EFFECT OF INTERMEDIATE DRAINS AND LEAKAGE THROUGH SHEETPILE LINES ON THE UPLIFT PRESSURES

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By

A. S. CHAWLA

WATER RESOURCES DEVELOPMENT TRAINING CENTRE
UNIVERSITY OF ROORKEE
ROORKEE, INDIA

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CERTIFICATE

This is to certify that the thesis entitled " Effect of Intermediate Drains and leakage through sheetpile lines on the uplift pressures", submitted by Sri A.S.Chawla to the water Resources Development Training Centre, University of Roorkee, Roorkee for the award of degree of Doctorate of Philosophy in Water Resources Development is a bonafide record of research work carried out by him from July, 1969 to May 1973 under our supervision and guidance.

The results embodied in this thesis have not been submitted to any other University for the award of any Degree or Diploma.

Dr. K.V.Mittal
Professor in the Deptt.
of Mathematics
University of Roorkee
Roorkee.

Bharat Singh

Dr. Bharat Singh
Professor (Designs)
Water Resources
Development Training
Centre University of
Roorkee, Roorkee.

ABSTRACT

The stability of a hydraulic structure founded on permeable soil has to be ensured against uplift pressures and piping. The problem of seepage under such structures belongs to a general class of confined flow problems for which the method of solution by conformal mapping was indicated by Pavlovsky (17) and Khosla (12). These solutions are based on the assumption that the cutoffs are absolutely impervious and flow takes place around them. In practice, however, it has been found impossible to provide a leakage free cutoff pile below the foundation, unless it is made of concrete and constructed monolithically with the apron. In steel sheetpiles leakage may be due to improper interlocking or corrosion of sheetpile. In case of cast insitu diaphragms the leakage may take place through faulty joints. Imperfections may also occur in case of grout curtains constructed using lines of grout holes. Brahma (3) studied the effect of single opening in a fully penetrating single cutoff resting on permeable soil of finite depth. Dachler (8) and Ambraseys (1) determined the effect of leakage through a fully penetrating cutoff with several equally spaced openings.

Intermediate filters or drains can be provided below hydraulic structures founded on permeable soil to reduce uplift pressures resulting in appreciable savings. Melenshchenko (14) and Numerov (16) determined the effect of drainage hole below an impervious floor. Zamerin (21) obtained the effect of plane drainage on the flow below a flat floor or a single overfall founded on infinite depth of permeable soil.

In the present work the problem of seepage below flat floor with two asymmetric cutoffs, founded on permeable soil of infinite depth

with a filter of any dimension has been solved with the help of conformal mapping. In addition solution based on the same method has been obtained for seepage below apron with a cutoff and a deep drainage anywhere. The cutoff may be either on the upstream or downstream end of floor. The conformal mapping in these problems has a special feature in that there are more than one continuous equipotential and impervious surfaces. The solution indicates that provision of filter or drainage of even small dimensions reduces pressures considerably along the entire profile of the structure. Further reduction in pressure with increase in length of filter or depth of drain is less as compared to initial reduction. The uplift pressures decrease on the downstream side and increase on the upstream side as the filter or drain is shifted towards downstream. The filter should be located such that total uplift on the floor downstream of the gateline is minimum. With the increase in the depth of downstream cutoff uplift pressures increase only along the floor downstream of the filter. The observed pressures, below the floor with two end cutoffs, in the portion upstream of the drain are very close to those theoretically calculated for this portion neglecting downstream cutoff. The observed pressures along the floor downstream of the drain are slightly less than those theoretically calculated neglecting upstream cutoff. The uplift pressures for floor with two end cutoffs and a deep drain can, therefore, be calculated for upstream portion neglecting downstream cutoff and for downstream portion neglecting the upstream cutoff.

The equations obtained have been used for computation of pressures at key points. The results have been plotted in the form of design curves.

The solution of seepage below a flat floor with central leakey cutoff founded on permeable soil of infinite depth has been obtained with the help of conformal mapping. The cutoff is assumed to have single opening anywhere. It is seen that uplift pressures increase along the downstream portion of the floor and decrease along the upstream portion of the floor with the existence of opening in the cutoff, resulting in smaller drop in pressures across the cutoff. The pressure drop reduces as the opening approaches the top of the cutoff. When the opening is at the top of cutoff pressure drop across the cutoff is nil. The solution indicates that even for a small leakage at any level, the uplift pressures along the floor approach values close to those obtained neglecting portion of cutoff below the opening. The seepage discharge passing through the opening is maximum when it is located at about 0.4 of the depth of cutoff. The seepage discharge increases with increase in the area of the opening and with decrease in the length of floor.

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CONTENTS

NOTATION

The following symbols are used:

a = transformation parameter

b = length of floor with end cutoff or cutoffs.
= half floor length in case of central cutoff

b_1 = distance of drain or filter from upstream cutoff or downstream cutoff, if no upstream cutoff.

d = depth of central cutoff.

d_1 = depth of upstream cutoff
= depth to the top of opening in central cutoff.

d_2 = depth of downstream cutoff

d_2 = depth to the bottom of opening in central cutoff.

$E(\varphi, m)$ = incomplete elliptic integral of second kind.

$E = E(\pi/2, m)$ = complete elliptic integral of second kind

$E' = E(\pi/2, m')$ = associated complete elliptic integral of second kind

$F(\varphi, m)$ = elliptic integral of first kind.

g_1, g_2 = constant

H_1 = Head in the filter or drain measured above downstream water level.

H_2 = Head in the upstream bed measured above downstream water level.

h = head at any point measured above downstream water level

$K = F(\pi/2, m)$ = complete elliptic integral of first kind

$K' = F(\pi/2, m')$ = Associated complete elliptic integral of first kind.

k = coefficient of percolation.

M = Constant

m = modulus of elliptic integrals

$m' = \sqrt{1 - m^2}$ = co-modulus of elliptic integral

N = Constant

p, p_1 = transformation parameters

q = discharge per unit width normal to the direction of flow

Sn u = elliptic sine of argument u

t = $r + is$ = complex variable representing auxiliary semi-infinite plane.

w = $\Phi + i\phi$ = complex variable representing rectangular flow field.

z = $x + iy$ = complex variable representing physical plane

α^2 = Parameter of elliptic integral of third kind

β, r, δ = transformation parameters

$\zeta = \xi + i\eta$ = complex variable representing auxiliary semi-infinite plane

$\lambda_1, \lambda_2, \mu$ = transformation parameters

$\Pi(\varphi, \alpha^2, m)$ = incomplete elliptic integral of third kind (φ = am u)

$\Pi = \Pi(\pi/2, \alpha^2, m)$ = complete elliptic integral of third kind

$\Pi' = \Pi(\pi/2, \alpha^2, m')$ = associated complete elliptic integral of third kind

ρ, σ = transformation parameters.

Φ = potential function

φ = argument of elliptic functions.

ϕ = stream function

CHAPTER I

INTRODUCTION

CHAPTER-I

1. INTRODUCTION.

1.0 The design of hydraulic structures founded on permeable soils presents problems of complex nature. Besides testing such structures for forces due to surface flow, their stability against forces caused by percolating water has to be ensured. Requirements of seepage control determine the final design and dimensions of hydraulic structures. Water seeps under the foundations of these structures, and past their abutments on the flanks. Seepage flow exerts a pressure on the structure and tends to wash away the soil under it leading to piping. Excessive uplift pressures and piping are often the cause of damage to structures. A study of the causes of failures of structures founded on permeable soils would indicate that most of the failures may be attributed to the destructive effects of seepage. The importance of proper design and planning of hydraulic structures founded on permeable soils is, therefore obvious.

1.1 The problem of seepage under a hydraulic structure belongs to a general class of confined flow problems for which a method of solution by conformal transformations was indicated by Pavlovsky (17) and Khosla (12). Based on conformal transformation, general solutions for various boundaries have been given by Muskat (15), Harr (11), Aravin and Numerov(2), Polubarinova Kochina (18), and Garg and Chawla (6,7,9). These solutions are applicable for flat floor with an asymmetric cutoff, inclined floor or inclined cutoff, depressed floor with symmetric or asymmetric cutoff, and finite pervious reaches. These cases have been solved by assuming finite and infinite depth of permeable soil. Some of these solutions have been obtained for anisotropic soil also.

1.2 It is seen that the design engineer has sometimes to provide pervious floor or plane drainage in order to economise in the design of structure. Also even though a cutoff is meant to be as perfectly impervious as possible, due to constructional uncertainties involved in the field, there may be some leakage through it. The leakage in sheetpiling cutoff may be due to improper interlocking or corrosion of sheetpiles. Performance of such sheetpiles has been observed on five completed dams of Missouri River and has been found relatively ineffective for controlling under seepage. The head loss across the sheetpiling has been initially low but this has increased with time as the piling gradually tightened. (13). In case of cast insitu diaphragms the leakage may take place through faulty construction joints. Imperfection may also occur in case of grout curtains constructed using lines of grout holes (1). The design engineer must also take into account the possibility of leakage.

1.3 A solution of considerable importance was obtained by Meleshcherko (14) and Numerov (16) wherein they included the effect of one or two drainage holes in the otherwise impervious floor. The effect of plane drainage connected to downstream bed in case of seepage below a flat apron or a single overfall founded on infinite depth of permeable soil was obtained by Zamarin (21). Sangal (20) determined the extent of reduction in pressure affected by a flat and deep filter of particular dimensions below the foundation of a barrage with the help of electrical analogy model. No solution was till now available to determine the effect of plane drainage located anywhere between the two cutoffs.

1.4 As for the possibility of leakage through cutoffs, Dachler (8) and Ambraseys (1) determined the effect of leakage through a fully penetrating cutoff with several equally spaced openings under a very long floor resting on finite depth of permeable soil by an approximate analytical method. Ambraseys determined the effect of the leakage area and the width of the cutoff on the efficiency of the cutoff. The efficiency of the cutoff of negligible width drops to about 2% even for a leakage area less than 1%. Although the efficiency of the cutoff improves with the increase in its width, the overall efficiency falls below 20% for 8% open area even if the width of cutoff is 1/5 of its depth. Ambraseys did not study the effect of leakage on the uplift pressures below the floor on either side of the cutoff. Brahma (3) studied the effect of single opening in a fully penetrating single cutoff resting on permeable soil of finite depth. His studies indicate that the quantity of seepage through the opening of a particular size depends upon its location. The influence of location is more pronounced when the size of the opening is greater. The exit gradient drops rapidly when opening is shifted from top to slightly below it. Further lowering of the opening has little effect on the value of exit gradient. The drop in value of exit gradient is again comparatively rapid when opening is shifted down near the impermeable boundary. Ramdurgaiah (19) also studied the effect of leakage through a central cutoff founded on finite depth of permeable stratum on three dimensional electrical analogy model. The study indicates that uplift pressure on the downstream of the cutoff increase as a result of leakage through cutoff (10). No solution was till now available to determine the effect of leakage through a cutoff in an infinite depth of permeable soil.

1.5 In this thesis an attempt has been made to study both these effects (drainage in floor and leakage through cutoffs) separately in a more general form. The cases with the following boundary conditions have been studied:

(i) Flat floor with two unequal cutoffs and a plane drainage of any dimensions located anywhere between the two cutoffs founded on infinite depth of permeable soil (Chapter II).

(ii) Flat floor with a single cutoff and a deep drainage founded on infinite depth of permeable soil. The cutoff may be either on the upstream or downstream side of the drainage. (Chapter III).

(iii) Flat floor with a central leaky cutoff founded on infinite depth of permeable soil. The cutoff is assumed to have a single opening anywhere (Chapter IV).

The solution to these problems is obtained with the help of conformal mapping. In the course of integration of transformation equations standard results given by Byrd and Friedman(4) have been made use. of. These integrations resulted into elliptic functions of all the three kinds. These functions were evaluated on digital computer for which a subroutine programme was developed.

With a view to verify the analytical results, the uplift pressures along the profile of a hydraulic structures with the above boundary condition were also determined on an electrical analogy model.(Chapter V).

CHAPTER II

THEORETICAL SOLUTION FOR INTERMEDIATE FILTERS

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THEORETICAL SOLUTION
FOR INTERMEDIATE FILTERS

2.1 LAYOUT, BOUNDARY CONDITIONS AND METHOD OF SOLUTION.

Consider an impervious floor AB of length b founded on a semi-infinite homogeneous permeable soil structure as shown in Fig. 2.1(a). The floor has a cutoff C' D' of depth d_1, at the upstream end of the floor, and a cutoff E D, of depth d_2, at the downstream end. The floor is underlain by an intermediate filter F G of length f, at a distance b_1 from the upstream cutoff (i.e. $AG=b_1$). On the upstream and downstream of the floor is pervious bed extending upto infinity. The profile is represented in z-plane as shown in Fig. 2.1(a).

The velocity potential Φ of the flow through the pervious medium under this floor satisfies the two dimensional Laplace's equation:

$$\nabla^2 \Phi = 0 \qquad ..(2.01)$$

In the present case $\Phi = - kh$ where h is the head, and k is coefficient of percolation.

Without loss of generality we shall take h = 0 on the downstream water level, and measured above it $h = H_1$ as the head in the filter and $h = H_2$ as the head in the upstream bed. Then on the upstream bed AM, $\Phi = - kH_2$ and on the downstream bed BM, $\Phi = 0$.

The foundation profile A D' C' F forms the inner boundary of the upstream flow and therefore can be taken as the streamline $\psi = 0$, where ψ is the stream function. The lower profile of the filter FG is an equipotential $\Phi = - kH_1$. The foundation profile GEDB forms the inner boundary of the downstream flow and may be taken as the streamline $\psi = q$, where q is the discharge per unit width normal to

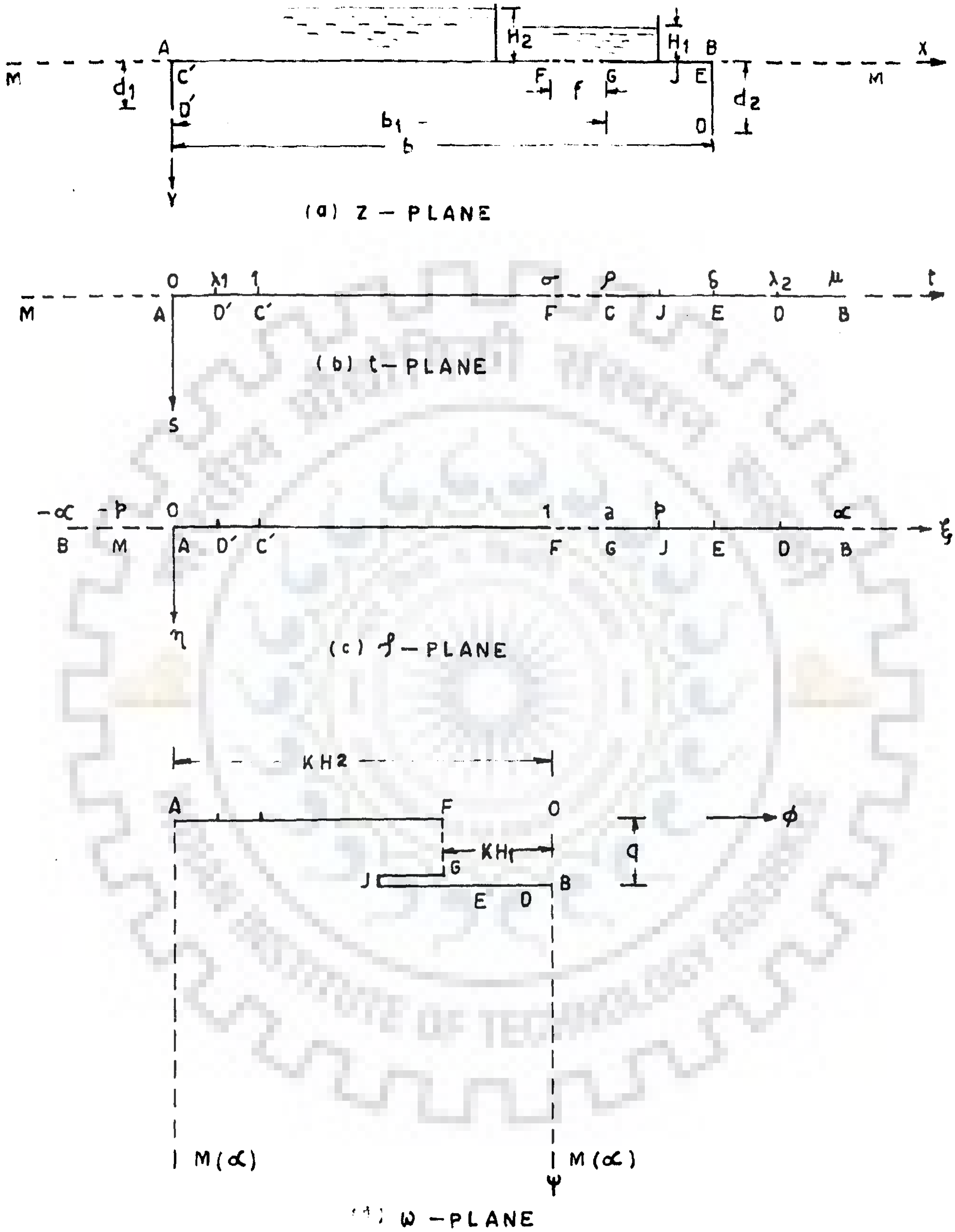


FIG. 2·1 – TRANSFORMATION LAYOUT

the direction of flow, drained through the intermediate filter. Starting from somewhere at the upstream end the streamline $\psi = q$ would meet the floor GEDB in some point J where it would divide into two streamlines, one along JG emerging at G and the other along JEDB emerging at B. The potential along the floor GJEDB would be maximum at J. The location of the point J would depend upon the relative values of H_1 and H_2. As the value of H_1 increases the point J moves towards G and for a particular value of H_1 , J may coincide with G. In case H_1 is increased beyond this value part of the seepage water emerging in the filter would percolate downstream below the foundation profile GEDB. Intermediate filters which are provided to reduce uplift pressures on the downstream portion of the floor would seldom be drained at such a high level. The analysis is, therefore, limited to the former case.

$w\ (= \Phi + i\psi)$ represents the complex potential. Conditions in the w-plane are as shown in Fig. 2.1 (d). The region of complex seepage potential is the area in the w-plane between the two vertical lines AM ($\Phi = -kH_2$) and BM ($\Phi = 0$) and bounded above by the lines AF($\psi = 0$), FG ($\Phi = -kH_1$), and GJEDB ($\psi = q$).

To obtain the mapping from the z-plane to the w-plane both the profile of structure in the z-plane and the complex seepage potential in the w-plane have been transformed into the lower half of the same semi-infinite t-plane using Schwarz-Christoffel transformation. The transformation of the w-plane onto semi-infinite t-plane has been obtained through an auxiliary ζ-plane with the help of bilinear transformation.

The following relations are thus obtained

$$z = f\ (t) \tag{2.02}$$

$$w = F_1(\zeta) \tag{2.03}$$

and $$\zeta = F_2(t) \tag{2.04}$$

Combining Eqs. 2.02, 2.03 and 2.04

$$z = f(t) = f\ F_0^{-1}\ F_1^{-1}\ (w) \tag{2.05}$$

and
$$w = F_1\ F_0\ f^{-1}(z) \tag{2.06}$$

where $z = x + iy$ represents the physical plane, $t = r + i\,s$ represents the intermediate semi-infinite plane, $\zeta = \xi + i\eta$ represents an auxiliary semi-infinite plane, and $w = \Phi + i\phi$ represents the flow field.

2.2 THEORETICAL SOLUTION

2.2.1 First Operation $z = f(t)$

In this operation the profile of the hydraulic structure in z-plane is transformed onto the real axis of the t-plane. On the t-plane the points A and C' are placed at zero and + 1. The points D', F, G, E, D and B lie at λ_1, σ, ρ, δ, λ_2 and μ respectively. These six positions are to be determined.

The Schwarz-Christoffel transformation that gives the above mapping is,

$$\frac{dz}{dt} = M \frac{(t - \lambda_1)(\lambda_2 - t)}{\sqrt{t(t-1)(\delta - t)(\mu - t)}}$$

$$\frac{dz}{dt} = M \left[\frac{-\lambda_1\lambda_2 + (\lambda_1 + \lambda_2)t - t^2}{\sqrt{t(t-1)(\delta - t)(\mu - t)}} \right] \tag{2.07}$$

<u>Along the upstream cutoff AD'C', $0 \leq t < 1$</u>

Integrating Eq. 2.07 in this region

$$-iy = \frac{M}{i}\left[-\lambda_1\lambda_2 \int_t^1 \frac{dt}{\sqrt{t(1-t)(\delta-t)(\mu-t)}} + (\lambda_1+\lambda_2)\int_t^1 \frac{t\,dt}{\sqrt{t(1-t)(\delta-t)(\mu-t)}} - \int_t^1 \frac{t^2\,dt}{\sqrt{t(1-t)(\delta-t)(\mu-t)}} \right]$$

Making use of standard formulae (4, Eqs. 253.11 ,340 and 336)

$$\frac{y}{M} = g_1 \Big[-\lambda_1\lambda_2 V_0 + (\lambda_1+\lambda_2) \left\{ \delta V_0 - (\delta - 1)V_1 \right\} - \left\{ \delta^2 V_0 - 2\,\delta(\delta-1)\, V_1 + (\delta - 1)^2 V_2 \right\} \Big]_t^1 \tag{2.08}$$

where $V_0 = F(\varphi, m_1)$ = elliptic integral of the first kind

$V_1 = \Pi(\varphi\, , \alpha_1^2\, , m_1)$ = elliptic integral of the third kind

$$V_2 = \frac{1}{2(\alpha_1^2 - 1)(m_1^2 - \alpha_1^2)} \Big[\alpha_1^2 E(\varphi, m_1) + (m_1^2 - \alpha_1^2) V_0 + (2\alpha_1^2 m_1^2 + 2\alpha_1^4 - 3m_1^2) V_1 - \frac{\alpha_1^4 \, \mathrm{sn}\, u \, \mathrm{cn}\, u \, \mathrm{dn}\, u}{1 - \alpha_1^2 \, \mathrm{sn}^2 \, u} \Big]$$

$E(\varphi\, , m_1)$ = elliptic integral of the second kind

$$\mathrm{Sn}\, u = \sqrt{\frac{(1-t)\delta}{\delta - t}}$$

$$\mathrm{Cn}\, u = \sqrt{\frac{t(\delta - 1)}{\delta - t}} \tag{2.09}$$

$$\mathrm{dn}\, u = \sqrt{\frac{(\mu - t)(\delta - 1)}{(\mu - 1)(\delta - t)}}$$

$$\varphi = \mathrm{Sin}^{-1} \sqrt{\frac{\delta(1 - t)}{\delta - t}} \tag{2.10}$$

$$m_1 = \sqrt{\frac{(\mu - \delta)}{\delta(\mu - 1)}} \tag{2.11}$$

$$\alpha_1^2 = \frac{1}{\delta} \tag{2.12}$$

$$g_1 = \frac{2}{\sqrt{\delta(\mu - 1)}} \tag{2.13}$$

on substituting for m_1, α_1^2, Sn u, Cn u and dn u

$$V_2 = \frac{1}{2(\delta-1)^2}\left[\delta(\mu-1)E(\varphi,m)-\delta(\delta-1)V_0-(\delta-1)(\mu-3\delta+1)V_1 - \delta(\mu-1)\sqrt{\frac{t(1-t)(\mu-t)}{\delta(\mu-1)(\delta-t)}}\right] \quad (2.14)$$

substitution of values of V_0, V_1 and V_2 in Eq. 2.08 yields

$$\frac{y}{Mg_1} = \left[-\lambda_1\lambda_2 F(\varphi,m_1)+(\lambda_1+\lambda_2)\left\{\delta F(\varphi,m_1)-(\delta-1)\Pi(\varphi,\alpha_1^2,m_1)\right\} - \frac{1}{2}\delta(\delta+1)F(\varphi,m_1) - \frac{1}{2}\delta(\mu-1)E(\varphi,m_1) + \frac{1}{2}(\delta-1)(\delta+\mu+1)\Pi(\varphi,\alpha_1^2,m_1)+\frac{1}{2}\delta(\mu-1)\sqrt{\frac{t(1-t)(\mu-t)}{\delta(\mu-1)(\delta-t)}}\right]_t^1 \quad (2.15)$$

At point D', $t=\lambda_1$, $y=d_1$, therefore,

$$\frac{d_1}{Mg_1} = -\lambda_1\lambda_2 F(\varphi_{D'}, m_1) + (\lambda_1+\lambda_2)\left\{\delta F(\varphi_{D'}, m_1) - (\delta-1)\Pi(\varphi_{D'}, \alpha_1^2, m_1)\right\} - \frac{1}{2}\delta(\delta+1)F(\varphi_{D'}, m_1) - \frac{1}{2}\delta(\mu-1)E(\varphi_{D'}, m_1) + \frac{1}{2}(\delta-1)(\mu+\delta+1)\Pi(\varphi_{D'}, \alpha_1^2, m_1) + \frac{1}{2}\delta(\mu-1)\sqrt{\frac{\lambda_1(1-\lambda_1)(\mu-\lambda_1)}{\delta(\mu-1)(\delta-\lambda_1)}} \quad (2.16)$$

where $\varphi_{D'} = \sin^{-1}\sqrt{\frac{\delta(1-\lambda_1)}{(\delta-\lambda_1)}}$

Also at point A, $y = 0$, $t = 0$, therefore

$$\varphi = \pi/2$$

and

$$0 = -\lambda_1\lambda_2 K_1+(\lambda_1+\lambda_2)\left\{\delta K_1-(\delta-1)\Pi_1\right\} - \frac{1}{2}\delta(\delta+1)K_1-\frac{1}{2}\delta(\mu-1)E + \frac{1}{2}(\delta-1)(\mu+\delta+1)\Pi_1 \quad (2.17)$$

where $K_1 = F(\pi/2, m_1)$ = complete elliptic integral of the first kind.

$E = E(\pi/2, m_1)$ = complete elliptic integral of the second kind.

$\Pi_1 = \Pi(\pi/2, \alpha_1^2, m_1)$ = complete elliptic integral of third kind.

Along the floor between the two cutoffs C' FGJE, $1 < t \leq \delta$

Integrating Eq. 2.07 along the floor between the two cutoffs

$$\frac{x}{M} = -\lambda_1\lambda_2 \int_1^t \frac{dt}{\sqrt{t(t-1)(\delta-t)(\mu-t)}} + (\lambda_1+\lambda_2) \int_1^t \frac{t \cdot dt}{\sqrt{t(t-1)(\delta-t)(\mu-t)}} - \int_1^t \frac{t^2 \cdot dt}{\sqrt{t(t-1)(\delta-t)(\mu-t)}}$$

Making use of standard formulae (4, Eqs. 254.10,340 and 336)

$$\frac{x}{M} = \Big[-\lambda_1\lambda_2 V_0 + (\lambda_1+\lambda_2) V_1 - V_2 \Big]_1^t \tag{2.18}$$

where $V_0 = F(\varphi, m_1)$

$V_1 = \Pi(\varphi, \alpha_2^2, m_1)$

$$V_2 = \frac{1}{2(\alpha_2^2-1)(m_1'^2 - \alpha_2^2)} \Big[\alpha_2^2 E(\varphi, m_1') + (m_1'^2 - \alpha_2^2) V_0 + (2\alpha_2^2 m_1'^2 + 2\alpha_2^2 - \alpha_2^4 - 3m_1'^2) V_1 - \frac{\alpha_2^4 \, Sn\,u\, Cn\,u\, dn\,u}{1-\alpha_2^2 Sn^2 u} \Big]$$

$$Sn\,u = \sqrt{\frac{\delta(t-1)}{t(\delta-1)}}$$

$$Cn\,u = \sqrt{\frac{\delta - t}{t(\delta-1)}} \tag{2.19}$$

$$dn\,u = \sqrt{\frac{\mu - t}{t(\mu-1)}}$$

$$\varphi = \sin^{-1}\sqrt{\frac{\delta\,(t-1)}{t(\delta-1)}} \tag{2.20}$$

$$m_1' = \sqrt{1-m_1^2} = \sqrt{\frac{\mu(\delta-1)}{\delta(\mu-1)}} \tag{2.21}$$

$$\alpha_2^2 = \frac{\delta-1}{\delta} \tag{2.22}$$

On substituting the values of m_1', α_2^2, Sn u, Cn u, and dn u

$$V_2 = -\frac{1}{2}\,\delta(\mu-1)E(\varphi,m_1') - \frac{1}{2}\,\delta\,F(\varphi,m_1') + $$

$$+\frac{1}{2}(\mu+\delta+1)\Pi(\varphi,\alpha_2^2,m_1') + \frac{1}{2}\sqrt{\frac{\delta(\mu-1)(t-1)(\mu-t)(\delta-t)}{t}} \tag{2.23}$$

substitution of values of V_0, V_1 and V_2 in Eq. 2.18 yields

$$\frac{x}{Mg_1} = -\lambda_1\lambda_2\,F(\varphi,m_1') + (\lambda_1+\lambda_2)\Pi(\varphi,\alpha_2^2,m_1') + \frac{1}{2}\delta\,(\mu-1)E(\varphi,m_1')$$

$$+\frac{1}{2}\,\delta\,F(\varphi,m_1') - \frac{1}{2}(\mu+\delta+1)\Pi(\varphi,\alpha_2^2,m_1')$$

$$-\frac{1}{2}\sqrt{\frac{\delta(\mu-1)(t-1)(\mu-t)(\delta-t)}{t}} \tag{2.24}$$

At point E, $x = b$, $t=\delta$ and therefore $\varphi = \pi/2$,

and $$\frac{b}{Mg_1} = \left(\frac{\delta}{2}-\lambda_1\lambda_2\right)K_1' + \frac{1}{2}\,\delta(\mu-1)\,E' + \frac{1}{2}(2\lambda_1+2\lambda_2-\mu-\delta-1)\Pi_2' \tag{2.25}$$

where $K_1' = F(\pi/2\,,\,m_1')$

$E' = E(\pi/2,\; m_1')$

and $\Pi_2' = \Pi(\pi/2,\,\alpha_2^2,m_1')$

<u>Along downstream cutoff EDB $\delta \;\; t \;\; \mu$</u>

Integrating Eq. 2.07 along downstream cutoff,

$$iy = \frac{M}{i}\left[-\lambda_1\lambda_2\int_\delta^t \frac{dt}{\sqrt{(\mu-t)(t-\delta)(t-1)t}} + (\lambda_1+\lambda_2)\int_\delta^t \frac{t\,dt}{\sqrt{(\mu-t)(t-\delta)(t-1)t}}\right.$$

$$\left. - \int^t \frac{t^2.\,dt}{\sqrt{(\mu-t)(t-\delta)(t-1)t}}\right]$$

Making use of standard formulae (4, Eqs. 256.11, 340, 336)

$$\frac{-y}{Mg_1} = \Big[-\lambda_1\lambda_2 \; V_0 + (\lambda_1+\lambda_2)\big\{ (\delta-1)V_1 + V_0 \big\} - \big\{ V_0 + 2(\delta-1)V_1 + (\delta-1)^2 V_2 \big\} \Big]_\delta^t \tag{2.26}$$

where $V_0 = F(\varphi, m_1)$

$V_1 = \Pi(\varphi, \alpha_3^2, m_1)$

$$V_2 = \frac{1}{2(\alpha_3^2-1)(m_1^2-\alpha_3^2)} \Big[\alpha_3^2 E(\varphi, m_1) + (m_1^2 - \alpha_3^2) V_0 + (2\alpha_3^2 m_1^2 + 2\alpha_3^2 - \alpha_3^4 - 3m_1^2) V_1 - \frac{\alpha_3^4 \; \text{sn}\, u \; \text{cn}\, u \; \text{dn}\, u}{1 - \alpha_3^2 \, \text{sn}^2 u} \Big]$$

$$\text{Sn}\; u = \sqrt{\frac{(\mu-1)(t-\delta)}{(\mu-\delta)(t-1)}}$$

$$\text{Cn}\; u = \sqrt{\frac{(t-\mu)(1-\delta)}{(\mu-\delta)(t-1)}} \tag{2.27}$$

$$\text{dn}\; u = \sqrt{\frac{t(\delta-1)}{\delta(t-1)}}$$

$$\varphi = \text{Sin}^{-1} \sqrt{\frac{(\mu-1)(t-\delta)}{(\mu-\delta)(t-1)}} \tag{2.28}$$

$$\alpha_3^2 = \frac{\mu-\delta}{\mu-1} \tag{2.29}$$

with these values of parameters

$$V_2 = \frac{1}{2(\delta-1)^2} \Big[\delta(\mu-1) E(\varphi, m_1) - (\delta-1)(\mu-1) V_0 + (\delta-1)(\mu+\delta-3) V_1 - \delta(\mu-1) \sqrt{\frac{(t-\delta)t(\mu-t)}{\delta(t-1)(\mu-1)}} \Big] \tag{2.30}$$

On substituting for V_0, V_1 and V_2, Eq 2.26 reduces to

$$\frac{y}{Mg_1} = \Big[\lambda_1\lambda_2 F(\varphi, m_1) - (\lambda_1+\lambda_2)\left\{(\delta-1)\Pi(\varphi,\alpha_3^2, m_1) + F(\varphi, m_1)\right\}$$
$$+ \frac{1}{2}\delta(\mu-1)E(\varphi, m_1) - \left\{\frac{1}{2}(\delta-1)(\mu-1)-1\right\}F(\varphi, m_1)$$
$$+ \frac{1}{2}(\delta-1)(\mu+\delta+1)\Pi(\varphi,\alpha_3^2, m_1) - \frac{1}{2}\sqrt{\frac{\delta(\mu-1)t(t-\delta)(\mu-t)}{t-1}}\Big] \quad (2.31)$$

At point D, $t = \lambda_2$, $y = d_2$, therefore

$$\frac{d_2}{Mg_1} = \lambda_1\lambda_2 F(\varphi_D, m_1) - (\lambda_1+\lambda_2)\left\{(\delta-1)\Pi(\varphi_D,\alpha_3^2, m_1) + F(\varphi_D, m_1)\right\}$$
$$+ \frac{1}{2}\delta(\mu-1)E(\varphi_D, m_1) - \left\{\frac{1}{2}(\delta-1)(\mu-1)-1\right\}F(\varphi_D, m_1)$$
$$+ \frac{1}{2}(\delta-1)(\mu+\delta+1)\Pi(\varphi_D,\alpha_3^2, m_1) - \frac{1}{2}\sqrt{\frac{\delta\lambda_2(\mu-1)(\lambda_2-\delta)(\mu-\lambda_2)}{\lambda_2-1}} \quad (2.32)$$

$$\text{in which } \varphi_D = \sin^{-1}\sqrt{\frac{(\mu-1)(\lambda_2-\delta)}{(\mu-\delta)(\lambda_2-1)}} \quad (2.33)$$

Also at point B, $y = 0$, $t = \mu$, therefore $\varphi = \pi/2$

$$0 = \lambda_1\lambda_2 K_1 - (\lambda_1+\lambda_2)\left\{(\delta-1)\Pi_3 + K_1\right\} + \frac{1}{2}\delta(\mu-1)E - \left\{\frac{1}{2}(\delta-1)(\mu-1)-1\right\}K_1$$
$$+ \frac{1}{2}(\delta-1)(\mu+\delta+1)\Pi_3 \quad (2.34)$$

where $\Pi_3 = \Pi(\pi/2, \alpha_3^2, m_1)$

Adding Eqs. 2.17 and 2.34

$$\lambda_1 + \lambda_2 = \frac{\mu+\delta+1}{2} \quad (2.35)$$

Substituting Eq. 2.35 in Eq. 2.34

$$\lambda_1\lambda_2 = \frac{\delta}{2}\left[\mu\left(\frac{K_1-E}{K_1}\right) + \frac{E}{K_1}\right] \quad (2.36)$$

From Eqs. 2.35 and 2.36

$$\lambda_1 - \lambda_2 = -\left[\left(\frac{\mu+\delta+1}{2}\right)^2 - 2\delta\left\{\mu\left(\frac{K_1-E}{K_1}\right) + \frac{E}{K_1}\right\} \right]^{\frac{1}{2}} \quad (2.37)$$

From Eqs. 2.35 and 2.37

$$\lambda_1 = \frac{(\mu+\delta+1)}{4} - \frac{1}{2}\left[\frac{1}{4}(\mu+\delta+1)^2 - 2\delta\left\{\mu\left(\frac{K_1-E}{K_1}\right) + \frac{E}{K_1}\right\} \right]^{1/2} \quad (2.38)$$

$$\lambda_2 = \frac{(\mu+\delta+1)}{4} + \frac{1}{2}\left[\frac{1}{4}(\mu+\delta+1)^2 - 2\delta\left\{\mu\left(\frac{K_1-E}{K_1}\right) + \frac{E}{K_1}\right\} \right]^{1/2} \quad (2.39)$$

Substituting the values of λ_1 and λ_2 in Eqs. 2.16 and 2.24 and combining

$$\frac{x}{d_1} = \frac{\left(\frac{E}{K_1}-1\right)F(\varphi, m_1') + E(\varphi, m_1') - \sqrt{\frac{(t-1)(\mu-t)(\delta-t)}{t\,\delta(\mu-1)}}}{\frac{E}{K_1}F(\varphi_{D'}, m_1) - E(\varphi_{D'}, m_1) + \sqrt{\frac{\lambda_1(1-\lambda_1)(\mu-\lambda_1)}{\delta(\mu-1)(\delta-\lambda_1)}}} \quad (2.40)$$

where $\varphi = \sin^{-1}\sqrt{\frac{(t-1)\delta}{t(\delta-1)}}$

$$\varphi_{D'} = \sin^{-1}\sqrt{\frac{\delta(1-\lambda_1)}{(\delta-\lambda_1)}}$$

From Eqs. 2.16 and 2.25 after substituting the values of λ_1 and λ_2

$$\frac{b}{d_1} = \frac{\left(\frac{E}{K_1}-1\right)K_1' + E'}{\frac{E}{K_1}F(\varphi_{D'}, m_1) - E(\varphi_{D'}, m_1) + \sqrt{\frac{\lambda_1(1-\lambda_1)(\mu-\lambda_1)}{\delta(\mu-1)(\delta-\lambda_1)}}} \quad (2.41)$$

Similarly from Eq. 2.16 and 2.32 after substituting the values of λ_1 and λ_2.

$$\frac{d_2}{d_1} = \frac{E(\varphi_D, m_1) - \frac{E}{K_1} F(\varphi_D, m_1) - \sqrt{\frac{\lambda_2(\lambda_2 - \delta)(\mu - \lambda_2)}{\delta(\mu - 1)(\lambda_2 - 1)}}}{\frac{E}{K_1} F(\varphi_D', m_1) - E(\varphi_D', m_1) + \sqrt{\frac{\lambda_1(1 - \lambda_1)(\mu - \lambda_1)}{\delta(\mu - 1)(\delta - \lambda_1)}}} \tag{2.42}$$

At point F, $x = b_1 - f$, $t = \sigma$, therefore from Eq. 2.40

$$\frac{b_1 - f}{d_1} = \frac{(\frac{E}{K_1} - 1) F(\varphi_F, m_1') + E(\varphi_F, m_1') - \sqrt{\frac{(\sigma - 1)(\mu - \sigma)(\delta - \sigma)}{\delta \sigma (\mu - 1)}}}{\frac{E}{K_1} F(\varphi_D', m_1) - E(\varphi_D', m_1) + \sqrt{\frac{\lambda_1(1 - \lambda_1)(\mu - \lambda_1)}{\delta(\mu - 1)(\delta - \lambda_1)}}} \tag{2.43}$$

in which $\varphi_F = \sin^{-1}\sqrt{\frac{\delta(\sigma - 1)}{\sigma(\delta - 1)}}$

Similarly at point G, $x = b_1$, $t = \rho$, therefore from Eq. 2.40

$$\frac{b_1}{d_1} = \frac{(\frac{E}{K_1} - 1) F(\varphi_G, m_1') + E(\varphi_G, m_1') - \sqrt{\frac{(\rho - 1)(\mu - \rho)(\delta - \rho)}{\delta \rho (\mu - 1)}}}{\frac{E}{K_1} F(\varphi_D', m_1) - E(\varphi_D', m_1) + \sqrt{\frac{\lambda_1(1 - \lambda_1)(\mu - \lambda_1)}{\delta(\mu - 1)(\delta - \lambda_1)}}} \tag{2.44}$$

in which $\varphi_G = \sin^{-1}\sqrt{\frac{\delta(\rho - 1)}{\rho(\delta - 1)}}$

Eqs. 2.38, 2.39, 2.41 to 2.44 enable to determine the values of λ_1, λ_2, δ, μ, σ and ρ respectively.

2.2.2 Second Operation $w = f_1(\zeta)$

In this operation the flow field in w-plane [Fig. 2.1(d)] is transformed onto the semi-infinite ζ-plane [Fig. 2.1 (c)]. The points B, A and F are mapped in ζ-plane at $\pm\infty$, 0 and + 1 respectively. The points M, G and J lie at $-p$, a and p_1 respectively.

The transformation of polygon MAFGJBM in w-plane onto ζ-plane is giben by

$$\frac{dw}{d\zeta} = N \frac{(p_1 - \zeta)}{(\zeta + p)\sqrt{\zeta(1-\zeta)(a-\zeta)}} \tag{2.45}$$

Along the portion AD' C' F, $0<\zeta\leq 1$.

Integrating Eq. 2.45 from 0 to ζ in this region.

$$\Phi + kH_2 = N\int_0^{\zeta} \frac{(p_1 - \zeta)\,d\zeta}{(\zeta + p)\sqrt{\zeta(1-\zeta)(a-\zeta)}}$$

Making use of standard formulae (4, Eqs. 233.19 and 340.01)

$$\Phi + kH_2 = \frac{Ng_2}{p}\left[(p_1+p)\,\Pi(\varphi, \alpha_4^2, m_2) - pF(\varphi, m_2)\right] \tag{2.46}$$

$$\text{where } \varphi = \sin^{-1}\sqrt{\zeta} \tag{2.47}$$

$$m_2 = \frac{1}{\sqrt{a}} \tag{2.48}$$

$$\alpha_4^2 = -\frac{1}{p} \tag{2.49}$$

$$g_2 = \frac{2}{\sqrt{a}} \tag{2.50}$$

At point, F, $\Phi = -kH_1$ and $\zeta = 1$, therefore

$$-kH_1 + kH_2 = \frac{Ng_2}{p}\left[(p_1+p)\Pi_4 - pK_2\right] \tag{2.51}$$

in which $\Pi_4 = \Pi(\pi/2, \alpha_4^2, m_2)$

$K_2 = F(\pi/2, m_2)$

From Eq. 2.51

$$Ng_2 = \frac{pk(H_2 - H_1)}{(p_1+p)\Pi_4 - pK_2} \tag{2.52}$$

Along the portion F G, $1 < \zeta \leq a$.

Integrating Eq. 2.45 in the portion from 1 to ζ

$$i\psi = N \int_1^{\zeta} \frac{(p_1 - \zeta)\, d\zeta}{(\zeta + p)\sqrt{\zeta(1-\zeta)(a-\zeta)}}$$

$$= \frac{N}{i} \int_1^{\zeta} \frac{(p_1 - \zeta)\, d\zeta}{(\zeta + p)\sqrt{\zeta(\zeta-1)(a-\zeta)}}$$

Making use of standard formulae (4, Eqs. 235.18 and 340.01)

$$i\psi = \frac{-Ng_2}{i}\left[\frac{p_1+p}{1+p}\Pi(\varphi, \alpha_5^2, m_2') - \frac{p_1}{p} F(\varphi, m_2')\right] \tag{2.53}$$

where

$$\varphi = \sin^{-1}\sqrt{\frac{a(\zeta - 1)}{\zeta(a-1)}} \tag{2.54}$$

$$\alpha_5^2 = \frac{(a-1)p}{a(1+p)} \tag{2.55}$$

$$m_2' = \sqrt{1 - m_2^2} = \sqrt{\frac{a-1}{a}} \tag{2.56}$$

At point G, $\psi = q$ and $\zeta = a$, therefore 2.53 reduces to

$$q = Ng_2\left[\frac{p_1 + p}{1+p}\Pi_5' - \frac{p_1}{p} K_2'\right] \tag{2.57}$$

where, q is the discharge through the filter,

$\Pi_5' = \Pi(\pi/2, \alpha_5^2, m_2')$

and $K_2' = F(\pi/2, m_2')$

Substituting the value of Ng_2 from Eq. 2.52 in Eq. 2.57

$$\frac{q}{k(H_2 - H_1)} = \frac{p(p_1+p)\Pi_5' - p_1(1+p)K_2'}{(1+p)\left[(p_1+p)\Pi_4 - pK_2\right]} \tag{2.58}$$

Along the portion G J E D B, $a < \zeta \leq \infty$.

Integrating Eq. 2.45 in this portion from a to ζ

$$\Phi + kH_1 = N\int_a^{\zeta} \frac{(p_1 - \zeta)\ d\zeta}{(\zeta+p)\sqrt{\zeta(\zeta-1)(\zeta-a)}} \tag{2.59}$$

Making use of standard formulae (4, Eqs. 237.18 and 340.01)

$$\Phi + kH_1 = Ng_2\left[\frac{p_1-1}{1+p} F(\varphi, m_2) - \frac{(a-1)(p_1+p)}{(1+p)(a+p)} \Pi(\varphi, \alpha_6^2, m_2)\right] \tag{2.60}$$

in which $\varphi = \sin^{-1}\sqrt{\frac{\zeta - a}{\zeta - 1}}$ (2.61)

$$\alpha_6^2 = \frac{1+p}{a+p} \tag{2.62}$$

At point B, $\Phi = 0$, $\zeta = \infty$ and $\varphi = \pi/2$, therefore

$$kH_1 = Ng_2\left[\frac{p_1 - 1}{1 + p} K_2 - \frac{(a-1)(p_1 + p)}{(1+p)(a + p)} \Pi_6\right] \tag{2.63}$$

where $\Pi_6 = \Pi(\pi/2, \alpha_6^2, m_2)$

From Eqs. 2.52 and 2.63

$$\frac{H_1}{H_2 - H_1} = \frac{p\left[(p_1-1)(a+p)K_2 - (a-1)(p_1+p)\Pi_6\right]}{(1+p)(a+p)\left[(p_1 + p)\Pi_4 - pK_2\right]} \tag{2.64}$$

The value of p_1 can be determined from Eq. 2.64

$$\frac{p_1}{p} = \frac{(a+p)K_2 - C_1 r K_2 + C_1 r \Pi_4 + p(a-1)\Pi_6}{p(a+p)K_2 - C_1 r \Pi_4 - p(a-1)\Pi_6} \tag{2.65}$$

in which $r = \frac{H_1}{H_2 - H_1}$ (2.66)

and $C_1 = (1+p)(a+p)$ (2.67)

2.2.3 Third Operation $\zeta = F_2(t)$

The transformation of ζ-plane onto the t-plane is obtained with the help of the following bilinear transformation

$$\zeta = \frac{t(\mu - \sigma)}{\sigma(\mu - t)} \qquad (2.68)$$

or

$$t = \frac{\zeta \mu \sigma}{\zeta\sigma + \mu - \sigma} \qquad (2.69)$$

At point G, $\zeta = a$ and $t = \rho$, therefore

$$a = \frac{\rho(\mu - \sigma)}{\sigma(\mu - \rho)} \qquad (2.70)$$

At point M, $\zeta = -p$ and $t = \infty$, therefore

$$p = \frac{\mu - \sigma}{\sigma} \qquad (2.71)$$

2.3 UPLIFT PRESSURES

2.3.1 Uplift Pressures below upstream floor, AD' C' F :

The uplift pressures below upstream floor can be determined from Eq. 2.46 after substituting the value of N_{g2}

$$\frac{\Phi + kH_2}{k(H_2 - H_1)} = \frac{(p_1+p)\Pi(\varphi, \alpha_4^2, m_2) - pF(\varphi, m_2)}{(p_1+p)\Pi_4 - pK_2} \qquad (2.72)$$

or

$$\frac{\Phi}{kH_2} = \frac{p\left[K_2 - \frac{1}{1+r}F(\varphi, m_2)\right] - (p_1+p)\left[\Pi_4 - \frac{1}{1+r}\Pi(\varphi, \alpha_4^2, m_2)\right]}{(p_1+p)\Pi_4 - pK_2} \qquad (2.73)$$

where $\varphi = \sin^{-1}\sqrt{\zeta}$

and r is given by Eq. 2.66 (2.74)

In case the release in the filter is provided at the downstream water level, $H_1 = 0$, therefore $r = 0$ and Eq. 2.73 reduces to

$$\frac{\Phi}{kH_2} = \frac{p\left[K_2 - F(\varphi, m_2)\right] - (p_1+p)\left[\Pi_4 - \Pi(\varphi, \alpha_4^2, m_2)\right]}{(p_1+p)\Pi_4 - pK_2} \qquad (2.75)$$

The values of p, p_1 , m_2 , φ and ζ are given by Eqs. 2.71, 2.65, 2.48, 2.74 and 2.68 respectively.

At point D', $t = \lambda_1$, therefore from Eq. 2.68

$$\zeta_{D'} = \frac{\lambda_1(\mu - \sigma)}{\sigma(\mu - \lambda_1)} \qquad (2.76)$$

and so from Eq. 2.74

$$\varphi_{D'} = \sin^{-1}\sqrt{\frac{\lambda_1(\mu - \sigma)}{\sigma(\mu - \lambda_1)}} \qquad (2.77)$$

and from Eq. 2.73

$$\frac{\Phi_{D'}}{kH_2} = \frac{p\left[K_2 - F(\varphi_{D'}, m_2)\right] - (p_1+p)\left[\Pi_4 - \Pi(\varphi_{D'}, \alpha_4^2, m_2)\right]}{(p_1+p)\Pi_4 - pK_2}$$

At point C' , $t = 1$, so from Eq 2.68

$$\zeta_{C'} = \frac{\mu - \sigma}{\sigma(\mu - 1)} \qquad (2.79)$$

$$\varphi_{C'} = \sin^{-1}\sqrt{\frac{\mu - \sigma}{\sigma(\mu - 1)}} \qquad (2.80)$$

$$\frac{\Phi_{C'}}{kH_2} = \frac{p\left[K_2 - F(\varphi_{C'}, m_2)\right] - (p_1+p)\left[\Pi_4 - \Pi(\varphi_{C'}, \alpha_4^2, m_2)\right]}{(p_1+p)\Pi_4 - pK_2} \qquad (2.81)$$

2.32 Uplift pressures below downstream floor, G J E D C:

The uplift pressures below the downstream floor can be determined from Eq. 2.60 after substituting the value of Ng_2

$$\frac{\Phi + kH_1}{k(H_2 - H_1)} = \frac{p}{(1+p)(a+p)} \frac{(p_1-1)(a+p)F(\varphi, m_2) - (a-1)(p_1+p)\Pi(\varphi, \alpha_6^2, m_2)}{(p_1+p)\Pi_4 - pK_2} \quad (2.82)$$

where $$\varphi = \sin^{-1}\sqrt{\frac{\zeta - a}{\zeta - 1}} \quad (2.83$$

In case the release in the filter is provided at the downstream water level, $H_1 = 0$, Eq 2.82 reduces to

$$\frac{\Phi}{kH_2} = \frac{p}{(1+p)(a+p)} \frac{(p_1-1)(a+p)F(\varphi, m_2) - (a-1)(p_1+p)\Pi(\varphi, \alpha_6^2, m_2)}{(p_1+p)\Pi_4 - pK_2} \quad (2.84$$

The values of p, p_1, m_2, φ and ζ are given by Eqs. 2.71, 2.65, 2.48, 2.83 and 2.68 respectively.

At point E, $t = \delta$

$$\zeta_E = \frac{\delta(\mu - \sigma)}{\sigma(\mu - \delta)} \quad (2.85$$

$$\varphi_E = \sin^{-1}\sqrt{\frac{\zeta_E - a}{\zeta_E - 1}} \quad (2.86$$

$$\frac{\Phi_E}{kH_2} = \frac{p}{(1+p)(a+p)} \frac{(p_1-1)(a+p)F(\varphi_E, m_2) - (a-1)(p_1+p)\Pi(\varphi_E, \alpha_6^2, m_2)}{(p_1+p)\Pi_4 - pK_2} \quad (2.87$$

At point D, $t = \lambda_2$

$$\zeta_D = \frac{\lambda_2(\mu - \sigma)}{\sigma(\mu - \lambda_2)} \quad (2.88$$

$$\varphi_D = \sin^{-1}\sqrt{\frac{\zeta_D - a}{\zeta_D - 1}} \quad (2.89$$

$$\frac{\Phi_D}{kH_2} = \frac{p}{(1+p)(a+p)} \frac{(p_1-1)(a+p)F(\varphi_D, m_2) - (a-1)(p_1+p)\Pi(\varphi_D, \alpha_6^2, m_2)}{(p_1+p)\Pi_4 - pK_2} \quad (2.90)$$

The maximum pressure below the downstream floor occurs at point J, where $\zeta = p_1$. The pressure at J can be determined from Eq 2.84 after substituting the value of φ_J and other parameters. The value of φ_J is determined from Eq. 2.83

$$\varphi_J = \sin^{-1}\sqrt{\frac{p_1 - a}{p_1 - 1}} \tag{2.91}$$

$$\frac{\Phi_J}{kH_2} = \frac{p}{(1+p)(a+p)} \frac{(p_1-1)(a+p)F(\varphi_J, m_2) - (a-1)(p_1+p)\Pi(\varphi_J, \alpha_6^2, m_2)}{(p_1+p)\Pi_4 - pK_2} \tag{2.9}$$

In case the water level at the filter is increased above the downstream bed the quantity of seepage into the intermediate filter decreases and the point of maximum pressure J moves towards G, and the two may ultimately coincide. At this stage the maximum uplift pressure is equal to the head in the filter. If the head in the filter increases seepage takes place from the intermediate filter to the downstream bed, the maximum uplift pressure continuing to be the same.

The critical value of H_1/H_2 at which J coincides with G is obtained from Eq 2.64 by substituting $p_1 = a$

$$\frac{H_1}{H_2} = \frac{p(a-1)\left[K_2 - \Pi_6\right]}{p(a-p-2)K_2 + (1+p)(a+p)\Pi_4 - p(a-1)\Pi_6} \tag{2.93}$$

2.4 RESULTS.

The equations derived above have been used for computations of uplift pressures. These calculations involve the use of elliptic functions of all the three kinds which were computed on the digital computer (Appendix II) because tables for the required range were not available. In order to facilitate the use of above equations for finding uplift pressures at key points, values have been computed for different combinations of variables involved and plotted in the form of curves. The analysis has been made for a floor with two end cutoffs and intermediate filter. The actual profile of a structure mostly conforms to this geometry. It is, therefore, not necessary to break up the actual profile into elementry profiles, and the results obtained have can be directly used for practical design. As will be seen interpolation of values is very easy and accurate and can be safely adopted for design.

The uplift pressures at key points were calculated for assumed values of δ and μ and various values of σ and ρ so as to cover all the possible dimensions of a structure. The curves cover the values $\frac{b}{d_1}$ = 5,10,15 and 20, $\frac{d_2}{d_1}$ = 1, 2 and 3, f/d_1 from 0.0 to 1.0 anywhere between the two cutoffs. The uplift pressures at D, E, C', D' and J are given. The location of J is also given.

The uplift pressures at D, E, C', D' and J have been plotted in Figs. 2.2, 2.3, 2.4, 2.5, and 2.6. The location of point J has been given in Fig. 2.7.

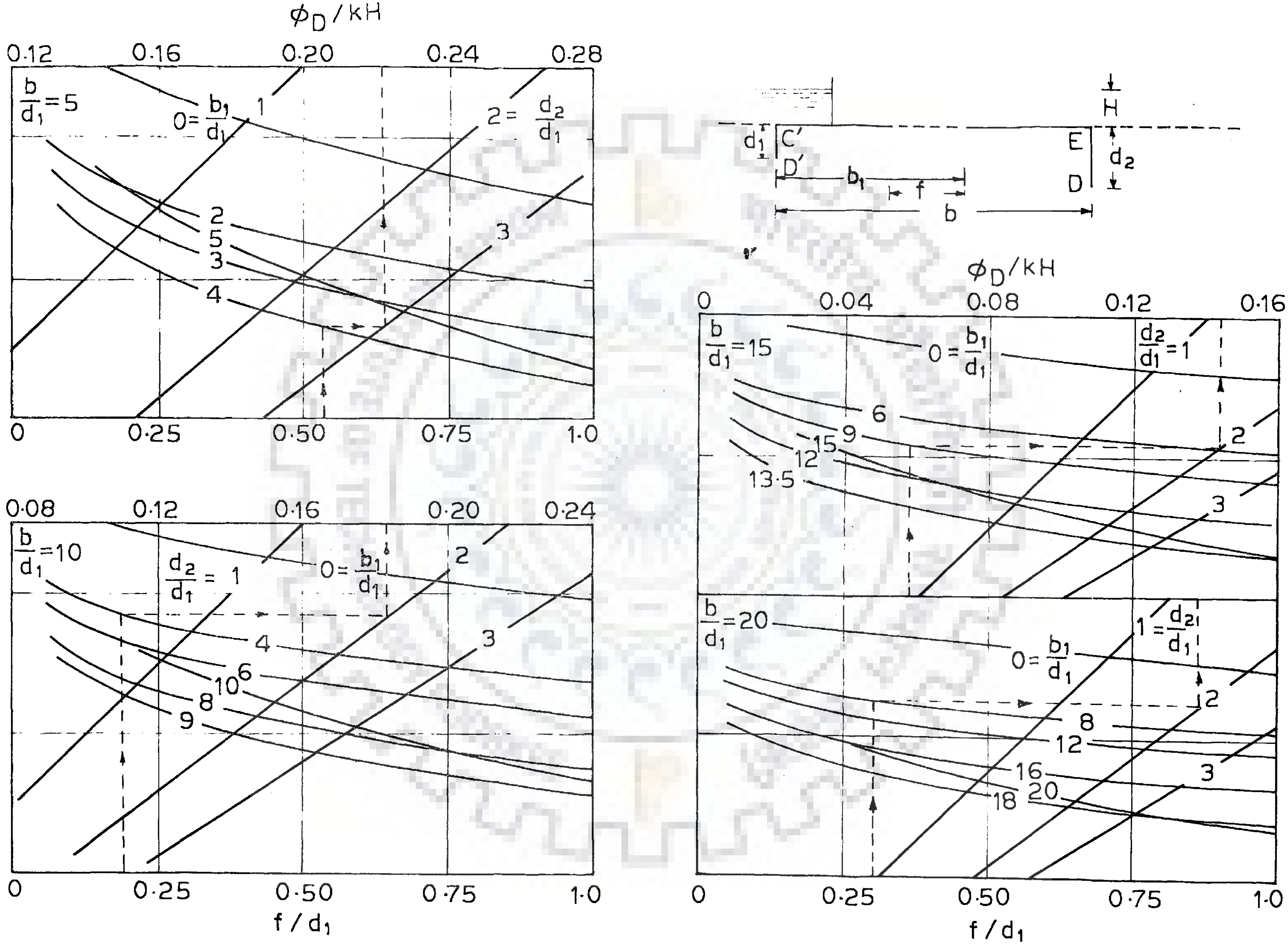


FIG.2.2--UPLIFT PRESSURE AT D

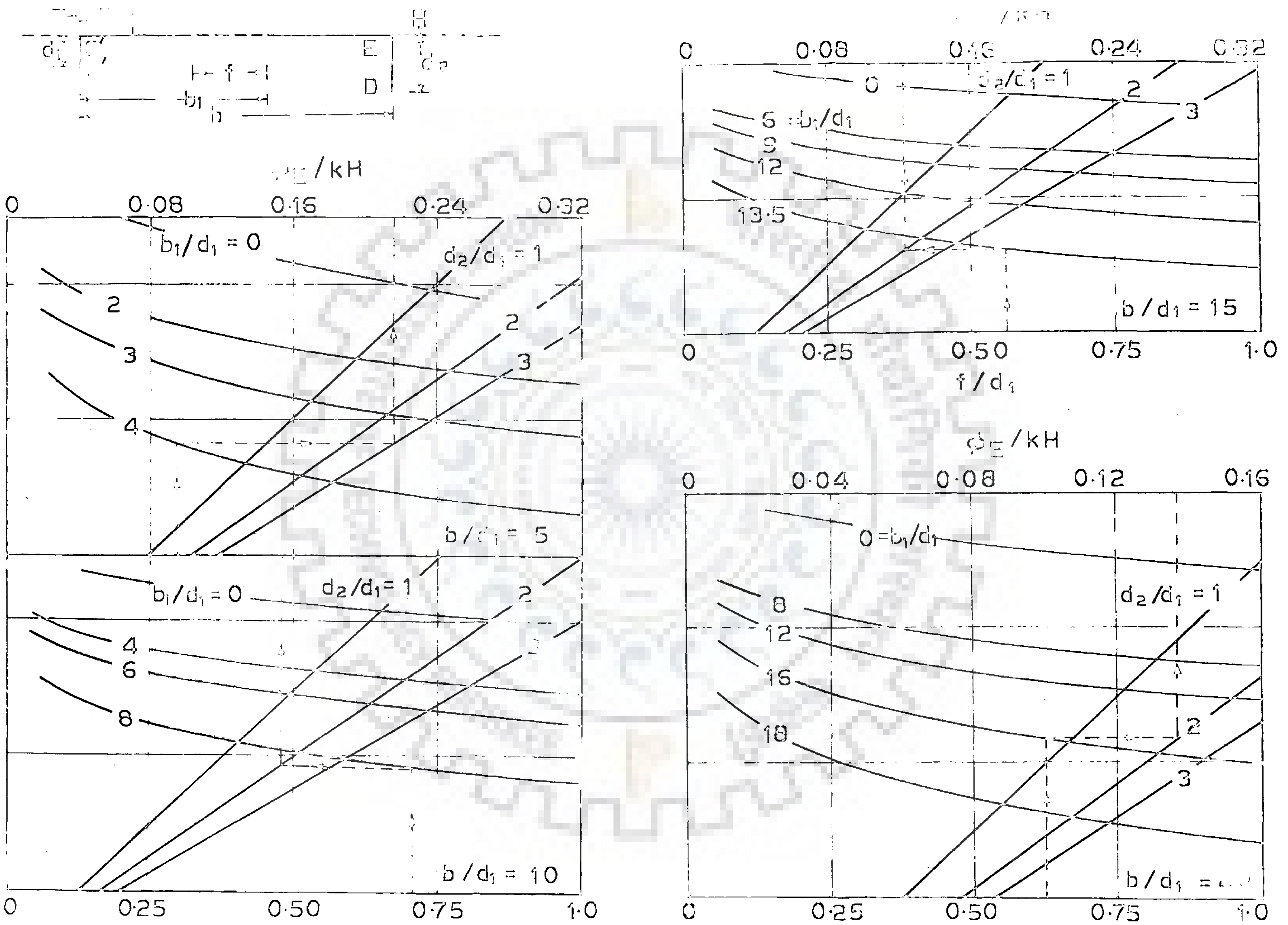
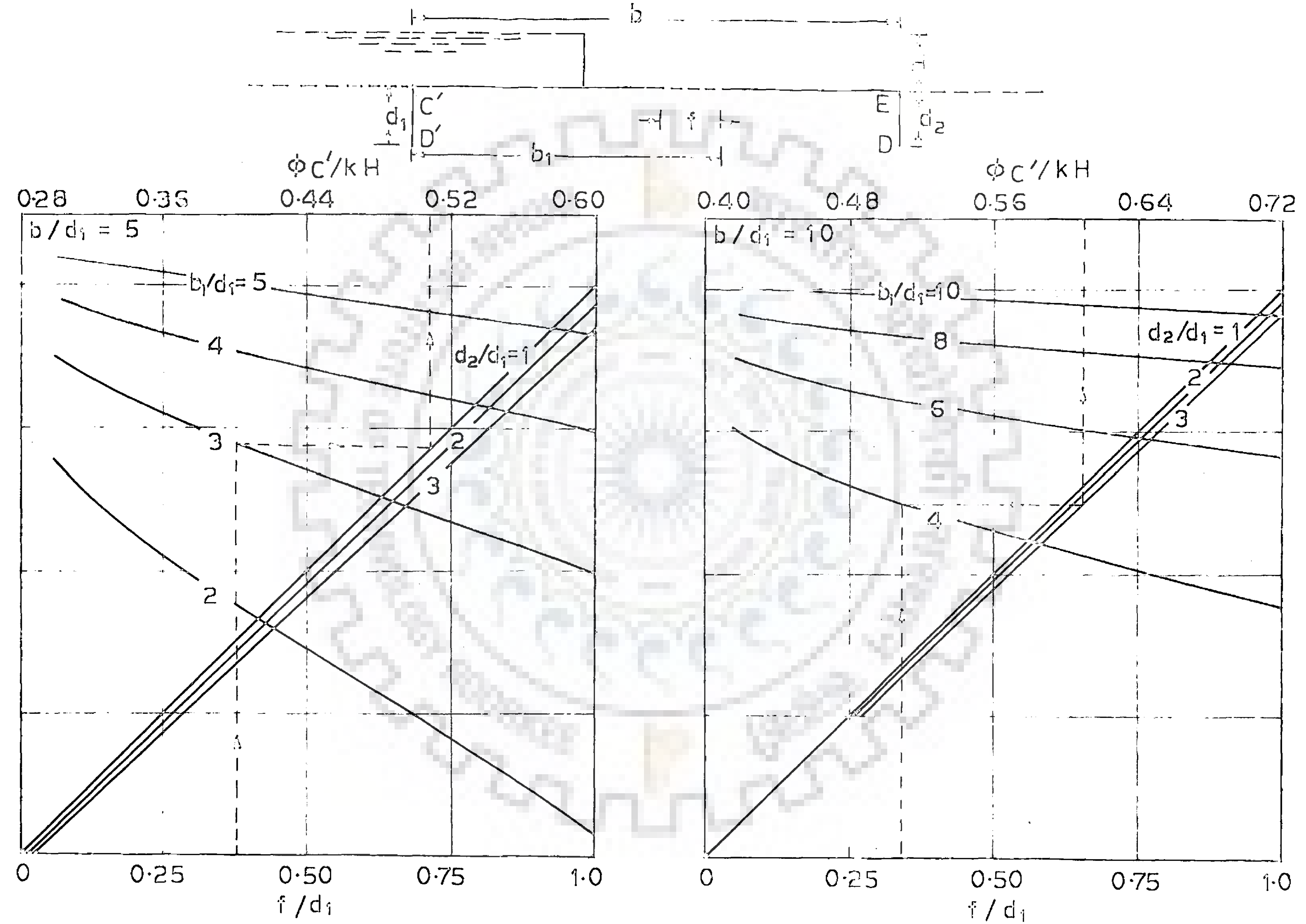
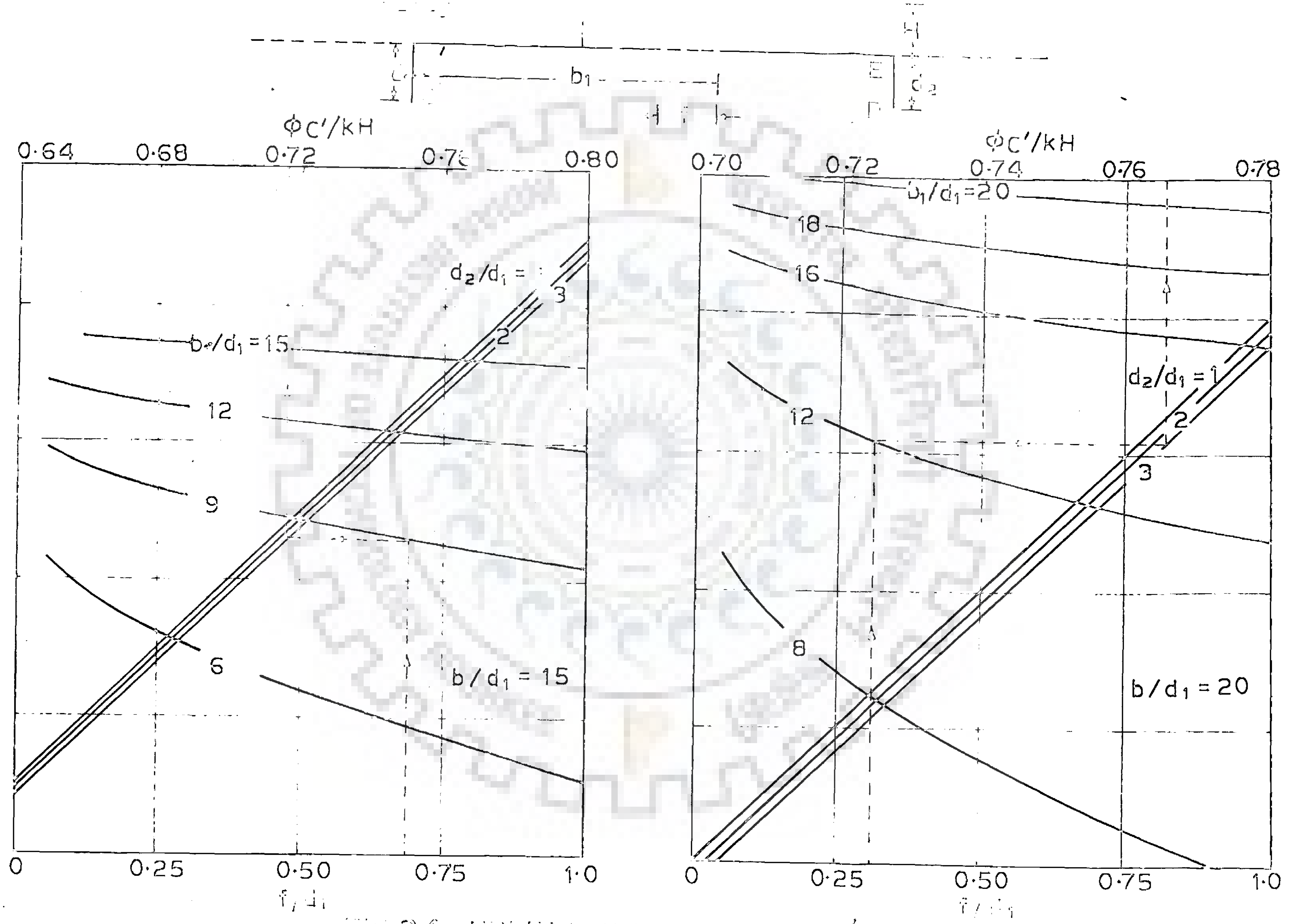


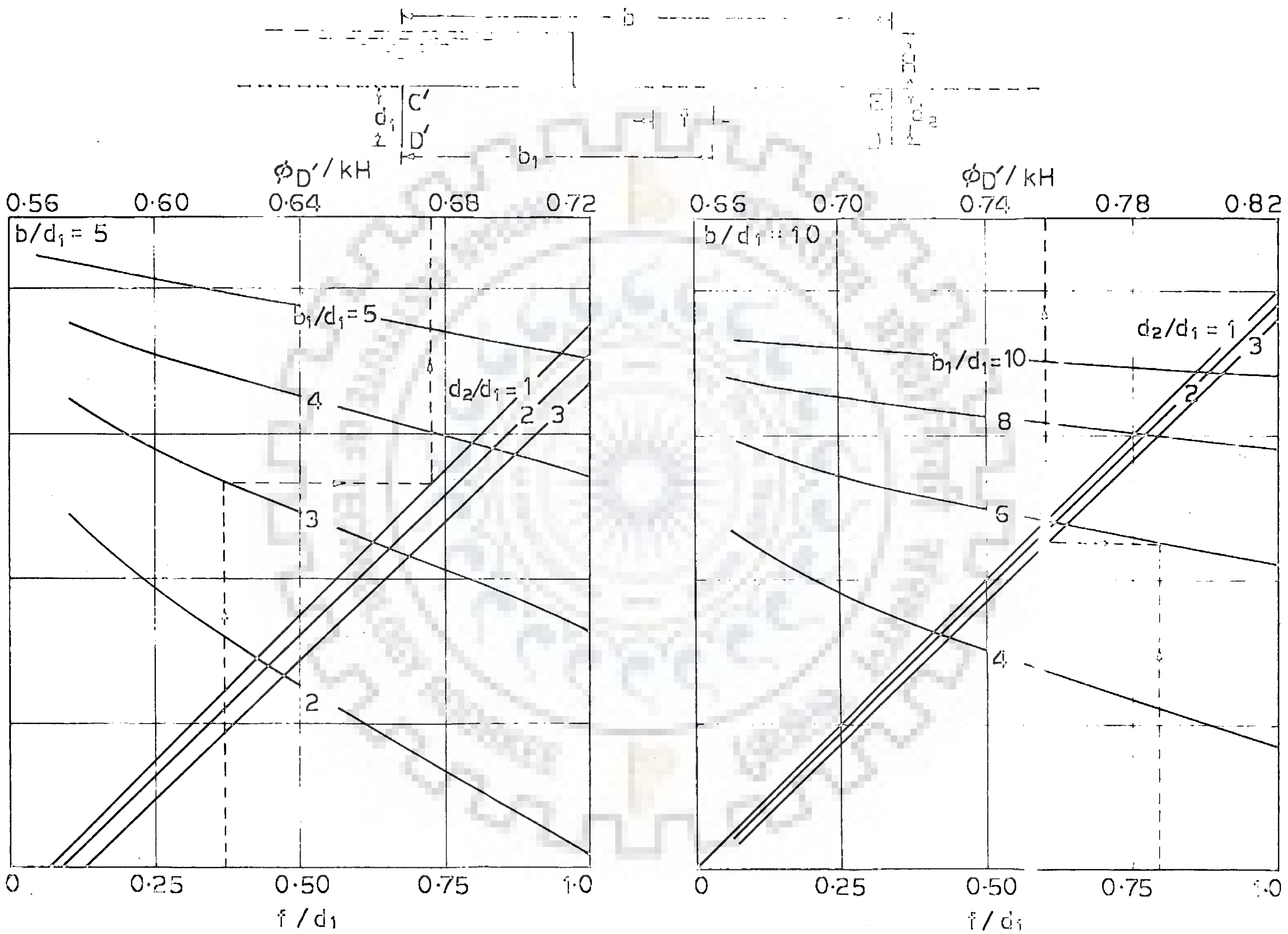
FIG. 2.3 – UPLIFT PRESSURE AT E



b
d1
C′
D′
b1
f
E
D
d2
ΦC′/kH
0·28
0·36
0·44
0·52
0·60
b / d1 = 5
b1/d1 = 5
4
3
2
d2/d1 = 1
2
3
0
0·25
0·50
0·75
1·0
f / d1
ΦC′/kH
0·40
0·48
0·56
0·64
0·72
b / d1 = 10
b1/d1 = 10
8
6
4
d2/d1 = 1
2
3
0
0·25
0·50
0·75
1·0
f / d1



b1
d2
φc'/kH
0·64
0·68
0·72
0·80
d2/d1 = 1
3
2
b1/d1 = 15
12
9
6
b/d1 = 15
0
0·25
0·50
0·75
1·0
0·70
0·74
0·76
0·78
b1/d1 = 20
18
16
8
b/d1 = 20


b
b₁
d₁
C′
D′
d₂
f
$\phi_{D'}/kH$
0·56
0·60
0·64
0·68
0·72
$b/d_1 = 5$
$b_1/d_1 = 5$
4
3
2
$d_2/d_1 = 1$
2
3
0
0·25
0·50
0·75
1·0
f/d_1
$\phi_{D'}/kH$
0·66
0·70
0·74
0·78
0·82
$b/d_1 = 10$
$b_1/d_1 = 10$
8
6
4
$d_2/d_1 = 1$
2
3
0
0·25
0·50
0·75
1·0
f/d_1

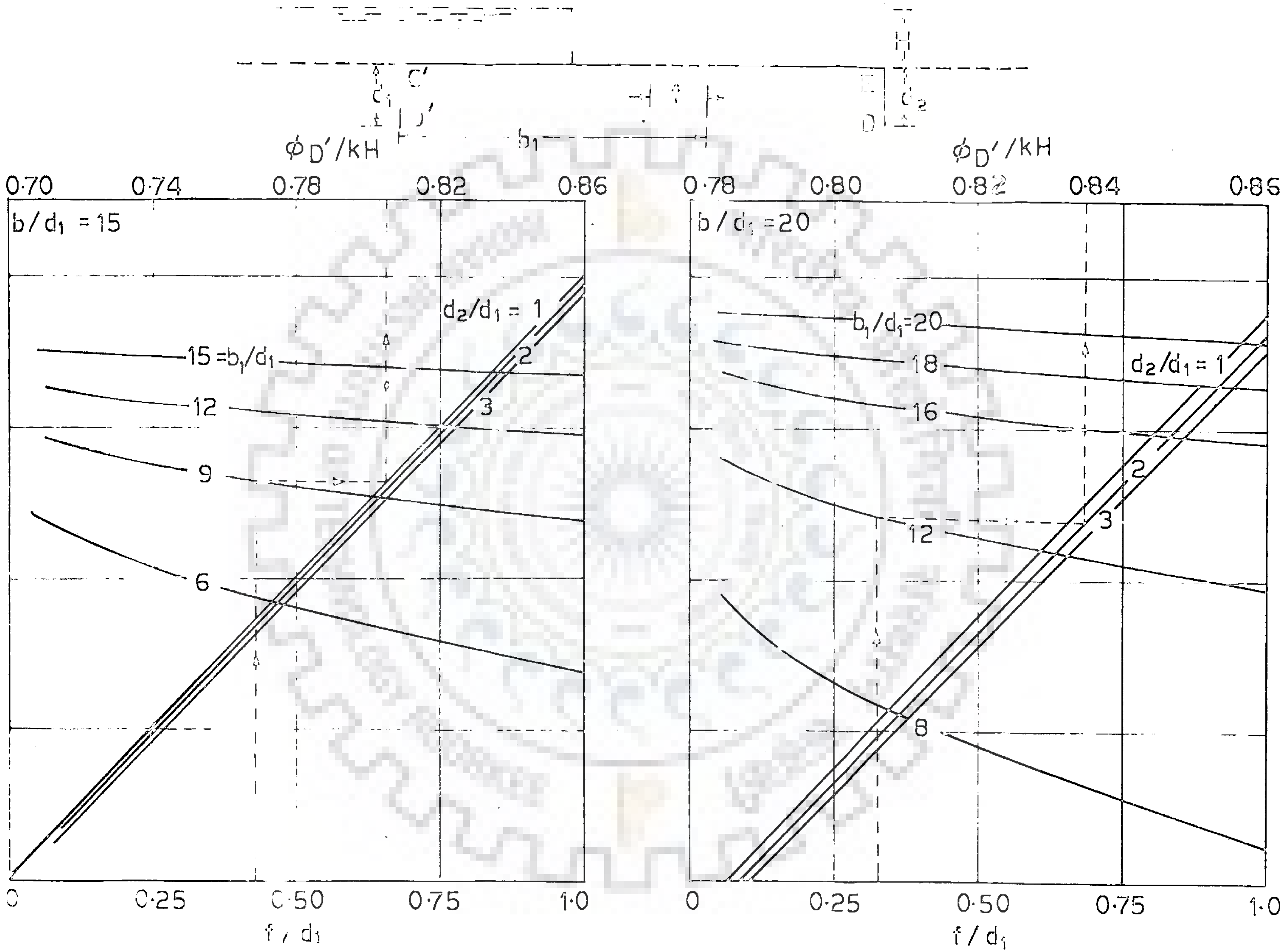


FIG. 2.8 UPLIFT PRESSURE AT D′

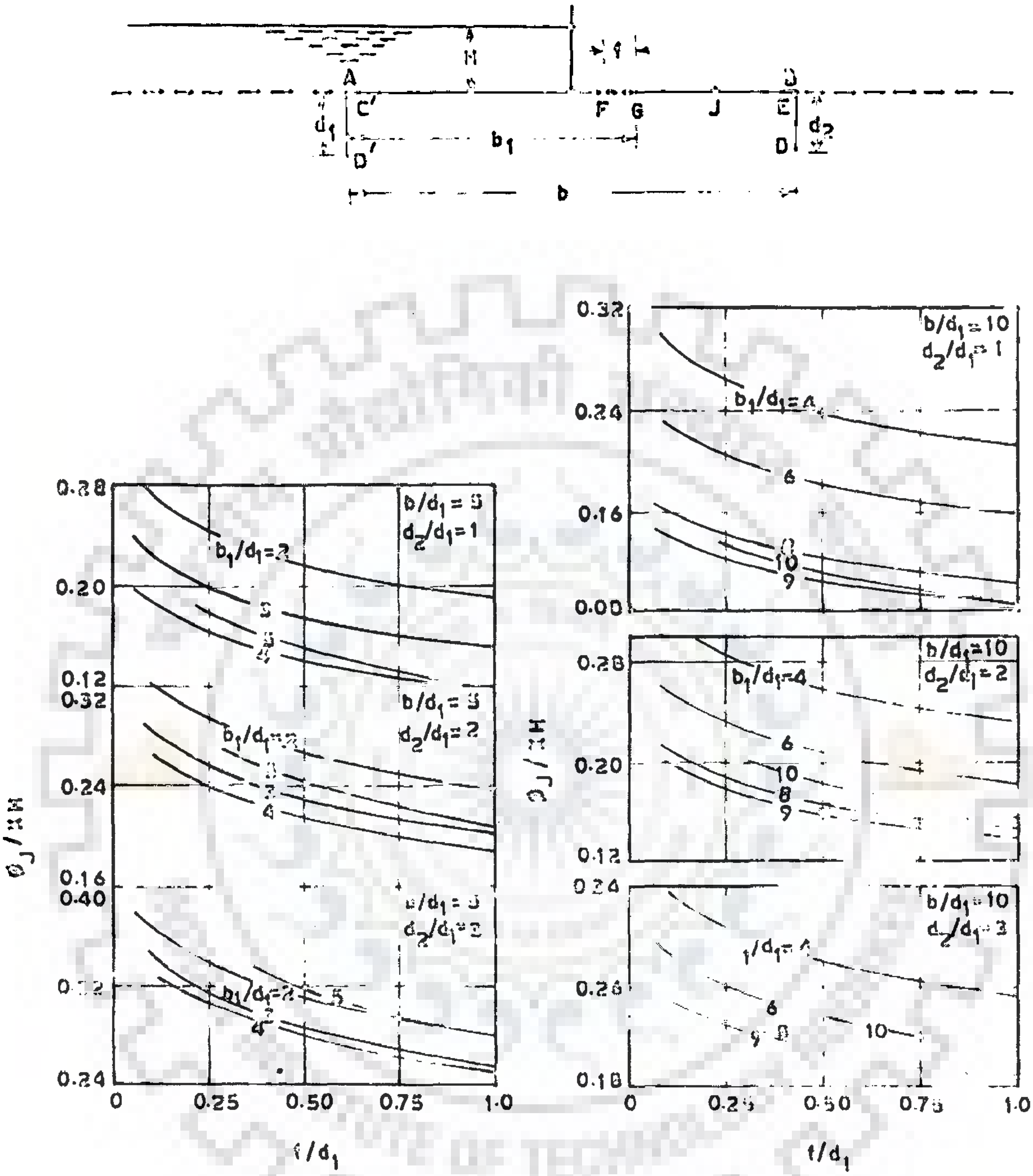


FIG. 2.6 – UPLIFT PRESSURE AT J

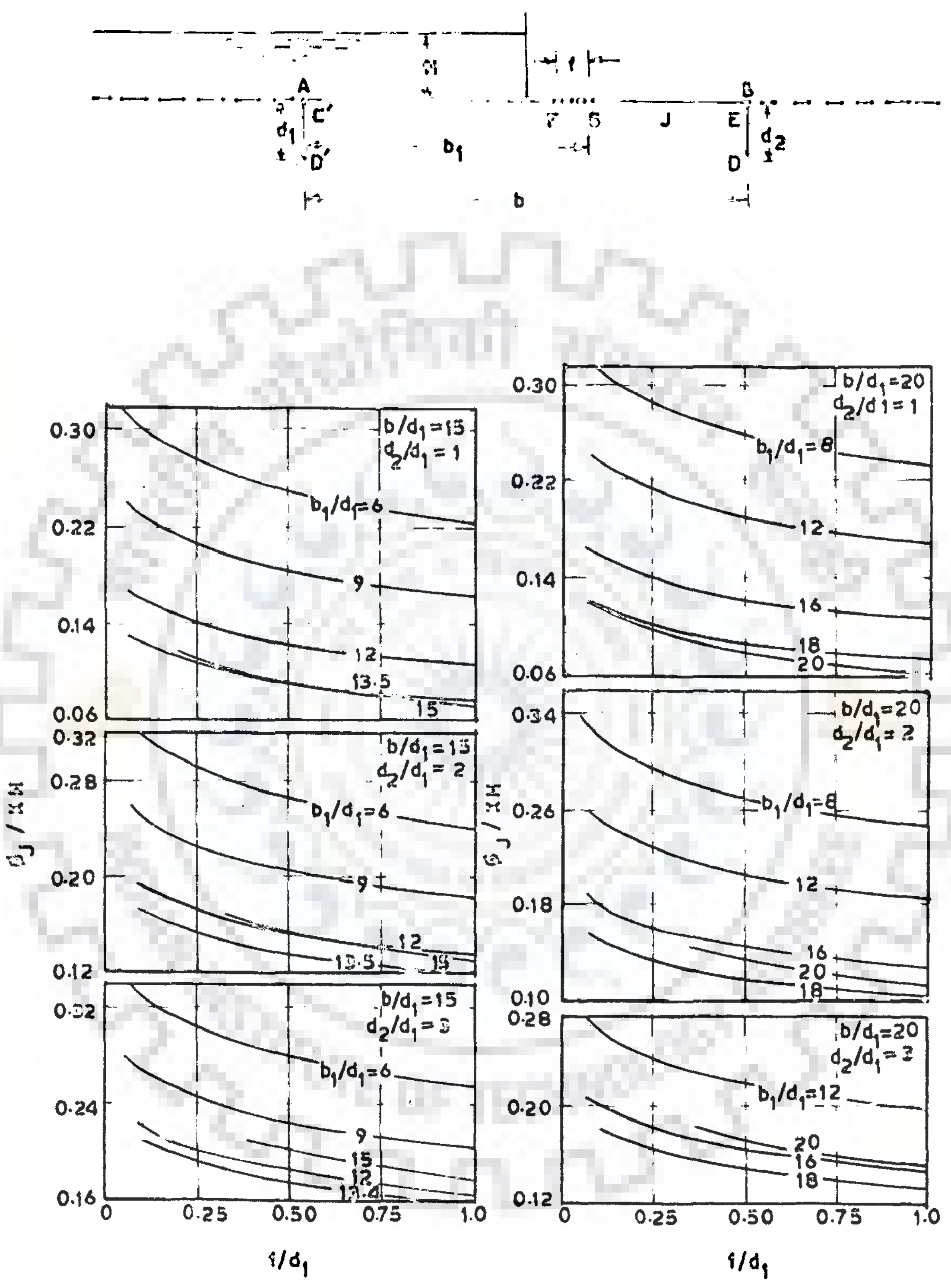


FIG. 2.6—UPLIFT PRESSURE AT J

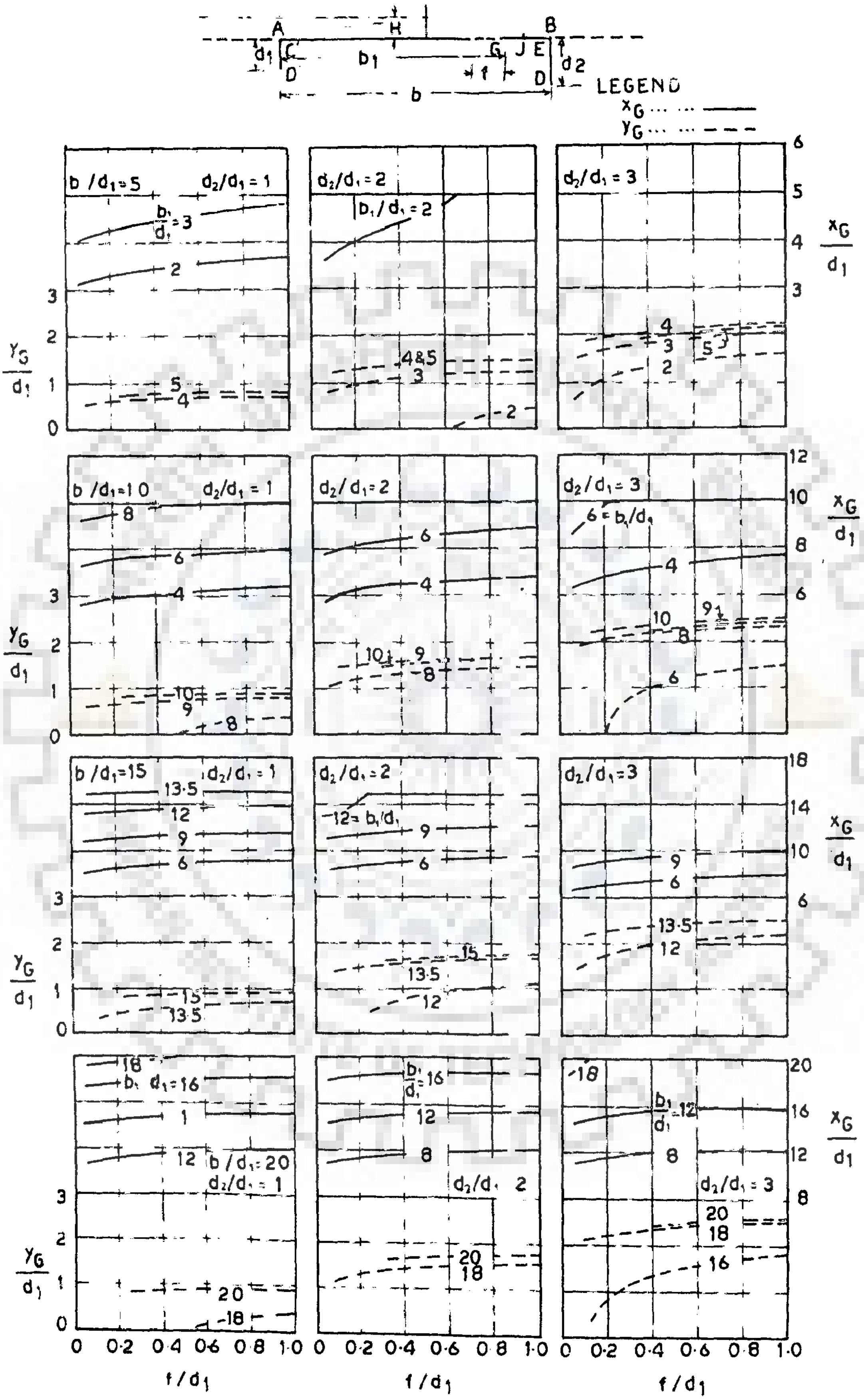


FIG. 2·7 – LOCATION OF J

CHAPTER III

THEORETICAL SOLUTION FOR DEEP FILTER DRAIN

CHAPTER III

THEORETICAL SOLUTION FOR DEEP FILTER DRAIN

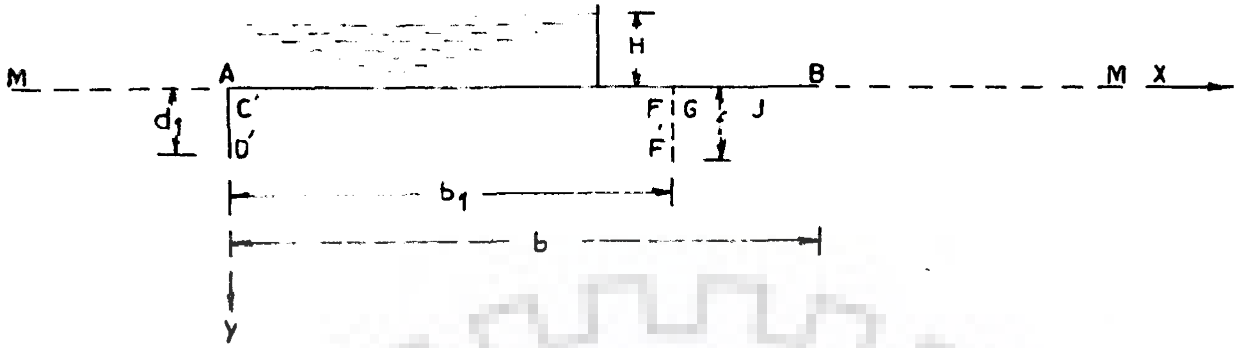
3.1 LAYOUT AND BOUNDARY CONDITIONS.

Consider an impervious floor A B of length b founded on a semi-infinite homogeneous permeable soil structure with a single cutoff. A deep filter of depth f has been provided at a distance $\bar{b}_1$ from the cutoff. Two cases have been considered:

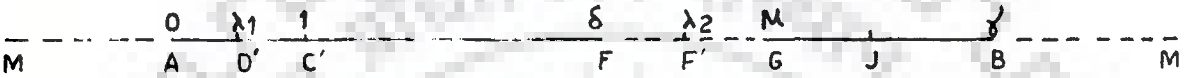
(i) cutoff of depth d_1 at the upstream end [Fig. 3.1(a)]

(ii) cutoff of depth d_2 at the downstream end [Fig. 3.2(a)]. On either side of the floor, pervious bed extends upto infinity. Along the downstream bed B M we assume the potential $\Phi = 0$. Then along the upstream bed A M the potential is $\Phi = -kH_2$ were H_2 is the upstream head measured above the downstream water level.

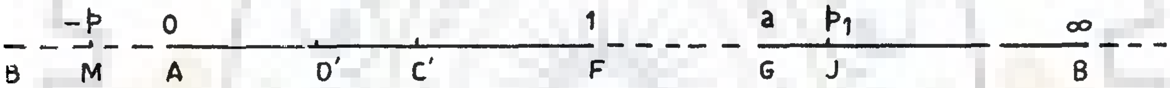
The foundation profile A D' C' F G J B forms the inner boundary in case of floor with upstream cutoff (Fig. 3.1) and A F G J E D B forms the inner boundary in case of floor with downstream cutoff (Fig. 3.2). This inner boundary ~~boundary~~ represents stream-line $\psi = 0$ upto filter drain i.e. between A and F and streamline $\psi = q$ between G and B where q is the discharge per unit length normal to the direction of flow, drained through the filter drain. Either face of the filter F F' or F' G, is an equi-potential with $\Phi = -k\bar{H}_1$, where $\bar{H}_1$ is the head in filter measured above the downstream water level. T-he potential along the floor downstream of the filter drain is maximum at some point J because the flow along the floor upstream of this point is towards the filter whereas on the other side it is towards downstream. As already indicated in chapter II, the location of the point J would



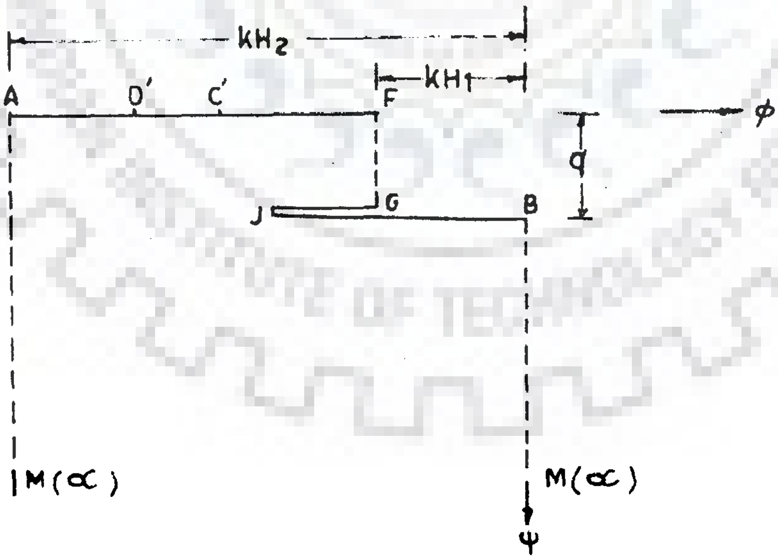
(a) Z-PLANE



(b) t-PLANE



(c) ζ-PLANE



(d) W-PLANE

FIG 3.1 – TRANSFORMATION LAYOUT

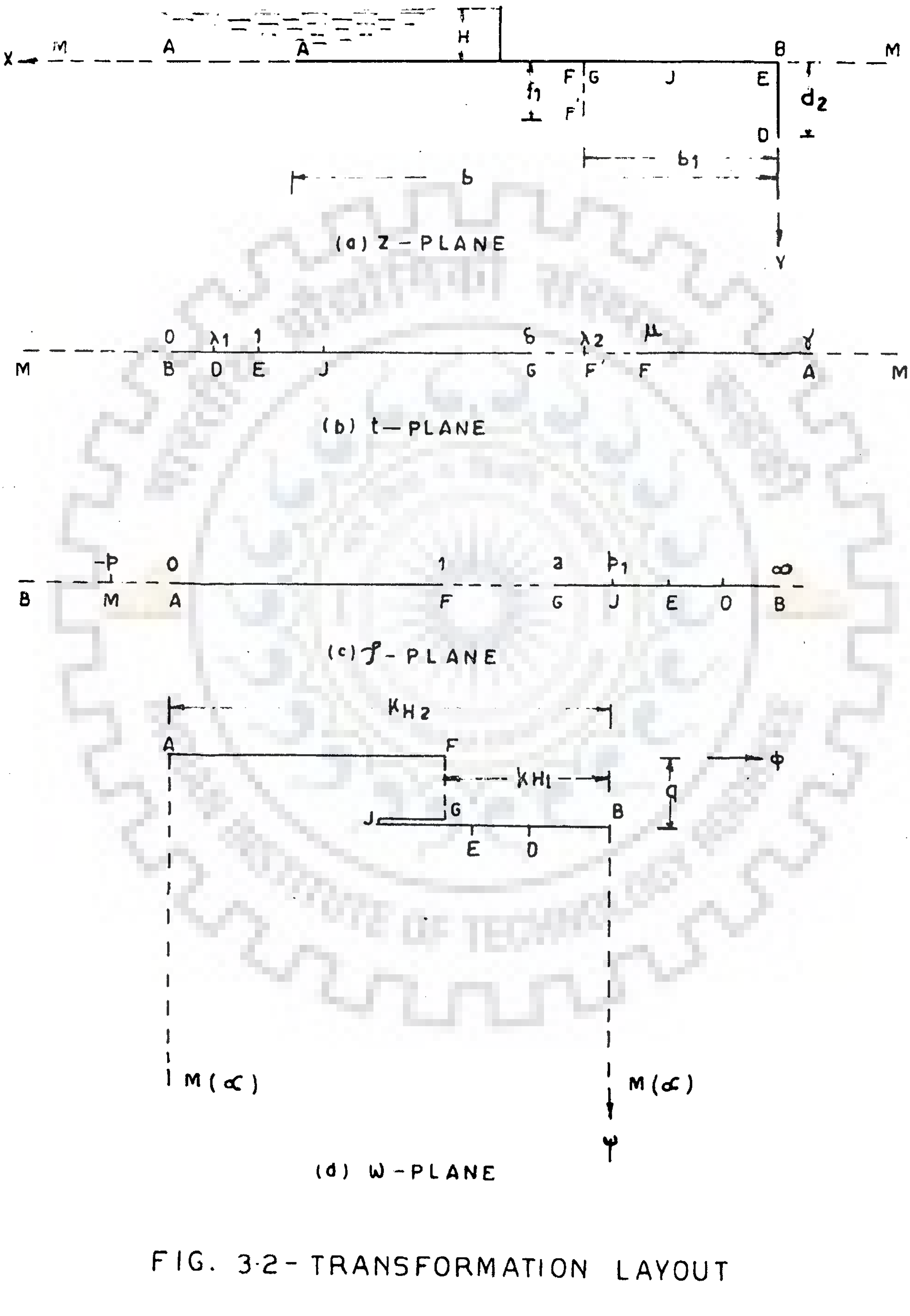


FIG. 3·2 - TRANSFORMATION LAYOUT

depend upon the relative values of H_1 and H_o . As the value of H_1 increases the point J moves towards G and for a particular value of H_1, J may coincide with G. In case H_1 is increased beyond this value part of the seepage water emerging in the filter would percolate downstream below the foundation profile. Intermediate drains which are provided to reduce uplift pressures on the downstream portion of the floor would seldom be drained at such a high level. The analysis is, therefore, limited to the former case. The method as indicated in chapter II has been adopted for solution.

3.2 THEORETICAL SOLUTION FOR FLOOR WITH UPSTREAM CUTOFF.

3.2.1 First operation $z = f(t)$

In this operation the profile of the structure in the z-plane is transformed onto the real axis of the t-plane (Fig. 3.1). The points A and C' are placed at O and + 1. The points D', F, F', G and B lie at λ_1, δ, λ_2, μ and γ. The values of these five transformation parameters have to be determined.

The transformation mapping of z-plane onto the t-plane is given by Eq. 2.07. The mapping from A to G is identical with the mapping in chapter II from A to B except that we have to substitute b_1 for b, and f for d_o. Also the points F, F' and G correspond respectively to points E, D and B in chapter II. Eqs. 2.41 and 2.42 take the form

$$\frac{b_1}{d_1} = \frac{\left(\frac{E}{K_1} - 1\right) K_1' + E'}{\frac{E}{K_1} F(\varphi_{D'}, m_1) - E(\varphi_{D'}, m_1) + \sqrt{\frac{\lambda_1(1-\lambda_1)(\mu-\lambda_1)}{\delta(\mu-1)(\delta-\lambda_1)}}} \tag{3.01}$$

$$\frac{f}{d_1} = \frac{E(\varphi_{F'}, m_1) - \frac{E}{K_1} F(\varphi_{F'}, m_1) - \sqrt{\frac{\lambda_2(\lambda_2-\delta)(\mu-\lambda_2)}{\delta(\mu-1)(\lambda_2-1)}}}{\frac{E}{K_1} F(\varphi_{D'}, m_1) - E(\varphi_{D'}, m_1) + \sqrt{\frac{\lambda_1(1-\lambda_1)(\mu-\lambda_1)}{\delta(\mu-1)(\delta-\lambda_1)}}} \tag{3.02}$$

where $\varphi_{F'} = \sin^{-1} \sqrt{\dfrac{(\mu-1)(\lambda_2-\delta)}{(\mu-\delta)(\lambda_2-1)}}$

<u>Along the floor G B, $\mu < t \leq \gamma$</u>.

Integrating Eq. 2.07 along this portion

$$\frac{x-b_1}{M} = \lambda_1\lambda_2 \int_{\mu}^{t} \frac{dt}{\sqrt{t(t-1)(t-\delta)(t-\mu)}} - (\lambda_1+\lambda_2) \int_{\mu}^{t} \frac{t.\ dt}{\sqrt{t(t-1)(t-\delta)(t-\mu)}}$$

$$+ \int_{\mu}^{t} \frac{t^2\ dt}{\sqrt{t(t-1)(t-\delta)(t-\mu)}}$$

Making use of standard formulae (4, Eqs. 258.11, 340,336)

$$\frac{x-b_1}{Mg_1} = \Big[\lambda_1\lambda_2 V_0 - (\lambda_1+\lambda_2)\left\{(\mu-\delta)V_1 + \delta V_0\right\} + \left\{\delta^2 V_0 + 2\delta(\mu-\delta) V_1 + (\mu-\delta)^2 V_2\right\}\Big] \qquad (3.03)$$

where $V_0 = F(\varphi, m_1')$

$V_1 = \Pi(\varphi, \alpha_7^2, m_1')$

$$V_2 = \frac{1}{2(\alpha_7^2-1)(m_1'^2-\alpha_7^2)}\Big[\alpha_7^2 E(\varphi, m_1') + (m_1'^2-\alpha_7^2) V_0 + (2\alpha_7^2 m_1'^2 + 2\alpha_7^2 - \alpha_7^4 - 3m_1'^2) V_1 - \frac{\alpha_7^4 \, Sn\,u\ Cn\,u\ Dn\,u}{1-\alpha_7^2 Sn^2 u}\Big]$$

$$Sn\ u = \sqrt{\frac{\delta(t-\mu)}{\mu(t-\delta)}}$$

$$Cn\ u = \sqrt{\frac{t(\mu-\delta)}{\mu(t-\delta)}} \qquad (3.04)$$

$$dn\ u = \sqrt{\frac{(\mu-\delta)(t-1)}{(\mu-1)(t-\delta)}}$$

$$\varphi = \sin^{-1}\sqrt{\frac{\delta\,(t-\mu)}{\mu\,(t-\delta)}} \tag{3.05}$$

$$m_1' = \sqrt{\frac{\mu\,(\delta-1)}{\delta\,(\mu-1)}}$$

$$\alpha_7^2 = \frac{\mu}{\delta} \tag{3.06}$$

On substituting the values of m_1', α_7^2, Sn u, Cn u, and dn u

$$V_2 = \frac{1}{2(\mu-\delta)^2}\Big\{\delta(\mu-\delta)F(\varphi,m_1') - \delta(\mu-1)E(\varphi,m_1') - (\mu-\delta)(3\delta-\mu-1)\,\Pi(\varphi,\alpha_7^2,m_1') + \sqrt{\frac{\delta(\mu-1)(t-\mu)(t-1)t}{t-\delta}}\Big\} \tag{3.07}$$

Substitution of values of V_0, V_1 and V_2 in Eq. 3.03 yields

$$\frac{x-b_1}{Mg_1} = \lambda_1\lambda_2 F(\varphi,m_1') - (\lambda_1+\lambda_2)\Big\{(\mu-\delta)\Pi(\varphi,\alpha_7^2,m_1') + \delta F(\varphi,m_1')\Big\} + \frac{1}{2}\delta(\mu+\delta)F(\varphi,m_1') - \frac{1}{2}\delta(\mu-1)E(\varphi,m_1') + \frac{1}{2}(\mu-\delta)(\mu+\delta+1)\,\Pi(\varphi,\alpha_7^2,m_1') + \frac{1}{2}\sqrt{\frac{\delta(\mu-1)(t-\mu)(t-1)t}{(t-\delta)}} \tag{3.08}$$

Substituting the values of λ_1, λ_2 and Mg_1 from Eqs. 2.38, 2.39 and 2.16 respectively, we obtain.

$$\frac{x-b_1}{d_1} = \frac{(1-\frac{E}{K_1})F(\varphi,m_1') - E(\varphi,m_1') + \sqrt{\frac{t(t-\mu)(t-1)}{\delta(\mu-1)(t-\delta)}}}{\frac{E}{K_1}F(\varphi_{D'},m_1) - E(\varphi_{D'},m_1) + \sqrt{\frac{\lambda_1(1-\lambda_1)(\mu-\lambda_1)}{\delta(\mu-1)(\delta-\lambda_1)}}} \tag{3.09}$$

At point B, $t = \gamma$ and $x = b$, therefore

$$\frac{b-b_1}{d_1} = \frac{(1-\frac{E}{K_1})F(\varphi_B,m_1') - E(\varphi_B,m_1') + \sqrt{\frac{\gamma(\gamma-\mu)(\gamma-1)}{\delta(\mu-1)(\gamma-\delta)}}}{\frac{E}{K_1}F(\varphi_{D'},m_1') - E(\varphi_{D'},m_1) + \sqrt{\frac{\lambda_1(1-\lambda_1)(\mu-\lambda_1)}{\delta(\mu-1)(\delta-\lambda_1)}}} \tag{3.10}$$

where $\varphi_B = \sin^{-1}\sqrt{\dfrac{\delta(\gamma-\mu)}{\mu(\gamma-\delta)}}$

3.2.2 Second operation $w = F_1(\zeta)$

The layouts of the w-plane and ζ-plane in this case are identical with the corresponding layouts in chapter II and therefore the transformation is identical. All the equations derived in section 2.2.2 remain valid in the present case.

3.2.3 Third operation $\zeta = F_2(t)$

The transformation of ζ-plane onto t-plane is obtained with the help of bilinear transformation.

$$\zeta = \frac{t(\gamma-\delta)}{\delta(\gamma-t)} \tag{3.11}$$

or

$$t = \frac{\delta\gamma\zeta}{\zeta\delta+\gamma-\delta} \tag{3.12}$$

At point G, $\zeta = a$ and $t = \mu$, therefore,

$$a = \frac{\mu(\gamma-\delta)}{\delta(\gamma-\mu)} \tag{3.13}$$

At point M, $\zeta = -p$ and $t = \infty$, therefore

$$p = \frac{\gamma-\delta}{\delta} \tag{3.14}$$

3.2.4 Uplift Pressures

The uplift pressures below the upstream floor, A D' C' F can be determined from Eq. 2.73 or 2.75. The values of p, P_1, m_2, φ and ζ are given by Eqs. 3.14, 2.65, 2.48, 2.74 and 3.11 respectively. The pressures at D' and C' for the case $H_1 = 0$ are given below

$$\frac{\Phi_{D'}}{kH_2} = \frac{p\left[K_2 - F(\varphi_{D'}, m_2)\right] - (p_1+p)\left[\Pi_4 - \Pi(\varphi_{D'}, \alpha_4^2, m_2)\right]}{(p_1+p)\Pi_4 - pK_2} \tag{3.15}$$

$$\text{where } \varphi_0 = \text{Sin}^{-1}\sqrt{\frac{\lambda_1(\gamma - \delta)}{\delta(\gamma - \lambda_1)}} \tag{3.16}$$

$$\frac{\Phi_{c'}}{kH_2} = \frac{p\left[K_2 - F(\varphi_{c'}, m_2)\right] - (p_1 + p)\left[\Pi_4 - \Pi(\varphi_{c'}, \alpha_4^2, m_2)\right]}{(p_1 + p)\Pi_4 - pK_2} \tag{3.17}$$

$$\text{where } \varphi_{c'} = \text{Sin}^{-1}\sqrt{\frac{\gamma - \delta}{\delta(\gamma - 1)}} \tag{3.18}$$

The uplift pressures below the downstream floor G J B can be determined from Eq. 2.84. The pressure at J is given by Eq.2.92.

3.3 THEORETICAL SOLUTION FOR FLOOR WITH DOWNSTREAM CUTOFF.

3.3.1 First operation z = f (t)

In this operation the profile of the structure in the z-plane is transformed onto the real axis of the t-plane (Fig. 3.2). The points B and E are placed at O and + 1. The points D, G, F', F and A lie at λ_1, δ, λ_2, μ and γ. The values of these five transformation parameters are to be determined.

The mapping of the z-plane onto the t-plane is given by Eq. 2.07 and is identical to that obtained in 3.2.1 except that direction of flow in z-plane in relation to boundaries has been reversed or keeping the direction of flow the same, the profile of the structure has been reversed. Also the points A, D', C' and F in section 3.2 are replaced by B, D, E and G respectively. The points G and B are replaced by F and A respectively. Eqs. 3.01, 3.02, 3.09 and 3.19 take the form.

$$\frac{b_1}{d_2} = \frac{\left(\frac{E}{K_1} - 1\right)K_1' + E'}{\frac{E}{K_1}F(\varphi_D, m_1) - E(\varphi_D, m_1) \pm \sqrt{\frac{\lambda_1(1 - \lambda_1)(\mu - \lambda_1)}{\delta(\mu - 1)(\delta - \lambda_1)}}} \tag{3.19}$$

$$\frac{f}{d_2} = \frac{E(\varphi_{F'}, m_1) - \frac{E}{K_1} F(\varphi_{F'}, m_1) - \sqrt{\frac{\lambda_2(\lambda_2 - \delta)(\mu - \lambda_2)}{\delta(\mu - 1)(\lambda_2 - 1)}}}{\frac{E}{K_1} F(\varphi_D, m_1) - E(\varphi_D, m_1) + \sqrt{\frac{\lambda_1(1 - \lambda_1)(\mu - \lambda_1)}{\delta(\mu - 1)(\delta - \lambda_1)}}} \tag{3.20}$$

$$\frac{x - b_1}{d_2} = \frac{\left(1 - \frac{E}{K_1}\right) F(\varphi, m_1') - E(\varphi, m_1') + \sqrt{\frac{t(t - \mu)(t - 1)}{\delta(\mu - 1)(t - \delta)}}}{\frac{E}{K_1} F(\varphi_D, m_1) - E(\varphi_D, m_1) + \sqrt{\frac{\lambda_1(1 - \lambda_1)(\mu - \lambda_1)}{\delta(\mu - 1)(\delta - \lambda_1)}}} \tag{3.21}$$

$$\frac{b - b_1}{d_2} = \frac{\left(1 - \frac{E}{K_1}\right) F(\varphi_A, m_1') - E(\varphi_A, m_1') + \sqrt{\frac{\gamma(\gamma - \mu)(\gamma - 1)}{\delta(\mu - 1)(\gamma - \delta)}}}{\frac{E}{K_1} F(\varphi_D, m_1) - E(\varphi_D, m_1) + \sqrt{\frac{\lambda_1(1 - \lambda_1)(\mu - \lambda_1)}{\delta(\mu - 1)(\delta - \lambda_1)}}} \tag{3.22}$$

where $\varphi_{F'} = \sin^{-1}\sqrt{\frac{(\mu - 1)(\lambda_2 - \delta)}{(\mu - \delta)(\lambda_2 - 1)}}$

$$\varphi_D = \sin^{-1}\sqrt{\frac{\delta(1 - \lambda_1)}{(\delta - \lambda_1)}}$$

$$\varphi = \sin^{-1}\sqrt{\frac{\delta(t - \mu)}{\mu(t - \delta)}}$$

$$\varphi_A = \sin^{-1}\sqrt{\frac{\delta(\gamma - \mu)}{\mu(\gamma - \delta)}}$$

The remaining parameters are defined in section 2.2.1

3.3.2 Second Operation $w = F_1(\zeta)$

The layouts of w-plane and the ζ-plane in this case are identical with corresponding layouts in chapter II and therefore the transformation is identical. All the equations derived in section 2.2.2

remain valid in the present case also.

3.3.3 Third Operation $\zeta = F_2(t)$

The transformation of ζ-plane onto t-plane, in this case is obtained with the help of bilinear transformation.

$$\zeta = \frac{\mu(\gamma - t)}{t(\gamma - \mu)} \tag{3.23}$$

or

$$t = \frac{\mu\gamma}{\zeta(\gamma-\mu)+\mu} \tag{3.24}$$

At point G, $\zeta = a$ and $t = \delta$, therefore

$$a = \frac{\mu(\gamma - \delta)}{\delta(\gamma - \mu)} \tag{3.25}$$

At point M, $\zeta = -p$ and $t = \infty$, therefore

$$p = \frac{\mu}{\gamma - \mu} \tag{3.26}$$

3.3.4 Uplift Pressures

The uplift pressures below the upstream floor A F for the case $H_1 = 0$ can be determined from Eq. 2.75. The values of p, p_1, m_2, φ and ζ are given by Eqs. 3.26, 2.65, 2.48, 2.74 and 3.23 respectively. The corresponding pressures below the downstream floor G J E D B is given by Eq. 2.84. The pressures at J, E, and D are given by

$$\frac{\phi_J}{kH_2} = \frac{p}{(1+p)(a+p)} \; \frac{(p_1-1)(a+p)F(\varphi_J, m_2)-(a-1)(p_1+p)\Pi(\varphi_J, \alpha_6^2, m_2)}{(p_1+p)\Pi_4 - pK_2} \tag{3.27}$$

$$\frac{\phi_E}{kH_2} = \frac{p}{(1+p)(a+p)} \; \frac{(p_1-1)(a+p)F(\varphi_E, m_2)-(a-1)(p_1+p)\Pi(\varphi_E, \alpha_6^2, m_2)}{(p_1+p)\Pi_4 - pK_2} \tag{3.28}$$

$$\frac{\Phi_D}{kH_2} = \frac{p}{(1+p)(a+p)} \frac{(p_1-1)(a+p)F(\varphi_D, m_2)-(a-1)(p_1+p)\Pi(\varphi_D, \alpha_6^2, m_2)}{(p_1+p)\Pi_4 - pK_2} \quad (3.29)$$

where $\varphi_J = \sin^{-1}\sqrt{\frac{p_1-a}{p_1-1}}$

$$\varphi_E = \sin^{-1}\sqrt{\frac{\xi_E - a}{\xi_E - 1}}, \quad \xi_E = \frac{\mu(\gamma-1)}{(\gamma-\mu)}$$

$$\varphi_D = \sin^{-1}\sqrt{\frac{\xi_D - a}{\xi_D - 1}}, \quad \xi_D = \frac{\mu(\gamma-\lambda_1)}{\lambda_1(\gamma-\mu)}$$

3.4 RESULTS

The equations derived in paras 3.2 and 3.3 have been used for computations of uplift pressures. The values of uplift pressures have been computed for different combinations of variables involved and plotted in the form of design curves. The curves cover the values of b/d_1 or b/d_2 upto 20, $f/d_1 = 0.2$ to 3.0, $f/d_2 = 0.2$ to 1.0, $b_1/d_1=4$ to 10 and $b_1/d_2 = 1$ to 4. The pressures at J, and D' and C' or D and E have been plotted. The location of J is also given.

These curves have been plotted for simple profiles consisting of a flat floor with a cutoff at upstream or downstream end and a deep drain anywhere along the floor. In an actual problem of hydraulic structure, the foundation profile is more complex. But it can be reduced to simple elementary profile and pressures at key points determined.

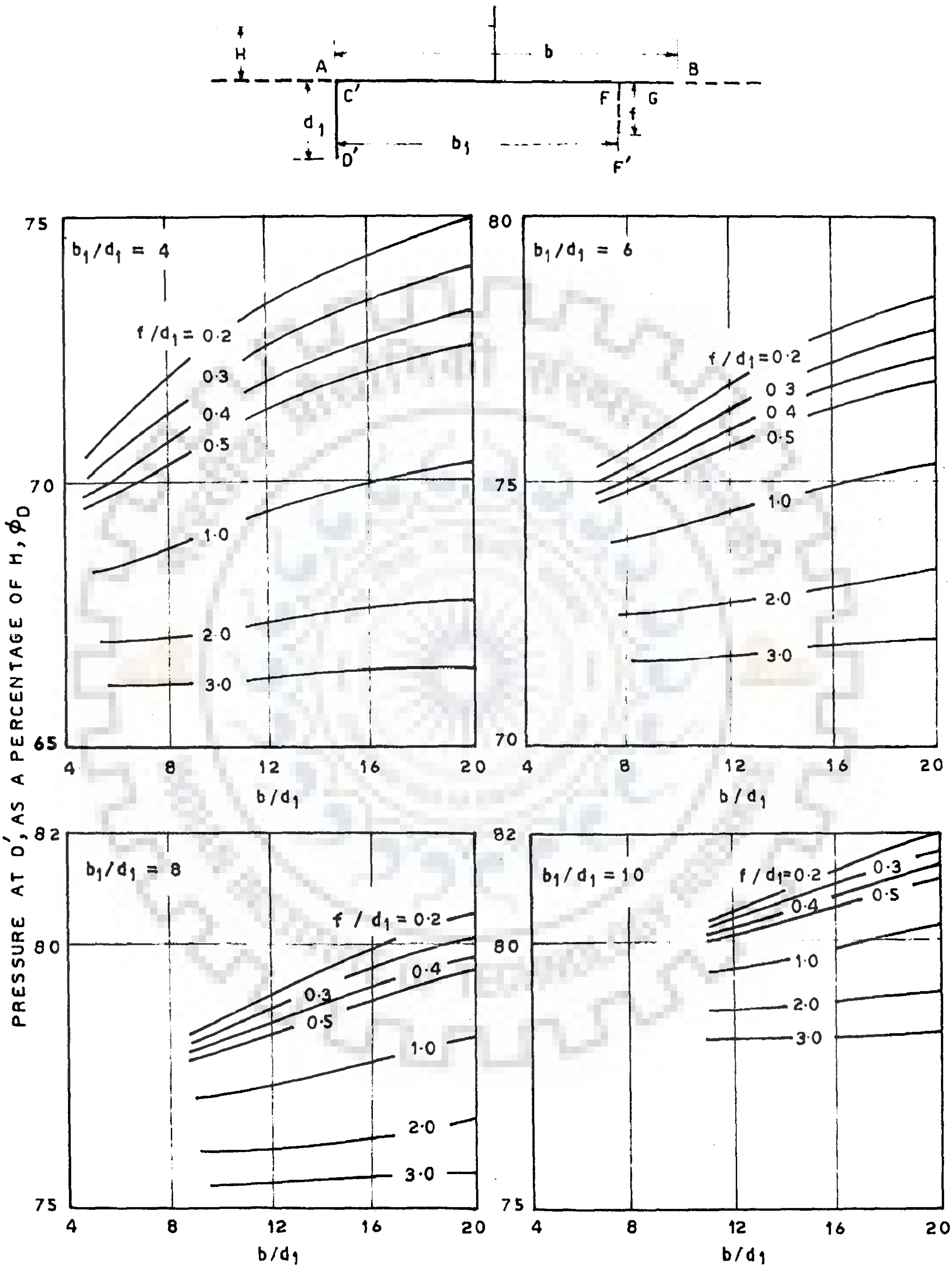
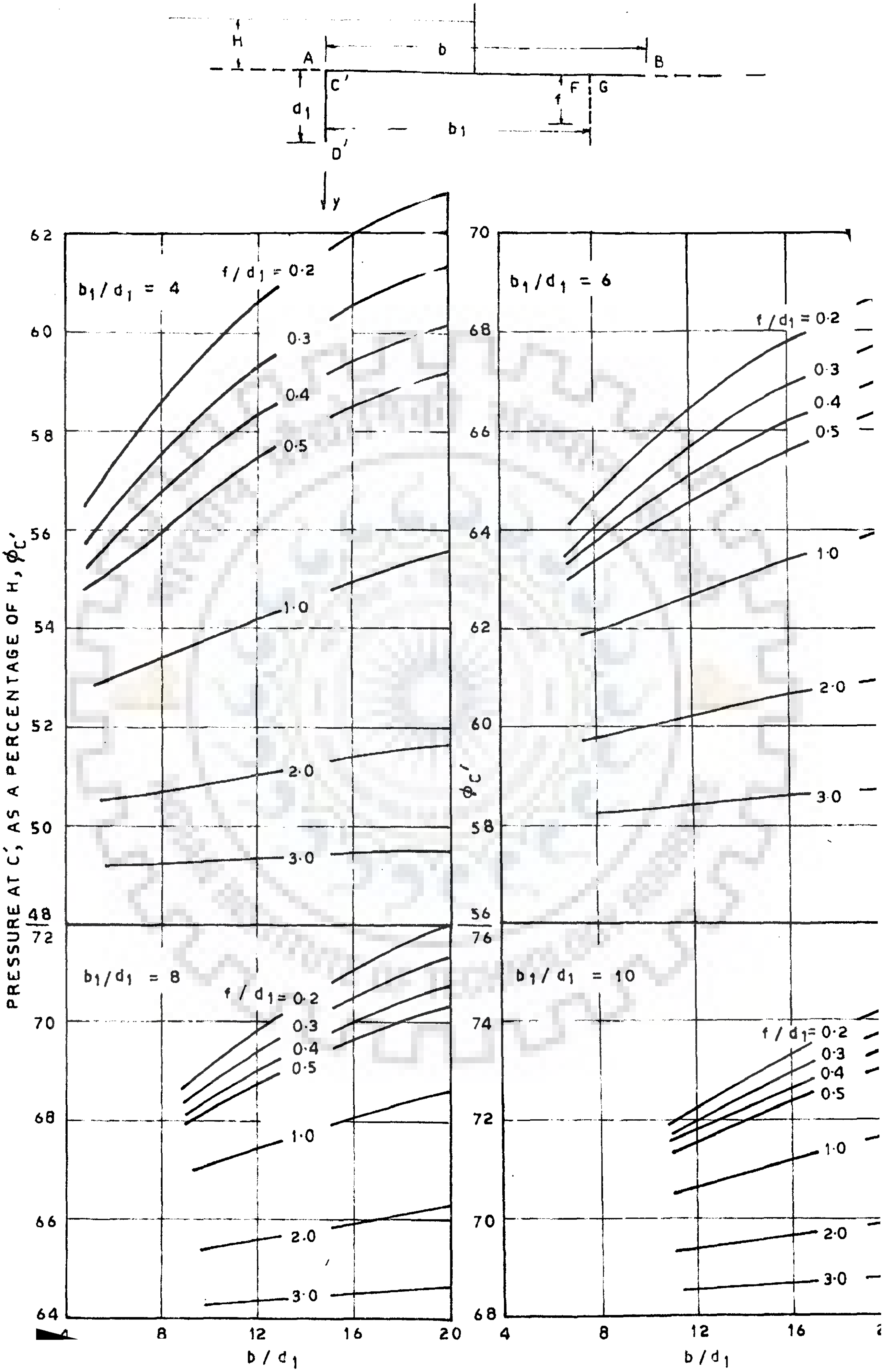


FIG. 3·3 – UPLIFT PRESSURE AT D′


H
A
b
B
C'
F
G
f
d_1
b_1
D'
y
PRESSURE AT C', AS A PERCENTAGE OF H, $\phi_{C'}$
$b_1/d_1 = 4$
$f/d_1 = 0.2$
0.3
0.4
0.5
1.0
2.0
3.0
$b_1/d_1 = 6$
$f/d_1 = 0.2$
0.3
0.4
0.5
1.0
2.0
3.0
$\phi_{C'}$
$b_1/d_1 = 8$
$f/d_1 = 0.2$
0.3
0.4
0.5
1.0
2.0
3.0
$b_1/d_1 = 10$
$f/d_1 = 0.2$
0.3
0.4
0.5
1.0
2.0
3.0
b/d_1
b/d_1

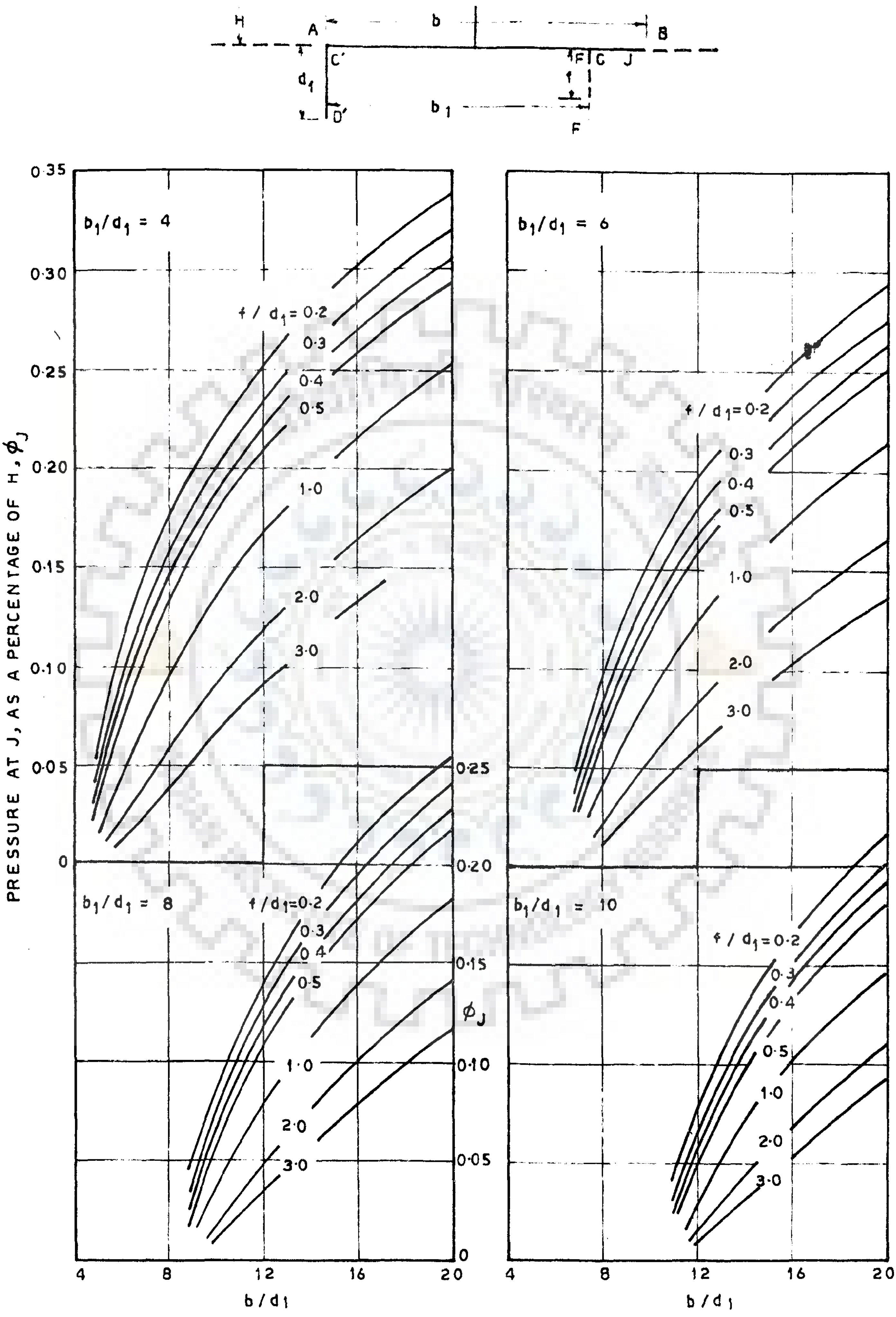


FIG. 3·5 — UPLIFT PRESSURE AT J

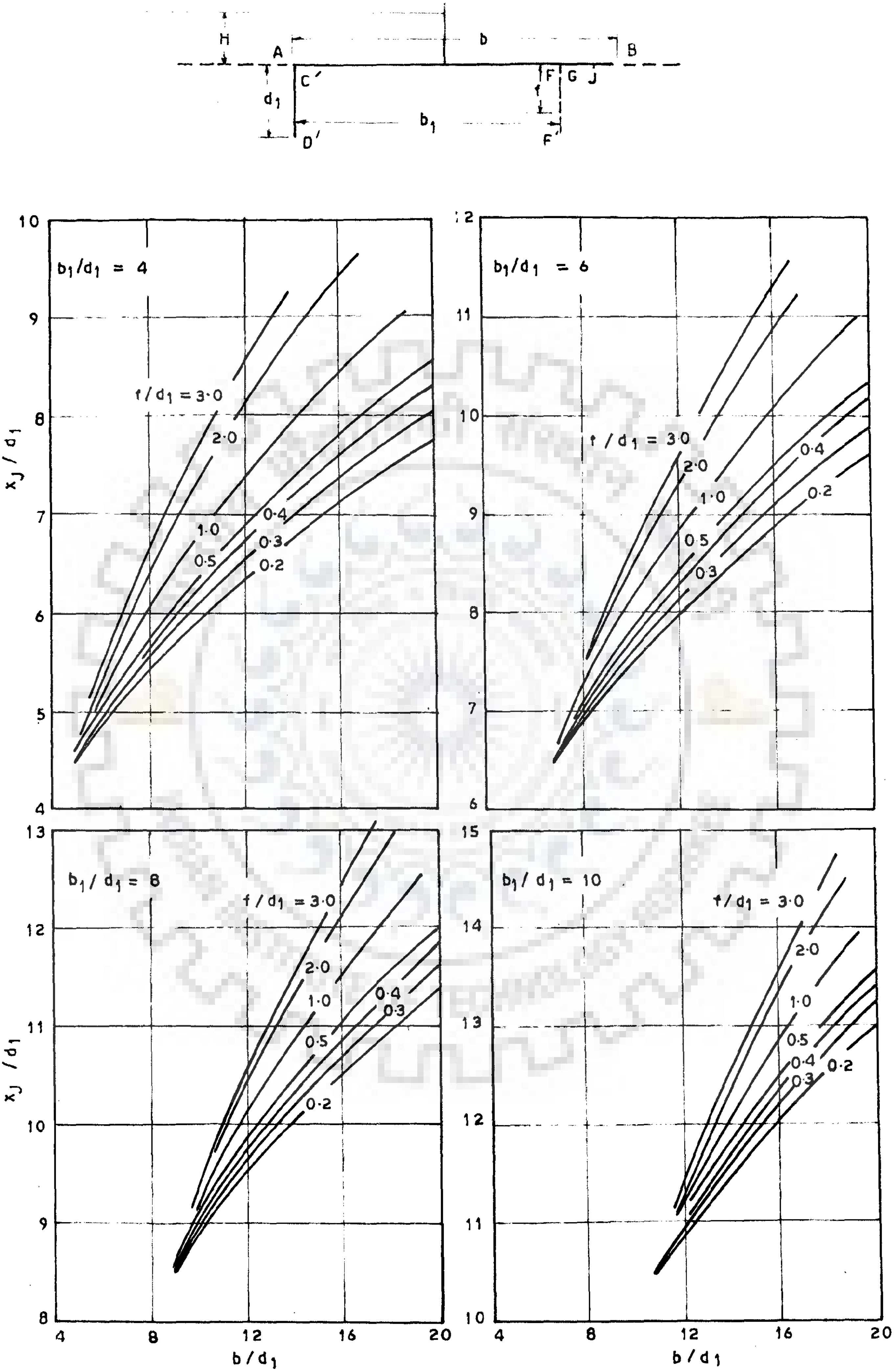


FIG.3·6 — LOCATION OF J

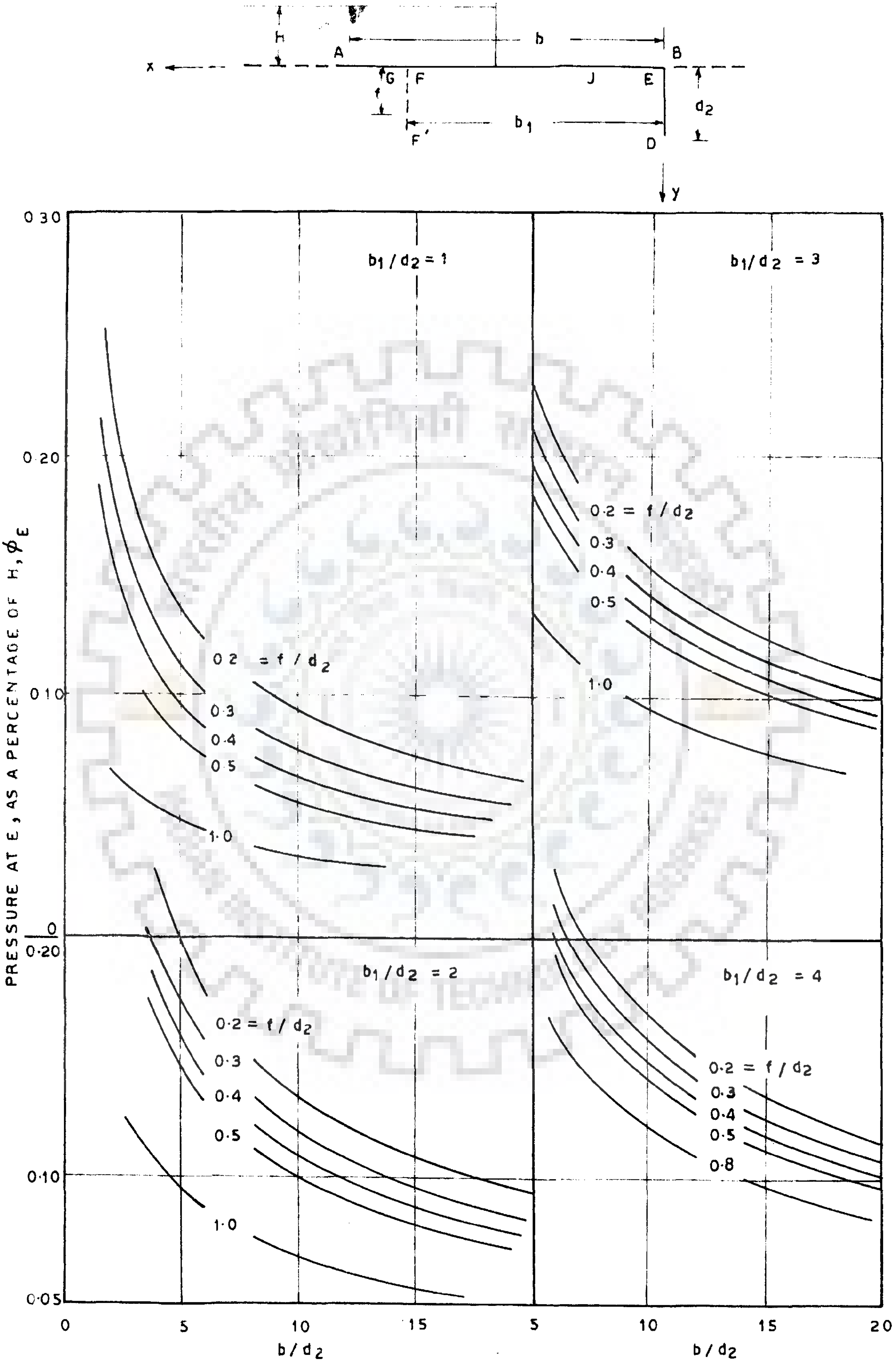


FIG. 3·7 —UPLIFT PRESSURE AT E

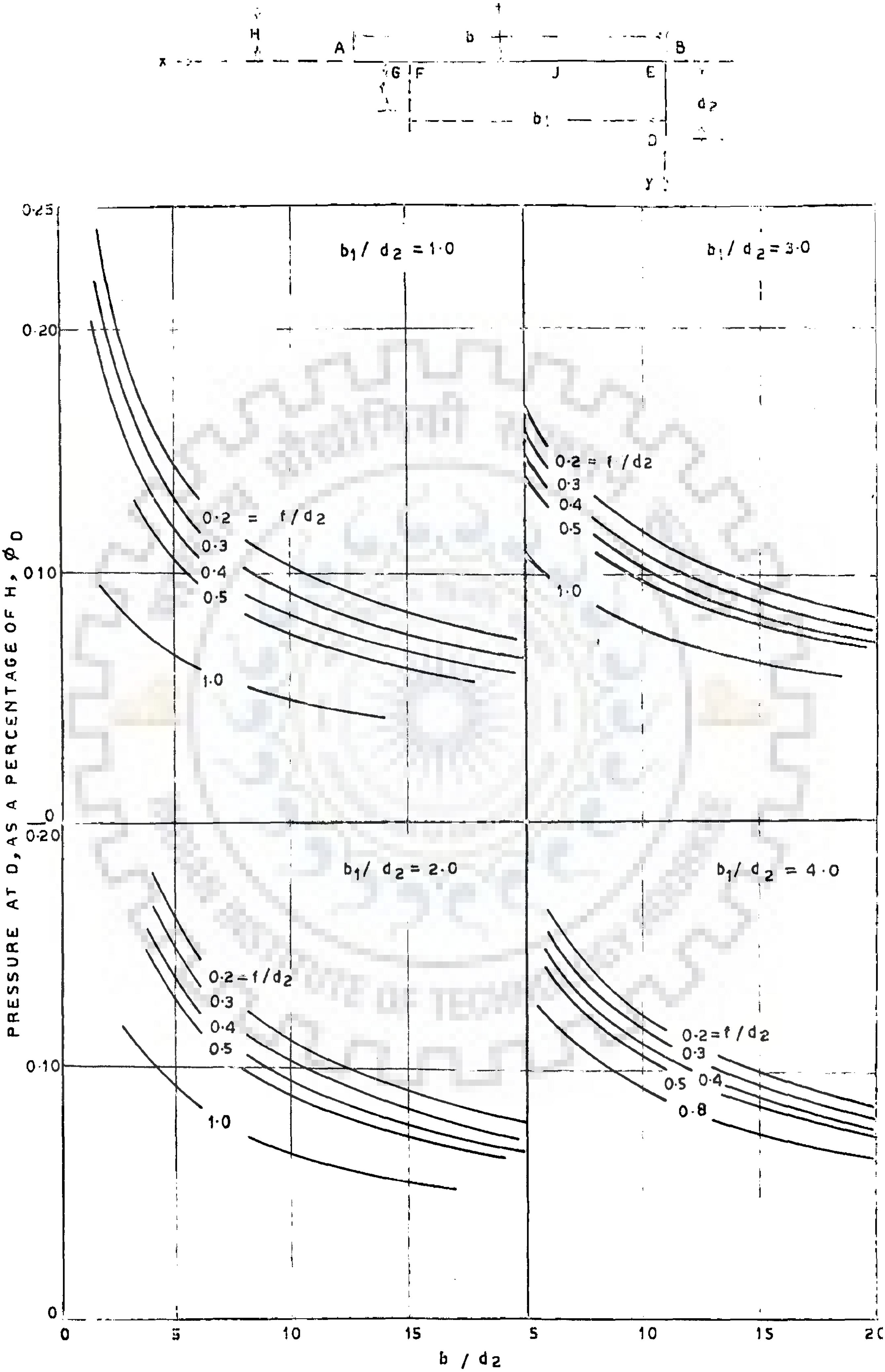


FIG 3·8 — UPLIFT PRESSURE AT D

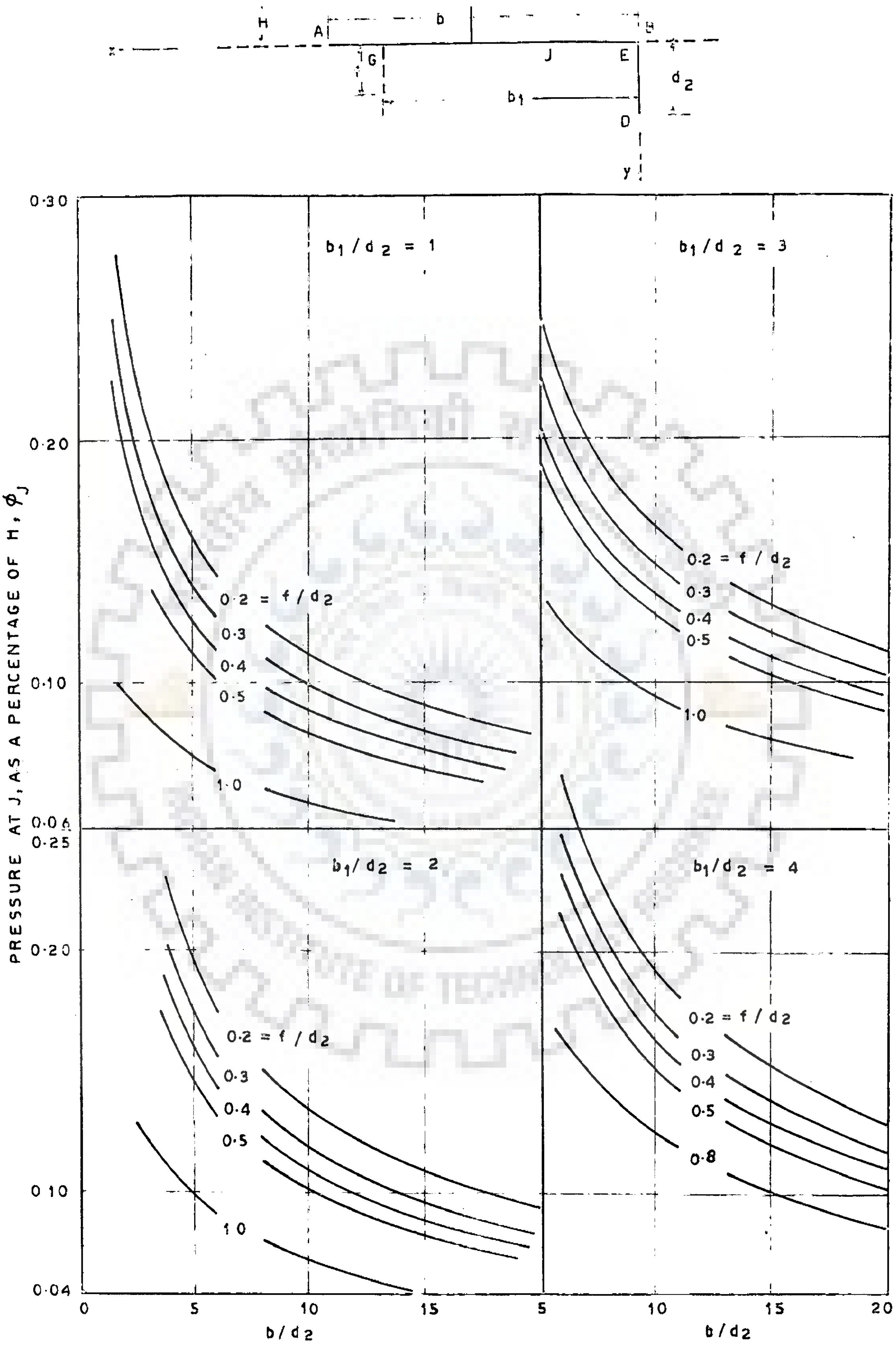


FIG. 3.9 - UPLIFT PRESSURES AT J

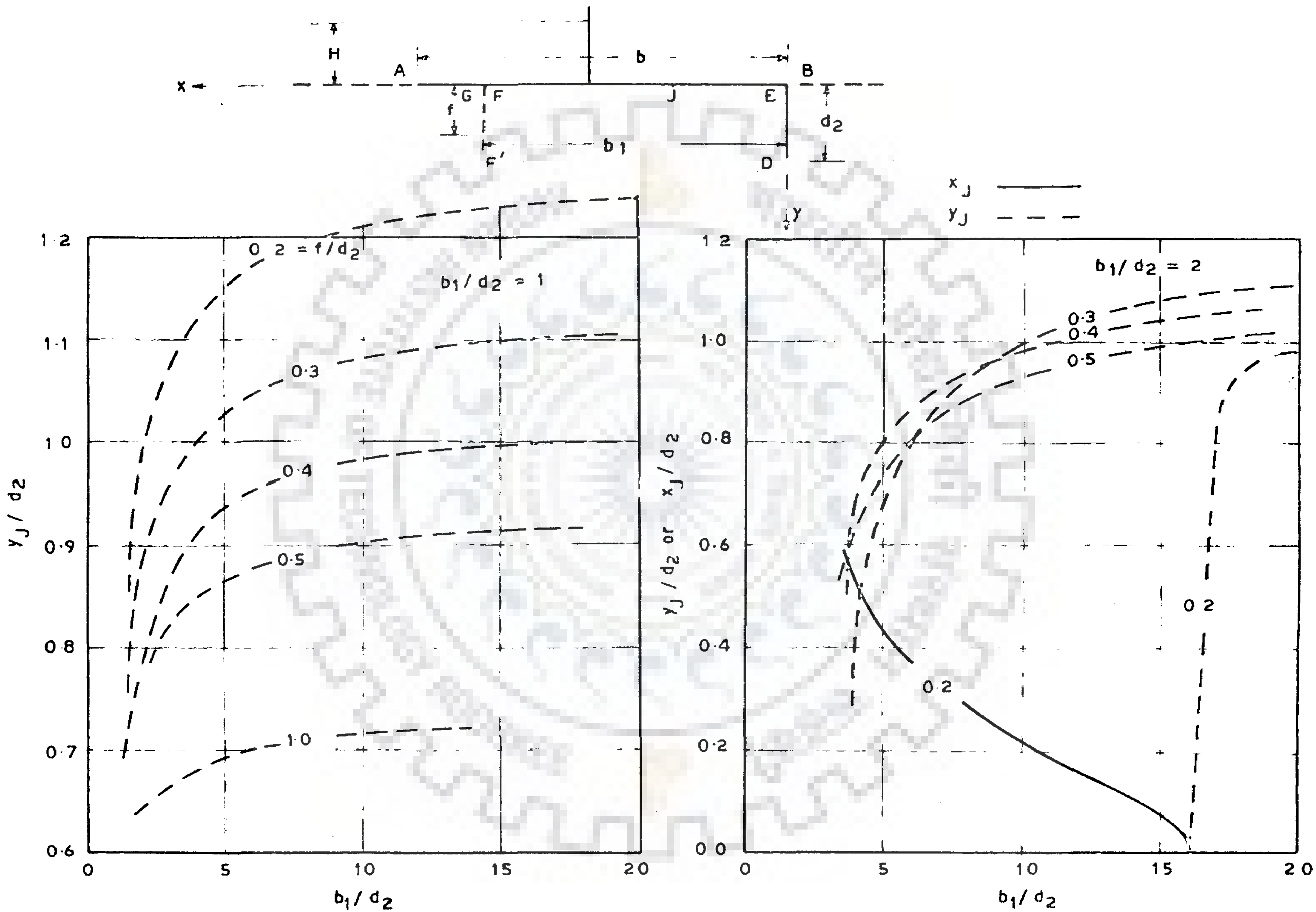


FIG.3·10 (a)– LOCATION OF J

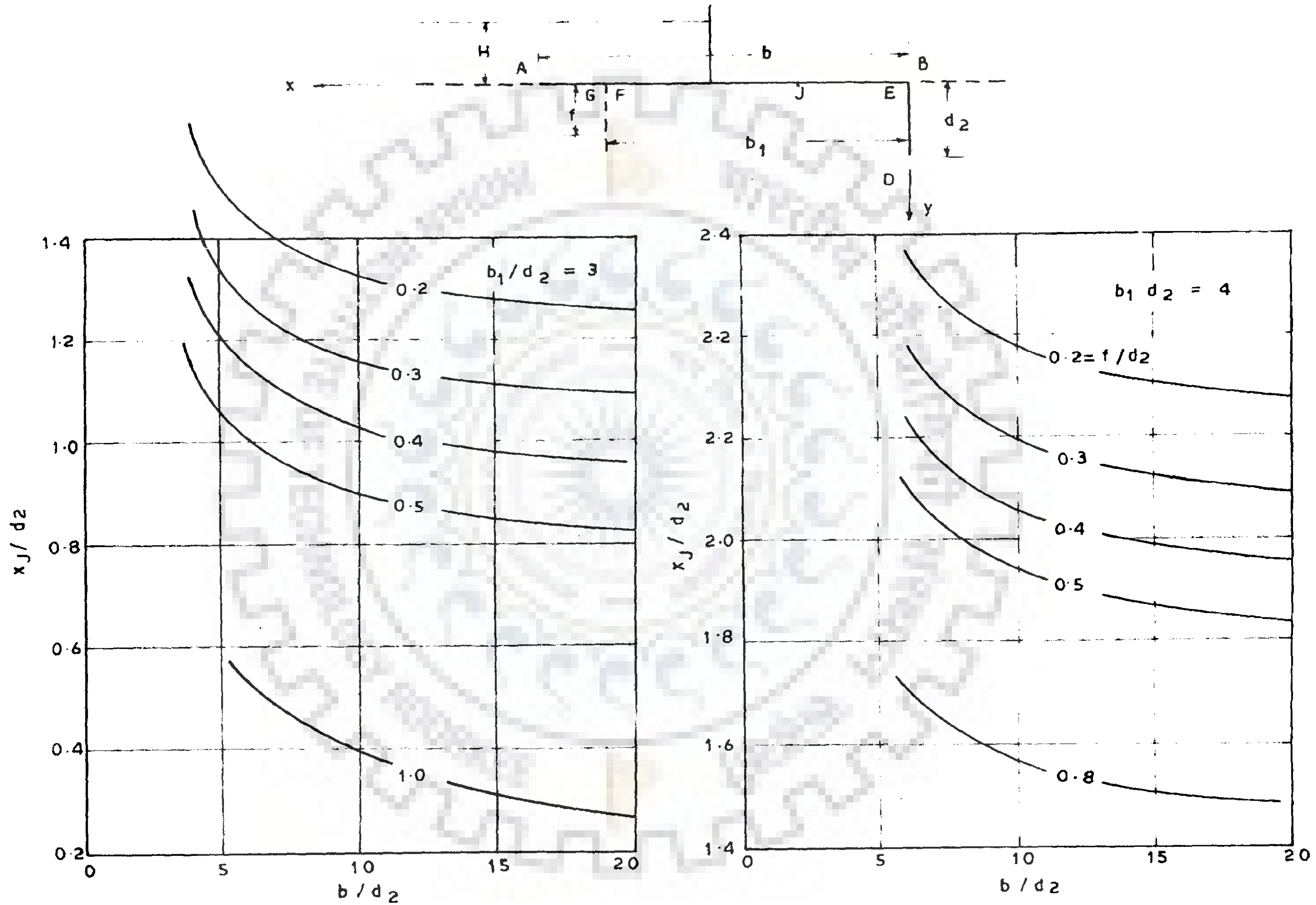


FIG.3·10(b) - LOCATION OF J

CHAPTER IV

LEAKAGE THROUGH CUTOFF

CHAPTER IV

LEAKAGE THROUGH CUTOFF

4.1 FORMULATION OF THE PROBLEM, LAYOUT, AND BOUNDARY CONDITIONS.

Consider an impervious floor A B of length 2b with a central cutoff C D of depth d, resting on a permeable soil of infinite depth. There is a continuous opening D_1 D_2 in the cutoff due to reasons mentioned in para 1.2, at a depth d_1 and the width of opening is d_2-d_1 . On the upstream and downstream of the floor is pervious bed extending upto infinity. The profile is represented in z-plane as shown in Fig. 4.1 (a).

The head on the downstream water level is assumed as zero and measured above it, H is taken as the head on the upstream bed. Therefore $\Phi = -kH$ on the upstream bed A M and $\Phi = 0$ on the downstream bed, where Φ is potential function and k is coefficient of percolation. The foundation profile AED_1 C B forms the inner boundary of the flow and represents the streamline $\psi = 0$. The opening D_1 D_2 in the cutoff is an equipotential $\Phi = -\frac{kH}{2}$ due to symmetry. D_2 D is impervious cutoff below the opening and represents a streamline $\psi = q$, where q is the discharge passing through the opening. Due to symmetry potential at D is also -kH/2. The foundation profile and other boundaries have been shown in w-plane in Fig. 4.1 (c).

Both the profile of the structure in the z-plane and the flow field in the w-plane can be transformed on the lower half of ζ-plane. The problem can be solved by combining the two relations obtained thus.

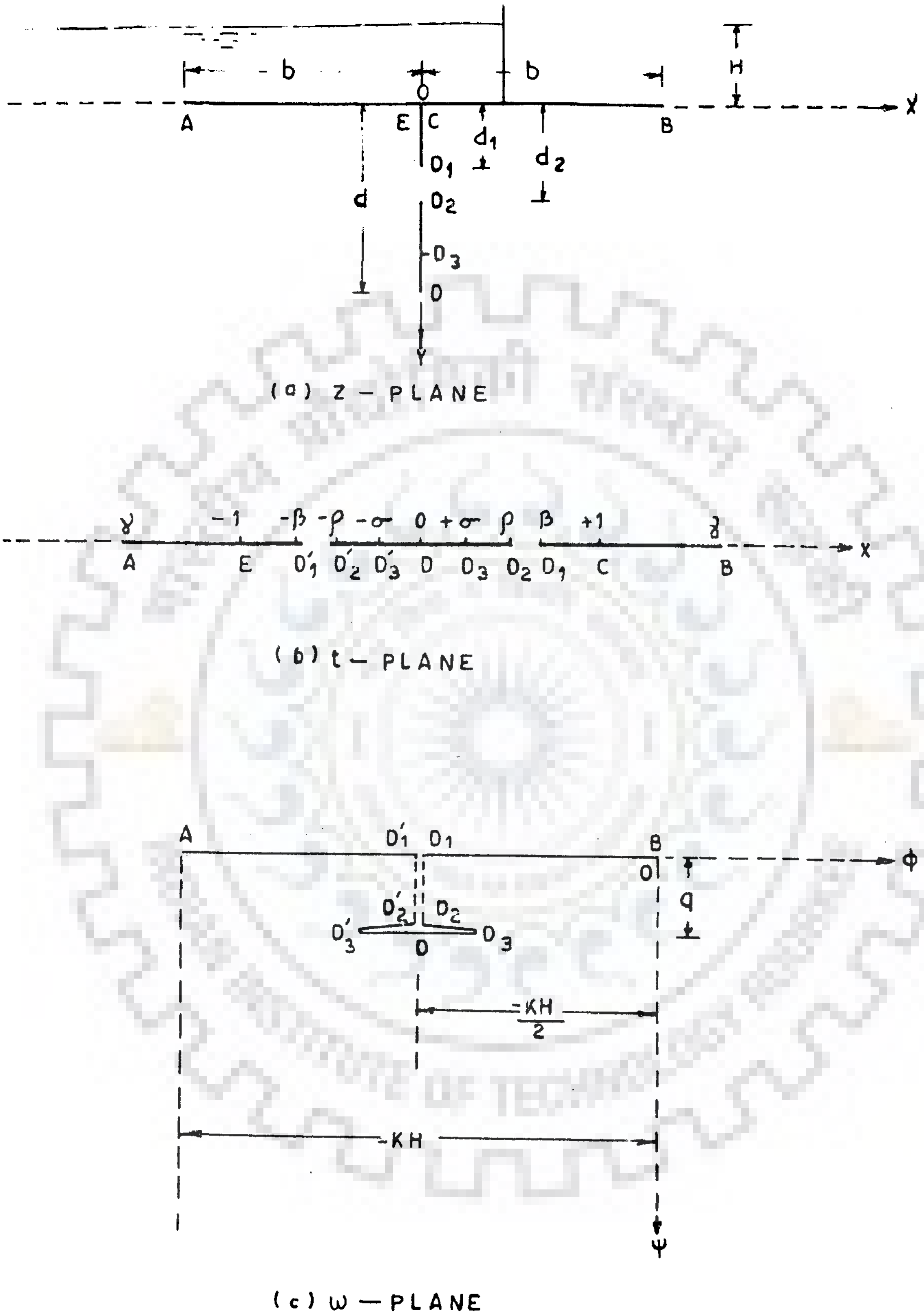


FIG. 4.1 — TRANSFORMATION LAYOUT

4.2 THEORETICAL SOLUTION

4.2.1 First Operation $z = f(\zeta)$

In this operation the profile of the structure in z-plane is transformed onto the real axis of the ζ-plane. The points c, D and E are placed at + 1 , 0 and - 1, due to symmetry and the points D_2, D_1 and B lie at ρ , β and γ.

The transformation equation that maps z-plane onto the ζ-plane is given by

$$z = d\sqrt{\zeta^2 - 1} \tag{4.01}$$

or

$$\zeta = \pm\sqrt{1 + (\frac{z}{d})^2} \tag{4.02}$$

This gives $\rho^2 = 1 - (\frac{d_2}{d})^2$ (4.03)

$$\beta^2 = 1 - (\frac{d_1}{d})^2 \tag{4.04}$$

$$\gamma^2 = 1 + (\frac{b}{d})^2 \tag{4.05}$$

where $\gamma > \beta > \rho > 0$

4.2.2 Second Operation $w = f(\zeta)$

In this operation the w-plane is transformed onto the ζ-plane. The points of stagnation D_3 and D_3' are placed at + σ and - σ. The transformation equation is given by

$$\frac{dw}{d\zeta} = M \frac{(\zeta^2 - \sigma^2)}{\sqrt{(\zeta^2 - \rho^2)(\zeta^2 - \beta^2)(\gamma^2 - \zeta^2)}} \tag{4.06}$$

Substituting $\zeta^2 = t$

Eq 4.06 reduces to

$$\frac{dw}{dt} = \frac{M}{2} \frac{(t - \sigma^2)}{\sqrt{(t - \rho^2)(t - \beta^2)(\gamma^2 - t)t}} \tag{4.07}$$

Along the lower impervious portion of cutoff D D_2 , $0 < \zeta \leq \rho$.

Integrating Eq. 4.07 in this region

$$\Phi + kH/2 = \frac{M}{2}\left[\int_0^t \frac{\sqrt{t}\,dt}{\sqrt{(\rho^2-t)(\beta^2-t)(\gamma^2-t)}} - \sigma^2\int_0^t \frac{dt}{\sqrt{t(\rho^2-t)(\beta^2-t)(\gamma^2-t)}}\right]$$

Making use of standard formulae (4, Eqs. 252.14 and 252.00)

$$\Phi + kH/2 = \frac{M_g}{2}\left[-\gamma^2\left\{\Pi(\varphi, \alpha_1^2, m) - F(\varphi, m)\right\} - \sigma^2 F(\varphi, m)\right] \quad (4.08)$$

$$\text{where } \varphi = \sin^{-1}\sqrt{\frac{(\gamma^2-\rho^2)(\zeta^2)}{\rho^2(\gamma^2-\zeta^2)}} \quad (4.09)$$

$$m = \sqrt{\frac{(\gamma^2-\beta^2)\rho^2}{(\gamma^2-\rho^2)\beta^2}} \quad (4.10)$$

$$\alpha_1^2 = -\frac{\rho^2}{\gamma^2-\rho^2} \quad (4.11)$$

$$g = \frac{2}{\sqrt{(\gamma^2-\rho^2)\beta^2}} \quad (4.12)$$

At point D_2 , $\Phi = -kH/2$, $\zeta = +\rho$ and $\varphi = \pi/2$ therefore Eq 4.08yields

$$\sigma^2 = \gamma^2\frac{K - \Pi_1}{K} \quad (4.13)$$

where $K = F(\pi/2, m)$

$$\Pi_1 = \Pi(\pi/2, \alpha_1^2, m)$$

Along the gap in cutoff $-D_2\ D_1$, $\rho < \varsigma \leq \beta$.

Integrating Eq 4.07 in this region

$$i(\phi - q) = \frac{M}{2i}\left[\int_{\rho^2}^{t}\frac{\sqrt{t}\,dt}{\sqrt{(t-\rho^2)(\beta^2-t)(\gamma^2-t)}} - \sigma^2\int_{\rho^2}^{t}\frac{dt}{\sqrt{t(t-\rho^2)(\beta^2-t)(\gamma^2-t)}}\right]$$

Making use of standard formula (4, Eqs. 254.02 and 254.00)

$$\phi - q = -\frac{M_g}{2}\left[\rho^2\,\Pi(\varphi, \alpha_2^2, m') - \sigma^2 F(\varphi, m')\right] \quad (4.14)$$

$$\text{where } \varphi = \sin^{-1}\sqrt{\frac{\beta^2(\varsigma^2-\rho^2)}{(\beta^2-\rho^2)\varsigma^2}} \quad (4.15)$$

$$m' = \sqrt{1-m^2} \quad (4.16)$$

$$\alpha_2^2 = \frac{\beta^2-\rho^2}{\beta^2} \quad (4.17)$$

At point D_1 , $\phi = 0$, $\varsigma = +\beta$ and $\varphi = \pi/2$ therefore Eq. 4.14 yields.

$$q = \frac{M_g}{2}\left[\rho^2\,\Pi_2' - \sigma^2 K'\right] \quad (4.18)$$

where $K' = F(\pi/2, m')$

$$\Pi_2' = \Pi(\pi/2, \alpha_2^2, m')$$

Along the upper impervious portion of cutoff and floor D_1CB $\beta < \varsigma \leq \gamma$.

Integrating Eq. 4.07 in this region

$$\Phi + kH/2 = \frac{M}{2}\left[\int_{\beta^2}^{t}\frac{\sqrt{t}\,dt}{\sqrt{(t-\rho^2)(t-\beta^2)(\gamma^2-t)}} - \sigma^2\int_{\beta^2}^{t}\frac{dt}{\sqrt{t(t-\rho^2)(t-\beta^2)(\gamma^2-t)}}\right]$$

Making use of standard formulae (4, Eqs. 256.13 and 256.00)

$$\Phi + kH/2 = \frac{M_q}{2}\left[\rho^2 F(\varphi, m) + (\beta^2 - \rho^2)\,\Pi(\varphi, \alpha_3^2, m) - \sigma^2 F(\varphi, m)\right] \quad (4.19)$$

$$\text{where } \varphi = \sin^{-1}\sqrt{\frac{(\gamma^2 - \rho^2)(\zeta^2 - \beta^2)}{(\gamma^2 - \beta^2)(\zeta^2 - \rho^2)}} \quad (4.20)$$

$$\alpha_3^2 = \frac{\gamma^2 - \beta^2}{\gamma^2 - \rho^2} \quad (4.21)$$

At point B , $\Phi = 0$, $\zeta = +\gamma$ and $\varphi = \pi/2$, therefore Eq. 4.19 yields

$$\frac{kH}{2} = \frac{M_q}{2}\left[(\rho^2 - \sigma^2)K + (\beta^2 - \rho^2)\Pi_3\right] \quad (4.22)$$

where $\Pi_3 = \Pi(\pi/2, \alpha_3^2, m)$

Eliminating Mg between Eqs. 4.22 and 4.18 , we get

$$\frac{q}{kH} = \frac{\rho^2 \Pi_2' - \sigma^2 K'}{2\left[(\rho^2 - \sigma^2) K + (\beta^2 - \rho^2)\Pi_3\right]} \quad (4.23)$$

4.3 UPLIFT PRESSURES

Uplift pressure along the portion D_1 C B is given by

$$\frac{\Phi}{kH} = \frac{(\rho^2 - \sigma^2)F(\varphi, m) + (\beta^2 - \rho^2)\,\Pi(\varphi, \alpha_3^2, m)}{2\left[(\rho^2 - \sigma^2)K + (\beta^2 - \rho^2)\Pi_3\right]} - \frac{1}{2} \quad (4.24)$$

in which φ, m and α_3^2 are defined in Eqs. 4.20 , 4.10 and 4.21 .

Pressure at point C is given by

$$\frac{\Phi_c}{kH} = \frac{(\rho^2 - \sigma^2)F(\varphi_c, m) + (\beta^2 - \rho^2)\,\Pi(\varphi_c, \alpha_3^2, m)}{2\left[(\rho^2 - \sigma^2)K + (\beta^2 - \rho^2)\Pi_3\right]} - \frac{1}{2} \quad (4.25)$$

in which $\varphi_c = \sin^{-1}\sqrt{\dfrac{(\gamma^2-\rho^2)(1-\beta^2)}{(\gamma^2-\beta^2)(1-\rho^2)}}$ (4.26)

Pressure along lower impervious portion of cutoff $D D_3$, is obtained from Eq. 4.8

$$\frac{\Phi}{kH} = \frac{(\gamma^2-\sigma^2)F(\varphi, m) - \gamma^2\Pi(\varphi, \alpha_1^2, m)}{2\left[(\rho^2-\sigma^2)K + (\beta^2-\rho^2)\Pi_3\right]} - \frac{1}{2} \qquad (4.27)$$

where φ is defined by Eq. 4.09. At point D_3 where pressure is minimum on the downstream side of the cutoff, it is given by

$$\frac{\Phi_{D_3}}{kH} = \frac{(\rho^2-\sigma^2)F(\varphi_{D3}, m) - \gamma^2\Pi(\varphi_{D3}, \alpha_1^2, m)}{2\left[(\rho^2-\sigma^2)K + (\beta^2-\rho^2)\Pi_3\right]} - \frac{1}{2} \qquad (4.28)$$

where $\varphi_{D3} = \sin^{-1}\sqrt{\dfrac{(\gamma^2-\rho^2)\,\sigma^2}{\rho^2(\gamma^2-\sigma^2)}}$ (4.29)

4.4 RESULTS

The equations derived in paras 4.2 and 4.3 can be used for computations of uplift pressures and the seepage discharge through the opening in the cutoff. The values of uplift pressures at key points and the seepage discharge through the opening have been computed for different values of variables involved and plotted in the form of curves.

Pressures at the junction of floor and cutoff and seepage through the opening have been plotted in Figs. 4.2 and 4.3 for various values of d/b, locations of opening, and the ratios of opening in the cutoff to its total depth = 0.01, 0.050 and 0.10.

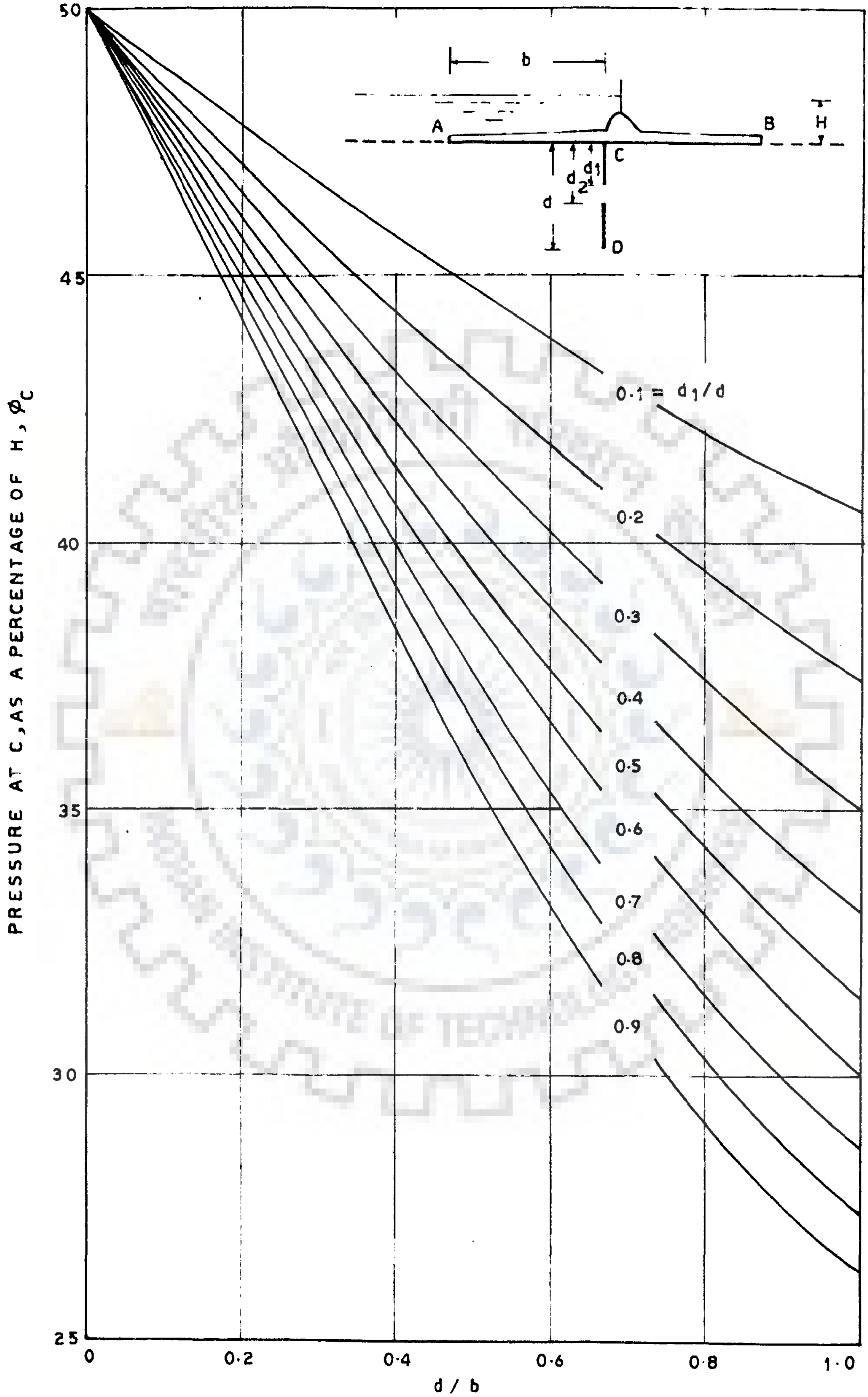


FIG. 4·2(a)– PRESSURES AT C FOR OPENING $d_2 - d_1 = 0{\cdot}01d$

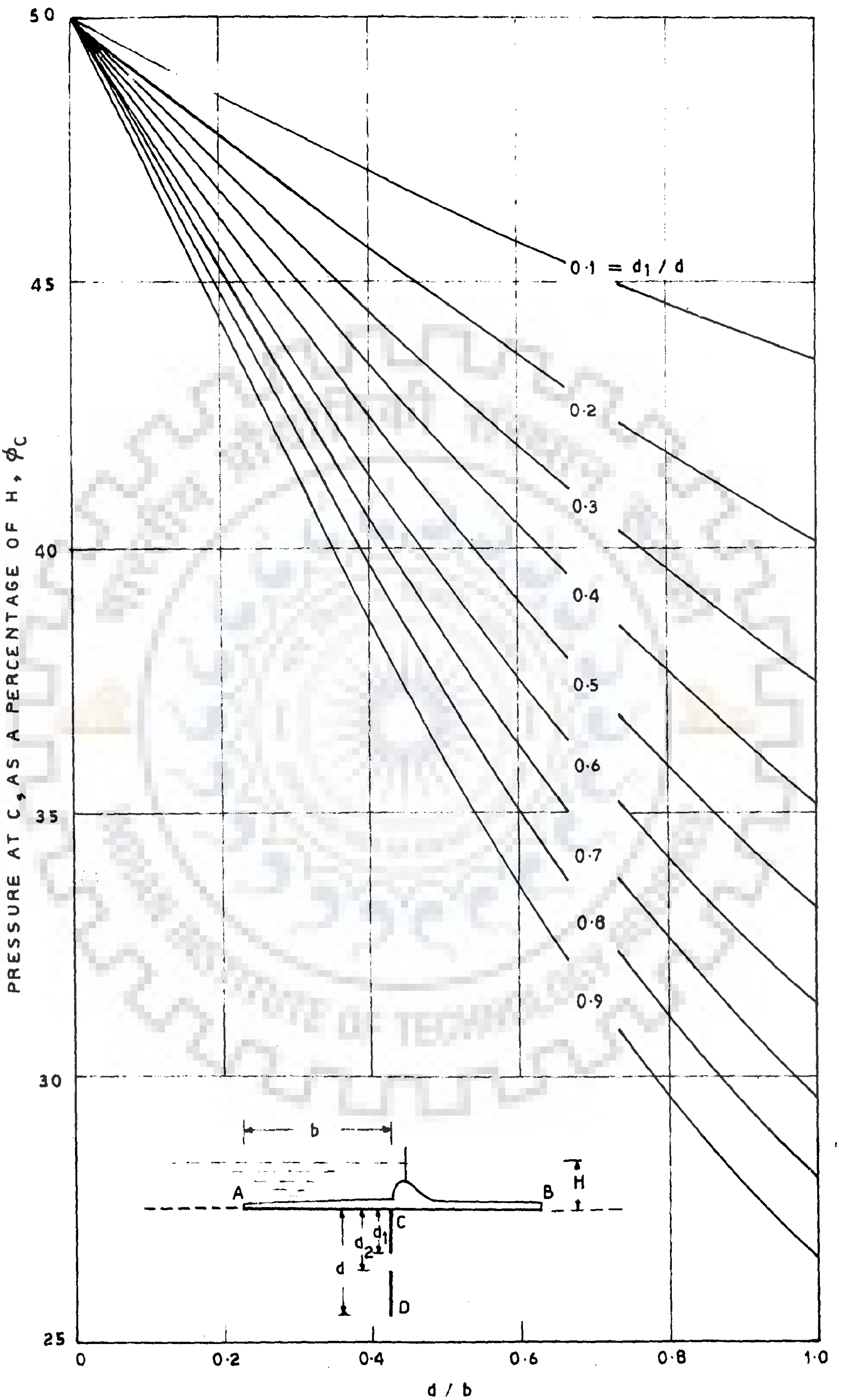


FIG 4·2 (b) —PRESSURES AT C FOR OPENING, $d_2 - d_1 = 0{\cdot}05\,d$

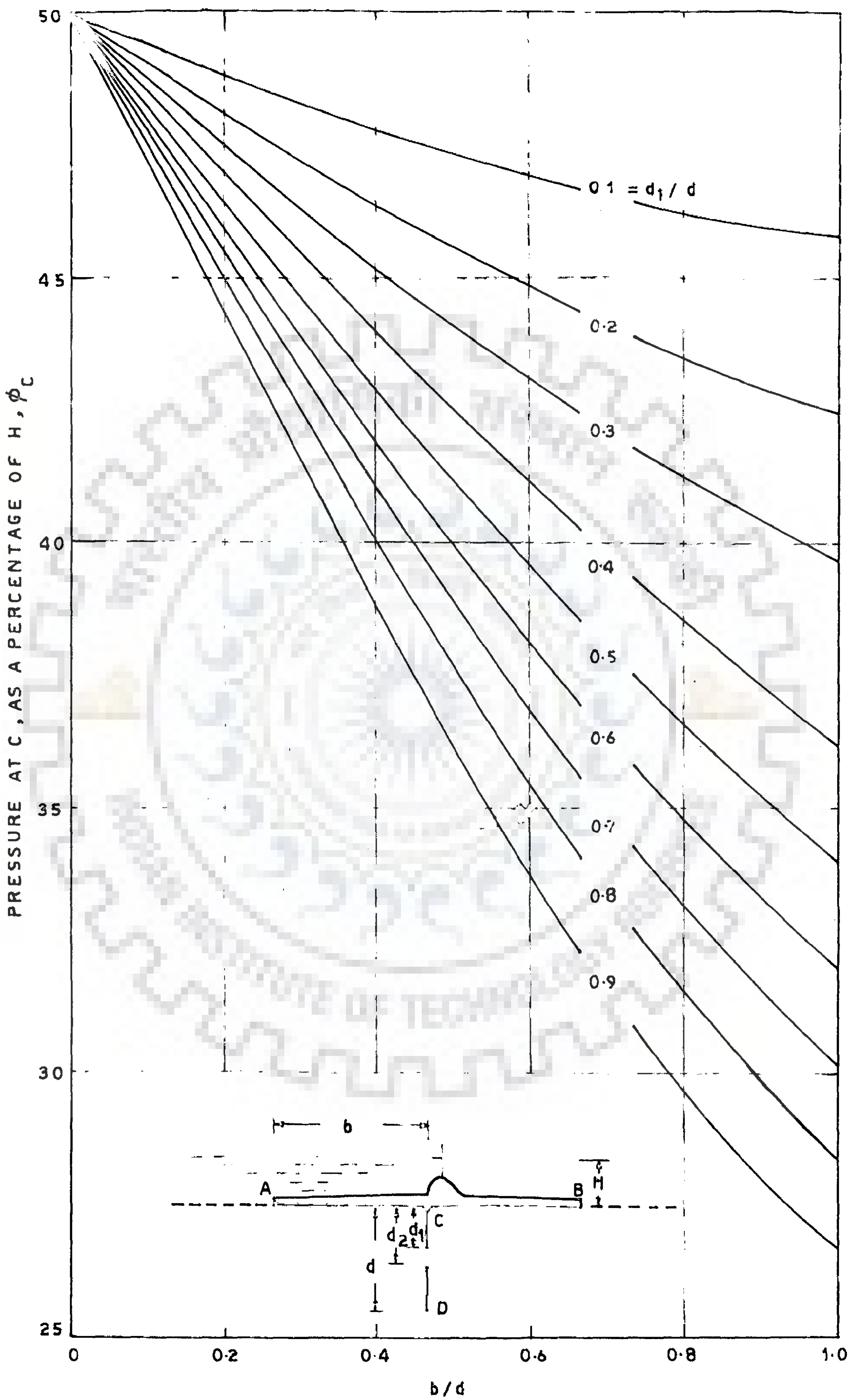


FIG 4·2 (c) - PRESSURES AT C FOR OPENING, $d_2 - d_1 = 0{\cdot}1\,d$

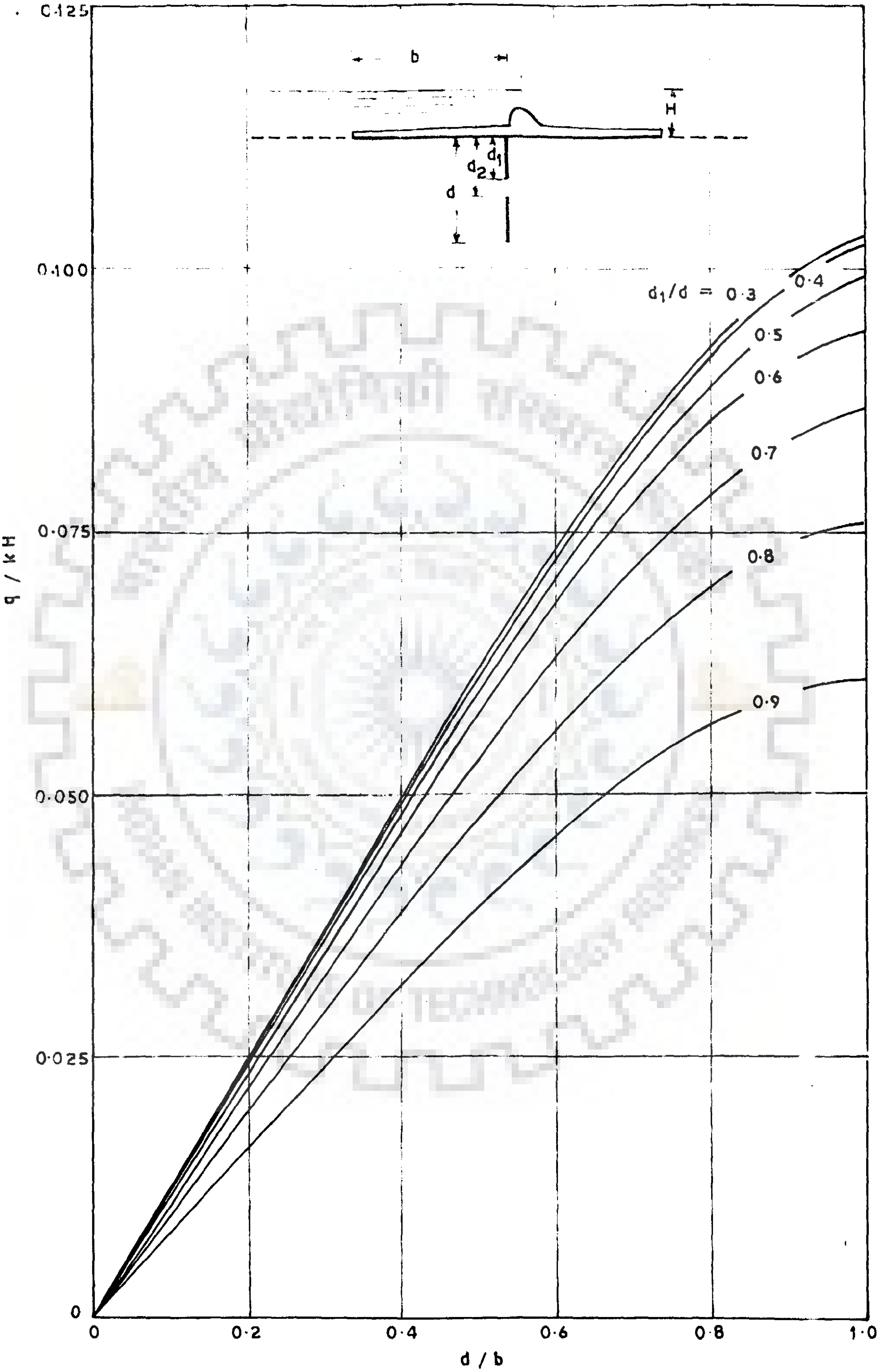


FIG. 4·3 (a) — SEEPAGE DISCHARGE FOR OPENING $d_2 - d_1 = 0{\cdot}01\,d$

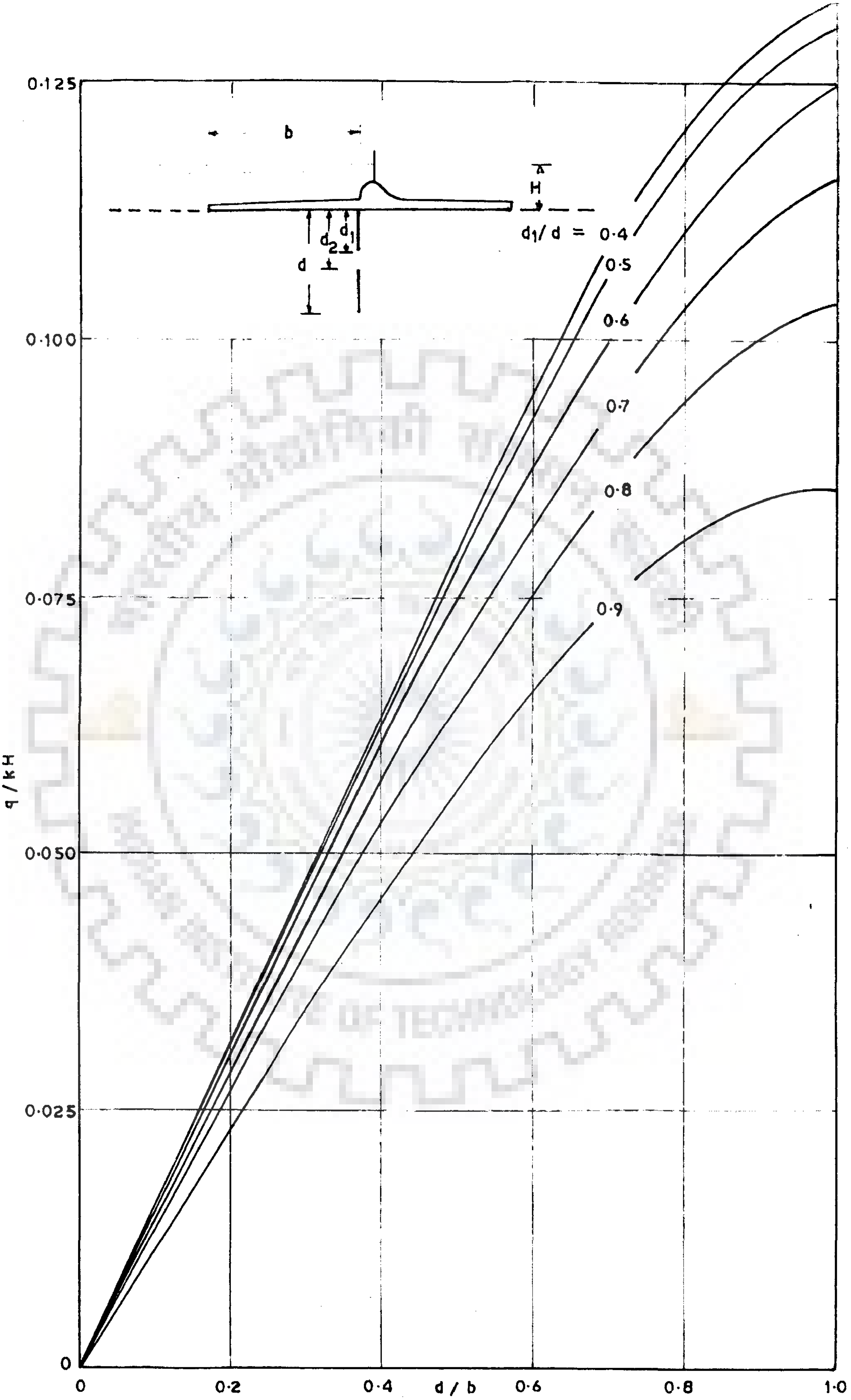


FIG. 4·3 (b)– SEEPAGE DISCHARGE FOR OPENING, $d_2 - d_1 = 0{\cdot}05\,d$

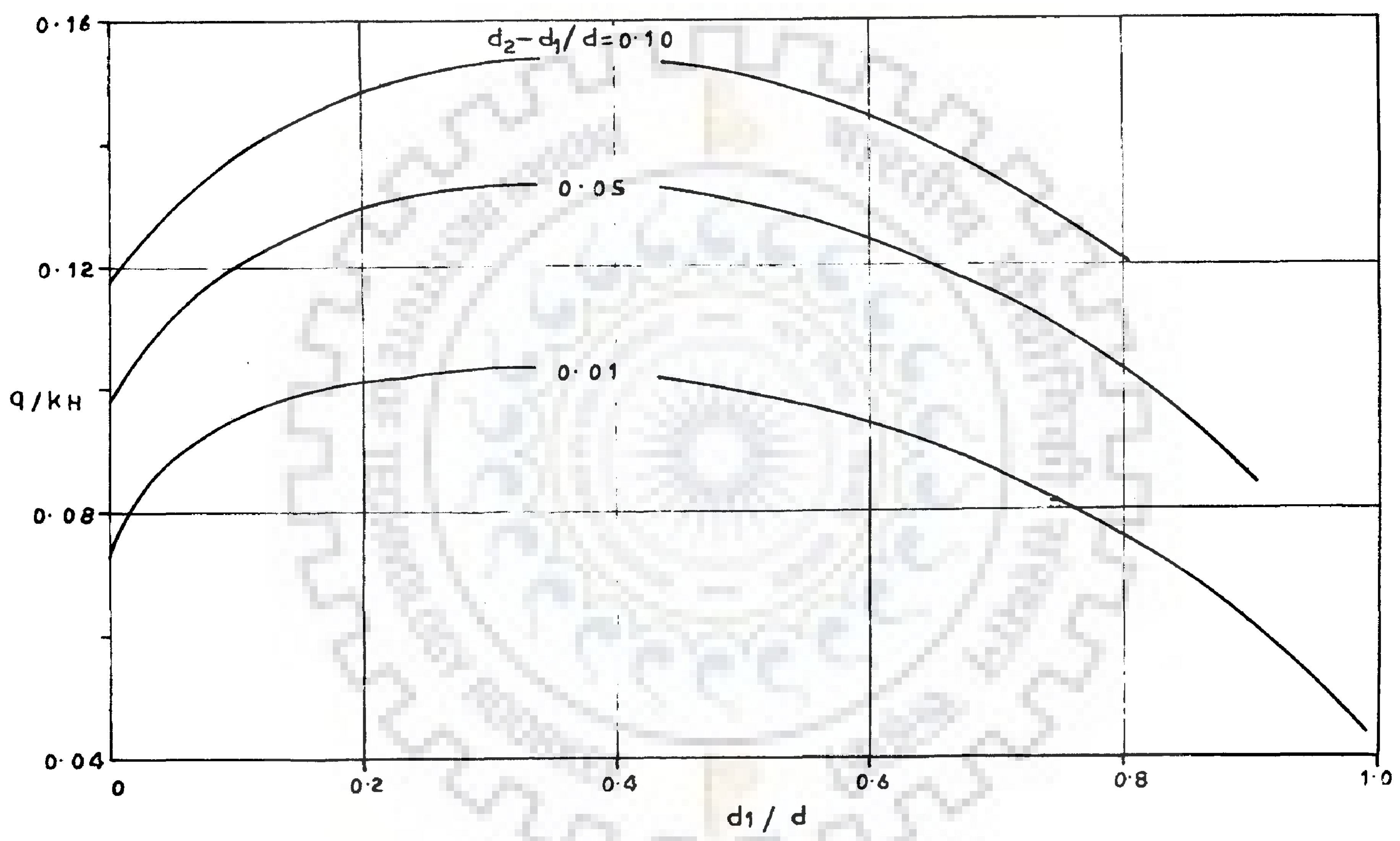


FIG. 4·4 – EFFECT OF LOCATION OF OPENING ON SEEPAGE DISCHARGE

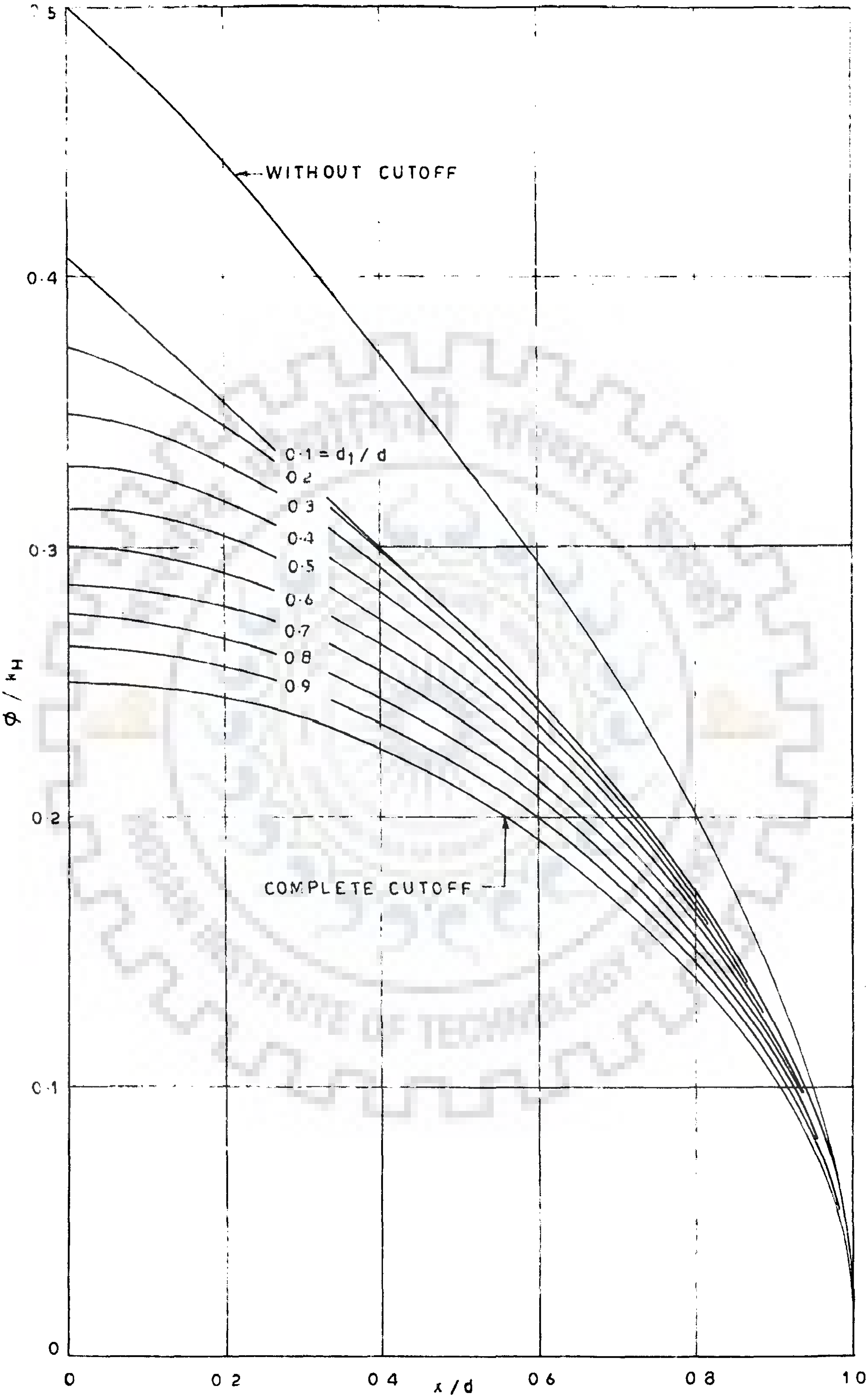


FIG. 4·5 – PRESSURE ALONG DOWN-STREAM FLOOR, $b/d = 10$, $d_2 - d_1 = 0{\cdot}1\, d$

CHAPTER V

EXPERIMENTAL VERIFICATION

CHAPTER V

EXPERIMENTAL VERIFICATION

5.1 TEST CONDITIONS

The results obtained from theoretical solution have been verified on two dimensional electrical analogy model for various boundary conditions to determine the validity of theoretical results and extend their applicability. The theoretically calculated uplift pressures were compared with those observed on model for $b/d_1 = 10$, 15 and 20, $d_2/d_1 = 1$ and 2, for various locations and lengths of filter. Since the agreement between the theoretically calculated pressures and those obtained on the model is very close, only the following illustrative examples have been given:

A. INTERMEDIATE FILTER

i)	Depth of upstream cutoff	$= d_1$
ii)	Depth of downstream cutoff	$= d_1$ and $2d_1$
iii)	Length of floor between two cutoffs	$= 10\ d_1$
iv)	Length of filter	$= 0.5 d_1$
v)	Distance of downstream end of the filter from upstream cutoff.	$= 5.98\ d_1$

B. DEEP FILTER DRAIN WITH UPSTREAM CUTOFF

i)	Depth of upstream cutoff	$= d_1$
ii)	Length of floor	$= 10\ d_1$
iii)	Depth of drain	$= 0.5\ d_1$
iv)	Distance of drain from upstream end	$= 5\ d_1$

C. DEEP FILTER DRAIN WITH DOWNSTREAM CUTOFF

i)	Depth of downstream cutoff	$= d_2$
ii)	Length of floor	$= 10\ d_2$
iii)	Depth of drain	$= 0.5\ d_2$
iv)	Distance of drain from downstream cutoff	$= 5\ d_2$

D. DEEP FILTER WITH TWO END CUTOFFS

i)	Depth of upstream cutoff	$= d_1$
ii)	Depth of downstream cutoff	$= d_2$
iii)	Length of floor between two cutoffs	$= 10\ d_1$
iv)	Depth of drain	$= 0.5\ d_1$
v)	Distance of drain from upstream cutoff	$= 5\ d_1$

5.2 EXPERIMENTAL SET UP.

5.2.1 The experimental set up consisted of two dimensional electrical analogy tray of size 1.15m x 0.56m. The model of the structure consisted of a flat floor with two end cutoffs in case A and an end cutoff at upstream or downstream end in cases B and C. The model of the structure was prepared to scale and fitted in the tray. The floor profile was made of seasoned wood and cutoffs were made of perspex. The upstream and downstream pervious reaches were simulated by 1.5mm thick copper plates. The filter or drainage was represented by copper plate fixed in the corresponding position. The boundary of the tray had a diameter of 3b so that it represented considerable depth of permeable strata below foundation and the theoretical results corresponding to the infinite depth of permeable strata could be compared with the experimental results. The depth of electrolyte was kept 2.5cm. Simple tap water was used to represent permeable foundation.

5.2.2 The surfaces of the copper plates representing upstream and downstream pervious reaches were carefully cleaned before experiments to reduce contact resistance. The probe was also made of copper to reduce the galvanic effect and any reversible potential difference. The plates were fed a low voltage (about 20V) to attenuate some of the side effects. Amplifier system was added to increase the sensitivity of the set up. The frequency of the current was raised to audio range (1000 cycles/sec) to increase the inductive reactance and the capacitive susceptance, and to improve measurement accuracy.

Moreover, audio frequencies eliminate electrolysis and reduce the effects of polarization, contact resistances on the plates, and other side phenomena. The model and experimental set up are shown in Fig.5.1.

5.3 RESULTS

5.3.1 The theoretically calculated uplift pressures and those observed on electrical analogy model for the case A, when the floor has two end cutoffs and an intermediate filter are given in Table 5.1 and plotted in Fig. 5.2. The calculated and observed uplift pressures for the cases B and C, when the floor has cutoff at the upstream or at the downstream end and a deep drainage are given in Table 5.2 and plotted in Fig. 5.3. A perusal of Table 5.1 and 5.2 and Figs. 5.2 to 5.3 indicates that the observed pressures are very close to the theoretically calculated values.

5.3.2 The uplift pressures have also been measured for the case when the floor has two end cutoffs and a deep drainage at a distance of 5 d_1 from the upstream end and given in Table 5.2. No theoretical solution has been worked out for these boundary conditions when floor has two end cutoffs and a deep drainage in between. However, it has been seen that the theoretically calculated pressures along the floor and cutoff upstream of the drainage without downstream cutoff, are close to those observed at the corresponding points with two end cutoffs. (See Table 5.2 Fig. 5.4). The calculated pressures along the floor and cutoff downstream of the drainage without upstream cutoff, differ slightly from those observed at the corresponding points with two end cutoffs.

TABLE 5.1

VERIFICATION OF THEORETICAL PRESSURES
INTERMEDIATE FILTER

Test Conditions:

i)	Depth of upstream cutoff	$= d_1$
ii)	Depth of downstream cutoff	$= d_2$
iii)	Length of floor between cutoffs	$= 10\ d_1$
iv)	Length of filter	$= 0.5\ d_1$
v)	Distance of filter from upstream end	$= 5.98\ d_1$

Points	Coordinates with origin at upstream floor end		Pressures with $d_2/d_1 = 1.0$		Pressures with $d_2/d_1 = 2.0$	
	x/d_1	y/d_1	Theoretically calculated	Observed on model	Theoretically calculated	Observed on model
	0.0	0.435	0.926	0.920	0.927	0.925
	0.0	0.599	0.895	0.890	0.896	0.890
D'	0.0	1.000	0.760	0.744	0.763	0.747
	0.0	0.601	0.670	0.666	0.674	0.666
	0.0	0.437	0.660	0.660	0.664	0.660
C'	0.0	0.000	0.650	0.635	0.654	0.638
	3.9	0.000	0.361	0.353	0.366	0.353
	5.27	0.000	0.143	0.135	0.143	0.145
	6.18	0.000	0.099	0.100	0.111	0.111
	8.75	0.000	0.170	0.160	0.209	0.193
E'	10.00	0.000	0.156	0.150	0.207	0.210
	0.00	0.436	0.1526	0.150	0.206	0.190
	0.00	0.600	0.1493	0.146	0.200	0.19
D	0.00	1.000	0.1149	0.117	0.162	0.160
	0.00	0.600	0.0525	0.050	0.074	0.074
	0.00	0.414	0.0353	0.035	0.050	0.050

TABLE 5.2

VERIFICATION OF THEORETICAL PRESSURES
DEEP DRAIN

Points	Coordinates with origin at upstream floor end		Pressures with upstream end cutoff $b/d_1=10, b_1/d_1=5, f/d_1=0.5$		Pressures with downstream end cutoff $b/d_2=10, b_1/d_2=5, f/d_2=0.5$		Observed pressures wi' two end cutoffs for $b/d_1=10$, $b_1/d_1=5, d_2/d$ $f/d_1=0.5$
	x/d_1 or x/d_2	y/d_1 or y/d_2	Calculated	Observed on model	Calculated	Observed on model	
	0.0	0.436	0.918	0.913	-	-	0.910
	0.0	0.600	0.884	0.875	-	-	0.880
	0.0	0.714	0.858	0.848	-	-	0.845
D'	0.0	1.000	0.734	0.713	-	-	0.741
	0.0	0.714	0.644	0.630	-	-	0.620
	0.0	0.600	0.632	0.616	-	-	0.602
	0.0	0.436	0.621	0.603	-	-	0.592
C'	0.0	0.000	0.609	0.594	1.000	1.000	0.587
	1.0	0.000	0.553	0.553	0.700	0.713	0.542
	3.0	0.000	0.368	0.340	0.427	0.430	0.350
	4.0	0.000	0.227	0.240	0.280	0.280	0.211
	5.0	0.000	0.000	0.000	0.000	0.000	0.000
	6.5	0.000	0.140	0.130	0.163	0.164	0.139
	7.0	0.000	0.146	0.135	0.180	0.174	0.149
	8.85	0.000	0.110	0.100	0.160	0.163	0.137
	9.85	0.000	0.042	0.035	0.153	0.150	0.128
E	10.00	0.000	0.000	0.000	0.153	0.148	0.128
	10.00	0.436	-	-	0.150	0.144	0.126
	10.00	0.600	-	-	0.147	0.140	0.124
	10.00	0.714	-	-	0.143	0.135	0.121
D	10.00	1.000	-	-	0.113	0.107	0.097
	10.00	0.714	-	-	0.063	0.059	0.049
	10.00	0.600	-	-	0.051	0.048	0.041
	10.00	0.436	-	-	0.036	0.034	0.029

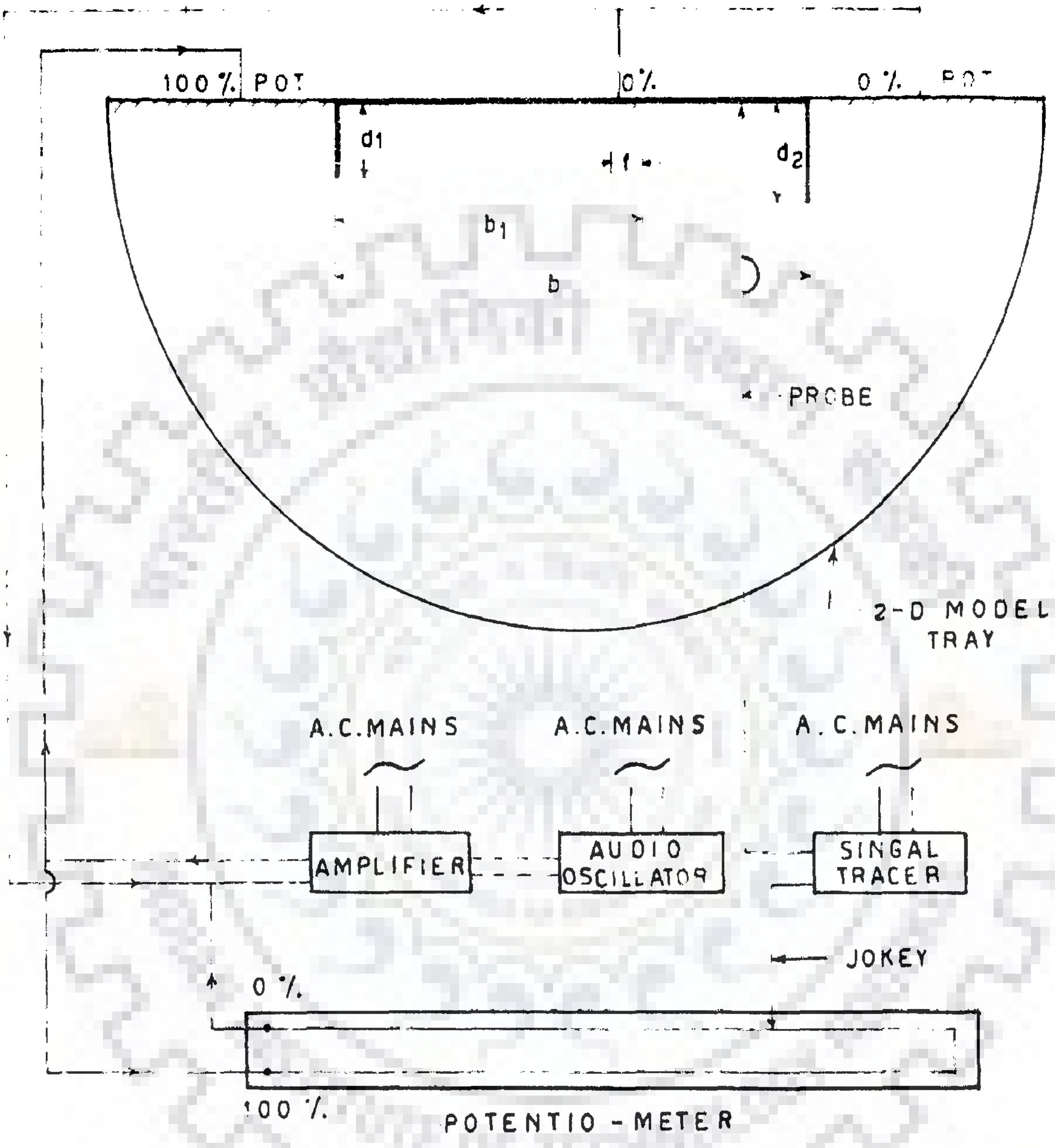


FIG. 5·1 - EXPRIMENTAL SETUP

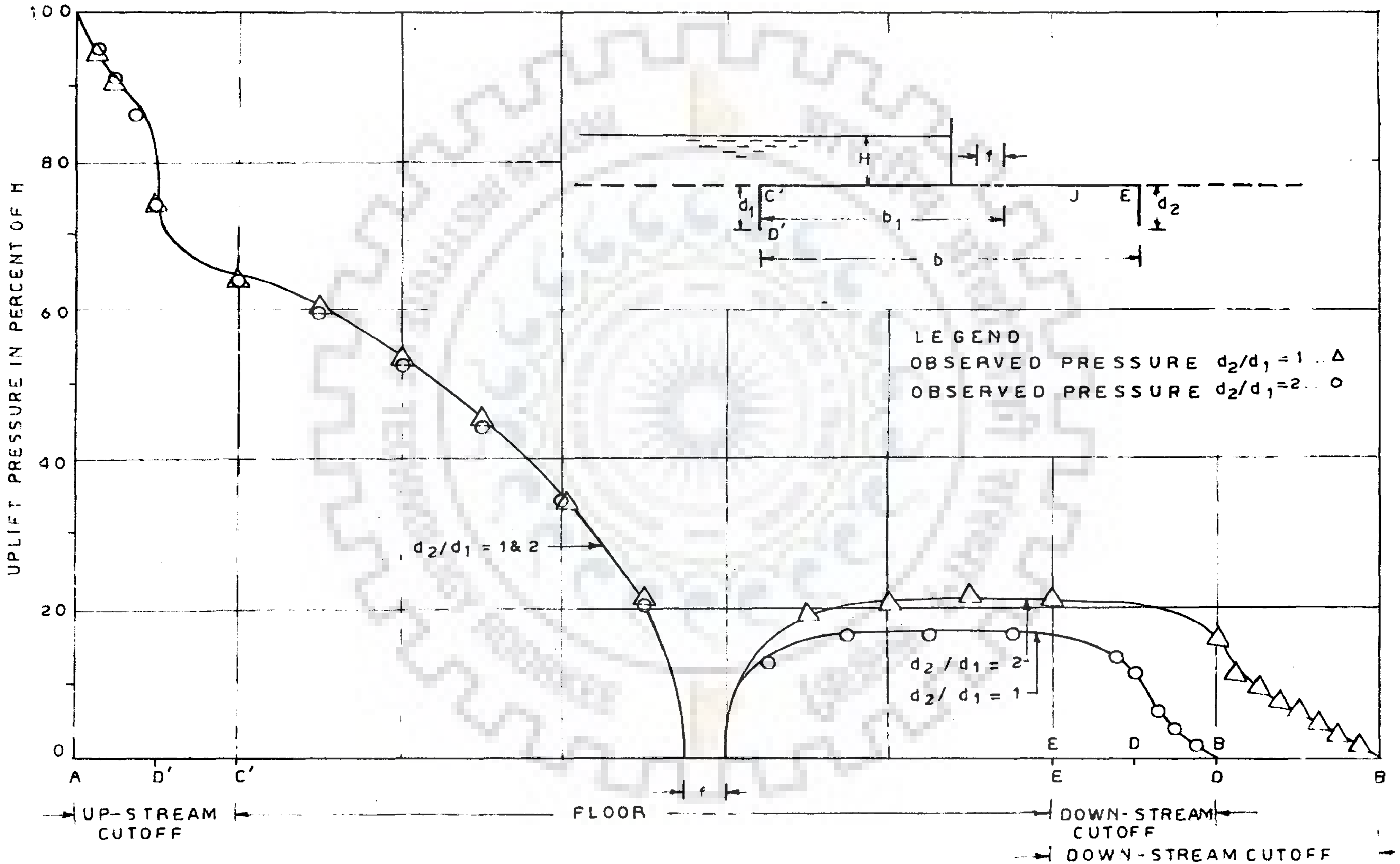


FIG 5.2 - COMPARISON OF CALCULATED AND OBSERVED PRESSURES

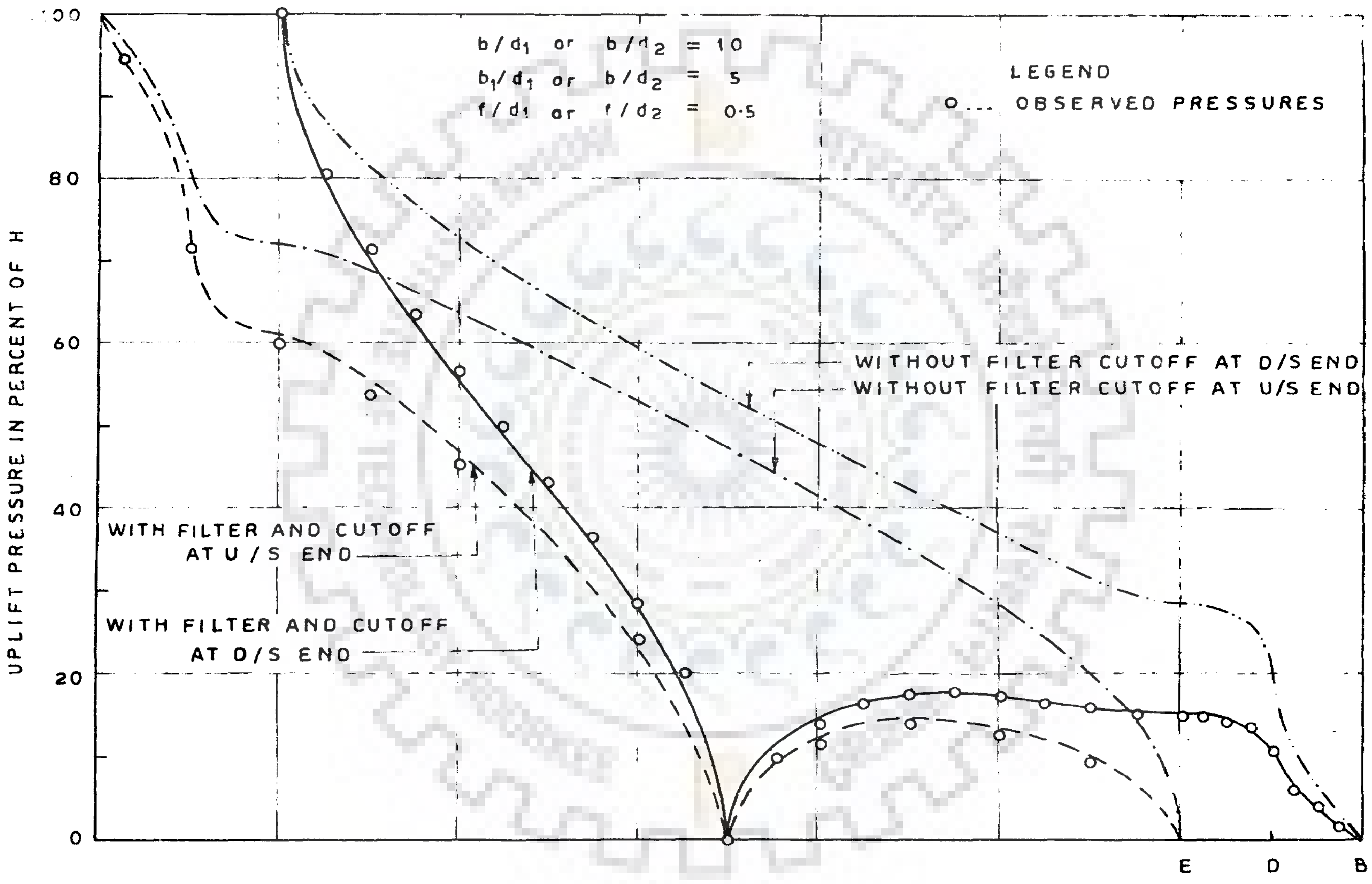


FIG. 5·3 — COMPARISON OF CALCULATED AND OBSERVED PRESSURES

CHAPTER VI

DISCUSSIONS

TABLE 6.1

EFFECT OF LENGTH OF FILTER

	Coordinates with origin at the junction of upstream cutoff and floor		Uplift pressures with filter $b/d_1=10, b_1/d_1=6.0, d_2/d_1=1.0$			Uplift pressures without filter
	x/d_1	y/d_1	$f/d_1=0.05$	0.5	1.0	
	0.00	0.436	0.932	0.926	0.922	0.942
		0.600	0.903	0.895	0.889	0.919
		0.714	0.881	0.871	0.863	0.899
D'		1.000	0.780	0.760	0.745	0.814
		0.714	0.709	0.681	0.660	0.755
		0.600	0.699	0.670	0.649	0.748
		0.436	0.690	0.660	0.638	0.741
C'	0.00	0.000	0.681	0.650	0.627	0.735
	2.834	0.000	0.520	0.464	0.421	0.611
	4.923	0.000	0.341	0.227	0.084	0.504
	6.000	0.000	0.000	0.000	0.000	0.449
	6.168	0.000	0.167	0.094	0.075	0.441
	7.871	0.000	0.232	0.180	0.157	0.351
E'	10.000	0.000	0.190	0.156	0.140	0.265
		0.436	0.186	0.153	0.137	0.258
		0.600	0.181	0.149	0.134	0.251
D		1.000	0.137	0.115	0.104	0.186
		0.600	0.062	0.052	0.048	0.082
		0.414	0.041	0.035	0.032	0.055
		0.312	0.031	0.026	0.024	0.041

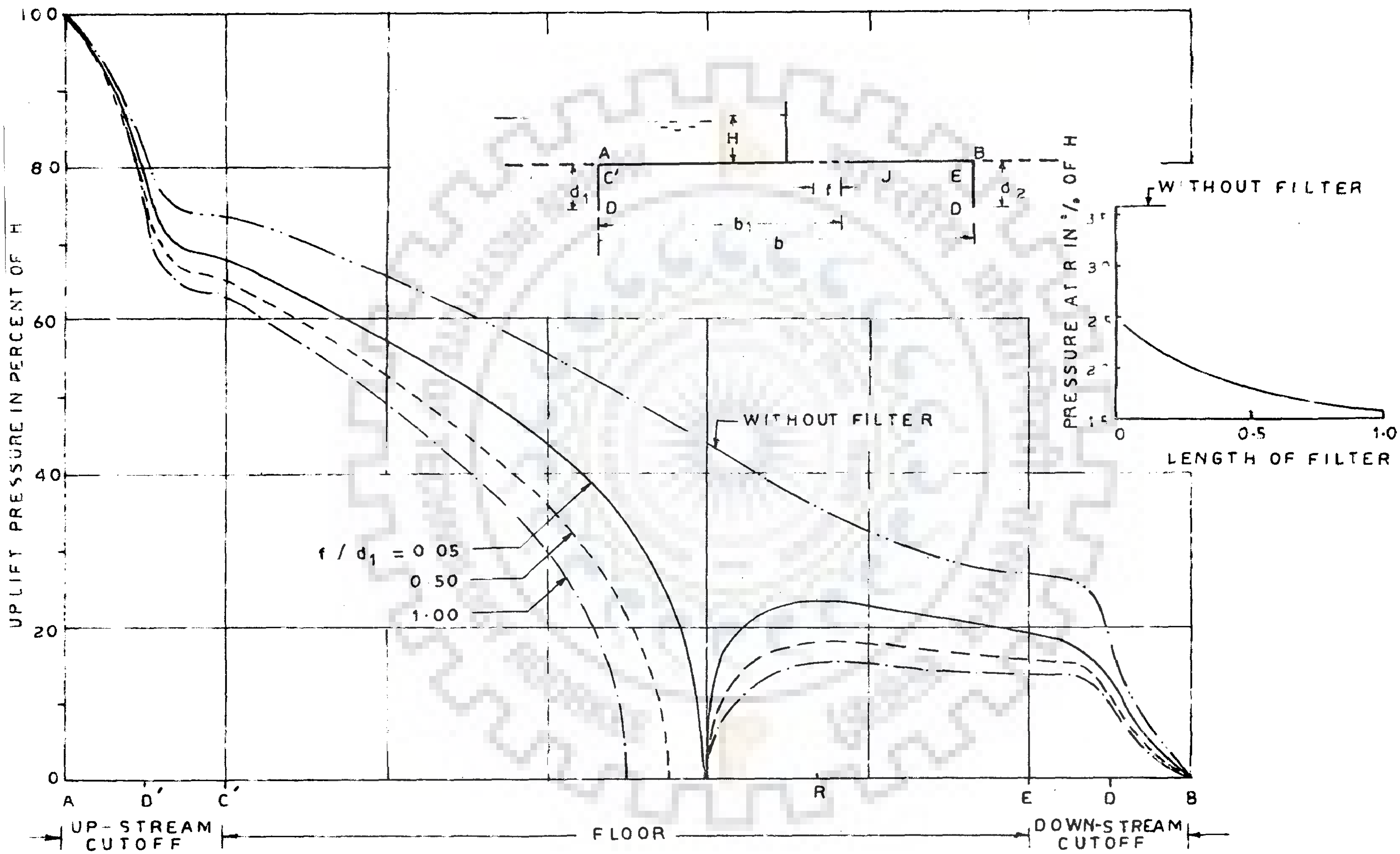


FIG. 6·1 — EFFECT OF LENGTH OF FILTER $b/d_1 = 10$, $b_1/d_1 = 5$, $d_2/d_1 = 1$

and 29.0 to 12.52%, of the total differential head respectively. The decrease in pressure at a distance of 7.5 d_1 from upstream cutoff (the approximate location of maximum rise of pressure downstream of the filter) has been plotted against length of filter in Fig. 6.1(a), which indicates that with the provision of filter, pressures reduce considerably from those without filter but the decrease in pressures with subsequent increase in the length of filter is less. The uplift pressures also decrease along the floor upstream of the filter.

6.1.2 Effect of filter location: In order to determine the effect of location of filter the uplift pressures for b/d_1 =10 d_2/d_1=1.0, f/d_1=0.5, and b_1/d_1=4.0, 5.0 and 5.77 have been given in Table 6.2 and plotted in Fig. 6.2. The uplift pressures without filter have also been given. A perusal of this figure indicates that uplift pressures decrease on the downstream side and increase on the upstream side as the filter is moved from upstream to the downstream side. For examples, in the case presented in Fig. 6.2, the maximum pressure downstream of the filter changes from 23.7% to 17.99% as the filter is moved from 4.0d_1 to 5.77 d_1 from the upstream end. The pressures along the outer faces of the cutoffs are not affected appreciably with the change in location of filter. The filter should, therefore, be located downstream of the gateline such that the total area of uplift on the floor downstream of the gateline is minimum. A few trial locations would give the optimum location of the filter. A perusal of Fig. 2.2 to 2.5 also indicates that uplift pressures at the tip of downstream cutoff decrease with the movement of filter towards E; however, this trend continues upto a certain point beyond which the pressure at D starts increasing if filter is moved further downstream.

6.1.3 Effect of downstream cutoff depth: In order to determine the effect of depth of downstream cutoff, the uplift pressures have been

TABLE 6.2

EFFECT OF LOCATION OF FILTER

	Coordinates with origin at the junction of upstream cutoff and floor		Calculated pressures with filter $b/d_1=10$, $d_2/d_1=1.0$, $f/d_1 = 0.5$			Uplift pressures without filter
	x/d_1	y/d_1	$b_1/d_1 = 4.0$	5.0	6.0	
	0.00	0.436	0.915	0.921	0.926	0.942
		0.600	0.879	0.888	0.895	0.919
		0.714	0.851	0.862	0.871	0.899
D'		1.000	0.720	0.743	0.760	0.814
		0.714	0.623	0.656	0.681	0.755
		0.600	0.610	0.645	0.670	0.748
		0.436	0.597	0.634	0.660	0.741
C'	0.00	0.000	0.584	0.623	0.650	0.735
	2.834	-	0.293	0.404	0.464	0.611
	4.000	-	0.000	0.234	0.361	0.575
	5.040	-	0.218	0.051	0.220	0.495
	6.000	-	0.236	0.100	0.000	0.449
	6.168	-	0.237	0.170	0.094	0.441
	7.871	-	0.215	0.200	0.180	0.351
E	10.00	0.000	0.172	0.165	0.156	0.265
		0.436	0.168	0.161	0.153	0.258
		0.600	0.164	0.157	0.149	0.251
D		1.000	0.124	0.120	0.115	0.186
		0.600	0.056	0.054	0.052	0.082
		0.414	0.037	0.036	0.035	0.055
		0.312	0.028	0.027	0.026	0.041

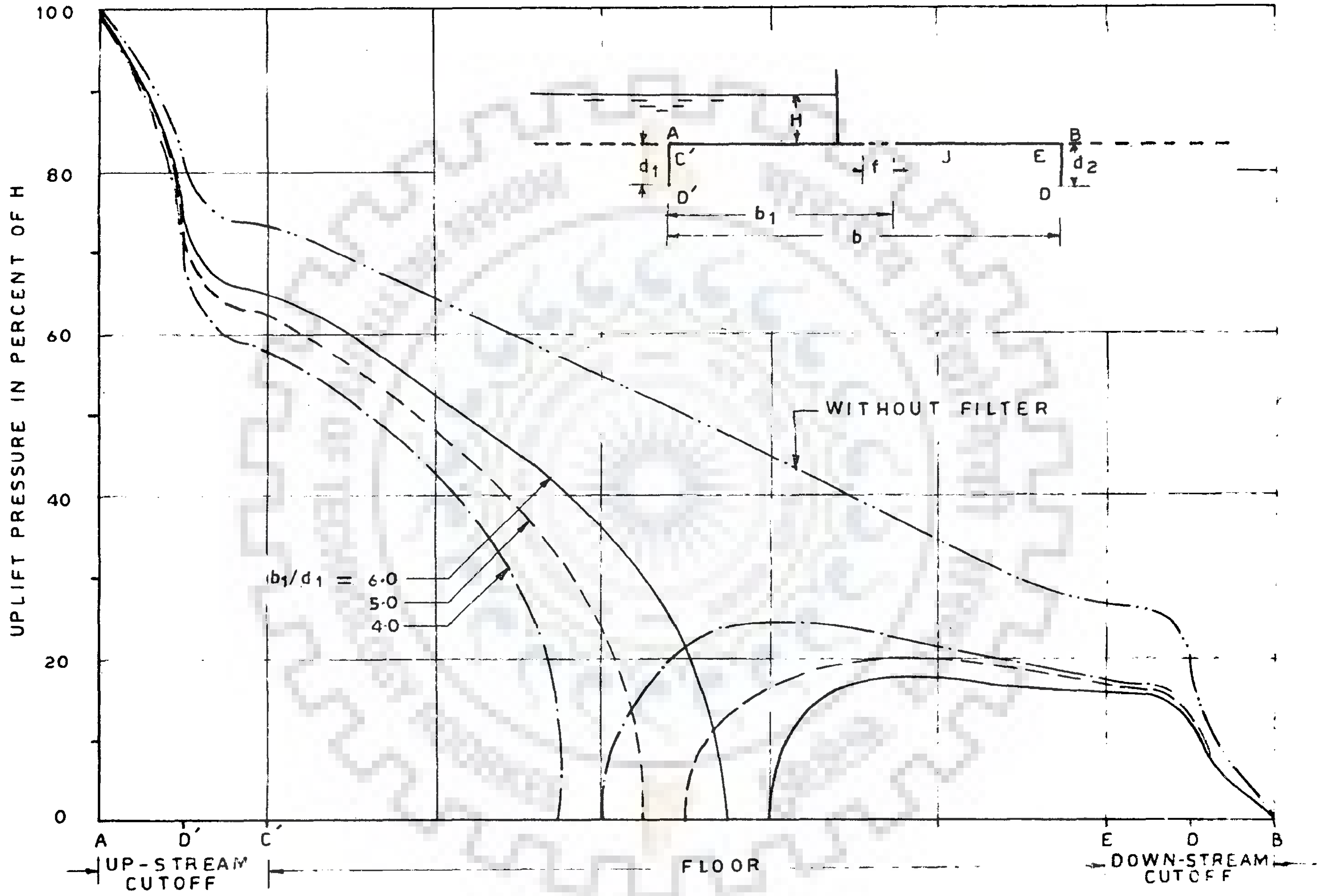


FIG. 6·2 —EFFECT OF LOCATION OF FILTER $b/d_1 = 10$, $f/d_1 = 0{\cdot}5$, $d_2/d_1 = 1{\cdot}0$

calculated for $b/d_1=10$, $f/d_1= 0.5$, $b_1/d_1 = 6.0$, and two different values of d_2/d_1 viz. $d_2/d_1=1.0$ and 2.0. The calculated pressures with filter and without filter have been given in Table 6.3 and plotted in Fig. 6.3. A perusal of the figure indicates that uplift pressures increase with the increase in the depth of downstream cutoff only along the floor downstream of the filter. The uplift pressures along the floor and cutoff upstream of the filter are practically uneffected by the increase in the depth of downstream cutoff. With the increase in depth of downstream cutoff from d_1 to $2d_1$, the uplift pressures along the downstream portion of the floor increase by upto 5.1%, whereas the maximum difference upstream of the filter is 0.4% only. This indicates that the cutoff on the otherside of the filter can be neglected for computing uplift pressures below floor.

6.1.4 The maximum pressures along the downstream portion of floor and the downstream cutoff occur at J. The maximum pressure reduces intially when filter is moved towards E. But after a certain location of filter it starts increasing as in case of pressure at D. However, this trend does not have design significance as the reversal occures when the point of maximum pressure has already shiffed from the floor to the inner face of the cutoff.

6.2 DEEP DRAIN

A deep drain would consist of a continuous trench of small width filled back with pervious material. As in case of intermediate filter, the pressures rise again downstream of the deep drain depending on its depth and location. The influence of both these factors has been studied

The theoretical study has been made for one cutoff with deep drain either at the upstream end or at the downstream end. When there are are two cutoffs, the results can be superimposed with negligible error as shown later.

TABLE 6.3

EFFECT OF DEPTH OF DOWNSTREAM CUTOFF

Key points	Coordinates with origin at the junction of upstream cutoff and floor		Calculated Pressures $b/d_1=10, b_1/d_1=5.98$			
			$d_2/d_1=10$		$d_2/d_1=2.0$	
	x/d_1	y/d_1	without filter	with filter $f/d_1=0.4985$	without filter	with filter $f/d_1=0.4985$
	-	0.435	0.942	0.926	0.945	0.927
		0.599	0.919	0.895	0.922	0.896
D'		1.000	0.814	0.760	0.825	0.763
		0.601	0.748	0.670	0.762	0.674
		0.437	0.741	0.660	0.755	0.664
C'	0.000	0.000	0.735	0.650	0.749	0.654
	3.900	-	0.555	0.361	0.583	0.366
	5.270	-	0.486	0.143	0.525	0.143
	6.180	-	0.439	0.099	0.483	0.111
	8.750	-	0.310	0.170	0.385	0.209
E	10.000	-	0.265	0.156	0.370	0.207
D	10.00	1.0 or 2.0	0.186	0.115	0.256	0.162

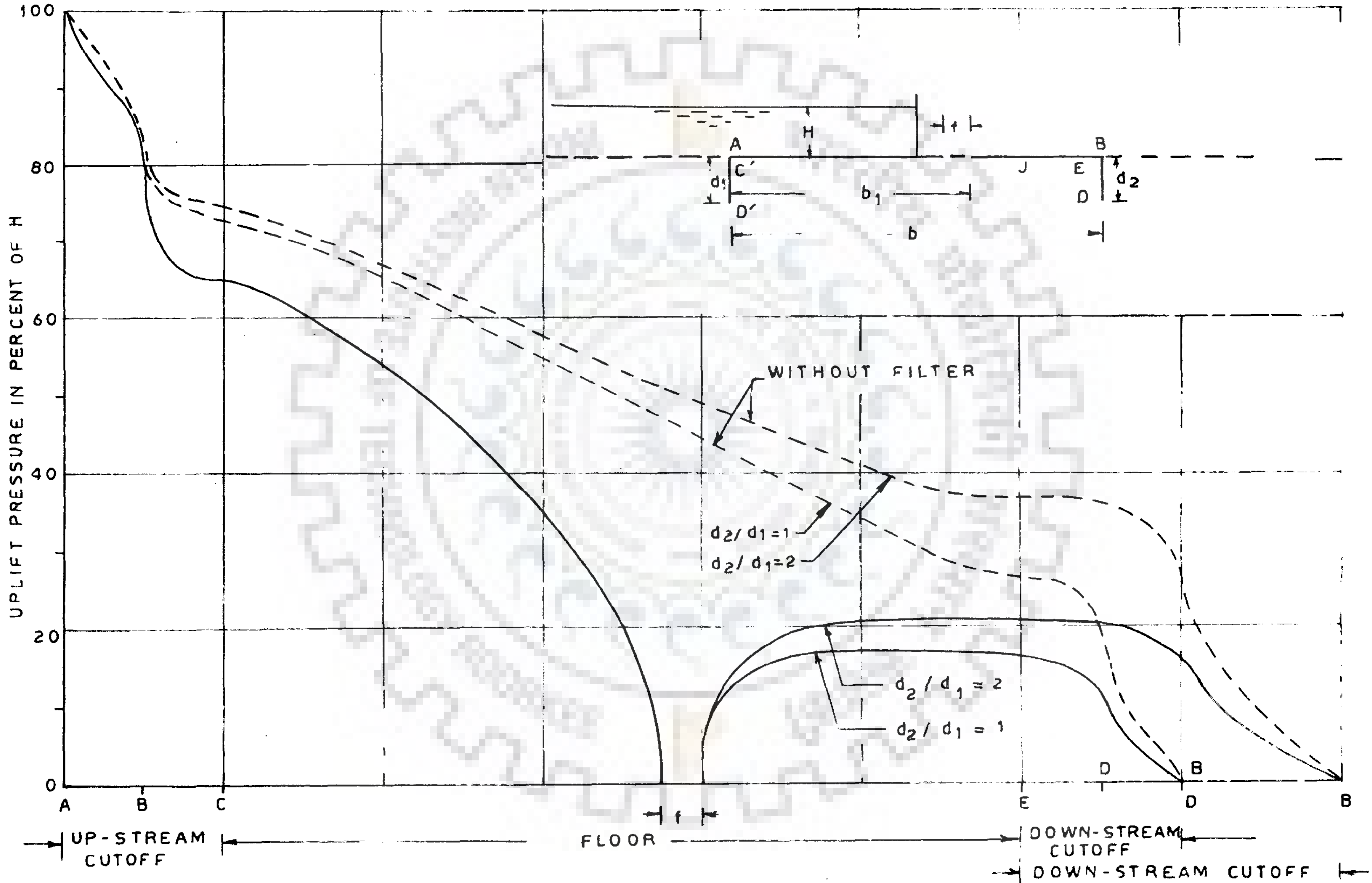


FIG. 6·3 – EFFECT OF DEPTH OF DOWN-STREAM CUTOFF
$b/d_1=10$, $b_1/d_1=6$, $f/d_1=0.5$

A. Upstream cutoff.

6.2.1 Effect of depth of drain: It is seen from Figs. 3.3 and 3.4 that the uplift pressures at C' and D' i.e. at the junction of floor and upstream cutoff and at the tip of the cutoff decrease with the increase in the depth of drainage. The uplift pressures along the upstream cutoff and floor, for b/d_1 =10, b_1/d_1 = 5.0, and f/d_1 = 0.2, 0.5 and 1.0 have been given in Table 6.4 and plotted in Fig.6.4 to determine the effect of increase in the depth of drainage over the entire length of floor. A perusal of Fig. 6.4 indicates that with the increase in the depth of drainage the pressures decrease both along the floor and the cutoff. The reduction in pressures is more in the downstream portion of floor than in the upstream portion. As in the case of horizontal filter the provision of drainage reduces pressures considerably from those when no drainage is provided. The decrease in pressures with subsequent increase in depth of drainage in less. This is clearly indicated by the relation between f/d_1, and the decrease in pressures plotted in Fig. 6.4 (a).

6.2.2 Effect of location: The uplift pressures along the floor and the cutoff for b/d_1 = 10, f/d_1= 0.5 and b_1/d_1= 4.0, 5.0 and 5.77 have been given in Table 6.5 and plotted in Fig. 6.5. It is seen that uplift pressures increase on the floor upstream of the drainage and decrease on the floor downstream of it, with the shifting of drainage towards downstream. The increase in the upstream portion is more than the decrease on the downstream portion.

B. Downstream cutoff:

6.2.3 Effect of depth: The uplift pressures along the floor and cutoff for the case when cutoff is provided at the downstream end of the floor, for b/d_2 = 10, b_1/d_2 = 5.0 and different depths of drain, f/d_2 = 0.2, 0.5 and 1.0 have been given in Table 6.6 and plotted in Fig. 6.6. A perusal o

TABLE 6.4

EFFECT OF DEPTH OF DRAINAGE WITH UPSTREAM CUTOFF

	Coordinates with origin at the junction of upstream cutoff and floor		Calculated pressures with drainage $b/d_1=10$, $b_1/d_1 = 5.0$			Calculated pressures with out drainage
	x/d_1	y/d_1	$f/d_1=0.2$	0.5	1.0	
	0.000	0.436	0.922	0.918	0.914	0.939
		0.600	0.890	0.884	0.878	0.914
		0.714	0.865	0.857	0.850	0.849
D'		1.000	0.748	0.734	0.720	0.806
		0.714	0.663	0.644	0.625	0.769
		0.600	0.652	0.632	0.612	0.736
		0.436	0.642	0.621	0.600	0.728
C'	0.000	0.000	0.631	0.609	0.588	0.721
	3.000	-	0.410	0.368	0.333	0.579
	4.000	-	0.267	0.227	0.194	0.523
	5.000	-	0.000	0.000	0.000	0.467
	6.500	-	0.185	0.140	0.103	0.382
	7.000	-	0.186	0.146	0.113	0.353
	8.850	-	0.130	0.110	0.090	0.205
	9.850	-	0.043	0.042	0.040	0.057
B	10.00	-	0.000	0.000	0.000	0.000

TABLE 6.5

EFFECT OF LOCATION OF DRAINAGE WITH UPSTREAM CUTOFF

	Coordinates of points with origin at the junction of upstream cutoff and floor		Calculated pressures with drainage $b/d_1 = 10$, $f/d_1 = 0.5$			Calculated pressures without drainage
	x/d_1	y/d_1	$b_1/d_1 = 4.02$	5.0	5.77	
	0.000	0.436	0.911	0.918	0.922	0.939
		0.600	0.874	0.884	0.890	0.914
		0.714	0.845	0.857	0.865	0.849
D'		1.000	0.710	0.734	0.749	0.806
		0.714	0.609	0.644	0.665	0.769
		0.600	0.595	0.632	0.654	0.736
		0.436	0.582	0.621	0.644	0.728
C'	0.000	0.000	0.569	0.609	0.633	0.721
	3.000	-	0.263	0.368	0.420	0.576
	4.000	-	0.000	0.227	0.316	0.523
	5.000	-	0.133	0.000	0.183	0.467
	6.500	-	0.170	0.140	0.900	0.382
	7.000	-	0.168	0.146	0.110	0.353
	8.850	-	0.118	0.110	0.105	0.205
	9.850	-	0.045	0.042	0.042	0.057
B	10.000	-	0.000	0.000	0.000	0.000

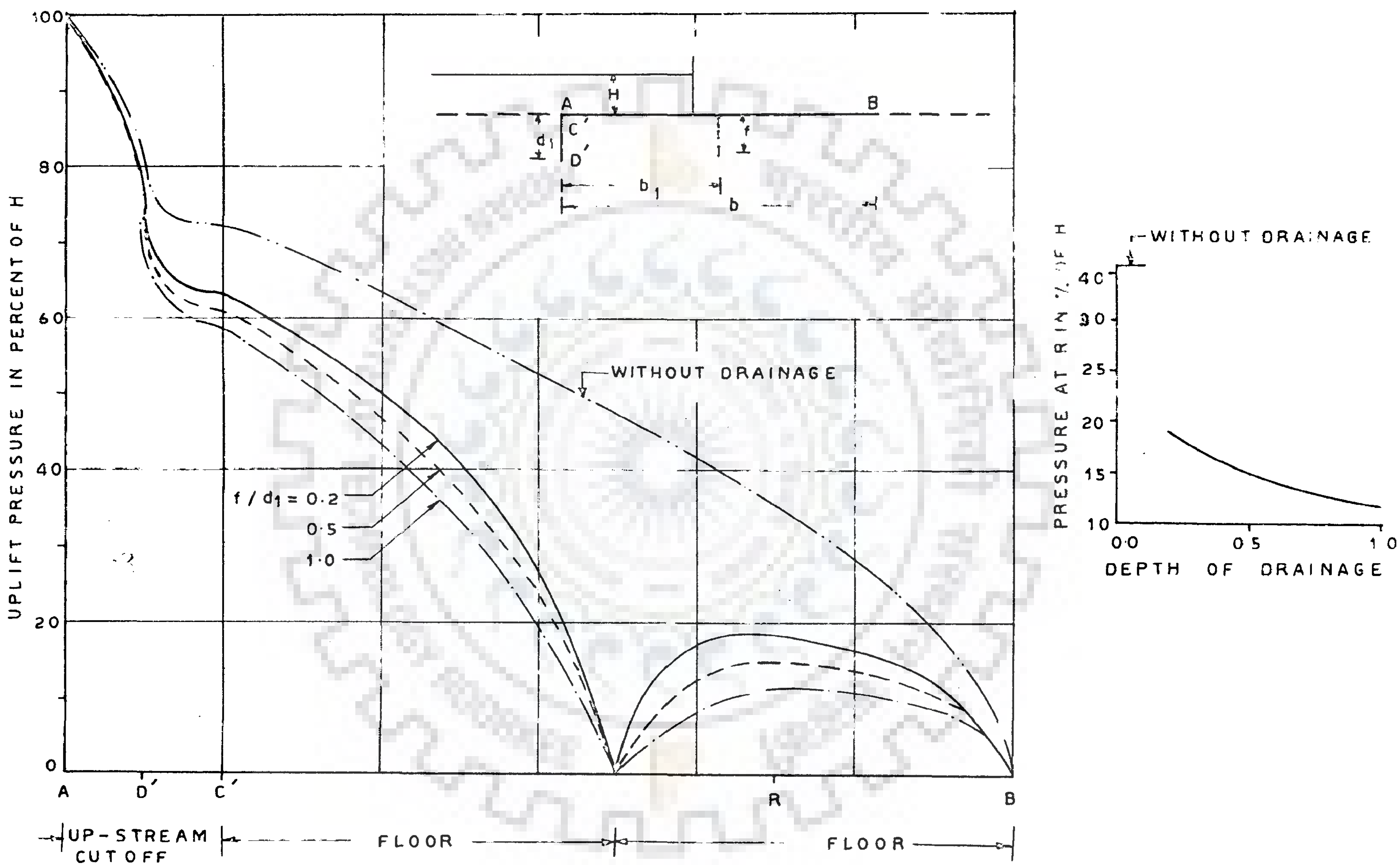


FIG. 6·4 — EFFECT OF DEPTH OF DRAINAGE $b / d_1 = 10$, $b_1/d_1 = 5$

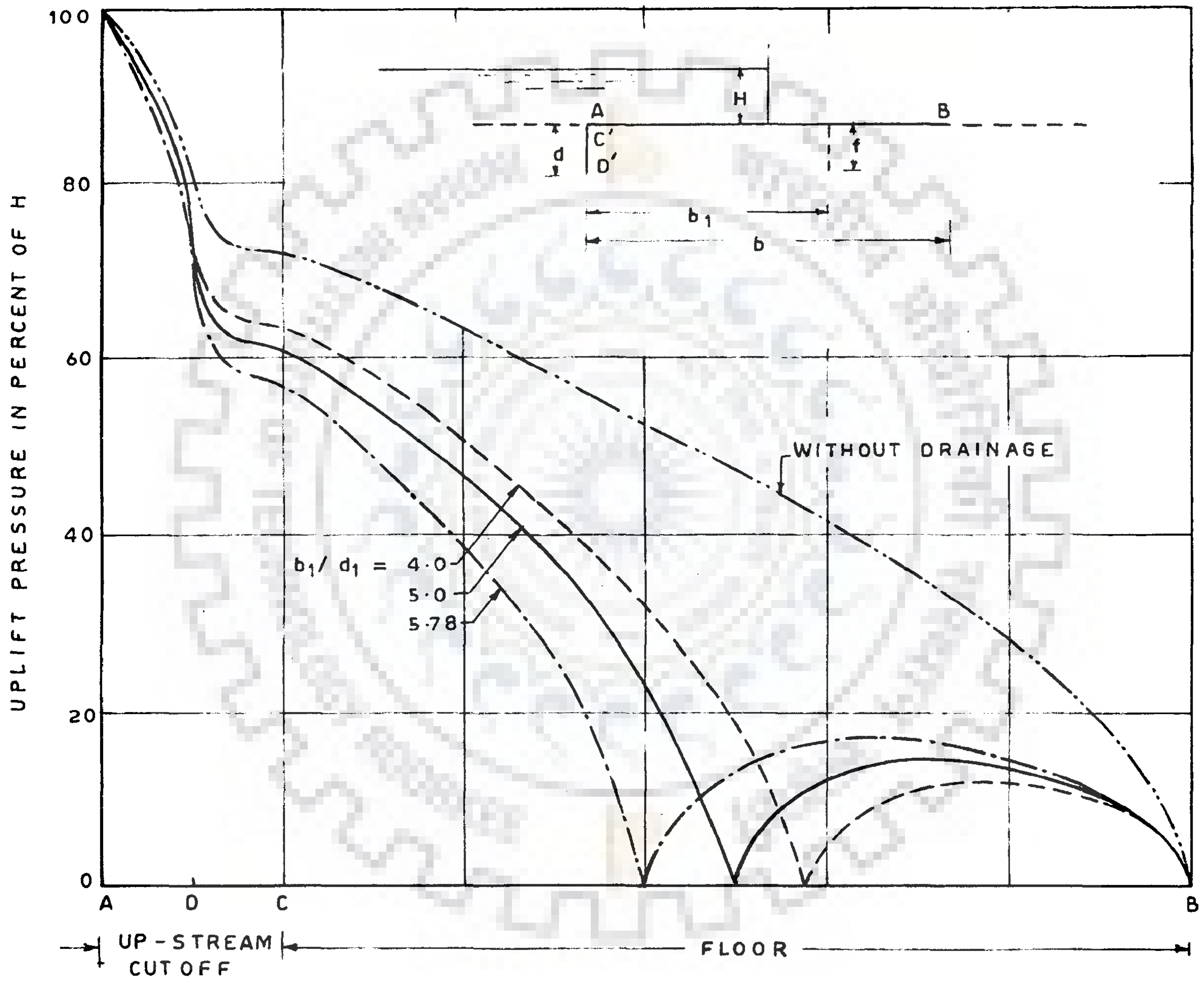


FIG. 6·5 —EFFECT OF LOCATION OF DRAINAGE $b / d_1 = 10$, $f / d_1 = 0{\cdot}5$

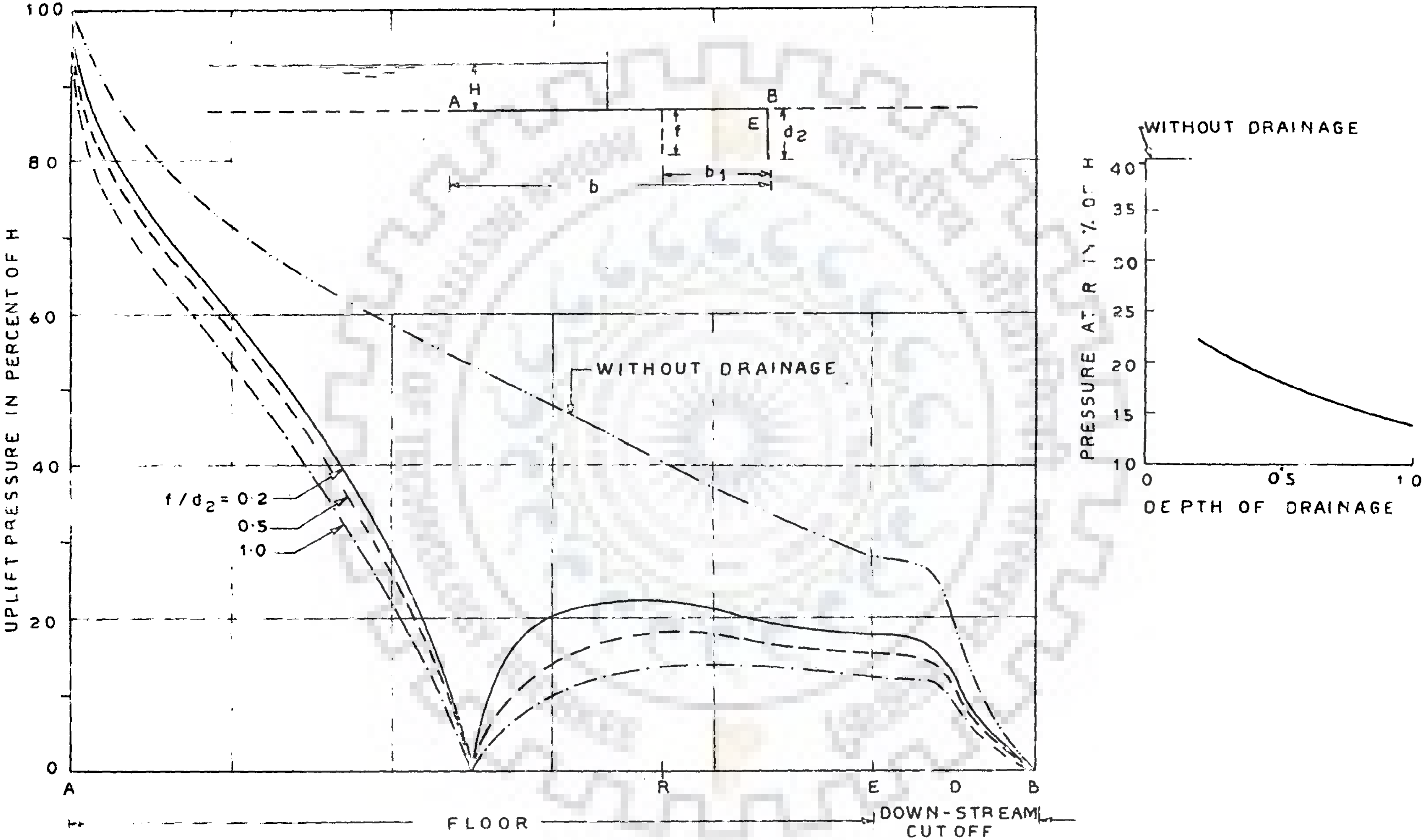


FIG. 6·6 – EFFECT OF DEPTH OF DRAINAGE $b/d_2 = 10$, $b_1/d_2 = 5$

Fig. 6.6 indicates that uplift pressures decrease throughout the length of floor with increase in depth of drainage. The reduction in uplift pressures with increase in the depth of drainage is more on the downstream side than on the upstream side.

6.2.4 Effect of location: The uplift pressures alon-g the floor for $b/d_2 = 10$, $f/d_2 = 0.5$ and various locations of drain, $b_1/d_2 = 4.0$, 5.0 and 5.77 have been given in Table 6.7 and plotted in Fig. 6.7. A perusal of the figure indicates that the pressures decrease on the downstream and increase on the upstream with the shifting of filter to the downstream.

As before, the drain location should be such that the total uplift on the floor downstream of the gateline is minimum.

6.2.5 Deep drain with two cutoffs: The uplift pressures along the floor and cutoff for the foundation profile having two end cutoffs and a deep drainage as observed on the model have been plotted in Fig. 5.4. The theoretically calculated pressures for single cutoff at upstream or at downstream end of the floor with a deep drainage have also been plotted in the same figure. It is seen that uplift pressures observed along the upstream cutoff and along floor upto the drainage are very close to those theoretically calculated for this portion neglecting downstream cutoff. In case of intermediate filter also it was seen that the increase in the depth of downstream cutoff does not affect the uplift pressures on the other side of the filter.

The observed uplift pressures along the downstream cutoff and the floor downstream of the drain differ only slightly from those calculated theoretically neglecting upstream cutoff. It is, therefore, seen that provision of downstream cutoff has practically no effect on the pressures upstream of drainage whereas provision of upstream cutoff

TABLE 6.7

EFFECT OF LOCATION OF DRAINAGE WITH DOWNSTREAM CUTOFF

	Coordinates of points with origin at the upstream floor end		Calculated pressures with drainage $b/d_2=10$, $f/d_2=0.5$			Calculated pressures without filter
	x/d_2	y/d_2	$b_1/d_2=4.02$	5.0	5.77	
	0.100	0.000	0.933	0.915	0.890	0.964
	1.000	0.000	0.730	0.708	0.667	0.806
	3.000	0.000	0.482	0.415	0.330	0.640
	3.500	0.000	0.430	0.364	0.223	0.606
	5.000	0.000	0.246	0.000	0.143	0.528
	6.000	0.000	0.000	0.145	0.195	0.473
	7.000	0.000	0.110	0.173	0.203	0.423
E	10.000	0.000	0.141	0.158	0.161	0.279
		0.436	0.139	0.154	0.157	0.272
		0.600	0.136	0.147	0.153	0.264
		0.714	0.133	0.143	0.149	0.231
D		1.000	0.106	0.113	0.117	0.194
		0.414	0.060	0.628	0.065	0.151
		0.600	0.049	0.051	0.053	0.086
		0.436	0.035	0.036	0.037	0.061
B	10.000	0.000	0.000	0.000	0.000	0.000

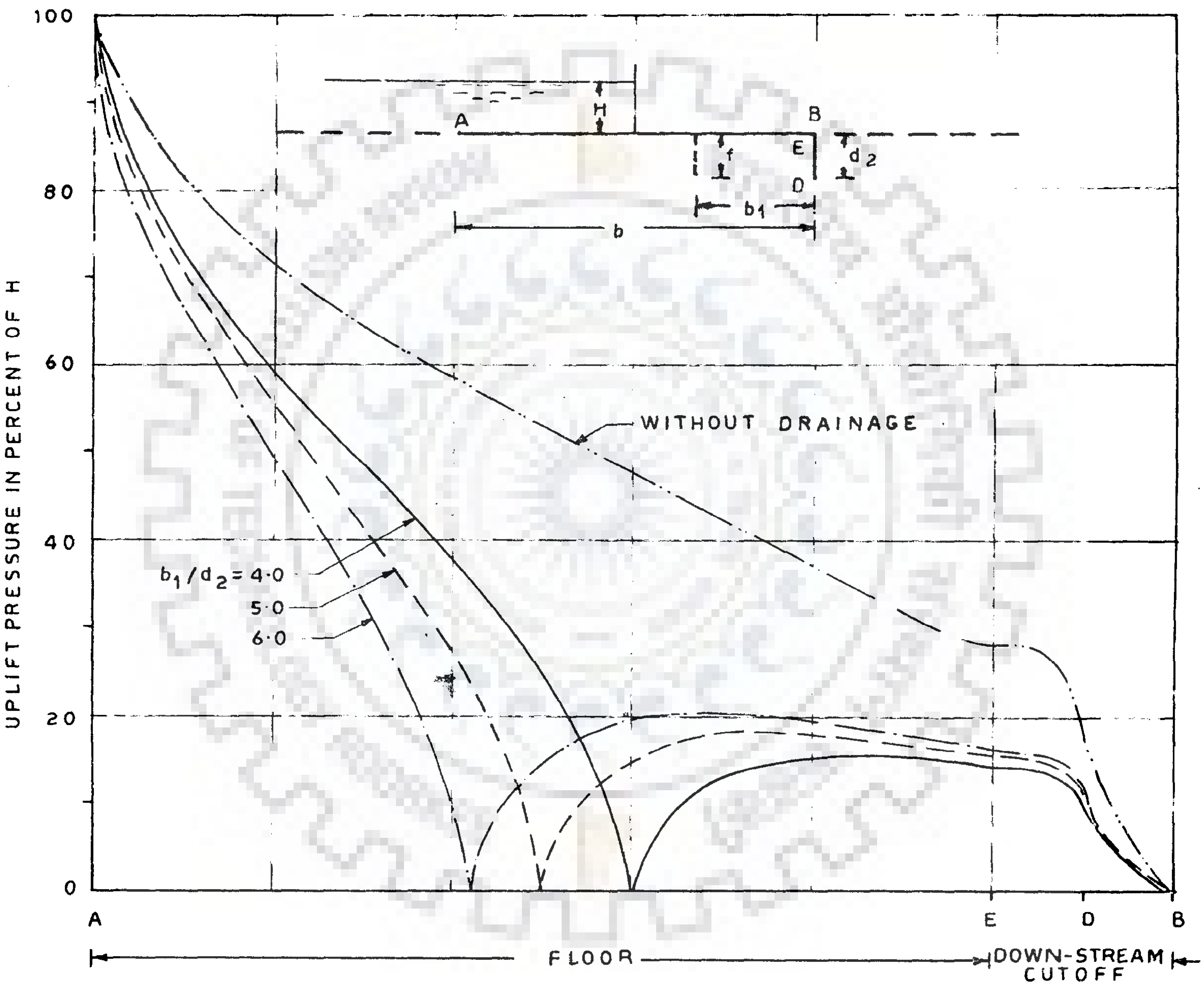


FIG. 6·7 — EFFECT OF LOCATION OF DRAINAGE $b/d_2 = 10$ & $f/d_1 = 0{\cdot}5$

reduces the pressures downstream of the drainage to a small extent. The effect of upstream cutoff on pressures downstream of drainage has been studied in some detail. The decrease in pressures at D, E and J with the provision of cutoff at upstream end from those without upstream cutoff have been plotted for $b/d_2 = 10$, $b_1/d_2 = 5$, $f/d_2 = 0.5$, and various depths of upstream cutoff viz. $d_1/d_2 = 0.50$, 1.0 and 2.0 in Fig. 6.8 A perusal of this figure indicates that the reduction in pressures at D, E and J increases with increase in the values of d_1/d_2. The decrease in pressures at D, E and J for $b/d_2 = 10$, $f/d_2 = 0.5$, $d_1/d_2 = 1.0$ and $b_1/d_2 = 1, 2, 3$ 4 and 5 have also been plotted in Fig. 6.8. It is seen that as the drainage approaches the downstream cutoff the effect of upstream cutoff in reducing the pressures at D, E and J reduces. It is also seen that reduction in pressures at D, E and J is not more than 4% of the total differential head when upstream cutoff upto twice the depth of downstream cutoff is provided. Actually, the depth of upstream cutoff is seldom likely to exceed that of the downstream cutoff in which case the error at J is limited to 2.5% of the differential head. The uplift pressures for floor with two end cutoffs and intermediate deep drainage can, therefore, be calculated for upstream portion neglecting downstream cutoff and for downstream portion by neglecting upstream cutoff. This may result in an error, less than 2.5% of the total differential head, in the uplift pressures along the downstream floor only. The error would give pressures on the higher side, and would thus be on the safe side.

6.2.6 The uplift pressures for intermediate filter and deep drain with two end cutoffs have been compared in Fig. 6.9. The length of the filter has been kept equal to the depth of the deep drain. The former values are theoretical while the latter are experimental. A perusal of this figure indicates that for these conditions the deep drainage is more

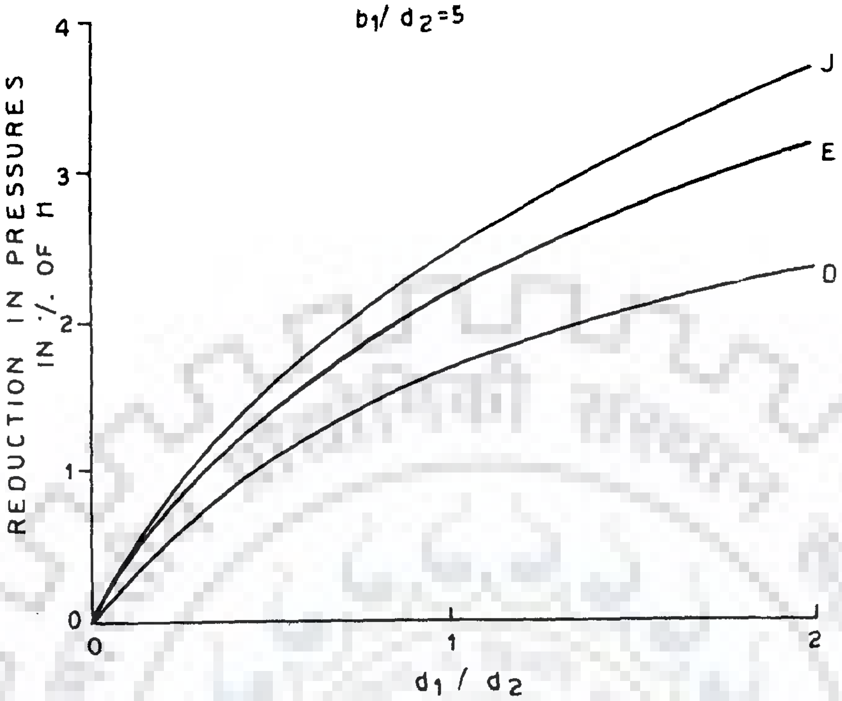


FIG. 6·8(a) –EFFECT OF U/S CUTOFF DEPTH ON PRESSURES AT E, D AND J

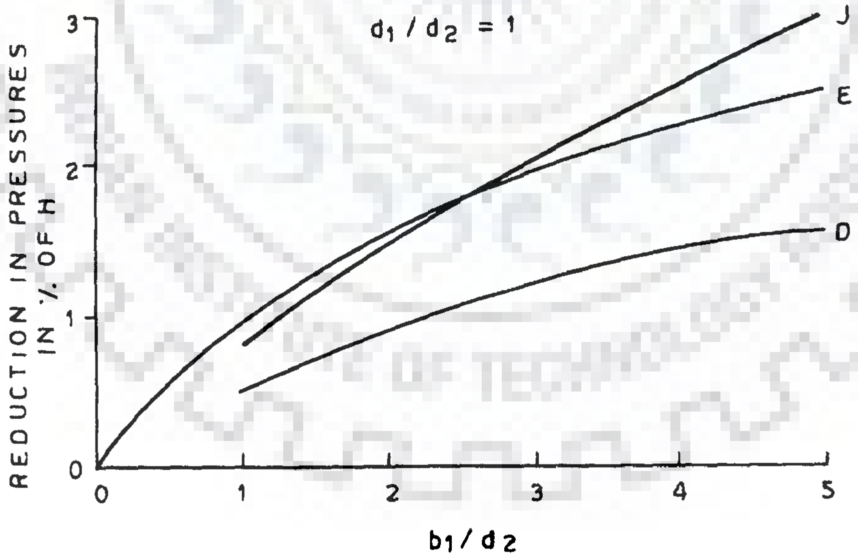


FIG. 6·8 (b)–EFFECT OF LOCATION OF DRAINAGE ON PRESSURES AT E, D AND J

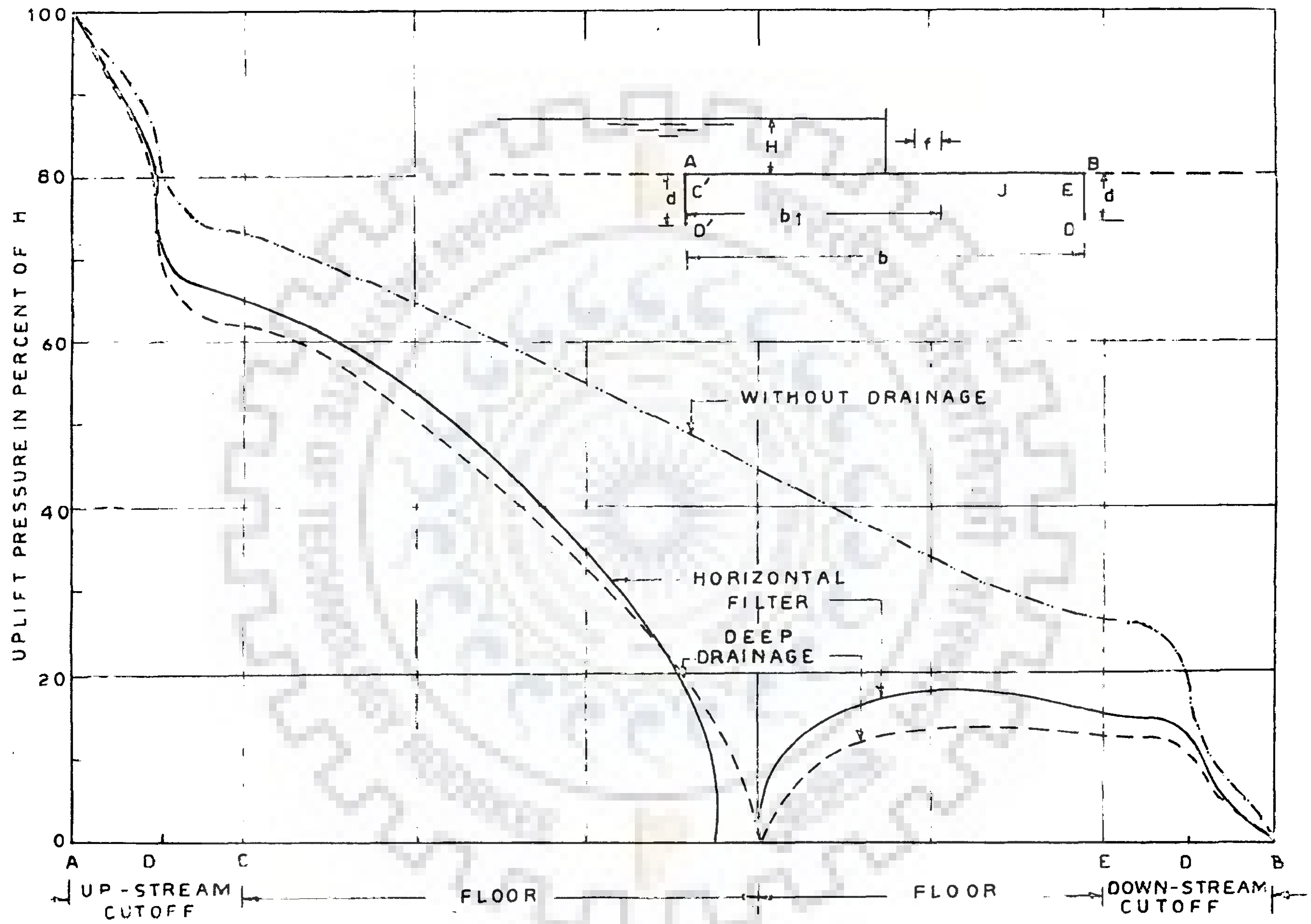


FIG. 6·9 —COMPARISON OF PRESSURES WITH HORIZONTAL FILTER AND DEEP DRAINAGE $b/d_1 = 10$, $b_1/d_1 = 6$, $f/d_1 = 0.5$, $d_2/d_1 = 1$

effective in reducing uplift pressures on the downstream and upstream portion of the floor except in the portion where filter is provided.

6.3 LEAKY CUTOFF

This problem has been studied for a central cutoff under a flat floor with a single opening of varying size and location along the cutoff. As in other cases, an infinite depth of permeable stratum has been assumed. This helps in quantitative assessment of the effect of leakage on uplift pressures, though it is recognised that these conditions do not represent the more complex situation in actual works.

6.3.1 Uplift Pressures: It is seen from Figs 4.2(a) to 4.2(c) that with the existence of an opening in the cutoff the uplift pressures increase below the floor on the downstream side and decrease below the floor on the upstream side. The increase in the pressures on the downstream and decrease on the upstream is more if gap is nearer the floor. For d/b = 0.2, the pressure at C is 43.85% of the differential head when there is no opening. The pressures at C, for 1% opening at a depth 0.3, 0.6 and 0.9 of the total cutoff depth rise to 46.55, 45.3 and 44.2% of differential head respectively. The pressure at C, for an opening of width 10% of the total depth of cutoff located at a depth of 0.3, 0.6 and 0.9 of the total depth of cutoff depth rise to 47.5, 45.9 and 44.4% of total differential head. The pressure at C for any size of opening at the junction of floor and the cutoff rises to 50% of the differential pressures. In case the portion of the cutoff below the opening is completely neglected or considered ineffective the pressures at C for d/b= 0.2 and d_1/d = 0.3, 0.6 and 0.9 work out to 48.1, 46.2 and 44.3% of differential head. The pressures with perfect cutoff, with cutoff having 1, 5 and 10% open area at d_1/d = 0.3, 0.6 and 0.9 and for the case when the portion of cutoff below the opening is neglected have been plotted in Fig. 6.10. A perusal of the figure indicates that even a small opening brings pressures at C close the value which would be obtained when cutoff below the opening is neglected It is seen that even 10% opening at any level results in uplift pressures rising to values close to those obtained neglecting portion of cutoff below opening. It is therefore necessary to ensure the imperviousness of at least upper part of the cutoff to ensure the stability of the structure.

The effect of leakage through the opening is more pronounced when the length of floor increases.

6.3.2 Seepage Discharge: A perusal of Figs. 4.4 indicates that the maximum seepage discharge passes through the opening when it is located at about 0.4 of the depth of cutoff. The leakage discharge is minimum when the opening is at the lowest end of the cutoff. The seepage discharge also reduces with decrease in the size of the opening and with increase in length of floor.

6.3.3 Cutoff Efficiency: The influence of location, area of leakage and the length of floor on efficiency of the cutoff has been shown in Fig. 6.11. The efficiency of the leaky cutoff has been taken as the ratio of the head drop across the leaky cutoff to that across a perfect cutoff, other variables remaining the same. A perusal of Fig. 6.1 indicates that efficiency of the cutoff reduces as the opening approaches the top of cutoff. The floor length does not affect the cutoff efficiency significantly unless it is very small. The cutoff efficiency reduces with the increase in the size of the opening.

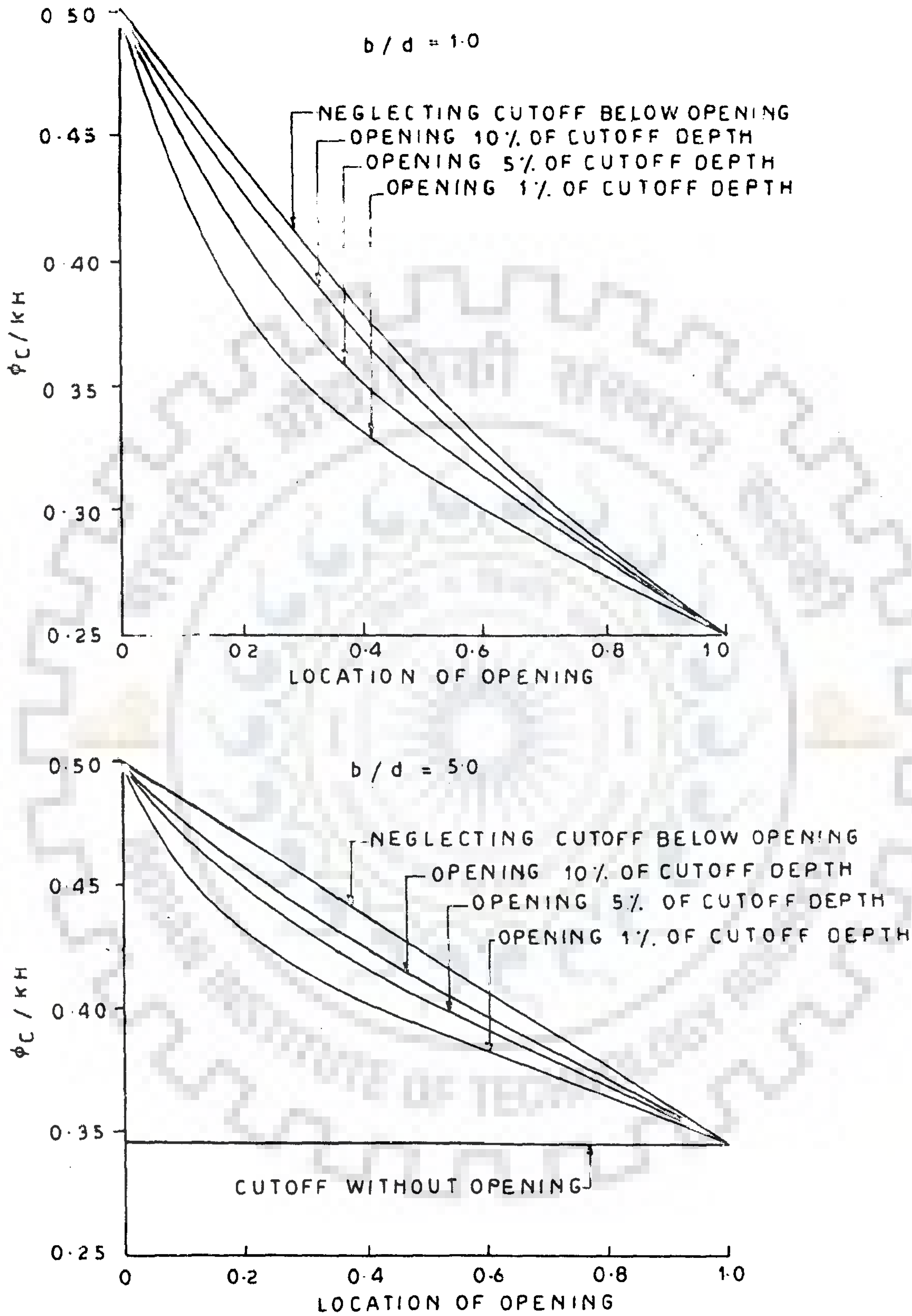


FIG. 6.10 —EFFECT OF LEAKAGE ON UPLIFT PRESSURES

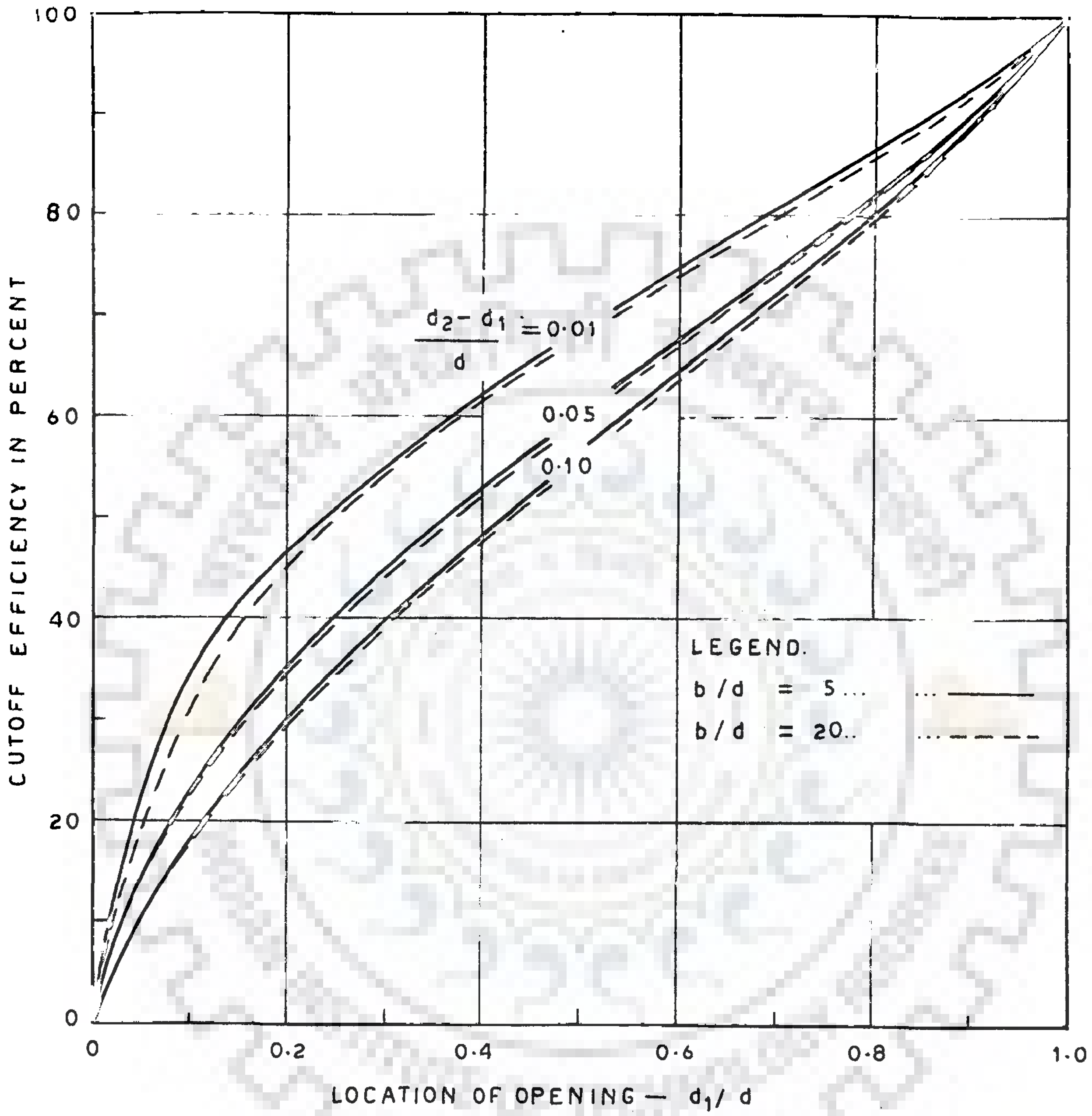


FIG. 6·11 —EFFECT OF LEAKAGE ON CUTOFF EFFICIENCY

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

7.0 Exact solutions have been obtained for the problems of two dimensional seepage flow below a hydraulic structure founded on permeable soil of infinite depth with the help of conformal mapping for the following boundary conditions:

(a) Flat floor with two end cutoffs and a horizontal filter of any length located anywhere between the two cutoffs.

(b) Flat floor with a single cutoff and a deep drain. The cutoff may be either on the upstream or downstream end of the floor.

(c) Flat floor with a central leaky cutoff. The cutoff is assumed to have a single opening anywhere.

The equations derived have been used for computation of pressures at the key points. The results have been plotted in the form of design curves. The effect of various parameters on the uplift pressures has been studied and is indicated below.

7.1 INTERMEDIATE FILTER.

7.1.1 As discussed in Chapter VI the uplift pressures reduce considerably along the entire profile of the structure with the provision of filter of even very small length. Further reduction in pressures with the increase in the length of filter is less as compared to the initial reduction. The maximum reduction in pressure takes place near the filter (Para 6.1.1 and Fig. 6.1).

7.1.2 The uplift pressures decrease on the downstream side and increase on the upstream as the filter is moved from upstream to the downstream side. The filter should be located such that the total uplift on the floor downstream of the gateline is minimum (Para 6.1.2 and Fig. 6.2).

7.1.3 With the increase in the depth of downstream cutoff, the uplift pressures increase only along the floor downstream of the filter. The uplift pressures along the floor upstream of the filter are uneffected by the increase in the depth of downstream cutoff. (Para 6.1.3 and Fig. 6.3)

7.2 DEEP DRAIN

7.2.1 With the increase in the depth of the drain the uplift pressures decrease along the entire floor length. As in the case of intermediate filter, the provision of deep drain of even a small depth reduces pressures considerably from those when no drainage is provided. Further reduction in the uplift pressures with the increase in the depth of drain is less as compared to initial decrease (Para 6.2.1 and 6.2.3, and Figs. 6.4 and 6.6).

7.2.2 The uplift pressures increase along the floor upstream of the drain and decrease along the floor downstream of the drain with the shifting of drain towards downstream. The drain should be located such that the total uplift on the floor downstream of the gateline is minimum (Para 6.2.2 and 6.2.4, and Figs. 6.5 and 6.6).

7.2.3 In the case of floor with two end cutoffs and a deep drain, the observed pressures along the floor upstream of the drain are very close to those theoretically calculated for this portion neglecting downstream cutoff. The observed pressures along the floor downstream of the drain are slightly less than those theoretically calculated for the portion neglecting upstream cutoff. The uplift pressures for floor with two end cutoffs and a deep drain can, therefore, be calculated for upstream portion neglecting downstream cutoff and for downstream portion neglecting the upstream cutoff. (Para 6.2.5 and Figs 5.4, 6.8 and 6.9).

This will not involve errors exceeding 2.5% of the total head.

7.3 LEAKY CUTOFF.

7.3.1 The uplift pressures increase along the downstream portion and decrease along the upstream portion of the floor with the existence of opening in the central cutoff. Even for small opening at any level, the uplift pressures along the floor approach values close to those obtained neglecting portion of cutoff below the opening. (Para 6.3.1). It is, therefore, evident that a leak near the top of the pile would make it more or less ineffective. A leak near the bottom would have much less effect in the present case of infinite depth of pervious medium. The influence of location and area of leakage on " efficiency" of cutoff has been shown in Fig. 6.11. The efficiency of the leaky cutoff has been taken as the ratio of the head drop across the leaky cutoff to that across a perfect cutoff, other variables remaining the same.

7.3.2 The maximum seepage discharge passes through the opening when it is located at about 0.4 of the depth of cutoff. The seepage discharge increases with the increase in the area of the opening and with decrease in the length of the floor.(Para 6.3.2).

7.4 RECOMMENDATIONS.

7.4.1 Intermediate filter or deep drainage should be provided below hydraulic structures to reduce uplift pressure and thereby afford reduction in the cost of structure. The filter should be relied upon for release of uplift pressures below hydraulic structures founded on permeable soils. In fact, a lot of reliance is placed on proper functioning of filters in case of earth and rockfill dams, even though in case of earth dams, it is not possible to take remedial measures if a filter gets choked up, whereas remedial measures can be taken up in case of filters below hydraulic structures. Constant watch can be kept on the behaviour of filter by providing piezometeric pipes and keeping a record of the seepage flow through the filter. In case some holes get choked it would be possible to drill fresh holes. Failure of an earth dam due to choking of filter inside it would normally lead to much greater loss of property and possibly even human lives, as compared to the loss due to failure of a hydraulic structure on pervious foundations. If the filters are properly designed and constructed and normal vigilance is maintained, there should, of course, be no possibility of failure. But the risk, if any, being much less than that being accepted in case of earth dams should be acceptable for these structures.

7.4.2 In actual practice the filter would neither be horizontal without depth nor deep without any width as assumed in the theoretical solution. However, for design purposes it would be safe to assume the filter to be horizontal only in case its depth is small in comparison to the depth of the cutoff. In case the depth of filter or drain is comparable to the depth of cutoff it may be considered as a deep drain.

7.4.3 Leakage of any size through the cutoff results in pressures nearly equal to those obtained neglecting the portion of cutoff below the opening. The impermeability of the upper part of the cutoff should, therefore, be ensured.

7.4.4 The design procedure based on the present work has been illustrated by an actual example in appendix I. This procedure is recommended for the design of similar Hydraulic structures.

7.5 SCOPE FOR FURTHER WORK

The work presented in thesis may be extended as follows:

(a) Theoretical solution of seepage below a flat floor with two end cutoffs and intermediate filter resting on permeable soil of finite depth.

(b) Analytical solution for seepage below a flat floor with two end cutoffs and a deep drain founded on finite or infinite depth of permeable subsoil.

(c) The effect of single opening in cutoff at the end of a flat floor resting on finite and infinite depth of permeable soil, on the uplift pressures and exit gradient.

(d) Effect of several openings in an end cutoff on the uplift pressures and exit gradients.

APPENDIX I

ILLUSTRATIVE EXAMPLE

APPENDIX I

ILLUSTRATIVE EXAMPLE.

Consider a low dam founded on an infinite depth of pervious material. Length of impervious floor is 60.0m, depths of upstream and downstream cutoffs are 6.0m. The gateline is located at 30m from the upstream end of floor (Fig.I-1). The design procedure both for horizontal filter and for deep drain is given below:

Horizontal Filter:

The uplift pressures were determined from Figs. 2.2 to 2.6 and plotted in Fig.I-2 for 3.0m long filter located at 24,30,36 and 48m from the upstream end of the floor. The total uplift pressures downstream of the gate line were worked out for these locations of the filter. These have been plotted against locations in Fig.I-4. The total uplift downstream of the gate line is seen to be minimum when the filter is located at a distance of about 36m from the upstream end of the floor.

The uplift pressures were also determined for various lengths filter located at 36.0m from the upstream end of the floor and plotted in Fig. I-4. Although pressures reduce with the increase in the length of the filter but the decrease in the pressures beyond 3.0m length of filter is not appreciable.

The uplift pressures without filter have also been plotted in Fig. I-2. It is seen that provision of 3.0m long filter reduces total uplift pressures on the floor downstream of the gate line to 39.35% of the total uplift in this portion without filter. The provision of filter would, therefore, result in very considerable saving

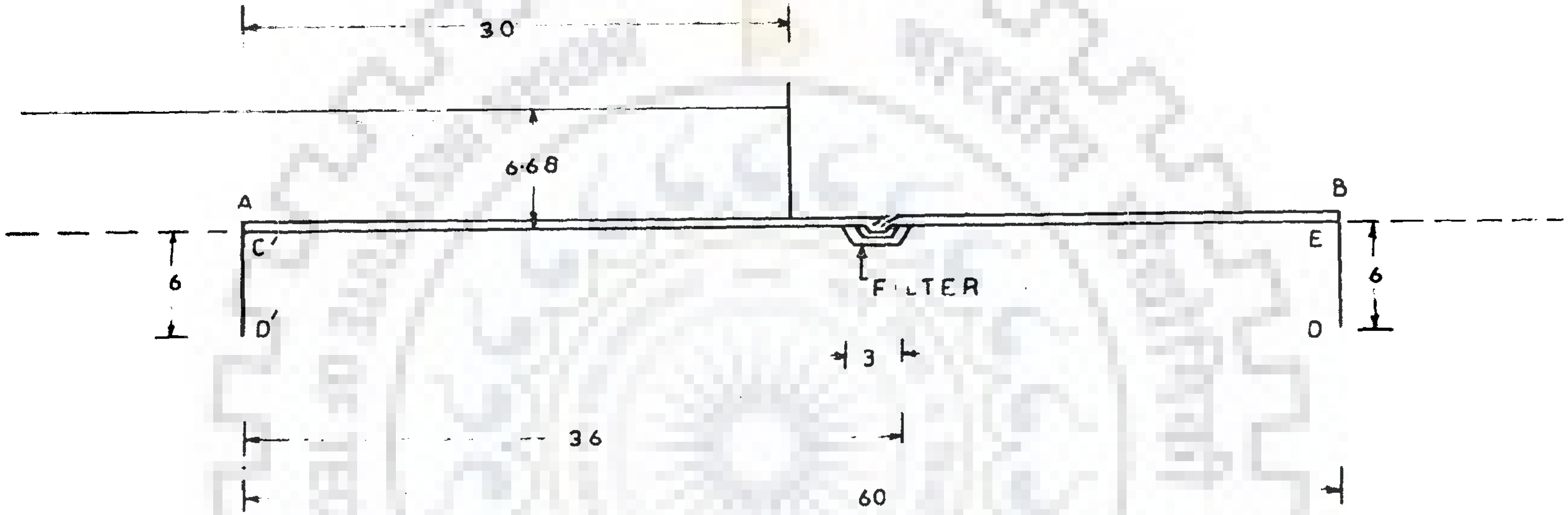


FIG. I-1 PROFILE OF HYDRAULIC STRUCTURE
(ALL DIMENSIONS IN METRES)

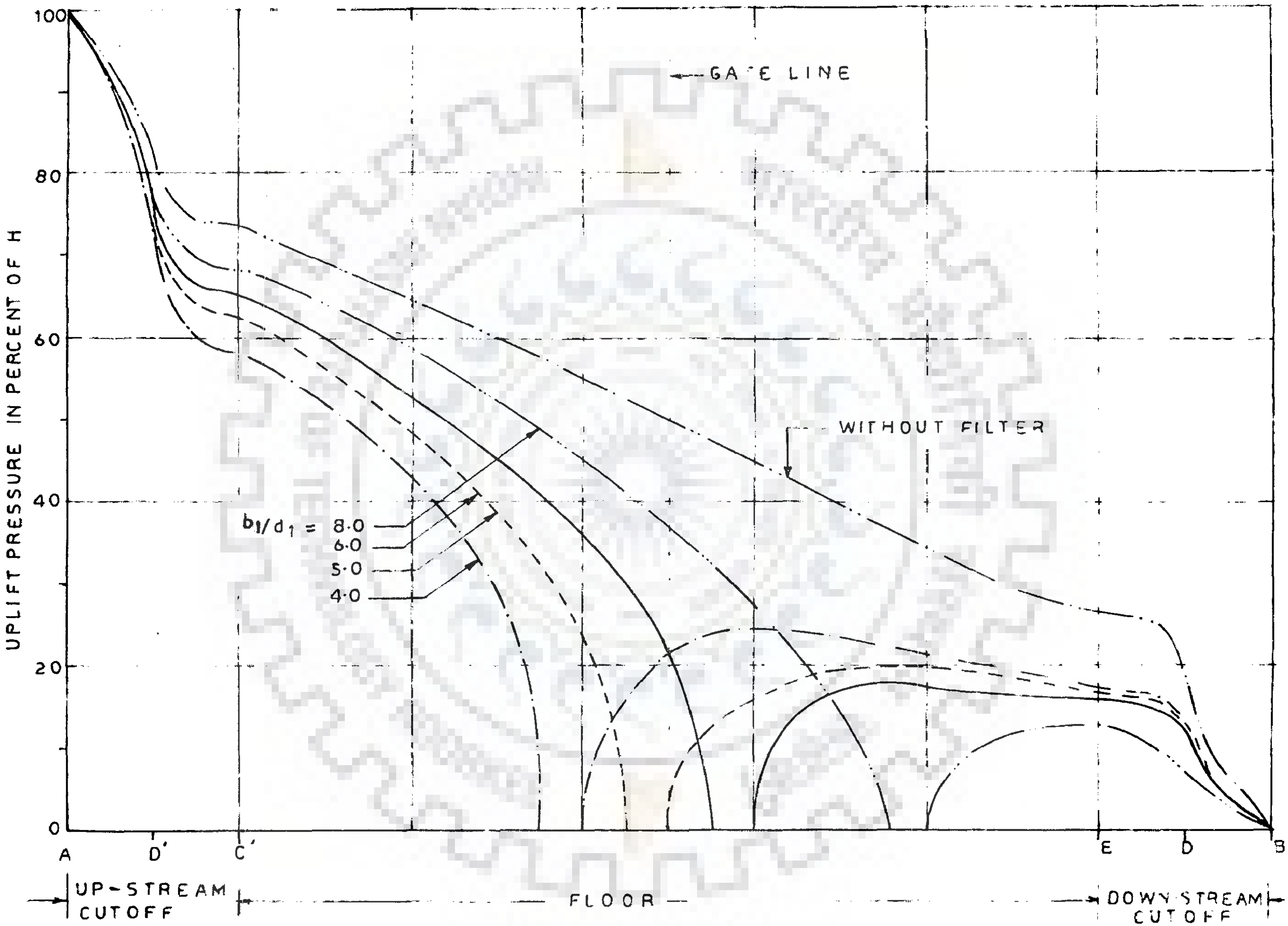


FIG. I-2 UPLIFT PRESSURE WITH HORIZONTAL FILTER

Deep Drain:

The uplift pressures at D' and C' were also determined from Figs. 3.3, 3.4 respectively neglecting downstream cutoff and pressures at D, E and J were determined from Figs. 3.7, 3.8 and 3.9 respectively neglecting upstream cutoff for 3.0m deep drain located at 24, 30, 36 and 48m from the upstream end of the floor (Fig.I-3). The total uplift pressures downstream of the gateline were worked out for these locations and plotted in Fig. I-4. The total uplift is seen to be minimum when the filter is located at a distance of about 36m from the upstream end of the floor.

The uplift pressures were also determined for various depths of drain located at 36.0m from the upstream end of the floor and plotted in Fig.I-4. Although the total uplift reduces with the increase in the depth of the drain but the reduction in uplift beyond 3.0m depth of drain is not significant. Moreover the excavation greater than 3.0m may present dewatering problem in the field.

With the provision of 3.0m deep drain at 36.0 m from upstream end of the floor of the total uplift downstream of the gateline reduces to 37.89% of the total uplift without drainage. It is seen that reduction in the total uplift downstream of the gateline is slightly more in case 3.0m deep drain as compared to that with horizontal filter of 3.0m length. But the convenience and economy of construction would normally favour a horizontal drain in comparison to a deep drain.

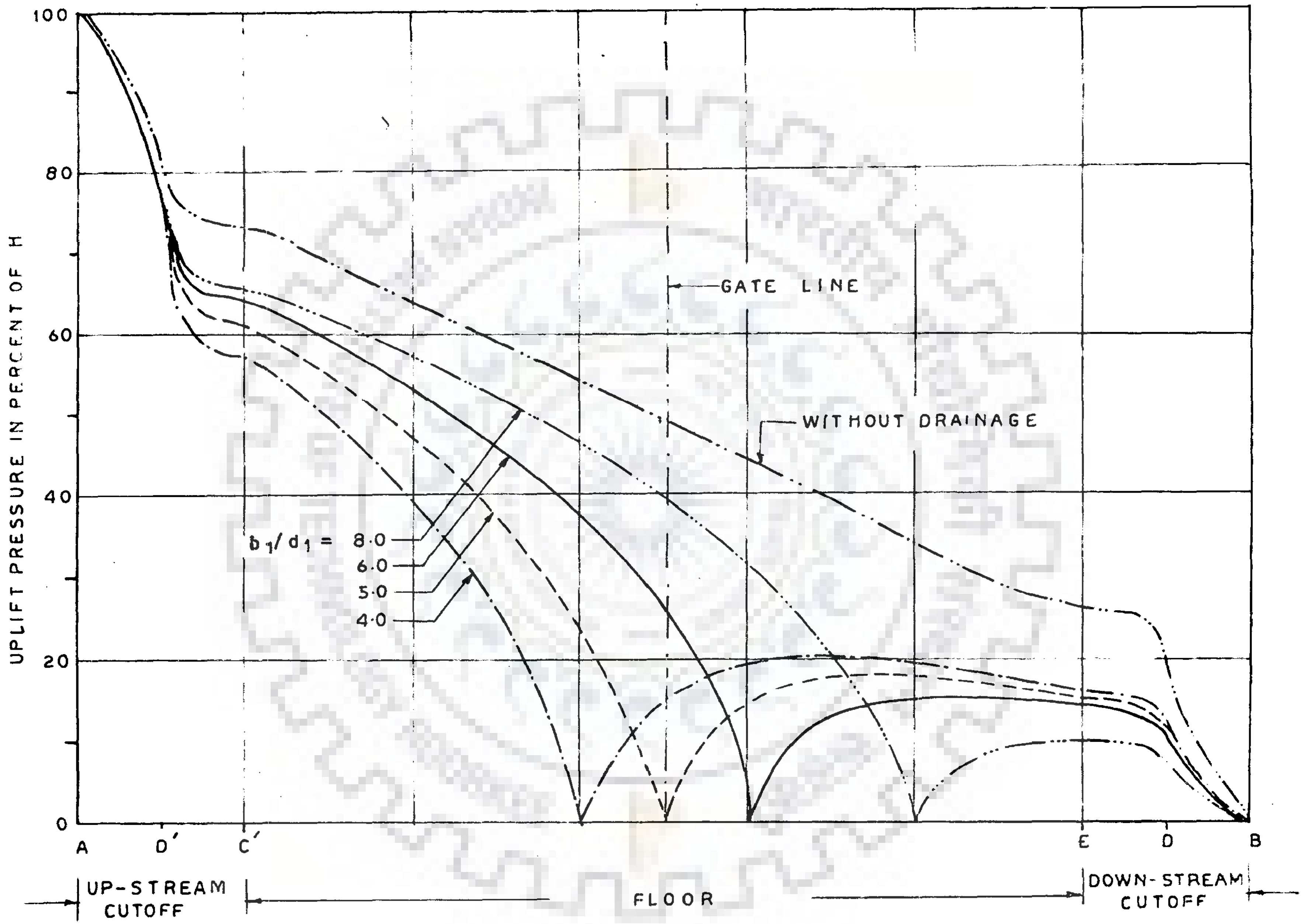


FIG I-3 UPLIFT PRESSURE WITH DEEP DRAINAGE

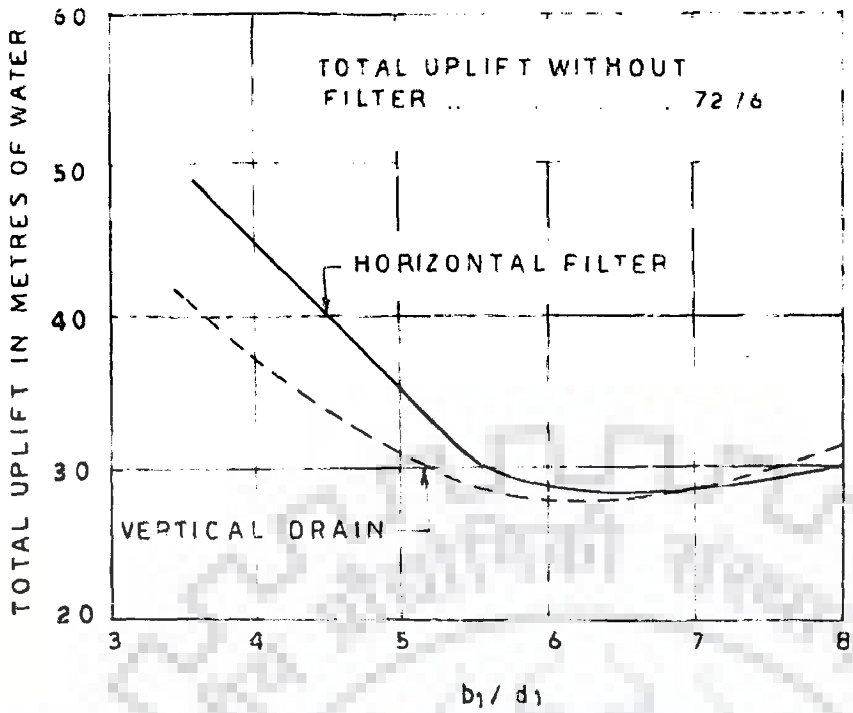


FIG. I-4 (a) UPLIFT FOR DIFFERENT LOCATIONS
$b / d_1 = 10$, $d_2 / d_1 = 1$, $f / d_1 = 0.5$

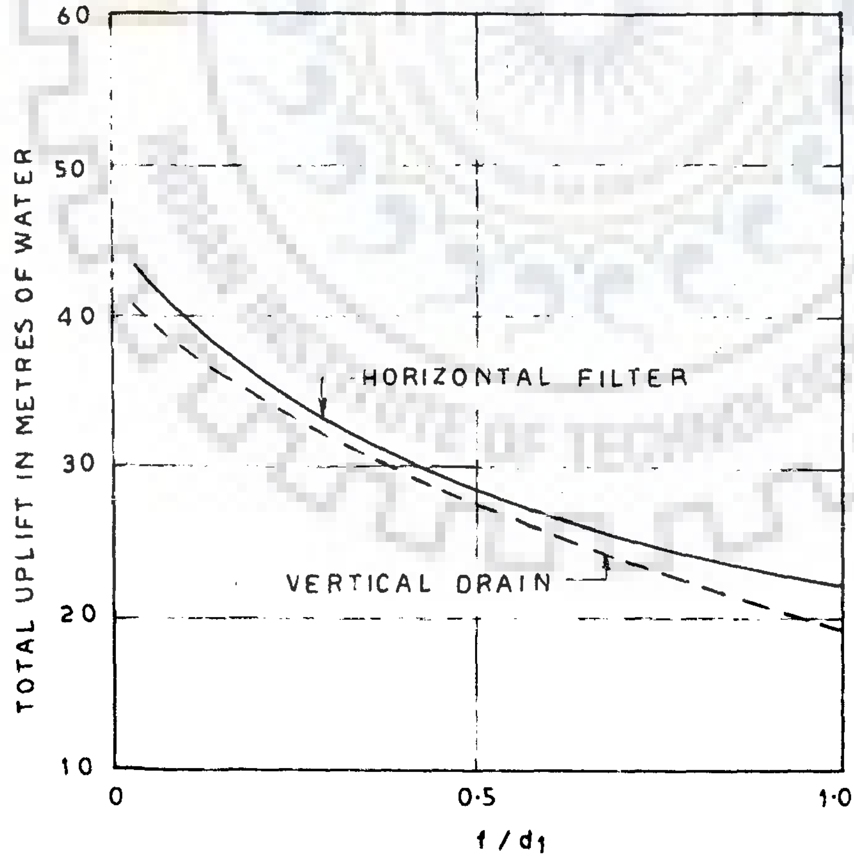


FIG. I-4 (b) UPLIFT FOR DIFFERENT FILTER SIZES
$b / d_1 = 10$, $b_1 / d_1 = 5$, $d_2 / d_1 = 1$

A P P E N D I X II

COMPUTER PROGRAMME

APPENDIX II

FORTRAIN SOURCE PROGRAMMES

Following computer programmes, used for the computation of uplift pressures and location of the point in physical plane for assumed values of parameters, are listed:

(i) Intermediate filter with two end cutoffs
(ii) Deep drain with upstream cutoff
(iii) Deep drain with downstream cutoff
(iv) Leaky central cutoff

In addition following subroutines were developed for the computation of elliptic integrals of first, second and third kinds.

(v) Subroutine DEFUN (for elliptic integrals of first and second kinds).
(vi) Subroutine EFUN (for elliptic integrals of all the three kinds)

The elliptic integrals of first and second kinds are calculated by suming up certain terms of infinite series (4,Eqs. 902.00 and 903.00) such that desired accuracy was obtained.

The computation of elliptic integral of the third kind involves Lambda function, Zeta function and the Theta functions. The latter requires evaluation by means of infinite series. In addition to the argument φ and the modulus m, this integrals depends also on the parameter α^2. Following are the possible cases accordingly to the values of α^2.

Circular cases

Case I : $0 < -\alpha^2 < k$ and $k < -\alpha^2 < \infty$
Case II: $k < \alpha^2 < 1$ and $k^2 < \alpha^2 < k$

Hyperbolic cases

Case III: $0 < \alpha^2 < k^2$
Case IV : $1 < \alpha^2 < \infty$

The programme includes all the above cases including special values. The relations given by Byrd and Friedman (4,Eqs. 431.01, 432.01,433.01,434.01,435.01, and 430.01) have been utilised.

```
C  C  INTERMEDIATE FILTER WITH TWO END CUTOFFS
C  C  DEL=δ ,EMU=μ ,TE1=σ ,TE2=ρ
 201  READ 202,DEL,EMU,TE1,TE2
 202  FORMAT(4F10.5)
      G=0.0001
      PI=3.1415926
      C1=(DEL+EMU+1.0)/2.0
      C2=C1*C1
      XK=(EMU-DEL)/(DEL*(EMU-1.0))
      CXK=1.0-XK
      PHI=PI/2.0
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      C3=2.0*DEL*(EMU*(CK-CE)+CE)/CK
      ELD1=0.5*(C1-SQRTF(C2-C3))
      ELD2=0.5*(C1+SQRTF(C2-C3))
      PHI=ATANF(SQRTF((DEL*(1.0-ELD1))/(ELD1*(DEL-1.0))))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      DD1=(DEL*(EMU-1.0)*(DEL-ELD1))
      DD=CE*U/CK-EU+SQRTF(ELD1*(1.0-ELD1)*(EMU-ELD1)/DD1)
      BD1=(CKD*(CE/CK-1.0)+CED)/DD
      PHI=ATANF(SQRTF((EMU-1.0)*(ELD2-DEL)/((DEL-1.0)*(EMU-ELD2))))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      DD2=SQRTF(ELD2*(ELD2-DEL)*(EMU-ELD2)/(DEL*(EMU-1.0)*(ELD2-1.0)))
      D2D1=(CE*U/CK-EU+DD2)/DD
      XK=CXK
      PHI=ATANF(SQRTF(DEL*(TE1-1.0)/(DEL-TE1)))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      XD=SQRTF((TE1-1.0)*(EMU-TE1)*(DEL-TE1)/(TE1*DEL*(EMU-1.0)))
      X1D1=((CED/CKD-1.0)*U+EU-XD)/DD
      PHI=ATANF(SQRTF(DEL*(TE2-1.0)/(DEL-TE2)))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      XD1=SQRTF((TE2-1.0)*(EMU-TE2)*(DEL-TE2)/(TE2*DEL*(EMU-1.0)))
      X2D1=((CED/CKD-1.0)*U+EU-XD1)/DD
      FD1=X2D1-X1D1
      PRINT203,DEL,EMU,TE1,TE2,BD1,D2D1,X1D1,X2D1,FD1
 203  FORMAT(1X,9F10.5)
      A=TE2*(EMU-TE1)/(TE1*(EMU-TE2))
      P=(EMU-TE1)/TE1
      XK=1.0/A
      ALFA=(1.0+P)/(A+P)
      PHI=PI/2.0
      CALL  EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
      DP=(A+P)*CK-(A-1.0)*CPYE
      P1=(P*(A-1.0)*CPYE+CK*(A+P))/DP
 210  READ 211,T,NN
 211  FORMAT(F10.5,I5)
      ZI=T*(EMU-TE1)/(TE1*(EMU-T))
      IF(TE1-T)251,251,252
 251  IF(TE2-T)253,271,271
```

```
 252  PHI=ATANF(SQRTF(ZI/(1.0-ZI)))
      XK=1.0/A
      ALFA=-1.0/P
      CALL  EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
      C7=(P+P1)*CPYE-P*CK
      PHIX=((P+P1)*(CPYE-PYE)-P*(CK-U))/C7
      GO TO 260
 253  C4=P/((A+P)*(1.0+P))
      PHI=ATANF(SQRTF((ZI-A)/(A-1.0)))
      XK=1.0/A
      ALFA=(1.0+P)/(A+P)
      CALL  EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
      PHIX=C4*((A+P)*(P1-1.0)*U-(A-1.0)*(P1+P)*PYE)/C7
C     LOCATION Z-PLANE
 260  IF(T-1.0)254,255,256
 254  PHI=ATANF(SQRTF(DEL*(1.0-T)/(T*(DEL-1.0))))
      XK=(EMU-DEL)/(DEL*(EMU-1.0))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      YD1=(CE*U/CK-EU+SQRTF(T*(1.0-T)*(EMU-T)/(DEL*(EMU-1.0)*(DEL-T))))
      YD1=YD1/DD
      GD1=0.0
      GO TO 269
 255  YD1=0.0
      GD1=0.0
      GO TO 269
 256  IF(DEL-T)259,258,257
 257  YD1=0.0
      PHI=ATANF(SQRTF(DEL*(T-1.0)/(DEL-T)))
      XK=(EMU-DEL)/(DEL*(EMU-1.0))
      XK=1.0-XK
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      GD=SQRTF((T-1.0)*(EMU-T)*(DEL-T)/(T*DEL*(EMU-1.0)))
      GD1=((CED/CKD-1.0)*U+EU-GD)/DD
      GO TO 269
 258  YD1=0.0
      GD1=BD1
      GO TO 269
 259  PHI=ATANF(SQRTF((EMU-1.0)*(T-DEL)/((DEL-1.0)*(EMU-T))))
      XK=(EMU-DEL)/(DEL*(EMU-1.0))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      YD1=(CE*U/CK-EU+SQRTF(T*(T-DEL)*(EMU-T)/(DEL*(EMU-1.0)*(T-1.0))))
      YD1=YD1/DD
      GD1=BD1
 269  PRINT 270,T,PHIX,GD1,YD1
 270  FORMAT(1X,4F10.5)
 271  IF(NN-1)210,201,201
      END
```

```
C   C   DEEP DRAIN WITH UPSTREAM CUTOFF
C   C   DEL=δ,EMU=μ,ZEB=γ
 201    READ 202,DEL,EMU,ZEB
 202    FORMAT(3F10.5)
        PRINT 204,DEL,EMU,ZEB
  204   FORMAT(1X,3F10.5)
        G=0.00001
        PI=3.1415926
        C1=(DEL+EMU+1.0)/2.0
        C2=C1*C1
        XK=(EMU-DEL)/(DEL*(EMU-1.0))
        CXK=1.0-XK
        PHI=PI/2.0
        CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
        C3=2.0*DEL*(EMU*(CK-CE)+CE)/CK
        ELD1=0.5*(C1-SQRTF(C2-C3))
        ELD2=0.5*(C1+SQRTF(C2-C3))
        PHI=ATANF(SQRTF((DEL*(1.0-ELD1))/(ELD1*(DEL-1.0))))
        CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
        DD1=(DEL*(EMU-1.0)*(DEL-ELD1))
        DD=CE*U/CK-EU+SQRTF(ELD1*(1.0-ELD1)*(EMU-ELD1)/DD1)
        BD1=(CKD*(CE/CK-1.0)+CED)/DD
        PHI=ATANF(SQRTF((EMU-1.0)*(ELD2-DEL)/((DEL-1.0)*(EMU-ELD2))))
        CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
        DD2=SQRTF(ELD2*(ELD2-DEL)*(EMU-ELD2)/(DEL*(EMU-1.0)*(ELD2-1.0)))
        D2D1=(CE*U/CK-EU+DD2)/DD
        ENP=SQRTF(ZEB*(ZEB-1.0)*(ZEB-EMU)/(DEL*(EMU-1.0)*(ZEB-DEL)))
        XK=CXK
        PHI=ATANF(SQRTF(DEL*(ZEB-EMU)/(ZEB*(EMU-DEL))))
        CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
        B2D1=((CED/CKD-1.0)*U+EU-ENP)/DD
        B2D1=ABSF(B2D1)+BD1
        A=EMU*(ZEB-DEL)/(DEL*(ZEB-EMU))
        P=(ZEB-DEL)/DEL
        XK=1.0/A
        ALFA=(1.0+P)/(A+P)
        PHI=PI/2.0
        CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
        DP=(A+P)*CK-(A-1.0)*CPYE
        P1=(P*(A-1.0)*CPYE+CK*(A+P))/DP
        PRINT 207,BD1,D2D1,B2D1
 207    FORMAT(1X,3F10.5)
 210    READ211,T,NN
 211    FORMAT(F10.5,I5)
        IF(T-DEL)251,253,253
C       PRESSURE UPSTREAM PORTION
251     XK=1.0/A
        ALFA=-1.0/P
        ZI=T*(ZEB-DEL)/(DEL*(ZEB-T))
        PHI=ATANF(SQRTF(ZI/(1.0-ZI)))
        CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
        C7=(P+P1)*CPYE-P*CK
        PHIX=((P+P1)*(CPYE-PYE)-P*(CK-U))/C7
        GO TO 260
```

```
C     PRESSURE  DOWNSTREAM PORTION
 253  C4=P/((A+P)*(1.0+P))
      PHI=PI/2.0
      XK=1.0/A
      ALFA=-1.0/P
      CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
      C7=)P.P1**CPYE-P*CK
      ZI=T*(ZEB-DEL)/(DEL*(ZEB-T))
      PHI=ATANF(SQRTF((ZI-A)/(A-1.0)))
      XK=1.0/A
      ALFA=(1.0+P)/(A+P)
      CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
      PHIX=C4*((A+P)*(P1-1.0)*U-(A-1.0)*(P1+P)*PYE)/C7
C     LOCATION IN Z-PLANE
 260  IF(T-1.0)254,255,256
 254  PHI=ATANF(SQRTF(DEL*(1.0-T)/(T*(DEL-1.0))))
      XK=(EMU-DEL)/(DEL*(EMU-1.0))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      YD1=(CE*U/CK-EU+SQRTF(T*(1.0-T)*(EMU-T)/(DEL*(EMU-1.0)*(DEL-T))))
      YD1=YD1/DD
      XD1=0.0
      GO TO 269
 255  YD1=0.0
      XD1=0.0
      GO TO 269
256   IF(DEL-T)259,258,257
257   YD1=0.0
      XK=CXK
      PHI=ATANF(SQRTF(DEL*(T-1.0)/(DEL-T)))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      GD=SQRTF((T-1.0)*(EMU-T)*(DEL-T)/(T*DEL*(EMU-1.0)))
      XD1=((CED/CKD-1.0)*U+EU-GD)/DD
      GO TO 269
 258  YD1=0.0
      XD1=BD1
      GO TO 269
 259  ENP=SQRTF(T*(T-1.0)*(T-EMU)/(DEL*(EMU-1.0)*(T-DEL)))
      XK=CXK
      PHI=ATANF(SQRTF(DEL*(T-EMU)/(T*(EMU-DEL))))
      CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      XD1=((CED/CKD-1.0)*U+EU-ENP)/DD+BD1
      YD1=0.0
 269  PRINT 270,T,PHIX,XD1,YD1
 270  FORMAT(1X,4F10.5)
 271  IF(NN-1)210,201,201
      END
```

```
C  C   DEEP DRAIN WITH DOWN STREAM CUTOFF
C  C   DEL=δ,EMU=μ,ZEB=γ
 201   READ 202,DEL,EMU,ZEB
 202   FORMAT(3F10.5)
       PRINT 204,DEL,EMU,ZEB
  204  FORMAT(1X,3F10.5)
       G=0.00001
       PI=3.1415926
       C1=(DEL+EMU+1.0)/2.0
       C2=C1*C1
       XK=(EMU-DEL)/(DEL*(EMU-1.0))
       CXK=1.0-XK
       PHI=PI/2.0
       CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
       C3=2.0*DEL*(EMU*(CK-CE)+CE)/CK
       ELD1=0.5*(C1-SQRTF(C2-C3))
       ELD2=0.5*(C1+SQRTF(C2-C3))
       PHI=ATANF(SQRTF((DEL*(1.0-ELD1))/(ELD1*(DEL-1.0))))
       CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
       DD1=(DEL*(EMU-1.0)*(DEL-ELD1))
       DD=CE*U/CK-EU+SQRTF(ELD1*(1.0-ELD1)*(EMU-ELD1)/DD1)
       BD1=(CKD*(CE/CK-1.0)+CED)/DD
       PHI=ATANF(SQRTF((EMU-1.0)*(ELD2-DEL)/((DEL-1.0)*(EMU-ELD2))))
       CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
       DD2=SQRTF(ELD2*(ELD2-DEL)*(EMU-ELD2)/(DEL*(EMU-1.0)*(ELD2-1.0)))
       D2D1=(CE*U/CK-EU+DD2)/DD
       ENP=SQRTF(ZEB*(ZEB-1.0)*(ZEB-EMU)/(DEL*(EMU-1.0)*(ZEB-DEL)))
       XK=CXK
       PHI=ATANF(SQRTF(DEL*(ZEB-EMU)/(ZEB*(EMU-DEL))))
       CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
       B2D1=((CED/CKD-1.0)*U+EU-ENP)/DD
       B2D1=ABSF(B2D1)+BD1
       A=EMU*(ZEB-DEL)/(DEL*(ZEB-EMU))
       P=EMU/(ZEB-EMU)
       XK=1.0/A
       ALFA=(1.0+P)/(A+P)
       PHI=PI/2.0
       CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
       DP=(A+P)*CK-(A-1.0)*CPYE
       P1=(P*(A-1.0)*CPYE+CK*(A+P))/DP
       PRINT 207,BD1,D2D1,B2D1
 207   FORMAT(1X,3F10.5)
 210   READ211,T,NN
 211   FORMAT(F10.5,I5)
C      PRESSURE UPSTREAM PORTION
       IF(T-DEL)253,253,251
251    XK=1.0/A
       ALFA=-1.0/P
       ZI=EMU*(ZEB-T)/(T*(ZEB-EMU))
       PHI=ATANF(SQRTF(ZI/(1.0-ZI)))
       CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
       C7=(P+P1)*CPYE-P*CK
       PHIX=((P+P1)*(CPYE-PYE)-P*(CK-U))/C7
       GO TO 260
```

```
C      PRESSURE  DOWNSTREAM PORTION
 253   C4=P/((A+P)*(1.0+P))
       PHI=PI/2.0
       XK=1.0/A
       ALFA=-1.0/P
       CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
       C7=)P.P1**CPYE-P*CK
       ZI=EMU*(ZEB-T)/(T*(ZEB-EMU))
       PHI=ATANF(SQRTF((ZI-A)/(A-1.0)))
       XK=1.0/A
       ALFA=(1.0+P)/(A+P)
       CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
       PHIX=C4*((A+P)*(P1-1.0)*U-(A-1.0)*(P1+P)*PYE)/C7
C      LOCATION IN Z-PLANE
 260   IF(T-1.0)254,255,256
 254   PHI=ATANF(SQRTF(DEL*(1.0-T)/(T*(DEL-1.0))))
       XK=(EMU-DEL)/(DEL*(EMU-1.0))
       CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
       YD1=(CE*U/CK-EU+SQRTF(T*(1.0-T)*(EMU-T)/(DEL*(EMU-1.0)*(DEL-T))))
       YD1=YD1/DD
       XD1=0.0
       GO TO 269
 255   YD1=0.0
       XD1=0.0
       GO TO 269
256    IF(DEL-T)259,258,257
257    YD1=0.0
       XK=CXK
       PHI=ATANF(SQRTF(DEL*(T-1.0)/(DEL-T)))
       CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
       GD=SQRTF((T-1.0)*(EMU-T)*(DEL-T)/(T*DEL*(EMU-1.0)))
       XD1=((CED/CKD-1.0)*U+EU-GD)/DD
       GO TO 269
 258   YD1=0.0
       XD1=BD1
       GO TO 269
 259   ENP=SQRTF(T*(T-1.0)*(T-EMU)/(DEL*(EMU-1.0)*(T-DEL)))
       XK=CXK
       PHI=ATANF(SQRTF(DEL*(T-EMU)/(T*(EMU-DEL))))
       CALL DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
       XD1=((CED/CKD-1.0)*U+EU-ENP)/DD+BD1
       YD1=0.0
 269   PRINT 270,T,PHIX,XD1,YD1
 270   FORMAT(1X,4F10.5)
 271   IF(NN-1)210,201,201
       END
```

```
C   C  LEAKY CUTOFF
1      READ 2, BOD
    2  FORMAT (F10.5)
       G=0.0001
       PI=3.1415926
       GAMA=1.0+BOD*BOD
       DO 23 I=1,10
       XI=I
       D1D=1.0-XI/10.0
       BETA=1.0-D1D*D1D
       GAP=0.01
    3  D2D=GAP+D1D
       RO=1.0-D2D*D2D
       XK=(GAMA-BETA)*RO/(BETA*(GAMA-RO))
       ALFA=-RO/(GAMA-RO)
       PHI=PI/2.0
       CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
       SIG=GAMA*(CK-CPYE)/CK
       ALFA=(GAMA-BETA)/(GAMA-RO)
       XK=(GAMA-BETA)*RO/(BETA*(GAMA-RO))
       PHI=ATANF(SQRTF((GAMA-RO)*(1.0-BETA)/((GAMA-1.0)*(BETA-RO))))
       CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
       EMGH=0.5/(CK*(RO-SIG)+CPYE*(BETA-RO))
       PHIC=EMGH*((RO-SIG)*(CK-U)+(BETA-RO)*(CPYE-PYE))
       XK=(BETA-RO)*GAMA/(BETA*(GAMA-RO))
       ALFA=(BETA-RO)/BETA
       PHI=PI/2.0
       CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
       QOKH=EMGH*(SIG*CK-RO*CPYE)
       PRINT11,SIG,RO,BETA,GAMA,BOD,D1D,D2D,GAP,PHIC,QOKH,EMGH
11     FORMAT(1X,11F10.5)
       DO 20 N=1,5
       XN=N
       XOD=(XN/5.0)*BOD
       T=1.0+XOD*XOD
       PHI=ATANF(SQRTF((GAMA-RO)*(T-BETA)/((GAMA-T)*(BETA-RO))))
       XK=(GAMA-BETA)*RO/(BETA*(GAMA-RO))
       ALFA=(GAMA-BETA)/(GAMA-RO)
       CALL EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
       PHIF=EMGH*((RO-SIG)*(CK-U)+(BETA-RO)*(CPYE-PYE))
       PRINT 18,N,XOD,PHIF
   18  FORMAT(1X,I5,2F15.5)
   20  CONTINUE
       IF(GAP-0.05)21,22,23
   21  GAP=0.05
       GO TO 3
   22  GAP=0.10
       GO TO 3
   23  CONTINUE
       GO TO 1
       END
```

```
      SUBROUTINE DEFUN(PHI,XK,G,U,CK,CKD,EU,CE,CED)
      SK=SQRTF(XK)
      PI=3.1415926
      M=-1
      P=PHI
      EK=XK
      EKD=1.0-EK
      IF(PHI-PI/2.0)1,2,1
    1 M=-1
      GO TO 17
2     M=0
      U=0.0
      EU=0.0
   17 P=PHI
   3  A=1.0
      B=1.0
      S=P
      F=P
      E=P
      N=1
   4  XN=N
      YN=2*N-1
      ZN=2*N
      S=S*YN/ZN-COSF(P)*(SINF(P))**YN/ZN
      A=A*EK*YN/ZN
      B=B*EK*(XN-1.5)/XN
      F=F+A*S
      E=E+B*S
      CHECK=ABSF(A*S)
      IF(CHECK-G)6,6,5
   5  N=N+1
      GO TO 4
   6  IF(M)7,8,9
   7  U=F
      EU=E
      P=PI/2.0
      M=M+1
      GO TO 3
    8 CK=F
      CE=E
      EK=EKD
      M=M+1
      GO TO 3
   9  CKD=F
      CED=E
      RETURN
      END
```

```
      SUBROUTINE EFUN(PHI,XK,ALFA,G,U,CK,CKD,EU,CE,CED,PYE,CPYE)
      SK=SQRTF(XK)
      PI=3.14159265
      M=-1
      P=PHI
      EK=XK
      EKD=1.-EK
3     A=1.
      B=1.
      S=P
      F=P
      E=P
      N=1
4     XN=N
      YN=2*N-1
      ZN=2*N
      S=S*YN/ZN-COSF(P)*(SINF(P))**YN/ZN
      A=A*EK*YN/ZN
      B=B*EK*(XN-1.5)/XN
      F=F+A*S
      E=E+B*S
      CHEK=ABSF(A*S)
      IF(CHEK-G)      6,6,5
5     N=N+1
      GO TO 4
6     IF(M)  7,8,9
7     U=F
      EU=E
      P=PI/2.
      M=M+1
      GO TO 3
8     CK=F
      CE=E
      EK=EKD
      M=M+1
      GO TO 3
9     IF(M-1)  10,10,11
10    CKD=F
      CED=E
      IF(ALFA)   101,104,109
101   IF(-ALFA-SK)   102,105,103
102   J=-2
      TH=ATANF(SQRTF(ALFA/(-XK)))
      GO TO 118
103   J=-1
      TH=ATANF(SQRTF(1.0/(-ALFA)))
      GO TO 118
104   J=4
      PYE=U
      CPYE=CK
      GO TO 20
```

```
105     J=6
       IF(PHI-PI/2.0)    106,107,107
106     AT=SQRTF(1.-XK*SINF(PHI)*SINF(PHI))
        PYE=U/2.+ATANF((1.-ALFA)*SINF(PHI)/(COSF(PHI)*AT))/(2.*(1.-ALFA))
        GO TO 108
107     PYE=CK/2.+PI/(4.*(1.-ALFA))
108     CPYE=CK/2.+PI/(4.*(1.-ALFA))
        GO TO 20
109    IF(ALFA-XK)  110,111,112
110     J=0
        TH=ATANF(SQRTF(ALFA/(XK-ALFA)))
        EK=XK
        GO TO 118
111     J=5
        AT=SQRTF(1.-XK*SINF(PHI)*SINF(PHI))
        PYE=(EU-XK*SINF(PHI)*COSF(PHI)/AT)/EKD
        CPYE=CE/EKD
        GO TO 20
112    IF(ALFA-SK)   113,105,114
113     J=1
        TH=ATANF(SQRTF((ALFA-XK)/(XK*(1.-ALFA))))
        GO TO 118
114    IF(ALFA-1.)  115,116,117
115     J=2
        TH=ATANF(SQRTF((1.-ALFA)/(ALFA-XK)))
        GO TO 118
116     J=7
        AT=SQRTF(1.-XK*SINF(PHI)*SINF(PHI))
        PYE=U-(EU-SINF(PHI)*AT/COSF(PHI))/EKD
        CPYE=1000.0
        GO TO 20
117     J=3
        TH=ATANF(1./SQRTF(ALFA-1.))
        EK=XK
118     P=TH
        M=M+1
        GO TO 3
11      EK=XK
        UT=F
        EUT=E
        V=PI*U/(2.*CK)
        W=PI*UT/(2.*CK)
        PE=PI*CKD/(2.*CK)
        Q=EXPF(-2.*PE)
        SUMN=0.0
        SUMD=0.0
        SUM=0.0
        N=1
12      XN=N
        NS=N*N
        XV=2.*XN*V
       IF(J)   121,124,125
```

```
121    IF(J+1)  122,123,124
122     XW=2.*XN*W
        GO TO 128
123     XW=2.*XN*(PE-W)
        GO TO 128
124     XW=2.*XN*W
        XP=2.*XN*PE
        SINH=(EXPF(2.*XP)-1.)/(2.*EXPF(XP))
        SUM=SUM+SINF(XV)*SINF(XW)/(XN*SINH)
        CHEK=ABSF(SINF(XV)*SINF(XW)/(XN*SINH))
       IF(CHEK-G)   16,16,15
125    IF(J-2)   126,127,134
126     XW=2.*XN*W
        GO TO 128
127     XW=2.*XN*(PE-W)
128     SINH=(EXPF(2.*XW)-1.)/(2.*EXPF(XW))
        COSH=(EXPF(2.*XW)+1.)/(2.*EXPF(XW))
       IF(J)  129,132,132
129     NH=N/2
       IF(N-2*NH)  130,130,131
130     SUMN=SUMN-Q**NS*SINF(XV)*SINH
        SUMD=SUMD+Q**NS*COSF(XV)*COSH
        GO TO 133
131     SUMN=SUMN+Q**NS*SINF(XV)*SINH
        SUMD=SUMD-Q**NS*COSF(XV)*COSH
        GO TO 133
132     SUMN=SUMN+Q**NS*SINF(XV)*SINH
        SUMD=SUMD+Q**NS*COSF(XV)*COSH
133     CHEK=ABSF(Q**NS*SINF(XV)*SINH)
       IF(CHEK-G)  14,14,15
14      CHEK=ABSF(Q**NS*COSF(XV)*COSH)
       IF(CHEK-G)  16,16,15
15      N=N+1
        GO TO 12
134     XW=2.*XN*W
        XP=2.*XN*PE
        SINH=(EXPF(2.*XP)-1.)/(2.*EXPF(XP))
        SUM=SUM+Q**XN*SINF(XW)*SINF(XV)/(XN*SINH)
        CHEK=ABSF(Q**XN*SINF(XW)*SINF(XV)/(XN*SINH))
       IF(CHEK-G)      16,16,15
16     IF(J)    136,135,136
135     QN=SUM
        ELM=EUT-CE*UT/CK
        D=SQRTF(ALFA*(1.-ALFA)*(XK-ALFA))
        PYE=U+ALFA*(U*ELM-QN)/D
        CPYE=CK+ALFA*(CK*ELM)/D
        GO TO  20
136    IF(J-3)  138,137,138
```

```
137     WPV=W+V
        WMV=W-V
        QN=0.5*ALOG(SINF(WPV)/SINF(WMV))+SUM
        ELM=EUT-CE*UT/CK
        D=SQRTF(ALFA*(ALFA-1.)*(ALFA-XK))
        PYE=-ALFA*(U*ELM-QN)/D
        CPYE=-ALFA*(CK*ELM)/D
        GO TO  20
138     D=2.*CK*SQRTF(ALFA*(1.-ALFA)*(ALFA-XK))
        ELM=(CE*UT+CK*EUT-CK*UT)*2./PI
        SUM=2.*SUMN/(1.+2.*SUMD)
        IF(J+1)  139,140,141
139     QN=2.*CK*ATANF(SUM)/PI
        PYE=U*XK/(XK-ALFA)-PI*ALFA*(U*ELM+QN)/D
        CPYE=CK*XK/(XK-ALFA)-PI*ALFA*(CK*ELM)/D
        GO TO  20
140    QN=U+2.*CK*ATANF(SUM)/PI
        PYE=U/(1.-ALFA)+PI*ALFA*(U*ELM-QN)/D
        CPYE=CK/(1.-ALFA)+PI*ALFA*(CK*ELM-CK)/D
        GO TO  20
141    IF(J-1)  142,142,143
142     QN=2.*CK*ATANF(SUM)/PI
        PYE=ALFA*PI*(U*ELM-QN)/D
        CPYE=ALFA*PI*(CK*ELM)/D
        GO TO 20
143     QN=U-2.*CK*ATANF(SUM)/PI
        PYE=U+PI*ALFA*(QN-U*ELM)/D
        CPYE=CK+PI*ALFA*(CK-CK*ELM)/D
20     RETURN
       END
```

APPENDIX III

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