"Some Aspects of Surface Wave Propagation in Elastic and Anelastic Media"

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of
MASTER OF ENGINEERING
in
EARTHQUAKE ENGINEERING

By
KRISHNA GOPAL BHATIA

SCHOOL OF RESEARCH AND TRAINING IN
EARTHQUAKE ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE

October 1968

CERTIFICATE

CERTIFIED that the dissertation entitled 'SOME ASPECTS OF SURFACE WAVE PROPAGATION IN ELASTIC AND ANELASTIC MEDIA' which is being submitted by Sri KRISHNA GOPAL BHATIA in partial fulfilment for the award of the Degree of MASTER OF ENGINEERING in EARTHQUAKE ENGINEERING, University of Roorkee, Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for about 7 months from March to October 1968 in preparing this thesis for Master of Engineering at this University.

VKGaur

(V.K.Gaur)
Professor,
Department of Geology and
Geophysics, University of
Roorkee, Roorkee. U.P.

Roorkee,

Dated October ,1968

ACKNOWLEDGEMENTS

The author wishes to express deep sense of gratitude to his supervisor Dr. V.K.Gaur, Professor, Department of Geology and Geophysics, University of Roorkee, Roorkee, for providing necessary facilities and for the invaluable guidance rendered to him from time to time.

The author is also thankful to Sri P.K.Swamee, Lecturer in Civil Engineer, Malviya Regional Engineering College, Jaipur for helping in Computer Programming. The author is also thankful to the staff of Structural Engineering Research Centre, Roorkee for their help while working on IBM 1620 Digital Computer.

SYNOPSIS

It is well known that dissipation accompanies vibrations in solids due to the presence of internal friction. In general, the effect of internal friction is to produce attenuation and dispersion of elastic waves.

Wave propagation in anelastic solids provide an interesting field for the application of mathematical techniques and in addition is of practical importance both to engineers and seismologists. The present work deals with the numerical computation of the variation of amplitudes for surface waves showing elastic and anelastic behaviour.

CONTENTS

Chapter			Page
	CERTIFICATE		
	ACKNOWLEDGEMENTS		
	SYNOPSIS		
	LIST OF SYMBOLS	..	1-2
I-	INTRODUCTION	..	3-7
II-	IMPERFECTIONS OF ELASTICITY	..	8-15
III-	RAYLEIGH AND LOVE WAVE EQUATIONS IN ELASTIC AND ANELASTIC MEDIA	..	16-26
IV-	REDUCED RAYLEIGH WAVE	..	27-29
V-	ANALYSIS OF RAYLEIGH AND LOVE WAVE EQUATIONS	..	30-43
VI-	DISCUSSION OF RESULTS AND CONCLUSION	..	
	BIBLIOGRAPHY		
	APPENDIX		
	A- GRAPHS		
	B- COMPUTOR PROGRAMME		
	C- RESULTS		

LIST OF SYMBOLS

ν	dashpot constant,
E	elasticity,
μ	coefficient of rigidity,
K	coefficient of incompressibility
V_P	velocity of P waves,
V_s	velocity of S wave,
ρ	density,
λ	Lame's constant,
ϕ	Scalor potential of displacement
ψ	Vector potential of displacement,
L	wave length
α	compressional wave velocity,
β	shear wave velocity,
c	phase velocity,
θ	cubical dialatation,
$R\|z\|$	real part of z,
$I\|z\|$	imaginary part of z,
σ	Poisson's ratio,
w	angular frequency,
k	parameter for wave equation,
X,Y,Z	body forces,
U,V,W	displacement in x,y and z directions respectively.

V_R Rayleigh wave velocity,

T time period,

τ rate of decrease of amplitude,

p stress at a point,

η ratio of shear wave velocity to compressional wave velocity,

ζ ratio of phase velocity to shear wave velocity.

CHAPTER - I

INTRODUCTION

It is shown in treatises on Theory of Elasticity that two types of wave motions are propogated in an isotropic homogeneous and unbounded elastic medium. These are the longitudinal or the compressional (P) waves and the transverse or the shear (S) waves. Their velocities are respectively given by $\sqrt{(K+\frac{4}{3}\mu)/\rho}$ and $V_S = \sqrt{\mu/\rho}$ where 'K' and 'μ' are the coefficients of compressibility and rigidity and 'ρ' the density. In any media the 'P' waves travel faster than the 'S' waves and are the first to arrive at a given point. They were therefore, called the 'Primary' or its short form the P wave. Earthquake records invariably show the arrival of these two waves but in addition to these they also show the presence of other slow travelling waves with large initial amplitudes which appear to have travelled along the earth's surface. The latter consists of approximately harmonic waves of varying amplitudes and progressively diminishing periods and belongs to two main groups known as the 'Rayleigh' and the 'Love' waves. The theory of Rayleigh waves was first given by Lord Rayleigh |11| in 1887. He showed that the free surface of an elastic homogeneous medium can support a disturbance involving displacements in a vertical plane, along the direction of propagation in the form of a retrograde ellipse, and propagating with a velocity which was about 0.9 times the shear wave velocity in that medium. Jeffreys and Stoneley and later Ewing and Press greatly extended the theory to cover cases of layered semi-infinite media. The results indicate that in all but the simplest case assumed by Lord Rayleigh, these waves suffered dispersion. The theory of Love waves were given by A.E.H.Love |8| to account for the presence of transverse component surface waves in earthquake records. He showed that on the free surface of an elastic layer resting on a semi-infinite elastic and homogeneous substratum, a wave of the SH type was possible. These waves suffered dispersion depending on the thickness of the upper layer.

The surface waves comprising the Rayleigh and Love waves are distinguished from the bodily P and S waves in being more or less confined to the surface. Thus, their amplitudes decrease very rapidly with depth below the surface. The variations of the horizontal and the vertical amplitudes of surface wave displacements with depth is a matter of great interest both in Seismology and Earthquake Engineering. When seismographs are installed in bore-holes, the amplitudes of earthquake waves may be measured directly and the same considerations apply to microseisms and to the recording of explosions. Further the ratio of the vertical and horizontal displacements (w/u) which are related to the nature of layering in the semi-infinite medium can be compared with observed values. Also according to a dynamical theorem earthquake foci lying at a depth corresponding to a node of the surface wave will not excite that mode. The observed amplitudes of different modes can therefore be discussed in terms of the depth of the ~~forces~~ focus. The results can also be applied to the problems of vibrations of roads and other structures. Stoneley(1958)|14| and Stoneley and Hochstrasser(1957)|15| have treated this problem for two uniform surface layers overlying another of great depth but in all cases conditions of perfect elasticity have been assumed.

The present work investigates numerically the variation of amplitudes for surface waves propogated over the surface of solids showing anelastic behaviour.

DEPARTURE FROM PERFECT ELASTICITY

The behaviour of a body is said to be elastic when Hook's law holds good i.e. the restoring force is proportional to the amplitude of vibrations. The equations of motion of an elastic solid are obtained by equating the products of the masses and accelerations to the elastic forces and it is assumed that no other forces came into play.

When dissipation accompanies vibrations in solid media, the behaviour is said to be anelastic. Some of the elastic energy of the vibrating body is always converted into heat, and the various mechanisms by which this takes place are collectively termed internal friction. Thus, when a solid specimen vibrates, its free oscillations decay even when it is isolated from its environments due to the presence of internal friction. We can define internal friction as the ratio $\frac{\nabla W}{W}$ where ∇W is the energy dissipated in taking a specimen through a stress cycle and W is the elastic energy stored in the specimen when the strain is maximum. This ratio is sometimes called the , "specific damping capacity" and can be measured for a stress cycle without any assumption being made about the nature of internal friction.

If we assume that restoring force is proportional to the amplitude and the dissipating force is proportional to the velocity then the ratio between the successive free oscillation is constant and the natural logarithm of this ratio, which is called the logarithmic decrement, is taken as a measure of the internal friction.

Another method of investigating internal friction which is more closely related to the problem is the measurement of the attenuation of a stress wave when it travels through a solid. For a plane sinusoidal wave of small amplitude the attenuation is found to be exponential $|\exp(-\tau x)|$, where τ represents an attenuation constant.

CHAPTER - II

IMPERFECTIONS OF ELASTICITY

Imperfections of elasticity are usually generalized in terms of perfect elasticity and viscosity. The ideal linear elastic and viscous elements thus become mathematically tractable and physically represent a spring and a dashpot respectively.

The microscopic structure of a linear anelastic material is mechanically equivalent to a linear viscous and elastic element. The assumption made is that the properties of the smallest portion are the same as those of the substance in bulk. The extension in each element is small. Also the energy per unit volume dissipated as heat everywhere is sufficiently small to warrant any variation in mechanical properties due to temperature change to be neglected.

Let us consider a spring of stiffness E and a dashpot of constant ν . |see Fig. (A)|. If P is the force and x is the extension, i.e. the increase in the distance between the ends, then,

for the spring $P = Ex$, and

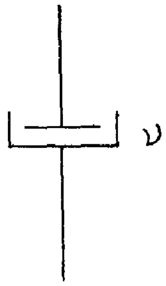
for the dashpot $P = \frac{dx}{dt} = \dot{x}$

To represent a particular type of anelastic behaviour, the basic elastic and viscous elements can be combined in various ways. If a spring and dashpot

1 SINGLE ELEMENT

SPRING-THE ELASTIC ELEMENT

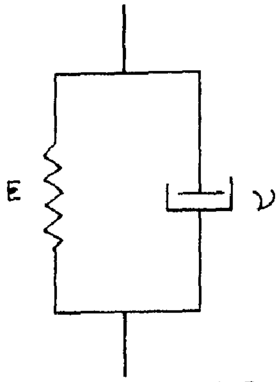
FIG. A (a)



VISCOUS ELEMENT-THE DASHPOT

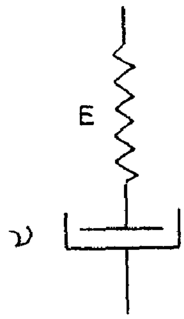
FIG. A (b)

2-TWO ELEMENT MODEL



THE VOIGT SOLID

FIG. B (a)



THE MAXWELL SOLID

FIG. B (b)

3-THREE ELEMENT MODEL

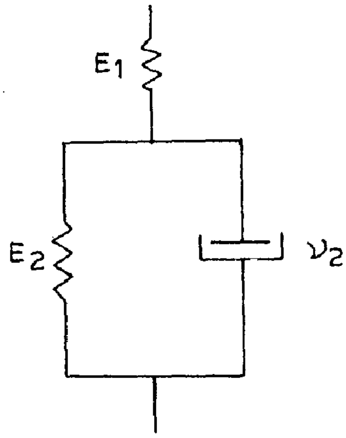


FIG. C (a)

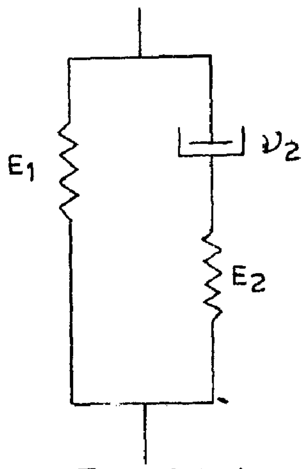


FIG. C (b)

4-FOUR ELEMENT MODEL

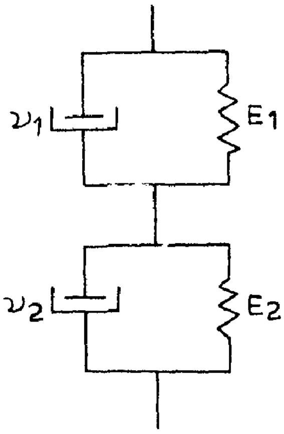


FIG D (a)

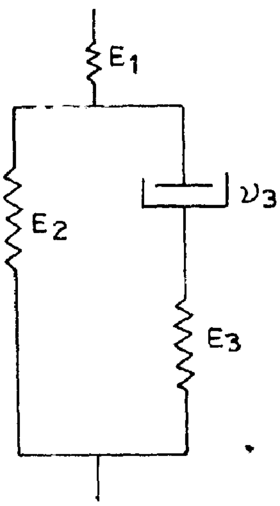


FIG. D (b)

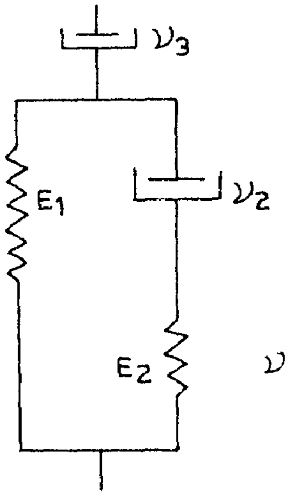


FIG. D (C)

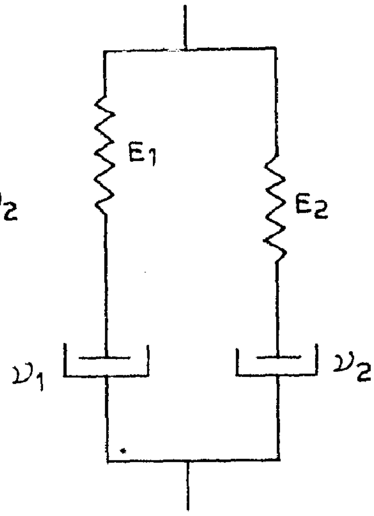


FIG. D (d)

are considered in series, the resulting model is known as a Maxwell element. If these elements are in parallel, the model is known as **Voigt** |17| or the **Kelvin** Model. |see fig. (B)|, Fig.(C) and Fig.(D) represent three and four element models.

CORRESPONDENCE PRINCIPLE

"If the elastic solution for any dependent variable in a particular problem is of the form $f = R|\bar{f}_E \exp(iwt)|$ and if the elastic moduli in $\bar{f}_E$ are replaced by the corresponding complex moduli $\bar{f}_{VE}$ then the anelastic solution for that variable in the corresponding problem is given by $f=R|\bar{f}_{VE}\exp(iwt)|$" This is known as the correspondence principle. This can only be used if (i) the elastic solution is known, (ii) there would be no operation in the anelastic solution corresponding to the operation in the elastic solution involving separation of either complex modulus into real and imaginary parts, with the exception of the final determination of f and $\bar{f}$ and (iii) the boundary conditions for the two cases are identical.

COMPLEX MODULI

Let the model be subjected to a sinusoidally oscillating force of radian frequency w. Also let sufficient time elapse for the effect of initial conditions to be negligible. The extension also will be of radian frequency w.

$$\left.\begin{aligned} \dot{P} &= R\left|P_o(w)\ \exp(iwt)\right| \\ \text{and}\qquad x &= R\left|x_o(w)\ \exp(iwt)\right| \end{aligned}\right\} \quad .. (1)$$

where, R(Z) denotes the real part of (Z) and $P_o(w)$ and $x_o(w)$ are independent of time and in general complex.

For spring $R\left|P_o(w)\ \exp(iwt)\right| = E.R\left|x_o(w)\ \exp(iwt)\right|$

For this equation to be valid for all 't'

$$\frac{P_o(w)}{x_o(w)} = E$$

E is the modulus of spring.

When the model is a dashpot,

$$P_o(w)\ \exp(iwt) = \nu R\left|\dot{x}_o(w)\ \exp(iwt)\right|$$

and therefore, $\dfrac{P_o(w)}{x_o(w)} = iw\nu$

<u>For Maxwell solid</u> |Fig. B(a)|

Since spring and dashpot are in series, let the extension of spring be x' and that of dashpot be x", then,

$$P_o \exp(iwt) = E\ x_o'(w)\ \exp(iwt)$$

Therefore,

$$\frac{P_0(w)}{x_0(w)} = (E + Iw\gamma)$$

or $$Y_T = (E + iw\gamma) \qquad .. (3)$$

Y_T is the complex modului of Voigt solid. Complex moduli for three and four element models can also be obtained by proceeding on the same lines.

Stress analysis in continua mechanics is concerned with the simultaneous solution of three sets of equations subjected to given boundary conditions. The first set comes from analysis of strain. The second set comes from analysis of stress. The third set is the stress-strain equations. The first set depends solely on geometry of the body. The second set depends on geometry and on Newton's laws and are known as equations of motion. The third set i.e the stress strain equations, are different for different material.

The equations of the first two sets are,

$$\left.\begin{array}{ll} \text{i)} & e_{ij} = \frac{1}{2}\left(\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y}\right) = e_{xy} \\ \text{ii)} & p_{ij,j} + \rho X_i = \frac{\partial^2 S_i}{\partial t^2} \end{array}\right\} \qquad .. (4)$$

Considering the propogation of sinusoidal waves, we look for the solutions of equation 4(i) and (ii) for anelastic material in which all the dependent variables vary sinusoidally with time.

and this also equals to

$$\nu \frac{d}{dt} \left| x_o''(w) \exp(iwt) \right|$$

$$\therefore \; E\, x_o'(w) \exp(iwt) = \nu \frac{d}{dt}\left| x_o''(w) \exp(iwt) \right|$$

$$= P_o(w) \exp(iwt)$$

Therefore, $$x_o'(w) + x_o''(w) = \frac{P_o(w)}{E} + \frac{P_o(w)}{iw\nu}$$

or, $$x_o(w) = P_o(w) \left| \frac{1}{E} + \frac{1}{iw\nu} \right|$$

Therefore, complex moduli,

$$\frac{P_o(w)}{x_o(w)} = \left| \frac{1}{E} + \frac{1}{iw\nu} \right|^{-1}$$

Therefore,

$$Y_T = \left| \frac{1}{E} + \frac{1}{iw\nu} \right|^{-1} \quad .. \; (2)$$

where, Y_T is said to be complex moduli.

VOIGT SOLID |Fig.B(b)|

Spring and dashpot are in parallel. Therefore,

$$P_o'(w) \exp(iwt) = E\, x_o(w) \exp(iwt)$$

$$P_o''(w) \exp.(iwt) = \nu x_o iw \exp(iwt)$$

or $$P_o = P_o' + P_o''$$

$$P_o = x_o(w) \left| E + iw\nu \right|$$

Therefore,

$$\left.\begin{aligned} p_{ij} &= R\left|\bar{p}_{ij}\ \exp(iwt)\right| \\ e_{ij} &= R\left|\bar{e}_{ij}\ \exp(iwt)\right| \end{aligned}\right\} \quad \cdots (5)$$

where,

$\bar{p}_{ij}$ and $\bar{e}_{ij}$ etc. are functions in general complex of the spacial coordinates only.

Introducing the deviatoric components of strain and stress ϵ_{ij} and S_{ij} respectively,

$$\left.\begin{aligned} \epsilon_{ij} &= e_{ij} - \frac{1}{3}\,\delta_{ij}\,e_{kk} \\ S_{ij} &= p_{ij} - \frac{1}{3}\,\delta_{ij}\,p_{kk} \end{aligned}\right\} \quad \cdots (6)$$

Therefore,

$$\bar{S}_{ij} = Y_s \bar{\epsilon}_{ij} \quad \cdots (7)$$

$$\bar{p}_{kk} = Y_n\,\bar{e}_{kk} \quad \cdots (8)$$

From equations (6), (7) and (8),

$$\begin{aligned} \bar{e}_{ij} &= \bar{\epsilon}_{ij} + \frac{1}{3}\,\delta_{ij}\,\bar{e}_{kk} \\ &= \frac{1}{Y_s}\bar{S}_{ij} + \frac{1}{3Y_n}\,\bar{p}_{kk}\,\delta_{ij} \\ &= \frac{1}{Y_s}\,\bar{p}_{ij} - \frac{1}{3}\left(\frac{1}{Y_s} - \frac{1}{Y_n}\right)\bar{p}_{kk}\,\delta_{ij} \end{aligned} \quad \cdots (9)$$

In particular,

$$\bar{e}_{11} = \frac{1}{Y_s}\bar{p}_{11} - \frac{1}{3}\left(\frac{1}{Y_s} - \frac{1}{Y_n}\right)(\bar{p}_{11}+\bar{p}_{22}+\bar{p}_{33})$$

$$= \left[\frac{1}{Y_s} - \frac{1}{3}\left(\frac{1}{Y_s} - \frac{1}{Y_n}\right)\right]\bar{p}_{11} - \frac{1}{3}\left(\frac{1}{Y_s} - \frac{1}{Y_n}\right).$$

$$(\bar{p}_{22} + \bar{p}_{33})$$

$$= \left[\frac{1}{3}\left|\frac{2}{Y_s} + \frac{1}{Y_n}\right|\bar{p}_{11}\right] - \frac{1}{3}\left(\frac{1}{Y_s} - \frac{1}{Y_n}\right)(\bar{p}_{22}+\bar{p}_{33}) \qquad .. (10)$$

For an elastic material,

$$e_{11} = \frac{1}{E}p_{11} - \frac{\sigma}{E}(p_{22} + p_{33}) \qquad .. (11)$$

Comparing (10) and (11)

$$E = \left|\frac{1}{3}\left(\frac{2}{Y_s} + \frac{1}{Y_n}\right)\right|^{-1}$$

$$= Y_T \qquad .. (12)$$

$$\sigma = E\left|\frac{1}{3}\left(\frac{1}{Y_s} - \frac{1}{Y_n}\right)\right|$$

$$= \frac{\frac{1}{3}\left(\frac{1}{Y_s} - \frac{1}{Y_n}\right)}{\frac{1}{3}\left(\frac{2}{Y_s} + \frac{1}{Y_n}\right)}$$

$$= \left|\frac{Y_n - Y_s}{2Y_n + Y_s}\right| \qquad .. (13)$$

Again from (9)

$$\bar{e}_{12} = \frac{1}{Y_s}\bar{p}_{12} \qquad .. (14)$$

and for elastic case,

$$e_{12} = \frac{1}{2\mu} p_{12} \qquad .. \ (15)$$

Therefore by comparing

$$\mu = \frac{1}{2} Y_s \qquad .. \ (16)$$

Also,

$$K = \frac{1}{3} Y_n \qquad .. \ (17)$$

$$\left.\begin{aligned} \lambda &= K - \frac{2}{3} \\ &= \frac{1}{3} (Y_n - Y_s) \end{aligned}\right\} \qquad .. \ (18)$$

Equations (12), (13), (16), (17), and (18) give the corresponding complex moduli for anelastic solid.

CHAPTER-III

RAYLEIGH AND LOVE WAVE EQUATIONS IN ELASTIC AND ANELASTIC MEDIA

When waves propagate into an extended solid for which the component of displacement in one direction is zero and that in the other two directions are independent of the previous one, the solid under these conditions is in a state of plane strain. Assume a simple harmonic wave train travelling in the x-direction such that

1) the disturbance is independent of Y coordinate and
2) it decreases rapidly with distance Z from the free surface. Waves satisfying the second condition are called surface waves.

In an unbounded isotropic solid two and only two types of elastic wave can be propagated. Where there is bounding surface, however, elastic surface waves may also occur.

Consider the boundary to be x-y plane with Z axis positive towards the interior of the solid.

Let U, V and W be the displacements in x,y and z directions respectively. The distance D in the medium can be represented by the sum of gradient of a scaler and curl of a vector i.e.,

$$D = \phi + \nabla \times \psi \qquad .. (1)$$

where ϕ is a scalor potential and ψ is a vector potential and D is a function of U, V and W.

Since the displacement is independent of y

$$\left.\begin{aligned} U &= \frac{\partial \phi}{\partial x} - \frac{\partial \psi_2}{\partial z} \\ V &= \frac{\partial \psi_1}{\partial z} - \frac{\partial \psi_3}{\partial x} \\ \text{and} \quad W &= \frac{\partial \phi}{\partial z} + \frac{\partial \psi_2}{\partial x} \end{aligned}\right\} \qquad .. (2)$$

Stress at a point is given as,

$$p_{ij} = \lambda \theta \delta_{ij} + 2\mu e_{ij} \qquad .. (3)$$

where,

θ is cubical dialatation, λ and μ are Lame's constants and δ_{ij} is cronecker delta, i.e.

$$\delta_{ij} = 1 \text{ for } i = j$$

$$= 0 \text{ for } i \neq j$$

$$\theta = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}$$

$$\lambda = \frac{E}{(1+\sigma)(1-2\sigma)}$$

$$\mu = \frac{E}{2(1+\sigma)} \qquad .. 3(a)$$

where,

σ is the Poisson's ratio and E is the modulus of elasticity.

Equation of motion is,

$$\left.\begin{aligned}\frac{\partial^2 U}{\partial t^2} &= \rho X + \frac{\partial}{\partial x} T_{xx} + \frac{\partial}{\partial y} T_{xy} + \frac{\partial}{\partial z} T_{xz} \\ \frac{\partial^2 V}{\partial t^2} &= \rho Y + \frac{\partial}{\partial x} T_{yx} + \frac{\partial}{\partial y} T_{yy} + \frac{\partial}{\partial z} T_{yz} \\ \text{and} \quad \frac{\partial^2 W}{\partial t^2} &= \rho Z + \frac{\partial}{\partial x} T_{zx} + \frac{\partial}{\partial y} T_{zy} + \frac{\partial}{\partial z} T_{zz}\end{aligned}\right\} \quad .. (4)$$

where, X,Y,Z are body forces in x,y and z directions respectively and ρ is the density.

From (4) and (3) we get,

$$\left.\begin{aligned}\frac{\partial^2 U}{\partial t^2} &= (\lambda+\mu)\frac{\partial\theta}{\partial x} + \mu\nabla^2 u \\ \frac{\partial^2 V}{\partial t^2} &= (\lambda+\mu)\frac{\partial\theta}{\partial y} + \mu\nabla^2 v \\ \frac{\partial^2 W}{\partial t^2} &= (\lambda+\mu)\frac{\partial\theta}{\partial z} + \mu\nabla^2 w.\end{aligned}\right\} \quad .. (5)$$

From (1)

$$\theta = \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}\right) = \nabla . D = \nabla^2 \phi \quad .. (6)$$

From (5) and (6)

$$\frac{\partial^2 D}{\partial t^2} = (\lambda+\mu)(\nabla(\nabla^2\phi)) + \mu\nabla^2(\nabla\phi + \nabla \times \psi)$$

or $$\frac{\partial^2}{\partial t^2}(\nabla\phi + \nabla x\psi) = (\lambda+\mu)\,\nabla(\nabla^2\phi) + \mu\nabla^2(\nabla\phi + \nabla x\psi) \qquad .. (7)$$

It can easily be seen that equation (7) is satisfied only if ϕ and ψ are the solutions and they take the form,

$$\left.\begin{aligned} \nabla^2\phi &= \frac{1}{\alpha^2}\frac{\partial^2\phi}{\partial t^2} \\ \nabla^2\psi_i &= \frac{1}{\beta^2}\frac{\partial^2\psi_i}{\partial t^2} \\ \nabla^2 V &= \frac{1}{\beta^2}\frac{\partial^2 V}{\partial t^2} \end{aligned}\right\} \qquad .. (8)$$

where,

$$\alpha = \sqrt{\frac{\lambda+2\mu}{\rho}}$$

and $$\beta = \sqrt{\frac{\mu}{\rho}} \qquad .. (9)$$

Equation (9) indicates that two types of disturbances with velocity α and β may be propagated through an elastic solid.

Solution of (8) can be written as,

$$\left.\begin{aligned} \phi &= A \exp\left|\pm qz + i(wt - kx)\right| \\ \psi &= B \exp\left|\pm Sz + i(wt-kx)\right| \\ V &= C \exp\left|\pm Sz + i(wt - kx)\right| \end{aligned}\right\} \qquad .. (10)$$

where,

$$\left.\begin{aligned} q^2 &= k^2 - k_\alpha^2 \\ k_\alpha &= w/\alpha \\ w &= c.k \\ S^2 &= k^2 - k_\beta^2 \\ k_\beta &= w/\beta \end{aligned}\right\} \quad .. \quad (11)$$

RAYLEIGH WAVE

According to the theory given by Rayleigh for surface waves on the free surface of a semi-infinite elastic solid, the motion becomes negligible at a distance of a few wave lengths from the surface.

Solution (10) takes the form,

$$\left.\begin{aligned} \phi &= A \exp|-qz+i(wt-kx)| \\ \psi &= B \exp|-Sz+i(wt-kx)| \end{aligned}\right\} \quad .. \quad (12)$$

provided $c<\beta<\alpha$

In a similar way the SH component takes the form,

$$V = C \exp|-Sz+i(wt-kx)| \quad .. \quad (12a)$$

provided $c<\beta<\alpha$

The sign has been chosen so that the potentials approach zero as z tends ∞.

In order to obtain constants A,B,C apply the boundary conditions.

i) All stresses must vanish at $z=0$

i.e. $p_{zz} = p_{zx} = p_{zy} = 0$ at $z=0$ .. (13)

From (3)

$$\left.\begin{aligned} p_{zx} &= 2\mu e_{ij} = 2\frac{\partial^2\phi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2} = 0 \\ \text{AND} \quad p_{zz} &= \lambda\nabla^2\phi + 2\mu\left(\frac{\partial^2\phi}{\partial z^2} + \frac{\partial^2\psi}{\partial x\partial z}\right) \\ \text{and} \quad p_{zy} &= 2\mu e_{zy} = \mu\left|\frac{\partial V}{\partial z}\right| \end{aligned}\right\} \quad .. (14)$$

Equating these stresses to zero at z=0 we obtain,

$$\left.\begin{aligned} &\text{i)}\ C = 0 \\ &\text{ii)}\ A(2iqk) - B(k^2+S^2) = 0 \\ &\text{iii)}\ A\left|(\lambda+2\mu)q^2 - \lambda k^2\right| + B(2\mu ikS) = 0 \end{aligned}\right| \quad .. (14a)$$

Eliminating A and B from 14(a) and putting for q and S we get,

$$\xi^2\left|\xi^6 - 8\xi^4 + \xi^2(24-16\eta^2) + 16(\eta^2-1)\right| = 0 \quad .. (15)$$

where,

$$\xi = \frac{c}{\beta} = \frac{k_\beta}{k}$$

and $$\eta = \frac{\beta}{\alpha} = \frac{k_\alpha}{k_\beta}$$

This represents Rayleigh wave.

One root of ξ is that $\xi = 0$ i.e. c=0, this gives **that equation** (10) is independent of time and U=W=0. Hence, this solution is not of interest. Therefore,

$$\xi^6 - 8\xi^4 + \xi^2(24-16\eta^2) + 16(\eta^2-1) = 0 \quad .. (16)$$

In (10) if we put $\zeta = 0$ we get negative value because $\eta < 1$ and if $\zeta = 1$ we get positive value. Thus there is always a root of ζ if $0<c<\beta<\alpha$ and under these conditions surface waves can exist.

LOVE WAVE

When a 'homogeneous layer is bounded above by a plane horizontal free surface and below by a parallel boundary at a finite distance away, surface waves of SH type can occur at free surface. This was shown by Love in 1911.

Since only SH component occurs, Equation (10) takes the form for medium M,

$$V = C \exp|-Sz + i(wt-kx)| \text{ for } 0 \leqslant z < \alpha \quad \ldots (17)$$

For medium M_1

$$V = C_1 \exp|-iS_1 z + i(wt-kx)| + F_1 \exp|iS_1 z + i(wt-kw)| \quad \ldots (18)$$
$$\text{for } 0 \leqslant z \leqslant H_1$$

where,

$$S_1^2 = -k^2 + k_{\beta_1}^2$$

$$k_{\beta_1} = \frac{w}{\beta_1}$$

β_1 is shear wave velocity in M_1.

Constants C, C_1, F_1 are determined by boundary conditions which are,

(a) The displacement at the boundary surface must be continuous at all times and places.

(b) Stress across the boundary surface between M and M_1 shall be continuous at all times and places.

(c) All the stresses at free surface will be zero.

Or we may write

from (19a),

$$V_M = V_{M_1} \text{ at } z = 0$$

This gives $C = C_1 + F_1$ .. (20)

from (19b),

$$\mu_1 \left. \frac{\partial V}{\partial z} \right|_{M_1} = \mu \left. \frac{\partial V}{\partial z} \right|_{M} \text{ at } z = 0$$

or $$-\mu SC = -\mu_1 \, i \, S_1 C_1 + \mu_i \, i \, S_1 F_1 \quad .. (21)$$

From (19c)

$$\mu_1 \left. \frac{\partial V}{\partial z} \right|_{M_1} = 0 \qquad \text{at } z = -H_1$$

or $$-i\mu_1 S_1 C_1 \exp\left[+iH_1 S_1 + i(wt-kx)\right] + i\mu_1 S_1 F_1 \exp\left[-iH_1 S_1 + i(wt-kx)\right] = 0$$

or $$C_1 \exp|iH_1 S_1 + i(wt-kx)| = F_1 \exp|-iH_1 S_1 + i(wt-kx)|$$

.. (22)

To eliminate C, C_1, F_1 from 20, 21, and 22,

from (20) and (21)

$$\frac{C_1}{F_1} = \frac{\mu_1 i S_1 + S}{\mu_1 i S_1 - S} \qquad .. (23)$$

Again from (21) and (22),

$$\frac{C_1}{F_1} = \frac{\exp\left[-iH_1S_1 + i(wt-kx)\right]}{\exp\left[iH_1S_1 + i(wt-kx)\right]}$$

$$= \frac{\exp|-iH_1S_1|}{\exp|iH_1S_1|} \qquad .. (24)$$

From (23) and (24),

$$\frac{\mu_1 i S_1}{\mu S} = \frac{\exp(-iH_1S_1) + \exp(iH_1S_1)}{\exp(-iH_1S_1) - \exp(iH_1S_1)}$$

or $\mu S = \mu_1 S_1 \tan(H_1S_1)$

or $\mu S - \mu_1 S_1 \tan(H_1S_1) = 0$ .. (25)

Substituting for S and S_1 we obtain,

$$\mu\sqrt{1-\frac{C^2}{\beta^2}} - \mu_1\sqrt{\frac{C^2}{\beta_1^2} - 1}\ \tan\left(kH_1\sqrt{\frac{C^2}{\beta_1^2} - 1}\right) = 0 \qquad .. (26)$$

Therefore from (26), SH surface wave can exist if this equation is satisfied i.e.

$$\beta_1 < C < \beta$$

or in other words the velocity of S bodily wave in the lower medium M must be greater than that in M_1 and the wave velocity c of the Love waves must lie between β and β_1. We can also see from (26) that c is dependent on the k, i.e. dispersion of Love wave will take place.

It is evident from (26) that for k to be very large c approaches β_1 and for k to be small c approaches β. For anelastic media solution of Rayleigh and Love waves can be obtained by Correspondence principle. For anelastic case we have to replace E by Y_T.

Rayleigh Wave Equation

Equation remains the same as for elastic case i.e.

$$\xi^6 - 8\xi^4 + \xi^2(24 - 16\eta^2) + 16(\eta^2 - 1) = 0$$

Here, $\eta = \frac{\beta}{\alpha} = \sqrt{\frac{\mu}{\lambda + 2\mu}} = \sqrt{\frac{3Y_s}{2Y_n + 4Y_s}}$

But $\sigma = \frac{Y_n - Y_s}{2Y_n + Y_s}$

Therefore $\eta = \sqrt{\frac{1 - 2\sigma}{2(1-\sigma)}}$ .. (27)

Also $\xi^2 = \frac{c^2}{\beta^2} = \frac{w^2}{k^2}$

Therefore $k = \frac{w}{\xi}\sqrt{\frac{2\rho}{Y_s}}$ ..(28)

We have to replace Y_s in terms of Y_T, Y_T being the complex moduli as given in eqn. (2-12),

$$\left.\begin{aligned} Y_T &= \left[\frac{1}{3}\left(\frac{2}{Y_s}+\frac{1}{Y_n}\right)\right]^{-1} = (1+\sigma)Y_s \\ Y_T &= (1-2\sigma)Y_n \end{aligned}\right\} \quad .. (29)$$

Hence,

$$k = \frac{w}{\zeta}\sqrt{\frac{2\zeta(1+\sigma)}{Y_T}} \quad .. (30)$$

For Maxwell solid,

$$Y_T = \left|\frac{1}{E}+\frac{1}{iw\nu}\right|^{-1} \quad .. (31)$$

and for Voigt solid,

$$Y_T = (E+iw\nu) \quad .. (32)$$

LOVE WAVE EQUATION

Considering the case when media M_1 is anelastic let us denote it by suffix 2 (e.g. M_2). Thickness of the layer is H_2. Then eqn. (2-26) takes the form,

$$\mu\sqrt{1-\frac{c^2}{\beta^2}} - \mu_2\sqrt{\frac{c^2}{\beta_2^2}-1}\left[\tan\left|kH_2\sqrt{\frac{c^2}{\beta_2^2}-1}\right|\right] = 0 \quad .. (33)$$

where μ_2 is complex moduli for medium M_2 and k is also complex.

$$\text{Now } \mu_2 = \frac{1}{2}Y_s = \frac{1}{2(1+\sigma)}Y_T \quad .. (34)$$

Corresponding values of Y_T from eqn. |3-31| and |3-32| for Maxwell and Voigt solid are substituted in |3-34| and then in |3-33| to give Love wave equation.

CHAPTER IV

REDUCED RAYLEIGH WAVE EQUATION

Rayleigh wave equation as derived in last chapter equation (3-16) is

$$\xi^6-8\,\xi^4+\;{}^2(24-16\,\eta^2)+16(\eta^2-1) = 0 \qquad ..\ (1)$$

We may write it as,

$$\eta^2(16-16\,\xi^2)+\xi^6-8\,\xi^4+24\,\xi^2-16 = 0$$

or $$\eta^2 = \frac{16-24\,\xi^2+8\,\xi^4-\xi^6}{16(1-\xi^2)} \qquad ..\ (2)$$

Therefore,

$$\eta = \sqrt{\left|\frac{16-24\,\xi^2+8\,\xi^4-\xi^6}{16(1-\xi^2)}\right.} \qquad ..\ (3)$$

Since c is always less than β, is always less than unity.

From (3) we can obtain a plot of η vs. ξ. We can also solve this equation (1) for ξ for different values of η. The computor programme is written for this and the results are obtained.

ξ_0 = .95529

η	ξ	$\xi - \xi_0$
0.00	0.95529	0.00
0.05	0.95520	-0.00009
0.10	0.95465	-0.00064
0.15	0.95393	-0.00136
0.20	0.95274	-0.00255
0.25	0.95111	-0.00418
0.30	0.94904	-0.00625
0.45	0.94634	-0.00895
0.40	0.94286	-0.01243
0.45	0.93835	-0.01694
0.50	0.93249	-0.02280
0.55	0.92473	-0.03056
0.60	0.91416	-0.04113
0.65	0.89936	-0.05593
0.70	0.87789	-0.07740

Maximum value of η has been taken as 0.70 because for η higher than this, Poisson's ratio becomes negative and no material yet has been obtained with negative Poisson's ratio. |4|

In order to make the first value equal to zero, ξ_0 = .95529 has been subtracted from each value. A Log Log plot is obtained for $(\xi - \xi_0)$ vs. η.

For $0 \leqslant \eta \leqslant .40$, we get one straight line and for values $.40 \leqslant \eta \leqslant .7$, we get another straight line. Each straight line will give an equation of the type $Y = kx^m$ i.e. $\xi - \xi_0 = k\,\eta^m$. put x=1, we obtain k from the graph and the slope of the line gives the value of m. Exact results are obtained by adjusting the values of constant k.

From graph,

for $0 < \eta < .4$

$$\xi = .95529 - (.1 + .05\,\eta)\,\eta^{2.275} \qquad .. (4)$$

and for $.4 < \eta \leqslant .7$

$$\xi = .95529 - (.12 + .1\,\eta)\,\eta^{2.83} \qquad .. (5)$$

Equations (4) and (5) are reduced Rayleigh wave equation and these give results correct to third decimal place.

CHAPTER-V

ANALYSIS OF RAYLEIGH AND LOVE WAVE EQUATIONS

Equations of Rayleigh and Love wave have been derived in Chapter II and are given by the equations(3-16) and eqns.(3-26). Values of λ and μ are given in terms of modulus of elasticity and Poisson's ratio in equation |(3-3(a)|.

Rewriting them we have,

$$\lambda = \frac{E}{(1-2\sigma)(1+\sigma)}$$

$$\mu = \frac{E}{2(1+\sigma)} \qquad .. (1)$$

Therefore,

$$\lambda + 2\mu = \frac{E(1-\sigma)}{(1-2\sigma)(1+\sigma)}$$

Therefore,

$$\alpha = \sqrt{\frac{\lambda+2\mu}{\rho}} = \sqrt{\frac{E(1-\sigma)}{(1-2\sigma)(1+\sigma)} \times \frac{1}{\rho}}$$

$$\text{and} \quad \beta = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{E}{2(1+\sigma)} \times \frac{1}{\rho}} \qquad .. (2)$$

Let us represent γ as the ratio of β/α

$$\gamma = \frac{\beta}{\alpha} = \sqrt{\frac{1-2\sigma}{2(1-\sigma)}} \qquad .. (3)$$

RAYLEIGH WAVE

(1) Elastic Case

Rewriting Rayleigh wave equation

$$\xi^6 - 8\xi^4 + \xi^2(24-16\eta^2) + 16(\eta^2-1) = 0 \qquad .. \quad (4)$$

From equation (3), values of η are obtained for different values of Poisson's ratio (σ). Poisson's ratio is varied from zero to 0.45 at an interval of 0.05, and for each value of η, value of ξ is computed from equation (4).

Computor programme of this is written[1] and the values are computed[2]. A graph is plotted between (1) ξ and σ and (2) η and σ, as shown in Fig. (1). (App. A) Since $\xi = \frac{C}{\beta}$, the velocity of propagation of surface waves **is** thus independent of the frequency for elastic case and depends only on the elastic constants of the material. Thus, there is no dispersion of these waves and a plane surface wave will travel without change in the form.

In order to calculate the displacements U and W from equation (2)

$$\left.\begin{aligned} U &= \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z} \\ \text{and} \quad W &= \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \end{aligned}\right\} \qquad .. \quad (5)$$

Here, ψ_2 is taken as ψ because other two components

---

1. Appendix B-I
2. For results see Appendix C

of ψ do not come into play. Substituting for ϕ and ψ from equation (3-10)

$$\left.\begin{aligned} U &= -\left| A\,ki\,e^{-qz} - B\,S\,e^{-Sz} \right| \exp\left| i(wt - kx) \right| \\ \text{and} \quad W &= -\left| A\,q\,e^{-qz} + Bik\,e^{-Sz} \right| \exp\left| i(wt-kx) \right| \end{aligned}\right\} \quad .. (6)$$

From |3-14(a)|

$$B = A\left| \frac{2iqk}{k^2+S^2} \right| \quad .. (7)$$

Substituting in (6) and taking only the real part,

$$U = Ak\left| e^{-qz} - \frac{2qS}{k^2 + S^2}\, e^{-Sz} \right| \sin\,(wt - kx) \quad .. (8)$$

and $$W = -Aq\left| e^{-qz} - \frac{2k^2}{k^2+S^2}\, e^{-Sz} \right| \cos(wt-kx) \quad .. (9)$$

The rate at which the amplitude of the displacement along the direction of propagation decreases with depth depends upon the factor,

$$\tau_x = \left| e^{-qz} - \frac{2qS}{k^2+S^2}\, e^{-Sz} \right| \quad .. (10)$$

Similarly the rate at which the amplitude of the motion in a direction normal to the surface decreases with depth depends upon the factor.

$$\tau_z = \left| -e^{-qz} + \frac{2k^2}{k^2+S^2}\, e^{-Sz} \right| \quad .. (11)$$

$$q^2 = k^2 - k_\alpha^2 .$$

$$k\alpha = \frac{w}{\alpha} ;$$

$$k_\beta = \frac{w}{\beta} ;$$

$$\xi = \frac{c}{\beta}$$

Therefore,

$$q^2 = k^2 \left| 1 - \frac{k\alpha^2}{k\beta^2} + \frac{k_\beta^2}{k^2} \right|$$

$$= k^2 \left| 1 - \eta^2 \xi^2 \right| \quad .. (12)$$

Similarly,

$$S^2 = k^2 - k_\beta^2$$

$$= k^2 (1 - \xi^2) \quad .. (13)$$

$$k = \frac{w}{c} = \frac{2\pi}{L}$$

where,

L - wave length of Rayleigh wave,

Therefore,

$$\frac{2k^2}{k^2+S^2} = \frac{2}{2-\xi^2} \quad .. (14)$$

and

$$\frac{2qS}{k^2+S^2} = \frac{2\sqrt{1-\eta^2\xi^2}\sqrt{1-\xi^2}}{2-\xi^2} \quad .. (15)$$

Therefore, we can write eqn.(10) and (11) as-

$$\tau_x = \exp\left| -2\pi\sqrt{1-\eta^2\xi^2} \cdot \frac{z}{L} \right| - \frac{2\sqrt{1-\eta^2\xi^2}\sqrt{1-\xi^2}}{2-\xi^2} .$$

$$\exp\left| -2\pi\sqrt{1-\xi^2} \cdot \frac{z}{L} \right| \quad .. (16)$$

and

$$\tau_z = -\exp\left[-2\pi\sqrt{1-\eta^2\xi^2}\cdot\frac{z}{L}\right] + \frac{2}{2-\xi^2}\exp\left[-2\pi\sqrt{1-\xi^2}\cdot\frac{z}{L}\right] \quad .. (17)$$

Since η and ξ are known for values of Poisson's ratio, τ_x and τ_z from equation (16) and (17) is calculated for each value of Poisson's ratio. For a particular value of z/L which varies from zero to 2.00 with an increment of 0.1. Results are plotted as shown in fig. 2 and 3. (Appendix A).

Let U_o and W_o be the values of displacement at z=0, i.e.

$$U_o = Ak\left(1-\frac{2qS}{k^2+S^2}\right)$$

$$= Ak\left(1-\frac{2\sqrt{1-\eta^2\xi^2}\sqrt{1-\xi^2}}{2-\xi^2}\right) \quad .. (18)$$

and

$$W_o = Aq\left|\frac{2}{2-\xi^2} - 1\right| \quad ... (19)$$

Then,

$$\frac{U}{U_o} = \left[\frac{e^{-qz} - \frac{2qS}{k^2+S^2}e^{-Sz}}{1-\frac{2qS}{k^2+S^2}}\right] \quad ... (20)$$

$$\frac{W}{W_o} = \left|\frac{e^{-qz} - \frac{2k^2}{k^2+S^2}e^{-Sz}}{\left|\frac{2k^2}{k^2+S^2} - 1\right|}\right| \quad ... (21)$$

$$\frac{U}{W_o} = \frac{k}{q}\left|\frac{e^{-qz} - \frac{2qS}{k^2+S^2}e^{-Sz}}{\left(\frac{2k^2}{k^2+S^2} - 1\right)}\right| \quad .. (22)$$

$$\frac{U}{W} = -\frac{k}{q}\left|\frac{e^{-qz} - \frac{2qS}{k^2+S^2} e^{-Sz}}{e^{-qz} - \frac{2k^2}{k^2+S^2} e^{-Sz}}\right| \quad .. (23)$$

From equations (20), (21), (22), and (23), we obtain U/U_0, W/W_0, U/W_0 and U/W for different values of Poisson's ratio and different values of z/L. z/L varies from 0.0 to 2.00 with an increment of 0.10.

Results[3] are plotted as shown in Fig. 4,5,6,7(App.A).

Computer programme is written[4] to obtain τ_x, τ_z, U/U_0, U/W_0, W/W_0, U/W against z/L for different values of Poisson's ratio.

RAYLEIGH WAVE IN MAXWELL SOLID

From equation (2-2), for maxwell solid, complex moduli is given as

$$Y_T = \left|\frac{1}{E} + \frac{1}{iw\nu}\right|^{-1}$$

Also from equation (3-29),

$$Y_T = (1+\sigma)Y_s$$

$$\text{Therefore, } Y_s = \frac{1}{1+\sigma}\left|\frac{1}{E} + \frac{1}{iw\nu}\right|^{-1} \quad .. (24)$$

We know that,

$$\xi = \frac{c}{\beta} = \frac{w}{k} \Big/ \sqrt{\mu/\rho}$$

$$\text{Therefore, } k = \frac{w}{\xi}\sqrt{\rho/\mu} \quad .. (25)$$

3. See Appendix C-
4. See Appendix B-III, IV, and V.

But $\mu = \frac{1}{2} Y_s$

Therefore,

$$k = \frac{w}{\xi} \sqrt{\frac{2\rho}{Y_s}}$$

$$= \frac{w}{\xi} \sqrt{\frac{2\rho(1+\sigma)}{\left(\frac{1}{E} + \frac{1}{iw\nu}\right)^{-1}}}$$

$$= \frac{w}{\xi} \sqrt{\frac{2\rho(1+\sigma)}{E}} \left|1 + \frac{E}{iw\nu}\right|^{1/2}$$

$$= \frac{w}{\xi} \sqrt{\frac{\rho}{\mu}} \left(1 + \frac{E}{iw\nu}\right)^{1/2}$$

Since $\frac{E}{w\nu}$ is less than unity

$$k = \frac{w}{\xi} \sqrt{\frac{\rho}{\mu}} \left(1 + \frac{1}{2}\frac{E}{iw\nu} + \dots\right)$$

The higher powers of $\frac{E}{iw\nu}$ are neglected as $\frac{E}{iw\nu} < 1$

Therefore,

$$k = \frac{w}{\xi} \sqrt{\frac{\rho}{\mu}} \left(1 - \frac{1}{2}\frac{iE}{w\nu}\right) \quad .. \quad (26)$$

Therefore,

$$q = \frac{w}{\xi} \sqrt{\frac{\rho}{\mu}} \left(1 - \frac{1}{2}\frac{iE}{w\nu}\right) \sqrt{1 - \eta^2 \xi^2}$$

Taking real part of q

$$q = R\left|\frac{w}{\xi} \sqrt{\frac{\rho}{\mu}} \left(1 - \frac{1}{2}\frac{iE}{w\nu}\right) \sqrt{1 - \eta^2 \xi^2}\right|$$

where,

$f = R|z|$ denotes the real part of z.

Therefore,

$$q = \frac{w}{\xi} \sqrt{\frac{\rho}{\mu}} \sqrt{1 - \eta^2 \xi^2} \quad .. \quad (27)$$

Similarly,

$$S = \frac{w}{\xi}\sqrt{\frac{2(1+\sigma)}{E}}\left(1 - \frac{1}{2}\frac{iE}{w\nu}\right)\sqrt{1-\xi^2}$$

Taking real part,

$$S = \frac{w}{\xi}\sqrt{\frac{\rho}{\mu}}\sqrt{1-\xi^2} \quad .. \quad (28)$$

From (27) and (28) we see that these values are same as for elastic case. Therefore the values of U/U_0, U/W_0, W/W_0, U/W, τ_x and τ_z will remain same as have been obtained for elastic ase.

From (26), the velocity of Rayleigh wave is obtained as,

$$\frac{w}{R|k|} = \xi\sqrt{\frac{\rho}{\mu}} \quad .. \quad (29)$$

This equation indicates that dispersion does not occur even the media is made of maxwell solid.

The imaginary part of (26) gives the attenuation which indicates that attenuation depends upon the ratio of $\frac{E}{\nu}$ i.e. the ratio of spring constant to the dashpot constant.

RAYLEIGH WAVE IN VOIGT SOLID

From equation (2-3), for Voigt solid,

$$Y_T = (E+iw\nu)$$

Therefore,

$$Y_s = \frac{1}{1+\sigma}(E+iw\nu)$$

Therefore,

$$k = \frac{w}{\xi}\sqrt{\frac{2\rho}{Y_s}} = \frac{w}{\xi}\sqrt{\frac{2\rho\,(1+\sigma)}{(E + iw\nu)}}$$

or $$k = \frac{w}{\xi}\sqrt{\frac{2\rho\,(1+\sigma)}{E^2+w^2\nu^2}\,(E-iw\nu)}$$

$$= \frac{w}{\xi}\sqrt{\frac{2\rho\,(1+\sigma)E}{E^2+w^2\nu^2}}\,\left(1 - \frac{iw\nu}{E}\right)^{1/2}$$

For Voigt solid $\frac{w\nu}{E} < 1$

Therefore,

$$k = \frac{w}{\xi}\sqrt{\frac{2\rho\,(1+\sigma)}{E\left(1+\frac{w^2\nu^2}{E^2}\right)}}\quad\left(1-\frac{1}{2}\,\frac{iw\nu}{E}\right) \qquad ..\ (30)$$

the higher powers of $\frac{iw\nu}{E}$ have been neglected.

The phase velocity of Rayleigh wave is thus,

$$V_R = \frac{w}{R|k|} = \xi\sqrt{\frac{E}{2(1+\sigma)\rho}\left(1+\frac{w^2\nu^2}{E^2}\right)}$$

or $$V_R = \xi\sqrt{\frac{\mu}{\rho}}\ \sqrt{1+\frac{4\pi^2}{T^2}\left(\frac{\nu}{E}\right)^2} \qquad ..\ (31)$$

Equation (31) is solved[5] for given values of ξ and ν/E with time period varying from 0.0 to 1 sec. at an interval of 0.1 sec. Hence, we obtain a graph between V_k and Time period for a fixed value of Poisson's ratio and for E/ν varying from 20 to 100

---

5. Results in Appendix C- and computer programme in Appendix B-VII

with an increment of 20. Similar results are obtained for different values of Poisson's ratio. |See Fig.8 and 9(Appendix A)|.

Rewriting equation (30); taking real part only,

$$k = \frac{w}{\xi}\sqrt{\frac{2\rho(1+\sigma)}{E\left(1+\frac{w^2\nu^2}{E^2}\right)}}$$

$$= \frac{w}{\xi}\sqrt{\frac{\rho}{\mu}}\sqrt{\frac{1}{1+\frac{w^2\nu^2}{E^2}}}$$

$$= \frac{2\pi}{L}\sqrt{1/\left(1+\frac{w^2\nu^2}{E^2}\right)} \qquad .. (32)$$

Therefore,

$$q = \frac{2\pi}{L}\sqrt{\frac{1}{1+\frac{w^2\nu^2}{E^2}}}\sqrt{1-\eta^2\xi^2} \qquad .. (33)$$

and $$S = \frac{2\pi}{L}\sqrt{\frac{1}{1+\frac{\omega^2\nu^2}{E^2}}}\sqrt{1-\xi^2} \qquad .. (34)$$

Therefore,

$$U = Ak\left|e^{-qz} - \frac{2qS}{k^2+S^2}e^{-Sz}\right|\sin(wt-kx) \qquad .. (35)$$

and $$W = Aq\left|e^{-qz} - \frac{2k^2}{k^2+S^2}e^{-Sz}\right|\cos(wt-kx) \qquad .. (36)$$

U_o and W_o = value of U and W at z=0

or $$U_o = Ak\left|1 - \frac{2qS}{k^2+S^2}\right| \qquad .. (37)$$

and

$$W_o = -Aq \left| 1 - \frac{2k^2}{k^2+S^2} \right| \quad .. (38)$$

From (35), (36), (37), and (38), U/U_o, W/W_o, U/W is calculated for a particular value of Poisson's ratio, which gives values of η and ξ, and for a particular value of $(\frac{w\nu}{E})$ with $\frac{z}{L}$ varying from 0 to 2.00 with an increment of 0.1. Similar sets are obtained for different values of σ and $\frac{w\nu}{E}$. A program[6] is written to compute the values. |see Appendix B- | and results are plotted |See Fig. 10,11,12 (Appendix A)|.

LOVE WAVES

Elastic Solid-

Eqn. (3-26) gives the equation for Love waves in elastic solid.

Rewriting the equation, we have

$$\mu \sqrt{1 - \frac{c^2}{\beta^2}} - \mu_1 \sqrt{\frac{c^2}{\beta_1^2} - 1} \tan(kH_1 \sqrt{(c^2/\beta_1^2) - 1} = 0 \quad .. (39)$$

In order to calculate the displacement V,

$$V = \left\{ \left[C_1 \exp(-ikz \sqrt{\frac{c^2}{\beta_1^2} - 1} \right] + \left\{ F_1 \exp(ikz \sqrt{\frac{c^2}{\beta_1^2} - 1}) \right| \exp \left| i(wt - kx) \right| \right\} \quad .. (40)$$

Let V_o be the displacement at z=0

$$V_o = (C_1 + F_1) \exp \left| i(wt - kx) \right| \quad .. (41)$$

6. Appendix B-VI
7. Appendix C-

Therefore,

$$\frac{V}{V_o} = \frac{C_1}{C_1+F_1}\exp\left(-ikz\sqrt{\frac{c^2}{\beta_1^2}-1}\right)$$

$$+\frac{F_1}{C_1+F_1}\exp\left(ikz\sqrt{\frac{c^2}{\beta_1^2}-1}\right) \quad .. \ (42)$$

From equation 3-

$$\frac{C_1}{F_1} = \frac{\exp(-iH_1S_1)}{\exp(iH_1S_1)} \quad .. \ (43)$$

Therefore, by componendo and dividendo,

$$\frac{C_1}{C_1+F_1} = \frac{\cos H_1S_1 - i\sin H_1S_1}{2\cos H_1S_1}$$

$$= \frac{1}{2} - \frac{1}{2}\, i\tan H_1S_1 \quad .. \ (44)$$

Similarly,

$$\frac{F_1}{C_1+F_1} = \frac{\operatorname{Cos} H_1S_1 + i\sin H_1S_1}{2\cos H_1S_1}$$

$$= \frac{1}{2} + \frac{1}{2}\, i\tan H_1S_1 \quad .. \ (45)$$

Taking real part of V/V_o, we have;

$$\frac{V}{V_o} = \cos\left(kz\sqrt{\frac{c^2}{\beta_1^2}-1}\right) - \tan H_1S_1 \sin\left(kz\sqrt{\frac{c^2}{\beta_1^2}-1}\right)$$

or

$$\frac{V}{V_o} = \cos\left(2\pi\frac{z}{L}\sqrt{\frac{c^2}{\beta_1^2}-1}\right) - \left|\tan\left(2\pi\frac{H_1}{L}\sqrt{\frac{c^2}{\beta_1^2}-1}\right)\right.$$

$$\left.\sin\left(2\pi\frac{z}{L}\sqrt{\frac{c^2}{\beta_1^2}-1}\right)\right| \quad .. \ (46)$$

Equation (46) is solved for a particular value of c/β_1 and H_1/L and for different values of z/L. Similar sets are obtained by varying first the ratio c/β_1 and then the ratio H_1/L.

Computer programme[8] is written to compute the values |see Appendix B }, and results[9] are obtained and plotted|see fig. 13,14,15,16,17 -App.A|.

Anelastic Case

Let us take the case when overlying media is anelastic. Let us call it as media M_2 with a layer thickness H_2. Love wave equation for such a case is obtained by using μ_2 in place of μ_1, β_2 in place of β_1 and H_2 in place of H_1 in equation (39). Therefore, we obtain,

$$\mu\sqrt{1-\frac{c^2}{\beta^2}} - \frac{1}{2} Y_s \sqrt{\frac{c^2}{\beta_2^2}-1} \left|\tan\left(kH_2\sqrt{\frac{c^2}{\beta_2^2}-1}\right)\right| = 0$$

because $\mu_2 = \frac{1}{2} Y_s$ .. (47)

Now $$Y_s = \frac{Y_T}{(1+\sigma)}$$

where, Y_T is the complex moduli for anelastic medium.

Maxwell Solid

When M_2 is maxwell solid,

$$Y_T = \left(\frac{1}{E} + \frac{1}{iw\nu}\right)^{-1}$$

therefore,

$$Y_s = \frac{E}{(1+\sigma)}\left(1+\frac{E}{iw\nu}\right)^{-1} \quad .. (48)$$

8. Appendix B-II
9. Appendix C-

and when M_2 is Voigt solid,

$$Y_s = \frac{E}{(1+\sigma)}\left(1+\frac{iw\nu}{E}\right) \qquad .. (49)$$

Equation (47) becomes, for Voigt solid,

$$\mu\sqrt{1-\frac{c^2}{\beta^2}} - \frac{1}{2}\frac{E}{(1+\sigma)}\left(1+\frac{iw\nu}{E}\right)\sqrt{\frac{c^2}{\beta_2^2}-1}\left|\tan kH_2\sqrt{\frac{c^2}{\beta_2^2}-1}\right| = 0$$

$$.. (50)$$

and for maxwell solid,

$$\mu\sqrt{1-\frac{c^2}{\beta^2}} - \frac{E}{2(1+\sigma)}\left(1+\frac{E}{iw\nu}\right)^{-1}\sqrt{\frac{c^2}{\beta_2^2}-1}\left|\tan kH_2\sqrt{\frac{c^2}{\beta_2^2}-1}\right|$$

$$= 0$$

$$.. (51)$$

Equation (50) and (51) represent Love wave in Voigt and Maxwell solid respectively.

CHAPTER VI

RESULTS AND CONCLUSION

The study made so far relates to both Rayleigh and Love waves. Special attention has been given to visualise the variation of horizontal and vertical components of the amplitude of displacement with respect to depth when the media is i) elastic and ii) anelastic.

Rayleigh Wave

The theory of Rayleigh waves was first given by Lord Rayleigh in 1887. He showed that the free surface of an elastic homogeneous medium can support a disturbance involving displacements in a vertical plane along the direction of propagation in the form of a retrograde ellipse and propagating with a velocity which was about 0.9 times the shear wave velocity in that medium. The theory was extended by various other authors to cover the cases of layered semi-infinite media. The results indicate that in all but the simplest case assumed by Lord Rayleigh, these waves suffered dispersion.

For the elastic case the following numerical results have been obtained for Rayleigh waves.

1. Reduction of Rayleigh wave equation into a simple form.

2. Variation of horizontal and vertical components of displacement with reference to depth $|U/U_0$ vs. $z/L|$.

3. Variation of the ratio of horizontal to vertical components of displacements with respect to depth.

The Rayleigh wave equation which is a sixth degree equation in c/β, is reduced to two simple equations for $0<\eta<.4$ and $.4<\eta<.7$. Since η is purely a function of Poisson's ratio even for the anelastic case, given η for a particular value of σ, the values of ζ can be obtained which give values correct upto three decimal places.

From figure 1 it is observed that c/β varies directly with Poisson's ratio. c/β increases with increase in Poisson's ratio whereas $\eta = \beta/\alpha$ decreases with increase in Poisson's ratio.

It is observed from the plot of U/U_0 vs z/L |Fig. 5 App.A| that the horizontal component of the displacement initially decreases with increase in depth and then attains a negative value. At Z=0.4L, the direction is reversed and approaches zero as Z increases further. It is also observed that the rate of change of amplitude increases with increase in the Poisson's ratio. Beyond Z=1.5 L the amplitude is negligible.

The plot of W/W_0 vs Z/L |Fig.7 Appendix A| indicates that the vertical component first increases with increase in depth and thereafter it decreases rapidly and approaches zero at about Z=2.00 L. Also the initial increase in amplitude increases with increase in Poisson's ratio, i.e. with increase in the Rayleigh wave velocity the amplitude increases upto a depth given by Z=0.1L.

From Fig. 2 (App.A) it is clear that rate of change of amplitude decreases with increase in the Poisson's ratio, i.e. with increase in phase velocity, the amplitude gradient decreases. One very remarkable point to note is that for nearly all the values of Poisson's ratio , the amplitude is the same at Z=0.1 times the wave length, irrespective of the initial amplitude. The horizontal components changes sign between Z=0.15 and Z=0.25 times the wave length and increase rapidly with increase in depth upto Z=.4L; thereafter they gradually decrease approaching zero.

From Fig.4 (App.A) it is observed that the ratio of the horizontal and vertical component decreases with increase in the Poisson's ratio. This ratio becomes negative for Z>0.15 but negative values are nearly equal for 0.4<Z/L<0.6 and thereafter the gradient decreases with increase in Poisson's ratio.

Analysis and computation have been carried out for anelastic media also represented in particular by the Maxwell and Voigt solids. The results indicate that there occurs no change for Maxwell sôlid so far as the variation of horizontal and vertical components of displacements are concerned but the attenuation becomes dependent on frequency.

For Voigt solid from fig. (8) and (9) |App.A| it is observed that the phase velocity is dependent upon values of E/ν for a fixed value of Poisson's ratio. The rate of change of velocity with frequency increases with increase in E/ν. Also the velocity increases with increase in Poisson's ratio.

So far as the variation of vertical and horizontal components is concerned, it is of the same nature as for the elastic case. From Fig.12 |App.A| U/W increases with decrease in depth. For all the values of Poisson's ratio, which have been tried, U/W attains a negative equal value at about Z=0.4L.

LOVE WAVES

The study of Love waves here represents the variation of amplitude with respect to depth |See Fig. (13 to 18) Appendix A|. When the overlying layer is elastic and homogeneous, the variation depends upon the value of c/β i.e. the amplitude increases with increase in the value of c/β. Since for Love wave to exist , $\beta < c < \beta_1$, a number of curves have been plotted for different values of the layer thickness.

The following observations are made.

i) With increase in c/β the amplitude of the displacement increases.

ii) The total number of nodal plane increases with increase in the layer thickness.

iii) The Love wave equation shows that phase velocity is dependent on the particular value of k, so that in the present boundary conditions, there will be dispersion of a general wave form. It is seen from the equation that if k is small, c approaches β i.e. the velocity of the longer Love waves approach the velocity of S Bodily waves in M. Also if k is large, c tends to β_1 i.e. the velocities of very short waves approach the velocity of S bodily waves in the upper medium M_1.

When the upper medium is anelastic, the k becomes complex and the variation of amplitude of displacement depends on its real part.

REFERENCES

1. Bullen, K.E., 'An Introduction to Theory of Seismology', Camb. Uni. Press, 1963.

2. Bland, D.R., 'The Theory of Linear Visco-Elasticity' Pergaman Press, 1960.

3. Ewing, Press and Jardetzky., 'Elastic Waves in Layered media', McGraw-Hill Book Company, 1957.

4. Huang, Y.T., Bull. Seis.Soc.Ame., pp.497,1968.

5. Jeffreys, H., 'The Earth', Camb. Uni.Press.1959.

6. Kolsky, H., 'Stress Waves in Solids', Dover Publications, 1963.

7. Kolsky, H., Proc. Phy.Soc.B, 62,pp.676, 1949.

8. Love, A.E.H., 'Some problems of Geodynamics', Cambridge University Press, London, 1926.

9. Maxwell, C., Scientific Papers, Vol.II, pp.26, Camb. Univ.Press., 1890.

10. Morse and Freshback, 'Methods of Theoretical Phy.I, McGraw-Hill Book Co., pp.52, 1953.

11. Rayleigh, London. Math. Soc. Proc., 17, 1887.

12. Sezwa, K., and K.Kanai., 'Damping of periodic Visco-elastic waves, with increase in Focal Distance, Bull.Earthquake Res.Inst. (Tokyo), Vol.16, pp.491-503, 1938.

13. Stoneley, R., Proc.Roy.Soc.(London),A, Vol.106, pp.416-428, 1924.

14. Stoneley, R., 'Contributions in Geophysics,I, International Series of Monographs on earth science, Pergaman Press, 1958.

15. Stoneley and Hochstrasser., 'Mon.Notices Royal Astr.Soc.Vol.7, No.5., 1957.

16. Usher, M.J., 'The Elastic Behaviour of Rocks at low frequencies, Thesis for Ph.D., Department of Geophysics, Imperial College, London, 1961.

17. Voigt, W., Ann.d.Phy. 47, 671, 1892.

APPENDIX -A

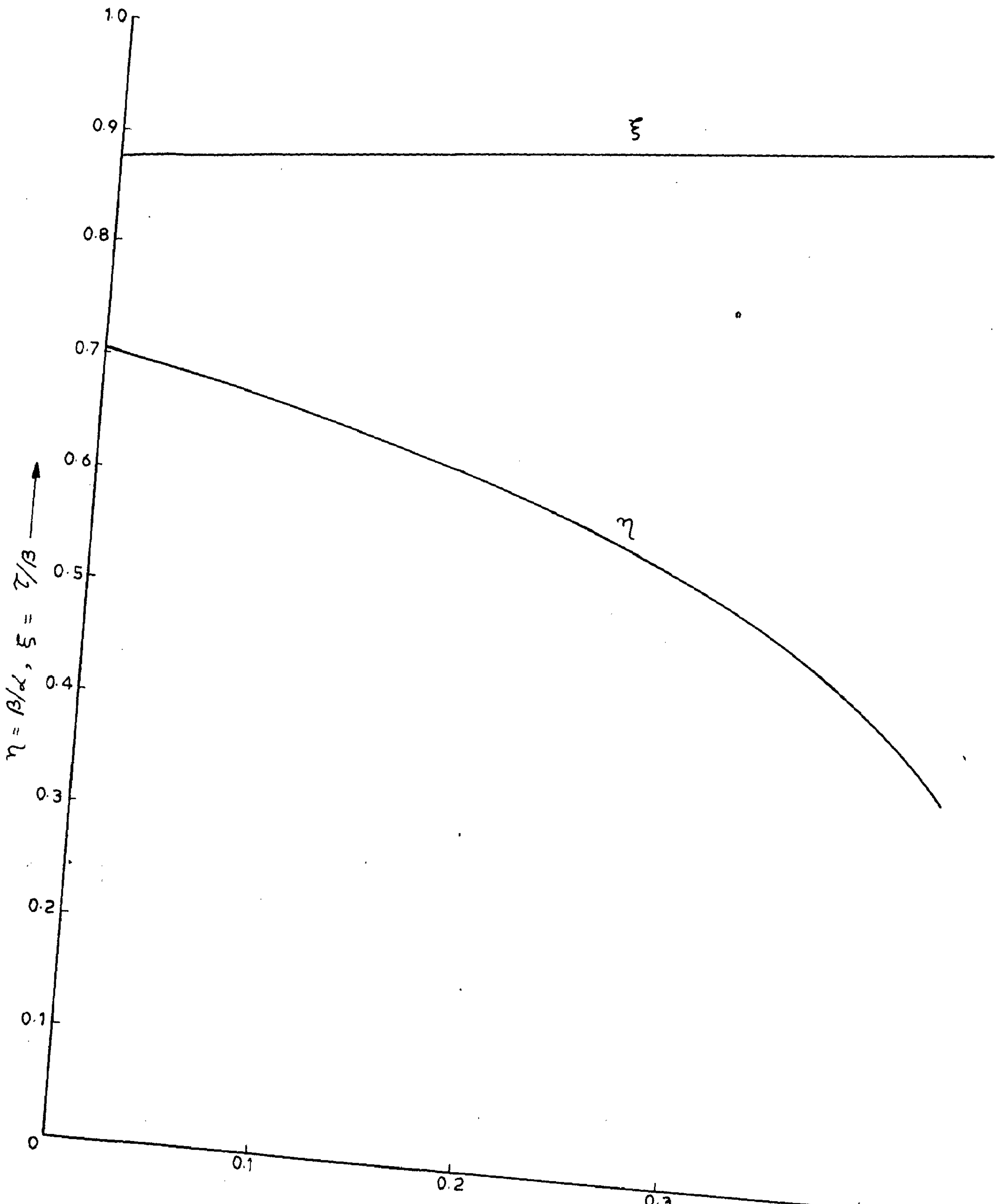


FIG. 1 - RAYLEIGH WAVE IN ELASTIC MEDIA

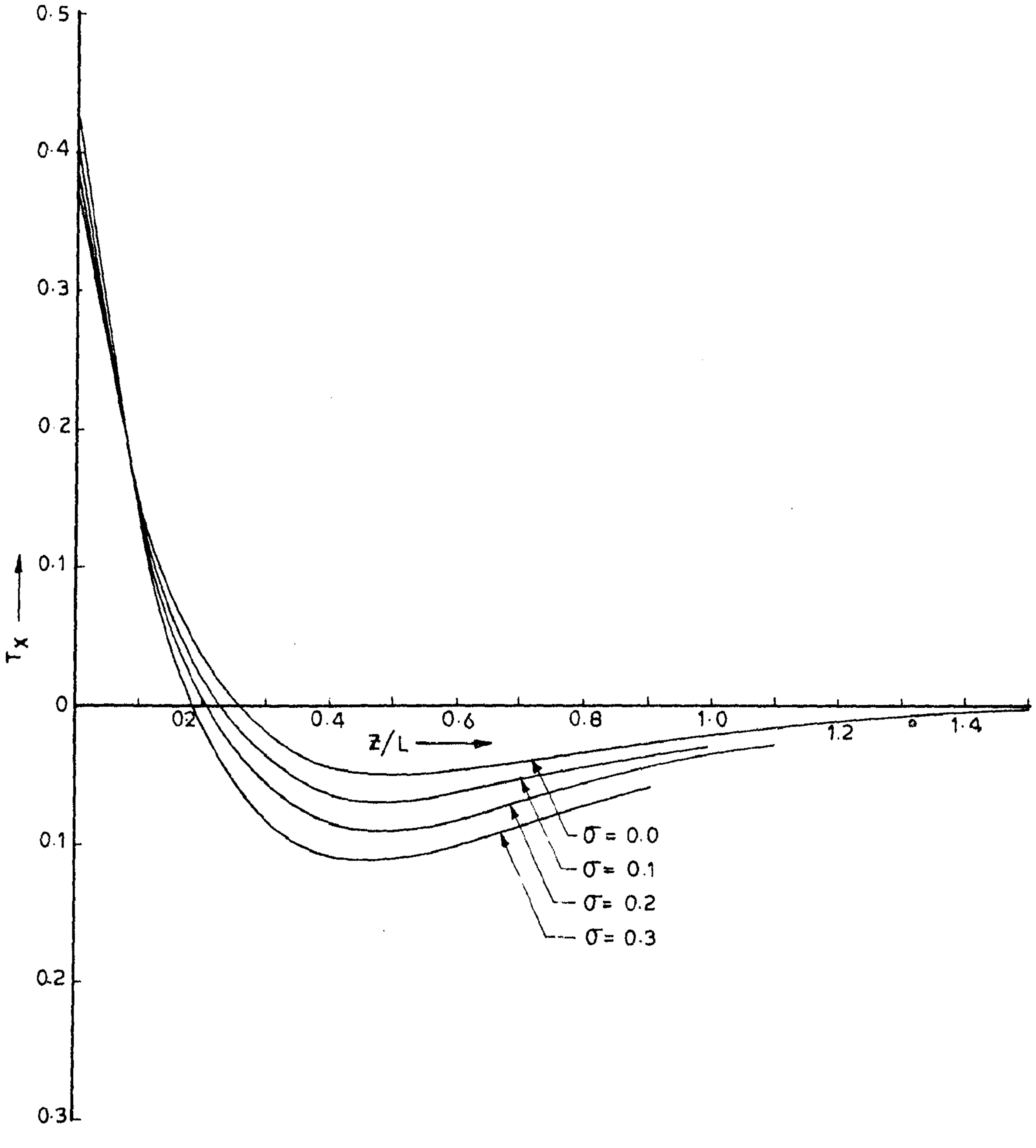


FIG.2. RAYLEIGH WAVE IN ELASTIC MEDIA FOR DIFFERENT VALUES OF POISSON'S RATIO

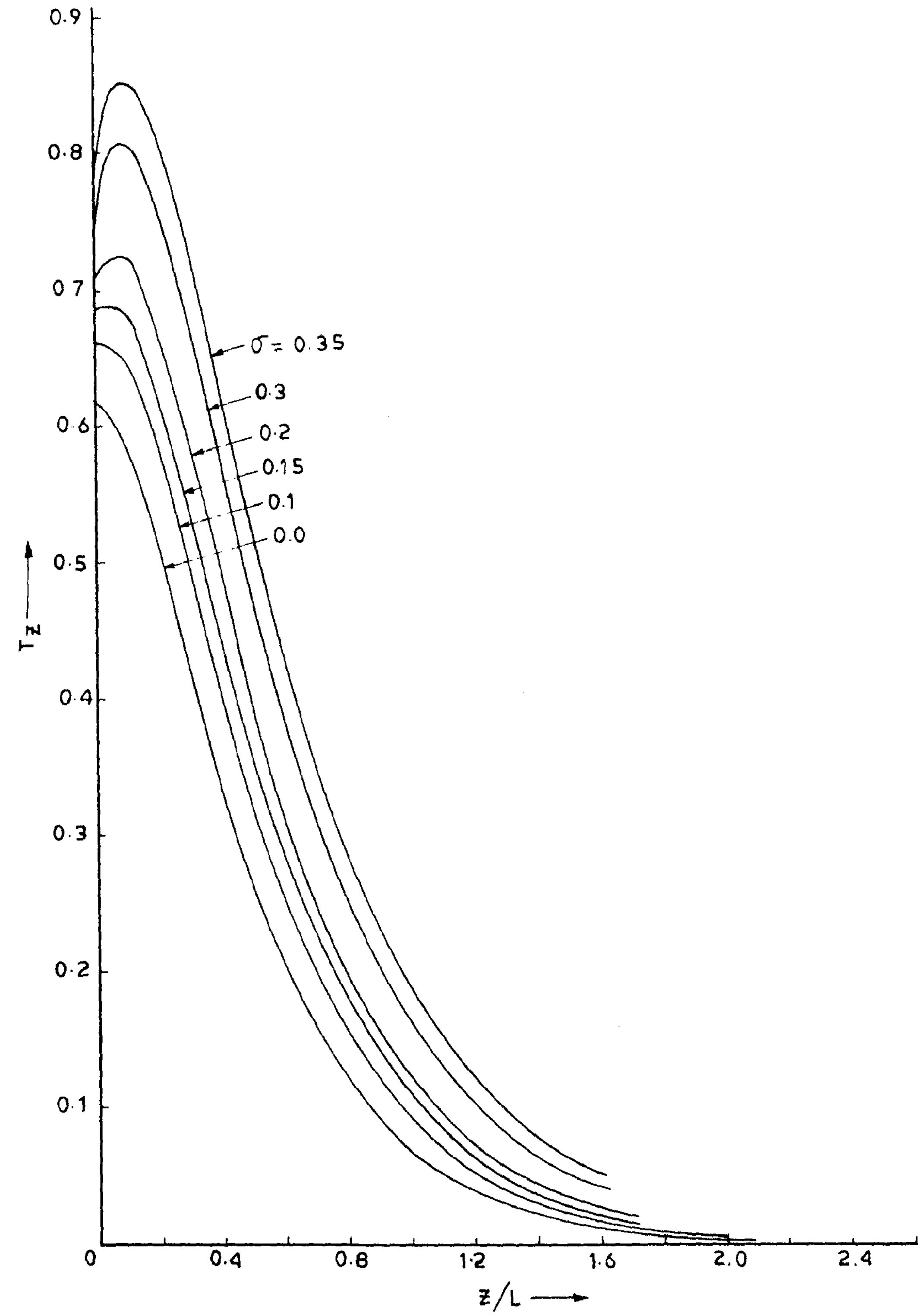


FIG. 3. RAYLEIGH WAVE IN ELASTIC MEDIA FOR DIFFERENT VALUES OF POISSON'S RATIO

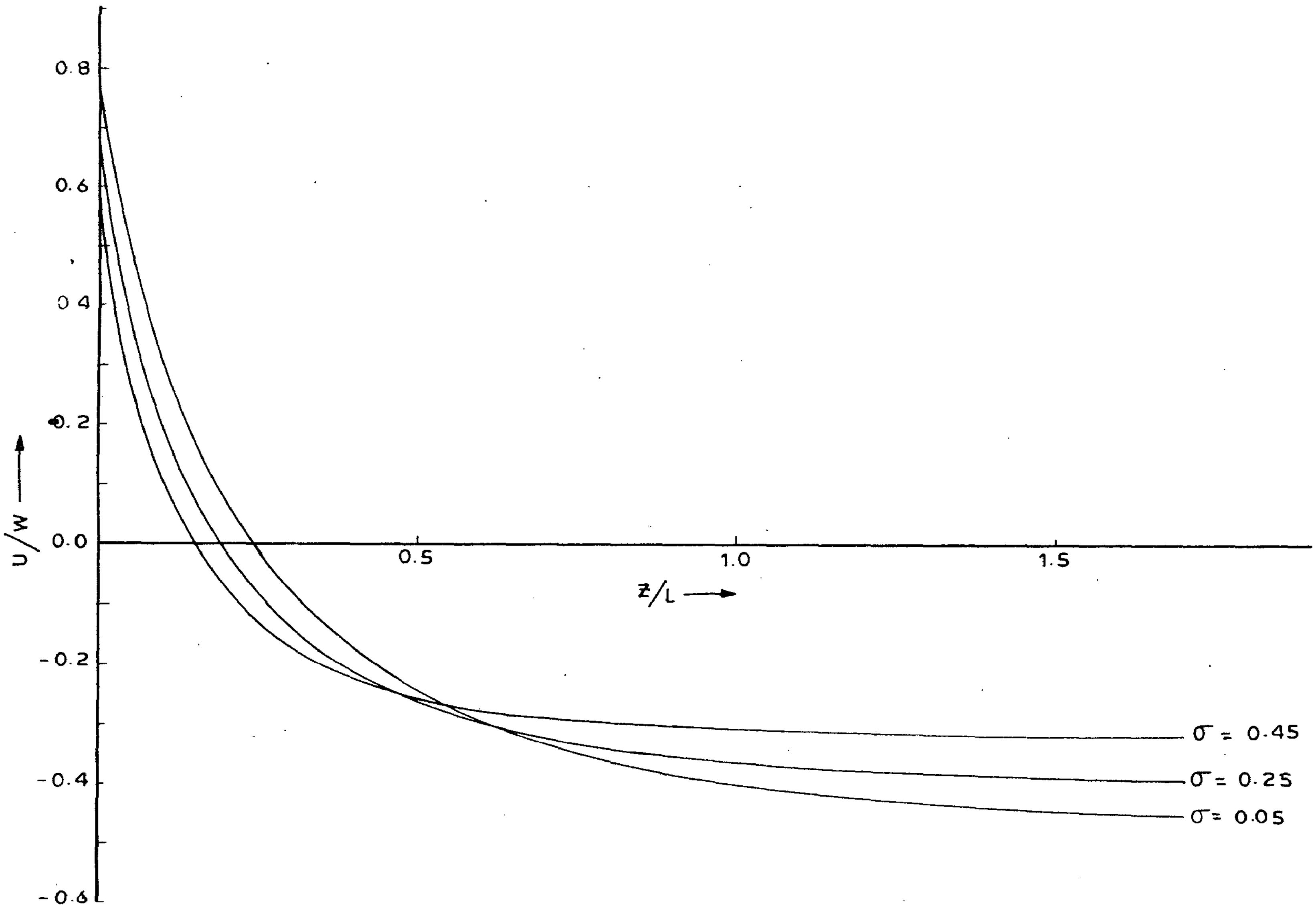


FIG. 4_RAYLEIGH WAVE IN ELASTIC MEDIA FOR DIFFERENT VALUES OF POISSONS RATIO.

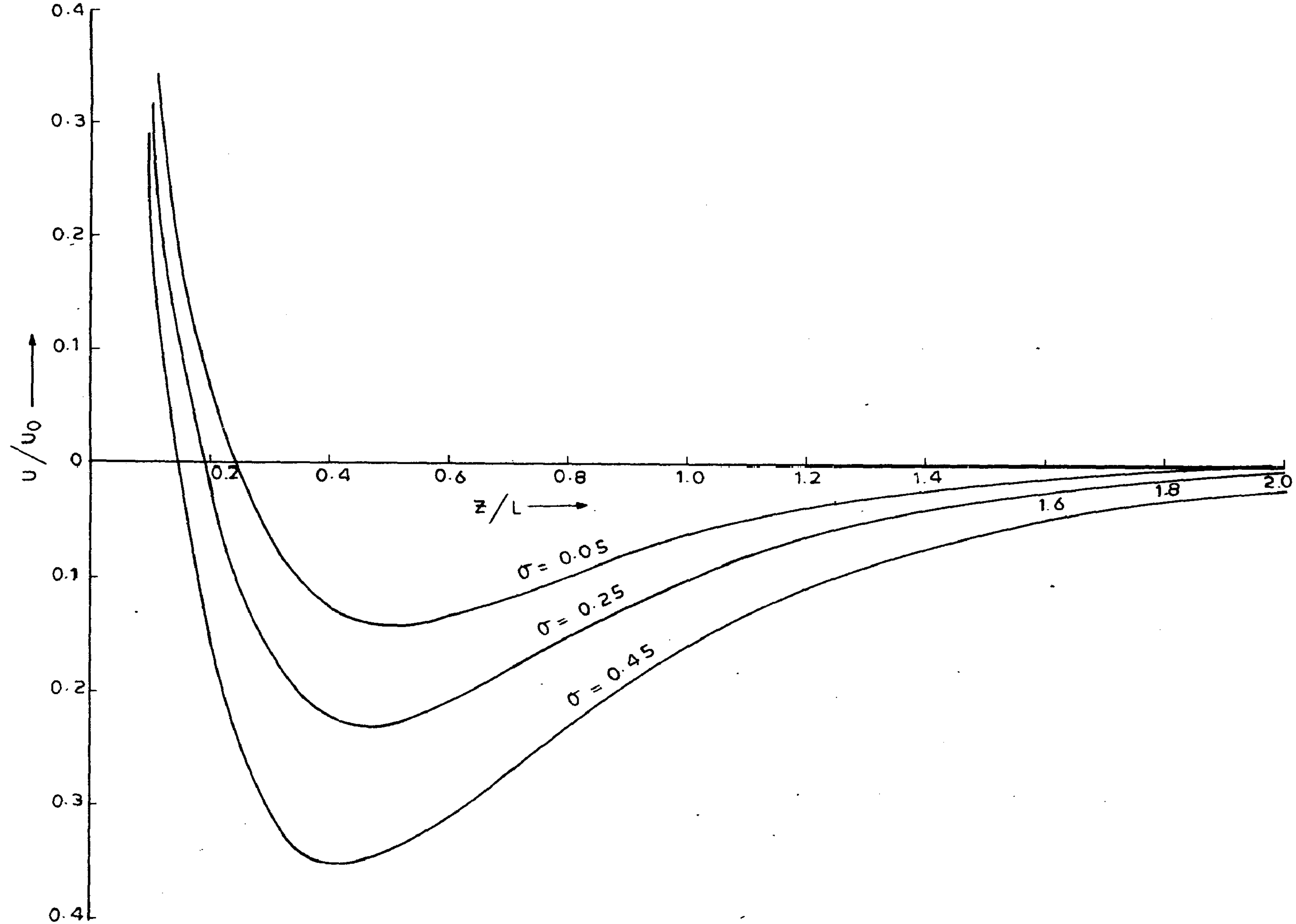


FIG.5_ RAYLEIGH WAVE IN ELASTIC MEDIA FOR DIFFERENT VALUES OF POISSON'S RATIO

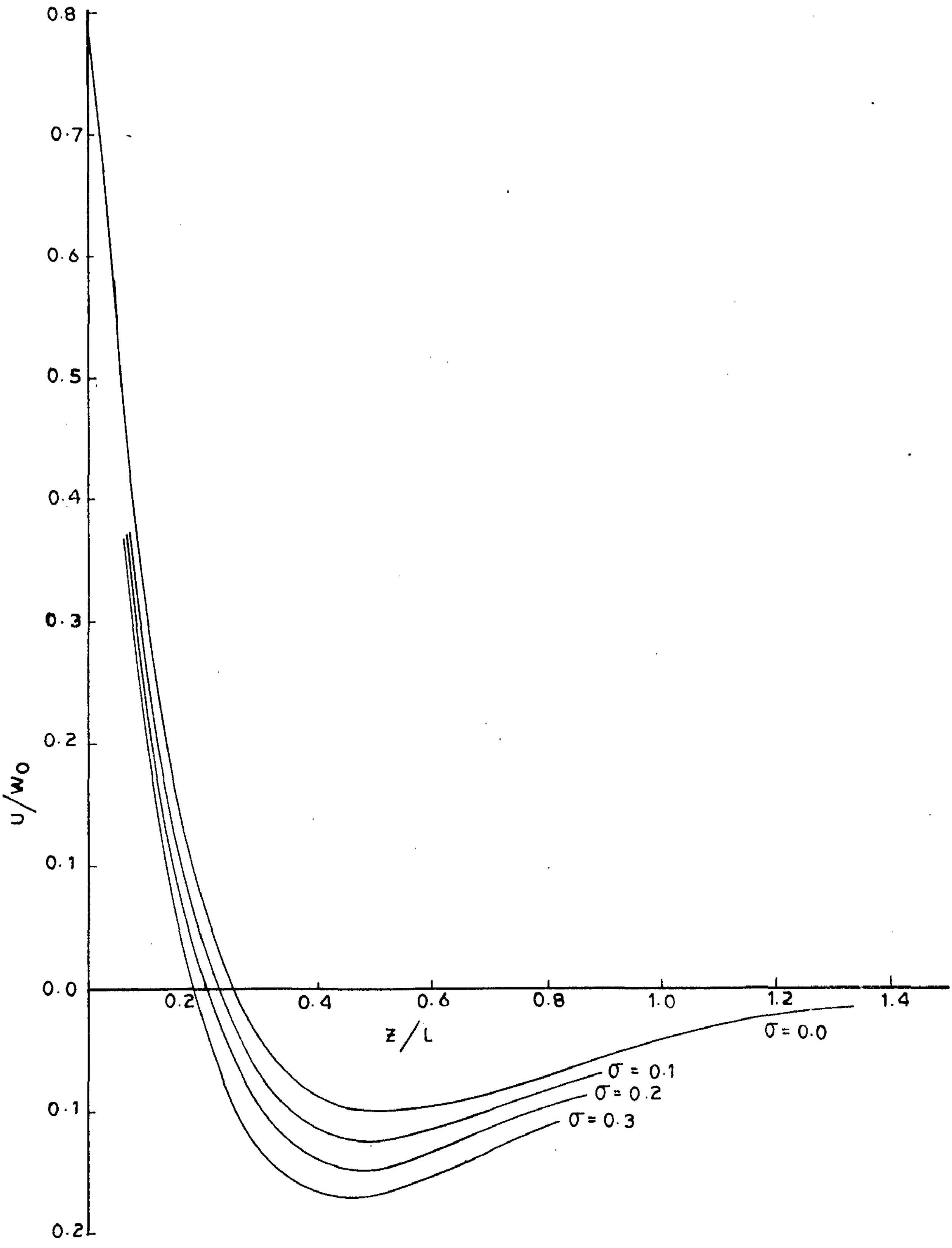


FIG.6_RAYLEIGH WAVE IN ELASTIC MEDIA FOR DIFFERENT VALUES OF POISSON'S RATIO

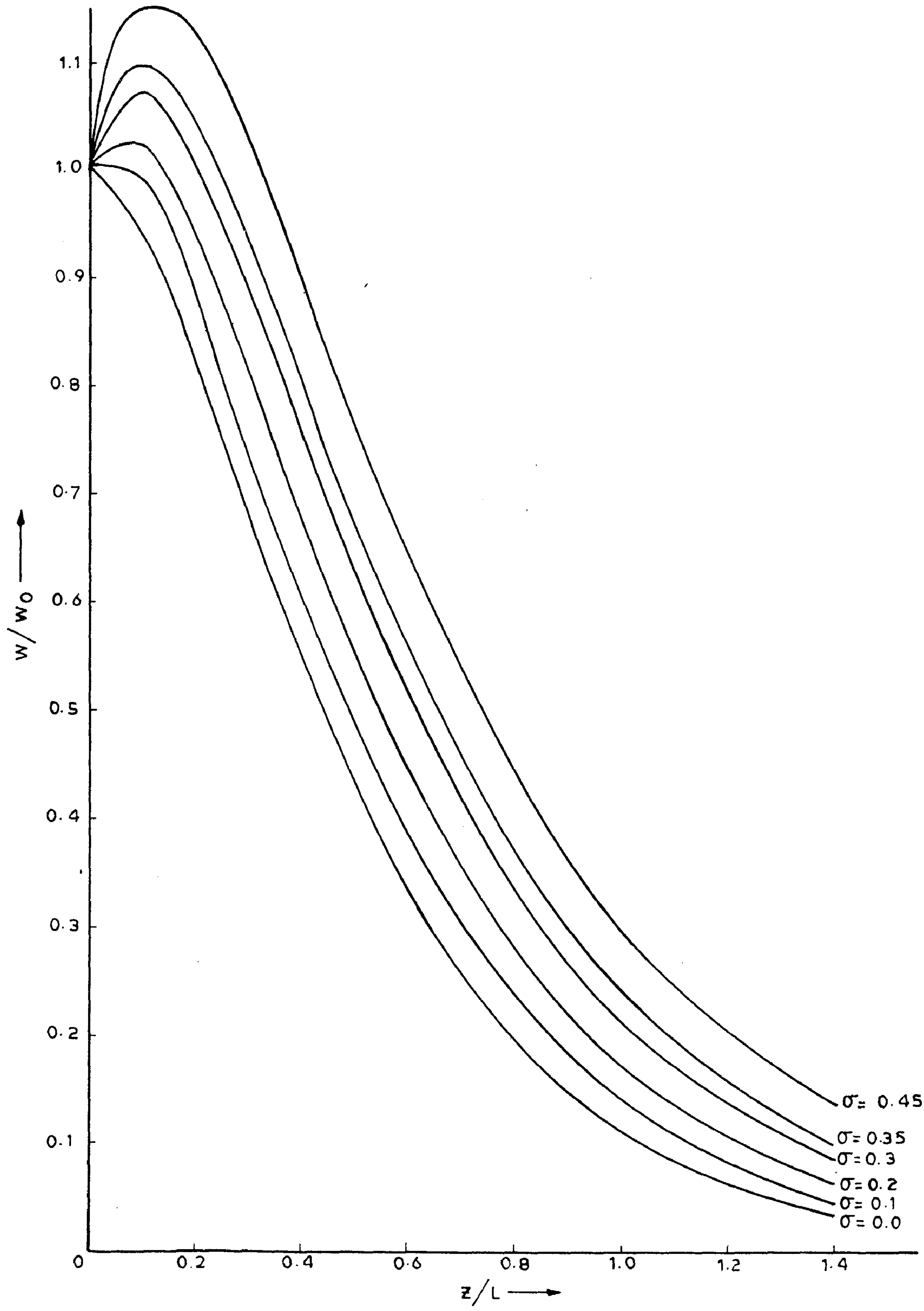


FIG 7. RAYLEIGH WAVE IN ELASTIC MEDIA FOR DIFFERENT VALUES OF POISSON'S RATIO

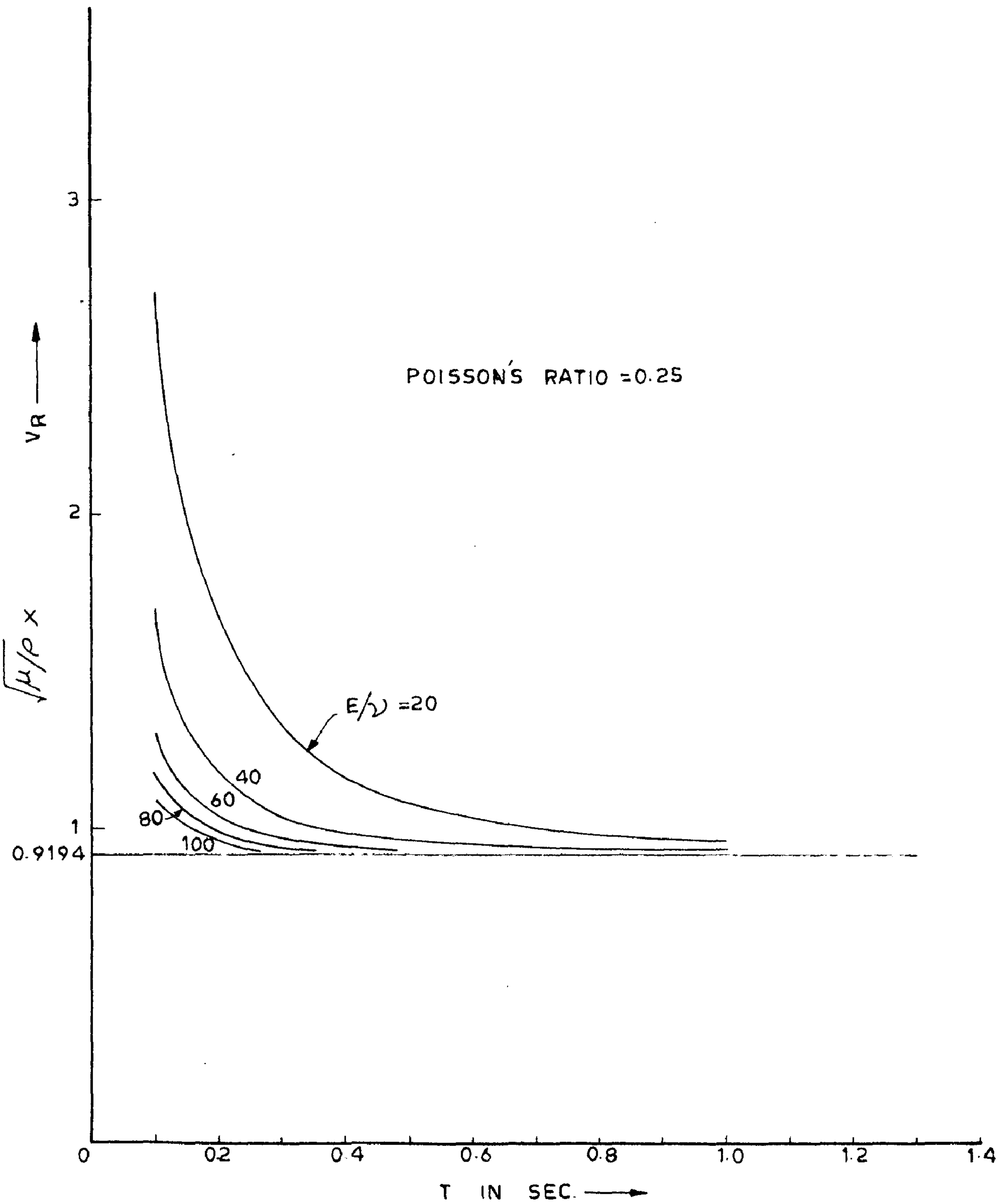


FIG. 8 - RAYLEIGH WAVE DISPERSION IN VOIGT SOLID FOR DIFFERENT VALUES OF E/ν

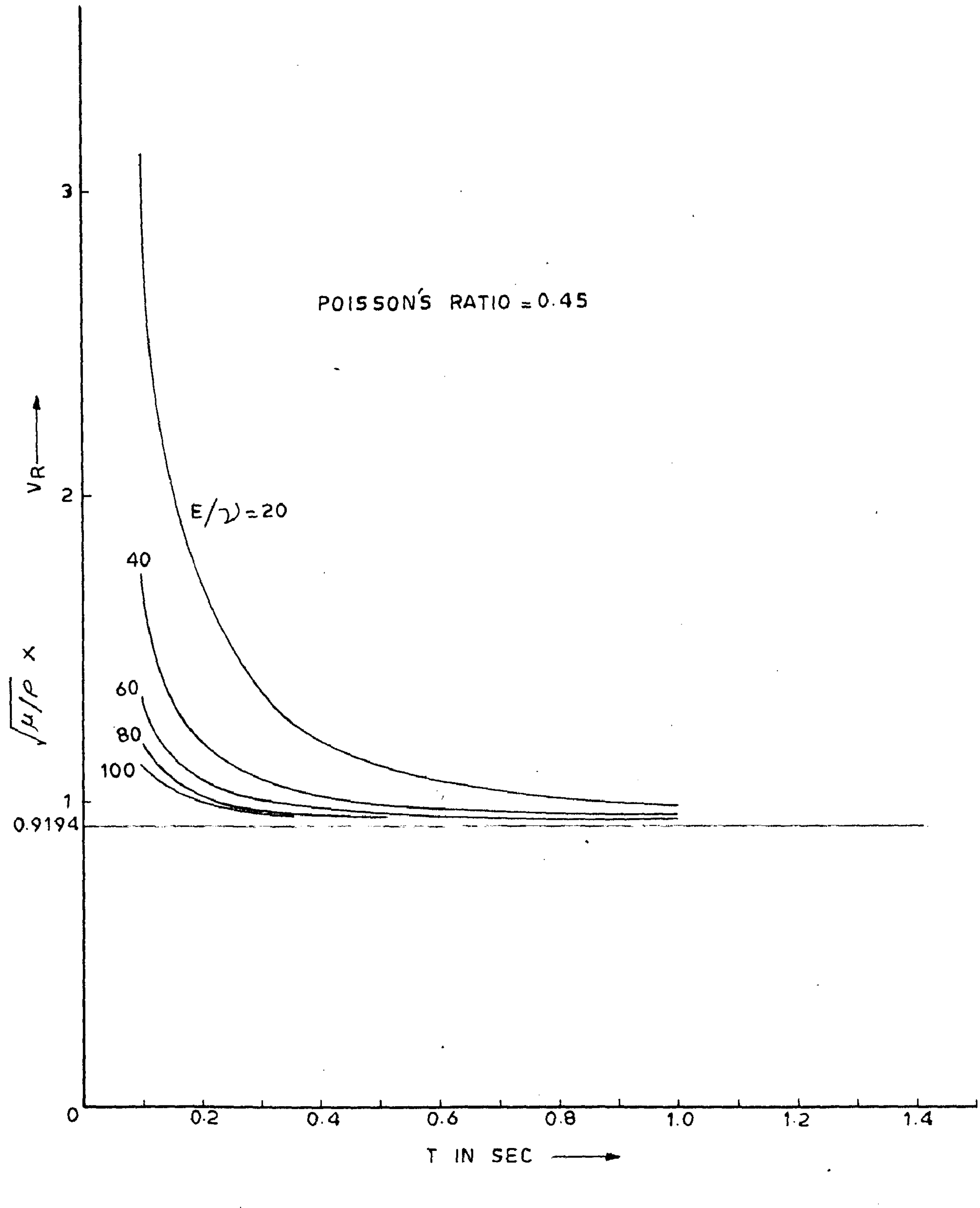


FIG. 9 - RAYLEIGH WAVE DISPERSION IN VOIGT SOLID FOR DIFFERENT VALUES OF E/ν

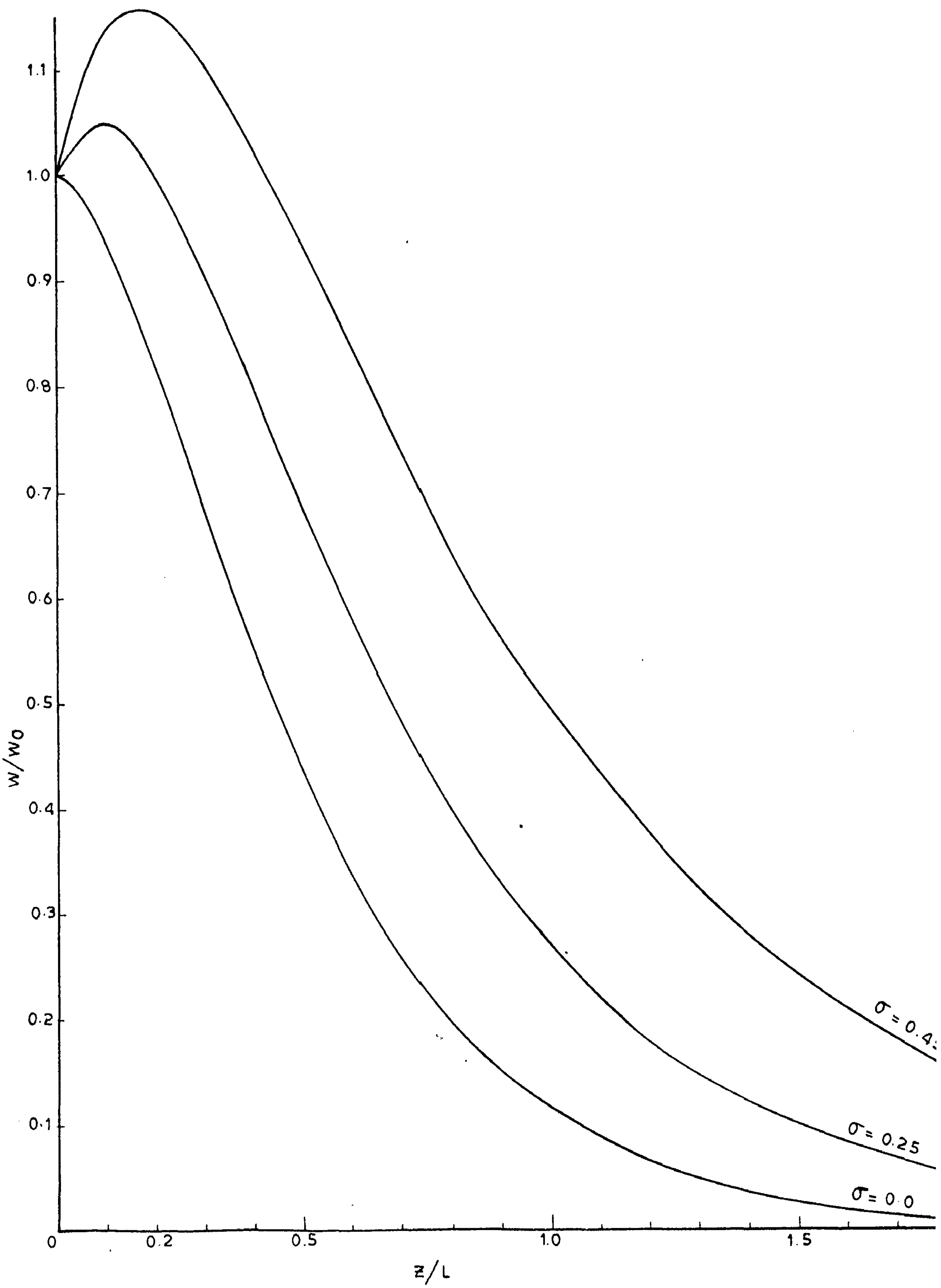


FIG 10-RAYLEIGH WAVE IN ANELASTIC MEDIA(VOIGT SOLID)

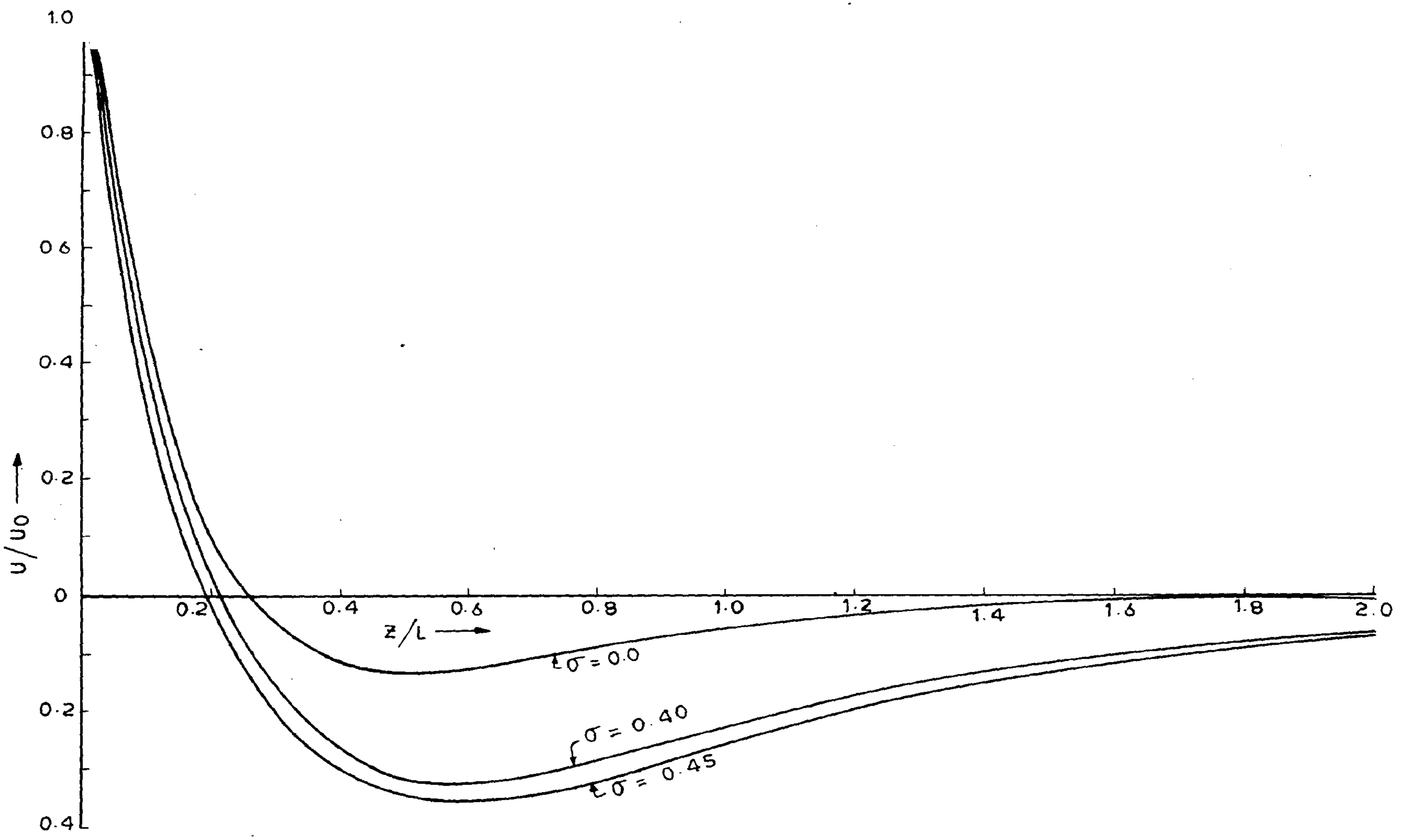


FIG.11_RAYLEIGH WAVE IN ANELASTIC MEDIA (VOIGT SOLID)

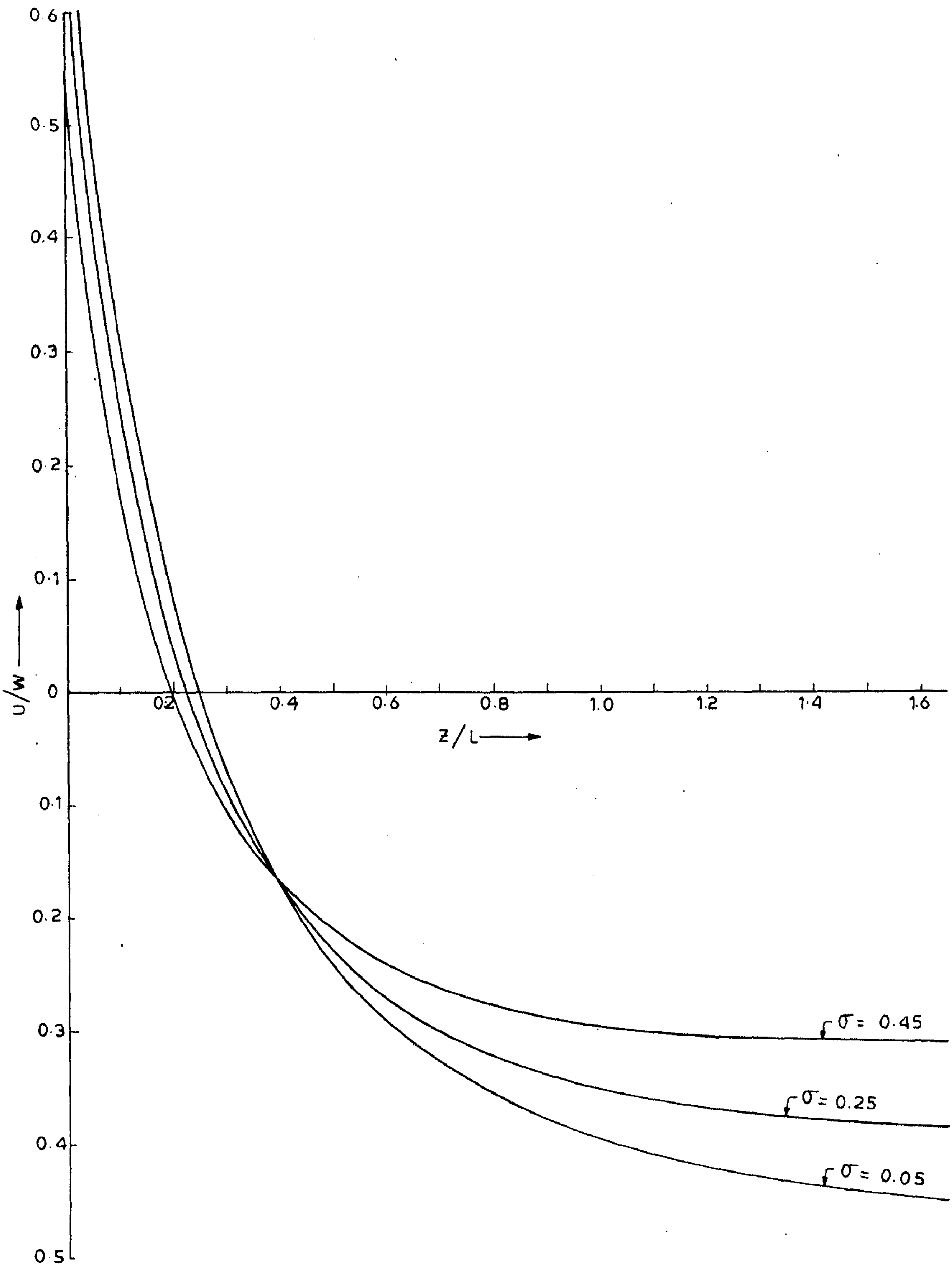


FIG.12-RAYLEIGH WAVE IN ANELASTIC MEDIA (VOIGT SOLID)

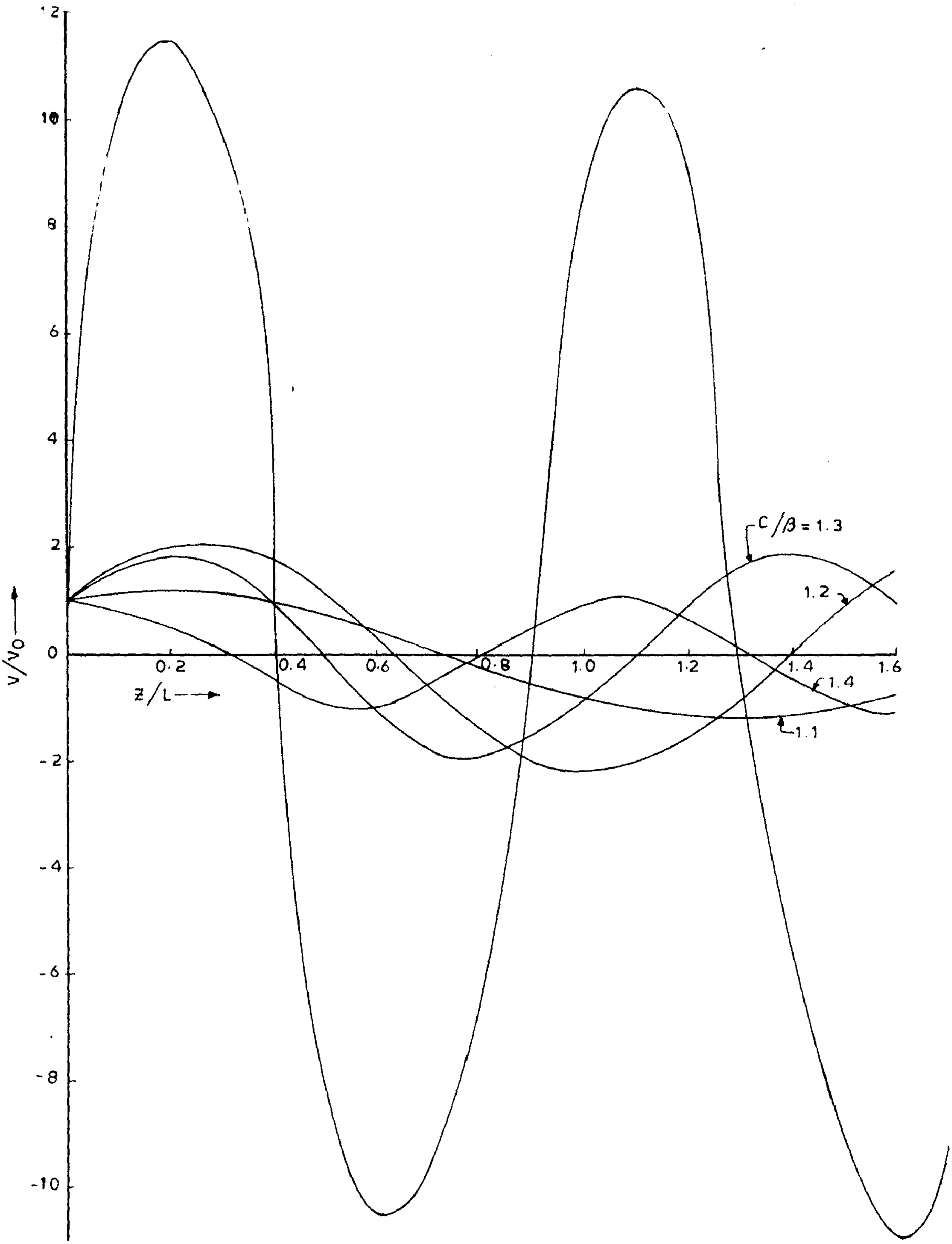


FIG.13-LOVE WAVE IN ELASTIC MEDIA (H/L =2.00)

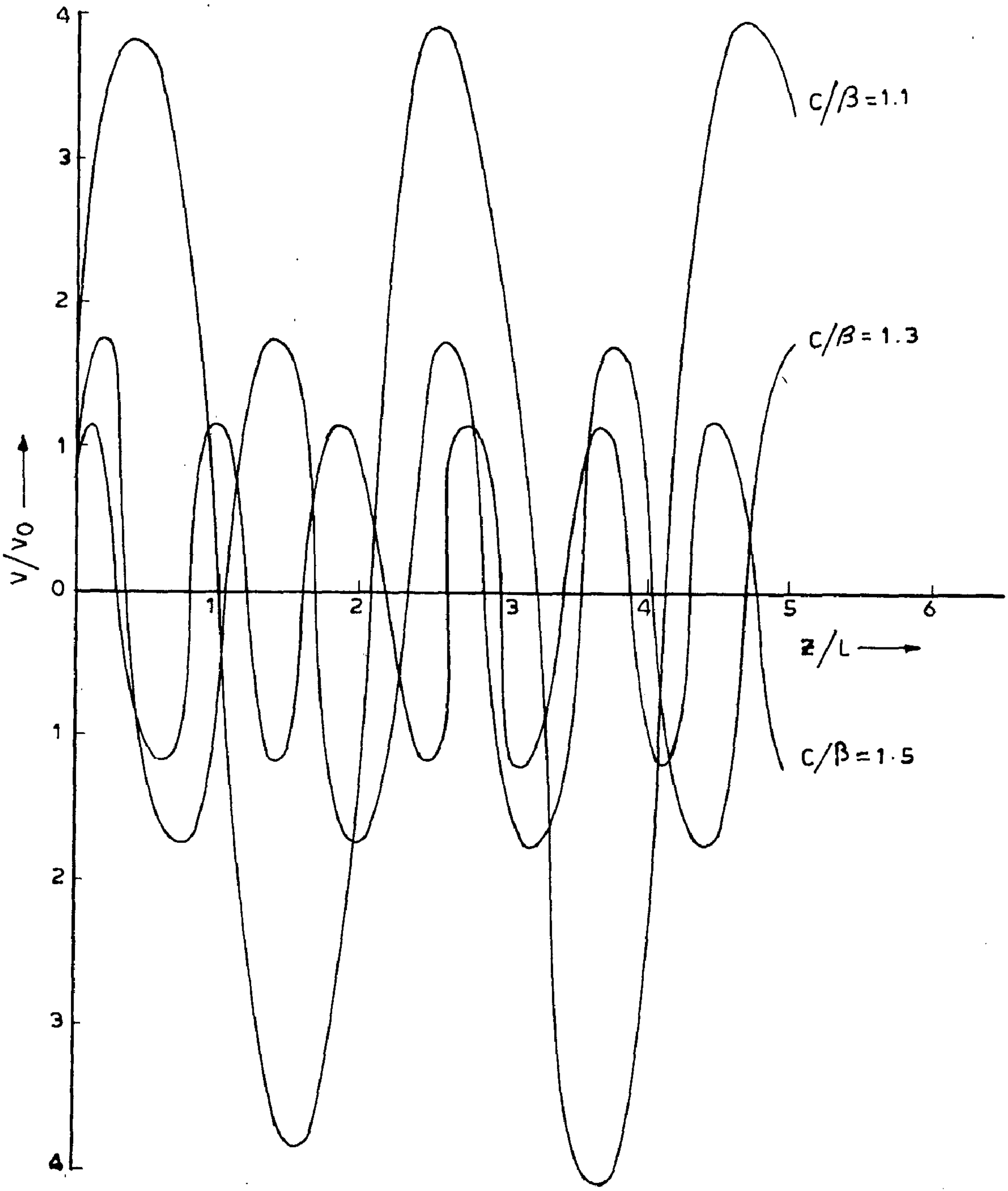


FIG.14-LOVE WAVE IN ELASTIC MEDIA (H/L = 5.00)

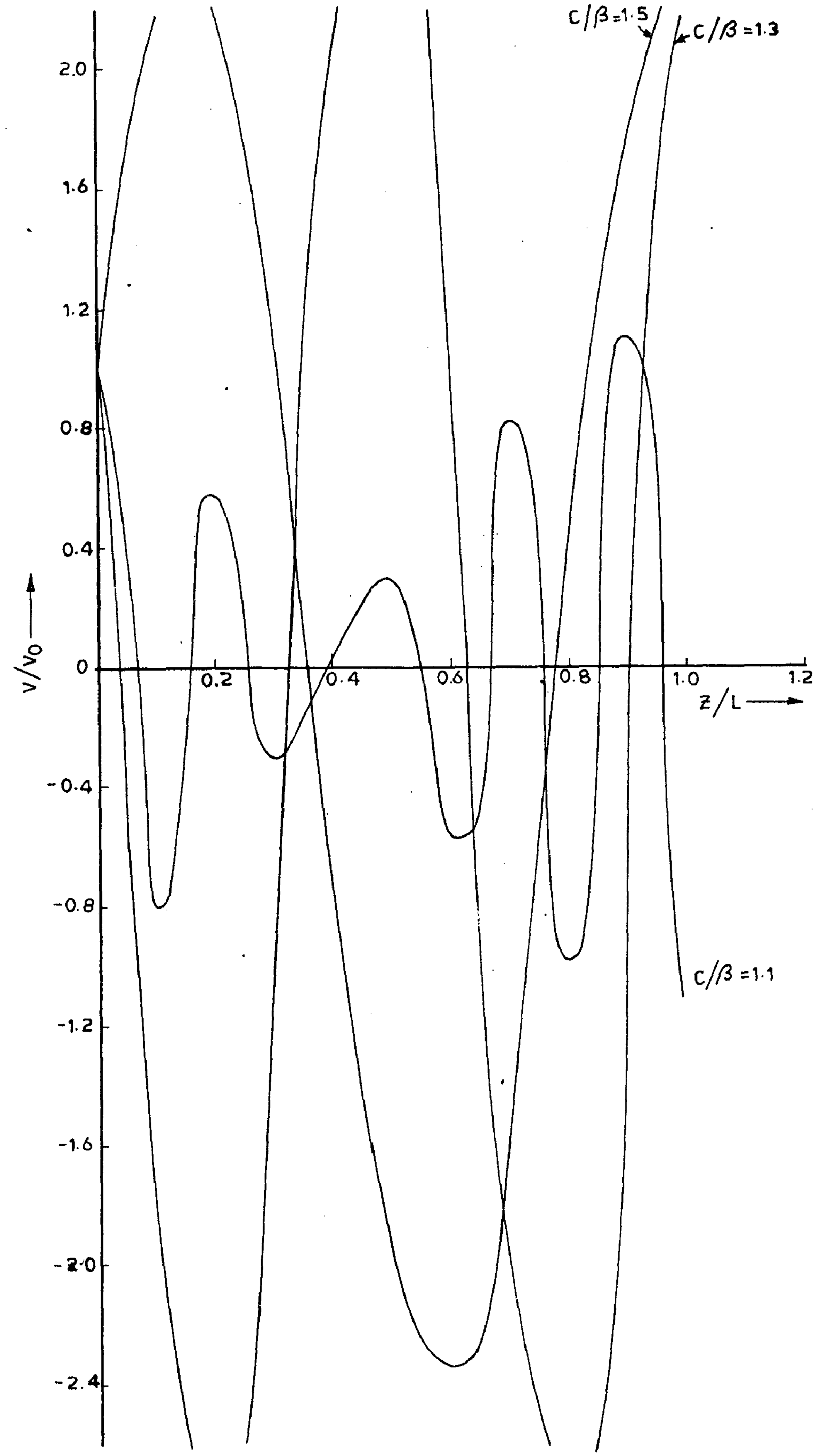


FIG.15- LOVE WAVE IN ELASTIC MEDIA (H/L = 10.00)

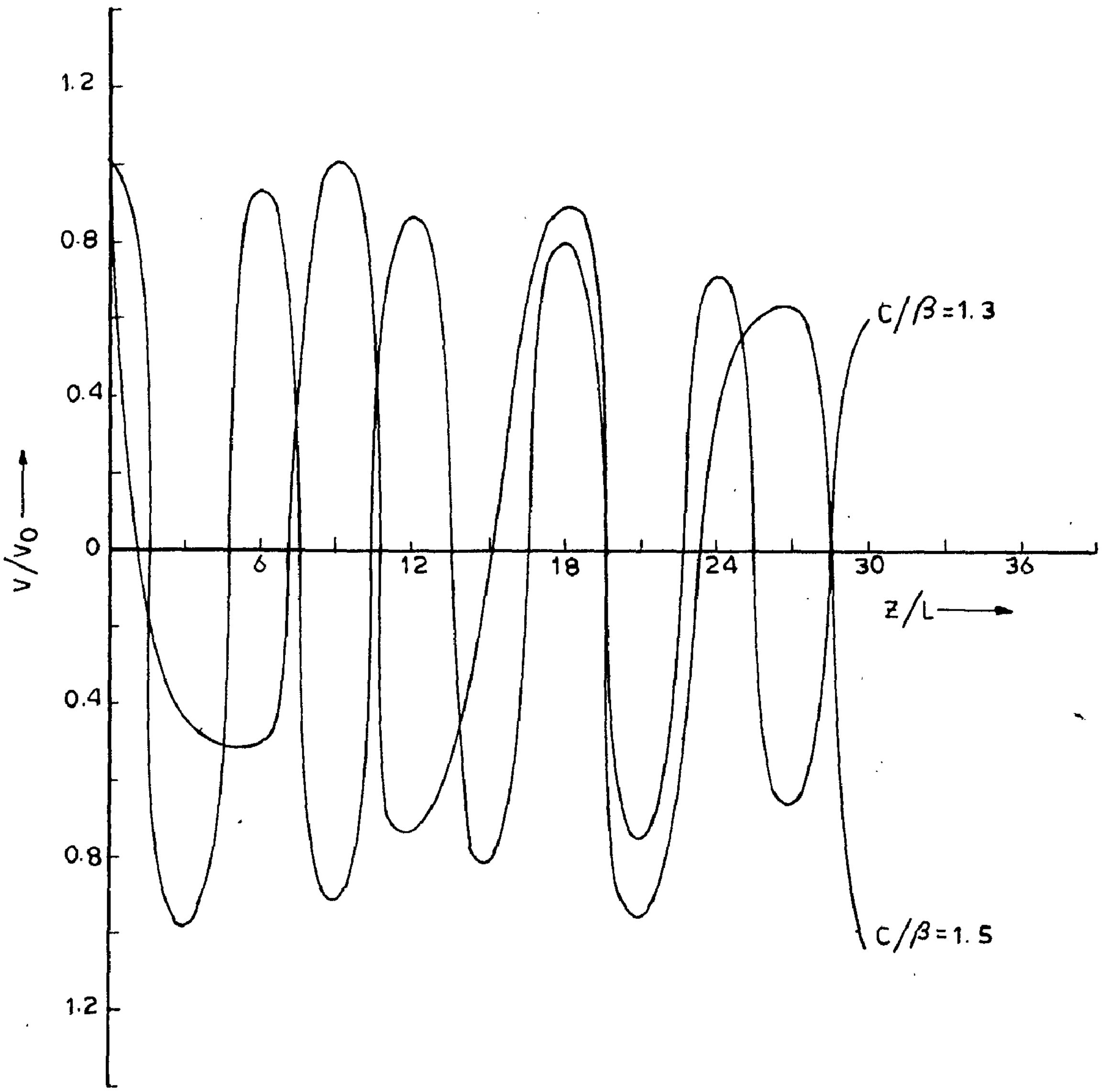


FIG.16_LOVE WAVE IN ELASTIC MEDIA (H/L=30.00)

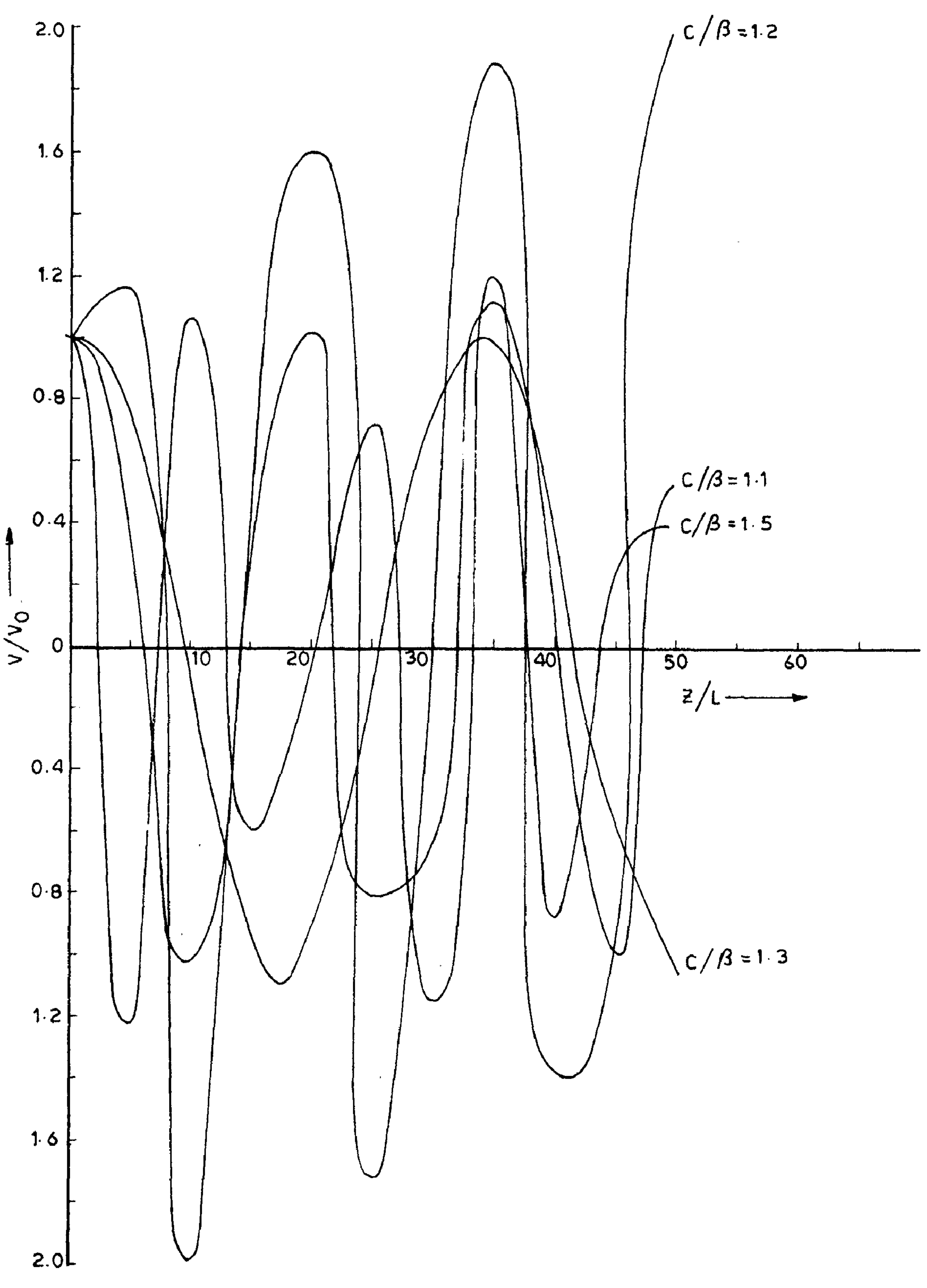


FIG.17-LOVE WAVE IN ELASTIC MEDIA (H/L = 50.00)

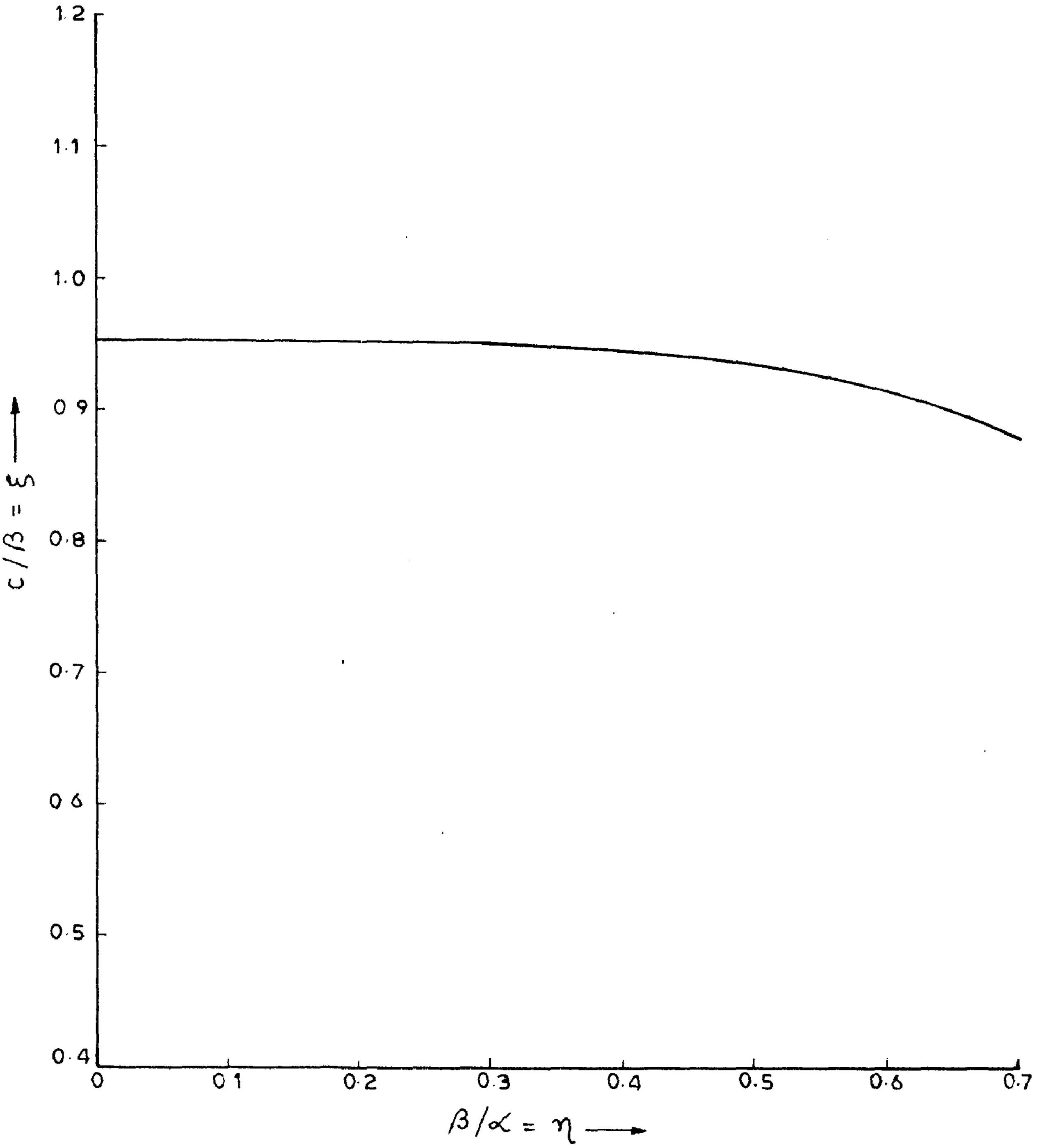


FIG.19-RAYLEIGH WAVE IN ELASTIC MEDIA

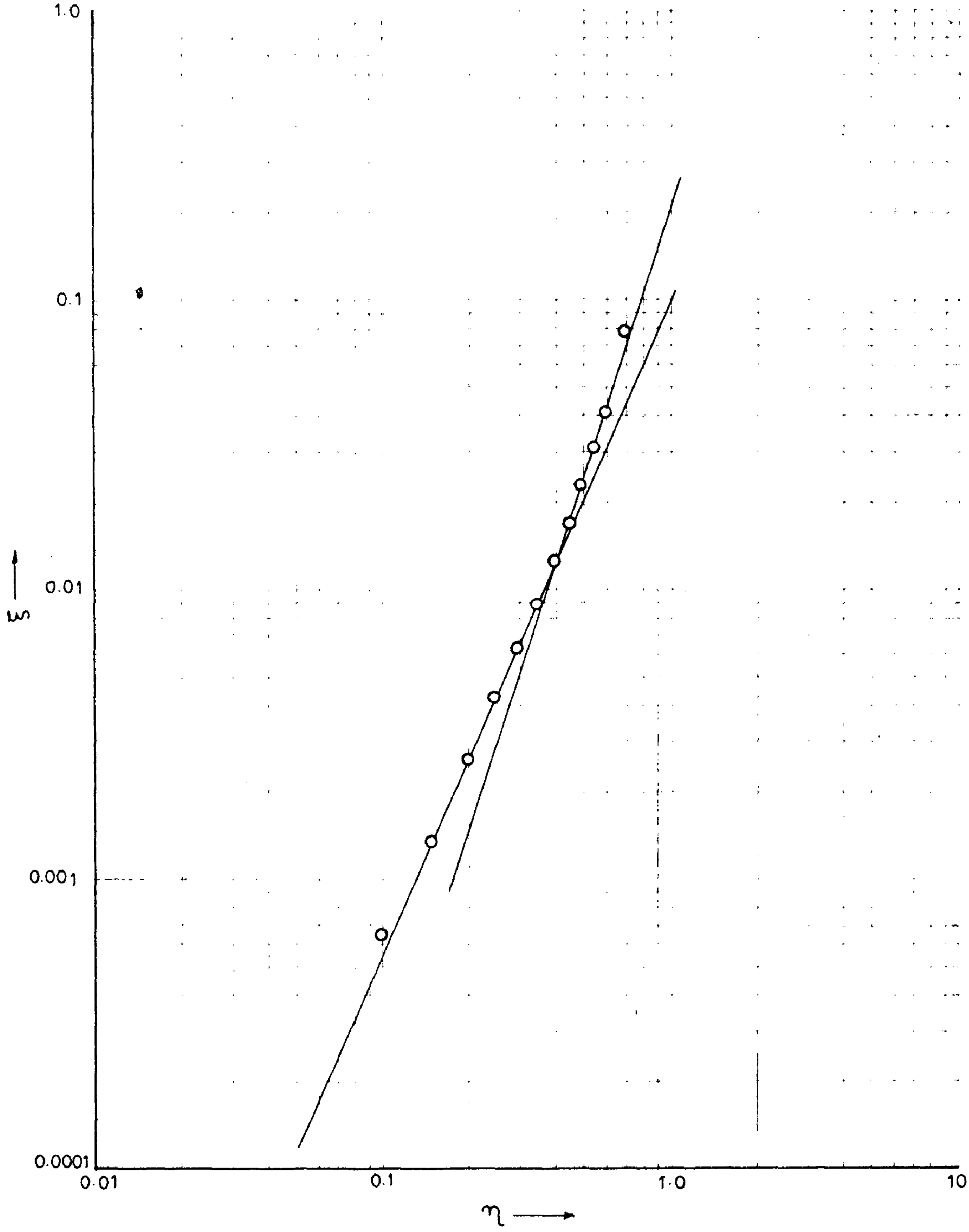


FIG 18 - REDUCED RAYLEIGH WAVE EQUATION

APPENDIX -B

```
C  C    RAYLEIGH WAVE SIGMA ETA ZY PLOT
        PI=3.1415926
        SI=0.
32      P=SQRTF((1.-SI-SI)/(2.-SI-SI))
        Q=P*P $ PIT=(PI/3.)*2. $ TA=1./3.
        A=16.*(1.-Q)
        B=-(24.-16.*Q)/3.
        C=8./3.   $   D=-1.
        H=A*C-B*B
        G=A*A*D-3.*A*B*C+2.*(B**3)
        E=G*G/4.+H**3
        IF(E)20,20,21
 20     F=-G/(2.*SQRTF(-H**3))
        IF(1.+F)22,22,23
 23     R=(1.-F)/(1.+F)
        ZA=2.*SQRTF(-H)
        S=SQRTF(R)
        TH=2.*ATANF(S)   $   THT=TH/3.
        GO TO 50
 22     TH= PI   $   ZA=2.*SQRTF(-H)        $   THT=TH/3.
 50     Z1=COSF(THT)*ZA
            Z3=ZA*COSF(THT-PIT)
        Z2  =ZA* COSF(PIT+THT)
        X1=(Z1-B)/A
        X2=(Z2-B)/A
       X3=(Z3-B)/A
        IF(X1)24,24,25
 25     XA=SQRTF(X1)
        ZX=1./XA
        PUNCH 200,SI,ZX,P
200     FORMAT(5X,3HSI=,F10.3,3HZX=,F15.5,2HP=,F15.5)
        GO TO 51
 24     PUNCH 201,Q,X1
 201    FORMAT(5X,2HQ=,F10.3,3HX1=,F15.4)
 51     IF(X2)26,26,27
 27     XB=SQRTF(X2)
        ZY=1./XB
        PUNCH 202,SI,ZY,P
202     FORMAT(5X,3HSI=,F10.3,3HZY=,F15.5,2HP=,F15.5)
        GO TO 52
 26     PUNCH 203,Q,X2
```

CONTD

```
 203      FORMAT(5X,2HQ=,F10.3,3HX2=,F15.4)
 52       IF (X3)28,28,29
 29       XC=SQRTF(X3)
          ZZ=1./XC
          PUNCH 204,SI,ZZ,P
204       FORMAT(5X,3HSI=,F10.3,3HZZ=,F15.5,2HP=,F15.5)
          GO TO 53
 28       PUNCH 205,Q,X3
 205      FORMAT(5X,2HQ=,F10.3,3HX3=,F15.4)
          GO TO 53
 21       T=SQRTF(E)
          TT=-0.5*G
          U=TT+T
          V=TT-T
          Z=U**TA+V**TA
          X=(Z-B)/A  $  XX=X
          IF(X)30,30,31
 31       XR=SQRTF(X)
          ZR=1./XR
          PUNCH 206,SI,ZR,P
206       FORMAT(5X,3HSI=,F10.3,3HZR=,F15.5,2HP=,F15.5)
          GO TO 53
 30       PUNCH 207,Q,XX
 207      FORMAT(5X,2HQ=,F10.3,3HXX=,F15.4)
53        SI=SI+.05
          IF(SI-.5)32,33,33
 33       STOP
          END
```

```
C  C   LOVE WAVE PROPAGATION ELASTIC SOLID
              PI=3.1415926 $ TPI=2.*PI
23     READ 100,HL,AI
100    FORMAT(2F10.3)
       A=1.1
22     ZL=0.
       B=SQRTF(A*A-1.)*TPI
       AA=COSF(HL*B)
       AO=1./(AA*AA)
       C=SQRTF(AO-1.)
20           RR=COSF(B*ZL)
           G=SINF(B*ZL)
         VVO=RR+C*G
       PUNCH 200,ZL,VVO,A,HL
       ZL=ZL+AI
       IF(ZL-HL)20,20,21
21     A=A+.1
       IF(A-1.5)22,22,23
200    FORMAT(5X,3HZL=,F10.3,5X,4HVVO=,F10.3,5X,3HCB=,F5.3,5X,F5.
       END
10.00      1.00
20.00      2.00
30.00      3.00
50.00      5.00
100.00     10.00
```

```
C  C   RAYLEIGH WAVE PROPAGATION MAXVELSOLID ZL UUO WWO ZI ETA SIG
21       READ 100, SI, ZI, ETA
100    FORMAT(8X,F10.3,3X,F15.5,2X,F15.5)
       ZL=0. $ PI=3.1415926 $ TPI=PI+PI
       A=SQRTF(1.-ETA*ETA*ZI*ZI)
       B=SQRTF(1.-ZI*ZI)
20     C=EXPF(-A*ZL*TPI)
       D=2.*A*B/(2.-ZI*ZI)
       E=EXPF(-B*ZL*TPI)
       UUO=(C-D*E)/(1.-D)
       F=2./(2.-ZI*ZI)
       WWO=(C-F*E)/(1.-F)
       PUNCH200,ZL,UUO,WWO,ZI,ETA,SI
200    FORMAT(3F10.5,5X,3F10.5)
       ZL=ZL+.1
       IF(ZL-2.1)20,21,21
       STOP
       END
    SI=      0.000ZX=          .87403P=          .70711
    SI=       .050ZX=          .88369P=          .68825
    SI=       .100ZX=          .89311P=          .66667
    SI=       .150ZX=          .90222P=          .64169
    SI=       .200ZX=          .91100P=          .61237
    SI=       .250ZX=          .91940P=          .57735
    SI=       .300ZR=          .92741P=          .53452
    SI=       .350ZR=          .93501P=          .48038
    SI=       .400ZR=          .94220P=          .40825
    SI=       .450ZR=          .94896P=          .30151
```

```
C   C     RAYLEIGH WAVE PROPAGATION MAXVELSOLID  UW-ZL  PLOT
21          READ 100, SI, ZI, ETA
100       FORMAT(8X,F10.3,3X,F15.5,2X,F15.5)
          ZL=0. $ PI=3.1415926 $ TPI=PI+PI
          A=SQRTF(1.-ETA*ETA*ZI*ZI)
          B=SQRTF(1.-ZI*ZI)
20        C=EXPF(-A*ZL*TPI)
          D=2.*A*B/(2.-ZI*ZI)
          E=EXPF(-B*ZL*TPI)
          F=2./(2.-ZI*ZI)
          UW=(C-D*E)/(A*(F*E-C))
          PUNCH 201,ZL,UW,SI
201       FORMAT(5X,F10.5,5X,F10.5,5X,F10.5)
          ZL=ZL+.1
          IF(ZL-2.1)20,21,21
          STOP
          END
```

```
SI=       0.000ZX=           .87403P=          .70711
SI=        .050ZX=           .88369P=          .68825
SI=        .100ZX=           .89311P=          .66667
SI=        .150ZX=           .90222P=          .64169
SI=        .200ZX=           .91100P=          .61237
SI=        .250ZX=           .91940P=          .57735
SI=        .300ZR=           .92741P=          .53452
SI=        .350ZR=           .93501P=          .48038
SI=        .400ZR=           .94220P=          .40825
SI=        .450ZR=           .94896P=          .30151
```

```
C   C     RAYLEIGH WAVE PROPAGATION TX TZ VS. ZL PLOT
          PI=3.1415926  $ TP=PI+PI
21        READ100,SI,ZI,ETA
100       FORMAT(8X,F10.3,3X,F15.5,2X,F15.5)
          ZL=0.
          ZZ=ZI*ZI
          A=SQRTF( 1.-ETA*ETA*ZZ)
          B=SQRTF(1.-ZZ)
          C=2./(2.-ZZ)
20        D=EXPF(-A*ZL*TP)
          E=EXPF(-B*ZL*TP)
          F=C*A*B
          TZ=-D+C*E
          TX=D-F*E
          PUNCH200,ZL,TZ,TX,SI,ZI,ETA
200       FORMAT(X,F10.3,5F12.5)
          WWO=(C*E-D)/(C-1.)
          UWO=TX/(A*(C-1.))
          PUNCH201,ZL,WWO,UWO,SI
201       FORMAT(3HZL=,F10.3,3HWO=,F15.5,3HUO=,F15.5,3HSI=,F15.5)
          ZL=ZL+.1
          IF(ZL-2.1)20,21,21
          STOP
          END
     SI=          0.000ZX=              .87403P=              .70711
     SI=           .050ZX=              .88369P=              .68825
     SI=           .100ZX=              .89311P=              .66667
     SI=           .150ZX=              .90222P=              .64169
     SI=           .200ZX=              .91100P=              .61237
     SI=           .250ZX=              .91940P=              .57735
     SI=           .300ZR=              .92741P=              .53452
     SI=           .350ZR=              .93501P=              .48038
     SI=           .400ZR=              .94220P=              .40825
     SI=           .450ZR=              .94896P=              .30151
```

```
C   C    RAYLEIGH WAVE PROPAGATION VOIGT SOLID
21       READ 100,SI,ETA,ZI,WEN
100      FORMAT(4F15.5)
         ZL=0. $ PI=3.1415926 $ TPI=PI+PI
         A=SQRTF(1.-ETA*ETA*ZI*ZI)
         B=SQRTF(1.-ZI*ZI)
         R=WEN*WEN
         S=SQRTF(1./(1.+R))
20       C=EXPF(-A*ZL*TPI*S)
         D=2.*A*B/(2.-ZI*ZI)
         E=EXPF(-B*ZL*TPI*S)
         F=2./(2.-ZI*ZI)
         UUO=(C-D*E)/(1.-D)
         WWO=(C-F*E)/(1.-F)
         UW=(C-D*E)/(A*(F*E-C))
         PUNCH200,ZL,UUO,WWO,UW,ZI,ETA,SI
200      FORMAT(4F10.5, X,3F10.5)
         ZL=ZL+.1
         IF(ZL-2.1)20,21,21
         STOP
         END
```

0.00000	.70711	0.87403	0.10000
0.050	0.68825	0.88369	0.20000
0.10000	0.66667	0.89311	0.30000
0.15000	0.64169	0.90222	0.40000
0.20000	0.61237	0.91100	0.50000
0.25000	0.57735	0.91940	0.60000
0.30000	0.53452	0.92741	0.7000
0.3500	0.48038	0.93522	0.8
0.40000	0.40825	0.94220	0.9
0.4500	0.30151	0.94306	0.90000

```
C  C    RAYLEIGH WAVE DISPERSION
        PI=3.1415926
22      READ 100,ZI
100     FORMAT(F10.5)
        EN=20.
23      T=.1
20      A=(4.*PI*PI)/(T*T*EN*EN)
        B=SQRTF(1.+A)
        VR=ZI*B
        PUNCH200 ,VR,T
200     FORMAT( 5X,F10.5,5X,F10.5)
        T=T+.1
        IF(T-1.1)20,21,21
21      EN=EN+20.
        IF(EN-100.)23,23,22
        END
.88369
.90222
.91940
.93501
.94896
```

APPENDIX...C

I

C C WAVE PROPAGATION Z K.G.BHATIA 4.10.68

	σ	ξ	η
SI=	0.000ZX=	.87403P=	.70711
SI=	.050ZX=	.88369P=	.68825
SI=	.100ZX=	.89311P=	.66667
SI=	.150ZX=	.90222P=	.64169
SI=	.200ZX=	.91100P=	.61237
SI=	.250ZX=	.91940P=	.57735
SI=	.300ZR=	.92741P=	.53452
SI=	.350ZR=	.93501P=	.48038
SI=	.400ZR=	.94220P=	.40825
SI=	.450ZR=	.94896P=	.30151

APPENDIX...C
III

C C RAYLEIGH WAVE PROPAGATION MAXVELSOLID K.G.BHATIA

$\frac{z}{L}$	$\frac{U}{U_0}$	$\frac{W}{W_0}$	ξ	η	σ
0.00000	1.00000	1.00000	.87403	.70711	0.00000
.10000	.40519	.94193	.87403	.70711	0.00000
.20000	.09617	.81923	.87403	.70711	0.00000
.30000	-.05265	.68004	.87403	.70711	0.00000
.40000	-.11417	.54771	.87403	.70711	0.00000
.50000	-.13013	.43204	.87403	.70711	0.00000
.60000	-.12396	.33572	.87403	.70711	0.00000
.70000	-.10847	.25799	.87403	.70711	0.00000
.80000	-.09038	.19657	.87403	.70711	0.00000
.90000	-.07298	.14880	.87403	.70711	0.00000
1.00000	-.05767	.11206	.87403	.70711	0.00000
1.10000	-.04487	.08404	.87403	.70711	0.00000
1.20000	-.03452	.06283	.87403	.70711	0.00000
1.30000	-.02632	.04685	.87403	.70711	0.00000
1.40000	-.01994	.03485	.87403	.70711	0.00000
1.50000	-.01502	.02589	.87403	.70711	0.00000
1.60000	-.01127	.01920	.87403	.70711	0.00000
1.70000	-.00843	.01423	.87403	.70711	0.00000
1.80000	-.00628	.01053	.87403	.70711	0.00000
1.90000	-.00468	.00779	.87403	.70711	0.00000
2.00000	-.00347	.00576	.87403	.70711	0.00000
0.00000	1.00000	1.00000	.88369	.68825	.05000
.10000	.39199	.96050	.88369	.68825	.05000
.20000	.07761	.84652	.88369	.68825	.05000
.30000	-.07243	.71023	.88369	.68825	.05000
.40000	-.13309	.57749	.88369	.68825	.05000
.50000	-.14722	.45963	.88369	.68825	.05000
.60000	-.13888	.36030	.88369	.68825	.05000
.70000	-.12121	.27930	.88369	.68825	.05000
.80000	-.10109	.21469	.88369	.68825	.05000
.90000	-.08187	.16397	.88369	.68825	.05000
1.00000	-.06498	.12461	.88369	.68825	.05000
1.10000	-.05083	.09433	.88369	.68825	.05000
1.20000	-.03934	.07119	.88369	.68825	.05000
1.30000	-.03021	.05359	.88369	.68825	.05000
1.40000	-.02305	.04026	.88369	.68825	.05000
1.50000	-.01750	.03021	.88369	.68825	.05000
1.60000	-.01324	.02263	.88369	.68825	.05000
1.70000	-.00999	.01694	.88369	.68825	.05000
1.80000	-.00752	.01267	.88369	.68825	.05000
1.90000	-.00565	.00947	.88369	.68825	.05000
2.00000	-.00424	.00707	.88369	.68825	.05000
0.00000	1.00000	1.00000	.89311	.66667	.10000
.10000	.37727	.98015	.89311	.66667	.10000
.20000	.05711	.87546	.89311	.66667	.10000
.30000	-.09414	.74239	.89311	.66667	.10000
.40000	-.15377	.60939	.89311	.66667	.10000
.50000	-.16590	.48940	.89311	.66667	.10000
.60000	-.15523	.38705	.89311	.66667	.10000
.70000	-.13522	.30270	.89311	.66667	.10000

(CONTD...

.80000	-.11292	.23478	.89311	.66667	.10000
.90000	-.09176	.18097	.89311	.66667	.10000
1.00000	-.07317	.13882	.89311	.66667	.10000
1.10000	-.05757	.10609	.89311	.66667	.10000
1.20000	-.04486	.08085	.89311	.66667	.10000
1.30000	-.03469	.06147	.89311	.66667	.10000
1.40000	-.02668	.04666	.89311	.66667	.10000
1.50000	-.02043	.03536	.89311	.66667	.10000
1.60000	-.01560	.02677	.89311	.66667	.10000
1.70000	-.01187	.02025	.89311	.66667	.10000
1.80000	-.00902	.01531	.89311	.66667	.10000
1.90000	-.00684	.01156	.89311	.66667	.10000
2.00000	-.00518	.00873	.89311	.66667	.10000
0.00000	1.00000	1.00000	.90222	.64169	.15000
.10000	.36086	1.00089	.90222	.64169	.15000
.20000	.03450	.90605	.90222	.64169	.15000
.30000	-.11790	.77650	.90222	.64169	.15000
.40000	-.17631	.64342	.90222	.64169	.15000
.50000	-.18622	.52139	.90222	.64169	.15000
.60000	-.17303	.41601	.90222	.64169	.15000
.70000	-.15054	.32828	.90222	.64169	.15000
.80000	-.12594	.25696	.90222	.64169	.15000
.90000	-.10271	.19992	.90222	.64169	.15000
1.00000	-.08233	.15484	.90222	.64169	.15000
1.10000	-.06518	.11950	.90222	.64169	.15000
1.20000	-.05114	.09199	.90222	.64169	.15000
1.30000	-.03986	.07066	.90222	.64169	.15000
1.40000	-.03091	.05419	.90222	.64169	.15000
1.50000	-.02388	.04151	.90222	.64169	.15000
1.60000	-.01840	.03177	.90222	.64169	.15000
1.70000	-.01414	.02429	.90222	.64169	.15000
1.80000	-.01085	.01857	.90222	.64169	.15000
1.90000	-.00832	.01418	.90222	.64169	.15000
2.00000	-.00637	.01083	.90222	.64169	.15000
0.00000	1.00000	1.00000	.91100	.61237	.20000
.10000	.34260	1.02279	.91100	.61237	.20000
.20000	.00964	.93836	.91100	.61237	.20000
.30000	-.14380	.81264	.91100	.61237	.20000
.40000	-.20076	.67966	.91100	.61237	.20000
.50000	-.20823	.55570	.91100	.61237	.20000
.60000	-.19234	.44736	.91100	.61237	.20000
.70000	-.16722	.35623	.91100	.61237	.20000
.80000	-.14020	.28144	.91100	.61237	.20000
.90000	-.11481	.22107	.91100	.61237	.20000
1.00000	-.09254	.17290	.91100	.61237	.20000
1.10000	-.07374	.13480	.91100	.61237	.20000
1.20000	-.05830	.10484	.91100	.61237	.20000
1.30000	-.04581	.08139	.91100	.61237	.20000
1.40000	-.03584	.06309	.91100	.61237	.20000
1.50000	-.02795	.04886	.91100	.61237	.20000
1.60000	-.02174	.03781	.91100	.61237	.20000
1.70000	-.01688	.02924	.91100	.61237	.20000
1.80000	-.01309	.02260	.91100	.61237	.20000
1.90000	-.01014	.01746	.91100	.61237	.20000
2.00000	-.00784	.01349	.91100	.61237	.20000

(CONTD...

0.00000	1.00000	1.00000	.91940	.57735	.25000
.10000	.32227	1.04591	.91940	.57735	.25000
.20000	-.01766	.97241	.91940	.57735	.25000
.30000	-.17195	.85080	.91940	.57735	.25000
.40000	-.22716	.71811	.91940	.57735	.25000
.50000	-.23194	.59235	.91940	.57735	.25000
.60000	-.21316	.48113	.91940	.57735	.25000
.70000	-.18528	.38663	.91940	.57735	.25000
.80000	-.15575	.30835	.91940	.57735	.25000
.90000	-.12812	.24457	.91940	.57735	.25000
1.00000	-.10387	.19321	.91940	.57735	.25000
1.10000	-.08336	.15220	.91940	.57735	.25000
1.20000	-.06642	.11963	.91940	.57735	.25000
1.30000	-.05265	.09388	.91940	.57735	.25000
1.40000	-.04157	.07358	.91940	.57735	.25000
1.50000	-.03273	.05763	.91940	.57735	.25000
1.60000	-.02572	.04510	.91940	.57735	.25000
1.70000	-.02018	.03528	.91940	.57735	.25000
1.80000	-.01582	.02758	.91940	.57735	.25000
1.90000	-.01238	.02156	.91940	.57735	.25000
2.00000	-.00969	.01685	.91940	.57735	.25000
0.00000	1.00000	1.00000	.92741	.53452	.30000
.10000	.29965	1.07035	.92741	.53452	.30000
.20000	-.04757	1.00829	.92741	.53452	.30000
.30000	-.20243	.89105	.92741	.53452	.30000
.40000	-.25554	.75886	.92741	.53452	.30000
.50000	-.25736	.63146	.92741	.53452	.30000
.60000	-.23552	.51748	.92741	.53452	.30000
.70000	-.20478	.41969	.92741	.53452	.30000
.80000	-.17265	.33792	.92741	.53452	.30000
.90000	-.14273	.27070	.92741	.53452	.30000
1.00000	-.11645	.21607	.92741	.53452	.30000
1.10000	-.09416	.17201	.92741	.53452	.30000
1.20000	-.07565	.13667	.92741	.53452	.30000
1.30000	-.06051	.10845	.92741	.53452	.30000
1.40000	-.04824	.08597	.92741	.53452	.30000
1.50000	-.03837	.06810	.92741	.53452	.30000
1.60000	-.03047	.05391	.92741	.53452	.30000
1.70000	-.02417	.04266	.92741	.53452	.30000
1.80000	-.01915	.03376	.92741	.53452	.30000
1.90000	-.01517	.02670	.92741	.53452	.30000
2.00000	-.01201	.02112	.92741	.53452	.30000
0.00000	1.00000	1.00000	.93501	.48038	.35000
.10000	.27446	1.09622	.93501	.48038	.35000
.20000	-.08030	1.04609	.93501	.48038	.35000
.30000	-.23535	.93345	.93501	.48038	.35000
.40000	-.28593	.80193	.93501	.48038	.35000
.50000	-.28449	.67309	.93501	.48038	.35000
.60000	-.25943	.55653	.93501	.48038	.35000
.70000	-.22572	.45556	.93501	.48038	.35000
.80000	-.19097	.37039	.93501	.48038	.35000
.90000	-.15872	.29972	.93501	.48038	.35000
1.00000	-.13038	.24175	.93501	.48038	.35000
1.10000	-.10626	.19454	.93501	.48038	.35000
1.20000	-.08613	.15630	.93501	.48038	.35000

(CONTD...

1.30000	-.06955	.12543	.93501	.48038	.35000
1.40000	-.05601	.10058	.93501	.48038	.35000
1.50000	-.04503	.08060	.93501	.48038	.35000
1.60000	-.03615	.06457	.93501	.48038	.35000
1.70000	-.02900	.05171	.93501	.48038	.35000
1.80000	-.02324	.04140	.93501	.48038	.35000
1.90000	-.01862	.03315	.93501	.48038	.35000
2.00000	-.01491	.02653	.93501	.48038	.35000
0.00000	1.00000	1.00000	.94220	.40825	.40000
.10000	.24639	1.12371	.94220	.40825	.40000
.20000	-.11605	1.08594	.94220	.40825	.40000
.30000	-.27077	.97809	.94220	.40825	.40000
.40000	-.31833	.84745	.94220	.40825	.40000
.50000	-.31331	.71739	.94220	.40825	.40000
.60000	-.28486	.59846	.94220	.40825	.40000
.70000	-.24815	.49451	.94220	.40825	.40000
.80000	-.21077	.40604	.94220	.40825	.40000
.90000	-.17620	.33199	.94220	.40825	.40000
1.00000	-.14580	.27066	.94220	.40825	.40000
1.10000	-.11983	.22023	.94220	.40825	.40000
1.20000	-.09804	.17895	.94220	.40825	.40000
1.30000	-.07996	.14528	.94220	.40825	.40000
1.40000	-.06508	.11787	.94220	.40825	.40000
1.50000	-.05290	.09558	.94220	.40825	.40000
1.60000	-.04295	.07749	.94220	.40825	.40000
1.70000	-.03485	.06281	.94220	.40825	.40000
1.80000	-.02826	.05090	.94220	.40825	.40000
1.90000	-.02291	.04125	.94220	.40825	.40000
2.00000	-.01857	.03342	.94220	.40825	.40000
0.00000	1.00000	1.00000	.94896	.30151	.45000
.10000	.21493	1.15299	.94896	.30151	.45000
.20000	-.15522	1.12796	.94896	.30151	.45000
.30000	-.30888	1.02500	.94896	.30151	.45000
.40000	-.35280	.89541	.94896	.30151	.45000
.50000	-.34384	.76438	.94896	.30151	.45000
.60000	-.31185	.64337	.94896	.30151	.45000
.70000	-.27213	.53669	.94896	.30151	.45000
.80000	-.23216	.44513	.94896	.30151	.45000
.90000	-.19532	.36780	.94896	.30151	.45000
1.00000	-.16289	.30316	.94896	.30151	.45000
1.10000	-.13508	.24947	.94896	.30151	.45000
1.20000	-.11160	.20506	.94896	.30151	.45000
1.30000	-.09198	.16844	.94896	.30151	.45000
1.40000	-.07569	.13829	.94896	.30151	.45000
1.50000	-.06221	.11351	.94896	.30151	.45000
1.60000	-.05110	.09314	.94896	.30151	.45000
1.70000	-.04195	.07642	.94896	.30151	.45000
1.80000	-.03443	.06269	.94896	.30151	.45000
1.90000	-.02826	.05143	.94896	.30151	.45000
2.00000	-.02318	.04219	.94896	.30151	.45000

OEVISED COHESION AND SURCHARGE FACTOR SWAMI SARAN Z

ERROR IN-1 IN STATEMENT 0021 + 00 LINES

APPENDIX...C

IV σ

C C RAYLEIGH WAVE PROPAGATION MAXVELSOLID UW-ZL PLOT K.G.BHATIA

Z/L	U/W	σ
0.00000	.78615	0.00000
.10000	.33818	
.20000	.09229	0.00000
.30000	-.06086	0.00000
.40000	-.16387	0.00000
.50000	-.23678	0.00000
.60000	-.29026	0.00000
.70000	-.33054	0.00000
.80000	-.36148	0.00000
.90000	-.38559	0.00000
1.00000	-.40462	0.00000
1.10000	-.41976	0.00000
1.20000	-.43191	0.00000
1.30000	-.44171	0.00000
1.40000	-.44965	0.00000
1.50000	-.45612	0.00000
1.60000	-.46139	0.00000
1.70000	-.46571	0.00000
1.80000	-.46925	0.00000
1.90000	-.47216	0.00000
2.00000	-.47455	0.00000
0.00000	.76789	.05000
.10000	.31338	.05000
.20000	.07040	.05000
.30000	-.07832	.05000
.40000	-.17697	.05000
.50000	-.24596	.05000
.60000	-.29600	.05000
.70000	-.33327	.05000
.80000	-.36157	.05000
.90000	-.38339	.05000
1.00000	-.40040	.05000
1.10000	-.41378	.05000
1.20000	-.42438	.05000
1.30000	-.43283	.05000
1.40000	-.43959	.05000
1.50000	-.44501	.05000
1.60000	-.44938	.05000
1.70000	-.45291	.05000
1.80000	-.45576	.05000
1.90000	-.45806	.05000
2.00000	-.45994	.05000
0.00000	.74828	.10000
.10000	.28802	.10000
.20000	.04881	.10000
.30000	-.09489	.10000
.40000	-.18882	.10000
.50000	-.25365	.10000
.60000	-.30010	.10000

(CONTD..

.70000	-.33427	.10000
.80000	-.35990	.10000
.90000	-.37940	.10000
1.00000	-.39441	.10000
1.10000	-.40606	.10000
1.20000	-.41516	.10000
1.30000	-.42231	.10000
1.40000	-.42795	.10000
1.50000	-.43240	.10000
1.60000	-.43594	.10000
1.70000	-.43874	.10000
1.80000	-.44098	.10000
1.90000	-.44276	.10000
2.00000	-.44418	.10000
0.00000	.72728	.15000
.10000	.26221	.15000
.20000	.02769	.15000
.30000	-.11043	.15000
.40000	-.19930	.15000
.50000	-.25976	.15000
.60000	-.30250	.15000
.70000	-.33350	.15000
.80000	-.35644	.15000
.90000	-.37365	.15000
1.00000	-.38670	.15000
1.10000	-.39667	.15000
1.20000	-.40434	.15000
1.30000	-.41027	.15000
1.40000	-.41487	.15000
1.50000	-.41844	.15000
1.60000	-.42123	.15000
1.70000	-.42340	.15000
1.80000	-.42511	.15000
1.90000	-.42644	.15000
2.00000	-.42748	.15000
0.00000	.70494	.20000
.10000	.23613	.20000
.20000	.00724	.20000
.30000	-.12474	.20000
.40000	-.20822	.20000
.50000	-.26415	.20000
.60000	-.30308	.20000
.70000	-.33090	.20000
.80000	-.35116	.20000
.90000	-.36612	.20000
1.00000	-.37727	.20000
1.10000	-.38565	.20000
1.20000	-.39199	.20000
1.30000	-.39679	.20000
1.40000	-.40045	.20000
1.50000	-.40324	.20000
1.60000	-.40537	.20000
1.70000	-.40701	.20000
1.80000	-.40826	.20000
1.90000	-.40922	.20000
2.00000	-.40996	.20000
0.00000	.68125	.25000

(CONTD..

.10000	.20991	.25000
.20000	-.01237	.25000
.30000	-.13768	.25000
.40000	-.21549	.25000
.50000	-.26674	.25000
.60000	-.30182	.25000
.70000	-.32647	.25000
.80000	-.34411	.25000
.90000	-.35689	.25000
1.00000	-.36625	.25000
1.10000	-.37314	.25000
1.20000	-.37825	.25000
1.30000	-.38204	.25000
1.40000	-.38487	.25000
1.50000	-.38699	.25000
1.60000	-.38857	.25000
1.70000	-.38976	.25000
1.80000	-.39065	.25000
1.90000	-.39131	.25000
2.00000	-.39181	.25000
0.00000	.65626	.30000
.10000	.18372	.30000
.20000	-.03096	.30000
.30000	-.14909	.30000
.40000	-.22099	.30000
.50000	-.26747	.30000
.60000	-.29869	.30000
.70000	-.32020	.30000
.80000	-.33530	.30000
.90000	-.34602	.30000
1.00000	-.35370	.30000
1.10000	-.35924	.30000
1.20000	-.36325	.30000
1.30000	-.36616	.30000
1.40000	-.36828	.30000
1.50000	-.36983	.30000
1.60000	-.37096	.30000
1.70000	-.37179	.30000
1.80000	-.37239	.30000
1.90000	-.37283	.30000
2.00000	-.37316	.30000
0.00000	.62999	.35000
.10000	.15773	.35000
.20000	-.04836	.35000
.30000	-.15884	.35000
.40000	-.22463	.35000
.50000	-.26628	.35000
.60000	-.29367	.35000
.70000	-.31215	.35000
.80000	-.32482	.35000
.90000	-.33362	.35000
1.00000	-.33977	.35000
1.10000	-.34409	.35000
1.20000	-.34714	.35000
1.30000	-.34931	.35000

(CONTD...

1.40000	-.35084	.35000
1.50000	-.35193	.35000
1.60000	-.35271	.35000
1.70000	-.35326	.35000
1.80000	-.35365	.35000
1.90000	-.35393	.35000
2.00000	-.35413	.35000
0.00000	.60250	.40000
.10000	.13211	.40000
.20000	-.06439	.40000
.30000	-.16679	.40000
.40000	-.22632	.40000
.50000	-.26313	.40000
.60000	-.28678	.40000
.70000	-.30234	.40000
.80000	-.31275	.40000
.90000	-.31978	.40000
1.00000	-.32456	.40000
1.10000	-.32783	.40000
1.20000	-.33008	.40000
1.30000	-.33162	.40000
1.40000	-.33268	.40000
1.50000	-.33342	.40000
1.60000	-.33392	.40000
1.70000	-.33427	.40000
1.80000	-.33451	.40000
1.90000	-.33468	.40000
2.00000	-.33479	.40000
0.00000	.57372	.45000
.10000	.10695	.45000
.20000	-.07895	.45000
.30000	-.17289	.45000
.40000	-.22605	.45000
.50000	-.25808	.45000
.60000	-.27810	.45000
.70000	-.29091	.45000
.80000	-.29923	.45000
.90000	-.30468	.45000
1.00000	-.30827	.45000
1.10000	-.31066	.45000
1.20000	-.31224	.45000
1.30000	-.31329	.45000
1.40000	-.31399	.45000
1.50000	-.31446	.45000
1.60000	-.31477	.45000
1.70000	-.31498	.45000
1.80000	-.31512	.45000
1.90000	-.31521	.45000
2.00000	-.31527	.45000

APPENDIX...C
V-A

C C RAYLEIGH WAVE PROPAGATION VOIGT SOLID K.G.BHATIA

0.00000	1.00000	1.00000	.87403	.70711	0.00000
.10000	.40731	.94243	.87403	.70711	0.00000
.20000	.09830	.82058	.87403	.70711	0.00000
.30000	-.05121	.68210	.87403	.70711	0.00000
.40000	-.11349	.55020	.87403	.70711	0.00000
.50000	-.13007	.43468	.87403	.70711	0.00000
.60000	-.12432	.33832	.87403	.70711	0.00000
.70000	-.10908	.26040	.87403	.70711	0.00000
.80000	-.09110	.19873	.87403	.70711	0.00000
.90000	-.07372	.15068	.87403	.70711	0.00000
1.00000	-.05837	.11366	.87403	.70711	0.00000
1.10000	-.04551	.08538	.87403	.70711	0.00000
1.20000	-.03507	.06393	.87403	.70711	0.00000
1.30000	-.02679	.04774	.87403	.70711	0.00000
1.40000	-.02033	.03558	.87403	.70711	0.00000
1.50000	-.01534	.02647	.87403	.70711	0.00000
1.60000	-.01153	.01966	.87403	.70711	0.00000
1.70000	-.00864	.01459	.87403	.70711	0.00000
1.80000	-.00645	.01082	.87403	.70711	0.00000
1.90000	-.00481	.00801	.87403	.70711	0.00000
2.00000	-.00358	.00593	.87403	.70711	0.00000
0.00000	1.00000	1.00000	.88369	.68825	.05000
.10000	.40046	.96221	.88369	.68825	.05000
.20000	.08612	.85160	.88369	.68825	.05000
.30000	-.06669	.71824	.88369	.68825	.05000
.40000	-.13045	.58735	.88369	.68825	.05000
.50000	-.14712	.47028	.88369	.68825	.05000
.60000	-.14052	.37090	.88369	.68825	.05000
.70000	-.12387	.28929	.88369	.68825	.05000
.80000	-.10421	.22375	.88369	.68825	.05000
.90000	-.08508	.17195	.88369	.68825	.05000
1.00000	-.06804	.13148	.88369	.68825	.05000
1.10000	-.05362	.10014	.88369	.68825	.05000
1.20000	-.04180	.07603	.88369	.68825	.05000
1.30000	-.03231	.05758	.88369	.68825	.05000
1.40000	-.02482	.04353	.88369	.68825	.05000
1.50000	-.01897	.03285	.88369	.68825	.05000
1.60000	-.01445	.02476	.88369	.68825	.05000
1.70000	-.01097	.01864	.88369	.68825	.05000
1.80000	-.00831	.01402	.88369	.68825	.05000
1.90000	-.00628	.01054	.88369	.68825	.05000
2.00000	-.00474	.00792	.88369	.68825	.05000
0.00000	1.00000	1.00000	.89311	.66667	.10000
.10000	.39621	.98323	.89311	.66667	.10000
.20000	.07618	.88594	.89311	.66667	.10000
.30000	-.08128	.75962	.89311	.66667	.10000
.40000	-.14795	.63113	.89311	.66667	.10000
.50000	-.16586	.51331	.89311	.66667	.10000
.60000	-.15920	.41121	.89311	.66667	.10000

(CONTD...

.70000	-.14159	.32580	.89311	.66667	.10000
.80000	-.12044	.25600	.89311	.66667	.10000
.90000	-.09953	.19989	.89311	.66667	.10000
1.00000	-.08065	.15532	.89311	.66667	.10000
1.10000	-.06444	.12023	.89311	.66667	.10000
1.20000	-.05095	.09280	.89311	.66667	.10000
1.30000	-.03997	.07145	.89311	.66667	.10000
1.40000	-.03118	.05492	.89311	.66667	.10000
1.50000	-.02420	.04215	.89311	.66667	.10000
1.60000	-.01872	.03231	.89311	.66667	.10000
1.70000	-.01444	.02474	.89311	.66667	.10000
1.80000	-.01112	.01893	.89311	.66667	.10000
1.90000	-.00854	.01448	.89311	.66667	.10000
2.00000	-.00655	.01107	.89311	.66667	.10000
0.00000	1.00000	1.00000	.90222	.64169	.15000
.10000	.39405	1.00498	.90222	.64169	.15000
.20000	.06806	.92276	.90222	.64169	.15000
.30000	-.09520	.80534	.90222	.64169	.15000
.40000	-.16608	.68078	.90222	.64169	.15000
.50000	-.18634	.56327	.90222	.64169	.15000
.60000	-.18044	.45906	.90222	.64169	.15000
.70000	-.16240	.37005	.90222	.64169	.15000
.80000	-.14001	.29588	.90222	.64169	.15000
.90000	-.11740	.23512	.90222	.64169	.15000
1.00000	-.09659	.18596	.90222	.64169	.15000
1.10000	-.07840	.14655	.90222	.64169	.15000
1.20000	-.06301	.11517	.90222	.64169	.15000
1.30000	-.05026	.09030	.90222	.64169	.15000
1.40000	-.03987	.07069	.90222	.64169	.15000
1.50000	-.03150	.05525	.90222	.64169	.15000
1.60000	-.02480	.04314	.90222	.64169	.15000
1.70000	-.01947	.03366	.90222	.64169	.15000
1.80000	-.01526	.02624	.90222	.64169	.15000
1.90000	-.01194	.02045	.90222	.64169	.15000
2.00000	-.00933	.01593	.90222	.64169	.15000
0.00000	1.00000	1.00000	.91100	.61237	.20000
.10000	.39339	1.02700	.91100	.61237	.20000
.20000	.06127	.96124	.91100	.61237	.20000
.30000	-.10868	.85440	.91100	.61237	.20000
.40000	-.18485	.73537	.91100	.61237	.20000
.50000	-.20849	.61945	.91100	.61237	.20000
.60000	-.20418	.51401	.91100	.61237	.20000
.70000	-.18628	.42192	.91100	.61237	.20000
.80000	-.16302	.34358	.91100	.61237	.20000
.90000	-.13887	.27810	.91100	.61237	.20000
1.00000	-.11615	.22409	.91100	.61237	.20000
1.10000	-.09589	.17994	.91100	.61237	.20000
1.20000	-.07842	.14410	.91100	.61237	.20000
1.30000	-.06368	.11515	.91100	.61237	.20000
1.40000	-.05144	.09188	.91100	.61237	.20000
1.50000	-.04138	.07321	.91100	.61237	.20000
1.60000	-.03319	.05828	.91100	.61237	.20000
1.70000	-.02655	.04636	.91100	.61237	.20000
1.80000	-.02120	.03685	.91100	.61237	.20000
1.90000	-.01691	.02928	.91100	.61237	.20000

(CONTD...

2.00000	-.01347	.02326	.91100	.61237	.20000
0.00000	1.00000	1.00000	.91940	.57735	.25000
.10000	.39354	1.04892	.91940	.57735	.25000
.20000	.05520	1.00056	.91940	.57735	.25000
.30000	-.12205	.90573	.91940	.57735	.25000
.40000	-.20434	.79374	.91940	.57735	.25000
.50000	-.23220	.68078	.91940	.57735	.25000
.60000	-.23023	.57523	.91940	.57735	.25000
.70000	-.21310	.48088	.91940	.57735	.25000
.80000	-.18941	.39886	.91940	.57735	.25000
.90000	-.16400	.32891	.91940	.57735	.25000
1.00000	-.13951	.27004	.91940	.57735	.25000
1.10000	-.11719	.22096	.91940	.57735	.25000
1.20000	-.09754	.18035	.91940	.57735	.25000
1.30000	-.08065	.14691	.91940	.57735	.25000
1.40000	-.06635	.11949	.91940	.57735	.25000
1.50000	-.05437	.09707	.91940	.57735	.25000
1.60000	-.04443	.07879	.91940	.57735	.25000
1.70000	-.03622	.06390	.91940	.57735	.25000
1.80000	-.02948	.05180	.91940	.57735	.25000
1.90000	-.02396	.04197	.91940	.57735	.25000
2.00000	-.01946	.03400	.91940	.57735	.25000
0.00000	1.00000	1.00000	.92741	.53452	.30000
.10000	.39386	1.07051	.92741	.53452	.30000
.20000	.04925	1.04012	.92741	.53452	.30000
.30000	-.13565	.95843	.92741	.53452	.30000
.40000	-.22462	.85482	.92741	.53452	.30000
.50000	-.25737	.74620	.92741	.53452	.30000
.60000	-.25840	.64179	.92741	.53452	.30000
.70000	-.24263	.54620	.92741	.53452	.30000
.80000	-.21899	.46128	.92741	.53452	.30000
.90000	-.19270	.38737	.92741	.53452	.30000
1.00000	-.16666	.32392	.92741	.53452	.30000
1.10000	-.14241	.27000	.92741	.53452	.30000
1.20000	-.12062	.22451	.92741	.53452	.30000
1.30000	-.10151	.18634	.92741	.53452	.30000
1.40000	-.08502	.15444	.92741	.53452	.30000
1.50000	-.07096	.12786	.92741	.53452	.30000
1.60000	-.05906	.10577	.92741	.53452	.30000
1.70000	-.04905	.08744	.92741	.53452	.30000
1.80000	-.04068	.07225	.92741	.53452	.30000
1.90000	-.03369	.05967	.92741	.53452	.30000
2.00000	-.02788	.04927	.92741	.53452	.30000
0.00000	1.00000	1.00000	.93522	.48038	.35000
.10000	.39419	1.09198	.93522	.48038	.35000
.20000	.04338	1.08006	.93522	.48038	.35000
.30000	-.14929	1.01247	.93522	.48038	.35000
.40000	-.24532	.91852	.93522	.48038	.35000
.50000	-.28350	.81560	.93522	.48038	.35000
.60000	-.28816	.71363	.93522	.48038	.35000
.70000	-.27438	.61795	.93522	.48038	.35000
.80000	-.25138	.53110	.93522	.48038	.35000
.90000	-.22469	.45395	.93522	.48038	.35000
1.00000	-.19749	.38643	.93522	.48038	.35000
1.10000	-.17157	.32795	.93522	.48038	.35000

(CONTD..

1.20000	-.14781	.27767	.93522	.48038	.35000
1.30000	-.12656	.23470	.93522	.48038	.35000
1.40000	-.10787	.19812	.93522	.48038	.35000
1.50000	-.09163	.16707	.93522	.48038	.35000
1.60000	-.07764	.14078	.93522	.48038	.35000
1.70000	-.06566	.11855	.93522	.48038	.35000
1.80000	-.05545	.09979	.93522	.48038	.35000
1.90000	-.04678	.08398	.93522	.48038	.35000
2.00000	-.03942	.07064	.93522	.48038	.35000
0.00000	1.00000	1.00000	.94220	.40825	.40000
.10000	.39252	1.11244	.94220	.40825	.40000
.20000	.03505	1.11849	.94220	.40825	.40000
.30000	-.16536	1.06511	.94220	.40825	.40000
.40000	-.26834	.98142	.94220	.40825	.40000
.50000	-.31195	.88514	.94220	.40825	.40000
.60000	-.32036	.78673	.94220	.40825	.40000
.70000	-.30878	.69214	.94220	.40825	.40000
.80000	-.28668	.60447	.94220	.40825	.40000
.90000	-.25986	.52509	.94220	.40825	.40000
1.00000	-.23178	.45435	.94220	.40825	.40000
1.10000	-.20441	.39199	.94220	.40825	.40000
1.20000	-.17884	.33745	.94220	.40825	.40000
1.30000	-.15556	.29003	.94220	.40825	.40000
1.40000	-.13473	.24897	.94220	.40825	.40000
1.50000	-.11632	.21352	.94220	.40825	.40000
1.60000	-.10019	.18299	.94220	.40825	.40000
1.70000	-.08614	.15675	.94220	.40825	.40000
1.80000	-.07397	.13421	.94220	.40825	.40000
1.90000	-.06345	.11488	.94220	.40825	.40000
2.00000	-.05439	.09831	.94220	.40825	.40000
0.00000	1.00000	1.00000	.94896	.30151	.45000
.10000	.36597	1.13630	.94896	.30151	.45000
.20000	-.00190	1.15534	.94896	.30151	.45000
.30000	-.20475	1.10875	.94896	.30151	.45000
.40000	-.30653	1.02827	.94896	.30151	.45000
.50000	-.34754	.93306	.94896	.30151	.45000
.60000	-.35299	.83443	.94896	.30151	.45000
.70000	-.33855	.73878	.94896	.30151	.45000
.80000	-.31386	.64951	.94896	.30151	.45000
.90000	-.28473	.56817	.94896	.30151	.45000
1.00000	-.25459	.49522	.94896	.30151	.45000
1.10000	-.22538	.43051	.94896	.30151	.45000
1.20000	-.19813	.37353	.94896	.30151	.45000
1.30000	-.17331	.32364	.94896	.30151	.45000
1.40000	-.15105	.28013	.94896	.30151	.45000
1.50000	-.13131	.24228	.94896	.30151	.45000
1.60000	-.11393	.20943	.94896	.30151	.45000
1.70000	-.09871	.18096	.94896	.30151	.45000
1.80000	-.08544	.15631	.94896	.30151	.45000
1.90000	-.07389	.13499	.94896	.30151	.45000
2.00000	-.06387	.11655	.94896	.30151	.45000

APPENDIX...C
V-B

C C RAYLEIGH WAVE PROPAGATION VOIGT SOLID K.G.BHATIA

$\frac{Z}{L}$	$\frac{U}{U_0}$	$\frac{W}{W_0}$	$\frac{U}{W}$	ξ	η	σ
0.00000	1.00000	1.00000	.78615	.87403	.70711	0.00000
.10000	.40731	.94243	.33977	.87403	.70711	0.00000
.20000	.09830	.82058	.09417	.87403	.70711	0.00000
.30000	-.05121	.68210	-.05902	.87403	.70711	0.00000
.40000	-.11349	.55020	-.16216	.87403	.70711	0.00000
.50000	-.13007	.43468	-.23524	.87403	.70711	0.00000
.60000	-.12432	.33832	-.28889	.87403	.70711	0.00000
.70000	-.10908	.26040	-.32932	.87403	.70711	0.00000
.80000	-.09110	.19873	-.36039	.87403	.70711	0.00000
.90000	-.07372	.15068	-.38463	.87403	.70711	0.00000
1.00000	-.05837	.11366	-.40377	.87403	.70711	0.00000
1.10000	-.04551	.08538	-.41902	.87403	.70711	0.00000
1.20000	-.03507	.06393	-.43126	.87403	.70711	0.00000
1.30000	-.02679	.04774	-.44114	.87403	.70711	0.00000
1.40000	-.02033	.03558	-.44915	.87403	.70711	0.00000
1.50000	-.01534	.02647	-.45568	.87403	.70711	0.00000
1.60000	-.01153	.01966	-.46101	.87403	.70711	0.00000
1.70000	-.00864	.01459	-.46538	.87403	.70711	0.00000
1.80000	-.00645	.01082	-.46896	.87403	.70711	0.00000
1.90000	-.00481	.00801	-.47191	.87403	.70711	0.00000
2.00000	-.00358	.00593	-.47433	.87403	.70711	0.00000
0.00000	1.00000	1.00000	.76789	.88369	.68825	.05000
.10000	.40046	.96221	.31959	.88369	.68825	.05000
.20000	.08612	.85160	.07766	.88369	.68825	.05000
.30000	-.06669	.71824	-.07130	.88369	.68825	.05000
.40000	-.13045	.58735	-.17055	.88369	.68825	.05000
.50000	-.14712	.47028	-.24022	.88369	.68825	.05000
.60000	-.14052	.37090	-.29092	.88369	.68825	.05000
.70000	-.12387	.28929	-.32879	.88369	.68825	.05000
.80000	-.10421	.22375	-.35765	.88369	.68825	.05000
.90000	-.08508	.17195	-.37996	.88369	.68825	.05000
1.00000	-.06804	.13148	-.39741	.88369	.68825	.05000
1.10000	-.05362	.10014	-.41118	.88369	.68825	.05000
1.20000	-.04180	.07603	-.42213	.88369	.68825	.05000
1.30000	-.03231	.05758	-.43087	.88369	.68825	.05000
1.40000	-.02482	.04353	-.43790	.88369	.68825	.05000
1.50000	-.01897	.03285	-.44355	.88369	.68825	.05000
1.60000	-.01445	.02476	-.44812	.88369	.68825	.05000
1.70000	-.01097	.01864	-.45182	.88369	.68825	.05000
1.80000	-.00831	.01402	-.45483	.88369	.68825	.05000
1.90000	-.00628	.01054	-.45727	.88369	.68825	.05000
2.00000	-.00474	.00792	-.45926	.88369	.68825	.05000
0.00000	1.00000	1.00000	.74828	.89311	.66667	.10000
.10000	.39621	.98323	.30154	.89311	.66667	.10000
.20000	.07618	.88594	.06435	.89311	.66667	.10000
.30000	-.08128	.75962	-.08006	.89311	.66667	.10000
.40000	-.14795	.63113	-.17541	.89311	.66667	.10000
.50000	-.16586	.51331	-.24178	.89311	.66667	.10000
.60000	-.15920	.41121	-.28970	.89311	.66667	.10000

(CONTD..

.70000	-.14159	.32580	-.32521	.89311	.66667	.10000
.80000	-.12044	.25600	-.35203	.89311	.66667	.10000
.90000	-.09953	.19989	-.37260	.89311	.66667	.10000
1.00000	-.08065	.15532	-.38855	.89311	.66667	.10000
1.10000	-.06444	.12023	-.40102	.89311	.66667	.10000
1.20000	-.05095	.09280	-.41083	.89311	.66667	.10000
1.30000	-.03997	.07145	-.41860	.89311	.66667	.10000
1.40000	-.03118	.05492	-.42478	.89311	.66667	.10000
1.50000	-.02420	.04215	-.42971	.89311	.66667	.10000
1.60000	-.01872	.03231	-.43364	.89311	.66667	.10000
1.70000	-.01444	.02474	-.43680	.89311	.66667	.10000
1.80000	-.01112	.01893	-.43933	.89311	.66667	.10000
1.90000	-.00854	.01448	-.44137	.89311	.66667	.10000
2.00000	-.00655	.01107	-.44300	.89311	.66667	.10000
0.00000	1.00000	1.00000	.72728	.90222	.64169	.15000
.10000	.39405	1.00498	.28517	.90222	.64169	.15000
.20000	.06806	.92276	.05364	.90222	.64169	.15000
.30000	-.09520	.80534	-.08597	.90222	.64169	.15000
.40000	-.16608	.68078	-.17742	.90222	.64169	.15000
.50000	-.18634	.56327	-.24060	.90222	.64169	.15000
.60000	-.18044	.45906	-.28587	.90222	.64169	.15000
.70000	-.16240	.37005	-.31918	.90222	.64169	.15000
.80000	-.14001	.29588	-.34415	.90222	.64169	.15000
.90000	-.11740	.23512	-.36314	.90222	.64169	.15000
1.00000	-.09659	.18596	-.37773	.90222	.64169	.15000
1.10000	-.07840	.14655	-.38905	.90222	.64169	.15000
1.20000	-.06301	.11517	-.39789	.90222	.64169	.15000
1.30000	-.05026	.09030	-.40481	.90222	.64169	.15000
1.40000	-.03987	.07069	-.41026	.90222	.64169	.15000
1.50000	-.03150	.05525	-.41457	.90222	.64169	.15000
1.60000	-.02480	.04314	-.41798	.90222	.64169	.15000
1.70000	-.01947	.03366	-.42068	.90222	.64169	.15000
1.80000	-.01526	.02624	-.42283	.90222	.64169	.15000
1.90000	-.01194	.02045	-.42454	.90222	.64169	.15000
2.00000	-.00933	.01593	-.42590	.90222	.64169	.15000
0.00000	1.00000	1.00000	.70494	.91100	.61237	.20000
.10000	.39339	1.02700	.27002	.91100	.61237	.20000
.20000	.06127	.96124	.04493	.91100	.61237	.20000
.30000	-.10868	.85440	-.08967	.91100	.61237	.20000
.40000	-.18485	.73537	-.17720	.91100	.61237	.20000
.50000	-.20849	.61945	-.23726	.91100	.61237	.20000
.60000	-.20418	.51401	-.28001	.91100	.61237	.20000
.70000	-.18628	.42192	-.31124	.91100	.61237	.20000
.80000	-.16302	.34358	-.33448	.91100	.61237	.20000
.90000	-.13887	.27810	-.35202	.91100	.61237	.20000
1.00000	-.11615	.22409	-.36540	.91100	.61237	.20000
1.10000	-.09589	.17994	-.37568	.91100	.61237	.20000
1.20000	-.07842	.14410	-.38364	.91100	.61237	.20000
1.30000	-.06368	.11515	-.38983	.91100	.61237	.20000
1.40000	-.05144	.09188	-.39466	.91100	.61237	.20000
1.50000	-.04138	.07321	-.39844	.91100	.61237	.20000
1.60000	-.03319	.05828	-.40140	.91100	.61237	.20000
1.70000	-.02655	.04636	-.40373	.91100	.61237	.20000
1.80000	-.02120	.03685	-.40556	.91100	.61237	.20000
1.90000	-.01691	.02928	-.40700	.91100	.61237	.20000

(CONTD...

2.00000	-.01347	.02326	-.40814	.91100	.61237	.20000
0.00000	1.00000	1.00000	.68125	.91940	.57735	.25000
.10000	.39354	1.04892	.25560	.91940	.57735	.25000
.30000	-.12205	.90573	-.09180	.91940	.57735	.25000
.20000	.05520	1.00056	.03759	.91940	.57735	.25000
.40000	-.20434	.79374	-.17538	.91940	.57735	.25000
.50000	-.23220	.68078	-.23236	.91940	.57735	.25000
.60000	-.23023	.57523	-.27266	.91940	.57735	.25000
.70000	-.21310	.48088	-.30190	.91940	.57735	.25000
.80000	-.18941	.39886	-.32350	.91940	.57735	.25000
.90000	-.16400	.32891	-.33969	.91940	.57735	.25000
1.00000	-.13951	.27004	-.35194	.91940	.57735	.25000
1.10000	-.11719	.22096	-.36129	.91940	.57735	.25000
1.20000	-.09754	.18035	-.36846	.91940	.57735	.25000
1.30000	-.08065	.14691	-.37399	.91940	.57735	.25000
1.40000	-.06635	.11949	-.37827	.91940	.57735	.25000
1.50000	-.05437	.09707	-.38159	.91940	.57735	.25000
1.60000	-.04443	.07879	-.38416	.91940	.57735	.25000
1.70000	-.03622	.06390	-.38617	.91940	.57735	.25000
1.80000	-.02948	.05180	-.38773	.91940	.57735	.25000
1.90000	-.02396	.04197	-.38895	.91940	.57735	.25000
2.00000	-.01946	.03400	-.38991	.91940	.57735	.25000
0.00000	1.00000	1.00000	.65626	.92741	.53452	.30000
.10000	.39386	1.07051	.24145	.92741	.53452	.30000
.20000	.04925	1.04012	.03107	.92741	.53452	.30000
.30000	-.13565	.95843	-.09288	.92741	.53452	.30000
.40000	-.22462	.85482	-.17244	.92741	.53452	.30000
.50000	-.25737	.74620	-.22635	.92741	.53452	.30000
.60000	-.25840	.64179	-.26423	.92741	.53452	.30000
.70000	-.24263	.54620	-.29152	.92741	.53452	.30000
.80000	-.21899	.46128	-.31156	.92741	.53452	.30000
.90000	-.19270	.38737	-.32646	.92741	.53452	.30000
1.00000	-.16666	.32392	-.33765	.92741	.53452	.30000
1.10000	-.14241	.27000	-.34612	.92741	.53452	.30000
1.20000	-.12062	.22451	-.35257	.92741	.53452	.30000
1.30000	-.10151	.18634	-.35750	.92741	.53452	.30000
1.40000	-.08502	.15444	-.36128	.92741	.53452	.30000
1.50000	-.07096	.12786	-.36418	.92741	.53452	.30000
1.60000	-.05906	.10577	-.36642	.92741	.53452	.30000
1.70000	-.04905	.08744	-.36815	.92741	.53452	.30000
1.80000	-.04068	.07225	-.36948	.92741	.53452	.30000
1.90000	-.03369	.05967	-.37051	.92741	.53452	.30000
2.00000	-.02788	.04927	-.37131	.92741	.53452	.30000
0.00000	1.00000	1.00000	.63055	.93522	.48038	.35000
.10000	.39419	1.09198	.22762	.93522	.48038	.35000
.20000	.04338	1.08006	.02533	.93522	.48038	.35000
.30000	-.14929	1.01247	-.09298	.93522	.48038	.35000
.40000	-.24532	.91852	-.16841	.93522	.48038	.35000
.50000	-.28350	.81560	-.21918	.93522	.48038	.35000
.60000	-.28816	.71363	-.25462	.93522	.48038	.35000
.70000	-.27438	.61795	-.27998	.93522	.48038	.35000
.80000	-.25138	.53110	-.29846	.93522	.48038	.35000
.90000	-.22469	.45395	-.31209	.93522	.48038	.35000
1.00000	-.19749	.38643	-.32226	.93522	.48038	.35000
1.10000	-.17157	.32795	-.32989	.93522	.48038	.35000

(CONTD...

1.20000	-.14781	.27767	-.33564	.93522	.48038	.35000
1.30000	-.12656	.23470	-.34001	.93522	.48038	.35000
1.40000	-.10787	.19812	-.34332	.93522	.48038	.35000
1.50000	-.09163	.16707	-.34585	.93522	.48038	.35000
1.60000	-.07764	.14078	-.34777	.93522	.48038	.35000
1.70000	-.06566	.11855	-.34925	.93522	.48038	.35000
1.80000	-.05545	.09979	-.35037	.93522	.48038	.35000
1.90000	-.04678	.08398	-.35123	.93522	.48038	.35000
2.00000	-.03942	.07064	-.35190	.93522	.48038	.35000
0.00000	1.00000	1.00000	.60250	.94220	.40825	.40000
.10000	.39252	1.11244	.21259	.94220	.40825	.40000
.20000	.03505	1.11849	.01888	.94220	.40825	.40000
.30000	-.16536	1.06511	-.09354	.94220	.40825	.40000
.40000	-.26834	.98142	-.16474	.94220	.40825	.40000
.50000	-.31195	.88514	-.21234	.94220	.40825	.40000
.60000	-.32036	.78673	-.24534	.94220	.40825	.40000
.70000	-.30878	.69214	-.26879	.94220	.40825	.40000
.80000	-.28668	.60447	-.28575	.94220	.40825	.40000
.90000	-.25986	.52509	-.29817	.94220	.40825	.40000
1.00000	-.23178	.45435	-.30736	.94220	.40825	.40000
1.10000	-.20441	.39199	-.31419	.94220	.40825	.40000
1.20000	-.17884	.33745	-.31931	.94220	.40825	.40000
1.30000	-.15556	.29003	-.32315	.94220	.40825	.40000
1.40000	-.13473	.24897	-.32604	.94220	.40825	.40000
1.50000	-.11632	.21352	-.32822	.94220	.40825	.40000
1.60000	-.10019	.18299	-.32987	.94220	.40825	.40000
1.70000	-.08614	.15675	-.33112	.94220	.40825	.40000
1.80000	-.07397	.13421	-.33207	.94220	.40825	.40000
1.90000	-.06345	.11488	-.33279	.94220	.40825	.40000
2.00000	-.05439	.09831	-.33333	.94220	.40825	.40000
0.00000	1.00000	1.00000	.57372	.94896	.30151	.45000
.10000	.36597	1.13630	.18478	.94896	.30151	.45000
.20000	-.00190	1.15534	-.00094	.94896	.30151	.45000
.30000	-.20475	1.10875	-.10595	.94896	.30151	.45000
.40000	-.30653	1.02827	-.17103	.94896	.30151	.45000
.50000	-.34754	.93306	-.21370	.94896	.30151	.45000
.60000	-.35299	.83443	-.24270	.94896	.30151	.45000
.70000	-.33855	.73878	-.26291	.94896	.30151	.45000
.80000	-.31386	.64951	-.27724	.94896	.30151	.45000
.90000	-.28473	.56817	-.28751	.94896	.30151	.45000
1.00000	-.25459	.49522	-.29495	.94896	.30151	.45000
1.10000	-.22538	.43051	-.30036	.94896	.30151	.45000
1.20000	-.19813	.37353	-.30432	.94896	.30151	.45000
1.30000	-.17331	.32364	-.30722	.94896	.30151	.45000
1.40000	-.15105	.28013	-.30936	.94896	.30151	.45000
1.50000	-.13131	.24228	-.31094	.94896	.30151	.45000
1.60000	-.11393	.20943	-.31210	.94896	.30151	.45000
1.70000	-.09871	.18096	-.31296	.94896	.30151	.45000
1.80000	-.08544	.15631	-.31359	.94896	.30151	.45000
1.90000	-.07389	.13499	-.31406	.94896	.30151	.45000
2.00000	-.06387	.11655	-.31441	.94896	.30151	.45000

0 ERROR LC-2 IN STATEMENT 0021 + 00 LINES

APPENDIX...C

VI

C C WAVE PROPAGATION ATTENUATION Z K.G.BHATIA

	$\frac{z}{L}$	τ_z	τ_x	σ	ξ	η
C C	WAVE PROPAGATION		ATTENUATION Z K.G.BHATIA			
	0.000	.61803	.38196	0.00000	.87403	.70711
ZL=	0.000WO=		1.00000UO=	.78615SI=		0.00000
	.100	.58214	.15477	0.00000	.87403	.70711
ZL=	.100WO=		.94193UO=	.31854SI=		0.00000
	.200	.50631	.03673	0.00000	.87403	.70711
ZL=	.200WO=		.81923UO=	.07561SI=		0.00000
	.300	.42028	-.02011	0.00000	.87403	.70711
ZL=	.300WO=		.68004UO=	-.04139SI=		0.00000
	.400	.33850	-.04361	0.00000	.87403	.70711
ZL=	.400WO=		.54771UO=	-.08975SI=		0.00000
	.500	.26702	-.04970	0.00000	.87403	.70711
ZL=	.500WO=		.43204UO=	-.10230SI=		0.00000
	.600	.20749	-.04735	0.00000	.87403	.70711
ZL=	.600WO=		.33572UO=	-.09745SI=		0.00000
	.700	.15944	-.04143	0.00000	.87403	.70711
ZL=	.700WO=		.25799UO=	-.08527SI=		0.00000
	.800	.12149	-.03452	0.00000	.87403	.70711
ZL=	.800WO=		.19657UO=	-.07106SI=		0.00000
	.900	.09196	-.02788	0.00000	.87403	.70711
ZL=	.900WO=		.14880UO=	-.05738SI=		0.00000
	1.000	.06925	-.02203	0.00000	.87403	.70711
ZL=	1.000WO=		.11206UO=	-.04534SI=		0.00000
	1.100	.05194	-.01714	0.00000	.87403	.70711
ZL=	1.100WO=		.08404UO=	-.03528SI=		0.00000
	1.200	.03883	-.01318	0.00000	.87403	.70711
ZL=	1.200WO=		.06283UO=	-.02714SI=		0.00000
	1.300	.02895	-.01005	0.00000	.87403	.70711
ZL=	1.300WO=		.04685UO=	-.02069SI=		0.00000
	1.400	.02154	-.00761	0.00000	.87403	.70711
ZL=	1.400WO=		.03485UO=	-.01567SI=		0.00000
	1.500	.01600	-.00574	0.00000	.87403	.70711
ZL=	1.500WO=		.02589UO=	-.01181SI=		0.00000
	1.600	.01187	-.00430	0.00000	.87403	.70711
ZL=	1.600WO=		.01920UO=	-.00886SI=		0.00000
	1.700	.00879	-.00322	0.00000	.87403	.70711
ZL=	1.700WO=		.01423UO=	-.00663SI=		0.00000
	1.800	.00651	-.00240	0.00000	.87403	.70711
ZL=	1.800WO=		.01053UO=	-.00494SI=		0.00000
	1.900	.00481	-.00179	0.00000	.87403	.70711
ZL=	1.900WO=		.00779UO=	-.00368SI=		0.00000
	2.000	.00356	-.00133	0.00000	.87403	.70711
ZL=	2.000WO=		.00576UO=	-.00273SI=		0.00000
	0.000	.64057	.39045	.05000	.88369	.68825
ZL=	0.000WO=		1.00000UO=	.76789SI=		.05000
	.100	.61526	.15305	.05000	.88369	.68825
ZL=	.100WO=		.96050UO=	.30100SI=		.05000
	.200	.54225	.03030	.05000	.88369	.68825
ZL=	.200WO=		.84652UO=	.05959SI=		.05000
	.300	.45495	-.02828	.05000	.88369	.68825
ZL=	.300WO=		.71023UO=	-.05562SI=		.05000
	.400	.36992	-.05196	.05000	.88369	.68825
ZL=	.400WO=		.57749UO=	-.10220SI=		.05000
	.500	.29442	-.05748	.05000	.88369	.68825

(CONTD...)

```
ZL=      .500WO=        .45963UO=         -.11305SI=         .05000
      .600      .23080      -.05423      .05000       .88369        .68825
ZL=      .600WO=        .36030UO=         -.10665SI=         .05000
      .700      .17891      -.04733      .05000       .88369        .68825
ZL=      .700WO=        .27930UO=         -.09308SI=         .05000
      .800      .13752      -.03947      .05000       .88369        .68825
ZL=      .800WO=        .21469UO=         -.07763SI=         .05000
      .900      .10503      -.03196      .05000       .88369        .68825
ZL=      .900WO=        .16397UO=         -.06286SI=         .05000
     1.000      .07982      -.02537      .05000       .88369        .68825
ZL=     1.000WO=        .12461UO=         -.04989SI=         .05000
     1.100      .06042      -.01985      .05000       .88369        .68825
ZL=     1.100WO=        .09433UO=         -.03903SI=         .05000
     1.200      .04560      -.01536      .05000       .88369        .68825
ZL=     1.200WO=        .07119UO=         -.03021SI=         .05000
     1.300      .03433      -.01179      .05000       .88369        .68825
ZL=     1.300WO=        .05359UO=         -.02320SI=         .05000
     1.400      .02579      -.00900      .05000       .88369        .68825
ZL=     1.400WO=        .04026UO=         -.01770SI=         .05000
     1.500      .01935      -.00683      .05000       .88369        .68825
ZL=     1.500WO=        .03021UO=         -.01344SI=         .05000
     1.600      .01450      -.00517      .05000       .88369        .68825
ZL=     1.600WO=        .02263UO=         -.01017SI=         .05000
     1.700      .01085      -.00390      .05000       .88369        .68825
ZL=     1.700WO=        .01694UO=         -.00767SI=         .05000
     1.800      .00811      -.00294      .05000       .88369        .68825
ZL=     1.800WO=        .01267UO=         -.00577SI=         .05000
     1.900      .00606      -.00220      .05000       .88369        .68825
ZL=     1.900WO=        .00947UO=         -.00434SI=         .05000
     2.000      .00453      -.00165      .05000       .88369        .68825
ZL=     2.000WO=        .00707UO=         -.00325SI=         .05000
     0.000      .66340       .39883      .10000       .89311        .66667
ZL=     0.000WO=       1.00000UO=          .74828SI=         .10000
      .100      .65023       .15047      .10000       .89311        .66667
ZL=      .100WO=        .98015UO=          .28231SI=         .10000
      .200      .58078       .02278      .10000       .89311        .66667
ZL=      .200WO=        .87546UO=          .04273SI=         .10000
      .300      .49250      -.03755      .10000       .89311        .66667
ZL=      .300WO=        .74239UO=         -.07044SI=         .10000
      .400      .40427      -.06133      .10000       .89311        .66667
ZL=      .400WO=        .60939UO=         -.11507SI=         .10000
      .500      .32467      -.06617      .10000       .89311        .66667
ZL=      .500WO=        .48940UO=         -.12414SI=         .10000
      .600      .25677      -.06191      .10000       .89311        .66667
ZL=      .600WO=        .38705UO=         -.11615SI=         .10000
      .700      .20081      -.05393      .10000       .89311        .66667
ZL=      .700WO=        .30270UO=         -.10118SI=         .10000
      .800      .15576      -.04504      .10000       .89311        .66667
ZL=      .800WO=        .23478UO=         -.08450SI=         .10000
      .900      .12005      -.03659      .10000       .89311        .66667
ZL=      .900WO=        .18097UO=         -.06866SI=         .10000
     1.000      .09209      -.02918      .10000       .89311        .66667
ZL=     1.000WO=        .13882UO=         -.05475SI=         .10000
     1.100      .07038      -.02296      .10000       .89311        .66667
ZL=     1.100WO=        .10609UO=         -.04308SI=         .10000
                                                                 (CONTD...)
```

	1.200	.05364	-.01789	.10000	.89311	.66667
ZL=	1.200WO=		.08085UO=	-.03357SI=		.10000
	1.300	.04078	-.01384	.10000	.89311	.66667
ZL=	1.300WO=		.06147UO=	-.02596SI=		.10000
	1.400	.03095	-.01064	.10000	.89311	.66667
ZL=	1.400WO=		.04666UO=	-.01997SI=		.10000
	1.500	.02346	-.00815	.10000	.89311	.66667
ZL=	1.500WO=		.03536UO=	-.01529SI=		.10000
	1.600	.01776	-.00622	.10000	.89311	.66667
ZL=	1.600WO=		.02677UO=	-.01167SI=		.10000
	1.700	.01343	-.00474	.10000	.89311	.66667
ZL=	1.700WO=		.02025UO=	-.00889SI=		.10000
	1.800	.01016	-.00360	.10000	.89311	.66667
ZL=	1.800WO=		.01531UO=	-.00675SI=		.10000
	1.900	.00767	-.00273	.10000	.89311	.66667
ZL=	1.900WO=		.01156UO=	-.00512SI=		.10000
	2.000	.00579	-.00207	.10000	.89311	.66667
ZL=	2.000WO=		.00873UO=	-.00388SI=		.10000
	0.000	.68634	.40700	.15000	.90222	.64169
ZL=	0.000WO=		1.00000UO=	.72728SI=		.15000
	.100	.68695	.14687	.15000	.90222	.64169
ZL=	.100WO=		1.00089UO=	.26245SI=		.15000
	.200	.62186	.01404	.15000	.90222	.64169
ZL=	.200WO=		.90605UO=	.02509SI=		.15000
	.300	.53294	-.04799	.15000	.90222	.64169
ZL=	.300WO=		.77650UO=	-.08575SI=		.15000
	.400	.44160	-.07176	.15000	.90222	.64169
ZL=	.400WO=		.64342UO=	-.12823SI=		.15000
	.500	.35785	-.07579	.15000	.90222	.64169
ZL=	.500WO=		.52139UO=	-.13544SI=		.15000
	.600	.28553	-.07042	.15000	.90222	.64169
ZL=	.600WO=		.41601UO=	-.12584SI=		.15000
	.700	.22531	-.06127	.15000	.90222	.64169
ZL=	.700WO=		.32828UO=	-.10948SI=		.15000
	.800	.17636	-.05126	.15000	.90222	.64169
ZL=	.800WO=		.25696UO=	-.09159SI=		.15000
	.900	.13721	-.04180	.15000	.90222	.64169
ZL=	.900WO=		.19992UO=	-.07470SI=		.15000
	1.000	.10627	-.03351	.15000	.90222	.64169
ZL=	1.000WO=		.15484UO=	-.05987SI=		.15000
	1.100	.08202	-.02653	.15000	.90222	.64169
ZL=	1.100WO=		.11950UO=	-.04740SI=		.15000
	1.200	.06313	-.02081	.15000	.90222	.64169
ZL=	1.200WO=		.09199UO=	-.03719SI=		.15000
	1.300	.04850	-.01622	.15000	.90222	.64169
ZL=	1.300WO=		.07066UO=	-.02899SI=		.15000
	1.400	.03720	-.01258	.15000	.90222	.64169
ZL=	1.400WO=		.05419UO=	-.02248SI=		.15000
	1.500	.02849	-.00972	.15000	.90222	.64169
ZL=	1.500WO=		.04151UO=	-.01737SI=		.15000
	1.600	.02180	-.00749	.15000	.90222	.64169
ZL=	1.600WO=		.03177UO=	-.01338SI=		.15000
	1.700	.01667	-.00576	.15000	.90222	.64169
ZL=	1.700WO=		.02429UO=	-.01029SI=		.15000
	1.800	.01274	-.00442	.15000	.90222	.64169

(CONTD...)

```
ZL=      1.800WO=          .01857UO=          -.00789SI=          .15000
     1.900       .00973      -.00338       .15000       .90222       .64169
ZL=      1.900WO=          .01418UO=          -.00605SI=          .15000
     2.000       .00743      -.00259       .15000       .90222       .64169
ZL=      2.000WO=          .01083UO=          -.00463SI=          .15000
     0.000       .70929       .41497       .20000       .91100       .61237
ZL=      0.000WO=         1.00000UO=           .70494SI=          .20000
      .100       .72545       .14217       .20000       .91100       .61237
ZL=       .100WO=         1.02279UO=           .24151SI=          .20000
      .200       .66557       .00400       .20000       .91100       .61237
ZL=       .200WO=          .93836UO=           .00679SI=          .20000
      .300       .57639      -.05967       .20000       .91100       .61237
ZL=       .300WO=          .81264UO=          -.10137SI=          .20000
      .400       .48208      -.08331       .20000       .91100       .61237
ZL=       .400WO=          .67966UO=          -.14152SI=          .20000
      .500       .39415      -.08641       .20000       .91100       .61237
ZL=       .500WO=          .55570UO=          -.14679SI=          .20000
      .600       .31731      -.07981       .20000       .91100       .61237
ZL=       .600WO=          .44736UO=          -.13559SI=          .20000
      .700       .25267      -.06939       .20000       .91100       .61237
ZL=       .700WO=          .35623UO=          -.11788SI=          .20000
      .800       .19962      -.05818       .20000       .91100       .61237
ZL=       .800WO=          .28144UO=          -.09883SI=          .20000
      .900       .15680      -.04764       .20000       .91100       .61237
ZL=       .900WO=          .22107UO=          -.08094SI=          .20000
     1.000       .12264      -.03840       .20000       .91100       .61237
ZL=      1.000WO=          .17290UO=          -.06523SI=          .20000
     1.100       .09561      -.03060       .20000       .91100       .61237
ZL=      1.100WO=          .13480UO=          -.05199SI=          .20000
     1.200       .07436      -.02419       .20000       .91100       .61237
ZL=      1.200WO=          .10484UO=          -.04110SI=          .20000
     1.300       .05773      -.01901       .20000       .91100       .61237
ZL=      1.300WO=          .08139UO=          -.03229SI=          .20000
     1.400       .04475      -.01487       .20000       .91100       .61237
ZL=      1.400WO=          .06309UO=          -.02527SI=          .20000
     1.500       .03466      -.01160       .20000       .91100       .61237
ZL=      1.500WO=          .04886UO=          -.01970SI=          .20000
     1.600       .02682      -.00902       .20000       .91100       .61237
ZL=      1.600WO=          .03781UO=          -.01533SI=          .20000
     1.700       .02074      -.00700       .20000       .91100       .61237
ZL=      1.700WO=          .02924UO=          -.01190SI=          .20000
     1.800       .01603      -.00543       .20000       .91100       .61237
ZL=      1.800WO=          .02260UO=          -.00923SI=          .20000
     1.900       .01239      -.00421       .20000       .91100       .61237
ZL=      1.900WO=          .01746UO=          -.00715SI=          .20000
     2.000       .00957      -.00325       .20000       .91100       .61237
ZL=      2.000WO=          .01349UO=          -.00553SI=          .20000
     0.000       .73205       .42264       .25000       .91940       .57735
ZL=      0.000WO=         1.00000UO=           .68125SI=          .25000
      .100       .76565       .13620       .25000       .91940       .57735
ZL=       .100WO=         1.04591UO=           .21954SI=          .25000
      .200       .71185      -.00746       .25000       .91940       .57735
ZL=       .200WO=          .97241UO=          -.01203SI=          .25000
      .300       .62282      -.07267       .25000       .91940       .57735
ZL=       .300WO=          .85080UO=          -.11714SI=          .25000
                                                                   (CONTD...)
```

```
      .400       .52569       -.09601        .25000      .91940      .57735
ZL=     .400WO=              .71811UO=        -.15475SI=             .25000
      .500       .43363       -.09803        .25000      .91940      .57735
ZL=     .500WO=              .59235UO=        -.15801SI=             .25000
      .600       .35221       -.09009        .25000      .91940      .57735
ZL=     .600WO=              .48113UO=        -.14522SI=             .25000
      .700       .28303       -.07831        .25000      .91940      .57735
ZL=     .700WO=              .38663UO=        -.12622SI=             .25000
      .800       .22572       -.06583        .25000      .91940      .57735
ZL=     .800WO=              .30835UO=        -.10610SI=             .25000
      .900       .17904       -.05415        .25000      .91940      .57735
ZL=     .900WO=              .24457UO=        -.08728SI=             .25000
     1.000       .14144       -.04390        .25000      .91940      .57735
ZL=    1.000WO=              .19321UO=        -.07076SI=             .25000
     1.100       .11142       -.03523        .25000      .91940      .57735
ZL=    1.100WO=              .15220UO=        -.05679SI=             .25000
     1.200       .08757       -.02807        .25000      .91940      .57735
ZL=    1.200WO=              .11963UO=        -.04525SI=             .25000
     1.300       .06872       -.02225        .25000      .91940      .57735
ZL=    1.300WO=              .09388UO=        -.03587SI=             .25000
     1.400       .05387       -.01757        .25000      .91940      .57735
ZL=    1.400WO=              .07358UO=        -.02832SI=             .25000
     1.500       .04218       -.01384        .25000      .91940      .57735
ZL=    1.500WO=              .05763UO=        -.02230SI=             .25000
     1.600       .03301       -.01087        .25000      .91940      .57735
ZL=    1.600WO=              .04510UO=        -.01752SI=             .25000
     1.700       .02582       -.00853        .25000      .91940      .57735
ZL=    1.700WO=              .03528UO=        -.01375SI=             .25000
     1.800       .02019       -.00668        .25000      .91940      .57735
ZL=    1.800WO=              .02758UO=        -.01077SI=             .25000
     1.900       .01578       -.00523        .25000      .91940      .57735
ZL=    1.900WO=              .02156UO=        -.00844SI=             .25000
     2.000       .01234       -.00410        .25000      .91940      .57735
ZL=    2.000WO=              .01685UO=        -.00660SI=             .25000
     0.000       .75452        .43004        .30000      .92741      .53452
ZL=    0.000WO=             1.00000UO=         .65626SI=             .30000
      .100       .80760        .12886        .30000      .92741      .53452
ZL=     .100WO=             1.07035UO=         .19664SI=             .30000
      .200       .76078       -.02046        .30000      .92741      .53452
ZL=     .200WO=             1.00829UO=        -.03122SI=             .30000
      .300       .67232       -.08705        .30000      .92741      .53452
ZL=     .300WO=              .89105UO=        -.13285SI=             .30000
      .400       .57258       -.10989        .30000      .92741      .53452
ZL=     .400WO=              .75886UO=        -.16770SI=             .30000
      .500       .47645       -.11067        .30000      .92741      .53452
ZL=     .500WO=              .63146UO=        -.16889SI=             .30000
      .600       .39045       -.10128        .30000      .92741      .53452
ZL=     .600WO=              .51748UO=        -.15456SI=             .30000
      .700       .31666       -.08806        .30000      .92741      .53452
ZL=     .700WO=              .41969UO=        -.13439SI=             .30000
      .800       .25497       -.07425        .30000      .92741      .53452
ZL=     .800WO=              .33792UO=        -.11331SI=             .30000
      .900       .20425       -.06138        .30000      .92741      .53452
ZL=     .900WO=              .27070UO=        -.09367SI=             .30000
     1.000       .16303       -.05008        .30000      .92741      .53452
                                                                  (CONTD...)
```

```
ZL=     1.000WO=        .21607UO=        -.07642SI=        .30000
      1.100      .12978     -.04049      .30000      .92741      .53452
ZL=     1.100WO=        .17201UO=        -.06179SI=        .30000
      1.200      .10312     -.03253      .30000      .92741      .53452
ZL=     1.200WO=        .13667UO=        -.04965SI=        .30000
      1.300      .08183     -.02602      .30000      .92741      .53452
ZL=     1.300WO=        .10845UO=        -.03971SI=        .30000
      1.400      .06486     -.02075      .30000      .92741      .53452
ZL=     1.400WO=        .08597UO=        -.03166SI=        .30000
      1.500      .05138     -.01650      .30000      .92741      .53452
ZL=     1.500WO=        .06810UO=        -.02518SI=        .30000
      1.600      .04068     -.01311      .30000      .92741      .53452
ZL=     1.600WO=        .05391UO=        -.02000SI=        .30000
      1.700      .03219     -.01039      .30000      .92741      .53452
ZL=     1.700WO=        .04266UO=        -.01586SI=        .30000
      1.800      .02547     -.00824      .30000      .92741      .53452
ZL=     1.800WO=        .03376UO=        -.01257SI=        .30000
      1.900      .02015     -.00652      .30000      .92741      .53452
ZL=     1.900WO=        .02670UO=        -.00995SI=        .30000
      2.000      .01593     -.00516      .30000      .92741      .53452
ZL=     2.000WO=        .02112UO=        -.00788SI=        .30000
      0.000      .77658      .43711      .35000      .93501      .48038
ZL=     0.000WO=       1.00000UO=         .62999SI=        .35000
       .100      .85131      .11997      .35000      .93501      .48038
ZL=      .100WO=       1.09622UO=         .17290SI=        .35000
       .200      .81237     -.03510      .35000      .93501      .48038
                                              -------             -----
ZL=      .200WO=       1.04609UO=
       .300      .72490     -.10287      .35000      .93501      .48038
ZL=      .300WO=        .93345UO=        -.14827SI=        .35000
       .400      .62277     -.12499      .35000      .93501      .48038
ZL=      .400WO=        .80193UO=        -.18014SI=        .35000
       .500      .52271     -.12436      .35000      .93501      .48038
ZL=      .500WO=        .67309UO=        -.17923SI=        .35000
       .600      .43219     -.11340      .35000      .93501      .48038
ZL=      .600WO=        .55653UO=        -.16344SI=        .35000
       .700      .35378     -.09867      .35000      .93501      .48038
ZL=      .700WO=        .45556UO=        -.14220SI=        .35000
       .800      .28764     -.08348      .35000      .93501      .48038
ZL=      .800WO=        .37039UO=        -.12031SI=        .35000
       .900      .23276     -.06938      .35000      .93501      .48038
ZL=      .900WO=        .29972UO=        -.09999SI=        .35000
      1.000      .18774     -.05699      .35000      .93501      .48038
ZL=     1.000WO=        .24175UO=        -.08214SI=        .35000
      1.100      .15108     -.04645      .35000      .93501      .48038
ZL=     1.100WO=        .19454UO=        -.06694SI=        .35000
      1.200      .12138     -.03765      .35000      .93501      .48038
ZL=     1.200WO=        .15630UO=        -.05426SI=        .35000
      1.300      .09741     -.03040      .35000      .93501      .48038
ZL=     1.300WO=        .12543UO=        -.04381SI=        .35000
      1.400      .07811     -.02448      .35000      .93501      .48038
ZL=     1.400WO=        .10058UO=        -.03529SI=        .35000
      1.500      .06260     -.01968      .35000      .93501      .48038
ZL=     1.500WO=        .08060UO=        -.02837SI=        .35000
      1.600      .05014     -.01580      .35000      .93501      .48038
ZL=     1.600WO=        .06457UO=        -.02277SI=        .35000
                                                                (CONTD...)
```

	1.700	.04016	-.01267	.35000	.93501	.48038
ZL=	1.700WO=		.05171UO=	-.01827SI=		.35000
	1.800	.03215	-.01016	.35000	.93501	.48038
ZL=	1.800WO=		.04140UO=	-.01464SI=		.35000
	1.900	.02574	-.00814	.35000	.93501	.48038
ZL=	1.900WO=		.03315UO=	-.01173SI=		.35000
	2.000	.02060	-.00652	.35000	.93501	.48038
ZL=	2.000WO=		.02653UO=	-.00940SI=		.35000
	0.000	.79814	.44388	.40000	.94220	.40825
ZL=	0.000WO=		1.00000UO=	.60250SI=		.40000
	.100	.89688	.10937	.40000	.94220	.40825
ZL=	.100WO=		1.12371UO=	.14845SI=		.40000
	.200	.86674	-.05151	.40000	.94220	.40825
ZL=	.200WO=		1.08594UO=	-.06992SI=		.40000
	.300	.78065	-.12019	.40000	.94220	.40825
ZL=	.300WO=		.97809UO=	-.16314SI=		.40000
	.400	.67638	-.14130	.40000	.94220	.40825
ZL=	.400WO=		.84745UO=	-.19179SI=		.40000
	.500	.57258	-.13907	.40000	.94220	.40825
ZL=	.500WO=		.71739UO=	-.18877SI=		.40000
	.600	.47765	-.12644	.40000	.94220	.40825
ZL=	.600WO=		.59846UO=	-.17163SI=		.40000
	.700	.39469	-.11015	.40000	.94220	.40825
ZL=	.700WO=		.49451UO=	-.14951SI=		.40000
	.800	.32408	-.09356	.40000	.94220	.40825
ZL=	.800WO=		.40604UO=	-.12699SI=		.40000
	.900	.26497	-.07821	.40000	.94220	.40825
ZL=	.900WO=		.33199UO=	-.10616SI=		.40000
	1.000	.21603	-.06472	.40000	.94220	.40825
ZL=	1.000WO=		.27066UO=	-.08785SI=		.40000
	1.100	.17577	-.05319	.40000	.94220	.40825
ZL=	1.100WO=		.22023UO=	-.07220SI=		.40000
	1.200	.14283	-.04352	.40000	.94220	.40825
ZL=	1.200WO=		.17895UO=	-.05907SI=		.40000
	1.300	.11595	-.03549	.40000	.94220	.40825
ZL=	1.300WO=		.14528UO=	-.04818SI=		.40000
	1.400	.09407	-.02889	.40000	.94220	.40825
ZL=	1.400WO=		.11787UO=	-.03921SI=		.40000
	1.500	.07629	-.02348	.40000	.94220	.40825
ZL=	1.500WO=		.09558UO=	-.03187SI=		.40000
	1.600	.06185	-.01906	.40000	.94220	.40825
ZL=	1.600WO=		.07749UO=	-.02588SI=		.40000
	1.700	.05013	-.01547	.40000	.94220	.40825
ZL=	1.700WO=		.06281UO=	-.02100SI=		.40000
	1.800	.04063	-.01254	.40000	.94220	.40825
ZL=	1.800WO=		.05090UO=	-.01703SI=		.40000
	1.900	.03292	-.01017	.40000	.94220	.40825
ZL=	1.900WO=		.04125UO=	-.01381SI=		.40000
	2.000	.02668	-.00824	.40000	.94220	.40825
ZL=	2.000WO=		.03342UO=	-.01119SI=		.40000
	0.000	.81905	.45026	.45000	.94896	.30151
ZL=	0.000WO=		1.00000UO=	.57372SI=		.45000
	.100	.94435	.09678	.45000	.94896	.30151
ZL=	.100WO=		1.15299UO=	.12331SI=		.45000
	.200	.92386	-.06989	.45000	.94896	.30151

(CONTD...)

```
ZL=      .200WO=        1.12796UO=        -.08905SI=        .45000
     .300       .83953        -.13908      .45000       .94896       .30151
ZL=      .300WO=        1.02500UO=        -.17721SI=        .45000
     .400       .73338        -.15885      .45000       .94896       .30151
ZL=      .400WO=        .89541UO=         -.20241SI=        .45000
     .500       .62606        -.15482      .45000       .94896       .30151
ZL=      .500WO=        .76438UO=         -.19727SI=        .45000
     .600       .52695        -.14042      .45000       .94896       .30151
ZL=      .600WO=        .64337UO=         -.17892SI=        .45000
     .700       .43958        -.12253      .45000       .94896       .30151
ZL=      .700WO=        .53669UO=         -.15613SI=        .45000
     .800       .36458        -.10453      .45000       .94896       .30151
ZL=      .800WO=        .44513UO=         -.13319SI=        .45000
     .900       .30125        -.08795      .45000       .94896       .30151
ZL=      .900WO=        .36780UO=         -.11206SI=        .45000
    1.000       .24830        -.07334      .45000       .94896       .30151
ZL=     1.000WO=        .30316UO=         -.09346SI=        .45000
    1.100       .20433        -.06082      .45000       .94896       .30151
ZL=     1.100WO=        .24947UO=         -.07750SI=        .45000
    1.200       .16796        -.05025      .45000       .94896       .30151
ZL=     1.200WO=        .20506UO=         -.06403SI=        .45000
    1.300       .13796        -.04142      .45000       .94896       .30151
ZL=     1.300WO=        .16844UO=         -.05277SI=        .45000
    1.400       .11327        -.03408      .45000       .94896       .30151
ZL=     1.400WO=        .13829UO=         -.04342SI=        .45000
    1.500       .09297        -.02801      .45000       .94896       .30151
ZL=     1.500WO=        .11351UO=         -.03569SI=        .45000
    1.600       .07629        -.02301      .45000       .94896       .30151
ZL=     1.600WO=        .09314UO=         -.02932SI=        .45000
    1.700       .06259        -.01889      .45000       .94896       .30151
ZL=     1.700WO=        .07642UO=         -.02407SI=        .45000
    1.800       .05135        -.01550      .45000       .94896       .30151
ZL=     1.800WO=        .06269UO=         -.01976SI=        .45000
    1.900       .04212        -.01272      .45000       .94896       .30151
ZL=     1.900WO=        .05143UO=         -.01621SI=        .45000
    2.000       .03455        -.01044      .45000       .94896       .30151
ZL=     2.000WO=        .04219UO=         -.01330SI=        .45000
OGGRADATION--K G BHATIA EQS 3.10.68                                Z
```

APPENDIX...C

VII

C C RAYLEIGH WAVE DISPERSION K.G.BHATIA

V_R	T	E/ν	σ
2.91344	.10000	20	.05
1.64552	.20000		
1.27956	.30000		
1.12366	.40000		
1.04365	.50000		
.99750	.60000		
.96861	.70000		
.94939	.80000		
.93598	.90000		
.92627	1.00000		
1.64552	.10000	40	
1.12366	.20000		
.99750	.30000		
.94939	.40000		
.92627	.50000		
.91347	.60000		
.90567	.70000		
.90056	.80000		
.89705	.90000		
.89453	1.00000		
1.27956	.10000	60	
.99750	.20000		
.93598	.30000		
.91347	.40000		
.90286	.50000		
.89705	.60000		
.89352	.70000		
.89123	.80000		
.88965	.90000		
.88852	1.00000		
1.12366	.10000	80	
.94939	.20000		
.91347	.30000		
.90056	.40000		
.89453	.50000		
.89123	.60000		
.88923	.70000		
.88794	.80000		
.88705	.90000		
.88641	1.00000		
1.04365	.10000	100	
.92627	.20000		
.90286	.30000		
.89453	.40000		
.89064	.50000		
.88852	.60000		
.88724	.70000		
.88641	.80000		
.88584	.90000		

.88543	1.00000		
2.97454	.10000	20	.15
1.68002	.20000		
1.30639	.30000		
1.14722	.40000		
1.06553	.50000		
1.01841	.60000		
.98892	.70000		
.96929	.80000		
.95561	.90000		
.94570	1.00000	40	
1.68002	.10000		
1.14722	.20000		
1.01841	.30000		
.96929	.40000		
.94570	.50000		
.93263	.60000		
.92466	.70000		
.91945	.80000		
.91586	.90000		
.91328	1.00000	60	
1.30639	.10000		
1.01841	.20000		
.95561	.30000		
.93263	.40000		
.92180	.50000		
.91586	.60000		
.91226	.70000		
.90992	.80000		
.90831	.90000		
.90715	1.00000		
1.14722	.10000	80	
.96929	.20000		
.93263	.30000		
.91945	.40000		
.91328	.50000		
.90992	.60000		
.90788	.70000		
.90656	.80000		
.90565	.90000		
.90500	1.00000	100	
1.06553	.10000		
.94570	.20000		
.92180	.30000		
.91328	.40000		
.90932	.50000		
.90715	.60000		
.90585	.70000		
.90500	.80000		
.90442	.90000		
.90400	1.00000		
3.03118	.10000	20	
1.71201	.20000		
1.33127	.30000		
1.16907	.40000		

1.08582	.50000	
1.03780	.60000	
1.00775	.70000	
.98775	.80000	
.97380	.90000	
.96370	1.00000	
1.71201	.10000	40
1.16907	.20000	
1.03780	.30000	
.98775	.40000	
.96370	.50000	
.95039	.60000	
.94226	.70000	
.93696	.80000	
.93330	.90000	
.93067	1.00000	
1.33127	.10000	60
1.03780	.20000	
.97380	.30000	
.95039	.40000	
.93935	.50000	
.93330	.60000	
.92963	.70000	
.92724	.80000	
.92560	.90000	
.92443	1.00000	
1.16907	.10000	80
.98775	.20000	
.95039	.30000	
.93696	.40000	
.93067	.50000	
.92724	.60000	
.92517	.70000	
.92382	.80000	
.92289	.90000	
.92223	1.00000	
1.08582	.10000	100
.96370	.20000	
.93935	.30000	
.93067	.40000	
.92663	.50000	
.92443	.60000	
.92310	.70000	
.92223	.80000	
.92164	.90000	
.92121	1.00000	
3.08264	.10000	20 35-
1.74108	.20000	
1.35387	.30000	
1.18892	.40000	
1.10426	.50000	
1.05543	.60000	
1.02486	.70000	
1.00452	.80000	
.99034	.90000	

.98007	1.00000	
1.74108	.10000	40
1.18892	.20000	
1.05543	.30000	
1.00452	.40000	
.98007	.50000	
.96652	.60000	
.95826	.70000	
.95286	.80000	
.94914	.90000	
.94647	1.00000	
1.35387	.10000	60
1.05543	.20000	
.99034	.30000	
.96652	.40000	
.95530	.50000	
.94914	.60000	
.94541	.70000	
.94299	.80000	
.94132	.90000	
.94012	1.00000	
1.18892	.10000	80
1.00452	.20000	
.96652	.30000	
.95286	.40000	
.94647	.50000	
.94299	.60000	
.94088	.70000	
.93951	.80000	
.93856	.90000	
.93789	1.00000	
1.10426	.10000	100
.98007	.20000	
.95530	.30000	
.94647	.40000	
.94236	.50000	
.94012	.60000	
.93877	.70000	
.93789	.80000	
.93729	.90000	
.93685	1.00000	
3.12863	.10000	20
1.76705	.20000	
1.37407	.30000	
1.20665	.40000	
1.12073	.50000	
1.07117	.60000	
1.04015	.70000	
1.01951	.80000	
1.00511	.90000	
.99469	1.00000	
1.76705	.10000	40
1.20665	.20000	
1.07117	.30000	
1.01951	.40000	

.99469	.50000	
.98094	.60000	
.97256	.70000	
.96708	.80000	
.96331	.90000	
.96060	1.00000	60
1.37407	.10000	
1.07117	.20000	
1.00511	.30000	
.98094	.40000	
.96955	.50000	
.96331	.60000	
.95952	.70000	
.95706	.80000	
.95536	.90000	
.95415	1.00000	80
1.20665	.10000	
1.01951	.20000	
.98094	.30000	
.96708	.40000	
.96060	.50000	
.95706	.60000	
.95491	.70000	
.95352	.80000	
.95257	.90000	
.95188	1.00000	100
1.12073	.10000	
.99469	.20000	
.96955	.30000	
.96060	.40000	
.95642	.50000	
.95415	.60000	
.95278	.70000	
.95188	.80000	
.95127	.90000	
.95083	1.00000	