MODELLING OF WASHING OF BROWN STOCK ON ROTARY VACUUM WASHER

A THESIS

submitted in fulfilment of the
requirements for the award of the degree
of
DOCTOR OF PHILOSOPHY
in
PULP AND PAPER TECHNOLOGY

By

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Candidate's Declaration

I hereby certify that the work which is being presented in the thesis entitled, **"MODELLING OF WASHING OF BROWN STOCK ON ROTARY VACUUM WASHER"**, in fulfillment of the requirements for the award of the Degree of **Doctor of Philosophy** submitted at **University of Roorkee**, is an authentic record of my own work carried out during the period May 1993 to November 1996, under the supervision of Dr. A. K. Ray, Dr. V. P. Singh and Dr. N. J. Rao.

The matter embodied in this thesis has not been submitted by me for the award of any other degree or diploma of any other University.

Dated : December 2 , 1996
Place : Saharanpur

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This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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ABSTRACT

Brown stock washing is the key operation controlling the cleanliness of pulp with least carryover of dissolved solids on one hand and separating the maximum black liquor at as high a concentration as possible on the other. Any carryover of black liquor solids with the washed pulp leaving for bleaching and paper making is a source of BOD, COD and other toxic pollution parameters harmful to the environment and subsequent processing. The washing process needs optimization with respect to equipment, operating cost and environmental factors. To achieve these objectives the primary step is to optimize the parameters for a desired degree of separation and to control them if any deviation occurs from the set values. For this development of mathematical modelling is imperative necessity. Several washing equipments are in use for pulp washing. But, rotary vacuum filters are most important equipments used for brown stock washing.

Though some mathematical models are available in the literature for different pulp washing systems, still there is enough scope to deal with this system with a view to achieve more realistic models. The present investigation deals with pulp washing under the influence of longitudinal dispersion coefficient(D_L) and accumulation capacity of fibers, which leads to adsorption and desorption dynamics in a rotary vacuum washer.

In a rotary vacuum washer several zones are formed during one rotation of drum. No two zones of a rotary vacuum washer have similar mechanism. Therefore mathematical model for each zone varies significantly. In this investigation a systematic approach for developing a mathematical model for all zones of a rotary vacuum filter, considering both macroscopic and microscopic interpretations has been followed.

For cake formation zone mathematical models in terms of pertinent parameters are available in literature. However, models for the prediction of filtrate flow rate of a rotary filter from constant pressure filtration data are rarely available. In this investigation filtrate flow rate is proposed with or without the consideration of filter medium resistance. The solution of the equations are given. The derived equations can be used for designing such filter from laboratory data on constant pressure filtration as well as for scaling up.

For cake washing zone non homogeneous, non linear, first order, second degree, partial differential equations are developed. Eight washing models are taken in hand with or without consideration of longitudinal dispersion coefficient (D_L) and different boundary conditions based on the earlier worker's models. Before solving, these differential equations are converted into dimensionless form by using Peclet number, dimensionless concentration, dimensionless time and dimensionless thickness. These equations are solved by applying Laplace transform technique. Inverse has been taken by using the method of residues. Dimensionless expressions are given for exit concentration of solute leaving the bed, average concentration of solute in discharged pulp and mean concentration of filtrate collected through the washing zone. The solution technique for models 1 to 6 are identical (except boundary conditions and adsorption isotherm) but the solution technique for models 7 and 8 are different.

The proposed model for filtrate flow rate has been validated by using the data of two investigators, Peck and Chand[59] and Perron and Lebeau[65]. The developed washing models have been validated by using the experimental data of Grahs[20], Perron and Lebeau[65] and Turner et al.[84]. The results obtained form present model are found in close agreement, indicating the reliability of the model.

Laboratory experiments were carried out by collecting blown unwashed pulp from a paper mill on an laboratory EIMCO-KCP rotary vacuum filter of size 18" X 12". Inlet vat consistency, rpm, fractional submergence of drum and wash water rate were varied during the experiments. The experimental data from laboratory were subjected to validate the models. The results calculated from the model also agrees quite well with the experimental data from a single stage rotary vacuum filter.

LIST OF RESEARCH PAPERS

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1. Kukreja V.K., Ray A.K., Singh V.P. & Rao N.J., "*Evaluation of filtration flow rates of continuous filters*", **IPPTA Convention Issue**, December, pp.41-45 (1994).

2. Kukreja V.K., Ray A.K., Singh V.P. & Rao N.J.,"*Different approaches to evaluate brownstock washer efficiency*", **IPPTA, 7(4)**, December, pp.61-65 (1995).

3. Kukreja V.K., Ray A.K., Singh V.P. & Rao N.J.,"*A mathematical model for pulp washing on different zones of a rotary vacuum filter*", **Indian Chemical Engineer**, Section A, **37(3)**, July-September, pp.113-124 (1995). Received Second Best Technical Paper award for the year 1995.

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CHAPTER 1
INTRODUCTION

1.1 Objective, Mechanism and Description of Brown Stock Washer

Pulp and paper industry is highly chemical, labour, pollution, energy and water intensive industry. It consumes approximately 10-17 tonnes of low pressure steam, 1300-1950 kwh electric energy, 250-350 m^3 of water and generates a pollution load of 300-400 ppm BOD, 1100-1300 ppm COD per tonne of paper. During the process of manufacture of paper various operations are involved like, chipping, pulping, washing, bleaching, paper making, chemical recovery and effluent treatment. Brown stock washing is one of the above steps, which inducts a direct impact on the economy of the mill as well as the environment.

The basic goal of brown stock washing is to remove black liquor solids from the pulp by spraying of water or weak wash liquor. Black liquor solids are removed mainly due to the following reasons:

(a) to obtain clean pulp for further processing.
(b) to recover expensive inorganic Na-salts in recovery section.
(c) to recover lignin based organic solutes for their heating values.
(d) to reduce consumption of bleaching chemicals in bleach plant.
(e) to reduce the cost of effluent treatment.
(f) strict pollution regulations forbid the sewering of chemicals.

Black liquor consists of dissolved organic (lignin derivatives) and inorganic (Na, Mg, Ca and K ions) solids. Aqueous organics (primarily lignin), if not separated from the pulp stream before bleaching, consumes excess bleaching chemicals which generates more undesirable effluents such as AOX, colour, BOD, COD and other toxic emissions especially dioxins and chlorinated furans during bleaching[31]. It may be pointed out that 1 kg of COD consumes about 0.4 to 0.8 kg of active chlorine. The lower the target Kappa number in the bleaching, the more Chlorine per kg of COD has to be used to get this Kappa number[76,77]. Estimated cost of removing the AOX was $84/kg of AOX[44].

Ideally there should be no overflow of black liquor solutes with the washed pulp and no overflow of fibers with the filtrate leaving the washing plant. But these ideal stipulations can never be met in the industry. Hence, it is desirable that the pulp should be washed optimally, i.e., using minimum amount of wash water and with maximum removal of black liquor solute.

The washing methodology is based on the following operations,
(a) displacement
(b) dilution - agitation - extraction
(c) dilution - displacement - extraction

The mechanism of pulp washing involves fluid mechanics superimposed by mass transport phenomena like diffusion. This is further complicated by dispersion, adsorption, desorption, foaming and channeling. Further, various studies[25,39,62,63] have shown that foam affects the brown stock performance to some extent. Time allowed for diffusion of washable substance out of fibers is also an important factor.

Grahs[18] has presented a simple model of a packed bed, which is given in figure 1. In this diagram the packed bed of cellulose fibers is divided into three different zones namely, flowing liquor, stagnant liquor and fibers. Mass transfer is assumed to take place between fibers to stagnant liquor and then from stagnant liquor to flowing liquor. This clearly indicates that the porosity value varies significantly from zone to zone. Therefore multiporosity values must be considered in the mathematical analysis of pulp washers. Majority of investigators, however, neglected the above aspects.

A variety of equipments are available in the market for brown stock washing. Notable among them are rotary vacuum washer, pressure washer, belt washer, wash press, screw press, diffusion washer, pressure diffusion washer and fiberfuge continuous centrifuge washer. Of all these equipments, rotary vacuum washer is the oldest and is very commonly used equipment in the industry. It has under gone continuous modification and development in its seven decades

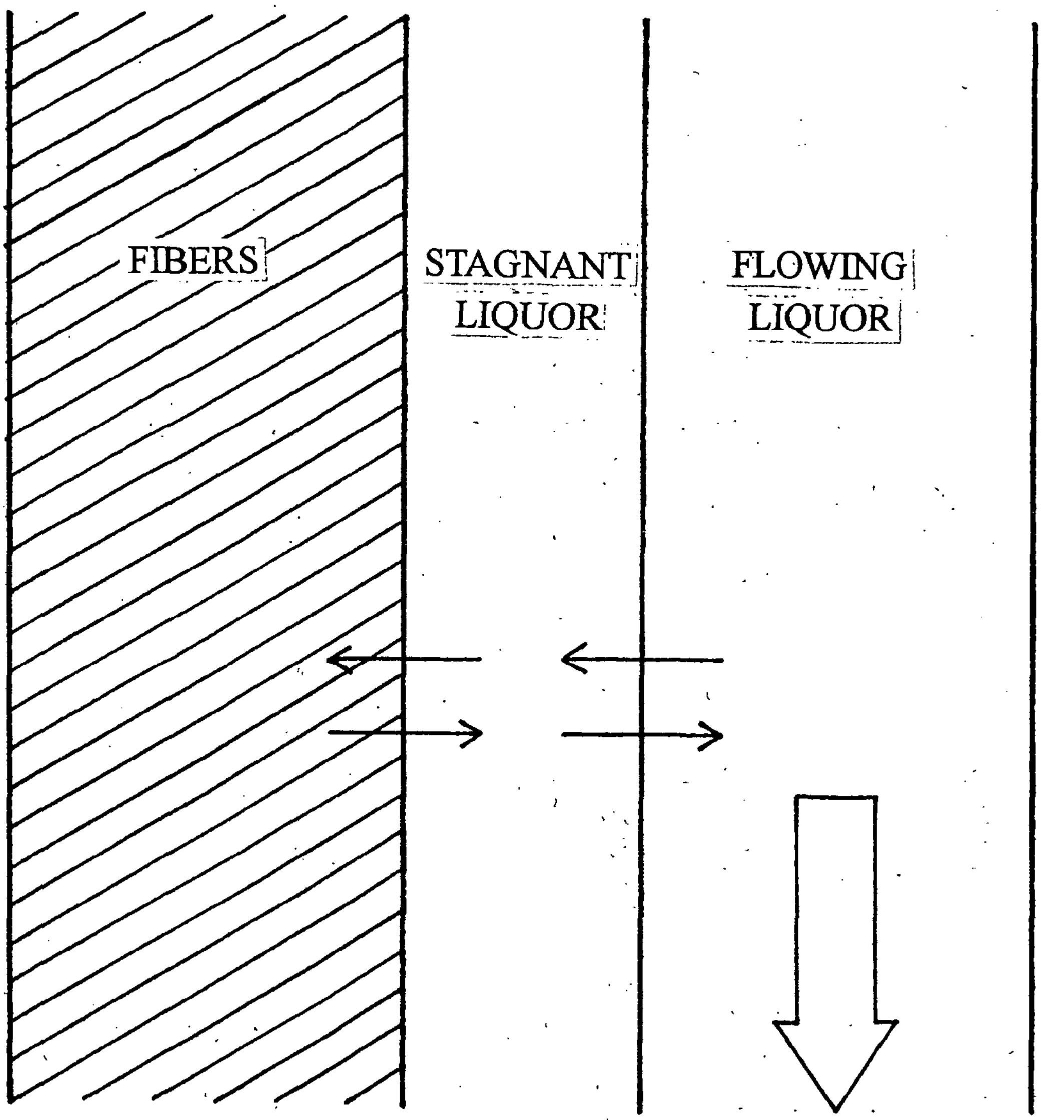

Figure 1 : A packed bed of cellulose fibers

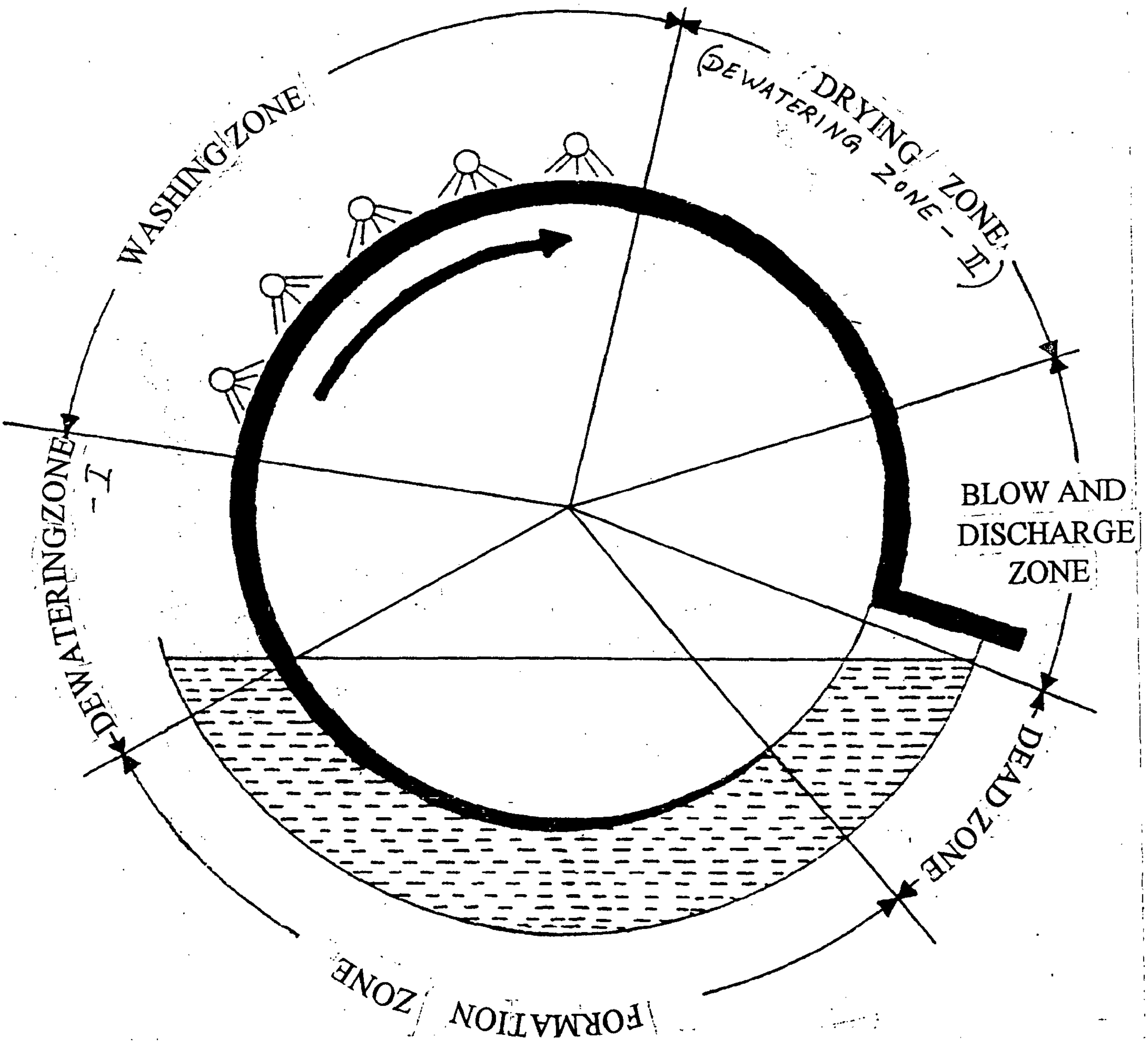

Figure 2 : Different zones of a rotary vacuum filter

of application. In industry a battery of 3 to 4 rotary vacuum washers connected in a counter current manner is used to wash the pulp.

A rotary vacuum washer consists of a wire mesh covered cylinder which rotates in a vat containing slurry. As the drum rotates inside the vat various zones are formed. These are cake formation, dewatering, washing, drying, blow, discharge and dead zones. Cross sectional view of a rotary vacuum washer is given in figure 2. The mechanism of each zone is explained hereunder.

Pulp of approximately 1% consistency is fed inside the vat. A layer of fibers is deposited on the outer surface of the drum due to vacuum inside the drum. This layer of fibers becomes progressively thicker and thicker till at the end of cake formation zone. At the outlet of the cake formation zone the mat consistency is approximately 10%. On the upper part of the drum, water/weak wash liquor is applied with the help of nozzles at a pre determined temperature and quantity. Certain portion of the drum before and after the washing zone is left out under the influence of the driving force (vacuum) to assist the removal of black liquor trapped inside the fibers. Black liquor, water and air collected through different zones is withdrawn with the help of pipes connected with the respective zones. Pressure differential is not applied after the drying zone which causes the cake to lift up from the wire mesh. The cake is removed with the help of a doctor blade. In a counter current washing system cleanest wash water is applied only at the showers of the final washer for economic reasons. Effluent from any stage is used for two purposes, to dilute the pulp coming from the previous washer and as a shower liquor for previous washer.

During washing the speed at which the diffusion of black liquor solids inside the fiber voids and the surrounding liquor takes place is dependent on the concentration difference between black liquor solutes at the inside and at the outside of the fibers, temperature, turbulence around the fibers and the size of the molecule. The larger the concentration difference, the higher the temperature and turbulence and the smaller the molecule size, the faster the diffusion will be and the faster the equilibrium will be reached[74].

According to Lindsay[41] dispersion is a convective mixing process that arises because of velocity profiles in individual pores and because of the complex branching and intertwining of flow paths in a porous medium.

The formation of cake during cake formation zone is highly dependent upon the inlet vat consistency, fractional submergence of drum, speed of drum, pressure drop across the cake and specific resistance of cake. During cake washing zone porosity, cake thickness, time of washing, liquor speed inside the cake pores, amount of wash water applied and mass transfer rate are major variables. Whereas in cake dewatering and drying zones cake saturation is a key variable because of simultaneous flow of liquor and air. Hence it can be concluded that the conditions and mechanism involved in each zone varies significantly and a single model can not be used for each zone. Therefore individual models are required for each zone to assess the pulp washing behaviour on a rotary vacuum washer. Normal range of different parameters is given in **Table 1.**

Table 1 : Normal range of different parameters

Parameter	Unit	Value
Inlet vat consistency	%	1 - 2
Outlet vat consistency	%	10 - 12
Cake thickness	m	0.025 - 0.040
Dilution factor	kg/kg	1 - 5
Fractional submergence of drum	-	0.25 - 0.45
Liquor speed in cake pores	m/s	0.003 - 0.006
Pressure drop across the cake	Pascal	20000 - 50000
Pulp temperature	°C	60 - 90
rpm	-	1 - 4

Mathematical model is an imperative necessity to characterize the pulp washing behaviour. For the research work a model can be highly complicated to obtain higher accuracy. But in actual practice there is a compromise between the accuracy and the complicity of the model. A mathematical model can be macroscopic, microscopic or semi quantitative in nature. Macroscopic / empirical /black box models gives a general outside description (in terms of material balances) of a brown stock washer. Microscopic /physical /gray box models gives a deep inside description (in terms of fundamental parameters) of a brown stock washer. Semi quantitative models are intermediate models between macroscopic and microscopic models.

A variety of parameters are available to describe the performance of a rotary vacuum washer. These can be mainly divided into three categories namely, wash liquor usage (dilution factor, wash liquor ratio, weight liquor ratio), solute removal (wash yield, displacement ratio, solid reduction ratio) and efficiency parameters (soda loss, Norden's and modified Norden efficiency factor, % efficiency, equivalent displacement ratio). These parameters are not employed by all investigators.

To analyze the performance of washers in different plants or in a same plant using different pulping techniques, in a uniform and coherent manner, interrelations between various parameters are required. Some relations between various efficiency parameters are, however, already known. More broad base correlations are still essential to define the effect of one variable on the others.

Different investigators have put forward nomenclature according to their own convenience. In this investigation a uniform nomenclature is followed to avoid confusion and difficulty in understanding various mathematical expressions. Each parameter is defined clearly and then a corresponding mathematical expression is given. Where ever possible an alternative expression is also given.

1.2 Aim of Present Investigation

Keeping the above considerations in view present investigation is planned to critically analyse the brown stock washer in details with the following distinct objectives.

-To develop simple realistic mathematical models for different zones of a brown stock washer based on phenomenological concept.

-To determine the possible solution of the individual model.

-To correlate the model with the conventional parameters employed by the industry to predict the washer performance in mills.

-To validate the model by the data reported by previous investigators.

-To extend the model for multiple stage brown stock washing system.

-To validate the model by experimental data obtained from a single stage laboratory brown stock washer.

CHAPTER 2
LITERATURE REVIEW

2.1 Review of Washing Conditions

A variety of experimental techniques and conditions have been employed by different researchers to study the pulp washing behaviour. Experiments were carried out by using different pulp samples like, Pine[19,22,23,25,27,28,32, 69,74, 87], White Oak[69], Birch[28], Spruce[28,78,79], Douglas fir[73] and a mixture of hardwood and softwood[40]. No experimental results are available for a mixture of Indian wood and nonwood (Bamboo) based pulps.

Equipments like displacement washing cell[19,20,22,23,31,32,40,74,80], buchner funnel[69, 87], equilibrium apparatus[78, 79], pulp tester[25] and brown stock washer[27,65] were used.

In **Table 2** a summary of the experimental conditions used by previous investigators is given. Some contradictions of different studies are summerised below.

Temperature was found to have no effect on sorbed sodium [69,78] whereas according to other investigators[20,40,73] more amount of Na can be removed by increasing the temperature.

Sodium concentration in wash liquor has no effect[69], little effect[73,87] on sorbed Na, while[19,28,78] have found Langmuir type relationship between sorbed Na and the Na in wash liquor.

Increase in mat consistency is not beneficial[40]. Norden efficiency factor increases by increasing the mat consistency [25]. According to [80] washing efficiency increases if mat consistency is less than 13% and it decreases if mat consistency is more than 13%.

Washing efficiency is slightly affected by velocity of wash liquor[40]. Norden efficiency factor decreases[25]. Displacement ratio decreases[23]. Washing efficiency factor increases if mat consistency is less than 3% and decreases if

Table 2 : A summary of different studies

Investigator		Kappa number	Temperature (°C)	pH	Concentration of solutes (ppm)	Time of leaching (hr)	Cake thickness (m)	Mat consistency (%)	Wash liquor velocity (m/s)
Gren et al.	[23]	17-81	-	-	-	-	0.040-0.090	6-11	0.1-0.5 $\times 10^{-3}$
Rosen	[69]	15-80	38-80	3-12	10-1300	48	-	30	-
Hartler et al.	[28]	19-55	-	-	0-400	48	-	10	-
Grahs	[19]	22.4-43.4	21	-	0-1000	-	0.102-0.166	6-10	0.1-1.1 $\times 10^{-3}$
Grahs	[20]	-	21-50	-	-	-	0.100	8-10	0.07 $\times 10^{-3}$
Xuan et al.	[87]	35-41	-	0-12	250-452	-	-	-	-
Lee	[40]	-	20-55	-	-	-	0.0254	3.5-17.5	0.072-0.75 $\times 10^{-3}$
Hakamaki et al.	[25]	27-32	30-90	-	-	-	0.020	7-12	2-12 $\times 10^{-3}$
Gren et al.	[23]	31-34	20	2-11	-	-	0.020-0.080	5-21	0.04-3.2 $\times 10^{-3}$
Trinh et al.	[78]	-	30-50	7-9	0-1000	48	-	-	-
Han et al.	[27]	-	-	-	3800	-	0.0295-0.0511	13.6-19.2	-
Trinh et al.	[80]	27-35	29-90	-	580-3800	-	0.025-0.085	3.2-17.4	0.03-0.99 $\times 10^{-3}$
Smith et al.	[73]	30-48	20-90	7-13	-	24	-	-	-

mat consistency is greater than 15% by increasing the wash liquor velocity[80].

To eliminate these contradictions it is required to determine the complex relationship between these key variables and washing efficiency. This necessitates the development of a plausible mathematical model based on the complex interactions between dependent and independent variables.

2.2 Mathematical Models of Previous Investigators

Models proposed by different investigators can be classified as macroscopic, microscopic and semi quantitative. In macroscopic models[42,50,51,53,54,55, 56,58,64,67,81,82] process is described in terms of material balances around the washing stage. These models can not account for the sorption and mass transfer phenomena. Microscopic models[5,18,23,26,27,34,38,41,45,61,65,72, 86] primarily define the mechanism of the process and are based on the filtration theory of flow through porous media. In semi quantitative approach[10,11,52, 56, 85] stage contact process is described in terms of fundamental parameters.

Different investigators have solved their equations by using various mathematical techniques. Laplace transform technique [4,5,13,26,33,34,37,38, 45,46,47,61,65,72,73] has been the most popular technique for solution while Orthogonal Collocation[18,48,49], Galerkin finite element method[1] and Finite difference method[41] are equally popular. Different models have been reviewed by [3,9, 60,68].

According to Edwards and Rydin[14], though empirical parameters such as displacement ratio and Norden's efficiency factor are simple to use for the approximate calculation of the material balance of the washing operation and serves the purpose qualitatively, these cannot account for sorption and mass transfer phenomena in a rational manner, which is a decided disadvantage. This disadvantage, however, can be eliminated by means of microscopic models.

According to Gren[21] and Gren and Grahs[22], for better and more detailed description of washing, a microscopic model with the influence of fluid velocity, pulp type, concentration of dissolved solids, fiber consistency, dispersion, non linear adsorption, time dependence of mass transfer and presence of stagnant zone during washing have to be accounted for. These factors play an important role at the end of the washing operation where the solute concentration is low and a macroscopic model would become less accurate.

Some of the mathematical models proposed by previous investigators are presented below along with their respective initial and boundary conditions.

Lapidus and Amundson[38] have studied the effect of longitudinal diffusion in ion exchange and chromatographic columns by using the following differential equation.

$$D_L\,(\partial^2 c/\partial z^2) = u\,(\partial c/\partial z) + (\partial c/\partial t) + (1/\varepsilon_t)(\partial n/\partial t) \qquad (2.1)$$

Two cases were used to study the mechanism of adsorption, i.e.,

$$n = k'c + k'' \qquad (2.2)$$

$$\partial n/\partial t = k_1 c - k_2 n \qquad (2.3)$$

Initial and boundary conditions were,

$$c(z,0) = C_i \quad \text{and} \quad n(z,0) = n_i \qquad (2.4)$$

$$c(0,t) = C_s \qquad (2.5)$$

Kuo[34] neglected the longitudinal dispersion coefficient to study Sodium Chloride washing and considered following differential equation for the wash liquor,

$$\partial c/\partial t = k_1(n-c) - u\,(\partial c/\partial z) \qquad (2.6)$$

and for the stagnant film, $\partial n/\partial t = k_2(c - n)$ (2.7)

At the boundary the solute concentration in the wash liquor was,

$$c(0,t) = C_s \qquad (2.8)$$

Initial solute concentration in the filtrate film was taken as,

$$n(z,t) = C_i, \quad \text{for } ut < z < L \tag{2.9}$$

Brenner[5] studied the washing of filter cake by neglecting the accumulation capacity of fibers and assumed that the phenomena of longitudinal mixing is governed by the following equation,

$$D_L\,(\partial^2 c/\partial z^2) = u\,(\partial c/\partial z) + (\partial c/\partial t) \tag{2.10}$$

Initial condition was $c(z,0) = C_i$ and boundary conditions were,

$$uc - D_L(\partial c/\partial z) = uC_s, \text{ at } z = 0, \; t > 0 \tag{2.11}$$

$$\partial c/\partial z = 0, \text{ at } z = L, \; t > 0 \tag{2.12}$$

Sherman[72] has described the overall movement of solute in the bed of non porous granular material with the diffusion like differential equation by replacing molecular diffusion coefficient (D_V) with longitudinal dispersion coefficient (D_L). D_V was found very small as compared to D_L and was neglected. An additional term was used to account for the accumulation (or depletion) capacity of material sorbed by the solids. Washing experiments were performed by preparing thin packed beds of glass beads, Dacron fibers and viscose fibers.

$$D_L\,(\partial^2 c/\partial z^2) = u\,(\partial c/\partial z) + (\partial c/\partial t) + [(1-\varepsilon_t)/\varepsilon_t](\partial n/\partial t) \tag{2.13}$$

Sherman expressed n in terms of c by using a linear relation,

$$n = k\,c \tag{2.14}$$

In order to avoid experimental difficulties involved in obtaining a satisfactory step function following initial and boundary conditions were used,

$$c(z,0) = C_i \tag{2.15}$$

$$c(0,t) = f(t) = C_i\,(k_0 + k_1 t + k_2 t^2 + k_3 t^3 + k_4 t^4)\, e^{-\gamma t} \tag{2.16}$$

Where γ is a constant. Constants k_0, k_1, k_2, k_3 and k_4 were adjusted so that the equation(2.16) agrees with the experimentally determined values of c vs t.

Pellett[61] has studied the longitudinal dispersion of solute, intra particle diffusion of solute and liquid phase mass transfer for the particles of cylindrical and spherical geometry by using a modified step function input. All skin viscose yarns of 1,4,16,64 denier cut to uniform length were used as the fiber bed. Three coloured solutes were used to obtain different diffusion characteristics. The following equation was used.

$$D_L (\partial^2 c/\partial z^2) - u (\partial c/\partial z) = (\partial c/\partial t) + [(1-\varepsilon_t)/\varepsilon_t](\partial n/\partial t) \qquad (2.17)$$

This equation is solved by using the initial condition,

$$c(z,0) = C_i \qquad (2.18)$$

and the following inlet boundary condition was used,

$$c(0,t) = C_i (k_0 + k_1{}'t + k_2{}'t^2 + k_3{}'t^3 + \ldots) e^{-\gamma' t} \qquad (2.19)$$

This equation is a modified step function that Sherman[72] introduced to describe experimentally observed inputs.

Grahs[18] has divided the packed bed of cellulose fibers into three different zones namely zone of flowing liquor, stagnant liquor and fibers. Longitudinal dispersion and mass transfer in the flowing liquor zone is characterized by the following equation,

$$\varepsilon_d D_L \frac{\partial^2 c}{\partial z^2} - \varepsilon_d u \frac{\partial c}{\partial z} = \varepsilon_d \frac{\partial c}{\partial t} + \varepsilon_s \frac{\partial c_s}{\partial t} + C_F \frac{\partial n}{\partial t} \qquad (2.20)$$

Here it was assumed that D_L is directly proportional to the interstitial velocity u, which implies that $D_L \varepsilon_d$ = constant.

For the stagnant liquor zone following equation applies,

$$\varepsilon_s(\partial c_s/\partial t) = k_1 (c-c_s) - C_F (\partial n/\partial t) \qquad (2.21)$$

and for the fibers following equation was used,

$$C_F\,(\partial n/\partial t) = k_2\,(c_s - c_s^*) \qquad (2.22)$$

n and c^* are assumed to be related by the Langmuir adsorption isotherm,

$$n = f(c_s^*) = A\,c_s^*\,/(1 + B\,c_s^*) \qquad (2.23)$$

The initial and boundary conditions were,

$$c(z,0) = c_s(z,0) = c_s^*(z,0) = C_i \quad \text{and} \quad n(z,0) = n_i \qquad (2.24)$$

$$uc_{in} = uc(0+) - D_L\,(\partial c/\partial z)\,, \quad \text{at } z = 0+ \qquad (2.25)$$

$$\partial c/\partial z = 0, \quad \text{at } z = L- \qquad (2.26)$$

Perron and Lebeau[65] have basically followed the model of Kuo[34] for pulp washing. They have proposed static as well as dynamic models for a rotary vacuum washer. They have considered following differential equation for the liquor,(neglecting the longitudinal dispersion coefficient)

$$\varepsilon_d u\,(\partial c/\partial z) + \varepsilon_d(\partial c/\partial t) + \varepsilon_s(\partial n/\partial t) = 0 \qquad (2.27)$$

and for stagnant film, $\partial n/\partial t = -k\,(n - c)$ (2.28)

Initial and boundary conditions were,

$$c(z,t) = C_i\,,\ n(z,t) = C_i \quad \text{for } 0 \le t \le z/u \qquad (2.29)$$

$$c(0,t) = C_s \qquad (2.30)$$

Viljakainen[85] has used following dispersion model for pulp washing,

$$D_L\,(\partial^2 c/\partial z^2) = u\,(\partial c/\partial z) + (\partial c/\partial t) + [(1-\varepsilon_t)/\varepsilon_t](\partial n/\partial t) \qquad (2.31)$$

He has solved this equation for the equilibrium isotherm n = c.

Wong and Reeve[86] have carried out experiments on nylon and various modified pulp fibers by using Potassium Chloride. They have studied the unsteady state diffusion in fiber beds by using following equation,

$$D_L(\partial^2 c/\,\partial z^2) = \partial c/\partial t \qquad (2.32)$$

Lindsay[41] used the following radial equation for dye transport through porous medium by assuming that effective dispersion coefficient (D) is constant.

$$(\partial c/\partial t) = D\ [(\partial^2 c/\partial r^2) + (1/r)(\partial c/\partial r)\ ] - (B/r)(\partial c/\partial r) \qquad (2.33)$$

Initial dye concentration is C_0.

2.3 Industrial Practice in Evaluation of Filter Performance

A model will be useful only when the same can predict the performance or efficiency of a brown stock washing system in the real situations. Therefore the model predicted data can control various parameters of a brown stock washer to get optimal results. However, the industry uses a number of washers (3 to 5) interconnected with each other and working concurrently. Therefore the model developed for a single washer must be applied in each and every washer individually. The interconnecting flow streams can be found out by material balance.

The prediction of performance of a pulp washing system is indicated by the quantum of black liquor solids removed with the amount of wash water added. Different investigators have put forward various parameters to estimate the performance of a washing plant. The parameters for brown stock washing control are broadly classified into three categories namely, wash liquor usage, solute removal and efficiency parameters. The nomenclature put forward by different investigators are also varying widely. As a consequence their results appear different for the same set of fundamental parameters, even for the same plant.

In this investigation a uniform nomenclature is attempted to avoid confusion and to alleviate difficulty in understanding various mathematical expressions. Each parameter is defined and a mathematical expression is given. Where ever possible an alternative expression is also cited.In figure 3 various streams related to a single stage rotary vacuum filter are shown.

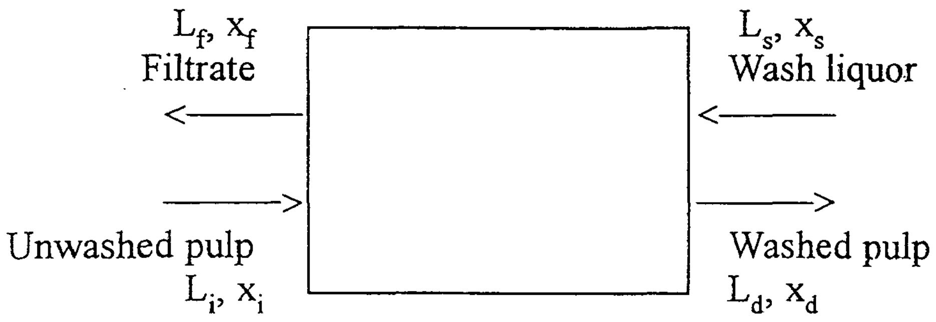

Figure 3 : A single stage rotary vacuum washer

2.3.1 Wash liquor usage parameters

These parameters describe the amount of wash water used in pulp washing and measures the evaporator load. The amount of wash water should be kept as low as possible otherwise an excess quantum of liquor is to be treated in evaporator which consumes excessive energy or steam.

2.3.1(a) Dilution factor

The difference between wash liquor entering and wash liquor in the washed pulp is known as dilution factor or excess wash water.

$$DF = L_s - L_d \qquad (2.34)$$

2.3.1(b) Wash liquor ratio

The ratio of wash liquor entering to the liquor leaving with washed pulp is known as wash liquor ratio.

$$WR = L_s/L_d \qquad (2.35)$$

Service and Seymour[71] have used the term **Dilution ratio** instead of wash liquor ratio. Tomiak[81] put forward a new term called **Effective wash ratio** (WR_e) as,

$$WR_e = (C_i - C_d)/(C_d - C_s) \qquad (2.36)$$

when $WR_e < 1$, complete recovery of solute is not possible even with an infinite number of stages. In practice, for good recovery of solute, it is necessary to use $WR_e > 1$. $WR_e = 1$ is known as **Minimum effective wash ratio**, it corresponds to an ideal case in which the concentration of the wash water is same as that of filtrate in unwashed cake.

Cullinan[11] has defined **Minimum wash ratio** (WR_{min}) as the value of wash ratio at which an infinite number of Norden stages would be required to attain the specified wash loss. In practice, wash ratio can be specified as some multiple of minimum wash ratio.

$$WR_{min} = (C_b - C_d)/(C_b - C_s) \tag{2.37}$$

Edwards and Rydin[14] have referred **Adjusted wash liquor ratio** (WR') as a multiple of wash liquor ratio with a correction for sorption effect as,

$$WR' = WR / [1+(K'/L_d)] \tag{2.38}$$

where K' is slope of sorption curve $AB/(1+Bc)^2$. A, B are constants of Langmuir isotherm $ABc/(1+Bc)$. The term $1/[1+(K'/L_d)]$ is significant at low liquor concentration, i.e., K' is large.

2.3.1(c) Weight liquor ratio

It is the ratio of filtrate liquor leaving the washer with washed pulp to the amount of liquor entering with unwashed pulp.

$$WL = L_f/L_i \tag{2.39}$$

where $L_i = (100 - C_{yi})/C_{yi}$; $L_f = L_i + DF$

When there is no change in the consistency through the washers, WL and WR are same, provided the changes in the liquor densities are small. The term **Liquor withdrawal ratio** used by Tomiak[83] is nothing but the weight liquor ratio.

2.3.1(d) Filter entrainment

During washing the displacement of black liquor is never complete and there is always a certain amount of black liquor left in the pulp mat. The concentration of entrained liquor is identical to the liquor inside the vat. As a result of entrainment, certain amount of wash liquor passes out as filtrate without displacing the black liquor. Fitch and Pitkin[15] have defined filter entrainment as weight of undisplaced vat liquor remaining in the washed pulp per unit of dry fibers.

$$FE = L_d(1\text{-}DR)m \qquad (2.40)$$

where $L_d = (100\text{-}C_{yd})/C_{yd}$ and DR is displacement ratio.

2.3.1(e) Thickening factor

It is defined as follows,

$$TF = \frac{\text{liquor in stock entering - liquor in washed pulp}}{\text{liquor in stock entering}} = \frac{L_i - L_d}{L_i} \qquad (2.41)$$

2.3.2 Solute removal parameters

These parameters describes the amount of dissolved solids removed during a washing stage or washing operation and can be used to predict the amount of bleach chemical consumption. These parameters increase when the wash liquor usage parameters are increased.

2.3.2(a) Wash yield

The ratio of dissolved solids removed (going with filtrate) to the dissolved solids entering with unwashed pulp is known as wash yield of any stage.

$$Y = L_f x_f / L_i x_i = (L_i + DF) x_f / L_i x_i \qquad (2.42)$$

For dilution/extraction washing operation, the wash yield can be expressed in terms of the extraction step input and discharge consistency as,

$$Y = [(C_{yd}\text{-}C_{yi})/\{C_{yd}(100\text{-}C_{yi})\}]100 \qquad (2.43)$$

2.3.2(b) Displacement ratio

Perkins et al.[64] have defined displacement ratio of any stage as the ratio of actual reduction of dissolved solids to the maximum possible reduction of dissolved solids.

$$DR = (x_i - x_d)/(x_i - x_s) \quad (2.44)$$

The effect of sorption is of course not included in the DR value. DR depends upon the nature of substances to be removed and hence a function of the solutes present in black liquor/waste liquor and is also influenced by pulping processes. Josephson et al.[32] have defined DR interms of the concentration of Na based (inorganic) and lignin based (organic) solutes as,

$$DR_i = (C_{ii} - C_{di})/(C_{ii} - C_{si}) \quad (2.45)$$

$$DR_o = (C_{io} - C_{do})/(C_{io} - C_{so}) \quad (2.46)$$

The subscripts 'i' and 'o' stands for inorganic and organic substances respectively. DR_i can be measured directly from conductivity measurements while DR_o comes from UV analysis.

2.3.2(c) Solid reduction ratio

The ratio of the solids in the washed pulp (or pulp going for bleaching section) to the solids in the vat (or blow tank) is known as solid reduction ratio of a washing stage (or washing system).

For a single stage, $SR = x_d/x_i$ (2.47)

For the entire washing system, $SR = x_c/x_b$ (2.48)

2.3.2(d) % solids to evaporator

Perkins et al.[64] have proposed following expression to calculate the percentage of black liquor solids going to evaporator as,

$$\% \; SE = [L_b x_b - L_c x_c] / [L_b + DF] \quad (2.49)$$

2.3.2(e) Stage loss

The ratio of dissolved solids going with the washed pulp to the dissolved solids entering with the unwashed pulp is known as stage loss of any stage.

Tomiak[83] has given following expression for stage loss,

$$ST = L_d x_d / L_i x_i \tag{2.50}$$

Alternatively, $ST = 1\text{-}Y$ (2.51)

2.3.2(f) Overall system loss ratio

Cullinan[11] has defined overall system loss ratio as the ratio of the solute concentration in washed pulp leaving for bleaching section to the solute concentration in the blow liquor.

$$SLR = C_d / C_b \tag{2.52}$$

2.3.3 Efficiency parameters

This section covers some efficiency parameters available in literature. The efficiency of a washer is directly proportional to the amount of solids removed during the washing operation.

2.3.3(a) Soda loss

Solids lost during the washing operation are termed as soda loss. It is usually expressed in terms of salt cake loss and measured in terms of Na_2SO_4. According to Phillips and Nelson[67] salt cake loss can be found by using following expression,

$$\text{Salt cake loss} = [(100 - \%WE)/100]\ \text{salt cake charged} \tag{2.53}$$

where,

$$\%WE = \left[1 - \frac{DF\, C_{yd} / (100 - C_{yd})}{[\{DF\, C_{yd} / (100 - C_{yd})\} + 1]^{MNEF+1} - 1}\right] 100$$

$$\underset{(kg/T\ OD\ pulp)}{\text{Salt cake charged}} = \frac{(142)(\text{actual AA charged})(1.667)(1000)}{(62)(\text{pulp yield})}$$

1.667 is the ratio of total titrable alkali (TTA) to the active alkali (AA) charged (Phillips and Nelson[67]). All measurements are made on OD basis.

$$\text{Actual solid loss (kg/T OD pulp)} = \frac{(100\text{-}\%WE)(142)(\text{actual AA charged})(1.667)(1000)(1.75)}{(100)(62)(\text{pulp yield})} \quad (2.54)$$

2.3.3(b) % efficiency

Percent of black liquor solids removed during the washing operation is known as % efficiency of the system.

According to Smook[75], $\% E = [TF + (1\text{-}TF)DR]\ 100$ (2.55)

According to Lafreniere et al.[36], $\% E = [(L_p x_p - L_d x_d) / L_p x_p]\ 100$ (2.56)

According to Luthi[42] **Overall % efficiency** ($\%E_o$) of a washing system can be found as,

$$\%E_o = [(L_b x_b - L_c x_c) / L_b x_b]\ 100 \quad (2.57)$$

In terms of DR value of each stage, $\%E_o$ can be found as,

$$\%E_o = [\,1 - (1\text{-}DR_1)(1\text{-}DR_2)\ldots(1\text{-}DR_n)]\ 100 \quad (2.58)$$

The suffix 1, 2, ..., n indicates the DR of that stage.

Lafreniere et al.[36] have found **Theoretical washing efficiency** (%TWE) of a subsystem operating for recycled pulps by assuming same concentration of contaminants in inlet, outlet and filtrate streams, with no fiber loss as,

$$\%TWE = [(C_{yd} - C_{yi})/C_{yd}(1 - C_{yi})]100 \quad (2.59)$$

For a multiple stage system the **Overall theoretical washing efficiency**($\%TWE_o$) of contaminants can be found as,

$$\%TWE_o = [\,1 - (1\text{-}E_1)(1\text{-}E_2)\ldots(1\text{-}E_n)]\ 100 \quad (2.60)$$

The suffix 1, 2, ..., n indicates the TWE of that stage.

2.3.3(c) Efficiency

Cullinan[10,11] has proposed a different concept of brown stock washing by combining the fundamental and empirical parameters. The performance or design equations are free of unnecessary algebraic complexity. Three type of efficiency namely, local, stage and overall is considered. These efficiencies are related to one another.

Local efficiency (Eff_l) can be found by,

$$Eff_l = [\,1 - \exp(-k^*L/u)]\,[1+(u/k^*L)] \qquad (2.61)$$

By neglecting dispersion and assuming transverse plug flow of liquor above equation reduces to,

$$Eff_l = 1 - \exp(-k^*L/u) \qquad (2.62)$$

Stage efficiency (Eff_s) is defined by,

$$Eff_s = (C_f - C_s)/(C_d - C_s) \qquad (2.63)$$

Overall efficiency (Eff_o) is the ratio of the required number of Norden (perfect) stages to the required number of actual stages.

$$Eff_o = \log[\,1 + Eff_s(WR-1)]/\log WR \qquad (2.64)$$

Cullinan[10] has proposed displacement ratio in terms of local efficiency and wash ratio as,

$$DR = 1 - \exp[-Eff_l WR] \qquad (2.65)$$

Local and stage efficiencies are correlated as,

$$Eff_s = [(1+RR)\exp\{Eff_l WR\} - 1]/[RR+WR] \qquad (2.66)$$

where RR : **Recycle ratio** = L_l/L_d (2.67)

When Eff_l=0, Eff_s has a sizable value. This means that if rate of mass transfer is zero, separation of black liquor solids would be caused by pure dilution.

2.3.3(d) Norden's efficiency factor

Norden's efficiency factor(NEF) of a washing system without side streams can be defined as the number of mixing stages in series with complete mixing of underflow and overflow required to achieve the same departing underflow and overflow as those of the washing system, when the entering flows of the mixing stage system are the same as those of the washing system. Mathematically NEF for a single stage can be written as,

$$NEF = \log[\{L_i(x_i-x_f)\}/\{L_d(x_d-x_s)\}] / \log(L_s/L_d) \quad (2.68)$$

If NEF=1, one has ideal mixing and if NEF= ∞, plug flow results. When $L_s=L_d$, NEF is calculated by using the following expression,

$$NEF = L_d(x_f-x_s)/L_i(x_i-x_d) \quad (2.69)$$

NEF is also known as **Displacement efficiency number**. According to Tomiak[81], NEF method is directly applicable to cases where solute sorption is not involved and it offers an insight into the difficulties encountered when solute sorption does occur. For a single stage, Oxby et al.[58] have expressed NEF interms of only concentration and consistency measurements as follows,

$$NEF = \log[L_i(x_i-x_f)/L_d(x_d-x_s)]/\log[\{L_i(x_i-x_f)+L_d(x_f-x_d)\}/L_d(x_f-x_s)] \quad (2.70)$$

NEF for whole washing system can be found as,

$$NEF = \log[\{L_b(x_b-x_f)\}/\{L_c(x_c-x_s)\}]/\log(L_s/L_d) \quad (2.71)$$

NEF for a system consisting of n stages (with no side streams) can also be found if the efficiency factor and wash ratio of each individual stage is known, by using the following equation,

$$NEF \log WR_n = NEF_1 \log WR_1 + NEF_2 \log WR_2 + ... + NEF_n \log WR_n = \sum_{i=1}^{n} NEF_i \log WR_i \quad (2.72)$$

Again if wash liquor ratio is 1, following expression results,

$$NEF/L_{dn} = NEF_1/L_{d1} + NEF_2/L_{d2} + ... + NEF_n/L_{dn} = \sum_{i=1}^{n} NEF_i/L_{di} \quad (2.73)$$

Where L_{d1}, L_{d2}, ..., L_{dn} indicates the amount of liquor in the pulp discharged from 1, 2, ..., n^{th} stage.

2.3.3(e) Modified Norden efficiency factor

Phillips and Nelson[67] have developed modified Norden efficiency factor (MNEF) for a stage, as the number of ideal countercurrent mixing stages equivalent to a washing system operating at standard discharge consistency (C_{yst}) of 10 % or 12 % and same dilution factor.

$$MNEF = \log[\{(L_i(x_i - x_f)\}/\{L_d(x_d - x_s)\}]/\log[1+(DF/L_{st})] \tag{2.74}$$

where, $L_{st} = (100-C_{yst})/C_{yst}$. Different type of washers or washers operating at the same DF but over different consistency ranges can be compared on the basis of their respective MNEF values. Total MNEF of the entire system can be found by adding the MNEF value for each individual stage.

2.3.3(f) Equivalent displacement ratio

The concept of equivalent displacement ratio(EDR) was introduced by Luthi[42]. EDR is a useful tool for comparing the performance of two washers of different designs. In this the actual washer is compared with a hypothetical washer operating at standard inlet consistency of 1% and outlet of 12%. EDR for the hypothetical washer is calculated by using following formula,

$$(1-EDR) = (1-DR)(DCF)(ICF) \tag{2.75}$$

DCF = discharge correction factor = $L_d/7.333$

ICF = inlet correction factor = $99(L_i+DF)/[L_i(99+DF) - L_d(99-L_i)(1-DR)]$

For dilution extraction washers, $ICF = 99/(99+DF+L_d)$

Here the term equivalent means that hypothetical washer has the same dilution factor as the actual washer. It also has the same loss as the actual washer. The loss is calculated by subtracting the weight of solids entering with shower liquor from the solids leaving with the discharge, i.e., loss = $L_d x_d - L_s x_s$.

Luthi[43] correlated EDR, WR and MNEF for displacement washing as,

$$EDR = 1 - [(WR - 1)/\{WR^{(MNEF+1)} - 1\}] \tag{2.76}$$

The above definitions can also be used for multistage washers.

2.4 Conclusions

It is very difficult to compare the results given in **Table 2**, because of the different type of raw materials, pulping techniques and a variety of equipments employed by the investigators. Pulps of different Kappa numbers were used to carry out the experiments at different temperature and pH values.

In the literature there is abundant of information regarding the material balances, however, there is reticent silence about the interaction of various operational and design parameters. While in operation very few investigators have attempted to throw light on these issues.

Majority of the investigators have not taken into account the parameters related to diffusion, dispersion, adsorption, desorption, multiporosity values for inter particle and intra particle voids and reinforced parameters like temperature, pressure drop and consistency.

Although some mathematical models are available in literature. Limited studies are carried out for pulp washing under the influence of longitudinal dispersion coefficient and accumulation capacity of fibers. Besides, there are significant variations noted among the models of many investigators in their adsorption desorption isotherm equations. The solution techniques are also remarkably different. No investigator has ever compared the results evolved out by assuming different adsorption isotherms that too for different boundary conditions.

In addition to above, a detailed review has also been done on rational definitions of parameters pertinent to brown stock washing. The interrelations between the different parameters are also reported. This is an important area as these can help the designer and the practicing engineers to predict, to compare, to optimize and to design the washing system and equipment for mill practice.

Majority of parameters are found to be a function of two variables only namely, concentration of dissolved solids and amount of liquor present in different streams. Expressions for % efficiency, actual soda loss and NEF for entire system have been reported. Equivalent displacement ratio, a modification of conventional displacement ratio, is a simple, useful and versatile concept. The rotary vacuum washers used in various industries handling different pulp of varying Kappa number and pH can be compared on the basis of their EDR values. This will be further helpful to distinguish the performance for different set up of washers in a same industry.

The parameters listed here cannot account for the sorption of Sodium and lignin. Hence to measure the amount of sorbed Na and lignin a detailed investigation is required, otherwise the results obtained would not be precisely reliable.

CHAPTER 3
MATHEMATICAL MODELS OF DIFFERENT ZONES

A systematic approach for developing a mathematical model of a rotary drum vacuum washer has been attempted considering both macroscopic and microscopic interpretations. The complete mathematical modelling of the said system requires in-depth knowledge of the washing operation as a whole and the function of various zones involved with it.

During each rotation of the drum various zones are formed (figure 4). Out of these zones the overall performance of the washer depends mainly upon the cake formation and washing zone. In a rotary vacuum washer working for brown stock washing and bleach washing the total area combining these two zones is more than 50 %. Therefore the maximum amount of filtrate (both thick and thin) from these zones will contribute maximum to the total whereas other zones, such as dewatering and drying, are engaged in discharging comparatively much less amount of filtrate. Major amount of air is also extracted in these zones along with the filtrate exhibiting a two phase flow situation. Contribution of air towards the displacement of black liquor is not so high. Dead and discharge zones have practically little or no contribution to the generation of filtrate.

3.1 Models for Cake Formation Zone

While the model of cake formation zone is supposed to be well established, the different parameters connected with this model appears to complicate the equations. It has been observed that no consideration of static pressure drop has been taken into consideration, as it might have less influence on the overall pressure drop.

In the present investigation an attempt has been made to develop general equations for cake formation zone. Equivalent expressions for some parameters are compiled and shown to be similar in nature. The parameters are discussed as follows :

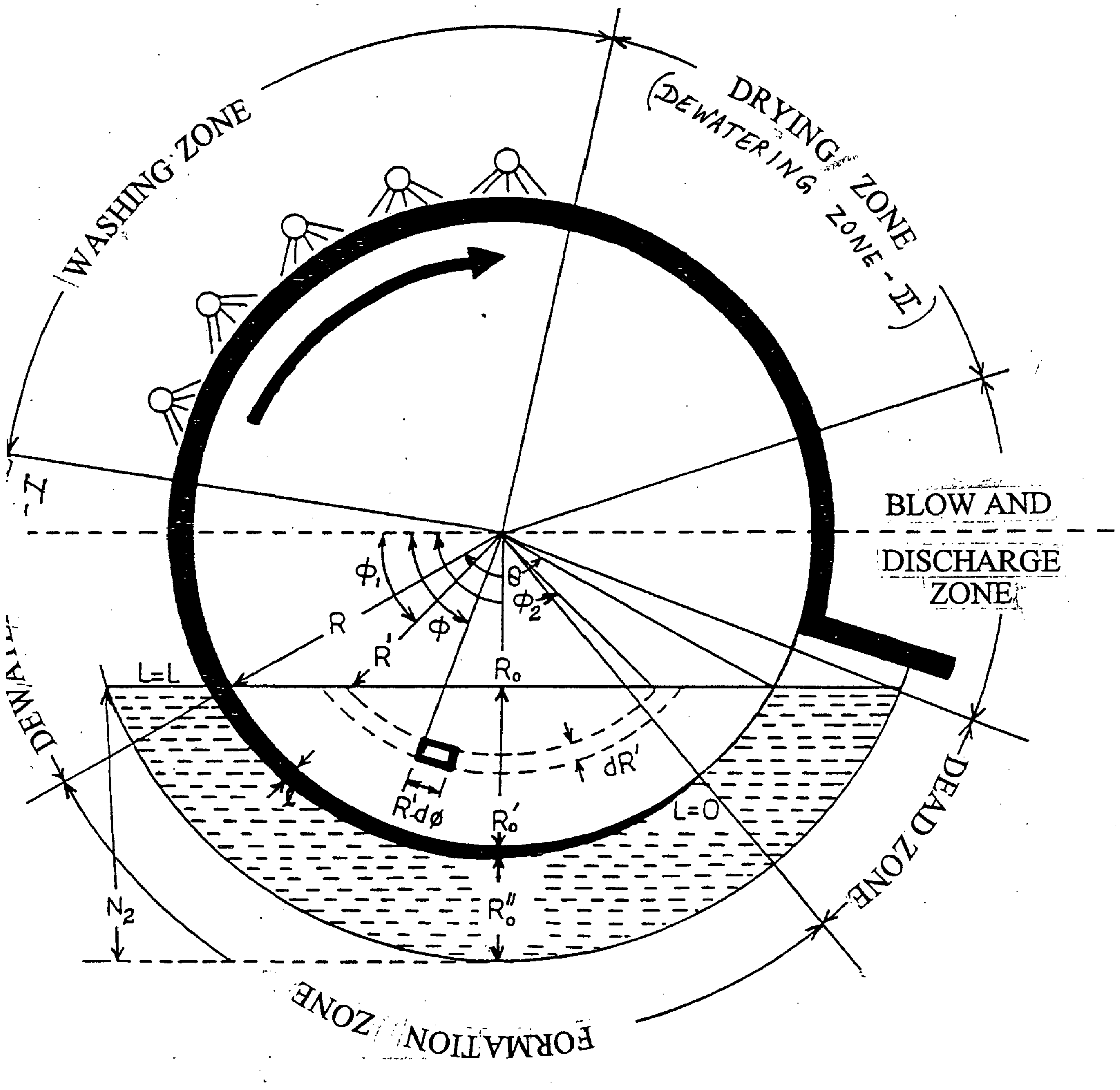

Figure 4 : Cross sectional view of a rotary vacuum washer

3.1.1 Filtrate flow rate

One of the important parameters for predicting the performance of a working system is the filtrate flow rate. Filtrate flow rate is a complex function of angle of submergence, radius of drum, pressure drop, rpm and properties of slurry and cake. For constant pulp suspension it depends upon radius of drum, rpm and angle of submergence.

Considering laminar flow of filtrate and negligible filter medium resistance, the filtrate flow rate V_f, through a surface area A, of a rotary drum in the direction of z can be written as,

$$dV/dt = V_f = K\Delta PA/\eta l \quad (3.1)$$

The microscopic material balance for the pulp and solute adsorbed in the cake for the area A and any arbitrary thickness of the pulp mat l, in any location of the filtrate zone is given by,

$$Al\rho_f(1-\varepsilon_t)+Al\rho_f(1-\varepsilon_t)\{ABC/(1+BC)\} = (V+\varepsilon_t Al)\rho X_i+Al\rho_f(1-\varepsilon_t)\{ABC/(1+BC)\}$$

During cake formation, the adsorption of solutes along with pulp fibers can be neglected. Hence

$$Al(1 - \varepsilon_t)\rho_f = (V + \varepsilon_t Al)\rho X_i$$

After rearranging one gets, $l = \rho X_i V / [A \{(1 - \varepsilon_t)\rho_f - \varepsilon_t \rho X_i\}$ (3.2)

Eliminating l from equations (3.1) and (3.2),

$$V(dV/dt) = [K\Delta PA^2 \{(1-\varepsilon_t)\rho_f - \varepsilon_t\rho X_i\}]/\eta\rho X_i \quad (3.3)$$

After integration, $V = [AL\{(1-\varepsilon_t)\rho_f - \varepsilon_t\rho X_i\}]/\rho X_i$ (3.4)

Equation (3.4) gives the total volume of the filtrate collected through the formation zone. L is cake thickness given in section 3.1.2.

$$V_f = NAL\{(1-\varepsilon_t)\rho_f - \varepsilon_t\rho X_i\}/\rho X_i \quad (3.5)$$

$$V_f = A[2K\Delta P\{(1-\varepsilon_t)\rho_f - \varepsilon_t \rho X_i\}\psi N/\eta\rho X_i]^{1/2} \tag{3.6}$$

$$= A[2K\Delta P\{(1-\varepsilon_t)(1-C_{yi}) - \varepsilon_t \rho C_{yi}\}\psi N/\eta\rho X_i]^{1/2} \tag{3.7}$$

Ruth and Kempe[70] for the first time estimated the filtrate flow rate in rotary leaf filters with the simplifying assumption, that the leaf is submerged into the slurry upto the axis, which is different from the actual plant practice. This was perhaps adopted because of operational difficulties. Thus their design equation does not hold good for optimal design.

Peck and Chand[59] later conceived this idea and derived an equation by extending the Ruth and Kempe's mathematical formula. Peck and Chand[59] have shown that constant pressure filtration data can be used to calculate the performance of a rotary leaf vacuum filter. Thus the procedure developed by them has been very useful for designing such filter from laboratory data on constant pressure filtration. However, they have solved their equation graphically.

Keeping in view the usefulness of the model and the uncertainties of certain design and process parameters in actual operation, their equation is redeveloped analytically for rotary drum vacuum filter using pulp suspension as slurry. The solution has been given for both the systems, i.e., with or without filter medium resistance considerations.

3.1.1(a) Filter medium resistance considered

Rewriting equation(3.3) as, $V(dV/dt) = A^2C'/2$ (3.8)

where, $C' = 2K\Delta P\{(1-\varepsilon_t)\rho_f - \varepsilon_t \rho X_i\}/\eta\rho X_i$ (3.9)

Integrating equation(3.8) between V=0 to V=V+V' and t=0 to t=t+t' one gets,

$$(V+V')^2 = A^2C'(t+t')$$

After differentiating, $dV/dt = (A/2)[C'/(t+t')]^{1/2}$

Specific rate can be written as, $(1/A)(dV/dt) = (1/2)[C'/(t+t')]^{1/2}$

The filtrate flow rate through the mat formation zone becomes,

$$V_f = \int_{R_0}^{R}\int_{\phi_1}^{\phi_2} (1/\psi A)(dV/dt)2R'd\phi\, dR' = \int_{R_0}^{R}\int_{\phi_1}^{\phi_2} (1/\psi)[C'/(t+t')]^{1/2} R'd\phi\, dR'$$

(for R, R_0, ϕ_1, ϕ_2 see figure 3)

putting $t = (\phi - \phi_1)/2\pi N$ in above integral, one gets,

$$V_f = (1/\psi)\int_{R_0}^{R} R'\left[\int_{\phi_1}^{\phi_2} \{2\pi NC'/(\phi - \phi_1 + 2\pi Nt')\}^{1/2} d\phi\right] dR'$$

On integrating with respect to ϕ, one gets,

$$V_f = (2/\psi)(2\pi NC')^{1/2}\int_{R_0}^{R} R'(\phi_2 - \phi_1 + 2\pi Nt')^{1/2} dR' - (4\pi N/\psi)(C't')^{1/2}\int_{R_0}^{R} R'dR'$$

Simplifying the first integral by using, $\phi_2 - \phi_1 = 2\cos^{-1}(R_0/R')$ & $2\pi Nt' = C''$. The filtrate flow rate becomes,

$$V_f = (2/\psi)(2\pi NC')^{1/2}\int_{R_0}^{R} R'\{2\cos^{-1}(R_0/R') + C''\}^{1/2} dR' - (2\pi N/\psi)(C't')^{1/2}(R^2 - R_0^2) \quad (3.10)$$

Simplifying the integral in equation(3.10) by substituting, $R = R_0 \operatorname{cosec} z$. The filtrate flow rate can be rewritten as,

$$V_f = (2R_0^2/\psi)(2\pi NC')^{1/2}\int_{\sin^{-1}(R_0/R)}^{\pi/2} (\pi + C'' - 2z)^{1/2} \operatorname{cosec}^2 z \cot z\, dz - (2\pi N/\psi)(C't')^{1/2}(R^2 - R_0^2)$$

Retaining the terms upto first order by using Binomial theorem and integrating by using $R_0 = R\cos(\theta/2)$. The final result on simplification is,

$$V_f = (R^2/\psi)\left[\{NC'/(2+4Nt')\}^{1/2}\{\pi(1+4Nt')\sin^2(\theta/2) + \theta - \sin\theta\} - 2\pi N(C't')^{1/2}\sin^2(\theta/2)\right] \quad (3.11)$$

Replacing θ by $2\pi\psi$, following expression results,

$$V_f = (R^2/\psi)\left[\{NC'/(2+4Nt')\}^{1/2}\{\pi(1+4Nt')\sin^2\pi\psi + 2\pi\psi - \sin 2\pi\psi\} - 2\pi N(C't')^{1/2}\sin^2\pi\psi\right] \quad (3.12)$$

To know the values of t' and V' a relationship between time and the volume of the filtrate is required.

3.1.1(b) Filter medium resistance neglected

In this case, the time to deposit a layer of cake with a resistance equal to that of the cloth can be neglected. By inserting $t' = 0$ in equations(3.11) and (3.12), the filtrate flow rate can be rewritten as,

$$V_f = (R^2/\psi)(NC'/2)^{1/2}\,[\pi \sin^2(\theta/2)+\theta-\sin\theta] \quad (3.13)$$

$$= (R^2/\psi)(NC'/2)^{1/2}\,[\pi \sin^2\pi\psi + 2\pi\psi - \sin 2\pi\psi] \quad (3.14)$$

The above expression can also be obtained from equation(3.8) by integrating it between the limits $V = 0$ to V and $t = 0$ to t.

Perron and Lebeau[65] referred the expression for the filtrate flow rate through cake formation zone as,

$$V_f = NAL\,\{(1-\varepsilon_t)\rho_f - \varepsilon_t \rho X_i\}/\rho X_i \quad (3.15)$$

Where, $L = [2K\Delta P \rho X_i \psi / \{(1-\varepsilon_t)\rho_f - \varepsilon_t \rho X_i\}\eta N\}]^{1/2}$

Peters and Timmerhaus[66] referred the above expression as,

$$V_f = A[2K\Delta P(1-\varepsilon_t)(1-C_{yi})\rho_f \psi N/\eta\rho C_{yi}]^{1/2} \quad (3.16)$$

3.1.2 Cake thickness

As the drum rotates inside the vat a layer of fibers is deposited on the outer surface of the drum due to vacuum inside the drum. When the drum emerges from the vat, the thickness of the mat in the transverse direction of the drum is taken as the cake thickness. The cake thickness depends upon many factors like, pressure drop, angle of submergence, concentration of slurry, mat resistance, cake resistance, rpm and permeability.

Eliminating V from equation (3.1) and (3.2),

$$l(dl/dt) = K\Delta P \rho X_i / [\eta\,\{(1-\varepsilon_t)\rho_f - \varepsilon_t \rho X_i\}]$$

Integrating by taking ΔP as constant, between $t = 0$ to $t = \psi/N$, where ψ/N corresponds to cake formation time, following expression is obtained,

$$L = [2K\Delta P\rho X_i\psi/\eta N\{(1-\varepsilon_t)\rho_f - \varepsilon_t\rho X_i\}]^{1/2} \tag{3.17}$$

$$= [2K\Delta P\rho C_{yi}\psi/\eta N\{(1-\varepsilon_t)(1-C_{yi})\rho_f - \varepsilon_t\rho C_{yi}\}]^{1/2} \tag{3.18}$$

$$= [2K\Delta P\rho C_{yi}\psi/\eta N(1-\varepsilon_t)(1-mC_{yi})\rho_f]^{1/2} \tag{3.19}$$

3.1.3 Fiber production rate

Mass of cake produced per unit time is known as fiber production rate. It is highly dependent upon the area of the drum and the cake thickness. Fiber production rate can be calculated by the following equations by assuming that the amount of adsorbed solute is negligible.

$$FPR = (1-\varepsilon_t)NAL\rho_f \tag{3.20}$$

$$= (1-\varepsilon_t)A\rho_f[2K\Delta P\rho X_i\psi N/\eta\{(1-\varepsilon_t)\rho_f - \varepsilon_t\rho X_i\}]^{1/2} \tag{3.21}$$

$$= (1-\varepsilon_t)A\rho_f[2K\Delta P\rho C_{yi}\psi N/\eta\{(1-\varepsilon_t)(1-C_{yi})\rho_f - \varepsilon_t\rho C_{yi}\}]^{1/2} \tag{3.22}$$

$$= A[2K\Delta P\rho\rho_f(1-\varepsilon_t)C_{yi}\psi N/\eta(1-mC_{yi})]^{1/2} \tag{3.23}$$

$$= A[2\Delta P\rho C_{yi}\psi N/\eta\alpha(1-C_{yi})]^{1/2} \tag{3.24}$$

$$= A[2(\Delta P)^{1-s}\rho C_{yi}\psi N/\eta\alpha_o(1-mC_{yi})]^{1/2} \tag{3.25}$$

According to Peters and Timmerhaus[66],

$$FPR = A[2K\Delta P(1-\varepsilon_t)C_{yi}\rho\rho_f\psi N/\eta(1-\varepsilon_t)]^{1/2} \tag{3.26}$$

Mathematically equation(3.26) is not true because $\varepsilon_t C_{yi}\rho$, has been neglected. It might have insignificant influence on the fiber production rate.

3.1.4 Specific loading factor

Fiber production rate per unit area of drum is known as specific loading factor. The parameter has been treated as an indispensable tool for designing a rotary vacuum filter.

$$SLF = (1-\varepsilon_t)\, NL\rho_f \tag{3.27}$$

$$= (1-\varepsilon_t)\rho_f\, [2K\Delta P\rho X_i\psi N / \eta\{(1-\varepsilon_t)\rho_f - \varepsilon_t\rho X_i\}]^{1/2} \tag{3.28}$$

$$= (1-\varepsilon_t)\rho_f\, [2K\Delta P\rho C_{yi}\psi N / \eta\{(1-\varepsilon_t)(1-C_{yi})\rho_f - \varepsilon_t\rho C_{yi}\}]^{1/2} \tag{3.29}$$

$$= [2K\Delta P\rho\rho_f(1-\varepsilon_t)C_{yi}\psi N / \eta(1-mC_{yi})]^{1/2} \tag{3.30}$$

$$= [2\Delta P\rho C_{yi}\psi\, N / \eta\alpha(1-C_{yi})]^{1/2} \tag{3.31}$$

$$= [2(\Delta P)^{1-s}\rho C_{yi}\psi N / \eta\alpha_o(1-mC_{yi})]^{1/2} \tag{3.32}$$

According to Peters and Timmerhaus[66],

$$SLF = [2K\Delta P(1-\varepsilon_t)C_{yi}\rho\rho_f\psi\, N / \eta(1-\varepsilon_t)]^{1/2} \tag{3.33}$$

To calculate the value of above parameters, certain parameters like porosity, permeability, angle of submergence, pressure drop across the cake and area of drum are frequently encountered. Detailed mathematical expressions for these parameters are given below.

3.1.5 Porosity

Porosity is defined as the ratio of volume available for flow to the total volume. Porosity of the mat is an important factor as the hydrodynamics of the filtration is highly influenced by the porous path through which the fluid will move. The porosity has been modelled based on average consistency of mat during the cake formation zone. The effect of air on porosity has not been considered because the saturation of cake is approximately 1 in the cake formation zone. Porosity of the pulp suspension varies from position to position during rotation of drum. Local total porosity can be defined as,

$$\varepsilon_t = \rho_f(1-C_y)/[\rho C_y + \rho_f(1-C_y)] \tag{3.34}$$

Alternatively, $\varepsilon_t = 1 - C_{yi}\rho_{sus} / \rho_f$ (3.35)

where, $\rho_{sus} = (1-\varepsilon_t)\rho_f + \varepsilon_t\rho$ (3.36)

Also, $\rho_{sus} = \rho\rho_f/[\rho C_y + (1-C_y)\rho_f]$ (3.37)

Grahs[19] correlated ε_t with fiber consistency(C_F) through an empirical equation(3.38). This predicts the actual values within an error of ±1% .It is true for any zone provided two phase flow does not exist. Therefore for the formation zone it is perfectly true and within reasonable accuracy during washing zone. However, it may deviate for dewatering zone.

$$\varepsilon_t = 1.00 - 6.80x10^{-4}\, C_F \qquad (3.38)$$

Grahs[17] has expressed ε_d as, $\varepsilon_d = 1.00 - 1.75x10^{-3}\, C_F$ (3.39)

For any zone, however, $\varepsilon_t = \varepsilon_d + \varepsilon_s$ (3.40)

3.1.6 Permeability

Permeability is a factor which influences the flow of fluid through a porous media and is a function of porosity and specific surface area. It is also a strong function of size, shape and orientation of the particles. The permeability K is evaluated by using the Kozeni's model,

$$K = \varepsilon_t^3 / [k_1 S_o^2 (1-\varepsilon_t)^2] \qquad (3.41)$$

For cellulose fibers Kozeni's constant k_1 = 5.55. In pulp washing, porosity value is usually very high and is of the order of $\varepsilon_t > 0.8$. As a matter of fact the solids (fibers) are cylindrical in shape and compressible in nature. For higher porosity values ($\varepsilon_t > 0.8$) equation(3.41) gives poor agreement with the experimental data.

For high porosity range Kozeni's model was modified by Davis[12]. The equation(3.42) is applicable to any system of cylindrical geometry such as natural and synthetic fibers. For fibers of vegetable origin like wood, bamboo and bagasse straw, the values of constants K_1 and K_2 differ. Average values of all these pulp making fibers have been experimentally determined by Ingmanson et al.[29] as 3.5 and 57 respectively.

$$K = [K_1 S_o^2 (1-\varepsilon_t)^{3/2} \{1 + K_2 (1-\varepsilon_t)^3\}]^{-1} \qquad (3.42)$$

3.1.7 Angle of Submergence

The angle subtended by the slurry level in the vat at the central axis of the drum is taken as angle of submergence. Angle of submergence depends upon many factors like, type, shape, geometry, orientation and the concentration of suspended solid (fibers) in the vat as well as area of drum and pressure drop across the cake. From figure 4, the angle of submergence is given in terms of slurry level, radius of drum and distance between drum and vat (R_0'') as,

$$\theta = 2\cos^{-1}\{(R_0''+R-N_2)/R\} \tag{3.43}$$

By taking $R_0'' = 0$, $\theta = 2\cos^{-1}\{(R-N_2)/R\}$. This corresponds to the expression of Perron and Lebeau[65]. In actual practice, R_0'' varies between 9 to 10 cm and the angle of submergence lies with in 120 to 150°, particularly in pulp washing system.

If ψ is the fractional submergence of the drum, then $\theta = 2\pi\psi$. Substituting value of θ in equation(3.43), one gets,

$$\psi = (1/\pi)\cos^{-1}[(R_0''+R-N_2)/R] \tag{3.44}$$

3.1.8 Pressure Drop

For rotary vacuum filter, certain pressure difference is necessary between pulp suspension and inlet from vat to the filtrate outlet through the vacuum tubes. The filtrate must pass through several resistances joined in series. The total pressure drop is the sum of all the individual pressure drops. The total pressure drop is caused by the pressure drops due to inlet connections, cake thickness, filter medium, outlet connections, kinetic and frictional pressure drops and hydrostatic pressure drops. These are influenced by the centrifugal force due to the rotation of the drum. However, the latter will influence the other pressure drops in some form or the other.

$$\Delta P_t = \Delta P_i + \Delta P_c + \Delta P_m + \Delta P_v + \Delta P_k + \Delta P_f$$

For well designed filter, the pressure drops due to inlet and outlet (kinetic and friction) are usually small provided single phase flow situation exists, which is however, true for formation zone and reasonably valid for washing zone. Therefore,

$$\Delta P_c = \Delta P_t + \Delta P_m + \Delta P_v = (P_a + P_h) - (P_a - P_v) - \Delta P_m = P_v + P_h - \Delta P_m$$

Prediction of filter medium pressure drop is not reliable without experimentation because the amount of fibers embedded in the meshes of the wire net is not known exactly. Therefore for simplification purpose, according to Orr[57], one can write,

$$\Delta P_c = P_v + P_h = P_v + \rho_{sus} gR\ [\cos\beta - \cos(\theta/2)]$$

Taking average value of the pressure for hydrostatic head,

$$\Delta P_c = P_v + \rho_{sus} gR\ [2\sin(\theta/2) - \theta\cos(\theta/2)]/\theta \qquad (3.45)$$

This equation can predict the extent of hydrostatic head contributing to the total pressure drop.

3.1.9 Area of drum

Area of the drum also has significant influence on the filtrate flow rate collected through the cake formation zone. The designing of a rotary vacuum filter consists of determining the required filtration area of the drum and then selecting the appropriate filter available to the manufacturer. Area of a filter depends upon the diameter to length ratio. However, parameters like pressure drop, rpm and angle of submergence affect the required area of drum for a particular capacity of any paper mill. Assuming the cake to be non compressible, area of the drum can be found out as under :

$$A = V_t\ [\ m_p\eta\ /2K\Delta P\psi\ N\ (1\text{-}\varepsilon_t)\rho_f]^{1/2} = V_t\ [m_p\eta\alpha\ /2\Delta P\psi N]^{1/2} \qquad (3.46)$$

$$= V_t\ [m_p\eta\alpha_o\ /2(\Delta P)^{1\text{-}s}\psi\ N]^{1/2} \qquad (3.47)$$

Where, $C_{sl} = \rho C_{yi}/(1\text{-}C_{yi})$; $m = \{\varepsilon_t\rho + (1 - \varepsilon_t)\rho_f\}/\ (1\text{-}\varepsilon_t)\rho_f$;

$V_t = L_e C_{sl}/m_p$; $m_p = C_{sl}\rho/[\rho\text{-}(m\text{-}1)C_{sl}]$

3.2 Models for Cake Washing Zone

The black liquor solutes occupy the space (interstices) between the fibers (as displaceable solutes) and in the pores of the fibers (as non displaceable solutes). These solutes are removed by water/weak wash liquor. The mechanics involved is the sum of **displacement** of the liquor by movement of water plug controlled by fluid mechanics, **dispersion** due to back mixing, **diffusion** due to concentration gradient and **adsorption - desorption** due to relative affinity of various solutes towards the fiber surface.

To develop the model for this zone is rather complex. In fact most of the investigators differ in their proposed models for this zone. In this investigation an attempt has been made to derive the detailed model, compatible with the practical system.

The mat of pulp fibers can be assumed to be stationary packed bed of homogeneous symmetrical cylindrical fibers. Instantaneous behavior of any system of this type can only be expressed by an equation involving the variables and their partial derivatives. For setting up a differential equation, consider a thin slice of a filter cake (pulp mat) as shown in figure 5, through which filtrate or wash water flows.

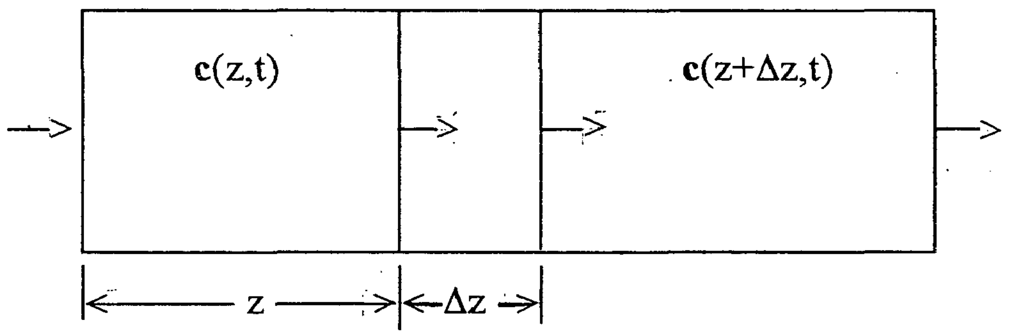

Figure 5 : A simple shell balance

Material balance across the simple shell given in figure 5, in the z direction can be written as,

$$(uc\varepsilon_t A')_{z,t} - (uc\varepsilon_t A')_{z+\Delta z,t} = [(\partial/\partial t)\{c\varepsilon_t A'\Delta z + n(1-\varepsilon_t)A'\Delta z\}]_{\bar{z},t}$$

where $z < \bar{z} < z+\Delta z$. Taking ε_t and A' as constant and taking the limit as $\Delta z \rightarrow 0$, one can obtain the following expression,

$$-\varepsilon_t c\,(\partial c/\partial z) = \varepsilon_t u\,(\partial c/\partial z) + \varepsilon_t\,(\partial c/\partial t) + (1-\varepsilon_t)(\partial n/\partial t)$$

The above equation contains principally two accumulation terms, one related to dispersion, diffusion and another related to adsorption-desorption. Other terms are velocity gradient and convective flow terms. Using Fick's second law of diffusion, i.e.,

$$-c\,(\partial u/\partial z) = (D_L + D_V)(\partial^2 c/\partial z^2)$$

the following equation is obtained,

$$(D_L + D_V)(\partial^2 c/\partial z^2) = u\,(\partial c/\partial z) + (\partial c/\partial t) + [(1-\varepsilon_t)/\varepsilon_t](\partial n/\partial t)$$

According to Sherman[72] the longitudinal dispersion coefficient D_L is a function of flow pattern within the bed (unless very low flow rates are used). The molecular diffusion coefficient D_V is very small compared to D_L and may be neglected. Writing $(1-\varepsilon_t)/\varepsilon_t$ as μ for convenience, the equation becomes,

$$D_L\,(\partial^2 c/\partial z^2) = u\,(\partial c/\partial z) + (\partial c/\partial t) + \mu\,(\partial n/\partial t) \qquad (3.48)$$

This is a non homogeneous, non linear, first degree, second order, parabolic, partial differential equation. Here u, ε_t and D_L are functions of z. While c and n are functions of both z and t. As the lumen of the fiber is porous and the same is true with the wall of the fiber, the porosity values for these cases are different from the porosity of the interfiber mass. Therefore three porosity values are required to represent the pulp mat system. It is extremely difficult to distinguish precisely between the values of porosity at the lumen and at the wall. Therefore, for practical calculations these are assumed to be the same. Hence, to describe the system two porosity values are assumed, one for the interfibers ε_d and another for intrafibers ε_s, so that $\varepsilon_d+\varepsilon_s = \varepsilon_t$, the total porosity for the entire system.

Various models have been developed which are plausible to the actual system with or without consideration of the effect of the longitudinal dispersion coefficient and simple adsorption isotherm models both finite and linear in nature. The models are also deferred by subjecting to various initial and boundary conditions. These possible models are summarized in **Table 3**.

Table 3 : Mathematical models for washing zone used in the present investigation

Model No.	Equation	Isotherm	B.C. (at z = 0)
1.	$D_L \frac{\partial^2 c}{\partial z^2} = u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \mu\frac{\partial n}{\partial t}$	$\frac{\partial n}{\partial t} = k_1 c - k_2 n$	$c = C_s$
2.	$D_L \frac{\partial^2 c}{\partial z^2} = u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \mu\frac{\partial n}{\partial t}$	$\frac{\partial n}{\partial t} = k^* (c - n)$	$c = C_s$
3.	$D_L \frac{\partial^2 c}{\partial z^2} = u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \mu\frac{\partial n}{\partial t}$	$\frac{\partial n}{\partial t} = k_1 c - k_2 n$	$uc - D_L \frac{\partial c}{\partial z} = uC_s$
4.	$D_L \frac{\partial^2 c}{\partial z^2} = u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \mu\frac{\partial n}{\partial t}$	$\frac{\partial n}{\partial t} = k^* (c - n)$	$uc - D_L \frac{\partial c}{\partial z} = uC_s$
5.	$D_L \frac{\partial^2 c}{\partial z^2} = u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \mu\frac{\partial n}{\partial t}$	$n = kc$	$c = C_s$
6.	$D_L \frac{\partial^2 c}{\partial z^2} = u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \mu\frac{\partial n}{\partial t}$	$n = kc$	$uc - D_L \frac{\partial c}{\partial z} = uC_s$
7.	$u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \mu\frac{\partial n}{\partial t} = 0$	$\frac{\partial n}{\partial t} = k_1 c - k_2 n$	$c = C_s$
8.	$u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \mu\frac{\partial n}{\partial t} = 0$	$\frac{\partial n}{\partial t} = k^*(c - n)$	$c = C_s$

Initial condition is $c(z,t) = n(z,t) = C_i$ for $0 < t < L/u$, where L/u corresponds to displacement time. Boundary condition at $z = L$ is $\partial c/\partial z=0$. It is important to solve all the models to get the concentration as a function of time and thickness of cake. These are depicted as follows:

3.2.1 Solution of the Washing Models

Before solving, these models are converted into dimensionless form by using certain variables like Peclet number (or Bodenstein number), dimensionless time, dimensionless thickness and dimensionless concentration. Then a linear partial differential equation is obtained. As most of the investigators have used the Laplace transform technique, the present investigation has also attempted for the same technique. The step by step solution procedure for different models (1-8) is described below:

3.2.1(a) Solution of washing model 1

The models is converted into dimensionless form by using,

$$C=(c-C_s)/(C_i-C_s), \quad N=(n-C_s)/(C_i-C_s), \quad Pe=uL/D_L, \quad Z=z/L,$$

$$T=ut/L, \quad K'=k_1/k_2, \quad G=k_2L/u \quad H=(K'-1)C_s/(C_i-C_s)$$

Then,
$$\frac{\partial c}{\partial t}=\frac{\partial c}{\partial C}\frac{\partial C}{\partial T}\frac{\partial T}{\partial t}=\frac{u(C_i-C_s)}{L}\frac{\partial C}{\partial T}$$

$$\frac{\partial c}{\partial z}=\frac{\partial c}{\partial C}\frac{\partial C}{\partial Z}\frac{\partial Z}{\partial z}=\frac{C_i-C_s}{L}\frac{\partial C}{\partial Z}$$

$$\frac{\partial^2 c}{\partial z^2}=\frac{\partial}{\partial z}\left[\frac{(C_i-C_s)}{L}\frac{\partial C}{\partial Z}\right]=\frac{(C_i-C_s)}{L^2}\frac{\partial^2 C}{\partial Z^2}$$

$$\frac{\partial n}{\partial t}=\frac{\partial n}{\partial N}\frac{\partial N}{\partial T}\frac{\partial T}{\partial t}=\frac{u(C_i-C_s)}{L}\frac{\partial N}{\partial T}$$

Then the following equations are obtained,

$$(\partial^2C/\partial Z^2)=Pe\,[(\partial C/\partial Z)+(\partial C/\partial T)+\mu(\partial N/\partial T)] \qquad (3.49)$$

$$\partial N/\partial T=G\,[H+K'C-N] \qquad (3.50)$$

After changing initial and boundry conditions into dimensionless form, one gets,

$$C(Z,0)=N(Z,0)=1 \qquad (3.51)$$

$$C(0,T)=0 \;\; \& \;\; \partial C/\partial Z=0 \text{ at } Z=1 \qquad (3.52)$$

Taking Laplace transform of equation(3.49) and (3.50) w.r.t. T,

$$(d^2\bar{C}/dZ^2) - Pe\,(d\bar{C}/dZ) - Pe\,f_1\,\bar{C} = Pe\,f_2 \qquad (3.53)$$

Where, $\bar{C}=L\{C(Z,T)\}$, $f_1=p(p+G+\mu K'G)/(p+G)$ and $f_2=[\mu G(H-1)-(p+G)]/(p+G)$

For **Complementary solution**(C.S.) put $\bar{C} = U\,e^{mZ}$ in equation(3.53),

$$U\,e^{mZ}\,(m^2 - Pem - Pef_1) = 0 \qquad \Rightarrow\ m = [Pe \pm \sqrt{Pe^2 + 4Pef_1}]/2$$

$$C.S. = C_1 \exp[A1Z] + C_2 \exp[A2Z] \qquad (3.54)$$

Where C_1 and C_2 are constants and the A1 and A2 are expressed as,

$$A1 = [Pe + \sqrt{Pe^2 + 4Pef_1}]/2 \quad \text{and} \quad A2 = [Pe - \sqrt{Pe^2 + 4Pef_1}]/2$$

Particular integral (P.I.) of equation(3.53) is given by,

$$P.I. = \frac{1}{D^2 - PeD - Pef_1}(Pef_2) = \frac{p+G+\mu G-\mu GH}{p(p+G+\mu K'G)} \qquad (3.55)$$

Solution of equation(3.53) = C.S. + P.I.

$$\bar{C}=C_1\exp[A1Z] + C_2\exp[A2Z]+[(p+G+\mu G-\mu GH)/p(p+G+\mu K'G)] \qquad (3.56)$$

Determination of C_1 and C_2

Taking Laplace transform of equation(3.52),

$$\bar{C}(0,T) = 0 \ \text{and} \ d\bar{C}/dZ = 0 \ \text{at} \ Z = 1 \qquad (3.57)$$

On solving equations(3.56) and (3.57), one gets,

$$C_1 = \frac{A2e^{A2}(p+G+\mu G-\mu GH)}{p(A1e^{A1}-A2e^{A2})(p+G+\mu K'G)}, \quad C_2 = \frac{-A1e^{A1}(p+G+\mu G-\mu GH)}{p(A1e^{A1}-A2e^{A2})(p+G+\mu K'G)}$$

Substituting the values of A1, A2, C_1 and C_2 in equation(3.56)

$$\bar{C}=\frac{(p+G+\mu G-\mu GH)}{p(p+G+\mu K'G)}\left[1 + \frac{e^{PeZ/2}[Pe\sinh\{(Z-1)P_1/2\} - P_1\cosh\{(Z-1)P_1/2\}]}{Pe\sinh(P_1/2) + P_1\cosh(P_1/2)}\right] \qquad (3.58)$$

Where, $P_1 = \sqrt{(Pe^2+4Pef_1)}$

Inverse of $\overline{C}(Z,p)$

To find the value of $C = [(c-C_s)/(C_i-C_s)]$, Laplace inverse of equation(3.58) is required, for which method of residues, especially proposed by Edeskuty and Amundson[13], is used.

The inverse transform is the sum of residues of the function $e^{pT}\overline{C}(Z,p)$ at the poles of $\overline{C}(Z,p)$. The poles will occur at the zeros of the denominator of equation(3.58).

If the pole at p = a is a simple pole, then the residue is given by the formula,

$$\lim_{p \to a} \left[(p-a)\, e^{pT}\, \overline{C}(Z,p) \right] \qquad (3.59)$$

If $\overline{C}(Z,p)$ has the fractional form q(p)/s(p), then the inverse transform of $\overline{C}(Z,p)$, provided q(p) is analytic, is given by,

$$\sum_{n=1}^{\infty} \left[q(p_n)/s'(p_n) \right] e^{p_n T} \qquad (3.60)$$

Where p_n is the root of s(p)=0. $s'(p_n)$ is the derivative of s(p) evaluated at $p=p_n$ and summation is taken over all the roots of s(p)=0.

In equation(3.58) simple poles occurs at p=0 and $p=-G-GK'\mu$. By using equation(3.59), the residue at these two poles is found to be as 0.

Roots of s(p) are obtained by equating s(p)=0, i.e., from equation(3.58), one can write,

$$Pe \sinh[\sqrt{(Pe^2+4Pef_1)}/2 + \sqrt{Pe^2+4Pef_1}\cosh[\sqrt{(Pe^2+4Pef_1)}/2] = 0$$

After solving, the above expression can be written as,

$$\beta\cot\beta + (Pe/2) = 0 \qquad (3.61)$$

Where $\beta = \sqrt{(Pe^2+4Pef_1)}/2i$

$$= (1/2i)\sqrt{[Pe^2(p+G)+4Pep(p+G+\mu K'G)]/(p+G)} \qquad (3.62)$$

Equation(3.61) is a transcendental equation. Its roots are β_n, where n is any integral number . For each value of β equation(3.62) is solved to get the corresponding value of p's, i.e., the zeros of s(p).

$$s'(p) = -\,Pe \sin\beta\,(Pe^2+4\beta^2+2Pe)[(p+G)^2+\mu K'G^2]/4i\beta^2(p+G)^2 \qquad (3.63)$$

Residue at the poles of s(p), i.e., at $p=p_n$ is obtained by using the formula given by equation(3.60) and is equal to

$$\sum_{n=1}^{\infty} \frac{4\beta_n^2\, e^{p_n T}\, e^{PeZ/2}\,(p_n+G+\mu G-\mu GH)[Pe \sin\{(Z-1)\beta_n\} - 2\beta_n \cos\{(Z-1)\beta_n\}]}{Pe\, p_n \sin\beta_n\,(p_n+G+\mu K'G)(Pe^2+4\beta_n^2+2Pe)\left[1+\dfrac{\mu K'G^2}{(p_n+G)^2}\right]} \qquad (3.64)$$

$$C(Z,T) = (c-C_s)/(C_i-C_s) = \text{Sum of residues}$$

$$\frac{c-C_s}{C_i-C_s} = \sum_{n=1}^{\infty} \frac{4\beta_n^2\, e^{p_n T} e^{PeZ/2}\,(p_n+G+\mu G-\mu GH)[Pe \sin\{(Z-1)\beta_n\} - 2\beta_n \cos\{(Z-1)\beta_n\}]}{Pe\, p_n \sin\beta_n (p_n+G+\mu K'G)(Pe^2+4\beta_n^2+2Pe)\left[1+\dfrac{\mu K'G^2}{(p_n+G)^2}\right]} \qquad (3.65)$$

Equation(3.65) gives the concentration of solute at any location and time in dimensionless form in the packed bed of fibers.

Exit concentration of solute leaving the bed at any time is obtained by setting Z=1 in equation(3.65),

$$\frac{C_e-C_s}{C_i-C_s} = \sum_{n=1}^{\infty} \frac{-8\,\beta_n^3\, e^{p_n \tau}\, e^{Pe/2}\,(p_n+G+\mu G-\mu GH)}{Pe\, p_n \sin\beta_n\,(p_n+G+\mu K'G)(Pe^2+4\beta_n^2+2Pe)\left[1+\dfrac{\mu K'G^2}{(p_n+G)^2}\right]} \qquad (3.66)$$

Average concentration of solute in the discharge pulp is obtained from the relation,

$$(C_d - C_s)/(C_i-C_s) = \int_0^1 C(Z,T)\, dZ$$

Putting the value of C(Z,T) from equation(3.65),

$$\frac{C_d - C_s}{C_i - C_s} = \sum_{n=1}^{\infty} \frac{-8\beta_n^2 e^{p_n \tau} (p_n + G + \mu G - \mu G H)[(Pe^2 + 4\beta_n^2)\sin\beta_n + 4Pe\, \beta_n e^{Pe/2}]}{Pe\, p_n \sin\beta_n (p_n + G + \mu K'G)(Pe^2 + 4\beta_n^2)(Pe^2 + 4\beta_n^2 + 2Pe)\left[1 + \frac{\mu K' G^2}{(p_n + G)^2}\right]} \quad (3.67)$$

Mean concentration of the filtrate collected through the washing zone can be calculated by the following method,

$$C_m - C_s = \frac{1}{T_w} \int_0^{T_w} (c - C_s)\, dt = \frac{1}{T_w} \left[\int_0^{L/u} (c - C_s)\, dt + \int_{L/u}^{T_w} (c - C_s)\, dt \right]$$

For the first integral, $c = C_i$ (bécause $0 \le t \le L/u$). For the second integral value of $(c - C_s)$ is substituted from equation(3.65). After performing the integration, final solution is given by,

$$\frac{C_m - C_s}{C_i - C_s} = \frac{L}{uT_w} + \sum_{n=1}^{\infty} \frac{8\beta_n^3 e^{Pe/2} (e^{p_n} - e^{p_n \tau})(p_n + G + \mu G - \mu G H)}{\tau\, Pe\, p_n^2 \sin\beta_n (p_n + G + \mu K'G)(Pe^2 + 4\beta_n^2 + 2Pe)\left[1 + \frac{\mu K' G^2}{(p_n + G)^2}\right]} \quad (3.68)$$

Where, $\tau = uT_w/L$

Since the solution technique for models 2 to 6 are identical to model 1, detailed solutions are not shown. Only initial dimensionless parameters and the various concentration terms are given.

3.2.1(b) Solution of washing model 2

The model is converted into dimensionless form by using,

$C=(c-C_s)/(C_i-C_s)$, $N=(n-C_s)/(C_i-C_s)$, $Pe=uL/D_L$, $Z=z/L$,

$T=ut/L$, $K'=1$, $G=k^*L/u$ $H=0$

The solution technique is same as in model 1. The concentration of solute in the dimensionless form for any location and time can be found as,

$$\frac{c-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{4\,\beta_n^2 e^{p_n T} e^{PeZ/2}\,[Pe\sin\{(Z-1)\beta_n\}-2\beta_n\cos\{(Z-1)\beta_n\}]}{Pe\,p_n\sin\beta_n\,(Pe^2+4\beta_n^2+2Pe)\left[1+\dfrac{\mu G^2}{(p_n+G)^2}\right]} \quad (3.69)$$

Expression for **Exit concentration** is

$$\frac{C_e-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{-8\,\beta_n^3\,e^{p_n\tau}\,e^{Pe/2}}{Pe\,p_n\sin\beta_n\,(Pe^2+4\beta_n^2+2Pe)\left[1+\dfrac{\mu G^2}{(p_n+G)^2}\right]} \quad (3.70)$$

Expression for **Average concentration** is,

$$\frac{C_d-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{-8\,\beta_n^2 e^{p_n\tau}-[(Pe^2+4\beta_n^2)\sin\beta_n+4Pe\beta_n e^{Pe/2}]}{Pe\,p_n\sin\beta_n(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+2Pe)\left[1+\dfrac{\mu G^2}{(p_n+G)^2}\right]} \quad (3.71)$$

Expression for **Mean concentration** is,

$$\frac{C_m-C_s}{C_i-C_s}=\frac{L}{uT_w}+\sum_{n=1}^{\infty}\frac{8\,\beta_n^3\,e^{Pe/2}\,(e^{p_n}-e^{p_n\tau})}{\tau\,Pe\,p_n^2\sin\beta_n\,(Pe^2+4\beta_n^2+2Pe)\left[1+\dfrac{\mu G^2}{(p_n+G)^2}\right]} \quad (3.72)$$

Where $\tau = uT_w/L$ and β_n are roots of the transcendental equation,

$$\beta\cot\beta + (Pe/2) = 0$$

Where $\beta = (1/2i)\sqrt{[Pe^2\,(p+G)+4Pep(p+G+\mu G)]/(p+G)}$

3.2.1(c) Solution of washing model 3

The model is converted into dimensionless form by using,

$C=(c-C_s)/(C_i-C_s)$, $N=(n-C_s)/(C_i-C_s)$, $Pe=uL/D_L$, $Z=z/L$,

$T=ut/L$, $K'=k_1/k_2$, $G=k_2L/u$ $H=(K'-1)C_s/(C_i-C_s)$

The solution technique is same as in model 1. The concentration of solute in the dimensionless form for any location and time can be found as,

$$\frac{c-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{16Pe\,\beta_n^2 e^{p_nT}e^{PeZ/2}(p_n+G+\mu G-\mu GH)[Pe\sin\{(Z-1)\beta_n\}-2\beta_n\cos\{(Z-1)\beta_n\}]}{p_n\sin\beta_n(p_n+G+\mu K'G)(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+4Pe)\left[1+\frac{\mu K'G^2}{(p_n+G)^2}\right]} \quad (3.73)$$

Expression for **Exit concentration** is,

$$\frac{C_e-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{-32Pe\,\beta_n^3 e^{p_n\tau}e^{Pe/2}(p_n+G+\mu G-\mu GH)}{p_n\sin\beta_n(p_n+G+\mu K'G)(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+4Pe)\left[1+\frac{\mu K'G^2}{(p_n+G)^2}\right]} \quad (3.74)$$

Expression for **Average concentration** is,

$$\frac{C_d-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{-128\,Pe^2\beta_n^3 e^{p_n\tau}e^{Pe/2}(p_n+G+\mu G-\mu GH)}{p_n\sin\beta_n(p_n+G+\mu K'G)(Pe^2+4\beta_n^2)^2(Pe^2+4\beta_n^2+4Pe)\left[1+\frac{\mu K'G^2}{(p_n+G)^2}\right]} \quad (3.75)$$

Expression for **Mean concentration** is,

$$\frac{C_m-C_s}{C_i-C_s}=\frac{L}{uT_w}+\sum_{n=1}^{\infty}\frac{32Pe\beta_n^3 e^{Pe/2}(p_n+G+\mu G-\mu GH)(e^{p_n}-e^{p_n\tau})}{\tau p_n^2\sin\beta_n(p_n+G+\mu K'G)(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+4Pe)\left[1+\frac{\mu K'G^2}{(p_n+G)^2}\right]} \quad (3.76)$$

Where $\tau=uT_w/L$ and β_n are roots of the transcendental equation,

$$\beta\cot\beta+(Pe^2-4\beta^2)/4Pe=0$$

Where $\beta=(1/2i)\sqrt{[Pe^2(p+G)+4Pep(p+G+\mu K'G)]/(p+G)}$

3.2.1(d) Solution of washing model 4

The model is converted into dimensionless form by using,

$C=(c-C_s)/(C_i-C_s)$, $N=(n-C_s)/(C_i-C_s)$, $Pe=uL/D_L$, $Z=z/L$,

$T=ut/L$, $K'=1$, $G=k^*L/u$, $H=0$

The solution technique is same as in model 1. The concentration of solute in the dimensionless form for any location and time can be found as,

$$\frac{c-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{16\,Pe\,\beta_n^2\,e^{p_n T}\,e^{PeZ/2}\,[Pe\sin\{(Z-1)\beta_n\}-2\beta_n\cos\{(Z-1)\beta_n\}]}{p_n\sin\beta_n\,(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+4Pe)\left[1+\frac{\mu G^2}{(p_n+G)^2}\right]} \quad (3.77)$$

Expression for **Exit concentration** is,

$$\frac{C_e-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{-32\,Pe\,\beta_n^3\,e^{p_n\tau}\,e^{Pe/2}}{p_n\sin\beta_n(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+4Pe)\left[1+\frac{\mu G^2}{(p_n+G)^2}\right]} \quad (3.78)$$

Expression for **Average concentration** is,

$$\frac{C_d-C_s}{C_i-C_s}=\sum_{n=1}^{\infty}\frac{-128\,Pe^2\,\beta_n^3\,e^{p_n\tau}\,e^{Pe/2}}{p_n\sin\beta_n\,(Pe^2+4\beta_n^2)^2\,(Pe^2+4\beta_n^2+4Pe)\left[1+\frac{\mu G^2}{(p_n+G)^2}\right]} \quad (3.79)$$

Expression for **Mean concentration** is,

$$\frac{C_m-C_s}{C_i-C_s}=\frac{L}{uT_w}+\sum_{n=1}^{\infty}\frac{32\,Pe\,\beta_n^3\,e^{Pe/2}\,(e^{p_n}-e^{p_n\tau})}{\tau\,p_n^2\sin\beta_n(Pe^2+4\beta_n^2)\,(Pe^2+4\beta_n^2+4Pe)\left[1+\frac{\mu G^2}{(p_n+G)^2}\right]} \quad (3.80)$$

Where $\tau=uT_w/L$ and β_n are roots of the transcendental equation,

$$\beta\cot\beta+(Pe^2-4\beta^2)/4Pe=0$$

Where $\beta=(1/2i)\sqrt{[Pe^2\,(p+G)+4Pep(p+G+\mu G)]/(p+G)}$

3.2.1(e) Solution of washing model 5

By coupling the adsorption isotherm with the equation of flow of liquor, one gets the following equation,

$$D_L\,(\partial^2 c/\partial z^2) = u\,(\partial c/\partial z) + (1+\mu k)(\partial c/\partial t)$$

The model is converted into dimensionless form by using,

$$C=(c-C_s)/(C_i-C_s), \quad Pe=uL/D_L, \quad Z=z/L, \quad T=ut/(1+\mu k)L$$

The solution technique is same as in model 1. The concentration of solute in the dimensionless form for any location and time can be found as,

$$\frac{c-C_s}{C_i-C_s} = \sum_{n=1}^{\infty} \frac{4\,\beta_n^2\, e^{p_n T}\, e^{PeZ/2}\,[Pe \sin\{(Z-1)\beta_n\} - 2\beta_n \cos\{(Z-1)\beta_n\}]}{Pe\, p_n \sin\beta_n\,(Pe^2+4\beta_n^2+2Pe)} \tag{3.81}$$

Expression for **Exit concentration** is,

$$\frac{C_e-C_s}{C_i-C_s} = \sum_{n=1}^{\infty} \frac{-8\,\beta_n^3\, e^{p_n \tau}\, e^{Pe/2}}{Pe\, p_n \sin\beta_n\,(Pe^2+4\beta_n^2+2Pe)} \tag{3.82}$$

Expression for **Average concentration** is,

$$\frac{C_d-C_s}{C_i-C_s} = \sum_{n=1}^{\infty} \frac{-8\,\beta_n^2\, e^{p_n \tau}\,[4Pe\beta_n\, e^{Pe/2} + (Pe^2+4\beta_n^2)\sin\beta_n]}{Pe p_n \sin\beta_n\,(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+2Pe)} \tag{3.83}$$

Expression for **Mean concentration** is,

$$\frac{C_m-C_s}{C_i-C_s} = \frac{L}{uT_w} + \sum_{n=1}^{\infty} \frac{8\,\beta_n^3\, e^{Pe/2}\,(e^{p_n/(1+\mu k)} - e^{p_n \tau})}{\tau\, Pe p_n^2 \sin\beta_n\,(Pe^2+4\beta_n^2+2Pe)} \tag{3.84}$$

Where $\tau = uT_w/(1+\mu k)L$ and β_n are roots of the transcendental equation,

$$\beta\cot\beta + Pe/2 = 0$$

Where $\beta = \sqrt{(Pe^2 + 4Pep)}/2i$

3.2.1(f) Solution of washing model 6

By coupling the adsorption isotherm with the equation of flow of liquor, one gets the following equation,

$$D_L\,(\partial^2 c/\partial z^2) = u\,(\partial c/\partial z) + (1+\mu k)(\partial c/\partial t)$$

The model is converted into dimensionless form by using,

$$C=(c-C_s)/(C_i-C_s), \quad Pe=uL/D_L\,, \quad Z=z/L\,, \quad T=ut/(1+\mu k)L$$

The solution technique is same as in model 1. The concentration of solute in the dimensionless form for any location and time can be found as,

$$\frac{c-C_s}{C_i-C_s} = \sum_{n=1}^{\infty} \frac{16\,Pe\beta_n^2\, e^{p_n T} e^{PeZ/2}\,[Pe\,\sin\{(Z-1)\beta_n\} - 2\beta_n\cos\{(Z-1)\beta_n\}]}{p_n \sin\beta_n\,(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+4Pe)} \qquad (3.85)$$

Expression for **Exit concentration** is,

$$\frac{C_e-C_s}{C_i-C_s} = \sum_{n=1}^{\infty} \frac{-32\,Pe\,\beta_n^3\, e^{p_n\tau}\, e^{Pe/2}}{p_n \sin\beta_n\,(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+4Pe)} \qquad (3.86)$$

Expression for **Average concentration** is,

$$\frac{C_d-C_s}{C_i-C_s} = \sum_{n=1}^{\infty} \frac{-128\,Pe^2\,\beta_n^3\, e^{p_n\tau}\, e^{Pe/2}}{p_n \sin\beta_n\,(Pe^2+4\beta_n^2)^2(Pe^2+4\beta_n^2+4Pe)} \qquad (3.87)$$

Expression for **Mean concentration** is,

$$\frac{C_m-C_s}{C_i-C_s} = \frac{L}{uT_w} + \sum_{n=1}^{\infty} \frac{32\,Pe\,\beta_n^3\, e^{Pe/2}\,(e^{p_n/(1+\mu k)} - e^{p_n\tau})}{\tau\, p_n^2 \sin\beta_n\,(Pe^2+4\beta_n^2)(Pe^2+4\beta_n^2+4Pe)} \qquad (3.88)$$

Where $\tau = uT_w/(1+\mu k)L$ and β_n are roots of the transcendental equation,

$$\beta\cot\beta + (Pe^2 - 4\beta^2)/4Pe = 0$$

Where $\beta = \sqrt{(Pe^2 + 4Pep)}/2i$

3.2.1(g) Solution of washing model 7

In this model longitudinal dispersion coefficient (D_L) is not considered. Initial condition for concentration of solute on fibers is taken as, $n(z,t) = N_i$ for $0 \le t \le L/u$

The model is converted into dimensionless form by using,

$$Z = k_1 \mu z/u, \qquad T = (k_2/u)(ut-z) \qquad \text{and} \qquad K'' = k_2/k_1$$

Then, $\partial c/\partial t = k_2\, \partial c/\partial T$, $\partial n/\partial t = k_2\, \partial n/\partial T$ and $\partial c/\partial z = (k_1 \mu/u)\, \partial c/\partial Z - (k_2/u)\, \partial c/\partial t$

Then the following equations are obtained,

$$(\partial c/\partial Z) + K''(\partial n/\partial T) = 0 \tag{3.89}$$

$$\text{and } K''(\partial n/\partial T) = c - K''n \tag{3.90}$$

Taking Laplace transform of equations(3.89) and (3.90) w.r.t. T,

$$(d\overline{C}/dZ) + p\overline{C} / (p+1) = K''N_i / (p+1)$$

Where $\overline{C}(Z,p) = L\{c(Z,T)\}$. This equation is a first order linear differential equation. After solving this equation following expression results,

$$\overline{C} = (K''N_i/p) + (C_s - K''N_i)\, e^{-Z}\, e^{Z/(p+1)}\, [\, 1/(p+1) + 1/p(p+1)\,] \tag{3.91}$$

Taking inverse transform of equation(3.91) by using the following expression,

$$L^{-1}\{e^{k/(p+a)} / (p+a)\} = e^{-at}\, I_0(2\sqrt{kt})$$

Since the second term in the bracket of equation(3.91) contains 'p' in the denominator, it is therefore the integral of the inverse of the first term. After mathematical manipulations the following expression in dimensionless form is obtained,

$$I = (c-C_s)/(K''N_i-C_s) = 1 - e^{-(\xi+\tau)}\, I_0(2\sqrt{\xi\tau}) - \int_0^{\tau} e^{-(\xi+x)}\, I_0(2\sqrt{\xi x})dx \tag{3.92}$$

Where $\xi = k_1 \mu L/u$ and $\tau = (k_2/u)(ut-L)$. Equation(3.92) represents the concentration of solute in the liquor in the dimensionless form.

The concentration of solute in the stagnant film is obtained by taking the Laplace transform of equation(3.90) with respect to T. The resulting expression is obtained as follows :

$$(p+1)\,N = N_i + (\overline{C}/K'')$$

where N(Z,p) = L{n(Z,T)}. Substituting the value of $\overline{C}$ from equation(3.91), one gets,

$$N = (N_i/p) + (C_s - K''N_i)\, e^{-Z}\, e^{Z/(p+1)}\,[\,1/p(p+1)\,] \qquad (3.93)$$

Following equation, in dimensionless form, results for z = L and t > L/u, by taking Laplace inverse of equation(3.93),

$$I' = (n - C_s)/(K''N_i - C_s) = 1 - \int_0^{\tau} e^{-(\xi+x)}\, I_0(2\sqrt{\xi x})\, dx \qquad (3.94)$$

Equations(3.92) and (3.94) are the most general dimensionless expressions for the concentration of solute in the liquor and in the stagnant film respectively.

In the present investigation it is assumed that c(z,t) = n(z,t) = C_i for $0 \le t \le L/u$. Then $I = (c-C_s)/(K''C_i-C_s)$ and $I' = (n-C_s)/(K''C_i-C_s)$.

Value of the integral $\int_0^{\tau} e^{-(\xi+x)}\, I_0(2\sqrt{\xi x})dx$ can be approximated in terms of error function and I_0 by using the following expression based on the guidelines of Goldstein[16],

$$\int_0^{\tau} e^{-(\xi+\tau)}\, I_0(2\sqrt{\xi x})\, dx = \{1-\mathrm{erf}(\sqrt{\xi}-\sqrt{\tau})\}/2 - e^{-(\xi+\tau)}\, I_0(2\sqrt{\xi\tau})[1/\{1+(\xi/\tau)^{(1/8)}\}] \qquad (3.95)$$

By using the asymptotic expansion given by equation(3.95), equations(3.92) and (3.94) reduces to the following expressions,

$$I = \{1 + \mathrm{erf}(\sqrt{\xi}-\sqrt{\tau})\}/2 - e^{-(\xi+\tau)}\, I_0(2\sqrt{\xi\tau})\,[\,(\xi/\tau)^{(1/8)}/\,\{1 + (\xi/\tau)^{(1/8)}\}\,] \qquad (3.96)$$

$$I' = \{1 + \mathrm{erf}(\sqrt{\xi}-\sqrt{\tau})\}/2 + e^{-(\xi+\tau)}\, I_0(2\sqrt{\xi\tau})\,[\,1\,/\,\{1 + (\xi/\tau)^{(1/8)}\}\,] \qquad (3.97)$$

Expression for **Exit concentration** is,

$$(C_e-C_s)/(K''C_i-C_s)=[1+\mathrm{erf}(\sqrt{\xi}-\sqrt{\tau})]/2 - e^{-(\xi+\tau)} I_0(2\sqrt{\xi\tau})[\ (\xi/\tau)^{1/8}/\{1+(\xi/\tau)^{1/8}\}] \quad (3.98)$$

Expression for **Average concentration** is,

$$(C_d-C_s)/(K''C_i-C_s)=(1/\xi)[\ \xi - 2 + (2+\xi)\ e^{-\xi} + e^{-\tau} - (1+\xi)\ e^{-(\xi+\tau)}\ ] \quad (3.99)$$

Expression for **Mean concentration** is,

$$(C_m-C_s)/(K''C_i-C_s)=(1/\tau)\ [\tau - \{\tau - (L/u)\}(1+\xi)e^{-\xi} - (\xi e^{-\xi-k_2\{\tau-(L/u)\}}/k_2)+(\xi e^{-\xi}/k_2)] \quad (3.100)$$

3.2.1(h) Solution of washing model 8

The model is converted into dimensionless form by using,

$Z = k^*\mu z/u, \quad T = (k^*/u)(ut-z) \quad$ and $\quad K'' = 1$

The solution technique is same as in model 7. The concentration of solute in the dimensionless form for any location and time can be found by using following expression,

$$(c-C_s)/(C_i-C_s)=[1+\mathrm{erf}(\sqrt{\xi}-\sqrt{\tau})]/2 - e^{-(\xi+\tau)} I_0(2\sqrt{\xi\tau})[\ (\xi/\tau)^{1/8}/\{1+(\xi/\tau)^{1/8}\}] \quad (3.101)$$

Expression for **Exit concentration** is,

$$(C_e-C_s)/(C_i-C_s)=[1+\mathrm{erf}(\sqrt{\xi}-\sqrt{\tau})]/2 - e^{-(\xi+\tau)} I_0(2\sqrt{\xi\tau})[\ (\xi/\tau)^{1/8}/\{1+(\xi/\tau)^{1/8}\}] \quad (3.102)$$

Expression for **Average concentration** is,

$$(C_d-C_s)/(C_i-C_s)=(1/\xi)[\ \xi - 2 + (2+\xi)\ e^{-\xi} + e^{-\tau} - (1+\xi)\ e^{-(\xi+\tau)}\ ] \quad (3.103)$$

Expression for **Mean concentration** is,

$$(C_m-C_s)/(C_i-C_s)=(1/\tau)\ [\tau - \{\tau - (L/u)\}(1+\xi)e^{-\xi} - (\xi e^{-\xi-k^*\{\tau-(L/u)\}}/k^*)+(\xi e^{-\xi}/k^*)] \quad (3.104)$$

Where $\xi = k^*\mu L/u$ and $\tau = (k^*/u)(ut-L)$

3.2.2 Filtrate flow rate through washing zone

Filtrate flow rate through the cake washing zone can be estimated in terms of mean concentration of filtrate collected through the cake washing zone as,

$$V_w = NAL\,\varepsilon_t\,[(C_i - C_d)/(C_m - C_s)] \tag{3.105}$$

3.3 Models for Cake Drying Zone (or Dewatering Zone)

Although, the liquor from the cake is removed continuously from the washing zone, a considerable amount of dilute liquor is still left in the cake. The remaining liquor is removed in drying zone. In the drying zone, besides the liquor, major amount of air is also sucked in. Thus drying zone consists of simultaneous flow of two fluids : liquor (wetting) and air (non wetting). In two phase flow situation through the porous medium, it is necessary to consider the degree of saturation of the cake because the amount of air entering depends upon the degree of saturation of cake.

Mathematical models representing the pre washing zone and cake drying zone basically bear the same mechanism. Therefore deducing the equations for cake drying zone can serve the purpose for pre drying zone also. Han and Edwards[27] proposed following equation which takes care of two phase flow through porous media,

$$(\varepsilon_t L)(dS_s/dt) = q - (K_e K \Delta P/\eta L) \tag{3.106}$$

Where $K_e = (S_e)^y$. Rewriting equation(3.106) in terms of S_e, one can obtain,

$$(\varepsilon_t L)(dS_s/dt) = q - [K\Delta P (S_e)^y/\eta L] \tag{3.107}$$

However, in practice one frequently uses the real saturation instead of effective saturation. Real saturation is related to S_e and S_r as,

$$S_s = (S_e - 2S_eS_r + S_r)/(1 - S_eS_r) \tag{3.108}$$

Where, $S_r = 0.025[K\Delta P/\sigma L]^{-0.26}$ (3.109)

S_r is constant at constant ΔP. Therefore, differentiating equation(3.108) with respect to t and rewriting equation(3.106) in terms of S_e as,

$$[(1-S_r)/(1-S_eS_r)]^2 (dS_e/dt) = (q/\varepsilon_t L) - [\, K\Delta P(S_e)^y/\eta\varepsilon_t L^2 \,] \qquad (3.110)$$

The term $[(1-S_r)/(1-S_eS_r)]^2$ equals unity when S_e equals unity and becomes $(1-S_r)^2$ when S_e equals zero. Therefore this term is always close to unity. Brown et al.[7] have replaced this term by the mean value term $[(1-S_r)^2+1]/2$. The exponent y, which is a function of particle size, is approximately equal to 2 for cellulose fibers, as proposed by Carman[8].

For drying zone q = 0. After incorporating these assumptions in equation(3.110), it reduces to,

$$dS_e/S_e^{\,2} = -\,[\, 2K\Delta P/\, \eta\varepsilon_t L^2\{1+(1-S_r)^2\} \,]\, dt$$

Integrating between $S_e=1$ to any value of S_e and t=0 to t_d, one can obtain,

$$S_e = [\, 1 + 2K\Delta P t_d \,/\, [\eta\varepsilon_t L^2 \{ 1 + (1-S_r)^2 \}] \,]^{-1} \qquad (3.111)$$

According to Perron and Lebeau[65] drying time is given by $t_d = l_d P/AN$. Real saturation S_s is calculated by using equation(3.108). S_r and S_e are calculated from equation(3.109) and (3.111) respectively.

3.3.1 Filtrate flow rate through drying zone (or dewatering zone)

It is calculated by the following expression,

$$V_d = NAL\varepsilon_t(1-S_s) \qquad (3.112)$$

Perron and Lebeau[65] proposed the following expression for drying zone by inserting value of S_r obtained differently,

$$V_d = l_d NALPK\Delta P(1-S_r)\varepsilon_t \,/\, \{\eta NAL^2(1-S_r)\varepsilon_t + l_d PK\Delta P \} \qquad (3.113)$$

Similar equation can be used for estimating the flow through the **dewatering zone**, only by inserting S_r value.

The above static models for cake drying and prewashing zone will subsequently be used for evaluating total filtrate flow rate through the filter and to examine the relative importance of these zones.

3.4 Prediction of Practical Performance Parameters Using the Basic Parameters Developed Through Filter Models

In this chapter complex mathematical models are presented along with their solutions. These are of little use to practicing engineers. These mathematically developed models must be narrowed down to the commonly used parameters. Therefore the parameters like dilution factor, wash ratio, displacement ratio and % efficiency are expressed in terms of fundamental parameters. Value of these fundamental parameters can be evaluated from the models.

Dilution factor can be written as,

$$DF = 1 + (V_w \rho_s/FPR) - (100/C_{yd}) \qquad (3.114)$$

$$= [(C_i-C_d)\,\varepsilon_t \rho_s/(C_m-C_s)(1-\varepsilon_t)\rho_f] + (C_{yd}-100)/C_{yd} \qquad (3.115)$$

Wash ratio in terms of fundamental parameters can be written as,

$$WR = V_w \rho_s C_{yd}/[FPR(100-C_{yd})] \qquad (3.116)$$

$$= (C_i-C_d)\,\varepsilon_t C_{yd} \rho_s/[(C_m-C_s)(1-\varepsilon_t)(100-C_{yd})\,\rho_f] \qquad (3.117)$$

Displacement ratio can be expressed in terms of concentration of different liquor streams as,

$$DR = \{(C_i/\rho_i) - (C_d/\rho_d)\} / \{(C_i/\rho_i) - (C_s/\rho_s)\} \qquad (3.118)$$

However, by assuming that the density of all the streams leaving and entering the washing stage is same ($\rho_i=\rho_s=\rho_d$) one gets an expression similar to that proposed by Grahs[19],

$$DR = (C_i-C_d)/(C_i-C_s) \qquad (3.119)$$

% efficiency of any stage is given by,

$$\%E = [\{(C_d\text{-}C_s)(100\text{-}C_{yd})\rho_i\}/\{(C_i\text{-}C_s)(100\text{-}C_{yi})\rho_s\}]\ 100 \qquad (3.120)$$

An expression in terms of concentration and consistency is obtained by assuming that density of all streams leaving and entering the washer is same.

$$\%E = [\{(C_d\text{-}C_s)(100\text{-}C_{yd})\}/\{(C_i\text{-}C_s)(100\text{-}C_{yi})\}]100 \qquad (3.121)$$

3.5 Relationship Between Efficiency Parameters

Relationships between different parameters are useful to predict and to compare the performance of existing brown stock washing system and can be used to optimize and to design the washing process or the equipment in a new plant. With this objective in mind, an attempt has been made in this investigation to streamline the concept of various variables, presently known, pertaining to the system. These are discussed as follows:

3.5.1 Relation between DF and WR

$$DF = [\,(100\text{-}C_{yd})/C_{yd}\,]\,(WR\text{-}1) \qquad (3.122)$$

When the volume of the wash liquor is equal to the volume of the liquor leaving with washed pulp, then there is no excess of water/wash liquor, i.e., when WR=1, DF=0.

3.5.2 Relation between %E and WR

Following relation can be found between %E and WR.

$$\%E = \left[\,1 - \left[\,1 - \frac{(C_m\text{-}C_s)(1\text{-}\varepsilon_t)(100\text{-}C_{yd})\rho_f}{(C_i\text{-}C_s)\varepsilon_t C_{yd}\rho}\,WR\right]\frac{\rho_i(100\text{-}C_{yd})}{\rho_s(100\text{-}C_{yi})}\right]100 \qquad (3.123)$$

3.5.3 Relation between DR and %E

DR and %E are correlated by assuming that the densities of all liquor streams entering and leaving the washer are same. Ideally, when DR=1, E=100%. But it is not possible to meet this stipulation in industry.

$$\% E = [1 - \{(1\text{-}DR)(100\text{-}C_{yd})/(100\text{-}C_{yi})] 100 \quad (3.124)$$

3.5.4 Relation between Y, WL, WR, NEF and %loss

Yield of the recovered product has been expressed interms of weight liquor ratio, wash ratio, NEF by Gullichsen[24] as,

$$Y = 1 - [(WL\text{-}1)/\{WL(WR)^{NEF}\text{-}1\}] \quad (3.125)$$

Therefore, %loss can be expressed as,

$$\% \text{ loss} = (WL\text{-}1)/\{WL(WR)^{NEF}\text{-}1\} \quad (3.126)$$

If WR's for each stage and the corresponding NEF's are expressed as, WR_1, WR_2, ..., WR_n and E_1, E_2, ..., E_n respectively, then

$$\% \text{ loss} = 100(WL\text{-}1)/[WL(WR_1)^{E_1}(WR_2)^{E_2}...(WR_n)^{E_n} - 1] \quad (3.127)$$

and $Y = 100(1 - \%\text{loss})$ (3.128)

If $WR_1 = WR_2 = ... = WR_n = WR$, then yield is given by,

$$Y = 1 - (WL\text{-}1)/[WL(WR)^{E_1+E_2+...+E_n} - 1] = 1 - (WL\text{-}1)/[WL(WR)^{\Sigma E} - 1] \quad (3.129)$$

However, for multistage, material balance calculations are prerequisite for evaluation of different parameters. The general material balance equations and the procedure for estimation are discussed below.

3.6 Material Balances

This method is widely practiced by the industry to determine the amount of soda loss during the washing operation. This method serves well despite being an approximate one. The quantity of the solutes adsorbed on the fibers, i.e., chemically bound solute can not be found by this method. Material balances around three washers are explained with the help of figure 6.

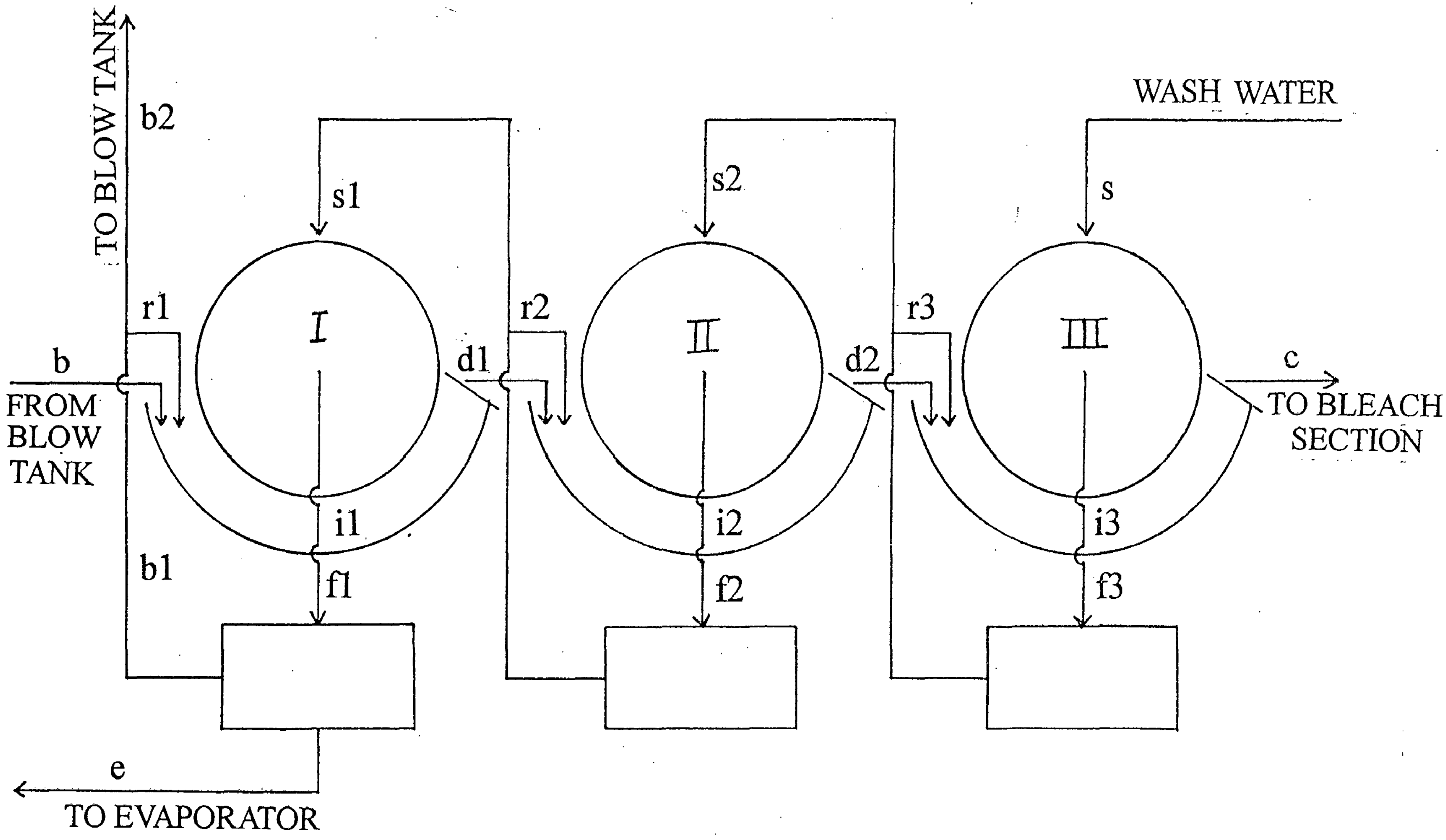

Figure 6 : Three stage counter current brown stock washing system

First washer mass balance

Liquor : $L_b + L_{r1} + L_{s1} = L_{f1} + L_{d1}$, $L_{f1} = L_{b1} + L_e$, $L_{b1} = L_{b2} + L_{r1}$, $L_{i1} = L_{r1} + L_b$

Solids : $L_b x_b + L_{r1} x_{r1} + L_{s1} x_{s1} = L_{f1} x_{f1} + L_{d1} x_{d1}$, $L_{f1} x_{f1} = L_{b1} x_{b1} + L_e x_e$,
$L_{b1} x_{b1} = L_{b2} x_{b2} + L_{r1} x_{r1}$, $L_{i1} x_{i1} = L_{r1} x_{r1} + L_b x_b$

Fiber : $L_b C_{yb} = L_{d1} C_{yd1}$

Water : $L_b(1-C_{yb})(1-x_b) + L_{r1}(1-x_{r1}) + L_{s1}(1-x_{s1}) = L_{f1}(1-x_{f1}) + L_{d1}(1-C_{yd1})(1-x_{d1})$

Second washer mass balance

Liquor : $L_{r2} + L_{d1} + L_{s2} = L_{f2} + L_{d2}$, $L_{f2} = L_{r2} + L_{s1}$, $L_{i2} = L_{r2} + L_{d1}$

Solids : $L_{r2} x_{r2} + L_{d1} x_{d1} + L_{s2} x_{s2} = L_{f2} x_{f2} + L_{d2} x_{d2}$,
$L_{f2} x_{f2} = L_{r2} x_{r2} + L_{s1} x_{s1}$, $L_{i2} x_{i2} = L_{r2} x_{r2} + L_{d1} x_{d1}$

Fiber : $L_{d1} C_{yd1} = L_{d2} C_{yd2}$

Water : $L_{d1}(1-C_{yd1})(1-x_{d1}) + L_{r2}(1-x_{r2}) + L_{s2}(1-x_{s2}) = L_{f2}(1-x_{f2}) + L_{d2}(1-C_{yd2})(1-x_{d2})$

Third washer mass balance

Liquor : $L_{r3} + L_{d2} + L_s = L_{f3} + L_c$, $L_{f3} = L_{r3} + L_{s2}$, $L_{i3} = L_{r3} + L_{d2}$

Solids : $L_{r3} x_{r3} + L_{d2} x_{d2} + L_s x_s = L_{f3} x_{f3} + L_c x_c$,
$L_{f3} x_{f3} = L_{r3} x_{r3} + L_{s2} x_{s2}$, $L_{i3} x_{i3} = L_{r3} x_{r3} + L_{d2} x_{d2}$

Fiber : $L_{d2} C_{yd2} = L_c C_{yc}$

Water : $L_{d2}(1-C_{yd2})(1-x_{d2}) + L_{r3}(1-x_{r3}) + L_s(1-x_s) = L_{f3}(1-x_{f3}) + L_c(1-C_{yc})(1-x_c)$

3.7 Conclusions

In this investigation mathematical models are derived for cake formation, washing, drying and dewatering zones of a rotary vacuum washer. It is an established fact that cake blow, discharge and dead zones have practically little contribution towards the brown stock washing operation.

The present mathematical model, given by equation (3.11), (3.12), (3.13) and (3.14), for filtrate flow rate through cake formation zone is simpler than the model proposed by Peck and Chand[59], which uses graphical evaluation technique. The present model predicts nearly the same value without going into the cumbersome calculations.

For cake washing zone, models have been derived from basic continuity equation of flow through porous media. These models have been solved for different adsorption isotherms alongwith varying initial and boundary conditions. Laplace transform technique is used to solve these models. Inverse has been performed by using the method of residues. Expressions are given for exit concentration of solute leaving the bed at any time, average concentration of solute in the discharged pulp and mean concentration of filtrate collected through the cake washing zone.

The asymptotic expressions(3.96) and (3.97) can be used to predict the values of the dimensionless concentration in the liquor and the stagnant layer for any value of ξ and τ. This is in excellent agreement with those of Kuo and Barrett[34] as can be found later.

Model for the cake drying zone is developed by using Darcy's law and in terms of real saturation of cake. Though the process agrees in principle with those developed by Brown et al.[7] and Perron and Lebeau[65], the solution obtained are distinctly different.The equations have been extended for multistage washers consisting of three stages joined in series. Material balance calculation procedure has been followed to find out the concentration in between the stages. The procedure appears to be simple and accurate.

CHAPTER 4
VALIDITY OF MATHEMATICAL MODELS

All the models of **Chapter 3** developed for different zones requires validation through experimental data or reported data already available in literature. These are shown in different paragraphs as follows :

4.1 Validity of the Model for Filtrate Flow Rate

For the validity of the proposed model of filtrate flow rate through the cake formation zone given by equations (3.11), (3.12), (3.13) and (3.14), the experimental data of two different investigators Peck and Chand[59] and Perron and Lebeau[65] is used.

4.1.1 Validity by using the data of Peck and Chand[59]

Data of a plate and frame filter press of filtration area 0.139 m^2, reported by Peck and Chand[59] is given in **Table 4**. The accuracy of the model is checked through the prediction of the size of the radius of rotary leaf vacuum filter. The reported radius of the filter is 0.84 m. Equation(3.12) predicts the value on the order of 0.81 m, with the % deviation of only 3.5 %, from which it appears that the developed model is correct.

The main advantage of the present model is as under :
It does not need the elaborate graphical evaluation. Besides the procedure developed in this investigation is simple and easier to use. Thus it claims superiority over the previous ones.

Table 4 : Data of Peck and Chand[59]

Design and process data	Calculated data
$N = 0.167X10^{-1}$ rev/s $R_0/R = 0.500$ $V_f = 0.631X10^{-3}$ m^3/s	$C' = 0.133X10^{-4}$ m^2/s $C'' = 0.900X10^{-1}$ $t' = 0.860$ s $\theta = 0.210X10^{1}$ Radian $V_c = 0.338X10^{-2}$ m^3/m^2

4.1.2 Validity by using the data of Perron and Lebeau[65]

For validity of the filtrate flow rate model, data of a washing plant, reported by Perron and Lebeau[65] is also used. This data is given in **Table 5**. The value of filtrate flow rate predicted by Perron and Lebeau[65] is examined with the value of filtrate flow rate obtained from present model.

The value of filtrate flow rate has been calculated according to equation(3.14), (3.15) and (3.16). The results are 0.222 m^3/s, 0.236 m^3/s and 0.251 m^3/s respectively. The % deviation between the values obtained from equations(3.14) and (3.15) is 5.5 %, which validates the present model.

Table. 5 : Data of Perron and Lebeau[65]

Parameter	Unit	Value	Parameter	Unit	Value
A	m^2	0.430×10^2	ε	-	0.940
K	m^2	0.789×10^{-11}	η	kg/ms	0.620×10^{-3}
N	1/s	0.250×10^{-1}	θ	Radian	0.214×10^1
ΔP	Pascal	0.200×10^5	ρ	kg/m^3	0.100×10^4
R	m	0.175×10^1	ρ_f	kg/m^3	0.156×10^4
S_0	m^2/m^3	0.156×10^7	ψ	-	0.340
X_i	kg/kg	0.119×10^{-1}			

4.2 Results and Discussion

The models for cake formation zone have been expressed in terms of simple physical parameters of solid and liquid system. These models have been used to calculate the effect of one parameter on the other parameters. For this purpose the data of **Table 5** has been chosen as the initial set of data. Some data are also varied within the normal ranges followed by the industry. The results obtained are plotted in figures 7 to 30.

4.2.1 Effect of N_2 and R on θ

Figures 7 and 8 represent the effect of slurry level(N_2) and radius of drum(R) on angle of submergence(θ). As it is evident from figure 7, with the increase of N_2, θ increases whereas it decreases with the increase of R (figure 8) as expected. From practical standpoint the submergence should not go beyond 50 % otherwise there will be less area left for subsequent zones and the unit becomes inoperable. Therefore the vat level and the drum radius must be estimated based on a fixed degree of submergence; for 30-35 % submergence and slurry level of the 0.9 m, the drum radius should be around 1.5 to 2.0 m.

4.2.2 Effect of θ and R on ΔP

As shown in model development the total pressure drop across the cake is influenced by the hydrostatic head or the level of suspension in the vat, which in turn is a function of degree of submergence. Interdependence of angle of submergence(θ), radius of drum(R) and pressure drop(ΔP) is explained in figures 9 and 10 respectively. As evident ΔP across the cake increases linearly with θ and R. Equation(3.45) predicts that the hydrostatic head has a major contribution of about 20-30 % to the total pressure drop. Therefore it can not be neglected. It may be pointed out that this factor is generally not taken into consideration in the design of brown stock washing system.

4.2.3 Effect of C_{yi}, ΔP and ψ on L

From figure 11, 12 and 13 it is observed that cake thickness(L) increases by increasing inlet vat consistency(C_{yi}), pressure drop(ΔP) and fractional submergence of drum(ψ) respectively. This can be attributable to higher deposition of fibers on the drum by increasing these parameters. Within the normal ranges of operating conditions the inlet vat consistency should be around 1.2 %. An increased C_{yi} is accompanied by increased production rate but is associated with increased dissolved solids carry over resulting in higher bleach plant load and greater environmental impact.

4.2.4 Effect of ΔP and ψ on A

The effective area of drum(A) decreases by increasing pressure drop across the cake(ΔP) and fractional submergence of drum(ψ) as can be seen from figures 14 and 15 respectively. Requisite area of the drum has been calculated by assuming that the cake is non compressible, because the compressibility of the pulp fiber cake is relatively unknown. Pressure drop of 18000 to 22000 Pascal is usually maintained across the cake.

4.2.5 Effect of ΔP, ψ, rpm, C_{yi}, R, and L on V_f

The parameters like pressure drop(ΔP), fractional submergence of drum(ψ), rpm, inlet vat consistency(C_{yi}), radius of drum(R) and cake thickness(L) are varied within the normal ranges followed by industry and their effects are calculated from equation(3.14) and are compared with those from equations(3.15) and (3.16).

From figure 16 it is observed that ΔP almost linearly enhances V_f. Present model(3.14) gives slightly lower values than those predicted from equations(3.15) and (3.16). The maximum deviation is of the order of 6% indicating an excellent agreement among all the investigators(figure 17). Equations(3.15) and (3.16) predict a linear variation of V_f and ψ, where as equation(3.14) exhibits that V_f increases upto 50 % submergence and beyond which it decreases. This fact was also proposed by Perkins et al.[64].

As the submergence increases from 0 to 50% the projected area on which the pressure is acting increases while beyond 50% this value will decrease indicating lower filtrate flow rate. Hence 50% is taken as the practical limit for submergence.

From figure 18, V_f as a function of rpm is drawn. It is reflected that the nature of the curves in all the three cases are found to be identical. From figure 19, V_f decreases sharply by increasing C_{yi} because the production rate increases. As a result, a thicker layer of cake is deposited outside the mesh, which resists the flow of liquor through the mat to a higher proportion. Equations(3.15) and (3.16) predicts a linear relationship between V_f and R and are nearly parallel to each other as observed from figure 20. On the other hand, the value of V_f obtained from equation(3.14) increases very sharply with the increase in R. Figure 21 shows that increase in cake thickness resists the flow through it.

4.2.6 Effect of A, ΔP, ψ , rpm and C_{yi} on FPR

Fiber production rate(FPR) increases by increasing the area of drum(A), pressure drop (ΔP), fractional submergence of drum(ψ), rpm and inlet vat consistency(C_{yi}) as shown in figures 22 to 26. The values obtained from equation(3.20) are compared with those of equation(3.26). The nature of graph is similar in both cases. The minimum deviation is 5.2% and maximum deviation is 6.6%.

4.2.7 Effect of ΔP, ψ , rpm and C_{yi} on SLF

As observed from figures 27 to 30, specific loading factor(SLF) increases by increasing pressure drop(ΔP), fractional submergence of drum(ψ), rpm and inlet vat consistency(C_{yi}) respectively. The results obtained from equations(3.28) and (3.33) are compared and the % deviation lies in the interval 5.2 to 6.6%.

4.3 Validity of Washing Models

In **Table 3** of **Chapter 3**, 8 models of washing zones are derived. The solutions of the individual models are also developed. Though all the models are individualistic due to varying adsorption and boundary conditions. The solution techniques basically are of two types. Models 1 to 6 follow one method of solution but models 7 to 8 take help of the other techniques. Perron and Lebeau[65], Kuo and Barrett[35] solved their models without considering longitudinal dispersion coefficient. Therefore the solutions of model 7 and 8 are similar to their solution. For the sake of comparison these are treated first.

4.3.1 Validity of the asymptotic expressions

While developing an expression for dimensionless concentration from models 7 and 8 a term has appeared whose solution introduces asymptotic expressions. The asymptotic expressions for the dimensionless concentration of solute in the washed pulp and stagnant layer given by equations(3.96) and (3.97) have not been reported earlier. Therefore it becomes necessary to validate these expressions. For this purpose the values of dimensionless concentration of solute in the washed pulp (I) and dimensionless concentration of solute in the stagnant layer (I') are calculated by giving different values to ξ and τ. The results obtained are plotted in form of figures 31 and 32. These graphs are similar to those obtained by Kuo and Barrett[35] for the same range of ξ and τ. They have plotted their graphs by taking values of I_0 from Jahnke and Emde[30] and the value of $\int_0^{\tau} e^{-(\xi+x)} I_0(2\sqrt{\xi x})dx$ was taken from Brinkley[6].

The results obtained from the present model are compared with those of Kuo and Barrett[35] in figure 31 and 32. Having compared the asymptotic expressions it is now imperative to compare all the models 1 to 8. These are given below.

Validity of the proposed washing models 1 to 8 have been carried out by using the data of three previous investigators namely Grahs[20], Perron and Lebeau[65] and Turner et al.[84].

4.3.2(a) Validity by using the data of Grahs[20]

Grahs[20] has obtained data from a Dorr Oliver type washer (length 3.6 m and diameter 3 m) from a Sulphate pulp mill. This data is given in **Table 6.**

Table 6 : Data of Grahs[20]

Parameter	Unit	Value	Parameter	Unit	Value
L	m	2.000×10^{-1}	C_{Fm}	kg/m^3	1.064×10^{2}
V_w	m^3/s	2.150×10^{-2}	A_c	m^2	6.100
ε_{dm}	-	8.800×10^{-1}	ε_{tm}	-	9.280×10^{-1}
C_i	kg/m^3	5.700×10^{-1}	C_s	kg/m^3	5.000×10^{-3}
k_1	1/s	7.000×10^{-3}	k_2	1/s	2.000×10^{-3}

Interstitial speed of liquor (u) has been calculated by using following expression.

$$u = V_w / (A_c \, \varepsilon_{dm}) \qquad (4.1)$$

Total time of washing (T_w) has been found by using,

$$T_w = (A_c \,.\, \text{Time of 1 revolution}) / (2\pi R \,.\, \text{Width of drum}) \qquad (4.2)$$

Under normal operating conditions time of one revolution varies between 30 to 40 seconds. Therefore in this investigation two extreme cases are considered, i.e., when the drum takes 30 s and when it takes 40 s to complete 1 revolution. Considering both the cases the results are given in **Tables** 7 and **8**.

Mass transfer coefficient k^* has been taken as the mean of k_1 and k_2 and is found to be 35 x 10^{-4} 1/s. Also at equilibrium $\partial n/\partial t = 0$ which means $k_1c - k_2n = 0$. This equation reduces to $n = kc$, where $k = (k_1/k_2)$. k is dimensionless and by using the data of Grahs[20] its value comes out to be 3.5.

Table 7 : Results from present washing models (Time of washing = 30 s)

Washing Models	C_e	C_d	C_m
Model 1	0.159	0.051	0.541
Model 2	0.159	0.051	0.541
Model 3	0.112	0.046	0.537
Model 4	0.111	0.046	0.537
Model 5	0.299	0.101	0.552
Model 6	0.126	0.074	0.536
Model 7	0.001	0.005	0.151
Model 8	0.006	0.005	0.528

Table 8 : Results from present washing models (Time of washing = 40 s)

Washing Models	C_e	C_d	C_m
Model 1	0.051	0.018	0.430
Model 2	0.051	0.018	0.430
Model 3	0.049	0.019	0.422
Model 4	0.048	0.019	0.422
Model 5	0.136	0.043	0.466
Model 6	0.104	0.041	0.433
Model 7	0.007	0.005	0.115
Model 8	0.022	0.005	0.398

4.3.2(b) Validity by using the data of Perron and Lebeau[65]

Validity of the model is also carried out by using the data of Perron and Lebeau[65]. They have collected data from the fourth washer of an industrial plant. This data is given in **Table 9**. The results obtained from different models are given in **Table 10**. Perron and Lebeau[65] have reported the values of mean concentration of liquor as 7.328 kg/m^3 for a fixed set of operational conditions and design parameters.

Table 9 : Data of Perron and Lebeau[65]

Parameter	Unit	Value	Parameter	Unit	Value
L	m	2.957×10^{-1}	V_w	m^3/s	2.500×10^{-2}
K	m^2	6.905×10^{-12}	ΔP	Pascal	2.000×10^{4}
A	m^2	4.300×10^{1}	C_i	kg/m^3	1.105×10^{1}
C_s	kg/m^3	0.000	η	kg/ms	6.200×10^{-4}
ε_d	-	5.000×10^{-1}	ε_t	-	9.400×10^{-1}

Liquor speed in cake pores has been estimated by using the Darcy's law,

$$u = K\Delta P/\eta\varepsilon_t L \tag{4.3}$$

Total time of washing has been found by using following expression,

$$T_w = V_w/u\varepsilon_d NA \tag{4.4}$$

Table 10 : Results from present washing models

Washing Models	C_e	C_d	C_m
Model 1	0.649	0.314	7.512
Model 2	0.653	0.316	7.513
Model 3	1.653	0.970	8.034
Model 4	1.659	0.974	8.037
Model 5	1.254	0.606	7.841
Model 6	2.685	1.581	8.496
Model 7	0.058	0.003	2.008
Model 8	0.435	0.005	7.025

Values of the mass transfer coefficients k_1, k_2, k^*, k and longitudinal dispersion coefficient D_L has been taken same as in case of Grahs[20]. The ratio of u/D_L is taken as 1.30 cm^{-1}, which has been assumed by Han and Edwards[27] and Turner et al.[84] for their investigations. All these values are assumed the same while validating the models by the data of Turner et al.[84] in this present investigation.

4.3.2(c) Validity by using the data of Turner et al.[84]

Turner et al.[84] have presented the steady state operating conditions for two individual washing stages. They have reported the concentration of solids left in the discharged cake as 2.50% and 0.83% and the concentration of solids in the filtrate as 5.15% and 1.83% respectively for washer 1 and washer 2. The input data of both washers (length 4.88 m, diameter 3.51 m) are given in **Table 11**. The results obtained from the present models for washer 1 and 2 are given in **Tables 12** and **13** respectively.

Table 11 : Data of Turner et al.[84]

Parameter	Unit	Value of Washer 1	Value of Washer 2
L	m	2.500 X 10^{-1}	2.820 X 10^{-1}
FPR	ODT/D	4.600 X 10^{2}	4.600 X 10^{2}
DF	kg/kg	2.280	2.280
rpm	-	1.930	1.870
C_i	%	5.290	1.890
C_s	%	1.830	5.900 X 10^{-1}
C_d	%	2.500	8.300 X 10^{-1}
C_{yi}	%	1.140	1.120
C_{yd}	%	1.230 X 10^{-1}	8.300 X 10^{-1}

Usually the angle of washing zone is 60°. The total time of washing is calculated by dividing the angle of washing zone by the time taken to complete one revolution. Porosity ε_t is calculated by using the following equation.

$$\varepsilon_t = \rho_f(1-C_y)/[\rho C_y + \rho_f(1-C_y)] \qquad (4.5)$$

Liquor flow rate through the washing zone is calculated by using,

$$V_w = (FPR/\rho_s)[DF - 1 + (100/C_{yd})] \quad (4.6)$$

Liquor speed has been calculated by using following expression,

$$u = V_w/(A_c\, \varepsilon_t) \quad (4.7)$$

Table 12 : Results from present washing models (washer 1)

Washing Models	C_e	C_d	C_m
Model 1	2.245	2.039	4.778
Model 2	2.247	2.041	4.778
Model 3	2.786	2.433	4.883
Model 4	2.790	2.435	4.883
Model 5	2.447	2.142	4.818
Model 6	3.082	2.623	4.937
Model 7	1.828	1.830	1.567
Model 8	1.940	1.831	4.685

Table 13 : Results from present washing models (washer 2)

Washing Models	C_e	C_d	C_m
Model 1	0.807	0.696	1.790
Model 2	0.807	0.697	1.790
Model 3	1.016	0.847	1.810
Model 4	1.017	0.848	1.811
Model 5	0.892	0.739	1.798
Model 6	1.128	0.917	1.821
Model 7	0.590	0.590	0.545
Model 8	0.616	0.590	1.767

4.4 Conclusions

For a fixed amount of filtrate flow rate, the radius of filter estimated from equation(3.13) is in close approximately with that reported by Peck and Chand[59]. This indicates that the present model can even accept the data from a constant pressure filtration equipment like plate and frame filter press.

For the filtrate flow rate the data predicted from the present investigation (equation 3.14), agrees quite well with the data estimated from the models of Perron and Lebeau(equation 3.15) and Peters and Timmerhaus(equation 3.16).

Based on the developed models for cake formation zone various parametric effects are discussed with a view to analyse the performance of a brown stock washer. It has been found that increase in pressure drop decreases the effective area requirement. Pressure drop is a strong linear function of hydrostatic head or slurry level in the vat vis-a-vis degree of submergence.

Radius of drum significantly influences the filtrate flow rate. Linear variation occurs between filtrate flow rate and fractional submergence upto 50 % submergence and beyond which filtrate flow rate decreases. Increase in inlet vat consistency causes cake thickness to increase which in turn offers a resistance to the filtrate flow rate.

The asymptotic expressions reported by equations(3.96) and (3.97) can be used to predict the concentration of solute in the wash liquor and the stagnant layer for any value of ξ and τ. The values predicted by these models are nearly matching with the values reported by Kuo and Barrett[35] though the systems are completely different.

The washing models 1 to 8 have been validated by using the data of Grahs[20], Perron and Lebeau[65] and Turner et al.[84]. From the results given in different tables it can be concluded that the values obtained by model 3 and 4 are close enough to the values reported by different investigators in all the three cases. It is also observed that the model 7 is giving very low predictions.

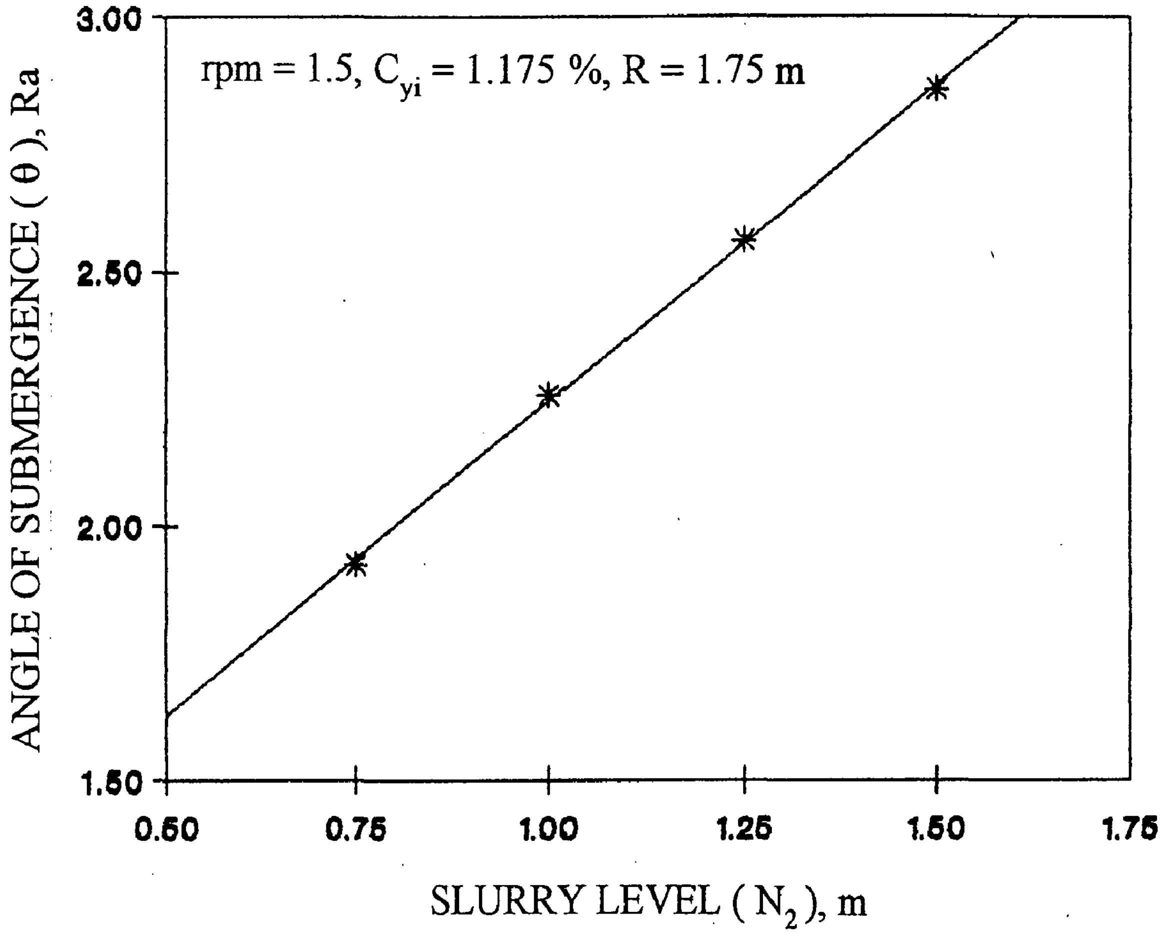

Figure 7 : Effect of slurry level on angle of submergence

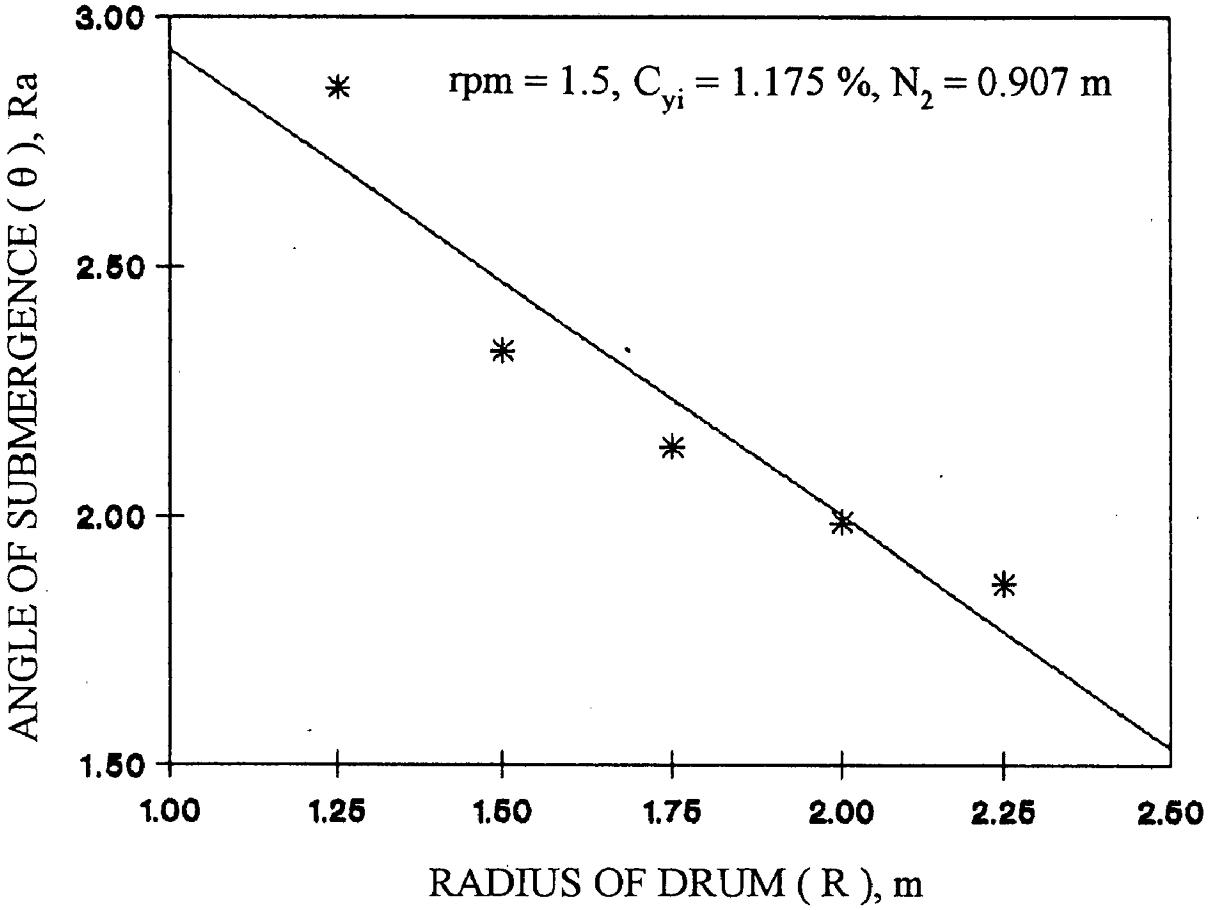

Figure 8 : Effect of radius of drum on angle of submergence

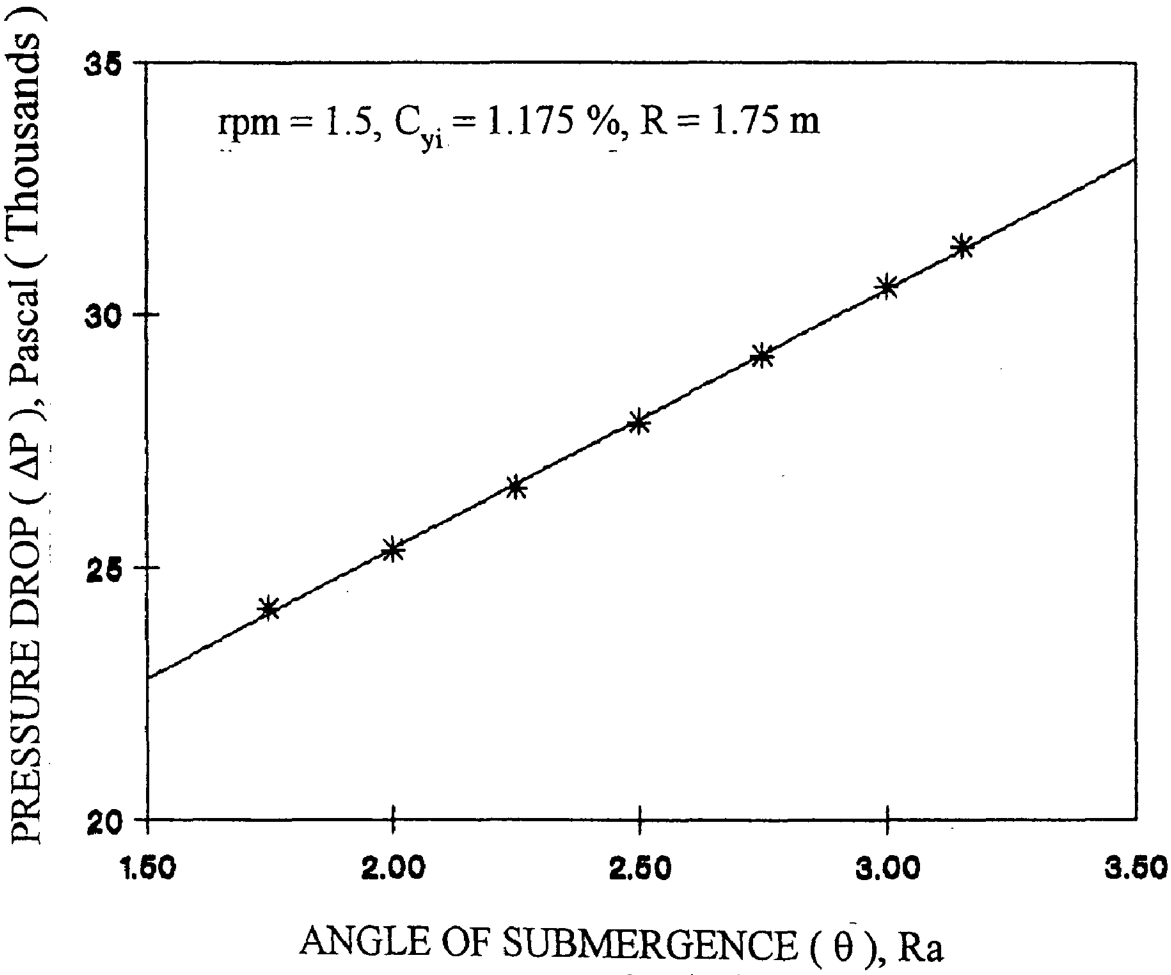

Figure 9 : Effect of angle of submergence on pressure drop

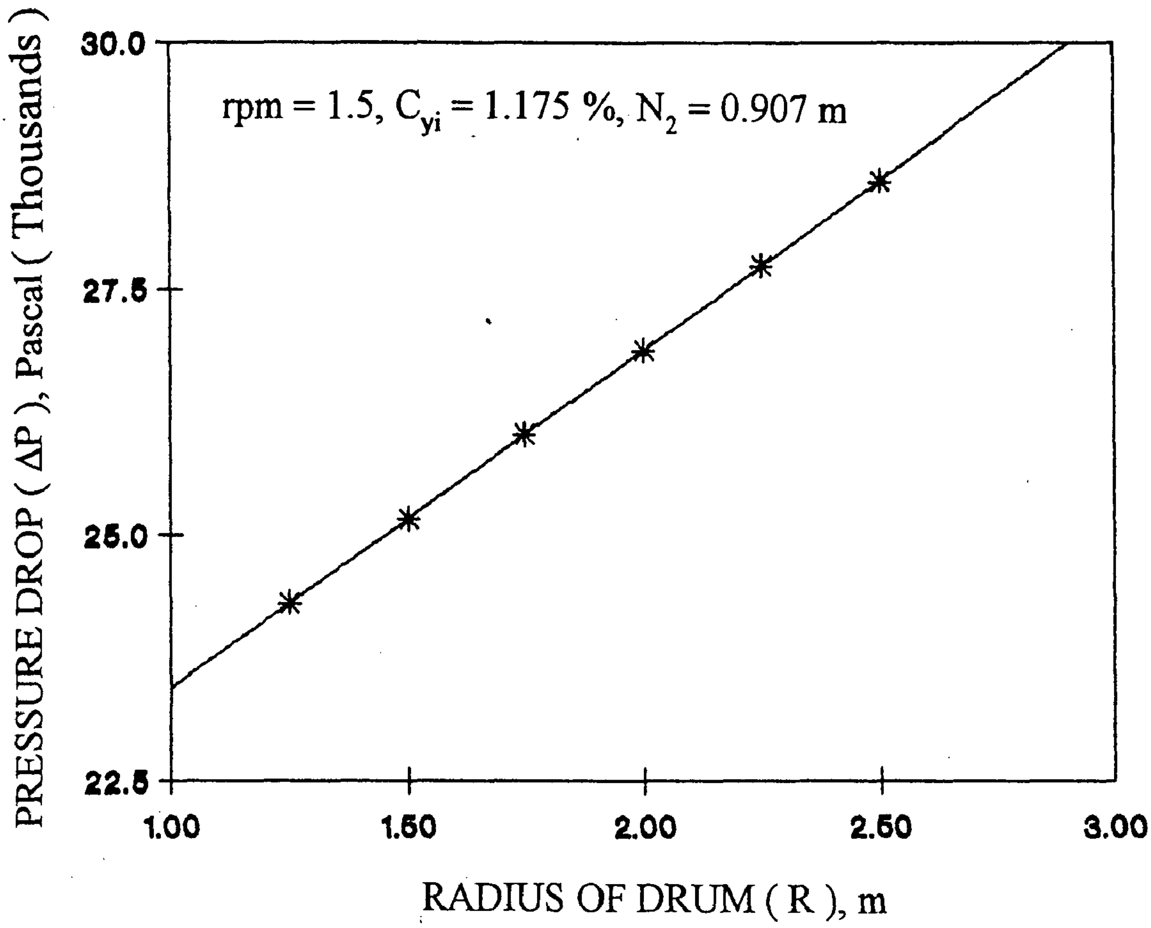

Figure 10 : Effect of radius of drum on pressure drop

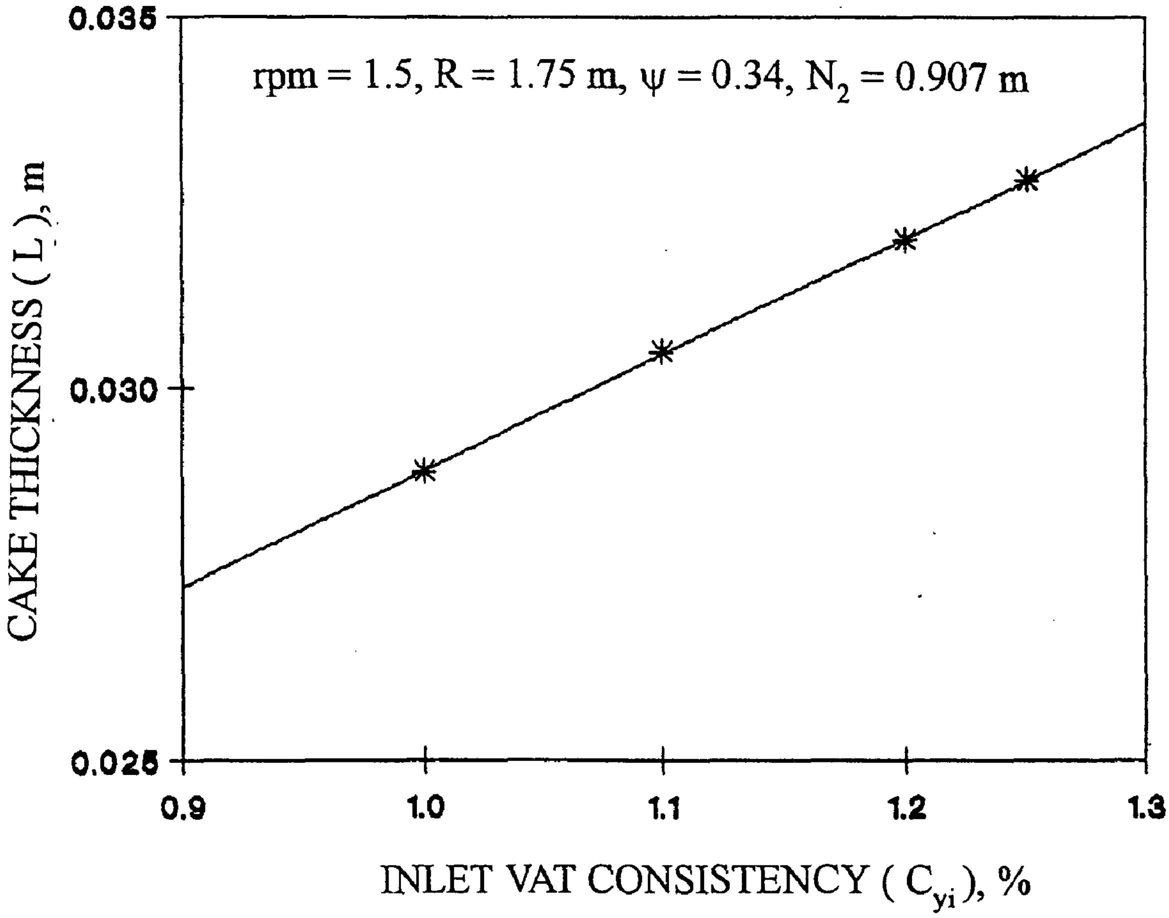

Figure 11 : Effect of inlet vat consistency on cake thickness

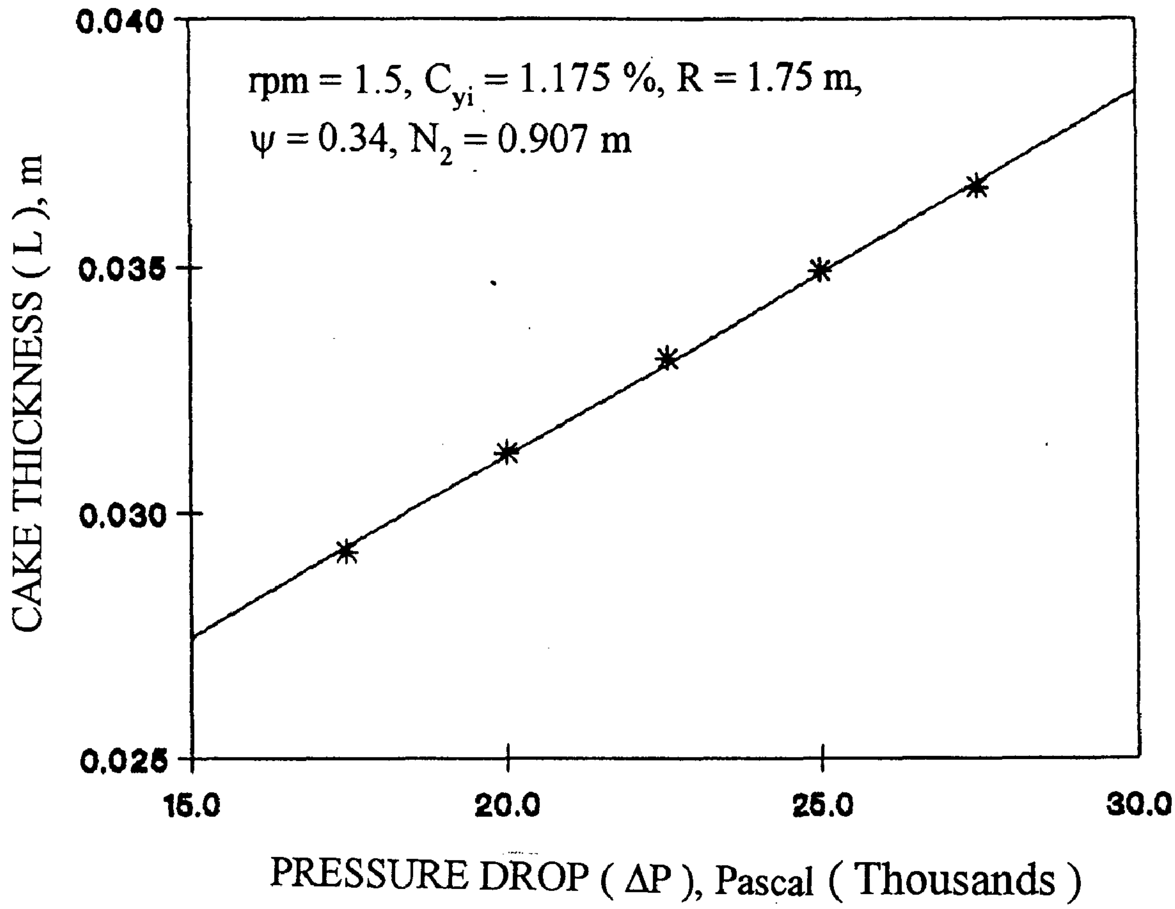

Figure 12 : Effect of pressure drop on cake thickness

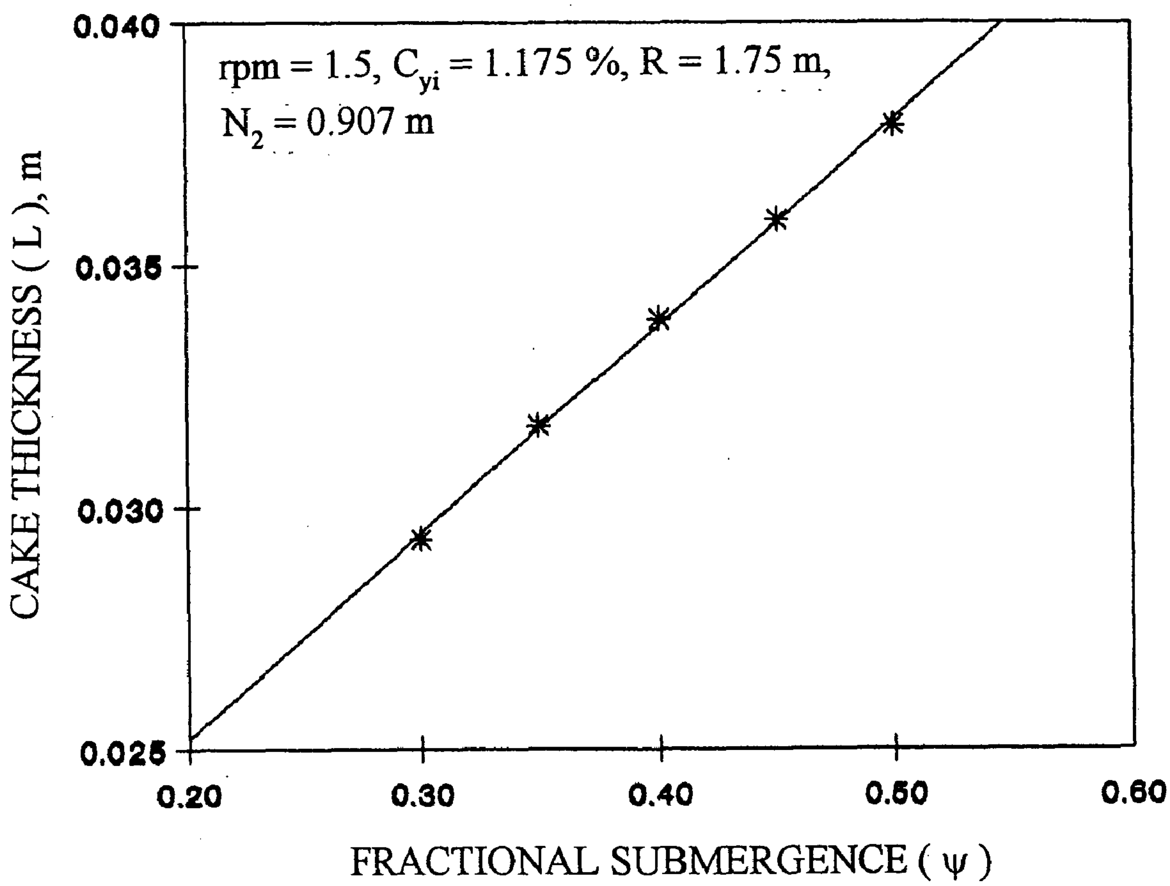

Figure 13 : Effect of fractional submergence on cake thickness

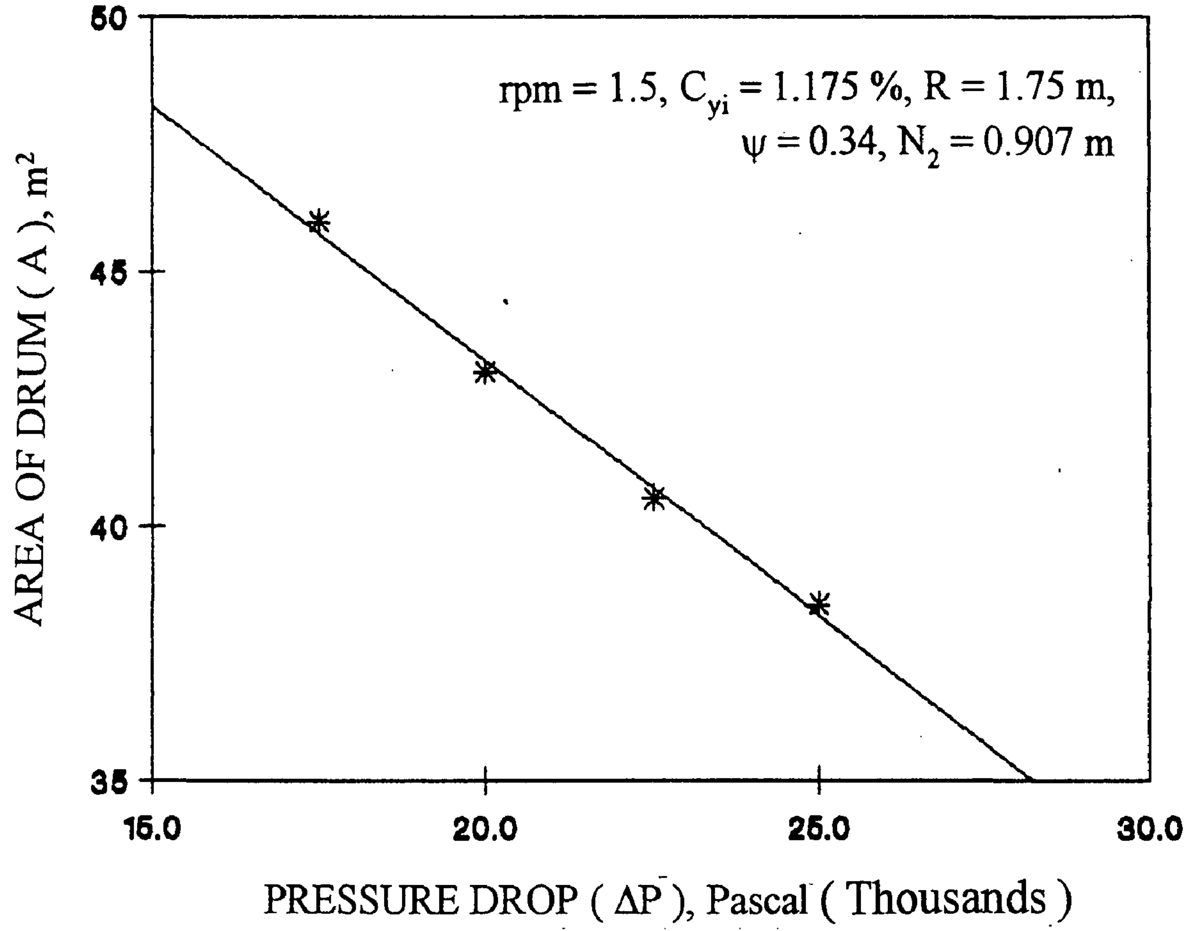

Figure 14 : Effect of pressure drop on area of drum

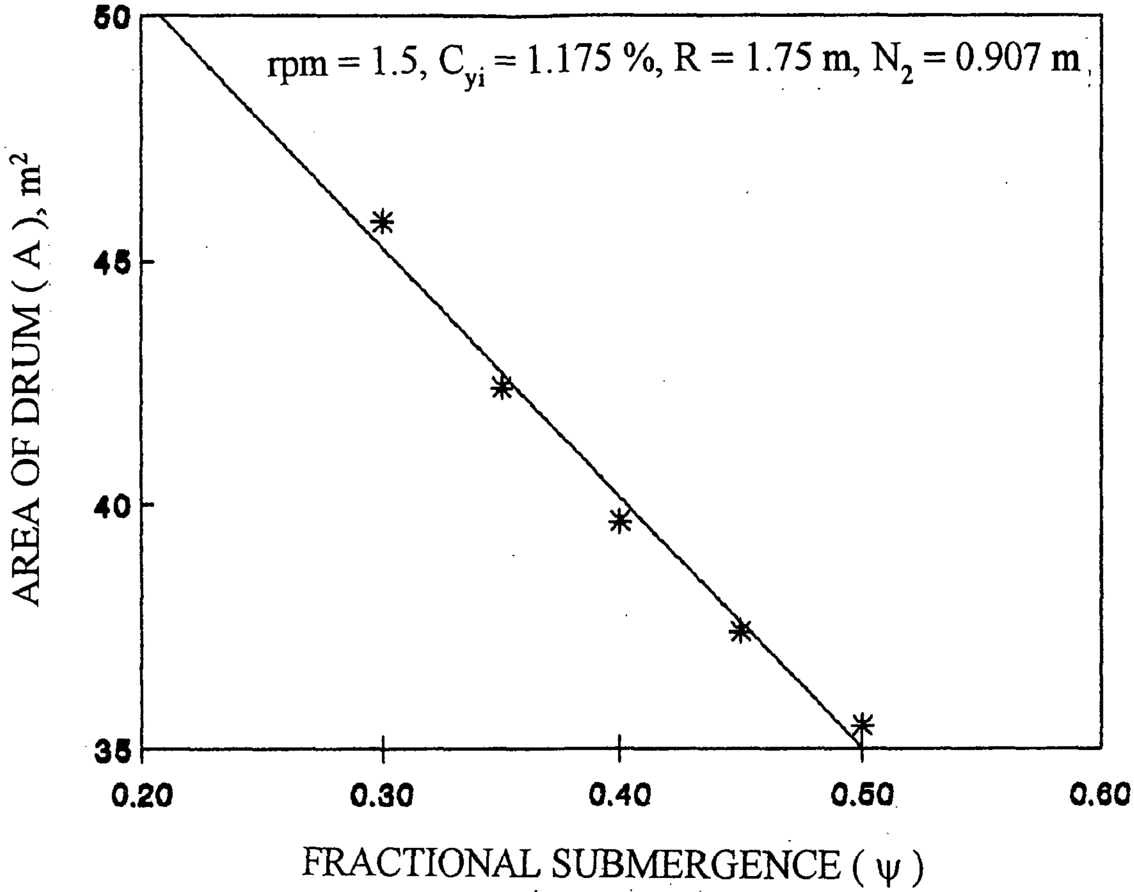

Figure 15 : Effect of fractional submergence on area of drum

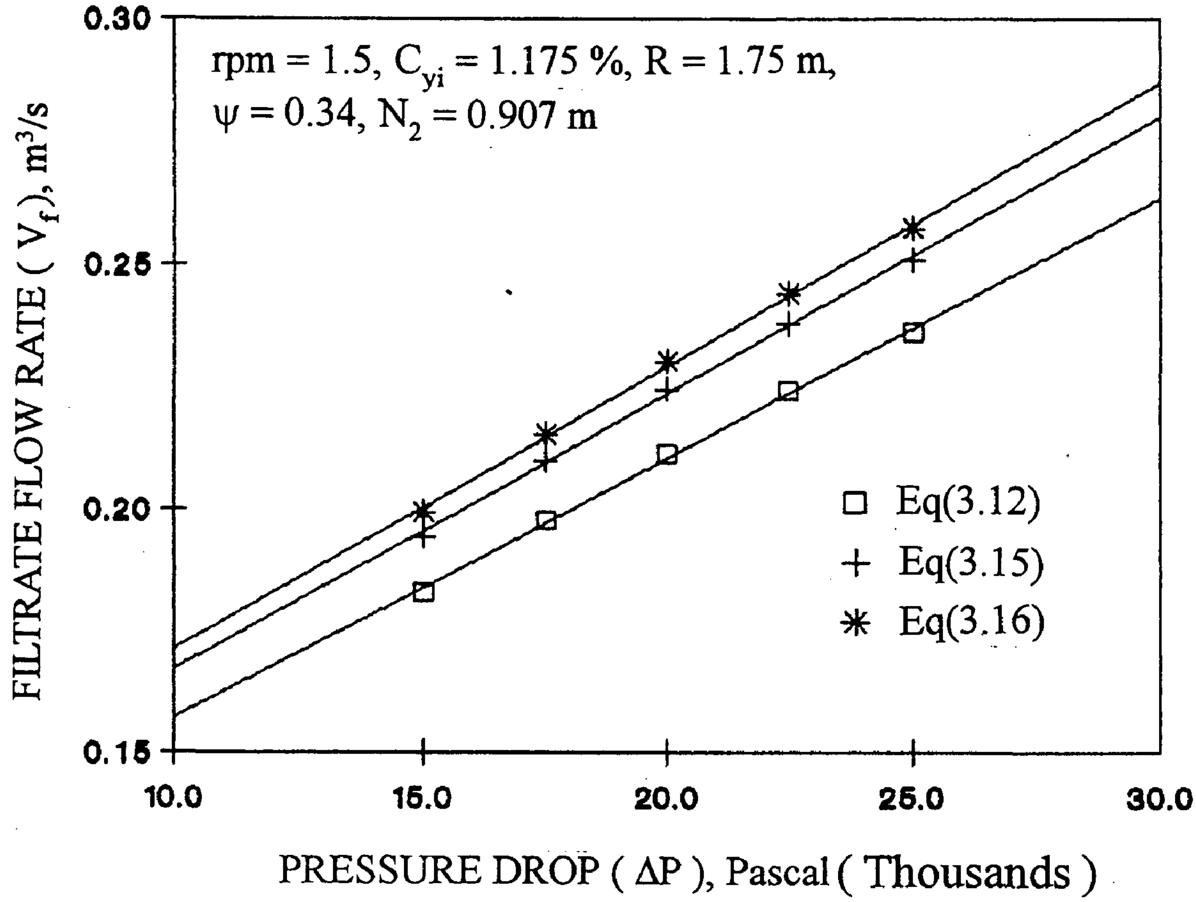

Figure 16 : Effect of pressure drop on filtrate flow rate

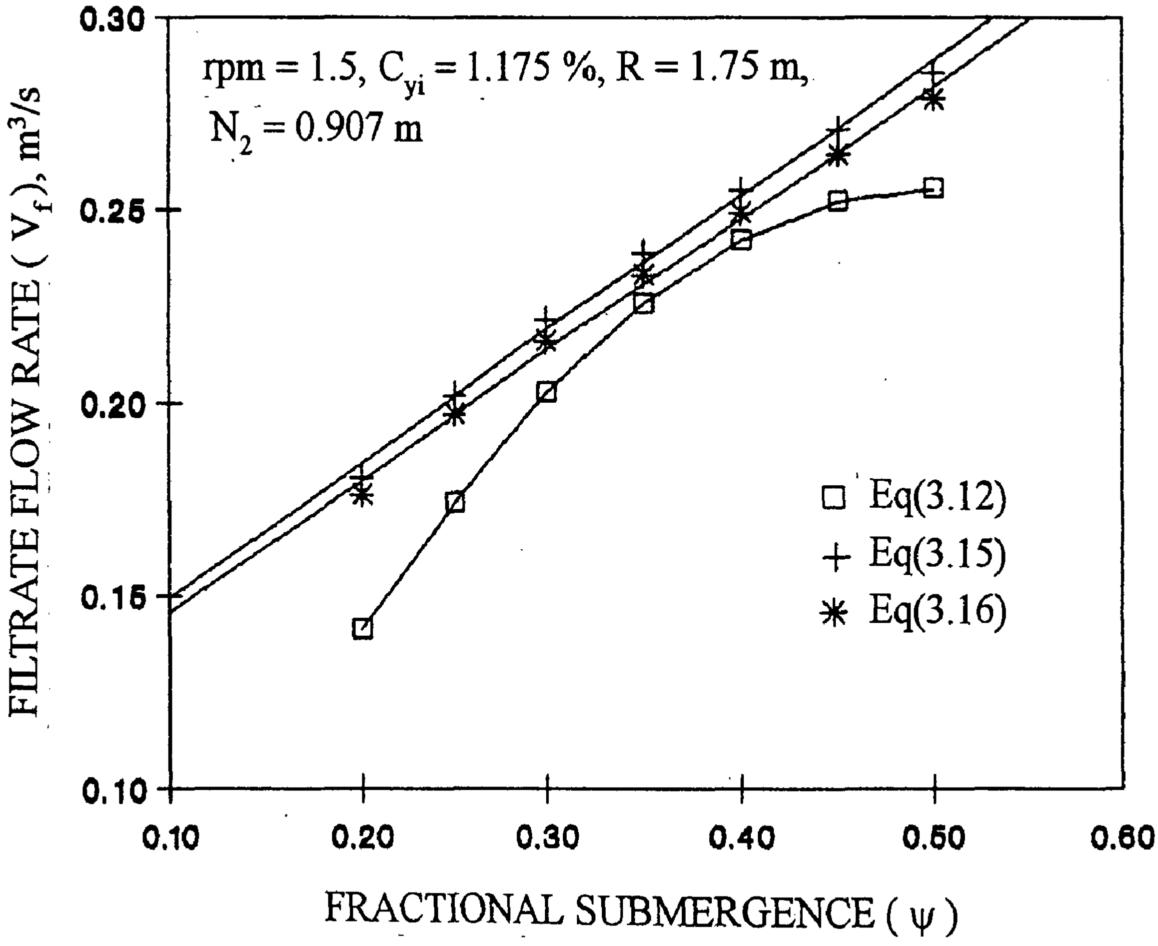

Figure 17 : Effect of fractional submergence on filtrate flow rate

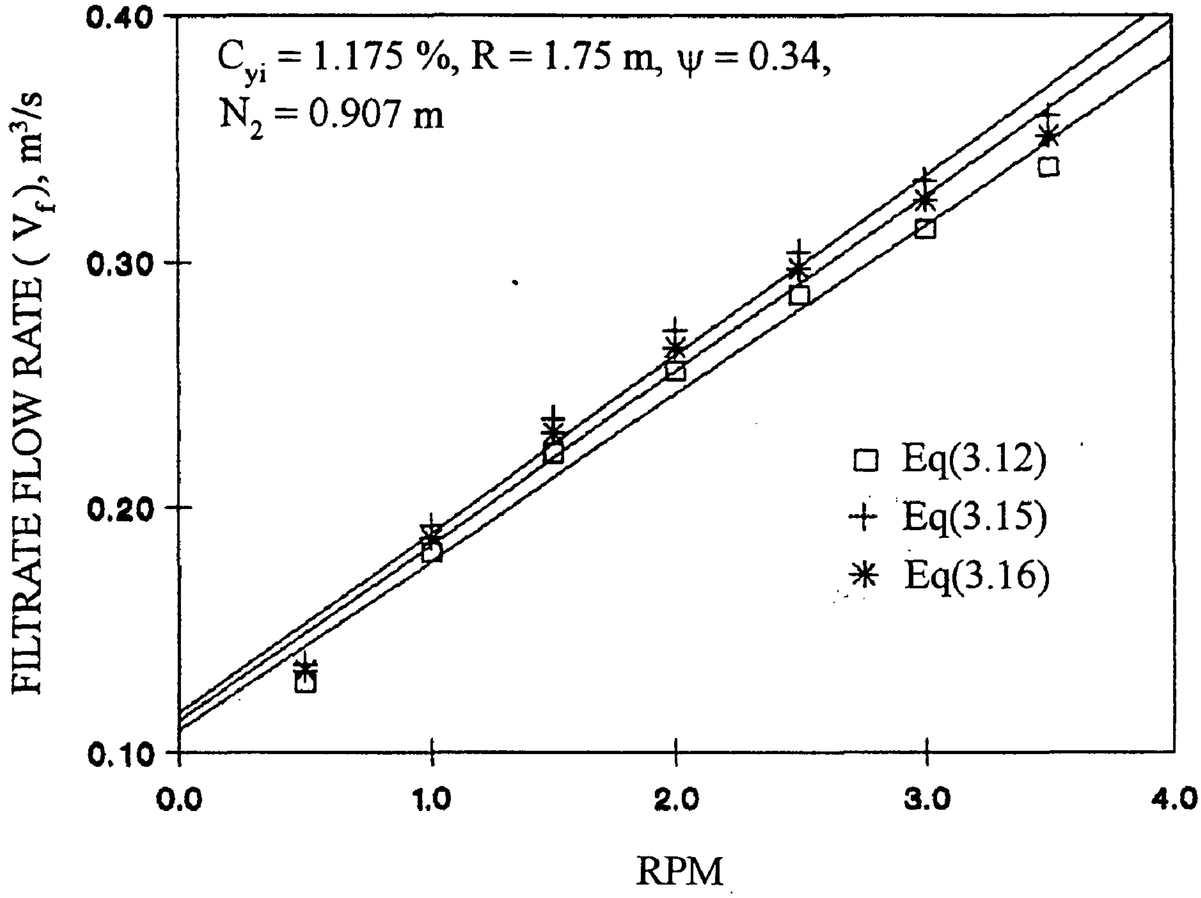

Figure 18 : Effect of rpm on filtrate flow rate

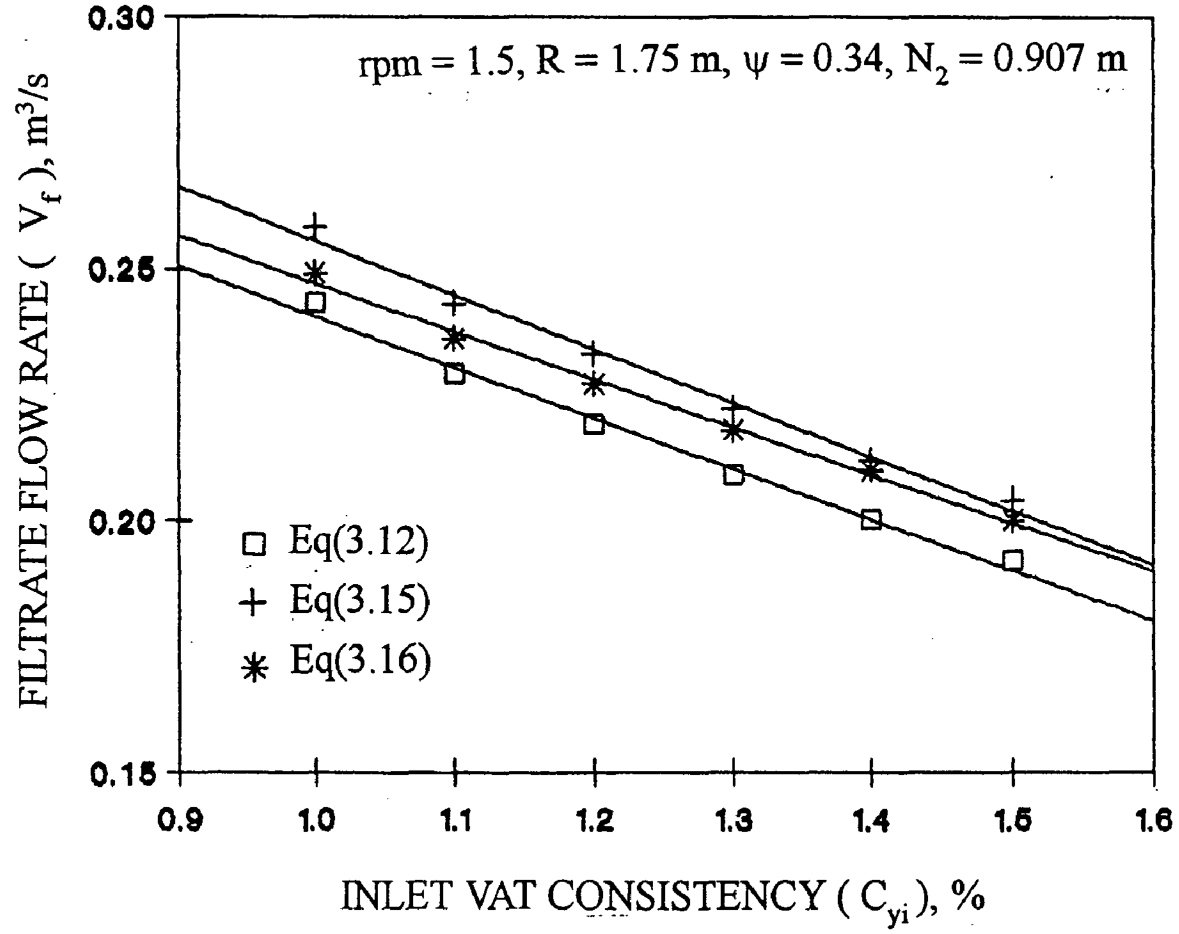

Figure 19 : Effect of inlet vat consistency on filtrate flow rate

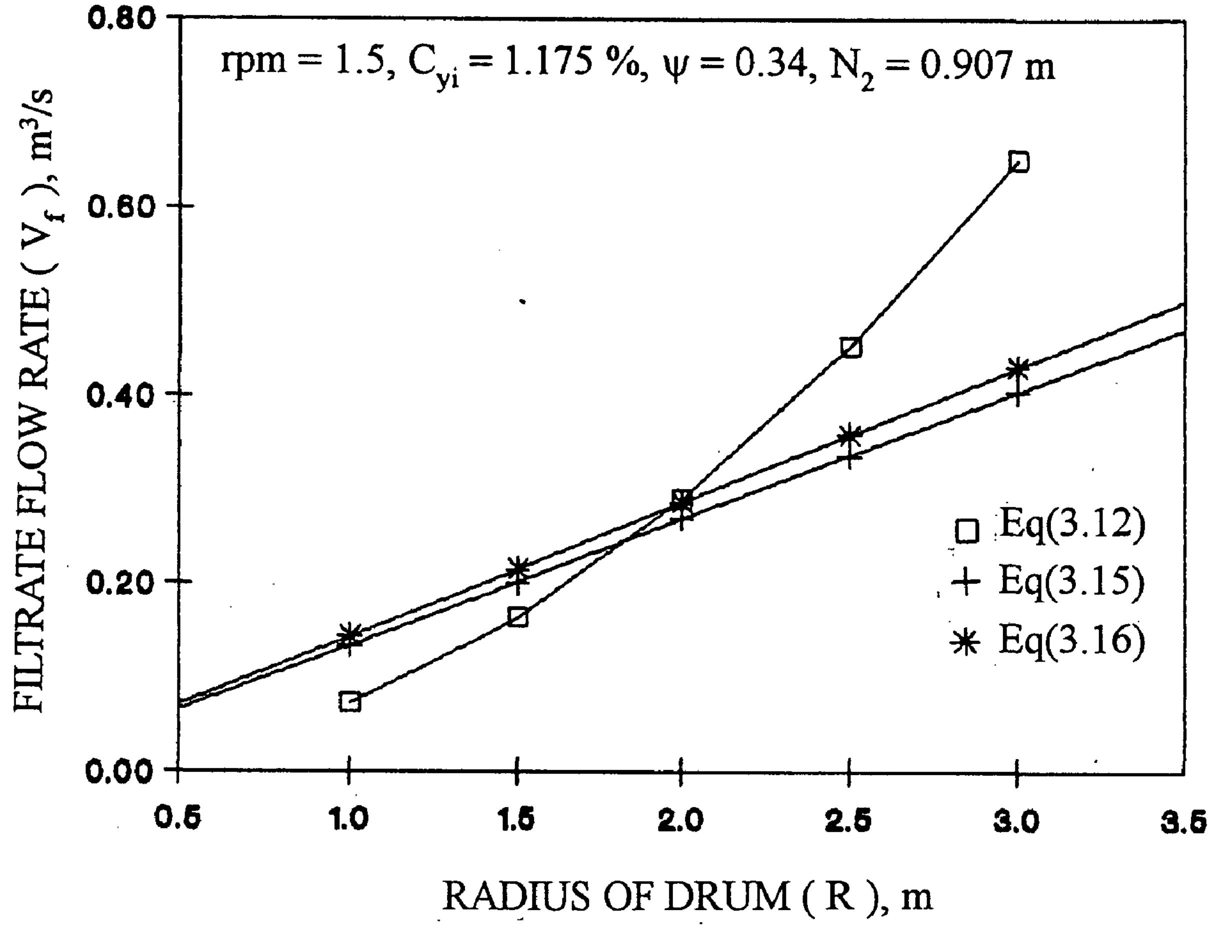

Figure 20 : Effect of radius of drum on filtrate flow rate

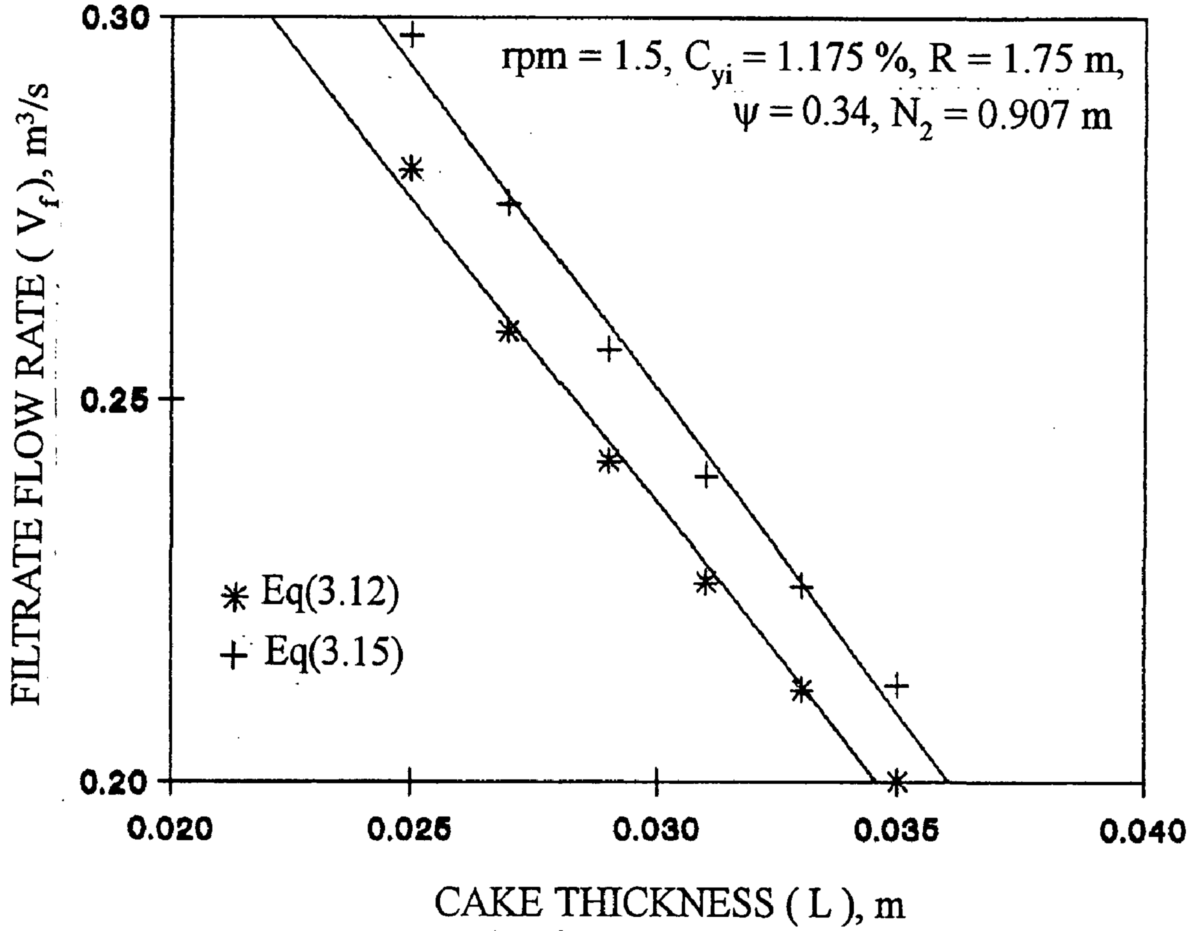

Figure 21 : Effect of cake thickness on filtrate flow rate

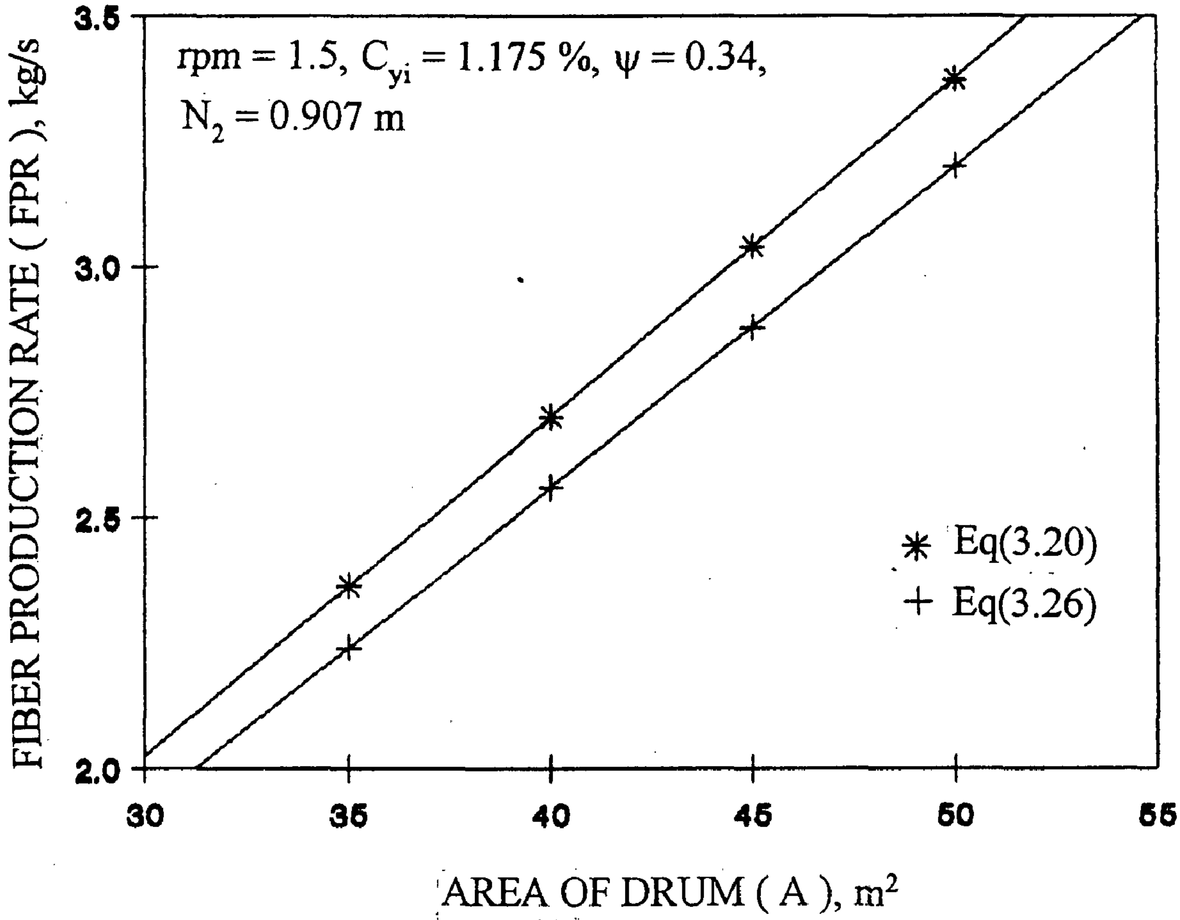

Figure 22 : Effect of area of drum on fiber production rate

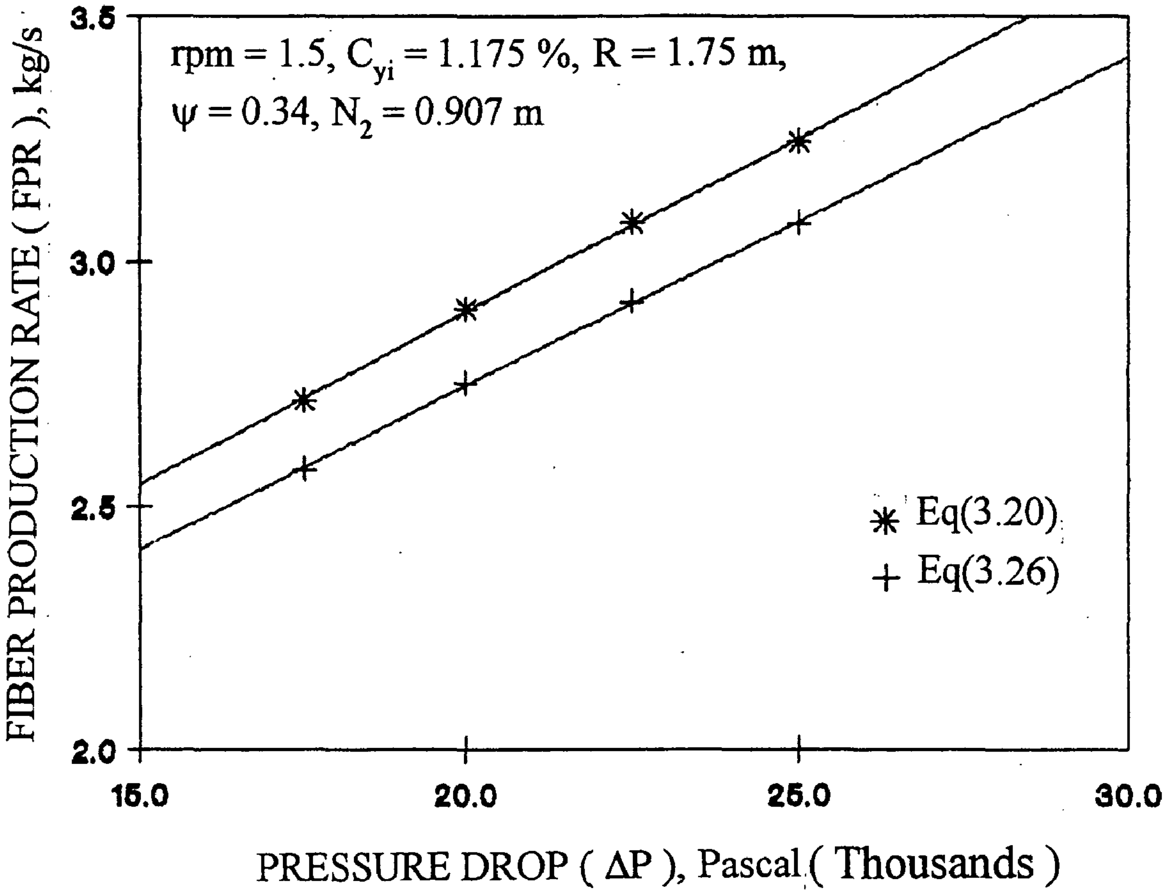

Figure 23 : Effect of pressure drop on fiber production rate

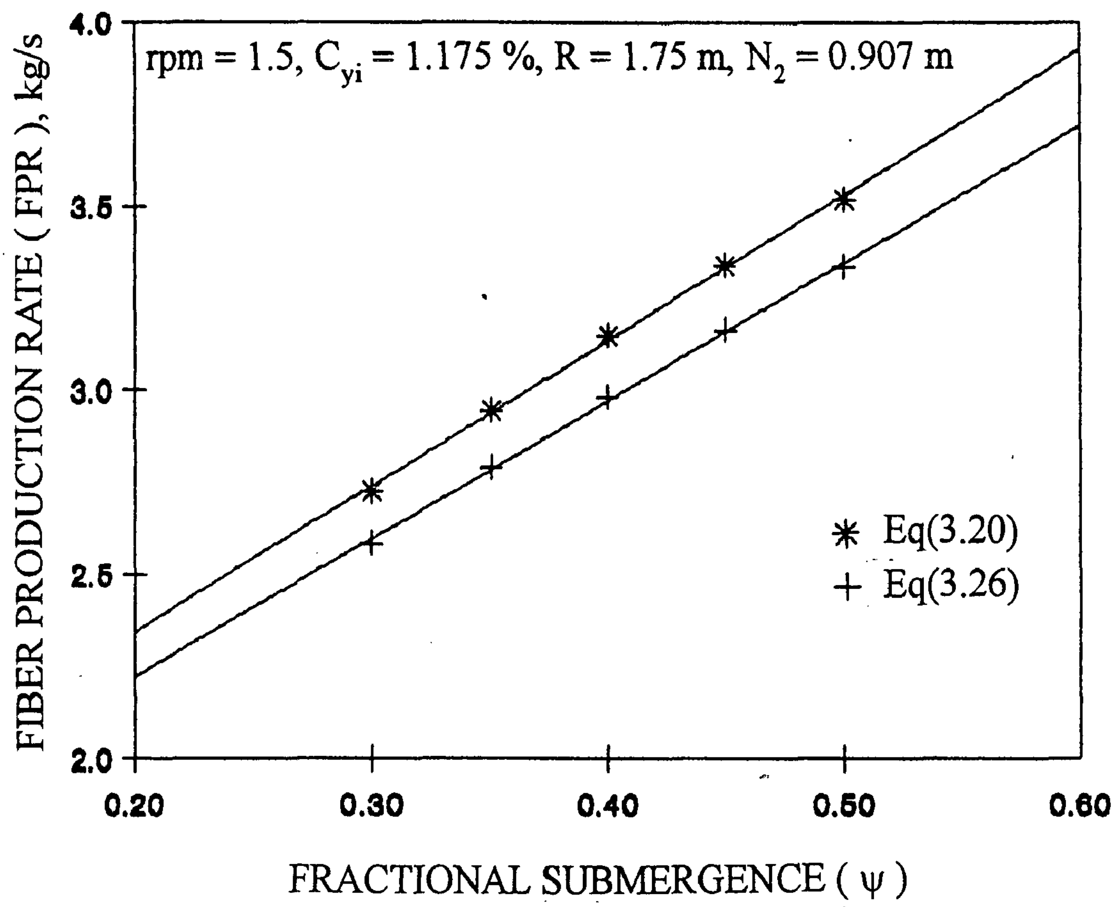

Figure 24 : Effect of fractional submergence on fiber production rate

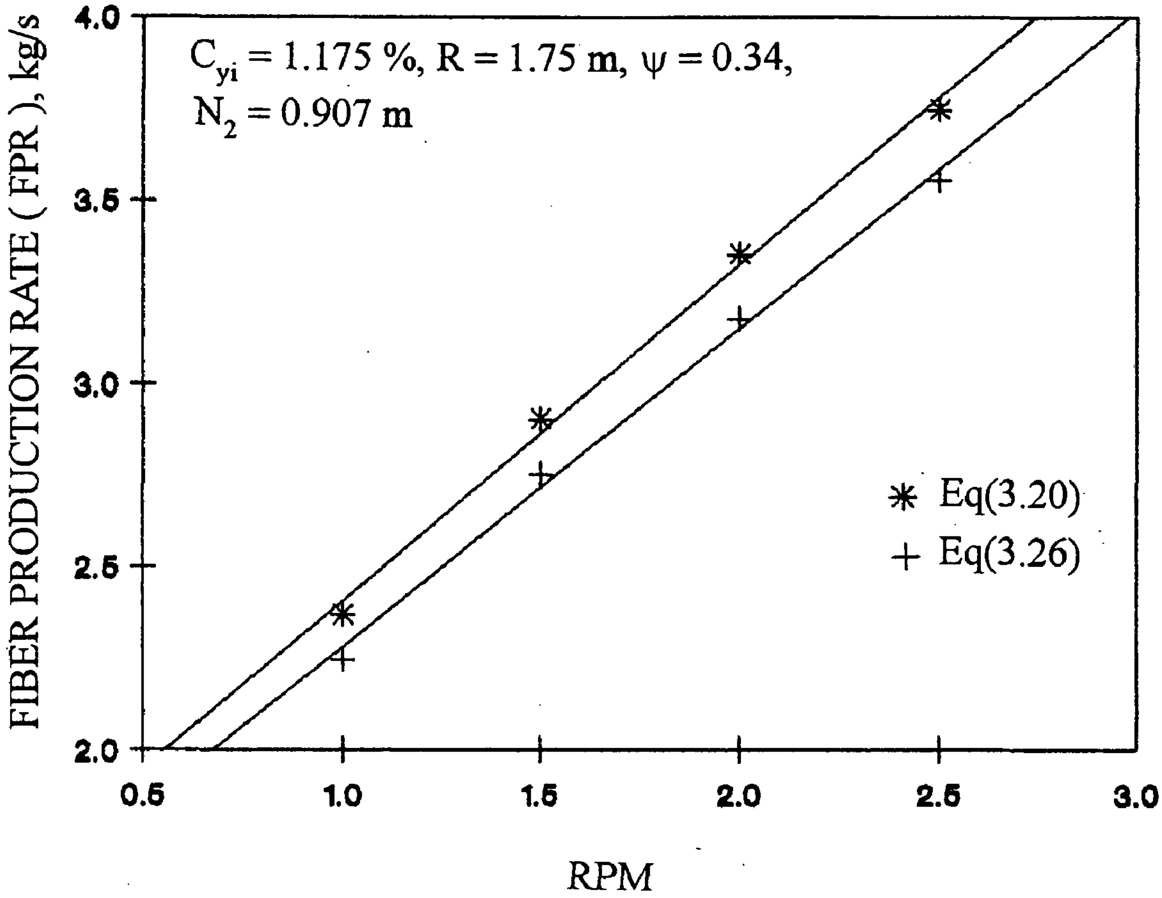

Figure 25 : Effect of rpm on fiber production rate

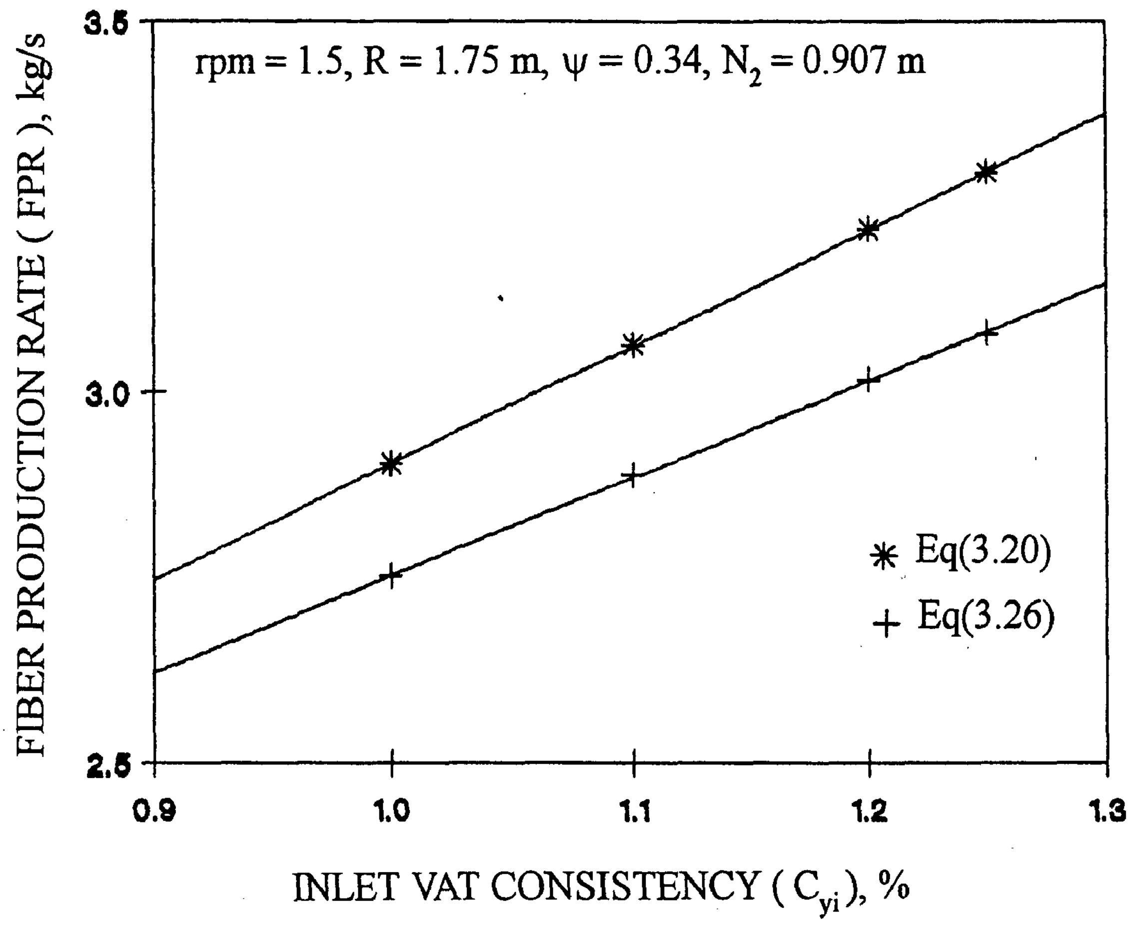

Figure 26 : Effect of inlet vat consistency on fiber production rate

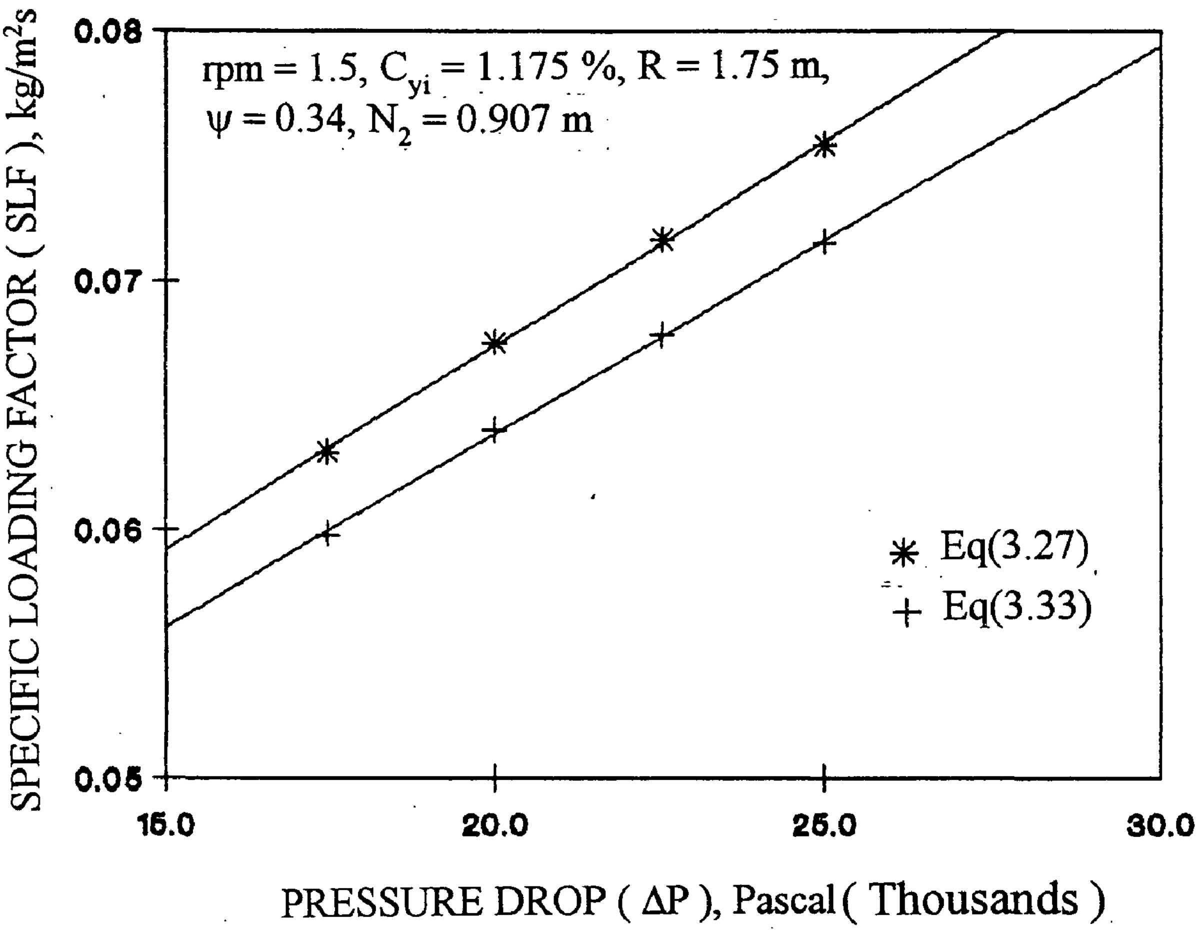

Figure 27 : Effect of pressure drop on specific loading factor

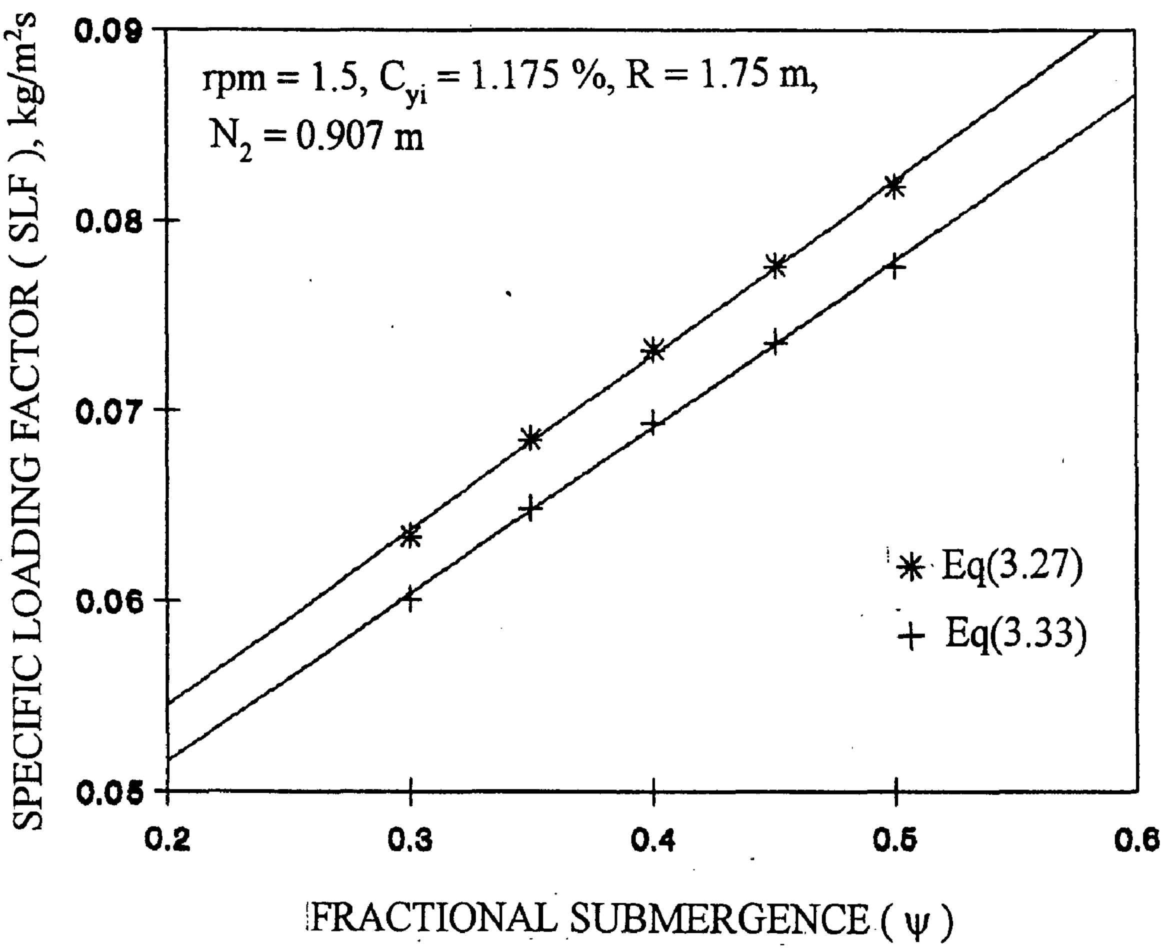

Figure 28 : Effect of fractional submergence on specific loading factor

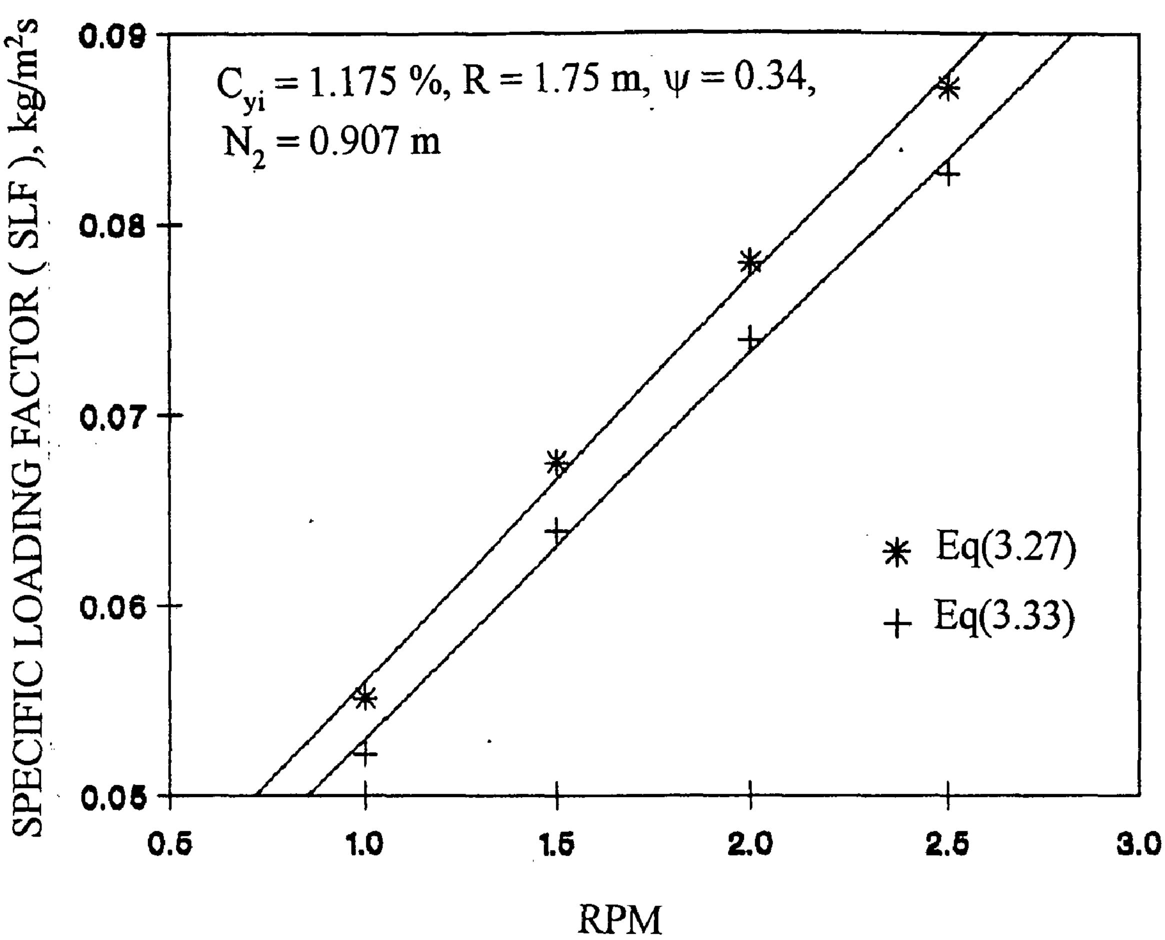

Figure 29 : Effect of rpm on specific loading factor

In **Chapter 4** the mathematical models developed in **Chapter 3** were validated with the existing data from industry, reported data from literature and the parallel models developed by various earlier investigators.

However, it is more apppropriate if the models are validated through the experimental data evaluated in the laboratory. But, laboratory prototype model of a 3 to 4 stage brown stock washer interconnecting the feed tank, vacuum pump/ barometric leg, recycle pipes, control and instrumentation are rarely met with success. Their availability is also a question. Instead, experimetation with laboratory model single stage brown stock washer is more common.

Proposed model is validated by collecting the data from a single stage laboratory scale EIMCO-KCP make rotary vacuum drum filter. The pictorial diagram of the equipment is given in figure 33 and some relevant design parameters of the washer are given in **Table 15**. Some characteristics of unwashed kraft pulp, collected from the mill, are given in **Table 16**.

Table 15 : Design parameters of lab scale washer

Parameter	Unit	Value
Radius of drum	m	2.346×10^{-2}
Width of drum	m	3.048×10^{-2}
Length of different zones		
Formation + Dewatering	m	6.000×10^{-2}
Washing	m	3.000×10^{-2}
Drying	m	2.500×10^{-2}
Discharge + Dead	m	3.246×10^{-2}

Table 16 : Characteristics of mill pulp

Composition of pulp	
Eucalyptus 81 %, Bamboo 17 %, Pine 2 %	
Kappa number	20
pH	12
Temperature of pulp	395 K
Active alkali (as Na_2O)	14.5 %

5.1 Experimental Procedure

About 25-30 kg of blown unwashed pulp, at 348 K, collected from the outlet of the blow tank is at first cooled to room temperature. This was then taken in a feed tank (provided with agitator) and diluted with water or weak wash liquor to different consistencies to carry out the experiments. Cake was washed by using fresh water. The other process and design parameters and their ranges for the filter operation are depicted in **Table 17**.

Rpm of the drum was controlled with the help of a knob. Fractional submergence of the drum was varied by placing the plates in the pipe carrying the overflow of pulp. The flow of wash water was controlled and measured by the rotameters attached to the wash line.

There was no scope for changing the vacuum pump capacity and as a result it remained constant throughout the experiment. The change of pressure drop has been due to change in other parameters like, degree of submergence, cake thickness, rpm and consistency of pulp within narrow ranges. Thus the pressure drop varied independently in each experiment and the data were noted down from the vacuum gauge mounted with the vacuum filter. Moist cake thickness (wet condition) was measured during each experiment.

Table 17 : Range of different parameters

Parameter	Unit	Range
Inlet vat consistency	%	0.53 - 1.99
rpm	-	0.60 - 1.09
Fractional submergence	-	0.22 - 0.28
Amount of wash water	lph	60 - 100
Pressure drop	Pascal	3999 - 10664
Cake thickness	m	0.0045 - 0.0070

The solid content, density, viscosity and surface tension of the black liquor were determined in the laboratory. Some of the parameters of the pulp samples like consistency, density were also evaluated. These are shown in **Table 18**.

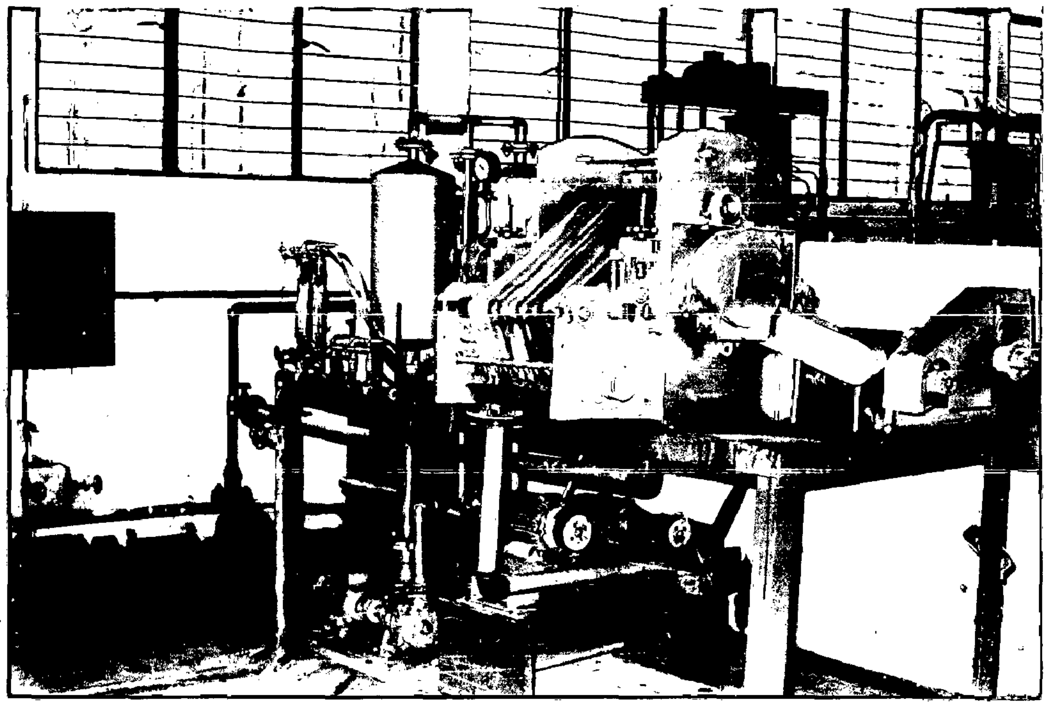

Figure 33: **EIMCO-KCP**
laboratory drum washer

Table 18 : Value of different parameters

Parameter	Unit	Value
Viscosity of black liquor	kg/ms	1.00 X 10^{-3}
Surface tension of black liquor	kg/m^3	3.30 X 10^{-2}
Density of fibers	kg/m^3	1.56 X 10^4
Mass transfer coefficients		
k_1 (same as in **Chapter 4**)	1/s	7.00 X 10^{-3}
k_2 (same as in **Chapter 4**)	1/s	2.00 X 10^{-3}
k^* (same as in **Chapter 4**)	1/s	3.50 X 10^{-3}
k (same as in **Chapter 4**)	-	3.50
Ratio u/D_L	1/cm	1.30

Some input data from a few experiments are given in **Table 19**. Data of some parameters measured during the experiments is given in **Table 20**. With the help of input and output data certain performance parameters like, dilution factor, wash ratio, displacement ratio, % efficiency, Norden's efficiency factor, modified Norden efficiency factor and equivalent displacement ratio are calculated for each experiment and are reported in **Table 21**.

Table 19 : Input data of some parameters in the experiments

SN	C_{yi} %	rpm	WW lph	ψ	C_i kg/m^3	L_i kg/kg	x_i %	L_s kg/kg
1	0.53	0.833	70	0.28	77.30	187.18	7.64	28.01
2	0.76	0.779	60	0.28	94.27	129.82	9.21	19.78
3	1.02	0.882	80	0.22	125.31	97.54	12.08	19.17
4	1.02	1.000	100	0.28	125.31	97.54	12.08	14.83
5	1.02	0.800	70	0.22	125.31	97.54	12.08	16.07
6	1.23	0.750	80	0.22	133.57	80.04	12.72	16.20
7	1.23	0.750	80	0.28	133.57	80.04	12.72	13.35
8	1.23	1.091	100	0.28	133.57	80.04	12.72	13.68
9	1.52	1.091	100	0.22	138.94	64.67	13.03	14.57
10	1.52	0.600	60	0.22	138.94	64.67	13.03	13.89
11	1.52	0.750	80	0.28	138.94	64.67	13.03	12.64
12	1.75	0.750	100	0.22	143.04	56.03	13.21	12.61
13	1.75	0.706	80	0.22	143.04	56.03	13.21	12.87
14	1.99	0.750	80	0.22	158.95	49.14	14.50	11.26
15	1.99	0.750	100	0.22	158.95	49.14	14.50	9.82

Table 20 : Output data of some parameters in the experiments

SN	C_{yd} %	L m	ΔP Pascal	ε_t	C_d kg/m^3	x_d %	L_i kg/kg	V_f x10^5 m^3/s
1	3.86	0.0050	5332	0.986	18.36	1.83	24.89	9.92
2	5.54	0.0045	3999	0.979	22.51	2.22	17.06	8.37
3	5.87	0.0050	6665	0.977	28.34	2.78	16.04	8.07
4	7.13	0.0060	9331	0.973	27.44	2.70	13.03	13.89
5	6.88	0.0050	6665	0.974	24.04	2.36	13.53	8.88
6	7.43	0.0055	6665	0.972	30.46	2.98	12.45	7.97
7	8.38	0.0060	7998	0.968	32.83	3.20	10.94	10.06
8	8.43	0.0050	7998	0.968	33.69	3.28	10.86	12.29
9	8.57	0.0045	5332	0.967	35.48	3.43	10.67	8.77
10	8.86	0.0050	3999	0.966	34.14	3.31	10.29	5.59
11	10.57	0.0050	5332	0.960	34.93	3.38	8.46	8.67
12	10.87	0.0060	7998	0.958	29.05	2.82	8.20	9.11
13	10.86	0.0050	5332	0.958	29.56	2.87	8.21	7.14
14	11.54	0.0050	5332	0.949	38.97	3.74	7.66	7.02
15	11.85	0.0070	10664	0.954	34.97	3.36	7.44	10.15

Table 21 : Some efficiency parameters

SN	FPR x10^4 kg/s	DF kg/kg	WR	DR	EDR	% E %	NEF	MNEF
1	6.94	3.11	1.13	0.76	0.22	76.76	3.4	1.1
2	8.42	2.72	1.16	0.76	0.45	76.34	3.3	1.5
3	11.59	3.14	1.19	0.77	0.50	77.68	3.3	1.6
4	18.73	1.80	1.14	0.78	0.60	78.68	3.6	2.1
5	12.10	2.54	1.18	0.80	0.64	81.27	3.8	2.2
6	13.71	3.75	1.30	0.77	0.60	77.56	3.0	1.9
7	16.65	2.41	1.22	0.75	0.62	76.06	3.0	2.1
8	20.30	2.83	1.26	0.74	0.61	75.44	2.9	2.0
9	19.07	3.90	1.36	0.74	0.60	74.72	2.6	1.9
10	11.99	3.60	1.35	0.75	0.63	75.75	2.7	2.0
11	17.58	4.18	1.49	0.74	0.69	75.66	2.5	2.2
12	22.03	4.40	1.54	0.79	0.75	80.05	2.7	2.5
13	17.27	4.66	1.57	0.78	0.74	79.70	2.7	2.5
14	19.74	3.60	1.47	0.74	0.72	75.74	2.5	2.4
15	28.28	2.38	1.32	0.77	0.76	78.33	2.9	2.2

5.2 Results and Discussion

With the help of data given in **Table 19** and **20**, interrelationship between various parameters like fractional submergence, rpm, filtrate flow rate, cake thickness, inlet vat consistency and fiber production rate are discussed. The results are plotted in figures 34 to 46.

5.2.1 Effect of ψ on ΔP

From figure 34 the increase in fractional submergence(ψ) increases the observed pressure drop(ΔP) linearly at the given consistency. This trend is similar to the proposed model (figure 9). Though the experimental range is of much lower values than the one shown in figure 9. Increase in inlet vat consistency results in increased pressure drop at a given fractional submergence, indicating thicker mat formation which is again as per the model predictions.

5.2.2 Effect of C_{yi} on L

The increased inlet vat consistency(C_{yi}) gives increased cake thickness(L) as observed from figure 35. This is again as per the predictions of the model given in figure 11. The increased submergence has not been able to give any clear trend on the cake thickness increase essentially due to the variation of rotational speed of drum(rpm) on one hand and variation in wash water flow rate on the other.

5.2.3 Effect of ΔP on L

The cake thickness(L) increases with the increase in pressure drop(ΔP) at a given fractional submergence as shown in figure 36. In the present experiments both cake thickness and pressure drop are measured variables for any given setting of inlet vat consistency, rpm and fractional submergence. It is observed that all the points of cake thickness and pressure drop fall on the same straight line and this is again an expected trend from the model given by figure 20.

5.2.4 Effect of ψ on L

From figure 37, it is observed that the increase in fractional submergence(ψ) results in increased cake thickness(L), a trend similar to the one proposed by the model figure 13. Further increased inlet vat consistency results in increased cake thickness at any given fractional submergence. The variation obtained in case of 1.23% inlet vat consistency is essentially due to variation in rpm.

5.2.5 Effect of ΔP and ψ on V_f

From figure 38, the increased pressure drop(ΔP) results in increased flow rate of filtrate(V_f). An increase in inlet vat consistency at a given pressure drop results in increased filtrate flow rate. Figure 39 shows that the increase in fractional submergence (ψ) results in increased flow rate at a given inlet vat consistency and increase in filtrate flow rate for a given fractional submergence at lower consistency. These trends are similar to those predicted by model figures 16 and 17 resectively.

5.2.6 Effect of rpm on V_f

Figure 40 shows the variation in filtrate flow rate(V_f) as a function of rpm of drum at constant cake thickness. The results indicate that at higher rpm greater filtrate flow takes place. A trend which is similar to the one predicted by the model figure 18.

5.2.7 Effect of C_{yi} on V_f

Increased inlet vat consistency(C_{yi}) at a given cake thickness results in decreased filtrate flow rate(V_f) as seen from figure 41. A trend similar to the one predicted by model as shown in figure 19.

5.2.8 Effect of L on V_f

Increased cake thickness(L) shows reduced filtrate flow rate(V_f) in the experimental runs(figure 42). Trends are similar to the one predicted by the model figure 21. As observed from figure 42, the variation of pressure drop, or variation of fractional submergence or variation of rpm will have influence on filtrate flow rate at a given cake thickness.

5.2.9 Effect of ΔP on FPR

Fiber production rate(FPR) increases with the increase in pressure drop(ΔP) as shown in figure 43. The effect of rpm has not been considered,as its effect is assumed to have been included in pressure drop. Fiber production rate is very sensitive to inlet vat consistency where as it is comparatively less sensitive to fractional submergence and least sensitive to rpm in the range studied. These trends are similar to the predictions made by earlier model in figure 23.

5.2.10 Effect of ψ on FPR

Figure 44 shows the variation of fiber production rate(FPR) as a function of fractional submergence(ψ). Higher fractional submergence has resulted in higher fiber production rate for any given inlet vat consistency and rpm. The inlet vat consistency has greater influence on fiber production rate compared to rpm. These trends are similar to those predicted by model figure 24.

5.2.11 Effect of rpm on FPR

Fiber production rate(FPR) increases with the increase in rpm for a given inlet vat consistency and given submergence of filter as indicated by the trend in figure 45. Increase in submergence results in higher fiber production rate when other conditions are comparable. Similarly increased inlet vat consistency results in increased fiber production rate at same rpm when other conditions are identical.

5.2.12 Effect of C_{yi} on FPR

Increase in inlet vat consistency(C_{yi}) results in higher fiber production rate (FPR) at constant fractional submergence and rpm(figure 46). A trend well predicted by figure 26. Fiber production rate at a given inlet vat consistency is strongly dependent on fractional submergence and less dependent on rpm.

5.3 Comparison Between Experimental and Model Predicted Values

Value of certain efficiency parameters like displacement ratio, % efficiency, Norden's efficiency factor, modified Norden efficiency factor and equivalent displacement ratio has also been found out by using the washing models 1 to 8. The results obtained by using these models are plotted in figures 47 to 51. The model predicted results are compared with the experimentally obtained values for different tests. The results are discussed below :

5.3.1 Displacement ratio

In figure 47 the results obtained for displacement ratio(DR) from different tests are plotted. It is observed that models 3, 4 and 6 have followed the experimentally predicted results quite well. The agreement between the predicted and the experimentally obtained value is found to be within reasonable limits. The results from model 3 and 4 are identical. The % deviation between the models 3, 4 and experimental results ranges from 2.5 to 7.4%. It can also be concluded that DR of the laboratory washer lies between 0.70 to 0.80. However the models 1, 2, 5, 7, 8 gave over predictions (nearly 1.0 DR), which are impracticable.

5.3.2 % efficiency

For different tests the experimental and model predicted results of % efficiency are compared in figure 48. Models 3 and 4 give almost similar values. The nature of the curves in models 3, 4 and 6 is the same. The minimum % deviation between experimental results and model 3, 4 is 2.3 % and the maximum % deviation is 7 %. The remaining models gave approximately 100 % efficiency which is again not possible in a single stage brown stock washer and even if it is carried out with unlimited number of stages in a multistage washer. % efficiency of the lab washer can be anticipated near about 75 %.

5.3.3 Norden's efficiency factor

Norden's efficiency factor(NEF) values are compared in figure 49 for different models with the experimentally obtained data. The NEF values of the models 1,2,5, 7 and 8 for different tests are very high and are not plotted.There is again agreement between the results predicted by the experiments and models 3, 4 and 6. The % deviation between the results of experiments and model 3, 4 varied between 6% to 17 %. A value of 2.7 to 3 can be assigned as the NEF for the laboratory brown stock washer.

5.3.4 Modified Norden efficiency factor

In figure 50 the modified Norden efficiency factor(MNEF) values for different tests are compared. The results from models 3 and 4 are exhibiting almost same results, whereas the values from other models(1,2,5,7,8) varied widely. A % deviation of 5.8 to 17 % is observed between models 3, 4 and the experimental results. For the laboratory brown stock washer value of MNEF ranges between 2 to 2.5

5.3.5 Equivalent displacement ratio

Figure 51 gives the comparison between the experimental and the model predicted results of equivalent displacement ratio(EDR) for different tests. Model 3 and 4 followed the experimental results more closely than the model 6. In majority of cases the % deviation between the data from the model 3, 4 and experimental results is between 2.4 to26%. EDR of the laboratory washer comes out between 0.65 to 0.70.

5.4 A Sample Calculation of an Industrial Brown Stock Washer

A sample calculation is given for a system consisting of 4 rotary vacuum filters operating in countercurrent manner. For this purpose the actual mill data of a near by pulp and paper mill is used. The input data is given below.

(a) moisture content of pulp	=	10	% (i.e. m = 0.9)
(b) actual AA charged	=	15	% as Na_2O
(c) pulp yield (on OD basis)	=	47	%
(d) consistency of blown pulp	=	13	%
(e) solids in blown pulp	=	22	%
(f) liquor in blown pulp	=	6.69	kg of liquor/kg of pulp
(g) standard consistency	=	12	%
(h) dilution factor	=	3	kg/kg

Table 22 : Data of a 4 stage industrial brown stock washer

Input Parameters		Washer 1	Washer 2	Washer 3	Washer 4
C_{yi}	(%)	1.25	1.25	1.25	1.25
C_{yd}	(%)	12.00	12.00	13.00	14.00
x_i	(%)	15.59	7.29	2.62	0.96
x_s	(%)	7.00	2.50	0.90	0.00
x_f	(%)	15.00	7.00	2.50	0.90
x_d	(%)	10.12	3.77	1.58	0.30
x_r	(%)	15.00	7.00	2.50	0.90
L_i	(kg/kg)	79.00	79.00	79.00	79.00
L_s	(kg/kg)	10.33	10.33	9.69	9.14
L_f	(kg/kg)	82.00	82.00	82.00	82.00
L_d	(kg/kg)	7.33	7.33	6.69	6.14
L_r	(kg/kg)	72.31	71.67	71.67	72.31

Table 23 : Value of some efficiency parameters

Parameters	Washer 1	Washer 2	Washer 3	Washer 4	Eq. No.
DF	3.00	3.00	3.00	3.00	2.34
WR	1.41	1.41	1.45	1.49	2.35
W	1.04	1.04	1.04	1.04	2.39
FE	2.39	1.76	2.64	1.73	2.40
TF	0.91	0.91	0.92	0.92	2.41
Y	0.99	0.99	0.99	0.98	2.42
DR	0.64	0.73	0.61	0.69	2.44
SR	0.65	0.52	0.60	0.31	2.47
ST	0.06	0.05	0.05	0.02	2.50
	0.001	0.003	0.009	0.02	2.51
% E	96.63	97.53	96.65	97.56	2.55
	93.98	95.19	94.89	97.56	2.56
NEF	2.09	2.61	1.95	2.28	2.68
	2.09	2.61	1.95	2.26	2.70
MNEF	2.09	2.61	2.10	2.64	2.74
EDR	0.63	0.73	0.63	0.67	2.75

Overall solid reduction ratio	=	0.014	%	2.48
Solids going to evaporator	=	15.000	%	2.49
Actual soda loss	=	16.621	kg/TOD pulp	2.54
Overall % Efficiency of the sytem	=	98.747	%	2.57
	=	98.804	%	2.58
NEF of entire system	=	10.906		2.71
(using data of 2.68)	=	8.143		2.72
(using data of 2.70)	=	8.129		2.72
MNEF of entire system	=	9.444		

5.5 Conclusions

The models developed in **Chapter 3** have been validated by carrying out some experiments on a laboratory scale single stage EIMCO-KCP rotary vacuum washer. A mixture of wood and nonwood pulp collected from a paper mill was used for the trials. Experiments were performed by varying inlet vat consistency, rpm, fractional submergence of drum and amount of wash water.

Data generated from these experiments were processed in the laboratory to get the values of different conventional efficiency parameters like, dilution factor, wash ratio, displacement ratio, % efficiency, Norden's efficiency factor, modified Norden efficiency factor and equivalent displacement ratio. Also by using the models 1 to 8 values of some parameters like displacement ratio, % efficiency, Norden's efficiency factor, modified Norden efficiency factor and equivalent displacement ratio were obtained.

The interrelationship between different parameters like, fractional submergence, rpm, filtrate flow rate, cake thickness, inlet vat consistency and fiber production rate are discussed with the help of various graphs. From the figures 47 to 51 it is observed that the models 3 and 4 agrees the experimental results more closely than those from model 6. Rest of the models 1, 2, 5, 7 and 8 predicts widely varying results for the laboratory scale brown stock washer with much higher spread of the data. Therefore the results from these models are not plotted in the figures.

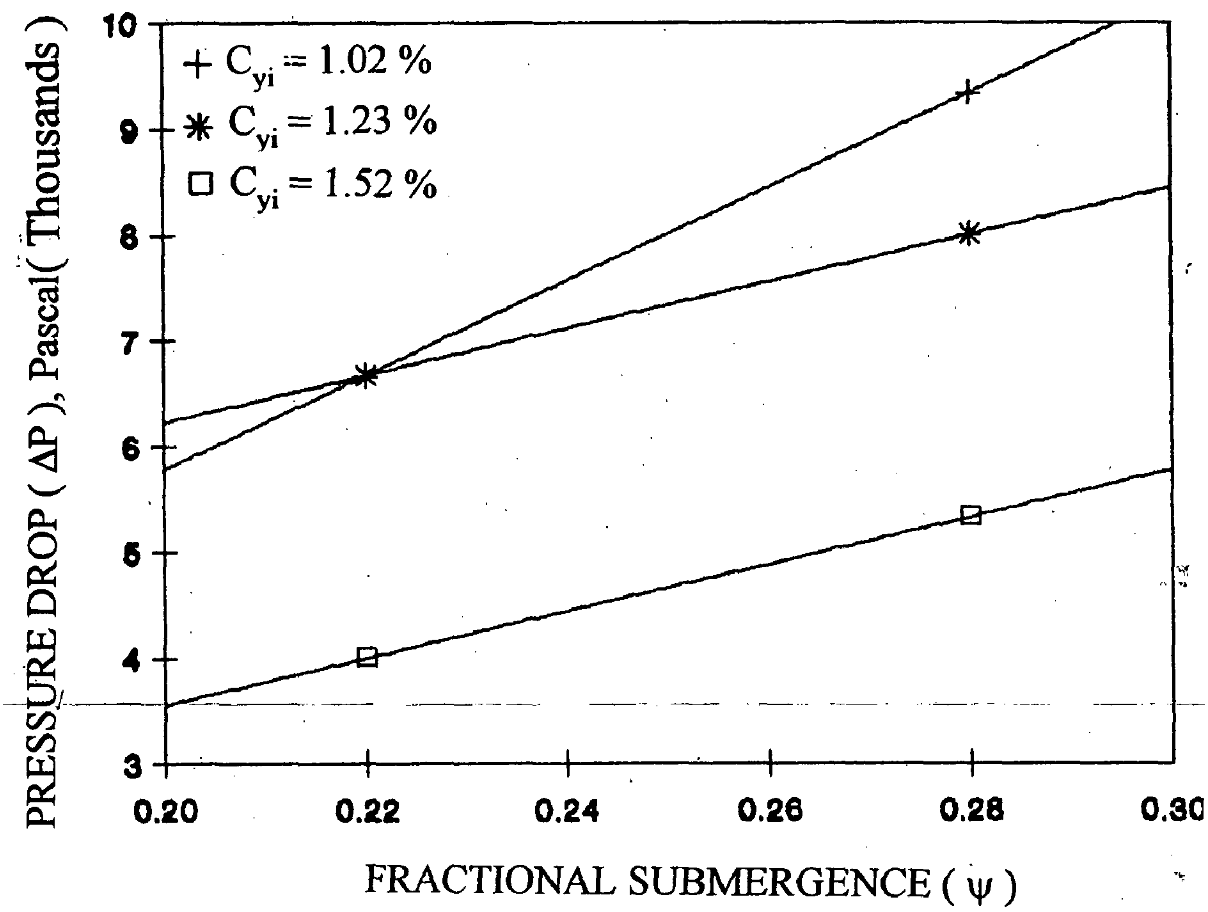

Figure 34 : Effect of fractional submergence on pressure drop

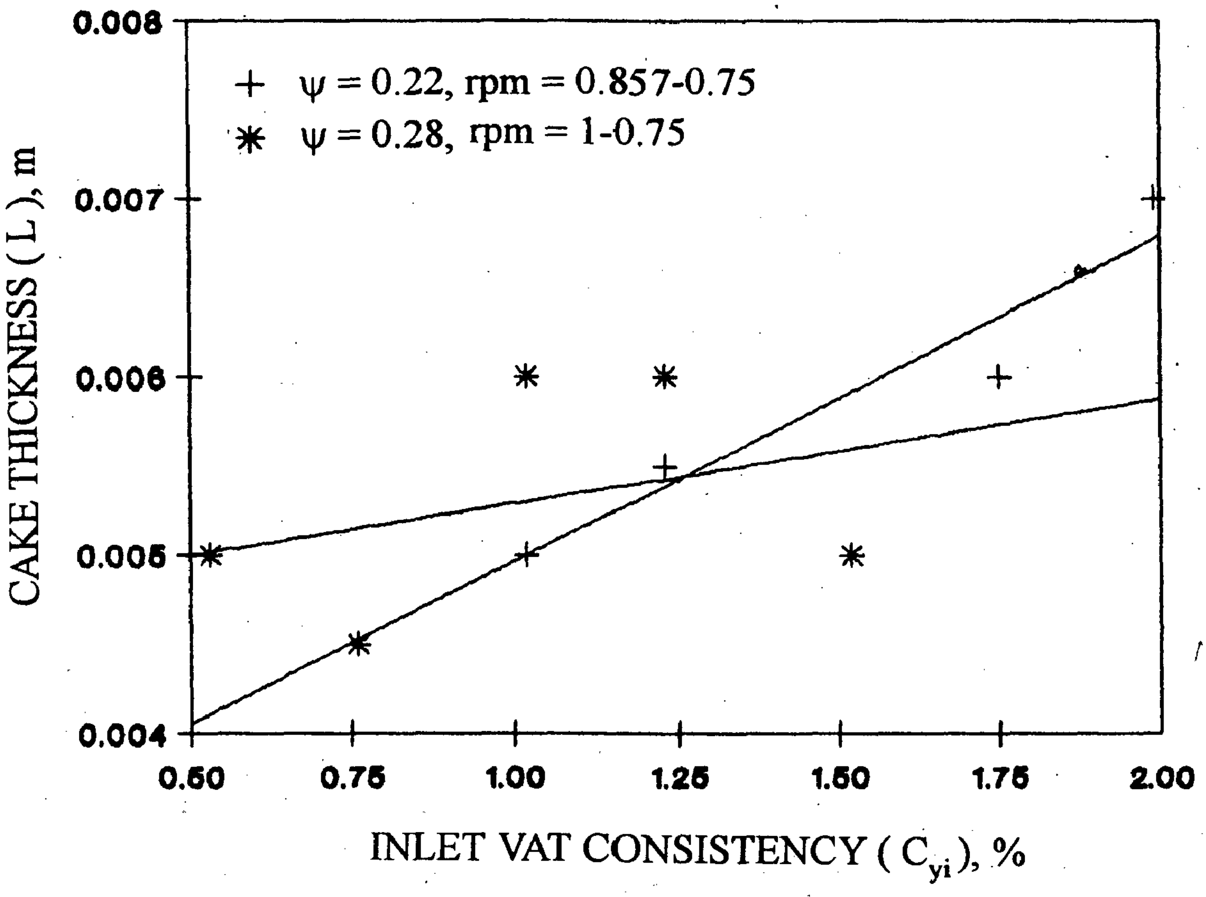

Figure 35 : Effect of inlet vat consistency on cake thickness

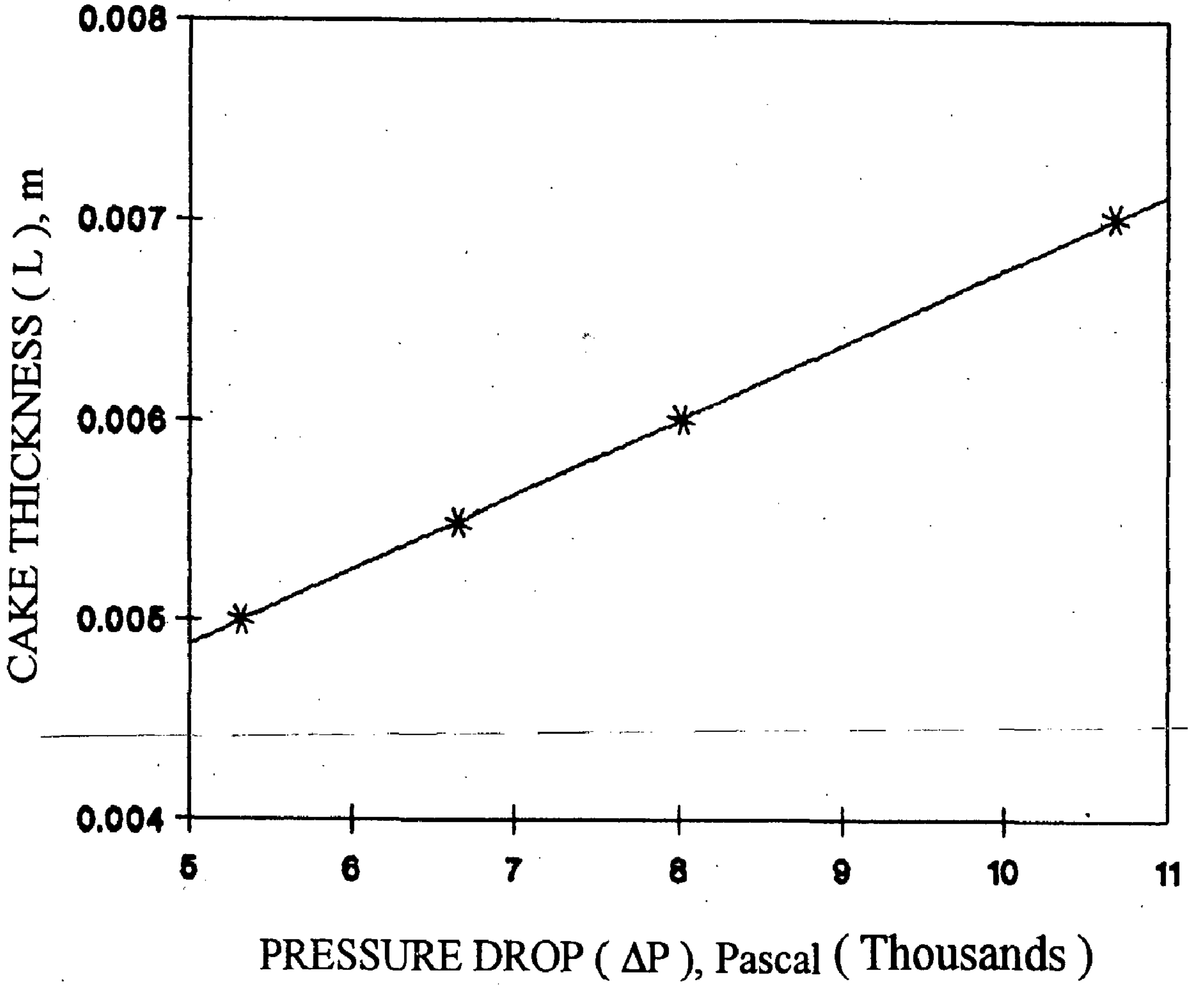

Figure 36 : Effect of pressure drop on cake thickness

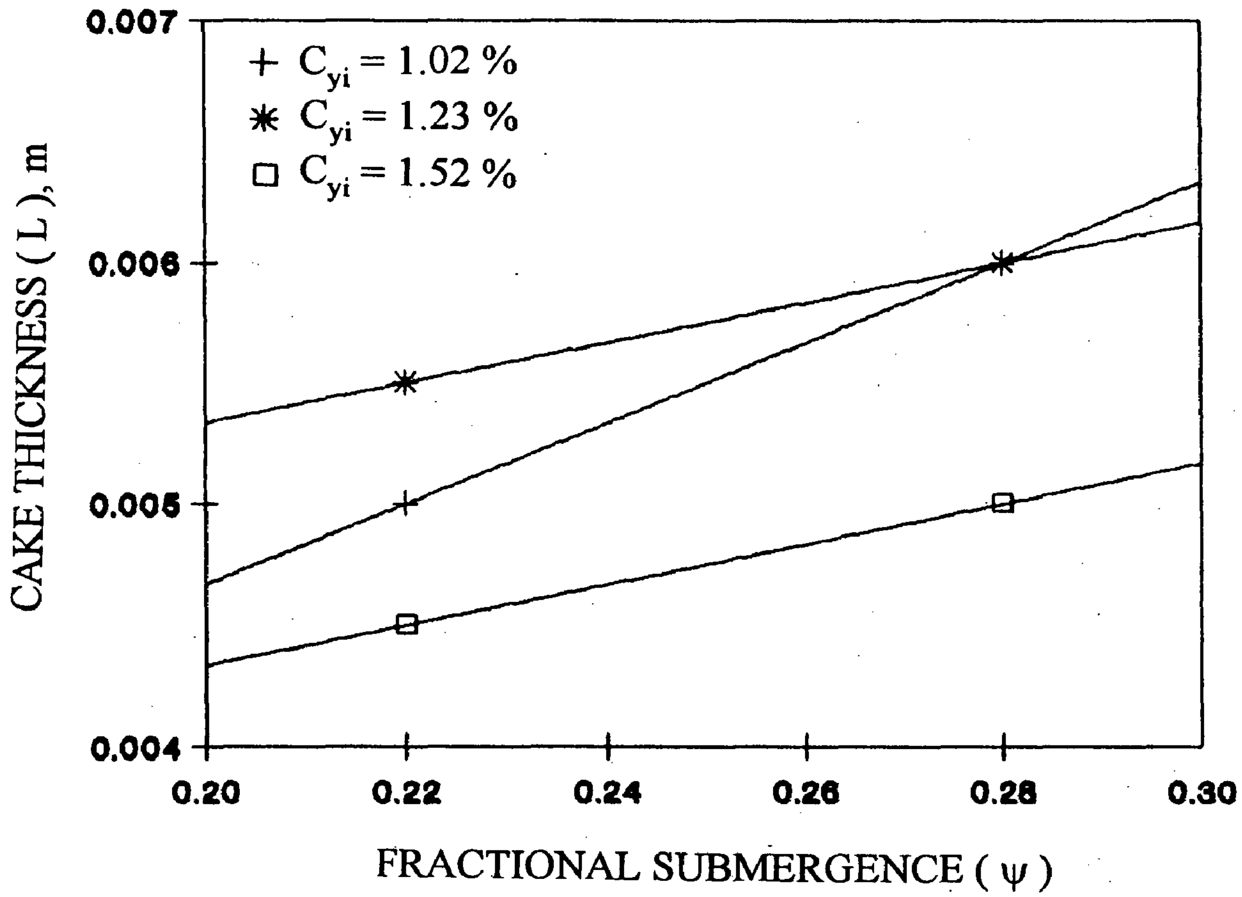

Figure 37 : Effect of fractional submergence on cake thickness

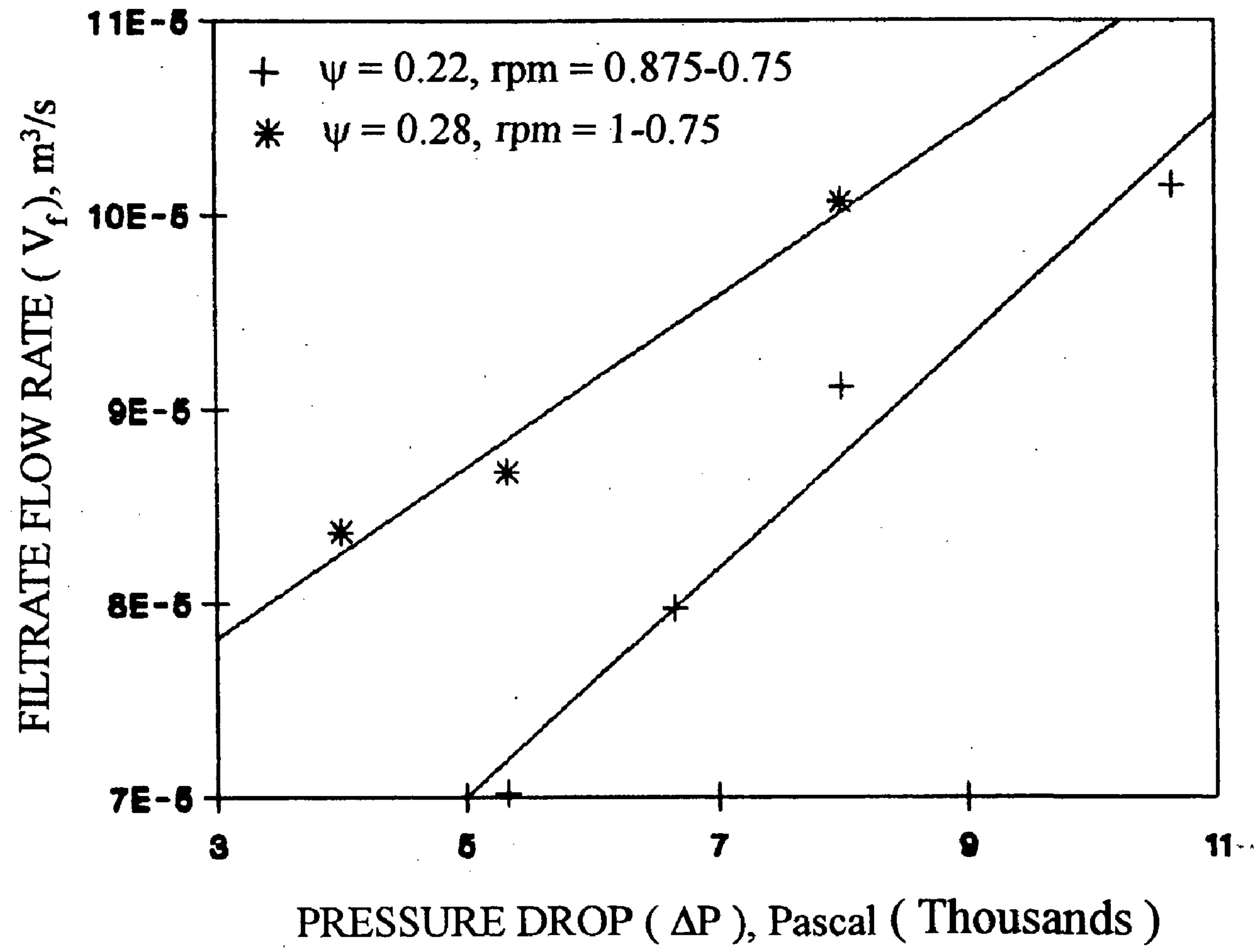

Figure 38 : Effect of pressure drop on filtrate flow rate

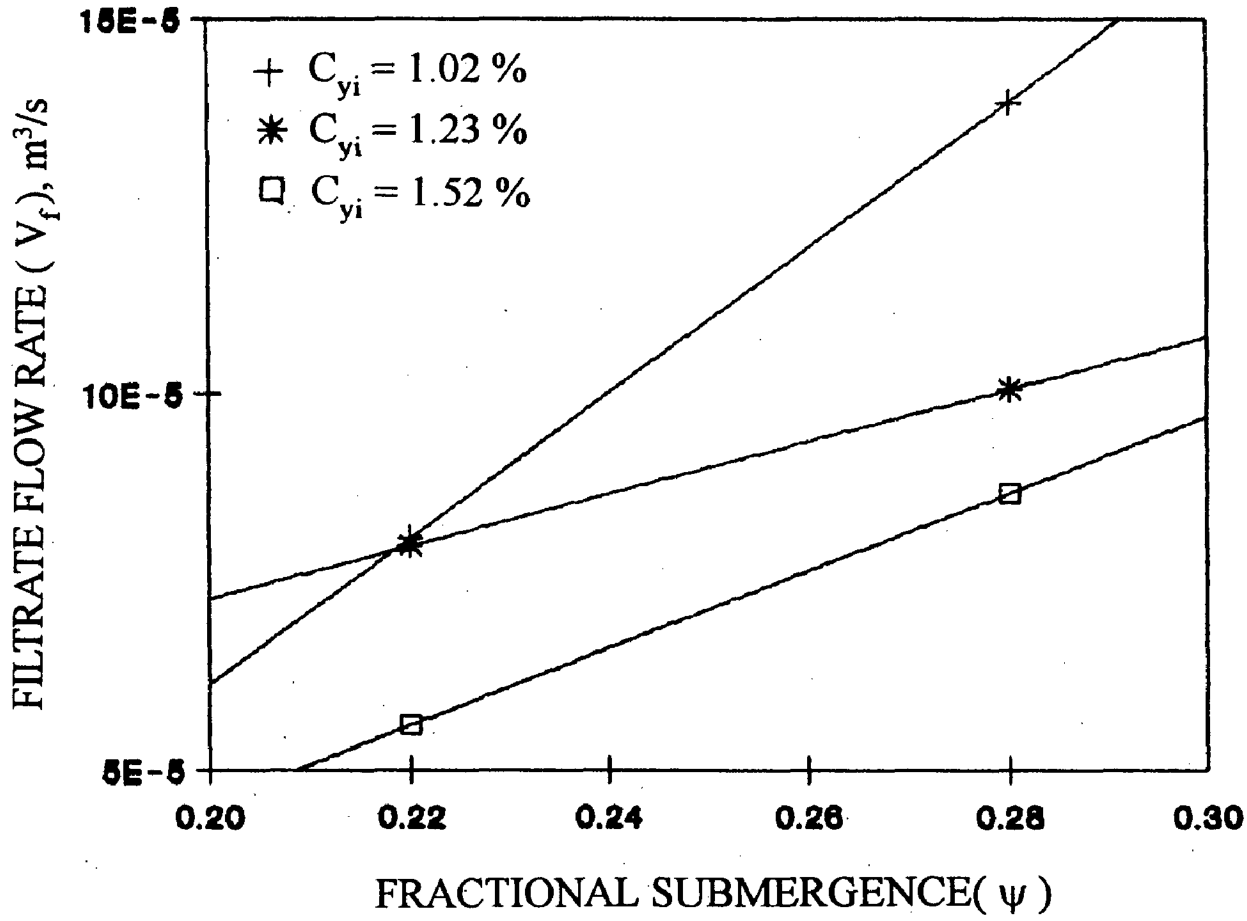

Figure 39 : Effect of fractional submergence on filtrate flow rate

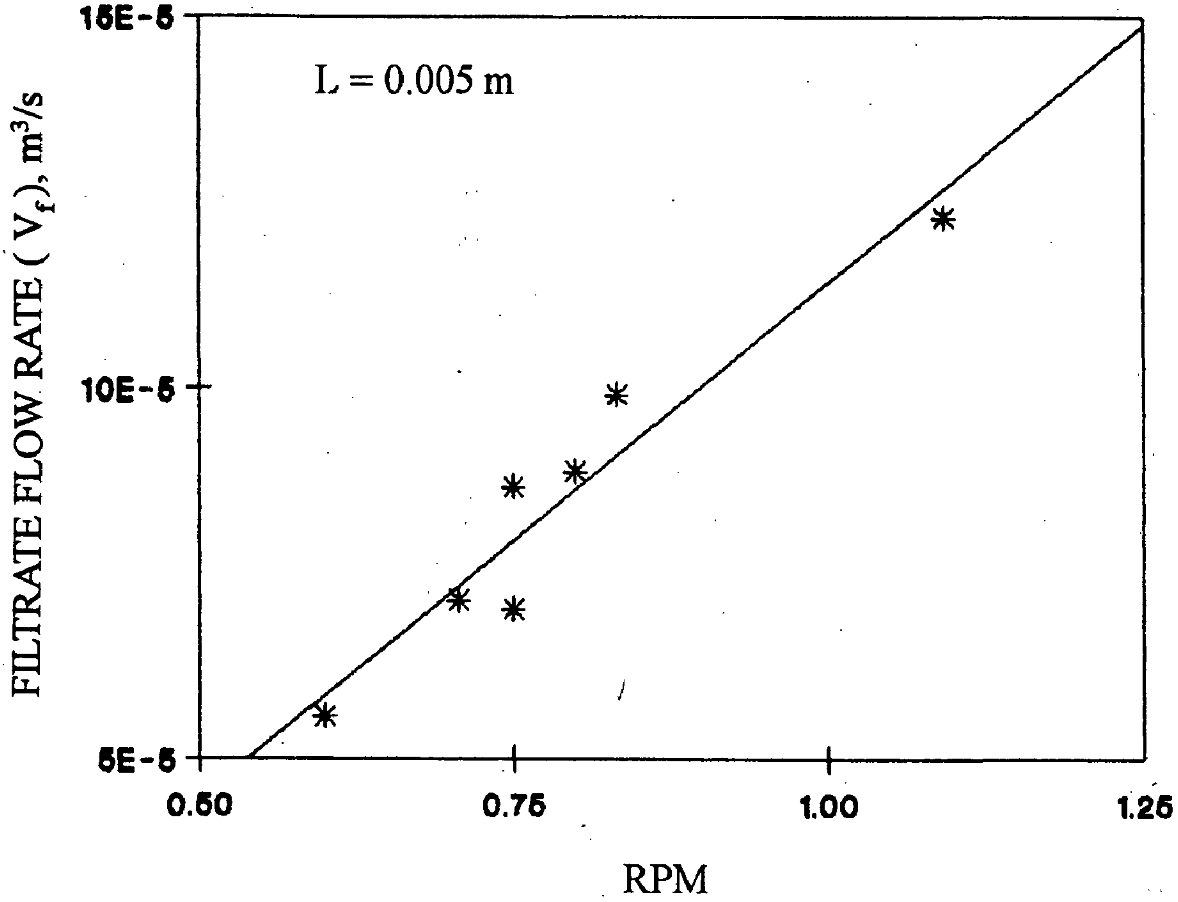

Figure 40 : Effect of rpm on filtrate flow rate

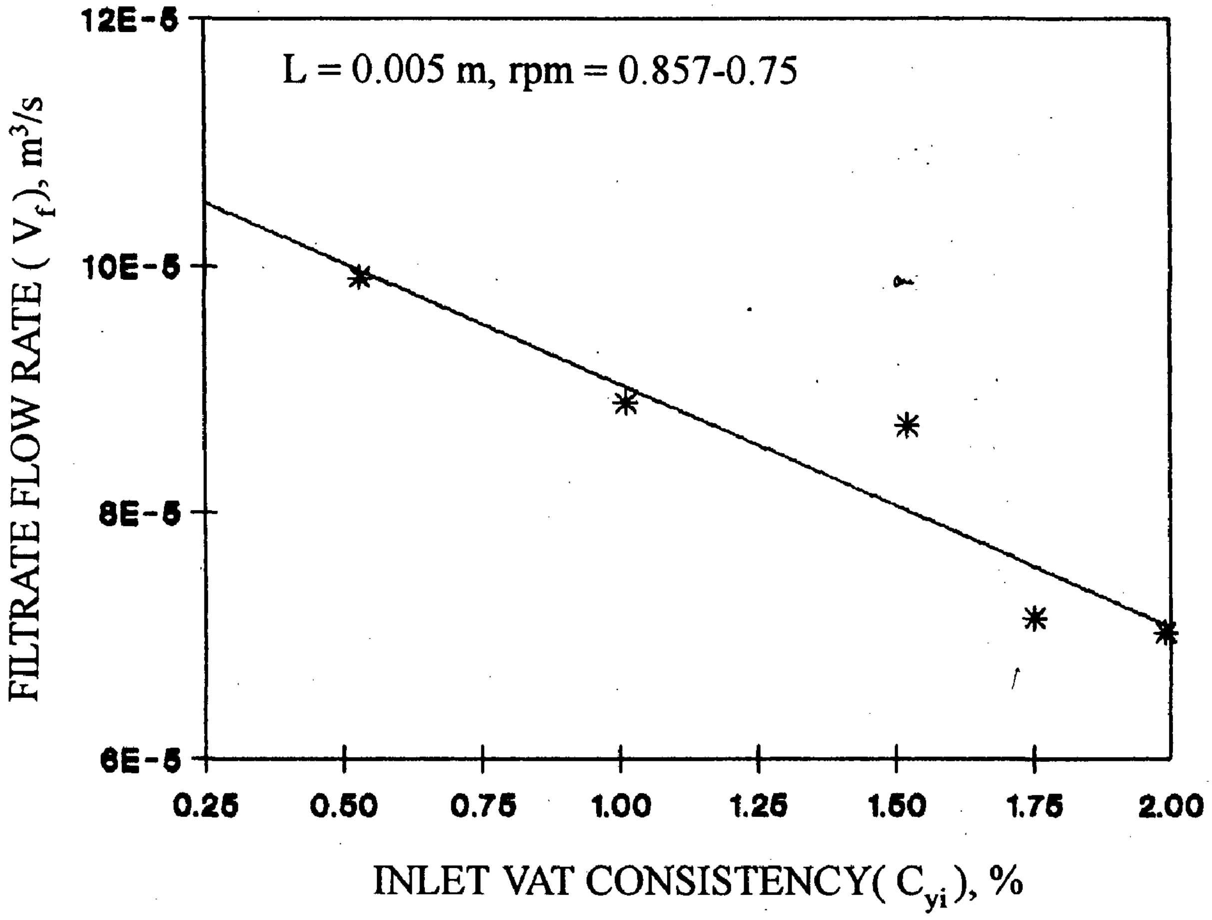

Figure 41 : Effect of inlet vat consistency on filtrate flow rate

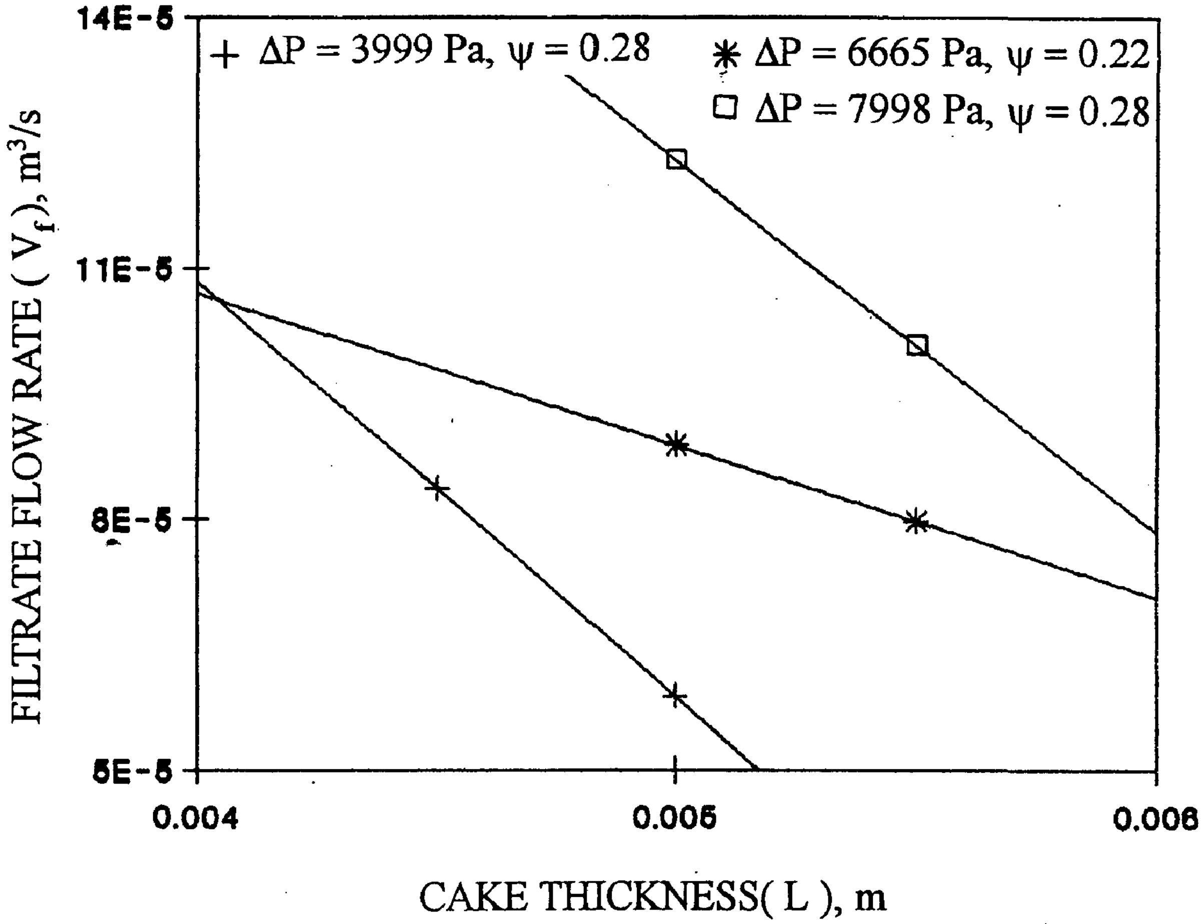

Figure 42 : Effect of cake thickness on filtrate flow rate

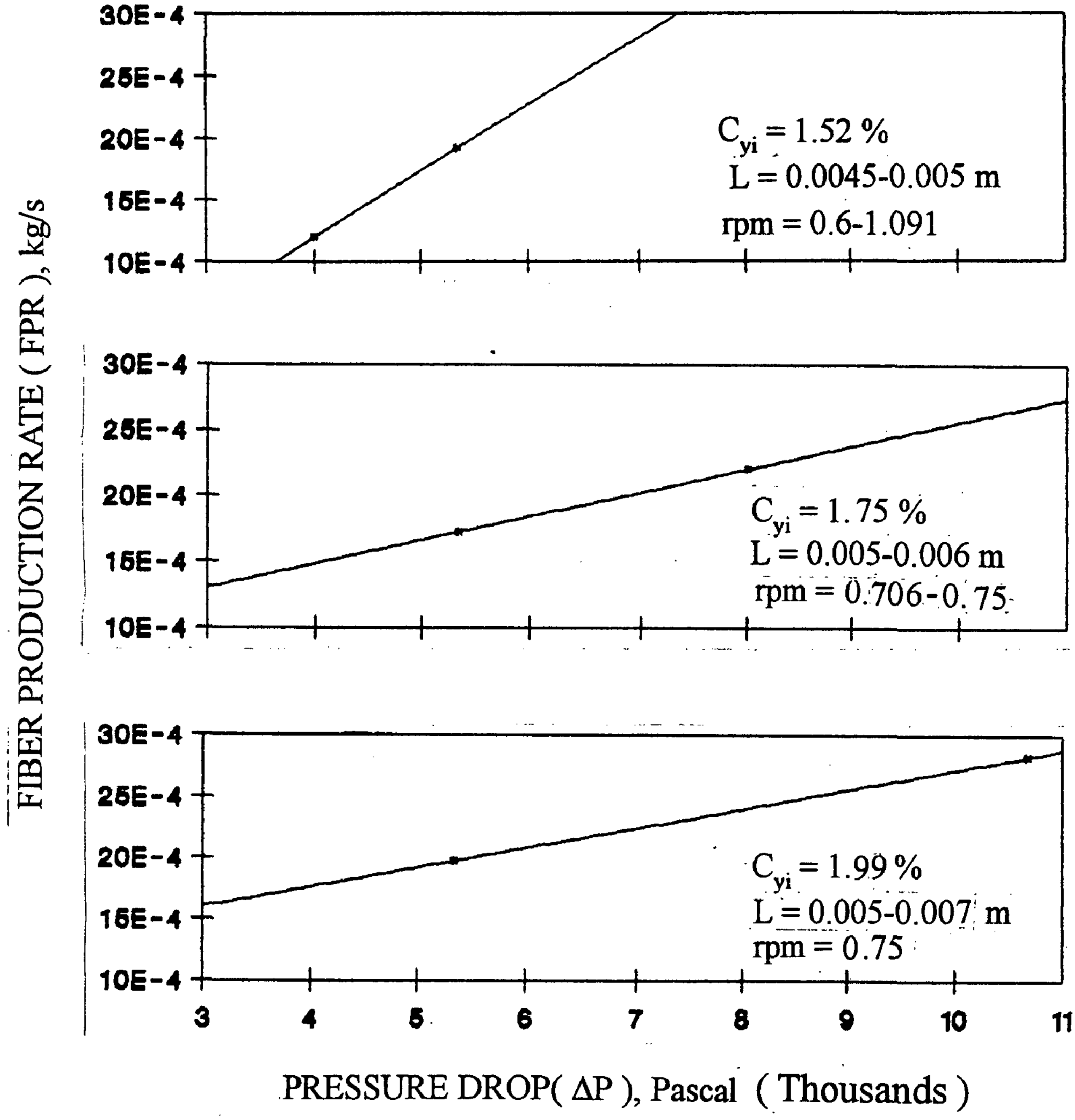

Figure 43 : Effect of pressure drop on fiber production rate

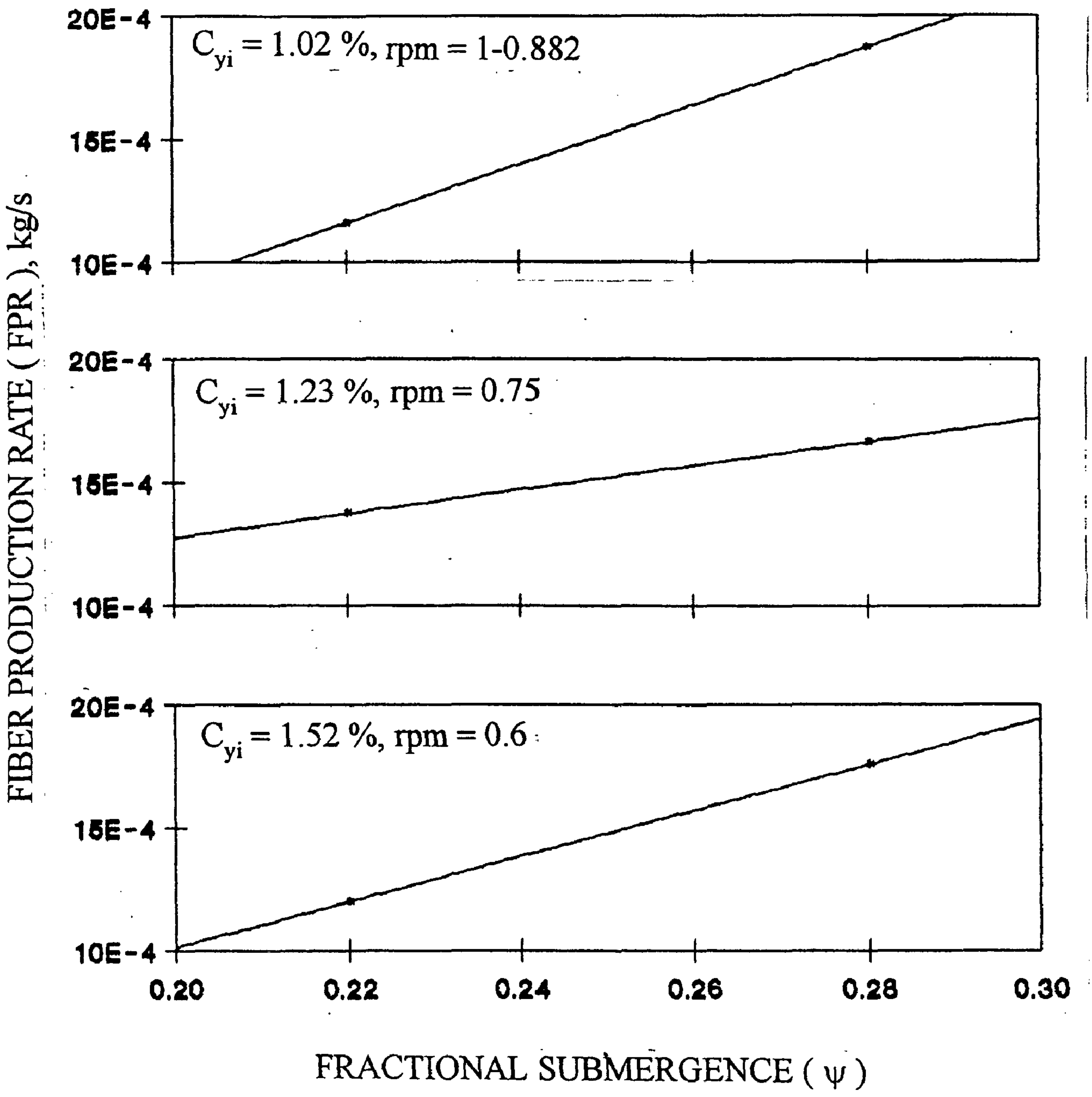

Figure 44 : Effect of fractional submergence on fiber production rate

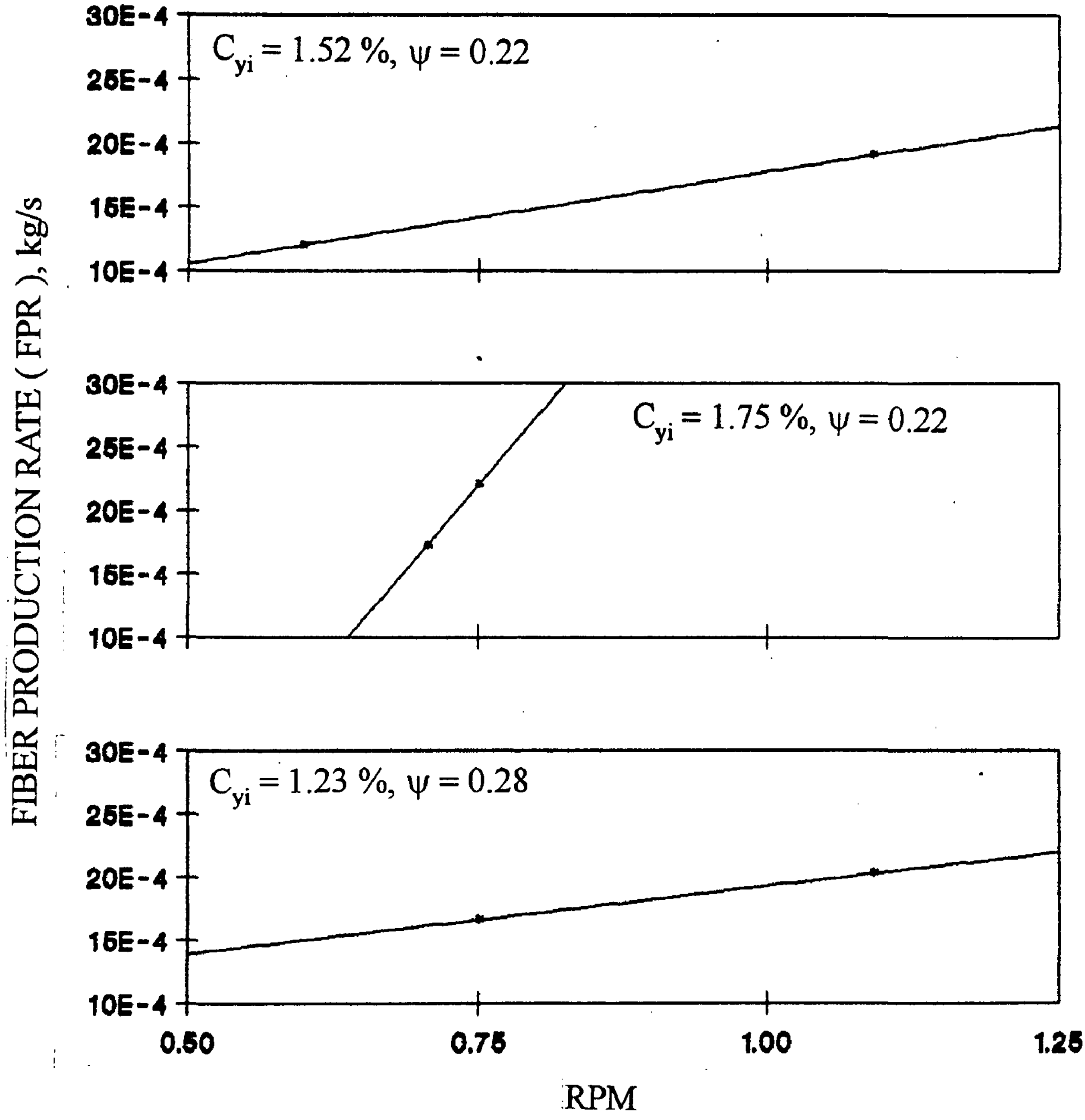

Figure 45 : Effect of rpm on fiber production rate

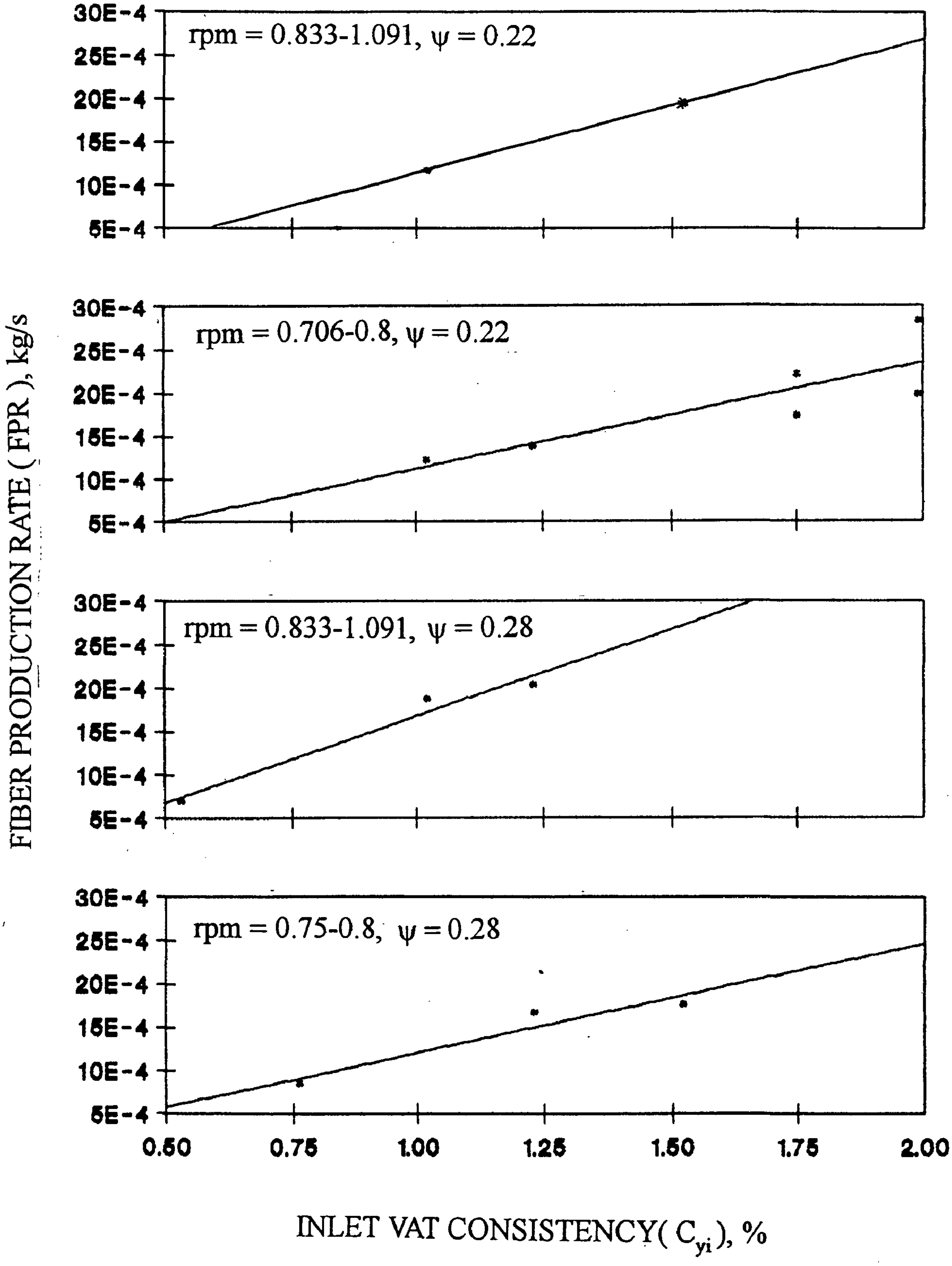

Figure 46 : Effect of inlet vat consistency on fiber production rate

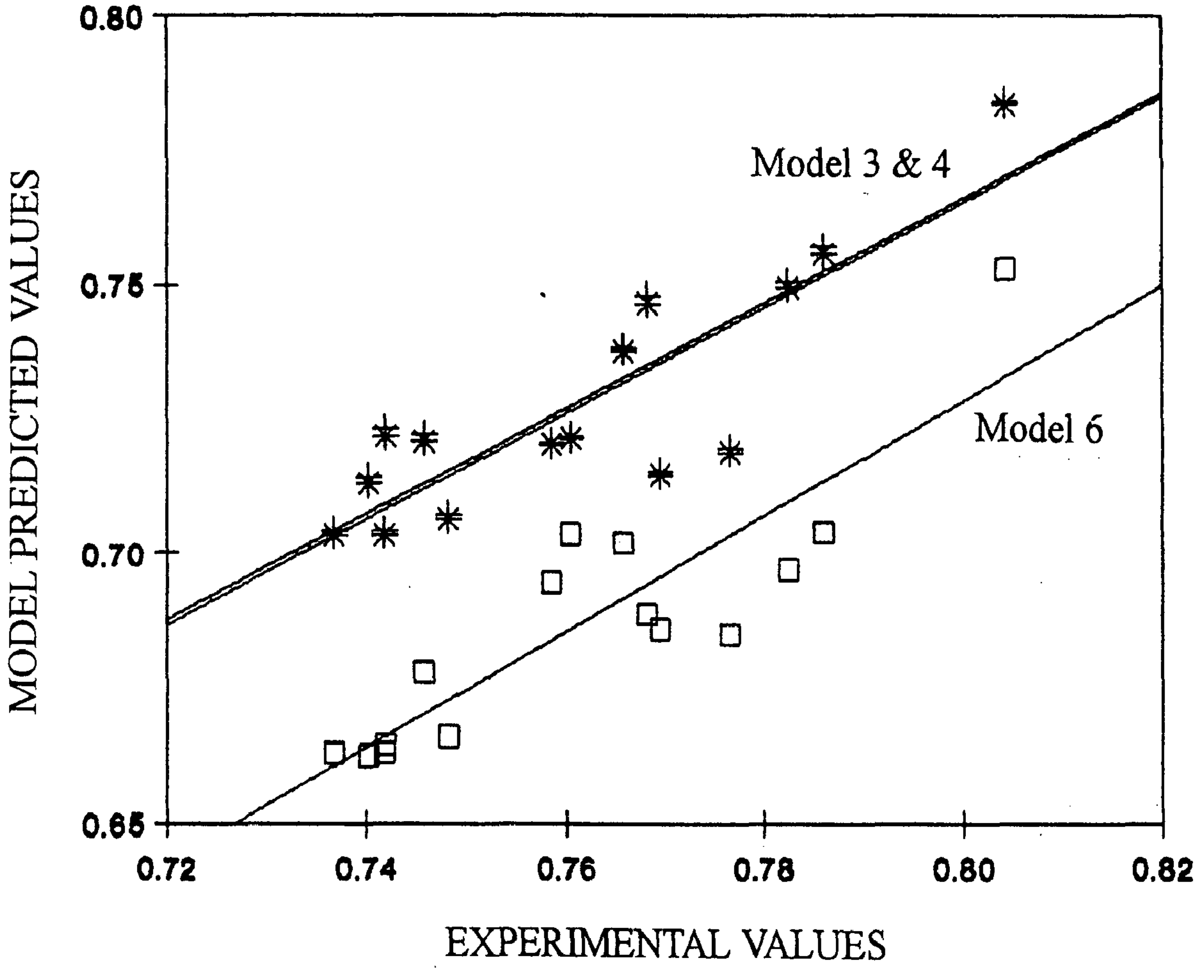

Figure 47 : Experimental and model predicted values of displacement ratio

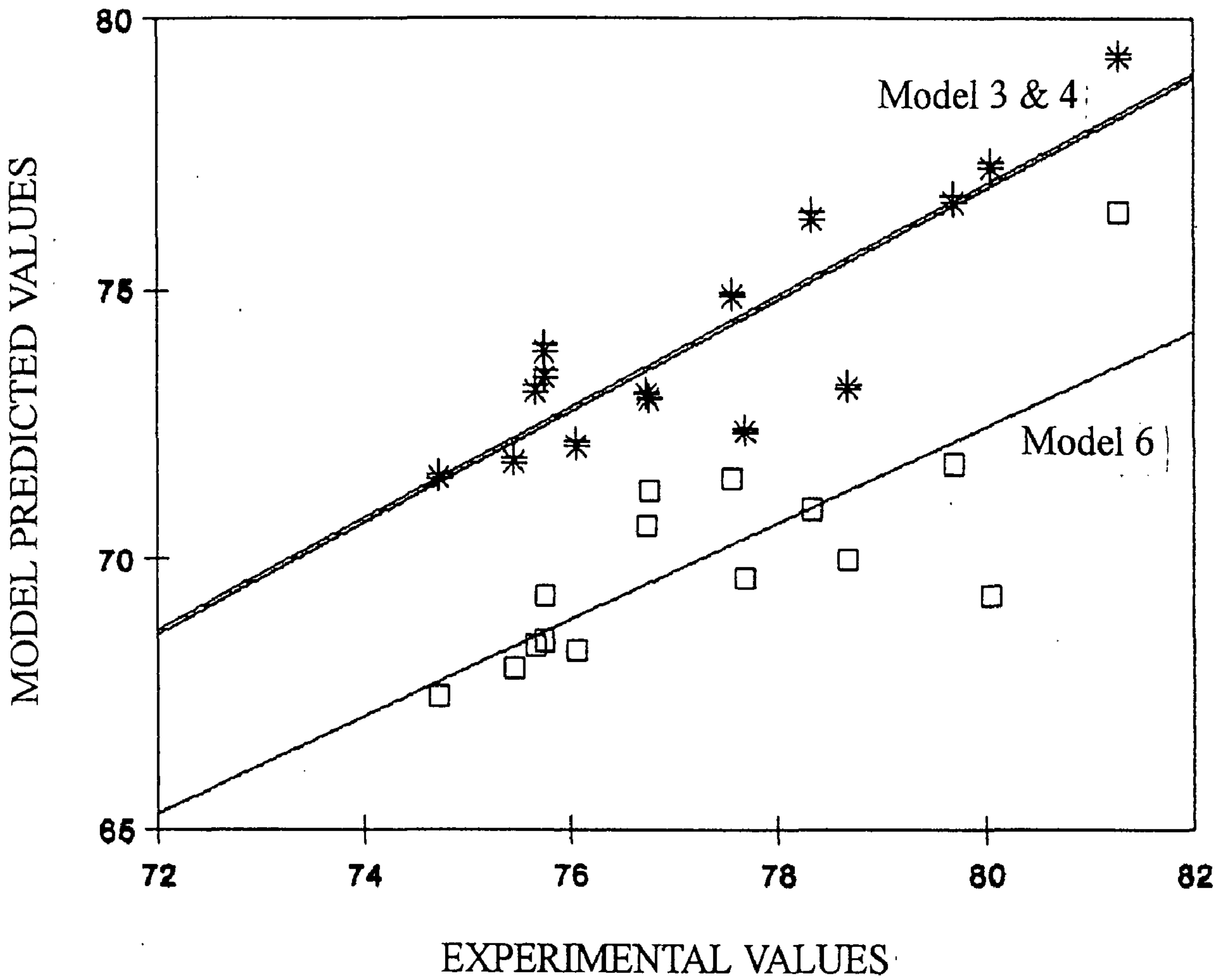

Figure 48 : Experimental and model predicted values of % efficiency

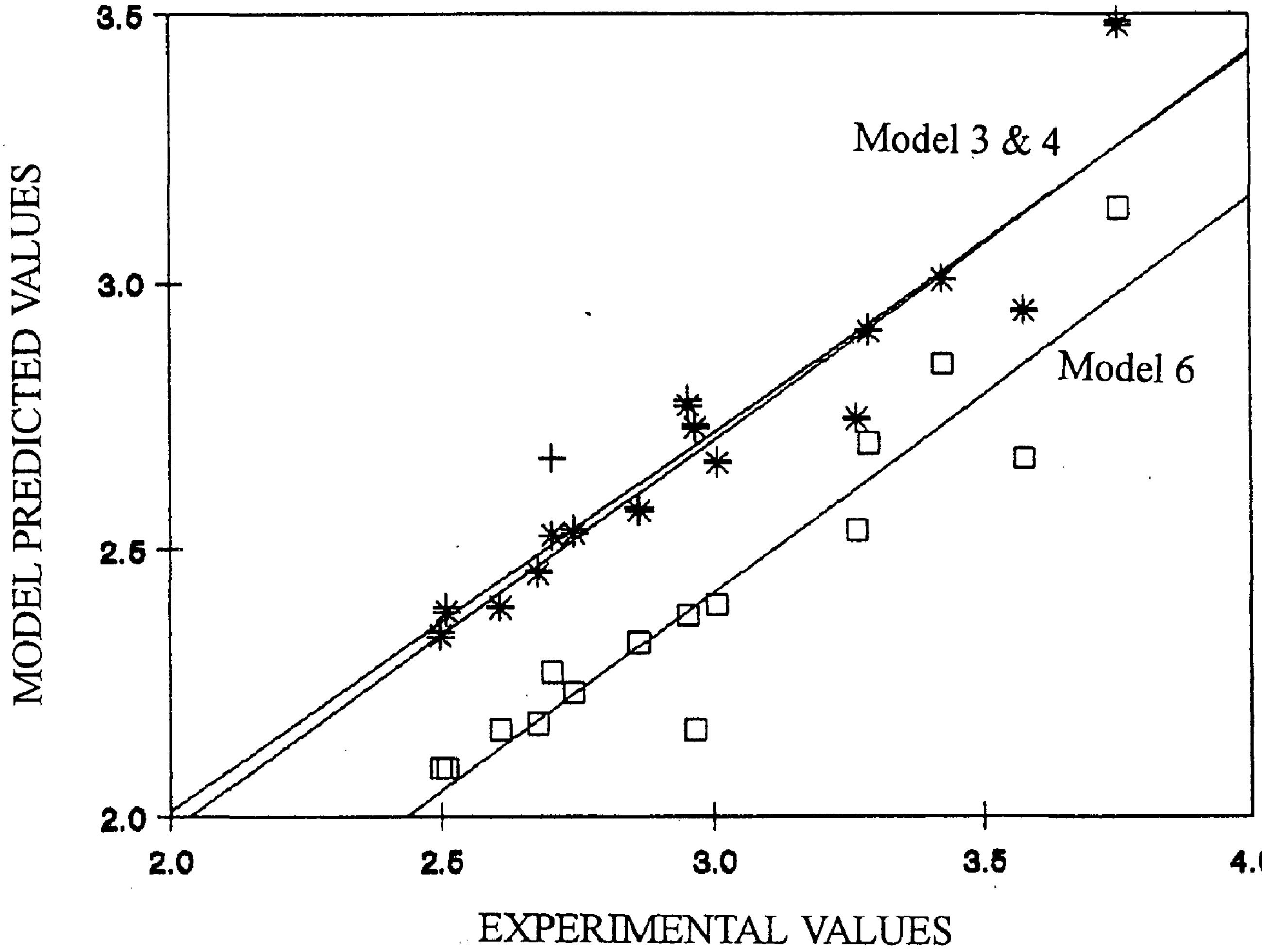

Figure 49 : Experimental and model predicted values of Norden's efficiency factor

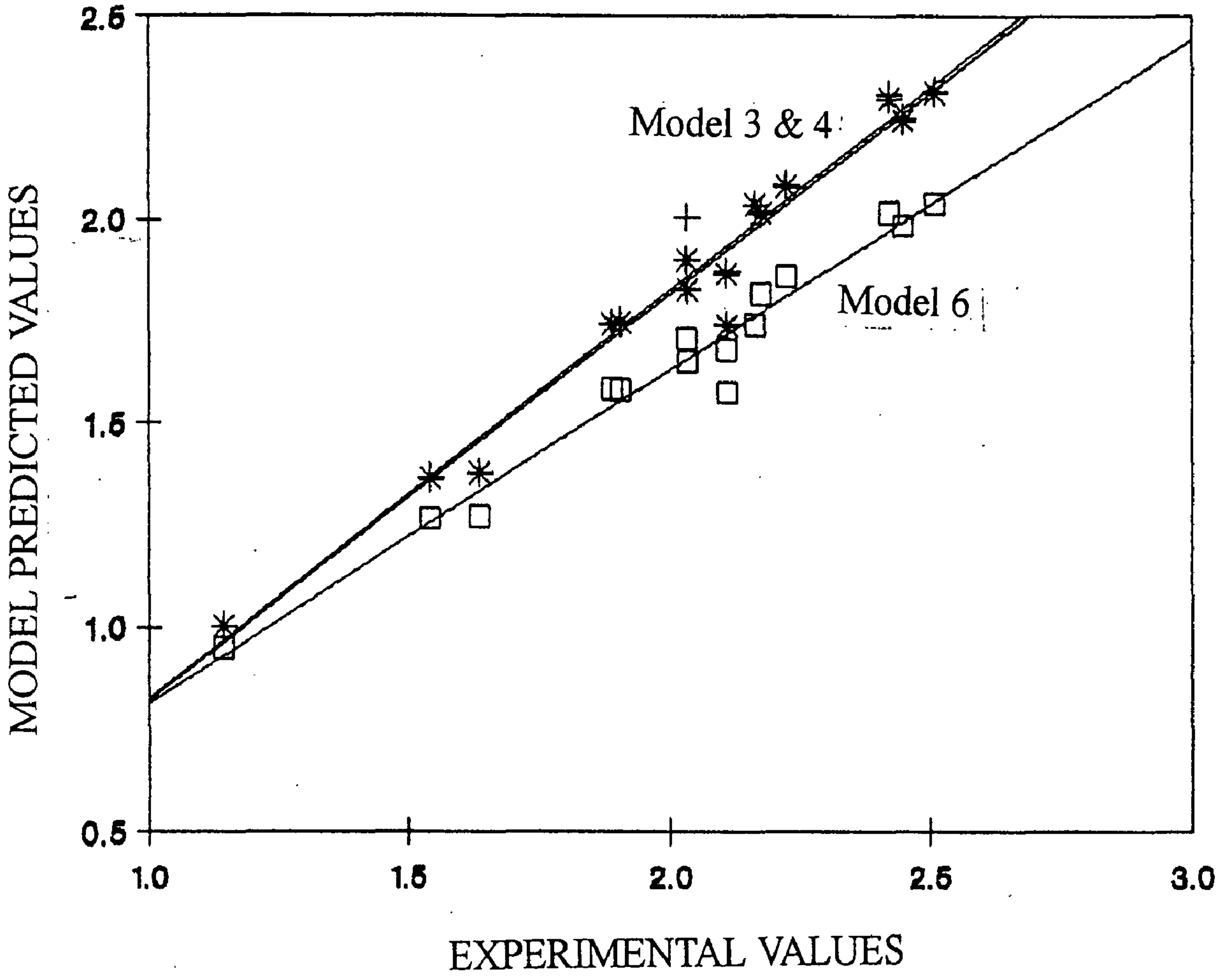

Figure 50 : Experimental and model predicted values of modified Norden efficiency factor

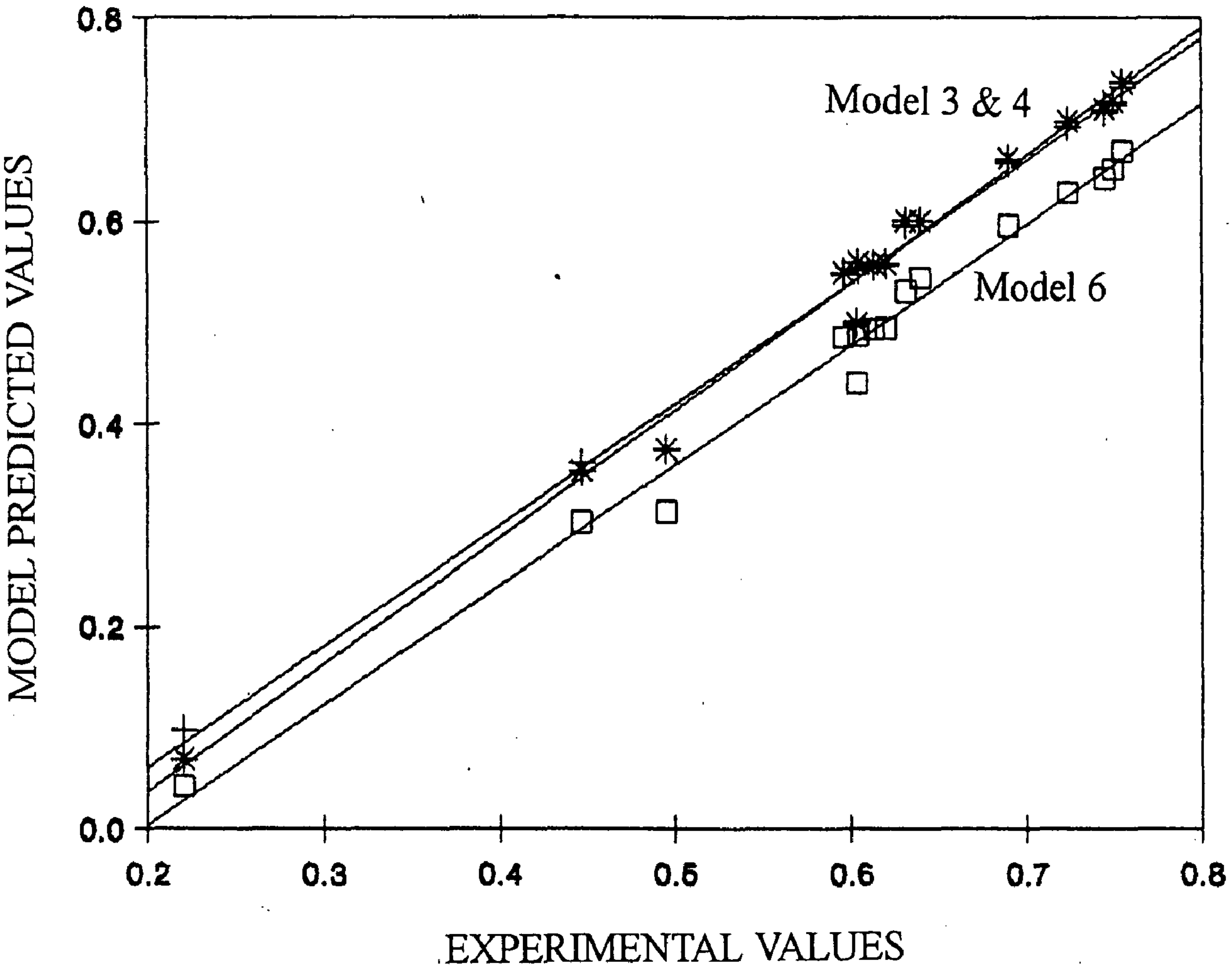

Figure 51 : Experimental and model predicted values of equivalent displacement ratio

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions of the Present Study

An investigation has been carried out on rotary vacuum filter used for washing of brown stock in a paper industry. The approach for evaluating the performance has been multi directional. Detailed literature survey indicated that there was non homogeneity of definitions and imperfections in defining the widely accepted parameters used in practice.

In this investigation an attempt has been made to define possible applicable performance evaluation parameters used by various investigators and to compare each other.

The most important parameter is the efficiency of washing expressed in terms of the solute concentration in the discharged as well as the extracted liquor and the filtrate flow rate with or without the consideration of the filter medium resistance. Effect of various design and process parameters like rpm,inlet vat consistency, radius of drum, fractional submergence of drum, pressure drop (which also included hydrostatic pressure drop, not considered earlier for brown stock washer designing in pulp mill) have been studied on cake thickness, filtrate flow rate, fiber production rate, specific loading factor.

For cake washing zone, available models are inspected in terms of their discrepancies. A mathematical formulation has been made in the same manner as has been done by different workers[5,17,26,27,34,45,61,65,72] for pulp washing system along with the effect of dispersion - diffusion as used by some workers[5,38,45,72,85].Two different porosity values used by some researchers [18,65] for displaceable solutes and non displaceable solutes and adsorption-desorption isotherms have been considered. It may be mentioned that some authors[5,86] neglected adsorption desorption isotherms. Many investigators [34,61,72,85] did not consider the difference of porosity values in different layers(stagnant and movable) during washing. While some other school of thoughts have neglected dispersion effects [34,35,65]. Even in adsorption-desorption isotherms investigators used different type of equations[18,38].

To alleviate from all these problems, 8 models have been considered in the present investigation with varying adsorption isotherms of finite and linear type only and varying boundary conditions. Solution of all the models have been attempted using Laplace transform technique. Inverse has been taken by using method of residues. Peclet number has been included as a parameter in the solution of the models.

The models for cake drying and dewatering zones have been assumed the same, as has been done by other investigators[27,65]. The common models have been formulated in the same manner as Han and Edwards[27] but, finally solved with effective saturation and real saturation as parameters following the concepts of Brown et al.[7] and Perron and Lebeau[65]. Final mathematical simplification, however, has become different from others. The volumetric flow rate in all the zones are summed up for the static model of the filter.

All the models have been tested by two methods. In the first attempt literature data on which earlier model predictions were based, were followed and then the data are generated based on certain Indian mill data and the models are subjected to validation of the data. The models are tested and results are reported for the single as well as multistage washers.

It has been felt necessary to redefine widely industry usage parameters in terms of the parameters obtained from the mathematical equations like mean concentration and discharge concentration. Results from some models 3, 4 and 6 are found closer to the earlier predicted equations, largely employed for process and design of industrial set up. The results from few rival models are departed from the earlier predictions. Therefore further comparisons are not made with the above models.

Laboratory EIMCO-KCP filter is used to generate data from the washing of mill based pulp of the following composition Eucalyptus 81%, Bamboo 17% and Pine 2%.Its Kappa number was 20 and pH 12.

The parameters like inlet vat consistency, water flow rate, rpm, fractional submergence have been varied to get thickness of the cake, filtrate flow rate, solids in filtrate and solids in discharged pulp. Vacuum is automatically varied within the narrow ranges (3999-10664 Pascal). The data are used for verification of the models for a single stage washer. It has been found that like the literature and plant data, the experimental data also confirms the accuracy of the model.

All the above experimental and theoretical findings suggest that the present model agrees quite well with the experimental data and takes care of many of the important aspects of diffusion, dispersion, adsorption - desorption and different porosity values in the system.

6.2 Recommendations Based on Present Study

Based on the entire investigation the following recommendations can be made for future work in this area :

1. The present model for washing zone has taken into consideration the longitudinal diffusion coefficient (D_L) as a parameter and as a consequence the model has been able to predict actual washing zone performance more accurately than most of the other models which have neglected D_L. Thus it is important to use models with D_L for predicting the washing zone performance.

2. Laplace transform technique has been found to be suitable for solving the model equations developed in the washing zone of a rotary vacuum filter. This method introduces the solution of difficult model equations after linearizing them through formation of dimensionless parameters.

3. The models should be solved by some other numerical techniques especially by Orthogonal Collocation method and Galerkin method and to examine the approximations and see the effect of other approximations to be introduced in the numerical method.

4. In the present investigation, the model did not consider the Langmuir adsorption isotherm which has been considered by Grahs[17]. The model did not consider the sorption effect of Sodium and lignin separately. These need to be looked into.

5. Effect of temperature and pH should be considered in future work.

6. The experiment should be performed in a laboratory and pilot plant model having 3 - 4 washers joined in series to validate the proposed model.

7. The model should be further extended for the other washers like pressure washer, horizontal washer, belt washer, screw press etc.

8. Experiments should be conducted with the wood fiber pulp, nonwood fiber pulp and their blends at different pH values and Kappa numbers and performance be checked against the model prediction for validating the model.

APPENDICES

APPENDIX I

NOMENCLATURE

A	:	Surface area of drum, m^2
A_c	:	Area of cross section of washing zone, m^2
$ABC/(1+BC)$	:	Langmuir adsorption isotherm
B	:	Constant for a given flow condition, geometry, fluid and pressure drop
c	:	Concentration of the solute in the liquor, kg/m^3
c_s	:	Solute concentration in stagnant zone, kg/m^3
c_s^*	:	Solute concentration in stagnant zone at equilibrium, kg/m^3
C_b	:	Concentration of solute in blow liquor, kg/m^3
C_d	:	Concentration of solute in the discharged pulp, kg/m^3
C_e	:	Exit concentration of solute leaving the bed, kg/m^3
C_f	:	Concentration of solute in the filtrate, kg/m^3
C_i	:	Concentration of solute inside the vat, kg/m^3
C_m	:	Mean concentration of the filtrate collected through the washing zone, kg/m^3
C_s	:	Concentration of solute in the wash liquor, kg/m^3
C_{sl}	:	Concentration of the pulp slurry, kg/m^3
C_F	:	Fibre consistency, kg/m^3
C_{Fm}	:	Mean fibre consistency, kg/m^3
C_y	:	Pulp consistency, kg of fibers/kg of liquor
C_{yb}	:	Blow consistency of pulp, kg of fibers/kg of liquor
C_{yd}	:	Discharged consistency of pulp, kg of fibers/kg of liquor
C_{yi}	:	Inlet vat consistency of pulp, kg of fibers/kg liquor
C_{yst}	:	Standard consistency typically 10% or 12%.
D_L	:	Longitudinal dispersion coefficient, m^2/s
D_V	:	Molecular diffusion coefficient, m^2/s
erf	:	Error function
g	:	Gravitational acceleration, m/s^2
I_o	:	Zero order Bessel function with imaginary argument
k, k', k''	:	Mass transfer coefficients, dimensionless
k_1, k_2, k^*	:	Mass transfer coefficients, 1/s
K	:	Permeability constant of cake, m^2;
K_e	:	Relative permeability of cake
L	:	Cake thickness, m

l : Arbitrary cake thickness, m
l_d : Length of dewatering zone, m
l_w : Length of washing zone, m
L_b : Amount of liquor in the pulp coming from blow tank, kg of liquor/kg of pulp
L_c : Amount of liquor in the pulp leaving for bleaching section, kg of liquor/kg of pulp
L_d : Amount of liquor in discharged pulp, kg of liquor/kg of pulp
L_e : Liquor speed inside the vat, m^3/s
L_f : Amount of filtrate, kg of liquor/kg of pulp
L_i : Amount of liquor inside the vat, kg of liquor/kg of pulp
L_p : Amount of liquor in the pulp coming from previous stage, kg of liquor/kg of pulp
L_r : Amount of liquor recyled to previous washer, kg of liquor/kg of pulp
L_s : Amount of wash water, kg of water/kg of pulp
L_{st} : Amount of liquor in the pulp at standard consistency, kg of liquor/kg of pulp
m : Mass of wet cake/mass of dry cake, kg/kg
m_p : Mass of particles deposited in filter per unit volume of filtrate, kg/m^3
N : Speed of the drum, rpm/60, 1/s
N_2 : Level of slurry in the drum, m
N_i : Concentration of solute on the fibers, kg/m^3
n : Concentration of solute on the fibres, kg/m^3
P : Width of the drum, m
Pe : Peclet number, dimensionless
P_a : Atmospheric pressure, Pascal
P_h : Hydrostatic pressure, Pascal
P_v : Vacuum, Pascal
ΔP : Pressure drop, Pascal
ΔP_c : Pressure drop across the cake, Pascal
ΔP_f : Pressure due to friction, Pascal
ΔP_i : Inlet pressure, Pascal
ΔP_k : Kinetic pressure, Pascal
ΔP_m : Pressure due to medium, Pascal

ΔP_t : Total pressure, Pascal
ΔP_v : Pressure due to vacuum, Pascal
q : Local shower flow (zero for drying zone), m/s
r : Radial distance from the centerline of the capillary, m
R : Radius of the drum, m
R_o : Perpendicular distance between axis of rotation and surface of slurry, m
R' : Arbitrary distance between axis of rotation and surface of slurry, m
R_o'' : Vertical distance between drum and vat, m
s : Constant for a particular cake
S_o : Specific surface of fibres, m^2/m^3
S_e : Effective saturation, %
S_r : Residual saturation, %
S_s : Real saturation, %
t : Time, s
t_d : Time of drying zone, s
t_w : Time of washing zone, s
t' : Time to deposit a cake layer with a resistance equal to that of the filter cloth, s
u : Liquor speed in cake pores, m/s
V : Filtrate volume, m^3
V' : Filtrate collected during deposition of a pulp mat with a resistance equal to filter cloth, m^3
V_c : Volume of filtrate collected during deposition of a cake of resistance equivalent to the cloth per m^2, m^3/m^2
V_d : Filtrate flow rate through cake drying zone, m^3/s
V_f : Filtrate flow rate through cake formation zone, m^3/s
V_w : Filtrate flow rate through cake washing zone, m^3/s
V_t : Total filtrate flow rate, m^3/s
WW : Amount of wash water added, liters per hour
x : Dummy variable of integration
x_b : Dissolved solids in pulp coming from blow tank, %
x_c : Dissolved solids in pulp going for bleaching section, %
x_d : Dissolved solids in discharged pulp, %
x_f : Dissolved solids in the filtrate, %

x_i : Dissolved solids inside the vat, %
x_p : Dissolved solids in pulp coming from previous stage, %
x_s : Dissolved solids in the washed pulp, %
X_i : Mass of fibres/liquor in the vat, $C_{yi}/(1-C_{yi})$, kg/kg
y : Constant depending on particle size
z : Variable cake thickness, m
Δz : Small increament in cake thickness, m

Greek Symbols

α : Specific resistance, kg/m
α_o : Constant for a particular cake
ß : Variable angle in the cake formation zone, Radian
$ß_n$: Roots of transcendental equation
γ, γ' : Constants
ε_d : Interfiber porosity of cake, dimensionless
ε_s : Intrafiber porosity of cake, dimensionless
ε_t : Total porosity of cake, dimensionless
ε_{dm} : Mean interfiber porosity of cake, dimensionless
ε_{tm} : Mean total porosity of cake, dimensionless
η : Viscosity of the liquor, kg/ms
θ : Angle of submergence, Radian
ρ : Density of water, kg/m^3
ρ_f : Density of fibres, kg/m^3
ρ_{sus} : Density of suspension, kg/m^3
ρ_d : Density of liquor in washed pulp, kg/m^3
ρ_i : Density of liquor inside the vat, kg/m^3
ρ_s : Density of wash water, kg/m^3
σ : Surface tension of the liquor, kg/m^3
ϕ : Angle subtended by point area with horizontal, Radian
ϕ_1 : Angle subtended by point area with horizontal when the drum enters the slurry, Radian
ϕ_2 : Angle subtended by point area with horizontal when the drum leaves the slurry, Radian
ψ : Fractional submergence of drum, dimensionless
ξ : Dimensionless distance, dimensionless
τ : Dimensionless time, dimensionless

APPENDIX II

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APPENDIX III

COMPUTER PROGRAMMES

```
C       PROGRAM TO FIND ROOTS OF TRANSCENDENTAL EQUATION ßcotß+(Pe/2)=0
        DIMENSION R(6),F(6),DF(6),RR(6),D(6),H(6)
        DOUBLE PRECISION R,DF,H,RR,D,F,Pe
        READ(*,*) Pe,R2
        R(1)=R2
        DO 50 I = 1,6
10      F(I)=R(I)/TAN(R(I))+(Pe/2.)
        DF(I)=-1.*R(I)/((SIN(R(I)))**2)+1./TAN(R(I))
        H(I)=F(I)/DF(I)
        RR(I)=R(I)-H(I)
        D(I)=ABS (RR(I)-R(I))
        IF(D(I).LE.0.00001) GO TO 20
        R(I)=RR(I)
        GO TO 10
20      R(I+1)=R(I)+3.1416
50      CONTINUE
60      FORMAT (6F13.6)
        WRITE(*,60) (R(I),I=1,6)
        STOP
        END
```

```
C     PROGRAM TO FIND ROOTS OF TRANSCENDENTAL EQUATION ßcotß+(Pe^2-4ß^2)/4Pe=0
      DIMENSION R(6),F(6),DF(6),RR(6),D(6),H(6)
      DOUBLE PRECISION F,DF,H,RR,D,R,R2,Pe
      READ(*,*)Pe,R2
      R(1)=R2
      DO 50 I = 1,6
10    F(I)=R(I)/TAN(R(I))+(Pe/4.)-R(I)**2/Pe
      DF(I)=-1.*R(I)/((SIN(R(I)))**2)+1./TAN(R(I))-2.*R(I)/Pe
      H(I)=F(I)/DF(I)
      RR(I)=R(I)-H(I)
      D(I)=ABS(RR(I)-R(I))
      IF(D(I).LE.0.00001) GO TO 20
      R(I)=RR(I)
      GO TO 10
20    R(I+1)=R(I)+3.1416
50    CONTINUE
60    FORMAT(6F13.6)
      WRITE(*,60)(R(I), I=1,6)
      STOP
      END
```

```
C       PROGRAM TO FIND CONCENTRATION FROM MODEL 1
C       μ=A,K'=R,k1=R1,k2=R2,L=CT,εt=PT,τ=T,Ce=CONCEX,Cd=CONCAV,Cm=CONCME
        DIMENSION P1(6),X1(6),Y1(6),A1(6),A2(6),U1(6),V1(6),W1(6),E1(6)
        DIMENSION P2(6),X2(6),Y2(6),A3(6),A4(6),U2(6),V2(6),W2(6),E2(6)
        DIMENSION S(6),Z(6),SUMEX(6),SUMAV(6),SUMME(6)
        DOUBLE PRECISION A,A1,A2,A3,A4,R,T,G,H,E,F,EN,SUMEX,SUMAV,SUMME
        DOUBLE PRECISION S,Z,P1,P2,E1,E2,X1,X2,Y1,Y2,U1,U2,V1,V2,W1,W2
        WRITE(*,*)'Pe,CT,PT,CI,CS,TW,R1,R2,U,R(I)'
        READ(*,*)Pe,CT,PT,CI,CS,TW,R1,R2,U,(R(I), I=1,6)
        A=(1.-PT)/PT
        R=R1/R2
        T=U*TW/CT
        G=R2*CT/U
        H=(R-1.)*CS/(CI-CS)
        E=G+A*G-A*G*H
        F=G+A*R*G
        SUMEX(0)=0.0
        SUMAV(0)=0.0
        SUMME(0)=0.0
        DO 100 I=1,6
        A1(I)=Pe**2+4.*R(I)**2
        A2(I)=A1(I)+4.*Pe*G*(1.+A*R)
        A3(I)=SQRT(A2(I)**2-16.*Pe*G*A1(I))
        A4(I)=A1(I)+2.*Pe
        P1(I)=(-1.*A2(I)+A3(I))/(8.*Pe)
        P2(I)=(-1.*A2(I)-A3(I))/(8.*Pe)
        E1(I)=EXP(P1(I)*T)
        E2(I)=EXP(P2(I)*T)
        S(I)=SIN(R(I))
        EN=EXP(Pe/2.)
        Z(I)=A1(I)*S(I)+4.*Pe*R(I)*EN
        X1(I)=-8.*(R(I)**2)*(P1(I)+E)/(Pe*P1(I)*S(I)*(P1(I)+F)*A4(I))
        Y1(I)=1.+(A*R*(G**2)/((P1(I)+G)**2))
        U1(I)=R(I)*X1(I)*EN*E1(I)/Y1(I)
        V1(I)=X1(I)*Z(I)*E1(I)/(Y1(I)*A1(I))
        W1(I)=-1.*R(I)*X1(I)*EN*(1.-E1(I))/(P1(I)*T*Y1(I))
        X2(I)=-8.*(R(I)**2)*(P2(I)+E)/(Pe*P2(I)*S(I)*(P2(I)+F)*A4(I))
        Y2(I)=1.+(A*R*(G**2)/((P2(I)+G)**2))
        U2(I)=R(I)*X2(I)*EN*E2(I)/Y2(I)
        V2(I)=X2(I)*Z(I)*E2(I)/(Y2(I)*A1(I))
        W2(I)=-1.*R(I)*X2(I)*EN*(1.-E2(I))/(P2(I)*T*Y2(I))
        SUMEX(I)=SUMEX(I-1)+U1(I)+U2(I)
        SUMAV(I)=SUMAV(I-1)+V1(I)+V2(I)
        SUMME(I)=SUMME(I-1)+W1(I)+W2(I)
100     CONTINUE
        CONCEX=SUMEX(6)*(CI-CS)+CS
        CONCAV=SUMAV(6)*(CI-CS)+CS
        CONCME=SUMME(6)*(CI-CS)+CS
120     FORMAT(3 E15.8)
        WRITE(*,*)'CONCEX,CONCAV,CONCME'
        WRITE(*,120)CONCEX,CONCAV,CONCME
        STOP
        END
```

```
C       PROGRAM TO FIND CONCENTRATION FROM MODEL 2
C       μ=A,k*=R,L=CT,ε_t=PT,τ=T,C_e=CONCEX,C_d=CONCAV,C_m=CONCME
        DIMENSION P1(6),X1(6),Y1(6),A1(6),A2(6),U1(6),V1(6),W1(6),E1(6)
        DIMENSION P2(6),X2(6),Y2(6),A3(6),A4(6),U2(6),V2(6),W2(6),E2(6)
        DIMENSION S(6),Z(6),SUMEX(6),SUMAV(6),SUMME(6)
        DOUBLE PRECISION A,R,T,G,H,E,F,A1,A2,A3,A4,EN,SUMEX,SUMAV,SUMME
        DOUBLE PRECISION S,Z,P1,P2,E1,E2,X1,X2,Y1,Y2,U1,U2,V1,V2,W1,W2
        WRITE(*,*)'Pe,CT,PT,CI,CS,TW,R,U,R(I)'
        READ(*,*)Pe,CT,PT,CI,CS,TW,R,U,(R(I), I=1,6)
        A=(1.-PT)/PT
        T=U*TW/CT
        G=R*CT/U
        H=(R-1.)*CS/(CI-CS)
        E=G+A*G-A*G*H
        F=G+A*R*G
        SUMEX(0)=0.0
        SUMAV(0)=0.0
        SUMME(0)=0.0
        DO 100 I=1,6
        A1(I)=Pe**2+4.*R(I)**2
        A2(I)=A1(I)+4.*Pe*G*(1.+A*R)
        A3(I)=SQRT(A2(I)**2-16.*Pe*G*A1(I))
        A4(I)=A1(I)+2.*Pe
        P1(I)=(-1.*A2(I)+A3(I))/(8.*Pe)
        P2(I)=(-1.*A2(I)-A3(I))/(8.*Pe)
        E1(I)=EXP(P1(I)*T)
        E2(I)=EXP(P2(I)*T)
        S(I)=SIN(R(I))
        EN=EXP(Pe/2.)
        Z(I)=A1(I)*S(I)+4.*Pe*R(I)*EN
        X1(I)=-8.*(R(I)**2)*(P1(I)+E)/(Pe*P1(I)*S(I)*(P1(I)+F)*A4(I))
        Y1(I)=1.+(A*R*(G**2)/((P1(I)+G)**2))
        U1(I)=R(I)*X1(I)*EN*E1(I)/Y1(I)
        V1(I)=X1(I)*Z(I)*E1(I)/(Y1(I)*A1(I))
        W1(I)=-1.*R(I)*X1(I)*EN*(1.-E1(I))/(P1(I)*T*Y1(I))
        X2(I)=-8.*(R(I)**2)*(P2(I)+E)/(Pe*P2(I)*S(I)*(P2(I)+F)*A4(I))
        Y2(I)=1.+(A*R*(G**2)/((P2(I)+G)**2))
        U2(I)=R(I)*X2(I)*EN*E2(I)/Y2(I)
        V2(I)=X2(I)*Z(I)*E2(I)/(Y2(I)*A1(I))
        W2(I)=-1.*R(I)*X2(I)*EN*(1.-E2(I))/(P2(I)*T*Y2(I))
        SUMEX(I)=SUMEX(I-1)+U1(I)+U2(I)
        SUMAV(I)=SUMAV(I-1)+V1(I)+V2(I)
        SUMME(I)=SUMME(I-1)+W1(I)+W2(I)
100     CONTINUE
        CONCEX=SUMEX(6)*(CI-CS)+CS
        CONCAV=SUMAV(6)*(CI-CS)+CS
        CONCME=SUMME(6)*(CI-CS)+CS
120     FORMAT(3 E15.8)
        WRITE(*,*)'CONCEX,CONCAV,CONCME'
        WRITE(*,120)CONCEX,CONCAV,CONCME
        STOP
        END
```

```
C      PROGRAM TO FIND CONCENTRATION FROM MODEL 3
C      μ=A,k1=R1,k2=R2,L=CT,εt=PT,τ=T,Ce=CONCEX,Cd=CONCAV,Cm=CONCME
       DIMENSION P1(6),X1(6),Y1(6),A1(6),A2(6),U1(6),V1(6),W1(6),E1(6)
       DIMENSION P2(6),X2(6),Y2(6),A3(6),A4(6),U2(6),V2(6),W2(6),E2(6)
       DIMENSION R(6),S(6),A5(6),SUMEX(6),SUMAV(6),SUMME(6)
       DOUBLE PRECISION A,R,T,G,H,E,F,A1,A2,A3,A4,A5,SUMEX,SUMAV,SUMME
       DOUBLE PRECISION S,EN,P1,P2,E1,E2,X1,X2,Y1,Y2,U1,U2,V1,V2,W1,W2
       WRITE(*,*)'Pe,CT,PT,CI,CS,TW,R1,R2,U,R(I)'
       READ(*,*)Pe,CT,PT,CI,CS,TW,R1,R2,U,(R(I), I=1,6)
       A=(1.-PT)/PT
       R=R1/R2
       T=U*TW/CT
       G=R2*CT/U
       H=(R-1.)*CS/(CI-CS)
       E=G+A*G-A*G*H
       F=G+A*R*G
       SUMEX(0)=0.0
       SUMAV(0)=0.0
       SUMME(0)=0.0
       DO 100 I = 1,6
       A1(I)=Pe**2+4.*R(I)**2
       A2(I)=A1(I)+4.*Pe*G*(1.+A*R)
       A3(I)=SQRT(A2(I)**2-16.*Pe*G*A1(I))
       A4(I)=A1(I)+4.*Pe
       P1(I)=(-1.*A2(I)+A3(I))/(8.*Pe)
       P2(I)=(-1.*A2(I)-A3(I))/(8.*Pe)
       S(I)=SIN(R(I))
       A5(I)=S(I)*A1(I)*A4(I)
       EN=EXP(Pe/2.)
       E1(I)=EXP(P1(I)*T)
       E2(I)=EXP(P2(I)*T)
       X1(I)=-32.*Pe*(R(I)**3)*(P1(I)+E)*EN/(P1(I)*(P1(I)+F)*A5(I))
       Y1(I)=1.+(A*R*(G**2)/((P1(I)+G)**2))
       U1(I)=X1(I)*E1(I)/Y1(I)
       V1(I)=4.*Pe*U1(I)/A1(I)
       W1(I)=-1.*(1.-E1(I))*X1(I)/(T*P1(I)*Y1(I))
       X2(I)=-32.*Pe*(R(I)**3)*(P2(I)+E)*EN/(P2(I)*(P2(I)+F)*A5(I))
       Y2(I)=1.+(A*R*(G**2)/((P2(I)+G)**2))
       U2(I)=X2(I)*E2(I)/Y2(I)
       V2(I)=4.*Pe*U2(I)/A1(I)
       W2(I)=-1.*(1.-E2(I))*X2(I)/(T*P2(I)*Y2(I))
       SUMEX(I)=SUMEX(I-1)+U1(I)+U2(I)
       SUMAV(I)=SUMAV(I-1)+V1(I)+V2(I)
       SUMME(I)=SUMME(I-1)+W1(I)+W2(I)
100    CONTINUE
       CONCEX=SUMEX(6)*(CI-CS)+CS
       CONCAV=SUMAV(6)*(CI-CS)+CS
       CONCME=SUMME(6)*(CI-CS)+CS
120    FORMAT(3 E15.8)
       WRITE(*,*)'CONCEX,CONCAV,CONCME'
       WRITE(*,120)CONCEX,CONCAV,CONCME
       STOP
       END
```

```
C       PROGRAM TO FIND CONCENTRATION FROM MODEL 4
C       μ=A,k*=R,L=CT,εt=PT,τ=T,Ce=CONCEX,Cd=CONCAV,Cm=CONCME
        DIMENSION P1(6),X1(6),Y1(6),A1(6),A2(6),U1(6),V1(6),W1(6),E1(6)
        DIMENSION P2(6),X2(6),Y2(6),A3(6),A4(6),U2(6),V2(6),W2(6),E2(6)
        DIMENSION R(6),S(6),A5(6),SUMEX(6),SUMAV(6),SUMME(6)
        DOUBLE PRECISION A,T,G,H,E,F,A1,A2,A3,A4,A5,SUMEX,SUMAV,SUMME
        DOUBLE PRECISION S,EN,P1,P2,E1,E2,X1,X2,Y1,Y2,U1,U2,V1,V2,W1,W2
        WRITE(*,*)'Pe,CT,PT,CI,CS,TW,R,U,R(I)'
        READ(*,*)Pe,CT,PT,CI,CS,TW,R,U,(R(I), I=1,6)
        A=(1.-PT)/PT
        T=U*TW/CT
        G=R*CT/U
        H=(R-1.)*CS/(CI-CS)
        E=G+A*G-A*G*H
        F=G+A*R*G
        SUMEX(0)=0.0
        SUMAV(0)=0.0
        SUMME(0)=0.0
        DO 100 I = 1,6
        A1(I)=Pe**2+4.*R(I)**2
        A2(I)=A1(I)+4.*Pe*G*(1.+A*R)
        A3(I)=SQRT(A2(I)**2-16.*Pe*G*A1(I))
        A4(I)=A1(I)+4.*Pe
        P1(I)=(-1.*A2(I)+A3(I))/(8.*Pe)
        P2(I)=(-1.*A2(I)-A3(I))/(8.*Pe)
        S(I)=SIN(R(I))
        A5(I)=S(I)*A1(I)*A4(I)
        EN=EXP(Pe/2.)
        E1(I)=EXP(P1(I)*T)
        E2(I)=EXP(P2(I)*T)
        X1(I)=-32.*Pe*(R(I)**3)*(P1(I)+E)*EN/(P1(I)*(P1(I)+F)*A5(I))
        Y1(I)=1.+(A*R*(G**2)/((P1(I)+G)**2))
        U1(I)=X1(I)*E1(I)/Y1(I)
        V1(I)=4.*Pe*U1(I)/A1(I)
        W1(I)=-1.*(1.-E1(I))*X1(I)/(T*P1(I)*Y1(I))
        X2(I)=-32.*Pe*(R(I)**3)*(P2(I)+E)*EN/(P2(I)*(P2(I)+F)*A5(I))
        Y2(I)=1.+(A*R*(G**2)/((P2(I)+G)**2))
        U2(I)=X2(I)*E2(I)/Y2(I)
        V2(I)=4.*Pe*U2(I)/A1(I)
        W2(I)=-1.*(1.-E2(I))*X2(I)/(T*P2(I)*Y2(I))
        SUMEX(I)=SUMEX(I-1)+U1(I)+U2(I)
        SUMAV(I)=SUMAV(I-1)+V1(I)+V2(I)
        SUMME(I)=SUMME(I-1)+W1(I)+W2(I)
100     CONTINUE
        CONCEX=SUMEX(6)*(CI-CS)+CS
        CONCAV=SUMAV(6)*(CI-CS)+CS
        CONCME=SUMME(6)*(CI-CS)+CS
120     FORMAT(3 E15.8)
        WRITE(*,*)'CONCEX,CONCAV,CONCME'
        WRITE(*,120)CONCEX,CONCAV,CONCME
        STOP
        END
```

```
C       PROGRAM TO FIND CONCENTRATION FROM MODEL 5
C       μ=A,K=R,L=CT,ε_t=PT,τ=T,C_e=CONCEX,C_d=CONCAV,C_m=CONCME
        DIMENSION A1(6),A2(6),P1(6),X1(6),Y1(6),U1(6),V1(6),W1(6)
        DIMENSION R(6),S(6),SUMEX(6),SUMAV(6),SUMME(6),E1(6)
        DOUBLE PRECISION A,T,A1,A2,SUMEX,SUMAV,SUMME
        DOUBLE PRECISION S,EN,P1,E1,X1,Y1,U1,V1,W1
        WRITE(*,*)'Pe,CT,PT,CI,CS,TW,R,U,R(I)'
        READ(*,*)Pe,CT,PT,CI,CS,TW,R,U,(R(I), I=1,6)
        A=(1.-PT)/PT
        T=U*TW/(CT*(1.+R*A))
        SUMEX(0)=0.0
        SUMAV(0)=0.0
        SUMME(0)=0.0
        DO 100 I=1,6
        A1(I)=Pe**2+4.*R(I)**2
        A2(I)=A1(I)+2.*Pe
        P1(I)=-1.*A1(I)/(4.*Pe)
        E1(I)=EXP(P1(I)*T)
        S(I)=SIN(R(I))
        EN=EXP(Pe/2.)
        X1(I)=-8.*(R(I)**2)/(P1(I)*Pe*S(I)*A2(I))
        Y1(I)=4.*Pe*R(I)*EN+A1(I)*S(I)
        U1(I)=R(I)*X1(I)*EN*E1(I)
        V1(I)=X1(I)*E1(I)*Y1(I)/A1(I)
        W1(I)=-1.*R(I)*X1(I)*EN*(1.-E1(I))/(P1(I)*T)
        SUMEX(I)=SUMEX(I-1)+U1(I)
        SUMAV(I)=SUMAV(I-1)+V1(I)
        SUMME(I)=SUMME(I-1)+W1(I)
100     CONTINUE
        CONCEX=SUMEX(6)*(CI-CS)+CS
        CONCAV=SUMAV(6)*(CI-CS)+CS
        CONCME=SUMME(6)*(CI-CS)+CS
120     FORMAT(3 E15.8)
        WRITE(*,*)'CONCEX,CONCAV,CONCME'
        WRITE(*,120)CONCEX,CONCAV,CONCME
        STOP
        END
```

```
C       PROGRAM TO FIND CONCENTRATION FROM MODEL 6
C       μ=A,K=R,L=CT,ε_t=PT,τ=T,C_e=CONCEX,C_d=CONCAV,C_m=CONCME
        DIMENSION A1(6),A2(6),P1(6),X1(6),Y1(6),U1(6),V1(6),W1(6)
        DIMENSION R(6),S(6),SUMEX(6),SUMAV(6),SUMME(6),E1(6)
        DOUBLE PRECISION A,T,A1,A2,SUMEX,SUMAV,SUMME
        DOUBLE PRECISION S,EN,P1,E1,X1,Y1,U1,V1,W1
        WRITE(*,*)'Pe,CT,PT,CI,CS,TW,R,U,R(I)'
        READ(*,*)Pe,CT,PT,CI,CS,TW,R,U,(R(I), I=1,6)
        A=(1.-PT)/PT
        T=U*TW/(CT*(1.+R*A))
        SUMEX(0)=0.0
        SUMAV(0)=0.0
        SUMME(0)=0.0
        DO 100 I = 1,6
        A1(I)=Pe**2+4.*R(I)**2
        A2(I)=A1(I)+4.*Pe
        P1(I)=-1.*A1(I)/(4.*Pe)
        E1(I)=EXP(P1(I)*T)
        S(I)=SIN(R(I))
        EN=EXP(Pe/2.)
        X1(I)=-32.*(R(I)**3)*Pe*EN/(P1(I)*S(I)*A1(I)*A2(I))
        U1(I)=X1(I)*E1(I)
        V1(I)=4.*Pe*U1(I)/A1(I)
        W1(I)=-1.*X1(I)*(1.-E1(I))/(T*P1(I))
        SUMEX(I)=SUMEX(I-1)+U1(I)
        SUMAV(I)=SUMAV(I-1)+V1(I)
        SUMME(I)=SUMME(I-1)+W1(I)
100     CONTINUE
        CONCEX=SUMEX(6)*(CI-CS)+CS
        CONCAV=SUMAV(6)*(CI-CS)+CS
        CONCME=SUMME(6)*(CI-CS)+CS
120     FORMAT(3 E15.8)
        WRITE(*,*)'CONCEX,CONCAV,CONCME'
        WRITE(*,120)CONCEX,CONCAV,CONCME
        STOP
        END
```

```
C       PROGRAM TO FIND CONCENTRATION FROM MODEL 7
C       μ=A,k1=R1,k2=R2,L=CT,εt=PT,I0(X)=OI,ERF(X)=ERF,ξ=Z,τ=T
        DOUBLE PRECISION A,A1,A2,A3,OI,ERF,Z,T,Z1,T1,T2,SUMEX,SUMAV,SUMME
        WRITE(*,*)'CT,PT,CI,CS,TW,R1,R2,U'
        READ(*,*)CT,PT,CI,CS,TW,R1,R2,U
        A=(1.-PT)/PT
        Z=R2*A*CT/U
        T=(R1/U)*(U*TW-CT)
        R=R2/R1
        Z1=EXP(-1.*Z)
        T1=EXP(-1.*T)
        T2=T-(CT/U)
        A1=((Z/T)**0.125)/(1.+(Z/T)**0.125)
        A2=EXP(-1.*(Z+T))
        A3=SQRT(Z)-SQRT(T)
        OI=1.+(Z*T)+((Z*T)**2)/4.
        ERF=1.1283792*(A3-((A3**3)/3.)+((A3**5)/10.))
        SUMEX=(1.+ERF)/2.-A2*OI*A1
        SUMAV=(Z-2.+(2.+Z)*Z1+T1-(1.+Z)*A2)/Z
        SUMME=(1./T)*(T-T2*(1.+Z)*Z1-Z*Z1*EXP(-1.*R2*T2)/R2+Z*Z1/R2)
        CONCEX=SUMEX*R*(CI-CS)+CS
        CONCAV=SUMAV*R*(CI-CS)+CS
        CONCME=SUMME*R*(CI-CS)+CS
100     FORMAT(3 E15.8)
        WRITE(*,*)'CONCEX,CONCAV,CONCME'
        WRITE(*,100)CONCEX,CONCAV,CONCME
        STOP
        END
```

```
C       PROGRAM TO FIND CONCENTRATION FROM MODEL 8
C       μ=A,K*=R,L=CT,εt=PT,I0(X)=OI,ERF(X)=ERF,ξ=Z,τ=T
        DOUBLE PRECISION A,A1,A2,A3,OI,ERF,Z,T,Z1,T1,T2,SUMEX,SUMAV,SUMME
        WRITE(*,*)'CT,PT,CI,CS,TW,R,U'
        READ(*,*)CT,PT,CI,CS,TW,R,U
        A=(1.-PT)/PT
        Z=R*A*CT/U
        T=(R/U)*(U*TW-CT)
        Z1=EXP(-1.*Z)
        T1=EXP(-1.*T)
        T2=T-(CT/U)
        A1=((Z/T)**0.125)/(1.+(Z/T)**0.125)
        A2=EXP(-1.*(Z+T))
        A3=SQRT(Z)-SQRT(T)
        OI=1.+(Z*T)+((Z*T)**2)/4.
        ERF=1.1283792*(A3-((A3**3)/3.)+((A3**5)/10.))
        SUMEX=(1.+ERF)/2.-A2*OI*A1
        SUMAV=(Z-2.+(2.+Z)*Z1+T1-(1.+Z)*A2)/Z
        SUMME=(1./T)*(T-T2*(1.+Z)*Z1-Z*Z1*EXP(-1.*R*T2)/R+Z*Z1/R)
        CONCEX=SUMEX*(CI-CS)+CS
        CONCAV=SUMAV*(CI-CS)+CS
        CONCME=SUMME*(CI-CS)+CS
100     FORMAT(3 E15.8)
        WRITE(*,*)'CONCEX,CONCAV,CONCME'
        WRITE(*,100)CONCEX,CONCAV,CONCME
        STOP
        END
```