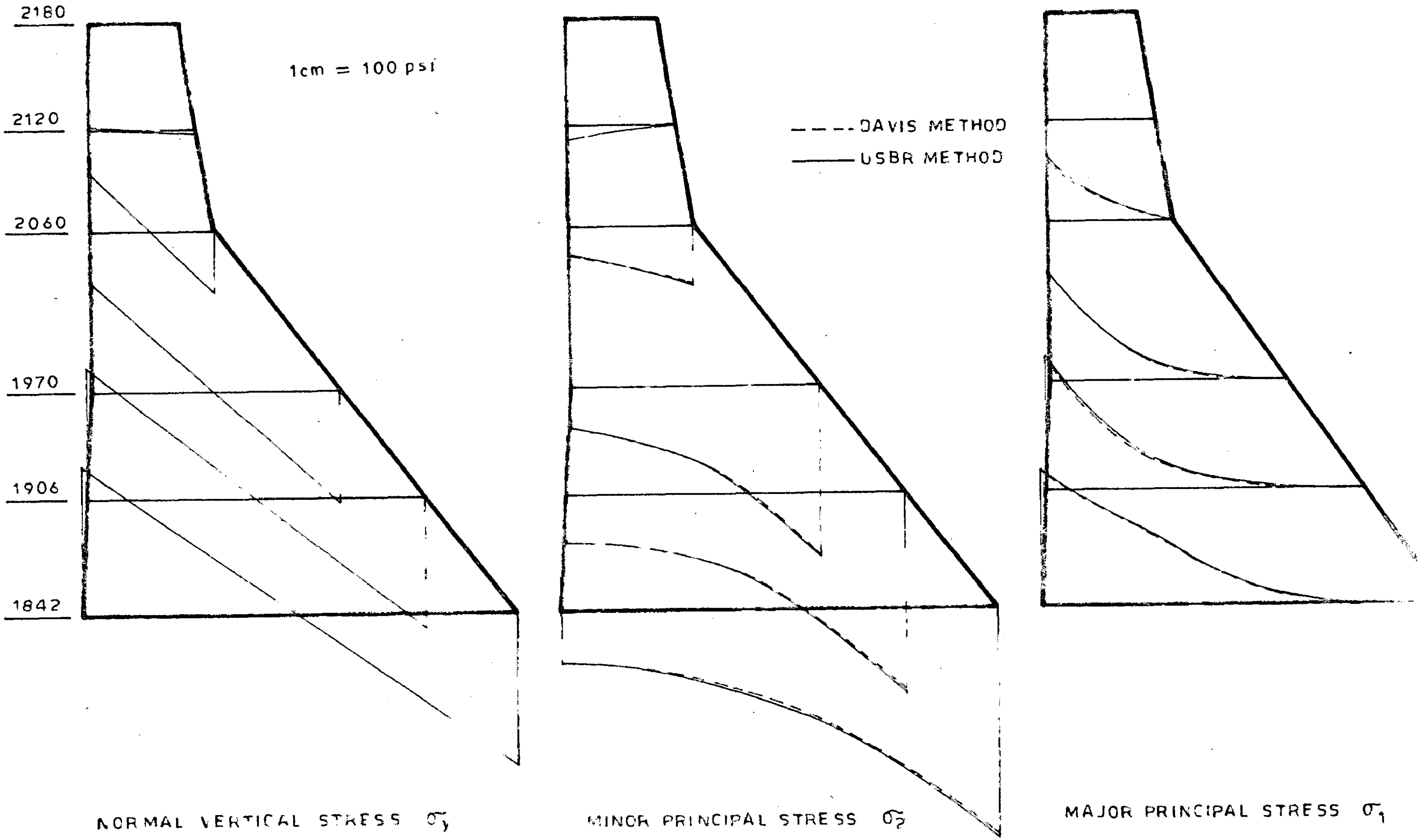


FIG. 19 _ STRESSES UNDER HYDRODYNAMIC PRESSURE ONLY
(KOYNA DAM)

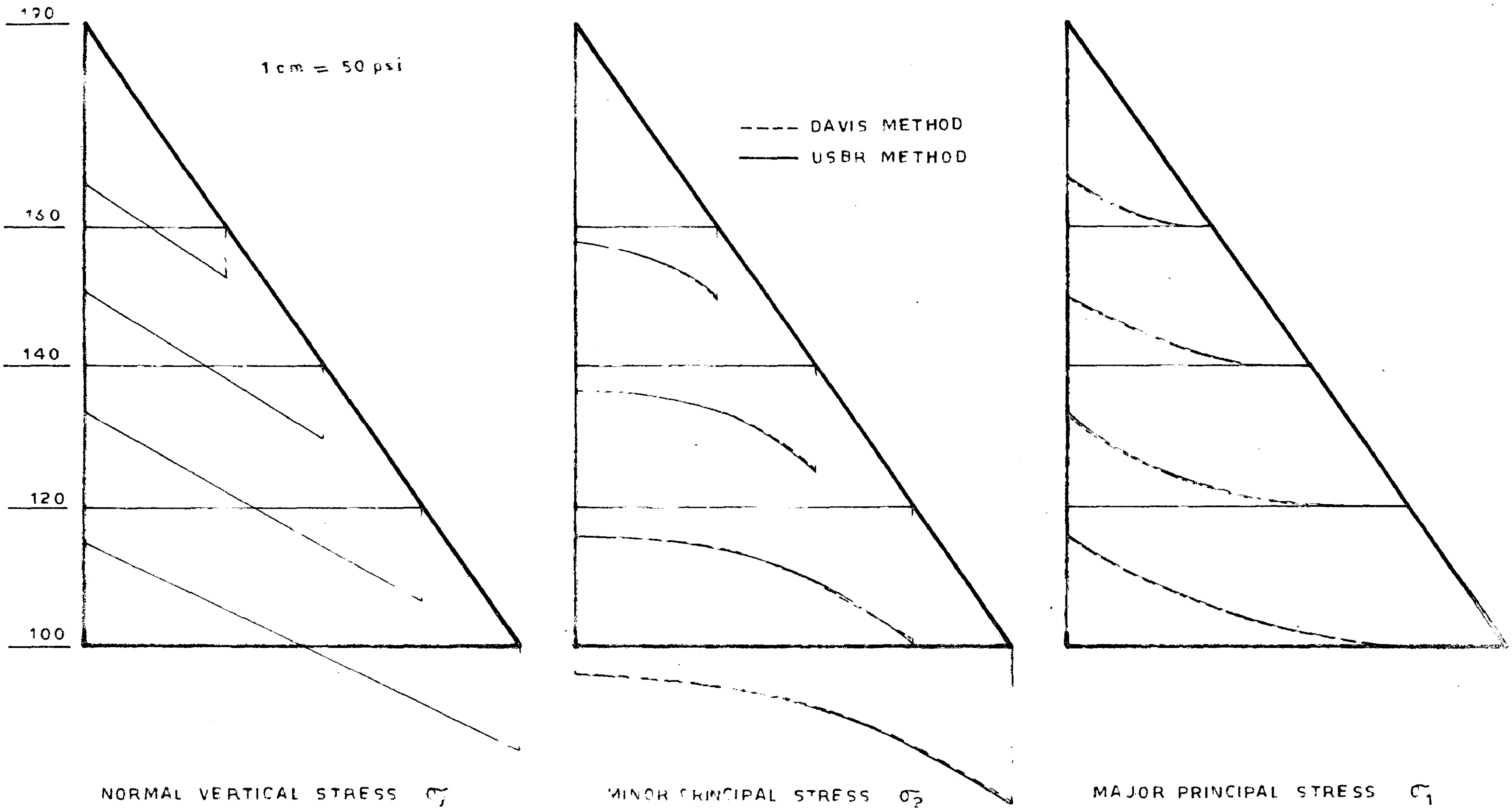


FIG. 20 _ STRESSES UNDER HYDRODYNAMIC PRESSURE ALONE

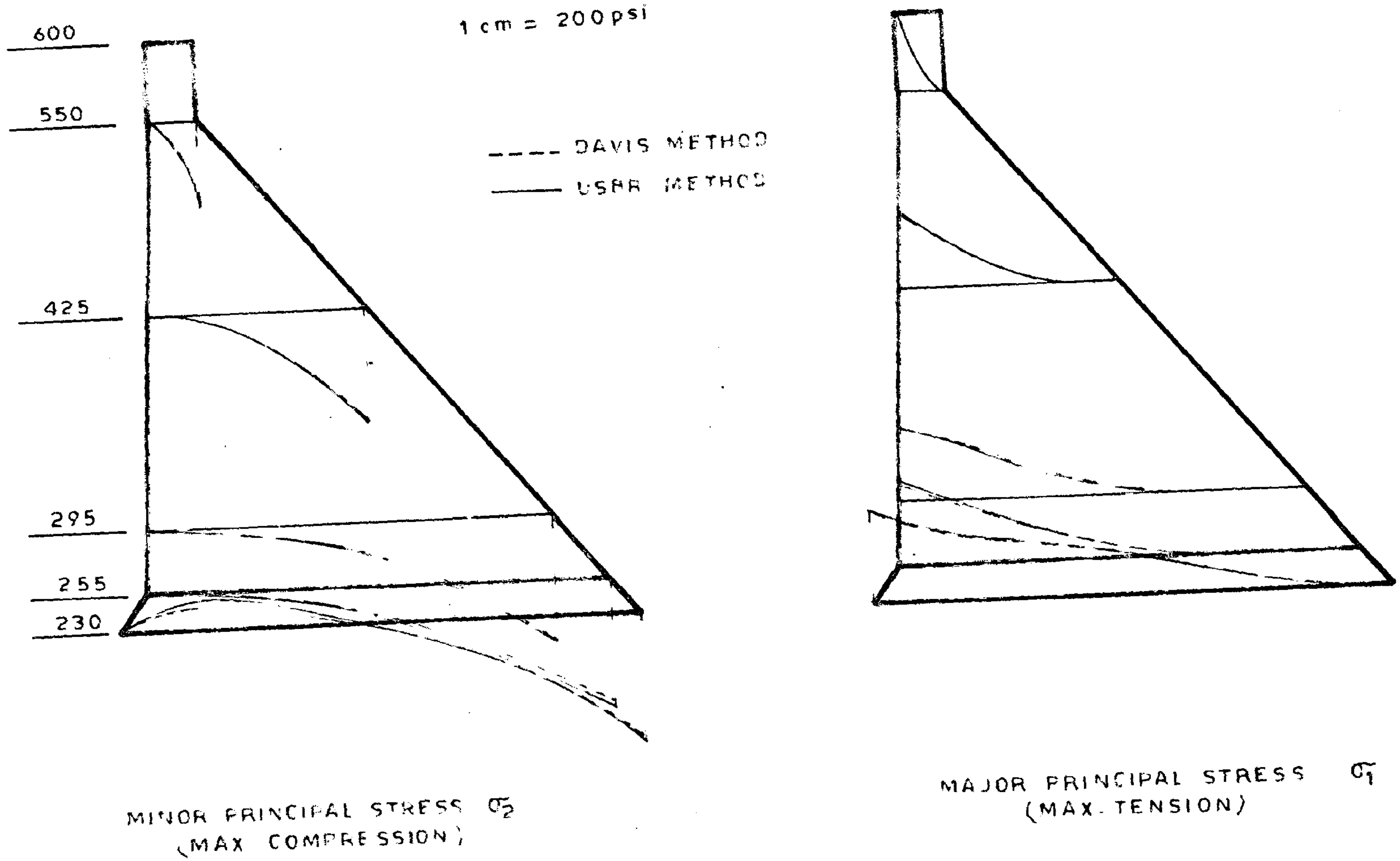


FIG. 21 _ STRESSES UNDER INERTIAL FORCE (LINEAR SEISMIC COEFFICIENT)
(NAGARJUNASAGAR DAM)

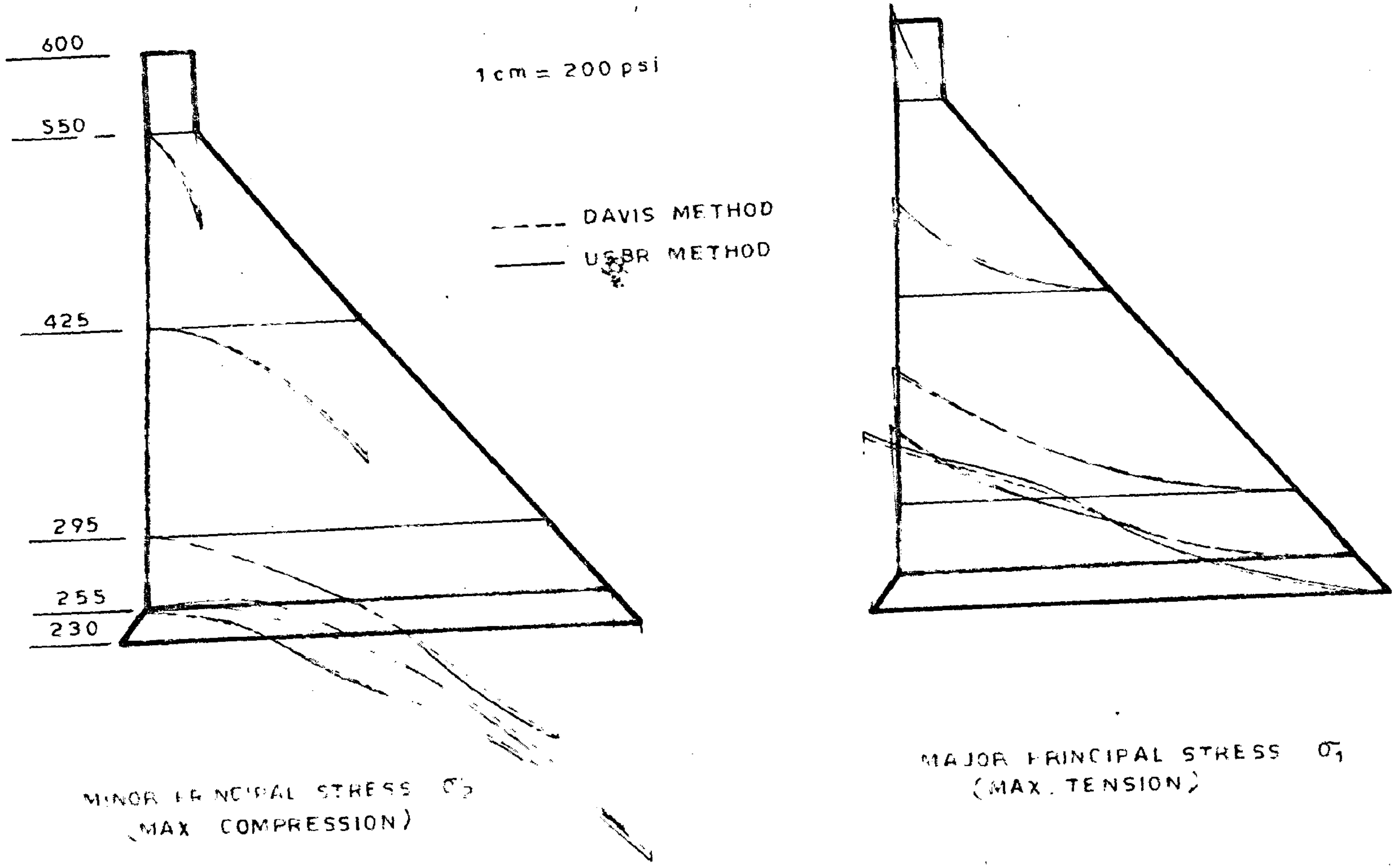


FIG. 22_STRESSES UNDER INERTIAL FORCE (UNIFORM SEISMIC COEFFICIENT)
(NAGARJUNASAGAR DAM)

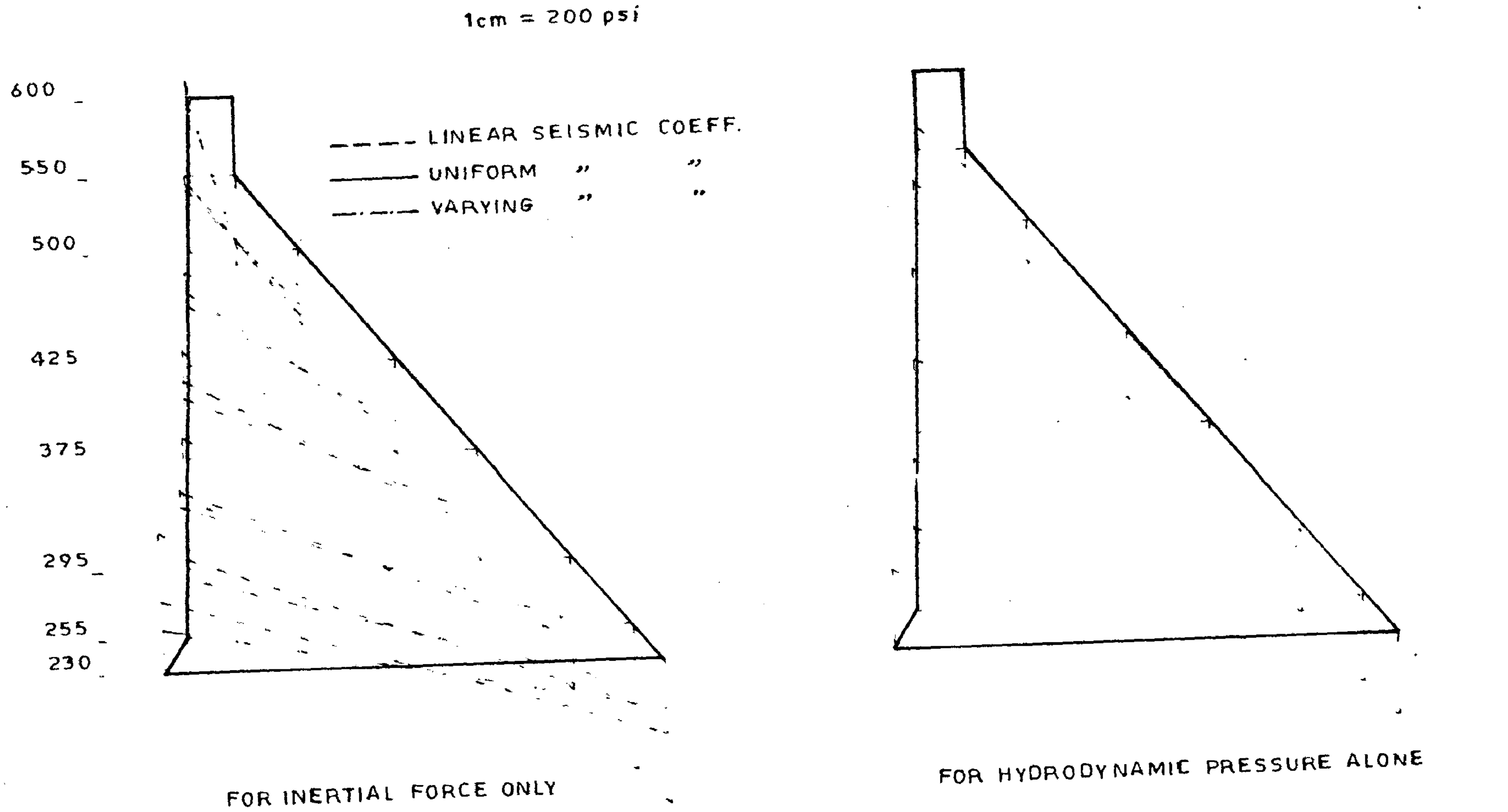


FIG. 23 – NORMAL VERTICAL STRESS σ_y UNDER INERTIAL FORCE AND HYDRODYNAMIC PRESSURE RESPECTIVELLY (NAGARJUNASAGAR DAM)

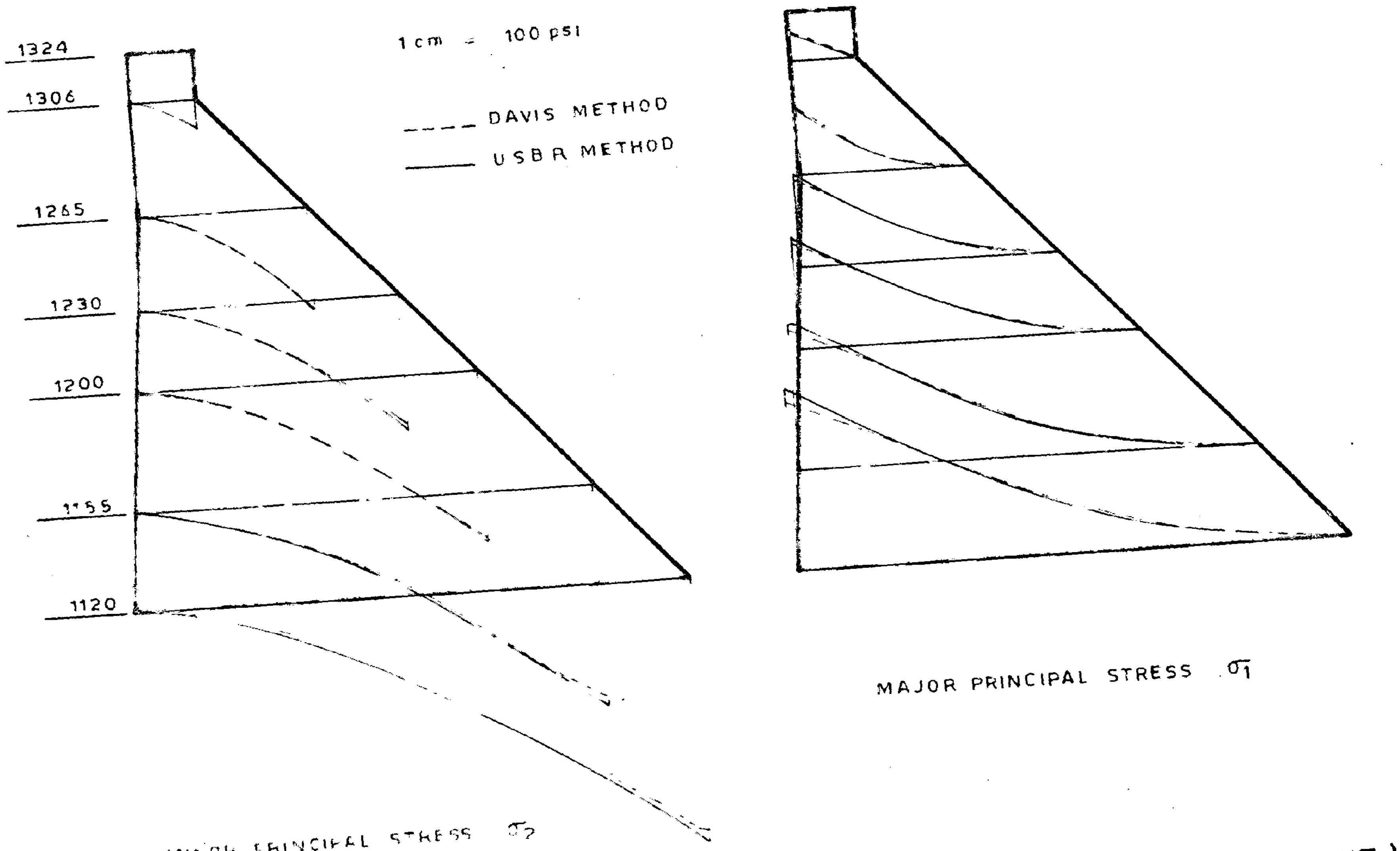


FIG. 24_STRESSES DUE TO INERTIAL FORCE (UNIFORM SEISMIC COEFFICIENT)
GANDHISAGAR DAM

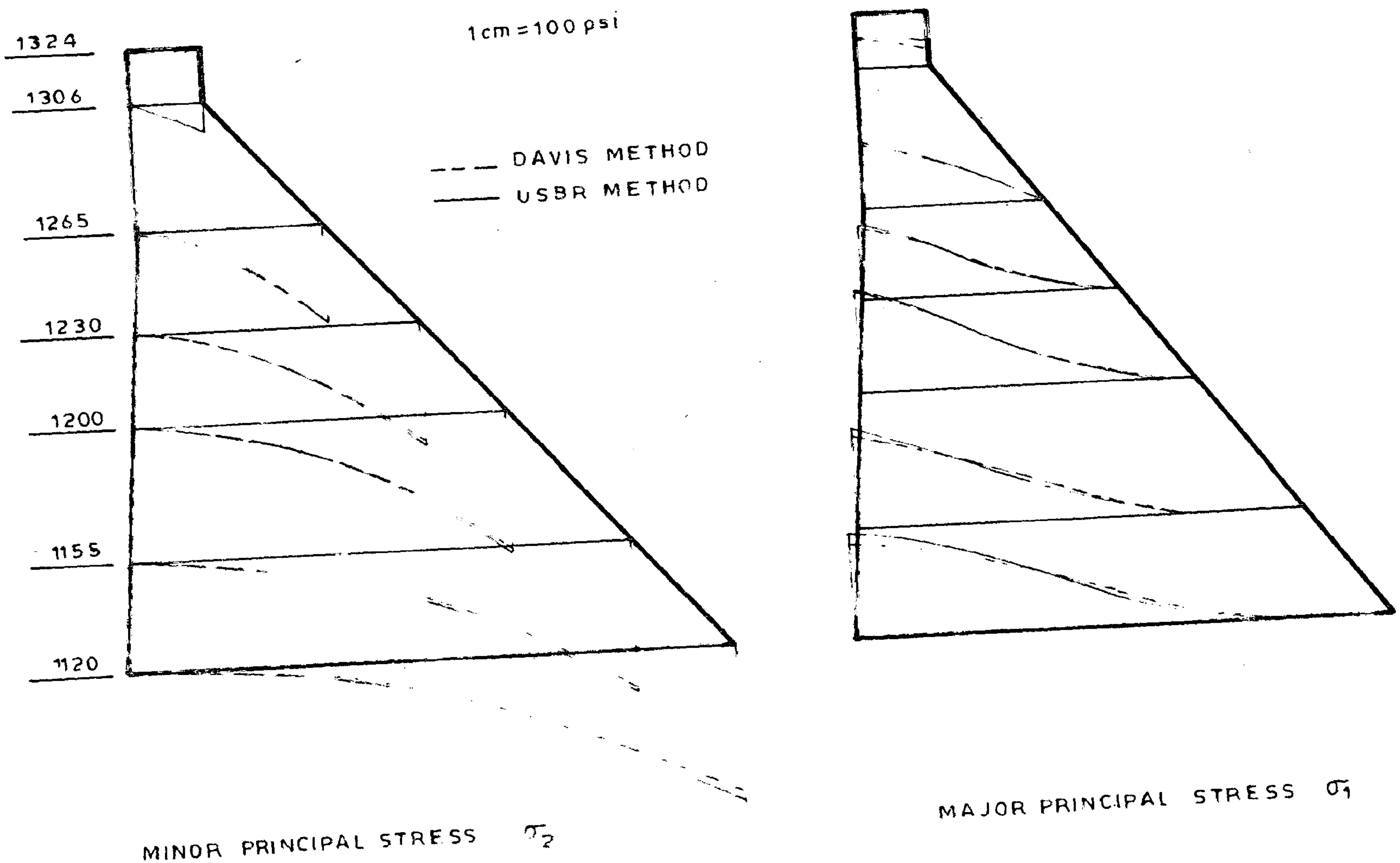


FIG. 25 – STRESSES DUE TO INERTIAL FORCE (LINEAR SEISMIC COEFF.) GANDHISAGAR DAM

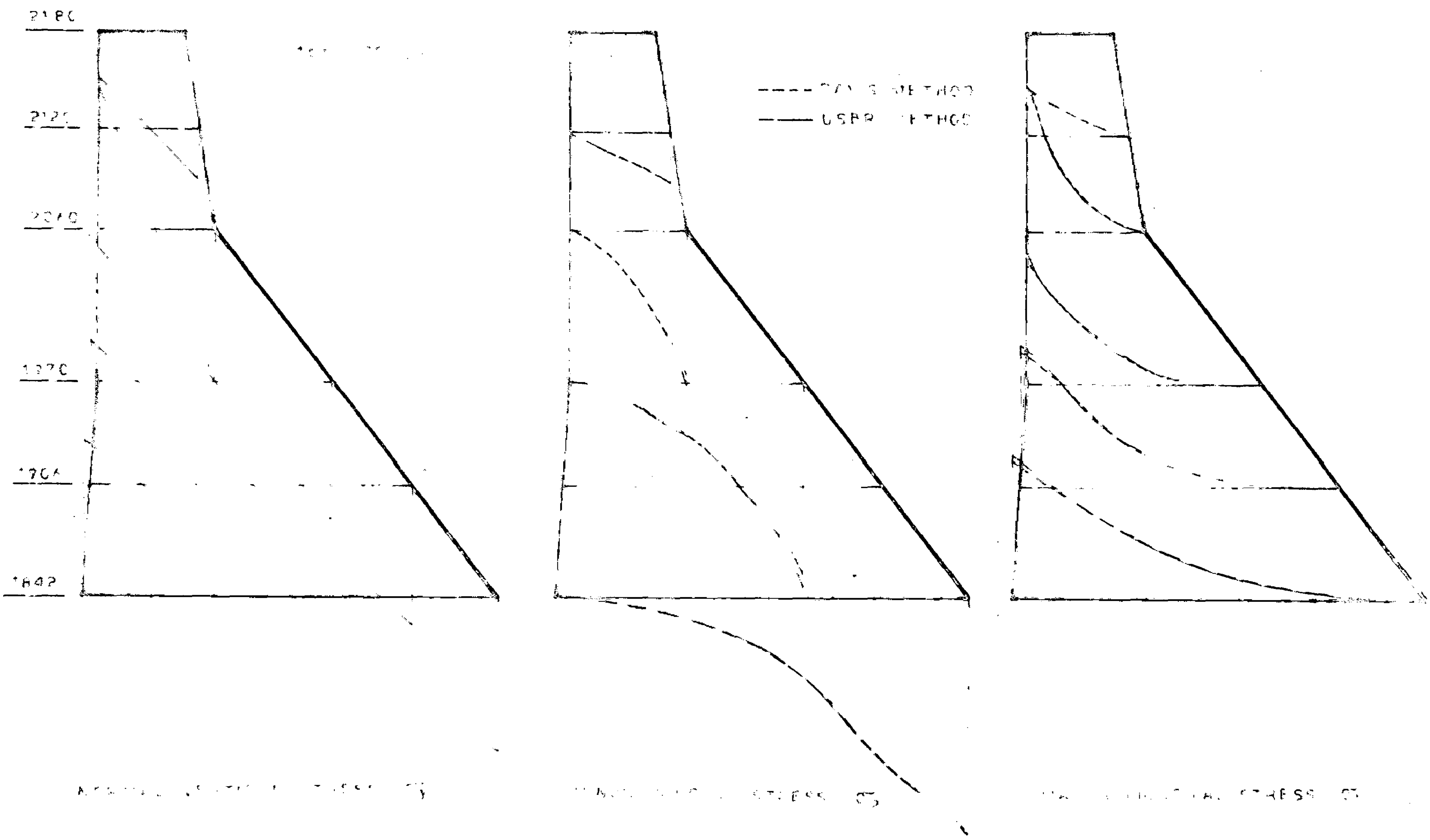


FIG. 2? – STRESSES UNDER INERTIAL FORCE (UNIFORM SEISMIC COEFFICIENT)
(KOYNA DAM)

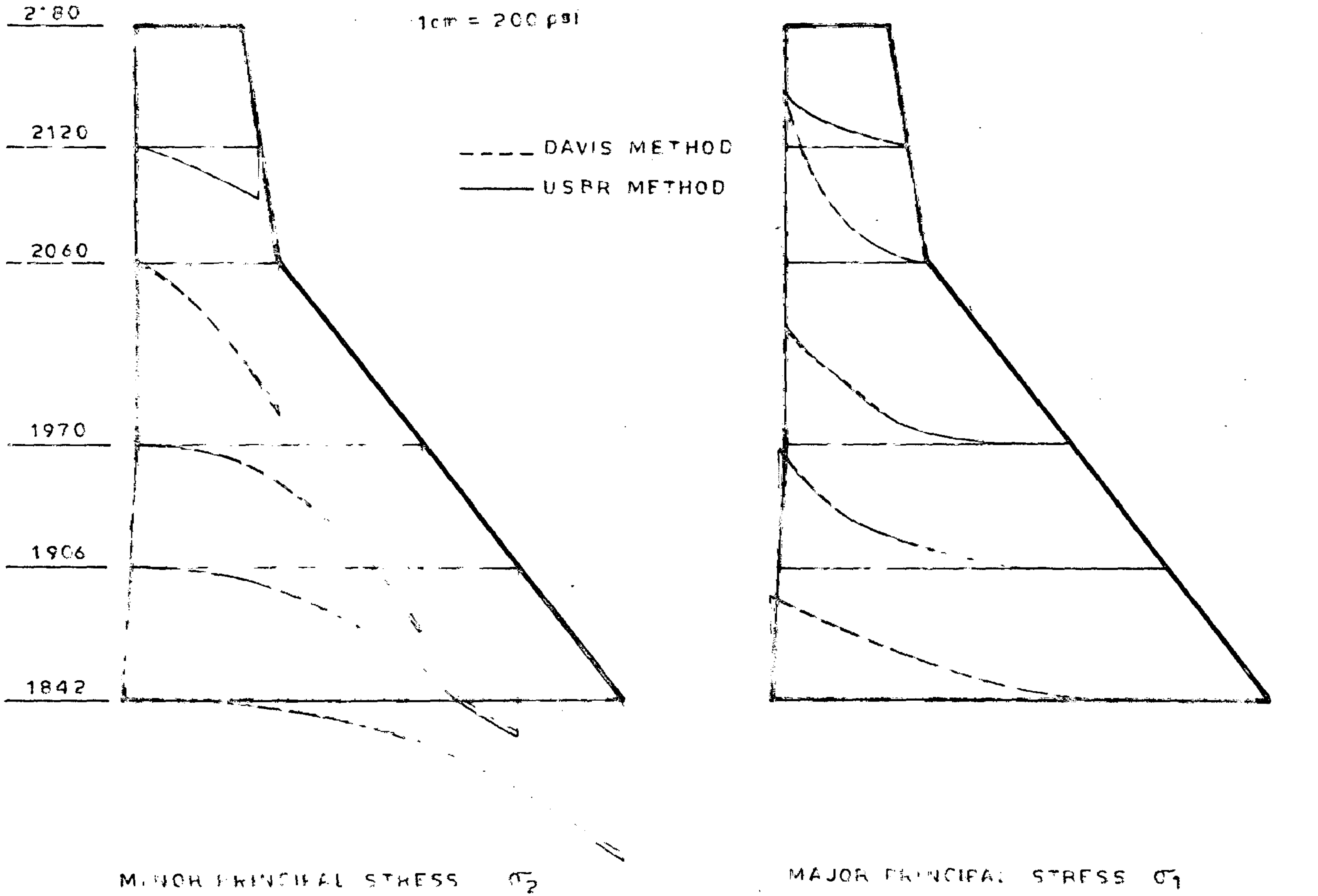


FIG.27 _ STRESSES UNDER INERTIAL FORCE (LINEAR SEISMIC COEFFICIENT)
(KOYNA DAM)

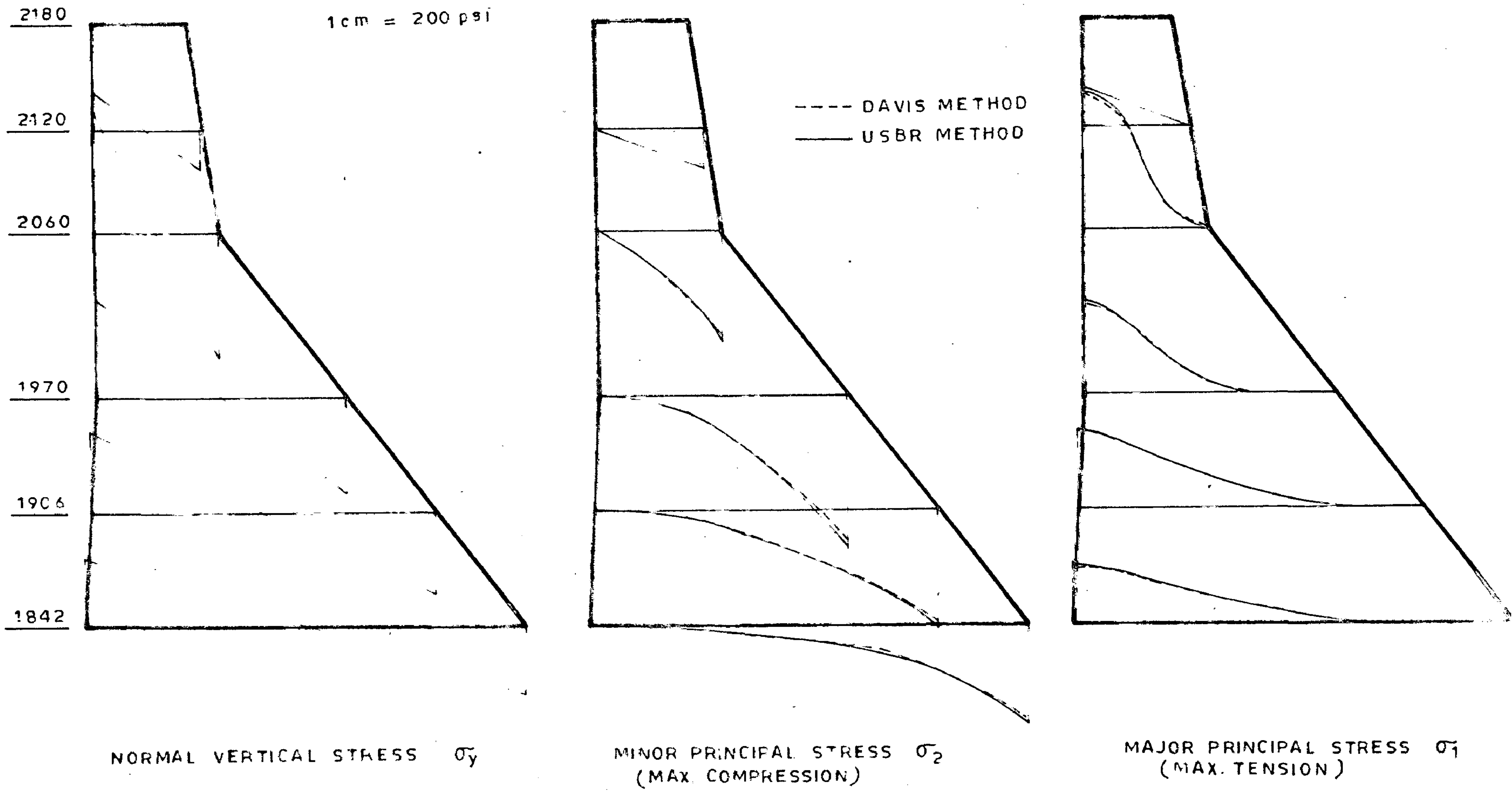


FIG. 28 _ STRESSES DUE TO INERTIAL FORCE ONLY (VARRYING SEISMIC COEFFICIENT)
KOYNA DAM

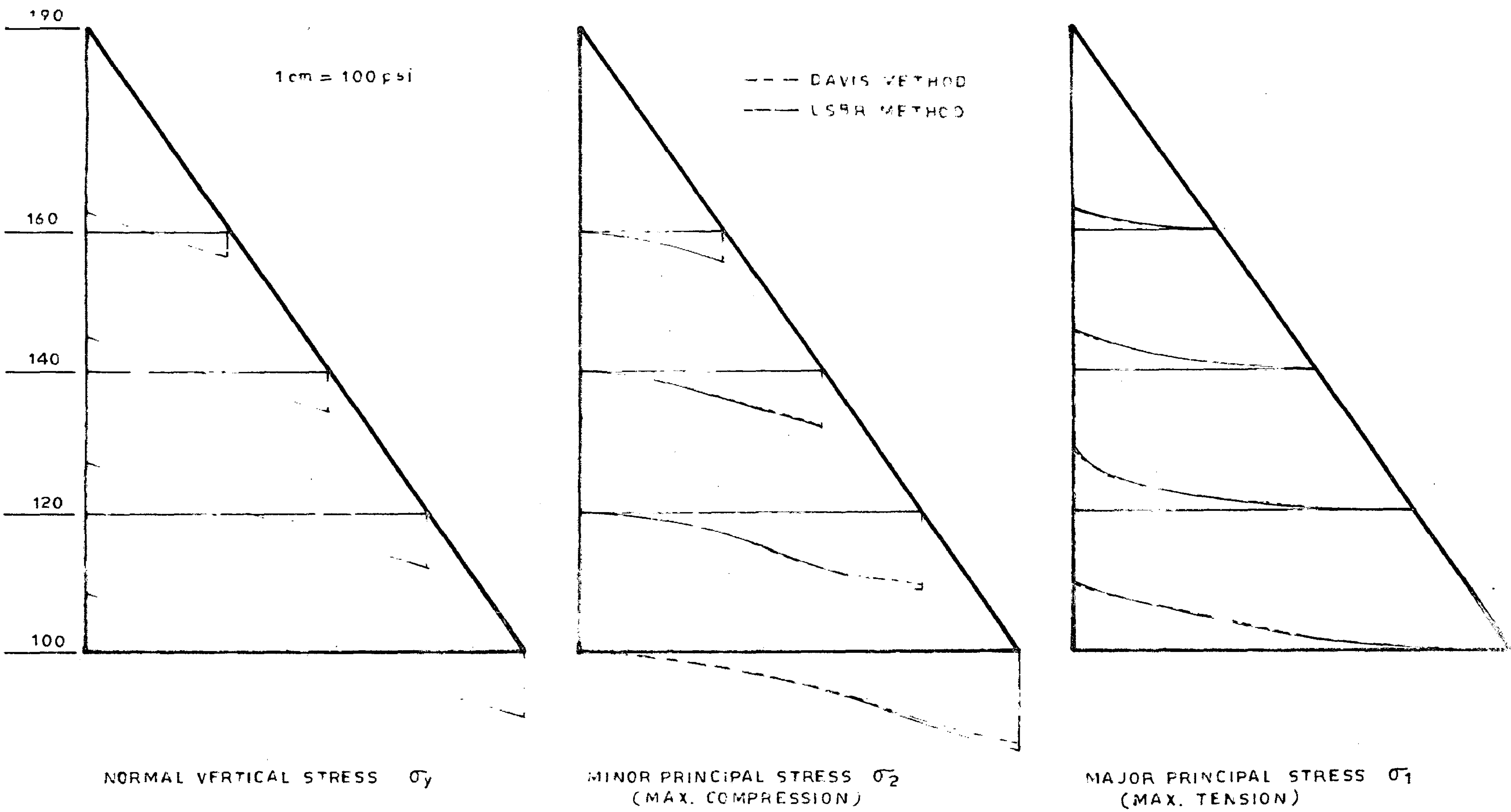


FIG. 29 _ STRESSES UNDER INERTIAL FORCE (UNIFORM SEISMIC COEFFICIENT)

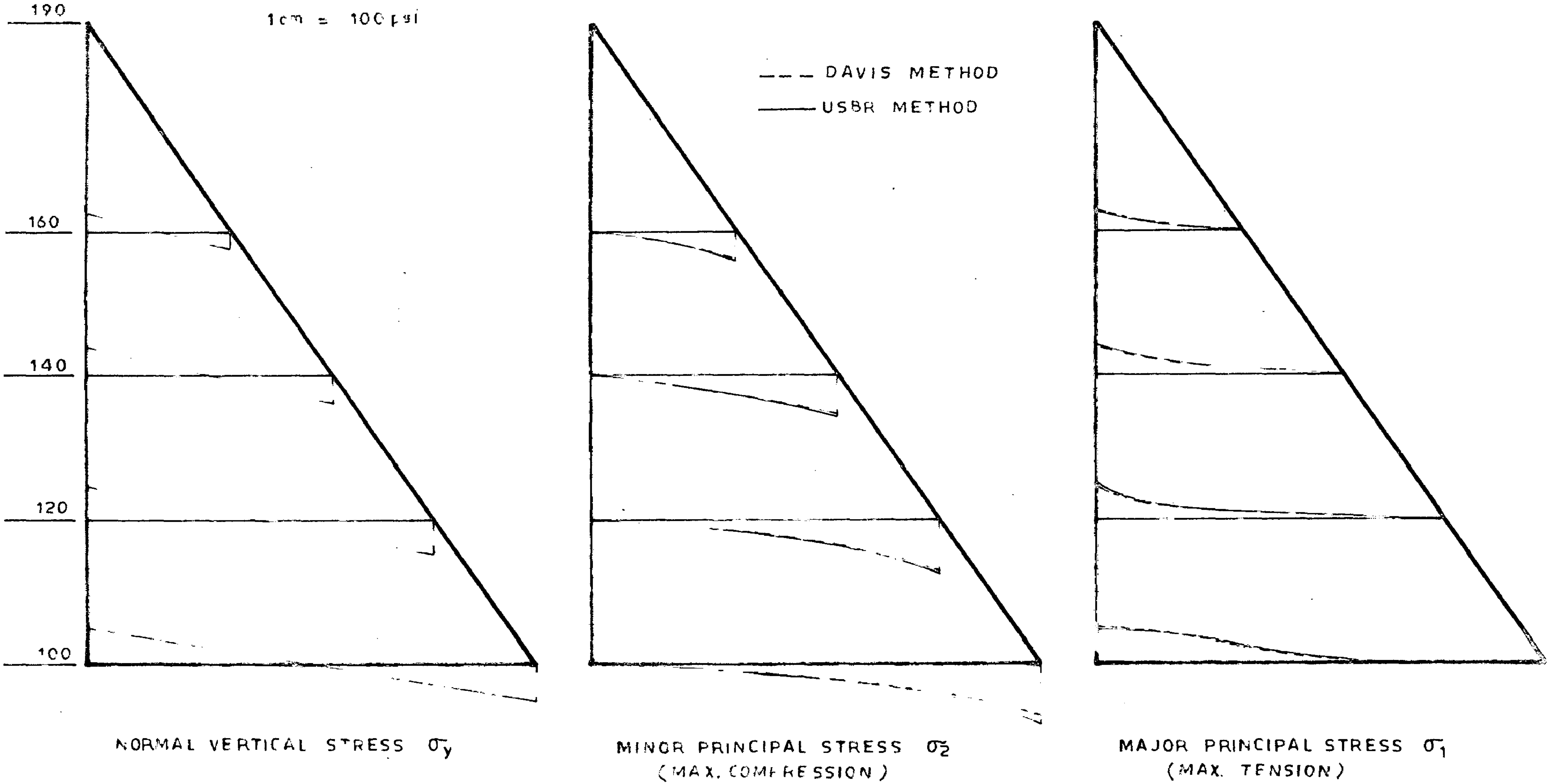


FIG.30_ STRESSES UNDER INERTIAL FORCE (LINEAR SEISMIC COEFFICIENT)

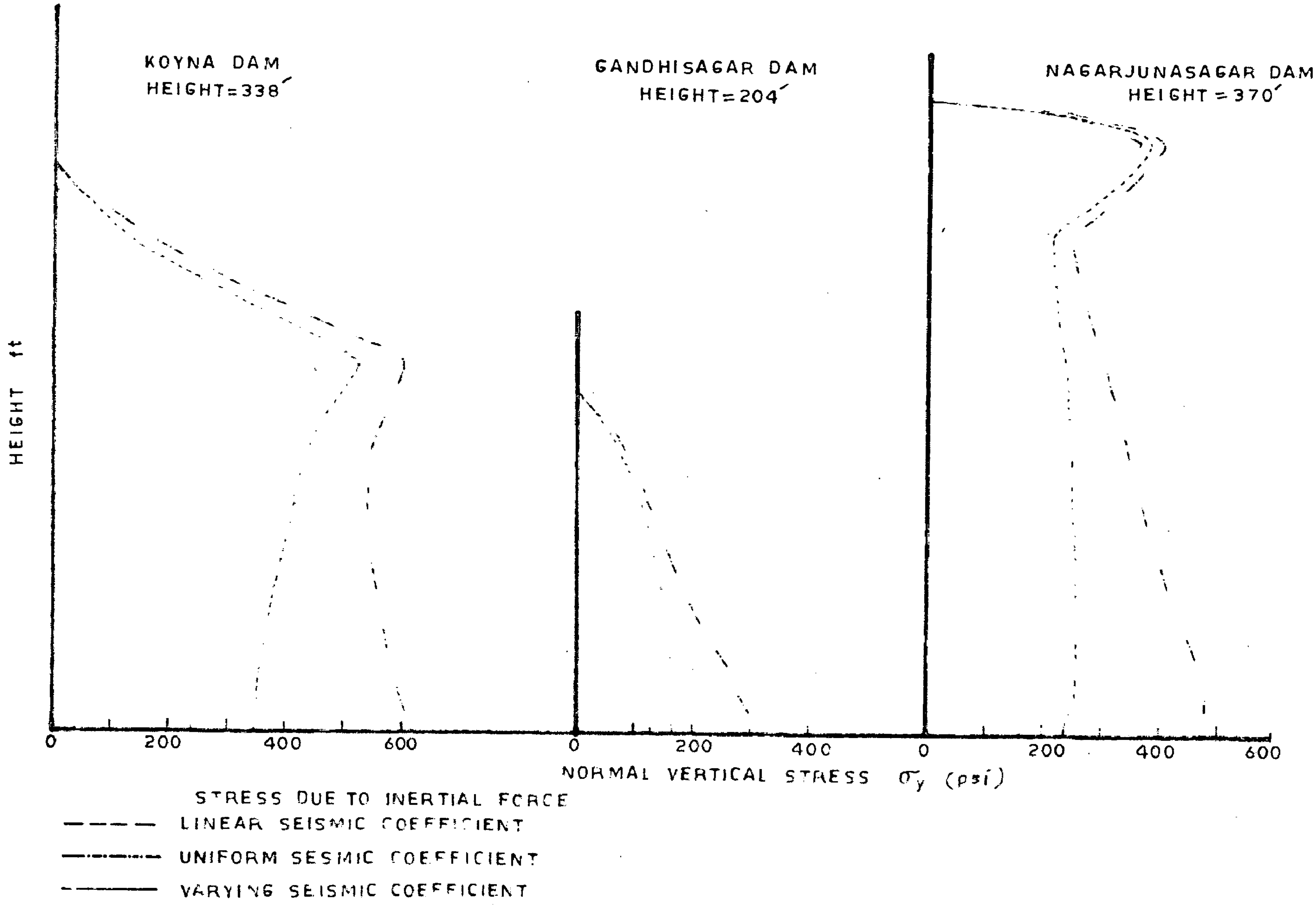
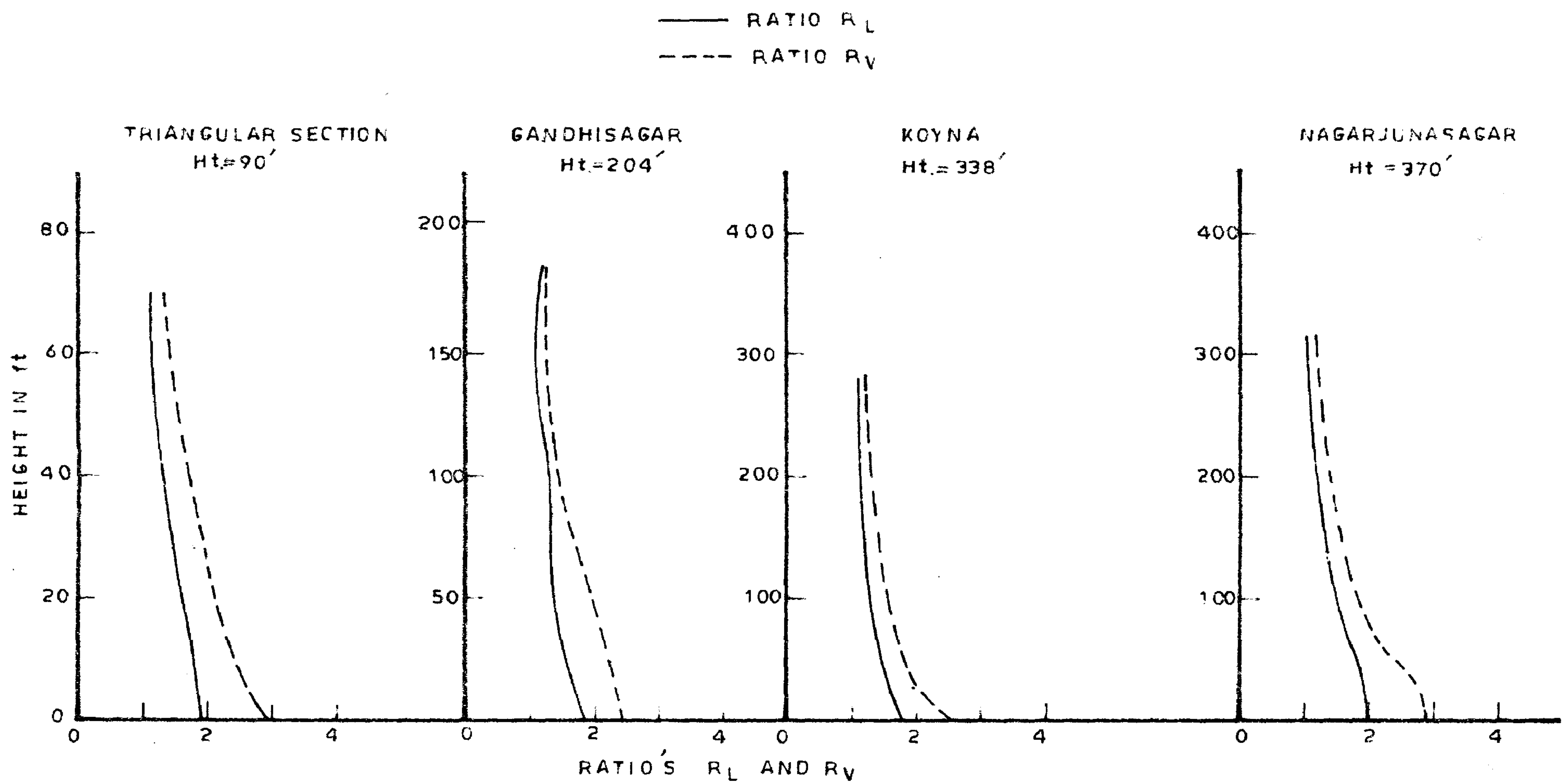

KOYNA DAM
HEIGHT=338'
GANDHISAGAR DAM
HEIGHT=204'
NAGARJUNASAGAR DAM
HEIGHT=370'
HEIGHT ft
0
200
400
600
0
200
400
0
200
400
600
NORMAL VERTICAL STRESS σ_y (psi)
STRESS DUE TO INERTIAL FORCE
LINEAR SEISMIC COEFFICIENT
UNIFORM SESMIC COEFFICIENT
VARYING SEISMIC COEFFICIENT

FIGURE _ 31



$$R_L = \frac{\text{STRESSES DUE TO UNIFORM SEISMIC COEFFICIENT}}{\text{STRESSES DUE TO LINEAR VARIATION OF SEISMIC COEFF.}}$$

$$R_V = \frac{\text{STRESSES DUE TO UNIFORM SEISMIC COEFF.}}{\text{STRESSES DUE TO VARYING SEISMIC COEFF.}}$$

FIG. 32 _ RATIO OF NORMAL VERTICAL STRESS (σ_y) DUE TO INERTIAL FORCE WITH THREE VARIATIONS OF SEISMIC COEFFICIENT

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APPENDIX 'A'

Method based on Theory of Elasticity (3)

Since the assumption of linear normal stress is found incompatible with theory of elasticity for the regions at and near the base, therefore, the stress function is so assumed that it satisfies compatibility of stress and strain and also the boundary conditions at the infinite limits of plane of the foundation.

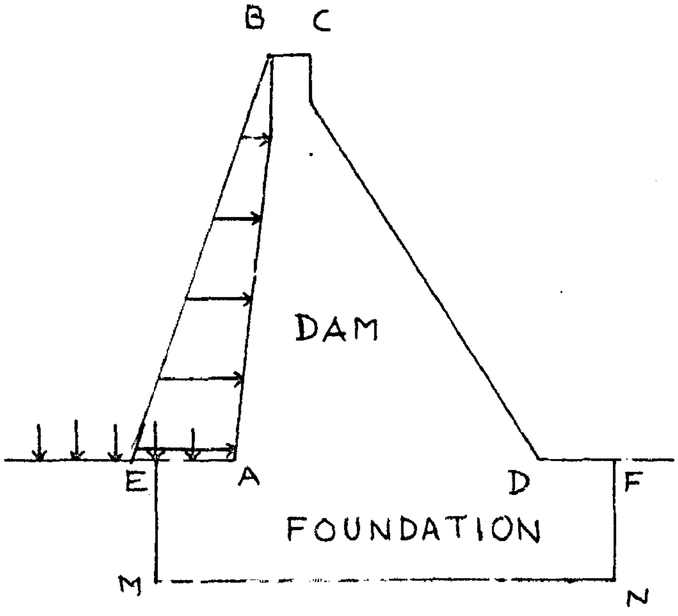
Consider a dam as shown in Figure (33a)

Fig. 33a shows the type of loading when reservoir is full. If the dam is cut along AD, the equivalent static loading can be represented as in Fig. 33b.

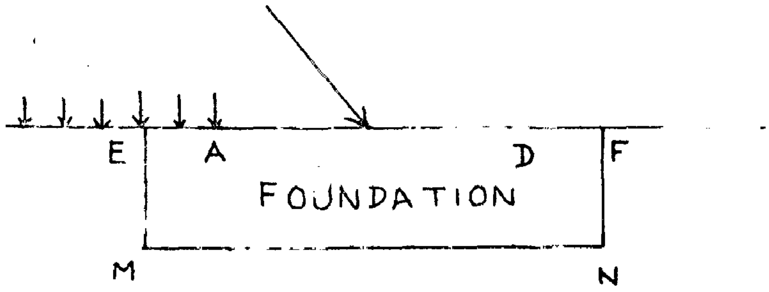
Then by St. Venants principle (4) stresses at distances far from the boundary will be same for the two cases of loading (Fig. 33a and 33b).

Taking origin at A, and the axes as shown consider the equilibrium of an element inside the dam.

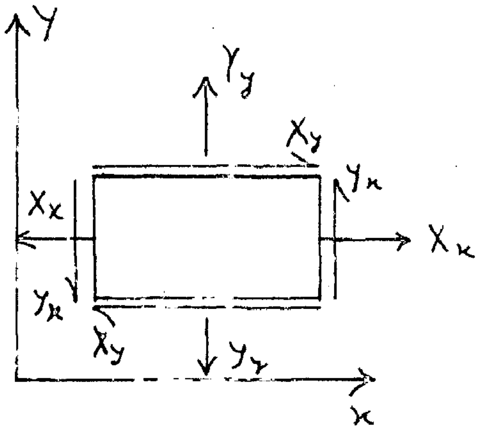
Positive direction of stresses on an element are shown in Fig. 33.


B
C
DAM
E
A
D
F
FOUNDATION
M
N

(a)


E
A
D
F
FOUNDATION
M
N

(b)


Y
Yy
Xy
Yx
Xx
Xx
Yx
Xy
Yy
x

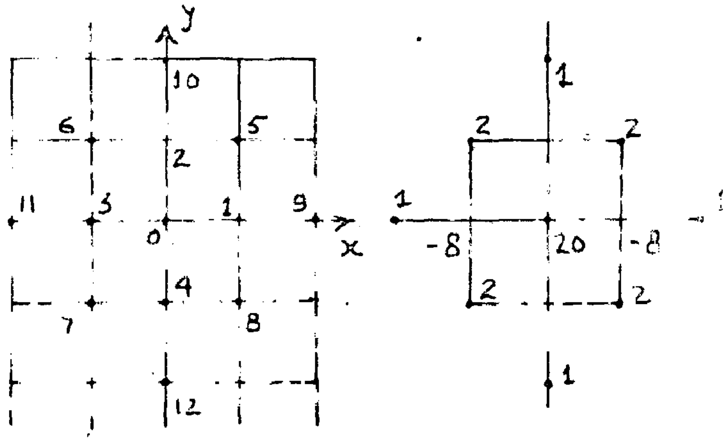

y
10
6
5
2
11
3
0
1
9
x
4
7
8
12
1
2
2
1
-8
20
-8
2
2
1

FIGURE_ 33

Assume Airy's stress function 'ϕ' (4) such that

$$X_x = \frac{\partial^2 \phi}{\partial y^2} \quad \ldots \quad \ldots \quad (1)$$

$$Y_y = \frac{\partial^2 \phi}{\partial x^2} + w \rho g y \quad \ldots \quad (2)$$

$$X_y = \frac{\partial^2 \phi}{\partial x\, y} \quad \ldots \quad \ldots \quad (3)$$

where,

w = density of water

ρ = Sp. density of material

g = accl. due to gravity

X_x and Y_y = Normal stresses, first letter denote direction of normal stress and subscript denote the direction of normal to the plane on which these stresses are acting.

$X_y = Y_x$ = Shear stresses

ϕ = Stress function

This stress function satisfies eqbm. equations for the case of plane stress. Eqbm. Equations are (4)

$$\frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + F_x = 0$$

$$\frac{\partial Y_x}{\partial x} + \frac{\partial Y_y}{\partial y} + F_y = 0 \qquad \ldots (4)$$

where, (F_x & F_y) are body forces.

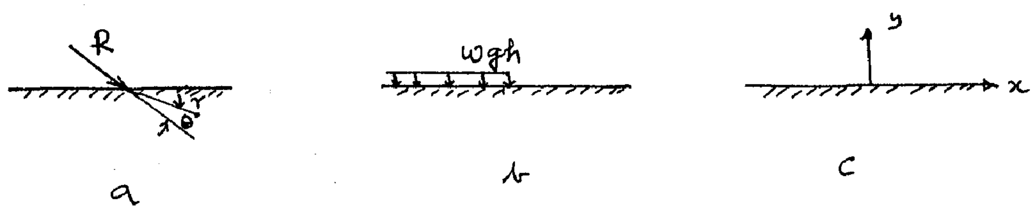
Compatibility of Strains requires (4),

$$\nabla^4 \phi = \frac{\partial^4 \phi}{\partial x^4} + \frac{\partial^4 \phi}{\partial x^2\, y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0 \qquad \ldots (5)$$

Now if ϕ satisfies Equation (5) and also the b.c. at boundary EABCDFMN, the assumed stress function will be correct.

BOUNDARY CONDITIONS (4)

The value of stress function along boundary AEMNFD can be found from the usual formulae (4). These are due to the three types of loading as shown.



(A) An inclined concentrated line load R, gives a stress function (4)

$$\phi' = -\frac{R}{\pi} \; r\,\theta\, \sin\theta \qquad \ldots \quad (6)$$

r and θ polar coordinates measured from line of action of load.

(B) A distributed uniform pressure Wgh starting at A and going to infinity (4).

$$\phi'' = -\frac{Wgh}{2\pi}\left[(x^2 + y^2) + \tan^{-1}\left(-\frac{y}{x}\right) + xy\right] \ldots (7)$$

(C) Weight of the Foundation Material giving (4)

$$\phi''' = -\frac{\nu}{1-\nu}\, wg\, \rho \,\frac{y^3}{6} \qquad \ldots \qquad \ldots \qquad (8)$$

ν = poissions ratio.

Total value of stress is sum of these three values as each of these is linear

i.e., $\phi = \phi' + \phi'' + \phi'''$

When Reservoir is empty

(i) $\phi''' = 0$

(ii) Load R in ϕ' is vertical.

<u>'ϕ' and its gradients along dam surface ABCD</u>

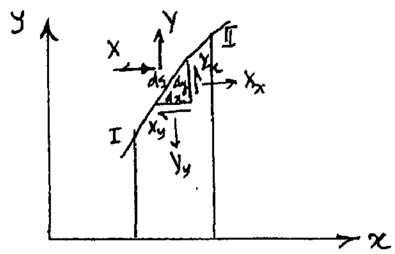


FIG A.1

Let x andy represent the component boundary forces per unit length of the boundary 'S'. Consider the Eqbm. of an surface element bounded by dx, dy and ds.

Along x - directions :

$$x\ ds + X_x . dy - X_y . dx = 0 \quad \ldots \quad (9)$$

Along Y - direction :

$$Y\ ds - Y_y\ dx + X_y\ dy = 0 \quad \ldots \quad (10)$$

Substituting for X_xm X_y and Y_y in terms of ϕ from equns. (1), (2) and (3).

$$Xds + \frac{\partial^2 \phi}{\partial y^2} dy + \frac{\partial^2 \phi}{\partial x\ y} dx = 0$$

and,

$$Yds = \left(\frac{\partial^2 \phi}{\partial x^2} + w \rho g y\right) dx - \frac{\partial^2 \phi}{\partial x\ y} dy = 0$$

or

$$Xds = -\left(\frac{\partial^2 \phi}{\partial y^2} dy + \frac{\partial^2 \phi}{\partial x\ y} dx\right)$$

$$= -d\left(\frac{\partial \phi}{\partial y}\right) \quad \ldots \quad \ldots \quad (11)$$

and,

$$Yds = d\left(\frac{\partial \phi}{\partial x}\right) + w \rho g\ y\ dx \quad \ldots \quad (12)$$

Integrating (11) and (12) from I to II on boundary

$$\left[\frac{\partial \phi}{\partial x}\right]_{I}^{II} = \int_{I}^{II} Y ds - \int_{I}^{II} w \rho g \; y \; dx \quad \ldots \quad (14)$$

$$\left[\frac{\partial \phi}{\partial y}\right]_{I}^{II} = - \int_{I}^{II} X \; ds \quad \ldots \quad (13)$$

Thus the value of $\frac{\partial \phi}{\partial y}$ and $\frac{\partial \phi}{\partial x}$ can be determined for all points along boundary as these are known at A.

Note that RHS of (13) represents the total horizontal force acting between points I and II of the boundary, and the RHS of (14) represents the total vertical force between the points I and II of boundary, and the weight of portion under this boundary but above X axis.

In order to determine the changes in 'ϕ' between points I and II, consider M, the clockwise moment of all the boundary forces between these limits and about point II.

From Fig. (A.1)

$$dM = X \, (y - Y_{II}) \, ds - Y(x - x_{II}) \, ds \quad \ldots \quad (15)$$

Using (11) and (12), we get

$$dM = - d \left(\frac{\partial \phi}{\partial y} \right) (y - y_{II}) - d\left(\frac{\partial \phi}{\partial x}\right) (x-x_{II})$$

$$- w \rho g \; h \, (x - x_{II}) \, dx \quad \ldots \quad (16)$$

Integrating (16) by parts between limits I and II we get,

$$M = \frac{\partial \phi}{\partial y} (y_{II} - y) + \frac{\partial \phi}{\partial y} dy +$$

$$\frac{\partial \phi}{\partial x} (x_{II} - x) + \frac{\partial \phi}{\partial x} dx + w \rho g y (x_{II}-x) dx$$

or, Since

$$\int_{I}^{II} \frac{\partial \phi}{\partial x} dx + \int_{I}^{II} \frac{\partial \phi}{\partial y} dy = \int_{I}^{II} d\phi = \left[\phi\right]_{I}^{II}$$

$$\left[M\right]_{I}^{II} = \frac{\partial \phi}{\partial y_I} (y_I - y_{II}) + (\frac{\partial \phi}{\partial x_I}) (x_I - x_{II})$$

$$+\left[\phi\right]_{I}^{II} + \int_{I}^{II} w \rho g h (x_{II} - x) dx$$

Rearranging,

$$\left[\phi\right]_{I}^{II} = \frac{\partial \phi}{\partial y_I} (y_{II} - y_I) + (\frac{\partial \phi}{\partial x_I}) (x_{II} = x_I) + \left[M\right]_{I}^{II}$$

$$+ \int_{I}^{II} w \rho g y (x - x_{II}) dx. \quad \ldots \quad (17)$$

So that the value of ϕ and all its derivatives along the boundary ABCD, can be found as value of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ are already known.

It may be noted that the last two terms in equn. (17) represent the clockwise moment about point II due to the boundary forces between points I and II and the weight of the material bounded by the x-axis and that portion of the boundary.

Equations (13), (14) and (17) take on a simple form if the boundary between I and II is a straight line. So that if the curved profile of a dam is divided into a series of short straight lines, close enough to represent it to any degree of accuracy desired, the value of ϕ and its gradient can be determined at all points by a step by step process.

Initial values of these functions are known at poin A and D.

Finite Difference Approximation

Having determined the boundary values and gradients of ϕ, it now remains to find their function at all points inside the dam such that governing equation (5) is satisfied.

For this purpose a square mesh (fig. 33) is drawn and the values of function is considered only at mesh points.

Taking O as origin ϕ can be explained in double Taylors series.

$$\phi = \phi_o + A_{1,0}\, x + A_{o,1}\, y + A_{2,o}\, x^2 + A_{1,1}\, xy$$
$$+ A_{0,2}\, y^2 + A_{3,0}\, x^3 + A_{2,1}\, x^2y + A_{1,2}\, xy^2$$
$$+ A_{o,3}\, y^3 + \ldots + A_{5,0}\, x^5 + A_{4,1}\, x^4y + A_{3,2}\, x^3y^2$$
$$+ A_{2,3}\, x^2y^3 + A_{1,4}\, xy^4 + A_{0,5}\, y^5 + \ldots + \text{higher}$$
$$\text{degree terms} \qquad \ldots \quad (18)$$

Coefficient $A_{n,m}$ of x^n, y^m stands for

$$\frac{1}{nm} \; \frac{\delta^{n+m}}{\delta x^n,\, y^m}\, \phi$$

Substituting appropriate coordinates, and neglecting higher degree terms, it can be shown

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\,\phi = 2a^2(A_{2,0} + A_{o,2})$$
$$+ 2a^4(A_{4,0} + A_{o,4}) \quad \ldots \qquad (19)$$

$$\phi_9 + \phi_{10} + \phi_{11} + \phi_{12} - 4\phi_o = 8a^2(A_{2,0} + A_{0,2})$$
$$+ 32a^4 + (A_{4,0} + A_{0,4}) \quad \ldots \qquad (20)$$

$$\phi_5 + \phi_6 + \phi_7 + \phi_8 - 4\phi_o = 4a^2(A_{2,0} + A_{o,2})$$
$$+ 4a^4(A_{4,0} + A_{0,4}) + 4A_{2,2}\, a^4 \quad \ldots \qquad (21)$$

Governing Equation :

$$\nabla^4 \phi = 24(A_{4,0} + A_{0,4}) + 8\ A_{2,2} \qquad \ldots (22)$$

using (19, 20, 21)

$$\nabla^4 \phi = a^4 \Big[20\ \phi_o + 2\ (\phi_5 + \phi_6 + \phi_7 + \phi_8) + (\phi_6 + \phi_{10} + \phi_{11} + \phi_{12}) - 8\ (\phi_1 + \phi_2 + \phi_3 + \phi_4) \Big] \ldots \qquad \ldots (23)$$

which is a finite diff. approximation to equation (17).

Approx. can be made as good as desired by taking smaller value of 'a'. Errors involved being of the order of a 6.

Similar equation can be written for all the mesh points in the region.

A solution can now be obtained by solving the system of n equations, obtained for n points considered, in the interior of region.

Method of relaxation developed by Prof. Southwell, enables a rapid solution of such system of equations.

<u>Method of Relaxation</u> (13, 3)

Suppose some initial values to be given to ϕ at all points of the mesh, chosen as a rough guess of the solution, then equation (23) takes as a form

$$20\,\phi_0 + 2(\phi_5 + \phi_6 + \phi_7 + \phi_8) + (\phi_9 + \phi_{10} + \phi_{11} + \phi_{12}) - 8\,(\phi_1 + \phi_2 + \phi_3 + \phi_4) = F_0 \quad ,\ldots(24)$$

F_0 in general is different from zèro and is called residual at 0. If ϕ_0 is changed by unity residual is changed by 20 and at other points as shown in fig. 33.

If the correction is applied at the point where largest residual occur so as to cancel it; the residual at surrounding points will change by smaller corresponding amount. This process of continual correct-ion is convergent and desired accuracy can be obtained.

<u>Orders of Errors Involved</u>

Approximations made are :

(1) Stresses on the line EMNF, which is at a fixed distance from the dam, are fixed.

(2) The finite difference equations are not solved completely, as some small resi-duals usually, remain.

(3) Finite difference equations do not represent exactly the differential equations.

These sources of errors can be reduced as much as desired by taking the line EMNF farther away and taking the small mesh size.

APPENDIX - B

Listing of Computer Programmes

```
$IBJOB
$IBFTC
C*************************************************************************
C     THIS PROGRAM CALCULATES THE INTERNAL STRESSES IN CONCRETE
C     GRAVITY DAMS. THE METHOD OF ANALYSIS IS USBR METHOD. STRES-
C     SES ARE CALCULATED FOR DEAD WT., HYDROSTATIC PRESSURE,
C     UPLIFT PRESSURE, HYDRODYNAMIC PRESSURE AND INERTIAL FORCES
C     SEPARATELY.
C     THE INPUT INFORMATION CONSIST OF THE MATERIAL AND GEOMETR-
C     ICAL PROPERTIES OF THE DAM.
C     THE OUTPUT INFORMATION CONSISTS OF ELEVATION OF THE PLANE
C     AT WHICH THE STRESSES ARE DESIRED,THE THREE COMPONENTS OF
C     STRESSES WITH THE PRINCIPAL STRESSES AT DIFFERENT POINTS
C     ON THAT PLANE.
C     THE PROGRAM CAN BE USED ON EITHER OF THE IBM-1620 OR IBM-7044
C     THE OUTPUT FORMATS MAY BE CHANGED TO SUIT THE COMPUTER.
C*************************************************************************
      DIMENSION Z(200),E(200),F(200),BW(200),AL(200)
      READ101,IL,JL,INDEX
      READ100,TW,DC,DW,H,FB,DD,ALS,CM,YY1
      READ100,(Z(I),I=2,IL)
      READ100,(E(I),I=2,IL)
      READ100,(F(I),I=2,IL)
      READ100,(AL(I),I=2,IL)
      PRINT101,IL,JL,INDEX
      PRINT103,TW,DC,DW,H,FB,DD,ALS,CM,YY1
      PRINT103,(Z(I),I=2,IL)
      PRINT103,(E(I),I=2,IL)
      PRINT103,(F(I),I=2,IL)
C     CONSTANTS
      AT=0.666666667
      BT=0.33333333
      PI=3.141592653589
      PA=90./PI
      PSI=1./144.
      DEE=0.5*DC
      ALP=.5*CM*ALS*DW*(H-FB)
C     INITIALIZATION
   80 AM=0.
      AL(1)=1.0
      GB=DD
      WH=0.
      SIGH=0.
      SIGHH=0.
      SIGHI=0.
      SIGMD=0.
      SIGMI=0.
      SIGWD=0.
      SIGWH=0.
      SIGMH=0.
      SIGHD=0.
      SIMD=0.
      WU=0.
      BMU=0.
      P=0.
```

```
      PEH1=0.
      PEH=0.
      HHT=0.
      VH=0.
      HHY=0.
      VHY=0.
      ALH=0.
      DPH=0.
      E(1)=0.
      F(1)=0.
      Z(1)=0.
      BW(1)=TW
      DO1I=2,IL
      ZZ=Z(I)-Z(I-1)
      EI=E(I)
      FI=F(I)
      BW(I)=BW(I-1)+ZZ*(EI+FI)
      T=BW(I)
      BB=BW(I-1)+T
      W1=DEE*ZZ*BB
      GO TO (411,412,413,414,415,416,418,1234),INDEX
C     DEAD WEIGHT
  411 AM1=BT*DC*ZZ**3*EI**2
      AM2=ZZ*DC*BW(I-1)*(.5*BW(I-1)+ZZ*EI)
      AM3=DEE*FI*(T-AT*ZZ*FI)*ZZ**2
      AM=AM+AM1+AM2+AM3+SIGWD*ZZ*EI
      SIGWD=SIGWD+W1
      ECC=.5*T-AM/SIGWD
      SIGMD=SIGWD*ECC
C     MOMENT IS CLOCKWISE AND FORCE IS DOWN WARDS
      GO TO 500
C     INERTIAL FORCE UNIFORM ACCLN.
  412 ALH=ALS
      GO TO 417
C     INERTIAL FORCE WITH LINEAR VARIATION OF ACCLN.
  413 ALH1=1.-Z(I)/H
      ALH2=1.-Z(I-1)/H
      CG1=BT*ZZ*(ALH1+2.*ALH2)/(ALH1+ALH2)
      ALH=ALS*(H+CG1-Z(I))/H
      GO TO 417
C     INERTIAL FORCE VARRYING SEISMIC COEFFICIENT
  418 ALH1=AL(I-1)
      ALH2=AL(I)
      CG1=BT*ZZ*(2.*ALH1+ALH2)/(ALH1+ALH2)
      ALH=ALH2+CG1/ZZ
  417 WH1=W1*ALH
      CGI=BT*ZZ*(BB+BW(I-1))/BB
      SIGMI=SIGMI+WH1*CGI+SIGHI*ZZ
      SIGHI=SIGHI+WH1
C     HORZ. FORCE ACTS IN D/S DIRECTION , MOMENT IS CLOCKWISE
      GO TO 500
C     HYDROSTATIC  PRESSURE
  414 ZBF=Z(I)-FB
      IF(ZBF)302,302,303
  303 P=DW*ZBF
      ZBF1=ZBF-ZZ
```

```
      IF(ZBF1)304,304,305
  304 HHY=.5*P*ZBF
      VHY=HHY*EI
      AMHY=BT*HHY*ZBF-VHY*(.5*T-EI*ZBF*BT)
C     MOMENT IS CLOCKWISE
      GO TO 302
  305 HHY=.5*DW*ZZ*(ZBF+ZBF1)
      VHY=HHY*EI
      CGHH=BT*ZZ*(ZBF+2.*ZBF1)/(ZBF+ZBF1)
      CGHV=.5*T-EI*ZZ*BT
      AMHY=HHY*CGHH-VHY*CGHV
  302 CG2=.5*ZZ*(FI-EI)
      SIGMH=SIGMH+AMHY+SIGHH*ZZ+SIGWH*CG2
      SIGHH=SIGHH+HHY
      SIGWH=SIGWH+VHY
C     MOMENTIS CLOCKWUSE, FORCE IN D/S DIRECTION
      GO TO 500
C
C     UPLIFT PRESSURE
  415 GD=GD+EI*ZZ
      ZBF=Z(I)-FB
      IF(GD-.5*T)306,307,307
  306 WU=DW*ZBF*(.5*GD+.25*T)
      BMU1=.75*GD*ZBF*(.5*T-4.*GD/9.)
      BMU2=.25*ZBF*(T-GD)*(.5*T-AT*(T-GD))
      BMU=DW*(BMU1-BMU2)
C     MOMENT IS CLOCKWISE , FORCE IS UPWARDS
      GO TO 500
  307 WU=.5*DW*ZBF*T
      BMU=WU*T/6.
      GO TO 500
C     HYDRODYNAMIC PRESSURE
  416 ZBF=Z(I)-FD
      ZBF1=ZBF-ZZ
      IF(ZBF)308,308,309
  309 IF(ZBF1)310,310,311
  310 EHY=ZBF*(2.-ZBF/(H-FB))/(H-FB)
      PEH=ALP*(EHY+SQRT(EHY))
      DPH=(PEH-PEH1)/ZZ
      HDY=.5*ZBF*(PEH+PEH1)
      PEH1=PEH
      AMHD=HDY*ZBF*BT
      GO TO 308
  311 EHY=ZBF*(2.-ZBF/(H-FB))/(H-FB)
      PEH=ALP*(EHY+SQRT(EHY))
      HDY=.5*ZZ*(PEH+PEH1)
      CGHD=BT*ZZ*(PEH+2.*PEH1)/(PEH+PEH1)
      AMHD=HDY*CGHD
      DPH=(PEH-PEH1)/ZZ
      PEH1=PEH
C     MOMRNT IS CLOCKWISE.   FORCE IN D/S
  308 SIMD=SIMD+AMHD+SIGHD*ZZ
      SIGHD=SIGHD+HDY
```

```
C
C     SUMMATION OF FORCES AND  MOMENTS
C
  500 SIGW=SIGWD+SIGWH-WU
      SIGH=-(SIGHI+SIGHH+SIGHD)
      SIGM=SIGMD-SIGMI-SIGMH-BMU-SIMD
C     MOMENT IS POSITIVE ANTICLOCKWISE AND IS ABOUT C,G. OF THE SECTION
C     HORZ. FORCES POSITIVE IN UPSTREAM DIRECTION
C     VERTICAL FORCES POSITIVE DOWNWARDS
      TT=T**2
      TTT=T**3
      SIGZU=SIGW/T+6.*SIGM/TT
      SIGZD=SIGW/T-6.*SIGM/TT
      A=SIGZD
      B=12.*SIGM/TTT
      A1=A*FI
      TAUZU=-EI*(SIGZU-P-PEH)
      B1=-(6.*SIGH/T+2.*TAUZU+4.*A1)/T
      C1=(6.*SIGH/T+3.*TAUZU+3.*A1)/TT
      A2=A1*FI
      X1=DC+(12.*SIGM/TTT+2.*SIGW/TT-2.*P/T-2.*PEH/T)*EI
      X2=FI*(B-4.*SIGW/TT)-6.*SIGH/TT
      X3=(X1+X2)*FI+SIGZD*(FI-F(I-1))/ZZ
      B2=B1*FI+X3-ALH*DC
      X4A=DC+EI*4.*(P/T+PEH/T-SIGW/TT-.25*B)
      X4B=FI*(2.*SIGW/TT-B)+6.*SIGH/TT
      X4=X4A+X4B
      X5=EI*(DW-X4+DPH)
      X5=X5+(P+PEH-SIGZU)*(EI-E(I-1))/ZZ
      X6=EI+FI
      X7=-(P+ALH*DC*T+PEH)
      X8=-(6.*X7-X6*(12.*SIGH/T+2.*TAUZU+4.*A1))/TT
      X8=X8-(2.*X5+4.*X3)/T
      C2=C1*FI+0.5*X8
      X9=(6.*X7-X6*6.*(3.*SIGH/T+TAUZU+A1))/TTT
      X9=X9+3.*(X5+X3)/TT
      D2=BT*X9
      PRINT206,X1,X2,X3,X4,X5,X6,X7,X8,X9
      PRINT201,A,B,A1,B1,C1,A2,B2,C2,D2
      PRINT215,Z(I)
      Y=-YY1
      DO 2 J=1,JL
      Y=Y+YY1
      IF(Y.GT.T)Y=T
      YY=Y**2
      YYY=Y**3
      SIGZ=(A+B*Y)*PSI
      TAUZY=A1+B1*Y+C1*YY
      TAUZY=TAUZY*PSI
      SIGY=A2+B2*Y+C2*YY+D2*YYY
      SIGY=SIGY*PSI
      QQ=SQRT((((SIGZ-SIGY)*.5)**2)+TAUZY**2)
      Q=0.5*(SIGY+SIGZ)
      IF(SIGZ-SIGY)30,30,31
   30 SIGP1=Q-QQ
      SIGP2=Q+QQ
```

```
      GO TO 60
   31 SIGP1=Q+QQ
      SIGP2=Q-QQ
   60 DIFF=SIGZ-SIGY
      IF(DIFF-0.00001)61,61,62
   61 PHIP1=90.0
      GO TO 63
   62 PHIP1=PA*ATAN(-2.*TAUZY/DIFF)
   63 PRINT200,Y,SIGZ,TAUZY,SIGY,SIGP1,SIGP2,PHIP1
      IF(Y.EQ.T)GO TO 1
    2 CONTINUE
    1 CONTINUE
      INDEX=INDEX+1
      GO TO 80
 1234 STOP
  101 FORMAT(20I4)
  100 FORMAT(8F10.4)
  206 FORMAT(1X,7HXVALUES,9E11.4)
  200 FORMAT(1X,7E13.6)
  201 FORMAT(1X,18HCONSTANTSA,B,CETC.,9E11.4)
  103 FORMAT(1X,9E13.6)
  215 FORMAT(40X,3HZ=  ,F10.4)
      END
$ENTRY
```

```
$IBJOB
$IBFTC MAIN
C***********************************************************************************
C      THIS PROGRAM CALCULATES THE INTERNAL STRESSES IN CONCRETE
C      GRAVITY DAMS, THE METHOD OF ANAL8SIS IS DAVIS METHOD. THE
C      STRESSES ARE CALCULATED FOR DEAD WT., UPLIFT PRESSURE, HY-
C      DROSTATIC PRESSURE, HYDRO( N M.  PRESSURE, AND INERTIAL
C      FORCE SEPARATLY. THE PROGRAM CAN BE USED ON IBM 1620
C      AND IBM7044 COMPUTER.
C***********************************************************************************
      DIMENSION DS(400),US(400),B(400),E(400),F(400),M(100)
      SQRTF(X)=SQRT(X)
   40 READ1,AB,H,Q,DC,DW,X,Y,DD,CM,ALPHA
      READ2,N,K,NS,INDU
       READ2,(M(J),J=1,NS)
      READ1,(E(I),I=3,N)
      READ1,(F(I),I=3,N)
      PRINT111,N,K,NS,INDU,AB,H,Q,DC,DW,X,Y,DD,CM,ALPHA
      PRINT112,(M(J),J=1,NS)
  455 PRINT3,(E(I),I=3,N)
      PRINT3,(F(I),I=3,N)
      B(1)=0.0
      B(2)=AB
      PSI=1./144.
  208 AM=0.0
      GD=DD
      WH=0.
      SX=0.0
      W=0.0
      HHT=0.0
      VH=0.
      BMH=0.
      BMVH=0.
      BM=0.0
      DO 100I=3,N
      SX=SX+X
      B(I)=B(I-1)+X*(E(I)+F(I))
      XI=B(I)**3/12.0
      GD=GD+X*E(I)
      C=SX+Q-H
      C1=C-X
      GO TO (201,202,203,204,205,207),INDU
C     DEAD WT.
  201 W1=.5*DC*X*(B(I)+B(I-1))
      AM1=.3333*DC*(X**3)*E(I)**2
      AM2=X*DC*B(I-1)*(.5*B(I-1)+X*E(I))
      AM3=.5*DC*F(I)*(B(I)-.667*X*F(I))*X**2
      AM=AM+AM1+AM2+AM3+W*X*E(I)
      W=W+W1
      ECC=.5*B(I)-AM/W
      BMW=W*ECC
      BM=-BMW
C     MOMENT IS ANTICLOCKWISE
      GO TO 206
C     INERTIAL FORCE
```

```
  202 W1=.5*DC*(B(I)+B(I-1))
      CGI=.3333*X*(B(I)+2.*B(I-1))/(B(I)+B(I-1))
      WH1=W1*ALPHA
      BM=BM+WH1*CGI+WH*X
      WH=WH+WH1
C     MOMENT IS CLOCKWISE
      GO TO 206
C     HYDROSTATIC
  203 IF(C)101,101,102
  101 VH=0.0
      BMH=0.
      C1=0.0
      BM=0.0
      GO TO 45
  102 IF(C1)103,104,104
  103 C1=0.
  104 VH1=.5*X*E(I)*(C+C1)*DW
      VH=VH+VH1
      VECC=.3333*X*E(I)*(2.*C1+C)/(C1+C)
      BMVH=BMVH+VH1*(.5*B(I)-VECC)+VH*(E(I)*X+.5*(B(I)-B(I-1)))
C     MOMENT IS ANTICLOCKWISE
      HH=.5*DW*C**2
      BMH=.3333*HH*C
      BM=BMH-BMVH
 45   CONTINUE
C     MOMENT IS CLOCKWISE
      GO TO 206
C     UPLIFT
204   IF(GD-.5*B(I))10,11,11
  11  W=-DW*C*(.5*GD+.25*B(I))
C     W ACTS DOWNWARDS BUT SIGN IS TAKEN NEGATIVE FOR UT
      BM1=.75*GD*(.5*B(I)-4.*GD/9.)
      BM2=.25*C*(B(I)-GD)*(.5*B(I)-.6667*(B(I)-GD))
      BM=DW*(BM1-BM2)
C     MOMENT IS CLOCKWISE
      GO TO 12
  10  W=-DW*C*B(I)*.5
      BM=-W*B(I)/6.0
C     MOMENT CLOCKWISE LOAD DOWNWARDS
  12  CONTINUE
      GO TO 206
C     HYDRODYNAMIC
  205 IF(C)105,105,106
  105 C1=0.
      BM=0.
      GO TO 108
  106 IF(C1)107,107,108
  107 C1=0.
  108 EHY=C1*(2.-C1/Q)
      FHY=SQRT(ABS(EHY))
      GHY=.5*CM*(EHY+FHY)
      HHY=GHY*ALPHA*DW*Q
      AHY=C*(2.-C/Q)
      BHY=SQRT(ABS(AHY))
      CHY=.5*CM*(AHY+BHY)
      DHY=ALPHA*CHY*DW*Q
      HHH=.5*(DHY+HHY)*(C-C1)
      HCG=.3333*(DHY+2.*HHY)*(C-C1)/(DHY+HHY)
      BM=BM+HHH*HCG+HHT*X
      HHT=HHT+HHH
C     MOMENT CLOCKWISE
```

```
206   DS(I)=W/B(I)+.5*BM*B(I)/XI
      US(I)=W/B(I)-.5*BM*B(I)/XI
100   CONTINUE
      GO TO (710,711,712,713,714),INDU
 710  PRINT701
      GO TO 715
 711  PRINT702
      GO TO 715
 712  PRINT703
      GO TO 715
 713  PRINT704
      GO TO 715
 714  PRINT705
 715  CONTINUE
      CMF=.5*DC*X
      IF(INDU.NE.1)CMF=0.
 800  DO200JJ=1,NS
      I=M(JJ)+2
      VU=I
      SX=(VU-2.)*X
      BI=B(I)
      B1=B(I-1)
      B2=B(I-2)
      B3=B(I+1)
      B4=B(I+2)
      EI=E(I)
      E1=E(I-1)
      E2=E(I-2)
      E3=E(I+1)
      E4=E(I+2)
      FI=F(I)
      F1=F(I-1)
      F2=F(I-2)
      F3=F(I+1)
      F4=F(I+2)
      X1=X*EI
      X2=X1+X*E1
      X3=X*E3
      X4=X3+X*E4
      Y1=X*FI
      Y2=Y1+X*F1
      Y3=X*F3
      Y4=Y3+X*F4
      BX=BI-Y2
      DSI=DS(I)
      USI=US(I)
      DS1=DS(I-1)
```

```
US1=US(I-1)
DS2=DS(I-2)
US2=US(I-2)
DS3=DS(I+1)
US3=US(I+1)
DS4=DS(I+2)
US4=US(I+2)
DLW1=CMF*Y1
DLW2=CMF*Y3
DLW3=CMF*(Y4-Y3)
DLW4=CMF*X3
AID=(DSI-USI)*(1.-Y1/BI)+USI
A3D=(DS3-US3)*(1.-Y3/B3)+US3
A3U=(DS3-US3)*X3/B3+US3
A4D=(DS4-US4)*(1.-Y4/B4)+US4
FID=.5*Y1*(AID+DSI)
F3D=.5*Y3*(A3D+DS3)
F3U=.5*X3*(A3U+US3)
F4D=.5*(Y4-Y3)*(A4D+DS4)
TXY1D=(DLW1-FID)/X
TXYID=(DLW2-F3D)/X
TXYIU=(DLW4-F3U)/X
TXY3D=(DLW3-F4U)/X
SY=0.
DO 300J=1,K
BB=BI-SY
DW1=CMF*(B1+B2-2.*SY+X1+X2)
DW2=CMF*(BI+B1-2.*SY+X1)
DW3=CMF*(BI+B3-2.*SY-X3)
DW4=CMF*(B3+B4-2.*SY-X4-X3)
AI=(DSI-USI)*SY/BI+USI
A1=(DS1-US1)*(SY-X1)/B1
A1=A1+US1
A2=(DS2-US2)*(SY-X2)/B2+US2
A3=(DS3-US3)*(SY+X3)/B3+US3
A4=(DS4-US4)*(SY+X4)/B4+US4
FSI=.5*(AI+DSI)*BB
FS1=.5*(BB-Y1)*(A1+DS1)
FS2=.5*(BB-Y2)*(A2+DS2)
FS3=.5*(BB+Y3)*(A3+DS3)
FS4=.5*(BB+Y4)*(A4+DS4)
V1=FS2+DW2-FS1
V2=FS1+DW1-FSI
V3=FSI+DW3-FS3
V4=FS3+DW4-FS4
TXYI=.5*(V2+V3)/X
TXY1=.5*(V1+V2)/X
TXY3=.5*(V3+V4)/X
```

```
      SUM1=.5*(TXY1D+TXY2)*(B1-SY+X2)
      SUM3=.5*(TXY3D+TXY3)*(B3-SY-X3)
      TX=.5*(SUM1-SUM3)/X
      IF(SY)211,211,212
 211  TXYI=TXYIU
      SUMI=.5*(TXYID+TXYI)*(BI-SY)
      TX=(SUMI-SUM3)/X
      GO TO 220
  212 IF(SY.LT.X2)SY=X2-Y
  217 IF(SY.EQ.(X2-Y))GO TO 300
  213 IF(SY.LE.BX)GO TO 220
  214 IF(SY-BI)215,216,215
  215 SY=BI-Y
      GO TO 300
  216 TXYI=TXYID
      TX=-SUM3/X
  220 AI=AI*PSI
      TXYI=TXYI*PSI
      TX=TX*PSI
      VIMI=(AI-TX)**2+4.*TXYI**2
      VAM=.5*SQRT(VIMI)
      SAM=.5*(AI+TX)
      PMAJ=SAM+VAM
      PMIN=SAM-VAM
      ANGLE=.5*ATAN(2.*TXYI/(AI-TX))
      ANGLE=ANGLE*180./3.14159265
      PRINT5,SX,SY,AI,TXYI,TX,PMAJ,PMIN,ANGLE
      IF(SY.GE.BI)GO TO 200
  300 SY=SY+Y
  200 CONTINUE
      INDU=INDU+1
      GO TO 208
  207 STOP
    1 FORMAT(8F10.4)
    2 FORMAT(20I4)
    3 FORMAT(1X,32F4.2)
    5 FORMAT(1X,8E15.6)
  111 FORMAT(1X,4I4,10F10.4)
  112 FORMAT(1X,30I4)
  701 FORMAT( * STRESSES DUE TO  SELF WEIGHT      ONLY *)
  702 FORMAT(* STRESSES DUE TO INERTIAL FORCE ONLY *)
  703 FORMAT(* STRESSES DUE TO HYDROSTATIC PRESSURE ONLY*)
  704 FORMAT(* STRESSES DUE TO UPLIFT ONLY*)
  705 FORMAT(* STRESSES DUE TO HYDRODYNAMIC PRESSURE ONLY *)
      END
$ENTRY
```