ANALYSIS OF PILE FOUNDATIONS UNDER STATIC AND DYNAMIC LOADS

Ph.D. THESIS

by

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SYNOPSIS

Critical evaluation of the existing literature on dynamic and earthquake forces on pile foundations embedded in soils, reveals the need for further studies on this subject.

This thesis presents the theoretical and experimental studies which were performed, to investigate the behaviour of pile foundations under dynamic loads, particularly earthquakes. The investigations have been carried out in a logical and sequential manner.

The soil-pile (physical) system has been idealised as a lumped-mass-spring system. The vibration characteristics of this model has been determined with the aid of, a transfer solution approach. Suitable numerical techniques and computer programmes, have been evolved for this purpose.

The performance of the mathematical model and the method of analysis have been tested, by predicting the dynamic response of piles embedded in soils in which :

1. Soil modulus can be considered to remain constant with depth.
2. Soil modulus can be considered to vary proportional to depth.

Using these approaches, several pile cases of carefully varied soil-pile parameter values have been analysed.

Analysis of the dynamic response of such piles problems resulted in the development of several non-dimensional design curves of practical value.

Using these non-dimensional design curves it would be possible, to predict, upto significant modes of vibrations:

(i) the natural frequencies of soil-pile systems.

(ii) the normalised model quantities of; deflection, rotation, bending moment and shear along the entire length of the piles.

The non-dimensional curves have been obtained for the following cases of practical significance :

1. Pile top free to rotate conditions.
2. Pile top fixed against rotation conditions.
3. Piles with non-dimensional maximum depth factor, Z_{max} = 1, 2, 3, 5, 10 and 15.

For the case of piles embedded in soils with soil modulus remaining constant with depth, the soil-pile system has also been idealised as a continuous system model. Using the above model, independent solutions and computer programmes have been developed, for evaluating the dynamic response of piles under pile top free to rotate condition. Each of the pile cases analysed with lumped mass solutions, has also been studied with these procedures. Thus the adequacy and correctness of the lumped mass solutions were established.

Also, the dynamic behaviour of piles have been studied through carefully conducted lateral vibration tests on full size prototype piles. These studies provide information on different types of piles embedded in varying soil-types. For in-situ determination of material constants under dynamic conditions, a logical method of interpreting the lateral vibration test results has been given. Certain guide lines for material constant values for use in preliminary designs have been provided.

The validity of the theoretical solutions have been verified by comparing experimentally observed quantities with the predicted ones.

The use of non-dimensional curves have been demonstrated by solving two practical problems. Also, the shortcomings and further applications of the theoretical solutions and models have been discussed in detail.

Based on these theoretical and experimental studies, the dynamic behaviour of the soil-pile system and the factors which influence the dynamic response have been discussed in a detailed manner. Based on these studies, logical conclusions have been drawn and detailed.

Thus for the first time (as of 1974) the dynamic behaviour of the piles have been investigated in a systematic manner to benefit the practising engineer to the maximum. The non-dimensional solutions which are based on logical idealisations and methods of analysis, could be used to obtain solutions, to practically, any type of pile embedded in any soil type. By using these solutions, there is no need to enter the complexities of any dynamic analysis, since, they are backed up by realistic consideration of soil-pile interaction mechanism, the advancements in structural dynamics and above all the available information of soil-pile interaction phenomena.

All the qualifying variables which control the dynamic response of piles embedded in soils have been taken into account.

Thus, the presented work provides a reasonable solution to this complex design problem and offers several advantages over those solutions which exist in the literature.

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Note : The units are expressed in units of force (F), length (L) and time (T).

Symbol		Units
c	Viscous damping coefficient	
d	Diameter of the pile	L
e_{max}	Maximum void ratio	
e_{min}	Minimum void ratio	
f_n	Natural frequency in cycles per sec.: subscripts, if used, denotes the mode numbers; with prime for identifying pile top fixed against rotation condition	
g	Acceleration due to gravity	LT^{-2}
i	Denotes ith station or ith mass point	
j	Denotes jth station or jth mass point	
k	Soil modulus	FL^{-2}
k_x	Soil modulus at any depth, x	FL^{-2}
m	Mass of a segment ; subscripts, if used, identify the mass location	$FL^{-1}T^2$
n	Number of division points	
n_h	Constant of horizontal subgrade reaction	FL^{-3}
p	Circular natural frequency in radians per sec.; subscripts, if used; identify the vibration mode number	

Symbol		Units
r	Denotes rth. station or mass at any division point r.	
t	Indicates time function	
u	Particle velocity	FL^{-1}
v	Velocity	FL^{-1}
v(t)	Displacement variation	L
w	Forcing frequency	
w_n	Circular natural frequency in radians per sec.; subscripts if used denotes the mode number with prime for identifying pile top fixed against rotation condition	
x	Depth co-ordinate ; depth x	L
Δx	Length of a segment	L
y	Total deflection	L
y_b	Deflection due to bending deformation	L
z	Non-dimensional depth factor; depth devided by relative stiffness factor	
A	Area of cross section of the pile	L^2
A_p	Projected area of the pile perpendicular to the stream velocity	L^2
A_w	Projected area of the pile perpendicular to the soil reaction	L^2
B	Width of the pile	L

Symbol		Units
C_D	Coefficient of drag	
C_M	Inertial coefficient	
$C_{11}, C_{21}, C_{31}, C_{41}$	Coefficients dependent on 'p'	
E	Modulus of elasticity of pile material	FL^{-2}
F	Indicates force	
F_o	Maximum dynamic force under vibratory loading condition	F
F_D	Drag force for piles embedded in a fluid	F
F_M	Force due to inertial effects of the moving fluid	F
G	Modulus of rigidity of pile structural material	FL^{-2}
I	Moment of inertia of pile section	L^4
K	Overall-stiffness of the soil-pile system	L
K_1	Identifies the spring attached to the top mass and indicates the spring constant value	FL^{-1}
K_r	Identifies the spring attached to any intermediate station or mass point, 'r', can take values from 2 to n, n being any number. The notation also indicates the spring constant value at the defined mass point	FL^{-1}

Symbol		Units
L	Length of the pile	L
L_s	Embedded length of the pile	L
M	Bending moment at any section	FL
M_t	Mass lumped at top of the pile = $\frac{W}{g}$	$FL^{-1} T^2$
M_{max}	Maximum earthquake induced moments considering root mean square addition of contributions from different modes	FL
$M_{(i)}^{(r)}$	Earthquake induced bending moment of the ith point in the rth. mode	FL
N	Standard penetration test values	•
R	Relative stiffness factor; for the case of piles embedded in soils assuming soil modulus to remain constant with depth. Relative stiffness factor is defined as $\sqrt[4]{\frac{EI}{k}}$	
S_a	Spectral acceleration	LT^{-2}
S_d	Spectral displacement; subscripts if used, identify the S_d values, corresponding to the period of the indicated vibration mode	L
S_v	Spectral velocity	
T	Relative stiffness factor for the case piles embedded in soils assuming soil modulus to vary proportional to depth; $k_x = n_h \cdot x$. 'T' is defined as $\sqrt[5]{\frac{EI}{n_h}}$	L

Symbol		Units
T_1	Time period corresponding to first mode of vibration, pertaining to pile top free to rotate condition. Prime used to identify pile top fixed against rotation condition.	T
T_2	Time period corresponding to second mode of vibration, pertaining to pile top free to rotate condition. Prime used to identify, pile top fixed against rotation condition.	T
T_3	Time period corresponding to third mode of vibration	T
V	Shear force	F
$\dot{S}_{max}$	Maximum earthquake induced shears considering root mean seuare addition of contributions from different modes	F
$S_{(i)}^{(r)}$	Earthquake induced shear of the ith. point in the rth. mode	F
W	Safe load carrying capacity of piles	F
Y	Deflection, co-ordinate perpendicular to pile axis	L
$Y_{(i)}^{(r)}$	Earthquake induced deflection of the ith. point in the rth. mode	L
Y_{max}	Maximum earthquake induced deflection considering root mean square addition of contributions from different modes	L
Z_{max}	Non-dimensional maximum depth factor. Defined as $\frac{L_s}{R}$, for piles embedded in soils assuming constant values of soil modulus with depth. Defined as $\frac{L_s}{T}$ for assuming soil modulus to vary proportional to depth	

Symbol		Units
	List of Non-dimensional coefficients based on lumped mass analysis; applicable to piles embedded in clay type soils, assuming soil modulus to remain constant with depth, $k_x = k$ (constant).	

Note:

(i) Subscripts, if used in a symbol, identifies the vibration mode number

(ii) Symbols without prime have been used to identify - pile top free to rotate condition

(iii) Symbols with prime have been used to identify pile top fixed against rotation conditions

A_{y1}	Non-dimensional normalised modal deflection coefficient corresponding to first mode of vibration	
A_{y2}	Non-dimensional normalised modal deflection coefficient corresponding to second mode of vibration	
A_{y3}	Non-dimensional normalised modal deflection coefficient corresponding to third mode of vibration	
$A_{\theta 1}$	Non-dimensional normalised modal rotation coefficient corresponding to first mode of vibration	
$A_{\theta 2}$	Non-dimensional normalised modal rotation corresponding to second mode of vibration	
$A_{\theta 3}$	Non-dimensional normalised modal rotation coefficient corresponding to third mode of vibration	

Symbol		Units
A_{m1}	Non-dimensional normalised modal bending moment coefficient corresponding to first mode of vibrations	
A_{m3}	Non-dimensional normalised modal bending moment coefficient corresponding to third mode of vibration	
A_{s1}	Non-dimensional normalised modal shear coefficient corresponding to first mode of vibration	
A_{s3}	Non-dimensional normalised modal shear coefficient corresponding to third mode of vibration	
F_{eL1}	Dimensionless frequency factor corresponding to first mode of vibration. Piles embedded in clay type soils. Solutions based on lumped mass analysis.	
F_{cL2}	Dimensionless frequency factor corresponding to second mode of vibration. Piles embedded in clay type soils. Solutions based on lumped mass analysis	
F_{cL3}	Dimensionless frequency factor, corresponding to third mode of vibration. Piles embedded in clay type soils. Solutions based on lumped mass analysis.	
Z	Non-dimensional depth factor equals x/R ; $R = \sqrt[4]{\frac{EI}{k}}$	
Z_{max}	Non-dimensional maximum depth factor equals L_s/R; L_s embedded length of the pile	

Symbol | Units

List of non-dimensional coefficients based on continuous system analysis (model)| applicable to piles embedded in clay type soils, assuming soil modulus to remain constant with depth, $k_x = k$ (constant)

A_{yc1} Non-dimensional normalised modal deflection coefficient corresponding to first mode of vibration

A_{yc2} Non-dimensional normalised modal deflection coefficient corresponding to second mode of vibration

$A_{\theta c1}$ Non-dimensional normalised modal rotation coefficient corresponding to first mode of vibration

$A_{\theta c2}$ Non-dimensional normalised modal rotation coefficient corresponding to second mode of vibration

A_{mc1} Non-dimensional normalised modal bending moment coefficient corresponding to first mode of vibration

A_{mc2} Non-dimensional normalised modal bending moment coefficient corresponding to second mode of vibration

A_{sc1} non-dimensional normalised modal shear coefficient corresponding to first mode of vibration

A_{sc2} Non-dimensional normalised modal shear coefficient corresponding to second mode of vibration

F_{cc1} Dimensionless frequency factor corresponding to first mode of vibration. Piles embedded in clay type soils. Solutions based on Continuous system analysis

Symbol		Units
F_{ce2}	Dimensionless frequency factor corresponding to second mode of vibration. Piles embedded in clay type soils. Solutions based on continuous system analysis	

List of non-dimensional coefficients based on lumped mass analysis; applicable to piles embedded in granular soils, assuming soil modulus to vary proportional to depth, $k_x = n_h \cdot x$.

Note:

(i) Subscripts, if used in a symbol, identifies the vibration mode number

(ii) Symbols without prime have been used to identify pile top free to rotate condition.

(iii) Symbols with prime have been used to identify pile top fixed against rotation condition.

B_{y1}	Non-dimensional normalised modal deflection coefficient corresponding to first mode of vibration	
B_{y2}	Non-dimensional normalised modal deflection coefficient corresponding to second mode of vibration	
$B_{\theta 1}$	Non-dimensional normalised modal rotation coefficient corresponding to first mode of vibration	
$B_{\theta 2}$	Non-dimensional normalised modal rotation coefficient corresponding to second mode of vibration	
B_{m1}	Non-dimensional normalised modal bending moment coefficient corresponding to first mode of vibration	

Symbol		Units
B_{m2}	Non-dimensional normalised modal bending moment coefficient corresponding to second mode of vibration	
B_{s1}	Non-dimensional normalised modal shear coefficient corresponding to first mode of vibration	
B_{s2}	Non-dimensional normalised modal shear coefficient corresponding to second mode of vibration	
F_{SL1}	Dimensionless frequency factor corresponding to first mode of vibration. Piles embedded in granular soils. Solutions based on lumped mass analysis.	
F_{SL2}	Dimensionless frequency factor corresponding to second mode of vibration. Piles embedded in granular type soils. Solutions based on lumped mass analysis	
z	Non-dimensional depth factor defined as x/T, where $T = \sqrt[5]{\frac{EI}{n_h}}$	
Z_{max}	Non-dimensional maximum depth factor defined as $\frac{L_s}{T}$	
δ	Logarithmic decrement	
γ	Weight density of the pile structural material	FL^{-3}
γ_d	Dry density of the soil	FL^{-3}
ξ	Damping coefficient or damping ratio	
ρ	Mass density of the pile structural material	$FL^{-4}T^2$

Symbol		Units
σ	Shape factor	
θ	Rotation at any point	
$\theta_{(i)}^{(r)}$	Earthquake induced rotation of the ith. point in the rth. mode	
θ_{max}	Maximum earthquake induced rotation, considering root mean squre addition of contribution from different modes	
$\Phi(M_{(r)})$	Normalised modal bending moment in the rth. mode, without prime identifies pile top free to rotate condition; with prime identifies pile top fixed against rotation condition	
$\Phi(S_{(r)})$	Normalised shear in the rth mode; without prime identifies pile top free to rotate condition; with prime identifies pile top fixed against rotation condition	
$\Phi(y_{(r)})$	Normalised modal deflection in the rth. mode. Without prime identifies pile top free to rotate condition. With prime identifies pile top fixed against rotation condition	
$\Phi(\theta_{(r)})$	Normalised modal rotation in the rth mode; without prime identifies pile top free to rotate condition; with prime identifies pile top fixed against rotation condition	
$\Upsilon_{(r)}$	Mode participation factor in the rth. mode	

NOTE

		Unit
		FL
$M^{(r)}_{(i)}$	$= \Phi \, (M_{(r)}) \quad S_{d(r)}$	F
$S^{(r)}_{(i)}$	$= \Phi \, (S_{(r)}) \quad S_{d(r)}$	L
$Y^{(r)}_{(i)}$	$= \Phi \, (Y_{(r)}) \quad S_{d(r)}$	-
$\theta^{(r)}_{(i)}$	$= \Phi \, (\theta_{(r)}) \quad S_{d(r)}$	•

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CHAPTER - I

INTRODUCTION

1.1 GENERAL

There is increased awareness to-day (1974), amongst engineering profession, of the importance of foundation support conditions in controlling the behaviour of structures during earthquakes. Analysis of damages to engineering structures in the past earthquakes such as Bihar-Nepal (1934), Mexico (1958), Niigata (1964) and Alaska (1964) have, clearly demonstrated that, for safe as well as economical earthquake resistant designs, understanding of the behaviour of foundation-soil system as a primary consideration. Hence it is obvious that the extent of damage to the structures are dependent predominantly on the type of foundations and soil conditions.

In this context, pile foundation is one of the common type of foundation adopted in unfavourable soil conditions. Behaviour of such foundations under dynamic loads has not been fully studied and offers considerable scope for investigations. Considering increasing construction activities in seismic regions there is an emphasised need to understand the behaviour of pile foundations under dynamic loads through experimental and theoretical investigations.

During earthquake excitations an element of soil-pile system in the ground is subjected to a complex system of stresses resulting from the erratic sequence of ground motion. In many earthquakes the major part of the dynamic stresses and deformations may be attributed to the upward propagation of shear waves from the underlying soil layers. These dynamic stresses are time-dependent and change in magnitude and reverse in direction many times during an earthquake.

Also, these dynamic stresses or loads caused by the earthquakes may vary along the embedded length of the pile. This would result in varying deformations, slopes, bending moments and shear forces with depth of embedment. The magnitude of these stresses, and deformations are controlled mainly by the interaction effects of soil with the pile.

1.2 CURRENT PRACTICES

Currently, aseismic designs of piles are performed at best by determining the natural frequency of piles to avoid quasi-resonance and by evaluating the induced stresses through pseudo-static analysis. The latter part would result in an easy and interesting solution. However, in order to achieve this effectively, at first there should be a clear knowledge of the effect of earthquakes on pile foundations. Unfortunately a solution to this problem is not available as yet (1974). Hence, the equivalent static loads are in general taken arbitrarily as some percentage of the sustained vertical loads.

Alternatively, the total lateral load applied to the foundation is taken to be the base shear computed in the dynamic analysis of super-structure considering its fixity with the base.

Determination of the natural frequencies of soil-pile system was attempted by Hayashi et al (1965), Prakash and Sharma (1969) and Gray (1964) through so called "equivalent cantilever" methods. Herein, the pile-soil system is idealised as a massless equivalent cantilever with a single concentrated mass at the top. The natural frequencies of the cantilever is determined using Raleigh's theory. Different approaches have been adopted to determine the equivalent cantilever lengths. In general, they are taken to be the distance from the top of the pile to the first point of zero deflection when the pile is analysed as a beam on elastic foundation subjected to a horizontal static load. The frequency of the idealised system so determined is checked for resonance against the exciting frequency.

The current practice, based upon above concepts is more or less arbitrary and there is a strong case for scientific investigation of the problem, which has been attempted in this thesis.

1.3 REQUIREMENTS OF SOLUTIONS

Unfortunately there are no recorded data of measurements on pile foundations during earthquakes nor have exhaustive vibration tests on piles been conducted. However, Fukuoka (1966) has provided a very interesting information regarding the deformed shapes of steel piles subjected to Niigata (1964) earthquake. The steel piles were of 60 cm diameter with a wall thickness of 9 to 6 mm. The piles were supporting Showa bridge, the profile of which is given in Fig 1.1. The pile of pier no. P4 was taken out after the earthquake. Fig 1.2 illustrates the deformed shape of the pile, together with the soil conditions near the site. The deformation as it can be seen is of a bending type. The above information is perhaps the only recorded information on the mode of deformation of piles during earthquakes.

Therefore considering such action of earthquakes, any logical solution of the problem, must include along the entire length of the pile determination of:

(i) deformations namely deflections and rotations.

(ii) induced bending moment and shear forces.

In other words it is necessary to determine the total response of pile subjected to earthquake excitation. Now, as the frequency of vibration during an earthquake is a varying phenomenon it is also necessary to determine the above quantities for different modes of vibration. Further, in order to avoid resonance or quasi reasonance conditions, it is also necessary to determine the natural frequencies of the soil-pile system under these modes of vibration.

The response of the pile during earthquakes depends upon the nature and type of soil, and the restraints offered by the super structure at the pile top. Also, the load carried by the pile, the pile end conditions as well as the nature of the ground motion seem to control the behaviour. Critical evaluation of the existing literature on the subject emphasise the importance of the following aspects

1. Choice of a proper mathematical model to idealise the real system.
2. To develop an easy but sufficiently accurate method of analysis for determining the pile response so that they can be used conveniently by practising engineers.

In his critical review on dynamic and earthquake forces on deep foundations Nair (1968) has also clearly emphasised the need for such an approach.

For the problem of present nature there seems to be need for dynamic analysis repeatedly, since piles are one of the common type of foundations in use. Therefore it would be advantageous to develop non-dimensional solutions based on the determined response of wide varieties of pile foundations of different characteristics. This, if achieved would enable the designer to determine the response of the pile without having required to perform dynamic analysis at every instant. Considering the complex nature of the dynamic problem formidable computational effort may be needed for such an approach. Herein, for such an attempt digital computers have been used for ease in computational efforts.

1.4 SCOPE OF STUDY :

Broad outline of the investigation carried out on the dynamic behaviour of piles is given below. Detailed discussions on various aspects would appear in the relevant chapters.

(1) A lumped mass mathematical model for idealising the mass distribution of the pile has been chosen. The masses are connected by elastic weightless bars possessing the same elastic properties as that of the pile section. The interaction effects of the soil are considered by treating the soil as independent closely spaced elastic springs connected at mass points. In reality the soil idealisation is a Winkler model. Evaluation of spring characteristics is considered through concept of modulus of subgrade reaction theory. The vibration characteristics of the pile were then determined generating transfer solutions. For executing the analysis computer programmes were prepared.

(2) The workability of the idealisation and method of analysis in determining the response of the piles have been

tested for:

(i) piles embedded in soils assuming soil modulus to remain constant with depth (typical of pre-loaded clays)

(ii) piles embedded in soils assuming soil modulus to vary proportional to depth (typical of normally consolidated clays and granular soils).

By, considering the above two forms of variation of soil modulus with depth, practically, information of response of piles embedded in almost any type of soil can be obtained.

(3) With the technique exhaustive parametric studies have been carried out to analyse the vibration characteristics of wide variaties of piles in the significant modes of vibration. Piles of different sectional properties embedded to different lengths in different soil types have been studied. The effect of variations in sustained loads has also been investigated.

(4) The above parametric studies have been carried out for pile top free to rotate conditions and pile top fixed against rotation conditions.

(5) The effect of soil strength has been considered by way of carefully varied values of soil modulus in both the above types of variations.

(6) For the case of piles embedded in soils assuming soil modulus to remain constant with depth, the soil-pile system has been idealised as a continuous system model. With the above model, independent solutions for pile top free to rotate conditions have been developed. Each of the pile cases analysed with the lumped mass solutions have been studied here also. This enabled in assessing the adequacy of lumped mass models as well as resulted in a better understanding of the pile behaviour.

(7) Detailed experimental investigations on full scale field piles embeded in clay and sand have been executed. A method has been proposed for determining the material constants of soil-pile system as required in any dynamic analysis.

(8) A review of all pertinent literature in the field of pile foundations subjected to earthquakes and other dynamic loads is also given.

1.5 CONCLUDING REMARKS

The direct result of the studies reported in the thesis is a better understanding of the dynamic characteristics of soil-pile system.

More importantly, for the first time (as of 1974), easy and practical solutions to this complicated problem, have been provided by way of non-dimensional design curves for predicting the dynamic characteristics of soil-pile systems.

These non-dimensional solutions have been possible because of:

1. the adopted logical and realistic mathematical idealisation of the soil-pile systems.

2. the performed analysis and the adopted numerical techniques.

3. the formidable computation effort for enormous pile cases with the help of digital computers.

With the presented solutions practically any type of pile embedded in any soil could be analysed, without going into the complexities of dynamic analysis.

Also, a procedure for determining the material constants of soil-pile system in-situ have been provided.

In the reported study almost all the qualifying variables of practical significance, which control the dynamic response have been taken into account.

CHAPTER - II

REVIEW OF LITERATURE

2.1 INTRODUCTION

2.1.1 GENERAL

Especially in poor soil conditions pile foundations are used to transfer effectively the loads of engineering structures to the surrounding soil. In general pile foundations are subjected to vertical loads, lateral loads and moments. Sufficient information is available to understand and estimate the behaviour of piles subjected to sustained vertical loads, (Mayerhoff (1951), Terzaghi and Peck (1967) and Vesic (1968)).

2.1.2 INFORMATION ON SUSTAINED LATERAL LOADS

While examining the effects of earthquakes and other types of dynamic loads on the soil pile system, it would be advantageous to know the available information on the subject of pile foundations subjected to static lateral loads. Though, exhaustive information is available in this regard, herein, certain salient aspects would be examined. Informative review in this direction has been detailed by Davisson (1960), Prakash (1962) and Srivastava (1970).

When a pile is subjected to lateral loads there is reaction offered by the surrounding soil, commonly termed as soil reaction. The soil reaction resisting the lateral load is a function of deformations and influences the stressess developed in the pile section. The detrimental effects of the lateral load may not necessarily mean the failure of the soil but may be dependent on the developed bending and shear stresses along the pile length.

Different catagories of solutions are available for analysing the piles subjected to lateral loads.

The procedure adopted by Davisson and Robinson (1964), Kosics (1968) is to assume the pile to be fixed at some point below the ground line. The point of fixity being dependent on the type of soil and are computed based on the theory of elasticity. The soil above the point of fixity is completely ignored and the piles are treated as pure structural members. Different fixity conditions at the top are considered and non-dimensional curves for deflection, slope, moment and shear have been presented. Obviously these methods adopt unrealistic characteristics of the pile-soil system.

Certain theories are developed based on the ultimate resistance offered by the soil. This ultimate resistance is presumed to act against the pile. Investigations in these directions based on the pile behaviour have been presented by Prakash (1960), Broms (1964 and 1965). These solutions assume shear failure in the soil in the case of stiff rigid piles; bending mode in the case of long piles treated as flexible ones. Certain equations have been proposed to check the ultimate resistance of the soil and the developed values of maximum bending moment. With these methods it is not possible to determine deflections along the length of the pile. Further, it is presumed that the soil modulus remains constant with depth. Though the main contention of the method is to determine the ultimate soil resistance, it is not possible to know whether they are fully mobilised or not.

Yet another catagory of solutions are based on the beam on elastic foundation theory proposed by Biot (1937) and further developed by Hetenyi (1946). In these methods the pile is treated as a beam resting on an elastic bed. They essentially involved Winkler's assumption that the soil can be replaced by independant closely spaced elastic springs. The use of such characterisation results in the definition of soil modulus while describing the reaction offered by the soil, for envisaged deflections of the pile under the lateral load. With such assumption equations for slope, deflection, bending moment and shear are easily developed. Non-Dimensional solution for the above quantities have been proposed by Reese and Matlock (1956) and Davission and Gill (1963) for linear variation and constant values of soil stiffness with depth. The above solutions have been used very widely by the practising engineer and has the advantage of both simplicity and accuracy. But herein, non-linear effects are not taken into account. Detailed discussion on the methods based on beam on elastic foundation concept has been presented by Davisson (1960), Prakash (1962) and Chandrasekaran (1967).

3.1.3 PILE FOUNDATION SUBJECTED TO DYNAMIC LOADS

Apart from the static loads pile foundations may be subjected cyclic and dynamic loads. These loads may act in addition to the sustained loads which are imposed on them.

Compared to the subject of pile foundation subjected to sustained lateral load lesser information is available concerning dynamic loads. The discussion herein, is classified in the following heads:

1. Nature of dynamic loads and their estimation.
2. Available procedures for considering the effects of such loads.
3. Experimental investigations on the study of pile foundations under dynamic loads.

2.2 NATURE OF DYNAMIC LOADS

Non availability of coherent information on the different types of dynamic loads that could act on pile foundation has been a concern of the engineer. Attempt is made to discuss the different types of dynamic loads and the literature available for estimating them.

The nature of dynamic loads depends upon the problem under consideration. Nair (1968) has made broad classifications of the various types of dynamic loads which could act on pile foundations.

The type of loads are grouped as follows:

1. The loads applied directly to the piles as in the case of piles supporting machines and off-shore structures. The loads introduced during pile driving also fall in this catagory.

2. The dynamic loads introduced due to earthquake and blast occurances.

2.2.1 MACHINE LOAD

Depending upon the characteristics of the machines, foundations supporting them may be subjected to periodic unbalanced forces along any of the co-ordinate axis. These may be either coupled or uncoupled motions. These periodic dynamic forces may be created by virtue of unbalcnced rotating parts.

In the case of rotating and reciprocating machines steady state vibrations are created. In the case of Forge hammer type of machines impact loads are applied. Normally, manufacturers of the machines provide data on these unbalanced forces. Denhortog (1950), Newcomb (1951), Barkan (1962), Major (1962) have discussed at great length the principles involved in calculation of the dynamic loads for various types of machines. The force levels and their frequencies are well defined in each case so that the foundation designer could check the performance of foundation against these forces. In the case of hammer foundation it would be necessary to base the data on actual measurements. Considering the huge varieties of machines in use the details are not discussed herein.

2.2.2 WAVE FORCES

In the case of pile foundations supporting off-shore structures dynamic loads are applied directly by virtue of the wave forces. Recent development in the off-shore technology permit the determination of these forces using established hydro-dynamic principles and empirical techniques

Two methods are available for determining the forces exerted by waves on piles. The method proposed by Morison et al (1954) presumes that two types of forces can be introduced by wave action. One due to inertial effects of the mass of fluid participating in wave action, and the other created due to drag effects. The drag forces are dependent on the viscosity of the fluid and roughness of the pile surface.

The total forces are obtained by superposition of the drag and inertia forces.

Considering linear wave theory the expression for drag forces is given as under:

$$F_D = \frac{1}{2} C_D \; A \; u \; u \qquad \ldots 2.1$$

F_D = drag force

A_p = projected area perpendicular to stream velocity u

C_D = coefficient of drag

ρ_f = mass density of fluid.

The inertia force is considered proportional to fluid density, the volume of the object and the particle acceleration. The expression for inertia force given by Morison et al (1954) is

$$F_M = (M_o + M_s) . \frac{\partial u}{\partial t} \qquad \ldots 2.2$$

M_o = mass of displaced fluid

M_s = added mass dependant on shape and flow characteristics.

The second method proposed by Crooks (1955) is based on the study by Inversson and Balent (1948) of the force exerted on a moving body through a fluid. Linear relation between velocity and acceleration is presumed.

$$dF = C_M \; V \frac{\partial u}{\partial t} + \frac{1}{2} C_D \; A \; u \; u \quad dz \qquad \ldots 2.3$$

Herein, the force is a product of one coefficient, projected area of the body and square of particle velocity.

In the above formulae uncertainty exists regarding the values of C_D and C_M.

Several field and laboratory investigations have been carried out by Morison (1951), Morison et al (1954) Weigel et al (1965) to assess the correct values of C_D and C_M, the drag and inertial coefficients. Of particular importance herein, is the work carried out by Weigel et al (1965).

Pile sections of four different dia.of 6.625", 11.75", 2'-0 and 4'-0 were tested by them in a wave trough. Precise electronic instruments were used for determining forces and strain.

Based on such studies the commonly suggested values of C_M and C_D lie between 1.5 and 2.5 and 0.8 and 1.5 respectively.

Apart from selecting the coefficients of C_M and C_D, the wave velocity and acceleration should also be assessed.

The expressions for total horizontal force on pile as developed by Morison et al (1954) is given in equation 2.4. The various components of the equation have been illustrated in Fig 2.1. We have in these equations:

horizontal component of water velocity, u given by:

$$u = \frac{g\,HT}{2L} \quad \frac{\text{Cosh}\, 2\pi\,(y+d)/L}{\text{Cosh}\, 2\pi\, d/L} \cdot \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right)$$

Vertical component of water velocity, V ... 2.4

$$V = \frac{\pi H}{T} \quad \frac{\text{Sinh}\, 2\pi\,(y+a)/L}{\text{Sin h}\, 2\pi\, a/L} \cdot \text{Sin}\, 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) \quad ...2.5$$

Fig 2.1 illustrates the various quantities appearing in the force and velocity equations. The above procedures suffer from the following drawbacks:

1. The expressions are derived presuming steady state rectilinear flow. But wave action on a submerged body may fall under turbulant category.
2. The flow may not be unidirectional as assumed.
3. The drag and inertia forces may act out of phase and linear addition may not be valied.

While utilising these formulae for determining the wave forces it should be borne in mind that the wave action is essentially a statistical problem. Therefore ultimate force determination must combine the use of formulae, statistical determination to-gether with engineering judgement.

2.2.3 LOADS APPLIED DURING PILE DRIVING

Enormous amount of literature is available discussing energy applied to the pile during driving. Majority of these works concern the determination of pile load capacity using dynamic formulae and wave equations. However, assessment of the acceleration time and force-time variations while determining the energy imparted to the pile during driving may be required for testing the performance of the driven pile sections.

The normal practice of pile driving is through the force applied by way of impact of a hammer. Usually pile driving hammers are rated according to equivalent potential energy that is available at the begining of their stroke. The energy imparted to the pile is normally equated to this potential energy. They are taken to be impulse loading meaning thereby to act for infinitesimal length of time.

But the force time measurements taken on piles during driving indicate the the force acts for sufficient length of time. Fig 2.2a shows a typical force time plot as obtained by Davisson and M.C. Donald (1965) for diesel hammer driven pile. Fig 2.2b gives a typical force-deflection plot. The area under this curve gives the net and gross energy.

Normally the energy imparted to the pile during hammer driving is considered as per the of weight of the hammer and height of fall or as per the manufacturers energy rating. This may not result in actual estimation of the load quantitie because many types of losses are incurred during the process of driving. In general it can be considered that at impact steam hammers generate 80 to 85 % of the rated energy. Whereas in the case of diesel hammers transmitted energy could be in the order of 100 percent of rated energy at combustion impact event. The energy transmitted to the pile would be 73 percent of the manufacturers rated energy for near refusal conditions. J.J. Tomko (1968) and Davisson and M.C. Donald (1968) have provided detailed discussion on the subject.

Consequent to the available information it is felt that stock of information on force and acceleration time measurements of different pile driving hammers, while driving different piles in variety of soil conditions need be collected. Depending upon the encountered situation such appropriate records may used to check the detrimental effects on pile section.

In recent years vibratory pile driving has become a common practice. Under such cases piles would be required to resist the steady state dynamic loads. Accurate estimation of the force-time relationships could be obtained from the vibrator specifications. The force equations are usually of the form

$$F(t) = F_o \text{ Sin } wt \qquad \dots 2.6$$

2.2.4 EARTHQUAKE LOADS

Earthquakes introduce the most hazardous type of dynamic loads on an engineering structure. The shaking of the surface of the ground during an earthquake is produced by the passage of seismic stress from the underlying rock strata due to release of stored strain energy. Detailed discussion on the mechanism of earthquake occurence, their measurements and classifications are available elsewhere (Richter (1958); Allen et al (1965); Ryal et al (1966); Hudson (1963); Halverson (1965)).

The basic data of earthquake engineering are the recording of ground acceleration-time variations (Hudson (1963), Halverson 1965)), a typical such record of the NS component of El centro (1940) earthquake is shown in Fig 2.3. The intensity and strong phase of the shaking is characterised by the size and shape of the pulses and the number of pulses. These informations have a special significance in deciding the detrimental effects of earthquakes on the structures. It would be ideal, that for seismic regions of different countries such records are accumulated. So that the design earthquake would involve selection of one such records considering the propriety and ground conditions of the relevant site. Though, since recent times, many countries maintain a net work of seismographs, unfortunately rich supply of recorded ground accelerations of destructive earthquakes are not available.

However, enough data is available on the earthquakes to indicate the magnitude, the distance from fault and the maximum acceleration of past earthquakes. Under these circumstances the selection of design earthquakes should be based on the accumulated data of these types.

Herein, the magnitude is defined as

$$M = \log_{10} A_w/A_o \qquad \ldots 2.7$$

where M is the magnitude of earthquake, A_w is the maximum amplitude recorded with a Wood Anderson seismograph at a distance of 100 km. from the center of disturbance and A_o is the amplitude of 1/100 th of a millimetre.

The magnitude of the earthquake depends upon the length of the slippage fault, epicentral and focal depths.

While deciding the size of the earthquake in a general area the frequency of occurrence of the strong motion earthquakes must be based on the seismicity and tectonic conditions.

Also while deciding the form of motion the soil condition at and near the site must be carefully considered. At many instances the influence of soil conditions on the ground motion is totally ignored. It has been observed that in a same seismic region within a distance of 10 km the recorded ground motion at surface might vary by 200 percent. Methods are available Idriss and Seed (1969) for considering the influence of soil condition on ground motion.

Thus the actual earthquake design criteria must be based on the following considerations:

1. The probability of occurrence of strong motion.
2. The nature of deformation to structures during earthquakes.
3. Seismicity geology and tectonic activity of the region.
4. Soil conditions at a site.
5. Past records of earthquakes.

2.3 METHOD OF ANALYSIS

2.3.1 ANALYSIS FOR MACHINE LOADS

Statifactory performance of foundations of high and low speed machines requires their natural frequency to be at least twice that of the operating frequency of the machines. When the foundation is to rest on a very soft soil the natural frequency of the foundation soil system could be increased to a certain extent by reducing the weight or by increasing the stiffness of the soil by chemical injection and compaction. However, under all circumstances this may not be possible.

Under these conditions often piles are used to provide the required stiffness increase.

The main design requirement of pile response subjected to steady state machine loads is the determination of resonant frequencies and amplitude of vibrations. In order to achieve this piles are normally idealised as a single mass spring dashpot systems. The amplitude of vibration and resonant frequencies are obtained using the procedures postulated by Barkan (1962), Richart and Woods (1970), for shallow foundations.

Such an idealisation has been utilised by Maxwel et al (1968) for predicting the pile response under vertical vibration condition, Fig 2.4.

The results have been compared favourably with forced vibration tests conducted in field.

The main features of the model are a lumped mass m_o, constrained to move in a vertical direction x_1 and subjected to sinusoidally varying load Q_1. The motion is resisted by linear spring and dashpot in parallel. The phase angle Φ relates the phase lag between peak force Q_1 and displacement amplitude, x.

The amplitude and resonant frequency equations for such a model are well known.

Though the method of analysis is simple, difficulty lies in the selection of stiffness and damping properties of the soil pile system. For this the investigators suggest forced vibration test on actual piles.

Further they suggest the use of static stiffness values for predicting the natural frequency within 20% limits.

Extending the solution of Timoshenko (1955) based on theory of elasticity Richart (1962) has asupplied reference Fig 2.5, to estimate the influence of length and load on the natural frequency of piles subjected to longitudinal vibrations.

For the analysis of piles subjected to machine loads under lateral direction, no established procedures are available. However many practical relationships are in vogue and they have been utilised successfully. The success of these approximate formulae are the result of safe performance of the past foundation designed using these formulae. But no analytical proof could be put forward to underline their validity.

Irish and Walker (1969) have provided appriximate formulae and design charts for estimating the natural frequencies of piles for use in preliminary designs. These informations can be used to predict the natural frequencies under vertical and lateral direction.

Using Fig 2.6 the natural frequencies of vibration under vertical mode can be obtained. This chart considers piles of different lengths and material properties. Using Fig 2.7 the natural frequencies of the piles under lateral vibration conditions can be predicted. Here also the material properties and the length of the piles have been varied. For the purpose of compiling these charts the dynamic modulus of elasticity, E for each material has been assumed to be:

Steel : 30,000,000 lb per sq. in (2,100,000 kg. per sq. cm.)
Concrete: 3,000,000 lb. per sq. in (210,000 kg. per sq. cm.)
Timber : 1,200,000 lb. per sq. in (84,000 kg. per sq. cm.)

In Fig 2.6 ,

$$\text{Stress} = \frac{W}{a.n}$$

$$= \frac{\text{Weight of foundation and machine (lb or kg)}}{\text{(Cross sectional area of one pile (sq. in. sq cm)) x (number of piles)}}$$

In Fig 2.7, for fixed ends $k_1 = \frac{W}{12\ I_p n}$

For pinned ends $k_1 = \frac{W}{3\ I_p n}$

Where, I_p = Second moment of area of one pile (in^4 or cm^4)

k_1 = Coefficient (lb per in^4 or kg. per cm^4)

2.3.2 ANALYSIS FOR LOADS DURING PILE DRIVING

The reported literature on the analysis of piles during driving concentrates mainly on the prediction of pile bearing capacities. For this, Newtons laws of motion form the basis and resistance is equated to the energy ratings accounting for losses during driving.

Equations developed on the above mentioned principles are catagorieed as dynamic formulae and there are a number of them available in the published work.

However little work is reported for predicting the pile response during driving. The analysis proposed by Issac (1931), Fox (1938) formed a basis for utilising the longitudinal impact and wave theory for analysing the response of pile during driving.

But Smith (1962) was the first to present a numerical solution to the wave equation by dividing the pile into discrete elements. Pile is divided into a spring mass system and the ram and dolly has been idealised as separate units attached to the pile systems. The side and the base resistance has also been accounted for, Fig 2.8. The basic one dimensional wave equation (Smith (1962)) has been used to predict the acceleration response and ultimate resistence of the pile.

Finite difference solution has been adopted for easiness in computation. Lapay (1966) has tried experimenta verification of Smith's analysis and obtained poor correlati between observed and predicted quantities. The uncertaintie involved in the side and base restraint idealization and consideration of one dimensional solutions could be the reason for poor correlations.

One of the exhaustive experimental and theoretical works on piles during driving has been reported by Tomko (1968). Two types of theoretical solutions have been proposed.

The first method assumes the pile to be a rigid body and Newton's second law and Pencelots resistance rule have been applied.

In the second model elastic solution are assumed.

Again the basic one dimensional wave equations are used but Laplace solution are utilised for solving the equations.

2.4 AVAILABLE SOLUTIONS FOR CONSIDERING EARTHQUAKES LOADS

Evaluation of the available solutions for predicting the effects of earthquakes on pile foundations is of greater concern to the present study hence, herein, more emphasis is laid on this aspect.

2.4.1 PSEUDO_STATIC SOLUTIONS

For considering the effects of the earthquakes mainly stability of the piles to withstand the lateral loads are investigated. The most widely used procedure to achieve this has been to replace the action of earthquakes by an equivalent static lateral load. The equivalent static loads are taken as a percentage of vertical load. Sometimes, they are considered to be a product of the seismic coefficient and the weight of the vibrating structure. The seismic coefficient values are fixed in different arbitrary ways in different countries. Normally the, seismic regions of the countries are divided into different zones and the assigned values of the seismic coefficients are based on the past earthquake records and seismicity of the areas.

Alternatively, certain times the total lateral load is taken to be the base shear, which can be computed from the dynamic analysis of the super structure. Herein, it is assumed that the structure is rigidly fixed to the foundatio

Once the value of these pseudo static loads are evaluated they are presumed to act in addition to the existi sustained loads. Thereafter any of the static methods of analysis such as Reese and Matlock (1956), Davisson and Gill (1963) are used to determine the displacements and stresses under these combined loads. If these quantities are found to be within safe limits the piles are considered to withstand the earthquake effects.

Obviously such procedures have no rational basis. They fail to consider the dynamic nature of the problem. Such equivalent techniques are possible in a logical sense only if the total solution regarding the dynamic response of the soil-pile system subjected to earthquake loads are well understood. Adoption of such procedures would at best give false sense of security to the designer.

2.4.2 EQUIVALENT CANTILEVER METHODS

The next catagory of solutions for the analysis of piles contend in predicting the natural frequency of the piles-soil system in addition to checking the stresses with pseudo-static methods.

Considering the complex nature of the problem very simple structural idealisations are resorted to. The most commonly used structural idealisation is the massless equivalent cantilever with or without a concentrated mass, at the free end, Fig 2.9. The length of the cantilever is normally taken as the distance from the top of the pile to the first point of zero contraflexure. The contraflexure points are determined using static methods of analysis for the applied lateral load at the ground surface.

Prakash and Sharma (1969) determine the equivalent cantilever length by equating deflections at the free end for a beam on elastic foundation and a cantilever under a static load.

Certain deviations in these procedures treat, instead the cantilever to be continuous structure having distributed mass.

Another model similar to the cantilever for determining the response of single pile subjected to dynamic load has been proposed by Hayashi et al (1965). This is a noval analytical solution for predicting the natural frequency of piles. The actual soil-pile system has been analysed as a pendulum model. The length of the pendulum is determined as in the previous case. The idealised model is shown in Fig 2.9.

As the pile-soil vibration is considered to possess non-linear effects, they are accounted for with the help of spring and dash-pot system attached to the top. The mass distribtion of the pendulum is similar to that of prototype pile.

The equation of motion of the model is written as

$$\Phi \frac{d^2y}{dt^2} + F\left(y, \frac{dy}{dt}, Y\right) = G(t)$$

$\Phi = \frac{I_o}{H_o^2}$ where I_o is the moment of inertia about the hinged end.

y = displacement of top of pile

Y = maximum displacement at top of pile.

It is assumed that

$$F\left(Y, \frac{dy}{dt}, Y\right) = F(y, Y) + c\frac{dy}{dt}$$

$$\frac{I_o}{L_e^2} \frac{d^2y}{dt^2} + c \frac{dy}{dt} + F(y, Y) = G(t)$$

The restoring force displacement relationship is obtained from the alternating load test result.

Neglecting the damping effects the resonant frequencies are obtained by solving the equation and are found to have good agreement with the observed test results.

Ishi, and Fujita (1965) have attempted certain simplified procedures for determining the natural frequencies.

One of their models treat the pile as an inverted single degree freedom oscillator with an attached spring and dash-pot. The natural frequency of this model can be easily determined.

In their other model they consider the pile as a lumped mass spring system. The bottom conditions are assumed as fixed. Ishi and Fujita (1965) in their paper argue that for a distributed system as that of the pile it would require infinite number of mass idealisation. This would require infinite number of equations to define the equilibrium of the complete structure. In order to reduce the analysis to practical proportions the authors reduce the structure to four limited masses and springs attached to the bottom of the two masses.

The spring constant are determined from the cyclic load tests and damping from forced vibration tests.

2.4.3 MISCELLANEOUS SOLUTIONS

Saul (1968) has proposed an approximate method for finding out the natural frequency of pile groups. Resolving the forces of piles on the pile cap along three of the co-ordinate axis equilibrium equation in terms of these forces and inertia forces are developed. Equating the forces along one of the co-ordinate axis to the inertia force in that direction equilibrium equation are derived

$$Q_i = m \Delta_i$$

m = mass of the vibrating system

Δ = acceleration

Assuming harmonic motion simple expressions for natural frequency is given by the Saul. In reality herein, the pile group is reduced to cantilever of defined length similar to the approach proposed by Davisson and Robinson (1965). Though it is argued that mass of the soil vibrating along with the pile must be taken into account, no guide line as such is given for this purpose.

The prevelent practice of analysis of piles subjected earthquake loads, seems to be, assessment of the natural frequencies idealising the soil pile systems to any of the above mentioned simplified structural systems. Once the natural frequencies are determined they are compared with the exciting frequencies to check for resonance conditions. Though these procedures are the starting points in the actual requirement of the design the validity of these procedures are not fully established.

2.4.4 DYNAMIC ANALYSIS

In the literature unfortunately, very few methods are available for predicting the pile behaviour based on logical dynamic analysis of realistic idealisation.

The method proposed by Penzien at al (1964), follows this approach. A lumped mass-spring dash-pot model has been chosen to idealise the soil-pile system. The response of this model for one component of earthquake acceleration applied to bed rock has been determined, based on advanced structural dynamics principles.

The analysis consists of two parts:

(i) to determine the dynamic response of the clay medium without the super-structure being present. Since the deformations produced in this medium by a horizontal excitation are essentially pure shear. The real system is idealised as shown in Fig 2.10. The response of the column of soil having unit cross sectional area and of constant depth equivalent to the depth of the soil-layer is determined.

The linkage connecting the adjacent masses consists of bilinear hysteritic springs and non-linear dash-pot connected in parallel. By the analysis of clay medium response the acceleration variation with time at various elevations of the clay medium are easily computed.

(ii) In the second part the soil-pile response are determined. The idealisation and assumptions involved in determining the response of pile-soil system is done in three steps concerning (1) Structural (2) Soil and (3) Interaction effects. The physical model chosen by Penzien is given in Fig 2.11.

The basic assumption in the structural idealisation is that the structural members display linear elastic behaviour.

The mass of the bridge super structure and piles are concentrated at various elevations and the elastic properties of the system are obtained by standard structura methods which involve necessary stiffness and flexibility matrices.

The interaction effects of the soil are taken care by idealising the soil as spring dash-pot system. The springs are connected to the descretised pile structure as a simple couple system. The springs are presumed to have bilinear force displacement characteristics. The dash-pots includes non-linear time dependant quantities representing the damping characteristics of the surrounding clayey soil. Apart from this the creep effects of the clay medium is also accommodated in the analysis.

The spring constants at various elevations are determined using the Midlin's elastic half space theory with simplyfying assumptions. Considering certain characteristics of the elastic half space, the interaction effects of the piles and clay medium is approximated to method similar to classical beam on elastic foundation theory.

The stiffness properties of the soil under such approximations are determined based on cyclic static loading tests on clay samples. Based on such cyclic tests bilinear hysteritic representation of the non-linearity are accounted for.

For determining the damping characteristics of the soil certain approximate dynamic tests are carried out.

In these tests the acceleration decay phenomenon under sudden impact loading is recorded. With the help of such a curve the damping characteristics are determined.

The analysis of group of piles is no way different from that of single piles except the stiffness values are reduced depending on the pile spacing. The reduced stiffness of individual piles are summed up depending upon number of piles in a row and the analysis is carried out as for a single pile.

The analysis proposed by Penzien et al takes full advantage of the advancements made in the field of structural dynamics.

But it is not in general use by the practising engineers. This is mainly due to the complexities involved in the analysis and the tedious iterative procedure which are required for obtaining the solutions.

The performance of the analysis mainly depends upon the characterisation of soil interaction effects. The procedure adopted by Penzien utilising the earlier mentioned Mindlin's technique involves complicated mathematical solutions. But finally utilising Mindlins solutions the interaction effects of the soil are accounted for in a manner similar to Winkler idealisation. By this idealisation the soil medium is replaced by infinite number of closely spaced independant springs. While considering the characteristic behaviour of the springs, non-linearity is accounted for.

For such characterisation the numerical values of stiffness depends upon the modulus of elasticity values of the clay medium, the accurate determination of which is required. The modulus of elasticity is not a unique property of soils. They are sensitive to especially moisture content and type of soil. Further the determination of the material properties based on laboratory tests may not be appropriate.

The method of analysis as reported may be applicable to only one type os soil, namely soft sensitive clays. The variation in soil type and in each type, the variation of soil stiffness with depth cannot be considered as such, without incorporating modifications in the analysis.

Moreover the encountered strain level under sustained loading conditions has not been taken into account. And as such principle of superposition may not be found valid. .

Nair (1968), has discussed in detail the state of the art on Dynamic and Earthquake Forces on Deep Foundations. As a concluding remark, he very rightly points out that it is extremely desirable to develop a simplified but sufficiently accurate analysis for use in routine design and the method proposed by Penzien is not widely used because of the complexities involved in computation and design.

Another mathematical model used for representing the soil-pile interattion effects in the Discrete beam - column element idealisation Fig 2.12 shows the salient features of a typical beam column element as approached by Matlock and Ingram (1963). This model and the analysis developed for determining the stresses and displacements along the pile length has been successfully used for static problems.

Tucker (1964) was first to try this model for dynamic anzlysis. Agrawal (1971) has extended this work for determining the response of single piles subjected harmonic excitation.

The physical model of the discrete-element beam-column as utilised by the investigator is shown in Fig 2.1 The model consists of a number of infinitely stiff bars connected end to end with pin joints. It is assumed that plane cross sections remain plane during and after bending The beam is assumed to have elastic behaviour. Considering free body diagram and the forces acting on the discrete elements the equation describing the response of the model is derived, taking into account the continuity of the secti and the equilibrium of the various forces acting on the elements.

Motohiko Hakuno (1973) has proposed a dynamic analy of the pile based on wave dissipation theory. Lumped mass spring - dashpot idealisation as proposed by Penzien has b used to determine the response of the pile. But herein th effect of frequency on the spring stiffness and the dampir coefficient including a part of loss caused from the wave dissipation is considered.

Mindlin's elastic half space solutions have been utilised. This has been done in two stages.

In the first stage the displacements produced by one point sinusoidal force in an elastic half space is evaluated. In the second stage the horizontal displacement produced by the vertical excitation on the surface of an elastic half space is determined. The vibration displacement at each place was evaluated superposing the two. Once the displacement matrix was evaluated the stiffness matrix was the inverse of the same.

The fundamental fourth order differential equation of motion was expressed by Hakuno (1973) in a finite difference form and solution for equation of motion of the pile was determined considering the effect of frequency of motion on the stiffness characteristics.

The predicted displacement response was checked with the actual field tests carried out by the Hakuno (1973). For close agreement,he arbitratily assumes the shear wave velocity of the soil.

Though consideration of effect of frequency on stiffness seems to be logical analytically,there is no experimental evidence available to prove this point and further the significance of this effect on the response is debatable.

A new approach for evaluating the seismic response of steel piles considering the restoring force characteristics upto yield point has been reported by Hayashi (1973). The soil-pile system has been treated as a single-degree freedom system and the general hysteritic restoring force characteristics was expressed in the manner proposed by P.C. Jennings (1965). The restoring force characteristics was defined based on actual static and dynamic tests carried out on piles upto yield point. Based on these approach new safety factors resulting in the economy of construction is suggested. Though economy should be the primary criteria in the earthquake resistant design, the idealisation of soil-pile system characteristics as those proposed for buildings may not prove to be effective.

2.5 EXPERIMENTAL STUDIES

Very few experimental studies are reported in the available literature discussing the behaviour of piles subjected to dynamic loads. Therefore no conclusive contention regarding earthquakes and other dynamic loads can be drawn.

Gaul (1958) reported for the first time the tests conducted on model piles embedded in bentonite clay. The tested piles were instrumented with tm SR-4 electrical resistance strain gauges.

A noval experimental set up was used to apply the dynamic loads. For applying the dynamic lateral load a mechanical oscillator driven by a motor was used, controlled by a speed control unit. Suitable crank and guide arrangements were attached to the driving system for applying the lateral dynamic load to the piles.

The strain induced in the piles due to the applied loads were recorded using a suitable amplifiers and oscilloscope having photo-graphic arrangements.

From the tests it was concluded that

1. The pile vibrates in the form of standing wave, which is in phase with the oscillating load. There is negligible amount of damping in the soil.
2. At low frequency, maximum bending moment is not altered.
3. Under dynamic loads the soil modulus for montmorillonite clay may be considered to remain constant with depth.
4. Under static load applications the maximum bending moment is not dependent upon the magnitude of lateral loads unless pile deflection becomes large enough to stress the soil beyond its elastic range.
5. Over-burden reduces bending moment but the shape and location of the maximum bending moment remains the same.

Though as a starting point the experimental observation of Gaul (1958) are valuable, it pertains to only one type of clay and the tests have been conducted at a particular frequency. No general conclusion could be drawn from the study.

Hayashi and Miyajima (1965) report tests conducted on on vertical steel H piles embedded in sand. The dimensions of the piles were 300 x 305 x 15 mm and of length 14 m and 16 m. Both free and forced vibration tests were conducted, using mechanical oscillators. From the results of the tests of the observed that damping coefficient, natural frequency and resonant frequency depends upon the soil conditions.

Hayashi et al (1965) reported results of several static cyclic and dynamic load tests on H piles of width 305 mm, embedded in sandy soil to the depth of 10-15 m. Both forced and free vibration tests were conducted by them and acceleration measurements at the pile head was recorded by them.

Interesting conclusions were drawn from their test results:

1. The natural frequency decreases as the initial displacement increases under free vibration conditions.
2. Sharp resonant peaks are observed under forced vibration conditions.
3. With increase in N-value of the soil the natural frequency and resonant frequency increases.
4. The comparison of predicted and observed quantieies reveal that natural periods could be predicted to reasonable accuracy using linear sub grade modulus theory. The test results of Ishi and Fujita (1965) also revealed sharp resonant peaks under forced vibration

test conditions. The piles were of 1200 mm dia 12 mm wall thickness and 34,000 mm length. Not many details regarding the testing is reported by them.

Both the above investigators suggest the use of Hayashi (1965) model for determining the dynamic characteristics of the soil-pile system. They state that determining the stiffness of the soil-pile system based on static or cyclic load tests if substituted in the Hayashi (1965) analysis would be able to produce the natural frequency and amplitude of vibration within reasonable limits of accuracies.

Prakash and Agarwal (1965) report one of the detailed investigations studying the behaviour of vertical piles subjected to dynamic lateral loads. The tests were conducted on small sized aluminium piles of 15 mm outer diameter 2.5 mm wall thickness. These piles were embedded in a tank containing uniform dry sand placed at medium density conditions. Steady state dynamic lateral load was applied with suitable connecting mechanism coupled to a horizontal steady state shake table. The steady state dynamic loads of varying amplitude and frequencies were applied to the pile at various elevations from the ground level. Acceleration, and displacement measurements were made and the change in surface of soil around the pile was also observed. Apart from this transient loads were also applied to the piles.

The study revealed that

1. The soil around the pile gets compacted under the applied dynamic load, creating a depression around the pile.

2. The displacements depend upon the frequency of dynamic load application.

3. Pile vibrates in phase with the dynamic load.

4. The zone of influence of the dynamically loaded pile extends below that of statically loaded pile.

5. The transient strength of the pile is greater than steady state strength, whereas static strength is less than both.

6. If moment is applied along with shear the displacement under static and dynamic conditions increases.

The extensive model studies performed by Gupta (196 on small sized aluminium piles embedded in uniform dry san revealed the following:

1. The natural frequency of the piles are dependent on the length of the piles. They are increased as the length of the pile increases.

2. The natural frequency is dependent on the sustained vertical load and decreases as the load level increases

The natural frequencies in his case was determined from the free vibration records of piles displaced from initial equilibrium position.

Prakash Chandrasekaran and Bhargava (1973) have extended the study of Gupta (1967) in order to investigate the various factors which control the natural frequency of isolated single piles and piles placed in clusters. The experimental work was performed on aluminium piles of 16 mm outer diameter with a wall thickness of 1.25 mm. The length of the piles were 70 cm, enabling them to be treated as long piles. These piles were driven in a tank containing uniform dry sand placed in dense state of deposition. The piles were allowed to vibrate freely by displacing them from equilibrium position.

On the basis of the study the following conclusions were drawn :

1. The natural frequency decreases with the increase of lateral deflection rapidly at first and very gradually at later stages. Beyond a certain value of deflection the natural frequency becomes constant.
2. The natural frequency is dependent on the sustained vertical load. As the sustained vertical load increases the natural frequency decreases.

3. In the case of pile groups the natural frequency is dependent on the pile spacing. The effect of pile spacing is felt only upto a spacing of six times the pile diameter.

4. The natural frequency of pile groups can be reasonably predicted on the basis of the behaviour of single piles.

5. The natural frequency of an isolated pile can be predicted by assuming it as a single degree freedom system. The stiffness of the soil-pile system can be taken to be defined by the tangent modulus which is derived from the load deflection plot of single piles.

6. In order to consider the effect of spacing on the free vibration characteristics of pile groups, reduction in the coefficient of horizontal sub-grade reaction, n_h, is suggested. Consequently the relative stiffness factor T, is increased. The suggested ratio of relative stiffness factor T, for a pile in a group to that of an isolated sing single pile is as under :

   a. 1.25 at a spacing of four pile width.

   b. 1.30 at a spacing of three pile width.

   c. At any other spacing linear interpolation can be made.

7. The method suggested by Saul (1968) with the above inclusions predict the natural frequencies on a higher side.

Maxwel et al (1968) have reported extensive field tests on full scale prototype piles embedded in silty sand.

Both static, cyclic and dynamic load tests were performed on single piles and pile groups with different sustained load levels.

Acceleration, frequency and phase measurements were made using acceleration pickups and suitable recording instruments.

The study revealed certain very interesting conclusion.

1. It is possible to test single pile under forced vertical vibrations and obtain information under resonance condition.
2. There is difference in the resonant frequency levels between piles, with caps resting on the soil and not resting on the soil.
3. Settlement of piles take place when the dynamic load acts in addition to the static vertical load.
4. The stiffness of the pile is increased when used in groups rather than when tested under isolated conditions.
5. The stiffness and damping properties of the soil-pile are sensitive to frequency of vertical vibration.
6. The resonant frequencies and amplitude of vibration can be determined based on the stiffness properties of the soil-pile system determined from static tests.

The experimental investigations reported by Hakuno (1973) is of great significance. He has tested steel piles of 60 cm dia, 16 mm wall thickness and 60 m lengths. The piles were embedded in predominantly fine sand. Piles were subjected to lateral vibrations using machanical exciter, capable of producing a force of 40 T at 12 Hz and with a frequency range of 1-12 HZ.

The vibration measurements were made using semi-conductor acceleration pickups placed on the surface and inside the ground at various distances from the pile. The tests were conducted at various force and frequency levels in order to get the resonant characteristics.

The author derived the following conclusions from hi tests.

1. The pile and the surrounding soil go into resonance almost simultaneously.
2. Even to the extent of ten times the pile diameter the soil around the pile was vibrating in phase with the pile. This was true for measurements on pickups place at surface and below ground level.

Hakuno (1973) catagorically states that the contenti of soil mass participating in vibrations or any equivalent soil mass concepts are completely meaningless. He conside that in any dynamic analysis there is absolutely no need t consider the soil mass participating in vibration because as per the experimental observation, on the surface of the ground and on those inside suggest the necessity to consider soil mass enclosed upto a distance of 100 m, which in reality may lead to erroneous prediction.

Dynamic response of buildings supported on piles was experimentally investigated by Ohata et al (1973). The vibration mode shapes plotted showed that in the first mode the super structure and the surrounding soil column are in same phase and of opposite phase in second mode. But in the third mode, there is a rocking mode experienced. Very important informations regarding the shearing modulli of the soil has been presented by the authors. They suggest that the shearing modulli can be obtained from dynamic triaxial compression tests and shear wave velocity measurements at the site. An interesting and useful result of shear modulus variation with N-values for different types of soils have been presented by the authors.

The piles have been idezlised in the mammer proposed by Penzien et al (1964) and the authors conclude that the predicted and observed quantities have closer agreement.

2.6 CONCLUDING REMARKS

Based on the examination of the available literature on the behaviour of piles subjected to dynamic loads the following broad conclusions can be drawn:

1. It is important to understand and estimate the behaviour of pile foundations subjected to dynamic loads. Pile foundations can be subjected to various types of dynamic loads, the definition of the loads depend upon the type of situation. Estimation of these loads may be based on theoretical consideration of mechanism of load applications and weightage must be given to statistical informations.

2. The method of analysis to be adopted for predicting the dynamic behaviour essentially depends upon the type of envisaged loads.

3. The dynamic behaviour of piles embedded in the soil is a soil pile interaction problem.

4. In the case of piles subjected to direct vertical vibrations reasonable estimate of their behaviour, if necessary, could be obtained through vertical vibration tests on the design pile sections. Estimation of dynamic amplitude and frequencies of vibrations can be done idealising the soil pile system as a single degree freedom system. The stiffness values of the soil-pile system could be based on static, cyclic load tests on the piles. Damping may be neglected.

5. The reported work on pile behaviour during driving, seem to concentrate on the prediction of static bearing capacity from dynamic results. The fitting principle and the method

of analysis suggested by Tomko (1968) seems to be a good procedure for predicting the capacities.

6. There is a need for methods for estimation of the effects of pile driving load on the pile section. The discrete-element model offers the best scope for further work in this direction. The method proposed by Agrawal (1971) offers limited scope and may be suitable for loading frequencies below natural frequencies of the system. The method needs experimental verification.

7. Wave forces tend to introduce dynamic loads on piles. The long term dynamic fatigue stresses and the estimation of them through proper model tests and method of analysis is much needed. The discrete-element model and the procedure putforth by Agrawal (1971) offers good scope for prediction.

8. Earthquake forces cause one of the most hazardous type of dynamic loads. The behaviour of piles definitely controls the performance of structures supported on piles.

9. Winkler model seems to be an useful and practical tool for idealising the soil-pile interaction phenominon.

10. Penzien's method results in useful solution for the analysis of structures (bridges) supported on long piles subjected to earthquakes. But the method is too complicated and may not be used by designer's concerned with analysis piles. The method may be suitable for only one type of soil.

11. There seems to be a need for easy and workable method of analysis and non dimensional solutions for predicting the dynamic characteristics of piles exclusively.

12. Judiciously conducted lateral forced vibration tests on piles could result in useful information concerning dynamic characteristics of piles.

13. There seems to be a need for standard vibration testing procedure and methods of insitu determination of dynami properties of soil-pile system.

CHAPTER - III

LUMPED MASS ANALYSIS AND DYNAMIC CHARACTERISTICS OF PILES EMBEDDED IN SOILS ASSUMING SOIL MODULUS CONSTANT WITH DEPTH

3.1 INTRODUCTION

3.1.1 GENERAL

Currently, aseismic designs of pile foundations are performed at best by evaluating the induced stresses and displacements through pseudo-static analysis and simultaneously determining their natural frequencies to check against resonance. For determining the natural frequencies, the soil-pile system is idealised as a cantilever. Though, this practice is recognised as unrealistic, the profession continues to follow the same, since no practical alternative solution is available to-day (1974) for predicting the dynamic response of piles.

In this chapter an analysis for evaluating the dynamic response of the pile foundations has been presented. The soil-pile system has been idealised by a logical and realistic mathematical model.

With the help of such an analysis, the dynamic response of a large number of pile cases of practical significance has been evaluated. Based on these results, design curves in non-dimensional form have been developed, with the help of which the response of practically any soil-pile system can be easily determined.

3.1.2 LUMPED-MASS SYSTEMS

In reality pile foundations embedded in soils are continuous systems. Therefore, mathematical idealisation of the soil-pile system as a continuous system model would be more appropriate.

However, in practice, pile cross sections, soil conditions and loading conditions vary to a great extent. But for these conditions it may be impracticable to obtain closed form solutions, for evaluating the dynamic response of piles.

For such situations, reasonable approximations can usually be made by lumping the mass of the piles at various convenient points. The interaction effects of the surrounding soil may also be discretised. This reduces the number of degrees of freedom of the system and the dynamic characteristics could be evaluated using suitable numerical techniques.

3.2 CHARACTERISATION OF SOIL PILE SYSTEM

The characterisation of soil-pile system is done in two parts. one, structural idealisation of the pile section and the other characterisation of the interaction effects of soil with the pile.

3.2.1 PILE STRUCTURAL IDEALISATION

Herein, the pile structural unit is idealised as a lumped mass system. The pile is divided into convenient number of segments of lengths Δx (say), Fig 3.1. If the pile section is divided into 'n' such segments it is considered that the division would result in n+1 number of masses, including the top mass.

A single pile or a pile in a group would be required to carry sustained vertical loads. It is presumed that this sustained vertical load (generally the safe load carrying capacity of pile) would be concentrated at the pile top. This vertical load concentrated at the top is considered to include : (i) a part of super-structure load (ii) pile cap weight and (iii) the weight of half segment length of the pile.

Thus the mass concentrated at top, $M_t = \frac{W}{g}$ ... 3.1

Where, W is the safe load carrying capacity of the pile.

The mass m_r, at any intermediate division point r, comprises of mass included within half the segment on either side of the division point r.

$$m_r = \frac{\gamma A}{g} \cdot \Delta x \qquad \text{... 3.2}$$

Where, γ is the weight density of the pile material and A, area of pile cross section. At the last division point, n only, mass of half the segment length would be lumped:

$$m_n = \frac{\gamma A}{g} \cdot \Delta x/2 \qquad \text{... 3.3}$$

Herein, it is recognised that the distribution of mass and flexibility above the pile cap would control the response of piles. However, this effect may not remain the same for the varities of structures in which piles may be used. In order to obtain generalised solutions, the primary factor mass is considered, by lumping at the pile top. Detailed discussion on this aspect appears in article 7.

3.2.2 SOIL INTERACTION IDEALISATION

3.2.2.1 Concept of Soil Modulus:

When a pile is subjected to lateral movements, the surrounding soil offers some resistance. This interaction force acts at every point along the pile length. The most convenient way of handling this phenomena is to consider th pile as a beam resting on elastic medium (soil). Replacing this continuous reaction, with infinite number of closely spaced independent elastic springs, we have the Winkler moc Fig 3.2a. The Winkler model presumes that the reaction at any point on the beam is proportional to the deflections of the beam at that point. The reaction of the soil per unit deflection over a unit area defines the soil modulus. Thus horizontal soil modulus, k_x is defined as: $k_x = p/y$ ... 3.

Where y, is the lateral deflection of the pile and p represents the soil resistance expressed as force per un length of the pile.

For a given soil type the soil modulus may assume any form of variation along the pile length. The probable and real form of variations in the case of stiff clays is shown in Fig 3.2b. In the case of granular soils, the soil modulus increases almost directly with depth, Fig 3.2c. In the present investigations, only these two forms of variations have been considered.

3.2.2.2 Discretisation of Soil Interaction Effects:

In the mathematical model used herein, the Winkler reaction offered by the soil is discretised as springs connected at convenient points. For discretisation and assigning the values of the spring constants at various elevations, the concept of subgrade modulus has been utilised, to-gether with the technique presented by Newmark (1943).

In order to achieve this, the soil reaction is assumed to act as a distributed loading intensity. Treating these distributed loads to be acting on a beam of length L_s (equal to length of the pile), the reactions at the mass points are easily evaluated treating this beam to be simply supported at the mass points. Herein, as assumed for convenience, the mass points are the division points and thus, the distance between the simply supported points would be Δx, the segment length.

The procedure that was followed in assigning the spring constant values, for the case of piles embedded in soils; assuming soil modulus to vary linearly with depth, is illustrated in Fig 3.3 (i).

For this linear form of variation, the soil modulus k_x, at any depth x, is defined by $k_x = n_h x$, where n_h is the constant of horizontal subgrade reaction (FL^{-3}). Thus, the continuous loading intensity, is bounded by a beam parallel to the axis of the pile and the said variation of soil modulus, with depth. The reactions and hence the spring constant values at various mass points would be as under:

$$K_1 = \frac{\Delta x}{6}(2 \times 0 + n_h \Delta x) = \frac{n_h (\Delta x)^2}{6} \qquad \ldots 3.5$$

K_1 is the spring constant value at mass M_t

$$K_2 = \frac{\Delta x}{6}(0 + 4\, n_h\, \Delta x + n_h\, 2\, \Delta x)$$

$$K_2 = n_h\, \Delta x^2 \qquad \ldots 3.6$$

$$K_3 = \frac{\Delta x}{6}(n_h\, \Delta x + 4\, n_h\, 2\, \Delta x + n_h\, 3\Delta x)$$

$$= \frac{n_h\, \Delta x^2}{6}(1 + 8 + 3)$$

$$= 2\, n_h\, \Delta x^2 \qquad \ldots 3.7$$

K_r, the spring constant value at any mass point r,

$$K_r = (r-1)\, n_h\, \Delta x^2 \qquad \ldots 3.8$$

The bottom most spring attached to the last mass, m_n would have a stiffness of:

$$K_n = \frac{\Delta x}{6} (n_h n \quad \Delta x \; 2 + n_h(n-1) \quad \Delta x)$$

$$= \frac{n_h \; \Delta x^2}{6} (2n + n-1)$$

$$K_n = \frac{n_h \; \Delta x^2}{6} (3n - 1) \qquad \ldots\ldots \; 3.9$$

where, n is the number of masses.

In Fig 3.3 (i), the above steps have been illustrated. Herein, it should be noted that the spring constants K have the usual units of FL^{-1}.

In the case of piles embedded in soils in which the form of variation of soil modulus are assumed to remain constant with depth, k_x = constant, the adopted discretisation procedure is as follows:

The value of the spring constant attached to the top mass M_t:

$$K_1 = \frac{1}{2} \cdot k_x \quad \Delta x \qquad \ldots \; 3.10$$

for any intermediate location, r (say)

$$K_r = k_x \; \Delta x \qquad \ldots \; 3.11$$

for the last mass m_n, again

$$K_n = \frac{1}{2} \cdot k_x \cdot \Delta x \qquad \ldots \; 3.12$$

herein, the unit of k_x, the soil modulus is, FL^{-2} and the spring constant K has the unit of FL^{-1}. The above steps are illustrated in Fig 3.3(ii).

3.3 MATHEMATICAL MODEL

3.3.1 COMPONENTS OF MODEL

In Fig 3.4a the discretised mathematical model of the actual soil-pile system is shown, to-gether with the structural and soil-interaction idealisation.

The parameters characterising this idealised syste are the following:

1. Mass M_t, includes the superimposed safe load, the mass c pile cap and a portion of the pile mass at top.
2. m_r, the lumped mass at the intermediate division point,
3. m_n, the lumped mass at the nth or the last division poir
4. K_1, the linear spring, having stiffness K_1, attached to the top mass M_t at one end and immovable support at the other end.
5. K_r, any intermediate linear spring attached to the rth mass m_r, at one end and immovable support at the other end.
6. K_n, the last linear spring attached to the mass m_n at o end and an immovable support at the other end.

3.3.1.1 Assumptions:

The adopted mathematical idealisation of the physical system involves the following assumptions.

1. The soil-pile system is idealised as a one dimensional model.
2. The pile material exhibits linear elastic behaviour.
3. The springs exhibit linear force displacement characteristics.
4. The discretised model represents the overall interaction mechanism of the physical (soil-pile) system.
5. The superstructure influence is considered by lumping the safe carrying capacity to-gether with end condition at top.

3.3.1.2 End Conditions:

While using the mathematical model for evaluating the dynamic characteristics of the piles; it is necessary to adopt proper end conditions. The adopted end conditions should be compatible with those existing in the physical system.

Under the applied static or dynamic lateral loads, if no restraint is imposed at the top of an isolated pile, it would be free to have lateral deflections and rotations. At the bottom, normally, the piles embedded in soil are known to experience negligible bending moment and shear. For the cases of piles subjected to sustained lateral loading conditions, ample evidence is available to corroborate this point (Prakash (1962), Davisson (1960)).

Therefore for pile top free to rotate condition , the following end conditions have been adopted, Fig 3.4b:

(1) Bending moment M_T and Shear V_T are zero at top.

(2) Bending moment M_B and shear V_B are zero at the bottom.

Fixity at the top of the pile member influences the pile behaviour under static and or dynamic load applications. In the case of pile groups, generally the pile top is completely or partially fixed against rotation. Unfortunately, there is no definite way of deciding the degree of fixity at the pile top.

In this investigation, solutions have also been obtained for pile top fixed against rotation conditions (100 % degree of fixity at the pile top), Fig 3.4b.

For 100 % degree of fixity at the pile top the end conditions adopted in the model are as under:

1. Rotation θ_T and shear V_T at pile top are considered as zero.

2. Moment M_B and shear V_B at the pile bottom are considered as zero.

For degree of fixity between 0% and 100 % suitable interpulation may be made .

3.4 NUMERICAL TECHNIQUE FOR DYNAMIC ANALYSIS

3.4.1 APPROACH

The dynamic response of any system subjected to earthquake excitation depends upon the natural periods, mode shapes, damping characteristics and the form of variations of acceleration with time. In order to evaluate the elastic response of any structure under earthquake excitation two approaches are usually available:

(1) Time-wise or modal superposition of response in various modes of vibration.

(2) Direct integration of simultaneous differential equations of motion.

The former approach has the merit because the first few modes have dominant contribution to the total response.

The dynamic characteristics of the soil-pile system and the various factors which control them are evaluated, analysing the idealised mathematical model, subjected to base motions. The approach adopted herein to determine the dynamic response, considers the free vibration characteristics of the system. With the free vibration analysis the time periods and mode shapes for different quantities are obtained. Then the superposition of response in different modes of vibrations are carried out to obtain the overall response of the syst

3.4.1.1 Assumptions:

While adopting such an approach the following assumptions are considered imperative:

1. The pile vibrates in its own plane. Only one dimension vibrations need be considered.
2. The pile material exhibits linear elastic behaviour.
3. Both shear and bending deformations take place.
4. Plane cross sections remain plane during and after bend:
5. Axial deformations are of negligible quantity.
6. Deformations of the pile sections are small.
7. The springs exhibit linear force displacement characteri tics.
8. The discretised model represents the overall interactior mechanism of the physical (soil-pile) system.

3.4.2 METHOD OF ANALYSIS:

In order to obtain the solutions, let us consider the model to be displaced from the equilibrium position anc released. The system would then be vibrating in the classi normal mode with a form:

$y = y(x) \sin p\,t$, where p is the circular natural frequency.

Considering, an element in a segment of the model, as illustrated in Fig 3.5a and its equilibrium for the rotational and translational elastic and inertia forces we get:

$$V = -\sigma AG \frac{dy_s}{dx} \qquad \ldots\ 3.13$$

$$M = EI \frac{d^2y_b}{dx^2} \qquad \ldots\ 3.14$$

$$\frac{dM}{dx} = V - \rho I_p p^2 \theta_b \qquad \ldots\ 3.15$$

$$\frac{dV}{dx} = mp^2y - Ky \qquad \ldots\ 3.16$$

$$y = y_b + y_s \qquad \ldots\ 3.17$$

where,

V is the shear force, due to shear deformation in F units

σ ratio of the average shear stress on a section to the product of shear modulus and the angle of shear at the neutral axis, termed as shape factor, 1.10 for circular section, a dimensionless quantity.

I_p Moment of inertia of the section, with units of L^4.

M Bending moment in a section, FL.

m	mass of the element of a segment.
y_s	Deflection due to shear deformation, L.
y_b	Deflection due to bending deformation, L.
θ_b	Rotation due to bending deformation.
ρ	Mass Density of the material, $FL^{-4} T^2$.
y	total deflection, sum of deflections due to bendin and shear, L.
G	Shear modulus of the structural material, FL^{-2}.
EI	Flexural stiffness, FL^2.
A	Area of cross section in L^2.
p	Natural frequency of vibration of the system in any mode.

Let us consider three mass locations and section drawn at mass point 1, Fig 3.5b.

A finite change of shear force occurs at each mass whicl is equal to the algebraic sum of the inertia force of the mass and the spring (soil) reaction. Each of these quantities are dependent on the deflection of the mass point,

Therefore we have:

$$\Delta V = mp^2y - Ky \qquad \ldots 3.18$$

Assuming that the quantities V_o, M_o, θ_{bo}, y_{bo} and y_{so} are known at the left of the section then,

$$V_1 = V_o + m_o p^2 y_o - k y_o \qquad \ldots 3.19$$

$$M_1 = M_o + V_1(\Delta x) - (\rho I)_1 (\Delta x)_1 p^2 \theta_{bo} \qquad \ldots 3.20$$

$$y_{s1} = y_{so} - \left(\frac{V}{\sigma AG} \right)_1 (\Delta x)_1 \qquad \ldots 3.21$$

Now, bending moment M at any distance x, from the left side of section, O, is:

$$M = M_o + \frac{M_1 - M_o}{(\Delta x)_1} \cdot x \qquad \ldots 3.22$$

$$\text{Slope } \theta_b = \frac{1}{(EI)_1} \int M\, dx + B \qquad \ldots 3.23$$

$$= \frac{1}{(EI)_1} \quad M_o x + \frac{M_1 - M_o}{(\Delta x)_1} \cdot \frac{x^2}{2} + \theta_{bo} \qquad \ldots 3.24$$

and deflection

$$y_b = \int \theta\, dx + B \qquad \ldots 3.25$$

$$= \frac{1}{EI} \left(M_o \cdot \frac{x^2}{2} + \frac{M_1 - M_o}{(\Delta x)_1} \quad \frac{x^3}{6} \right) + \theta_{bo}\, x + y_{bo} \qquad \ldots 3.26$$

For a distance $x = (\Delta x)_1$

$$\theta_{b1} = \frac{(\Delta x)_1}{EI} \left(\frac{M_o}{2} + \frac{M_1}{2} \right) + \theta_{bo} \qquad \ldots 3.27$$

$$\text{and } y_{b1} = \frac{(\Delta x)_1}{EI} \left(\frac{M_o}{3} + \frac{M_1}{6} \right) (\Delta x)_1 \qquad \ldots 3.28$$

$$+ \theta_{bo} (\Delta x)_1 + y_o \qquad \ldots 3.29$$

These results can be generalised and the expressions for the jth section in terms of the (j-1)th section or mass as illustrated in Fig 3.6, would be:

$$V_j = V_{j-1} + m_{j-i}\, p^2\, y_{j-1} - K_j\, y_{j-1} \qquad \ldots\ 3.30$$

$$M_j = M_{j-1} + V_j(\Delta x)_j - (\rho I)_j (\Delta x)_j\, p^2\, \theta_{b_{j-1}} \qquad \ldots\ 3.31$$

$$\theta_{b_j} = \frac{(\Delta x)_j}{2(EI)_j}\,(M_{j-1} + M_j) + \theta_{b_{j-1}} \qquad \ldots\ 3.32$$

$$y_{b_j} = \frac{(\Delta x)^2_j}{3(EI)_j}\,(0.5\, M_j + M_{j-1}) \qquad \ldots\ 3.33$$

$$+\ \theta_{b_{j-1}} (\Delta x)_j + y_{b_{j-1}} \qquad \ldots\ 3.34$$

$$y_{s_j} = y_{s_{j-1}} - \left(\frac{\sigma \Delta x}{GA}\right)_j V_j \qquad \ldots\ 3.35$$

$$y_j = y_{b_j} + y_{s_j} \qquad \ldots\ 3.36$$

Where,

V_j - Shear force at any section j, in the element

M_j - Bending moment at any division point j acting on the segment

θ_j - Slope of the element at the division point j

y_j - deflection of the mass at j

y_{b_j} - deflection due to bending of the mass at j

y_{s_j} = deflection due to shear of the mass at j.

$(\Delta x)_{j-1}$ = length of the element between points j-1 and j

A_{j-1} = cross sectional area of the element

I_{j-1} = moment of inertia of cross section of the element

$\frac{2\pi}{p}$ = natural period of vibration of the system

From the above equations it is seen that for any frequency 'p', once we know the values of V, M, θ_b, y_s, and y_b at a particular section or mass point we can find the corresponding values at all other points. In order to achive this, the aid of the defined end conditions are taken.

For the pile top free to rotate condition, we know that the bending moment M_T and shear V_T at the top of the pile are zero. Similarly at the bottom the bending moment M_B and shear V_B are also known to be zero. Therefore the unknown quantitites are deflection y_T and rotation θ_T at the top of the pile and deflection y_B and rotation θ_B at the bottom of the pile.

Once we know the quantities at pile top, the unknown quantities at pile bottom may be determined by starting from the pile top and proceeding towards pile bottom.

But at pile top y_T and θ_T are also unknown. Therefore, for convenience if we assume $y_T = 1$ and $\theta_T = 0$ we get the other quantities at bottom for this assumed conditions in terms of y_T and θ_T.

Thus

$$y_B' = C_{11}\, y_T \qquad \dots\ 3.37$$

$$\theta_B' = C_{21}\, y_T \qquad \dots\ 3.38$$

$$V_B' = C_{31}\, y_T \qquad \dots\ 3.39$$

$$M_B' = C_{41}\, y_T \qquad \dots\ 3.40$$

in which C_{11}, C_{21}, C_{31} and C_{41} are the constants dependent on 'p'.

Similarly for the condition of $y_T = 0$ and $\theta_T = 1$ at top we get

$$y_B'' = C_{12}\, \theta_T \qquad \dots\ 3.41$$

$$\theta_B'' = C_{22}\, \theta_T \qquad \dots\ 3.42$$

$$V_B'' = C_{32}\, \theta_T \qquad \dots\ 3.43$$

$$M_B'' = C_{42}\, \theta_T \qquad \dots\ 3.44$$

In which C_{12}, C_{22}, C_{32} and C_{42} are constants dependent on p. Therefore in general for any translation y_T and rotation θ_T, the quantities at the pile bottom would be:

$$y_B = C_{11}\, y_T + C_{12}\, \theta_T \qquad \dots\ 3.45$$

$$\theta_B = C_{21}\, y_T + C_{22}\, \theta_T \qquad \dots\ 3.46$$

$$V_B = C_{31}\, y_T + C_{32}\, \theta_T \qquad \dots\ 3.47$$

$$M_B = C_{41}\, y_T + C_{42}\, \theta_T \qquad \dots\ 3.48$$

Now, at the pile bottom we know that the quantities V_B and M_B are zero.

Therefore we have:

$$V_B = C_{31}\, y_T + C_{32}\, \theta_T = 0 \qquad \ldots\ 3.49$$

$$M_B = C_{41}\, y_T + C_{42}\, \theta_T = 0 \qquad \ldots\ 3.50$$

i.e.

$$\begin{bmatrix} C_{31} & C_{32} \\ C_{41} & C_{42} \end{bmatrix} \begin{bmatrix} y_T \\ \theta_T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \ldots\ 3.51$$

However, in the above equation, the condition for y_T and θ_T to be non-zero, is the vanishing of the determinant.

$$\begin{vmatrix} C_{31} & C_{32} \\ C_{41} & C_{42} \end{vmatrix} = 0 \qquad \ldots\ 3.52$$

It can be noted herein, that the terms of the determinant involve quantities which have bearing on 'p' alone. Thus, if the assumed values of 'p' is such that, it is one of the natural frequencies of the system then the determinant would be equal to zero.

Knowing the natural frequency of vibration each of the quantities given in Eq. (3.30) through Eq. (3.36) can be obtained starting from the top mass and proceeding successively to the bottom mass. By this the quantities : deflection (y), rotation (θ), bending moment (M) and shear (V) are known at each mass point for didferent modes of vibrations, defined by different natural frequencies of vibrations. Thus the variation of the said quantities along the entire length of the piles are established.

For the case of pile top fixed against rotations the procedure of obtaining the modal values are the same. However the required change in the end conditions would be $\theta_T = 0$ and $V_T = 0$ at the top of the piles. Thus the unknown quantities at the top would be y_T and M_T. As before certain arbitrary values for these quantities could be assigned at the top and rest of the procedure remains the same.

The change in the equations 3.45 to 3.48 would be:

$$y_B = C_{11}\, y_T + C_{12}\, M_T \qquad \ldots\ 3.53$$

$$\theta_B = C_{21}\, y_T + C_{22}\, M_T \qquad \ldots\ 3.54$$

$$V_B = C_{31}\, y_T + C_{32}\, M_T \qquad \ldots\ 3.55$$

$$M_B = C_{41}\, y_T + C_{42}\, M_T \qquad \ldots\ 3.56$$

3.5 DYNAMIC RESPONSE

In order to obtain the earthquake response of the pile soil system, (earthquake) response spectrum technique was used (Housner 1964). According to this technique the response of a general single-degree-of freedom system may be obtained by the application of Duhamel integral. The integral expression for the earthquake response of a damped system is given by:

$$v(t) = \frac{1}{p} \int_0^t v_g(\tau)\, e^{-p\xi(t-\tau)} \sin p(t-\tau)\, dt \quad \ldots 3.57$$

Denoting the integral by symbol $V(t)$,

$$V(t) = \int_0^t v_g(\tau)\, e^{-\xi p(t-\tau)} \sin p(t-\tau)\, d\tau \quad \ldots 3.58$$

Where, $v(t)$ is the displacement variation with time, $v_g(\tau)$ acceleration variation, t time for which the random vibration acts, $V(t)$ velocity response , damping coefficient.

The earthquake response for a lumped mass system becomes :

$$v(t) = \frac{1}{p} V(t) \quad \ldots 3.59$$

The above equation represents the displacement response to any ground motion input for which the earthquake respomse function $V(t)$ is evaluated. In a similar manner the forces developed in the system during earthquakes can also be evaluated.

The displacement response at any time 't' of any single degree system to earthquake excitation is defined completely by the equation. But for a pile-soil system assessment of such an entire time history of displacement and forces would involve a tedious computational problem to the designer.

Therefore in a preatical problem of the present type it is sufficient to determine only the maximum response quantities. The maximum displacement response can be obtained by introducing the maximum value of the response function V(t) into the equation. This maximum value of the function is commonly called as spectral velocity, S_v

$$S_v = \left[\int_0^t v_g(\tau)\ e^{-\xi p(t-\tau)} \sin\ p(t-\tau)\ d\tau \right]_{max} \quad \ldots\ 3.60$$

The spectral displacement would then be S_v/p and spectral acceleration $S_v \cdot p$.

The relationship of S_d, S_v and S_a for any given earthquake, for systems with different periods and damping is the response spectrum. A typical set of design spectrum f(the above quantities with period has been given in Fig 3.7.

It is possible to evaluate the dynamic response of t multi degree system considering the combined equation of moti(However, herein, for the elastic system as adopted, it is considered that, once the mode shapes and frequencies of the system are known the system could be treated as uncoupled system, and the superposition of the individual modes could be done to determine the overall response.

It is also considered that the total maximum is not given by the sum of the individual maximum. An approximation to the total maximum was based on probability considerations as obtained by the rootmean-square addition otherwise known as quadratic superposition which was proposed by Housner and Jennings (1964).

In this process the contribution of each mode by considering their participation factors has also been incorporated.

Therefore the adopted procedure for determining the dynamic response is as under:

1. For each mode the dynamic deflection Y is calculated by

$$Y_{(i)}^{(r)} = \Phi_{(i)}^{(r)}(y)\ \Upsilon_{(r)}\quad S_d(r) \qquad \ldots\ 3.61$$

$Y_{(i)}^{(r)}$ = the dynamic deflection of the i th point in the 'r'th mode.

$\Upsilon_{(r)}$ = The mode participation factor for the 'r'th mode given by

$$\Upsilon_{(r)} = \frac{\sum_{i=1}^{i=n} m_i y_i}{\sum_{i=1}^{i=n} m_i y_i^2}$$

$S_d(r)$ = The spectral displacement $S_a(r)/p^2$ where $S_a(r)$ - Spectral acceleration corresponding to the period T of the 'r'th mode.

$\Phi_{(i)}^{(r)}(y)\ \Upsilon_{(r)}$ = Normalised modal deflection

p_r - the natural frequency of the 'r'th mode.

Now, for each of the mode shapes there are associated values of shear, moment and rotation. The above equation (3.61) was used to calculate these by replacing $\Phi(y)$ by the desired quantities at various points.

The maximum value of the modal quantities was calculated by the root-mean seuare addition.

$$Y_{max} = \sqrt{Y_{(1)}^2 + Y_{(2)}^2, + \quad} \qquad \ldots \; 3.62$$

The variation of Y_{max} at different points gives the dynamic deflected shape of the pile due to the earthquake considered. Similarly, the dynamic rotation, bending moment and shear can also be obtained.

3.6 COMPUTER PROGRAMMES

For determining the natural frequencies and mode shap at different frequencies of vibrations a generalised compute programme was written in Fortran IV language.

For the lumped mass-spring system idealising the physical system, it would not be possible to guess the value of 'p' correctly. In order to achieve this, a lower bound value of 'p' was initially assumed and for that assumed 'p' value the free vibration analysis was performed to evaluate the value of the determinant. The programme incorporated the process of plotting 'p' versus determinant, Δ relationship. When this plot changed sign, suitable interpolation technique was adopted to catch the exact value of 'p' corresponding to the near zero value of the determinant. Once the exact 'p' value corresponding to first mode frequency was obtained, the mode shapes were computed.

Then a suitable increment to the first mode frequency was given, and in a similar manner the second and third mode frequencies and mode shapes were obtained.

The other salient features of the programme are as under:

1. Piles of varying cross sections can be analysed.
2. Any desired end conditions can be incorporated.
3. The programme could get out-put for soil modulus remaining constant or varying linearly with depth.
4. The mode shapes and natural frequencies could be obtained to any desired numbers.
5. For the desired earthquakes the programme is capable of obtaining the dynamic response considering the combination of each mode and the probable maximum by root mean square addition.

3.7 VARIABLES

Using the computer programme the dynamic respo sse of variaties of soil-pile systems was determined. The response assessment was based on the analysis of the proposed mathematical model following the principles of dynamic analysis which was discussed in article 3.4 and 3.5.

In this process to understand and estimate the dynamic characteristics of piles embedded in soil, the various factors which control the dynamic behaviour have been examined for the following broadly classified cases.

Information has been obtained for piles embedded in soils in which:

(i) the soil modulus can be considered to remain constant with depth.

(ii) the soil modulus can be considered to vary linearly with depth.

In both the above soil types the following two conditions have been considered:

(i) pile top free to rotate conditions(with 0 % degree of fixity at the top).

(ii) Pile top fixed against rotation conditions (with 100% degree of fixity at the top).

In each of the above four cases the influence of the following factors have been analysed:

1. The soil stiffness.
2. The flexural stiffness, EI of the pile.

3. Sustained vertical loads.

4. Pile lengths.

3.7.1 PROCESS OF VARIABLE SELECTION

3.7.1.1 Definition of Maximum Depth Factor

For determining the pile response subjected to sustained lateral loads and moments, Reese and Matlock (1956) and Davisson and Gill (1963) have proposed non-dimensional solutions. These solutions define, the relative stiffness factors and maximum non-dimensional depth factors as under:

(i) For the case of piles embedded in soil in which the soil modulus remains constant with depth, relative stiffness factor R, is defined as, $R = \sqrt[4]{EI/k}$. Where EI is the flexural stiffness, k the soil modulus, FL^{-2}.

(ii) For the case of soils with linear variation of soil modulus with depth, relative stiffness factor, T is defined as, $T = \sqrt[5]{EI/n_h}$, where n_h is the constant of horizontal subgrade reaction, FL^{-3}.

(iii) The factor Z_{max}, termed as maximum depth factor is obtained by dividing the embedded length L_s by Relative Stiffness Factor (R or T). This results in a dimensionless number which is indicative of the flexibility of the pile member relative to soil.

3.7.1.2 Adopted Practice:

Before deciding upon the numerical values for the material constants of the soil-pile system a systematic study of the adopted piling practice in the country was made. The study revealed the following:

1. Normally adopted pile diameters vary between 0.25 m to 0.75 m.
2. The pile length range between 5m to 60m.
3. The area ratio of steel is between 0.2 % and 0.4 %.
4. The generally adopted concrete mix is M150 or M200.

3.7.1.3 Numerical Values and Pile Cases:

In order to obtain information of practical significance, dynamic response of 180 pile cases was evaluated. In each of these pile cases the soil-pile parameter values were carefully varied. For the cases of piles embedded in the type of soils, in which the soil modulus can be considered to remain constant with depth, the list of analysed pile cases with their salient features has been given in Table 3.1.

From this table the following points can be observed.

1. The considered pile diameters are: 0.3, 0.4, 0.5, 0.6 and 0.7 m.

Table 3.1

Details of Analysed Pile Cases Considering Soil Modulus To Remain Constant With Depth

Relative Stiffness Factor, R in metres	Diameter in metres	Soil Modulus in T/m^2	Flexural Stiffness EI Tm^2	Remarks
1.0	0.30	477.13	0.477×10^3	1. In each case the maximum depth factor, Z_{max} = 1, 2, 3, 5, 10 and and 15 were considered.
1.25	0.30	195.43	0.477×10^3	
1.5	0.30	94.25	0.477×10^3	
1.0	0.40	1507.96	0.151×10^4	2. The sustained vertical load was varied in each case. The value was calcultaed considering frictional and end bearing resistance using Terzaghi's (1943) theory.
1.25	0.40	617.66	0.151×10^4	
1.50	0.40	297.87	0.151×10^4	
1.0	0.50	3681.55	0.368×10^4	
1.25	0.50	1507.96	0.368×10^4	
2.0	0.50	230.10	0.368×10^4	3. Each was analysed for both pile top free to rotate and fixed against rotation conditions.
1.5	0.60	1507.96	0.763×10^4	
2.0	0.60	477.13	0.763×10^4	
1.25	0.60	3126.92	0.763×10^4	
1.50	0.70	2793.69	0.141×10^5	
2.0	0.70	883.94	0.141×10^5	
3.0	0.70	174.61	0.141×10^5	

2. In each of these diameters, the pile lengths have been varied to obtain information on cases with Z_{max} = 1,2, 3, 5, 10 and 15.

3. For each of these pile sectional properties and lengths three different values of relative stiffness factors have been considered.

4. Three relative stiffness factors, yielded information regarding piles embedded in soils of different stiffnes In the case of clayey soils the above may be considered to cover soft to stiff consistencies.

3.8 NEED FOR NON-DIMENSIONAL SOLUTIONS

The analysis of the dynamic behaviour of piles inclu

1. The examination of natural frequencies in different mod of vibrations.

2. Study of mode shapes at different modes of vibrations. That is the assessment of variation of deflection, slop bending moment and shear along the entire length of the piles at each mode of vibrations.

3. The analysis of various soil-pile parameters which influence the above two factors.

Based on the results of different pile cases analyse it would be advantageous, to develop non-dimensional curves for logical explanation of the basic quantities of pile response and the factors which influence them. In addition to this, if such solutions are developed, it would help the designer in predicting the response of any soil-pile system under the dynamic loads.

3.9 NON-DIMENSIONAL CURVES FOR NATURAL FREQUENCIES

The overall response of the system to earthquakes depends upon the natural frequencies in different modes of vibrations. In order to obtain the significant assessment of the various factors that influence the natural frequencies, each individual factors (of soil-pile parameters) have been varied keeping the others as a constant. In this manner, the overall picture as to how the different variables influence the behaviour was obtained. Such an analysis of the results of pile cases embedded in clay (given in Table 3.1) resulted in a set of curves between a factor termed as FREQUENCY FACTOR, F_{CL} and relative stiffness factors of the piles.

The variables constituting, F_{CL} the non-dimensional frequency factor under different modes of vibrations are defined below.

Mode	Identification	Components
First	F_{CL1}	$w_{n1} \cdot \sqrt{\frac{W}{g\ k\ R}}$
Second	F_{CL2}	$w_{n2} \cdot \sqrt{\frac{\gamma d^2}{gk}}$

Third F_{CL3} $w_{n3} \cdot \sqrt{\frac{\gamma d^2}{gk}}$

In the above list,

w_n, is the circular natural frequency of the systems in the subscript identified modes given in radians per sec.

$\frac{W}{g}$, is the mass at top M_t (FT^2L^{-1})

k , soil modulus values in FL^{-2}

d , dia of the pile section (L)

R , relative stiffness factor (L)

γ , weight density of the pile (FL^{-3})

F_{CL1}, frequency factor a dimensionless number. Letter C denotes the clay case and L, identifies the use of lumped mass model. The numerals identify the mode number. Prime used for pile top fixed against rotation conditions.

It is to be noted that the definition of frequency factors hold good for both, pile top free to rotate (0% degree of fixity) and pile top fixed against rotation conditions (100% degree of fixity).

In Fig 3.8 the variation of F_{CL1} with relative stiffness factor, R has been drawn for different identified, Z_{max} cases.

In Figures 3.9 and 3.10 the variation of F_{CL2} and F_{CL3} is provided in a similar manner, for second and third modes respectively. For a particular Z_{max}, each of these curves cover sixteen pile cases of varying soil-pile parameters and vertical loads.

The above plots pertain to the pile top free to rotate conditions.

In order to indicate the uniqueness of the above plots, as an example, the dimensionless frequency factor values F_{CL1}, F_{CL2} and F_{CL3} as obtained have been tabulated in Table 3.2. These results pertain to the case of piles with $Z_{max} = 5$ and for pile top free to rotate conditions. The soil-piles parameter values and the sustained vertical loads pertaining to each pile case have been provided in this Table. Such tables were prepared for each Z_{max} case.

Similar process was found to be valid for the conditions of pile top fixed against rotations. The variation of these factors (identified as F'_{CL1}, F'_{CL2}) with relative stiffness factors for piles with different Z_{max} cases are given in Fig 3.11 and Fig 3.12 respectively. In Figure 3.13 to 3.16 the variation of frequency factors with maximum depth factor Z_{max} has been provided for different modes and for both pile top free to rotate and fixed against rotation conditions.

Table 3.2

Frequency Factor Values For Different Pile Problems With Z_{max} = 5 - Pile Top Free To Rotate Conditions

›b- n	Relative Stiffness Factor R in metres	Sustained Vertical load W, in Tonnes	Soil Modulus k in T/m^2	Diameter of pile in metres	First Mode Natural Frequency w_{n1} in radians per sec	Frequency Factor for First Mode of vibrations F_{CL1} $w_{n1} \frac{W}{g} \cdot \frac{1}{k\ F}$
	2	3	4	5	6	7
	1.5	15.0	297.87	0.4	14.27	0.8347
	1.0	140.0	3681.55	0.5	13.48	0.8392
	1.25	70.0	1507.96	0.5	13.63	0.83860
	2.0	20.0	230.1	0.5	12.49	0.83132
	1.5	100.0	1507.96	0.6	12.49	0.83846
	2.0	40.0	477.13	0.6	12.76	0.83408
	1.25	175.0	3126.91	0.6	12.42	0.83905
	1.5	225.0	2793.69	0.7	11.34	0.83894
	2.0	90.0	883.94	0.7	11.61	0.83635
	3.0	30.0	174.61	0.7	10.75	0.82136
	1.0	10.0	477.13	0.3	18.12	0.83752
	1.25	5.0	195.43	0.3	18.2	0.83132
	1.5	5.0	94.25	0.3	13.8	0.83858
	1.0	45.0	1507.96	0.4	15.22	0.83943
	1.25	25.0	617.66	0.4	14.59	0.83822

Contd --

Table 3.2 (Contd.)

Problem No.	Second Mode natural frequency w_{n2} in radians per sec	Frequency factor for second Mode of vibrations $F_{CL2}=w_{n2}\sqrt{\frac{d^2}{k.g}}$	Third Mode natural frequency w_{n3}, in radians per sec	Frequency factor for Third Mode of vibrations $F_{CL3}=w_{n3}\sqrt{\frac{d^2}{k.g}}$
1	8	9	10	11
36	98.39	1.12783	115.3	1.32166
37	276.5	1.12654	324.098	1.321
38	177.0	1.12641	207.359	1.3206
39	69.18	1.12786	81.16	1.32237
40	147.5	1.12718	172.9308	1.3215
41	83.01	1.12752	97.25	1.32094
42	212.4	1.12591	248.83	1.32058
43	172.10	1.12722	201.631	1.32085
44	96.83	1.127556	113.3	1.31934
45	43.05	1.12795	50.57	1.32498
31	166.0	1.12707	194.31	1.3200
32	106.3	1.12798	124.7	1.3231
33	73.8	1.12795	86.66	1.32069
34	221.0	1.12471	259.180	1.32048
35	141.7	1.12737	165.863	1.32046

3.10 NON-DIMENSIONAL CURVES FOR NORMALISED MODAL QUANTITIES

3.10.1 NORMALISED MODAL QUANTITIES

Each of the pile cases given in Table 3.1 when analysed with the lumped mass analysis gave output of modal quantities of deflection, rotation, bending moment and shear at the mass points along the pile length. As mentioned earlier the various modal quantities were obtained by giving a unit displacement to the pile top article 3.4.2. For each mode of vibration along with the modal quantities the values of mode participation factor (defined in equation 3.61) was also obtained. Such factors when multiplied by the modal quantities at every point along the pile length take into account the vibration mass distribution effects at each point For each of the analysed pile cases the normalised modal quantities explained above have been obtained. In the subsequent sections the normalised modal quantities of deflection, rotation, bending moment and shear have been identified as $\Phi(y)$, $\Phi(\theta)$, $\Phi(M)$ and $\Phi(s)$ respectively. Numerals in subscrip have been used to indicate the mode numbers. Primes have bee used to identify solutions of pile top fixed against rotation conditions. Considering the enormous amount of such information non-dimensional curves for these normalised modal quantities of deflection, slope, bending moment and shear were obtained for both pile top free to rotate and fixed against rotation conditions. The procedure of obtaining the non-dimensional curves for each of these normalised modal quantities has been explained below.

3.10.2 NON-DIMENSIONAL CURVES FOR NORMALISED MODAL DEFLECTION

In Fig 3.17 and Fig 3.18 the normalised modal values of deflection along the pile length (i.e. at various absolute depths) have been plotted for different pile cases with Z_{max} = 1 and 5 respectively. The particulars of each of the five pile cases are as under :

No.	Dia in metres	Soil modulus k, in T/m²	Relative Stiffness Factor, R in metres	Z_{max}
1	0.4	1507.96	1.00	1.0
2	0.5	1507.96	1.25	1.0
3	0.6	1507.96	1.50	1.0
4	0.6	3126.92	1.25	1.0
5	0.6	477.17	2.0	1.0
1	0.4	1507.96	1.0	5.0
2	0.5	1507.96	1.25	5.0
3	0.6	1507.96	1.50	5.0
4	0.5	3681.55	1.0	5.0
5	0.5	230.10	2.0	5.0

In these curves the normalised modal deflection $\Phi(y)$ is a dimensionless quantity. At any depth the values of $\Phi(y)$ is dependent on the relative stiffness factor of the soil-pile system. When the normalised modal quantities are plotted against x/R, the dimensionless depth factor, each of the five curves of Fig 3.17 merges into a single unique curve as shown in Fig 3.19. Similarly the curves in Fig 3.18 results in curve shown in Fig 3.20. The above pile cases pertaining to Z_{max}=1 and 5 have been particularly chosen for presentation, considering the widely differing modes of deformations in these two cases. Thus it was observed that for a particular Z_{max} case (of 1 and 5 at present) whatever be the pile-soil parameter, there exists a unique non-dimensional curve for normalised modal deflection. Such a non-dimensional curve for Z_{max} = 2, 3, 5, 10 and 15 between A_y and x/R have been presented in Fig 3.21. These curves pertain to the case of piles embedded in clay under first mode of vibrations with pile top free to rotate condition. It was observed that the above process of obtaining non-dimensional unique curves held valid for second and third moc of vibration also. Thus for any soil-pile system with a particular Z_{max}, a unique non-dimensional curve existed between x/R and the normalised modal deflection. For pile top free to rotate conditions and for the second mode of vibration the variations of non-dimensional normalised modal deflection A_{y2} against x/R for different Z_{max} have been plotted in Fig 3.22.

In Fig 3.23 and 3.23a similar such non-dimensional cu between non-dimensional normalised modal deflection A_{y3} and depth factor x/R have been presented for Z_{max} = 2, 3, 5, 10 and 15. These curves pertain to third mode of vibration for the pile top free to rotate conditions.

For pile top fixed against rotation conditions the similar process of obtaining non-dimensional curves was foun to agree.

Fig 3.24 gives the variation of non-dimensional normalised modal deflection Λ'_{y1} with depth factor x/R, for the first mode of vibrations in the above conditions.

In Fig 3.25 the above curves for the second mode of vibrations have been presented as Λ'_{y2} versus x/R. In all these plots it is emphasised that for a particular Z_{max}, irrespective of the variations in soil pile parameter values and the sustained loads there exists a unique non-dimensional plot.

3.10.3 NON-DIMENSIONAL CURVES FOR NORMALISED MODAL ROTATION:

In Fig 3.26 and 3.27 the normalised modal values of rotation $\Phi(\theta)$ have been plotted against depth factor x/R for first mode of vibration. The five curves in each of these figures pertain to the pile cases given in article 3.10.2 for $Z_{max} = 1$ and $Z_{max} = 5$ respectively.

As shown in Fig 3.28 and Fig 3.29 the product of respective relative stiffness factors and the normalised modal rotation $\Phi(\theta)$ when plotted against x/R merges to a single unique dimensionless curve for each of the five curves of Fig 3.26 and 3.27. This process was found to hold good for any soil-pile conditions with a particular Z_{max} case.

The above process was found to be valid for the second mode of vibrations also. In Fig 3.30, 3.31, 3.32 and 3.3 the non-dimensional normalised modal rotations A_{θ_1}, A_{θ_2} and A_{θ_3} have been plotted against x/R for first, second and third mode respectively. These curves correspond to the pile top free to rotate conditions. The results have been plotted for Z_{max} = 2, 3, 5, 10 and 15. Thus for any particular Z_{max} whatever be the pile-soil condition, the normalised modal rotation can be easily read. Followin the procedure for pile top free to rotate conditions, in Fig 3.33 and Fig 3.34 normalised non-dimensional modal rotation values $A'_{\theta 1}$ and $A'_{\theta 2}$ for first and second mode have been plotted against x/R. The above plots pertain to the pile top fixed against rotation conditions and the results are given for different identified Z_{max} cases of 2, 3, 5, 10 and 15. As before for any given Z_{max}, whatever be the pile-soil parameter, the values of normalis modal rotation in first and second mode could be obtained from these figures for the pile top fixed against rotation conditions.

3.10.4 NON-DIMENSIONAL CURVES FOR NORMALISED MODAL BENDING MOMENT

For the same five pile cases with Z_{max} = 1 and 5 t values of normalised bending moment Φ (M) at various x/R have been plotted in Fig 3.35 and Fig 3.36 respectively.

From these figures it is seen that at the same depth factors for a particular Z_{max} (say $Z_{max} = 1$, in Fig 3.35) the values of Φ (M) are in the ratios of the product of squares of the corresponding relative stiffness factors and the soil modulus, k.

Thus a non-dimensional curve is obtained by plotting the products of Φ (M) and $1/kR^2$ against x/R. In Fig 3.37 and 3.38 the quantity Φ (M)/kR^2 identified as Am_1 has been plotted against x/R for the five different pile cases with $Z_{max} = 1$ and $Z_{max} = 5$ respectively. As is seen, the five cases have converged to a single unique non-dimensional curve for the respective Z_{max} cases.

The variation of non-dimensional modal bending moment Am_1 with x/R for different Z_{max} values, have been given in Fig 3.39 for the first mode of vibration and pile top free to rotate conditions.

For the third mode following the similar principles the variation of Am_3 against x/R have been given in Fig 3.40 and 3.40 a. These non-dimensional curves have been obtained for the pile cases with $Z_{max} = 2$, 3, 5, 10 and 15. Herein, Am_3 is the quantity obtained by the product of normalised modal bending moment with $1/kR^2$ for the third mode.

Similarly, for the pile top fixed against rotation conditions the non-dimensional normalised bending moment Am_1

in the first and $A'm_2$ in the second modes of vibrations have been plotted in Fig 3.41 and Fig 3.42.

From these figures for any given soil-pile system the normalised bending moment variation in the first and second mode can be easily assessed.

3.10.5 NON-DIMENSIONAL CURVES FOR NORMALISED MODAL SHEAR:

In a similar manner as for other modal quantities it was observed that non-dimensional plots are obtained by plotting the product of normalised modal shear, Φ (S) with the quantity $1/k.R.$, against x/R. The process of non-dimensional curves so obtained for the cases examined have been presented in Fig 3.43 to Fig 3.46.

In Fig 3.47, 3.48 and 3.48a the quantity $\Phi(S)/k.R$ against x/R, identified as A_{S1} and A_{S3} have been plotted for the first and third modes respectively.

In Fig 3.49 and Fig 3.50 the variation of A'_{S1} and A'_{S2} with x/R have been plotted for the pile top fixed aga rotation conditions. The quantity A'_{s1}, A'_{s2} identify the first and second mode of vibrations respectively.

3.10.6 LIST OF NON-DIMENSIONAL CURVES FOR PILES EMBEDDED IN CLAY

For ready reference the various non-dimensional curves for different conditions have been listed below:

1. Pile Embedded in clay-Pile Top Free to Rotate (0% degree of fixity) - First Mode

Normalised Modal	Processing Factors	Identification of non-dimensional normalised modal quantities	Figure No.
(1)	(2)	(3)	(4)
Deflection $\Phi(y_1)$	$\Phi(y_1)$	A_{y1}	3.21
Rotation $\Phi(\theta_1)$	$\Phi(\theta_1).R.$	$A_{\theta 1}$	3.30
Bending Moment $\Phi(M_1)$	$\Phi(M_1).\frac{1}{kR^2}$	A_{m1}	3.39
Shear $\Phi(S_1)$	$\Phi(S_1)\ \frac{1}{kR}$	A_{s1}	3.47

2. Piles Embedded in clay-Pile Top Free to Rotate (0% Degree of Fixity)-Second Mode

Deflection $\Phi(y_2)$	$\Phi(y_2)$	A_{y2}	3.22
Rotation $\Phi(\theta_2).R$	$\Phi(\theta_2).R$	$A_{\theta 2}$	3.31

3. Piles Embedded in Clay-Pile Top Free to Rotate (0% Degree of Fixity) - Third Mode

Deflection $\Phi(y_3)$	$\Phi(y_3)$	A_{y3}	3.23 and 3.23a
Rotation $\Phi(\theta_3)$	$\Phi.(\theta_3).R$	$A_{\theta 3}$	3.32 and 3.32a
Bending Moment $\Phi(M_3)$	$\Phi(M_3)\frac{1}{kR^2}$	A_{m3}	3.40 and 3.40a
Shear $\Phi(S_3)$	$\Phi(S_3)\frac{1}{kR}$	A_{s3}	3.48 and 3.48a

(1)		(2)	(3)	(4)
4. Piles Embedded in Clay-Pile Top Fixed Against Rotation (100 % Degree of Fixity) - First Mode				
Deflection	$\Phi'(y_1)$	$\Phi'(y_1)$	A'_{y1}	3.24
Rotation	$\Phi'(\theta_1)$	$\Phi'(\theta_1).R$	$A'_{\theta 1}$	3.33
Bending Moment	$\Phi'(M_1)$	$\Phi'(M_1).\frac{1}{kR^2}$	A'_{m1}	3.41
Shear	$\Phi'(S_1)$	$\Phi'(S_1)\frac{1}{kR}$	A'_{S1}	3.49
5. Piles Embedded in clay-Pile Top Fixed Against Rotation-(100 % Degree of Fixity)-Second Mode				
Deflection	$\Phi'(y_2)$	$\Phi'(y_2)$	A'_{y2}	3.25
Rotation	$\Phi'(\theta_2)$	$\Phi'(\theta_2).R$	$A'_{\theta 2}$	3.34
Bending Moment	$\Phi'(M_2)$	$\Phi'(M_2).\frac{1}{kR^2}$	A'_{m2}	3.42
Shear	$\Phi'(S_2)$	$\Phi'(S_2)\frac{1}{kR}$	A'_{S2}	3.50

In the above list:

(i) $\Phi(y)$, $\Phi(\theta)$, $\Phi(M)$ and $\Phi(s)$ are the normalised modal quantities of deflection, rotation, bending moment an shear at any point along the pile length.

(ii) The normalised modal quantities are the product of mo values and the mode participation factor in a particu mode.

(iii) k is the soil modulus as defined for the case of soil modulus remaining constant with depth, having units of FL^{-2}.

(iv) R is the relative stiffness factor for the soil conditions pertaining to (iii) defined as $R = 4\sqrt{\frac{EI}{k}}$ having units of length.

(v) A_y, A_θ, A_m, A_s are the dimensionless normalised modal quantities. Each of these factors plotted against x/R result in a non-dimensional curve. Primes are used to identify the case of pile top fixed against rotation. Numerals in the subscript denote the mode numbers.

3.10.7 SPECIMEN OUT PUT

The complete out-put of the dynamic analysis for assessing the response of an example problem, with $Z_{max} = 3$ has been provided in Appendix I. The results pertain to pile top free to rotate conditions and the values of soil- pile parameters and other details are mentioned in the table itself.

Also, in Appendix 1, the computed values of A'_{y1} and A'_{m1} for fifteen pile cases with $Z_{max} = 3$

have been provided along with the soil-pile parameter and sustained vertical load values.

The above two information have been provided as examples, to bring out the effort involved in determining these non-dimensional coefficients for different modes of vibrations. Also, they show the unequiness of the computed values of non-dimensional quantities applicable to any soil-pile system.

3.11 DYNAMIC CHARACTERISTICS AND THE INFLUENCING FACTORS

3.11.1 NATURAL FREQUENCIES

3.11.1.1 First Mode of Vibration

In Fig 3.8 and Fig 3.11 the variations of frequency factor have been given for pile top free to rotate and pile top fixed against rotation conditions respectively. These curves have been drawn for different identified Z_{max} cases. In Fig 3.13 the variation of F_{CL1} and F'_{CL1} with Z_{max} have been given. Each circle in these figure represent fifteen different pile cases of varying soil-pile parameters.

Examination of these figures shows that the frequency factors F_{CL1} and F'_{CL1} are mainly dependent on Z_{max}, pile length in relation to the relative stiffness factor. The absolute length does not govern the behaviour singularly.

There is also increase in F_{CL1} and F'_{CL1} from short pile range to long pile range. Herein, for piles with $Z_{max} \geqslant 5$ there is no appreciable difference in the frequency factor values.

Considering the analogy, of cantilever structural idealisation the frequency factor should increase for shorter piles. However, the realistic end conditions and rigid body deformations of short pile ranges, disagree with such contentions.

From the examination of the above figures it is seen that for any particular Z_{max} case there is not much of variation in the frequency factor values with change in relative stiffness factors.

For both pile top free to rotate and fixed against rotation conditions we have the frequency factor defined as under.

$$F_{CL1} = w_{n1} \sqrt{\frac{W}{g} \cdot \frac{1}{k.R}} \quad \ldots 3.63$$

$$F'_{CL1} = w'_{n1} \sqrt{\frac{W}{g} \cdot \frac{1}{k.R}} \quad \ldots 3.64$$

where $\frac{W}{g} = M_t$, is the mass lumped at top, k and R are the soil-pile parameters.

both, w_{n1} and w'_{n1} are $\propto \frac{k^{0.5} R^{0.5}}{M_t^{0.5}}$ ... 3.65

$$\propto \frac{k^{0.375} EI^{0.125}}{M_t^{0.5}} \quad \text{... 3.66}$$

From the above relationship the following points are seen to be of significance.

1. Long Pile Ranges:

(i) For a given pile section increase in k, results in increase in natural frequencies. Physically a pile section embedded in stiffer soil would have greater natural frequency of vibration compared to the one in softer soil.

(ii) By definition $R = \sqrt[4]{\frac{EI}{k}}$ and $Z_{max} = \frac{L_s}{R}$. Now increase in k would result in reduction in R and hence increase of Z_{max} values of a pile of given section and length. However, the change in F_{CL1} and F'_{CL1} for $Z_{max} \geqslant 5$ is not appreciable. Therefore for long pile ranges practically the increase in 'k' results in increase of natural frequencies.

(iii) For a pile of given length embedded in given soil, increase in flexural stiffness, that is, in the

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pile section and reinforcements, would increase w_{n1} by $EI^{0.125}$ times. This would be valid only if, despite of increase in El the Z_{max} values fall in the long pile range.

(iv) However for a given pile length if sectional properties are improved there would be increase in value of R. This may result in the reduction of Z_{max} values. It is to be noted that with the reduction of Z_{max} the natural frequency also reduces.

(v) The increase in top mass reduces w_{n1} by $1/\sqrt{M_t}$ times.

For practical significance it may be concluded that to increase the natural frequency of long piles the soil stiffness may be increased and top mass should be reduced. Increase in pile sectional properties may not result in the required appreciable increase of w_{n1}.

If the piles are such that $2 \geqslant Z_{max} \leqslant 5$ the increase in stiffness of the soil does result in increase of w_{n1} to an appreciable degree. However in both these cases the increase in sectional prpperties (and hence EI) may not result in the increase of w_{n1} values to an appreciable extent.

3.11.1.2 Second Mode of Vibtation

Pile Top Free To Rotate Conditions:

In Fig 3.9 and 3.14 variations of frequency factor F_{CL2} with relative stiffness factor and maximum depth factor, Z_{max} have been given respectively.

The frequency factor in this mode has been defined as under:

$$F_{CL2} = w_{n2} \sqrt{\frac{\gamma d^2}{kg}} \qquad \ldots \; 3.67$$

Therefore we have

$$w_{n2} \propto \sqrt{\frac{kg}{d^2 \gamma}} \qquad \ldots \; 3.68$$

From these figures it may be seen that the values of F_{CL2} do not alter appreciably with changes in relative stiffness factor and Z_{max}. In fact the dynamic behaviour of piles in this mode under pile top free to rotate conditior has been rigid body type of motion and has been separately dealt with in a later section of this chapter and in Chapter

From the above relationship, it is seen that:-

1. The natural frequency w_{n2} increases $\sqrt{k}$ times as the stiffness of the soil is increased.
2. The natural frequency w_{n2} decreases with increase of pile section because of increase in weight per unit length of the pile.

3. The second mode frequency is independant of flexural stiffness EI of the pile.

<u>Pile Top Fixed Against Rotation Conditions</u>: In Fig 3.12 and 3.16 the variation of non-dimensional frequency F'_{CL2} with relative stiffness factor and Z_{max} have been given respectively.

As per definition of F'_{CL2} we have:

$$F'_{CL2} = w'_{n2} \sqrt{\frac{\nu d^2}{gk}} \qquad \ldots 3.69$$

$$\text{i.e., } w'_{n2} \text{ is } \propto \sqrt{\frac{g\,k}{\nu d^2}} \qquad \ldots 3.70$$

From the above relationship and the figures the following points are of significance:

1. In the second mode of vibrations F'_{CL2} values decrease with increase in Z_{max}, for same pile-sections. This is reverse of the trend under first mode of vibrations.

2. For piles with $Z_{max} \geqq 5$ there is no appreciable change in the F'_{CL2} values.

3. For any Z_{max}, the natural frequency decreases with the increase of weight per unit length of the pile.

4. Maintaining constant Z_{max}, the increase in soil stiffness results in the increase of w'_{n2} values.

5. For long pile ranges increase in soil stiffness results in the increase of w'_{n2}.

6. For short pile ranges increase in soil stiffness would increase w'_{n2}. However, the increase of Z_{max} by virtue of increase in 'k' would offset the net increase of w'_{n2} values.

2.11.1.3 Third Mode of Vibrations:

Pile Top Free To Rotate Conditions: Each of the pile cases in Table 3.1 have been analysed upto third mode of vibrations for the pile top free to rotate condition. In Fig 3.10 and 3.15 the variation of non-dimensional frequency factor F_{CL3} with relative stiffness factor and maximum depth factor Z_{max} have been given respectively. The factors influencing the natural frequency values, w_{n3} and the general trend in the analysed behaviour are similar to those for second mode vibrations under pile top fixed against rotation condition.

Apart from the above, presented, discussions it may also be noted that, for any soil pile conditions the natural frequencies under pile top fixed agianst rotation conditions are always greater than pile top free to rotate conditions.

3.11.2 FACTORS INFLUENCING DYNAMIC DISPLACEMENT

3.11.2.1 First Mode of Vibration:

The variation of non-dimensional normalised modal deflection A_{y1} and A'_{y1} have been plotted against depth factor x/R in Fig 3.21 and 3.24. These results pertain to pile top free to rotate and fixed against rotation conditions, respectively.

From these two figures it is seen that piles with $Z_{max} \leq 2$ display rigid body deformations, whereas piles with $Z_{max} \geq 5$ display flexural bending deformations. Moreover in the case of piles with $Z_{max} \geq 5$, there is no appreciable difference in the deformed shapes. The non-dimensional normalised rotation coefficients $A_{\theta 1}$ and $A'_{\theta 1}$ plotted against depth factor x/R in Fig 3.30 and Fig 3.33 respectively emphasise these points further.

From these figures it can be concluded that the modal displacements are primarily dependent on the length of the pile in relation to the relative stiffness factor (i.e., Z_{max}) It is also evident that the deflected shapes are similar to the deformed shapes of the piles subjected to static lateral loads apllied at the ground surface. The Z_{max} values govern the deformed shapes under the static conditions also. In the first mode of vibration, it is possible that the top mass controls the pile vibrations. The pile section acts as a massless elastic member deriving reactions from the surrounding soil depending on the movements at various points. In both these figures it is seen that at the bottom the displacements suffered by short piles are more than those of long piles. Especially for the pile top free to rotate conditions greater rotations are experience by short piles.

From the knowledge of normalised displacement mode shapes it is easy to estimate the various factors which govern the dynamic displacements.

We have from equation 3.61 the dynamic deflection as under:

$$Y_{(i)}^{(r)} = \Phi_{(i)}^{(r)}(y) \quad \Upsilon_{(r)} \quad Sd_{(r)} \qquad \ldots 3.61$$

where $Y_{(i)}^{(r)}$ - the dynamic deflection of the ith point in rth mode

$\Upsilon_{(r)}$ - the mode participation factor

$Sd_{(r)}$ - the spectral displacement corresponding to the period in the rth mode.

The product of $\Phi_{(i)}^{(r)}(y) . \Upsilon_{(r)}$ is the normalised modal deflection.

Now, from article 3.10.6, the processing factors of non-dimensional deflection coefficients A_{y1} and A'_{y1} are the normalised modal deflection quantities corresponding to the respective cases.

Therefore, we have, the dynamic deflection Y_1 in the first mode as:

$Y_1 = A_{y1} \; S_{d1}$ (for pile top free to rotate)

$Y_1 = A'_{y1} \; S'_{d1}$ (for pile top fixed against rotation)

Therefore it follows that:

$$Y \propto S_{d1} \qquad \ldots 3.72$$

Herein, for all practical purposes S_d can be considered to be proportional to the time period, Fig 3.7 (Newmark 1970).

From article 3.9, we have:

$$w_{n1} = F_{CL1} \sqrt{\frac{k.R.g}{W}}$$ (for pile top free to rotate)

$$w'_{n1} = F'_{CL1} \sqrt{\frac{kRg}{W}}$$ (for pile top fixed against rotation).

Thus we have for first mode of vibration the dynamic deflection Y_1 to be:

$$Y_1 \propto \sqrt{\frac{M_t}{kR}} \qquad \ldots 3.73$$

$$Y_1 \propto \frac{M_t^{0.5}}{k^{0.5} R^{0.5}} \qquad \ldots 3.74$$

$$\propto \frac{M_t^{0.5}}{k^{0.375} EI^{0.125}} \qquad \ldots 3.75$$

From the examination of the above equations and Fig 3.21 and Fig 3.24 the factors incluencing the dynamic deflections are discussed as below.

1. For piles falling under long pile ranges we have:

   (i) As the top mass increases the dynamic deflection is increased by $\sqrt{M_t}$ times.

(ii) Increase in 'k', stiffness of the soil results in reduction of dynamic deflection by $k^{0.375}$ times.

(iii) Increase in pile section or the flexural stiffnes EI results in the reduction of dynamic deflection by $EI^{0.125}$ times.

However, under border line cases it has to be borne in minc that increase of EI, though should decrease dynamic deflecti the reduction may not be appreciable because of reduction i Z_{max} values and hence w_{n1}. When w_{n1} is reduced the peric is increased to enhance S_{d1} values.

2. For piles falling under intermediate and short pile ranc we have:

   (i) The dynamic deflection increases $M_t^{0.5}$ times wit increase of top mass.

   (ii) The increase of soil stiffness k results in reduction of dynamic deflection.

However, in these length ranges it has to be kept in mind that increase of EI may not result in appreciable reductic of dynamic deflection. This is because of the reduction in Z_{max} values which follow the increase in EI. In these cases the most effective way is to increase the stiffness c the soil because this has the additional advantage of incre ing the Z_{max} value to increase w_{n1}., Which in turn reduces the values of S_{d1} and hence the Y_1.

From article 3.10.6 the non-dimensional rotation coefficients $A_{\theta 1}$ and $A'_{\theta 1}$ are as under:

$$A_{\theta 1} = \Phi(\theta_1) . R \qquad \ldots \ 3.76$$

$$A'_{\theta 1} = \Phi'(\theta_1) . R. \qquad \ldots \ 3.77$$

where $\Phi(\theta_1)$ is the normalised modal rotation in the first mode of vibration.

Therefore from equation 3.61 we have the dynamic rotation under the first mode as under:

$$\theta_1 = \Phi(\theta_1) . S_{d1} \qquad \ldots \ 3.78$$

$$\theta_1 = \frac{A_{\theta 1}}{R} . S_{d1} \qquad \ldots \ 3.79$$

That is $\theta_1 \propto \frac{S_{d1}}{R}$ ... 3.80

$$\propto \frac{M_t^{0.5}}{k^{0.5} . R^{1.5}} \qquad \ldots \ 3.81$$

$$\propto \frac{M_t^{0.5}}{k^{0.125} \ EI^{0.375}} \qquad \ldots \ 3.82$$

From the above equations and Fig 3.30 and Fig 3.33, we see that the general trend of the factors influencing dynamic rotation are same as those of dynamic deflections discussed in the earlier article.

However, the reduction in the dymamic rotation with increase in soil stiffness, k, is lesser compared to the previous case. The increase of EI reduces the dynamic rotation by $EI^{0.375}$ times. But as before the manifested increase in time period, and hence Sd_1 need be borne in mind.

3.11.2.2 Second Mode of Vibrations:

Pile Top Free To Rotate Condition: In Fig 3.22 the normali sed modal deflection has been plotted against depth factor for piles with Z_{max} = 2, 3, 5, 10 and 15. The above curves pertain to the pile top free to rotate conditions. Under these conditions, as it is seen from the figure a unic rigid body type mode has been displayed. In Fig 3.31 the normalised modal rotation coefficients have been plotted against depth factor x/R. As it is seen there is negligik slope difference between successive points. This type of unique mode shape has been a peculiarity of the piles embedded in clay with free head conditions. Irrespective of the Z_{max} values the form of mode shapes was similer. However the slope of the deformed shapes vary with Z_{max}. The motion is such that the piles rotate about the top mass It is possible that in these modes the piles are excited by virtue of the soil reaction forces and the flexural stiffn of the piles are not brought into effect at all. In Fig 3. and 3.14 the variations of frequency factors with R and Z_{max} further emphasise these points. As it was seen in the figures, there is negligible variation in F_{CI2} with R and Z_{max}. Hereafter, this mode for 0 % degree of fixity conditions shall be identified as a rigid body mode. Considering such rigid body motion, it is seen that the dynamic bending moments are not flexural moments. They are simply overturning moments and need not be considered in the dynamic stress effects.

<u>Pile Top Fixed Against Rotation Conditions</u>: In Fig 3.25, the non-dimensional normalised modal deflection coefficient A'_{y2} has been plotted against depth factor. In Fig 3.34 the non-dimensional normalised modal rotation coefficient $A'_{\theta 2}$ have been drawn against depth factor. These curves pertain to second mode of vibration for the case of pile top fixed against rotation. The solutions have been obtained for different identified Z_{max} cases of 2, 3, 5, 10 and 15.

The factors which influence the dynamic displacements under the above pile top fixity conditions can be assessed from Fig 3.25 and Fig 3.34 and the definition of A'_{y2} and $A'_{\theta 2}$, article 3.10.6.

3.11.2.3 Third Mode of Vibration

<u>Pile Top Free To Rotate Conditions</u>: In Fig 3.23, 3.23a the non-dimensional modal deflection coefficient A_{y3} have been plotted against depth factor x/R for the third mode for pile top free to rotate conditions. These curves pertain

to $Z_{max} = 2, 3, 5, 10$ and 15. This mode has been of flexural deformations for piles with $Z_{max} \geqslant 3$, unlike the earlier one where the deformations were purely rigid body type irrespective of the Z_{max} cases. As it is seen from the above figures there is a nodal point for each of the Z_{max} cases. The value of the coefficient at the pile top is far less than at the depths and maximum deflection is e experienced at the pile bottom in each Z_{max} case.

For a particular Z_{max} case irrespective of the soil-pile parameter values unique mode shapes are obtained.

In Fig 3.32 and 3.32a the non-dimensional normalised tion coefficient $A_{\theta 3}$ with depth factor has been given. As is seen in these figures for a particular Z_{max}, unique non-dimensional curves are obtained. Considering the variation of slope between different points along the pile it can be concluded that flexural deformations does take place in these modes.

From the figures presented above the influence of pile length on the dynamic displacements can be assessed. As it is seen both A_{y3} and $A_{\theta 3}$ increase with the increase in Z_{max} values. Thus rest of the factors remaining same the increase of pile length results in the increase of A_{y3} and at any x/R. But the behaviour for $Z_{max} = 10$ and 15 are sligh different.

The dynamic deflection and rotation in the third mode of vibration for any particular earthquake is given by:

$$Y_3 = A_{y3} \cdot S_{d3} \qquad \ldots 3.83$$

$$\theta_3 = \frac{A_{\theta 3}}{R} \cdot S_{d3} \qquad \ldots 3.84$$

In the above two equations A_{y3} and $A_{\theta 3}$ are dimensionless numbers.

Therefore,

$$Y_3 \propto S_{d3} \qquad \ldots 3.85$$

$$Y_3 \propto \frac{1}{w_{n3}} \qquad \ldots 3.86$$

$$\sqrt{\frac{\gamma}{g} \cdot \frac{d^2}{k}} \qquad \ldots 3.87$$

From the above relation we have:

1. The dynamic deflection increases with the increase in weight per unit length of the piles.
2. The dynamic deflection is reduced with the increase in soil stiffness by $\sqrt{k}$ times.
3. With the increase in pile length the dynamic deflection is increased, in two ways. As it is seen from Fig 3.23 the coefficient A_{y3} increases with increase in pile length, rest of the factors remaining same. Also as seen from Fig 3.10 and 3.15 with the increase in Z_{max} the w_{n3} also get reduced. This results in the increase of time period and hence S_{d3}.

4. However for piles with $Z_{max} \geqslant 5$, the deflections remai practically unaltered. Hence for long pile ranges rest of the factors remaining same the increase in Z_{max} woul not alter the dynamic displacements to any appreciable extent of practical significance.

5. Therefore, for long pile ranges increase of soil stiffne would reduce dynamic deflection.

From equation 3.84 the factors influencing dynamic rotation can be assessed, as follows:

$$\theta_3 \propto \frac{1}{R} \cdot S_{d3} \qquad \ldots 3.88$$

$$\propto \frac{1}{R} \cdot w_{n3} \qquad \ldots 3.89$$

$$\propto \frac{1}{R} \sqrt{\frac{\nu d^2}{gk}} \qquad \ldots 3.90$$

$$\propto \sqrt{\frac{d^2 \nu}{g}} \cdot \frac{1}{k^{0.25} \cdot EI^{0.25}} \qquad \ldots 3.91$$

From the above equation it is seen that:

1. The dynamic rotation in the second mode increases as th weight per unit length is increased.
2. For any particular Z_{max} the dynamic rotation is redu as the soil stiffness and flexural stiffness are increa
3. For long pile ranges increase in EI and k results in reduction of dynamic rotation.

3.11.3 FACTORS INFLUENCING DYNAMIC BENDING MOMENT AND SHEAR

3.11.3.1 First Mode of Vibrations:

The variation of non-dimensional normalised modal bending moment coefficient A_{m1} and A'_{m1} have been plotted in Fig 3.39 and Fig 3.41, respectively. These results pertain to pile top free to rotate and pile top fixed against rotation respectively.

From these two figures the following points can be inferred:

1. For pile top free to rotate conditions greater bending moments are experienced by long pile than short piles.
2. Under the above conditions the difference in the maximum bending moment values between $Z_{max} = 2$ and $Z_{max} = 3$ are greater, compared to those between $Z_{max} = 3$ and $Z_{max} = 5$.
3. In any pile case with $Z_{max} \geqslant 5$ there is no appreciable difference in bending moment values. Therefore $Z_{max} = 5$ is practically a long pile case.
4. In the case of pile top free to rotate the maximum bending moment for $Z_{max} = 2$, $Z_{max} = 3$, $Z_{max} = 5$ and $Z_{max} \geqslant 5$ occurs at depths of 0.650R, 1.0 R, 1.15 R and 1.20 R respectively.

5. After a depth of about 5 R the bending moment values are negligible.

6. In the case of pile top fixed against rotation the maximu bending moment occurs at the pile top.

From the knowledge of the normalised bending moment values it is easy to estimate the dynamic bending moment.

We have from equation 3.61 the dynamic bending moment in the first mode as under:

$$M_1 = \Phi(M_1).\ S_{d1} \qquad \ldots\ 3.92$$

where $\Phi(M_1)$ is the normalised bending moment (product of modal values and MPF).

From article 3.10.6, the processing factor for the non-dimensional bending moment coefficients A_{m1} and A'_{m1} are as under:

$$A_{m1} = \frac{\Phi(M_1)}{k\ R^2} \qquad \ldots\ 3.93$$

$$A'_{m1} = \frac{\Phi'(M_1)}{kR^2} \qquad \ldots\ 3.94$$

Therefore the dynamic bending moment is given by

$$M_1 = A_{m1} \times k\ R^2.\ S_{d1} \qquad \ldots\ 3.95$$

But A_{m1} is dimensionless, hence we have:

$$M_1 \propto k\ R^2\ S_{d1} \qquad \ldots\ 3.96$$

$$M_1 \propto k\, R^2 \frac{1}{w_{n1}} \quad \ldots\ 3.97$$

$$M_1 \propto k\, R^2 \cdot \frac{M_t^{0.5}}{k^{0.5}\, R^{0.5}} \quad \ldots\ 3.98$$

$$M_1 \propto k^{0.5}\, R^{1.5}\, M_t^{0.5} \quad \ldots\ 3.99$$

$$M_1 \propto k^{0.125} \cdot EI^{0.375} \cdot M_t^{0.5} \quad \ldots\ 3.100$$

From the examination of the above equations and Figs 3.39 and 3.41 the factors influencing the dynamic bending moments can be discussed as below:

1. Increase in the top mass increase the dynamic bending moment $\sqrt{M_t}$ times.

2. For long piles i.e $Z_{max} \geqslant 5$ we have:

   (i) Increase in soil stiffness k, results in the increase of dynamic bending moments by $k^{0.125}$ times. This is possible because at any pile section the dynamic bending moment is made up of inertia forces and soil reactions. Now for same pile section for same movement the reaction offered by stiffer soil would be greater than those of softer ones.

   (ii) Increase in EI results in the increase of dynamic bending moment in two eways. As per equation the dynamic bending moment would increase by $EI^{0.375}$ times. In addition the increase in EI results in the decrease of w_{n1} and hence increase of S_d values. However herein, it should be noted that as Z_{max} get reduced the maximum bending moment values also decrease.

In the above condition it has to be noted that though with softer soils the dynamic bending moment may decrease the displacements would be greater.

3. For piles falling under intermediate range lengths we have:

   (i) the dynamic bending moment increases with increase in soil stiffness. This occurs in two ways. Firstly by virtue of the influence of various parameters as shown in equation (3.100) and secondly because, increase in k results in increase in Z_{max} and hence greater values of A_{m1} as seen from Fig. 3.39.

   (ii) However, increase of EI causes M_1 values to reduce because of decrease in Z_{max} but at the same increase in M_1 may also result as given in Eq. 3.100.

In Fig 3.47 and 3.49 the variations of non-dimensional normalised modal shear coefficients A_{s1} and A'_{s1} with x/R have been provided. These curves pertain to pile top free to rotate and fixed against rotation conditions respectively.

For the first mode of vibrations the dynamic shear is given by :

$S_1 = A_{s1} . kR. S_{d1}$ ... (0% degree of fixity)

$S_1 = A'_{s1} . k.R. S'_{d1}$ ... (100% degree of fixity)

From the above relation we have:

$$S_1 \propto \frac{1}{w_{n1}} . k R \quad \ldots 3.101$$

$$\propto \frac{M_t^{0.5}}{k^{0.5} , R^{0.5}} . k. R. \quad \ldots 3.102$$

$$\propto M_t^{0.5} . k^{0.5} R^{1.5} \quad \ldots 3.103$$

$$\propto M_t^{0.5} k^{0.125} EI^{0.375} \quad \ldots 3.104$$

The influence of the various soil pile parameters on the dynamic shear are similar to those of bending moments discussed in the previous articles.

3.11.3.2 Higher Modes of Vibrations:

The values of non-dimensional normalised bending moment coefficients A_{m3} and A'_{m2} at various x/R have been given in Fig 3.40 , 3.40a and 3.42 respectively; pertaining to pile top free to rotate (third mode) and fixed against rotation conditions (for second mode).

The dynamic bending moment for these conditions is given by:

$M_3 = A_{m3} . k.R^2. S_{d3}$ (pile top free to rotate)

$M_2 = A'_{m2}.\ k.R^2\ S'_{d2}$ (pile top fixed against rotation)

Herein A_{m3} and A'_{m2} are dimensionless coefficients.

Therefore, we have for both these conditions:

$$\text{Both}\quad M_3 \text{ and } M'_2 \propto k\ R^2\ \frac{1}{w_{n2}} \qquad \ldots\ 3.105$$

$$\text{i.e.,}\quad \ldots \propto k\ R^2.\ \sqrt{\frac{\nu d^2}{g} \cdot \frac{1}{k}} \qquad \ldots\ 3.106$$

$$\ldots \propto k.\ \frac{EI^{0.5}}{k^{0.5}}\ \sqrt{\frac{d^2\nu}{g} \cdot \frac{1}{k}} \qquad \ldots\ 3.107$$

$$\propto \sqrt{\frac{\nu d^2}{g}} \cdot \frac{EI^{0.5}}{k^{0.5}} \cdot \frac{1}{k^{0.5}} \cdot k \qquad \ldots\ 3.108$$

$$\propto \sqrt{\frac{\nu d^2}{g}}\quad EI^{0.5} \qquad \ldots\ 3.109$$

From the above relation and the respective figures, we have:

1. The dynamic bending moment increases with weight per unit length.

2. Increase in EI results in increase of dynamic bending moment, which may be due to the increase of weight density

3. In the above conditions for a particular Z_{max} case dynamic bending moment is independent of soil stiffness. However increase in k results in increase of Z_{max} to reduce dynamic bending moment in second mode.

The variations of non-dimensional normalised modal shear coefficients λ_{s3} and λ'_{s2} have been plotted in Fig 3.48, 3.48a and 3.50 respectively. These curves pertain to pile top free to rotate (third mode) and pile top fixed against rotation (second mode) conditions.

We have the dynamic shear for these conditions as under:

$$S_3 = \lambda_{s3} . k.R. \; S_{d3} \qquad \ldots \; 3.110$$

$$S'_3 = \lambda'_{s2} . k.R. \; S'_{d2} \qquad \ldots \; 3.111$$

Herein, λ_{s3} and λ'_{s2} are dimensionless coefficients.

Therefore we have:

$$S_3 \text{ or } S'_2 \propto kR. \frac{1}{w_{n2}} \qquad \ldots \; 3.112$$

$$\propto k.R. \sqrt{\frac{\gamma d^2}{g} . \frac{1}{k}} \qquad \ldots \; 3.113$$

$$\propto k^{0.25} \, EI^{0.25} \sqrt{\frac{\gamma d^2}{g}} \qquad \ldots \; 3.114$$

Herein it can be seen that:

1. Increase in weight per unit length of pile increases the dynamic shear.
2. With the increase in soil stiffness dynamic shear is increased.
3. With the increase flexural stiffness of the pile dynamic shear is increased.

3.12 REMARKS ON THE METHOD OF ANALYSIS

It has been demonstrated that the physical(soil-pile)system could be idealised as a lumped mass-spring sys and that the dynamic characteristics of the physical syst could be conveniently assessed based on the dynamic analys of such an idealised model. The procedure adopted for ide lising and characterising the soil-pile interaction mechan has proved to be an effective tool. The chosen model is versatile in the sense that any pile cross section and so type could be easily accommodated.

The adopted transfer solution approach for determ the dynamic characteristics of the soil-pile system has be found to be a convenient procedure.

While performing the analysis there should be a p choice of, lumping procedure and the number of division po for lumping the physical system. For different pile cases the mass points were located at the mid points of each seg division. The number of masses were more than thirty in case. The above two precautions, it is believed, had minimised the possibilities of errors in lumping (Duncan (Karmon and Biot (1940)).

The computer programmes were executed in IBM 360- system and the execution was performed under double precis For such beam on elastic foundations problems it is necess to work in double precision to avoid rounding of errors. All the computation works concerning the present investigations in this chapter and the subsequent ones were always executed under double precision.

The adopted technique and the mathematical model could be considered to possess the following short comings:

1. Considering the non-linear behaviour of the physical systems, idealisation of the interaction effects through Winkler model may not be appropriate.
2. The real systems has far coupled mechanism whereas the adopted procedures consider only simple coupled systems.
3. The soil mass participating in vibrations has not been taken into account.
4. Time wise response has not been evaluated.

While adopting the presented procedure of analysis the above short comings have been fully recogonised but they have been ignored for the following reasons:

1. The primary concern in the present investigations has been to propose an easy but sufficiently accurate method of analysis for determining the dynamic characteristics of piles. By this the practising engineer would be greatly benefited.
2. In the given discritised model non-linear effects could be incorporated. But development of non-dimensional

solutions based on the dynamic analysis of so many pile cases could have been difficult considering the amount of computer time by such considerations.

3. The comparison of lumped mass analysis and continuous system solutions have been made (in Chapter IV) to asses the significance of far coupled systems.

4. The soil mass participating in vibrations has not been considered based on the work of Penzien et al (1964), Hakuno (1973) and Prakash et al (1973).

5. Though time-wise response computations predict the over all dynamic response to a better degree, they fail to give insight to the contribution of higher modes in the overall response of the systems. The assessment of individual modal contributions has been considered to b a matter of greater significance. Because, the greater contribution of first mode response supports the adoptio of pseudo-static design procedures.

3.13 CONCLUDING REMARKS

In this Chapter it has been shown that the dynamic response of pile foundations could be predicted successfully by idealising the soil-pile systems and the interaction effects by discretised mathematical models.

The transfer solution approach and the numerical technique thereoff has been found to be a successful procedure for predicting the pile response for desired end conditions.

In this chapter based on such analysis non-dimensional solutions have also been developed for predicting the dynamic response of piles embedded in clay type soils, assuming soil modulus to remain constant with depth.

These non-dimensional solutions cover the following broadly classified problems of practical significance:

1. Pile top free to rotate conditions.
2. Pile top fixed against rotation conditions.
3. Piles with non-dimensional depth factor, Z_{max} = 1, 2, 3, 5, 10 and 15.

Using these non-dimensional curves, it is possible to predict the dynamic response of any given pile section of known length embedded in soils in which the soil modulus remain constant with depth. The pile can be subjected any desired sustained load along with desired fixity conditions at the top.

These non-dimensional design curves have been developed for significant modes of vibrations and they facilitate:

1. Determination of natural frequencies of vibrations.
2. Determination of normalised modal quantities of deflection, rotation, bending moment and shear along the entire length of the pile.

Based on the investigations of the dynamic behaviour of piles embedded in soils, in which soil modulus remain constant with depth, the following points are considered as significant:

1. The dynamic behaviour of the piles are dependent on the

   (i) relative stiffness of the pile soil system (structural stiffness of the pile in relation to the soil stiffness).

   (ii) length of the pile in relation to the relative stiffness factor.

   The absolute length of the piles do not govern the behaviour singularly. The above is true for natural frequencies of vibrations as well as normalised modal values of displacements, bending moment and shear along the entire length of the piles.

2. Under first mode of vibrations the variations of normali modal deflection along the length of the pile follows th static deflection shape.

3. The first mode of vibrations contribute significantly to the overall response.

4. Under dynamic conditions piles with $Z_{max} \leq 2$ display rigid body deformations wheeeas piles with $Z_{max} \geq 5$ display flexural bending.

5. For a given soil-pile parameters $Z_{max} = 5$ can be considered as a limiting value of long pile range. Increase in pile length beyond this length loses significance as far as dynamic behaviour is concerned.

6. For pile top fixed against rotation conditions the envisaged dynamic displacements may be smaller than under pile top free to rotate conditions, because of the smaller period of vibrations in the former case.

7. For pile top free to rotate conditions the maximum bending moment occurs at some point below the ground surface whereas it occurs at top for pile top fixed against rotation conditions.

8. For long pile ranges under pile top free to rotate conditions the maximum bending moment occurs at a depth of 1.20 R from the ground surface.

9. With the increase in lumped mass at top or the super structure load:

   (i) the natural freauencies under first mode of vibrations are reduced by $\frac{1}{\sqrt{M_t}}$ times.

   (ii) the induced values of dynamic displacements, bending moments and shear are increase by $\sqrt{M_t}$ times.

10. Similar is the effect of weight per unit length of the pile under higher modes of vibrations.

11. For any mode the increase in soil stiffness results in the reduction of pile displacements.

12. Under pile top free to rotate conditions there is a possibility of a peculiar rigid body motion. The natural frequencies for a given soil pile system, during the rigid body motion, is independant of pile length and hence the alterations in Z_{max}.

    Along the pile length, under this mode the normalised modal deflection values vary in a straight line fashion and the rotation difference between any two points are near zero.

13. The correctness of the lumped mass solutions need be checked by some other independent solutions such as continuous system analysis.

CHAPTER - IV

METHOD OF ANALYSIS WITH CONTINUOUS SYSTEM MODELS FOR PILES EMBEDDED IN CLAY

4.1 INTRODUCTION

In this chapter the solutions for the dynamic characteristics of the piles embedded in soils in which the soil modulus remain constant with depth have been presented, treating the soil pile system as a continuous system model. In the previous chapter III, this problem has been solved by considering the system as ne with lumped masses. Since, no data is available on the actual behaviour of the piles subjected to dynamic loads, this solution shall serve as a check on the solution already obtained and vice versa. Herein, solutions have been presented only for the pile top free to rotate conditions.

4.2 APPROACH AND ASSUMPTIONS

The adopted mathematical model treating the piles as a continuous system is shown in Fig 4.1.

The dynamic characteristics of soil-pile systems are determined considering the free vibration characteristics of such idealised model. The mode of operation is similar to the lumped-mass model, but exact solutions of the frequency determinants, the various modal quantities and the mode participation factors have been developed. The adopte end conditions have been compatible with those of the physic system. Numerical technique has not been resorted to obtain the solutions. The dynamic response has been evaluated treating the ground motions to be applied at the base of the model. The response of the various quantities in each mode has been considered separately, and statistical root mean square addition has been performed, to assess the overal response. Such proposed procedures and the solutions involve the following assumptions:

1. The pile vibrates in its own plane.
2. The pile material exhibits linear elastic behaviour.
3. Plane cross sections remain plane during and after bendi
4. Axial deformations are of negligible quantity.
5. The pile mass and the fluxural stiffness are considered to be distributed.
6. The soil is treated as a homogeneous elastic medium.
7. The soil stiffness is considered to be distributed uniformly and continuously along the pile length.
8. The modulus of subgrade reaction concept is considered to be valid.

4.3 DIFFERENTIAL EQUATION AND SOLUTIONS

The differential equation describing the flexural vibrations of piles embedded in soil is given by:

$$EI \frac{\partial^4 y}{\partial x^4} + \frac{\nu A}{g} \frac{\partial^2 y}{\partial t^2} + ky = 0 \qquad \ldots 4.1$$

where

y , is the displacement perpendicular to the pile axis

x , is the depth co-ordinate

ν , is the weight density of the pile

A , the uniform area of cross section of the pile

EI , uniform flexural stiffness of the pile section

k , soil modulus for $k_x = k$ constant case

Considering the system to be displaced from its equilibrium position, it would be vibrating freely in classical normal mode of vibration.

Considering the free vibration to be of the form:

$$y = X(x) \text{ Sin } pt \qquad \ldots 4.2$$

Substitution in eqn. 4.1 results

$$EI \frac{d^4X}{dx^4} - \frac{\nu A}{g} p^2 X + kX = 0 \qquad \ldots 4.3$$

where 'p' is the circular natural frequency in radians per sec.

Assuming a solution

$$X = A' e^{mx} \qquad \ldots 4.4$$

where m = characteristic root and A' certain constant. Substituting equation 4.4 in equation 4.3 the following characteristic equation is obtained:

$$m^4 - \beta^4 + \frac{k}{EI} = 0 \qquad \ldots\ 4.5$$

Where $\beta^4 = \frac{\nu A}{g.EI} . p^2$

Equation 4.5 will give four values of root m.

It is seen that the quantity β^4 is a function of the frequency. The deflected shape function would vary with the frequency under consideration.

That is, depending upon the values of the natural frequencies of vibration the following three cases would arise.

<u>Case</u> (i) $\beta^4 > \frac{k}{EI}$, resulting in two real and two immaginary unequal roots.

<u>Case</u> (ii) $\beta^4 = \frac{k}{EI}$, resulting in all four equal roots.

<u>Case</u> (iii) $\beta^4 < \frac{k}{EI}$, resulting in four complex roots.

The solutions for each case has to be considered separately.

4.3.1 POSITI CASE SOLUTIONS

Using the symbol $\lambda = \beta^4 - \frac{k}{EI}$, for the case of $\beta^4 > \frac{k}{EI}$, we have λ as positive quantity. Identifying this case as the positi case, we have the solutions as developed in the following sections.

Rewriting equation 4.5 we have

$$m^4 = \beta^4 - \frac{k}{EI}$$

$$m^4 = \lambda$$

Noting λ as a positive real quantity we have four values for 'm':

$$m_{1,2} = \pm \sqrt{\lambda} \qquad \ldots 4.6$$

$$m_{3,4} = \pm i \sqrt{\lambda} \qquad \ldots 4.7$$

Knowing the four roots of the characteristic equations, the general deflected shape can be expressed in the form

$$X = A \operatorname{Cosh} \sqrt{\lambda}\, x + B \operatorname{Sinh} \sqrt{\lambda}\, x + C \operatorname{Cos} \sqrt{\lambda}\, x + D \operatorname{Sin} \sqrt{\lambda}\, x \qquad \ldots 4.8$$

Where, A, B, C, D are the four undetermined coefficients.

Replacing $\sqrt{\lambda}$ by α we have

$$X = A \operatorname{Cosh} \alpha x + B \operatorname{Sinh} \alpha x + C \operatorname{Cos} \alpha x + D \operatorname{Sin} \alpha x \qquad \ldots 4.9$$

4.3.1.1 Boundary Conditions and Frequency Determinant :

Applying the four boundary conditions for the bending moment and shear at the top and bottom of the pile, we have :

(i) At $x = o$, bending moment = 0; $\left(\frac{d^2X}{dx^2}\right)_{x = o} = 0$

(ii) At $x = L$, bending moment = 0; $\left(\frac{d^2X}{dx^2}\right)_{x = L} = 0$

(iii) At $x = o$, Shear force equals the inertia force at the to

The inertia force at the top, is mass at top multiplied by acceleration.

That is, $- EI \left(\frac{d^3X}{dx^3}\right)_{x = o} = - M_t p^2 (X)_{x = o}$

Where, M_t is the top mass.

(iv) At $x = L$, the shear at the bottom is zero which gives

$$- EI \left(\frac{d^3X}{dx^3}\right)_{x = L} = 0$$

These boundary conditions, when applied to the equation 4.9, would result in the homogenous equations in terms of the undetermined coefficients, the natural frequency and the pile-soil parameters.

It is easily seen that the application of the boundary conditions results in the following two homogeneous equations:

$$A (\text{Cosh}\,\alpha L - \text{Cos}\,\alpha L + \epsilon\,\text{Sin}\,\alpha L) + B (\text{Sinh}\,\alpha L - \text{Sin}\,\alpha L) = 0 \quad \ldots 4.10$$

$$A(\sinh \alpha L + \sin \alpha L + \epsilon \cos \alpha L) + B(\cosh \alpha L - \cos \alpha L) = 0 \quad \ldots 4.11$$

The determinant of the coefficients of A and B should vanish for the natural frequency of vibrations of the model. Thus we have

$$DT = (\cosh \alpha L - \cos \alpha L + \epsilon \sin \alpha L)(\cosh \alpha L - \cos \alpha L)$$
$$- (\sinh \alpha L - \sin \alpha L)(\sinh \alpha L + \sin \alpha L + \epsilon \cos \alpha L) \quad \ldots 4.12$$

where

$\epsilon = \frac{2 M_t p^2}{EI \alpha^3}$ and α is a function of 'p', and pile-soil properties.

For a given soil-pile system of known, EI, ϵ A, L and soil-modulus k, in equation 4.12 the determinant value could be obtained for any assumed values of p. If the assumed value of p, is one of the natural frequencies as mentioned earlier the determinant - DT would be zero. The curve of p and DT could be plotted in a computer and the various natural frequencies in different modes of vibration can be easily determined.

4.3.1.2 Modal Quantities :

Once the correct value of the natural frequencies are determined, the unknown quantity 'α' and the other constants are easily evaluated. Then the pile is divided into very small segments and the deflection, rotation or slope bending moment and shear at each point along the pile length is easily evaluated using the following equations.

$$\Phi(y) = (\cosh \alpha x + \cos \alpha x - \epsilon \sin \alpha x) + \mu (\sinh \alpha x + \sin \alpha x) \quad \ldots 4.13$$

$$\Phi(\theta) = \alpha(\sinh \alpha x - \sin \alpha x - \epsilon \cos \alpha x) + \mu (\cosh \alpha x + \cos \alpha x) . \alpha \quad \ldots 4.14$$

$$\Phi(M) = -EI \alpha^2 \left[(\cosh \alpha x - \cos \alpha x + \epsilon \sin \alpha x) + \mu (\sinh \alpha x - \sin \alpha x) \right] \quad \ldots 4.15$$

$$\Phi(S) = -EI \alpha^3 \left[(\sinh \alpha x + \sin \alpha x + \epsilon \cos \alpha x) + \mu (\cosh \alpha x - \cos \alpha x) \right] \quad \ldots 4.16$$

$$\text{Where } \mu = \frac{-\cosh \alpha L - \cos \alpha L + \epsilon \sin \alpha L}{\sinh \alpha L - \sin \alpha L}$$

In the above equation for the given pile length, known soil-pile parameters, the top mass and the natural frequencies each of the modal quantities have been evaluated.

4.3.1.3 Mode Participation Factors :

The adopted procedure of assessing the dynamic characteristics of the piles has the advantage of evaluating the individual contribution of each mode to the overall response of the pile-soil-systems subjected to dynamic loads. In order to achieve this the mode participation factor has been determined in each mode of vibration.

The mode participation factor is defined as under :

$$C_{(r)} = \frac{\int_o^L \frac{\gamma A}{g} X \, dx + M_t \, (X)_{x=0}}{\int_o^L \frac{\gamma A}{g} \; X^2 \, dx + M_t (X^2)_{x=0}} \qquad \ldots \; 4.17$$

For the positi case under consideration the mode participation factor in any rth mode is:

$$C_{(r)} = \frac{\int_o^L \frac{\gamma A}{g} X \, dx + 2 \, M_t}{\int_o^L \frac{\gamma A}{g} X^2 \, dx + 4 \, M_t} \qquad \ldots \; 4.18$$

Where,

$$X = (\cosh \alpha x + \cos \alpha x - \epsilon \, \sin \alpha x) + \mu \, (\sinh \alpha x + \sin \alpha x) \qquad \ldots \; 4.19$$

$$X^2 = \left[(\cosh \alpha x + \cos \alpha x - \epsilon \, \sin \alpha x) + \mu \, (\sinh \alpha x + \sin \alpha x) \right]^2 \qquad \ldots \; 4.20$$

The various integral quantities were separately evaluated and incorporated in the programme.

4.3. 4.3.2 EQUAL ROOT CASE

From equation 4.5 we see

$$m^4 = \beta^4 - \frac{k}{EI}$$

There exists a possibility that for a particular 'p' value

$$\beta^4 = \frac{k}{EI} \qquad \ldots \; 4.21$$

This means the roots of the characteristic equations to be as under :

$$m_{1,2,3,4} = 0$$

For such conditions of the four roots of the characteristics equations, we have the general deflected shape in the form

$$X = A + Bx + Cx^2 + Dx^3 \qquad \ldots 4.22$$

Applying the boundary conditions we have

$$\frac{dX}{dx} = B + 2Cx + 3Dx^2$$

$$\frac{d^2X}{dx^2} = 2C + 6Dx; \quad \left(\frac{d^2X}{dx^2}\right)_{x=0} = 0; \quad C = 0$$

$$\frac{d^3X}{dx^3} = 6D \; .$$

$$\left(EI \frac{d^3X}{dx^3}\right)_{x=0} = M_t p^2 A$$

$$A = \frac{6DEI}{M_t p^2}$$

$$\left(\frac{d^2X}{dx^2}\right)_{x=L} = 0; \quad D = 0, \quad A = 0$$

We have, therefore

$$X = Bx \qquad \ldots 4.23$$

4.3.3 NEGATI CASE

In this case the possible condition of $\beta^4 < \frac{k}{EI}$ is considered.

This results in

$$m^4 = -\lambda \quad , \text{ where } \lambda = \frac{k}{EI} - \beta^4$$

Applying De Moivre's theorem and letting :

$$r\,(\text{Cos}\,\theta + i\ \text{Sin}\ \theta) = -\lambda \qquad \ldots\ 4.24$$

where $r\ \text{Cos}\ \theta = -\lambda$ ... 4.25

$r\ \text{Sin}\ \theta = 0$ ... 4.26

We get the roots of the characteristic equations for case (iii) as under :

$$m_1 = \lambda^{1/4}\ (\text{Cos}\ \pi/4 + i\ \text{Sin}\ \pi/4) \qquad \ldots\ 4.27$$

$$m_2 = \lambda^{1/4}\ (\text{Cos}\ 3\pi/4 + i\ \text{Sin}\ 3\,\pi/4) \qquad \ldots\ 4.28$$

$$m_3 = \lambda^{1/4}\ (\text{Cos}\ 5\,\pi/4 + i\ \text{Sin}\ 5\,\pi/4) \qquad \ldots\ 4.29$$

$$m_4 = \lambda^{1/4}\ (\text{Cos}\ 7\,\pi/4 + i\ \text{Sin}\ 7\,\pi/4) \qquad \ldots\ 4.30$$

Simplification of the above equations result in:

$$m_1 = \alpha\,(1+i) \qquad \ldots\ 4.31$$

$$m_2 = \alpha\,(1-i) \qquad \ldots\ 4.32$$

$$m_3 = -\alpha\,(1+i) \qquad \ldots\ 4.33$$

$$m_4 = -\alpha\,(1-i) \qquad \ldots\ 4.34$$

Where, $\alpha = 0.707 \times \lambda^{1/4}$

For the above four roots of the characteristics equations, the general deflected shape can be expressed in the form :

$$X = A \operatorname{Cosh} \alpha x \operatorname{Cos} \alpha x + B \operatorname{Cosh} \alpha x \operatorname{Sin} \alpha x + C \operatorname{Sinh} \alpha x \operatorname{Cos} \alpha x + D \operatorname{Sinh} \alpha x \operatorname{Sin} \alpha x \quad \ldots \; 4.35$$

Where, A, B, C, D are the four undetermined coefficients

4.3.3.1 Boundary Conditions and Frequency Determinant :

Applying the four boundary conditions at the top and bottom of the pile (as given under positive case) in equation 4.35, the result is two undetermined coefficients, the natural frequency and the pile soil parameters.

The two homogenous equations are as under:

$$B \left(\operatorname{Sinh} \alpha L \operatorname{Cos} \alpha L - \frac{1}{\epsilon} \operatorname{Sinh} \alpha L \operatorname{Sin} \alpha L\right) + C \left(\frac{1}{\epsilon} \operatorname{Sinh} \alpha L \operatorname{Sin} \alpha L - \operatorname{Cosh} \alpha L \operatorname{Sin} \alpha L\right) = 0 \quad \ldots \; 4.36$$

$$B \left(\operatorname{Cosh} \alpha L \operatorname{Cos} \alpha L - \operatorname{Sinh} \alpha L \operatorname{Sin} \alpha L - \frac{1}{\epsilon} \operatorname{Cosh} \alpha L \operatorname{Sin} \alpha L - \frac{1}{\epsilon} \operatorname{Sinh} \alpha L \operatorname{Cos} \alpha L \right) + C \left(-\operatorname{Cosh} \alpha L \operatorname{Cos} \alpha L - \operatorname{Sinh} \alpha L \operatorname{Sin} \alpha L + \frac{1}{\epsilon} \operatorname{Cosh} \alpha L \operatorname{Sin} + \frac{1}{\epsilon} \operatorname{Sinh} \alpha L \operatorname{Cos} \alpha L \right) = 0 \quad \ldots \; 4.37$$

where $$\epsilon = \frac{M_t \, p^2}{2EI\alpha^3} \quad \ldots \; 4.38$$

and L = the embedded length of pile.

Putting $(\text{Sinh}\,\alpha L\,\text{Cos}\,\alpha L - \frac{1}{\epsilon}\,\text{Sinh}\,\alpha L\,\text{Sin}\,\alpha L) = A_1$

$$(\frac{1}{\epsilon}\,\text{Sinh}\,\alpha L\,\text{Sin}\,\alpha L - \text{Cosh}\,\alpha L\,\text{Sin}\,\alpha L) = B_1$$

$$(\text{Cosh}\,\alpha L\,\text{Cos}\,\alpha L - \text{Sinh}\,\alpha L\,\text{Sin}\,\alpha L - \frac{1}{\epsilon}\,\text{Cosh}\,\alpha L\,\text{Sin}\,\alpha L - \frac{1}{\epsilon}\,\text{Sinh}\,\alpha L\,\text{Cos}\,\alpha L) = C_1$$

$$(-\text{Cosh}\,\alpha L\,\text{Cos}\,\alpha L - \text{Sinh}\,\alpha L\,\text{Sin}\,\alpha L + \frac{1}{\epsilon}\,\text{Cosh}\,\alpha L\,\text{Sin}\,\alpha L + \frac{1}{\epsilon}\,\text{Sinh}\,\alpha L\,\text{Cos}\,\alpha L) = D_1$$

Putting the coefficients of B and C in equation 4.36 as A_1 and B_1 respectively and the coefficients of B and C in equation 4.37 as C_1 and D_1 respectively, we get the determinant as under :

$$A_1\,D_1 - B_1\,C_1 = DT \qquad \ldots\ 4.39$$

For a chosen pile in equation 4.39 all the other quantities are known except the circular natural frequency, 'p'. If 'p', is one of the natural frequencies the determinant would have zero value. As before the curve between p and DT could be plotted in a computer and the natural frequencies in different modes of vibrations have been obtained.

4.3.3.2 Modal Quantities :

Once the correct value of the natural frequency is determined for a particular mode, the unknown quantity α, and the other constants are easily evaluated. Then the pile is divided into very small segments and the quantities viz. deflection, rotation, bending moment and shear at each point along the pile length is easily evaluated for diffe: modes using the following equations :

$$\Phi(y) = \left(-\frac{\mu+1}{\epsilon}\,\mathrm{Cosh}\,\alpha x\,\mathrm{Cos}\,\alpha x - \mu\,\mathrm{Cosh}\,\alpha x\,\mathrm{Sin}\,\alpha x + \mathrm{Sinh}\,\alpha x\,\mathrm{Cos}\,\alpha x\right) \quad \ldots\ 4.40$$

$$\Phi(\theta) = \left(-(\mu+1)\,\mathrm{Sinh}\,\alpha x\,\mathrm{Sin}\,\alpha x + (1-\mu)\,\mathrm{Cosh}\,\alpha x\,\mathrm{Cos}\,\alpha x - \frac{\mu+1}{\epsilon}\,\mathrm{Sinh}\,\alpha x\,\mathrm{Cos}\,\alpha x + \frac{\mu+1}{\epsilon}\,\mathrm{Cosh}\,\alpha x\,\mathrm{Sin}\,\alpha x\right)\alpha \quad \ldots\ 4.41$$

$$\Phi(M) = \left(-\mu\,\mathrm{Sinh}\,\alpha x\,\mathrm{Cos}\,\alpha x - \mathrm{Cosh}\,\alpha x\,\mathrm{Sin}\,\alpha \quad + \frac{\mu+1}{\epsilon}\,\mathrm{Sinh}\,\alpha x\,\mathrm{Sin}\,\alpha x\right)(-2EI\,\alpha^2) \quad \ldots\ 4.42$$

$$\Phi(S) = \left(-(\mu+1)\,\mathrm{Cosh}\,\alpha x\,\mathrm{Cos}\,\alpha x + (\mu-1)\,\mathrm{Sinh}\,\alpha x\,\mathrm{Sin}\,\alpha x + \frac{\mu+1}{\epsilon}\,\mathrm{Cosh}\,\alpha x\,\mathrm{Sin}\,\alpha x + \frac{\mu+1}{\epsilon}\,\mathrm{Sinh}\,\alpha x\,\mathrm{Cos}\,\alpha x\right)(-2EI\,\alpha^3) \quad \ldots\ 4.43$$

where,

$$\mu = \frac{\left(\frac{1}{\epsilon}\,\mathrm{Sinh}\,\alpha L\,\mathrm{Sin}\,\alpha L - \mathrm{Cosh}\,\alpha L\,\mathrm{Sin}\,\alpha L\right)}{\left(\mathrm{Sinh}\,\alpha L\,\mathrm{Cos}\,\alpha L - \frac{1}{\epsilon}\,\mathrm{Sinh}\,\alpha L\,\mathrm{Sin}\,\alpha L\right)} \quad \ldots\ 4.44$$

In the above equation for the given pile length, known soi. pile parameters, the top mass and the predicted natural frequencies each of the modal quantities are easily evaluat

4.3.3.3 Mode Participation Factors :

For the Negati case under consideration the mode participation factor, $C_{(r)}$, in any rth mode is as under:

$$\frac{\int_0^L \frac{\gamma A}{g} \cdot X \, dx - M_t \cdot \frac{\mu+1}{\epsilon}}{\int_0^L \frac{\gamma A}{g} \cdot X^2 \, dx + M_t \left(\frac{\mu+1}{\epsilon}\right)^2} \quad \ldots \; 4.45$$

where $X = -\frac{\mu-1}{\epsilon} \cdot \text{Cosh}\, \alpha x \;\; \text{Cos}\, \alpha x - \mu \;\text{Cosh}\, \alpha x \;\; \text{Sin}\, \alpha x$

$$+ \text{Sinh}\, \alpha x \; \text{Cos}\, \alpha x \,) \quad \ldots \; 4.46$$

4.4 COMPUTER PROGRAMMES

The determination of the dynamic response of the soil-pile system with the above model and for the mentioned technique involve the following steps:

(i) Evaluating the characteristic roots of the equation and checking whether the quantity is positive or negative for the assumed natural frequency.

(2) If positive, the solutions suggested in positive case, case (i) has been followed to obtain:

(a) the frequency determinant and the natural frequency.

(b) the modal quentities at each of the natural frequencies.

(c) the mode participation factors at each mode of vibration.

(3) If the frequency levels or such that the quantity is negative, the solutions suggested in the negative has been followed to obtain again the dynamic characteristic of the piles.

All the above three operations have been programmed in Fortran IV language. The programme comprised of one main and two sub routines to do the operations of the positi and negati case when needed. The process was quite complicated because at each and every stage of frequency increment, the applicability of the positi or negati solution have to be checked.

The convergence as well as evaluation of the modal quantities have been quite rapid. For one pile case the time taken was around 25 sec.

As the process involves exponential and hyperbolic functions and as the problem is a beam on elastic foundation problem, it is necessary to work in double precision.

4.5 NON-DIMENSIONAL SOLUTIONS

4.5.1 VARIABLES

The different pile cases which were analysed using continuous system analysis (model) are listed in Table 4.1. The dynamic response for each pile problem was evaluated only for pile top free to rotate conditions. Solutions for these problems have also been obtained using lumped

Table 4.1

Details of Analysed Pile Cases Considering Soil Modulus To Remain Constant With Depth

Relatice stiffness factor, R in metres	Diameter in metres	Soil Modulus in T/m^2	Flexural Stiffness EI Tm^2	Remarks
1.0	0.30	477.13	0.477×10^3	1. In each case the maximum depth factor, Z_{max} = 1,2,3,5, 10 and 15 were considered. 2. The sustained vertical load was varied in each case. The value was calculated considering frictional and end bearing resistance using Terzaghi's (1943) theory. 3. Each pile was analysed for pile top free rotate conditions.
1.25	0.30	195.43	0.477×10^3	
1.5	0.30	94.25	0.477×10^3	
1.0	0.40	1507.96	0.151×10^4	
1.25	0.40	617.66	0.151×10^4	
1.50	0.40	297.87	0.151×10^4	
1.0	0.50	3681.55	0.368×10^4	
1.25	0.50	1507.96	0.368×10^4	
2.0	0.50	230.10	0.368×10^4	
1.5	0.60	1507.96	0.763×10^4	
2.0	0.60	477.13	0.763×10^4	
1.25	0.60	3126.92	0.763×10^4	
1.50	0.70	2793.69	0.141×10^5	
2.0	0.70	883.94	0.141×10^5	
3.0	0.70	174.61	0.141×10^5	

mass analysis and detailed discussions regarding these results has already been presented in Chapter III.

The dynamic response of the different pile cases based on continuous analysis include:

1. Determination of natural frequencies for significant modes of vibrations.
2. Under these modes of vibrations assessment of normalised modal quantities of deflection, rotation, bending moment and shear along the entire length of the pile.

In order to achieve the above, the derived solutions (article 4.3) of : (i) Positi case, (ii) Equal Root case (iii) Negati case and the pertaining computer programmes, were utilised.

4.5.2 NON-DIMENSIONAL CURVES FOR NATURAL FREQUENCIES:

The examination of the dynamic response of all the ninety pile case of Table 4.1, resulted in the definition of non-dimensional frequency factors in different modes of vibrations. It was seen that the components of the frequency factors in different modes of vibrations were similar to article 3.9 of lumped mass solutions.

For the case of continuous system solutions, the non-dimensional frequency factors have been defined below.

Mode No.	Identification	Components
First	F_{cc1}	$w_{n1} \sqrt{\frac{W}{g} \cdot \frac{1}{k.R}}$
Second	F_{cc2}	$w_{n2} \sqrt{\frac{}{g} \cdot \frac{\gamma d^2}{k}}$

In the above list:

w_n, in radians per sec., is the circular natural frequency of the systems as obtained using continuous system analysis. Given subscripts identify the mode under consideration.

F_{cc1} Frequency factor, a dimensionless number. The first letter 'c' denotes the clay case and the second identifies the use of continuous system models. The numerals in the subscript indicate the mode numbers.

In Fig 4.2 and Fig 4.3 the variations F_{cc1} and F_{cc2} with relative stiffness factor, R have been provided. The variations of these frequency factors with non-dimensional depth factor Z_{max} have been provided in Fig 4.4 and Fig 4.5 for first and second modes of vibrations respectively.

It is to be noted herein, that the frequency factors F_{cc1} and F_{cc2} are based on the soil-pile parameters (Table 4.1) and the solutions obtained with positi case and Negati case.

The natural frequencies which were obtained for the equal root case have been identified and treated as critical frequency case. The frequencies under this condition has been treated separately.

4.5.3 NON-DIMENSIONAL CURVES FOR NORMALISED MODAL QUANTITIES

The analysis of each of the pile cases of Table 4.1 resulted in the values of normalised modal: deflection, rotation, bending moment and shear along the entire length of the pile under different modes of vibrations. These quantities have been evaluated for piles with Z_{max} = 1, 2, 3, 5, 10 and 15. It was observed that as in the case of lumped mass solutions, the following quantities when plotted against depth factor, x/R resulted in a non-dimensional unique plot under any desired mode of vibration:

1. The normalised modal deflection values.
2. The product of relative stiffness factor and the normalisec modal rotation values.
3. The product of $\frac{1}{k.R^2}$ with the normalised bending moment.
4. The product of $\frac{1}{k.R}$ with the normalised modal shear.

The non-dimensional curves as obtained using continuous system analysis (model) for different modal quantities for first and second mode of vibrations have been listed below:

First Mode of Vibrations:

Normalised Modal:	Identi-fication	Components	Figure No.
Deflection Φ (y_1)	A_{yc1}	Φ (y_1)	4.6
Rotation Φ (θ_1)	$A_{\theta c1}$	$\Phi(\theta_1).R$	4.7
Bending Moment Φ(M_1)	A_{mc1}	$\Phi(M_1).\frac{1}{k.R^2}$	4.8
Shear Φ (S_1)	A_{sc1}	$\Phi(S_1).\frac{1}{k.R}$	4.9
Second Mode of Vibrations:			
Deflection Φ(y_2)	A_{yc2}	$\Phi(y_2)$	4.10 and 4.10a
Rotation Φ(θ_2)	$A_{\theta c2}$	$\Phi(\theta_2).R$	4.11 and 4.11a
Bending Moment Φ(M_2)	A_{Mc2}	$\Phi(M_2).\frac{1}{k.R^2}$	4.12 and 4.12a
Shear Φ(S_2)	A_{Mc2}	$\Phi(S_2)\ \frac{1}{k.R}$	4.13 and 4.13a

4.6 COMPARISON OF CONTINUOUS SYSTEM AND LUMPED MASS SOLUTIONS

4.6.1 NATURAL FREQUENCY OF VIBRATIONS

Comparing the definitions of frequency factors in different modes of vibrations based on continuous solutions (system analysis, article 4.5.2) and those of lumped mass analysis (article 3.9); it is seen that though the solutions are based on two different approaches, the resulting definitions are identical. Similarly, the general trend in the variations of F_{cc1} and F_{cc2} with R and Z_{max} (Figures 4.2, 4.3, 4.4 and 4.5) are similar to those of F_{cL1} and F_{cL3} with R and Z_{max} (Figures : 3.8, 3.10, 3.13 and 3.15).

Therefore, the discussions (appearing in article 3.11.1) regarding the factors influencing the natural frequencies of vibrations, pertaining to solutions based on lumped mass analysis; can also be considered, applicable to the corresponding solution based on continuous system analysis.

In table 4.2 the comparison of frequency factors F_{CL1} and F_{CL2} of lumped mass analysis with F_{cc1} and F_{cc2} of continuous system analysis has been provided. It is emphasised herein, that the identified second mode frequencies of lumped mass solution corresponds to the critical frequencies of continuous system analysis (equal root case, article 4.3.2). From From Table 4.2 it can be seen that the percentage difference in the frequency factor values for any Z_{max} is about 1 % for first mode of vibration and a maximum of 8 % for second mode of vibrations. Thus for similar soil-pile systems, practically, both the lumped mass and continuous system analysis, predict identical values of natural frequencies under different modes of vibrations.

4.6.1.1 Critical Frequency Case :

From equation 4.21 of equal root case (article 4.2) we have :

$$\beta^4 = \frac{k}{EI}$$

Table 4.2

Comparison of Frequency Factors of Lumped Mass and Continuous System Analysis

Pile max	Case Relative stiffness factor, R in metres	Lumped Mass Model F_{CL1}	Lumped Mass Model F_{CL3}	Continuous System Model F_{CC2}	Continuous System Model F_{CC2}	Remarks
.0	1.0	0.495	16.40	0.490	17.55	The third mode Frequency Factor, F_{CL1} corresponds to the second mode frequency factor F_{CC2} of continuous system analysis. Because the second mode frequency with lumped mass analysis has been identified as critical frequency in continuous system analysis
.0	1.5	0.490	16.40	0.480	17.55	
.0	2.0	0.480	16.40	0.470	17.55	
.0	1.0	0.680	4.55	0.685	4.50	
.0	1.5	0.675	4.55	0.680	4.50	
.0	2.0	0.670	4.55	0.675	4.50	
.0	1.0	0.795	2.25	0.790	2.15	
.0	1.5	0.790	2.25	0.785	2.15	
.0	2.0	0.785	2.25	0.775	2.15	
.0	1.0	0.840	1.25	0.840	1.20	
.0	1.5	0.835	1.25	0.835	1.20	
.0	2.0	0.830	1.25	0.830	1.20	
.0	1.0	0.840	1.25	0.840	1.20	
0	1.5	0.835	1.25	0.835	1.20	
0	2.0	0.831	1.25	0.830	1.20	
0	1.0	0.841	1.25	0.840	1.15	
0	1.5	0.835	1.25	0.835	1.15	
0	2.0	0.830	1.25	0.830	1.15	

We have from equation 4.5:

$$\beta^4 = \frac{\gamma_A \, p^2}{g.\,EI}$$

Therefore we have:

$$\frac{\gamma_A \, p^2}{gEI} = \frac{k}{EI}$$

$$p^2 = \frac{k.\,g}{\gamma_A} \quad \ldots \; 4.47$$

This equation is similar to the one resulting from the definition of second mode frequency factor of lumped mass analysis (article 3.9). It is emphasised herein, that for piles embedded in clay type of soils (assuming soil modulus to remain constant with depth) the natural frequencies at higher mode of vibrations (except first mode) are independent of flexural stiffness EI of the pile member. However the change in pile section affects the values by way of change in weight per unit length of the pile.

But the soil-pile systems display an additional peculiarity under the critical frequency (and second mode frequency of lumped mass analysis) of vibration conditions. That is the critical frequency values are idependent of change in length of the piles and hence the maximum depth factor, Z_{max}.

The results of the lumped mass analysis in Fig 3.9 and 3.14 indicate such conclusions for second mode of

Table 4.3

Comparison of Critical Frequencies as Obtained with continuous System Analysis with Those of Second Mode Frequencies Obtained by Lumped Mass Solutions

Z_{max}	Pile Details: Relative stiffness factor R in metres	Pile Details: Soil Modulus k in T/m²	Pile Details: Diameter in metres	Critical frequency with continuous system analysis in radians per sec	Second Mode frequency with lumped mass analysis in radians per sec
1.0	1.0	477.13	0.30	166.10	164.70
1.0	1.5	94.25	0.30	73.81	73.54
1.0	2.0	230.10	0.50	69.20	68.80
2.0	1.0	477.13	0.30	166.10	165.70
2.0	1.5	1507.96	0.60	147.62	147.10
2.0	2.0	230.10	0.50	69.20	69.10
3.0	1.0	477.13	0.30	166.10	165.90
3.0	1.5	94.25	0.30	73.81	73.78
3.0	2.0	230.10	0.50	69.20	69.10
5.0	1.0	3681.55	0.50	276.80	276.50
5.0	1.5	297.87	0.40	98.41	98.39
5.0	2.0	230.10	0.50	69.20	69.18
10.0	1.0	477.13	0.30	166.10	165.70
10.0	1.5	1507.96	0.60	147.62	147.60
10.0	2.0	230.10	0.50	69.20	77.20
15.0	1.0	477.13	0.30	166.10	169.90
15.0	1.5	727.22	0.50	123.00	123.5
15.0	2.0	477.13	0.60	83.04	84.97

vibrations. The details of Table 3.2, the frequency factor values of lumped mass analysis for different pile cases with $Z_{max} = 5$ also emphasise this point. If the tabulated frequency factor values for second mode of vibrations are examined it would be found that the maximum difference betwee any two pile case is in the order of 2.55 %.

In addition to this in Table 4.3 the values of critical frequencies obtained by continuous system analysis for different soil-pile conditions have been compared with those of second mode frequencies based on lumped-mass solutions.

Critical evaluation of this Table would reveal:

1. For any identical soil-pile systems the critical frequencies obtained by tcontinuous system analysis and the second mode frequencies based on lumped mass analysis are practically same.
2. Mass per unit length of the pile remaining same the critical frequencies are dependent on soil-stiffness alone.
3. Changes in pile length do not alter the frequency values under these conditions.

4.6.2 NORMALISED MODAL QUANTITIES

Comparison of the variations with depth factor, x/R of the non-dimensional normalised modal quantities: A_{ycl},

$A_{\theta cl}$, A_{mcl}, A_{scl} with the corresponding quantities based on lumped mass analysis show that:

1. At any x/R both lumped mass and continuous system analysis yield practically same values of the normalised modal quantities. This is true for any pile length of given soil-pile parameter values.
2. Both these analysis yield similar mode of deformations for long and short pile ranges.
3. The observed trend in the dynamic behaviour of the piles are the same.

As an example, in Table 4.4 the non-dimensional modal quantities at different depth factors have been provided for piles with $Z_{max} = 5$. These values are based on the continuous system analysis. Comparison of these quantities with those obtained by lumped mass analysis would further emphasise that both these solutions yield practically similar results.

Similarly, comparison of non-dimensional normalised modal quantity values, for second mode of vibrations of continuous system analysis; with those for third mode of vibrations of lumped mass analysis indicate practically, similar form of results in both the approaches.

Table 4.4

Table of Computed Non - Dimensional Coefficients
From Continuous System Analysis

DETAILS OF DATA SUPPLIED

PROB NO = 36 ZMAX = 5.0 R = 1.5 ALTH = 7.5 DIA = 0.4 k = 297.
W = 15.0 NO OF MASSES = 30 NO OF MODES = 3 EI = 0.15080D 04
AREA = 0.12566D 00

CRITICAL FREQ. = 0.98415D 02

TABLE OF COMPUTED MODE SHAPES FROM NEGATI CASE

MODE NO = 1 p = 0.1425D 02 FREQ = 2.26758 PERIOD = 0.4410
MPF = 0.1005D 01 XH = 0.250 DT = -0.461D-04

PT	x/R	A_{ycl}	A cl	A_{mcl}	A_{scl}
1	0.0	-0.1001D 01	0.7036D 00	0.0	0.6953D 00
3	0.333	-0.7698D 00	0.6707D 00	0.1816D 00	.0.4069D 00
5	0.667	-0.5585D 00	0.5918D 00	0.2793D 00	0.1910D 00
7	1.000	-0.3777D 00	0.4912D 00	0.3161D 00	0.3925D-01
9	1.333	-0.2316D 00	0.3857D 00	0.3114D 00	-0.5915D-01
11	1.667	-0.1198D 00	0.2864D 00	0.2813D 00	-0.1156D 00
13	2.000	-0.3925D-01	0.1996D 00	0.2379D 00	-0.1407D 00
15	2.333	0.1496D -01	0.1283D 00	0.1899D 00	-0.1440D 00
17	2.667	0.4804D-01	0.7283D-01	0.1434D 00	-0.1333D 00
19	3.000	0.6515D-01	0.3211D-01	0.1020D 00	-0.1144D 00
21	3.333	0.7086D-01	0.4062D-02	0.6753D-01	-0.9199D-01
23	3.667	0.6899D-01	-0.1376D-01	0.4071D-01	-0.6902D-01
25	4.000	0.6254D-01	-0.2391D-01	0.2135D-01	-0.4748D-01
27	4.333	0.5364D-01	-0.2875D-01	0.8768D-01	-0.2849D-01
29	4.667	0.4372D-01	-0.3040D-01	0.2009D-01	-0.1260D-01

$$X = A + B_x$$

This would mean certain modal deflection values at the top of the piles.

It is believed that in the absence of continuous system analysis, the creditability of such mode of vibration could not have been established. In the available literatur (as of 1974) on the dynamic behaviour of piles no indication to such possibilities has been provided.

4.8 CONCLUDING REMARKS

For piles embedded in clay type soils (assuming soil modulus to remain constant with depth) the dynamic analysis can be performed treating the soil pile system by a continuous system model.

For pile top free to rotate conditions, as an example it has been demonstrated that both lumped mass and continuous system analysis result in near identical solutions.

Therefore the lumped-mass analysis approach, as giver in Chapter III can be considered as an effective tool in determining the dynamic response of piles.

This check is considered as essential because, the soil-pile system in reality are continuous systems. But in practice, along the length of the pile the pile cross section and the soil conditions may vary. Therefore, under these conditions it may be impracticable to perform the continuous system analysis. Because of this demonstration, it is believed, that greater confidence can be placed on the proposed lumped mass solution which can be adopted to varieties of situations.

Based on the discussions of continuous system analysis and the results, thereoff, the following points are considered as significant:

1. Depending on the soil-pile parameter values and the natural frequency of vibrations of the system three different solutions are possible to define the mode shapes and the other related parameters. While predicting the dynamic response, at each stage the applicability of the related solutions need be checked.

2. The proposed analysis and the adopted end conditions perform well in predicting the dynamic behaviour of piles.

3. More than the absolute lengths. The dynamic behagiour of pile-soil system is essentially dependent on the Z_{max} values.

4. Piles having $Z_{max} \leq 2$ display rigid body deformations whereas piles with $Z_{max} \geq 5$ display flexural-bending deformation.

5. Increase in pile length beyond $Z_{max} = 5$ has little effect on the dynamic behaviour of piles, especially in the first mode of vibrations. Thus $Z_{max} = 5$ may be considered as a limiting long pile condition.

6. With the increase in super-structure load or the mass lumped at top

   (i) the natural frequencies under firts mode of vibrations are reduced by $\sqrt{1/M_t}$ times.

   (ii) At any depth the induced dynamic displacements, bending moment and shear are increased by $\sqrt{M_t}$ times

7. For any mode and any soil type increase in soil stiffness results in the reduction of pile displacements and increase in natural frequencies of vibrations.

8. For pile top free to rotate conditions the maximum bendin moment occurs below the ground level. For long pile ranges they occur at a depth of 1.20 R from top.

9. There is a possibility of critical frequency of vibrations and a rigid body motion under such conditions. This may occur for all pile-soil conditions of any pile lengths.

More importantly it has been demonstrated that it is possible to obtain non-dimensional solutions for predicting the dynamic response of piles.

With the help of these non-dimensional curves (i) the natural frequencies of vibrations and (ii) the normalised modal quantities at every point along the pile length can be obtained upto significant modes of vibrations. In order to facilitate the above, series of non-dimensional plots have been provided for piles with Z_{max} = 1, 2, 3, 5, 10 and 15. With such solutions, the dynamic behaviour of any soil-pile system embedded in clay type soils (assuming soil modulus to remain constant with depth) under pile top free to rotate conditions can be predicted. These solutions are based on continuous system analysis.

CHAPTER - V

DYNAMIC CHARACTERISTICS OF PILES EMBEDDED IN SOIL ASSUMING SOIL MODULUS TO VARY PROPORTIONAL TO DEPTH

5.1 INTRODUCTION

The dynamic analysis of the piles idealising the soil-pile system as lumped-mass-spring systems have been successfully developed in Chapter III. The performance and correctness of the solutions have been checked by an independent technique considering the soil-pile system as a continuous model. It has been demonstrated in Chapter IV tha both the above techniques result in near identical solutions for all practical purposes.

Thus, having established the performance of the above approach in soils, assuming soil modulus constant with depth (clay type soils); solutions have been presented in this Chapter for the case of piles embedded in granular type soils Herein, the soil modulus has been assumed to vary linearly with depth. The mathematical model and the characterisation of soil pile interaction effects for these types of variation have been discussed in article 3.2 of Chapter III. The adopt numerical technique and the method of analysis to obtain dynamic response, are the same as us d in clay case

5.2 NON-DIMENSIONAL SOLUTIONS

5.2.1 VARIABLES

As in the case of piles embedded in clayey soil in order to obtain information of practical significance, non-dimensional solutions have been obtained in the present case also. For achieving this, solutions have been obtained by determining the response of piles for the following conditions:

1. For piles with pile top free to rotate condition.
2. For piles in which pile top is fixed against rotation
3. In both the above cases the maximum non-dimensional depth factors considered are : $Z_{max} = 1, 2, 3, 5, 10$ and 15. For granular soils assuming soil modulus to vary linearly with depth Z_{max} is defined as: $Z_{max} = Ls/T$ where $T = 5\sqrt{EI/n_h}$

In all the above three cases the following soil-pile parameters were varied.

1. Flexural stiffness EI of the pile
2. Relative stiffness factor and soil stiffness
3. Sustained vertical loads
4. Pile lengths

The different pile cases for which dynamic analysis have been performed are given in Table 5.1. As it is seen from the table, a total number of 180 pile cases have been examined.

From the Table 5.1, the following points are seen to be of significance.

Details of Analysed Pile Cases Assuming Soil Modulus to Vary Linearly With Depth ($k_x = n_h \cdot x$)

No.	Relative stiffness factor T, in metres	Diameter in metres	Constant of Subgrade Reaction, n_h in T/m^3	Flexural Stiffness EI in Tm^2	Remarks
1	0.75	0.30	2010.62	0.477×10^3	1. In each case piles with Z_{max} = 1, 2, 3, 5, 10 and 15 were considered.
2	1.0	0.30	477.13	0.477×10^3	
3	1.25	0.30	156.35	0.477×10^3	2. The sustained vertical loads were computed considering skin friction and point bearing.
4	1.0	0.40	1507.96	0.151×10^4	
5	1.25	0.40	494.13	0.151×10^4	
6	1.5	0.40	198.58	0.151×10^4	3. Each of the pile cases for each Z_{max} was analysed for (i) pile top free to rotate condition and (ii) pile top fixed against rotation condition.
7	1.0	0.50	3681.55	0.368×10^4	
8	1.25	0.50	1206.37	0.368×10^4	
9	1.5	0.50	484.81	0.368×10^4	
10	2.0	0.50	115.05	0.368×10^4	
11	1.25	0.60	2501.53	0.763×10^4	
12	1.5	0.60	1005.31	0.763×10^4	
13	2.0	0.60	238.56	0.763×10^4	
14	1.25	0.70	4634.397	0.141×10^5	
15	1.5	0.70	1862.46	0.141×10^5	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

1. The considered pile diameters vary between 0.3 metre to 0.7 metre.

2. In each of these diameters the pile lengths considered in the study covered ranges of lengths to obtain information for cases with Z_{max} = 1, 2, 3, 5, 10 and 15.

3. For each of these pile sectional properties three ranges of relative stiffness factors were considered.

4. These values of relative stiffness factors yield information on piles embedded in soils with different soil stiffness. For granular soils this covered ranges between loose to dense state of depositions.

5.2.2 NON-DIMENSIONAL CURVE FOR NATURAL FREQUENCIES

Examination of the results of the analysis of all the one hundred eighty pile cases of Table 5.1, resulted in understanding the various factors which influence the natural frequencies of soil-pile system. Analysis of each individual factors resulted in a set of non-dimensional curves to explain the vibrating frequencies of piles. The non-dimensional frequency factor F_{SL} is defined as below for different modes of vibration .

Mode No.	Identification	Components	Remarks
First	F_{SL1}	$w_{n1}\sqrt{\frac{W}{g}\frac{1}{n_h T^2}}$	Pile Top Free to Rotate
Second	F_{SL2}	$w_{n2}\sqrt{\frac{\gamma}{g}\frac{d^2}{n_h T}}$	Pile Top Free to Rotate
First	F'_{SL1}	$w'_{n1}\sqrt{\frac{W}{g}\frac{1}{n_h T^2}}$	Pile top Fixed Against Rotation
Second	F'_{SL2}	$w'_{n2}\sqrt{\frac{\gamma}{g}\frac{d^2}{n_h T}}$	Pile Top Fixed Against Rotation

In the above list:

w_n is the circular natural frequency of the system in radians per second for the subscript-identified modes.

$\frac{W}{g}$ is the concentrated mass at top, M_t FT^2L^{-1}

n_h constant of horizontal subgrade reaction FL^{-3}

d diameter of pile section L

T relative stiffness factor L

γ weight density of piles FL^{-1}

F_{SL} frequency factor.

The numerals in the subscrip identify the mode number and prime used for fixed head conditions.

In Fig 5.1 and 5.2 the variation of frequency factors F_{SL1} and F_{SL2} with relative stiffness factor, T has been provided. These figures pertain to pile top free to rotate conditions, for the first and second modes of vibrations respectively. The curves have been drawn for piles with different Z_{max} values. For each Z_{max} case in these figures fifteen pile cases of varying values of soil-pile parameters and sustained vertical loads have been considered. In Fig 5.3 and 5.4 similar variation of F'_{SL1} and F'_{SL2} for first and second modes of vibration have been provided. These figures pertain to pile top fixed against rotation conditions.

In Figures 5.5 and 5.6 the variations of frequency factor, F_{SL1} and F_{SL2} with Z_{max} have been given for the case of pile top free to rotate conditions. In Fig 5.7 and 5.8 the variation of F'_{SL1} and F'_{SL2} with Z_{max} for pile top fixed against rotation conditions have been given.

5.2.3 NON-DIMENSIONAL CURVES FOR NORMALISED MODAL QUANTITIES

The analysis of each of the pile cases in Table 5.1 (180 cases) gave output of the variation of normalised modal quantities along the pile length. It was observed that for any soil-pile system with a particular Z_{max}, unique plots exist between the depth factor x/T and certain non-dimensional coefficients for each of the normalised modal quantities. These quantities include normalised modal deflection, rotation, bending moment and shear in different

The following quantities for a given Z_{max} when plotted against x/T resulted in a non-dimensional unique plot under all modes of vibration:

1. The normalised modal deflection values.
2. The product of relative stiffness factor and the normali modal rotation.
3. The product of $\frac{1}{n_h \cdot T^3}$ with the normalised bending moment.
4. The product of $\frac{1}{n_h T^2}$ with the normalised modal shear.

The above non-dimensional plots were found to be valid for the first and second modes of vibrations for both pile top free to rotate and fixed against rotation conditior

The various non-dimensional curves for different modal quantities for various conditions have been listed below.

I Piles Embedded in granular Soils-Pile Top Free To Rotate Conditions First Mode of Vibration.

Normalised Modal	Identi-fication	Components	Figure No.
1	2	3	4
Deflection $\Phi(y_1)$	B_{y1}	$\Phi(y_1)$	5.9
Rotation $\Phi(\theta_1)$	$B_{\theta 1}$	$\Phi(\theta_1) \cdot T$	5.10
Bending Moment $\Phi(M_1)$	B_{M1}	$\Phi(M_1) \frac{1}{n_h T^3}$	5.11
Shear $\Phi(S_1)$	B_{S1}	$\Phi(S_1) \frac{1}{n_h T^2}$	5.12

II Piles Embedded in granular Soils Piles Top Free To Rotate Conditions-Second Mode of Vibration

1	2	2	3	4
Deflection $\Phi(y_2)$		B_{y2}	$\Phi(y_2)$	5.13
Rotation $\Phi(\theta_2)$		$B_{\theta 2}$	$\Phi(\theta_2) . T$	5.14
Bending Moment $\Phi(M_2)$		B_{M2}	$\Phi(M_2) \frac{1}{n_h T^3}$	5.15
Shear $\Phi(S_2)$		B_{S2}	$\Phi(S_2) \frac{1}{n_h T^2}$	5.16

III Piles Embedded in Granular Soils Pile Top Fixed Against Rotation Condition- First Mode of Vibration

1	2	3	4
Deflection $\Phi'(y_1)$	$B'y_1$	$\Phi'(y_1)$	5.17
Rotation $\Phi'(\theta_1)$	$B'_{\theta 1}$	$\Phi'(\theta_1) T$	5.18
Bending Moment $\Phi'(M_1)$	B'_{M1}	$\Phi'(M_1) \frac{1}{n_h T^3}$	5.19
Shear $\Phi'(S_1)$		$\Phi'(S_1) \frac{1}{n_h T^2}$	5.20

IV Piles Embedded in Granular Soils-Pile Top Fixed Against Rotation Condition-Second Mode of Vibration

1	2	3	4
Deflection $\Phi'(y_2)$	B'_{y2}	$\Phi'(y_2)$	5.21
Rotation $\Phi'(\theta_2)$	$B'_{\theta 2}$	$\Phi'(\theta_2) . T$	5.22
Bending Moment $\Phi'(M_2)$	B'_{M2}	$\Phi'(M_2) \frac{1}{n_h T^3}$	5.23
Shear $\Phi'(S_2)$	B'_{S2}	$\Phi'(S_2) \frac{1}{n_h T^2}$	5.24

5.3 FACTORS INFLUENCING NATURAL FREQUENCIES

In Fig 5.1 and Fig 5.3 the variation of frequency factors F_{SL1} and F'_{SL1} with relative stiffness factors T have been shown. These curves pertain to first mode of vibration for pile top free to rotate and fixed against rotation conditions respectively. In the same order the variations of F_{SL1} and F'_{SL1} with Z_{max} have been shown in Fig 5.5 and Fig 5.7. These figures have been obtained by plotting the analysed results of fifteen pile cases for each Z_{max} shown in Table 5.1. As seen from the figures for a particular Z_{max} unique curves are obtained between F'_{SL1} and F_{SL1} with T and Z_{max}. Herein, F_{SL1} and F'_{SL1} are dimensionless numbers.

From the examination of these figures the following points may be concluded:

1. There is a reduction in frequency factor value with increase in T values. The reduction is more pronounced particularly for cases where T is greater than 2.

2. For a particular T increase in length results in increase of frequency factor values. However, for $Z_{max} \geq 5$ the change is insignificant.

3. For any T, the variation of frequency factors with Z_{max} is a straight line. However for $Z_{max} \geq 5$ the variation is negligible.

From the above Figures for a particular Z_{max} for both pile top free to rotate and fixed against rotation condition we have:

$$w_{n1} = F_{SL1} \sqrt{\frac{n_h T^2}{M_t}}$$

$$w_{n1} \propto \frac{n_h^{0.5} \; T}{M_t^{0.5}} \qquad \ldots 5.1$$

$$\propto \frac{n_h^{0.3} \; EI^{0.2}}{M_t^{0.5}} \qquad \ldots 5.2$$

From equation 5.2 and the related figures we have:

1. For any given soil-pile case the first natural frequency reduces with increase of sustained vertical load.
2. For long pile ranges increase in soil stiffness results in increase of first mode frequency. Also, increase in flexural stiffness EI, of pile, results in increase of w_{n1}. However, increase in EI, results in increase of T, which would offset the said increase in w_{n1} values.
3. For piles other than long pile ranges increase in soil stiffness would result in increase of w_{n1} values. But , the increase of EI may not result in absolute increase of w_{n1}, because of increase in T and reduction in Z_{max}. Hence under these conditions careful consideration of these effects are necessary.

In Fig 5.2 and 5.4 the variations of frequency factors F_{SL2} and F'_{SL2} with relative stiffness factors have been given for pile top free to rotate and fixed against rotation conditions respectively. For the same conditions the variation with Z_{max} has been given in Fig 5.6 and Fig 5.8.

For a given Z_{max} the natural frequency this mode has been defined as.

$$w_{n2} = F_{SL2} \sqrt{\frac{n_h T}{d^2} \frac{g}{\gamma}} \qquad \ldots 5.3$$

$$w_{n2} \propto \frac{EI^{0.1} \, n_h^{0.4}}{d} \cdot \left(\frac{g}{\gamma}\right)^{0.5} \qquad \ldots 5.4$$

From the above relation and the pertaining figures we have:

1. For any Z_{max}, unlike in first mode the value of frequency factors is not altered appreciably with changes in relative stiffness factor values.
2. The frequency factor values increase with increase of Z_{max}. However for $Z_{max} \geqslant 5$ there is no appreciable change.
3. The conclusion 2 is contrary to the clay case wherein, in the second mode of vibration, with increase in Z_{max} there was reduction in frequency factor values.

4. The increase of soil stiffness results in increase of second mode frequencies.

5. Compared to the first mode frequency the influence of flexural stiffness of the pile is greater in the present case.

6. Increase of weight density results in reduction of natural frequencies.

5.4.1 FACTORS INFLUENCING DYNAMIC DISPLACEMENTS

5.41 FIRST MODE OF VIBRATION

In Fig 5.9 and 5.17 the variation of non-dimensional normalised modal deflections B_{y1} and B'_{y1} has been plotted against depth factor x/T. These results pertain to pile top free to rotate and fixed against rotation conditions respectively. In Fig 5.10 and 5.18 for the same conditions the non-dimensional normalised rotation coefficients $B_{\theta 1}$ and $B'_{\theta 1}$ have been plotted.

From these figures the following points are seen to be of significance:

1. Piles with $Z_{max} \leqslant 2$ display rigid body type deformations.
2. The displacements are primarily dependent on Z_{max} rather than the absolute lengths.
3. For piles with $Z_{max} \geqslant 5$, the form of variation of displacements are practically the same, indicating that piles with $Z_{max} \geqslant 5$ may be treated as infinitely long.

4. At depths greater than 4T insignificant displacements are experienced by long piles.

5. The rotation at bottom ends are greater for short piles ranges than for long piles.

From the knowledge of normalised modal displacemen it is easy to estimate the various factors which influence the dynamic displacements.

We have from equation 3.61 the dynamic deflection as under.

$$Y_{(i)}^{(r)} = \Phi_{(i)}^{(r)}(y) \; \Upsilon(r) \; S_{d(r)}$$

The explanation to the various quantities of the above equation has already been defined in article 3.5.

For the first mode of vibration thus we have:

$$Y_1 = B_{y1} \cdot S_{d1} \quad \ldots (\text{pile top free to rotate}) \quad \ldots 5.5$$

$$Y'_1 = B'_{y1} \cdot S'_{d1} \quad (\text{pile top fixed against rotation}) \quad \ldots 5.6$$

Therefore it follows that:

$$Y \propto S_d$$

From article 5.3 we have:

$$w_{n1} = F_{SL1}\sqrt{\frac{n_h T^2}{M_t}} \quad \ldots \quad \text{(pile top free to rotate)}$$

$$w'_{n1} = F'_{SL1}\sqrt{\frac{n_h T^2}{M_t}} \quad \text{(pile top fixed against rotation)}$$

Since spectral displacement is proportional to the time period we have:

$$Y_1 \propto \frac{M_t^{0.5}}{n_h^{0.3}\, EI^{0.2}} \quad \ldots \quad 5.7$$

From the above relation and Fig 5.9 and 5.17 it can be concluded that:

1. As the top mass is increased, dynamic deflection is increased by $\sqrt{M_t}$ times.
2. Increase in soil stiffness results in reduction of dynamic deflection.
3. Though there is apparent reduction in the dynamic deflection with increase of flexural stiffness of piles, the increase may be offset by reduction in Z_{max}, because as Z_{max} reduces the natural period and hence S_d increases to enhance the values of dynamic deflection.
4. Rest of the soil-pile parameter values remaining same reduction in pile length increase dynamic deflection.

The non-dimensional rotation coefficients $B_{\theta 1}$ and $B'_{\theta 1}$ have been defined as under:

$$B_{\theta 1} = \Phi(\theta_1).\ T \quad \ldots \text{ (pile top free to rotate)}$$

$$B'_{\theta 1} = \Phi'(\theta_1)\ T \quad \ldots \text{ (pile top fixed against rotation)}$$

where $\Phi(\theta_1)$ is the normalised modal rotation in the fir mode of vibration.

Therefore we have:

Dynamic rotation $\theta_1 = \Phi(\theta_1)\ S_{d1}$ ... 5.8

$$\theta_1 = \frac{B_{\theta 1}}{T} \cdot Sd_1 \qquad \ldots\ 5.9$$

Herein, both $B_{\theta 1}$ and $B'_{\theta 1}$ are dimensionless numbers

Therefore;

Dynamic rotation $\theta_1 \propto \frac{S_{d1}}{T}$

$$\text{or } \theta_1 \propto \frac{M_t^{0.5}}{n_h^{0.3}\ EI^{0.2}} \qquad \frac{n_h^{0.2}}{EI^{0.2}} \qquad \ldots\ 5.10$$

$$\text{or } \theta_1 \propto \frac{M_t^{0.5}}{n_h^{0.1}\ EI^{0.4}} \qquad \ldots\ 5.11$$

From the above relation and the non-dimensional rotation curves we have:

1. For any given pile the dynamic rotation increases with increase in top mass.

2. With the increase of soil stiffness dynamic rotation is reduced. However, the effect here is lesser compared to the influence of n_h on Y_1.

3. There is greater influence of EI on dynamic rotation.

5.4.2 SECOND MODE OF VIBRATION

In Fig 5.13 and 5.21 the variations of non-dimensional normalised modal deflections B_{y2} and B'_{y2} with depth factor have been provided. These curves correspond to second mode of vibration for pile top free to rotate and fixed against rotation conditions. For the same conditions the variations of normalised modal rotation $B_{\theta 2}$ and $B'_{\theta 2}$ have been provided in Fig 5.14 and 5.22.

As is seen from these figures, the displacements in the second modes vary with the increase of Z_{max}. For piles with $Z_{max} \geq 5$ the variations are negligible.

The dynamic deflection and rotation in the second mode of vibrations are given by:

$$Y_2 = B_{y2} \; S_{d2} \qquad \ldots \; 5.12$$

$$\theta_2 = \frac{B_{\theta 2}}{T} \cdot S_{d2} \qquad \ldots \; 5.13$$

In the above equations B_{y2} and $B_{\theta 2}$ are dimensionless numbers.

Therefore, the factors influencing dynamic deflections can be assessed based on the definition of B_{y2} in article 5.2.3.

Therefore we have

$$\therefore \quad Y_2 \propto S_{d2} \qquad \ldots \; 5.14$$

$$\text{ie} \quad Y_2 \propto \frac{1}{w_{n2}} \qquad \ldots \; 5.15$$

$$\propto \sqrt[4]{\frac{d^2 \gamma}{g\, n_h T}} \qquad \ldots \; 5.16$$

The above relation has been found to agree for both pile top free to rotate and fixed against rotation conditions.

Therefore we have:

1. The dynamic deflection (i) increases with weight per unit length of the pile (ii) decreases with increase i soil stiffness.

2. Rest of the factors remaining constant dynamic deflecti increases with increase in pile length.

3. For long pile ranges with increase in pile length the dynamic deflection remain practically unaltered.

The dynamic rotation in the second mode of vibration is given by

$$\theta_2 = \frac{B_{\theta 2}}{T} \cdot S_{d2} \qquad \ldots \; 5.17$$

$$\theta_2 \propto \frac{1}{w_{n2}} \frac{1}{T} \quad \ldots 5.18$$

$$\propto \sqrt{\frac{d^2 \gamma}{g\, n_h T}} \frac{1}{T} \quad \ldots 5.19$$

$$\propto \sqrt{\frac{\gamma d^2}{g}} \frac{1}{n_h^{0.5}} \frac{1}{T^{1.5}} \quad \ldots 5.20$$

$$\propto \sqrt{\frac{\gamma d^2}{g}} \frac{1}{n_h^{0.2}} \frac{1}{EI^{0.3}} \quad \ldots 5.21$$

From the above relation it is seen that

1. The dynamic rotation increases as weight per unit length is increased.
2. For any particular Z_{max} the dynamic rotation is reduced with increase in soil stiffness and flexural stiffness EI.
3. For long pile ranges increase in EI and soil stiffness results in reduction of dynamic rotation.

5.5 FACTORS INFLUENCING DYNAMIC BENDING MOMENT AND SHEAR

5.5.1 FIRST MODE OF VIBRATION.

The variation of non-dimensional normalised bending moment coefficients B_{m1} and B'_{m1} have been plotted in Fig 5.11 and Fig 5.19. These figures pertain to pile top free to rotate and fixed against rotation conditions respectively.

From these two figures the following points can be inferred:

1. For pile top free to rotate conditions greater bending moment coefficients are applicable for long pile ranges than for short piles.
2. Under the above conditions the difference in maximum bending moment values between $Z_{max} = 2$ and $Z_{max} = 3$ are greater compared to those between $Z_{max} = 3$ and $Z_{max} \geq 5$.
3. For any condition for piles with $Z_{max} \geq 5$ there is no appreciable difference in the bending moment values. Hence, $Z_{max} \geq 5$ is practically a long pile case.
4. In the case of pile top free to rotate condition the maximum bending moment for $Z_{max} = 2$, 3 and $Z_{max} \geq 5$ occur at depths of 0.8T, 1.15T and 1.30T respectively.
5. For pile top fixed against rotation condition maximum bending moment occurs at top.

With the knowledge of normalised modal values of bending moments it is easy to assess the various factors which influence the dynamic bending moment.

We have from equation 3.61 the dynamic bending moment in the first mode as under:

$$M_1 = \Phi(M_1)\, S_{d1} \qquad \ldots\ 5.22$$

Now from article 5.2.3 the non-dimensional bending moment coefficients for the cases of pile top free to rotate and fixed against rotation condition are defined as:

$$B_{ml} = \frac{\Phi(M_l)}{n_h T^3} \quad \ldots 5.23$$

$$B'_{ml} = \frac{\Phi'(M_l)}{n_h T^3} \quad \ldots 5.24$$

Therefore the dynamic bending moment M_l is given by

$$M_l = B_{ml} \times n_h T^3 \times S_{dl} \quad \ldots 5.25$$

$$M_l \propto n_h T^3 S_{dl} \quad \ldots 5.26$$

$$\propto n_h T^3 \frac{1}{w_{nl}} \quad \ldots 5.27$$

$$\propto n_h \frac{EI^{0.6}}{n_h^{0.6}} \cdot \frac{M_t^{0.5}}{n_h^{0.3} EI^{0.2}} \quad \ldots 5.28$$

$$\propto M_t^{0.5} EI^{0.5} n_h^{0.1} \quad \ldots 5.29$$

The examination of the above equation reveals that:

1. Increase in top mass increases dynamic bending moment by $\sqrt{M_t}$ times.

2. For long piles ($Z_{max} \geq 5$) increase in soil stiffness increases dynamic bending moment. However the displacements are reduced.

3. For any piles increase in EI results in increase of dynamic bending moment. However both deflections and rotation are reduced.

In Fig 5.12 and 5.20 the variations of non-dimensional normalised modal shear B_{S1} and B'_{S1} with depth factor have been provided. These curves pertain to pile top free to rotate and fixed against rotation conditior

The non-dimensional modal shear is defined as the product of $\Phi(S_1)$ or $\Phi'(S_1)$ with $\frac{1}{n_h T^2}$. Both B_{S1} and B'_{S1} are dimensionless numbers. Therefore dynamic shear S_1 in the first mode is given by:

$$S_1 = B_{S1} \cdot n_h T^2 \; S_d$$

..... (pile top free to rotate) ... 5.30

$$S'_1 = B'_{S1} \, n_h \, T^2 \, S'_d$$

...(pile top fixed against rotation)

... 5.31

We have

$$S_1 \propto \frac{1}{w_{n1}} \cdot n_h T^2 \qquad \text{... 5.32}$$

$$\propto \frac{M_t^{0.5}}{n_h^{0.3} \, EI^{0.2}} \cdot n_h \cdot \frac{EI^{0.4}}{n_h^{0.4}} \qquad \text{... 5.33}$$

$$\propto M_t^{0.5} \; n_h^{0.3} \; EI^{0.2} \qquad \text{... 5.34}$$

The influence of various factors on the dynamic shear are similar to those of dynamic bending moments discussed earlier in this article. However, the quantum of influence of both n_h and EI varies as $n_h^{0.3}$ and $EI^{0.2}$ respectively.

5.5.2 SECOND MODE OF VIBRATION

The variations of non-dimensional normalised bending moment with x/T, the depth factor have been provided in Fig 5.15 and Fig 5.23. These curves pertain to second mode of vibration for pile top free to rotate and fixed against rotation conditions.

The definitions of B_{m2} and B'_{m2} are as under:

$$B_{m2} = \Phi(M_2).\ n_h T^3 \quad \ldots 5.35$$

$$B'_{m2} = \Phi'(M_2)\ n_h T^3 \quad \ldots 5.36$$

Therefore the dynamic bending moment is given by:

$$M_2 = B_{m2}.\ n_h T^3 S_{d2}$$

...(pile top free to rotate)... 5.37

$$M'_2 = B'_{m2}\ n_h T^3 S'_{d2}$$

...(pile top fixed against rotation) ... 5.38

Herein B_{m2} and B'_{m2} are dimensionless coefficients.

Therefore we have for both pile top free to rotate and fixed against rotation conditions we have:

$$M_2 \propto n_h T^3 \frac{1}{w_{n2}} \qquad \ldots 5.39$$

$$\propto n_h T^3 \sqrt{\frac{\gamma d^2}{g}} \frac{1}{EI^{0.1} n_h^{0.4}} \qquad \ldots 5.40$$

$$\propto n_h \frac{EI^{0.6}}{n_h^{0.6}} \sqrt{\frac{\gamma d^2}{g}} \frac{1}{EI^{0.1} n_h^{0.4}} \qquad \ldots 5.41$$

$$\propto \sqrt{\frac{\gamma d^2}{g}} \cdot EI^{0.5} \qquad \ldots 5.42$$

The above relation is same as for piles embedded in clay type soils. From the examination of these relations we can conclude that:

1. For a particular Z'_{max}, dynamic bending moment in second mode of vibration is independent of soil stiffness.
2. Increase in EI results in increase of M_2 values.
3. Increase in weight per unit length of pile results in increase of dynamic bending moment.

For the second mode of vibration the variations of non-dimensional modal shear coefficients B_{S2} and B'_{S2} with x/T have been provided in Fig 5.16 and Fig 5.24. These figures pertain to pile top free to rotate and fixed against rotation conditions respectively.

The dynamic shear coefficients have been defined as the product of normalised shear and $\frac{1}{n_h T^3}$. Therefore for both pile top free to rotate and fixed against rotation conditions we have:

Dynamic shear $S_2 \propto n_h T^2 \cdot S_{d2}$ ... 5.41

$$S_2 \propto n_h T^2 \frac{1}{w_{n2}} \quad \ldots 5.44$$

$$\propto n_h T^2 \sqrt{\frac{\gamma d^2}{g}} \cdot \frac{1}{EI^{0.1} \; n_h^{0.4}} \quad \ldots 5.45$$

$$\propto n_h \frac{EI^{0.4}}{n_h^{0.4}} \sqrt{\frac{\gamma d^2}{g}} \cdot \frac{1}{EI^{0.1} n_h^{0.4}} \quad 5.46$$

$$\propto n_h \frac{EI^{0.4}}{n_h^{0.4}} \sqrt{\frac{\gamma d^2}{g}} \; \frac{1}{EI^{0.1} n_h^{0.4}} \quad \ldots 5.47$$

$$\propto EI^{0.3} \; n_h^{0.2} \; \sqrt{\frac{\gamma d^2}{g}} \quad \ldots 5.48$$

From these relation it is seen that:

Dynamic shear in the second mode increases with increase in

1. Weight per unit length of pile.
2. Increase in soil stiffness.
3. Increase in flexural stiffness of piles.

5.6 CONCLUDING REMARKS

In this chapter the dynamic behaviour of piles embedded in granular soils has been predicted using lumped mass analysis, which was developed in Chapter III and checked by continuous system analysis in Chapter IV. These solutions are applicable to the type of soils in which the soil-modulus can be considered to vary in proportion to depth.

In order to get a clear picture of the dynamic characteristics of the piles, the solutions have been obtain for the following cases of practical significance:

1. Pile top free to rotate conditions.
2. Pile top fixed against rotation conditions.
3. Piles having non-dimensional depth factor, Z_{max} = 1, 2, 3, 5, 10 and 15.

In each of the above cases the soil and pile stiffness, the pile length and the sustained vertical loads have been carefully varied to obtain solutions of practical significance.

Based on the solutions of several such pile problems non-dimensional design curves have been developed. These non-dimensional curves are capable of predicting the dynamic response upto significant modes of vibrations.

With these non-dimensional curves it is possible to assess:

1. the natural frequencies of vibrations under first two modes of vibrations.

2. the normalised modal quantities of deflection, rotation, bending moment and shear at every point along the pile length.

Based on these studies concerning the dynamic behaviour of piles embedded in granular soils, the following conclusions may be drawn:

1. Lumped mass analysis is an effective tool in predicting the dynamic behaviour of piles embedded in granular soils. In fact any form of soil modulus variation can be handled with this approach.

2. The dynamic behaviour of the piles are dependent on the:

   (i) relative stiffness of the pile and soil.

   (ii) length of the pile in relation to the relative stiffness factor.

The absolute length of the pile does not govern the behaviour singularly.

3. Under first mode of vibrations the variations of pile displacements, bending moment and shear along the length of the pile follow the pattern under static loading conditions.

4. First mode of vibrations contribute significantly to the overall response.

5. Under dynamic conditions piles with $Z_{max} \leq 2$ display rigid body deformations whereas piles with $Z_{max} \geq 5$ display bending deformations.

6. For a given soil-pile system $Z_{max} = 5$ can be considere as a limiting value of long pile range. Increase of pil length beyond this value oses significance as far as dynamic behaviour is concerned.

7. For similar pile-soil systems for pile top fixed against rotation conditions the envisaged dynamic displacemets may be smaller than under pile top free to rotate conditions, because of the smaller period of vibrations in th former case.

8. For pile top fixed against rotation conditions the maxim bending moment occurs at top. But under free head conditions, for long piles, it occurs at a depth of 1.30 T from the ground surface.

9. With increase in lumped mass at top we have:

   (i) the natural frequencies under first mode of vibrations are reduced by $\frac{1}{\sqrt{M_t}}$ times.

   (ii) the values of dynamic displacements and bending moment are increased by $\sqrt{M_t}$ times.

10. The increase in soil-stiffness results in the reduction of pile displacements.

Thus, based on the present investigations it can be concluded that the dynamic characteristics of the pile-soil systems are better understood. Further, utilising the results of the investigations on piles embedded in soils, in which soil modulus can be considered to vary in proportion to depth; the dynamic behaviour of any soil-pile system embedded in granular soils can be determined without entering into the complexities of dynamic analysis.

CHAPTER - VI

EXPERIMENTAL STUDIES

6.1 INTRODUCTION

Only very few experimental studies are reported in the available literature, discussing the behaviour of piles subjected to dynamic loads. Majority of them deal with the performance of small size piles tested in the laboratory under dynamic loading conditions. Out of this, greater stress has been paid to natural frequency of pile vibrations as a result of sudden pull and release.

The available informations on vibration testing of prototype piles are meagre. More importantly, till to-day (1974), no logical in-situ testing procedure is availabl for determining the material constants of the soil-pile system which is required in any dynamic analysis dealing with the assessment of pile response.

In this Chapter the details of the dynamic tests on full scale prototype piles have been reported.

The experimental studies were carried out with the following objectives:

(i) to understand the dynamic behaviour of piles.

(ii) to evaluate a logical procedure for determining the material constants of the soil-pile systems under dynamic conditions.

(iii) to compare the observed quantities with the predicted ones using the techniques given in Chapter III, IV and V.

6.2 TESTS PERFORMED

The experimental studies comprised of the following tests.

1. Static lateral load tests.
2. Lateral free vibration tests.
3. Lateral forced vibration tests.

The tests were executed on different types of full scale piles embedded in varying soil types. In Table 6.1 the details of the tests conducted have been explained.

6.2.1 SITE LOCATION

The piles VTP1 to VTP4 were a part of the foundation systems of the Haldia refinery project. The site is located on the west bank of river Hoogly at 40 km downstream from Calcutta, India. The site is situated in an esturian deltaic environment and consists of alluvial and marine deposits of soil.

The piles VTP5 and VTP6 were embedded in a loose silty sand area at the CBRI campus Roorkee, India.

Table 6.1

Tests Performed

I STATIC LATERAL LOAD TESTS

Sl. No.	Pile Identi-fication	Lateral Load Level in Kg	Incre-ment of load in kg	Rotation	Remarks
1.	VTP1	3000	250	Permitted	
2.	VTP2	3000	250	Permitted	With a Sustai vertical load of 55T
3.	VTP3	3000	250	Permitted	
4.	VTP4	3000	250	Permitted	
5.	VTP5	900	100	Permitted	
6.	VTP6	900	100	Permitted	

II FREE VIBRATION TESTS

Serial No.		Pile Identification	Remarks
1.	VTP1	VTP1	1. A particular pull was applied and load released suddenl
2.	VTP	VTP2	
3.	VTP	VTP3	
4.		VTP4	2. Each test was repeate three times
5-		VTP5	
6.		VTP6	

III LATERAL VIBRATION TESTS

Sl. No.	Pile Identi-fication	Acceleration Measurement at	Rotation	Remarks
1.	VTP1	Pile cap and ground level	Permitted	1. Observations wer taken for eccent cities of 8° to 9 in steps of 8°
2.	VTP2	Pile cap and ground level	Permitted	
3.	VTP3	Pile cap and ground level	Permitted	2. Acceleration mea ments were taken each eccentricit at 20 different frequencies.
4.	VTP4	Pile cap and ground level	Permitted	
5.	VTP5	Ground level	Permitted	
6.	VTP6	Ground level	Permitted	3. Sufficient time allowed to reach steady state con ditions.

6.2.2 TEST PILE AND SOIL

In Table 6.2 the details of soil-pile systems have been given.

The piles VTP1, VTP2 and VTP3 were Franki piles with enlarged bulbs at bottom. A typical Franki pile section is shown in Fig 6.1a. These piles were of 40 cm diameter with six 12 mm bars and nominal stirrup placing. In Fig 6.1b the soil conditions near the pile locations have been given. The soil upto a depth of 8m. consists of soft clay having N-value of two to four. Below 8 m upto 14 m is a loose deposit of clayey silt. Again, from 14 m to 18 m is soft clay deposit which is underlain by stiff to very stiff clay upto 22m. Below the stiff clay layer the dense sand strata has been encountered. Normally the piles were set at the sand deposit. The driving record of a typical Franki pile is given in Table 6.3.

The VTP4 pile was a Simplex friction pile of 50 cm diameter having six bars of 16 mm diameter as reinforcements. The soil around the piles were medium to stiff clay. Fig 6.2a and 6.2b show the pile section and the soil details.

The piles VTP5 and VTP6 were hollow steel casing pipes of internal diameter 10.2 cm and wall thickness 1.2 cm. The piles were driven to a depth of 6 m. The soil condition near the pile locations have been given in Fig 6.3. The soil

Table 6.2

Details of Soil and Pile

Pile Identi-fication	Details of Pile Section	Soil type
VTP1	40 cm Φ. Franki pile of 25 metres length with six bars of 12 mm dia.	Soft clay
VTP2	40 cm Φ franki pile of 24 metres length with six bars of 12 mm dia	Soft clay
VTP3	40 cm Φ franki pile of 22 metre length with six bars of 16 mm dia	Medium clay
VTP4	50 cm Φ simplex pile of 16 metre length with six bars of 16 mm dia	Stiff clay
VTP5	12.6 cm Φ steel pipe o six metre long 12.6 cm Φ steel pipe 5 metre	Silty sand
VTP6	12.6 cm Φ steel pipe 5 metre long	Medium silty sand

Table 6.3

Driving Results of Test Piles VIP_1 and VIP_2 Carried Out At I.O.C. Site

Drop of Hammer in ft.	Depth of drive in metre	Nature of Blows VIP_1	Nature of Blows VIP_2	Settlement in mm VIP_1	Settlement in mm VIP_2
Self wt					
4'-0"	0-1				
"	1-2	4	4		
"	2-3	4	4		
16'-0"	3-4	3	3		
"	4-5	3	4		
"	5-6	3	4		
"	6-7	3	4		
"	7-8	3	5		
"	8-9	4	5		
"	9-10	14	15		
"	10-11	13	17		
"	11-12	15	16		
"	12-13	13	16		125
"	13-14	17	13		
"	14-15	10	19		
"	15-16	9	15		
"	16-17	11	11		
"	17-18	13	11	44	90
"	18-19	16	18	20	15
"	19-19.5		11	25	18
"	19-19-20	29	9	16	21
"	20-20.5		10		22
"	20.5-21	25	16	15	14
"	21-21.5	23	17	19	14
"	21.5-22	20	22	10	11
"	(22-22.25)				
"	22-22.5	25	23	9	8
"	22.5-23	29	27	6	17
"	23-23.5		28		8

near pile locations consists of silty sand deposits. The top 1.0m layer was a dessicated soil. There was a large reduction in the strength characteristics during rainy seasor

6.3 TEST PROCEDURE

6.3.1 STATIC LATERAL LOAD TESTS

The lateral loads were applied at the ground level by jacking the piles with the help of a suitable hydraulic jack. The lateral displacements were measured with dial gauges, placed in the lateral load direction. Fig 6.4 illustrates the test set-up for piles VTP_1 and VTP_2. In this case the piles VTP_1 was loaded with a vertical load of 55T and VTP_2 was without any vertical load. One pile served as the reaction for the other. In each case the lateral load was applied in suitable increments. Each load increment was maintained till the rate of movement was 0.002 per hour. In a similar way rest of the piles were also teste

6.3.2 LATERAL FORCED VIBRATION TESTS:

The set-up for the lateral forced vibration tests is shown in Fig 6.5. The desired lateral vibration was generated with a Lazan type mechanical oscillator. The oscillator was driven with the help of a variable speed D.C. motor using pulleys and belt drives. Using a specially fabricated fixing arrangement the oscillator-motor assembly was mounted rigidly along the central axis of piles. The position of the oscillator was such that, only lateral vibrations were generated.

The oscillator consists of two equal eccentric masses rotating at the same angular frequency but in opposite directions. Thus a sinusoidal force is generated perpendicular to the plane passing through the two axes of rotation. In the Lazan type oscillator which was used, the eccentricity is altered by changing the relative positions of two semi-cylindrial equal masses. When the masses are in phase, the eccentricity setting (θ) is referred as 180°, and the force developed is maximum. On the other hand zero eccentricity means that the masses are exactly opposite to each other on the shaft. In this case the angular eccentricity as well as the unbalanced force equal zero values.

The oscillator is driven by a D.C. motor. The speed of the motor is controlled by an independent speed control unit. The oscillator is capable of developing forces over wide ranges of frequencies and eccentricities. The dynamic force generated by the oscillator (Puri (1969)) which was used is given by:

$$\text{Dynamic Force} = 4.52\,w^2 \,\text{Sin}\, \theta/2 \qquad \ldots\ 6.1$$

where, w is frequency in cps and θ eccentricity settings in degrees.

In each pile case, in order to develop a constant force over a wide range of frequency, the eccentricity and frequencies were varied in a broad range. The adjustment of the eccentricity was done in steps of 8^o (two complete revolutions of the eccentricity adjusting knob) and the frequency was varied by adjusting the control turns in the auto-transformer of the speed control unit.

Each pile was tested under eccentricities of 8^o to 96^o in steps of 8^o. The vibration measurements were made under each eccentricity for twenty different frequency level or more.

The vibrations were sensed with two acceleration pickups (inductance type) mounted on the plane of vibration. One pickup was mounted on the pile cap and the other to the pile at ground level. The pickups were securely attached to the concrete sections using chemical resins.

The time-history of acceleration measurements (varying signal response) were fed to a self recording oscillograph through suitable pre-amplifiers.

In Fig 6.6 the block-diagram of the instrument assembly has been illustrated.

While executing the lateral vibration test, at each instant care was taken to allow sufficient time for the system to reach steady state.

6.3.3 FREE VIBRATION TESTS

For executing the free vibration tests, the set-up shown in Fig 6.7 was used, to apply the necessary pulling force. By rotating the pulling screw, certain load is applied to the pile at the ground level. Then the load is released suddenly with the help of the clamp. The pile-soil system which has been displaced from the equilibrium position, when released, oscillates under its natural frequency of vibration. The vibration-time signals are sensed with the acceleration pickups and are recorded. Each of the pile was tested different times applying different level of pulls and the free vibration record was obtained for each test conditions.

6.4 TEST DATA

6.4.1 STATIC TESTS:

The load deflection characteristics of VTP_2 through VTP_6 have been plotted in Fig 6.8 and 6.9. The pile VTP1 which was tested under lateral load along with a vertical load of 55T experienced no lateral movement at all.

The static lateral load tests were carried out to determine the stiffness of the soil-pile systems under sustained load applications. The soil stiffness values have been determined based on the relationship provided by Reese and Matlock (1956) and Davisson and Gill (1963).

These solutions have been explained below.

The differential equation governing the deflection is given by

$$\frac{d^4y}{dx^4} + \frac{k_x y}{EI} = 0 \qquad \ldots 6.2$$

Where EI - flexural stiffness of the pile in kg cm²

y - is the deflection in lateral direction

x - is the depth co-ordinate.

The variation of the modulus of subgrade reaction would depend upon the type of soil. For granular soils and normally loaded clays the soil modulus varies linearly with depth, i.e. $k_x = n_h . x$, where n_h is the constant of horizontal subgrade sreaction, FL^{-3}. Solutions in terms of non-dimensional parameters are available for this differential equation (Reese and Matlock (1956)).

Using the notation:

$T = \sqrt[5]{\frac{EI}{n_h}}$ where T is called relative stiffness factor having units of length (defined in Chapter III). The non-dimensional depth factor $Z = x/T$ where x is the distance below the ground level along the pile axis. The maximum depth factor $Z_{max} = \frac{L_s}{T}$ where, L_s is the embedded length of the pile.

The solution for deflection y_g due to horizontal shear, Q_{hg} at ground level is obtained as:

$$y_g = \frac{Q_{hg}\, T^3}{EI} \times Ax \qquad \ldots\ 6.3$$

Where Ax is the non-dimensional deflection coefficient. $Ax \doteq 2.435$ at ground level for a long pile ($Z_{max} \geqslant 5$).

For piles embedded in pre loaded clays, Davisson and Gill (1963) have given the solutions. For such soils the soil modulus values remain constant with depth, $kx = k$, where, k and kx are having units of FL^{-2}.

For such conditions the solution for deflection y_g due to horizontal shear Q_{hg} at ground level is given by

$$y_g = \frac{Q_{hg}\, R^3}{EI} \times Az \qquad \ldots\ 6.4$$

Where $R = \sqrt[4]{\frac{EI}{k}}$, R is the relative stiffness factor having length units. The non-dimensional deflection coefficients Az at the ground surface has the value of 1.40. Using the above solutions, the soil constants for static loading conditions have been computed. These values for different piles have been given in Table 6.4. The soil surrounding the piles VTP1 to VTP4, have been considered to have constant values of soil modulus with depth. For VTP5 and VTP6, (granular soils) the soil modulus is considered to vary linearly with depth.

6.4.2 FREE VIBRATION TESTS:

Typical free vibration records for piles VTP1 to VTP6 have been shown in Fig 6.10. Knowing the paper speed

Table 6.4

Soil Constants Under Static Loading Conditions

Pile Code	Load kg	Deflection cm	EI kg cm²	R cm	K kg/cm²	n_h kg/cm²
2	3	4	5	6	7	8
VTP2	3000	0.063	0.151×10^{11}	61	1093.0	-
VTP3	3000	0.099	0.151×10^{11}	70.8	599.0	-
VTP4	3000	0.021	0.36816×10^{11}	56.8	3516.0	-
VTP5	750	1.0	6.21×10^{8}	69.8	-	0.375
VTP6	950	0.65	6.22×10^{8}	56	-	1.135

Table 6.5

Free Vibration Test Results

No.	Identi-fication	Natural frequency in cps	Damping coefficient
1	2	3	4
1	VTP1	6.25	15 %
2	VTP2	6.40	16%
3	VTP3	10.0	16 %
4	VTP4	31.2	12 %
5	VTP5	2.38	12 %
6	VTP6	2.4	10 %

in the oscilloscript recorder, the time period and hence the frequencies for one complete cycle is easily evaluated.

Using the logarithmic decay principle the, damping coefficient ξ, is given by:

$$\xi = \frac{2.303}{2\pi} \cdot \log_{10} \frac{A_1}{A_2} \qquad \ldots\ 6.5$$

where, A_1 and A_2 are the amplitude of vibrations in successive cycles and ξ is the damping factor.

In Table 6.5 the values of natural frequencies and damping coefficients for different piles have been provided. These values are based on the average test results of the several free vibration tests on each piles.

6.4.3 LATERAL FORCED VIBRATION TESTS:

A typical acceleration time record for each of the tested pile is given in Fig 6.11.

For sinusoidal motions, from the knowledge of acceleration and frequency, the amplitude of motion is given b

$$\text{Amplitude} = \frac{\text{Acceleration}}{(2\pi \times \text{frequency})^2} \qquad \ldots\ 6.6$$

For each eccentricities the acceleration of vibration and hence the amplitude were recorded for differen frequencies of vibrations. Under each eccentric setting and frequencies of vibrations the imparted dynamic force was also calculated knowing the characteristics of the oscillator.

From the amplitude-frequency data the response of piles at few eccentricities have been plotted in Fig 6.12, for typical pile test. Similar such plots have been obtained for piles VTP1, VTP2, VTP3, VTP4, VTP5 and VTP6. Each of the piles were excited at resonant ranges. In fact the test data is so much, that they are not tabulated, herein.

6.4.3 DYNAMIC BEHAVIOUR OF PILES

From the observed behaviour of piles under lateral vibration conditions, the dynamic behaviour could be quantitatively assessed. With the increase in frequency the dynamic force and hence the acceleration should increase. However, the increase in amplitude under resonant conditions is greatly indicative in the amplitude frequency plots. Greater amplifications under resonant conditions have been indicated since the forced vibration was of sinusoidal characteristics.

From these figures it is also seen that as the eccentricity level (the dynamic force) is increased the resonant frequencies get reduced. This indicates the strain softening phenomena which is typical of non-linear system.

The natural frequencies under free vibration conditions have been greater than the resonant frequencies under forced vibration conditions.

During testing it was observed that when the forcing frequencies were at or near the resonant frequencies there was a sudden increase in the amplitude of vibrations. However, when the vibration was maintained for sufficient time, even without any change in the forcing frequencies, the amplitude of vibration died down suddenly. The above phenomena was displayed in each of tested piles. The record of such obaervations for different piles have been given in Fig 6.13.

At each eccentricities despite of increase in the frequency of vibration beyond resonant ranges, there was an observed reduction of amplitudes, though with the increase in frequencies the force levels also increase.

6.5 DETERMINATION OF THE SOIL-PILE CONSTANTS

6.5.1 THE EXPERIMENTAL CASE:

In any dynamic analysis proper assessment of the values of the basic material constants of the soil-pile system is required. Nair (1968) has clearly emphasised the importance of the determination of material constants especially from the soil engineers point of view, so that there is a realistic estimate of pile-soil interaction mechanism under dynamic loads.

In order to determine the material constants, a logical interpretation of in-situ lateral vibration tests has been proposed. As shown earlier in Fig 6.5, the piles have been excited for lateral forced vibration conditions. The lateral dynamic loads have been applied to the pile top or the top mass.

The experimental case is shown in Fig 6.14(i). This experimental situation is idealised by treating the soil pile system as a single-mass-spring-dash pot system. The idealisation is given in Fig 6.14(ii). Herein, it is to be noted that the lateral forced vibration, F_0 Sin wt is applied directly to the mass.

But in the actual case the piles-soil systems are subjected to Lateral vibrations by virtue of base rock motions or the dynamic loads can be considered to be applied at base. The actual case and the single degree-mass-spring-dash pot idealisation with motions applied at the base have been shown in Fig 6.14 (iii) and Fig 6.14 (iv).

Now, the differential equation of motions governing the idealised experimental case and actual case are given below:

<u>Actual Case</u>

$$y = Y \text{ Sin } wt \qquad \ldots 6.7$$

$$\ddot{y} = -Yw^2 \text{ Sin } wt \qquad \ldots 6.8$$

$$m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0 \quad \ldots 6.9$$

$$m(\ddot{x}-\ddot{y}) + c(\dot{x}-\dot{y}) + k(x-y) = -m\ddot{y} \quad \ldots 6.10$$

$$m\ddot{z} + c\dot{z} + kz = mY w^2 \operatorname{Sin} wt \quad \ldots 6.11$$

where $z = x-y$

Experimental case

$$m\ddot{y} + ky + c\dot{y} = Fo \operatorname{Sin} wt$$

It is seen that the differential equations governing the motions are the same. Therefore it is appropriate to determine the materail constants based on the basis of latera forced vibration tests as performed.

6.5.2 SOIL PILE STIFFNESS:

The overall soil-pile stiffness under dynamic loading conditions was determined based on the observed dynamic behaviour of piles under lateral vibration conditions

For the idealised single degree freedom system subjected to sinusoidal forced vibration at mass point we have:

$$X_{dyn} = \frac{1}{\mu} \frac{F_o}{K} \quad \ldots 6.12$$

where

$$\frac{1}{\mu} = \frac{1}{\sqrt{\left[1-\left(\frac{w}{w_n}\right)^2\right]^2 + \left[2\xi\frac{w}{w_n}\right]^2}} \quad \ldots 6.13$$

$$\frac{F_o}{K} = \mu X_{dyn} \quad \ldots 6.14$$

$$\frac{F_o}{K} = \delta_{st} \quad \ldots 6.15$$

herein, w - is the forcing frequency

w_n - is the resonant frequency

ξ - is the damping coefficient

F_o - is the dynamic force

K - the overall stiffness of the system.

For each of the tested piles, the observed pile response has been plotted as

(i) Amplitude versus Frequency plots.

(ii) Dynamic Force versus Dynamic Amplitude plots.

The dynamic force versus amplitude plots for piles VTP1 through VTP6 have been plotted in Fig 6.15 through Fig 6.20 for different indicated forces frequency levels.

Based on such results using equation 6.12 it is possible to obtain a plot of δ_{st} ($= X_{dyn} \div \mu$) with dynamic force Fo.

The dynamic force Fo is a function of eccentric settings in the oscillator and the forcing frequency. Also, depending on the eccentricity the resonant frequency varies. Therefore, utilising the amplitude-frequency and dynamic force plots and considering the variation of resonant frequencies at each eccentricities, it is possible to obtain the δ_{st} values at different 'Fo' and 'w' values. The number of such calculated δ_{st} values for different 'F_o' and 'w' values when plotted result in Fig 6.21 to Fig 6.24. Herein, the values of damping coefficient have been based on the logar thmic decay record of free vibration tests.

It is seen from these figures that, for all practi purposes there is a unique variation of δ_{st} with F_o, irrespective of variations in forcing frequency w, and resonant frequency w_n.

The tangent modulus of this plot gives the overal stiffness 'K', in FL^{-1}, of the soil- pile system under dynamic conditions.

6.5.3 SOIL STIFFNESS

The soil-stiffness values under dynamic conditior are evaluated based on the knowledge of overall-stiffness, of the soil- pile systems.

Idealising the soil-pile system with a similar mechanical model as proposed by Reese and Matlock (1956) v have the deflection at the ground surface given in articl as under

$$y_g = \frac{Q_{hg} T^3}{EI} \cdot A_x$$

For long piles in the case of granular soils, A_x = 2.435, a ground surface. Q_{hg}/y_g is the overall stiffness 'K' of t soil-pile system under dynamic loads.

Substituting the values of overall-soil-pile stiffness, 'K' in the above equation we can assess the values of soil constants under dynamic conditions by following the procedure given below:

1. We have :

$$T^3 = \frac{EI}{A_x} \cdot \frac{1}{K} \quad \text{... 6.16}$$

2. After determining the value of T, the constant of horizontal subgrade reaction, n_h is calculated from the relation :

$$T = \sqrt[5]{\frac{EI}{n_h}}$$

Hence, the soil constant value under dynamic condition is established.

For piles embedded in clay type soils, with soil modulus remaining constant with depth we have the deflection at ground level given by equation 6.4 reproduced below

$$y_g = \frac{Q_{hg} R^3}{EI} A_z$$

For long piles and at ground surface, $A_z = 1.40$ under free head conditions.

For piles embedded in clayey soils, knowing the values overall-soil-pile stiffness 'K', they may be substituted in the above equation to get the value of relative stiffness factor, R under dynamic conditions.

We have by definition :

$$R = \sqrt[4]{\frac{EI}{k}}$$

Using the above relation for dynamic conditions, the value of soil-modulus k, can be determined.

Thus, based on the lateral vibration tests and logical interpretation of the resulting test data, the values of the relevant soil-constants under dynamic conditions can be easily evaluated.

In Table 6.6, the soil-constants under dynamic conditions, have been provided for the test piles VTP1 through VTP6.

In the same table comparison of soil-pile stiffness and soil-constants under static and dynamic conditions have been provided.

Now, knowing the values of soil modulus, k_x under dynamic conditions, the discretisation procedure of article 3.2 may be used, to evaluate the spring constant values at various mass locations.

Thus, with the knowledge of, soil modulus k_x, the damping factor ξ, flexural stiffness EI and modulus of rigidity G the required material constants for use in

Table 6.6

Soil-Pile Constants Under Static And Dynamic Conditions

Pile No.	Soil-Pile Stiffness 'K' kg/cm		$\frac{K_{static}}{K_{dynamic}}$	Soil Constants		$\frac{(k_x)_{static}}{(k_x)_{dynamic}}$	Remarks
	Dynamic	Static		Dynamic	Static		
VTP1	3.8×10^3	4.7621×10^4	7.61	$k = 72.95$ kg/cm²	$k = 1093.7$ kg/cm²	15.344	Assuming soil modulus constant with depth.
VTP3	3.8×10^3	3.0×10^4	7.888	$k = 37.58$ kg/cm²	$k = 599.0$ kg/cm²	15.93	Assuming soil modulus constant with depth.
VTP4	1.55×10^4	1.43×10^5	9.43	$k = 181.97$ kg/cm²	$k = 35160$ kg/cm²	19.2	Assuming soil Modulus properties to depth.
VTP5	185	750	4.06	$n_h = 0.036$ kg/cm³	$n_h = 0.3750$ kg/cm³	10.4	
VTP6	537	1770	3.38	$n_h = 0.215$ kg/cm³	$n_h = 1.6$ kg/cm³	7.3	Assuming Soil Modulus proportional to depth

Pile No.	ness 'K' kg/cm		$K_{static} / K_{dynamic}$	Soil Constants		$(k_x)_{static} / (k_x)_{dynamic}$	Remarks
	Dynamic	Static		Dynamic	Static		
VTP1	3.8×10^3	4.7621×10^4	7.61	k = 72.95 kg/cm²	k = 1093.7 kg/cm²	15.344	Assuming soil modulus constant with depth.
VTP3	3.8×10^3	3.0×10^4	7.888	k = 37.58 kg/cm²	k = 599.0 kg/cm²	15.93	Assuming soil modulus constant with depth.
VTP4	1.55×10^4	1.43×10^5	9.43	k = 181.97 kg/cm²	k = 35160 kg/cm²	19.2	
VTP5	185	750	4.06	n_h = 0.036 kg/cm³	n_h = 0.3750 kg/cm³	10.4	Assuming soil Modulus properties to depth.
VTP6	537	1770	3.38	n_h = 0.215 kg/cm³	n_h = 1.6 kg/cm³	7.3	Assuming Soil Modulus proportional to depth

any dynamic analysis are considered as evaluated.

6.6 COMPARISON OF OBSERVED AND PREDICTED QUANTITIES

The performance of the mathematical models and the methods of analysis, which are discussed in Chapters III, IV and V, have been checked herein, by comparing the observed and predicted quantities.

As discussed in articles 3.4.2 and 4.3 the entire response computations are primarily dependent on the predict values of the natural frequencies of the soil-pile system.

The relationship between frequency factors and rela stiffness factors, given in Figures 3.8, 4.2 and 5.1 are bas on the proposed models and methods of analysis.

The natural frequencies of the soil-pile systems we predicted using these design curves so that their validity also could be established.

In Table 6.7 the observed and predicted quantities of test piles VTP1, VTP2, VTP3 and VTP4 have been compared Fig 3.8 has been used for predicting the natural frequencies of piles under first mode of vibrations. The pile top is considered as free to rotate. Since the frequency factors of lumped-mass analysis and those of continuous system analy are practically the same, the predicted values using continu ous system analysis have not been tabulated separately.

In Table 6.8 the observed and predicted quantities of piles VTP5 and VTP6 have been tabulated. The predicted quantities are based on the Fig 5.1.

In addition to the above, the observed natural frequencies of single piles of Prakash et al (1973) have been compared with the present solutions. Detailed model tests on single piles and pile groups have been carried out by the investigators to understand the vibration characteristics.

The free vibration characteristics of the piles were observed by applying a desired pull (lateral load) and suddenly releasing the same.

The relevant soil-pile properties in the above studies are as under :

1.	Soil Type	: SP - poorly graded sand with little fines.
2.	Maximum void ratio	: 0.93
3.	Minimum void ratio	: 0.55
4.	Relative Density at test Condition	: 80%
5.	Angle of internal friction at 80% R_D	: 42.6
6.	Embedded length of the pile	: 70 cm
7.	Pile Material	: Aluminium (Al)
8.	External Dia	: 16 mm
9.	Wall thickness	: 1.25 mm
10.	Flexural stiffness, EI	: 1.25 x 105 kg cm^2

Table 6.7

Comparison of Observed and Predicted Quantities

Pile code	Relative stiffness Factor, R in cm	Soil Modulus k, in kg/cm²	Sustained vertical load in kg	Frequency Factor	Natural Frequency in cps Observed	Natural Frequency in cps Predicted
VTP1	153.18	27.42	1750	0.83	6.25	6.40
VTP2	119.95	72.95	3500	0.84	6.40	6.60
VTP3	141.60	37.58	1000	0.835	10.00	9.77
VTP4	119.20	181.97	500	0.84	31.22	27.56

Table 6.8

Comparison of Observed and Predicted Quantities

Pile code	Relative stiffness Factor, T in cm	n_h, in kg/cm³	Sustained vertical load in kg	Frequency Factor	Natural Frequency in cps Observed	Natural Frequency in cps Predicted
VTP5	111.63	0.036	800	0.65	2.38	2.42
VTP6	78.00	0.215	800	0.65	4.80	4.14

In Table 6.9 and 6.10 the observations of Prakas
et al (1973) have been tabulated. This particular aspec
of the study has been performed.

(i) to investigate the effect of sustained verti load on the observed values of natural frequ

(ii) to investigate the influence of lateral load on the observed values of natural frequencie

In the same tables the predicted values of the na frequencies of these piles, under the given conditions, u the present analysis have been provided. Fig 5.1 has bee used for predicting the natural frequencies.

Critical examination of the observed and predicte quantities of Tables 6.7, 6.8, 6.9 and 6.10 reveal the following :

(i) the proposed theory predicts the natural frequ cies to a reasonable degree of accuracy.

(ii) the proposed design curves for predicting the natural frequencies, can be used with confider since the predicted quantities are close to t observed quantities.

(iii) Solutions to practically any soil-pile system is possible with the presented methods of anal and the design curves. In fact the pile types soil types have been varied and even model pil

Table 6.9

Comparison of Observed and Predicted Quantities using Prakash et al (1973)

$T = 14.80$ cm , $n_h = 0.176$, $Z_{max} = 4.735$

Sustained vertical load in kg	Frequency factor	Natural Frequency in cps		Remarks
		Observed	Predicted	
4	0.64	12.0	9.4	1. Lateral Load Level = 5 kg.
8	0.64	8.8	6.61	2. Fig 5.1 has been used for comparison
12	0.64	7.0	5.45	3. With the increase in sustained vertical load the natural frequency is reduced by $\frac{1}{\sqrt{M_t}}$ times
16	0.64	5.8	4.7	
20	0.64	5.0	4.42	
24	0.64	4.5	4.0	

Table 6.10

Comparison of Observed and Predicted Quantities using Prakash et al (1973)

$T = 15.875$ cm $n_h = 0.1239$ $Z_{max} = 4.41$

Sustained vertical load in kg	Frequency factor	Natural Frequency in cps		Remarks
		Observed	Predicted	
4	0.64	11.0	8.91	1. Lateral load level = 10 kg.
8	0.64	9.0	6.30	2. Fig 5.1 has been use
12	0.64	6.3	5.14	3. With the increase in sustained vertical load the natural fre quency is reduced by $\frac{1}{\sqrt{M_t}}$ times
16	0.64	5.2	4.45	
20	0.64	4.5	3.985	
24	0.64	4.0	3.64	4. At same sustained ve tical load, $(f_n)_{6.9}/(f_n)_{6.10} = (\frac{0.176}{0.1239})^{0}$

have been covered in these comparisons.

From Tables 6.9 and 6.10 the following points are seen to be of significance.

1. For a particular lateral load, the natural frequencies, at different sustained vertical load levels M_t, are varying as $\frac{1}{\sqrt{M_t}}$ times. The experimental values of Table 6.9 a shown that:

   If the natural frequency, f_n at a sustained vertical load M_x is known.

   he natural frequency f_{ny} at any other increased sustained vertical load level of M_y, is given by :

$$f_{ny} = f_{nx} \cdot \sqrt{\frac{M_x}{M_y}}$$

   In a similar way the predicted quantities also show reduction in natural frequencies with increase in lumped mass at top.

2. In Table 6.10 the observed and predicted quantities of natural frequencies of the same pile system (of Table 6.9) with a different lateral load level have been tabulated. The effect of lateral load level has been investigated to see the influence of deflections and hence flexibility defined by relative stiffness factor,

and of soil stiffness (defined by, n_h) on the natural frequency characteristics of soil-pile system.

Comparison of the experimental natural frequencies for a particular sustained load level of Tables 6.9 and 6. show that :

$$\frac{(f_n)_{6.9}}{(f_n)_{6.10}} = \left[\frac{(n_h)_{6.9}}{(n_h)_{6.10}} \right]^{0.3}$$

where

$(f_n)_{6.9}$ - experimental natural frequency corresponding to Table 6.9 at any sustained vertical load M_x.

$(f_n)_{6.10}$ - experimental natural frequency corresponding to Table 6.10 at the same sustained vertical load of M_x.

$(n_h)_{6.9}$ - 0.176 kg/cm³

$(n_h)_{6.10}$ - 0.1239 kg/cm³

It may be recalled herein, that in article 5.2 the factors influencing the first natural frequency has been defined as under by equation 5.2.

$$w_{n1} \propto \frac{n_h^{0.3} \; EI^{0.2}}{M_t^{0.5}}$$

Thus the above verification further emphasises the correctness of the approach.

6.7 CONCLUDING REMARKS

Based on the limited experimental tests on full size prototype piles the following conclusions may be drawn :

1. By conducting lateral forced and free vibration tests, it is possible to assess the following material properties of the soil-pile system under dynamic conditions

   (i) overall soil-pile stiffness.

   (ii) soil modulus values at any depth.

   (iii) damping characteristics of the soil-pile systems.

2. The suggested method of testing and the material properties thereoff could be considered realistic since they are based on vibration tests on actual test piles.

3. The suggested testing and interpretation techniques could be standardised to collect useful data on varieties of soil-pile systems.

4. Under steady state vibrations quasi-resonant conditions are possible.

5. For preliminary design purposes the soil-pile properties suggested in Table 6.6 may be used.

It has also been demonstrated that the method of analysis presented in Chapter III and IV and the design curves of Chapters III, IV and V may be used with confidence for predicting the dynamic response of soil-pile systems.

CHAPTER - VII

USE OF DESIGN CURVES FOR ASSESSING DYNAMIC RESPONSE OF PILES AND DISCUSSIONS ON THE PRESENTED TECHNIQUE

7.1 INTRODUCTION

In this chapter the uses of the various non-dimensional curves for practical problems have been explained, by solving two typical problems. In one of the cases the pile has been embedded in a soil in which the soil modulus can be considered to vary in proportion to depth. In the secon case the pile is embedded in a soil in which the soil modul has been assumed to remain constant with depth. The dynami deflection, rotation, bending moment and shear along the leng of the piles have been obtained using the proposed non-dimensional curves. The adopted spectral displacement valu are based on the design spectrum proposed by Housner (1970) Fig 7.1. For any specified earthquake the calculated response spectrum may not be same as this design spectrum. If results for a specified earthquake is required the computed response spectrum of the specified earthquake must be used in the computation. However, the procedure outline for calculating the dynamic response remains same. Also, a the end of the chapter the salient features and the limitations of the techniques presented have been brought out.

7.2 METHOD OF USING NON-DIMENSIONAL CURVES

7.2.1 PILES EMBEDDED IN GRANULAR SOILS:

7.2.1.1. Problem

A multistoreyed building shown in Fig 7.2 is to be constructed at New Delhi in a predominantly sandy area.

The details of the soil at the site are given in Fig 7.3 and Fig 7.4 and Tables 7.1 and 7.2.

The details of the chosen pile sections and loading are as follows:

1. Piles are used in groups of four, five and six. Arrangements of different pile groups are shown in Fig 7.5a.
2. The details of the pile sections are given in Fig 7.5b. The piles are 48.2 cm in diameter with reinforcements of six bare of 18 mm dia and nominal binders. The piles are driven to a depth of 25 metres from the ground surface.
3. A typical heavily loaded column is subjected to a vertical load of 257 tonnes.
4. The safe vertical load carrying capacity of piles is 150 Tonnes.
5. The static lateral load test results of a single pile is given in Fig 7.6.

Since New Delhi is in a seismically active zone, earthquake resistant considerations of the chosen pile sections are to be examined.

7.2.1.2 Design Steps

1. From the chosen pile group arrangements it is seen that the critical case would be that of four pile groups. Therefore it is required to examine the safety of pile sections of the four pile groups.

2. The maximum vertical load in a column equals 257 tonnes. Neglecting group action, the load per pile = $\frac{257}{4}$ = 64.25 tonnes. Since this load is a sustained vertical load it shall be taken for top mass, M_t, lumped at top of the pile.

3. For the given section I_{xx} is calculated.

$$I_{xx} = \frac{\pi D^4}{64} + (m-1)\ n.\ \frac{\pi d^4}{64} + 4.\ A x_1^2$$

For the given pile section we have.

$$I_{xx} = \frac{\pi.\ 48.2^4}{64} + 17 \times 6 \times \frac{\pi.\ 1.8^4}{64} + \frac{4.\pi}{4}.\ 1.8^2 (17.24$$

$$= 31,50,00\ cm^4.$$

$$Ec\ \ I_{xx} = 3.71 \times 10^{10}\ Kg\ cm^2.$$

Where Ec I_{xx} is the flexural stiffness of the pile.

4. No dynamic test data has been given for determining the material constants under dynamic conditions. Therefore,

the static test result values are utilised for assigning the soil-constants under dynamic condition. From Fig 7.6, the static lateral load deflection plots, we have, for a load of 3000 kg the deflection of pile at ground surface is 0.15 mm.

Using the non-dimensional solutions proposed by Reese and Matlock (1956), for soil modulus varying proportional to depth, we have:

$$y_g = \frac{Q_{hg} \; T^3}{EI} \times A_x$$

herein, Q_{hg} = 3000 kg

y_g = 0.15 mm

A_x = 2.435 at ground surface

EI = 3.71×10^{10} kg cm²

Therefore relative stiffness factor, $T = 76.74$ cm

As per definition $T = \sqrt[5]{\frac{EI}{n_h}}$, where n_h is the constant of horizontal subgrade reaction, FL^{-3}.

Therefore, $n_h = 13.964$ kg/cm³.

For the dynamic conditions, we have:

$$(n_h)_{dynamic} = \frac{1}{10} \cdot (n_h)_{static}$$

Hence, $(n_h)_{dynamic} = 1.3964$ kg/cm³

Relative stiffness factor, T, under dynamic conditions

= 121.60 cm.

5. The four pile group given in Fig 7.5a is capped with a pile cap and the columns are connected in a fairly rigid fashion to the above. The pile top may not be free to rotate. Hence, it is required to obtain solutions for pile top fixed against rotation conditions as well as free to rotate conditions. Linear interpolation for partially fixed case is suggested.

6. The subsequent steps involve the determination of dynamic response for the following condition :

   (i) pile top free to rotate

   (ii) pile top fixed against rotations.

   (iii) pile top partially fixed against rotation.

7.2.1.3 Pile Top Free To Rotate Conditions:

(a) <u>Determination of Natural Frequencies</u>:

For the given problem, we have:

Embedded length of pile, L_s = 25.0 m.

Relative stiffness factor, T = 1.216 m.

Therefore maximum depth factor, $Z_{max} = \frac{25.0}{1.216} > 15$

The pile can be considered to be an infinitely long pile, since $Z_{max} > 15.0$. Therefore, the solutions pertaining to $Z_{max} = 15$ would be used for obtaining the dynamic respons for the given problem.

Therefore we have:

(i) From Fig 5.1 for T = 1.216, and $Z_{max} > 15$, $F_{SL1} = 0.65$

By definition:

$$w_{n1} = F_{SL1} \sqrt{\frac{g}{W} n_h T^2}$$

herein,

$W = 64.25$ Tonnes

$T = 1.216$ m.

$n_h = 1396.4$ T/m³

$g = 9.81$ m/sec²

$$w_{n1} = 0.65 \sqrt{\frac{9.81}{64.25} \times 1396.4 \times 1.216 \times 1.216}$$

$$= 11.5411 \text{ radian/sec.}$$

$$f_{n1} = \frac{11.5411}{2\pi} = 1.836 \text{ cps.}$$

Period of vibration in first mode, $T_1 = \frac{1}{f_{n1}}$

$= 0.5446$ sec.

From Fig 7.1, corresponding to the period of vibration in first mode we have the spectral displacement value, $S_{d1} = 1.24$ cm

from Fig 7.1

(ii) Using Fig 5.2 in a similar manner we have:

$F_{SL2} = 1.71.$

By definition $w_{n2} = F_{SL2} \cdot \sqrt{\dfrac{n_h \cdot T \cdot g}{d^2 \times \gamma}}$

herein $\gamma = 2.4 \ T/m^3$

$d = 0.482 \ m$

Therefore $w_{n2} = 1.71 \sqrt{\dfrac{1396.4 \times 1.216 \times 9.85}{2.40 \times 0.482 \times 0.482}}$

$= 293.8341$ radian/sec.

$f_{n2} = 46.742$ cps

So, period of vibration = 0.0213 sec.

The spectral displacement, S_{d2}, corresponding to this period equals 0.0.

(b) Determination of Dynamic Displacements:

At various x/T values the non-dimensional normalised modal deflection values B_{y1} and B_{y2} can be read from Fig 5.9 and Fig 5.13 for first and second modes respectively, using the curves pertaining to $Z_{max} = 15$.

Similarly from Fig 5.10 and 5.14 the non-dimensional modal rotation $B_{\theta 1}$ and $B_{\theta 2}$ are read for various x/T values.

By definition (article 5.2.3) the dynamic displacements are as under:

Dynamic deflection, Y_1 in the first mode = $B_{y1} \cdot S_{d1}$

Dynamic deflection, Y_2 in the first mode = $\dfrac{B_{y2}}{T} \cdot S_{d2}$

Dynamic rotation, θ_1 in the first mode = $\frac{B_{\theta 1}}{T} \times S_{d1}$

Dynamic rotation, θ_2 in the second mode = $\frac{B\theta_2}{T} \times S_{d2}$

For the given problem the dynamic displacement values have been tabulated in Table 7.3 and Table 7.4, for first and second modes of vibration respectively.

Table 7.3

Dynamic Displacements During First Mode of Vibrations - Pile Top Free To Rotate Condition

Depth in cm	$\frac{x}{T}$	B_{y1}	Dynamic Deflection $Y_1 = S_{d1} \times B_{y1}$ in cm	$B_{\theta 1}$	Dynamic Rotation $\theta_1 = \frac{B_{\theta 1}}{T}$ × S… in radians
0	0.0	1.0	1.24	-0.63	- 0.0063
100.	0.8223	0.48	0.594	-0.52	-0.0052
200.	1.6447	0.16	0.198	-0.28	-0.0028
300.	2.4671	0.00	0.00	-0.09	-0.0009
400.	3.2894	-0.02	-0.0248	0.0	0.00
500.	4.1118	-0.01	-0.0124	+0.02	+0.0002
600	4.9142	-0.01	-0.0124	+0.01	+0.0001
700	5.7565	0.00	0.00	0.00	0.00

Table 7.4

Dynamic Displacements During Second Mode of Vibrations-Pile Top Free To Rotate Conditions

Depth	$\frac{x}{T}$	B_{y2}	$Y_2=B_{y2}.S_{d2}$	$B_{\theta 2}$	$\theta_2=\frac{B_{\theta 2}}{T}.S_{d2}$
0.0	0.0	0.0	0.0	-1.0	0.0
100.	0.8223	-0.79	0.0	-0.67	0.0
200.	1.6447	-1.1	0.0	0.0	0.0
300.	2.4671	-0.91	0.0	-0.43	0.0
400.	3.2894	-0.47	0.0	+0.44	0.0
500.	4.1118	-0.15	0.0	+0.26	0.0
600.	4.9342	-0.08	0.0	+0.20	0.0
700.	5.7565	0.0	0.0	0.0	0.0

(c) <u>Determination of Dynamic Bending Moment and Shear:</u>

At various x/T values the non-dimensional normalised modal bending moment B_{m1} and B_{m2} are read from Fig 5.11 and 5.15 for first and second modes of vibrations respectively, using the curves pertaining to $Z_{max} = 15$.

Similarly the normalised modal shear values are read from Fig 5.12 and Fig 5.16.

By definition (article 5.2.3) at any point the dynamic bending moments are as under:

Dynamic bending moment, in the first mode $M_1 = B_{m1} \times n_h T^3 \times S_{d1}$

Dynamic bending moment, M_2 in the second mode $= B_{m2} \times n_h T^3 \times S_{d2}$

Similarly, the dynamic shear is given by:

Dynamic shear, S_1 in the first mode $= B_{S1} \times n_h T^2 \times S_{d1}$

Dynamic shear, S_2 in the second mode $= B_{s2} \times h_n T^2 \times S_{d2}$

For the given problem the non-dimensional coefficier the dynamic bending moments and shear values have been tabul ted in tables 7.5 and 7.6 respectively.

Table 7.5

Dynamic Bending Moment and Shear During First Mode of Vibrations-Pile Top Free To Rotate Conditions.

Depth in cm	$\frac{x}{T}$	B_{m1}	Dynamic Bending Moment $M_1 = B_{m1} \cdot n_h T^3 \cdot S_{d1}$ in kg cm x 10^4	B_{s1}	Dynamic Shear $S_1 = B_{S1} \cdot n_h$ x S_{d1} in x 10^4
0.0	0.0	0.0	0.00	0.46	1.1776
100.0	0.8223	0.262	81.57	0.22	0.5632
	1.30	$(0.314)_{max}$	97.76	0.0	0.0
200.0	1.6447	0.294	91.53	-0.07	-0.1792
300.0	2.4671	0.160	49.81	-0.16	-0.4096
400.0	3.2894	0.046	14.32	-0.11	-0.2816
500.0	4.1118	0.00	0.0	-0.03	-0.0768
600.0	4.9342	-0.008	2.48	-0.005	-0.0128
700.0	5.7565	0.0	0.0	0.0	0.0

Table 7.6

Dynamic Bending Moment and Shear During Second Mode of Vibrations - Pile Top Free To Rotate Conditions.

Depth in cm	$\frac{x}{T}$	B_{m2}	Dynamic Bending Moment: $M_2=B_{m2} \cdot n_h T^3 \cdot S_{d2}$ in kg cm x 10^4	B_{S2}	Dynamic shear: $S_2=B_{S2} \cdot n_h T \cdot S_{d2}$ in kg x 10^4
0.0	0.0	0.0	0.0	1.4	0.0
100.0	0.8223	0.82	0.0	0.55	0.0
200.0	1.6447	0.73	0.0	-0.25	0.0
300.0	2.4671	0.32	0.0	-0.55	0.0
400.0	3.2894	-0.15	0.0	-0.22	0.0
500.0	4.1118	-0.26	0.0	+1.0	0.0
600.0	4.9342	0.16	0.0	+0.06	0.0
700.0	5.7565	0.0	0.0	0.0	0.0

(d) Determination of the Overall-response of the System:

The overall response of the pile foundations for the design earthquake is given by the root-mean square addition of the individual modal responses.

Thus for the given problem for pile top free to rotate conditions we have:

(i) The deflection, Y_{max} suffered by the pile $= \sqrt{Y_1^2 + Y_2^2}$

(ii) The rotation, θ_{max} experienced by the piles $= \sqrt{\theta_1^2 + \theta_2^2}$

(iii) The bending moment, M_{max} on the pile section $= \sqrt{M_1^2 + M_2^2}$

(iv) The shear force, S_{max} of the piles $= \sqrt{S_1^2 + S_2^2}$

In table 7.7 the overall dynamic response of the piles in the first two modes have been tabulated.

Table 7.7

The Maximum Response of the System-Pile Top Free to Rotate Conditions.

Depth in cm	$\frac{x}{T}$	Y_{max}	θ_{max} in radian	M_{max} kg cm x104	S_{max} kg x10⁴	Remarks
0.0	0.00	1.24	-0.0063	0.00	1.1776	The contribution of second mode is insignificant. First mode frequency = 1.836 cps. First mode time period = 0.5446 sec. Second mode frequency = 46.742 cps. Second mode time period =0.0213 sec.
100.0	0.8223	0.592	-0.0052	81.57	0.5632	
	1.30			97.76	0.0	
200.0	1.6447	0.198	-0.0028	91.53	-0.1792	
300.0	2.4671	0.0	-0.0009	41.81	-0.4096	
400.0	3.2894	-0.0248	0.00	14.32	-0.28166	
500.0	4.118	-0.0124	+0.0002	0.00	-0.0768	
600.0	4.9342	0.00	+0.0001	-2.48	-0.0128	
700.0	5.7565	0.00	0.0	0.00	0.0	

(e) Examination of the Induced Soil Reactions:

For pile top free to rotate conditions the deflections suffered by the piles during the given earthquake have been tabulated in Table 7.7.

In the given problem the piles have been embedded in granular soils. For these types of soils the soil modulus is considered to vary linearly with depth.

At any depth, x, the soil modulus is given by:

$$k_x = n_h . x$$

where n_h is the constant of horizontal subgrade reaction.

The soil reaction, p at any depth is given by

$$p_x = k_x \; Y_{max} \text{ at } x$$

where, Y_{max} is the deflection of the pile at depth x.

The resistance which the soil can offer to the pile in lateral direction at any depth depends upon the passive resistance of the soil.

At a depth x, the passive resistance P_p is given b

$$P_p = K_p \gamma_1 x . A$$

where (i) K_p is the passive pressure coefficient and is equa to $\frac{1+\sin\Phi}{1-\sin\Phi}$ (ii) Φ the angle of internal friction, and (A) the projected area of pile per unit width.

For the given problem the variation of the induced soil reaction and the soil resistance at different depths have been tabulated in Table 7.8.

Table 7.8

Induced Soil Reaction and Available Soil Resistance.

Depth in cm	$\frac{x}{T}$	Maximum Induced Soil Reaction P $=n_h \cdot x \cdot Y_{max}$ at x kg/cm	Maximum Available Passive Resistance $P_p = K_p \cdot \gamma_d \cdot x \cdot A$ kg/cm	Remarks
0.00	0.0	0.00	0.0	$K_p = \frac{1+\sin\Phi}{1-\sin\Phi}$
100.0	0.8223	82.9461	25.164	$= \frac{1+\sin 30}{1-\sin 30}$
200.0	1.6447	55.2974	46.329	$= 3$
300.00	2.4671	0	69.49	$\gamma_d = 1.602$ gm/cm³
400.0	3.2894	-13.8522	92.65	$A = 48.2 \times 1$ cm²
500.0	4.1118	- 8.6576	115.82	
600.0	4.9342	0.0	138.99	
700.0	5.7565	0.0	0.0	

7.2.1.4 Pile Top Fixed Against Rotation Conditions

(a) <u>Determination of Natural Frequencies:</u>

From Fig 5.3 and 5.4 we have:

(i) The natural frequencies in the first and second mode are $f_{n1}' = 2.8529$ cps and $f_{n2}' = 53.8597$ cps respectively.

(ii) The time period for the above two natural frequencies are $T_1' = 0.351$ sec and $T_2' = 0.018$ sec respectively.

(iii) From the design spectrum curves in Fig 7.1 we have the spectral displacement values for the above periods as S'_{d1} = 0.635 cm and S_{d2} = 0.0

(b) Determination of Dynamic Displacements:

(i) For the first mode of vibration the non-dimensional displacement coefficients B'_{y1} and $B'_{\theta 1}$ are easily read for various x/T values from Fig 5.17 and 5.18. The dynamic displacements Y'_1 and θ'_1 are calculated from the definition. The above quantities are tabulated in Table 7.9

Table 7.9

Dynamic Displacements During First Mode of Vibration - Pile Top Fixed Against Rotation Conditions.

Depth in cm	$\frac{x}{T}$	B'_{y1}	Dynamic Deflection $Y'_1 = S'_{d1} \cdot B'_{y1}$ cm	$B_{\theta 1}$	Dynamic Rotation $\theta'_1 = \frac{B'_{\theta 1}}{T} \cdot S'_{d1}$ radian
0.0	0.0	1.0	0.635	0.00	0.0
100.0	0.8223	0.75	0.4762	-0.45	-0.0023
200.0	1.6447	0.37	0.2349	-0.42	-0.0021
300.0	2.4671	0.10	0.0635	-0.21	-0.0010
400.0	3.2894	0.0	0.0	-0.05	-0.0002
500.0	4.1118	0.02	-0.0127	+0.01	0.0
600.0	4.9342	-0.04	-0.00635	+0.02	-0.0001
700.0	5.7565	0.00	0.0	+0.02	-0.0001

(ii) For the second mode of vibrations since the $S'_{d2} = 0.0$ the dynamic displacement values have not been tabulated.

(c) <u>Determination of Dynamic Bending Moment and Shear</u>:

(i) For the first mode of vibration the non-dimensional bending moment and shear coefficients at various x/T values are easily read from Fig 5.19 and 5.20. Fr the definition, the dynamic bending moment and shear M_1' and S_1' are calculated. The above quantities are tabulated in Table 7.10.

Table 7.10

Dynamic Bending Moment and Shear During First Mode of Vibrations-Pile Top Fixed Against Rotation Condition

Depth in cm	$\frac{x}{T}$	B'_{m1}	Dynamic bending moment $M'_1 = n_h T^3 . B'_{m1} . S'_{d1}$ x10^6 kg/cm	B'_{s1}	Dynamic shear $B'_1 = n_h T^2 . S'_{d1}$ x B'_{s1} x10^4 kg
0.0	0.0	-0.93	-1.4827	1.15	1.507
100.0	0.8223	-0.22	-0.3507	0.90	1.1799
200.0	1.6447	+0.20	+0.3188	0.33	0.4326
300.0	2.4671	+0.25	+0.3986	-0.02	0.0262
400.0	3.2894	+0.12	0.191	-0.16	0.2097
500.0	4.1118	+0.02	0.0318	-0.10	0.1311
600.0	4.9342	0.0	0.0	-0.02	0.0262
700.0	5.7565	0.0	0.0	0.0	0.0

(ii) For the recond mode of vibration since $S'_{d2} = 0.0$ the dynamic bending moment and shear values have not been tabulated.

(d) Determination of Overall Response:

Since there is no contribution from the second mod of vibration the overall response is given by tables 7.9 and 7.10.

(e) Determination of induced soil reaction and check agains the available soil resistance:

The induced soil reaction and the available soil resistance for pile top fixed against rotation condition ha been given in Table 7.11.

Table 7.11

Induced Soil Reaction-Available Soil Resistance

Depth in cm	$\frac{x}{T}$	Maximum Induced Soil Reaction $P = n_h \cdot x \; Y_1'$ at x kg/cm	Maximum available passive resistanc $P_p = K_p \cdot \gamma_d \cdot A$ kg/cm
0.0	0.0	0.0	0.0
100.0	0.8223	66.4965	23.164
200.0	1.6447	65.603	46.29
300.0	2.4671	26.60	69.49
400.0	3.2894	0.0	92.65
500.0	4.1118	-8.867	115.82
600.0	4.9342	-5.278	138.99
700.0	5.7565	0.0	

7.2.1.5 For Partial Fixity Condition:

Depending on the discretion of the designer the degree of fixity at the pile top may be assigned.

Herein, the pile top is considered to have 50 % degree of fixity.

Linear interpolation between the obtained results for pile top free to rotate and fixed against rotation conditions is suggested. The values for partially fixed conditions have not been eseparately tabulated.

7.2.1.6 Design Check

1. The maximum earthquake induced displacements are as under:

(i)	Maximum deflection at top pile top free to rotate conditions	- 1.24 cm
(ii)	Maximum deflection at top for the conditions of pile top fixed against rotation	- 0.635 cm
(iii)	Maximum deflection at top for 50% degree of fixity	- 0.9375 cm
(iv)	Maximum rotation at top for pile top free to rotate conditions	- 0.0063 radians
(v)	Maximum rotation(at a depth of 0.8223 T) for pile top fixed against rotation condition	- 0.0023 radian
(vi)	Maximum rotation (at a depth of 0.8223T) for 50 % degree of fixity	- 0.00375 radian

The permissible displacements are to be checked agains the worst case. Herein, the maximum induced displacements are seen in case (i) and case (iv), as shown above.

The earthquake induced displacements are seen to be within permissible limits.

2. The maximum earthquake induced bending moments and are as under;

(i) The maximum induced bending moment for pile top free to rotate condition (occurs at a depth of 1.30m) = 97.76×10^4 kg cm

(ii) The maximum induced bending moment for pile top fixed against rotation condition (occurs at top) = 148.27×10^4 kg c

(iii) The maximum induced bending moment for 50% degree of fixity (occurs at top) = 74.135×10^4 kg c

The worst case in the above list is case (ii).

Therefore the pile section safety need be checked for the induced bending moment of 148.27×10^6 kg cm.

From the structural design details of the building the total vertical reaction per pile during the earthquake has been obtained as 134.3×10^3.

Induced stress in concrete $= \frac{P}{A} \pm \frac{M}{I} \cdot y$

For the given pile section we have :

Effective area A $= 2083\ cm^2$

Moment of inertia I_{xx} $= 31,50,00\ cm^4$

Distance of neutral axis, y $= 24.1$ cm

Compressive stress in concrete f_1

$$= \frac{134.3 \times 10^3}{2.083 \times 10^3} + \frac{1.48 \times 10^6}{0.315 \times 10^6} \times 24.1$$

$$= 64.47 + 101.9068$$

$$= 166.37\ kg/cm^2$$

Tensile stress in concrete $= 64.47 - 101.9008$

$= -37.4368\ kg/cm^2$

It is seen that the concrete section is subjected to large induced tensile stresses. Therefore the pile section may not be safe.

7.2.1.7 Recommendations:

If four pile groups are used the earthquake induced bending stresses on the pile section exceed safe values. Therefore it would be safer to use six pile group arrangements. However, the earthquake induced stresses and displacements under these groupings are to be rechecked.

7.2.2 PILES EMBEDDED IN SOILS, SOIL MODULUS REMAINING CONSTANT WITH DEPTH:

7.2.2.1 Problem

Examine the earthquake resistant considerations of pile foundations supporting storage oil tanks of a refinery area in an active seismic zone. In the absence of design earthquakes use any suitable design spectrum curves.

The details of loading, the chosen pile sections ar soil conditions are as follows:

1. Design pile sections are circular ones of 50 cm diamet with six bars of 12 mm diameter as reinforcements.
2. The piles are driven to a depth of 20 m.
3. The safe load carrying capacity of the piles are 90 tonnes.
4. From structural design it is seen that during the eart quakes the maximum vertical reaction per pile works out to be 120 tonnes.
5. The soil at site is uniform deposit of preloaded clay with unconfined compressive strength of 1.8 kg/cm^2.
6. The static and dynamic lateral load displacement curves are given in Fig 7.7. The dynamic tests were conducte a series of lateral forced vibration tests applied to pile at ground surface.
7. A total humber of 40 piles are supporting the oil tank and are capped with a pile cap.

7.2.2.2 Design Steps

Since the fixity conditions at pile top can not be assessed properly the solutions are obtained for both pile top free to rotate and fixed against rotation conditions separately.

Determination of Soil Modulus and Relative Stiffness Factor :

The dynamic load, F_o and s_t plot shows the overall stiffness of the pile as 15,500 kg/cm., using the non-dimensional solutions given by Davisson and Gill (1963) we have

$$y_g = \frac{Q_{hg} R^3 \times A_z}{EI}$$

where $A_z = 1.40$ at ground level.

$$R^3 = \frac{0.3681 \times 10^{11}}{15.500 \times 1.40}$$

$$R = 119.2 \text{ cm}$$

and $$R = \sqrt[4]{\frac{EI}{k}}$$

$$k_{dyn} = 181.97 \text{ kg/cm}^2$$

$$Z_{max} = \frac{L_s}{T} = \frac{20.00}{1.192} = 16.779$$

Determination of Natural Frequencies:

For the given pile sections the natural frequencies in different modes are obtained using the appropriate non-dimensional design curves given in Chapter III. In each of these figures, curves identified for $Z_{max} = 15$ have been used. The computed natural frequencies are tabulated below in Table 7.12.

Table 7.12

Natural Frequencies at Different Conditions

Mode No	Fixity conditions at top	Frequency factor dimensionless	Natural Frequency in cps	Time period in sec	Spectral displacement values in cm	Figure used
One	Free	0.84	3.0818	0.325	0.635	3.8
Two	Free	1.13	31.008	0.03	0.005	3.9
Three	Free	1.30	35.674	0.028	0.0048	3.10
One	Fixed	1.18	4.3298	0.23	0.33	3.11
Two	Fixed	1.14	31.30	0.0319	0.005	3.12

7.2.2.3 Pile Top Free To Rotate Conditions:

Determination of Pile Displacements

The non-dimensional deflection and rotation coefficients and the maximum induced displacements for the design earthquake have been computed.

The above quantities for pile top free to rotate condition are tabulated in table 7.13 and 7.14, at different depths.

Table 7.13

Dynamic Deflections For Different Modes of Vibrations-Pile Top Free To Rotate Conditions

Depth	$\frac{x}{R}$	A_{y1}	Y_1 cm	A_{y2}	Y_2	A_{y3}	Y_3	Remarks
0.0	0.0	1.0	0.6350	0.0	0.0	0.0	0.0	1. To determine A_{y1}
50.0	0.4194	0.70	0.445	0.04	0.0003	0.0	0.0	use Fig 3.21
100.0	0.8388	0.46	0.2921	0.08	0.0004	-0.05	0.0002	2. For A_{y2} use
150.0	1.2582	0.25	0.1587	0.12	0.0006	-0.10	-0.0004	Fig 3.22.
200.0	1.6776	0.12	0.0762	0.16	0.0008	-0.15	-0.0007	3. For A_{y3} use Fig
250.0	2.0970	0.03	0.019	0.20	0.0010	-0.19	-0.0009	3.23 and 3.23a
300.0	2.5164	-0.02	-0.0127	0.24	0.0014	-0.20	-0.0009	$Y = A_y \times S_d$
350.0	2.9358	-0.06	-0.038	0.28	0.0016	-0.22	-0.0010	
400.0	3.3552	-0.08	-0.0508	0.32	0.0018	-0.225	-0.0010	
450.0	3.7746	-0.07	-0.0440	0.36	0.0020	-0.225	-0.0010	
500.0	4.194	-0.06	-0.0381	0.40	0.0022	-0.22	-0.0010	
550.0	4.6134	-0.03	-0.0190	0.44	0.0024	-0.21	-0.0009	
600.0	5.0328	-0.02	-0.0126	0.48		-0.18	-0.0008	

Table 7.14

Dynamic Rotations For Different Modes of Vibrations - Pile Top Free to Rotate Conditions

Depth cm	$\frac{x}{R}$	$A_{\theta 1}$	θ_1 Radian	$A_{\theta 2}$	θ_2 radian	$A_{\theta 3}$	θ_3 radian	Remarks
0.0	0.0	-0.70	-0.0037	+0.008	0.0	-0.112	0.0	1. To determine
50.0	0.4174	-0.68	-0.0036	+0.08	0.0	-0.104	0.0	$A_{\theta 1}$ use Fig 3.30
100.0	0.8388	-0.55	-0.0029	+0.08	0.0	-0.098	0.0	2. For $A_{\theta 2}$ use
150.0	1.2582	-0.41	-0.0021	+0.09	0.0	-0.08	0.0	Fig 3.31
200.0	1.6776	-0.28	-0.0014	+0.09	0.0	-0.07	0.0	3. For $A_{\theta 3}$ use
250.0	2.0970	-0.17	-0.0009	+0.09	0.0	-0.454	0.0	Fig 3.32 and 3.32a
300.0	2.5164	-0.10	-0.0005	+0.09	0.0	-0.032	0.0	$\theta = \frac{A_\theta}{T} \cdot S_d$
350.0	2.9358	-0.03	-0.0001	+0.09	0.0	-0.010	0.0	
400.0	3.3552	0.0	0.0	0.1	0.0	+0.008	0.0	
450.0	3.7746	+0.02	0.0001	+0.1	0.0	+0.028	0.0	
500.0	4.194	+0.03	0.0001	+0.1	0.0	+0.05	0.0	
550.0	4.6134	+0.03	0.0001	+0.1	0.0	+0.066	0.0	
600.0	5.0328	+0.03	0.0001	+0.1	0.0	+0.082	0.0	

Determination of Earthquake Induced Bending Moment and Shear:

The non-dimensional bending moment and shear coefficients and the maximum induced moment and shear for the design earthquake for pile top free to rotate condition are tabulated in Table 7.15.

Table 7.15

Dynamic Bending Moment and Shear For Different Modes of Vibrations - Free To Rotate Conditions

Depth cm	$\frac{x}{R}$	A_{m1}	M_1	A_{m3}	M_3	A_{S1}	S_1	Remarks
0.0	0.0	0.0	0.0	0.01	0.000124	0.75	1.6267	
50.0	0.4194	0.20	0.3283	0.01	0.000124	0.55	1.1929	1. To determine A_{m1} use Fig 3.39.
100.0	0.8388	0.282	0.4629	0.02	0.000248	0.12	0.2602	
150.0	1.2582	0.316	0.5188	0.03	0.000372	-0.03	-0.065	2. For A_{m3} use Fig 3.40 and 3.40a
200.0	1.6776	0.280	0.4597	0.038	0.00047	-0.10	-0.2169	
250.0	2.0970	0.230	0.3776	0.043	0.00053	-0.13	-0.2819	3. For A_{S1} use Fig 3.47
300.0	2.5164	0.17	0.2741	0.047	0.000582	-0.12	-0.2819	
350.0	2.9358	0.136	0.174	0.05	0.000624	-0.10	-0.2602	4. A_{S3} for Z_{max} = 15 is near zero
400.0	3.3552	0.05	0.082	0.05	0.000620	-0.075	-0.2169	5. $M_2 = A_m KR^2 . S_d$
450.0	3.7741	0.022	0.036	0.048	0.000595	-0.05	-0.1626	6. $= A_S KR S_d$
500.0	4.194	0.006	0.0098	0.0045	0.000558	-0.02	-0.1084	7. Due to rigid body motion no flexural bending envisaged in second mode
550.0	4.6134	0.0	0.0	0.041	0.00050	-0.01	-0.0433	
600.0	5.0328	0.0	0.0	0.033	0.000412	0.0	-0.0216	

In Table 7.16 the maximum earthquake induced displacements and shears during the design earthquakes have been tabulated for pile top free to rotate conditions.

Table 7.16

Maximum Response of the System-Pile Top Free To Rotate Conditions

Depth	$\frac{x}{R}$	Y_{max} cm	θ_{max} radians	M_{max} kg cm x 10^6	S_{max} kg x 10^4	Remarks
0.00	0.0	0.6350	-0.0037	0.0	1.6267	$Y_{max} = \sqrt{Y_1^2 + Y_2^2}$
50.0	0.4194	0.445	-0.0036	0.3283	1.1929	
100.0	0.8388	0.2921	-0.0029	0.4624	0.2602	$\theta_{max} = \sqrt{\theta_1^2 + \theta_2^2}$
150.0	1.2582	0.1587	-0.0021	0.5188	-0.065	
200.00	1.6776	0.0762	-0.0014	0.4547	-0.2169	$M_{max} = \sqrt{M_1^2 + M_2^2}$
250.00	2.0970	0.019	-0.0009	0.3776	-0.2819	$S_{max} = \sqrt{S_1^2 + S_2^2}$
300.00	2.5164	-0.0127	-0.0005	0.2791	-0.2819	
350.00	2.9358	-0.038	-0.0001	0.174	-0.2602	
400.00	3.3552	-0.0508	0.0	0.082	-0.2164	
450.00	3.7746	-0.0440	+0.0001	0.036	-0.1626	
500.00	4.194	-0.0381	+0.0001	0.0098	-0.1084	
550.00	4.6134	-0.0190	+0.0001	0.0	-0.0433	
600.00	5.0328	-0.0126	+0.0001	0.0	-0.0238	

It is neceaary to check the earthquake induced soil reaction against the available soil resistance. For purely cohesive soils the available passive resistance is given by:

Passive resistance P_p = 2c x projected area of pile

The induced soil reaction at any depth, x = $k_x . x . Y_{max}$ at x.

In table 7.17 the two quantities have been compared.

Table 7.17

Induced Soil Reaction and Available Soil Resistance

Depth	$\frac{X}{R}$	Induced soil Reaction in kg/cm	Available Soil Resistance in kg/cm
0.0	0.0	115.55	100.0
50.0	0.4194	82.80	100.0
100.0	0.8388	53.15	100.0
150.0	1.2582	28.88	100.0
200.0	1.6776	13.866	100.0
250.0	2.0970	3.4574	100.0
300.0	2.5164	-2.3110	100.0
350.0	2.9358	-6.92	100.0
400.0	3.3552	-9.244	100.0
450.0	3.7741	-8.01	100.0
500.0	4.194	-6.9330	100.0
550.0	4.6134	-3.4574	100.0
600.0	5.0328	-2.2928	100.0

7.2.2.4 Pile Top Fixed Against Rotation

The earthquake induced displacements, bending moments and shears along the pile length have been obtained for pile top free to rotate conditions also.

The natural frequencies in first two modes of vibration have been tabulated in Table 7.12.

The other response quantities have been tabulated in Table 7.18 for first mode of vibration. The contribution from the second mode of vibration have been found to be negligible.

The maximum earthquake induced pile response is the same as given in Table 7.18 since there is no contribution from higher modes. The induced soil reaction is lesser than the free haad case hence not tabulated.

7.2.2.5 Design Check

The safety of the piles in the group, supporting the tank is to be checked against the permissible values. The earthquake effects are as below:

Earthquake induced deflection = 0.6350 cm

Earthquake induced rotations = 0.0037 radian

Earthquake induced bending moment = 78.5×10^4 kg cm.

It is seen that the pile displacements are quite small.

Table 7.18

Dynamic Response During First Mode of Vibration - Pile Top Fixed Against Rotation Conditions

Depth in cm	$\frac{x}{R}$	A'_{y1}	Y_1 in cm	$A'_{\theta 1}$	θ_1 radian	A'_{m1}	M_1 in kg cmx10^6	A'_{S1}	S_1 in kgx10^4	Remarks
0.0	0.0	1.0	0.33	0.0	0.0	-0.92	-0.785	1.35	0.9663	For A'_{y1},
50.0	0.4194	0.91	0.30	-0.21	0.0005	-0.47	-0.401	1.05	0.7515	$A'_{\theta 1}$, A'_{m1}
100.0	0.8388	0.75	0.248	-0.41	0.0011	-0.16	-0.136	0.68	0.4867	and A'_{S1}
150.0	1.2582	0.56	0.185	-0.41	0.0011	+0.034	0.40	0.40	0.2863	refer to
200.0	1.6776	0.40	0.132	0.37	0.0009	0.15	0.128	0.20	0.1431	Figs 3.24
250.0	2.0970	0.24	0.079	-0.29	0.0009	0.19	0.162	0.05	0.0357	3.33, 3.41
300.0	2.5164	0.14	0.046	-0.21	0.0005	0.18	0.154	-0.05	-0.0286	and 3.49
350.0	2.9358	0.05	0.0165	-0.14	0.0003	0.16	0.136	-0.09	-0.0644	respectively
400.0	3.3552	0.0	0.000	-0.07	0.0001	0.11	0.0938	-0.11	-0.0787	
450.0	3.7746	-0.02	-0.006	-0.04	0.0001	0.08	0.0682	-0.10	-0.0715	
500.0	4.194	-0.04	-0.0132	-0.01	0.0	0.05	0.0426	-0.09	-0.0644	
550.0	4.6134	-0.03	0.009	-0.01	0.0	0.03	0.0255	-0.04	-0.0286	
600.0	5.0328	-0.02	0.0061	-0.02	0.0	0.02	0.0170	0.04	-0.0286	

The compressive stress in concrete $= 61.22 + 63.34$

$= 124.56\ kg/cm^2$

The tensile stress in concrete $= 2.12\ kg/cm^2$

The above tensile stress can be permitted since the pile top may not exhibit 100 % degree of fixity conditions. Hence the chosen design may be considered safe.

7.3 DISCUSSIONS ON THE METHOD OF ANALYSIS AND THE DESIGN CURVES

7.3.1 LUMPED MASS AT TOP

As shown in Tables 3.1, 4.1 and 5.1 for each Z_{max} fifteen numbers of pile cases of varying pile sections, leng and soil conditions have been analysed. In each case the assigned sustained vertical loads were based on the safe vertical load carrying capacities worked out by static formu proposed by Terzaghi (1943). For calculating the bearing capacities, the soil strength parameter values were based on the soil modulus table provided by Davisson (1970). Leonard (1970) has advocated use of these values for design purposes

As seen from the results of the analysis and the non-dimensional curves presented, the normalised mode shapes are independent of the sustained vertical loads acting on th system. For each pile case of a particular Z_{max}, though the assigned vertical loads (lumped at top) have been variec unique plots of non-dimensional normalised modal quantities were obtained.

However, the vertical load influences the natural frequency in the first mode of vibration. The natural frequencies were inversely proportional to $\sqrt{M_t}$. Because of this, the spectral displacement values and hence the maximum response during earthquakes are altered.

The safe load carrying capacity of the pile has been lumped at the top, with the view that any pile (isolated or in a group) would be designed to carry this load. Normally, this would be transmitted from the super-structure and would be acting at the top of the pile. However, if the actual transmitted loads are lesser, these may be lumped, instead of the safe load per pile.

Normally for engineering structures resting on piles the mass and the flexibility are distributed above the pile cap level. For particular super-structure - foundation-soil system, such distributions, if considered may result in a slightly different response computation. Though, the above factor is recognised, lumping of super structure mass at pile top, has been adopted to obtain generalised solutions, exclusively for pile foundation response. Considering the range of structures which can be supported on piles, it may not be possible to consider such distributions and also obtain solutions of practical significance.

The interaction between superstructure and foundation is however considered by obtaining solutions for two fixity conditions, namely pile top free to rotate and fixed against rotation conditions.

If exact solutions are needed, instead of using the design curves, separate analysis may be performed incorporating these distributions in the model.

7.3.2 REDUCTION IN VERTICAL LOAD CAPACITY:

Determination of pile foundation stability under lateral vibration conditions has been a primary concern in the analysis and the solutions thereoff. However, an indirect answer to the reduction in vertical load carrying capacity is seen to be available. This is evident from the two worked out examples. From the results of which it is seen that though the mass lumped at top have been lesser than the carrying capacities, still, the induced bending stress values workout to be larger. For the safety of the section it becomes imperative to increase the number of piles thereby reducing the mass lumped per pile. This result may be viewed as the resulting reduction in the vertical carrying capacity.

7.3.3 LOSS OF CONTACT BETWEEN SOIL AND PILE

During lateral vibration conditions there may be loss of contact between soil and pile under the following cases:

(i) Due to liquifaction of the upper layers of soil in the case of piles embedded in saturated granular soils.

(ii) In the case of piles embedded in clay type soils, air gap or loss of contact between soil and pile may be developed.

At certain times these factors may assume considerable importance since the piles may fail due to non-availability of sufficient soil support. With the solutions presented it may be possible to safeguard the foundation stability against these two conditions.

For granular soils prone to liquefaction hazards, it is possible to estimate reasonably, the depth upto which the liquefaction may propagate (Seed (1971)). While using the solutions provided herein, the length of the pile, should be considered as the length of embedment below this depth. For this modified pile length and the soil condition below the maximum depth of possible liquefaction, the relative stiffness factor T, and Z_{max} value need be determined. The new ground surface thus becomes the depth upto which complete liquefaction is expected.

Neglecting the pile length above this elevation and lumping the imposed vertical load and the neglected length weight as the top mass of the modified pile, analysis is done in the usual manner for the condition of pile top free to rotate.

With the knowledge of displacement at new ground level linear interpolation for pile cap level may be made. This would enable the designer to assess as to whether the depth of embedment in the dense soil is sufficient or not.

If still better solutions are warranted, instead of using the design curves, in the soil-pile model, the interaction effects may be taken to be zero upto the zone of expected liquefaction. Below this depth appropriate soil springs may be connected and the analysis performed.

Similarly in the case of clayey soils if assessment is possible, of depth upto which separation takes place; the influence may be studied with the presented solutions.

7.3.4 CONSIDERATION OF SOIL PILE INTERACTION EFFECTS

In order to consider the effect of dynamic loading on the soil-pile interaction effects, it is suggested that the estimate of the soil modulus must be made based on the lateral vibration test, results. In the absence of lateral vibration tests with the limited information available, ratio of static to dynamic soil modulus values have been suggested.

In the case of clay type soils considering the increase in strain due to pulsating load applications reductions in dynamic soil modulus compared to static values are considered necessary.

However in the case of granular soils, if p-y relationship at various depths are known, the convergence technique suggested by Matlock and Reese (1962) may be adopted. In this case the first trial value should be the one obtained from dynamic test result or $\frac{1}{10}$ th static values.

7.3.4.1 Other Forms of Soil Modulus Variations:

The solutions for the dynamic response of the piles have been obtained only for constant values and linear variations of soil modulus with depth. These two forms of variations have been adopted; considering their wide usage in solving problems of piles, subjected to sustained lateral loads.

However, the suggested mathematical model in Chapter III is of a general nature. In this model any form of variation of soil modulus with depth can be incorporated.

Once the exact form of variation of soil modulus with depth is defined, the procedure of discretisation is simple. This would mean a continuous loading intensity over a beam of length L, equal to the length of the pile. The piles may be divided into several segments as before. At the division points they may be considered to be simply supported. The reaction at the support points are easily evaluated by using the technique suggested by Newmark (1941).

7.3.5 INFLUENCE OF SUSTAINED LOADS

Since the solutions have been obtained for linear systems superposition of static and dynamic loads is considered to be valid. The pile displacements and bending moment under static conditions may be estimated using a suitable technique. The pile response during earthquakes should be estimated with the suggested procedures. The design should then be checked for superimposed values of the two.

7.3.6 EFFECT OF GROUP ACTION

The proposed solutions are applicable more to isolated piles than those in a group. Because, the action of pile in a group are affected due to the influence of one pile over the other. Prakash (1962) has suggested that for static lateral loading conditions group effect would be present, if, in the direction of loading, the spacing is less than 8-times the pile diameter and in the perpendicular direction 3-times the pile diameter. Even for static loading conditions exact estimation of the group effect has not been possible till today (1974) and the adopted procedures consider only individual action of the piles.

For considering the group loading effect recently Davisson and Sally (1970) suggest a reduction in the values of constant of horizontal subgrade reaction n_h and increase in the relative stiffness factor T.

The ratio of T, for piles in a group to that of individual piles are:

1. 1.25 at a spacing of four pile widths, 4d.
2. 1.30 at a spacing of three pile widths, 3d.

For spacings other than the above, linear interpolations are suggested.

Prakash et al (1973) have utilised the above suggestions for predicting the natural frequencies of pile groups.

Therefore, it is suggested that, if required, while analysing the individual piles in a group, the above said increase in flexibility may be adopted. Similar variations in flexibility may be adopted for clayey soils also.

However, it is emphasised that solutions presented in this thesis do not consider group effect in an exact manner.

7.3.7 SOLUTIONS FOR DYNAMIC LOADS OTHER THAN EARTHQUAKES

With the presented model and the solutions it is believed that with reasonable adjustment solutions of pile response under the following loadings are possible:

(1) Dynamic loads imparted to the top of the piles as in the case of piles supporting machines.

(2) Dynamic loads imparted by wave forces.

For the latter case spectrum curves for wave forces are available.

7.3.8 SOLUTIONS FOR STATIC LOADS

With the same mathematical model and the suggested numerical techniques solutions for sustained loading conditions are easy to obtain.

For this, only top mass is to be considered and the rest of the masses are assigned zero values, treating the pile section as a massless flexural member. The mathematical model becomes a mass resting on a massless flexural member to which soil springs are connected at various elevations. The dynamic response of this model, if evaluated gives the variations of displacements, bending moment and shear along the entire length of the pile. Convergence of solutions for such conditions have been found to be extremely rapid.

In Fig 7.8 (for a typical pile) the deflected shape obtained by the present technique has been compared with the static deflected shape obtained by Reese and Matlock (1956). The lateral load applied at the ground surface equals the inertia force at top. The results agree very closely, this further emphasises the correctness of the model and the method of analysis. Because the solutions presented by Reese and Matlock (1956) has been supported by by several field tests, Prakash (1962).

7.4 CONCLUDING REMARKS

For predicting the performance of pile foundations subjected to dynamic loads, including earthquakes, non-dimensional charts which are simple to use and of practical significance have been made available.

With the non-dimensional charts, mathematical model and the method of analysis, realistic estimation (engineering solutions) of seismic stability of soil-pile system supporting various types of structures is possible.

CHAPTER - VIII

SUMMARY AND CONCLUSIONS

The matter embodied in this thesis essentially deals with the prediction and understanding of the behaviour of pile foundations embedded in soils and subjected to dynamic loads, particularly earthquakes. This was achieved through theoretical and experimental studies.

Throughout the studies the singular persistant concern and objective had been to provide the practising engineer a simplified design procedure. Such a proceedure had to be backed up by consideration of soil-structure interaction phenomena, advancements in structural dynamics and the available knowledge of soil-pile interaction mechanism about which the designer is familiar.

The discussions in various preceeding Chapters describe as to how effective solutions based on such approache were made possible.

Chapter III describes the soil-pile system idealised as a lumped mass-spring system. The masses are connected by elastic weightless bar possessing the same elastic properties as that of the pile section. The interaction effects of the soil are considered by treating the soil as independant closely spaced elastic springs (Winkler model) connected at the mass points. Discretisation of the soil-pile interaction effects was achieved by considering the modulus of subgrade reaction concept. The vibration characteristics of the pile was determined with the aid of a transfer solution approach.

The performance of the idealisation and the method of analysis was tested by predicting the response of piles embedded in soils in which:

1. Soil Modulus can be considered to remain constant with depth.
2. Soil modulus can be considered to vary linearly with depth.

Then using such an approach the dynamic response of ninety pile cases was studied. These problems consider the variations of (i) pile flexural stiffness (ii) length of piles (iii) soil stiffness and (iii) sustained vertical loads.

For each of these above ninety pile cases solutions were also obtained for pile top (i) Free to rotate conditions and (ii) fixed against rotation conditions.

The different pile cases were chosen in such a manner as to obtain response of piles with maximum depth factor, Z_{max} = 1, 2, 3, 5, 10 and 15.

The above studies thus included information on piles embedded in granular soils and silts of various relative densities, peat and cohesive soils of normally and preloaded condition of varying consistencies.

Based on the results of such analysis, for differen modes of vibrations it was possible to define:

(i) Certain dimensionless frequency factors

(ii) Non-dimensional factors for normalised modal quanties of deflection, rotation bending moment and shear.

The frequency factor variation has been provided with (1) the relative stiffness factor for different identified Z_{max} cases (2) non-dimensional maximum depth factor Z_{max}

The variations of the various non-dimensional normalised model quantities with dimensionless depth factor (x/T or x/R) have also been made available.

In Chapter III the above mentioned studies for pile foundations embedded in soils considering constant values of soil modulus with depth have been discussed. Discussions for piles embedded in soils assuming linear variation of soil modulus with depth appear in Chapter V.

For the case of piles embedded in soils with constar values of soil modulus with depth, the soil-pile system has also been idealised as a continuous system model. In Chapte: IV with the above model independent solutions have been developed for pile top free to rotate conditions. Each of the pile cases analysed with lumped mass solutions has been studied here also. This enabled in assessing the adequacy of the earlier approach as well as resulted in a better understanding of the pile behaviour.

These studies showed that the adopted model and technique were sufficiently accurate and that they could predict the dynamic response of piles effectively. While utilising such approaches the importance of incorporating realistic end conditions compatible with the physical system behaviour has been brought out.

Based on these theoretical investigations, for any pile section with desired pile top conditions, and for piles embedded in any soil type, the following findings may be considered of particular significance.

1. The dynamic behaviour of piles are essentially dependent on the length of the pile in relation to the relative stiffness factor, Z_{max}. The absolute length of the pile does not govern the behaviour singularly. The above is true for natural frequencies of vibrations as well as values of displacements, bending moment and shear along the entire length of the pile.

2. The first mode of vibration has a significant contribution to the overall response of the piles.

3. The form of variations of modal quantities under first mode of vibrations essentially follow the investigated (Reese and Matlock (1956), Davisson and Gill (1963)), behaviour

of pile foundations subjected to sustained lateral loading conditions.

4. Piles embedded in any soil types and having $Z_{max} \leq 2$ display rigid mode of deformations. For $Z_{max} \geq 5$ flexural bending deformations are displayed. For Z_{max} values between 3 and 5 mixed modes are evidenced.

5. Especially under first mode of vibrations for piles with $Z_{max} \geq 5$ there is no appreciable difference in the deformed shapes as well as values of normalised modal quantities of deflection, rotation, bending moment and shear along the pile length.

   As such $Z_{max} = 5$ can be considered as a limiting case of infinitely long piles. Increase of pile length beyond this value loses significance as far as dynamic behaviour is concerned.

6. With the increase in lumped mass at top or the superstructure load:

   (i) the natural frequencies under first mode of vibrations is reduced by $\frac{1}{\sqrt{M_t}}$ times.

   (ii) the induced dynamic displacements bending moment and shear are increased by $\sqrt{M_t}$ times.

7. Similar is the effect of increase in the weight per unit length of the pile at higher modes of vibrations.

8. For any mode and any soil type increase in soil stiffness results in reduction of pile displacements and increase in natural frequencies of vibrations.

9. Increase in flexural stiffness of the pile does not have similar effect on the pile behaviour mainly because of the resulting alterations in Z_{max} values.

10. The natural frequencies under first mode of vibrations incrdase with increase in Z_{max} values. However for $Z_{max} \geqslant 5$ no significant difference is observed.

11. Since displacements are the governing parameters in design, it is seen that for any given soil-pile system the induced dynamic displacements get reduced when the pile length is increased. This is true upto limiting case of long pile ranges.

12. For similar pile-soil systems the induced dynamic deflections under fixed head conditions are smaller than those under free head conditions. Whereas the natural frequencies and bending moment values are greater in the former case.

13. For pile top fixed against rotation conditions the maximum bending moment occurs at top but for pile free to rotate conditions the maximum bending moment occurs at some depth below the ground surface.

14. In the case of clayey type soils for long pile ranges the maximum bending moment occurs at depths of 1.15R compared to depths of 1.30 T in granular soils.

15. For piles embedded in clay and under pile top free to rotate conditions a unique rigid body mode irrespectiv of the pile-soil conditions has been observed.

Both lumped mass and continuous system models support the above.

As a sequence to the above studies it has been argued as to how the present model as well as method of analysis is capable of obtaining solutions of:

1. Piles subjected to machine loads and wave forces.
2. Piles embedded in soils with soil modulus having any generatised form of variations with depth.
3. Piles embedded in layered soils.
4. Loss of contact between soil and pile as a result of vibration of piles.
5. Piles subjected to sustained loading conditions.

In the experimental studies described in Chapter VI the dynamic behaviour of piles have been investigated throug lateral vibration tests on full size prototype piles. Each pile has been tested under resonant conditions to record the increase in amplitudes of vibration despite of force reductions. This underlines the possibility of quasi-resonance state under dynamic conditions.

A logical procedure for in-situ determination of material constants under dynamic loading conditions has been presented. Thus such a procedure, if standardised would lead to useful informations.

With the limited information available the following material constant values are suggested for use in routine design:

1. The damping coefficient of soil-pile system equals 10%.
2. In clay type soils the ratio of static to dynamic soil modulus need be 15.
3. In granular soils the ratio of static to dynamic values of n_h, need be 10.

The solutions presented in the thesis very effectively bring out the draw-backs and inconsistencies that exist in the prevelant design practices such as pseudo-static methods and the equivalent structural system approaches of Prakash and Sharma (1969), Hayashi et al (1965) and Ishii and Fujita (1965).

The presented solutions are believed to have advantages compared to the rigorous one proposed by Penzien et al (1964) for the following reasons:

1. By providing exclusive solutions for predicting the dynamic behaviour of piles applicable to any pile-soil system.

2. By providing a simplified design procedure (engineering solution) considering in a realistic manner the soil-pile interaction mechanism. The practising engineer coul freely use these solutions.

3. By providing a logical and realistic testing procedures for in-situ determination of material constants.

Thus for the first time (as of 1974) the dynamic behaviour of the piles have been investigated in a logical and sequential manner. For the firt time non-dimensional solutions of practical significance are made available to the designer for predicting the dynamic response of piles. With these solutions practically any type of pile embedded in any soil type could be analysed, more importantly, without going into the complexities of dynamic analysis. All the qualifyir variables which control the dynamic response has been taken into account.

Thus, the procedure of analysis and the other studie described in this thesis, should be construed as a procedure effecting an approach that offers several advantages and it believed that the work presented provides a reasonable solut to this complex design problem.

CHAPTER - IX

SUGGESTIONS FOR FURTHER RESEARCH

The following aspects of the dynamic behaviour of piles may need further investigations. The suggested areas of research have been divided into three groups.

1. Theoretical Investigations :

(i) The soil-pile system may be idealised as a discrete-beam-column model and the dynamic behaviour may be investigated through suitable methods of analysis.

(ii) In the lumped mass idealisation, while idealising the soil-pile interaction phenomena, far coupled springs may be considered and dynamic analysis may be performed.

(iii) Suitable mathematical model can be selected to include non-linear and creep effects.

(iv) The effect of variation of ground motion along the length of the pile may be considered.

(v) Time wise response computations may be attempted.

(vi) Influence of different types of super-structures may be considered.

(vii) Dynamic analysis of pile groups considering the influence of group action may be attempted.

2. Experimental Investigations :

   (i) Dynamic response of model piles subjected to shake table motions may be investigated.

   (ii) Lateral vibration tests on instrumented piles may be performed.

   (iii) Dynamic flexibility of different piles in group of piles may be studied.

   (iv) Response of piles subjected to blast loading may be attempted.

3. Determination of Material Constants :

   (i) Lateral vibration tests on piles embedded in different soil types must be attempted to-gather information, on dynamic properties of varieties of soil pile systems.

   (ii) Correlation between modulus of elasticity and shear modulus with soil modulus may be attempted.

   (iii) Influence of sustained load level, pulsating load level and frequency of load applications on soil properties may be investigated.

   (iv) Use of pressero-meter and cyclic plate load tests for determining dynamic properties may be studied.

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A P P E N D I X

SPECIMEN-OUTPUT

SPECIMEN OUTPUT

RESPONSE OF PILE IN CLAY

ROB NO 18 ZMAX = 3.00 R = 1.50 LALTH = 4.5 DIA = 0.30
= 94.24778 W = 3.0 NO OF MASSES = 30 NO OF MODES = 3
I = 0.47713D 03 AREA = 0.70686D-01
ABLE OF COMPUTED MODE SHAPES
ODE NO = 1 p = 0.1683D 02 FREQ = 2.67794 PERIOD = 0.3734
PF = 0.1008D 01 X = 0.155172 DT = -0.3048D-05

PT	X/R	A_{y1}	A 1	A_{m1}	A_{s1}
1	0.0	0.1000D 01	-0.6895D 00	0.0	0.0
3	0.207	0.8577D 00	-0.6786D 00	0.1072D 00	0.4725D 0
5	0.414	0.7200D 00	-0.6479D 00	0.1796D 00	0.3111D 0
7	0.621	0.5900D 00	-0.6059D 00	0.2227D 00	0.1764D 0
9	0.828	0.4695D 00	-0.5576D 00	0.2419D 00	0.6669D-0
.1	1.034	0.3593D 00	-0.5073D 00	0.2420D 00	-0.1987D-0
.3	1.241	0.2595D 00	-0.4586D 00	0.2275D 00	-0.8533D-0
.5	1.448	0.1694D 00	-0.4140D 00	0.2024D 00	-0.1317D 0
.7	1.655	0.8804D-01	-0.3753D 00	0.1704D 00	-0.1609D 0
9	1.862	0.1392D-01	-0.3437D 001	0.1349D 00	-0.1744D 0
1	2.069	-0.5443D-01	-0.3196D 00	0.9873D-01	-0.1737D 0
3	2.276	-0.1185D 00	-0.3027D 00	0.6478D-01	-0.1599D 0
5	2.482	-0.1799D 00	-0.2924D 00	0.3563D-01	-0.1336D 0
7	2.690	-0.2397D 00	-0.2874D 00	0.1379D-01	-0.9535D-0
9	2.897	-0.2989D 00	-0.2859D 00	0.1662D-02	-0.4542D-0

PROB NO 18
TABLE OF COMPUTED [illegible]E SHAPES
MODE NO 2 p = 0.737D 02 FREQ = 11.74258 PERIOD = 0.0852
MPF = 0.1619D-03

PT	x/R	A_{y1}	A_{1}	A_{m1}	A_{s1}
1	0.0	0.1000D 01	0.3141D 04	0.0	0.0
3	0.207	0.6510D 03	0.3142D 04	0.8107D 00	0.1175D 02
5	0.414	0.1301D 04	0.3142D 04	0.1598D 01	0.1161D 02
7	0.621	0.1951D 04	0.3142D 04	0.2339D 01	0.1136D 02
9	0.828	0.2601D 04	0.3143D 04	0.3010D 01	0.1100D 02
11	1.034	0.3251D 04	0.3143D 04	0.3588D 01	0.1052D 02
13	1.241	0.3902D 04	0.3144D 04	0.4050D 01	0.9931D 01
15	1.448	0.45[illegible]D 04	0.3145D 04	0.4373D 01	0.9231D 01
17	1.655	0.5203D 04	0.3146D 04	0.4533D 01	0.8489D 01
19	1.862	0.5854D 04	0.3147D 04	0.4507D 01	0.7495D 01
21	2.069	0.6504D 04	0.3148D 04	0.4272D 01	0.6459D 01
23	2.276	0.7157D 04	0.3149D 04	0.3804D 01	0.5311D 01
25	2.483	0.7808D 04	0.3149D 04	0.3082D 01	0.4050D 01
27	2.690	0.8460D 04	0.3150D 04	0.2080D 01	0.2678D 01
29	2.897	0.9112D 04	0.3150D 04	0.7775D 00	0.1193D 01

TABLES OF

COMPUTED NON - DIMENSIONAL COEFFICIENTS FOR DIFFERENT PILE CASES WITH Zmax = 3

PILES EMBEDDED IN CLAY; ASSUMING SOIL MODULUS TO REMAIN CONSTANT WITH DEPTH

PILE TOP FIXED AGAINST ROTATION CONDITION

R=1.0 L_s=3.0 DIA=0.3 k=477.1294 W=6.0 EI=0.47713D 03

R=1.25 L_s=3.75 DIA=0.3 k=195.4322 W=3.0 EI=0.47713D 03

PT	x/R	A'_{y1}	A'_{m1}	A'_{y1}	A'_{m1}
1	0.0	0.1000D 01	-0.9450D 00	0.1000D 01	-0.9237D 00
3	0.207	0.9794D 00	-0.7019D 00	0.9806D 00	-0.6899D 00
5	0.414	0.9289D 00	-0.4991D 00	0.9317D 00	-0.4943D 00
7	0.621	0.8572D 00	-0.3346D 00	0.8617D 00	-0.3351D 00
9	0.828	0.7714D 00	-0.2055D 00	0.7774D 00	-0.2096D 00
11	1.034	0.6769D 00	-0.1081D 00	0.6842D 00	-0.1145D 00
13	1.241	0.5779D 00	-0.3866D-01	0.5861D 00	-0.4603D-01
15	1.448	0.4774D 00	0.6939D-02	0.4861D 00	-0.5234D-03
17	1.655	0.3772D 00	0.3284D-01	0.3861D 00	0.2598D-01
19	1.862	0.2785D 00	0.4317D-01	0.2872D 00	0.3738D-01
21	2.069	0.1817D 00	0.4201D-01	0.1899D 00	0.3755D-01
23	2.276	0.8680D-01	0.3334D-01	0.9420D-01	0.3029D-01
25	2.483	-0.6704D-02	0.2108D-01	-0.1652D-03	0.1935D-01
27	2.690	-0.9931D-01	0.9101D-02	-0.9370D-01	0.8405D-02
29	2.897	-0.1915D 00	0.1216D-02	-0.1869D 00	0.1126D-02

.5 L_s=4.5 DIA=0.7 k=2793.690 35.0 EI=0.14143D 05

R=2.0 L_s=6.0 DIA=0.7 k=883.941 W=54.0 EI=14143D 05

T	x/R	A'_{y1}	A'_{m1}	A'_{y1}	A'_{m1}
	0.0	0.1000D 01	-0.9453D 00	0.1000D 01	-0.9391D 00
	0.207	0.9762D 00	-0.6999D 00	0.9787D 00	-0.6979D 00
	0.414	0.9232D 00	-0.4967D 00	0.9279D 00	-0.4967D 00
	0.621	0.8497D 00	-0.3304D 00	0.8561D 00	-0.3335D 00
	0.828	0.7626D 00	-0.2010D 00	0.7703D 00	-0.2053D 00
	1.034	0.6675D 00	-0.1038D 00	0.6759D 00	-0.1086D 00
3	1.241	0.5684D 00	-0.3470D-01	0.5771D 00	-0.3951D-01
5	1.448	0.4682D 00	0.1035D-01	0.4769D 00	0.5892D-02
7	1.655	0.3689D 00	0.3562D-01	0.3770D 00	0.3177D-01
9	1.862	0.2713D 00	0.4531D-01	0.2786D 00	0.4220D-01
1	2.069	0.1759D 00	0.4352D-01	0.1821D 00	0.4121D-01
3	2.276	0.8242D-01	0.3431D-01	0.7836D-01	0.3277D-01
5	2.483	-0.9502D-02	0.2160D-01	0.5900D-02	0.2074D-01
7	2.690	-0.1005D 00	0.9300D-02	0.9827D-01	0.8957D-02
9	2.897	-0.1912D 00	0.1241D-02	-0.1903D 00	0.1194D-02

R=2.0 L_s=6.0 DIA=0.6 k=477.13
W=24.0 EI=0.76341D 04

R=1.25 L_2=3.75 DIA=0.6 k=3126.
W=105.0 EI=0.76341D 04

PT	x/R	A'_{y1}	A'_{m1}	A'_{y1}	A'_{m1}
1	0.0	0.1000D 01	-0.9291D 00	0.1000D 01	-0.9456D 0
3	0.207	0.9797D 00	-0.6926D 00	0.9759D 00	-0.6999D 0
5	0.414	0.9300D 00	-0.4950D 00	0.9227D 00	-0.4954D 0
7	0.621	0.8592D 00	-0.3343D 00	0.8489D 00	-0.3300D 0
9	0.828	0.7743D 00	-0.2078D 00	0.7617D 00	-0.2005D 0
11	1.034	0.6806D 00	-0.1121D 00	0.6665D 00	-0.1033D 0
13	1.241	0.5823D 00	-0.4351D-01	0.5674D 00	-0.3423D-0
15	1.448	0.4822D 00	0.1923D-02	0.4673D 00	-0.1077D-0
17	1.655	0.3823D 00	0.2815D-01	0.3680D 00	0.3597D-0
19	1.862	0.2836D 00	0.3919D-01	0.2705D 00	0.4558D-0
21	2.069	0.1867D 00	0.3891D-01	0.1752D 00	0.4372D-0
23	2.276	0.9142D-01	0.3121D-01	0.8193D-01	0.3444D-0
25	2.483	-0.2463D-02	0.1986D-01	-0.9818D-02	0.2168D-0
27	2.690	-0.9550D-01	0.8603D-02	-0.1006D 00	0.9329D-0
29	2.897	-0.1882D 00	0.1148D-02	-0.1912D 00	0.1245D-0

R=2.0 L_s=6.0 DIA=0.5 k=230.097
W=12.0 EI=0.36816D 04

R=1.5 L_s=4.5 DIA=0.6 k=1507.964
W=60.0 EI=0.76341D 04

PT	x/R	A'_{y1}	A'_{m1}	A'_{y1}	A'_{m1}
1	0.0	0.1000D 01	-0.9191D 00	0.1000D 01	-0.9460D 00
3	0.207	0.9806D 00	-0.6871D 00	0.9777D 00	-0.7013D 00
5	0.414	0.9318D 00	-0.4930D 00	0.9257D 00	-0.4975D 00
7	0.621	0.8620D 00	-0.3349D 00	0.8530D 00	-0.3324D 00
9	0.828	0.7780D 00	-0.2101D 00	0.7664D 00	-0.2029D 00
11	1.034	0.6850D 00	-0.1154D 00	0.6715D 00	-0.1055D 00
13	1.241	0.5871D 00	-0.4726D-01	0.5725D 00	-0.3624D-01
15	1.448	0.4872D 00	-0.1828D-02	0.4721D 00	0.9070D-02
17	1.655	0.3872D 00	0.2474D-01	0.3724D 00	0.3461D-01
19	1.862	0.2884D 00	0.3632D-01	0.2743D 00	0.4455D-01
21	2.069	0.1911D 00	0.3672D-01	0.1782D 00	0.4300D-01
23	2.276	0.9543D-01	0.2971D-01	0.8414D-01	0.3398D-01
25	2.483	0.1022D-02	0.1901D-01	-0.8483D-02	0.2143D-01
27	2.690	-0.9257D-01	0.8268D-02	-0.1002D 00	0.9233D-02
29	2.897	-0.1858D 00	0.1108D-02	-0.1915D 00	0.1232D-02

$R=1.0$ $L_s=3.0$ DIA=0.5 k=3681.554
W=84.0 EI=0.36816D 04

$R=1.25$ $L_s=3.75$ DIA=0,5 k=1507.
W=42.0 EI=0.36816D 04

PT	x/R	A'_{y1}	A'_{m1}	A'_{y1}	A'_{m1}
1	0.0	0.1000D 01	-0.9454D 00	0.1000D 01	-0.9477D 00
3	0.207	0.9754D 00	-0.6995D 00	0.9776D 00	-0.7023D 00
5	0.414	0.9218D 00	-0.4948D 00	0.9256D 00	-0.4979D 00
7	0.621	0.8478D 00	-0.3293D 00	0.8528D 00	-0.3324D 00
9	0.828	0.7604D 00	-0.1998D 00	0.7661D 00	-0.2026D 00
11	1.034	0.6651D 00	-0.1036D 00	0.6711D 00	-0.1051D 00
13	1.241	0.5660D 00	-0.3367D-01	0.5720D 00	-0.3571D-01
15	1.448	0.4659D 00	0.1124D-01	0.4716D 00	0.9616D-02
17	1.655	0.3668D 00	0.3635D-01	0.3718D 00	0.3512D-01
19	1.862	0.2695D 00	0.4587D-01	0.2738D 00	0.4498D-01
21	2.069	0.1744D 00	0.4392D-01	0.1777D 00	0.4333D-01
23	2.276	0.8133D-01	0.3457D-01	0.8367D-01	0.3421D-01
25	2.483	-0.1017D-01	0.2175D-01	-0.8905D-02	0.2156D-01
27	2.690	-0.1008D 00	0.9358D-02	-0.1006D 00	0.9288D-02
29	2.897	-0.1910D 00	0.1249D-02	-0.1919D 00	0.1240D-02

=1.25 L_s=3.75 DIA=0.4 k=617.662   R=1.5 L_s=4.5 DIA=0.4 k=297.8695
=15.0 EI=0.15080D 04   W=9.0 EI=0.15080D 04

PT	x/R	A'_{y1}	A'_{m1}	A'_{y1}	A'_{m1}
1	0.0	0.1000D 01	-0.9458D 00	0.1000D 01	-0.9340D 00
3	0.207	0.9791D 00	-0.7022D 00	0.9801D 00	-0.6958D 00
5	0.414	0.9283D 00	-0.4990D 00	0.9305D 00	-0.4967D 00
7	0.621	0.8564D 00	-0.3342D 00	0.8597D 00	-0.3350D 00
9	0.828	0.7704D 00	-0.2049D 00	0.7747D 00	-0.2077D 00
11	1.034	0.6758D 00	-0.1075D 00	0.6808D 00	-0.1115D 00
13	1.241	0.5768D 00	-0.3803D-01	0.5823D 00	-0.4257D-01
15	1.448	0.4762D 00	0.7523D-02	0.4820D 00	0.2994D-02
17	1.655	0.3761D 00	0.3335D-01	0.3819D 00	0.2922D-01
19	1.862	0.2776D 00	0.4358D-01	0.2831D 00	0.4012D-01
21	2.069	0.1809D 00	0.4231D-01	0.1870D 00	0.3967D-01
23	2.276	0.8614D-01	0.3354D-01	0.9064D-01	0.3274D-01
25	2.483	-0.7196D-02	0.2119D-01	-0.3345D-02	0.2017D-01
27	2.690	-0.9962D-01	0.9144D-02	-0.9647D-01	0.3735D-02
29	2.897	-0.1917D 00	0.122[illegible]D-02	-0.1892D 00	0.1169D-02

R=1.5 L_s=4.5 DIA=0.3 k=94.2478 W=3.0 EI=0.47713D 03

R=1.0 L_s=3.0 DIA=0.4 k=1507.9 W=27.0 EI=0.15080D 04

PT	x/R	A'_{y1}	A'_{m1}	A'_{y1}	A'_{m1}
1	0.0	0.1000D 01	-0.9174D 00	0.1000D-01	-0.9494D 00
3	0.207	0.9811D 00	-0.6864D 00	0.9776D 00	-0.7033D 00
5	0.414	0.9328D 00	-0.4930D 00	0.9255D 00	-0.4983D 00
7	0.621	0.8635D 00	-0.3355D 00	0.8526D 00	-0.3324D 00
9	0.828	0.7798D 00	-0.2110D 00	0.7658D 00	-0.2024D 00
11	1.034	0.6870D 00	-0.1165D 00	0.6707D 00	-0.1046D 00
13	1.241	0.5891D 00	-0.4839D-01	0.5715D 00	-0.3519D-01
15	1.448	0.4893D 00	-0.2892D-02	0.4711D 00	0.1015D-01
17	1.655	0.3892D 00	0.2382D-01	0.3713D 00	0.3561D-01
19	1.862	0.2902D 00	0.3557D-01	0.2732D 00	0.4540D-01
21	2.069	0.1928D 00	0.3616D-01	0.1773D 00	0.4366D-01
23	2.276	0.9681D-01	0.2935D-01	0.8321D-01	0.3443D-01
25	2.483	0.2121D-02	0.1882D-01	-0.9314D-02	0.2169D-01
27	2.690	-0.9176D-01	0.8197D-02	-0.1009D 00	0.9342D-02
29	2.897	-0.1853D 00	0.1102D-02	-0.1922D 00	0.1247D-02

R=3.0 L_s=9.0 DIA=0.7 k=174.606
W=18.0 EI=0.14143D 05

PT	x/R	A'_{y1}	A'_{m1}
1	0.0	0.1000D 01	-0.8911D 00
3	0.207	0.9816D 00	-0.6645D 00
5	0.414	0.9347D 00	-0.4825D 00
7	0.621	0.8672D 00	-0.3335D 00
9	0.228	0.7855D 00	-0.2152D 00
11	1.034	0.6946D 00	-0.1245D 00
13	1.241	0.5984D 00	-0.5841D-01
15	1.448	0.4997D 00	-0.1345D-01
17	1.655	0.4004D 00	0.1385D-01
19	1.862	0.3018D 00	0.2699D-01
21	2.069	0.2043D 00	0.2947D-01
23	2.276	0.1081D 00	0.2472D-01
25	2.483	0.1294D-01	0.1614D-01
27	2.690	-0.8151D-01	0.7110D-02
29	2.897	-0.1757D 00	0.9562D-03

F I G U R E S

A P P E N D I X

SPECIMEN-OUTPUT

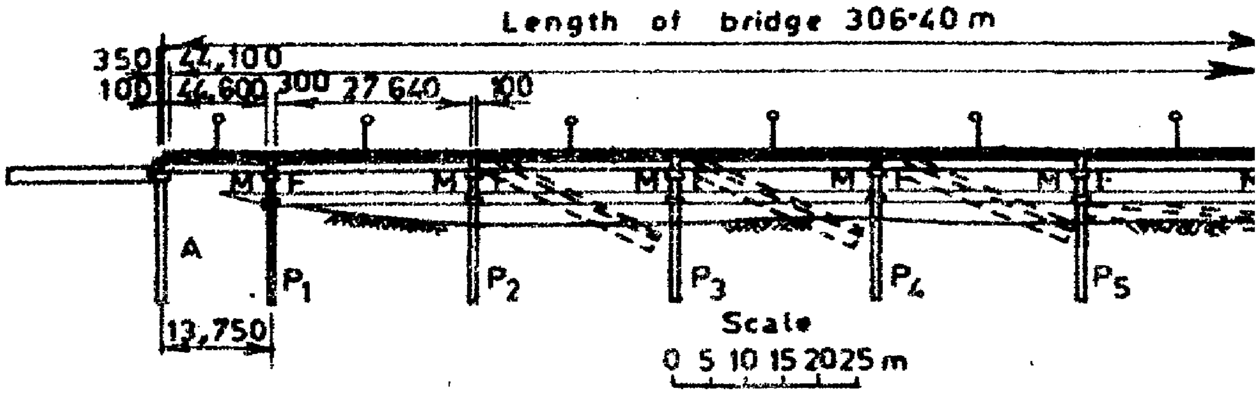


Fig. 1·1 Profile of showa — bridge

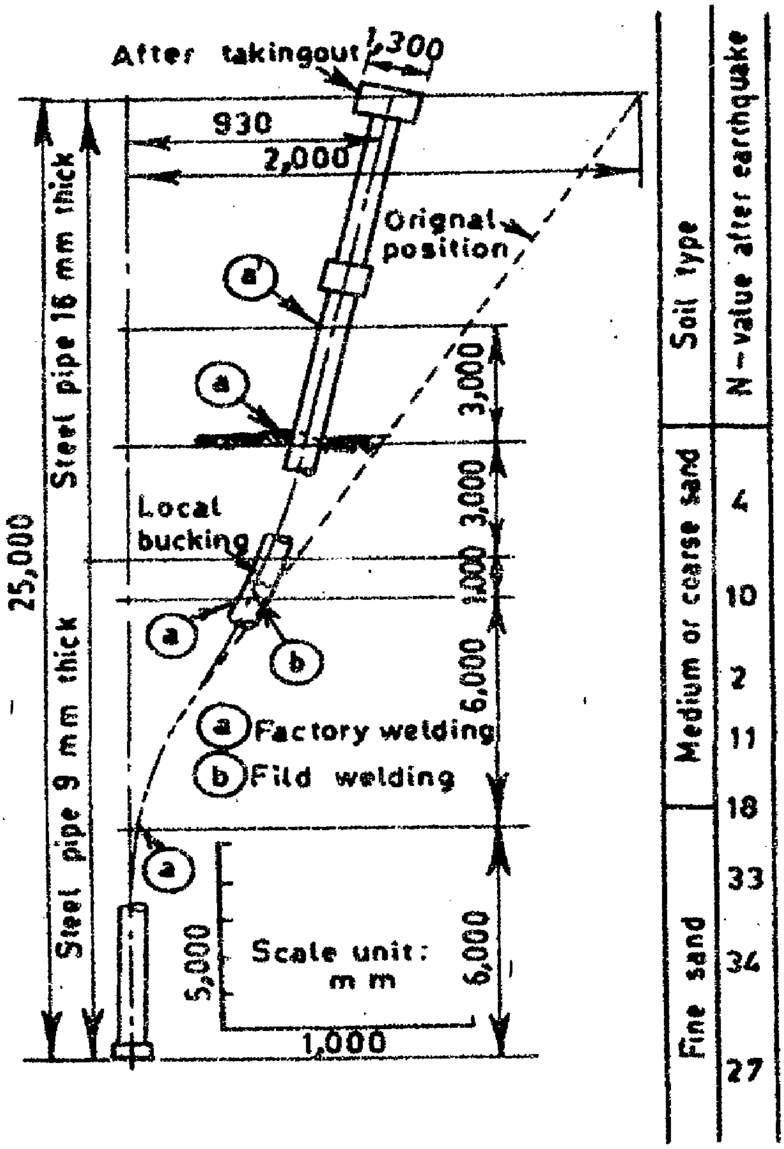


Fig. 1·2 Pipe pier no. 4, taken out fr
ground after niigata earthquake

(After Fukuoka 1966)

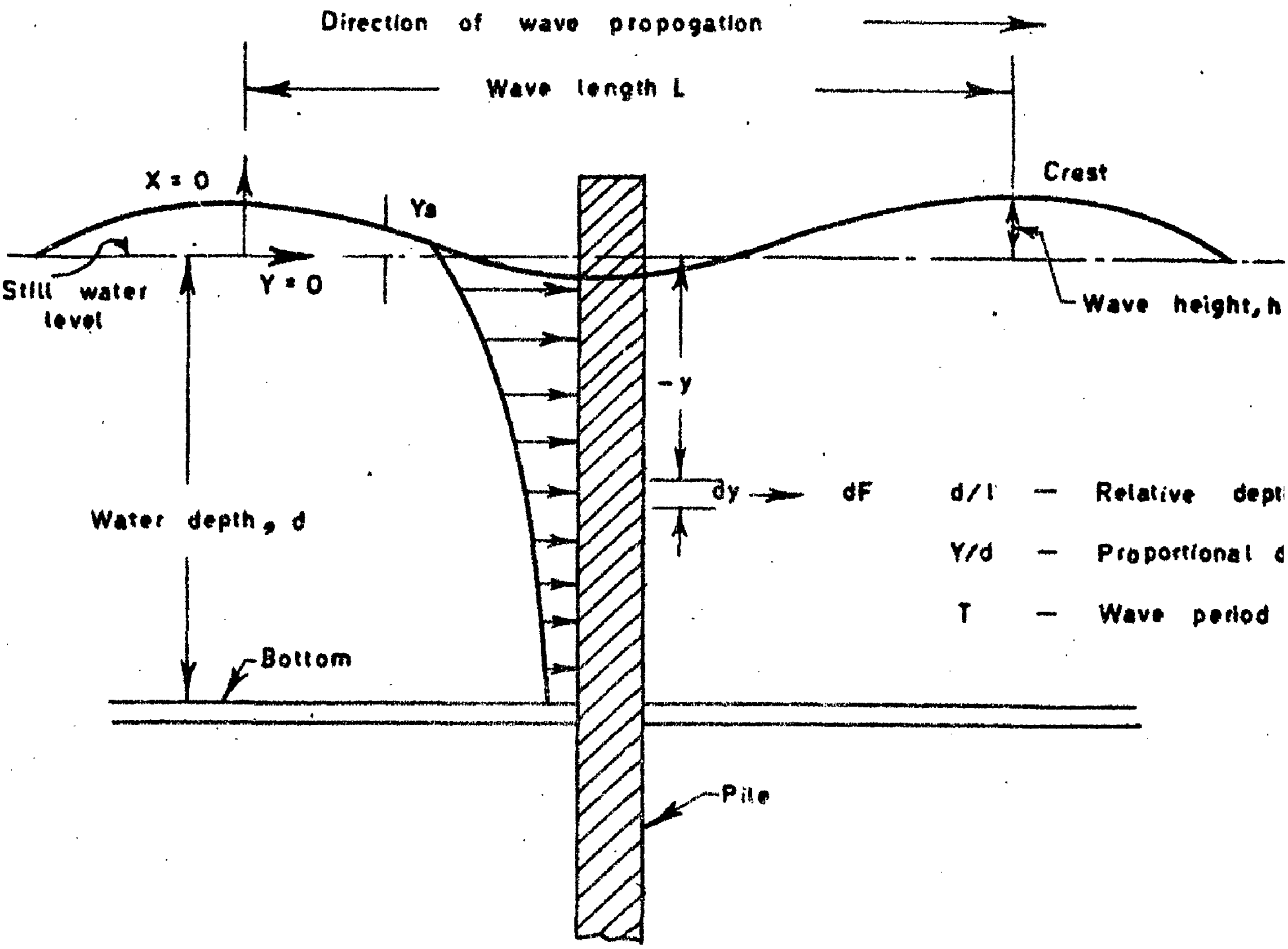


Fig. 2·1 Wave forces on a pile

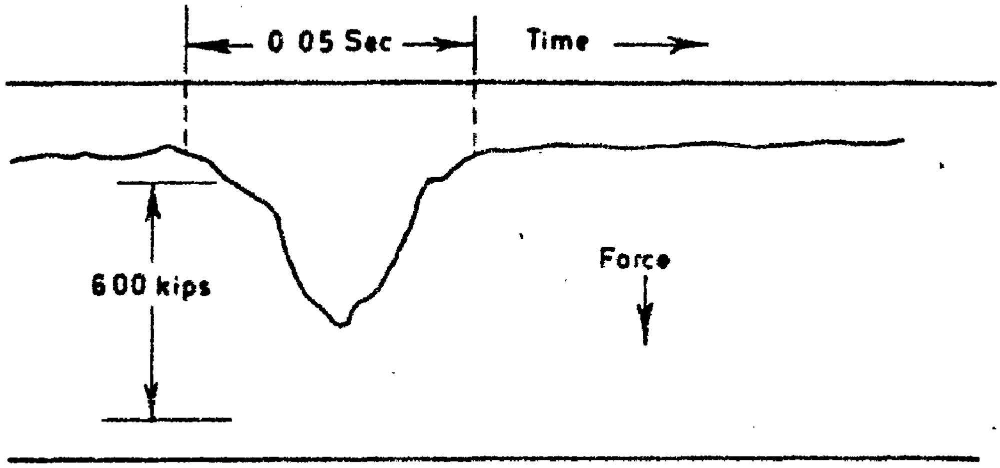


Fig. 2·2 a Force time plot

Fig. 2·2 b Pile energy determination force deflection plot

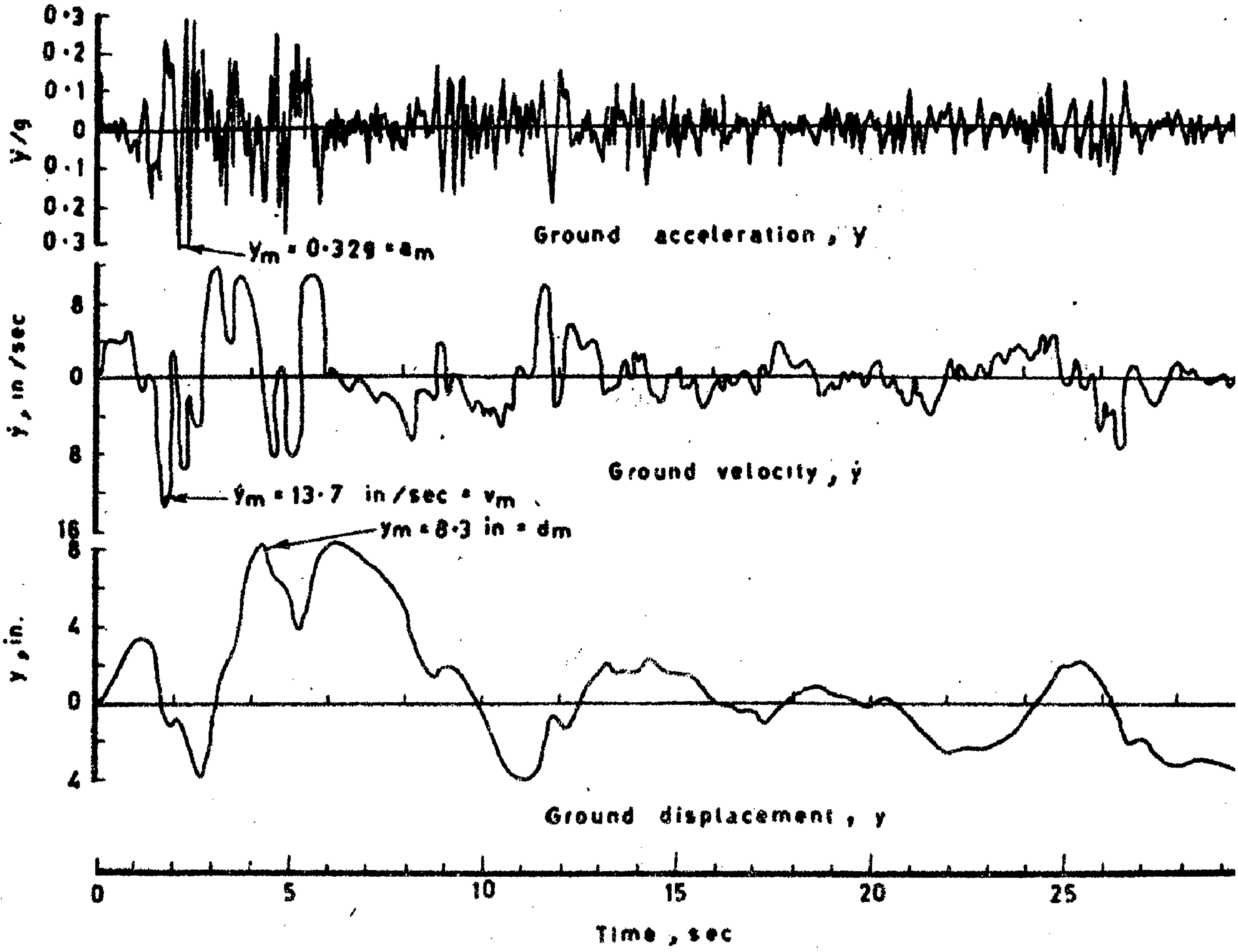


Fig. 2·3 El centro , california earthquake of may 1940 N-S component

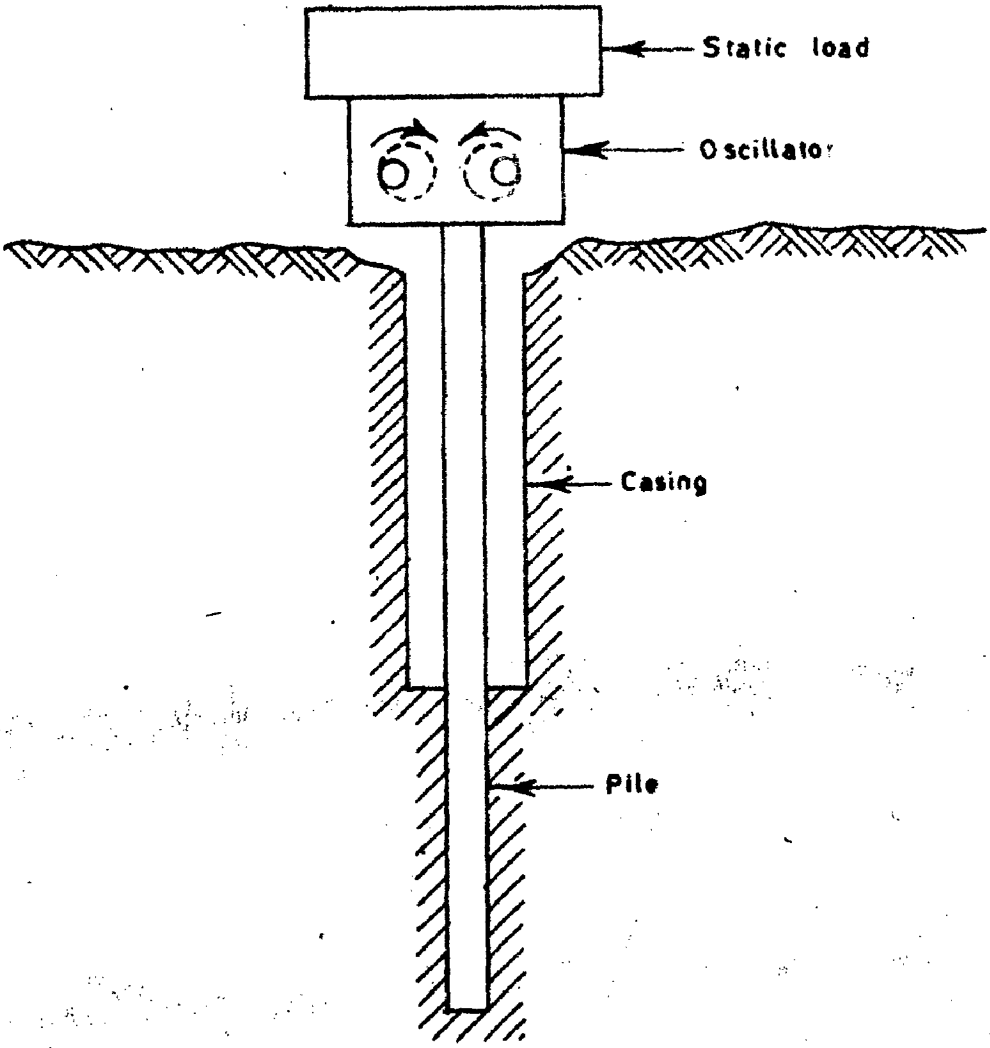


Fig. 2·4 a Pile soil system

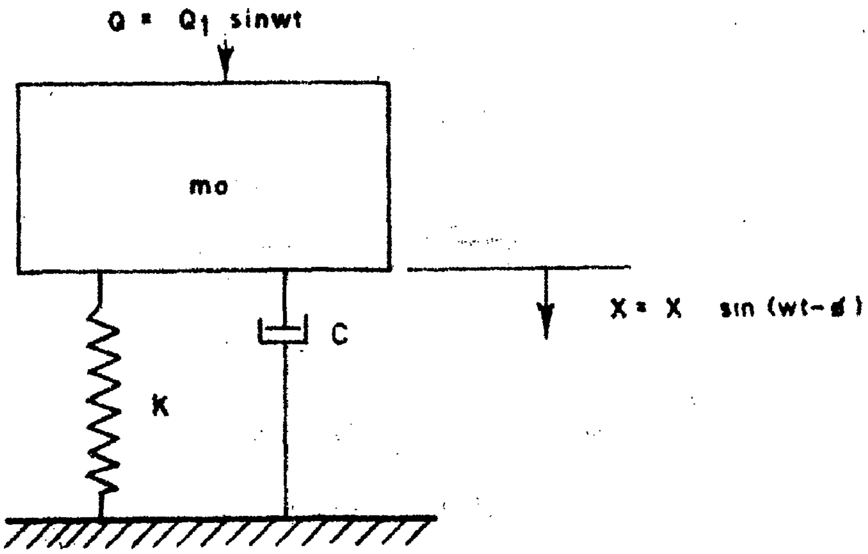


Fig. 2·4 b Mathematical model

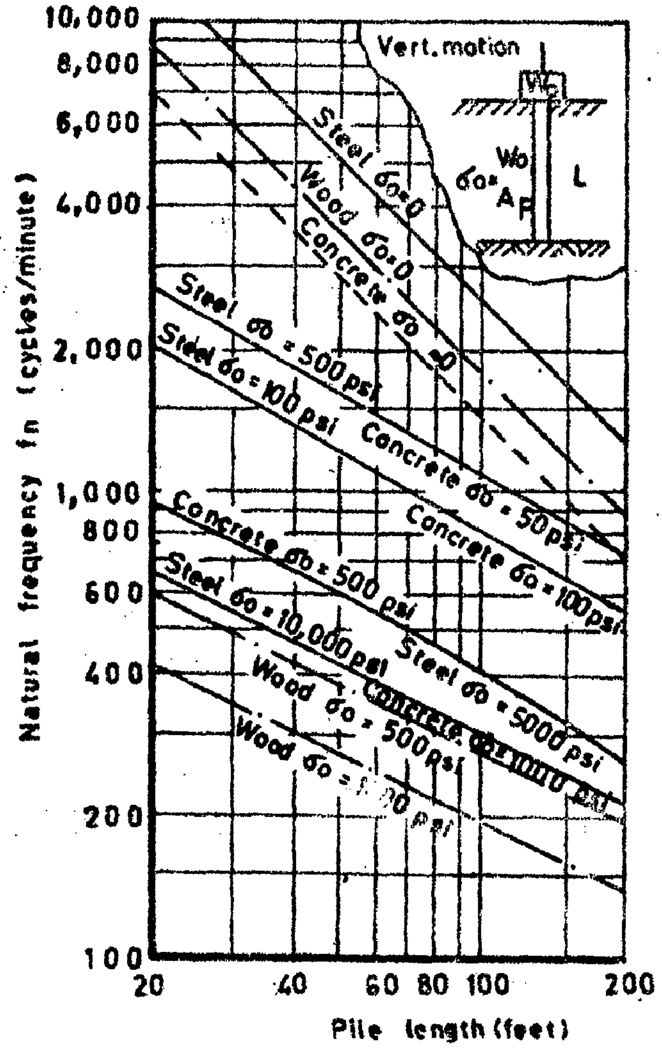


Fig. 2·5 Resonant frequency under vertical oscillation of a pile

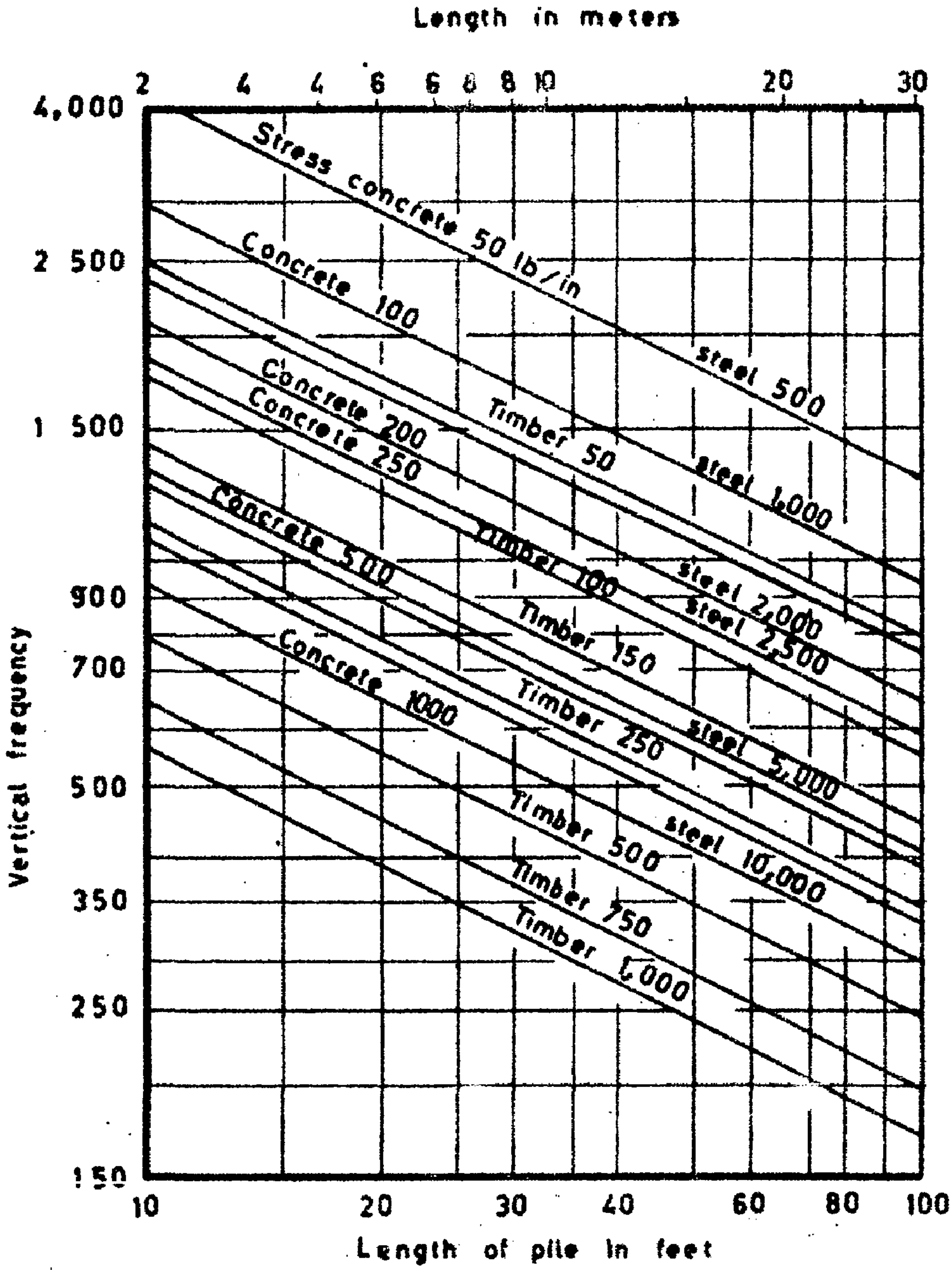
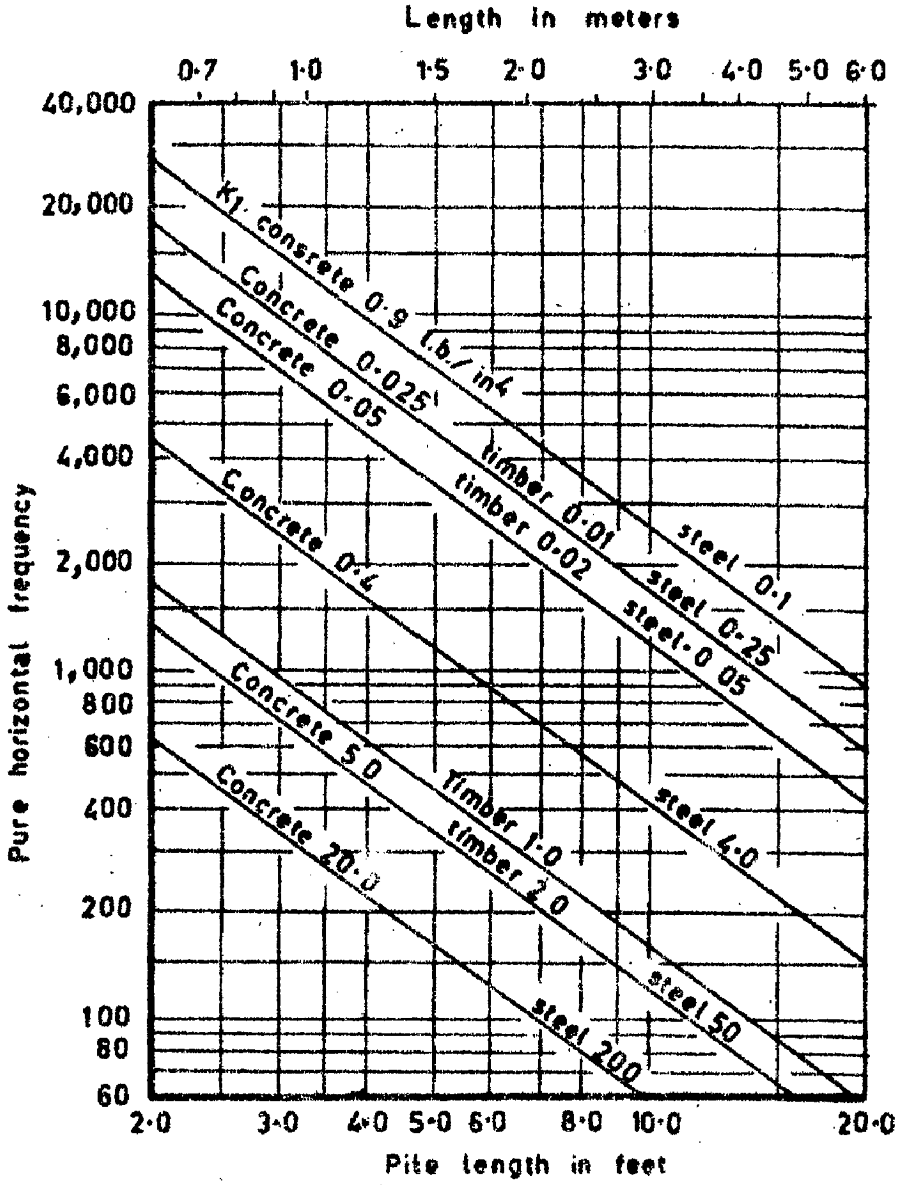


Fig. 2·6 Chart for vertical frequency of piles



Units :— This chart is applicable to problem in pound – inch units or metric units so long as units for the various terms are constant.

Fig. 2·7 Chart for determining horizontal frequen of piles

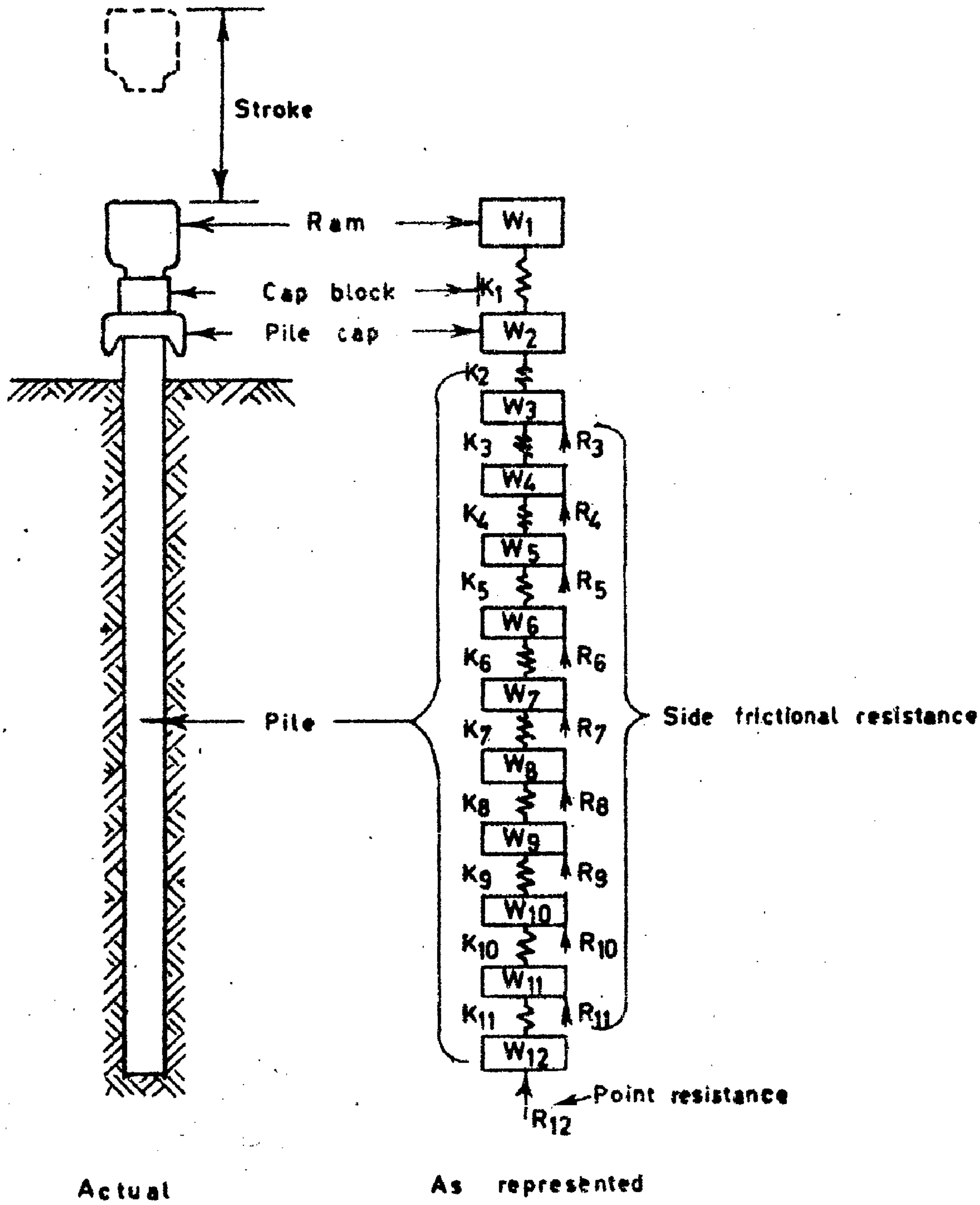


Fig. 2·8 Soil pile model for determining forces during pile driving

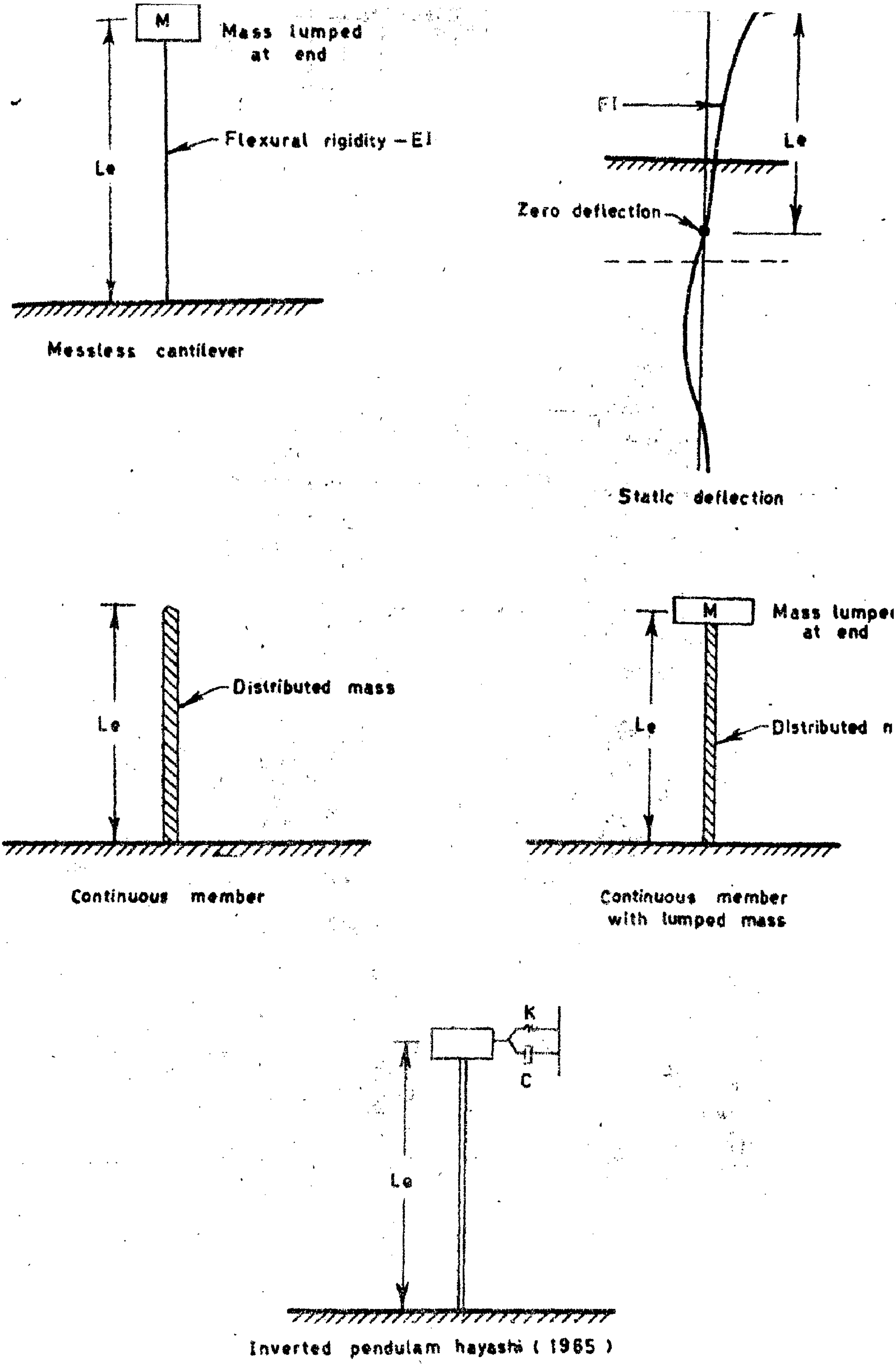


Fig. 2·9 Simplified structural systems equivalent cantilevers

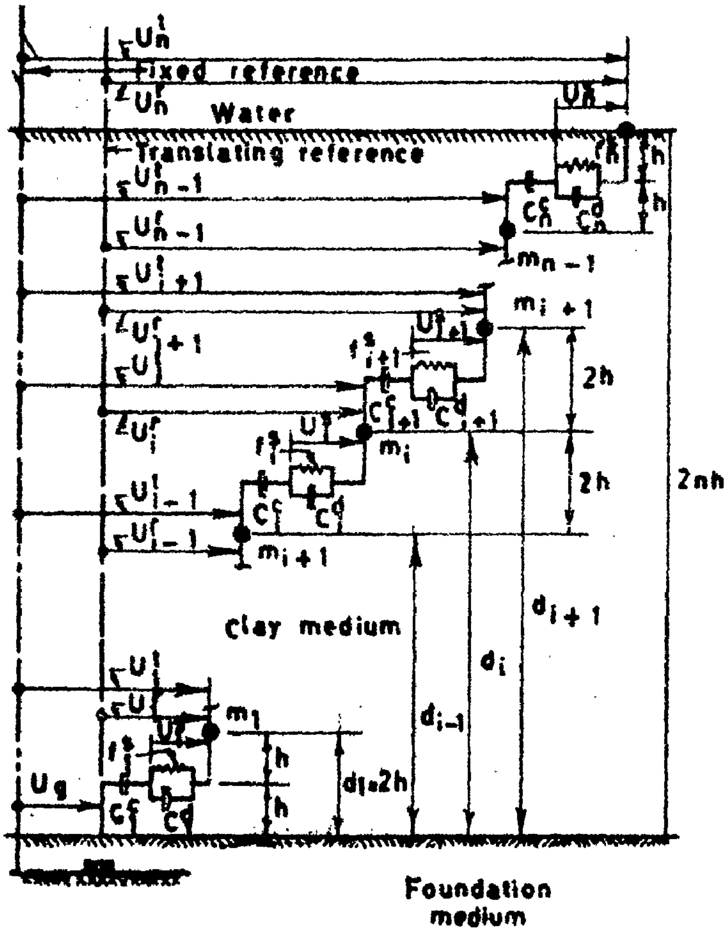


Fig. 2·10 Idealized clay medium

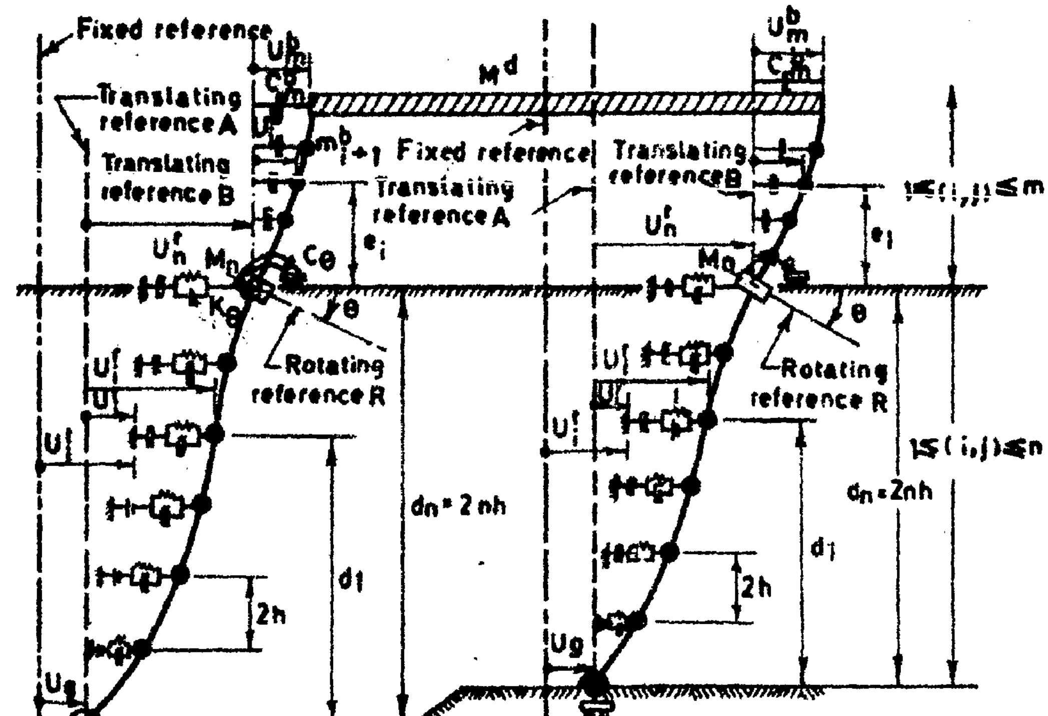


Fig. 2·11 Idealized structural system

(Penzien et al. 1964)

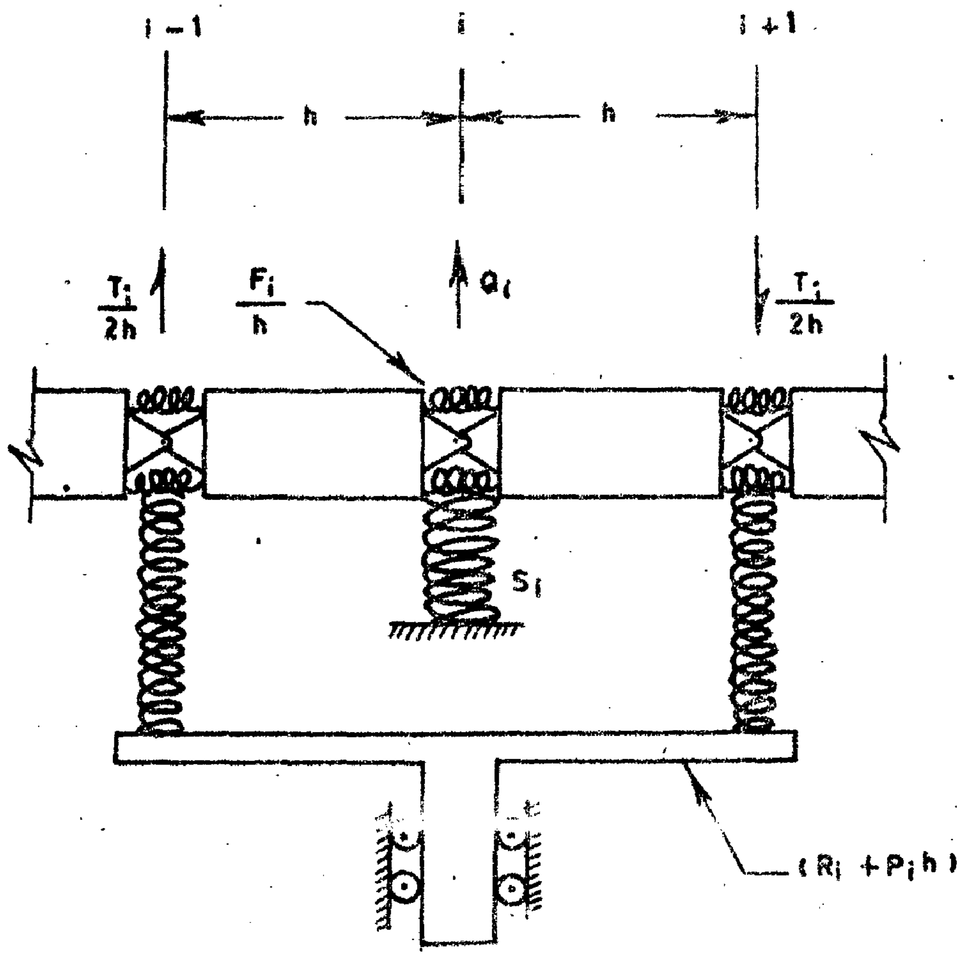


Fig. 2·12 Mechanical model for beam - column idealisation

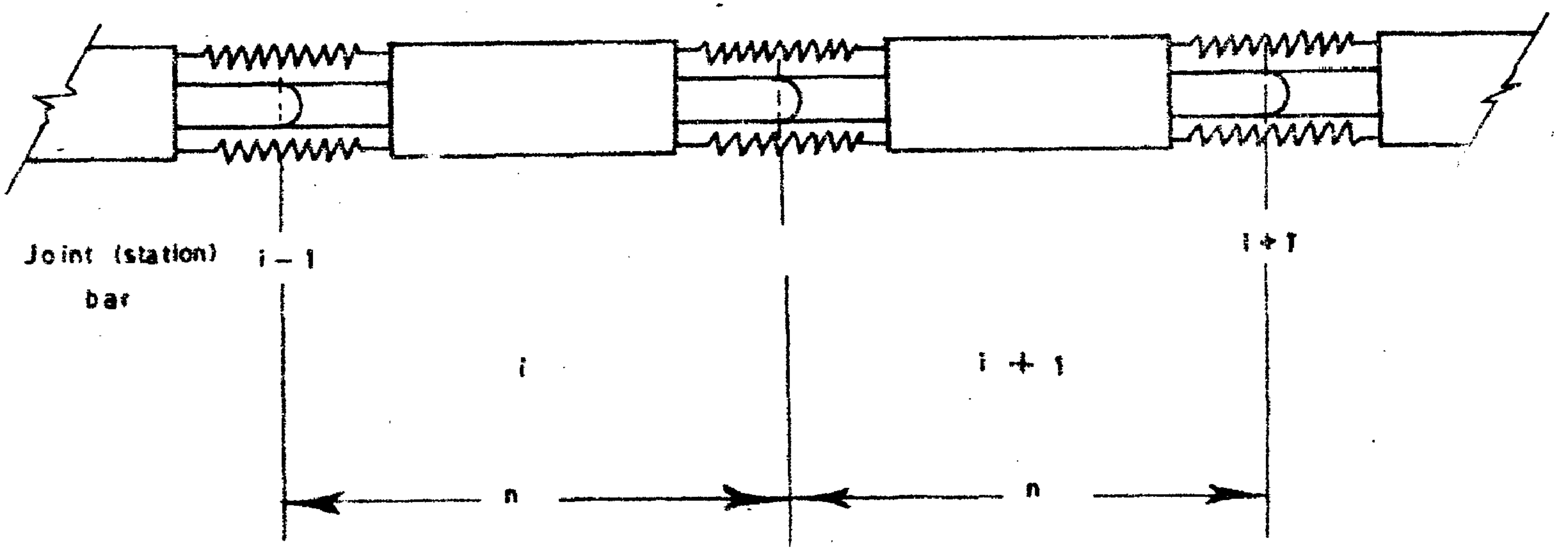


Fig. 2·13 Discrete element beam column model

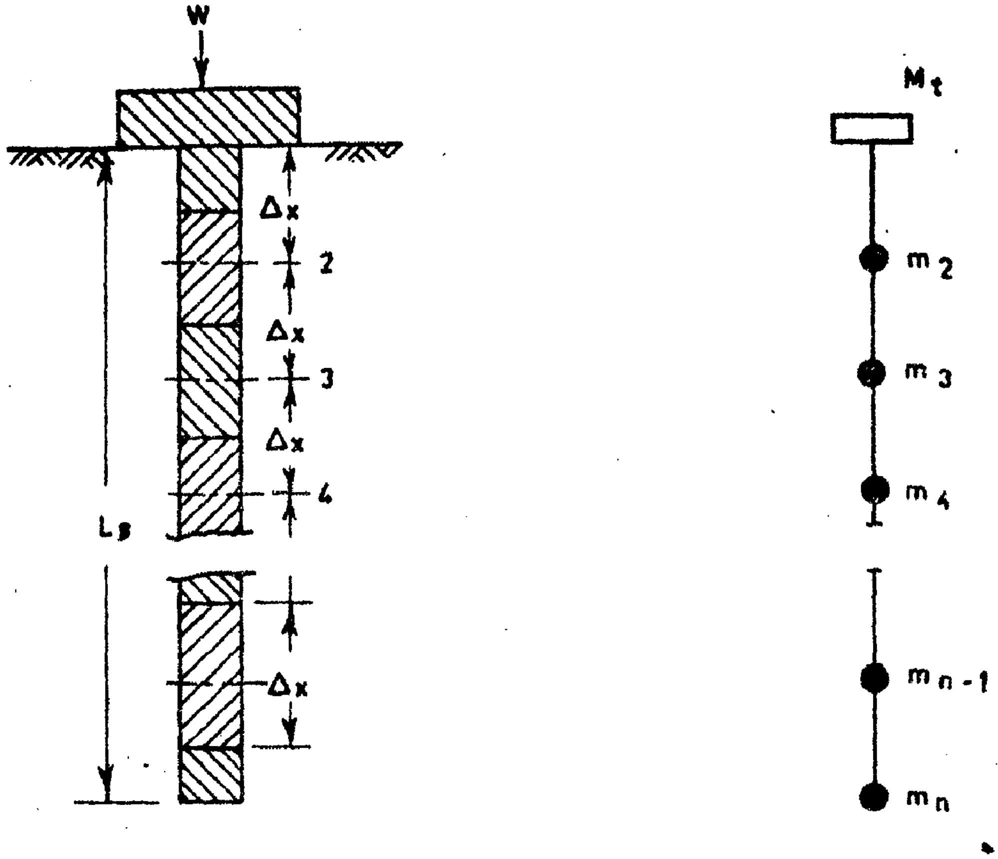


Fig. 3·1 Pile structural idealisation

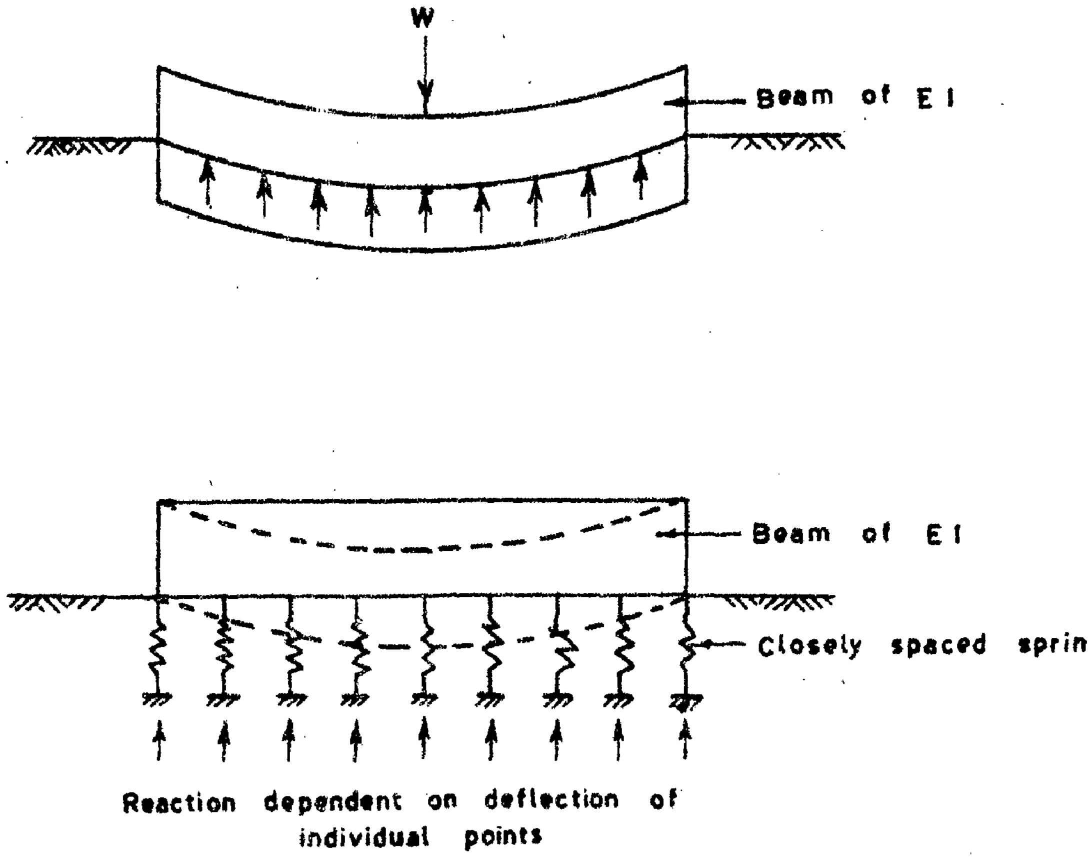


Fig. 3·2a Winkler idealisation

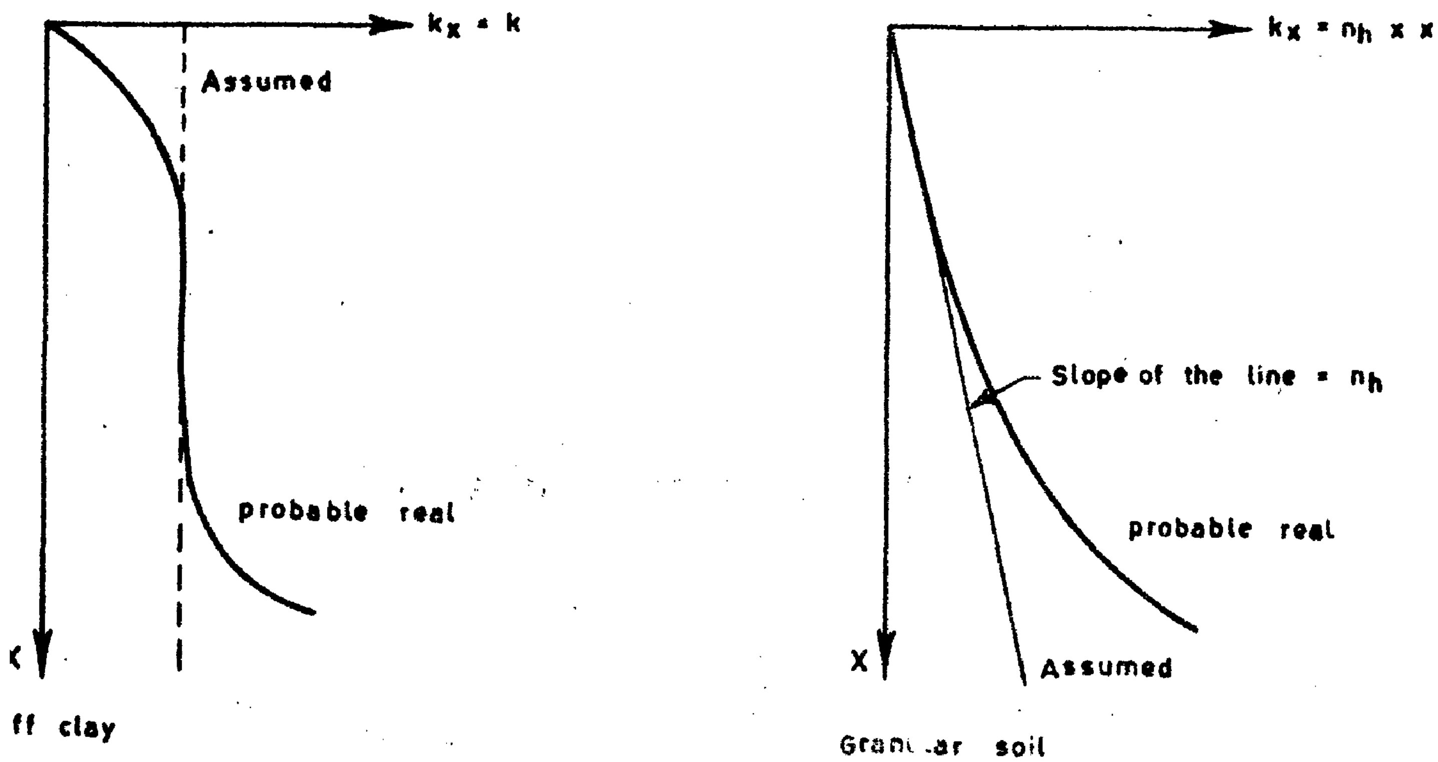


Fig. 3·2b Variation of soil modulus

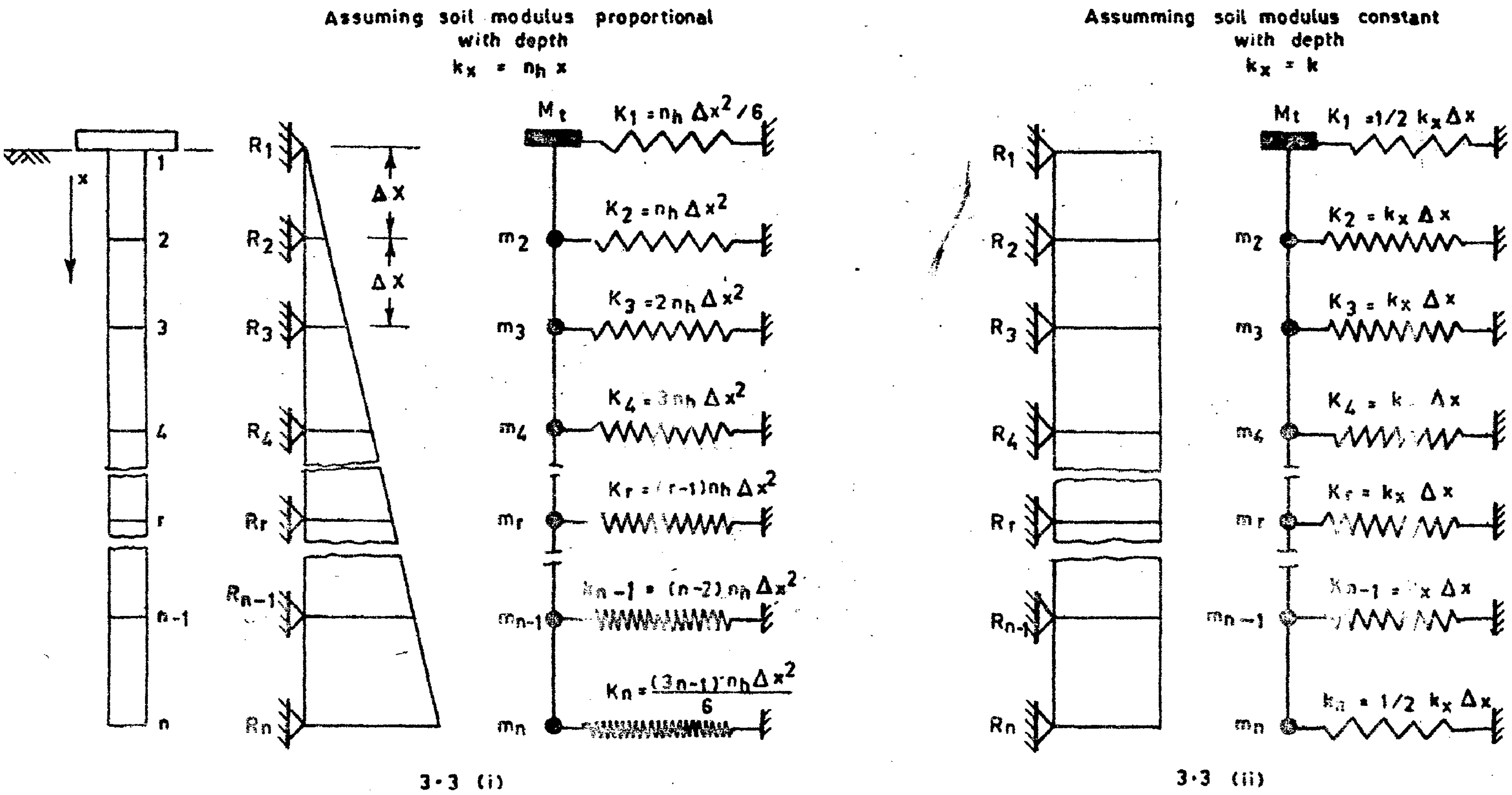


Fig. 3·3 Discretisation of soil-pile interaction effects

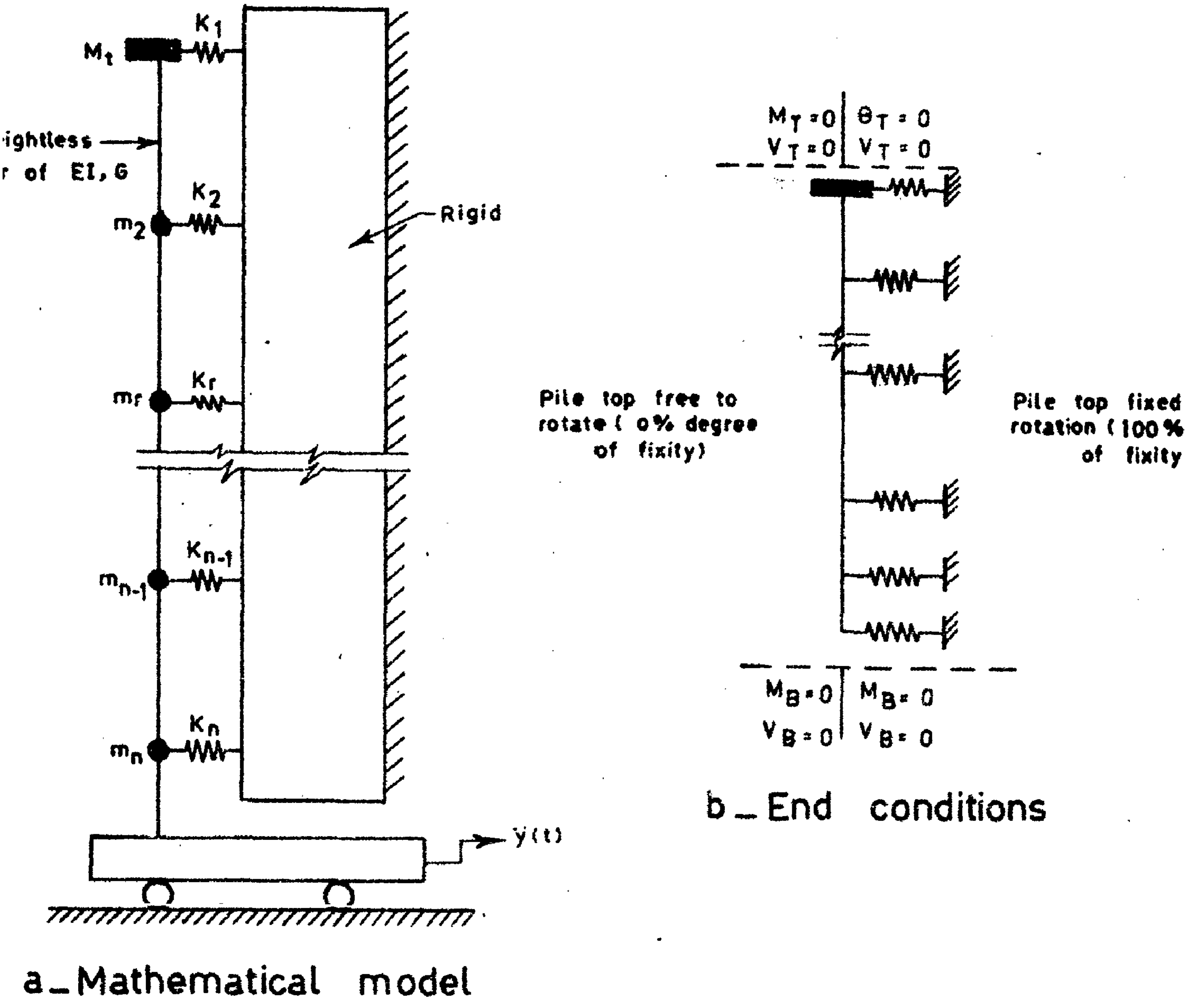


Fig. 3-4 Mathematical model with end conditions

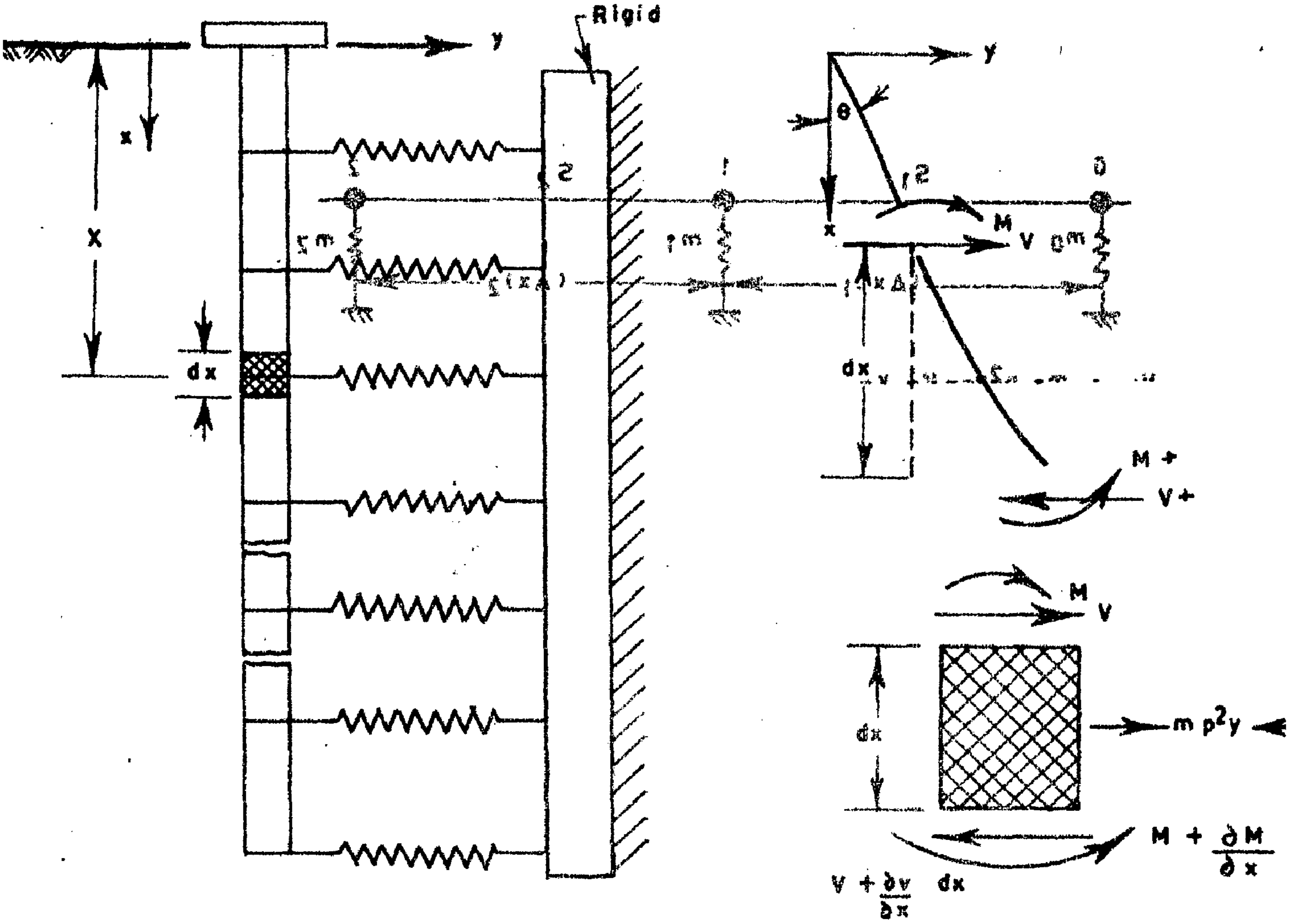


Fig. 3·5a Elastic and inertia forces on an element

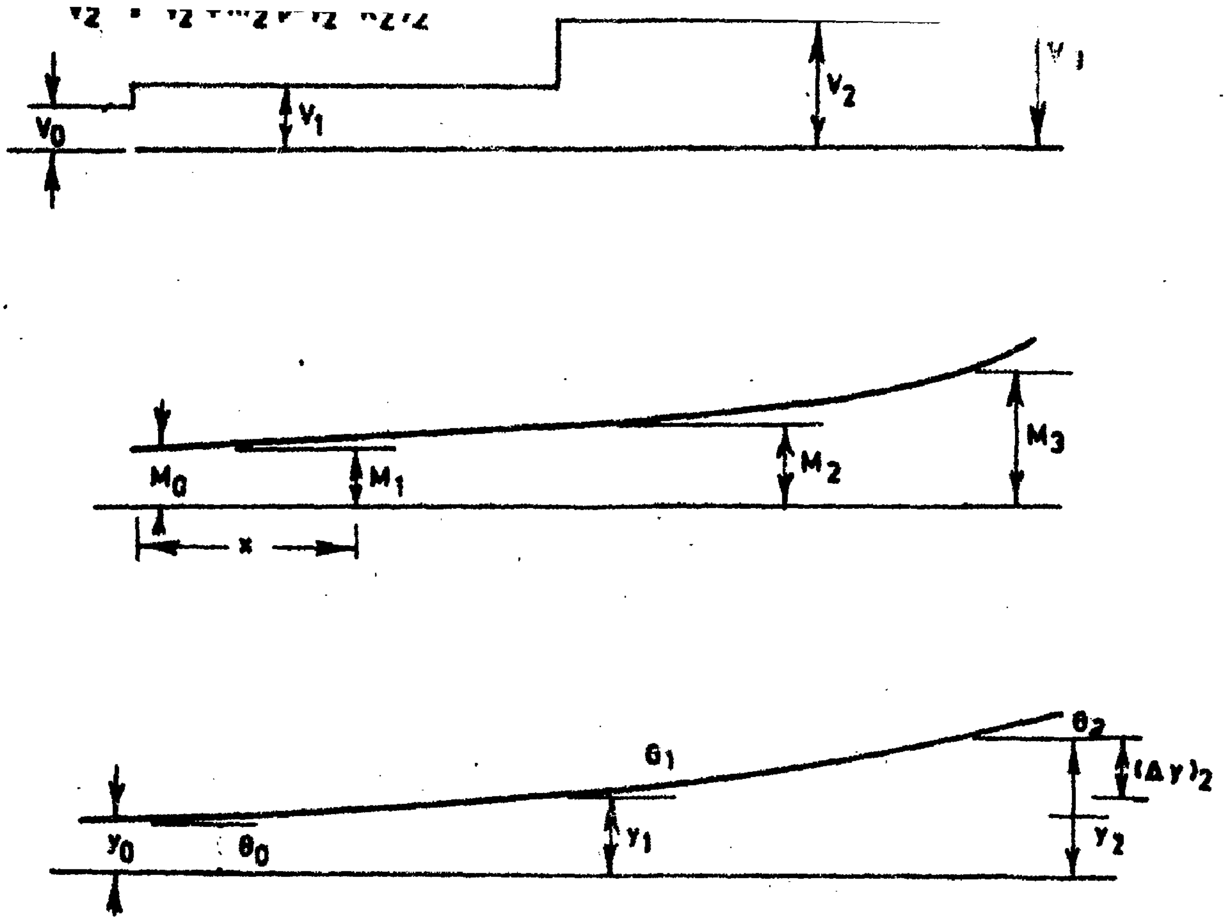


Fig. 3·5b Transfer operation

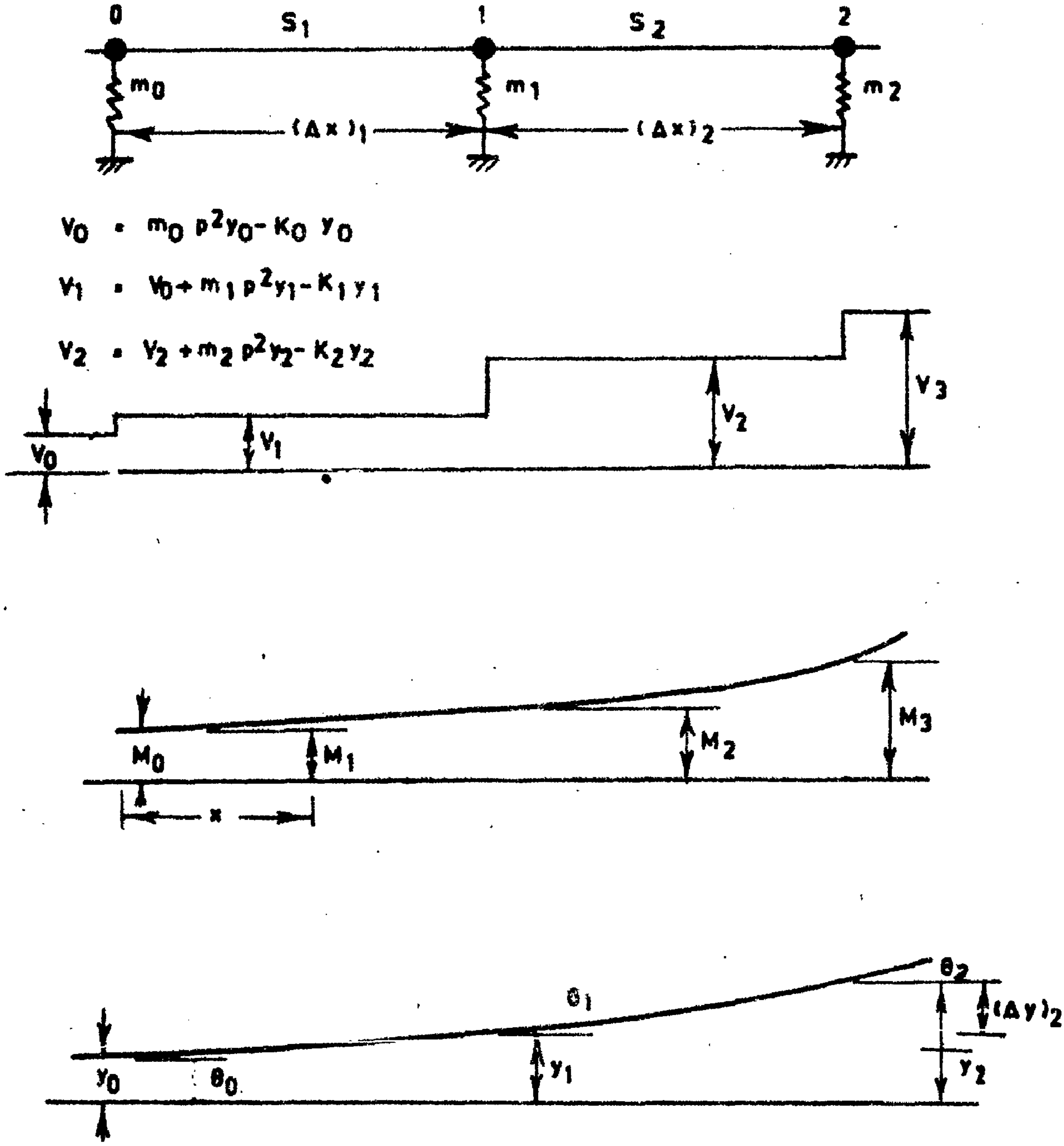


Fig. 3·5b Transfer operation

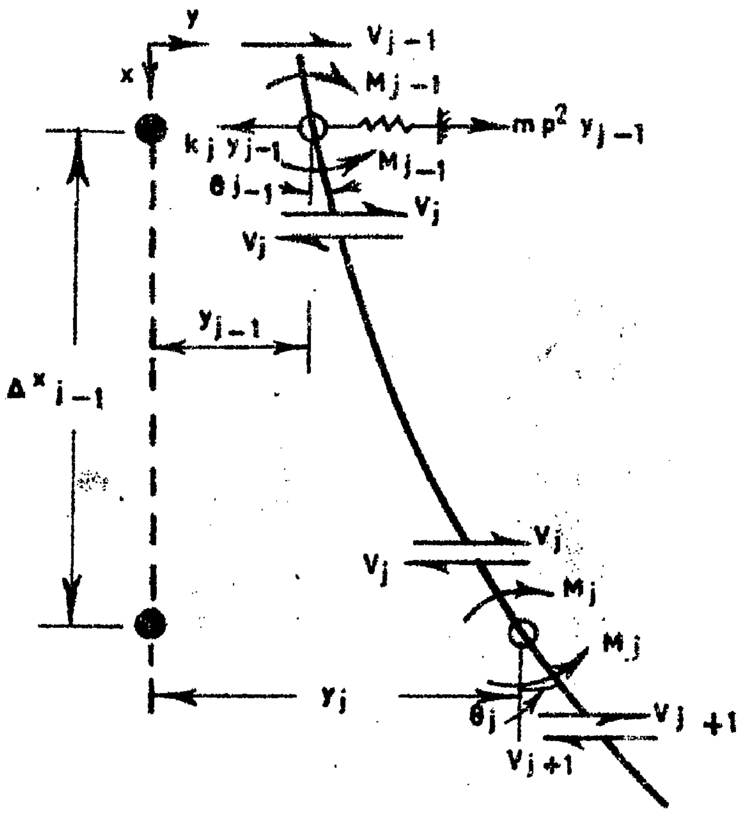


Fig. 3·6 Deflection and forces in a segment

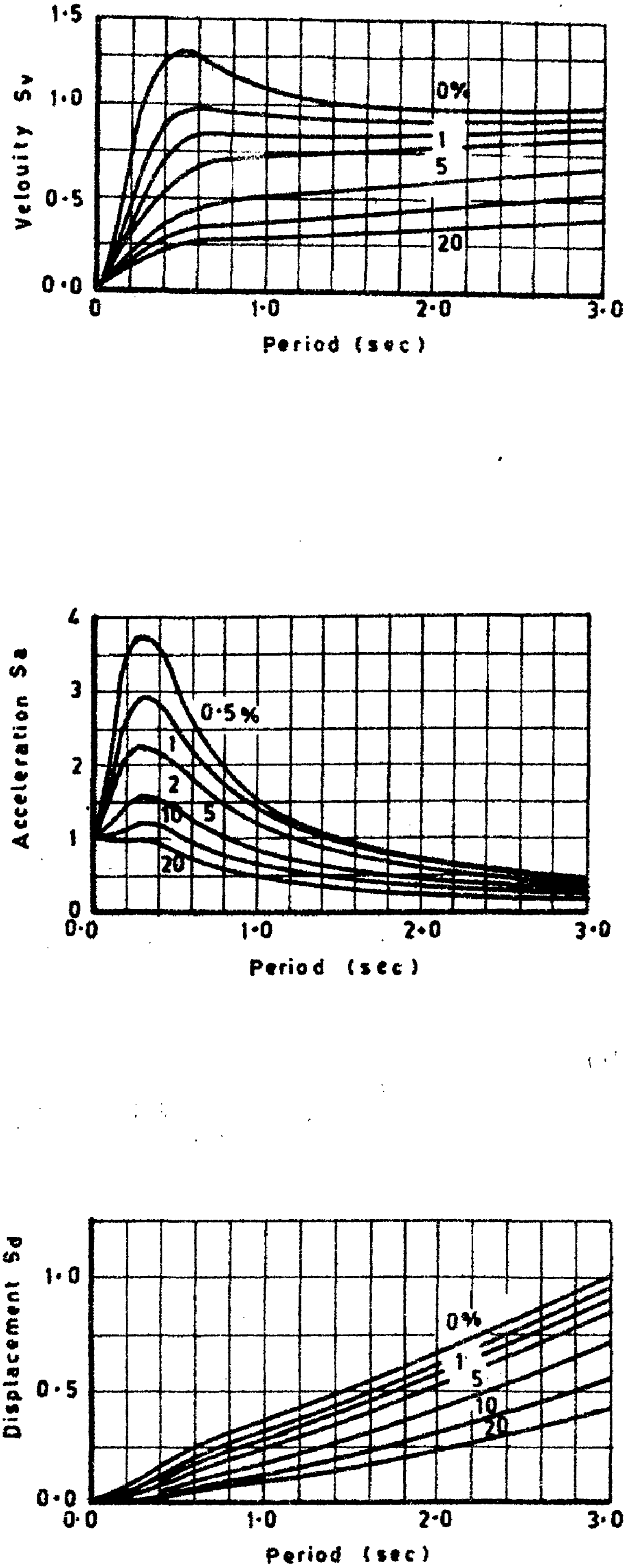


Fig. 3·7 Design spectrum curves — arbitrary scale

(Housner 1970)

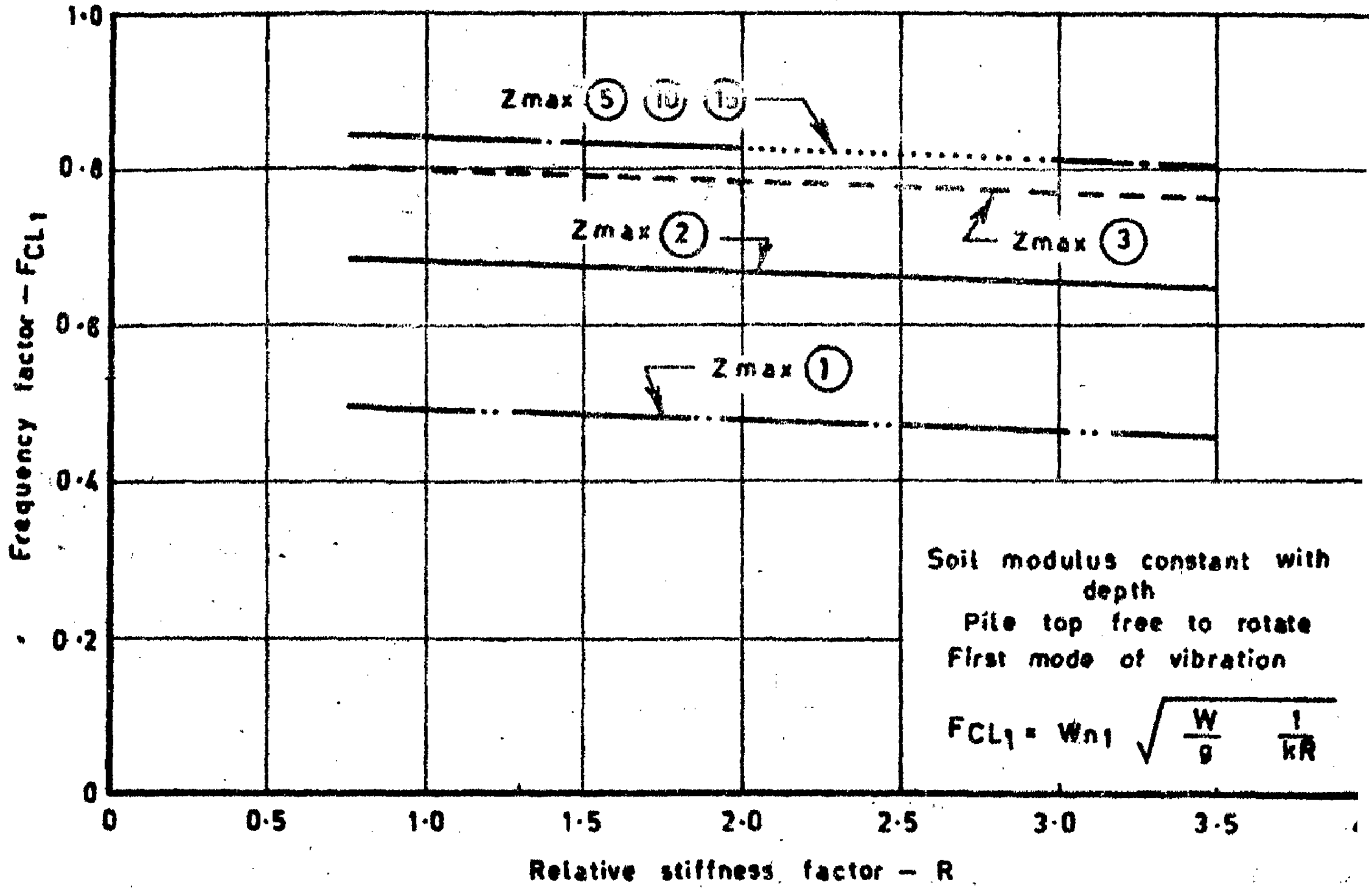


Fig. 3·8 Non – dimensional frequency factor versus relati stiffness factor

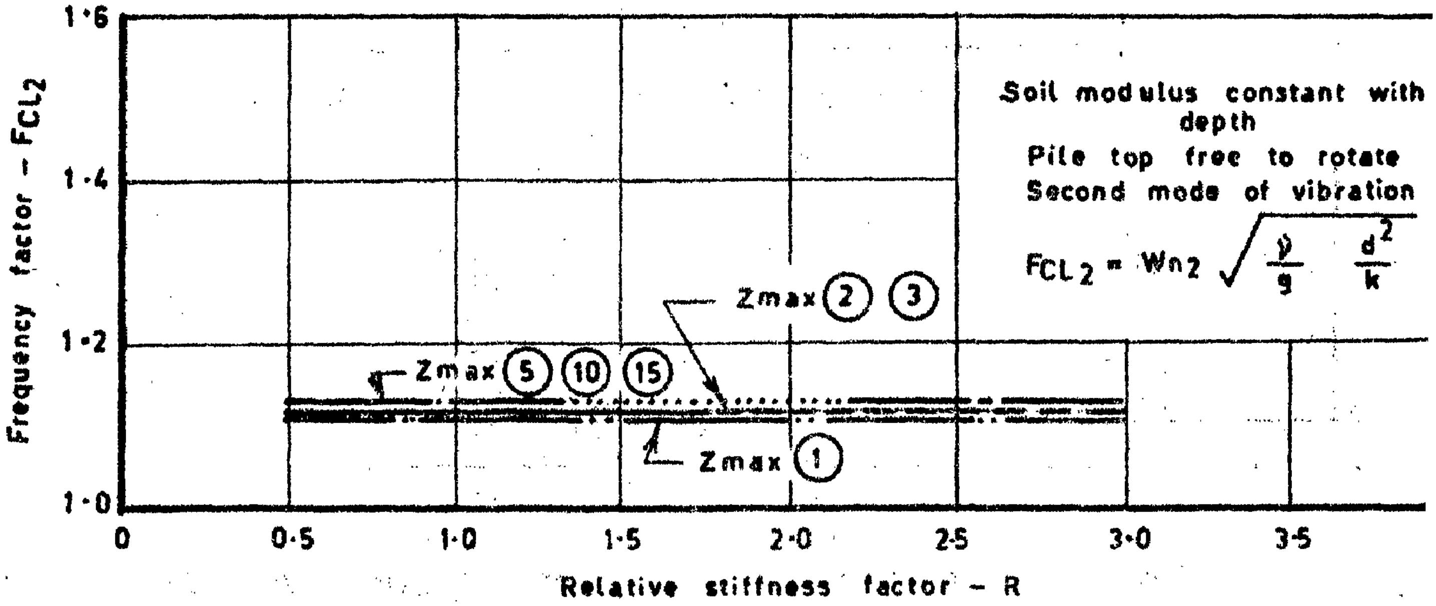


Fig. 3·9 Non – dimensional frequency factor versus relat stiffness factor

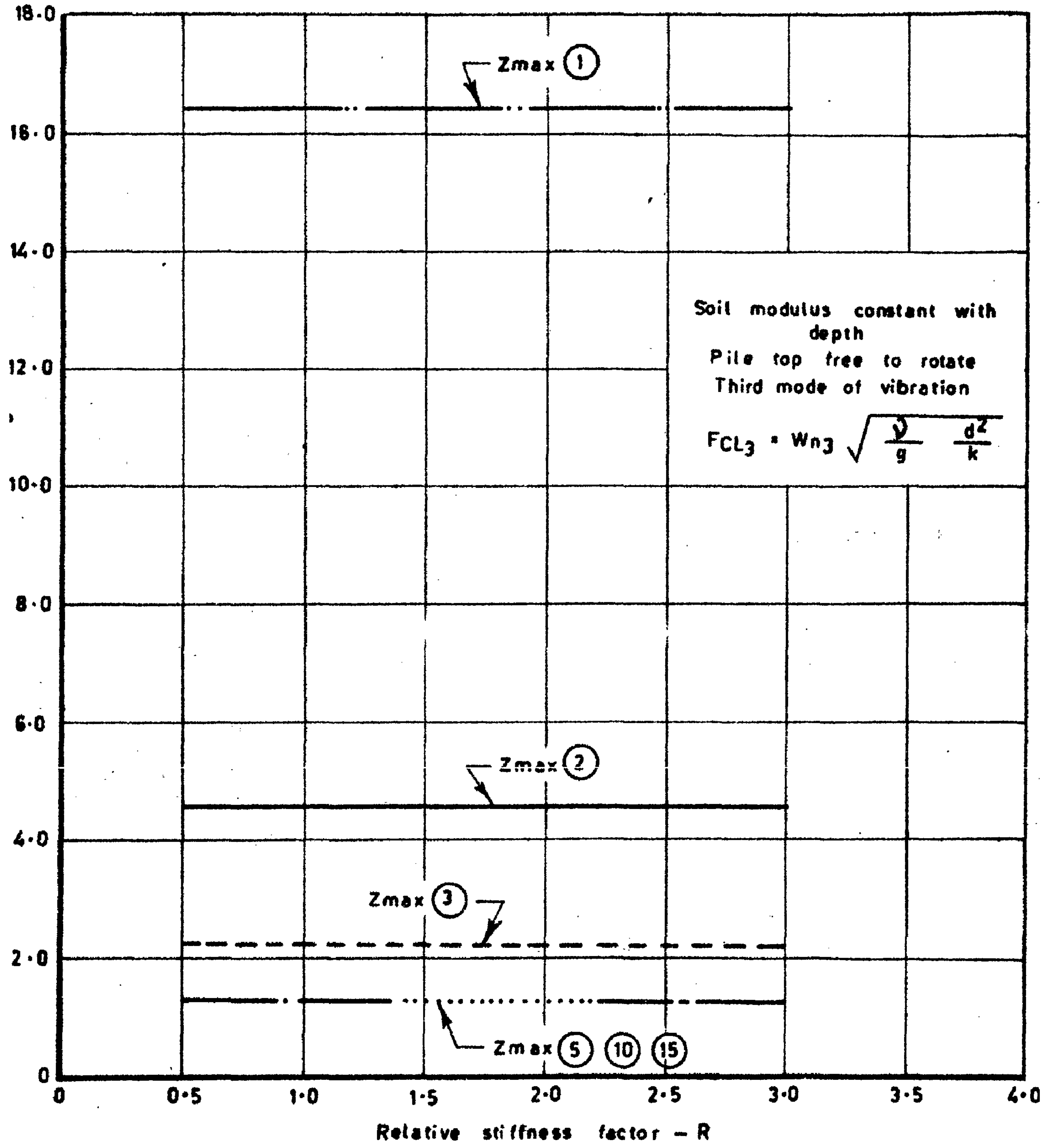


Fig. 3·10 Non – dimensional frequency factor versus relativ stiffness factor

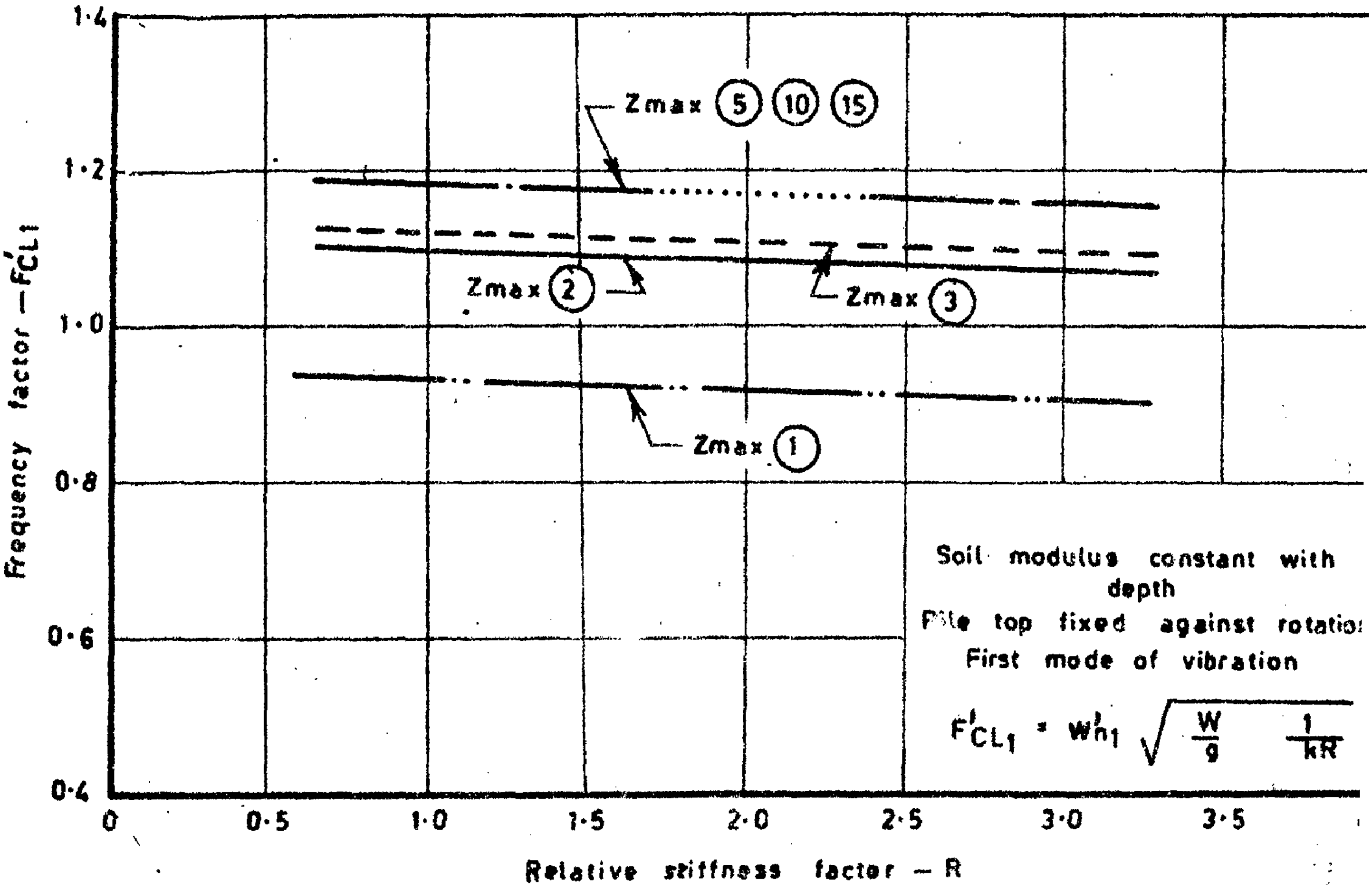


Fig. 3·11 Non – dimensional frequency factor versus relat stiffness factor

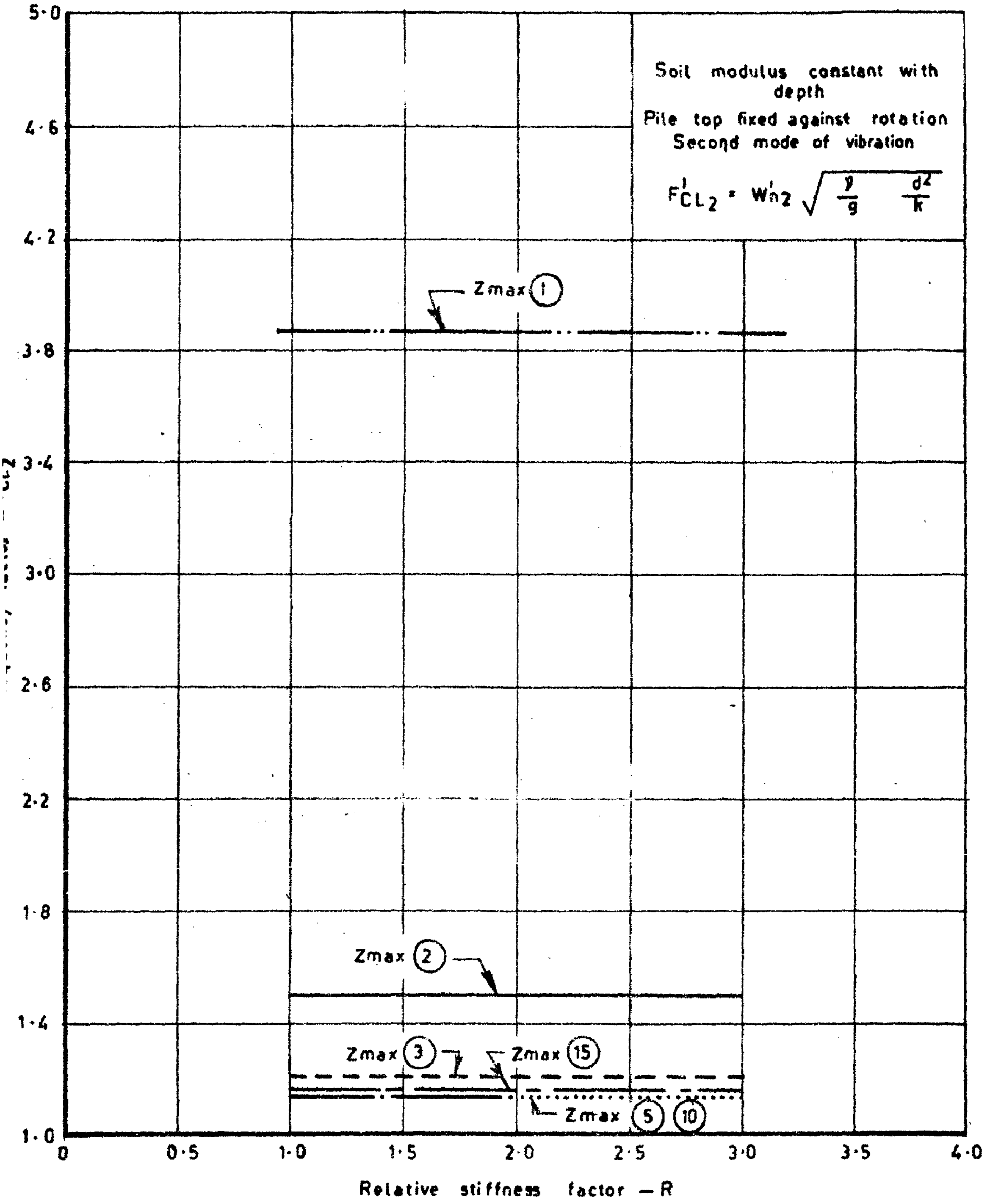


Fig 3·12 Non — dimensional frequency factor versus relative stiffness factor

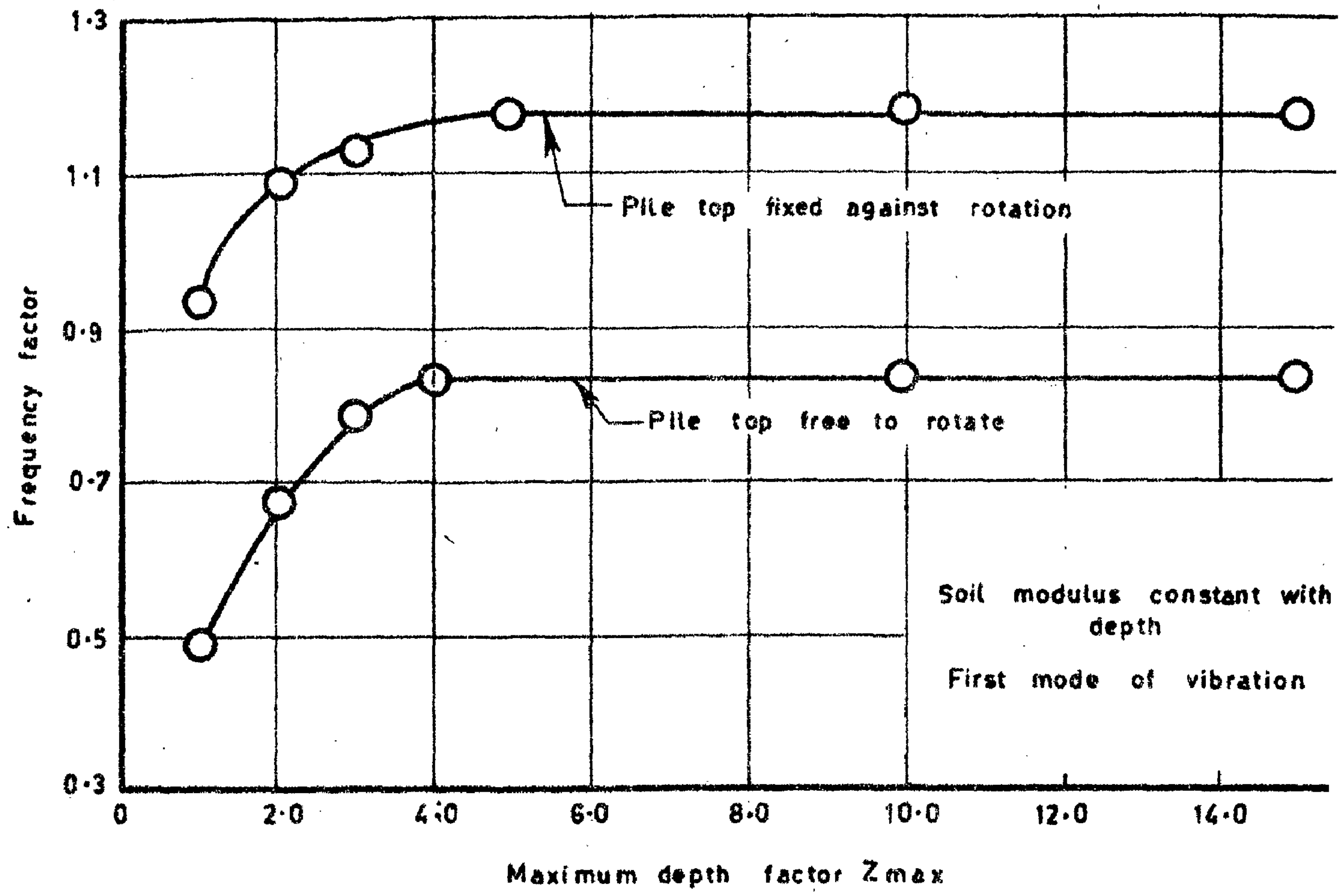


Fig. 3·13 Non – dimensional frequency factor versus ma depth factor Zmax

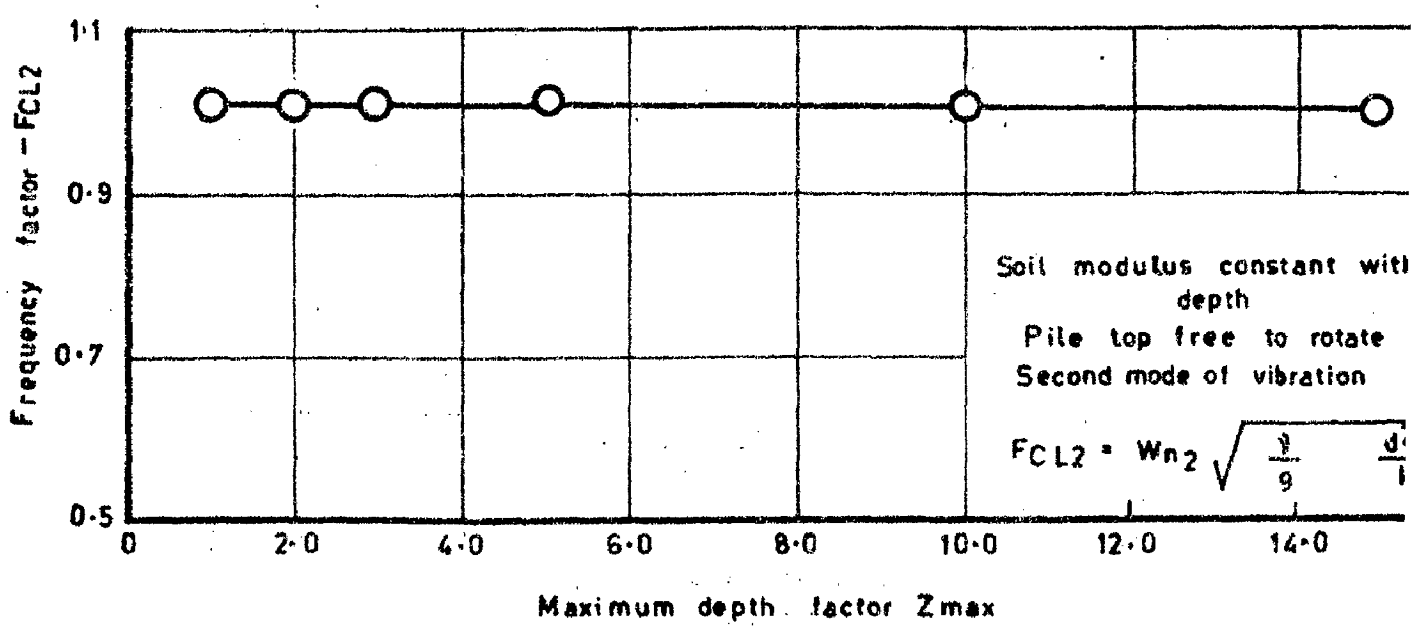


Fig. 3·14 Non – dimensional frequency factor versus ma depth factor Zmax

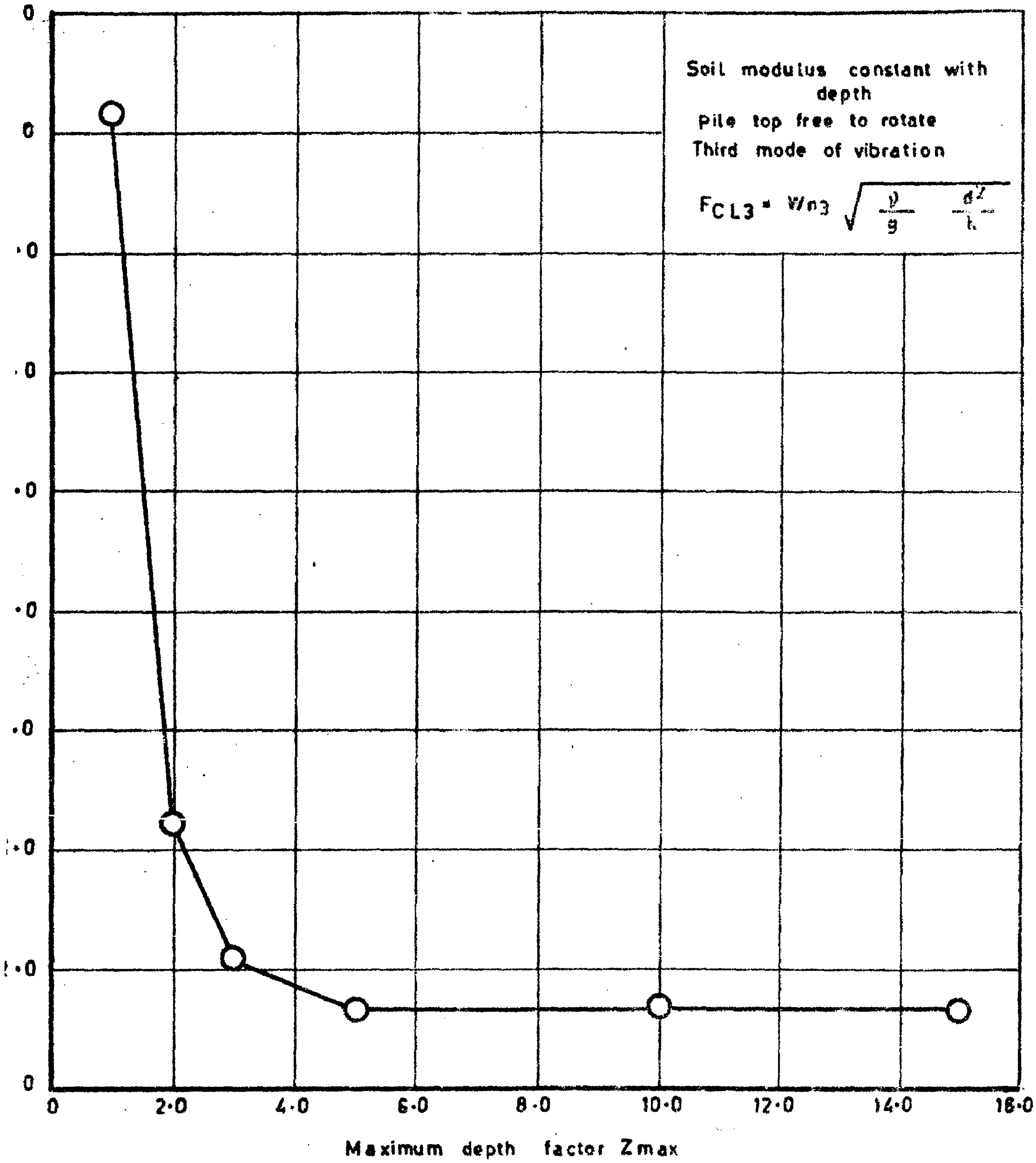


Fig. 3·15. Non – dimensional frequency factor versus maxim depth factor Zmax

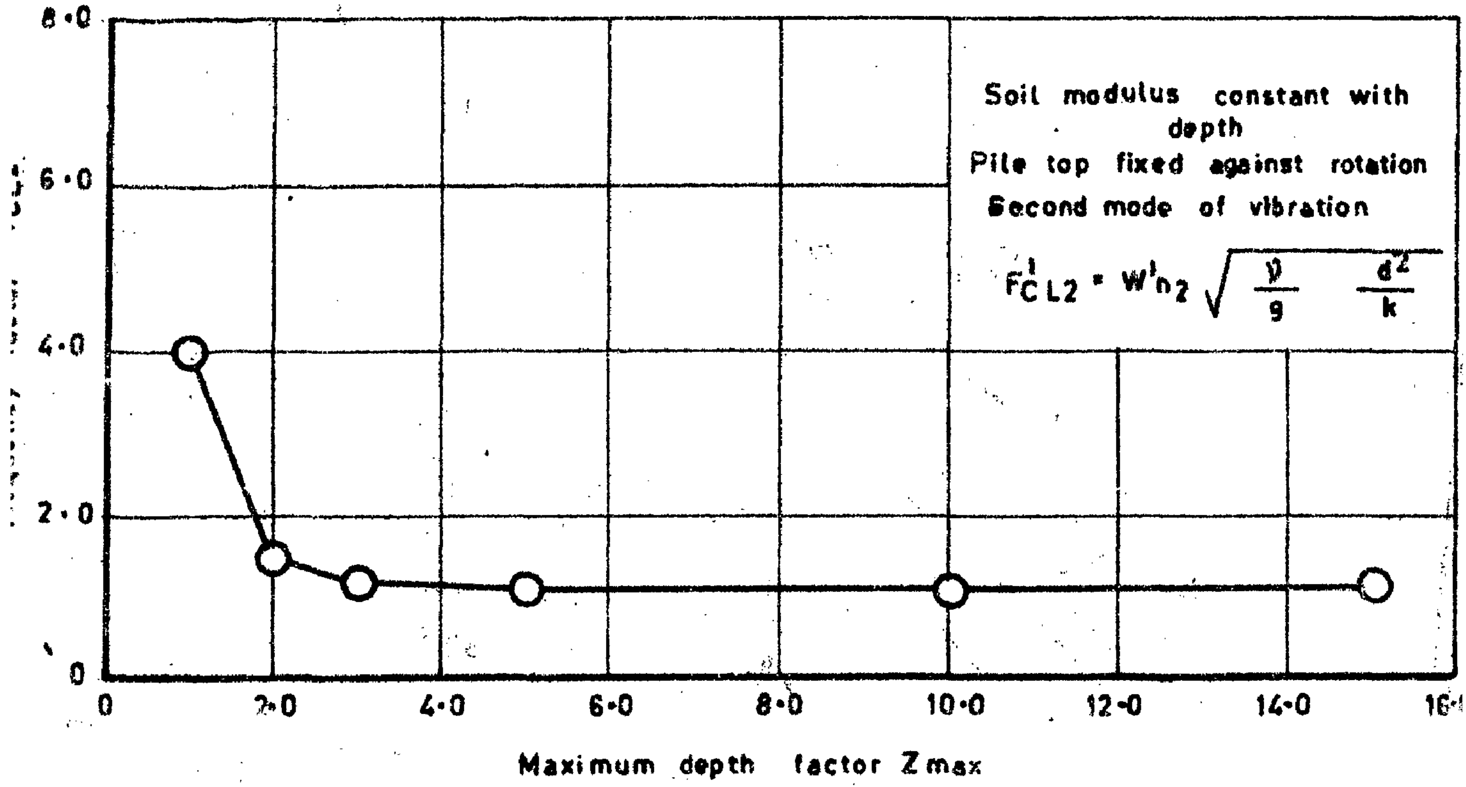


Fig. 3·16 Non - dimensional frequency factor versus maxin
depth factor Zmax

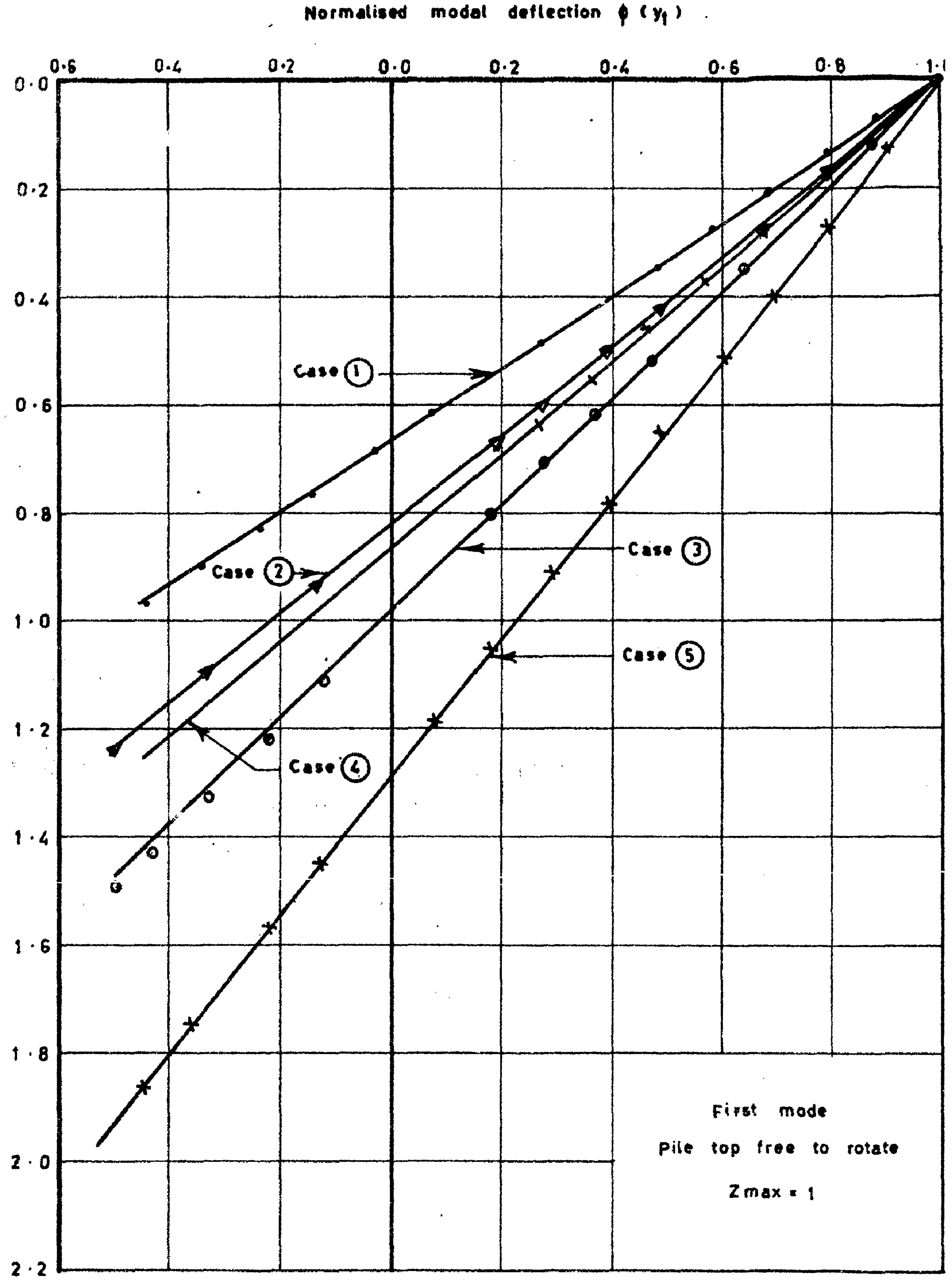


Fig. 3·17 Modal deflection versus depth assuming soil modulus constant with depth

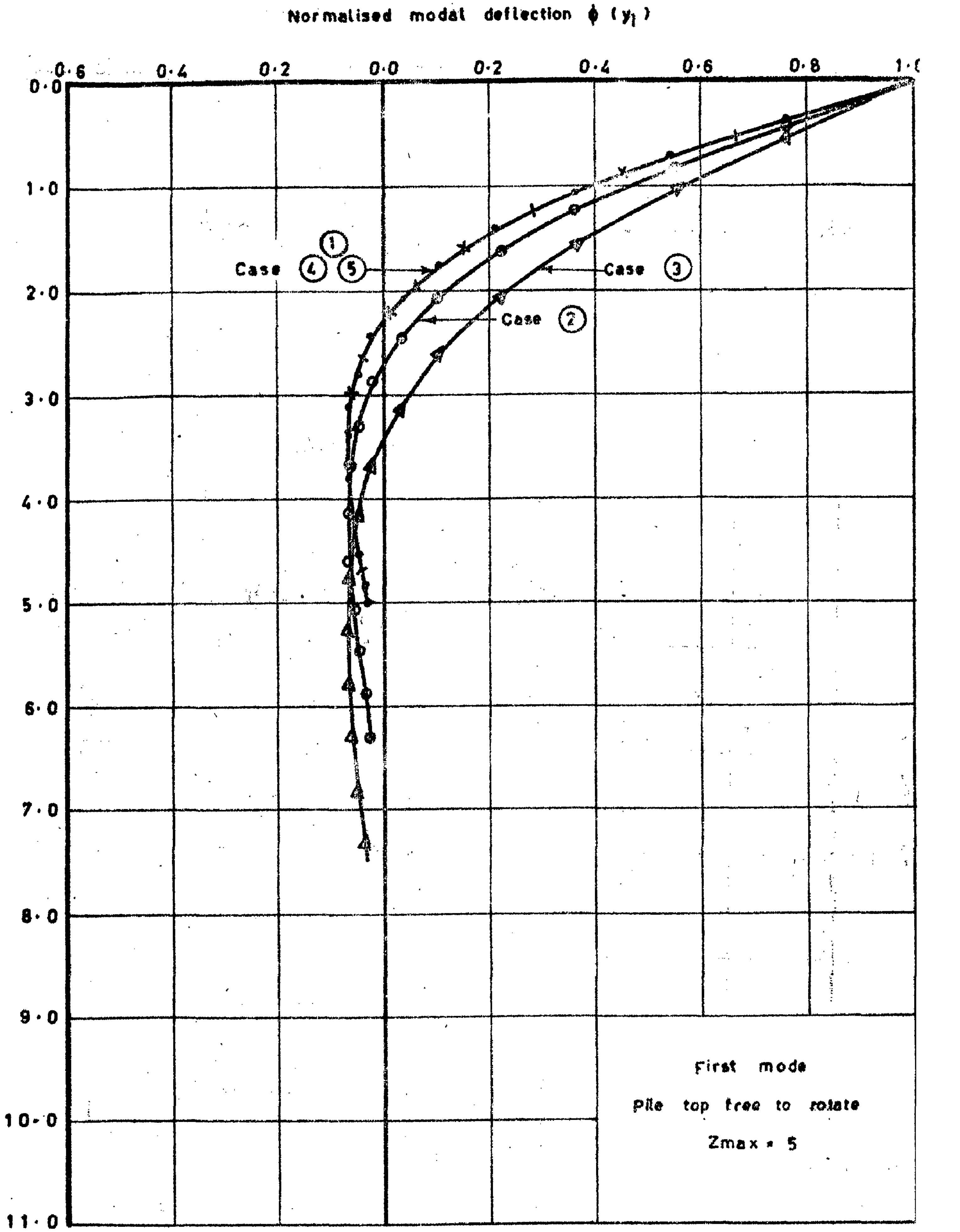


Fig. 3·18 Modal deflection versus depth assuming soi
modulus constant with depth

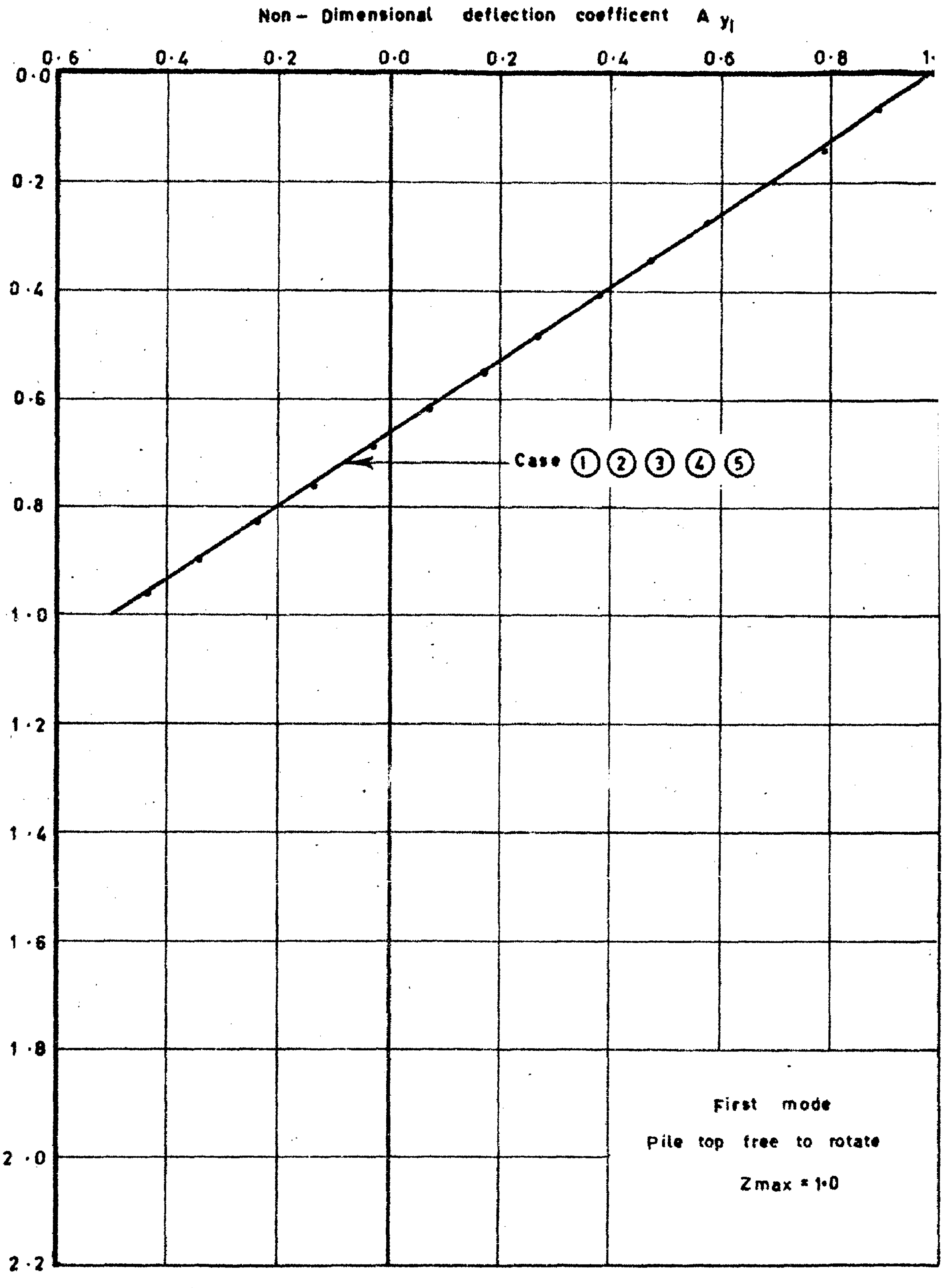


Fig. 3·19 Non – dimensional deflection versus depth fac
assuming soil modulus constant with depth

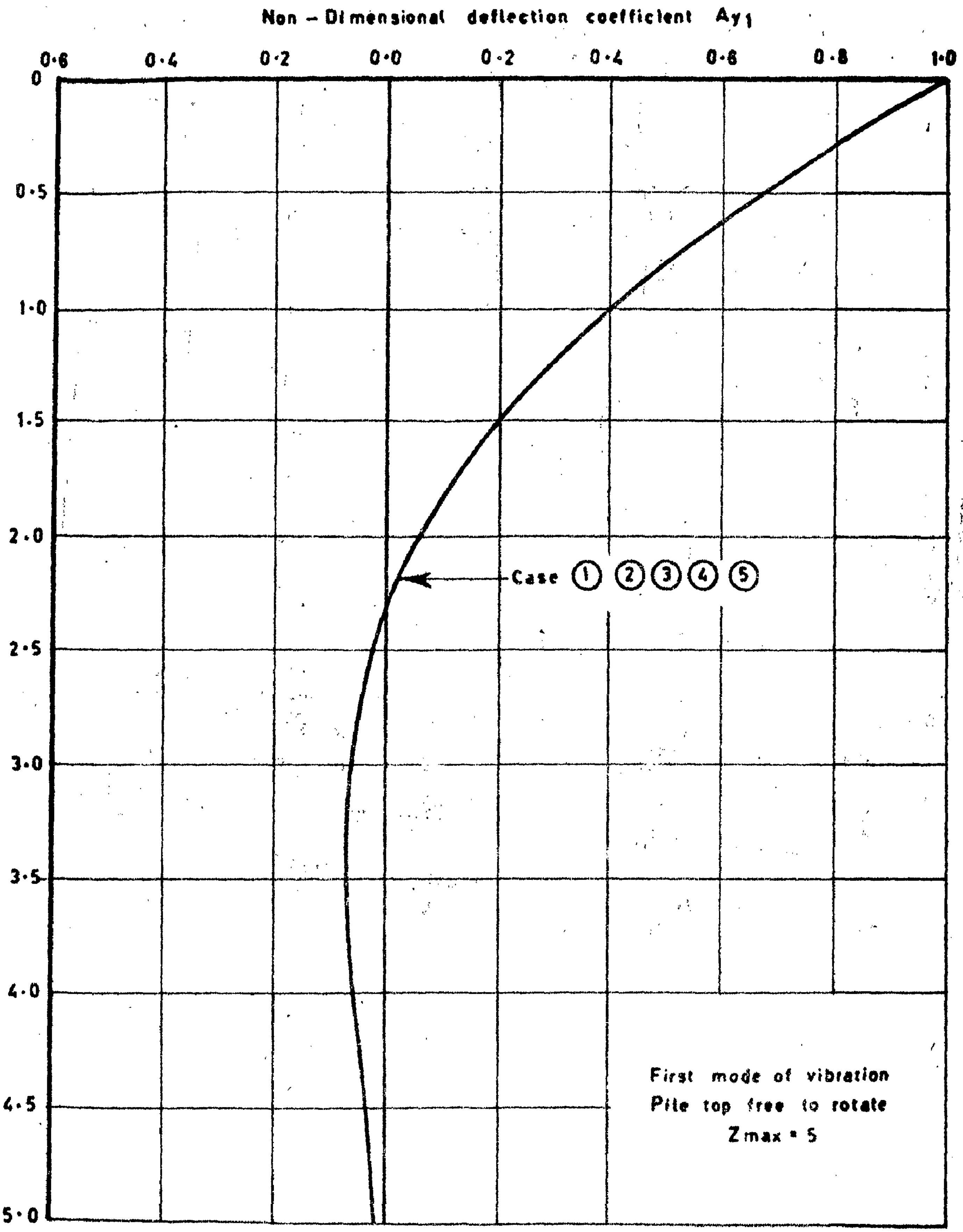
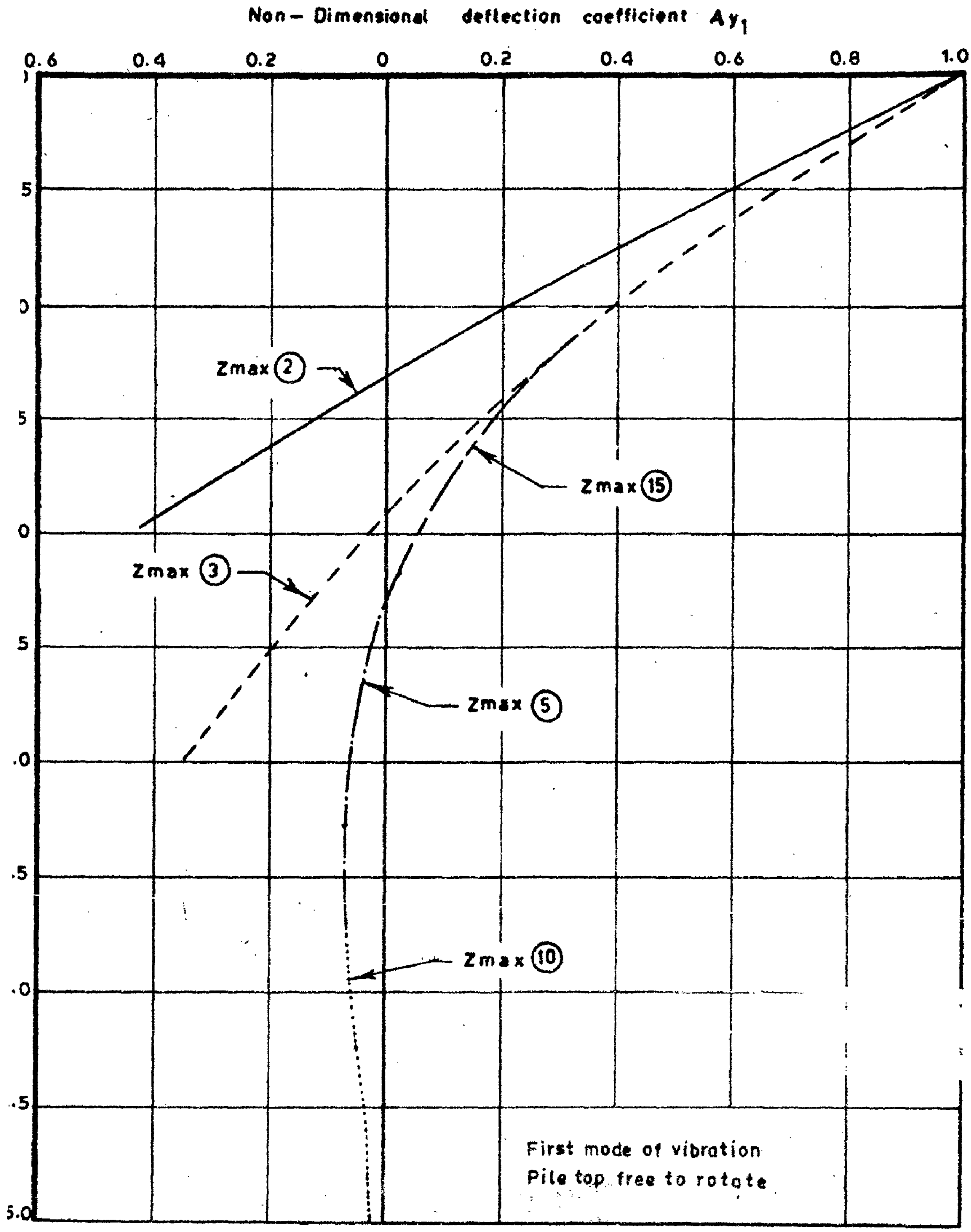


Fig. 3·20 Non - dimensional deflection coefficient assum
soil modulus constant with depth



g. 3·21 Non-dimensional deflection coefficient assuming soil modulus constant with depth

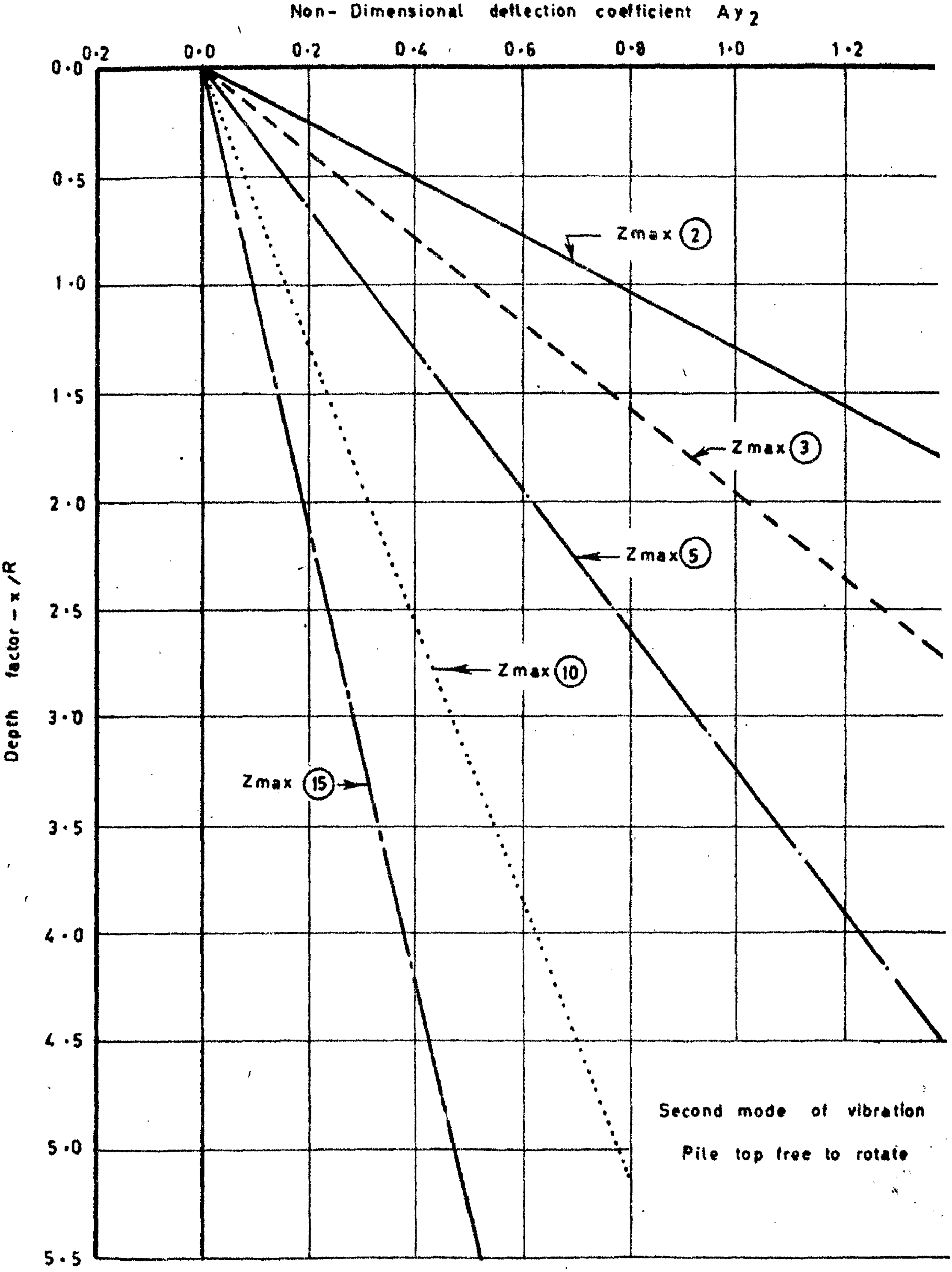


Fig. 3·22 Non - dimensional deflection coefficient assuming s
modulus constant with depth

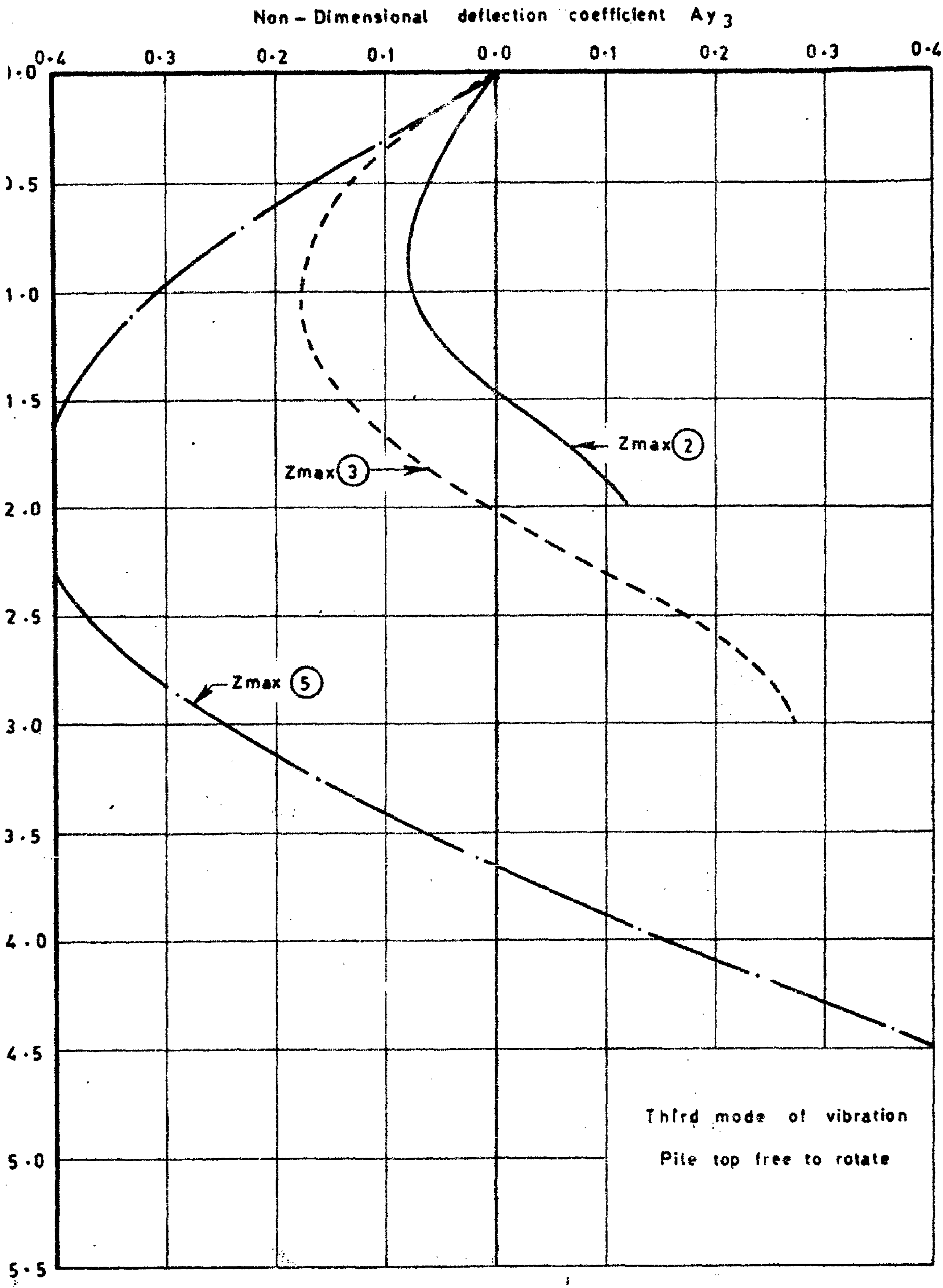


Fig. 3·23 Non- dimensional deflection coefficient assuming s[illegible]
modulus constant with depth

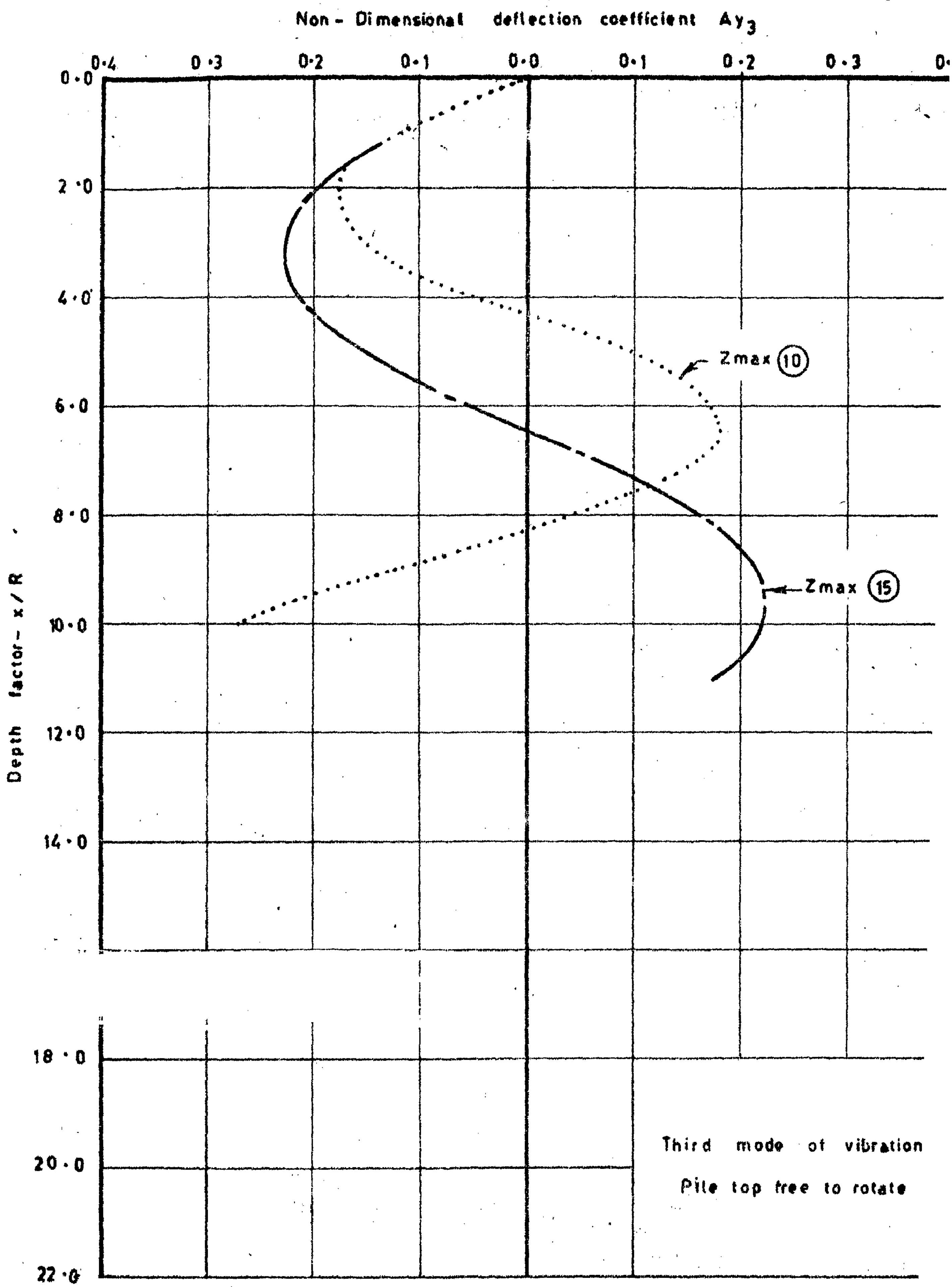


Fig. 3·23a Non - dimensional deflection coefficient assuming modulus constant with depth

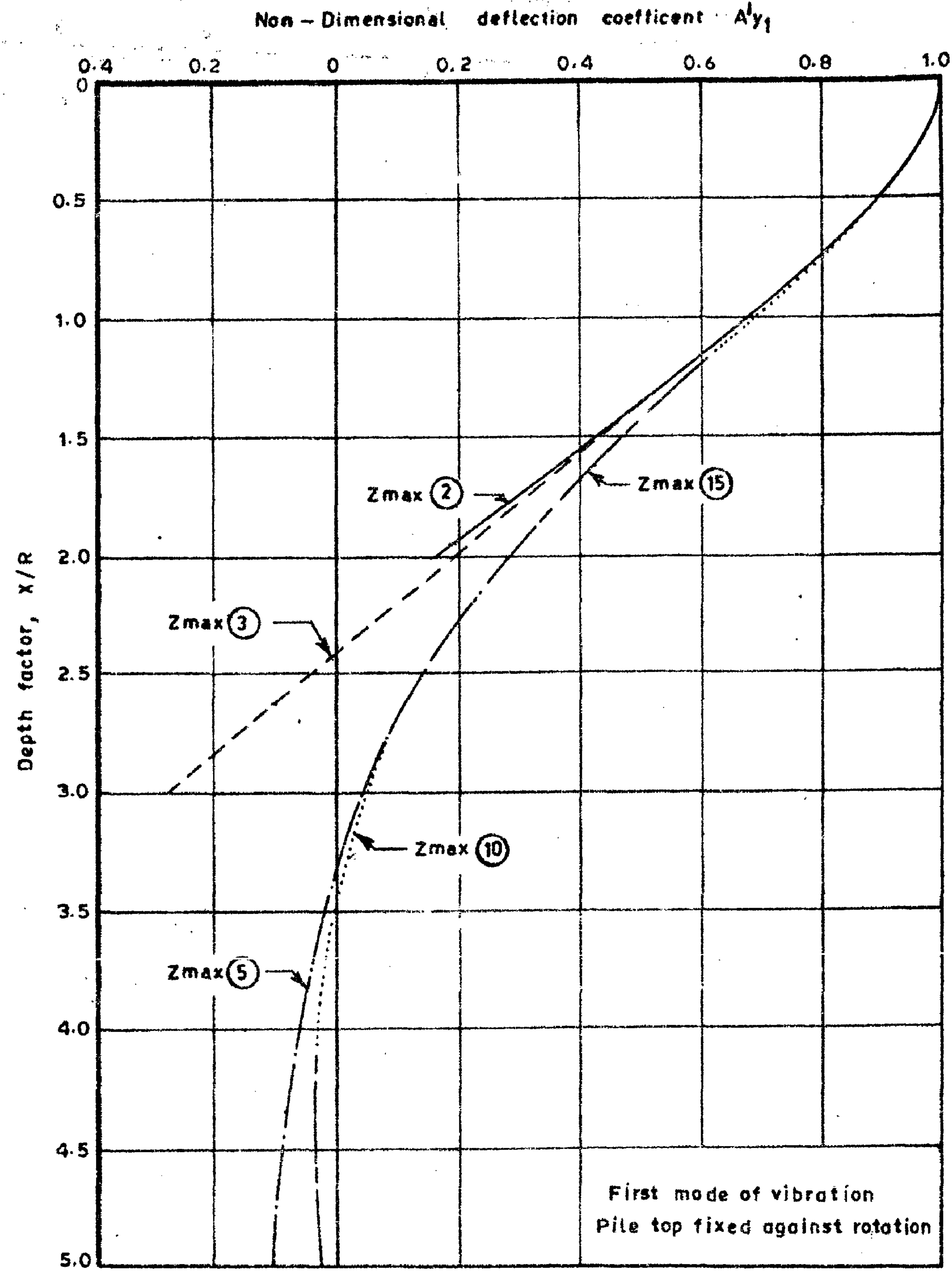


Fig. 3·24 Non-dimensional deflection coefficient assuming soil modulus constant with depth

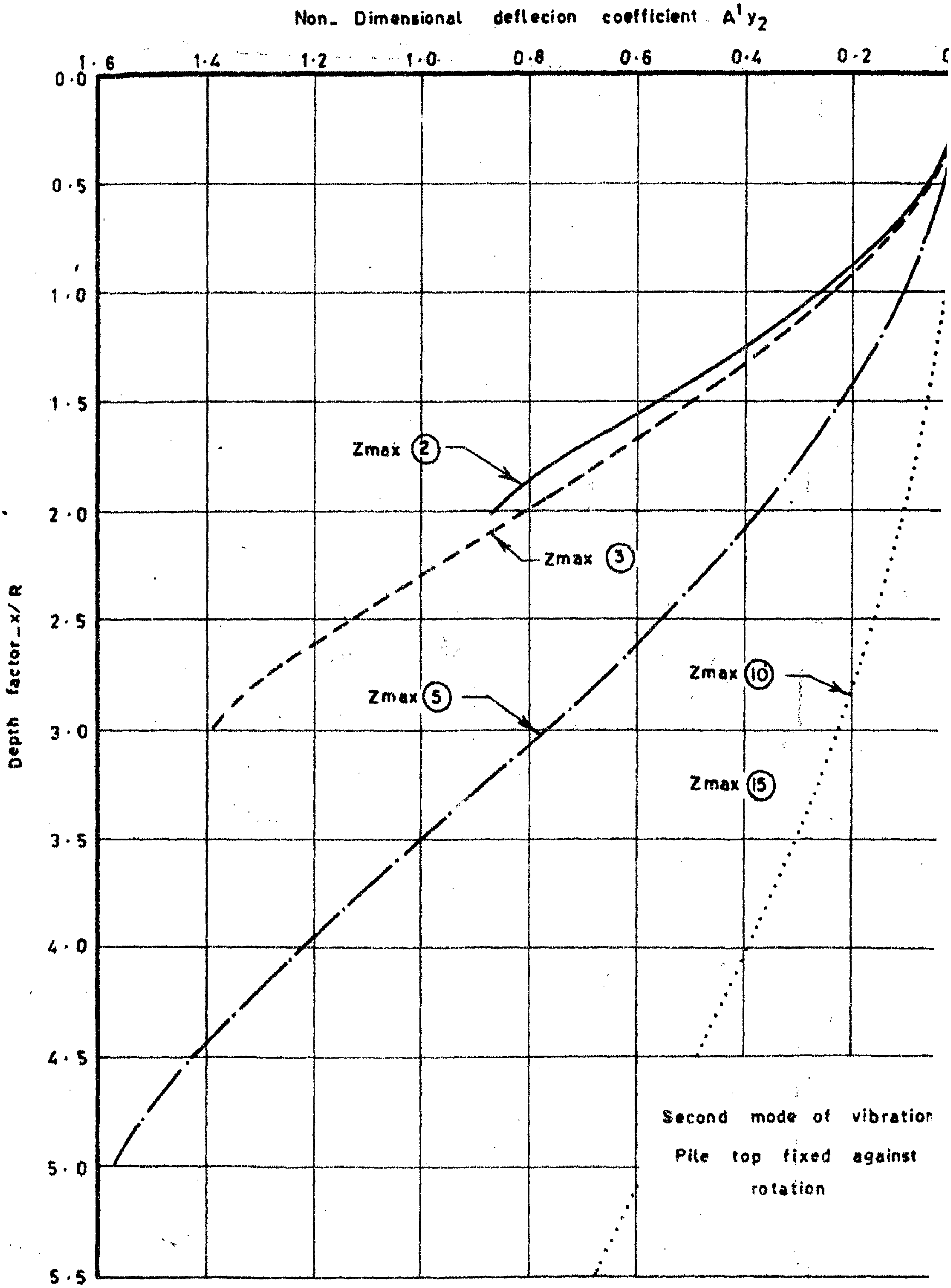


Fig.3·25 Non—dimensional deflection coefficient assuming s

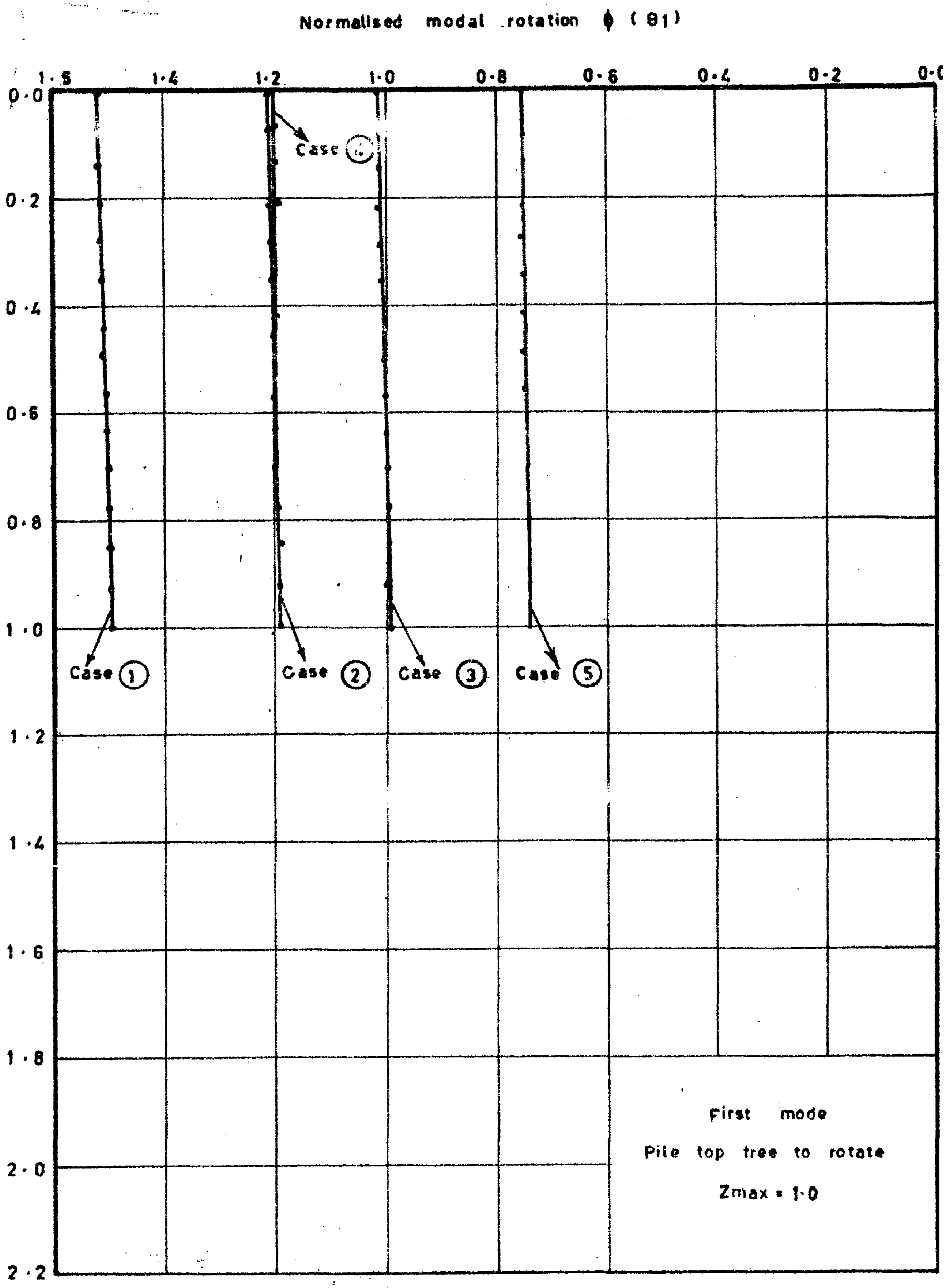


Fig. 3·26 Modal rotation versus depth factor assuming

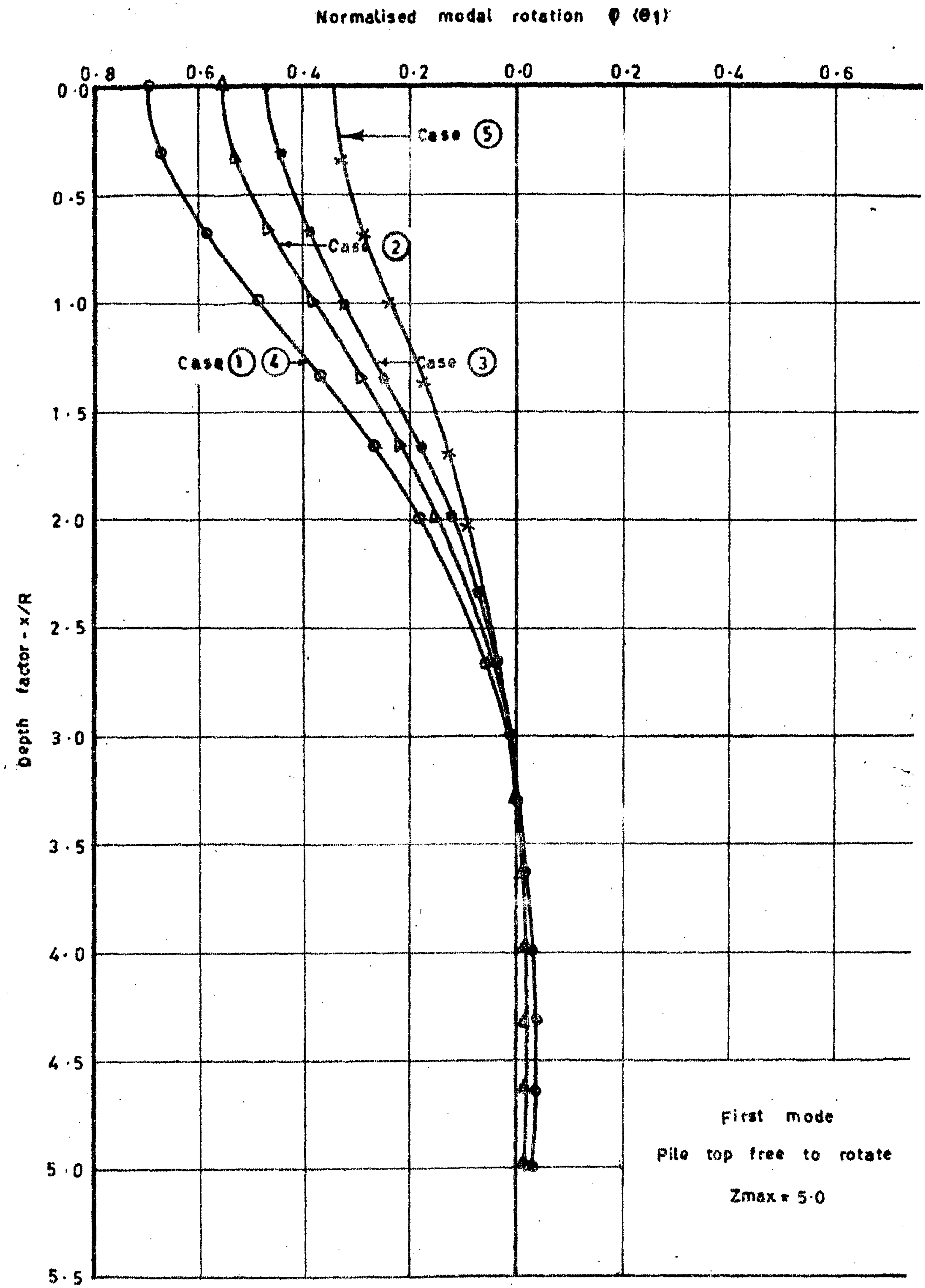


Fig. 3·27 Modal rotation versus depth factor assumir
soil modulus constant with depth

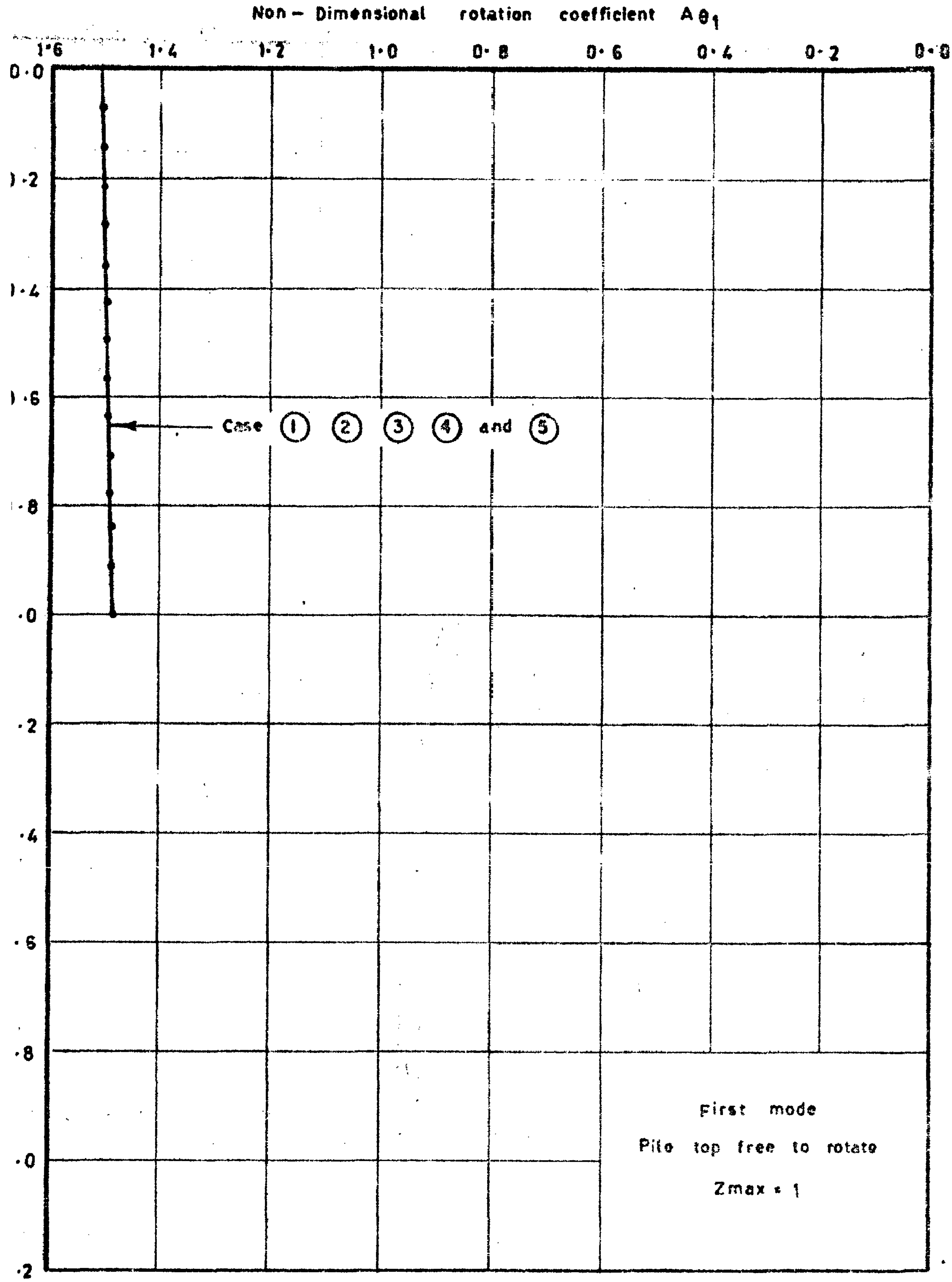


Fig. 3·28 Non. dimensional rotation coefficient versus dept

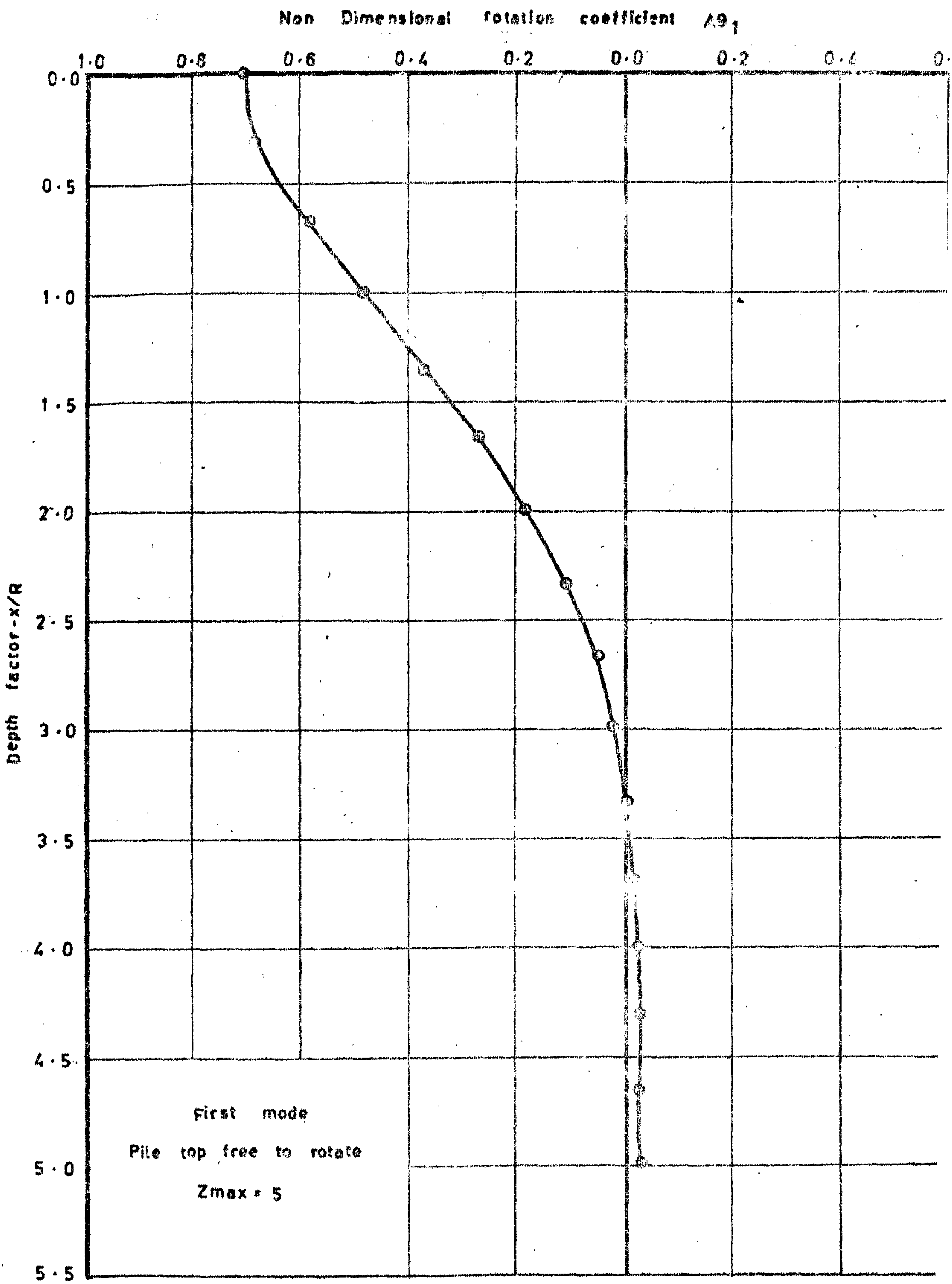


Fig. 3 29 Non – dimensional rotation coefficient versus
factor assuming soil modulus constant with depth

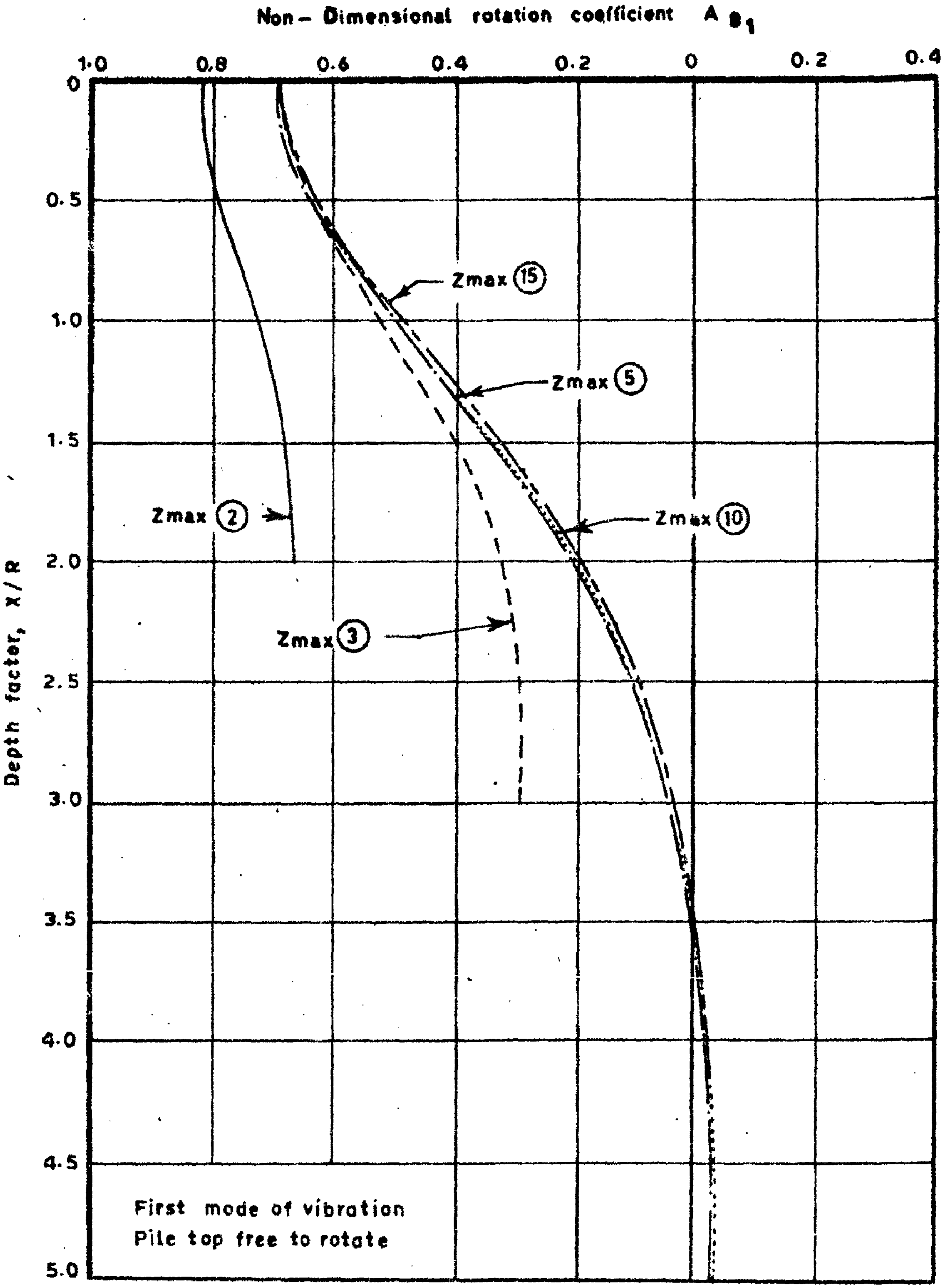


Fig. 3·30 Non-dimensional rotation coefficient assuming soil modulus constant with depth

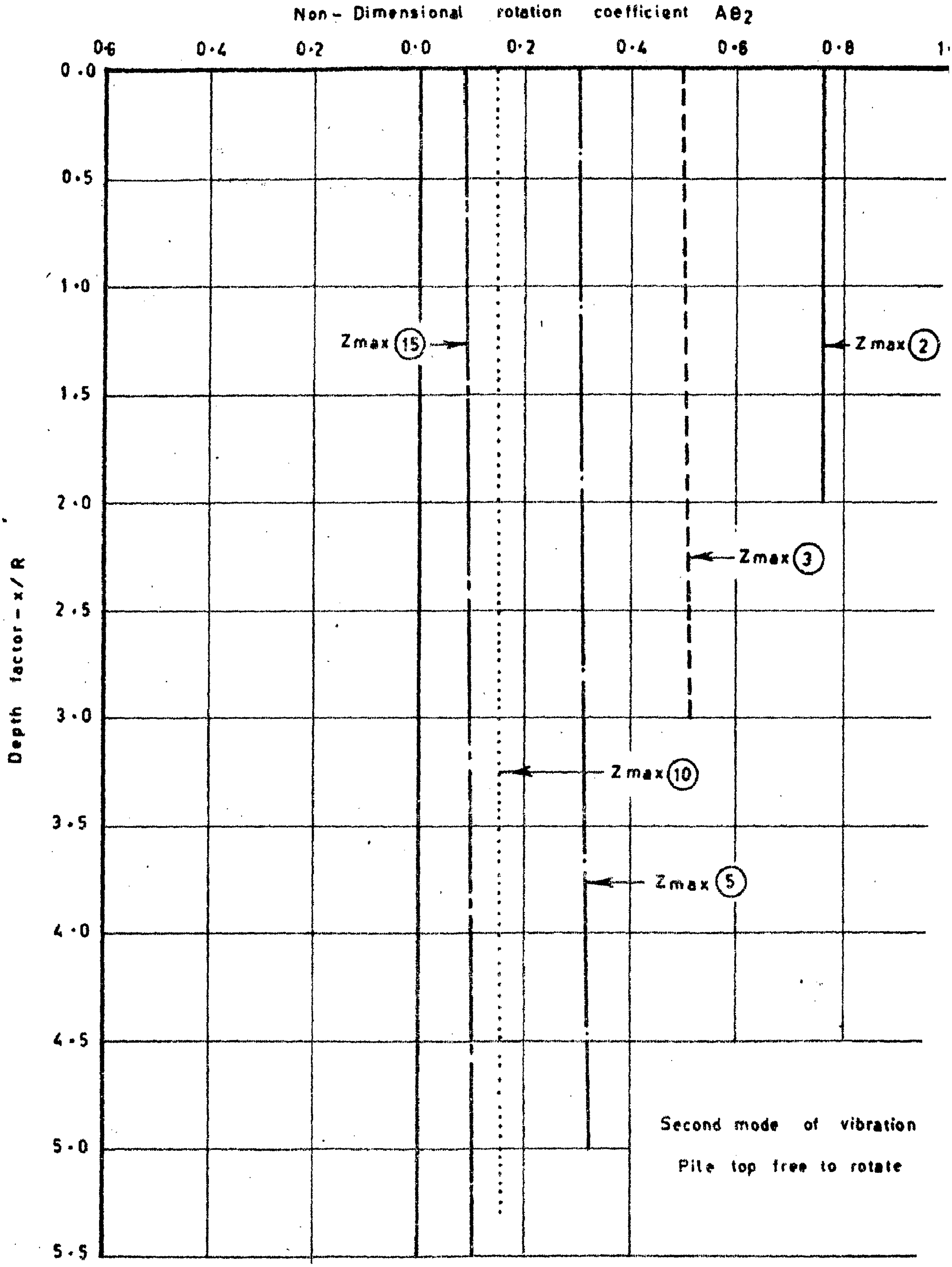


Fig. 3·31 Non-dimensional rotation coefficient assuming modulus constant with depth

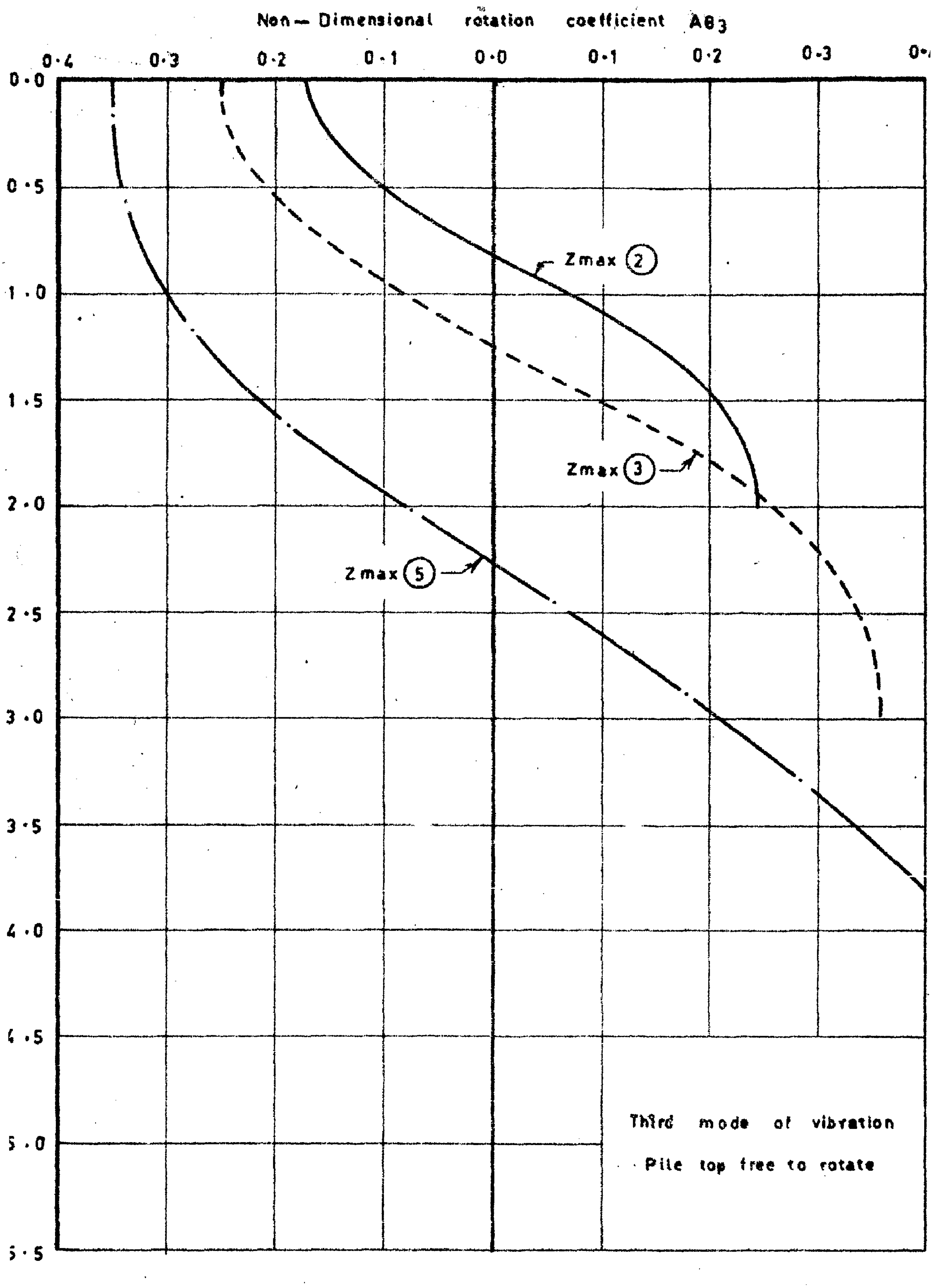


Fig. 3·32 Non-dimensional rotation coefficient assuming soil modulus constant with depth

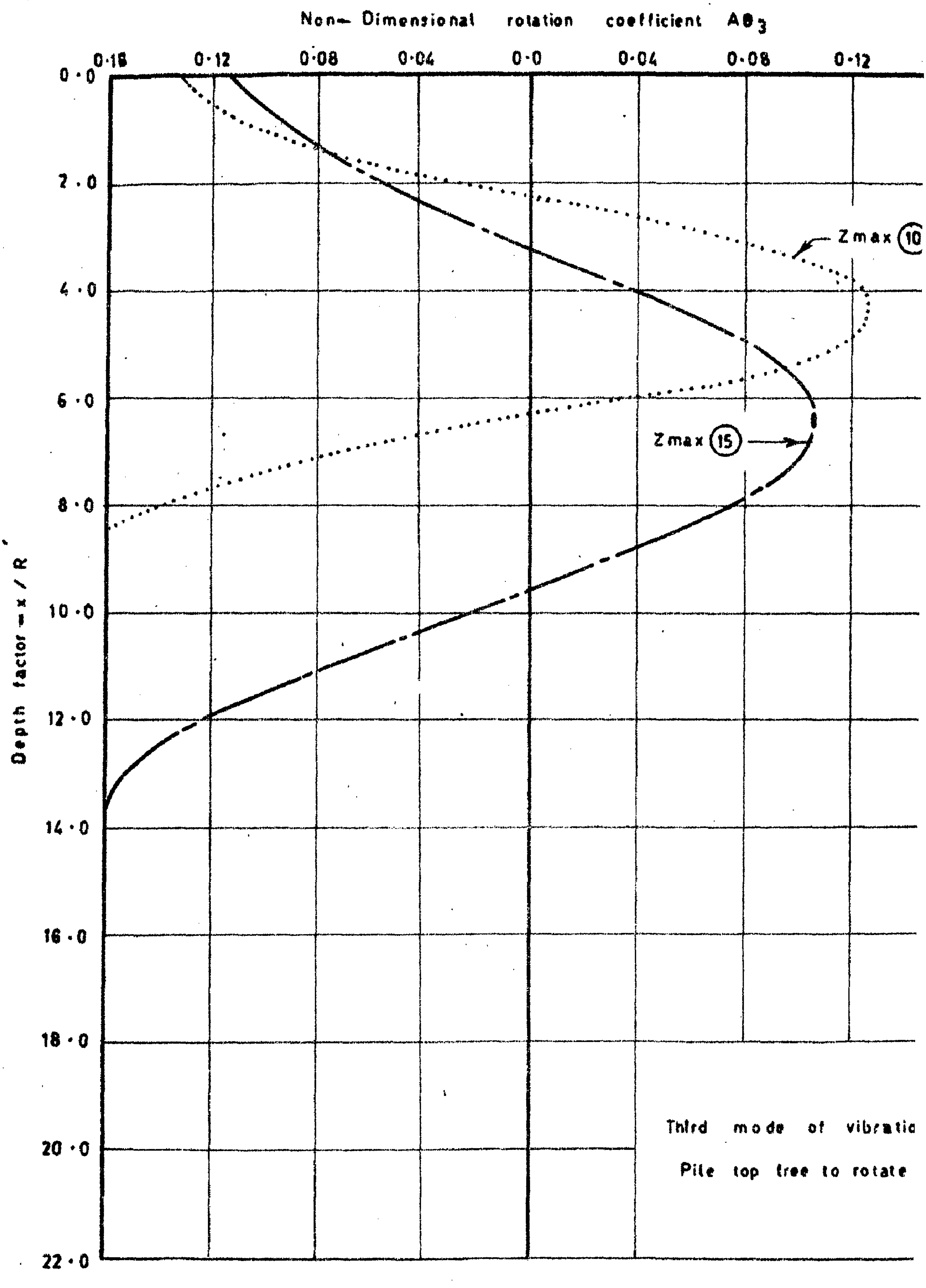


Fig. 3·32a Non - dimensional rotation coefficient assuming modulus constant with depth

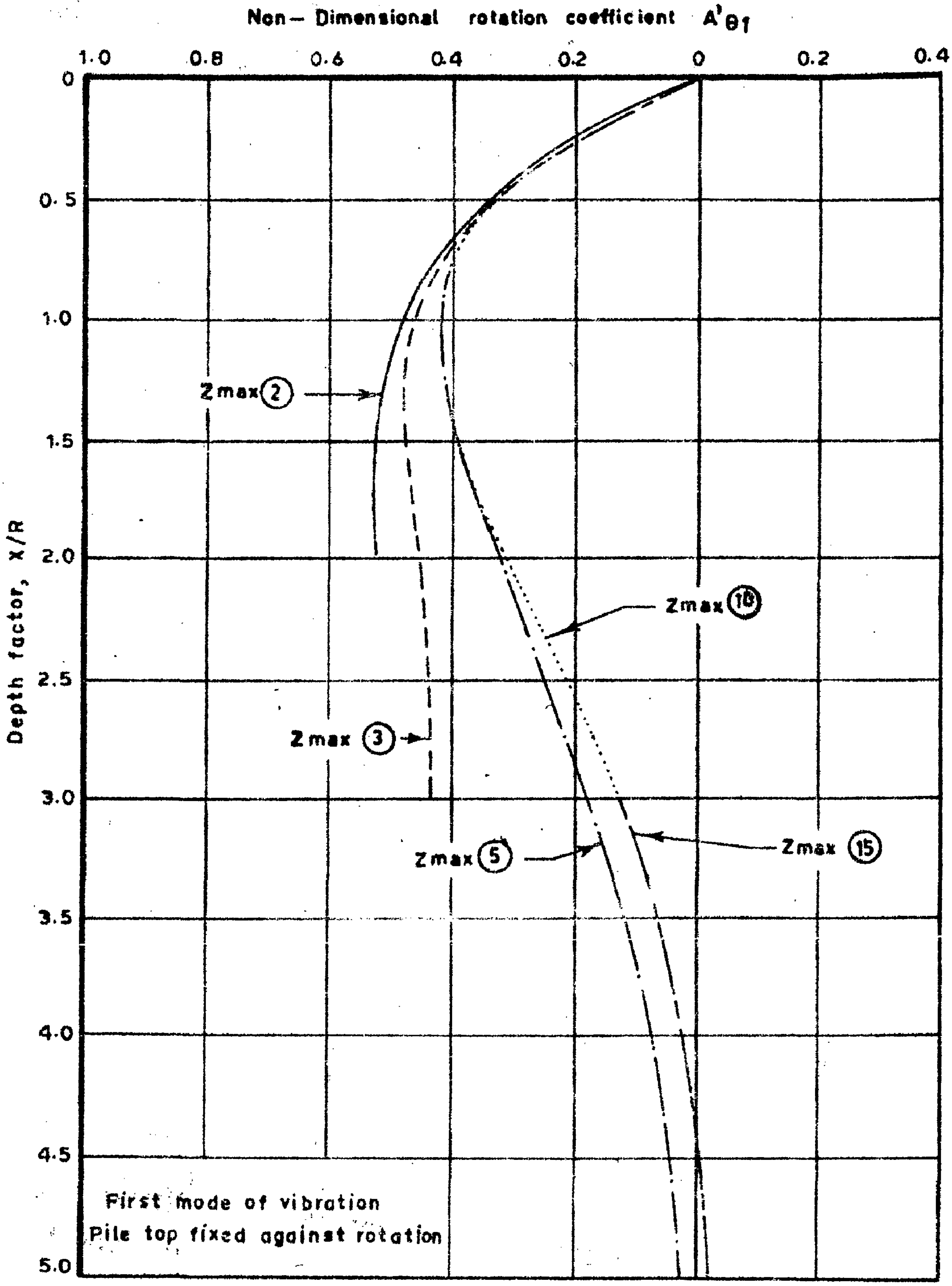


Fig. 3·33 Non-dimensional rotation coefficient assuming soil modulus constant with depth

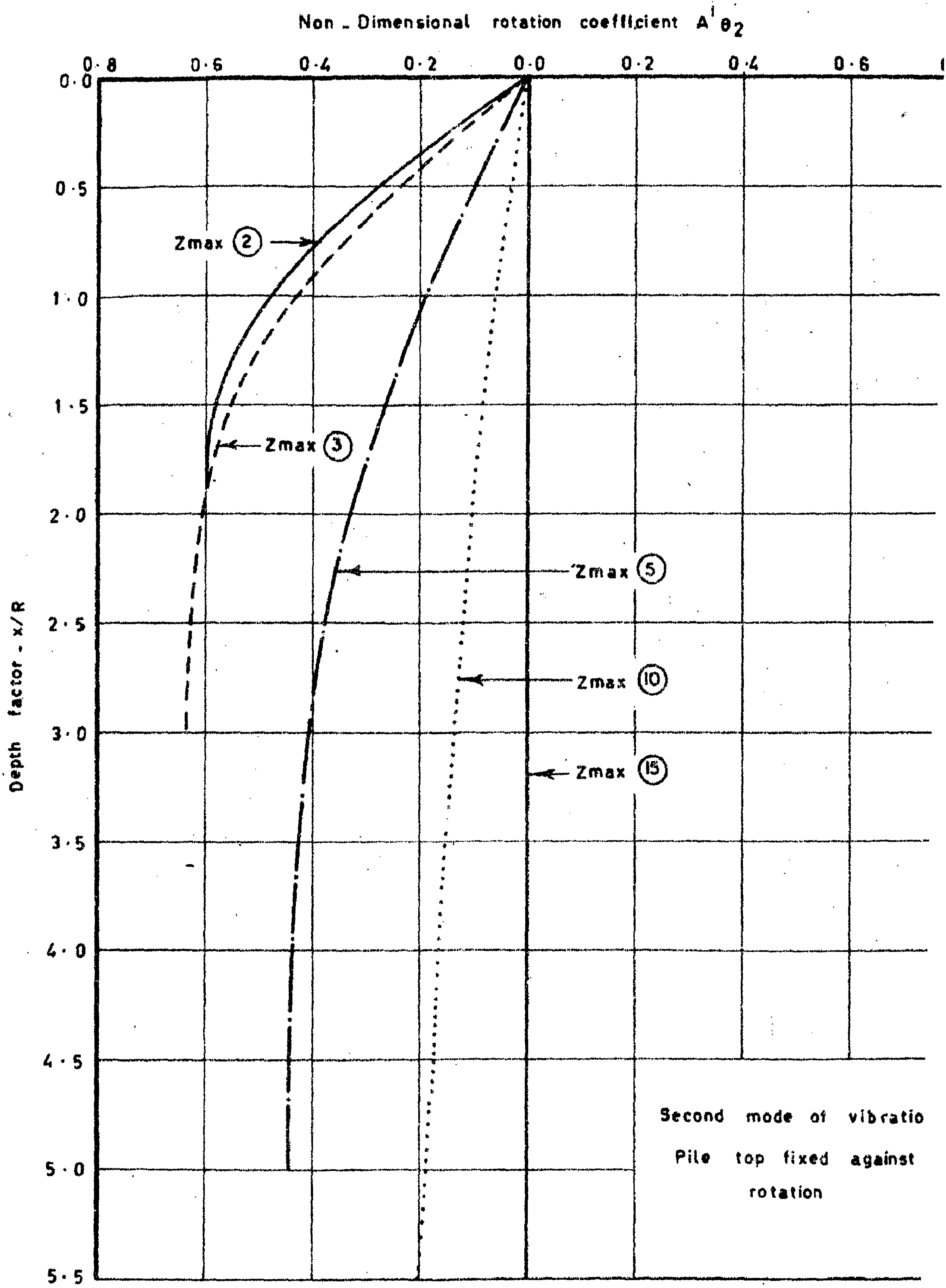


Fig. 3·34 Non-dimensional rotation coefficent assuming modulus constant with depth.

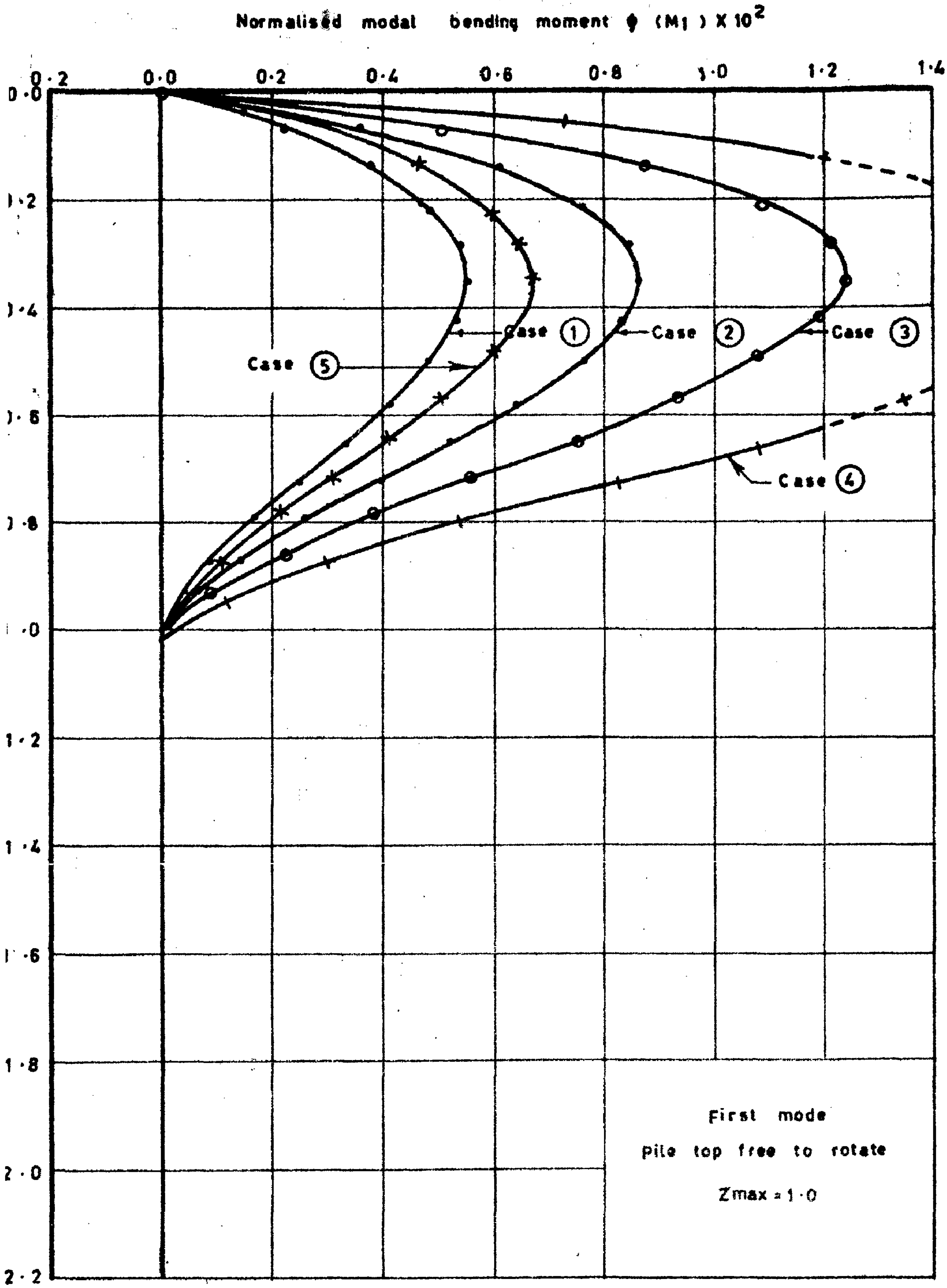


Fig. 3·35 Modal bending moment versus depth factor

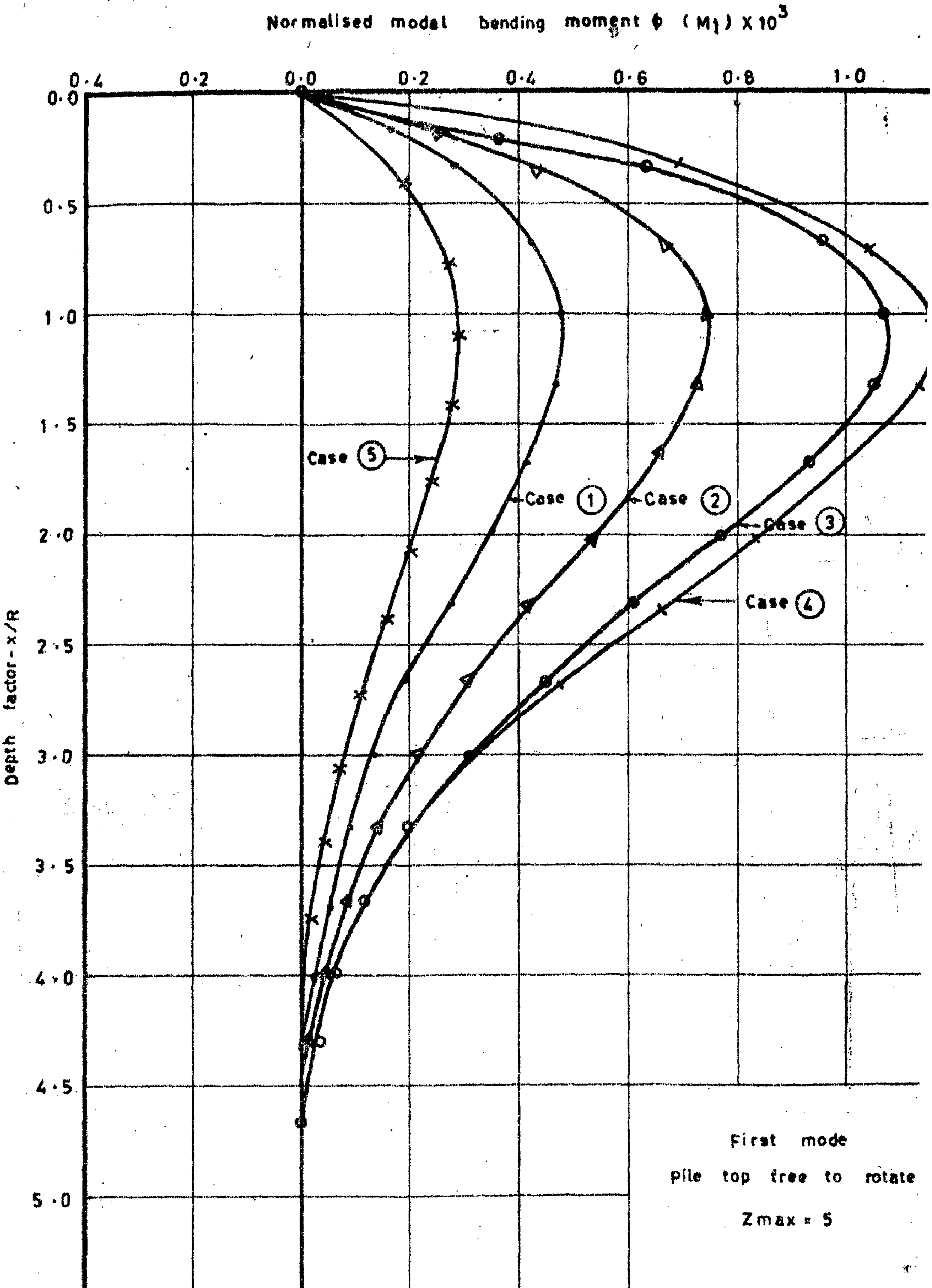


Fig. 3·36 Modal bending moment versus depth facto

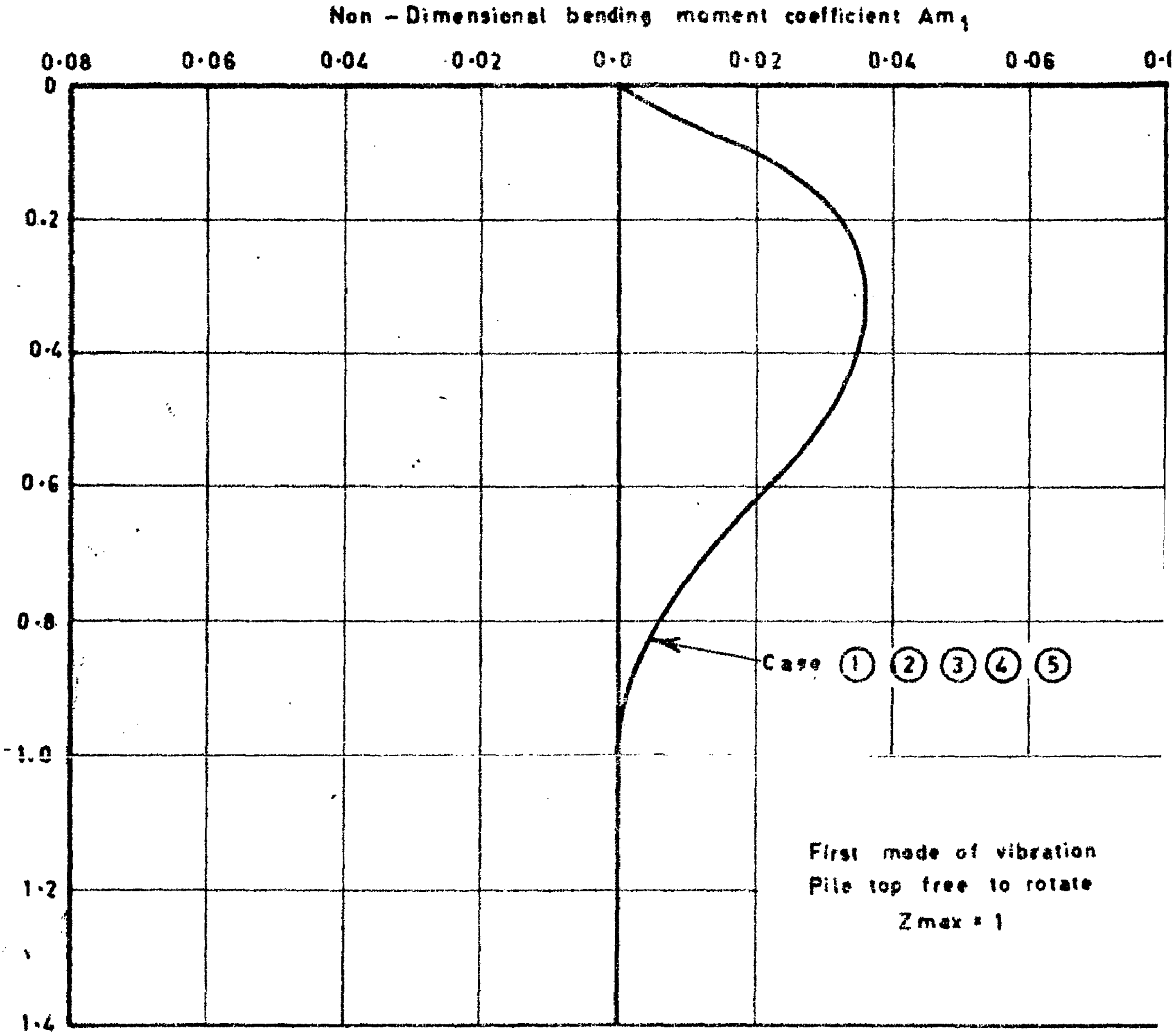


Fig. 3·37 Non – dimensional bending moment coefficient assur
soil modulus constant with depth

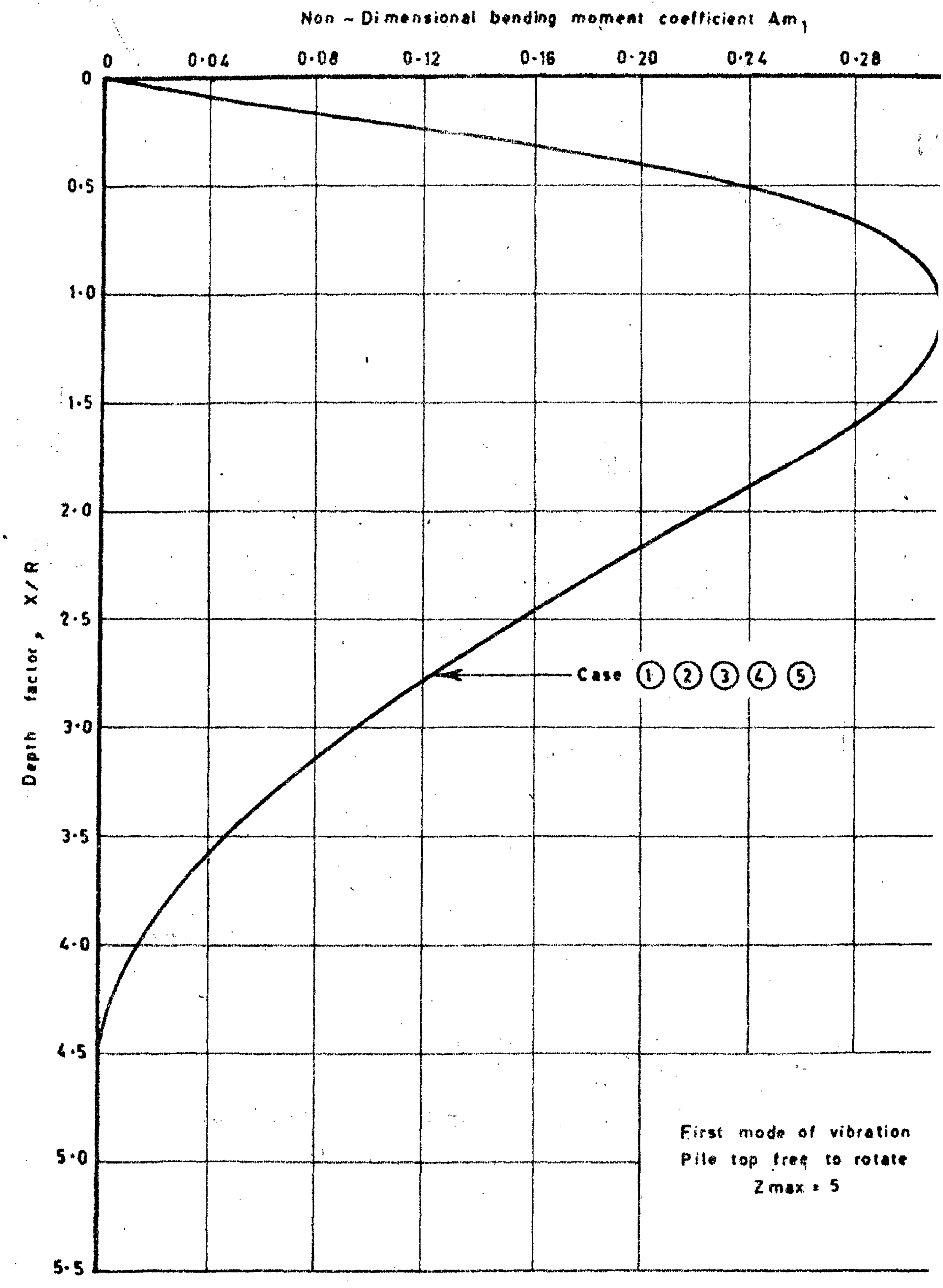
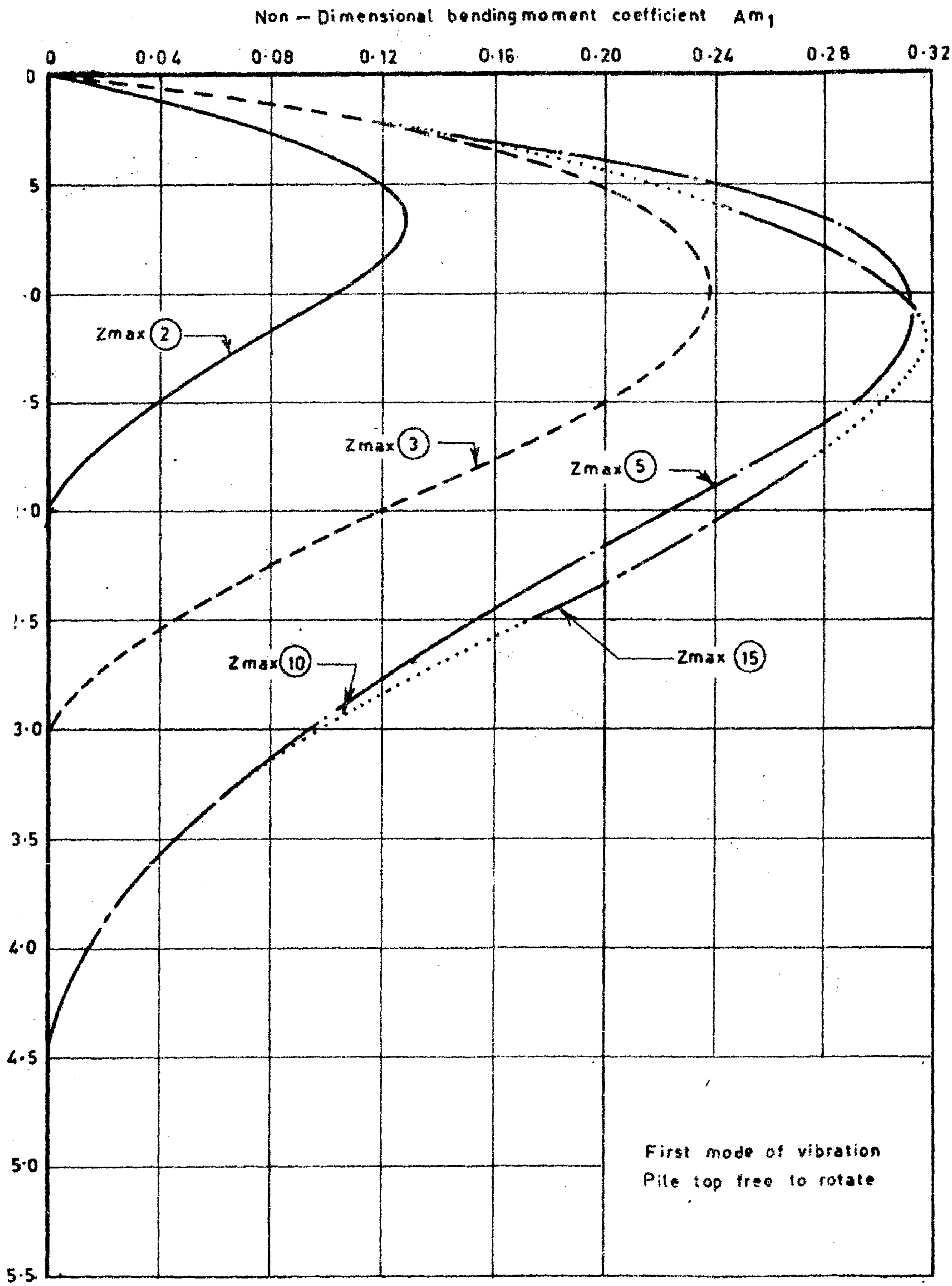


Fig. 3:38 Non - dimensional bending moment coefficient assu
soil modulus constant with depth



j. 3·39 Non – dimensional bending moment coefficient assumir
soil modulus constant with depth

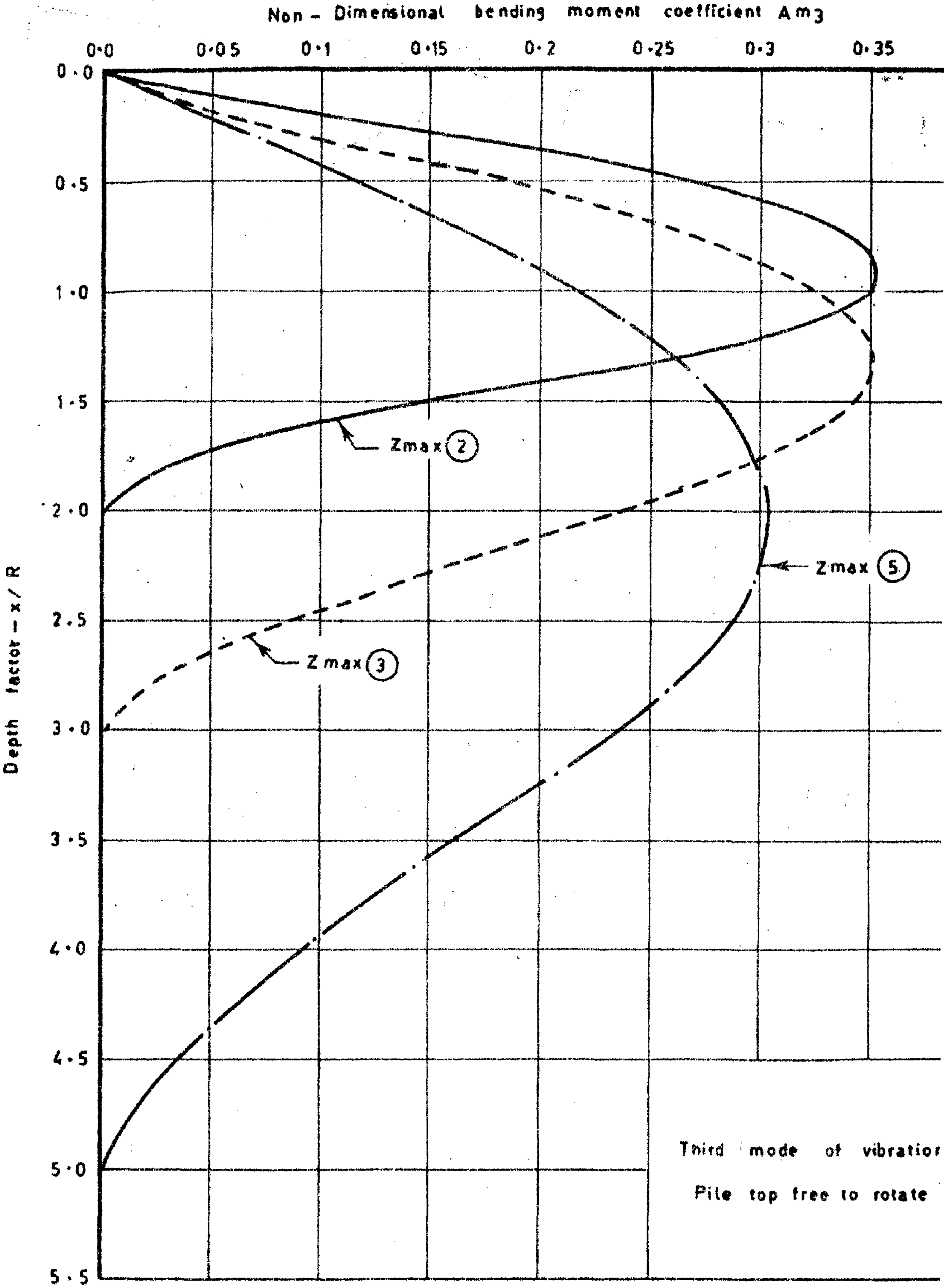
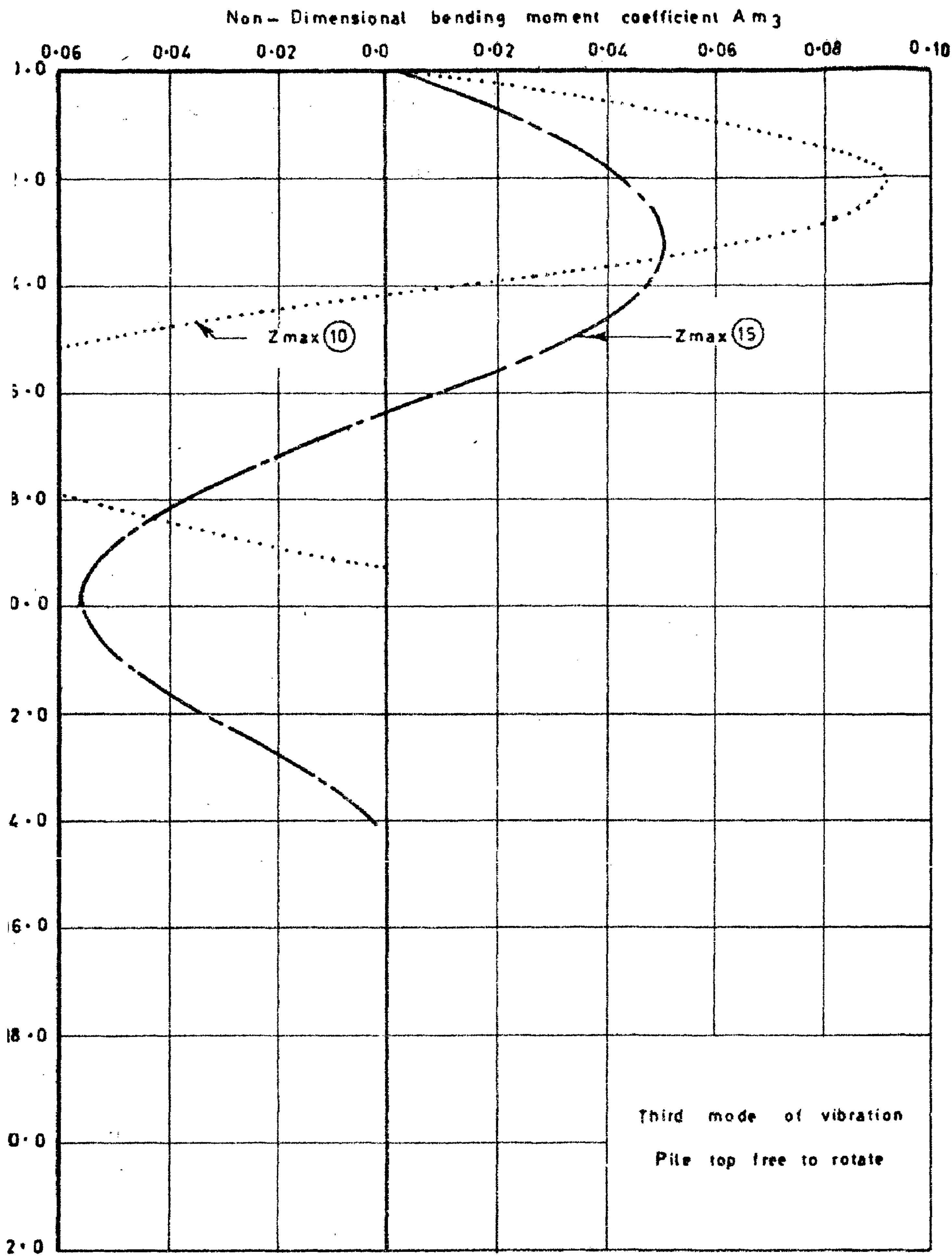


Fig. 3·40 Non – dimensional bending moment coefficient assu
soil modulus constant with depth



ig. 3·40a Non - dimensional bending moment coefficient assumi
soil modulus constant with depth

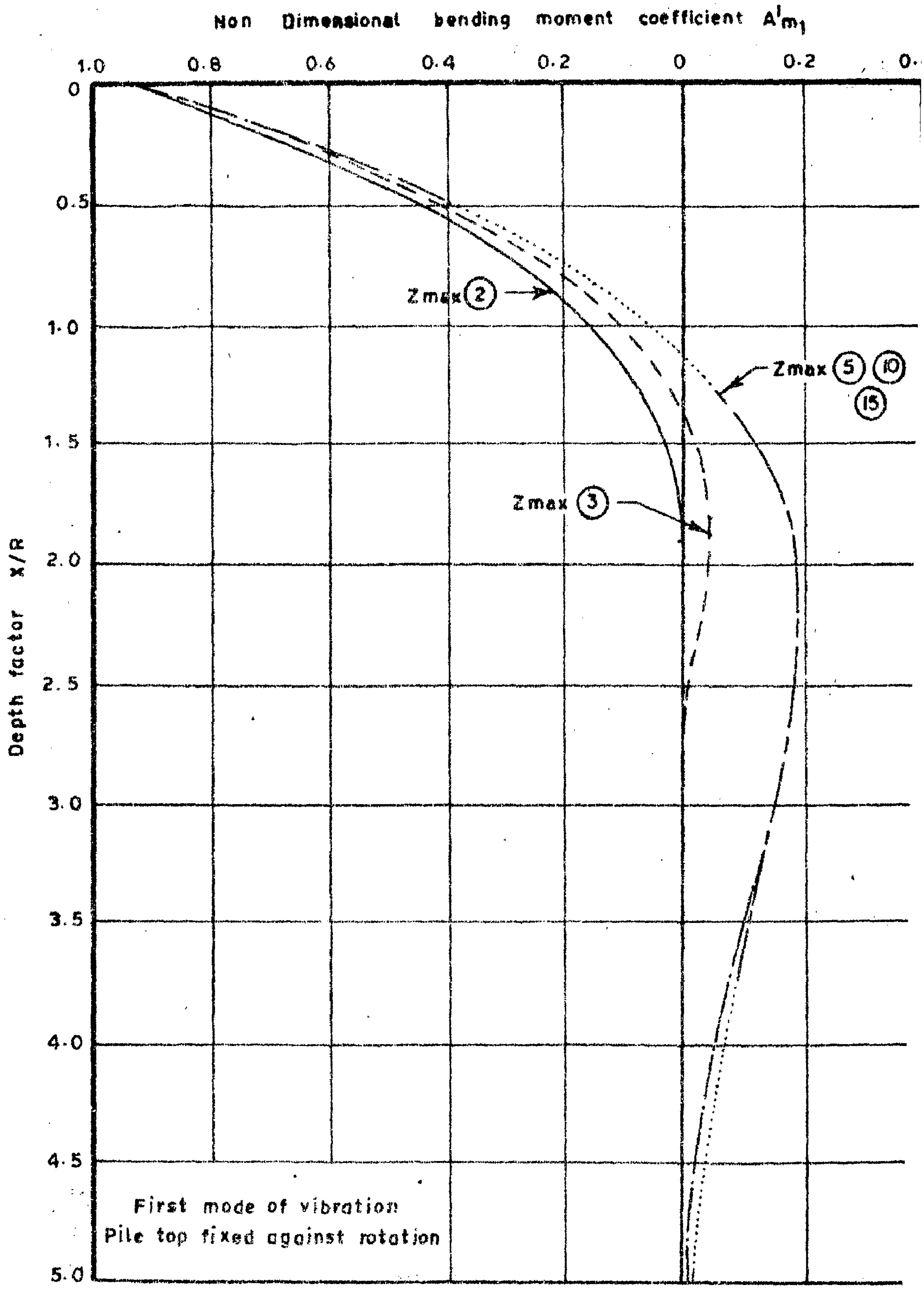


Fig. 3·41 Non-dimensional bending moment coefficient assuming soil modulus constant with depth

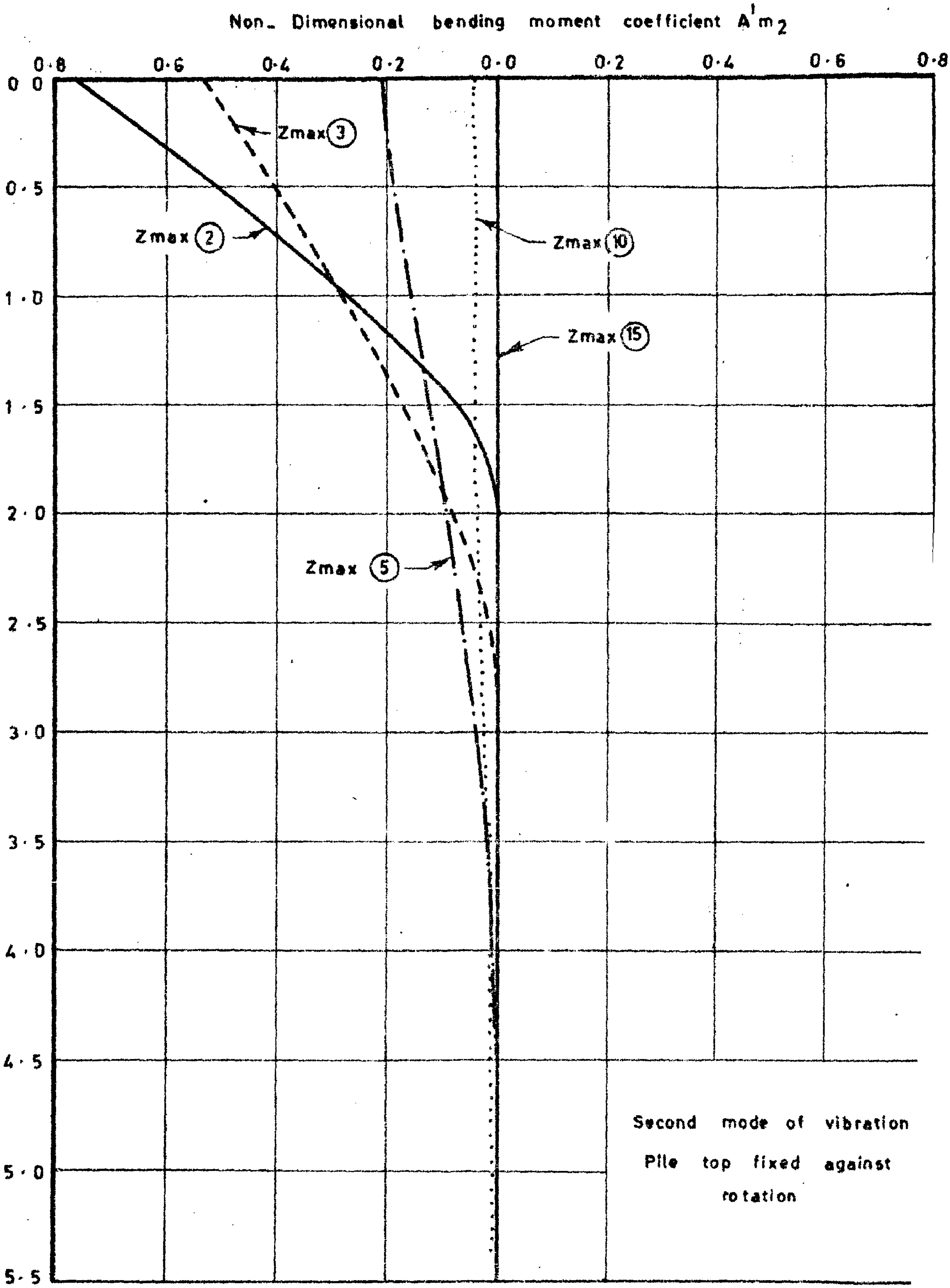


Fig. 3·42 Non-dimensional bending moment coefficient ass
soil modulus constant with depth.

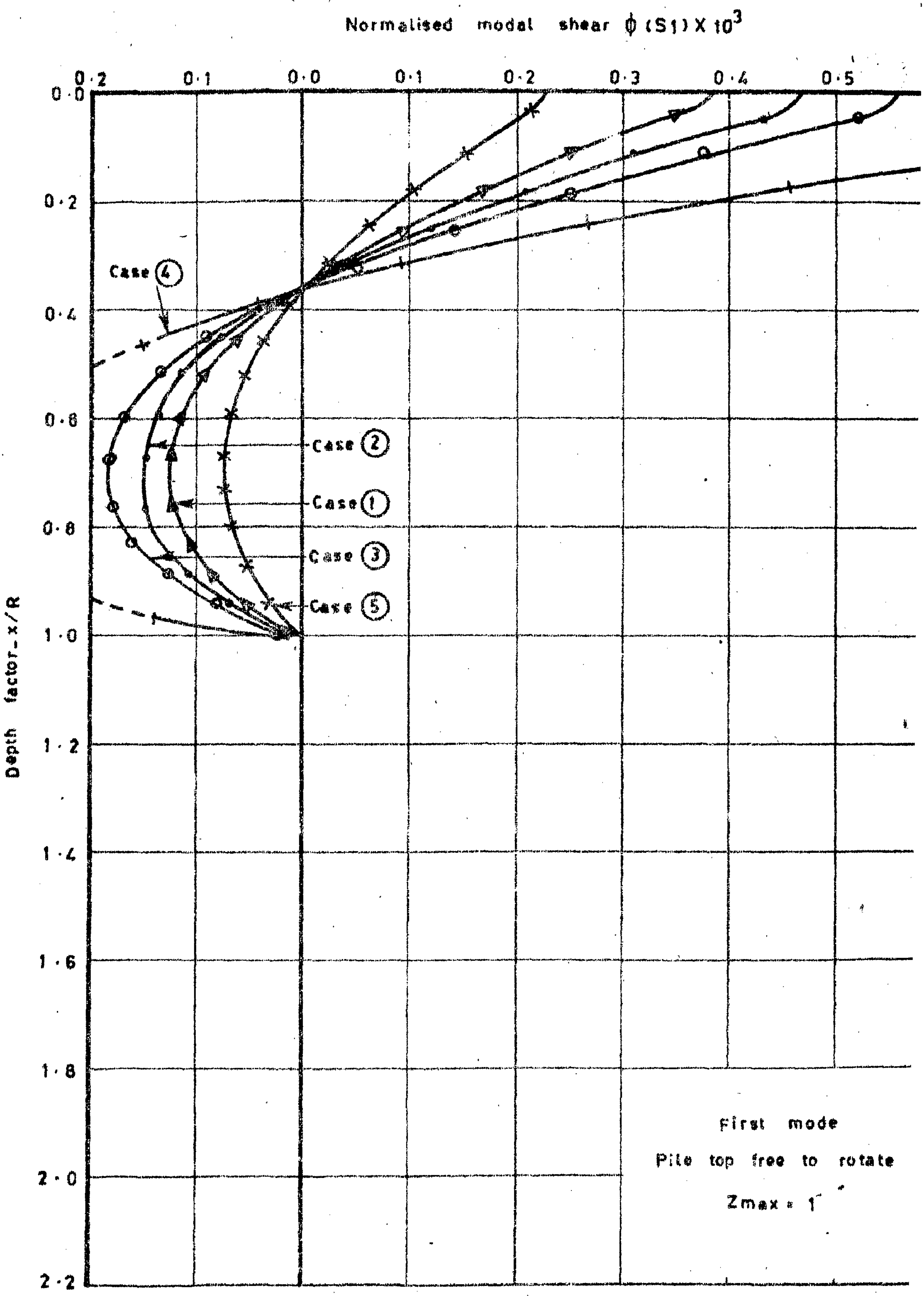


Fig. 3·43 Modal shear versus depth factor assumi

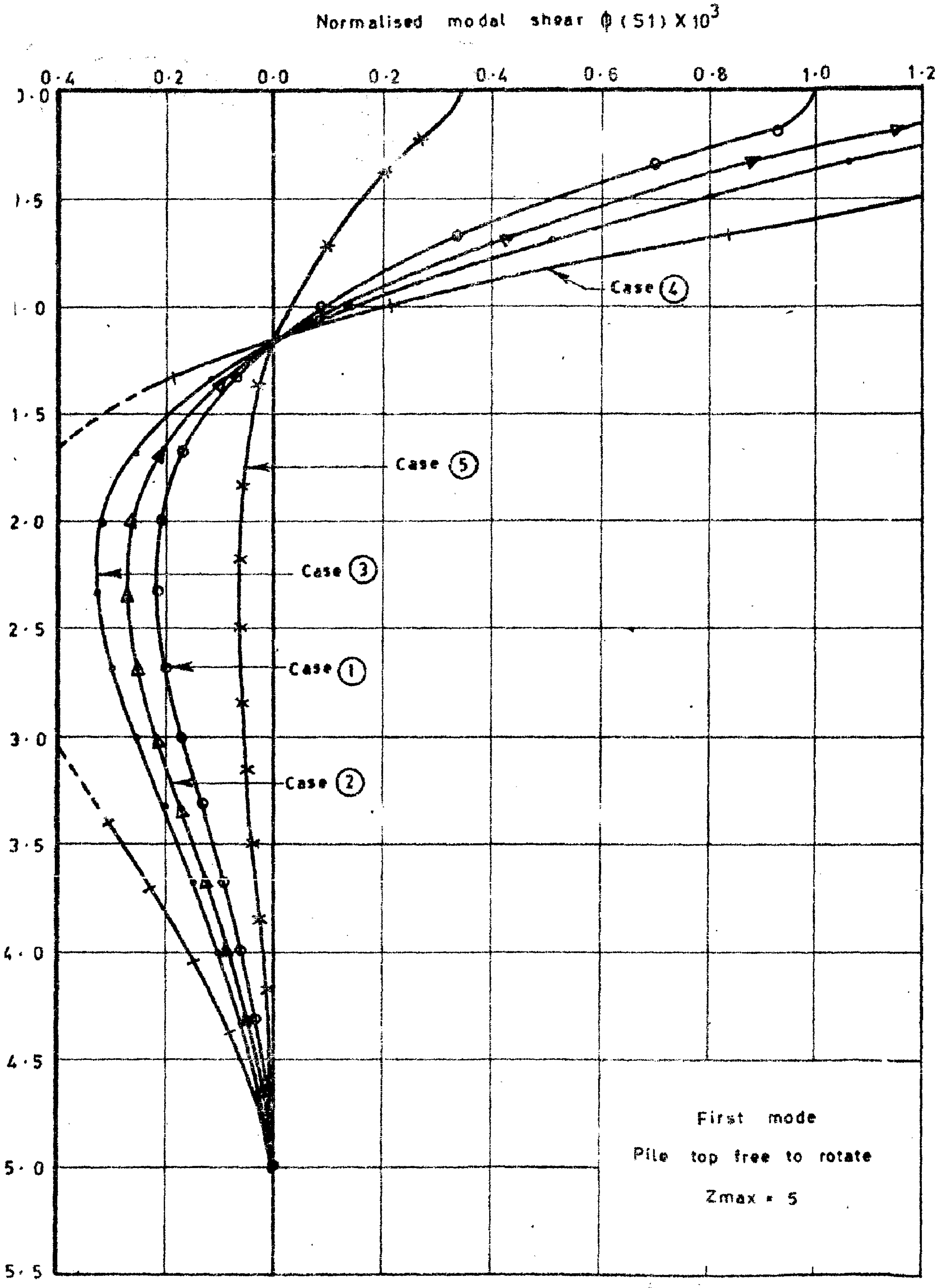


Fig. 3·44 Modal shear versus depth factor assuming soil modulus constant with depth

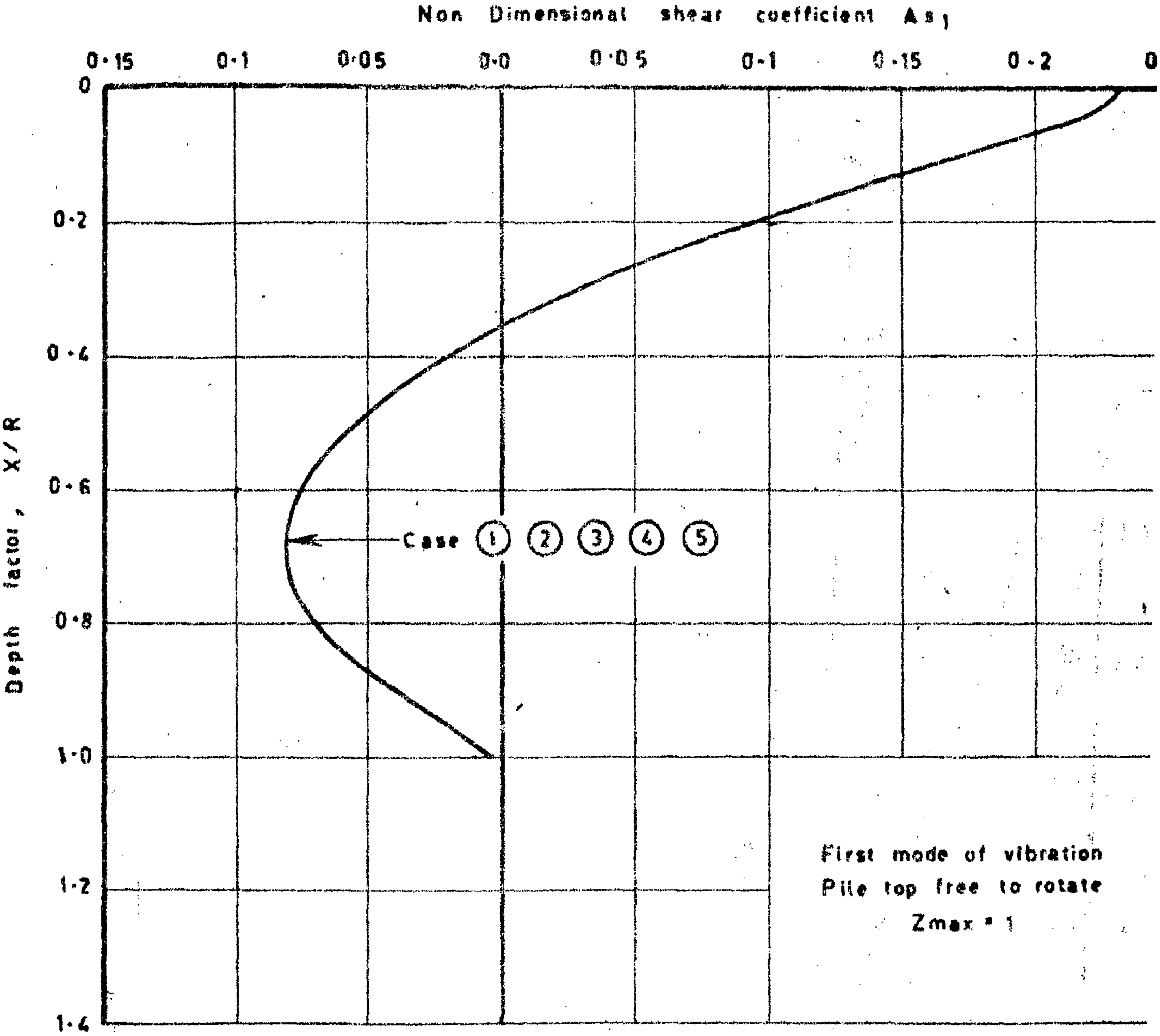


Fig. 3·45 Non - dimensional shear coefficient assumir soil modulus constant with depth

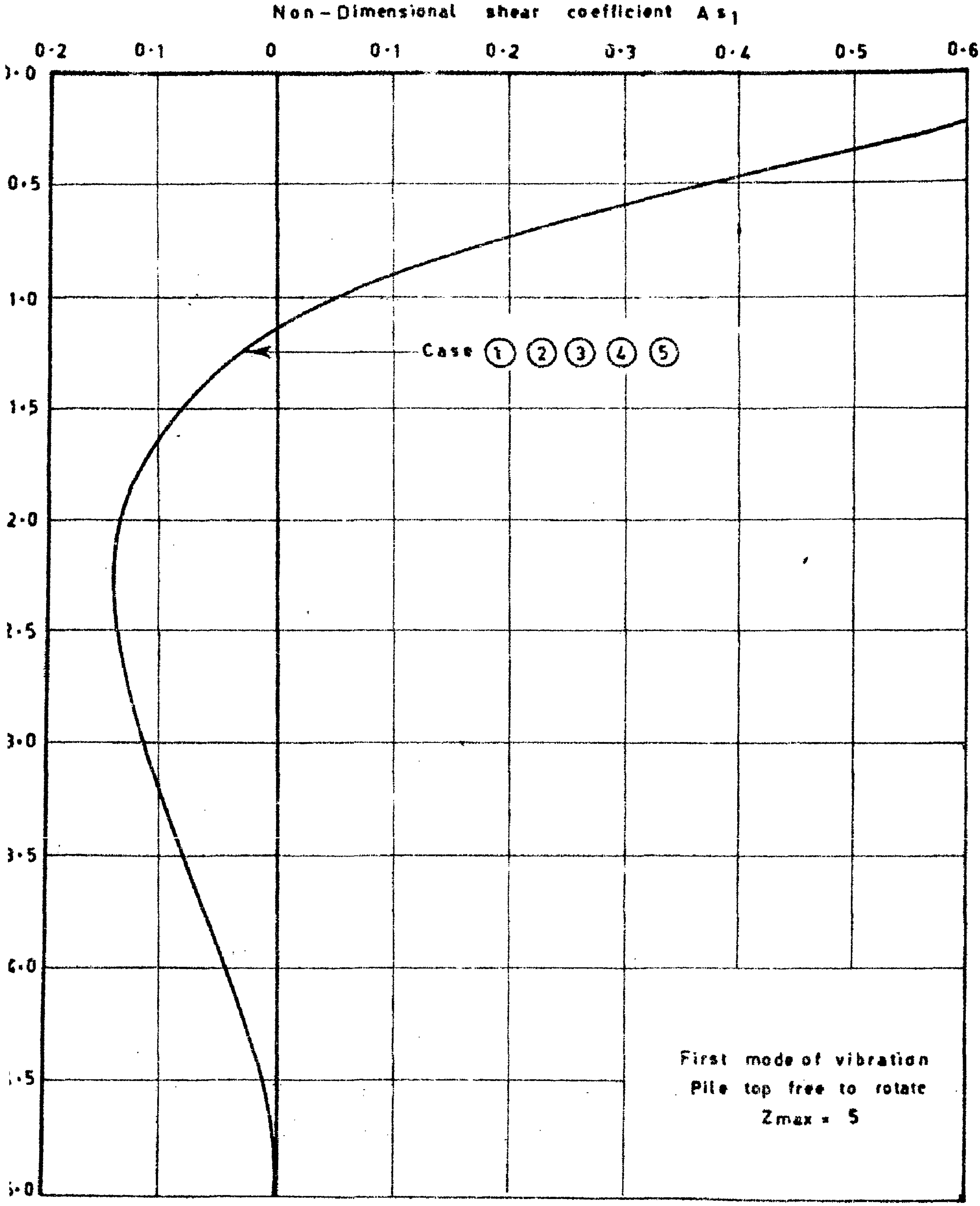
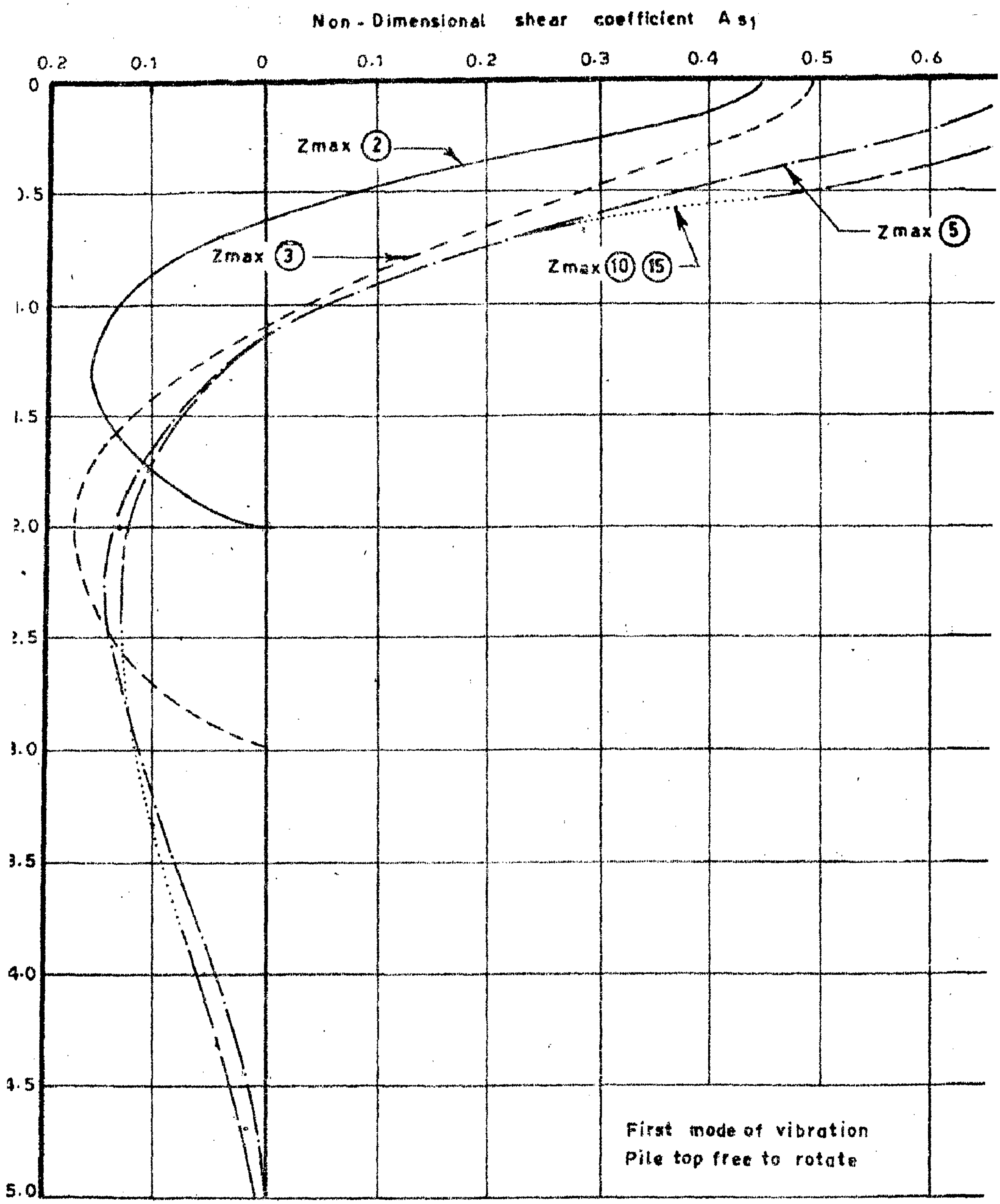


Fig. 3·46 Non - dimensional shear coefficient assuming soil modulus constant with depth



]· 3·47 Non-dimensional shear coefficient assuming soil mod constant with depth .

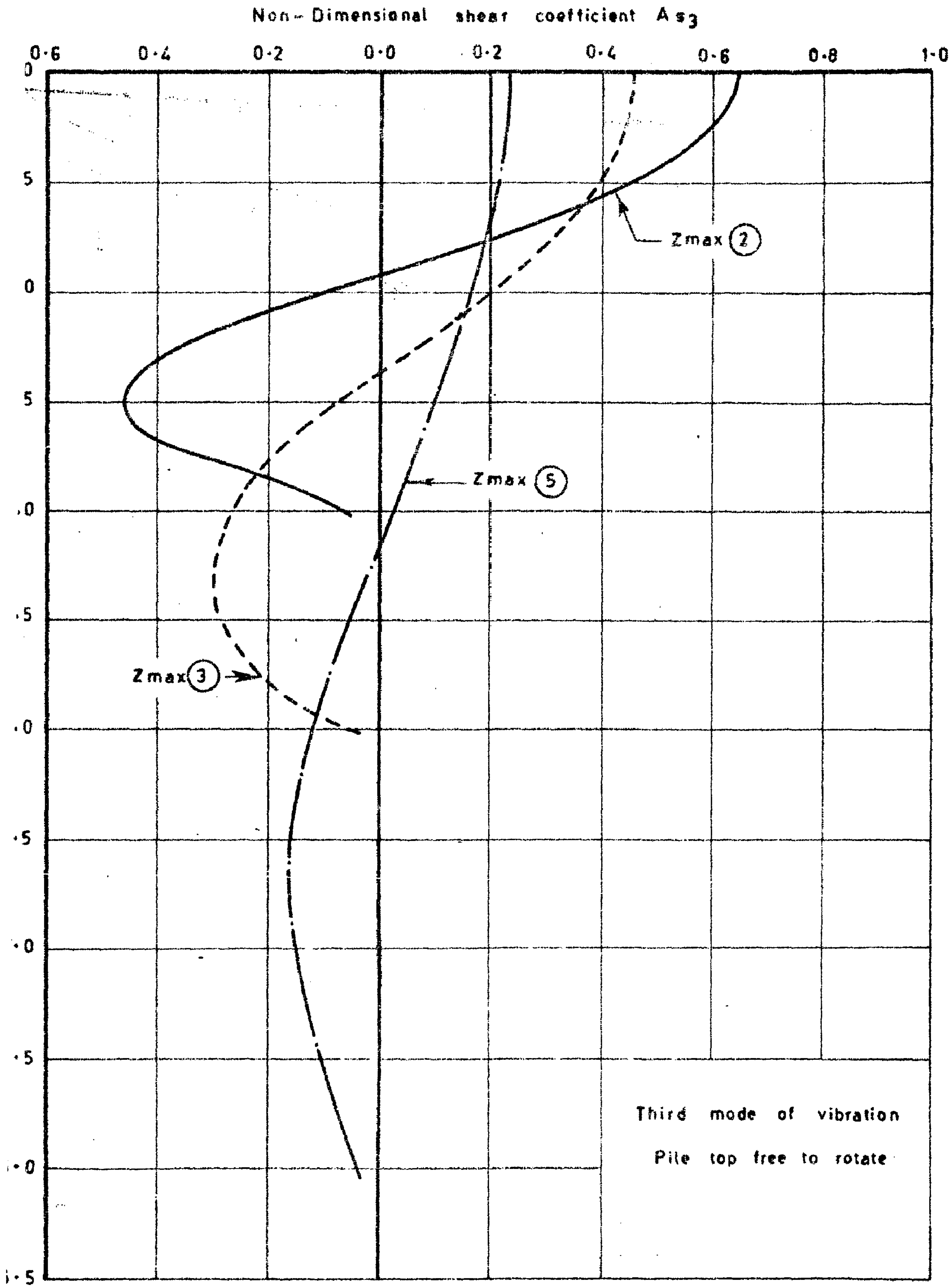


Fig. 3·48 Non-dimensional shear coefficient assuming soi modulus constant with depth

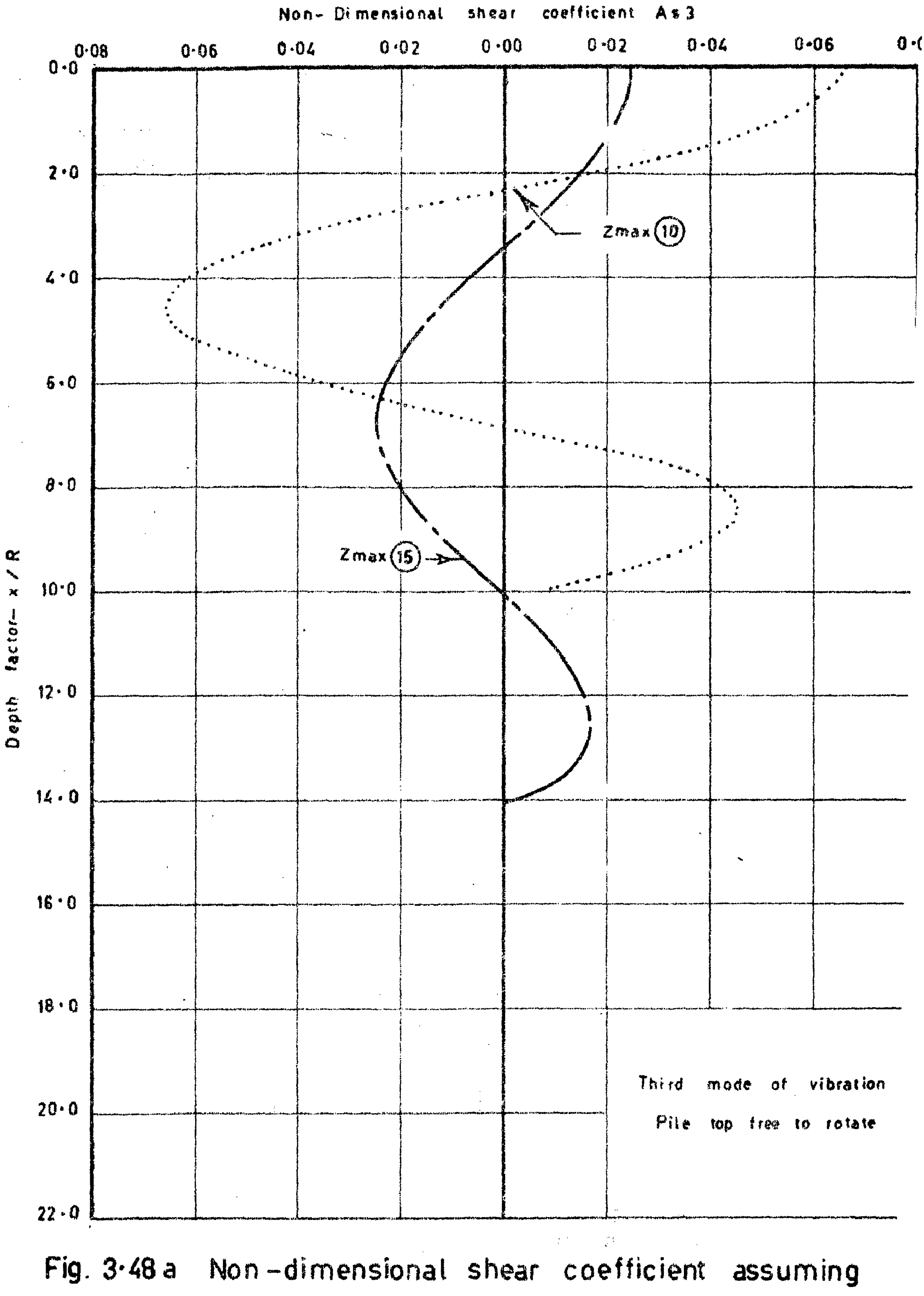


Fig. 3·48 a Non-dimensional shear coefficient assuming modulus constant with depth

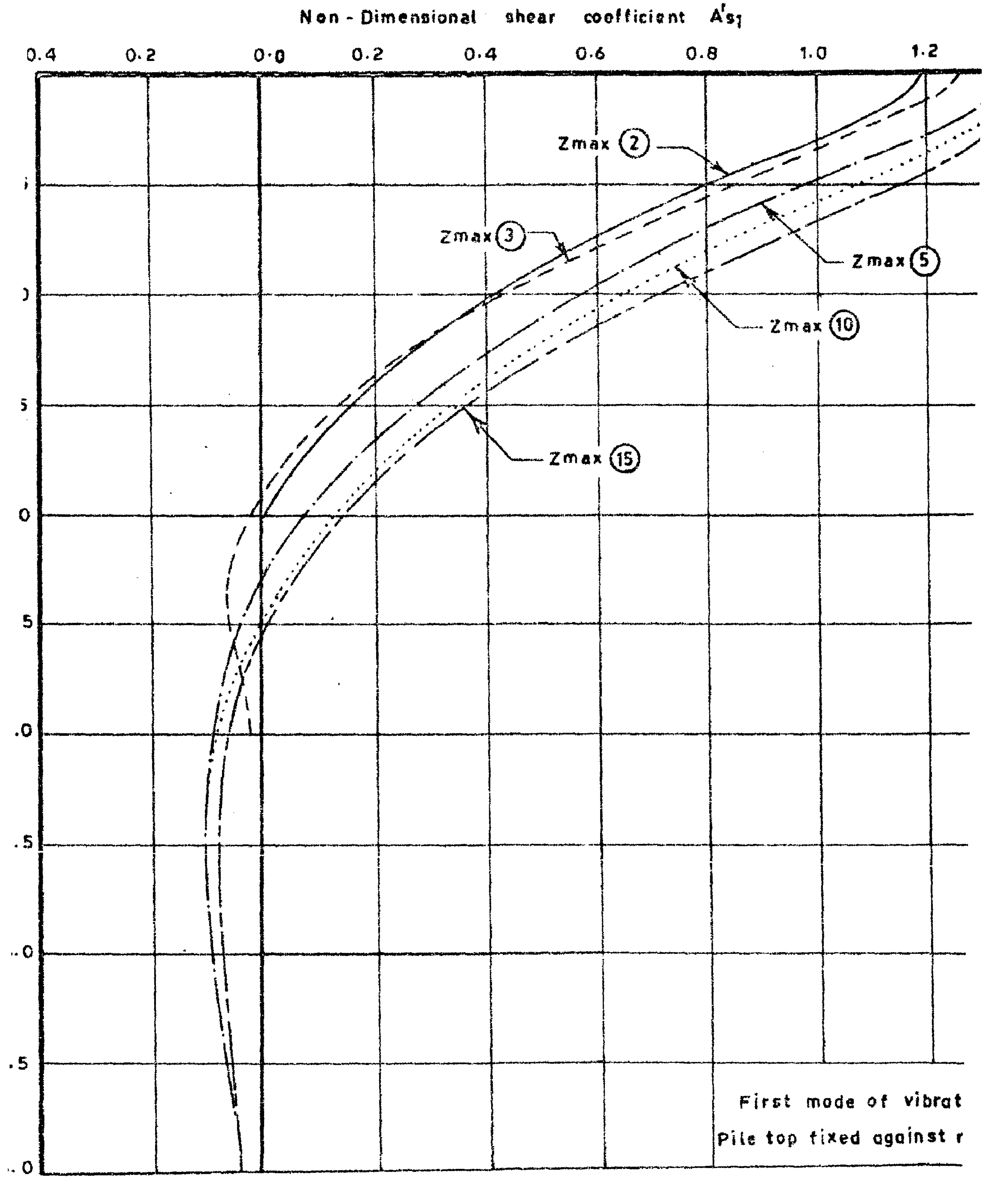


Fig. 3·49 Non-dimensional shear coefficient assuming soil modulus constant with depth

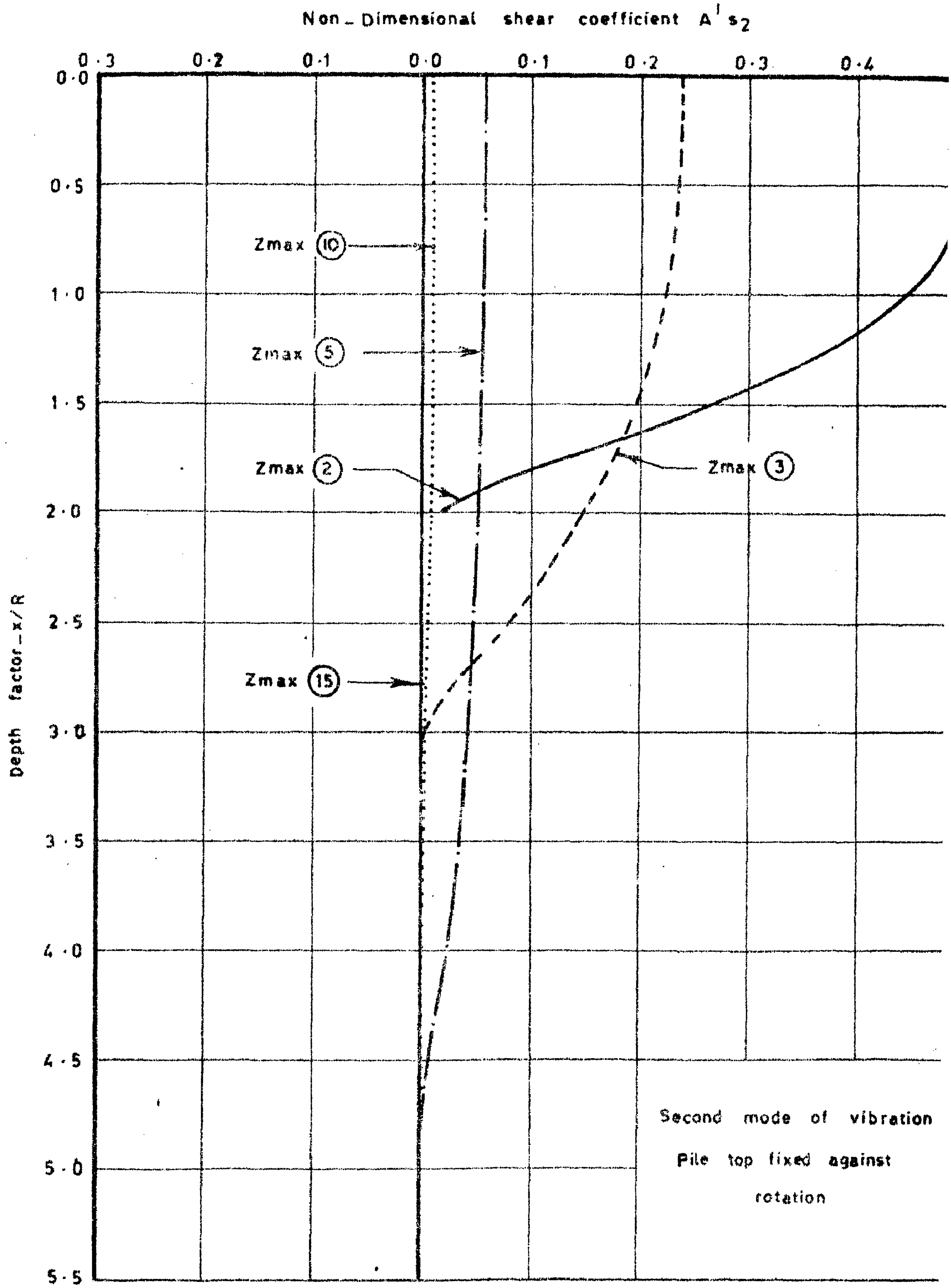


Fig. 3·50Non-dimensional shear coefficient assuming s modulus constant with depth

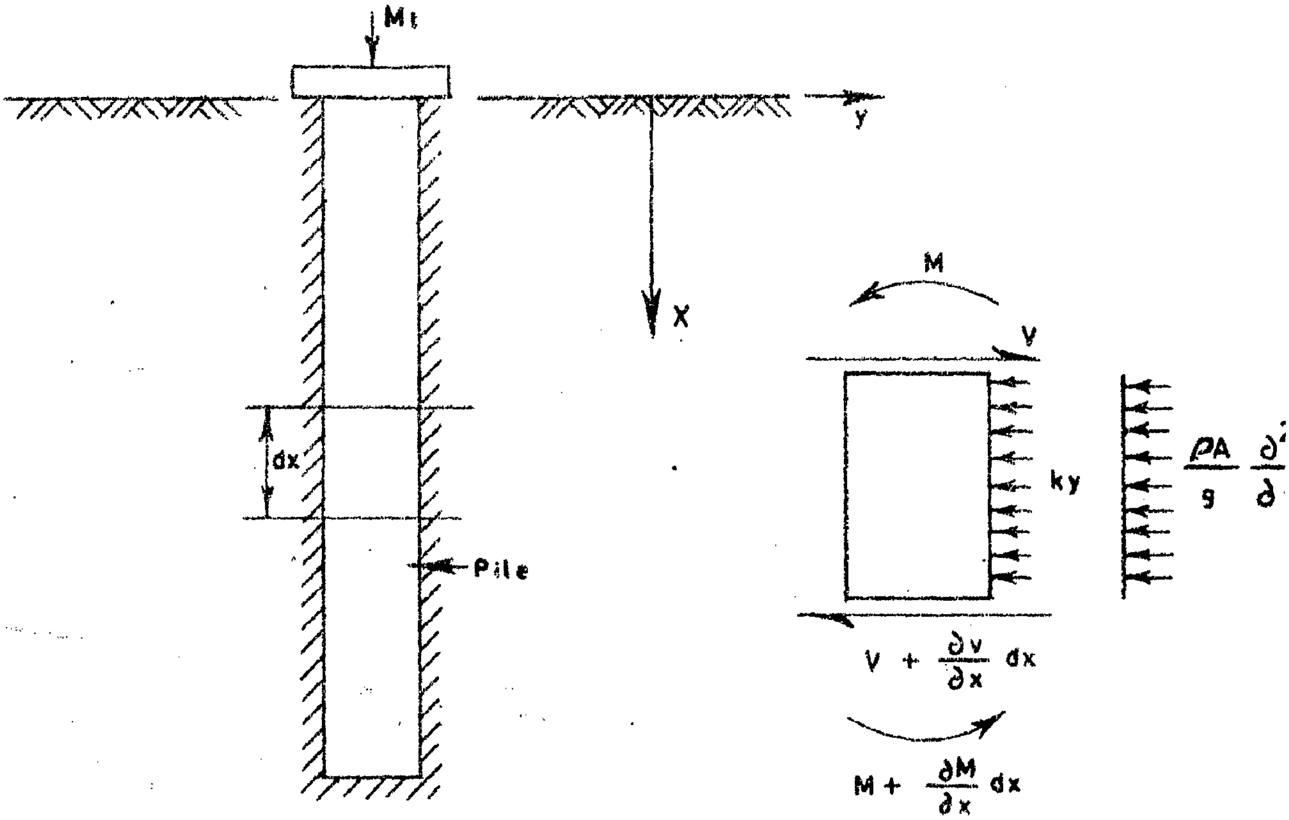


Fig. 4·1 Continuous system model

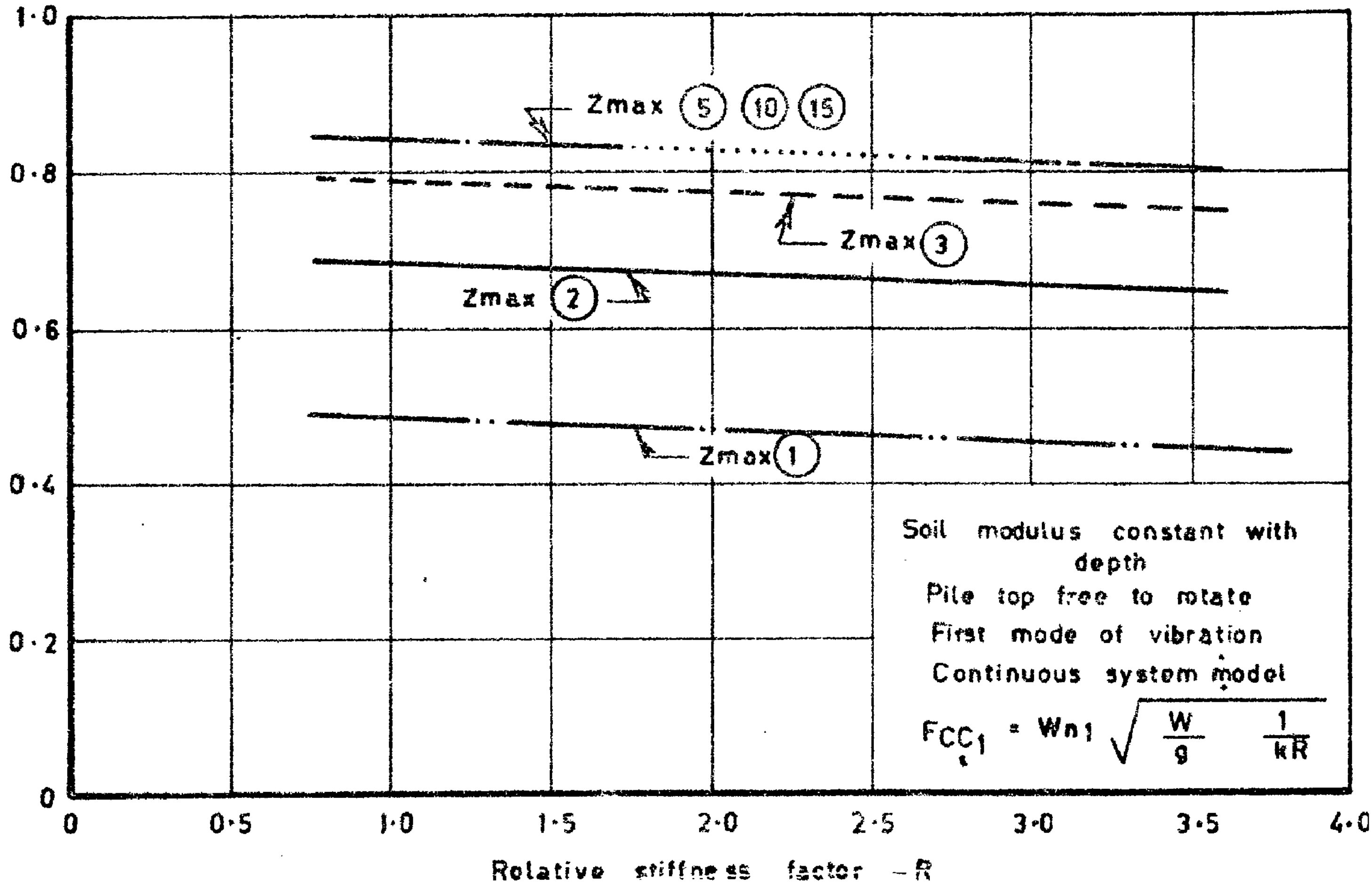


Fig. 4·2 Non – dimensional frequency factor versus relativ stiffness factor

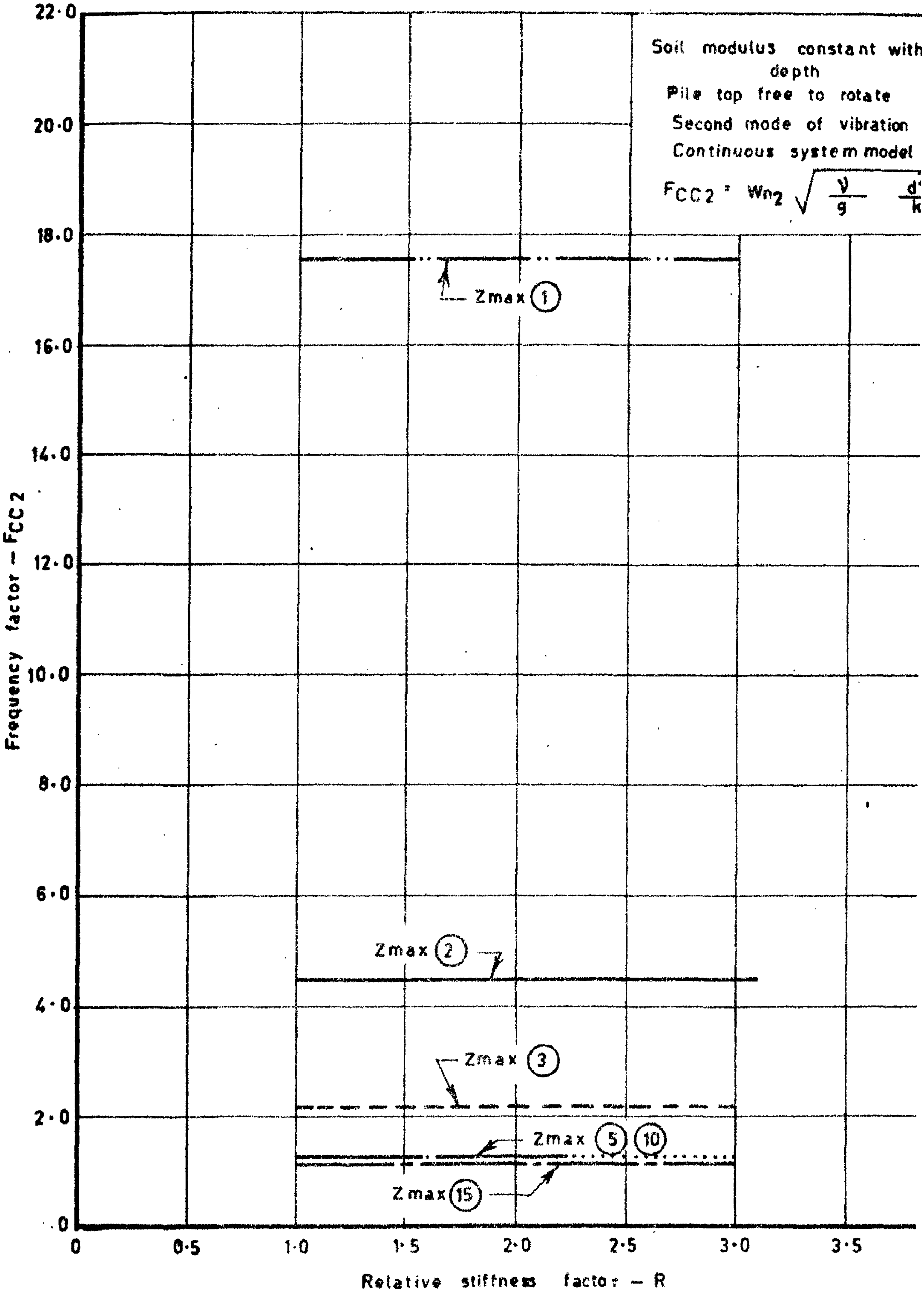


Fig. 4·3 Non – dimensional frequency factor versus rel

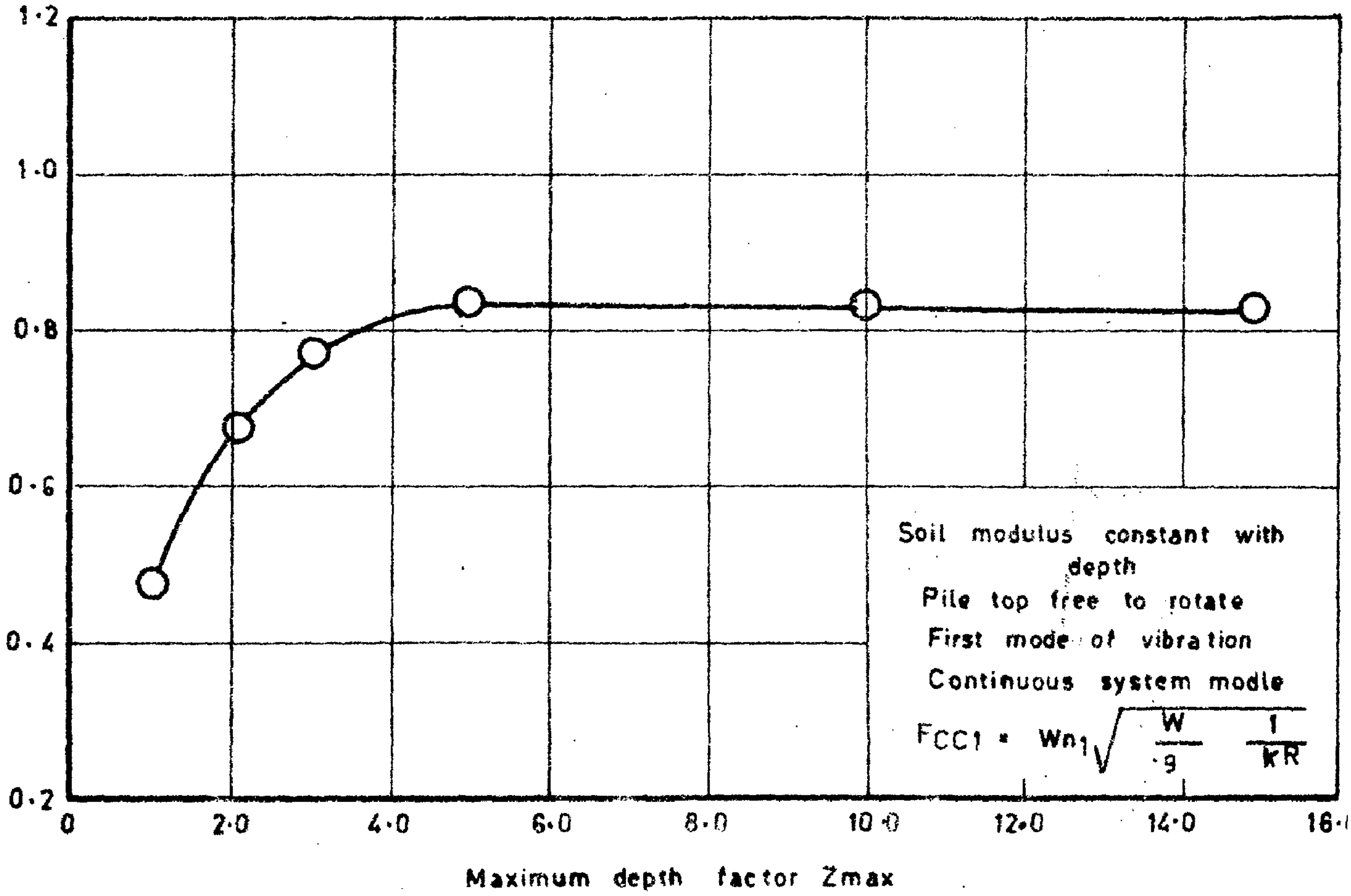


Fig. 4·4 Non – dimensional frequency factor versus maxim depth factor Zmax

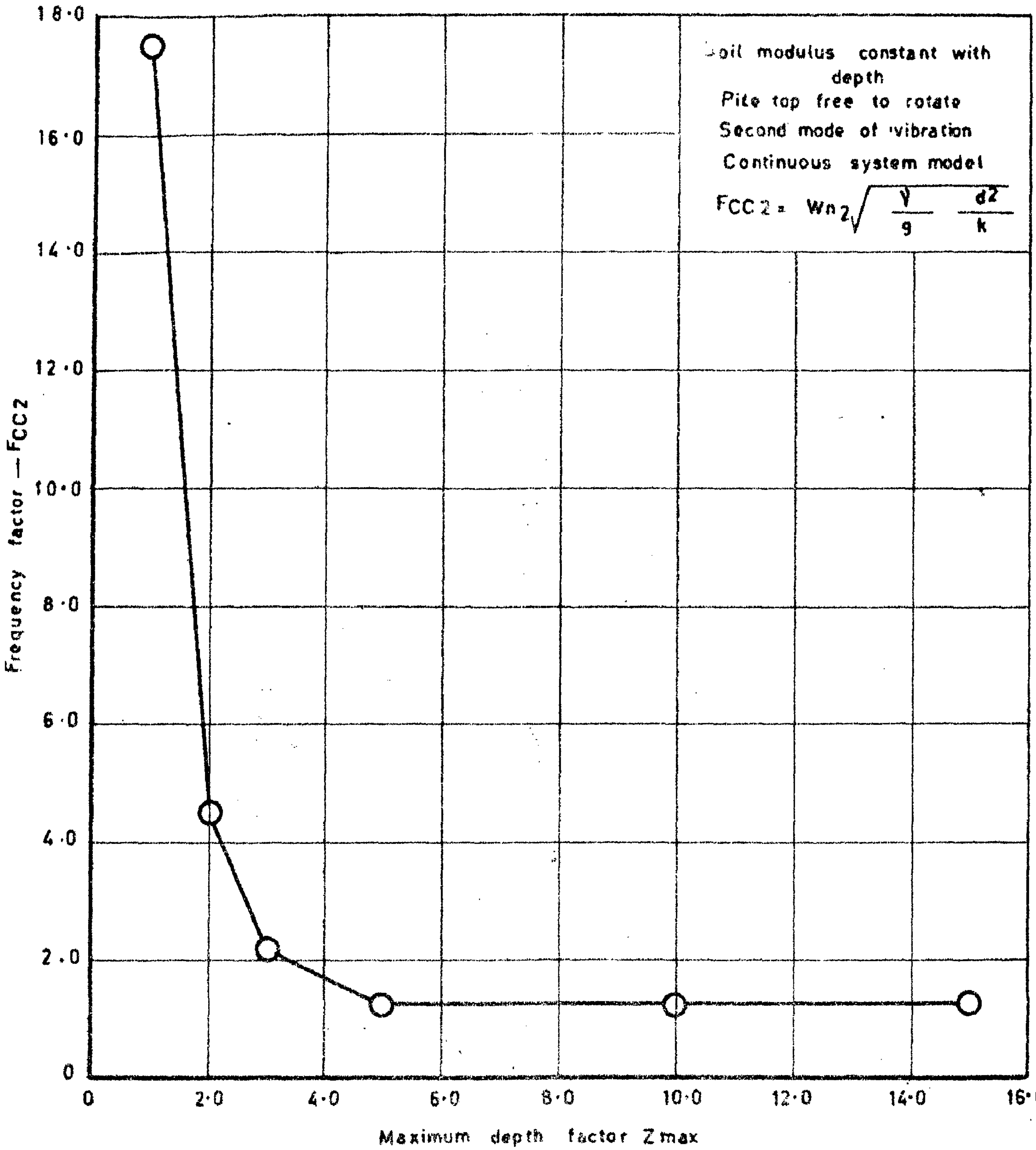


Fig. 4·5 Non – dimensional frequency factor versus maxim
depth factor Zmax

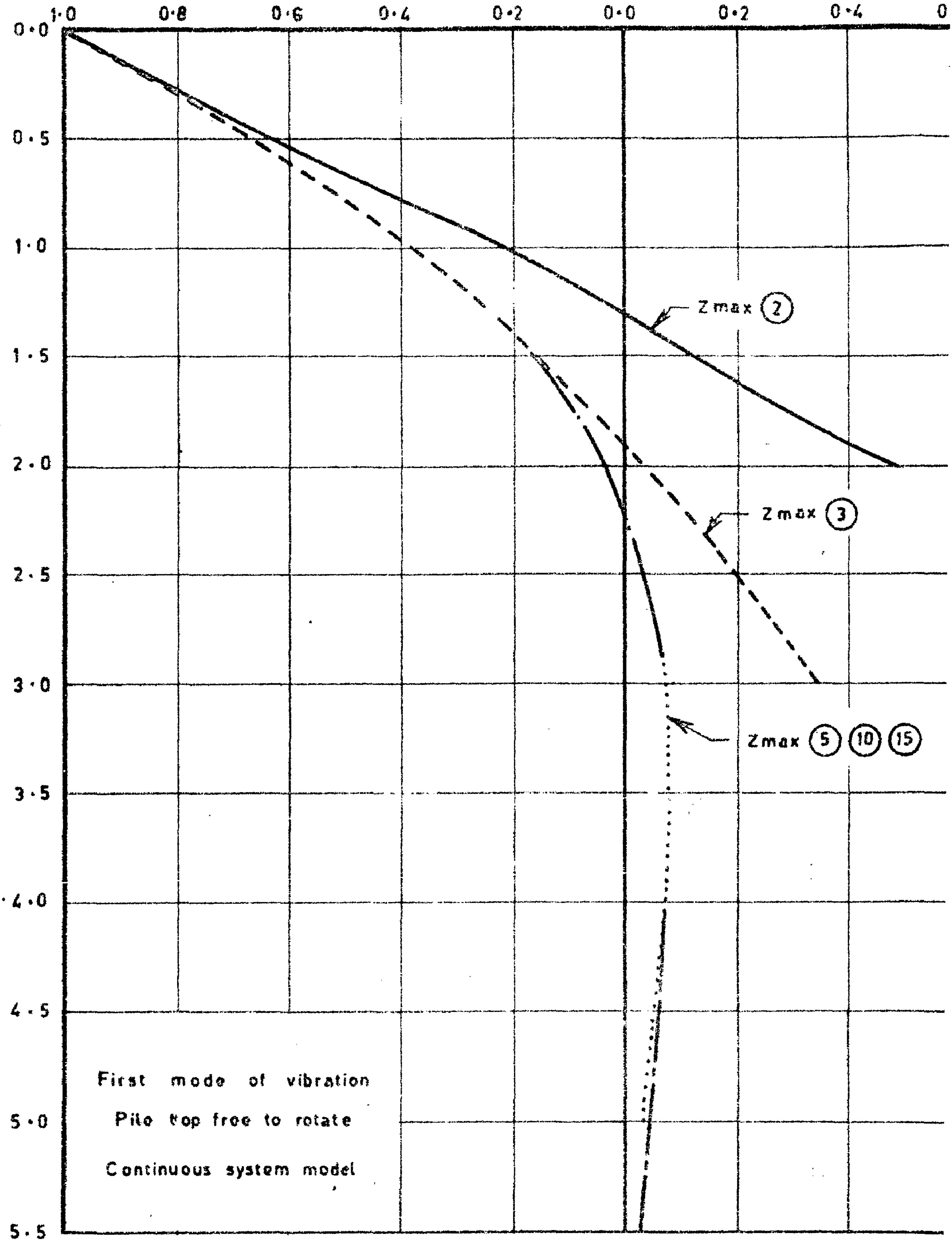


Fig. 4·6 Non-dimensional deflection coefficient assuming s
modulus constant with depth

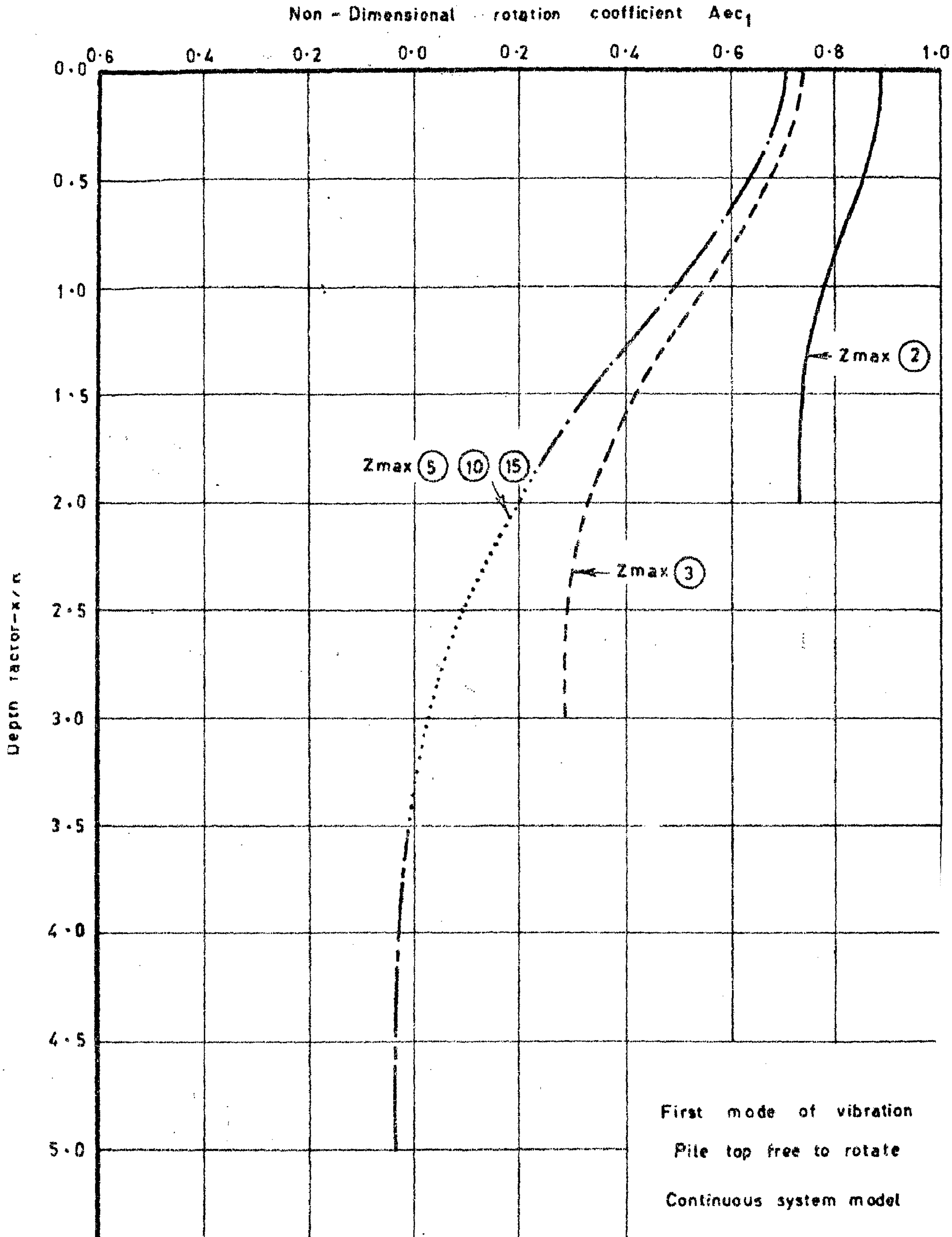


Fig. 4·7 Non-dimensional rotation coefficient assuming s
modulus constant with depth

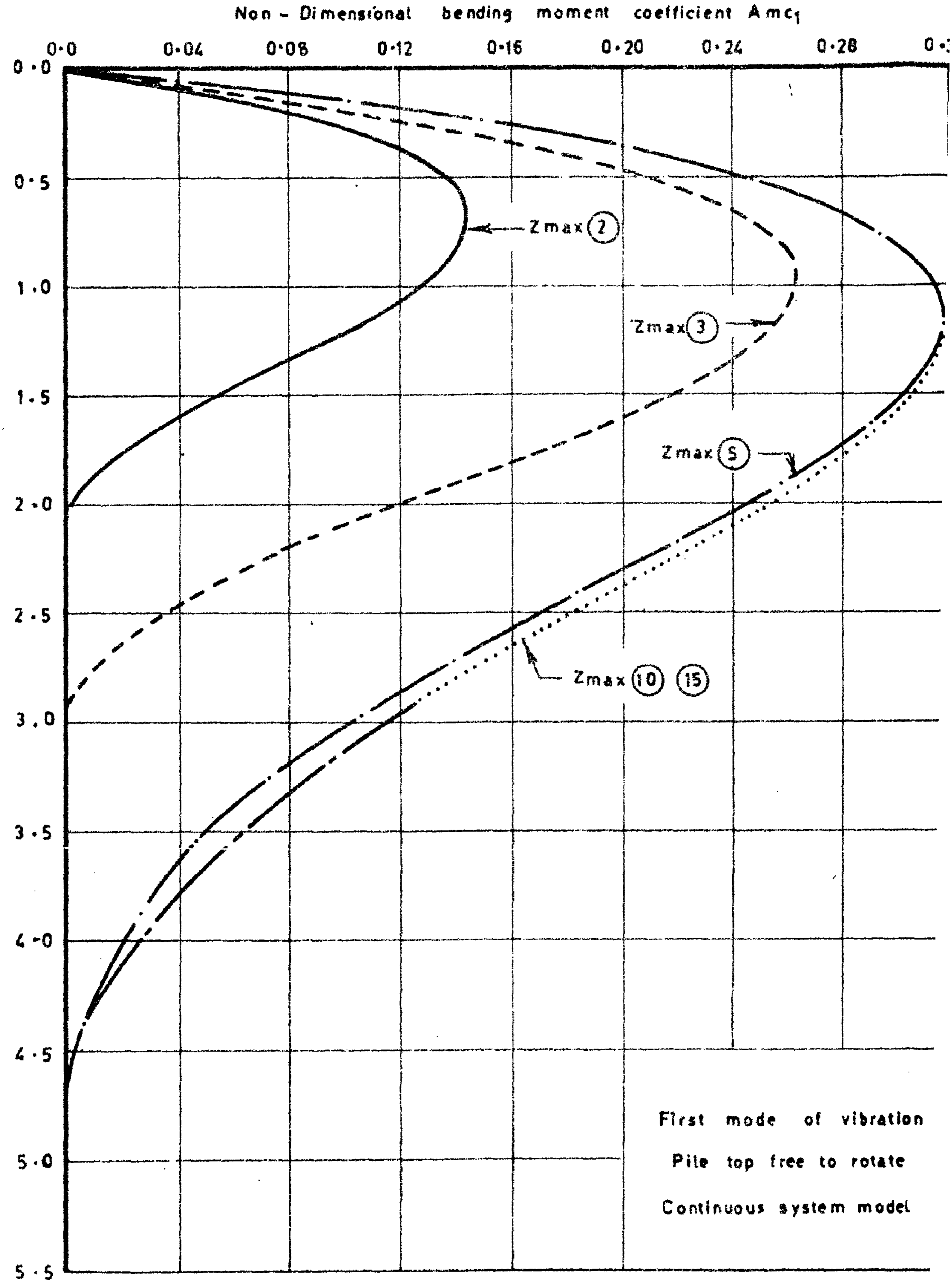


Fig. 4·8 Non – dimensional bending moment coefficient assu[illegible]
soil modulus constant with depth

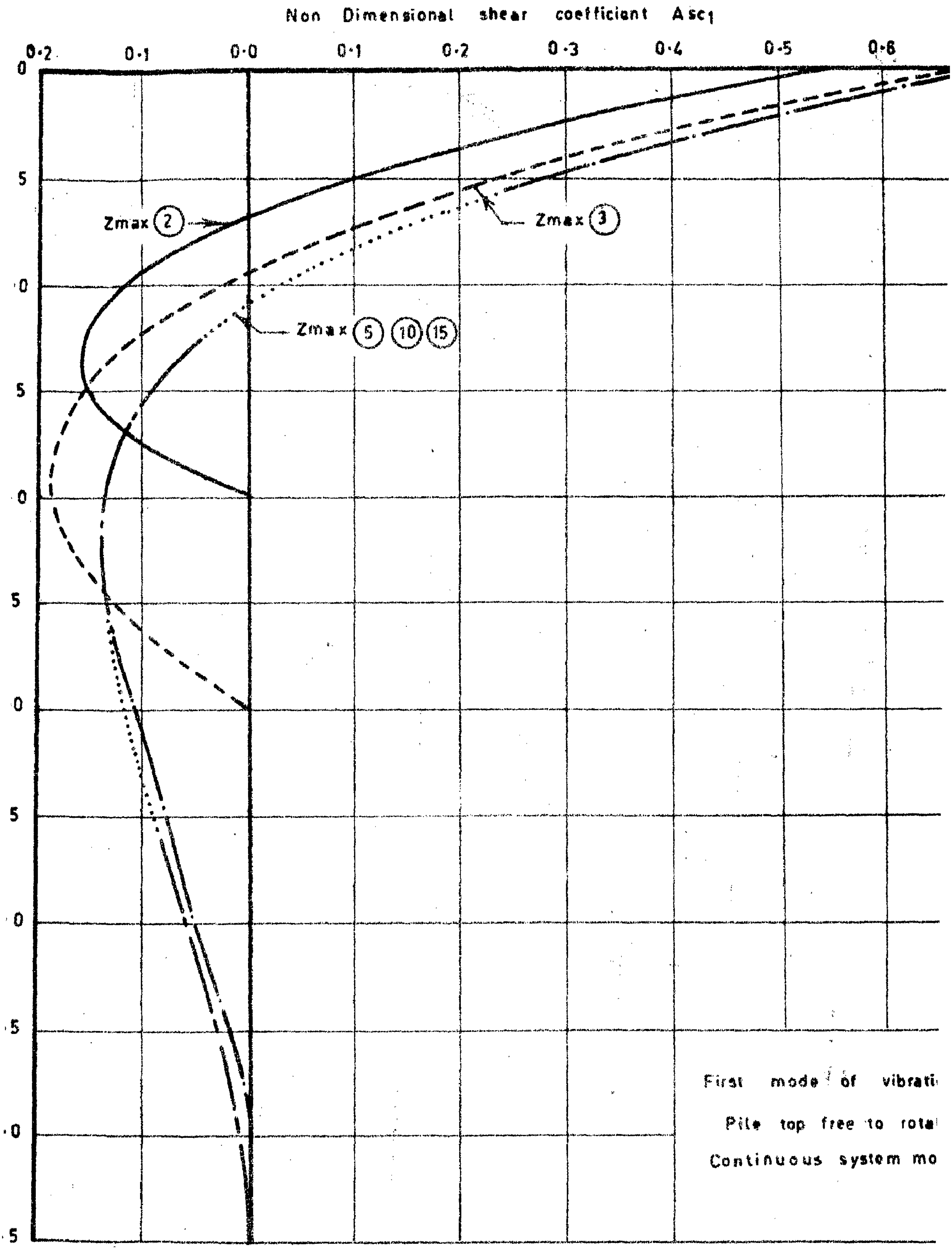


Fig. 4·9 Non-dimensional shear coefficient assuming s
modulus constant with depth

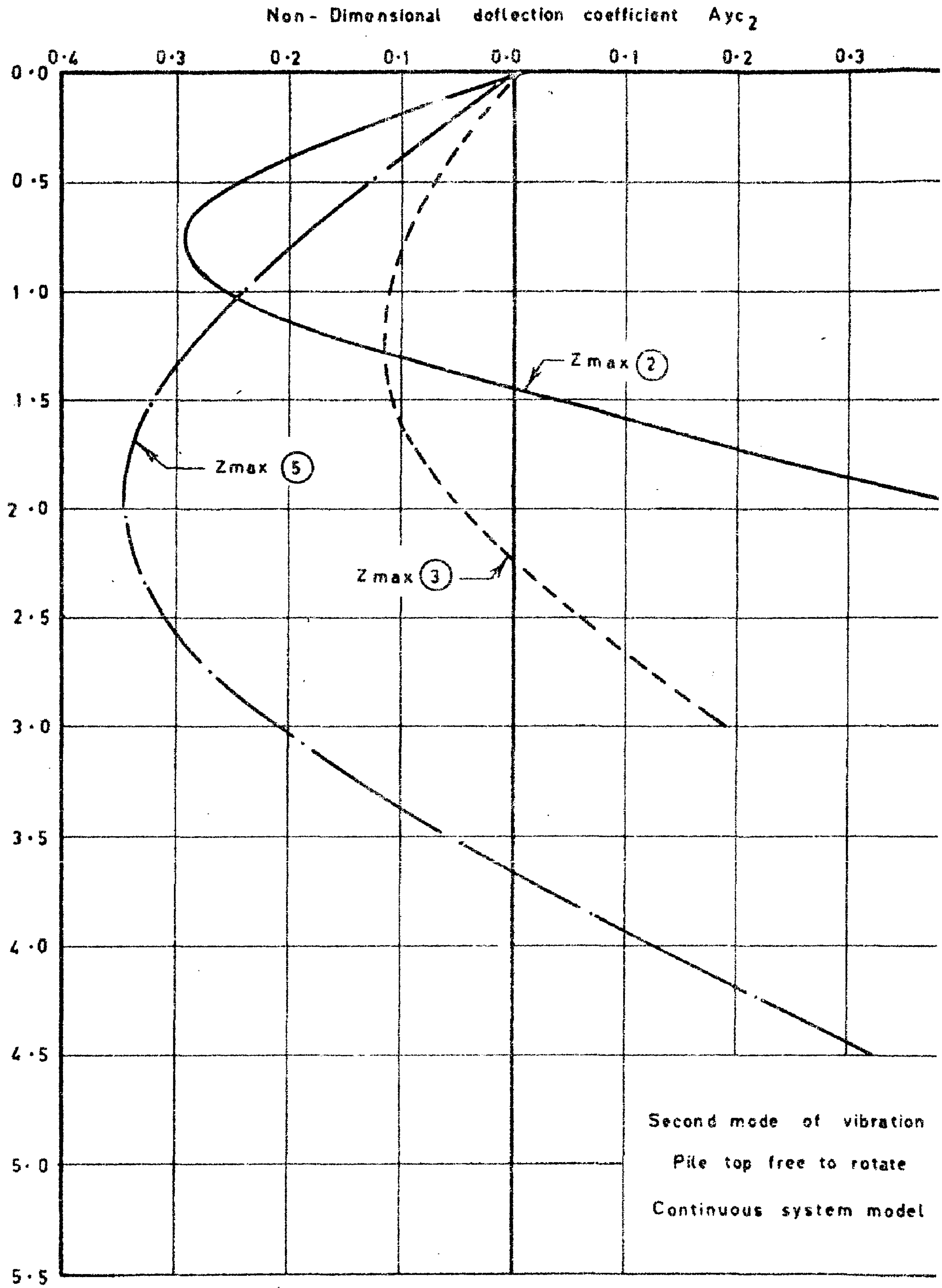


Fig.4·10 Non- dimensional deflection coefficient assuming modulus constant with depth

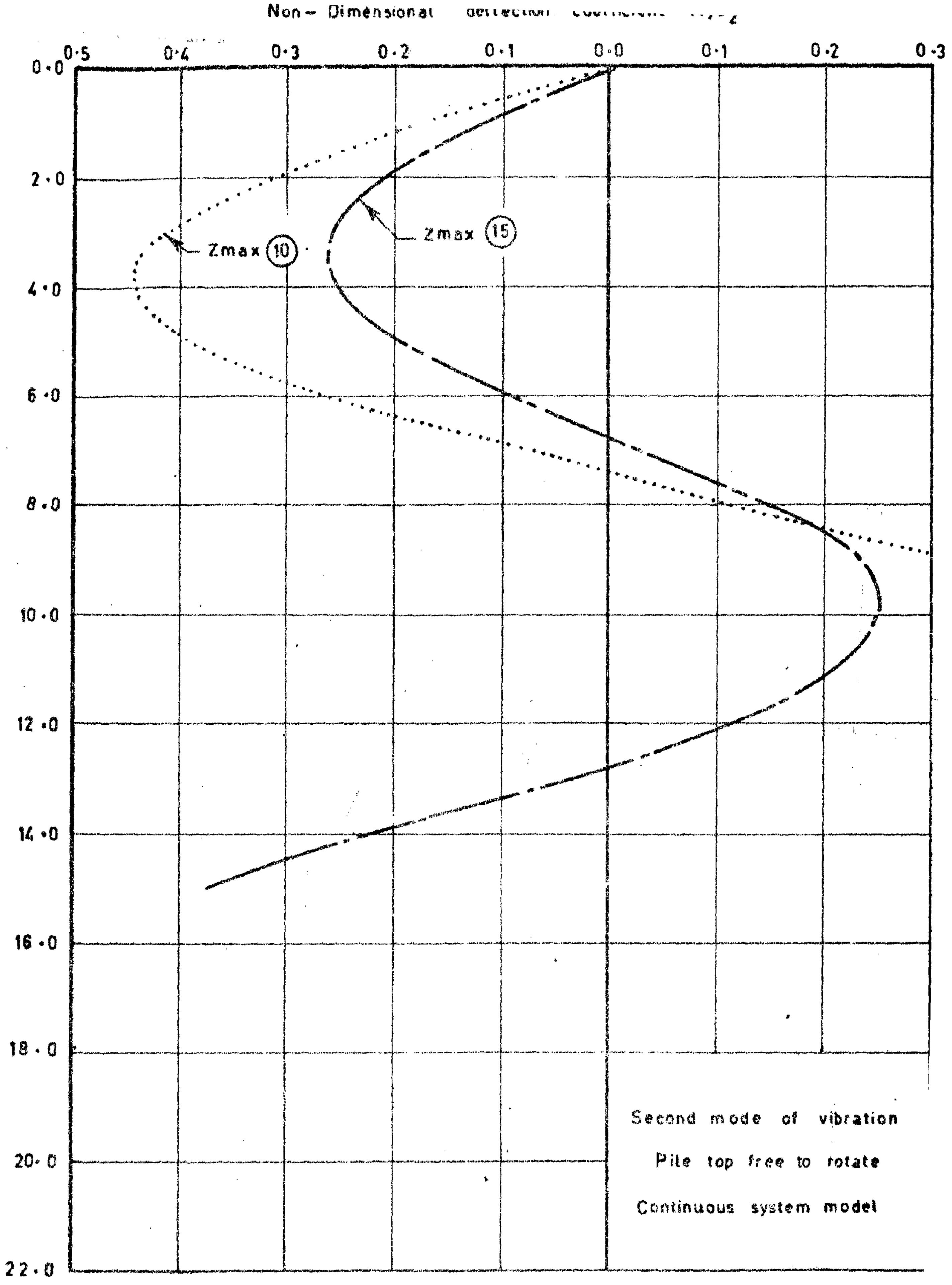


Fig. 4·10a Non-dimensional deflection coefficient assuming s
modulus constant with depth

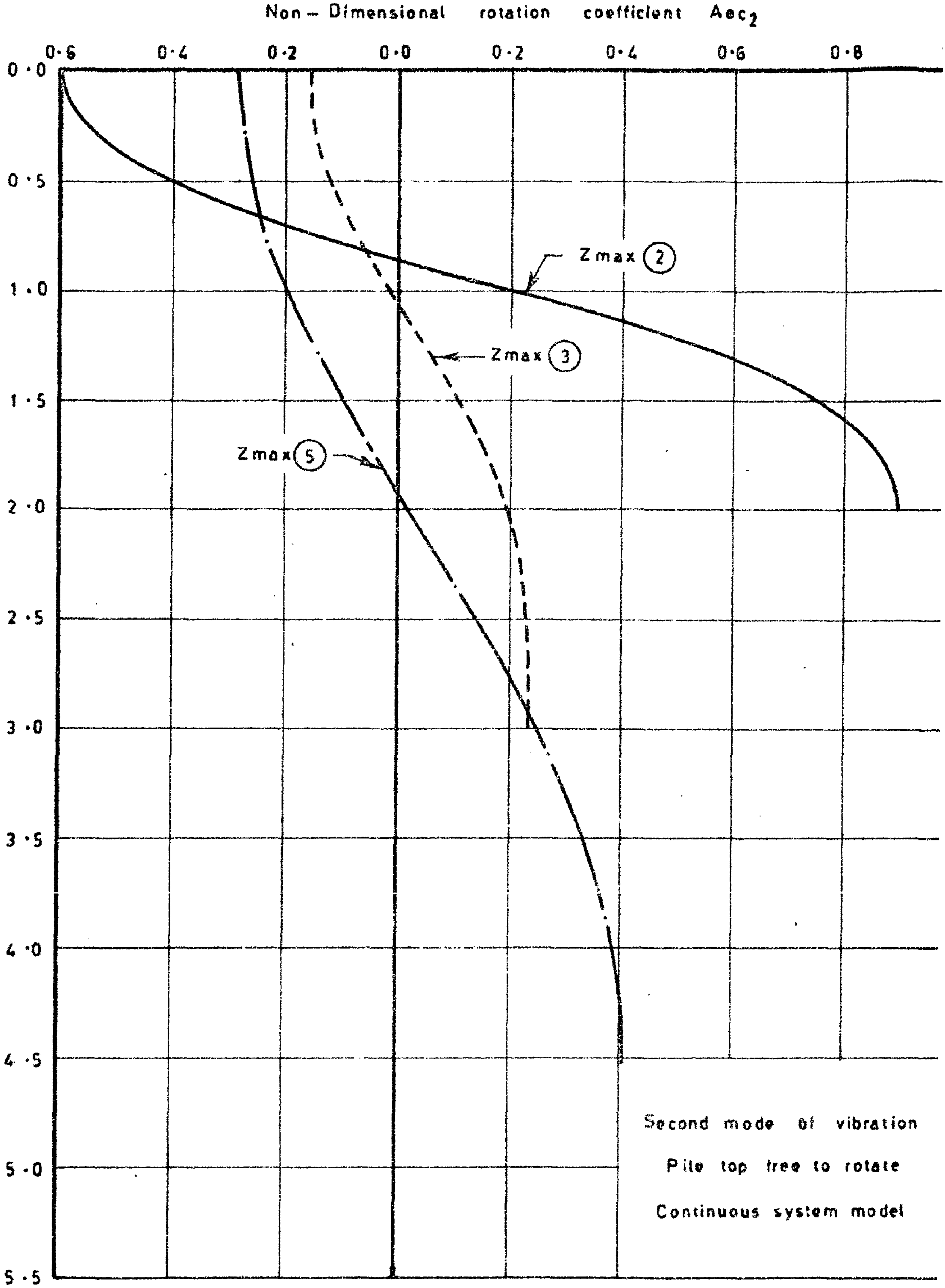


Fig. 4·11 Non-dimensional rotation coefficient assuming s
modulus constant with depth

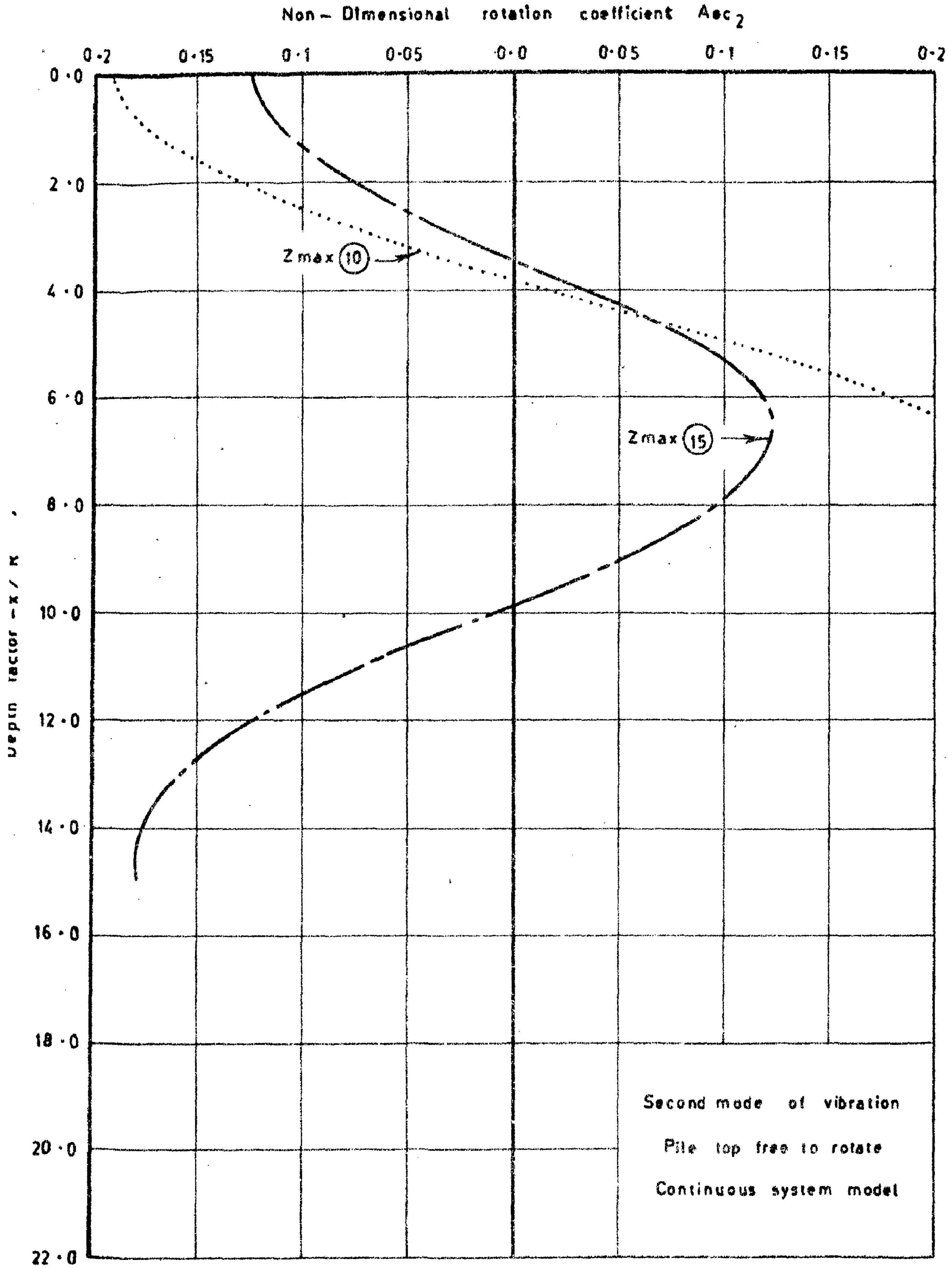
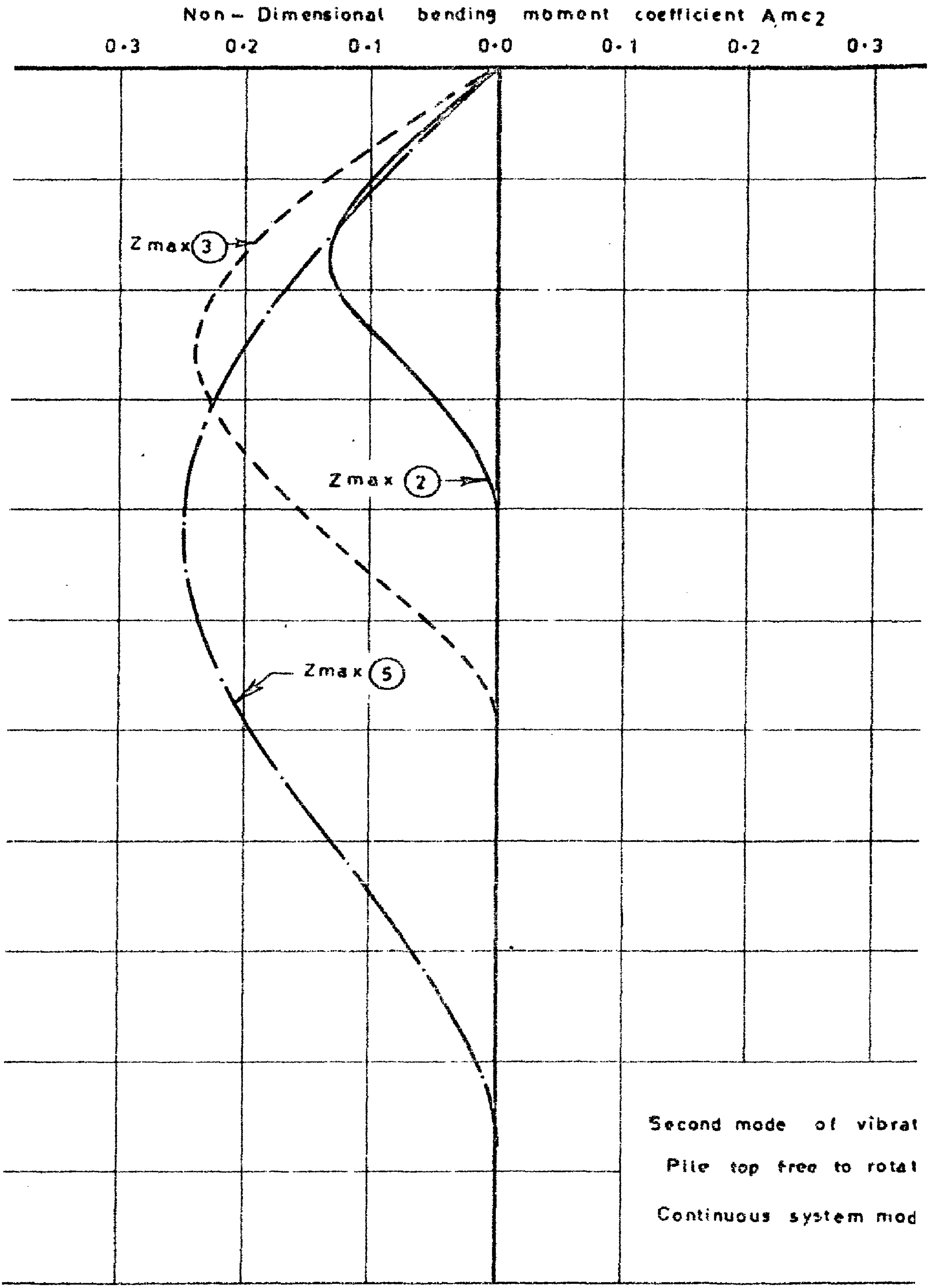


Fig. 4·11 a Non-dimensional rotation coefficient assuming soi modulus constant with depth



·12 Non – dimensional bending moment coefficient as
soil modulus constant with depth

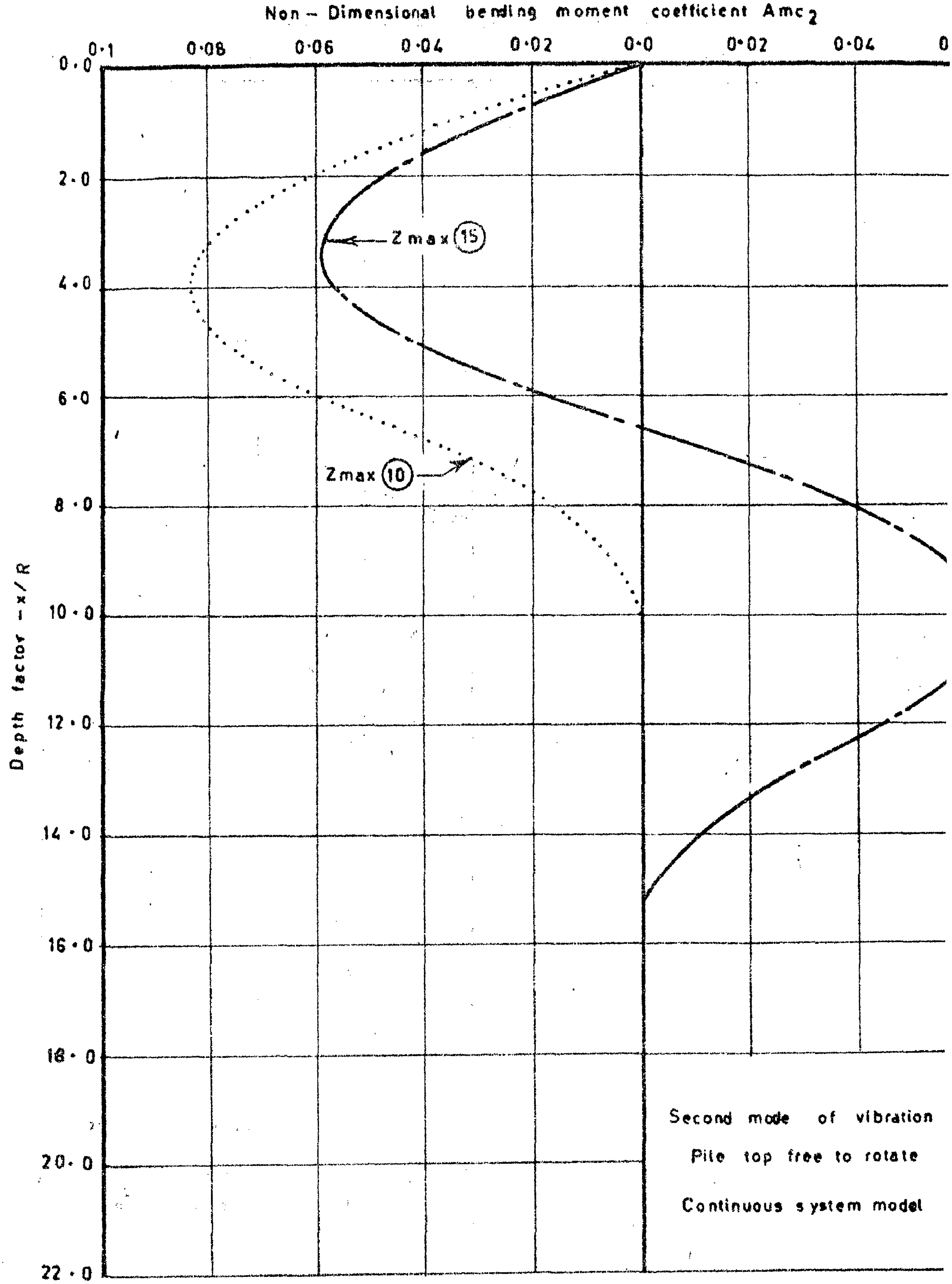


Fig. 4·12a Non – dimensional bending moment coefficient assu
soil modulus constant with depth

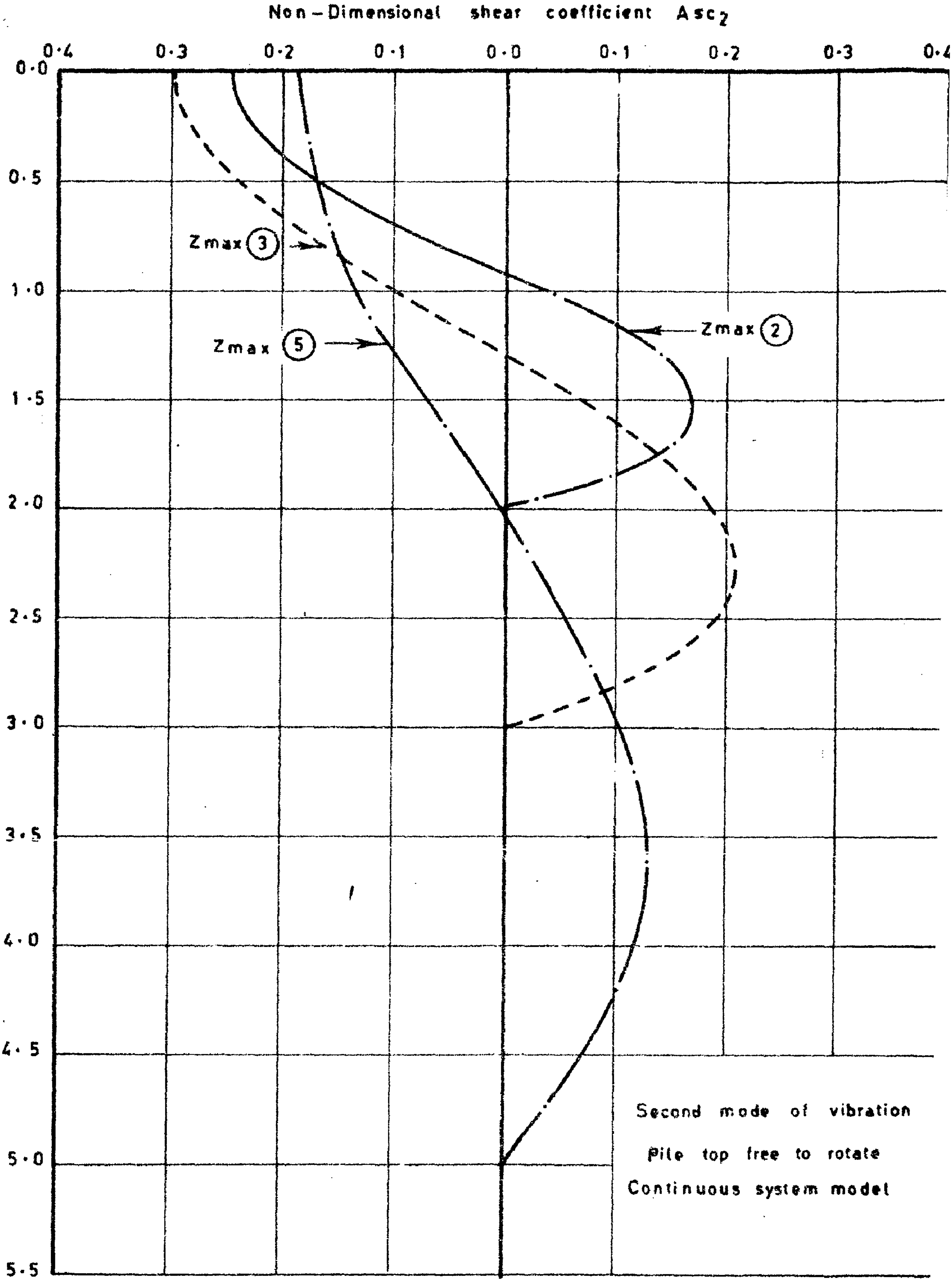


Fig. 4·13 Non-dimensional shear coefficient assuming modulus constant with depth

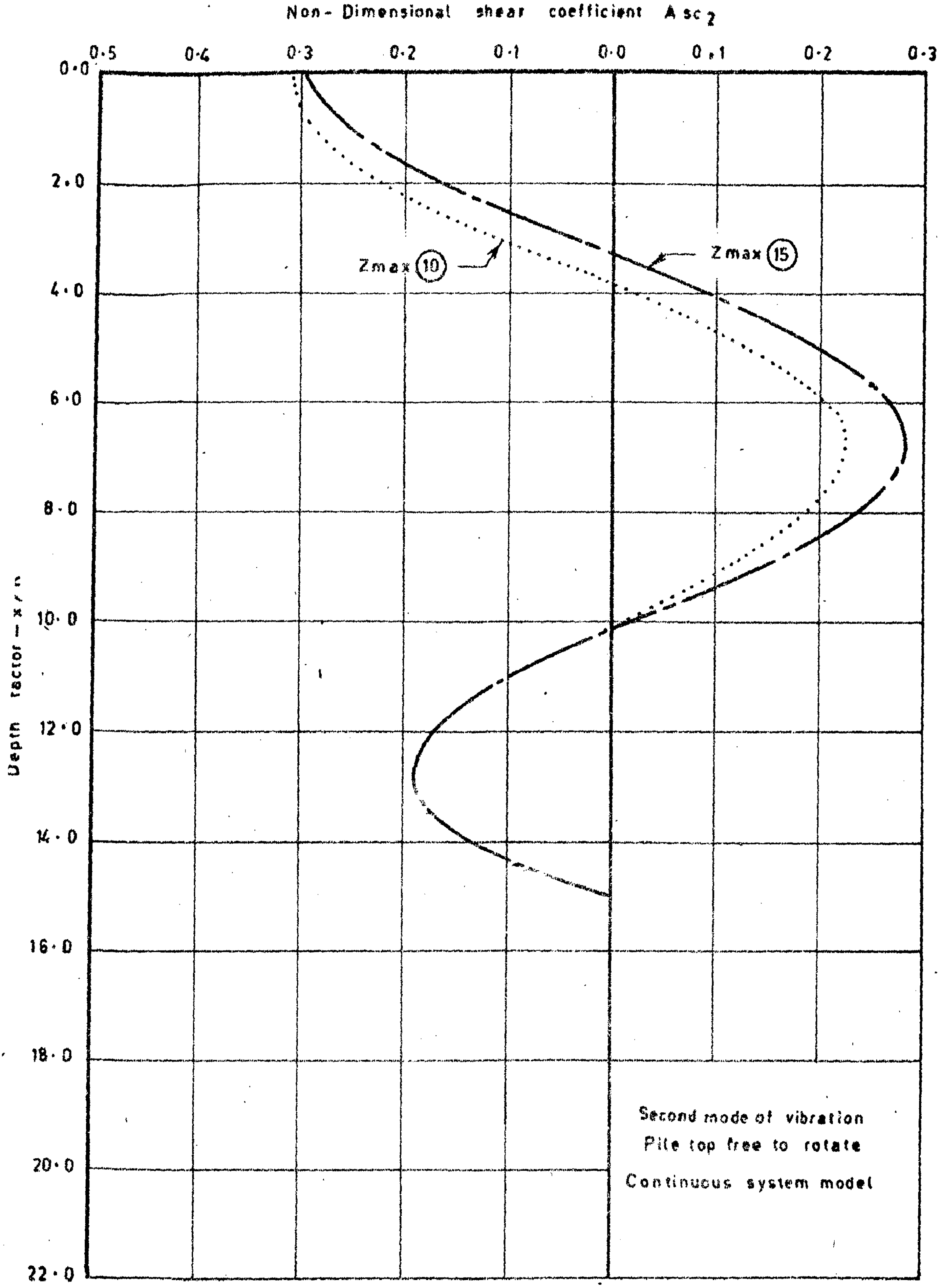


Fig. 4·13a Non-dimensional shear coefficient assuming sc
modulus constant with depth

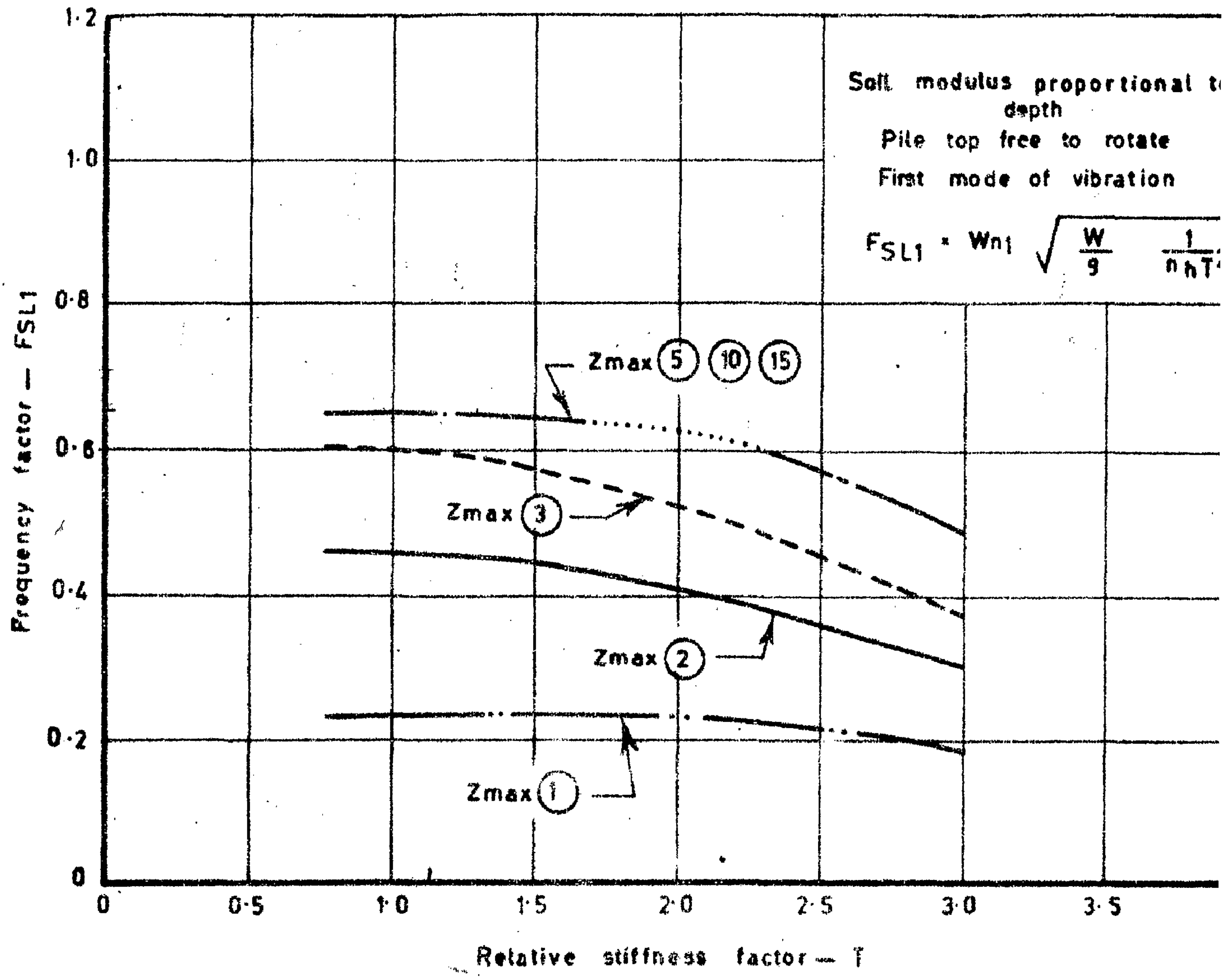


Fig. 5·1 Non – dimensional frequency factor versus relat stiffness factor

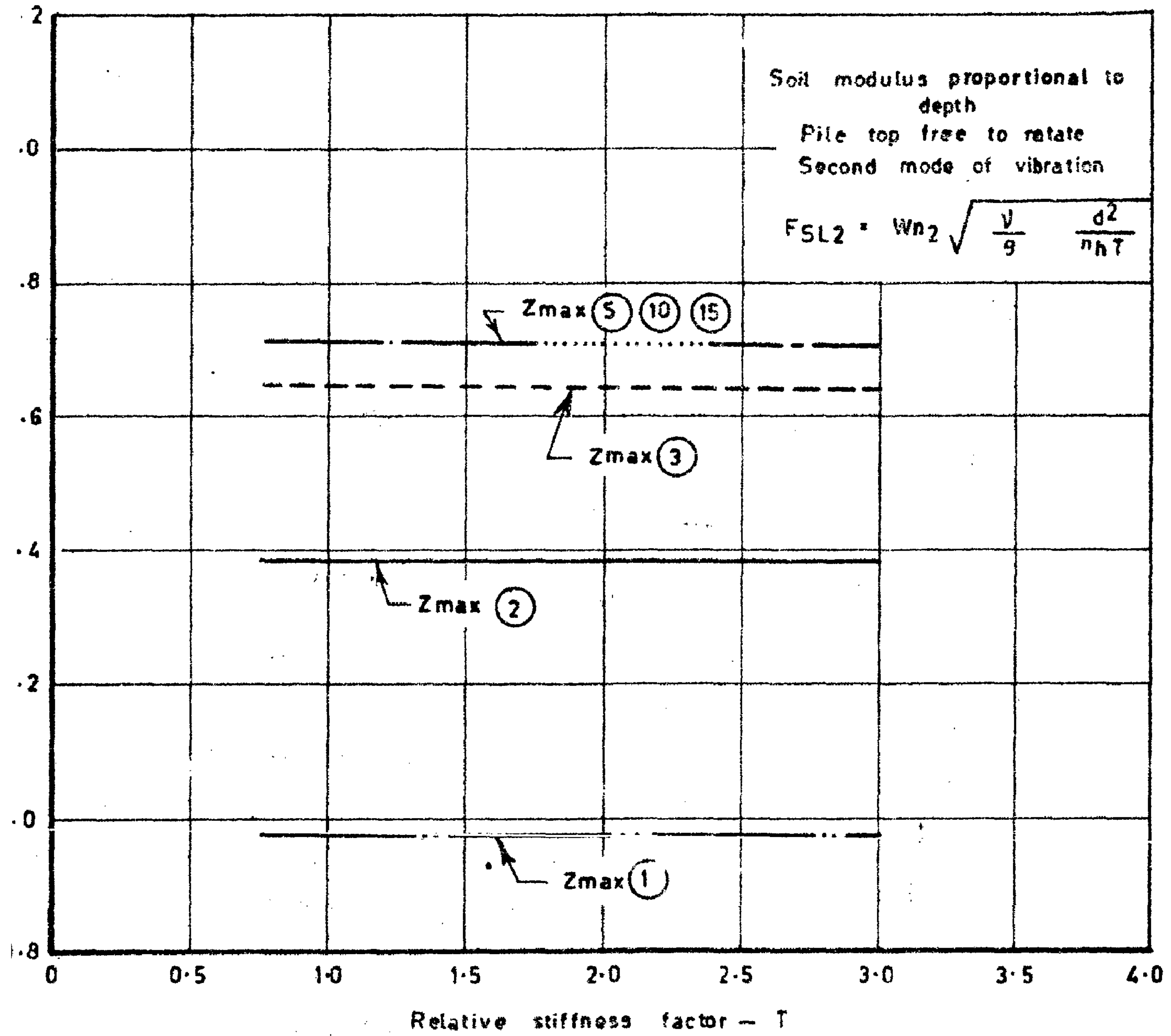


Fig. 5·2 Non – dimensional frequency factor versus relative stiffness factor

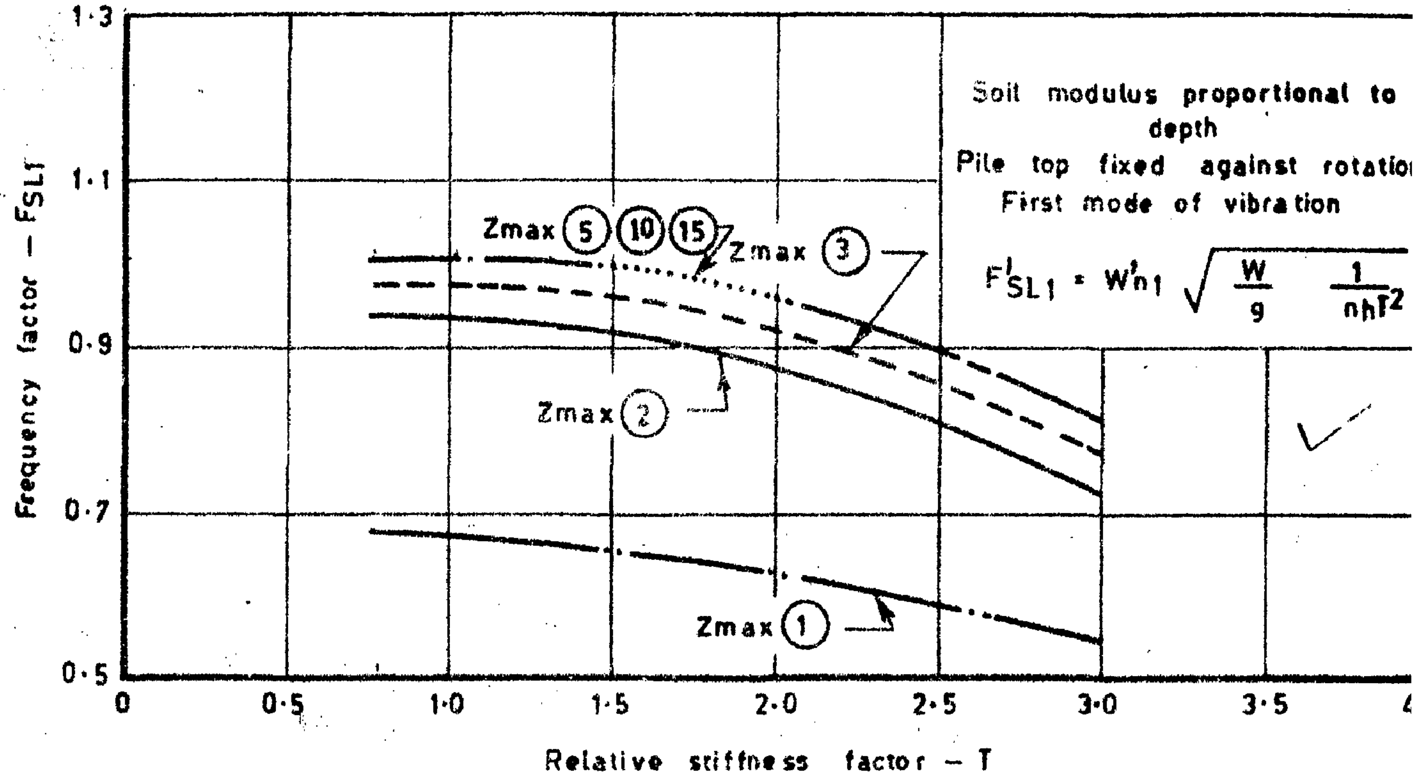


Fig. 5·3 Non – dimensional frequency factor versus relat stiffness factor

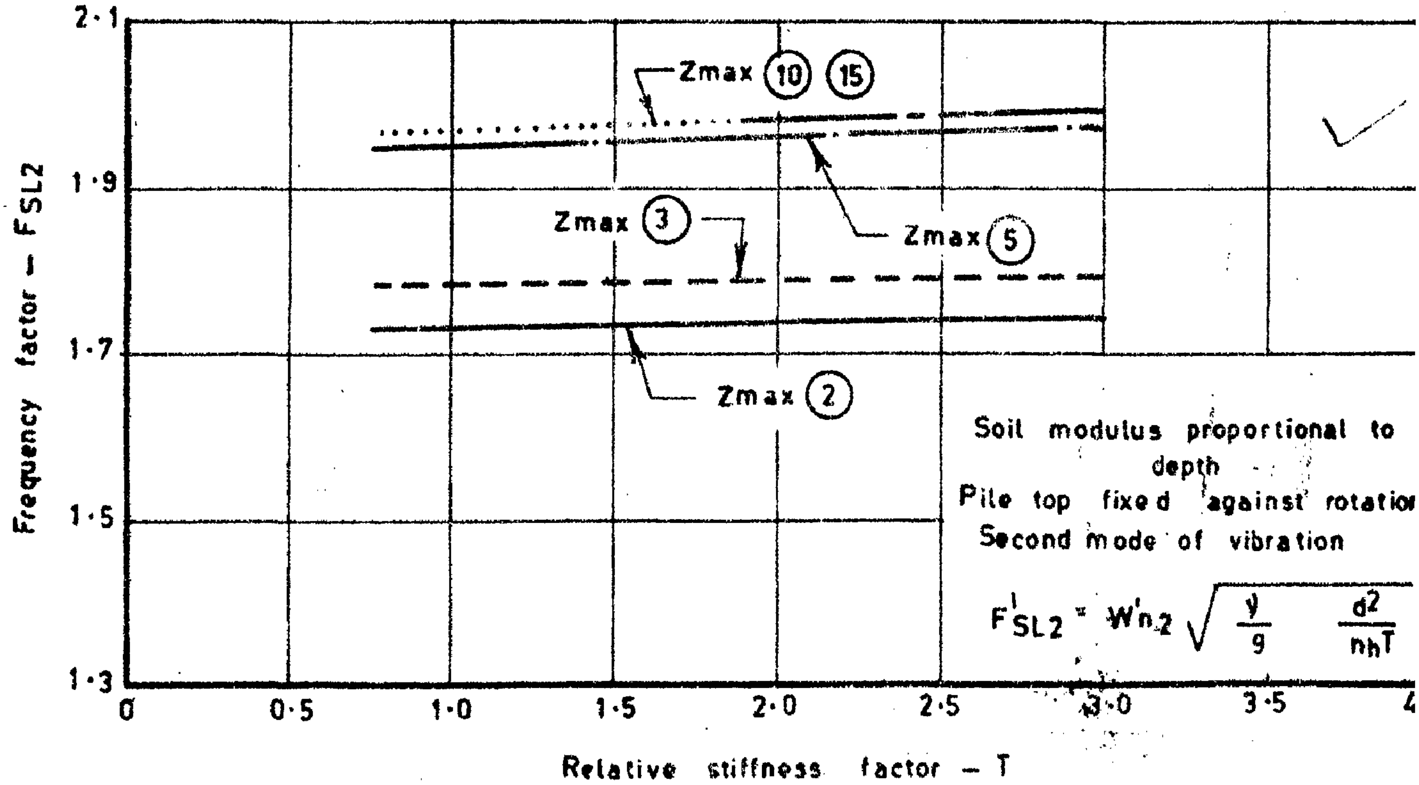


Fig. 5·4 Non – dimensional frequency factor versus relat stiffness factor

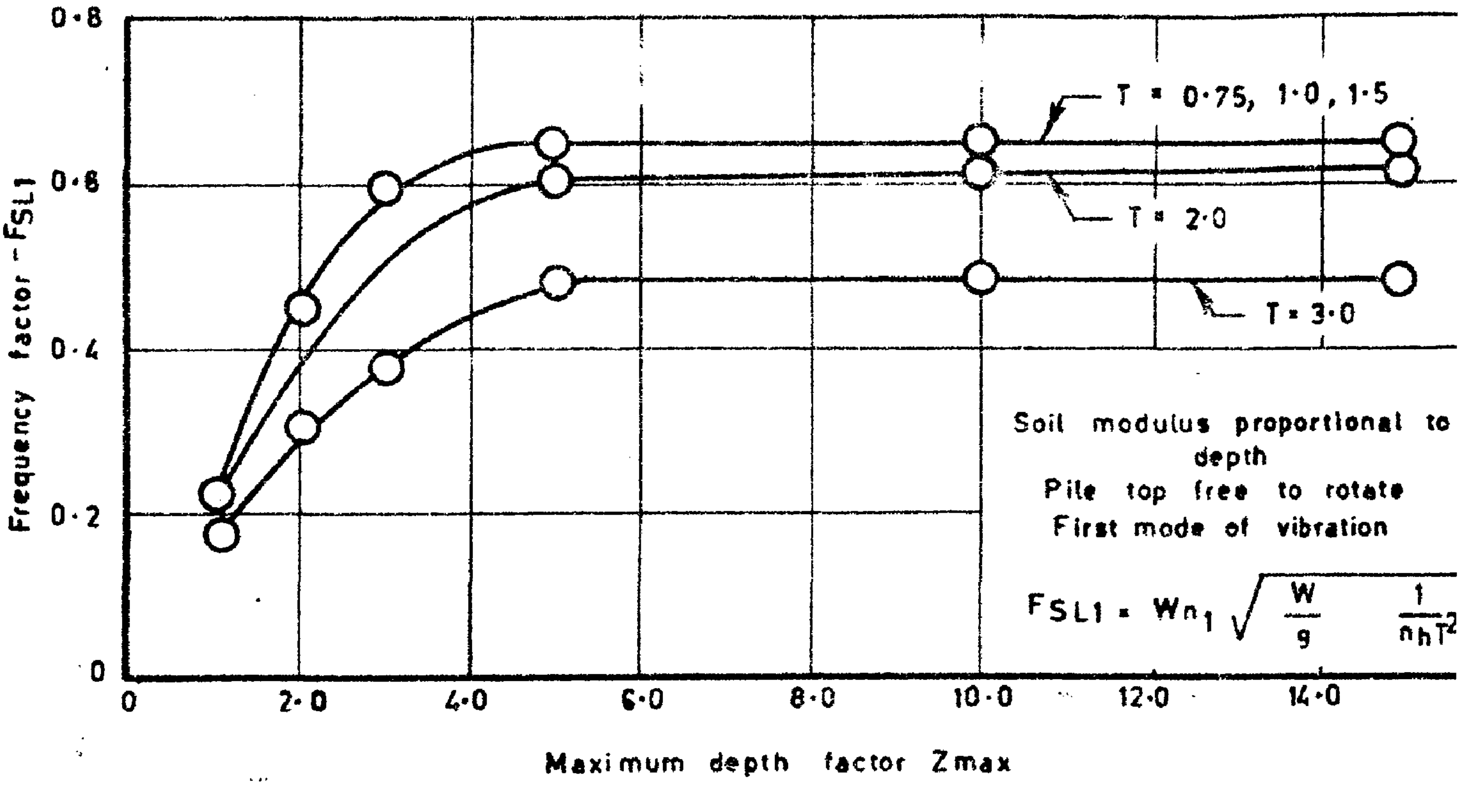


Fig. 5·5–Non-dimensional frequency factor versus max depth factor Zmax

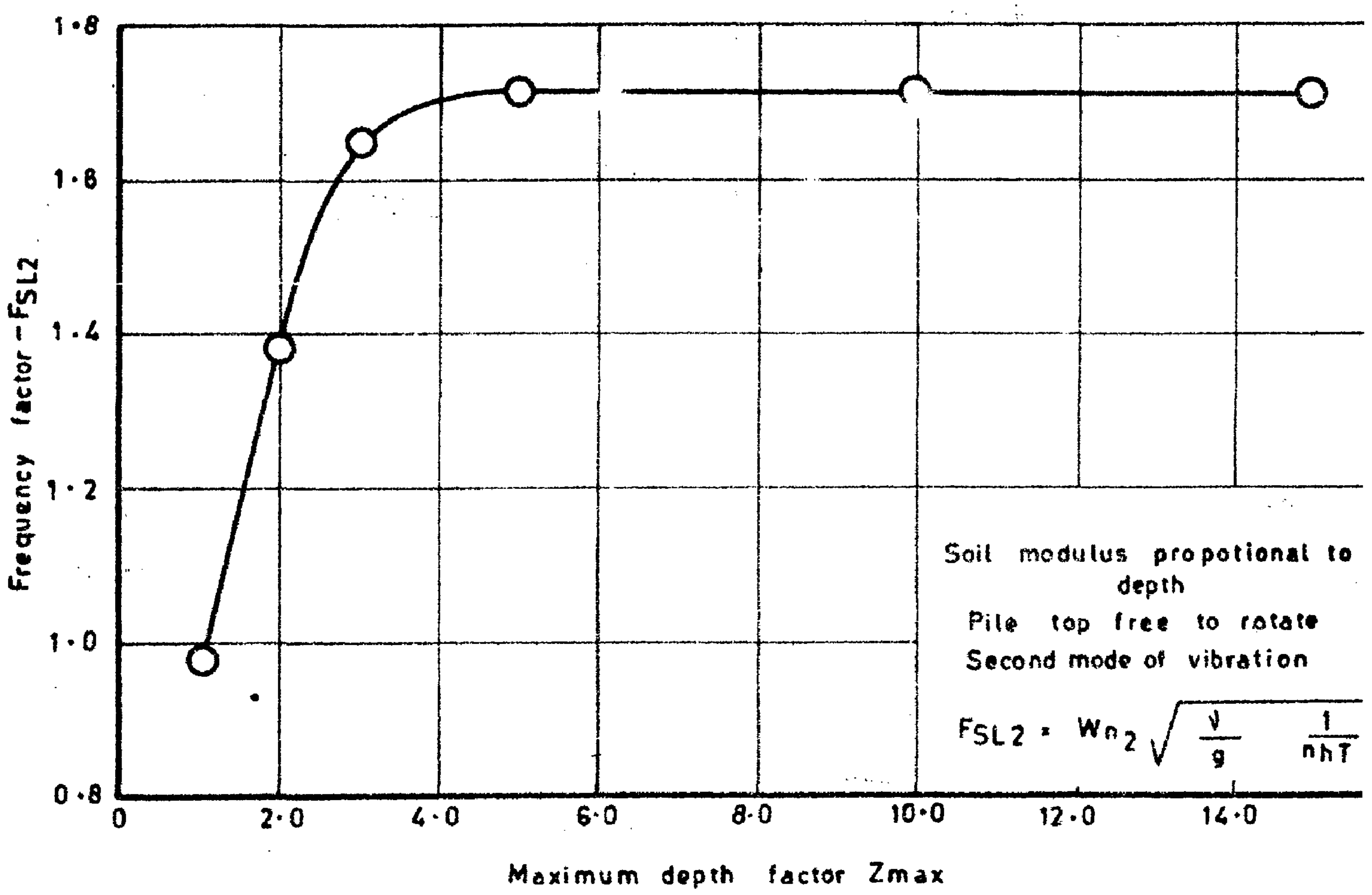


Fig. 5·6 Non-dimensional frequency factor versus max depth factor Zmax

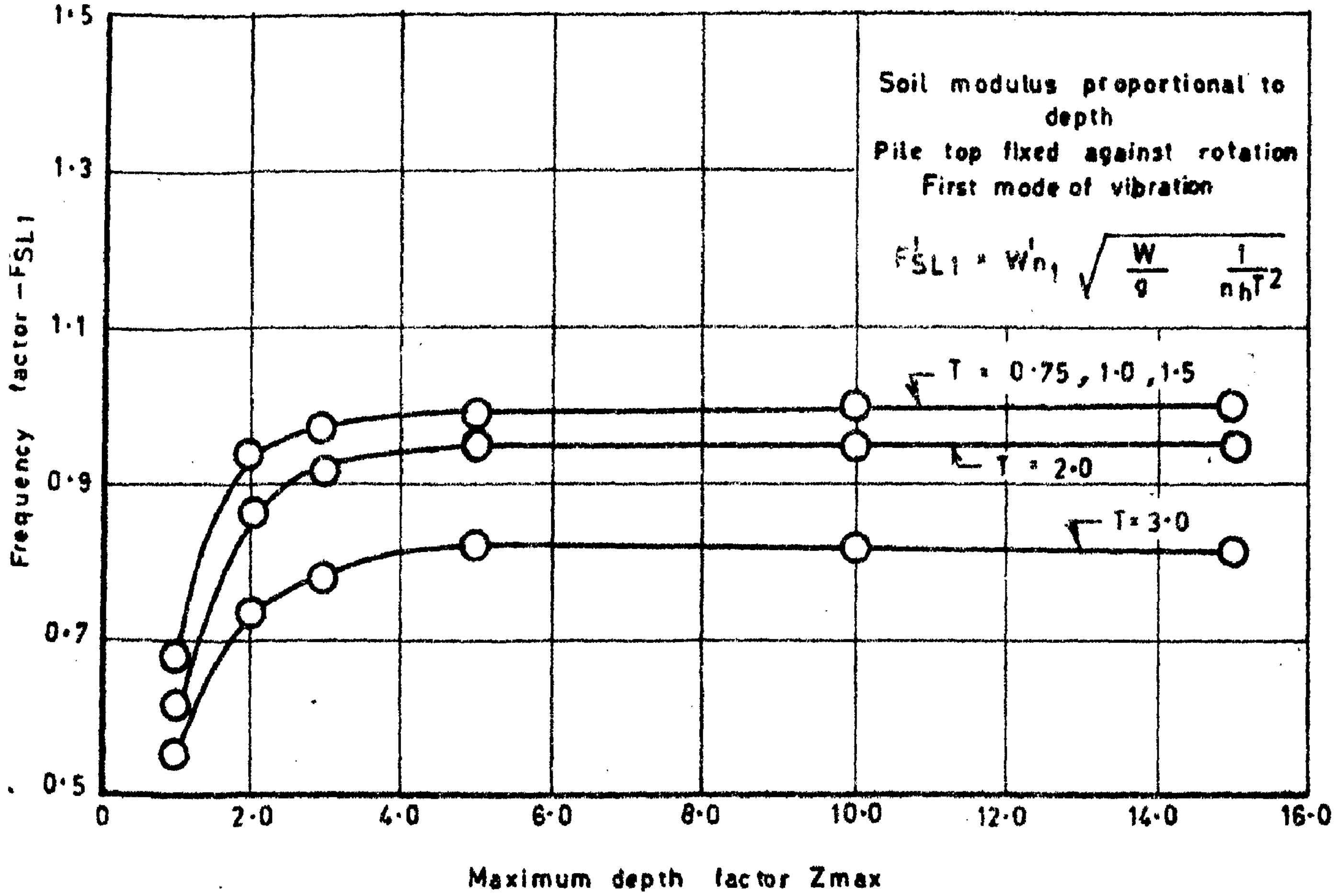


Fig. 5·7 Non – dimensional frequency factor versus maximu depth factor Zmax

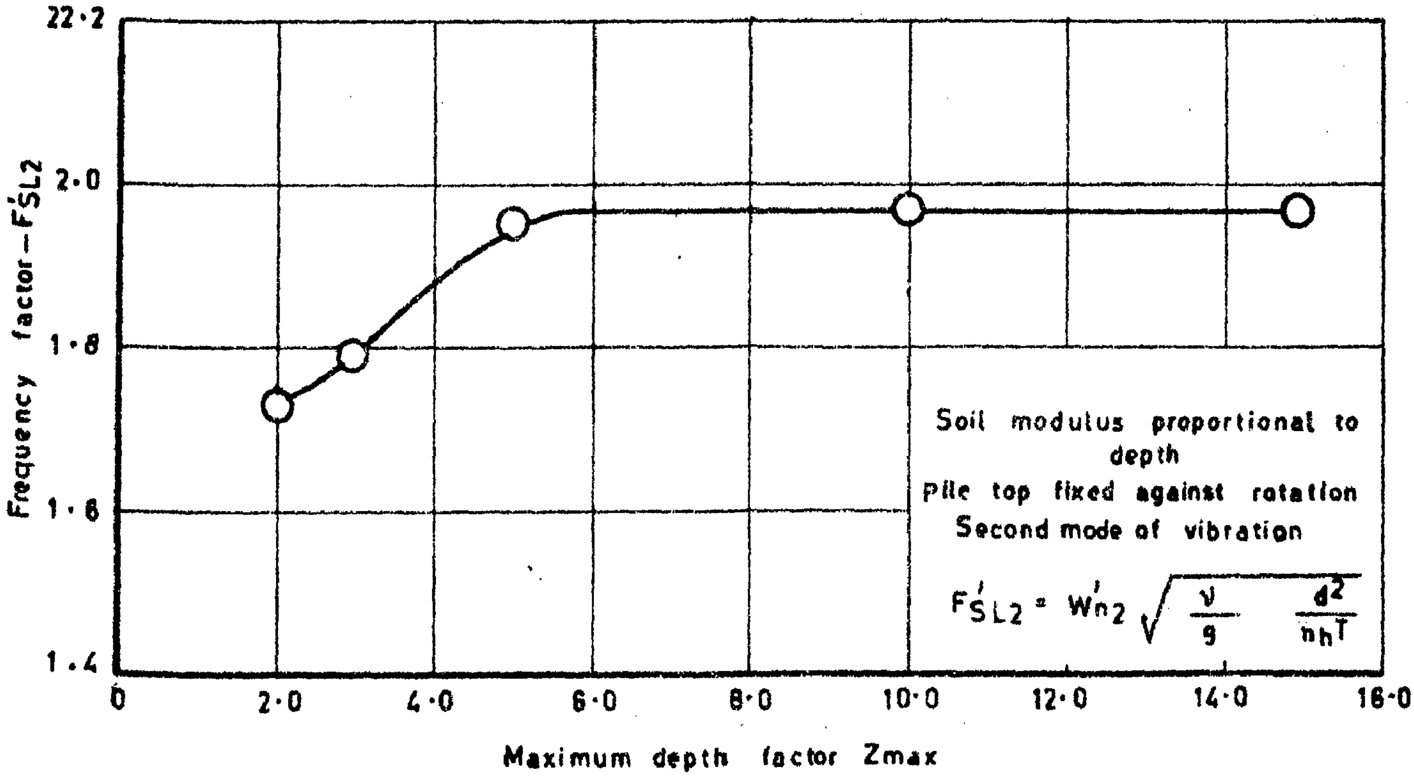
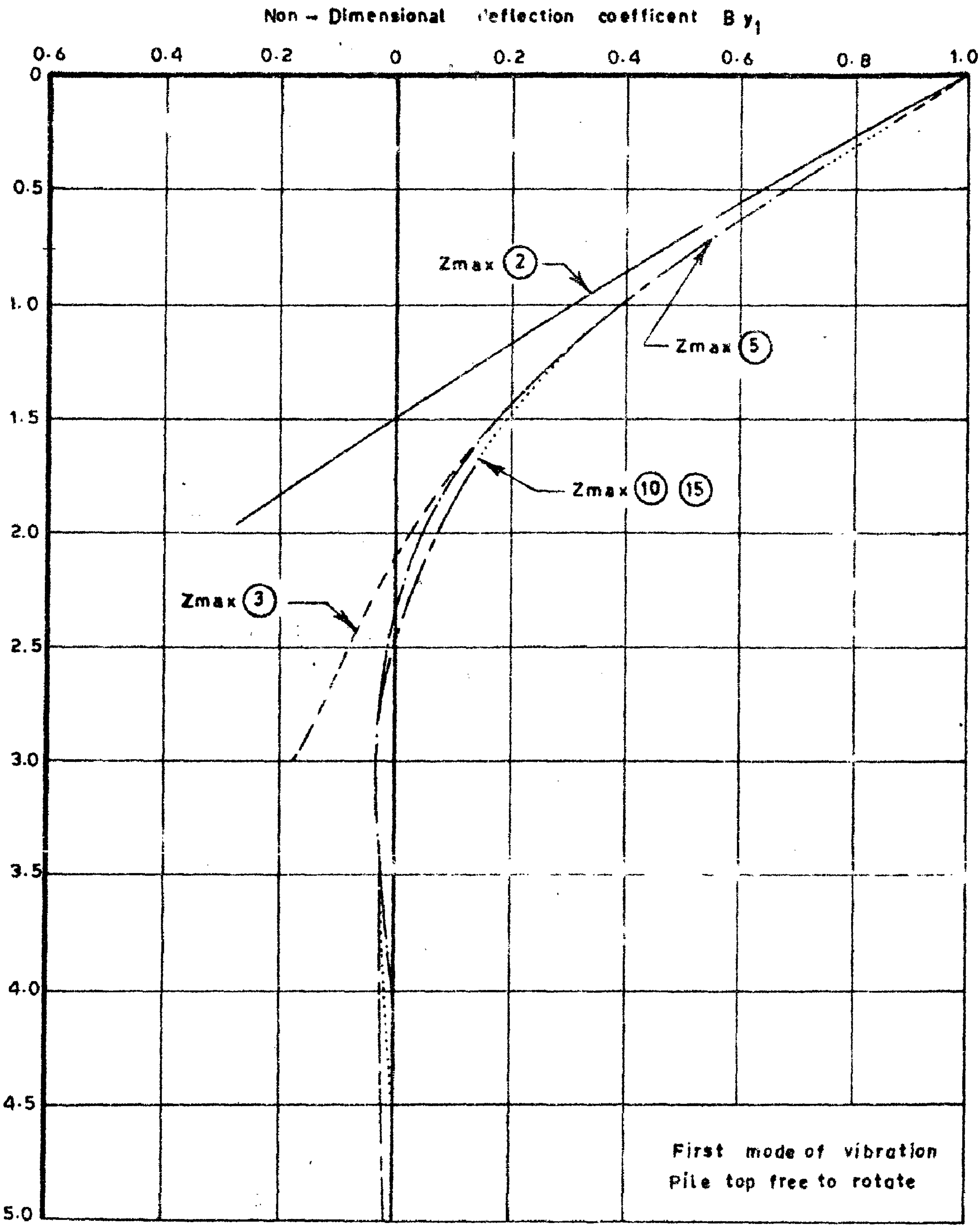


Fig. 5·8 Non – dimensional frequency factor versus maximu depth factor Zmax



ig. 5·9 Non-dimensional deflection coefficient assuming soil modulus proportional to depth

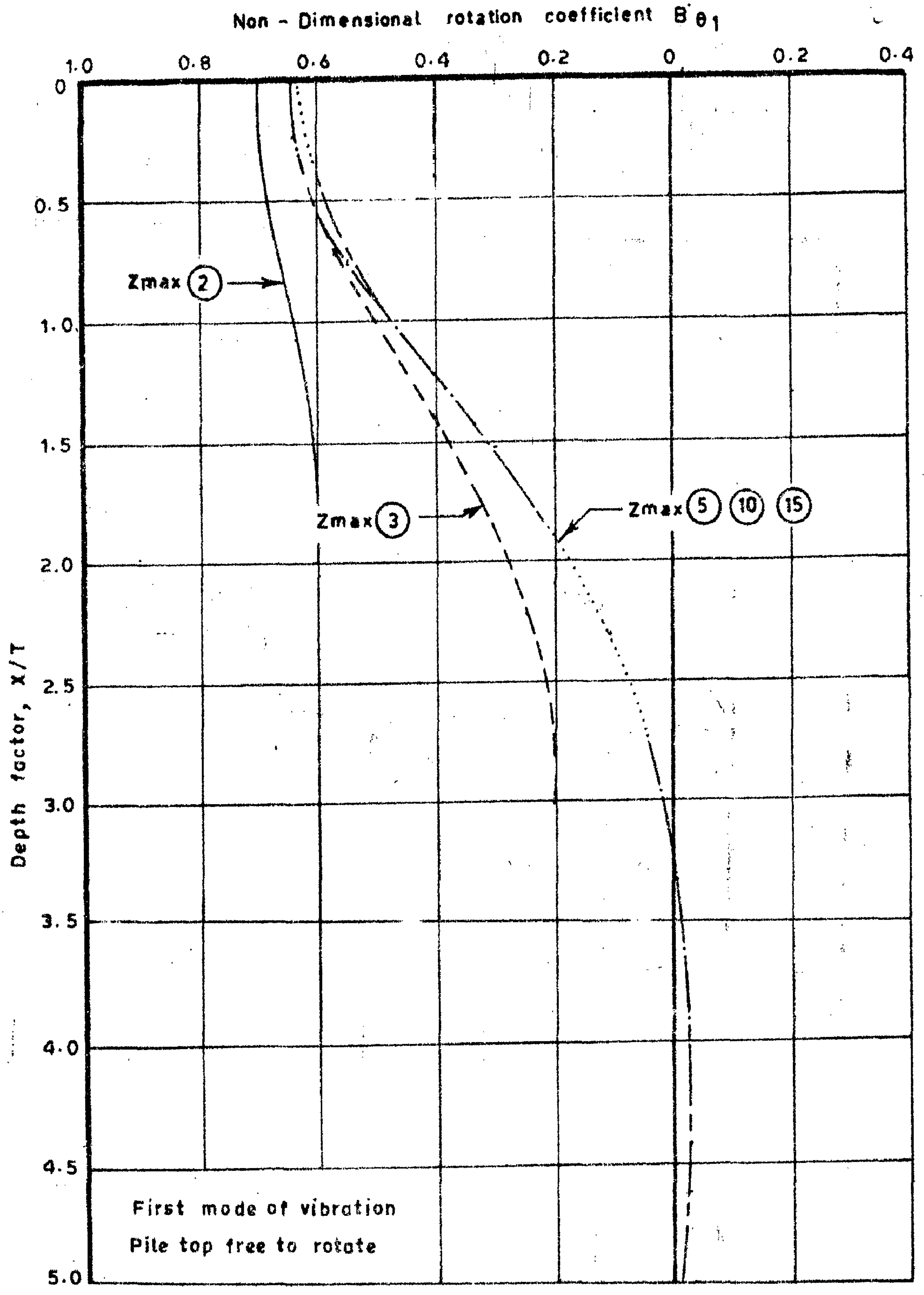
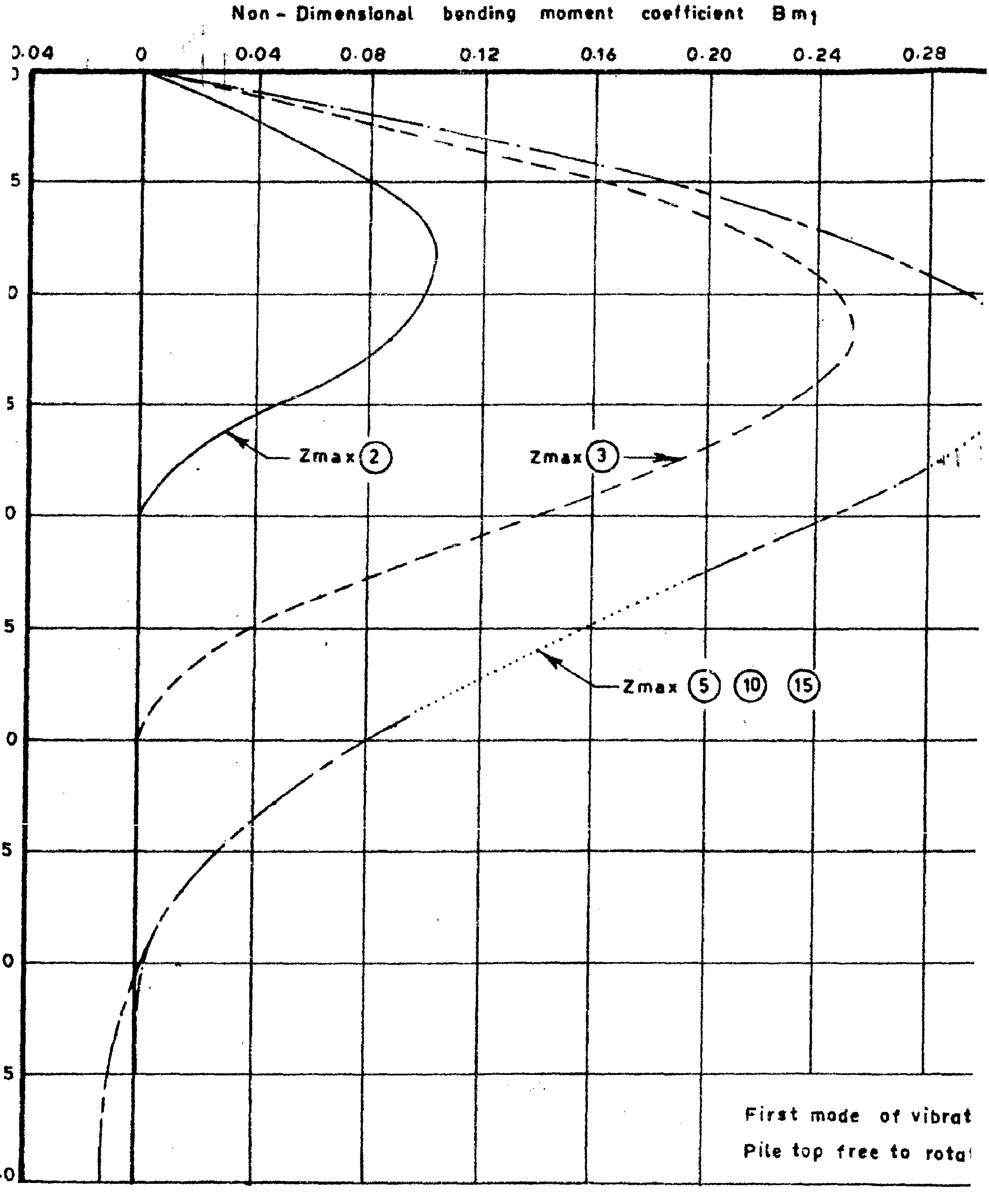


Fig. 5·10 Non-dimensional rotation coefficient assuming soil modulus proportional to depth



). 5·11 Non-dimensional bending moment coefficient assumi
soil modulus proportional to depth

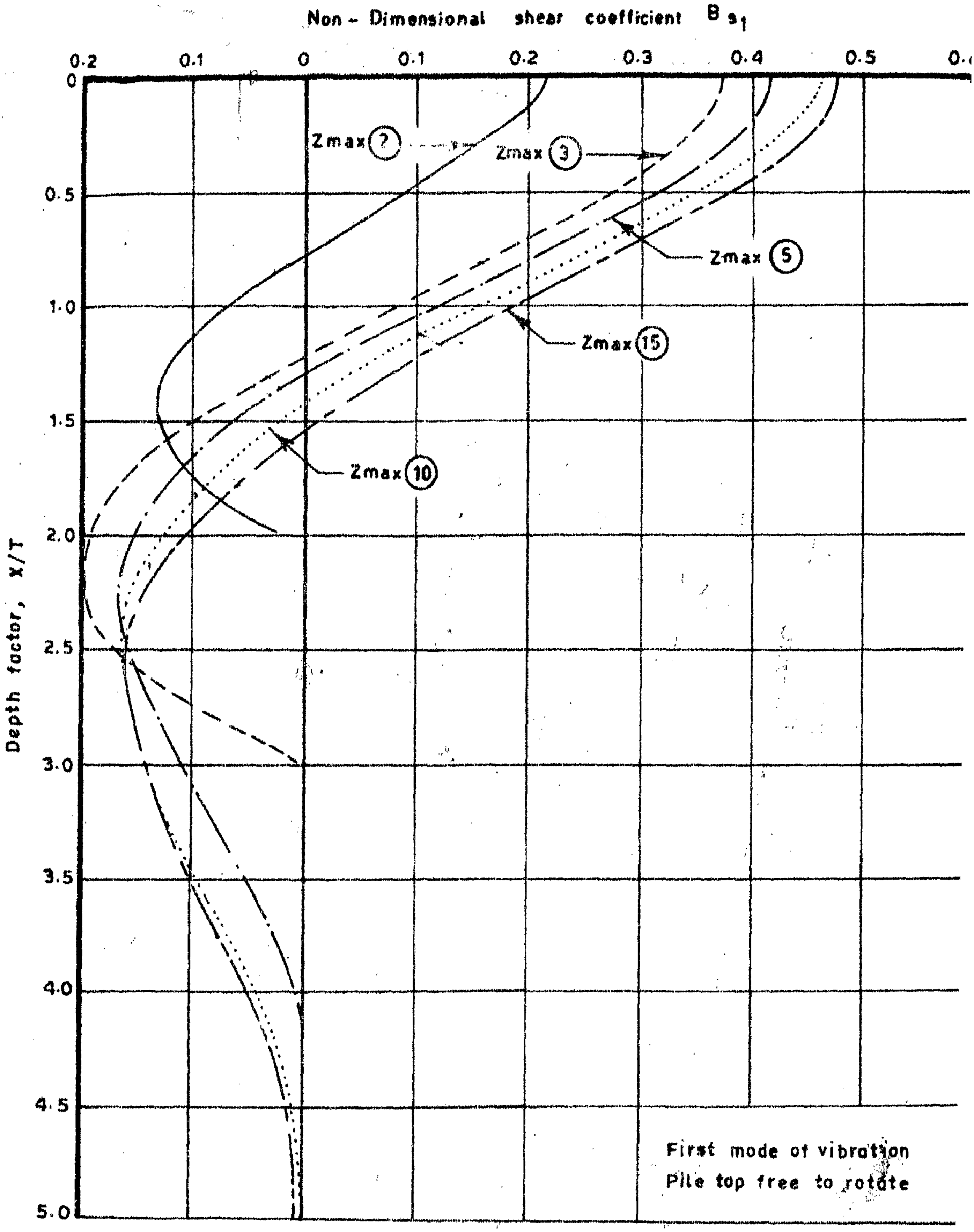


Fig. 5·12 Non-dimensional shear coefficient assuming soil modulus proportional to depth

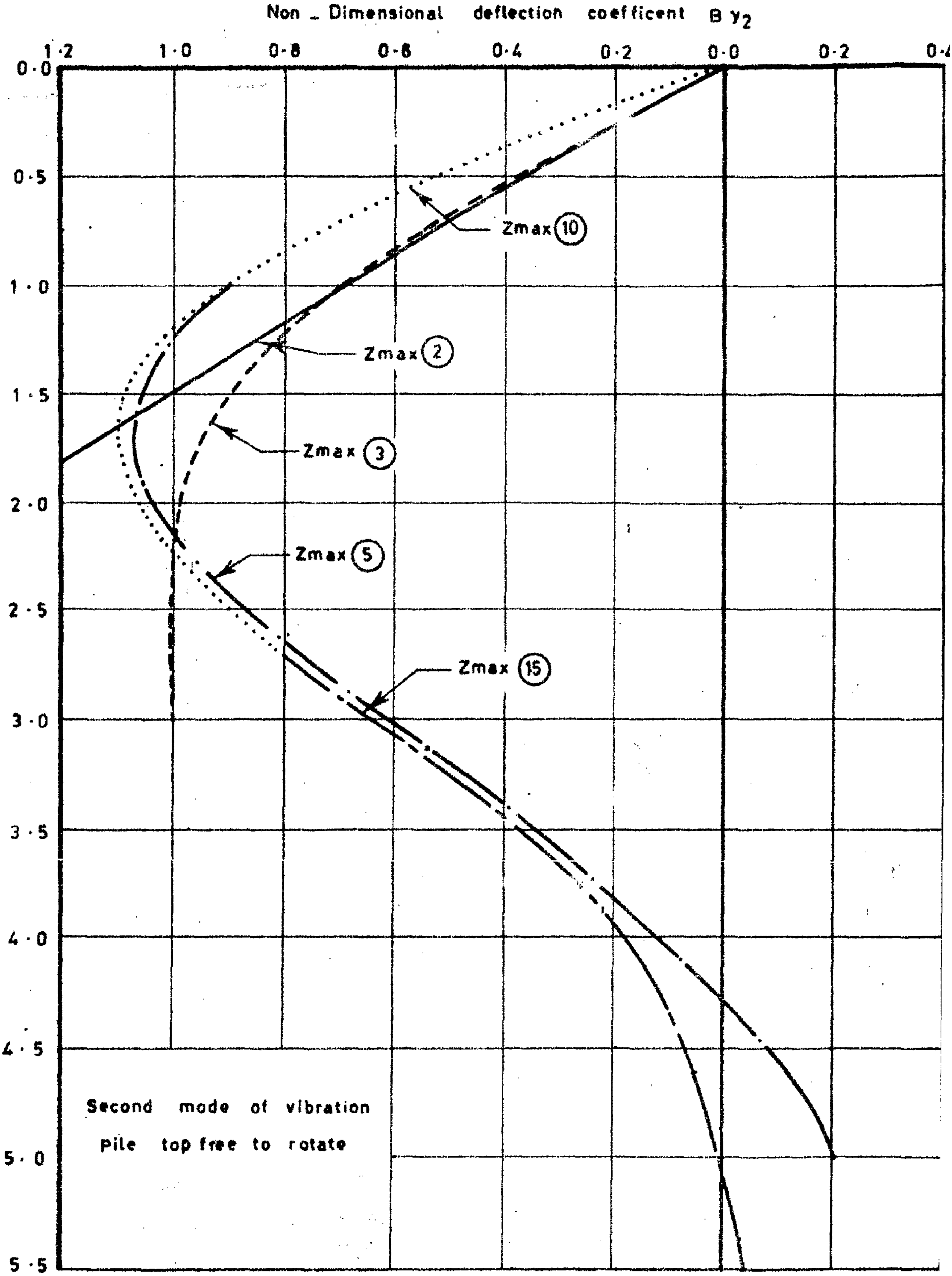


Fig.5·13 Non. dimensional deffection coefficient assuming soi
modulus proportional to depth

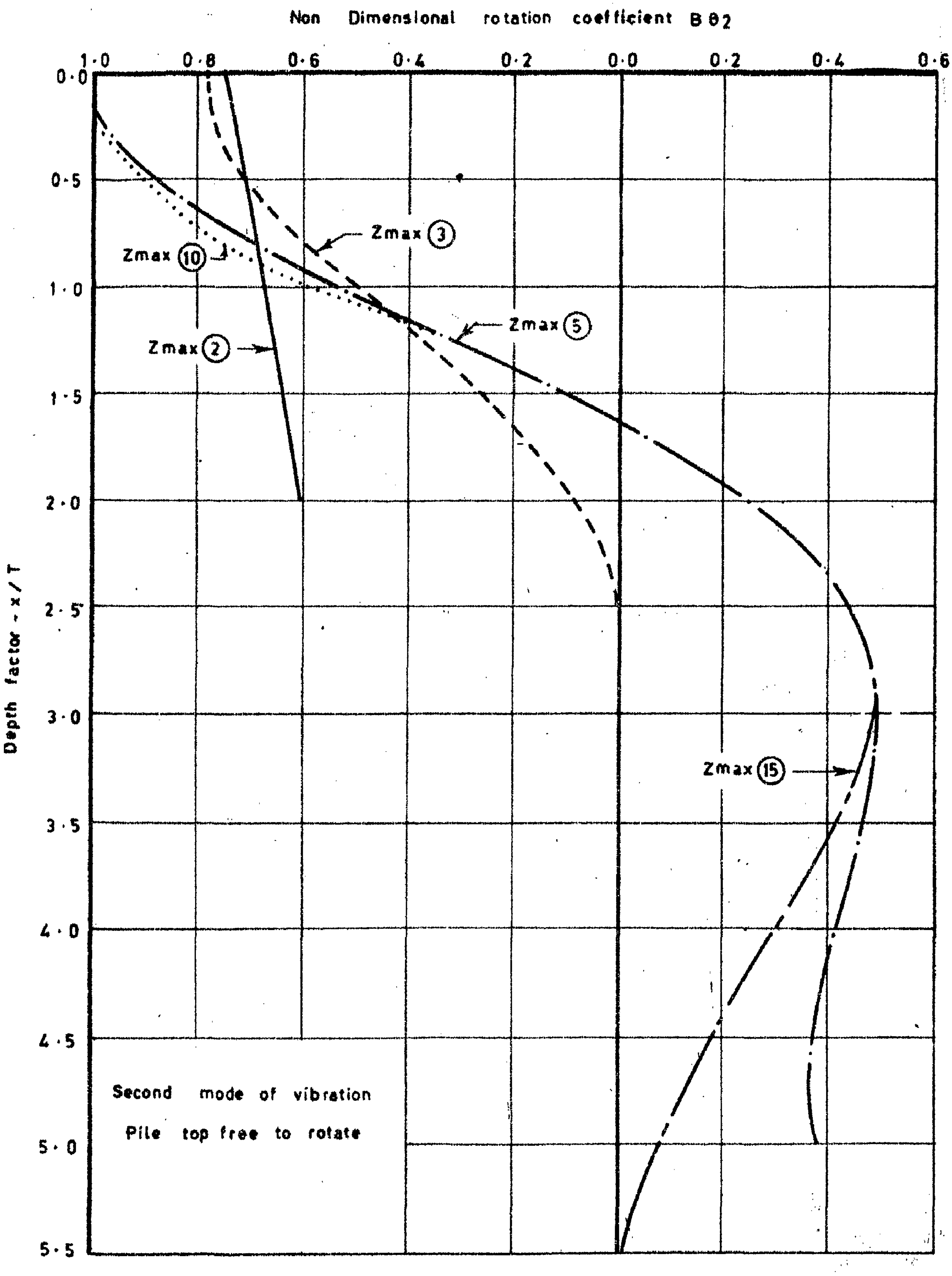


Fig. 5·14 Non. dimensional rotation coefficient assuming soil modulus proportional to depth

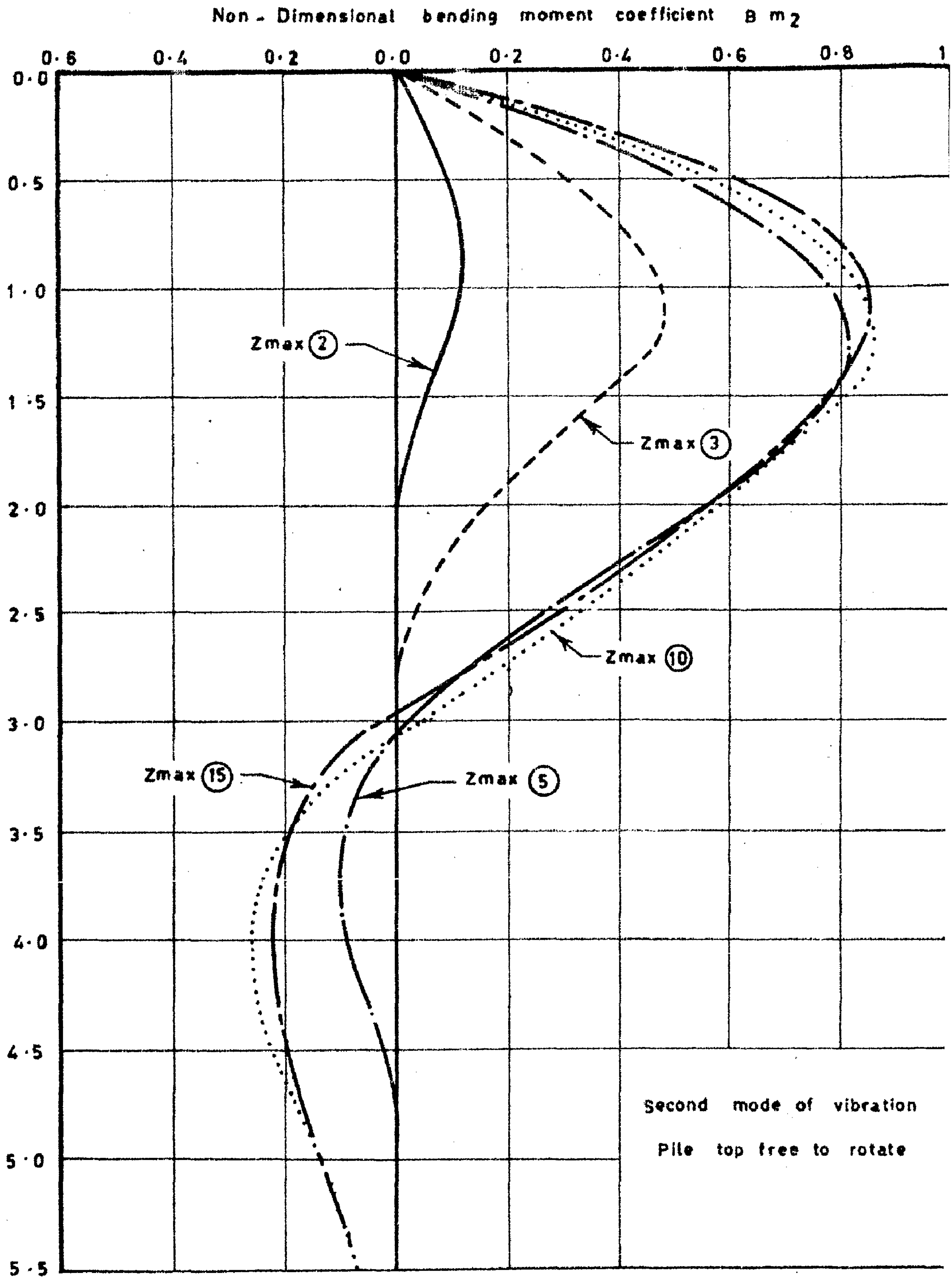


Fig. 5·15 Non. dimensional bending moment coefficient assu
soil modulus proportional to depth

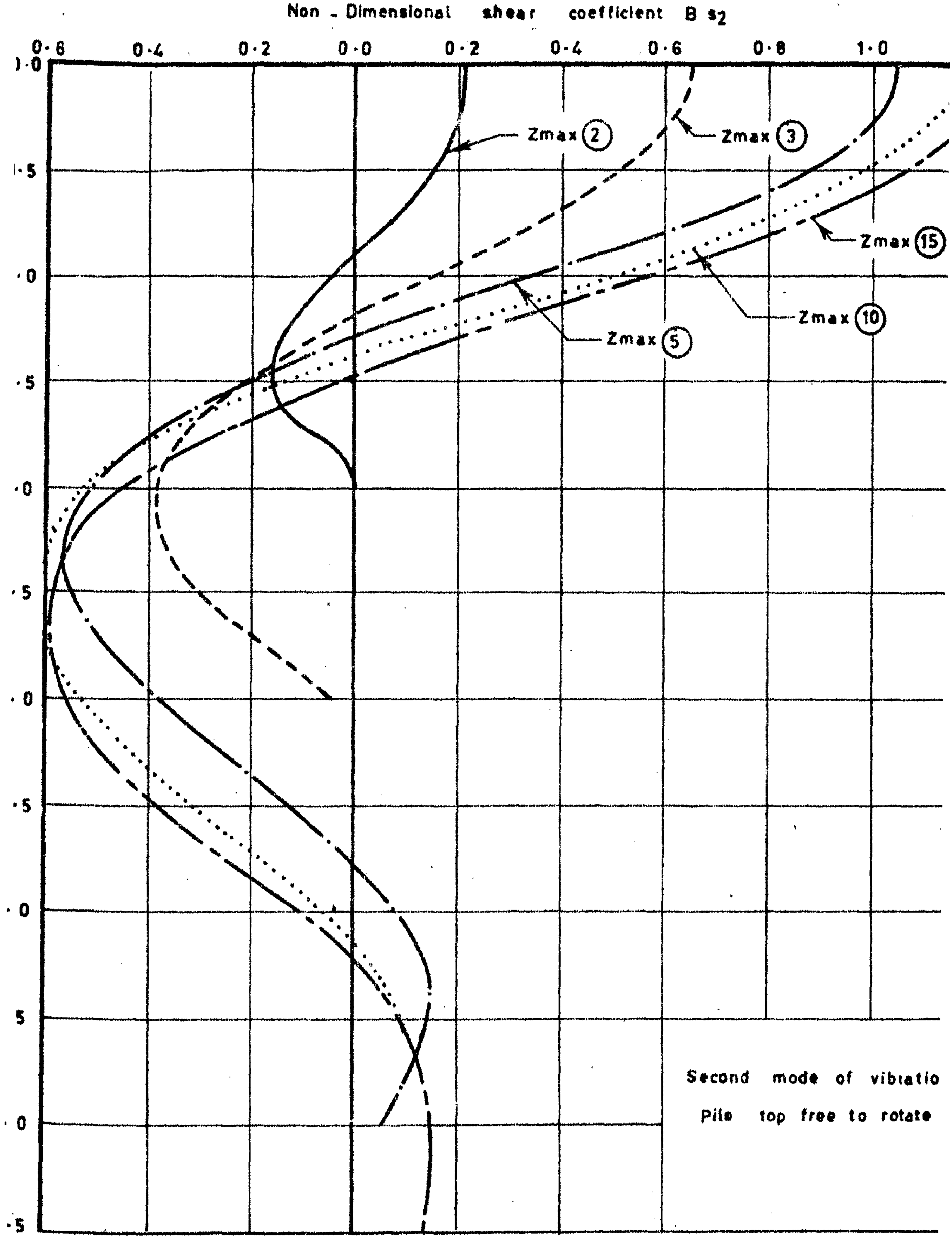
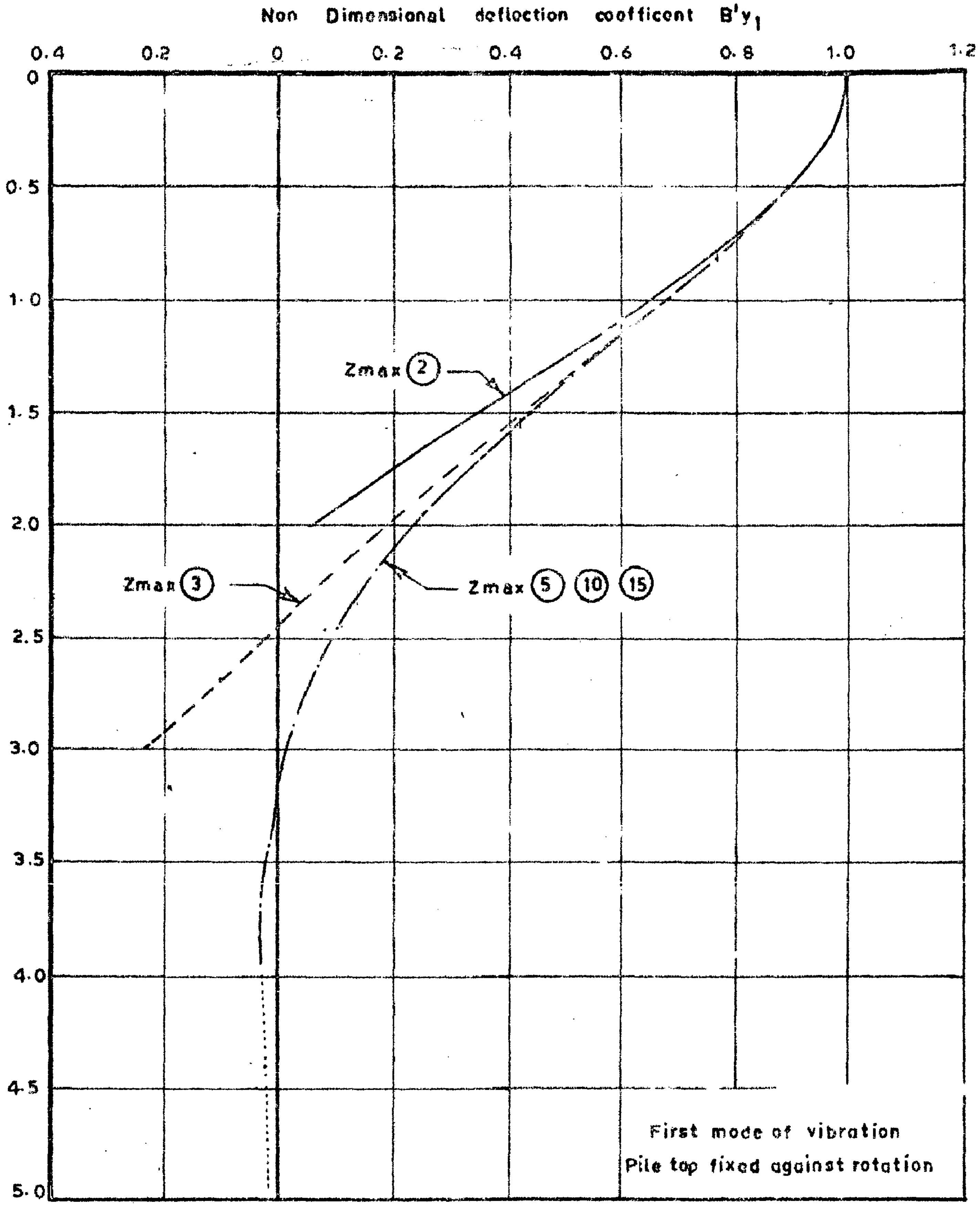


Fig. 5·16 Non. dimensional shear coefficient assuming soi modulus proportional to depth



g. 5·17 Non-dimensional deflection coefficient assuming soil modulus proportional to depth

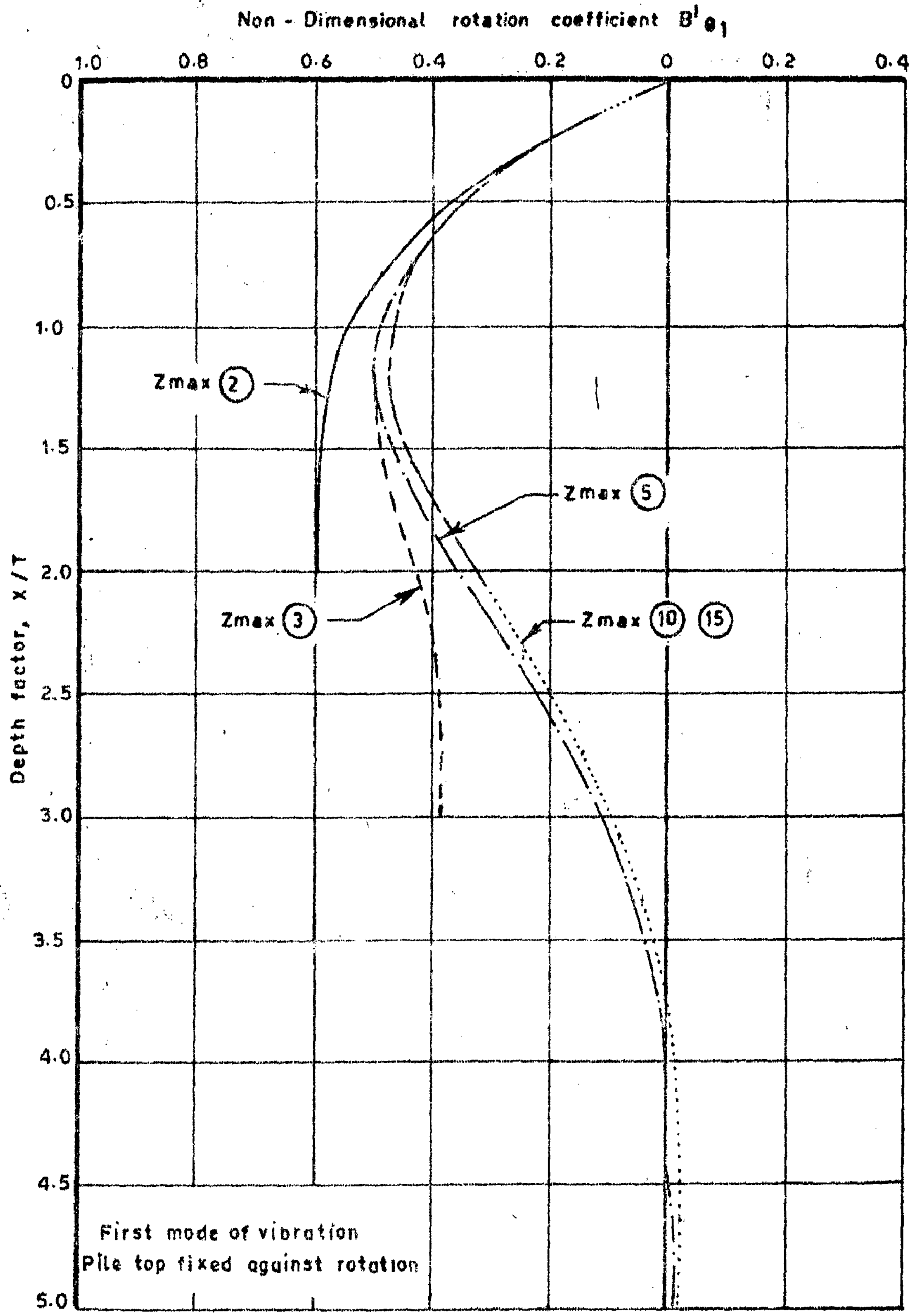
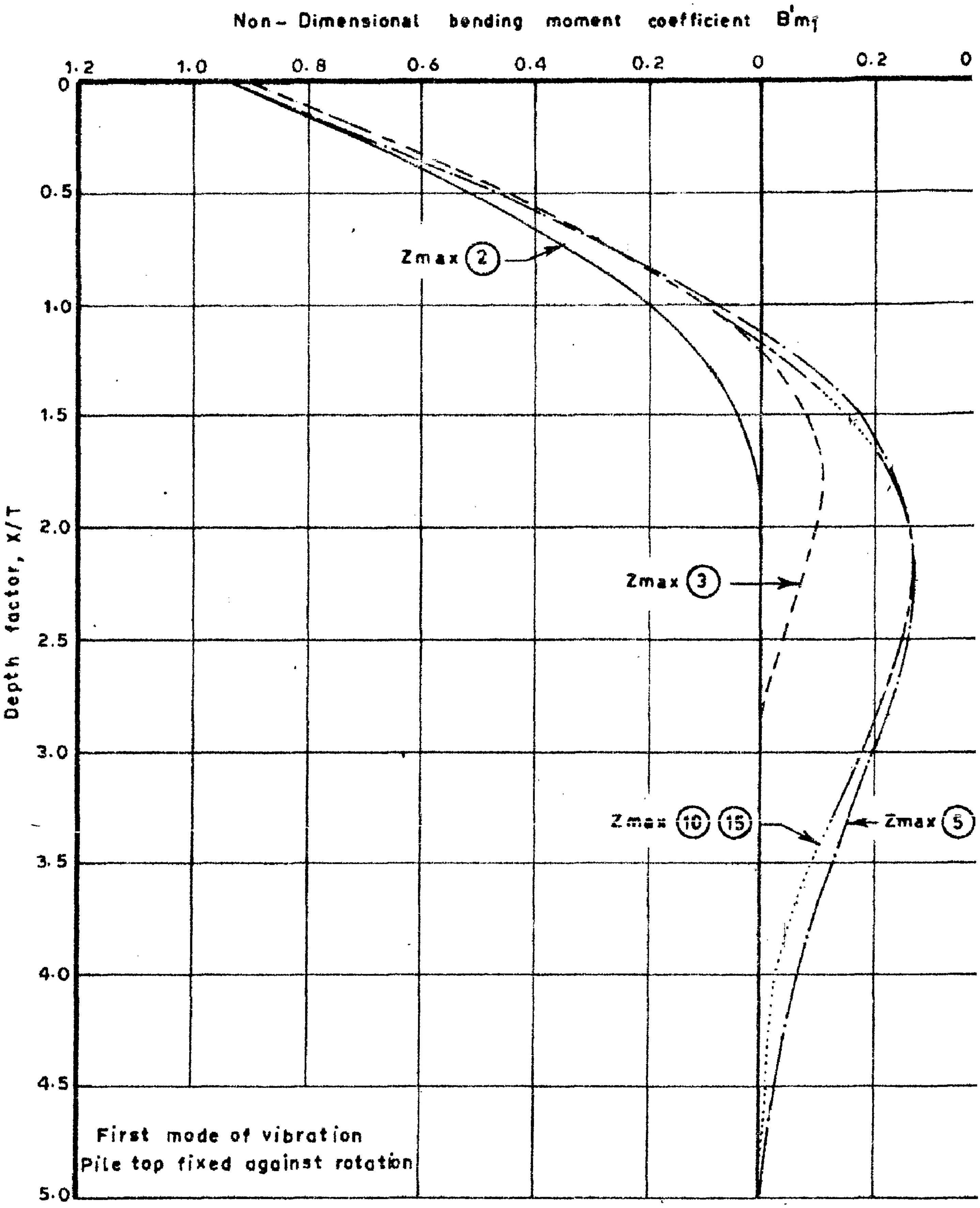


Fig. 5·18 Non-dimensional rotation coefficient assuming soil modulus proportional to depth



g. 5·19 Non-dimensional bending moment coefficient assum
soil modulus proportional to depth

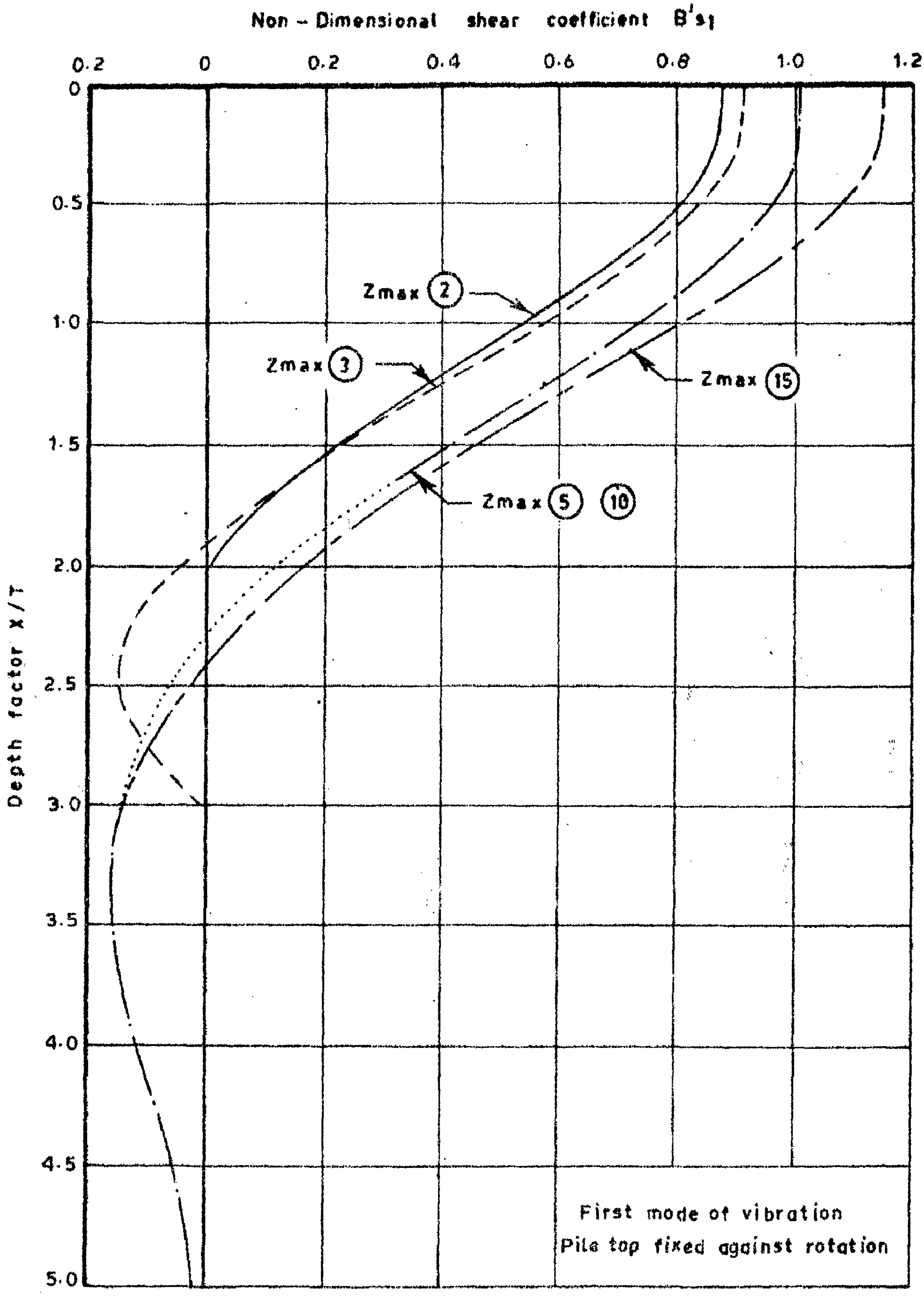


Fig. 5·20 Non-dimensional shear coefficient assuming soil modulus proportional to depth

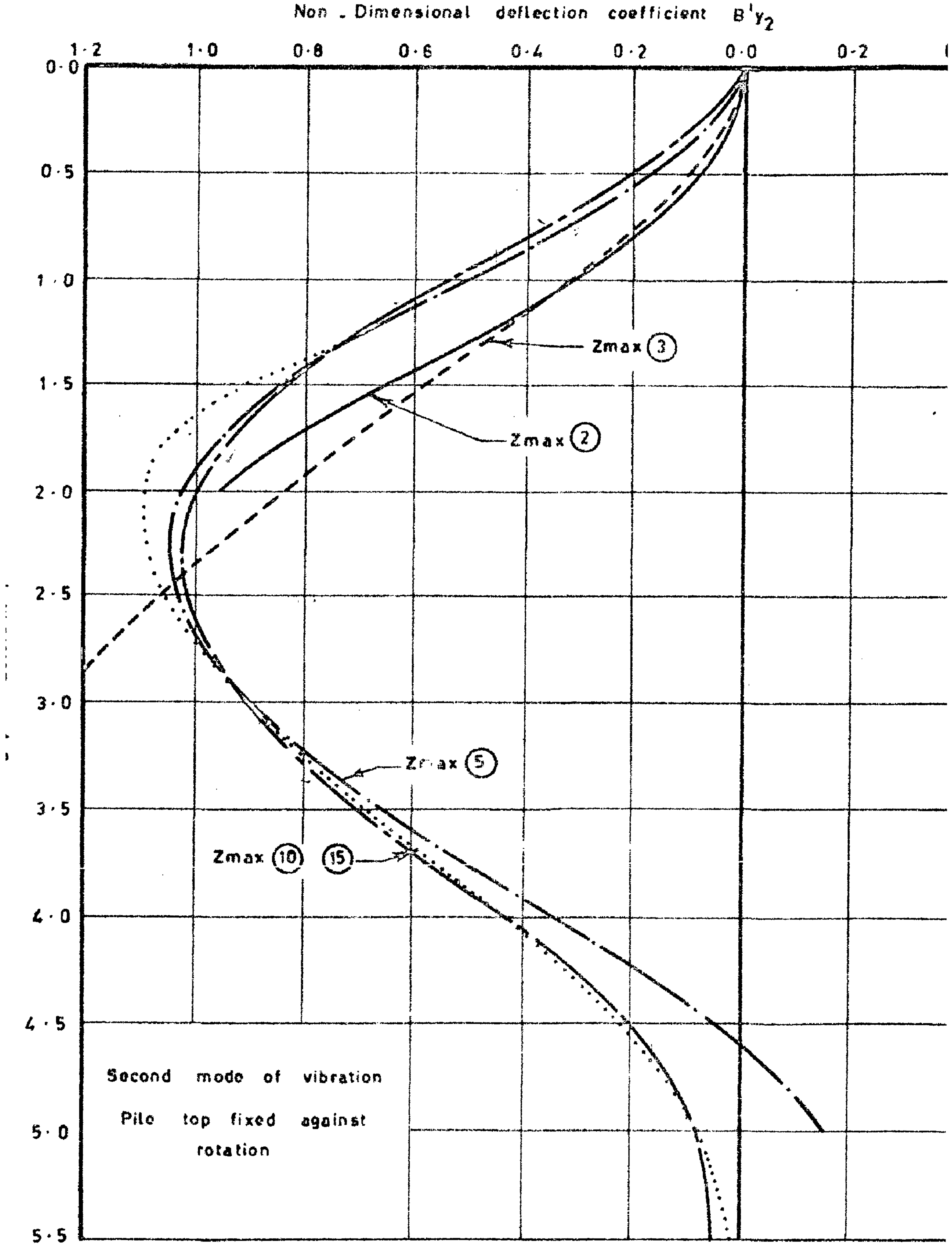


Fig. 5·21 Non. dimensional deflection coefficient assuming modulus proportional to depth

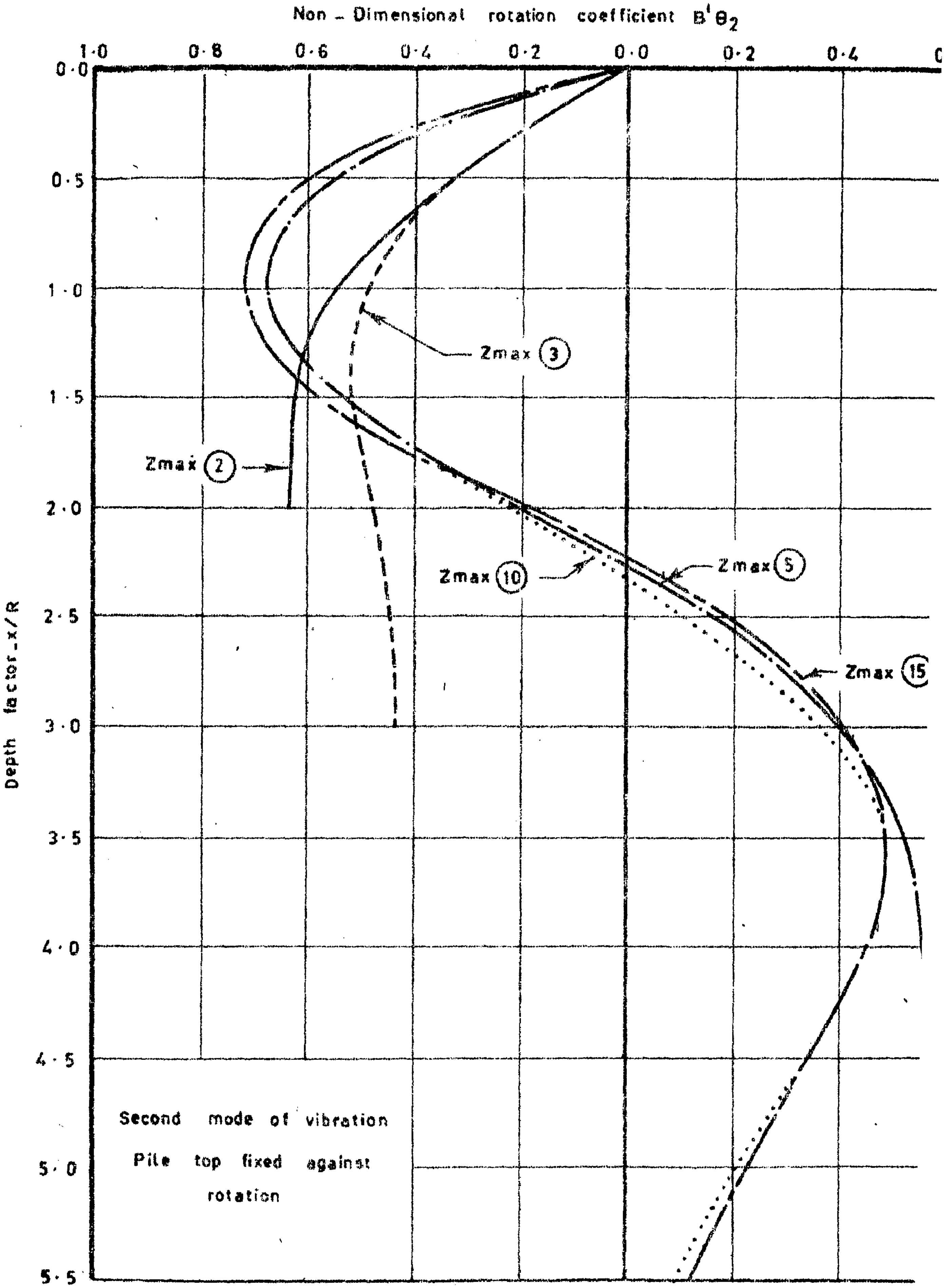


Fig. 5·22 Non. dimensional rotation coefficient assuming modulus proportional to depth

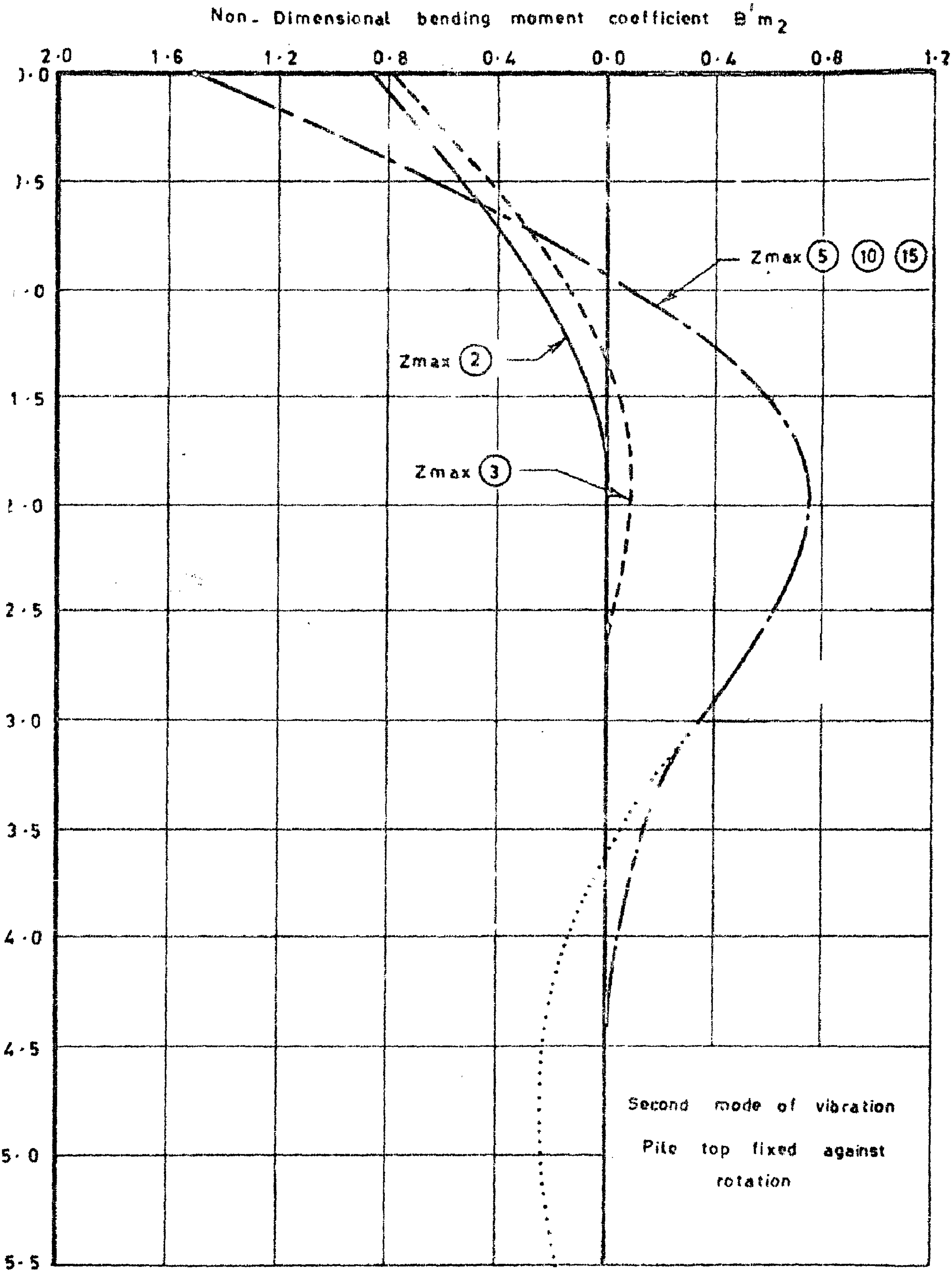


Fig. 5·23 Non. dimensional bending moment coefficient assu
soil modulus proportional to depth

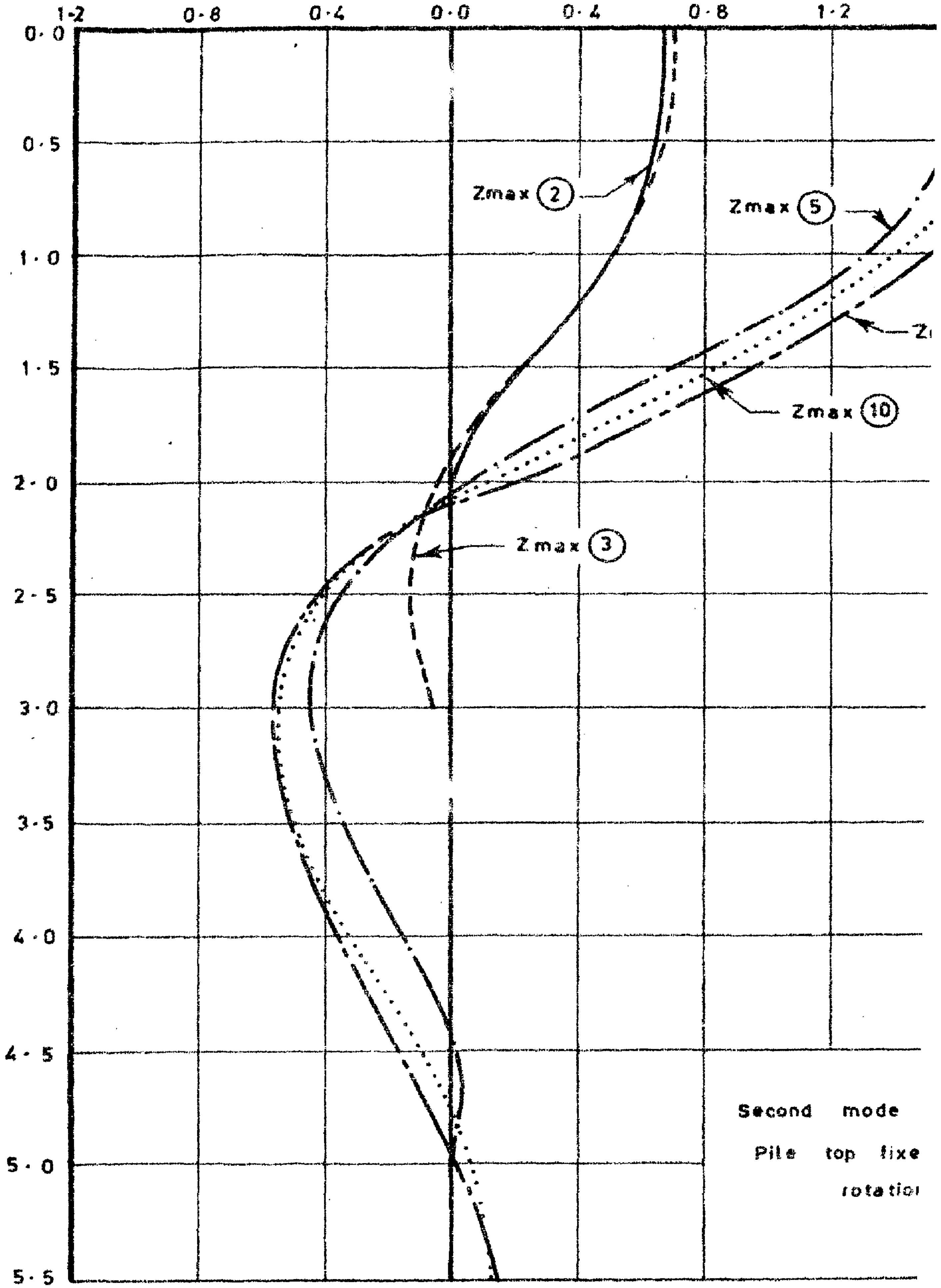


Fig. 5·24 Non. dimensional shear coefficient assur
modulus proportional to depth

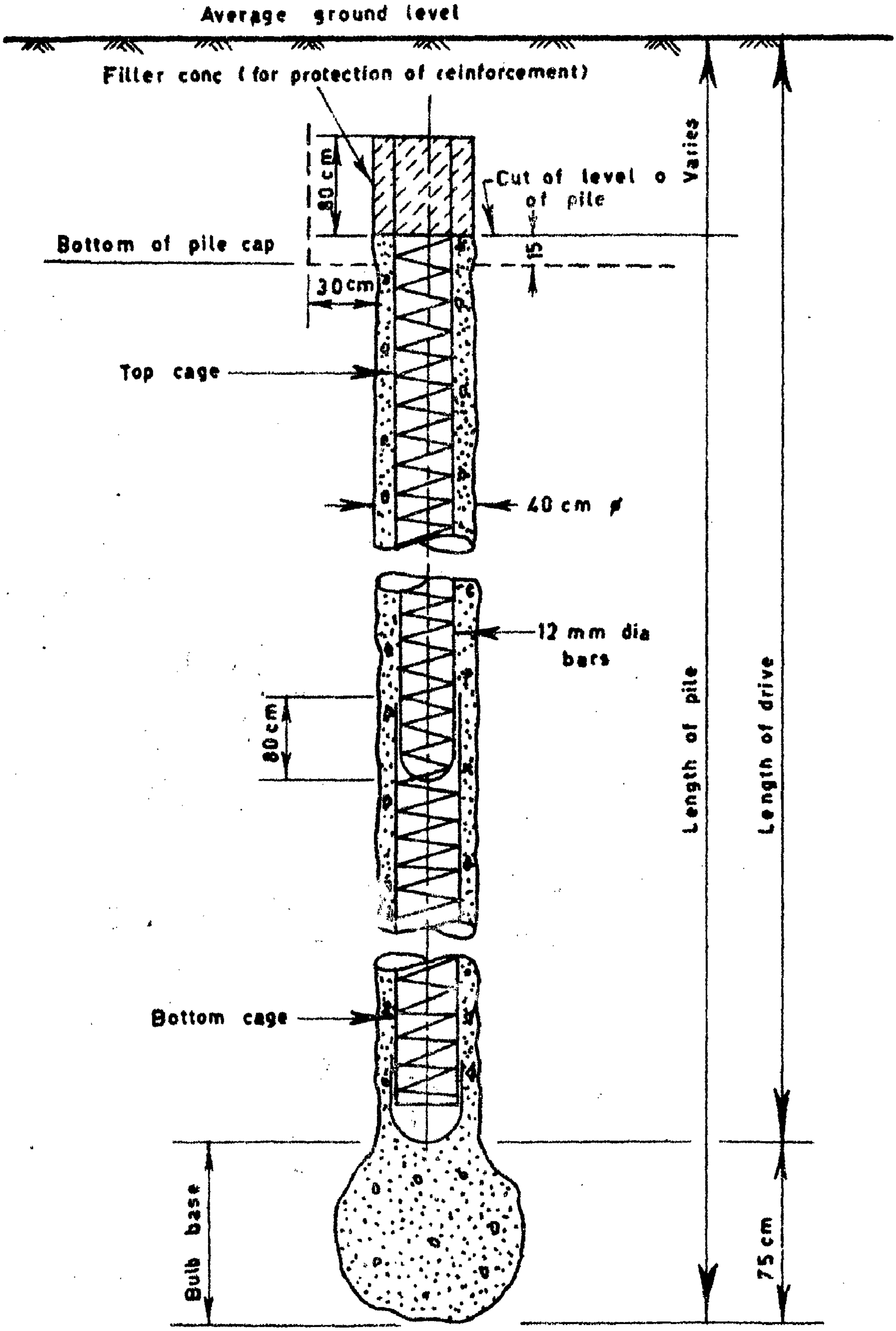


Fig. 6·1a Franki pile sectional details

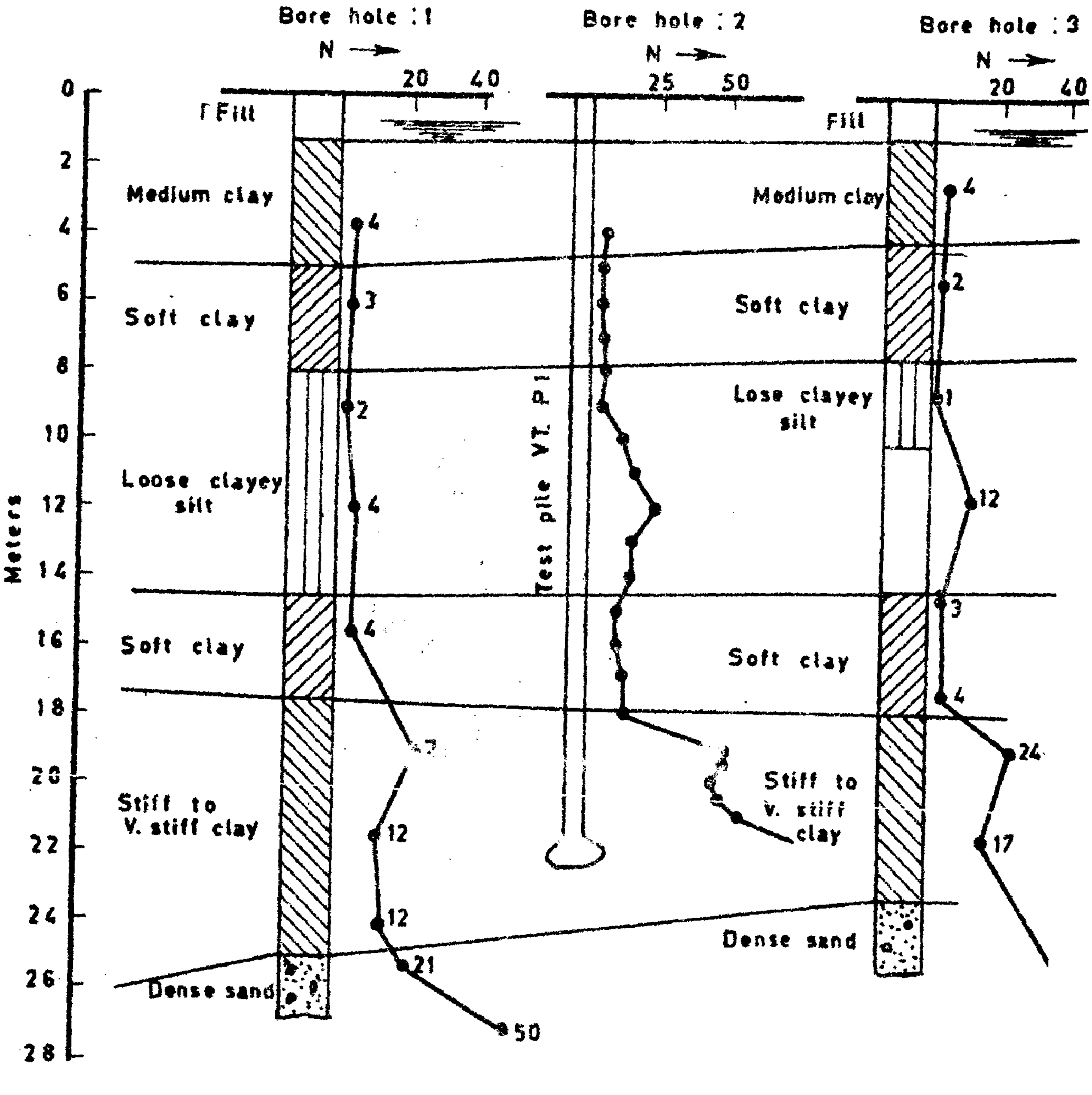


Fig. 6·1b Soil condition near franki piles

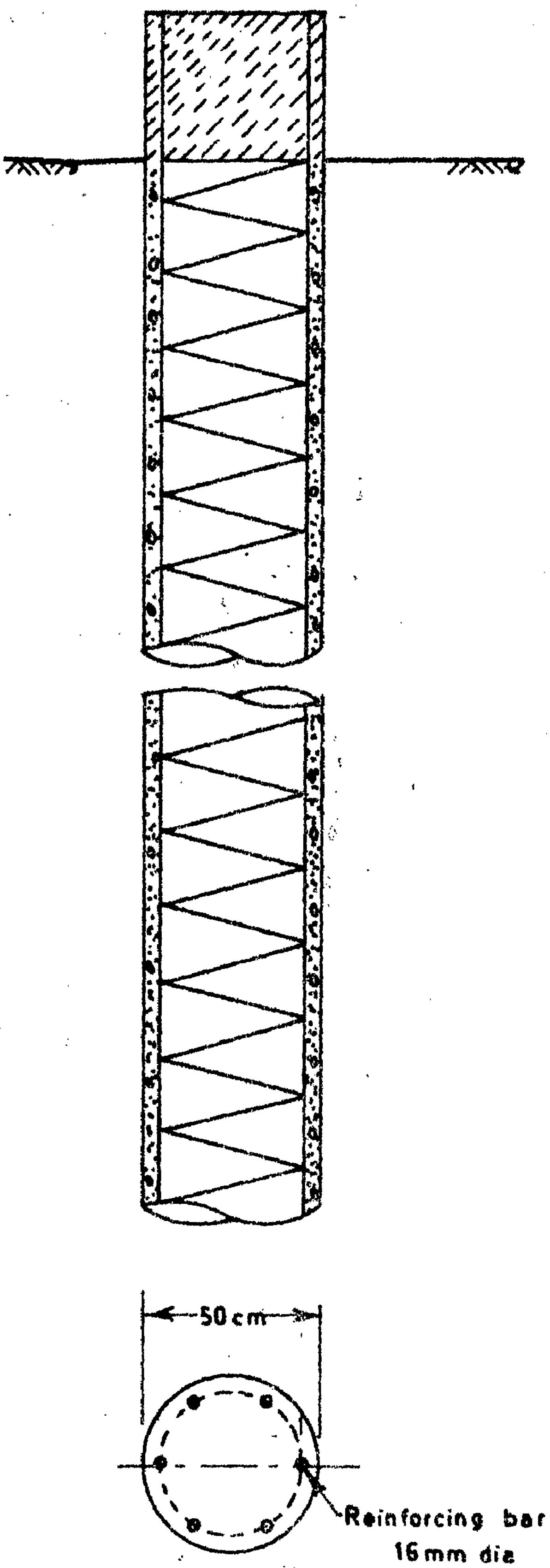


Fig. 6·2a Simplex pile section

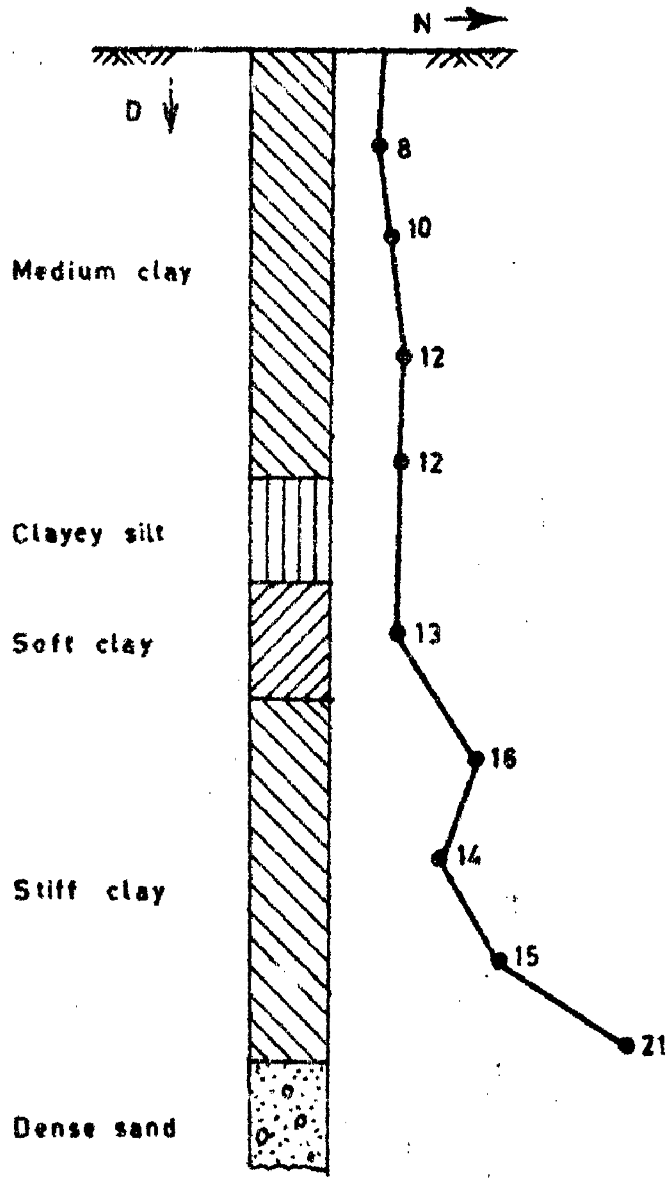


Fig. 6·2b Soil condition near pile section

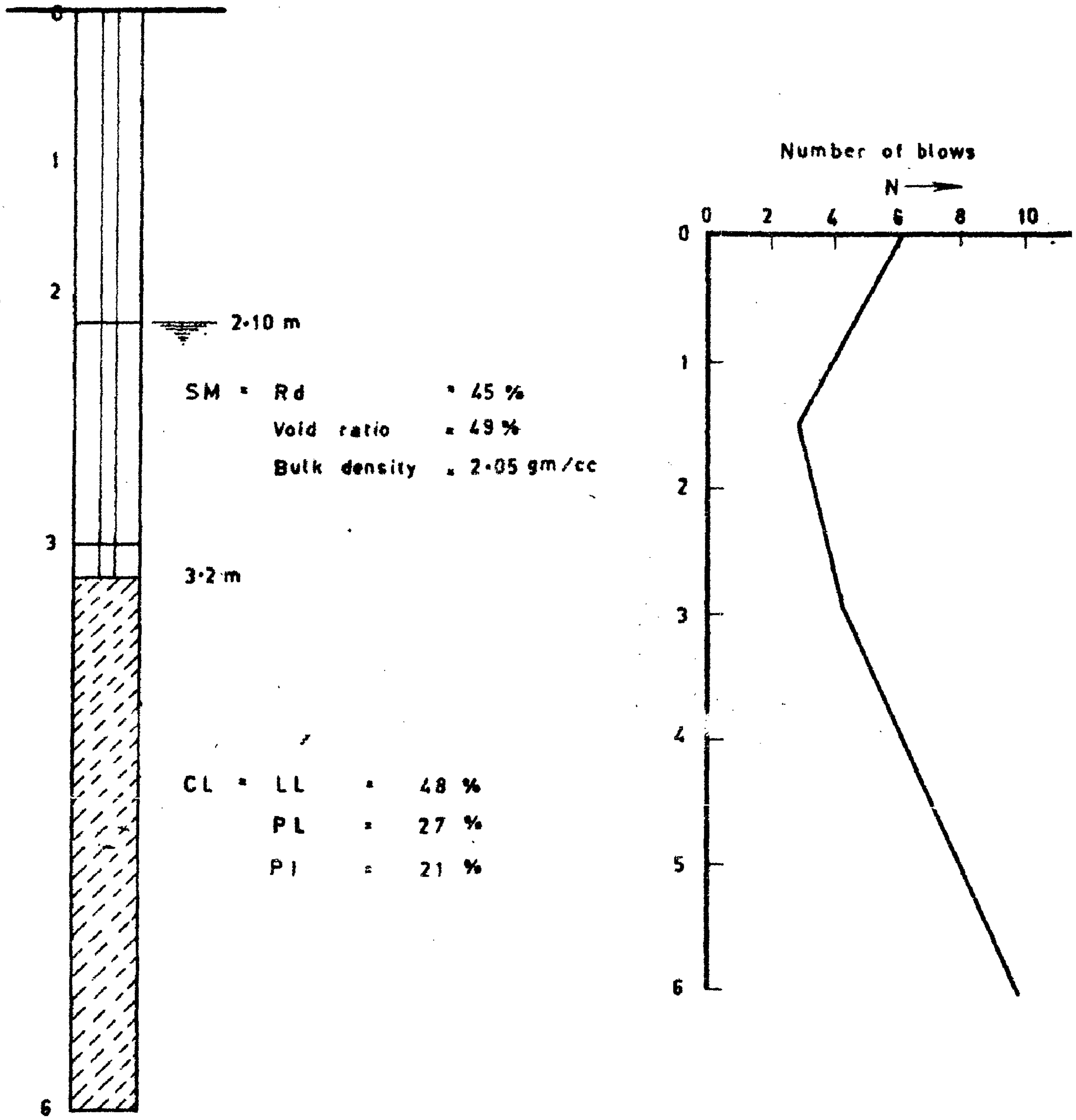


Fig. 6·3 Soil conditions near piles VTP 5 & VTP

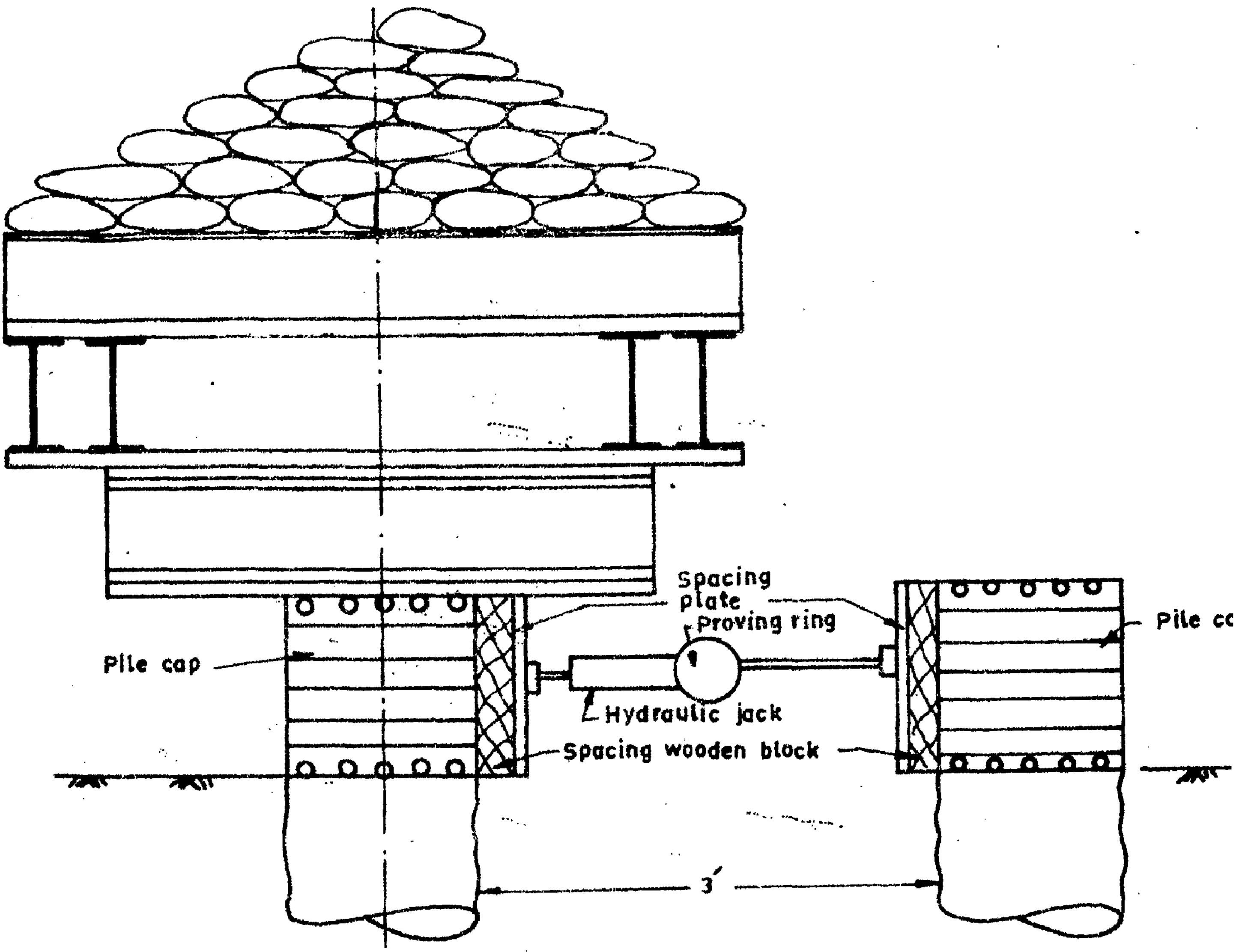


Fig. 6·4 Lateral load test set-up

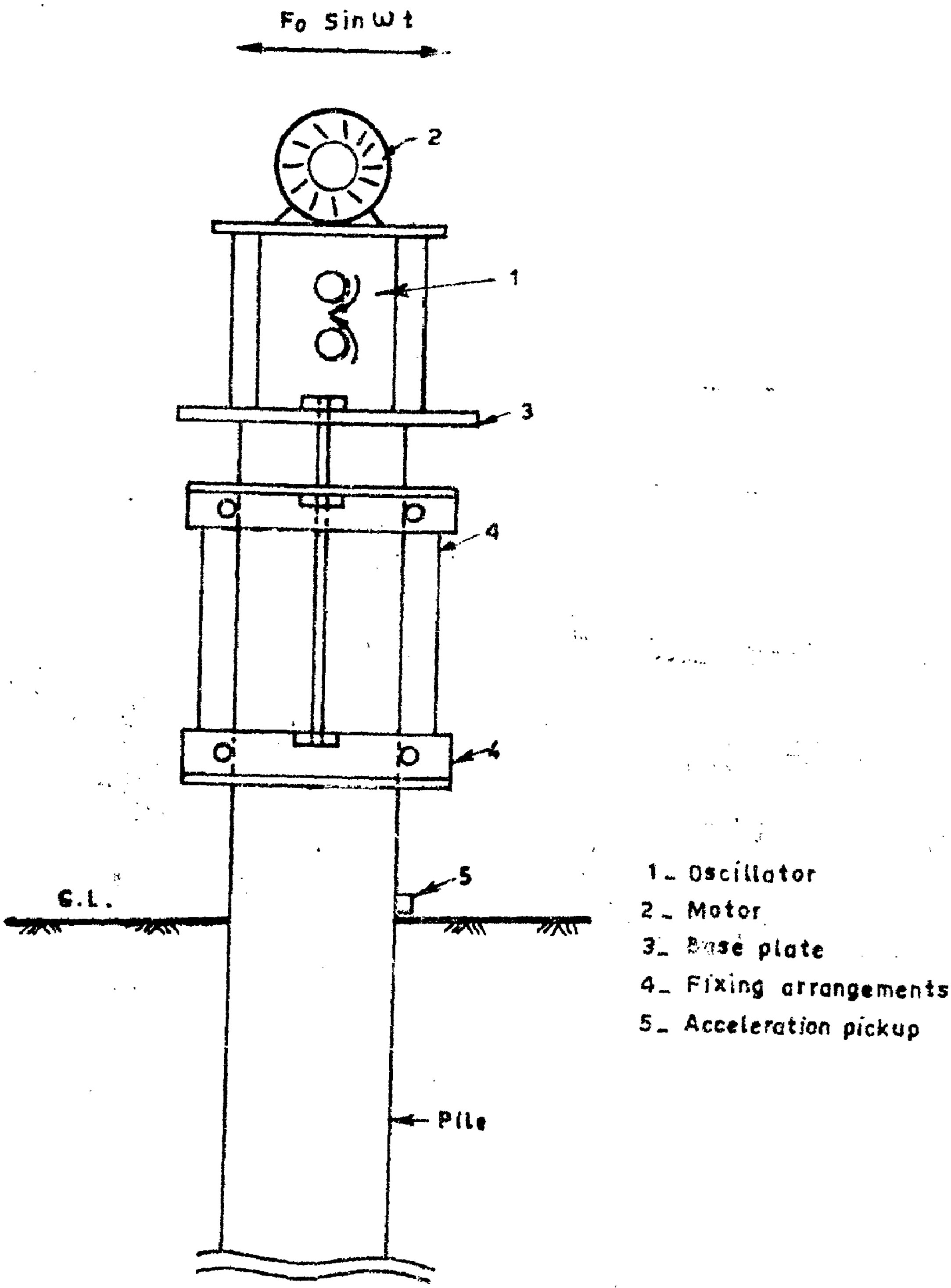


Fig. 6·5 Lateral forced vibration test set-up

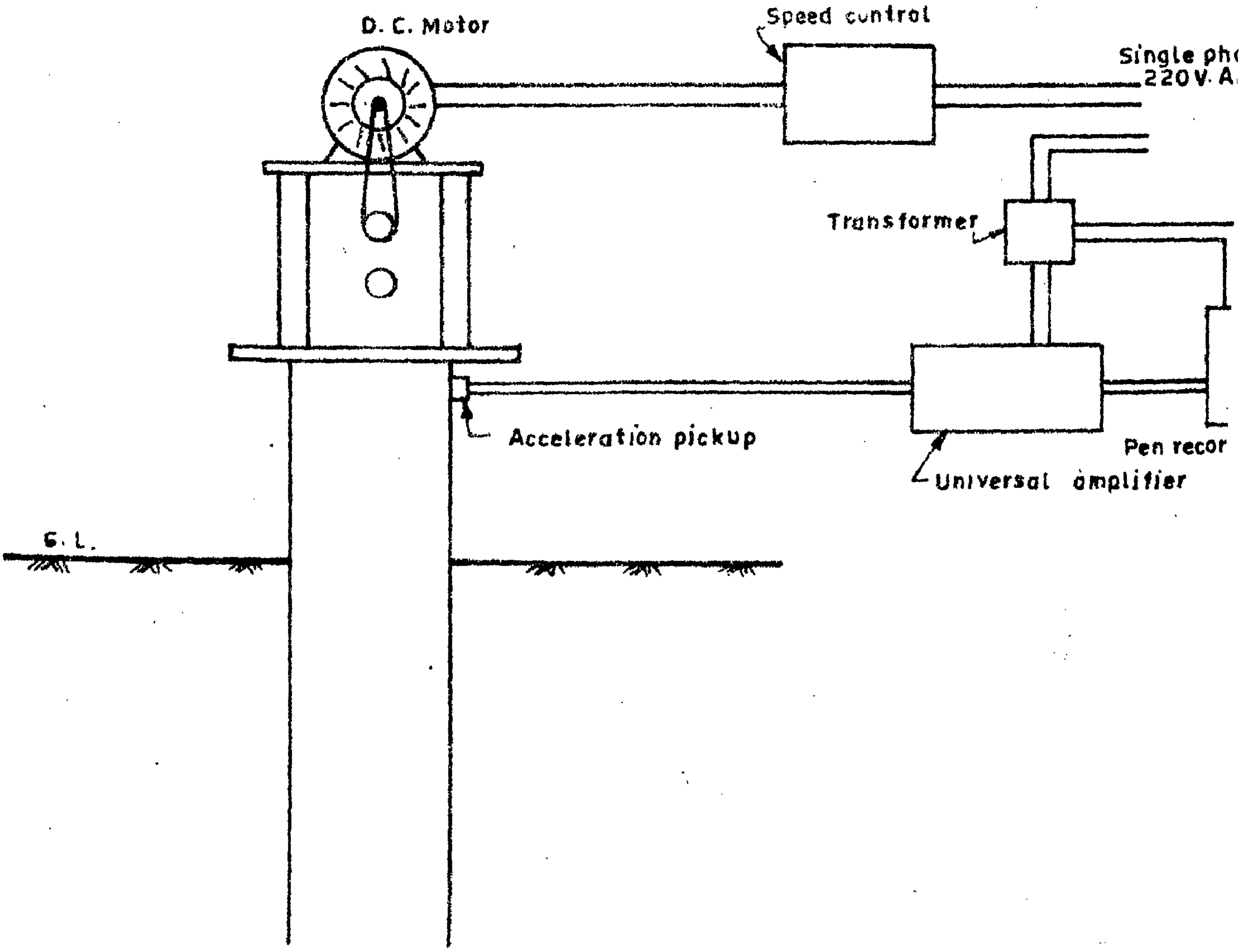


Fig. 6·6 Block diagram for instrumentation

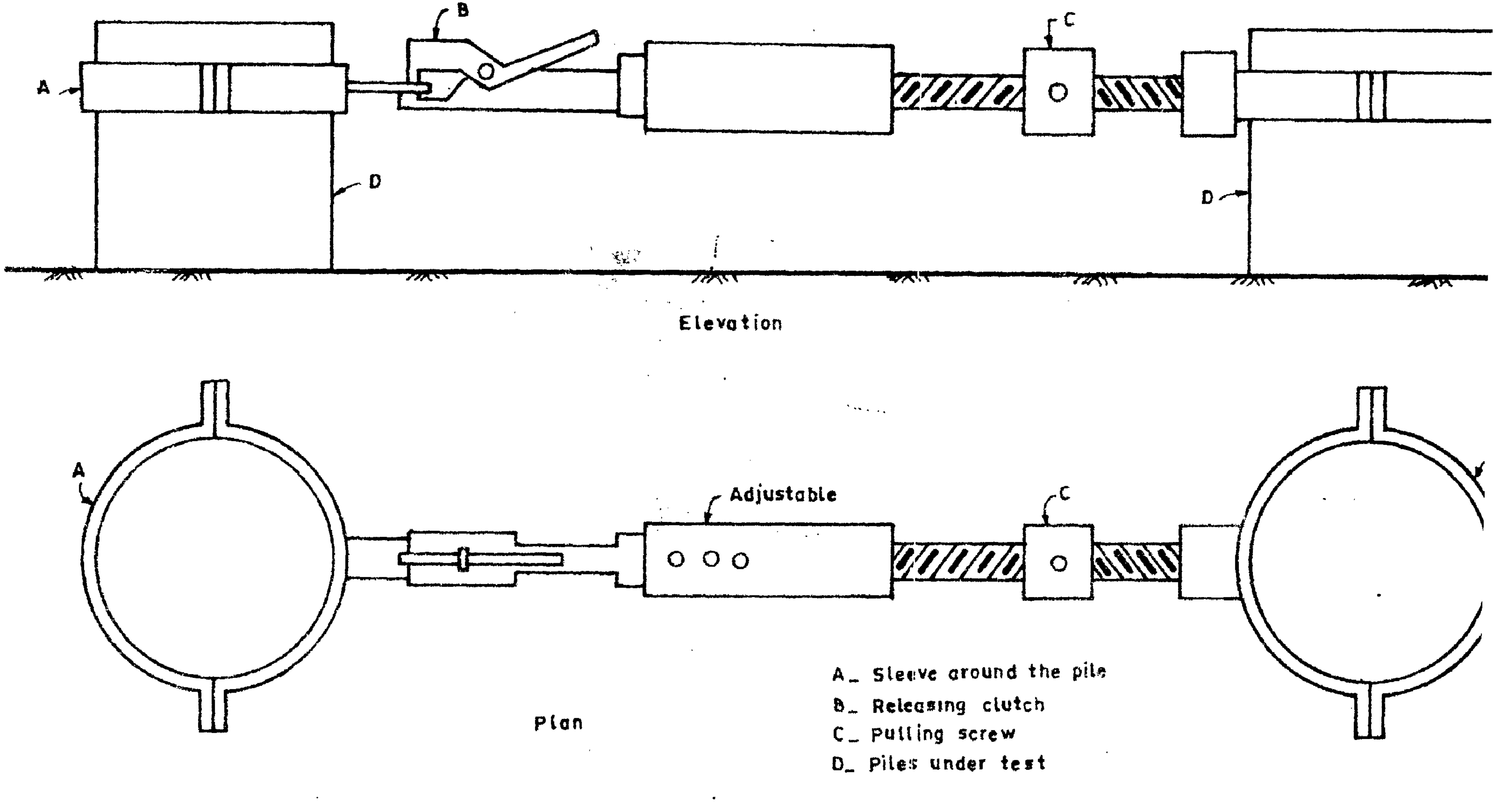


Fig. 6.7 Release arrangement for free vibration tests

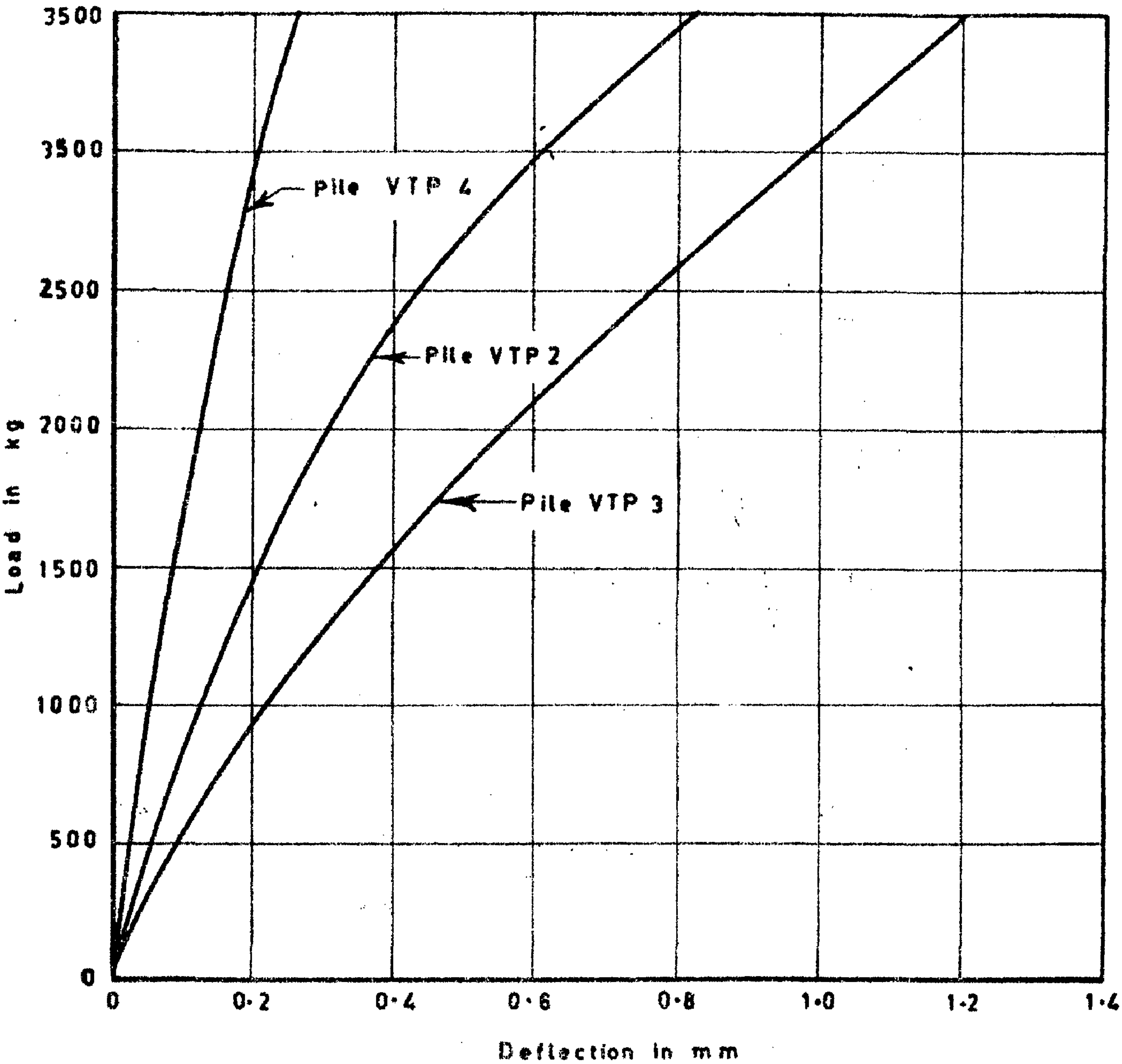


Fig. 6·8 Lateral load deflection plots of piles VTP 2, VTP 3 & VTP 4

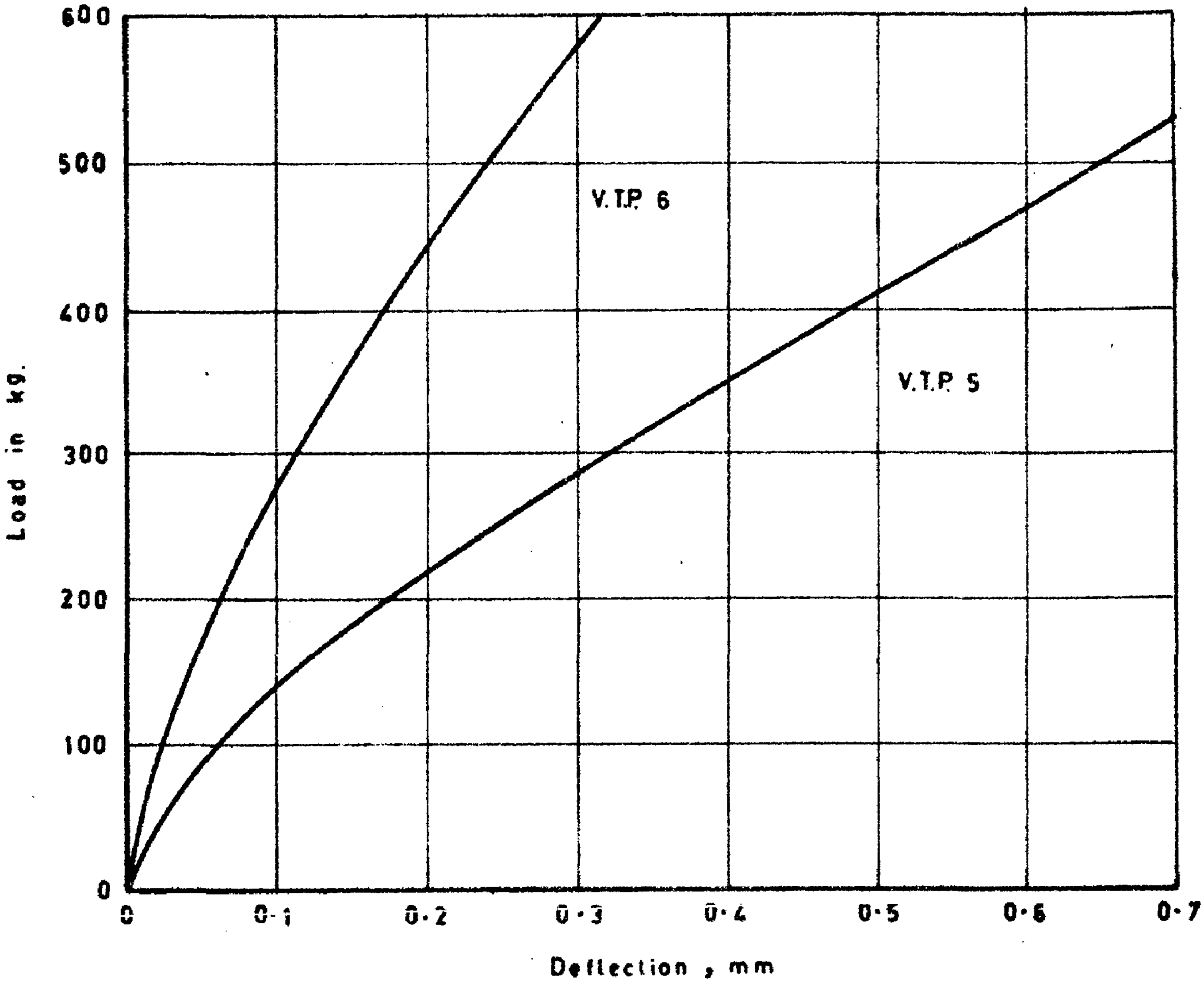
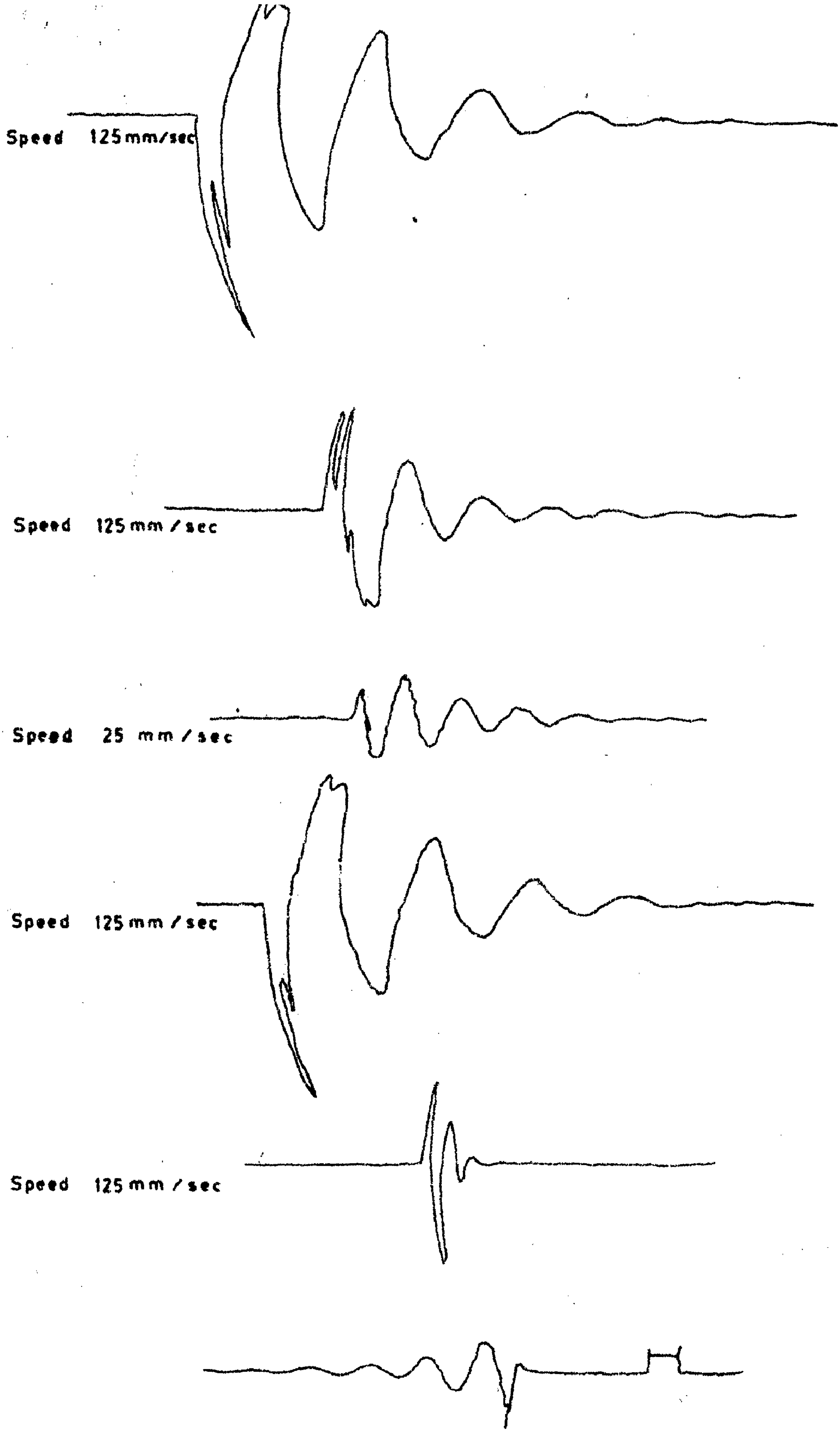


Fig. 6·9 Lateral load deflection plots of piles VTP 5 and VTP 6



[illegible] Typical free vibration test records

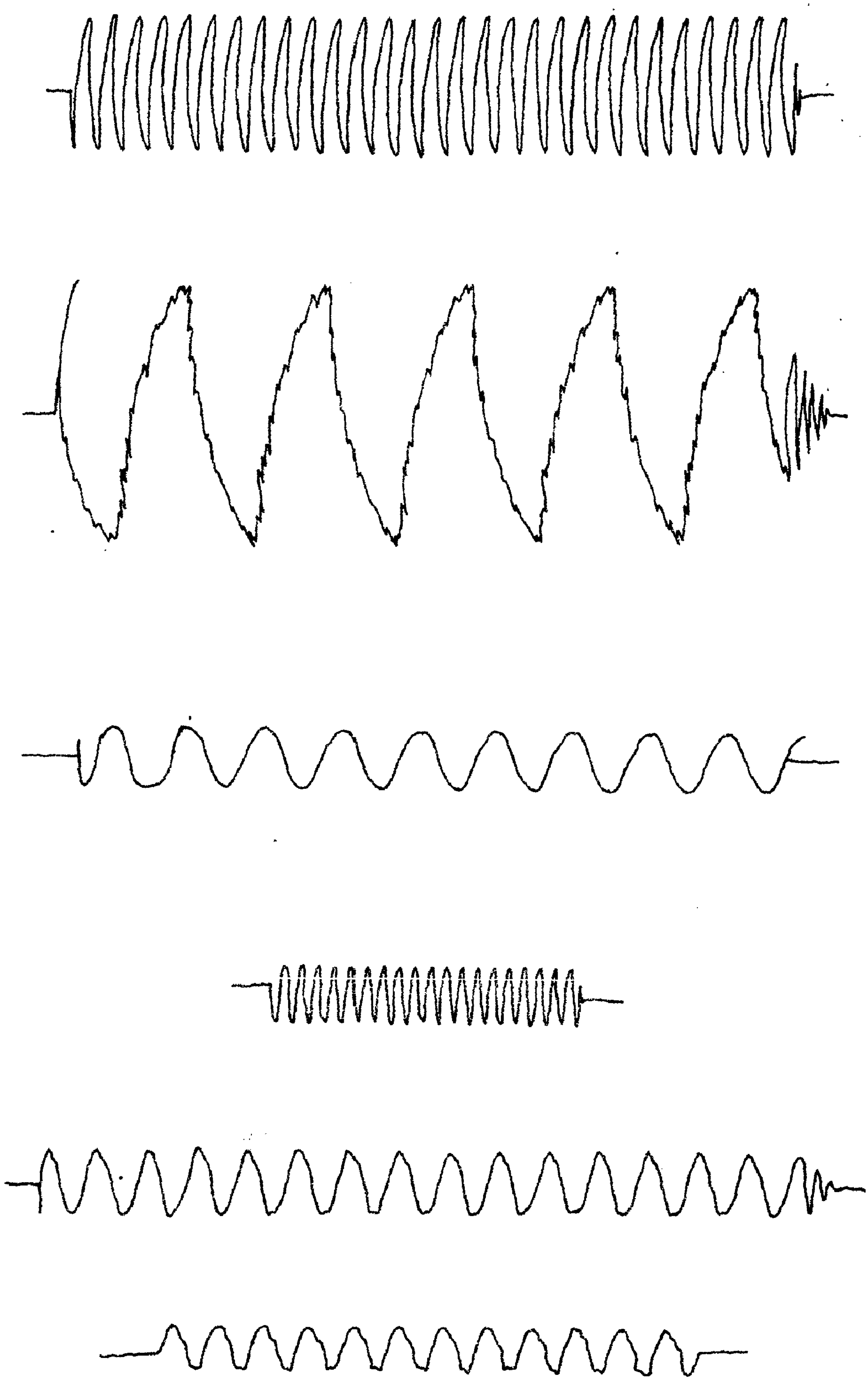


Fig. 6·11 Typical acceleration time records

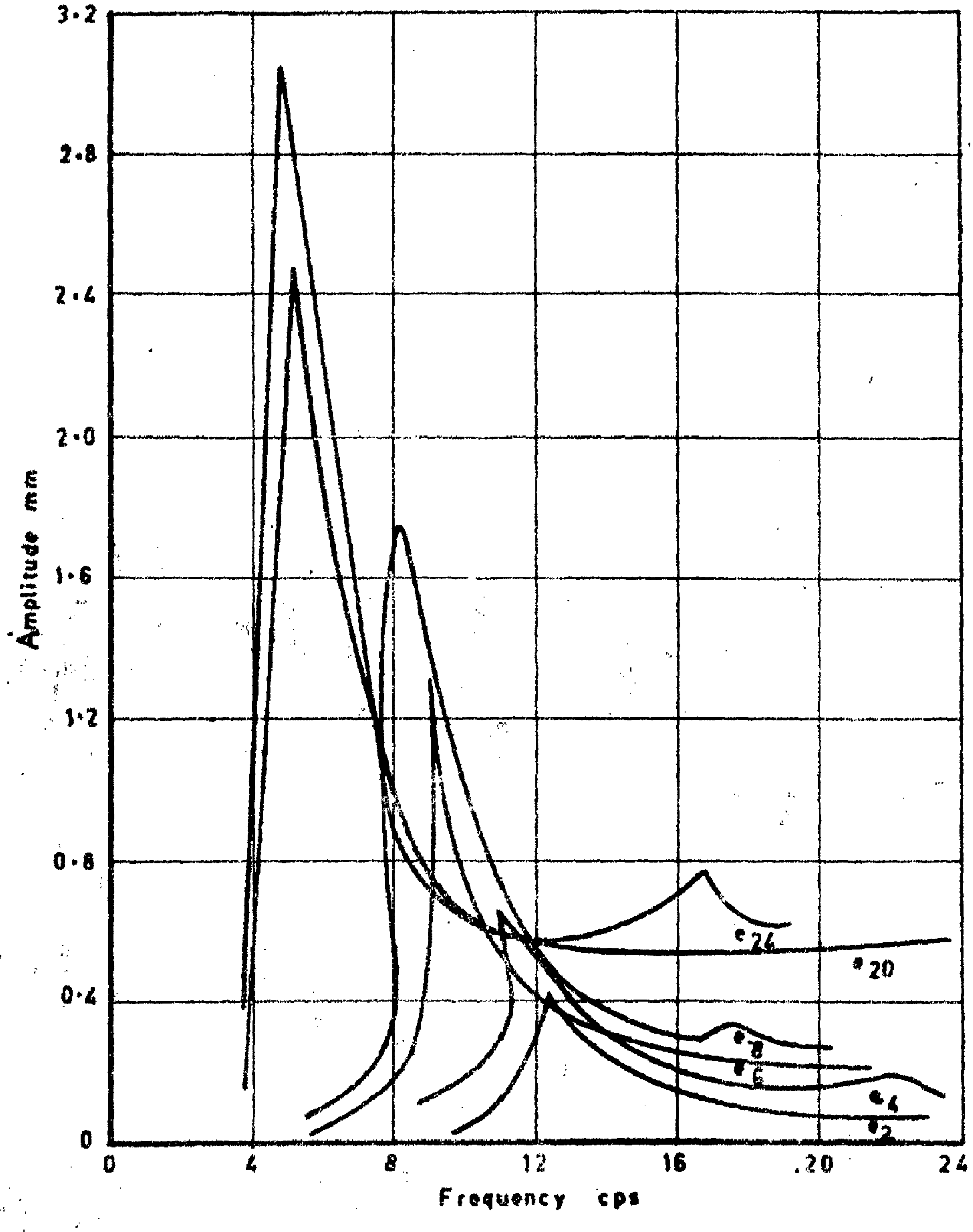


Fig. 6·12 Amplitude frequency plot of a typical pile test

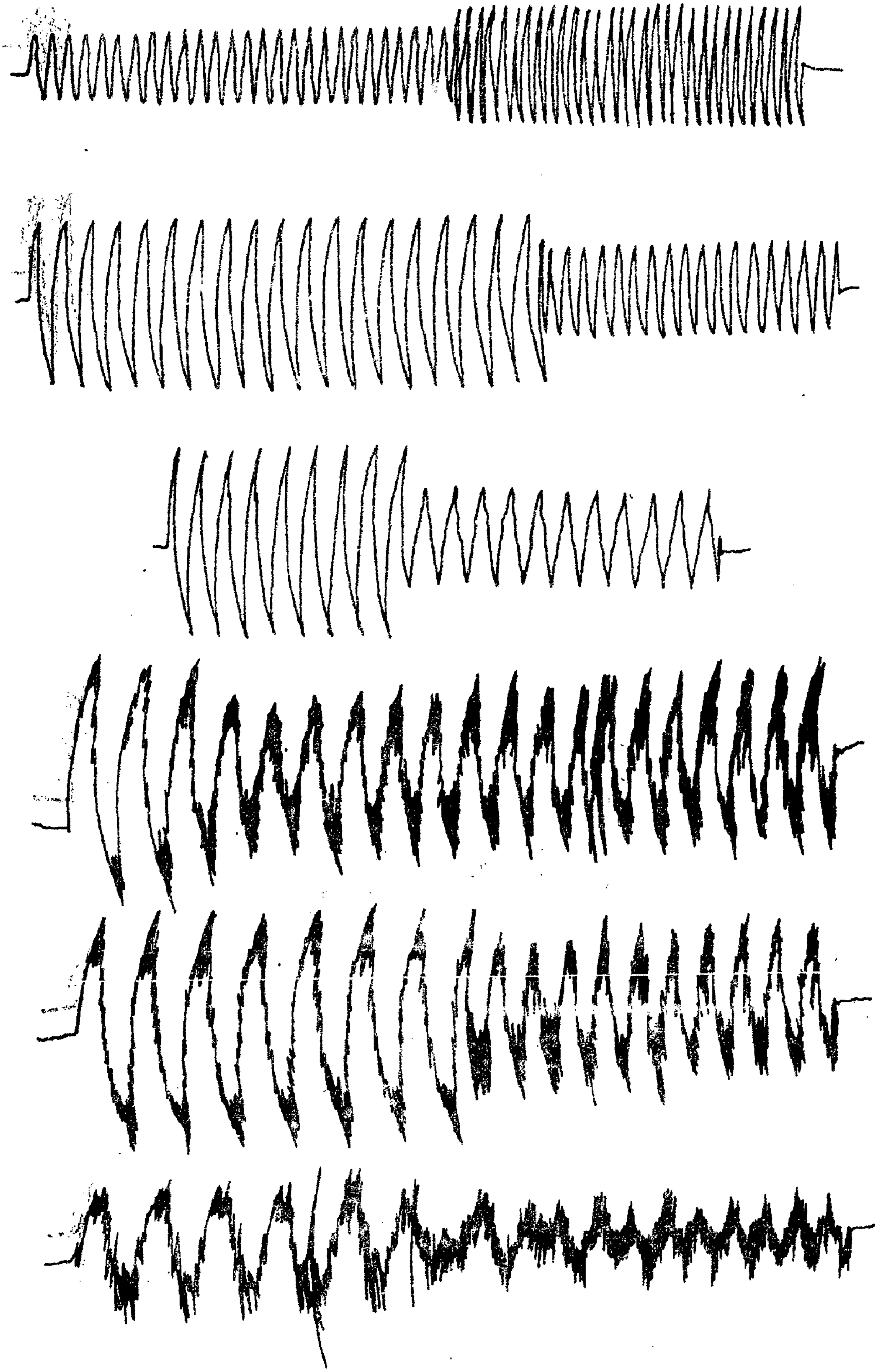


Fig. 6·13 Sudden die down phenomena under resonance condition

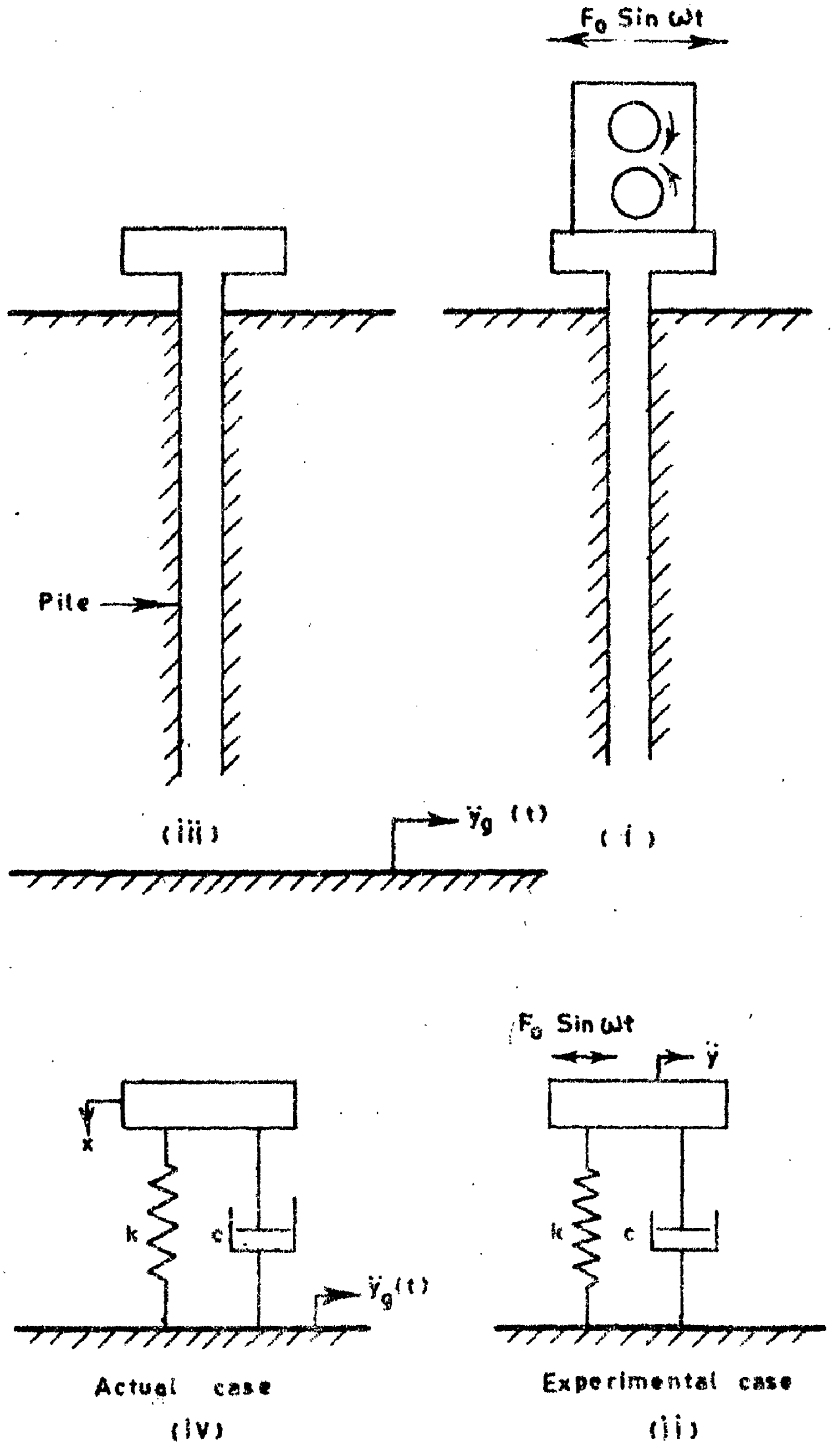


Fig. 6·14 Idealised actual and experimental case for determining ovrall response

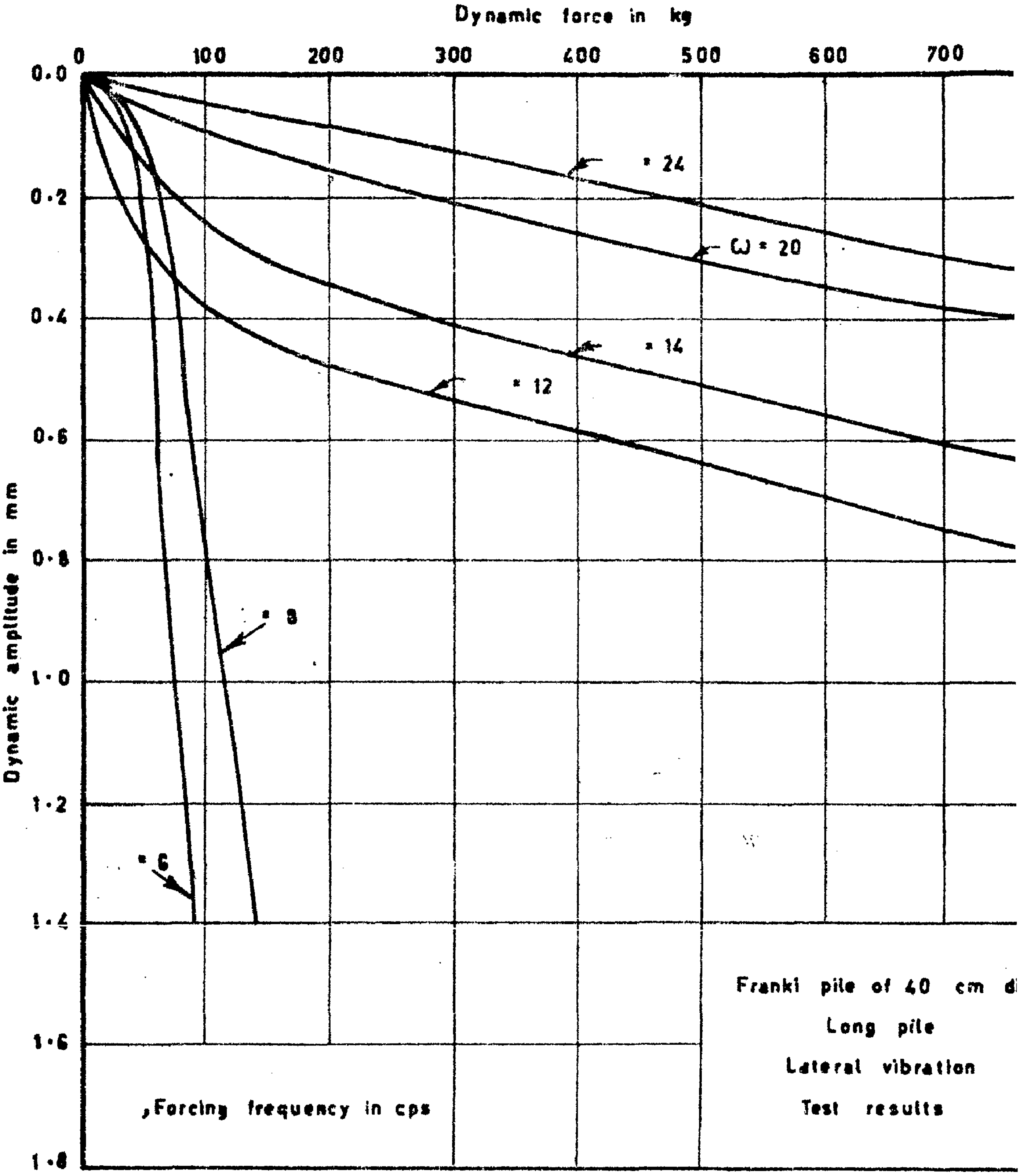


Fig. 6·15 Dynamic force versus amplitude for p

VTP 1

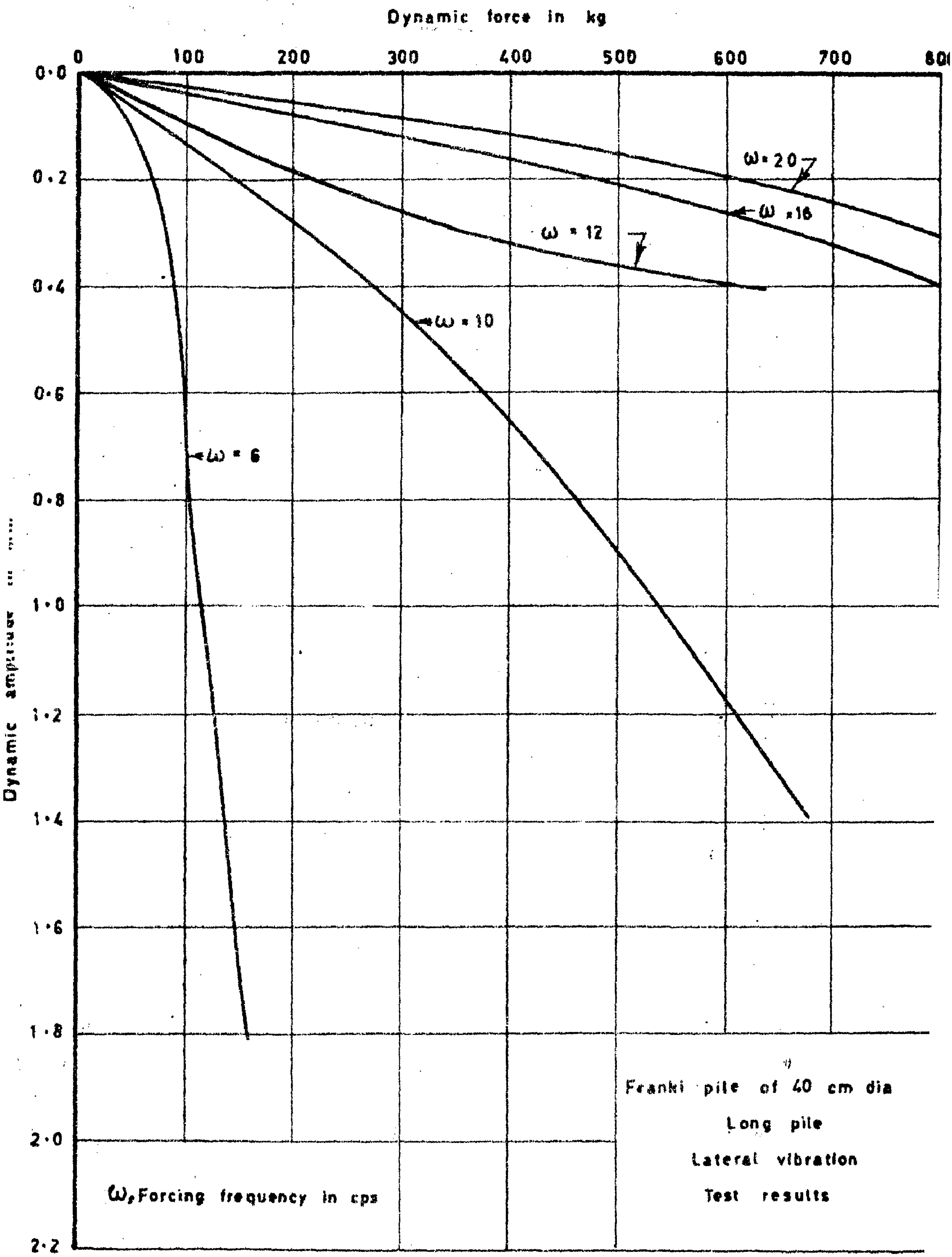


Fig. 6·16 Dynamic force versus amplitude for pil

VTP 2

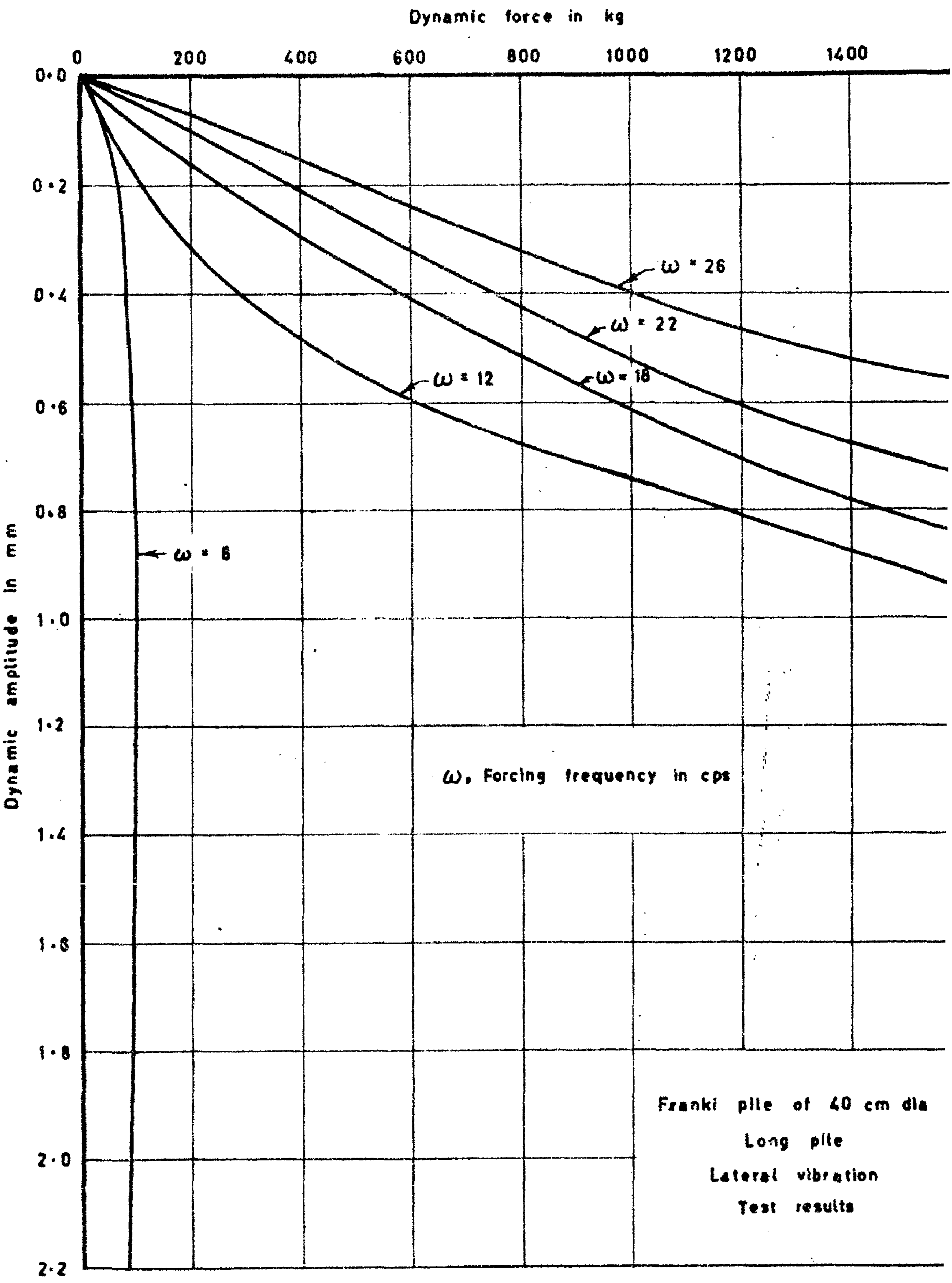


Fig. 6·17 Dynamic force versus amplitude for pi

VTP 3

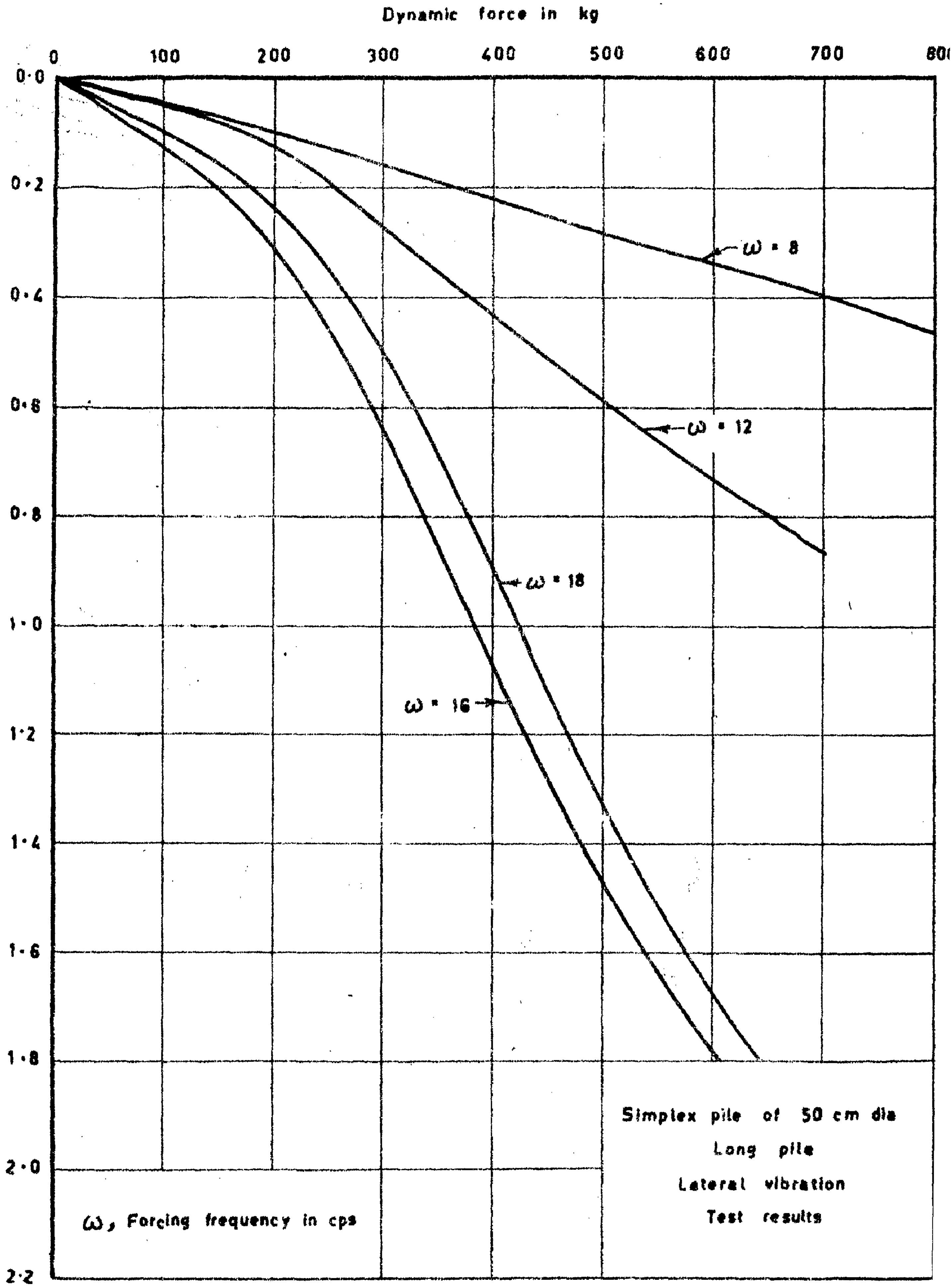


Fig. 6·18 Dynamic force versus amplitude for pile

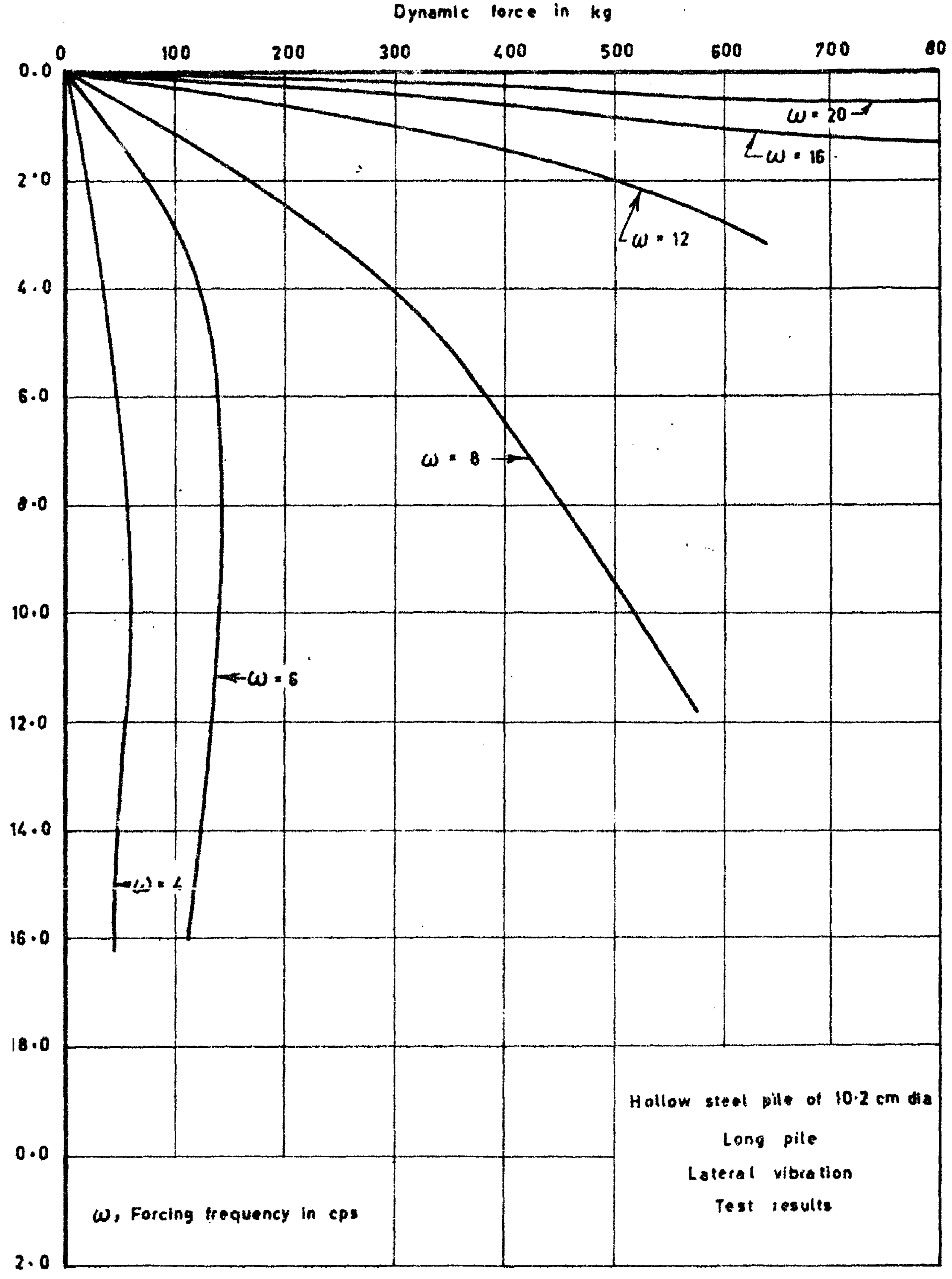


Fig. 6·19 Dynamic force versus amplitude for pile VTP 5

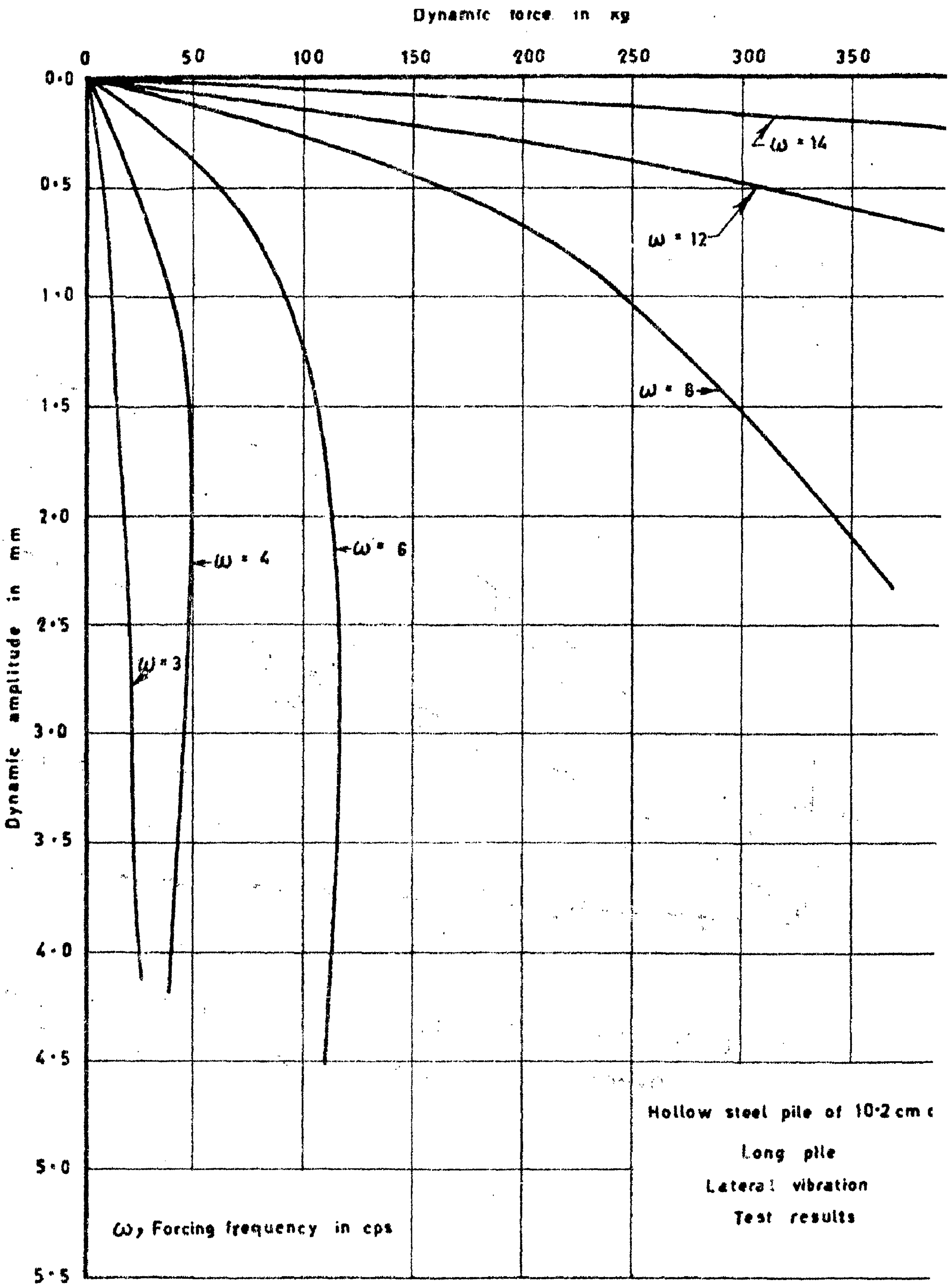


Fig. 6·20 Dynamic force versus amplitude for p

VTP 6

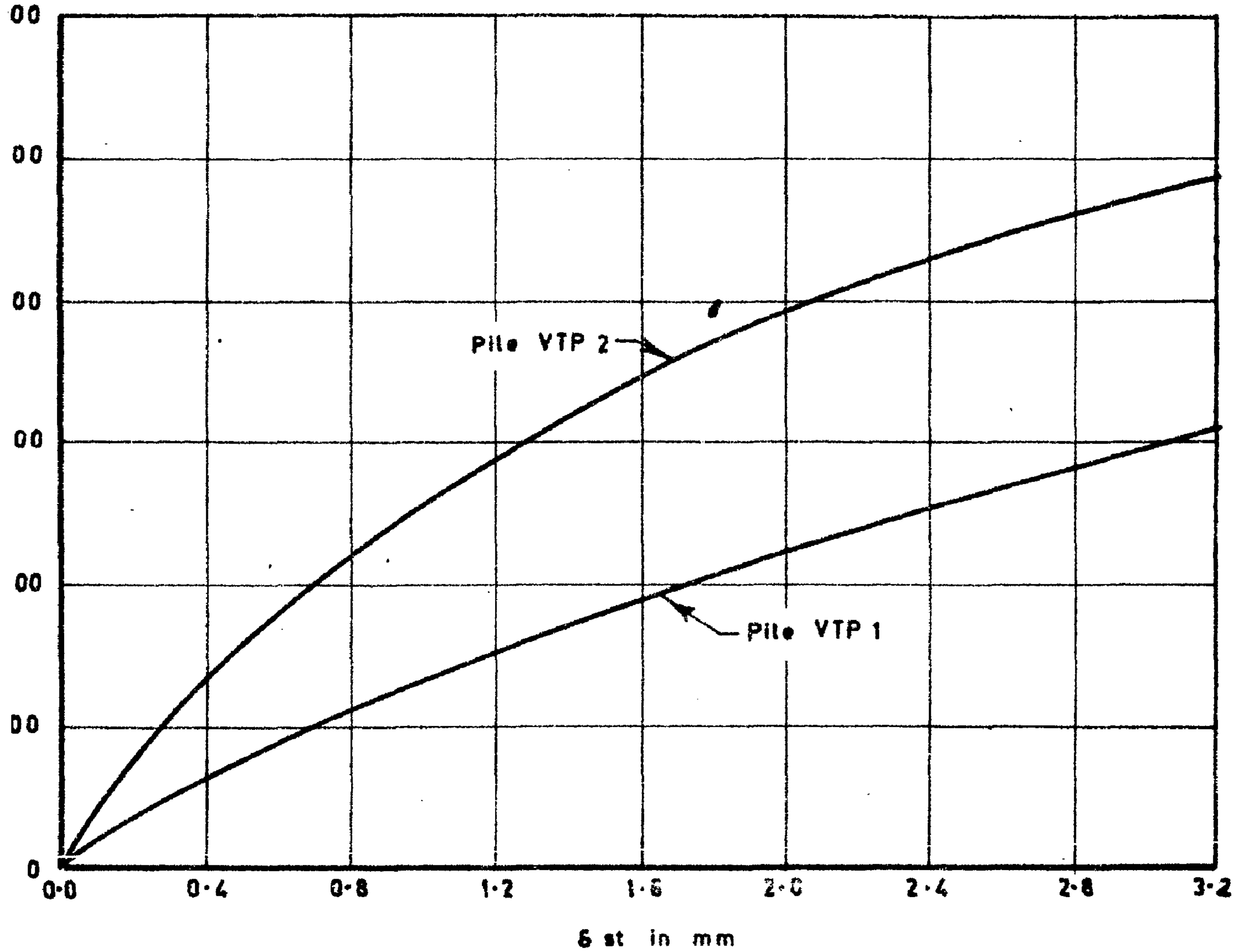


Fig. 6·21 Dynamic force versus δ_{st} for determining overall stiffness of pile VTP1 and VTP2

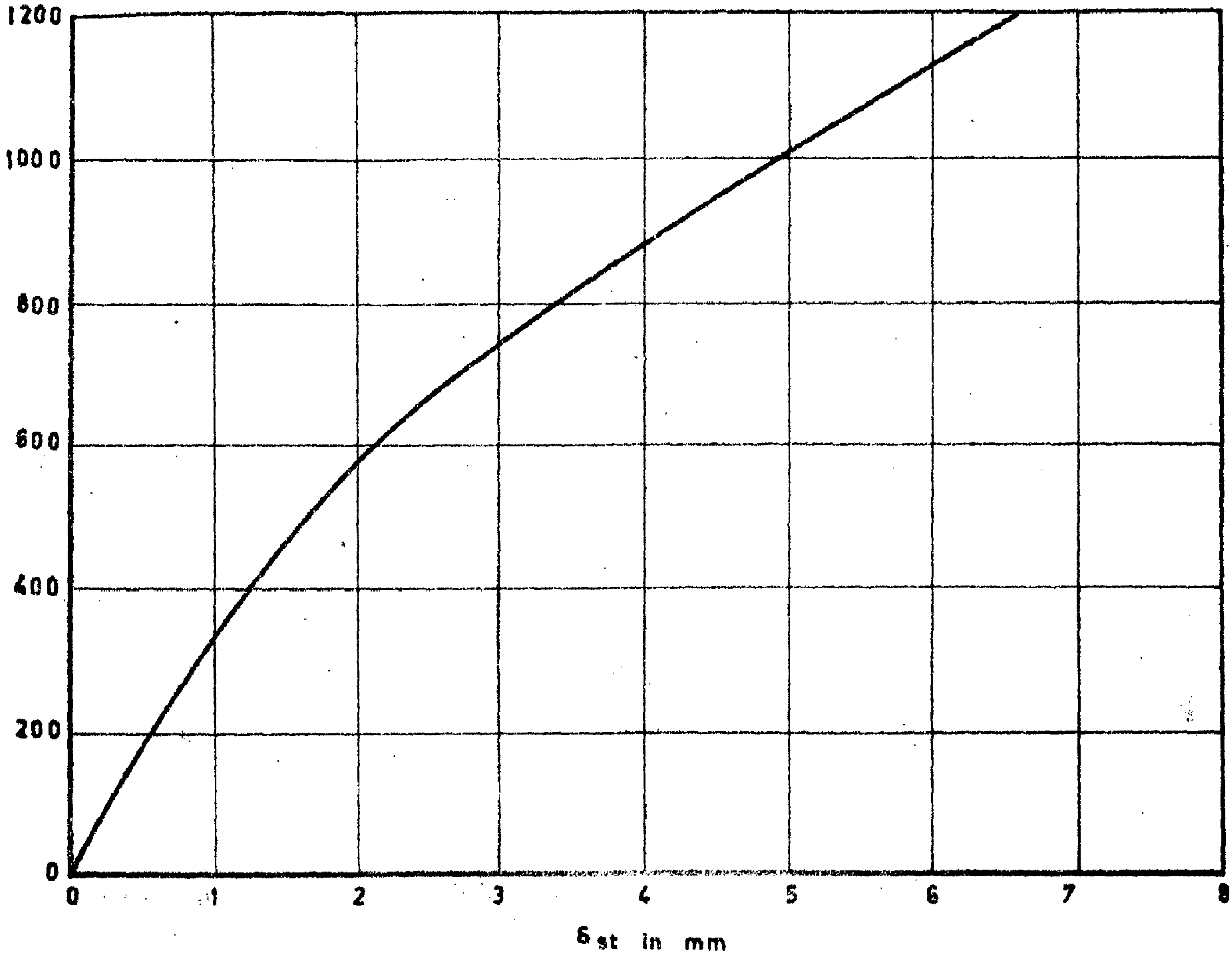


Fig. 6·22 Dynamic force versus δ_{st} for determining overall stiffness of pile VTP 3

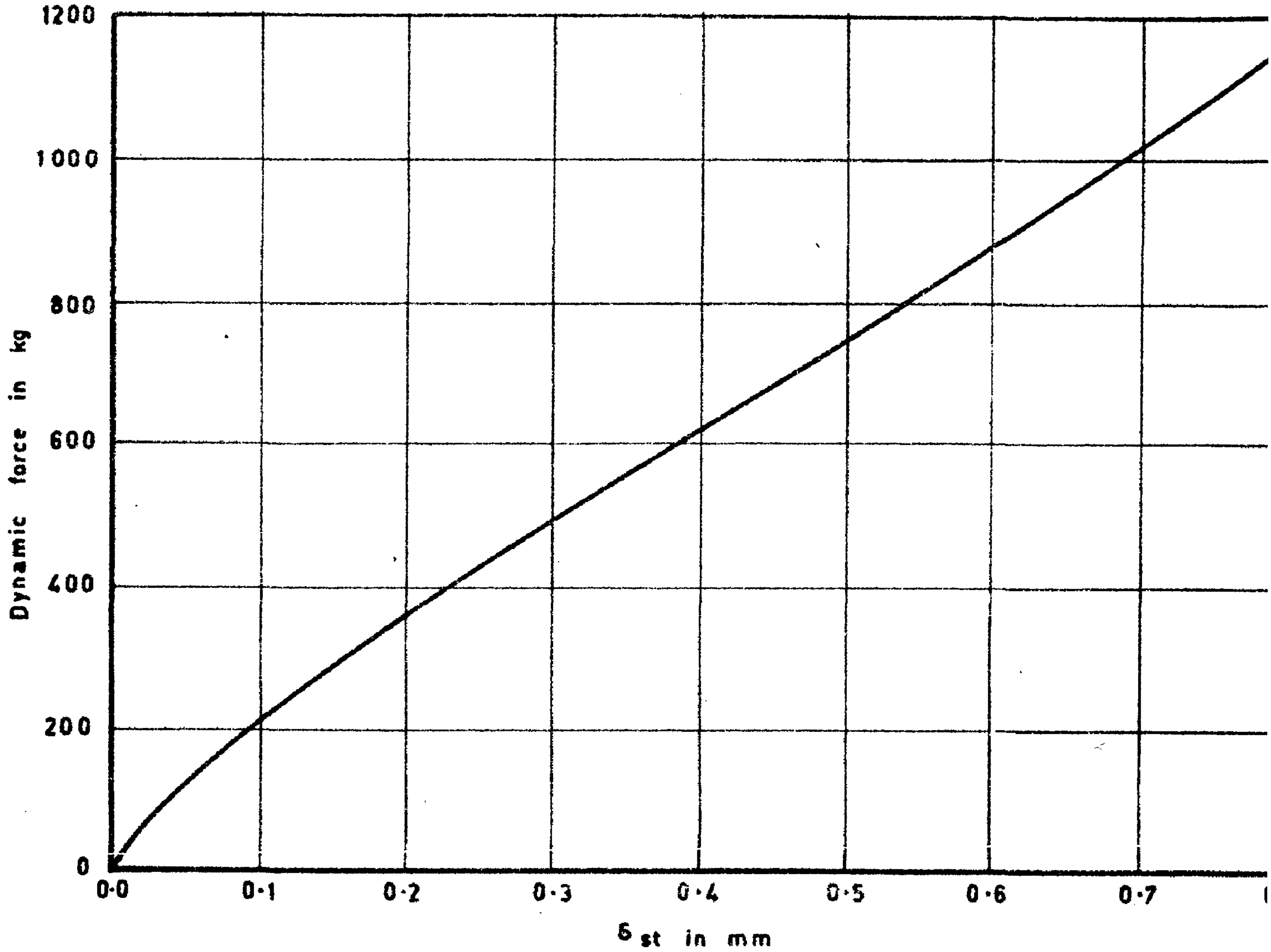


Fig. 6·23 Dynamic force versus δ_{st} for determining overall stiffness of pile V T P 4

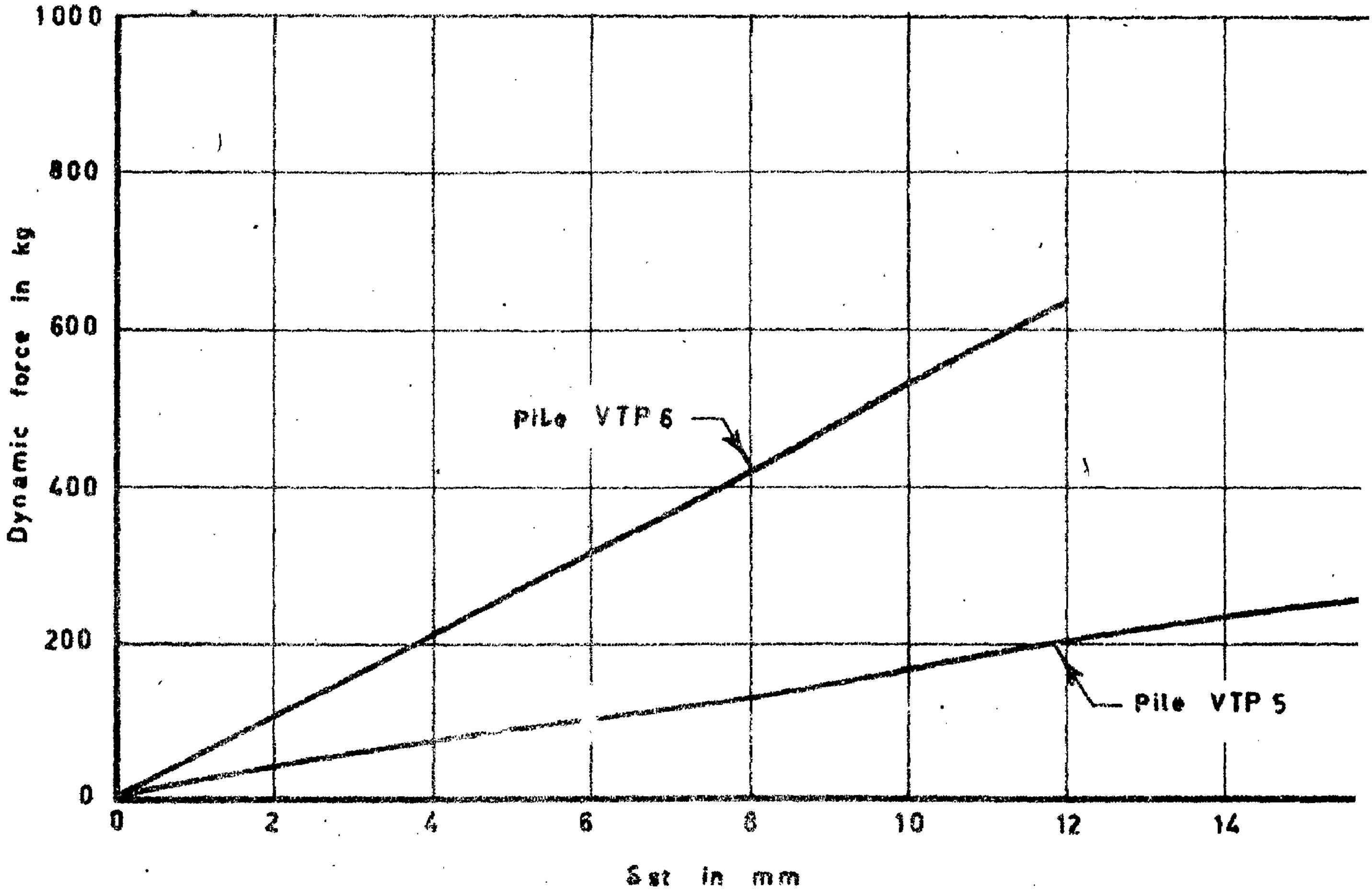


Fig. 6·24 Dynamic force versus δ_{st} for determini overall stiffness of pile VTP 5 and VTP 6

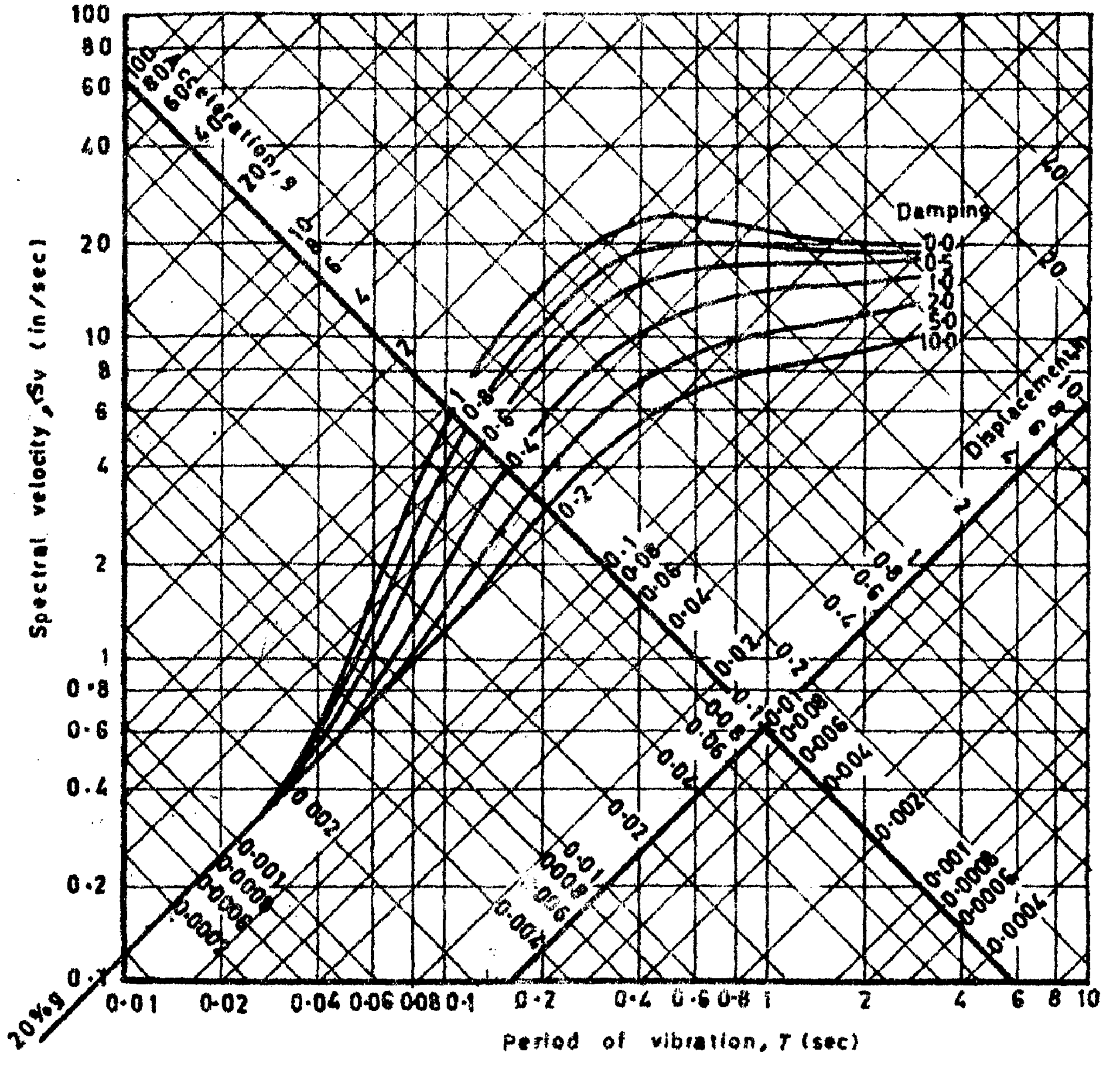


Fig. 7·1 Combined earthquake response spectra

(Housner 1959)

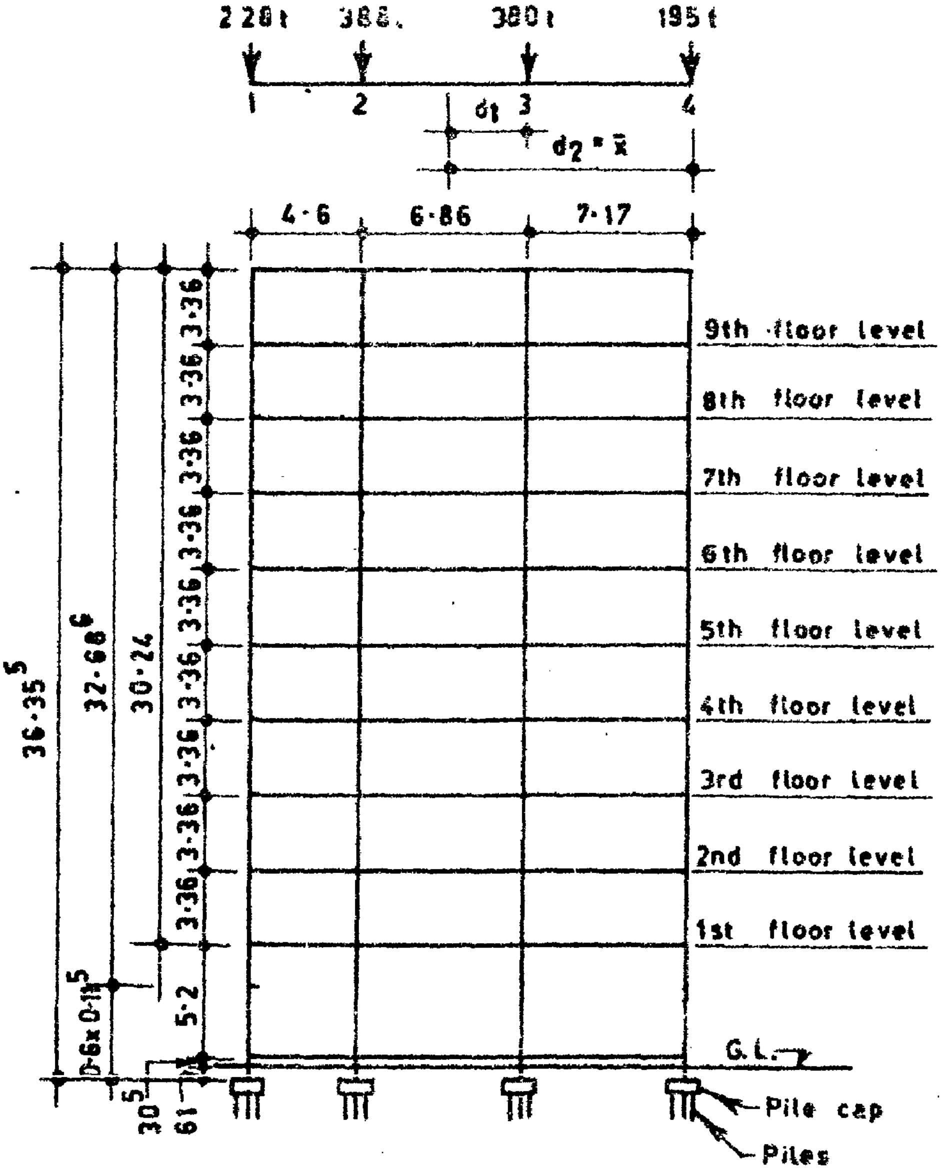


Fig. 7·2 Sketch showing different floor heights and loads at a typical section

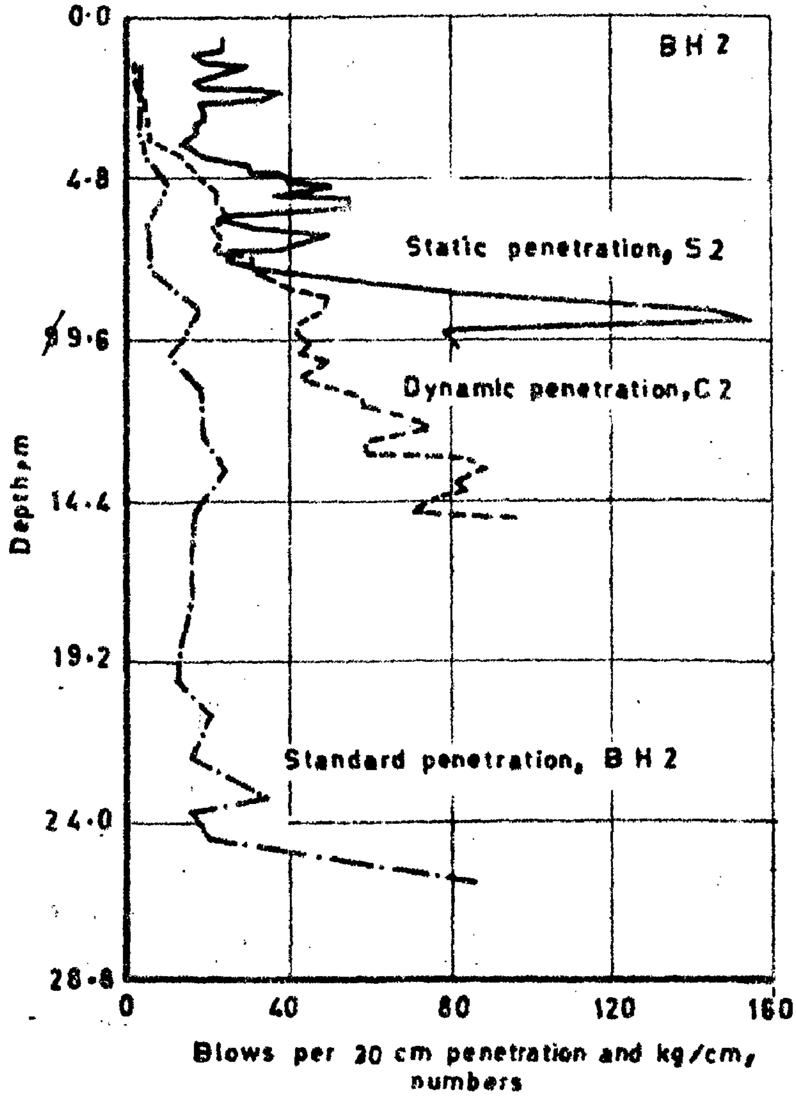


Fig. 7·3 Typical bore-hole data

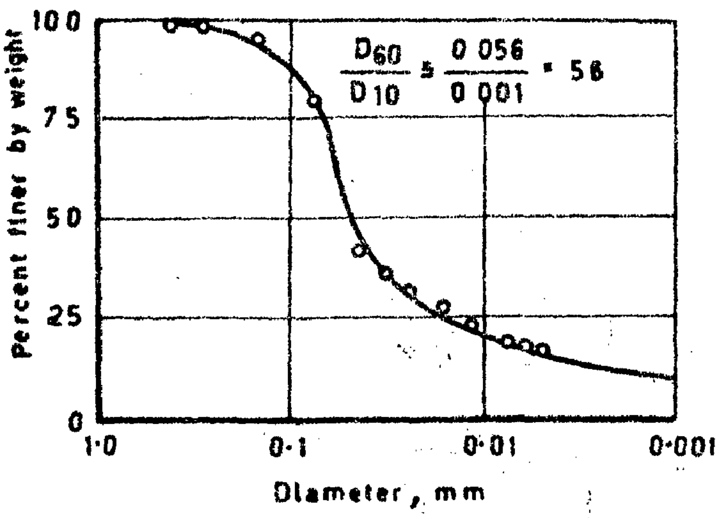
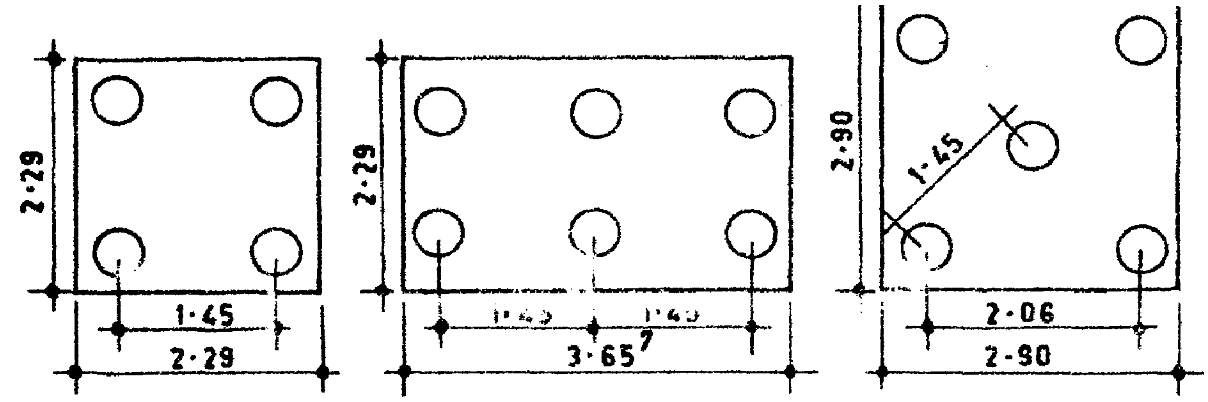
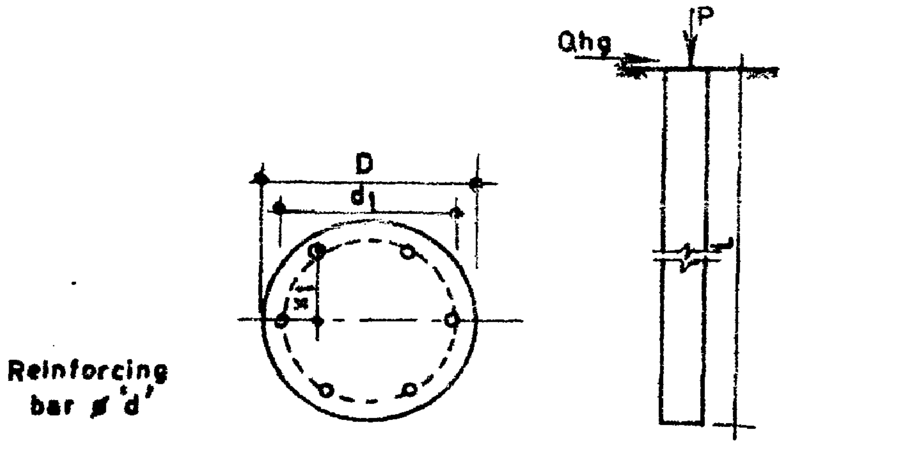


Fig. 7·4 Grain size distribution of soil

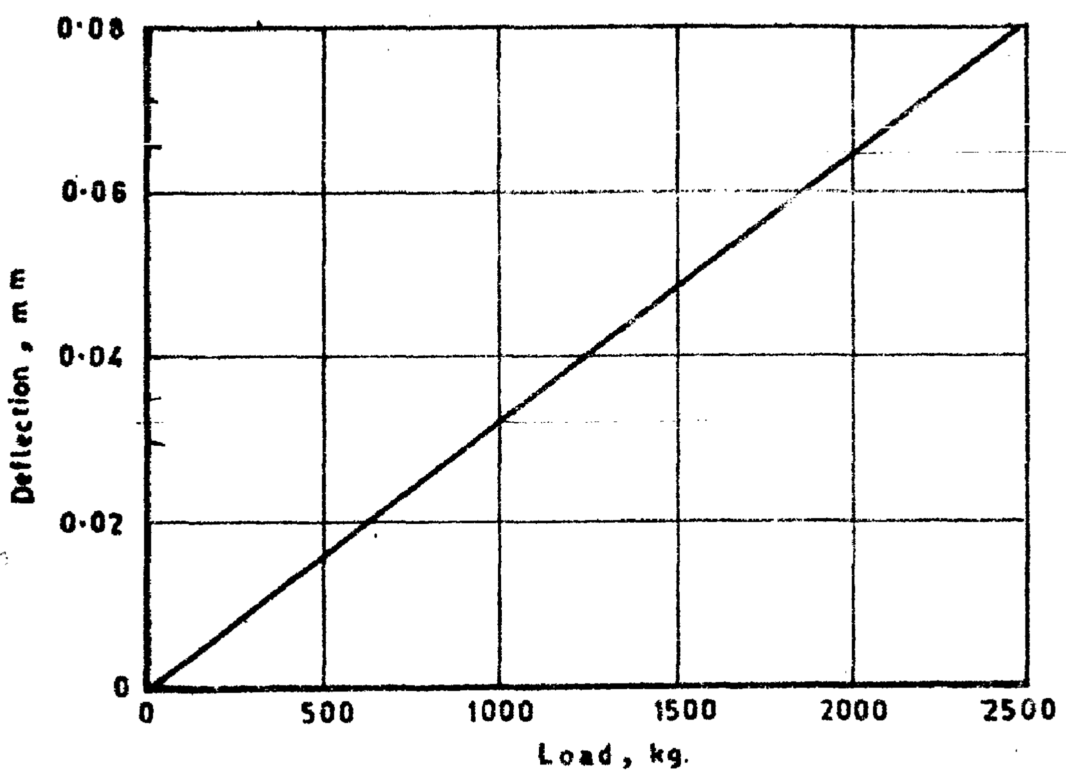


ig. 7·5a Arrangement of piles under different pile groups



(a) Pile section (b) Pile subjected axial and lateral load

Fig. 7·5 b Details of pile section



ig. 7·6 Lateral load deflection curves for static tests

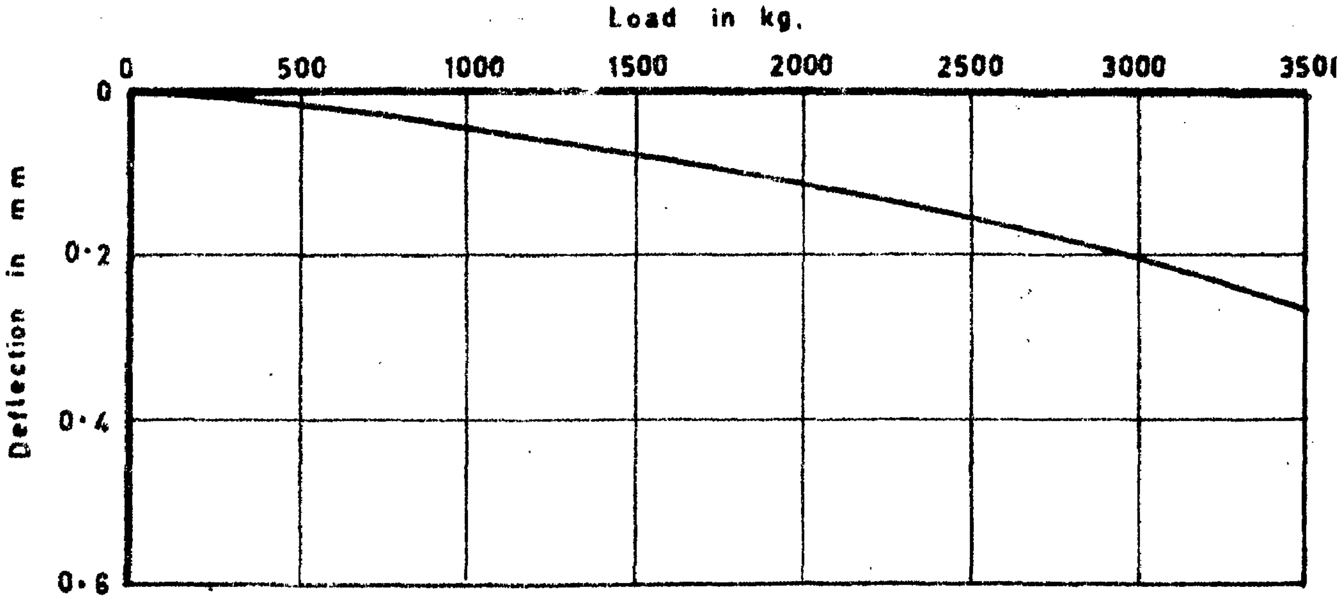


Fig. 7·7a Static load deflection plot

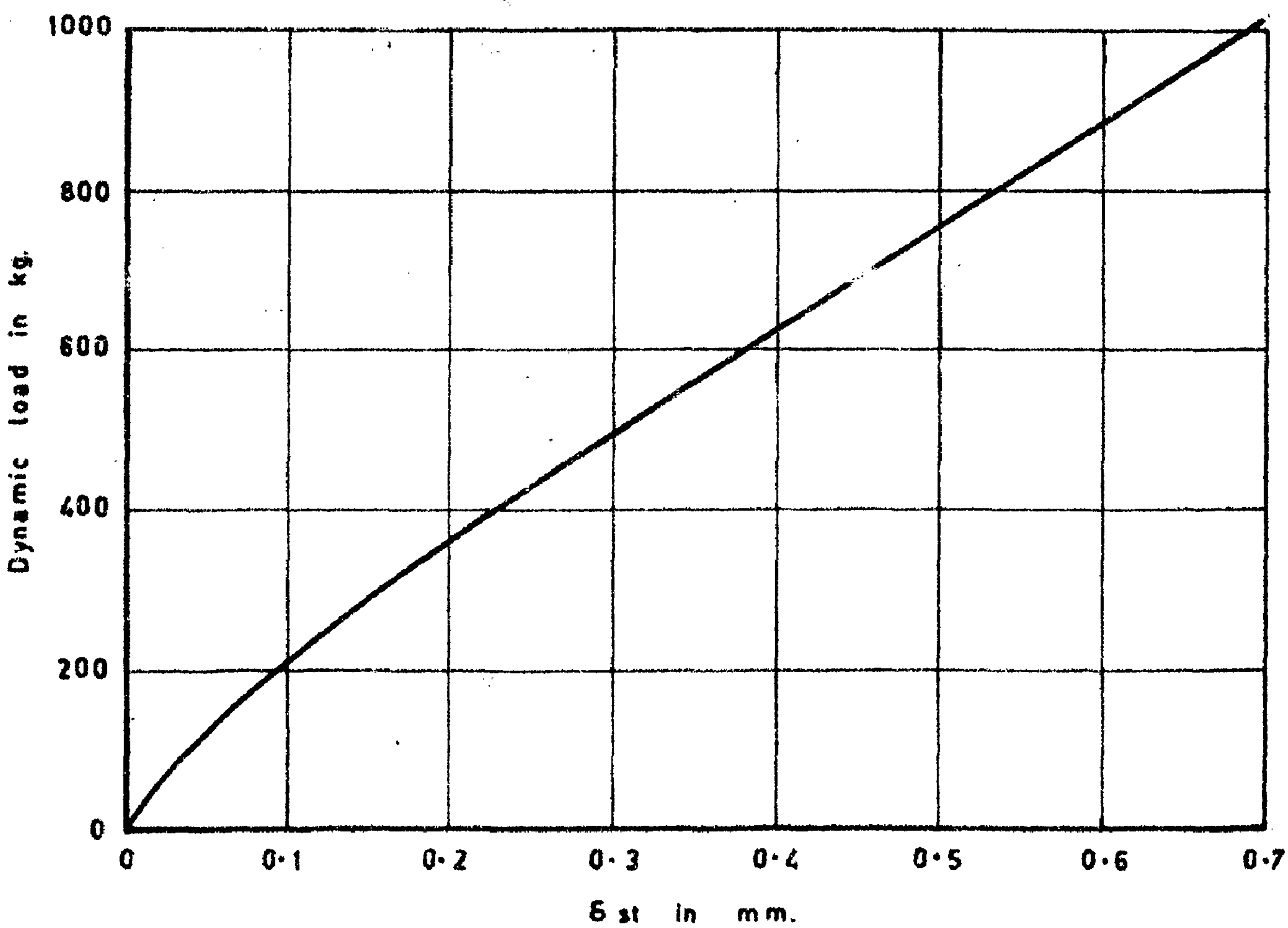


Fig. 7·7b Dynamic load deflection plot

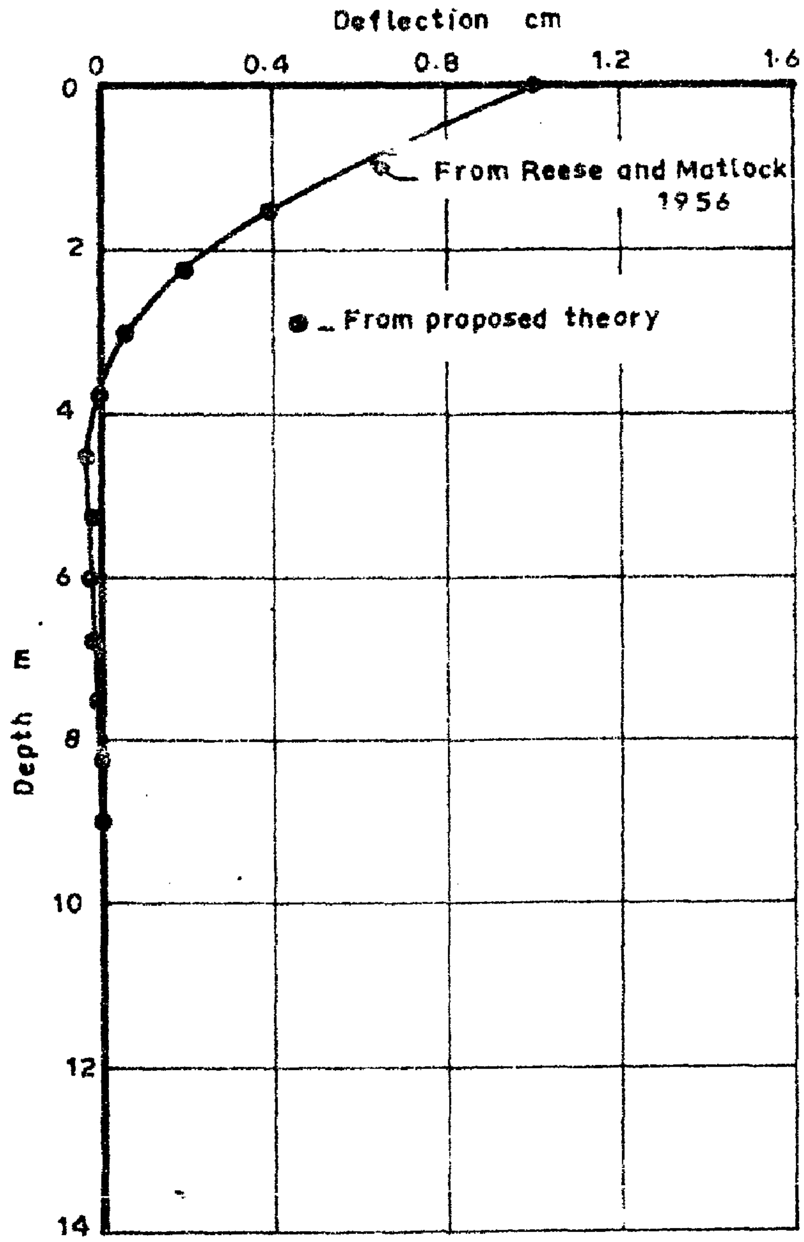


Fig. 7·8 Comparison of static deflection shape with first mode shape of a massless pile section

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Publications

"Battered Piles Subjected to Lateral Loads", Institution of Engineers India, Roorkee Center, 1967.

with Prakash, S. and Mani Kant.

"Investigation of Saturated Sand Masses for Earthquakes", Symposium on Earth and Rock-Fill Dams, Indian National Society of Soil Mechanics and Foundation Engineeing, Talwara, Nov. 1968.

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"Deflections of Battered Pile Groups Under Cyclic Lateral Loads", Second Sout East Asian Regional Conference in Soil Engineering, 1969.

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"Importance of Soil and Foundation During Earthquakes", Bulletin Indian Society of Earthquake Technology, March, 197Q

with Prakash, S., and Nandakumaran, P.,

"Behaviour of Battered Piles Under Lateral Loads", International Geotechnical Conference, Shiraz, Iran, Sept. 197.

with Prakash, S., and Nandakumaran, P.

"Stability of Slopes During Earthquakes, Fourth Symposium on Earthquake Engineering, Roorkee, Nov. 197Q

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"Earthquake Considerations For Foundation Design Around Delhi Region", Proceedings Seminar on Foundation Problems in and Around Delhi Region, Indian Geotechnical Society, January, 1971.

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"Behaviour of Foundations on Saturated Sand Masses, Indian Geotechnical Journal Vol. 1, No. 4, September, 1971.

with Prakash, S., and Nandakumaran, P.,

"Model Studies on Rock-fill Dams at Pandoh", Proceedings 42nd Annual Research Session, Central Board of Irrigation and Power, Madras, Vol. 1, Publcn. No. 116, June, 1972.

with Prakash, S., and Nandakumaran, P.,
"Some Oscillatory Tests on Clay", Third South-East Asian Conference on Soil Mechanics and Foundation Engineering, Hongkong, Nov. 1972.

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"Dynamic Behagiour of Pile Foundations During Earthquakes", Symposium on Earth and Earth Structures Subjected to Earthquakes and other Dynamic Loads, Roorkee, March, 1973.

with Prakash, S.,
"Pilr Foundations Under Lateral Dynamic Loads", Proceedings of the Eighth International Conference on Soil Mechanics and Foundation Engineering, Moscow 1973.