COMPARATIVE STUDY OF SYNTHETIC UNIT HYDROGRAPH METHODS

A DISSERTATION

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By

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this Dissertation entitled **"COMPARATIVE STUDY OF SYNTHETIC UNIT HYDROGRAPH METHODS"** in partial fulfillment of the requirement for the award of degree of **Master of Technology** in Water Resource Development and submitted in the Department of Water Resources Development and Management, Indian Institute of Technology, Roorkee is an authentic record of my own work carried out during the period from July 2009 to June 2010 under the supervision and guidance of Dr. S.K. Mishra, Associate Professor, Department of Water Resources Development and Management, Indian Institute of Technology Roorkee, India.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree.

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ABSTRACT

The synthetic unit hydrograph (SUH) methods are widely used for estimating design flood for unguaged catchments, and for partial data availability conditions, as in developing countries for majority of small watersheds are unguaged. Several SUH methods are available in literature. Most of them involve manual, subjective fitting of a hydrograph through few data points. Because it is difficult, the generated unit hydrograph (UH) is often left unadjusted for unit runoff volume. Therefore, probability distribution function (pdf) based methods are favored to derive an SUH more conveniently and accurately than the traditional methods of Snyder and Soil conservation Service. pdfs as SUH is accepted because of the similarity between pdf of a distribution with area under the pdf curve and a conventional UH being unity, an important feature of a pdf. This study explores the potential and suitability of the parametric expressions of geomorphological instantaneous unit hydrograph (GIUH)-coupled probability models of One-parameter Chi-square (1PCSD), Two-parameter Fréchet (2PFD), Two-parameter Inverse Gamma (2PIGD) and Two-parameter Gamma (2PGD) (Rosso, 1989) distributions for limited data availability condition compared with the widely used traditional methods of Snyder and SCS for SUH derivation. UH features, peak discharge, time to peak, etc. are derived using Horton order ratios given by Rodriguez-Iturbe and Valdés (1979). Geomorphologic characteristics of study watershed have been extracted from ASTER data (resolution 30 m) using ILWIS (version 3.31) GIS software. Analytical procedure is proposed for estimation of parameters of the used four distributions.

The applicability of the four pdf models-coupled with GIUH and traditional methods of Snyder and SCS for SUH derivation is tested on both text and field data. Their suitability is compared with the observed UH of the four study catchments (area = 27.93 km^2 - 4000 km^2) located in different parts of the country. Finally, their performance is evaluated based on STDER and RE in peak flow rate. 2PFD performed marginally better than 2PIGD, followed by 2PGD and 1PCSD models for mid-sized study catchments (Myntdu-Leska and Gagas). However, for the large size catchment (Burhner) as well as for the smallest (Kothuwatari), the 2PGD model performed better than 2PIGD, followed by 2PFD and 1PCSD models. The four density functions coupled with GIUH except 1PCSD compared well with the observed UH, indicating good potentiality for SUH derivation.

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LIST OF SYMBOLS

$\overline{L}_w$	Mean Length of stream of order w
$\overline{A}_w$	Area of basin of order w
β	Non-dimensional term
(1-C)	Routing Coefficients
Γ	Gamma Function
A_w	Area of the watershed (square km)
b	Parameter of 1Parameter Chi-Square Distribution general equation
C	Routing Coefficients
c, α	Parameters of the 2Parameter Fréchet distribution
C_P	Non-dimensional Constants
C_T	Non-dimensional Constants
D	Duration of Rainfall (hr)
e	Exponential
I_i	i^{th} ordinate of time-area diagram
k	Scale parameter
K	Storage Coefficient (hr)
k , α	Parameters of Two-parameter Inverse Gamma Distribution
K, n	Parameters of Two-parameter Gamma Distribution
K_1	Storage coefficient of the first reservoirs (hr)
K_2	Storage coefficient of the second reservoirs (hr)
L	Hydraulic length of watershed (m)
L	Length of main stream (km)
L_{CA}	The distance from the watershed outlet to a point on the main stream nearest to the center of the area of the watershed (km)
m	Parameter of 1-Parameter Chi-Square Distribution
m_1	First moments of the IUH about the origin
m_2	Second moments of the IUH about the origin
n	Shape parameter
N_w	Number of streams of the order w
P_R	The time from beginning of surface runoff to the occurrence of peak discharge (minutes)
P_R	Period of Rise

$Q_2(t)$	Output from the second hybrid unit (mm / hr / mm)
Q_p	Peak flow rate (m^3 /s)
q_p	Peak discharge ($m^3/s/km^2$)
$Q_{p[COM]}$	Peak flow rate of the computed unit hydrograph (m^3/s)
$Q_{p[OBS]}$	Peak flow rate of the observed unit hydrograph (m^3/s)
R_A	Area ratio
R_B	Bifurcation ratio
R_L	Length ratio
S_{av}	Average Catchment Slope (m/m)
S_M	Slope of Main Stream (%)
S_o	Overland Slope
Δt	Computational Interval (hr)
t_1	Point of inflection
t_a	Base time (hours)
t_c	Time of concentration
T_D	Rainfall Excess Duration (hr)
t_L	Lag time to peak (hr)
t_p	Time to peak (hr)
t_r	Time to recession (hr)
u_1	Functions of non-dimensional term β
U_i	i^{th} ordinate of IUH
v	Average peak velocity
v_1	Functions of non-dimensional term β
V_R	Runoff volume (mm or inch)
W_{50}	Widths of SUH at 50% of Q_P (hr)
W_{75}	Widths of SUH at 75% of Q_P (hr)
γ	Scale Parameter
γ'	Dimensionless parameter
λ'	Shape parameter
μ	Mean of the distribution
σ^2	Variance of the distribution
τ	Parameter of 1Parameter Chi-Square Distribution special substituted equation

Abbreviations

1PCSD	One-Parameter Chi-square Distribution
2PFD	Two-parameter Fréchet distribution
2PGD	Two-parameter Gamma distribution
2PIGD	Two-parameter Inverse Gamma distribution
3PGD	Three Parameter Beta Distribution
ASTER	Advanced Space borne Thermal Emission and Reflection Radiometer
ASTERDEM	ASTER Global Digital Elevation Model
CN	Curve Number
CWC	Central Water Commission
DEM	Digital Elevation Model
DVC	Damodar Valley Corporation
EOS	Earth Observing System
ERDAS	Earth Remote Sensing Data Analysis Center
FPWS	Flood Prediction and Warning Systems
GIS	Geographic Information System
GIUH	Geomorphologic Instantaneous Unit Hydrograph
HEC	Hydrologic Engineering Center
HES	Hydrological and Earth Sciences
IAHS	International Association of Hydrological Sciences
IGBP	Indo-German Bilateral Project
ILWIS	Integrated Land and Water Information System
IUH	Instantaneous Unit Hydrograph
METI	Ministry of Economy, Trade and Industry
Msl	Mean sea level
Pdf	Probability Density Functions
PUB	Predictions in Ungauged Basins
RE	Relative error
RS	Remote Sensing
SCS	Soil Conservation Service
STDER	Standard error
SUH	Synthetic unit hydrograph
SW	South West
TF	Transfer Function

UH	Unit hydrograph
USACE	United State Army Corps of Engineers
USDA	United States Department of Agricultural

CHAPTER-1
INTRODUCTION

Rainfall–runoff process is of paramount importance in watershed hydrology. However, many components of this process are difficult to observe routinely and unambiguously, and require costly measuring facilities. Due to economic and other constraints, such facilities are scarce, particularly in developing countries. In the developing countries like India, majority of mid-size catchments are ungauged, and thus posing the problem of accurate estimation of floods from these catchments. Moreover, the problem is further aggravated by the impacts of human-induced changes to the land surface and climate, occurring at the local, regional and global scales and thus making the predictions of ungauged or poorly gauged basins highly uncertain. Notably, an accurate estimation of floods in ungauged catchments is frequently required in hydrological practice and is of great economic significance. The estimates of flood magnitudes are required for economical planning and design of river basin projects meant for conservation and utilization of water for various purposes. A catchment's flood response to rainfall may have to be quantified for a variety of reasons. Among the most common are peak flow and flow volume estimation, flood duration, flood warning and the design of hydraulic structures. The catchment characteristics, used in the estimation of flood parameters at ungauged sites, are more difficult and at the same time, any flood estimation procedure is only as good as the data used in its construction.

The unit hydrograph (UH) theory introduced by Sherman (1932) is a potential powerful tool in watershed hydrology for development of Flood Prediction and Warning Systems (FPWS). More than 75 years, since its inception, it is still one of the most widely used methods for estimating the storm runoff hydrograph at the gauging site in a catchment corresponding to a rainfall hyetograph. Also it is one of the first tools available to hydrologic and water resources community to determine the complete hydrograph shape rather than the quantum of peak discharge only (Todini, 1988). However, this data driven traditional approach limits the derivation of UH from only from gauged watersheds. Hence, the synthesis of the UHs from physical basin characteristics is a pressing need of the time for extension of the theory to ungauged basins. Probably, Sherman was the first to see the possibilities of extending the UH theory he had developed. He listed out the physical basin characteristics he thought would be reflected in a unit hydrograph and could be used to estimate the stream flow for an ungauged basin from given rainfall data. These characteristics were drainage area, size and shape, distribution of water courses, slope of main stream, slope of valley sides, and pondage due to surface or channel obstructions. Notably, the Sherman's idea has been the basis of many synthetic unit hydrograph procedures (Hoffmeister and Weisman, 1977). In order, most procedures seek to establish relationships between parameters used to describe the UH and the basin. The procedures differ either in the relationships established or in the methodology employed.

As discussed above, the UH concept needs the observed rainfall-runoff data at the gauging site for hydrograph generation, the paucity of these data sparkled the idea of synthetic unit hydrograph (SUH) concept. The term "*synthetic*" in synthetic unit hydrograph denotes the unit hydrograph (UH) derived from watershed characteristics rather than rainfall-runoff data. The need for a synthetic method to develop UHs has inspired many studies as the drainage basins in many parts of the world are ungauged or poorly gauged, and in some cases existing measurement networks are not working properly. The beginning of SUH concept can be traced back to the model (distribution graph) proposed by Bernard (1935) to synthesize the UH from watershed characteristics, rather than the rainfall runoff data. These synthetic or artificial unit hydrographs are characterized by their simplicity and ease in construction. They require fewer amounts of data and yield a smooth and single–valued shape corresponding to unit runoff volume, which is essential for UH derivation. Accepting this challenge, for the first time, the *International Association of Hydrological Sciences* (IAHS) decided to launch a new initiative to devote a decade (2003-2012) on *Predictions in Ungauged Basins* (PUB), for formulating and implementing appropriate science programmes to actively engage and re-energize the hydrologic and water resources community, in a coordinated manner, towards achieving major advances in hydrologic models/tools to make predictions in ungauged basins successfully. In order to expand the information on hydrological inputs new efforts are to be needed in the form of establishment of better gauging facilities and at the same better RS and GIS facilities to provide efficient and accurate information about DEM, land-cover and land-use, soil type, soil moisture, traditional meteorological parameters, snow, and related societal data. Although there are several traditional methods of SUH derivation, e.g., Snyder (1938) and Soil Conservation Service Method (SCS) (SCS, 1957) from ungauged catchments, however, these are greatly criticized for their inefficiency to serve the purpose.

Estimation of geomorphologic parameters of a catchment using manual procedures from toposheets is a quite tedious and time consuming process. However, with the recent advancements in geo-spatial techniques like Geographical Information System (GIS) and Remote Sensing (RS), Digital Elevation Model (DEM) has gained much impetus in Hydrological and Earth Sciences (HES) since last two decades. In a general practice, DEM is prepared from the digitization of the contours from concerned toposheets or mosaic of toposheets of study area, which is very painstaking and time consuming process especially when the area of interest is very large. Furthermore, readily available and probably cost free ASTER-DEM data plays a vital role to extract the catchment's geomorphological parameters using GIS and image processing softwares.

Similarly, the use of probability distribution functions as SUH has a long successful hydrologic history. Due to similarity in the shape of the statistical distributions and a conventional unit hydrograph, several attempts have been made in the past to use their probability density functions (pdfs) for derivation of the SUH. Strong mathematical perception and conceptual basis of pdfs coupled with quantitative geomorphology of drainage basins successfully fills the technological niche for flood estimation from ungauged catchments. Recently, the hydrologists are finding it more convenient to couple the distribution function based approach with the classical geomorphologic instantaneous unit hydrograph (GIUH) approach for development of SUH models by harvesting the geographic information systems (GIS) and remote sensing (RS) technologies.

1.1 OBJECTIVE

Keeping in view the aforementioned facts and a critical diagnosis of literature review, the present study is undertaken for further improvements in development of SUH methods for ungauged catchments with the following specific objectives:

1. To explore the parametric expressions of probability models of One-parameter Chi-square distribution (1PCSD), Two-parameter Fréchet

distribution (2PFD), Two-parameter Inverse Gamma distribution (2PIGD), and Two-parameter Gamma distribution (2PGD) (Rosso, 1984) to describe the complete shape of SUH for limited data availability condition using Horton order ratios of a catchment on the basis of a geomorphologic model of catchment response (GIUH approach).

2. To evaluate the applicability of the above four probability models to the four Indian catchments categorized as small, medium and large with an area ranging from 27.93 km^2 to 4000 km^2 having different hydro-climatological characteristics and terrain.
3. To develop simple analytical procedures for estimation of the distribution parameters.

4. To evaluate the workability of the widely used traditional methods of Snyder (1938) and Soil Conservation Service Method (SCS) (SCS, 1957) for SUH derivation with the above GIUH coupled probability distribution-based approach for limited data availability condition.

5. To generate the digital elevation model (DEM) followed by catchment and drainage network extraction map of the study catchments to compute to Horton's ratios from the most recent ASTER-DEM data using ILWIS 3.31 version GIS software rather than the manual procedures.

1.2 ORGANIZATION OF THE THESIS

The thesis is arranged in six chapters as follows:

Chapter One: The first chapter introduces the problem, briefly describes the present state-of- the-art knowledge, and outlines the research objectives and the organization of the thesis.

Chapter Two: This chapter presents a critical review of literature available in the field of SUH methods related with the present study. To accomplish this, the chapter is divided mainly into three sections dealing with (i) popular SUH methods; (ii) conceptual methods of SUH; (iii) geomorphologic unit hydrograph (GIUH) based synthetic SUH methods; and (iv) probability distribution function based SUH methods

Chapter Three: This chapter describes the study area and data acquirement. The types of data are the geomorphologic characteristics of the study catchments and UH characteristics. The most recent ASTER-DEM has been for the purpose and processed using ILWIS GIS 3.31 version.

Chapter Four: In this chapter the parametric expressions of the probability models of One-parameter Chi-square distribution (1PCSD), Two-parameter Fréchet distribution (2PFD), Two-parameter Inverse Gamma distribution (2PIGD), and Two-parameter Gamma distribution (2PGD) (Rosso, 1984) are diagnosed for their suitability to SUH derivation.

Chapter Five: In this chapter, the traditional methods of SUH derivation namely Snyder (1938) and Soil Conservation Service Method (SCS) (SCS, 1957) and GIUH coupled probability models as discussed above have applied to the study catchments and their comparative performance is evaluated using different goodness-of-fit criteria on the data of small to larger study catchments.

Chapter Six: This chapter presents summary, important conclusions drawn from the study, major research contributions of the study, and scope for future research work.

CHAPTER-2

LITERATURE REVIEW

The process of rainfall–runoff is of immense interest in hydrology. However, many components of this process are difficult to observe routinely and unambiguously, and require costly measuring facilities. Due to economic and other constraints, such facilities are scarce, particularly in developing countries. Use of unit hydrograph (UH) for predicting storm runoff is a criticized, but widely used and accepted, tool in hydrologic analysis and synthesis. The UH at a specific point on the stream (gauging site) in a catchment is generally determined by using effective rainfall and surface runoff data observed for the gauging site. Since then, numerous techniques have been developed to determine runoff when limited data are available. Among these, the synthetic unit hydrograph (SUH) is of great significance for determining runoff volume with respect to time, especially from ungauged catchments. Synthetic unit hydrographs (SUHs) are used in determination of flood peak and runoff volume, especially from ungauged watersheds. Singh (1988) provided a comprehensive review of several methods dealing with the SUH derivation. The qualifier ''synthetic'' denotes that the unit hydrographs are obtained without using rainfall-runoff data from a watershed. Their simplicity and ease in development can characterize these synthetic or artificial unit hydrographs. These require less data and yield a smooth and single valued shape corresponding to one unit runoff volume, which is essential for UH derivation. The Flood Estimation Handbook (IH, 1999) provides a good review of several methods of SUH derivation; there are two distinct approaches. The first uses an empirical method, e.g. McCarthy (1938) and Snyder (1938), where functional relationships of catchment characteristics are used for this purpose. One such relationship was proposed by Bernard (1935), who accomplished the transformation of rainfall into runoff through distribution graphs; it was assumed to be a function of catchment characteristics. Similar prominent approaches are later given by the Soil Conservation Service (SCS) (Hydrology 1972), Clark (1945), Gray (1961), Edson (1951), Gray (1961) and Haan et al. (1984), among others. All these methods begin by obtaining the salient points of the UH, and a smooth curve is fitted through these points to obtain a SUH; a degree of subjectivity is involved in such manual fitting, as this requires simultaneous adjustments for the area under the UH to represent unit runoff volume.

Several methods for synthetic unit hydrographs are available in literature. Most of them involve manual, subjective fitting of a hydrograph through few data points. Because it is difficult, the generated unit hydrograph is often left unadjusted for unit runoff volume. To circumvent this problem, a simplified version of the existing two-parameter gamma distribution was introduced to derive a synthetic hydrograph more conveniently and accurately than the popular Gray, Soil Conservation Service, and Synder methods. Thereafter, Due to similarity in shapes, several attempts have been made in the past to use pdf for UH and its derivation (Gray, 1961; Sokolov et al., 1976; Ciepielowski, 1987). The pdfs of the Gamma and Beta distributions to represent the UH shape, were used by Croley (1980) and Haktanir and Sezen (1990), respectively. Yue et al. (2002) extended the use of these pdfs to predict the shape of flood hydrograph for a T-year return period. Singh (2000) transmuted the popular SUHs, such as those of Snyder, SCS, and Gray into the Gamma distribution. However, using the concept of instantaneous unit hydrograph (IUH) Nash (1959) and Dooge (1959) were the first to derive the two parameter Gamma distribution from a cascade of linear reservoirs. Since then, the Gamma distribution is most commonly used in various forms depending on the values of peak flow rate and time to peak, because it yields a smooth and single-valued shape of the hydrograph of unit runoff volume, essential for UH derivation. Depending on the data availability, Croley (1980) and Aron and White (1982) derived the two parameters of the Gamma distribution: (t_p,q_p), (t_p, t_I) or (q_p, t_I), where t_I is the point of inflection after the peak [T], q_p is the peak discharge per unit area per unit effective rainfall [T^{-1}], and t_p is the time to peak [T]. These parameters are useful in dimensionless hydrograph derivation (SCS, 1986; Hann et al., 1994). The Gamma distribution yielding the complete shape of IUH from the non-dimensional shape factor (=$q_p t_p$) is derivable from geomorphological characteristics of the watershed, and therefore, is useful for ungauged catchments (Bhunya et al., 2003). Bhunya et al. (2003b) introduced a simplified version of two-parameter gamma distribution to drive a SUH more conveniently and accurately than the popular Snyder, SCS, and Gray methods.

Similarly, the three-parameter Beta distribution provides all possible shapes depending on the magnitude of its parameters (Johnson and Kotz, 1970), therefore, it was used by Haktanir and Sezen (1990) for SUH derivation, and Bhunya et al. (2004) demonstrated its flexibility in UH prediction for Turkish and Indian catchments. However, for reasons of non-availability of an explicit parameter estimation procedure, the pdf parameters are determined using the least square approach or any other optimization procedure with suitable error criteria. Since the synthetic methods are still widely practiced in developing countries such as India (CWC, 1993) and Turkey (Haktanir and Sezen, 1990) among others, Bhunya et al. (2007a) explored the potential of four pdfs, i.e., two-parameter Gamma, three-parameter Beta, Weibull, and Chi-square distributions for SUH derivation and simple expressions are derived for computing the distribution parameters in terms of salient points of UH, e.g., q_p and t_p, using both analytical and numerical methods.

Recently, Bhunya et al. (2009) explored the potential of using the density functions of the one-parameter chi-square distribution (1PCSD) and the two-parameter Fréchet distribution (2PFD) as parametric expressions for describing synthetic unit hydrographs (SUH) using geomorphological parameters of the catchment and, in particular, Horton ratios. Since the beginning of SUH concept proposed by Bernard (1935), several attempts have been made to explore different synthetic unit hydrograph methods. Apart from their merits and demerits, these methods are used separately & many efforts are reported in literature to synthesize different types of SUHs.

2.1 SYNTHETIC UNIT HYDROGRAPH (SUH) METHODS

The SUH methods are of great significance in determination of flood peak and runoff volume, especially from ungauged watersheds. The qualifier "synthetic" here denotes that the ordinate of UH is obtained without using watershed's rainfall – runoff data (Bhunya et al., 2003b). These synthetic or artificial unit hydrographs are characterized by their simplicity and ease in construction. They require less amount of data and yield a smooth and single – valued shape corresponding to unit runoff volume, which is essential for UH derivation.

2.1.1 Background

The unit hydrograph (UH) concept developed by Sherman (1932) for estimating the storm runoff hydrograph at the gauging site in a watershed corresponding to a rainfall hyetograph is still a widely accepted and admired tool in hydrologic analysis and synthesis. This was one of the first tools available to hydrologic community to determine the complete shape of the hydrographs rather than the peak discharges only (Todini, 1988). As the unit hydrograph concept needed the observed rainfall-runoff data at the gauging site for hydrograph generation, the paucity of these data sparkled the idea of SUH concept. Numerous techniques have been developed to determine runoff when limited data are available. Among these, SUH is of great significance for determining runoff volume with respect to time, especially from ungauged catchments. The beginning of SUH concept can be traced back to the model (distribution graph) proposed by Bernard (1935) to synthesize the UH from watershed characteristics, rather than the rainfall runoff data (Singh, 1988). The methods used for derivation of unit hydrographs in catchments where there is a limited amount of data and for catchments with no data i.e. ungauged catchments will be discussed here under four sections: (i) popular SUH methods; (ii) conceptual methods of SUH; (iii) geomorphologic unit hydrograph (GIUH) based synthetic SUH methods; and (iv) probability distribution function based SUH methods.

2.1.2 POPULAR SYNTHETIC UNIT HYDROGRAPH (SUH) METHODS

The UH obtained for a gauged catchment by analyzing observed rainfall and runoff data is applicable for the gauging site at which the runoff data were measured. SUH is a tool to derive UH for the other gauging stations in the same catchment or for the other similar catchments for which runoff data are not available. In practice, an SUH is derived from a few salient points of the UH by fitting a smooth curve manually. The methods of Synder (1938), Taylor and Schwarz (1952), Soil Conservation Services (SCS, 1957), Gray (1961), and Espey and Winslow (1974) are a few examples among others, which utilize empirical equations to estimate salient points of the hydrograph, such as peak flow (Q_p), lag time (t_L), time base (t_B), and UH widths at $0.5Qp$ and $0.75Qp$. A greater degree of subjectivity and labor is involved in fitting a smooth curve manually over a few points to get an SUH and at the same time to adjust the area under the SUH to unity. In spite of their limitations, these methods are widely used for SUH derivations in ungauged watersheds. Popular methods of obtaining an SUH are discussed below.

2.1.2.1 Snyder's Method

Based on a study of a large number of catchments in the Appalachian Highlands of eastern United States, Snyder (1938) developed & established a set of empirical equations for SUHs in those areas, which relate the watershed characteristics, such as A_w = area of the watershed (square km); L = length of main stream (km); and L_{CA} = the distance from the watershed outlet to a point on the main stream nearest to the center of the area of the watershed (km) with the three basic parameters of the UH (i.e. t_L= lag time to peak (hr); Q_p = peak flow rate (m^3 /s); and t_a = base time (hours), to describe the shape of the UH, expressed as:

$$t_L = C_T(LL_{CA})^{0.3} \tag{2.1}$$

$$Q_P = 2.78\left(\frac{A_W C_P}{t_L}\right) \tag{2.2}$$

$$t_B = 5(t_L + t_D/2) \tag{2.3}$$

where C_T and C_P are non-dimensional constants, varying from 0.5 to 1.2 and 0.56 to 0.96, respectively. Eqs. (2.1) to (2.2) hold good for rainfall-excess duration (or unit duration = T_D (hr)) as:

$$t_D = \frac{t_L}{5.5} \tag{2.4}$$

If the duration of rainfall – excess, say D (hr), is different from T_D, a revised lag time t_{LR} (hr) is estimated from

$$t_{LR} = t_L + \frac{(D - T_D)}{4} \tag{2.5}$$

These relationships (Eqs. 2.1 to 2.5) provide a complete shape of SUH. Again the U.S. Army Corps of Engineers (USACE, 1940) developed empirical equations between widths of SUH at 50% and 75% of Q_P i.e. W_{50} and W_{75} respectively as a function of $(Q_p/A_w) = q_p$ expressible as:

$$W_{50} = \frac{5.87}{q_P^{1.08}} \tag{2.6}$$

$$W_{75} = \frac{W_{50}}{1.75} \tag{2.7}$$

where W_{50} and W_{75} are in units of hour. Thus, one can sketch a smooth curve through the seven points (t_L, t_B, Q_p, W_{50} and W_{75}) relatively in an easier way with less degree of ambiguity and to have the area under the SUH unity. However, the procedure followed is tedious and involves great degree of subjectivity and errors due to manual fitting of the points and simultaneous adjustments for the area under SUH. In summary, the major inconsistencies associated with the method are:

1. the manual fitting of the characteristic points involves great degree of subjectivity and trial and error, and may involve error.
2. the constants C_T and C_P vary over wide range and from region to region, and may not be equally suitable for all the regions. Some investigators have shown that value of C_T may ranging from 0.3 to 6.0 have been reported and C_P range from 0.31 to 0.98 have been reported.
3. Time base t_B (Eq. 2.3) is always greater than three days (Raudkivi, 1979) and is applicable to large watersheds only (Langbein, 1947; Taylor and Schwarz, 1952; Gray 1961).

2.1.2.2 SCS Method

The SCS method (SCS, 1957, 1972) of the United States Department of Agricultural (USDA) uses a specific average dimensionless unit hydrograph derived from the analysis of large number of natural UHs for the watersheds of varying size and geographic locations, to synthesize the UH (Singh, 1988). The method assumes the triangular shape of dimensionless unit hydrograph in order to define time base, t_B, in terms of time to peak, t_p, and time to recession, t_r, and to compute runoff volume (V_R) and peak discharge q_p as:

$$V_R = \frac{(q_p t_B)}{2} = \frac{1}{2} q_p (t_p + t_r);\ t_r = 1.67 t_p \tag{2.8}$$

$$q_P = 0.749 \left(\frac{V_R}{t_P}\right) \tag{2.9}$$

where q_p is in mm/hr/mm (or inch/hr/inch) and can be related to Q_p as equal to Q_p/A_w per unit depth of rainfall excess; V_R is in mm (or inch); t_p and t_r are in hrs. To determine the SUH shape from the non-dimensional (q/q_p vs t/t_p) hydrograph, the time to peak and peak flow rate are computed as:

$$t_p = t_L + \left(\frac{D}{2}\right) \tag{2.10}$$

$$Q_p = 2.08 \left(\frac{A_W}{t_p}\right) \tag{2.11}$$

$$t_p = \left(\frac{D}{2}\right) + 0.6tc \tag{2.12}$$

where t_L = lag time from centroid of excess – rainfall to peak discharge (Q_p) (hour); D = the excess – rainfall duration (unit duration) (hour); Q_p = peak discharge in m^3/s; and A_w = area in square km.

Further on the basis of a large number of small rural watersheds, SCS found that $t_L \approx 0.6 t_c$, where t_c = time of concentration. The lag time (t_L) can be estimated from the watershed characteristics using curve number (CN) procedure as:

$$t_L = \frac{L^{0.8}(2540 - 22.86CN)^{0.7}}{14104 CN^{0.7} S_{av}{}^{0.5}} \tag{2.13}$$

where t_L = in hours; L = hydraulic length of watershed (m); CN = curve number (50, 95); and S_{av} = average catchment slope in (m/m). Thus, with known Q_p, t_p, and specified dimensionless UH, the SUH can be developed smoothly. However, the inconsistencies associated with the method can be enumerated as follows:

1. Since the SCS method fixes the ratio for time base to time to peak (t_B/t_p) for triangular UH equal to 2.67 (or 8/3), ratios other than this may lead to other shapes of UH. In particular, the larger ratio implies the greater catchment storage. Therefore, since the SCS method fixes the ratio (t_B/t_p), it should be limited to mid-size watersheds in the lower end of the spectrum (Ponce, 1989).
2. This is one of the popular methods for synthesizing UH for only small watersheds of less than 500 sq miles (Wu, 1969; Wang and Wu, 1972; and McCuen and Bondelid, 1983).
3. Snyder (1971) recommended that a single SCS dimensionless unit hydrograph should not be used for watersheds greater than 20 mi^2.

2.1.2.3 Gray's Method

Gray (1961) developed a dimensionless graph (empirical in nature) procedure based on two-parameter gamma distribution function and watershed characteristics to derive a SUH. The geometry of dimensionless graph is expressed as:

$$Q_{t/P_R} = \frac{25.0(\gamma')^{\lambda'}}{\Gamma(\lambda')}\left(e^{-\lambda' t/P_R}\right)\left(\frac{t}{P_R}\right)^{(\lambda'-1)} \qquad (2.14)$$

where $Q_{t/PR}$ = percent flow / 0.25 P_R at any given t/P_r value; P_R = the time from beginning of surface runoff to the occurrence of peak discharge (minutes); γ' = a dimensionless parameter = γP_R; λ' = shape parameter = $1+\gamma'$; γ = scale parameter; Γ =gamma function.

In words of Gray (1961), "Each graph was adjusted with the ordinate values expressed in percentage flow based on a time increment equal to one-fourth the period of rise, P_R. The empirical graphs described in this manner were referred to as dimensionless graphs". He defined the ratio $1/\lambda = P_R/\lambda'$ as the storage factor, a measure of the storage property of watershed or the travel time required for water to pass through a given reach, and related it with the watershed characteristics in the form of a power equation as:

$$\frac{P_R}{\gamma'} = a\left(\frac{L}{\sqrt{S_M}}\right)^b \qquad (2.15a)$$

where a and b are the coefficient and exponent of the power equation. Eq. (2.15a) was applied to 33 watersheds comprising of three regional groups: (i) Nebraska-Western Iowa; (ii) Central Iowa-Missouri-Illinois and Wisconsin; and (iii) Ohio, to estimate a and b. Finally, for each group Eq. (2.15a) is expressed as:

For Nebraska-Western Iowa: $$\frac{P_R}{\gamma'} = 7.40\left(\frac{L}{\sqrt{S_M}}\right)^{0.498} \quad (2.15b)$$

For Central Iowa-Missouri-Illinois and Wisconsin: $$\frac{P_R}{\gamma'} = 9.27\left(\frac{L}{\sqrt{S_M}}\right)^{0.562} \quad (2.15c)$$

For Ohio: $$\frac{P_R}{\gamma'} = 11.40\left(\frac{L}{\sqrt{S_M}}\right)^{0.531} \quad (2.15d)$$

where the ratio P_R/γ' is in minutes; L = length of main stream in miles; S_M is the slope of main stream in %. Finally, Gray developed a regression relationship between the period of rise P_R and dimensionless parameter γ' as:

$$\frac{P_R}{\gamma'} = \frac{1}{\frac{2.676}{P_R}+0.0139} \quad (2.16)$$

Thus, Eqs (2.14 to 2.16) are used to develop the dimensionless UH, and consequently, the SUH. One of the best findings of the study is that the two-parameter gamma distribution can be used successfully to describe the SUH. However, the empirical relationships (Eqs. 2.15 to 2.16) are watershed size specific and should be used within the area limits for which these are developed (Gray, 1961).

2.1.3 Conceptual Synthetic Unit Hydrograph Methods

In this section the popular conceptual models of Clark (1945); Nash (1958, 1959); and Hybrid model (HM) of Bhunya et al. (2005) are discussed.

2.1.3.1 Clark's Model

Clark (1945) proposed that a SUH could be obtained by routing 1 inch of direct runoff into the channel in proportion to the time-area curve and routing the runoff entering the channel through a linear reservoir. Based on this concept, IUH can be derived by routing unit excess rainfall in the form of a time area diagram through a single linear reservoir. For derivation of IUH the Clark model uses two parameters viz. time of concentration (T_c) in hours and storage coefficient (K) in hours of a single linear reservoir in addition to the time-area diagram. The governing equation of the Clark IUH model is expressed as (Kumar et al., 2002):

$$u_i = CI_i + (1 - C)u_{i-1} \quad (2.17)$$

where u_i = i^{th} ordinate of IUH; C and (1-C) = the routing coefficients; and C = $\Delta t/(K+0.5\,\Delta t)$; Δt = computational interval in hours; I_i = i^{th} ordinate of time-area diagram. Finally, a unit hydrograph of desired duration (D) can be derived using the following equation:

$$U_i = \frac{1}{N}(0.5_{i-N} + u_{i-N+1} + \ldots\ldots\ldots + u_{i-1} + 0.5u_i) \quad (2.18)$$

where U_i = i^{th} ordinate of unit hydrograph of D-hour duration and computational interval Δt hours; N = number of computational intervals in D-hours = $D/\Delta t$. Eq. (2.18) can also be used to derive flood hydrograph in ungauged catchments. One of the approaches popularly used by field engineers is through regionalization of Clark parameters K and C. For example, HEC-1 (1990) evaluates these parameters for determining the representative UH for a catchment. The computed parameters are given in the form of $K(T_c+K)$, which can be used for developing a regional relationship by relating it to physical characteristics of different catchments in a homogeneous region. This regional relationship can then be used to compute the Clark model parameters for an ungauged catchment which is then used to derive UH. Alternatively, data of gauged catchments in a region can be used to develop regional relationship that yields either of the parameters for an ungauged catchment in the region. However, some of the inconsistencies associated with Clark's model are of concern such as (i) the entire hydrograph recession is represented by a single recession constant, while a recession constant that varies with time is implicitly incorporated into Nash model (Nash, 1958). HEC-1 uses Snyder's C_p and t_p to optimize the parameters (T_c and K) of Clark's UH, which are required for application. This is a limitation of Clark's UH to be used as an SUH.

2.1.3.2 Nash Model

In a series of publications, Nash (1957, 1958, 1959, 1960) developed a conceptual model based on a cascade of n equal linear reservoirs with equal storage coefficient K for derivation of the IUH for a natural watershed. The analytical form of the model is expressed as:

$$q(t) = \frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{-\frac{t}{K}} \qquad (2.19)$$

where q(t) is the depth of runoff per unit time per unit effective rainfall and K is the storage coefficient of the reservoirs in units of hours. The parameters n and K are often termed, respectively, as the shape and scale parameters. It is noteworthy that parameter n is dimensionless and K has the unit of time. The area under the curve defined by Eq. (2.19) is unity. Thus the rainfall – excess and direct surface runoff depths are equal to unity. The IUH (Eq. 2.19) is used to derive the resultant flood hydrograph for a given input rainfall. To estimate n and K, Nash (1960) related the first and the second moments of the IUH with important physical characteristics for some English catchments as follows:

$$m_1 = 27.6{A_W}^{0.03}{S_0}^{-0.3} = \frac{1}{K\Gamma(n)}\int_0^{\infty}\left(\frac{t}{K}\right)^{n-1} e^{-\frac{t}{K}}(t)dt = nK$$

(2.20a)

$$m_2 = 1.0{m_1}^{-0.2}{S_0}^{-0.1} = \frac{1}{K\Gamma(n)}\int_0^{\infty}\left(\frac{t}{K}\right)^{n-1} e^{-\frac{t}{K}}(t^2)dt = n(n+1)\,K^2$$

(2.20b)

where m_1 and m_2 are the first and the second moments of the IUH about the origin, A_w is the catchment area in square miles, and S_o is the overland slope. Eqs. (2.20) can be used compute the parameters of Eq. (2.19). Once the parameters are evaluated from available A_w and S_o, the complete IUH can be derived using Eq. (2.19). Thus, is one of the approaches for deriving IUH for ungauged catchments. It may be noted here that IUH can be extended to a UH for the catchment using existing conventional procedures (Ponce, 1989; Bras, 1990; Singh, 1992). It is observed that Eq. (2.19) is nothing but the two-parameter gamma distribution (2PGD). Use of two-parameter gamma distribution for representing the SUH has long hydrologic history that started with Edson (1951) and subsequently followed by Croley (1980), Aron and White (1982), Haan et al. (1994), and Bhunya et al. (2003b, 2004, 2007a). A detail review of these studies on gamma distribution along with some popular probability distributions as SUH is discussed in the forthcoming section.

2.1.3.3 Hybrid Model (HM)

To overcome the inconsistencies associated with the Nash model such as (i) the number of linear reservoirs 'n' should desirably be an integer value, generally comes out to be a fractional value when derived from observed data (Singh, 1988); and (ii) a single linear reservoir of Nash model (n=1) yields as IUH that follows extreme Poisson distribution without a rising limb, or $t_p = 0$. Hence, to simulate a complete IUH with rising limb (or $t_p > 0$) the Nash model requires a minimum of two reservoirs connected in series. Building on this idea, Bhunya et al. (2005) developed a hybrid model for derivation of synthetic unit hydrograph by splitting Nash's single linear reservoir into two serially connected reservoirs of unequal storage coefficient (one hybrid unit) to have a physically realistic response. The hybrid unit concept is similar to one generally used in chemical engineering for defining a unit of chemical system (Kafarov, 1976). The analytical form of the model for two hybrid units in series is expressed as:

$$Q_2(t) = \frac{1}{(K_1-K_2)^2}\left[\left(te^{-\frac{t}{K_1}} + te^{-\frac{t}{K_2}}\right) - \frac{2K_1K_2}{(K_1-K_2)}\left(e^{-\frac{t}{K_1}} - e^{-\frac{t}{K_2}}\right)\right] \qquad (2.21)$$

where $Q_2(t)$ = output from the second hybrid unit (mm / hr / mm); and K_1 and K_2 = storage coefficient of the first and the second reservoirs (hr), respectively, of each hybrid unit. From Eq. (2.21) one can get easily the expression for time to peak flow rate (t_p) for the condition at $t = t_p$ $Q_2(t) = Q_p$ or $dQ_2(t)/dt = 0$. Eq. (2.21) is nothing but the output response function for the second hybrid unit due to a unit impulse perturbation at the inlet of the first hybrid unit and defines the complete shape of IUH. Eq. (2.21) has two parameters, K_1 and K_2. They developed empirical relationships to estimate K_1 and K_2 from known peak flow rate (q_p) and time to peak (t_p). However, for ungauged conditions q_p and t_p were estimated through Snyder method (Snyder, 1938) and SCS method (SCS, 1957). The hybrid model was found to work significantly better than the most widely used methods such as Snyder, SCS, and Nash model (two parameter gamma distribution function) when tested on the data of Indian and Turkey catchments for partial (known q_p and t_p) and no data availability (ungauged) conditions.

2.1.4 Geomorphologic Instantaneous Unit Hydrograph (GIUH) Based SUH Methods

Recently, the hydrologists are finding it more convenient to couple the distribution function based approach with the classical geomorphologic instantaneous unit hydrograph (GIUH) approach for development of SUH models by harvesting the remote sensing (RS) and geomorphologic information systems (GIS) technologies. Hence it would be better to get familiarized the readers with GIUH approach in a very simplified and lucid manner. Following this, the next section deals with GIUH approach right from its origin to the most recent developments and applications in hydrological sciences. There must not be any ambiguity to accept the fact that the GIUH approach has gone/is going under fabulous applications with the recent advancements in geo-spatial and geo-sciences techniques such as RS and GIS. As discussed in the previous section, Nash (1957) built a theory on an analogy between the basin and a series of linear reservoirs and proposed a gamma law type analytical expression of the UH. The main point of this model is the gamma law type, since Nash considered it to be the characteristic basin UH shape. For the same *a priori* reason, much work in hydrological sciences is based on this model only. Thereafter, the idea of identifying a basin-scale transfer function (TF) from some geomorphological characteristics emerged in order to give physical basis to the UH. Thus, linking quantitative geomorphology with basin hydrologic characteristics can provide a simple way to understand the hydrologic behavior of different basins, particularly the ungauged ones. These characteristics relate to the physical characteristics of the drainage basin as well as the drainage network. The physical characteristics of the drainage basin include drainage area, basin shape, ground slope, and centroid. On the other hand, in a drainage network the important channel characteristics include number of channels of different orders, their lengths and slopes, etc.

Dooge (1959) initiated a kind of quest by affirming that such a theory would be gratifying and 'should help to remove many of the subjective elements from unit hydrograph analysis and also to release the problem of synthesis from its present dependence on empirical relationships derived from localized data'. Regarding the geomorphological basement itself for UH identification, Rodriguez-Iturbe and Valdés (1979) formulated the GIUH trying to reach the universality 'with the conviction that the search for a theoretical coupling of quantitative geomorphology and hydrology is an area which will provide some of the most exciting and basic developments of hydrology in the future'. The roots can be traced back to Horton (1945) who originated the quantitative study of channel networks and developed a system for ordering streams networks and derived laws relating the stream numbers (N), stream lengths (L), and catchment area (A) associated with streams of different order. The quantitative expressions of Horton's laws are expressed as (Rodriguez-Iturbe and Valdés, 1979):

Law of stream number: $$N_w / N_{w+1} = R_B \qquad (2.22)$$

Law of stream length: $$\bar{L}_w / \bar{L}_{w-1} = R_L \qquad (2.23)$$

Law of stream areas: $$\bar{A}_w / \bar{A}_{w-1} = R_A \qquad (2.24)$$

where N_w is the number of streams of the order w, $\bar{L}_w$ is the mean length of stream of order w, and $\bar{A}_w$ is the mean area of basin of order w. R_B, R_L, and R_A represent the bifurcation ratio, length ratio, and area ratio whose values in nature are normally between 3 and 5 for R_B, between 1.5 and 3.5 for R_L, and between 3 and 6 for R_A.

Several attempts have been made to establish relationships between the parameters of the models of ungauged catchments and the physically measurable watershed characteristics (Bernard, 1935; Snyder, 1938; Taylor and Schwarz, 1952;

Gray, 1961; and Boyd et al., 1979 and 1987). In this regard, the pioneering works of Rodriguez-Iturbe and Voldes (1979), Valdés et al. (1979), and Rodriguez-Iturbe et al. (1979), which explicitly integrate the geomorphology details and the climatological characteristics of a basin, in the framework of travel time distribution, are a boon for stream flow synthesis in ungauged framework of travel time distribution. Gupta et al. (1980) parameterized the Nash model in terms of Horton order ratios of a catchmnet based on the geomphorlogic model of a catchment. Rinaldo and Rodriguez – Iturbe (1996) and Rodrigue-Iturbe and Rinaldo (1997) expressed the pdf of travel times as a function of the basin forms characterized by the stream networks and other landscape features. Chutha and Dooge (1990) reformulated the GIUH on a deterministic platform rather than on Markov and statistical mechanics approach. Kirshen and Bras (1983) studied the effect of linear channel on GIUH. Al-Wagandy and Rao (1997) investigated the dependency of average velocity of GIUH on climatic and basin geomorphologic parameters and found that the average velocity varies inversely with effective rainfall depth. Cudennec et al. (2004) provided the geomorphological explanation of the UH concept based on the statistical physics reasoning that considers a hydraulic length symbolic space, built on self similar lengths of the components, and derived the theoretical expressions of the probability density functions of the hydraulic length and of the lengths of all the components in form of gamma pdf in terms of geomorphological parameters. Allam and Balkhair (1987) discussed several issues related to the probabilistic and hydraulic structure of the GIUH concept. Jain et al. (2000), Jain and Sinha (2003), Sahoo et al. (2006), Kumar et al. (2007) applied geographic information system (GIS) supported GIUH approach for estimation of design flood. Similarly the works of Bérod et al. (1995), Sorman (1995), Bhaskar et al. (1997), Yen and Lee (1997), Hall et al. (2001), and Fleurant et al. (2006) based on GIUH approach for estimation of design flood from gauged as well ungauged basins are noteworthy.

2.1.5 Probability Distribution Function Based SUH Methods

Due to similarity in the shape of the statistical distributions and a conventional unit hydrograph, several attempts have been made in the past to use their probability density functions (pdfs) for derivation of the SUH. For example, Gray (1961), Sokolov et al. (1976), Croley (1980), Aron and White (1982), Haktanir and Sezen

(1990), Yue et al. (2002), and Nadarajah (2007) are to name but are only a few of them. Singh (2000) transmuted the popular SUHs, such as those of Snyder, the SCS, and Gray into the Gamma distribution. Singh presented a simple method for transmuting, the calculations for which can be performed on a calculator. It gives a smooth shape for the SUH, and guaranteed the area under the hydrograph to be unity.

Bhunya et al. (2003b & 2004) utilized two-parameter Gamma distribution (2PGD) and three parameter Beta distribution function (3PGD) in deriving SUH for Indian as well as Turkey catchments. More recently, Bhunya et al. (2007a) explored the potential of four popular pdfs, i.e. two-parameter Gamma, three-parameter Beta, two- parameter weibull, and one parameter Chi-square distribution to derive SUH. Here, simple formulae are derived using analytical and numerical schemes to compute the distribution parameters, and their validity is checked with simulation of field data.

Croley (1980) developed SUH by fitting two-parameter gamma distribution for different sets of boundary conditions: (t_p, q_p), (t_p, t_l) or (q_p, t_l). These boundary conditions are used to estimate the parameters n and K of the distribution. The general expression for the synthetic hydrograph is expressed as:

$$q(t) = \frac{V_R}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{-\frac{t}{K}} \qquad (2.25)$$

where V_R is defined as:

$$\int_0^{\infty} q(t)dt = V_R \qquad (2.26)$$

where t_l is the point of inflection [T], q_p is the peak discharge per unit area per unit effective rainfall [T^1] and t_p is the time to peak [T]. It is interesting to note that if V_R corresponds to the volume of runoff produced by a unit depth of rainfall-excess uniformly applied both spatially over the watershed area and temporarily over the storm duration, then q(t) (Eq. 2.25) is by definition, the 'unit hydrograph' for that area and for that storm duration. It can be converted easily into hydrographs corresponding to other rainfall excess depths and storm durations by using the available linear superposition techniques (Linsley et al., 1975; Croley, 1977). The methodology provides a line of initiation to work with probability distribution functions for SUH derivation for ungauged catchments.

Haktanir and Sezen (1990) explored the suitability of two-parameter gamma and three-parameter beta distributions as SUHs for Anatolia catchments in Turkey. Yue et al. (2002) extended the use of these pdfs to predict the shape of flood hydrograph for a T-year return period. Bhunya et al. (2003b) introduced a simplified version of two-parameter gamma distribution to drive a SUH more conveniently and accurately than the popular Snyder, SCS, and Gray methods. The analytical form of the distribution is represented by Eq. (2.19). They also defined a non-dimensional term $\beta = q_p t_p$ same as to Rosso (1984) to relate n and β. Since the exact solution of n in terms of β is not possible, they developed simpler relationships between n and β to obtain the simplified versions of gamma distribution. The developed relationships are given as:

$$n = 5.53\beta^{1.57} + 1.04 \qquad \text{for} \qquad 0.01<\beta<0.35;\ \text{COD}=1 \qquad (2.27a)$$

and

$$n = 6.29\beta^{1.998} + 1.157 \qquad \text{for} \qquad \beta \geq 0.35;\ \text{COD}=1 \qquad (2.27b)$$

Thus, for known values of β, n can be estimated from Eq. (2.27) and K from $K = t_P/(n-1)$. In addition, it eliminates the cumbersome trial and error solution to estimate n and K. One thousand sets of (n, β) values with n ranging from 1 to 40.0 and β ranging from 0.01 to 2.5 were considered for developing the relationships (Eqs. 2.27a, b). The major findings are: (i) n can be expressed mathematically in terms of β in a simple but accurate form; (ii) the parameter n and dimensionless term β are dependent not only on the physical characteristics of the watershed, but also on its storage characteristics; and (iii) the present approach worked better than the Snyder, SCS, and Gray methods.

Bhunya et al. (2007a) explored the potential of four popular pdfs, viz., two-parameter Gamma, three-parameter Beta, two-parameter Weibull, and one-parameter Chi-square distributions to derive SUH. They developed simple analytical and numerical relationships to compute the distribution parameters, and checked their validity using simulation and field data. Some of the important conclusions drawn from the study are as follows:

1. given two points on the UH, e.g., time to peak and peak flow, these pdfs can be used to describe the shape of the unit hydrograph, and they perform

better than the existing synthetic methods, such as those by Snyder (1938), SCS (1957), and Gray (1961).

2. the proposed analytical solutions for parameter estimations are simple to use and give accurate results of the actual pdf parameters.
3. among the four pdfs analyzed in the study, the Beta and Weibull distributions are more flexible in description of SUH shape as they skew on both sides similar to a UH, and on the basis of their application to field data.

2.1.6 Remarks

The SUH approach is a powerful tool to estimate flood peak, time to peak, and the complete shape of unit hydrograph for ungauged catchments. Central Water Commission (CWC) of India has carried out extensive study on derivation of SUHs for various regions in India. Two approaches, short term and long-term were adopted by CWC to develop methodologies of design flood discharges applicable to small and medium catchments (25-1000 ha) of India. The SUH methods developed so far can be categorized as (i) the empirical methods of Snyder, SCS, and Taylor and Schwartz methods; (ii) conceptual models of Clark and Nash, and Bhunya et al. (2005); (iii) GIUH based methods of Rodriguez-Iturbe and Valdés (1979),Gupta et al. (1980), Rosso (1984), etc.; and (iv) the probability distribution functions based methods of Gray, (1961), Sokolov et al., (1976), and Croley (1980), Haktanir and Sezen (1990), Bhuyan et al. (2003b, 2004, 2007a), and Nadarajah (2007), etc. Though the empirical methods of Snyder and SCS are very common and widely used for SUH derivation, but have several limitations. Due to similarity in shapes, several attempts have been made in the past to use pdf and recently some exploration of pdfs has been done successfully for SUH derivation. Pdfs based methods yield a smooth and single valued shape UH corresponding to one unit runoff volume and lastly gives better results than other methods for derivation of SUH.

CHAPTER-3

STUDY AREA AND DATA ACQUIREMENT

In the present study, the storm rainfall-runoff data, comprising of UH data and geomorphological characteristics, of four Indian watersheds is used for testing the workability of the traditional SUH methods of Snyder (1938) and Soil Conservation Service (SCS) (SCS, 1957) and GIUH coupled probability models of One-parameter Chi-square (1PCSD), Two-parameter Fréchet (2PFD), Two-parameter Inverse Gamma (2PIGD), and Two-parameter Gamma (2PGD) distributions for SUH derivation for limited data availability condition. Following four watersheds varying in area from 27.93 km^2 to 4000 km^2 having different hydro-climatological and terrain characteristics have been considered in this study.

- **Kothuwatari watershed**
- **Myntdu-Leska watershed**
- **Gagas watershed**
- **Burhner watershed**

A brief description of each study watershed is given here.

3.1 Kothuwatari watershed

The Kothuwatari catchment (area = 27.93 km^2) is a sub-catchment of Tilaiya dam catchment of upper Damodar Valley Corporation (DVC), Hazaribagh, India. The catchment is situated at the South-Eastern part of the Tilaiya dam catchment between 24° 12′ 27″ and 24° 16′ 54″ North latitudes and 85° 24′ 18″ and 85 °28′ 10″ East longitudes. The watershed is irregular in shape with a maximum elevation of 577 m above mean sea level. The general slope of land in the watershed varies from 1 to 10 percent. The climate of the watershed varies from sub-tropical to sub-temperate. Most of the land area in the watershed is used for agricultural and horticultural productions. In the year 1991, the catchment was selected for 'Watershed Management' under the "Indo-German Bilateral Project (IGBP)" for assessing the effects of soil conservation measures on runoff and wash load. The UH data for the catchment were taken from Singh (2003). Geomorphologic characteristics of study watershed have been extracted from ASTER data, resolution 30 m, using ILWIS (version 3.31) GIS software. The extraction procedure is being briefly summarized in the coming section of this chapter. The extracted digital elevation model (DEM) and the drainage network map of the study watershed is shown in Fig. 3.1 and 3.2, respectively. The details of catchment and unit hydrograph characteristics are given in Table 3.1.

Table 3.1 Summary of the UHs and catchment characteristics of the study area

Catchments	Catchment Characteristics							UH Characteristics	
	A(km2)	L(km)	L_{ca}(km)	Order	R_A	R_B	R_L	$q_p(h^{-1})$	t_p(h)
Kothuwatari	27.93	11.7	6.2	4	4.06	3.57	2.43	0.429	1

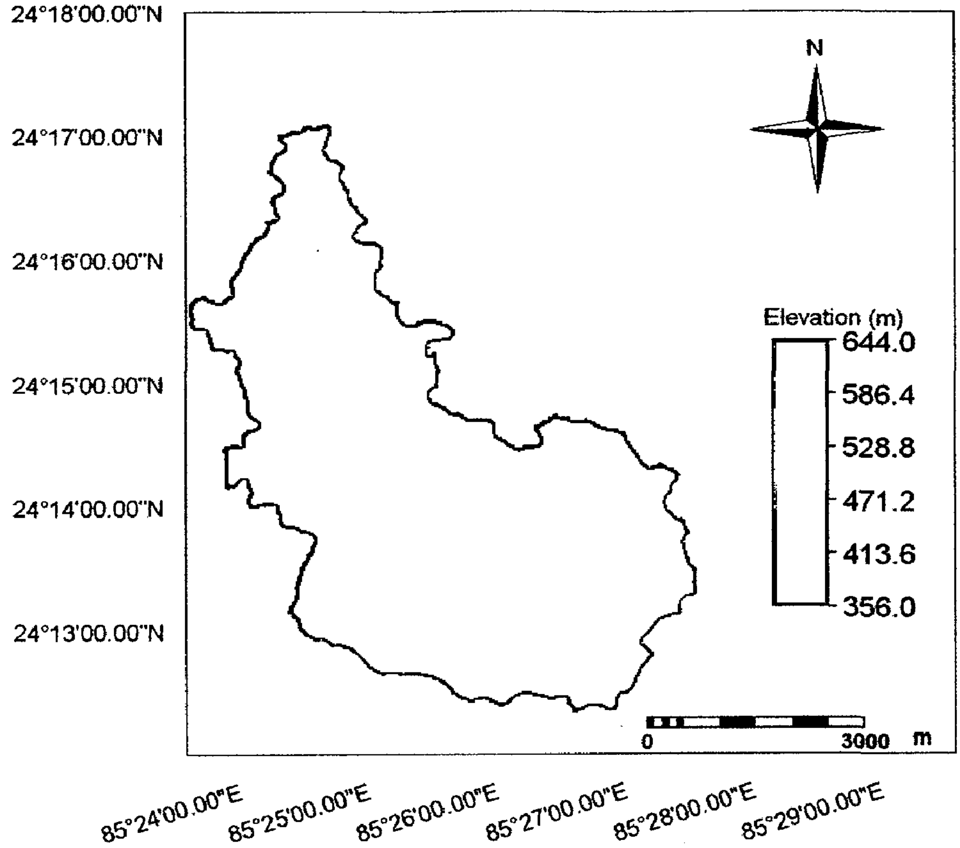

Figure 3.1: Digital Elevation Model (DEM) of Kothuwatari Catchment

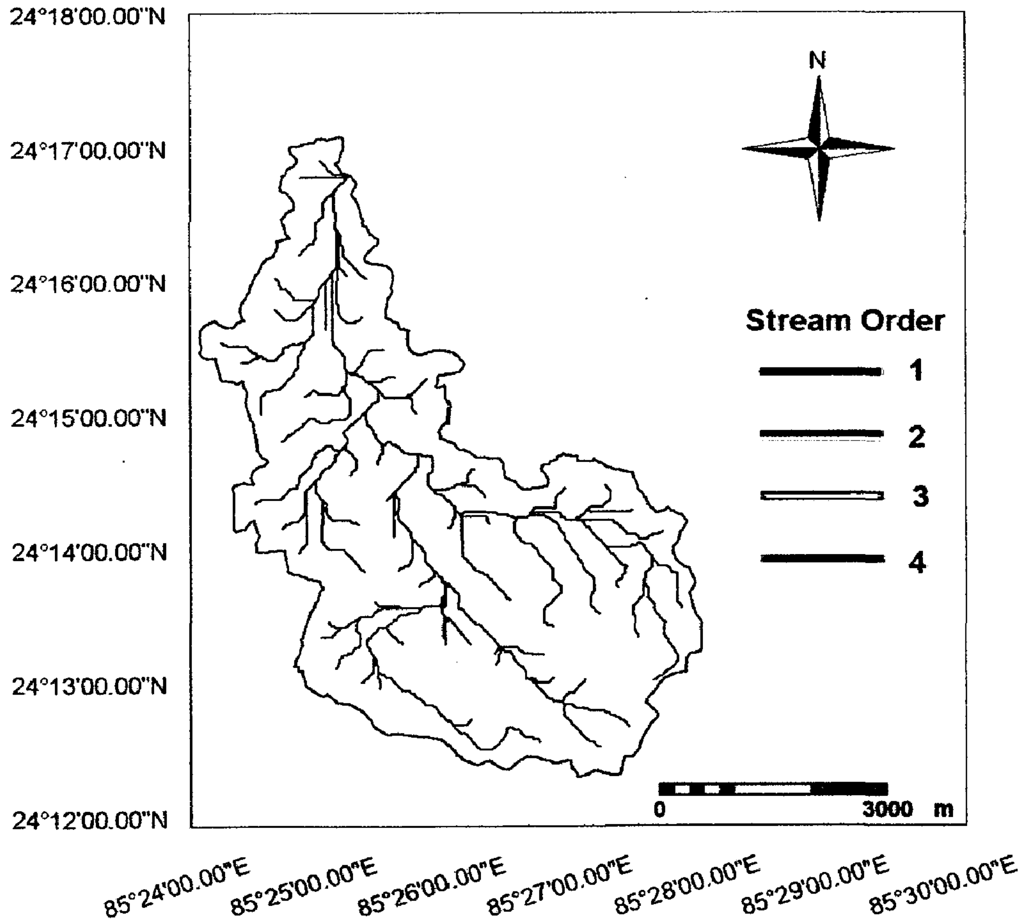

Figure 3.2: Drainage Network Map of Kothuwatari Catchment.

3.2 Myntdu-Leska watershed

The Myntdu-Leska river catchment (Fig. 3.3) is located in Jaintia hills district of Meghalaya, in the northeastern part of India, in the southern slope of the State adjoining Bangladesh. Its geographic location extends from 92° 05' to 92° 20' E longitude and 25° 15' to 25° 30' N latitude. The area is narrow and steep, lying between central upland falls of the hills of Meghalaya. The main season of this area is from May to October with a maximum monthly rainfall of 715 mm. The catchment area is about 350 sq. km and elevations range from about 1372 m to 595 m above mean sea level (msl). The UH data for the catchment were taken from Bhunya (2005). Its data were also used by Mani & Panigrahy (1998) and Bhunya *et al.* (2003, 2004, 2008). Geomorphologic characteristics of study watershed have been extracted from ASTER data, resolution 30 m, using ILWIS (version 3.31) GIS software. The extraction procedure is being briefly summarized in the coming section of this chapter. The extracted digital elevation model (DEM) and the drainage network map of the study watershed is shown in Fig. 3.3 and 3.4, respectively. The details of catchment and unit hydrograph characteristics are given in Table 3.2.

Table 3.2 Summary of the UHs and catchment characteristics of the study area

Catchments	Catchment Characteristics							UH Characteristics	
	A(km2)	L(km)	L_{ca}(km)	Order	R_A	R_B	R_L	$q_p(h^{-1})$	t_p(h)
Myntdu-Leska	350	51.8	27.8	5	4.61	4.27	2.12	0.122	5

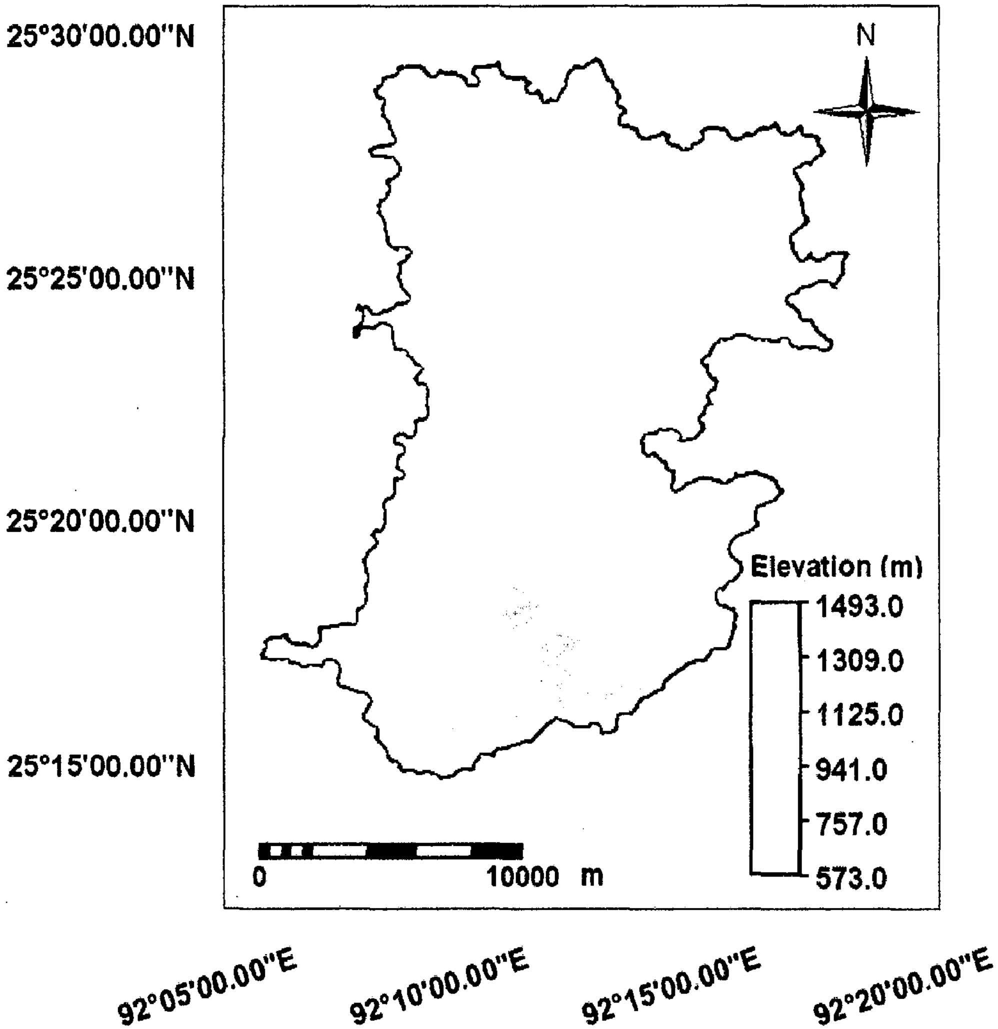

Figure 3.3: Digital Elevation Model (DEM) of Myntdu-Leska Catchment.

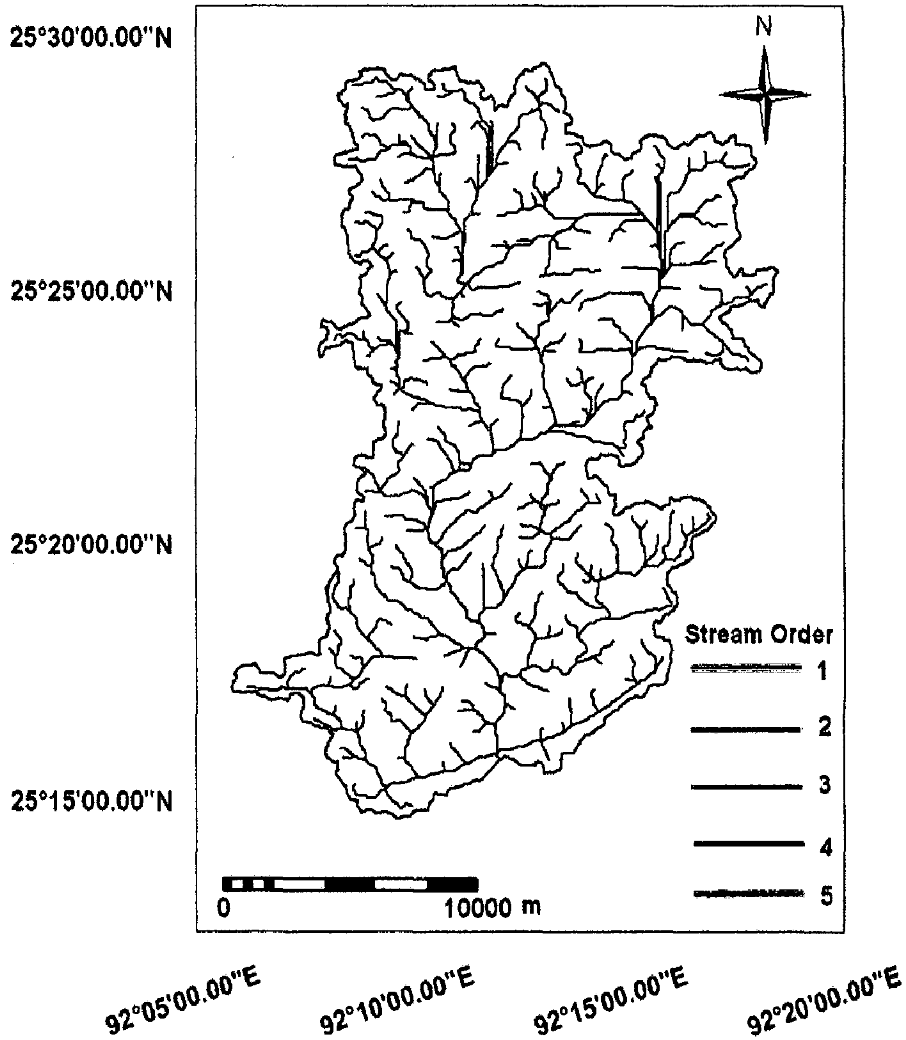

Figure 3.4: Drainage Network Map of Myntdu-Leska Catchment.

3.3 Gagas watershed

A mountainous catchment Gagas is a hilly catchment of river Gagas having an area of 506 km^2 located in the Himalayan region of India lies between latitudes 29^0 35' 20" N and 29^0 51'N, and longitudes 79^0 15'E and 79^0 35' 30"E as shown in Fig.3.5 has been selected for the present study. The Gagas Watershed lies in the middle and outer range of the Himalayas in Almora district of Uttarakhand State of India. The Gagas catchment is one of the sub-catchments of the Ramganga river catchment having total catchment area of 3134 km^2. The catchment is approximately rectangular in shape with a minimum elevation of 772 m at the outlet e.g., Bhikiasen and a maximum of 2744 m above mean sea level at the upstream end of the catchment. The catchment area in general has a hilly terrain with undulating and irregular slopes ranging from relatively flat in narrow river valley to steep towards ridge. About 35 percent of watershed area is under agriculture, 34 per cent under pastures lpld 26 per cent under forests. The climate of the region is Himalayan sub-tropical to sub-temperate and the mean annual rainfall varies from 903 to 1281 mm with a mean value of 1067 mm (Kumar and Kumar, 2008). About 75 per cent of the annual rainfall occurs during mid June to mid September. The soils of the catchment are highly coarse textured, varying from coarse sand to gritty sandy loam, stony, highly erodible and slightly acidic to neutral in nature. The hydrologic data has been gauged by Divisional Forest Office Ranikhet, the state of Uttarakhand, India. The UH data for the catchment were taken from Kumar and Kumar (2008). Geomorphologic characteristics of study watershed have been extracted from ASTER data, resolution 30 m, using ILWIS (version 3.31) GIS software. The extraction procedure is being briefly summarized in the coming section of this chapter. The extracted digital elevation model (DEM) and the drainage network map of the study watershed is shown in Fig. 3.5 and 3.6, respectively. The details of catchment and unit hydrograph characteristics are given in Table 3.3

Table 3.3 Summary of the UHs and catchment characteristics of the study area

Catchments	Catchment Characteristics							UH Characteristics	
	A(km2)	L(km)	L_{ca}(km)	Order	R_A	R_B	R_L	$q_p(h^{-1})$	t_p(h)
Gagas	506	59.6	31.6	5	5.37	4.82	2.39	0.373	2

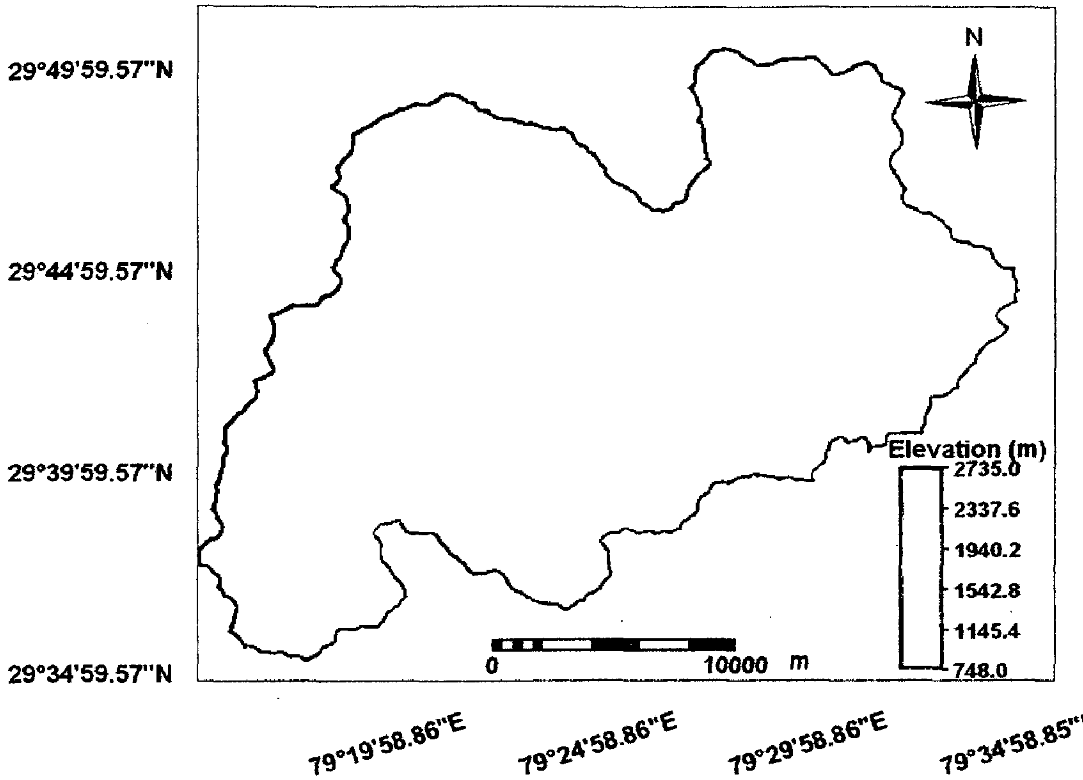

Figure 3.5: Digital Elevation Model (DEM) of Gagas Catchment.

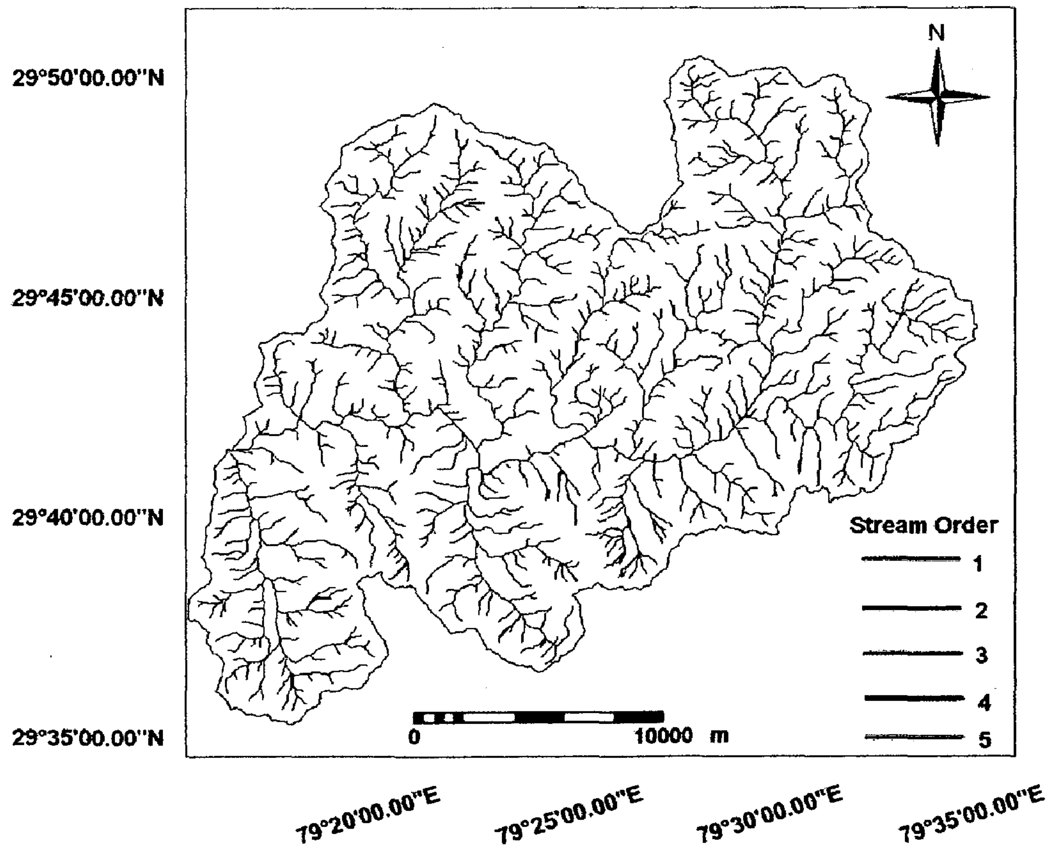

Figure 3.6: Drainage Network Map of Gagas Catchment.

3.4 Burhner watershed

The Burhner catchment falls under hydro-meteorological sub-zone 3(c) and is comprised of Upper Narmada and Tapi basins. It lies between 80° 36' and 81° 23'E longitude and 22° 0' and 22° 56'N latitude (Figure 3.7). The catchment is irregular in shape having minimum elevation of 509 m at the outlet e.g., Mohegoan and a maximum of 895 m above mean sea level (msl) at the upstream end of the catchment.

It has a continental type of climate with hot summers and cold winters and receives most rainfall from the SW monsoon during the period from June to October. Mean annual rainfall varies approximately from 800 to 1600 mm with an average annual rainfall of 1,547 mm. The catchment area comprises both flat and undulating lands covered with forest and cultivated lands. Forest and agricultural lands share nearly 58 and 42% of the catchment area, respectively. The main soil group is black soil, comprised of different varieties viz., deep black soil, medium black soil, and shallow black soil. The catchment area is about 4103 km^2. The data has been taken from Bhunya et al. (2008) for this study. Geomorphologic characteristics of study watershed have been extracted from ASTER data, resolution 30 m, using ILWIS (version 3.31) GIS software. The extraction procedure is being briefly summarized in the coming section of this chapter. The extracted digital elevation model (DEM) and the drainage network map of the study watershed is shown in Fig. 3.7 and 3.8, respectively. The details of catchment and unit hydrograph characteristics are given in Table 3.4.

Table 3.4 Summary of the UHs and catchment characteristics of the study area

Catchments	Catchment Characteristics							UH Characteristics	
	A(km2)	L(km)	L_{ca}(km)	Order	R_A	R_B	R_L	$q_p(h^{-1})$	t_p(h)
Burhner	4103	361	189	6	3.94	3.52	1.79	0.061	11

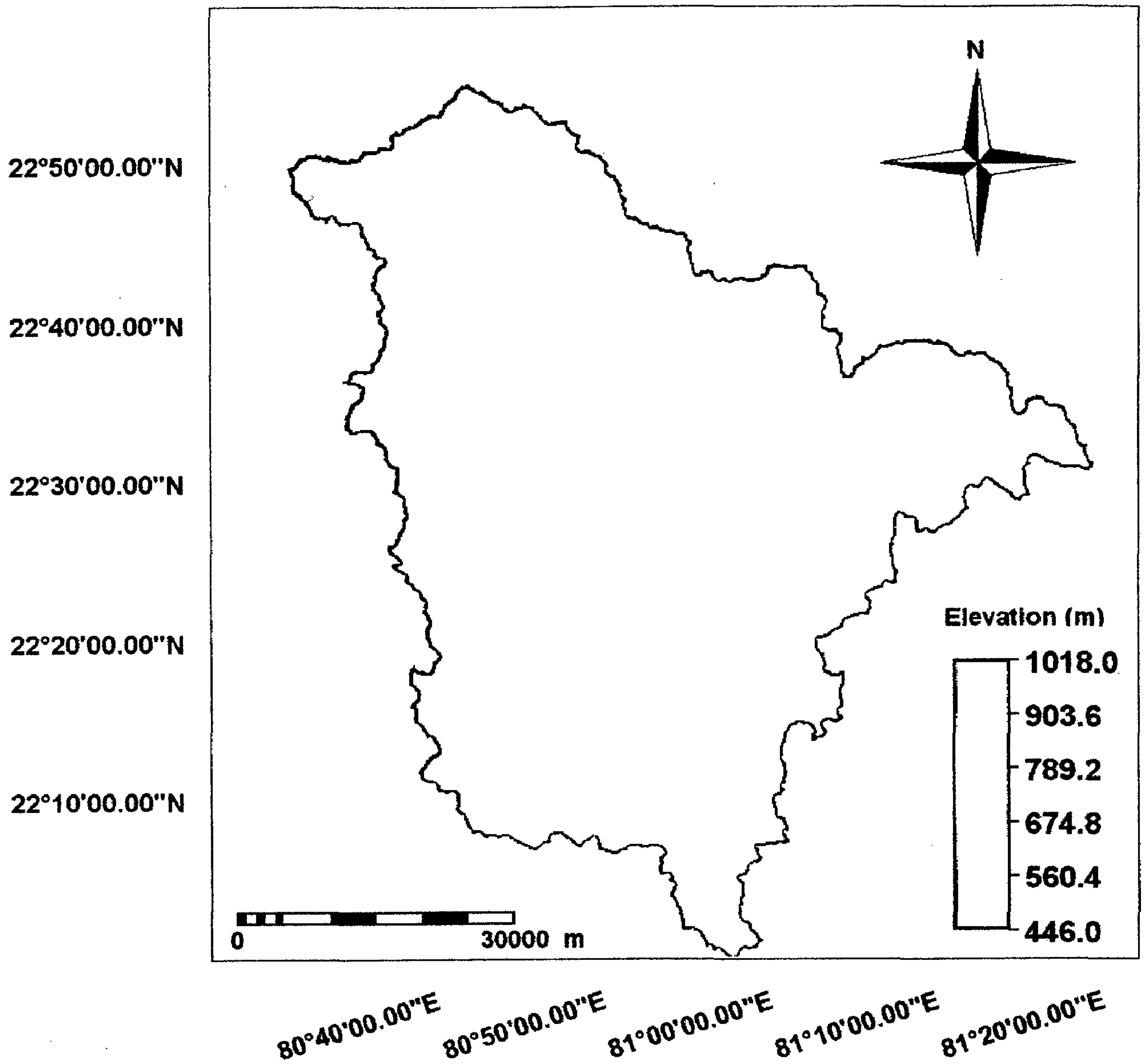

Figure 3.7: Digital Elevation Model (DEM) of Burhner Catchment.

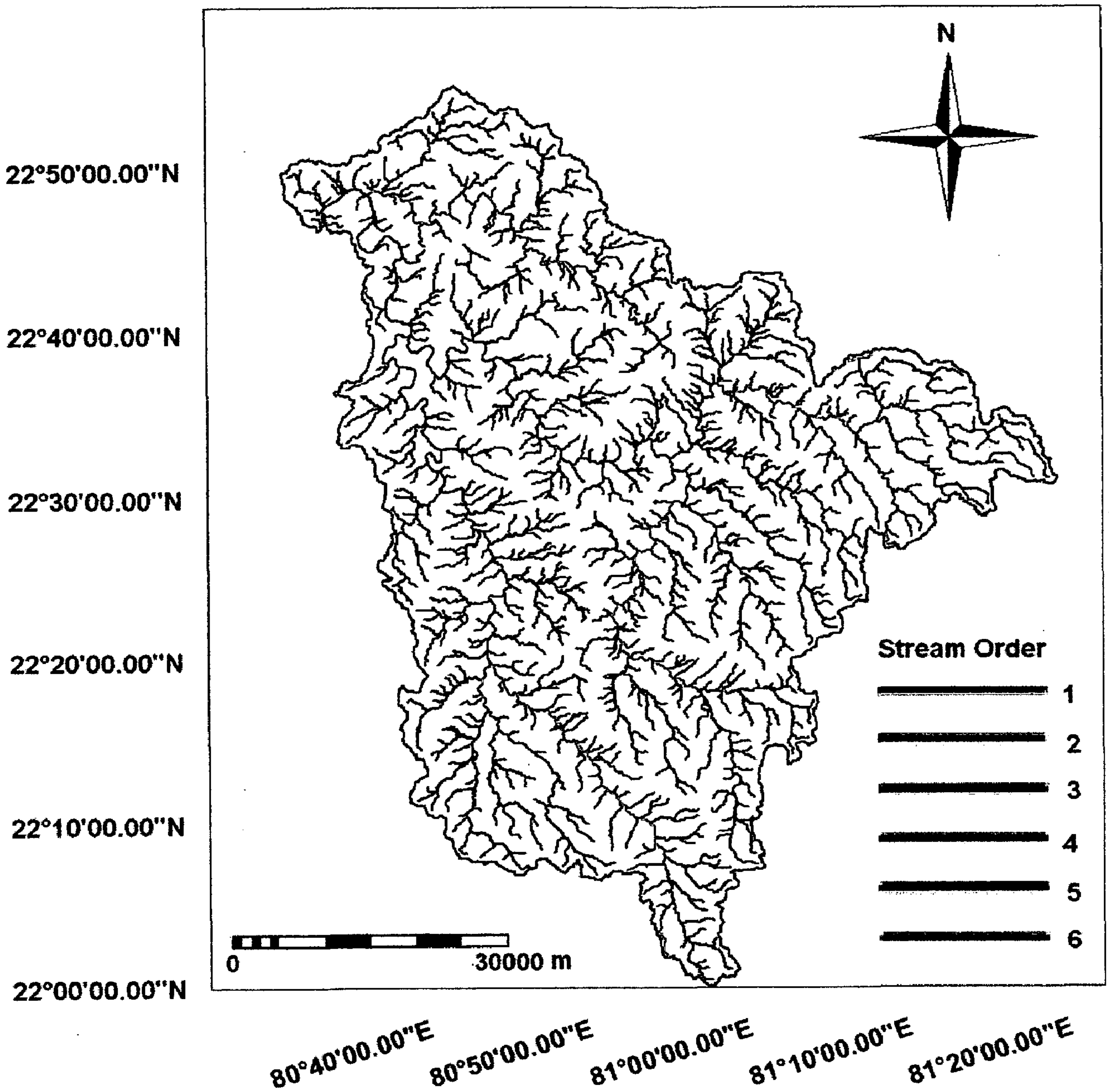

Figure 3.8: Drainage Network Map of Burhner Catchment.

3.5 Extraction of Geomorphologic Characteristics of Watersheds from ASTER data using ILWIS GIS

In general practice, the geomorphological characteristics of the watersheds were computed by manual process using toposheets. However, it was very tedious and time consuming task and at the same time, there are always some chances of errors. With the recent advancements in Remote Sensing and GIS tools and techniques, extraction of land-surface parameters and objects, and assessment of their properties can be done today easily.

For geomorphologic analysis, a detailed DEM of the catchment was prepared using ASTER data having fineness of 1-arc second spatial resolution, which was downloaded from the website ASTER Global Digital Elevation Model (ASTER GDEM) (www.gdem.aster.ersdac.or.jp/).

ASTER (Advanced Space borne Thermal Emission and Reflection Radiometer) is an imaging instrument flying on Terra, a satellite launched in December 1999 as part of NASA's Earth Observing System (EOS). ASTER is a cooperative effort between NASA, Japan's Ministry of Economy, Trade and Industry (METI) and Japan's Earth Remote Sensing Data Analysis Center (ERSDAC). The ASTER data is available as 1 arc second (approx. 30m resolution) DEMs, which is used here in the study. The elevation models are arranged into tiles, each covering one degree of latitude and one degree of longitude.

3.5.1 DEM Processing Using ILWIS 3.31

In this study, a PC-based GIS and Remote Sensing software Integrated Land and Water Information System (ILWIS) has been used for extraction of geomorphological parameters of the catchment. ILWIS comprises a complete package of image processing, spatial analysis, digital mapping and hydrological processing. The ASTER data having fineness of 1-arc second spatial resolution, which was downloaded from the website ASTER Global Digital Elevation Model (ASTER GDEM) (www.gdem.aster.ersdac.or.jp/) and imported into ILWIS through "import via Geo-gateway". Due care should be taken into account to make ASTER derived DEMs free from undefined pixels. It should be resolved from interpolation of surrounding pixels.

To delineate the catchment boundary and consistent drainage network of the Study catchments, the ASTER mosaic were passed through subsequent process of modules which are well embedded in the ILWIS under "DEM Hydro-Processing" operation, however at some steps user interference is required to schematize and parameterize more realistic drainage network. DEM hydro-processing in ILWIS are done by running series of operations to get the significant information. The hydrological analysis is done by the various operation steps described below:

3.5.2 DEM-hydro processing operations steps:

Fill sinks

We may wish to clean up our Digital Elevation Model (DEM), so that local depressions (sinks) are removed from our DEM. The Fill sinks operation 'removes' local depressions (of single pixels and of multiple pixels) from a Digital Elevation Model (DEM). After using the Fill sinks operation, but before using the Flow direction and Flow accumulation operations, we can use the DEM optimization operation on the output DEM of the Fill sinks operation, to further improve our DEM.

Flow direction

In a (sink-free) Digital Elevation Model (DEM), the Flow direction operation determines into which neighbouring pixel any water in a central pixel will flow naturally and drain in to outlets. The operation can be used to find the drainage pattern of a terrain. Flow direction is calculated for every central pixel of input blocks of 3 by 3 pixels, each time comparing the value of the central pixel with the value of its 8 neighbours. You can choose option whether we wish to calculate the flow direction for the central pixels by steepest slope or by lowest height

Flow accumulation

The Flow accumulation operation performs a cumulative count of the number of pixels that naturally drain into outlets. The operation can be used to find the drainage pattern of a terrain. The Flow direction operation determines the natural drainage direction for every pixel in a Digital Elevation Model (DEM). Based on the output Flow direction map, the Flow accumulation operation counts the total number of pixels that will drain into outlets.

Drainage network extraction

The Drainage Network Extraction operation extracts a basic drainage network (boolean raster map). The output raster map will show the basic drainage as pixels with value True, while other pixels have value false. Subsequently, here we can choose option to use a map containing either stream threshold (no. of pixels) value or stream threshold map.

Drainage network ordering

The Drainage network ordering operation examines all drainage lines in the drainage network map, finds the nodes where two or more streams meet, and assigns a unique ID to each stream in between these nodes, as well as to the streams that only have a single node. To limit the number of output streams and reduce calculation time for the Drainage network ordering operation, there is a option in which we can specify the minimum drainage length (in meters).

The operation delivers an output raster map, a segment map and an attribute table that all use a newly created ID domain. The attribute table contains information on each stream related to Strahler ordering number, Shreve ordering number, stream length, and slope values in degrees and in percentages, sinuosity of the drainage path as a measure of meandering, total upstream drainage length etc.

Catchment extraction

The Catchment extraction operation constructs catchments; a catchment will be calculated for each stream found in the output map of the Drainage network ordering operation. The operation delivers an output raster map, an output polygon map and an output attribute table. In the attribute table, we will find information on each catchment related to area and perimeter of the catchment, total upstream area etc.

Catchment merge

The Catchment merge operation is able to merge adjacent catchments, as found by a previous Catchment extraction operation. From a point map that contains locations of stream outlets within a catchment, merge all adjacent catchments that drain into such outlets. In fact, new catchments will be created on the basis of the Drainage network ordering output map and its attribute table. We can merge catchments in two manners: (i) by specifying a point map that contains locations of stream outlets (ii) by simply specifying a Strahler or Shreve ordering value. The operation delivers an output raster map, an output polygon map and an output attribute table. The attribute table contains information on the new catchments, similar to the output attribute table of the Catchment

Extraction operation, but we will also find information on total drainage length, total upstream area, drainage density, longest flow path length and longest drainage length etc.

Optionally, we can also obtain longest flow path segment map and extract stream segments and attributes. Finally, this operation also has an option to include undefined pixels (from the Flow direction map) into a catchment.

Horton statistics

The Horton Statistics operation calculates the followings for each (Strahler) stream order number and for each merged catchment:

- the number of streams,
- the average stream length,
- the average area of catchments, and
- expected values for these by means of a least squares fit.

For each merged catchment all streams are found, and sorted by their Strahler stream order value. Connected streams with the same order number are joined and are counted as a single stream. Values are summed for each Strahler stream order number within each catchment.

The output is stored in a table which can be used to construct so-called Horton plots. Horton plots enable us to inspect the regularity of our extracted stream network and may serve as a quality control indicator for the entire stream network extraction process. Finally, the bifurcation ratio R_B, the length ratio R_L, or the area ratio R_A value for subsequent stream orders i can be calculated.

CHAPTER-4

DIAGNOSIS OF PARAMETRIC EXPRESSIONS OF PROBABILITY MODELS

The need for a synthetic method to develop UHs has inspired many studies as the drainage basins in many parts of the world are ungauged or poorly gauged, and in some cases existing measurement networks are not working properly. Moreover, the problem is being further aggravated by the impacts of human-induced changes to the land surface and climate, occurring at the local, regional and global scales and thus making the predictions of ungauged or poorly gauged basins highly uncertain. However, accepting this challenge, for the first time, the *International Association of Hydrological Sciences* (IAHS) decided to launch a new initiative to devote a decade (2003-2012) on *Predictions in Ungauged Basins* (PUB), for formulating and implementing appropriate science programs to actively engage and re-energize the hydrologic and water resources community, in a coordinated manner, towards achieving major advances in hydrologic models/tools to make predictions in ungauged basins successfully. Thus, the time has come to identify the new techniques/models of SUH derivation.

Various attempts have been made in the past to derive synthetic unit hydrograph (SUH) using the parametric expressions of probability distribution functions (pdfs), as evident from Chapter 2. In SUH derivation, one of the important steps is the estimation of one or two key points on UH (or IUH), through which the hydrograph is fitted. To achieve this objective, relationships are sought from the salient points of UH and selected catchment characteristics which can be obtained from ASTERDEM data using DEM hydro-processing module of ILWIS 3.31 GIS software and generalized rainfall statistics. Moreover, the linking of salient points of UH to the catchment characteristics provides a scientific basis for the hydrograph fitting to yield a smooth and single-valued shape corresponding to unit runoff volume (Bhunya et al., 2007b).

Keeping in view the aforementioned discussions, this study explores the potential of the parametric expressions of One-parameter Chi-square (1PCSD), Two-parameter Fréchet (2PFD), Two-parameter Inverse Gamma (2PIGD) and Two-parameter Gamma

(2PGD) distributions for fitting UH. Notably, analytical solutions are developed to estimate the distribution parameters. The UH parameters, viz., peak discharge, time to peak, etc. are accomplished using Horton order ratios given by Rodriguez-Iturbe and Valdés (1979). Finally, the workability of this approach in SUH derivation is demonstrated using data of four Indian catchments having different areas and terrains.

STATISTICAL DISTRIBUTIONS

In this section, the parametric expressions of four probability distributions namely One-parameter Chi-square (1PCSD), Two-parameter Fréchet (2PFD), Two-parameter Gamma (2PGD) and Two-parameter Inverse Gamma (2PIGD) distributions are diagnosed and simple analytical solutions are developed to estimate the distribution parameters.

4.1 One-Parameter Chi-Square Distribution

The one-parameter Chi-square distribution (1PCSD) (Fig. 4.1) is a special case of gamma distribution and the parametric expression of this pdf is given as (Montgomery and Runger, 1994):

$$h(t) = \frac{1}{2^{b/2}\Gamma(b/2)} t^{b/2-1} e^{-t/2} \qquad \text{for } b>0,\ t>0 \qquad (4.1)$$

The mean and variance are given by

$$\mu = b \text{ and } \sigma^2 = 2b \qquad (4.2)$$

The salient properties of the Chi-square distribution are:

(i) It is skewed to the right and its random variate is non-negative (Fig.4.1); as b tends to infinity, it approaches the normal distribution.

(ii) $h(t) = 0$ at $t = 0$ and $h(t) \approx 0$ as $t \to \infty$.

(iii) $\int_{0}^{\infty} h(t)dt = 1$

which makes this distribution fit for describing a UH shape.

In spite of its resemblance to the gamma pdf, the distribution was considered in this study because estimating a single parameter of the Chi-square distribution is simple and should involve less error compared to estimates of two parameters of the gamma distribution (Bhunya et al., 2007a).

Taking τ = b/2, the form of the Chi-square distribution of Eq. (4.1) can be written as:

$$q(t) = \frac{1}{2^{\tau}\Gamma(\tau)} t^{\tau-1} e^{-\frac{t}{2}} \tag{4.3}$$

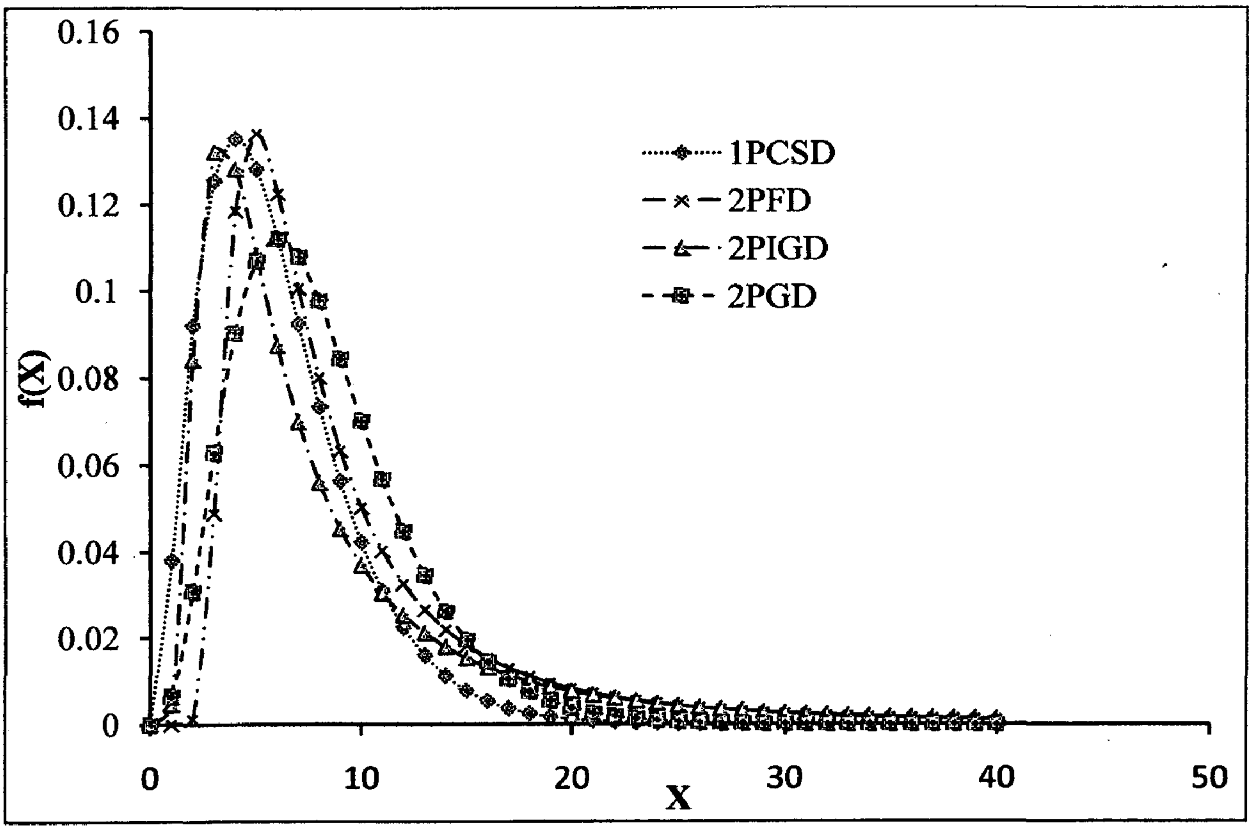

Figure 4.1: pdf shapes for One-parameter Chi-square distribution (1PCSD) (□ = 3), Two-parameter Fréchet distribution (2PFD) (c = 2, α= 6), Two-parameter Inverse Gamma distribution (2PIGD) (α = 4, k = 2) and Two-parameter Gamma distribution (2PGD) (n = 4, K = 2).

Analytical Solution for Parameter Estimation:

Now applying the condition at time to peak ($t = t_p$), $dq(t)/dt = 0$, from Eq. (4.3) yields (Appendix A):

$$t_p = 2(\tau-1)\,; \quad q_p t_p = \frac{(\tau-1)^{\tau} e^{-(\tau-1)}}{\Gamma(\tau)} \tag{4.4a,b}$$

A substitution of $m = \tau-1$ in Eq. (4.4b) yields:

$$q_p t_p = \frac{m^{m+1} e^{-m}}{\Gamma(m+1)} \tag{4.5}$$

Further, simplification to Eq. (4.5) yields:

$$q_p t_p = \frac{m^{m} e^{-m}}{\Gamma(m)} \tag{4.6}$$

Defining the non dimensional term $\beta = q_p t_p$, Eq. (4.6) reduces to

$$\beta = \frac{m^{m} e^{-m}}{\Gamma(m)} \tag{4.7}$$

Solution of Eq. (4.7) can be approximately given as (Appendix B):

$$m = \pi\beta^2 \pm \beta\sqrt{(\pi^2\beta^2 + \pi/3)} \tag{4.8}$$

Taking +ve sign only, Eq. (4.8) reduces to

$$m = \pi\beta^2 + \beta\sqrt{(\pi^2\beta^2 + \pi/3)} \tag{4.9}$$

For given q_p and t_p, Eqs. (4.3) - (4.9) describe the complete shape of the SUH.

4.2 Two-Parameter Fréchet Distribution

The parametric expression of Two-parameter Fréchet distribution (2PFD) (Fig. 4.1) is given as (Ayyub and McCuen, 1997):

$$f(x) = (c/\alpha)(\alpha/x)^{c+1} e^{-(\alpha/x)^c} \qquad \text{for } x > 0 \qquad (4.10)$$

where the location parameter $\alpha > 0$ and the shape parameter $c > 0$.
The mean (μ) and variance (σ^2) of the distribution are, respectively, given as:

$$\mu = \alpha\Gamma(1-1/c); \ \sigma^2 = \alpha^2\left[\Gamma(1-2/c) - \Gamma^2(1-1/c)\right] \qquad (4.11)$$

and the cumulative distribution function (cdf) is given as:

$$F(x) = e^{-(\alpha/x)^c} \qquad (4.12)$$

As $x \to \infty$, F(x) = 1. This condition meets the criterion for UH description (Sherman, 1932). Now, taking Fréchet distribution pdf (Eq. 4.10) as the discharge ordinates q (t) of UH and x as time t, one gets

$$q(t) = (c/\alpha)(\alpha/t)^{c+1} e^{-(\alpha/t)^c} \qquad \text{for } t>0 \qquad (4.13)$$

Analytical Solution for Parameter Estimation:

Now applying the condition at time to peak ($t = t_p$), dq (t)/dt = 0 to Eq. (4.13) yields (Appendix C):

$$t_p = \alpha[c/(c+1)]^{1/c}; \alpha = t_p\left(\frac{c+1}{c}\right)^{\frac{1}{c}}; \text{and } q_p = (c/\alpha)(1+1/c)^{(1+1/c)} e^{-(1+1/c)} \qquad (4.14 \text{ a, b, c})$$

Hence the non-dimensional term β can simply be expressed as:

$$\beta = (1 + c)e^{-(1+1/c)} \tag{4.15}$$

Since, for a given UH, the dimensionless factor β is always positive, the shape parameter of the Fréchet distribution is always greater than zero i.e. $c > 0$.

On expanding the exponential term up to second order in Eq. (4.15), it simplifies to the following form:

$$c^3 + (1 - e\beta)c^2 - (e\beta)c - (e\beta)/2 = 0 \tag{4.16}$$

Following Abramowitz and Stegun (1964), the solution of Eq. (4.16) can be expressed as:

$$c = (u_1 + v_1) - A_1/3 \tag{4.17}$$

where A_1, u_1, and v_1 are the functions of β and are defined as:

$A_1 = (1 - e\beta)$; $B_1 = -e\beta$; $C_1 = -e\beta/2$, and $u_1 = \left[r_1 + \left(p_1^3 + r_1^2\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}$;

$$v_1 = \left[r_1 - \left(p_1^3 + r_1^2\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} \tag{4.18}$$

where $p_1 = B_1/3 - A_1^2/9$; $r_1 = (A_1B_1 - 3C_1)/6 - A_1^3/27$.

Thus, the parameter c of the 2PFD can be estimated using Eq. (4.17), and corresponding to this α parameter can be estimated from Eq. (4.14b) for a known value of t_p. Estimated parameters are substituted in Eq. (4.13) to get the complete shape of UH.

4.3 Two-Parameter Inverse Gamma Distribution

The parametric expression of two-parameter Inverse Gamma Distribution (2PIGD) is given as:

$$f(x) = \frac{k^{\alpha} x^{-\alpha-1}}{\Gamma(\alpha)} e^{-\left(k/t\right)} \quad ;\text{for } x>0 \qquad (4.19)$$

The shape parameter, $\alpha>0$, and the scale parameter, $k>0$. The mean (μ) and variance (σ^2) for the distributions are, respectively, given by

$$\mu = \frac{k}{\alpha-1};\ \sigma^2 = \frac{k^2}{(\alpha+1)^2(\alpha-2)} \quad \text{where } \alpha>1 \qquad (4.20)$$

and the cumulative distribution function (cdf) is given as:

$$F(x, \alpha, k) = \frac{\Gamma(\alpha, k/x)}{\Gamma(\alpha)} \qquad (4.21)$$

where, $\Gamma(\alpha, k/x)$ is incomplete gamma function and $\Gamma(\alpha)$ is gamma function.

Taking the Inverse gamma density function (Eq. 4.19) as the discharge ordinates q(t) of UH and x as time t, one obtains

$$q(t) = \frac{1}{k\Gamma(\alpha)} \left(k/t\right)^{\alpha+1} e^{-\left(k/t\right)} \quad \text{for } t>0 \qquad (4.22)$$

Analytical Solution for Parameter Estimation:

Now applying the condition at time to peak ($t = t_p$), $dq(t)/dt = 0$, Eq. (4.22) yields (Appendix D):

$$t_p = \frac{k}{\alpha+1};\ k = t_p(\alpha+1);\ \text{and}\ q_p = \frac{1}{k\Gamma\alpha}(\alpha+1)^{(\alpha+1)} e^{-(\alpha+1)} \qquad (4.23a,b,c)$$

Hence the non-dimensional term $\beta = q_p t_p$, can be expressed as:

$$\beta = qptp = \frac{(\alpha+1)^{\alpha} e^{-(\alpha+1)}}{\Gamma\alpha} \tag{4.24}$$

Or

$$\beta = \frac{\alpha(\alpha+1)^{\alpha+1} e^{-(\alpha+1)}}{(\alpha+1)\Gamma(\alpha+1)} \tag{4.25}$$

Substitution of $m = (\alpha+1)$ in Eq. (4.25) yields

$$\beta = \frac{(m-1)m^{m} e^{-m}}{m\Gamma m} \tag{4.26}$$

On expanding the exponential term up to second order and from Appendix E, Eq. (4.26) simplifies to following form:

$$3m^2 - 6m - 6\pi m\beta^2 - \pi\beta^2 + 3 = 0 \tag{4.27}$$

Lastly, the solution of this above equation can be approximately given as:

$$m = (1 + \pi\beta^2) \pm \beta\sqrt{\pi^2\beta^2 + \frac{7}{3}\pi} \tag{4.28}$$

Taking the +ve sign only, Eq. (4.26) reduce to

$$m = (1 + \pi\beta^2) + \beta\sqrt{\pi^2\beta^2 + \frac{7}{3}\pi} \tag{4.29}$$

Thus for given q_p and t_p, Eqs (4.22) - (4.29) describe the complete shape of the SUH.

4.4 GIUH MODEL

Rodriguez-Iturbe and Valdés (1979) expressed the initial state probability of one droplet of rainfall in terms of geomorphological parameters as well as the transition state probability matrix. The final probability density function of droplets leaving the highest order stream into the trapping state is nothing but the GIUH. The model was parameterized in terms of Horton's order laws (Horton 1945) of drainage network composition and Strahler's (1957) stream ordering scheme. An exponential holding time mechanism, equivalent to that of a linear reservoir, was assumed. Further, they suggested that it is adequate to assume a triangular instantaneous unit hydrograph and the expressions for peak flow (q_p) and time to peak (t_p) of the GIUH are given as:

$$q_p = 1.31 R_L^{0.43} v L^{-1} \qquad (4.30)$$

and

$$t_p = 0.44 (R_B / R_A)^{-0.55} R_L^{-0.38} L v^{-1} \qquad (4.31)$$

where L is the length of main channel or length of highest order stream in kilometers, v is the average peak flow velocity or characteristic velocity in m/s, q_p and t_p are in units of hr^{-1} and hr, respectively.

They defined a non-dimensional term β as the product of q_p (Eq. 4.30) and t_p (Eq. 4.31) as:

$$\beta = 0.584 (R_B / R_A)^{0.55} R_L^{0.05} \qquad (4.32)$$

It is observed from Eq. (4.32) that β is independent of velocity v and length of highest order stream or scale variable L, thereby, on the storm characteristics and hence is a function of only the catchment characteristics. The expressions (Eqs. 4.30 & 4.31) were obtained by regression of the peak as well as time to peak of IUH derived from the analytic solutions for a wide range of parameters with that of the geomorphologic characteristics and flow velocities. Alternatively, Eqs. (4.30) & (4.31) can be expressed as (Rosso, 1984):

$$q_p = 0.364 R_L^{0.43} v L^{-1} \qquad (4.33)$$

and

$$t_p = 1.584 (R_B / R_A)^{0.55} R_L^{-0.38} v^{-1} L \qquad (4.34)$$

where q_p, t_p, L and v must be in coherent units.

4.5 GIUH-COUPLED TWO-PARAMETER GAMMA DISTRIBUTION FUNCTION

The possibility of preserving the form of the SUH through a Two-parameter gamma distribution (2PGD) was analyzed by Rosso (1984), where Nash model parameters were related to Horton ratios as discussed here. The gamma probability density function (Fig. 4.1) is given as:

$$q(t) = \frac{1}{k\Gamma n}\left(\frac{t}{k}\right)^{n-1} e^{\frac{-t}{k}} \quad (4.35)$$

where k is the scale parameter [T], n is the shape parameter equal to m_2^{-1} , where m_2 is the second dimensionless moment about the centre of area of the IUH, and Γ ( ) is the gamma function. The mean, variance, and skewness of the 2PGD are described as:

$$\text{Mean } (\mu) = n\,k;\ \text{variance } (\sigma^2) = n\,k^2;\ \text{skewness } (\gamma) = 2/\sqrt{n} \quad (4.36)$$

For the condition at time to peak ($t = t_p$), dq (t)/dt = 0, Eq. (4.35) yields following expression relating n and k as:

$$k = t_p / (n-1) \quad (4.37)$$

The expression for dimensionless product $\beta = q_p\, t_p$ can be obtained into the following simpler form as

$$\beta = q_p t_p = \frac{(n-1)^{(n-1)} e^{-(n-1)}}{\Gamma(n-1)} \quad (4.38)$$

Rosso (1984) equated both the expressions of $q_p t_p$ (Eqs. 4.32 & 4.38), and used an iterative computing scheme, and proposed the following equations for n and k

$$n = 3.29(R_B / R_A)^{0.78} R_L^{0.07} \tag{4.39}$$

$$k_* = 0.70[R_A /(R_B R_L)]^{0.48} \tag{4.40}$$

where $k_* = kvL^{-1}$ is a dimensionless scale parameter. Thus, for an observed v, the parameters of the 2GPD and the shape of the UH can be computed from the geomorphological parameters of the catchment.

CHAPTER-5

APPLICATION OF TRADITIONAL AND GIUH BASED PROBABILITY MODELS FOR SUH DERIVATION

5.1 GENERAL

In this chapter, the workability of the traditional methods of Snyder (1938) and Soil Conservation Service Method (SCS) (SCS, 1957) and GIUH coupled probability models of One- parameter Chi-square (1PCSD), Two-parameter Fréchet (2PFD), Two-parameter Inverse Gamma (2PIGD), and Two-parameter Gamma (2PGD) (Rosso, 1989) distributions is tested for SUH derivation for limited data availability condition. For application of these models, four study watersheds categorized as small, medium and large with an area ranging from 27.93 km^2 to 4000 km^2, as discussed in chapter 3, are used.

In a nutshell, the main aphorism of this study is to test the suitability of the parametric expressions of probability models of 1PCSD, 2PFD, 2PIGD, and 2PGD for SUH derivation in comparison with traditional SUH methods of Snyder and SCS. This chapter has been divided into four sub-sections as per the area of study catchments. In each sub-section, the six models (two traditional and four GIUH coupled pdf models) are applied to the study catchments and finally their performance is compared with the observed UH to adjudge their suitability SUH derivation.

5.2 Application on Kothuwatari Watershed

The Kothuwatari watershed, as discussed in chapter 3, is the smallest study catchment having an area of 27.93 km^2. The geomorphologic characteristics as well as the UH characteristics are given in Table 3.1. The application procedure has been divided into two steps as discussed here: (i) the traditional SUH methods of Snyder and SCS are applied and their performance is compared with the observed UH. Similarly, at the second step, (ii) the GIUH coupled pdf based models, as discussed above, are applied and their performance is compared with the observed UH. Finally, to have an overall assessment of the relative performance of these models, the SUHs developed at the steps (i) and (ii) are compared with observed UH. A detailed step-to-step procedure followed is described here.

5.2.1 Traditional SUH Methods of Snyder and SCS

For the determination of SUH by Snyder method, the geomorphologic watershed characteristics and UH characteristics, as shown in Table 3.1 are used. The non-dimensional constants C_T and C_P are assumed as: $C_t = 0.6$ and $C_p=0.6$. However, it is to be noted here that the selection of these constants largely depends upon size, slope, storage effects, and terrain characteristics of the watersheds and on the user's experience. The following procedure is adopted to derive SUHs.

Using Eqs. (2.1) and (2.2) and L, L_{ca}, C_t compute time to peak t_p = 2.17 hr and peak flow rate q_p=2.15m^3/s; and the widths of the UH at $0.5q_p$ (Eq.2.6) and $0.75q_p$ (Eq.2.7) are computed as: W_{50}=7.80 h and W_{75}=4.45 h, respectively. Finally, the SUH is computed using these salient points as shown in Fig. 5.1 in comparison with observed UH. Secondly, for application of SCS method, Eqs. (2.10) and (2.11) are used to compute t_p and q_p as t_p= 5 hr and q_p=1.162 m^3/s. Using these salient points, SUH is drawn as shown in Fig. 5.1 to have a comparison with Snyder's method. It can be observed from Fig. 5.1 that SUHs due to Snyder and SCS method deviate largely from the observed UH, underestimating the peak flow rates and have more than 50% of the area as non-matching.

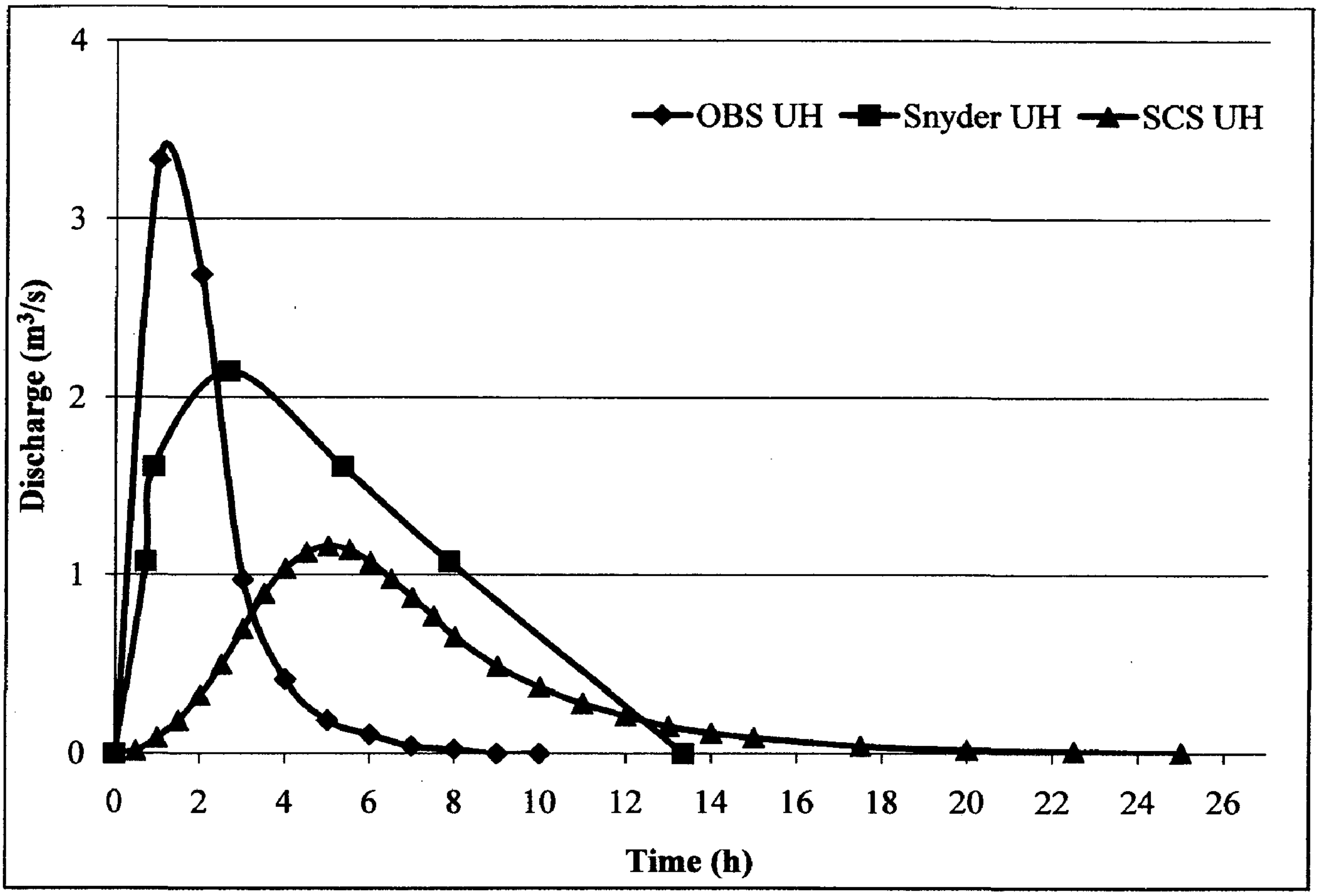

Figure 5.1: Comparison of observed and computed UHs using Snyder and SCS method for Kothuwatari catchment.

5.2.2 GIUH Coupled Probability Models

The GIUH based probability models of 1PCSD, 2PFD, 2PIGD and 2PGD are applied to derive SUHs for Kothuwatari catchment, where q_p is considered to be known; t_p, β, and the parameters of these models are derived as follows:

I. At first step use Eq. (4.33) and substitute the value of q_p and R_L (Table 3.1) to get the value of vL^{-1} as:

$q_p = Q_P/A_w = (3.328x1000x3600)/(27.93x10^6) = 0.429$ mm/hr/mm $= 0.364 \times (2.43)^{0.43}\, v\, L^{-1}$. Hence, $v\, L^{-1} = 0.429 / [0.364\, (2.43)^{0.43}] = 0.805$ hr^{-1};

II. Now substitute the values of vL^{-1} (Step I) and R_A, R_B, and R_L (Table 3.1) into Eq. (4.34) to get t_p as:

$t_p = 1.584\, (3.57/4.06)^{0.55}\, 2.43^{-0.38}\, v^{-1}L = 1.309$ hrs;

III. Get the dimensionless product $\beta = q_p x t_p = 0.429 \times 1.309 = 0.5616$.

IV. Taking these values (at Steps I-III), estimate the parameters of the 1PCSD, 2PFD, 2PIGD and 2PGD (Rosso, 1984). For 1PCSD, use Eq. (4.9) for m or τ, Eqs. (4.17) and (4.14) for c and α, Eqs. (4.29) and (4.23) for m or α and k, and Eqs. (4.39) and (4.40) for n and K, respectively for 2PFD, 2PIGD, and 2PGD models. The estimated parameters values are given in Table 5.1.

V. Finally, derive the SUHs using the above four methods, viz., Eq. (4.3) for 1PCSD, Eq. (4.13) for 2PFD, Eq. (4.22) for 2PIGD, and Eq. (4.35) for 2PGD. The derived SUHs are shown in Fig. 5.2.

Table 5.1: Parameters of the four probability models for the partial data availability condition for Kothuwatari watershed

Catchment	Parameter estimates of						
	1PCSD	2PFD		2PIGD		2PGD	
	τ	c	α	α	k	n	K
Kothuwatari	3.14	1.69	1.72	2.81	4.98	3.17	0.6

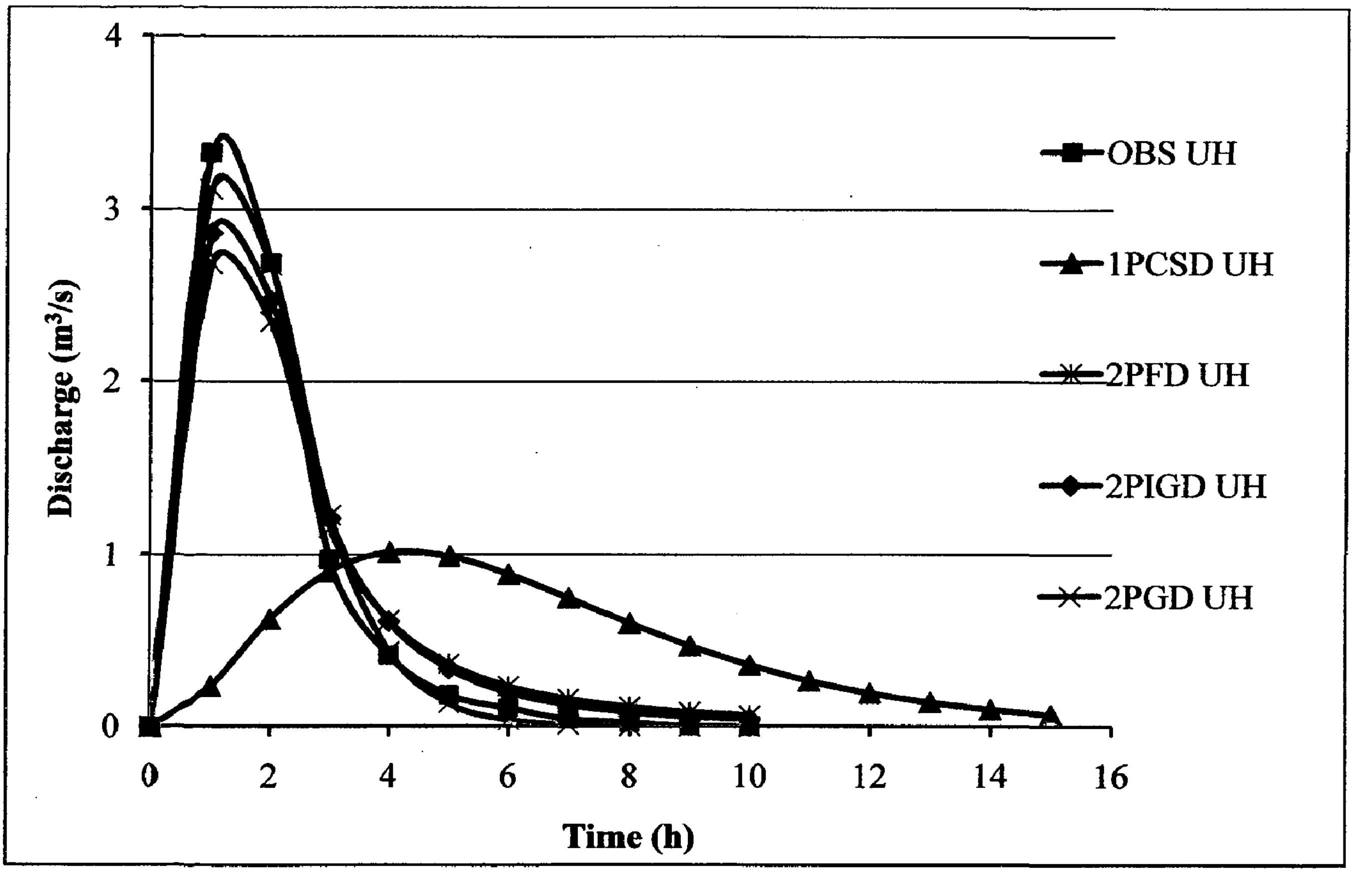

Figure 5.2: Comparison of observed and computed UHs using four different pdfs for Kothuwatari catchment.

5.2.3 PERFORMANCE OF MODELS

The performance of the models was assessed using the goodness-of-fit between the UHs in terms of standard error (STDER) and relative error (RE) in peak flow rate. The goodness-of-fit in terms of standard error (STDER) (USACE, 1990) can be expressed as:

$$\text{STDER} = \left[\frac{\sum_{i=1}^{N} (Q_{oi} - Q_{ci})^2 w_i}{N} \right]^{\frac{1}{2}} \tag{5.1}$$

$$w_i = \frac{(Q_{oi} + Q_{av})}{2Q_{av}} \tag{5.2}$$

where w_i = weighted value of the i^{th} UH ordinate, Q_{oi} = i^{th} ordinate of the observed UH; Q_{ci} = i^{th} ordinate of the computed UH, and N = total number of UH ordinates. It represents the ratio of the absolute sum of non-matching areas to the total hydrograph area. A low value of STDER-value represents a good-fit, and vice versa; and STDER equal to zero represents a perfect fit. Table 5.2 shows the STDERs due to GIUH coupled probability models for Kothuwatari catchment. It can be observed from the Table 5.2 that the STDERs due to the probability models of 1PCSD, 2PFD, 2PIGD and 2PGD are found to be 1.89, 0.38, 0.28 and 0.14, respectively. These results indicate that the SUH derived by 2PGD (Rosso, 1984) model performs marginally better than 2PIGD and 2PFD, and much better than 1PCSD.

Further, the relative error (RE) in peak flow rate is expressed as:

$$\text{RE}\ (\%) = \frac{Q_{P[OBS]} - Q_{P[COM]}}{Q_{P[OBS]}} \times 100 \tag{5.3}$$

where $Q_{p[OBS]}$ = peak flow rate of the observed unit hydrograph (m^3/s), $Q_{p[COM]}$ = peak flow rate of the computed unit hydrograph (m^3/s). The results are given in Table 5.2. It can be observed from Table 5.2 that the RE (%) in Q_p due to 2PGD is lowest (6.32) followed by 2PIGD (14.04), 2PFD (19.37) and 1PCSD (0.491) models. Similar inferences can also be drawn from Fig. 5.2, which shows the comparison between the SUHs computed by four probability models with the observed UH.

Finally, for an overall appraisal of these models, the SUHs computed by the traditional methods of Snyder and SCS and GIUH coupled probability models of 1PCSD, 2PFD, 2PIGD and 2PGD are further compared with the observed UH as shown in Fig. 5.3. It can be observed from the figure that all the probability models perform much better than the traditional methods of SUH, except 1PCSD, which has lower peak flow rate as compared to Snyder method. It can also be noted that the traditional methods have much larger deviation from the observed UH with respect to q_p and t_p and area under their UH curve is hard enough to get unity with larger portion (more than 50%) of non-matching area. Thus for Kothuwatari watershed (the smallest study watershed), the 2PGD model, performs better than 2PIGD followed by 2PFD, 1PCSD, Snyder and SCS methods.

Table 5.2: Error in using different probability models for Kothuwatari catchment

Error	Error Estimates of Probability Models			
	1PCSD	**2PFD**	**2PIGD**	**2PGD**
STDER	1.89	0.38	0.28	0.14
Relative Error, Q_p(%)	69.53	19.37	14.04	6.32

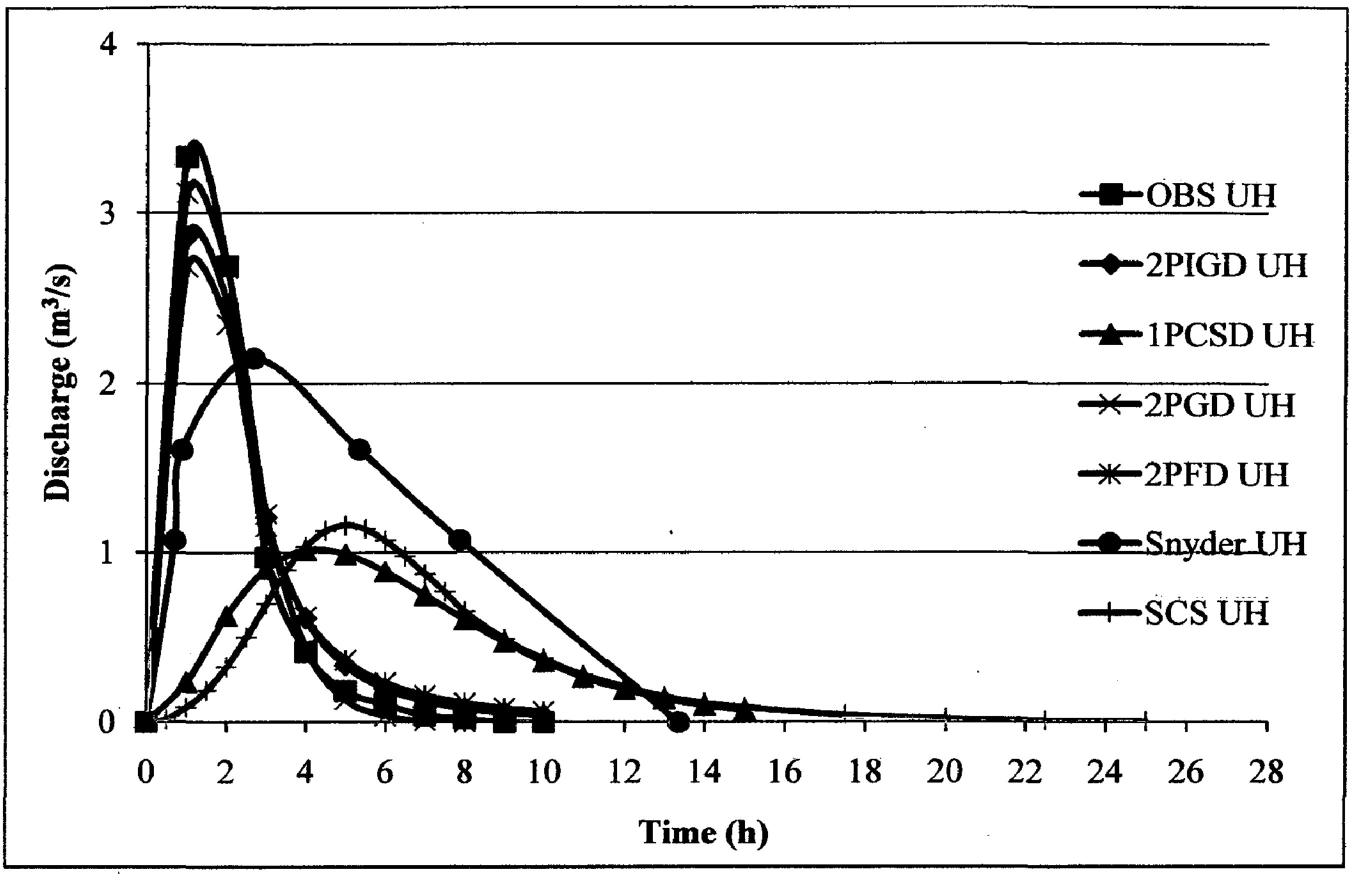

Figure 5.3: Comparison of observed and computed UHs using four different pdfs, Snyder and SCS method for Kothuwatari catchment.

5.3 Application on Myntdu-Leska Catchment

The Myntdu-Leska watershed, as discussed in chapter 3, is the medium size catchment having an area of 350 km^2. The geomorphologic characteristics and UH characteristics are given in Table 3.2. The application procedure has been divided into two steps as: (i) the traditional SUH methods of Snyder and SCS, as discussed above, are applied and their performance is compared with the observed UH. Similarly, at the second step, (ii) the GIUH coupled pdf based models, as discussed above, are applied and their performance is compared with the observed UH. Finally, the SUHs developed at the steps (i) and (ii) are compared with observed UH to have an overall assessment of the relative performance of these models. The detailed steps followed in this procedure are described below.

5.3.1 Traditional SUH Methods of Snyder and SCS

For the determination of SUH by Snyder method, the geomorphologic watershed characteristics and UH characteristics, as shown in Table 3.2 are used. The non-dimensional constants C_T and C_P are assumed as: $C_t = 1.1$ and $C_p = 0.8$. The following procedure is adopted to derive SUH.

Using Eqs. (2.1) and (2.2) and L, L_{ca}, C_t compute time to peak t_p = 10.25 hr and peak flow rate $q_p = 7.98$ m^3/s; and the widths of the UH at $0.5q_p$ (Eq. 2.6) and $0.75q_p$ (Eq. 2.7) are computed as: $W_{50} = 28.96$ h and $W_{75} = 16.55$ h, respectively. Finally, the SUH is computed using these salient points as shown in Fig. 5.4 in comparison with observed UH. For application of SCS method, Eqs. (2.10) and (2.11) are used to compute t_p and q_p as: t_p= 5 hr and $q_p = 14.56$ m^3/s. Using these salient points, SUH is drawn as shown in Fig. 5.4 to have a comparison with Snyder's method. It can be observed from Fig. 5.4 that SUHs due to both the traditional methods deviate largely from the observed UH. Secondly, the Snyder's method underestimates the peak flow rate, whereas the SCS overestimate. As well, the SUHs due to both traditional methods have large portion of the area as non- matching.

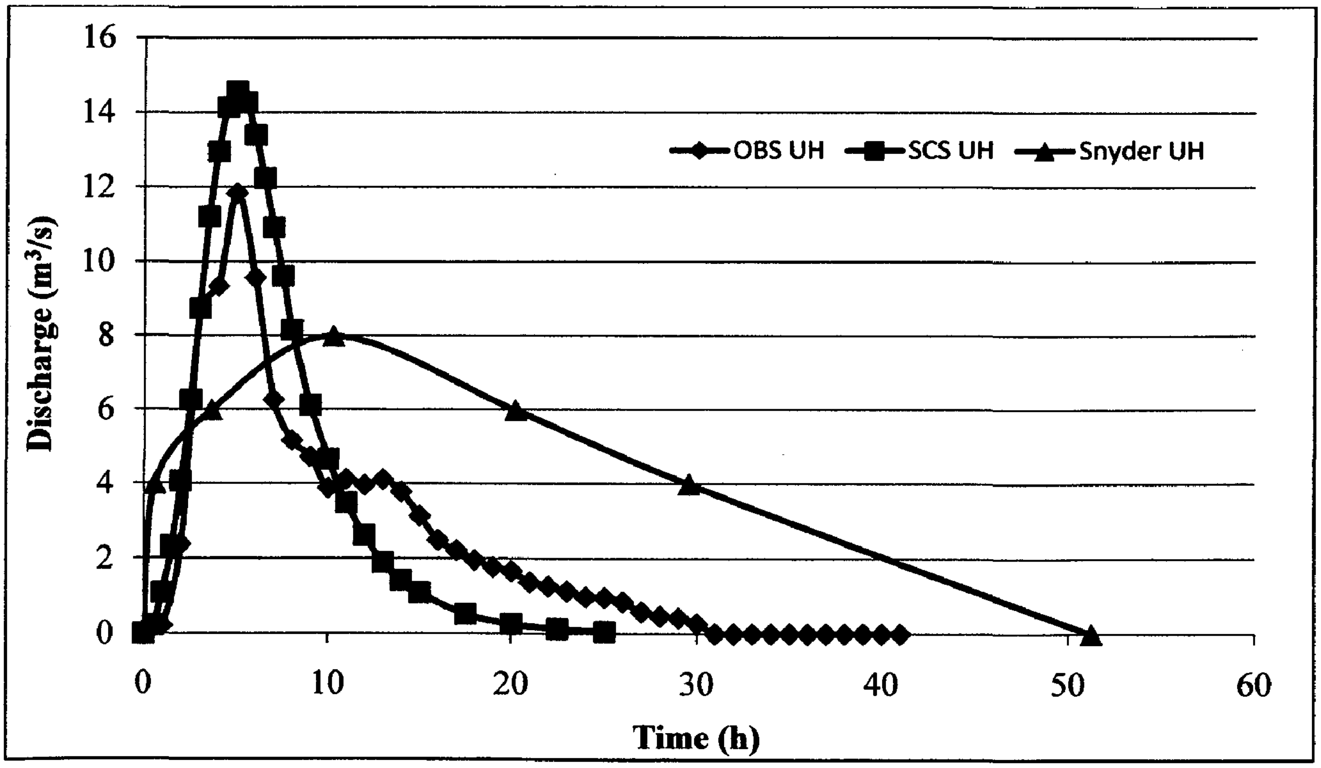

Fig. 5.4 Comparison of observed and computed UHs using Snyder and SCS method for Myntdu-Leska catchment.

Table 5.3: Parameters of the four probability models for the partial data availability condition for Myntdu-Leska watershed

Catchment	Parameter estimates of						
	1PCSD	2PFD		2PGD		2PIGD	
	τ	c	α	n	k	α	K
Myntdu-Leska	3.22	1.73	6.15	3.27	2.09	2.92	18.49

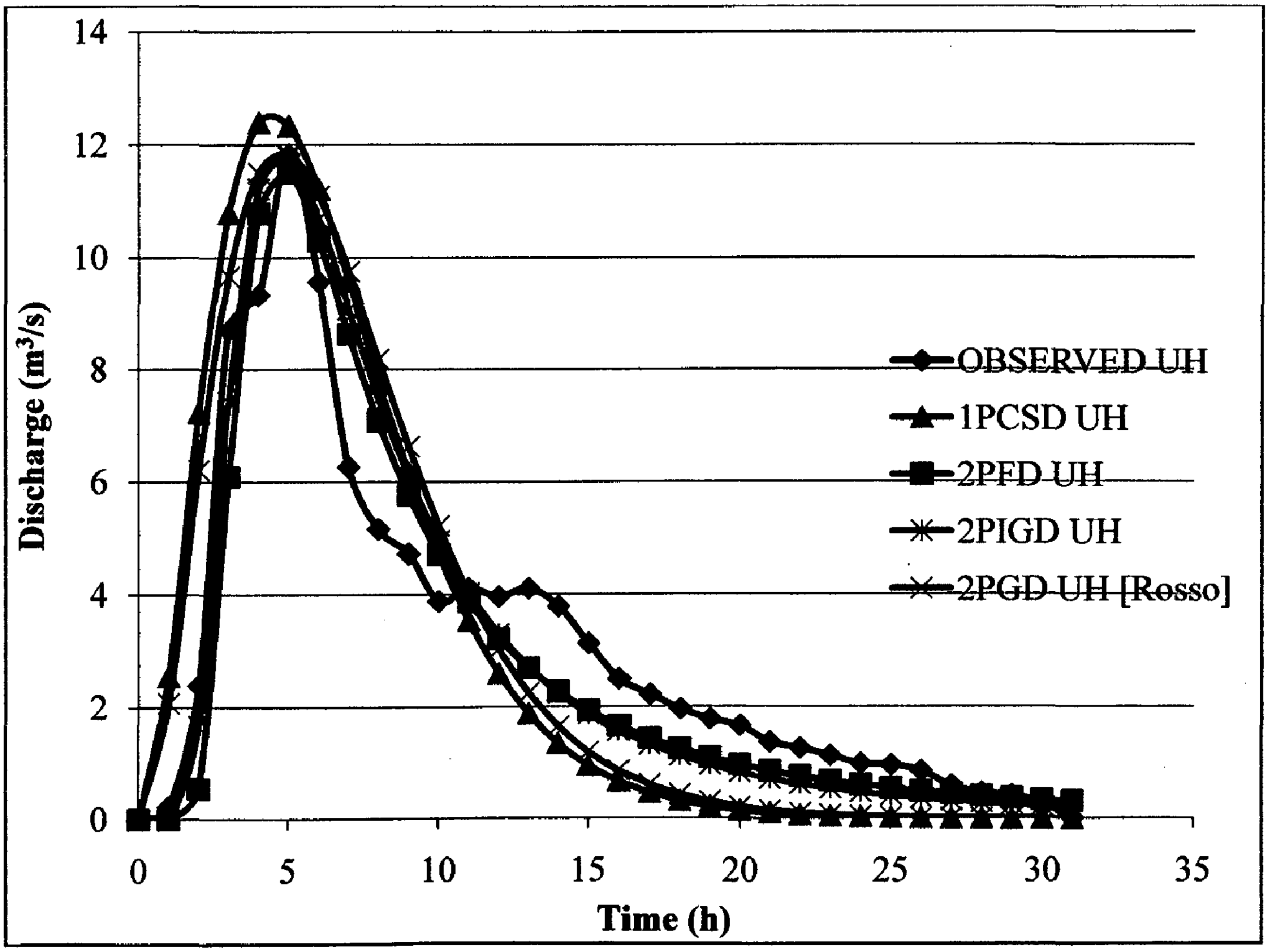

Figure 5.5 Comparison of observed and computed UHs using four different pdfs for Myntdu-Leska catchment.

5.3.1 Traditional SUH Methods of Snyder and SCS

For the determination of SUH by Snyder method, the geomorphologic watershed characteristics and UH characteristics, as shown in Table 3.2 are used. The non-dimensional constants C_T and C_P are assumed as: $C_t = 1.1$ and $C_p = 0.8$. The following procedure is adopted to derive SUH.

Using Eqs. (2.1) and (2.2) and L, L_{ca}, C_t compute time to peak t_p = 10.25 hr and peak flow rate $q_p = 7.98$ m^3/s; and the widths of the UH at $0.5q_p$ (Eq. 2.6) and $0.75q_p$ (Eq. 2.7) are computed as: $W_{50} = 28.96$ h and $W_{75} = 16.55$ h, respectively. Finally, the SUH is computed using these salient points as shown in Fig. 5.4 in comparison with observed UH. For application of SCS method, Eqs. (2.10) and (2.11) are used to compute t_p and q_p as: t_p= 5 hr and $q_p = 14.56$ m^3/s. Using these salient points, SUH is drawn as shown in Fig. 5.4 to have a comparison with Snyder's method. It can be observed from Fig. 5.4 that SUHs due to both the traditional methods deviate largely from the observed UH. Secondly, the Snyder's method underestimates the peak flow rate, whereas the SCS overestimate. As well, the SUHs due to both traditional methods have large portion of the area as non- matching.

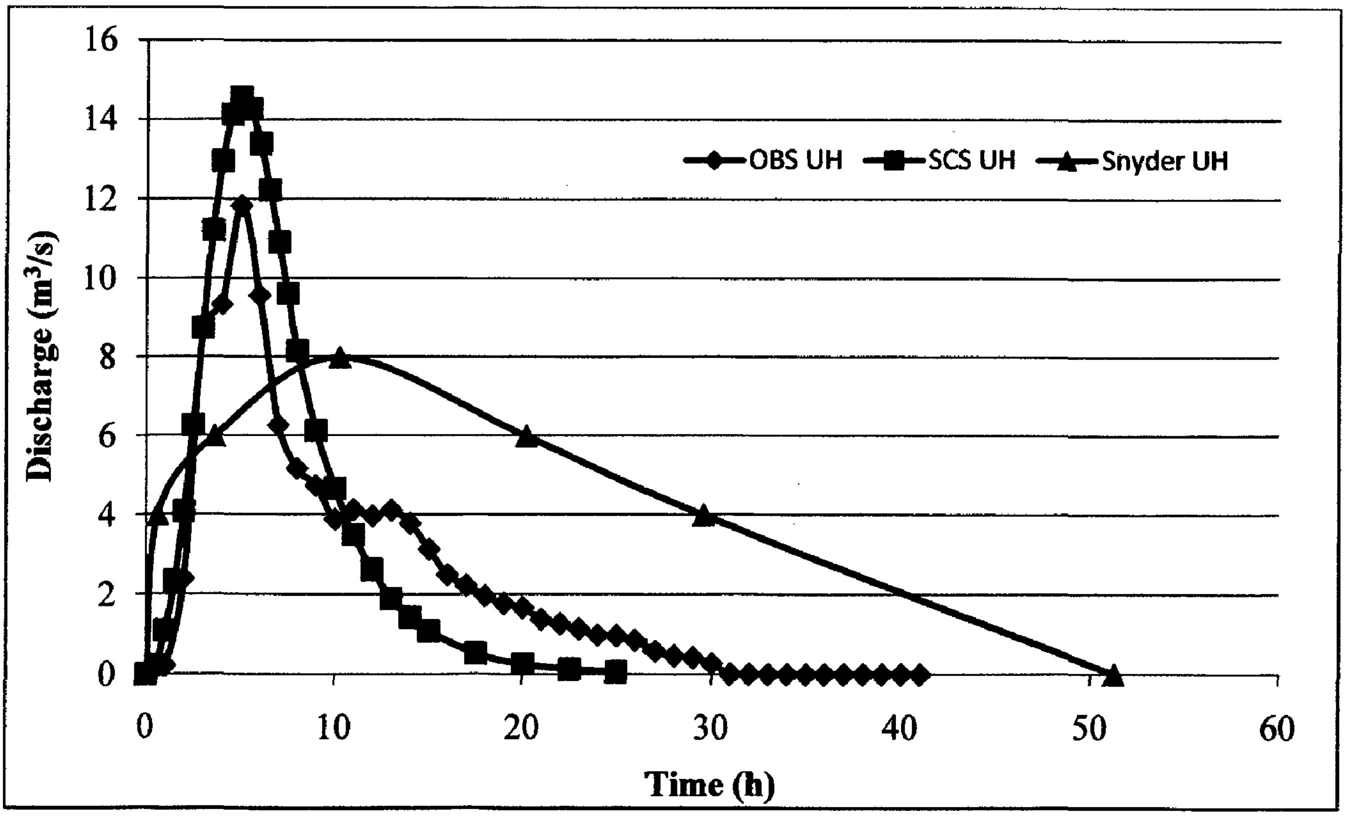

Fig. 5.4 Comparison of observed and computed UHs using Snyder and SCS method for Myntdu-Leska catchment.

5.3.2 GIUH Coupled Probability Models

The GIUH based probability models of 1PCSD, 2PFD, 2PIGD and 2PGD are applied to derive SUHs for Myntdu-Leska catchment, where q_p is considered to be known, t_p, β, and the parameters of the models are derived as follows:

I. At first step use Eq. (4.33) and substitute the value of q_p and R_L (Table 3.2) to get the value of vL^{-1} as:

$q_p = Q_P/A_w = (11.829 \times 1000 \times 3600)/(350 \times 10^6) = 0.122$ mm/hr/mm $= 0.364 \times (2.12)^{0.43}\, v\, L^{-1}$. Hence, $v\, L^{-1} = 0.122 / [0.364\, (2.12)^{0.43}] = 0.243\ hr^{-1}$;

II. Now substitute the values of vL^{-1} (Step I) and R_A, R_B, and R_L (Table 3.2) into Eq. (4.34) to get t_p as:

$t_p = 1.584\, (4.27/4.61)^{0.55}\, 2.12^{-0.38}\, v^{-1}L = 4.72$ hrs

III. Get the dimensionless product $\beta = q_p x t_p = 0.122 \times 4.72 = 0.574$.

IV. Taking these values (at Steps I-III), estimate the parameters of the 1PCSD, 2PFD, 2PIGD and 2PGD (Rosso, 1984). For 1PCSD, use Eq. (4.9) for m or τ, Eqs. (4.17) and (4.14) for c and α, Eqs. (4.29) and (4.23) for m or α and k, and Eqs. (4.39) and (4.40) for n and K and respectively for 2PFD, 2PIGD, and 2PGD models. The estimated parameters values are given in Table 5.3.

V. Finally, derive the SUHs using the above four methods, viz., Eq. (4.3) for 1PCSD, Eq. (4.13) for 2PFD, Eq. (4.22) for 2PIGD, and Eq. (4.35) for 2PGD. The derived SUHs are shown in Fig. 5.5.

Table 5.3: Parameters of the four probability models for the partial data availability condition for Myntdu-Leska watershed

Catchment	Parameter estimates of						
	1PCSD	2PFD		2PGD		2PIGD	
	τ	c	α	n	k	α	K
Myntdu-Leska	3.22	1.73	6.15	3.27	2.09	2.92	18.49

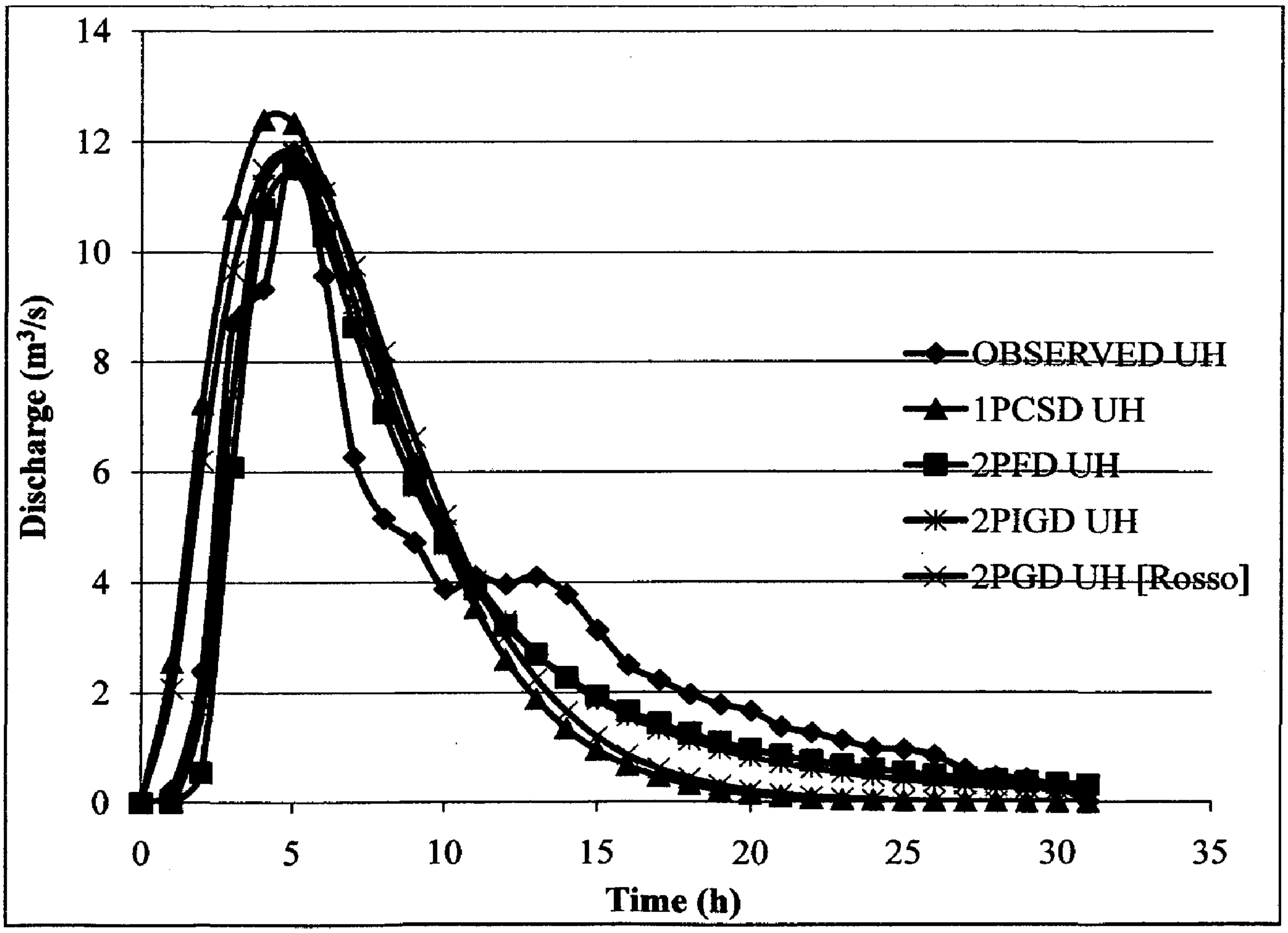

Figure 5.5 Comparison of observed and computed UHs using four different pdfs for Myntdu-Leska catchment.

5.3.3 PERFORMANCE OF MODELS

As discussed in sub-section 5.2.3, the performance of the models was assessed using the goodness-of-fit between the UHs in terms of standard error (STDER) and relative error (RE) in peak flow rate. Table 5.4 shows the STDERs due to GIUH coupled probability models. It can be observed from Table 5.4 that the STDERs due to four probability models, i.e. 1PCSD, 2PFD, 2PIGD and 2PGD are found to be 1.851, 1.162, 1.163 and 1.642, respectively. It means that the SUH derived by 2PFD model performs marginally better than 2PIGD and 2PGD (Rosso, 1984), and much better than 1PCSD.

Further, the RE (%) in peak flow rate was evaluated using Eq.(5.3) as shown in Table 5.4. It is observed from Table 5.4 that the RE in Q_p due to 2PGD and 2PIGD is almost same, followed by 2PFD and 1PCSD models. Similar inferences can also be drawn from Fig. 5.5, which shows the comparison between the observed UH and the SUHs computed by four probability models.

Finally, for an overall appraisal, the SUHs computed by the traditional methods of Snyder and SCS and four probability model coupled with GIUH, i.e., 1PCSD, 2PFD, 2PIGD and 2PGD are also compared as shown in Fig. 5.6. It can be observed from the figure that all the probability models perform much better than the traditional methods of SUH. It can also be noted that the traditional methods mainly Snyder have much larger deviation from the observed UH with respect to q_p and t_p and area under their UH curve is hard enough to get unity with larger portion (more than 50%) of non-matching area. In case of SCS, tp is almost same but, there is high value of q_p with respect to observed UH. Thus for Myntdu-Leska watershed (the medium watershed) the 2PFD model performs better than 2PIGD followed by 2PGD, 1PCSD, Snyder and SCS methods.

Table 5.4: Error in using different probability models for Myntdu-Leska catchment

Error	Error Estimates of Probability Models			
	1PCSD	**2PFD**	**2PIGD**	**2PGD**
STDER	1.851	1.162	1.163	1.642
Relative Error, Qp(%)	-4.77	2.70	0.28	-0.08

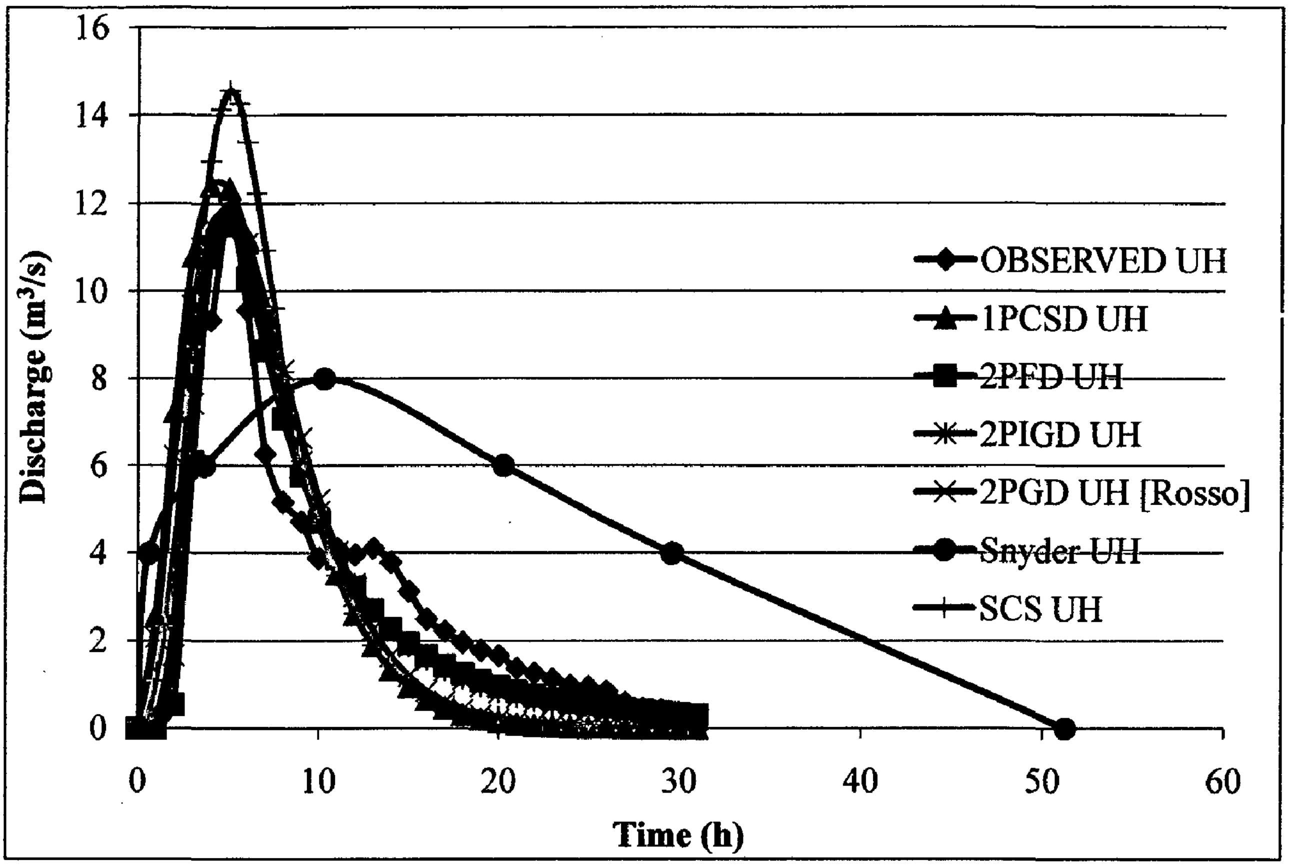

Figure 5.6 Comparison of observed and computed UHs using four different pdfs, Snyder and SCS method for Myntdu-Leska catchment.

5.4 Application on Gagas Catchment

The Gagas watershed, as discussed in chapter 3, is the medium size catchment having an area of 506 km^2. The geomorphologic characteristics and UH characteristics are given in Table 3.3. Similar to the above, the application procedure has been divided into two steps as: (i) the traditional SUH methods of Snyder and SCS, as discussed above, are applied and their performance is compared with the observed UH. Similarly at the second step, (ii) the GIUH coupled pdf based models are applied and their performance is compared with the observed UH. Finally, the SUHs developed at the steps (i) and (ii) are compared with observed UH to have an overall assessment of the relative performance of these models. The detailed steps followed in this procedure are described below.

5.4.1 Traditional SUH Methods of Snyder and SCS

For the determination of SUH by Snyder method, the geomorphologic watershed characteristics and UH characteristics as shown in Table 3.3 are used. The non-dimensional constants C_T and C_P are assumed as: C_t =1.024 and C_p=0.8. Notably, the selection of these constants largely depends upon size, slope, storage effects, and terrain characteristics of the watersheds and on the user's experience. The following procedure is followed to derive SUH.

Using Eqs. (2.1) and (2.2) and L, L_{ca}, C_t compute time to peak t_p = 10.34 hr and peak flow rate q_p=11.44 m^3/s; and the widths of the UH at 0.5q_p (Eq.2.6) and 0.75q_p (Eq.2.7) are computed as: W_{50}=29.24 h and W_{75}=16.70 h, respectively. Finally, the SUH is computed using these salient points as shown in Fig. 5.7 in comparison with observed UH. Secondly, for application of SCS method, Eqs. (2.10) and (2.11) are used to compute t_p and q_p as t_p= 5 hr and q_p=21.05 m^3/s. Using these salient points, SUH is drawn as shown in Fig. 5.7 to have a comparison with Snyder's method. It can be observed from Fig. 5.7 that SUHs due to both the traditional methods of Snyder and SCS deviate largely from the observed UH and much underestimate the peak flow rates and have more than 50% of the area as non- matching.

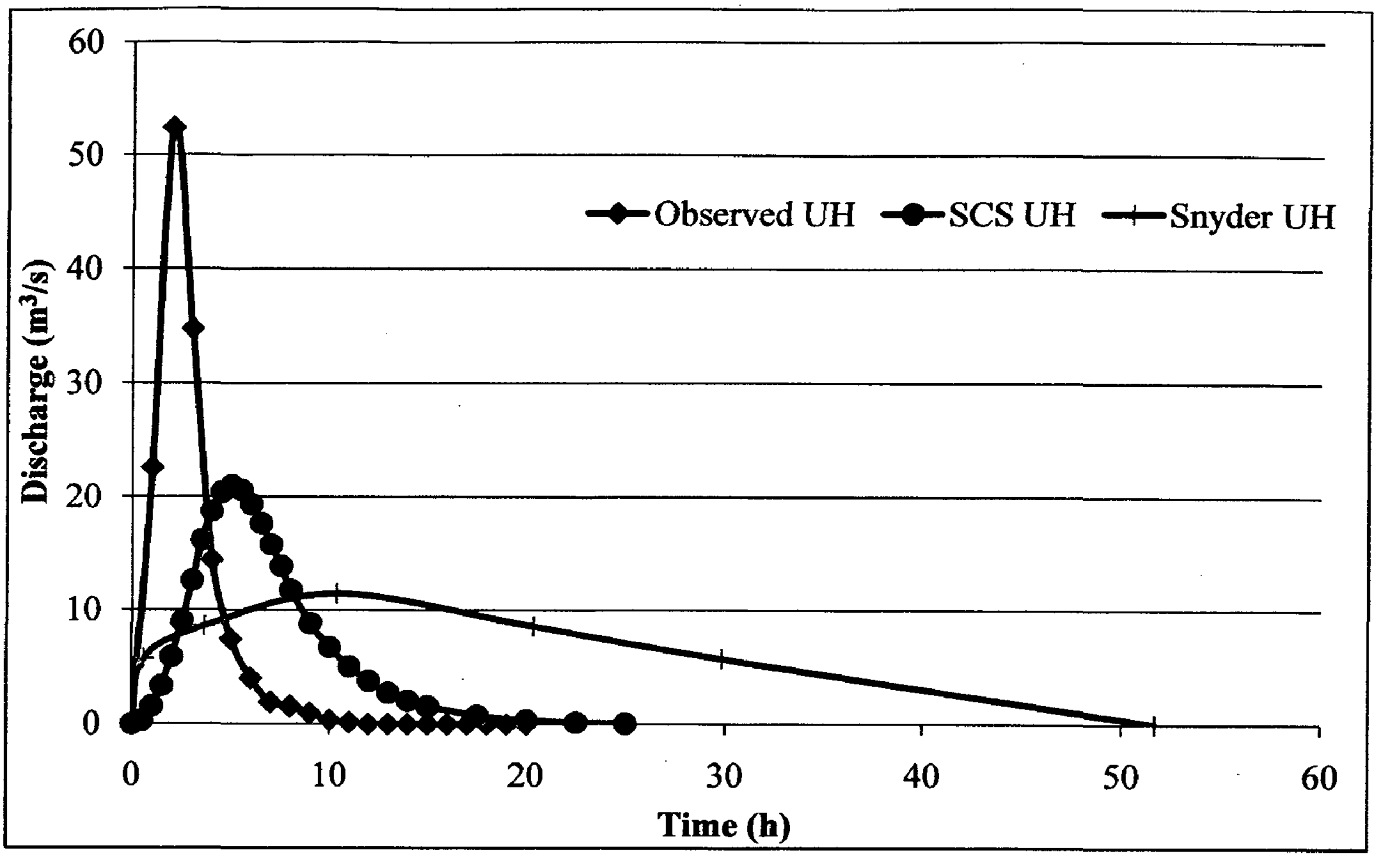

Fig. 5.7 Comparison of observed and computed UHs using Snyder and SCS method for Gagas catchment.

5.4.2 GIUH Coupled Probability Models

The GIUH based probability models of 1PCSD, 2PFD, 2PIGD and 2PGD are applied to derive SUHs for Gagas catchment, where q_p is considered to be known, t_p, β, and the parameters of the models are derived as follows:

I. At first step use Eq. (4.33) and substitute the value of q_p and R_L (Table 3.3) to get the value of vL^{-1} as:

$q_p = Q_P/A_w = (52.4\text{x}1000\text{x}3600)/(506\text{x}10^6) = 0.373$ mm/hr/mm $= 0.364 \text{ x } (2.39)^{0.43}\, v\, L^{-1}$. Hence, $v\, L^{-1} = 0.373 / [0.364\, (2.39)^{0.43}] = 0.704\ \text{hr}^{-1}$;

II. Now substitute the values of vL^{-1} (Step I) and R_A, R_B, and R_L (Table 3.3) into Eq. (4.34) to get t_p as:

$$t_p = 1.584\ (4.82/5.37)^{0.55}\ 2.39^{-0.38}\ v^{-1}L = 1.522 \text{ hrs;}$$

III. Get the dimensionless product $\beta = q_p x t_p = 0.373 \times 1.522 = 0.5675$.

IV. Taking these values (at Steps I-III), estimate the parameters of the 1PCSD, 2PFD, 2PIGD and 2PGD (Rosso, 1984). For 1PCSD, use Eq. (4.9) for m or τ, Eqs. (4.17) and (4.14) for c and α, Eqs. (4.29) and (4.23) for m or α and k, and Eqs. (4.39) and (4.40) for n and K and respectively for 2PFD, 2PIGD, and 2PGD models. The estimated parameters values are given in Table 5.5.

V. Finally, derive the SUHs using the above four methods, viz., Eq. (4.3) for 1PCSD, Eq. (4.13) for 2PFD, Eq. (4.22) for 2PIGD, and Eq. (4.35) for 2PGD. The derived SUHs are shown in Fig. 5.8

Table 5.5: Parameters of the four probability models for the partial data availability condition for Gagas watershed

Catchment	Parameter estimates of						
	1PCSD	2PFD		2PIGD		2PGD	
	τ	c	α	α	k	n	K
Gagas	3.18	1.71	1.99	2.85	5.86	3.21	0.69

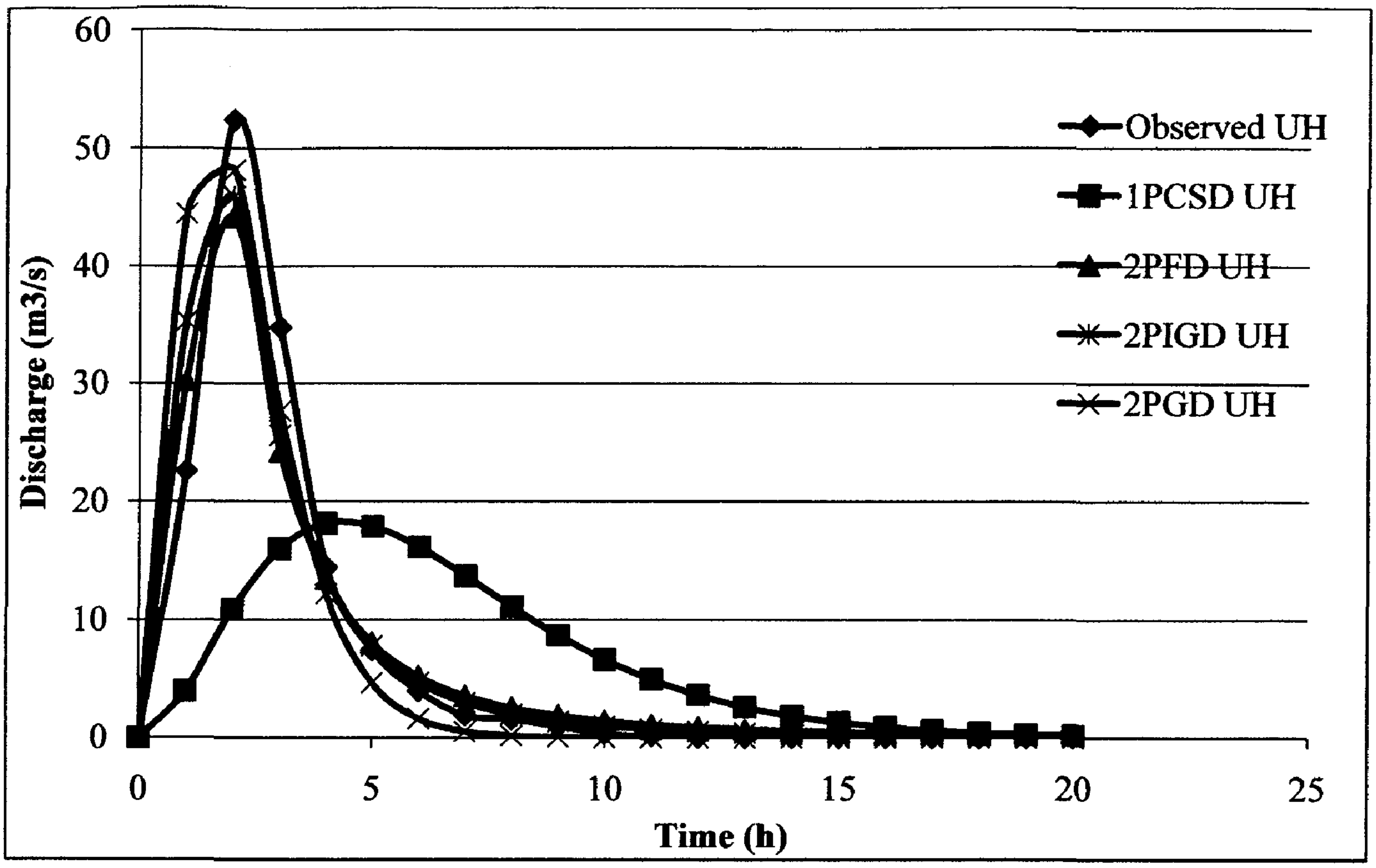

Fig. 5.8 Comparison of observed and computed UHs using four different pdfs for Gagas catchment.

5.4.3 PERFORMANCE OF MODELS

As discussed in the previous sections, the performance of the models on Gagas watershed was assessed using the goodness-of-fit between the observed and computed UHs in terms of STDER (Eqs. (5.1) & (5.2)) and RE in peak flow rate (Eq. 5.3) as shown in Table 5.6. It can be observed from Table 5.6 that STDERs due to four probability models, i.e. 1PCSD, 2PFD, 2PIGD and 2PGD are found to be 21.75, 6.09, 6.18 and 7.89, respectively. It means that the SUH derived by 2PFD model performs marginally better than 2PIGD and 2PGD (Rosso, 1984), and much better than 1PCSD.

Further, the RE in peak flow rate is evaluated Eq.(5.3) as shown in Table 5.6. It is observed from the Table 5.6 that the relative error in Q_p due to 2PGD is lowest, followed by 2PIGD, 2PFD and 1PCSD models. Similar inferences can also be drawn from Fig. 5.8, which shows the comparison between the observed UH and the SUHs computed by the four probability models.

Finally, for an overall appraisal, the SUHs computed by the traditional methods of Snyder and SCS and GIUH coupled probability models of 1PCSD, 2PFD, 2PIGD and 2PGD are also compared with the observed UH as shown in Fig. 5.9. It can be observed from the figure that all the probability models perform much better than the traditional methods of SUH, except 1PCSD, which has lower peak flow rate as compared to SCS method. It can also be noted that the traditional methods have much larger deviation from the observed UH with respect to q_p and t_p and area under their UH curve is hard enough to get unity with larger portion (more than 50%) of non-matching area. Thus for Gagas watershed (the medium size watershed) the 2PFD model performs better than 2PIGD followed by 2PGD, 1PCSD, Snyder and SCS method.

Table 5.6: Error in using different probability models for Gagas catchment

Error	Error Estimates of Probability Models			
	1PCSD	**2PFD**	**2PIGD**	**2PGD**
STDER	21.75	6.09	6.18	7.89
Relative Error, Q_p(%)	65.37	15.68	12.35	8.04

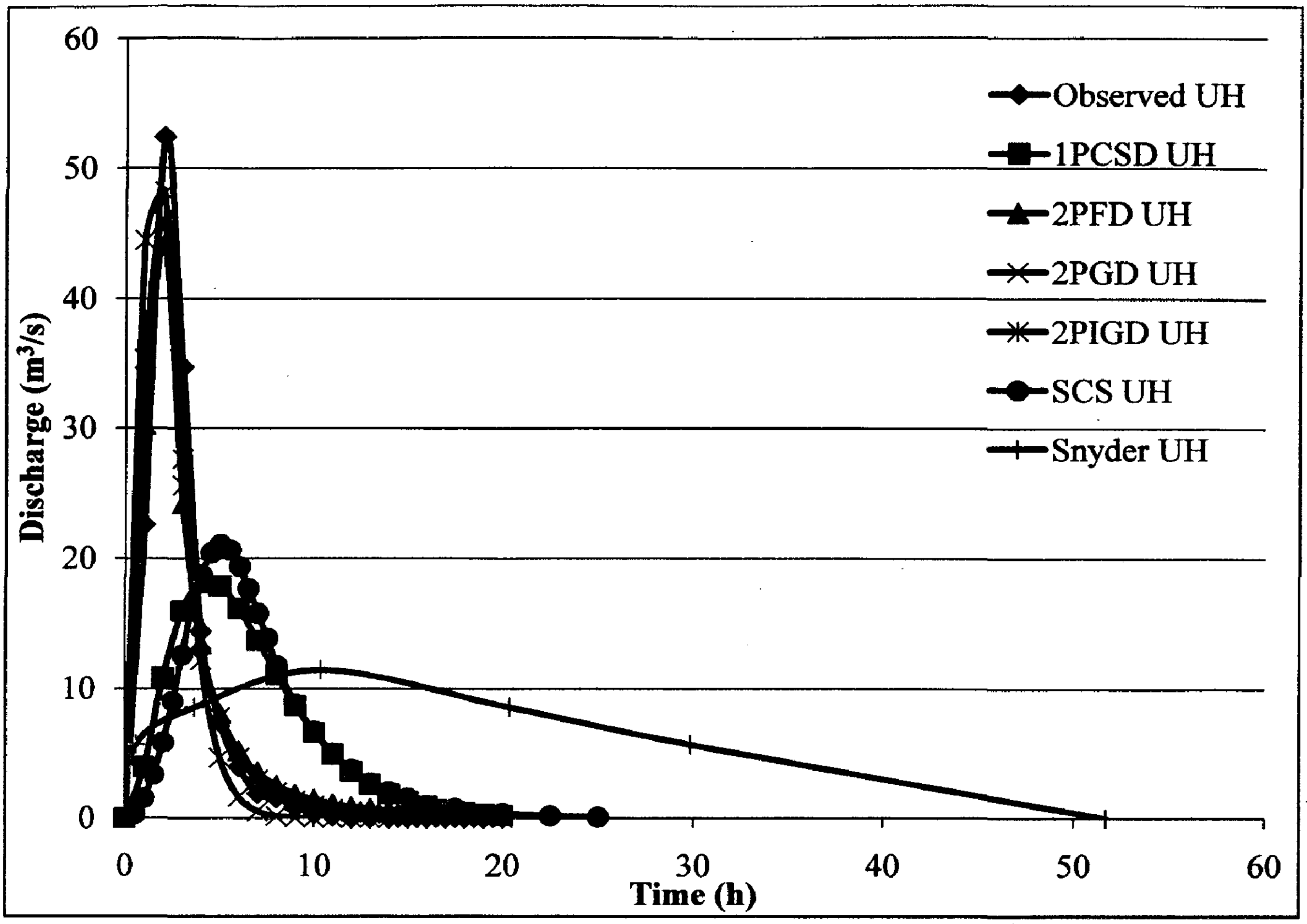

Fig. 5.9 Comparison of observed and computed UHs using four different pdfs, Snyder and SCS method for Gagas catchment.

5.5 Application on Burhner Catchment

The Burhner watershed, as discussed in chapter 3, is the large size catchment having an area of 4103 km^2. The geomorphologic characteristics and UH characteristics are given in Table 3.4. For applications of the models, a similar procedure as discussed in the previous sections has been adopted, i.e., at the step (i) the traditional SUH methods of Snyder and SCS, as discussed above, are applied and their performance is compared with the observed UH. And at the second step, (ii) the GIUH coupled pdf based models are applied and their performance is compared with the observed UH. Finally, the SUHs developed at the steps (i) and (ii) are compared with observed UH to have an overall assessment of the relative performance of these models. The detailed steps followed in this procedure are described below.

5.5.1 Traditional SUH Methods of Snyder and SCS

For the determination of SUH by Snyder method, the geomorphologic watershed characteristics and UH characteristics as shown in Table 3.4 are used. The non-dimensional constants C_T and C_P are assumed as: C_t =0.5 and C_p=0.93. The following procedure is adopted to derive SUH.

Using Eqs. (2.1) and (2.2) and L, L_{ca}, C_t compute time to peak t_p = 14.6 hr and peak flow rate q_p=75.23 m^3/s; and the widths of the UH at 0.5q_p (Eq.2.6) and 0.75q_p (Eq.2.7) are computed as: W_{50}=36.67 h and W_{75}=20.95 h, respectively. Finally, the SUH is computed using these salient points as shown in Fig. 5.10 in comparison with observed UH. Similarly, for application of SCS method, Eqs. (2.10) and (2.11) are used to compute t_p and q_p as t_p= 5 hr and q_p=170.685 m^3/s. Using these salient points, SUH is drawn as shown in Fig. 5.10 to have a comparison with Snyder's method.

It can be observed from Fig. 5.10 that SUHs due to both the traditional methods deviate largely from the observed UH and largely overestimate the peak flow rates with more than 50% of the area as non- matching.

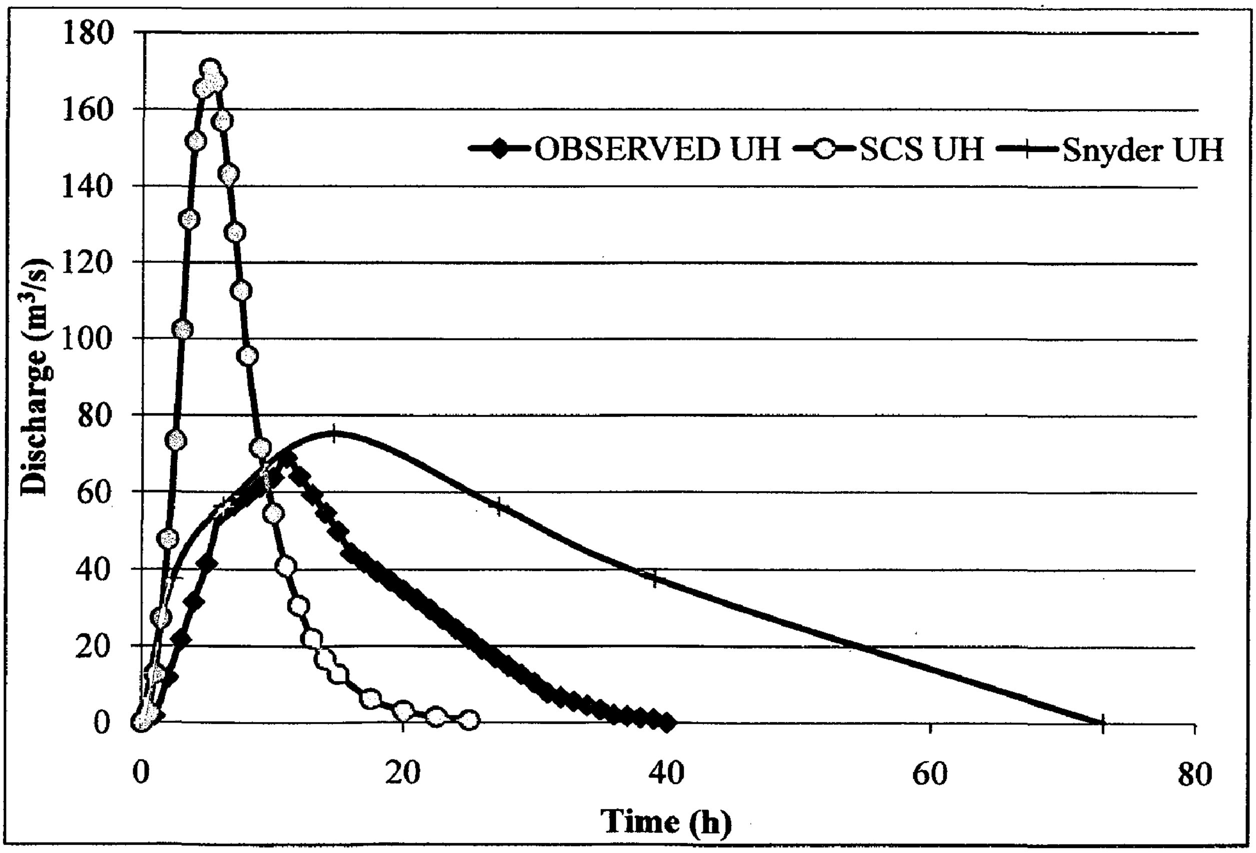

Fig. 5.10 Comparison of observed and computed UHs using Snyder and SCS method for Burhner catchment.

5.5.2 GIUH Coupled Probability Models

The GIUH based probability models of 1PCSD, 2PFD, 2PIGD and 2PGD are applied to derive SUHs for Burhner catchment, where q_p is considered to be known, t_p, β, and the parameters of the models are derived as follows:

I. At first step use Eq. (4.33) and substitute the value of q_p and R_L (Table 3.4) to get the value of vL^{-1} as:

$q_p = Q_P/A_w = (69.02x1000x3600)/(4103x10^6) = 0.061$ mm/hr/mm $= 0.364 \times (1.787)^{0.43}$ v L^{-1}. Hence, v $L^{-1} = 0.061 / [0.364 (1.787)^{0.43}] = 0.13$ hr^{-1};

II. Now substitute the values of vL^{-1} (Step I) and R_A, R_B, and R_L (Table 3.4) into Eq. (4.34) to get t_p as:

$t_p = 1.584 (3.52/3.94)^{0.55} 1.79^{-0.38} v^{-1}L = 9.210$ hrs;

III. Get the dimensionless product $\beta = q_p x t_p = 0.0606 \times 9.2163 = 0.5582$.

IV. Taking these values (at Steps I-III), estimate the parameters of the 1PCSD, 2PFD, 2PIGD and 2PGD (Rosso, 1984). For 1PCSD, use Eq. (4.9) for m or τ, Eqs. (4.17) and (4.14) for c and α, Eqs. (4.29) and (4.23) for m or α and k, and Eqs. (4.39) and (4.40) for n and K and respectively for 2PFD, 2PIGD, and 2PGD models. The estimated parameters values are given in Table 5.7.

V. Finally, derive the SUHs using the above four methods, viz., Eq. (4.3) for 1PCSD, Eq. (4.13) for 2PFD, Eq. (4.22) for 2PIGD, and Eq. (4.35) for 2PGD. The derived SUHs are shown in Fig. 5.11.

Table 5.7: Parameters of the four models for the partial data availability condition for Burhner watershed

Catchment	Parameter estimates of						
	1PCSD	2PFD		2PIGD		2PGD	
	τ	c	α	α	k	n	K
Burhner	3.11	1.68	10.76	2.78	34.83	3.14	4.31

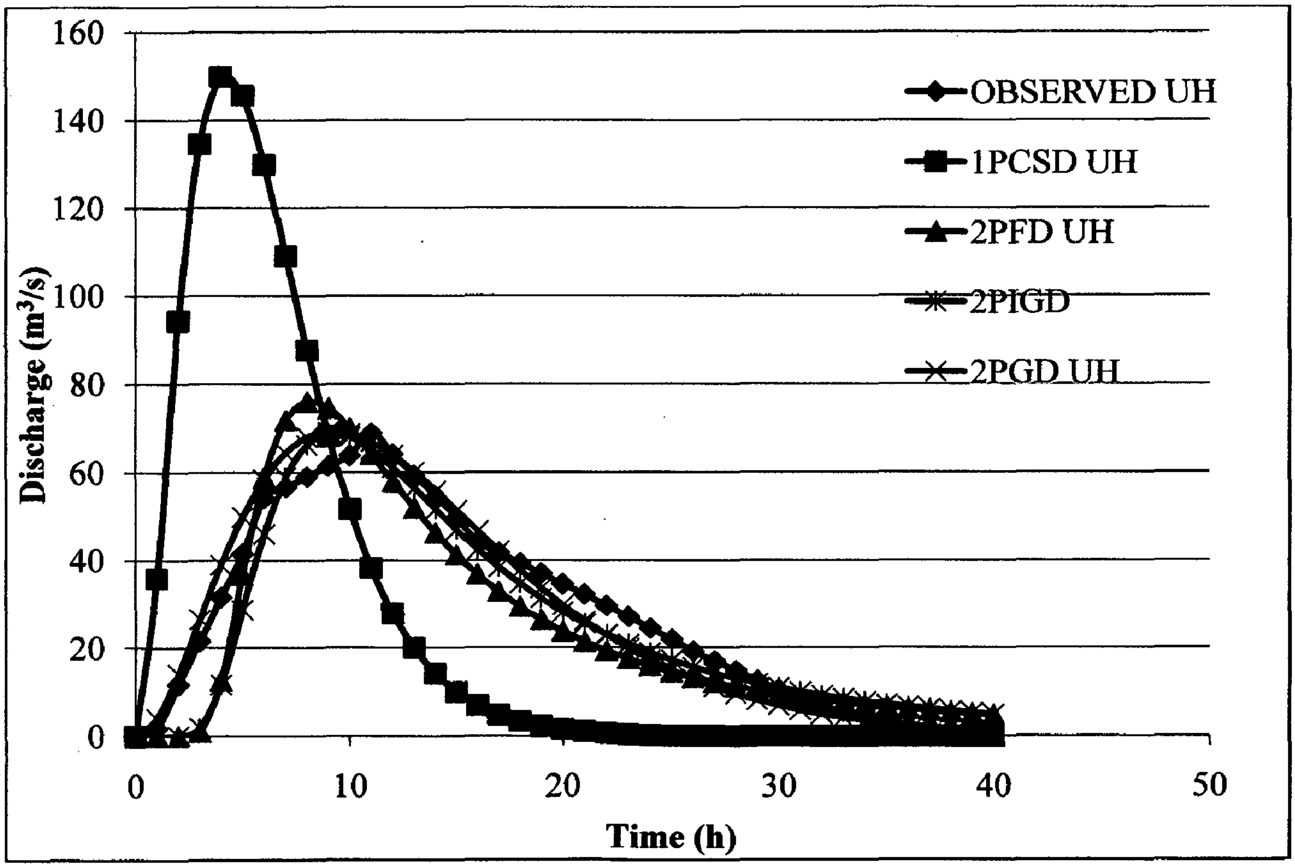

Fig. 5.11 Comparison of observed and computed UHs using four different pdfs for Burhner catchment.

5.5.3 PERFORMANCE OF MODELS

Following the same procedure as discussed in the previous sections, the performance of the models was assessed using the goodness-of-fit between the observed and computed UHs in terms of STDER (Eqs. (5.1) and (5.2)) and RE in peak flow rate (Eq. 5.3) as shown in Table 5.8. It can be observed from Table 5.8 that the STDERs due to 1PCSD, 2PFD, 2PIGD and 2PGD are found to be 44.91, 9.33, 6.73 and 4.85 respectively. Thus the results indicate that the SUH derived by 2PGD (Rosso, 1984) model performs marginally better than 2PIGD and 2PFD, and much better than 1PCSD. Further, the RE in peak flow rate is evaluated using Eq. (5.3) as shown in Table 5.8. It can be observed from these results that 2PGD model performs better than 2PIGD followed by 2PIGD, 2PFD, and 1PCSD models. Similar inferences can also be drawn from Fig. 5.11, which shows the comparison between the observed UH and the SUHs computed by the GIUH coupled probability models.

Finally, for an overall appraisal of the models performance, the SUHs computed by the traditional methods of Snyder and SCS and GIUH coupled probability models, i.e., 1PCSD, 2PFD, 2PIGD and 2PGD are also compared as shown in Fig. 5.12. It can be observed from the figure that all the probability models perform much better than the traditional methods of SUH, except 1PCSD, which has higher peak flow rate as compared to Snyder method. It can also be noted that the traditional methods have much larger deviation from the observed UH with respect to q_p and t_p and area under their UH curve is hard enough to get unity with larger portion (more than 50%) of non-matching area. Thus for Burhner watershed (the largest watershed) the 2PGD model performs better than 2PIGD followed by 2PFD, 1PCSD, Snyder and SCS methods.

Table 5.8: Error in using different probability models for Burhner catchment

Error	Error Estimates of Probability Models			
	1PCSD	**2PFD**	**2PIGD**	**2PGD**
STDER	44.91	9.33	6.73	4.85
Relative Error, Q_p(%)	-117.27	-10.25	0.14	-0.54

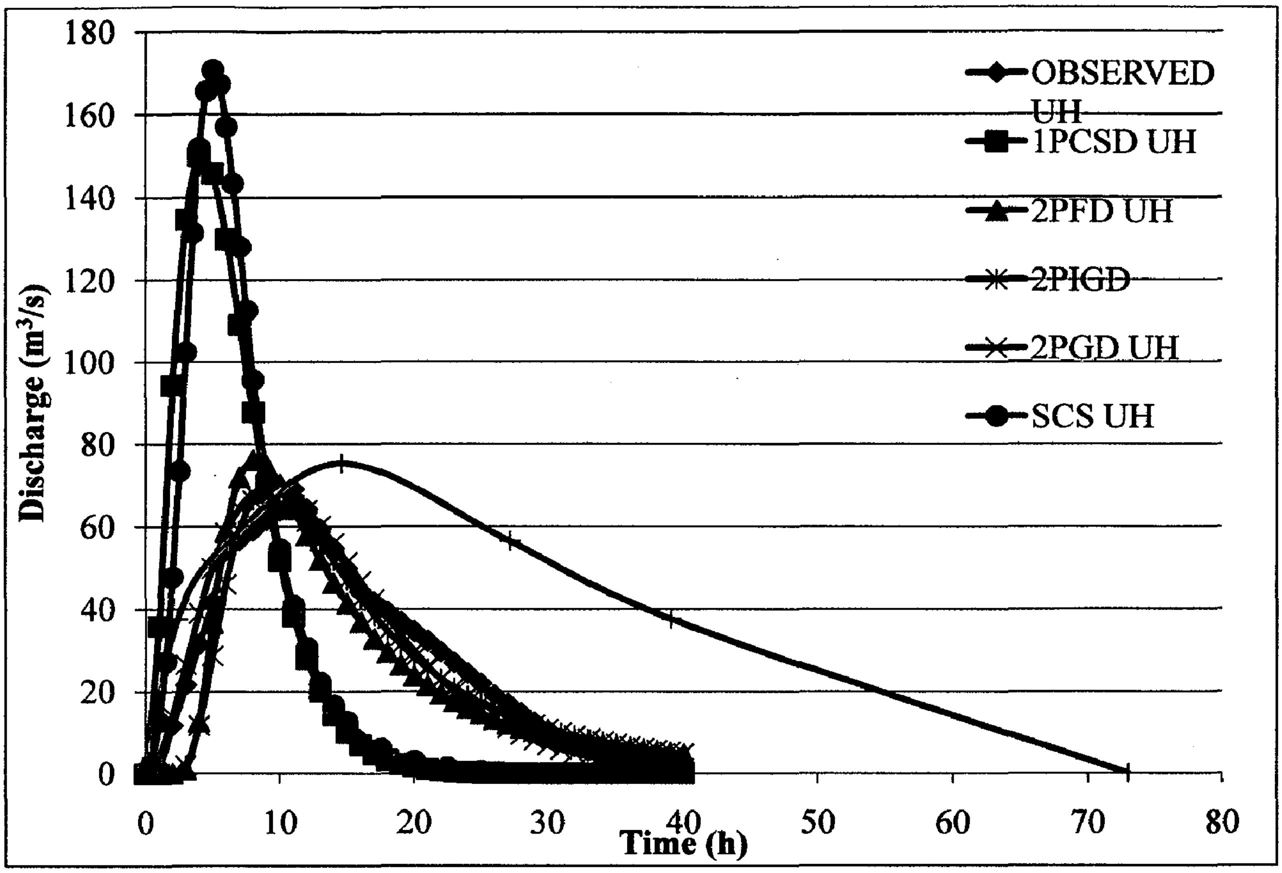

Fig. 5.12 Comparison of observed and computed UHs using four different pdfs, Snyder and SCS method for Burhner catchment.

5.6 SUMMARY

Various attempts have been made in the past to derive synthetic unit hydrograph (SUH) using the parametric expressions of the probability models of density functions (pdfs) using GIUH, as evident from chapter 2. In this chapter, an attempt was performed to explore the potential of GIUH coupled probability models of One- parameter Chi-square distribution (1PCSD), Two-parameter Fréchet distribution (2PFD), Two-parameter Inverse Gamma distribution (2PIGD), and Two-parameter Gamma distribution (2PGD) (Rosso, 1989) functions for limited data availability condition in comparison with the widely used traditional methods of Snyder and SCS for SUH derivation. For application of these models, four study watersheds categorized as small, medium and large with an area ranging from 27.93 km^2 to 4000 km^2 were used. For limited data conditions, SUH parameters, viz., peak discharge, time to peak, etc. were computed using Horton order ratios given by Rodriguez-Iturbe and Valdes (1979).

In general the results show that the GIUH coupled probability models performs much better than the traditional methods of Snyder and SCS. It was found that for mid-sized study catchments (Myntdu-Leska and Gagas), the 2PFD model perform better than 2PGD, followed 2PIGD and 1PCSD models. However, for the large size catchment (Burhner) as well as for the smallest (Kothuwatari), the 2PGD model was found to perform better than 2PIGD, followed by 2PFD and 1PCSD models. Notably, for all the study catchments, the traditional methods perform poor as these have much larger deviation from the observed UH and area under their curve is hard enough to get unity with larger portion (more than 50%) of non-matching area.

Chapter-6

SUMMARY AND CONCLUSIONS

The need for a synthetic method to develop UHs has inspired many studies as the drainage basins in many parts of the world are ungauged or poorly gauged, and in some cases existing measurement networks are not working properly. Most available methods for synthetic unit hydrograph (SUH) derivation involve manual, subjective fitting of a hydrograph through a few data points. Because of this tedious procedure, the generated unit hydrograph is often left unadjusted for unit runoff volume. During recent decades, use of probability distribution functions (pdfs) in developing SUH has received much attention because of its similarity with unit hydrograph properties. Here, GIUH based probability models are applied to derive SUHs for partially data availability condition, where q_p is considered to be known; t_p, β, and the parameters of these models are derived.

In order to expand the information on hydrological inputs the recent advancements in geo-spatial techniques like Geographical Information System (GIS) and Remote Sensing (RS) has gained much impetus in Hydrological and Earth Sciences (HES) since last two decades. The geomorphologic characteristics of study watershed have been extracted from ASTER data, resolution 30 m, using ILWIS (version 3.31) GIS software. For geomorphologic analysis, a detailed DEM of the catchment was prepared using ASTER data having fineness of 1-arc second spatial resolution, which was downloaded from the website ASTER Global Digital Elevation Model (ASTER GDEM) (www.gdem.aster.ersdac.or.jp/). The hydrological analysis is done through the various operation steps of "DEM Hydro-Processing" within ILWIS (version 3.31) GIS software which facilitate us to extract the digital elevation model (DEM), drainage network map, Horton's order ratio etc. of the study watershed.

This study focused on exploring the potential of GIUH coupled probability models of One-parameter Chi-square distribution (1PCSD), Two-parameter Fréchet distribution (2PFD), Two-parameter Inverse Gamma distribution (2PIGD), and Two-parameter Gamma distribution (2PGD) (Rosso, 1989) functions for limited data availability condition in comparison with the widely used traditional methods of Snyder and SCS for SUH derivation. The parametric expressions of four probability distributions namely One-parameter Chi-square (1PCSD), Two-parameter Fréchet (2PFD), Two-parameter Gamma (2PGD) and Two-parameter Inverse Gamma (2PIGD) distributions are diagnosed and simple analytical solutions are developed to estimate the distribution parameters. For application of these models, four Indian catchments categorized as small, medium and large with an area ranging from 27.93 km^2 to 4000 km^2 having different hydro-climatological characteristics and terrain were considered. In each study catchments, six models (two traditional and four GIUH coupled pdf models) are applied. For the determination of SUH by traditional methods of Snyder and SCS, the geomorphologic watershed characteristics and UH characteristics of the given study area are used. Here, time to peak t_p, peak flow rate q_p , and other salient points of the hydrograph are computed and used to derive SUH. For limited data conditions, UH parameters, e.g. peak discharge and time-to-peak, were determined using Horton ratios through the relationships given by Rodriguez-Iturbe & Valdes (1979). The GIUH based probability models of 1PCSD, 2PFD, 2PIGD and 2PGD are applied to derive SUHs for each study area, where q_p is considered to be known; t_p, β, and the parameters of these models are calculated. The parameters of the 1PCSD, 2PFD, 2PIGD and 2PGD (Rosso, 1984) are estimated using the value of dimensionless β. Finally, the SUHs are derived using these four pdf based methods. Then, assessment of the relative performance of the GIUH coupled pdf models was made using the goodness-of-fit between the UHs in terms of standard error (STDER) and relative error (RE) in peak flow rate to adjudge their suitability for SUH derivation. Finally, for an overall appraisal of these models, the results of the four pdfs based methods coupled with GIUH and traditional methods of Snyder and SCS were compared with the observed UH. The following conclusions can be drawn from this study:

1. In general, GIUH-coupled probability models perform much better than the traditional methods of Snyder and SCS on both text and field data.
2. The pdf-based methods avoid manual, subjective fitting of a hydrograph through few data points and, in turn, eliminate the problem of subjective adjustments. Therefore, it is easier and more accurate to apply in the field than other available methods.
3. In the description of SUH shape with limited data conditions, the 2PFD model performed better than 2PIGD for mid-sized study catchments, followed by 2PGD and

1PCSD models. However, for the large size catchment like Burhner as well as for the smaller size catchment, the 2PGD (Rosso, 1984) model performed better than 2PIGD followed by 2PFD and 1PCSD models.

4. Accurate and easier to generate the digital elevation model (DEM) followed by catchment and drainage network extraction map of the study catchments to compute the Horton's ratios from the most recent ASTER-DEM data using ILWIS 3.31 version GIS software rather than the manual procedures.
5. Analytical diagnosis of 1PCSD, 2PFD, 2PGD & 2PIGD indicates a similar behavior of the parameters and statistical properties of UH. The proposed parameter estimations methods are simple to use and provide accurate results of the actual pdf parameters, as verified using simulation and field data.
6. Among the four pdfs analyzed in this study, the frechet and gamma distributions are more flexible in description of SUH shape as they skew on both sides similar to a UH, and on the basis of their application to field data. The 2PFD for mid size and 2PGD for small and large size catchments may be a preferred method for deriving SUH.

APPENDIX A

Derivation for time to peak, t_p in the parametric expression 2PCSD

For applying the condition at time to peak ($t = t_p$), $dq(t)/dt = 0$, to the general equation of Two Parameter Chi-square density function as in term of discharge ordinates q(t) of UH (Eq. (4.3)) i.e., $q(t) = \frac{1}{2^{\tau}\Gamma(\tau)} t^{\tau-1} e^{-\frac{t}{2}}$

On applying condition,

$$\frac{dq(t)}{dt} = \frac{d}{dt}\left[\frac{1}{2^{\tau}\Gamma(\tau)} t_p{}^{\tau-1} e^{-\frac{t_p}{2}}\right] \quad \text{(A1)}$$

Or,

$$0 = \frac{1}{2^{\tau}\Gamma(\tau)}\left[t_p{}^{\tau-1} e^{-\frac{t_p}{2}} * \frac{-1}{2} + e^{-\frac{t_p}{2}}(\tau-1)t_p{}^{\tau-2}\right]$$

Or,

$$0 = \frac{1}{2^{\tau}\Gamma(\tau)} t_p{}^{\tau-2} e^{-\frac{t_p}{2}}\left[\frac{-t_p}{2} + (\tau-1)\right]$$

Or,

$$0 = \frac{1}{2^{\tau}\Gamma(\tau)} t_p{}^{\tau-2} e^{-\frac{t_p}{2}}\left[\frac{-t_p+2\tau-2}{2}\right]$$

Or,

$$0 = t_p{}^{\tau-2}(-t_p + 2\tau - 2)$$

Or,

$$0 = -t_p + 2\tau - 2 \qquad \text{Or,} \qquad \boxed{t_p = 2(\tau-1)} \quad \text{(A2)}$$

Again,

$$q_p t_p = \frac{1}{2^{\tau}\Gamma(\tau)} t_p{}^{\tau-1} e^{-\frac{t_p}{2}} * 2(\tau-1)$$

Or,

$$q_p t_p = \frac{1}{2^{\tau-1}(\tau-1)\Gamma(\tau-1)}[2(\tau-1)]^{\tau-1} e^{-\frac{[2(\tau-1/2)]}{2}} * (\tau-1)$$

Or,

$$q_p t_p = \frac{(\tau-1)^{\tau} e^{-(\tau-1)}}{\Gamma(\tau)} \quad \text{(A3)}$$

APPENDIX B

Simplification for β

Using, Stirling's formula (Abramowitz and Stegun, 1964) for expansion of gamma function is expressed as:

$$\Gamma(m) = \sqrt{2\pi m}\,(m^{m-1}e^{-m})\left(1+\frac{1}{12m}+\frac{1}{288m^2}-\frac{139}{51840m^3}-\frac{571}{2488320m^4}\ldots\right) \quad \text{(B1)}$$

Considering first two terms of Eq. (B1) in the parenthesis, Eq. (4.7) simplifies to the following form

$$\beta = \frac{m^m e^{-m}}{e^{-m}m^{m-1}\left(1+\frac{1}{12m}\right)\sqrt{2\pi m}} \quad \text{(B2)}$$

[1+1/ (12m)] in the denominator of (A2) can be approximated to $[1+1/(6m)]^{1/2}$. In such case (B2) simplify to the following form

$$\beta^2 \cong \frac{m^2}{(2\pi m+\pi/3)} \quad \text{(B3)}$$

Alternatively Eq. (B3) can be expressed as

$$3m^2 - 6\pi m\beta^2 - \pi\beta^2 = 0 \quad \text{(B4)}$$

Since, Eq. (B4) is a quadratic expression in m, the roots of Eq. (B4) can be expressed as

$$m = \pi\beta^2 \pm \beta\sqrt{(\pi^2\beta^2+\pi/3)} \quad \text{(B5)}$$

APPENDIX C

Derivation for time to peak, t_p in the parametric expression 2PFD

For applying the condition at time to peak ($t = t_p$), $dq(t)/dt = 0$, to the general equation of Two Parameter Fréchet density function as in term of discharge ordinates q(t) of UH (Eq. (4.13)) i.e., $q(t) = (c/\alpha)(\alpha/t)^{c+1} e^{-(\alpha/t)^c}$ for $t>0$

On applying condition,

$$\frac{dq(t)}{dt} = \frac{d}{dt}\left[(c/\alpha)(\alpha/t)^{c+1} e^{-(\alpha/t)^c}\right] \tag{C1}$$

Or,

$$0 = (c/\alpha)\alpha^{c+1}\frac{d}{dt}\left[(1/t)^{c+1} e^{-(\alpha/t)^c}\right]$$

Or,

$$0 = (c/\alpha)\alpha^{c+1}\left[(1/t)^{c+1}\frac{d}{dt}\left(e^{-(\alpha/t)^c}\right) + e^{-(\alpha/t)^c}\frac{d}{dt}\left((1/t)^{c+1}\right)\right]$$

Or,

$$0 = c\alpha^c\left[(1/t)^{c+1} e^{-(\alpha/t)^c} c\alpha^c\left(\frac{1}{t}\right)^{c+1} + e^{-(\alpha/t)^c}(-(c+1))(1/t)^{c+2}\right]$$

Or,

$$0 = c\alpha^c e^{-(\alpha/t)^c}(1/t)^{c+2}\left[c\alpha^c(1/t)^c + (-(c+1))\right]$$

Or,

$$0 = c\alpha^c(1/t)^c + (-(c+1))$$

Or,

$$c+1 = c\alpha^c(1/t)^c$$

Or,

$$\left(t_p\right)^c = \alpha^c \left(\frac{c}{c+1}\right)$$

Or,

$$t_p = \alpha[c/(c+1)]^{1/c} \tag{C2}$$

So, again from above equation, we can obtain

$$\alpha = t_p \left(\frac{c+1}{c}\right)^{1/c} \tag{C3}$$

and

$$q_p = (c/\alpha)(1+1/c)^{1+1/c} e^{-(1+1/c)} \tag{C4}$$

APPENDIX D

Derivation for time to peak, t_p in the parametric expression 2PIGD

For applying the condition at time to peak ($t = t_p$), $dq(t)/dt = 0$, to the general equation of Inverse gamma density function as in term of discharge ordinates q(t) of UH (Eq. (4.22))

i.e. $q(t) = \frac{1}{k\Gamma(\alpha)} \left(k/t\right)^{\alpha+1} e^{-\left(k/t\right)}$

On applying condition,

$$\frac{dq(t)}{dt} = \frac{d}{dt}\left[\frac{1}{k\Gamma(\alpha)}\left(\frac{k}{t}\right)^{\alpha+1} e^{-\left(\frac{k}{t}\right)}\right] \tag{D1}$$

Or,

$$0 = \frac{1}{k\Gamma(\alpha)}\left[\left(\frac{k}{t_p}\right)^{\alpha+1}\frac{d}{dt}\left(e^{-\left(\frac{k}{t_p}\right)}\right) + e^{-\left(\frac{k}{t_p}\right)}\frac{d}{dt}\left(\frac{k}{t_p}\right)^{\alpha+1}\right]$$

Or,

$$0 = \frac{1}{k\Gamma(\alpha)}\left[\left(\frac{k}{t_p}\right)^{\alpha+1} e^{-\left(\frac{k}{t_p}\right)}\left(\frac{k}{{t_p}^2}\right) + e^{-\left(\frac{k}{t_p}\right)}\left(-(\alpha+1)k^{(\alpha+1)}\frac{1}{{t_p}^{\alpha+2}}\right)\right]$$

Or,

$$0 = \frac{1}{k\Gamma(\alpha)} e^{-\left(\frac{k}{t_p}\right)}\left[\frac{k^{\alpha+2}}{{t_p}^{\alpha+3}} + \left(-(\alpha+1)\frac{k^{(\alpha+1)}}{{t_p}^{\alpha+2}}\right)\right]$$

Or,

$$0 = \frac{1}{k\Gamma(\alpha)} e^{-\left(\frac{k}{t_p}\right)}\left[\frac{k^{\alpha+2} + \left(-(\alpha+1)k^{(\alpha+1)}t_p\right)}{{t_p}^{\alpha+3}}\right]$$

Or,

$$(\alpha+1)k^{(\alpha+1)}t_p = k^{\alpha+2}$$

Or,

$$t_p = \frac{k}{(\alpha+1)} \tag{D2}$$

Or,

$$k = t_p(\alpha+1) \tag{D3}$$

APPENDIX E

Simplification for β

Using, Stirling's formula (Abramowitz and Stegun, 1964) for expansion of gamma function is expressed as:

$$\Gamma(m) = \sqrt{2\pi m}\,(m^{m-1}e^{-m})\left(1 + \frac{1}{12m} + \frac{1}{288m^2} - \frac{139}{51840m^3} - \frac{571}{2488320m^4}\cdots\right) \qquad \text{(E1)}$$

Considering first two terms of Eq. (E1) in the parenthesis, Eq. (4.26) simplifies to the following form

$$\beta = \frac{(m-1)m^{m-1}e^{-m}}{e^{-m}m^{m-1}\left(1+\frac{1}{12m}\right)\sqrt{2\pi m}} \qquad \text{(E2)}$$

[1+1/ (12m)] in the denominator of (E2) can be approximated to $[1+1/(6m)]^{1/2}$. In such case (E2) simplify to the following form

$$\beta^2 \cong \frac{(m-1)^2}{(2\pi m + \pi/3)} \qquad \text{(E3)}$$

Alternatively Eq. (E3) can be expressed as

$$3m^2 - (6 + 6\pi\beta^2)m + (3 - \pi\beta^2) = 0 \qquad \text{(E4)}$$

Since, Eq. (E4) is a quadratic expression in m, the roots of Eq. (E4) can be expressed as

$$m = (1 + \pi\beta^2) \pm \beta\sqrt{(\pi^2\beta^2 + 7\pi/3)} \qquad \text{(E5)}$$

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