LOAD DISTRIBUTION IN GIRDER BRIDGES BY DIFFERENT METHODS

A DISSERTATION

Submitted in partial fulfilment of the
requirements for the award of the degree
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CANDIDATE'S DECLARATION

I hereby certify that the work which is presented in the dissertation entitled 'LOAD DISTRIBUTION IN GIRDER BRIDGES BY DIFFERENT METHODS' in partial fulfilment of the requirements for the award of Master of Engineering in Civil Engineering with specialisation in Structural Engineering, University of Roorkee, is an authentic record of my own work carried out during the period from August 1987 to June 1988, under the supervision of Dr. Jagdish Prasad, Lecturer in Structural Engineering Section, Civil Engineering Department, University of Roorkee, Roorkee.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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(ANIL KUMAR BINJOLA)

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SYNOPSIS

Girder bridges which are generally simply supported have been found to suit the small and medium span coverage both from point of view of structural efficiency of the deck behaviour as well as ease of construction and economy. Both conventional as well as computer based methods of analysis for girder bridges are available. Some of the conventional methods are briefly presented in the various chapters of this dissertation. These methods are (1) Courbon's Method (2) Orthotropic Plate theory based (a) Morice, Little and Rowe curves ; and (b) Cusens and Pama curves ; and (3) Harmonics method. The assumptions and limitations with regard to the mathematical modeling of girder bridges by these methods have been discussed. From the application point of view a typical 4-girder right bridge having a span of 19.4 m and carriage width of 7.5 m has been analyzed for two critical load positions of IRC class-A loading. A comparative study of the results for girder bending moments and deflections as obtained by the above mentioned methods has been carried out and certains conclusions arrived at.

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CONTENTS

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NOTATIONS

1. COURBON'S METHOD

I_i	M.I. of girder i about its own bending axis
R_i	Share of loading by girder i
X_i	distance of girder i from bridge axis
$\bar{X}$	C.G./ Bridge axis location from girder 1

2. ORTHOTROPIC PLATE THEORY

a	Half span of bridge deck
b	Half width of bridge deck
D_1, D_2	Coupling rigialities with respect to x and y directions
D_x, D_y	Flexural rigidities in x and y directions
D_{xy}, D_{yx}	Torsional rigidities in x and y directions
E_x, E_y	Direct moduls of eleusticity in x and y directions
G	Shear modulus of elasticity
H	Torsional plate rigidity
I	Moment of inertia
i	M.I. per unit width
I	Polar moment of inertia
j	Polar moment of inertia per unit width

K	Distribution coefficient
L	Span (L = 2 a)
M_x, M_y	Bending moments in x and y directions
M_{xy}, M_{yx}	Torsional moment about x and y axes
γ	Roots of characteristic equation
Q_x, Q_y	Shearing forces related to x and y directions
α	Torsional Parameter $[\frac{G(i_o+j_o)}{2E\sqrt{ij}}]$
α_n	$n\pi/L$
β	$n\pi br/L$
γ_{xy}	shearing strain in x-y plane
θ	Flexural parameter $(\frac{b}{2a}(i/j)^{1/4})$
ν_x, ν_y	Poisson's Ratio in x and y directions
σ_x, σ_y	Directe stress in x and y directions
τ_{xy}	Shear stress normal to x direction in x - y plane
ϕ_n	Roots of characteristics equation

3. HARMONICS METHOD

A_j, B_j, C_j	Deflection Coefficients of girder j due to applied load w_{ja}
a_j, b_j, c_j	Deflection coefficients of girder j due to retained load w_{jr}

d_j, e_j, f_j Rotation coefficients for girder j

E Young's modulus of elasticity of material

G Shear modulus of elasticity of material

G_j Girder j

$F_{j1}(k), F_{j2}(k)$ First, second and third harmonic coefficients of girder j due to the total shear force generated as a result of deformations of transverse system.

$F_{14}(k), F_{24}(k)$ $F_{34}(k)$ Torque equilibrium expressions for girders 1,2 and 3.

$F_{j5}(k), F_{j6}(k)$ Torque-twist expressions for girder j

h Transverse spacing of girders

I_j Moment of inertia of cross-section of girder j

I_T Moment of inertia of transverse system

J_j Torsional constant of girder, j

J_T Torsional constant of transverse system

L Skew span of bridge

m_{12} Bending moment per unit length of transerse system connecting girder 1 and 2 at the interface of girder 1 (similarly m_{21}, m_{23} and m_{32})

M_{ja} Moment function as applied load on girder j

M_{js} B.M. due to shear force generated at the interface of transverse system with girder j

M_j B.M. function for girder j

$M_{ij}(x_i)$ B.M.function for girder i due to loading on girder j

M_{ij}^{m}	mth harmonic coefficient of B.M. function for girder i due to loading on girder j
$M_i(x_i)$	B.M. function for girder i due to loads simultaneously acting on more than one girder
p_{ij}^{mn}	Design coefficient defined as the mth harmonic of B.M. in girder i due to n th harmonic of moment loading $(\sin(n\pi x/L))$, n = 1,2,3) on girder j
B.M.	Bending moment
S_{jf}	Shear force generated at interface with girder j due to flexural deformation of transverse system
S_{jt}	Shear force generated at girder j due to torsional deformation of transverse system
T_{jt}	Torsion generated at girder j due to torsional deformation of transverse system acting as B.M. (M_{jt}) on girder j
w_{ja}	Applied load on girder j
w_{jr}	Retained load on girder j
w_j^1, w_j^2, w_j^3	First, second and third harmonic coefficients respectively of load w on girder j
x_j	Distance along length of girder j
y_1, y_2, y_3	Deflection of girder 1,2 and 3 respectively
$\theta_1, \theta_2, \theta_3$	Rotation of girder 1.2 and 3 respectively at any cross-section of bridge deck
λ	Skew angle

$\phi_j = \left(\frac{dy}{dx}\right)_j$	slope of girder j
α, β, γ, K	Non-dimensional structural parameters

$$\alpha_j = (12/\pi^4)(L/h)^3(EI_T/EI_J)$$

$$\beta_j = (\pi^2/2)(h/L)(GJ_j/EI_T)$$

$$\gamma = \pi^2/12(h/L)^2(GJ_T/EI_T)$$

$$K = (h/L)\tan\lambda$$

$$\alpha_j' = \frac{\alpha_j}{(1-k)}$$

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CHAPTER - 1

INTRODUCTION AND LITERATURE REVIEW

1.1 GENERAL

Interconnected girder bridges are the most widely used one, despite its complexicity in analysis. The introduction of an increasingly vehicular loading has emphasised the need for a better understanding of the way, they function.

Girder bridges, in general, consist of several parallel longitudinal girders, connected through deck slab and if necessary, through cross beams or diaphragms. In almost all highway projects, the majority of bridges needed are of short span, for which girder bridge are found to be suitable. Girder bridges are generally simply supported and therefore, simply supported girder bridges have especially been chosen for the present study.

1.2 LOAD DISTRIBUTION THEORIES

The fundamental problem in present day bridge design is to determine the effect of a single concentrated load on the structure. Due to interconnection of bridge deck with longitudinal giruers and cross beams, loads acting on a girder or bridge deck get distributed to other girders. The increased need of economy in construction and modern practice of transporting very heavy loads by road bridges has given rise to the need for an easily applied and fairly accurate method of load distribution amongst various girders. The importance of this can be recognised by the fact that between the first idea of Engesser (1889) on the subject and the present day an extensive amount of literature has been published. However many of the theories are very complex to apply and require a fair amount of calculations. The best theory shall be one which will give a design formula in very simplified form, preferebly in the form of charts or graphs (16).

Various methods of girder bridge analysis basically fall into three categories based on the assumptions made with regard to their construction.

The first category covers those analyses which divide the structure into individual longitudinal and transverse member each possesing the appropriate flexural and torsional rigidities. For each point of intersection of members equations of deflection and slope compatibility can be set up and finally a set of simultaneous equations can be solved. This general approach is extremely combersome, involving a great deal of arithmetical work and can't be generalised.

The second category covers those analyses which separates the longitudinal members of structures and considers some form of secondary cross connection which represents the behaviour of transverse members. Hetenyi (14) assumed that there was no rotation of individual member at an intersection and used a sine series to represent the load and deflection of the grillage in the direction of longitudinal members. Pippard and Dewode (21) assumed that the longitudinal members did not rotate and replaced the transverse members by a continuous medium. Leonhardt (16) assumed that the transverse member could be replaced by a single member at mid span with zero torsional stiffness. Most of these assumptions are invalid in practical bridge structures, where the torsional stiffness of members, particularly in reinforced and prestressed concrete, may be considerable. Further, the methods again do not lend themselves to generalisation for an unspecified load position and are, as in the first category very combersome to use. Hendry and Jaeger (13) have developed considerably the basic approach outlined above based upon only one simplifying assumption that the transverse members can be replaced by uniform continuous transverse medium of equivalent stiffness.

The third and final category covers those analyses which are based on anisotropic or orthotropic plate theory. These analyses replace the actual bridge structure by an equivalent orthotropic plate which is then treated according to classical theory. This particular approach has the merit that a single set of distribution coefficients for two extreme cases of no torsion grillage and full torsion slab, enables the distribution behaviour of any type of bridge structure to be found.

Literature reviews of different methods commonly adopted for girder bridge analysis is briefly given here under. The various methods of girder bridge analysis are

(1) Courbon's Method

(2) Orthotropic Plate Theory

(3) Harmonics Method

(4) Grillage Analogy

(5) Finite Strip Method

(6) Finite Element Method

(1) Courbon's Method

This is one of the early methods which for its simplicity in application has been commonly used for analysing the girder bridges. This method assumes transverse members as rigid and hence bridge deck bends bodily and does not change its shape when loads are applied on deck. The formulation of mathamatical model is presented in Chapter - 2.

(2) Orthotropic Plate Theory

The basis for the analysis by this method is that the actual bridge deck is replaced by an equivalent orthotropic plate which is then treated according to classical plate theory. Guyon (10) was the first to solve a case of simply supported grillage beams with negligible torsional stiffness by this method. Later Massonet (14) extended the method to include the torsional stiffness of the deck. He has also given formulae for the distribution coefficients for any particular value of torsional parameter.

It must be taken as a contribution of Morice and Little (15) that they adopted the method of Guyon and Massonet. A considerable amount of work has been done at the research station of the Cement and Concrete Association of England by Morice, Little and Rowe (15,20). The works have confirmed the applicability of the method to a wide range of bridge types and indicated a high degree of accuracy. In the original paper of Guyon and Massonet a limited number of values for the distribution coefficient K were derived. However in a later publication Massonet presented some comprehensive tables giving the values of the distribution coefficient K for values of torsional parameter α, of zero and unity. Rowe presented these tables in the form of design curves.

Rowe (20) considered the effect of poisson's ratio on the load distribution. Massonat (14) introduced a new coefficient for calculation of the torsional moment. He also extended this theory to edge stiffened bridge neglecting torsional stiffness. The calculation for edge stiffened girder bridges were also dealt by Little and Rowe (15,20). Rowe has further given the load distribution theory for no torsional bridges with various support conditions.

Cusens and Pama (7) incorporated the coupling rigidities into account. They also gave the curves for torssionally stiff anf flexurally soft bridges.

The design curves developed on the basis of this method can be used for any simply supported right bridge or bridges having skew angle upto 20°. Due to its simplicity in application it is one of the most widely used method in design offices and is discribed in detail in Chapter - 3.

(3) Harmonics Method

Harmonics method lends a very powerful tool to the analysis of girder bridges (right or skew). Unlike other methods, the unique feature of this method is that it lends itself to develope design coefficients. These coefficients serves as a highly suitable aid for determining the girder bending moments, shear forces etc. due to any type of imposed loading on the deck. The development of these coefficients is possible through the dimensionless structural parameters α,β,γ and K introduced in the formulation of this method. These parameters uniquely combine for a particular skew girder bridge to identify it completely insofar as its structural behaviour is concerned. Thus taking advantage of this capability of the method, design coefficients have been made available for different combinations of these parameters covering all real-life bridges for ready use in design offices. These coefficients for all real life 4-girder skew bridges are given in reference (22).

The concept of Harmonics method was first introduced by Hendry and Jaeger . They applied it to three and four girder skew bridges, neglecting torsional stiffness and considering only the first term of the fourier series of loading. Later this method was modified by Surana, Agrawal and Prasad, to incorporate the torsional stiffness of transverse system and using more general deflection function (22,18,1).

In Harmonics method, applied loading on a girder is broken down into harmonic components, which are easily obtainable using fourier analysis (Appendix - A). Each harmonic component is distributed separately amongst the girders using the design coefficients. The bending moment for any girder is found by adding together fraction of harmonics so distributed. All the loads acting between the girders are first converted into equivalent girder loads, by assuming the deck as a continuous beam in transverse direction. The method lends itself to channelize the process of computing girder bending moments etc. for any imposed loading. This results in a systematic computation procedure making the calculation work simple and quick. The method has been were established for skew girder bridges (22) and demand a considerable amount of theoretical and experimental research to improve and test its applicability for other forms of decks such as, Box-girder, multispan skew girder bridges. The theoretical formulation of harmonics method for girder bridges is presented in the Chapter - 4 and 5.

(4) Grillage Analogy

Grillage analogy is probably the most popular computer aided method for analyzing bridge decks. This is because it is easy to comprehend and use, relatively inexpensive and has proved to be reasonably accurate for a wide variety or bridge types (7). Lazarides (1952) and Hendry and Jaeger (12) used grillage analysis but were severly limited in scope since hand methods to be used for solution of simultaneous equations. The method pioneered for computer used by Lightfoot and Sawko (17) involves the idalization of the bridge deck through its representation as a plane grillage of discrete interconnected beams.

Although the method is necessarily approximate, it has the great advantage of almost complete generality. At the joints of the grillage, any normal form of restraint to movement may be applied so that any support condition may be represented.

The dispersed bending and torsion stiffnesses in every region of the slab are assumed, for the purpose of analysis to be concentrated in the nearest equivalent grillage beam. The slab's longitudinal stiffnesses are concentrated in the longitudinal beams while the transverse stiffnesses are concentrated in the transverse beams. Ideally, the beam stiffnesses should be such that when prototype slab and equivalent grillage are subjected to identical loads, the two structure should deflect identically and the moments, shear forces and torsion in any grillage beam should equal the resultants of the stresses on the cross-section of the part of the slab the beam represents. For beam and slab decks, the logical selection of lingitudinal grillage beam is for them to coincide with the actual beams.

The problem is solved in this method by matrix method of structural analysis. The method, being completly computer oriented, becomes unsuitable where access to computer is absent. The method also fails to give high local moments and torques in the immediate neighbourhood of a load, which is concentrated in an area much smaller than grillage mesh.

(5) <u>Finite Strip Method</u>

The finite strip method is a hybrid procedure which combines some of the advantages of series solution of orthotropic plates with the finite element concept. The method can be applied both to slab and to cellular decks, which have the same form end to end. It was first forwarded by Cheung for rectangular slabs and suggested independently for the same problem by Powell and Ogden.

With the help of orthotropic plate theory it is possible to find a displacement function for simple support conditions, applicable to all region of the plate. When such a solution is not conveniently obtainable, the plate may be divided into discrete longitudinal strips spanning between supports. Simple displacement interpolation function may be used to represent displacement fields within and between individual strips.

The limitations of this method (7) are

(i) It is effectively applicable only to prismatic (right or circularly curved) structures with simply supported ends.

(ii) Each finite strip is assumed to have constant geometry and material properties in longitudinal (spanwise) direction.

(6) Finite Element Method

The most powerful of the techniques of analysis which arises from the direct stiffness approach is the finite element method. It employs an assemblage of discrete two and three dimensional elements to represent the structure. The elements are connected at nodal points which possess an appropriate number of degrees of freedom. Many shapes of element are available analysis time for the interpretation of results.

(ii) Expensive, as regard to computer time.

(iii) If the choice of element is incorrect the results can be far more inaccurate than those predicted by simple methods such as grillage method.

1.3 CONTENTS IN BRIEF

In Chapter - 1 a comprehensive literature review with regard to the various methods of bridge deck analysis commonly adopted is presented. Suitability or otherwise of a method of particular type of bridge is also indicated. Chapter - 2, deals exclusively with the mathematical formulation of the Courbon's method of analysis for a given bridge.

Chapter - 3 deals with orthotropic plate theory. Formulation of mathamatical model and preparation of design curves by Morice, Little and Rowe (Appendix-E). The improved applications of the orthotropic plate theory by Cusens and Pama is also presented in this chapter.

A generalized method of analysis for a three girder bridge deck aligned at a skew with the help of Harmonics Method is presented in Chapter - 4. The method involves four non dimensional structural parameters α, β, γ and K, which uniquely combine for a particular skew girder bridge insofar its structural behaviour is concerned (18).

Chapter - 5 presents a generalized mathamatical model for design coefficients under three standard sine harmonic loadings for any combination of α, β, γ and K. These design coefficients serve as a very useful design aid in evaluating girder moments for any configuration of the imposed loading. The systematic computation procedure has been developed in this chapter for computing girder moments (18).

Chapter - 6 deals with the application of various methods presented earlier, for analyzing a girder bridge. A single span 4-girder right bridge is analysed for live load moments under two configurations of IRC class - A loading.

Discussions and relevant conclusions draws are presented in Chapter - 7.

Appendix - A : Gives the fourier analysis of various loading cases encountered in practice.

Appendix - B : Explains the computation of torsional constant.

Appendix - C : Presents expressions for support moments based on three moment theorem which are used for equivalent load computation in Harmonics Method.

Appendix - D : Explains the procedure for picking up the design coefficients for a particular combination of α, β and K from already available design aid. Two sample tables of design coefficients are also presented herein.

Appendix - E : Contains the curves used for analysing the bridge deck with the help of orthotropic plate theory in Chapter - 6.

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CHAPTER - 2

COURBON'S METHOD

2.1 INTRODUCTION

Girder bridge decks are generally simply supported and interconnected in the transverse direction through slab, cross-beam and/or diaphragms. These transverse structural media, particularly the diaphragms and/or cross-beam are the agents of load distribution amongst the girders. Thus, the crux of the bridge girder design **lies** in ascertaining the share of the imposed load going to the various girders. Once loading on a girder is determined, the design is done in the conventional fashion as for a beam with simple supports at its ends. One of the earliest methods to determine the share of the imposed loading by various girders is Courbon's method which is based on comparatively gross assumptions but is quite simple in applications. This method is presented and discussed hereunder.

2.2 ASSUMPTIONS

The mathematical model for load distribution amongst the girders is based on assumptions primarily with regard to the deflection profile of the girders in the transverse direction.

1. Load distribution amongst the girders takes place along the transerse direction at right angle to the bridge axis.

2. Under a load acting at a point on the bridge axis,

the transverse section comprising all the girders deflect uniformly downward by the same amount.

3. Under a moment acting about the bridge axis, the transverse section comprising all the girders undergoes a rotation about the bridge axis causing linear deflection of the girders.

Assumption (1) cited above is quite close to the actual load distribution in a bridge deck. The stiffness of the transverse structural elements along the direction at ritht angle to the bridge axis is much higher than along any other directions and hence load transmission takes place chiefly along an axis at right angle to the bridge axis. However, assumptions (2) and (3) may be considered over simplified since the transverse deflection profile is non-linear. The non-linear transverse deflection profile induces complexity in the mathematical model for load distribution. These assumptions, though gross, achieve a lot of simplicity in the mathematical model and result in a little conservative and safe design. Indian Roads Congress (IRC) recommends the use of this method primarily for its simplicity in application and safety in design.

2.3 MATHEMATICAL MODEL :

Courbon's mathematical model based on the earlier mentioned assumtions is pictorially presented in Fig. 2.1(a),(b) and (c). In Fig. 2.1(a), the simply supported girder bridge deck is depicted with different moment of inertia for girder (I_i). The transverse system is notionally depicted by the broken transverse lines located at the actual locationns of cross-beam and/or diaphragms.

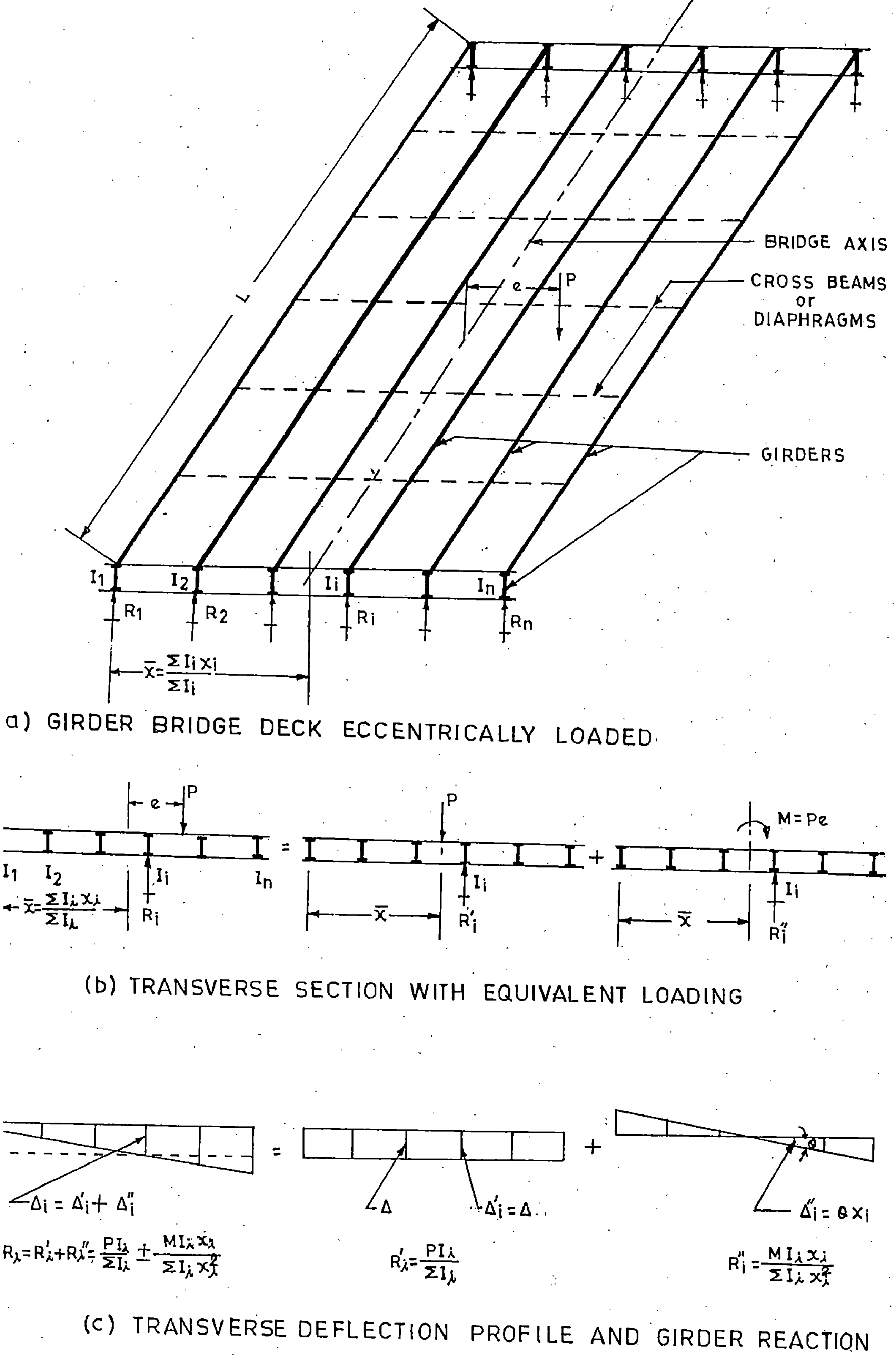

FIG. 2.1 – COURBON'S MODEL

A transverse section at right angle to the bridge axis and the location of the load is shown in Fig. 2.1(b) with the applied load acting eccentrically. The eccentric load is replaced by its equivalent load in terms of a direct load (P) and a moment ($M = P_e$) acting at the centre of gravity (C.G) of the section. The C.G. of the section is located at the bridge axis which is also the axis of rotation and neutral axis for the moment ($M = P_e$). The bridge axis may be located in the usual way i.e.,

$$\bar{X} = \frac{\Sigma\, I_i X_i}{\Sigma\, I_i} \qquad \ldots(2.1)$$

where in

$\bar{X}$ = C.G./Bridge axis locations from girder 1.

I_i = Moment of inertia of girder i about its own bending axis.

X_i = distance of girder i from girder 1.

The concentrated load (P) acting at the bridge axis causes uniform vertical deflection (Δ) in all the girders as per assumption (2). This is shown in Fig. 1(c). Since all the girders deflect by the same amount (Δ), each of the girders attracts a part of the load (P) according to its relative stiffness. Thus, girder i attracts its share of the applied load (P) as given below,

$$R_i' = \frac{P\, I_i}{\Sigma\, I_i} \qquad \ldots(2.2)$$

Along the load R_i' due to vertical deflection, the girder also attracts load due to rotation of the deck. The amount of this load depends upon the distance of the girder (X_i) from the bridge axis and relative stiffness of the girder. The magnitude of this load is given by,

$$R_i'' \pm \frac{M\ I_i X_i}{\Sigma\ I_i X_i^2} \qquad ...(2.3)$$

The positive sign for R_i'' is taken for girders which lie on the applied load (P) side of the bridge axis. Negative sign for R'' is taken for girders lying on the other of the bridge axis where the load (P) is not acting. The total load shared by girder i is then given by adding (2.2) and (2.3) i.e.,

$$R_i = R_i' \pm R_i'' = \frac{P\ I_i}{\Sigma\ I_i} \pm \frac{M\ I_i X_i}{\Sigma\ I_i X_i^2} \qquad ...(2.4)$$

After girder load R_i is determined from (2.4), it is designed for bending moment and shear due to this load R_i with the beam being simply supported at its ends. Expression (2.4) for girder load (R_i) becomes much simpler if all the girders have the same moment of inertia (I_i = I) and is given by

$$R_i = \frac{P}{N} \pm \frac{M\ X_i}{\Sigma\ X_i^2} \qquad ...(2.5)$$

where in N is the number of girders in the deck.

2.4 LIMITATIONS :

Apart from the gross assumption on which the mathematical model is based, Courbon's method is applicable only the following conditions are satisfied.

(1) The ratio of span to width is greater than 2 but less than 4.

(2) The longitudinal beams are interconnected by at least five symmetrically spaced cross girders/or diaphragms.

(3) The cross girders extend to a depth of at least 3/4th of the depth of the longitudinal girders.

The above three conditions are generally satisfied in all of the modern T-beam bridges.

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CHAPTER - 3

ORTHOTROPIC PLATE THEORY

3.1 INTRODUCTION

The concept of considering an actual bridge deck as an equivalent plate for the purpose of determining the distribution of stresses is well established. Ordinary flexibility and stiffness methods of analysis called open grillage become more complicated if the number of beams is more. Also, these methods turn more and more difficult if the torsional rigidity of elements is considered (20). Further, a bridge is never an open grillage, as there is a connecting slab. Therefore, it may be considered quite reasonable to replace the interconnected bridge deck by an orthotropic plate with the same total stiffness in two direction as the original structure.

An orthotropic plate is defined as one which has different specified elastic properties in two orthogonal directions. Most bridge decks are orthotropic because of shape orthotropy and it must be emphasized that reasonable results can only be obtained if the deck is made up of multiple longitudinal beams. For a right simply supported deck the number of transverse beams is less important. It is very difficult to give the precise number of beams to be present in the actual deck before it can be idealised as an orthotropic plate, but in normal circumstances five longitudinal beams may be required as safe minimum number (7).

3.2 BRIDGE DECK IDEALIZATION

The basic assumptions adopted in the development of mathamatical model are those commonly used in the theory of elastic, isotropic, thin plates (23). The main assumptions are as fallows

(i) The actual bridge deck is replaced by an equivalent orthotropic plate having same total stiffness in two directions as the original deck. The torisional stiffness is also kept same.

(ii) Plane section perpendicular to the neutral plane, remains plane and perpendicular to the deflected neutral plane.

(iii) Deflection of plate is very small as compared to the thickness of plate.

(iv) The neutral plane can be taken as x-y plane.

(v) $E_x \nu_y = E_y \nu_x$.

(vi) Material of the bridge deck is linearly elastic, homogeneous and isotropic.

(vii) Bridge deck is right angled and simply supported.

(viii) It is taken as plane stress problem.

(ix) Intersecting beams are assumed to be rigidly connected at their point of intersections.

3.3 MATHEMATICAL MODEL

The actual bridge deck is replaced by an equivalent orthotropic plate of effective width 2b and effective span

L = 2a. The plate is simply supported along x = 0 and x = 2a and free along y = ± b as shown in Fig. 3.1.

For the element of an orthotropic plate, plane stress and strain are related by

$$\sigma_x = \frac{E_x}{(1-\nu_x\nu_y)}\ (\epsilon_x+\nu_y\epsilon_y),\ \sigma_y = \frac{E_y}{(1-\nu_x\nu_y)}\ (\epsilon_y+\nu_x\epsilon_x) \qquad ...(3.1)$$

$$\tau_{xy} = G\gamma_{xy}$$

wherein,

σ_x, σ_y = normal stresses along x and y axes.

ϵ_x, ϵ_y = normal strains along x and y axes.

τ_{xy} = Shearing stress along y-axis on x-plane.

γ_{xy} = shear strain

E_x, E_y = moduli of elasticity along x and y-axes

ν_x, ν_y = poission's ratio along x and y-axes

G = shear modulus of elasticity

We know from Betti's theorem that

$$E_x\nu_y = E_y\nu_x \qquad ...(3.2)$$

Applying strain displacement relationship, the stress-strain equations can be expressed in terms of the transverse deflection w in the form

$$\sigma_x = -\frac{E_x Z}{(1-\nu_x\nu_y)}\ \left(\frac{\partial^2 w}{\partial x^2} + \nu_y\ \frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = -\frac{E_y Z}{(1-\nu_x\nu_y)}\ \left(\frac{\partial^2 w}{\partial y^2} + \nu_x\ \frac{\partial^2 w}{\partial x^2}\right) \qquad ...(3.3)$$

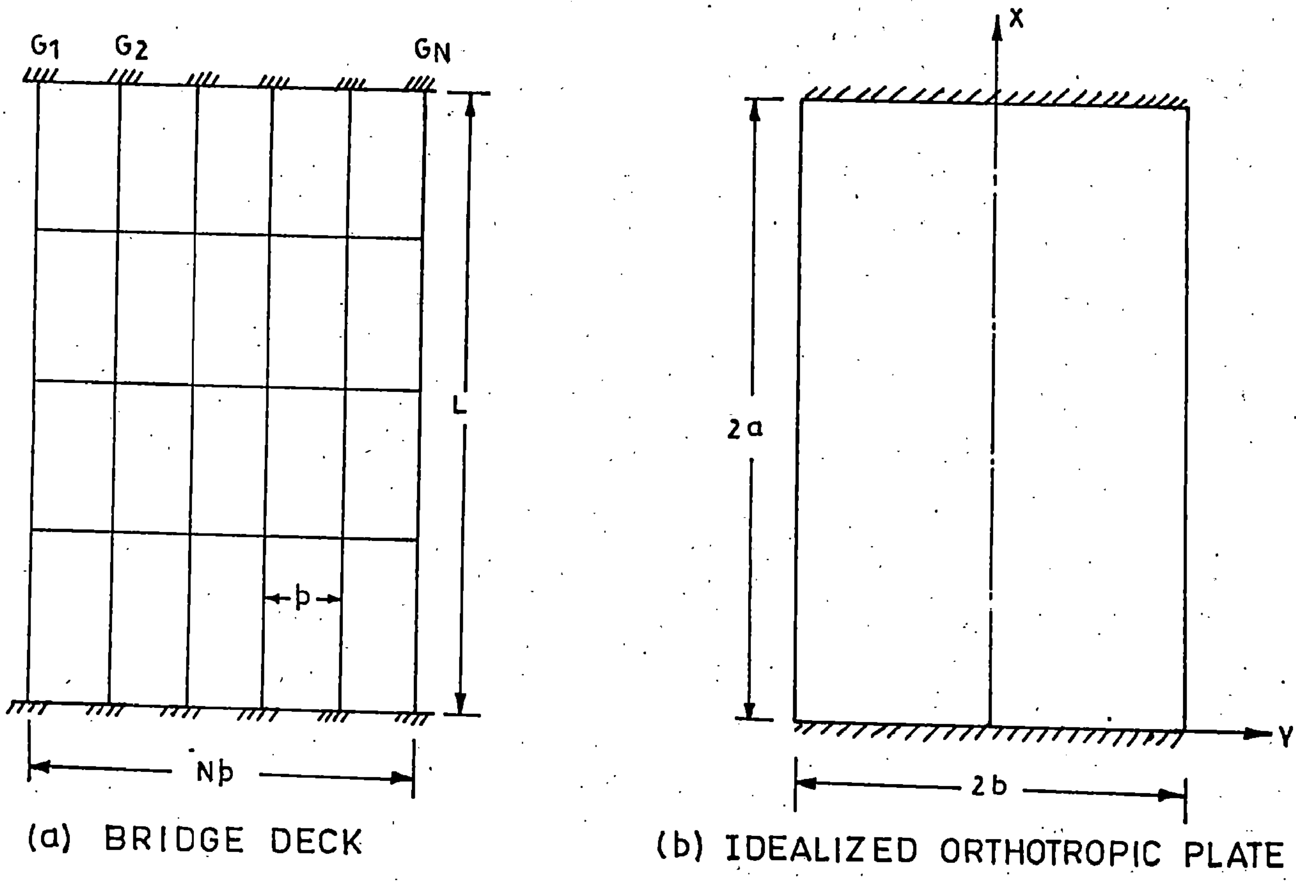


FIG. 3·1–BRIDGE DECK IDEALIZATION

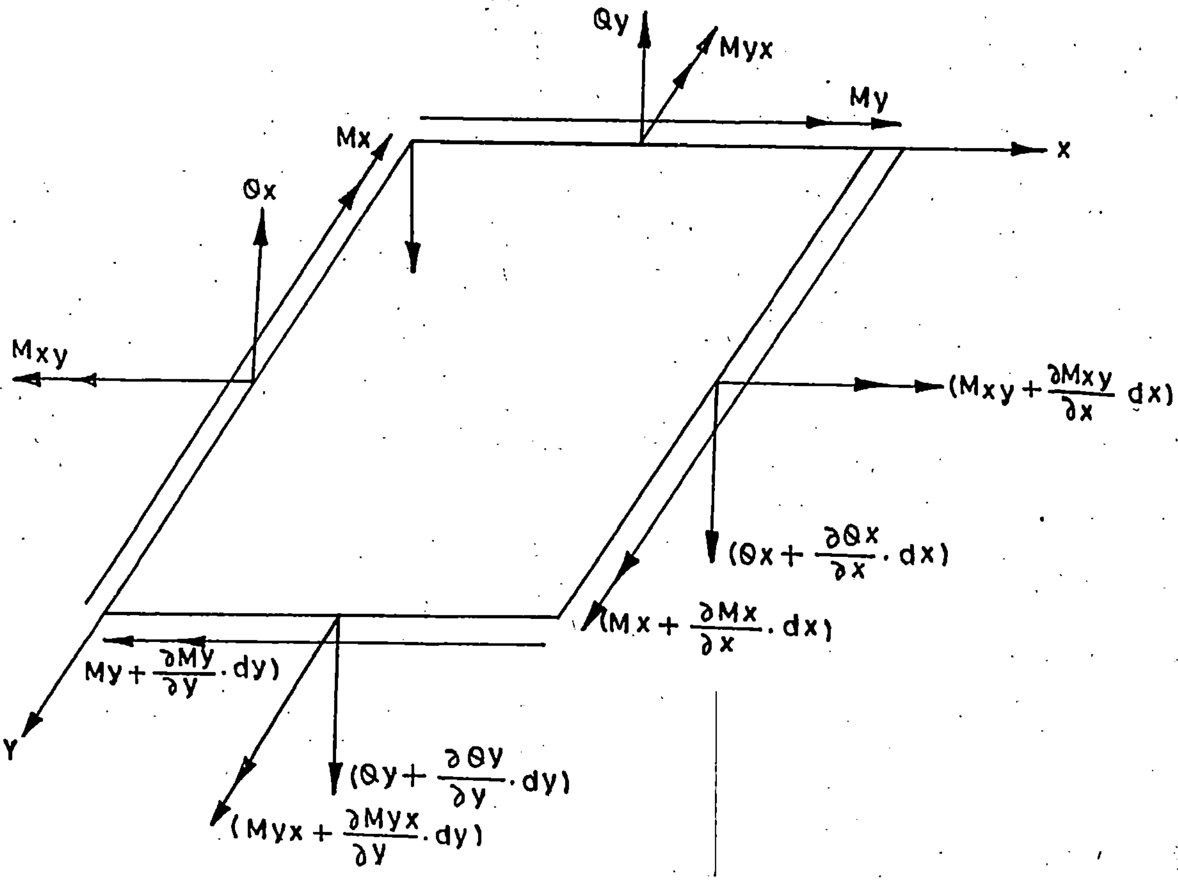


FIG. 3·2–STRESSED PLATE ELEMENT

$$\tau_{xy} = - 2GZ \frac{\partial^2 w}{\partial x \, \partial y}$$

The moment resultants are obtained as follows

$$M_x = \int_{A_x} \sigma_x . Z . dZ = - (D_x \frac{\partial^2 w}{\partial x^2} + D_1 \frac{\partial^2 w}{\partial y^2})$$

$$M_y = \int_{A_y} \sigma_y . Z . dZ = - (D_y \frac{\partial^2 w}{\partial y^2} + D_2 \frac{\partial^2 w}{\partial x^2}) \qquad ...(3.4)$$

$$M_{xy} = - \int_{A_x} \tau_{xy} . Z . dZ = - D_{xy} \frac{\partial^2 w}{\partial x \, \partial y}$$

$$M_{yx} = - \int_{A_y} \tau_{yx} . Z . dZ = - D_{yx} \frac{\partial^2 w}{\partial x \, \partial y}$$

wherein

D_x, D_y = flexural rigidities

D_1, D_2 = coupling rigidities

D_{xy},D_{yx}= torsional rigidities

Considering the equilibrium of forces and moments in Figure 3.2, We get the following equations

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \, \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x,y) \qquad ...(3.5)$$

wherein

$$2H = (D_{xy} + D_{yx} + D_1 + D_2)$$

The shearing forces Q_x and Q_y can be expressed as

$$Q_x = - [D_x \frac{\partial^3 w}{\partial y^3} + (D_{yx} + D_1) \frac{\partial^3 w}{\partial x \, \partial y^2}]$$

$$Q_y = - [D_y \frac{\partial^3 w}{\partial y^3} + (D_{xy} + D_2) \frac{\partial^3 w}{\partial y \, \partial x^2}] \qquad ...(3.6)$$

The reactions at free edge can be found by representing the twisting moment as vertical force

$$V_x = -\left[D_x \frac{\partial^3 w}{\partial x^3} + (D_{xy}+D_{yx}+D_1) \frac{\partial^3 w}{\partial x \, \partial y^2}\right] \quad \ldots(3.7)$$

$$V_y = -\left[D_y \frac{\partial^3 w}{\partial y^3} + (D_{yx}+D_{yx}+D_2) \frac{\partial^3 w}{\partial y \, \partial x^2}\right]$$

3.4 AN INFINITELY WIDE SIMPLY SUPPORTED BRIDGE DECK WITH SINUSOIDAL LOADING ALONG x-AXIS

The solution of non-homogeneous plate equation (3.5) can be obtained by adding the particular and homogeneous parts, thus

$$w = w_p + w_h \quad \ldots(3.8)$$

For particular solution Levy-Nadai solution will be adopted. This is done by considering an infinitely wide bridge deck (Fig. 3.3) subjected to a sinosodial loading along the x-axis,

$$p(x) = \sum_{n=1}^{\infty} H_n \sin \alpha_n x \, , \; \alpha_n = \frac{n\pi}{L} \quad \ldots(3.9)$$

The load function H_n can be derived for any particular loading case using Fourier Series. Some of them are given in Appendix - A.

Particular solution can be written in the form

$$w_p = \sum_{n=1}^{\infty} A \, e^{\phi_n y} \sin (\alpha_n x) \quad \ldots(3.10)$$

Which satisfies the simple support conditions. Substituting this value in equation 3.5 we get

$$D_y \phi_n^4 - 2H\alpha_n^2 \, \phi_n^2 + D_x \alpha_n^4 = 0 \quad \ldots(3.11)$$

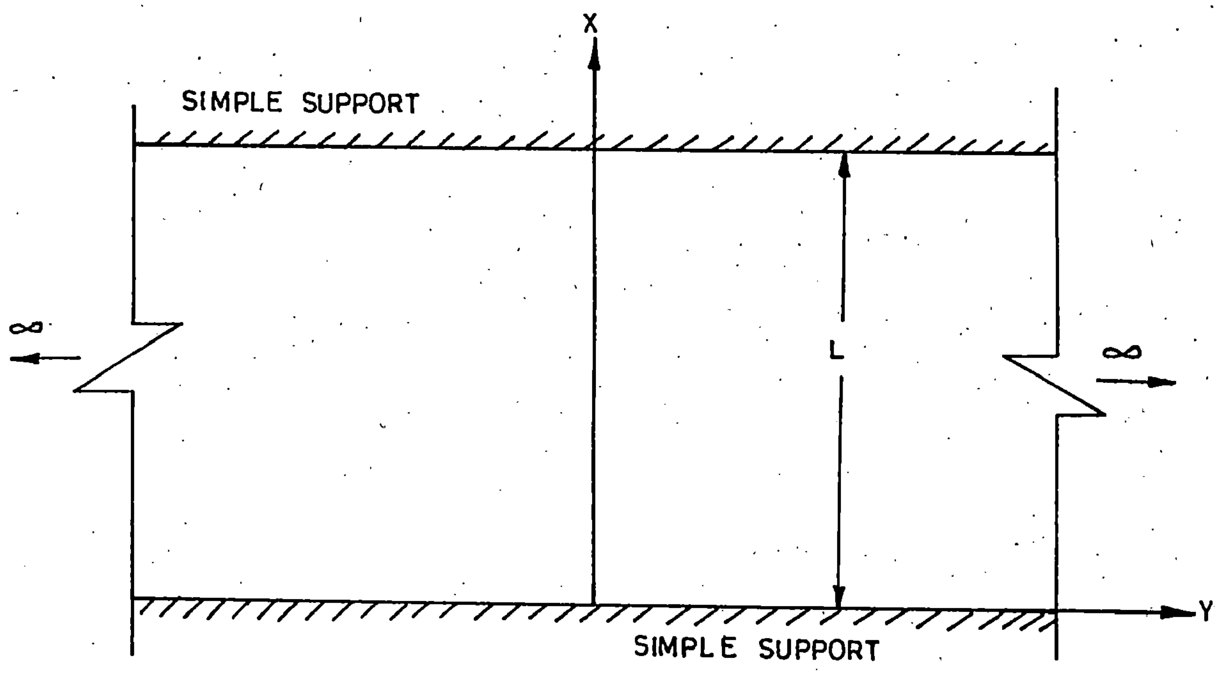


FIG 3·3 BRIDGE DECK OF INFINITE WIDTH

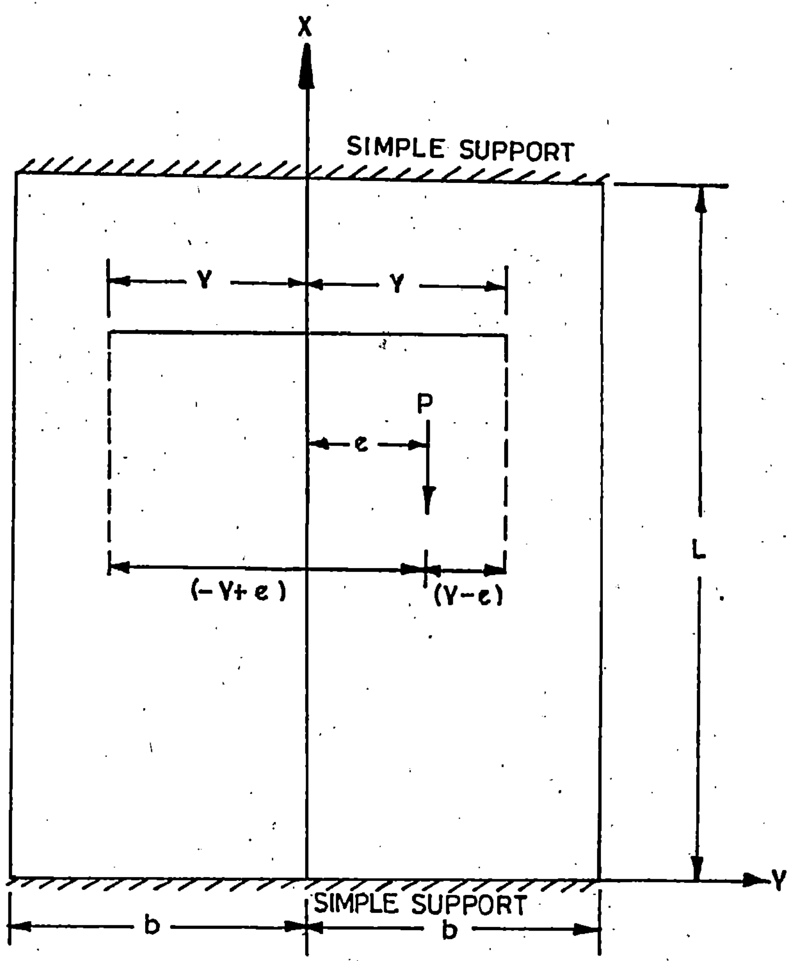


FIG. 3·4–BRIDGE DECK OF FINITE WIDTH

The roots of which are given by

$$\phi_{n_{1,2,3,4}} = \pm \left[\frac{H}{D_y} \pm \sqrt{\left(\frac{H}{D_y}\right)^2 - \left(\frac{D_x}{D_y}\right)}\right]^{1/2} \qquad \ldots(3.12)$$

Examining these roots it is observed that the following cases are possible

Case - I	$(H^2 > D_x D_y)$	Torsionally stiff and/or flexurally soft bridge decks
Case - II	$(H = D_x = D_y)$	Isotropic bridge decks
Case - III	$(H^2 < D_x D_y)$	Torsionally soft and/or flexurally stiff decks

Since most of the slab decks fall into the case-III, (7) only this particular category for the present study is taken. In this case, terms inside the inner most bracket of equation 3.12 will be imaginary. Hence roots of this equation can be written as

$$\phi_{1,2,3,4} = \pm \alpha_n\left[\left(\frac{\sqrt{(D_x/D_y)}+H/D_y}{2}\right)^{1/2} \pm i\left(\frac{\sqrt{(D_x/D_y)}-H/D_y}{2}\right)^{1/2}\right] \qquad \ldots(3.13)$$

Let

$$r_3 = \left[\frac{\sqrt{(D_x/D_y)} + H/D_y}{2}\right]^{1/2}$$

$$r_4 = \left[\frac{\sqrt{(D_x/D_y)} - H/D_y}{2}\right]^{1/2} \qquad \ldots(3.14)$$

Equation3.13 changes to

$$\phi_{1,2,3,4} = \pm \alpha_n[r_3 \pm i r_4] \qquad \ldots(3.15)$$

It is observed from Figure 3.3, that deflection w tends to zero as y tends to infinity, hence all the possitive roots have been discarded, and equation for deflection w_p takes the form as follows

$$w_p = \sum_{n=1}^{\infty} [A_n e^{-\alpha_n(r_3+ir_4)y} + B_n e^{-\alpha_n(r_3-ir_4)y}]\sin(\alpha_n x)$$

or

$$w_p = \sum_{n=1}^{\infty} [A_n \cos(\alpha_n r_4 y) + B_n \sin(\alpha_n r_4 y)] e^{-\alpha_n r_3 y} \sin(\alpha_n x)$$

Considering the symmetry of bridge deck, we get the boundary conditions

$$\left(\frac{\partial w_1}{\partial y}\right)_{y=0} = 0$$

and ...(3.16)

$$(V_y)_{y=0} + \frac{H_n}{2} \sin(\alpha_n x) = 0$$

Substituting these boundary conditions in equation (3.15) we get particular solution as

$$w_p = \sum_{n=1}^{\infty} \frac{H_n \sin(\alpha_n x)}{4D_y \alpha_n^3 (r_3^2 + r_4^2)} \left[\frac{1}{r_3} \cos(\alpha_n r_4 y) + \frac{1}{r_4} \sin(\alpha_n r_4 y)\right] e^{-\alpha_n r_3 y}$$

...(3.17)

The homogeneous solution of equation 3.5 can be found by Levy's method, which will be as follows

$$w_n = \sum_{n=1}^{\infty} [A_n \cosh(\alpha_n r_3 y) \cos(\alpha_n r_4 y) + B_n \cosh(\alpha_n r_3 y) \sin(\alpha_n r_4 y) + C_n \sinh(\alpha_n r_3 y) \cos \alpha_n r_4 y + D_n \sinh(\alpha_n r_3 y) \sin \alpha_n r_4 y] \sin \alpha_n x$$

...(3.18)

The complete solution will be sum of equation (3.17) and equation (3.18).

3.5 A BRIDGE OF FINITE WIDTH (2b) WITH A CONCENTRATED LOAD AT ECCENTRICITY (e) FROM x-AXIS

To generalize the expressions for deflection, we consider a bridge deck of finite width 2b, acted upon by a concentrated load p at eccentricity e. The solution is obtained by replacing y by $|y-e|$ in equation (3.17). The modulus value is used to ensure the symmetry of deflection for both positive and negative values of e. Now the complete solution can be written as follows

$$w = \sum_{n=1}^{\infty} \left[\frac{H_n \sin(\alpha_n x)}{4D_y \alpha_n^3 (r_3^2 + r_4^2)} \left(\frac{1}{r_3} \cos(\alpha_n r_4 |y-e|) + |y-e| + \frac{1}{r_4} \sin(\alpha_n r_4 |y-e|) \right) e^{-\alpha_n r_3 |y-e|} + (A_n \cosh(\alpha_n r_3 y) \cos(\alpha_n r_4 y) \right.$$

$$+ B_n \cosh(\alpha_n r_3 y) \sin(\alpha_n r_4 y) + C_n \sinh(\alpha_n r_3 y) \cos(\alpha_n r_4 y)$$

$$\left. + D_n \sinh(\alpha_n r_3 y) \sin(\alpha_n r_4 y)) \sin(\alpha_n x) \right] \qquad \ldots(3.19)$$

The four constants A, B, C and D are determined with the help of boundary conditions at edges, as given below

(a) $\quad M_y = 0 \quad$ at $y = \pm b$

or $\quad - \left(D_y \frac{\partial^2 w}{\partial y^2} + D_2 \frac{\partial^2 w}{\partial x^2} \right) = 0 \quad$ at $y = \pm b \qquad \ldots(3.20)$

(b) $\quad V_y = 0 \quad$ at $y = \pm b$

or $\quad - \left[D_y \frac{\partial^3 w}{\partial y^3} + (D_{xy} + D_{yx} + D_2) \frac{\partial^3 w}{\partial y \, \partial x^2} \right] = 0 \quad$ at $y = \pm b$

$\ldots(3.21)$

Value of deflection w as obtained in equation 3.19 is substituted in above equations to obtain these constants.

Cusens and Pama (7) found the values of these integration coefficients as given below

$$A = \frac{(S_1+S_2)\ d_3 - (S_3-S_4)\ d_1}{2(a_1 d_3 - a_3 d_1)} \qquad \ldots(3.22)$$

$$B = \frac{(S_1-S_2)\ c_3 - (S_3+S_4)\ c_1}{2(b_1 c_3 - c_1 b_3)} \qquad \ldots(3.23)$$

$$C = \frac{(S_3+S_4)\ b_1 - (S_1-S_2)\ b_3}{2(b_1 c_3 - c_1 b_3)} \qquad \ldots(3.24)$$

$$D = \frac{(S_3-S_4)\ a_1 - (S_1+S_2)\ a_3}{2(a_1 d_3 - a_3 d_1)} \qquad \ldots(3.25)$$

where,

$$S_1 = -\left(E_1 \cos(\beta_4\eta_1) + E_2 \sin(\beta_4\eta_1)\right) e^{-\beta_3\eta_1}$$

$$S_2 = -\left(E_1 \cos(\beta_4\eta_2) + E_2 \sin(\beta_4\eta_2)\right) e^{-\beta_3\eta_2}$$

$$S_3 = \left(E_3 \sin(\beta_4\eta_1) + E_4 \cos(\beta_4\eta_1)\right) e^{-\beta_3\eta_1}$$

$$S_4 = -\left(E_3 \sin(\beta_4\eta_2) - E_4 \cos(\beta_4\eta_2)\right) e^{\beta_b\eta_2}$$

$$a_1 = E_5 \cosh\beta_3 \cdot \cos\beta_4 + E_4 \sinh\beta_3 \cdot \sin\beta_4$$

$$b_1 = E_5 \cosh\beta_3 \cdot \sin\beta_4 - E_4 \sinh\beta_3 \cdot \cos\beta_4$$

$$c_1 = E_5 \sinh\beta_3 \cdot \cos\beta_4 + E_4 \cosh\beta_3 \cdot \sin\beta_4$$

$$d_1 = E_5 \sinh\beta_3 \cdot \sin\beta_4 - E_4 \cosh\beta_3 \cdot \cos\beta_4$$

$$a_3 = E_6 \sinh\beta_3 \cdot \cos\beta_4 - E_7 \cosh\beta_3 \cdot \sin\beta_4$$

$$b_3 = E_6 \sinh\beta_3 \cdot \cos\beta_4 + E_7 \cosh\beta_3 \cdot \cos\beta_4$$

$$c_3 = E_6 \cosh\beta_3 \cdot \cos\beta_4 - E_7 \sinh\beta_3 \cdot \cos\beta_4$$

$$d_3 = E_6 \cosh \beta_3 \cdot \sin \beta_4 + E_7 \sinh \beta_3 \cdot \cos \beta_4$$

and

$$E_1 = \frac{r_4}{(r_3^2 + r_4^2)} [D_2 + D_y(r_3^2 + r_4^2)]$$

$$E_2 = \frac{r_3}{(r_3^2 + r_4^2)} [D_2 - D_y(r_3^2 + r_4^2)]$$

$$E_3 = D_y(r_4^2 - r_3^2) + (D_2 + D_{xy} + D_{yx})$$

$$E_4 = 2D_y \; r_3 \; r_4$$

$$E_5 = D_2 - D_y(r_3^2 - r_4^2)$$

$$E_6 = r_3 \; (D_2 + D_{xy} + D_{yx}) - D_y(r_3^2 - 3r_3 \; r_4^2)$$

$$E_7 = r_4 \; (D_2 + D_{xy} + D_{yx}) + D_y \; (r_4^3 - 3r_4 r_3^2)$$

$$\alpha_n = n\pi/L$$

$$\beta_3 = \alpha_n b r_3$$

$$\beta_4 = \alpha_n b r_4$$

$$r_3 = [1/2\sqrt{(D_x/D_y)} + H/D_y]^{1/2}$$

$$r_4 = [1/2\sqrt{(D_x/D_y)} - H/D_y]^{1/2}$$

$$\eta_1 = (1 - e/b)$$

$$\eta_2 = (1 + e/b)$$

Now once the values of these coefficients are determined, these values are substituted in the equations (3.18), (3.4) and (3.6) to get the expressions for deflection, bending moments and shear forces. These stress resultants and deflection can be written as

(a) Deflection $w = \sum_{n=1}^{\infty} \frac{H_n \sin \alpha_n x}{\alpha_n^4 D_x \, 2b} K_1$ ...(3.26)

(b) Bending Moments

$$M_x = \sum_{n=1}^{\infty} \frac{H_n \sin(\alpha_n x)}{\alpha_n^2 \, 2b} \left(K_1 - \frac{D_1}{D_x} K_2\right) \qquad ...(3.27)$$

$$M_y = -\sum_{n=1}^{\infty} \frac{H_n \sin(\alpha_n x)}{\alpha_n^2 \, 2b} \left(\frac{D_y}{D_x} K_2 - \frac{D_2}{D_x} K_1\right) \qquad ...(3.28)$$

(c) Twisting Moments :

$$M_{xy} = -\sum_{n=1}^{\infty} \frac{H_n \cos(\alpha_n x)}{\alpha_n} \left(\frac{D_{xy}}{D_y} K_3\right) \qquad ...(3.29)$$

$$M_{yx} = \sum_{n=1}^{\infty} \frac{H_n \cos(\alpha_n x)}{\alpha_n} \left(\frac{D_{yx}}{D_y} K_3\right) \qquad ...(3.30)$$

(d) Shearing Forces

$$Q_x = \sum_{n=1}^{\infty} \frac{H_n \cos(\alpha_n x)}{\alpha_n \, 2b} \left[K_1 - \left(\frac{D_{yx} + D_1}{D_x}\right) K_2\right] \qquad ...(3.31)$$

$$Q_y = -\sum_{n=1}^{\infty} H_n \sin(\alpha_n x) \left[K_4 - \left(\frac{D_{xy} + D_2}{D_y}\right) K_3\right] \qquad ...(3.32)$$

(e) Reactions

$$V_x = \sum_{n=1}^{\infty} \frac{H_n \cos(\alpha_n x)}{\alpha_n \, 2b} \left[K_1 - \left(\frac{D_{xy} + D_{yx} + D_1}{D_x}\right) K_2\right] \qquad ...(3.33)$$

$$V_y = -\sum_{n=1}^{\infty} H_n \sin \alpha_n x \left[K_4 - \left(\frac{D_{xy} + D_{yx} + D_1}{D_y}\right) K_3\right] \qquad ...(3.34)$$

where in

$$K_1 = \frac{\alpha_n b D_x}{2 r_3 r_4 D_y} \Big[\frac{(r_4 \cos(\beta_4 S_1) + r_3 \sin(\beta_4 S_1)) \, e^{-\beta_3 S_2}}{(r_3^2 + r_4^2)}$$

$$+ A \cosh(\beta_3 S_1) \cos(\beta_4 S_1) + B \cosh(\beta_3 S_1) \sin(\beta_4 S_1)$$

$$+ C \sinh(\beta_3 S_1) \cos(\beta_4 S_1) + D \sinh(\beta_3 S_1) \sin(\beta_4 S_1) \Big] \qquad ...(3.35)$$

$$K_2 = \frac{\alpha_n bD_x}{\partial r_3 r_4 D_y} [r_3 \sin(\beta_4 S_2) - r_4 \cos(\beta_4 S_2)) e^{-\beta_3 S_1}$$

$$+A[(r_3^2-r_4^2)\cosh(\beta_3 S_1)\cos(\beta_4 S_1) - 2r_3 r_4 \sinh(\beta_3 S_1)\sin(\beta_4 S_1)]$$

$$+ B[(r_2^2-r_4^2)\cosh(\beta_3 S_1)\sin(\beta_4 S_1) + 2r_3 r_4 \sinh(\beta_3 S_1)\cos(\beta_4 S_1)]$$

$$+ C[(r_3^2-r_4^2)\sinh \beta_3 81 \cos(\beta_4 S_1) - 2r_3 r_4 \cosh(\beta_3 S_1)\sin(\beta_4 S_1)]$$

$$+ D[(r_3^2-r_4^2)\sinh(\beta_3 S_1)\sin(\beta_4 S_1) + 2r_3 \cosh(\beta_3 S_1)\cos(\beta_4 S_1)]] \quad ...(3.36)$$

$$K_3 = \frac{1}{4r_3 r_4} [\pm(-\sin(\beta_4 S_2))e^{-\beta_2 S_2}$$

$$+ A[r_3 \sinh(\beta_2 S_1)\sin(\beta_4 S_1) + r_4 \cosh(\beta_3 S_1)\cos(\beta_4 S_1)]$$

$$+ B[r_3 \sinh(\beta_3 S_1)\sin(\beta_4 S_1) + r_4 \cosh(\beta_3 S_1)\cos(\beta_4 S_1)]$$

$$+ C[r_3 \cosh(\beta_3 S_1)\cos(\beta_4 S_1) - r_4 \sinh(\beta_3 S_1)\sin(\beta_4 S_1)]$$

$$+ D[r_2 \cosh(\beta_3 S_1)\sin(\beta_4 S_1) + r_4 \sinh(\beta_3 S_1)\cos(\beta_4 S_1)]] \quad ...(3.37)$$

$$K_4 = \frac{1}{4r_3 r_4}[\pm [(r_4^2-r_3^2)\sin(\beta_4 S_2) + 2r_3 r_4 \cos(\beta_4 S_2)] e^{-\beta_3 S_2}$$

$$+ A[(r_3^2-3r_3 r_4^2)\sinh(\beta_3 S_1)\cos(\beta_4 S_1) + (r_4^3 r_3^2)\cosh(\beta_3 S_1)\sin(\beta_4 S_1)]$$

$$+ B[(r_3^2-3r_3 r_4^2)\sinh(\beta_3 S_1)\cos(\beta_4 S_1) + (r_4^3-3r_4 r_3^2)\cosh(\beta_3 S_1)\cos(\beta_4 S_1)]$$

$$+ C[(r_3^3-3r_3 r_4^2)\cosh(\beta_3 S_1)\cos(\beta_4 S_1) + (r_4^3-3r_4 r_3^2)\sinh(\beta_3 S_1)\sin(\beta_4 S_1)]$$

$$+ D[(r_3^3-3r_3 r_4^2)\cosh(\beta_3 S_1)\sin(\beta_4 S_1) - (r_4^3-3r_4 r_3^2)\sinh(\beta_3 S_1)\cos(\beta_4 S_1)] \quad ...(3.38)$$

and

$$S_1 = \frac{|y-e|}{b} \qquad S_2 = \frac{y}{b} \quad ...(3.39)$$

The positive and negative signs in K_3 and K_4 are for stations to the right and left of the load respectively.

3.6 DETERMINATION OF ELASTIC RIGIDITIES OF GIRDER BRIDGE DECKS

We consider an element of a T-beam bridge shown in Fig. 3.5. The intersecting beams are assumed to be rigidly connected at there point of intersection. The controid axes are denoted by $_x$ and $_y$ and both are measured from the centre of the slab for b_x and b_y are the spacing of long girder of cross beams respectively for the slab, the stress-strain relationship can be written as

$$\sigma_x = E'(\epsilon_x + \epsilon_y)$$

$$\sigma_y = E'(\epsilon_y + \epsilon_x) \qquad \ldots(3.40)$$

where

$$E' = \frac{E}{(1-\nu^2)}$$

As we know that strains at a distance Z from the neutral axis can be expressed as follows

$$E_x = - Z\frac{\partial^2 w}{\partial x^2} \; ; \; E_y = - Z \frac{\partial^2 w}{\partial y^2} \qquad \ldots(3.41)$$

Hence the stress curvature relationship for slab element at a distance Z from neutral axis is given by

$$\sigma_x = - E'Z \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = - E'Z \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2}\right) \qquad \ldots(3.42)$$

For the element of T-beam the arerage strain in the direction

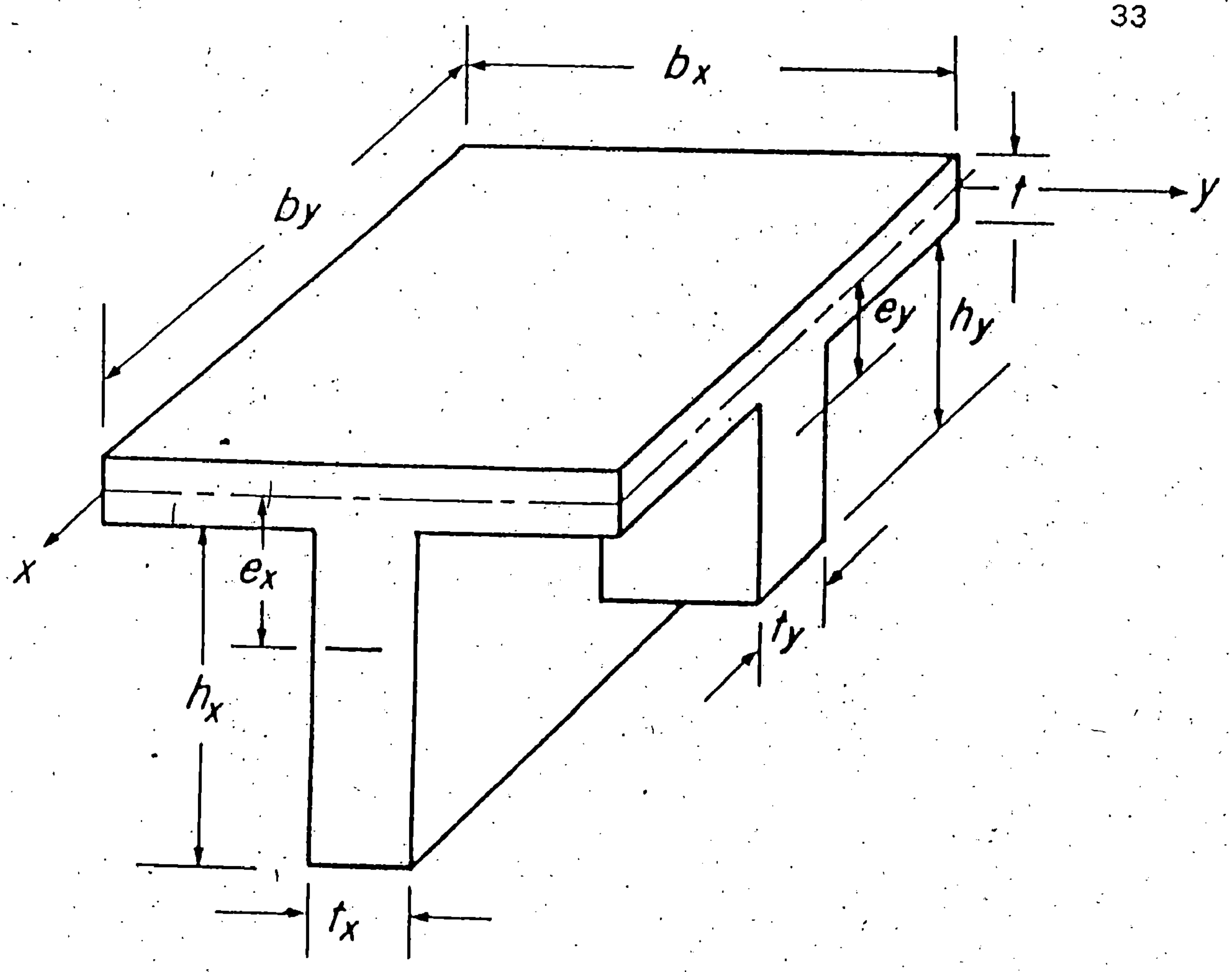


Figure 3. 5 Element of T-beam bridge deck

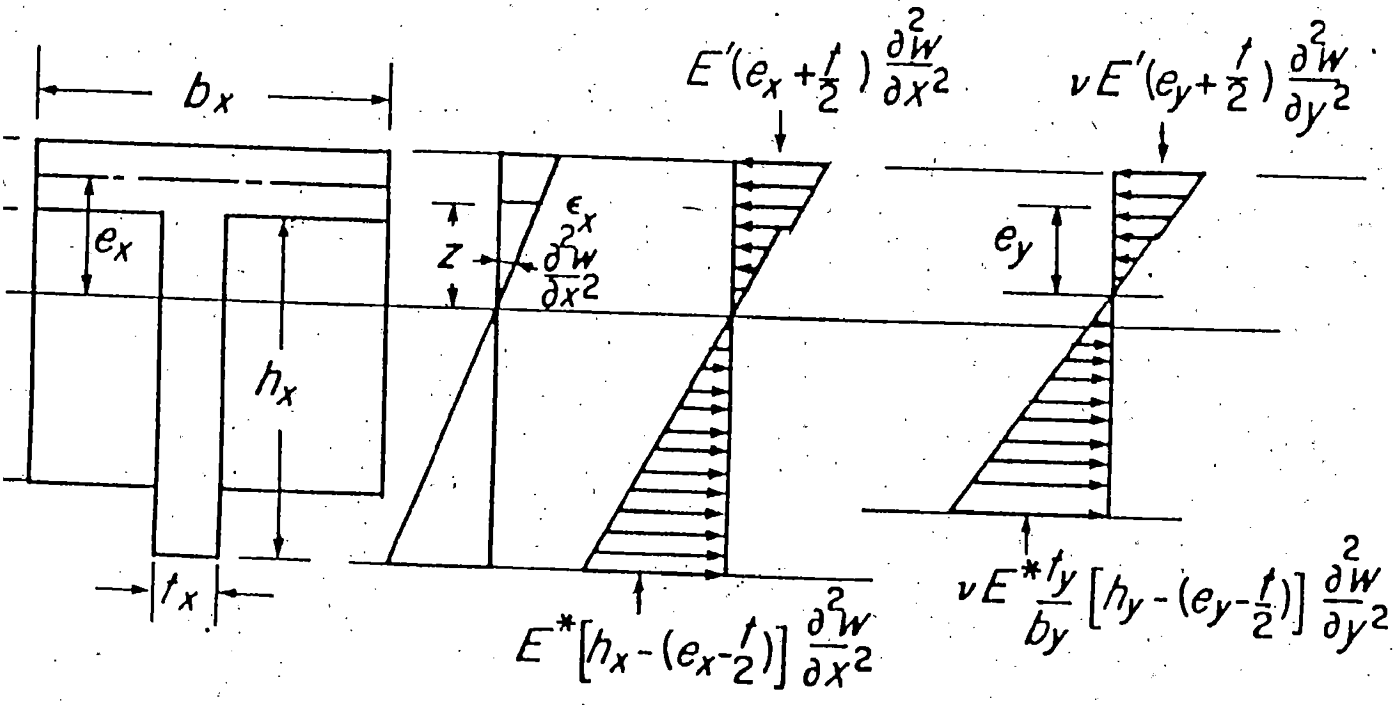


Figure 3. 6 Distributions of stresses and strains

of x and y axes can be written as

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\sigma_y}{E}\left(\frac{t_y}{b_y}\right)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\sigma_x}{E}\left(\frac{t_x}{b_x}\right) \qquad \ldots(3.43)$$

Solving these equations for σ_x and σ_y we get

$$\sigma_x = E^*\left(\epsilon_x + \epsilon_y \frac{t_y}{b_y}\right)$$

$$\sigma_y = E^*\left(\epsilon_y + \epsilon_x \frac{t_x}{b_x}\right) \qquad \ldots(3.44)$$

wherein

$$E^* = \frac{E}{\left(1 - {}^2\, \frac{t_x t_y}{b_x b_y}\right)}$$

Now we substitute the values of ϵ_x and ϵ_y from equation (3.41) to equation (3.44)

$$\sigma_x = -E^* Z\left(\frac{\partial^2 w}{\partial x^2} + \frac{t_x}{b_y}\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = -E^* Z\left(\frac{\partial^2 w}{\partial y^2} + \frac{t_x}{b_x}\frac{\partial^2 w}{\partial x^2}\right) \qquad \ldots(3.45)$$

The values of σ_x is expressed in terms of two stress blocks as shown in Fig. (3.6).

Now we find the values of bending moment M_x per unit length by taking the moments of individual forces about the slab

$$M_x = \int \sigma_x . Z . dZ$$

$$= -\frac{\partial^2 w}{\partial x^2}\left[\frac{E' t^3}{\partial x_y} + \frac{E^* t_x}{\partial b_x}\left([h_x - (e_x - \tfrac{t}{2})]^2 (2h_x + e_x + b)\right.\right.$$

$$\left.\left. - (e_x - \tfrac{t}{2})^2 (e_x + t)\right] - \frac{\partial^2 w}{\partial y^2}\left[\frac{E^* t^3}{12} + \frac{E^* t_x t_y}{\epsilon\, b_x b_y}\right.$$

$$\times \left([h_x - (e_x - \tfrac{t}{2})]^2 (2h_x + e_x + t) - (e_x - \tfrac{t}{2})^2 (e_x + b)\right]\Big] \qquad \ldots(3.46)$$

ilar procedure is followed to evaluate M_y and twisting ts and we get

$$- \frac{\partial^2 w}{\partial_y^2} \left[\frac{E' t^3}{12} + \frac{E^* t_y}{6 b_y} [\,] h_y - (e_y - \frac{b}{2})\,]^2 \, x (2h_y + e_y + t)\right.$$

$$\left. - (e_y - \frac{t}{e})^2 (e_y + t)\,]\right] - \frac{\partial^2 w}{\partial x^2} \left[\frac{E' t^3}{12} + \frac{E^* t_x t_y}{6 b_x b_y} [[h_y - (e_x - \frac{t}{2})]^2\right.$$

$$\left. x\ (2h_y + e_x + t) - (e_x - \frac{t}{2})^2 (e_x + t)]\right] \qquad \ldots(3.47)$$

$$(\beta_{xy} + \int_{slab} (_{xy} Z . dA) \frac{\partial^2 w}{\partial x \partial y}$$

$$(\beta_{xy} + 2\int_0^{\tau/L} 2G Z^2 dZ) \frac{\partial^2 w}{\partial x \partial y}$$

$$(\beta_{xy} + \frac{G t^3}{6}) \frac{\partial^2 w}{\partial x \partial y} \qquad \ldots(3.48)$$

$$- (\beta_{yx} + \frac{G t^3}{6}) \frac{\partial^2 w}{\partial x \partial y} \qquad \ldots(3.49)$$

ing these equations with the moment equations for an ropic plate equation (3.4) we get

$$\frac{E' t^3}{12} + \frac{E^* t_x}{6 b_x} [[h_x - (e_x - \frac{t}{2})]^2 x (2h_x + e_x + t) - (e_x - \frac{t}{2})^2 (e_x + t)] \qquad \ldots(3.50)$$

$$\frac{E' t^3}{12} + \frac{E^* t_y}{6 b_y} [[h_y - (e_y - \frac{t}{2})]^2 x (2h_y + e_y + t) - (e_y - \frac{t}{2})^2 (e_y + t)] \qquad \ldots(3.51)$$

$$\frac{E' t^3}{12} \quad \frac{E^* t_x t_y}{6 b_x b_y} [[h_y - (e_y - \frac{t}{2})]^2 x (2h_y + e_y + t) - (e_y - \frac{t}{2})^2 (e_y + t)] \qquad \ldots(3.52)$$

$$\frac{E' t^3}{12} + \frac{E^* t_x t_y}{6 b_x b_y} [[h_y - (e_x - \frac{t}{2})]^2 x (2h_y + e_x + t) - (e_x - \frac{t}{2})^2 (e_x + t)] \qquad \ldots(3.53)$$

$$\beta_{xy} + \frac{G t^3}{t} \qquad \ldots(3.54)$$

$$\beta_{yx} + \frac{G t^3}{6} \qquad \ldots(3.55)$$

where β_{xy} and β_{yx} are the torsional rigidities of longitudinal beams and cross beams respectively. Now we know that

$$2H = D_{xy} + D_{yx} + D_1 + D_2$$

or
$$2H = \beta_{xy} + \beta_{yx} + \frac{Et^3}{6(1-\nu^2)} + C \qquad ...(3.56)$$

where the value of C is

$$C = \frac{E^* t_x t_y}{2b_x b_y} h_y[(h_y+t)[)h_y+t)-(e_x+e_y)]+ \frac{1}{3} h_y^2]$$

Since the value of for concrete is small (0.15), $\frac{t_x t_y}{b_x b_y}$ is also small it can be assumed that

$$E^* = \frac{E}{(1- {}^2 \frac{t_x t_y}{b_x b_y})} \simeq E$$

Now total torsional rigidity 2H can be expressed as follows :

$$2H = \beta_{xy} + \beta_{yx} + \frac{Et^3}{6(1- {}^2)} + \frac{Et_x t_y}{2b_x b_y} h_y[H_y[H_y-(e_x+e_y)+\frac{1}{3} h_y^2] \qquad ...(3.57)$$

where $H_y = (h_y+t)$.

On an approximate analysis we can neglect the fourth term and above equation get simplify to

$$2H = \beta_{xy} + \beta_{yx} + \frac{Et^3}{6(1-\nu^2)} \qquad ...(3.58)$$

2.7 DESIGN CURVES FOR SIMPLE RIGHT DECKS WITH CONCENTRATED LOADS

The direct analysis of load distribution in bridge decks using the equations derived in previous articles becomes a laborious procedure if a computer is not readily accesible. The availabelity of approximate results in the form of design curves is clearly desirable at the preliminary design stage. Design curves based on orthotropic plate theory were first prepared by Morice, Little and Rowe (1958). A summary of design technique using these curves has been presented by Row (20). Later on these curves were modified by Cusens and Pama (7).

3.7.1 Morice, Little And Rowe Curves

These design curves are based on a calculation of the partial defferential equation (3.5) i.e.

$$D_x \frac{\partial^4 w}{\partial x^4} + \partial H \frac{\partial^4 w}{\partial x^2\, \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x,y)$$

The torsional rigidity 2H is written in the form

$$2H = 2\alpha \sqrt{D_x D_y}$$

wherein

$$\alpha = \frac{D_{xy} + D_{yx} + D_1 + D_2}{2\sqrt{D_x D_y}} \qquad \text{...(3.59)}$$

They considered orthotropic decks with $\alpha < t$, and omitted the coupling rigidities D_1 and D_2 in equation (3.59). Thus the contribution of bending to torsional rigidity is neglected and as bridge deck approaches the isotropic case, α values approach $(1-\nu)$ and not unity. The value of W is given by equation 3.26, the m^{th} term of which is

$$W_m = \frac{H_m}{2b} \frac{16a^4}{D_x m^4 \pi^4} \sin\left(\frac{m\pi x}{a}\right) K_m \text{ or } W_m = K_m W_m \qquad \text{...(3.60)}$$

where W_m is the deflection produced if the applied loads were uniformly distributed over the entire width. The value of K_m is given by equation (3.35) which is independent of x and hence the value of K_m remains same along the span. The complete expression for the deflection of the bridge is a Fourier series, namely

$W = K_1W_1+K_2W_2+ \ldots K_mW_m + \ldots$

and the actual mean deflection is **given by**

$W = W_1 + W_2 + \ldots W_m + \ldots.$

The true distribution coefficient K', **therefore, given by**

Since W_m is inversly proportional to m^4, both the series are rapidly convergent, and for all practical purposes it is sufficiently accurate to consider the first term only, thus

$$K' = K_1.$$

For the calculation of bending moments,these authers neglected the poisson's ratio **and obtained as follows**

$$M_{xm} = - D_x \frac{\partial^2 w_m}{\partial x^2}$$

$$= K_m \frac{H_m}{2b} \frac{4a^2}{m^2\pi^2} \sin \frac{m\pi x}{a}$$

$$M_{xm} = M_mK_m \qquad \ldots(3.61)$$

wherein M_m is the mean longitudinal moment. Considering all terms,

$$M_x = K_1M_1 + K_2M_2+\ldots K_mM_m+\ldots$$

$$M = M_1 + M_2 + \ldots M_m + \ldots$$

$$K' = \frac{M_x}{M} = \frac{K_1M_1+K_2M_2+ \ldots K_mM_m}{M_1 + M_2 + \ldots M_m} \qquad \ldots(3.62)$$

Since M_m is inversly proportional to m^2, both the series in equation 3.61 are convergent though not so rapidly as the series in equation (3.62). However,in practical applications it will be sufficiently accurate to consider the first term only of each series, thus $K' = K_1$, provided some increase in moment so derived is assumed. For all design purposes it is seen that the appropriate percentage increase is 10 (24). Thus we can write

$$M_x = 1.1\ K_1 M \qquad \text{...(3.63)}$$

Hence a single set of distribution coefficient K_1 is sufficient to determine both the deflection and longitudinal moments in the bridge structure and it is common to denote it by K.

From equation (3.35) , value of K can be determined for any value of the torsional parameter. In the two limiting cases of $\alpha = 0$ and $\alpha = 1$, these values are called K_o and K_1 respectively. It was shown by Massonet that for any intermediate value of α distribution coefficient can be determined with sufficient accuracy from the interpolation formula

$$K_\alpha = K_o + (K_1 - K_o)\sqrt{\alpha} \qquad \text{...(3.64)}$$

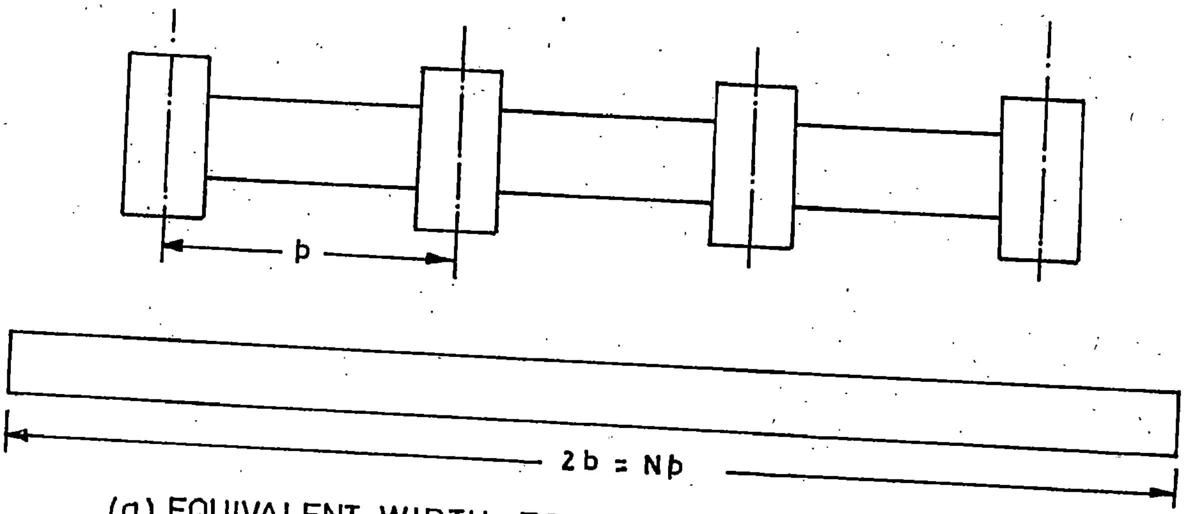
The transvese bending moment M_y is given by

$$M_y = -\ D_y \frac{\partial^2 w}{\partial y^2}$$

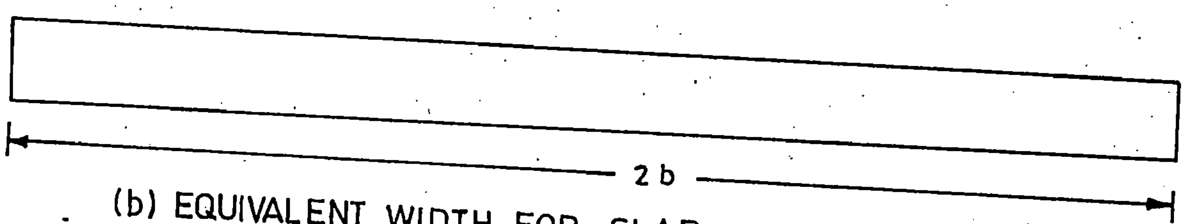
$$M_y = \sum_{n=1}^{\infty} \mu_n\ b\ H_n \sin \frac{n\pi x}{L} \qquad \text{...(3.65)}$$

where μ is a distribution coefficient for transverse moment dependent as θ, α, $\frac{y}{b}$ an $\frac{e}{b}$ and H_n. H_n is the amplitude of the terms in the fourier series for the load.

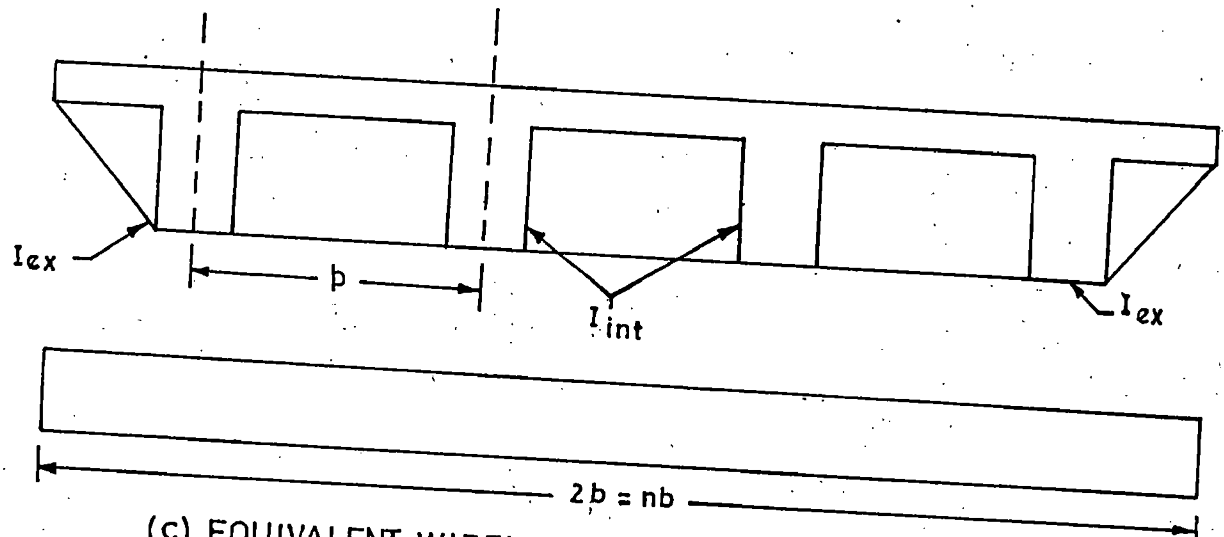
For preparation of desin curves , bridge deck is assumed as an orthotropic plate of width 2b and span of 2a. The relation between actual and equivalent width are shown in Fig. 3.7 . This effective width of the deck is divided into eight equal parts as shown in Fig. 3.8. The nine points



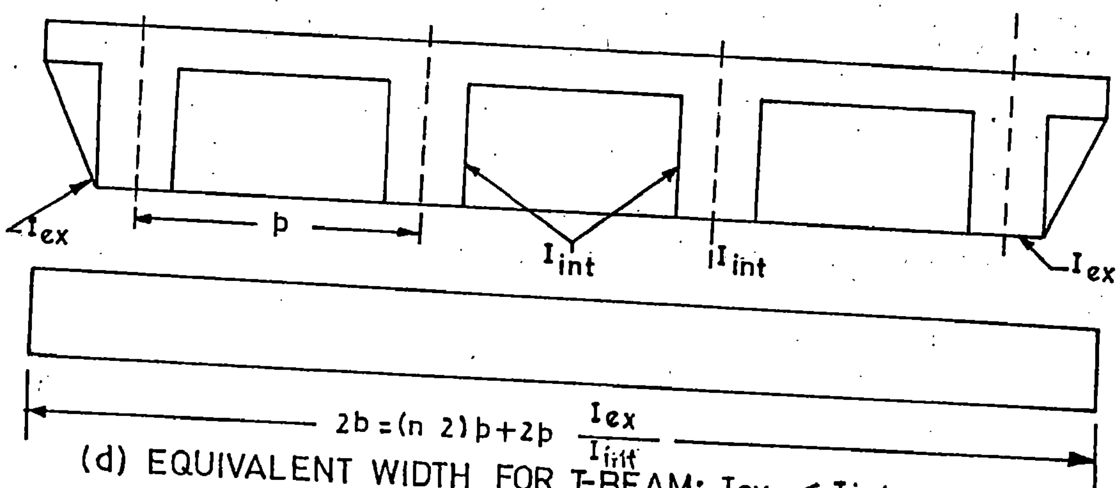
(a) EQUIVALENT WIDTH FOR GRILLAGE



(b) EQUIVALENT WIDTH FOR SLAB



(c) EQUIVALENT WIDTH FOR T-BEAM ; Iex. = Iint



(d) EQUIVALENT WIDTH FOR T-BEAM; Iex. ≤ Iint

FIG. 3·7 — EQUIVALENT ORTHOTROPIC PLATE WIDTH (2b) thus obtained are called reference stations or standard

-b   -3b/4   -b/2   -b/4   0   -b/4   +b/2   +b/4   +b

FIG. 3.8 STANDARD POSITIONS OR REFERENCE STATIONS

positions. Loads acting in between two reference stations are converted in the equivalent loads at these stations, considering the deck simply supported between two reference stations. Design curves for distribution coefficients at these points are drawn by these authors. Some parameters are defined to draw these curves which are

(i) <u>Flexural Parameter</u>

$$\theta = \frac{b}{2a} \sqrt[4]{\frac{D_x}{D_y}} = \frac{b}{2a} \sqrt[4]{\frac{i}{j}} \qquad \ldots(3.66)$$

where,

i = Moment of inertia per unit or width of girder.

u = Moment of inertia per unit or width of cross girder

(ii) <u>Torsional Parameter</u>

$$\alpha = \frac{D_{xy} + D_{yx}}{2\sqrt{D_x D_y}}$$

$$= \frac{G(i_o + j_o)}{2E\sqrt{G\,j}} \qquad \ldots(3.67)$$

wherein

i_o = torsional constant per unit width

j_o = torsional constant per unit length

Method of calculating torsional constant is given in Appendix - B.

(a) <u>Distribution Coefficient K</u>

It can be seen from equation (3.35) that K is a function of

(i) Flexural parameter, θ

(ii) location of concentrated load, e

(iii) torsional parameter, α

(iv) reference station considered

(i) is taken into account by plotting the curves as a function of θ.

(ii) is taken into account of by plotting separate curves for each independent load position, the load positions considered corrosponding to the standard position already defined.

(iii) is taken account of by plotting a separa te set of curves for each individual reference station.

(iv) is taken account by plotting a separate set of curves for each individual reference station.

Rowe gave 11 curves for finding the value of K curves 1 to 5 are distribution coeff. K_o at reference stations $0, \frac{b}{4}, \frac{b}{2}, \frac{3b}{4}$ and b respectively for various load eccentricities. Curve 6 is for large range distribution coefficient. K_o. Curves 7 to 11 are for distribution coefficient. K_1 at reference stations $0, \frac{b}{4}, \frac{3b}{4}$ and b respectively. These curves are shown in Appendix - E.

The distribution coefficients for a specific value of the torsional parameter α may be obtained by using the interpalation formula (3.64) i.e.

$$K_\alpha = K_o + (K_1 - K_o)\sqrt{\alpha}$$

The use of these distribution coeff. is illustrated by an example in section 6 of this report.

(b) Distribution Coefficient μ

Design curves are prepared in the same methods as per the distribution coeff. K for the standard position 0, $\frac{b}{4}$, $\frac{b}{2}$ and $\frac{3b}{4}$ with load position of 0, $\pm\frac{b}{2}$, $\pm\frac{3b}{4}$ $\pm$ b ,(24). However for most design purposes only the design curves relevant to the standard position 0 are required. These are given in reference (20).

For values of the torsional parameter between 0 and 1 the interpalation formula is used.

$$\mu_\alpha = \mu_o + (\mu_1 - \mu_o)\sqrt{\alpha} \qquad ...(3.68)$$

3.7.2 Cusens and Pama Curves

Cusens and Pama (7) improved upon the above design curves by eleminating the shortcomings of Morice, Little and Rowe Curves, which are as follows

(i) In the preparation of design curves Morice Little and Rowe neglected the coupling rigidities. Torsional parameter, α could be modified to include coupling rigidities D_1 and D_2.

(ii) As we have seen in equation (3.62) terms after first in the series are neglected. The value of K, based on first term of a series representation of deflection is used for the calculation of longitudinal B.M. In the estimation of maximum values an additional ten percent is added to allow for poisson's ratio effects and for slower convergence of the series for moment.

(iii) The determination of distribution coefficients for transverse moment entails a fairly tedious calculation. The series expression employes a limited number of terms. If a new distribution coeff. could be formulated in terms of the mean longitudinal moment the calculation would be simplified. A greate number of terms in the series could also be added.

(iv) The range of the design curves should be extended to cover the case of the torsionally stiff or Flexurally soft bridge deck ($\alpha > 1$).

Refering to Fig.3.1, if a load p acts at a distance C from the support x = O, the expression for deflection (3.26) charges to

$$w = \frac{pL^3}{\pi^4 bD_x} \sum_{n=1}^{\infty} \frac{1}{n^4} \sin\left(\frac{n\pi C}{L}\right) \sin\left(\frac{n\pi x}{L}\right) . K_1 \qquad \ldots(3.69)$$

The longitudinal moment per unit width is given by equation (3.4) i.e.

$$M_x = -\left(D_x \frac{\partial^2 w}{\partial x^2} + D_1 \frac{\partial^2 w}{\partial y^2}\right)$$

The second term $D_1 \frac{\partial^2 w}{\partial y^2}$ is normally small in relation to the formulation of distribution coefficient. It has its maximum effect in an isotropic slab i.e. ($\alpha = 1$) and atthis value of α the term may redily be included in the expression. For the values of $\alpha = 0$ and 2 the effect of it is small and this term has been omitted. Now M_x can be written as

$$M_x = \frac{PL}{\pi^2 b} \sum_{n=1}^{\infty} \frac{1}{h^2} \sin \frac{n\pi C}{L} \sin \frac{n\pi x}{L} K_1^1 \qquad \ldots(3.70)$$

In design the critical values of longitudinal bending moment for a simply supported span occurs at or near mid span when the load is at or near mid span. For the preparation of design curves it has been assumed that

$$\frac{C}{L} = \frac{x}{L} = 0.5$$

Nine terms of the series have been considered to give reasonable accuracy so that,

$$M_x = \frac{PL}{\pi^2 b} \sum_{n=1}^{9} \frac{L}{n^2} \sin^2 \frac{n\pi}{2} K_1^1 \qquad \ldots(3.71)$$

The mean longitudinal moment at mid span is

$$M_{x\ mean} = \frac{PL}{\pi^2 b} \sum_{n=1}^{9} \frac{1}{n^2} \sin^2 \frac{n\pi}{2} \qquad \ldots(3.72)$$

Hence distribution coeff.

$$K_{mx} = \frac{M_x}{M_{x\ mean}} \frac{\sum_{n=1}^{9} \frac{1}{n^2} \sin^2 \frac{n\pi}{2} K'_1}{\sum_{n=1}^{9} \frac{1}{n^2} \sin^2 \frac{n\pi}{2}} \qquad \ldots(3.73)$$

The transverse B.M. per unit span is

$$M_y = -(D_y \frac{\partial^2 w}{\partial y^2} + D_2 \frac{\partial^2 w}{\partial w^2}) \qquad \ldots(3.74)$$

which may be expressed as :

$$M_y = \frac{PL}{\pi^2 b} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi c}{L} \sin \frac{n\pi x}{L} \sqrt{\frac{D_y}{D_x}} K_2^1 \quad \ldots(3.75)$$

The coeff K_2^1 is given in (3.36). It contains a poisson's ratio term which is of significance only when the torsional parameter $\alpha = 1$. The term $D_2 \frac{\partial^2 w}{\partial x^2}$ has been included only in the value computed for design curves for $\alpha = 1$. In a similar way to the formation of K_{mx} if

$$\frac{C}{L} = \frac{x}{L} = 0.5$$

$$K_{my} = \frac{\sum_{n=1}^{9} \frac{1}{n^2} \sin^2 \frac{n\pi}{2} K_2^1}{\sum_{n=1}^{9} \frac{1}{n^2} \sin^2 \frac{n\pi}{2}} \quad \ldots(3.76)$$

If values of K_{mx} and K_{my} can be found from design curves the moments may be determined as

Longitudinal moment $M_x = K_{mx} M_{x\ mean}$

$$M_y = \sqrt{\frac{D_y}{D_m}} K_{my} M_{m\ mean}$$

Thus both bending moments M_x and M_y are now expressed in terms of the product of a distribution coefficient and the mean longitudinal moment. Design curves drawan by curens and Pama are having same values of flexural parameter θ, effective width 2b, and same interpolation formula for α as given by morice, Little and Rowe.

The curves for distribution coefficient K_{mx} are given in Appendix -E and those for K_{my} are given in reference (7).

These curves are drawn for mid span section when the load is acting at mid span. Bakht (1973) has suggested that a 45 degree spread can be assumed to convert the load, not acting at mid span into equivalent loads at mid span and for $\alpha > 0.5$ this spread can be neglected. Application of this method is illustrated in Chapter-6 of this report.

**

CHAPTER - 4

HARMONICS METHOD

4.1 INTRODUCTION

The concept of Harmonics method was first introduced by Hendry and Jaeger in 1958 (12). They applied it to the three and four girder skew girder bridges neglecting torsional stiffnesses and considering only first term of the harmonic component of the applied loading. Later the method was modified to incorporate the torsional stiffness of the transverse system by Surana, Agarwal and Prasad (18,1,22). They also assumed more generalised displacement functions for the girders.

The applied load on a bridge deck gets distributed amongst the girders through transverse system. It is therefore pertinent to ascertain the share of the imposed load by the various girders of the bridge deck.

This is achieved by establishing the equations of equilibrium of forces, moments and torsion at interfaces of the transverse system with the girders and the overall statics of the bridge deck. The applied load is represented by harmonic components using fourier analysis. This forms the basis of mathematical model for the bridge deck system considered herein.

4.2 BRIDGE DECK IDEALIZATION

Mathematical model developed for this method is based on the following assumptions

(i) Material of bridge deck is elastic, homogeneous and isotropic.

(ii) The interconnecting transverse system is replaced by an equivalent uniform thick continuum, which does not affect the actual structural behaviour of the bridge deck.

(iii) Length of the transverse system which is effective in distributing loads amongst the girders is considered between two planes at right angle to the axes of the girders.

(iv) Any load applied in between the girders is replaced by an equivalent system of coplaner loads acting along a plane normal to the axes of girders.

(v) Interconnection of transverse system with the girders is assumed to be perfectly rigid.

(vi) All the girders are equidistant, simply supported and free to rotate about their longitudinal axis.

(vii) Shear deformations of the girders are negligibly small and hence ignored.

4.3 MATHEMATICAL MODEL

Bridge deck considered here for the development of mathematical model consists of three girders. This mathematical model developed for a three girder bridge is also valid for multiple girder bridge. Equations for the extreme left and right girders are exactly the same as for girders 1 and 3, and the intermediate girders have similar equations as the centre girder in a three girder bridge.

The applied load is expressed in terms of the first three hormonics of the sine series using fourier analysis. Figure 4.1(a) shows the general deformed shape of a simply supported bridge deck under applied loading.

Refering to Fig. (4.1) and using fourier analysis for the first three hormonics the applied load can be written as

$$W_{ia} = W_{i1}(\text{Sin}\pi\ x_i/L) + W_{i2}(\text{Sin}\ 2\ \pi\ x_i/L) + W_{i3}\ \text{Sin}(3\pi\ x_i/L) \quad ...(4.1)$$

Load-deflection relationship can be obtained by integrating four times equation (4.1) and dividing by girder flexural rigidity EI_1 as follows

$$Y_{ia} = (1/EI_i)(\tfrac{L}{\pi})^4 (W_{i1}\ \text{Sin}(\pi\ x_i/L) + \frac{W_{i2}}{2^4}\ \text{Sin}\ (2\pi\ x_i/L) + \frac{W_{i3}}{3^4}\ \text{Sin}(3\pi\ x_i/L))$$

$$= A_i\ \text{Sin}(\pi\ x_i/L) + B_i\ \text{Sin}(2\pi\ x_i/L) + C_i\ \text{Sin}(3\pi\ x_i/L) \quad ...(4.2)$$

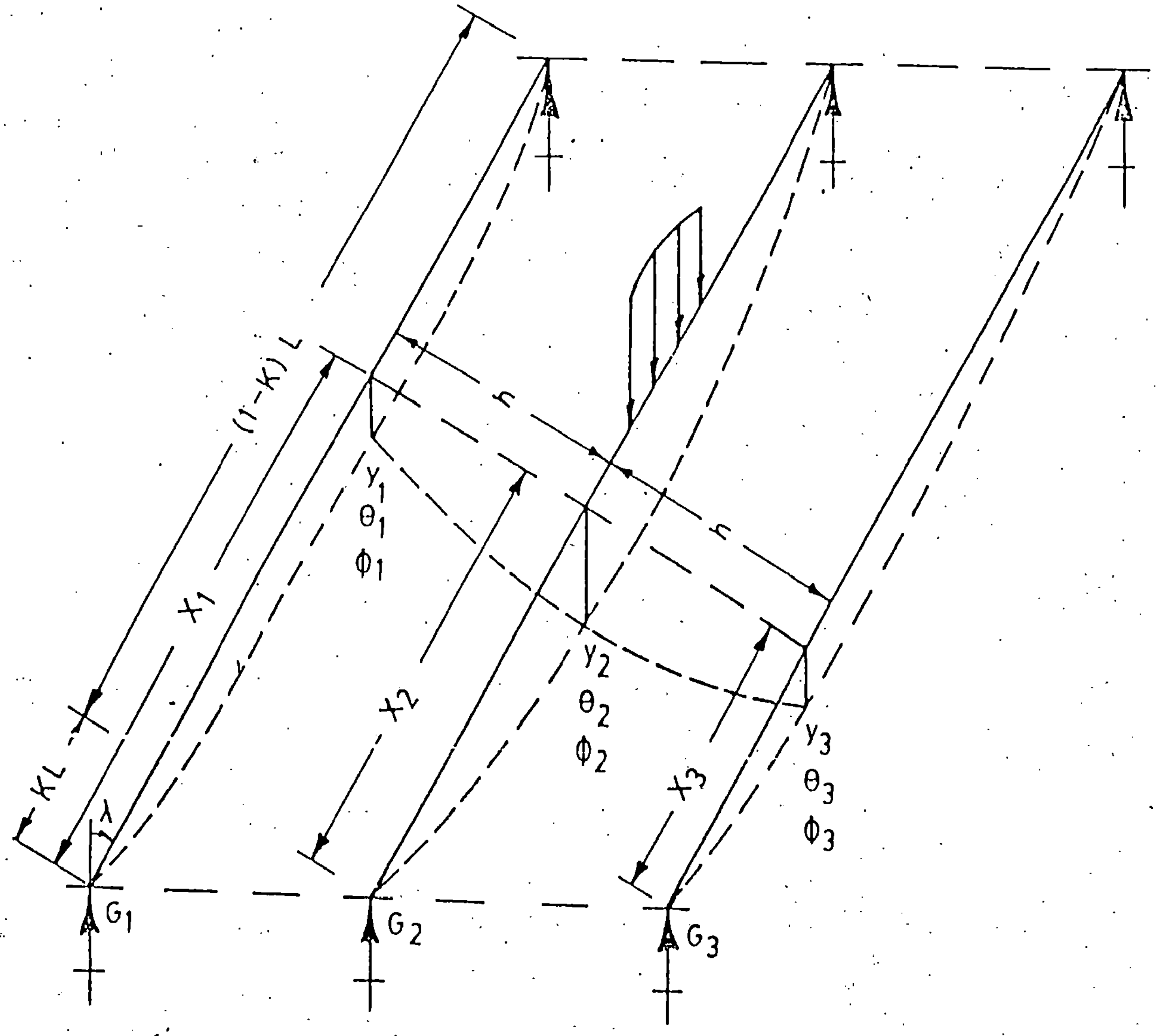
The deflection coefficient can therefore be written as

$$A_i = (W_{i1}/EI_i)(L/\pi)^4 \qquad B_i = (W_{i2}/EI_i)(L/2\pi)^4$$

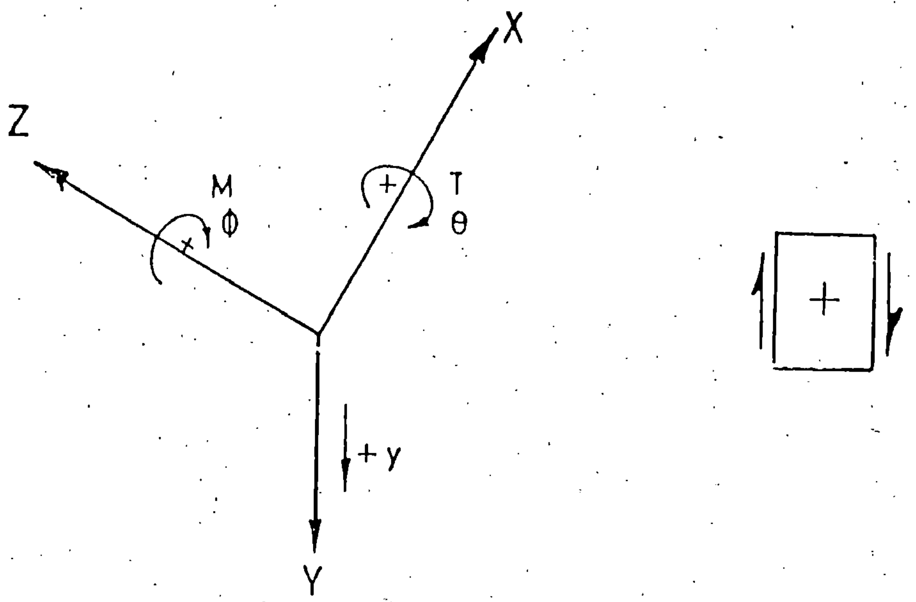
$$C_i = (W_{i3}/EI_i)(L/3\pi)^4 \qquad \text{where } i = 1,2,3$$

The applied load on bridge deck is distributed among all the girders depending upon their relative stiffnesses, and hence the problem is to ascertain how much load is carried by each individual girder in terms of assumed displacement functions. The effective loading on girder i is equal to the applied load W_{ia} minus the load transmitted to the other girders.

The deflection function Y_{ir} and rotation function θ_{ir} due to the load finally retained in girder i can be written as follows



(a) BRIDGE DECK UNDER LOADING



(b) COORDINTE SYSTEM & SIGN CONVENTION

FIG. 4·1

$$y_{ir} = a_i \text{ Sin}(\pi\, x_i/L) + b_i \text{ Sin}(2\pi\, x_i/L) + c_i \text{ Sin}(3\pi\, x_i/L) \quad \ldots(4.3)$$

$$h\, \theta_{ir} = d_i + c_i \cos(\pi\, x_i/L) + f_i \cos(2\pi\, x_i/L) \quad \ldots(4.4)$$

The length of interface of transverse system structurally effective with various girders can be expressed as follows

(1)	Left interface of the transverse system with girder 1	$kL \leq x \leq L$
(2)(a)	Right interface of the transverse system with girder 2	$0 \leq x \leq (1-k)L$
(b)	Left interface of the transverse system with girder 2	$kL \leq x \leq L$
(3)	Right interface of the transverse system with girder 3	$0 \leq x \leq (1-k)L$

After imposition of loads on bridge deck, transverse system deforms and hence shear force will be generated at the interface of the girder with transverse system. This will act as loading on the girders.

Refering to the Fig. 4.1(a) for deformation and Fig. 4.1(b) for sign convention, bending moment developed at interfaces of the transverse system with girders per unit length can be written as indicated in equation (4.5) below Fig. 4.2. Shear force developed at interfaces of the transverse system with girders 1,2 and 3, can be written as indicated in equations (4.6) below the same Figure.

Due to variation of rotation along Z-axis of the transverse system, torsion is generated at interfaces with various girders.

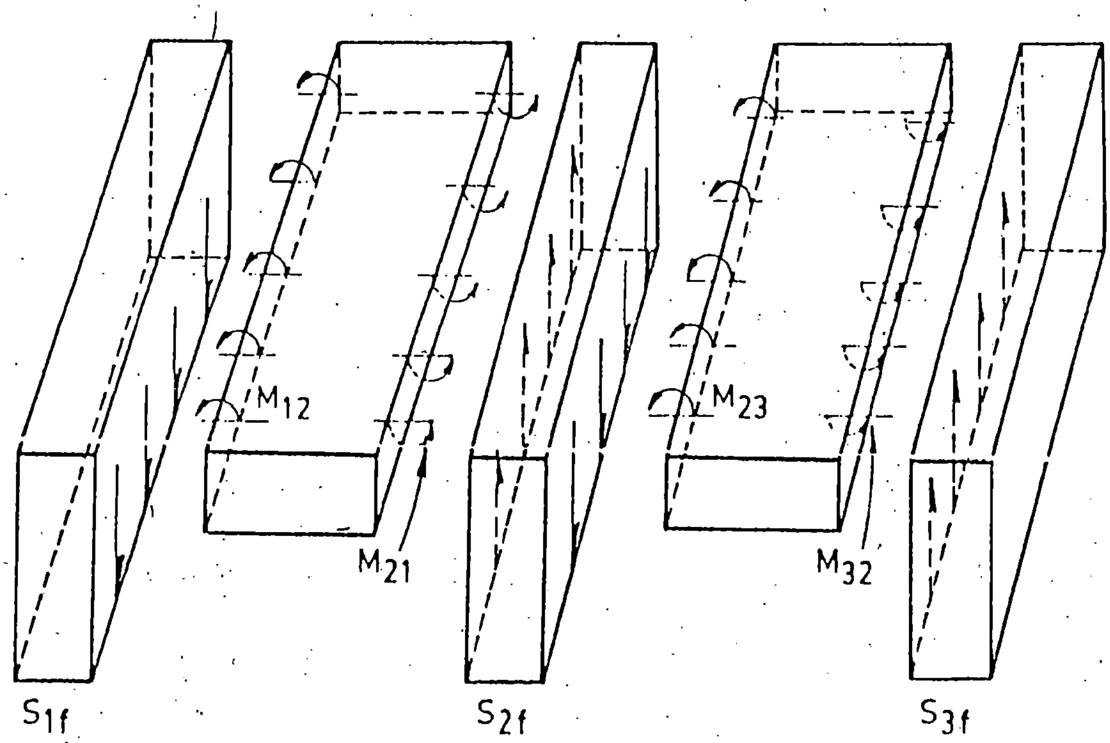


FIG. 4·2—SHEAR FORCE DUE TO FLEXURE

BENDING MOMENT AT INTERFACES

$$M_{12} = Q[-2h\theta_1 - h\theta_2 + 3(Y_2 - Y_1)]$$
$$M_{21} = Q[-2h\theta_2 - h\theta_1 + 3(Y_2 - Y_1)]$$
$$M_{23} = Q[-2h\theta_2 - h\theta_3 + 3(Y_3 - Y_2)]$$
$$M_{32} = Q[-2h\theta_3 - h\theta_2 + 3(Y_3 - Y_2)] \quad \text{------------} \quad [4\cdot5]$$

WHEREIN $Q = \dfrac{2EI_T}{(1-K)Lh^2}$ AND $K = \dfrac{h}{L}\tan\lambda$

SHEAR FORCE AT INTERFACES DUE TO TRANSVERSE MOMENT

$$S_{1f} = (m_{12} + m_{21})/h$$
$$S_{2f} = -(m_{12} + m_{21})/h + (m_{23} + m_{32})/h \quad \text{------------} \quad [4\cdot6]$$
$$S_{3f} = -(m_{23} + m_{32})/h$$

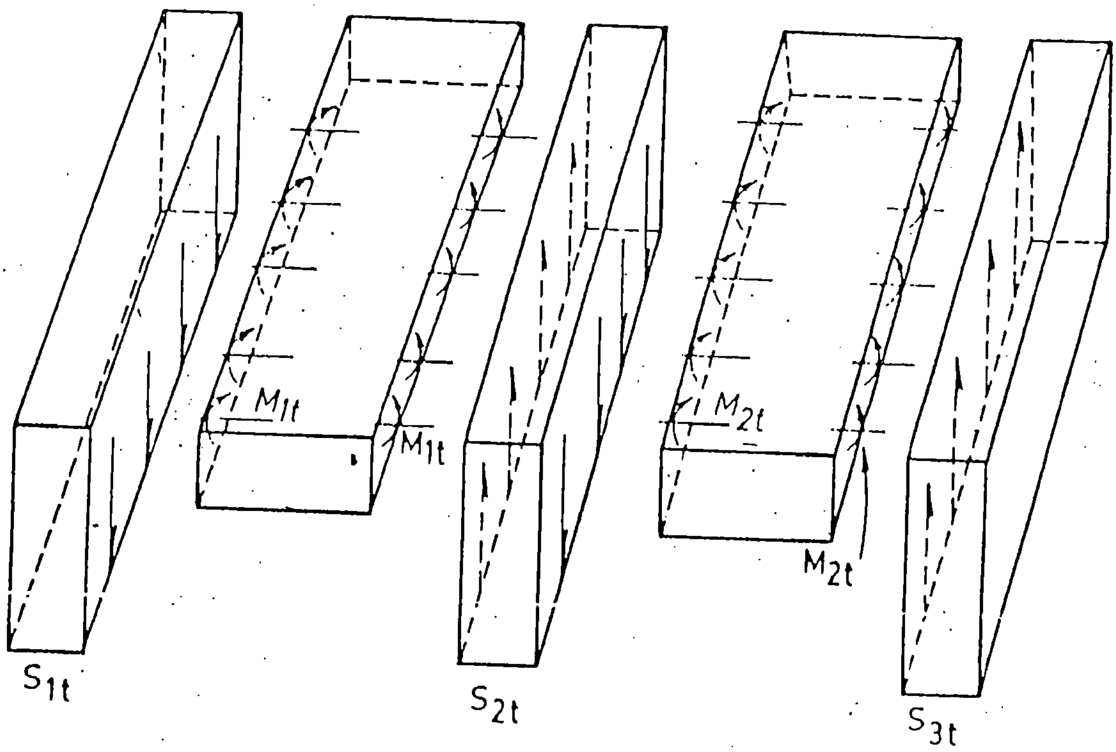


FIG. 4·3—SHEAR DUE TO TORSION

SHEAR DUE TO TORSION

$$T_{1t} = -[GJ_T/(1-K)L](\phi_2-\phi_1)/h = M_{1t}$$

$$S_{1t} = \frac{d}{dx}(M_{1t}) = -[GJ_T/(1-K)hL]\frac{d}{dx}(\phi_2-\phi_1) \quad [4\cdot7]$$

$$S_{2t} = \frac{d}{dx}(M_{2t}-M_{1t})$$

$$= [GJ_T/(1-K)hL][\frac{d}{dx}(\phi_2-\phi_1)-\frac{d}{dx}(\phi_3-\phi_2)]$$

$$S_{3t} = \frac{d}{dx}(-M_{2t}) = [GJ_T/(1-K)hL][\frac{d}{dx}(\phi_3-\phi_2)]$$

IN WHICH $\phi_i = (\frac{dy}{dx})_i$

This torsion acts as moment loading on the girders and variation of moment loading causes shear force in the girders as shown in Fig. (4.3) and presented in equation (4.7) below Fig. (4.2).

Retained load (W_{ir}) thus can be determined by summing up applied loading, flexural shear force and torsional shear force as follows

$$W_{ir} = W_{ia} \pm S_{if} \pm S_{it} \qquad \ldots(4.8)$$

from Fig.4.2 and 4.3 and by virtue of sign convention adopted it is clear that the shear force at the right of interfaces of the transverse system is subtractive while at the right of it is additive. Flexural shear force S_{if} and torsional shear force S_{it} put together can be expressed in terms of the basic deformation coefficients a_i, b_i, c_i, etc. with the help of equations for the deflection, rotation and moment that is equations (4.3), (4.4) and (4.5) given in next page.

$$S_{1f}+S_{1t} = (6Q/h)(-1/2(d_1+e_1 \cos(\pi x_1/L) + f_1 \cos(2\pi x_1/L) + d_2$$
$$+ e_2 \cos \pi/L(x_1-kL) + f_2 \cos 2\pi/L(x_1-kL))+(a_2\gamma_1 \sin \pi/L$$
$$\times (x_1-kL) + b_2\gamma_4 \sin 2\pi/L(x_1-kL) + c_2\gamma_9 \sin 3\pi/L(x_1-kL))$$
$$- (a_1\gamma_1 \sin \pi x_1/L + b_1\gamma_4 \sin 2\pi x_1/L + c_1\gamma_9 \sin 3\pi x_1/L)) \quad ...(4.9)$$

$$S_{2f}+S_{2t} = (6Q/h)(1/2(d_1+e_1 \cos \pi/L(x_2 + kL) + f_1 \cos 2\pi/L(x_2+L)$$
$$+ d_2+e_2 \cos \pi x_2/L + f_2 \cos 2\pi x_2/L) + (a_1 \gamma_1 \sin \pi/L(x_2+kL$$
$$+ b_1\gamma_4 \sin 2\pi/L(x_2+kL) + c_1\gamma_9 \sin 3\pi/L(x_2+kL))$$
$$- (a_2\gamma_1 \sin \pi x_2/L + b_2\gamma_4 \sin 2\pi x_2/L + c_2\gamma_9 \sin 3\pi x_2/L)$$
$$- 1/2(d_2+e_2 \cos \pi x_2/L + f_2 \cos \pi x_2/L + d_3+e_3 \cos \pi/L$$
$$\times (x_2-kL) + f_3 \cos 2\pi/L(x_2-kL)- (a_2\gamma_1 \sin \pi x_2/L$$
$$+ b_2\gamma_4 \sin 2\pi x_2/L + c_2\gamma_9 \sin 3\pi x_2/L) +(a_3\gamma_1 \sin \pi/L(x_2-kL)$$
$$+ b_3\gamma_4 \sin 2\pi/L(x_2-kL) + c_3\gamma_9 \sin 3\pi/L(x_2-kL))) \quad ...(4.10)$$

$$S_{3f}+S_{3t} = (6Q/h)(1/2(d_2+e_2 \cos \pi/L(x_3+kL) + f_2 \cos 2\pi/L(x_3+kL)$$
$$+ d_3+e_3 \cos \pi x_3/L + f_3 \cos 2\pi x_3/L) + (a_2\gamma_1 \sin \pi/L$$
$$\times (x_3+kL) + b_2\gamma_4 \sin 2\pi/L(x_3+kL) +c_2\gamma_9 \sin 3\pi/L(x_3+kL))$$
$$- (a_3\gamma_1 \sin \pi x_3/L + b_3 r_9 \sin 2\pi x_3/L)) \quad ...(4.11)$$

in which $\gamma_i = 1 + i\gamma$ and $i = 1,2,3. ....9$

For superposition of the combined shear with applied loads on each girder, it is necessary to express each term of the combined shear force in terms of the first three harmonic components, using fourier analysis. After rearranging we can write

$$S_{if}+S_{it} = 6Q/h(F_{i1}(k)\sin(\pi x_i/L) + 16F_{i2}(k)\sin(2\pi x_i/L) + 81F_{i3}(k)\sin(3\pi x_i/L)),\ i= 1,2,3. \quad ...(4.12)$$

Intergrating four times the above equation and further dividing by Flexural rigidity (EL_i) girder deflection correspon-ding to shear force will be obtained as follows

$$y_{is} = \alpha_i'(F_{i1}(k)\sin(\pi x_i/L) + F_{i2}(k)\sin(2\pi x_i/L) + F_{i3}(k)\sin(3\pi x_i/L)) \quad ...(4.13)$$

Thus the retained girder deflection can be written as follows

$$y_{ir} = y_{ia} + y_{is}$$

$$y_{ir} = (A_i+\alpha_i' F_{i1}(k))\sin(\pi x_i/L) + (B_i+\alpha_i' F_{12}(k))\sin(2\pi x_i/L) + (C_i+\alpha_i' F_{i3}(k))\sin(3\pi x_i/L) \quad ...(4.14)$$

in which $\alpha_i' = \alpha_i/(1-k)$

Comparing equation (4.3) and (4.14) we will get

$$a_i = A_i + \alpha_i' F_{i1}(k) \qquad b_i=B_i+\alpha_i' F_{12}(k)$$

$$c_i = C_i + \alpha_i' F_{i3}(k) \quad ...(4.15)$$

The expression for the various coefficients are given below (18).

$$F_{11}(k) = (1/\pi)((-d_1+d_2)(1+\delta_1)+(e_1/4)(1-\delta_2)-(\pi/2)k_1e_2\mu_1+(f_1/6)(2+3\delta_1-\delta_3) + (f_2/3)(\delta_1-\delta_2)-(a_1/2)\gamma_1(2\pi k_1+\mu_2)+b_1\gamma_4(3\mu_1-\mu_3)+(c_1/4)\gamma_9(2\mu_2-\mu_4) + a_2\gamma_1(\pi k_1 S_1+\mu_1)+(2b_2/3)\gamma_4(2\mu_1+\mu_2)+(c_2/4)\gamma_9(3\mu_1-\mu_3))$$

$$F_{12}(k) = (1/16\pi)[(1/2)(d_1+d_2)(1-\delta_2)-(e_1/6)(4+3\delta_1+\delta_3)+(f_1/8)(1-\delta_4)$$

$$-(2e_2/3)(\delta_1+\delta_2)-(\pi/2)k_1\mu_2 f_2+(a_1/3)\gamma_1(3\mu_1-\mu_3)+(b_1/4)\gamma_4(4\pi k_1+\mu_4)$$

$$+(c_1/5)\gamma_9(5\mu_1-\mu_5)+(2/3)a_2\gamma_1(2\mu_1+\mu_2)+(b_2/2)\gamma_4(2\pi k_1 S_2+\mu_2)$$

$$+(2/5)c_2\gamma_9(3\mu_2+2\mu_3)]$$

$$F_{13}(k) = 1/(81\pi)[(-1/3)(1+\delta_3)(d_1+d_2)+(e_1/8)(3-2\delta_2-\delta_4)-(f_1/10)(6+5\delta_1+\delta_5)$$

$$+(3/8)e_2(\delta_1-\delta_3)-(3/5)(\delta_1+\delta_3)f_2+(a_1/4)\gamma_1(2\mu_2-\mu_4)+(b_1/5)\gamma_4(5\mu_1-\mu_5)$$

$$-c_1/6\gamma_9(6\pi k_1+\mu_6)+a_2/4\gamma_1(3\mu_1-\mu_3)-2/5\gamma_4(3\mu_2+2\mu_3)b_2$$

$$+c_2/3\gamma_9(3\pi k_1 S_3+\mu_3)]$$

$$F_{21}(k) = \frac{1}{\pi}[(1+\delta_1)(d_1-d_3)-(\pi/2)k_1\mu_1(e_1+e_3)-(1/3)(S_1+S_2)(f_1-f_2)+(e_2/2)(1-S_2)$$

$$+(\pi k_1\delta_1+\mu_1)\gamma_1(a_1+a_3)-(2\pi k_1+\mu_2)\gamma_1 a_2-(2/3)(2\mu_1+\mu_2)\gamma_4(b_1-b_3)$$

$$+(1/4)(3\mu_1-\mu_3)\gamma_9(c_1+c_3)+(1/2)(2\mu_2-\mu_4)\gamma_9 c_2)]$$

$$F_{22}(k) = (1/16\pi)[(1/2)(1-S_2)(d_1+2d_2+d_3)+(2/3)(\delta_1+\delta_2)(e_1-e_3)(-\pi/2k_1\mu_2$$

$$\times(f_1+f_3)+f_2/2(1-\delta_4)+2/3(2\mu_1+\mu_2)\gamma_1(a_1-a_3)+(\gamma_4/2)(2\pi k_1\delta_2+\mu_2)$$

$$\times(b_1+b_3)-(\gamma_4/2)(4\pi k_1+\mu_4)b_2-(2/5)(3\mu_2+2\mu_3)\gamma_9(c_1-c_3)$$

$$F_{23}(k) = (1/81\pi)[(1/3)(1+\delta_3)(d_1-d_3)+(3/8)(\delta_1-\delta_3)(c_1+c_3)+(3-2\delta_2-\delta_4)e_2$$

$$+(3/5)(\delta_2+\delta_3)(f_1-f_3)+(\gamma_1/4)(3\mu_1-\mu_3)(a_1+a_3)+(\gamma_1/2)(2\mu_2-\mu_4)a_2$$

$$+(2/5)(3\mu_2+2\mu_3)\gamma_4(b_1-b_3)+(\gamma_9/3)(3\pi k_1\delta_3+\mu_3)(c_1+c_3)$$

$$-\gamma_9/3(6\pi k_1+\mu_6)c_2)]$$

$$F_{31}(k) = (1/\pi)[(1+\delta_1)(d_2+d_3)-(\pi/2)k_1\mu_1 e_2+(e_3/4)(1-\delta_2)-(f_2/3)(\delta_1+\delta_2)+(f_3/6)$$

$$\times(-2-3\delta_1+\delta_3)+(\pi k_1 S_1+\mu_1)\gamma_1 a_2-(\gamma_1/2)(2\pi k_1+\mu_2)a_3-(2/3)(2\mu_1+\mu_2)\gamma_4 b_2$$

$$-(\gamma_4/4)(3\mu_1-\mu_3)b_3+(\gamma_9/4)(\mu_1-\mu_3)c_2+(\gamma_9/4)(2\mu_2-\mu_4)c_3]$$

$$F_{32}(k) = (1/16\pi)[(1/2)(1-\delta_2)(d_2+d_3)+(2/3)(\delta_1+\delta_2)e_2+(e_3/6)(4+3\delta_1+\delta_3)$$

$$+(f_3/8)(1-\delta_4)-(\pi/2)k_1\mu_2 f_2+(2/3)(2\mu_1+\mu_2)\gamma_1 a_2-\frac{\gamma_1}{3}(3\mu_1-\mu_3)a_3+(\gamma_4/2)$$

$$\times\ (2\pi k_1\delta_2+\mu_2+)b_2-(\gamma_4/4)(4\pi k_1+\mu_4)b_3-(2/5)(3\mu_2+2\mu_3)\gamma_9 c_2-(\gamma_9/5)$$

$$\times\ (5\mu_1-\mu_5)c_3]$$

$$F_{33}(k) = ((1/81\pi)[(1/3(1+\delta_3)(d_2+d_3)+(3/8)(\delta_1-\delta_3)e_2+(e_3/8)(3-2\delta_2-\delta_4)$$

$$+(3/5)(\delta_3+\delta_2)f_2+(f_3/10)(6+5\delta_1+\delta_5)+(\gamma_1/4)(3\mu_1-\mu_3)a_2+(\gamma_1/4)(2\mu_2-\mu_4)$$

$$\times\ a_3+(2/5)(3\mu_2+2\mu_3)\gamma_4 b_2-(\gamma_4/5)(5\mu_1-\mu_5)b_3+\gamma_9/3(3\mu K_1\delta_3(3\pi k_1\delta_3+\mu_3)$$

$$\times\ c_2-(\gamma_9/6)(6\pi k_1+\mu_6)c_3]$$

$$\alpha_i = (12/\pi^4)(L/h)^3(EI_T/EL_i) \qquad i = 1,2,3$$

$$\gamma = (\pi^2/12)(h/L)^2(G\,J_T/El_T)$$

$$S_i = \cos(j\pi k),\ \mu_j = \sin(j\pi k), \qquad \gamma_j = 1+j\gamma \qquad j = 1,2\ldots9$$

$$k = (h/L)\tan\lambda$$

$$k_1 = (1-k)$$

Equations (4.15) provides nine independent relationships involving girder displacement coefficients for three girders, These relationships are obtained from consideration of equilibrium of vertical forces.

Since **each** girder has six coefficients, hence for a three girder bridge deck, eighteen such coefficients exist. Therefore nine additional coefficient are required to get a set of eithteen simultaneous equations, from which the unknown displacement coefficients can be found out.

Bending moment acting at interface of the transverse system act as torque on the girders. Since girders are free to rotate about their two orthogonal axes hence the total torque acting on girders or total bending moment acting at interface of transverse system must be equal to zero.

$$\int_{kL}^{L} m_{12}dx_1 = \int_{0}^{(1-k)L} m_{21}dx_2 + \int_{kL}^{L} m_{23}dx_2 = \int_{0}^{(1-k)L} m_{32}dx_3 = 0 \quad ...(4.16)$$

Substituting for m_{12}, m_{21}... and carrying out the integration, a three independent relationship for three girders in terms of the girder displacement coefficient are obtained as follows :

$$F_{14}(k) = F_{24}(k) = F_{34}(k) = 0$$

or

$$F_{i4}(k) = 0 \quad ...(4.17)$$

in which

$$F_{14}(k) = -\pi k_1(2d_1+d_2) + (2e_1-e_2)\mu_1 + (\mu_2/2)(2f_1+f_2) - 3(1+\delta_1)(a_1-a_2) + (3/2)(1-\delta_2)(b_1+b_2) - (1+\delta_3)(c_1-c_2)$$

$$F_{24}(k) = -\pi k_1(d_1+4d_2+d_3) + (e_1-e_3)\mu_1 + (\mu_2/2)(f_1+4f_2+f_3) - 3(1+\delta_1)(a_1-a_3) + (3/2)(1-\delta_2)(b_1+2b_2+b_3) - (1+\delta_3)(c_1-c_3)$$

$$F_{34}(k) = -\pi k_1(d_2+2d_3) + (e_2-2e_3)\mu_1 + (\mu_2/2)(f_2-2f_3) - 3(1+\delta_1)(a_2-a_3) + (3/2)(1-\delta_2)(b_2+b_3) - (1+\delta_3)(c_2-c_3)$$

Equation (4.17) is the torque equilibrium equations for each of the girders due to bending moment at interfaces of the torsional rigidity GJ_i. The torque-rotation relationship is expressed as

$$T_i = -GJ_i \frac{d\,\theta_i}{dx_i} \qquad i = 1,2,3 \quad ...(4.18)$$

Differentiating the rotation functions (4.4) with respect to distance along the girder axis and subsequently substituting in equation (4.18), torque-rotation relationships are obtained as follows :

$$T_i = (\pi GJ_i/hL)(e_i \sin(\pi x_i/L) + 2f_i \sin(2\pi x_i/L)) \quad ...(4.19)$$

Torque in girders at any cross-section of the bridge deck is given as

$$T_i = \int_0^x m_{12} dx_i \qquad \text{or } m_{12} = dT_1/dx_1 \text{ for } i = 1 \quad ...(4.20)$$

Substituting expression for m_{12} in above equation in which deflection and rotation are involved and expressing them in terms of basic deformation coefficients, transverse moment function m_{12} can be written as :

$$\begin{aligned} m_{12} = Q(&-2d_1-d_2-2e_1 \cos(\pi x_1/L)-e_2 \cos(x_1-kL)\pi/L-2f_1 \cos(2\pi x_1/L) \\ &- f_2 \cos 2(x_1-kL)\pi/L-3a_1 \sin(\pi x_1/L)-3b_1 \sin(2\pi x_1/L) \\ &- 3b_i \sin(2\pi x_1/L) + 3a_2 \sin(x_1-kL)\pi/L + 3b_2 \sin 2(x_1-kL)\pi/L \\ &+ 3c_2 \sin 3(x_1-kL)\pi/L) \end{aligned} \quad ...(4.21)$$

Similarly expressions for transverse moment functions $(m_{12}+m_{23})$ and m_{32} can be written. In order to simplify the intergration involved in equation (4.20), each term of the transverse moment expression (4.21), is replaced by the first two harmonics of its cosine series. This is consistent with the assumed rotation functions (4.4). Since the second derivative of rotation function $d^2\theta/dx^2$ is proportional to the moment. Thus the transverse moment expression for m_{12} can be written as

$$m_{12} = R_1 \cos(\pi x_1/L) + S_1 \cos(2\pi x_1/L) \quad ...(4.22)$$

in which R_1 and S_1 are fourier coefficients to be obtained from fourier transforms as given below

$$R_1 = (2/L) \int_{kL}^{L} m_{12} \cos(\pi x_1/L)\, dx_1$$

$$S_1 = (2/L) \int_{kL}^{L} m_{12} \cos(2\pi x_1/L)\, dx_1 \qquad \ldots(4.23)$$

Coefficients R_1 and S_1 are evaluated carrying out the integrations (4.23) after substituting the expression (4.21) for the transverse moment m_{12}. These coefficients for girder 1 are given as

$$R_1 = Q\, F_{15}(k)$$

$$S_1 = Q\, F_{16}(k) \qquad \ldots(4.24)$$

The transverse moment equation now becomes :

$$m_{12} = Q(F_{15}(k) \cos(\pi\, x_1/L) + F_{16}(k) \cos(2\pi\, x_1/L) \qquad \ldots(4.25)$$

Substituting the equation (4.25) in (4.26) the torque equation will be obtained as follows

$$T_1 = \int_{0}^{x_1} m_{12}\, dx_1$$

$$= (QL/\pi)(F_{15}(k) \sin(\pi\, x_1/L) + (1/2)F_{16}(k)\sin(2\pi\, x_1/L)) \qquad \ldots(4.26)$$

Equations (4.19) and (4.26) are written for torque T_1 in girder 1 at a distance X_1 from left support and hence are indentical whose corresponding coefficients must be equal. Thus, equating the coefficient of the two sine harmonics, the following relationships are obtained

$$F_{15}(k) = (1-k)\ \beta_1 e_1$$

$$F_{16}(k) = 4(1-k)\ \beta_1 f_1 \qquad \ldots(4.27)$$

Torque rotation relationships for girders 2 and 3 can similarly be derived. The above relationships for any of the three girders can be written as

$$F_{i5}(k) = (1-k)\ \beta_i e_i$$

$$F_{i6} = 4(1-k)\ \beta_i f_i \qquad i = 1,2,3 \qquad \ldots(4.28)$$

in which

$$\beta_i = (\pi^2 b/2L)(GJ_i/EI_T) \qquad i = 1,2,3$$

$$F_{15}(k) = (1/\pi)(2(2d_1+d_2)\mu_1-(2\pi k_1-\mu_2)e_1 + (2/3)(3\mu_1+\mu_3)f_1-(\pi k_1\delta_1-\mu_1)e_2$$
$$-(2/3)(\mu_1+2\mu_2)f_2 + (3/2)(1-\delta_2)a_1-(4+3\delta_1+\delta_3)b_1$$
$$+(3/4)(3-2S_2-S_4)c_1 - 3\pi k_1\mu_1 a_2+4(S_1+S_2)b_2$$
$$+(9/4)(\delta_1-\delta_3)c_2)$$

$$F_{16}(k) = (1/\pi)((2d_1+d_2)\mu_2 + (2/3)(3\mu_1+\mu_3)e_1 + (f_1/2)(\pi k_1-\delta_4)$$
$$+ (2/3)(\mu_1+2\mu_2)e_2 - (f_2/2)(2\pi k_1 S_2-\mu_2) + (2+3S_1-S_3)a_1$$
$$+ (3/4)(1-S_4)b_1-(3/5)(6+5S_1+5S_1+S_5)c_1 - 2(S_1+S_2)a_2$$
$$- 3\pi k_1\mu_2 b_2 + (18/5)(\delta_2+\delta_3)c_2)$$

$$F_{25}(k) = (1/\pi)((-2d_1+d_3)\mu_1 - (\pi k_1 S_1-\mu_1)(e_1+e_3)-2(2\pi k_1-\mu_2)e_2$$
$$+ (2/3)(\mu_1+2\mu_2)(f_1-f_3)-3\pi k_1\mu_1(a_1+a_3) +3(1-S_2)a_2$$
$$- 4(S_1+S_2)(b_1-b_3) + (9/4)(S_1-S_3)(c_1+c_3) + (3/2)(3-2S_2-S_4)c_2)$$

$$F_{26}(k) = ((1/\pi)((d_1+4d_2+d_3)\mu_2 - (2/3)(2\mu_2+\mu_1)(e_1-e_3) - ((1/2)(2\pi k_1 S_1-\mu_2)$$

$$\times (f_1+f_3) - (4\pi k_1-\mu_4)f_2+2(S_1+S_2)(a_1-a_3)-3\pi k_1\mu_2(b_1+b_3)$$

$$+((3/2)(1-S_4)b_2 - 18/5(S_2+S_3)(c_1-c_3)$$

$$F_{35}(k) = (1/\pi)((-2d_2+4d_3)\mu_1-(\pi k_1 S_1-\mu_1)e_2 - (2\pi k_1-\mu_2)e_3$$

$$+((2/3)(\mu_1+2\mu_2)f_2 - (2/3)(\mu_3\mu 3\mu_1)f_3-3\pi k_1\mu_1 a_2+(3/2)(1-S_2)a_3$$

$$- 4(S_1+S_2)b_2 + (4+3S_1+S_3)b_3 + 9/4(S_1-S_3)c_2$$

$$+((3/4)(3-2S_2-S_4)c_3)$$

$$F_{36}(k) = ((1/\pi)((d_2+2d_3)\mu_2-(2/3)(2\mu_2+\mu_1)e_2 - (2/3)(\mu_3+3-\mu_1)e_3$$

$$-((1/2)(2\pi k_1 S_2-\mu_2)f_2 - (1/2)(4\pi k_1-\mu_4)f_3+2(S_1+S_2)a_2$$

$$- (2+3S_1-S_3)a_3 - 3\pi k_1\mu_2 b_2+(3/4)(1-S_4)b_3$$

$$-((18/5)(S_2+S_3)c_2 + (3/5)(6+5S_1+S_5)c_3)$$

Thus, three load deflection (4.15), one torque equilibrium (4.17) and two torque rotation (4.28) equations involving 6-displacement coefficients provide sufficient conditions for the complete analysis of a skew girder bridge deck.

These eithteen linear simultaneous equations obtained for a three girder bridge deck are solved for the deformation coefficients on a digital computer. These deformation coefficient are back substituted to get the value of girder deflection, rotation, shear force, bending moment and torsion for the applied loading.

Bending moment at the interface of the transverse system can be found with the help of slope deflection equations (4.5) after knowing the deformation functions.

4.4 RIGHT BRIDGE DECK

Right bridge is a special case of skew bridges, wherein the skew angle and thus skew parameter (K) vanishes. The theory assumptions and equations developed in this chapter for skew girder bridges are equally applicable to right bridges as well, by puting the value of, K, as zero. Thus in case of **right bridge** entire length of transverse system becomes effective.

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CHAPTER - 5

GIRDER MOMENT COEFFICIENTS BY HARMONICS METHOD

5.1 INTRODUCTION :

In Harmonics method girder moment expression can be written forthwith for a trial section of bridge under imposed loading of a certain fashion. This inherent capability of Harmonics method can be utilize to develope design coefficients. Thus the method lends itself to writing the girder moment expression in terms of sine series with the help of design coefficients developed in this chapter. These design coefficients are the functions of non dimensional structural parameters, α,β,γ and K, defined previously. The actual load harmonics of a particular loading are appropriately combined with the design coefficients corrosponding to a particular combination of α,β,γ and K, to give the moment coefficients for various girders. These moment coefficients are used to get moment expressions for various girders. A mathamatical model for these design coefficients and systematic computation procedure for calculating girder moments is presented hereunder.

5.2 MATHAMATICAL MODEL :

If we express girder moment for any imposed loading in terms of first three terms of an appropriate sine series, the coefficients of these terms are called as girder moment coefficients. These moment coefficients are evaluated corrosponding to a standard loading $(\frac{n\pi}{L})^2 \sin (\frac{n\pi x}{L})$; n = 1,3,3 applied one at a time on a girder for a particular combinations of structural parameters. The coefficients corrosponding to above loading thus became basic in nature as they help in computing bending moment corrosponding to any configuration of imposed loading. These moment coefficients therefore for various combination of structural parameters can be called as design coefficients. These design coefficients are denoted by a generallized symbol (p_{ij}^{mn}), representing the m^{th} harmonic of the bending moment in girder, i due to the n^{th} standard load harmonic $(\frac{n\pi}{L})^2 \sin (\frac{n\pi x}{L})$ on girder j. Three design coefficients for each of the three harmonic loading result in 9 coefficients for each of the girders and hence 9xN coefficients in total for a bridge deck with N girders.

For the development of mathamatical model the applied, loading on a particular girder, j in Fig. 4.1 is expressed as first three terms of its sine series, i.e.,

$$W_{ja} = (\frac{\pi}{L})^2 (\sin(\pi x_j/L) + 4 \sin(2\pi x_j/L) + 9 \sin(3\pi x_j/L)) \quad \ldots(5.1)$$

Integrating this loading twice we get corrosponding moment loading as :

$$M_{ja} = \sin(\pi x_j/L) + \sin(2\pi x_j/L) + \sin(3\pi x_j/L) \quad \ldots(5.2)$$

Let girder j be subjected to the first harmonic of imposed loading i.e.,

$$W_j = (\frac{\pi}{L})^2 \sin(\frac{\pi x_j}{L}) \text{ or } M_j = \sin (\pi x_j/L).$$

This imposed loading causes a generalized deformation of its constitute members, which give rise to the development of shear loading at the interface of transverse system and girders. This shear loading given by equation (4.12) at the interface of transverse system with girder j can be written as

$$S_{jf}+S_{jt} = \frac{6Q}{h} (F_{j1}(K) \sin(\pi x_j/L) + 16F_{j2}(K) \sin(2\pi x_j/L) + 81F_{j3}(K) \sin(3\pi x_j/L)) \quad ...(5.3)$$

Moment loading corrosponding to above loading can be written as

$$M_{js} = \frac{6Q}{h} (\frac{L}{\pi})^2[F_{j1}(K) \sin(\pi x_j/L) + 4F_{j2}(K) \sin(2\pi x_j/L) + 9F_{j3}(K) \sin(3\pi x_j/L)] \quad ...(5.4)$$

Thus the final moment function for girder j under the action of first load harmonic, after rearranging the terms can be written as

$$M_j = M_{ja}+M_{js} =[\alpha'_jF'_{j1}(K)+1]\sin(\pi x_j/L)+\alpha'F'_{j2}(K)\sin(2\pi x_j/L) + \alpha'F'_{j3}(K)\sin(3\pi x_j/L) \quad ...(5.5)$$

Deflection function for girder j under the action of first harmonic of retained loading is similar to that given by equation (5.3) and can be written as

$$Y_j = a_j\sin(\pi x_j/L) + b_j\sin(2\pi x_j/L) + C_j\sin(3\pi x_j/L) \quad ...(5.6)$$

If we multiply the flexural rigidity of girder (EI_j) with second derivative of equation (5.6) we get the expression of girder moment as

$$M_j = a_j' \sin(\pi x_j/L) + b_j' \sin(2\pi x_j/L) + C_j'(3\pi x_j/L) \quad ...(5.7)$$

In which $a_j' = (\frac{\pi}{L})^2 \, EI_j a_j$

$b_j' = (\frac{2\pi}{L})^2 \, EI_j b_j$, $C_j' = (\frac{3\pi}{L})^2 \, EI_j C_j$

Since equations (5.5) and (5.7) are identical comparing these two we get following relationships

$$\alpha_j' F_{j1}'(K) - a_j' = -1$$
$$\alpha_j' F_{j2}'(K) - b_j' = 0 \quad ...(5.8)$$
$$\alpha_j' F_{j3}'(K) - C_j' = 0$$

Similar relationships for girder j can be derived under the action of 2nd and 3rd load harmonics on girder j.

The corrosponding relationship for the unloaded girder $i(i \neq j)$ for each of the three load harmonics are obtained as :

$$\alpha_i' F_{i1}'(K) - a_i' = 0$$
$$\alpha_i' F_{i2}'(K) - b_i' = 0 \quad ...(5.9)$$
$$\alpha_i' F_{i3}'(K) - C_i' = 0$$

The girder torque equilibrium equation (4.17) and two girder rotation relationships (4.27) as obtained earlier remains unaffected irrespective of whether girder is loaded or unloaded. These three relationships for any girder j with slight modification can be written as

$$F_{j4}'(K) = 0$$
$$F_{j5}'(K) = (1-K)\beta_j e_j \quad ...(5.10)$$
$$F_{j6}'(K) = 4(1-K)\beta_j f_j'$$

wherein

$$F'_{j4}(K) = (\frac{\pi}{L})^2 EI_j F_{j4}(K), \; d'_j=(\frac{\pi}{L})^2 EI_j d_j$$

$$F'_{j5}(K) = (\frac{\pi}{L})^2 EI_j F_{j5}(K), \; e'_j=(\frac{\pi}{L})^2 EI_j e_j$$

$$F'_{j6}(K) = (\frac{2\pi}{L})^2 EI_j F_{j6}(K), f'_j =(\frac{2\pi}{L})^2 EI_j f_j$$

Thus under each load harmonic, six independent equations in terms of six displacement coefficients a',b',C'',d',e' and f' are available. For each of the girders loaded or unloaded. These equations provides sufficient conditions to uniquely determine displacement coefficients for any girder. The first three of these harmonics represent the bending moment design coefficients and are easily obtained corrosponding to each of the three standard load harmonics $(\frac{n\pi}{L})^2 \sin (\frac{h\pi x}{L})$ n = 1,2,3 for any combination of structural parameters α,β,and K. Some sample values of these design coefficients are given in appendix-D . For the efficient use of these design coefficients a generalised symbol (p_{ij}^{mn}) as explained earlier is adopted. For ready use in design office these coefficients are available in reference 6 for various values of α,β and K covering the entire range of real life 4-girder bridges.

Using this generalised notation for design coefficients, the moment function for girder i due to the first load harmonic $(\pi/L)^2 \sin (\pi x_j/L)$ or its equivalent moment loading $\sin(\pi x_j/L)$ on girder j can be written as :

$$M_{ij}^{(1)} (X_i) = p_{ij}^{11} \sin(\pi x_i/L) + p_{ij}^{21} \sin(2\pi x_i/L)$$

$$+ p_{ij}^{31} \sin(3\pi x_i/L) \qquad ...(5.11)$$

Moment functions for girder i due to second and third load

harmonic acting separately ...

as :

$$M_{ij}^{(2)}(X_i) = p_{ij}^{12} \sin(\pi x_i/L) + p_{ij}^{22} \sin(2\pi x_i/L) + p_{ij}^{32} \sin(3\pi x_i/L) \quad \ldots(5.12)$$

$$M_{ij}^{(3)}(X_i) = p_{ij}^{13} \sin(\pi x_i/L) + p_{ij}^{23} \sin(2\pi x_i/L) + p_{ij}^{33} \sin(3\pi x_i/L) \quad \ldots(5.13)$$

Moment expression for girder i for all the three harmonics of loading acting simultaneously on girder j is obtained by super-imposing the moment function under each harmonic loading separately and can be written as

$$M_{ij}(X_i) = \sum_{m=1}^{3} M_{ij}^{m} \sin(m\pi x_i/L) \quad \ldots(5.14)$$

in which $M_{ij}^{m} = \sum_{n=1}^{3} p_{ij}^{mn}$ $\qquad m = 1,2,3$

or in the matrix form we can write -

$$M_{ij}(X_i) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T \begin{bmatrix} p_{ij}^{11} & p_{ij}^{21} & p_{ij}^{31} \\ p_{ij}^{12} & p_{ij}^{22} & p_{ij}^{32} \\ p_{ij}^{13} & p_{ij}^{23} & p_{ij}^{33} \end{bmatrix} \begin{bmatrix} \sin(\pi x_i/L) \\ \sin(2\pi x_i/L) \\ \sin(3\pi x_i/L) \end{bmatrix} \quad \ldots(5.15)$$

For the actual loading cases when expressed by the first three harmonic components of their sine series the load harmonic coefficients are going to be different than those considered above. These harmonic coefficients are appropriately combined with the design coefficients to obtain corrosponding moment expression of girder for various loading conditions.

5.3 GIRDER MOMENT EXPRESSIONS :

Girder moment expressions with the help of design coefficients are developed hereunder for various cases of imposed loading.

(a) Single Load on a Girder :

Let W_{j1} be the concentrated load acting on girder j at a distance x_{j1} from the left support. Using the Fourier analysis (Appendix A) for expressing this load by its Sine series and is given as :

$$W_{j1} = \sum_{n=1}^{3} W_{j1}^{n} \sin(n\pi x_j/L) \qquad \ldots(5.16)$$

in which, $W_{j1}^{n} = 2W_{j1}/L \sin(n\pi x_{j1}/L)$

The above loading expressions (5.16) is equivalent to a moment loading on the girder as given below :

$$M_j = (L/\pi)^2 \sum_{n=1}^{3} (W_{j1}^{n}/n^2)\sin(n\pi x_j/L) \qquad \ldots(5.17)$$

Combining the harmonic coefficients of (5.17) with the design coefficients (obtained already with respect to standard load harmonics with harmonic coefficients as unity) the bending moment expression for girder i due to a conc. load on girder j is written in matrix form as follows :

$$M_{ij}(x_i) = \left(\frac{L}{\pi}\right)^2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^{T} \begin{bmatrix} (w_{j1}^{1} p_{ij}^{11} & w_{j1}^{1} p_{ij}^{21} & w_{j1}^{1} p_{ij}^{31} \\ \frac{w_{j1}^{2}}{4} p_{ij}^{12} & \frac{w_{j1}^{2}}{4} p_{ij}^{22} & \frac{w_{j1}^{2}}{4} p_{ij}^{32} \\ \frac{w_{j1}^{3}}{9} p_{ij}^{13} & \frac{w_{j1}^{3}}{9} p_{ij}^{23} & \frac{w_{j1}^{3}}{9} p_{ij}^{33} \end{bmatrix} \begin{bmatrix} \sin(\pi x_i/L) \\ \sin(2\pi x_i/L) \\ \sin(3\pi x_i/L) \end{bmatrix} \qquad \ldots(5.18)$$

or $M_{ij}(x_i) = (L/\pi)^2 \sum_{m=1}^{3} M_{ij}^{m} \sin(m\pi x_i/L)$ ...(5.19)

in which, $M_{ij}^{m} = \sum_{n=1}^{3} (w_{j1}^{n}/n^2)\, p_{ij}^{mn}$ $\quad m = 1,2,$ or 3

(b) Multiple Load on a Single Girder :

Let $w_{j1}, w_{j2}, \ldots w_{jp}$ are the concentrated loads acting at $x_{j1}, x_{j2}, \ldots x_{jp}$ respectively on girder j or an N-girder bridge. This load system can be expressed in terms of first three Sine harmonics (Appendix-A) and is given as :

$$w_j = \sum_{n=1}^{3} W_j^{n} \sin(n\pi x_j/L) \qquad \ldots(5.20)$$

where, in $W_j^{n} = \frac{2}{L} \sum_{1}^{p} w_{jp} \sin(n\pi x_{jp}/L)$

Integrating twice the above load function, the equivalent moment load function on girder j is obtained as

$$M_j = (L/\pi)^2 \sum_{n=1}^{3} (w_j^{n}/n^2) \sin(n\pi x_j/L) \qquad \ldots(5.21)$$

Again combining the design coefficients with the harmonic coefficients of moment loading (5.21) on girder j appropriately, the corresponding moment expression for girder i is obtained in matrix form as :

$$M_{ij}(x_i) = \left(\frac{L}{\pi}\right)^2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T \begin{bmatrix} w_j^1 p_{ij}^{11} & w_j^1 p_{ij}^{21} & w_j^1 p_{ij}^{31} \\ \frac{w_j^2 p_{ij}^{12}}{4} & \frac{w_j^2 p_{ij}^{22}}{4} & \frac{w_j^2 p_{ij}^{32}}{4} \\ \frac{w_j^3 p_{ij}^{13}}{9} & \frac{w_j^2 p_{ij}^{23}}{9} & \frac{w_j^3 p_{ij}^{33}}{9} \end{bmatrix} \begin{bmatrix} \sin(\pi x_i/L) \\ \sin(2\pi x_i/L) \\ \sin(3\pi x_i/L) \end{bmatrix} \qquad \ldots(5.22)$$

or

$$M_{ij}(x_i) = (L/\pi)^2 \sum_{m=1}^{3} M_{ij}^m \sin(m\pi x_i/L) \qquad \ldots(5.23)$$

in which, $M_{ij}^m = \sum_{n=1}^{3} (w_j^n/n^2)\ p_{ij}^{mn} \qquad m = 1,2 \text{ or } 3$

(c) Multiple Loads on Multiple Girders :

If there are multiple loads on more than one girder, the girder moment expressions are initially obtained corresponding to each one of the loaded girders, loaded one at a time. The total effect due to all the loaded girders loaded simultaneously is obtained by using principle of superposition. Thus, for multiple loadings simultaneously applied on N_1 girders ($N_1 < N$) of an N-girder bridge, the moment expression for girder i is written as follows :

$$M_i(x_i) = (L/\pi)^2 \sum_{m=1}^{3} M_i^m \sin(m\pi x_i/L) \qquad \ldots(5.24)$$

in which, $M_i^m = \sum_{j=1}^{N_1} M_{ij}^m \qquad m = 1,2,3$

(d) Uniformly Distributed Load on a Girder :

Let w_j be the uniformly distributed load per unit length acting over entire length L or girder j. It is expressed by its Sine series (Appendix -A) and is given as :

$$w_j = (4w_j/\pi)\ [\sin(\pi x_j/L) + \tfrac{1}{3}\sin(3\pi x_j/L)] \qquad \ldots(5.25)$$

The above load functions corresponds to an equivalent moment load function on girder j given as follows :

$$M_j = (4w_j L^2/\pi^3)[\sin(\pi x_j/L) + \tfrac{1}{27}\sin(3\pi x_j/L)] \qquad \ldots(5.26)$$

Combining the harmonic coefficients of (5.26) with the design coefficients, the moment expression for girder i due to applied u.d.l on girder j is obtained as :

$$M_{ij}(x_i) = (4w_j L^2/\pi^3) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^T \begin{bmatrix} p_{ij}^{11} & p_{ij}^{21} & p_{ij}^{31} \\ 0 & 0 & 0 \\ \frac{1}{27}p_{ij}^{13} & \frac{1}{27}p_{ij}^{23} & \frac{1}{27}p_{ij}^{33} \end{bmatrix} \begin{bmatrix} \sin(\pi x_i/L) \\ \sin(2\pi x_i/L) \\ \sin(3\pi x_i/L) \end{bmatrix} \quad \ldots(5.27)$$

or $M_{ij}(x_i) = (4w_j L^2/\pi^3) \sum_{m=1}^{3} M_{ij}^m \sin(m\pi x_i/L)$ ...(5.28)

in which, $M_{ij}^m = p_{ij}^{m_1} + \frac{1}{27} p_{ij}^{m_3}$ $m = 1,2,$ or 3 ...(5.28a)

From expression (5.28a), it is evident that girder moment coefficient for this loading case can be obtained without requiring the load harmonic coefficients, thus reduces here the computation work.

(e) Patch Loading on a Girder :

Let w_j represents the uniformly distributed load over a small segment $x_1 \leq x \leq x_2$ of the span L of girder, using Fourier series the load expansion function can be written as:

$$w_j = \sum_{n=1}^{3} w_n \sin(n\pi x_j/L) \quad \ldots(5.29)$$

where in, $w_n = (2w_j/n\pi)[\cos(n\pi x_1/L) - \cos(n\pi x_2/L)]$

The equivalent moment loading on girder j is obtained as

$$M_j = (L/\pi)^2 \sum_{n=1}^{3} (w_n/n^2)\sin(n\pi x_j/L) \quad \ldots(5.30)$$

Combining the harmonic coefficients of (5.30) with design coefficients appropriately, the moment expression for girder i due to patch loading on girder j is obtained as follows :

$$M_{ij}(x_i) = (L/\pi)^2 \sum_{m=1}^{3} M_{ij}^{m} \sin(m\pi x_i/L) \qquad \ldots(5.31)$$

where in, $M_{ij}^{m} = \sum_{n=1}^{3} (w_n/n^2)\, p_{ij}^{mn}$ $m = 1, 2$ or 3

Bending moment in all the girders of a bridge deck can thus be computed with the help of the expressions developed above for various types of loadings. A systematic procedure for computation of the girder bending moment is presented in the section that follows.

5.4 GIRDER MOMENT COMPUTATIONS :

The procedure presented hereunder makes the entire process of computations systematic wherein all calculations are carried out in tabular form. The procedure also facilitates quick rechecking and detection of errors. The various steps are as :

(i) From the known dimensions and material properties of a girder bridge to be designed, compute the structural parameters α, β, γ and K of the bridge.

(ii) Pick up the design coefficients corresponding to above structural parameters obtained under step (i) from the already available design aid tables (18). These remain same for all the loading cases and are arranged in Tabular form as shown in Table 6.19 through Table 6.22 . In the development of these design coefficients γ is ignored for its insignificant influence on the structural behaviour of real girder bridge decks. Design coefficients for values of α and K other than those available in design coefficient tables (18) may be computed by assuming a linear interpolation, but for the value of β other than the ones given directly in the design tables following interpolation function is used

$$(p_{ij}^{mn})_{\beta} = (p_{ij}^{mn})_{o} + z[(p_{ij}^{mn})_{10} - (p_{ij}^{mn})_{o}] \qquad \ldots(5.32)$$

where in, $z = \left[\frac{\beta\sqrt{\alpha}}{(3+\beta\sqrt{\alpha})} \right]^{1/2}$

The computations of design coefficients corresponding to actual value of β are accomplished in tabular from using above interpolation function and are shown in the lower half of Table 6.19 through Table 6.22

(iii) Compute the equivalent loading directly acting on dirders as continuous reactions in case applied loading on the deck is lying between the girders.

(iv) Load harmonic coefficients corresponding to a given loading case are computed in a tabular form as is shown, e.g., for the multiple loads on a girder in Table 6.23 and Table 6.24.

(v) These load harmonic coefficients obtained in (iv) are appropriately combined with the design coefficients obtained in (ii) to yield girder moment coefficients. This process is accomplished in a tabular form and is shown in Table 6.25 through Table 6.34.

(vi) Once girder moment coefficients corresponding to applied loading are obtained, the moment values at various desired location along the length of girder can easily be computed in tabular form as shown in Table 6.35 . Bending moment for all the girders at any number of desired locations can thus be obtained for any type of loading on a bridge deck.

The scheme for girder moment computation presented above is quite general in nature and devoid of any constraint with regard to the number of loads on a girder or number of girders loaded simultaneously.

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CHAPTER - 6

APPLICATION OF LOAD DISTRIBUTION THEORIES

6.1 INTRODUCTION

In the previous chapters, theoretical formulations of various load distribution theories were discussed in detail. This chapter deal with the application of these load distribution theories for structurally analysing a girder bridge. A four girder right bridge has been chosen for this purpose. This bridge has been analysed for two loading cases of IRC class-A loading. In the first loading case, transverse position of this loading is kept at extreme left i.e., at minimum clear distance from the kerb of the bridge deck, to obtain maximum moment in outer girder. In the second case, transverse position of loading is kept symmetrical with respect to the bridge axis.

Live load bending moments and deflections are computed for the longitudinal girders with the help of theories discussed earlier. These moments and deflections obtained with the help of Courbon's method, Orthotropic plate theory and Harmonics method are plotted and compared at the end of this chapter. In the case of Orthotropic plate theory , design curves are available only for right bridges and therefore, a right bridge has been chosen for analysis. To make this chapter comprehensive, each computation step is preceeded by the relevent explanation required for easy comprehension.

6.2 RIGHT GIRDER BRIDGE

The cross section and longitudinal section of the 4-girder right bridge chosen for demonstrating the application of various theories are shown in Fig. 6.1. The relevant bridge data are given below.

6.2.1 Dimensions

The various dimensions of the bridge elements are assumed as given hereunder

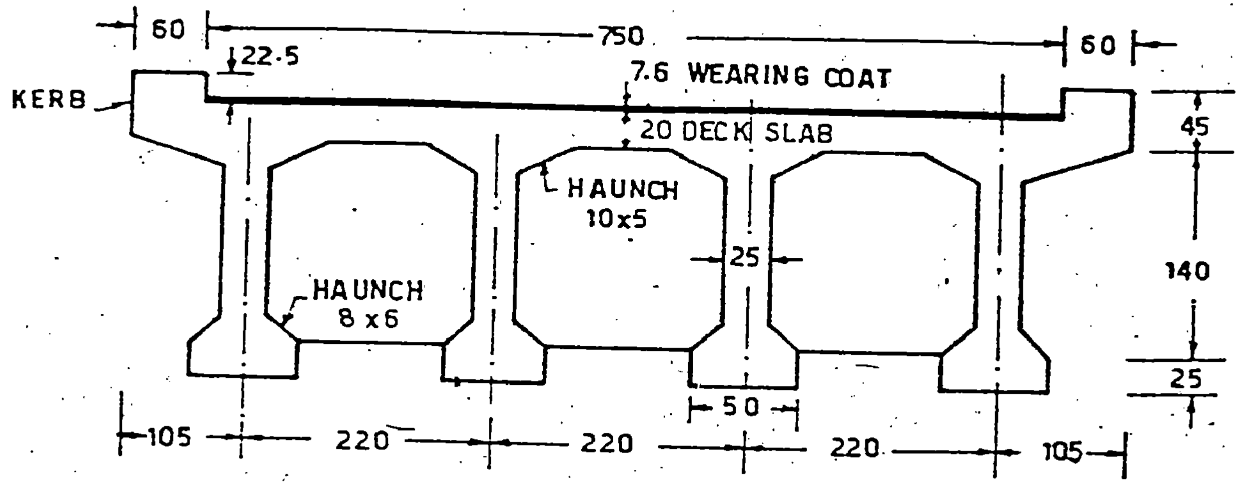
Clear road way (B) (cm)	= 750
Effective span (L) (cm)	= 1940
Width of bearing assumed (cm)	= 100
Number of longitudinal girders (N)	= 4
Spacing of longitudinal girders (h) (cm C/C)	= 220
Depth of girder (cm)	= 180
Width of rib of girder (cm)	= 25
Spacing of cross beams (q) (cm C/C)	= 485
Depth of cross beam (cm)	= 155
Width of rib of cross beam (cm)	= 20
Slab thickness (t) (cm)	= 20

6.2.2 Material Properties

M 20 concrete is assumed to be used for construction of bridge members which has modulus of elasticity approximately equal to 2.5×10^5 kg/cm^2 and possion's ratio is assumed as 0.15

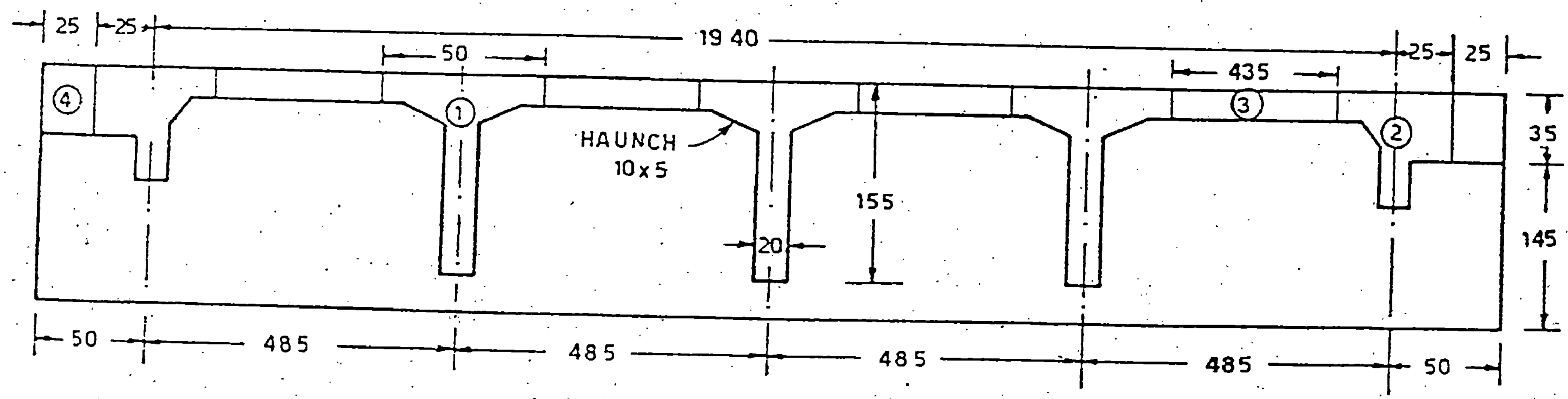
6.2.3 Design Loading

IRC class-A two lane loading as shown in Fig. 6.2 and Fig. 6.3 is taken for analysis.



(a) CROSS-SECTION

ALL DIMENSIONS IN-cm



(b) LONGITUDINAL-SECTION

FIG. 6·1 — 4-GIRDER RIGHT BRIDGE

Impact factor (I) = $\frac{4.5}{6+L}$ = 0.177

6.2.4 Live Load Positions

Two typical transverse dispositions of IRC class-A loading have been considered herein, as shown in Fig. 6.2 and Fig. 6.3, to explain the computation of live load bending moments and deflection of girders.

In the first case as shown in Fig. 6.2, two trains of Class-A loading are placed close to the kerb with minimum clearance as specified by IRC (15 cm). It gives the maximum bending moment in edge girder.

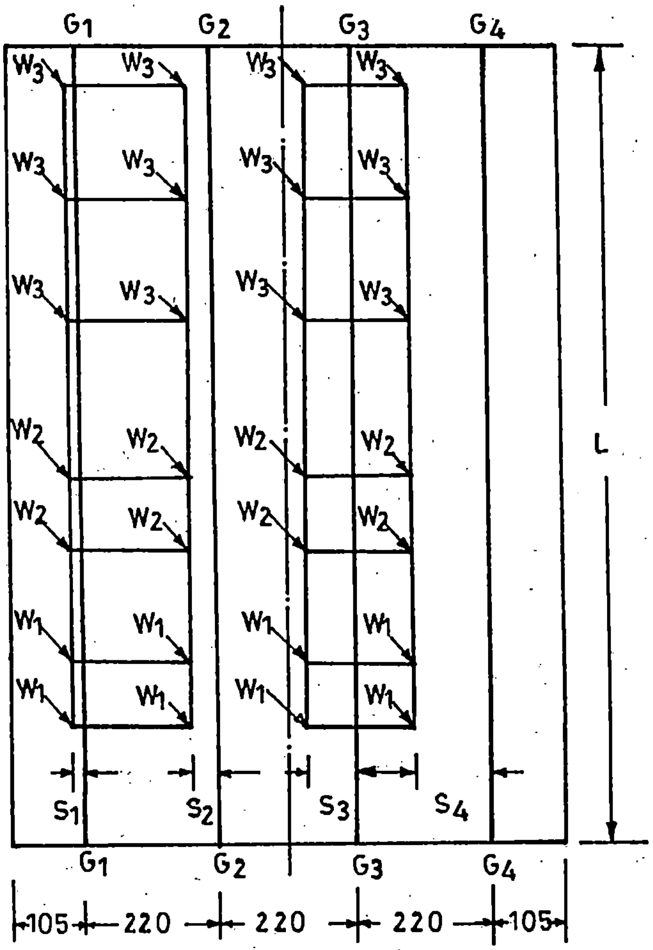
In the second case shown in Fig. 6.3, two trains of IRC class-A loading are placed symmetrical with respect to the bridge axis. In the longitudinal direction, first seven loads of trains are placed in such a manner that the C.G. of the load system and the load nearest to the mid-span section of the bridge are equidistant from the mid-span section of the bridge to give maximum moment under that load.

6.3 COURBON'S METHOD

Courbon's method, as discussed in chapter - 2 has been applied to find the longitudinal girder moments for the right girder bridge. Part of the total load (4W as shown in Fig.6.2(b) and Fig.6.3(b)),shared by each of the girders is found with the help of equation (2.5) i.e.

$$R_i = \frac{P}{N} \left[1 + \frac{N}{\Sigma X_i^2} \cdot e X_i \right]$$

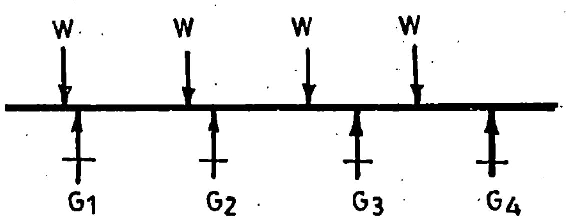
LOAD SYSTEM : EXTREME LEFT (15 Cm FROM KERB)



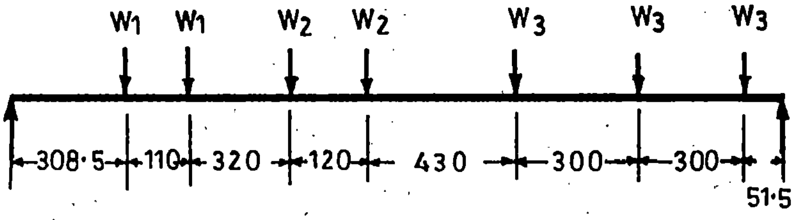
L = 1940
h = 0220
N = 0004
S_1 = 0005
S_2 = 0045
S_3 = 0095
S_4 = 0135
W_1 = 1350
W_2 = 5700
W_3 = 3400

NOTE :-
1. ALL DIMENSIONS IN Cm
2. LOADS IN Kg.

(d) LOAD POSITION IN PLAN

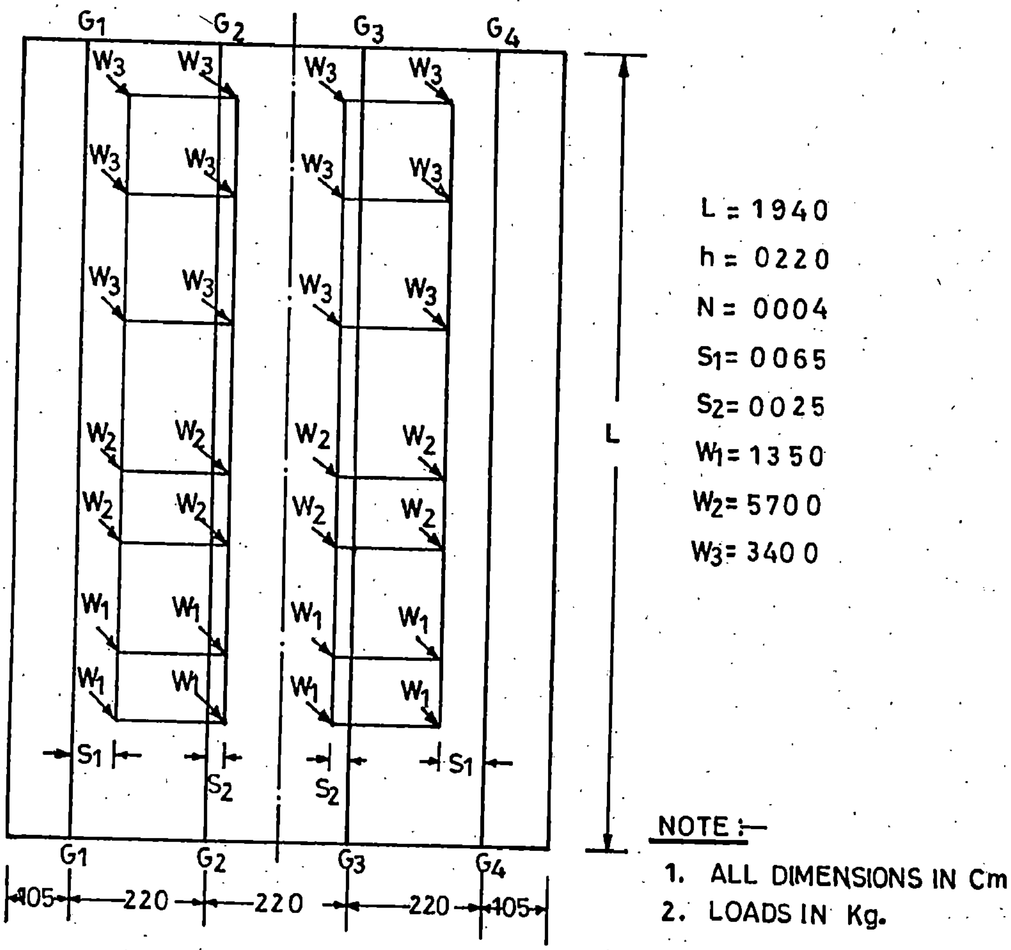


(b) TRANSVERSE LOAD POSITION

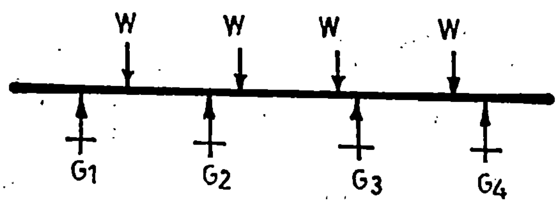


(c) LONGITUDINAL LOAD POSITION

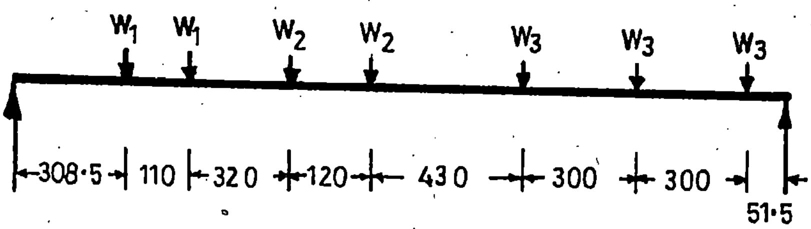
FIG. 6·2–LOADED BRIDGE DECK.



(a) LOAD POSITION IN PLAN



(b) TRANSVERSE LOAD POSITION



(c) LONGITUDINAL LOAD POSITION

FIG. 6·3—LOADED BRIDGE DECK

for bridge deck shown in Fig. 6.1

$$N = 4 \; ; \qquad \Sigma X_i^2 = 24.2$$

Hence, $R_i = P[1 + \frac{4}{24.2} e X_i]$

where e is the eccontricity of C.G. of loads from the bridge axis.

This equation gives the load sha red by various girders as given below

(i) Load system extreme left (15 cm from kerb)

$R_1 = 1.381$ W, $R_3 = 0.873$ W

$R_2 = 1.127$ W, $R_4 = 0.618$ W

(ii) Load system symmetrically place (e = 0)

$R_1 = R_2 = R_3 = R_4 = W.$

6.3.1 Girder Moment Computation

Girder moments for both the load systems are computed in Table 6.1 and Table 6.2. In the second column of the tables mean bending moment for each girder obtained by considering each girder under the action of loading as shown in Fig. 6.2(c) and Fig. 6.3(c) with impact loading. Since there are four girders and bridge is subjected to two lane loading i.e. four wheel loads along the width of the bridge, mean bending moment for each girder will be corrosponding to above loading.

TABLE - 6.1 : LIVE LOAD B.M. AT VARIOUS LOCATIONS.
LOAD SYSTEM : Extreme Left.

X/L	M_x(mean) (t.m.)	Girder moments at various locations (t.m.)			
		G_1 M_x(mean)xR_1	G_2 M_x(mean)xR_2	G_3 M_x(mean)xR_3	G_4 M_x(mean)xR_4
0.20	47.84	66.07	53.92	41.76	29.56
0.40	82.58	114.04	93.07	72.09	51.03
0.44	84.86	117.19	95.63	74.08	52.44
0.50	80.46	111.12	90.68	70.23	49.72
0.60	73.81	101.93	83.18	64.44	45.61
0.80	46.97	64.87	52.94	41.00	29.02

TABLE - 6.2 : LIVE LOAD B.M. AT VARIOUS LOCATIONS.
LOAD SYSTEM : Symmetrically Placed.

X/L	M_x(mean) (t.m.)	Girder moments at various locations (t.m.)			
		G_1 M_x(mean)xR_1	G_2 M_x(mean)xR_2	G_3 M_x(mean)xR_3	G_4 M_x(mean)xR_4
0.20	47.84	47.84	47.84	47.84	47.84
0.40	82.58	82.58	82.58	82.58	82.58
0.44	84.86	84.86	84.86	84.86	84.86
0.50	80.46	80.46	80.46	80.46	80.46
0.60	73.81	73.81	73.81	73.81	73.81
0.80	46.97	46.97	46.97	46.97	46.97

Now actual bending moments due to load distribution are obtained by multiplying these mean moments with the share of each girder as obtained in previous article.

6.4 ORTHOTROPIC PLATE THEORY

Girder bending moments and deflections have been calculated with the help of both Morice, Little and Rowe Curves and Cusens and Pama curves. These design curves are given in references (7 and 20) and are also given at the end of this report in Appendix - E. Various steps for analysing bridge deck are shown hereunder.

6.4.1 Geometric Properties

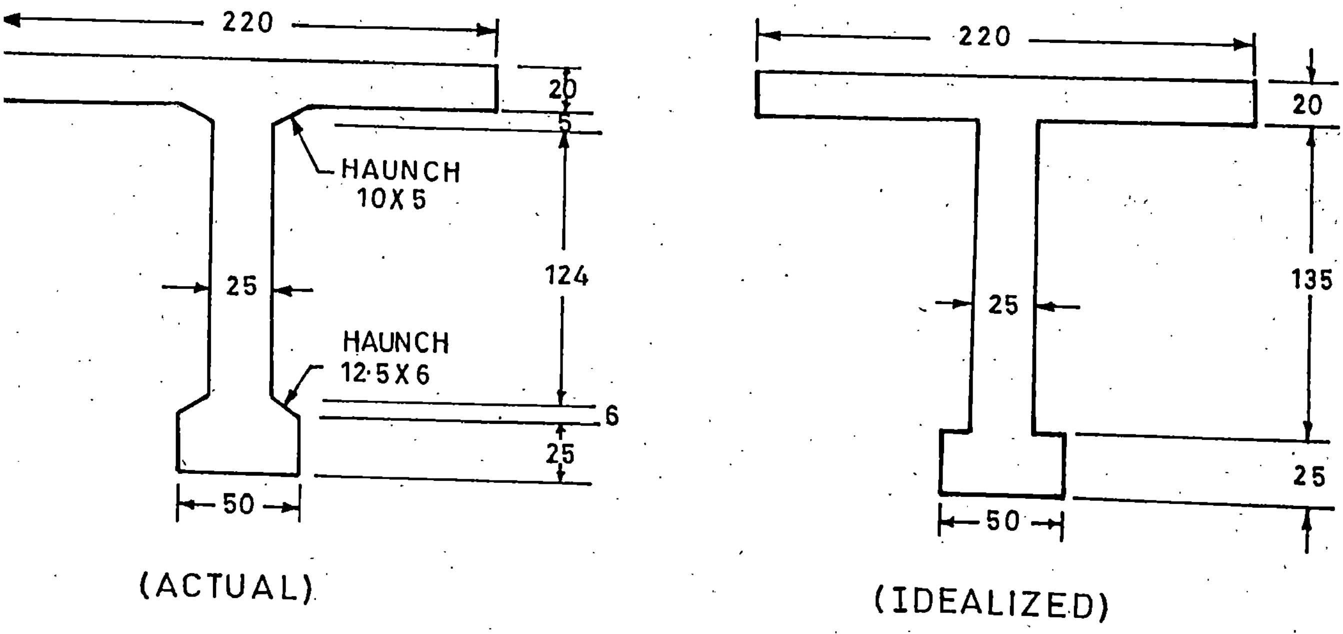
For the calculation of geometric properties flange width of I-beam is taken as the spacing of girders, as specified by Rowe (20), cross sections and idealized sections of longitudinal girders and cross beams are shown in Fig. 6.4.

For longitudinal girders (Fig. 6.4 a)

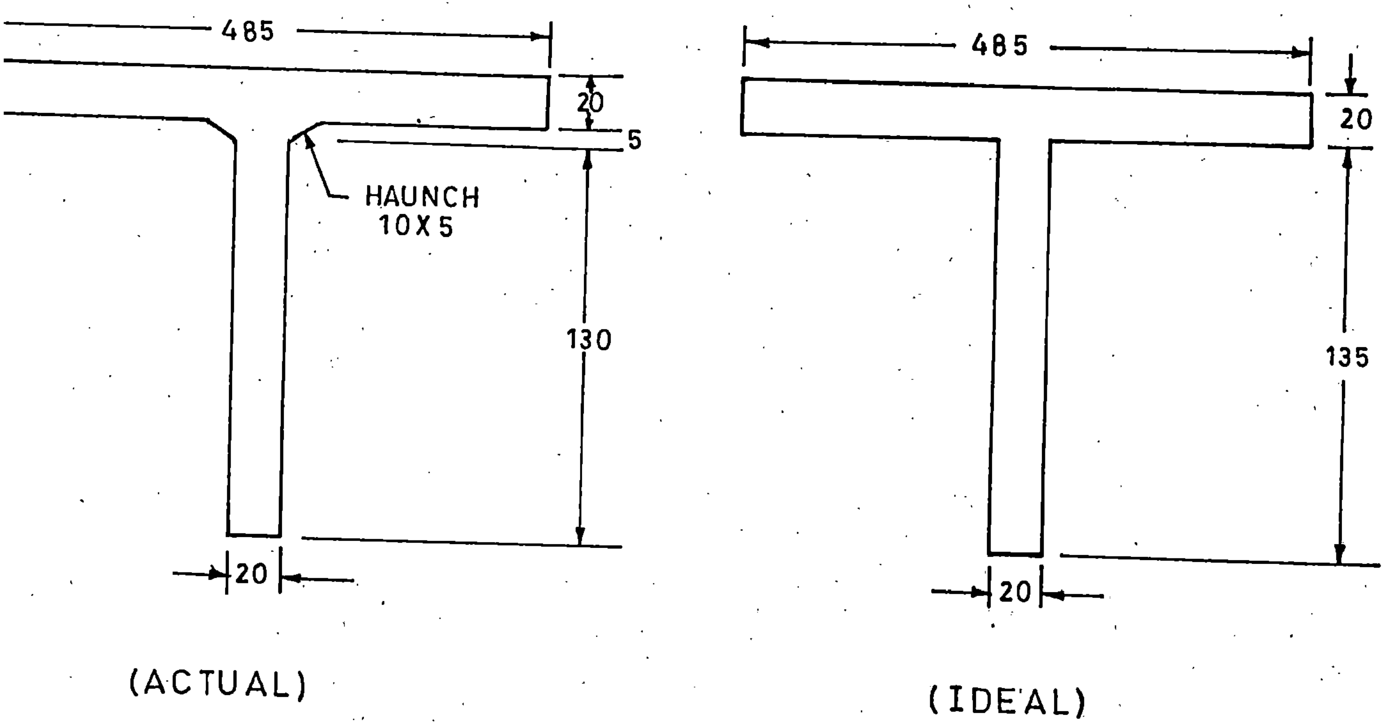
Depth of H.A. ($\bar{Y}$)	= 0.608 m
Moment of inertia (I)	= 0.3329 m^4
Torsional inertia (I_o)	= 10.937×10^{-3} m^4
M.I. per unit length ($i = \frac{I}{p}$)	= 0.1515 m^4/m
T.I. per unit length ($i_o = \frac{I_o}{g}$)	= 4.97×10^{-3} m^4/m

For cross girders (Fig. 6.4 (b))

Moment of inertia (J)	= 0.2049 m^4
Torsional inertia (J_o)	= 12.64×10^{-3} m^4
M.I.per unit length ($j = \frac{J}{q}$)	= 0.042 m^4/m



(a) CROSS SECTION OF LONGITUDINAL GIRDER



(b) CROSS SECTION OF CROSS BEAM

FIG. 6·4—CROSS SECTION OF GIRDERS IN ORTHOTROPIC PLATE THEORY

Method of calculating torsional inertia is shown in Appendix - B.

Effective width (2b=Np) = 8.8 m

Effective span (2a = L) = 19.40 m

6.4.2 Morice, Little and Rowe Curves

Structural parameters defined by above authores are

Flexural Parameter, $\theta = \frac{b}{2a}\left(\frac{i}{j}\right)^{\frac{1}{4}} = 0.312$

Rorsional Parameter, $\alpha = \frac{G(i_o+j_o)}{2E\sqrt{ij}} = 0.0206$

Corrosponding to these values of α and θ, values of distribution coefficients K_o, K_1 and K_α are obtained at reference stations from the design curves (Appendix - E) and are shown in Tables 6.3, 6.4 and 6.5. From Maxwell's reciprocal theorem these coefficients are symmetrical about the marked diagonal. Also each row in these tables represents a number of points on the influence line for a load moving across the transverse width of the bridge from - b to +b. If a uniformly distributed line load were applied over the entire width of the bridge, the distribution coefficients at any reference point would be unity, since the entire bridge deflects uniformly. Therefore for a uniformly distributed load the area between the influence line for K and the axis must be unity. This area is found at the end column of table by Simpson's rule.

$$K_{mean} = \frac{1}{3} \times \frac{b}{4} \times \frac{1}{2b} [K{-b}+4(K{-3b}+4\ Kb/4 + K3b/4)+2(-Kb/2)+Kb]=1$$

Equivalent Load Coefficient (λ)

For computation of equivalent load at reference stations,

TABLE - 6.3 : VALUES OF K_o.

Position of section load at	Reference Station									Row Integral
	-b	-3b/4	-b/2	-b/4	0	+b/4	+b/2	+3b/4	+b	
0	0.85	0.94	0.99	1.09	1.12	1.09	0.99	0.94	0.85	1.00
+b/4	0.19	0.40	0.64	0.88	1.09	1.23	1.38	1.52	1.67	1.00
+b/2	-0.54	-0.17	0.24	0.64	0.99	1.35	1.74	2.10	2.50	0.99
+3b/4	-1.14	-0.62	-0.17	0.40	0.94	1.52	2.10	2.74	3.32	1.00
+b	-1.77	-1.14	-0.54	0.19	0.85	1.67	2.50	3.32	4.10	1.00

TABLE - 6.4 : VALUES OF K_1.

Position of section load at	Reference Station									Row Integral
	-b	-3b/4	-b/2	-b/4	0	+b/4	+b/2	+3b/4	+b	
0	0.95	0.97	1.00	1.03	1.05	1.03	1.0	0.97	0.95	1.00
b/4	0.84	0.88	0.94	0.97	1.03	1.06	1.06	1.06	1.07	0.99
b/2	0.76	0.80	0.88	0.94	1.00	1.06	1.13	1.20	1.24	1.00
3b/4	0.68	0.72	0.80	0.88	0.97	1.06	1.20	1.31	1.43	1.00
b	0.60	0.68	0.76	0.84	0.85	1.07	1.24	1.43	1.65	1.01

TABLE - 6.5 : VALUES OF $K_\alpha = K_o + (K_1 - K_o)\sqrt{\alpha}$; $\alpha = 0.0206$

Position of section load at	Reference Station									Row Integral
	-b	-3b/4	-b/2	-b/4	0	+b/4	+b/2	+3b/4	+b	
0	0.86	0.94	0.99	1.08	1.11	1.08	0.99	0.94	0.86	1.00
+b/4	0.28	0.47	0.68	0.90	1.08	1.21	1.33	1.45	1.58	1.01
+b/2	-0.35	-0.03	0.03	0.68	0.99	1.33	1.65	1.97	2.32	0.99
+3b/4	-0.88	-0.43	-0.03	0.47	0.94	1.45	1.97	2.53	3.05	1.00

the deck portion between two reference stations is considered as simply supported at those reference stations and reactions at each reference stations due to applied loading are obtained. These equivalent loads at various reference stations, for both loading cases, are shown in Fig. 6.5(a) and Fig. 6.5 (b).

Derivation of Distribution Coefficient Profile

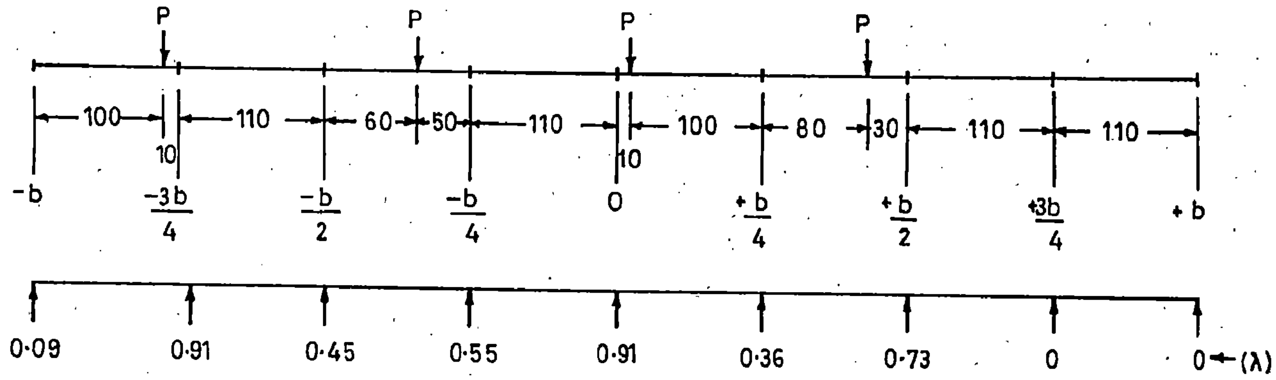
Distribution coefficients at different reference stations are derived in Table 6.6. Various computational steps are shown in table itself. Distribution coefficients at girder locations can be obtained by drawing distribution coefficient profile. Since this particular bridge is having girders at reference stations -3b/4, -b/4, +b/4 and +3b/4, the distribution coefficients are taken directly from Table 6.6 and are given below in Table A.

TABLE - A : DISTRIBUTION COEFFICIENTS FOR GIRDERS

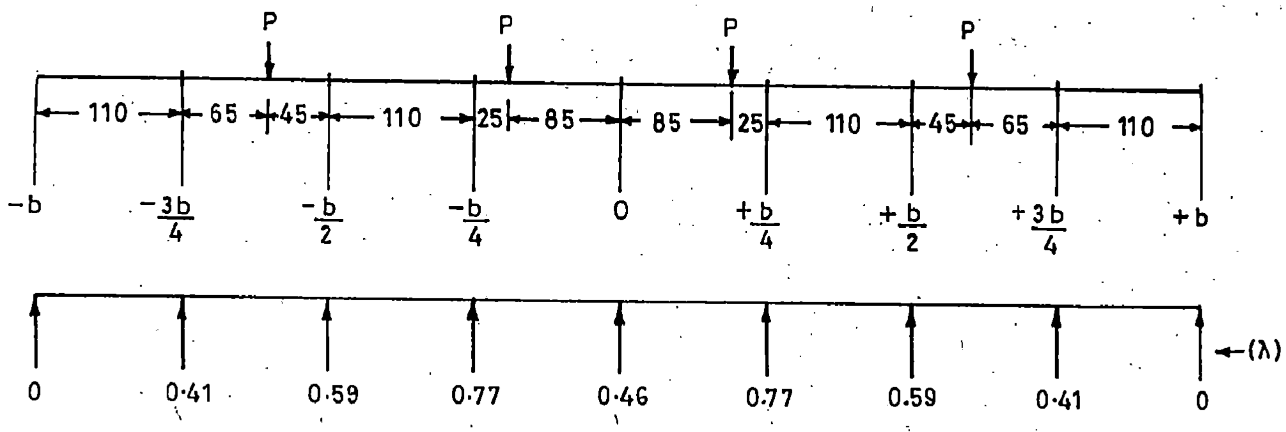
Girder	Distribution Coefficient K' (Case I)	(Case II)
G_1	1.29	0.978
G_2	1.13	1.024
G_3	0.91	1.024
G_4	0.65	0.978

Girder Moment and Deflection Computation

In theoritical formulation bridge deck was considered as an orthotropic plate. Values of distribution coefficients at various reference stations have already been found. These values are not changing along the span of the bridge. For the computation of longitudinal bending moments, loading is assumed to be



(a) LOAD POSITION: EXTREME LEFT



(b) LOAD POSITION: EXTREME LEFT

FIG. 6·5 — EQUIVALENT LOAD COEFFICIENT (λ)

TABLE - 6.6 : DERIVATION OF DISTRIBUTION COEFFICIENT PROFILE

CASE (I) : LOAD SYSTEM : Extreme Left (15 cm FROM KERB)

Load at	Equi. Load Coeffi. (λ)	Values of λK_α at reference stations								
		$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$
-b	0.09	0.34	0.28	0.21	0.14	0.08	0.03	-0.03	-0.08	-0.13
-3b/4	0.91	2.77	2.30	1.79	1.31	0.86	0.43	-0.03	-0.39	-0.80
-b/2	0.45	1.04	0.89	0.74	0.60	0.44	0.30	0.15	-0.01	0.16
-b/4	0.55	0.87	0.80	0.72	0.67	0.59	0.49	0.37	0.26	0.15
0	0.91	0.78	0.85	0.90	0.98	1.01	0.98	0.80	0.85	0.78
+b/4	0.36	0.10	0.17	0.24	0.32	0.39	0.44	0.47	0.52	0.57
+b/2	0.73	-0.26	-0.02	0.24	0.49	0.72	0.97	1.20	1.44	1.69
$K' = \frac{\Sigma \lambda K_\alpha}{4}$		1.41	1.29	1.22	1.13	1.03	0.91	0.81	0.65	0.61

TABLE – 6. 6 : DERIVATION OF DISTRIBUTION COEFFICIENT PROFILE

CASE (II) : LOAD SYSTEM : Symmetrically placed

Load at	Equi. Load Coeffi. (λ)	Values of λK_α at reference stations								
		$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$
−3b/4	0.41	1.251	1.037	0.808	0.595	0.385	0.193	−0.012	−0.176	−0.361
−b/2	0.59	1.369	1.162	0.974	0.785	0.584	0.401	0.195	−0.018	−0.205
−b/4	0.77	1.216	1.162	1.023	0.935	0.831	0.693	0.523	0.362	0.216
0	0.46	0.396	0.432	0.455	0.4960	0.511	0.496	0.455	0.432	0.396
+b/4	0.77	0.216	0.361	0.524	0.693	0.832	0.932	0.024	1.116	1.216
+b/2	0.59	−0.206	−0.018	0.195	0.401	0.584	0.785	0.975	1.162	1.369
+3b/4	0.41	−0.361	−0.176	−0.012	0.193	0.385	0.595	0.808	1.037	1.251
$K' = \frac{\Sigma \lambda K_\alpha}{4}$		0.984	0.978	0.991	1.024	1.028	1.024	0.991	0.998	0.984

uniformuly distributed along the width of the bridge deck. This loading gives the mean bending moments along the span of the equivalent orthotropic plate. Mean bending moment for a particular girder will be obtained by multiplying mean bending moment obtained above by flange width of that girder, which is same as in Courbon's Method. This girder mean moment when multiplied by distribution coefficient for a particular girder will give the actual bending moment along that girder. An additional 10% increase, as suggested by Rowe is given to the values of bending moments, to incorporate the slower convergence of moment series. Bending moments and deflections at various location of girders are shown in Table 6.7 through Table 6.10.

6.4.3 Cusens and Pama Curves

Bridge deck is analysed with the help of Cusens and Pama curves in a similar way as in case of Morice Little and Rowe curves. The values of i,j and θ will remain same. Only the value of torsional parameter, α will change which is given by Equation (3.58) i.e.

$$2H = B_{xy}+B_{yx}+ \frac{Et^3}{6(1- \quad ^2)} = (4.97+2.606)10^{-3}G+ \frac{0.28E}{6(1-0.15^2)}$$

$$= (7.576G + 1.364E)\ 10^{-3}$$

$$\text{and } \alpha = \frac{2H}{2\sqrt{D_x D_y}} = \frac{2H}{2E\sqrt{ij}} = 0.0291$$

values of distribution coefficients K_o, K_1 and K_α corrosponding to above values of θ and α are calculated and checked in the similar way and are shown in Tables 6.11, 6.12 and 6.13.

TABLE - 6.7 : GIRDER MOMENTS AT VARIOUS LOCATIONS
LOAD SYSTEM : Extreme Left.

X/L	M_x(mean) (t.m.)	Girder moments at various locations (t.m.)			
		G_1 M_x(mean) x K'_1	G_2 M_x(mean) x K'_2	G_3 M_x(mean) x K'_3	G_4 M_x(mean) x K'_4
0.20	47.84	67.87	59.46	47.89	34.20
0.40	82.58	117.16	102.63	82.62	59.00
0.44	84.86	120,42	105.48	84.91	60,63
0.50	80,46	114.16	99.99	80.50	57.48
0.60	73.81	103.30	90.49	72.84	52.00
0.80	46.97	66.64	58.37	46.99	33.55

TABLE - 6.8 : GIRDER MOMENTS AT VARIOUS LOCATIONS
LOAD SYSTEM : Symmetrically placed.

X/L	M_x(mean) (t.m.)	Girder moments at various locations (t.m.)			
		G_1 M_x(mean) x K'_1	G_2 M_x(mean) x K'_2	G_3 M_x(mean) x K'_3	G_4 M_x(mean) x K'_4
0.20	47.84	51.47	53.89	53.89	51.47
0.40	82.58	88.84	93.02	93.02	88.84
0.44	84.86	95.38	99.87	99.87	95.38
0.50	80.46	86.56	90.63	90.63	86.56
0.60	73.81	78.33	82.01	82.01	78.33
0.80	46.97	50.53	52.91	52.91	50.53

TABLE - 6.9 : GIRDER DEFLECTION AT VARIOUS LOCATIONS
LOAD SYSTEM : Extreme Left.

X/L	w(mean)(cm)	Deflection after load distribution			
		G_1 w(mean)xK'_1	G_2 w(mean)xK'_2	G_3 w(mean)xK'_3	G_4 w(mean)xK'_4
0.20	0.219	0.2430	0.2479	0.1997	0.1426
0.40	0.355	0.4576	0.4009	0.3228	0.2306
0.50	0.371	0.4782	0.4189	0.3374	0.2409
0.60	0.350	0.4520	0.3959	0.3188	0.2277
0.80	0.216	0.2785	0.2439	0.1964	0.1403

TABLE - 6.10 : GIRDER DEFLECTIONS AT VARIOUS LOCATIONS
LOAD SYSTEM : Symmetrically Placed.

X/L	w(mean)(cm)	Deflection after load distribution			
		G_1 w(mean)xK'_1	G_2 w(mean)xK'_2	G_3 w(mean)xK'_3	G_4 w(mean)xK'_4
0.20	0.219	0.2146	0.2247	0.2247	0.2146
0.40	0.355	0.3469	0.3633	0.3633	0.3469
0.50	0.371	0.3626	0.3796	0.3796	0.3626
0.60	0.350	0.3427	0.3588	0.3588	0.3427
0.80	0.216	0.2111	0.2210	0.2210	0.2111

TABLE 6.11 : VALUES OF K_o.

Position of section load at	Reference Station -b	-3b/4	-b/2	-b/4	0	+b/4	+b/2	+3b/4	+b	Row Integral
0	0.71	0.84	0.97	1.16	1.40	1.16	0.97	0.04	0.71	1.00
+b/4	0.12	0.35	0.57	0.82	1.16	1.00	1.45	1.40	1.40	0.97
+b/2	-0.44	-0.12	0.20	0.57	0.97	1.45	1.95	2.09	2.19	0.99
+3b/4	-1.02	-0.58	-0.12	0.35	0.84	1.40	2.09	2.90	3.36	1.01
+b	-1.60	-1.02	-0.46	0.12	0.71	1.40	2.19	3.36	5.17	0.99

TABLE -- 6.12 : VALUES OF K_1.

Position of section load at	Reference Station -b	-3b/4	-b/2	-b/4	0	+b/4	+b/2	+3b/4	+b	Row Integral
0	0.860	0.89	0.96	1.08	1.27	1.08	0.96	0.89	0.86	0.99
+b/4	0.740	0.78	0.84	1.93	1.08	1.30	1.13	1.05	1.01	1.00
+b/2	0.66	0.70	0.76	0.84	0.96	1.13	1.38	1.27	1.24	0.99
+3b/4	0.58	0.65	0.70	0.78	0.89	1.05	1.27	1.60	1.62	1.01
+b	0.55	0.58	0.66	0.74	0.86	1.01	1.24	1.62	2.30	1.01

TABLE - 6.13 : VALUES OF $K_\alpha = K_1 + (K_1 + K_o)\sqrt{\alpha}$ $\alpha = 0.0291$

Position of section load at	Reference Station -b	-3b/4	-b/2	-b/4	0	+b/4	+b/2	+3b/4	+b	Row Integral
0	0.74	0.85	0.97	1.15	1.38	1.15	0.97	0.85	0.74	1.00
+b/4	0.23	0.42	0.62	0.84	1.15	1.47	1.40	1.34	1.33	1.00
+n/2	-0.27	0.02	0.30	0.62	0.97	1.40	1.85	1.95	2.03	0.99
+3b/4	-0.75	-0.27	0.02	0.42	0.85	1.34	1.95	2.68	3.06	1.00
+b	-1.23	-0.75	-0.27	0.23	0.74	1.33	2.03	3.06	4.68	0.99

TABLE - 6. 14 : DERIVATION OF DISTRIBUTION COEFFICIENT PROFILE

CASE (I) : LOAD SYSTEM : Extreme Left (15 cm From Kerb)

Load at	Equi. Load Coeffi. (λ)	Values of λK_α at reference stations								
		$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$
-b	0.09	0.42	0.27	0.18	0.12	0.07	0.02	-0.02	-0.07	-0.11
-3b/4	0.91	2.78	2.44	1.77	1.22	0.77	0.38	0.02	-0.34	-0.68
-b/2	0.45	0.91	0.88	0.83	0.63	0.44	0.28	0.14	0. 00	-0.12
-b/4	0.55	0.73	0.74	0.77	0.81	0.63	0.46	0.341	0.23	0.13
0	0.91	0.67	0.77	0.88	1.05	1.26	1.05	0.88	0.77	0.67
+b/4	0.26	0.08	0.15	0.22	0.30	0.41	0.53	0.50	0.48	0.48
+b/2	0.73	-0.20	0.01	0.22	0.45	0.71	1.02	1.35	1.42	1.48
$K' = \frac{\Sigma \lambda K_\alpha}{4}$		1.348	1.315	1.20[illegible]	1.145	1.055	0.935	0.803	0.663	0.463

TABLE - 6.14 : DERIVATION OF DISTRIBUTION COEFFICIENT PROFILE

CASE (II) : LOAD SYSTEM : Symmetrically placed.

Load at	Equi. Load Coeffi. (λ)	Values of λK_α at reference stations								
		$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$
-3b/4	0.41	1.25	1.10	0.80	0.55	0.35	0.17	0.01	-0.11	-0.31
-b/2	0.59	1.20	1.15	1.09	0.83	0.57	0.37	0.18	0.01	-0.16
-b/4	0.77	1.02	1.03	1.08	1.13	0.88	0.65	0.48	0.32	0.18
0	0.46	0.34	0.39	0.45	0.53	0.63	0.53	0.45	0.39	0.34
+b/4	0.77	0.18	0.32	0.48	0.65	0.88	1.13	1.08	1.03	1.02
+b/2	0.59	-0.16	0.01	0.18	0.37	0.57	0.83	1.09	1.15	1.20
+3b/4	0.41	-0.31	-0.11	0.01	0.17	0.35	0.55	0.80	1.10	1.25
$K' = \frac{\Sigma \lambda K_\alpha}{4}$		0.880	0.973	1.020	1.058	1.058	1.058	1.020	0.973	0.880

TABLE - 6.15 : GIRDER MOMENTS AT VARIOUS LOCATIONS
LOAD SYSTEM : Extreme Left.

X/L	M_x(mean) (t.m.)	Girder moments at various locations (t.m.)			
		G_1 M_x(mean)xK'_1	G_2 M_x(mean)xK'_2	G_3 M_x(mean)xK'_3	G_4 M_x(mean)xK'_4
0.20	47.84	62.91	59.78	44.70	31.74
0.40	82.58	108.60	94.55	77.16	54.78
0.44	84.86	115.59	97.16	79.28	56.29
0.50	80.46	105.80	92.13	75.17	53.37
0.60	73.81	97.06	84.51	68.96	48.96
0.80	46.97	61.76	53.78	43.88	31.16

TABLE - 6.16: GIRDER MOMENTS AT VARIOUS LOCATIONS
LOAD SYSTEM : Symmetrically placed.

X/L	M_x(mean) (t.m.)	Girder moments at various locations (t.m.)			
		G_1 M_x(mean)xK'_1	G_2 M_x(mean)xK'_2	G_3 M_x(mean)xK'_3	G_4 M_x(mean)xK'_4
0.20	47.84	46.55	50.60	50.60	46.55
0.40	82.58	80.35	87.34	87.34	80.35
0.44	84.86	82.57	89.75	89.75	82.57
0.50	80.46	78.29	85.10	85.10	78.29
0.60	73.81	71.82	78.06	78.06	71.82
0.80	46.97	45.70	49.68	49.68	45.70

TABLE - 6.17 : GIRDER DEFLECTIONS AT VARIOUS LOCATIONS
LOAD SYSTEM : Extreme Left.

X/L	w(mean)(cm)	Girder deflections after load distribution			
		G_1 w(mean)xK_1'	G_2 w(mean)xK_2'	G_3 w(mean)xK_3'	G_4 w(mean)xK_4'
0.20	0.219	0.2886	0.2513	0.2052	0.1455
0.40	0.355	0.4665	0.4062	0.3317	0.2352
0.50	0.371	0.4875	0.4245	0.3466	0.2458
0.60	0.350	0.4608	0.4012	0.3776	0.2323
0.80	0.216	0.0838	0.2472	0.2018	0.1431

TABLE - 6.18 : GIRDER DEFLECTION AT VARIOUS LOCATIONS
LOAD SYSTEM : Symmetrically Placed.

X/L	w(mean)(cm)	Girder deflections after load distribution			
		G_1 w(mean)xK_1'	G_2 w(mean)xK_2'	G_3 w(mean)xK_3'	G_4 w(mean)xK_4'
0.20	0.219	0.2135	0.2322	0.2322	0.2135
0.40	0.355	0.3452	0.3754	0.3754	0.3452
0.50	0.371	0.3607	0.3922	0.3922	0.3607
0.60	0.350	0.3409	0.3707	0.3707	0.3409
0.80	0.216	0.2100	0.2284	0.2284	0.2100

Equivalent loads will be same as calculated for Rowe curves. Distribution profile is derived in Table 6.14, which gives the distribution coefficients at girder location as follows

TABLE - B : DISTRIBUTION COEFFICIENTS FOR GIRDERS

Girder	Distribution Coefficients K'	
	Case (I) Load position Extreme Left	Case (II) Load position symmetrically placed
G_1	1.315	0.973
G_2	1.145	1.058
G_3	0.935	1.058
G_4	0.663	0.973

Girder moments and deflections are obtained in Tables 6.15 through Table 6.18. Here, 10 percent increase is not given in calculation of bending moments because Cusens and Pama considered nine terms of the series. Although these curves are derived for mid-span section when the load is also at mid-span, but for comparison we have assumed the same distribution profile along the span as higher terms have less effect on distribution coefficients.

6.5 HARMONICS METHOD

Design coefficients $(p_{ij})^{min}$ for the design purposes are taken from reference (18) and are given in Appendix - D. Various geometric properties of the deck such as moment of inertia and torsional constant of constituent members required for the determination of structural parameters are obtained hereunder.

6.5.1 Geometric Properties

In determination of geometric properties for calculating structural parameters it is assumed that deck slab is cast monolithic with longitudinal and cross beams, thus in transfer of loads slab acts in concert with these beams as their top flanges. The effective flange width are obtained according to IRC code of practice and are evaluated for the given bridge as given below

(a) Flange width of main girders is least of the following.

(i) 1/4 (L) = 1/4(1940) (cm)	= 485
(ii) c/c spacing of girders (cm)	= 220
(iii) br+12 ts = 25+12(20) (cm)	= 265
Thus effective flange width taken (cm)	= 220

(b) Flange width of cross beams is least of the following.

(i) 1/4 (195) (cm)	= 49
(ii) c/c spacing of cross beams (cm)	= 485
(iii) 20 + 12(20) (cm)	= 260
Thus effective flange width adopted (cm)	= 50
The width of the slab element left out between the cross beams (cm)	= 435
and that left out at the ends (cm)	= 25

The various structural elements of the deck thus formed are as shown in Fig. 6.6.

Neutral axis depth of girder, $\bar{y}$ (cm) = 60.80

Flexural moment of inertia of girder I (cm^4) = 333.79×10^5

Torsional constant of girder I, (cm^4) = 13.87×10^5 (Appendix-B)

for simulating actual bridge deck by an equivalent uniform thick medium we add the moments of inertia and torsional

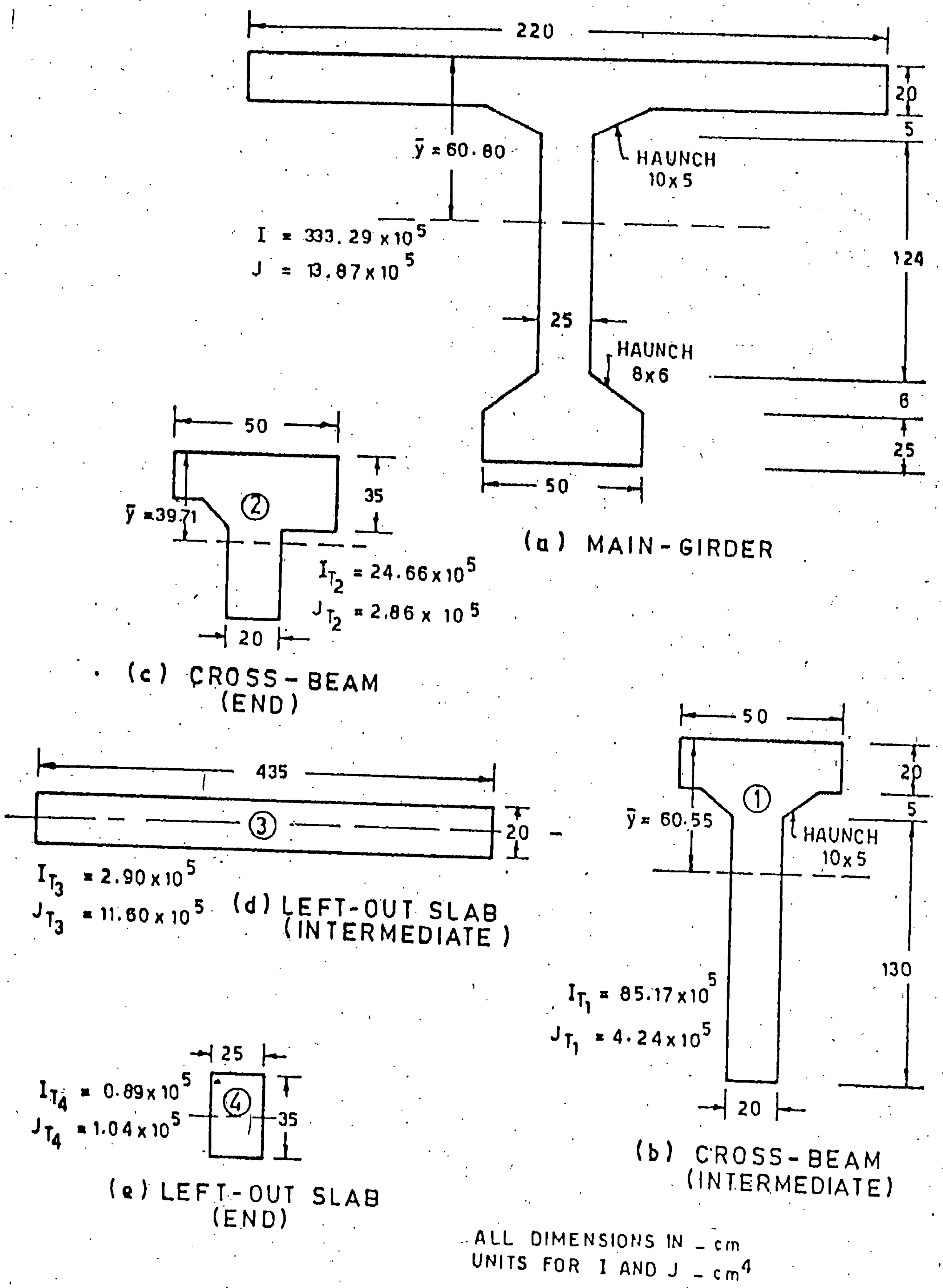


FIG. 6·6 — STRUCTURAL ELEMENTS

constants respectively of the various transverse structural elements indicated by (1) (2), (3) and (4) as shown in Fig. 6.6. The geometrical properties for these elements and hence for the equivalent uniform transverse media are obtained below

Neutral axis depth of intermediate cross beam, $\bar{y}$ (cm) = 66.55

Neutral axis depth of end cross beam $\bar{y}$ (cm) = 39.71

Flexural moment of inertia of intermediate cross beam, I_{T1} (cm^4) = 85.17×10^5

Flexural moment of inertia of end cross beam, I_{T2}(cm^4)= 24.66×10^5

Flexural moment of inertia of intermediate left out slab, I_{T3} (cm^4) = 2.90×10^5

Flexural moment of inertia of left out slab, I_{T4}(cm^4) = 0.89×10^5

Total flexural moment of inertia of equivalent uniform transverse media

$I_T = 3I_{T1}+2I_{T2}+4I_{T3}+2I_{T4} = 318.21 \times 10^5\ cm^4$

Torsional constants are obtained in accordance of Appendix - B, and are given below

Torsional constant of intermediate cross beam J_{T1}(cm^4) =4.24×10^5

Torsional constant of end cross beam J_{T2} (cm^4) =2.86×10^5

Torsional constant of intermediate left out slab J_{T3}(cm^4)=11.60×10^5

Torsional constant of left out slab J_{T4} (cm^4) =1.04×10^5

Total torsional constant of equivalent uniform transverse media, $J_T(cm^4)=J_{T1}+2J_{T2}+4J_{T3}+2J_{T4}=66.92 \times 10^5$

6.5.2 Structural Parameters

In the formulation of Harmonics method, certain non-dimensional parameters were defined. These parameters which are denoted by α, β, γ and K are called structural parameters. These structural parameters completly identify a girder bridge insofar its structural behaviour is concerned and are the functions of only geometrical and material properties of the deck.

(a) Structural parameter, α which is a relative measure of flexural regedity of transverse system (EI_T) to that of the girder (EI), is the most important structural parameter as it plays most vital role in the distribution of imposed loading amongst the girder load distribution capacity increases with increase in α but the improvement become in significant for $\alpha>100$, thus design coefficients are made available only upto α equal to 100 (18). Thus, the structural parameter, α of the given bridge deck may be computed as given below

$$\alpha = \frac{12}{\pi 4} \left(\frac{L}{h} \right)^3 \frac{E_c\ I_T}{E_c\ I}$$

$$= \left(\frac{12}{\pi^4} \right) \left(\frac{1940}{220} \right)^3 \frac{(1.5\text{x}10^5)(318.21\text{x}10^5)}{(1.5\text{x}10^5)(333.29\text{x}10^5)} = 80.6 \simeq 80$$

(b) Structural parameter β is the relative measure of girder torsional rigidity (GJ) to that of flexural rigidity of transverse system (EI_T). For real bridges β generally has a small value yet it plays a significant role in the load distribution amongst the girders. The maximum possible distribution of loads is obtained if girders posses very high torsional stiffness. The accurate evaluation of β is difficult as regards the computation of torsional constant J of a T-shaped girder section.

Thus, the structural parameter β of the given bridge deck may be computed as done hereunder

$$\beta = (\pi^2/2)(h/L)\ GJ/(E_c\ I_T)$$

$$= \frac{\pi^2}{2}\left(\frac{220}{1940}\right)\frac{(0.652\times10^5)(12.8\times10^5)}{(1.5\times10^5)(318.21\times10^5)} = 0.0106 \simeq 0.01$$

(c) Structural parameter, γ is a relative measure of torsional rigidity of transverse system (GJ_T) to its own flexural rigidity (EI_T). The value of γ for practical girder bridges is always found to be very very small, compared to α and β values. Its influence on load distribution is quite insignificant and hence has been dropped for the sake of case of computation. This excercise of taking $\gamma = 0$ for computing design coefficients results in a little conservative estimate of load distribution amongst the girders. Thus, the structural parameter, γ of the given bridge deck,

$$\gamma = \frac{\pi^2}{12}\left(\frac{h}{L}\right)^2 \frac{GJ_T}{E_c\ I_T}$$

$$= \frac{\pi^2}{12}\left(\frac{220}{1940}\right)^2 \left(\frac{(0.652\times10^5)(66.9\times10^5)}{(1.5\times10^5)(318.21\times10^5)}\right) = 0.0009 \simeq 0$$

(d) Skew parameter K is a measure of the length of transverse system interacting in transfering imposed loads. With the increase in K values, the interconnecting length $(1-K)L$ decreases, thus affecting the load distribution. It has been found that Harmonics method results are acceptable for K values upto 0.40 and majority of practical skew girder bridgs have K values lying within 0.40 (18). Skew parameter, K of the given bridge deck,

$$K = (h/L)\tan\lambda = 0 \text{ (Since } \lambda = 0 \text{ for right bridge).}$$

6.5.3 Design Coefficients

The design coefficients which are given in reference (18) and also in Appendix - D, are used to obtain girder moment coefficient. These design coefficients are basic in nature and are used as a mean to determine the girder moment coefficients corresponding to any type of imposed loading that can be expressed as fourier sine series. These coefficients are function of three structural parameters α, β, γ and K. Design coefficients available in reference (18) are the direct computer output (with $\gamma = 0$) of the program prepared therin for this purpose. The sample tables along with the procedure for referring these tabulated design coefficients is presented in Appendix - D.

For the computed values of α and K the corrosponding design coefficients (p_{ij}^{mn}) are picked up from appendix for $\beta = 0$ and $\beta = 10$ and are then entered in appropriate places in Table 6.19 to Table 6.22. The coefficients corrosponding to actual value of β are then computed using the interpolation function given earlier and are shown in the lower half of Table 6.19 to Table 6.22.

It gives all the design coefficients corrosponding to the values of α, β and K which are used for analysis of given girder bridge under any type of imposed loading.

6.5.4 Equivalent Load Computation

Expressions for the girder moments, developed in chapter - 5 are applicable only if the loads are acting directly on one or more girders. It is thus necessary to ascertain the

TABLE - 6. 19: DESIGN MOMENT COEFFICIENTS(p_{ij}^{mn}) FOR GIRDER i = 1

$\alpha = 80$; $\beta = 0.01$; $z = 0.17$; $K = 0$

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
	1	0.591	0.344	0.128	-0.064	0.313	0.262	0.225	0.201
i = 1	2	0	0	0	0	0	0	0	0
	3	-0.088	-0.111	0.025	0.075	0.057	0.001	-0.021	-0.038
$(p_{ij}^{11})_\beta$		0.544		0.330		0.145		-0.019	
$(p_{ij}^{12})_\beta$		0		0		0		0	
$(p_{ij}^{13})_\beta$		-0.063		-0.009		0.017		0.056	

INDUCED FIRST MOMENT HARMONIC (m = 1)

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
	1	0	0	0	0	0	0	0	0
i = 1	2	0.551	0.269	0.120	0.059	0.414	0.268	0.174	0.140
	3	0	0	0	0	0	0	0	0
$(p_{ij}^{21})_\beta$		0		0		0		0	
$(p_{ij}^{22})_\beta$		0.528		0.269		0.130		0.073	
$(p_{ij}^{23})_\beta$		0		0		0		0	

INDUCED SECOND MOMENT HARMONIC (m = 2)

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
	1	0.088	-0.020	-0.033	0.075	0.057	0.005	-0.025	-0.038
i = 1	2	0	0	0	0	0	0	0	0
	3	0.802	0.196	0.016	-0.014	0.701	0.224	0.060	0.010
$(p_{ij}^{31})_\beta$		0.063		-0.016		0.023		0.056	
$(p_{ij}^{32})_\beta$		0		0		0		0	
$(p_{ij}^{33})_\beta$		0.785		0.202		0.023		-0.010	

INDUCED THIRD MOMENT HARMONIC (m = 3)

TABLE - 6.20 : DESIGN MOMENT COEFFICIENTS(p_{ij}^{mn}) FOR GIRDER i = 2

$\alpha = 80$; $\beta = 0.01$; $z = 0.17$; $K = 0$

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
i = 1	1	0.344	0.304	0.224	0.128	0.262	0.266	0.247	0.225
	2	0	0	0	0	0	0	0	0
	3	-0.020	-0.014	0	0.033	0.005	0.016	0.003	-0.025
$(p_{ij}^{11})_\beta$			0.330		0.298		0.228		0.145
$(p_{ij}^{12})_\beta$			0		0		0		0
$(p_{ij}^{13})_\beta$			-0.016		- 0.009		0		0.023

INDUCED FIRST MOMENT HARMONIC (m = 1)

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
i = 1	1	0	0	0	0	0	0	0	0
	2	0.269	0.375	0.236	0.120	0.268	0.324	0.229	0.179
	3	0	0	0	0	0	0	0	0
$(p_{ij}^{21})_\beta$			0		0		0		0
$(p_{ij}^{22})_\beta$			0.269		0.366		0.235		0.130
$(p_{ij}^{23})_\beta$			0		0		0		0

INDUCED SECOND MOMENT HARMONIC (m = 2)

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
i = 1	1	-0.011	-0.014	0	0.025	0.001	0.016	0.003	-0.021
	2	0	0	0	0	0	0	0	0
	3	0.196	0.583	0.205	0.016	0.224	0.511	0.200	0.060
$(p_{ij}^{31})_\beta$			-0.009		-0.009		0		0.017
$(p_{ij}^{32})_\beta$			0		0		0		0
$(p_{ij}^{33})_\beta$			0.202		0.571		0.204		0.023

INDUCED THIRD MOMENT HARMONIC (m = 3)

TABLE - 6.21 : DESIGN MOMENT COEFFICIENTS(p_{ij}^{mn}) FOR GIRDER i = 3

$\alpha = 80$; $\beta = 0.01$; $z = 0.17$; $K = 0$

m=1	n	β = 0				β = 10			
j→	↓	1	2	3	4	1	2	3	4
	1	0.128	0.224	0.304	0.344	0.225	0.247	0.266	0.262
i = 1	2	0	0	0	0	0	0	0	0
	3	0.033	0	-0.014	-0.020	-0.025	0.003	0.016	0.005
$(p_{ij}^{11})_\beta$			0.144		0.228		0.298		0.330
$(p_{ij}^{12})_\beta$			0		0		0		0
$(p_{ij}^{13})_\beta$			0.023		0		-0.009		-0.016

INDUCED FIRST MOMENT HARMONIC (m = 1)

m=1	n	β = 0				β = 10			
j→	↓	1	2	3	4	1	2	3	4
	1	0	0	0	0	0	0	0	0
i = 1	2	0.120	0.236	0.375	0.269	0.179	0.229	0.324	0.268
	3	0	0	0	0	0	0	0	0
$(p_{ij}^{21})_\beta$			0		0		0		0
$(p_{ij}^{22})_\beta$			0.130		0.235		0.366		0.269
$(p_{ij}^{23})_\beta$			0		0		0		0

INDUCED SECOND MOMENT HARMONIC (m = 2)

m=1	n	β = 0				β = 10			
j→	↓	1	2	3	4	1	2	3	4
	1	-0.021	0.003	0.016	0.001	0.025	0	-0.014	-0.011
i = 1	2	0	0	0	0	0	0	0	0
	3	0.060	0.200	0.511	0.229	0.016	0.205	0.583	0.146
$(p_{ij}^{31})_\beta$			0.017		0		-0.009		-0.009
$(p_{ij}^{32})_\beta$			0		0		0		0
$(p_{ij}^{33})_\beta$			0.023		0.204		0.571		0.202

TABLE - 6. 22: DESIGN MOMENT COEFFICIENTS(p_{ij}^{mn}) FOR GIRDER i = 4

$\alpha = 80$; $\beta = 0.01$; $z = 0.17$; $K = 0$

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
	1	-0.064	0.128	0.344	0.591	0.201	0.225	0.262	0.313
i = 1	2	0	0	0	0	0	0	0	0
	3	0.075	0.025	-0.011	-0.088	-0.038	-0.021	0.001	0.057
$(p_{ij}^{11})_\beta$			-0.019		0.144		0.330		0.544
$(p_{ij}^{12})_\beta$			0		0		0		0
$(p_{ij}^{13})_\beta$			0.056		0.017		-0.009		-0.063

INDUCED FIRST MOMENT HARMONIC (m = 1)

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
	1	0	0	0	0	0	0	0	0
i = 1	2	0.059	0.120	0.269	0.551	0.140	0.179	0.268	0.414
	3	0	0	0	0	0	0	0	0
$(p_{ij}^{21})_\beta$			0		0		0		0
$(p_{ij}^{22})_\beta$			0.073		0.130		0.269		0.528
$(p_{ij}^{23})_\beta$			0		0		0		0

INDUCED SECOND MOMENT HARMONIC (m = 2)

m=1 j→	n ↓	β = 0				β = 10			
		1	2	3	4	1	2	3	4
	1	0.075	0.033	-0.020	-0.088	-0.038	-0.025	0.005	0.057
i = 1	2	0	0	0	0	0	0	0	0
	3	-0.014	0.016	0.196	0.802	0.010	0.060	0.229	0.701
$(p_{ij}^{31})_\beta$			0.056		0.023		-0.016		-0.063
$(p_{ij}^{32})_\beta$			0		0		0		0
$(p_{ij}^{33})_\beta$			-0.010		0.023		0.202		0.785

INDUCED THIRD MOMENT HARMONIC (m = 3)

equivalent loading on girders if the situation is otherwise. The imposed loads not acting directly on a girder are replaced by a system of coplaner loads acting on all the girder of the bridge. This vertical plane containing the load is orthagonal to the girder axis. The equivalent system of loads acting on the girders are thus the reactions obtained by treating the transverse structural element as simply supported continuous beam. Thus all the loads acting directly on the girders become available. Any conventional methods such as slope deflection method or theorem of three moments can be used for computing the reactions.

The generalized expression for support moments under a general loading deposition have been obtained herin to facilitate the computation of support reactions. The well known theorem of three moments has been applied for this purpose, which is

$$M_1L_1+2M_2(L_1+L_2)+M_3L_2 = \frac{-6A_1\bar{X}_1}{L_1} - \frac{6A_2\bar{X}_2}{L_2}$$

wherein

M_1, M_2, M_3 = Moments at supports 1,2 and 3 respectively

A_1 = Area of free span B.M.D. over span (1-2)

$\bar{X}_1$ = Distance of C.G. of free span B.M.D. from support 1.

L_1 = Span (1-2)

$A_2, \bar{X}_2$ and L_2 are the corrosponding parameters for span (2-3).

The final expressions based on the above theorem give the support moments under that loading condition. These expressions are given in Appendix - C.

Equivalent girder loads under two cases of loading are given below.

Case - I

Live Load System : Extreme left refering to Fig. 6.2(b) for loading and Appendix - C for girder moment expressions we have

$M_1 = -5P$; $M_2 = -25.17\ P$

$M_3 = -35.90P$; $M_4 = 0$

Now, girder reactions can easily be found, which are

$R_1 = 1.115P$; $R_2 = 1.267\ P$

$R_3 = 1.196P$; $R_4 = 0.222\ P$

Case - II

Load Position : Symmetrically Placed : Refering to Fig. 6.3(b) for loading, we get

$M_1 = M_4 = 0$; $M_2 = M_3 = -25.17\ P$

which gives,

$R_1 = R_4 = 0.591\ P$; $R_2 = R_3 = 1.524\ P$

6.5.5 Load Harmonic Coefficients

Multiple loads on multiple girders as obtained in section 6.5.3 are expressed in terms of the first three harmonic components of their sine series on respective girders. The coefficients of these three harmonics, called load harmonic coefficients are easily obtained using fourier series (Appendix-A). The expression for these coefficients are also available in chapter - 5. These coefficients for loading cases I and II are computed in Tables 6.23 and 6.24 respectively.

6.5.6 Girder Moment Coefficients

Girder moment for any imposed loading is also expressed by first three harmonics of its sine series with reasonable accuracy. The coefficients of these three terms called as girder moment coefficients are the functions of design coefficients and load harmonic coefficients for all loading cases. There is no expression which can directly give moment coefficients for multiple loads on multiple girders. Thus, it is important to note here that when more than one girder is loaded simultaneously, the moment coefficients for all the girders are initially obtained corrosponding to actual loading condition wherein all girders are loaded simultaneously are then obtained using principle of superposition.

Girder moment coefficients for multiple loads on multiple girders obtained corrosponding to loading configuration shown in Fig. 6.2 are computed in Table 6.25 through Table 6.29. Similarly the girder moment coefficients corrosponding to loading configuration shown in Fig. 6.3 are computed in Table 6.30 through Table 6.34. In the computation process design coefficients from Table 6.19 through Table 6.22 are entered in the appropriate positions of above tables. Load harmonic coefficient of imposed

TABLE - 6. 23 : HARMONIC LOAD COMPUTATIONS.

LOAD SYSTEM : Extreme Left.

Girder	X/L	$W_j (x10^3 Kg)$	$W^1_{jp} = \frac{2W_j}{L} \sin\frac{\pi x}{L}$	$W^2_{jp} = \frac{2W_j}{L} \sin\frac{2\pi x}{L}$	$W^3_{jp} = \frac{3W_j}{L} \sin\frac{3\pi x}{L}$
1	0.159	1.77	87.40	153.35	181.88
	0.216	1.77	114.46	178.21	162.99
	0.381	7.25	705.45	515.26	-328.85
	0.443	7.35	745.61	265.96	-650.89
	0.664	4.46	400.02	-394.04	-11.49
	0.819	4.46	247.37	-417.03	455.65
	0.973	4.41	39.08	- 77.70	115.87
		$W^1_j = \Sigma W^1_{jp}$	2366.14	$W^2_j = \Sigma W^2_{jp}$ = 237.24	$W^3_j = \Sigma W^3_{jp}$ = - 91.99
2	0.159	1.47	97.28	170.80	202.48
	0.216	1.47	127.54	198.42	202.68
	0.381	8.35	801.43	585.36	-373.60
	0.443	8.35	847.05	302.15	-739.45
	0.664	5.07	454.73	-447.94	- 13.07
	0.819	5.07	281.20	-474.07	-517.98
	0.973	5.07	44.43	- 88.33	131.72
		$W^1_j = \Sigma W^1_{jp}$	2688.74	$W^2_j = \Sigma W^2_{jp}$ = 269.58	$W^3_j = \Sigma W^3_{jp}$ = -104.54
3	0.159	2.22	109.63	192.48	228.18
	0.216	2.22	143.73	223.60	228.41
	0.381	9.20	883.01	644.95	-411.63
	0.443	9.20	933.28	332.91	-814.72
	0.664	5.59	501.37	-493.87	- 14.41
	0.819	5.59	310.04	-522.69	571.11
	0.973	5.59	49.98	- 97.39	145.22
		$W^1_j = \Sigma W^1_{jp}$ =	2963.50	$W^2_j = \Sigma W^2_{jp}$ = 297.04	$W^3_j = \Sigma W^3_{jp}$ = -115.18
4	0.154	0.35	17.28	30.35	35.97
	0.216	0.35	22.66	35.25	36.01
	0.381	1.46	140.13	102.35	- 65.37
	0.443	1.46	148.11	52.83	-129.29
	0.664	0.888	79.20	- 78.45	- 2.28
	0.819	0.888	48.97	- 83.03	90.72
	0.973	0.888	7.47	- 15.47	23.07
		$W^1_j = \Sigma W^1_{jp}$ =	471.14	$W^2_j = \Sigma W^2_{jp}$ = 47.22	$W^3_j = \Sigma W^3_{jp}$ = - 18.36

TABLE - 6.24 : HARMONIC LOAD COMPUTATIONS.

LOAD SYSTEM : Symmetrically Placed.

Girder	X/L	$W_j (x10^3 Kg)$	$W^1_{jp} = \frac{2W_j}{L} \sin\frac{\pi x}{L}$	$W^2_{jp} = \frac{2W_j}{L} \sin\frac{2\pi x}{L}$	$W^3_{jp} = \frac{3W_j}{L} \sin\frac{3\pi x}{L}$
1	0.159	0.939	46.37	81.41	96.55
	0.216	0.939	60.76	94.60	96.61
	0.381	3.96	380.05	277.59	-177.30
	0.443	3.96	401.72	143.18	-350.74
	0.664	3.36	211.71	-208.51	- 6.11
	0.819	2.36	131.01	-220.74	241.08
	0.973	2.36	20.61	- 41.08	61.031
		$W^1_j = \Sigma W^1_{jp}$ =	1252.23	$W^2_j = \Sigma W^2_{jp}$ = 26.32	$W^3_j = \Sigma W^3_{jp}$ = - 38.66
2	0.159	2.42	119.50	209.82	248.74
	0.216	2.42	156.68	243.75	248.48
	0.381	10.22	980.91	716.45	-457.26
	0.443	10.22	1036.75	369.82	-905.05
	0.664	6.10	547.11	-538.94	- 15.72
	0.814	6.10	338.33	-570.38	623.21
	0.473	6.10	53.45	-106.28	158.47
		$W^1_j = \Sigma W^1_{jp}$ =	3232.73	$W^2_j = \Sigma W^2_{jp}$ = 324.24	$W^3_j = \Sigma W^3_{jp}$ = - 98.63
3	0.159	2.42	114.50	209.82	248.74
	0.216	2.42	156.68	242.75	248.48
	0.381	10.22	980.91	716.45	-457.26
	0.443	10.22	1036.75	369.82	-905.05
	0.664	6.10	547.11	-538.94	- 15.72
	0.819	6.10	338.33	-570.38	623.21
	0.973	6.10	53.45	-106.28	158.47
		$W^1_j = \Sigma W^1_{jp}$ =	3232.73	$W^2_j = \Sigma W^2_{jp}$ = 324.24	$W^3_j = \Sigma W^3_{jp}$ = - 98.63
4	0.159	0.939	46.37	81.41	96.55
	0.216	0.439	60.76	94.60	96.61
	0.381	3.96	380.05	277.59	-177.61
	0.443	3.96	401.72	43.10	-350.74
	0.664	2.36	211.71	-208.51	- 6.11
	0.819	2.36	131.01	-220.74	241.08
	0.473	2.36	20.61	- 41.08	61.75
		$W^1_j = \Sigma W^1_{jp}$ =	1252.23	$W^2_j = \Sigma W^2_{jp}$ = 26.32	$W^3_j = \Sigma W^3_{jp}$ = - 38.66

TABLE - 6.25 : GIRDER MOMENT COEFFICIENTS FOR LOAD ON GIRDER j = 1

LOAD SYSTEM : Extreme Left.

LOADING : $1770G_1(0.159L, 0.216L) + 7350G_1(0.381L, 0.443L) + 4460G_1(0.664L, 0.819L, 0.973L)$

j=1 i=1 n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.544	- 0	-0.063	0	0.528	0	-0.065	0	0.788
w_j^n/n^2	2366.14	59.31	-10.22	2366.14	59.31	-10.22	2366.14	59.31	-10.22
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	1287.18	0	-0.64	0	31.32	0	-149.07	0	-8.02
M_i^m = 2		1286.54			31.32			-157.09	

MOMENT COEFFICIENTS FOR GIRDER i = 1

j=1 i=1 n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.330	0	-0.016	0	0.269	0	-0.009	0	0.202
w_j^n/n^2	2366.14	59.31	-10.22	2366.14	59.31	-10.22	2366.14	59.31	-10.22
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	780.83	0	0.16	0	15.91	0	-21.29	0	-2.06
M_i^n = 2		780.99			15.95			-23.35	

MOMENT COEFFICIENTS FOR GIRDER i = 2

j=1 i=1 n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.144	0	0.023	0	0.130	0	0.017	0	0.023
w_j^n/n^2	2366.14	59.31	-10.22	2366.14	59.31	-10.22	2366.14	54.31	-10.22
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	340.72	0	-0.24	0	7.71	0	40.22	0	-0.24
M_i^n = 2		340.48			7.71			39.98	

MOMENT COEFFICIENTS FOR GIRDER i = 3

j=1 i=1 n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	-0.019	0	0.056	0	0.073	0	0.056	0	-0.010
w_j^n/n^2	2366.14	59.31	-10.22	2366.14	59.31	-10.22	2366.14	59.31	-10.22
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	-44.96	0	-0.57	0	4.53	0	132.50	0	0.10
M_i^m = 2		-45.53			4.53			132.60	

MOMENT COEFFICIENTS FOR GIRDER i = 4

TABLE - 6.26 : GIRDER MOMENT COEFFICIENTS FOR LOAD ON GIRDER j = 2

LOAD SYSTEM : Extreme Left.

LOADING : $1970G_2(0.159L, 0.216L) + 8350G_2(0.381L, 0.443L) + 5070G_2(0.664L, 0.819L, 0.973L)$.

j=1, i=1	m = 1			m = 2			m = 3		
n →	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.330	0	-0.009	0	0.269	0	-0.016	0	0.202
w_j^n/n^2	2688.74	67.40	-11.62	2688.74	67.40	-11.62	2688.74	67.40	-11.62
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	887.28	0	0.10	0	18.31	0	-43.02	0	- 2.35
M_i^m = 2	887.38			18.13			-45.37		

MOMENT COEFFICIENTS FOR GIRDER i = 1

j=1, i=1	m = 1			m = 2			m = 3		
n →	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.298	0	-0.009	0	0.366	0	-0.009	0	0.571
w_j^n/n^2	2688.74	67.40	-11.62	2688.74	67.40	-11.62	2688.74	67.40	-11.62
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	801.24	0	0.10	0	24.67	0	-24.20	0	- 6.63
M_i^n = 2	801.34			24.67			-30.83		

MOMENT COEFFICIENTS FOR GIRDER i = 2

j=1, i=1	m = 1			m = 2			m = 3		
n →	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.228	0	0	0	0.235	0	0	0	0.204
w_j^n/n^2	2688.74	67.40	-11.62	2688.74	67.40	-11.62	2688.74	67.40	-11.62
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	613.03	0	0	0	15.84	0	0	0	- 2.75
M_i^n = 2	613.03			15.84			-2.79		

MOMENT COEFFICIENTS FOR GIRDER i = 3

j=1, i=1	m = 1			m = 2			m = 3		
n →	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.144	0	0.017	0	0.130	0	0.023	0	0.023
w_j^n/n^2	2688.74	67.40	-11.62	2688.74	67.40	-11.62	2688.74	67.40	-11.62
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	387.18	0	-0.20	0	8.76	0	61.84	0	- 0.27
M_i^m = 2	386.98			8.76			61.57		

MOMENT COEFFICIENTS FOR GIRDER i = 4.

TABLE - 6.27 : GIRDER MOMENT COEFFICIENTS FOR LOAD ON GIRDER j = 3
LOAD SYSTEM : Extreme Left.
LOADING : $2220G_3(0.159L,0.216L)+9200G_3(0.381L,0.443L)$
$+5590G_3(0.664L,0.819L,0.97L)$.

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.145	0	0.017	0	0.130	0	0.023	0	0.023
w_j^n/n^2	2962.50	74.26	-12.80	2962.50	74.26	-12.80	2962.50	74.26	-12.80
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	429.56	0	-0.22	0	9.65	0	68.14	0	-2.29
$M_i^m = 2$		429.34			9.65			67.85	

MOMENT COEFFICIENTS FOR GIRDER i = 1

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.228	0	0	0	0.235	0	0	0	0.204
w_j^n/n^2	2962.50	74.26	-17.80	2962.50	74.26	-12.80	2962.50	74.26	-12.80
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	675.45	0	0	0	17.45	0	0	0	- 2.61
$M_i^n = 2$		675.45			17.45			-2.61	

MOMENT COEFFICIENTS FOR GIRDER i = 2

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.298	0	-0.009	0	0.366	0	0.009	0	0.571
w_j^n/n^2	2962.50	74.26	-12.80	2962.50	74.26	-12.80	2962.50	74.26	-12.80
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	882.82	0	0.12	0	27.18	0	-26.66	0	- 7.31
$M_i^n = 2$		882.94			27.18			-37.97	

MOMENT COEFFICIENTS FOR GIRDER i = 3

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.330	0	-0.009	0	0.269	0	-0.016	0	0.202
w_j^n/n^2	2962.50	74.26	-12.80	2962.50	74.26	-12.80	2962.50	74.26	-12.80
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	977.62	0	0.12	0	19.98	0	-47.40	0	-2.58
$M_i^m = 2$		977.74			19.88			-49.98	

MOMENT COEFFICIENTS FOR GIRDER i = 4.

TABLE - 6.28 : GIRDER MOMENT COEFFICIENTS FOR LOAD ON GIRDER $j = 4$

LOAD SYSTEM : Extreme Left.

LOADING : $350G_4(0.159L, 0.216L)+1460G_4(0.381L,0.443L)$ $+888G_4(0.664L,0.819L,0.973L)$

j=1 i=1 n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	-0.019	0	0.056	0	0.073	0	0.056	0	-0.010
w_j^n/n^2	471.14	11.81	-2.04	471.14	11.81	-2.04	471.14	11.81	-2.04
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	-8.95	0	-0.11	0	0.86	0	26.38	0	0.02
$M_i^m = 2$	-9.06			0.86			26.40		

MOMENT COEFFICIENTS FOR GIRDER $i = 1$

j=1 i=1 n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.145	0	0.023	0	0.130	0	0.017	0	0.023
w_j^n/n^2	471.14	11.81	-2.04	471.14	11.81	-2.04	471.14	11.81	-2.04
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	68.32	0	-0.05	0	1.53	0	8.01	0	-0.05
$M_i^n = 2$	67.27			1.53			7.96		

MOMENT COEFFICIENTS FOR GIRDER $i = 2$

j=1 i=1 n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.330	0	-0.016	0	0.269	0	-0.009	0	0.202
w_j^n/n^2	471.14	11.81	-2.04	471.14	11.81	-2.04	471.14	11.88	-2.04
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	155.48	0	0.63	0	3.18	0	-4.24	0	-0.41
$M_i^n = 2$	155.51			3.18			-4.65		

MOMENT COEFFICIENTS FOR GIRDER $i = 3$

j=1 i=1 n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.544	0	-0.063	0	0.528	0	-0.063	0	0.785
w_j^n/n^2	471.14	11.81	-2.04	471.14	11.88	-2.04	471.14	11.81	-2.04
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	256.30	0	0.13	0	6.23	0	-29.68	0	-1.60
$M_i^m = 2$	256.43			6.23			-21.28		

MOMENT COEFFICIENTS FOR GIRDER $i = 4$.

TABLE – 6.29 : GIRDER MOMENT COEFFICIENTS FOR LOAD ACTING ON GIRDER G_1, G_2, G_3 AND G_4 SIMULTANEOUSLY.
LOAD SYSTEM : Extreme Left.

LOADING : $(1770G_1+1970G_2+2220G_3+35G_4)(0.159L.0.216L)+(7350G_1+8350G_2+9200G_3+1460G_4)(0.381L,0443L)(4460G_1+5070G_1+5590G_3+89G_4)(0.664L,0.819L,0.973L)$.

i = 1	j	m = 1	m = 2	m = 3
M_{ij}^m	1	1286.54	31.32	-157.09
	2	887.38	18.17	- 45.37
	3	429.34	9.65	67.85
	4	- 9.66	0.86	26.40
M_i^m		2594.20	59.96	-108.25

MOMENT COEFFICIENTS FOR GIRDER i = 1

i = 2	j	m = 1	m = 2	m = 3
M_{ij}^m	1	780.99	15.05	-23.35
	2	801.34	24.67	-30.83
	3	675.45	12.45	- 2.61
	4	67.27	1.53	7.96
M_i^m		2325.05	58.70	-48.83

MOMENT COEFFICIENTS FOR GIRDER i = 2

i = 3	j	m = 1	m = 2	m = 3
M_{ij}^m	1	340.48	7.71	30.08
	2	612.03	15.84	- 2.79
	3	882.94	27.18	-33.97
	4	155.51	3.18	- 4.65
M_i^m		1991.96	53.91	- 1.43

MOMENT COEFFICIENTS FOR GIRDER i = 3

i = 4	j	m = 1	m = 2	m = 3
M_{ij}^m	1	-45.53	4.33	132.60
	2	386.98	8.76	61.57
	3	977.74	19.98	-49.98
	4	255.43	6 .73	-31.29
M_i^m		1575.62	39.30	112.91

MOMENT COEFFICIENTS FOR GIRDER i = 4.

TABLE - 6.30 : GIRDER MOMENT COEFFICIENTS FOR LOAD ON GIRDER j = 1

LOAD SYSTEM : Symmetrically Placed.

LOADING : $9390G_1(0.159L, 0.216L) + 3960G_1(0.381L, 0.44.5L) + 2360G_1(0.664L, 0.819L, 0.973L)$.

j=1 i=1 n→	m = 1 1	2	3	m = 2 1	2	3	m = 3 1	2	3
p_{ij}^{mn}	0.544	0	-0.063	0	0.528	0	-0.063	0	0.785
w_j^n/n^2	1252.23	6.58	-4.30	1252.23	6.58	-4.30	1252.23	6.58	-4.30
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	681.21	0	0.27	0	3.47	0	-78.89	0	-3.38
M_i^m = 2	681.48			3.47			-82.27		

MOMENT COEFFICIENTS FOR GIRDER i = 1

j=1 i=1 n→	m = 1 1	2	3	m = 2 1	2	3	m = 3 1	2	3
p_{ij}^{mn}	0.330	0	-0.016	0	0.269	0	-0.009	0	0.202
w_j^n/n^2	1252.23	6.58	-4.30	1252.23	6.58	-4.30	1252.23	6.58	-4.30
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	413.21	0	0.07	0	1.77	0	-11.27	0	-0.87
M_i^n = 2	413.31			1.77			-12.14		

MOMENT COEFFICIENTS FOR GIRDER i = 2

j=1 i=1 n→	m = 1 1	2	3	m = 2 1	2	3	m = 3 1	2	3
p_{ij}^{mn}	0.144	0	0.023	0	0.130	0	0.017	0	0.023
w_j^n/n^2	1252.23	6.58	-4.30	1252.23	6.58	-4.30	1252.23	6.58	-4.30
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	180.32	0	-0.10	0	0.86	0	21.29	0	-0.10
M_i^n = 2	180.22			0.86			21.19		

MOMENT COEFFICIENTS FOR GIRDER i = 3

j=1 i=1 n→	m = 1 1	2	3	m = 2 1	2	3	m = 3 1	2	3
p_{ij}^{mn}	-0.019	0	0.056	0	0.073	0	0.056	0	-0.010
w_j^n/n^2	1252.23	6.58	-4.30	1252.23	6.58	-4.30	1252.23	6.58	-4.30
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	-23.79	0	-0.24	0	0.48	0	70.12	0	0.04
M_i^m = 2	-24.03			0.48			70.16		

MOMENT COEFFICIENTS FOR GIRDER i = 4.

TABLE – 6.31 : GIRDER MOMENT COEFFICIENTS FOR LOAD ON GIRDER $j = 2$

LOAD SYSTEM : Symmetrically Placed.

LOADING: $2420\ G_2(0.159L,\ 0.216L) + 10220\ G_2(0.391L,\ 0.443L)$
$+6100\ G_2(0.664L,\ 0.819L,\ 0.973L)$

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.330	0	-0.09	0	0.269	0	-0.016	0	0.202
w_j^n/n^2	3232.73	81.06	-10.96	3232.73	81.06	-10.96	3232.73	81.06	-10.96
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	1066.80	0	0.10	0	21.81	0	-51.72	0	-2.21
$M_i^m = 2$	1066.90			21.81			-53.93		

MOMENT COEFFICIENTS FOR GIRDER $i = 1$

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.248	0	-0.009	0	0.366	0	-0.009	0	0.571
w_j^n/n^2	3232.73	81.06	-10.96	3232.73	81.06	-10.96	3232.73	81.06	-10.96
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	963.35	0	0.10	0	29.67	0	-29.09	0	-6.26
$M_i^n = 2$	963.45			29.67			-35.35		

MOMENT COEFFICIENTS FOR GIRDER $i = 2$

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.228	0	0	0	0.235	0	0	0	0.204
w_j^n/n^2	3232.73	81.06	-10.96	3232.73	81.06	-10.96	3232.73	81.06	-10.96
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	737.06	0	0	0	19.05	0	0	0	-2.24
$M_i^n = 2$	737.06			19.05			-2.24		

MOMENT COEFFICIENTS FOR GIRDER $i = 3$

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.144	0	0.017	0.	0.130	0	0.023	0	0.023
w_j^n/n^2	3232.73	81.06	-10.96	3232.73	81.06	-10.96	3232.73	81.06	-10.96
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	465.51	0	-0.19	0	10.54	0	74.35	0	-0.25
$M_i^m = 2$	465.32			10.54			74.10		

MOMENT COEFFICIENTS FOR GIRDER $i = 4$.

TABLE 6.32: GIRDER MOMENT COEFFICIENTS FOR LOAD ON GIRDER j = 3

LOAD SYSTEM : Symmetrically Placed.

LOADING: 2420 G_3(0.159L, 0.216L) + 10220 G_3(0.381L, 0.443L)

+ 6100 G_3(0.664L, 0.819L, 0.973L)

j=1, i=1	m = 1			m = 2			m = 3		
$n \rightarrow$	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.145	0	0.017	0	0.130	0	0.023	0	0.023
w_j^n/n^2	3232.73	81.06	-10.96	3232.73	81.06	-10.96	3232.73	81.06	-10.96
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	465.51	0	-0.19	0	10.54	0	74.35	0	-0.25
M_i^m = 2		465.32			10.54			74.10	

MOMENT COEFFICIENTS FOR GIRDER i = 1

j=1, i=1	m = 1			m = 2			m = 3		
$n \rightarrow$	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.228	0	0	0	0.235	0	0	0	0.204
w_j^n/n^2	3232.73	81.06	-10.96	3232.73	81.06	-10.96	3232.73	81.06	-10.96
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	737.06	0	0	0	19.05	0	0	0	-2.24
M_i^n = 2		737.06			19.05			-2.24	

MOMENT COEFFICIENTS FOR GIRDER i = 2

j=1, i=1	m = 1			m = 2			m = 3		
$n \rightarrow$	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.298	0	-0.009	0	0.366	0	-0.009	0	0.571
w_j^n/n^2	3232.73	81.06	-10.96	3232.73	81.06	-10.96	3232.73	81.06	-10.96
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	962.35	0	0.10	0	29.67	0	-29.09	0	-6.26
M_i^n = 2		963.45			29.67			-35.35	

MOMENT COEFFICIENTS FOR GIRDER i = 3

j=1, i=1	m = 1			m = 2			m = 3		
$n \rightarrow$	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.330	0	-0.009	0	0.269	0	-0.016	0	0.202
w_j^n/n^2	3232.73	81.06	-10.96	3232.73	81.06	-10.96	3232.73	81.06	-10.96
$p_{ij}^{mn}\frac{w_j^n}{n^2}$	1066.80	0	0.10	0	21.81	0	-51.72	0	-2.21
M_i^m = 2		1066.90			21.81			-53.93	

MOMENT COEFFICIENTS FOR GIRDER i = 4.

TABLE 6.33 : GIRDER MOMENT COEFFICIENTS FOR LOAD ON GIRDER j = 4

LOAD SYSTEM : Symmetrically Placed.

LOADING: 939 G_4(0.159L, 0.216L) + 3960 G_4(0.381L, 0.443L) + 2360 G_4 (0.664L, 0.819L, 0.973L).

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	-0.019	0	0.056	0	0.073	0	0.056	0	-0.010
w_j^n/n^2	1252.23	6.58	-4.30	1252.23	6.58	-4.30	1252.23	6.58	-4.30
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	-23.79	0	-0.24	0	0.48	0	70.12	0	0.04
M_i^m = 2	-24.03			0.48			70.16		

MOMENT COEFFICIENTS FOR GIRDER i = 1

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.145	0	0.023	0	0.130	0	0.017	0	0.023
w_j^n/n^2	1252.23	6.58	-4.30	1252.23	6.58	-4.30	1252.23	6.58	-4.30
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	180.32	0	-0.10	0	0.86	0	21.29	0	-0.10
M_1^n = 2	180.22			0.86			21.19		

MOMENT COEFFICIENTS FOR GIRDER i = 2

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.330	0	-0.016	0	0.269	0	-0.009	0	0.202
w_j^n/n^2	1252.23	6.58	-4.30	1252.23	6.58	-4.30	1252.23	6.58	-4.30
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	413.24	0	0.07	0	1.77	0	-11.27	0	-0.87
M_i^n = 2	413.31			1.77			-12.14		

MOMENT COEFFICIENTS FOR GIRDER i = 3

j=1, i=1, n→	m = 1			m = 2			m = 3		
	1	2	3	1	2	3	1	2	3
p_{ij}^{mn}	0.544	0	-0.063	0	0.528	0	-0.063	0	0.785
w_j^n/n^2	1252.23	6.58	-4.30	1252.23	6.58	-4.30	1252.23	6.58	-4.30
$p_{ij}^{mn} \frac{w_j^n}{n^2}$	681.21	0	0.27	0	3.47	0	-78.89	0	-3.38
M_1^m = 2	681.48			3.47			-82.27		

MOMENT COEFFICIENTS FOR GIRDER i = 4.

TABLE - 6.34 : GIRDER MOMENT COEFFICIENTS FOR LOAD ACTING ON GIRDER G_1, G_2, G_3 AND G_4 SIMULTANEOUSLY.

LOAD SYSTEM : Symmetrically placed.

LOADING : $(939G_1+2420G_2+2420G_3+939G_4)(0.159L, 0.216L)+(3960G_1+10220G_2+10220G_3+3960G_4)(0.381L+0.443L)+(2360G_1+6100G_2+6100G_3+2360G_4)(0.664L, 0819L, 0.973L)$.

i = 1	j	m = 1	m = 2	m = 3
M_{ij}^m	1	681.48	3.47	-82.27
	2	1066.90	21.81	-53.93
	3	465.32	10.54	74.10
	4	-24.03	0.48	70.16
M_i^m		2189.67	36.30	8.06

MOMENT COEFFICIENTS FOR GIRDER i = 1

i = 2	j	m = 1	m = 2	m = 3
M_{ij}^m	1	413.31	1.77	-12.14
	2	963.45	29.67	-35.35
	3	737.06	19.05	- 2.24
	4	180.22	0.86	21.19
M_i^m		2294.04	51.35	-28.54

MOMENT COEFFICIENTS FOR GIRDER i = 2

i = 3	j	m = 1	m = 2	m = 3
M_{ij}^m	1	180.22	0.86	21.19
	2	737.06	19.05	- 2.24
	3	968.45	29.67	-35.35
	4	413.31	1.77	-12.16
M_i^m		2294.04	51.35	-28.54

MOMENT COEFFICIENTS FOR GIRDER i = 3

i = 4	j	m = 1	m = 2	m = 3
M_{ij}^m	1	-24.03	0.48	70.16
	2	465.32	10.54	74.10
	3	1066.90	21.81	-53.93
	4	681.48	3.47	-82.27
M_i^m		2189.67	36.30	8.06

MOMENTS COEFFICIENTS FOR GIRDER i = 4.

loading as computed in Table 6.23 and Table 6.24 for the two load positions are then entered in a row below the design coefficient, simple arithmatic operations as indicated in various Tables are carried out to obtain the girder moment coefficients.

6.5.7 Girder Moment And Deflection

Moment coefficients derived in previous section are used to obtain the girder moments and deflections at various locations. These bending moments are calculated for all the four girders under two transverse disposition of IRC Class - A loading. Various computation steps and final girder moments and deflections at different locations are presented in Tables 6.35 through 6.38.

6.6 COMPARISON OF RESULTS

Based on the computations explained and performed in earlier sections of this chapter, girder bending moment and deflection diagrams have been plotted for the two typical loadings adopted. These diagrams have been plotted on the same sheet using different symbols for comparative study. Solid line diagrams represent the Harmonics method values for bending moments and deflections in Fig. 6.7 through Fig. 6.9. Courbon's method values have been shown by open circles at certain locations of the girders. Orthotropic plate values have been computed using (a) Rowe curves and (b) Cusens and Pama curves. Bending moment and deflections values based on Rowe curves have been represented by open triangles whereas those based on Cusens and Pama curves have been shown by filled-in circles.

TABLE - 6.35 : LIVE LOAD GIRDER MOMENTS AT VARIOUS LOCATIONS
TRANSVERSE LOAD SYSTEM : EXTREME LEFT.

GIRDER i	X/L	A	B	C	$M_i=(L^2/\pi^2)(A+B+C)$ (t.m.)
	0.20	1524.83	57.02	-102.91	56.40
	0.40	2467.23	35.24	63.60	97.85
1	0.50	2594.20	0	108.21	103.05
	0.60	2497.23	-35.24	63.60	95.16
	0.80	1524.84	-57.02	-102.91	52.05
	0.20	1366.63	55.83	-46.44	52.47
	0.40	2211.25	34.50	23.70	86.73
2	0.50	2325.05	0	48.83	90.52
	0.60	2211.25	-34.50	28.70	84.10
	0.80	1366.63	-55.83	-46.44	48.21
	0.20	1170.84	51.27	- 1.36	46.55
	0.40	1894.42	31.69	0.84	73.48
3	0.50	1991.96	0	1.43	76.01
	0.60	1894.47	-31.64	0.84	71.06
	0.80	1170.84	-51.27	- 1.36	42.64
	0.20	926.13	38.38	107.38	40.84
	0.40	1498.5o	23.10	-66.37	55.49
4	0.50	1575.62	0	-112.91	55.78
	0.60	1499.50	-23.10	- 66.37	53.73
	0.80	926.13	-37.38	107.38	37.99

$A = M_i^1 \sin(\pi x_i/L)$; $B = M_i^2 \sin(2\pi x_i/L)$; $C = M_i^3 \sin(3\pi x_i/L)$

TABLE - 6.36 : LIVE LOAD GIRDER MOMENTS AT VARIOUS LOCATIONS TRANSVERSE LOAD SYSTEM : SYMMETRICALLY PLACED.

GIRDER i	X/L	A	B	C	$M_i=(L^2/\pi^2)(A+B+C)$ (t.m.)
1	0.20	1287.06	34.52	7.66	50.70
	0.40	2082.50	21.34	-4.74	80.00
	0.50	2189.67	0	-8.06	83.20
	0.60	2082.50	-21.34	-4.74	78.40
	0.80	1287.06	-34.52	7.66	48.10
2	0.20	1348.41	49.01	-27.14	50.30
	0.40	2181.76	30.29	16.78	85.00
	0.50	2294.04	0	28.54	88.60
	0.60	2181.76	-30.29	16.78	82.70
	0.80	1248.41	-49.01	-27.14	48.50
3	0.20	1348.41	49.01	-27.14	52.30
	0.40	2181.76	30.29	16.78	85.00
	0.50	2294.04	0	28.54	88.60
	0.60	2181.76	-30.29	16.78	82.70
	0.80	1348.41	-49.01	-27.14	48.50
4	0.20	1287.06	34.52	7.66	50.70
	0.40	2082.50	21.34	- 4.74	80.00
	0.50	2189.67	0	- 8.06	83.20
	0.60	2082.50	-21.34	- 4.74	78.40
	0.80	1287.06	-34.52	7.66	48.10

$A = M_i^1 \sin(\pi x_i/L)$; $B = M_i^2 \sin(2\pi x_i/L)$; $C = M_i^3 \sin(3\pi x_i/L)$.

TABLE 6. 37 : GIRDER DEFLECTIONS AT VARIOUS LOCATIONS.

LOAD SYSTEM : Extreme Left.

GIRDER	X/L	$A=M_i^1 \sin(\frac{\pi x}{L})$	$B=M_i^2 \sin(\frac{2\pi x}{L})$	$C=M_i^3 \sin(\frac{3\pi x}{L})$	$W_i = 1/EI (\frac{L}{\pi})^4 (A+\frac{1}{4}B+\frac{1}{9}C)$ (cm.)
	0.20	1524.83	57.02	-102.91	0.2669
	0.40	2467.23	35.24	63.60	0.4338
1	0.50	2594.20	0	108.21	0.4554
	0.60	2497.23	-35.24	63.60	0.4308
	0.80	1524.84	-57.02	-102.91	0.2619
	0.20	1366.63	55.83	-46.44	0.2403
	0.40	2211.25	34.50	23.70	0.3884
2	0.50	2325.05	0	48.83	0.4071
	0.60	2211.25	-34.50	28.70	0.3854
	0.80	1366.63	-55.83	-46.44	0.2354
	0.20	1170.84	51.27	- 1.36	0.207
	0.40	1894.42	31.69	0.84	0.332
3	0.50	1991.96	0	1.43	0.348
	0.60	1894.47	-31.64	0.84	0.329
	0.80	1170.84	-51.27	- 1.36	0.202
	0.20	926.13	38.38	107.38	0.1656
	0.40	1498.50	23.10	-66.37	0.261
4	0.50	1575.62	0	-112.91	0.273
	0.60	1499.50	-23.10	- 66.37	0.254
	0.80	926.13	-37.38	107.38	0.1633

TABLE 6. 38 : GIRDER DEFLECTIONS AT VARIOUS LOCATIONS.

LOAD SYSTEM : Symmetrically placed.

GIRDER	X/L	$A=M_i^1 \sin(\frac{\pi x}{L})$	$B=M_i^2 \sin(\frac{2\pi x}{L})$	$C=M_i^3 \sin(\frac{3\pi x}{L})$	$W_i = (1/EI)(\frac{L}{\pi})^4 (A+\frac{1}{4}B+\frac{1}{9}C)$ (Cm.)
	0.20	1287.06	34.52	7.66	0.226
	0.40	2082.50	21.34	-4.74	0.365
1	0.50	2189.67	0	-8.06	0.382
	0.60	2082.50	-21.34	-4.74	0.363
	0.80	1287.06	-34.52	7.66	0.223
	0.20	1348.41	49.01	-27.14	0.237
	0.40	2181.76	30.29	16.78	0.383
2	0.50	2294.04	0	28.54	0.401
	0.60	2181.76	-30.29	16.78	0.380
	0.80	1348.41	-49.01	-27.14	0.233
	0.20	1348.41	49.01	-27.14	0.237
	0.40	2181.76	30.29	16.78	0.383
3	0.50	2294.04	0	28.54	0.401
	0.60	2181.76	-30.29	16.78	0.380
	0.80	1348.41	-49.01	-27.14	0.233
	0.20	1287.06	34.52	7.66	0.226
	0.40	2082.50	21.34	- 4.74	0.365
4	0.50	2189.67	0	- 8.06	0.382
	0.60	2082.50	-21.34	- 4.74	0.363
	0.80	1287.06	-34.52	7.66	0.223

LIVE LOAD SYSTEM : EXTREME LEFT (15 Cm FROM KERB)

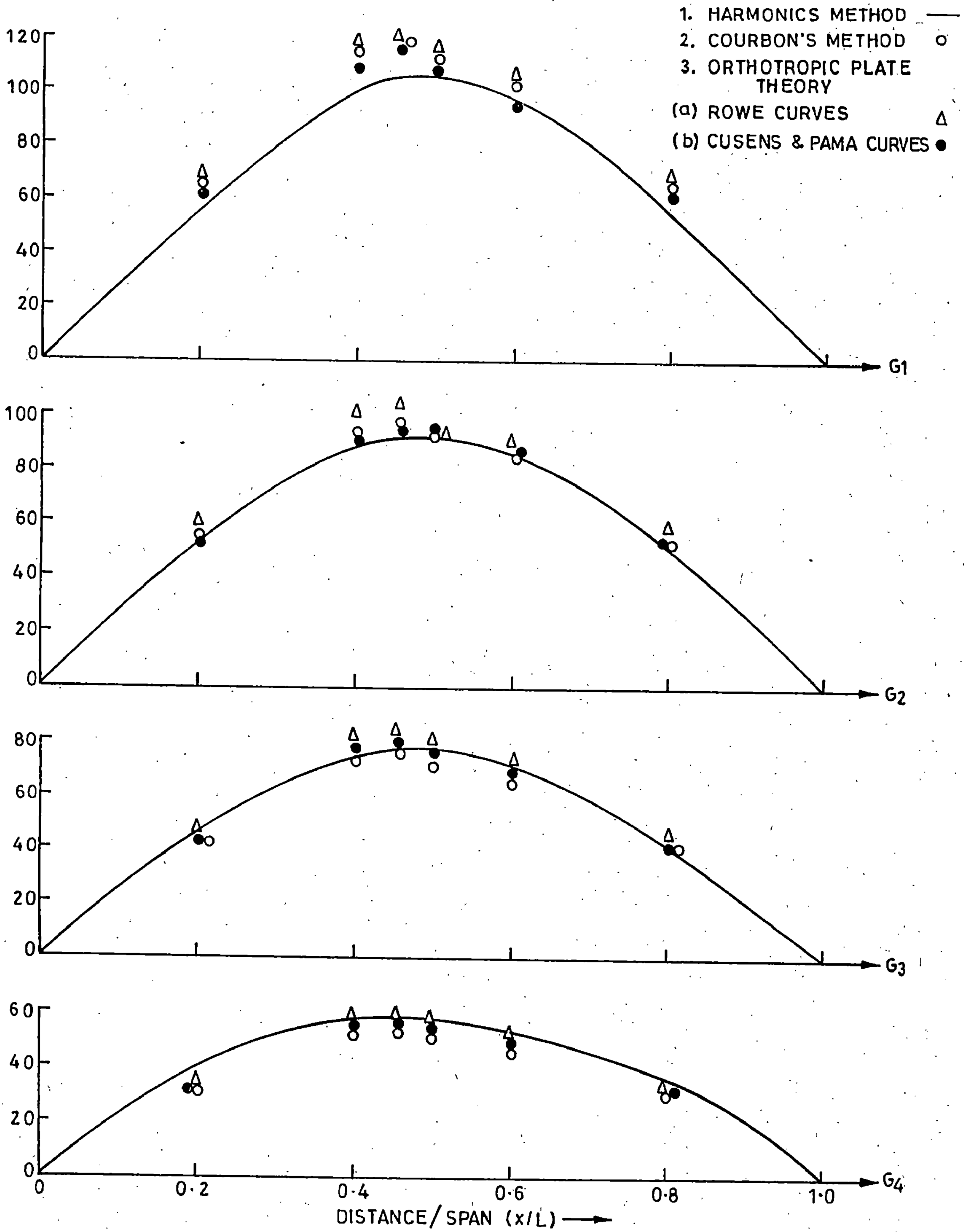


FIG 6·7 — GIRDER MOMENTS AT DIFFERENT LOCATIONS

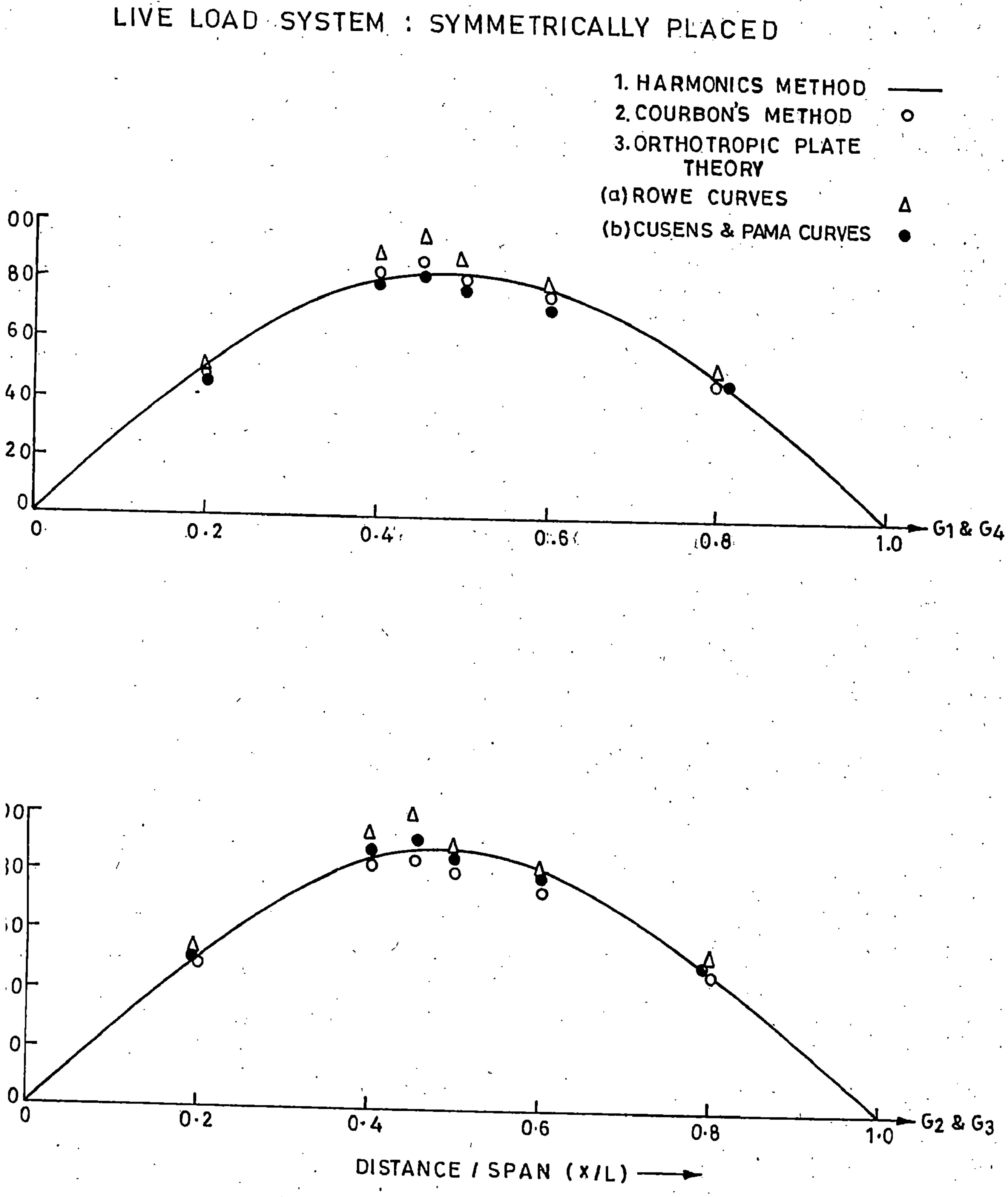


FIG. 6·8 — GIRDER MOMENTS AT DIFFERENT LOCATIONS

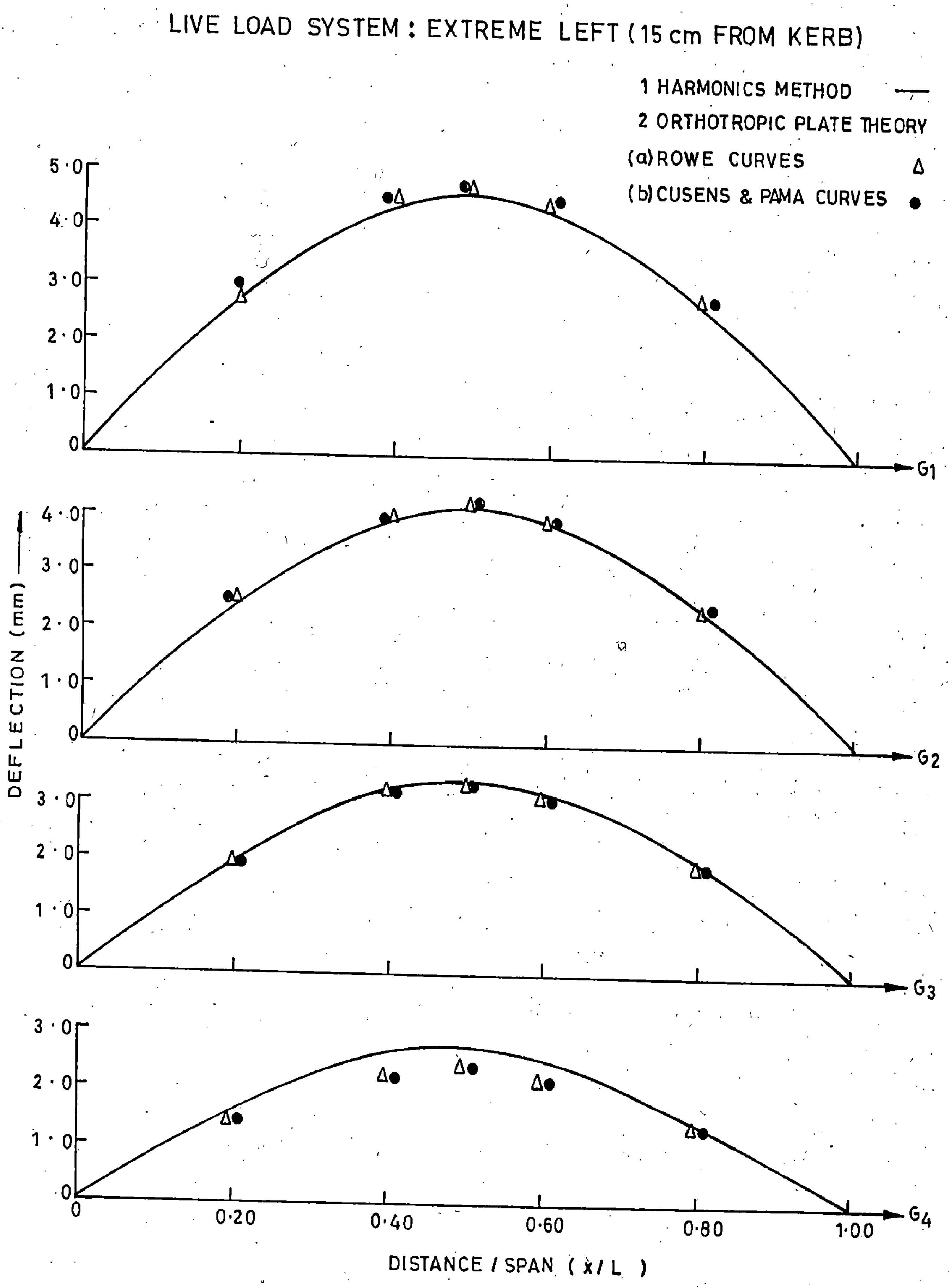


FIG. 6·9 – GIRDER DEFLECTIONS AT VARIOUS LOCATIONS

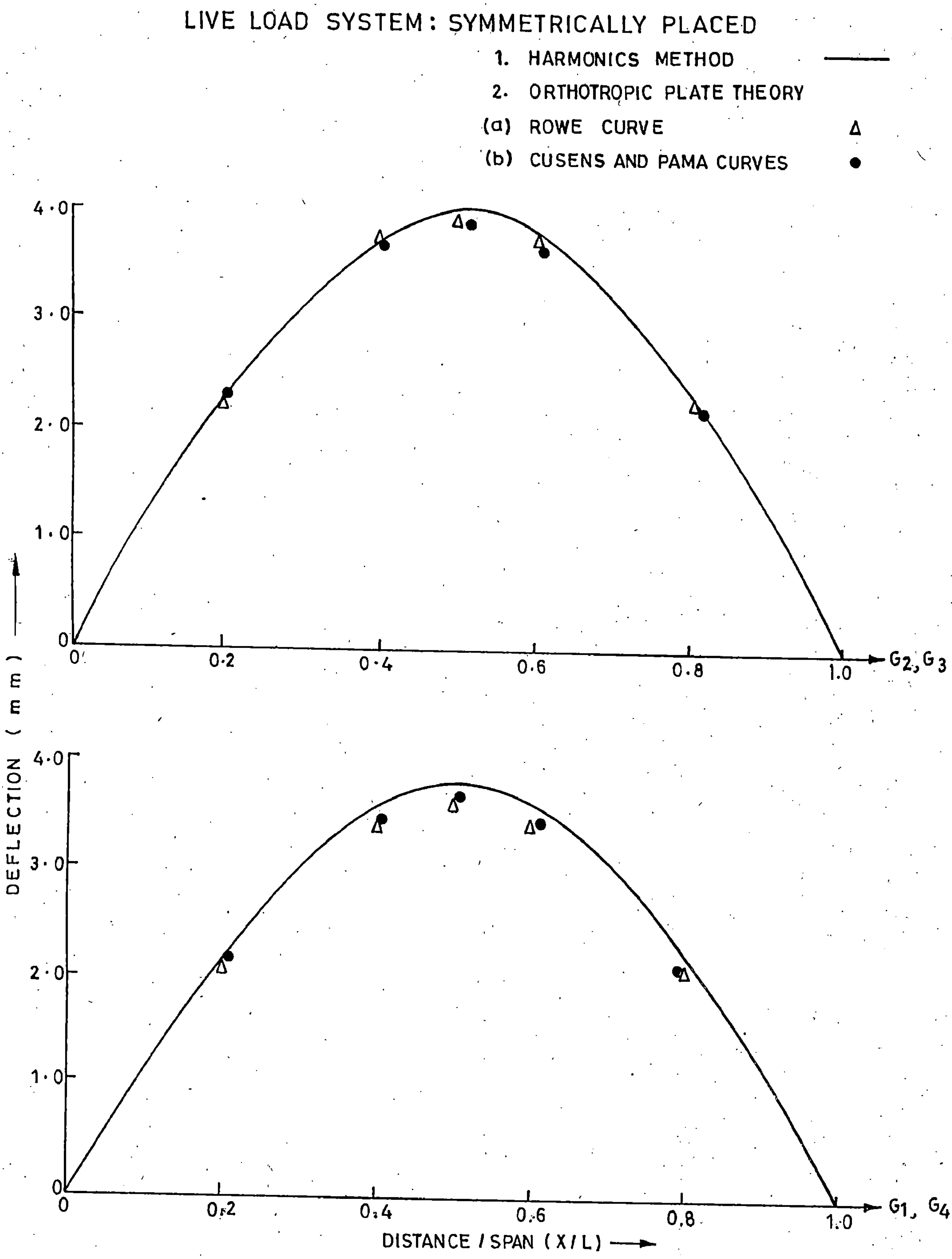


FIG. 6·10—GIRDER DEFLECTION AT VARIOUS LOCATIONS

Girder bending moment diagrams for the extreme left load positions (15 cm from kerb) have been presented in Fig. 6.7. A close study of the bending moment values which are critical from the design point of view indicates that Harmonics method values are the lowest (top diagram of Fig. 6.7). Rowe curves method which allows for a general increase in values by 10 percent due to slow convergence results in higher critical values. Courbon's method values are also higher. This method being based on arbitary and gross assumptions does not appeal to theoreticians. However, owing to its simplicity in actual use and being conservative in design values, it is readily adopted by bridge designers. Cnsens and Pama method yields-in bending moment values a little higher than the Harmonics method values (top diagram of Fig. 6.7). Similar conclusions may be arrived at from the study of bending moment diagrams presented in Fig. 6.8 and deflection diagrams presented in Fig. 6.9 although the values differ by small magnitude in some situations. Deflection values being quite small in magnitude, it becomes difficult to conclude from the study of diagrams. However, the computed values in tabular form lead to the same conclusion as above.

Based on the study presented above it may be concluded that

(1) Harmonics method results in the lowest critical girder moment values in comparison with (a) Orthotropic plate theory (i) Rowe curves , (ii) Cusens and Pama curves and (b) Courbon's method values (Fig.6.7 and Fig.6.8).

(2) Harmonics method can easily be applied to skew girder bridge where as Rowe curves, Cusens and Pama curves and Courbon's Method apply to the right bridge only. Right bridge is a special case in Harmonics method where-in the skew parameter K is taken equal to zero (Section 6.5.3(d)).

(3) Harmonics method lends itself to perform computation in tabular forms where-in computations in each sheet may be performed or checked independently. It offers a definite advantage in a design office where-in a group of persons is frequently required to do computations independently (Table 6.23 through Table 6.34)

(4) With the shifting of load positions or change of bridge cross-sections only a part of the computations involved in Harmonics method need to be changed or redone. This becomes evident from computation tables for the two loading cases adopted. Tables 6.19 through 6.22 have remained unchanged. Tables 6.23 and 6.24 have been affected to incorporate changes due to shifting of load system.

(5) Cusens and Pama curves are derived for mid-span section of a right bridge with load also located there. This is alright from the design point of view. There is no such limitation or restriction in Harmonics method.

CHAPTER - 7

SUMMARY AND CONCLUSIONS

Girder bridges are found to be quite suitable for small and medium span negotiations. A number of methods both conventional as well as computer based for the analysis of girder bridges are available. Some of the methods which do make use of digital computers for the determination and development of distribution coefficients in tabular or chart form may be clubbed with conventional methods since they no longer need any access to computers for complete analysis and design of girder bridges. These methods are (1) Orthotropic Plate Theory based (a) Morice, Little and Rowe curves ; and (b) Cusens and Pama curves ; and (2) Harmonics Method. Courbon's method is another conventional method which is quite popular with bridge designers primarily owing to its simplicity in application. Courbon's method is not a favourite with theoreticians for reasons of gross assumptions associated with its mathematical modelling. These conventional methods have been briefly but lucidly presented in the various chapters of this dissertation. From the theory point of view, the assumptions and limitations associated with these methods have been discussed. From the application point of view, a typical 4-girder right bridge having a span of 19.4 m and carriage width of 7.5 m has been analyzed for two critical load positions of IRC class-A loading. A comparative study of the results for girder bending moments and deflections as obtained by the above mentioned methods has been carried out and certains conclusions arrived at which may be stated as follows

(1) Harmonics method results-in the lowest critical girder moment values in comparison with (a) Orthotropic plate theory (i) Rowe curves (ii) Cusens and Pama curves and (b) Courbon's method values (Fig. 6.7 and Fig. 6.8).

(2) Harmonics method can easily be applied to skew girder bridge where as Rowe curves, Cusens and Pama curves and Courbon's Method apply to the right bridge only. Right bridge is a special case in Harmonics method where-in the skew parameter K is taken equal to zero (Section 6.5.2(d)).

(3) Harmonics method lends itself to perform computation in tabular forms where-in computations in each sheet may be performed or checked independently. It offers a definite advantage in a design office where-in a group of persons is frequently required to do computations independently (Table 6.23 through Table 6.34).

(4) With the shifting of load positions or change of bridge cross-sections only a part of the computations involved in Harmonics method need to be changed or redone. This becomes evident from computation tables

for the two loading cases adopted. Tables 6.19 through 6.22 have remained unchanged. Tables 6.23 and 6.24 have been affected to incorporate changes due to shifting of load system.

(5) Cusens and Pama curves are derived for mid-span section of a right bridge with load also located there. This is alright from the design point of view. There is no such limitation or restriction in Harmonics method.

(6) Harmonics method gives girder moment instead of share of imposed loading taken by individual girder ; which is highly advantageous from the design point of view.

**

APPENDIX - A

FOURIER SERIES OF LOAD FUNCTIONS :

Fourier Sine series of a general load function $w(x)$ acting on a simply supported girder of length L is given by -

$$w(x) = \sum_{n=1}^{\infty} W_n \sin\left(\frac{n\pi x}{L}\right)$$

First three terms of this series for same loading cases are given below

(i) For a single conc. load w at a distance x_1 from the left support of girder of length L, the coefficients are

$$W_n = \frac{2}{L} \int_o^L f(x) \sin\left(\frac{n\pi x}{L}\right) = \frac{2w_1}{L} \sin \frac{4\pi x}{L}$$

$$W(x) = \frac{2w}{L} \sum_{n=1}^{3} \sin\left(\frac{n\pi x_1}{L}\right) \sin\left(\frac{n\pi x}{L}\right)$$

(ii) For multiple loads $w_1, w_2 \ldots w_p$ simultaneously acting on a girder of length L, at $(x_1, x_2 \ldots x_p)$.

$$w_n = \frac{2}{L} \sum_{i=1}^{p} W_i \sin\left(\frac{n\pi x_i}{L}\right)$$

and $$w(x) = \frac{2}{L} \sum_{i=1}^{3} \left[W_i \sin\left(\frac{n\pi x_i}{L}\right) \sin\left(\frac{n\pi x}{L}\right)\right]$$

(iii) For patch load w per unit length over a segment $x_i < x < x_2$.

$$w_n = \frac{2w}{n\pi} \left[\cos\left(\frac{n\pi x_1}{L}\right) - \cos\left(\frac{n\pi x_2}{L}\right)\right]$$

or $$w(x) = \sum_{n=1}^{3} \frac{2w}{n\pi} \left[\cos\left(\frac{n\pi x_1}{L}\right) - \cos\left(\frac{n\pi x_2}{L}\right)\right] \sin\left(\frac{n\pi x}{L}\right)$$

(iv) For u.d.l., w per unit length over entire span of girder,

$$w_n = \frac{2}{L}\ \frac{2w}{n\pi}\,(1-\cos(n\pi))$$

or $$w(x) = \sum_{n=1}^{3} \frac{2w}{n\pi}\,(1-\cos(n\pi))\sin\frac{n\pi x}{L}$$

**

APPENDIX - B

TORSIONAL CONSTANT (J)

It is not a simple geometric property of section like flexural constant I. Approximate procedure for J is explained for various shapes hereunder.

(a) Rectangular Section :

Torsional constant J for rectangular shown in Fig. B-1(b) with sides b and a is given by $J = K\ b\ a^3$. Where K is called the Timoshenko torsion coefficient and is a cunction of aspect ratio b/a as shown in figure B-1(a).

(b) T-beam Section :

Fig. B-2(a) shows a practical T-beam section and Figure B-2(b) shows an idealized section consisting of three rectangular areas. In a section comprising of a numbers of rectangules it is logical that the overall torsional stiffness is equal to the sum of stiffnesses of individual rectangles. This is perfectly true but in load distribution procedure what is required is the equivalent torsional stiffness of orthotropic plate and torsional parameter, α as function of this torsional stiffness. Hence it is not correct to isolate an individual T-beam in a T-beam bridge and determine α in this way. If this is done the value of α so derived will be greater than unity, which is impossible. This is due to the fact that top flange of the T-beam, which is a part of the top slab does not satisfy the equation ($J = Kb\ a^3$) as

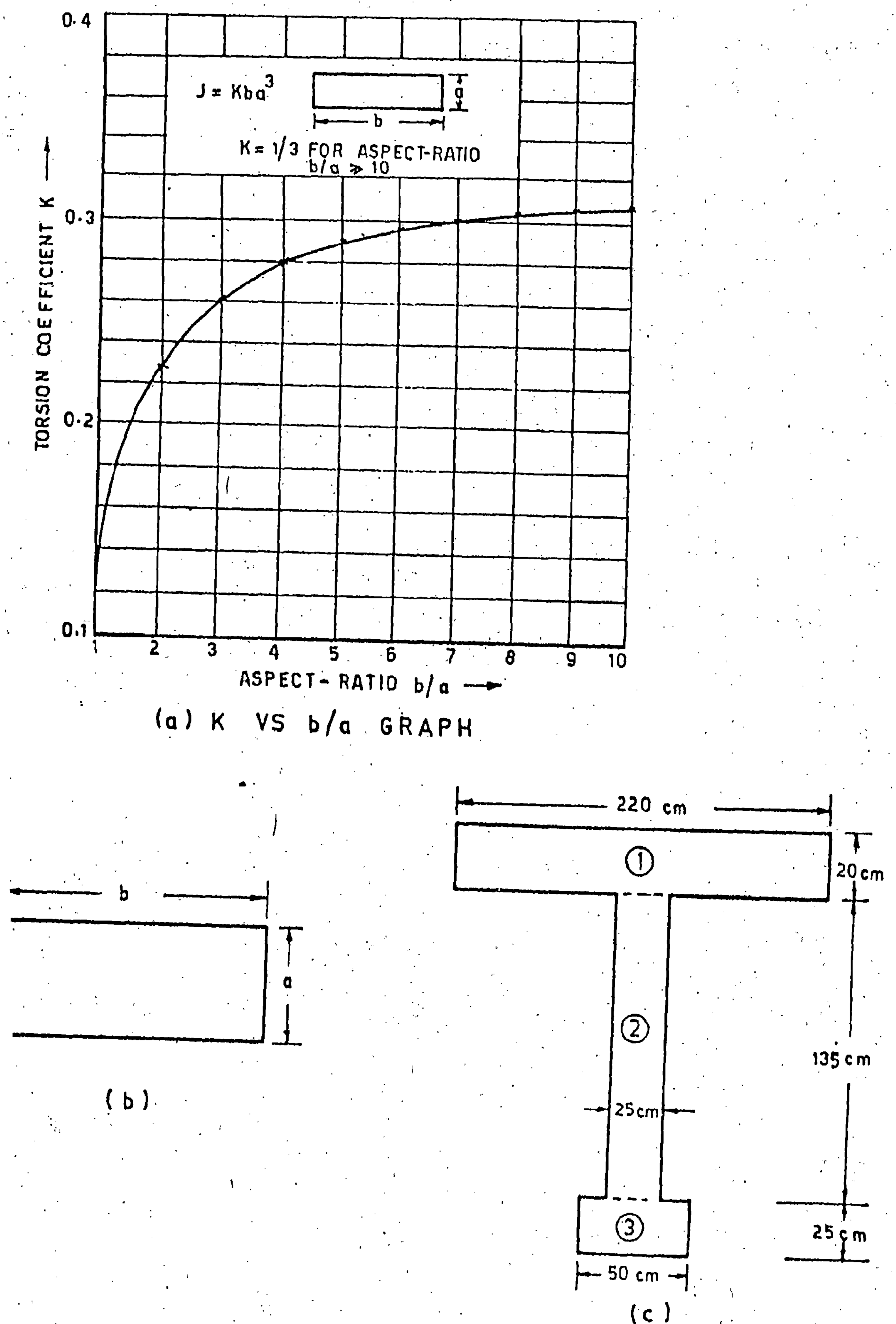


FIG. B-1 - TORSIONAL CONSTANT COMPUTATION

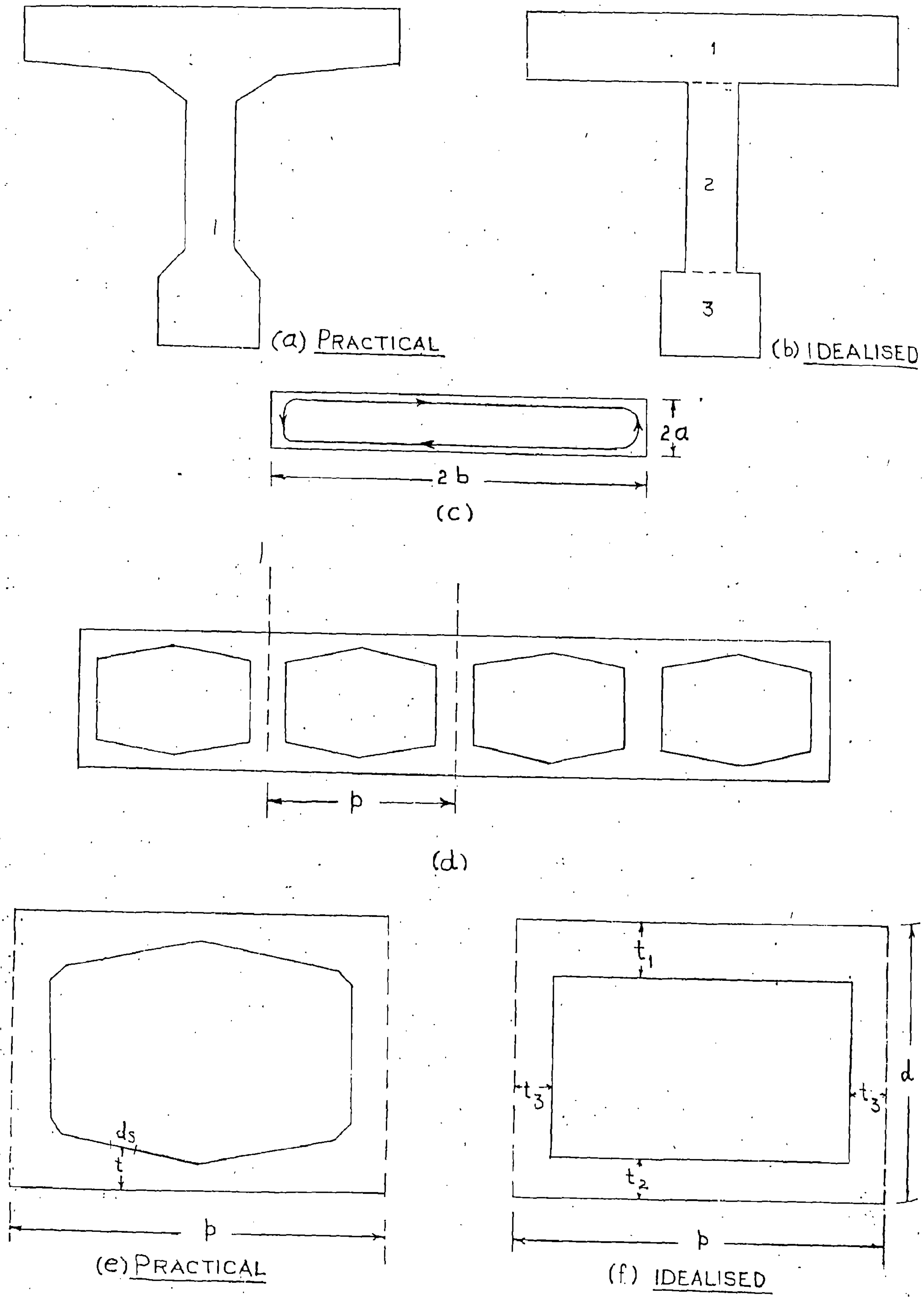


FIG B-2 - PRACTICAL & IDEALISED SECTIONS FOR DERIVATION OF TORSIONAL PARAMETER it assumes the shear flow in the direction as shown in Fig. B-2(c) If the shear stress at the ends of the rectangle which have a large lever arm are neglected, the value of torsional stiffness will be half of that given by the above equation.

In a T-beam bridge in each individual T-beam only the shear stress parellel to the top surface can exist and if an individual T-beam is isolated, then top flange contributes only 50% of torsional stiffness. Thus torsional inertia is given by

$$J = \frac{1}{2} K_1 (a_1)^3 \; 2b_1 + K_2 \; (2a_2)^3 \; 2b_2 + K_3(2a_3)^3 \; 2b_3.$$

Further in reducing a practical section to idealised section, it is sufficiently accurate to make the two sections of equivalent area and neglect the small fillet as shown in Fig. B-2(b).

(c) Hollow Box Section :

Consider a bridge with a cross-section shown in Fig. B-2(d), to deduce the torsional inertia of this section, an individual cell is considered as shown in Fig. B-2(e) by dividing the webs at their centreline. By using the membrane analogy J is given by

$$J = \frac{4A^2}{\oint \frac{ds}{t}}$$

where,

A = Area of hole in Section

ds = An element of perimeter of the hole.

t = Thickness of wall at the element.

If the perimeter of hole consists of a series of straight lines as shown in Fig. B-2(f), J can be deduced as :

$$J = \frac{4\,A^2}{\left[\frac{p-2t_3}{t_1} + \frac{p-2t_3}{t_2} + \frac{2(d-t_1-t_2)}{t_3}\right]}$$

ILLUSTRATION

Torsional constant, in case of Harmonics Method is calculated by taking full contribution of flange (18).

J for T-section shown in Fig. B-1(c)

From Fig. B-1(a)

K_1 for element (1) = 1/3

K_2 for element (2) = 0.295

K_3 for element (3) = 0.228

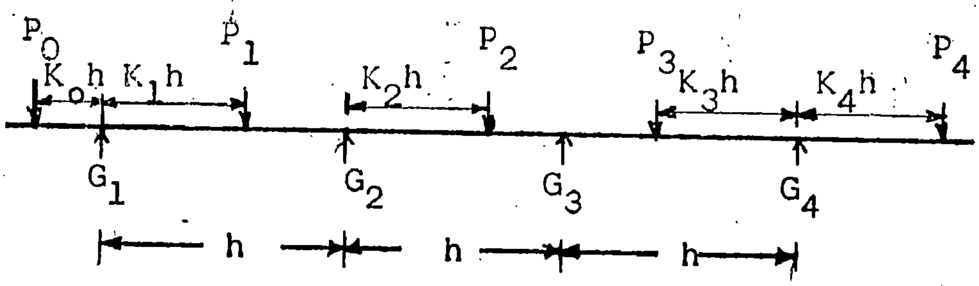
$$J = K_1(220)(20)^3 + K_2(135)(25)^3 + K_3(50)(25)^3$$

$$= 13.87 \times 10^5 \text{ cm}^4.$$

**

APPENDIX - C

Expressions for bending moments at various girder locations. For four girder bridge acted upon by a loading shown in figure are given below.



$$M_1 = -hp_oK_o$$

$$M_2 = \frac{h}{15}\left[4P_oK_o(1-K_o^2)-4P_1K_1(1-K_1^2)-P_2K_2\right.$$

$$\left.(5-K_2^2-12K_2+7) + P_3K_3(1-K_3^2)\right]$$

$$M_3 = \frac{h}{15}\left[-P_oK_o(1-K_o^2) + P_1K_1(1-K_1^2) + P_2K_2(5K_2^2-3K_2-2)\right.$$

$$\left.- 4P_3K_3(1-K_3^2) + 4P_4K_4(1-K_4^2)\right]$$

$$M_4 = -4P_4K_4$$

Here positive moment gives sagging moment.

**

APPENDIX - D

The procedure for reading the design coefficients from tabulated values is explained hereunder. Linear interpolation is applicable for the values of α and K but for β interpolation function as given earlier is used. The sample design coefficient tables presented herein may be referred as follows

- Column 1 indicated by G stands for girders 1,2,3,4
- Column 2 indicated by H stands for load harmonics 1,2,3
- Values in Column 3 through column 6 are the design coefficients of first moment harmonic $\sin(\pi x/L)$ for girders 1,2,3,4
- Values in column 7 through column 10 are the design coefficients of second harmonic $\sin(2\pi x/L)$ for girders 1,2,3,4
- Values in column 11 through column 14 are the design coefficients of 3rd harmonic $\sin(3\pi x/L)$ for girders 1,2,3,4.

Thus there are 3 x 4 design coefficients for the 4-girders in each row corresponding to a particular imposed load harmonic on a particular girder. For load on all the four girders there are 12 x 12 design coefficients for a particular combination of α, β and K.

**

G H K= 0.00000 BE= 0.00 GA=0.00 AL= 50.00

1	1	0.632	0.347	0.110	-0.089	0.000	0.000	0.000	0.000	-0.068	-0.003	0.019	0.053
1	2	-0.000	-0.000	0.000	0.000	0.626	0.257	0.087	0.030	0.000	0.000	-0.000	-0.000
1	3	-0.068	-0.012	0.027	0.053	0.000	0.000	0.000	0.000	0.849	0.158	0.000	-0.007
2	1	0.347	0.317	0.226	0.110	0.000	0.000	0.000	0.000	-0.012	-0.013	-0.003	0.027
2	2	-0.000	-0.000	0.000	0.000	0.257	0.424	0.232	0.087	0.000	0.000	-0.000	-0.000
2	3	-0.003	-0.013	-0.003	0.019	0.000	0.000	0.000	0.000	0.158	0.660	0.182	0.000
3	1	0.110	0.226	0.317	0.347	0.000	0.000	0.000	0.000	0.027	-0.003	-0.013	-0.012
3	2	-0.000	-0.000	0.000	0.000	0.087	0.232	0.424	0.257	0.000	0.000	-0.000	-0.000
3	3	0.019	-0.003	-0.013	-0.003	0.000	0.000	0.000	0.000	0.000	0.182	0.660	0.158
4	1	-0.089	0.110	0.347	0.632	0.000	0.000	0.000	0.000	0.053	0.019	-0.003	-0.068
4	2	-0.000	-0.000	0.000	0.000	0.030	0.087	0.257	0.626	0.000	0.000	-0.000	-0.000
4	3	0.053	0.027	-0.012	-0.068	0.000	0.000	0.000	0.000	-0.007	0.000	0.158	0.849

G H K= 0.00000 BE= 0.00 GA=0.00 AL= 64.00

1	1	0.611	0.346	0.119	-0.077	0.000	0.000	0.000	0.000	-0.078	-0.007	0.022	0.064
1	2	-0.000	-0.000	0.000	0.000	0.587	0.264	0.104	0.044	0.000	0.000	-0.000	-0.000
1	3	-0.078	-0.016	0.030	0.064	0.000	0.000	0.000	0.000	0.825	0.178	0.008	-0.011
2	1	0.346	0.310	0.225	0.119	0.000	0.000	0.000	0.000	-0.016	-0.013	-0.002	0.030
2	2	-0.000	-0.000	0.000	0.000	0.264	0.397	0.234	0.104	0.000	0.000	-0.000	-0.000
2	3	-0.007	-0.013	-0.002	0.022	0.000	0.000	0.000	0.000	0.178	0.619	0.195	0.008
3	1	0.119	0.225	0.310	0.346	0.000	0.000	0.000	0.000	0.030	-0.002	-0.013	-0.016
3	2	-0.000	-0.000	0.000	0.000	0.104	0.234	0.397	0.264	0.000	0.000	-0.000	-0.000
3	3	0.022	-0.002	-0.013	-0.007	0.000	0.000	0.000	0.000	0.008	0.195	0.619	0.178
4	1	-0.077	0.119	0.346	0.611	0.000	0.000	0.000	0.000	0.064	0.022	-0.007	-0.078
4	2	-0.000	-0.000	0.000	0.000	0.044	0.104	0.264	0.587	0.000	0.000	-0.000	-0.000
4	3	0.064	0.030	-0.016	-0.078	0.000	0.000	0.000	0.000	-0.011	0.008	0.178	0.825

G H K= 0.00000 BE= 0.00 GA=0.00 AL= 80.00

1	1	0.591	0.344	0.128	-0.064	0.000	0.000	0.000	0.000	-0.088	-0.011	0.025	0.075
1	2	-0.000	-0.000	0.000	0.000	0.551	0.269	0.120	0.059	0.000	0.000	-0.000	-0.000
1	3	-0.088	-0.020	0.033	0.075	0.000	0.000	0.000	0.000	0.802	0.196	0.016	-0.014
2	1	0.344	0.304	0.224	0.128	0.000	0.000	0.000	0.000	-0.020	-0.014	0.000	0.033
2	2	-0.000	-0.000	0.000	0.000	0.269	0.375	0.236	0.120	0.000	0.000	-0.000	-0.000
2	3	-0.011	-0.014	0.000	0.025	0.000	0.000	0.000	0.000	0.196	0.583	0.205	0.016
3	1	0.128	0.224	0.304	0.344	0.000	0.000	0.000	0.000	0.033	0.000	-0.014	-0.020
3	2	-0.000	-0.000	0.000	0.000	0.120	0.236	0.375	0.269	0.000	0.000	-0.000	-0.000
3	3	0.025	0.000	-0.014	-0.011	0.000	0.000	0.000	0.000	0.016	0.205	0.583	0.196
4	1	-0.064	0.128	0.344	0.591	0.000	0.000	0.000	0.000	0.075	0.025	-0.011	-0.088
4	2	-0.000	-0.000	0.000	0.000	0.059	0.120	0.269	0.551	0.000	0.000	-0.000	-0.000
4	3	0.075	0.033	-0.020	-0.088	0.000	0.000	0.000	0.000	-0.014	0.016	0.196	0.802

G H K= 0.00000 BE= 0.00 GA=0.00 AL=100.00

1	1	0.572	0.341	0.136	-0.049	0.000	0.000	0.000	0.000	-0.098	-0.015	0.028	0.086
1	2	-0.000	-0.000	0.000	0.000	0.517	0.272	0.136	0.076	0.000	0.000	-0.000	-0.000
1	3	-0.098	-0.024	0.037	0.086	0.000	0.000	0.000	0.000	0.779	0.214	0.026	-0.018
2	1	0.341	0.299	0.224	0.136	0.000	0.000	0.000	0.000	-0.024	-0.014	0.002	0.037
2	2	-0.000	-0.000	0.000	0.000	0.272	0.356	0.237	0.136	0.000	0.000	-0.000	-0.000
2	3	-0.015	-0.014	0.002	0.028	0.000	0.000	0.000	0.000	0.214	0.547	0.213	0.026
3	1	0.136	0.224	0.299	0.341	0.000	0.000	0.000	0.000	0.037	0.002	-0.014	-0.024
3	2	-0.000	-0.000	0.000	0.000	0.136	0.237	0.356	0.272	0.000	0.000	-0.000	-0.000
3	3	0.028	0.002	-0.014	-0.015	0.000	0.000	0.000	0.000	0.026	0.213	0.547	0.214
4	1	-0.049	0.136	0.341	0.572	0.000	0.000	0.000	0.000	0.086	0.028	-0.015	-0.098
4	2	-0.000	-0.000	0.000	0.000	0.076	0.136	0.272	0.517	0.000	0.000	-0.000	-0.000
4	3	0.086	0.037	-0.024	-0.098	0.000	0.000	0.000	0.000	-0.018	0.026	0.214	0.779

G	H	K= 0.00000			BE=10.00			GA=0.00			AL= 50.00		
1	1	0.341	0.266	0.213	0.180	0.000	0.000	0.000	0.000	0.062	-0.003	-0.022	-0.037
1	2	-0.000	-0.000	0.000	0.000	0.476	0.268	0.152	0.105	-0.000	0.000	0.000	0.000
1	3	0.062	0.003	-0.027	-0.037	0.000	0.000	0.000	0.000	0.767	0.196	0.035	0.002
2	1	0.266	0.275	0.246	0.213	0.000	0.000	0.000	0.000	0.003	0.019	0.005	-0.027
2	2	-0.000	-0.000	0.000	0.000	0.268	0.359	0.221	0.152	-0.000	-0.000	0.000	0.000
2	3	-0.003	0.019	0.005	-0.022	0.000	0.000	0.000	0.000	0.196	0.587	0.182	0.035
3	1	0.213	0.246	0.275	0.266	0.000	0.000	0.000	0.000	-0.027	0.005	0.019	0.003
3	2	-0.000	-0.000	0.000	0.000	0.152	0.221	0.359	0.268	-0.000	-0.000	0.000	0.000
3	3	-0.022	0.005	0.019	-0.003	0.000	0.000	0.000	0.000	0.035	0.182	0.587	0.196
4	1	0.180	0.213	0.266	0.341	0.000	0.000	0.000	0.000	-0.037	-0.022	-0.003	0.062
4	2	-0.000	-0.000	0.000	0.000	0.105	0.152	0.268	0.476	-0.000	-0.000	-0.000	0.000
4	3	-0.037	-0.027	0.003	0.062	0.000	0.000	0.000	0.000	0.002	0.035	0.196	0.767

G	H	K= 0.00000			BE=10.00			GA=0.00			AL= 64.00		
1	1	0.325	0.264	0.220	0.192	0.000	0.000	0.000	0.000	0.060	-0.001	-0.021	-0.038
1	2	-0.000	-0.000	0.000	0.000	0.442	0.268	0.167	0.123	-0.000	-0.000	0.000	0.000
1	3	0.060	0.004	-0.026	-0.038	0.000	0.000	0.000	0.000	0.733	0.214	0.048	0.005
2	1	0.264	0.270	0.247	0.220	0.000	0.000	0.000	0.000	0.004	0.018	0.004	-0.026
2	2	-0.000	-0.000	0.000	0.000	0.268	0.339	0.226	0.167	-0.000	-0.000	0.000	0.000
2	3	-0.001	0.018	0.004	-0.021	0.000	0.000	0.000	0.000	0.214	0.546	0.192	0.048
3	1	0.220	0.247	0.270	0.264	0.000	0.000	0.000	0.000	-0.026	0.004	0.018	0.004
3	2	-0.000	-0.000	0.000	0.000	0.167	0.226	0.339	0.268	-0.000	-0.000	0.000	0.000
3	3	-0.021	0.004	0.018	-0.001	0.000	0.000	0.000	0.000	0.048	0.192	0.546	0.214
4	1	0.192	0.220	0.264	0.325	0.000	0.000	0.000	0.000	-0.038	-0.021	-0.001	0.060
4	2	-0.000	-0.000	0.000	0.000	0.123	0.167	0.268	0.442	-0.000	-0.000	0.000	0.000
4	3	-0.038	-0.026	0.004	0.060	0.000	0.000	0.000	0.000	0.005	0.048	0.214	0.733

G	H	K= 0.00000			BE=10.00			GA=0.00			AL= 80.00		
1	1	0.313	0.262	0.225	0.201	0.000	0.000	0.000	0.000	0.057	0.001	-0.021	-0.038
1	2	-0.000	-0.000	0.000	0.000	0.414	0.268	0.179	0.140	-0.000	-0.000	0.000	0.000
1	3	0.057	0.005	-0.025	-0.038	0.000	0.000	0.000	0.000	0.701	0.229	0.060	0.010
2	1	0.262	0.266	0.247	0.225	0.000	0.000	0.000	0.000	0.005	0.016	0.003	-0.025
2	2	-0.000	-0.000	0.000	0.000	0.268	0.324	0.229	0.179	-0.000	-0.000	0.000	0.000
2	3	0.001	0.016	0.003	-0.021	0.000	0.000	0.000	0.000	0.229	0.511	0.200	0.060
3	1	0.225	0.247	0.266	0.262	0.000	0.000	0.000	0.000	-0.025	0.003	0.016	0.005
3	2	-0.000	-0.000	0.000	0.000	0.179	0.229	0.324	0.268	-0.000	-0.000	0.000	0.000
3	3	-0.021	0.003	0.016	0.001	0.000	0.000	0.000	0.000	0.060	0.200	0.511	0.229
4	1	0.201	0.225	0.262	0.313	0.000	0.000	0.000	0.000	-0.038	-0.021	0.001	0.057
4	2	-0.000	-0.000	0.000	0.000	0.140	0.179	0.268	0.414	-0.000	-0.000	0.000	0.000
4	3	-0.038	-0.025	0.005	0.057	0.000	0.000	0.000	0.000	0.010	0.060	0.229	0.701

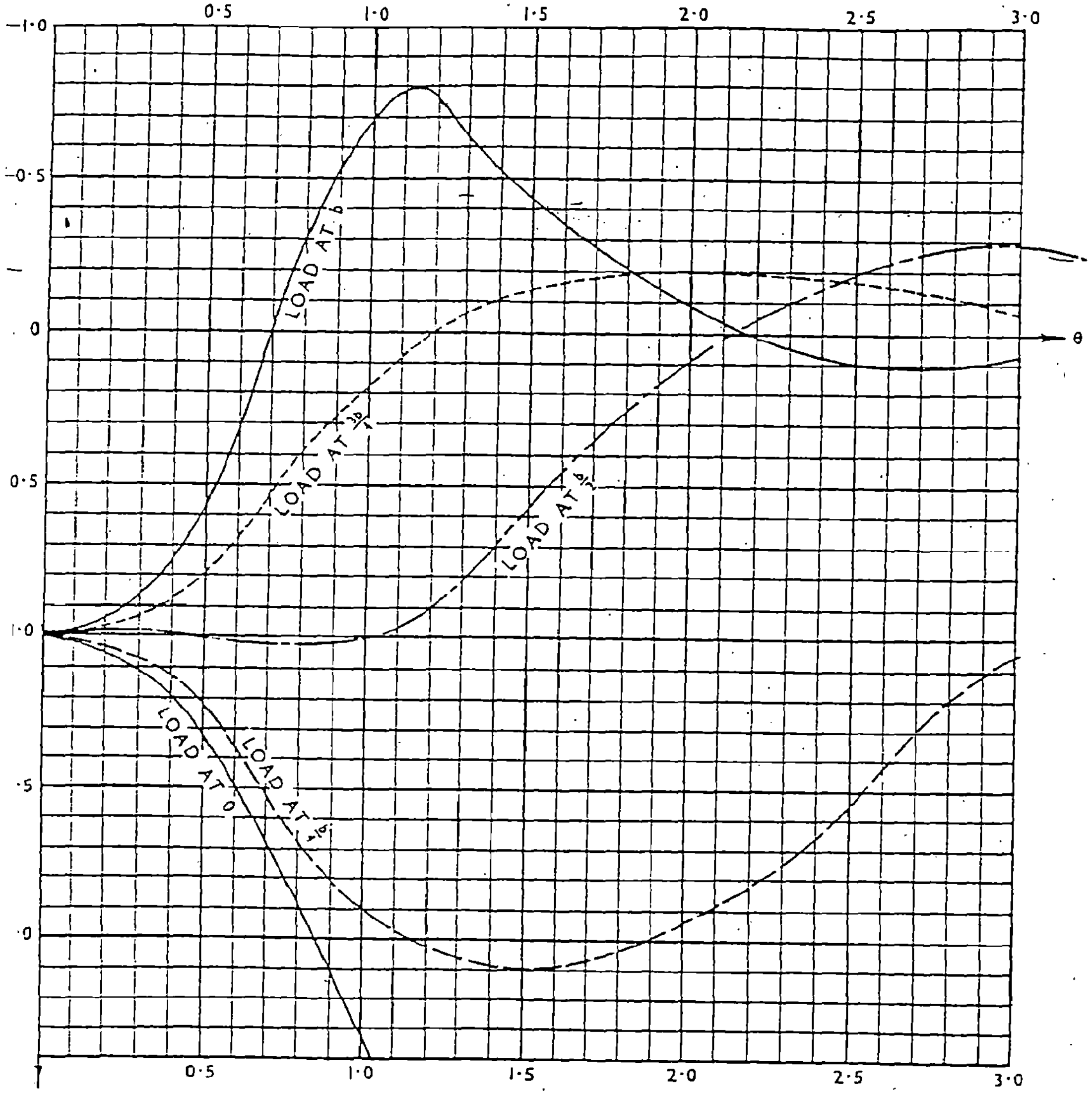
G	H	K= 0.00000			BE=10.00			GA=0.00			AL=100.00		
1	1	0.302	0.260	0.229	0.209	0.000	0.000	0.000	0.000	0.054	0.002	-0.020	-0.037
1	2	-0.000	-0.000	0.000	0.000	0.388	0.267	0.190	0.155	-0.000	-0.000	0.000	0.000
1	3	0.054	0.006	-0.023	-0.037	0.000	0.000	0.000	0.000	0.667	0.242	0.074	0.017
2	1	0.260	0.264	0.248	0.229	0.000	0.000	0.000	0.000	0.006	0.015	0.002	-0.023
2	2	-0.000	-0.000	0.000	0.000	0.267	0.311	0.232	0.190	-0.000	-0.000	0.000	0.000
2	3	0.002	0.015	0.002	-0.020	0.000	0.000	0.000	0.000	0.242	0.478	0.207	0.074
3	1	0.229	0.248	0.264	0.260	0.000	0.000	0.000	0.000	-0.023	0.002	0.015	0.006
3	2	-0.000	-0.000	0.000	0.000	0.190	0.232	0.311	0.267	-0.000	-0.000	0.000	0.000
3	3	-0.020	0.002	0.015	0.002	0.000	0.000	0.000	0.000	0.074	0.207	0.478	0.242
4	1	0.209	0.229	0.260	0.302	0.000	0.000	0.000	0.000	-0.037	-0.020	0.002	0.054
4	2	-0.000	-0.000	0.000	0.000	0.155	0.190	0.267	0.388	-0.000	-0.000	0.000	0.000
4	3	-0.037	-0.023	0.006	0.054	0.000	0.000	0.000	0.000	0.017	0.074	0.242	0.667

APPENDIX - E

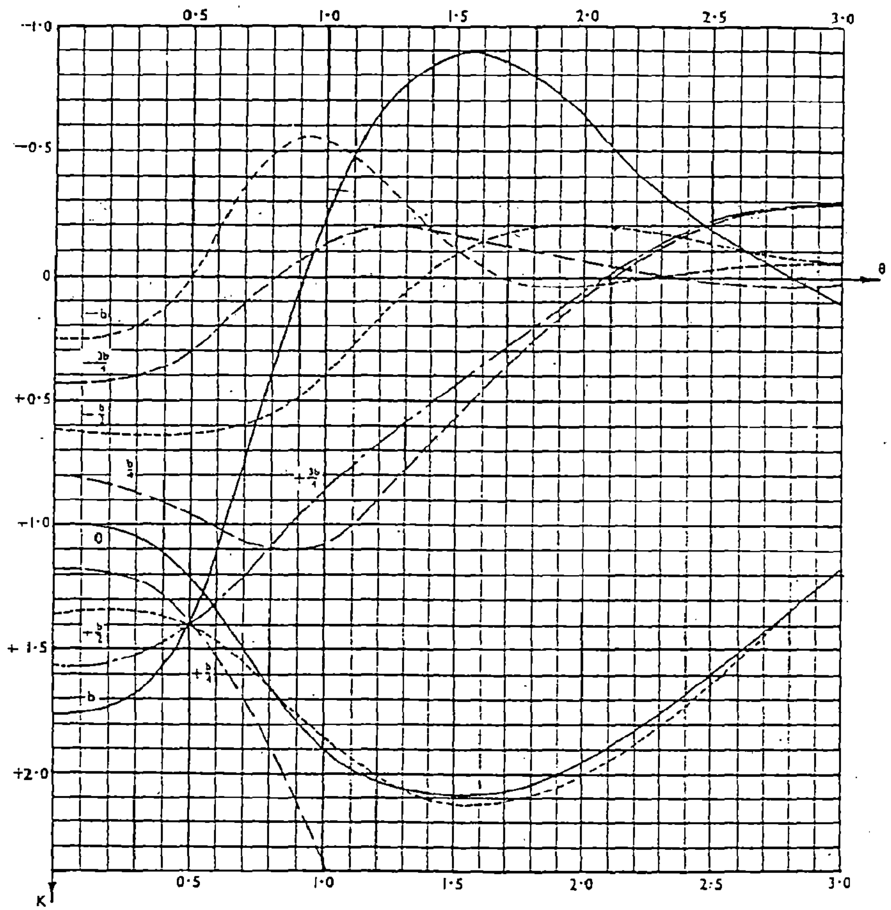
Design curves used for obtaining distribution coefficients in Orthotropic Plate Theory, given by Morice, Little and Rowe (20) and Cusens and Pama (7) are presented here. These cureve are drawn between flexural parameter (θ) and distribution coefficient (K), for various load eccentricities.

First five curves gives distribution coefficients for no torsion grillage($\alpha = 0$), design curve six is for large range distribution coefficients, curves seven through curve eleven are for full torsion grillage($\alpha = 1$). In Cusens and Pama curves one additional graph for large range distribution coefficients ($\alpha = 1$) is also given.

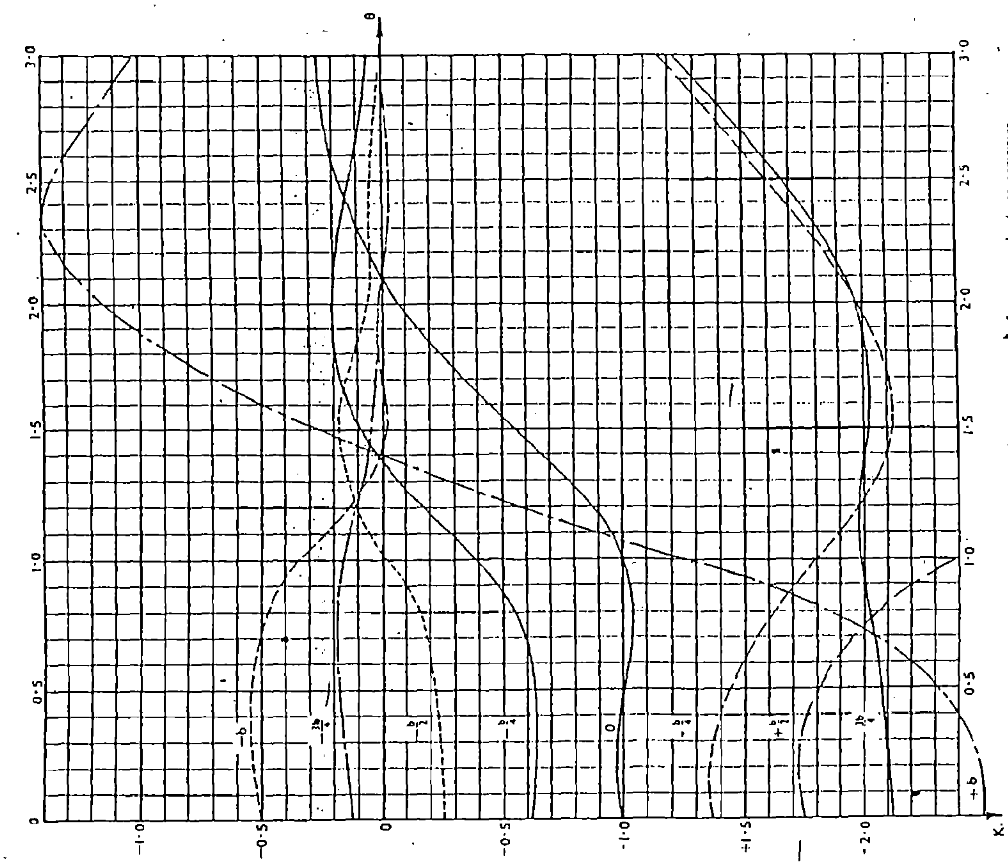
**



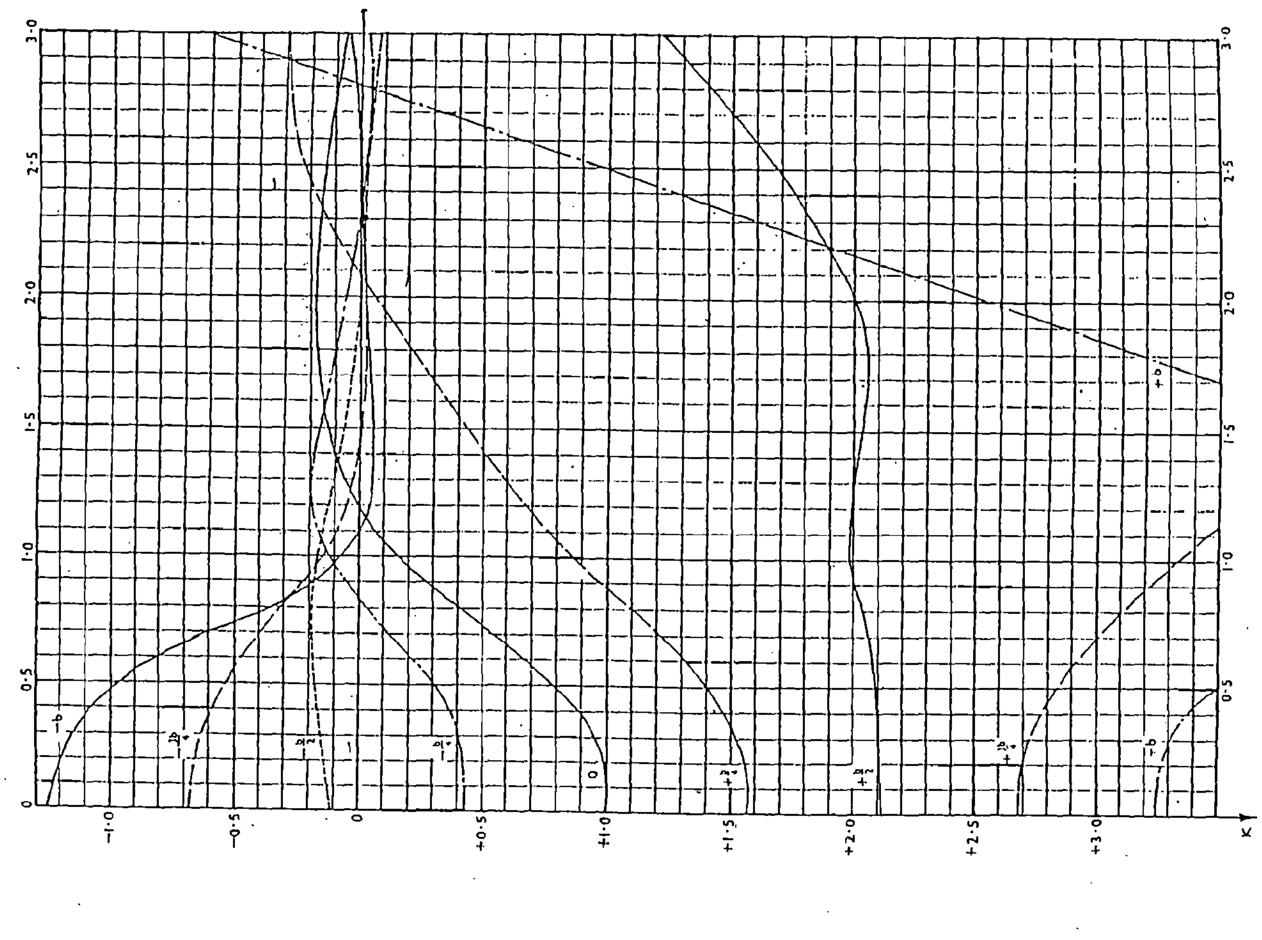
Graph 1. Distribution coefficients K_o at reference station 0 for various load eccentricities.



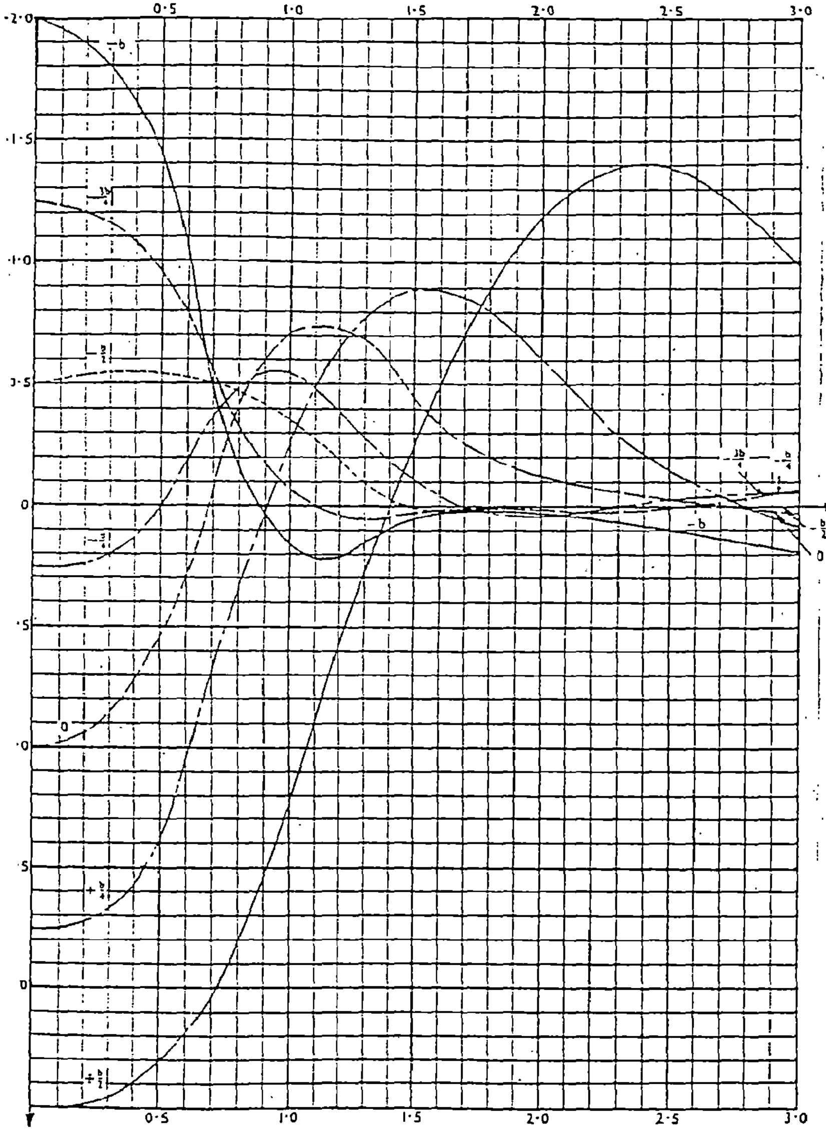
Graph 2. Distribution coefficients K_o at reference station $\frac{b}{4}$ for various load eccentricities.



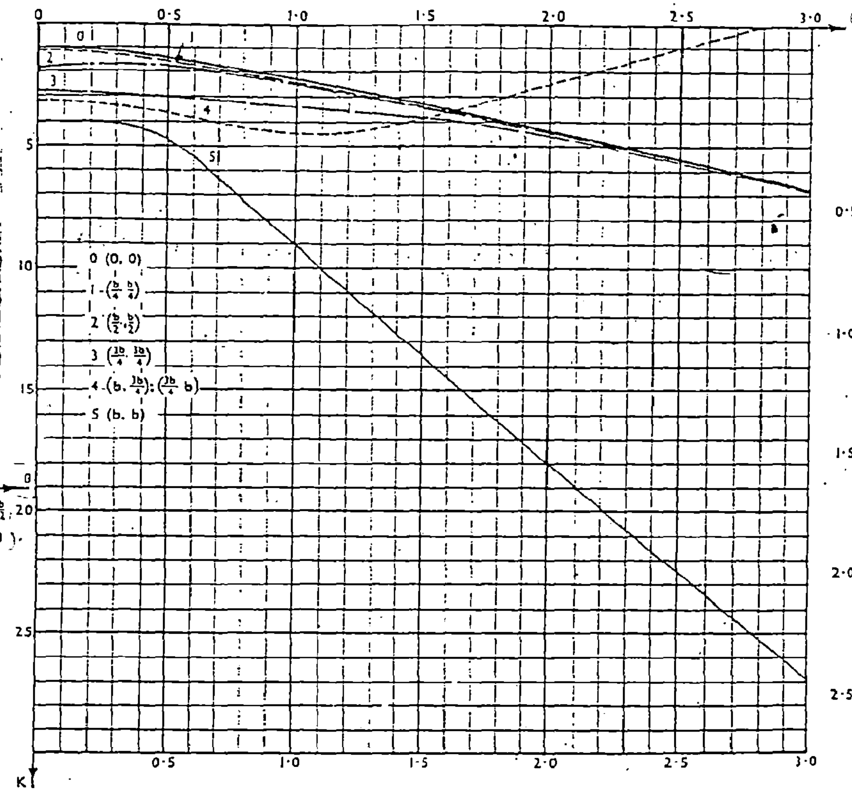
Graph 3. Distribution coefficients K_α at reference station $\frac{b}{2}$ for various load eccentricities.



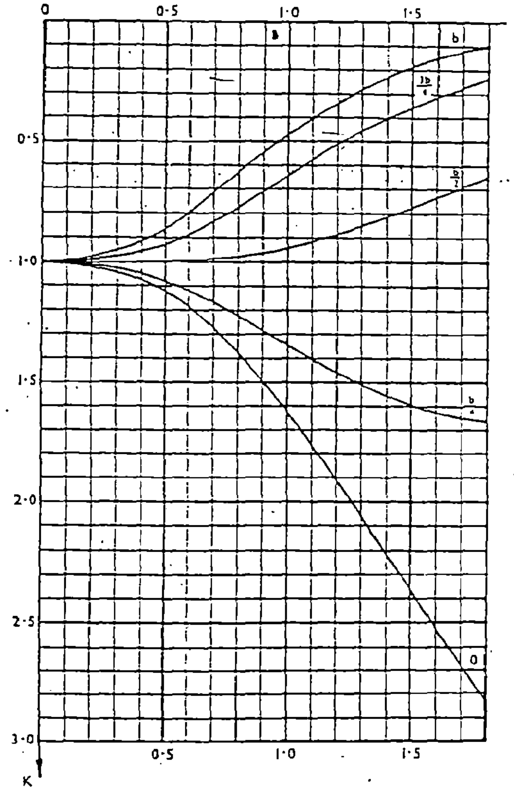
Graph 4. Distribution coefficients K_α at reference station $\frac{3b}{4}$ for various load eccentricities.



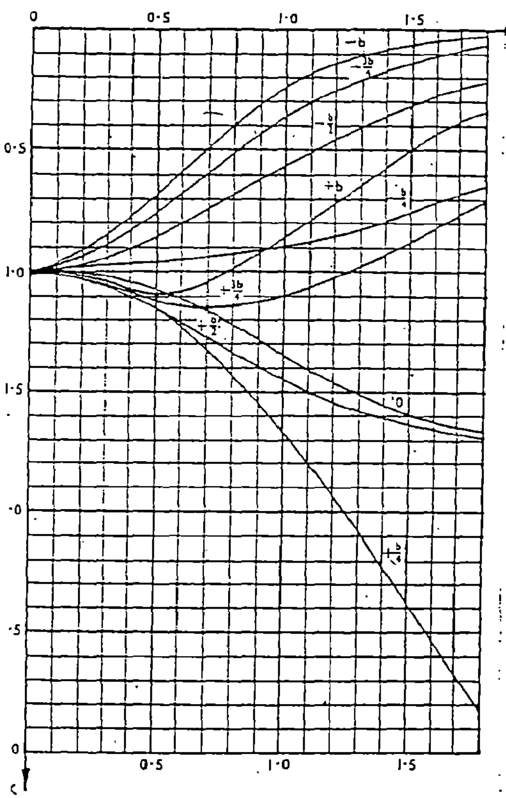
Graph 5. Distribution coefficients K_0 at reference station b for various load eccentricities.



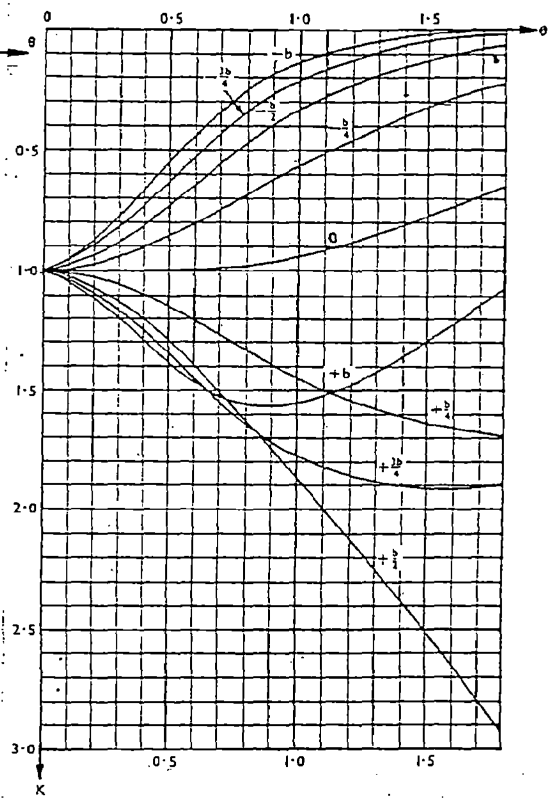
Graph 6. Large range distribution coefficients K_∞.



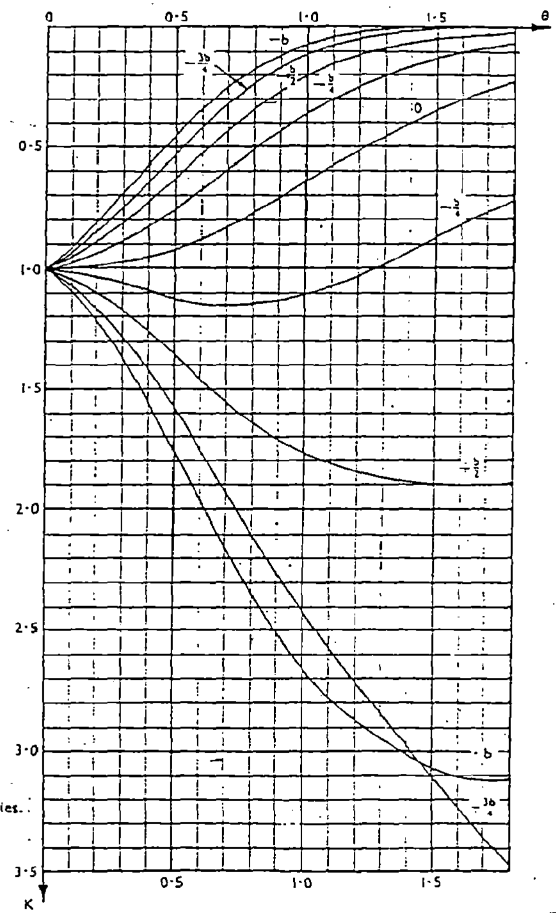
Graph 7. Distribution coefficients K_1 at reference station 0 for various load eccentric



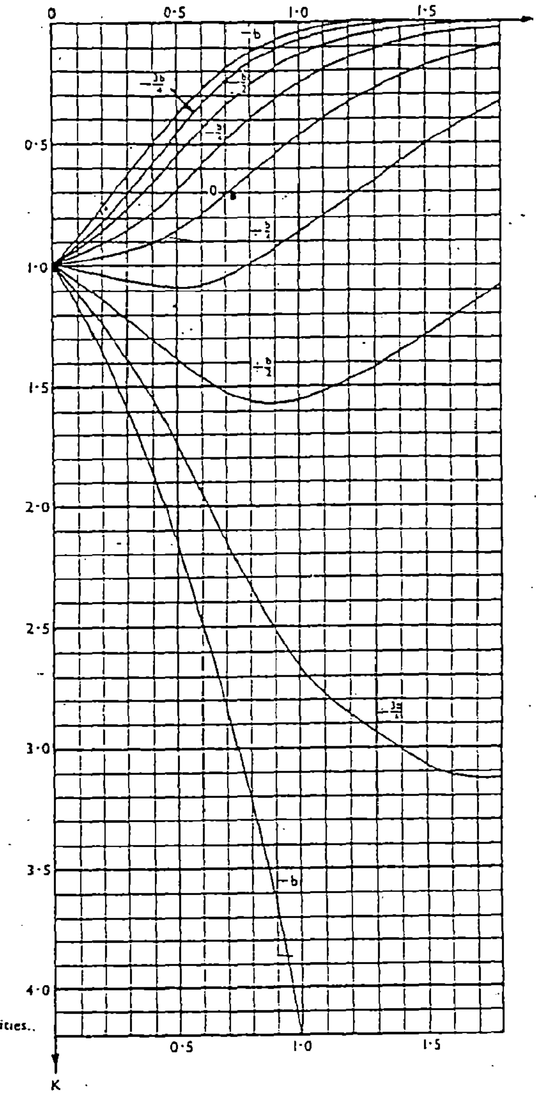
ıph 8. Distribution coefficients K_1 at reference station $\frac{b}{4}$ for various load eccentricities.



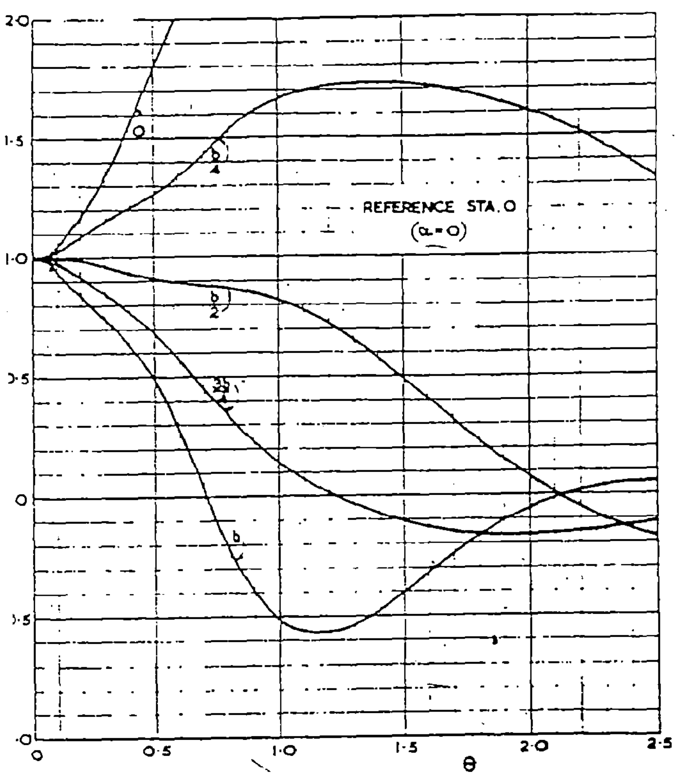
Graph 9. Distribution coefficients K_1 at reference station $\frac{b}{2}$ for various load eccentricities.



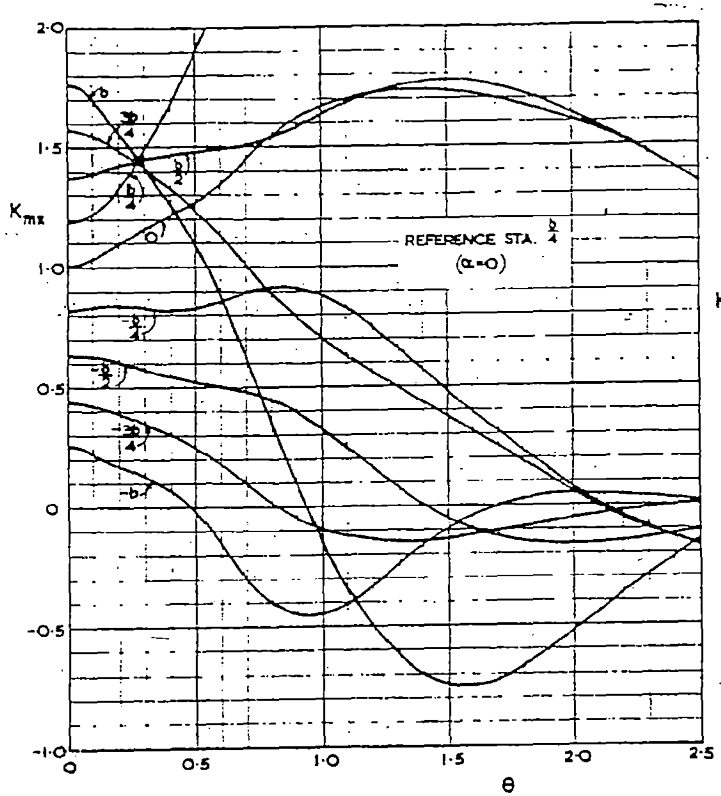
Graph 10. Distribution coefficients K_1 at reference station $\frac{3b}{4}$ for various load eccentricities.



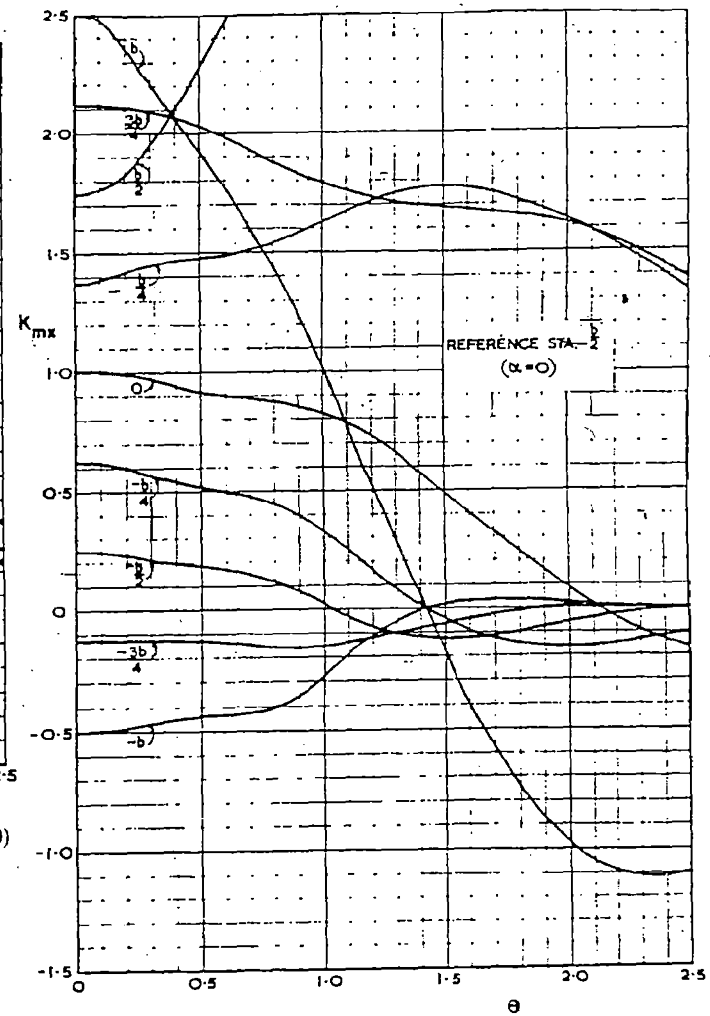
Graph 11. Distribution coefficients K_1 at reference station b for various load eccentric



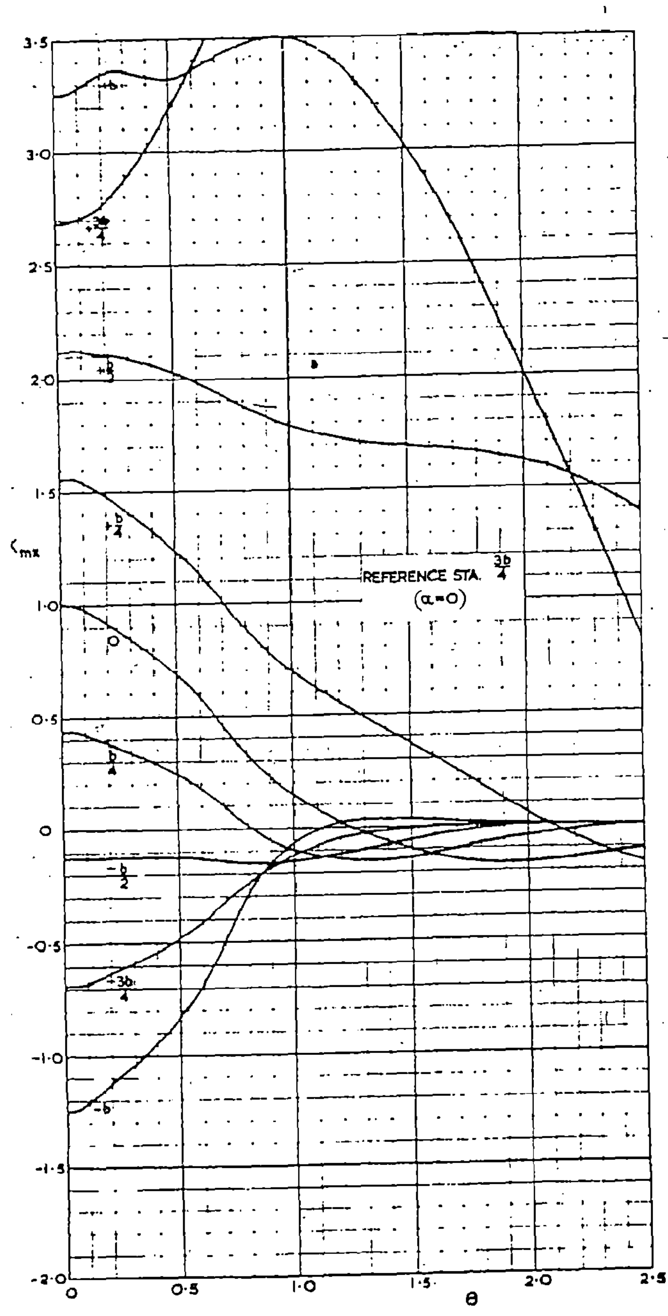
Design Curve 1 Values of K_{mx} at reference station 0 ($\alpha = 0$)



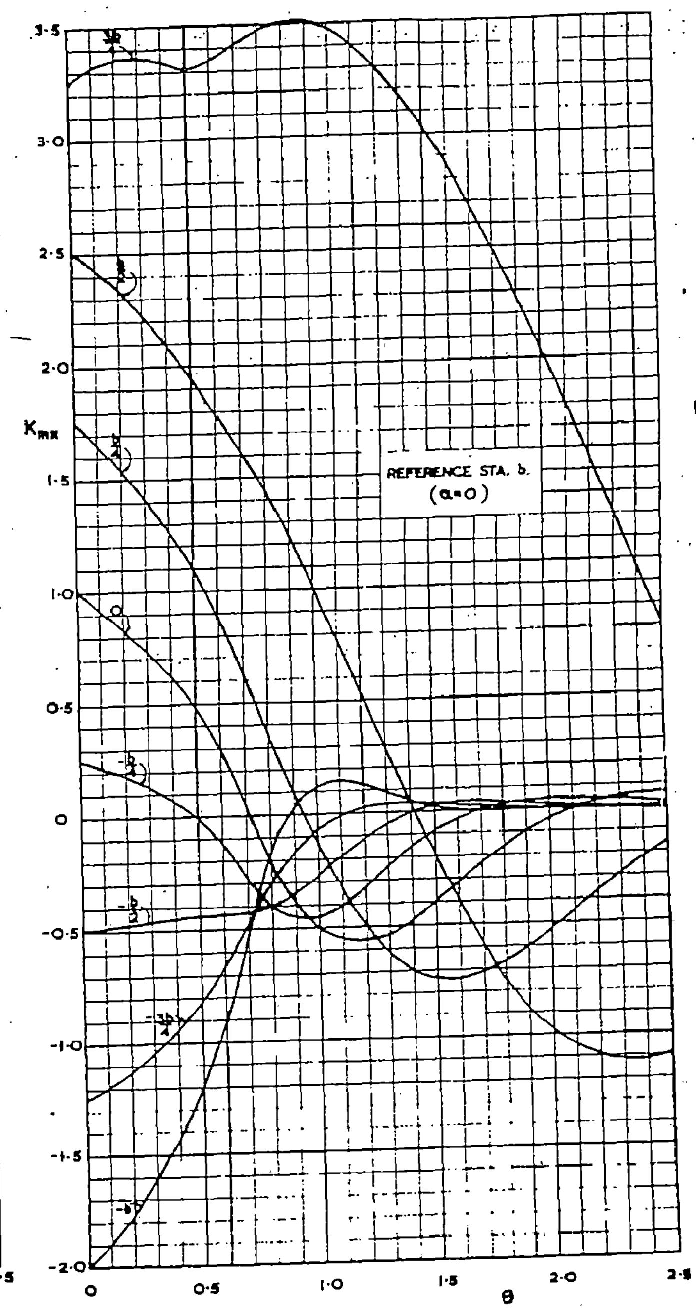
Design Curve 2 Values of K_{mx} at reference station $b/4$ ($\alpha = 0$)



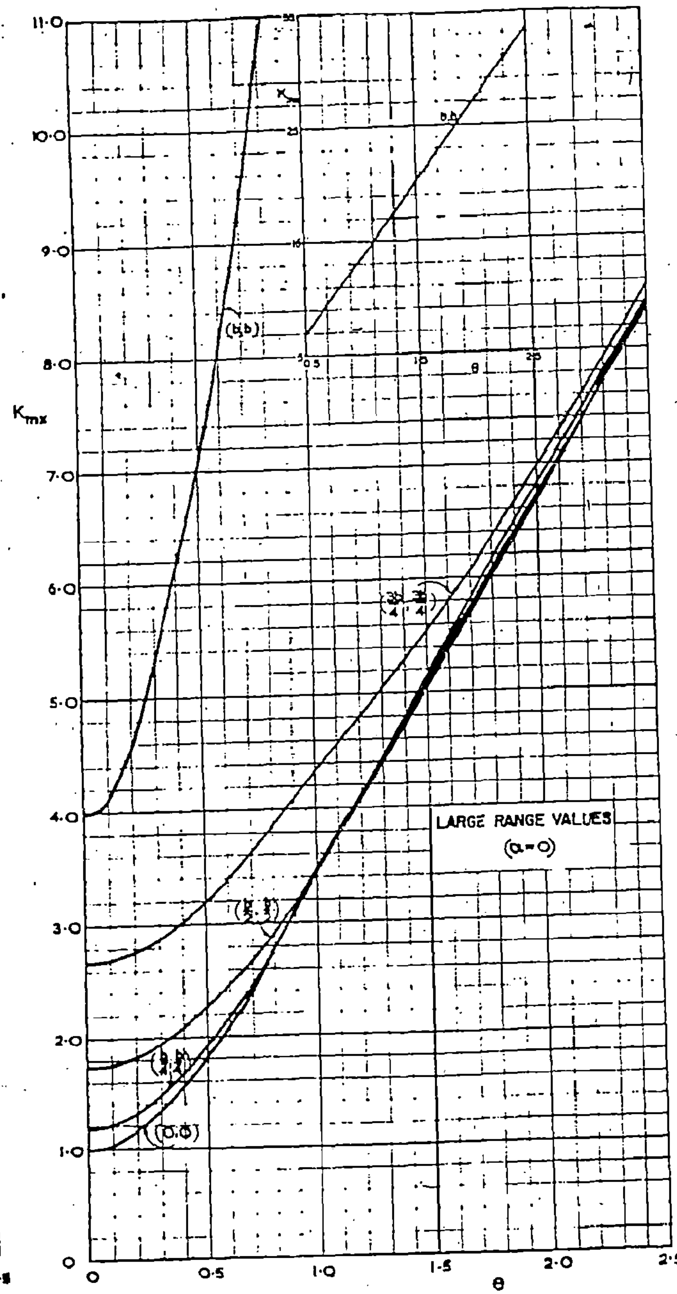
Design Curve 3 Values of K_{mx} at reference station $b/2$ ($\alpha = 0$)



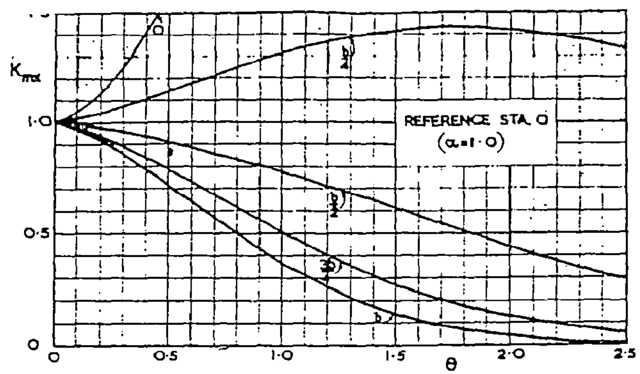
Design Curve 4 Values of K_{mx} at reference station $3h/4$ ($\alpha = 0$)



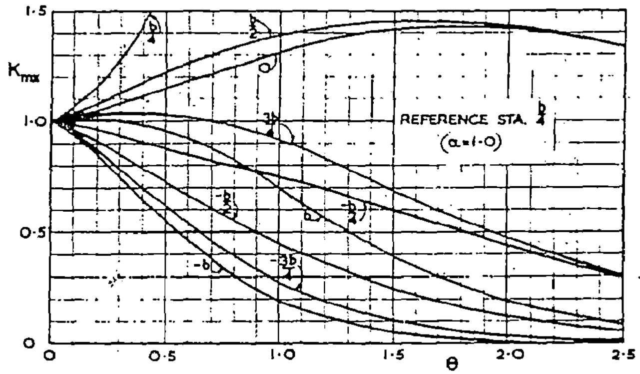
Design Curve 5 Values of K_{mx} at reference station h ($\alpha = 0$)



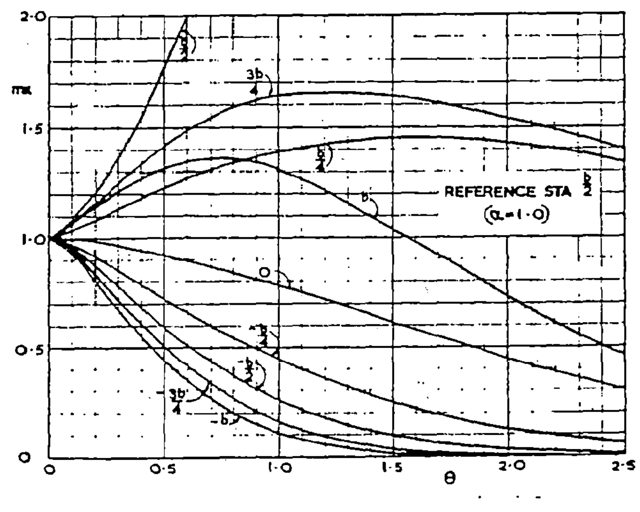
Design Curve 6 Large range values of K_{mx} ($\alpha = 0$)

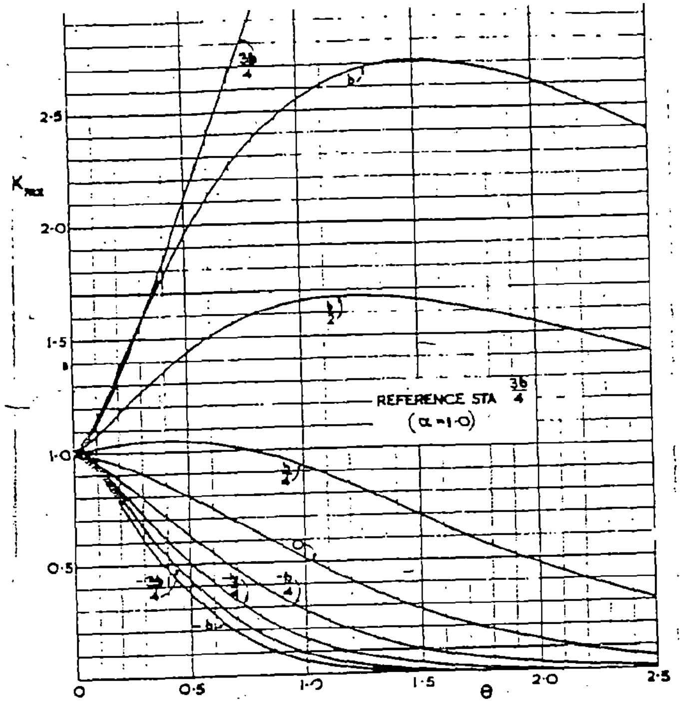


Design Curve 7   Values of K_{mx} at reference station 0 ($\alpha = 1$)

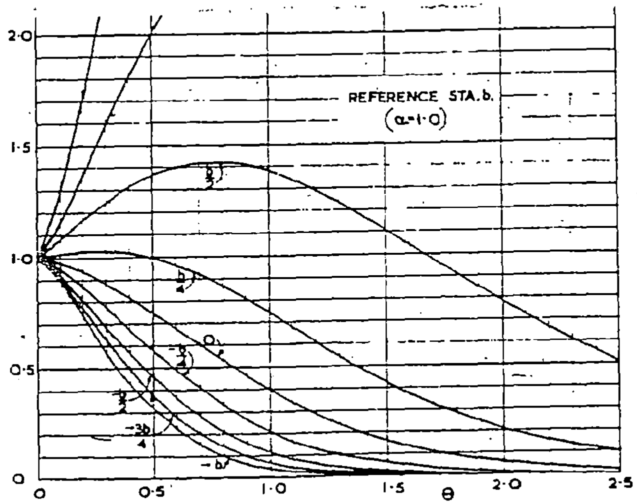


Design Curve 8   Values of K_{mx} at reference station $b/4$ ($\alpha = 1$)

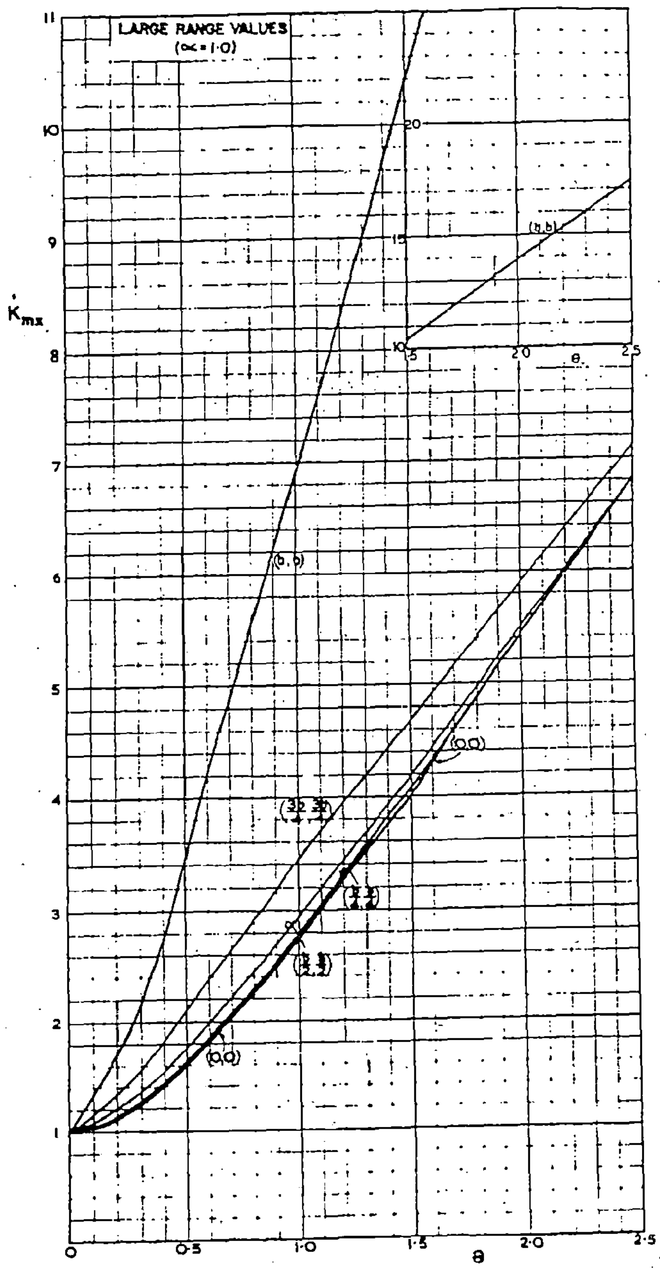




Design Curve 10   Values of K_{mx} at reference station $3b/4$ ($\alpha = 1$)



Design Curve 11   Values of K_{mx} at reference station b ($\alpha = 1$)



Design Curve 12   Large range values of K_{mx} ($\alpha = 1$)

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